cardinal priest of the title of St. Cecilia, legate of the apostolic see, made after separate deliberation of the nations, and who adhere expressly and entirely to the opinion of the seven judges appointed by the same legate in the same statute, greeting in the Saviour of all. All should know that we masters, each and all, from the preceding abundant and considered advice and deliberation of good men concerning this, wishing with all our power to avoid present and future dangers which by occasion of this sort might in the future befall our faculty, by common consent, no one of us contradicting, on the Friday preceding the Sunday on which is sung *Rejoice Jerusalem,* the masters one and all being convoked for this purpose in the church of Ste. Geneviève at Paris, decree and ordain that no master or bachelor of our faculty should presume to determine or even to dispute any purely theological question, as concerning the Trinity and incarnation and similar matters, since this would be transgressing the limits assigned him, for the Philosopher says that it is utterly improper for a non-geometer to dispute with a geometer.[2]

But if anyone shall have so presumed, unless within three days after he has been warned or required by us he shall have been willing to revoke publicly his presumption in the classes or public disputation where he first disputed the said question, henceforth he shall be forever deprived of our society. We decree further and ordain that, if anyone shall have disputed at Paris any question which seems to touch both faith and philosophy, if he shall have determined it contrary to the faith, henceforth he shall forever be deprived of our society as a heretic, unless he shall have been at pains humbly and devoutly to revoke his error and his heresy, within three days after our warning, in full congregation or elsewhere where it shall seem to us expedient. Adding further that, if any master or bachelor of our faculty reads or disputes any difficult passages or any questions which seem to undermine the faith, he shall refute the arguments or text as far as they are against the faith or concede that they are absolutely false and entirely erroneous, and he shall not presume to dispute or lecture further upon this sort of difficulties, either in the text or in authorities, but shall pass over them entirely as erroneous. But if anyone shall be rebellious in this, he shall be punished by a penalty which in the judgment of our faculty suits his fault and is due. Moreover, in order that all these may be inviolably observed, we masters, one and all, have sworn on our personal security in the hand of the rector of our faculty and we all have spontaneously agreed to be so bound. In memory of which we have caused this same statute to be inscribed and so ordered in the register of our faculty in the same words. Moreover, every rector henceforth to be created in the faculty shall swear that he will cause all the bachelors about to incept in our faculty to bind themselves to this same thing, swearing on their personal security in his hand. Given at Paris the year of the Lord 1271, the first day of April.[3]

13 THE CONDEMNATION OF 1277: A SELECTION OF ARTICLES RELEVANT TO THE HISTORY OF MEDIEVAL SCIENCE

Translation, introduction, and annotation by Edward Grant[1]

Ever since the introduction of Aristotle's natural philosophy into the Latin West in the twelfth century, ecclesiastical authorities had feared its impact on theological studies and Christian belief. The Parisian prohibition of Aristotle's natural books in 1210 and the demand for their expurgation in 1231 (see Selections 9 and 10) reflect the apprehension of some Church authorities. But the scientific and philosophic riches in the natural books of Aristotle made inevitable their inclusion into the medieval university curriculum, and by 1255 Aristotle's works formed the core of medieval university education (see Selection 11) which was so heavily oriented toward logic and natural philosophy. Aristotelian natural philosophy and metaphysics provided students and teachers of the thirteenth century with philosophical tools of analysis that were applied with great fervor to all areas of human

2. [Perhaps a reference to Aristotle's remark in *Physics* I.2.185a.1–3: "For just as the geometer has nothing more to say to one who denies the principles of his science—this being a question for a different science or for one common to all—so a man investigating *principles* cannot argue with one who denies their existence."—*Ed.*]

3. As Easter fell at that time after the first of April, the year is 1272 according to our reckoning.

1. This translation has been made from the *Chartularium Universitatis Parisiensis,* ed. H. Denifle and E.

thought. Aristotle and his Arabian commentators, especially Averroes, who was himself known as "the Commentator," reigned supreme during the middle decades of the thirteenth century and were zealously studied at the University of Paris and elsewhere. It was not long before the views held by some masters of arts on a number of controversial issues became obnoxious to the theologians at Paris. Being thoroughly aroused, they persuaded the bishop of Paris, Étienne Tempier, to condemn in 1270 thirteen articles or propositions, among which were the doctrines of the eternity of the world and the unicity of the intellect (which ascribed to all men a single common intelligence). In 1272 they compelled the masters of arts to swear an oath that they would not treat theological questions (see the preceding selection). The controversy forms the central theme of the treatise of Giles of Rome (Aegidius Romanus), who sometime between 1270 and 1274 published, in defense of his faith, a treatise called *Errors of the Philosophers (Errores philosophorum)*, in which the so-called errors of Aristotle, Averroes, Avicenna, Algazali, Alkindi, and Moses Maimonides were listed.[2] Finally, in 1277, Pope John XXI (formerly Peter of Spain, author of the *Summulae logicales*), concerned over the uproar, instructed the bishop of Paris to investigate the controversies besetting the University of Paris. Not only did Tempier investigate but in only three weeks, on his own authority, he issued a condemnation of 219 propositions drawn from many sources, including, apparently, the works of Thomas Aquinas, some of whose ideas found their way onto the list. The Pope appears to have acquiesced in the actions of the bishop of Paris, and he accepted the penalty of excommunication for all who upheld even a single proposition.

There is little doubt that the condemnation influenced the course of philosophy, for under penalty of excommunication, many deterministic arguments drawn from, or based on, Aristotle's philosophy had of necessity to be modified and qualified. Alternatives, previously thought to be silly or absurd, had now to be entertained as at least possible—even if only by virtue of God's infinite and absolute power. Because of the condemnation, it became a characteristic feature of fourteenth-century scholastic discussion for authors to declare that although something was naturally impossible, it was supernaturally possible. Thus while it was naturally impossible for more than one world to exist, or for a vacuum to exist, God could achieve both of these effects if He so desired. Theologians expanded the domain where God's absolute and unpredictable power was operative and severely reduced the domain of certain and demonstrated knowledge. The God who lived in obedience to His own laws in the Thomistic interpretation was little in evidence in the fourteenth century.

It was because the condemnation was, in effect, a frontal assault on Aristotelian metaphysics and philosophy that it seemed important and significant to Pierre Duhem, the eminent historian of medieval science, who saw it as an instrument liberating medieval science from bondage to Aristotelian cosmological and metaphysical assumptions and conclusions. It was fortunate, according to Duhem, that propositions denying God's power to move the universe with a rectilinear motion (see Proposition 49) or to create more than one world (see Proposition 34) were condemned, because it was a happy consequence that thereafter discussions on the possibility of a void and the existence of other worlds were forthcoming and served to stimulate scientific imagination and investigation. Indeed, Duhem was moved to declare that "if we must assign a date for the birth of modern science, we would, without doubt, choose the year 1277 when the bishop of Paris solemnly proclaimed that several worlds could exist, and that the whole of the heavens could, without contradiction, be moved with a rectilinear motion."[3] Although this claim is extreme and it is also doubtful whether the new intellectual vistas opened up by the condemnation were helpful to medieval science (Alexandre Koyré, for example, denied it completely), it is a fact that the questions which Duhem thought so monumental were indeed widely discussed in the fourteenth century. It is also true to assert that fourteenth-century physicists and natural philosophers departed on many specific points from Aristotelian solutions and mechanisms of explanation.

On February 14, 1325, the condemnation was annulled because of a great sentiment in favor of the teachings of Thomas Aquinas, a number of whose ideas and positions had been included in the condemnation. In the document of nullification (printed in *Chartularium*, II, 280–281), the bishop of Paris declares that "on the basis of certain knowledge held at present, we wholly annul the aforementioned condemnation of articles and judgments of excommunication as they touch, or are said to touch, the teaching of blessed Thomas, mentioned above; and because of this we neither approve nor disapprove of these articles, but leave them for free scholastic discussion." From this it would appear that the articles condemned in 1277 no longer carried the penalty of excommunication. It is of interest, however, that, in annulling the condemnation of 1277, the bishop of Paris adopted a position of neutrality toward the articles themselves, which perhaps explains why they continued to be cited in the latter part of the fourteenth century by men of the stature of Jean Buridan, Nicole Oresme, Albert of Saxony, and others.

To all who shall examine these words, Stephen, by divine permission unworthy minister of the Church of Paris, sends greetings in the [name of the] Son of the glorious Virgin. Frequent reports from great and important people aflame with the zeal of faith have made known that some who study at Paris in the arts exceed the proper bounds of the faculty and in the schools presume to write about, and dispute, certain obvious and abominable errors—indeed, rather, "vanities and lying follies"[4] (these are listed on the roll following this letter)—as if they were [merely] doubtful.[5] They seem not to understand what Gregory said, namely that one who endeavors to speak wisely should be very apprehensive lest the unity of his audience be confounded by his discourse, especially since they support the aforesaid errors by the writings of the gentiles, which—oh! for shame—they declare in their ignorance, that these have such force that they do not know how to answer them. However, that they should not seem to assert what they hint at, they declare such feeble responses that while they are eager to avoid Scylla, they fall into Charybdis. For, indeed, they say that things are true according to philosophy but not according to the Catholic faith; as if there could be two contrary truths,[6] and as if contrary to the truth of Sacred Scripture there could be truth in the statements of the damned gentiles of whom it was written: "I will destroy the wisdom of the wise"[7] because true wisdom destroys false wisdom. Oh that such people might understand the advice of the wise man who says: "If thou have understanding, answer *thy* neighbour: but if not, let thy hand be upon thy mouth, lest thou be surprised in an unskilful word, and be confounded."[8] Therefore, lest dangerous discourse should draw the innocent into error, we strictly forbid, on the advice communicated to us by doctors of sacred theology and other prudent men, that such things and similar things be done, and wholly condemn these things, excommunicating all those who shall have taught some or all of the said errors, or shall have presumed to defend or support them in any way whatever, and also those who listen to these things, unless it be disclosed that within seven days they have come forward to us or to the Chancellor of Paris, notwithstanding other punishments that might be inflicted upon them for the extent of their guilt, as the law shall dictate. By our same judgment, we also condemn the book *De amore*,[9] or *De Deo amoris*,[10] which begins as follows: *Cogit me multum*, etc., and ends with these words: *Cave igitur, Galtere, amoris exercere mandata*, etc.; likewise the book of Geomancy, which begins this way: *Estimaverint Indi*, etc., and ends this way: *Ratiocinare ergo super*

Chatelain (Paris, 1889–1897), I, 543–555. The original order of the propositions as printed in the *Chartularium* has been retained. A complete translation of all 219 condemned propositions following the regrouping and renumbering of P. Mandonnet, *Siger de Brabant et l'averroisme latin au xiii^me siècle*, 2d ed. (Louvain: Institut supérieur de philosophie de l'Université, 1908), Part II, has been made by Ernest L. Fortrin and Peter D. O'Neill in *Medieval Political Philosophy: A Sourcebook*, ed. Ralph Lerner and Muhsin Mahdi (New York: The Free Press of Glencoe, 1963), pp. 337–354.

2. This treatise has been edited and translated by Josef Koch and John O. Reidl (Milwaukee, Wis.: Marquette University Press, 1944).

3. *Études sur Léonard de Vinci* (Paris: A. Hermann, 1906–1913), II, 412. The translation is mine from my article "Late Medieval Thought, Copernicus, and the Scientific Revolution," *Journal of the History of Ideas*, XXIII (1962), 200, n. 8.

4. Psalms 39:5.

5. Errors that should have been identified unequivocally were instead treated as if they were only doubtful.

6. This is the doctrine of the double truth attributed to the Latin Averroists in the thirteenth century. Not only is it doubtful that Averroes himself ever held the view described here but no blatant adherents of the doctrine can be identified in the Latin West—not even Siger of Brabant, so often alleged to have been its most famous proponent. As presented here, the doctrine implies that two contrary statements could be maintained if it were merely argued that one was true by virtue of natural reason (and could, therefore, be false supernaturally) and the other by virtue of faith and dogma (and perhaps false by natural reason; see Proposition 90). When faced with such contrary statements, an adherent of this doctrine would have argued, presumably, that he accepted the truth of faith as a higher truth, but that this did not conflict with his acceptance of the truth of the contrary statement arrived at through natural reason from the premises of physics and metaphysics.

7. 1 Corinthians 1:19.

8. *Ecclesiasticus* 5:14.

9. *On Love*.

10. *On the God of Love*.

eum, et invenies, etc.; likewise [we condemn] books, rolls, or pamphlets *(quaternos)* on magic containing experiences of prophesying, invocations of demons, or conjurations on the peril of souls, or [writings] in which such things and similar things are evidently treated in ways that are adverse to the good customs of the orthodox faith. On all those who shall have taught or heard the said rolls, books, [or] pamphlets, we set forth the judgment of excommunication, unless, as expressed above, within seven days they shall have revealed themselves to us or the aforementioned Chancellor of Paris, notwithstanding other punishments that might be exacted according to the seriousness of the crime. Given at the curia of Paris, in the year of the Lord, 1276,[11] on the Sunday in which *Rejoice Jerusalem* is sung.

The letter ends. The errors listed on the roll follow:

. . . . .

6. That when all celestial bodies have returned to the same point—which will happen in 36,000 years—the same effects now in operation will be repeated.[12]

9. That there was no first man, nor will there be a last; on the contrary, there always was and always will be generation of man from man.

21. That nothing happens by chance, but all things occur from necessity and that all future things that will be will be of necessity, and those that will not be it is impossible for them to be; and that upon considering all causes, [it will be seen that] nothing happens contingently. [This is an] error because by definition a concourse of causes occurs by chance, as Boethius says in his book *On Consolation [of Philosophy].*[13]

34. That the first cause could not make several worlds.[14]

35. That without a proper agent, as a father and a man, a man could not be made by God [alone].

37. That nothing should be believed unless it is self-evident or could be asserted from things that are self-evident.

38. That God could not have made prime matter without the mediation of a celestial body.

48. That God cannot be the cause of a new act [or thing], nor can he produce something anew.

49. That God could not move the heavens [that is, the world] with rectilinear motion; and the reason is that a vacuum would remain.[15]

52. That that which is self-determined, as God, either always acts or never acts; and that many things are eternal.[16]

66. That there are several first movers.

74. That a motive intelligence of the heavens influences the rational soul just as a celestial body influences the human body.

87. That the world is eternal as to all the species contained in it; and that time is eternal, as are motion, matter, agent, and recipient; and because the world is [derived] from the infinite power of God, it is impossible that there be novelty in an effect without novelty in the cause.

88. That nothing could be new unless the sky were varied with respect to the matter of generable things.

90. That a natural philosopher ought to deny absolutely the newness [that is, the creation] of the world because he depends on natural causes and natural reasons. The faithful, however, can deny the eternity of the world because they depend upon supernatural causes.[17]

91. That the argument of the Philosopher[18] dem-

11. Although the year 1276 is recorded here, the date March 7, 1277, precedes the document published in the *Chartularium.* During this period in France, the new year began at Easter.

12. See Selection 51.2, last paragraph, and note 86, where Nicole Oresme is probably alluding to this very proposition.

13. *De consolatione philosophiae,* Book V, Prose I.

14. This is Aristotle's position in *On the Heavens (De, caelo),* Book I, chapter 9. A detailed discussion of this widely considered question is given below, in Selection 71, by Nicole Oresme (see also next note). Here we have one of a number of propositions which limited God's power and aroused the ire of the theologians (for similar restrictions on God's power, see propositions 35, 38, 48, 49, and 141).

15. On the basis of this condemned proposition, it was henceforth respectable to argue that a vacuum was possible by divine action, and scholastics would concede this routinely. But it did not produce any proponents of an actually existent void space within the confines of the universe (concerning the existence of void beyond the world, see the dialogue between Hermes Trismegistus and Asclepius, Selection 72. Indeed, acceptance of motion in a hypothetical void antedates the Condemnation, for we find Aquinas asserting it in his *Commentary on the Physics,* quoted in Selection 55.1. Condemnation of this proposition could hardly have had the consequences ascribed to it by Duhem (see the introduction to this selection). However, at least five scholastics (Richard of Middleton, Thomas Bradwardine, Jean Buridan, Nicole Oresme, and Jean Hennon) found occasion to mention it.

16. Cited by Thomas Bradwardine in *De causa Dei contra Pelagium* (ed. Henry Savile [London, 1618], p. 177) and included in Selection 73.1.

17. Here is an illustration of the doctrine of the double truth.

18. That is, Aristotle.

onstrating that the motion of the sky is eternal is not sophistical; and it is amazing that profound men do not see this.

92. That celestial bodies are moved by an internal principle, which is soul; and that they are moved by a soul and by an appetitive power *(per virtutem appetitivam)* [that is, by force of desire] just as an animal; for just as an animal is moved by desire, so also is the sky.

93. That celestial bodies have eternity of substance but not eternity of motion.

94. That there are two eternal principles, namely the body of the sky and its soul.

95. That there are three principles in celestial bodies: (1) a subject of eternal motion, (2) a soul of a celestial body, and (3) the prime mover as that which is desired.—The error concerns only the first two.

98. That the world is eternal because that which has a nature by [means of] which it could exist through the whole future [surely] has a nature by [means of] which it could have existed through the whole past.

99. That the world, though it was made from nothing, was not, however, made anew; and although it came into being from nonbeing, nevertheless nonbeing did not precede being in duration, but only in nature.

100. That theologians who say that the sky [or heavens] sometimes rests argue from a false assumption; and that to say that the sky exists and is not moved is to utter contradictories.[19]

101. That an infinite [number] of celestial revolutions have preceded which it was not impossible for the first cause [that is, God] to comprehend, but [which are impossible of comprehension] by a created intellect.

102. That the soul of the sky is an intelligence and the celestial orbs are not instruments of the intelligences, but organs, just as the ear and the eye are organs of the sensitive power.

106. That the immediate effective cause of all forms is an orb.

107. That the elements are eternal. However, they have been made [or created] anew in the relationship which they now have.

110. That the celestial motions occur because of an intellective soul; but an intellective soul or intellect cannot be produced except by means of a body.

111. That no form coming from outside can become one with matter. For what is separable does not make [or become] one with what is corruptible.

137. That although the generation of men might become deficient, it does not because of the power of the first orb, which not only moves to generate the elements but also to generate men.

140. That to make an accident exist without a subject is an impossible argument implying a contradiction.

141. That God cannot make an accident exist without a subject nor make more [than three] dimensions exist simultaneously.

143. That from the different [zodiacal] signs of the sky diverse conditions are assigned in men, both with respect to spiritual gifts and temporal things.

145. That no question is disputable by reason which a philosopher ought not to dispute and determine, since arguments *(rationes)* are taken from [or based on] things. Moreover, philosophy has to consider all things according to its diverse parts.[20]

147. That the absolutely impossible cannot be done by God or another agent.—An error, if impossible is understood according to nature.

148. That by nutrition a man can become another numerically and individually.[21]

150. That on any question, a man ought not to be satisfied with certitude based upon authority.[22]

151. That for a man to have certitude of any conclusion, it is necessary that he found it on self-

19. It is tempting to speculate that perhaps this condemned proposition stimulated Buridan and Oresme to argue that the heavens may rest and the earth move (see Selections 67. 4, 5). But neither cites this proposition, and it is more than likely that the fourteenth century discussions derived from Simplicius' *Commentary on Aristotle's De caelo,* which was translated by Moerbeke in 1271 and used by Thomas Aquinas and Nicole Oresme both of whom (Selections 67.3,5) cite Heraclides of Pontus, the ancient Greek proponent of the earth's axial rotation, whose name and views are mentioned by Simplicius.

20. This and other propositions reflect the desire of the philosophers, whose numbers came largely from the teaching masters in the arts faculty at Paris, to subject all knowledge and ideas to philosophical and scientific scrutiny (usually along Aristotelian lines). As far back as 1270 and 1272 (see the preceding selection, on the oath of the arts faculty in 1272) the theologians had sought persistently to control and confine these activities.

21. Among the many problems raised by this preposterous proposition is the obvious one of the fate of souls when one person becomes another in all respects.

22. Although they accepted the articles of their faith, the overwhelming number of scholastics were not content to argue from authority.

evident principles.—An error, because it speaks in a general way about both certitude of understanding and [certitude of] adhesion *(adhesionis)*.[23]

152. That theological discussions are based on fables.

153. That nothing is known better because of knowing theology.

154. That the only wise men of the world are philosophers.[24]

161. That the effects of the stars on free will are hidden.

162. That our will is subject to the power of the celestial bodies.

185. That it is not true that something could be made from nothing, and also not true that it was made in the first creation.

186. That the sky never rests because the generation of the lower things, which is the end purpose of celestial motion, ought not to cease; another reason is because the heaven has its being and power from its mover which things are preserved by its motion. Whence if its motion should cease, its existence would cease.

199. That in efficient causes when the first cause [God (?)] ceases [to act] the second [or secondary] cause does not cease its operation since it could operate in accordance with nature.

201. That He who generates the whole world assumes a vacuum because place necessarily precedes what is generated in that place; therefore, before the generation of the world there was a located place which is a vacuum.[25]

202. That the elements have been made in a previous generation from chaos; but they are eternal.

. . . . .

Here ends the roll [or list] of errors containing two hundred and nineteen articles condemned at Paris by Stephen, faithworthy minister and bishop of the same place; [issued] in the curia of Paris in the year of the Lord 1276, on the Sunday in which *Rejoice Jerusalem* is sung.

14 AN OBJECTION TO THEOLOGICAL RESTRICTIONS IN THE DISCUSSION OF A SCIENTIFIC QUESTION

John Buridan (ca. 1300—ca. 1358)

Translated and annotated by Edward Grant[1]

In the eighth [question] we inquire whether it is possible that a vacuum exist by means of any power.

It is argued that this cannot be . . . . [After citing a few arguments of others in behalf of this position, Buridan presents his own views.—*Ed.*]

The opposite [position] is argued because God could annihilate everything under the lunar orb with the magnitude and figure of the lunar orb preserved. Then the concave orb of the moon, which is now a plenum in the lower world, would be a vacuum,[2] just as a pitcher would be a vacuum if God annihilated the wine in it while preserving the pitcher and where no other body enters or is made in the pitcher. And thus some of my lords and masters in theology have reproached me on this, [saying] that sometimes in my physical questions

23. How *adhesio* should be translated in this context is unclear. But the term seems to be used here by way of contrast with understanding on the basis of self-evident principles, suggesting a reference to articles of faith.

24. The hostility and tension that had developed between theologians and philosophers is well illustrated by propositions 152–154.

25. Who the proponents of this decidedly un-Aristotelian proposition were is a puzzle. In the fourteenth century, however, Thomas Bradwardine himself utilized substantially the same position in his *De causa Dei contra Pelagium* (Savile edition, p. 178) in seeking to show that an imaginary infinite void space exists outside the cosmos. Curiously, in the very section where this condemned proposition is accepted and utilized by Bradwardine, he cites propositions 49 and 52 from the Condemnation of 1277, which he identifies as having been issued by the bishop of Paris. It should be noted, however, that in Bradwardine's version of condemned Proposition 201, God himself, who is ubiquitous, occupies the place in which eventually He created the world. Hence, the place which the world was to occupy was void of matter prior to the creation, but not void of God. In Proposition 201 the impression is given that the preexistent vacuum was not created by God, but was coexistent and coeternal with Him.

1. This selection constitutes a part of Book IV, Question 8, of John Buridan's *Questions on the Eight Books of the Physics of Aristotle* and has been translated from *Acutissimi philosophi reverendi Magistri Johannis Buridani subtilissime questiones super octo phisicorum libros Aristotelis diligenter recognite et revise a Magistro Johanne Dullaert de Gandavo antea usque impresse* (Paris, 1509; Frankfurt: Minerva, 1964), fols. 73 verso, col. 2, to 74 recto, col. 1.

2. From Buridan's time and thereafter this particular illustration became almost commonplace. For Albert of Saxony's use of it, see Selection 53.1.

I intermix some theological matters which do not pertain to the artists [that is, Masters of Arts]. But with [all] humility I respond that I very much wish not to be restricted [with respect] to this, namely that all masters beginning in the arts swear that they will dispute no purely theological question,[3] nor [dispute] on the incarnation; and they swear further that if it should happen that they dispute or determine some question which touches faith and philosophy, they will determine it in favor of the faith and they will destroy the arguments *(rationes)* as it will be seen that they must be destroyed. Now it is evident that if any question touches faith and theology, this is one of them, namely whether it is possible that a vacuum exist.[4] And so, if I wish to dispute it, it is necessary that I say about it what appears to me must be said according to theology, or to perjure myself and avoid the arguments on the opposite side insofar as this will seem possible for me. But I could not resolve these arguments [on the opposite side] unless I produce them. Therefore, I am compelled to do these things. I say, therefore, that "vacuum" can be imagined in two ways . . . .

15 AN ASSESSMENT OF BURIDAN'S OBJECTIONS

Ludovicus (Luis) Coronel (fl. 1511)

Translation and introduction by Lynn Thorndike[1]

Annotated by Edward Grant

The following passage from the *Perscrutationes physicales* or commentary on the eight books of the *Physics* of Aristotle by Ludovicus Coronel, composed between 1506 and 1511,[2] shows that, while the above statute was still enforced in the days of Buridan in the middle of the fourteenth century, it had become obsolete before the close of the fifteenth century.

After stating that God could supernaturally produce a vacuum, Coronel cites Buridan's commentary on the *Physics,* book IV, question 8.

For he says that many of our masters blamed him for sometimes mingling certain theological matters with physical questions, of which sort is whether this or that which is beyond the power of nature lies within divine power, since to discuss this does not pertain to artists. But he replying humbly said bidden for a Master of Arts to do this. (Actually, in this connection, the statute says only that "if anyone shall have disputed at Paris any question which seems to touch faith and philosophy," it must not be determined contrary to the faith; since Buridan does not seek a defense by appeal to the statute itself, it may possibly have been altered and made more stringent by his time.) Perhaps Buridan was one of the first to challenge the theologians on this matter (see next selection, n.3). Buridan's sensitivity about the reactions of theologians is evident in another part of his *Questions on the Physics* (Bk. VIII, Question 12), where after suggesting the existence of a celestial impetus, he declares, "This I do not say assertively, but [rather tentatively] so that I might seek from the theological masters what they might teach me in these matters as to how these things take place" (see Selection 48 for the context of this quotation). In his *Questions on De caelo,* Book I, Question 20, we observe an even more extreme deference: "Thirdly, I say that there is no body beyond the heaven or world, namely beyond the outermost heaven; and Aristotle assumes this as obvious. But you ought to have recourse to the theologians [in order to learn] what must be said about this according to the truth of faith or constancy" (translated from *Iohannis Buridani Quaestiones super libris quattuor De caelo et mundo,* ed. E. A. Moody [Cambridge, Mass.: Mediaeval Academy of America, 1942], p.93).

3. A reference to the Statute of 1272 (Selection 12), which had been in force for many years but which, judging by Ludovicus Coronel's remarks (see next selection), had apparently been abandoned by the end of the fifteenth century.

4. The theological implications of this question on the vacuum were concerned largely with the absolute power of God and the nature of existence prior to the creation of the world (see Selection 73.1 and n. 13). To deny the *possibility* of vacuum was to deny that God could create one supernaturally, a position that no one at Paris would have upheld during the fourteenth century, when the impact of Article 49 of the Condemnation of 1277 was very great indeed. Although no theologians would have quarreled with Buridan's resolution of this question, apparently some were annoyed at his audacity in introducing theological matters into a physical question, since by their interpretation of the Statute of 1272 it was for-

1. Reprinted by permission of the Columbia University Press from Lynn Thorndike, *University Records and Life in the Middle ages* (New York: Columbia University Press, 1944), pp. 86–88.

2. Of the edition from which he made his translation, Thorndike says: "In the edition of Lyons, 1530, which I have used, three other forms of the title are given: *Perscrutationes physices, Physica Coronel,* and *Physice perscrutationes egregii interpretis magistri Ludovici Coronel Hispani Segoviensis diligenter castigate.* Prostant Luguduni in edibus Jacobi Giunti in vico Mercuriali, 1530. The passage to be quoted occurs at fols. xcii verso, col. 2—xciii recto, col. 1."

that he would prefer not to be restricted to this. But all the masters, he says, when they incept in arts, swear that they will dispute no purely theological question such as concerning the Trinity and incarnation, and they further swear that, if they chance to dispute or determine a question which touches faith and philosophy, they will determine it in favor of the faith. And they will overthrow the reasons to the contrary as they shall seem to them able to be overthrown. Moreover, it is clear, he says, that if any question touches the faith it is this one whether there can be a vacuum; therefore, if he is going to debate it, he must state what he thinks should be said about it according to theology or perjure himself.

These remarks of Buridan have astonished me, first, that our masters blamed him, for from the declaration of this term, *vacuum,* to conclude that it cannot be produced naturally but can happen supernaturally seems in no way blameworthy.[3] In the second place, Buridan's method of reply, granting that our masters were justified in blaming him, was not satisfactory, for Buridan was not forbidden absolutely to treat of theology, and they did not blame him for this, that where he had treated something concerning the faith he had determined it in favor of the faith, because he was bound to do so apart from his oath. Thirdly, that oath does not seem reasonable when it compels a man to overthrow arguments, for there might be some loyal teacher for whom some argument worked out contrary to the determination of the church, and how would he overthrow it? But to this it seems it should be said that, if the instructor does not know how to overthrow such an argument, he ought not to formulate it in public to his students. But if he should do otherwise, he would remain perjured if he had taken the oath, and will sin although no oath has preceded. Therefore, let Parisian teachers of artists who touch on theological problems look out for themselves. In the fourth place, I, inadequate and unworthy as I am, do not recall that when I was promoted to the degree in arts I took, or knew of any of my fellows taking, such an oath, but, alas, that laudable custom of the university along with others had become obsolete.[4] In the fifth place, where two catholic masters and teachers disagree about any matter in natural or moral philosophy touching the faith in any way, one of them would not be immune from perjury, which seems to be awkward. For example, if one held that the infinite is possible and the other the opposite; and one that matter can exist without form and the other the opposite; one that, given two things neither of which is God nor either part of the other, God could annihilate one of them and leave the other, and the other, without excepting respective entities, should be unwilling with the Subtle Doctor to concede this. And whether there was an oath or not, I would strive where occasion offered to proceed conformably to the intention of the person who ordered that oath be taken. I therefore assert that God can render any place vacant of content . . . .[5]

3. Indeed, it was common for masters of arts to declare repeatedly that while a natural vacuum was impossible, God could create one supernaturally. In light of the Condemnation of 1277, it would have been foolhardy to deny it at Paris—even, apparently, long after the Condemnation had been set aside. Buridan's complaint may not have been unwarranted, however, for it is quite possible that he was one of the first masters of arts to challenge the Statute of 1272 (Selection 12) forbidding the artists from discussing theological matters. In order to treat many topics properly—especially the possible existence of a vacuum—Buridan fought for the right to introduce theological considerations where necessary. Perhaps it was due to his courageous, and apparently successful, efforts that men like Albert of Saxony, Marsilius of Inghen and other eminent masters who followed aroused no opposition from the theologians when they came to discuss the very same problems.

4. Perhaps because of the efforts of men like Buridan.

5. Interestingly, the locus of Coronel's remarks is his own lengthy discussion on the vacuum.

Mathematics

19 ON THE IMPORTANCE OF STUDYING MATHEMATICS

Roger Bacon (ca. 1219—1292)

Translated by Robert B. Burke[1]

Annotated by Edward Grant

CHAPTER 2
IN WHICH IT IS PROVED BY AUTHORITY THAT EVERY SCIENCE REQUIRES MATHEMATICS

As regards authority I so proceed. Boetius says in the second prologue to his Arithmetic, "If an inquirer lacks the four parts of mathematics, he has very little ability to discover truth." And again, "Without this theory no one can have a correct insight into truth." And he says also, "I warn the man who spurns these paths of knowledge that he cannot philosophize correctly." And again, "It is clear that whosoever passes these by, has lost the knowledge of all learning." He confirms this by the opinion of all men of weight saying, "Among all the men of influence in the past, who have flourished under the leadership of Pythagoras with a finer mental grasp, it is an evident fact that no one reaches the summit of perfection in philosophical studies, unless he examines the noble quality of such wisdom with the help of the so-called quadrivium." And in particular Ptolemy and Boetius himself are illustrations of this fact. For since there are three essential parts of philosophy, as Aristotle says in the sixth book of the Metaphysics, mathematical, natural, and divine, the mathematical is of no small importance in grasping the knowledge of the other two parts, as Ptolemy teaches in the first chapter of the Almagest, which statement he also explains further in that place. And since the divine part is twofold, as is clear from the first book of the Metaphysics, namely, the first philosophy, which shows that God exists, whose exalted properties it investigates, and civil science, which determines divine worship, and explains many matters concerning God as far as man can receive them. Ptolemy likewise asserts and declares that mathematics is potent in regard to both of these branches. Hence Boetius asserts at the end of his Arithmetic that the mathematical means are discovered in civil polity. For he says that an arithmetic mean is comparable to a state that is ruled by a few, for this reason, that in its lesser terms is the greater proportion; but he states that there is a harmonic mean in an aristocratic state for the reason that in the greater terms the greater proportionality is found. The geometrical mean is comparable to a democratic state equalized in some manner; for whether in their lesser or greater terms they are composed of an equal proportion of all. For there is among all a certain parity of mean preserving a law of equality in their relations. Aristotle and his expositors teach in the morals in many places that a state cannot be ruled without these means. Concerning these means an exposition will be given with an application to divine truths. Since all the essential parts of philosophy, which are more than forty sciences distinct in their turn, may be reduced to these three, it suffices now that the value of mathematics has been established by the authorities mentioned.

Now the accidental parts of philosophy are grammar and logic. Alpharabius makes it clear in his book on the sciences that grammar and logic cannot be known without mathematics. . . . But not only does a knowledge of logic depend on mathematics because of its end, but because of its middle and heart, which is the book of Posterior Analytics, for that book teaches the art of demonstration. But neither can the fundamental principles of demonstration, nor conclusions, nor the subject as a whole be learned or made clear except in the realm of mathematics, because there alone is there true and forceful demonstration, as all know and as we shall explain later. Therefore of necessity logic depends on mathematics.

What has been said is applicable likewise because

1. Reprinted from *The Opus Majus of Roger Bacon*, translated by Robert Belle Burke (Philadelphia: University of Pennsylvania Press, 1928; reprinted New York: Russell and Russell, 1962), pp. 117–127. The translation was based on the Latin edition of Bacon's *Opus maius* by John Henry Bridges (London, 1900). Precise citations for the numerous references to Aristotle, Averroes, Boethius, and others have not been furnished. In many instances, this would be superfluous, since Bacon himself frequently cites his sources accurately.

of its beginning and not only because of its middle and end. For the book of Categories is the first book of logic according to Aristotle. But it is clear that the category of quantity cannot be known without mathematics. For the knowledge of quantity belongs to mathematics alone. Connected with quantity are the categories of when and where. For when has to do with time, and where arises from place. The category of habit cannot be known without the category of place, as Averroes teaches in the fifth book of the Metaphysics. But the greater part of the cagetory of quality contains the attributes and properties of quantities, because all things that are in the fourth class of quality are called qualities in quantities. And all the attributes of these which are absolutely essential to them are qualities, with which a large part of geometry and arithmetic is concerned, such as straight and curved and other essential qualities of the line, and triangularity and other figures belonging to surface or to a solid body; and the prime and non-factorable in numbers, as Aristotle teaches in the fifth book of the Metaphysics, as well as other essential attributes of numbers. Moreover, whatever is worthy of consideration in the category of relation is the property of quantity, such as proportions and proportionalities, and geometrical, arithmetical, and harmonic means, and the kinds of greater and lesser inequality. Moreover, spiritual substances are known by philosophy only through the medium of the corporeal, and especially the heavenly bodies, as Aristotle teaches in the eleventh book of the Metaphysics. Nor are inferior things known except through superior ones, because the heavenly bodies are the causes of things that are lower. But the heavenly bodies are known only through quantity, as is clear from astronomy. Therefore all the categories depend on a knowledge of quantity of which mathematics treats, and therefore the whole excellence of logic depends on mathematics.

CHAPTER 3
IN WHICH IT IS PROVED BY REASON THAT EVERY SCIENCE REQUIRES MATHEMATICS

What has been shown as regards mathematics as a whole through authority, can now be shown likewise by reason. And I make this statement in the first place, because other sciences use mathematical examples, but examples are given to make clear the subjects treated by the sciences; wherefore ignorance of the examples involves an ignorance of the subjects for the understanding of which the examples are adduced. For since change in natural objects is not found without some augmentation and diminution nor do these latter take place without change; Aristotle was not able to make clear without complications the difference between augmentation and change by any natural example, because augmentation and diminution go together always with change in some way; wherefore he gave the mathematical example of the rectangle which augmented by a gnomon increases in magnitude and is not altered in shape. This example cannot be understood before the twenty-second proposition of the sixth book of the Elements. For in that proposition of the sixth book it is proved that a smaller rectangle is similar in every particular to a larger one and therefore a smaller one is not altered in shape, although it becomes larger by the addition of the gnomon.

Secondly, because comprehension of mathematical truths is innate, as it were, in us. For a small boy, as Tullius states in the first book of the Tusculan Disputations, when questioned by Socrates on geometrical truths, replied as though he had learned geometry.[2] And this experiment has been tried in many cases, and does not hold in other sciences, as will appear more clearly from what follows. Wherefore since this knowledge is almost innate, and as it were precedes discovery and learning, or at least is less in need of them than other sciences, it will be first among sciences and will precede others disposing us toward them; since what is innate or almost so disposes toward what is acquired.

Thirdly, because this science of all the parts of philosophy was the earliest discovered. For this was first discovered at the beginning of the human race. Since it was discovered before the flood and then later by the sons of Adam, and by Noah and his sons, as is clear from the prologue to the Construction of the Astrolabe according to Ptolemy, and from Albumazar in the larger introduction to astronomy, and from the first book of the Antiquities,[3] and this is true as regards all its parts, geometry, arithmetic, music, astronomy. But this would not have been the case except for the fact that this science is earlier than the others and naturally precedes them. Hence it is clear that it should be studied first, that through it we may advance to all the later sciences.

Fourthly, because the natural road for us is from

2. An illustration derived ultimately from Plato's *Meno*.

3. Flavius Josephus, the Jewish historian.

what is easy to that which is more difficult. But this science is the easiest. This is clearly proved by the fact that mathematics is not beyond the intellectual grasp of any one. For the people at large and those wholly illiterate know how to draw figures and compute and sing, all of which are mathematical operations. But we must begin first with what is common to the laity and to the educated; and it is not only hurtful to the clergy, but disgraceful and abominable that they are ignorant of what the laity knows well and profitably. Fifthly, we see that the clergy, even the most ignorant, are able to grasp mathematical truths, although they are unable to attain to the other sciences. Besides a man by listening once or twice can learn more about this science with certainty and reality without error, than he can by listening ten times about the other parts of philosophy, as is clear to one making the experiment. Sixthly, since the natural road for us is to begin with things which befit the state and nature of childhood, because children begin with facts that are better known by us and that must be acquired first. But of this nature is mathematics, since children are first taught to sing, and in the same way they can learn the method of making figures and of counting, and it would be far easier and more necessary for them to know about numbers before singing, because in the relations of numbers in music the whole theory of numbers is set forth by example, just as the authors on music teach, both in ecclesiastical music and in philosophy. But the theory of numbers depends on figures, since numbers relating to lines, surfaces, solids, squares, cubes, pentagons, hexagons, and other figures, are known from lines, figures, and angles. For it has been found that children learn mathematical truths better and more quickly, as is clear in singing, and we also know by experience that children learn and acquire mathematical truths better than the other parts of philosophy. For Aristotle says in the sixth book of the Ethics that youths are able to grasp mathematical truths quickly, not so matters pertaining to nature, metaphysics, and morals. Wherefore the mind must be trained first through the former rather than through these latter sciences. Seventhly, where the same things are not known to us and to nature, there the natural road for us is from the things better known to us to those better known to nature, or known more simply; and more easily do we grasp what is better known to ourselves, and with great difficulty we arrive at a knowledge of those things which are better known to nature. And the things known to nature are erroneously and imperfectly known by us, because our intellect bears the same relation to what is so clear to nature, as the eye of the bat to the light of the sun, as Aristotle maintains in the second book of the Metaphysics; such, for example, are especially God and the angels, and future life and heavenly things, and creatures nobler than others, because the nobler they are the less known are they to us. And these are called things known to nature and known simply. Therefore, on the contrary, where the same things are known both to us and to nature, we make much progress in regard to what is known to nature and in regard to all that is there included, and we are able to attain a perfect knowledge of them. But in mathematics only, as Averroes says in the first book of the Physics and in the seventh of the Metaphysics and in his commentary on the third book of the Heavens and the World, are the same things known to us and to nature or simply. Therefore as in mathematics we touch upon what is known fully to us, so also do we touch upon what is known to nature and known simply. Therefore we are able to reach directly an intimate knowledge of that science. Since, therefore, we have not this ability in other sciences, clearly mathematics is better known. Therefore the acquisition of this subject is the beginning of our knowledge.

Likewise, eighthly, because every doubt gives place to certainty and every error is cleared away by unshaken truth. But in mathematics we are able to arrive at the full truth without error, and at a certainty of all points involved without doubt; since in this subject demonstration by means of a proper and necessary cause can be given. Demonstration causes the truth to be known. And likewise in this subject it is possible to have for all things an example that may be perceived by the senses, and a test perceptible to the senses in drawing figures and in counting, so that all may be clear to the sense. For this reason there can be no doubt in this science. But in other sciences, the assistance of mathematics being excluded, there are so many doubts, so many opinions, so many errors on the part of man, that these sciences cannot be unfolded, as is clear since demonstration by means of a proper and necessary cause does not exist in them from their own nature because in natural phenomena, owing to the genesis and destruction of their proper causes as well as of the effects, there is no such thing as necessity. In metaphysics there can be no demonstration except through effect, since spiritual facts are discovered through corporeal effects and the creator through the creature, as is clear in that science. In morals there cannot be demonstrations from proper causes, as Aristotle teaches. And likewise neither in matters pertaining to logic nor in grammar, as is clear, can there be very convincing demonstrations because of the weak nature of the material concerning which those sciences treat. And therefore in mathematics alone are there demonstrations of the most convincing kind through a necessary cause. And therefore here alone can a man arrive at the truth from the nature of this science. Likewise in the other sciences there are doubts and opinions and contradictions on our part, so that we scarcely agree on the most trifling question or in a single sophism; for in these sciences there are from their nature no processes of drawing figures and of reckonings, by which all things must be proved true. And therefore in mathematics alone is there certainty without doubt.

Wherefore it is evident that if in other sciences we should arrive at certainty without doubt and truth without error, it behooves us to place the foundations of knowledge in mathematics, in so far as disposed through it we are able to reach certainty in other sciences and truth by the exclusion of error. This reasoning can be made clearer by comparison, and the principle is stated in the ninth book of Euclid. The same holds true here as in the relation of the knowledge of the conclusion to the knowledge of the premises, so that if there is error and doubt in these, the truth cannot be arrived at through these premises in regard to the conclusion, nor can there be certainty, because doubt is not verified by doubt, nor is truth proved by falsehood, although it is possible for us to reason from false premises, our reasoning in that case drawing an inference and not furnishing a proof; the same is true with respect to sciences as a whole; those in which there are strong and numerous doubts and opinions and errors, I say at least on our part, should have doubts of this kind and false statements cleared away by some science definitely known to us, and in which we have neither doubts nor errors. For since the conclusions and principles belonging to them are parts of the sciences as a whole, just as part is related to part, as conclusion to premises, so is science related to science, so that a science which is full of doubts and besprinkled with opinions and obscurities, cannot be rendered certain, nor made clear, nor verified except by some other science known and verified, certain and plain to us, as in the case of a conclusion reached through premises. But mathematics alone, as was shown above, remains fixed and verified for us with the utmost certainty and verification. Therefore by means of this science all other sciences must be known and verified.

Since we have now shown by the peculiar property of that science that mathematics is prior to other sciences, and is useful and necessary to them, we now proceed to show this by considerations taken from its subject matter. And in the first place we so conclude, because the natural road for us is from sense perception to the intellect, since if sense perception is lacking, the knowledge related to that sense perception is lacking also, according to the statement in the first book of the Posterior Analytics, since as sense perception proceeds so does the human intellect. But quantity is especially a matter of sense perception, because it pertains to the common sense and is perceived by the other senses, and nothing can be perceived without quantity, wherefore the intellect is especially able to make progress as respects quantity. In the second place, because the very act of intelligence in itself is not completed without continuous quantity, since Aristotle states in his book on Memory and Recollection that our whole intellect is associated with continuity and time. Hence we grasp quantities and bodies by a direct perception of the intellect, because their forms are present in the intellect. But the forms of incorporeal things are not so perceived by our intellect; or if such forms are produced in it, according to Avicenna's statement in the third book of the Metaphysics, we, however, do not perceive this fact owing to the more vigorous occupation of our intellect in respect to bodies and quantities. And therefore by means of argumentation and attention to corporeal things and quantities we investigate the idea of incorporeal things, as Aristotle does in the eleventh book of the Metaphysics. Wherefore the intellect will make progress especially as regards quantity itself for this reason, that quantities and bodies as far as they are such belong peculiarly to the human intellect as respects the common condition of understanding. Each and every thing exists as an antecedent for some result, and this is true in higher degree of that which has just been stated.[4]

Moreover, for full confirmation the last reason can be drawn from the experience of wise men;[5]

4. At this point, Burke remarks: "The Latin of this sentence is obscure."

5. "Wise men" replaces Burke's "men of science," which is too narrow a rendition of *sapientes*.

for all the wise men in ancient times labored in mathematics, in order that they might know all things, just as we have seen in the case of men of our own times, and have heard in the case of others who by means of mathematics, of which they had an excellent knowledge, have learned all science. For very illustrious men have been found, like Bishop Robert of Lincoln[6] and Friar Adam de Marisco, and many others, who by the power of mathematics have learned to explain the causes of all things, and expound adequately things human and divine. Moreover, the sure proof of this matter is found in the writings of those men, as, for example, on impressions such as the rainbow, comets, generation of heat, investigation of localities on the earth and other matters, of which both theology and philosophy make use. Wherefore it is clear that mathematics is absolutely necessary and useful to other sciences.

These reasons are general ones, but in particular this point can be shown by a survey of all the parts of philosophy disclosing how all things are known by the application of mathematics. This amounts to showing that other sciences are not to be known by means of dialectical and sophistical argument as commonly introduced, but by means of mathematical demonstrations entering into the truths and activities of other sciences and regulating them, without which they cannot be understood, nor made clear, nor taught, nor learned. If any one in particular should proceed by applying the power of mathematics to the separate sciences, he would see that nothing of supreme moment can be known in them without mathematics. But this simply amounts to establishing definite methods of dealing with all sciences, and by means of mathematics verifying all things necessary to the other sciences. But this matter does not come within the limits of the present survey.

6. That is, Robert Grosseteste.

7. It is evident throughout this selection that Bacon's interest in mathematics stems from his conviction that it is essential for a proper understanding of natural philosophy and all the sciences. As A. G. Molland has observed, the study and pursuit of geometry for its own sake was little in evidence during the Middle Ages. Few theorems were added to the sum total of pure geometry. The latter was valued primarily for its use in comprehending the world. See A. G. Molland, "The Geometrical Background to the 'Merton School': An Exploration into the Application of Mathematics to Natural Philosophy in the Fourteenth Century," *The British Journal for the History of Science*, Vol. IV, Part II, No. 14 (December 1968), pp. 109–110.

Arithmetic

20 ARABIC NUMERALS AND ARITHMETIC OPERATIONS IN THE MOST POPULAR ALGORISM OF THE MIDDLE AGES

John of Sacrobosco (or Holywood) (d. ca. 1244—1256)

Translated and annotated by Edward Grant[1]

Everything that has proceeded from the earliest beginning of things has been formed in a pattern *(ratione)* of numbers, and the manner in which these exist must be understood as follows: that in the whole comprehension of things, the art of numbering *(ars numerandi)* is concerned with combination [or union] *(cooperativa)*. A certain philosopher named Algus wrote this brief science of numbering, for which reason it is called *Algorismus*,[2] which is understood to be the art of numbering *(ars numerandi)* or the introductory art into number *(ars introductoria in numerum)*.

1. Translated from Maximilian Curtze's edition of Sacrobosco's *Algorismus vulgaris (Common Algorism)* in *Petri Philomeni de Dacia Algorismum vulgarem Johannis de Sacrobosco Commentarius una cum Algorismo ipso edidit* (Copenhagen, 1897). During the Middle Ages and into the sixteenth century, this was the most popular treatise describing operations with Arabic numerals. Sometimes known as the *De arte numerandi*, it was translated into Middle English, and somewhat expanded, under the title *The Art of Nombrynge* (ed. Robert S. Steele, *The Earliest Arithmetics in English* [London: Oxford University Press, 1922]). Sacrobosco's treatise drew upon al-Khwārizmī's *Algorism* (ninth century) from the Latin translation titled *De numero Indorum (On Hindu Numerals)*.

2. "Algus" is probably a corruption of "al-Khwārizmī," the name of the author of a book on Hindu numerals (our Arabic numerals). It is from a corruption of his name that we get the term "algorism." For an extract from al-Khwārizmī's algebra, see Selection 22.

A number is made known *(notificatur)* in two ways: materially and formally. Materially *(materialiter)* a number is a collection of units; formally *(formaliter)* it is the multitude of units extended [or spread out].[3] A unit *(unitas)* is that by which anything is said to be one. Included among numbers are the digit *(digitus)*, article *(articulus)*, and composite number *(numerus compositus)*. A digit is every number smaller than 10; an article is every number divisible into ten equal parts with no remainder;[4] a composite number is mixed, consisting of digit and article.[5] And it must be understood that every number between [any] two proximate [or successive] articles is composite.[6] Moreover, there are nine species of this art [of Algorism]: (1) numeration, (2) addition, (3) subtraction, (4) halving *(mediatio)*, (5) doubling *(duplatio)*, (6) multiplication, (7) division, (8) progression, and (9) extraction of roots, which is twofold, since [it applies] to square numbers and cube numbers. Among these, numeration will be considered first and then the others will follow in order.

ON NUMERATION

The numeration of any number by any suitable figures is an artificial representation. However, figure *(figura)*, difference *(differentia)*, place *(locus)*, and limit *(limes)* suppose the same thing but they are imposed for different reasons. "Figure" is so called because of the drawing of a line: "difference" is shown in this way: how the figure following differs from the figure preceding; "place" is so called by reason of the space in which a number is written; "limit" is the path [or system] that has been organized for the representation of any number. Therefore, it should be understood that alongside the nine "limits" are found nine significative "figures" representing nine digits, which are these:

9. 8. 7. 6. 5. 4. 3. 2. 1.

The tenth [figure], 0, is called *teca*,[7] circle *(circulus)*, cipher *(cyfra)*,[8] or "the figure of nothing" *(figura nichilis)*, since it signifies nothing. It holds a place and signifies for others, for without a cipher or ciphers, a pure article can not be formulated. And so it happens that any number can be represented by these nine significative figures, with a cipher or ciphers added whenever needed, rendering it unnecessary to find more significative figures [than these]. Therefore, it must be noted that every digit must be written with only one figure that is appropriate to it; every article, however, must be represented by a cipher placed in the first position and a digit by which this article is denominated, since every article is denominated by some digit—as ten *(denarius)* by unity, twenty *(vigenarius)* by two, and so on. Indeed, every number that is a digit has to be placed in a first [category of] difference, every article in a second [category of] difference.[9] For every number from ten to one hundred, excluding one hundred, must be written by two figures; and if it is an article *(articulus)*, [it is written] by a cipher, placed in the first position, and a figure written to the left, which signifies the digit by which the article is denominated. If it is a composite number, the digit which is part of it is written first *(praescribatur)*, and as before, the article is placed to the left.[10] Again, every number from one hundred

3. The *material* aspect of number appears to be understood as a collection of units assembled and united to form each number, thereby, in a sense, constituting the "matter" of each number. The formal aspect of number consists of the units themselves conceived as a multiplicity. It is probable that Sacrobosco adopted this twofold definition from Boethius' *Arithmetic*, Book I, chapter 3, where we read: "Number is a collection of units or a multitude of a quantity brought together from units" (see Boethius, "On Arithmetic," Bk. I, ch. 3, in Selection 2. Following Aristotelian philosophical terminology, Sacrobosco then labeled the first part "material," the second part "formal."

4. Thus an "article" would be 10 and any multiple or power of 10.

5. Any whole number can be expressed as a multiple or power of 10 plus digits.

6. For example, the numbers between 100 and 110, and so on.

7. The meaning of this term is unknown to me, but Peter of Dacia, in his commentary on Sacrobosco's *Common Algorism*, says, "*Teca* is iron of round figure," and that the roundness of the cipher was called *teca* by virtue of its similarity to the roundness of the iron (see Curtze, p. 26).

8. A transliteration of the Arabic term *sifr*, meaning "empty."

9. By definition, a "difference" signifies "how the figure following differs from the figure preceding" and appears to be Sacrobosco's way of describing place value in the decimal system.

10. If the passage means what it seems to say, one must conclude that Sacrobosco would write a number like 532 by first writing 2, then to the left of it 3, and finally, 5 to the left of 3. Many Arabs write their numbers in this manner. It must be emphasized, however, that although Arabic text is written and read from right to left, numerals in Arabic are read from left to right. In the early years of the introduction of Arabic numerals, Sacrobosco urged his readers to write the numbers from right to left, believing that he was following the Arabic custom.

The peculiarity in Arabic of reading everything except numerals from right to left may derive from the fact

25 A PROPOSITION ON MATHEMATICAL PROBABILITY

Nicole Oresme (ca. 1325—1382)

Translated and annotated by Edward Grant[1]

Proposition X. It is probable that two proposed unknown ratios are incommensurable because if many unknown ratios are proposed it is most probable that any [one] would be incommensurable to any [other].

As found in the first chapter, there are three types of ratios.[2] Some, indeed, are rational ratios, others irrational with denominations, i.e., commensurable to rationals, and perhaps there is a third type, namely irrational ratios which have no denomination because they are not commensurable to any rationals.

Let there be two unknown ratios. Now each might be rational, namely of the first type, and the argument would proceed as follows: with any whatever number of rational ratios forming one or more series of denominations, those which are mutually commensurable are much fewer than those which are incommensurable; and therefore it is likely that any two proposed unknown ratios are incommensurable.

The antecedent is [now] demonstrated.[3] Let 100 ratios of the multiple genus be taken according to the sequence of their denominations, as 2/1, 3/1, 4/1, 5/1, etc., up to 101/1, and let them be as 100 terms mutually compared. Then by comparing any one of these terms to any other of them there would be 4,950[4] ratios which are ratios of ratios and, of these, 25—and no more—are rational and all the others are irrational, as I shall show afterwards.[5] Now if more rational ratios were taken as terms, [for example] 200 or 300, and if ratios of these were taken, the ratio of irrational to rational ratios would be much greater. And if the ratios were taken in a genus other than multiple there will be even fewer mutually commensurable ratios. Indeed, all superparticular ratios are mutually incommensurable, as was seen above.[6] Furthermore, if some ratios are taken from one genus and others from another, this will result in the fewest ratios being mutually commensurable because, as is evident from the third proposition of this [chapter], all ratios in the multiple genus are incommensurable to [ratios of other genera], just as 10/1 is not commensurable to any rational before 100/1, and 7/1 is [commensurable] to [only] one [ratio before 100/1], and 12/1, 13/1, etc., are not commensurable to any below 100/1. For this reason if all the multiple ratios below 100/1 were taken, only 16 would be commensurable to any rational below 100/1, as will be seen afterward.[7] Thus the antecedent has now been stated and all the propositions beyond the ninth of this chapter are designed to make it evident.

I now state the principal consequence[8] With regard to numbers, we see that however many numbers are taken in series, the number of perfect[9] or cube numbers is much less than other numbers and as more numbers are taken in the series the greater is the ratio of noncube to cube numbers or nonperfect to perfect numbers. Thus if there were some number and such information as what it is or how great it is, and whether it is large or small, were wholly unknown—as is the case for the number of all the hours which will pass before anti-Christ—it will be likely that such an unknown number would not be a cube number. A similar situation is found in games where, if one should inquire whether a hidden number is a cube number, it is safer to reply in the negative since this seems more probable and likely.

Now what has been said here about numbers may be applied to ratios of rational ratios, as was shown before, since there are many more irrationals than others, understood in the previous sense [of ratios of ratios]. What is more, [it is even more applicable to ratios, for] if one reflected carefully he would find that between ratios of rational ratios those which are rational are fewer than cube numbers in an aggregate of numbers. Therefore, if any unknown ratio of ratios were sought, it is probable that it would be irrational and its ratios incommensurable. And all this applies if the unknown ratios which are sought should be rational. . . .

And so it is clear that with two proposed unknown ratios—whether they are rational or not—it is probable that they are incommensurable, which was proposed in the first place.[10] Therefore, if many [unknown ratios] are proposed, it is [even] more probable that any one of them would be incommensurable to any other, which was proposed in the second instance. Now the more there are, the more one must believe that any one is incommensurable to any other, for if it is probable that one proposed ratio of ratios is irrational, it is more probable when many are proposed that any one would be irrational, just as could be shown in the example involving cube numbers.

26 INFINITE SERIES

Nicole Oresme (ca. 1325—1382)

Translated and annotated by Edward Grant[1]

QUESTION 1

Concerning the book of Euclid [that is, the exponentially related ratios are both irrational but have rational exponents, as for example, $(4/1)^{1/3}$ and $(2/1)^{1/2}$, which are related by a rational exponent (that is, $(4/1)^{1/3} = [(2/1)^{1/2}]^{1/3}$), or $(3/1)^{1/4}$ and $(2/1)^{1/2}$, which are related by an irrational exponent; and (3) "ratios of ratios" where the two exponentially related ratios are irrational with irrational exponents, as for example, $(8/1)^{\sqrt{2}}$ and $(2/1)^{\sqrt{2}}$, which are themselves related by a rational exponent (that is, $(8/1)^{\sqrt{2}} = [(2/1)^{\sqrt{2}}]^{3/1}$), or $(8/1)^{\sqrt{2}}$ and $(5/1)^{\sqrt{2}}$, which are not only irrational ratios with irrational exponents but are also relatable by an irrational exponent.

I have reproduced here only Oresme's discussion of the first type of "ratios of ratios" and omitted his brief discussions of types (2) and (3). By analogy with, and inference from, type (1), however, Oresme declares that for any finite number of ratios related two at a time in categories (2) and (3) there would be more irrational than rational "ratios of ratios." Without additional information about specific sequences of ratios, his claim cannot be properly evaluated.

1. This selection is from Oresme's *Questions on the*

1295, since any arbitrary value could be taken for *l*. However, the presence of a denominator of 7 in the values for *y*, *z*, *m*, *n*, and *p* allows one to suppose that the initial value of *l* was 7, or a multiple of 7. However, the four small lacunae cannot be determined with certainty.

1. Reprinted with permission of the copyright owners, The Regents of the University of Wisconsin, from *Nicole Oresme: De proportionibus proportionum and Ad pauca respicientes*, edited with Introductions, English translations, and Critical Notes by Edward Grant (Madison, Wis.: University of Wisconsin Press, 1966), pp. 247–255. The notes presented here are based upon my analysis and summary in the introduction to the edition.

In this selection, we may have one of the earliest, if not the earliest, reasonably formal propositions in probability theory.

2. These are described in Selection 29, notes 2, 3, and 4.

3. The antecedent demonstrated in this paragraph declares that when a finite sequence of rational ratios is taken, the number of irrational ratios of ratios will exceed the number of rational ratios of ratios.

4. By taking 100 consecutive rational ratios from 2/1 *to* 101/1 and relating them two at a time, there would result $100 \cdot 99 = 9900$ possible *ratios of ratios* (for a definition of this expression, see Selection 51. 2, n. 51). Since Oresme is interested here only in relations involving ratios of greater inequality— that is, ratios of the form $n/1$, where n is an integer between 2 and 101, 9900 must be divided by 2, resulting in 4950 ratios of ratios of greater inequality. These may be represented generally by $n/1 = (m/1)^{p/q}$, where n and m are integers and p/q may be rational or irrational.

5. In a practical conclusion, which is really chapter 3, Proposition 11, Oresme provides the details of this particular example (see Grant, *Nicole Oresme: De proportionibus*, pp. 257, 259) and shows that of the 4950 ratios of ratios already mentioned, only 25 would be "rational ratios of ratios." These 25 ratios are derived from all the geometric series contained within the sequence of given terms. Thus the series $(2/1)^n$, where $n = 1, 2, 3, 4, 5, 6$, will produce 15 rational ratios of ratios when any two are taken at a time (that is, $(6 \cdot 5)/2 = 15$). For example, one could take the ratios $(2/1)^2$ and $(2/1)^5$, where $(2/1)^5 = [(2/1)^2]^{5/2}$. Thus the rational exponent, 5/2, relates the two rational ratios 32/1 and 4/1. Similarly the series $(3/1)^n$, where $n = 1, 2, 3, 4$, yields 6 rational ratios of ratios; and one each will result from $(5/1)^n$, $(6/1)^n$, $(7/1)^n$, and $(10/1)^n$, where in each case $n = 1, 2$. The total number is therefore 25. The remaining 4925 ratios of ratios will be irrational—that is, the exponent relating the two rational ratios will be irrational. For example, taking 3/1 and 2/1, we see that $3/1 \neq (2/1)^{p/q}$ if p/q is a rational exponent. At the end of his practical conclusion, Oresme declares that the number of irrational to rational ratios of ratios is 197 to 1 (that is, 4925/25). For this reason, he insists, in the next two paragraphs of this selection, that if one were asked to guess whether or not an unknown ratio of ratios is irrational or rational, the mathematical probabilities would clearly dictate the choice of an irrational ratio.

6. That is, $(n + 1)/n \neq [(m + 1)/1]^{p/q}$, where n and m are integers greater than 2 and p/q is a rational exponent.

7. The 16 ratios less than 100/1 are all those cited above in note 5 except 10/1 and 100/1.

8. The principal consequence is the enunciation of Proposition X. With the antecedent demonstrated (see n.3), Oresme arouses that any unknown ratio of ratios would probably be irrational.
This is now shown by analogy with perfect and nonperfect numbers and cube and noncube numbers.

9. A perfect number is one whose sum of factors is equal to it (for example, $6 = 1 + 2 + 3$).

10. Oresme claims that his "probability theorem" applies to "ratios of ratios" of all three types that could be formed from the three types of single ratios described at the beginning of this selection—that is, (1) "ratios of ratios" where the ratios that are exponentially related are both rational, as for example, 2/1 and 3/1; or 2/1 and 16/1, and so on; (2) "ratios of ratios" where the two

Elements], we inquire first about a certain statement by Campanus asserting that a magnitude decreases into infinity.[2] First we inquire *whether a magnitude decreases into infinity according to proportional parts.*

In the first place, it is argued that it does not [increase into infinity according to proportional parts]. In a continuum there are not infinite parts of the same [finite] quantity, therefore there are not infinite parts of the same proportion. The antecedent is obvious, for then it would be infinite;[3] the consequent is clear because any proportional part is part of the same quantity with any other [proportional part]. Therefore, the parts of the same proportion and of the same quantity are the same.[4]

In the comment by Campanus the opposite [of this] is obvious.

For this question, these things must be noted: firstly, what has [already] been said, namely that proportional parts are[5] parts of the same proportion; secondly, in how many ways can such parts be imagined; thirdly, how something can be divided into such parts; fourthly, assumptions and propositions.

As to the first, it must be noted that proportional parts are said to be in continued proportion and such a proportion is a similitude of ratios, as it is called in the comment [by Campanus] on the ninth definition of the fifth [book of Euclid],[6] where it is said that such a [relation or similitude] is had between at least two ratios; and for this reason Euclid says that the least number of terms in which it [that is, similitude of ratios] is found is three, but he does not give a maximum number because it is a process [that continues] into infinity. It follows from this that it is not proper to speak of [a single] proportional part, nor of two proportional parts, but it is necessary that there be at least three[7] and there can be an infinite number. Proportional parts are said to be continually proportional because the first [proportional part] is related to the second as the second to the third, and so on if more are taken.

As to the second point, I reply that a division can be made into such proportional parts in as many ways as there are continuous proportionals; and there are just as many continuous proportionals as there are ratios, namely infinitely many. For example, it can happen that the first [proportional part] is double the second, and the second double the third, and so on,[8] just as is commonly said about the division of a continuum; and it could happen that the first [proportional part] is triple the second and the second triple the third, and so on.[9]

As to the third point, it is held that a line and any continuum can be divided into such [proportional] parts. A line [can be so divided] in two ways because there are two extremities of it and such parts

Geometry of Euclid, written perhaps around 1350. The translation, containing all but a few lines of the first two questions, is based upon the Latin text edited by H. L. L. Busard, *Nicole Oresme: Quaestiones super geometriam Euclidis* (Leiden: Brill, 1961), pp. 1–6, and the many emendations and corrections to that text made by John E. Murdoch in his long review of Busard's text in *Scripta Mathematica*, XXVII (1964), 67–91.

The subject of infinite series, with its associated paradoxes, exerted a strong fascination for medieval natural philosophers and mathematicians. Many series were formulated and utilized by Oxford (Merton College) scholastics such as Swineshead, Dumbleton, Heytesbury, and others in the first half of the fourteenth century, and then by Parisian scholastics such as Nicole Oresme and, much later, Alvarus Thomas, to name only two; for some of the infinite series proposed by Alvarus Thomas, see H. Wieleitner, "Zur Geschichte der unendlichen Reihen im christlichen Mittelalter," *Bibliotheca Mathematica*, XIV (1914), 150–168.

2. Immediately following the axioms *(communes animi conceptiones)* and just before Book I of Euclid's *Elements*, Campanus of Novara remarks that "magnitude decreases into infinity, but in numbers this is not so."

3. Presumably, a defender of this position would argue that if a continuous quantity consisted of an infinite number of parts, it would be infinite.

4. Since there cannot be an infinite number of parts in a finite quantity, there cannot be an infinite number of proportional parts, because "the parts of the same proportion and of the same quantity are the same." In the concluding paragraph of this question, Oresme interprets this argument to mean that all proportional parts are equal, an absurd position which he promptly denies without elaboration, for it is contrary to all that has been shown in the question itself.

5. Although the text has *vel* (or), the sense seems to require "are."

6. For a translation of this definition, see Selection 27, Definition 9.

7. If three terms, a, b, and c, are in continued proportion, then $a/b = b/c$. In the same way, a proportion of proportional parts must consist of at least three proportional parts in continued proportion, as, for example, $(1:a)/(1:b) = (1:b)/(1:c)$. Hence at least three such ratios are required before we can speak of proportional parts. Murdoch remarks (p. 68), "The key notion in his analysis is correctly seen by Oresme to be that of 'proportional parts,' parts determined, he adds, by a given continued proportion. Thus, his fundamentals for an investigation of infinite geometric series are in fine order. Moreover, when he probes further his success is, by fourteenth-century standards, considerable."

8. That is, $(1/2)^n$, where $n = 1, 2, 3, 4 \ldots$.

9. That is, $(1/3)^n$, where $n = 1, 2, 3, 4 \ldots$.

can begin from either one. A surface [can be divided into proportional parts] in infinite ways, and similarly for a body.

As to the fourth point, a first supposition is assumed such that if any ratio were increased to infinity with the greater term unchanged, the smaller term would be diminished into infinity. This is obvious, since a ratio between two [terms] can be increased into infinity in two ways: either by the augmentation to infinity of the greater term or by the diminution to infinity of the lesser term.[10]

A second supposition is that if to any ratio there is added so much, and again so much, and so on into infinity, this ratio will be increased into infinity; and this is common to all quantities.

A third supposition is this: that to any quantity an addition can be made by proportional parts; and by the same [supposition] a diminution can be made by proportional parts.

The first proposition is that if an aliquot part should be taken from some quantity, and from the first remainder such a part is taken, and from the second remainder such a part is taken, and so on into infinity, such a quantity would be consumed exactly—no more, no less—by such a mode of subtraction.[11] This is proved because the whole that was originally assumed, and the first remainder, and the second remainder, and the third, and so on, are continually proportional, as could be proved when arguing with an altered ratio.[12] Therefore, there is a certain ratio, and then so much [of it], and so on endlessly; consequently, such a ratio of the whole to the remainder increases to infinity, because, by the second supposition, it is composed of these [ratios].[13] And the other term, say the whole, is imagined as unchanged, so that, by the first supposition, the remainder is diminished into infinity, and consequently, the whole quantity is consumed exactly.

This corollary follows: If from any foot [length] there should be taken away a half foot, and then half of the remainder of this quantity, and then half of the next remainder, and so on into infinity, the foot length will be removed exactly by this [procedure].[14]

A second corollary is that if one-thousandth part of a foot were taken away [or removed], then [if] one-thousandth part of the remainder of this foot [were removed], and so on into infinity, exactly one foot would be subtracted from this [original foot].[15]

But this is doubted. Since exactly half of one foot and then half of the remainder of that foot, and so on into infinity, make one foot, let this whole [foot] be a; similarly, by the second corollary, one-thousandth part of this one foot, and one-thousandth of another [that is, the next] remainder, and so on into infinity, make one foot, let this be b. It is then obvious that a and b are equal. But it can be proved that they are not equal because the first part of a is greater than the first part of b; and the second part of a [is greater] than the second [part] of b, and so on to infinity, Therefore, the whole a is greater than the whole b. And this is confirmed, [for] if Socrates were moved over a for one hour and Plato over b [for one hour], and [if] they divide the hour by proportional parts and traverse a and b, respectively, then Socrates is moved quicker than Plato in the first proportional part; and similarly in the second [proportional part], and so beyond. Socrates will therefore traverse a greater distance than Plato, so that a is a greater distance than b.[16] In response to this, I deny the antecedent,

10. Oresme does not say but this supposition can apply only to ratios of greater inequality. He makes this quite explicit in his *On Ratios of Ratios (De proportionibus proportionum,)* ch. 1, *ll.* 106–111 (Grant, *Nicole Oresme: De proportionibus* [Madison, Wis.: University of Wisconsin Press, 1966], pp. 144–147.

11. Murdoch (p. 68) represents this series as:
$$\frac{a}{n}+\frac{a}{n}\left(1-\frac{1}{n}\right)+\frac{a}{n}\left(1-\frac{1}{n}\right)^2+\cdots+\frac{a}{n}\left(1-\frac{1}{n}\right)^m+\cdots=a.$$

12. By subtracting from a each successive proportional part in the series described in note 11, we obtain a, $a - a/n$, $a - 2a/n + a/n^2$, $a - 3a/n + 3a/n^2 - a/n^3$, . . ., which are the successive remainders in continued proportion. These successive remainders, as Oresme tells us a few lines below, are diminished to infinity and "the whole quantity is consumed exactly."

13. The ratio of a, the whole, to the successive remainders forms a sequence of ratios increasing to infinity. Thus the successive ratios are $a/(a - a/n) < a/(a - 2a/n + a/n^2) < a/(a - 3a/n + 3a/n^2 - a/n^3) < \cdots$.

14. Oresme expresses here the convergent series $1/2 + 1/4 + 1/8 + \cdots + 1/2^n + \cdots = 1$.

15. Here we have the convergent series
$$\frac{1}{1000}+\frac{1}{1000}\left(1-\frac{1}{1000}\right)+\frac{1}{1000}\left(1-\frac{1}{1000}\right)^2+\cdots+\frac{1}{1000}\left(1-\frac{1}{1000}\right)^n+\cdots=1.$$

16. This paragraph brings to the fore a doubt that arises from a comparison of corresponding terms in the series described in notes 14 and 15. Since the successive terms of the first series, called a, are greater than the corresponding terms in the second series, called b—and we see this by inspection where $1/2 > 1/1000$; $1/4 > (1/1000)[1 - (1/1000)]$, and so on—it is concluded that the whole magnitude in series a is greater than that of series b, making it impossible for them both to sum at 1. Oresme denies this (see remainder of paragraph) by insisting rightly that eventually the terms of series b will become greater than their corresponding counterparts in series a.

namely that the first part of a is greater [than the first part of b, and the second part of a is greater than the second part of b], and so on. The reason [for denying the antecedent] is that although the first part of a is greater than the first part of b, and the second part of a is greater than the second part of b, this is not so into infinity, since one part will eventually be reached that will not be greater than the part of b to be compared, but smaller. In this way the response to the question is clear, because in the imagination any continuum can have infinite proportional parts; similarly, the first [part] can really be separated from the others in thought[17] and then the second, and so on into infinity.

As for the argument to the contrary, I deny the consequent, and for proof I say that although any proportional part belongs to the same quantity as any other [proportional part], it is nevertheless not of the same quantity with that [part] with which it is of the same proportion. They are not equals, since it follows [that if] they are equals, therefore they are mutually equal to themselves.[18]

QUESTION 2

Next we inquire *whether an addition to any magnitude could be made into infinity by proportional parts*. . . .[19]

For this question, it must be noted in the first place that there is a ratio of equality and it is between equals; another [kind of ratio] is a ratio of greater inequality, which is of greater to smaller, as 4 to 2; and another [kind] is [a ratio] of lesser inequality, which is of smaller to greater, as 2 to 4. And these names differ with respect to relative superposition and subposition,[20] as is obvious in what has been said before; and by this there are three ways in which an addition can be made to any quantity.

Secondly, it must be noted that if an addition were made to infinity by proportional parts in a ratio of equality or of greater inequality, the whole would become infinite;[21] if, however, this addition should be made [by proportional parts] in a ratio of lesser inequality, the whole would never become infinite, even if the addition continued into infinity.[22] As will be declared afterward, the reason is because the whole will bear a certain finite ratio to the first [magnitude] assumed [or taken] to which the addition is made.

Finally, it must be noted that every term smaller than another which bears to it [that is, to the greater] a fixed ratio is called a fraction or fractions, or part or parts; and this is obvious in the principles of the seventh book of Euclid, and it is denominated by two numbers, one of which is the numerator and the other the denominator, as is clear in the same place [that is, in the principles of Euclid's seventh book].[23] For example, one is smaller than two and is called one-half of two and one-third of three, and so on; and two is called two-thirds of three and two-fifths of five, and they ought to be written in this way;[24] and the two is called the numerator, the five the denominator.

The first proposition is that if a one-foot quantity should be assumed and an addition were made to it into infinity according to a subdouble [that is, one-half] proportion so that one-half of one foot

17. Although the text reads *per corrupcionem*, it seems to require emendation to *per concepcionem*, which I have translated as "in thought."

18. See note 4.

19. Eighteen lines, offering a few objections to the affirmative of this question, have been omitted from the translation.

20. I have emended Busard's text from *relativi* <sc. *termini*> *posicionis* to *relative* <*super*> *posicionis* and altered my translation accordingly. The terms 2 and 4 can be related so that one is relatively "superposed" or "subposed" to the other. Thus 4 "superposed" with respect to 2 produces a ratio of greater inequality; if "subposed"—that is, placed under 2—it produces a ratio of lesser inequality. Almost identical terminology was used by Oresme in his *On Ratios of Ratios*, ch. 1, *ll.* 201–204 (Grant, *Nicole Oresme: De proportionibus*, pp. 154, 155).

21. Should the proportional parts be represented as $(a/b)^n$, where $a > b$ and $n = 1, 2, 3, 4, \ldots$, an addition to infinity would produce an infinite whole (that is, a divergent series).

22. Here $a < b$.

23. In Book VII of Campanus' edition of Euclid's *Elements*, fractions are not mentioned. While Campanus speaks of *denomination* of ratios, he does not assert that two numbers are required to denominate a ratio nor does he identify these numbers as numerator and denominator. By "fraction or fractions, or part or parts," Oresme probably meant to distinguish between unit and non-unit fractions. That is, if a and b are numbers in their lowest terms and $a < b$, then if $a = 1$ and $b > 1$, we have a (unit) fraction or part; but if $b > a > 1$, we have fractions or parts.

24. Apparently Oresme intended to convey the precise manner in which fractions were to be written, but the Latin text, and presumably the manuscripts used to establish that text, fail to reveal this. It seemed best to verbalize them in full. The form desired by Oresme for expressing simple fractions is preserved in Part 1 of his *Algorismus proportionum*, as the reader can see in Selection 28. But just as the manuscripts of the *Questions on the Geometry of Euclid* display no unanimity, so also do the many manuscripts of the *Algorismus* present a bewildering variety of forms for representing irrational ratios.

is added to it, then one-fourth [of one foot], then one-eighth, and so on into infinity by squaring the halves,[25] the whole will be exactly double the first [magnitude] assumed.[26] This is obvious, because if these [very] parts were taken from any quantity, exactly double the first quantity would be taken, as is clear from the first question preceding. By a parity of reasoning, then, if the parts were added [the whole would be exactly double the first].[27]

The second proposition is this: If any quantity were assumed, say one foot, then let one-third as much be added [to it], and then [let] one-third of what was added [be added to the sum], and so on into infinity, the whole will be exactly $1\frac{1}{2}$ feet, namely in a sesquialterate ratio to the first quantity assumed.[28] Furthermore, this rule should be known: We must see how much the second part falls short of the first part, and [how much] the third [falls short] of the second, and so on with the others, and to denominate this [difference] by its denomination, and then the ratio of the whole aggregate to the quantity [first] assumed will be just as a denominator to a numerator. For example, in what was [just] proposed, the second part, which is one-third of the first, falls short by two-thirds of the [quantity of the] first, so that the ratio of the whole to the first part, or what was assumed, is as 3 to 2 and this is a sesquialterate [ratio].[29]

The third proposition is this: It is possible that an addition could be made, though not proportionally, to any quantity by ratios of lesser inequality, and yet the whole would become infinite; but if it were done proportionally, it would be finite, as was said. For example, let a one-foot quantity be assumed to which one-half of a foot is added during the first proportional part of an hour, then one-third of a foot in another [or next proportional part of an hour], then one-fourth [of a foot], then one-fifth, and so on into infinity following the series of [natural] numbers, I say that the whole would become infinite,[30] which is proved as follows: There exist infinite parts of which any one will be greater than one-half foot and [therefore] the whole will be infinite. The antecedent is obvious, since 1/4 and 1/3 are greater than 1/2; similarly [the sum of the parts] from 1/5 to 1/8 [is greater than 1/2] and [also the sum of the parts] from 1/9 to 1/16, and so on into infinity. . . .[31]

25. "Squaring the halves" translates *duplando subduplos*.

26. That is, $1 + 1/2 + 1/4 + 1/8 + \cdots + 1/2^n + 1/2^{n+1} + \cdots = 2$.

27. If from any magnitude parts were taken in the order $1 + 1/2 + 1/4 + 1/8 + \cdots + 1/2^n + \cdots$, the total magnitude taken would be double the first given quantity (one foot in this case). Reversing the process and reconstituting the whole by adding the successive parts, a quantity double (that is, two feet) the first given quantity (that is, one foot) would be produced.

28. We have here the convergent series $1 + 1/3 + 1/9 + 1/27 + \cdots + 1/3^n + 1/3^{n+1} + \cdots = 3/2$.

29. The difference between the first and second terms is $1 - 1/3 = 2/3$. "To denominate this [difference] by its denomination" is, in medieval mathematics, to represent the quantity two-thirds by a ratio of numbers, namely 2/3. Then, when 2/3 is related as denominator to numerator, we get 3/2, the sum of the series. From the fact that Oresme offered only this example, it would appear that he intended that it apply to any two successive terms in the series. But if the second and third parts—or any other two successive parts—are taken, the concept of "denomination" cannot have its usual meaning for Oresme. For $1/3 - 1/9 = 2/9$, where the difference, two-ninths, is a ratio whose numerical denomination is 2/9, and the ratio of its denominator to its numerator, representing the final step, is 9/2, a result Oresme could not have intended. Obviously, $(2/9):(1/3) = 2/3$ is required where the reciprocal yields 3/2. But "to denominate a difference by its denomination" cannot mean representing 2/9, the difference, by the ratio $(2/9):(1/3)$ unless, perhaps, Oresme extended the meaning of denomination to include the case where the greater of two ratios (in this instance 1/3) can serve as the denominator and denomination of the difference (that is, 2/9). Nevertheless, one is left with the strong impression that Oresme used the expression "to denominate this [difference] by its denomination" correctly in the only instance he gives and improperly assumed that it applied equally to all instances.

30. The series $1 + 1/2 + 1/3 + 1/4 + \cdots + 1/n + \cdots$ is divergent and is offered as illustration of the case in which ratios of lesser inequality are not added proportionally and can constitute an infinitely large whole.

31. Omitted here are the remaining twenty lines of Question 2, in which Oresme replies to the objections mentioned in note 19.

you AB greater, or smaller, or equal, indifferently, as I wish. Therefore, either he does not demonstrate it, or it is circular and the principles are less known than the conclusions. With Euclid, however, the principles must be assumed as known and they must not be demonstrated from propositions, but [rather] propositions must be demonstrated from them.

28 AN ALGORISM OF RATIOS: MANIPULATION OF RATIONAL EXPONENTS

Nicole Oresme (ca. 1325—1382)

Translated and annotated by Edward Grant[1]

THE PROLOGUE OF MASTER NICOLE ORESME
ON THE TREATISE
ALGORISM OF RATIOS

To the Most Excellent Reverend Phillip of Meaux,[2] whom I would call Pythagoras if it were possible to believe in the opinion about the return of souls, I present this *Algorism of Ratios*, so that if it is agreeable to your Excellency, you may correct that which I put before you. For should it be approved by the authority of so great a man and corrected after his examination [of it], everything that has been revised by your correction would be an improvement. Then, if a disparager should open his mouth and set his teeth to rend [my work] into pieces, he would not find [what he seeks].

PART I

One half is written as 1/2, one third as 1/3, and two thirds as 2/3, and so on. The number above the crossbar is called the numerator, that below the crossbar the denominator.

A double ratio [2/1] is written as 2^p, a triple ratio [3/1] as 3^p, and so forth. A sesquialterate ratio [3/2] is written as 1^p 1/2, and a sesquitertian ratio [4/3] as 1^p 1/3. A superpartient two-thirds ratio [5/3] is written as 1^p 2/3, a double superpartient three-fourths [11/4] as 2^p 3/4, and so on. Half of a double ratio $[(2/1^{1/2}]$ is written as 1/2 2^p and a fourth part of a double sesquialterate $[(5/2)^{1/4}]$ as 1/4 2^p 1/2, and so on.[3] But sometimes a rational ratio is written in its least terms or numbers just as a ratio of 13 to 9, which is called a superpartient four-ninths [1 4/9]. Similarly, an irrational ratio such as half of a superpartient two-thirds $[(5/3)^{1/2}]$ is written as half of a ratio of 5 to 3.

Every irrational ratio—and these shall now be considered—is denominated by a rational ratio in such a manner that it is said to be a part or parts of the rational ratio, as [for example] half of a double $[(2/1)^{1/2}]$, a third part of a quadruple $[(4/1)^{1/3}]$, or two thirds of a quadruple $[(4/1)^{2/3}]$.[4] It is clear that there are three things [or elements] in the denomination of such an irrational ratio: [1] a numerator, [2] a denominator, and [3] a rational ratio by which the irrational ratio is denominated, that is, a rational ratio of which that irrational ratio is said to be a part or parts, as, [for example,] in half of a double ratio $[(2/1)^{1/2}]$ the unit is the numerator, or represents the numerator, two is the denominator,[5] and the double ratio is that by which the irrational ratio is denominated.[6] And this can easily be shown for other ratios.

Rule One. [How] to add a rational ratio to a rational ratio.

Assuming that each ratio is in its lowest terms, multiply the smaller term, or number, of one ratio by the smaller number of the other ratio; and [then] multiply the greater [number of one ratio] by the greater [number of the other], thereby producing the numbers or terms of the ratio composed of the two given ratios. In this way, three or [indeed] any number [of ratios] can be added by adding two of them at a time and then adding a third to the whole composed of those two; then, if you wish add a fourth ratio, and so on.

For example, I wish to add sesquitertian [4/3] and quintuple [5/1] ratios. The prime numbers of a sesquitertian are 4 and 3, of the other 5 and 1. And so, as already stated, I shall multiply 3 by 1 and 4 by 5 obtaining 20 and 3, which is a sextuple superpartient two-thirds ratio [6⅔]. In this way a ratio can be doubled, tripled, and quadrupled, as many times as you please.[7] And this can be demonstrated and is [indeed] adequately shown in the sixth proposition of the fifth book of the *Arithmetic* of Jordanus [de Nemore].[8]

Rule Two [How] to subtract a rational ratio from [another] rational ratio.

As before, assume any [two] rational ratios in their lowest numbers. The smaller number of one ratio is then cross-multiplied *(ducantur contradictorie)* by the greater number of the other ratio, and the same is done with the remaining numbers, thus producing the terms of a ratio in which a greater term exceeds a smaller term. The greater of the given ratios was that whose greater term when multiplied by the smaller term of the other given ratio produces the greater number [of the resultant ratio].

For example, let a sesquiteritan [4/3] be subtracted from a sesquialterate ratio [3/2]. The prime numbers, or terms, of a sesquitertian are 4 and 3, of a sesquialterate 3 and 2. I shall multiply 4 by 2 and obtain 8, and then multiply 3 by 3 to obtain 9. A sesquialterate ratio is, therefore, greater than a sesquitertian by a ratio of 9 to 8, that is, greater by

1. This selection and the bulk of the notes were first published under the title "Part I of Nicole Oresme's *Algorismus proportionum*," translated and annotated by Edward Grant in *Isis*, LVI (1965), 327–341. It is reprinted here by permission of the editors of *Isis*. Although the translation has been left intact, the notes have been altered slightly and reduced in extent (for instance, information about manuscripts has been omitted, as have all Latin quotations; however, the English translations of these quotations have been included, and unless otherwise specified, are my own).

The present translation is made from my edition of Part I, based on thirteen manuscripts, which appears in "The Mathematical Theory of Proportionality of Nicole Oresme," Ph. D. Dissertation, University of Wisconsin, 1957, pp. 331–339. *The Algorismus proportionum (Algorism of Ratios)* of Nicole Oresme consists of a prologue and three parts; only the prologue and first part—the most important—are translated here. In Part I, Oresme formulates a series of rules for operations with exponents, which he subsequently applies to various physical problems in Parts II and III. All three parts—but not the prologue—were edited by Maximilian Curtze in his *Der Algorismus Proportionum des Nicolaus Oresme; zum ersten Male nach der Lesart der Handschrift R. 4° 2 der königlichen Gymnasialbibliothek zu Thorn* (Berlin, 1868).

The importance of the first part of Nicole Oresme's *Algorismus proportionum* has long been recognized and appreciated. As far as we know now, it appears to be the first extant systematic attempt to describe operational rules for multiplication and division (called by Oresme addition and subtraction) of ratios involving integral and fractional exponents. Despite the apparent popularity of the *Algorismus* (there exist many manuscript copies of it), its subsequent impact, if any, on the history of mathematics is largely unknown.

2. This is the eminent Phillip de Vitri (1291–1361), renowned musical scholar and friend of Petrarch, who was Bishop of Meaux from January 3, 1351, to his death in Paris on June 9, 1361. Since Phillip did not have an ecclesiastical post at Meaux prior to 1351, Oresme's reference to him as "Most Excellent Reverend Phillip of Meaux" *(Reverende Presul Meldensis Phillipe, . . .)* indicates that Oresme composed the *Algorismus* sometime between 1351 and 1361, the period during which Phillip was Bishop of Meaux.

3. In writing irrational ratios, Oresme places the exponent first, followed by the ratio that expresses the rational base. Since the manuscripts reveal that Oresme followed the pattern consistently, we have here an early attempt at a mathematical notation. However, the rational bases were written with considerable variation. Thus 1/4 2^p 1/2 might also appear as 1/4 2 1/2; 1/4 $p2/12$; 1/4 2^{p1a} 1/2; and so on. Often enough, Oresme followed the usual medieval practice and verbalized the entire expression. With the exception of exponents, the first part of the *Algorismus* is concerned exclusively with ratios of greater inequality—that is, with ratios of the form A/B, where $A > B$.

4. For Oresme the irrational ratio $(4/1)^{1/3}$ is a *part* of the rational ratio 4/1 because $(4/1)^{1/3} < 4/1$ *and* the exponent, 1/3, is a unit fraction. The irrational ratio $(4/1)^{2/3}$ is *parts* of 4/1 because $(4/1)^{2/3} < 4/1$ *and* the exponent is a proper fraction in its lowest terms, where both integers are greater than 1. In a later work, the *De proportionibus proportionum* (*On Ratios of Ratios;* see Selection 29, n. 3), Oresme related the concept of exponential parts to that of commensurability.

5. Oresme is referring here to the numerator and denominator of the exponent.

6. When an irrational ratio has a rational base, the latter is said to denominate the former. Thus if $(A/B)^{p/q}$ is irrational, A/B rational, and p, q are integers with $p < q$, then A/B will denominate $(A/B)^{p/q}$. In his *De proportionibus proportionum*, Oresme says that such an irrational ratio is *immediately denominated* by the rational ratio A/B and *mediately denominated* by a number—that is, by the exponent p/q when it is a ratio of numbers. Irrational ratios in the form cited above were always expressed by Oresme as exact parts of some rational ratio (see Selection 29, n. 3). The bare distinction, without any elaboration, between immediate and mediate denomination appears as early as 1328 in Thomas Bradwardine's *Treatise On Proportions (Tractatus proportionum seu de proportionibus velocitatum in motibus)*; the published edition is cited in Selection 51.1, n. 1.

7. If we have a rational ratio A^n, where n is an integer, Oresme would expand this ratio by stages. Thus $A \cdot A = A^2$; $A^2 \cdot A = A^3$; $A^3 \cdot A = A^4$, and so on until $A^{n-1} \cdot A = A^n$. Note that Oresme says a ratio is "to be doubled" *(duplari)* and "tripled" *(triplari)* when he obviously means "squared" and "cubed." This ambiguous terminology was quite common but usually not troublesome, since context would almost always reveal an author's meaning.

8. Book V, Proposition 6, of the *Arithmetica* of Jordanus de Nemore (propositions from this work are given in Selection 21) is not relevant to the "addition" of ratios in this rule, for in that proposition Jordanus considers how to reduce proportions to their lowest terms (Cambridge, Peterhouse 277 [the codex is in Magdalene College], Bibliotheca Pepysiana 2329, fol. 13 verso, cols. 1–2). But Jacques Le Fèvre d'Estaples (Jacobus Faber Stapulensis [1455–1536]) explains the addition of two ratios in his edition of Jordanus' *Arithmetica* (Paris, 1496; this includes Jordanus' enunciations and Le Fèvre's demonstrations and comments; for a description of this edition see David Eugene Smith, *Rara Arithmetica* [Boston: Ginn, 1908], pp. 62–63). Before actually commencing the demonstration of Book V, Proposition 3, Le Fèvre says (sig. b6, col. 1; the folios are unnumbered):

a sesquioctavan ratio.[9] This can be shown by the twenty-seventh proposition of the second book of the *Arithmetic* of Jordanus.[10]

Rule Three. If an irrational ratio is parts of any rational ratio, it is possible to designate it as a part

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"Before we demonstrate what has been proposed, I wish to show how one ratio is added to another. Now, I say that the ratio of the products of the first term of one ratio by the first term of the other, and of the second [term of one] by the second [term of the other] is composed of these two ratios. For let A to B and C to D be the two ratios that I wish to add and to show what is composed from them. I multiply C by A and get E; and I multiply D by B and get G. I say that ratio E to G is composed of ratios A to B and C to D. Now, again, I multiply C by B and get F, and by the seventh [proposition] of the second [book] ratio A to B is as E to F; and by the eighth [proposition] of the same [book] ratio C to D is as F to G and ratio E to G is composed of ratios E to F and F to G, and, therefore, ratio E to G is composed of ratios A to B and C to D, which was asserted."

We see that "addition" of ratios is simply multiplication. Why, or when, the multiplication of such ratios as 5/1 and 4/3 came to be designated "addition" is unknown to me (for instance, it does not appear in Boethius' *Arithmetica*, a very widely used treatise). Where Oresme applies this terminology to exponents a plausible explanation can be offered (see n. 38). In the *De proportionibus proportionum*, Oresme speaks briefly of adding (that is, multiplying) ratios of greater inequality (ch. 1, *ll.* 75–83; Grant, *Nicole Oresme: De proportionibus · · ·* [Madison, Wis.: University of Wisconsin Press, 1966], pp. 142, 143). The procedure is couched exclusively in terms of continuous proportionality where extreme terms are assigned and the ratios composed. In the *Algorismus*, however, the addition of ratios is effected by multiplication performed directly on the prime numbers, or numerical denominations, of the ratios; extreme terms are not assigned.

9. $(3/2):(4/3) = 9/8$.

10. The enunciation of this proposition in Jordanus' *Arithmetica* reads (Bibliotheca Pepysiana 2329, fol. 6 recto, col. 1): "If the ratio of the first to the second term is greater than the ratio of the third term to the fourth, then the product of the first and fourth terms is greater than the product of the second and third terms. And if the product is greater the ratio of the first term to the second will be greater."

In his proof Jordanus assumes that $A/B > C/D$, so that $AD = E$, $CB = F$, and $BD = G$. Since $E/G = A/B$ and $F/G = C/D$, it follows that $E/G > F/G$ and, consequently, that $E > F$ (that is, $AD > CB$).

Although this proposition follows the steps outlined by Oresme, nowhere does Jordanus speak of subtracting one ratio from another. But in Book V, Proposition 1, he approximates rather closely the ideas expressed by Oresme, again without using any form of the term *subtrahere*. The enunciation of this proposition reads (Bibliotheca Pepysiana 2329, fol. 13 recto, col. 1): "That ratio by which the ratio of the first to the second term exceeds the ratio of the third to the fourth term is the ratio formed by the product of the second and third terms."

Jordanus assumes that A, B, C, and D are four successive terms, where $A/B > C/D$. Thus if $DA = E$ and $BC = F$, then $(A/B):(C/D) = E/F$, and Jordanus would say that A/B exceeds C/D by ratio E/F just as Oresme says that 3/2 exceeds 4/3 by 9/8, a sesquioctavan ratio. Although Jordanus did not refer to this as subtraction of ratios, Jacques Le Fèvre, in commenting on this proposition, says (sig. b6, col. 1): "This [proposition] shows how to subtract a ratio from a ratio. And after the lesser ratio has been subtracted from the greater ratio, the ratio that is left, howsoever much it be, is what we call here the difference between one ratio and another."

For Oresme, as for Jacques Le Fèvre and others, "subtraction" of ratios of greater inequality is actually division by cross-multiplication. But why was the quotient of (3/2):(4/3) called a "difference" and the whole process of division designated "subtraction"? In our concluding note we shall see that where exponents are concerned a certain rationale may be supplied for this terminology. But, as with "addition" of ratios, its origin is unknown to me and can only be conjectured (see n. 38). In the *De proportionibus* (ch. 1, *ll.* 72–74) Oresme mentions briefly the subtraction (that is, division) of one ratio of greater inequality from another. In that treatise, subtraction is performed by assigning a mean term between the terms of the greater ratio and subsequently composing the ratios. Thus, if ratio E is to be subtracted from ratio B/C, it follows that $B/C > E$. Assign a mean term, D, between B and C so that either $D > C$ and $D/C = E$ in which event $(B/C):(D/C) = B/D$; or $D < B$ and $B/D = E$, so that $(B/C):(B/D) = B/C \cdot D/B = D/B \cdot B/C = D/C$. Here the *modus operandi* is to produce continuous proportionality by assigning mean terms and composing the two ratios. This is a wholly different emphasis than in the *Algorismus*, where the operation is effected directly by cross-multiplying the prime numbers of the given rational ratios; hence no means need be assigned.

Perhaps with his own *Algorismus* in mind, Oresme informs the readers of the *De proportionibus* that ratios may be added and subtracted in a manner different than that which is appropriate to the latter treatise *(De proportionibus proportionum*, ch. 1, *ll.* 84–89): "If, however, you wish to add a ratio of greater inequality to another by means of algorism *(per artem)*, it is necessary to multiply the denomination of one ratio by the denomination of the other. And if you wish to subtract one ratio from another, you do this by dividing the denomination of one ratio by the denomination of the other. The [method] of finding denominations will be taught afterward. Multiplication and division of denominations are done by algorism."

Although we are told here that the addition of ratios can be performed by multiplication of denominations (that is, multiplication of the prime numbers of the two ratios) and that subtraction of ratios can be performed by division of denominations (that is, division of the prime numbers of one ratio into the prime numbers of the greater ratio), Oresme in the ninth rule of his *Algorismus* emphasizes that the multiplication of the denominations of ratios must, strictly speaking, be called addition, not multiplication. Similarly, the division of the numerical denominations of two ratios must be called subtraction, not division (see n. 38 for a possible explanation of this.)

of yet another rational ratio, so that it might more appropriately be called a part rather than parts.

Let B, an irrational ratio, be parts of A, a rational ratio. Without changing the denominator [of the exponent], I say that B will be a part of some ratio [obtained by expanding A by the numerator of the exponent] and this [expanded] ratio will be multiple to A. Any irrational—and it is of these we speak—will be a part of some rational ratio, since a multiple ratio can be found for every rational ratio.[11]

As an example, let us take a ratio which is two-thirds of a quadruple $[(4/1)^{2/3}]$. Since 2 is the numerator [of the exponent], we shall have one-third of a quadruple ratio squared $[((4/1)^2)^{1/3}]$, namely [one-third of] a sedecuple ratio $[(16/1)^{1/3}]$. The same applies to other ratios of this kind. The justification for this lies in the general truth that one-third of a whole $[(A)^{1/3}]$ equals two-thirds of its half or subdouble[12] $[(A^{1/2})^{2/3}]$, and conversely, two-thirds of a subdouble $[(A^{1/2})^{2/3}]$ equals one-third of a double $[(A)^{1/3}]$. The same reasoning is applicable to any other parts.

Rule Four. [How] to assign the most appropriate denomination of an irrational ratio.

Here it must be understood that a rational ratio is called *primary* when it cannot be divided into equal rational ratios, and no mean proportional number or numbers can be assigned between its least numbers, as is the case with double [2/1], triple [3/1], or sesquialterate [3/2] ratios. But a rational ratio is called *secondary* when it can be so divided and a mean proportional number or numbers can be assigned between its [least] numbers.[13] As examples, take a quadruple ratio [4/1], which is divisible into two doubles [4/2 · 2/1]; an octuple ratio [8/1], divisible into three doubles [8/4 · 4/2 · 2/1]; a nonacuple ratio [9/1], into two triples [9/3 · 3/1]; and so on.

From all this we see that if any proposed irrational ratio is denominated by parts, that ratio, by the preceding rule, could be transformed and called a part. However, it must [first] be seen if the rational ratio denominating the irrational is a *primary* ratio. If it is, let it stand, for then the irrational ratio, which is our topic of discussion, is most appropriately denominated, as in one-third of a sextuple $[(6/1)^{1/3}]$ or double ratio $[(2/1)^{1/3}]$, and so forth.

But if the rational ratio that denominates the irrational ratio is *secondary*, one must determine how many primary rational ratios are contained by it, where each primary is an equal part of it. Should the number representing these parts [or primary ratios] be incommunicant or prime to the denominator of the proposed irrational ratio, the [initial] denomination must be left as it is. The denomination of half of an octuple $[(8/1)^{1/2}]$, for example, is proper because an octuple ratio [8/1] has three equal rational parts, namely three doubles $[8/4 \cdot 4/2 \cdot 2/1$ or $(2/1)^3]$, and 2 is the denominator of the proposed irrational ratio. But 3 and 2^{14} are incommunicant [or prime to one another], so that half of an octuple is not a part of any rational ratio smaller than an octuple, although it could certainly be parts since half of an octuple is three-fourths of a quadruple $[(4/1)^{3/4}]$, but this would not constitute an appropriate denomination.

However, if the number representing the smallest, or primary, parts of such a secondary rational ratio by which an irrational ratio is denominated *and* [the number representing] the denominator [of the exponent] of this irrational ratio were communicant [or mutually nonprime] numbers, the greatest

11. If the irrational ratio is $B = A^{m/n}$, where A is a rational ratio and m/n is a ratio of integers in its lowest terms with $n > m > 1$, then $A^{m/n}$ can be expanded to $D^{1/n}$ so that $D = A^m$. Initially, then, $A^{m/n}$, or B, is an irrational ratio that is exponentially *parts* of rational ratio A, since the exponent m/n is a proper fraction whose numerator is an integer greater than 1. By expanding A^m we obtain a rational ratio D which is multiple to A in an exponential sense, since $D = A^m$. (Thus when Oresme says that "a multiple ratio can be found for every rational ratio" he means that any rational ratio A can be expanded by any integral exponent m, and the rational ratio which is the product of that expansion, namely D, is called the multiple of A.) Hence $A^{m/n} = D^{1/n}$ and has been transformed from an irrational ratio that was *parts* of rational ratio A, to one that is a *part*—that is, where the exponent is a unit fraction—of another rational ratio D. The most appropriate form of irrational ratio $A^{m/n}$ is, therefore, $D^{1/n}$. This is illustrated by the example in the next paragraph of the text: $(4/1)^{2/3} = [(4/1)^2]^{1/3} = (16/1)^{1/3}$.

12. *Medietas* and *subduplus* are the Latin terms rendered here as "half" and "subdouble" and clearly mean square root. More commonly, however, they signified "half" in the arithmetic sense. The context usually reveals the meaning without ambiguity.

13. In the *De proportionibus proportionum*, Oresme does not use the terms "primary" *(primaria)* and "secondary" *(secundaria)*, but, at the conclusion of the first chapter, these concepts constitute the first two of seven conceivable ways in which rational ratios are divisible (see ch. 1, *ll.* 385–389). In the first way rational ratios are divisible into smaller equal rational ratios, which corresponds to secondary ratios in the *Algorismus*; all rational ratios not divisible in the first way constitute the second group, which makes this category equivalent to the primary ratios of the *Algorismus*.

14. The numbers 3 and 2 refer to the exponent in $(2/1)^{3/2}$.

number [or common factor] in which they are communicant must be taken and each of them divided by it. By dividing the number representing the parts of the secondary ratio we arrive at the number of parts or [primary] ratios composing the rational ratio that will most suitably denominate the [proposed] irrational ratio; and by dividing the denominator of the [initially] proposed [irrational ratio] by the same greatest number [or common factor], the most appropriate denominator of the irrational ratio is found.

For example, let a ratio of three-fourths of a quadruple $[(4/1)^{3/4}]$ be proposed. By utilizing the third rule it is clear that it equals one-fourth of 64/1 $[(64/1)^{1/4}]$. But 64/1 is composed of six double ratios $[64/1 = (2/1)^6]$, where 6, which signifies the number of primary parts in 64/1, and 4, which represents the denominator of the proposed ratio,[15] are communicant[16] [or have a common measure] in 2, so that dividing 2 into 6 gives 3, signifying that the proposed ratio is a part of three doubles, namely part of 8/1.[17] Dividing 2 into 4 yields 2, so that the proposed ratio is one-half—that is, the rule shows that the proposed ratio is one-half of 8/1, which is written as 1/2 8^p $[(8/1)^{1/2}]$, and this is its most proper denomination. In the same way, one-twelfth of four triple ratios $[((3/1)^4)^{1/12}]$, namely 81/1 $[(81/1)^{1/12}]$, is one-third of 3/1 $[(3/1)^{1/3}]$; and similarly one-fourth of six triple ratios $[((3/1)^6)^{1/4}]$ is one-half of three triple ratios $[((3/1)^3)^{1/2}]$, namely [one-third of] 27/1 $[(27/1)^{1/3}]$, and so on.[18]

Rule Five. [*How*] *to add*[19] *an irrational ratio to a rational ratio.*

Let B, an irrational ratio, be added to A, a rational ratio, and assume that B is a part[20] of rational ratio D, and in accordance with previous rules, this constitutes the most proper denomination of B.[21] Then, by the first rule, add A to ratio D a number of times equal to the value of the denominator of B, and let C be the total result.[22] I say, therefore, that the ratio composed of [ratios] B and A will be part of C and [it] will be denominated by the same denominator which denominated [or denoted] the part which B was of its denominating ratio, namely D.[23]

For example, let one-third of a double ratio $[(2/1)^{1/3}]$ be added to a sesquialterate ratio [3/2]. Now join three sesquialterate ratios with a double ratio to yield a sextuple superpartient three-fourths ratio $[6\frac{3}{4}]$, which is a ratio of 27 to 4. The ratio produced from [the addition of] one-third of a double ratio and a sesquialterate ratio is one-third of the ratio of 27 to 4 $[(27/4)^{1/3}]$, which is written as 1/3 6^p 3/4.[24]

Rule Six. [*How*] *to subtract an irrational from a rational ratio, or the converse.*

Let A be a rational ratio and B an irrational ratio

15. Oresme refers here to the denominator of the exponent in the ratio $(2/1)^{6/4}$.

16. In Campanus of Novara's thirteenth-century edition of Euclid's *Elements* (for full title of the edition, see Selection 27, n. 1), Book VII, Definition 8, "communicant" numbers are defined as follows (p. 168): "Numbers are said to be mutually composite or communicant which are measured by a number other than unity, and none of them is prime to any other of them."

17. That is, $[(2/1)^3]^{1/2} = (8/1)^{1/2}$.

18. In the fourth rule Oresme presents criteria for determining the final form of an irrational ratio. If possible, all irrational ratios should have a primary rational ratio as base and a unit fraction for an exponent, as in ratios $(6/1)^{1/3}$ and $(3/1)^{1/4}$. If, however, the base is a secondary rational ratio, Oresme outlines the procedure for determining its most proper denomination or representation, that is, the most proper rational ratio that will serve as base. If $B^{1/q}$ is a given irrational ratio, where B is a secondary rational ratio, one must decompose B into its constituent primary ratios. Let A represent the primary ratios so that $B = A^m$, where m is an integer. Therefore, $B^{1/q} = (A^m)^{1/q}$. Should m and q be unequal mutually prime numbers greater than 1, the original form of the ratio, namely, $B^{1/q}$, will be the most appropriate denomination, since it expresses the irrational ratio as a part of rational ratio B. For example, $(8/1)^{1/2}$ is most properly denominated because the alternatives, $(2/1)^{3/2}$ and $(4/1)^{3/4}$, fail to represent the relation in terms of a single part, or unit fraction.

But if m and q are not mutually prime, they must then be reduced to their lowest terms. Then either $m/q = 1/n$ and the final form is $A^{1/n}$; or $m/q = p/n$, where p and n are greater than 1; and the final form is $D^{1/n}$ (rather than $A^{p/n}$), where $D = A^p$. An example of the first case is $(81/1)^{1/12} = (3/1)^{4/12} = (3/1)^{1/3}$, the last expression being the most suitable final form; an example of the second case is $(64/1)^{1/4}$ (given initially as $(4/1)^{3/4}$ but expanded to $(64/1)^{1/4}$ by the third rule), which equals $(2/1)^{6/4} = (2/1)^{3/2} = (8/1)^{1/2}$, the last expression being the most appropriate. The two examples given by Oresme in which $(8/1)^{1/2}$ is the final form are both represented here.

19. We have already seen that "to add" *(addere)* signifies multiplication in the context of this treatise.

20. Here $B = D^{1/q}$, where q is an integer.

21. That is, $D^{1/q}$ is the most proper denomination of B.

22. The denominator of B is q, so that $D \cdot A^q = C$. In accordance with the first rule, A is expanded in stages. Thus if $q = 3$, then $A \cdot A = A^2$ and $A^2 \cdot A = A^3$.

23. Since $D \cdot A^q = C$, $B < D$, and $A < A^q$, it follows that $(B \cdot A) < C$. Also, $(B \cdot A)$ is an exponential part of C, since $(B \cdot A) = (D \cdot A^q)^{1/q} = C^{1/q}$. But $B = D^{1/q}$, so that $D^{1/q}$ and $C^{1/q}$ have the same denominator in their exponents, namely q. It is in this sense that ratios B and $(B \cdot A)$ are denominated, or represented, by the same denominator.

24. The two given ratios are first cubed, then fully expanded, after which the cube root is taken. Thus $[(2/1)^{1/3} \cdot 3/2]^3 = 2/1 \cdot (3/2)^3 = 27/4$, whose cube root is $(27/4)^{1/3}$ but which Oresme writes as $(6\frac{3}{4})^{1/3}$.

denominated by rational ratio D. Then, whether A is greater or smaller than B, let it be multiplied by the denominator of B. This is the same thing as taking a ratio composed of a number of A's equivalent to [the number representing] B's denominator. The method for accomplishing this is shown in the first rule. Now, let this composed, or produced, ratio be called C. By the second rule one can determine whether C is greater or smaller than D. If it is greater, then A was greater than B; but if D is greater than C, B was greater than A. Whatever the given ratios, namely C and D, the lesser must be subtracted from the greater. This is done by the second rule. Let the remainder be F. And so I say that if A is subtracted from B, or conversely, the remainder will be the same total part of F as B was of D.[25]

For example, subtract a sesquitertian ratio [4/3] from half of a double $[(2/1)^{1/2}]$. I join [or combine] two sesquitertian ratios and obtain a ratio of 16 to 9, which is smaller than a double and must therefore be subtracted from [that is, divided into] a double ratio, leaving a sesquioctavan [9/8] ratio. Hence half of a sesquioctavan ratio $[(9/8)^{1/2}]$ will remain after the subtraction of a sesquitertian from half of a double ratio.

Similarly, should you wish to subtract a third part of a double ratio $[(2/1)^{1/3}]$ from a sesquialterate [3/2], join [or combine] three sesquialterate ratios $[(3/2)^3]$ to obtain a ratio of 27 to 8, from which the double ratio is [then] subtracted, leaving a ratio of 27 to 16. Therefore, if a third part of a double is subtracted from a sesquialterate, there will remain a third [part] of ratio 27 to 16 $[(27/16)^{1/3}]$, which is a third part of what would remain if the whole double ratio were subtracted from three sesquialterate ratios.[26] This rule can be demonstrated easily, since, generally, if a first ratio is subtracted from [that is, divided into] a second, a third ratio is left, so that if a third part of the [same] first ratio is subtracted from [that is, divided into] a third part of the [same] second ratio, there is left a third part of the [same] third ratio. This applies to half of any ratio, or a fourth part, or a fifth, and so forth.[27]

General Rule: In the addition of an irrational to an irrational ratio, and in the subtraction of an irrational from an irrational ratio, there are general rules applicable to any quantities whatever.

Let a known part of a known quantity be added to a known part of a known or measurable quantity. For example, let C be part of quantity A, and D part of quantity B; and suppose that C is denominated by number E, and D by number F. I then multiply A by F—that is, I take A continuously F number of times—and produce G; in the same way I multiply B by E and obtain H. Hence C will be a part of G, which is represented by [one part of] the number produced from the multiplication of E by F; and the same number will represent the part that D is of H. Therefore, as C is to G, so is D to H.[28]

25. We are given rational ratio A and irrational ratio $B = D^{1/n}$, where n is an integer (in the enunciation of Rule Six, Oresme does not specify that the numerator of the exponent must be 1; but the rule itself and the examples indicate this unmistakably). Now, whether $A \gtrless B = D^{1/n}$, we must expand A exponentially by n, the denominator of the exponent of B in $D^{1/n}$. The expansion of A is carried out by the first rule, that is, $A \cdot A = A^2$; $A^2 \cdot A = A^3$; $A^3 \cdot A = A^4$; . . . $A^{n-1} \cdot A = A^n$; and let $A^n = C$. We can now determine whether $C \gtrless D$. Let rational ratio $C = p/q$ (where $p > q$ and both are integers) and let rational ratio $D = s/t$ (where $s > t$ and both are integers). By the second rule, $(p/q) : (s/t) = pt/qs$; and if $pt > qs$, then $p/q > s/t$ and $C > D$; but if $qs > pt$, then $s/t > p/q$ and $D > C$. Furthermore, if $C > D$, then $A > B$; and if $D > C$, then $B > A$. This being determined, the lesser ratio must then be subtracted from (that is, divided into) the greater, and this is also carried out by the second rule. If $C > D$, then let us assume that $C: D = F$ (or $A^n : B^n = F$); and since $A = C^{1/n}$ and $B = D^{1/n}$, it follows that $A: B = F^{1/n}$; but if $D > C$, then assume that $D: C = F$ and $B: A = F^{1/n}$. Hence $A: B$ (or $B: A$) $= F^{1/n}$ and $B = D^{1/n}$, which is what Oresme wished to show. Thus $A: B$, or $B: A$, is the same exponential part of F, namely $1/n$, as B is of D.

In general, the original ratios are expanded by a power equal to the denominator n of the exponent of the irrational ratio. The desired ratio is obtained by taking the nth root of the quotient resulting from the division of the expanded ratios. This is illustrated by the first example of the next paragraph, where Oresme subtracts (that is, divides) $A = 4/3$ from (that is, into) $B = (2/1)^{1/2}$. Here $D = 2/1$. Expanding both ratios by the denominator of the exponent, we obtain $[(2/1)^{1/2} : 4/3]^2 = 2/1 : 16/9$, where $16/9 = (4/3)^2 = C$; subtracting (that is, dividing) the lesser from (that is, into) the greater ratio by cross-multiplying (as we are told in Rule Two), we get $2/1 : 16/9 = 18/16 = 9/8$, which shows that $2/1 > 16/9$ (or $D > C$), since $2 \cdot 9 > 16 \cdot 1$, and indicates that the lesser ratio has been properly divided into the greater. Reverting to the original ratios, we see that $(2/1)^{1/2} : 4/3 = (9/8)^{1/2}$.

26. In this example we have $3/2 : (2/1)^{1/3}$. Now, $[3/2 : (2/1)^{1/3}]^3 = 27/8 : 2/1 = 27/16$. Therefore, $3/2 : (2/1)^{1/3} = (27/16)^{1/3}$.

27. Oresme concludes the sixth rule with a general statement that if A, B, and C are ratios and $A/B = C$, then $A^{1/n}/B^{1/n} = C^{1/n}$.

28. The steps outlined in Oresme's general rule are as follows: Let $C = A^{1/E}$, where A is a known quantity or ratio and C is a known part of quantity or ratio A; and let $D = B^{1/F}$, where, similarly, B is a known quantity or ratio and D is a known part of quantity or ratio B. Next take the integral exponent F and expand A so that A^F

Rule Seven. By addition [that is, multiplication], it follows that as C is to G and D to H, so also is the total [or product] *(aggregatum)* of C and D to the total [or product] *(aggregatum)* of G and H. Consequently, the total, [or product,] CD is a part of the total [or product] GH and is represented by [one part of] the number produced by [the multiplication of] E by F.[29]

Rule Eight. By subtraction, however, it follows that if G is subtracted from H, or conversely, and C is subtracted from D, or conversely, the remainder will be the same part of the remainder as part C was of G, or D was of H, and this part is represented by [one part of] the number resulting from the multiplication of E by F.[30]

As an example in the addition of irrational ratios, let half of a double $[(2/1)^{1/2}]$ be added to a third part of a triple ratio $[(3/1)^{1/3}]$. On the one hand, I join or multiply three double ratios just as shown by the first rule. I do this because the other ratio is denominated by a three and is called a third part. On the other hand, and for the same reason and in the same way, I join [or multiply] two triples and then multiply the denominators of the parts, namely 2 by 3, and produce 6. Half of a double is, therefore, a sixth part of three doubles; similarly, a third part of a triple is a sixth part of two triples. Thus the total [or product] of half of a double and a third part of a triple is a sixth part of the total [or product] of three doubles and two triples. By the first rule it is obvious that such a product is a 72^{p1a} ratio, namely 72 to 1. It follows, then, that from the addition [multiplication] of half of a double and a third part of a triple we obtain a sixth part of a 72^{p1e} ratio $[(72/1)^{1/6}]$.

As an example in subtraction [that is, division] of irrational ratios, let half of a double ratio $[(2/1)^{1/2}]$ be subtracted from a third part of a triple $[(3/1)^{1/3}]$. In the first place, and in accordance with the second rule, the product of three doubles $[(2/1)^3]$ should be subtracted from [that is, divided into] the product of two triple ratios $[(3/1)^2]$, leaving a sesquioctavan ratio [9/8]. Then, by subtracting a sixth part from a sixth part—that is, half of a double from a third part of a triple ratio—there will be left a sixth part of a sesquioctavan ratio $[(9/8)^{1/6}]$, since half of a double is a sixth part of three doubles $[((2/1)^3)^{1/6}]$ and a third part of a triple ratio is a sixth part of two triples $[((3/1)^2)^{1/6}]$. Thus subtracting a sixth [part of a ratio] from a sixth [part of another ratio] leaves a sixth [part] of what remains after the subtraction of [one] whole [ratio] from [the other] whole [ratio], and this can be demonstrated easily.

Rule Nine. Now, if the parts should have the same denomination, then it is easier to posit a special rule apart from the general rule already stated, so that if a third part of A $[(A)^{1/3}]$ is added to a third part of B $[(B)^{1/3}]$ we obtain a third part of the result produced by the addition [multiplication] of A and B.[31] In a similar manner, if a third part of A is subtracted from [divided into] a third part of B, there will remain a third part of the remainder that was left after the subtraction of A from B.[32] Thus, [for example,] if a double ratio is added to a triple ratio, a sextuple ratio is produced, so that if half of a double $[(2/1)^{1/2}]$ is added to [multiplied by] half of a triple $[(3/1)^{1/2}]$, the result will be half of a sextuple $[(6/1)^{1/2}]$.[33] Similarly, if a double ratio is subtracted from [divided into] a triple, a sesquialterate ratio is left, so that if a third part of a double ratio $[(2/1)^{1/3}]$ is subtracted from [divided into] a third part of a triple $[(3/1)^{1/3}]$, there remains a third part of a sesquialterate ratio $[(3/2)^{1/3}]$;[34] and the same holds in other cases.

Moreover, just as in other things, addition always proves subtraction, and conversely.[35] A ratio can be doubled, tripled, and multiplied—even sesquialterated— or increased proportionally as much as you wish by the addition [multiplication] of a ratio to a ratio. Thus, if someone should want a ratio which is the sesquitertian of a double ratio, it would be necessary, by the fifth rule, to add a third part of a double ratio $[(2/1)^{1/3}]$ to the double ratio so that a third part of a sedecuple $[(16/1)^{1/3}]$ is obtained, which is a ratio that is the sesquitertian of a double ratio.[36] In the same way, subtraction allows a ratio to be subdoubled, subtripled, subsesquialterated,[37] and so on.

Indeed, one ratio cannot be multiplied *(multiplicatur)* or divided *(dividitur)* by another except improperly, as when two doubles are multiplied by two doubles to obtain four doubles. But this is nothing other than a multiplication of numbers, since the multiplication of two doubles $[(2/1)^2]$ by two triples $[(3/1)^2]$ comes to nothing, just as does the multiplication of a man by an ass. The same reasoning applies to division. Thus only addition and subtraction are appropriate types of algorism [that is, operational procedures] for dealing with ratios[38] and we have discoursed sufficiently about them. The first tractate ends here.

= G; in the same manner take integral exponent E and expand B so that $B^E = H$. Since $A = G^{1/F}$, it follows that $C = (G^{1/F})^{1/E}$, or $C = G^{1/EF}$, where C is an exponential part of ratio G. Similarly, since $B = H^{1/E}$, it follows that $D = (H^{1/E})^{1/F}$, or $D = H^{1/EF}$, where D is an exponential part of H. Therefore, ratio C is the same exponential part of ratio G that ratio D is of H. In the seventh rule, this general rule is applied to addition (that is, multiplication) of two irrational ratios.

29. Since the general rule has shown that C is to G as D is to H, by which Oresme means $C = G^{1/EF}$ and $D = H^{1/EF}$, in the seventh rule he multiplies C and D, obtaining the product $C \cdot D = (G \cdot H)^{1/EF}$. In an example directly following Rule Eight, Oresme adds (that is, multiplies) two irrational ratios.

Let $C = A^{1/E} = (2/1)^{1/2}$ and $D = B^{1/F} = (3/1)^{1/3}$. Now, $A^E = G = (2/1)^3$, so that $A = G^{1/F} = [(2/1)^3]^{1/3}$ and $C = A^{1/E} = (G^{1/F})^{1/E} = (2/1)^{1/2} = ([(2/1)^3]^{1/3})^{1/2} = [(2/1)^3]^{1/6}$. Similarly, $B^E = H = (3/1)^2$, so that $B = H^{1/E} = [(3/1)^2]^{1/2}$ and $D = B^{1/F} = (H^{1/E})^{1/F} = (3/1)^{1/3} = ([(3/1)^2]^{1/2})^{1/3} = [(3/1)^2]^{1/6}$. The product of $(2/1)^{1/2} \cdot (3/1)^{1/3} = [(2/1)^3]^{1/6} \cdot [(3/1)^2]^{1/6} = [(2/1)^3 \cdot (3/1)^2]^{1/6}$. Expanding each ratio by the first rule, we obtain $8/1 \cdot 9/1 = 72/1$, which yields as a final product $(72/1)^{1/6}$. In brief, Oresme shows that $A^{1/E} \cdot B^{1/F} = [(A^F)^{1/EF} \cdot (B^E)^{1/EF}] = (A^F \cdot B^E)^{1/EF}$ or $(G \cdot H)^{1/EF}$.

30. Here we have subtraction (that is, division) of the ratios $D/C = (H/G)^{1/EF}$. In an example given immediately before the ninth rule, Oresme subtracts the same irrational ratios that were added in the previous example. Thus $(3/1)^{1/3} : (2/1)^{1/2} = [(3/1)^2]^{1/6} : [(2/1)^3]^{1/6} = (9/1)^{1/6} : (8/1)^{1/6} = (9/8)^{1/6}$. Generally, the eighth rule asserts that $B^{1/F} : A^{1/E} = (B^E)^{1/EF} : (A^F)^{1/EF} = (B^E : A^F)^{1/EF}$ or $(H : G)^{1/EF}$.

31. $A^{1/3} \cdot B^{1/3} = (A \cdot B)^{1/3}$; or generally, $A^{1/n} \cdot B^{1/n} = (A \cdot B)^{1/n}$.

32. $B^{1/3} : A^{1/3} = (B/A)^{1/3}$; or generally, $B^{1/n} : A^{1/n} = (B : A)^{1/n}$.

33. Since $3/1 \cdot 2/1 = 6/1$, it follows that $(3/1)^{1/2} \cdot (2/1)^{1/2} = (6/1)^{1/2}$.

34. Since $(3/1) : (2/1) = 3/2$, it follows that $(3/1)^{1/3} : (2/1)^{1/3} = (3/2)^{1/3}$.

35. As shown in the next two notes, this converse relation probably applies to addition and subtraction of exponents having the same base. In *Algorismus de integris*, ascribed to Master Gernardus, Rule 12 reads: "Addition and subtraction are proved mutually." Gernardus tells us that if numbers $a + b = c$, and then if $c - b \neq a$, the addition was necessarily faulty; similarly if $c - a = b$ and $a + b \neq c$, the subtraction was improperly performed. See G. Eneström, "Der 'Algorismus de integris' des Meisters Gernardus," *Bibliotheca Mathematica* XIII (1912/1913), 301–302. Oresme's statement may be an application of this arithmetic relation to addition and subtraction of exponents. Unfortunately, he offers no specific example to substantiate this interpretation.

36. Here Oresme wishes to find ratio $(2/1)^{4/3}$ and invokes the fifth rule, in which an irrational ratio is added to a rational ratio. Thus $(2/1) \cdot (2/1)^{1/3} = (2/1)^{4/3} = (16/1)^{1/3}$; or generally, $A^{n/n} \cdot A^{1/n} = A^{(n+1)/n}$. If we now wish to prove the addition, we would "subtract" $(2/1)^{1/3}$ from $(2/1)^{4/3}$ and obtain $(2)^{4/3} : (2)^{1/3} = (2/1)$, the double ratio to which we initially added $(2/1)^{1/3}$.

37. For subtraction (that is, division) $A^{n/n} : A^{m/n} = A^{(n-m)/n}$, where $m < n$ and both are integers. Addition proves this subtraction of ratios when we take $A^{m/n} \cdot A^{(n-m)/n} = A^{n/n}$. The terms "subdouble," "subtriple," and "subsesquialterate" are, in order, the fractional exponents 1/2, 1/3, and 2/3.

38. In this final paragraph Oresme explains why he believes it proper to speak only of addition and subtraction, rather than multiplication and division, of ratios—and here he actually uses verb forms of *multiplicatio* and *divisio*. The distinction seems connected with the fact that exponents cannot be multiplied—except improperly, as in $(2/1)^2 \cdot (2/1)^2 = (2/1)^4$, where, coincidentally, the correct result is produced. But we cannot multiply the ratios $(2/1)^2$ and $(3/1)^2$ directly, for by analogy with $(2/1)^2 \cdot (2/1)^2$ we ought to obtain four of something. However, such a multiplication would yield something as unintelligible as the union of a man and an ass.

In adopting such terminology, perhaps Oresme was guided by the fact that operations performed with exponents having the same base—as for example, $A^m \cdot A^n = A^{m+n}$ and $A^m : A^n = A^{m-n}$—required the addition and subtraction of exponents. It was then natural to think in terms of adding and subtracting ratios, which are the very entities represented by the exponents. The altered terminology was then applied to cases involving exponents with different bases.

Multiplication, addition, subtraction, and division could be performed on numbers, whether or not those numbers denominated, or represented, ratios. But operations on ratios were restricted to multiplication and division, which are called addition and subtraction respectively. Although evidence is lacking, it would come as no great surprise to discover that Oresme himself originated this terminology and created a new genre of mathematical treatise, an *algorism of ratios*, patterned after, but very different from, an *algorism of numbers*. Treatises of the latter kind, explaining and elaborating the four arithmetic operations as they are performed on numbers in the usual manner, were popular long before Oresme's time (see Selection 20). But I am not aware of the existence of any algorism of ratios in the Oresmian sense prior to Oresme's *Algorismus proportionum*. The absence of such terminology in Jordanus' *Arithmetica* and its use by Jacques Le Fèvre in a much later commentary on that treatise point to a post-Jordanian origin with Oresme himself as the possible originator. He may have derived his new, and peculiar, terminology from addition and subtraction of exponents, and then extended it to the simplest nonexponential cases (represented by the first two rules) in order to make the operational terminology for handling ratios as consistent as possible. If, on the other hand, Oresme inherited this terminology for the simplest cases, he may have extended it to operations involving integral and fractional exponents. The novel manner in which Oresme extended traditional Euclidean terminology to embrace exponential relations in the *De proportionibus* renders such a conjecture perfectly credible (see Selection 29, n. 3).

29 RATIONAL AND IRRATIONAL EXPONENTS DISTINGUISHED

Nicole Oresme (ca. 1325—1382)

Translated and annotated by Edward Grant[1]

Every rational ratio is immediately denominated by some number, either with a fraction, or fractions, or without a fraction.[2] The determination of these denominations will be shown below.

An irrational ratio is said to be mediately denominated by some number when it is an aliquot part or parts of some rational ratio, or when it is commensurable to some rational ratio, which is the same thing, as [for example], the ratio of a diagonal to its side is half of a double ratio.[3]

I say, therefore, that it does not seem true that every irrational ratio is commensurable to some rational ratio.[4] And the reason is that every ratio is just like a continuous quantity with respect to division, which is obvious by the last supposition. Therefore, by the ninth proposition of the tenth [book of Euclid, every ratio] can be divided into two [ratios] any of which is incommensurable to the whole. Thus there will be some ratio which will be part of a double ratio and yet will not be half of a double, nor a third part, or fourth part, or two-thirds part, etc., but it will be incommensurable to a double[5] and, consequently [incommensurable] to any [ratio] commensurable to this double ratio (by the comment on the eighth proposition of the tenth book of Euclid). And further, by the same reasoning there could be some ratio incommensurable to a double and also to a triple ratio and [consequently incommensurable] to any ratios commensurable to these, as [for example], half of a sesquitertian ratio.[6] And the same may be said of other ratios.

And there might be some irrational ratio which is incommensurable to any rational ratio. Now the reason for this seems to be that if some ratio is incommensurable to two [rational ratios], and some ratio is incommensurable to three rational ratios,

2. That is, $A/B = n$, where n is an integer or ratio of integers.

3. In this treatise Oresme employs the terms *part (pars)*, *multiple (multiplex)*, and *commensurable (commensurabilis)* in a special and restricted exponential sense. Here is a brief summary of this special usage (Grant, *Nicole Oresme: De proportionibus*, p. 26): "If A and B are two ratios and m and n are integers, then if $A > B$, (1) ratio B is a *part* of ratio A when $B = A^{1/n}$ and (2) A is commensurable and multiple to B, since $A = B^n$; (3) B is *parts* of A when $B = A^{m/n}$, where $n > m > 1$ and m and n are in their lowest terms; so that (4) A is commensurable to B, since they have a common measure in the unit ratio $A^{1/n}$. However, A is not multiple to B, since $A \neq B^n$."

As an example representing this type of proportion, we are given "the ratio of a diagonal to its side" which is "half of a double ratio, that is, $(2/1)^{1/2}$. For Oresme this is a *part* of 2/1 because the latter is its multiple in the special sense that $2/1 = [(2/1)^{1/2}]^2$. Therefore, Oresme would hold that 2/1 is the *immediate denomination* of $(2/1)^{1/2}$, since it is its rational base (indeed, 2/1 would also be the immediate denomination of $(2/1)^{3/2}$, even though the latter is greater than 2/1). But what does Oresme mean when he insists that an irrational ratio is *mediately denominated* by some number? Almost certainly he meant the exponent itself, by means of which we can distinguish between all the different irrational ratios having the same base. Thus the exponent is the mediate numerical denomination of (2/1).

4. In the remainder of this selection Oresme considers a third category of ratio, namely an irrational ratio with an irrational exponent, thereby distinguishing two types of irrational ratio: one with a rational exponent, the other with an irrational exponent.

5. Oresme's "pattern of thought seems to be as follows: Any ratio is like a continuous quantity and therefore divisible in any conceivable mathematically logical manner; now, one conceivable way is to divide a rational ratio into smaller ratios one of which, at least, is irrational and no part, or parts, of the given rational, and hence incommensurable to it; therefore, such a conceivable ratio, logically possible, must correspond to some real category of ratios even though we can not express a single instance of it." (Grant, *Nicole Oresme: De proportionibus*, p. 36.) Because he could not symbolize or otherwise explicitly express an irrational exponent, Oresme could only convey negatively his intuitive sense about such ratios. Thus it could not be $(2/1)^{1/2}$ (that is, "half of a double" ratio), nor $(2/1)^{1/3}$ ("a third part" of a double), nor $(2/1)^{1/4}$ ("a fourth part" of a double), nor $(2/1)^{2/3}$ ("a two-thirds part" of a double), and so on. Indeed, it would be incommensurable to a double ratio because it would be inexpressible in the form $(2/1)^{m/n}$, where m/n is rational and $m < n$. It seems correct to express what Oresme had in mind by $(2/1)^{1/\sqrt{q}}$, since raising this to the qth power will never raise this ratio to equality with 2/1.

6. That is, $(4/3)^{1/2}$

1. Reprinted with permission of the copyright owners, the Regents of the University of Wisconsin, from *Nicole Oresme: De proportionibus proportionum and Ad pauca respicientes*, edited with Introductions, English translations, and Critical Notes by Edward Grant (Madison, Wis.: University of Wisconsin Press, 1966), pp. 161, 163. In this selection from Oresme's treatise *On Ratios of Ratios (De proportionibus proportionum)* we may have the first explicit, though intuitive, sense of an irrational exponent emerging from a persistent analysis of the fourteenth-century concept of a "ratio of ratios" (for the meaning of this expression, see Selection 51.2, n. 51). His treatment of rational exponents can be seen in the preceding selection.

and so on, then there might be some ratio incommensurable to any rational ratio whatever,[7] though this does not follow from the form of the argument as [it does follow when we say that] some continuous quantity is incommensurable to all quantities of one [geometric] series. However, I do not know how to demonstrate this; but if the opposite should be true, it is indemonstrable and unknown. . . .

Geometry

30 ON THE DIVISION OF FIGURES

Leonardo of Pisa (Fibonacci) (b. ca. 1179; d. after 1240)

Translation, introduction, and annotation by Edward Grant[1]

In his commentary on the first book of Euclid's *Elements*, Proclus (410–485) mentions a *Book on Divisions (of Figures)*, which he ascribed to Euclid. In this treatise Euclid apparently divided given figures sometimes into "like" figures (that is, triangles into triangles, and so on) and sometimes into unlike figures (for example, a triangle into a triangle and a quadrilateral). Although the treatise has been lost in Greek, an Arabic text of it containing enumerations of thirty-six propositions but only four proofs was found by F. Woepcke in 1851. Later it was found that in Leonardo Pisano's *Practica geometriae (The Application of Geometry)*, composed in 1220, "no less than twenty-two of Woepcke's propositions are practically identical in statement with propositions in Leonardo; the solutions of eight more of Woepcke are either given or clearly indicated by Leonardo's methods, and all six of the remaining Woepcke propositions (which are auxiliary) are assumed as known in the proofs which Leonardo gives of propositions in Woepcke."[2] The propositions included below appear in Distinction 4 of Leonardo's *Practica geometriae* and are some of those used by Archibald in his reconstruction of Euclid's *Book on Divisions*. It is noteworthy, however, that if Leonardo derived the basis of this section, and a number of its propositions, from Euclid's lost work, he probably supplied additional proofs for some propositions, perhaps rearranged others, and added numerical examples, since he was composing an "applied geometry." Thus, while "his *Practica Geometriae* contains many references to Euclid's *Elements* and many uncredited extracts from this work," and "similar treatment is accorded works of other writers," it seems true to say that "in the great elegance, finish and rigour of the whole, originality of treatment is not infrequently evident."[3]

The popularity of dividing geometrical figures into other figures stems perhaps from the importance of dividing differently shaped plots of land into a variety of figures for purposes of inheritance and crops. It would be surprising, however, if Euclid himself had such concerns when he wrote his lost treatise.

[1.] And if you wish to divide any triangle by a line parallel *(equidistantem)* to one of the sides of the triangle, the manner of doing this will be demonstrated in the following triangle *ABG* [Fig. 1],

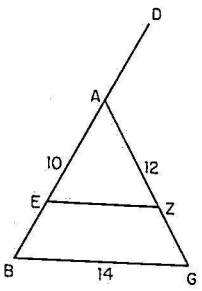

Fig. 1

7. If there is an irrational ratio incommensurable to $(2/1)^n$, there might also be an irrational ratio incommensurable to $(2/1)^n$ and $(3/1)^n$, and so on inductively. By this approach, Oresme was convinced of the existence of one or more irrational ratios that are incommensurable to all irrational ratios, although he readily admitted that he was unable to demonstrate this.

1. The propositions are translated from Leonardo Pisano's *Practica geometriae (The Application of Geometry)* in the edition of Baldassare Boncompagni, *Scritti di Leonardo Pisano* (Rome, 1862), Vol. II, Distinction 4, "On the Division of Fields between Partners," pp. 119–125, 128–129, 138. Translations from Euclid's *Elements* in the following notes are from Thomas Heath, *The Thirteen Books of Euclid's Elements*, 2d. ed. (New York: Dover, 1956).

2. R. C. Archibald, *Euclid's Book on Divisions of Figures, with a Restoration Based on Woepcke's Text and on the Practica Geometriae of Leonardo Pisano* (Cambridge, England: Cambridge University Press, 1915), p. 11.

3. Quotations from Archibald, pp. 11–12. Archibald conjectures (p. 12) that Leonardo may have had a Latin translation of Euclid's treatise made by Gerard of Cremona.

whole quadrant *ac* be known and let it be required to find the sinus of any arc greater than the quadrant. Then if this arc is less than a semi-circle I shall have to subtract it from the semi-circle, 180 degrees, and find the sinus rectus of the residuum by a preceding (theorem). This is the desired sinus because the general law is that the sinus of any arc less than a semi-circle is the same as the sinus of its supplement, and conversely. Whence if I wish to find the sinus of arc *ad* I shall subtract arc *ad* from arc *ao* and find the sinus *md* of the remainder *do*. But *md* can easily be known since it is equal to *hb*. For if I subtract *ac* from *ad*, *cd* will remain and if I subtract *cd* from *ac*, the remainder will be *ab*. But *ob* equals *ad*. Therefore if their common part *bd* be taken away there will remain equal arcs *ab* and *do*. Hence their sinus recti, and their sinus versi, are equal. This is evident also by III 35. Again if I wish to find the sinus of arc *ace*, from *ace* I subtract *ao* and there remains *oe* whose sinus *em* is equal to *md* and hence to *hb* by a preceding (theorem). Therefore the sinus of *ade* is known. Again if I wish to know the sinus of *adg* I shall subtract it from the whole circle and the remainder is *ag* whose sinus, *gh*, is equal to *hb*. So the proposition is proved. All this about equality can also be proved. If I draw lines *ab*, *ag*, *od*, *oe* the demonstration is evident from right triangles by Q 5.[49]

Thirteenth Proposition : *If equal arcs be taken at the beginning and at the end of the quadrant the sinus rectus and the sinus versus of the one are equal respectively to the sinus versus and the sinus rectus of the other.*

This is evident. For if *ar* (Fig. 13) equals *cd* in the circle *adtg* the corda recta *sr* of arc *ar* equals the line *qc* which is the corda versa of arc *dc*, as is evident enough. From this observe that since the sinus versus of the first kardaga equals the sinus rectus of the last kardaga then the sinus versus of the second kardaga will equal the sinus rectus of the fifth kardaga, and so on for the rest. Observe also that if I wish to find the sinus versus of an arc greater than a quadrant I must take for the quadrant the sinus totus, that is the radius, and for the excess over a quadrant the sinus of the excess measured from the beginning of the quadrant and combine them. To one with insight this operation is sufficiently evident.

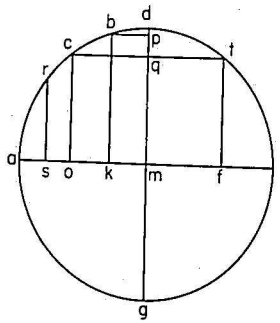

Fig. 13

From this follows the corollary: The sinus rectus of an arc less than a quadrant and the sinus versus of its complement together equal the radius, and the sinus rectus of the quadrant together with the sinus rectus of the part of the semi-circle in excess of the quadrant equals the sinus versus of the sum of the arcs. In the circle *adtg* (Fig. 13) it is seen that sinus rectus *oc* of arc *ac* and sinus versus *qd* of arc *cd* together equal the radius *md*. Conversely sinus versus *ka* of arc *ab* and sinus rectus[50] *bp* of the complementary arc *bd* together equal the radius *am*. And *md* together with corda (recta) *qt* of arc *dt* equals the corda versa *af* of the whole arc *at*.[51]

49. Bond's representation:
"sin $(360° - a) = \sin (180° + a)$
$= \sin (180° - a) = \sin a$,
only absolute values being considered."

50. The Latin text and Bond's translation have *br*, which is erroneous. I have, therefore, added the letter *p* to Fig. 13 and called *bp* the sinus rectus of arc *bd*.

51. Bond represents the two major points of this proposition as follows (since *r* occurs in Fig. 13, only where *r* is used alone does it represent the radius of the circle):
I. $sr = r - ao$; $as = r - oc$;
II. sinus $sr = r -$ sinus versus ao,
sinus versus $as = r -$ sinus oc;
III. $\sin (90° - a) = 1 -$ versin a,
versin $(90° - a) = 1 - \sin a$.

I. $md + qt = af$;
II. $r +$ sinus $qt =$ sinus versus af;
III. $1 + \sin a =$ versin $(90° + a)$.

Typical Scientific Questions Based on Aristotle's Major Physical Treatises

INTRODUCTION

Edward Grant

Questiones, or problems, based on Aristotelian physical treatises constituted a highly significant part of the medieval genre of question literature, which embraced theological, medical, and scientific problems, indeed almost any disputed question. Since the whole or a part of a number of such *questiones* have been translated in this source book, it will be useful to describe the rigidly consistent patterns to which nearly every question was made to conform. Following the enunciation of the question, the author presented an affirmative or negative solution. If affirmative, it was very likely that the author himself adhered to the negative side of the issue. If he presented the negative argument first, he would ultimately underwrite the affirmative position. The central arguments, those the author later rejected, were called the "principal arguments." Following this, the author would often describe his mode of procedure and sometimes further clarify and qualify the question, or define and explain particular terms that occurred in its formulation. He then usually presented a number of conclusions reflecting his opinions and viewpoints. Then he might raise doubts about his own position and subsequently resolve them. Finally, he would respond to the "principal arguments," and, hopefully, by answering them, tie up the remaining loose ends. Although scientific problems had been presented in question form since antiquity (for example, the *Problems* ascribed to Aristotle, and Seneca's *Natural Questions* and in the twelfth century, Adelard of Bath), the form described above was a peculiar product of the late middle ages and was the most significant and widespread form of scientific literature in the thirteenth and fourteenth centuries. While the questions sometimes varied, many were in vogue for centuries, and the long list presented below is wholly typical for each of the four physical treatises represented.

35 QUESTIONS ON THE EIGHT BOOKS OF ARISTOTLE'S *PHYSICS*

Albert of Saxony (ca. 1316–1390)

Translated and annotated by Edward Grant[1]

BOOK I

[1.] In the first [question] we inquire whether *(queritur primo utrum)*[2] natural science considers mobile being as its proper and adequate subject.

[2.] In the second [question] we inquire whether scientific understanding *(cognitio scientifica)* of what has been caused depends on the knowledge of its causes.

[3.] We inquire whether for perfectly understanding something it is necessary to know all its causes.

[4.] We inquire whether the same things are better known to us and nature.[3]

[5.] Next we inquire whether universals are prior to singulars or less universal in the understanding.

[6.] In the sixth [question] we inquire whether every extended thing is a quantity.

[7.] Whether a whole is [composed] of its parts.

1. This complete list of questions is translated from Albert of Saxony's *Questions on the Eight Books of the Physics*, published in *Questiones et decisiones physicales insignium virorum: Alberti de Saxonia in octo libros Physicorum; tres libros De celo et mundo; duos libros De generatione et corruptione; Thimonis in quatuor libros Meteororum. . . Buridani in librum De sensu et sensato . . . Aristotelis. . . recognitae rursus et emendatae summa accuratione. . .Magistri Georgii Lokert. . .* (Paris, 1518).

2. Here we see the common formula for medieval questions. Often the ordinal number of the question is omitted, leaving only *queritur utrum*, literally "it is sought whether," which I have rendered as "we inquire whether." The cardinal numbers have been added for convenience.

3. Aristotle denied this at the beginning of Book I of the *Physics:* "For the same things are not 'knowable relatively to us' and 'knowable' without qualification. So in the present inquiry we must follow this method, and advance from what is more obscure by nature, but clearer to us, towards what is more clear and more knowable by nature" (184a.18–21).

[8.] In the eighth [question] we inquire whether from the addition of some whole to some whole another whole is made; similarly, [whether] by the removal of some whole from some whole, another whole is made.

[9.] Whether a minimum [piece of] matter can be assigned from whose potentiality a natural form could be brought forth.

[10.] In the tenth [question] we inquire whether in any species a minimum of matter can be assigned under which some individual of this species can exist.

[11.] We inquire whether there are contrary principles of any natural transmutation whatever.

[12.] In the twelfth [question] we inquire whether there are only two or three principles of natural things.

[13.] Whether it would be possible that something be made anew *(de novo)* with no subject presupposed.

[14.] We inquire whether matter could be a principle of any transmutation whatever.

[15.] In the fifteenth[4] [question] we inquire whether prime matter could be knowable per se.

[16.] In the sixteenth, we inquire whether privation is a thing distinct from matter.

[17.] We inquire whether form is the internal principle of any transmutation.

[18.] We inquire whether something of a form that will be generated preexists in matter before its [that is, the form's] generation.

[19.] Finally, we inquire whether every being would like to be permanent.

BOOK II

[1.] We inquire whether every natural being has within itself active and passive principles of natural motion and rest.

[2.] In the second [question] we inquire whether any mixed [that is, compound] *(mixtum)* or simple body could have within itself an internal principle of its alteration.

[3.] In the third [question] we inquire whether a figure is a thing distinct from the figured thing; for example, whether a face and its figure [or shape] are distinct things.

[4.] We inquire whether artificial things could be distinguished from natural things.

[5.] We inquire whether the definition of nature assumed by Aristotle in this second [book] is sound *(bona)*—namely that nature is a source *(principium)* and cause of moving and resting of that in which it is primarily per se and not by accident.[5]

[6.] In the sixth [question] we now inquire whether nature is properly divided into matter and form.

[7.] Next we inquire whether the difference which Aristotle assigns between natural science and mathematics is well conceived—namely that natural science defines by means of motion and mathematics does not.[6]

[8.] In the eighth [question] we inquire whether the middle sciences[7] between pure mathematics and the pure natural sciences are more natural [sciences] than mathematics.

[9.] Next, in the ninth [question] we inquire whether there are four causes—no more nor less—of every natural thing.

[10.] Whether every effecting thing is the cause of that which it is effecting.

[11.] In the eleventh [question] we inquire whether a proportion of effects is mutually similar to the proportion of acting causes [which] mutually [produced those effects].

[12.] Whether an end is a cause.[8]

[13.] We inquire whether Aristotle's definition of *chance (fortuna)* is sound—namely that *chance* is an accidental cause *(causa per accidens)* always external *(extra semper)* and frequently with respect to those things that come to pass for the sake of something.[9]

[14.] We inquire whether anything can happen by chance *(fortuna)* and accident *(casu)*.

[15.] We inquire whether nature endeavors to produce a monster.

[16.] Finally, we inquire whether necessity in natural dispositions and operations comes from the end[10] or from matter.

BOOK III

[1.] Next, with respect to the third book of the *Physics*, we inquire whether it is necessary that ignorance of a motion [entails] ignorance of the nature [of the thing in motion].

[2.] In the second [question] we inquire whether [Aristotle's] definition of motion is sound, namely

4. The text has, mistakenly, "sixteenth" *(sextodecimo)*.
5. See Aristotle, *Physics* II.1.192b.21–23.
6. Aristotle, *Physics* II.2.193b.31–35.
7. For example, optics, harmonics, astronomy (see *Physics* II.2).
8. That is, whether the goal or end toward which a thing strives is to be construed as a (final) cause.
9. See Aristotle, *Physics* II.5.197a.32–35 and II.6.197b.18–22.
10. That is, from the final cause, namely "that for the sake of which" the natural dispositions and operations are performed.

that motion is the actualization of what exists in potentiality insofar as it exists in potentiality.[11]

[3.] In the third [question] I inquire *(quero)* whether of all the things that exist, some are permanent and others successive.[12]

[4.] Now, in the fourth [question] we inquire whether a motion of alteration is a thing distinct from the quality that is acquired and from the quality that is lost and [distinct from] the alterable thing which acquires or loses such a quality.

[5.] Next I inquire whether according to Aristotle and his Commentator something that is moved locally requires something that is a certain flux *(fluxus)* [or flow] distinct from the mobile and place.[13]

[6.] Whether upon admitting [certain] divine events, it would be necessary to concede that local motion is something other than the mobile and [its] place.[14]

[7.] In the seventh [question] we inquire whether motion is subjectively in the mobile or in the mover.

[8.] In the eighth [question] we inquire whether every action is a passion.

[9.] In the ninth, we inquire whether determinations about the infinite belong to the natural philosopher [physicist].

[10.] Whether any actual infinite body would be sensible.

[11.] Whether there could be an infinite dimension.

[12.] Whether it is possible for an actual infinite magnitude to exist.

[13.] In the thirteenth [question] we inquire whether there is an infinite [number of] parts in a continuum.

[14.] In the final question we inquire whether for every given magnitude, a greater magnitude could be assigned; and for every given number, a greater number could be assigned.

BOOK IV

Now that the first three books of the *Physics* have been treated, we must next consider the fourth book, and since this book deals principally with *place*, let the first question be this:

[1.] Whether place *(locus)* is a surface.

[2.] In the second [question] we inquire whether place is equal to the thing [or body] that is placed [or located].

[3.] In the third [question] we inquire whether place is immobile.

[4.] In the fourth [question] we inquire whether the definition of place is sound, namely that place is the terminus of the first immobile containing body.

[5.] In the fifth [question] we inquire whether the natural and proper place of earth is in water or inside the concave surface of water.

[6.] In the sixth [question] we inquire whether the concave [surface] of the moon is the natural place of fire.

[7.] Next, in the seventh [question], we inquire whether every being is in a place.

[8.] In the eighth [question] we inquire whether the existence of a vacuum is possible.[15]

[9.] Next we inquire whether in its downward motion a heavy simple body has an internal resistance; and similarly, [whether] in its upward motion a light [simple] body [has an internal resistance].[16]

[10.] Whether a resisting medium is required in every motion of heavy and light bodies.

[11.] In the eleventh [question] we now inquire whether, if a vacuum did exist, a heavy body could move in it.[17]

[12.] Whether, if a vacuum existed, something could be moved in it with a finite velocity or local motion, or with a motion of alteration.[18]

[13.] Whether condensation and rarefaction are possible.

[14.] Concerning the chapter on time, we inquire whether time is the motion of the sky [or heavens].

[15.] We inquire whether the definition of time is sound, [namely where] time is said to be the number *(numerus)* [or measure] of motion ac-

11. The definition appears in *Physics* III.1.201a.10–11.
12. As Albert expresses it (*Questiones et decisiones Physicales*. . ., fol. 33 verso, col. 1), Aristotle "insists that some things are permanent, as stone and wood, and some successive, as the day and year."
13. For a discussion of this problem by William of Ockham, see Selection 40.
14. The "[certain] divine events" *(casus divinos)* involve the assumption that God annihilates everything except one mobile—say the sphere of the moon—which rotates from east to west. Under these conditions the moon's motion cannot be related to anything external, from which Albert concludes that it must be related in a continually varying manner to something internal or intrinsic to the mobile itself. This he identifies as a "flux," or the motion itself, and argues that motion is therefore distinct from the mobile and the places it occupies (see the enunciation of the immediately preceding, or fifth, question).
15. See Selection 53.1 for a translation of this question.
16. A substantial part of this question is translated in Selection 47.
17. For a translation, see Selection 55.2.
18. *Ibid.*

cording to before and after.[19]

[16.] We inquire whether every motion is measured or measurable by time.

[17.] In the last question we inquire whether rest is measured by time.

BOOK V

[1] We inquire next about the text of the fifth book of the *Physics:* whether every transmutation to a substance is successive.[20]

[2.] In the second [question] we inquire whether generation is motion.

[3.] In the third, we inquire whether every motion is from contrary to contrary.

[4.] In the fourth, we inquire whether there is motion per se according to the three categories of quantity, quality, and place *(ubi)*.

[5.] In the fifth, we inquire whether any motion is one [whole unity].

[6.] Next, in the sixth [question], we inquire whether some motions differ in species.

[7.] In the seventh, we inquire whether a motion could be contrary to a motion.

[8.] In the eighth, we inquire whether rest is contrary to motion.

[9.] In the ninth, we inquire whether contrary qualities could be suffered [or tolerated] in the same subject; for example, whether some degree of hotness could exist simultaneously in the same subject with some degrees of coldness.

[10.] Now, in the tenth [question] we inquire whether in the intension of a quality, the quality that is acquired is acquired all at once [and] simultaneously; or [whether] it is acquired part after part.

[11.] In the final [question], concerning the intension of any quality, we inquire whether what is acquired first remains [or is preserved] with [parts of] the [same] quality acquired afterwards.

BOOK VI

[This is] the sixth book, treating of indivisibles and their motions, in that order, and this difficulty [or problem] is posed:

[1.] Whether a continuum is composed of indivisibles;[21] for example, as a line might be composed of points, time of instants, and motion of mutations [that is, instantaneous changes].

[2.] In the second [question] we inquire whether a continuum is divisible into things that are always [further] divisible.

[3.] Next, in the third [question], we inquire whether an indivisible entity could be moved.

[4.] In the fourth, we inquire whether, in local motion, velocity is measured according to distance traversed.

[5.] In the fifth [question] we inquire in what way velocity of circular motion should be measured.

[6.] In the sixth, we inquire in what sense velocity is measured as if it were an effect in motion of augmentation.[22]

[7.] In the seventh, we inquire whether motion could be accelerated *(velocitari)* to infinity.

[8.] In the eighth, we inquire whether every thing which is moved [or in motion] was moving before.[23]

[9.] In the ninth, we inquire whether an infinite magnitude could be traversed in a finite time; and whether a finite magnitude could be traversed in an infinite time.

[10.] In the final [question] we inquire whether during the time a mobile is moved continuously, it is in a place greater than itself.

BOOK VII

As to the text of the seventh book of the *Physics*, we inquire about this question:

[1.] Whether the same thing could act on itself.

[2.] In the second [question] we inquire whether the mover *(movent)* and the thing moved should be [conjoined] together.

[3.] In the third [question] we inquire whether alteration per se occurs only in qualities of the third species.[24]

[4.] In the fourth [question] we inquire whether it is the case that for motions to be comparable they must be of the same particular species.

19. Aristotle's definition in *Physics* IV.11.220a.24–25.

20. That is, whether every change of substance occurs in a finite time.

21. See Selections 52.1–4 for a discussion of this problem.

22. By "motion of augmentation" is meant the process of increasing a quantity.

23. See Aristotle, *Physics* VI.6.236b.34–35.

24. Albert distinguishes four species of alteration, or change of quality, of which the third is alteration of sensible quality, such as colors, tastes, and so on. His affirmative response to the question agrees with Aristotle's conclusion "that things that are undergoing alteration are altered in virtue of their being affected in respect of their so-called affective qualities, since that which is of a certain quality is altered insofar as it is sensible, and the characteristics in which bodies differ from one another are sensible characteristics: for every body differs from another in possessing a greater or lesser number of characteristics or in possessing the same sensible characteristics in a greater or lesser degree" (*Physics* VII.2. 244b.5–7; in VII.3 Aristotle considers "alterations" that are not really alterations; the four species of alteration discussed by Albert were enumerated by Aristotle, *Categories*, chapter 8).

[5.] In the fifth [question] we inquire whether any motion is comparable to any [other] motion.

[6.] Now, in the sixth [question] we inquire whether a ratio *(proportio)*[25] of velocities in motion varies as a ratio of ratios of the motive powers to their resistances.[26]

[7.] In the final [question] we inquire whether those rules are true which Aristotle assumes in the seventh book on the comparison of mobiles and movers.[27]

BOOK VIII

As to the text of the eighth book of the *Physics*, we inquire about this:

[1.] Whether there always was and always will be motion.

[2.] Next, in the second [question], whether some motion could be eternal.

[3.] In the third, we inquire whether time is eternal.

[4.] Next, in the fourth [question], we inquire whether every body could be moved.

[5.] In the fifth, we inquire whether an animal could be self-moved *(ex se)*.

[6.] We inquire whether inanimate heavy and light bodies could be self-moved.

[7.] In the seventh, we inquire whether a heavy body is self-moved after removal of whatever prevents its motion.

[8.] In the eighth [question] we inquire whether the first mover is immobile.

[9.] In the ninth, we inquire whether local motion is the first of motions.[28]

[10.] Whether it is possible that a mover of finite power move [or act] through an infinite time

[11.] Now we inquire whether an infinite power [or force] could exist in a finite magnitude.

[12.] Whether it is necessary that a moment of rest should intervene between any reflected [that is, converse] motions *(motus reflexos)*.[29]

[13.] In the final [question] we inquire what it is that moves a projected body upwards after separation from what has projected it.[30]

36 QUESTIONS ON THE FOUR BOOKS OF ARISTOTLE'S *ON THE HEAVENS (DE CAELO)*

John Buridan (ca. 1300—ca. 1358)

Translated and annotated by Edward Grant[1]

BOOK I

On the book *On the Heavens and the World (De caelo et mundo)* we inquire:

[1.] Whether the science *(scientia)* of *On the [Heavens and the] World (De mundo)* ought to be distinct from the science of the book of the *Physics*.

[2.] Next we inquire whether in the same body the dimensions length, width, and depth are mutually distinct.

[3.] Next we inquire whether there are three species of magnitude and no more.

[4.] Next we inquire whether every natural body is naturally mobile with respect to place.

[5.] Next we inquire whether there are three simple motions and no more, namely upward motion, downward motion, and circular motion.

[6.] Next we inquire whether by nature there is to one simple body one simple motion only, and also whether by nature there is one simple motion only to one simple body. The sense of this is that by nature, there could not be several simple motions for one simple body; nor by nature could there be one simple motion of several simple bodies.

[7.] Next we inquire whether a mixed *(mixtum)* [or compound] body is moved by the nature of the dominant element.[2]

[8.] Next we inquire whether circular motion has

25. For a brief discussion of the term *proportio*, see Selection 27, n. 11.

26. This is actually a description of "Bradwardine's function" (see Bradwardine's enunciation of it in Selection 51.1 and also n. 29) or "ratio of ratios" (for an explanation of this expression, see Selection 51.1, n. 51), as it was usually designated in the second half of the fourteenth century.

27. In the Middle Ages two rules were usually formulated to represent Aristotle's position. Both are refuted by Oresme in Proposition I, Selection 51.2.

28. That is, whether local motion (or motion from one place to another) is prior to, and more fundamental than, motion of alteration, augmentation, and diminution. See Aristotle, *Physics* VIII.9.265b.16–226a.5.

29. Marsilius of Inghen's response to this question has been translated in Selection 50.1.

30. Among other things, Albert discusses impetus theory here. For medieval impetus theory, see Selection 48.

1. Translated from *Iohannis Buridani Quaestiones super libris quattuor De caelo et mundo*, ed. E. A. Moody (Cambridge, Mass.: Mediaeval Academy of America, 1942).

2. See Albert of Saxony's fifth doubt in Book I, Question 19, of his *Questions on Generation and Corruption* in Selection 77.2 (also n. 41 thereto) and the response to that doubt in the same selection.

a contrary or whether to circular motion there is another contrary motion.

[9.] Next we inquire whether the sky is heavy or light.

[10.] Next we inquire whether the sky [or heaven] is generable and corruptible, augmentable and diminishable, and alterable.

[11.] Next we inquire whether the sky [or heaven] has matter.

[12.] Next we inquire whether the world is perfect.

[13.] If a body that is moved circularly were infinite, we inquire next whether the distance between lines extended from its center would be infinite.

[14.] Next we inquire whether an infinite body, if such existed, would have some active power or even a passive power.

[15.] Next we inquire whether it is possible that a body moved circularly could be infinite.

[16.] Next we inquire whether it is possible that a body moved rectilinearly could be infinite.

[17.] Furthermore, we inquire whether an infinite body is possible.

[18.] This question is next: If there were several worlds, whether the earth of one would be moved naturally to the middle of another.

[19.] Next we inquire whether it is possible that there are several worlds.[3]

[20.] Next we inquire whether there is something beyond the sky [or heaven].[4]

[21.] Next we inquire whether a power [or force] ought to be defined by its maximum capability.[5]

[22.] Next we inquire whether a power's maximum capability can be assigned.

[23.] Next we inquire whether the power to be and not to be of everything which can sometimes be and not be is for a definite, and not infinite, time.

[24.] Next we inquire whether every corruptible thing is corrupted of necessity.

[25.] Next we inquire whether every generable thing will be generated.

[26.] Next we inquire whether *generable* and *corruptible*, as well as *ungenerable* and *incorruptible*, are mutually convertible, so that every generable thing would be corruptible, and conversely; and every ungenerable thing would be incorruptible, and conversely.

BOOK II

Concerning the second book of *On the Heavens and World*, we inquire first:

[1.] Whether or not the sky [or heaven] is moved with any fatigue.

[2.] In the second [question] we inquire whether in the sky there are up and down, front and behind, right and left.

[3.] Next, in the third [question], we inquire whether in the sky up and down should be taken as the extension which stretches from pole to pole, so that one pole ought to be called up and the other down.

[4.] Next, in the fourth, we inquire whether the north *(arcticus)* pole, namely the one appearing to us, is down, and the south *(antarcticus)* pole is up.

[5.] Next, in the fifth, we inquire whether in the sky [or heavens], right and left and front and behind are [part] of the [very] nature of the sky, or are only relative to us.

[6.] In the sixth [question] we inquire whether beyond the heavens that are moved, there should be assumed a heaven that is resting or unmoved.

[7.] Next, in the seventh, we inquire whether the whole earth is habitable.

[8.] Next, in the eighth [question], we inquire whether everything doing work exists for the sake of that work; that is, whether for any working thing, the work is its end [or goal].

[9.] In the ninth, we inquire whether it is appropriate to demonstrate that the plurality of motions and spheres in the heavens is from God.

[10.] Next, in the tenth [question], we inquire whether this consequence is sound: If it is necessary that generations and corruptions be here below, it is [therefore] necessary that there be several celestial motions.

[11.] In the eleventh, we inquire whether the sky [*caelum*] is always moved regularly.

[12.] In the twelfth, we inquire whether natural motion ought to be quicker at the end than at the beginning.

[13.] In the thirteenth, we inquire whether projected things are moved quicker in the middle [of their motions] than in the beginning or end.[6]

[14.] In the fourteenth, we inquire about the discussion of the stars: whether all celestial spheres and all stars are mutually of the same ultimate species *(speciei specialissimae)*.[7]

3. See Selection 71.

4. For a discussion on infinite space beyond the finite cosmos, see Selections 72 and 73.

5. This and the next question are translated in Selection 57.

6. In terms of the Aristotelian distinction between natural and violent motion, this question is concerned with the latter.

7. "Ultimate species" signifies an individual species not further divisible into other species.

[15.] In the fifteenth [question] we inquire whether by their light *(lumen)* the celestial bodies are generative of heat.

[16.] Next we inquire whether local motion can produce heat.

[17.] Next we inquire whether by its motion the sphere of the sun makes lower things hotter than do other spheres.[8]

[18.] Next we inquire about the chapter [in *On the Heavens and the World*] concerning the motion of stars: whether the stars are self-moved or moved by the motion of their spheres.[9]

[19.] Next we inquire whether spots appearing in the moon arise from differences in parts of the moon or from something external.

[20.] We inquire [now] about the chapter on the order [of the planets];[10] whether the lower spheres of the planets should be moved quicker in their proper motions than the superior spheres.

[21.] Next we inquire whether the sun and moon ought to be moved with fewer motions than other planets.

[22.] Next we inquire whether the earth always rests in the middle [or center] of the world.[11]

[23.] In the next and last question on the second book [of *On the Heavens*] we inquire whether the earth is spherical.

BOOK III

As to the third book of *On the Heavens and the World*, we inquire:

[1.] Whether in terms of heaviness and lightness it could be demonstrated that bodies are not composed of indivisibles.

[2.] In the next and last [question] we inquire whether after departure from a projector, a stone that is projected, or an arrow that is shot from a bow, and so on in similar cases, [these] are moved by an internal principle or an external principle.[12]

BOOK IV

As to the fourth book of *On the Heavens and the World*, we inquire first:

[1.] Whether there is something absolutely heavy and something absolutely light; and also [whether there is] something [both] heavy and light which is neither absolutely heavy or light.

[2.] Next we inquire whether the natural places of heavy and light bodies are the causes of their motions.

[3.] Next we inquire whether a heavy or light body is moved naturally downward or upward by its generator *(generans)*,[13] or by the thing which removes the impediment [to its motion].[14]

[4.] Next we inquire whether heavy and light bodies are actively moved by heaviness and lightness, [respectively].[15]

[5.] Next we inquire whether heaviness and lightness are substantial forms of heavy and light bodies.

[6.] Next we inquire whether it might suffice to assume two motive natures, and mixtures of them, in order to save [or explain] the appearances of natural motions of heavy and light bodies.

[7.] Next we inquire whether, in its proper region [or place], air is heavy or light; or neither heavy nor light.

[8.] In the last [question] we inquire whether the number of four elements could be derived in terms of [or by arguing from] heaviness and lightness.

37 QUESTIONS ON THE TWO BOOKS OF ARISTOTLE'S *ON GENERATION AND CORRUPTION (DE GENERATIONE ET CORRUPTIONE)*

Albert of Saxony (ca. 1316–1390)

Translated and annotated by Edward Grant[1]

BOOK I

[1.] First we inquire whether it is possible that

8. The question is concerned primarily with the action of the sun on all things between the earth and moon.

9. In Book II, chapter 8, of *De caelo*, Aristotle considers whether the stars are free, self-moving bodies or whether they are attached to spheres which rotate and carry them. It is the latter position that he adopts.

10. See Aristotle, *De caelo* II.10.

11. Burjdan's treatment of this very question is given in Clagett's translation in Selection 67.4. Because it was desirable to translate the enunciation of this question in conformity with the pattern used throughout, my translation differs slightly from Clagett's.

12. This is the locus of one of Buridan's famous discussions of impetus theory. For his views, see Selections 48 and 49.

13. That is, the thing which caused or produced the heavy or light body.

14. For a discussion of this question by St. Thomas Aquinas, see Selection 46.

15. Questions 2, 3, and 4 present three quite different causal explanations of natural motion.

1. This treatise occupies folios 128 verso to 155 recto of the same edition cited in Selection 35, n. 1, which also contains Albert of Saxony's *Questions on the Physics*.

something be generated absolutely or corrupted absolutely.

[2.] In the second [question] we inquire whether generation is alteration.

[3.] We inquire whether, if generation were impossible, alteration would [also] be impossible.

[4.] We inquire whether the generation of one [thing] is the corruption of another.

[5.] We inquire whether in generation there occurs an absolute resolution [or decomposition] into prime matter.

[6.] We inquire whether any quality that was previously in a corrupted [substance] could persist in the [substance] generated [from what was corrupted].

[7.] We inquire whether any generable thing that may be given could be generated from any of several agents. For example, if *A* were a given generable thing which could be produced by agent *B*, [the question is] whether *A* could be generated or produced by an agent other than *B*.

[8.] We inquire whether any generable thing that may be given could be generated indifferently in any one of several instants.

[9.] We inquire whether augmentation is generation.

[10.] We inquire whether that which is increased in augmentation remains the same before and after.

[11.] We inquire whether any part of something that is increased could [itself] be increased.

[12.] We inquire whether augmentation occurs by means of formal, and not material, parts.

[13.] We inquire whether augmentation is a continuous motion.

[14.] We inquire whether every action and passion occurs by contact.

[15.] We inquire whether a similar thing can act on a thing similar to itself.

[16.] We inquire whether in acting [on something] every agent is [also] acted upon *(repatiatur)*.

[17.] We inquire whether an indivisible [thing] could be altered.

[18.] We inquire whether substantial forms of elements can be intended and remitted.

[19.] We inquire whether elements remain [or persist] formally in a compound [or mixed] body.

[20.] We inquire whether a compound is possible.[2]

[21.] We inquire whether a compound *(mixtio)* is natural. By "compound" we understand what Aristotle understands in the first book of this *[On Generation and Corruption]*, namely a "compound" properly so called is that whose every part is [also] said to be a compound [or mixed] body.

BOOK II

As for the second book, we inquire firstly:

[1.] Whether there are four primary qualities, no more nor less.

[2.] We inquire whether of the four primary qualities (hot, cold, [dry, and moist]) two are active, namely hot and cold, and two passive, namely moist and dry.

[3.] We inquire whether there are four elements, no more nor less.

[4.] We inquire whether fire is primarily hot, air primarily moist, water primarily cold, and earth primarily dry.

[5.] We inquire whether the hotness of fire and the hotness of air are of the same species, differing only by more and less; similarly we ask [the same question] about the dryness of fire and earth, the moistness of air and water, and the coldness of earth and water.

[6.] We inquire whether any element is pure.

[7.] We inquire whether one element could be generated directly from another, so that water could be generated directly from air without something else being generated from it previously; and [the same question can be asked] of the other elements.

[8.] We inquire whether [elements] having a common quality *(symbolum)* are more easily [and quickly] transmuted into one another.[3]

[9.] We inquire whether a third element can be generated from two elements lacking a common quality *(symbolum)*.

[10.] We inquire whether every compound body located around the middle [or center of the world] *(circa locum medium)* is composed of all the simple [or elemental] bodies.[4]

[11.] We inquire whether a compound body can be [equally or] duly proportioned from the [four] elements or qualities of the elements.

[12.] We inquire whether generation is perpetual.

[13.]We inquire whether every corruptible thing has a definite period of duration.

[14.] We inquire whether the process in the generation of things continues into infinity.[5]

2. For translations of Questions 19 and 20, see Selection 77.2.

3. See Aristotle, *On Generation and Corruption* II. 4.

4. See *On Generation and Corruption* II. 8.

5. Aristotle distinguishes between an eternal process conceived linearly and circularly. Rejecting the former, he argues for a cyclical infinite process.

38 QUESTIONS ON THE FOUR BOOKS OF ARISTOTLE'S *METEOROLOGICA*

Themon, Son of the Jew (fl. 1349–1361)

Translated and annotated by Edward Grant[1]

BOOK I

[1.] In the first [question] we inquire whether this whole sensible world is subject to, and continuous[2] with, the superior motions *(superioribus lationibus)*, so that the power of this whole [sensible] world is regulated [and governed] from there.[3]

[2.] We inquire whether the mass *(moles)* of the whole earth—that is, its quantity *(quantitas)* or magnitude *(magnitudo)*—is much less than certain stars.[4]

[3.] We inquire whether the sky has the nature of fire.

[4.] We inquire whether the same opinions are repeated by men an infinite number of times.[5]

[5.] We inquire whether the four elements are continuously proportional or have an equality of a common ratio *(communis analogie)*, that is, whether they form a continuous proportionality.[6]

[6.] In the sixth [question] we inquire whether celestial motions cause hotness here below on lower [or inferior] things.

[7.] In the seventh [question] we inquire whether the middle region of air is always cold.

[8.] In the eighth, we inquire whether every luminous body causes heat by its light.

[9.] In the ninth, we inquire whether the matter of all meteorological phenomena *(impressiones)* is a vapor or exhalation.

[10.] In the tenth [question] we inquire whether burning phenomena *(impressiones ignite)* generated in the upper air occur naturally.

[11.] In the eleventh, we inquire whether clefts *(hyatus)* and chasms *(voragines)* can appear [in the sky] on a clear night.[7]

[12.] In the twelfth, we inquire whether a comet is of a celestial nature or [whether it is] of an elementary nature, say of a fiery exhalation.[8]

[13.] In the thirteenth [question] we inquire whether a comet or stellar comet signifies the death of princes, drynesses, winds, and other evils.[9]

[14.] In the fourteenth, we inquire whether the milky way *(galaxia)* is of a celestial or elementary [that is, sublunary] nature.[10]

[15.] In the fifteenth, we inquire whether the middle region of air is the place where rain is generated.

[16.] In the sixteenth, we inquire whether moisture *(pluvia ros)* and hoar frost [or dew] *(pruina)* differ as ultimate species *(specie specialissima)*.[11]

[17.] In the seventeenth, we inquire whether hail occurs more in spring and autumn.[12]

[18.] In the eighteenth, we inquire whether a redness in the morning signifies future rains.

[19.] In the nineteenth [question] we inquire

1. My translation was made from the Latin text *(Questiones super quatuor libros Meteororum compilate per doctissimum philosophii professorem Thimonem)* found in folios 155 verso to 214 verso of the same edition cited in Selection 35, n. 1.

2. The text has *contiguus* instead of *continuus*.

3. Aristotle, *Meteorologica* I.2.339a.21–23.

4. *Meteorologica* I.3.339b.7–9.

5. A question based upon Aristotle's claim that "the same opinions appear in cycles among men not once or twice, but infinitely often" (*Meteorologica* I.3.339b.28–29).

6. If we represent the four elements by *A*, *B*, *C*, and *D*, Themon asks whether the four elements are related in continued proportion so that $(A/B)\cdot(B/C)\cdot(C/D) = (A/D)^3$ and $A/B = B/C = C/D$. In his lengthy discussion of this question Themon cites Euclid's definition of continued proportion (*Elements*, Bk. V, Def. 5), which is translated in Selection 27.

7. See *Meteorologica* I.5. Themon conveys a vague sense of these terms when he observes (fol. 164 recto, col. 1): "The first experience is that sometimes the sky appears perforated or opened so that in these openings a depth appears which is so small and obscure that it is called a cleft *(hyatus)* and if large is called a chasm *(vorago)*."

8. In *Meteorologica* I.7, Aristotle denies that comets are celestial, or supralunar, phenomena, and in I.8 argues that they are fiery elemental exhalations (a second class of comets, however, is associated with the stars). Themon accepts Aristotle's opinions, which are quoted in the introduction to Selection 70.

9. Aristotle says nothing of omens such as the death of princes. His discussion is strictly scientific.

10. Following Aristotle, Themon argues that the milky way is in the celestial regions.

11. That is, whether moisture, on the one hand, and hoar frost or dew on the other, are separate, and not further divisible, species. Themon, once again in accord with Aristotle, holds that all such watery phenomena are of the same species, differing only in possessing more or less water.

12. Discussed by Aristotle in *Meteorologica* I. 12.

whether the waters of springs and rivers are generated in the concavities of the earth.[13]

BOOK II

Next we inquire about the second book of Aristotle's *Meteors*.

[1.] In the first [question] we inquire whether the sea, which is in the natural place of water, is generable and corruptible or perpetual.

[2.] In the second [question] we inquire whether the sea flows at one time and ebbs at another.[14]

[3.] In the third, we inquire whether the whole sea must be salty and bitter.[15]

[4.] We inquire whether river and spring waters could be salty.

[5.] We inquire whether wind is a hot and dry exhalation.[16]

[6.] In the sixth [question] we inquire whether the sun makes the winds cease and moves them.

[7.] In the seventh, we inquire whether a motion of the earth is possible.[17]

[8.] In the eighth, we inquire whether the tranquility of the air is a sign of the earth's motion to come.

[9.] In the ninth, we inquire whether thunder is caused by fire extinguished in a cloud.[18]

[10.] In the tenth, we inquire whether a typhoon and a hurricane are made from a hot and dry exhalation.

[11.] In the eleventh [question] we inquire whether lightning is fire descending from a cloud.

BOOK III

[1.] In the first [question] we inquire whether every visual ray is refracted *(refrangatur)*[19] in meeting a denser or rarer medium.

[2.] We inquire whether a visual ray proceeding from a luminous body of a particular density is refracted *(frangatur)* toward or away from the perpendicular when it meets a medium of a different density.[20]

[3.] In the third [question] we inquire whether every visual ray is reflected *(reflectatur)* when it meets a denser medium.

[4.] In the fourth [question] we inquire whether any visual ray could fall upon mirrors or bodies and be reflected so that it represents color only but not the shape [or figure of the body].[21]

[5.] In the fifth, we inquire whether a halo appears because of the refraction of rays in the vapor interposed between the eye *(visum)* and a luminous body[22] around which it appears.

[6.] In the sixth, we inquire whether a halo could be produced by refraction of visual rays.

[7.] In the seventh, we inquire whether at the time of the appearance of a halo, there is a vapor lying between the eye *(visum)* and a star *(astrum)*.

[8.] In the eighth, we inquire whether the vapor in which a halo appears is of a spherical figure.

[9.] In the ninth [question] we inquire whether a halo must appear as the periphery or circumference of a circle.[23]

[10.] In the tenth, we inquire whether, without the aid of mirrors, a visual ray could be reflected by homogeneous, uncondensed vapor-free air to the same eye [from which the visual ray issued],[24] and which [eye] is [located] in the same air.

[11.] In the eleventh, we inquire whether the colors appearing in the rainbow are where they seem to be and are true colors.

[12.] In the twelfth, we inquire whether a rainbow is a real form impressed on a cloud; or whether it is only an imaginary form.

13. In agreement with Aristotle, Themon adopts the affirmative side of this question (see *Meteorologica* I.13).

14. Based upon *Meteorologica* II.1.354a.6–12. In his Loeb Classical Library translation of the *Meteorologica* (Cambridge, Mass: Harvard University Press, 1962), H. D. P. Lee remarks (pp. 128–129) that "it is not clear exactly what Aristotle means by this ebb and flow (lit. swinging to and fro) of the sea, for he had no real knowledge of the tides." By the Middle Ages, thanks to observations by Bede (ca. 673–735), it was known that tides could be established as roughly constant for a given port (see ch. 29 of his *On the Reckoning of Times [De temporum ratione]*).

15. A topic discussed by Aristotle in *Meteorologica* II.3.

16. *Meteorologica* II.4.

17. Although Themon lists a number of ways in which the earth can be conceived to move (including rotation on its axis, which he rejects), this question is concerned primarily with earthquakes and tremors, the subject matter of II.7 and II.8 of the *Meteorologica*. For a discussion of the earth's possible axial rotation, see Selections 67.3–6.

18. Aristotle, as also Themon, denies this in II.9.

19. In the *Meteorologica*, Aristotle discusses only reflection of visual rays.

20. Since Aristotle did not consider refraction, he could not have raised this question.

21. An affirmative response is given by Aristotle in *Meteorologica* III.2.372a.30—372b.8 and III.4.373b.17–33.

22. Such as sun, moon, or stars.

23. Aristotle answers in the affirmative (*Meteorologica* III.3.372b.34—373a.18).

24. This bracketed addition is warranted from the nature of the question. Although, in the *Meteorologica*, Aristotle speaks as if visual rays issue from the eye, it was not his considered opinion and is denied elsewhere (for example, *Topics* I.14.105b.6). For medieval discussions on visual rays, see Selections 62.14–17.

[13.] In the thirteenth, we inquire whether a rainbow is a translucent *(diaphanalis)* or mirror-like form; or whether it occurs by reflection or refraction, which is the same thing.[25]

[14.] On the supposition that a rainbow can occur by reflection of rays, we inquire whether such reflection occurs in a cloud or whether it occurs in tiny dewdrops or raindrops.[26]

[15.] We inquire whether every rainbow must be three-colored.[27]

[16.] In the sixteenth [question] we inquire whether a rainbow can appear double only, but no more [than double].

[17.] We inquire whether on the appearance of two rainbows, the upper one must always have two colors located in opposite positions [to the colors of the lower bow].[28]

[18.] We inquire whether the colors of the upper or secondary rainbow should necessarily appear weaker *(remissior)* than [the colors] of the principal bow.

[19.] In the nineteenth, we inquire whether a rainbow, if [its formation is] unhindered, ought to appear around the periphery [or circumference] of a circle.

[20.] In the twentieth, we inquire whether, at the time of a rainbow's appearance, it is necessary that the center of the sun, the center of the horizon, the center of the rainbow, and the poles or pole of the rainbow, be in a straight line.

[21.] In the twenty-first [question] we inquire whether at the time of appearance of a rainbow and a halo, the diameter of the rainbow is exactly double the diameter of the halo.[29]

[22.] In the twenty-second, we inquire whether to an observer *(oculo)* placed on the surface of the horizon, some portion of a rainbow would appear during every hour of an artificial [or arbitrary] day, wherever the man or eye of the viewer might be located [on the surface of the horizon].[30]

[23.] In the twenty-third, we inquire whether a rainbow of the moon could appear more than twice in fifty years with colors and properties like the solar rainbow.[31]

[24.] In the twenty-fourth, we inquire whether a rainbow that is seen when the sun or a star is elevated considerably above the horizon appears as a smaller part *(portio)* of a great circle than it would appear if the sun were along the horizon.

[25.] Next we inquire whether rainbows appearing around candles really do exist around those candles.

[26.] In the twenty-sixth [question] we inquire whether rods *(virge)* and mock suns *(paralleli)* are produced by refraction of rays, just as with a rainbow or a halo.[32]

[27.] We inquire [now] about things made under the earth; whether metals could be produced with the aid of art just as a rainbow and a halo are sometimes made artificially.

BOOK IV

[1.] In the first [question] on the fourth book of the *Meteors* we inquire whether putrefaction is possible, or whether it can be done or exist.

[2.] In the second [question] on this fourth book we inquire whether digestion is a perfection caused by proper and natural heat [acting] on passive opposites.[33]

[3.] In the third, we inquire whether there are only three species of digestion [or concoction].[34]

[4.] In the fourth, we inquire whether digestion

25. For Themon, reflection and refraction differ, but both are required in the formation of a rainbow (see Selection 63.2 for Theodoric of Freiberg's qualitatively correct explanation of the formation of the rainbow), which is simultaneously mirror-like (because reflection occurs) and diaphanous or translucent (because of refraction).

26. Before Themon, Robert Grosseteste (Selection 61.2) had claimed that reflection (and refraction) occurred in a cloud, whereas Theodoric of Freiberg had argued correctly that it (and refraction) occurred in each raindrop (Selection 63.2).

27. For Aristotle, red, green, and blue were the three basic colors of the rainbow (*Meteorologica* III.2.371b.33—372a.11 and III. 4.374b.7—375b.8). Themon will argue that rainbows can be three-, four-, and even six-colored.

28. Aristotle answered in the affirmative, since he held that the upper or secondary bow consisted of three arcs arranged from outer to inner arc in the order blue, green, and red. The lower, or primary, bow had its colors arranged from outer to inner arc in the order red, green, blue. Hence in the two bows, red and blue are reversed, with green retaining a middle position in each bow.

29. This question is not discussed by Aristotle.

30. Aristotle (and Themon) deny the occurrence of rainbows at any time of day during the year. Indeed, Themon, perhaps relying ultimately on Roger Bacon, denies the occurrence of rainbows whenever the sun rises more than 42° above the horizon.

31. Although Aristotle mentions moon rainbows (*Meteorologica* III.4.375a.18), he does not raise this question.

32. In *Meteorologica* III. 6, Aristotle discusses the formation of rods and mock suns.

33. See *Meteorologica* IV.1.378b.26—379a.2.

34. "The effect of heat is concoction and there are three species of concoction, ripening, boiling, and roasting" (*Meteorologica* IV.2.379b.13–14; trans. H. D. P. Lee).

[or concoction] of blood and organic bodies *(nascentium)* by natural heat is [a process of] ripening *(pepansis)*.

[5.] In the fifth, we inquire whether a simple element could be digested or undergo digestion.

[6.] In the sixth, we inquire whether things that have been boiled are drier than things that have been roasted.

[7.] In the seventh, we inquire whether things that have been fried *(frixata)* are boiled or roasted.

[8.] In the eighth [question] we inquire whether the definition of simple generation is sound, namely where it is said that simple and natural generation is a change *(permutatio)* [effected] by those powers which bear a ratio *(rationem)* [or relation] to the subject matter of any nature.[35]

35. This question is based on Aristotle's discussion of hot and cold as active factors in change; and moist and dry as passive factors. The definition is found in *Meteorologica* IV.1.378b.32—379a.1. If the edition of Themon's text used here is complete, then he chose to terminate his treatise after drawing questions from chapters 1 to 3 only, ignoring chapters 4 to 12.

Physics

Statics, or "The Science of Weights"

INTRODUCTION[1]

Edward Grant

On the basis of extant manuscripts, three types of treatises must be distinguished in the history of medieval statics.

1. Those translated into Latin from Greek or Arabic and originating largely in late antiquity in Alexandria. Included are the *De ponderoso et levi* ascribed to Euclid *(The Book of Euclid Concerning the Heavy and Light)*, the *De insidentibus in humidum* ascribed to Archimedes *(The Book of Archimedes on Floating Bodies)*, the anonymous *Liber de canonio (The Book Concerning the Balance)*, and the *Liber Karastonis (The Book of the Roman Balance)* edited by Thabit ibn Qurra in the ninth century.

2. A group of treatises ascribed to, and some of which were written by, Jordanus de Nemore in the thirteenth century. Jordanus undoubtedly wrote the *Elementa Jordani super demonstrationem ponderum (The Elements of Jordanus on the Demonstration of Weights)* and perhaps the most significant of the treatises ascribed to him, *Liber Jordani de Nemore de ratione ponderis (The Book of Jordanus de Nemore on the Theory of Weight)*.[2] A third treatise in this group, *Liber Jordani de ponderibus (The Book of Jordanus on Weights)*, is not by Jordanus.

3. A third group of treatises, written, for the most part, by fourteenth-century authors, were composed as commentaries on, or revisions of, the treatises in the second group.[3]

In the treatise represented by our selection, we find some of the most significant contributions to medieval and early modern statics. Combining the purely Archimedean geometrical tradition in statics with the Aristotelian dynamical tradition derived ultimately from the Pseudo-Aristotelian *Mechanical Problems*, Jordanus and his followers developed not only the form of the principle of virtual displacements but made other conceptual advances as well. Here is Clagett's summation[4] of these medieval contributions:

> Heir of Hellenistic mechanics, the medieval mechanician sharpened the concept of virtual displacements, which had had its origin in the Pseudo-Aristotelian treatise *Mechanica*. The form of the principle of virtual displacements used by the thirteenth-century mathematician Jordanus and expressed by one of his thirteenth- or fourteenth-century commentators (in the so-called *Aliud commentum* published in 1533 by Peter Apianus [ed. and trans. J. E. Brown; see n. 2—*Ed.*]) was essentially of this form:
>
> 5. *What suffices to lift a weight W through a vertical distance H will lift a weight kW through a vertical distance H/k or a weight W/k through a vertical distance kH.*
>
> This principle was employed by Jordanus to prove the law of the lever as applied to both a straight and bent lever. It was also used in the elegant proof of the proposition concerning the equilibrium of inter-connected weights on oppositely inclined planes whose lengths vary directly as the weights. Furthermore, in the proposition concerning the bent lever, what is essentially the concept of static moment was conceived:
>
> 6. *The effective force of a weight on any lever arm, straight or bent, depends on both its weight and its horizontal distance from the vertical line passing through the fulcrum.*

1. The following introduction is drawn from the general introduction by E. A. Moody and M. Clagett to their volume *The Medieval Science of Weights (Scientia de ponderibus), Treatises Ascribed to Euclid, Archimedes, Thabit ibn Qurra, Jordanus de Nemore, and Blasius of Parma* with Introductions, English Translations, and Notes (Madison, Wis.: University of Wisconsin Press, 1952) and from M. Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Wis.: University of Wisconsin Press, 1959).

2. Joseph E. Brown inclines to the view that a pupil of Jordanus was the author of this lengthy treatise, which comprised forty-five propositions. See his *"The Scientia de ponderibus in the Later Middle Ages,"* Ph. D. dissertation, University of Wisconsin, 1967, pp. 64–87. For convenience, we shall ascribe the treatise to Jordanus.

3. A few of the important ones have been edited and translated by Joseph Brown in the work cited in note 2.

4. *The Science of Mechanics in the Middle Ages*, pp. 675–676.

Still another fertile statical concept was used by the author of the *De ratione ponderis*. This was the concept of positional gravity used to determine the component of natural gravity acting along an inclined plane. This concept can be expressed as follows:

7. *The ratio of the effective weight (positional gravity) of a body in the direction of the inclined plane on which it rests to the free natural gravity is equal to the ratio of the vertical component of any given potential trajectory along that plane to that trajectory.*

This concept is equivalent to the modern formulation $F = W \sin a$, where F is the force along the plane, W is the free weight and a is the angle of inclination of the plane. It was employed by the medieval author to establish the law of equilibrium of interconnected weights on oppositely inclined planes. This concept and its use marked an important step in the rise of a vectorial analysis of forces.

Indeed, the medieval achievement in statics is best measured in terms of the four principles of statics distinguished by Ernst Mach in his *Science of Mechanics* and deemed essential to that discipline:

(1) The general lever principle; (2) the inclined plane principle; (3) the principle of composition of forces; and (4) the principle of virtual displacements. The statics of Archimedes, which steered clear of all entanglements with dynamics, attained only the first of these principles; the other three, involving dynamical considerations, characterize modern statics. In the treatises ascribed to Jordanus de Nemore, however, these last three principles appear, and provide a dynamical foundation for the proof of the lever principle.[5]

39 ON THE THEORY OF WEIGHT

Jordanus of Nemore (fl. 1230–1260)

Translated and annotated by Ernest A. Moody[1]

PART 1

Postulates[2]

R1.001. The movement of every weight[3] is toward the center (of the world), and its force is a power of tending downward and of resisting movement in the contrary direction.

5. Moody and Clagett, *The Medieval Science of Weights*, p. 15.

1. [This is a translation of the *De ratione ponderis* ascribed to Jordanus of Nemore (see the introduction, n.2), and edited with introduction, translation, and notes by Ernest A. Moody with the assistance of Raymond Clements, Arthur Ditzel, and Jason Lewis Saunders. It is reprinted with permission of the Regents of the University of Wisconsin from *The Medieval Science of Weights (Scientia de ponderibus), Treatises Ascribed to Euclid, Archimedes, Thabit ibn Qurra, Jordanus de Nemore, and Blasius of Parma*, edited with Introductions, English Translations, and notes by E. A. Moody and Marshall Clagett (Madison, Wis.: University of Wisconsin Press, 1952). The notes, which were cued to line numbers in the Latin text and placed at the end of the book, have been rearranged and assigned as regular footnotes. In what follows, Moody uses the letter *E* to represent *The Elements of Jordanus On the Demonstration of Weights (Elementa Jordani super demonstrationem ponderum)*, a treatise containing nine propositions and undoubtedly written by Jordanus. The present treatise, *De ratione ponderis*, referred to by the letter *R*, may have been written by Jordanus himself or a pupil (see the introduction, note 2). It is divided into four parts containing a total of forty-five propositions. Although based on *E*, the *De ratione ponderis* corrects mistakes in *E* and adds much that is original. It is perhaps the finest of the medieval statical treatises and was published in 1565 at Venice from a manuscript once owned by Niccolo Tartaglia.—*Ed.*]

2. The seven postulates are the same as those of the *Elementa Jordani*, and our comments on the latter hold equally for these. [These comments are referenced to the postulates to which they refer.—*Ed.*]

3. The terminology introduced in the Postulates requires a few comments. *Pondus*, which we have translated by "weight," is to be understood in the substantival sense, as that which *has* a natural tendency to move toward the center of the world. This tendency itself is called the *virtus, gravitas*, or *ponderositas* of the heavy body *(grave)* or of the weight *(pondus)*. Thus *pondus* is treated as a scalar quantity, analogous to our notion of mass, while *virtus* or *gravitas* is a vector quantity, analogous to our notion of force. The simple gravitational *virtus* is assumed to be proportional to the scalar weight or *pondus*, just as in Newtonian physics the force of gravity acting on a body in a given region is proportional to its mass. The additional clause, added in the first Postulate by our manuscript *C*, and by most of the manuscripts of the *De ratione ponderis* of Jordanus, makes this plain: *virtutemque ipsius potentiam ad inferiora tendendi et motui contrario resistendi*. This also

R1.002. That which is heavier descends more quickly.

R1.003. It is heavier in descending, to the degree that its movement toward the center (of the world) is more direct.[4]

R1.004. It is heavier in position when in that position its path of descent is less oblique.[5]

R1.005. A more oblique descent is one which, in the same distance, partakes less of the vertical.[6]

R1.006. One weight is less heavy in position, than another, if it is caused to ascend by the descent of the other.[7]

R1.007. The position of equality is that of equality of angles to the vertical, or such that these are right angles, or such that the beam is parallel to the plane of the horizon.[8]

Theorems

R1.01. Between any heavy bodies, the forces are directly proportional to the weights.

indicates that the same *virtus* which measures a body's downward directed force, in falling, measures its resistance to any force acting to lift it vertically upward. This is a basic principle in the consideration of weights in equilibrium, and it is invoked in Postulate *E.06* and in the second part of Theorem *E.1*.

The "heaviness" of a weight, or its *gravitas* or *ponderositas*, is equal to its *virtus* only insofar as it is directed vertically downward. If a weight descends obliquely, as along an inclined plane, its "heaviness" in descending along the oblique path (or the force which it exerts along the oblique path) will be less than its natural *virtus* as directed toward the center of the world. This variable "heaviness," or force exerted by a body in descending along a path other than the vertical, is called *gravitas in descendendo*.

4. The second and third Postulates state that the "heaviness" of a body in its descent, along whatever path it descends, is proportional to the speed with which it descends along that path, and both its heaviness and its speed depend on the degree to which its path of descent approaches the vertical. Postulate *E.02* might be given a more general interpretation, as stating that a body of greater natural weight *(pondus)* will fall in free vertical descent faster than a body of smaller natural weight. But in the Aristotelian context, even this thesis would be understood in terms of a descent against a resistance—either a resistant medium, or a resistance such as that of a counterweight on the other arm of a balance, which will be raised more rapidly by a larger weight on the descending arm, than by a smaller weight. Postulates *E.02* and *E.03* seem, however, to be concerned with the restricted case of variability of the force and speed with which a given weight descends along paths which deviate from the vertical—in our language, the concern is with the component of gravity effective in producing motion along a path compatible with a constraint system such as a balance.

5. Postulate *E.04* introduces the expression *gravitas secundum situm*, or "positional gravity." This plays a major part in the statical theorems which follow. Whereas "gravity in descending" is measured by the obliquity of the path *actually* traversed in the movement of a heavy body along a path compatible with the constraints, "positional gravity" is defined by a merely *virtual* movement along a possible path of descent, in a constraint system which is in equilibrium. In modern language, "positional gravity" is a component in a system of forces whose vector sum is zero. It is measured by the work which would be done if the system permitted the element in question to undergo motion independently of the rest of the system; but as taken in the constraint system, in equilibrium, it actually does no work. The measure of "positional gravity" of a body, relative to its natural gravity or *virtus*, is determined by the ratio of the obliquity of the path it would traverse in a "virtual displacement" of the system, to the vertical path along which its natural gravity is directed.

6. Postulate *E.05* defines "obliquity" as the deviation of a path of descent from the vertical, or as its approach to the horizontal. The contrary of "more oblique" is "more direct," so that the positional gravity varies inversely with the degree of obliquity of the path of virtual movement. Jordanus' definition is precise: the descent is more oblique if the path of descent has a smaller component of the vertical. In Proposition *R1.10* of the *De ratione ponderis* (see below) this notion is explicitly explained, for the case of a rectilinear descent along an inclined plane. Thus, taking an inclined plane as path of possible descent, we project a given length of the inclined plane onto a vertical line, forming a right-angled triangle. Then the ratio of the altitude of this triangle to its hypothenuse varies inversely with the obliquity, or directly with the "positional gravity." The positional gravity, or component of force directed along the inclined plane, is therefore measured by $\sin a \cdot pondus$, when a is the angle of hypothenuse to base.

7. Postulate *E.06* states that one body has less positional gravity than another, if the descent of the other suffices to cause it to ascend. Obviously it is assumed that the two bodies are connected by a balance beam, or by a pulley arrangement. It is not wholly clear whether the author is also assuming equality of the two weights compared, or whether he is taking into consideration the case where the weights are unequal in their natural gravity or mass. On the first assumption, the postulate would state that if two equal weights are attached so that the descent of one involves the ascent of the other, then if one of them is able to descend and to raise the other, it possesses this advantage by reason of position—i.e., because its path of possible descent, within the constraint system, is less oblique. But if the postulate is understood to hold for unequal weights also, then it asserts that if the descent of one weight causes the ascent of the other, the product of the *pondus* and its "positional" coefficient, in the case of the weight which descends, is greater than the product of the *pondus* and positional coefficient of the other weight.

8. Postulate *E.07* defines the "position of equality," obviously referring to a balance beam capable of rotation on an axis of support. The *situs equalitatis* [that is, position of equality—*Ed.*] is the horizontal position of the beam, such that its arms form equal (and right) angles with the perpendicular to the plane of the horizon passing through the axis of support.

Let there be two weights, *ab* and *c*, of which *c* is the lighter [Fig. 1]. And let *ab* descend to D, and let

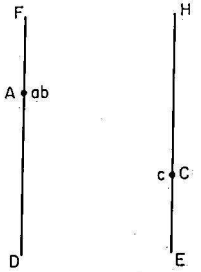

Fig. 1

c descend to E. Again, let *ab* be raised to F, and *c* raised to H. I then say that the distance AD is to the distance CE, as the weight *ab* is to the weight *c;* for the velocity of descending is as great as the force of the weight. But the force of the combined weight consists of the forces of its components. Let the weight *a* then be equal to the weight *c*, so that *a*'s force is the same as that of *c*. If then the ratio of the weight *ab* to the weight *c* is less than the ratio of the force of *ab* to the force of *c*, the ratio of the weight *ab* to (its component weight) *a* will likewise be less than the ratio of the force of *ab* to the force of *a*. And therefore the ratio of the force of *ab* to that of *b* will likewise be less than the ratio of the weight *ab* to the weight *b*. Consequently the ratio of the same weights will be both greater and less than the ratio of their forces. Since this is absurd, the proportion must be the same in both cases. Hence the weight *ab* is to the weight *c*, as the distance AD is to the distance CE, and conversely as the distance CH is to the distance AF.[9]

RI.02. When the beam of a balance of equal arms is in the horizontal position, then, if equal weights are suspended from its extremities, it will not leave the horizontal position; and if it is moved from the horizontal position, it will revert to it. But if unequal weights are suspended, the balance will fall on the side of the heavier weight until it reaches the vertical position.

A balance is said to be of equal arms, when the arms of the beam, measured from the axis of rotation, are equal. Let the axis, then, be A, and the beam BAC [Fig. 2] and let the suspended weights be *b* and *c*, and let FA be the line of the vertical. If then we describe a circle through B and C, the mid

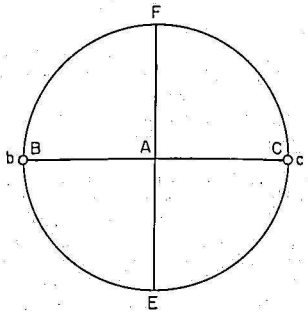

Fig. 2

9. [Since this theorem and its diagram are very similar to Theorem *E.1* and its diagram, Moody's remarks on *E.1* (*The Medieval Science of Weights*, p. 376) in the first paragraph are relevant; what follows is quoted from his discussion of *RI.01* on pages 390–391.—*Ed.*]:

"At first sight the theorem appears to be an assertion that unequal weights, in free fall, descend with velocities proportional to their weight. The diagram normally supplied on the margins of the manuscripts, and which we have reproduced, seems to support such an interpretation. Yet the use of the term 'velocity of descent' [in *RI.01, virtus* is substituted for *velocitas* and the equivalent expression is "forces of descent"—*Ed.*], which in postulates *E.02* and *E.03* [these are equivalent to *RI.002* and *RI.003* of the selection reprinted here—*Ed.*] is associated with direction of descent and with "heaviness" directed along an oblique path, suggests that the author is thinking of movements along oblique paths, as between bodies in a connected system, and their "gravities" in the relative sense determined by the paths of movement compatible with the system."

. . . The entire theorem bears on the relative capacities of unequal weights to lift a given counterweight on the other arm of the balance, or to resist being lifted by the given counterweight. Only as so interpreted is the theorem of value in establishing the general lever principle for which it is invoked here (in *RI.06*) just as it was invoked in E.8 of the *Elementa Jordani*. On this interpretation, the theorem is an enunciation of the principle of work, in this form: If a weight *w*, descending through a vertical distance *d*, can lift a counterweight *x* through a certain vertical height *h*, then another weight k·*w*, by descending the vertical distance *d*/k, can raise the same counterweight *x* the same height *h*; and also, another weight *w*/k can raise *x* through the height *h* by descending the vertical distance k·*d*. The second part of the theorem will then assert that if *w*, descending the vertical distance *d*, can raise *x* through the height *h*, then k·*w*, descending the same distance *d*, can raise k·*x* the same height *h*, or it can raise *x* the height k·*h*. The three crucial theorems of Book I of the *De ratione ponderis* (*RI.06* on the general lever principle, *RI.08* on the bent lever, and *RI.10* on the inclined plane) rest on this principle of work; and at least in the case of *RI.06* the

point of its lower half being E, it is evident that the descent of both *b* and *c* will be along the circumference of the circle, toward E. And since the descents along these paths are of equal obliquity, and since *b* and *c* are equal weights, therefore neither of them will move.

Let it now be supposed that the balance is pushed down on the side of *b*, and elevated correspondingly on the side of *c* [Fig. 3]. I say that it will revert to

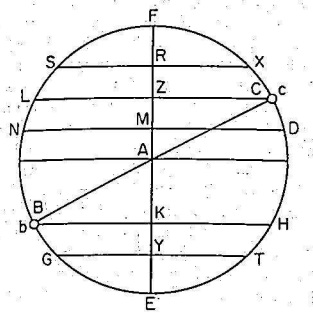

Fig. 3

the horizontal position. For the descent from C toward the horizontal position is less oblique than the descent from B toward E. For let there be taken equal arcs, as small as you please, which we will call DC and BG; and let the lines CZL and DMN, and also BKH and GYT, be drawn parallel to the horizontal. And let fall, vertically, the diameter FRZMAKYE. Then ZM will be greater than KY, because if an arc CX, equal to CD, is taken in the direction of F, and if the line XRS is drawn horizontally, then RZ will be smaller than ZM; and since RZ is equal to KY, ZM will be greater than KY. Since therefore any arc you please, which is beneath C, has a greater component of the vertical than an arc equal to it which is taken beneath B, the descent from C is more direct than the descent from B; and hence *c* will be heavier in its more elevated position, than *b*. Therefore it will revert to the horizontal position.[10]

proof is explicitly referred to the principle that the heights through which weights can be lifted by the same counterweight are in inverse ratio to their weights, as a principle which had been established *(ut ostensum est)*. Now it is only this first theorem that could possibly be construed as having established this principle; consequently it seems proper to interpret the first theorem in this sense.

The method of proof here employed is similar to that used in *E.1*, depending on Euclid V, Prop. 30; i.e., if $(a + b)/a < (AE + ED)/AE$, then $(a + b)/b > (AE + ED)/ED$. Here a + b is the greater of the two weights, and its part *a* is assumed to be the same as the lesser weight *c*, while the distance of a + b's descent is assumed to be divisible proportionately to its components *a* and *b* in such manner that the distance AE corresponding to *a*'s part of the descent is equal to the whole descent of the lesser weight *c*, and the distance ED represents *b*'s part in the whole descent of a + b.

10. Theorem *RI.02:* The first two parts of this theorem correspond exactly to *E.2*, the second part committing the same error in assuming stable equilibrium to obtain. [Moody's remarks on the two parts of *E.2* (*The Medieval Science of Weights*, pp. 378–379) follow.—*Ed.*]

"This theorem, like the first, gave rise to repeated discussion and criticism in the later Middle Ages and Renaissance. It is discussed by Roger Bacon in his *Opus Maius* (Part 4, Dist. 3, Ch. 16), a work written between 1266 and 1268. The theorem has two parts, the first of which seeks to demonstrate, by means of the concept of positional gravity, what Archimedes had laid down as his initial postulate—that equal weights on equal lever arms are in equilibrium. Since it was shown in *E.1* that movement (or 'velocity of descent'), as between weights on a balance, is determined either by inequality of *pondera* or by inequality of positional gravities, or by both together, and since the present theorem assumes equality of the *pondera*, the condition determining equilibrium reduces to that of equality of positional gravities—i.e., of the 'obliquities' of virtual descents of the two weights. The first part of the theorem shows that when the balance beam is in horizontal position, equal arcs cut off just below the horizontal diameter of the circumference of rotation, will have equal components of the vertical; hence the obliquities of virtual descent, for the two weights, will be the same.

"The second part of the theorem, which is erroneous, is what gave rise to the criticisms discussed by Bacon and later writers. Jordanus here attempts to show that the equilibrium of equal weights on equal lever arms is a case of stable equilibrium. The attempted proof involves an erroneous application of the concept of positional gravity. Jordanus first shows that a small finite arc cut off just below the position of the weight on the lever arm that is elevated above the horizontal has a greater component of the vertical than an equal arc cut off just below the position of the weight on the depressed arm. From this he concludes that the elevated weight has greater positional gravity than the depressed weight, so that the balance beam will return to the horizontal position. His error is to compare the positional gravities of the two weights *b* and *c* directly, according to the ratios of vertical component to arc of *descent* in both cases—as if the descent of *b* could be treated independently of the ascent of *c*. He should have compared the ratio of the obliquity of a small descent of *b* to the obliquity of an equal *ascent* of *c*, with the ratio of the obliquity of an equal small descent of *c* to the obliquity of a corresponding ascent of *b*; he would then have seen that *these* ratios would remain equal under any displacement one way or the other, so that no advantage would be gained by either side through any displacement. While Jordanus could

Now let *b* be heavier than *c*, and let the balance be in the horizontal position. Then, since the descent on each side is of equal obliquity, it is evident that *b* will descend. For let *b* be placed in any position you please, below, with *c* above [Fig. 4]. I say that in this position *b* will still be heavier.

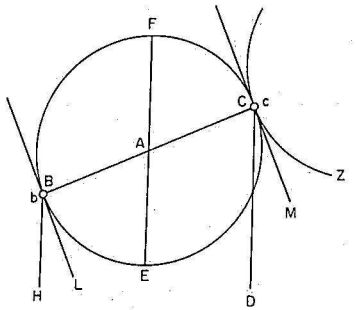

Fig. 4

For let the vertical lines CD and BH be drawn; and let the lines BL and CM be tangents to the circle; and let the arc CZ be drawn, similar and equal to the arc BE, and similarly placed, so that the line CM is tangent to it as well as to the circle. Then the obliquity of the arcs BE and CZ is measured by the angle DCZ, and the obliquity of the arc CE by the angle DCM; and the proportion of the angle DCZ to the angle DCM is smaller than any ratio that can be assigned between a greater and a lesser quantity. Therefore it will also be less than the ratio of the weight *b* to the weight *c*. Since then *b* exceeds *c* to a greater extent than the obliquity exceeds the obliquity, *b* will be heavier in this position than *c*. For this reason *b* will not cease to descend, and *c* to ascend, until the beam is in the position FE.[11]

R1.06. If the arms of a balance are proportional to the weights suspended, in such manner that the heavier weight is suspended from the shorter arm, the weights will have equal positional gravity.

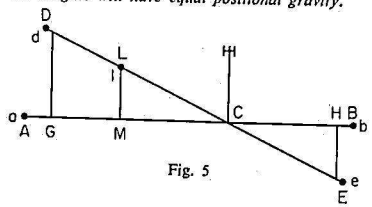

Fig. 5

Let the balance beam be ACB, as before, and the suspended weights *a* and *b* [Fig. 5] and let the ratio of *b* to *a* be as the ratio of AC to BC. I say that the balance will not move in either direction. For let it be supposed that it descends on the side of B; and let the line DCE be drawn obliquely to the position of ACB. If then the weight *d*, equal to *a*, and the weight *e* equal to *b*, are suspended, and if the line DG is drawn vertically downward and the line EH vertically upward, it is evident that the triangles DCG and ECH are similar, so that the proportion of DC to CE is the same as that of DG to EH. But DC is to CE as *b* is to *a;* therefore DG is to EH as *b* is to *a*. Then suppose CL to be equal to CB and to CE, and let *l* be equal in weight to *b;* and draw the perpendicular LM. Since then LM and EH are shown to be equal, DG will be to LM as *b* is to *a*, and as *l* is to *a*. But, as has been shown, *a* and *l* are inversely proportional to their contrary (upward) motions. Therefore, what suffices to lift *a* to D, will suffice to lift *l* through the distance LM. Since therefore *l* and *b* are equal, and LC is equal to CB, *l* is not lifted by *b;* and consequently *a* will not be lifted by *b*, which is what is to be proved.[12]

have avoided his error within the principles and methods of his own theory, it is probable that the complexity introduced into the problem, by the constantly changing obliquities involved in displacements along the circular path, led him to misapply his own principles in this case."

11. The third part of *R1.02*, not found in *E.2*, shows that if one of the weights is the least bit heavier than the other, its finite excess of natural weight will offset the infinitesimal advantage due to difference in the obliquities of virtual descent, so that the arm on which the heavier weight hangs will continue to fall until it is vertically beneath the axis of support. The argument is of interest because of its glimpse of the infinitesimal aspect of the concept of a virtual displacement. But Duhem's claim (*Les Origines de la Statique*, I, p. 140) that this constitutes "une refutation concluante" of the error committed in the second part of the proof, is untenable; for part of the argument in this third section is the assumption that there is an advantage in positional gravity on the side of the weight which is above the horizontal position, which *needs* to be offset by some finite addition to the weight on the other arm in order to keep the balance from reverting to the horizontal.

12. Theorem *R1.06:* This states the general lever principle, and is in all respects identical with *E. 8*. The principle of work, according to Duhem, underlies the proof; and if this is so, we must assume that the first theorem *(R1.01)*, here as in the *Elementa*, is Jordanus' statement of the principle of work. [What follows is Moody's comment on *E.8* (*Medieval Science of Weights*, pp. 382–383).—*Ed.*]

"Theorem *E.8:* This is the crucial theorem—the

R1.08. If the arms of a balance are unequal, and form an angle at the axis of support, then, if their ends are equidistant from the vertical line passing through the axis of support, equal weights suspended from them will, as so placed, be of equal heaviness.

Let the axis be C, the longer arm AC, and the shorter arm BC [Fig. 6]. And draw the vertical line

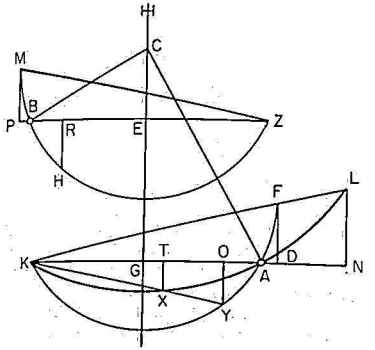

Fig. 6

CEG; and let the lines AG and BE, perpendicular to this vertical, be equal. When, therefore, equal weights are suspended at A and B, they will not change from this position. For let AG and BE be extended by a distance equal to their own length, to K and to Z; and on them let the arcs of circles, MBHZ and KXAL, be drawn; and let the arc KYAF be made similar and equal to arc MBHZ;[13] and let the arcs AX and AL be equal to each other, and similar to the arcs MB and BH. And let the arcs AY and AF also be equal and similar. If then *a* is heavier in this position than *b*, let it be supposed that *a* descends to X and that *b* is raised to M. Then draw the lines ZM, KXY, KFL; and let MP be erected perpendicularly on ZBP, and XT and FD on KAD. And because MP is equal to FD, while FD is greater than XT—on account of similar triangles—, MP will also be greater than XT. Hence *b* will be lifted vertically more than *a* will descend vertically, which is impossible since they are of equal weight. Again, let it be supposed that *b* descends to H and lifts *a* to L; and let HR fall perpendicularly on BZ, and LN and YO on KAN. Then LN will be greater than YO, and consequently greater than HR; so that in the same way the impossible will result.

general principle of the lever. The method of proof used by Jordanus is wholly different from that of Archimedes, though it is perhaps vaguely implicit in the *Mechanical Problems* ascribed to Aristotle (850a 30—b 10), and also in the Third Proposition of Thabit ibn Qurra's *Liber Karastonis*. But whereas Aristotle, and Thabit as well, had argued that the lesser weight on the longer arm balances the greater weight on the shorter arm, because it would traverse a proportionately greater distance *along the arc* of its movement, Jordanus states that the comparison is determined according to the lengths of *vertical descent or ascent* accomplished in the movements along the curvilinear path. As Duhem pointed out, Jordanus' proof rests on the principle of work, in this form: What can lift a weight W through a height H, can lift a weight W/*k* through a height *k*·H, or a weight *k*·W through a height H/*k*. Duhem held, that this principle was 'implicitly' invoked by Jordanus in his proof; and B. Ginzberg, arguing against Duhem, made much of the fact that a man who fails to make his principle explicit is not likely to be very conscious of the fact that his proof depends on that principle. But neither Duhem nor Ginzberg seems to have noticed that Jordanus *does* invoke, as principle of his demonstration, an explicit theorem—namely, the second part of his first theorem *E.1*. This stated that the 'velocities of descent and of ascent' are inversely proportional to the weights. Hence the whole problem of the interpretation of Jordanus' proof of the general lever principle, given here in *E.8*, depends on the meaning to be given to *E.1*. This is clearly seen by the 14th-century commentator whose text was edited by Petrus Apianus, and whose discussion of *E.1* we have printed and translated in our Appendix III. He points out that Jordanus invoked the second part of *E.1* in his proof of *E.8*, and states that *E.1* would only be of value for the proof of *E.8* on condition of being interpreted as a statement that two unequal weights, so placed on a balance that their vertical descents for equal degrees of rotation of the balance on its axis are unequal in inverse proportion, have equal power to sustain or to lift a weight on the other arm of the balance. This interpretation would make *E.1* an explicit statement of the principle of work on which *E.8* depends.

"The whole problem of 'explicit interpretation,' however, is undecidable in an historical sense; for the differentiations which we make among alternative possibilities, through the distinctions developed in Newtonian mechanics, were certainly not made in the thirteenth century. Hence, though we must concede that many of the Newtonian principles were included 'virtually' in the general conception of *potentia motiva* as a function of time, distance, and resistance, the different relationships among these factors such as are expressed by the set of sharply differentiated formulas of Newtonian mechanics, were not distinguished and separated in the thirteenth century, but were included in the general conception in undifferentiated manner. Nevertheless, Jordanus' use of his general dynamic principles, for demonstrating the special theorem of unequal weights on unequal lever arms, contributed to the clarification of the dynamical analysis by determining one explicit sense for the concept of work, as involved in the general lever principle."

13. [I have added the material between the semicolons, which was left untranslated.—*Ed.*]

To make this more evident, let us draw a different figure, as follows [Fig. 7]. Let there be a vertical

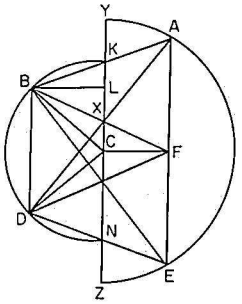

Fig. 7

line YKCNZ, and around the center C let there be drawn two semicircles, YAEZ and KBDN; and let the lines AFE and BD be drawn at equal distances from the diameter, and from these let there be drawn the equal perpendiculars BL and CF. Then draw the lines CB, CA, CE, and CD. If we then suppose that equal weights are suspended at A, B, D, E, and F, they will be of equal positional gravity. For if the lines BA, BXF, BE, DA, DF, and DE are drawn, all of them will be bisected by the diameter—as for instance BXF. For since BL and CF are equal, and the triangles BLX and CFX are similar, BX will be equal to XF. And in the same manner the others will be divided at their mid points. Therefore, since the mid points of all the lines are placed at a common center, so likewise the weights are placed; therefore they will be of equal heaviness.

A more subtle variant may, however, be added, if we suppose that *a* is heavier than *b*, *b* heavier than *f*, *f* heavier than *d*, and *d* heavier than *e*. Yet *d* is not able to lift *e;* for the segment of the line DE on the side of E would immediately become greater. But if *a* is given an impulse downward, it is able to raise *b;* and similarly *b* can raise *a;* and *d* can raise *a* and *a* can raise *d;* and *b* can raise *f* and *f* can raise *b;* until they make a complete revolution, and hang in such manner that the angle with the axis is beneath them. For when *b* is moved downward, the segment of the line BA, on the side of B, will become steadily longer, and *b* will become heavier. [14]

R1.09. Equality of the declination conserves the identity of the weight.

Only on a rectilinear path is equality of declination conserved. Let this path be on the line AB, and let the line AC descend vertically [Fig. 8].

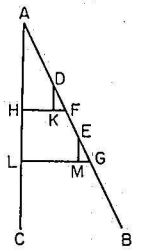

Fig. 8

And let two points, D and E, be assigned on AB. Any weight you please, then, whether it descends from D, or from E, will retain the same heaviness. For equal segments of AB, taken beneath D and E, will have equal components of the vertical. This is shown, if we draw perpendiculars to the line AC, from these points, namely FH and GL, and if we let the lines DK and EM fall perpendicularly on them. Thus, whether a weight is extended along AB, or placed there all at once, it will be of the same heaviness.[15]

14. Theorem *R1.08:* There is no antecedent in the *Elementa Jordani* for this theorem on the bent lever, except that the two erroneous theorems *E.6* and *E.7* involved the problem of equilibrium in a bent lever. The proof given here is rather intricate in its geometrical development, as can be seen by a glance at the diagram. But the proof is valid, and the principle which it invokes is clear enough—namely, that the reason why a balance of bent arms will be in stable equilibrium where the weights are equidistant from the vertical passing through the axis of support, is because in any displacement from this position a weight would be raised some vertical distance, by an equal weight descending less than that distance. Thus the principle of work underlies this demonstration, just as it does in the proofs of the general lever principle *(R1.06)* and of the inclined plane theorem *(R1.10)*.

15. Theorems *R1.09* and *R1.10:* These two theorems, not found in the *Elementa Jordani* or in any of the antecedent literature, Greek or mediaeval, present the first correct statement and proof of the condition of equilibrium of unequal weights on diversely inclined planes, in the history of mechanics. The demonstration is clear and elegant, and requires no comment in explication of the reasoning. The basis of the proof is indicated, in the last two lines, as the principle of work—namely, a force which would suffice to lift the weight *g* through the vertical height ZN, could lift the weight *h* through the height XM, where $g : h = XM : ZN$, or, equivalently,

R1.10. If two weights descend along diversely inclined planes, then, if the inclinations are directly proportional to the weights, they will be of equal force in descending.

Let there be a line ABC parallel to the horizon, and let BD be erected vertically on it [Fig. 9] and

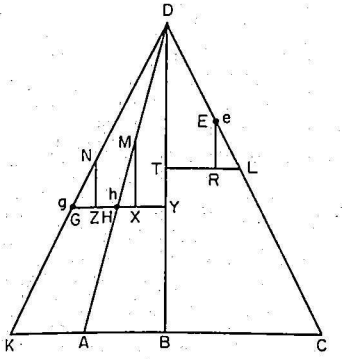

Fig. 9

from D draw the lines DA and DC, with DC of greater obliquity. I then mean by proportion of inclinations, not the ratio of the angles, but of the lines taken to where a horizontal line cuts off an equal segment of the vertical. Let the weight *e*, then, be on DC, and the weight *h* on DA; and let *e* be to *h* as DC is to DA. I say that those weights are of the same force in this position. For let DK be a line of the same obliquity as DC, and let there be on it a weight *g*, equal to *e*. If then it is possible, suppose that *e* descends to L, and draws *h* up to M. And let GN be equal to HM, which in turn is equal to EL. Then let a perpendicular on DB be drawn from G to H, which will be GHY; and another from L, which will be TL. Then, on GHY, erect the perpendiculars NZ and MX; and on LT, erect the perpendicular ER. Since then the proportion of NZ to NG is as that of DY to DG, and hence as that of DB to DK, and since likewise MX is to MH as DB is to DA, MX will be to NZ as DK is to DA—that is, as the weight *g* is to the weight *h*. But because *e* does not suffice to lift *g* to N, it will not suffice to lift *h* to M. Therefore they will remain as they are.[15]

Here ends the first part.

PART 2

R2.01. When there is a balance beam of uniform weight and thickness throughout, and its weight is known, if it is divided into unequal segments and if a body of known weight, suspended from the shorter arm, holds the beam in equilibrium, then the lengths of the arms on each side of the axis of rotation will also be determined.

Let the beam be ABC, of a given weight and of uniform thickness [Fig. 10]. Let a body, *d*, of

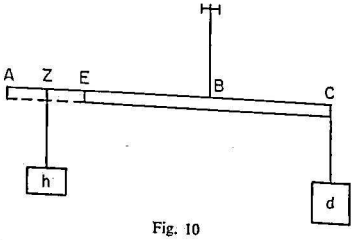

Fig. 10

known weight, hang from the end C, and let BE be equal to BC. From the mid point of AE, designated as Z, let there be suspended a body, *h*, equal in weight to the segment of the beam AE;

where the products of the weights by the heights through which they are raised, are equal. In this way the theorem shows that the force required to sustain a body on an inclined plane (i.e., the force directed upward along the plane) is measured by the weight of the body sustained, multiplied by the ratio of the altitude to the length of incline corresponding to it. And this is of course our modern formula.

What is essential, and apparently original, in Jordanus' whole treatment of the statical problems, is his recognition that it is the *vertical* descent or ascent of a weight, in relation to the length of oblique trajectory, which is to be taken into account. It is this which differentiates Jordanus' definition of "positional gravity" from that offered by the author of the so-called "Peripatetic commentary" (our version "P"), who seems to associate positional gravity with degree of curvature of the trajectory, or with the ratio of vertical to *horizontal* displacement. In the case of the curved trajectory of a weight on a balance arm, Jordanus sees that it is necessary to determine the positional gravity at each point of the curve by the ratio of the tangent to its projection on the vertical; since this is, in effect, a reduction of the more difficult problem of the curved trajectory, to the simpler case of the inclined plane, it seems likely that the inclined plane formulation was present to Jordanus' thought from the start even though it was only in his later work that he introduced a separate theorem in this formulation.

For the history of the attempts to solve the inclined plane problem, from Pappus to Descartes, see Pierre Duhem, *Les Origines de la Statique* [2 vols.; Paris: Hermann, 1905–1906], I, 27–29, 49–51, 182–193, 257–258, 272–279, 303–311, 315–316, and 329–332.

and in this position it will also be of equal heaviness. Since therefore h and d are equally heavy in this position, the proportion of d to h will be that of ZB to BC. And by alternation, the proportion of d to ZB will be that of AE (i.e., of h) to BC. And by composition, the proportion of d plus twice ZB (i.e., AC) to ZB, will be that of AE plus twice BC (i.e., EC) to BC. If therefore the whole weight ABC is multiplied by its half, and the product is divided by the sum of the weights of d and of AC—all these being given—, the weight of the segment BC is thereby determined.[16]

PART 3

R3.01. If the axis is on a perpendicular above the beam of the balance, then, however great a weight be suspended from either of the arms, it will not be possible for it to descend to a position directly below the axis.

Let the balance beam, for example, be ABC [Fig. 11], the perpendicular BDE, with the axis at

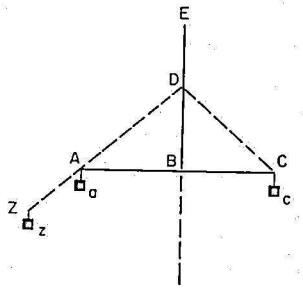

Fig. 11

D, and a greater weight a (at A) than the weight c (at C). Then draw the lines DA and DC, and let DA be extended to Z in such manner that DAZ is to DA as the weight a is to the weight c. Let there be a weight z (at Z) equal to c. When, therefore, the three weights a, c, and z are in this manner attached to the beam ABC, their revolution is around the axis D, just as if they were hanging on the lines DAZ and DC. But as so placed, z will tend to be at the same distance from the vertical passing through D, as c will; and a likewise will be at a proportional distance from the vertical. Therefore a will not be able to descend as far as the vertical.[17]

R3.02. When the ratio is given, of the distance from the axis to the mid point of the beam, to the length of the beam, and if the ratio of the suspended weights to the weight of the beam is given, then the angle of declination of the perpendicular to the beam, from the vertical, will be determined.

16. Theorem *R2.01*: As the diagram indicates, the argument supposes that the segment AE of the material balance beam is replaced by a weightless line from whose midpoint at Z there is hung a weight h equal to the weight of the material segment AE. In this way the problem of the Roman balance is reduced to that of the ideal balance to which the lever principle, as developed in *R1.06* [above], applies directly. The problem of this theorem is to determine the ratio of the shorter arm BC to the whole beam AC, when the weight of AC and the weight of d are given. Obviously, in dealing with the symmetrical balance beam of uniform material, divisions in its length can be treated as equivalent to divisons of its weight. We suppose that B is the point of division at which the axis of rotation is placed, such that the beam will be in equilibrium when d is suspended from the shorter arm BC. Then we mark off a point E, such that EB = BC. Since EB and BC, considered alone, are in equilibrium (since each part of BC has a corresponding equal part of EB at an equal distance from the axis), it follows that the weight of AE must be what balances the weight d. We then suppose that the weight of AE is replaced by a weight, h, equal to the weight of AE, but concentrated at the mid point of the ideal line AE. The proof then proceeds as follows:

$d : h = ZB : BC$ (by theorem *R1.06*).

Therefore, $d : ZB = h : BC$ (Euclid V, 12: "alternation").

But h = AE (in weight), by hypothesis; therefore $d : ZB = AE : BC$.

Dividing both sides by 2, $d : 2 \cdot ZB = AE : 2 \cdot BC$; but $2 \cdot ZB = AC$, and $2 \cdot BC = EC$; therefore $d : AC = AE : EC$.

But $d + AC : AC = AE + EC$: EC (Euclid V, 13; "composition").

Therefore $d + AC : (AC/2) = AE + EC : (EC/2)$; but $EC/2 = BC$; and $AE + EC = AC$.

Therefore $d + AC : (AC/2) = AC : BC$; therefore $BC = (AC \cdot \frac{1}{2}AC)/(d + AC)$.

Since AC and d are known, the value of BC is determined by this equation.

17. Theorem *R3.01*: This demonstration clearly presupposes the principle established in *R1.08* [above] of the first book—namely, that equal weights, suspended from unequal lever arms which form an angle at the axis of support, will be in equilibrium when the extremities of the two arms are equidistant from the vertical passing through the axis of support. Even more than *R1.08*, this theorem represents a correction of the error involved in *E.6* and *E.7* of the *Elementa Jordani*.

The balance beam is initially conceived as a straight beam which is attached rigidly to a rigid perpendicular rod extending upwards, this perpendicular being in turn supported at its upper end by a cord which permits it to swing freely from an axis at D. By drawing the lines DAZ and DC, Jordanus reduces the problem to that of the bent lever of unequal arms supporting equal weights, and invokes *R1.08* to show that a weight at Z, equal to the weight at C, would determine equilibrium

Let the rod which stands vertically be HDLZ, and the rest as before [Fig. 12]. And let the beam be

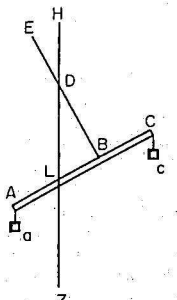

Fig. 12

depressed on the side of A until the line HDLZ cuts it at L. With the beam in this position, it is as if its axis were at L. Since then the suspended weights, and the weight of the balance beam, are given, the segments of the beam, AL and LC, are given. Hence the ratio of the length of each segment, to BD, will be given; and also the ratio of LB to BD. Consequently the angle LDB is determined, and it is the same as the angle EDH. But this is the angle of declination of the perpendicular to the beam, from the vertical.[18]

R3.03. If however the axis is beneath the beam, the weights can scarcely be stabilized in this position.

Let the beam, as before, be ABC, and the perpendicular to it DBE [Fig. 13]. And let E be the

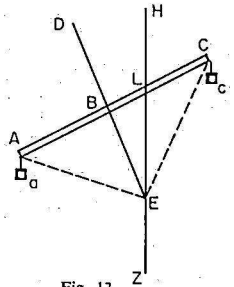

Fig. 13

axis beneath the balance, and let the weights be a and c. If we draw the lines EA and EC, the weights are so placed as if they were suspended on these lines. If they are in equilibrium in this position, then if any little tilt occurs in either direction, as for instance on the side of A, the segment of the beam from A to the vertical HLZ will increase, so that the beam will intersect the vertical at L. And this segment will thus become continuously heavier, until the beam rotates to a position beneath the axis E.[19]

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in such manner that the horizontal distance of Z from the vertical passing through D, would be the same as the horizontal distance of C from that vertical. Since, by construction, the ratio of DZ to DA is the same as the ratio of the weight a to the weight c, it follows that the distance from the vertical at which a will reach equilibrium will bear the same ratio to the distance from the vertical at which z will reach equilibrium, as the ratio of the weight c to the weight a. Hence, as long as there is some finite weight, however small, on the opposite side, the heavier weight, no matter how great, can never descend to a position vertically below the axis of support. And this is what was to be proved.

18. Theorem *R3.02*: This theorem, and the next, strongly reflect the passage in Aristotle's *Mechanical Problems* (850 a 2–29) dealing with stable and unstable equilibrium. When the axis of rotation is above the center of the straight balance beam, the equilibrium will be stable; if below, it will be unstable. The Aristotelian text involves some obscurities, leading Duhem to state that Jordanus corrected some implicit errors involved in the discussion of the *Mechanical Problems*. In any case, these two theorems constitute a correction by Jordanus of the obvious fallacy involved in the second part of *R1.02* and *E.2*, which had misapplied the theory of positional gravity in an attempt to show that equal weights on an equal armed balance beam (of no thickness) would be in stable equilibrium.

The argument of *R3.02* is as follows: Given the ratio DB : AC, and the ratio of the weights a and c to the total weight of the beam AC, it follows that since the balance is in equilibrium, $a : c = CL : AL$ (by *R1.06*, the general lever principle). But since the ratio of DB to AC (which is the same as AL + CL) is given, the ratios of DB to AL, and of DB to CL, are thereby given. But LB = (AC/2) − AL; and consequently the ratio of LB to DB is known. But this ratio LB : DB measures the angle LDB; and this angle is equal to the angle EDH, since they are vertical angles. Therefore the ratio LB : DB measures the declination of the perpendicular ED from the vertical line HZ.

19. Theorem *R3.03*: The proof here used is very similar to that given in the *Mechanical Problems* ascribed to Aristotle (850 a 20–29); i.e., when the material beam is supported from below, any movement of the beam involves an increase in the quantity of beam on one side of the vertical, and hence increases the weight on that side and decreases the weight on the other. Jordanus however uses the method of reduction to the case of weights supported by bent lever arms, considering the weight of the straight balance beam as if it were attached to the ends of the bent arms in such manner that the part of the beam on the left side of the vertical through the axis of support is supported by the left-hand bent lever arm, and the remainder by the right-hand arm.

R3.04. If the beam is in horizontal equilibrium, it is possible to suspend, from either of the arms, as great a weight as you please, in such a way that the balance beam will not depart from its horizontal position.

Let ABC be the beam, B its axis, and DBE a line designating the vertical [Fig. 14]. And suppose

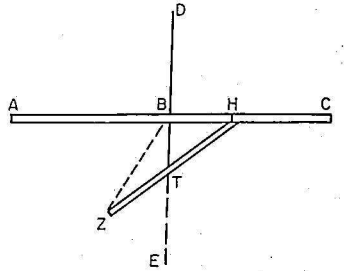

Fig. 14

the beam to be in horizontal equilibrium, by its own weight. Then take some other bar of equal thickness and weight, which we will call HTZ, T being its mid point. And take a segment of the beam, BH, on whichever arm is desired, such that it is of less length than HT. And attach this bar rigidly at H, in such manner that T is vertically beneath B, cutting the line of the vertical at T. I say then that this bar, so suspended, will not cause the beam to change position. For it is placed as if the line BZ were drawn, and as if it were hung on the line BH, in such manner that all its parts equidistant from T would be of equal heaviness. For they are at equal distance from the line of the vertical; hence ZT balances TH, and thus AB plus ZT is as heavy as BC plus TH. Therefore no movement will occur. But beyond this, if any weight desired be suspended from T, it will not cause any inclination.[20]

R3.05. If a heavy bar is raised from a horizontal to a vertical position, it is possible, given the height of the man raising it, to determine how heavily it will weigh on him in each position.

Let the heavy bar be AB [Fig. 15] and let its weight be uniformly distributed throughout its length. Then, the end B remaining in a fixed place, let it be raised at the end A until it reaches the vertical position CB, by its movement describing a

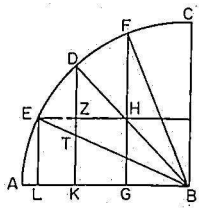

Fig. 15

quarter circle from A to C. And let the horizontal position be the first position, and the vertical position the last. And when the arc AC is divided into equal segments, let this position be called BD, or the middle position. And when the bar is raised to the height of the man lifting it, let this be BE, with the perpendicular EL representing the height of the man lifting it; and let this be the second position. Let the third position be at BF, with the arc FD equal to the arc DE.

I then say that the bar will become continuously lighter from A to E, and then continuously heavier as far as D; then however it will become lighter as far as F, and at F it will be of the same heaviness as at E; and then it will again become continuously lighter as far as C. Yet it is possible for it to be lighter at A than at D, or heavier, or equally heavy, depending on the quantity EL. For let GH, equal to EL, be erected vertically so that it touches DB

20. Theorem *R3.04:* Pierre Duhem, in his *Études sur Léonard de Vinci* (3 vols.; Paris: Hermann, 1906–1913), I, 306–307, discusses this theorem and compares it with a similar one stated by Leonardo da Vinci. It is an application of the basic principle of statical moment as measured by equidistance of weights from the vertical passing through the axis of support, to the somewhat tricky case in which the weight of the bar HTZ, though actually divided evenly on each side of the vertical through the axis, seems to be applied only to one arm because of its rigid attachment to the arm BC at the point H. Children, today, enjoy performing this stunt; e.g., they fix a short stick and a much longer one onto a cork, to form an acute angle, and then show that if the free end of the short stick is set on the very edge of a table, with the other end projecting horizontally out into the air, it will not fall off. Jordanus here seems to take a certain childish delight in the apparently paradoxical character of the theorem; as in his final statement, "But beyond this, if any weight you please is suspended from T, it will not cause any inclination one way or the other." This "experiment" is described in a mediaeval work, the *Secretum philosophorum*, which perhaps comes from an Arabic source; cf. L. Thorndike, *History of Magic and Experimental Science*, Vol. II, N. Y. 1929, p. 789.

at H; and let DK fall perpendicularly on AB. If then G should be at the mid point of AB, in which case GH would be equal to half of AB, then, since the weight DB, at D, is to the weight AB, as the line BK is to the line BA, and since its weight at D is to its weight at H, as BG is to BK, and since BG is to BK as BK is to BA, the weight DB, at H, will be as the weight of AB. But if G is nearer to B, the weight at H will be greater than at A; but if nearer to A, less.

Again, let MN be a perpendicular equal to EL, touching BF at N [Fig. 16]. And draw the line

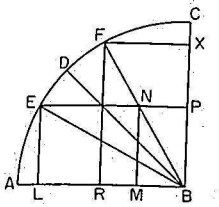

Fig. 16

ENP, and drop the perpendiculars FR and FX to AB and CB respectively. Since therefore the ratio of the weight EB to the weight FB, is as the ratio of LB to RB, or of XB to PB, and since the weight of FB at N is to its weight at F, as FB is to NB, or as RB is to MB, and since XB is to PB as RB is to MB, the weight EB is to the weight FB as the weight of FB at N is to its weight at F. The weight of EB at E is therefore as great as that of FB at N. But that it is lighter at E than at H, is proved, because DH is longer than ET. For DZ is greater than EZ, and the angle BEZ is less than the angle DHZ.[21]

R3.06. A weight not suspended at the middle, makes the shorter part heavier, according to the ratio of the longer part to the shorter part.

Let ABC be that on which the weight *e* is hung [Fig. 17]. Then let the weight *e* be divided into two

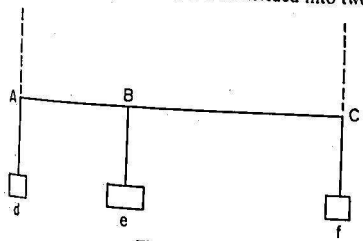

Fig. 17

weights, *d* and *f*, such that *d* is to *f* as the length AB is to the length BC. If then the weight *d* is hung at C, and *f* at A, each of them will be of as much heaviness as the weight *e*, if we imagine in each case that the opposite end is the fulcrum. Hence for those who, at A and at C, are carrying the weight *e* suspended at B, the heaviness at A will be to the heaviness at C, as the length CB is to the length BA.[22]

21. Theorem *R3.05:* [Moody notes that the last two sentences involve reference to Fig. 15—*Ed.*] The kind of experience that this theorem seeks to analyze and explain is illustrated when a man attempts to raise a long ladder from the ground. He first lifts it up at one end, to the extent of his own height, and then he walks toward the end which remains on the ground until he has the ladder standing vertically on that end. A marginal note on our manuscript H gives just such an illustration [the Latin text is omitted here—*Ed.*]:

(Translation): In this fifth theorem the author supposes that the stature of the man who does the lifting, is of the quantity EL; so that when this man commences to raise the weight AB at its end A, he will be bent down; and when he becomes erect he is as the line EL and then, holding up the said weight, he will walk toward B; and when he stands at B, then the said weight will be vertically on end, as the line CB.

There is some obscurity in the text of this theorem, as given in the manuscripts; this led Duhem to conjecture that Jordanus' original theorem had been mutilated by some careless copyist—though he cites this theorem (*Les Origines de la Statique* I, p. 143) as evidence of the author's clear grasp of the principle of statical moment. Presumably the author is concerned to exhibit the way in which the positional gravity of the weight of AB varies in function of two different conditions: (1) with the movement of the supporting bar AB from the horizontal to the vertical position, which steadily decreases its positional gravity or statical moment relative to the axis at B; and (2) with the shift of the point of application of the support EL as it moves from A toward B. What Jordanus wishes to determine is the relative "heaviness" bearing on the support EL as it shifts its point of application to the bar AB. As he formulates the problem, in abstraction from accidental considerations (such as torque) which would introduce considerable complexity into the actual "experiment," the analysis appears to be substantially sound, and the proof offered on lines 116–131 [that is, the second part of the proof associated with Fig. 16—*Ed.*] exhibits an ingenious application of the principle of statical moment to the problem.

22. Theorem *R3.06:* The source of this theorem may have been a passage in the *Mechanical Problems* attributed to Aristotle, where we find (857 b 9) the following:

"Why is it that when two men carry a weight between them on a plank or something of the kind, they do not feel the pressure equally, unless the weight is midway between them, but the nearer carrier feels it more? Surely it is because in these circumstances the plank becomes a lever, the weight the fulcrum, and the nearer of the two carrying the weight is the object moved, and

PART 4

R4.01. Every medium resists what is moved in it.

Let *ab* be that which is moved, and let the medium obstructing it be called *c* [Fig. 18]. And

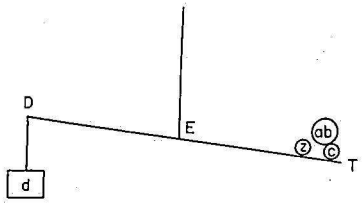

Fig. 18

suppose *c* to be as if on a balance, which we will call TED. Now if *c* is assumed to be of no gravity, then, if it does not resist the movement of *ab* descending, when it is pushed by *ab* it will be forced to descend; and thus it will be just like something having weight. Therefore it will be able, as it descends on the side of T, to raise some weight on the side D; it will thus turn out that *ab*, in its descent, will be able to raise *d* just as easily as it is able to push *c* down. Hence what lifts *d* will not be resisted by *d*'s velocity, which is impossible.

But if *c* should be heavy, then, if it is not in motion, whatever is impeding it will have to impede *ab* to some extent; but if *c* is in motion, then, since *ab* follows after it, *ab* will be heavier to the extent that it is moving faster. Now let *z* be equal in weight to *ab*. It is possible, then, for *z*, placed on the side T of the balance, to descend with the movement of *c*, and to raise some weight on the side D. And then *z* will become just like *c* in weight. If therefore *ab* encounters no resistance in pushing *c* down, it will encounter no resistance in pushing *z* down at the same time. Therefore, when *ab* and *z* are moved with their natural motion, they will encounter no resistance in raising *d*, all of which is impossible.[23]

the other carrier is the mover of the weight." (Loeb Library ed., Aristotle, *Minor Works* I, trans. by W.S. Hett; London 1936, p. 403.)

This same problem was discussed by Vitruvius, *De architectura* X, Ch. 3; and by Hero of Alexandria in his *Mechanics*. These earlier treatments all contain an erroneous application of the lever principle, just as Aristotle's explanation does. Jordanus, however, corrects the error involved in treating the weight as the fulcrum, by treating the opposite end as a fulcrum relative to the man whose share of the burden is being considered. On this basis he argues that the positional gravity of *d* at C, is to that of *e* at B, as that of *f* at A is to that of *e* at B. Duhem (*Études sur Léonard de Vinci*, I, 299) remarks: "Ce court passage mérite grandement, par lui-même, d'attirer l'attention de l'historien des sciences; pour la première fois, en effet, depuis que les hommes s'occupent de Mécanique une force de liaison s'y trouve déterminée par une méthode exacte."

23. Theorem *R4.01:* The most distinctive principle of Aristotelian dynamics, that motion and velocity essentially involve a finite proportion between the power by which the body moves through the medium and the power by which the medium resists this movement, is involved in this theorem. The proof has special interest because of its attempt to reduce the case of free fall in a resistant medium to the case of one weight raising another on the opposite arm of a balance.

The argument, which is somewhat obscure, may be construed as follows: We suppose the opposite of what is asserted by the theorem—i.e., we suppose that the medium offers no resistance to the weight falling through it. The medium itself, then, will either be of some weight, or it will be weightless; and if it is of some weight, it will either be at rest or in motion. We first suppose it to be weightless, though nevertheless capable of being set in motion by the heavy body *ab* as it descends in the medium. The author then argues: If this weightless body is set in motion, it will be "as if having weight," since a body in motion has a capacity to do work by reason of its state of movement. This is supported by supposing that the weightless body *c* is on one arm of a balance, on whose other arm is a weight *d;* clearly, if *c* descends, *d* will be raised, and since *d* has weight, work will be done in raising it. But if *c*'s descent involves no expenditure of force on the part of *ab*—as would be the case if the medium *c* offered no resistance to the fall of *ab* in it—, it would then occur that the weight *d* would be lifted without any force being expended in doing this work. But this is impossible, and in violation of the principle of work, which is fundamental to Jordanus' whole treatment of weights.

The second case is where the medium is assumed to have some weight of its own. Then, if it is assumed to be at rest when *ab* descends on it, this could only be so if there is something supporting *c* and preventing it from falling by its own weight. But in that case, whatever is resisting *c*'s descent, will resist *ab*'s descent also.

If however the medium is assumed to be in motion (by reason of *c*'s weight and the absence of any impediment to its fall), then *ab*, if not held back by any resistance from *c*, will be accelerated and overtake *c;* and its effective force will increase with its velocity *(erit AB gravius quo velocius)*. This will then reduce to the same situation as before—namely, *c* would be able to lift a counterweight *d* in virtue of the velocity given to it by *ab* pushing against it, so that if *c* offered no resistance to this acceleration by *ab*, work would be done on *d* without the expenditure of any force.

The last part of the argument, which is rather obscure, is apparently designed to reinforce the above conclusion. The author supposes a weight *z*, whose scalar value (at rest) is the same as that of *ab;* and he then supposes that *z* descends with the speed that *c* was assumed to have

R4.02. The heavier the medium through which a body passes, the more difficult is its descent in passing through it.

The kinds of medium through which such passage occurs, are air and water and other fluids. Let the heavier medium, then, be ABC, and the lighter one DEF [Fig. 19], and let the body traversing it

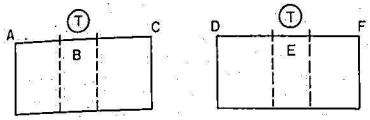

Fig. 19

be T, which, passing through it, comes up against B and E. B, however, is heavier than E; and since these media are also hindered from descending, because when they have to descend they are at rest, that which hinders B is of greater weight than that which hinders E. And because T has to encounter the same impediment, it is hindered more when it is in B. Similarly, if from beneath B and E it is propelled upwards in equal manner, its movement in B will be slower.[24]

R4.06. The longer a heavy body falls, the faster it becomes in descending.

In air, indeed, more, and in water less. For air is related to all movements. When therefore a heavy body is falling, in its first movement it will draw along those parts of the medium which are behind it, and it will move the parts just beneath it; and these, set in motion, move the parts next to them, so that these in turn, being set in motion, offer less resistance to the gravity of the falling body. Hence it becomes heavier, and pushes the receding parts of the medium still more, so that they presently cease to be pushed, and even pull. And so it comes about that the gravity of the falling body is aided by the traction of those parts of the medium, and their movement in turn is aided by the body's gravity. Hence its velocity also is observed to be continuously multiplied.[25]

R4.14. The greater the impulsion given to a body, the more its parts cohere.

This impulsion is produced from behind; and the posterior parts, having been pushed, have to push the front parts ahead; and these, which resist to some extent because of their weight, have to compress the middle parts. Hence these are sometimes driven out at the side. And in this way it also happens that when the lower parts of an object are attached to the upper ones by being stuck into them, if a downward impulsion is given to the

when *ab* was falling against *c* and *c* was merely moving along with it. It is then argued that *z*, falling at *c*'s speed on the side T of the balance, could raise some weight *d* on the other side. But, says Jordanus, if *ab* is not impeded in moving *c* at that speed, it will not be impeded in moving *z* along with it; hence we would have to suppose that when *ab* and *z* are moved by their natural weight, they lift *d* without being resisted, or without expending any force. So again, from the assumption opposed to the theorem, we derive the impossible consequence of work being done without force being expended.

24. Theorem *R4.02:* This theorem expresses the Aristotelian principle that a given heavy body falls more slowly in proportion to the relative "thickness" or density of the medium. But it is interesting to note that Jordanus here assumes that if a body of some mass, such as air or water, is at rest in any place outside the center of the world, this is because some other body is exerting force in holding it up—as if "natural rest" were similar to the "violent rest" of a weight on a balance. He then argues that since whatever is keeping the heavier medium from descending must be of more gravity *(pluris gravitatis)* than what is holding up the lighter medium, the body falling in the heavier medium is as if raising a greater counterweight on a balance. So again we have the effort to reduce the analysis of free fall through a resistant medium, to the case of weights on opposite arms of a balance.

The author seems to make the assumption that all bodies are *per se* heavy, and that "lightness" is to be reduced to an upward pressure caused mechanically by the downward force of other heavy bodies, or of a medium heavier than the body in question.

25. Theorem *R4.06:* This rather fantastic explanation of the natural acceleration of falling bodies had considerable vogue during the last decades of the thirteenth century and at the beginning of the fourteenth century. It represents an intermediate stage between the teleological explanation in terms of increasing "eagerness" due to proximity of the body to its natural place, and the later fourteenth century explanation in terms of cumulative retention of *impetus* or of acquired velocity. A mechanical explanation was being sought, but at first it was sought in a perturbation of the corporeal medium through which the body falls, on the basic assumption that if the medium were at rest it would offer a uniform resistance throughout the fall, and thereby would determine a uniform speed of fall, since the natural gravity of the falling body is a constant.

For the history of attempts to explain the acceleration of falling bodies, from antiquity up to the fourteenth century, cf. P. Duhem, *Études sur Léonard de Vinci*, III, pp. 57–96. As Duhem pointed out in *Les Origines de la Statique*, I, pp. 138–9, the type of explanation offered by Jordanus of Nemore had many defenders in the 16th century. Leonardo da Vinci adopted it, as did Cardanus. It was defended by Card. Contarini in his *De elementis* (published 1548), by Benito Pereira in 1576, and as late as 1640 by Gassendi.

upper parts, the lower ones are driven into them more deeply.[26]

R4.15. If a body, having parts which cohere, is directly obstructed in its motion, it will recoil directly.

This occurs both because of the medium in which the body is carried, whether it be air or water, and also by reason of the rarity of its own parts. Let the medium in which it is moved be B, and the moved body A; and let that against which it strikes be C [Fig. 20]. Since then A moves B,

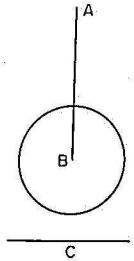

Fig. 20

when A departs from its place and expels B from its place, B has to reverse its motion in order to fill up the places at the rear. Hence it is both pushed ahead, and turned back to the rear, by the same impulsion. But it is carried back with all the more weight, when A strikes C, since it is then unable to proceed further because it is stopped by the mass which is in front of it. And since the impulsive force of A has been broken up on C, and A is now inclined only by its own weight, it must be carried back by B's movement unless its own weight prevails against this; and it will recoil in a direct path, since B recedes equally in every direction.

The rarity of its own parts, however, brings about the same result. For the front parts of A, when they first encounter the obstruction C, are pressed upon by the mass and impulsion of the rear parts, and are compressed on themselves; then, when the impulsion is expended, they revert to their places and in so doing push back the other parts. If the parts are separable, then when they are compressed into a smaller place, they will rebound.[27]

26. Theorem *R4.14:* Here we find the conception of "impulsion" or *impetus* applied to the internal elastic structure of bodies. The argument invites us to contemplate the fall of a piece of wood, into whose under side nails have been very lightly driven. When the wood falls with great speed, or is given a sudden downward push, the inertial resistance offered by the nails will be sufficient to cause them to be driven more deeply into the wood. We have a vague suggestion here of the conception of the innate resistance of a body to motion, as being a resistance to acceleration; but it is far from explicit. Jordanus is primarily interested in the problem of communication of an impulsive force through the parts of elastic bodies; the example of the nails driving deeper into the wood when the latter moves suddenly, is designed to represent the manner in which the elastic structure of a body permits a compression of its parts generating a recoil—as in a bouncing ball.

27. Theorem *R4.15:* This theorem develops the idea of internal compression, introduced in *R4.14*, in explanation of the rebound of an elastic body. Again the notion of "impulsio" (here also called *impetus*) plays an important part, applied in a quasi-atomistic analysis of elasticity. In the works of Robert Grosseteste we find a somewhat similar use of the concept, in explanation of the vibratory motion of a stretched string in its production of sound. In his treatise *On the Liberal Arts* (L. Baur, *Die philosophische Werke des Robert Grosseteste*, Münster, 1912, pp. 2–3), Grosseteste speaks as follows:

"For when a body is struck violently, the parts which have been struck and compressed recede from their natural positions. But the natural force, tending toward the natural position, makes these parts pass beyond their proper bounds. . . . And thus they again leave their natural position, and by the same natural inclination they return and pass across that position. And in this way, in the smallest parts of the struck body there is generated a vibration, until finally the natural inclination no longer impels them beyond their proper position."

It is of interest to compare this passage from Grosseteste, as well as the present theorem of Jordanus, with another part of the same Question 12, on the eighth book of Aristotle's *Physics*, written by Jean Buridan:

"There are many things such as can be curved back on themselves or compressed by violence, which naturally revert with great velocity to their straightness or natural condition; and, in thus reverting, are able by their impetus to push or pull something else that is attached to them—as is seen in the case of the bow and arrow. Thus also a resilient ball bounced on hard ground is compressed on itself by reason of the impetus of its motion, and immediately after it strikes it reverts with great velocity to its spherical shape, pushing itself upward from the ground; and in so doing it acquires an impetus which moves it for a long distance. In this way also the chord of the zither, tightly strung, and struck hard, remains for a long time in a certain vibration, so that for a perceptible period of time its sound continues. And this happens because, by reason of the initial blow, the string is stretched violently in one direction, and it reverts so rapidly to its straightness that it passes beyond the straight position to the opposite side, and then it returns again, and thus many times."

As these quotations reveal, the fourth book of Jordanus' *De ratione ponderis* takes its place in an historically continuous line of development, leading from early thirteenth century efforts to apply mathematics to physical phenomena, found in such men as Robert Grosseteste, to the developed "impetus mechanics" associated with Buridan, Albert of Saxony, and Nicole Oresme [see Selection 49—*Ed.*]. This line of development is found in the liberal arts tradition, in the disciplines of the *quadrivium;* and it was within this tradition that Jordanus de Nemore lived and worked.

R4.16. A liquid which is continuously poured, forms a narrower stream at its lower end, to the degree that it falls further.

Let the hole through which the liquid escapes be AB, and let the first portion of the liquid be C [Fig. 21] and when C shall have descended as far

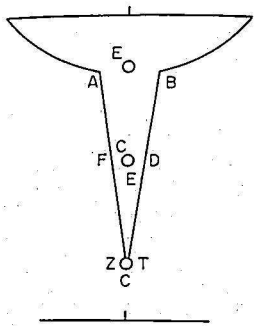

Fig. 21

as DF, let the portion E be at the opening. And then, when E has reached DF, let C be at ZT. Since therefore a body becomes heavier the more it falls, C will be heavier when at DF than when at AB; therefore it will be heavier at DF than E is at AB. But because, when E arrives at DF, C reaches ZT, the distance FZ will be longer than the distance AF; hence the stream will be more slender. And thus it becomes ever thinner, because the front parts are faster, and so, finally, it breaks up into drops.[28]

R4.17. If a body of non-uniform weight is given an impulsion in any given direction, the heavier part will come to be in the front position.

Let AB be the body which is projected, and its heavier part A [Fig. 22]. If then the impulsion is given on the side of A, B will also be impelled;

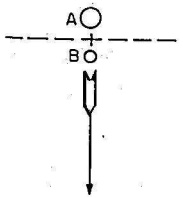

Fig. 22

and since it is lighter, it will more easily yield to the impulsion; and since A does not follow it with such ease, it will be held back in its own motion and will aid the gravity of A. And thus the whole impulsive force will pass back into A; therefore A will move out in front, and with its impetus pull B. If however the part B receives the impulsion from the rear, and A is in front, then B, having received the impulsion, will impel A; and the lightness of B will be retarded in the moving of A, and therefore A will receive more impulsion because it offers more resistance to motion. And being then impelled with the whole of the original impulsive force, it will have to pull B along.[29]

Here ends the fourth part, and with it there ends the book of Jordanus on the theory of weight.

28. Theorem *R4.16:* This little demonstration goes back to a passage in Simplicius' *Commentary on the Physics*, where it is said that Strato of Lampsacus, pupil of Theophrastus, had offered this "experiment" of pouring water from a spout as evidence of the fact that falling bodies are accelerated. Jordanus seeks to explain the fact through the principle that acquired velocity increases the "ponderosity" of a body falling, so that it is able to move continuously faster as it falls. Here again we have a vague anticipation of the "impetus" explanation of acceleration given later on by Buridan; for Buridan states that the effect of the natural gravity of a body, in free fall, is to continuously add new degrees of impetus to the body, thereby continuously increasing the power of the body to overcome the resistance of the medium, and consequently giving it continuously greater speed.

Since Simplicius' commentary had not been translated into Latin in the period when Jordanus wrote the *De ratione ponderis*, we must suppose that this experiment of the poured water, as associated with the problem of acceleration in free fall, had been described by some intermediary source accessible to Jordanus, which had in turn derived it from Simplicius.

29. Theorem *R4.17:* This theorem, though expressed in a most obscure fashion, applies the concept of "impetus" to the case where a body is of unequal composition—i.e., with respect to density, and is projected or thrown from the hand or from a machine. The denser portion of the missile will acquire a greater impulsion or impetus from the initial velocity given to the projectile, than will the rarer portion, as was previously indicated in *R4.10*. Consequently the denser portion will more easily overcome the resistance of the medium, than the rarer portion and it will therefore move out to the front, even if at first it was in the rear. Again we have indication of the connection between Jordanus' dy-

Motion

40 WHAT IS MOTION?

William of Ockham (ca. 1280—ca. 1349)

Translation, introduction, and annotation by John E. Murdoch[1]

The medieval natural philosopher, Aristotelian that he was, expended considerable effort and care in analyzing and expounding fundamentals. The precise nature of all the entities, of all the variables, as it were, that one is to encounter in natural philosophy must, the scholastic felt, be determined with all possible accuracy. And since no one of these variables was more central to the preoccupations of the medieval natural philosopher than motion, it was essential that an adequate understanding of its nature be a primary concern. Indeed, few issues within medieval science received more attention than the problem of the nature or, in other terms, of the definition of motion. And few problems appear to have received a greater variety of answers.

The seeds of the problem are in Aristotle himself, but were more explicitly brought to the fore in Averroes' comment on a particular passage of Book III of the *Physics* (200b.32—201a.3):

> Again, there is no such thing as motion over and above the things.[2] It is always with respect to substance or to quantity or to quality or to place that what changes changes. But it is impossible, as we assert, to find anything common to these which is neither "this" nor *quantum* nor *quale* nor any of the other predicates. Hence neither will motion and change have reference to something over and above the things mentioned, for there is nothing over and above them.

Averroes' presumed resolution of the puzzle[3] he saw in this text did more to excite and extend the discussion of the problem than it did to solve it. For between the absorption of the *Physics* (together with Averroes' commentary) and the fourteenth century one can uncover a considerable spectrum of opinion concerning the nature of motion.[4] Almost all such opinions, however, held that, whatever else might be true, at least motion was certainly something other than the body undergoing motion. It is relative to this point that Ockham's analysis of the problem struck home. For motion was not, he urged, something distinct from the mobile doing the moving. With a single application of his well-known ontological "Razor" he cut away all superfluous entities and reduced motion to, as he puts it, permanent things. In the case of local motion, he tells us, we need only posit a moving body and its continuous, successive occupation of diverse places. Although Ockham's theory is undeniably an improvement upon that of his less perspicacious forebears, we must note, however, that he falls short of being successful in that he fails to account for the relation of the moving body being in, or occupying, the various places. But then his theory of relations itself was unsatisfactory.

Ockham's criticism of the then current views of motion is a particularly good example of a consequence of his belief that the proper subject matter of science consisted in propositions and the terms out of which they were constructed.[5] Motion as a "scientific concept" was a connotative term. To reveal the nature of such a "concept" was to reduce it—or more correctly any proposition in which it occurs—to a proposition containing only absolute terms, terms which required no other thing save what they primarily signify in order to be significant. It is, in effect, this logical reduction that Ockham occupies himself with in establishing his doctrine of the nature, or definition, of motion.

Since it is the common opinion that motion, time, and place are certain things other than the movable body and the placed thing, we must inquire of the opinion of the Philosopher and Commentator, firstly concerning motion, secondly place, and thirdly time.[6]

[THE TREATISE ON MOTION]

With respect to the first of these matters, we must note that according to the Philosopher in Book III of the *Physics* together with the Commentator in his fourth comment [to that Book]: "Motion does not have a definition stated univocally because it is not named univocally.[7] Rather, it has a definition according to prior and posterior. None the less, such definitions do enter into the natural sciences."[8] In the passage cited, the Commentator says that the Philosopher intends to show that motion belongs to more than one category.[9] The Commentator proves this as follows: Transmutation is only found in that which is transmuted, but a moved thing can be found under more than one category; therefore so can motion.[10] And in the same comment the Commentator brings up the puzzle as to how it can elsewhere be maintained that motion belongs to only one category (namely that of passivity), or even that [motion of itself] forms a single category. However, it is also said in this very same comment that there are four genera of motion. The Commentator answers [this puzzle] in the passage which begins with the words: "To this however we respond. . . ." See for yourself the solution he gives in this place.[11]

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namics and the "impetus physics" of Buridan; where he says that the heavier part A "will receive more impulsion because it offers more resistance to motion," he is giving crude expression to the truth that a body of greater mass, which is therefore a body that offers greater resistance to an accelerating force, will when set in motion at a given velocity have a greater momentum, which in turn will make it more resistant to the decelerating action of the medium. In Buridan, this insight achieves more explicit formulation though it was not reduced to exact quantitative laws until the 17th and 18th centuries. But the historical continuity of the development of modern mechanics, from the small beginnings found in Jordanus de Nemore through the fourteenth century "impetus physics" and down to Galileo and his contemporaries, is discernible at each stage. In attempting to apply the mathematical method, which he had used so successfully in the field of statics, to the difficult realm of general dynamics, Jordanus took a first step in a direction which led slowly and painfully toward the modern science of mechanics.

1. The following selections were translated from *The Tractatus de successivis attributed to William Ockham*, edited by Philotheus Boehner (St. Bonaventure, N. Y.: Franciscan Institute, 1944), pp. 32–39, 42, 43, 45–49. Although this work was admittedly not composed by Ockham himself, it may nevertheless be considered a true expression of his contentions as it is a compilation drawn directly from his indisputably genuine *Expositio in libros physicorum*.

2. Although Aristotle's τὰ πράγματα, here rendered as "things," does not mean the mobile or the subject undergoing change but rather, as Ross and others have noted, the "respects in which things may change" (which Aristotle specifies in the next sentence), such a refinement was not present in the medieval translations. There τὰ πράγματα was rendered by *res* (for example, the present sentence was rendered as: *Non est autem motus preter res*). Thus the above English translation, at least at this point, adequately reflects what the scholastic had before him. [For the Latin text of this paragraph in Aristotle's *Physics*, see *Aristotelis opera cum Averrois commentariis* (Venice, 1562–1574; Frankfurt: Minerva, 1962), Vol. IV, Book III, Text 4, fol. 86 verso, col. 2.—*Ed.*]

3. This is given in note 11.

4. For a history of some of these opinions see Anneliese Maier, *Die Vorläufer Galileis im 14. Jahrhundert*, 2d ed. (Rome: Edizioni di storia e letteratura, 1966), pp. 9–25, and *Zwischen Philosophie und Mechanik* (Rome: Edizioni di storia e letteratura, 1958), pp. 59–186.

5. See Ockham's discussion of the nature of *scientia demonstrativa* in his prologue to his *Expositio super VIII libros Physicorum* as translated by Philotheus Boehner in *Ockham, Philosophical Writings* (Edinburgh and London: Nelson, 1957), ch. 1.

6. Ockham's discussions of place and time constitute the second and third parts of the treatise at hand. Selections from these subsequent parts have not been included.

7. Averroes is referring to Aristotle's *Categories*, 1.1a.6–12: ". . . Things are said to be named "univocally" which have both the name and the definition answering to the name in common. A man and an ox are both "animal" and these are univocally so named, inasmuch as not only the name, but also the definition, is the same in both cases."

8. Ockham here quotes Averroes verbatim. For the passage directly preceding this quote, see note 10. [For Book III, Comment 4, see Vol. IV, fol. 87 recto of the edition cited in n. 2 above—*Ed.*]

9. Specifically, Averroes interprets this intent to reveal that motion belongs to more than one category as the fourth basic point Aristotle sets forth in his attempt to define motion, a task which occupies all the earlier chapters of Book III of the *Physics*.

10. The text from which Ockham draws this argument is the following: "Motion is not found outside the ten categories. For a thing which has undergone change is always changed either relative to the category of substance, of quantity, of quality, of place, or of some other category; yet change *(transmutatio)* is only found in a thing which has undergone change, all of which implies that motion exists in more than one category. The argument is constituted in the following manner: Motion exists in the moved thing; but the moved thing exists in more than one category; therefore motion exists in more than one category. 'Moreover,' [Aristotle goes on], 'nothing is found, etc.,' that is to say, it is impossible that some one thing be found which is common to those categories in which motion is found, for example something common to those four categories [mentioned above] which is no one of these four nor of the other categories" (Averroes, *Physics* III, comment 4 [see Vol. IV, fol. 87 recto of the edition cited in n.2.—*Ed.*]).

11. "To this however we respond that, insofar as it does not differ, save by more or less, from the perfection toward which it is directed, motion necessarily belongs to the genus of the perfection in question. For motion is nothing other than a generation, part by part, of the perfection toward which it is directed, [a generation which continues] until the perfection is reached and actually exists. Thus it is necessary that a motion which takes place in substance is to be found in the genus of substance, and that a motion which occurs in quantity, in the genus of quantity, and, similarly with that which occurs with respect to place and quality. However,

Several things must be examined and proved concerning the grounds for his solution. In the first place one must see and know that motion is sometimes taken in a strict sense insofar as it entails succession, while at other times motion is taken in a general sense to cover all change, be it immediate *(mutatio subita)* or successive. Firstly, therefore, one must prove that immediate change is not another thing, taken by itself as a whole, distinct from the mobile or the moving body and from the terminus acquired or lost [by the change] and from other permanent things. Secondly, one must prove that no motion is some one thing in itself as a whole distinct from all permanent things, and that motion is not some one thing outside the essence of permanent things, and that no permanent thing derives from the essence of motion (just as whiteness is totally outside the essence of substance and substance is totally outside the essence of whiteness).

[THAT IMMEDIATE CHANGE IS NOT SOMETHING DISTINCT FROM PERMANENT THINGS]

Point One: That immediate change is not some thing distinct from permanent things is shown, to begin with, as follows:

When a medium is first illuminated by the sun, I inquire whether or not there might be in this medium several things, distinct from the light itself and from other permanent things, which come into existence in the medium due to the occurrence of the illumination. If not, one has attained that proposed, namely that immediate change is not something other than, and so on. If, on the other hand, there are [such distinct things in the medium], I argue against this as follows: Every single thing is either substance or quality—as has been shown elsewhere and is now assumed.[12] But it is certain that such a thing is not substance, for it is neither matter nor form nor anything composed out of them. Nor is it a quality, as can be inductively proved by running through all species of quality. Nor is it quantity, since it is certain that the quantity of the air [which is the medium in question] is in no way changed. Therefore, and so on, [there is no thing distinct from permanent things in this case]. . . .

Furthermore, it is futile to accomplish with a greater number of things what can be accomplished with fewer *(frustra fit per plura, quod potest fieri per pauciora)*.[13] But, if everything other than matter, form, an agent, and other permanent things be set aside, and if it be assumed that some matter first does not have a [given] form and then later does have this form (but does not acquire it part by part) then, indeed, that matter is changed. Therefore, in order to account for change, it is vain to assume the existence of some thing other than matter, form, an agent, and other permanent things.

This is confirmed because in order to account for change, it is sufficient that matter have some form it did not previously have. But given the fact that matter has some form, nothing other than the form and the matter is assumed. Nor does the fact that the matter does not previously possess this form necessitate that we assume something in addition to the matter, form, an agent, and other permanent things. Consequently, above and beyond permanent things, there is no other thing which is change. . . .

[OCKHAM'S OWN VIEW]

Therefore, it must be said that immediate change is not some thing distinct from permanent things that is destroyed following the first instant in which the subject suffers immediate change. On the contrary, when matter suffers change toward the accession of a substantial form and the substantial form is introduced into the matter, no thing is in this case afterwards lost, but everything which exists in the first instant exists afterward, or at least can so exist. But for a subject to undergo immediate change involves nothing further than that the same subject have a form which it did not previously have, or that it lack a form which it previously did have. However, [this acquisition or loss of a form] does not occur part by part (that is, in such a way that the subject has one part of the form before another, or loses one part before another), but the whole form is simultaneously acquired or simultaneously lost.

insofar as motion is a process *(via)* toward a perfection and is different from that perfection, it is necessary that motion belong to a genus per se. For a process toward a thing is not the same as that thing, and in view of this [motion as a process] has been set down as a category per se. This manner [of treating our problem] is better known, while that referred to above is truer. Therefore, Aristotle has introduced the better known view in the *Categories*, and the true one in this book" (Averroes, *Physics* III, Comment 4 [see Vol. IV, fol. 87 recto of the edition cited in n.2.—*Ed.*]).

12. In Ockham's view quantitative terms signified nothing distinct from substances and instances of qualities (see his *Commentaria sententiarum*, Bk. I, Distinction 2, Questions 1–8, and his *Summa logicae*, Part I, chs. 14–18).

13. This is, perhaps, the most frequent form one finds in Ockham for the expression of his so-called "Razor."

And yet [in objection to this view] it may be asked whether or not change is some one thing. If it is not a thing, it is nothing, and therefore nothing is changed by real change. If, however, it is a thing, and is not something other than a permanent thing—as has been proved—then change is a permanent thing, and consequently it is either form or substratum *(subjectum)*. And be it the one or the other, it follows that as long as the substratum or the form would exist, the change would exist, and so something would be changed by such a change.

One must reply [to this objection] that nouns which are derived from verbs, and even nouns derived from adverbs, conjunctions, prepositions, and also from syncategorematic terms (be they syncategorematic nouns or verbs or any other sort or of another part of speech) are only introduced for the sake of conciseness of speech or embellishment of locution. And many of these [nouns], when they are not standing for those words from which they derive, are, in their significative use, equivalent to propositions. And consequently they signify nothing else except those words which they are derived from and the things signified by those words. However, all of the following nouns are of this sort: "negation," "privation," "condition," "perseity," "contingency," "universality," "quantity" (that which is called the "quantity" of a proposition), "action," "passion," "calefaction," "frigefaction," "change," "motion," and in general all verbal nouns derived from verbs which fall under the categories of action and passion, and many other nouns which will not presently be dealt with.

For the noun "change" does not signify some single absolute thing (as is the case with the noun "man" or "ass" or the noun "whiteness"), but it is introduced (1) sometimes for the sake of embellishing locution, as it is sometimes used for the purpose of embellishment quite suitably in a place where the verb from which it is derived would not be used in an embellishing way, (2) sometimes for the sake of brevity, just as this brief sentence, "Every change results from an agent," is equivalent to this whole sentence, "Everything which is changed, is changed by some agent." And so it is with many other sentences, just as this, "Every transmutation is from something and in something and into something," is equivalent to this whole sentence, "Whenever something is transmuted, in such a case there is something which is acquired or lost by that which is transmuted and there is some other thing which causes it to acquire or lose that something."

And since such nouns are invented for the aforesaid reasons, they are often interpreted equivocally by authors and in different places in different ways. For (1) sometimes such nouns in the writings of various authors supposit or stand for the very words from which they are derived, or for the concepts in one's mind corresponding [to such words]. Yet (2) sometimes they supposit or stand for that which is signified by one part of that complex expression to which they are equivalent; (3) at other times they stand for that signified by another part; and (4) at still other times for an aggregate of such signified things. Lastly, (5) sometimes they stand for that very thing for which the participle of such a verb as that from which it [that is, such a noun] is derived stands (for in such a case it is used in place of such a participle, together with other elements expressing the *quid nominis*[14] of the verb in question).

Example of the first case: If the following be said: "Change is truly predicated of Socrates," one sense is that the verb "is changed" is truly predicated of Socrates, as in saying "Socrates is changed." Similarly, if one says, "Perseity is suitable to this proposition: 'Man is an animal,'" one sense is "'Man is an animal' is a proposition per se." And so on with respect to other examples.

Example of the second case: If I say, "Change is capable of change," in this case "change" stands for that signified by the term "something." Whence the definition which expresses the *quid nominis* of the term "to be changed" is the following: "Something, which now has something for the first time which it did not have previously, lacks something which it had previously." And therefore sometimes "change" stands for that which has something which it did not previously have, and so on.

Example of the third case: "Change is really the same as the terminus [involved in the change]," since this is equivalent to this: "That which is acquired or lost by something is really the same as

14. Nominal definitions *(diffinitiones quid nominis)* only tell us what is meant by a term or name. They can be given to all terms whether there is some existing thing for which the terms directly stand or not. On the other hand, a real definition *(diffinitio quid rei)* of a term can be given only if there is some existing thing for which the term primarily stands. Such definitions thus express the essential nature of the thing for which the term in question stands as well as define the term itself.

that which is possessed or is not possessed by that thing."

An example of the fourth case is evident if I say, "Change is both the changeable thing *(mobile)* and the terminus or form existing in this changeable thing," or something of this sort.

An example of the fifth case occurs when I say, "Change is that which proceeds from prior to posterior," which is equivalent to this: "When something is changed, it proceeds from prior to posterior."

Nevertheless, it must be known that the noun "change" is frequently used in place of a complex expression, as are other similar nouns, and that which follows [such a noun] is used in place of a verb. Thus, for example, in saying, "Change is the acquisition or loss of something," the noun "change" is used in place of this complex expression: "When something is changed"; and the nouns "acquisition" and "loss" are used in place of the verbs "it acquires," "it loses"; and the noun (which is in an oblique case) "of something" is used in place of the case which should follow those verbs. For example, the sentence, "Change is the loss or acquisition of something," is used in place of this temporal [sentence]: "When something is changed, it acquires or loses something," or in place of this conditional [sentence]: "If something is changed, and so on". . . .

Therefore, different propositions [involving the term "change"] are to be verified and expounded in diverse ways. Yet it is difficult to discover some one general manner of expounding such propositions, and particularly so because philosophers use the noun "change" equivocally and in different ways. Nevertheless, it must be maintained that, insofar as things are concerned, change is not some thing other than a permanent thing and that the noun "change" does not signify some thing which is not permanent or does not last through time. On the contrary, it signifies that some thing acquires or loses some whole thing simultaneously and not part before part, nor one after another. . . .[15]

[THAT MOTION IS NOT SOMETHING OTHER THAN PERMANENT THINGS]

Now that we have revealed the foregoing with respect to change *(mutatio)*, it remains to prove that no motion *(motus)* is some thing, in itself as a whole, distinct from a permanent thing. . . .

Consequently, it must be said that motion is not some thing in itself wholly distinct from permanent things. For it is vain to accomplish with many things that which can just as well be accomplished by fewer. And we can save both motion and all things said about motion without recourse to any such [separately existing] entity. Therefore, it is superfluous to posit such a thing. The fact that we can save motion and all things said about it without any such added entity is clear if one enumerates, one by one, the various parts of motion.[16]

This is evident with respect to local motion. For if it be assumed that a body is first in one place and then in another place, and that it proceeds in this manner without rest and without there being any thing other than the body and the agent which moves it, it is clear that we have local motion. Thus, it is vain to assume the existence of any additional things.

It may be objected that body and place are not sufficient to account for local motion because, if this were the case, then whenever there were body and place there would be motion and thus a body would always be in motion. To this we must reply that body and place are not sufficient to account for the existence of motion in the sense that the following consequence is not a formal one: "Body and place exist, therefore motion exists." Nonetheless, no other thing is required in addition to body and place. It is only required that a body is first in one place, later in another place, and so on continuously, in such a way that the body never rests at any place throughout the whole time [of its motion]. And it is clear that in addition to all these things, nothing is posited other than permanent things. Thus, by the fact that a body is first in *A*, nothing else except *A* [and the body] is posited; similarly, by the fact that it was not first in *B*, nothing is posited aside from *B* and the body; and similarly by the fact that the body is secondly in *B* nothing is posited other than *B* and the body. And if we proceed with respect to the other [places occupied by the body] in a similar manner, it is clearly evident that above and beyond body and the parts of space and other permanent things it is not necessary to assume the existence of any other thing. On the contrary, it is only necessary to assume that a body at one time is in this place, or in any part of space, and that another time it is not. This is what

15. The qualification that the acquisition or loss in question must be of some whole and must occur simultaneously is required, one will recall, by the fact that Ockham is presently analyzing immediate change *(mutatio subita)*.

16. By "parts of motion" Ockham means the various kinds of motion: local, alteration, and augmentation and diminution.

being moved locally amounts to: for a body first to occupy one place, and with no other thing assumed, at a later time to occupy another place without any intervening rest and without any entity other than place, the body, and other permanent things, and to proceed in this manner continuously. Consequently, beyond these permanent things we need consider nothing else, but alone must add that a body is not simultaneously in [all] these places and does not rest in any of them. By these negative statements no entity aside from permanent things is assumed to exist. Consequently, without any other thing whatsoever, the whole nature of motion can be accounted for by the fact that a body is successively in distinct places and does not rest in any.

The same thing is evident with respect to motion of alteration (that is, that one need not assume anything but permanent things). For, the sole fact that the subject acquires the parts of a form one after another and not simultaneously accounts for motion of alteration. Thus, it is not necessary to posit anything other than the subject and the parts of a form, but it is sufficient to posit only the subject and the parts of a form in such a way that these parts are not simultaneously acquired. However, the fact that these parts are not acquired simultaneously does not mean that some thing other than these parts is posited, but rather merely that some parts of a form exist at different times and not at the same time. Therefore, accordingly, the existence of no other thing is assumed, but rather something is denied, namely that not any one, but that many, parts of a form exist simultaneously.

And if, when we say that "the parts do not exist simultaneously" it be objected that this non-simultaneity of parts is something, we must answer that such an abstract noun fiction constructed out of adverbs, conjunctions, prepositions, verbs, and syncategorematic terms[17] causes many difficulties and leads many into error. For many thinkers imagine that just as there are distinct nouns there are distinct things corresponding to them, so that insofar as there is a distinction between things signified, to such an extent there is a distinction between the signifying nouns. This, however, is not true. For sometimes the same things are signified when there is a difference in the logical or grammatical manner of signifying. Therefore, non-simultaneity is not some thing in addition to those things which can exist simultaneously, but it merely signifies that [such] things do not exist simultaneously. Thus, in modern times, because of the errors arising from the use of such abstract nouns, it would be better in philosophy, for the sake of simplicity, not to use them but only the verbs, adverbs, conjunctions, prepositions, and syncategorematic terms as they were instituted in the first place. This is preferable to the invention of such abstract nouns and their use. Indeed, were it not for the use of such abstract terms as "motion," "mutation," "mutability," "simultaneity," "succession," "rest," and the like, there would be little difficulty with respect to motion, mutation, time, instants, and so forth.

Moreover, it is clear that no other thing, above and beyond permanent things, is required in the case of motion of augmentation and diminution. For in such a case one can account for motion by the sole fact that there is, successively, more and more, or less and less, quantity. However, when one says that there is more and more quantity it is not necessary to posit anything aside from permanent things, unless someone would wish to form an abstract noun out of the conjunction "and" and thus say that and-ness or and-eity is something other than the parts of quantity. But this is absurd. Therefore, by the fact that in the case of motion of augmentation or diminution there is more and more, or less and less, quantity, one is not required to assume the existence of something in addition to permanent things. Permanent things alone suffice, in such a way that the same [permanent] thing is first of such-and-such a quantity and later is not of this quantity, and thus proceeding without end. Moreover, I have elsewhere said[18] that in this way

17. "Categorematic terms have a definite and fixed signification, as for instance the word "man" (since it signifies all men) and the word "animal" (since it signifies all animals) and the word "whiteness" (since it signifies all occurrences of whiteness). Syncategorematic terms, on the other hand, as "every," "none," "some," "whole," "besides," "only," "insofar as," and the like do not have a fixed and definite meaning, nor do they signify things distinct from the things signified by categorematic terms. Rather, just as, in the system of numbers, zero standing alone does not signify anything, but when added to another number gives it a new signification; so likewise a syncategorematic term does not signify anything, properly speaking; but when added to another term, it makes it signify something or makes it stand for some thing or things in a definite manner, or has some other function with regard to a categorematic term. Thus the syncategorematic word "every" does not signify any fixed thing, but when added to "man" it makes the term "man" stand for all men actually, or with confused distributive *suppositio*." (Ockham, *Summa logicae*, Part I, ch. 4).

18. Ockham appears to be referring to his *Commentaria sententiarum*, Book II, Question 9, which is devoted to an exhaustive analysis of the problem of "whether

motion is composed of affirmations and negations. That is, that in order that there be motion it is sufficient that (1) there be parts or permanent things which do not exist simultaneously in the sense that for the truth of [the proposition] "motion exists" certain affirmative propositions and certain negative propositions [referring to these parts or permanent things] are sufficient, and (2) that no thing other than permanent things is posited or denoted by these affirmative or negative propositions.

In the case of motion of alteration it is sufficient that there is first one thing and not another, and that later there is the other thing and not the first, proceeding thus without anything other than permanent things. . . .

Turning to how one speaks [about motion, we should say that] if the noun "motion" is posited in a sentence, we must assert just those things we have asserted above concerning the noun "change." That is, sometimes "motion" stands for the verb "to be moved" and for its moods and tenses; sometimes it stands for the mobile itself; at other times it stands for the terminus or thing acquired when something is moved; at still other times it is used in place of such expressions as "that it is moved," or "when it is moved," or something of the sort. And similarly, those things which are added to the noun "motion" in a sentence, either in that part of the sentence in which this noun occurs or in the other part of the sentence, are to be expounded in different ways. Hence "motion exists in time" should be expounded as "when something is moved, it does not acquire or lose all that it acquires or loses at one and the same time, but rather part by part." And in such a way it is clear that the nouns "motion" and "time," just as other such abstract nouns, are invented for the purpose of brevity, that is, so that that which is signified by the long expression "that which is moved does not acquire or lose all that it acquires or loses simultaneously, but rather part by part" is also signified by the brief expression "motion exists in time." Similarly, the proposition "motion exists in the moved thing" should be expounded as "that which is moved acquires or loses something." The case is similar with other examples. . . .

Kinematics

41 THE REDUCTION OF CURVILINEAR VELOCITIES TO UNIFORM RECTILINEAR VELOCITIES

Gerard of Brussels (fl. thirteenth century)

Translation, introduction, and annotation by Marshall Clagett[1]

The *Liber de motu* is divided into three short "books." The first attempts to find uniform punctual rectilinear velocities which will make uniform the varying curvilinear velocities possessed by different lines (and perimeters of regular polygons) as they rotate or revolve. The second discovers similar uniform movements for surfaces in rotation, and the last seeks uniform movements for solids in rotation. Gerard's work, then, is probably the first Latin treatise to take the fundamental approach to kinematics that was to characterize modern kinematics, namely, to see in kinematics the basic objective of reducing variations in velocity to uniform velocity.

In proposition I.1 Gerard says that a segment of a rotating radius or the radius is moved "equally" as its middle point. By this he means that if we take the segment or the radius itself and allow it to move, not in a movement of rotation, but rather in one of translation always parallel to itself so that all of its points are moving rectilinearly with the velocity which the middle point had when it was

motion is a genuine thing outside the soul really different from the mobile and from the terminus [of the motion]."

1. [Reprinted with permission of the Regents of the University of Wisconsin from M. Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Wis.: University of Wisconsin Press, 1959), pp. 185–186, 187–189, 194–195. The introduction has been constructed from relevant passages on pages 185–186; the commentary has been located immediately after the translation, just as in Clagett's volume. Clagett believes that Gerard's *Book on Motion (Liber de motu)* was composed sometime between 1187 and 1260 and represents, among other things, a significant revival of the mathematical works of Euclid and Archimedes. Virtually nothing is known about the life of Gerard.—*Ed.*]

rotating, then the line segment or radius so moved traverses the same area in the same time as it did in revolution. And when Gerard says that the rotating lines move "equally" as its middle point, he means thereby that it is the velocity of the middle point which can convert or transform the varying motion of rotation into uniform motion of translation. This theorem may have stimulated by analogy the discovery at Merton College of the fundamental theorem of uniform acceleration—a theorem which held that with respect to the traversal of space in a given time a uniform acceleration is equal to a uniform motion at a velocity equal to the velocity at the middle instant of the time of acceleration [see Selection 42, Part VI *(continued)*, par. 1, and commentary—*Ed.*]. At any rate, Thomas Bradwardine, the founder of kinematic studies at Merton College, in his *Tractatus de proportionibus velocitatum* of 1328, mentions Gerard's proposition I.1 and uses the treatise as a point of departure for his chapter on kinematics, even while he disagrees with Gerard's conclusions.[2]

Finally, we should notice that, unlike the Greek authors, Gerard throughout his treatise seems to be assigning some given magnitude to velocity, a magnitude distinct from, although proportional to, the distance traversed in a given time. He thereby opens the way for a similar but more varied treatment of the proportionality of velocities in the fourteenth century.

BOOK I.

[SUPPOSITIONS:]

[1.] Those which are farther from the center or immobile axis are moved more [quickly]. Those which are less far are moved less [quickly].

[2.] When a line is moved "equally," "uniformly," and "equidistantly" (i.e., uniformly parallel to itself), it is moved equally in all of its parts and points.

[3.] When the halves [of a line] are moved equally and uniformly to each other, the whole is moved equally [fast] as its half.

[4.] Of equal straight lines moved in equal periods of time, that which traverses greater space and to more [distant] termini is moved more [quickly].

[5.] [That which traverses] less [space] and to less [distant] termini is moved less [quickly].

[6.] That which does not traverse more space nor to more distant termini is not moved more [quickly].

[7.] That which does not traverse less space nor to less distant termini is not moved less [quickly].

[8.] The proportion of the movements (i.e., speeds) of points is that of the lines described in the same time.

[Proposition] I.1: Any part as large as you wish of a radius describing a circle, which part is not terminated at the center, is moved equally as its middle point. Hence the radius is moved equally as its middle point. From this it is clear that the radii and the speeds are in the same proportion.

Proceed therefore: I say that *CF* (see Fig. 1) is

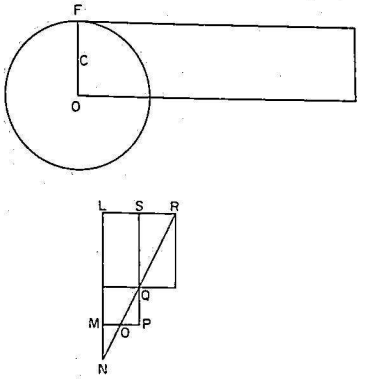


Fig. 1

moved equally as its middle point, it having been proved that the [annular] difference between [concentric] circles is equal to the product of the difference of the radii into half the sum of the circumferences.

For let lines *OF* and *RL* be equal, and let line *LN* be equal to the circumference of the circle with *OF* radius. It is evident by the first proposition of the *De quadratura circuli* [of Archimedes][3] that the circle *OF* and the triangle *RLN* are equal. Also, let lines *SL* and *CF* be equal, as well as lines *ON* and *OQ*. And let lines *SL* and *MP* be parallel. It is necessary, therefore, that triangle *RSQ* be equal to circle *OC* and line *SQ* be equal to the circumference of that circle. For, since triangles *RLN* and *RSQ* are similar, the proportion of *LR* to *SR* is as

2. [This part of Bradwardine's treatise is not included here, but a selection from the earlier chapters of that work appears in Selection 51.1.—*Ed.*]

3. [For two medieval versions of this proposition, see Selection 31.]

the proportion of *LN* to *SQ*. But the proportion of *LR* to *SR* is as that of *OF* to *OC*, and the proportion of *OF* to *OC* is as that of the circumference of *OF* circle to the circumference of *OC* circle, because the diameters and circumferences [of circles] are in the same proportion. Since, therefore, the circumference of *OF* circle is equal to the line *LN*, *SQ* will be equal to the circumference of circle *OC*. And the surface *SLNQ*, which is the difference of the triangles, will be equal to the difference of the circles *OF*, *OC*. Furthermore, the surface *SLNQ* is equal to the quadrangular surface *SLMP*. This is proved as follows: Triangles *OMN* and *OPQ* are equal and similar because *M* and *P* are right angles, *MP* being parallel to line *SL*, and the opposite angles at *Q* are equal. Hence angle *N* is equal to angle *Q*. Therefore, the sides are proportional. But *ON* is equal to *OQ*, so that *NQ* has been divided into two equal parts at point *O*. Hence *OM* and *OP* are equal, as are *MN* and *PQ*. Therefore, the triangles are equal. Consequently the surfaces *SLNQ* and *SLMP* are equal. But line *LN* plus *SQ* is equal to *LM* plus *SP* because *MN* and *PQ* are equal. But lines *LN* and *SQ* are equal to the circumferences of circles *OF* and *OC*. Hence the lines *LM* and *SP* together are equal to these circumferences together. Therefore, the surface *SLMP* is equal to the product of the difference of the radii by half of the sum of the circumferences, and this is equal to the difference of the circles. This same thing can be proved in another way. But the above proof suffices for the present.

Let *SL*, then, be moved through the surface *SLMP* and the line *CF* through the [annular] difference of the circles *OF* and *OC*. I say, therefore, that lines *SL* and *CF* are equally moved, for they traverse equal spaces and to equal termini, as is now clear from the [following] statements: *SL* is either equally moved as *CF* or it is moved more or less [rapidly]. It is not moved more [rapidly] because it does not describe more space to more [distant] termini [by supposition 5]. Nor is it moved less rapidly, for it does not describe less space to less [distant] termini [by supposition 6]. . . . Since, therefore, *SL* is not moved more [rapidly], and is not moved less [rapidly], it must be moved equally as *CF*.

But *SL* is moved equally as any of its points by the second supposition because it is moved equally and uniformly in all of its parts and points. Hence it is moved equally as its middle point. But the middle point of *SL* is moved equally as the middle point of *CF* because these points describe equal lines in equal times [by supposition 7]. That, moreover, these lines are equal you will prove in the same way that it was proved that line *SQ* is equal to the circumference of circle *OC*. And so, therefore, *SL* is moved equally as the middle point of *CF*. But *SL* is moved equally as *CF*, as was proved. Hence *CF* is moved equally as its middle point. The demonstration is the same for any part you wish of the radius *OF* not terminated at *O* . . . [and also for the radius itself].

COMMENTARY

I.1. The basic intent of this proposition we have already indicated.[4] But we must say something more about the lack of precision in the terminology used. I have purposely left some of the ambiguities, since it would be a mistranslation to give precision where precision is lacking. In the first place, the word "motion" *(motus)* is continually used as we would use the more precise term "speed." Thus the verb "to move" is used with the meaning of "to have a speed of motion." And "to move more" means "to have a greater speed of motion."

Similarly there is a lack of precision in the use of the word "equal" as applied to movement. Not only is it used to equate one speed to another, i.e., to identify the space traversed in the same time by one line or point with that traversed by another line or point. But it is also used as we use the term "uniform," namely, to describe the character of a single movement. Here it is employed to identify the space traversed by a line or point in any one part of the duration of a movement with the space traversed by a line or point in any other equal part of the duration of the movement. There is an element of both usages in the statement of the first proposition. Thus to say a segment of a radius is moved "equally" as its middle point means that were the segment to move "uniformly" with a speed "equal" to that of its middle point when rotating so that all of its points were moving uniformly with the speed of the middle point, then it would traverse the same total area in its uniform movement as when rotating.

The proof is simple enough. He finds a rectangular area *SLMP* equal to the area which the rotating segment *CF* describes. Then he lets *SL* equal to *CF* move uniformly over *SLMP* in the same time that

4. [See the introductory remarks at the beginning of this selection. The primary objective of this proposition, and most of the others in Gerard's treatise, is to compare rectilinear and curvilinear velocities.—*Ed.*]

CF sweeps out its annular area. The areas being equal, the "movements" or speeds of the two lines, *SL* and *CF*, are by definition equal. But what is the punctual speed of all the points of *SL*? Since *SL* is moving uniformly, the speeds of all of its points are the same. Hence the speed of all its points is the speed of any one of them, and thus of the speed of its middle point. But the speed of the middle point of *SL* is the same as the speed of the middle of *CF*, since these points describe equal lines in the same time. Therefore, *SL* and *CF* being called equal in movement or speed because they describe equal spaces in equal times, *CF* is said to move equally as its middle point.

42 UNIFORM AND NONUNIFORM MOTION AND THE MERTON COLLEGE MEAN SPEED THEOREM

William Heytesbury (fl. ca. 1350)

Translated by Ernest A. Moody
Introduction and annotation by Marshall Clagett[1]

[From the works of Thomas Bradwardine, William Heytesbury, Richard Swineshead, and John Dumbleton, all of whom were associated with Merton College, Oxford, in the first half of the fourteenth century, there] emerged some very important contributions to the growth of mechanics: (1) A clear-cut distinction between *dynamics* and *kinematics*, expressed as a distinction between the *causes* of movement and the spatial-temporal *effects* of movement.[2] (2) A new approach to speed or velocity, where the idea of an instantaneous velocity came under consideration, perhaps for the first time, and with it a more precise idea of "functionality." (3) The definition of a uniformly accelerated movement as one in which equal increments of velocity are acquired in any equal periods of time. (4) The statement and proof of the fundamental kinematic theorem which equates with respect to space traversed in a given time a uniformly accelerated movement and a uniform movement where the velocity is equal to the velocity at the middle instant of the time of acceleration. It was this last theorem in a somewhat different form that Galileo states and which lies at the heart of his description of the free fall of bodies.

[Of the contributions mentioned by Clagett, the final three appear in the selection from Heytesbury reproduced here.—*Ed.*]

[Part VI. Local Motion]

[PROLOGUE]

There are three categories or generic ways in which motion, in the strict sense, can occur. For whatever is moved, is changed either in its place, or in its quantity, or in its quality. And since, in general, any successive motion whatever is fast or slow, and since no single method of determining velocity is applicable in the same sense to all three kinds of motion, it will be suitable to show how any change of this sort may be distinguished from another change of its own kind, with respect to

1. [Reprinted with permission of the Regents of the University of Wisconsin from M. Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Wis.: University of Wisconsin Press, 1959), pp. 235–237, 270–277. The translation was supplied by Ernest A. Moody to M. Clagett, who made slight alterations so that Moody's translation might better conform to other translations in the same volume. Although the selection reproduced here constitutes a single consecutive portion of Heytesbury's *Rules for Solving Sophisms (Regule solvendi sophismata)*, composed in 1335 or earlier, Clagett published it as two separate selections in successive chapters of his book. Since each selection was followed immediately by Clagett's commentary, I have retained his order and arrangement, omitting only some of the references to other documents in the volume and occasionally adding a note from material in other parts of the volume. All omissions, as indicated by points of ellipsis, were made by Clagett.

The brief introduction is drawn from Clagett's remarks on page 205.—*Ed.*]

2 [This distinction is clearly made by Thomas Bradwardine in his *Treatise on the Proportions of Velocities in Movements* of 1328 (for an extract from this work, see Selection 51.1). "In the third chapter of the work, Bradwardine treats of the 'proportion of velocities in movements in relationship to the *forces (potentias)* of the movers and the things moved,' in short, to the dynamic considerations of velocity. On the other hand, the fourth chapter treats of velocities in 'respect to the magnitudes of the thing moved and of the *space traversed (spatii pertransiti)*,' i.e., to the kinematic measure of movement. . ." (Clagett, pp. 207–208). Authors like Swineshead, and others, also described the distinction as one between the treatment of motion *causally* by forces and resistances (that is, dynamics) and by *effect*, that is, kinematically in terms of distance traversed in a given time (see Clagett, pp. 208–209).—*Ed.*]

speed or slowness. And because local motion is prior in nature to the other kinds, as the primary kind, we will carry out our intention in this section, with respect to local motion, before treating of the other kinds.

[1. MEASURE OF UNIFORM VELOCITY]

Although change of place is of diverse kinds, and is varied according to several essential as well as accidental differences, yet it will suffice for our purposes to distinguish uniform motion from nonuniform motion. Of local motions, then, that motion is called uniform in which an equal distance is continuously traversed with equal velocity in an equal part of time. Nonuniform motion can, on the other hand, be varied in an infinite number of ways, both with respect to the magnitude, and with respect to the time.

In uniform motion, then, the velocity of a magnitude as a whole is in all cases measured *(metietur)* by the linear path traversed by the point which is in most rapid motion, if there is such a point. And according as the position of this point is changed uniformly or nonuniformly, the complete motion of the whole body is said to be uniform or difform (nonuniform). Thus, given a magnitude whose most rapidly moving point is moved uniformly, then, however much the remaining points may be moving nonuniformly, that magnitude as a whole is said to be in uniform movement. . . .

[2. MEASURE OF NONUNIFORM VELOCITY]

In nonuniform motion, however, the velocity at any given instant will be measured *(attendetur)* by the path which *would* be described by the most rapidly moving point if, in a period of time, it were moved uniformly at the same degree of velocity *(uniformiter illo gradu velocitatis)* with which it is moved in that given instant, whatever [instant] be assigned. For suppose that the point *A* will be continuously accelerated throughout an hour. It is not then necessary that, in any instant of that hour as a whole, its velocity be measured by the line which that point describes in that hour. For it is not required, in order that any two points or any other two moving things be moved at equal velocity, that they should traverse equal spaces in an equal time; but it is possible that they traverse unequal spaces, in whatever proportion you may please. For suppose that point *A* is moved continuously and uniformly at *C* degrees of velocity, for an hour, and that it traverses a distance of a foot. And suppose that point *B* commences to move, from rest, and in the first half of that hour accelerates its velocity to *C* degrees, while in the second half hour it decelerates from this velocity to rest. It is then found that at the middle instant of the whole hour point *B* will be moving at *C* degrees of velocity, and will fully equal the velocity of the point *A*. And yet, at the middle instant of that hour, *B* will not have traversed as long a line as *A*, other things being equal. In similar manner, the point *B*, traversing a finite line as small as you please, can be accelerated in its motion beyond any limit; for, in the first proportional part of that time, it may have a certain velocity, and in the second proportional part, twice that velocity, and in the third proportional part, four times that velocity, and so on without limit.

From this it clearly follows, that such a nonuniform or instantaneous velocity *(velocitas instantanea)* is not measured by the distance traversed, but by the distance which *would* be traversed by such a point, *if* it were moved uniformly over such or such a period of time at that degree of velocity with which it is moved in that assigned instant.

[3. MEASURE OF UNIFORM ACCELERATION]

With regard to the acceleration *(intensio)* and deceleration *(remissio)* of local motion, however, it is to be noted that there are two ways in which a motion may be accelerated or decelerated: namely, uniformly, or nonuniformly. For any motion whatever is *uniformly accelerated (uniformiter intenditur)* if, in each of any equal parts of the time whatsoever, it acquires an equal increment *(latitudo)* of velocity. And such a motion is uniformly decelerated if, in each of any equal parts of the time, it loses an equal increment of velocity. But a motion is *nonuniformly accelerated or decelerated*, when it acquires or loses a greater increment of velocity in one part of the time than in another equal part.

In view of this, it is sufficiently apparent that when the latitude of motion or velocity is infinite, it is impossible for any body to acquire that latitude uniformly, in any finite time. And since any degree of velocity whatsoever differs by a finite amount from zero velocity, or from the privative limit of the intensive scale, which is rest—therefore any mobile body may be uniformly accelerated from rest to any assigned degree of velocity; and likewise, it may be decelerated uniformly from any assigned velocity, to rest. And, in general, both kinds of change may take place uniformly, from any degree of velocity to any other degree.

COMMENTARY

Notice in this passage the definitions of uniform velocity, uniform acceleration, and instantaneous velocity. In the definition of uniform velocity, Heytesbury speaks of the traversal of an equal space in an equal part of the time. Thus he failed to say in *any* equal parts of the time. That this would appear to be understood by him is clear, however, from his definition of uniform acceleration in terms of the acquisition of an equal increment of speed in *any* equal parts of the time. Heytesbury's contemporary, Richard Swineshead, was careful to specify that uniform velocity is to be defined by the traversal of an equal distance in *every (omni)* equal period of time. Hence Swineshead, at least, was anticipating Galileo's admonition to include the word "any" in a proper definition of uniform motion.[3]

One final important point should be noticed about this selection from Heytesbury's *De motu*. For him instantaneous velocity is to be measured or determined by the path which *would* be described by a point if that point were to move during some time interval with a uniform motion of the velocity possessed at the instant.[4]

I. Part VI. Local Motion *(continued)*

In this connection, it should be noted that just as there is no degree of velocity by which, with continuously uniform motion, a greater distance is traversed in one part of the time than in another equal part of the time, so there is no latitude (i.e., increment, *latitudo*) of velocity between zero degree [of velocity] and some finite degree, through which a greater distance is traversed by uniformly accelerated motion in some given time, than would be traversed in an equal time by a uniformly decelerated motion of that latitude. For whether it commences from zero degree or from some [finite] degree, every latitude, as long as it is terminated at some finite degree, and as long as it is acquired or lost uniformly, will correspond to its mean degree [of velocity]. Thus the moving body, acquiring or losing this latitude uniformly during some assigned period of time, will traverse a distance exactly equal to what it would traverse in an equal period of time if it were moved uniformly at its mean degree [of velocity].

2. For of every such latitude commencing from rest and terminating at some [finite] degree [of velocity], the mean degree is one-half the terminal degree [of velocity] of that same latitude.

3. From this it follows that the mean degree of any latitude bounded by two degrees (taken either inclusively or exclusively) is more than half the more intense degree bounding that latitude.

4. From the foregoing it follows that when any mobile body is uniformly accelerated from rest to some given degree [of velocity], it will in that time traverse one-half the distance that it would traverse if, in that same time, it were moved uniformly at the degree [of velocity] terminating that latitude. For that motion, as a whole, will correspond to the mean degree of that latitude, which is precisely one-half that degree which is its terminal velocity.

3. [In the Third Day of the *Dialogues Concerning Two New Sciences (Discorsi)*, Galileo defines uniform motion as follows:

"On equable (i.e., uniform) motion (*De motu aequabili*). In regard to equable or uniform *(uniformis)* motion, we have need of a single definition, which I give as follows. I understand by equal *(equalis)* or uniform movement one whose parts *(partes)* gone through *(peracte)* during any *(quibuscumque)* equal times are themselves equal. We must add to the old definition—which defined equable motion simply as one in which equal distances *(spatia)* are traversed in equal times—the word "any" *(quibuscumque)*, i.e., in "all" *(omnibus)* equal periods of time. . . ." Clagett's translation (p. 251), while based in part upon the Crew and De Salvio translation, is made more literal to show the ultimate dependence of Galileo's terminology on the Merton College formulations. Indeed, the Latin terms included by Clagett are frequently the very same terms employed by Galileo's medieval predecessors.—*Ed.*]

4. [As Clagett observes, Galileo's definition of instantaneous velocity is much the same as Heytesbury's. Here, in Clagett's translation (p. 251), is how Galileo expresses it (the omissions are Clagett's):

"Thus we may conceive that a motion is uniformly and continually accelerated when in any equal time periods equal increments of swiftness are added. . . . To put the matter more clearly, if a moving body were to continue its motion with the same degree or moment of velocity (*gradus seu momentum velocitatis*) it acquired in the first time-interval, and continue to move uniformly with that degree of velocity, then its motion would be twice as slow as that which it would have if its velocity *(gradus celeritatis)* had been acquired in two-time intervals."

Of Galileo's definitions of motion, cited in this and the preceding note, Clagett remarks (p. 252): "Note once more that Galileo in this passage compared the instantaneous velocities at the end of the first time-period and at the end of the second time-period (in a uniformly accelerated movement) by imagining that the bodies were moving uniformly over some time-period with these respective instantaneous velocities. This, as we have seen, was precisely what Heytesbury and Swineshead recommended in their treatment of instantaneous velocity. Needless to add also, the definitions of uniform velocity and uniform acceleration given by Galileo have their almost exact Merton counterparts."—*Ed.*]

5. It also follows in the same way that when any moving body is uniformly accelerated from some degree [of velocity] (taken exclusively) to another degree inclusively or exclusively, it will traverse more than one-half the distance which it would traverse with a uniform motion, in an equal time, at the degree [of velocity] at which it arrives in the accelerated motion. For that whole motion will correspond to its mean degree [of velocity], which is greater than one-half of the degree [of velocity] terminating the latitude to be acquired; for although a nonuniform motion of this kind will likewise correspond to its mean degree [of velocity], nevertheless the motion as a whole will be as fast, categorematically, as some uniform motion according to some degree [of velocity] contained in this latitude being acquired, and, likewise, it will be as slow.

6. To prove, however, that in the case of acceleration from rest to a finite degree [of velocity], the mean degree [of velocity] is exactly one-half the terminal degree [of velocity], it should be known that if any three terms are in continuous proportion, the ratio of the first to the second, or of the second to the third, will be the same as the ratio of the difference between the first and the middle, to the difference between the middle and the third; as when the terms are 4, 2, 1; 9, 3, 1; 9, 6, 4. For as 4 is to 2, or as 2 is to 1, so is the proportion of the difference between 4 and 2 to the difference between 2 and 1, because the difference between 4 and 2 is 2, while that between 2 and 1 is 1; and so with the other cases.

Let there be assigned, then, some term under which there is an infinite series of other terms which are in continuous proportion according to the ratio 2 to 1. Let each term be considered in relation to the one immediately following it. Then, whatever is the difference between the first term assigned and the second, such precisely will be the sum of all the differences between the succeeding terms. For whatever is the amount of the first proportional part of any continuum or of any finite quantity, such precisely is the amount of the sum of all the remaining proportional parts of it.

Since, therefore, every latitude is a certain quantity, and since, in general, in every quantity the mean is equidistant from the extremes, so the mean degree of any finite latitude whatsoever is equidistant from the two extremes, whether these two extremes be both of them positive degrees, or one of them be a certain degree and the other a privation of it or zero degree.

But, as has already been shown, given some degree under which there is an infinite series of other degrees in continuous proportion, and letting each term be considered in relation to the one next to it, then the difference or latitude between the first and the second degree—the one, namely, that is half the first—will be equal to the latitude composed of all the differences or latitudes between all the remaining degrees—namely those which come after the first two. Hence, exactly equally and by an equal latitude that second degree, which is related to the first as a half to its double, will differ from that double as that same degree differs from zero degree or from the opposite extreme of the given magnitude.

And so it is proved universally for every latitude commencing from zero degree and terminating at some finite degree, and containing some degree and half that degree and one-quarter of that degree, and so on to infinity, that its mean degree is exactly one-half its terminal degree. Hence this is not only true of the latitude of velocity of motion commencing from zero degree [of velocity], but it could be proved and argued in just the same way in the case of latitudes of heat, cold, light, and other such qualities.

7. With respect, however, to the distance traversed in a uniformly accelerated motion commencing from zero degree [of velocity] and terminating at some finite degree [of velocity], it has already been said that the motion as a whole, or its whole acquisition, will correspond to its mean degree [of velocity]. The same thing holds true if the latitude of motion is uniformly acquired from some degree [of velocity] in an exclusive sense, and is terminated at some finite degree [of velocity].

From the foregoing it can be sufficiently determined for this kind of uniform acceleration or deceleration how great a distance will be traversed, other things being equal, in the first half of the time and how much in the second half. For when the acceleration of a motion takes place uniformly from zero degree [of velocity] to some degree [of velocity], the distance it will traverse in the first half of the time will be exactly one-third of that which it will traverse in the second half of the time.

And if, contrariwise, from that same degree [of velocity] or from any other degree whatsoever, there is uniform deceleration to zero degree [of velocity], exactly three times the distance will be traversed in the first half of the time, as will be traversed in the second half. For every motion as a whole, completed in a whole period of time, corresponds to its mean degree [of velocity]—namely, to the degree it will have at the middle instant of the time. And the second half of the motion in question will correspond to the mean degree of the second half of that same motion, which is one-fourth of the degree [of velocity] terminating that latitude. Consequently, since this second half will last only through half the time, exactly one-fourth of the distance will be traversed in that second half as will be traversed in the whole motion. Therefore, of the whole distance being traversed by the whole motion, three-quarters will be traversed in the first half of the whole motion, and the last quarter will be traversed in its second half. It follows, consequently, that in this type of uniform intension and remission of a motion from some degree [of velocity] to zero degree, or from zero degree to some degree, exactly three times as much distance is traversed in the more intense half of the latitude as in the less intense half.

8. But any motion can be uniformly accelerated or decelerated from some degree [of velocity] to another degree in an endless number of ways, because it may be from some degree to a degree half of that, or to a degree one-fourth of it, or one-fifth, or to a degree two-thirds of that degree, or three-quarters of it, and so on. Consequently there can be no universal numerical value by which one will be able to determine, for all cases, how much more distance would be traversed in the first half of this sort of acceleration or deceleration than in the second half, because, according to the diversity of the extreme degrees [of velocity], there will be diverse proportions of distance traversed in the first half of the time to distance traversed in the second half.

But if the extreme degrees [of velocity] are determined, so that it is known, for instance, that so much distance would be traversed in such or such a time by a uniform motion at the more intense limiting degree [of velocity], and if this is likewise known with respect to the less intense limiting degree [of velocity], then it will be known by calculation how much would be traversed in the first half and also how much in the second. For, if the extreme degrees [of velocity] are known in this way, the mean degree [of velocity] of these can be obtained, and also the mean degree between that mean degree and the more intense degree terminating the latitude. But a calculation of this kind offers more difficulty than advantage.

And it is sufficient, therefore, for every case of this kind, to state as a general law, that more distance will be traversed by the more intense half of such a latitude than by the less intense half—as much more, namely, as would be [the excess of distance] traversed by the mean degree [of velocity] of this more intense half, if it moved in a time equal to that in which this half is acquired or lost uniformly, over that [distance which] would be traversed by the mean degree [of velocity] of the less intense half, in the same time.

9. But as concerns nonuniform acceleration or deceleration, whether from some degree [of velocity] to zero degree or *vice versa*, or from one degree to some other degree, there can be no rule determining the distance traversed in such or such time, or determining the intrinsic degree to which such a latitude of motion, acquired or lost nonuniformly, will correspond. For just as such a nonuniform acceleration or deceleration could vary in an infinite number of ways, so also that motion as a whole could correspond to an infinite number of intrinsic degrees [of velocity] of its latitude—indeed, to any intrinsic degree whatsoever, of the latitude thus acquired or lost.

In general, therefore, the degree [of velocity] terminating such a latitude at its more intense limit is the most remiss degree [of velocity], beyond the other limit (i.e., the most remiss extreme) of the latitude, to which such a nonuniformly nonuniform motion as a whole *cannot* correspond; and the degree [of velocity] terminating that latitude at its more remiss limit is the most intense degree [of velocity] beneath the upper limit of the same latitude, to which such a nonuniformly nonuniform motion *cannot* correspond. Consequently, it is not possible for such a motion as a whole to correspond to such a remiss degree (as that of the lower limit); nor to such an intense degree (as the upper limit).

COMMENTARY

1. An able commentary on this whole section, comparing this passage to similar passages in the *Probationes conclusionum* attributed to Heytesbury, to the section of Swineshead's *Liber calculationum* entitled *Regule de motu locali*, and to the *De motu* of John of Holland, is given by Curtis Wilson in his *William Heytesbury* (Madison, 1956), Chapter 4.

The substance of this first passage is that it makes no difference whether a body is uniformly accelerating from V_0 to V_f or uniformly decelerating from V_f to V_0 in the same time, the distance traversed, ΔS, will be the same. The reason given (without

proof) is that regardless of whether ΔV is positive or negative, the equivalent uniform speed with which a body would traverse the same distance in the same time is the mean velocity, i.e., the velocity at the middle instant of the period of acceleration. This would appear to be the earliest definitely established statement of the mean speed theorem. . . . No formal proof of this theorem is given.

2. Furthermore, it can be shown that for uniform acceleration, if $V_o = 0$, then $V_m = V_f/2$, where V_f is the final velocity of the acceleration, V_m is its mean velocity and V_o is the initial velocity.

3. Hence if $V_o > 0$, then $V_m > V_f/2$, whether we include or exclude the terminal velocities in ΔV.

4. $S_a = S_m = S_f/2$, S_a being the distance traversed in the course of the uniform acceleration from rest, S_m the distance traversed by a body moving uniformly during the same time with the mean velocity, and S_f being the distance traversed by a body moving uniformly in the same time with the final velocity.

5. When $V_o > 0$, $S_a > S_f/2$.

6. For the next conclusion, Heytesbury feels it necessary to give a fuller treatment. It is clear that Heytesbury here characterizes a uniform acceleration from rest not only by equal increments of velocity in equal time periods [see p.239—*Ed.*] but also by the fact that if any degree of speed within such an acceleration is taken, we shall always find, on dividing the time period (to that degree of velocity) into continually proportional parts, that the degrees of velocities at the ends of those periods are continually proportional in the same way. . . . Such a definition of uniform acceleration was also assumed by Swineshead in one of his proofs of the mean speed theorem. The meaning of this alternate definition of uniform acceleration can be made clearer if we pretend for a moment that the geometric system of representing movements developed later at Paris by Nicole Oresme was in existence [see, section III.vii of Selection 43—*Ed.*]. In the accompanying figure (see Fig. 1), AC represents the time of acceleration and the perpendicular lines on AC making up *in toto* the triangle represent velocities at successive instants along AC. Heytesbury's alternate definition is illustrated by taking any velocity QD. Then at time $AD/2$ there is a velocity $QD/2$, at $AD/4$ a velocity $QD/4$, etc. The same would hold true if we took the final velocity FC instead of QD, namely, there would be a velocity $FC/2$ (= EG) at time $AC/2$ (= AG).

In Heytesbury's elaboration he indicates that

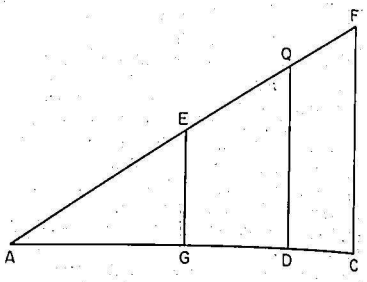


Fig. 1

EG is the mean or middle velocity, and thus there is equal distance in velocity increment (latitude) from the two extremes, namely FC and zero. Hence $FC = 2EG$. While this would appear to be obvious, Heytesbury would like to show this is so without directly reckoning with zero velocity. Assume

$\Delta V_1 = \Delta V_2$, where $\Delta V_1 = FC - EG$, and

$$\Delta V_2 = \left(EG - \frac{EG}{2}\right) + \left(\frac{EG}{2} - \frac{EG}{4}\right) + \cdots + \left(\frac{EG}{2^{n-1}} - \frac{EG}{2^n}\right) + \cdots$$

and rewriting $\Delta V_2 = \frac{EG}{2} + \frac{EG}{4} + \cdots + \frac{EG}{2^n} + \cdots$

It is clear then that, as n goes to infinity, ΔV_2 sums to EG, and thus $FC = 2EG$.

7. From the special case of uniform acceleration from rest, the mean speed theorem can be proved for uniform acceleration commencing from a finite velocity. Furthermore, from the special case of the theorem for acceleration from rest, it can be proved that the distance traversed in the first half of the uniform acceleration from rest is 1/3 of that traversed during the second half of the period of acceleration. Conversely, the distance traversed during the first half of the time of uniform deceleration to rest is three times that traversed in the second half. The proof is indicated for the case of deceleration. The deceleration of each half corresponds to its mean velocity (for the equivalent traversal of space). Thus in the second half, the mean velocity is 1/4 the initial velocity and in the first half it is 3/4 the initial velocity. Using modern formulas, $S_1 = (3\ V_f/4)\ (t/2)$ and $S_2 = (V_f/4)\ (t/2)$ and thus $S_1/S_2 = 3/1$.

8. No single universal relationship between the distances traversed in the first and second half of the time of uniform acceleration can be established if the acceleration commences with a finite velocity rather than rest. But, if we know the terminal

velocities, we can, by using the mean speed theorem, determine what the relationships are, for the proportion will be as that of the mean velocities of the respective halves of the acceleration.

9. In the case of nonuniform accelerations, no general rule is applicable for determining the equivalent uniform speed allowing for an equal traversal of space in the same time. The only evident conclusion is that the equivalent speed cannot be either of the terminal velocities. (See examples for a uniformly increasing or decreasing acceleration drawn from Swineshead's treatment [on pp. 292–294, 297 of *Science of Mechanics*—*Ed.*].)

43 THE CONFIGURATION OF QUALITIES AND MOTIONS, INCLUDING A GEOMETRIC PROOF OF THE MEAN SPEED THEOREM

Nicole Oresme (ca. 1325–1382)

Translation, introduction, and annotation by Marshall Clagett[1]

[After judging that Oresme probably composed his *Tractatus de configurationibus qualitatum et motuum (A Treatise on the Configuration of Qualities and Motions)* during the 1350's while at the College of Navarre, Clagett launches into a general description of the treatise.—*Ed.*][2]

In its entirety, the tract includes a proemium, a table of chapter titles, and ninety-three chapters divided into three parts. The first and second parts each have forty chapters and the third has thirteen. As I shall show later, there is a strong possibility that Oresme undertook the preparation of this work as an elaboration of the long section on the configuration doctrine appearing in his *Questiones super geometriam Euclidis*. Speaking generally, one can say that the first part of the *De configurationibus* establishes the tenets of the geometry of the figuration doctrine, applies the doctrine to qualities, i.e., to entities which are essentially permanent or enduring in time, and relates it to the intricacies of the internal configurations of qualities. In the course of elaborating the doctrine of internal configurations, he suggests how the theory might explain numerous physical and psychological phenomena. The second part describes how the configuration doctrine can be fruitfully applied to motion, i.e., to entities that are successive. Here again after describing the external, geometrical aspects of the doctrine, he goes on to a detailed analysis of how the actual natures of motions, possessing some kind of essential configuration, may well account for certain sonic and musical effects. And he concludes the second part with a discussion in many chapters of how these essential configurations of motion go far in explaining magical and psychological effects. Thus we see his broadly conceived doctrine of configurations giving him a physical basis to attack magic, just as his elaborate discussion of the ratios of ratios gave him a mathematical base to attack astrology.[3] Finally, in the third part, Oresme returns again to the external geometrical figures used to represent qualities and motions. He shows how in the comparison of the areas of these figures we have a basis for the comparisons of different qualities and motions.

Initially, I should point out that there are two keys to a proper understanding of the *De configurationibus*. The first is that Oresme uses the term *configuratio* with two distinguishable but related meanings, i.e., a primitive meaning and a derived meaning. In its initial, primitive meaning it refers to the fictional and imaginative use of geometrical figures to represent or graph intensities

1. [Reprinted with permission of the Regents of the University of Wisconsin, from *Nicole Oresme and the Medieval Geometry of Qualities and Motions, A Treatise on the Uniformity and Difformity of Intensities Known as Tractatus de configurationibus qualitatum et motuum*, edited with an Introduction, English Translation, and Commentary by M. Clagett (Madison, Wis.: University of Wisconsin Press, 1968), pp. 165–169, 173–175, 177–183, 191–195, 199–203, 271, 277, 283–285, 393–395, 409–411. The introduction and notes have been brought together from other parts of Clagett's volume.

For the origins and subsequent influence of Oresme's configuration doctrine, see Clagett's excellent chapter "The Configuration Doctrine in Historical Perspective," pp. 50–121. There Clagett describes the important influence of Aristotle and the fourteenth-century Merton College authors, one of whom, William Heytesbury, is represented in the preceding selection. The influence of Oresme's coordinate or graphing system, which he used to represent variations in qualities and motions, is traced through such later authors as Galileo, Thomas Hariot, John Wallis, and Christiaan Huygens.—*Ed.*]

2. [The introduction to this selection is drawn from *Nicole Oresme*. . ., pp. 14–16.—*Ed.*]

3. [For Oresme's discussion of "ratios of ratios" and his use of it to attack astrology, see Selection 51.2, notes 51 and 84, and Selection 69.—*Ed.*]

in qualities and velocities in motions. Thus the base line of such figures is the subject when we are talking about linear qualities or the time when we are talking about velocities, and the perpendiculars raised on the base line represent the intensities of the quality from point to point in the subject or represent the velocity from instant to instant in the motion. The whole figure, consisting of all the perpendiculars, represents the whole distribution of intensities in the quality, i.e., the quantity of the quality, or in case of motion the so-called total velocity, dimensionally equivalent to the total space traversed in the given time. A quality of uniform intensity is thus represented by a rectangle, which is its *configuration;* a quality of uniformly non-uniform intensity starting from zero intensity is represented as to its configuration by a right triangle. Similarly, motions of uniform velocity and uniform acceleration are represented respectively by a rectangle and a right triangle. There is a considerable discussion of other possible configurations.

Now we see that for Oresme differences in configuration taken in its primitive meaning reflect in a useful and suitable fashion internal differences in the subject. Thus we can say that the external configuration represents some kind of internal arrangement of intensities which we can call its essential internal *configuration.* In this way, we arrive at the second usage of the term configuration. Configuration in this second sense abandons the purely spatial or geometrical meaning, since one of the variables involved, namely, intensity, is not essentially spatial, although, Oresme tells us, variations in intensity can be represented by variations in the length of straight lines. . . .

Oresme suggests at great length how differences in internal configuration may explain many physical and even psychological phenomena, otherwise not simply explicable on the basis of the primary elements that make up a body. Thus two bodies might have the same amounts of primary elements in them and even in the same intensity but the configuration of their intensities may well differ, thus producing different effects in natural actions.

The second key to the understanding of the configuration doctrine of Oresme is that which we may call the suitability doctrine, and it pertains to the nature of the configurations in their primitive meaning of external figures. Briefly, it holds that any figure or configuration is suitable or fitting to describe a quality when its altitudes (ordinates, we would say in modern parlance) on any two points of its base or subject line are in the same ratio as the intensities of the quality at those points in the subject.

CHAPTERS OF PART I

I.i On the Continuity of Intensity

Every measurable thing except numbers is imagined in the manner of continuous quantity. Therefore, for the mensuration of such a thing, it is necessary that points, lines, and surfaces, or their properties, be imagined. For in them (i.e. the geometrical entities), as the Philosopher has it,[4] measure or ratio is initially found, while in other things it is recognized by similarity as they are being referred by the intellect to them (i.e., to geometrical entities). Although indivisible points, or lines, are nonexistent, still it is necessary to feign them mathematically for the measures of things and for the understanding of their ratios.[5] Therefore, every intensity which can be acquired successively ought to be imagined by a straight line perpendicularly erected on some point of the space or subject of the intensible thing, e.g., a quality. For whatever ratio is found to exist between intensity and intensity, in relating intensities of the same kind, a similar ratio is found to exist between line and line, and vice versa. For just as one line is commensurable to another line and incommensurable to still another, so similarly in regard to intensities certain ones are mutually commensurable and others incommensurable in any way because of their [property of] continuity. Therefore, the measure of intensities can be fittingly imagined as the measure of lines, since an intensity could be imagined as being infinitely decreased or infinitely increased in the same way as a line.

Again, intensity is that according to which something is said to be "more such and such," as "more white" or "more swift." Since intensity, or rather the intensity of a point, is infinitely divisible in the manner of a continuum in only one way, therefore there is no more fitting way for it to be imagined

4. The primary reference seems to be to Aristotle's *Metaphysics,* Bk. X, Chap. 1, 1052b. . . . [*Nicole Oresme*. . ., p. 438; on page 54, Clagett remarks that for Oresme it was probably this passage of the *Metaphysics* "that clinched the transference of the concept of ratios between lines to that of ratios between intensities." —*Ed.*]

5. Oresme suggests here an opinion that geometrical entities are really "nichil," i.e., they are mere mathematical fictions useful for the "measures of things" and "for the understanding of their ratios" [*Nicole Oresme* . . ., p. 438—*Ed.*].

than by that species of a continuum which is initially divisible and only in one way, namely by a line. And since the quantity or ratio of lines is better known and is more readily conceived by us—nay the line is in the first species of continua, therefore such intensity ought to be imagined by lines and most fittingly by those lines which are erected perpendicularly to the subject. The consideration of these lines naturally helps and leads to the knowledge of any intensity, as will be more fully apparent in chapter four below. Therefore, equal intensities are designated by equal lines, a double intensity by a double line, and always in the same way if one proceeds proportionally. And this is to be understood universally in regard to every intensity that is divisible in the imagination, whether it be an active or non-active quality, a sensible or non-sensible subject, object, or medium. For example, it is to be understood in regard to the light of the body of the sun, to the illumination of a medium, or to a species in the medium, to a diffused influence or power, and similarly to others, with the possible exception of curvature, concerning which we shall speak in a limited way in chapters twenty and twenty-one of this part [of our work].

Of course, the line of intensity of which we have just spoken is not actually extended outside of the point or subject but is only so extended in the imagination, and it could be extended in any direction whatever except that it is more fitting to imagine it standing up perpendicularly on the subject informed with the quality.[6]

I.iii On the Longitude of Qualities

. . . Let the extension of a quality be called its longitude and intensity its latitude or altitude.[7] But however this might be, it is obvious from the things said that certain moderns do not speak in the best way when they call the whole of the quality its latitude, just as it would be an abuse [of terminology] to understand by the breadth of a surface the whole surface or figure.[8] For just as the breadths of some unequal surfaces or figures are equal, so, as will be seen later, many latitudes of unequal qualities are equal, or vice versa.

I.iv On the Quantity of Qualities

The quantity of any linear quality is to be imagined by a surface whose length or base is a line protracted in a subject of this kind, as the preceding chapter says, and whose breadth or altitude is designated by a line erected perpendicularly on the aforesaid base in the way that the second chapter proposes. And I understand by "linear quality" the quality of some line in the subject informed with a quality. . . .

Now, reverting to the subject at hand, just as the quality of a point is imagined as a line, and the quality of a line by a surface, so the quality of a surface is imagined as a body whose base is the surface informed with the quality.[9] This will be more fully clarified as we go along. Moreover, since in any kind of a body there is an infinite number of equivalent[10] surfaces and the quality of any one of them is imagined as a body, it is not unfitting but necessary that one body be imagined to be at the same time in the place where another body—or even any other body whatever—is imagined to be. [We can think of this taking place] by penetration or by mathematical superposition or the simultaneous placing of the bodies so imagined. However, this penetration is not real. And although a surface quality is imagined by means of a body and it does not happen that a fourth dimension exists

6. The contrast between *secundum rem* [this expression is translated by "actually"—*Ed.*] and *secundum ymaginationem* [translated as "imagination" in this paragraph—*Ed.*] made here by Oresme is clear evidence that he considers the specific, external figure representing the quality as a fictional entity not existing as such in the matter (i.e., as a specific geometrical figure) [*Nicole Oresme*. . ., p. 439—*Ed.*].

7. [In I.ii Oresme had argued that—*Ed.*] lines of intensity. . . ought to be called the longitude of the quality, but generally and conventionally they are called the latitude of the quality. Now, the extent of the quality in the subject, if the quality is imagined as existing in a line, is to be imagined as a line on which the lines representing the intensity from point to point are to be erected perpendicularly. This line representing the extension of the quality in the subject, which ought to be called its latitude, is conventionally called its longitude [*Nicole Oresme*. . ., p. 16—*Ed.*]

8. ["In short, some call the whole quantity of quality 'latitude,' while it is more proper to apply it to one dimension alone (conventionally, intension)." Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Wis.: University of Wisconsin Press, 1959), p. 362. That is, some applied the term "latitude" to the whole figure representing the sum of all the intensions, a practice of which Oresme disapproved.—*Ed.*]

9. That is, Oresme is telling us that the surface quality can be represented by a three-dimensional figure that consists of the totality of perpendiculars rising on the surface informed with the quality [*Nicole Oresme*. . ., p. 17—*Ed.*].

10. One would suppose that Oresme would have conceived of them as being of infinitely small thickness, syncategorematically speaking, i.e., that they are thinner than any assignable quantity [*Nicole Oresme*. . ., p. 177, n. 4—*Ed.*].

or is imagined, still a corporeal quality is imagined to have a double corporeity: a true one with respect to the extension of the subject in every dimension and another one that is only imagined from the intensity of this quality taken an infinite number of times and dependent upon the multitude of surfaces of the subject.[11] The suitableness of this imagined concept has been touched upon before and will be more fully apparent in what follows.

I.v On the Figuration of Qualities

Every linear quality is "figured" (i.e., represented in figures) by means of a surface perpendicularly erected upon a subject line. For let *AB* be a line informed with a quality [see Fig. 1]. And since by

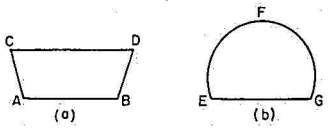


Fig. 1

the preceding chapter this quality is designated by a surface, it is necessary that it be imagined as "figured" by the surface by which it is designated or imagined. The latitude of this surface designates the intensity of this quality. It is necessary also that any point of this surface or figure outside of subject line *AB* stands perpendicularly above this same line *AB*, as is obvious in the first chapter, for otherwise the intensity and quality would be [laterally] outside the subject.[12] [This is true] because anything which according to our imagination is above this line is actually in the subject, and vice versa. Accordingly, if anything were imagined as being above the subject but not above it perpendicularly, then it would actually be outside of the subject. Thus it is obvious that no quality is to be imagined by a surface or figure having an angle at the base greater than a right angle, e.g., quadrangle *ABCD;* or by a segment of a circle that is greater than a semicircle, e.g., segment *EFG*. But some linear quality can be imagined by any other plane figure.

I.vi On the Clarification of the Figures

Although some linear quality can be correctly imagined by any plane figure other than those mentioned before, still not any quality can be imagined by any figure. Indeed no linear quality is imagined or designated by any figure except the ones in which the ratio of the intensities at any points of that quality is as the ratio of the lines erected perpendicularly in those same points and terminating in the summit of the imagined figure.[13]

For example [see Fig. 2], let line *AB* be divided

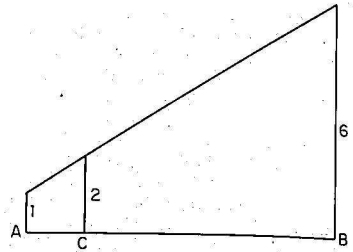


Fig. 2

in point *C* in any way such that the intensity in point *C* is double that in point *A;* and in point *B* let it be triple that in point *C*. Therefore, by the first chapter the line imagined as rising perpendicularly above point *C* and denoting the intensity at that point is double the line imagined as rising above point *A*, and the line imagined as rising above point *B* is three times the line imagined as rising above *C*. Therefore, this quality can be imagined only by the figure which at point *C* is twice as high as at point *A* or whose summit at *C* is double that at point *A*, and whose summit at point *B* is triple that at point *C*—with the further stipulation however that the figure of this sort could be varied in altitude according to the ratio of intensities in the other points of line *AB*. But

11. Hence, in effect, Oresme is telling us that we can represent a corporeal quality by an infinite number of interlacing bodies, each of which represents the quality of some surface or plane of the body (*Nicole Oresme . . .*, p. 17—*Ed.*].

12. The obvious meaning of this is that if the lines were not perpendicular, not only would they be outside of the subject vertically (and this he was prepared to grant in order to have some geometric representation of intensity which is not itself geometrical) but they would also have to be outside of the subject laterally, in the sense that the surface representing the whole quantity of the quality would not then be directly above the subject line. He had already admitted that it was quite possible to have the intensity lines intersecting the subject line at an angle other than a right angle. But it would be more fitting from the standpoint of what one might call physical intuition to have the quality kept within the lateral bounds of the subject [*Nicole Oresme. . .*, p. 441—*Ed.*].

13. This is the first statement of the suitability doctrine that lies at the heart of Oresme's system. It will be made more precise in the succeeding chapter [*Nicole Oresme . . .*, p. 441—*Ed.*].

from this it is apparent that a quality of this sort cannot be designated by a rectangle or by a semicircle; and similarly concerning an infinite number of other figures.

I.vii On the Suitability of the Figures

Any linear quality can be designated by every plane figure which is imagined as standing perpendicularly on the linear [extension of the] quality and which is proportional in altitude to the quality in intensity. Moreover, a figure erected on a line informed with a quality is said to be "proportional in altitude to the quality in intensity" when any two lines perpendicularly erected on the quality line as a base and rising to the summit of the surface or figure have the same ratio in altitude to each other as do the intensities at the points on which they stand.

For example [see Fig. 3], let there be line *AB* on

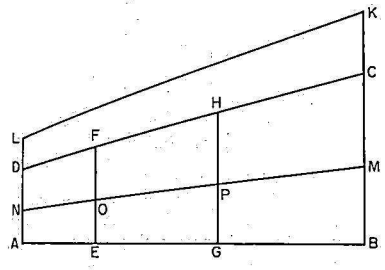


Fig. 3

which surface *ABCD* stands and let the two lines *EF* and *GH* be erected on the base. If, therefore, the ratio of *EF* to *GH* is as the ratio of the intensity in point *E* to the intensity in point *G*, and similarly for the other points and their corresponding lines, then I say that this surface or figure is "proportional in altitude to this quality in intensity," so that the altitude of the surface is similar to the intensity of this quality. Therefore, this quality is most fittingly designated by such a figure or surface.[14] Moreover, since on the same line *AB* a great number of surfaces can be erected which are proportional or similar in altitude—some of which are larger and some smaller than *ABCD*, as for example surface *ABKL* which is larger and surface *ABMN* which is smaller, and any number of others which would be of similar although unequal altitude—it follows that the quality of line *AB* can be designated by any one of them indifferently. There is however this provision: if the quality is imagined by some one of these designated figures, then with this figuration retained a quality which is double the original one in intensity and similar to it will be designated by a figure of similar altitude but twice as high.[15] The same thing holds proportionally for any greater or lesser quality, notwithstanding the fact that the first quality could have been imagined in the beginning by a greater or lesser surface or figure. Moreover these greater or lesser surfaces are unequal in area, dissimilar in figure and also unequal in altitude, and yet they are similar or proportional in altitude. Hence, if two points *O* and *P* are marked in the intersections as in the accompanying figure, then if $GH/EF = GP/EO$ and similarly in regard to any two lines erected in like fashion on the base *AB*, I say that surface *ABCD* and surface *ABMN* are of similar or proportional altitude.

I.xi On Uniform and Difform Quality

And so every uniform quality is imagined by a rectangle[16] and every quality uniformly difform terminated at no degree is imaginable by a right triangle.[17] Further, every quality uniformly difform

14. [Clagett explains (p. 441) that it is from this sentence and the title of chapter vii that he coined the expression "suitability doctrine"—*Ed.*].

15. This constitutes an important proviso to the general statement that any figure whose altitudes (ordinates) were in the same ratio as the intensities would be suitable. The proviso merely holds that when we are *comparing* qualities, we must take as the basis of our comparison some specific figure of the infinite possible figures of the same kind [*Nicole Oresme . . .*, p. 442—*Ed.*].

16. The quality that is uniform in intensity is obviously represented by a rectangle, since the altitude of the rectangle is uniform throughout; i.e., the perpendicular height above any point of the base is the same [*Nicole Oresme. . .*, p. 18—*Ed.*].

17. [Uniformly difform qualities are] qualities in which the intensity of the subject quality varies in a uniform way as it is distributed through the subject (see I. viii–I.ix). Oresme discusses two categories of uniformly difform qualities. The first one is a uniformly difform quality that begins from zero intensity and ends with a given intensity, or which begins at a given intensity and ends with zero intensity. Such a quality is to be represented by a right triangle. For this reason it is called in the title of the chapter (I.viii) a 'right-triangular quality.' Oresme is once more naming the quality from the figure by which it can be represented. Again Oresme applies the suitability doctrine and concludes that this quality could be equivalently represented without any difference by every triangle having a right angle on the base. This is proved by showing that the ratio of any two ordinates on the base of a right triangle on a given base is the same regardless of the height of the triangle. Therefore, some quality is assimilated to any one of these triangles, and, further, the *same* quality can be assimilated to any

terminated in both extremes at some degree is to be imagined by a quadrangle having right angles on its base and the other two angles unequal. Now every other linear quality is said to be "difformly difform" and is imaginable by means of figures otherwise disposed according to manifold variation. Some modes of the "difformly difform" will be examined later. The aforesaid differences of intensities cannot be known any better, more clearly, or more easily than by such mental images and relations to figures, although certain other descriptions or points of knowledge could be given which also become known by imagining figures of this sort: as if it were said that a uniform quality is one which is equally intense in all parts of the subject, while a quality uniformly difform is one in which if any three points [of the subject line] are taken, the ratio of the distance between the first and the second to the distance between the second and the third is as the ratio of the excess in intensity of the first point over that of the second point to the excess of that of the second point over that of the third point, calling the first of those three points the one of greatest intensity.[18]

Let us clarify this first with respect to a quality uniformly difform which is terminated at no degree and which is designated or imagined by $\triangle ABC$ [see Fig. 4 (a)]. With the three perpendicular

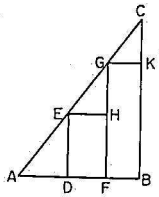


Fig. 4(a)

lines *BC*, *FG*, and *DE* erected, then let *HE* be drawn parallel to line *DF* and similarly *GK* parallel to line *FB*. Therefore, the two small triangles *CKG* and *GHE* are formed and they are equiangular. Hence, by [proposition] VI.4 of [the *Elements* of] Euclid, $GK|EH = CK|GH$, *CK* and *GH* being excesses. And since $GK = FB$ and similarly $EH = DF$, so $FB|DF = CK|GH$, *FB* and *DF* being the distances on the base of the three points and *CK* and *GH* being the excesses of altitude proportional to the intensity of these same points. Since, therefore, the quality of line *AB* is such that the ratio of the intensities of the points of the line is as the ratio of the altitudes of the lines perpendicularly erected on those same points, that which has been proposed is evidently clear, namely that the ratio of the excess in intensity of the first point over the second to the excess of the second over the third is the same as the ratio of the distance between the first and second points to the distance between the second and the third, and similarly for any other three points. Hence what we have premised in regard to a quality difform in this way is quite fitting, and so it (this quality) was well designated by such a triangle.

By the same method the aforesaid description or property can be demonstrated for a quality uniformly difform terminated in both extremes at [some] degree, and thus for one which we let be imagined by quadrangle *ABCD* in which line *DE* is drawn parallel to base *AB* forming $\triangle DEC$ [see Fig. 4 (b)]. Then let lines of altitude be drawn in the

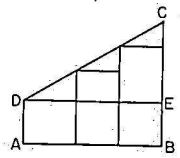


Fig. 4(b)

quadrangle and also transversals parallel to the base in this triangle, thus forming small triangles. And then one can easily argue concerning the excesses and the distances in this triangle just as was argued in the other one. This will be easily apparent to one who is observant.

Further, every quality which is disposed in [any] other way than those described earlier is said to be "difformly difform." It can be described negatively as a quality which is not equally intense in all parts of the subject nor in which, when any three points of it are taken, the ratio of the excess of the first over the second to the excess of the second over the third is equal to the ratio of their distances.

I.xiv On Simple Difform Difformity

We now treat of difform difformity; there are

other one of them and be imagined by it. There is this proviso, however: if some quality is designated by one triangle, another quality of similar but double intensity must be designated by a triangle that is twice as high, and similarly for the proportionally greater [intensities]. [*Nicole Oresme*. . ., p. 19—*Ed.*].

18. This is similar in modern parlance to saying that we have a figure of a uniformly difform quality when the slope of the summit line is a constant [*Nicole Oresme* . . ., p. 20—*Ed.*].

two modes of such difformity: simple and composite. We must first talk of the simple mode. Simple difform difformity is that which can be designated by a figure whose line of summit or line of intensity is a single line, i.e. not composed of several lines. It is necessary, therefore, that the line be a curve; because if it were straight, then it would be simply a uniformity or uniform difformity, as is clear from the preceding chapter. Furthermore, it is necessary that the curvature of the summit line does not attain that of a circular segment greater than a semicircle so that the angle[19] on the base is greater than a right angle, as was clear in chapter five. However, it can happen that the angle on the base is less than a right angle by any amount you please.

Therefore, for example, let there be line *AB*, whose quality can be designated by semicircle *ACB* [see Fig. 5]. This is possible, as is evident

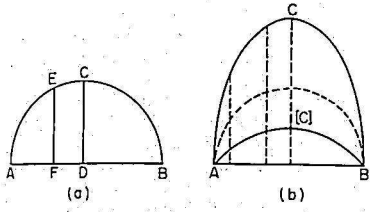


Fig. 5

from chapter seven. And so I now say that the same quality of line *AB* is imaginable or can be designated by a figure having an altitude greater or less than that of the semicircle by any amount you please. For let line *CD* be drawn as a perpendicular to center *D* and again let another line *EF* be drawn as a perpendicular to line *AB*. Therefore, since it is possible to construct on the same points two other perpendiculars less than *CD* and *EF* but having the same ratio between them as do *CD* and *EF* and in the same way to construct on all the points of line *AB* perpendiculars which are greater or less than the corresponding perpendiculars in semicircle *ACB* constructed on those points of *AB* and having between any two of them the same ratio as the corresponding perpendiculars on *AB* in semicircle *ACB*, it follows that there can be erected on base *AB* a figure of less height but which will be proportional in altitude to this semicircle and with equal reason a figure of greater height by any amount you wish. Therefore, by chapter seven the quality of line *AB* can be correctly imagined by any of these figures without it making any difference [which figure is used].[20]

For if it were not so that the quality of line *AB* imaginable by the semicircle could be imagined by a figure greater or less than the semicircle which is proportional [in altitude to the semicircle] it would follow (1) that the intensity of point *D* could not be correctly designated by a greater or lesser line than *DC*, and similarly for all the points, unless the intensity were varied, and thus (2) that any intensity would in itself determine the definite length of the line by which it would be imaginable, and then (3) an intensity would be equivalent and comparable to a line or to quantitative extension, and as a consequence (4) local motion would be comparable in velocity to [qualitative] alteration, all of which seems excessively absurd.

However, any figure by which this quality of line

19. In this case the angle would be a mixed angle composed of the curve and the straight base line [*Nicole Oresme*. . ., p. 199, n. 1—*Ed.*].

20. [In coping with the problem of the quality represented by a semicircle, Oresme altered an earlier view and—*Ed.*] now realized in the *De configurationibus* that such a quality could be represented by higher or lower figures whose altitudes (i.e., ordinates) are in the same ratio as those of the semicircle. He was puzzled as to what kinds of figures these would be. He definitely rejects the possibility [see the final paragraph of I.xiv—*Ed.*] that the figures of higher altitude could be segments of a circle, while he says that he will not consider the case of the figures of lower altitudes. Incidentally, the compositor of MS J of the *De configurationibus* independently proves the case of the figures lower than the semicircle and further suggests that a similar proof can be constructed for the case of the figures higher than the semicircle. [Clagett gives the Latin text and translation in Appendix III of his edition.—*Ed.*]. Unfortunately, Oresme had little or no knowledge of conic sections. For, in fact, the conditions he specifies for these curves comprise one of the basic ways of defining ellipses: if the ordinates of a circle $x^2 + y^2 = a^2$ are all shrunk (or stretched) in the same ratio a/b, the resulting curve is an ellipse whose equation is $x^2/a^2 + y^2/b^2 = 1$. Hence, without realizing it, Oresme has given conditions which show that the circle is merely one form of a class of curves we call elliptical. It should be remarked that even if Oresme had had some knowledge of Greek conic sections, he might well have had difficulty in applying the Greek formulation of the ellipse to his figures, which are like curves in Cartesian geometry. [*Nicole Oresme* . . ., pp. 442–443; in the next chapter Oresme elaborates four kinds of simple difform difformity: (1) rational convex, (2) rational concave, (3) irrational convex, and (4) irrational concave. In I.xv, these four are added to simple uniformity and uniform difformity to constitute six simple figurations of qualitative intensity from which sixty-three species of composite difformity can be formed—*Ed.*].

AB is imaginable is curved. But whether the figure less than a semicircle by which this quality can be imagined is a segment of a circle, I leave aside as a matter to be discussed. But I do say that it cannot be designated by a greater figure which is at the same time a segment of a circle. For this quality can be designated by no figure of which *AB* is not the base or chord. But *AB* cannot be the chord in a circle smaller than circle *ACB* if that circle were completed, for *AB* is the diameter of that circle. Therefore, this quality cannot be imagined by a greater figure which is a segment of a smaller circle than circle *ACB;* nor also of a greater figure which is a segment of a larger circle. [This last is evident,] for then that segment would either (1) be greater than half of its own circle, and therefore no quality could be designated by it, as is clear from chapter five, or (2) it would be less than half its own circle. [But in the case of the second possibility,] since the segment which is less than half of a larger circle would have the same chord as semicircle *ACB*, the segment would be less [in area] and would be a part of semicircle *ACB*, as is easily evident and can be proved by the last [proposition] of the sixth [book] of [the *Elements* of] Euclid. Therefore, this quality cannot be designated by a figure which is a segment of a circle and is [at the same time] greater [in altitude and area] than semicircle *ACB*, and yet it can be designated by [some] greater curved figure, as was proved before. Therefore, the curvature of the greater [curved] figure will not be circular but will bound a figure which in altitude is proportional to that which the circular curvature bounds; and so there will be two figures proportional in altitude, the curvature of one being circular and that of the other being non-circular.

HERE BEGINS THE SECOND PART OF THIS TRACT AND IT TREATS OF THE DIFFORMITY OF SUCCESSIVE THINGS

II.i On the Double Difformity of Motion

Every successive motion of a divisible subject has parts and is divisible in one way according to the division and extension or continuity of the mobile, in another way according to the divisibility and duration or continuity of time, and in a third way—at least in imagination—according to the degree and intensity of velocity. From its first continuity motion is said to be "great" or "small"; from its second, "short" or "long," and from its third, "swift" or "slow." . . .

II.iii On the Quantity of the Intensity of Velocity

Since each uniformity of motion posited in the first chapter consists in equality of intensity and each difformity arises from inequality [of intensity] we ought to set out first [the measure of gradual intensity, i.e. we ought to specify] with what the gradual intensity of the velocity is measured. However, in the matter of velocity three closely related ideas can be considered. One is the total quantity of the velocity taking into account both intensity and extension. I shall speak of this in the third part of this tract, which will be concerned with the measures of qualities and velocities. Another thing to be considered in connection with velocity is the denomination in terms of which a subject is said to become such a kind more quickly or more slowly. I shall also speak of this in the following chapter. Third, there is the gradual intensity [of velocity]. This is the subject which must now be considered. Therefore, I say universally that that degree of velocity is absolutely more intense or greater by means of which in an equal time more is acquired or lost of that perfection according to which the motion takes place. For example, in local motion that degree of velocity is greater and more intense by means of which more space or distance would be traversed. In alteration, similarly, that degree of velocity is greater by means of which more intensity of quality would be acquired or lost; and so in augmentation, by means of which more quantity is acquired, and in diminution, by means of which more quantity or extension is lost. And so generally [our definition would hold] wherever motion would be found.

II.v On Certain Other Successions in Motion

. . . . . .

There can be imagined one further succession,[21] for every velocity is capable of being increased in intensity and decreased in intensity. Now its continuous increase in intensity is called acceleration, and indeed this acceleration or augmentation of velocity can take place more quickly or more slowly. Whence it sometimes happens that velocity is increasing and acceleration is decreasing, while sometimes both are simultaneously increasing. Similarly acceleration of this sort sometimes takes place uniformly and sometimes non-uniformly and in diverse ways. But since every divisibility or succession which is found in acceleration of this sort is according to parts of the subject, or according to parts of the time, or according to gradual intensity, and from such threefold divisibility arises twofold uniformity or difformity, as was demonstrated in the first chapter, therefore, as before, every uniformity and difformity which can so arise is reduced to the two above-mentioned kinds, that is, to uniformity and difformity according to parts of the subject or uniformity and difformity according to parts of the time. And so let us first speak of that which is according to parts of the subject.

21. [The first part of II. v considers "succession according to inception," where successive parts of a body begin to be moved while yet others remain at rest, and succession according to quantitative parts—*Ed.*].

HERE BEGINS THE THIRD PART [OF THIS TREATISE]: ON THE ACQUISITION AND MEASURE OF QUALITIES AND VELOCITIES

III.i How the Acquisition of Quality Is To Be Imagined

Succession in the acquisition of quality can take place in two ways: (1) according to extension, (2) according to intensity, as was stated in the fourth chapter of the second part. And so extensive acquisition of a linear quality ought to be imagined by the motion of a point flowing over the subject line in such a way that the part traversed has received the quality and the part not yet traversed has not received the quality. An example of this occurs if point *c* were moved over line *AB* so that any part traversed by it would be white and any part not yet traversed would not yet be white [see Fig. 6(a)]. Further the extensive acquisition of

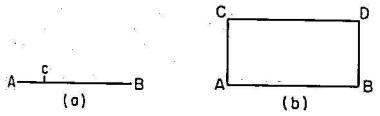


Fig. 6

a surface quality ought to be imagined by the motion of a line dividing that part of the surface that has been altered from the part not yet altered. And the extensive acquisition of a corporeal quality in a similar way is to be imagined by the motion of the surface dividing the part altered from the part not yet altered.[22]

The intensive acquisition of punctual quality is to be imagined by the motion of a point continually ascending over a subject point and by its motion describing a perpendicular line imagined [as erected] on that same subject point. But the intensive acquisition of a linear quality is to be imagined by the motion of a line perpendicularly ascending over the subject line and in its flux or ascent leaving behind a surface by which the acquired quality is designated. For example [see Fig. 6(b)], let *AB* be the subject line. I say, therefore, that the intension of point *A* is imagined by the motion, or by the perpendicular ascent, of point *C*, and the intension of line *AB*, or the acquisition of the intensity, is imagined by the ascent of line *CD*. Further, the intensive acquisition of a surface quality is in a similar way to be imagined by the ascent of a surface, which (by its motion) leaves behind a body by means of which that quality is designated. And similarly the intensive acquisition of a corporeal quality is imagined by the motion of a surface because a surface by its imagined flux leaves behind a body, and one does not have to pose a fourth dimension, as has been said in the fourth chapter of the first part.

One should speak and conceive of the loss of quality in the same way that we have now spoken of its acquisition, whether that loss is of extension or intensity. For such loss is imagined by movements which are the opposite of the movements described before. Furthermore, one ought to speak of the acquisition or loss of velocity, both in extension and intensity, in the same way we have just spoken of the acquisition or loss of quality.

III.vii On the Measure of Difform Qualities and Velocities

Every quality, if it is uniformly difform, is of the same quantity as would be the quality of the same or equal subject that is uniform according to the degree of the middle point of the same subject.[23] I understand this to hold if the quality is linear. If it is a surface quality, [then its quantity is equal to that of a quality of the same subject which is uniform] according to the degree of the middle line; if corporeal, according to the degree of the middle surface, always understanding [these concepts] in a conformable way. This will be demonstrated first for a linear quality. Hence let there be a quality imaginable by $\triangle ABC$, the quality being uniformly

22. For reasons of symmetry, Oresme also extends his concept to the production of surface and corporeal qualities by the movement of lines and surfaces. But neither here nor in the next chapter on the summits of the configurations representing surface or corporeal qualities does Oresme take up the difficulties of actually using his imagery. His knowledge of geometry was simply inadequate for any detailed discussions of the kind he presents for linear qualities [*Nicole Oresme. . .*, p. 491—*Ed.*].

23. This is, of course, the famous Merton College Rule of uniformly difform [motion—*Ed.*]. And this chapter with its geometric proof constitutes the most significant chapter of the work, historically speaking [*Nicole Oresme. . .*, p. 494—*Ed.*].

difform and terminated at no degree in point *B* [see Fig. 7(a)]. And let *D* be the middle point of the

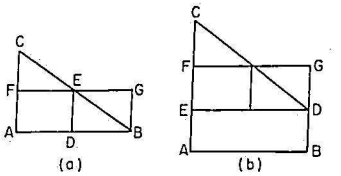


Fig. 7

subject line. The degree of this point, or its intensity, is imagined by line *DE*. Therefore, the quality which would be uniform throughout the whole subject at degree *DE* is imaginable by rectangle *AFGB*, as is evident by the tenth chapter of the first part. Therefore, it is evident by the 26th [proposition] of [Book] I [of the *Elements*] of Euclid that the two small triangles *EFC* and *EGB* are equal. Therefore, the larger $\triangle BAC$, which designates the uniformly difform quality, and the rectangle *AFGB*, which designates the quality uniform in the degree of the middle point, are equal. Therefore the qualities imaginable by a triangle and a rectangle of this kind are equal. And this is what has been proposed.[24]

In the same way it can be argued for a quality uniformly difform terminated in both extremes at a certain degree,[25] as would be the quality imaginable by quadrangle *ABCD* [see Fig. 7(b)]. For let line *DE* be drawn parallel to the subject base and $\triangle CED$ would be formed. Then let line *FG* be drawn through the degree of the middle point which is equal and parallel to the subject base. Also, let line *GD* be drawn. Then, as before, it will be proved that $\triangle CED = \square EFGD$. Therefore, with the common rectangle *AEDB* added to both of them, the two total areas are equal, namely quadrangle *ACDB*, which designates the uniformly difform quality, and the rectangle *AFGB*, which would designate the quality uniform at the degree of the middle point of the subject *AB*. Therefore, by chapter ten of the first part, the qualities designatable by quadrangles of this kind are equal.

It can be argued in the same way regarding a surface quality and also regarding a corporeal quality. Now one should speak of velocity in completely the same fashion as linear quality, so long as the middle instant of the time measuring a velocity of this kind is taken in place of the middle point [of the subject].[26] And so it is clear to which

24. It is clear, then, that what Oresme has done is to give a rudimentary geometric proof of the so-called Merton Rule of measuring uniform acceleration. [*Nicole Oresme*. . ., p. 46; see Heytesbury's statement, without formal proof, of the Merton Rule, or mean speed theorem, and Clagett's commentary, in the preceding selection—*Ed.*].

The question has often been raised in connection with this proposition of III.vii and its geometric proof whether Oresme realized that the areas of his figures—that is, the areas of the right triangle and rectangle in this proof—represented, or were in the same ratio as, the distances when the theorem was applied to velocities. I think that there can be little doubt that he did realize this, although he certainly does not make a central point of it. One has to remember that in the Merton Rule of uniformly difform motion, it is the distances of the uniform motion and uniform acceleration that are the same, and indeed this is so specified by Oresme in his *Questions on the Geometry of Euclid* (Appendix I, Question 13, lines 72–77). Hence when it is shown in this chapter of the *De configurationibus* that the areas are equal, it is evident that they must represent distances or, to put it more precisely, whatever ratio one area has to another one distance will have to another. This is even more apparent in III.viii of the *De configurationibus*, where after determining the ratios of the various areas over the proportional parts of the base, Oresme specifically states that the distances traversed are in the same ratios as those already specified for the areas. Thus it would appear that for Oresme, just as the simple denomination of surfaces was to be determined by attention to the dimensions of length and breadth, so the denomination of motions was to be determined by the distances which the areas represented or were proportional to, and thus by attention to the "latitudes" and "longitudes" of motion. Admittedly this is somewhat obscured by the method of calculation by means of the compound ratios of intensities and extensions outlined in III.vi; but this obscurity rises from the medieval and antique practice of using Euclidian proportionality statements in which the ratios of like quantities must be used, so that the ratios of the motions are determined by the form: Total velocity *a*/Total velocity $b = (v_1/v_2) \cdot (t_1/t_2)$, where v_1 and v_2 are the average velocities, while the modern expression would simply be: Distance *a*/Distance $b = v_1t_1/v_2t_2$. That the techniques of integral calculus are needed for more rigorous proofs concerning "areas under curves," as Oresme's figures would be called in analytic geometry, should not blind us to the apparent fact that for Oresme these figures were representative of, or proportional to, the distances traversed, or that in his terms the "total velocity" imagined by such a figure is strictly proportional to the distance traversed in the time imagined by the base line of the figure [*Nicole Oresme*. . . , p. 47—*Ed.*].

25. [This case is merely mentioned without discussion by Heytesbury; see above, Part VI *(continued)*, par. 7.—*Ed.*]

26. This statement shows that Oresme realized that the same geometric proof holds for uniformly difform motion, i.e., uniformly accelerated motion. Hence it is clear that this proof is essentially the same as Galileo's first theorem in the "Third Day" of the *Discorsi*. . . [*Nicole Oresme*. . . , p. 495—*Ed.*].

uniform quality or velocity a quality or velocity uniformly difform is equated. Moreover, the ratio of uniformly difform qualities and velocities is as the ratio of the simply uniform qualities or velocities to which they are equated. And we have spoken of the measure and ratio of these uniform [qualities and velocities] in the preceding chapter.

Further, if a quality or velocity is difformly difform, and if it is composed of uniform or uniformly difform parts, it can be measured by its parts, whose measure has been discussed before. Now, if the quality is difform in some other way, e.g. with the difformity designated by a curve, then it is necessary to have recourse to the mutual mensuration of the curved figures, or to [the mensuration of] these [curved figures] with rectilinear figures; and this is another kind of speculation. Therefore what has been stated is sufficient.

Dynamics

44 DOES FINITE AND TEMPORAL MOTION REQUIRE A RESISTANT MEDIUM? THE RESPONSES OF AVERROES AND AVEMPACE IN COMMENT 71

Averroes (1126–1198)

Translations, introduction, and annotation by John E. Murdoch and Edward Grant[1]

The comment by Averroes on the text of Aristotle from Book IV of the *Physics* presented here proved to be one of the most important in medieval physics. Here we find Averroes agreeing with Aristotle that finite motion could occur only in a resistant medium and arguing against Avempace (Ibn Bajja, a Spanish Muslim of the twelfth century), who insisted that temporal motion could occur in a resistance-less medium and therefore, presumably, in a void, though strictly speaking he does not mention void in the passages quoted by Averroes. So important was this particular text of Averroes (it was known in the Middle Ages as Comment 71 of Bk. IV) that it gave rise not only to discussions about the possibility of motion in a hypothetical void but also raised the question as to whether inorganic elemental bodies could be self-moved. (Although Averroes denied the self-motion of inorganic bodies, his explanation was ambiguous and seemed to allow it; in any event, Thomas Aquinas criticised him for invoking the form of a body as the mover in natural motion. See below, n. 35). A number of selections in this source book are concerned, wholly or in part, with the issues just described.

1. [The Text of Aristotle][2]

And that which we have asserted is also made apparent in the following way: We see that one and the same body is moved faster because of one or the other of two reasons: either (1) because the medium in which it is moved is different (for example, its motion may occur in water, or in earth, or in air, or in fire), or (2) because the moved thing [itself] is different.

If, therefore, other things, which are due to the difference in heaviness and lightness [of the moved body], are equal, the medium in which the motion will occur will be the cause.[3] For it resists the motion, either to a great extent, when the motion of that [medium] is contrary to the motion of that which is moved in it, or much less so, when it

1. Translated from Averroes' Comment 71 on Book IV of Aristotle's *Physics*, as found in *Aristotelis opera cum Averrois commentariis* [*The Works of Aristotle with the Commentaries of Averroes*] (Venice, 1562–1574; Frankfurt: Minerva, 1962), Vol. IV, fols. 158 recto, col. 1, to 162 recto, col. 1. Aristotle's discussion, which prompted Averroes' lengthy commentary and which immediately precedes it in the volume cited, has also been translated here from the version rendered from Arabic into Latin in the twelfth century.

2. The text is part of Book IV, chapter 8, of Aristotle's *Physics* (215a.24—215b.20).

3. It is interesting to compare this with the Oxford translation made directly from the modern Greek text of Aristotle. "We see the same weight or body moving faster than another for two reasons, either because there is a difference in what it moves through, as between water, air, and earth, or because, other things being equal, the moving body differs from the other owing to excess of weight or of lightness." For Aristotle, the phrase "other things being equal" (or "are equal") is a qualification establishing the possibility that a difference in weight may be the cause of a greater, or lesser, speed. That Averroes understands this passage to be a qualification which *excludes* the possible differences in weight, and hence refers to the alternative, that a variation in media is the cause, is clear from his gloss below on "other things are equal."

[that is, the medium] is at rest. And the [medium] offers greater [resistance] when it is not easily divided, that is, when it is such as that in which there is a density *(spissitudo)*.

Therefore, let body *A* be moved *(expellatur)* in medium *B* for time *C*, and for time *H* in medium *D*, which is more rare *(subtilius)* and, insofar as it is an impeding [or resisting] body, equal in length to *B*. The purpose of this is the following: Since *B* could be water and *D*, air, then to the extent that air is rarer and of lighter corporeity, to that extent will *A* be moved faster in medium *D* than in medium *B*. And let the ratio of speed to speed be as the ratio of the excess [in rarity] of air over water in such a way that if [the air] is twice as rare, then [*A*] will traverse space *B* in twice the time in which it traverses space *D*. And then the time *C* will be double time *H*. And always by so much as the medium in which there will be motion is of lighter corporeity and of less resistance and more easy to divide, so much faster will be the movement *(expulsio)*.

Indeed, a void has no ratio *(comparationem)* to a body by which that body exceeds it, just as that which is called zero[4] has absolutely no ratio to that which is called a number. For if 4 exceeds 3 by 1, and exceeds 2 by more, and further exceeds 1 by even more, still it does not have a ratio to zero *(non unum)* by which it exceeds zero. For that which exceeds is necessarily divisible into that which is exceeded and the excess *(augmentum)*. Therefore, 4 is divisible into these two things: into the excess [by which it exceeds zero] and into zero.[5] Thus it is that a line does not exceed a point, since it is not composed of points. Similarly, a void does not have a ratio to a plenum; therefore, there will be no motion in it.

2. [Averroes' *Expositio* on the Text]

This is another demonstration in which he [Aristotle] declares that it is impossible for motion to occur in a medium which is a void. And he has said: *"And that which we maintain is also apparent"*,[6] namely that it is impossible that motion occur in a void. And this is apparent above because we see that two heavy bodies differ in motion according to speed and slowness *(velocitatem et tarditatem)* for two reasons. And these are (1) If the body be the same or if there be two bodies equal in weight, then the difference will be due to that in which they are moved. For example, the same body is moved faster in air than in water (the case is similar with two bodies equal in weight, shape, and size). However, (2) If the bodies are different, and they are moved in one medium, then the difference in motion will be due to the difference in heaviness and lightness *(gravitas et levitas)*, that is, that the heavier of them will be faster. And, after he has asserted this proposition—namely that the cause of the speed and the slowness in movable bodies is either the difference in the density and rarity *(spissitudo et tenuitas)* of the medium, or the difference in heaviness and lightness of the bodies—he says, *"Other things. . . are equal."* That is, if the other things that exist in bodies [and which are] capable of causing speed or slowness (as the difference in weight, shape, and size of bodies) are the same in two different bodies, then, as we see that that which is moved in air is moved faster than that which is moved in water, it is quite clear that the cause of the diversity in motion is the medium in which the motion occurs. And the same thing is true in the case of one body, that is, we see that its motion in a rarer body [or medium] is faster than in a denser body [or medium].

Then he specifies the reason why this occurs in a medium and says, *"As it resists* [the motion] *to a great extent*, and so on." That is, a retardation is caused because that in which the motion will occur [that is, the medium] will resist the thing moved, and this resistance varies by degrees. For it will resist a great deal when the medium is moved with a motion contrary to that which is moved in it. And it will resist less when it is at rest.

Then [Aristotle] says, *"And* [the medium] *offers*

4. What has been translated here as "zero" is literally "that which is not called 1 *(quod non dicitur unum)*.

5. In his exposition of this sentence Averroes makes it clear that this division of 4 into an excess and zero is impossible. Therefore, 4 does not really exceed zero; nor, *mutatis mutandis*, does a plenum exceed a void in density, or a void a plenum in rarity.

6. Here, and in all subsequent instances in this translation, the cue words drawn from the Aristotelian text just translated are italicized in our translation of Averroes' exposition. Frequently, for greater ease of reference and intelligibility, more of the English translation is repeated than is warranted by the few cue words cited by Averroes. It should also be noted that the Latin cue words are not always those actually found in the medieval text of Aristotle and therefore, in those instances, will not be the very same words that appear in our translation.

greater [resistance], and so on." That is, a medium which, by its motion, resists a contrary motion, resists more when it is dense and difficult to divide. Therefore, the cause of the diversity of motions of the same moved thing—or of mobiles like one another—depends on the difference of the media with respect to two factors, namely in the difficulty and facility of the resistance offered by these media, be they in motion or at rest.

Then he tells us what [kind of medium] is difficult to divide and says, *"It is that in which there is great density."* And, when he has asserted this proposition, he begins to explain his contentions by using letters. He says, *"Therefore, let* [body *A*] *be moved* and so on." That is, therefore, let *A* be the body which is moved in a medium, and let *B* be the medium in which it is moved, and let *C* be the time during which it is moved. Moreover, let it be assumed that *A* is moved in another medium more rare than the first, and let this [second medium] be *D*, and the time during which *A* is moved in this medium *H*. And let *A* be moved through equal spaces in the two media, that is, in the more dense *B* and the more rare *D*. Therefore, it follows that the time in which [*A*] traverses these two spaces will be different. For we know that the swifter traverses the same space in less time. Moreover, since the cause of speed and slowness in these two motions [that is, the motion of *A* in *B* and in *D*] is the difference of the media in rarity and density, it follows that the ratio of time to time is as the ratio of the density which exists in one medium to that which exists in the other. Similarly, so stands the ratio of motion to motion.[7]

And since he [Aristotle] gives an example in letters, he [also] gives an example in terms of [elemental] matter so that the demonstration will be clearer and that what seems less [clear] in one example will appear in the other. And he says, *"The purpose of this is* [the following], and so on"; for example, water the denser medium, is assumed in place *B*, and air, the rarer medium, in place *D*.[8] Therefore, as air is rarer than water and of less resistance against something that is moved [in it], so will the velocity of motion *A* in it be greater than in water. Hence the ratio of speed to speed will be as the ratio of the excess of the rarity of air over the rarity of water. And indeed he says, *"Therefore the ratio of speed to speed will be,* and so on," that is, it follows from these propositions that a ratio of the speed of the motion of *A* in *B* to its motion in *D* is as the ratio of the excess of the rarity of air over the rarity of water. Then he says that *"If* [the air] *is twice as rare*, and so on," that is, it follows that if we assume that air is twice as rare as water, a thing moved in water traverses in twice the time a distance equal to the distance it traverses in air. Therefore, the time *C*, in which the moved thing traverses a distance in the denser body [or medium], is twice time *H*, in which it is moved an equal distance in the rarer medium, *D*. For it always traverses the same distance slower, [that is,] in a longer time. And although he does not express this proportion, it must, nevertheless, be understood from his discussion.

Then he says, *"And always by so much as the medium,* and so on," and this is also self-evident. And when he posited these propositions, he also joined to them another proposition, and he says, *"Indeed, a void* [has no ratio to a body by which that body exceeds it], and so on." That is, the rarity of a void has no ratio to the rarity of a body that is the medium where a motion occurs. And when he says, *"It has no ratio to a body,"* he means that the void has no ratio—for example, in rarity—by which it exceeds the body that is the medium. For in a void there is no rarity by which it could be

7. Here and elsewhere Averroes' text reads so that one cannot be absolutely certain whether the proportionality between motions and densities or resistances is direct or inverse. That is, if V is velocity, or speed, and R resistance, does he mean $V/V' = R/R'$ or $V/V' = R'/R$? Surely we would say the latter, inverse proportionality, is intended. However, there was good reason for confusion concerning this matter in the Middle Ages. One could use two kinds of terms to refer to the resistance of a medium: (1) One could speak of its density (*spissitudo* or *crassitudo*) and of one medium being more dense (*spissior* or *crassior*) than another. Thus, interpreting the resistance of a medium as its density, *inverse* proportionality would obtain between resistances and motions: $V/V' = R'/R$. For (other things being equal) an increase in speed follows from a decrease in resistance as density, and vice versa. But (2) one could also refer to the resistance of a medium as its rarity (*subtilitas* or *tenuitas*) and of one medium being more rare (*subtilior*) than another. Hence, following the Aristotelian–Scholastic notion of density and rarity as contraries, one could maintain that *direct* proportionality obtained between motions and resistances when the latter were interpreted as the respective rarities of media: $V/V' = R/R'$. For (other things being equal) an increase in speed follows from an increase in resistance as rarity and a decrease from a decrease. (Averroes says just this in the first paragraph of the exposition of his own opinion below.) This ambiguous use of "proportional" also occurs in the comparison of other quantities, for instance, resistances to retardations. It will be left to the reader to fill in "direct" or "inverse" when such equivocation occurs below.

8. The text mistakenly says "*C*."

said to be rarer than fire, for example, or air. Then he says, "*Just as* [that which is called zero] *has* [absolutely] *no ratio* [to that which is called a number], and so on." That is, there is no ratio between void and the body that is the medium (since there is no rarity in a void; but it is [by means of a ratio] that media are mutually compared with respect to rarity and density), just as there is no comparison or ratio between zero[9] and a number. For numbers are mutually compared because 1 is their common measure *(communicant in uno)*. And, universally, any two things which lack a common measure with respect to the same nature do not have a mutual ratio. And Aristotle intends this when he says, "*For if 4 exceeds 3* [by 1], and so on." That is, numbers exceed each other by the units in which they have their common measure. But zero cannot be compared to a number. Similarly, what lacks rarity cannot be compared to bodies possessing rarity. Indeed, it is in virtue of "more" or "less" that different things necessarily have a common measure with respect to the same nature.

Then he says, "*Still* [it does not have a ratio to] *zero*, and so on." That is, what is composed of zeroes [that is, things in which 1 is not found] does not bear a ratio to anything that contains 1, however much the excess between them diminishes. Then he says, "*For that which exceeds* [is necessarily divisible], and so on." That is, what is said to exceed something is necessarily divided into that which is exceeded and the excess. For example, since 4 exceeds 2, it is necessary that it be divided into the 2 which is exceeded and the 2 which exceeds. Therefore, when there is something which is not divided into something and there is another thing which is divided into something, it cannot be said that what is divided into something exceeds what is not divided into anything. For example, 4 cannot be said to exceed zero[10] because if it should exceed it, then 4 would be divisible into an excess and into zero, which is impossible. And this is what Aristotle intended when he said, "*Therefore, 4 is divisible* [into these two things], and so on." That is, since it is necessary that 4 be divided into that by which it exceeds something which contains 1 and into zero [that is, that which lacks 1 *(in quo non est unum)*], and because the ratio that he intended to declare is common to numbers and other things, and because he gave an example in numbers, he wished also to give an example in magnitudes, and so he says, "*Thus it is that a line does not exceed a point.*" That is, it is because of this that there is no ratio between a line and a point from which an excess could arise, because if an excess did arise then that which exceeds and that which is exceeded would be parts of a line so that a line would be composed of points which will afterwards be declared to be impossible.

And when he has asserted this proposition, saying (1) that everything which exceeds something should be divided into an excess and the thing exceeded, and (2) that a void is not divided into that by which it exceeds a body [that is, exceeds it in rarity]—since there is no rarity in a void—he sets down the conclusion which follows from this. That is, that a void has no ratio to a plenum by which it may be said to exceed the plenum in rarity. And he said, "*Similarly, a void* [does not have a ratio to a plenum], and so on." This syllogism is also in the second figure and what he has added[11] is the minor proposition [or premise]. When he has asserted this conclusion he reveals the impossibility that follows from it if something were to be moved in a void, and says, "*Therefore, there will be no motion in it.*" That is, since a void has no ratio to a plenum in rarity, and something is moved in it, it follows that motion in a void has no ratio to motion in a plenum, and thus [the former] will be an indivisible motion occurring in an indivisible time, namely in an instant. But this is impossible, as will be shown later. For the present, he takes for granted what is declared in book VI, to wit, that every motion takes time and that motion is divisible.

[AVEMPACE'S CRITICISM]

Avempace, however, here raises a good question. For he says that it does not follow that the ratio of the motion of one and the same stone in water to its motion in air is as the ratio of the density of water to the density of air, unless we assume that the motion of the stone takes time only because it is moved in a medium.[12] And if we make this assumption, it would imply that motion only takes time because of something resisting it—for the medium seems to impede the thing moved. And, if this were the case, then the heavenly bodies would be moved instantaneously as they have no medium resisting

9. Here again, and throughout the Latin text, what has been translated as "zero" is given literally as "that which is not 1" or "that which lacks 1" [that is, lacks unity] *(quod non est unum)*.

10. Literally, "something in which there is no 1" *(aliquid in quo non est unum)*.

11. "Subticuit" (*l*) has been altered to *subiaculi*.

12. Thus Avempace rejects the opinion of Aristotle and Averroes.

them. And he says that the ratio of the rarity of water to the rarity of air is as the ratio of the retardation *(tarditatis)* suffered by the moving body in water to the retardation suffered by it in air.

And these are his own words, in the Seventh Book of his work, where he says: "This resistance which occurs between the plenum and the body which is moved in it is that between which and the potency of the void Aristotle made a ratio in his Fourth Book.[13] And what is believed because of his opinion is not so. For the ratio of water to air in density is not as the ratio of the motion of the stone in water to its motion in air. On the contrary, the ratio of the cohesive power *(potentiae continuitatis)* of water to that of air is as the ratio of the retardation suffered by the moved thing due to the medium in which it is moved, namely water, to the retardation suffered by it when it is moved in air.[14]

"For, if what certain thinkers have believed were true, then natural motion would be violent. Thus, if there were no resistance present, how could there be motion? For it would necessarily be instantaneous. Moreover, what then shall be said concerning circular motion [that is, the circular motion of the heavenly spheres]? No resistance is there, since there is absolutely no division there, for the place of a circle [that is, of a celestial sphere] is always the same, so that it does not abandon one place and enter another. Therefore, it is necessary that circular motion occur instantaneously. Yet we observe in it the greatest slowness, as in the case of the motion of the fixed stars, and also the greatest speed, as in the case of the diurnal rotation.[15] And this is only due to a difference in perfection *(nobilitate)* between the mover and the thing moved. Therefore, when the mover is of greater perfection, that which is moved by it will be swifter; and when the mover is of less perfection, it will be nearer [in perfection] to the thing moved, and the motion will be slower."[16]

[AVERROES' CRITICISM OF AVEMPACE]

And these are his [Avempace's] words. And if that which he [Avempace] has said be conceded, then Aristotle's demonstration will be false. For, if the ratio of the rarity of one medium to the rarity of another medium is as the ratio of the retardation suffered by a moved thing in one of them to the retardation suffered by it in the other, and is not as the ratio of the motion itself, it will not follow that what is moved in a void is moved instantaneously. For if this [that is, Avempace's contention] were the case, then there would be subtracted from the mobile's motion only the retardation which affects it by reason of the medium, and its natural motion would remain. And as every motion involves time, that which is moved in a void is also necessarily moved in time and with a divisible motion.[17] If this is so, nothing impossible follows. This, therefore, is Avempace's question.[18]

13. This is a reference to the next passage (215b.23—216a.7) following the text of Aristotle translated above (see n.2).

14. It is clear that Avempace's formulation gives us only the ratio of retardations for bodies falling in resisting media. It says nothing, however, about the manner in which bodies would fall in resistanceless media or in a hypothetical void. Thus we are unable to determine from this passage whether for Avempace such fall was to be understood as proportional to weight, or to dimension, or perhaps to some indwelling motive power. Avempace's vagueness stands in contrast with Philoponus' discussion of the same problem (it is possible that Avempace derived his view point from Philoponus, a sixth-century Greek commentator on Aristotle), for the latter insists that fall in the void can be determined by the inverse proportional relation between the weights of the falling bodies and their respective times of fall over equal distances (that is, $W_2/W_1 = T_1/T_2$, where W is the weight of a body and T its time of fall) from which it would follow that $W_2/W_1 = V_2/V_1$ when $S_2 = S_1$ (V is velocity and S distance). We see, then, that Philoponus has actually adopted for fall in the void the same relation that Aristotle had employed to represent exclusively the fall of bodies in media. See M.R. Cohen and I. E. Drabkin, *A Source Book in Greek Science* (Cambridge, Mass.: Harvard University Press, 1948), p. 218.

15. Latin scholastics in agreement with Avempace frequently cited celestial motion as the most basic illustration of temporal motion in a resistanceless medium (for example, see Selections 55.1,2.)

16. What Avempace understood by greater or lesser perfection of movers is unclear; nor is it readily apparent from the context of the discussion that he intended this last sentenec as a general assertion applicable to both celestial and terrestrial motions (see also n. 24 and n. 25.)

17. The quotation from Avempace leads Averroes to infer that Avempace's general rule for motion is $V = V' - r$, where V is the velocity of a body in a resistant medium, V' its natural velocity in a void or resistanceless medium, and r the retardation of the natural motion caused by the resistance of the medium. This rule has sometimes been improperly formulated as $V = F - R$, where V is velocity, F the motive power or weight of a falling body, and R the resistance of the medium. Thus when $R = 0$, a body would fall in the void with a natural speed proportional to its motive power or weight (see n. 14 and n. 25). Indeed the confused manner of Averroes' presentation led to this very interpretation of it in the Middle Ages, an interpretation more akin to that of John Philoponus (for Thomas Bradwardine's repudiation of this form of the rule see the discussion of his first erroneous theory at the beginning of Selection 51.1; for

[AVERROES' OPINION]

However, we maintain that it is self-evident that if (1) we are given things equal in weight, shape, and size, and (2) these things be moved in one and the same medium, or, indeed, in two media of equal rarity and density, and (3) the cause of the equality of their motions is held to be the equality of the medium, *then* (4) it is also clear that the diversity of the medium, with respect to more, or less, rarity, is the cause of the diversity of motion with respect to fastness or slowness. Moreover, this diversity—whose cause is the diversity of the medium—essentially follows the rarity [of the medium], namely that an increase of this diversity follows from an increase in rarity, and a decrease follows a decrease. Therefore, it is obvious that the ratio of the motions varies as the ratio of the media in rarity and density.

These propositions are self-evidently clear: namely that when two bodies, equal in heaviness and lightness, in shape and size, are moved in one and the same medium, their motion will be equal in speed, and both will traverse an equal space in an equal time. And [secondly], when the two media [in which these bodies move] are of different rarity, their motion will be diversified with a proportional difference.

It is also universally clear that the cause of the diversity, and the equality, of motions, is the diversity and equality of the ratio of the mover to the thing moved.[19] Therefore, whenever there are two movers, and two things moved, and the ratio of the one mover to one of the things moved is as the ratio of the other mover to the other of the things moved, then the two motions will be equal in speed. And when the ratio is different, the motions are diversified according to that ratio.[20] And this is true in both natural and violent motion. But in the case of violent motion, if the two movers are equal in potency, the diversity [of the resulting motions] will be that of the resistance. In natural motion, however, the diversity, or equality, follows the diversity, or equality, of the ratio of the mover to the thing moved. But from this it does not follow that equality in natural motion follows from the equality of the movers in potency when the ratio [between the mover and moved] is changed—just as this does not occur in the case of violent motion when the resistance is changed. For [in both cases] the ratio will be changed by a change in resistance. Therefore, these propositions are self-evidently clear.

[CRITICISM OF AVEMPACE RESUMED]

If that which Avempace says were true—namely that in such motions the ratio of one motion to another does not follow the ratio of the density [of one medium] to another, but rather the ratio of the retardation [of one motion] to another—then this would, in the case of any violent motion, make what is said in the last chapter of Book VII [of the *Physics*[21]] false, namely that if a given body moves another body through a certain space in a certain time, then it will move half that body through the same space in half the time. For, according to Avempace, this would occur so that the ratio of one time to the other would only be that of one retardation to the other.[22] The cause of this error lies in the judgment that slowness and speed are motions added to, and subtracted from, a motion in the same manner as a line is added to, or subtracted from, a line. Therefore, when it is supposed that speed is an addition to, and slowness a subtraction from, natural motion, the cause of this addition and subtraction would be resistance. It follows from this that the ratio of resistance to resistance is as the ratio of retardation to retardation and not as the ratio of motion to motion.[23] And this is so because when the same quantity is added to, or subtracted from, proportional quantities, they no longer remain proportional.

Avempace is of the opinion that observable motion *(motus sensibilis)* is what remains of natural motion. He has judged that natural motion is, as it were, a single measure from which two measures are subtracted according to the ratio of two other quantities.[24] And he has seen that when this occurs the sequel makes clear, these are not mutually exclusive. For a resistance necessarily arises between a mover and a moved thing (see Averroes' "Resolution of the Objection," following). Thus to assert (c) implies that the ratio of motions follows a ratio between some resistances or other. However, it might be asked whether or not these latter resistances are due to the respective densities or rarities of media; in other words, can we directly relate (c) with (a) and (b)? To answer this question we must determine more exactly the function of media in the motion of bodies for Averroes. To do this we must anticipate something of what he has to say later on in the present selection. The following considerations are relevant: (1) In the case of the violent motion of bodies, the resistance arising from the relation of mover to moved is due either to the moved body—that is, to its tendency to seek its natural place—or to the resistance to penetration offered by the medium through which the body moves (see the beginning of "Exposition of Averroes' Opinion Resumed," following). (2) In the case of the motion of heavenly bodies and animals, the resistance which arises is due solely to the moved body itself. That is, it is due to the action of the form ("Intelligences" and "Souls," respectively) of such bodies acting on the matter of the bodies (see the final paragraph of the translation). In the case of the natural motion of inorganic bodies (both "simple" and compound) the resistance is due to the medium alone (see the final paragraph of the translation). But one remark must be made apropos this third, and most important, distinction. We must note that merely because this resistance is due to the medium itself *(ex ipso medio)* does not mean that there is not also a resistance obtaining between the mover and the thing moved. This puzzling contention may be cleared up if we consider (just after the beginning of "Resolution of the Objection," following) that in no sense are we to take Averroes to mean that the medium in the *natural* motion of inorganic bodies is merely an accidental impediment to their motion. The medium does not *directly resist* the natural motion of simple bodies, yet there is a resistance in such motions *due to* the medium. How is this so? This resistance is due to the medium, it seems, because the medium is a necessary condition in order that resistance obtain between the mover and the moved body. This is why Averroes repeatedly says that there can be no natural motion of simple bodies without a medium. (Further details on this point will be found in n. 30.) Furthermore, Averroes felt that the medium could not offer direct resistance to the natural motion of a simple body because then such bodies would be *observed* to move with something less than their natural motion. For they would then, as Avempace maintained, move with $V' - r$ (since $V' - V = r$, where V' stands for their natural motion, V for their motion in the resistant medium, and r represents the difference in velocities or accidental retardation due solely to the direct resistance of the medium; (see n. 28). To return now to the original problem, we may say that for all bodies the ratio of motions follows both the ratio of resistances involved and the ratio of the ratios between movers and things moved. For it is the relation between the mover and the moved which gives rise to the resistances concerned. Thus, even in the case of the motion of the heavenly bodies, where there is no material resistance of a medium, a resistance does come into the picture, a resistance arising solely from the mover-moved relation and having nothing at all to do with a medium. One final point: In of the case the motion of heavenly bodies—and in almost all cases of the motion of animals—it seems that we would have to exclude the possibility that the ratio of such motion could follow a ratio of the densities or rarities of media. For, as we have just seen, no external media are here involved. But, in the natural motion of simple bodies, and at times in their violent motion, the resistance between mover and moved is determined by the resistance of the medium (see Selection 45, n. 12). Thus "resistance" is used ambiguously by Averroes, at times in a general sense to stand for a relation between mover and moved, and at other times to stand for the impedance of a medium which in turn itself determines a resistance in the first sense of the word.

Galileo's acceptance of it and the special Archimedian interpretation which he placed upon it, see Selections 55.3–5 and n. 58. While many discussed it, few in the Latin West adopted and applied this rule to bodies falling in an actual corporeal resistant medium. However, largely as a consequence of this passage, it became commonplace during the Middle Ages to argue that a body would move through a hypothetical void with a finite and temporal motion and not instantaneously, as Aristotle and Averroes had insisted (for example, see Selection 55.1 and the quotation from Albert of Saxony in Selection 47, n. 6).

18. In this translation of Comment 71, the parts under "Avempace's Criticism" and "Averroes' Criticism of Avempace" were previously translated by E. A. Moody in his important article "Galileo and Avempace," *Journal of the History of Ideas*, XII (1951), 184–186. For the most part, our translation follows his.

19. A word should be said about Averroes' various statements about what the "diversity" or "ratio" of motions does follow. He tells us variously that the ratio of motions follows: (a) the ratio of the densities or rarities of the media involved, or (b) the proportion (ratio) of the resistances involved, or (c) the ratio of the ratios of the movers to the things moved. As a reading of

20. That is, the motions have to one another the same diversity of ratio as do the two differing ratios between the respective movers and mobiles. Loosely speaking, if we let A equal the mover, B equal the moved thing, and C equal the resulting motion, then this passage might be formulated as follows: (1) When $A/B = A'/B'$ then $C = C'$; and (2) when $A/B > A'/B'$ or $A/B < A'/B'$ then $C/C' = (A:B)/(A' : B')$.

21. *Physics* VII.5.

22. Here Averroes is confused. To begin with, Avempace does not say—to judge from the quotation given above—that the ratio of motions follows the ratio of retardations. On the contrary, he says that the ratio of retardations follows the ratio of resistances, and that the latter does *not* follow the ratio of motions, i.e., $V/V' \neq R'/R = r'/r$ (where V equals motion, R equals resistance, and r equals retardation). The only possible explanation is that here, and again later (see n. 26), Averroes uses retardation to mean the "amount by which the natural motion (V') in a void is slowed down by the accidental impedance (r) of a medium," that is, retardation $= V' - r$. But elsewhere he, and what is more important, Avempace, uses retardation $= r$ alone.

23. Here, the ambiguity referred to in note 7 is particularly striking, since the ratio of resistances is *directly* proportional to the ratio of retardations, but *inversely* proportional to the ratio of motions. But Averroes refers to both in a single sentence without differentiation.

24. Averroes' resumé of Avempace's contentions might be formulated as follows: (1) Natural motion is as a single measure; that is, holding the potency of the mover constant, there is a motion V' which every body moving in a void possesses. It appears, as a matter of fact, that V' is equal to the potency of the mover, even though neither Avempace nor Averroes expressly declares this to be the case. (2) ". . . two measures are subtracted [from this V' as a single measure] according to the ratio of two other quantities." That is, if A's speed (for a given mover) in a void is V', then if it is subsequently moved in medium B of a resistance R and in medium D of a resistance R', the following results: (a) In medium B its speed will be $V' - r$, where r stands

in quantities, it does not follow that the ratio of one of the subtracted quantities to the other is as the ratio of one remaining quantity to the other remaining quantity.[25] Moreover, he is of the opinion that a certain subtraction occurs in natural motion because of resistance; and this subtraction occurs according to the ratio of resistance to resistance. Further, he contends, as we have said, that observable motion is what remains of natural motion after this subtraction, in the same way as something remains of a [mathematical] magnitude after a subtraction. Consequently, he has concluded that it does not follow that the ratio of one observable motion to another is as the ratio of one resistance to another, but rather as the ratio of one retardation to another.[26] Analogously, it does not follow that if quantities are diminished by some ratio the remainders will be in that ratio. On the contrary, they differ, because when some actually existing quantity is subtracted from a mathematical quantity, an actually existing quantity remains. When this subtraction occurs in natural motion, then the natural motion has only potential being.[27] Consequently, observable motion can only be the non-natural motion which is what remains of natural motion [after the retardation is subtracted] and not the natural motion from which this subtraction occurs.[28] And similarly, when some moved thing is moved faster than its natural motion, in that case there will not be natural motion and an addition, but only an addition.[29] Nevertheless, we also say that the ratio of retardation to retardation is as the ratio of resistance to resistance, as Avempace maintains. But in this case there is nothing else but retardation.

The problem at hand is really sophistical, though difficult, for it is all based upon a confusion [of categories]. And this is so because, as he assimilates motion to a line, he contends that what occurs apropos a line, occurs apropos motion.

[EXPOSITION OF AVERROES' OPINION RESUMED]

Therefore, in this passage we have asserted that the ratio of resistance to resistance is as the ratio of motion to motion. This applies to the case of violent motion when the mover is held constant and the resistance is due either to the medium or the moved body itself. But in natural motion, since there is no impediment [or resistance] this varies according to the diversity of the ratios which obtain between movers and things moved; indeed [it varies] according to the diversity of the ratio of the excess of the first motive power over the thing moved to the excess of the second power over the thing moved.

[OBJECTION]

Yet, someone may object that when we assert that motion will occur in a medium resisted by that medium, it does not follow, if we maintain that the ratio that motions have to one another in that medium is as the ratio of resistance to resistance,

for a retardation due to the resistance R of B, and (b) in medium D its speed will be $V' - r'$, r' being the retardation due to the resistance R' of D. The retardations r and r' are the two quantities subtracted from the single measure V' of "natural motion." But they are subtracted "according to the ratio of two other quantities," that is, according to the ratio of the respective resistances R/R'. (3) The motions $V' - r$ and $V' - r'$ are the observable motions of A in two different media. They are what remain of its natural motion V' in those media.

25. Continuing our analysis of the foregoing note, we may say that Avempace realizes the ratio of the subtracted retardations r/r' is not equal to the ratio of the remaining observable motions $(V' - r)/(V' - r')$. This is as good a place as any to make two remarks about our analysis of Avempace's opinions. To begin with, he is not in any sense of the word attempting to discover the appropriate form of proportionality between motions and their respective forces and resistances. That is, he is not bent upon replacing the so-called Aristotelian $V = F/R$ by $V = F - R$. (Moody's article cited in n. 18 strongly suggests this interpretation of his work). On the contrary, it appears that Avempace is merely pointing out that the *natural* motion of terrestrial bodies occurs on the same basis as that of celestial bodies. This brings us to the second point. Avempace seems to be speaking only about the natural motion of bodies. Thus our analyses of Avempace as reported by Averroes should only be taken as aids in clarifying particular passages and not as implying any attempt to formulate a completely general law of motion.

26. In this last clause Averroes seems to misinterpret Avempace. In fact, what Averroes says here is inconsistent with his report in the preceding paragraph. For there he correctly maintains that Avempace holds that $r/r' = R/R'$. But if this is the case, and further, if $R/R' \neq (V' - r)/(V' - r')$, then we cannot conclude, as he does in the present passage, that $r/r' = (V' - r)/(V' - r')$. The first two formulas find support in Avempace's own words; the last does not (see n. 22).

27. That is, observable motion, which actually exists, is $V' - r$; the natural motion V' however, only potentially exists, since it never actually occurs without some r or other being subtracted from it.

28. That, consequently, observable motion would not be natural is unthinkable in view of Averroes' pronounced Aristotelianism, which requires that the natural be identified with the observable (see Moody, pp. 189–190).

29. That is, we observe in this case not a natural motion plus some additional motion but only their sum in which, apparently, the addends are indiscriminable.

that when the same thing is moved in a void it is moved instantaneously and with an indivisible motion. On the contrary, [they would say,] it necessarily follows that [in a void the same thing] is moved in time. And the ratio of this time [a] to the time [b] in which it is moved in a medium is as the ratio of the excess [a'] of the power of the mover over the thing moved insofar as the thing is moved without any extrinsic resistance [that is, in a void] to the excess [b'] of the power of the mover to the thing moved insofar as the thing is resisted [that is, is moved in a medium]. Hence it is even believed that although these propositions which are used may be conceded, one is, nevertheless, not led to the contradiction which one sought to produce.

[RESOLUTION OF THE OBJECTION]

To this, one must reply that those motions of simple bodies in the media of water and air are not motions resisted by media, as appears at first glance. For, if this were the case, natural motions would exist which have not yet been observed, and which never will be observed—unless it were possible that these bodies could move without a medium. But this would only be possible if a void existed. The reason for this is that such moved things require a medium; for a moved thing and its mover necessarily should give rise to some resistance by which the potency of the mover exceeds the potency of the moved thing. This is so in the case of natural motion. If it were not, any mover could move any body.

Furthermore, the motions of the heavenly bodies are absolutely differentiated according to the diversity of the ratio which arises out of this resistance between the mover and the thing moved. Thus certain [heavenly bodies] are swifter than others. And by this resistance the mover only moves the thing moved when its potency exceeds the potency of the thing moved.

[RELATION OF MOVER AND MOVED]

Yet a moved thing is universally similar to the mover in a certain way, and in another way contrary to it. This we must take into account with respect to the perfection *(nobilitas)* which Avempace speaks of in celestial bodies. For the mover whose potency exceeds [the potency of the thing moved] is more perfect than the mover whose potency is less than [the potency of the thing moved]. It is clear that a resistance is observed between the mover and the thing moved when the thing moved is distinct in itself [from the efficient mover]. This is the case in the celestial bodies.[30] Yet, in the elements [that is, in the motions of inorganic bodies within the spheres of fire, air, water, and earth], the moved thing potentially exists and the mover actually exists, since [simple, inorganic bodies] are composed out of prime matter and simple forms; the mover is a form, and the thing moved matter. And, since these bodies [that is, simple bodies] are not [actually] distinguished into a thing moved and an actually existing mover, it is impossible that they be moved without a medium being involved. Moreover, Aristotle has declared (in Book VIII of this work) that, accordingly, it is necessary that these bodies are not moved by themselves.[31] Thus, if these simple bodies were moved without there being a medium, there would be no resistance between the mover and the thing moved. On the contrary, the moved thing would in no way essentially exist.[32] Moreover,

30. That is, in celestial and organic bodies (animals) the mover and the moved are actually *distinct within* the body itself. It is the immaterial intelligence actually existent in the celestial sphere that is its mover. As such it is actually distinct from the celestial sphere *qua* moved thing. The case is similar in animals whose bodies are moved by a soul actually distinct from the body as moved. (We must note here that "mover" is taken as an efficient cause; we are not speaking of it as a final cause.) However—and this will become clearer in what follows—this is not the case in the motion of inorganic, or as Averroes calls them, "simple," bodies. Here, mover and moved are not actually distinct entities within the body. Averroes tells us, in the sentence following the present passage, that in such simple bodies the mover, as its form, actually exists, but that the body *qua* moved—that is, as matter—only potentially exists. Thus, within these bodies the form, or "essence," of the body as mover is not actually distinct from the body as a moved thing. But, according to Averroes, in order that there be any motion, there must be an *actual* distinction between the mover and the moved. Therefore, as the form of a simple body in natural motion is not actually distinct from the body *qua* moved, this form cannot be efficient cause of the motion. A simple body cannot, in short, move itself; its form cannot act upon its matter. The form only moves the body *per accidens* by overcoming the resistance of the medium in which the body moves. Thus a medium is required for any such motion. It is the form of the body plus the medium which affords the efficient cause of the motion. This is the substance of the present paragraph (see n. 19).

31. Book VIII, chapter 4.

32. That is, the body *qua* moved thing would not *actually* exist. For as we have seen, the body *qua* moved thing only potentially exists. It is only when a medium is involved and there arises a resistance between the mover and the moved that motion actually occurs. Without such a medium and the consequent resistance, the body would not actually be moved.

if this were the case, [simple bodies] would be moved with any motion whatsoever in no time at all. Because of this Aristotle has said that if these bodies were moved in a void, it would follow that they would be moved with an indivisible motion in an indivisible time (provided, of course, that it is natural for these bodies to be moved in a medium). The reason for all this is that [simple bodies] may be moved and yet not moved by themselves with an unnatural motion as Avempace believes.

[SUMMARY]

Therefore, Avempace raises doubts in this passage in two places. One occurs when he says that the ratio of motion to motion is not as the ratio of the density of one medium to the density of another. The second, however, is that these motions are resisted by the medium and are not natural. Yet nobody before him had arrived at these questions, and thus he was more profound than any others.[33] And in the copy [of Avempace's work] from which we have written, we found a certain page all by itself, and this was written on it.

However, we maintain that it is necessary that there be a resistance between the mover and the thing moved. For the mover moves the thing moved insofar as it is contrary [to it], and the thing moved is moved by it insofar as it is similar. And every motion will be according to the excess of the potency of the mover over the thing moved, and the diversity of motions in speed and slowness follows the ratio which exists between the two potencies.[34]

And this resistance [between the mover and the thing moved is due to one of three things]: (1) It will be due to the thing moved itself when that which is moved by its own will is divided into an actually existing mover and an actually existing thing moved (as is the case in animals and heavenly bodies); or (2) it will be due to the medium itself in which it is moved, and this will occur when the moved thing is not divided into a mover and an actually existing moved thing (as is the case in simple bodies); or (3) the resistance will be due to both, that is, to the thing moved and the medium (as is the case in animals which are moved in water). Moreover, those things in which a self-moved thing is divided into an actually existing mover and an actually existing moved thing do not necessarily require a medium, but if one is involved, it will be so accidentally. Yet, those things which move themselves[35] and are not divided into a mover and an actually existing moved thing, necessarily do require a medium. Of this sort are heavy and light bodies. And if there were no medium [for such bodies], they would be moved in no time at all. For there would be nothing present to resist the motive power, and it would be impossible that they have a natural motion and be always resisted. And thus it comes about that when we assume these bodies to be moved in a void, they are moved instantaneously, and that bodies of different heaviness are moved with equal speed, which is impossible. Therefore, what Avempace has maintained is that these simple bodies have a natural motion without a medium.

33. However, as Moody himself notes, criticisms of Aristotle's dynamics had preceded Avempace. He refers to the late antique commentator on Aristotle, John Philoponus. For more complete information on these earlier criticisms, see M. Clagett, *Greek Science in Antiquity* (New York: Abelard-Schuman, 1955), pp. 169–177; and Cohen and Drabkin, pp. 217–223.

34. That is, in each case, between the potency of the mover and the potency of the thing moved.

35. It is puzzling what Averroes means by this declaration. He has just cited Aristotle as believing that "it is necessary that these bodies are not moved by themselves." Perhaps Averroes meant to distinguish the natural motion of heavy and light bodies from the violent motion by referring to the former as things "which move themselves." He may have felt this justified because in such cases the indwelling form or essence of the body is *per accidens* the cause of its motion. Yet in order that this cause may actualize itself an *external* medium is needed. Thus, strictly speaking, the bodies do not move themselves since they possess no internal efficient cause of movement.

It is important to realize, however, that although Averroes may not have intended to assert that heavy and light bodies move themselves, this is what he seems to say here, thereby providing sufficient grounds for later Latin authors to believe that he disagreed with Aristotle on this point. The same ambiguity and lack of clarity appear in the next selection. It is probable that in Selection 46 (at the end of paragraph 1035) Thomas Aquinas has Averroes in mind when he says that "it is clearly contrary to the Philosopher's intention to say that there is an active principle in matter, which some maintain is necessary for natural motion."

45 THE MOVER OR CAUSE IN NATURAL MOTION

Averroes(1126–1198)

Translated and annotated by Edward Grant[1]

Then he [Aristotle] says "and so the mover does not accompany the body that is moved with an accidental [or unnatural] motion."[2] That is, it is not necessary that the mover accompany the body in motion until the motion terminates. But although the mover rests, the body in motion will be moved nonetheless, because the air will then move it, since the motion it receives at the beginning from the external mover will remain after it has been separated from its mover. This self-motion [of the air] is in accordance with a natural principle in it, namely a heaviness or lightness. For if this were not so, a stone would fall when separated from its mover since a stone does not have a principle of lightness within itself. . . .[3] And it can be understood that if there were no air, there would be no violent motion. . . .[4]

Moreover, as Aristotle says, air is also an aid to natural motion in the same manner, necessarily, as in violent motion.[5] But it also seems necessary that in natural motion there should be a proper mover; but there is difficulty in this. For indeed it was asserted about the elements that among them none are self-moved because they are moved to a place only by the agent which brought them into existence. Thus when something is brought [from potentiality] into actuality by an agent [or *generans*], all things, as well as all other proper accidents, which constitute it in its proper place exist in it,[6] unless something should impede it from moving to its proper place. Therefore, just as other accidents exist in a generated thing, they do exist in it only through the form of the generated thing which is produced by the agent that generated it. And this is true of motion. For example, when the element is fire, all the accidents existing in fire and which make it fire exist immediately in the first part of the fire that is produced by the generating fire. One of these accidents is upward motion, because insofar as lightness exists in the generated part [of the fire] it will have the place of a light body; and [the same may be said] about other proper accidents. But, nevertheless, since these accidents from the generating agent exist only in the generated body and motion occurs primarily and essentially by means of a mover, and in no other way, this motion is therefore not produced primarily or essentially from an external mover but [rather] from the form of the body that is moved.[7] And so this kind of motion [that is, natural motion] has the characteristic that by means of the form a thing is moved by itself when something does not hinder it from motion by violence, namely when it will rest after being hindered. Now, it is in this way that this kind of motion is likened [or compared] to the motion of things that are self-moved. And since a mover is distinguished essentially in being from that which is moved, as in animals, where the mover in them is distinct from the thing moved, since the mover is the soul and the thing moved is the body, so in simple bodies the mover and the thing moved are the same in species but differ in their modes. For a stone moves itself insofar as it is actually heavy, and it is moved insofar as it is a lower potency. The reason that this occurs in one way in [terms of] actuality and in another in [terms of] potentiality is that a stone is composed of matter and form. Therefore, its form moves [that is, a motive

1. Translated from Averroes' Comment 28 on Book III of Aristotle's *On the Heavens (De caelo)*, as found in *Aristotelis opera cum Averrois commentariis [The Works of Aristotle with the Commentaries of Averroes]* (Venice, 1562–1574; Frankfurt: Minerva, 1962), Vol. V, fols. 198 recto, col. 1, to 199 recto, col. 1.

2. *De caelo* III.2.301b.27–28. In the passage from which this quote was taken, Aristotle explains that the motive force which initially sets a body in motion does not itself accompany that body but rather communicates a motion to the air, which then possesses the capacity to serve as motive power and continue pushing the body.

3. That is, unless air moved a stone after the latter lost contact with its initial mover, the stone would fall immediately to the ground in a vertical downward natural motion.

4. *De caelo* III.2.301b.28–29.

5. *De caelo* III.2.301b.29–30.

6. Thomas Aquinas would have found this statement wholly acceptable. See the introduction to Selection 46.

7. This would appear to be the specific passage to which Thomas Aquinas strenuously objected (see Selection 46, n. 31). For it seems that Averroes posits that the forms of inorganic bodies are movers or efficient causes in natural motion. However, Averroes seems to qualify this elsewhere by insisting that in such natural motions the form alone of a simple body cannot move the body without the concomitant and necessary action of an external medium (see below, near the end of this selection). Nevertheless, his vagueness and lack of clarity on this important issue provoked much later controversy (see Selection 44, notes 30 and 35).

power] insofar as it is form; and it is moved insofar as it is in matter. And thus a stone is not essentially self-moved, but is [only] similar *(assimilatur)* to what is self-moved and therefore requires an external mover in its motion.[8] Nor is it even counted amongst those things that are moved by an external mover, since it is not immediately [or directly] moved with this motion by an external mover but [rather is moved] by means of its form. And this is one of those cases where Plato thought it necessary that the primary mover should be moved essentially—that is, in the sense that the mover in the stone is moved. But although the mover in the stone is identical with the thing moved, they do not, as Plato thought, exist in the same mode but [rather] differently, as it appears in the first demonstration at the beginning of the seventh book of the *Physics*, where it is said that every motion has a mover.[9] Now, as we said, since the stone is not divided into mover and thing moved where each exists in actuality, the stone is moved, then, by a principle that is in it accidentally, since it does not have its being except through that principle.[10] For there is no motion of a stone except as it is in an inferior potentiality and the mover is heavy. Therefore, if there is self-motion essentially insofar as it is heavy, and since we already assume that it [that is, the whole stone] does not move essentially except insofar as it is heavy, then the mover and the moved exist in essentially the same way, which is impossible.[11] And so it follows that the motion of the stone is here not an essential motion, and it does not move itself essentially. And since this is so, it is necessary that everything which is moved accidentally be moved because another thing moves the moved thing essentially by itself. For example, a man does not move himself accidentally in a ship unless he moves the ship essentially. And since this is so, a stone does not move essentially unless it moves the air in which it is, and it moves itself because what moves itself follows [or depends on] the motion of the air, just as the man with the ship.[12] Therefore, since this is so, air and water are necessary in the motion of a stone and in our exposition of the *Physics* this is what we promised to declare here [in our exposition of *De caelo*]; and this is the most appropriate place. . . .

8. It requires an external mover because it is not actually self-moved but is like something that is self-moved. But in all self-moved things (for instance animals) the mover and the thing moved can be distinguished in one and the same thing. Hence, by its similarity to self-moved things, a mover and moved thing must be distinguished in the stone. But Averroes now goes on to make further qualifications and seems to shift back to the form as mover. It is hardly surprising that his position gave rise to controversy.

9. Aristotle begins the seventh book of his *Physics* with this remark (VII.1.241b.24–25).

10. Here Averroes emphasizes that in a stone, mover and thing moved cannot be distinguished in actuality (see Selection 44, n. 30).

11. Here Averroes argues that the heaviness of the stone cannot be the motive cause of its motion, since it is also the case that heaviness is an essential property of the stone itself. In brief, *heaviness* would be common to mover and moved; but he argued above that mover and moved must exist in different modes.

12. The manner in which the rower analogy should be applied to the motion of the stone is left somewhat ambiguous. At least two interpretations seem plausible. First, the rower moves the ship essentially by directly overcoming the resistance of the water with his oars. Once the resistance is overcome, the ship moves and carries the rower with it *per accidens*. In the same way, the form of the stone directly overcomes the resistance of the air thus enabling the stone to move naturally through the air. As the stone moves, it carries its form with it *per accidens*. In this first interpretation, the medium serves only to resist the motion of the inorganic body. But unless this resistance were overcome, the stone could not move at all because *mover* and *moved* are not actually distinct within it. Hence the efficient cause of motion requires both the form of the inorganic body and a resistant medium (see Selection 44, notes 19, 30, 32, and 35). In this first interpretation the medium is construed only as a resistance, not as a mover. But a second interpretation would have the medium serve as the direct contactual mover of the inorganic body. In this view, the rower is said to move the ship essentially by agitating the water, which then moves the ship directly. Hence the rower is again carried by the ship *per accidens*. In the same manner, the form of the stone acts upon the air, which in turn pushes the stone directly with the latter carrying its form *per accidens*. On this second interpretation the medium would function as direct mover (and the form as indirect mover) and the inorganic body would serve as resistance, just as in violent motion. In one place, at least, Thomas Aquinas seems to understand Averroes in this way (see Selection 46, n. 31; see also Selection 46, introduction, where he denies that the natural form is a mover). In the first interpretation, the form of the stone is the (indirect) mover by virtue of overcoming the resistance of the medium. Once this is achieved the body simply moves, since the impediment to its motion has been overcome. Thus the mover is the form, the inorganic body is the thing moved, but the resistance is the medium, not the inorganic body. Because the first interpretation emphasizes the resistive function of the medium, it seems preferable.

46 THE MEDIEVAL ARISTOTELIAN PRINCIPLE OF MOTION: "WHATEVER IS MOVED IS MOVED BY ANOTHER"

St. Thomas Aquinas (ca. 1225–1274)

Translated by Richard J. Blackwell, Richard J. Spath, and W. Edmund Thirlkel[1]
Introduction and annotation by Edward Grant

The Aristotelian principle "whatever is moved is moved by another" (its usual medieval Latin form was *omne quod movetur ab alio movetur*) was applied to all animate and inanimate objects capable of motion. As applied to inanimate objects, it was at first widely held that in cases of violent motion, or motion away from a body's natural place, the mover could be easily identified since it would always be external to the body it moved. For example, when a stone is thrown, the initial cause of its motion would be the thrower and the continuing cause the ambient air, which is continually in contact with the stone. Physical contact between mover and moved is a prime characteristic of violent motion. When, however, many came to reject air as a direct mover in violent motions and invoked instead incorporeal impressed force or impetus, serious problems arose, since now it was no longer meaningful to speak of "physical" contact. Indeed, the agent in violent motion was no longer "obvious." Was it the initial projector, an external object, with the impetus merely an instrumental agent? Or, was it the impetus itself, an internal incorporeal entity?

In natural motion, Aristotle himself emphasized the difficulty in determining the mover when he wrote: "It is in these cases that difficulty would be experienced in deciding whence the motion is derived, e.g. in the case of light and heavy things. When these things are in motion to positions the reverse of those they would properly occupy, their motion is violent: when they are in motion to their proper positions— the light thing up and the heavy thing down—their motion is natural; but in this latter case it is no longer evident, as it is when the motion is unnatural, whence their motion is derived" (*Physics* VIII.4.255a.1–5). But subsequently, at the very end of the chapter, Aristotle suggests that in natural motion light and heavy things "... are moved either by that which brought the thing into existence as such and made it light and heavy [the medieval term for this causal agent was *generans—Ed.*], or by that which released what was hindering and preventing it [the scholastics called this *removens prohibens—Ed.*]. . . " (*Physics* VIII.4.256a.1–2). Although, in other places, Aristotle would suggest yet other causes of natural motion and free fall, such as an active principle and the gravity or heaviness of a body, in the passage just quoted he did not distinguish a separate motive force or internal principle to explain natural motion but instead accounted for it by a natural spontaneous action of the body, an action that it would always undergo when not otherwise impeded. The capacity for such action was assumed to be a natural property communicated by the agent (that is, the *generans*) that caused or produced it. It is this very position which Aquinas adopts in this selection and which earlier in this very treatise he expressed as a formal principle of motion: "However, in heavy and light bodies there is a formal principle of motion. . . . For just as other accidents are consequent upon substantial form, so also is place, and thus also 'to be moved to place.' However the natural form is not the mover. Rather the mover is that which generates and gives such and such a form upon which such a motion follows" (*Physics* II, Lecture 1, par. 144, on p. 71 of the translation by Blackwell, Spath, and Thirlkel). But even before

1. Included here are Lectures 7 and 8 of Book VIII of the *Commentary on Aristotle's Physics* by St. Thomas Aquinas, translated by Richard J. Blackwell, Richard J. Spath, and W. Edmund Thirlkel, with an Introduction by Vernon J. Bourke (New Haven: Yale University Press, 1963), pp. 503–512; reprinted by permission of the Yale University Press. These two lectures *(lectiones)* incorporate Aquinas' exposition of the whole of Book VIII chapter 4, of Aristotle's *Physics*.

The lucidity and intelligibility of Aquinas' exposition has required almost no additional commentary. Although slightly expanded and occasionally altered, the numerous cross-references and citations to other books supplied by the translators have been included here. The phrases between single quotes represent cue words to the text of Aristotle which Aquinas furnished in lieu of needlessly repeating the full text of Aristotle. They have been taken by the translators from the Oxford English translation of the *Physics* (by R. P. Hardie and R. K. Gaye) and followed in each instance by Bekker numbers, thus conveniently enabling the reader to locate the full Aristotelian discussion in any modern edition or translation. Paragraph numbers have been retained and will also appear in notes supplying cross-references.

Thomas wrote, Averroes, while seemingly arguing the same position which Thomas later adopted, namely that the *generans* is the sole cause of natural motion, appears to have actually distinguished the form of a natural body as an internal mover and the body as the thing moved. And although he still insisted that inanimate bodies are not self-moved in natural motion, his distinction (for Averroes' discussion, see his Commentary 28 on Bk. III of *De caelo*, translated in the preceding selection) may have formed the basis of an even more radical departure initiated by Duns Scotus near the beginning of the fourteenth century. In his *Sentence Commentary*, Book II, Distinction 2, Question 10, Scotus rejected the *generans* as a direct efficient cause of free fall (the latter was now considered as only a remote cause) and argued instead that light and heavy inorganic bodies are self-moved by an internal principle. Though variations on this theme were formulated, it became one of the basic positions in the later Middle Ages. Some who saw the heaviness *(gravitas)* of a body as the internal principle causing it to fall, came to argue that this heaviness produced successive and cumulative increments of impetus which, in turn, caused the falling body to accelerate (this is Buridan's opinion; see Selection 49, section 5). It seems evident that while Scotus' position bears a certain affinity with that of Averroes, it is wholly at variance with Aquinas' viewpoint.

LECTURE 7 (254 b 7–255 a 18)

WHATEVER IS MOVED IS MOVED BY ANOTHER

1021. After the Philosopher has explained[2] his intention, he here begins to develop it; namely, not everything is sometimes moved and sometimes at rest. Rather there is something which is absolutely immobile, and something which is always moved.

This discussion is divided into two parts. First he shows that the first mover is immobile. Secondly, where he says, "And further, if there is. . . " (259 b 32),[3] he shows that the first mobile object is always moved.

The first part is divided into two parts. First he shows from the order of movers and mobile objects that the first mover is immobile. Secondly, where he says, ". . . but also by considering. . ." (259 a 21),[4] he proves the same thing from the eternity of motion.

The first part is divided into two parts. First he shows that the first mover is immobile. Secondly, where he says, "Since there must always. . ." (258 b 10),[5] he shows that the first mover is eternal.

Concerning the first part he makes two points. First he explains something that is necessary for the proof of what follows; namely, whatever is moved is moved by another. Secondly, where he says, "Now this may come about. . ." (256 a 3),[6] he proves his position.

He has shown above at the beginning of Book VII[7] that whatever is moved is moved by another with a common argument taken from motion itself. But since he has begun to apply motion to mobile things, he shows that that which was proven universally above is universally verified in all movers and mobile objects.

Hence the first part is divided into two parts. First he gives a division of movers and mobile objects. Secondly, where he says, "The fact that a thing. . ." (254 b 24),[8] he explains the proposition in individual cases.

Concerning the first part he makes two points. First he divides movers and mobile objects. Secondly, where he says, "Thus in things that. . ." (254 b 15),[9] he explains this division.

1022. First, therefore, he gives three divisions of movers and mobile objects.

The first of these is that some movers and mobile objects move or are moved *per accidens*, and some *per se*. Here he uses *"per accidens"* in the broad sense, according to which it includes even that which is "according to a part." Hence, to explain the meaning of *'per accidens'*, he adds that a thing can be said to move or be moved *per accidens* in two ways. First, those things are said to move *per accidens* which are said to move because they are in certain movers. For example, it is said that music cures, because he who cures is musical. Similarly, those things are said to be moved *per accidens* which are in things which are moved, either as located in a place, as when we say that a man is moved because he is in a ship which is moved, or else as an accident in a subject, as when we say that white is moved because a body is moved. Things are said to move or be moved *per accidens* in another way insofar as they move or are moved according to a part. For example, a man is said to strike or be struck because his hand strikes or is struck. Those things are said to be moved or to

2. Lecture 6, 1016.
3. Lecture 13, 1083.
4. Lecture 13.
5. Lecture 12.
6. Lecture 9.
7. Book VII, Lecture 1.
8. 1024.
9. 1023.

move *per se* which are free of the two preceding modes. For they are not said to move or be moved because they are in other things which move or are moved, nor because some part of them moves or is moved.

Omitting things which move or are moved *per accidens*, he subdivides those which are moved *per se*. First, of those things which are moved *per se* some are moved by themselves, such as animals, and others are moved by others, such as inanimate things.

He gives a third division; namely, some things are moved according to nature, and others are moved outside of nature.

1023. Next where he says, "Thus in things that . . . " (254 b 15), he explains how motion according to nature and motion outside of nature is found in things which are moved by themselves and in things which are moved by another.

He says first that things which are moved by themselves (as are animals, which move themselves) are moved according to nature. He proves this from the fact that they are moved by an intrinsic principle. We say that those things whose principle of motion is in themselves are moved by nature. Hence it is clear that the motion of an animal by which it moves itself, if compared to the whole animal, is natural, because it is from the soul, which is the nature and form of the animal. But if it is compared to the body, it happens that motion of this kind is both natural and outside of nature. For this must be considered according to the diversity of motions and of elements from which the animal is constituted. If an animal is composed of predominantly heavy elements, as is the human body, and if it is moved upward, then this motion will be violent in respect to the body. But if it is moved downward, this motion will be natural to the body. If, however, there are some animal bodies which are composed of air, as some Platonists held, then the contrary must be said of them.

Secondly, he explains how violent and natural motion are found in things which are moved by another.

He says that some of these are moved according to nature; for example, fire is moved upward and earth downward. But some are moved outside of nature; for example, earth upward and fire downward. This latter is violent motion.

Thirdly, he gives another mode of unnatural motion in animals according to which the parts of animals are often moved outside of nature if one considers the natures *[ratio]* and modes of natural motion in the parts of animals. For example, a man bends his arms forward and his legs backward; but dogs and horses and such animals bend their front feet to the rear and their rear feet to the front. But if a contrary motion occurs in animals, the motion will be violent and outside of nature.

1024. Next where he says, "The fact that a thing . . ." (254 b 24), he proves that whatever is moved is moved by another.

He shows this first in cases in which it is obvious. Secondly, where he says, "The greatest difficulty . . ." (254 b 33),[10] he shows this in cases in which it is open to question.

He omits, however, things which are moved *per accidens*, because such things are not themselves moved, but are said to be moved because some other things are moved. Among things which are moved *per se*, it is especially clear that things which are moved violently and outside of nature are moved by another.

For it is clear from the very definition of violence that things which are moved by violence are moved by another. For as is said in *Ethics*, III,[11] the violent is that whose principle is outside, the patient contributing none of the force.

After he has shown that things which are moved by violence are moved by another, he shows that things which are naturally moved by themselves, as animals are said to move themselves, are also moved by another. For in these cases it is clear that something is moved by another. But there can be a difficulty of how one should designate the mover and the moved in these cases. At first sight it seems (and many think this) that that which moves and that which is moved are distinct in animals, just as they are in ships and in other artificial things which do not exist according to nature. For it seems that the soul which moves is related to the body which is moved as a sailor is related to a ship, as is said in *De Anima*, II.[12] Thus it seems that a whole animal moves itself insofar as one part of it moves another. Whether the soul is related to the body as a sailor to a ship he leaves for investigation in *De Anima*. He will explain later[13] how a thing is said to move itself insofar as one part of it moves and another part is moved.

1025. Next where he says, "The greatest difficulty

10. 1025.
11. *Ethics* III.1.1110a.1–3; St. Thomas, *On the Ethics*, Book III, Lecture 1, 386.
12. *De anima (On the Soul)* II.1.(413a.7–9; St Thomas *In de Anima*, Book II, Lecture 2, 243.
13. Lecture 10.

. . ." (254 b 33), he explains his position in cases in which there is greater difficulty.

Concerning this he makes three points. First in regard to cases in which there is greater difficulty he states that whatever is moved is moved by another. He is referring to heavy and light things when they are moved according to nature. Secondly, where he says, "It is impossible. . ." (255 a 5),[14] he shows that such things do not move themselves. Thirdly, where he says, "It is the fact that. . ." (255 a 19),[15] he explains how they are moved.

He says, therefore, first that it is rather clear that the proposition "whatever is moved is moved by another" applies to things which are moved by violence and to things which move themselves. The main difficulty arises in the remaining alternative of the last division, namely, in things which do not move themselves, but are nevertheless moved naturally.

He says the "last" division; namely, of things which are not moved by themselves but by another, some are moved outside of nature and some on the contrary are moved according to nature. In the latter cases the difficulty is: by what are they moved? For example, heavy and light things are moved to their contrary places by violence and to their proper places by nature, that is, the light is moved upward and the heavy downward. But what moves them is not clear when they are moved by nature as it is when they are moved outside of nature.

1026. Next where he says, "It is impossible. . ." (255 a 5), he proves with four arguments that such things do not move themselves.

The first of these is that to move oneself pertains to the nature *[ratio]* of life and is proper to living things. For by motion and sensation we distinguish the animate from the inanimate, as is said in *De Anima*, I.[16] Now it is clear that these things are not living or animated. Therefore they do not move themselves.

1027. He gives the second argument where he says, "Further, if it were. . ." (255 a 7). The argument is as follows.

Things which move themselves can also be the cause of their own rest. For example, we see that by their appetites animals are moved and stopped. Therefore, if heavy and light things move themselves by natural motion, they would be able to stop themselves, just as if a man is the cause of his own walking, he is also the cause of his non-walking. But we see that this is false, because such things are not at rest outside of their proper places unless some extrinsic cause prevents their motion. Therefore they do not move themselves.

But someone might say that even though such things are not the cause of their own rest outside of their proper places, nevertheless, they are the cause of their own rest in their proper places. Hence he adds a third argument where he says, ". . . and so, since on this supposition. . ." (255 a 9). The argument is as follows.

It is irrational to say that things which move themselves are moved by themselves with respect to only one motion and not a plurality of motions. For that which moves itself does not have its motion determined by another, but determines its own motion. Sometimes it determines for itself this motion, and sometimes another. Hence, that which moves itself has the power to determine for itself either this or that motion. Therefore, if heavy and light things move themselves, it follows that if fire has the power to be moved upward, it also has the power to be moved downward. But we never see this happen except through an extrinsic cause. Therefore they do not move themselves.

It should be noted, however, that these two arguments are probable with respect to those things which are apparent in things which move themselves among us. Such things are found to be moved sometimes by this motion, sometimes by another motion, and are sometimes at rest. And so he did not say "impossible" but "irrational," which is his custom when speaking of probables. For he will show below[17] that if there is some self-motion in which the mover is altogether immobile, then it is moved always and by one motion. But this cannot be said of heavy and light things in which there is not something which is not moved *per se* or *per accidens*, since they are generated and corrupted.

1028. He gives the fourth argument where he says, "Again, how can anything. . ." (255 a 12). The argument is as follows.

No continuous thing moves itself. But heavy and light things are continuous. Therefore, none of these moves itself.

He proves as follows that no continuous thing moves itself. The mover is related to the moved as agent to patient. When, however, the agent is contrary to the patient, a distinction is necessary between that which naturally acts and that which is

14. 1026.
15. Lecture 8.
16. *De anima* I.2.(403b.26); St. Thomas, *In de Anima*, Book I, Lecture 3, 32.
17. Lecture 13.

naturally acted upon. Therefore, insofar as some things are not in contact with each other, but are altogether one and continuous both in quantity and form, they cannot be acted upon by each other. Therefore, it follows that no continuous thing moves itself. Rather the mover is separated from that which is moved, as is clear when inanimate thing are moved by animate things, as a stone by a hand. Hence in animals which move themselves there is a certain collection of parts rather than a perfect continuity. For one part can be moved by another, but this is not found in heavy and light things.

LECTURE 8 (255 a 19–256 a 2)

HE EXPLAINS HOW HEAVY AND LIGHT THINGS ARE MOVED

1029. After he has shown that heavy and light things do not move themselves, he here explains how they are moved. First he explains how they are moved. Secondly, where he says, "If then the motion. . ."(255 b 31),[18] he concludes to his main point.

Concerning the first part he makes two points. First he shows that they are moved naturally by something. Secondly, where he says, "But the fact that. . ." (255 a 30),[19] he inquires into that by which they are moved.

He says, therefore, first that, although heavy and light things do not move themselves, nevertheless they are moved by something. This may be made clear by distinguishing between moving causes. For just as things which are moved are moved either according to nature or outside of nature, so also in things which move some move outside of nature, for example, a staff does not naturally move a heavy body like a stone, and some move according to nature, for example, that which is actually hot moves that which according to its nature is potentially hot. And the same is true of other such things. And just as that which is in act naturally moves, so that which is in potency is naturally moved, either with respect to quality, or quantity, or place.

He has said in Book II[20] that those things are moved naturally whose principle of motion is in them *per se* and not *per accidens*. From this it may seem that when that which is only potentially hot becomes hot, it is not moved naturally, since it is moved by an active principle existing exterior to it. As if to refute this objection, he adds, ". . .when it contains the corresponding principle in itself and not accidentally. . ." (255 a 25). This is as if he were to say that for motion to be natural, it is sufficient if "the corresponding principle," that is, potency, which he mentioned, be in that which is moved, *per se* and not *per accidens*. For example, a bench is potentially combustible, not insofar as it is a bench, but insofar as it is wood.

Explaining what he means when he says, ". . . and not accidentally. . ." (255 a 25), he adds that the same subject may be both quantified and qualified, but one of these is related to the other *per accidens* and not *per se*. Therefore, that which is potentially a quality is also potentially a quantity, but *per accidens*.

Since, therefore, that which is in potency is moved naturally by another which is in act, nothing can be in potency and act with respect to the same thing. It follows that neither fire nor earth nor anything else is moved by itself, but by another. Fire and earth are indeed moved by another, but through violence, when their motion is outside of their natural potency. But they are moved naturally when they are moved to their proper acts to which they are in potency according to their nature.

1030. Next where he says, "But the fact that . . ." (255 a 30), he explains that by which they are moved. And since that which is in potency is moved by that which is in act, he first distinguishes potency. Secondly, where he says, "As we have said. . ." (255 b 17),[21] he explains that by which such things are moved.

Concerning the first part he makes three points. First he shows that it is necessary to know in how many ways a thing is said to be in potency. Secondly, where he says, "One who is learning. . ." (255 a 32),[22] he explains these distinctions. Thirdly, where he says, "But, be it noted. . ." (255 b 14),[23] he answers a certain question.

He says, therefore, first that since being in potency is predicated in many ways, it is not clear what moves heavy and light things in natural motions, for example, fire upward and earth downward.

1031. Next where he says, "One who is learning . . ." (255 a 32), he distinguishes being in potency, first in the intellect, secondly in quality, where he says, "In regard to natural bodies. . ." (255 b 5),[24]

18. 1036.
19. 1030.
20. Book II, Lecture 1, 145.
21. 1035.
22. 1031.
23. 1034.
24. 1032.

and thirdly in local motion, where he says, "So, too, with heavy. . ." (255 b 8).[25]

He says, therefore, first that the potency for science in one who is learning and does not yet have the habit of science differs from the potency for science in one who already has the habit of science but is not using it.

A thing is reduced from first potency to second when something active is joined to its passivity. And then this passivity, through the presence of that which is active, comes to be in this kind of act, which up to this point was in potency. For example, the learner, through the act of the teacher, is reduced from potency to act, to which act is joined another potency. Thus, a thing existing in first potency comes to be in another potency. For one who has science but who is not contemplating it is in a certain way in potency to the act of science, but not in the same way as he was before he learned. Therefore he was reduced from first potency to act, to which is joined a second potency, through some agent, namely, a teacher.

But when one possesses the habit of science, it is not necessary for him to be reduced to second act by some agent. Rather he does this immediately by his own contemplation unless something else prevents him, for example, business, or sickness, or his will. But if he is not impeded so that he cannot contemplate, then he does not have the habit of science but its contrary, namely, ignorance.

1032. Next where he says, "In regard to natural bodies. . ." (255 b 5), he explains the same thing with respect to qualities.

He says that what was said above[26] about the potency for science in the soul also applies to natural bodies. For when a body is actually cold, it is potentially hot, just as one who is in ignorance is in potency for knowing. But when it has been changed so that it has the form of fire, then it is fire in act, having the power of acting. And it operates immediately by burning, unless something prevents it by a contrary action or unless it is in some other way impeded, for example, by a withdrawal of the combustible object, just as it was said that after someone has become a knower by learning, he immediately contemplates, unless something prevents him.

1033. Next where he says, "So, too, with heavy . . ." (255 b 8), he explains the same thing with reference to the local motion of heavy and light things.

He says that a heavy thing becomes light in a way similar to that in which a cold thing becomes hot. For example, air, which is light, comes from water, which is heavy. The water, therefore, is first in potency to become light, and afterwards it becomes light in act, and then it immediately possesses its operation, unless something prevents it. But the now existing light thing is compared to place as potency to act (for the act of a light thing, as such, is to be in some determined place, namely, up). But it is prevented from being up because it is in the contrary place, namely, down. For it cannot be in two places at once. Hence that which holds the light thing down prevents it from being up. And what is said about local motion must also be said of motion in respect to quantity or quality.

1034. Next where he says, "But be it noted. . ." (255 b 14), he answers a certain question about the foregoing.[27]

Granted that the act of a light thing is to be up, nevertheless some ask why heavy and light things are moved in their proper places. The reason for this is that they have a natural aptitude for such places. For to be light is to have an aptitude for that which is up. And the nature *[ratio]* of the heavy is to have an aptitude for that which is down. Hence, to ask why a heavy thing is moved downward is nothing other than to ask why it is heavy. The same thing which makes it heavy also makes it to be moved downward.

1035. Next where he says, "As we have said. . ." (255 b 17), he shows from the foregoing what moves heavy and light things.

He says that since that which is in potency is moved by that which is in act, as was asserted,[28] it must be realized that a thing is said to be potentially light or heavy in many ways.

In one way, while it is still water, it is in potency to become light. In another way, when air has already been made from water, it is still in potency to the act of the light, which is to be up, just as one who has the habit of science but is not contemplating it is still said to be in potency. For it happens that what is light may be prevented from being up.

But if that impediment is removed, it rises immediately so that it may be up. As was said[29] with respect to quality, when a quality is in act, it strives immediately toward its own operation, just as he who is a knower immediately contemplates, unless something prevents him. And the same applies to

25. 1033.
26. 1031.
27. 1033.
28. 1029, 1030.
29. 1032.

the motion of quantity. For when an addition of quantity has been made to quantity, extension follows immediately in the body which can be increased, unless something prevents it.

Therefore, it is clear that that which removes that which prevents and restrains in a certain sense moves and in another sense does not. For example, if a column supports something heavy, and so prevents it from falling, he who destroys the column in a certain sense is said to move the weight supported by the column. Similarly, one who removes a stone which stops water from flowing out of a vessel in a certain sense is said to move the water. He is said to move *per accidens*, and not *per se*. For example, if a sphere, that is, a ball, rebounds from a wall, it is moved by the wall *per accidens* and not *per se*. It is moved *per se* by the initial thrower. For the wall did not give it any impetus[30] toward motion, but the thrower did. It is *per accidens* because, when the ball was impeded by the wall, it did not receive a second impetus. Rather because of the same remaining impetus it rebounded with an opposite motion. And similarly, one who destroys a column does not give to the supported weight an impetus or inclination downward. For it has this from its first generator which gave to it the form which such an inclination follows. Therefore, the generator is the *per se* mover of heavy and light things. But that which removes an obstacle is a *per accidens* mover.

He concludes, therefore, that it is clear from what has been said that no heavy or light thing moves itself. Nevertheless the motion of these things is natural because they have a principle of motion within themselves, not, indeed, a motive or active principle, but a passive principle, which is the potency for such act.

From this it is clearly contrary to the Philosopher's intention to say that there is an active principle in matter, which some maintain is necessary for natural motion.[31] This passive principle, which is the natural potency for act, is sufficient.

1036. Next where he says, "If, then, the motion . . ." (255 b 31), he arrives at the conclusion principally intended in this whole chapter.

He says that if it is true that everything that is moved is moveed acording to nature or outside of nature and by violence, then it is clear that all things which are moved by violence are moved not only by some mover but by some other extrinsic mover. And further of things which are moved according to nature, some are moved by themselves, in which it is clear that they are moved by something, not indeed, extrinsic, but intrinsic, and some are moved according to nature but not by themselves, as heavy and light things. These latter are also moved by something, as was shown (because either they are moved *per se* by the generator which makes them heavy and light, or they are moved *per accidens* by that which removes what impedes or prevents their natural motion). Therefore, it is clear that whatever is moved is moved by some mover, either intrinsic or extrinsic, which he calls 'being moved by another'.

30. Although the English term "impetus," which occurs four times in the next few lines, represents a translation of the Latin *impetus*, this is in no sense an early version of the medieval impetus theory. For it is obvious that Thomas is here employing the term in its nontechnical sense of "tendency" or "inclination" to motion, rather than as an incorporeal impressed force, or as some type of separate and distinguishable entity producing or preserving motion. Elsewhere, however, in his *Commentary on the De caelo*, Book III.2, Lecture 7, Thomas did consider and reject impressed forces as motive powers (see M. Clagett, *The Science of Mechanics in the Middle Ages* [Madison, Wis.: University of Wisconsin Press, 1959], pp. 516–517).

31. A probable allusion to the opinion of Averroes in Commentary 28 of Book III of the *De caelo*, which is translated in Selection 45. In his commentary on Book III of *De caelo*, Lecture 7, No. 9, Thomas emphatically repudiates Averroes' alleged position that the natural form is a mover:

"Both arguments stem from the same error. [Averroes] thought that the form of heavy and light bodies is an active principle of motion after the manner of a mover needing some resistance contrary to the tendency of form, and that motion is not immediately due to the agent who conferred the form. But this is absolutely false. The form of heavy and light bodies is not a principle of motion as a generator of motion, but as a means by which the mover moves, just as color, a principle of sight, is a means by which something is seen. . . . Thus movement of heavy and light bodies does not come from the generator by the intervention of another moving power *(mediante alio principio movente)*. Nor even is there any need to look for resistance here other than that which exists between generator and generated. Thus it follows that air is not required for natural motion of necessity *(ex necessitate)*, as in the case of violent motion, since that which moves naturally has a force *(virtutem)* imparted to it which is a source of motion. Consequently there is no need for a body to be moved by any other force impelling it, as though it were a case of violent motion, having no implanted force from which motion springs."

The translation is by James A. Weisheipl, O. P., in his article "The Principle *Omne quod movetur ab alio movetur* in Medieval Physics," *Isis*, LVI (1965), 41. An examination of Averroes' discussion will reveal that Aquinas ignored the fact that Averroes would allow the form of an inorganic body to be a mover only indirectly, or *per accidens*, denying that it can function as a direct and essential mover.

47 EXTERNAL AND INTERNAL RESISTANCES TO MOTION

Albert of Saxony (ca. 1316–1390)

Translated and annotated by Edward Grant[1]

Next it is sought whether, in its downward motion, a heavy simple body has an internal resistance; and similarly, [whether] in its upward motion a light [simple] body [has an internal resistance]. [This statement of the question is followed by six arguments in favor of internal resistance in simple bodies (all six are subsequently rejected), after which Albert observes that Aristotle denied the idea of internal resistance in simple heavy bodies. It is at this point that Albert enters into an enumeration and description of the different types of resistance.—*Ed.*]

In this question we must first posit the different ways of imagining resistance in the motion of heavy and light bodies; and secondly, one way of imagining the internal resistance of simple bodies must be disproved. Then, in the following question, I shall disprove certain types of internal resistance of simple bodies with respect to their local motion and then we shall respond to what has been sought.

As to the first part [of this question], one must know that resistance is called an inclination [or tendency] not to be moved, [an inclination] to opposite motion, or [an inclination] to rest. For example, if there were a certain body that had four degrees of heaviness and two degrees of lightness, then, when that body should descend, those two degrees of lightness would resist those four degrees of heaviness because they would tend to an opposite motion, namely upward.[2]

In the second place, it must be understood that there is a difference between a simple heavy body and an absolutely heavy body. For having assumed that some [piece of earth] has suffered from fire so that some degrees of lightness have been induced into the earth by this fire, then that [piece of] earth is substantially simple and heavy and is called a simple heavy body. But it is not called absolutely heavy, since it has some degree of lightness [mixed] with degrees of heaviness. Now, this question [under discussion here] should be understood [as being concerned] with a simple heavy body which is absolutely simple.

In the third place, it is known that resistance in the motion of heavy and light bodies can be imagined as external or internal. External [resistance] can be imagined in many ways. In the first way as in balances; thus, having assumed that *a*, pure earth, was moved to one arm of a balance and *b*, [also] pure earth, to the other [arm], then none of them would descend because one would resist the other[3]—and this would be [resisting] externally. External resistance can be imagined in a second way, just as if a light body were connected to a heavy body; then the light body would resist the descent of this heavy body externally. In a third way, that which is connected sometimes resists by virtue of its shape, as if [for example] an extended [of flattened] sheet of paper were connected to some heavy body which would then descend slower than if it were not so connected. In a fourth way, sometimes pulling [or attracting] toward the opposite [direction serves to] resist externally, just as if, for example, a magnet were held up and under it there were posited [a piece of] iron so great that this magnet would not suffice to hold back the iron. Then that [piece of] iron would descend slower than if there were no magnet posited above, because the magnet sometimes attracts iron and by such an attraction the velocity of the descending iron is impeded. In such a case, therefore, the magnet can be called an external resistance. In a fifth way, a medium through which there is motion can be called an external resistance, for because of its thickness it resists a mobile.[4] It is for this reason that any mobile is moved slower through water than air. In a sixth way, a moving power can take

1. From the Latin text in Albert of Saxony's *Questions on the Physics of Aristotle*, Book IV, Question 9, published in *Questiones et decisiones physicales insignium virorum: Alberti de Saxonia in octo libros Physicorum; tres libros De celo et mundo . . . Thimonis in quatuor libros Meteororum . . . Buridani in librum De sensu et sensato . . . Aristotelis . . . recognitae rursus et emendatae summa accuratione . . . Magistri Georgii Lokert . . .* (Paris, 1518). Our selection occupies fols. 49 recto, col. 1, to 49 verso, col. 1.

2. This example illustrates the internal resistance of contrarily inclined elements in a heavy mixed body. Although Albert accepts this as a genuine kind of resistance, it is not discussed in this selection (but see Selection 55.2) because he is concerned here only with the motion of simple elemental—not mixed—bodies. The one type of "internal resistance" considered below is radically different.

3. Presumably because *a* equals *b* and the balance is equal-armed.

4. This was the fundamental and usual sense of resistance in natural motion.

the place of an external resistance, as if, [for example,] a moving power does not wish to move quickly, nor as quickly as it could.[5] A seventh way is the *incompossibility [or incompatibility] of the termini*, namely the termini from which, and the termini to which, can be called an external resistance.[6]

[Now], as to the second [part of this question] concerning internal resistance, some say that any natural agent is of finite power, that is, it is limited to some certain degree of velocity which it does not exceed however much the external resistance should be removed, so that they assume such a limitation as an internal resistance. Whence it is that such people say that if pure earth were assumed in a vacuum, it would descend with a finite velocity, say with that degree of velocity to which it would be limited.[7] Furthermore, they say that heavier bodies would be limited to a more intense degree of velocity than less heavy bodies; moreover, they say that a simple heavy body posited in a vacuum would not be moved in it instantaneously but [rather] successively because of the limitation and finitude of the motive power.

But this imagination [or conception] is contrary to Aristotle, for which reason I posit one conclusion contrary to it, which is that the limitation and finiteness of the power of the agent is not the cause of the succession of motion of a simple heavy body; nor is it a substitute for resistance. This is proved because if any natural agent should act successively, this would be because it is of finite power. It ought to follow, [therefore,] that a luminous body should produce its light in a medium successively. But this is false because illumination of a medium is instantaneous,[8] as has been seen elsewhere. The consequence holds because it is of finite power.[9]

Secondly, let it be proved [in this way]: By the same [reasoning as before], a visible object ought to send its species into the eye successively, and a nearer visible object ought to send its species into the eye more quickly than a visible object more remote from the eye; and thus it ought also to be seen more quickly. But this is false, because if Socrates is 4 feet from me and Plato 8 feet away, I see Plato just as quickly as I see Socrates when opening my eyes. But this would not be so if a visible object could send its species into the eye successively.

Thirdly, if the aforesaid imagination [or con-

5. A motive power which, for whatever reason, fails to exert its maximum force cannot produce its maximum effect and from this standpoint retards the maximum motion that it is capable of producing. It is not obvious under what part of his threefold definition of resistance Albert would have classified and explained this type of resistance.

6. In the very next question (Question X on fol. 49 verso, col. 2), Albert defines this type of resistance as follows: "There is an imagination [or conception] that if there were no resistance of a medium but a vacuum could be imagined, a simple heavy body would nevertheless descend successively and with a finite velocity because it is impossible that it be simultaneously in the terminus from which, and the terminus to which, [it moves]. And because this is impossible, it is necessary that it be moved successively from the terminus from which [it moves] to the terminus to which [it moves], so that the incompossiblity [or incompatibility] of being simultaneously in different places is the cause of the finite velocity of motion of simple heavy and light bodies when surrounded by the resistance of a medium. And those who are of this opinion say that it is the cause of the finite velocity of celestial motion which is without any of the aforementioned resistances." Albert then proceeds to reject this opinion (as did also John Buridan), which, as we have already seen, was put forth, perhaps for the first time, by St. Thomas Aquinas (see Selection 55.1 and n. 4) as justification for the view that motion in a hypothetical void must take a finite time. Since Thomas did not himself designate this argument as a form of resistance (nor indeed did he name it "incompossibility of the termini"), it must have been so designated sometime after.

7. This form of "internal resistance" may also be traceable to that very passage of Thomas Aquinas cited in the preceding note and containing the substance of the "incompossibility of the termini" argument which is ultimately traceable to Averroes' quotation of Avempace's opinion (see Selection 44.2) on motion in a resistanceless medium. As a consequence of the "incompossibility" argument Thomas insisted that finite motion in the void is possible and that "if motion in a void were assumed, it follows that no retardation would occur beyond the natural velocity; . . ." However, Thomas did not attribute this natural finite velocity to the finite power of the natural agent and did not conceive of the natural agent as a resistance. These were all later refinements.

8. Here Albert is probably following Aristotle who insisted that light moves instantaneously (*De anima* II. 7.418b.20–27, and *De sensu*, 6.446a.26–446b.3; the latter passage is cited in Selection 55.1, n. 4). Many, in antiquity and the Middle Ages, disagreed with Aristotle (see Roger Bacon's discussion in Selection 62.6).

9. The consequence in this argument declares that a natural agent acts successively because it is of finite power; therefore a luminous body should produce light successively in a medium. The consequence is upheld because every luminous body is of finite power and should diffuse light successively. Albert rejects the consequence, however, since he believed that light is diffused instantaneously, from which it follows that not every finite agent can produce a successive motion. Consequently, it is false to assume that finitude of power is the cause of successive motion.

ception] were true, then a greater force should be less limited and a smaller force more limited. But this is false, for then the stars would not produce their light through the whole medium as quickly as the moon, which, however, is false because any of these produces its light through the whole medium instantaneously. The consequence holds for this reason: Since the moon is of greater power, it would be less limited, and so should produce its effect more quickly; and since the stars are of lesser power, they should be more limited and produce their effect slower.

Fourthly, it follows universally that no mutation is naturally possible in an instant (the opposite of this will be seen afterward). The consequence holds, because any natural being is limited to a certain degree of velocity in producing its effect.[10]

Fifthly, it follows that after every external impediment has been removed,[11] a simple heavy body would not be moved to its natural place until its middle coincided with the middle [or center] of the world.[12] The consequent is false[13] and this is [now] proved: Let *a* be a uniformly [or homogeneously] simple heavy body whose total heaviness [or gravity *(gravitas)*] is as six and its limitation as three.[14] Now *a* descends until a third part of it is under the other side of the center [of the world]; and let this [part of *a*] be *c*, and let the remainder, which is still on the other side of the center, be *d* so that the center [of the world] is between *c* [and] *d*. It is clear, then, that not even the middle of *a* is the middle [or center] of the world. But I prove that it does not descend further, for *c*, which is on the other side of the center, resists *d* with its total limitation. Now because the total limitation is as three and *c* is as two, and since these are together five and its motive power is only as four, it follows that there is more resistance than motive power, and, as a consequence, *a* rests and is not moved further, even though its center of gravity is not the center of the world.[15] Therefore, et cetera, and consequently the conclusion is true.[16]

10. From the assumption that any natural being is limited, and of finite power, it follows universally that no mutation or change is naturally possible in an instant. As a counterinstance, however, Albert has already insisted that light illuminates a medium instantaneously, and says he will show later that there are instantaneous changes.

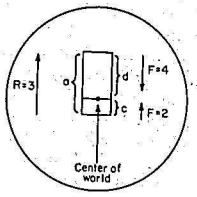
11. This is the antecedent of this conditional sentence.

12. This is the consequent.

13. The consequent is false because, as Albert shows below, by assuming the limitation theory as cause of finite successive motion and assuming the conditions just asserted, it can be shown that a body will come to its natural place when its center of gravity does not coincide with the geometric center of the world. For by the "natural place" of a simple heavy body in a universe where all impediments to its motion have been removed, Albert means primarily the place where it will come to rest. But, as his example shows, the place of rest can be such that the center of gravity of the body and the geometric center of the universe fail to coincide.

14. The ratio of motive force to resistance is therefore 6 to 3.

15. A figure may be useful in explaining Albert's example; *F* is motive force, or heaviness; *R* is total limitation, or resistance. We see that the total limitation, or resistance, of *a*, which equals 3, acts in opposition to the heaviness of *d*; and since part *c* of *a* has now moved to the other side of the geometric center of the world, and since, furthermore, all heavy things move naturally toward the center of the world, the heaviness, or motive force, of *c*, namely 2, tends to move toward the center of the world in opposition to *d*. Hence *c*'s motive force of 2 must be added to 3, the total limitation, or resistance, of *a*, resulting in a total resistance of 5 to the motive force, or heaviness, of *d*, namely 4. Since a force of 4 cannot move a resistance of 5, it follows that *a* will not descend further and its center of gravity will not coincide with the center of the world.



16. As already enunciated, the true conclusion is that "the limitation and finiteness of the power of the agent is not the cause of the succession of motion of a simple heavy body; nor is it a substitute for resistance." In the next question (Question 10 [fol. 50 verso, col. 1]: "Whether a resisting medium is required in every motion of heavy and light bodies"), where Albert continues to discuss absolutely heavy and light bodies, we discover his own position in the enunciation of these two conclusions: "The first conclusion: simple heavy and light bodies do not have internal resistance in their motion . . . .The second conclusion: an external resistance, as the resistance of a medium or some such thing, is required for the motion of heavy and light bodies. This is proved, because every motion follows a ratio of moving power to resistance, and, as a consequence, a resistance is required for every motion. Therefore, since [simple] heavy and light bodies do not have internal resistance, it follows that an external resistance is required for their motion. . . ."

48 THE IMPETUS THEORY OF PROJECTILE MOTION

John Buridan (ca. 1300—ca. 1358)

Translation, introduction, and annotation by Marshall Clagett[1]

In the same loose sense that Bradwardine was the founder of a school of mechanicians at Merton College, so Buridan was the "founder" of such a school at Paris. But Buridan's principal interest so far as mechanics was concerned was in dynamics. And his exposition of the impressed force (called by him *impetus*)[2] was the starting point of similar discussions of his principal successors at Paris in the second half of the century, namely Nicole Oresme, Albert of Saxony, and Marsilius of Inghen.

BOOK VIII, QUESTION 12.

1. It is sought whether a projectile after leaving the hand of the projector is moved by the air, or by what it is moved.

It is argued that it is not moved by the air, because the air seems rather to resist, since it is necessary that it be divided. Furthermore, if you say that the projector in the beginning moved the projectile and the ambient air along with it, and then that air, having been moved, moves the projectile further to such and such a distance, the doubt will return as to by what the air is moved after the projector ceases to move. For there is just as much difficulty regarding this (the air) as there is regarding the stone which is thrown.

Aristotle takes the opposite position in the eighth[3] [book] of this work (the *Physics*) thus: "Projectiles are moved further after the projectors are no longer in contact with them, either by antiperistasis, as some say, or by the fact that the air having been pushed, pushes with a movement swifter than the movement of impulsion by which it (the body) is carried towards its own [natural] place." He determines the same thing in the seventh and eighth [books] of this work (the *Physics*) and in the third [book] of the *De caelo*.

2. This question I judge to be very difficult because Aristotle, as it seems to me, has not solved it well. For he touches on two opinions. The first one, which he calls "antiperistasis," holds that the projectile swiftly leaves the place in which it was, and nature, not permitting a vacuum, rapidly sends air in behind to fill up the vacuum. The air moved swiftly in this way and impinging upon the projectile impels it along further. This is repeated continually up to a certain distance. . . .But such a solution notwithstanding, it seems to me that this method of proceeding was without value because of many experiences *(experientie)*.

The first experience concerns the top *(trocus)* and the smith's mill (i.e. wheel—*mola fabri*) which are moved for a long time and yet do not leave their places. Hence, it is not necessary for the air to follow along to fill up the place of departure of a top of this kind and a smith's mill. So it cannot be said [that the top and the smith's mill are moved by the air] in this manner.

1. [Reprinted with permission of the Regents of the University of Wisconsin from M. Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Wis.: University of Wisconsin Press, 1959), pp. 532–538. The introductory paragraph appears on page 522. For convenience, and following his order, Clagett's commentary (pp. 538–540), which is cued to the paragraph numbers of the selection, is reproduced after the translation. The editorial omissions are Clagett's, whose translation was made from the 1509 Paris edition of Buridan's *Questions on the Eight Books of the Physics* (for full title, see Selection 14, n. 1) with the modifications of A. Maier, *Zwei Grundprobleme der scholastischen Naturphilosophie*, 2d ed. (Rome: Edizioni di Storia e Letteratura, 1951), pp. 201–214.—*Ed.*]

2. [An impressed force theory explaining the motion of bodies projected through the air was enunciated at least as early as the sixth century A.D. by the Greek Aristotelian commentator, John Philoponus. In his *Commentary on the Physics of Aristotle*, Philoponus shows the absurdities of two related explanations (one is known as *antiperistasis*) proposed by Aristotle in which the air is invoked as the motive force continuing the motion of the projectile after it has lost contact with its initial mover (this selection is translated in M.R. Cohen and I. E. Drabkin, *A Source Book in Greek Science* [Cambridge, Mass.: Harvard University Press, 1948], pp. 221–223). Instead he argues that it is necessary "*to assume that some incorporeal motive force is imparted by the projector to the projectile*, and that the air set in motion contributes either nothing at all, or else very little to this motion of the projectile" (Cohen and Drabkin, p. 223). Undoubtedly familiar with the views of Philoponus, a number of Arabs discussed, and some even adopted, the concept of an impressed force (for instance, Avicenna and Abū l'Barakāt). Although it is believed that Arab discussions on impressed force were somehow transmitted to the Latin West, the path of transmission has not been discovered. See Clagett, pages 505–525, for a thorough discussion of the history of impetus theory—*Ed.*]

3. This is a statement from the fourth, rather than the eighth, book.

The second experience is this:[4] A lance having a conical posterior as sharp as its anterior would be moved after projection just as swiftly as it would be without a sharp conical posterior. But surely the air following could not push a sharp end in this way, because the air would be easily divided by the sharpness.

The third experience is this: a ship drawn swiftly in the river even against the flow of the river, after the drawing has ceased, cannot be stopped quickly, but continues to move for a long time. And yet a sailor on deck does not feel any air from behind pushing him. He feels only the air from the front resisting [him]. Again, suppose that the said ship were loaded with grain or wood and a man were situated to the rear of the cargo. Then if the air were of such an impetus that it could push the ship along so strongly, the man would be pressed very violently between that cargo and the air following it. Experience shows this to be false. Or, at least, if the ship were loaded with grain or straw, the air following and pushing would fold over *(plico)* the stalks which were in the rear. This is all false.

3. Another opinion, which Aristotle seems to approve, is that the projector moves the air adjacent to the projectile [simultaneously] with the projectile and that air moved swiftly has the power of moving the projectile. He does not mean by this that the same air is moved from the place of projection to the place where the projectile stops, but rather that the air joined to the projector is moved by the projector and that air having been moved moves another part of the air next to it, and that [part] moves another (i.e., the next) up to a certain distance. Hence the first air moves the projectile into the second air, and the second [air moves it] into the third air, and so on. Aristotle says, therefore, that there is not one mover but many in turn. Hence he also concludes that the movement is not continuous but consists of succeeding or contiguous entities.

But this opinion and method certainly seems to me equally as impossible as the opinion and method of the preceding view. For this method cannot solve the problem of how the top or smith's mill is turned after the hand [which sets them into motion] has been removed. Because, if you cut off the air on all sides near the smith's mill by a cloth *(linteamine)*, the mill does not on this account stop but continues to move for a long time. Therefore it is not moved by the air.

Also a ship drawn swiftly is moved a long time after the haulers have stopped pulling it. The surrounding air does not move it, because if it were covered by a cloth and the cloth with the ambient air were withdrawn, the ship would not stop its motion on this account. And even if the ship were loaded with grain or straw and were moved by the ambient air, then that air ought to blow exterior stalks toward the front. But the contrary is evident, for the stalks are blown rather to the rear because of the resisting ambient air.

Again, the air, regardless of how fast it moves, is easily divisible. Hence it is not evident as to how it would sustain a stone of weight of one thousand pounds projected in a sling or in a machine.

Furthermore, you could, by pushing your hand, move the adjacent air, if there is nothing in your hand, just as fast or faster than if you were holding in your hand a stone which you wish to project. If, therefore, that air by reason of the velocity of its motion is of a great enough impetus to move the stone swiftly, it seems that if I were to impel air toward you equally as fast, the air ought to push you impetuously and with sensible strength. [Yet] we would not perceive this.

Also, it follows that you would throw a feather farther than a stone and something less heavy farther than something heavier, assuming equal magnitudes and shapes. Experience shows this to be false. The consequence is manifest, for the air having been moved ought to sustain or carry or move a feather more easily than something heavier. . . .

4. Thus we can and ought to say that in the stone or other projectile there is impressed something which is the motive force *(virtus motiva)* of that projectile. And this is evidently better than falling back on the statement that the air continues to move that projectile. For the air appears rather to resist. Therefore, it seems to me that it ought to be said that the motor in moving a moving body impresses *(imprimit)* in it a certain impetus *(impetus)* or a certain motive force *(vis motiva)* of the moving body, [which impetus acts] in the direction toward which the mover was moving the moving body, either up or down, or laterally, or circularly. *And by the amount the motor moves that moving body more swiftly, by the same amount it will impress in it a stronger impetus.* It is by that impetus that the stone is moved after the projector ceases to move. But that impetus is continually decreased *(remittitur)* by the resisting air and by the gravity

4. [A similar argument was given by Philoponus. See Cohen and Drabkin, pp. 221–222.—*Ed.*]

of the stone, which inclines it in a direction contrary to that in which the impetus was naturally predisposed to move it. Thus the movement of the stone continually becomes slower, and finally that impetus is so diminished or corrupted that the gravity of the stone wins out over it and moves the stone down to its natural place.

This method, it appears to me, ought to be supported because the other methods do not appear to be true and also because all the appearances *(apparentia)* are in harmony with this method.

5. For if anyone seeks why I project a stone farther than a feather, and iron or lead fitted to my hand farther than just as much wood, I answer that the cause of this is that the reception of all forms and natural dispositions is in matter and by reason of matter. *Hence by the amount more there is of matter, by that amount can the body receive more of that impetus and more intensely* (intensius). *Now in a dense and heavy body, other things being equal, there is more of prime matter than in a rare and light one. Hence a dense and heavy body receives more of that impetus and more intensely, just as iron can receive more calidity than wood or water of the same quantity.* Moreover, a feather receives such an impetus so weakly *(remisse)* that such an impetus is immediately destroyed by the resisting air. *And so also if light wood and heavy iron of the same volume and of the same shape are moved equally fast by a projector, the iron will be moved farther because there is impressed in it a more intense impetus, which is not so quickly corrupted as the lesser impetus would be corrupted. This also is the reason why it is more difficult to bring to rest a large smith's mill which is moving swiftly than a small one, evidently because in the large one, other things being equal, there is more impetus.*[5] And for this reason you could throw a stone of one-half or one pound weight farther than you could a thousandth part of it. For the impetus in that thousandth part is so small that it is overcome immediately by the resisting air.

6. From this theory also appears the cause of why the natural motion of a heavy body downward is continually accelerated *(continue velocitatur)*. For from the beginning only the gravity was moving it. Therefore, it moved more slowly, but in moving it impressed in the heavy body an impetus. This impetus now [acting] together with its gravity moves it. Therefore, the motion becomes faster; and by the amount it is faster, so the impetus becomes more intense. Therefore, the movement evidently becomes continually faster.

[The impetus then also explains why] one who wishes to jump a long distance drops back a way in order to run faster, so that by running he might acquire an impetus which would carry him a longer distance in the jump. Whence the person so running and jumping does not feel the air moving him, but [rather] feels the air in front strongly resisting him.

Also, since the Bible does not state that appropriate intelligences move the celestial bodies, it could be said that it does not appear necessary to posit intelligences of this kind, because it would be answered that God, when He created the world, moved each of the celestial orbs as He pleased, and in moving them He impressed in them impetuses which moved them without his having to move them any more except by the method of general influence whereby he concurs as a co-agent in all things which take place; "for thus on the seventh day He rested from all work which He had executed by committing to others the actions and the passions in turn." And these impetuses which He impressed in the celestial bodies were not decreased nor corrupted afterwards, because there was no inclination of the celestial bodies for other movements. Nor was there resistance which would be corruptive or repressive of that impetus.[6] But this I do not say assertively, but [rather tentatively] so that I might seek from the theological masters

5. [Of the lines which he italicized, Clagett observes (p. 523) that "Buridan gives a quasi-quantitative definition of *impetus* at the time of its imposition. We must say 'quasi' because there is no formal discussion of its mathematical description. The exact nature of the impetus as conceived by Buridan is difficult to pin down. He spoke of it as motive force and as the reason for the continued movement, . . . It appears that while the impetus is unquestionably a force, it nevertheless seems close to being the effectiveness which the original force has on a particular body, an effectiveness measurable in terms of the velocity immediately supplied to the body and the quantity of matter in the body."—*Ed.*]

6. ["The characteristic of permanence which Buridan assigned to his impetus made it plausible for him to explain the everlasting movement of the heavens by the imposition of impetus by God at the time of the world's creation. . . .The use of impetus to explain the continuing movement of the heavens is the closest that Buridan comes to the inertial idea of Newton's mechanics. It can scarcely be doubted that impetus is analogous to the later inertia, regardless of ontological differences (Clagett, pp. 524–525)." Although Buridan used impetus to explain terrestrial and celestial motion, Clagett denies that he had sought to formulate a single mechanics for the whole universe. From other passages, it is clear that Buridan accepted the Aristotelian dichotomy between the behavior of celestial and terrestrial bodies. See the next selection, paragraph 7, where Buridan also mentions celestial impetus.—*Ed.*]

what they might teach me in these matters as to how these things take place. . . .[7]

7. The first [conclusion] is that that impetus is not the very local motion in which the projectile is moved, because that impetus moves the projectile and the mover produces motion. Therefore, the impetus produces that motion, and the same thing cannot produce itself. Therefore, etc.

Also since every motion arises from a motor being present and existing simultaneously with that which is moved, if the impetus were the motion, it would be necessary to assign some other motor from which that motion would arise. And the principal difficulty would return. Hence there would be no gain in positing such an impetus. But others cavil when they say that the prior part of the motion which produces the projection produces another part of the motion which is related successively and that produces another part and so on up to the cessation of the whole movement. But this is not probable, because the "producing something" ought to exist when the something is made, but the prior part of the motion does not exist when the posterior part exists, as was elsewhere stated. Hence, neither does the prior exist when the posterior is made. This consequence is obvious from this reasoning. For it was said elsewhere that motion is nothing else than "the very being produced" *(ipsum fieri)* and the "very being corrupted" *(ipsum corumpi)*. Hence motion does not result when it *has been* produced *(factus est)* but when it *is being* produced *(fit)*.

8. The second conclusion is that that impetus is not a purely successive thing *(res)*, because motion is just such a thing and the definition of motion [as a successive thing] is fitting to it, as was stated elsewhere. And now it has just been affirmed that that impetus is not the local motion.

Also, since a purely successive thing is continually corrupted and produced, it continually demands a producer. But there cannot be assigned a producer of that impetus which would continue to be simultaneous with it.

9. The third conclusion is that that impetus is a thing of permanent nature *(res nature permanentis)*, distinct from the local motion in which the projectile is moved. This is evident from the two aforesaid conclusions and from the preceding [statements]. And it is probable *(verisimile)* that that impetus is a quality naturally present and predisposed for moving a body in which it is impressed, just as it is said that a quality impressed in iron by a magnet moves the iron to the magnet. And it also is probable that just as that quality (the impetus) is impressed in the moving body along with the motion by the motor; so with the motion it is remitted, corrupted, or impeded by resistance or a contrary inclination.

10. And in the same way that a luminant generating light generates light reflexively because of an obstacle, so that impetus because of an obstacle acts reflexively. It is true, however, that other causes aptly concur with that impetus for greater or longer reflection. For example, the ball which we bounce with the palm in falling to earth is reflected higher than a stone, although the stone falls more swiftly and more impetuously *(impetuosius)* to the earth. This is because many things are curvable or intracompressible by violence which are innately disposed to return swiftly and by themselves to their correct position or to the disposition natural to them. In thus returning, they can impetuously push or draw something conjunct to them, as is evident in the case of the bow *(arcus)*. Hence in this way the ball thrown to the hard ground is compressed into itself by the impetus of its motion; and immediately after striking, it returns swiftly to its sphericity by elevating itself upwards. From this elevation it acquires to itself an impetus which moves it upward a long distance.

Also, it is this way with a cither cord which, put under strong tension and percussion, remains a long time in a certain vibration *(tremulatio)* from which its sound continues a notable time. And this takes place as follows: As a result of striking [the chord] swiftly, it is bent violently in one direction, and so it returns swiftly toward its normal straight position. But on account of the impetus, it crosses beyond the normal straight position in the contrary direction and then again returns. It does this many times. For a similar reason a bell *(campana)*, after the ringer ceases to draw [the chord], is moved a long time, first in one direction, now in another. And it cannot be easily and quickly brought to rest.

This, then, is the exposition of the question. I would be delighted if someone would discover a more probable way of answering it. And this is the end.

COMMENTARY

The reader's attention is first called to the refutation in passages 2 and 3 of the two theories presented by Aristotle. The first point worth noting is that

7. [Elsewhere in the same *Physics* treatise Buridan was more assertive and less deferential to the theologians (see Selection 14).—*Ed.*]

it is largely on the basis of experience that these two theories are shown to be inadequate. The *experientie* adduced against *antiperistasis*, i.e., the mechanical action of the air, are the following: (1) The spinning of a top or smith's wheel takes place without leaving its place of motion and the air can hardly be said to come behind the moving body to continue its motion. (2) The sharpening to a point of the posterior end of a lance does not thereby reduce its speed, as one would expect if this theory were correct. (3) In the course of the continuation of the movement of a ship in a river after the haulers have stopped pulling it, a sailor on deck does not feel the air pushing him from behind but rather feels it resisting him. Nor if behind some cargo would he be pushed against it; and similarly straws in the rear are not bent over. Similar "experiences" are brought against the second theory which held for a successive communication of motive power to the parts of the air.

In passage 4 Buridan states his acceptance of the theory which posited that the projectile motion is continued because the motor impresses in the projectile an *impetus* or motive force. He relates the intensity of impressed impetus to the velocity imparted by the original force to the projectile, i.e., the greater the velocity, the greater the impetus. Quite evidently this velocity is the speed of the projectile immediately after the original force of action has ceased. At the same time, he says that the impetus is made to decrease (i.e., is remitted and corrupted) in the same way that the motion is made to decrease, by the resistance and contrary inclination of the moving body. Perhaps Buridan might hold that the factors of impetus, resistance, and the continuing speed of the projectile are related as Bradwardine held generally for all cases of motion considered dynamically. However, he makes no such statement; and one might suppose from his arguments that, were there no resistance, not only would the *impetus* last indefinitely (as he states) but also that the movement maintained by the impetus would be both finite and uniform. However, such a conclusion would be difficult to fit in with the Aristotelian framework, which Buridan generally accepted. And hence our puzzlement as to just what kind of force this *impetus* really is.

Then Buridan in passage 5 relates the intensity of impetus imparted to the projectile with the quantity of prime matter in the projectile: This "quantity of prime matter" is a kind of analogue of the "mass" of early modern physics.

It is interesting to speculate on how it happened that Buridan seized upon velocity and quantity of matter as the two factors in determining the intensity of the *impetus*. It seems to be another case of the simultaneous consideration of extensive and intensive factors. The schoolmen had considered the effectiveness of a heat agent (i.e., its *potentia*) to be dependent on both the intensity of heat and the extent of the subject through which the heat is distributed, in short on a quantity of heat. Now *impetus* seems semantically to have the meaning of something like force of impact; its effectiveness, according to Buridan, would depend on how fast it is going (an intensive factor) and how much matter there was in motion (an extensive factor). The analogy is apparent.

Having outlined the measure of impetus in terms of the velocity imparted to the projectile and the quantity of matter in the projectile, Buridan then in passage 6 notes that the continuous impress of impetus in a falling body by gravity can account for its acceleration. This idea is taken up in greater length by Buridan in a question on the *De caelo*, . . .[8] The impetus theory also explains the fact that a broad jumper takes a long initial run before making his jump. Buridan incidentally adds that such a jumper never feels the air moving him along from behind but rather feels it resisting him. In this passage Buridan also presents his tentative hypothesis that the impetus theory could be used to account for the continuing motion of the heavenly bodies, thus eliminating the necessity of positing Intelligences as the movers. . . .

While not discussing here the ontology of movement, Buridan in his first conclusion of this question opposes the identification of impetus and the motion of the projectile, and initially on the ground that, since impetus produces the continuing motion something cannot produce itself. Buridan thus appears to accept the continuance of motion as a new effect for which a cause must be sought. And if the continuing motion is such an effect, it cannot be identical with its cause, namely, impetus. And so, while fundamentally in opposition to William Ockham in conceiving of the continuing motion as a new effect demanding a causal explanation, Buridan does not in this passage appear to be opposing specifically Ockham's view that the motion of a projectile is merely *secundum se*, for Buridan would appear to be arguing against those who would identify impetus with motion as an entity separate from the projectile.

8. [See the next selection.—*Ed.*]

In addition to rejecting the identification of impetus and motion, Buridan rejects the complementary view that impetus is a successive entity of the same kind as motion (see passage 8). Rather impetus must be something permanent, distinct from the local motion of the projectile. However, it is corrupted and diminished in the same way as the motion, i.e., by resistance and contrary inclination.

Finally, according to Buridan (passage 10), the impetus theory gives a satisfactory explanation of the phenomena of rebound, the vibration of a bow string or the string of a musical instrument, and the pendular swing of a bell.

49 ON THE CAUSE OF ACCELERATION OF FREE-FALLING BODIES

John Buridan (ca. 1300—ca. 1358)

Translated and annotated by Marshall Clagett[1]

BOOK II, QUESTION 12.

1. Whether natural motion ought to be swifter in the end than the beginning. . . . With respect to this question it ought to be said that it is a conclusion not to be doubted factually *(quia est)*, for, as it has been said, all people perceive that the motion of a heavy body downward is continually accelerated *(magis ac magis velocitatur)*, it having been posited that it falls through a uniform medium. For everybody perceives that by the amount that a stone descends over a greater distance and falls on a man, by that amount does it more seriously injure him.

2. But the great difficulty *(dubitatio)* in this question is why this [acceleration] is so. Concerning this matter there have been many different opinions. The Commentator (Averroës) in the second book [of his commentary on the *De caelo*] ventures some obscure statements on it, declaring that a heavy body approaching the end is moved more swiftly because of a great desire for the end and because of the heating action *(calefactionem)* of its motion. From these statements two opinions have sprouted.

3. The first opinion was that motion produces heat, as it is said in the second book of this [work, the *De caelo*], and, therefore, a heavy body descending swiftly through the air makes that air hot, and consequently it (the air) becomes rarefied. The air, thus rarefied, is more easily divisible and less resistant. Now, if the resistance is diminished, it is reasonable that the movement becomes swifter.

But this argument is insufficient. In the first place, because the air in the summer is noticeably hotter than in the winter, and yet the same stone falling an equal distance in the summer and in the winter is not moved with appreciably greater speed in the summer than in the winter; nor does it strike harder. Furthermore, the air does not become hot through movement unless it is previously moved and divided. Therefore, since the air resists before there has been movement or division, the resistance is not diminished by its heating. Furthermore, a man moves his hand just as swiftly as a stone falls toward the beginning of its movement. This is apparent, because striking another person hurts him more than the falling stone, even if the stone is harder. And yet a man so moving his hand does not heat the air sensibly, since he would perceive that heating. Therefore, in the same way the stone, at least from the beginning of the case, does not thus sensibly heat the air to the extent that it ought

1. [Reprinted with permission of the Regents of the University of Wisconsin from M. Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Wis.: University of Wisconsin Press, 1959), pp. 557–562. Clagett's commentary, which follows this selection, is reproduced here from pages 562–564. Clagett's translation was made from E. A. Moody's Latin edition of Buridan's *Questions on the Four Books On the Heavens and the World of Aristotle* (Cambridge, Mass.: Mediaeval Academy of America, 1942). The editorial omissions indicated in the selection were made by Clagett.

"Aristotle was by no means clear in his views on either the cause or the measure of the acceleration of falling bodies. He appears to have believed that acceleration depends on the increasing proximity of the body to its natural place, i.e., the center of the world. This increasing proximity produces additional weight, which in accordance to his dynamical rules would bring about the observed quickening. Yet, at the same time in refuting infinite locomotion [*De caelo* I.8.277a.27–277b.8—*Ed.*] Aristotle appears to imply two mutually incompatible ideas, namely that the speed of fall varies with the proximity to the center and that it also varies as the distance of fall" (Clagett, p. 542). Elaborations of this explanation, as well as additional explanations, were devised through the centuries. Some of these are repeated by Buridan. Along with the problem of explaining the continuation of projectile motion (see the beginning of the preceding selection), the problem of accounting for the acceleration of free-falling bodies was the most important problem in the history of physics.—*Ed.*]

to produce so manifest an acceleration *(velocitatio)* as is apparent at the end of the movement.

4. The other opinion which originated from the statements of the Commentator is this: Place is related to the thing placed as a final cause, as Aristotle implies and the Commentator explains in the fourth book of the *Physics*. And some say, in addition to this, that place is the cause moving the heavy body by a method of attraction, just as a magnet attracts iron. By whichever of these methods it takes place, it seems reasonable that the heavy body is moved more swiftly by the same amount that it is nearer to its natural place. This is because, if place is the moving cause, then it can move that body more strongly when the body is nearer to it, for an agent acts more strongly on something near to it than on something far away from it. And if place were nothing but the final cause which the heavy body seeks naturally and for the attainment of which the body is moved, then it seems reasonable that that natural appetite *(appetitus)* for that end is increased more from it as that end is nearer. And so it seems in every way reasonable that a heavy body is moved more swiftly by the amount that it is nearer to [its] downward place. But in descending continually it ought to be moved more and more swiftly.

But this opinion cannot stand up. In the first place, it is against Aristotle and against the Commentator in the first book of the *De caelo*, where they assert that, if there were several worlds, the earth of the other world would be moved to the middle of this world. . . .

Furthermore, this opinion is against manifest experience, for you can lift the same stone near the earth just as easily as you can in a high place if that stone were there, for example, at the top of a tower. This would not be so if it had a stronger inclination toward the downward place when it was low than when it was high. It is responded that actually there is a greater inclination when the stone is low than when it is high, but it is not great enough for the senses to perceive. This response is not valid, because if that stone falls continually from the top of the tower to the earth, a double or triple velocity and a double or triple injury would be sensed near the earth than would be sensed higher up near the beginning of the movement. Hence, there is a double or triple cause of the velocity. And so it follows that that inclination which you posit not to be sensible or notable is not the cause of such an increase of velocity.

Again, let a stone begin to fall from a high place to the earth and another similar stone begin to fall from a low place to the earth. Then these stones, when they should be at a distance of one foot from the earth, ought to be moved equally fast and one ought not be swifter than the other if the greater velocity should arise only from nearness to [their] natural place, because they should be equally near to [their] natural place. Yet it is manifest to the senses that the body which should fall from the high point would be moved much more quickly than that which should fall from the low point, and it would kill a man while the other stone [falling from the low point] would not hurt him.

Again, if a stone falls from an exceedingly high place through a space of ten feet and then encountering there an obstacle comes to rest, and if a similar stone descends from a low point to the earth, also through a distance of ten feet, neither of these movements will appear to be any swifter than the other, even though one is nearer to the natural place of earth than the other.

I conclude, therefore, that the accelerated natural movements of heavy and light bodies do not arise from greater proximity to [their] natural place, but from something else that is either near or far, but which is varied by reason of the length of the motion *(ratione longitudinis motus)*. Nor is the case of the magnet and the iron similar, because if the iron is nearer to the magnet, it immediately will begin to be moved more swiftly than if it were farther away. But such is not the case with a heavy body in relation to its natural place.

5. The third opinion was that the more the heavy body descends, by so much less is there air beneath it, and the less air then can resist less. And if the resistance is decreased and the moving gravity remains the same, it follows that the heavy body ought to be moved more swiftly.

But this opinion falls into the same inconsistency as the preceding one, because, as was said before, if two bodies similar throughout begin to fall, one from an exceedingly high place and the other from a low place such as a distance of ten feet from the earth, those bodies in the beginning of their motion are moved equally fast, notwithstanding the fact that one of them has a great deal of air beneath it and the other has only a little. Hence, throughout, the greater velocity does not arise from greater proximity to the earth or because the body has less air beneath it, but from the fact that that moving body is moved from a longer distance and through a longer space.

Again, it is not true that the less air in the afore-

mentioned case resists less. This is because, when a stone is near the earth, there is still just as much air laterally as if it were farther from the earth. Hence, it is just as difficult for the divided air to give way and flee laterally [near the earth] as it was when the stone was farther from the earth. And, in addition, it is equally difficult or more difficult, when the stone is nearer the earth, for the air underneath to give way in a straight line, because the earth, which is more resistant than the air, is in the way. Hence, the imagined solution *(imaginatio)* is not valid.

6. With the [foregoing] methods of solving this question set aside, there remains, it seems to me, one necessary solution *(imaginatio)*. It is my supposition that the natural gravity of this stone remains always the same and similar before the movement, after the movement, and during the movement. Hence the stone is found to be equally heavy after the movement as it was before it. I suppose also that the resistance which arises from the medium remains the same or is similar, since, as I have said, it does not appear to me that the air lower and near to the earth should be less resistant than the superior air. Rather the superior air perhaps ought to be less resistant because it is more subtle. Third, I suppose that if the moving body is the same, the total mover is the same, and the resistance also is the same or similar, the movement will remain equally swift, since the proportion of mover to moving body and to the resistance will remain [the same]. Then I add that in the movement downward of the heavy body the movement does not remain equally fast but continually becomes swifter.

From these [suppositions] it is concluded that another moving force *(movens)* concurs in that movement beyond the natural gravity which was moving [the body] from the beginning and which remains always the same. Then finally I say that this other mover is not the place which attracts the heavy body as the magnet does the iron; nor is it some force *(virtus)* existing in the place and arising either from the heavens or from something else, because it would immediately follow that the same heavy body would begin to be moved more swiftly from a low place than from a high one, and we experience the contrary of this conclusion. . . .

From these [reasons] it follows that one must imagine that a heavy body not only acquires motion unto itself from its principal mover, i.e., its gravity, but that it also acquires unto itself a certain impetus with that motion. This impetus has the power of moving the heavy body in conjunction with the permanent natural gravity. And because that impetus is acquired in common with motion, hence the swifter the motion is, the greater and stronger the impetus is. So, therefore, from the beginning the heavy body is moved by its natural gravity only; hence it is moved slowly. Afterwards it is moved by that same gravity and by the impetus acquired at the same time; consequently, it is moved more swiftly. And because the movement becomes swifter, therefore the impetus also becomes greater and stronger, and thus the heavy body is moved by its natural gravity and by that greater impetus simultaneously; and so it will again be moved faster; and thus it will always and continually be accelerated to the end. And just as the impetus is acquired in common with motion, so it is decreased or becomes deficient in common with the decrease and deficiency of the motion.[2]

And you have an experiment [to support this position]: If you cause a large and very heavy smith's mill [i.e., a wheel] to rotate and you then cease to move it, it will still move a while longer by this impetus it has acquired. Nay, you cannot immediately bring it to rest, but on account of the resistance from the gravity of the mill, the impetus would be continually diminished until the mill would cease to move. And if the mill would last forever without some diminution or alteration of it, and there were no resistance corrupting the impetus, perhaps the mill would be moved perpetually by that impetus.

7. And thus one could imagine that it is unnecessary to posit intelligences as the movers of celestial bodies since the Holy Scriptures do not inform us that intelligences must be posited. For it could be said that when God created the celestial spheres, He began to move each of them as He wished, and they are still moved by the impetus which He gave to them because, there being no resistance, the impetus is neither corrupted nor diminished.[3]

You should note that some people have called that impetus "accidental gravity" and they do so aptly, because names are for felicity of expression. Whence this [name] appears to be harmonious with Aristotle and the Commentator in the first [book] of this [work, the *De caelo*], where they say that gravity would be infinite if a heavy body were moved infinitely, because by the amount that it is moved more, by that same amount is it moved more swiftly; and by the amount that it is moved more swiftly, by that amount is the gravity greater. If this is true, therefore, it is necessary that a heavy body in moving acquires continually more gravity, and that gravity is not of the same constitution *(ratio)* or nature as the first natural gravity, because the first gravity remains always, even with the movement stopped, while the acquired gravity does not remain. All of these statements will appear more to be true and necessary when the violent movements of projectiles and other things are investigated. . . .

2. [For Galileo's explanation, see note 41 of the next selection.—*Ed.*]

3. [See paragraph 6 of the preceding selection, where Buridan also discusses this; also note 6.—*Ed.*]

COMMENTARY

This selection (passages 3–5) tells us of three common explanations of the cause of the acceleration of falling bodies in addition to the fourth, the impetus explanation which Buridan is supporting: (1) a heating of the medium which decreases its resistance and thus increases the velocity; (2) proximity to natural place which acts by some virtue or other (like that of the magnet) as a moving cause, this virtue being increased as the body comes closer. . . ; (3) as the body falls there is continually less air beneath it acting as resistance; hence the velocity increases. . . ;[4] (4) the impetus explanation, i.e., gravity continually introduces an impetus which acting as a supplementary increasing cause of movement, and acting with the gravity, produces the acceleration. Among the other explanations not mentioned by Buridan were two which centered around the supplementary action of the medium: (5) one which held that air stirred up by the movement is able to get behind the falling body and give it supplementary pushes (a theory taken over from Aristotle's explanation of the continuance of projectile motion. . .); and (6) the falling body not only draws the air behind, but in pushing the air beneath it, it sets it in motion, and this air sets other air in motion, and the drawing action of the air makes it less resistant and helps the gravity of the body (a theory found in the *Liber de ratione ponderis* attributed to Jordanus[5] . . .). One other opinion drawn from Simplicius (and supported later by Galileo in his early work *De motu*) held (7) that when a body starts to fall, it still has unnatural lightness which is a holdover from its previous violent movement, and this lightness slows up the body until it is dissipated by the action of the gravity.

Like Albert of Saxony later. . ., Buridan seems to believe that the increasing velocity is a simple function of the increasing distance of fall (see the second paragraph of passage 5). But he does say in one place, you will notice, that the increasing velocity is dependent on the "length of movement" (passage 4, last paragraph), which is somewhat ambiguous but sounds as if he is talking of time. Furthermore, the adverb he uses in his exposition of the impetus theory (passage 6) and its application to falling bodies is temporal, e.g., he speaks of the velocity *continually (continue)* increasing. I believe actually he made no clear distinction between the mathematical difference involved in saying that the velocity increases directly as the distance of fall and saying that it increases directly as the time of fall. The reader should compare this discussion with the discussions of Nicole Oresme. . . , Albert of Saxony. . . and Galileo. . . .[6]

The source of the continually increasing impetus acquired by the falling body clearly appears to be the continually acting gravity, and, furthermore, the increasing velocity caused by the gravity and continually impressed impetus is a measure of the increased impetus. Hence this whole passage seems to confirm his previous description of impetus as varying directly with the velocity it maintains.

The strongly empirical and observational character of Buridan's refutation of the "erroneous" opinions as well as of the exposition of his own theory should be noticed. For example (see passage 4), if the theory that held that the cause of acceleration lay in the proximity of the body to its natural place were tested by dropping a body from a high place and another body from a low place, according to the theory when these stones "should be at a distance of one foot from the earth, [they] ought to be moved equally fast and one ought not to be swifter than the other if the greater velocity should arise only from nearness to [their] natural place because they should be equally near to [their] natural place. Yet it is manifest to the senses that the body which should fall from the high point would be moved much more quickly than that which should fall from the low point, and it would kill a man while the other stone [falling from the

4. [See M. R. Cohen and I. E. Drabkin, *A Source Book in Greek Science* (Cambridge, Mass.: Harvard University Press, 1948), p. 210, where this is discussed in Simplicius' *Commentary on Aristotle's De caelo*.—*Ed.*]

5. [See above, Theorem *R4.06*, Selection 39. Later in the same treatise, Jordanus employed a crude version of impetus; see Theorems *R4.14–17* and notes 26–29.—*Ed.*]

6. [The omissions are references to these discussions given by Clagett on p. 563.—*Ed.*]

low point] would not hurt him." Notice then that, like Strato,[7] Buridan takes as his practical measure of velocity of fall the force of impact, as also did Leonardo and Galileo later.

50 IN OPPOSITION TO ARISTOTLE: CONTRARY MOTIONS CAN BE CONTINUOUS WITHOUT AN INTERVENING MOMENT OF REST

Introduction by Edward Grant

In *Physics*, Book VIII, Chapter 8, Aristotle argues that the only possible single and continuous infinite motion must be rotatory not rectilinear. After a lengthy discussion involving, among other things, one of Zeno's paradoxes, Aristotle presents the following passage which aroused considerable controversy and disagreement in the Middle Ages.

> Everything whose motion is continuous must, on arriving at any point in the course of its locomotion, have been previously also in process of locomotion to that point, if it is not forced out of its path by anything: e.g. on arriving at *B* a thing must also have been in process of locomotion to *B*, and that not merely when it was near to *B*, but from the moment of its starting on its course, since there can be no reason for its being so at any particular stage rather than at an earlier one. So, too, in the case of the other kinds of motion. Now we are to suppose that a thing proceeds in locomotion from *A* to *C* and that at the moment of its arrival at *C* the continuity of its motion is unbroken and will remain so until it has arrived back at *A*. Then when it is undergoing locomotion from *A* to *C* it is at the same time undergoing also its locomotion to *A* from *C*: consequently it is simultaneously undergoing two contrary motions, since the two motions that follow the same straight line are contrary to each other. With this consequence there also follows another: we have a thing that is in process of change from a position in which it has not yet been: so, inasmuch as this is impossible, the thing must come to a stand at *C*. Therefore the motion is not a single motion, since motion that is interrupted by stationariness is not single.
>
> Further, the following argument will serve better to make this point clear universally in respect of every kind of motion. If the motion undergone by that which is in motion is always one of those already enumerated, and the state of rest that it undergoes is one of those that are the opposites of the motions (for we found that there could be no other besides these), and moreover that which is undergoing but does not always undergo a particular motion (by this I mean one of the various specifically distinct motions, not some particular part of the whole motion) must have been previously undergoing the state of rest that is the opposite of the motion, the state of rest being privation of motion; then, inasmuch as the two motions that follow the same straight line are contrary motions, and it is impossible for a thing to undergo simultaneously two contrary motions, that which is undergoing locomotion from *A* to *C* cannot also simultaneously be undergoing locomotion from *C* to *A*; and since the latter locomotion is not simultaneous with the former but is still to be undergone, before it is undergone there must occur a state of rest at *C*; for this, as we found, is the state of rest that is the opposite of the motion from *C*. The foregoing argument, then, makes it plain that the motion in question is not continuous. (264a.7–35)

As will be seen in the notes, it would seem that some of the basic anti-Aristotelian arguments were passed on from Arabic sources to the Latin West, where they were utilized and added to by Marsilius and others in the fourteenth century, eventually passing in the sixteenth century to Benedetti and Galileo, who apparently found the question significant because it provided them with one more

7. [Clagett's reference is to Simplicius' *Commentary on Aristotle's De caelo*, where Simplicius attributes the following argument to Strato (fl. ca. 287 B.C.): "If one drops a stone or any other weight from a height of about an inch, the impact made on the ground will not be perceptible, but if one drops the object from a height of a hundred feet or more, the impact on the ground will be a powerful one. Now there is no other cause for this powerful impact. For the weight of the object is not greater, the object itself has not become greater, it does not strike a greater space of ground, nor is it impelled by a greater [external force]. It is merely a case of acceleration. And it is because of this acceleration that this phenomenon and many others take place." The translation appears in Cohen and Drabkin, pp. 211–212, and is quoted by Clagett, p. 546. Since William Moerbeke translated Simplicius' *Commentary on De caelo* from Greek to Latin in 1271 (Selection 8, no. 47), Buridan probably derived his argument in Passage 4 from Simplicius.—*Ed.*]

point of attack against Aristotle. The question is of interest because, incorporated into the responses, we find dynamic considerations involving impressed forces (*impetus*).

1. Marsilius of Inghen (ca. 1340–1396)

Translated and annotated by Edward Grant[1]

BOOK VIII, QUESTION 7

Whether an intervening [moment of] rest (quies media) is necessary between any contrary [or reverse] (reflexos) motions.

Now, it is argued affirmatively *(quod sic)* that between things resting in opposite places there is an intervening motion; therefore, between any contrary motions there is an interval of rest *(quies media)*. The consequence holds by similarity [or analogy];[2] the antecedent is obvious by experience.[3]

Secondly, [on the assumption of an intervening moment of rest,] Aristotle, in this eighth book, argues and proves that a rectilinear motion is not continuous and perpetual. Since a continuous and perpetual rectilinear motion could not occur except by breaking the motion,[4] it is necessary that a moment of rest intervene between any motions and the breaks *(reflexos)* in those motions. But motions which have an intervening [moment of] rest are not continuous.

Thirdly, unless this is so [namely that a moment of rest intervenes], it would follow that contrary motions are one continuous motion, which is impossible. The consequence[5] is proved, because there is no moment of rest between the contrary motions.

Fourthly, it would follow that something could be moved with contrary motions. This consequent is impossible, as is evident in the fourth book of [Aristotle's] *Metaphysics*.[6] [But] the consequence can be proved,[7] for if something should ascend and afterwards descend without an intervening moment of rest, then in the middle instant [of the motion], it ought to ascend with as much reason as descend. Therefore, either the body is moved [simultaneously] with each motion [that is, up and down] and the consequent is proven;[8] or it moves with neither motion, so that the proposition is upheld and the body rests there.[9]

Fifthly, it would follow that something *could both be* and *not be* in some place. This consequent is implied and the consequence is proved: After assuming that *A* is moved from *B* to *C* and then returned *(reflectatur)* over the same line, I inquire whether or not *A* recedes from *C* at the middle instant[10] [of the motion]. If not, then it rests in the same place and what has been proposed is had; if it does [recede from *C*] then it is not in *C*, because everything which recedes from any terminus is no longer in that terminus, as is evident in this eighth book; and yet it was previously posited in *C*.

Sixthly, with the exception of circular motion, every motion tends to come to rest, as if that were

1. From Book VIII, Question 7, of *Questiones subtilissime Johannis Marcilii Inguen (sic) super octo libros Physicorum secundum nominalium viam . . .* (Lyon, 1518); reprinted as *Johannes Marsilius von Inghen Kommentar zur Aristotelischen Physik* (Frankfurt: Minerva, 1964), fols. 83 verso, col. 2, to 84 recto, col. 1. The lengthy quotation from Aristotle's *Physics* in the Introduction was drawn from the Oxford translation of that work by R. P. Hardie and R. K. Gaye.

2. See Marsilius' response to the first principal argument below.

3. The antecedent declares that "between things resting in opposite places there is an intervening motion." For Marsilius it is only by experience that we know that a body can rest in two different places at two different times and only by virtue of some kind of intervening motion.

4. In a finite world, a body could move with continuous and perpetual rectilinear motion only if it oscillated between two fixed points separated by a finite distance. The text of this second argument appears corrupt in at least one place. In the first sentence, I have added "non" before "esse" in the statement "probat motum rectum esse continuum et perpetuum."

5. Namely, that without a moment of rest contrary motions are continuous.

6. Probably a reference to Aristotle's remark that it is impossible for contrary attributes—in this case up and down motion—to belong to the same subject at the same time (*Metaphysics* IV.3.1005b.25–26; also IV.4.1006a. 3–5).

7. As so frequently happens in scholastic argumentation, Marsilius rejects the consequent—namely that something could move with contrary motions—as impossible, but then shows further that if the antecedent is true—namely if a moment of rest does not intervene—the rejected consequent must follow logically. For this reason the antecedent must be rejected and there must be a moment of rest.

8. Namely that a body can move with contrary motions, which is impossible.

9. The proposition upheld is the claim that a moment of rest intervenes.

10. That is, the point at which the motion becomes oppositely directed.

its end, [or goal, or terminus] *(terminus)*. But if over the very same path *(per eandem lineam)* there occurred an upward motion and then a downward motion of the same mobile, it follows that rest will occur at the upward [terminus of the motion], and what has been proposed is shown.[11]

Seventhly, let it be assumed that *A* is moved from *B* to point *C* and then is returned *(reflectatur)* over the same line, I inquire whether it was in *C* at some time or never. If never, then it was not moved through the whole distance, which is contrary to what was assumed. Furthermore, *A* recedes from some terminus in which it was not. [But] if *A* was once in *C* through some [period of] time, *A* consequently rested through that time.[12]

Eighthly, let it be assumed that a certain magnitude *D* is conjoined with point *C*. I ask, then, whether or not *A* touches *D* at the point lying between the contrary motions, [namely point *C*]. If it does, and since every motion is temporal, it follows that *A* touches [*D*] and consequently rests in point *C* through [some interval of] time. [But] if *A* never touches *D* [in point *C*], it follows that it was not moved through the whole distance, which is contrary to the assumption. . . .[13]

[I argue for] the opposite position[14] because if a heavy body were projected upward, it would rest prior to descending, and then it would follow that a heavy body would rest [while high] up without being detained violently.[15] Secondly, [I argue for the opposite position] because there appears to be no reason why it should rest more [that is, through a greater length of time (?)] at one time than another, and thus it would rest either through no time at all or through any time whatever.[16]

It must be noted that it is possible that an upward motion occurs, and afterward a downward motion, without [an exact] return [over the same path], as [for example] if a motion occurs over a curved line, as is obvious in the motion of an arrow through the air. In another way there can first be ascent and then a descent by returning through an angle. And this can happen in two ways: in one way by returning *(reflectendo)* through an acute angle in a rectilinear or curvilinear path, or [a path] composed of a straight line and a curved line, or even [returning] through an obtuse or right angle by coming over one line [or path] and returning over another; in the other [or second] way, there is a return at a straight angle in such a way that the mobile comes over the same line [or path] by which it returns, and this motion is properly composed of two contrary motions and this question must be understood to be concerned [only] with such motions that turn back *(reflexos)*.[17]

Now these conclusions are posited. The first is that it is not necessary that there be a temporal moment of rest between any motions that turn back *(reflexos)* [over the same path]. The proof is that if a bean *(fabba)* were projected upward against a millstone *(molarem)* which is descending, it does not appear probable that the bean could rest before descending, for if it did rest through some time it would stop the millstone from descending, which

11. Here the upward motion and downward motion are assumed to be independent motions, each having rest as its goal. Obviously, then, on this argument, since a heavy body thrown into the air will have reached its goal or terminus at the uppermost point of its motion, it will come to rest there "naturally." This would turn Aristotelian physics upside down, with heavy bodies "naturally" suspended at rest, unsupported above the surface of the earth.

12. The point of this argument is that the first alternative—that *A* was never in *C*—is absurd and impossible. The second—that *A* was in *C* for some time—must be true, and we conclude that *A* rested momentarily in *C*.

13. Point *C* is the midpoint of the whole motion, being the last point of the upward leg and the first point of the downward movement. If a magnitude *D* is assumed to be in *C*, then when *A* is thrown upward either it touches *D* in *C* or not. If it does, then their contact in *C* constitutes a temporal interval of rest; if *A* does not touch *D*, it never reached *C* and its motion is incomplete, contrary to the assumption that it completed its up-down motion. This seems but a variation on the preceding (seventh) argument.

14. That is, that no temporal moment of rest intervenes.

15. This would be contrary to a fundamental tenet of Aristotelian physics, for it would make "up" the natural resting place of a heavy body. See also note 11.

16. In the absence of further details, two interpretations seem plausible here, both employing the principle of sufficient reason. If the body rests at the turning point of the motion, it ought to be able to rest at any other point of its up-down path. Furthermore, if it rests only through a durationless instant, there is no moment of rest; but if it rests through some temporal period, however small, then it ought to do so at any point of the path (since none are privileged), in which event it could possibly rest "through any time whatever." A second, and perhaps more likely, intepretation, takes the "interval of rest" as applicable only to the turning point of the motion where, if rest occurs at all, one must inquire about the length of the interval, which, in the absence of any reasons or explanations, may range from a durationless interval to one of any temporal duration, however great. Hence the body "would rest either through no time at all or through any time whatever." My own uncertainty is revealed by the query in square brackets.

17. The problem of an interval of rest is thus restricted to cases of up and down motion where the path is represented by a single straight line.

seems impossible.[18] Secondly, let it be assumed that Socrates *(Sortes)* is moved toward the west in a ship that is at rest. Then it is possible that Socrates might cease moving in any instant. Now let it be assumed that in the [very] same instant in which Socrates should cease to be moved [toward the west], the ship, with all its contents, begins to be moved toward the east. Hence, immediately before, Socrates was moved to the west, and immediately after, will be moved toward the east. Therefore, previously he was moved with one motion and afterward with another, and contrary, motion without a moment of rest.[19]

But we must see now the manner in which response is made to the first two arguments. To the first, it is said that a millstone forcefully pushes the air before it and that this air by its pushing stops the bean before the millstone reaches it.[20] But [I argue] against this because either the impetus of the air, which you say stops the bean, is equally as forceful *(fortis)* as the impetus of the bean for moving upward against the resistance of the medium, or it is stronger or weaker. If stronger, then it would make the bean descend and consequently the bean would not rest there; if weaker, the bean would ascend upward even further, and as a consequence, the air will not stop the bean. And if [the air] is equally as forceful, this will endure exactly for an instant[21] because as the millstone approaches closer [to the bean] so much more does it push the air. There is, consequently, no temporal rest in the middle [or midpoint of the bean's motion].

To the second argument, it is customary, in the first place, to respond that Socrates is self-moved and is moved [only] accidentally with the motion of the ship. Since one of them is self-moved and the other is moved accidentally, we do not have contrary motions and hence no rest at the middle of the motion. But concerning two self-moved [contrary] motions, it is impossible [that there should be no temporal rest]. Another response is that although it is possible that Socrates might cease to be moved in the very instant in which the ship begins to be moved, nevertheless it is impossible that these two [events] occur at the same instant. I argue against the first solution because our conception of motion requires that the same judgment be made about self-motion and accidental motion, even though there is a difference with respect to the movers [in each of these motions]. If, therefore, one of the contrary motions is self-moved and the other is accidental and there is no interval of rest, it follows that there is also no interval of rest between contrary motions when each motion is self-moved *(per se)*. Secondly, since any argument *(ratio)* drawn from Aristotle, and any other argument which might prove something about self-motion, can be utilized to derive the same thing about accidental motion, [it follows] therefore that what is conceded about the one [kind of motion] must be conceded about the other.

I argue against the second solution, because it does not seem [to offer] a [good] reason why the ship could not begin to be moved in the same instant in which Socrates ceases to be moved, so that each can occur in the same instant. In the second place, this can also be proved by varying the case. Let it be assumed that the ship is moved uniformly toward the east and that Socrates is moved difformly (that is, non-uniformly) on the ship with a proper motion toward the west, but quicker than the ship in the first half of the hour and slower in

18. In this example, we have the most fundamental experiential appeal of those who rejected Aristotle's interval of rest. The example itself, and its numerous trivial variations, can be traced back at least as far as Abū l'Barakāt al-Baghdādī (ca. 1080—ca. 1165) who, in order to reject Avicenna's arguments in defense of Aristotle's position, formulated it in terms of a falling millstone striking an ascending date pit. (See S. Pines, "Études sur Ahwad al Zamān Abū l'Barakāt al-Baghdādī," *Révue des études juives*, New Ser., IV [1938], 5.) Indeed, since Avicenna sought to repudiate it with an argument which Marsilius repeats (see n. 20), it probably antedates Avicenna. Marsilius and many others would utilize it, including Johannes Versor, Pico della Mirandola, Hasdai Crescas, who made a mountain out of a millstone, Giovanni Benedetti, and, finally, Galileo, who by that time could cite the argument "as the well-known one about a large stone falling from a tower" and striking an ascending pebble (see Selection 50.2).

19. Abūl'Barakāt offers analogous arguments where a motion becomes oppositely directed without a moment of rest. Since his works (in Arabic) were not translated into Latin, the manner in which his arguments may have reached the Latin West—if indeed they were not independently formulated—is unknown to me. Continuing with the first conclusion, Marsilius presents five more brief arguments (fol. 84 recto, col 1) which are omitted here. They are analogous to the two already presented but involve the motion of spheres, light, heat, augmentation and diminution. Immediately after, he returns, for further consideration, to the first two arguments translated above.

20. This very argument was formulated long before by Avicenna. The air pushed downward by the descending millstone would stop the ascent of the bean, which would remain motionless through an interval of time and then descend (see Pines, p. 5, n. 265).

21. As we see from the very next sentence, the instant is of no temporal duration.

the next half hour. In the middle instant of the time,[22] it is true to say that immediately before, Socrates was moving toward the west, and immediately after, will be moved toward the east. Therefore, previously he was moved with one motion and afterward with another and contrary motion without an interval of temporal rest.[23]

A second conclusion is this: It is not necessary that between any contrary motions there be rest during an indivisible or intervening instant. This is demonstrated because there is no rest in an instant, as was proved in the sixth [book] of this [treatise]. Secondly, since nothing can be moved locally in an instant,[24] therefore there can be no rest [in an instant] because there is no other measure [of time] in which rest can be produced but not motion.[25] Thirdly, because in no instant intervening between contrary motions is it true to say that a mobile is related otherwise [or differently] than it was immediately before and than it will be immediately after.[26] Therefore the mobile does not rest [in the intervening instant between contrary motions]. Fourthly, an argument that could prove that in this instant rest occurs, could also prove that in any instant of motion the mobile is at rest.[27] Obviously, then, it is not necessary that rest intervene either instantaneously or temporally between any contrary motions.[28]

A third conclusion is that between any *contrary natural motions*, it is necessary that there be an interval of rest. The proof of this is that since every natural motion, except celestial motion, is directed toward coming to rest as if that were its goal or end, therefore, if it does not rest when it reaches its terminus [or goal], it will have been frustrated [or prevented] from attaining its end [or goal]. Secondly, what is moved naturally has not only a natural inclination [or tendency] to motion, but also to rest in the terminus [or goal] of the motion. But this natural inclination [for rest] in the terminus of the motion cannot be lost instantaneously *(subito)*. It is necessary, therefore, that there be a time [lapse] before this inclination to contrary motion be acquired [again]. Consequently, it will rest through that interval of time.[29]

A fourth conclusion is that when something is moved upward violently and then descends downward over the same path, there is, necessarily, an interval of rest between those motions, unless it should be hindered by the resistance of something external. Let this be proved [as follows]: For just as long as anything is moved upward after it has been projected, just so long does its motive power exceed the natural heaviness of the projected body plus the resistance of the medium. But when the impetus moving the projected body is equal to the natural heaviness of the stone, the stone ceases to be moved upward. Nevertheless, it does not yet begin to be moved downward. This is proved because the natural heaviness scarcely exceeds the

22. That is, at the end of the first half hour.

23. During the first half hour the resultant or net motion of Socrates is westward, in the direction of his own proper motion. But at the very instant when Socrates' westward motion becomes slower than the eastward motion of the ship, his resultant motion will become eastward, although he continues to walk westward on the ship. Hence while his direction of motion changed instantaneously from west to east, he did not rest during the change. Marsilius has shown that, by varying the conditions of the first argument against an interval of rest, it is not essential to rely on the mere coincidence that Socrates ceases to be moved exactly at the instant when the ship begins to be moved in the opposite direction. Now Socrates continues walking westward while his direction of motion is changed to eastward. Much the same type of argument was used by Galileo (see the next selection).

24. In Book VI, Question 10, Marsilius asks "whether motion could occur in an instant"; it cannot, he argues.

25. Neither rest nor motion can be measured by "instants"; that is, if motion is unmeasurable by an "instant," so also is rest.

26. The intention here is to show that if the mobile's behavior is identical during the three successive instants of time embracing (1) the instant immediately prior to the intervening instant, (2) the instant intervening between the contrary motions, and (3) the instant immediately following the intervening instant, it follows that the mobile did not rest in the instant represented by (2).

27. If rest can be proven for one instant, it can be proven for any instant, and consequently, for every instant. Therefore, the mobile will be at rest in every instant.

28. Throughout this question, Marsilius interweaves these two notions of an "instant"—one durationless, the other of temporal length.

29. In the example above of the millstone and bean, the sequence of motions was violent to natural (the bean ascends by a violent motion, and after having been struck by the millstone, descends by natural motion) with no intervening moment of rest. But here the sequence is reversed, where a natural motion is followed by a violent motion and, it is now argued, an interval of rest must occur. This can be illustrated by a rubber ball (a heavy earthy body) falling naturally toward the earth. Since it has a natural tendency to rest when it reaches its natural place, and since "this natural inclination [for rest] in the terminus of motion cannot be lost instantaneously," we must suppose that before it bounces upward in a violent motion, it will rest for a temporal moment. Thus the curious expression "contrary natural motions" must not be taken literally (for then it would be unintelligible in Aristotelian terms; one and the same body cannot have "contrary natural motions") but rather must signify a natural motion followed by a violent one.

medium yet and it is necessary, therefore, that time should pass before it does.[30] Consequently, the body will rest in this time.

And so it is clear in what manner it is not necessary that an interval of rest occur between any contrary motions. However, between *contrary natural motions* of one and the same mobile, it is necessary that the mobile rest for an interval of time and [also] when one of the motions is natural and the other violent, except when there is an obstacle by the interference of another [external body].[31]

Now [let us turn] to the [principal] arguments. To the first, I deny the consequence, which is not similar [or analogous], since things resting in *opposite places* can do so only in different places, whereas *opposite motions* can occur over the same space.[32] Furthermore, by denying the antecedent, one can declare otherwise, for if the earth did not rest naturally and if by the power of the sun it became hot and light on one side, then there is now another center of gravity than before. But previously the earth rested naturally, but now it rests violently, and yet there is no intermediate motion [between the natural and violent rest].[33] That the earth now rests is obvious, because the lightness produced [on one side of the earth] is not sufficient to move the earth locally because of the resistance of the surrounding medium. To the second argument, I say that Aristotle understands that there is an interval of rest between any contrary natural motions and this is sufficient for his purpose, because if any motion were continuous and eternal it would necessarily be natural. . . .[34]

2. Galileo Galilei (1564–1642)

Translated by I. E. Drabkin[35]
Annotated by Edward Grant

CHAPTER [20]

In which, in opposition to Aristotle and the general view, it is shown that at the turning point [an interval of] rest does not occur.

Aristotle and his followers believed that two

30. Since the natural heaviness is balanced by the impetus pushing the body upward, the body will not actually move down until the balancing impetus, or incorporeal impressed force, will have become further corrupted, after which it is exceeded by the natural gravity or heaviness of the body. This fourth conclusion presents again the case of a violent motion followed by a natural motion, just as with the bean and millstone. But now Marsilius assumes a temporal interval of rest because there is no external body, such as the millstone, to interfere with the smooth flow of the up-down movement. For this same case, Avicenna argued for an interval of rest by insisting that the upward impressed force had to be wholly dissipated before the natural force could start the body downward. The interval of rest occurs between the disappearance of the one and the advent of the other. Abū l'Barakāt, on the other hand, denied Avicenna's arguments and the interval of rest for this case.

31. The cases distinguished by Marsilius may be summarized as follows:
I. *No interval of rest*
1. Vertical Contrary Motions: Violent motion followed by natural motion plus an external interference (millstone-bean)
2. Horizontal Contrary Motions (east-west) (Socrates on the ship)
II. *Interval of rest*
1. Vertical Contrary Motions: Natural motion followed by violent motion
2. Vertical Contrary Motions: Violent followed by natural motion with no external interference.

32. Because the proposition "between things resting in opposite places there is an intervening motion" is true, it does not follow, argues Marsilius, that the proposition "between any contrary motions there is an interval of rest" is true by analogy or similarity. Such an argument merely trades on the superficial similarity between the forms of the statements.

33. The antecedent of the first principal argument (on p. 285) is that "between things resting in opposite places there is an intervening motion." Marsilius denies this by positing a contrary case, namely by supposing that the earth does not rest naturally and that its center of gravity continually shifts because of the sun's heating action. But the earth does rest, therefore it rests violently as its center of gravity shifts. Hence the earth rests violently in different places but does so without intervening motion, since it was at violent rest—but rest nonetheless—throughout.

34. The remaining counterarguments, brief and unilluminating, have been omitted.

35. Reprinted with permission of the Regents of the University of Wisconsin from *Galileo Galilei "On Motion" and "On Mechanics" Comprising "De Motu"* (ca. 1590), translated with introduction and notes by I. E. Drabkin, and *"Le Meccaniche"* (ca. 1600), translated with introduction and notes by Stillman Drake (Madison Wis.: University of Wisconsin Press, 1960), pp. 94–100. The basic similarity between some of Galileo's arguments and those of Marsilius of Inghen in the preceding selection will be obvious. On this question, Galileo merely continues and elaborates a well-established medieval tradition.

contrary motions—he defines contrary motions as those which tend toward opposite goals—could in no way be continuous with each other. And therefore they believed that, when a stone is projected upward and then falls back over the same path, it must necessarily remain at rest at the turning point. The chief argument with which Aristotle tries to prove this is as follows. "Whatever moves by approaching some point and then moving away from that same point, using it as an end and as a beginning, will not move away from it, unless it has first stopped at it. But that which moves to the farthest point of a line and then moves back from that point, uses that point both as an end and as a beginning. It must therefore remain stationary between the motion toward and the motion away from the point."[36] Aristotle proves his major premise thus: "Whoever treats something both as beginning and as end, makes what is one in number two in logic, just as the person who in thought takes a point, which is one and the same numerically, and makes it two in logic, namely the end of one thing and the beginning of a second thing. But if something uses one thing as two it must necessarily remain stationary there; for there is [an interval of] time between the two."

Such is Aristotle's argument. But how weak it is will soon be clear. For, as he himself holds, the moving body makes use of a point on the line of its motion, i.e., one point, numerically, for what are two things in logic, for a beginning and an end. Yet there is no line between these two things, since they are only one in number. And why, similarly, will the same body not use the same instant (one, in number) as two in logic, namely, for the end of the time of moving toward [the turning point] and for the beginning of the time of moving away [from it], so that between these instants that are two in logic, no time intervenes, since they are only one in number?

There is no compelling reason why this should not be the case, especially since Aristotle himself holds that what is true of a line is true also of time and motion. If, then, on the same line the same point, numerically, is both the end of one motion and the beginning of a second, and if, nevertheless, it is not necessary that a line form a connection between this beginning and that end, then, in the same way, the same instant numerically, will, in logic, be the end of one time and the beginning of a second time, and it will not be necessary for time to intervene between the two. It is clear, then, that the refutation of Aristotle's argument can be neatly derived from the propositions of that same argument. Hence, since the argument no longer has compelling force for us, let us see whether we can construct arguments for the opposite view that are more sharply convincing.

So much in opposition to Aristotle. But in order for us to show by other arguments that [an interval of] rest does not occur at the turning point, and that there need not be such rest between contrary motions, consider these additional arguments.

Secondly, suppose that some continuum, such as the whole of line *ab*, moves in the direction of *b* in a motion like a forced motion which becomes continuously slower. And while the line so moves, suppose that a body, say *c*, moves on the same line in the opposite direction, from *b* to *a*. But let this motion be like a natural motion, that is, one that is accelerated. And let the motion of the line at the beginning be faster than the motion of *c* at the beginning. Now it is clear that at the beginning *c* will move in the same direction as that in which the line moves, because the motion by which it is carried in the opposite direction is slower than the motion of the line. And yet, since the motion of the line becomes slower and the [leftward component of the] motion of *c* becomes faster, at some moment *c* will actually move toward the left, and will thus make the change from rightward to leftward motion over the same line. And yet it will not be at rest for any [interval of] time at the point where the change occurs. And the reason for this is that it cannot be at rest unless the line moves to the right at the same speed as body *c* moves to the left. But it will never happen that this equality will continue over any interval of time, since the speed of one motion is continuously diminished, and that of the other continuously increased. Hence it follows that *c* will change from one motion to its contrary with no intervening state of rest.[37]

My third argument can be drawn from a certain rectilinear motion which Nicholas Copernicus in his *De Revolutionibus* compounds from two circular motions.[38] There are two circles [the center of] each of which is carried on the circumference of the other. When one circle moves more swiftly than the other, a point on the circumference of the first circle moves in a straight line continuously back and forth over the same path. And yet it cannot be said that the point is at rest at the extremities, since it is carried continuously by the circumference of the circle.[39]

36. *Physics* VIII.8.262b.2–8 and also the quotation at the beginning of this selection.

37. This argument is fundamentally the same as Marsilius of Inghen's, in the preceding section.

38. Book III, chapter 4.

My fourth argument is the well-known one about a large stone falling from a tower.[40] A little pebble is forcibly thrown up from below against it, but the stone will not be sufficiently blocked by the pebble so as to allow the pebble to be at rest for any interval of time. Hence the pebble will surely not remain at rest at the farthest point of its upward motion, and, despite what Aristotle said, it will use that farthest point as two termini, namely, of upward motion and of downward motion. And the last instant is taken twice, viz., as the end of one interval of time and as the beginning of the other.

But in order to escape from this argument my adversaries declare that the large stone is at rest, and so they believe that they have answered the argument.[41] But, so that they may not believe this in the future (unless they are thoroughly obstinate), I shall add the following to my argument. Suppose that those stones which move with contrary motions move not up and down but on a plane surface parallel to the horizon, one with great impetus, and the other more slowly. And suppose that they move in opposite directions from opposite parts and meet in the middle in an interacting motion. In that case, there is no doubt that the weaker will be thrust back by the stronger and forced to move back. But how can they say that at that point of impact an interval of rest occurs? For if once they remained at rest, they would thereafter always be at rest, since they would not have reason for moving. In the case of the large stone falling from a high point, even if it were stopped by the pebble, yet after the interval of rest, both would fall together, moving down by reason of their own weight. But when they are in a plane parallel to the horizon there exists no cause of motion after the [supposed] interval of rest.

Before expounding my last argument, I make these two assumptions. My first assumption is that only then is it possible for a body to be at rest outside its proper place, when the force that opposes its fall is equal to its weight, which exerts pressure downward. Surely this is clear: for if the impressed force was greater than the resistant weight, the body would continue to move upward; and if it were smaller, the body would fall. Secondly, I assume that the same body can be sustained in the same place over equal intervals of time by equal forces.

I then urge the following. If a state of rest lasting for some interval of time occurs at the turning point, e.g., when a stone changes from forced upward motion to [natural] downward motion, then over the same interval of time there will exist equality between the projecting force and the resisting weight. But this is impossible, since it was proved in the previous chapter that the projecting force is continuously diminished.[42] For the motion in which the stone changes from accidental lightness to heaviness is one and continuous, as when iron moves [i.e., changes] from heat to coldness. Therefore the stone will not be able to remain at rest.

Furthermore, suppose that the stone moves forcibly from *a* to *b*, and naturally from *b* to *a*. If, then, the stone is at rest at *b* for some interval of time, suppose that this time has as its end moments *c* and *d*. If, then, the body is at rest for time *cd*, the external projecting force will, through time *cd*, be equal to the weight of the body. But the natural

39. Drabkin notes (p. 97, n. 5) that "there is an analogous and simpler example in Benedetti, *Diversarum speculationum. . .liber*, p. 183 (cited by A. Koyré, *Études Galiléennes*, I, 51), in which one end of a rigid bar is attached to a point (other than the center) of a continuously rotating circle, and the other end moves back and forth along the same straight line." Abū l' Barakāt had formulated much the same argument long before (see my n. 19).

40. See Inghen section and note 18.

41. Probably a reference to the type of argument cited by Marsilius of Inghen but formulated long before by Avicenna (see Inghen section and n. 20). However, Galileo's version is distorted. It was not claimed that the large stone would rest, but rather that the pebble would have had its upward direction reversed (by the force of the air pushed downward by the large falling stone) before the large stone reached it. Perhaps Galileo's version represents a Renaissance variation on the traditional arguments.

42. Actually demonstrated in Chapter 18 ("In which it is shown that the motive force is gradually weakened in the moving body"). In chapter 19 upward deceleration and downward acceleration of bodies is explained in terms of a self-expending impressed force. (Also see Drabkin, p. 89). "In Galileo's exposition, the notion of a residual force plays the leading role. Initially, the mover imparts an impressed force to a stone that is hurled aloft. As the force diminishes, the body gradually decreases its upward speed until the impressed force is counterbalanced by the weight of the stone at which moment the stone commences to fall, slowly at first and then more quickly as the impressed force diminishes and gradually dissipates itself. The acceleration arises from the continual increase of the difference between the weight of the stone and the diminishing impressed force." (Edward Grant, "Bradwardine and Galileo: Equality of Velocities in the Void," *Archive for History of Exact Sciences*, II [1965], 360.)

weight is always the same. Therefore the [projecting] force at moment *c* is equal to that force at moment *d*. Now it is the same stone and the same place: hence the stone will be held there over equal intervals of time by equal forces. But the force at moment *c* sustains the body throughout time *cd*. Hence the force at moment *d* will sustain the same stone throughout an interval of time equal to interval *cd*. The body will therefore be at rest throughout twice time *cd*. But this is inconsistent: for it was assumed to be at rest only through time *cd*. Indeed, by continuing the same form of argument, we could prove that the stone would always be at rest at *b*.

But do not be confused by the argument that, if the weight and projecting force are equal at some time, then the body must be at rest for some time. For it is one thing to say that the weight of the body at some time comes to be equal to the projecting force; but it is another thing to say that it remains in this state of equality over an interval of time. This becomes clear from the following consideration. While the body is in motion, since (as has been shown) the projecting force is always being diminished, but the intrinsic weight always remains the same, it must follow that, before they arrive at a relation of equality, countless other ratios occur. Yet it is impossible for the force and the weight to remain in any of these ratios over any interval of time. For it has been proved that the projecting force never remains at the same level over an interval of time, since it is always diminishing.

And so, it is true that the [projecting] force and the weight pass through ratios of, let us say, 2 to 1, 3 to 2, 4 to 3, and countless other ratios; but it is false and impossible that they should remain for any interval of time in any one of these ratios. So, too, they arrive at equality at some moment, but they do not remain at equality. This being so, since local motion upward and downward is a consequence of that alterative motion of change from light *per accidens* to heavy *per se*, in such a way that upward motion flows from an excess of impressed force, downward motion from a deficiency thereof, and rest from equality, and since this equality does not persist over an interval of time, it follows that neither does the state of rest persist.

51 MATHEMATICAL REPRESENTATIONS OF MOTION

1. Thomas Bradwardine (ca. 1290–1349): "Bradwardine's Function" and the Repudiation of Four Opposition Theories on Proportions of Motion

Translated by H. Lamar Crosby, Jr.[1]
Annotated by Edward Grant

CHAPTER 2

Having looked into these introductory matters,[2] let us now proceed with the undertaking which was proposed at the outset. And first, after the manner of Aristotle, let us criticize erroneous theories, so that the truth may be the more apparent.

There are four false theories to be proposed as relevant to our investigation, the first of which holds that: *the proportion between the speeds with which motions take place varies as the difference whereby the power of the mover exceeds the resistance offered by the thing moved.*[3]

This theory claims in its favor that passage from Book I of the *De caelo et mundo* (in the chapter on the "infinite") in the text which reads:

1. Reprinted with permission of the copyright owners, the Regents of the University of Wisconsin, from *Thomas of Bradwardine: His Tractatus de Proportionibus; Its Significance for the Development of Mathematical Physics*, edited and translated by H. Lamar Crosby, Jr. (Madison, Wis.: University of Wisconsin Press, 1955). The following selection is from chapters 2 and 3, pp. 87–117. Bradwardine wrote the *Tractatus de proportionibus (Treatise on Proportions)* in 1328.

2. The introduction and chapter 1 were devoted to a description of the various types of ratios, proportionalities, and the enunciation of a number of axioms and theorems necessary for subsequent developments. Many of the preliminaries were drawn from the *Arithmetica* of Boethius and the *Elements* of Euclid (especially Bk. V).

3. This first erroneous theory is representable by $V_2/V_1 = (F_2 - R_2)/(F_1 - R_1)$, where V is velocity, F motive force, and R the resistance of the moving body (or, should the body fall with a natural downward motion, R would represent the resistance of the medium). It is likely that Bradwardine's version of this theory was derived ultimately from Avempace, a conjecture that gains support from the fact that a few lines further on, we find mentioned Averroes' Comment 71 on Book IV of Aristotle's *Physics*, the very place where Avempace's views were reported. If so, this could only have occurred by reading into Avempace's words a great deal more than was warranted, for it was noted (Selection 44, n. 17) that Avempace did not represent the natural motion of bodies by the formulation $V = F - R$.

"It is necessary that proportionally as the mover is in excess, etc.," together with Averroes' Comment[4] 71 on Book IV of the *Physics*, in which he says: "Every motion takes place in accordance with the excess of the power of the mover over that of the thing moved." In Comment 35, on Book VII of the *Physics*, he further states that: "The speed proper to any given motion varies with the excess of the power of the mover over that of the thing moved," and in the final Comment, Comment 39, he says that: "The speed of alteration and the quantity of time will vary in accordance with the amount whereby the power of that which is causing the alteration exceeds the resistance offered by what undergoes the alteration." Many other passages afford similar remarks.

The present theory may, however, be torn down in several ways:

First, according to this theory, it would follow that, if a given mover moved a given *mobile* through a given distance in a given time, half of that mover would not move half of the *mobile* through the same distance in an equal time, but only through half the distance. The consequence is clear, because, if the whole mover exceeds the whole *mobile* by the whole excess, then half the mover exceeds half the *mobile* by only half the former amount; for, just as 4 exceeds 2 by 2, half of 4 (namely, 2) exceeds half of itself (that is, 1) by 1, which is only half of the former excess.

That such a consequence is false is apparent from the fact that Aristotle proves, at the close of Book VII of the *Physics*, that: "If a given power moves a given *mobile* through a given distance in a given time, half that power will move half the *mobile* through an equal distance in an equal time" [250a.4–6—*Ed.*]. Aristotle's reasoning is quite sound, for, since the half is related to the half by the same proportion as the whole is to the whole, the two motions will, therefore, be of equal speed.

Secondly, it follows from this theory that, given two movers moving two *mobilia* through equal distances in equal times, the two movers, conjoined, would not move the two *mobilia*, conjoined, through an exactly equal distance in an equal time, but, instead, through double that distance. This consequence follows necessarily because the excess of the two movers, taken together, over the two *mobilia*, taken together, is twice the excess of each of them over its own *mobile;* for, just as anything having a value of 2 exceeds unity by 1, so two such "2's" (which make 4) exceed two "1's" (which make 2) by 2, which is twice the excess of 2 over 1. The foregoing holds in all cases in which two subtrahends are equally exceeded by two minuends.

That the above consequence is false is evident from the foregoing argument of Aristotle, in Book VII of his *Physics*, where he demonstrates the following conclusion: "If two powers move two *mobilia*, separately, through equal distances in equal times, those powers conjoined, will move the two *mobilia* conjoined, through an equal distance in a time equal to the former one" [250a.25–28—*Ed.*]. This argument of Aristotle is sufficient proof that the relation between a single mover and its *mobile*, and a compound mover and its *mobile*, is a proportional one.

In the third place, it would follow that a geometric proportion (that is, a similarity of proportions) of movers to their *mobilia* would not produce equal speeds, since it does not represent an equality of excesses; for, although the proportions of 2 to 1 and 6 to 3 are the same, the excess of the one term over the other is 1 in the first case and 3 in the second case.

The consequence to which we are thus led is, however, false and opposed to Aristotle's opinion, as expressed at the close of Book VII of the *Physics* and in many other places, where, from an equality of proportions of movers to their *mobilia*, he always argues equal speeds. Averroes supports the same view in his remarks on the passages just mentioned and also in his Comment 71 on *Physics* IV, Comment 63 on *De caelo et mundo* I, and in many other places.

Nor can it be legitimately maintained that, in the passages cited, Aristotle and Averroes understand, by the words "proportion" and "analogy," arithmetic proportionality (that is, equality of differences), as some have claimed. Indeed, in Book VII of the *Physics*, Aristotle proves this conclusion: "If a given power moves a given *mobile* through a given distance in a given time, half that power will move half the *mobile* through an equal distance in an equal time, because, 'analogically,' the relation of half the mover to half the *mobile* is similar to that of the whole mover to the whole *mobile*."[5]

4. References to the comments of Averroes on the *Physics* and *De caelo (On the Heavens)* of Aristotle may be found in the *Aristotelis opera cum Averrois commentariis* (Venice, 1562–1574; Frankfurt: Minerva, 1962), Vols. IV and V. Comment 71 is translated above, in Selection 44.

5. 250a.4–9. In this quotation, Bradwardine has omitted an inessential line.

Such a statement, interpreted as referring to arithmetic proportionality, is discernibly false (as has already been made sufficiently clear in the first argument raised against the present theory). Moreover, regarding this same passage, Averroes says that the proportion will be the same "in the sense that geometricians universally employ in demonstrations."

The above thesis of Aristotle may be demonstrated geometrically as follows: As is the whole mover to half the mover, so is the whole *mobile* to half the *mobile*. Therefore, permutatively (by Axiom 7 of Chapter I):[6] As is the whole mover to the whole *mobile*, so is half the mover to half the *mobile*. And this is what was to have been proved.

The reading of Aristotle proposed by the present theory does not stand up, moreover, because, in Book VII of the *Physics*, Aristotle also proves this conclusion: "If two movers, separately, move two *mobilia* through equal distances in equal times, those two movers, conjointly, will move those two *mobilia*, conjoined, through a distance and in a time equal to the former,"[7] if the medium through which the motions take place remains the same, "for the motions are 'analogous'." By "analogous" he means a proportional, but not in the sense of arithmetic proportionality, for the simple mover does not exceed the simple *mobile* by the same amount that the compound mover exceeds the compound *mobile*, (as was made clear in the second argument against this theory).

Averroes, commenting on this passage, proves that, "although the proportion will be the same, the excess will not." He does this by employing Proposition [Crosby has "Theorem"—*Ed.*] I of Book V of Euclid's *Elements*, which states that: "If the members of a given set of quantities are either equal multiples of, equally greater or less than,[8] or exact equals of the members of a corresponding set, it follows that the relations between corresponding individual terms of the two sets will be the same as the aggregate relation of the two sets." Therefore, the above-mentioned reading of Aristotle cannot be taken as valid.

The fourth criticism is as follows: It would follow, on the basis of the present theory, that a mixed body, possessing internal resistance, could move faster through a medium than through a vacuum. Let *A*, for example, be a heavy mixed body (possessing within itself both motive and resistive power)[9] and imagine it to descend of itself through some medium, *B*. Let *C* represent a quantity of pure earth, possessing less power than the excess of motive power over resistance in *A*. *A* will, of itself, move at a determinate speed in a vacuum. Now let the medium, *B*, be rarified to the point at which *C* moves in it with a speed equal to that of *A* in the vacuum. If *A* is now placed in the same medium with *C*, it should move faster than *C* (for it possesses a greater excess of motive power over resistance). *C* will move in that medium with a speed equal to that of *A*, moving in the vacuum. Therefore *A* will move faster through the medium than through the vacuum.[10]

6. The seventh axiom or assumption declares that if four quantities are related as $A/B = C/D$, then permutatively $A/C = B/D$.

7. 250a.25–28. This quotation is meant to be identical with one given above even though the Latin texts, and hence the translations, differ somewhat, most noticeably in the replacement of "powers" *(potentiae)* by "movers" *(motores)*.

8. The phrase "equally greater or less than" does not appear in the modern Greek text nor in the popular medieval Latin version by Campanus of Novara.

9. A heavy mixed body was one compounded of both heavy and light elemental bodies, where the light elements, with their natural upward inclination, were assumed to function as internal resistance to the prevailing heavy elements, whose natural downward inclination functioned as the motive force actually moving the body downward. Every such body was held to be capable of moving through a hypothetical vacuum with finite speed—rather than instantaneously, as Aristotle held—since its internal resistance guaranteed that the motion would take time and consequently be theoretically measurable. The innovator of the concept of internal resistance, a radical departure from Aristotle, is unknown to me. Mixed bodies and internal resistance play a fundamental role in Selections 55.2,4;5.

10. Let $(f - r)$ represent the difference between motive force and internal resistance in *A*, a heavy mixed body that can fall in a resistant medium *B* and also fall in the void with a determinate speed. Furthermore, let *C* be a quantity of pure elemental earth whose motive power is represented by F_C (because *C* is a pure elemental body, it has no internal resistance). Finally, it is assumed that $(f - r) > F_C$ (although the motive power of a body *C* is always relative to the resistive power of a corporeal medium, in this example Bradwardine seems to compare $(f - r)$ to the absolute power of *C* without reference to a medium; indeed, it is as if *C* were falling in a void).

Since *A* is taken as falling with a finite velocity in the void, it is possible to produce an equal velocity for *C* falling in medium *B* by properly adjusting the density of *B*. In terms of the first erroneous theory, we may represent this as: $f - r = F_C - B$, so that V_A in void $= V_C$ in *B*. But now if *A* is let fall in the same medium *B*, it will fall with a speed greater than *C*, since $(f - r) - B > F_C - B$ (for by assumption $f - r > F_C$) so that V_A in $B > V_C$ in *B*. Drawing together these two consequences, we see that V_A in void $= V_C$ in B, and V_A in $B > V_C$ in *B*, from which it follows that V_A in $B > V_A$ in void.

That it is, in fact, possible to rarify *B* to a point at which *C* would move in it with the speed just specified is evidently true, for, by rarefaction of the medium, local motion can be accelerated to any desired degree. This is shown to be the case in Book IV of the *Physics* (in the chapter on the "void" [215b.1–11—*Ed.*]), where it is stated that, with the moving power remaining constant, it is possible to arrive at any given speed of local motion by rarefying the medium.

Thus (positing that a local motion could take place in a void), we find that the same body could move at the same velocity in both a medium and a vacuum.[11]

Our fifth criticism is that it would also follow that, if a given mover exceeded its resistance by a lesser amount than another mover exceeded its resistance, the former motion would be the slower one. Let a large quantity of earth, possessing a given resistance which its downward force greatly exceeds, be supposed to fall. Let also another quantity of earth, possessing a lesser such excess of power, be supposed to fall. Letting the larger quantity of earth and the resistance associated with it remain constant, let the medium in which the smaller quantity of earth moves now be rarefied to the point at which its speed becomes equal to that of the larger quantity. The smaller quantity now moves its resistance with the same speed that the larger moves its own, and yet exceeds it by a smaller amount.[12]

Sixthly, it would also follow that, if a bit of pure earth were moving in some medium whose resistance its power exceeded by a ratio of two to one, or more, it could not move at double that speed in any other medium. It could not exceed any medium by double the first excess, for, in that case, the entire moving power would be excess, and, with the moving power remaining constant, it would consequently not be possible to increase the speed of the motion indefinitely by rarefaction of the medium. Such a consequence has already been established as false.[13]

In the seventh place, another consequence would be that, if a given mover were to exceed its resistance by a greater amount than another mover exceeded its own resistance, the former motion would be the faster. Then, since a strong man exceeds anything he moves by a greater excess of power than a weaker mover (such as a boy, or a fly, or something of that sort) exceeds what it moves, he should move it more rapidly.

Experience, however, teaches us the contrary, for we see that a fly carrying some small particle flies very rapidly, and that a boy also moves a small object rather rapidly. A strong man, on the other hand, moving some large object which he can scarcely budge, moves it very slowly, and even if there were added to what he moves a quantity larger than either the fly or the boy can move, the man will then move the whole not much more slowly than he did before.

From all these considerations, therefore, the

By the assumption of a subtractive relation between the motive force and internal resistance of *A* (that is, $f - r$), one derives the absurd consequence that one and the same heavy mixed body will fall more quickly in a plenum than in a vacuum.

11. A lapse in Bradwardine's thought seems to have occurred here. For the proof just presented (and summarized in the preceding note) demonstrates that the same body would move faster through a medium than through a void, and not that it would move with equal speeds in medium and vacuum (in *Physics* IV.8.215b.20—216a.7, Aristotle showed that one and the same body would, in the same time, fall equal distances in void and plenum).

12. It must be understood that Bradwardine is here speaking of heavy mixed bodies of earth possessing internal resistances as well as being opposed by an external resistance in the form of an ambient medium. Let A_2 and A_1 be heavy mixed bodies, with $A_2 < A_1$. If $f_2 - r_2 < f_1 - r_1$, where f is motive force and r internal resistance, then, as a consequence of the erroneous theory, $V_2 < V_1$ (where V_2 is the velocity of A_2 and V_1 of A_1). Taking R_1 as the initial resistance of the external medium and R_2 as the resistance of the medium after it has been rarefied to the point where $f_2 - r_2 - R_2 = f_1 - r_1 - R_1$, it follows that the velocities of A_2 and A_1 are equal—that is, the two forces f_2 and f_1 now move A_2 and A_1 with the same speeds even though $f_2 - r^2 < f_1 - r_1$. That Bradwardine should have taken seriously this criticism is surprising. An adherent of the theory in question would undoubtedly have replied that it is the total resistance—not just internal resistance—which must be subtracted from the motive force, and if the remainders are equal, so also are the resultant velocities.

13. On the assumption that a medium is infinitely rarefiable (this was assumed above), Bradwardine now argues that if $F - R = 2 - 1$, where F is the motive force of a piece of pure earth, say A, and R the resistance of the medium, then A could not double in speed in any whatever corporeal medium, but could do so only in void space, where $F - R = 2 - 0$. But if, as all believed, a medium is infinitely rarefiable, it ought to be possible to double any speed by holding the force constant and suitably rarefying the medium. Hence a universally accepted Aristotelian position is violated and the theory is held to collapse. Bradwardine's argument is wide of the mark, since a partisan of this theory was committed to a rejection of the Aristotelian position that velocity is inversely proportional to the density of the medium.

following negative conclusion is sufficiently well established:

The proportion of speeds in motions does not vary with the amount whereby the power of the mover exceeds that of the thing moved.

Objections to this conclusion are not difficult to dissolve, for Aristotle and Averroes, when they say that the speed of a motion varies in accordance with the amount by which the power of the mover excels or exceeds that of the thing moved, understand by "excellence," or "excess," a proportion of greater inequality whereby the power of the mover excels, or exceeds, the power of the thing moved.[14]

CHAPTER 2, PART 2
[THEORY II]

Let us now turn to the second erroneous theory, which supposes *the proportion of the speeds of motions to vary in accordance with the proportion of the excesses whereby the moving powers exceed the resisting powers.*[15] This idea is evidently based on Averroes' Comment 36 on Book VII of the *Physics*, for he there states that the speed of a motion is determined by the proportion whereby the power of the mover exceeds that of the thing moved.

This theory should, however, be refuted as false. For just imagine the case in which the excess of the power of the mover over that of the thing moved is equal to the power of the thing moved. No mover will be able to move any *mobile* either faster or slower than the speed produced by this proportion, because no other proportion can be either greater or smaller than that whereby the excess of power of this mover over its *mobile* is related to the power of the *mobile* as a whole (as is demonstrable by Theorem VII, Chapter I).[16]

In the second place, a moving power moves a whole *mobile* primarily by means of its total strength, and not by means of a residuum of its strength. A motion and its speed vary primarily and essentially, therefore, with the relation, or proportion, between the entire power of the mover and that of the thing moved, and not (except accidentally and secondarily) according to a proportion of excess.

This negative conclusion is therefore evident: *The proportion of the speeds of motions does not vary in accordance with the proportion*[17] *of the excess of the motive power over the power of the thing moved.* The above-mentioned statement by Averroes may, if anyone were so to desire, be interpreted in the same way as were the other authorities cited in support of Theory I.

CHAPTER 2, PART 3
[THEORY III]

There follows the third erroneous theory, which claims that: *(with the moving power remaining constant) the proportion of the speeds of motions varies in accordance with the proportion of resistances,*[18] *and (with the resistance remaining constant) that it varies in accordance with the proportion of moving powers.*[19]

With respect to its first part, this theory is seen to be founded on many passages of Aristotle's

14. That is, velocity varies as F/R, where $F > R$ (rather than as $F - R$, where $F > R$).

15. We may represent this as $V_2/V_1 = (F_2 - R_2) : R_2/(F_1 - R_1) : R_1$. I know of no supporters of this theory prior to Bradwardine, and can cite only Giovanni Marliani (d. 1483) as one who adopted it after Bradwardine. In Latin, the language describing this theory is almost indistinguishable from that of the previous theory. It is only by means of specific examples that one can tell them apart with any assurance.

16. If $V \propto (F_1 - R_1)/R_1$ and $F_1 - R_1 = R_1$, then $(F_1 - R_1)/R_1 = 1$ and we have a ratio of equality, which, by Theorem VII of chapter 1, can bear no exponential relation to ratios of greater or lesser inequality. In short, a ratio of equality, $(1/1)^n$, cannot be made equal to p/q, where $p \neq q$ and p and q are integers. Thus $(F_1 - R_1)/R_1$, where $F_1 - R_1 = R_1$, cannot be made equal to, or greater than, or be compared in any way with, $(F_2 - R_2)/R_2$, where $F_2 - R_2 > R_2$. As a consequence of this theory, then, it follows that whatever the speed with which F_2 moves R_2, it is not comparable to the speed with which F_1 moves R_1. While this consequence is an important feature of Bradwardine's "true" theory (see later), it seems strange to us that he should invoke Theorem VII of chapter 1 against this "erroneous" theory, where such a theorem would have been wholly irrelevant. However, it was fairly typical of medieval scholastic argumentation to repudiate opposing theories by deriving consequences from the basic presuppositions of one's own theory, which were then used against opposition theories subscribing to a quite different set of fundamental assumptions. (Nicole Oresme does much the same thing; see n. 59 below.)

17. I have substituted the remainder of this sentence for Crosby's version which reads: "whereby the excess of the moving power over its mobile is related to the power of that mobile." While this presents an accurate verbal description of the theory Bradwardine repudiates, it adds elements that do not appear in the Latin text, which differs very little from the first rejected theory.

18. $V_2/V_1 = R_1/R_2$ when $F_1 = F_2$.

19. $V_2/V_1 = F_2/F_1$ when $R_2 = R_1$. The two parts of this third theory are representable as $V \propto F/R$, which actually represents Aristotle's verbal formulation despite Bradwardine's attempt to associate Aristotle with his own theory.

writings. In Book IV of the *Physics* (in the chapter on the "void") he speaks as follows: "Let B represent a given quantity of water and D a given quantity of air. Now, by however much air is thinner and more incorporeal than water, by so much will A (that is, the moving body) move faster through D than through B. Let the one speed bear the same ratio, or proportion, to the other as that whereby air differs from water, and then, if air is twice as thin, the body will traverse B in twice the time required to traverse D" [215b.4–8—*Ed.*]. Furthermore, the text immediately following manifestly makes the supposition that, with the moving power remaining constant and the medium being varied, the proportion of the speeds of motions varies in accordance with the proportion of media, and that, conversely, the proportion of the times measuring those motions varies also in accordance with the proportion of media (namely, that the longer time corresponds to the motion through the denser medium and the shorter time to the motion through the rarer medium).

Further, in Book I of the *De caelo* (in the chapter on the "infinite") he speaks as follows: "It is held to undergo a greater and less effect by the action of the same agent, in a longer and shorter time, any such effects are divided proportionally to the time" [275a.32—275b.2—*Ed.*].

And, at the end of Book VII of the *Physics*, Aristotle wishes it to be understood that, if a given power moves a given *mobile* through a given distance in a given time, the same power will move half the same *mobile* through twice the distance in an equal time, and through the same distance in half the time [249b.30—250a.2—*Ed.*].

Thus much in favor of the first half of the present theory.

In support of the second part of this position, Aristotle holds, in Book IV of the *Physics* (in the chapter concerning the "void") that, other conditions remaining constant, heavy and light bodies differing in quantity will move through a given distance in the same medium more swiftly and more slowly in accordance with the proportion of the heavy and light bodies to each other [216a.14–16—*Ed.*].

According to Averroes' exposition, at the close of Book VII of the *Physics*, Aristotle intends that, if a given power moves a given *mobile* through a given distance in a given time, double the power will move that *mobile* through double the distance in an equal time.

At the close of Book VIII of the *Physics*, Aristotle maintains that a motive power which is double another such power will move the same *mobile* in half the time required by the lesser power [266b.10–11—*Ed.*], and that, universally, a motive power greater than another will move the same *mobile* which the smaller power moves in a time that is less by converse proportion (that is, that less time is required by the larger power and more time by the lesser power) [266b.17–19—*Ed.*].

Moreover, Aristotle intends the same thing in Book I of the *De caleo* (in the chapter on the "infinite"), where, speaking of heavy bodies that fall equal distances in the same medium, he writes as follows: "The 'analogy' (that is, the proportion) between the weights will be the contrary of that between the times. For example, if the whole weight in a given time, then double the weight in half that time" [273b.31—274a.2—*Ed.*].

Further, in Book III of the *De caelo*, where Aristotle proves every body to possess a rectilinear gravity or levity, he states that heavy bodies unequal in power will traverse, in the same medium and the same time, distances proportional to those powers [perhaps 301b.11–15—*Ed.*].

The same is evident from Theorem I of the *De ponderibus*, which states the following: "The proportion between the speeds of descent of any given heavy bodies is the same as that between their respective weights."[20]

The theory may also be set forth by the following reasoning: If one mover has exactly twice the power possessed by another, it can move the same *mobile* exactly twice as much, or move twice the *mobile* the same amount, for if it is exactly twice the power, it can accomplish exactly twice as much. If it could accomplish more than twice as much, it would be of more than twice the power; and if it were not capable of twice, but only of less, it would be of less than twice the power.

So much for the second part of this position, and thus we have seen what are the foundations of both parts of the theory.

The theory is, however, refutable on two grounds: first, on that of insufficiency, second, because it yields false consequences.

It is insufficient, because it does not determine the proportion of the speeds of motions except in cases where either the mover or the *mobile* are constant. Concerning motions in which the moving

20. This is from the *Liber Jordani de ponderibus (The Book of Jordanus on Weights)*, the third treatise mentioned in section 2 of the introduction to Selection 39. Despite the title, it is not by Jordanus of Nemore.

forces, as well as the *mobilia*, are varied, it tells us almost nothing.

The theory is, on the other hand, to be refuted on the ground of falsity, for the reason that a given motive power can move a given *mobile* with a given degree of slowness and can also cause a motion of twice that slowness. According to this theory, therefore, it can move double the *mobile*. And, since it can move with four times the slowness, it can move four times the *mobile*, and so on *ad infinitum*. Therefore, any motive power would be of infinite capacity.[21]

A similar argument may be made from the standpoint of the *mobile*. For any *mobile* may be moved with a given degree of slowness, with twice that degree, four times, and so on without end; and, therefore, by the given mover, and by half of it, one fourth of it, and so on, without end. Any *mobile* could, therefore, be moved by any mover.[22]

Nor is it legitimate to object that slowness of motion cannot be doubled indefinitely, for, supposing this to be true, let A represent some slowness of a *mobile* that cannot be doubled. Now imagine a sphere or cylinder, revolving about a fixed axis; then, at some point near the pole of the sphere, or the axis of the cylinder, there is a degree of slowness double that of A, as is quite clear and easy to demonstrate. Now, at this point let there be attached a strong, long cord, at whose end is affixed a given weight, B. Then the slowness of the motion of B is twice that of A, and this is what we wished to demonstrate.

Nor can some quibbler properly claim that the motion of B is accidental motion, merely motion *in potentia*, and that it really has no relevance to the question; for this motion possesses a mover, a thing moved, initial and final limits, a time and a space traversed. . . all *in actu*. Therefore, the motion is a real one. Nor can it be maintained that the mover is not *in actu*, but only *in potentia*, inasmuch as it is part of the sphere, or cylinder, and because it is the whole that moves primarily and the part by consequence. In that case, if a man were to pull that weight by hand, by means of the cord, it would move accidentally, because by virtue of a part of the man; it would then follow that no motion, extrinsically caused, could be a "real" motion, (one which is *in actu*), since no mover can apply itself wholly to the thing moved, but can only do so by means of a part.

Thirdly, the present theory is to be refuted on the ground of falsity, because sense experience teaches us the opposite. We see, indeed, that if, to a single man who is moving some weight which he can scarcely manage with a very slow motion, a second man joins himself, the two together can move it much more than twice as fast.[23] The same principle is quite manifest in the case of a weight suspended from a revolving axle, which it moves insensibly during the course of its own insensible downward movement (as is the case with clocks). If an equal clock weight is added to the first, the whole descends and the axle, or wheel, turns much more than twice as rapidly (as is sufficiently evident to sight).

Since the situation regarding retardation is closely similar, whether the thing moved be constant and the mover be diminished or the reverse we arrive at this negative conclusion: *With the mover remaining constant, the proportion of the speeds of motions does not vary in accordance with the proportion of resistances, nor, with the resistance remaining constant, does it vary in accordance with the proportion of movers.*

As for the reasons which seemed to support this theory, it should be pointed out that all authorities claiming that, with the mover remaining constant, the proportion of the speeds of motions varies in accordance with the proportion of resistances, really mean that the proportion of speeds

21. If $F/R \propto V$, where $F > R$ and produces motion, then $F/nR \propto V/n$, where $n = 2, 4, 8, 16, 32, \ldots$ Thus, when $R > F$, and however great R becomes, F will nevertheless be capable of producing a positive velocity in accordance with the ratio it bears to nR. Hence F will be of "infinite capacity," able to move any resisting weight, however large (Oresme draws the same consequence and then demonstrates that, although it should follow from the false theory, it actually does not; see Selection 51.2). The theory is rejected by Bradwardine as physically impossible, since it violated the generally accepted axiom that in order for motion to occur a motive power must exceed the resistance opposing it, whether this resistance be another body or a corporeal medium. Bradwardine's new theory remedied this defect (see n. 30). Aristotle, who first formulated and accepted this third theory, actually anticipated Bradwardine's criticism and would have countered by insisting that if after successive halvings of a velocity, R became equal to, or greater than, F, motion would cease and mathematical rules of proportionality would no longer apply (see his shiphauler argument in *Physics* VII.5.250a.10–20).

22. Here $F:nR \propto V/n$, where $n = 2, 4, 8, 16, 32, \ldots$ As the motive force, F, is successively halved, it will happen eventually that $R > F:n$. But however small and feeble F becomes it can yet produce a proportionate velocity in R. See note 30.

23. As can be seen further on (Theorem VI, chapter 3), Bradwardine's theory will mathematically yield such results.

varies with the proportion of the things affected to the things affecting them.[24]

As a matter of fact, in the case of the first authority cited (that of Book IV of the *Physics*) it should be realized that what Aristotle means is that, to whatever extent the proportion of air to a given body which moves through it is smaller than that of water to that same body (due to the greater thinness and incorporeality of air), to that extent the body will move faster through air than through water; for to whatever extent the proportion of air to a given body is smaller than the proportion of water to that body, to the same extent is the proportion of the body to air larger than its proportion to water, and, as will later be shown, the proportion of the speeds of motions varies in accordance with the proportion of movers to things moved.

The second authority cited should also be construed in the same sense, together with that passage, from Book I of the *De caelo et mundo*, which reads: "It is held to undergo a greater and less effect, by the action of the same agent, in a longer and shorter time, and any such effects are divided proportionally to the time." In other words, the proportions of any given effects are divided proportionally to the time.

In the case, moreover, of the theorem drawn from Book VII of the *Physics* (which states that, if a given power moves a given *mobile* through a given distance in a given time, the same power will move half of it through double the distance in an equal time), Aristotle understands, by "half the *mobile*," a part of the *mobile* possessing a proportional relation to the given moving power which is half that of the whole *mobile* to that power.

This quite clearly appears to be Averroes' interpretation, for, regarding the passage in question, he proves the above-mentioned theorem as follows: "When we divide the motion (or the thing moved), it follows necessarily that the proportion of the power of the mover to the motion (or thing moved) is twice the former proportion. This would, nevertheless, be untrue, unless understood in the sense previously indicated, for, although a given whole may bear a given proportion to some other whole, it does not follow that it bears to half of the latter whole a proportion double the first proportion [after "bears" Crosby has "a proportion to half of the latter whole that is half the original proportion"—*Ed.*] (as will be shown in what follows)." It is in this sense that the following authorities should be interpreted.

The theorem drawn from the *De ponderibus* must be read in the same way. "Between whatever heavy bodies, etc. . . .," that is, between whatever heavy bodies the proportion of speed of descent and that of the proportion of weight to resistance are taken in the same order. (And with the proviso that the resistance remains equal.)

The author of this work, however, proposes no principle in proof of this theorem, and it may well be objected against the above interpretation, that neither does any commentator prove the theorem in this sense, but rather in another, which the present theory supports (namely, that there is neither any proportion nor excess of motive power over resistive power) and which is, in fact, what the words of this theorem really mean.

The fact is that no commentator whom we have seen either proves this theorem according to our interpretation, or according to any other. One commentator, for example, takes two unequal weights and two unequal lines representing their descents, and then, first taking it as granted by his adversary that the proportion of the larger weight to the smaller is greater than the proportion of the longer line to the shorter, he argues from this that the proportion of the smaller weight to the larger is less than the proportion of the shorter line to the longer. From this he concludes that the proportion of weights is less than that of descents (the opposite of which had been stipulated).

This however, presents no obstacle, for it was admitted that the proportion between the weights was greater than that between the descents, in the first place, and it not only does not invalidate the theorem, but it follows (conversely) that the proportion of the smaller weight to the larger will be

24. Bradwardine argues here that partisans of the third erroneous theory have misinterpreted the authors cited in their favor. The references and quotations from Aristotle, Averroes, and the *Liber de ponderibus* do not, Bradwardine insists, uphold the position that when $F_2 = F_1$ the ratio of velocities will vary inversely as the ratio of resistances ($V_2/V_1 = R_1/R_2$) but rather that the ratio of velocities will vary as the relation between the whole ratio F_2/R_2 to the whole ratio F_1/R_1. These authors do not take V to vary with R alone when F is held constant; nor indeed do they take it to vary with F alone when R is held constant; but it will vary with alterations in the whole ratio F/R. Bradwardine sees these authors as proponents of a viewpoint that is akin to his own, although, as he declares, none of them have demonstrated it. In truth, none of these authors subscribed to the position attributed to them by Bradwardine.

smaller than that of the shorter descent, or line, to the longer.[25]

Another commentator, also taking two unequal weights and their unequal descents, adds to the smaller weight a second, such that the two together are equal to the larger. He now posits that the descent of the added weight, by itself, through a time equal to that of the previous descents, when added to the descent of the lesser weight, is equal to the descent of the larger weight.

This had neither been previously proved, nor is independently known, nor follows logically, nor is universally true. In many cases it is, in fact, false (as will appear from what follows).

From this supposition, at any rate, he concludes that the proportion of the larger weight to the weight that was added is less than that of the descent of the larger weight to the descent of the additional weight (the opposite of what he states to have been given). Yet this is not the case, for it was not previously laid down as universally true that, "of any given unequal weights, the proportion of the larger to the smaller is in the proportion of their descents, taken in the same order," but only specifically that, "of these two weights, which have been chosen, and of their descents." This is, therefore, not incompatible with the thesis that, in the case of certain other weights, the proportion of the larger to the smaller should be less than that of their respective descents; for, in the case of some weights, the proportion of the larger to the smaller is equal to that of their respective descents, in other cases it is greater, and in yet other cases less (as will be clearly demonstrated, later on).[26]

As for the argument which is most convincing to writers on this subject, it should be replied that, although the causal principle adduced in its proof is true, the first consequence drawn from it does not hold. Fundamentally, a power which is double another power can move a mobile which is twice that moved by the lesser power through an equal distance and time. Instead of it following from this that the greater power can move the lesser *mobile* twice as fast, it rather follows that the greater power can move the lesser *mobile* at a speed which is as much greater as that expressed by the proportion of double the resistance to the former speed, and that it requires twice the power to do it. That speed will, in some cases, be exactly double the former speed, in some cases more than double, and in some cases less than double (as will be clear from later portions of this work).[27]

CHAPTER 2, PART 4
[THEORY IV]

A fourth theory declares there to be *neither any proportion nor any relation of excess between motive and resistive powers.* It holds that, instead of there being some proportion or excess of motive power over thing moved, motions take place in accordance with some sort of "natural dominance" of mover over moved.

This contention may be seen as founded on the authority of Averroes, who (Comment 79, *Physics* VIII) in solution of the same problem, says that an incorporeal power is neither to be called finite nor infinite, because only bodies may be referred to in this way. Furthermore, one incorporeal power may not be referred to as greater than another, since the terms, greater and less, may be applied only to quantities. Moreover, powers separate from body can neither be proportionals nor possess proportional relations, since a proportion can only be between one magnitude and another.

From the above it is seen to follow that no motive power can be either finite or infinite, greater or lesser, or in any way proportional to the power of

25. If $W_2 > W_1$ (W is the weight of a body) and $A > B$, where A and B are lines representing the speeds of descent of the weights, and if it is further assumed that $W_2/W_1 > A/B$, it follows that $W_1/W_2 < B/A$. But Bradwardine insists that this does not controvert the initial assumption that $W_2/W_1 > A/B$, since the anonymous commentator has merely converted a relation between ratios of greater inequality to one between ratios of lesser inequality.

26. If W is the weight of a body and V its velocity, the commentator assumes initially that $W_2 > W_1$ and $V_2 > V_1$. He then says that $W_1 + w = W_2$ (w is the added weight) and assumes, therefore, that $V_1 + v = V_2$ (where v is the velocity of w), from which he apparently concluded that $W_2/w < V_2/v$. Although this conclusion does not seem to follow from the data, Bradwardine accepts it as proper but argues that he sees in it nothing that is incompatible with his own view, since this consequence does not invalidate any general rule to the effect that $W_2/W_1 = V_2/V_1$. In fact, there is no such universally proven rule, so that it cannot be said that this commentator has disproved it. Indeed his consequence is compatible with consequences that are derivable from Bradwardine's own theory, where generally $F_2/R_2 = (F_1/R_1)^{V_2/V_1}$. For example, in Theorem VI of chapter 3 (see later), $F_2/F_1 = 2/1$ (that is, the ratio of motive forces or weights are related as 2/1) and $V_2/V_1 > 2/1$, so that $F_2/F_1 < V_2/V_1$. Therefore, just as in the consequence drawn by the commentator, the ratio of forces or weights are not related as the ratio of velocities taken in the same order.

27. See in Theorems I–VII of chapter 3.

the thing it moves, because no motive power is a body, but is rather a form (either extensive within the body or separate from it).

This theory, together with Averroes' opinion, may be further confirmed by the definition of proportion, for a proportion is a comparison of two things of the same kind (as is evident on the basis of the definition of proportion set forth in Chapter I). It is obvious, however, that active and passive powers are not of the same kind.

Furthermore, if active and passive powers were to bear a proportion to each other, they would then be comparables. They would, therefore, be of exactly the same species and would consequently, have a subject or substance of exactly the same species. This consequence is false, because powers are divided into active and passive, as is a genus, by incompatible differences. That it nevertheless follows is shown by Aristotle, at the end of Book VII of the *Physics*, where he expresses the opinion that all things which are to be compared must be of the same individual species and entirely without difference with respect to the subject or substance of comparison, as well as to that regard or those regards in which the comparison is made.

Further, if there were some proportion of a motive power to the thing it moves, it would have to be one of greater inequality, since it would have to exceed the power of the thing moved. And since everything which exceeds something else is divisible into what exceeds and what is exceeded (as appears from Book IV of the *Physics*, in the chapter on the "void" [215b.12–18—*Ed.*]), it follows that any motive power may be so divided. This is false, for all incorporeal motive powers are fundamentally indivisible, and an embodied motive power is smaller in extension than the power of the thing moved.

Nor can it be claimed that Aristotle is here speaking of excesses only in the strictest sense (the sense in which they are found among quantities), for he is actually referring to the excess or rarity of one medium over another. To the same effect, in Book I, Chapter 7 of Aristotle's *Rhetoric*, where relative good and relative utility are under discussion, there appears the following: "So let the thing that surpasses be as much as and more than the exceeded thing contained within it." Aristotle's dictum is, therefore, not true merely of the strict usage of "excellence."

This theory is, however, capable of disproof, for if there were no proportion between powers, for the reason that they are not quantities of the same kind, neither could there be such a proportion between musical pitches, and the entire science of harmonics would collapse, accordingly. For the *epogdoös* or "tone" is constituted in the proportion of nine to eight, the *diatessaron* in the proportion of four to three, the *diapente* in the proportion of three to two, the *diapason* (composed of *diatessaron* and *diapente*) in the proportion of two to one, the *diapason* and *diapente* combined in the proportion of three to one, and the double *diapason* in the proportion of four to one. This is sufficiently evident from various passages of the *Music* [probably the *De Musica* of Boethius—*Ed.*].

Furthermore, Averroes, in Comments 36 and 38 on Book VII of the *Physics*, proves certain theorems concerning the proportion of the speeds of motions by means of geometric theorems, as has already appeared in the third argument against Theory I. In Comment 65 on Book I of the *De caelo*, he also proves this theorem: It being granted that the infinite can move the finite in a finite time and that a finite agent can, in the same time, move part of this finite resistance, no infinite can move a finite. What he does is take a second finite mover which bears the same relation to the former as the whole finite resistance bears to this part. He then argues permutatively, from Definition xii of Book V of Euclid's *Elements*, that the proportion of the larger finite mover to the whole resistance is equal to that of the lesser mover to the part. And from this he concludes that the larger finite mover moves the whole resistance in a time equal to that in which the lesser finite mover moves the part and also equal to that in which an infinite mover moves the whole resistance.[28]

Further, according to the present theory and

28. Averroes does little more than repeat the argument given by Aristotle in Book I of *De caelo* (275a.14–22), where the latter assumes that A is an infinite power, BF (mistakenly called B at the outset) a finite resistive power, and C the time of A's acting on BF. Now, if A should produce a finite motion in BF during time C, then during the same time a finite power, D, should be able to cause the same motion in F, a finite resistive power or mobile less than BF. Now, because $D/F = A/BF$, where $D < A$ and $F < BF$, it follows that $V_F = V_B$ in time C (V is velocity). Next, take another power E such that $E/D = BF/F$, so that by permutation of ratios $E/BF = D/F$, from which it follows that $E/BF = A/BF$ and, as Aristotle concludes, "the finite and the infinite effect the same alteration in equal times. But this is impossible; for the assumption is that the greater effects it in a shorter time." The relevance of this argument for Bradwardine is that Averroes (and, of course, Aristotle) has accepted as proper the relation between the finite movers and their respective resistances ($E/D = BF/F$) and would obviously oppose this fourth theory.

(as a matter of fact) in reality, the power of the mover "dominates" that of the thing moved. In many passages Averroes says that the power of the mover exceeds that of the thing moved, and that the mover is of greater power than the thing moved. If, therefore, it "dominates" and exceeds and is of greater power, this must necessarily take place according to some proportion, whether strictly or generally understood, and both Aristotle and Averroes, in many passages, suppose there to be some proportion of the power of the mover to that of the thing moved.

We thus arrive at this affirmative conclusion: *A proportion is found to exist between any motive power and the resistive power of the thing it moves.*

The first reasons which were brought forward in favor of the present theory are easily countered by means of the definition of proportion already given in Chapter I, for the proportion found to exist between motive and resistive power is not a strict, but only a general one.

The next argument concerning comparison is overthrown by a similar distinction in the meaning of the word, "comparison." The authority which was cited is to be understood as speaking of strict rather than general comparison. It is to be noted, indeed, that comparison is made: (1) within a genus (as, for example, indicated by the terms: "more virtuous," "wiser," etc.), (2) within the most general genus (for example, "form is more [Crosby omits "more" *(magis)—Ed.*] substance [after "substance" Crosby has "rather," which is omitted here—*Ed.*] than matter or a compound of the two"), and (3) also in transcendence of every genus (for example, "substance is more [Crosby omits "more" *(magis)* and has "rather" after "being"—*Ed.*] being than accident").

As for the final argument, it is to be replied that it is true that there is, in the general sense, a proportion of excess between motive and resistive power. To the authority cited as saying that everything that exceeds is divided into what exceeds and what is exceeded, it must be replied that, just as "what exceeds" may be taken in two senses, so also "to be divided into what exceeds and what is exceeded" may be taken in two senses (i.e., generally and strictly). Everything that exceeds is, therefore, strictly divisible in this manner.

In the general sense, on the other hand, what exceeds may be divided as follows: In a general sense everything that exceeds may be reduced or lessened until equal to what was exceeded; and thus may be understood the entire latitude whereby it exceeds and likewise the similarity or equality which is contained virtually and potentially in the thing that exceeds.

Or it may be carried out thus: Everything exceeding something else is divided, in the general sense, into excess and what is exceeded, not, of course, in itself, but in comparison to something else outside itself (for example, action, and passion or resistance). In this sense, the powers of mover, moved and resistance can be compared to each other in every way in terms of excess and what exceeds. And if one were to take the example of a motive power equal to a resistive power, that motive power is twice half of the resistive power, not because it can produce twice the motion, but because twice the halved resistance has precisely the power of resistance that the motive power has of moving. Concerning every other proportion of mover to moved this is proportionally true.

This theory, therefore, together with the former ones, is pronounced false.

CHAPTER 3

Now that these fogs of ignorance, these winds of demonstration, have been put to flight, it remains for the light of knowledge and of truth to shine forth. For true knowledge proposes a fifth theory which states that the proportion of the speeds of motions varies in accordance with the proportion of the power of the mover to the power of the thing moved.[29]

29. This fifth and "true" theory is Bradwardine's function, which we have already represented as $F_2/R_2 = (F_1/R_1)^{V_2/V_1}$. In general, to obtain n times any velocity arising from a ratio F/R, the latter must be raised to the nth power. So vague is the language used here and in Theorem I to describe this theory that without reliance on the subsequent theorems it would be almost impossible to discern Bradwardine's meaning. In what immediately follows, Bradwardine associates Averroes, and to a lesser extent Aristotle, with his own theory, but there is little doubt that their views are quite properly represented by Bradwardine's third erroneous theory.

The exponential function described here, and commonly called "Bradwardine's Function," probably had its origin in a pharmaceutical rule which sought to calculate the effects of hot and cold parts in compound medicines. On the formulation of this rule by al-Kindi and Arnald of Villanova, see Michael R. McVaugh, "Arnald of Villanova and Bradwardine's Law," *Isis* LVIII (1967), 56-64. For the suggestion that Bradwardine might have derived his function directly from Walter of Odington's *Icocedron*, in which the rule is applied to the quantitative composition of intensive qualities, see Donald Skabelund and Phillip Thomas, "Walter of Odington's Mathematical Treatment of the Primary Qualities," *Isis* LX (1969), 331–350.

This is what Averroes intends when he says, in Comment 71 on Book IV of the *Physics:* "It is manifest that, universally, the cause of the diversity and equality of motion is the equality and diversity of the proportion of mover to thing moved. If, therefore, there are two movers and two things moved, and the proportions between these movers and the things which they respectively move are equal, then the two motions are of equal speed. If the proportion is varied, the motion is also varied in that proportion."

Further on in the same comment, he also says: "The difference between motions with respect to slowness and fastness varies in accordance with the proportion between the two powers (namely, motive and resistive)."

In Comment 36, on Book II of the *De caelo,* he says: "Fastness and slowness do not occur otherwise than in accordance with the proportion of the power of the mover to that of the thing moved. By however much, therefore, the proportion is greater, by so much will the motion be faster; and by however much the proportion is less, by so much will the motion be slower."

In Comment 35, on Book VII of the *Physics,* from a doubling of the proportion of mover to moved he argues a doubling of the speed of the motion, as follows: "If we divide the *mobile* in two, it necessarily comes about that the proportion of the power of the mover to the thing moved becomes double the former proportion, and thus the speed will be twice what it was before."

Further on, in the final comment, he remarks that: "These two (that is, the speed of alteration and the quantity of time) vary in accordance with the proportion between that which causes the alteration and that which undergoes it. If, therefore, the proportion is great, the speed will be great and the time short, and conversely."

Concerning this same problem, both Aristotle and Averroes (as is evident in the third argument against Theory I) express, in many passages, the opinion that, from an equality of proportion between mover and moved, there follows equality of speed. Equality of the proportion of movers to *mobilia* is, therefore, the causal condition which, when fulfilled, posits an equal speed of motions and which, when not fulfilled, makes impossible an equality of speeds. Equality of the proportion of movers to *mobilia* is, thus, the primary and precise cause of equality of the speeds of motions, and to the variation of this cause there directly corresponds the variation of proportion between different motions.

Furthermore, there does not seem to be any theory whereby the proportion of the speeds of motions may be rationally defended, unless it is one of those already mentioned. Since, however, the first four have been discredited, therefore the fifth must be the true one.

We, therefore, arrive at the following theorem:

Theorem I. *The proportion of the speeds of motions varies in accordance with the proportion of motive to resistive forces, and conversely.* Or, to put it in another way, which means the same thing: *The proportion* [the words "of the proportion," which immediately follow, are omitted here, since they are not represented in the Latin text—*Ed.*] *of motive to resistive powers is equal to the proportion of their respective speeds of motion, and conversely.* This is to be understood in the sense of geometric proportionality.[30]

Theorem II. *If the proportion of the power of the mover to that of its mobile is that of two to one, double the motive power will move the same mobile exactly twice as fast.* This may be demonstrated by means of an example. Let A be a motive power that is twice B (its resistance), and let C be a motive power that is twice A. Then, (by Theorem I, Chapter I) the proportion of C to B is exactly double that of A to B. Therefore (by the immediately preceding theorem), C will move B exactly twice as fast as A does. This is what was to be proved.[31]

Theorem III. *If the proportion of the power of the*

30. This is Bradwardine's function, already cited in the preceding note. No formal proof of it is offered, but its truth is assumed on grounds that only five possibilities exist, and since four have already been "proven" false, Bradwardine's theory stands confirmed by a process of elimination. We can now see how Bradwardine avoided the major criticism which he directed against the third erroneous theory (see n. 21 and n. 22). If initially motion occurs, then necessarily $F > R$ and it is thereafter impossible for R to become equal to, or greater than, F. This is obvious, since in this function, successively halving a velocity arising from a given ratio F/R is achieved by taking $(F/R)^{1/n}$, where $n = 2, 4, 8, 16, 32, \ldots$. In this way Bradwardine remained faithful to contemporary Aristotelian physics by insisting that velocity is determined by a ratio of force to resistance, but avoided the mathematical difficulties.

31. Once again using F's, R's, and V's instead of A, B, and C, we see that if $F_1/R_1 = 2/1$, $F_2 = 2F_1$, and $R_2 = R_1$, then $F_2/R_2 = (F_1/R_1)^{2/1}$, so that $V_2 = 2V_1$, since 2/1, the exponent, represents the ratio of velocities V_2/V_1. Therefore, F_2 moves the same resistance ($R_2 = R_1$) with twice the velocity with which it is moved by F_1.

mover to that of its mobile is two to one, the same power will move half the mobile with exactly twice the speed. This you may demonstrate by an argument like that used for Theorem II.[32]

Theorem IV. *If the proportion of the power of the mover to that of its mobile is greater than two to one, when the motive power is doubled the motion will never attain twice the speed.* This may be demonstrated by means of Theorem IV, Chapter I; and Theorem I, Chapter III.[33]

Theorem V. *If the proportion of the power of the mover to that of its mobile is greater* [Crosby has "less" although the Latin reads *maior—Ed.*] *than two to one, when the resistance of the mobile is halved the motion will never attain twice the speed.*[34] This may be demonstrated by means of Theorem III, Chapter I and Theorem I, Chapter III.

Theorem VI. *If the proportion of the power of the mover to that of its mobile is less than two to one, when the power moving this mobile is doubled it will increase the speed to more than twice what it was.*[35] This is likewise easily demonstrable, from Theorem VI, Chapter I and Theorem I, Chapter III.

Theorem VII. *If the proportion of the power of the mover to that of its mobile is less than two to one, when the same mover moves half that mobile the speed of the motion will be more than doubled.*[36] This may be demonstrated clearly, from Theorem V, Chapter I and Theorem I, Chapter III.

Theorem VIII. *No motion follows from either a proportion of equality or one of lesser inequality, between mover and moved.*[37] With the addition of the following axiom, independently known:

Axiom 1. *All motions of the same species may be compared to each other with regard to slowness and fastness;* this theorem may be proved by means of Theorems VII and VIII of Chapter I and Theorem I, Chapter III.

Theorem IX. *Every motion is produced by a proportion of a greater inequality, and from every proportion of greater inequality a motion may arise.* The first part of this may be proved by Theorems I and VIII of Chapter III and the axiom just given. The second part is demonstrable from the fact that every excess of mover over moved suffices to produce motion, as will be shown elsewhere.

Theorem X. *Given any motion, one twice as fast and one twice as slow can be determined.* This may be proved by Theorem I and Theorem IX (Part 2) of Chapter III, with the help of the following axiom, independently known:

Axiom 2. *A proportion of greater inequality of mover to moved may be halved or doubled indefinitely.*[38]

Theorem XI. *An object may fall in the same medium both faster, slower, and equally with some other object that is lighter than itself.*

Let, for example, A represent a heavy mixed body composed of heavy and light and having a certain weight, and let B represent some pure heavy body, as small as you please. Now let a given medium be rarefied to the point at which B bears to it a proportion equal to, or greater than, that of the heaviness to the lightness in A. Then let both bodies be placed in the same medium. The heaviness of A will now be in a lesser proportion to its total intrinsic and extrinsic resistance than B is to its resistance. Therefore, by Theorem I, Chapter III, A moves more slowly than B.[39]

32. If $F_1/R_1 = 2/1$, $F_1 = F_2$, and $R_2 = R_1/2$, then $F_2/R_2 = (F_1/R_1)^{2/1}$, where $2/1 = V_2/V_1$, and it is shown that the same power moves half the original resistance with twice the speed of the whole resistance.

33. If $F_1/R_1 > 2/1$, $F_2 = 2F_1$, and $R_2 = R_1$, it follows that $F_2/R_2 < (F_1/R_1)^{2/1}$, so that $V_2 < 2V_1$.

34. If $F_1/R_1 > 2/1$, $F_2 = F_1$, and $R_2 = R_1/2$, it follows that $F_2/R_2 < (F_1/R_1)^{2/1}$, so that $V_2 < 2V_1$.

35. If $F_1/R_1 < 2/1$, $F_2 = 2F_1$, and $R_2 = R_1$, it follows that $F_2/R_2 > (F_1/R_1)^{2/1}$, so that $V_2 > 2V_1$.

36. If $F_1/R_1 < 2/1$, $F_2 = F_1$, and $R_2 = R_1/2$, it follows that $F_2/R_2 > (F_1/R_1)^{2/1}$, so that $V_2 > 2V_1$.

37. When $F \leq R$ no motion can occur. In medieval physics this was accepted as almost axiomatic, since it was held that motion could occur only when the motive force exceeded the resisting body or medium (that is, when $F > R$; this is made explicit in the next theorem). The proof of Theorem VIII is based on Theorems VII and VIII of Chapter I, where it is shown that a ratio of equality, A/A, is not relatable by any exponent whatever to a ratio of greater inequality A/B $(A > B)$ in the sense that $(A/A)^n$, where n may have any value however great, always remains equal to A/A and consequently cannot be made equal to, or greater than, A/B; similarly, a ratio of lesser inequality D/C $(D < C)$ is not relatable by means of any exponent to a ratio of greater inequality A/B, since whatever the value of n in the expression $(D/C)^n$ it will always remain less than A/B.

Although Bradwardine might have appealed to the accepted axiom that no motion could be produced when $F \leq R$, he chose instead to "demonstrate" this mathematically by showing that in the context of his function even if velocities could arise when $F \leq R$, such motions were not comparable to motions arising when $F > R$.

38. When $F > R$, the motion produced can always be doubled or halved by squaring or extracting the square root of F/R—that is, determining $(F/R)^2$ or $(F/R)^{1/2}$.

39. Let f/r represent the ratio of heavy to light (or motive force to internal resistance) in body A, and let B represent a pure heavy body as small as you please; and assume also that R is an external medium which is rarefied so that the ratio of $B/R \geq f/r$. Upon placing both bodies in R, we see that $B/R > f/(r + R)$ and therefore $V_A < V_B$ in R.

Conversely, let the medium be condensed to the point at which the proportion of B to it is less than the proportion of the heaviness of A to its entire intrinsic and extrinsic resistance. Then, by Theorem I of this chapter, A moves faster than B.[40]

Thirdly, let the medium be so determined that the proportion of B to it is equal to the proportion of the heaviness of A to its entire intrinsic and extrinsic resistance. Then, by Theorem I of the present chapter, A and B will move at equal speeds.[41]

Alternatively, let A be supposed to have a determinate speed, C, in a vacuum, and let some medium be rarefied until B falls in it with speed C or faster; then A, placed in the same medium, will fall more slowly than B.[42] Conversely, let the medium now be condensed as required, and the remaining two consequences will follow.[43]

Corollary 1. It is manifest, from the foregoing, that *the fastness and the slowness of any pure body and the slowness of any mixed body may be doubled indefinitely, but that the fastness of a mixed body may not be so doubled by rarefaction of the medium.*[44] This corollary is sufficiently well established on the basis of what has been said above.

Theorem XII. *All mixed bodies of similar composition will move at equal speeds in a vacuum.* In all such cases the moving powers bear the same proportion to their resistances. Therefore, by Theorem I of this chapter, all such bodies move at the same speed.[45] From this you must also understand that:

Corollary 2: *If two heavy mixed bodies of unequal weight, but similar composition, were balanced on a scale within a vacuum, the heavier would descend.*

Let A and B represent two such heavy bodies, A greater and B less; let C and D represent the heaviness and lightness of A, respectively, and let E and F represent the heaviness and lightness of B, respectively. Then C, D, E, and F are four proportionals, C being the greatest and F the smallest.[46] Therefore (by Axiom [Crosby has "Theorem" —*Ed.*] VIII, Chapter I) C and F, combined, exceed D and E, combined.[47] Since C and F tend to raise B and only D and E resist, therefore (by Part ii of Theorem IX of the present chapter) B ascends and A descends.[48]

40. Here $B/R < f/(r + R)$, so that $V_A > V_B$.

41. Here the density of the medium is such that $B/R = f/(r + R)$ and $V_A = V_B$.

42. In heavy mixed body A let f/r produce speed C in a vacuum and assume the density of medium R to be such that pure elemental body B is related to it as $B/R \geq f/r$, so that $V_B \geq V_A = C$. Subsequently dropping A in R makes V_A (or C) $< V_B$, since $B/R > f/(r + R)$.

43. By properly adjusting the density of R one can arrange to have A and B fall in R so that V_A (or C) $> V_B$, and $V_A = V_B$.

44. The speeds of a pure elemental body and a heavy mixed body can be halved indefinitely because theoretically a corporeal medium can be made infinitely dense. But only a pure elemental body can have its velocity doubled indefinitely, since it was almost universally assumed that a corporeal medium is rarefiable ad infinitum. But rarefaction of a medium ad infinitum will not permit the infinite doubling in speed of a mixed body, for when R becomes zero or void in the formulation $f/(r + R)$, a maximum finite speed would be determined by the ratio of motive force to internal resistance, namely f/r.

45. Since the two unequal mixed bodies are of homogeneous composition, the ratio of motive force to internal resistance (f/r) per unit of matter is equal in each body and consequently their velocities of fall in the void will be equal. Bradwardine relies here on a specific factor (namely equality of ratio per unit of matter) rather than utilizing an extensive factor, or gross weight, as did Aristotle. Indeed the distinction is analogous to that between specific and gross weight. This important conclusion, which was rejected by Aristotle as an absurdity (*Physics* IV. 8.216a. 14–21), is akin to that enunciated in the sixteenth century by Galileo in his *De motu (On Motion)* (and even earlier by Giovanni Benedetti), where it is declared that homogeneous bodies of different weight or size would fall in the void with equal finite velocities. Galileo, however, abandoned the medieval distinction between pure elemental bodies (for Bradwardine such bodies, devoid of internal resistance, could not fall in the void with finite velocities) and mixed bodies, and the dichotomy between absolute heaviness and lightness, so that his arguments in favor of essentially the same conclusion were quite different, despite a few striking similarities (see Selection 55.5 and n. 79 thereto).

46. C, D, E, and F are four proportionals, because A and B are homogeneous bodies.

47. Axiom VIII, Chapter I, states that if "four quantities are proportionals and the first the largest and the last the smallest, the sum of the first and last will be necessarily found greater than the sum of the other two" (trans. Crosby).

48. Upon suspending heavy mixed bodies A and B (with $A > B$) from a balance placed in a vacuum, we see

from the figure (added here for convenience; it is not in the text) that C and F (that is, the downward acting motive power of A and the upward acting resistive power or lightness of B) act conjointly to move B upward, while D and E (the upward acting resistive power or lightness of A and the downward acting motive power of B) act conjointly to move A downward. Since we have a constrained system and $C + F > D + E$ (by Axiom VIII, Chapter I, quoted in n. 47), we can form a ratio of greater inequality $(C + F)/(D + E)$, where $C + F$ acts as total motive power and $D + E$ as total resistance.

2. Nicole Oresme (ca. 1325–1382): Extended Application of "Bradwardine's Function"

Translated and annotated by Edward Grant[49]

Proposition 1. That the following rules are false: If a power moves a mobile with a certain velocity, double the power will move the same mobile twice as quickly. And this [rule]: If a power moves a mobile, the same power can move half the mobile twice as quickly.[50]

The falsity of the first [rule] is obvious: Now, let B be a power that moves mobile C with a certain velocity, and let A be a double power. Then if ratio B to C were a double ratio, it follows, by the third noteworthy point of the first chapter, that ratio A to C will be [equal to] ratio B to C squared. It certainly follows, by the first supposition, that the velocity with which A moves C is double the velocity with which B moves C, since a ratio of velocities is just like a ratio of ratios.[51]

But now, on the contrary, if ratio B to C were less than a double when ratio A to B is a double by assumption, it follows, by the second supposition,[52] that ratio A to C would be greater than ratio B to C squared because it is composed of ratio B to C, the lesser, and ratio A to B, the greater, namely a double. Consequently, by the first supposition, A will move C more than twice as quickly as B will move C.[53]

Again, if ratio B to C is greater than double and A to B is assumed to be a double ratio, it follows by the second supposition, that ratio A to C will be less than ratio B to C squared. Hence, by the first supposition, A will move C with a velocity that is less than twice the velocity with which B moves C.[54]

For example, let A be 8 and B, 4. Therefore, if ratio B to C is related as ratio A to B, so that C would be 2, then ratio A to C will be ratio B to C squared, and thus the velocity is doubled. Furthermore, if ratio B to C were less than double, so that C might be 3, then ratio A to C will be greater than ratio B to C squared, and the velocity is more than doubled. If, however, ratio B to C were greater than a double ratio, so that C might be a unit, then ratio A to C [will be less than the square of B to C], and similarly, the velocity will be less than doubled.

Thus it is clear that from a doubling of the power, a doubling of the velocity does not follow except in one case, namely when the power is first taken to be double the mobile. Hence the rule is false, because it is conditional and ought to be necessary. And the antecedent ought to be incapable of being true without the consequent [being true], but the truth of the consequent does not agree with the truth of the antecedent except in one case.

The falsity of the second rule can be shown by the same principles, because if a power moves a mobile with a certain velocity, the same power will not move half [the mobile] twice as quickly unless the first velocity should arise from a double ratio. But

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Now, by the second part of Theorem IX, Chapter 3 "from every proportion of greater inequality a motion may arise," so that motive power $C + F$ will cause A to descend, while $D + E$, the total resistance, will cause B to ascend.

49. The translation is reprinted with permission of the copyright owners, the Regents of the University of Wisconsin from *Nicole Oresme: De proportionibus proportionum* and *Ad pauca respicientes*, edited with Introductions, English Translations, and Critical Notes by Edward Grant (Madison, Wis.: University of Wisconsin Press, 1966). The propositions included here are taken from chapter 4 of his *De proportionibus proportionum (On Ratios of Ratios)* on pages 269–275, 279–283, 287–289, and 299–309. The annotations have been specially prepared for this volume but are based almost wholly on material in the edition and translation.

50. "The first false rule asserts that if $F/R \propto V$, then $(2F/R) \propto 2V$; in the second false rule, if $F/R \propto V$, then $F/(R:2) \propto 2V$" (p. 269). As in this instance, citations drawn directly from my edition will be placed within quotation marks, followed by the page number in parentheses. The two false rules under attack here are substantially the same as those represented by the third erroneous theory in Bradwardine's *Treatise on Proportions* (see chapter 2, Part 3, in the preceding selection).

51. "Using F's, R's, and V's for A's, B's, and C's, we see that if $F_1/R = 2/1$ and $F_2/F_1 = 4/2$, then $F_2/R = (F_1/R)^{2/1}$, where the exponent $2/1 = V_2/V_1$" (p. 269). By the expression "ratio of ratios" (Oresme calls his treatise "On Ratios of Ratios" from the technical expression *proportio proportionum*), Oresme means any exponent (expressed always as a ratio) relating two ratios. If the exponent is rational, we have a "rational ratio of ratios"; if irrational, the exponent is called an "irrational ratio of ratios." Since any ratio of velocities, V_2/V_1, serves as an exponent relating two ratios of force and resistance, it "is just like a ratio of ratios."

52. A reference to the second supposition of Chapter 4, where it is stated that if $A/C = A/B \cdot B/C$, and $A/B > B/C$, then $A/C < (A/B)^2$, and $A/C > (B/C)^2$.

53. Again using F's, R's, and V's, we may summarize this as: "If $F_1/R < 2/1$, and $F_2/F_1 = 4/2$, then $F_2/R < (F_1/R)^{2/1}$ (where $2/1 = V_2/V_1$), since $F_2/R = F_2/F_1 \cdot F_1/R$" (p. 269).

54. "If $F_1/R > 2/1$, and $F_2/F_1 = 4/2$, then $F_2/R < (F_1/R)^{2/1}$, where $2/1 = V_2/V_1$" (p. 271).

sometimes what this [power] moves twice as quickly would be exactly half the first [mobile], sometimes greater than half, [and] sometimes less [than half].

In the second place, I argue against the second rule thus: If it were true, it follows that any power, however feeble, can move any mobile, whatever its resistance.[55]

Now, let a power A be taken, which can move C. And let D be a mobile double to C, and E another mobile double to D, and F double to E, and G to F, and so on. Then this consequence is proved: A can move C; therefore it can move D. And similarly this will be proved: A can move D; therefore it can move E. And likewise it can move E, and therefore F, and so on. Moreover, since it is possible, let B be a power which could move D exactly twice as slowly as A can move C.

Thus if the rule were true, B can move C twice as quickly as this very same B can move D, since C is subdouble to D and half of it. Therefore, B can move C exactly as quickly as A can move C, since A moves C twice as quickly as B moves D, and B moves C twice as quickly as B can move D. Hence A and B can move C equally quickly, by the ninth [proposition] of the fifth [book of Euclid, which says]: If two [quantities] bear the same ratio to a third, they are equal. Consequently, by the third supposition,[56] A and B are equal powers. But by assumption, B can move D: therefore, by the fourth supposition, A can move D, which was to be proved.

And this consequence will be proved in the same way: A can move D; therefore it can move E by taking a power that can move E twice as slowly as A can move D. It can then be proved as before that A and this given power that can move E are equal, from which the proposed consequence follows just as it was deduced before.[57]

However, if A should move C in a quadruple ratio, then in virtue of this it surely follows that A moves C. Then that power, namely power B, which moves D twice as slowly, that is, in a double ratio, can move C, the half of D, exactly as quickly as A moves C. Thus they are equal powers and A can move D because, similarly, A, as well as B, is related to D in a double ratio. And this is so because it follows that B moves D in a double ratio; therefore, B moves C twice as quickly, as conceded before, because, like A, it moves it in a quadruple ratio.[58]

For this reason it was stated in the corollary to the fourth proposition of the third chapter that a ratio of a quadruple to a double ratio is just like a ratio of [their] denominations, but this is not found in other ratios. And because of this it does not follow further in the aforementioned case that [if] A moves D in a double ratio, therefore that power that can move E twice as slowly can move

55. As will be seen, Oresme uses a radically different approach than Bradwardine in repudiating this false rule. Where Bradwardine was content to reject the false rule because it yielded this consequence, which he deemed physically absurd, Oresme shows that although the physical consequence enunciated here is entailed by the false rules, it does not in fact follow mathematically. The major criticism of Oresme's procedure will be described below.

56. Supposition III asserts that "all powers are equal which can move the same mobile, or equal mobiles, with equal velocity" (p. 263).

57. Up to this point Oresme has sought to show that any motive power, say A, can move any resisting body whatever. This is accomplished by showing that if A moves a body D, it can also move $2D$, $4D$, $8D$, and so on ad infinitum. The basic move involves the postulation of another motive power that can move $E = 2D$ with half the speed with which A moves D. Should this other motive power be P, it is taken as obvious by Oresme that if P moves $2D$, it can also move D with twice the velocity with which it moves $2D$. But A also moves D with twice the velocity with which P moves $2D$, from which it follows that $A = P$ (by the third supposition quoted above in n. 56). Obviously, A will also move $E = 2D$. By use of this procedure, it can be shown that A can move $4D$, $8D$, and so on. The power of A is therefore of potentially infinite capacity (compare Bradwardine's discussion in the preceding selection. This completes the first part of the demonstration. Now Oresme seeks to demonstrate that despite the entailment of A's infinite capacity as a motive power, this consequence does not in fact follow mathematically.

58. If $A/C = 4/1$, $D = 2C$, and if $B/D = 4/2$, it follows that B can move C, so that $A/C = B/C$ and we see that $A = B$. Therefore, $A/D = B/D = 2/1$, and $A/D = (1/2)(A/C)$ so that $V_D = V_C/2$ when A is the motive power. Thus if A can move C, it can move $2C = D$. But, as Oresme will emphasize at the beginning of the next paragraph, A can move D with half the velocity with which it moves C only because the initial conditions of this particular case are $A/C = 4/1$ and $B/D = 4/2$, so that $A/C = 2(B/D)$, which signifies that A moves C with twice the velocity with which B moves D. This result, arising from the false rule, agrees with the result that would be produced from what Oresme accepts as the true theory (Bradwardine's function) where for the same data $A/C = (B/D)^{2/1}$, which equals $A/C = 2(B/D)$ in the false rule. In this case Oresme would say that the "ratio of denominations" (that is, 2/1, where 4 is the denomination of A/C and 2 of B/D) equals the "ratio of ratios" (that is 2/1, where 2/1, the "ratio of ratios," is the exponent in $4/1 = [2/1]^{2/1}$). But in all other cases there would be no agreement, as, for example, if we were to relate 9/1 and 3/1, where the ratio of denominations is $9/3 = 3/1$, while their ratio of ratios is 2/1, since $9/1 = (3/1)^{2/1}$.

D exactly as quickly as A moves D. On the contrary, it follows [that it will move D] more quickly, because that power would be B, and then ratio B to E will be half of a double ratio; and since ratio E to D is a double ratio it follows that ratio B to D will be composed of a double and half of a double ratio. It will, consequently, be greater than double, so that, by the first supposition, B will move D quicker than A moves D, since A was double to D. Thus it no more follows that [because] A can move D, therefore it can move E. But, as was shown, this would nevertheless follow if the rule were true.[59]

What, then, should we say to Aristotle, who seems to enunciate the repudiated rules in the seventh [book] of the *Physics*? It must be said that they are false unless [the following] is added to the first rule: If a power moves a mobile *in a double ratio*, a double power will move [the same mobile twice as quickly]. And likewise [this must be added] to the second rule: If a power [moves a mobile] *in a double ratio*, the same power will move [half of the mobile twice as quickly]. And so we can gloss [Aristotle] and say that these rules ought to be understood [in this way]. Perhaps Aristotle said this but has been poorly translated. But if he did not say it, perhaps he failed to understand [the rules] properly.

Proposition 3. The ratio of a power to each of [two] mobiles can be made known when [both] the ratio of the two mobiles is known and the ratio by which the same power moves the lesser mobile more quickly than the greater mobile.

Let A be the power, B the greater and C the lesser mobile, and since I assume that every motion arises from a ratio of greater inequality, A will be greater than B. And let E be ratio A to B, F ratio B to C, and D ratio A to C.

Then ratio D is composed of intermediate ratios, which are E and F.[60] Furthermore, let G be ratio D to E, so that G will be a ratio of ratios from which the velocities arise.[61] Now, since the ratio of velocities is known by hypothesis, the ratio of ratios, namely G, will be known, by the first supposition. [The first supposition is actually a statement of Bradwardine's function; see chapter 3, Theorem I, in the preceding selection.—*Ed.*] We will then have four ratios, D, E, F, and G, of which two are known, namely G and F, and two are unknown, namely D and E, which we wish to make known.[62]

Let it be argued as follows: The ratio of D, the whole, to E, its part, is known, and therefore the ratio of the same part, namely E, to the remainder, F, will be known, by the first part of the eighth supposition. And similarly, the ratio of the whole, namely D, to the remainder F, will be known, by the second part of the same eighth supposition.[63] Then, continuing, ratio E to F is known, and F is a known ratio or quantity, so that E will be known, by the ninth supposition.[64] Likewise, ratio D to F is known, and F is a known ratio or quantity, and consequently D will be known. Thus the two ratios, namely E and D, are now known, and this is what I desired.[65]

59. I quote here my own summary of this argument from the edition cited in note 49 above: "Thus according to the second false rule, if A moves D in a double ratio, i.e., $A/D = 2/1$, and $E = 2D$, then should another force B move E with half the velocity with which A moves D, it follows that $B/D = A/D$. But by Supposition I, which is Bradwardine's function, this is false, for according to that function to say that B moves E with half the velocity with which A moves D is to say that $B/E = (A/D)^{1/2}$ rather than $B/E = (1/2)(A/D)$, as required by the false rule. Now, $E/D = 2/1$ and $A/D = 2/1$. Consequently, $B/E = (A/D)^{1/2} = (2/1)^{1/2}$, and $B/D = B/E \cdot E/D = (2/1)^{1/2} \cdot (2/1)$, so that $B/D = (2/1)^{3/2}$. It is now evident that $B/D > A/D$, since $(2/1)^{3/2} > (2/1)$. Therefore, $B > A$ and $B/E > A/E$ (where $E = 2D$). Thus from the fact that $A < B$, it does not follow that because A can move D it can move $E = 2D$, even though B can move E and *a fortiori* can move D, which is half of E. Indeed, A may be incapable of producing any motion whatever in E. But the second false rule requires that $B/D = A/D$, making $B = A$, in which event A must necessarily be capable of moving D and E.

The success of Oresme's demonstration depends entirely on setting $B/E = (A/D)^{1/2}$ rather than $B/E = (1/2)(A/D)$. This move enables him to show that $B/D > A/D$ instead of $B/D = A/D$. The entire demonstration is improper, since Oresme has substituted his own function for the false rule and merely demonstrated that one obtains quite different results from the rival theories. He has produced no good reasons for repudiating the false rule, but has only shown that in terms of Bradwardine's function the consequence of the false rule—namely that any power, however weak, can move any resistance, however great—is contradicted."

60. $D = E \cdot F$, or $A/C = A/B \cdot B/C$.

61. That is, $D = E^G$.

62. That is, $D = E^G \cdot F$, where G and F are known.

63. In $D = E^G$, G is known by assumption, thus allowing us to determine that $E = (D)^{1/G}$, and $F = (D)^{(G-1)/G}$, from which it follows that $E = (F)^{1/(G-1)}$, and $D = (F)^{G/(G-1)}$. This is all derived from the eighth supposition of the fourth chapter.

64. Since F is known by assumption and exponent $1/(G-1)$ is known (since G is also known by assumption), $F^{1/(G-1)}$ is completely known, and consequently, so is E.

65. Here also F and exponent $G/(G-1)$ are both known, so that $F^{G/(G-1)}$ is completely known, and consequently, D is also known (see n. 63.) In the relation $D = E^G \cdot F$, all four quantities are now known.

For example, let B be double C and let the velocity with which C is moved be triple the velocity with which B is moved. By the first supposition, therefore, the ratio of ratios, namely G, will be a triple so that D will be triple to E. Hence E will be one third of D, and F, the remainder, two thirds of D. The ratio of D to F will then be as a denominating number to its numerator, namely 3 to 2, a sesquialterate. Now, by hypothesis, F is a double ratio, and therefore D would be composed of a double and half of a double ratio and will be three fourths of a quadruple ratio. And since D is composed of E and F, and F is a double ratio, E will be half of a double; or E will be one third [of D] and F two thirds [of D]. Thus their ratio is as a ratio of [their] numerators, so that E is related to F as 1 to 2.[66]

It must, however, be noted that if B were double C, it does not follow for this reason that A could move C twice as quickly as it moves B, because this is disproved by the first proposition of this chapter.[67]

It must also be understood that just as the ratio of a power to each of [two] mobiles can be ascertained [if] the ratio of those two mobiles and [the ratio of] the respective velocities produced by that same power are known,[68] so, contrarily, with the ratio of two powers known, as well as the [ratio of] velocities they produce with respect to the same mobile, the ratio of each power to that mobile can be made known. And thus the ratios from which velocities arise will be known, so that if it were assumed that C is the greater power, B the lesser power, and E the mobile which is moved by each [power] successively, and if the other things remain as before, one can carry on the argument exactly as above.[69]

Proposition 5. When any velocity is given and the ratio from which it arises is known, then the ratio that gives rise to any [other] velocity can be known, provided that the ratio of the velocities is known.

Let A be a known ratio from which a given velocity arises, and let there be another velocity, commensurable to this given velocity, which arises from ratio B. I say that B can be found, and argue as follows: A ratio of ratios, namely A to B, is like a ratio of velocities, by the first supposition; and the ratio of velocities is known, as is assumed; therefore, ratio A to B is known. But A is known by hypothesis, and consequently B can be made known, by the ninth supposition [of this chapter]; and this is what has been proposed.[70]

In this way we can find the ratio of a power to a resistance, namely the ratio from which a velocity arises, provided the power is not separated from its mobile or resistance, or [simultaneously] applied to diverse mobiles.

For example, let A be a double ratio from which a given velocity arises, and let some other velocity which arises from ratio B be quadruple to it. Therefore, ratio B is quadruple to A, namely to a double ratio. Then, as was taught in the first chapter, I shall increase the double ratio to quadruple of itself and shall get a sedecuple ratio. Therefore, B was a sedecuple ratio.[71]

Proposition 6. If a ratio from which a velocity arises is known and rational, give its prime numbers; but if irrational, find two lines the greater of which is as the power of the mover, the lesser as the resistance of the thing moved. . . .

66. To summarize this example, let $D = A/C$, $E = A/B$, and $F = B/C$, so that $D = E \cdot F$ (where A is motive force and B, C are resistances). If $B/C = 2/1$, and $A/C = (A/B)^{3/1}$, then $D = (E)^{3/1}$. Therefore, $E = (D)^{1/3}$, and $F = (D)^{2/3}$, so that $D = (F)^{3/2}$, and $E = (F)^{1/2}$. By hypothesis $F = B/C = 2/1$, which determines that $E = (2/1)^{1/2}$, and $D = E \cdot F = (2/1)^{1/2} \cdot (2/1) = (4/1)^{3/4}$. Ratios D and E are now known and are $(4/1)^{3/4}$ and $(2/1)^{1/2}$, respectively; furthermore, $E = (F)^{1/2}$. When applied to the ratios of force and resistance, we see that $A/C = (4/1)^{3/4}$ and $A/B = (2/1)^{1/2}$, from which it follows that the ratio of velocities, or "ratio of ratios," is $3/1$, since $(4/1)^{3/4} = (2/1)^{3/2}$, and $(2/1)^{3/2} = [(2/1)^{1/2}]^{3/1}$, or $A/C = (A/B)^{3/1}$, where $3/1 = V_C/V_B$ (V_C representing the velocity of C and V_B the velocity of B). Using F's, R's, and V's for force, resistance, and velocity, this proposition utilizes the basic relation $F_2/R_2 = F_1/R_1 \cdot R_1/R_2$, where $F_2 = F_1$, $R_1 > R_2$, and $F_2/R_2 = (F_1/R_1)^{V_2/V_1}$. Since ratios R_1/R_2 and V_2/V_1 are assumed known, it is easy to determine ratios F_2/R_2 and F_1/R_1.

67. Oresme insists here that simply because the ratio of mobiles or resistances is $B/C = 2/1$, it does not follow that A will move C with twice the velocity with which it moves B. This will happen only if $A/B = 4/2$ when $B/C = 2/1$, in which event $A/C = A/B \cdot B/C = 4/2 \cdot 2/1 = 4/1$ (see n. 58).

68. An example representing this part of the proposition is summarized in note 66.

69. Replacing C, B, and E with F's, R's, and V's (see n. 66), we are told here that if there is only one mobile or resistance and two motive powers or forces, the two unknown ratios of force to resistance can be determined. For if $F_2/R_2 = F_2/F_1 \cdot F_1/R_1$, with $R_2 = R_1$, $F_2 > F_1$, and $F_2/R_2 = (F_1/R_1)^{V_2/V_1}$, then should V_2/V_1 and F_2/F_1 be known, the two ratios F_2/R_2 and F_1/R_1 can be determined.

70. This can be formulated as $A = (B)^{m/n}$, where m/n and A are known, so that B is easily found. Since A and B represent ratios of force to resistance, the exponent, m/n, which is a "ratio of ratios," is also the ratio of velocities.

71. If $A = 2/1$, and $B = (A)^{3/1}$, then $B = (2/1)^3 = 16/1$.

Proposition 7. If any velocity should arise from a rational ratio that has no mean proportional number or numbers between its prime numbers, then every lesser velocity that is commensurable to it arises from an irrational ratio; and similarly every greater [velocity that is commensurable but] not multiple to it [arises from an irrational ratio].

Since, by the first supposition, a ratio of velocities is like a ratio of ratios, and, by the fifth proposition of the second chapter, any ratio [that has no mean proportional number or numbers] between its prime numbers is incommensurable to any lesser [rational] ratio and to any greater that is not multiple to it, then any velocity which arises from such a ratio will be incommensurable to any lesser velocity which arises from a rational ratio.[72] Therefore, if any smaller velocity is commensurable to such a velocity, it will have been produced by an irrational ratio;[73] and this also applies to any greater velocity which is not multiple to it.

For example, let B be a velocity produced by a double ratio. I say that every velocity commensurable to it arises from an irrational ratio; and every lesser velocity arising from a rational ratio is incommensurable to it. However, not every lesser [velocity] which is incommensurable to it arises from a rational ratio, since some irrational ratio less than a double ratio may be commensurable to it and another incommensurable.[74]

Furthermore, every velocity greater than B, but not multiple to it, that arises from a rational ratio is incommensurable to B; and every [greater velocity] that is commensurable, but not multiple, to B arises from an irrational ratio. For example, if a league were traversed in one day by velocity B, and a league-and-a-half by velocity C, I say that C arises from an irrational ratio.[75] For indeed these [velocities] are related as ratios of ratios, which is clear from the fifth proposition of the second chapter and other places in the third chapter. Thus, by means of the first supposition of this chapter and the propositions given in the third chapter, one who understands can demonstrate many propositions about velocities.

For example, every velocity that arises from a multiple ratio is incommensurable to any other velocity not arising from a multiple ratio.[76] But even if each velocity were produced by a multiple ratio, it does not follow that they are commensurable.[77] This is obvious from the first supposition and third proposition of the third chapter. Moreover, by the fifth proposition of the same third chapter, it is clear that every velocity that arises from a superparticular ratio is incommensurable to any other arising from a superparticular ratio.[78]

And so, in a like manner, one or more [additional] propositions about velocities can be demonstrated from any proposition of the third chapter,[79] but in order to be briefer so that I may move on, I omit all the propositions up to the tenth of the third chapter, from which latter proposition, however, such a proposition is [now] elicited.

72. In Proposition 5 of chapter 2, Oresme had shown that if A is a rational ratio that has no mean proportional number between its prime numbers and B is a smaller rational ratio, then $B \neq (A)^{m/n}$, where m/n is a rational exponent. As applied to ratios of force and resistance, let $A = F_2/R_2$, and $B = F_1/R_1$, so that $F_2/R_2 \neq (F_1/R_1)^{V_2/V_1}$, where $V_2/V_1 = m/n$. In the same fifth proposition, Oresme had also shown that if $C > A$, and $C \neq (A)^n$ (n is any integer), ratios C and A must be incommensurable, since they would be related by an irrational ratio. For Oresme, two ratios (both may be rational or irrational; or one rational, the other irrational) are commensurable when they can be related by a rational exponent; they are incommensurable when they cannot be related by a rational exponent (see also n. 51).

73. If $F_2/R_2 = (F_1/R_1)^{V_2/V_1}$, where V_2/V_1 is a rational exponent and F_2/R_2 is a ratio lacking a mean proportional number between its terms, then F_1/R_1 is irrational. For example, if $F_2/R_2 = 2/1$, then should $F_1/R_1 = (2/1)^{1/2}$, it follows that $V_2/V_1 = 2/1$, since $2/1 = [(2/1)^{1/2}]^{2/1}$.

74. In note 73 we saw that $F_1/R_1 = (2/1)^{1/2}$ was commensurable to $F_2/R_2 = 2/1$. But if $F_1/R_1 = (2/1)^{m/n}$, with m/n irrational, then F_1/R_1 is incommensurable to F_2/R_2.

75. Since $S_C/S_B = V_C/V_B = 3/2$ when $T_C = T_B$ (where S is distance, V, velocity, and T, time), it follows that $V_C = (3/2)\ (V_B)$. Now, in the preceding paragraph it was assumed that V_B arises from a ratio of force to resistance of 2/1, so that V_C must be generated by the irrational ratio $(8/1)^{1/2}$, since $(8/1)^{1/2} = (2/1)^{3/2}$.

76. A multiple ratio is of the form $n/1$, where n may be any integer. Here Oresme is saying that if p/q is a ratio of numbers in its lowest terms and $p > q > 1$, then $n/1 \neq (p/q)^{m/r}$, where m/r is a ratio of integers, that is, a rational exponent. Thus any two ratios of force to resistance related in this manner will produce a ratio of velocities that is irrational.

77. For example, 3/1 and 2/1 are multiple ratios but incommensurable, since no rational exponent can relate them. Any ratios of force to resistance related as these two ratios will produce incommensurable velocities.

78. A superparticular ratio is of the form $(n + 1)/n$, where n is any integer (see p. 6, n. 17). If in one superparticular ratio $n = 2$ and in another $n = 3$, those two ratios are not relatable by a rational exponent, and therefore any ratios of force to resistance related in the same way will give rise to incommensurable velocities represented by an irrational ratio.

79. There are ten strictly mathematical propositions in chapter 3 plus an eleventh practical proposition illustrating, by way of example, the tenth proposition.

When two velocities have been proposed whose ratio is unknown, it is probable that their ratio is irrational and that these velocities are incommensurable. And when more velocities are proposed, it is exceedingly probable that any [one of them] would be incommensurable to any [other of them]. And as more are proposed it must be considered even more probable, since it has frequently been said, with reference to the first supposition, that a ratio of velocities is as a ratio of ratios. But when one unknown ratio of ratios has been proposed, it is probable that it is incommensurable and that those ratios would be incommensurable, because if more ratios of ratios are proposed, it is most probable that any one of them would be irrational, since there are fewer rationals among ratios of ratios, just as there are fewer cube numbers among numbers, which was stated in that [very] tenth proposition of the third chapter. For this reason, the same thing must be said about ratios of velocities, namely that when two velocities have been proposed [whose ratio is unknown, it is probable that their ratio is irrational and that these velocities are incommensurable], which has been proposed.[80]

Now, any ratio of magnitudes [or distances] would be just like the ratio of velocities with which those magnitudes [or distances] were traversed in the same time or in equal times.[81] And a ratio of times is just like a ratio of velocities when it happens that equal distances are traversed in those times, and conversely, which is clear from the sixth [book] of the *Physics* [of Aristotle]. [From what has been said] this proposition follows: When there have been proposed any two things whatever acquirable [or traversable] by a continuous motion and whose ratio is unknown, it is probable that they are incommensurable. And if more are proposed, it is more probable that any [one of them] is incommensurable to any [other]. The same thing can be said of two times and of any continuous quantities whatever.

For example, let there be two unequal motions which last through an equal time and whose ratio is unknown. I say that it is probable that the magnitudes [or distances] traversed would be incommensurable,[82] as would any other magnitudes [or distances] that are traversed or traversable by these motions.

Now, if the ratio of times were unknown between two motions traversing equal magnitudes [or distances] in unequal times, it is probable that the times of this ratio would be incommensurable; and if there were more times [one would argue] as before. Therefore, it must be said that it is probable that a day and the solar year are incommensurable times.[83] And if this be so, it is impossible to discover the true length of the year, for it is just as if the year should last through a certain number of days and one part of a day which is incommensurable to a day. And one can make similar remarks about other things.

From all the things which have been said, this proposition also follows: When two motions of celestial bodies have been proposed, it is probable that they would be incommensurable, and most probable that any celestial motion would be incommensurable to the motion of any other [celestial] sphere,[84] but if the opposite of this were true, it could not be known. And this seems especially true since, as I shall declare afterward, harmony comes from incommensurable motions.

Now that I have declared that any celestial motion might be incommensurable to any other celestial motion, many very beautiful propositions

80. Since this paragraph represents a straightforward application of Proposition 10, chapter 3, to ratios of velocities, the reader is urged to examine that proposition in Selection 25. Oresme is saying here that in the proportional relation $F_2/R_2 = (F_1/R_1)^{V_2/V_1}$, if V_2/V_1, the ratio of velocities, is initially unknown, it is probable that it would be irrational and the velocities incommensurable, so that the ratios F_2/R_2 and F_1/R_1 would also be incommensurable. The mathematical basis for assuming the probability of their incommensurability was presented by Oresme in Proposition 10, chapter 3, of this same treatise.

81. In the basic relation $F_2/R_2 = (F_1/R_1)^{V_2/V_1}$ it is obvious that $V_2/V_1 = S_2/S_1$ when $T_2 = T_1$ (where S is distance and T, time), so that what was said about the probable incommensurability of the ratio of velocities (see n. 80) applies also to the ratio of distances traversed in equal times. At the end of this paragraph, Oresme extends his argument to any continuous magnitudes whatever.

82. Since $V_2/V_1 = S_2/S_1$ when $T_2 = T_1$, it follows that if the ratio of velocities is probably irrational so also is the ratio of distances traversed.

83. Oresme stated this in a number of other treatises.

84. Here Oresme extends to celestial motions what was said in preceding paragraphs about terrestrial motions. His mathematical arguments in support of the probable incommensurability of celestial motions were utilized to discredit astrological prediction, since he could argue that if two celestial motions are probably incommensurable, it would be impossible for astrologers to make exact predictions of celestial positions, and all consequences based upon imprecise knowledge of these positions would be necessarily uncertain and questionable. Not even the most accurate astronomical data could remedy this inherent mathematical indeterminacy.

that I arranged at another time follow, and I intend to demonstrate them more perfectly later, in the last chapter,[85] among which will be these.

One [of them] is: If a perfect [or total] eclipse of the moon should occur only once—and this could happen—it is impossible that a like conjunction should have happened at another time and impossible that it happen again during an eternal future time to come. And I always understand this "naturally speaking," and have even assumed an eternity of motion and the principles put forth by Aristotle in the second [book] of his *On the Heavens* and in other places.

Another proposition is: If two planets, with respect to longitude and latitude, should be conjuncted once in a point, they will never again be conjuncted.

Another proposition is: If three planets were conjuncted with respect to longitude so that they were on the same meridian, it is impossible for them to conjunct again even if they were moved eternally. Thus they were in conjunction in only one way through perpetual times.

Another proposition—I shall not relate any more for now—will be this: In any instant it is necessary that celestial bodies be so related that in any moment there will be a configuration such that there never was a similar one before, nor will there be one after in all eternity, just as it has been written in the twelfth [chapter] of Daniel: "And a time shall come such as never was from the time that nations began even until that time." And the Lord disposeth through these celestial bodies, [for] as the poet says: "The First [Cause] rules all things through the heavens as He wishes; [and the heavens] are the organs of the Prime [Being], the instruments of the Supreme [Being]."

Indeed, along with these I shall demonstrate many other no less beautiful propositions based on the same principle, [but] with a few other more probable principles assumed from those which have [already] been demonstrated. Many errors about philosophy and faith could be attacked by the use of these [propositions], as [for example], that [error] about the Great Year, which some assert to be 36,000 years, saying that celestial bodies were in an original state and then return [to it in 36,000 years] and that past aspects are arranged again as of old;[86] and other errors of this kind which people have been accustomed to reject not by demonstrations but rather by strife and verbosity. But it is better to attack philosophers with philosophy and mathematicians with mathematics, just as Goliath was struck dead by a suitable weapon, and so also truth is made manifest and falsity destroyed. This, then, ends the fourth chapter.

52 THE DEVELOPMENT AND CRITICISM OF ATOMISM IN THE LATER MIDDLE AGES

Translations, introductions, and annotation by John E. Murdoch

To Aristotle's critical eye, the atomism of Leucippus and Democritus was, although admittedly among the most encouraging of the pre-Socratic systems of natural philosophy, inadequate to the task of yielding a satisfactory explanation of the physical world. Yet his opposition to these two of his predecessors was not Aristotle's only criticism of atomistic conceptions. For in Book VI of the *Physics* he clearly demands that no continuum, be it space, time, or motion, be composed of indivisibles; the atomistic structure, not of sensible bodies but of magnitude in general, was, in Aristotle's view, not permissible. Thus, if in rejecting the hypothesis of Leucippus and Democritus, Aristotle had disavowed a physical atomism by arguing in Book VI of the *Physics* for the absolute continuity of all extensive magnitudes, he had also, in effect, rigorously disassociated himself from the possibility of a mathematical atomism, from, to cite the most crucial examples, the possible composition of lines out of points or of time intervals out of instants. It was against this, against, as it were, the very substance of Book VI of the *Physics*, and not against Aristotle's dissatisfaction with Democritus, that the atomism of the later Middle Ages reacted. Moreover, given such a context for the development of atomism in the earlier fourteenth century, it was quite natural that this atomism be, in essence, a mathematical one. Mathematical not in the sense that atomistic conceptions were employed in anything like a mathematical system of analysis, but mathematical merely in the philosophical sense of the atoms at hand always being points or extensionless indivisibles. Thus it is not surprising that this late medieval atomism was not intended, as was that of Democritus or Epicurus, as any kind of general system which might cover or explain the natural world [Nicholas of Autrecourt was among the few scholastics, if not the only one, to propose physical atomism as a more plausible alternative to Aristotle's explanation of the world; see Selection 56.3, n.12—*Ed.*]. It was intended rather as a single facet of natural philosophy, designed simply to explain the structure of magnitudes, and specifically of space, time, and motion as magnitudes. In medieval terms, this atomism proposed an anti-Aristotelian answer to the question of the composition of continua. Indeed, Aristotle's opposition to the views the medieval atomist wished to espouse made the latter's task a predominantly defensive one.

The first object of his dissent was, of course, to render lame the highly effective (so they were usually regarded) arguments of Aristotle against the indivisibilist composition of continuous magnitudes. For in the opening chapter of Book VI of the *Physics*, Aristotle had made it clear that the constitution of continua out of indivisibles must necessarily falter, not merely because indivisibles cannot be continuous but chiefly because there is no manner in which they can touch or be in contact. In other terms, if one is to maintain the indivisibilist structure of a continuum, then one must be able to explain exactly how the component indivisibles are "connected to" one another. But, Aristotle cautioned, no "connecting relation" is possible which will, at the same time, also account for the extended continuum such indivisibles are supposed to compose.[1] Viewed in such a light, the initial move of the fourteenth-century atomist was to circumvent Aristotle's argument by discovering and establishing a "relation of connection" for his indivisibles which did not fall prey to the by now traditional gambit of the Aristotelian attack.[2]

Should he be so fortunate as to have survived this stage of the battle (at least in his own eyes, if not those of his critics), the medieval atomist had no cause to rest or to expect eventual victory. Other arguments that were yet more formidable remained to be repulsed, and these were, at bottom, mathematical. For the scholastic, their origins could be found in the *Metaphysica* of Algazel. But they gained immeasurably in force and prestige when John Duns Scotus employed such mathematical arguments for the primary bastion of his criticism of indivisibilism in the second book of his *Opus Oxoniense*. From Scotus on, mathematical arguments against the atomist increased, in complexity as well as in number, at an almost exponential rate. In fact, the core of the substantial body of fourteenth-century literature comprising the treatises of these newly born atomists and their critics exemplifies the history of the debate, over such mathematical arguments and over the strictures pronounced by Aristotle in the sixth book of his *Physics* against mathematical atomism. The following selections are meant to illustrate this debate.[3]

85. These were never demonstrated as part of the *De proportionibus proportionum*, which terminates with the fourth chapter. However, the four specific propositions mentioned in the following paragraphs all appear in Oresme's *Ad pauca respicientes* and all but one in his *De commensurabilitate vel incommensurabilitate motuum cell* (*On the Commensurability or Incommensurability of the Celestial Motions*), which are two treatises devoted wholly to the subject of celestial commensurability and incommensurability (indeed there are also brief discussions of this subject in other of his works). For some propositions drawn from his *De commensurabilitate* see Selection 69. Oresme's objective in enunciating these four propositions in the following paragraphs was to emphasize that if the celestial motions are incommensurable, no celestial configuration or event can ever repeat exactly in all respects.

86. The concept of a Great Year as an unending sequence of equal time intervals in each of which the same exact celestial phenomena—and for some of its proponents even the very same terrestrial events—are repeated was rather popular (especially among some Stoics) during the Hellenistic and Roman Empire periods. Oresme calls it an error in philosophy and faith because in 1277 the bishop of Paris condemned the following proposition (one of 219 condemned; see Selection 13, no. 6). "That when all the celestial bodies have returned to the same point—which will happen in 36,000 years—the same effects now in operation will be repeated." To Christians, the unending cyclical repetition of events was repugnant, since it was contrary to their belief in a unique creation that must run its course but once.

1. This is, in effect, the substance of Aristotle's argument in *Physics* VI. 1.231a. 26–231b. 6. A medieval reformulation of just this argument of Aristotle is given below in the selection from William of Alnwick as the first of five arguments against the atomistic position of Henry of Harclay. The "confirmation" constructed by Averroes of this argument is also given in the passage from Alnwick, an addition which became as standard as the original argument itself in all medieval treatises dealing with the composition of continua.

2. Such a move is precisely what Harclay was concerned with when he replied to Aristotle's argument (see section 4 below).

3. Further details concerning other participants, both atomist and anti-atomist, in the fourteenth-century continuum debate, as well as with respect to the whole history of late medieval atomism, may be found in the following publications by John E. Murdoch: *Rationes Mathematicae: Un aspect du rapport des mathématiques et de la philosophie au Moyen Age* (Paris: Université de Paris, 1962); "Superposition, Congruence and Continuity in the Middle Ages" in *Mélanges Alexandre Koyré* (2 vols.; Paris, Hermann, 1964), Vol. I: *L'aventure de la science*, pp. 416–441; "Two Questions on the Continuum: Walter Chatton (?), O.F.M., and Adam Wodeham, O.F.M.," *Franciscan Studies*, XXVI (1966), 212–288 [with Edward A. Synan]. Critical editions of the complete texts of the works from which the following selec-

1. Thomas Bradwardine (ca. 1290–1349)

Easily the most impressive and the most exacting refutation of the incipient atomism of the fourteenth century was Bradwardine's *Tractatus De continuo* (written, it would appear, sometime between 1328 and 1335). In form the *Tractatus* is geometrical, modeled—as was Bradwardine's later major theological work *De causa Dei*—on Euclid's *Elements*. It opens with 24 definitions and 10 suppositions or postulates and then moves on to demolish atomism of all sorts in the formulation and demonstration of 151 conclusions. Unfortunately, the work is far too systematic and interconnected to allow selection of brief passages illustrative of the substance of its criticism of indivisibilism. Yet the very brief selection given below is instructive in revealing Bradwardine's awareness of the then rising current of atomism and his view concerning the various forms which such an atomism could, and did, assume. The present translation has been made from my own as yet unpublished edition of the *Tractatus De continuo*.

TRACTATUS DE CONTINUO

If one continuum is composed in a certain manner out of indivisibles, any [continuum] is so composed, and if one is not composed of atoms, none is.

2. Algazel (1058–1111)

Although the repetition and refinement of Aristotle's arguments against atomism naturally formed the basis of any criticism of the rising indivisibilism of the later Middle Ages, a second group of arguments that became standard fare in any such criticism were six drawn from the *Metaphysics* of Algazel (as he was known to the Latins). The *Metaphysics* was a Latin translation of part of al-Ghazzālī's *Maqasid*, a work which did not mean to give his own views but rather a summary of those of Avicenna. Thus, the arguments given below derive initially from Avicenna, although the Latin West knew them only under the name of *rationes Algazelis*. My translation has been made from J. T. Muckle's edition of *Algazel's Metaphysics* (Toronto: Institute of Medieval Studies, St. Michael's College, 1933), pp. 10–13, as well as from additional manuscripts.

Metaphysics
First Treatise

CHAPTER 3. ON THE DIVERSE VIEWS OF THE COMPOSITION OF A BODY

In order to understand this conclusion we must realize that there are five famous opinions concerning the composition of continua among ancient and modern philosophers. For certain [philosophers], like Aristotle, Averroes and most of the moderns, hold that a continuum is not composed of atoms, but rather of parts divisible without end. Others, however, hold with its composition out of indivisibles. But there are two variants [of this position]. For Democritus maintains that a continuum is composed of indivisible bodies. Others claim [it is composed] of points, and these [thinkers] fall into two groups. For Pythagoras—the father of this sect—Plato, and Walter the modern[4] contend that a continuum is composed of a finite number of indivisibles. Others, however, [believe in its composition] out of an infinite number [of indivisibles], and these [indivisibilists] are [again] twofold. For certain of them, like Henry the modern,[5] say a continuum is composed of an infinite number of indivisibles immediately joined [to one another]; others still, like [the Bishop of] Lincoln[6] [side with] an infinity of [indivisibles] which are mediate to one another.

Men have been of diverse opinions concerning the composition of bodies, and we can only learn the truth about this problem after we have revealed which of these opinions—and there are three of them—is the truest. Some have said that a body is composed of parts that are indivisible, both conceptually and physically, and are called atoms. Indeed, these thinkers have called these [indivisible parts] units and odd substances[7] and have claimed that a body is composed of them. However, others have said that a body is in no manner a composite thing but that it is a being that is one both in essence and by definition and that contains no multiplicity. And still others have held that a body is composed of matter and form.

tions are drawn, as well as of other works directly relevant to this particular segment of fourteenth-century science, have been prepared by Murdoch and will shortly appear in a volume setting forth the history of the debate about continuity and atomism in the fourteenth century.

4. That is, the Franciscan Walter Chatton, who, presumably in 1323, included an exhaustive examination of the problem of atomism and continuity in the second book of his *Commentary on the Sentences*.

5. That is, Henry of Harclay; see section 4 below.

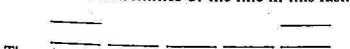
6. That is, Robert Grosseteste, who did not, strictly speaking, maintain the atomistic composition of continua, but who, on the grounds of several important implications within his work, was clearly held to have been an atomist by such thinkers of indivisiblist sympathy as Henry of Harclay.

[Refutation of the First View]

The first opinion of those who have said that a body is composed of odd substances is destroyed in six ways.

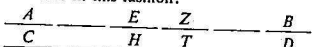
[1] The first[8] is that if one of these substances is placed between two others, then each of the two extreme ones [will either touch the same things in the middle one as the other [extreme one] does or something different [from that which the other extreme touches]. If it touches something different, then [the middle substance] is divisible, since that which one of the extremes occupies [in the middle substance] is not that which the other extreme occupies. On the other hand, it is, without doubt, impossible for [the two extremes] to touch the same thing [in the middle substance]. For if they did, it would follow that each of the extremes would penetrate the mean substance totally insofar as each has touched the whole [mean] because the latter has no totality [of parts] due to the fact that it is one. Yet one of the extremes has touched something belonging to the mean; therefore, it has touched the whole of it. Moreover, the other extreme has similarly touched the whole of it. Consequently, it follows that there is one place for three things and the mean substance. Alternatively, the mean substance lies separated between the two extremes and touches one of the extremes differently than it does the other, since it can only touch one in the same thing as it does the other if one of the [extremes] has penetrated the other. The same thing will occur if a third, fourth, or more [substances] be added. It necessarily follows, then, that a distance of a thousand parts is no greater than a distance of one part. But it is clear that this is inconsistent; for, on these grounds, nothing could be composed out of these [indivisible substances].

[2] The second [way of refuting this indivisibilist position] is to arrange five of the aforesaid substances in a line and to place two [other substances] over the two extremities of the line in this fashion:



There is no doubt that the intellect can conceive that these two superposed parts simultaneously begin to move toward one another with equal speeds [and continue to do so] until they meet. If this be assumed, then each of them will cut a part from the middle substance [in the line]. Consequently, the middle substance is divided. And all this is true because if it is not, then it follows that the substances moving with equal speeds cannot meet, but rather, after they have begun to move and one of them has reached the second part of the line, this [moving substance] will remain at rest in that place until the other one reaches the third part of the line. But from whence comes the variety in this unequal motion, and why does this occur on the right rather than on the left when either one [of the moving substances] is equally capable of accomplishing this and both are equally capable of receiving motion?

[3] The third [manner of refutation] is to take two lines, both composed of six parts, one of them *AB*, the other *CD*, and let them be parallel and opposite one another in this fashion:



Next, assume that two parts [of these lines] begin to move: one from *A* to *B*, the other from *D* to *C* until they are opposite one another, one over the other. Surely, [our two mobile parts] will first be opposite one another and one will be over the other; then, one will pass the other. Moreover, it is possible for them to move with equal speed. Then, if [each] substance [composing our two lines] is odd,[9] they cannot possibly meet. For their meeting can occur in only one of three ways: [a] They will

7. The Latin text reads *substantias impares* which is a mistranslation of what in Arabic was the standard term for atom or indivisible: *al-jawhar al-fard*. This should have been rendered *substantia simplex* (and hence whenever "odd substance" appears below, one should read "simple substance," or merely "atom"), but I have let the mistranslation stand, since it appears in an overwhelming majority of extant manuscripts, and since, consequently, it was just such an incorrect text that the Latins used and which, on occasion, in its erroneous form gave rise to criticism that would otherwise not have been possible. Thus, the Franciscan Gerard of Odo (apparently writing about 1326) had, as an atomist, grounds to object to the fourth argument (see below) of Algazel by claiming that: "He seems to contradict his own assumptions. For he explicitly says that an odd number of substances, that is, indivisibles, may be taken, and then goes ahead to take sixteen, an even, not odd, number of them."

8. This argument derives ultimately from Book VI of Aristotle's *Physics* (see n. 1).

9. See note 7.

meet in the two points E and H, and hence one traverses three parts of the line, the other two; or [b] they meet in the two points T and Z, and hence [again] one traverses three parts, the other two (and [in either of these two cases their] motion will not be equal); or [c] one will be in point T and the other in point E and both will traverse two parts; but E and T are not opposite. Therefore, their direct opposition and traversal by equal motion is destroyed, which is completely impossible. However, this impossibility arises only because we have assumed the substances to be indivisible and [our mobiles] come into opposition over a half of each of two [such indivisible substances]. For every length is susceptible of division into two equal halves. Therefore, the bisection of our line occurs in the middle of the substances and it is [only] in this way that [the mobiles] come to be directly opposed, one over the other.

[4] The fourth [manner of refuting this indivisibilist position] is to arrange sixteen continuously joined odd substances into a square figure, four in [each of] four lines, in this way [Fig. 1]:

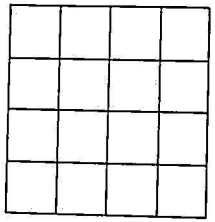


Fig. 1

Furthermore, though we illustrate these [substances] as being separate from one another, we nevertheless conceive them to be joined to each other in such a fashion that no space falls between them. Moreover, there is then no doubt that the four sides are equal; for each side is composed of four [equal] parts. Yet the diagonal is also composed of four. Hence, it follows that the diagonal is equal to each of the sides, which is impossible. For every diagonal cutting a square into two equal triangles is always greater than any of the sides. This is sensibly evident in all squares and is proved to be the case in geometry. Yet this becomes impossible if we assume the existence of odd substances.

[5] The fifth manner [of refutation] is that if we have placed a stick perpendicularly [in the earth] facing the rays of the sun, it will undoubtedly cause a shadow; then, by means of a ray, a straight line is projected from the end of the shadow cast by the top of the stick to the sun. Therefore, it is then necessary either for the shadow to move when the sun moves or it is not. A ray, moreover, is extended only in a straight line. If, then, the shadow does not move when the sun moves, a straight line will have two ends in a single direction, one in the place previously occupied by the sun and the other in the place to which the sun has subsequently been moved. But this is impossible. On the other hand, if the shadow does move, then, when the sun moves the distance of a single atom, the shadow will move less than an atom, which means an atom is divided. If, finally, the shadow moves the same extent as the sun, this is certainly an inconsistency. For the sun traverses a million miles while the shadow moves only so much as a hair's breadth.

[6] The sixth manner [of refutation] is the following: When a wooden or stone wheel revolves, the parts at its center clearly move less than the exterior parts because of the fact that a circle at the center [of a wheel] is smaller than a circle at its perimeter. However, when the exterior circle moves one atom, the center circle will either move less than an atom and hence an atom will be divided, or it will not move at all and it will follow that all parts of the wheel are separate so that some move, while others do not, but remain at rest. But our senses tell us this is false, for the parts of the wheel are not separated in any way at all. . . .

3. John Duns Scotus (ca. 1266–ca.1308)

The following two geometrical arguments against the composition of a continuum out of indivisibles are drawn from Scotus' rather lengthy examination of the problem of "whether an angel can move from place to place with a continuous motion." Once proposed by Scotus, these two arguments never failed to grasp the attention of all concerned, both atomist and anti-atomist, with the structure of continua. The translation has been made from my own edition, based largely on MS Assisi, Biblioteca communale, 137 and occasionally corrected by MS Padua Anton. 173 and the Luke Wadding edition of Scotus published at Lyons in 1639.

ON BOOK II OF THE SENTENCES: THE OPUS OXONIENSE DISTINCTION II, QUESTION 9

Nevertheless, this antecedent[10] is even more effectively proved by a pair of geometrical arguments or propositions. The first of them is the following: [1] In accordance with Postulate 2 of Book I of Euclid "one can construct a circle of any size one wishes about any center." Let there be constructed, then, two [unequal] circles about some given center A [Fig. 2]; call the smaller circle

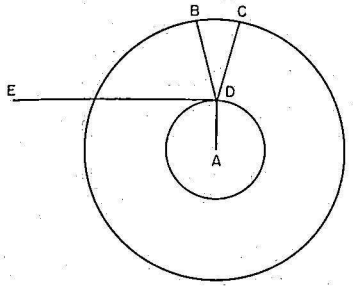


Fig. 2

D, the larger one B. If the larger circumference is composed of points, take two of them immediately next to each other and call them B and C. Next, draw straight lines from A to B and C in accordance with Postulate 1 of Book I of Euclid (which reads "To draw a straight line from point to point, etc."). The straight lines so drawn will pass through the circumference of the smaller circle at right angles. I ask, then, whether they will cut it in one and the same point or in different ones. If in different ones, there will then be as many points in the smaller circle as in the larger one. Yet it is impossible for two unequal quantities to be composed of parts equal in both magnitude and number. [And this applies to our case,] since one point does not exceed another in magnitude and there are the same number of points in the smaller circumference as there are in the circumference of the larger circle. Therefore, the smaller circumference is equal to the larger one, and consequently, the part to the whole. If, on the other hand, the two straight lines AB and AC cut the smaller circumference in the same point, call it D; then erect a straight line DE over AB, cutting it at point D, which, by III.15 of Euclid,[11] also touches the smaller circle. Now, by I.13 of Euclid,[12] DE forms two right angles, or [two angles] equal to two right angles, with AB; furthermore, by the same I.13, DE will also form two right angles, with line AC (which is assumed to be a straight line). Consequently, $\angle ADE$ together with $\angle BDE$ is equivalent to two right angles. By a parity of reasoning, $\angle ADE$ together with $\angle CDE$ is equivalent to two right angles. Yet any pairs of right angles are equal, on grounds of Postulate 3 of Book I of Euclid; therefore, if we take away [the angle these two pairs] have in common (that is, ADE) the remainders will be equal; consequently $\angle BDE$ will be equal to $\angle CDE$ and thus the part will be equal to the whole.

[Objection]

An opponent might reply to this that DB and DC do not enclose an angle, since if they did, a base could be drawn between B and C under the angle, which is contrary to that assumed, since B and C were held to be immediate points. Therefore, the claim that $\angle CDE$ functions as a whole relative to $\angle BDE$ as part is to be denied, since nothing from $\angle CDE$ is added to $\angle BDE$ because of the fact that there is no angle between B and C about their juncture at point D.

[Reply to the Objection]

This response seems absurd on two counts. First, because in denying that there is an angle at the point where two [straight] lines—which are extended over a surface and not directly applied to each other—meet, it contradicts the definition of an angle given in Book I of Euclid. Secondly, because in denying that a line can be drawn from one point to another, it simultaneously denies Postulate 1 of Book I of Euclid. Nonetheless, even though these absurdities ensue, they are not held to be inconsistencies, because they follow from that proposed [by our opponent]. In view of this, I argue against the response in another way: $\angle CDE$ includes the whole of $\angle BDE$ and, though you impudently claim that it adds no angle, adds at least a point [to BDE]; yet a point is, on your grounds, a part; therefore, $\angle CDE$ adds some part to $\angle BDE$ and functions, consequently, as a whole relative to the latter. Our assumption is evident no matter which way one interprets an angle. For either: (1) an angle

10. Namely, that a continuum cannot be composed of indivisibles.

11. That is III.16 of the Greek Euclid, the relevant portion of which reads: "The straight line drawn at right angles to the diameter of a circle from its extremity will fall outside the circle. . . ."

12. Which reads: "If a straight line set up on a straight line make angles, it will make either two right angles or angles equal to two right angles."

is held to be the space intercepted between the lines [enclosing it] without counting these lines themselves, and then the first point of line *DB* beyond the smaller circumference will not belong to $\angle BDE$ but will belong to $\angle CDE$; or (2) an angle includes, in addition to this intercepted space, the enclosing lines as well, and then the first point of line *DC* beyond the smaller circumference will not belong to $\angle BDE$ but will belong to $\angle CDE$. Consequently, from either point of view, $\angle CDE$ adds a point to $\angle BDE$.

Nor can the principal demonstration be opposed by pretending that the two [straight] lines do not begin to diverge from one another in the smaller circumference, but rather at some other point closer to, or further from, the center. For I shall draw the smaller circumference wherever you assume this occurs.

The second part [of our argument]—to wit, that the smaller circumference is not cut in just one point if it is cut by both of the lines—would need no proof were it not for the arrogant obstinacy of our opponent. For it is evident enough that if one continuously and directly extends one and the same line, it will never end in two points in a single direction. And if this is conceded to be manifestly true, that which is proposed is immediately evident from the chain of reasoning in the first part [of our argument].

[2] The second [geometrical] proof is based on Propositions 5 and 7 of Book X of Euclid. For the fifth proposition says that the ratio of all commensurable quantities to one another is as the ratio of some number to some number, and consequently, as the seventh proposition asserts, if several lines are commensurable, their squares will stand to one another as one square number to some other square number. However, the square of a diagonal [of a [square] does not stand to the square of the side as some square number to another square number; therefore, the line which was the diagonal of the square will not be commensurable to the side of the square. The minor premise is evident from the penultimate proposition of Book I [of Euclid's *Elements*], because the square of the diagonal is twice the square of the side insofar as it is equal to the squares of two sides; yet no square number is twice another square number, as is evident if one runs through all squares arising from any roots whatsoever.

From the foregoing, the following conclusion is evident: The diagonal is asymmetrical, that is, incommensurable, with the side. However, if these [two] lines were composed of points, they would not be incommensurable, for the points of one would not stand to the points of the other in any ratio of numbers. And it would not only follow that the lines would not be incommensurable, but that they would even be equal, which plainly runs contrary to our senses.

Proof of the consequence: Take [Fig. 3] two

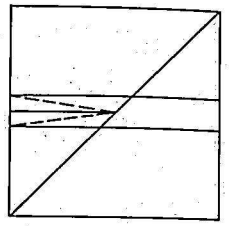


Fig. 3

immediate points in one side and two other immediate points directly opposite them in the facing side; draw two straight lines between these two pairs of points parallel to the base [of the square]; these lines will cut the diagonal. I ask, then, whether [they cut it] in immediate points or mediate ones. If in immediate points, then there are no more points in the diagonal than in the side and, hence, the diagonal is no greater than the side. If [they cut the diagonal] in mediate points, then I take a mean point falling between these two mediate points of the diagonal; it will belong to neither of the two originally given [parallel] lines. Then, by I.31 [of Euclid],[13] I draw a line from that mean point parallel to both of the given lines. This latter parallel is drawn continuously and directly in accordance with the second part of Postulate 1 of Book I [of the *Elements*]. It will cut the side, yet will do so in neither of its given points but rather between them both. If this were not the case, it would meet a line with which it is, by hypothesis, parallel, which is contrary to the definition of parallel (the last definition set down in Book I [of Euclid]). Therefore, there is a mean point between those two points in the side that were assumed to be immediate. This follows from the fact that it was claimed that there was a mean point between the points in the diagonal [arising from its intersection with out first two parallels]. Thus, the denial of the consequent implies the denial of the antecedent; therefore,

13. Which reads: "Through a given point to draw a straight line parallel to a given straight line."

and so on. Indeed, one should even say in general that the whole of Book X of Euclid overthrows the composition of lines out of points. For, given such, there would be no irrational or surd lines at all, which nevertheless, are Euclid's principal occupation in Book X, as is evident from the many species of irrational lines he points out in that place. . . .

4. Henry of Harclay (ca.1275–1317) and William of Alnwick (ca. 1270–1333)

Henry of Harclay, Chancellor of Oxford University in 1312, was, it appears, the first thoroughgoing atomist and, one can add, adherent of the existence of an actual infinite, in the later Middle Ages. Although his long-lost treatise on the infinite and the continuum has recently been discovered, his views concerning the latter are given below not from that work but rather from the second so-called *Determinatio* of one William of Alnwick, Franciscan, editor of the works of his teacher Duns Scotus and, it seems, first critic of Harclay's atomist utterances. Inasmuch as Alnwick reports Harclay's arguments almost verbatim (he even takes some of his objections to Harclay word for word from possible objections Harclay himself formulated), the substance of the latter's contentions are most accurately reported in the work of his Franciscan critic.

Both Harclay's treatise and Alnwick's *Determinatio II* are rather lengthy *quaestiones*, so only a brief portion of the latter has been presented in what follows. However, the selection has been made with a view to giving: (1) Harclay's two major positive arguments for his atomistic stance; (2) three of the most often employed arguments against any atomist doctrine; (3) Harclay's replies to these three frequently occurring arguments; and (4) Alnwick's criticism of the initial two positive arguments of his opponent. Harclay's replies to these criticisms of Alnwick, as well as Alnwick's counter-replies have been omitted; so also Alnwick's criticism of Harclay's response to the three arguments noted in (2) above. The translation has been made from my own as yet unpublished edition, based directly upon manuscript versions.

Determinatio II

ARTICLE TWO OF THE QUESTION

In the second article of the question[14] it remains to see whether continuous quantity is composed of indivisibles. And I shall proceed as follows in this article: First, I shall set forth the arguments of the aforesaid doctor [that is, Henry of Harclay] by means of which he holds a continuum to be composed of indivisibles, for example, a line of points. Secondly, I shall offer arguments which prove with greater evidence and certainty that a continuum is not composed of indivisibles. Thirdly, I shall give the replies on which he bases his refutation of these arguments and show that these refutations are unsatisfactory. Fourthly, I shall refute the arguments which he constructs to support his point of view.

The Opinion of Master Henry, Chancellor of Oxford: That a Continuum Is Composed of Indivisibles

(1) Relative to the first point [of our investigation], this doctor has used two arguments to show that a continuum is composed of indivisibles. These arguments have [even] moved him to asserting this fact. The first of them has the same force as the second argument given above among those set forth above in the [introductory] dispute. It is this: God actually perceives or knows the first inchoative point of a line and any other point capable of designation in the same line. Therefore, God either perceives that a line can fall between this inchoative point of the line and any other point in the same line, or He does not. If not, then He perceives point immediate to point, which is that proposed. If He does, then, since points can be assigned in the mean line [which falls between the first inchoative point and the other, random, point], these mean points would not be perceived by God, which is false. The consequence is evident: For, by hypothesis, a line falls between the first point and any other point of the same line perceived by God, and consequently there exists some mean point between this [first] point and any other point perceived by God; therefore, the mean point is not perceived by God.

(2) His second argument is the following: God can annihilate the end point of a line without corrupting the line (as will be proved). This done, the line actually possesses a terminus between which and the corrupted point there was no mean line.

14. The question itself reads: "Whether there are potentially more parts in a greater continuous quantity than in a smaller one," while the first article dealt with the problem of whether one infinite could be greater than another.

For otherwise some line would have been corrupted, the opposite of which will be shown. In the same way, He could corrupt another point without corrupting any line or part thereof, and it will necessarily follow that point is immediate to point. Proof of the major premise: Let some continuous line be broken at the middle, [the two halves] then create two actual points; let [these two halves] be joined to one another, then [these two points] will exist potentially. This accomplished, it is possible that numerically the same point which actually existed before and was the beginning of the right part of the line is now the end of the left of the line. Proof: A line can be broken at any one of its points in such a way that any point can indifferently be the beginning of one or the other [of the resultant parts], though not of [both] at the same time. Therefore, the point which was the beginning of some other line can now be the end of this line, and conversely. Hence, as one line can be corrupted while the other remains an integral whole, God can, if He corrupts one line, corrupt the point which was the beginning of the other line without any part of this [other] line being corrupted.

[*Five Arguments against Henry's Opinion*]

Secondly, we must set forth contrary arguments which prove that a continuum is not composed of indivisibles. There are five arguments.[15]

(1) The first of these is that of Aristotle in [Book] VI of the *Physics;* it is the following: If a continuum were composed of indivisibles, it would be necessary that the addition of indivisible to indivisible would cause an increase in size. The consequent is impossible, thereore [so also is] the antecedent. The consequence is self-evident, since, if the addition of indivisible to indivisible would not cause an increase in size, there would [upon such addition] result but a single indivisible and such a procedure would never yield a continuum composed of the [indivisibles in question]. Therefore, opposite follows from opposite. Aristotle proves the consequent to be false as follows: If indivisible added to indivisible were to create an increase in size, it would be necessary that one indivisible touch another; therefore, [they will touch] either according to their wholes or according to a part. Not according to a part, since [an indivisible] does not have a part (for if it did, it would be divisible and not indivisible). If [they touch] according to their wholes, then no increase in size will occur, since whole touching whole causes no increase in extension. And we must argue in the same fashion with respect to an infinity of additional indivisibles.

This reasoning is corroborated by Averroes' argument in the same place in [Book] VI of the *Physics,* Comment 2, where he argues as follows: "When one thing wholly touches another, superposition occurs, and no magnitude that did not exist previously results from superposition. Hence no magnitude in breadth is caused by the superposition of line to line, as they are only superposed breadthwise, in which [dimension] a line is indivisible; similarly, the superposition of surface to surface does not create depth. And since a point is indivisible in every dimension, [a point wholly touching, or superposed to, a point] does not create something of greater size that possesses parts." These are the words of Averroes.

(2) The second argument [against Harclay's view] is this: If a line were to be composed of points, the diagonal of a square would be equal to the side, which is false and contrary to our senses. Proof of the consequence: The sides of a square are equal and, you maintain, composed of points, hence there are an equal number of points in each. Therefore, let lines be drawn from each and every point (infinite in number) of one side to each and every facing point of the opposite side. These lines will be mutually parallel, and neither point nor line can fall between them since they are drawn parallel from points [in the side] that are immediately next to one another. Then [one argues] as follows: These lines pass through the diagonal and cut it. Hence, they either cut it in every one of its points, or they do not. If in every one, then there are an equal number of points in the diagonal and the side, and consequently the side and diagonal are equal. It may be claimed that the lines which pass through the diagonal do not touch every one of its points, and that thus some point of the diagonal falls between these lines. Yet, since these lines are parallel, it follows that howsoever much falls between them in one place just as much falls between them in another place, and hence a point can fall between these lines where they are joined to the side: but this is contrary to that assumed, since these lines exhaust all the points of [each] side. Therefore, no point can fall between two lines drawn [parallel] from two immediately situated points in the side.

(3) The third argument has to do with the indivisible instants in a continuous, successive time

15. The fourth and fifth arguments, here omitted, are those of Algazel dealing with the rotating wheel and the shadow-casting stick (see section 2 above, p. 316).

which is not composed of instants; for if it were, one 1-foot quantity would be greater than another 2-foot quantity, and a part would be greater than, or equal to, the whole. Proof of the consequence: Take a slow mobile which is moved for two days through a space of 1 foot, and let a quick mobile be moved for one day through a space of 2 feet. Then take the assertion of Aristotle in [Book] VI of the *Physics:* When a mobile is moved, it is in a space equal to itself in any designated or designatable instant, and from one instant to another it is in one and another space, for otherwise it would be at rest and not be continuously moved. Therefore, as many instants as one can assign in the time during which a mobile is moved through some space, so many spaces equal to the mobile can be assigned in the same magnitude, and so many points terminating these spaces. Hence, if time be composed of instants, then, as you claim there are more instants in two days than in one, there are more points in a 1-foot space traversed in two days than in a 2-foot space traversed in one day, which is impossible. . . .

Replies of the Aforesaid Doctor to these Arguments

The third task left to fulfill in this article is to note this doctor's replies to the aforesaid arguments and to show that these replies are unsatisfactory. . . .

[*To the first.*] To the first argument drawn from [Book] VI of the *Physics*—that the addition of indivisible to indivisible causes no increase in size, since whole touches whole—he replies that whole touching whole can occur in two ways: either in the same local position, or in distinct positions immediately next to one another. In the first way superposition occurs, and no increase in size results, as Averroes [himself] argues. This is also evident [from the following consideration]: An infinite number of points terminating an infinity of lines running from the circumference [of a circle] to its center do not, since they are superposed at the center in the same local position, cause an increase in size. In the same way, the points of two intersecting lines occupy the same position at the point of intersection and do not cause an increase in size. Hence, this [lack of increase in size] is not due to the fact that an indivisible is added to an indivisible in just any manner, but rather because they are added according to the same position. However, when they are added according to distinct positions, then [the addition] can create an increase in extension. Whence the points of two lines which touch in a common boundary of contact have distinct positions and therefore cause an increase in size.

He first explains this reply by means of an example dealing with numbers: A unit, even though it be indivisible, nonetheless causes an increase in numerosity when added to another unit, not insofar as the second unit is added to the first relative to the same numbered thing (for in such a case an infinite number of units do not make more than one), but when [one unit] is added to another relative to another numbered thing. But points are related to an increase in the genus of continuous quantity just as units are to an increase in the genus of discrete quantity. And thus point added to point in the same position causes no increase in size, but [if added] in a distinct position it does.

He also explains this by an appeal to the fact that if a surface or a body be added to another [surface or body] according to the same position, no increase in size results, but only [when the addition takes place] according to diverse positions. For a single body which possesses every dimension can be added or conjoined to another body without there occurring an increase in size. For he explains at length that "quantitative dimension no more prevents one body from simultaneously existing [in a single position] with another body than a point with a point." From these considerations he infers that not merely a point but also a line or a body creates an increase in extension only when it is applied [to another thing] according to diverse positions. Thus he claims that if two indivisibles—points, for example—be applied to one another according to diverse positions, they will cause an increase in size. "Yet the points or lines of two touching lines or surfaces, or the surfaces of two touching bodies, have corresponding distinct positions, just as do the bodies of which they are the surfaces [or the surfaces and lines of which they are, respectively, lines and points]. For just as the places and positions of bodies are distinct and separate, so are the termini of these bodies." Therefore, he says that "point added to point according to a distinct position does cause an increase in size, but that such addition of point to point will not attain a quantity perceptible or comprehensible by us unless there occurs an addition of an infinite number of points (just as not every part in a continuum creating an increase in size is perceptible to us)." With this, the reply to the argument of Aristotle and Averroes in [Book] VI of the *Physics* is evident. . . .

To the second argument. To the second argument about the side and diagonal of a square, this doctor replies that a point falls according to oblique position, and not according to right-angled position, between the parallel lines cutting the diagonal, and that therefore a point [in the side] does not fall between the lines drawn between the sides, because these points [in the side] have right-angled positions [relative to the parallel lines]. And this is the way in which there will be more points in the diagonal than in a side, since a point in the diagonal, but not in the side, falls between the [parallel] lines. . . .

To the third argument. He replies to the third argument dealing with time and the instants of time by making a distinction: that time can be considered as it is discrete, or as it is continuous. If [it be considered] as continuous, he claims that instants are multiplied in time according to the *mutata esse*[16] of a mobile [moving] over a magnitude. And, since there are more points in a greater magnitude than in a smaller one, it follows that if a magnitude of 2 feet is traversed in one hour, and a magnitude of 1 foot in a hundred years, there will be more instants in one hour than in a hundred years. But the converse holds in time [considered] insofar as it is discrete. . . .

[*Replies*] *to the Arguments Supporting* [*Henry's*] *View*

Fourthly, it remains in this article to reply to the arguments supporting his view.

(1) To the first—where, in arguing, it is asked whether or not there is a mean line between the first point of a line and every other point known by God. If not [it is replied] then God perceives this [first] point to be immediate to the other [points]. If there is, then, since points can be assigned in the [mean] line, these mean points will not be perceived by God, which is false. To this argument, I have replied at length in the question: "Whether God knows an infinity of things," while excluding the unsatisfactory replies of others who are ignorant of logic. However, I reply in brief that this is true: "Between the first point of the line and every other point of the same line known by God there is a mean line." For any singular [of this universal] is true, and moreover, its contradictory is false. And this is so because the term "mean line" in the predicate mediately following the universal sign has merely confused supposition.[17] On the other hand, this is false: "There is [some one] mean line between the first point and every other point of the same line perceived by God," since there is no [one] mean line between the first point and every other point

16. A *mutatum esse*, or alternatively *motum esse* (literally a "having been changed" or a "having been moved"), was what an indivisible of motion was called, just as, the Scholastics often explain, a point is an indivisible of space and an instant is an indivisible of time.

17. The doctrine of supposition was one of the most important, and certainly one of the most used, additions made by medieval logicians to the logic of Aristotle as they knew it. It is impossible to explain in brief compass the precise significance of this doctrine [the theory of supposition is discussed in Selection 18—*Ed.*], but for present purposes it is perhaps sufficient to note that, very roughly, the doctrine has to do with the logic of the discrimination of what a term, *within a proposition*, "stands for" *(supponit pro)*. Thus, William of Ockham defines supposition (*Summa logicae*, I, ch. 62) as follows: "It remains to speak of supposition, which is a property of the term, but only when it is in a proposition. . . . It is called supposition, in the sense that it is a positing for other things, such that when a term in a proposition stands for something, we use the term for that of which (or of a demonstrative pronoun indicating it) that term is verified." Ockham's meaning becomes clearer if we follow his discussion and division of supposition further. Thus, one of the major types (for our purpose the most significant one) of supposition is that called *personal*, when the term in question "stands for the objects it signifies." Personal supposition can, however, itself be divided into a variety of distinct types, each type being characterized, among other things, by the kind of "logical descent" one can, or cannot, make from the initial proposition containing the term whose supposition is in question to another proposition involving singular terms. Thus, Ockham tells us (ch. 68) that "*confused distributive* supposition occurs when it is licit to make a logical descent in some way to a *copulative* proposition if the term has many inferiors. . . .So it is with this proposition, 'every man is an animal.' The subject of this proposition has *confused distributive* supposition. For this follows: 'Every man is an animal, therefore this man is an animal and that one is (and so on for every individual)'." On the other hand, in the proposition "some man is an animal," the term "man" has *determinate* supposition, which means that a licit logical descent can be made to a *disjunctive* proposition, that is, to the proposition: "This man is an animal, or that man is an animal (and so on for every individual)." Similarly, if a term has, as in the passage in Alnwick now in question, *merely confused* supposition, then we know that, among other things, a logical descent to a disjunctive proposition *cannot* be made. Therefore, since the term "mean line" in the proposition "Between the first point of the line and every other point of the same line. . . there is a mean line" does have *merely confused* supposition, we cannot make a disjunctive descent to the proposition "Between the first point of the line and every other point of the same line. . .there is this mean line or that mean line (and so on for every individual)." This is, then, the point which Alnwick wishes to make, which is but another way of saying that although there is a mean line between the first point of the line and every other point, there is no *one* mean line between the first point and every other point.

perceived by God. For there cannot be any such mean line, for if there were, it would fall between the first point and itself; nor would that line be perceived by God. And therefore, when it is inferred: "If there is [such a mean line], then, as points can be assigned in the line, etc.," the term "line" there has particular supposition. And hence an inference is made affirmatively from a superior to an inferior, and thus the fallacy [of affirming] the consequent is committed.[18] Similarly, an inference is made from a term having merely confused supposition to the same term having determinate or particular supposition,[19] and *quale quid* is changed into *hoc aliquid*, and a fallacy of a figure of speech occurs.[20]

To the proof [of the consequence in Harclay's argument]: When he takes as that assumed "a line falls between this first point and any other point of the same line perceived by God," we must reply that this is not what is assumed or conceded. But this has been conceded if "mean line" is placed in the predicate, namely [if we assert]: "between the first point and any other point perceived by God there is a mean line." The other [proposition above] does not follow from this, because of the reason stipulated, as is evident. And thus it is clear that this doctor, howsoever subtle, has cozened by [committing] fallacies of the consequent and of a figure of speech.

(2) To the second argument—when it is argued that God can annihilate the point at the end of a line without corrupting the line or a part of it—I reply that this is not so. For that which only exists positively through the existence of another thing is only annihilated by means of annihilation of the existence of that [other] thing through which it exists. Hence, as a point possesses existence only through the existence of a line, it is only annihilated by the line's annihilation. And when you prove [your point] by breaking a line and in turn joining [the parts so obtained, and argue]: "then [the previous end points of the parts] exist potentially; this accomplished, it is possible that numerically the same point which actually existed before and was the beginning of the right part of the line is now the end of the left part of the line," I reject [the reasoning]. Not only because an actually existent point reduced to that which exists potentially does not return as numerically the same thing but [also because] if it be given that by divine power it were to return as numerically the same thing, it would return as the beginning or end of the same line to which it belonged before and cannot wander from one line to another, which this argument would prove [to be possible] even by means of the ability of a created being to divide [lines].

Furthermore, to the proof: When it is argued "a line can be broken at any one of its points in such a way that any point can indifferently be the beginning of one line or another, even though not of [both] at the same time," I reply: When it is claimed that a line can be broken at any one of its points, one may concede this with respect to any one of its points continuing the line, but one cannot take some [point] immediate to that [point] which existed before. And therefore that [point] which was previously at the end of one line, even though it were to be agreed that it can exist in another line, will nonetheless not be immediate [to the previous end point]; nay, rather there will be a mean line between it and the point which actually existed. Secondly, I say that when it is assumed that "a line can be broken at any one of its points," this is true of any one of its points which remains in the line according to the actuality

18. That is, *X* is inferior relative to *Y* as superior when *X* is related to *Y* as, for example, species to genus. Now, there is a valid consequence when one argues from an inferior to a superior (for instance, A man is bald, therefore an animal is bald), but not from a superior to an inferior (for instance, An animal is bob-tailed, therefore a man is bob-tailed). But Alnwick here claims that Harclay has the term "mean line" in his initial proposition as a superior, and "mean line" (or "line") in that which he *infers* from this proposition as an inferior,which means that he infers an inferior *from* a superior. Yet if inference *from* an inferior *to* a superior is that which is valid (call this, "if *I*, then *S*"), then to argue as Harclay does is to commit the fallacy of affirming the consequent (that is, given "if *I*, then *S*" to claim "*S*, therefore *I*").

19. As we have seen in note 17, if a term has *determinate* supposition, then we can make a logical descent from the proposition in which such a term occurs to a corresponding *disjunctive* proposition. This means that Alnwick is charging Harclay with fallaciously moving from a true proposition from which no disjunctive descent can be made to a false proposition where the corresponding disjunctive descent can be made. For, since in the proposition "There is a mean line between the first point and every other point of the same line" the term "mean line" has *determinate* supposition, one can licitly descend to the disjunctive proposition "there is this mean line or that mean line (and so on for every individual) between the first point and every other point of the same line." But this latter is clearly false.

20. That is, Harclay argues, claims Alnwick, from the same term ("line") functioning as a species (the *quale quid*, or a "what kind of a what") to this same term functioning as singular, to "that line" (the *hoc aliquid*, a "this something"), which is one kind of a "fallacy of a figure of speech," that is, in general, a fallacy involving the use of the same term to cover really different things.

of its presence, just as a part exists remaining in a whole. But a point which actually was before the beginning or end of a line does not remain in a line once again joined and united to another [line] unless it does so in the potency of matter. Therefore, it cannot be broken at that point.

As to the fact that it is also assumed that "any point can indifferently be the beginning of one line or another, even though not of [both] at the same time," we must reply that this is not true. For a point in an undivided line does not remain after the division of the line occurs at that point, since the point which exists in a continuous line would, if the line were divided at that point, have no greater reason to remain in one [resultant] line after division rather than in the other. And therefore it would either remain in both [resultant lines], which is impossible, or in neither, which is true. And therefore that point ceases to be a continuing [point] and withdraws into the potency of matter, and two points actually result each of which existed only in the potency of matter before division. . . .

On Vacuum

53 NATURE ABHORS A VACUUM

1. Albert of Saxony (ca.1316–1390): A Natural Vacuum Denied

Translated and annotated by Edward Grant[1]

QUESTIONS ON THE PHYSICS OF ARISTOTLE, BOOK IV, QUESTION 8

In the eighth [question] we inquire whether the existence of a vacuum is possible. . . .[2]

In this question, some things must be premised and, secondly, conclusions posited.

As to the first [part, namely the assertion of premises], it must be understood that this name "vacuum" is a privative term and valid in signifying only this sentence: "a place not filled by body." Therefore, when "vacuum" is mentioned it is "a place not filled by body." The said definition is posited as it signifies. But it does not follow that because "a vacuum is a place not filled by body," therefore "a vacuum exists. . . ."[3] Secondly, it must be understood that just as "place" is to be imagined [or conceived] in two ways, so also "vacuum" is to be imagined in two ways. Thus some have imagined that "place" is equal to all the dimensions of what is placed [or located] in it; others, however, as Aristotle and his followers, have imagined that place is not such a space but [rather] a body external to the body placed [or located and] equal to it in two dimensions, namely width and length. And "vacuum" must also be imagined in this twofold way: in one way as a separate space in which there is no other body conjointly; in the second way as a body between whose mutually closest sides there is nothing. As for the first way of speaking of a vacuum, it must be conceded that a vacuum would exist within the concave orb of the moon if God were to annihilate all local bodies within the concave orb of the moon;[4] however, it must not be conceded that the heaven or concave orb of the moon is a vacuum, although this would be conceded in the second way declared above. . . .[5]

1. Translated from Albert of Saxony's *Questions on the Physics of Aristotle*, Book IV, Question 8, published in *Questiones et decisiones physicales insignium virorum; . . . tres libros De celo et mundo . . . Thimonis in quatuor libros Meteororum . . . Buridani in librum De sensu et sensato . . . Aristotelis . . . recognitae rursus et emendatae summa accuratione . . . Magistri Georgii Lokert . . .* (Paris, 1518), fols. 48 recto, col. 2, to 48 verso, col. 2.

2. Since he will argue for a highly qualified affirmative position, Albert first presents unqualified affirmative arguments, which he will later repudiate. These have been omitted, and our translation commences with Albert's personal response to the question.

3. Merely because "vacuum" can be defined does not imply that it exists.

4. This constitutes perhaps the most popular illustration in discussions on the existence of vacuum (see Selection 14).

5. At this point, there is a hole or tear in the page of my copy of this book, rendering the next four lines unintelligible.

For the proper comprehension of the remainder of this question, the two ways in which "vacuum" is distinguished must be kept in mind. In the first, it is the emptiness or space *between* certain containers or boundaries. In the second way, however, it is the container itself which is taken to be the vacuum, provided that it is a container between whose sides there is only emptiness. To use Albert's example of the concave orb of the moon after God has annihilated everything within it, it would be the intervening space itself that is taken as the vacuum

As to the second [part of this question], let the first conclusion be that it is impossible for a vacuum to exist, when speaking of a vacuum in the first way.[6] This can be proved, because in the first way a vacuum is a separate dimension and this cannot be posited because a separate accident must not be assumed to exist without a subject.[7] Secondly, if such a separate space were posited, it follows that because it is a body (for it would have length, width, and depth)[8] an interpenetration of bodies would occur when it received something placed in it, which is impossible.[9] Thirdly, if such a vacuum were possible, it would follow that if it existed, the vacuum which it implies would not be a vacuum. The consequence is proved, because when all the locatable things within the [concave] surfaces of the sky have been annihilated, a separate void space would exist there, according to those who hold that place is such a space; hence, according to them, a vacuum exists there. But I prove that it does not exist there because *something* would be there, namely such a separate space.[10]

A second conclusion is that a vacuum is not possible by [the action of] any natural power, when taking "vacuum" in the second way. This is proved by [appeal to] certain experiences. First, if all the openings in a bellows were stopped up, no power could raise one handle *(asserem)* from the other[11] unless a break occurred somewhere through which air could enter. After this had been done, one of the handles could easily be raised [or parted] from the other, for then there would be something that could be received between the sides of the bellows. This [experience] seems to be a sign that nature abhors a vacuum. This same thing could be shown for a clepsydra.

A third conclusion, however, is that, in the second way, a vacuum is possible by means of supernatural power. This is obvious, because God could annihilate everything that is within the sides [or concave surface parts] of the sky, after which the sky would be a vacuum;[12] and in this case, the vacuum would be a good [and proper] circularly mobile thing. But, on the contrary, this is doubted, for having assumed the case just mentioned, either the sides of the sky would be distant [that is, separated] or not. If they are not separated by a distance, they would be conjoined as two leaves of a book or an empty purse; but then it ought not to be conceded that the sky is a vacuum, because in the aforesaid case the sides of the sky would be immediate [that is, in contact]. If, however, it should be stated in this case that the sides of the sky are yet distant [or separated], there would then be some dimension between them by which they would be distant [or separated]; hence there would be no vacuum. To this, one might respond that the sides [or concave surface parts] of the sky are not distant [or separated] by a straight line, although they may well be separated by a curved line. But if it is said that then the sides of the sky would be conjoined, I deny this, for the sky would remain spherical, just as now; and its sides would not be in direct contact, even if not separated by a [rectilinear] distance.[13]

in the first way, whereas in the second way the concave orb itself is the vacuum, for it is the container of that empty space. In the remainder of the question, Albert denies the first way and allows the second.

6. That is, as a separate and continuous space.

7. Although a separate vacuum is treated as a dimension, it is here also dissociated from a body or subject—that is, the dimension of the vacuum is a property or attribute of vacuum, but since vacuum is "nothing," the dimension cannot be said to inhere in anything. Although it was generally assumed that no accident or quality could exist naturally without inhering in a body or subject, it was also conceded that by his absolute power God could create an accident without a subject. To deny this would have meant running afoul of Articles 140 and 141 of the Condemnation of 1277 (see Selection 13).

8. In the preceding note we saw that Albert treated dimension as a quality or attribute without a subject or body in which it inheres. Now, however, for the sake of a new argument, he seems to shift ground and consider the dimension of a vacuum as a body, which would qualify it as a subject. Thus in one argument it serves as a free-floating quality and in the next as a body or subject.

9. This argument is drawn from Aristotle, *Physics* IV.8.216a.27–216b.11.

10. Here again, Albert trades on the "dimension" of a vacuum. For if it has dimension, it is a "something." Hence "vacuum" is a "something" and does not exist in the sense of "nothing."

11. That is, separate the sides of the bellows.

12. Since it would fuaction as the corporeal container of the emptiness that remains after the annihilation.

13. Although this conclusion and its arguments are hardly obvious, Albert does allow that if God annihilated all things within the concave surface of the outermost heaven, a vacuum would exist. Drawing upon the first two conclusions, Albert first formulates a counterargument in which the vacuum is denied under the conditions described and then proposes a solution to the counterargument. In essence, Albert argues that a vacuum of the second kind would exist, since the spherical shape of the celestial container would remain after the annihilation. He agrees that rectilinear distances cannot be measured across, or through, this void, for if they could be measured, this empty space would be a dimension and hence a "something" and not a vacuum; the first kind of vacuum is therefore impossible (see the third part of the first conclusion above). But it is nonetheless

A fourth conclusion is that by assuming the previously mentioned case [where God annihilates everything within the concavity of the sky], one pole of the world would not be near the other, and yet would not be distant [or separated] from it by a rectilinear distance. This is obvious, because distance is nothing but intervening *(intermediam)* dimension. But since, in the case assumed before, there could be no intervening rectilinear dimension, it follows because of this that the poles are not separated by a rectilinear distance; therefore, [one pole is not near the other, although they are not separated by a rectilinear distance].[14]

A fifth conclusion is that the poles may well be distant [or separated] by a curved distance. This is obvious, because a curved dimension would come between them, namely half the circumference of a great circle imagined in the sky. Therefore, and so on [that is, they may well be separated by a curved distance]. . . .

2. John Buridan (ca. 1300–ca. 1358): Experiments Demonstrating that Nature Abhors a Vacuum

Translated and annotated by Edward Grant[15]

Again, every universal proposition in natural science ought to be conceded as a principle which can be proved by experimental induction, just as in many particular [occurrences] something is clearly found to be so and in no instances did it fail to appear. For Aristotle puts it very well [when he says] that many principles must be accepted and known by sense, memory, and experience.[16] Indeed, at some time or other, we could not know that every fire is hot [except in this way].

[Now,] by such experimental induction it appears to us that no place is a vacuum, because everywhere we find some natural body, namely air, or water, or some other [body]. But now let us show by experience that we cannot separate one body from another unless another body intervenes. Thus if all the holes of a bellows were perfectly stopped up so that no air could enter, we could never separate their surfaces. Not even twenty horses could do it if ten were to pull on one side and ten on the other; they would never separate the surfaces of the bellows unless something were forced or pierced through and another body could come between the surfaces.[17]

[Another experiment] is with a [hollow] reed, one end of which you place in wine, and the other end in your mouth. By drawing up the air standing in the reed, you [also] draw up the wine by moving it above [even] though it is heavy. This happens because it is necessary that some body always follow immediately after the air which you draw upward in order to prevent the formation of a vacuum. And thus there are many other experiences of a mathematical kind.[18] We ought, therefore, to concede that a vacuum is not naturally possible, as is [indeed] known by that method [deemed] adequate for assuming and conceding principles in natural science. And by such induction it can be shown that there is no vacuum in any of the two ways mentioned before,[19] since we always see that natural bodies follow one another by touching [or contact] without there being any space between them devoid of a natural body such as air, or water, or some such body.

possible for the second kind of vacuum to exist, because it is defined as the physical container of an empty space, in this instance the outermost sphere itself. Distances between any two points on this spherical physical surface are indeed potentially measurable, albeit only by curvilinear distances traced along the material surface, since, as we have already seen, it is not possible to measure the distances between these points rectilinearly through the void space.

14. This and the fifth conclusion are merely extensions of his argument in the third conclusion (see n. 13).

15. Translated from John Buridan's *Questions on the Eight Books of the Physics of Aristotle*, Book IV, Question 8, fol. 73 verso, col. 1. The full Latin title is cited in Selection 14, n. 1.

16. *Posterior Analytics* II.19.100a.4–9.

17. In 1654 Otto von Guericke performed experiments demonstrating his air pump before the Imperial Diet at Ratisbon. These were subsequently described in Book III of von Guericke's *New (as they are called) Magdeburg Experiments on Void Space (Experimenta Nova (ut vocantur) Magdeburgica de Vacuo Spatio)* (Amsterdam, 1672); a selection from this treatise is given in Selection 73.4. "The most impressive of these was that of the celebrated 'Magdeburg Hemispheres' [Bk. III, ch. 23—*Ed.*]. Two hollow bronze hemispheres were fitted carefully edge to edge, and the interior was evacuated through a stop-cock in one of them which was then closed. A team of eight horses was harnessed to each hemisphere and the two teams were driven in opposite directions, but they were unable to pull the hemisphere asunder so long as the stop-cock was left closed." A. Wolf, *A History of Science, Technology and Philosophy*, 2d ed. (London: George Allen and Unwin, 1950), p. 101. Although no air pump is involved, Buridan's account bears an obvious physical resemblance to von Guericke's experiment. But where one team of horses labored mightily to demonstrate nature's abhorrence of a vacuum, the other labored with equal energy to demonstrate the creation of an artificial vacuum and the powerful pressure of the atmosphere.

18. Does Buridan mean to classify the experiments cited above as "mathematical"?

3. Marsilius of Inghen (ca. 1340–1396): Experiments Demonstrating that Nature Abhors a Vacuum

Translated and annotated by Edward Grant[20]

. . .In the second place, one may declare against the vacuum by experiments given in the treatise *On Emptiness and Void (De inani et vacuo)*,[21] since natural bodies are moved against their natural inclinations lest a vacuum occur.[22] The first experiment *(experientia)* is that of water rising to extinguish a candle covered by a vessel.[23]

A second experiment is this: If a vessel is made with two arms, one longer than the other, and the shorter is assumed to be in water, then if air is withdrawn through the longer arm the water rises through the shorter arm which it would not do unless it were preventing a vacuum.[24]

A third experiment is that a heavy body existing upwards without any obstacle [to its descent] does not descend because a vacuum might occur. Therefore, a vacuum is not possible in lower things by means of a natural agent. The antecedent is proved, because if a vessel having many small openings below and one large opening above was filled with water and the opening above was obstructed, the water would not descend through the lower openings because there would not be any other means of preventing a vacuum.[25]

Fourthly, we see that light elements, which descend into the depths of cisterns, descend downward. Here is an argument against this conclusion: Let there be a body that is everywhere concave within. . .and let this body be filled with air. This

19. A reference to an earlier section of the same question (fol. 73 recto, col. 2). The first way assumes that a vacuum is an entity or corporeal dimension having length, width, and depth. When this dimension is filled with a natural body it becomes a full space; when not occupied by any natural body it is a void place. A second kind of void is not itself an entity or dimension but can be imagined when a body occupying a place is annihilated and the place retains its shape or figure. There would be "nothing" in that place, where place is understood as the innermost surface of the containing body. Buridan's two-fold conception of vacuum is much like Albert of Saxony's in the preceding section.

20. I have translated from Book IV, Question 13 ("Whether it is possible that a vacuum exist"), of Marsilius' *Questions on the Eight Books of the Physics* as published in *Questiones subtilissime Johannis Marcilii Inguen (sic) super octo libros Physicorum secundum nominalium viam*. . . (Lyon, 1518); reprinted as *Johannes Marsilius von Inghen Kommentar zur Aristotelischen Physik* (Frankfort: Minerva, 1964), fol. 55 verso, col. 1.

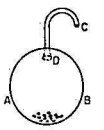
21. The identity of this treatise is uncertain. Perhaps it is a version of the *De ingeniis spiritualibus (Pneumatics)* of Philo of Byzantium (ca. third century B.C.), which exists in a medieval Latin translation. Another candidate is the *Pneumatics* of Hero of Alexandria (first century A.D.), which is somewhat similar to Philo's treatise and of which there appears to have been a fragmentary Latin version available in the Middle Ages. Or perhaps the *Tractatus de inani et vacuo* was a compilation from Arabic sources, as Pierre Duhem believed.

22. In this statement and the four "experiments" that follow, one sees the extremes to which medieval authors would go in order to deny a vacuum. They would readily sacrifice the fundamental Aristotelian principle that a heavy body (for instance, water) must descend in a lighter medium (for instance, air).

23. In the Latin text published by Wilhelm Schmidt (*Heronis Alexandrini Opera*. . . , Vol. I: *Pneumatica et Automata*. . . *accedunt*. . . *Philonis De ingeniis spiritualibus* [Leipzig, 1899]) this experiment is described in section VIII on page 476. It is translated in M. R. Cohen and I. E. Drabkin, *A Source Book in Greek Science* (Cambridge, Mass.: Harvard University Press, 1948), p. 256.

24. A similar experiment appears in section IX of Philo's *Pneumatics* (p. 478 of Schmidt's edition).

25. This effect is produced in a clepsydra and is described in the *Problems* XVI.8 attributed to Aristotle but probably compiled by his followers. It was apparently



a device for transferring liquids and "consists of a globular vessel [see fig.—*Ed.*] with small holes at the bottom and a pipe handle open at the top. When the clepsydra is placed in a vessel containing liquid, this liquid enters the clepsydra through the holes in the bottom. Then, if the hole at the top is covered with the thumb, the clepsydra may be removed without the loss of the liquid in it. When the thumb is taken from the opening, the liquid flows out of the holes at the bottom" (Cohen and Drabkin, pp. 245–246, n. 3). The same illustration appears in section XI of Philo's *Pneumatics* (pp. 480, 482 of Schmidt's edition), in Hero's *Pneumatica* I.7 (see Cohen and Drabkin, pp. 326–327; the figure has been reproduced from p. 327), and is also described by Nicholas of Autrecourt (see Selection 56.2).

body is then filled with air and placed in water or in something that is made intensely cold so that the air contained in the vessel will be condensed, since condensation follows upon coldness. Hence the air will occupy a smaller place than before, and as a consequence, there will be a vacuum. [But to this] it must be replied that it is impossible that the air in this body be condensed by coldness, however cold it may be, unless the vessel is broken or there is an opening in it through which another body enters it.

4. Galileo Galilei (1564–1642): Experiments Demonstrating that Nature Abhors a Vacuum

Translated by Henry Crew and Alfonso de Salvio[26]
Annotated by Edward Grant

[59]

Salv. A truly ingenious device![27] I feel, however, that for a complete explanation other considerations might well enter; yet I must not now digress upon this particular topic since you are waiting to hear what I think about the breaking strength of other materials which, unlike ropes and most woods, do not show a filamentous structure. The coherence of these bodies is, in my estimation, produced by other causes which may be grouped under two heads. One is that much-talked-of repugnance which nature exhibits towards a vacuum; but this horror of a vacuum not being sufficient, it is necessary to introduce another cause in the form of a gluey or viscous substance which binds firmly together the component parts of the body.[28]

First I shall speak of the vacuum, demonstrating by definite experiment the quality and quantity of its force [*virtù*]. If you take two highly polished and smooth plates of marble, metal, or glass and place them face to face, one will slide over the other with the greatest ease, showing conclusively that there is nothing of a viscous nature between them. But when you attempt to separate them and keep them at a constant distance apart, you find the plates exhibit such a repugnance to separation that the upper one will carry the lower one with it and keep it lifted indefinitely, even when the latter is big and heavy.[29]

This experiment shows the aversion of nature for empty space, even during the brief moment required for the outside air to rush in and fill up the region between the two plates. It is also observed that if two plates are not thoroughly polished, their contact is imperfect so that when you attempt to separate them slowly the only resistance offered is that of weight; if however, the pull be sudden, then the lower plate rises, but quickly falls back, having followed the upper plate only for that very short interval of time required for the expansion of the small amount of air remaining between the plates, in consequence of their not fitting, and for the entrance of the surrounding air. This resistance which is exhibited between the two plates is doubtless likewise present between the parts of a solid, and enters, at least in part, as a concomitant cause of their coherence.

[60]

Sagr. Allow me to interrupt you for a moment, please; for I want to speak of something which just occurs to me, namely, when I see how the lower plate follows the upper one and how rapidly it is lifted, I feel sure that, contrary to the opinion of many philosophers, including perhaps even Aristotle himself, motion in a vacuum is not instantaneous. If this were so the two plates mentioned above would separate without any resistance whatever, seeing that the same instant of time would suffice for their separation and for the surrounding medium to rush in and fill the vacuum between them. The fact that the lower plate follows the upper one allows us to infer, not only that motion in a vacuum is not instantaneous,[30] but also that, between the two plates, a vacuum really exists, at least for a very short time, sufficient to allow the surrounding medium to rush in and fill the vacuum; for if there were no vacuum there would be no need of any motion in the medium. One must admit then that a vacuum is sometimes produced by violent motion [*violenza*] or contrary to the laws of nature, (although in my opinion nothing occurs contrary to nature except the impossible, and that never occurs).

26. Reprinted with the kind permission of the Macmillan Company from the First Day of the *Dialogues Concerning Two New Sciences by Galileo Galilei*, translated from the Italian and Latin into English by Henry Crew and Alfonso de Salvio with an Introduction by Antonio Favaro (New York: Macmillan, 1914), pp. 11–12. As we shall see here and in another brief selection (Selection 56.3) Galileo did accept the existence of interstitial vacua but nevertheless maintained the traditional viewpoint that nature abhorred a separate vacuum and sought as quickly as possible to occupy it with matter. Not until Pascal explained the role of atmospheric pressure was this long-held opinion emphatically denied (for Pascal's exposition, see next selection).

27. A reference to a device for descending from a window at a controlled rate, described by Sagredo, one of the interlocutors in the dialogue.

28. Later, Galileo identifies this other cause as interstitial vacua, which he suggests function as a binding or attractive force *(forza del vacuo)* holding together the particles of a solid body (see Selection 56.3).

29. The opinions of Salviati and Sagredo represent Galileo's viewpoint on this subject. Admitting a momentary vacuum between the plates (see the last two paragraphs) represents a radically different response than was given in a typically medieval exposition by Roger Bacon to explain much the same phenomena.

After demonstrating the impossibility of void by rational argument, Bacon offers a series of supporting illustrations and experiments, the first of which is basically that presented by Galileo. "Let two plane tables be taken and joined together so that nothing lies between them. If one were raised above the other, the air would enter between the tables, first reaching the extreme parts, and then the central point. But in this raised position, and before the air reaches the central point, a void will exist there." In order to deny the formation of a void, Bacon says that one might suppose that when the tables are parted, air reaches all parts of the tables instantaneously. Finding this repugnant, he proposes another solution. "I reply to this by saying that the two round tables could not be raised if one were above the other unless there should be an inclination of some part. Hence it would be necessary that some part be inclined before it could be raised, for otherwise a void would be produced. This applies to all of nature. . . . It is evident, then, that anyone who wishes to raise [anything] must incline one part beforehand. . . . For this reason a vacuum should not be assumed." (My translation is from Bacon's *Questions on the Eight Books of the Physics of Aristotle*, Book IV ("Whether a vacuum can be assumed below the heavens"), in *Opera hactenus inedita Rogeri Baconi*, Fasc. XIII: *Questiones supra libros octo Physicorum Aristotelis*, edited by F. M. Delorme, O.F.M., with the aid of Robert Steele [Oxford: Clarendon Press, 1935], pp. 225–226.)

30. Some scholastics argued that motion in a hypothetical vacuum would not be instantaneous (Selection 47, n. 6, and Selection 55.1). Of great interest here is the fact that Galileo holds that an actual vacuum would exist momentarily before nature rushed to fill it.

54 NATURE DOES NOT ABHOR A VACUUM

Blaise Pascal (1623–1662)

Translated by I. H. B. Spiers and A. G. H. Spiers[1]
Annotated by Edward Grant

CONCLUSION OF THE TWO PRECEDING TREATISES

I have recorded in the preceding Treatise all the general effects which have been heretofore ascribed to nature's effort to avoid a vacuum, and have shown that it is utterly wrong to attribute them to that imaginary cause. I have demonstrated, on the contrary, by absolutely convincing arguments and experiments that the weight of the mass of the air is their real and only cause. Consequently, it is now certain that nature nowhere produces any effects in order to avoid a vacuum.

It is not difficult to demonstrate, furthermore, that nature does not abhor a vacuum at all. This manner of speaking is improper, since created nature, which is the nature under consideration, is

1. Reprinted with the kind permission of the Columbia University Press from the *Physical Treatises of Pascal: The Equilibrium of Liquids and the Weight of the Mass of the Air*, translated by I. H. B. Spiers and A. G. H. Spiers, with Introduction and Notes by Frederick Barry (New York: Columbia University Press, 1937), pp. 67–75. The present selection, translated from the original French, constitutes the complete conclusion in a volume by Pascal edited and published in 1663, one year after his death, by his brother-in-law F. Perier. The two treatises in the volume, referred to in the heading of the conclusion, are titled "On the Equilibrium of Liquids" and "On the Weight of the Mass of Air." As Frederick Barry remarks in a foreword (pp. v–vi), "to him we owe the first conclusive proof of the pressure of the atmosphere, and the final banishment from the minds of natural philosophers of that ancient and persistent conception of *horror vacui* which had long inhibited investigation in this field; a complete mechanical correlation of all the diverse phenomena of fluid equilibrium, without recourse to that or any other imaginary conception; and finally, the establishment of detailed analogies between the effects of pressure in liquids and in air which affected the unification of hydrostatics and aerostatics as one deductively organized and logically coherent discipline." In the preceding chapter of this source book the selections from John Buridan and Marsilius of Inghen typically represent the medieval conception of "abhorrence of a vacuum." Even Nicholas of Autrecourt, who argued for minute, interstitial vacua, reaffirmed nature's abhorrence of a separate or extended vacuum. Pascal's exposition is so simple and clear that it requires little comment.

not animated, and can have no passions. Such language is in fact metaphorical, and means nothing more than that nature makes the same efforts to avoid a vacuum as if she abhorred it. Those who use this phrase mean that it is the same thing to say that nature abhors a vacuum as to say that nature makes great efforts to prevent a vacuum. Now, since I have shown that nature does nothing at all to avoid a vacuum, the conclusion is that nature does not abhor it. To carry out the metaphor: just as we say of a man that a thing is indifferent to him when his actions never betray any movement of desire for, or of aversion to, that thing, so should we say of nature that it is supremely indifferent to a vacuum, since it never does anything either to seek or to avoid it. (I am here still using the word "vacuum" to mean a space empty of all bodies which our senses can apprehend.)[2]

It is perfectly true (and this is what misled the ancients) that water rises in a pump when the air has no access to it, that a vacuum would result if the water did not follow the piston, and that the water ceases to rise as soon as any cracks develop by which the air can get in to fill the pump. Thus it looks as though the water rose merely for the purpose of preventing a vacuum, since it rises only when otherwise there would be one.

Similarly it is a fact that a bellows is hard to open when its apertures are so carefully sealed that no air can enter it; and it is true that its opening would produce a vacuum. This resistance ceases when air can enter to fill the bellows, and since it is met with only when a vacuum would otherwise result, it seems to be due to nothing else than the fear of a vacuum.[3]

Finally, it is a fact that all bodies in general make great efforts to follow one another and to keep together whenever their separation, and nothing else, would produce a vacuum between them. This is why it has been inferred that this close adhesion is due to the fear of a vacuum.

To reveal the weakness of this reasoning the following example will serve. When a bellows is placed in water in the manner we have often described, with its nozzle at the end of a tube assumed to be twenty feet long which projects out of the water into the air, and with all its side apertures sealed so as to exclude the air, everyone knows that it is hard to open, and the more so the greater the amount of water above it; whereas if the vents in one of the wings are unsealed so that they admit the water freely, the resistance disappears.

If one wished to reason out this effect like the others, he might say: When the side vents are closed and when, therefore, if the bellows is to be opened, the air must enter through the tube, there is difficulty in opening it; but when water instead of air can enter to fill it, the resistance ceases. Therefore, since there is resistance only to the entrance of air, the resistance arises from an abhorrence of the air.

There is no one who will not laugh at this inference, seeing there may well be another cause of the resistance. It is evident indeed that the bellows cannot be opened without raising the water, because the water that would be pushed aside in the act of opening cannot enter the body of the bellows, and, compelled to find room for itself elsewhere, raises the whole body of the water and causes the resistance. This does not occur when the bellows has vents through which the water can enter; for whether it is opened or shut the water neither rises nor falls in consequence, since water enters the bellows just as fast as it is pushed aside, and thus offers no resistance to its opening. All this is clear, and consequently we must believe that the bellows cannot be opened without two results: first, the air does really enter, or second, the level of the water is raised. It is the latter action that causes the resistance and with this the former has nothing to do, although it occurs simultaneously.

Let us give the same explanation for the difficulty experienced in opening in the air a bellows sealed on all sides. If it were forced open two things would occur: first, a vacuum would really be formed; second, the whole mass of air would be raised and upheld. It is the latter action that causes the resistance felt; the former has nothing to do with it. This resistance also increases or diminishes in proportion to the weight of the air, as I have shown.

The same facts explain the resistance to separation offered by all bodies between which there is a vacuum: air cannot filter in, otherwise there would be no vacuum, and this being so, they cannot be separated except by raising and upholding the

2. The usual assumption made by Aristotle and his followers was that a vacuum, by definition, was absolutely devoid of matter. However, in discussing imaginary void space beyond the cosmos it was assumed that, although devoid of corporeal entities, it was wholly occupied by God or spirit (see Selection 73, introduction). Pascal, here, seems to allow for the possibility that nonperceptible, or non-sensible, matter might, in some way, occupy, or partially occupy, a vacuum.

3. Compare to Buridan's discussion of the bellows in the preceding selection.

whole mass of the air. It is this which occasions the resistance.

Such then is the real cause of the adhesion of bodies between which there exists a possible vacuum. It was for a long time not understood because erroneous opinions were entertained which were discarded only by degrees. There have been three different periods during which different opinions of this character were held; and these involved three generally prevailing errors which made it absolutely impossible to understand the cause of this adhesion of bodies.

The first is that, in nearly all times, it was believed that the air has no weight; for the ancients said so,[4] and their professed disciples followed them blindly. They would have remained forever wedded to that theory had not keener thinkers rescued them by the force of experimental evidence;[5] for it was impossible to believe that the weight of the air causes such adherence, so long as it was held that air had no weight.

The second error lay in the belief that the elements have no weight in themselves,[6] for the sole reason that the weight of water is not felt by those who are in it, that a bucket full of water immersed in the water is easy to lift so long as it stays there, and that its weight begins to be felt only when it is lifted out. As if these effects could not be due to another cause—or rather, as if this one was not wholly beyond all probability! For there is no sense in believing that water in a bucket has weight when out of the water, but has no weight left after it is poured back into the well; that it loses its weight when mixing with the rest, and recovers its weight when lifted above the surface. Strange are the means men employ in order to cloak their ignorance! Because they could not understand why the weight of water is not felt, and were loath to confess their ignorance, they declared it had no weight, for the satisfaction of their vanity and to the ruin of truth. Their views prevailed; and, of course, the weight of the air could not be accepted as the cause of these effects so long as this vain imagining had currency. Even had it been known that air has weight, the claim would still have been made that it has no weight when contained within itself, and consequently the belief would have persisted that it can effect nothing by its weight. That is why I have shown in the Equilibrium of Liquids that water weighs the same within itself as outside, and I have explained there why, in spite of that weight, a bucket is not hard to raise while it is in the water and its weight is scarcely felt.

And in the Treatise on the Weight of the Mass of

4. On the basis of an experiment in which an inflated and uninflated bladder were found to be of equal weight, the Greek commentator, Simplicius, in his *Commentary on Aristotle's De caelo*, concluded that air has no weight in air but apparently believed that it would have weight in the natural place of fire. Aristotle (*De caelo*, 311b.6–10), however, on the basis of the same experiment insisted that an inflated bladder weighs more than an empty one and concluded that air would have weight in its own natural place as well as in the natural place of fire. The measurements were too delicate for the instruments available and the issue could not be resolved in this manner. See M. R. Cohen and I. E. Drabkin, *A Source Book in Greek Science* (Cambridge, Mass.: Harvard University Press, 1948), pp. 247–248.

5. Frederick Barry observes (pp. 27–28, n.3): "The famous Milanese mathematician and philosopher of nature, Girolamo Cardano (1501–1576) appears to have been the first to demonstrate that air has weight and to attempt a determination of its specific gravity. After him, and before it was known how to produce a vacuum, many experiments were made to establish his principal conclusion as indubitable fact. The best of these were carried out by Galileo and described in his *Discorsi e dimonstrazioni mathematiche* (1638), in the dialogue of the First Day. With a syringe he forced into a properly valved bottle, previously weighed, an amount of air which at ordinary pressure would have been of two or three times its volume, and determined with precision an actual increase of weight; then he allowed the excess air to escape without loss through a tube which led into another bottle completely filled with water and so constructed that the water displaced by the entering air could be caught in a suitable weighed vessel, the increase in the weight of which, finally, he determined. Galileo remarked that if air were specifically light, the first flask when filled with condensed air should be lighter than before, not heavier; that air consequently had weight (as, indeed, previous experience had already proved); and that since the volume of the water displaced was that of the excess air introduced, in its normal condition, the ratio of the determined weight of this air to that of the displaced water was the specific gravity sought. In a second, more elegant experiment, he forced water, instead of air, into the valved flask first used and thus compressed the air within it; weighed it in this condition; then opened the valve, thus allowing to escape a volume of air which, in its normal condition, had previously occupied the space now filled by the water; and finally weighed again. In this case, the increase in weight after the water had been introduced was the weight of this water, the subsequent loss in weight being that of the same volume of air in its normal condition. See the English translation of the *Discorsi* by Henry Crew and Alfonso de Salvio, *Dialogues Concerning Two New Sciences by Galileo Galilei* (New York: Macmillan, 1914)."

6. Pascal is in error here, for Aristotle insisted that earth, water, and air have weight in their own natural places (*De caelo*, 311b.6–10; fire is an exception, for it was deemed to be weightless or absolutely light). See note 4.

the Air I have given the same demonstration in the case of the air, to clear up all doubts.

The third error is different in kind. It does not appear in connection with [the weight of] the air, but in connection with the effects which were ascribed to the abhorrence of a vacuum. Concerning these the most erroneous theories were entertained. For it had been imagined that a pump raises water not only to ten or twenty feet, which is true enough, but still farther—to a height of fifty, one hundred, or one thousand feet, or as high as you please, without limit. Likewise the belief was held that it is not only difficult, but actually impossible, to separate two polished bodies in close contact;[7] that not even an angel, or any created force, could do so, with hundreds of exaggerations which I scorn to repeat. And so with the rest.

This is an error of observation so ancient that it cannot be traced back to its source. Heron himself, who is one of the oldest and best of the authors who have written on the raising of water, states as a positive and uncontrovertible fact that the water of a river may be made to pass over a mountain ridge and to flow into the valley beyond, provided this valley be somewhat lower down, by means of a siphon placed on the summit with its legs stretching along the slopes, one into the river and the other on the farther side; and he asserts that the water will rise from the river over the mountain and drop down again into the other valley, however high the ridge between may be.[8]

All writers on the subject have said the same thing; and even at the present time our fountain builders guarantee that they can make suction pumps which will raise water as much as sixty feet if it be desired.

Neither Heron, nor those other writers, nor the artisans, and still less the natural philosophers, can have carried their tests very far; for had they tried to draw water to the height of forty feet, they would have failed. They had only seen suction pumps and siphons six, ten, or twelve feet high, which worked beautifully; and in all the experiments they had occasion to make, had observed no case in which water failed to rise. They never imagined, consequently, that there was a limit beyond which water behaved otherwise. They conceived that the facts they had noticed were the results of an invariable natural necessity; and since they believed that water rose by an invincible abhorrence of a vacuum, they concluded that as it rose at first, so it would continue to rise without limit, applying their interpretation of what they did observe to what they did not observe and declaring both statements to be equally true.

So positively was this believed that philosophers have made it one of the most general principles of their science and the foundation of the treatises on the vacuum. It is and has been didactically asserted every day in all the schoolrooms in the world, ever since books were written. Everyone has firmly believed it, and it has remained uncontradicted down to our own time.

This fact perhaps may open the eyes of those who dare not doubt an opinion which has always been universally entertained; for simple workmen have been able to prove in this instance that all the great men we call philosophers were wrong. Galileo declares in his dialogues[9] that Italian plumbers taught him that water rises in pumps only to a certain height: whereupon he himself confirmed the statement as others did also, afterward, first in Italy and later in France, by using quicksilver, which is easier to handle but provides merely several other ways of making the same demonstration.

Before men gained that knowledge, there was no incentive to prove that the weight of the air was the cause of water rising in pumps; since, the weight of the air being limited, it could not produce an unlimited effect.

But all these experiments were insufficient to show that the air does produce those effects: they had rid us of one error but left us in another. They taught us, to be sure, that water rises only to a certain height, but they did not teach us that it rises higher in low-lying places. On the contrary, the belief was held that it always rises to the same height, in every place on the earth. And since the weight of the air never entered anybody's head, it was vaguely thought that the nature of the pump

7. The problem of two plane surfaces in contact was discussed by Galileo (see Selection 53.4 and n. 29 thereto).

8. Barry remarks (pp. 71–72, n. 3) that the modern Greek edition of Hero's *Pneumatica* and the editions available to Pascal contain no such explicit statement and that Pascal either misread Hero or relied on hearsay. "On the other hand, Heron, in discussing the siphon. . . makes no mention of any limitation to its efficacy; neither does Cardan (*De subtilitate*, 1560: I.3.364a); and Galileo's discussion of pumps in the *Mathematical Discourses* of 1638. . . implies that it was then still taken for granted by the philosophers, if not by all workmen, that a siphon of any height would operate."

9. See the First Day of Galileo's *Dialogues Concerning Two New Sciences* in the translation of Crew and de Salvio, pp. 16–17.

was such that it lifted water to a limited height and no further. Indeed Galileo took that to be the natural height of a pump, and called it *la altessa limitatissima*. How indeed could it have been imagined that that height was different in different places? Certainly, it would seem improbable. Yet that last error again put out of the question the proof that the weight of the air causes these effects; since, because this weight would be greater at the foot than at the top of a mountain, its effects, obviously, would be proportionately greater there.[10]

That is why I decided that the proof could be obtained only by experimenting in two places, one some four or five hundred fathoms above the other. I chose for my purpose the Puy de Dôme mountain in Auvergne, for the reasons that I have set forth in a little paper which I printed as early as the year 1648, immediately after the experiment had proved successful.[11]

This experiment revealed that fact that water rises in pumps to very different heights, according to the variation of altitudes and weathers, but is always in proportion to the weight of the air. It perfected our knowledge of these effects, and put an end to all doubting; it showed their real cause, which was not abhorrence of a vacuum, and shed on the subject all the light that could be wished for.

Try now to explain otherwise than by the weight of the air why suction pumps do not raise water so high by one-quarter on the top of Puy de Dôme in Auvergne as at Dieppe; why the same siphon lifts water and draws it over at Dieppe and not at Paris; why two polished bodies in close contact are easier to separate on a steeple than on the street level; why a completely sealed bellows is easier to open on a house-top than in the yard below; why, when the air is more heavily charged with vapors, the piston of a syringe is harder to withdraw;[12] and lastly why all these effects are invariably proportional to the weight of the air, as effects are to their cause.

Does nature abhor a vacuum more in the highlands than in the lowlands? In damp weather more than in fine? Is not its abhorrence the same on a steeple, in an attic, and in the yard? Let all the disciples of Aristotle collect the profoundest writings of their master and of his commentators in order to account for these things by abhorrence of a vacuum if they can. If they cannot, let them learn that experiment is the true master that one must follow in Physics; that the experiment made on mountains has overthrown the universal belief in nature's abhorrence of a vacuum, and given the world the knowledge, never more to be lost, that nature has no abhorrence of a vacuum, nor does anything to avoid it; and that the weight of the mass of the air is the true cause of all the effects hitherto ascribed to that imaginary cause.

10. Barry supplies the following note (pp. 73–74. n, 5): "Pascal here refers, in the first place, to ideas current up to the time of Galileo, which were consistent with the theory of a limitless *horror vacui;* then to Galileo's observations on the pump which, though they permitted the retention of the general notion of a *horror vacui*, nevertheless proved that it was a finite and measurable force. But Galileo did not dogmatically assert, as Pascal implies, that it was an invariable force: it was contrary to his habit of thought, in the absence of experimental evidence, to do anything of the sort; indeed, for lack of sufficient knowledge, he left the phenomenon unexplained. Pascal's omission of all reference to the several physicists who, after Galileo, quite definitely conceived that this force was external and due in fact to atmospheric pressure—among whom were Mersenne and Descartes, whose ideas were almost certainly known to him—and particulary his failure to mention Torricelli, who verified this theory, leaves a false impression too favorable to himself. It was not his work that invalidated the conception of *horror vacui:* and his contradictory hypotheses were not only suggested but were almost completely verified by his predecessors. It is sufficient honor to him that he so brilliantly completed their work.

11. Perier included this paper in a later part of the same volume which contained the conclusion reproduced here. It describes Pascal's most famous experiment.

12. In an earlier note (pp. 51–52, n. 1) Barry explains, "although the density of moist air is less than that of dry air under the same conditions, barometric pressures at the same altitude and like temperatures are usually greater when the air is moist; primarily on account of its greater compressibility at ordinary temperatures in this condition, since this may occasion an increase of density that overcompensates, at the moderate altitudes where measurements are usually made, the decrease due to the mere presence of water vapor—especially when, as is usual, the upper air is dry. Perier observed this effect in a long series of observations made with the mercury barometer at one station in 1649–51. . .; and Pascal, in his experiment with the bellows. . . not only noted it, but made it the basis of several acute meteorological inferences. . . . In 1753 Bouguer remarked especially the strikingly greater difference of pressure between two fixed altitudes when the air is moist—an informing fact which had previously been missed by Perier for lack of a sufficient number of observations; but it was not until 1784 that Deluc, as the result of a long series of measurements made under widely variable conditions and carefully compared, finally explained the phenomenon satisfactorily. See Deluc: *Recherches sur les modifications de l'atmosphere*, nouvelle ed. 1784, III, 715, 716.

55 MOTION IN A HYPOTHETICAL VOID

1. St. Thomas Aquinas (ca.1225–1274): A Kinematic Argument for Finite Motion in a Hypothetical Void

Translated and annotated by Edward Grant[1]

But several difficulties arise against the opinion of Aristotle. The first of these is that it does not seem to follow that if a motion occurs in a void it would bear no ratio in speed to a motion made in a plenum.[2] Indeed, any motion has a definite velocity [arising] from a ratio of motive power to mobile—even if there should be no resistance. This is obvious by example and reason. An example is that of the celestial bodies, whose motions are not impeded by anything, and yet they have a definite speed in a definite time.[3] An appeal to reason is this: Just as there is a prior and posterior part in a magnitude traversed by a motion so also we understand that in the motion [itself] there is prior and posterior. From this it follows that motion takes place in a definite time.[4] But it is true that in virtue of some impediment [or resisting medium] something could be subtracted from this speed. It is not necessary, therefore, that a ratio of speeds be related as a ratio of resistance to resistance, for then, if there were no resistance, motion would occur instantaneously. But it is necessary that the ratio of retardation to retardation be as the ratio of resistant medium to resistant medium.[5] Thus if motion in a void were assumed, it follows that no retardation would occur beyond the natural velocity,[6] and it does not follow that motion in a void would bear no ratio to motion in a plenum.

1. My translation was made from St. Thomsa' *Commentary on the Eight Books of the Physics of Aristotle (Commentaria in octo libros Physicorum Aristotelis)*, Book IV, Lectio 12, in Vol. II of *Sancti Thomae Aquinatis . . . opera omnia* iussu impensaque Leonis XIII, P. M. edita (Rome, 1884), p. 186, col. 1. It first appeared in my article "Motion in the Void and the Principle of Inertia in the Middle Ages," *Isis*, Vol. LV, Part 3, No. 181 (1964), p. 269. The notes have been written especially for this source book. This brief selection is Thomas' comment on Text 71 of Book IV of Aristotle's *Physics*, and it is obvious that he was thoroughly familiar with the disagreement between Avempace and Averroes as set forth in Averroes' comment on Text 71 (see Selection 44). Without mentioning either Avempace or Averroes, Thomas emphatically sides with the former against Averroes and Aristotle.

2. A reference to Aristotle's argument in *Physics* IV.8.215b.12–216a.7.

3. This is one of Avempace's principal arguments (see Selection 44.2).

4. Later authors named this argument "the incompossibility [or incompatibility] of the termini" *(incompossibilitas terminorum* or *distantia terminorum)*, and some even designated as a resistance the distance traversed in a finite time (see Albert of Saxony's discussion of this in Selection 47 and n. 6 thereto). Perhaps St. Thomas was influenced in the formulation of this particular kinematic proposition by Aristotle's report in *De sensu* (446a.26–446b.3) of Empedocles' argument for the finite velocity of light. For just as Thomas denies instantaneous velocity in the void because intervening parts of a distance must be traversed before the *terminus ad quem* is reached, so Empedocles says that "light from the sun arrives first in the intervening space before it comes to the eye, or reaches the earth." In response, Aristotle concedes, "This might plausibly seem to be the case. For whatever is moved [in space] is moved from one place to another; hence there must be a corresponding interval of time also in which it is moved from the one place to the other." But Aristotle immediately rejects Empedocles' argument because "any given time is divisible into parts; so that we should assume a time when the sun's ray was not as yet seen, but was still traveling in the middle space." Since we cannot physically distinguish the sun's rays in the prior parts of its path from the posterior parts, Aristotle concludes that the transmission of light is instantaneous. However, Thomas, who commented on the *De sensu* and obviously knew Empedocles' argument, may have been sufficiently impressed to extend its application against another claim for instantaneous motion—this time in void space. A similar argument in favor of the successive and finite motion of light (and other bodies) in a void was given by Roger Bacon (see Selection 62.32; for yet another instance, see Selection 56.2 and n. 29 thereto).

5. Thus Aquinas rejects Averroes' opinion that $V/V' = R/R'$, where V is velocity and R is the *density* of the resistant media; or $V/V' = R/R'$, where R is the *rarity* of the resistant media (see Selection 44.2, n. 7). In place of it, he adopts Avempace's formulation that $r/r' = R/R'$ (here r is the retardation of motion resulting from the resistance of the density of medium R, and $r = V' - V$, where V' is the natural speed of a body in a resistanceless medium and V is its speed in a resistant medium (see above, Selection 44.2, n. 26).

6. For a brief discussion of this statement, see Selection 47 and n. 7 thereto. It may have prompted others to explain the finite velocity of a body in void space as the direct consequence of a natural limitation—a limitation that would sometimes be called a resistance—of the falling body.

Elsewhere Aquinas argued that while external forces were necessary in violent motion, they were superfluous for natural motion. In rejecting Averroes' claim that a separate motive force is necessary in natural motion, he insists that "movement of heavy and light bodies does not come from the generator by the intervention of another moving power. Nor even is there any need to look for resistance here other than that which exists between generator and generated. Thus it follows that air is not required for natural motion of necessity, as in the case of violent motion, since that which moves naturally has a force *(virtutem)* imparted to it which is a source of motion. Consequently there is no need for a body to be moved by any other force impelling it, as though it were a case of violent motion, having no implanted force from which motion springs." (This paragraph is repeated from note 31 to Selection 46).

2. Albert of Saxony (ca. 1316–1390): Dynamic Arguments Justifying Motion in a Hypothetical Void

Translated and annotated by Edward Grant[7]

BOOK IV, QUESTION 11

In the eleventh [question] we now inquire whether if a vacuum did exist, a heavy body could move in it.

And it is argued yes, because if a vacuum did exist and a heavy mixed [or compound] body *(mixtum)* were assumed in it, it would descend successively because of the internal resistance[8] it has. Secondly, every body that is outside its natural place is moved, or begins to be moved, toward it. Hence if any heavy body were assumed to be in a vacuum and outside its natural place, it would be moved toward it. Consequently, if a heavy body were moved in a vacuum—if there were a vacuum —it would move to its natural place if unimpeded.

Thirdly, everything that seeks some end [or goal] follows it unless impeded. But when a heavy body is outside its natural place, it seeks to be in it as an end. Therefore, it is moved to it if unimpeded, since it cannot pursue its end except by motion.

Fourthly, a heavy body is moved through air more quickly than through water because air is subtle and resists less than water, and since in a vacuum there would be absolutely no resistance, it seems that a heavy body ought to be moved most easily *(optime)* in a vacuum. Fifthly, if a heavy body were not moved in a vacuum—assuming a vacuum existed—it follows that this would be solely because of a lack of resistance. But this cannot be, since any luminous body could easily illuminate any space successively. This is obvious with respect to a luminous body moved locally, for nothing resists it. It also seems, therefore, that a simple heavy or light body could be moved in a vacuum without any resistance to it; similarly, the sky is moved successively and yet the sky offers no resistance to the mover, namely the Intelligence.[9] Therefore, and so on.

Aristotle seems to take the opposite position, arguing that if a vacuum existed, a heavy simple body could not be moved in it because there would be no reason why it should be moved to one different [place] than to another.[10] Furthermore, even if a vacuum existed, a heavy body assumed in it would be moved in an instant.[11] But that motion be made in an instant implies a contradiction. To this question we must first respond as it is expressed *(sicut verba sonant)*, and [then], secondly, by reforming the question we must respond to the conception [and purpose] of those who ask it.

As for the first [part, concerning the question as expressed], let this be the first conclusion: This [proposition] is false: if a vacuum existed, a heavy body could be moved in it. [The falsity of] it is proved because it is a conditional proposition whose antecedent can be true when the consequent is false; consequently, it is false. The antecedent is obvious, [for] if by the power of God the whole which lies between the sides of the sky were annihilated and no heavy body existed, then this [that is, the antecedent] is true: a vacuum exists; but yet it is false [to assert] that a heavy body is moved in it.

A second conclusion [is this]: Similarly this [proposition] is false: if a vacuum existed, a heavy body could not be moved in it. For it is also obvious that this is a conditional [statement] whose antecedent can be true when the consequent is false. Nor do such conditional propositions contradict [each other].[12]

7. This is virtually the whole of Book IV, Questions 11 and 12, of Albert of Saxony's *Questions on the Eight Books of the Physics* appearing on fols. 50 recto to 51 verso of the edition cited in note 1 to Selection 35.

8. For a discussion of *mixtum* and internal resistance, see Selection 51.1, n. 9, and Selection 77.2, n. 22.

9. Probably based on Avempace's argument that despite a lack of resistance in the celestial regions, the fixed stars and planets move successively and with finite speeds. See Selection 44.2 and note 15 thereto.

10. *Physics* IV.8.214b.28–33.

11. Probably a reference to *Physics* IV. 8.215b.12–24.

12. Toward the beginning of his *Questions on the Phys-*

As to the second [part of this], the question can be formed [in this way]: "If a vacuum exists, whether a heavy body could be moved in it."[13] But then concerning the vacuum, we must make the distinction about it which we declared above in the first article of the eighth question.[14]

Let this be the first conclusion: By taking "vacuum" in the second way,[15] a heavy simple body could be moved in it. This is obvious, for with earth and water having been destroyed, and if some heavy body were posited at the sphere of fire, it would descend successively through the air [perhaps "fire" is intended here—*Ed.*] to the concavity of the air, because in the air it would have a sufficient resistance [which it would not have in the] vacuum.

A second conclusion [is this]: By taking "vacuum" in the first way,[17] as it is commonly taken in this question, a heavy mixed body is easily moved in it successively. This is clear, for let there be a heavy mixed body whose heaviness *(gravitas)* is as 2 and lightness *(levitas)* as 1. And let it reach the concave [surface] of air and descend successively until its center of gravity *(medium gravitatis)* is the middle [or center] of the world *(medium mundi)*. [This will happen] because it has an internal resistance, for it has one degree of lightness inclining [or tending] upward and two degrees of heaviness inclining downwards.

A third conclusion [is this]: In some [particular] case a heavy mixed [or compound] body *(mixtum)* could be moved quicker in a plenum than in a vacuum. This can be demonstrated, for let a mixed body be composed of earth, which is as 3, and air, which is as 2. Now, should everything below the sphere of fire be annihilated, let us then assume that heavy body is at the sphere of fire which resists it with [a resistance of] 1.[18] Then, because both the earth and air [of the mixed body] are outside their natural places, and because both seek to descend through the fire, the motive power is as 5, since the earth is as 3 and the air as 2; and the resistance of the medium is as 1, as has been assumed in this case. Hence, this body is moved through fire, or through a plenum, in a quintuple ratio.[19] Now, when it reaches to where the natural place of air would be if there were no vacuum, the air [in the mixed body], which is as 2, begins to resist the earth, which is as 3, because the air strives to remain there. But since this mixed body is dominated by earth, it descends further so that only the earth in it moves while the air resists, whereas previously in the full medium [of fire] both were moving.[20] Thus the mixed body descends further into the vacuum by a ratio of 3 to 2, which is smaller than a quintuple ratio, as is the motion it produces. And so, the same mixed body has a much slower motion in a vacuum than in a plenum, namely fire. It follows from this that the natural motion of some heavy body can be quicker in the beginning than in the end. For example, if a mixed [or compound] body of four elements should have one degree of fire, one of air, one of water, and four of earth and if everything were annihilated within the sides of the sky except this mixed body, and if the mixed body were placed where the fire was, then this mixed body would descend more quickly through the vacuum of fire *(vacuum ignis)* than through the vacuum of air *(vacuum aeris)*,[21] and so on, as can easily be deduced from this case. But you [now] say, What should be said, therefore, about the common assertion that natural motion is quicker in the end than at the beginning? One can say that is universally true of the motion of heavy and light bodies but not of the motion of heavy and light mixed [or compound] bodies.

A fourth conclusion [is this]: If a vacuum existed, a heavy simple body could be moved in it successively. For example, assuming that everything within the concave [surface] of the lunar orb were annihilated except for a small uniformly heavy body 1 foot [in length], of which one fourth is on one side of the middle [or center] of the world *(medii mundi)* and the remainder is on the other side [of the center]. Then this heavy body would be moved until its middle [or center] becomes [or coincides with] the middle [or center] of the world and through the whole motion its smaller part, which is on one side of the center of the world, would resist the remainder.[22] All these things are understood on the assumption that in heavy bodies there is encompassed a tendency [or striving or appetite] *(appetitus)* by means of which they would seek to fill a vacuum if such existed and that they possess *(obtineant)* a natural tendency for this, namely a tendency for being in their proper places.

And let this be a fifth conclusion:

Assuming what has already been said and that everything within the sides of the sky is annihilated except one heavy simple body which touches the concave surface of the sky, [then] *in infinitum* quickly it [that is, the heavy simple body] would descend *(infinite velociter descenderet)* [because of what follows in this paragraph, I have translated *infinite* by *in infinitum*—*Ed.*]. This is proven, because [it will fall] more than twice as fast as any body descending with a definite [or finite] velocity; and more than four times as fast; and more than eight times as fast; and so on into infinity, because it would descend without any internal or external resistance. Secondly, if it should descend with any definite [or finite] velocity, it would fall with a definite ratio of motive power to resistance. But this is false. The falsity of it is obvious, because nothing would resist it; but the consequence holds, because every definite [or finite] velocity varies as

ics, Bk. IV, Question 10, fol. 77 recto, col. 1 (for full title, see Selection 14, n. 1), John Buridan offers virtually the same two conclusions. "And so for the second conclusion, I say that this is not a good consequence: 'A vacuum exists, therefore a heavy body is moved in it.' The reason for this is that having assumed the existence of a vacuum, which is possible, perhaps there is no heavy body in it; or, although there could be a heavy body in it, perhaps it rests either because of the divine power or for another reason. A third conclusion is that this is also not a good consequence: 'A vacuum exists, therefore a heavy body is not moved in it,' because it is possible that it is moved at least by the divine power, as was said. And so these two conditional statements must be denied: (1) 'If a vacuum exists, a heavy body could be moved in it:' [and] (2) 'If a vacuum exists, a heavy body could not be moved in it.' "

13. In the first part Albert showed that both conditional propositions are false because logically possible situations are imaginable which falsify them. Now he explores whether a body could move in a vacuum on the assumption that vacuum exists *and* that a heavy body is actually located within it.

14. Albert refers here to two types of vacuum that are distinguished in this eighth question, which is reproduced in Selection 53.1.

15. In this sense, a vacuum is conceived as a body between whose sides there is nothing (see Selection 53.1).

16. Assuming a correct text, Albert's intent is wholly unclear. Under the conditions described here, it seems that the body does not even enter the vacuum formed where earth and water formerly existed but remains in the sphere of air. In what sense, then, are we to conceive of it as having moved in a vacuum? Are we to suppose that the entire sublunar region, including the concavities of fire and air as well as the concavities where earth and water formerly existed, is conceived as a vacuum? This would indeed be a strange interpretation of the second type of vacuum. That Albert believes a heavy simple body would move instantaneously if it fell freely in a vacuum as usually understood emerges in what follows (see n. 17 and the fifth conclusion, below).

17. That is, as a separate space devoid of body (see Selection 53.1). As Albert tells us, this is the usual sense of vacuum. Simple (or pure elemental) heavy bodies would, it seems, move with infinite velocity in this kind of separate vacuum, since they do not have internal resistance and the void itself lacks external resistance. According to almost all adherents of Aristotelian physics in the Middle Ages, if such a body moved freely it would have an infinite velocity, since no ratio of motive force to resistance would be possible. This seems to be Albert's position in the fifth conclusion of this question. But as will be seen in the fourth conclusion of this question and especially in the fourth conclusion of the next (twelfth) question, Albert believes that force and resistance relations are distinguishable in suitably devised thought experiments, so that motion of simple heavy bodies in a vacuum would be possible. Heavy mixed bodies, however, which are compounds of varying proportions of the four elements, have internal resistances (understood in the sense described in Selection 51.1, n. 9) and could move in a hypothetical vacuum with finite velocities, as Albert shows in this conclusion.

18. In this imaginary situation, the concentric spheres of earth, water, and air are assumed to be annihilated, with only the outermost sphere of fire remaining as an external resistance.

19. That is, $F/R = 5/1$, where F represents the motive power and R the resistance of the fire.

20. The concept of the potency of "natural place," which was accepted by virtually all who discussed physics in the Middle Ages, is well illustrated here. Although all is void space below the sphere of fire, that portion of it formerly containing air causes the body to behave differently than those portions which formerly contained water or earth. Thus, although in one sense it would be absurd to say that one portion of void space "differs" from any other, it is yet true to say that for Albert the different intrinsic cosmic properties of each natural portion of sublunar space continue to exert their customary influence on the behavior of elements. The potency and behavior of each natural place is thus unalterable, and the air in the mixed body behaves as would any quantity of air in that portion of space—that is, it resists the motion of heavy bodies and acts as if it were an external medium.

21. That is, the respective vacua where formerly there were fire and air.

22. Compare Albert of Saxony's discussion at the end of Selection 47 and note 15 thereto. In this example we have a homogeneous body situated as in Fig. 1. Let *c*

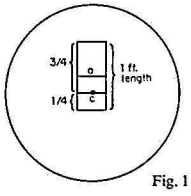


Fig. 1

represent the center of the world and *a* the center of the body. The body will move in a vacuum, because the fourth part of it on one side of the center (or diameter) of the world will resist the motion of the other three fourths, since both parts of this single body will seek the exact center of the world, thereby giving rise to contrary motions. But that part which is three fourths of it has a greater motive power than the resisting one-fourth part and will consequently cause the entire body to move until *a* coincides with *c*, at which time motion ceases.

some ratio of mover to resistance, at least in things moving involuntarily. But you say that if a heavy simple body would descend *in infinitum* quickly *(descenderet. . .infinite velociter)* in a vacuum, then a heavy simple body could be moved in a vacuum. But if it were moved, then it would be moved in time [that is, in a finite time] in a vacuum, since every motion occurs in time. Consequently, it would not be moved in an instant, which is the opposite of what Aristotle says. In replying to this, I concede the consequent, but deny the antecedent, for I did not say in the question that "it would descend *in infinitum* quickly," but I said that "*in infinitum* quickly it would descend."[23] The first of these statements is false and the second true; nor does the first imply the second, because in the first the words "it would descend" are taken determinately, but in the second they have merely confused supposition, because in this second statement the syncategorematic term "infinite" precedes [the words "it would descend"], while in the first statement it follows them.[24]

Yet, you might say, if a heavy simple body were moved infinitely quickly *(in infinitum velociter moveretur)* in a vacuum, it would follow that any resistance, however small, would be of infinite power. However, this is false. But the consequence is proved, because a very small resistance could reduce *(demere)* an infinite velocity with which a body is moved in a vacuum by taking from it a [single] simple degree [of velocity]. But to do this is to be of infinite power, because to diminish any degree of velocity by some [amount] is to be of some [resistive] power; and to diminish it by twice that amount is to be of [even] greater [resistive] power, and so on. Therefore, to take from it an infinite velocity is to be of infinite power. As the solution of this argument, I say that if something were moved infinitely quickly with a resistance, then I readily concede that that which could make it be moved with a finite speed would be of infinite power. If, however, something could be moved infinitely quickly without a resistance, then I say that that which makes it move with a finite speed need not necessarily be of infinite power, and from this to the argument. . . .[25]

BOOK IV, QUESTION 12

Whether, if a vacuum existed, something could be moved in it with a finite velocity or local motion, or with a motion of alteration.[26]

In the first place, it is argued that local motion [does] not [occur in a vacuum], for if it did this would happen especially in the local motion of mixed [or compound bodies]. But this would not

23. The antecedent which is denied is the assertion that a heavy simple body "would descend *in infinitum* quickly" in a vacuum; the conceded consequent declares that "a heavy simple body could be moved in a vacuum."

The intelligibility of this difficult paragraph depends on the distinction between the expressions (1) *descenderet in infinitum velociter*, or *descenderet infinite velociter*, and (2) *in infinitium velociter descenderet*, or *infinite velociter descenderet*. By the rules of Latin grammer and syntax, these phrases are equivalent and would ordinarily be translatable by the same expression. Not, however, in this paragraph, where word order becomes crucial. That *descenderet* ("it would descend") precedes *in infinitum* (or *infinite*, which is equivalent to *in infinitum*) in expression (1) and follows in expression (2) requires two quite different interpretations, which will be described in the next note. To emphasize the fundamental importance of word order, the equivalent Latin terms "in infinitum" and "infinite" have been rendered as "in infinitum" in the translation (ordinarily, they would be translated as "infinitely)." The words that precede and follow will thus be readily detectable.

24. In the first expression, *descenderet in infinitum velociter* ("it would descend *in infinitum* quickly"), the term "infinitum" follows the term it modifies, namely *descenderet* ("it would descend"), and thereby gives determinate supposition to *descenderet*. Under these circumstances, the term *infinitum* was often considered, as it is here, a categorematic infinite, which signifies that the heavy simple body descending from the concave surface of the sky would fall with a single, actual infinite speed that is greater than any other assignable speed. Albert rejects this in favor of the second mode of expression, *in infinitum velociter descenderet*. Here the term "infinitum," or "infinite," precedes the term it modifies, namely *descenderet*, and thus gives merely confused supposition to *descenderet*. In this instance, the term *infinitum* is considered a syncategorematic infinite, which signifies that the heavy simple body will fall infinitely quickly in the sense that however large the assigned speed, the body can always descend with yet a greater speed. On this interpretation, Albert holds that a heavy simple body could be conceived to move temporally in a vacuum only with a syncategorematic infinite speed. For the distinction between *determinate* and *merely confused suppositio*, see Selection 18, [II] 4, and Selection 52.4, reply to the first argument supporting Henry of Harclay's views and notes 17 and 19 thereto. I am indebted to John E. Murdoch for the interpretation in this and the preceding note, and for that part of the translation which they elucidate.

25. A paragraph containing Albert's reply to the five principal arguments enunciated at the beginning of this question, is omitted. Although the infinite speed discussed in this final paragraph of Question 11 seems to be syncategorematic, I have not indicated this by word order.

26. "Motion of alteration" signifies change of quality, as, for example, from hot to cold, white to black, and so on.

occur, because mixed bodies are moved only with the motion of the predominating element. But no element is moved with a finite velocity in a vacuum, as declared in another question,[27] because of a lack of internal and external resistance. Therefore, it would seem that no mixed body [could be moved in a vacuum]. This can be confirmed [as follows]: Take a portion of pure earth and take a portion of lead heavier than this earth. Now, this [piece of pure earth] would not descend in the vacuum with a finite velocity but [rather] with an infinite velocity. The piece of lead, therefore, could not descend with a finite velocity.

Secondly, if [there could be motion with a finite velocity in a vacuum], it would seem especially true with respect to motion of alteration [that is, change of quality], since no external resistance is required for this. But this is not so, because local motion, which is the primary motion, cannot occur in a vacuum.[28] Therefore, it seems that motion of alteration would not occur. Therefore, and so on [that is, no motion of finite velocity can occur in a vacuum].

I argue against this. For if a vacuum existed, men could, nonetheless, walk around on the earth—although birds could not fly. It is true that men could not walk around for long, since they could not live long without breathing. Similarly, if some heated water were assumed to exist in a vacuum, it would change itself to its prior state of coldness. It seems, therefore, that local motion as well as motion of alteration could be made in a vacuum, for as stated in the other question,[29] mixed [or compound] bodies could be moved in a vacuum because they have internal resistance.

In brief, I say that we have never experienced the existence of a vacuum, and so we do not readily know what would happen if a vacuum did exist. Nevertheless, we must inquire what might happen if it existed, for we see that natural beings undergo extraordinarily violent actions to prevent a vacuum. Indeed we see that if some tube *(fistula)* is put in water, and air is drawn from the tube, the water follows by ascending, striving to remain contiguous with the air lest a vacuum be formed.[30]

And so, let the first conclusion be that if God should annihilate everything between two walls, those two walls would become rarefied infinitely quickly [*in infinitum velociter*] in order to avoid a vacuum. This can be proven, because it has already been declared that natural things seek to avoid a vacuum and are inclined toward this action by their very natures. Now, they strive for, or are inclined to, this finitely or infinitely. If the latter, the sides [or walls] would meet infinitely quickly and the intent of this [conclusion] would be had. However, if the striving is only finite, it follows that a finite power (*virtus*) that is less than the power and inclination of these walls would be sufficient to hold them and prevent their meeting. Hence a finite power could conserve a vacuum and even increase it if it were a greater power [than the one that only conserves it] and could condense those walls. But this is false, because only God could conserve, increase, and make a vacuum.[31]

But you say [now] that there is no infinite striving (*appetitus infinitus*) in these lower beings, so that it does not follow that if a vacuum did exist these natural beings would seek to avoid it infinitely. I reply by conceding that in fact there is no infinite striving [or inclination] in these lower beings. Nevertheless, if God should make [or form] a vacuum, natural beings would then strive infinitely to avoid it if their natures could be directed; and this is what is commonly said: "Nature abhors a vacuum infinitely." Indeed, before the sky would allow a vacuum to remain, it would descend and fill it.

A second conclusion [is this]: If there were a vacuum between sky and earth, and if there should be a plane surface equidistant from the center on which there are two heavy spheres *A* and *B*, with *A* heavier than *B*, then any power (*virtus*) [or force], however small, could, with infinite ease, move any

27. Probably a reference to Aristotle's argument cited near the beginning of Question 11. As will be evident in some of the conclusions to follow in this selection, Albert does not accept this unqualifiedly, since he believes that under properly imagined circumstances pure elemental bodies could move with finite velocities in a vacuum (see also n. 16 and n. 17). Here, however, he is formulating an opposition argument which he will subsequently reject.

28. Proponents of this argument, which Albert will reject, would insist that a change of quality depends on spatial motion of the material parts of the body undergoing the change of quality.

29. See the second and third conclusions of Question 11, Selection 55.2.

30. Substituting wine for water, the same illustration is cited by Buridan (Selection 53.2); compare also Marsilius of Inghen's second experiment (Selection 53.3).

31. That only God could make a vacuum was a standard feature of medieval discussions of the void. Except for Nicholas of Autrecourt, and perhaps a few others, none believed that God had made, or would make, a vacuum within the physical cosmos.

of these spheres on this surface.[32] This can be proved, because any of these spheres would touch this surface at a point and thus any half would counterbalance equally the other half just as two weights in a balance.[33] But since any excess would suffice to produce motion, it follows that any [motive] power (*virtutem*) [however small] could move any one of these spheres with infinite ease.[34] However, it is true that this could not occur in a plenum, because then not only would one [half] of the sphere resist the pushing force [*pellenti*] but also air. And so in air not every force that is as small as you please could push and move these spheres.[35]

[Here is] a third conclusion: Although a small force (*virtus*), however small, could move these spheres in a vacuum, no force could move the large and the small heavy body with equal velocity. Let this be set forth in numbers. For it happens that in the greater sphere, namely in *A*, one half of it pulls against the other half, as [for example] 2 against 2; and in the small sphere, say *B*, one half pulls against the other, [say] as 1 against 1. And should there be a power as 1, it would move the small body in a greater ratio—and therefore more quickly—than the larger body. Thus it moves the small body with a double ratio [that is, $F/R = 2/1$] and the large body with a sesquialterate ratio [that is, $F/R = 3/2$]. For when this power begins to move the small body, one of those halves in moving the other [half] moves itself with it, and when any one of those halves is as 1, the whole power, the total motive power of the small body will be as 2 and the resistance as 1. But 2 to 1 is a double ratio. And in the motion of the great sphere, the total motive power is as 3, the resistance as 2, and the ratio of 3 to 2 is a sesquialterate ratio.

32. Since no figure accompanies this conclusion, the manner of conceiving this demonstration is not obvious. For, what does Albert mean by "a plane surface equi-

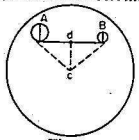


Fig. 2

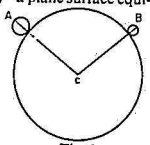


Fig. 3

distant from the center on which there are two spheres *A* and *B*" ("aliqua superficies plana eque distans a centro super quam essent due sphere graves, una *A* et alia *B*"; fol. 51 recto, col. 1)? Obviously all the points on a plane cannot be equidistant from the center of a sphere. Perhaps something like Fig. 2 is meant, where the two points of contact of sphere and plane are equidistant from *c*, and *d* is the midpoint of the surface. If, however, *plana* signifies merely an even or uniform, rather than a flat, surface, perhaps a curved surface is intended, as in Fig. 3, where the circumference of the circle would make all the points of the surface equidistant from the center *c*. Indeed any representation in which the two heavy spheres are virtually equidistant from the center of the world would be satisfactory, since "heaviness" of a body in Aristotelian physics varies with distance from the center of the world.

33. That is, one half of sphere *A* will counterbalance the other half of *A*, just as one half of *B* counterbalances the other half of *B*. This interpretation gains support from the illustration in the next (third) conclusion. Thus Albert imagines the two halves of each sphere as constituting a force-resistance relation. Since in each sphere $F = R$, where *F* is motive force and R resistance, neither will move, for it was an accepted principle in medieval physics that motion occurs only when $F > R$ but not when $F \leq R$.

34. In this interesting conclusion Albert remains faithful to Aristotelian physics in one sense and drastically departs from it in another. Should any force, however small, be applied to spheres *A* or *B*, a motion would be produced, since by assumption $F = R$ in each sphere, so that $F + f > R$, where *f* is any applied force. For, as stated in note 33, motion is possible only when the force is greater than the resistance, as required by the commonly accepted interpretation of Aristotelian physics. But if we ignore the arbitrary division of each sphere into halves representing motive force and resistance, and focus on the fact that a force as small as you please is deemed able to move either sphere, we realize that from this standpoint Albert daringly allows that motion in a hypothetical vacuum can arise when $F < R$. Generally, as long as the force is applied, a force as small as you please can move a body as large as you please. This second interpretation may be compared to Galileo's early discussion in *De motu (On Motion)*. There, in chapter 14, Galileo truly abandons the Aristotelian doctrine that motion can arise only when $F > R$. If a perfectly smooth, hard spherical body rests on a perfectly smooth and hard plane, and "if everything is arranged in this way, then any body on a plane parallel to the horizon will be moved by the very smallest force, indeed, by a force less than any given force"; *Galileo Galilei, On Motion*, trans. I. E. Drabkin (Madison, Wis.: University of Wisconsin Press, 1960), pp. 65–66. In addition to the fact that Galileo's discussion is not hypothesized in a void, the "plane parallel to the horizon" follows the earth's surface and circumference, so that the motion would be circular if continued indefinitely. Although it does not appear to be inertial in the *De motu*, it is indeed inertial (though still circular) in *Dialogues Concerning Two New Sciences (Discorsi)*, written many years later. Albert does not describe the path which *A* or *B* will follow. Also, their motions could not be inertial, since removal of *f* would make $F = R$ and the motion would cease immediately. Marsilius of Inghen, in his *Questions on the Eight Books of the Physics*, Book IV, Question 12, has virtually the same example (fol. 54 verso, col. 2, of the edition cited in Selection 53.3, n. 20.

35. That is, if $F = R$, and *f* is the applied force, however small, and *r* the resistance of the air, then motion arises in air only when $F + f > R + r$, and $f > r$.

A fourth conclusion [is this]: Similarly if any balances were suspended in a vacuum, the greater weight—even if it were a heavy simple body—would descend successively. This is obvious, for although it would not have internal resistance nor external resistance in the form of a medium, yet it would have an external resistance [in the form] of a counterweight.

A fifth conclusion [is this]: After suspending balances in a vacuum and hanging two equal weights, any heavy body, however small, could be added to [either] one [of the suspended weights] and cause it to descend. The reason for this is that any excess suffices to produce motion. But this would not be true in a plenum, as was said.

A sixth conclusion [is this]: If there were a table placed transversely, [or crosswise] in a vacuum and on it is posited a heavy simple spherical body, the body would descend on the table with a successive finite velocity. This is obvious, because it would not descend by a straight line [but] would descend by an uncertain path (*natando*); and one part [of the table(?)] would raise the other, and then that part which is raised violently would take the place of a resistance.[36]

[Here is] a seventh conclusion: Although a lead mass should weigh more on a balance than simple water, nevertheless if both were posited in a vacuum, water would move or descend infinitely quickly; lead, however, would descend successively. This is obvious, because lead, although it has no external resistance, yet because it is a mixed [or compound] body, has internal resistance;[37] water, however, has neither internal nor external resistance.[38]

An eighth conclusion [is this]: Mixed [or compound] bodies of homogeneous composition [*consimilis compositionis*] are moved with equal velocity in a vacuum but not in a plenum. The first part [concerned with fall in a vacuum] is obvious, because they are of homogeneous mixture. The ratio of motive power to total resistance in one body is the same as in another homogeneous [*consimilis*] body, because they both have only internal resistance.[39] The second part [that is, dealing with the plenum] is [also] obvious, because if there is a mixed [or compound] body, say *A*, whose heaviness (*gravitas*) is as 8 and lightness (*levitas*) as 4, and there is another mixed body, *B*, whose heaviness is as 4 and lightness as 2, then *A* and *B* are homogeneous mixed bodies.[40] [Finally] these bodies are posited in a resistant medium taken as 1. I [now] prove *A* descends quicker than *B*. This is obvious from [*A*'s] greater ratio of moving power to resistance. For by computing together the internal and external resistances, the total resistance of *A* is as 5 and the motive power as 8; and in *B* the total resistance is as 3 and the motive power as 4. Now, a ratio of 8 to 5 is greater than a ratio of 4 to 3.[41]

A ninth conclusion [is this]: In a vacuum—if it existed—an alteration would be possible. This is obvious, for if a vacuum existed and heated water were assumed in it, it would be altered from outside to its prior coldness.

36. In this ambiguous and fuzzily presented illustration of finite and temporal motion in a vacuum, it seems that the sphere is rolling down a table lying crosswise with respect to an observer at the center of the world (see Fig. 4). The force-resistance dichotomy is either

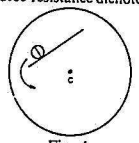


Fig. 4

conceived in the spherical body itself, as in the second and third conclusions, or the part of the table which rises as the sphere rolls down serves as the resistance to the sphere. In the first possibility, if half the sphere opposes the other half and $F = R$, how will the motion arise? Perhaps, then, the second alternative is more plausible. Realizing that a sphere would roll down an inclined table, Albert feels compelled to distinguish a force and resistance, so that he could meet the Aristotelian requirements for a finite motion.

37. In an obvious mistake, the text has "external" *(extrinseca)*.

38. See note 17.

39. Earlier, Bradwardine had formulated the same conclusion and undoubtedly for the very same reason (see Theorem XII in Selection 51.1 and note 45 thereto), whereas Galileo, who also reached the same conclusion about two centuries later, argued on a quite different basis (see the introduction to Selections 55.4, 5 and Selection 55.6, by Galileo.

40. Since their ratios of heaviness to lightness—or motive force to resistance—are equal (that is, $8/4 = 4/2$), Albert concludes that they are necessarily bodies of the same homogeneous composition.

41. While two unequal homogeneous mixed bodies would fall with equal speed in the void, their behavior in a plenum is made to depend on which particular equal ratio is assigned to it. In the example cited here *A* descends with greater speed than *B* because the ratio 8/4 was assigned to it and the equal ratio 4/2 was assigned to *B*. Thus *A* might be smaller than, equal to, or greater than *B*, and it would, under the conditions set forth, descend more quickly then *B*. It is painfully evident how bewildering were the consequences of the doctrine of internal resistance when interpreted in this arbitrary manner

A tenth [and final] conclusion [is this]: If a vacuum existed, an alteration would not be possible in it from the outside unless the agent and patient are immediate [that is, in direct contact], for otherwise one of them could not act on the other. And this could happen because something intermediate that could serve to receive and transmit (*deferentis*) the action is lacking. Thus, if a vacuum existed between us and the sky, we could not see the stars nor the sky. And this could be the result of a lack of a medium by means of which the visible species could be multiplied.[42] Similarly, if there were a vacuum between you and me, then however much I shouted you would neither hear nor see me; and all this would be the result of a lack of a medium.

[I now reply] to the principal arguments.[43] To the first, I say that although a mixed [or compound] body is moved with the motion of the predominant element in it, nevertheless it does not move with so great a velocity as that element [itself] if it were separated from the compound. The confirmation of this is obvious from the question. To the second [argument] I say that indeed any local motion could occur in a vacuum—if it existed or could be made. And so, what was assumed in this argument must be denied.[44] But also [it must be denied] that every local motion is the cause and first motion of all other [types of] motions. But [this can be said of] the circular motion of the sky only.[45] It is [certainly] not necessary that if a motion of alteration [or change of quality] could occur in a vacuum, that [therefore] any other local motions could occur. It follows, and so on [that the second principal argument is false].

42. That is, transmitted. For a discussion of multiplication of species, see Selection 62.1–4.

43. These are the two responses to the two opening arguments of Question 12.

44. In the second principal argument it was assumed that no local motion could occur in a vacuum, and consequently no motion of alteration, or change of quality, was possible.

45. Following Aristotle, it was generally accepted that the circular motions of the heavens were essential for all other terrestrial motions, since the annual motion of the sun determined the changes of season, the growth of crops, and life itself.

3. Galileo Galilei (1564–1642): His Earliest Law of Motion and His Arguments for Finite Velocity in a Void

Translated and annotated by I. E. Drabkin;[46] additional notes by Edward Grant[47]

CHAPTER [10]

IN WHICH, IN OPPOSITION TO ARISTOTLE, IT IS PROVED THAT, IF THERE WERE A VOID, MOTION IN IT WOULD NOT TAKE PLACE INSTANTANEOUSLY, BUT IN TIME.

Aristotle, in Book 4 of the *Physics*, in his attempt to deny the existence of a void adduces many arguments. Those that are found beginning with section 64[48] are drawn from a consideration of motion. For since he assumes that motion cannot take place instantaneously, he tries to show that, if a void existed, motion in it would take place instantaneoulsy; and, since that is impossible, he concludes necessarily that a void is also impossible. But, since we are dealing with motion, we have decided to inquire whether it is true that, if a void existed, motion in it would take place instantaneously. And since our conclusion will be that motion in a void takes place in time,[49] we shall first examine the contrary view and Aristotle's arguments.

In the first place, of the arguments adduced by Aristotle there is none that involves a necessary conclusion, but there is one which, at first sight, seems to lead to such a conclusion. This is the argument set forth in sections 71 and 72,[50] in which Aristotle deduces the following inconsistency—that, on the assumption that motion can take place in time in a void, then the same body will move in the same time in a plenum and in a void. In order to be better able to refute this argument, we have decided to state it at this point.

Thus, Aristotle's first assumption, when he saw that the same body moved more swiftly through the rarer than through the denser medium, was this: that the ratio of the speed of motion in one medium to the speed in the second medium is equal to the ratio of the rareness of the first medium to the rareness of the second. He then reasoned as follows. Suppose body *a* traverses medium *b* in time *c*, and that it traverses a medium rarer than *b*, namely *d*, in time *e*. Clearly, the ratio of time *c* to time *e* is equal to the ratio of the density of *b* to the density of *d*. Suppose, then, that there is a void *f* and that body *a* traverses *f*, if it is possible, not in an instant, but in time *g*. And suppose that the ratio of the density of medium *d* to the density of some new medium is equal to the ratio of time *e* to time *g*. Then, from what has been established, body *a* will move through the new medium in time *g*, since [the density of] medium *d* has to that of the new medium the same ratio as time *e* to time *g*. But in the same time *g* body *a* also moves through the void *f*. Therefore *a* will in the same time move over two equal paths, one a plenum, the other a void. But this is impossible. Therefore the body will not move through the void in time; and therefore the motion will be instantaneous.

Such is Aristotle's proof. And, indeed, his conclusions would have been sound and necessary, if he had proved his assumptions, or at least if these assumptions, even though unproved, had been true. But he was deceived in this, that he assumed as well-recognized axioms propositions which not only are not obvious to the senses, but have never been proved, and cannot be proved because they are completely false. For he assumed that the ratio of the speeds of the same body moving in different media is equal to the ratio of the rarenesses of the media. But that this is false has been fully proved above.[51] In support of that proof, I shall add only this. Suppose it is true that the ratio of the rareness of air to the rareness of water is equal to the ratio of the speed of a body moving in air to the speed of the same body in water. Then, when a drop or some other quantity of water falls swiftly in air, but does not fall at all in water, since the speed in air has no ratio to the speed in water, it follows, according to Aristotle himself, that there will be no ratio between the rareness of air and the rareness of water. But that is ridiculous.[52]

Therefore, it is clear that, when Aristotle argues in this way, we must answer him as follows. In the first place, as has been shown above, it is not true that differences in the slowness and speed of a given body arise from the greater or lesser density and rareness of the medium. But even if that were conceded, it is still not true that the ratio of the speeds of the motion of the body is equal to the ratio of the rarenesses of the media.

And as for Aristotle's statement in the same section that it is impossible for one number to have the same relation[53] to another number as a number has to zero, this is, of course, true of geometric ratios [viz., *a*/*b*], and not merely in numbers but in every kind of quantity. Since, in the case of geometric ratios, it is necessarily true that the smaller magnitude can be added to itself a sufficient number of times so that it will ultimately exceed any magnitude whatever, it follows that this smaller magnitude is something, and not zero. For zero, no matter how often it is added to itself, will exceed no quantity. But Aristotle's conclusion does not apply to *arithmetic* relations [viz., the difference *a* − *b*]. That is, in these cases, one number can have the same relation to another number as still another number has to zero. For, since [two pairs of] numbers are in the same arithmetic relation when the difference of the [two] larger is equal to

46. [Reprinted with permission of the Regents of the University of Wisconsin from *Galileo Galilei "On Motion" and "On Mechanics" Comprising "De Motu" (ca. 1590)*, translated with Introduction and Notes by I. E. Drabkin, and *"Le Meccaniche" (ca. 1600)*, translated with Introduction and Notes by Stillman Drake (Madison, Wis.: University of Wisconsin Press, 1960), pp. 41–50. Although possibly the earliest extant scientific essay by Galileo, it was left unpublished. It reveals an interesting mixture of traditional (that is, medieval) and new concepts.—*Ed.*]

47. [Since additional notes are interspersed with Drabkin's, the numbers of the notes as they appear in his translation have been altered. Four simple marginal figures, which contribute in no way to the understanding of the arguments, have been omitted.—*Ed.*]

48. [214b. 10ff. I have replaced Drabkin's citation of 214a.27ff., since it does not correspond to Galileo's reference to Text 64 of Book IV of Aristotle's *Physics*. Here, and in other places below, Galileo is using the traditional medieval system of reference to the texts of Aristotle. In all such cases Drabkin has converted the medieval citation to the modern system of reference based upon Bekker numbers.—*Ed.*]

49. [As Galileo seems to have acknowledged (see n. 69), this viewpoint was widely adopted in the Middle Ages and, as already stated, is traceable to Avempace and ultimately to Philoponus. See Selection 44.2 and n. 17 thereto.—*Ed.*]

50. [215a.24–216a.3. Drabkin gives 215b–216a, which does not precisely embrace Texts 71 and 72.—*Ed.*]

51. [In chapter 8. In the next few lines, Galileo describes Aristotle's law in terms of the rareness, or subtlety, of media, that is, $V_2/V_1 = R_2/R_1$, where R is the rareness of a medium. See Selection 44.2, n. 7.—*Ed.*]

52. There is the following marginal addition at this point: "If this were true, the ratio of the speed of motion in air to the speed of motion in water would be the same for all bodies. Hence the ratio of the speed of lead in air to its speed in water would be the same as the ratio of the speed of wood in air to its speed in water. But who cannot see that this is false? For lead sinks in water, wood does not."

53. Galileo's point depends on two senses of this word (*proportio*). Aristotle's conclusion is sound for what Galileo here calls a geometric *proportio* (i.e., the ratio *a*/*b*), but not for an arithmetic *proportio* (i.e., the difference *a*—*b*).

the difference of the [two] smaller,[54] it will, of course, be possible for one number to have the same [arithmetic] relation to another number, as still another number has to zero. Thus, we say that the [arithmetic] relation of 20 to 12 is the same as that of 8 to 0: for the excess of 20 over 12, i.e., 8, is equal to the excess of 8 over 0.

Therefore, if, as Aristotle held, the ratio of the speeds were equal to the ratio, in the geometric sense, of the rarenesses of the media, Aristotle's conclusion would have been valid, that motion in a void could not take place in time. For the ratio of the time in the plenum to the time in the void cannot be equal to the ratio of the rareness of the plenum to the rareness of the void,[55] since the rareness of the void does not exist.[56] But if the ratio of the speeds were made to depend on the aforesaid ratio, not in the geometric, but in the arithmetic sense [i.e., as a ratio of differences], no absurd conclusion would follow. And, in fact, the ratio of the speeds does depend, in an arithmetic sense, on the relation of the lightness of the first medium to that of the second. For the ratio of the speeds is equal, not to the ratio of the lightness of the first medium to that of the second, but, as has been proved, to the ratio of the excess of the weight of the body over the weight of the first medium to the excess of the weight of the body over the weight of the second medium.

So that this may be clearer, here is an example. Suppose there is a body *a* whose weight is 20, and two media unequal in weight, *bc* and *de*. Let the volume of *b* be equal to that of *a*, and the volume of *d* also equal to that of *a*. Since we are now discussing downward motion that takes place in a void, let the media be lighter than the body *a*, and let the weight of *b* be 12, and of *d* 6. It is clear, then, from what was proved above, that the ratio of the speed of body *a* in medium *bc* to the speed of the same body in medium *de* will be equal to the ratio of the excess of the weight of *a* over the weight of *b* to the excess of the weight of *a* over the weight of *d*, that is, as 8 is to 14. Thus if the speed of *a* in medium *bc* is 8, its speed in medium *de* would be 14. Now it is clear that the ratio of the speeds, 14 to 8, is not the same as the ratio (in the geometric sense) of the lightness of one medium to the lightness of the other. For the lightness of medium *de* is double that of medium *bc* (for since the weight of *b* is 12, and of *d* 6, i.e., since the weight of *b* is double the weight of *d*, the lightness of *d* will be double the lightness of *b*); but a speed of 14 is less than twice a speed of 8. Yet the speed 14 has to the speed 8 the same relation, in the arithmetic sense, as the lightness of *d* to the lightness of *b*, since the difference between 14 and 8 is 6, and 6 is also the difference between the lightness of *d* (12) and the lightness of *b* (6).[57]

Furthermore, if medium *de* should be lighter, so that the weight of *d* is 5, the speed *f* will be 15 (for 15 will be the difference between the weight of body *a* and the weight of the medium *d*). And again the relation [i.e., arithmetic difference] of speed 15 and speed 8 will be the same as between the weight of medium *b* (12) and the weight of medium *d* (5), that is, the same as the relation of the lightness of *d* and the lightness of *b*. For the difference in each case will be 7. Furthermore, if the weight of *d* is only 4, the speed *f* will be 16: and the relation of speed 16 and speed 8 (with a difference of 8) is the same arithmetic relation as between the weight of *b* (12) and the weight of *d* (4), i.e., between the lightness of *d* and the lightness of *b*, the difference being also 8. If, again, medium *de* becomes lighter, and the weight of *d* is only 3, the speed *f* will now be 17. And between the speed *f* (17) and the speed 8 (a difference of 9), the difference is the same as between the weight of *b* (12) and the weight of *d* (3), i.e., as between the lightness of *d* and the lightness of *b*. If, again, medium *de* becomes lighter, and the weight of *d* is only 2, the speed *f* will now be 18. And the arithmetic difference between that speed and the speed 8 will be the same as the difference between the weight of *b* (12) and the weight of *d* (2), i.e., between the lightness of *d* and the lightness of *b*. In each case the difference will be 10. If, again, medium *de* becomes lighter, and the weight of *d* is only 1, the speed *f* will now be 19. And there will be the same arithmetic difference between this speed and the speed 8 as between the weight of

54. Or, what is equivalent, the difference between the two members of each pair is equal.

55. There seems to be a slip here. Galileo may have meant to say "density" instead of "rareness" (see the next note). The same effect would have been achieved by saying "ratio of the rareness of the *void* to the rareness of the *plenum*" or by speaking of the ratio of *speeds*, not of times.

56. In the sense of being greater than any finite number. But Galileo may have meant to speak of density, in which case *nulla* would be "zero."

57. If the weights of *d* and *b* are 6 and 12, respectively, Galileo here speaks of their lightnesses as 12 and 6, respectively, with difference 6. And similarly in what follows.

b (12) and the weight of *d* (1), i.e., between the lightness of *d* and the lightness of *b*. In each case the difference will be 11. Now if, finally, the weight of *d* is 0, so that the difference between the weight of body *a* and of the medium *d* is 20, the speed *f* will be 20; and the arithmetic difference between the speed *f* (20) and the speed 8 will be the same as that between the weight of *b* (12) and the weight of *d* (0), the difference in each case being 12.

It is clear, therefore, that the relation of speed to speed is the same as the relation of the lightness of one medium to the lightness of the other, not geometrically [i.e., as a quotient] but arithmetically [i.e., as a difference]. And since it is not absurd for this arithmetic relation [i.e., difference] to be the same between one quantity and a second quantity as between a third quantity and zero, it will similarly not be absurd for the relation of speed to speed to be the same, in this arithmetic sense, as the relation of a given lightness [of medium] to zero.

Therefore, the body will move in a void in the same way as in a plenum. For in a plenum the speed of motion of a body depends on the difference between its weight and the weight of the medium through which it moves. And likewise in a void [the speed of] its motion will depend on the difference between its own weight and that of the medium.[58] But since the latter is zero, the difference between the weight of the body and the weight of the void will be the whole weight of the body. And therefore the speed of its motion [in the void] will depend on its own total weight.[59] But in no plenum will it be able to move so quickly, since the excess of the weight of the body over the weight of the medium is less than the whole weight of the body. Therefore its speed will be less than if it moved according to its own total weight.

From this it can clearly be understood that in a plenum, such as that which surrounds us, things do not weigh their proper and natural weight, but they will always be lighter to the extent that they are in a heavier medium. Indeed, a body will be lighter by an amount equal to the weight, in a void, of a volume of the medium equal to the volume of the body. Thus, a lead sphere will be lighter in water than in a void by an amount equal to the weight, in a void, of an aqueous sphere of the same size as the lead sphere. And the lead sphere is lighter in air than in a void by an amount equal to the weight, in a void, of a sphere of air having the same size as the lead sphere. And so also in fire, and in other media. And since the speed of a body's motion depends on the weight the body has in the medium in which it moves, its motion will be swifter, the heavier the body is in relation to the various media.

But the following argument is invalid: "A void is a medium infinitely lighter than every plenum; therefore motion in it will be infinitely swifter than in a plenum; therefore such motion will be instantaneous." For it is true that a void is infinitely lighter than any [nonvacuous] medium; but we must not say that such a [nonvacuous] medium is of infinite weight. We must instead understand [the applicability of the term "infinite"] in this way, that between the lightness of air, for example, and a void there may exist an unlimited number of media lighter than air and heavier than a void. And if we understand the matter in this way, there may also exist, between the speed in air and the speed in a void, an unlimited number of speeds, greater than the speed in air and less than the speed in a void. And so also between the weight of a body in air and its weight in a void, an unlimited number of intermediate weights may exist, greater than the

58. [Galileo's law of motion is $V = W_{body} - W_{medium}$, where V is velocity and W is weight per unit volume, or specific weight. Hence a difference in specific weights determines velocity, so that it does not matter whether a body falls in a plenum or a void. The subtractive relation is such that when $W_{medium} = 0$, velocity is proportional to the specific weight of the body. In this Galileo is apparently following the tradition stemming from Avempace (and beyond him probably to Philoponus; for references, see n. 49), where velocity was thought to be determined by the difference—not the ratio as Aristotle would have it—between the weight of a body and the resistance of the medium through which it fell. Resistance of a medium was something to be subtracted, since it served only to retard motion. In the absence of a resistant medium bodies were held to fall freely in the void with a natural finite velocity. Although numbers were arbitrarily assigned and subtractions made (see Bradwardine's criticism of the first erroneous theory at the beginning of Selection 51.1), to my knowledge no one in the Middle Ages, Avempace included, specified how this subtraction could be interpreted physically. This is not surprising when one realizes that it was usual to conceive of bodies in terms of gross, or total, weight and the resistance of a medium by its density (often vaguely understood). Under these circumstances, no proper subtraction was possible. By utilizing specific weight as the criterion for measuring the difference between body and medium, Galileo, and Giovanni Benedetti before him, made Avempace's law theoretically precise and intelligible even though it was as erroneous as the dynamic theories with which it contended.—*Ed.*]

59. I.e., its weight in the void, undiminished by any weight of medium.

weight of the body in air, but less than its weight in a void.

And the same is true of every continuum. Thus between lines *a* and *b*, of which *a* is greater, an unlimited number of intermediate lines, smaller than *a*, but greater than *b* may exist (for since the amount by which *a* exceeds *b* is also a line, it will be infinitely divisible). But we must not say that line *a* is infinitely greater than line *b*, in the sense that even if *b* were to be added to itself without limit, it would not produce a line greater than *a*. And by similar reasoning, if we suppose *a* to be the speed in a void, and *b* the speed in air, an unlimited number of speeds, greater than *b* and smaller than *a*, will be able to exist between *a* and *b*. Yet we must not conclude that *a* is infinitely greater than *b*, in the sense that the time in which [the motion with] speed *a* is accomplished, when added to itself any number of times without limit, can still never exceed the time corresponding to speed *b*, and that, therefore, the speed corresponding to time *a* is instantaneous.

It is therefore clear how the argument is to be understood. "The lightness of a void infinitely exceeds the lightness of a [nonvacuous] medium; therefore the speed in the void will infinitely exceed the speed in a plenum." All that is conceded. What is denied is the conclusion: "Therefore the speed [i.e., the motion] in the void will be instantaneous." For such motion can take place in time, but in a shorter time than the time corresponding to the speed in a plenum; so that between the time in the plenum and the time in the void an unlimited number of times, greater than the latter and smaller than the former, may exist. Hence it follows, not that motion in a void is instantaneous, but that it takes place in less time than the time of motion in any plenum.

Therefore, to put it briefly, my whole point is this. Suppose there is a heavy body *a*, whose proper and natural weight[60] is 1000. Its weight in any plenum whatever will be less than 1000, and therefore the speed of its motion in any plenum will be less than 1000. Thus if we assume a medium such that the weight of a volume of it equal to the volume of *a* is only 1, then the weight of *a* in this medium will be 999. Therefore its speed too will be 999. And the speed of *a* will be 1000 only in a medium in which its weight is 1000, and that will be nowhere except in a void.

This is the refutation of Aristotle's argument. And from this refutation it can readily be seen that motion in a void does not have to be instantaneous.

The other arguments of Aristotle are without force or cogency. To say, for example, that in a void the body will not move in one direction rather than in another, or up rather than down, because the void does not give way upward or downward but equally in all directions,[61] is childish. For I could say the same thing about air. That is, when a stone is in air, how does the air give way downward rather than upward, or to the left rather than to the right, if the rareness of the air is everywhere the same? At this point someone, quoting Aristotle,[62] might say that air has weight in its own place and therefore helps downward motion more. We shall examine these fantasies in the next chapter, where we shall investigate whether elements have weight in their own proper places. And similarly, when they say that in a void there is neither up nor down, who dreamt this up? If the air were a void, would not the void near the earth be nearer the center than the void which is near [the region of] fire?

Similarly lacking in force is the argument which Aristotle makes about projectiles when he says:[63] "Projectiles cannot move in a void, for projectiles, when they have left the hand of the thrower, are moved by the air or by some other corporeal medium that surrounds them and is set in motion. But this is not present in a void." For Aristotle assumes that projectiles are carried along by the medium; and in the proper place we shall show that this is false. And what he adds to his argument, about different bodies moving in the same medium, is also false. For he assumes that in a plenum heavier bodies move more swiftly because they cleave the medium more forcibly, and that this is the only reason for their speed; but since that resistance is not present in a void, he supposes that all motions in a void will take place in the same time and with the same speed—and this, he asserts, is impossible.[64]

Now, in the first place, Aristotle errs in that he does not prove that it is absurd for different bodies to move in a void with the same speed. But he makes an even greater error when he assumes that the speeds of different bodies depend on an ability of heavier bodies to divide the medium better. For, as we showed above, the speed of moving bodies does not depend on this, but on the size

60. I.e., its weight in a void.
61. Cf. *Physics* 215a.22–24.
62. Cf. *De Caelo* 311b.9.
63. Cf. *Physics* 215a.14–19.
64. *Physics* 216a. 12–21.

of the difference between the weight of the bodies and the weight of the medium. For the speeds are in the ratio of these differences. But the difference between the weights of different bodies and the weight of the same medium is not the same (for otherwise the bodies would be equally heavy). Therefore the speeds will not be equal. For example, in the case of a body whose weight is 8, the excess over the weight of the void (which is 0) is 8; hence its speed will be 8. But if the weight of a body is 4, the excess over the [weight of the] void will, in the same way, be 4; and hence its speed will be 4. Finally, using the same method of proof in the case of the void as we used in the case of the plenum,[65] we can show that bodies of the same material but of different size move with the same speed in a void. So much for that.

Such[66] is the force of truth that learned men, even Peripatetics, have recognized that Aristotle's view on this subject was mistaken, though none of them could properly refute his arguments. And, as for what is contained in *Physics* 4.71–72,[67] certainly no one was ever able to refute that argument, for up to now the fallacy in it has never been noticed. And though Scotus, Saint Thomas, Philoponus,[68] and some others hold a view opposed to Aristotle's, they arrive at the truth by belief rather than real proof or by refuting Aristotle.[69] And, indeed, if one were to accept Aristotle's assumption about the ratio of the speeds of the same body moving in different media, one could scarcely hope to be able to refute Aristotle and upset his proof. For Aristotle assumes that the speed in one medium is to the speed in the other, as the rareness of the first medium is to the rareness of the second. And no one up to now has ventured to deny this relation.[70]

Nor is there any validity in the assumption made by the aforesaid writers, namely, a twofold resistance to the motion of the body—one external, resulting from the density of the medium, the other internal, by reason of the determinate weight of the body.[71] For there is something artificial about this; since those two resistances do not, if we look at the matter carefully, differ from each other. For, as has been made clear above, the density or (to use a better term) the weight of the medium makes for lightness of the moving body, and the lightness of the medium is responsible for the heaviness of the body; and the same body is now heavier, now lighter, according as it is in a lighter or heavier medium.[72] And so, these writers add nothing new when they assume this twofold resistance, since it is

65. [For Galileo's proof, see section 6 below.—*Ed.*]
66. This section to the end of the chapter is on a separate sheet. Galileo merely indicated that it belongs in this chapter; and Favaro [the editor of the National Edition of Galileo's works—*Ed.*] placed it at the end of the chapter.
67. [215a.24—216a.3. Drabkin repeats the numbers cited in note 50.—*Ed.*]
68. Ioannes Duns Scotus, *Opera Omnia* III (Paris, 1891), 102–3 (on Arist. *Phys.* 4. Quaest 12.18,19); D. Thomas Aquinas, *Opera* III (Rome, 1884), 186. *Comm. on Arist. Phys.* IV. ch.8. lectio 12, no. 8; Ioannes Philoponus, *Comm. in Phys. Arist.*, pp. 678–84 (Vitelli).
69. [Duns Scotus, Aquinas, Philoponus, and Avempace (the last-named is mentioned in note 72) are cited because they held, in opposition to Aristotle, that bodies would move in a hypothetical void with finite, not instantaneous, velocities. None of these authors, however, held or asserted Galileo's other conclusion, namely that homogeneous bodies would fall with equal speed in the void. But a view similar to Galileo's was held by Thomas Bradwardine (see Theorem XII, Selection 51.1) and Albert of Saxony (see section 2 above, Question 12), and very likely by others.—*Ed.*]
70. But Philoponus, cited above by Galileo, *does* deny the relation as it is expressed in the form "time required in one medium is to the time required in the other medium as density of first medium is to density of the second." Cf. *Commentary on Aristotle's Physics*, p. 682.30–32 (Vitelli). [This section from Philoponus is translated in M. R. Cohen and I. E. Drabkin, *A Source Book in Greek Science* (Cambridge, Mass.: Harvard University Press, 1948), p. 219; Avempace, whom Galileo does not mention in this connection, agreed with Philoponus on this point.—*Ed.*] G. B. Benedetti also denies the relation (*Diversarum speculationum mathematicarum et physicarum liber* [Turin, 1585], p. 172; see A. Koyré, *Études Galiléennes*, I, 50). From what Galileo says one may question how much direct and detailed knowledge he had, at this time, of either Philoponus' or Benedetti's work. [The remainder of this note is omitted.—*Ed.*]
71. [The internal resistance described here by Galileo is not that frequently accepted in the Middle Ages, which was held to arise either from light elements acting in an opposite direction to the predominant heavy elements of a body, or vice versa (see Selection 51.1 n. 9). Nor is it properly attributable to Duns Scotus or to Thomas Aquinas, as Galileo declares. Indeed, since no explicit statement is made by either Philoponus or Avempace (see n. 72), it is also highly questionable whether it is proper to interpret them as construing the weight (or force) of a body as a kind of resistance which is instrumental in producing finite motions in the void. There is, however, little doubt that such a position was distinguished in the Middle Ages (see Albert of Saxony's discussion in Selection 47). It is worth noting that those who accepted the type of internal resistance arising from contrary tendencies of light and heavy elements would have rejected the assumption that a pure elemental body, say earth, could move with a finite speed in a hypothetical void. However, this assumption would probably have been adopted by those who accepted the internal resistance described by Galileo.—*Ed.*]
72. Galileo's meaning may be clarified by the memo-

merely increased or decreased according to the decrease or increase[73] in the heaviness or density of the medium. But if, on the other hand, they admit that the resistance increases or decreases in the *ratio* in which the weights of the medium vary, their attempts to upset Aristotle's argument will be in vain.

4. Thomas Bradwardine (ca. 1290–1349) and
5. Albert of Saxony (ca. 1316–1390): Unequal Homogeneous Bodies Fall with Equal Velocity in a Vacuum

Introduction by Edward Grant[74]

To arrive at the conclusion that two bodies of similar or homogeneous composition but unequal volume and weight will fall with equal speed in a vacuum, Thomas Bradwardine, Albert of Saxony, and Galileo, in his early work *De motu* (*On Motion*), used intensive rather than extensive factors of analysis. Since the bodies were assumed to be homogeneous, it followed that the parts of each body, however small or large, were identical. Consequently, their behavior was identical. The whole body, which was merely an aggregate of identical units, would behave exactly as every one of its constituent parts behaved. The smallest unit would fall with the same speed as an aggregate of such units formed into a large body. Subtraction or addition of identical units could neither diminish nor increase the speed of the body or its units. By contrast, Aristotle was concerned exclusively with the body as a gross or extensive object. Homogeneous bodies of different size, and therefore of unequal weight, would fall with different speeds because their total weights differed.

In coming to the same conclusion, however, the two medieval authors, Bradwardine and Albert, employed a ratio of force to resistance per unit of matter, whereas Galileo used weight per unit volume. For Galileo, the concept of effective weight rather than gross weight was the determinant of velocity. But the effective weight, or heaviness, of a body in a medium depended on the difference in the specific weights of the body and the medium through which it fell. Hence, it is actually a difference in specific weights which determines velocities. The free fall of a body may be represented as $V \propto$ *specific weight of body* minus *specific weight of medium*, where V is speed;[75] the velocity of a rising body would be $V \propto$ *specific weight of medium* minus *specific weight of body*. Clearly, then, if the specific weights of two unequal bodies are the same, they must fall with equal speeds in the same medium or in a vacuum. This reasoning underlies Galileo's argument at the conclusion of section 6 below, namely that if bodies *a* and *b* were equal in size and of the same composition they would fall with equal speed, but if joined together would not double that speed, as Aristotle held, but would fall with the same speed as when they fell individually, since bodies of the same material should fall with the same speed.

Despite a significant similarity in reliance on intensive rather than on gross or extensive factors, our two medieval authors formulated their interesting conclusions within the framework of Aristotelian physics, whereas, already in *De motu*, Galileo has moved outside of that framework and into early modern physics.

Where Bradwardine and Albert of Saxony accept without question the notion that heaviness and lightness are absolute properties of bodies and believed they operated as contrary forces—one as motive force, the other as internal resistance—in mixed or compound bodies, Galileo has completely abandoned absolute heaviness and lightness. He insists that everything has weight and that things are only less heavy, equal to, or heavier than other things; void alone is weightless, and it is not a substance. Adopting the notion of relative density, Galileo utilized it in its more precise form of specific weight. While the true weights of bodies could only be determined in the void, their effective weights in a given medium could be determined by comparing the difference in specific weights between each body and the medium.

By abandoning absolute heaviness and lightness and adopting the concepts of relative density and specific weight, Galileo could avoid many of the strange consequences, distinctions, and explanations that were unavoidably associated with the use of the absolute heaviness-lightness dichotomy. Where Bradwardine, Albert, and medieval Aristotelians generally, explained direction of motion in terms of absolute lightness and heaviness functioning as motive qualities, Galileo relied on the relation between the specific weight of body and medium. No longer was it necessary to distinguish the behavior of simple or pure elemental bodies from that of mixed bodies. The distinction forced upon most medieval physical speculators that only certain bodies (mixed) could fall with finite speed in a hypothetical void space and that all others (pure elemental bodies) could not became utterly meaningless in the physics of Galileo's *De motu*, where, by virtue of the concept of specific weight, *all* bodies were to be treated alike, regardless of composition, and *all* bodies would fall with finite speed in void and plenum. With Galileo, homogeneous Archimedean magnitudes replaced the simple and heavy mixed bodies of the later Middle Ages. The emptiness of this distinction between bodies in the analysis of motion and the rejection of absolute heaviness and lightness destroyed the need for the medieval concept of internal resistance. Internal resistance, which, in the Middle Ages, had been invoked to permit an explanation in dynamic terms for finite motion in the void, depended upon the contrary tendencies of elements distinguishable as light and heavy in a mixed body. In downward motion heavy and light functioned as motive force and internal resistance respectively; in upward motion their roles were reversed. Galileo, however, required neither internal nor external resistance for the production of finite speed in a void, where the speed of a falling body would be directly proportional to its specific weight.

Thus where specific weight provided Galileo with a consistent standard of comparison for the motions of bodies in void and plenum, those who relied on internal resistance in mixed bodies were led to peculiar and often bizarre conclusions. Thus Albert of Saxony would argue that a heavy mixed body might fall more slowly at the end of its natural motion than at the beginning;[76] that one and the same heavy mixed body could be moved quicker in a plenum than in a vacuum;[77] and that even if one compound heavy body were smaller than, or equal to, another of the same homogeneous composition, it would descend more quickly in a plenum.[78] Bradwardine deduced similar paradoxical consequences.

For the remarks of Bradwardine and Albert of Saxony on this subject, see Theorem XII in Selection 51.1, and the eighth conclusion of the second section above.

6. Galileo Galilei: Unequal Homogeneous Bodies Fall with Equal Velocity in a Vacuum

Translated by I. E. Drabkin[79]
Annotated by Edward Grant

Similarly, if two bodies move downward [in natural motion], one more slowly than the other, for example, if one is wood and the other an [inflated] bladder, both falling in air, the wood more swiftly than the bladder, our assumption is as follows: if they are combined, the combination will fall more slowly than the wood alone, but more swiftly than the bladder alone. For it is clear that the speed of the wood will be retarded by the slowness of the bladder, and the slowness of the bladder will be accelerated by the speed of the wood; and, as before, some motion will result intermediate between the slowness of the bladder and the speed of the wood.

randum (410.21–26): "Philoponus, Avempace, Avicenna, Saint Thomas, Scotus and others who try to maintain that motion takes place in time [i.e., not instantaneously] in the void, are mistaken when they assert a twofold resistance in the moving body, viz., one accidental and due to the medium, the other intrinsic and due to the body's own weight. These two resistances are clearly one, for the medium, insofar as it is heavier, both offers more resistance and [by that very fact] renders the body lighter."

That is, though one may, by abstracting, analyze the weight of a body into two factors (weight of the body in a void, and weight of the medium in a void), these factors do not act independently and are not separable in the actual case of a body in a medium.

73. "Decrease or increase" seems to be a slip for "increase or decrease."

74. This introduction is partly a rearrangement and partly a verbatim reproduction from pages 356–359 of my article "Bradwardine and Galileo: Equality of Velocities in the Void." *Archive for History of Exact Sciences*, II (1965), 344–364.

75. For the relation of Galileo's law to the tradition stemming from Avempace, see note 58 and Selection 44, n. 17.

76. *Questions on the Physics of Aristotle*, fol. 50 verso, col. 1., of the edition cited in Selection 35, n. 1.

77. See the third conclusion from Book IV, Question 11, Selection 55.2.

78. See the second part of the eighth conclusion of Book IV, Question 12, Selection 55.2.

79. Reprinted with the kind permission of the Regents of the University of Wisconsin from *Galileo Galilei "On Motion" and "On Mechanics" Comprising "De Motu" (ca. 1590)*, translated with Introduction and Notes by I. E. Drabkin, and *"Le Meccaniche" (ca. 1600)*, trans-

On the basis of this assumption, I argue as follows in proving that bodies of the same material but of unequal volume move [in natural motion] with the same speed. Suppose there are two bodies of the same material, the larger *a*, and the smaller *b*, and suppose, if it is possible, as asserted by our opponent, that *a* moves [in natural motion] more swiftly than *b*. We have, then, two bodies of which one moves more swiftly. Therefore, according to our assumption, the combination of the two bodies will move more slowly than that part which by itself moved more swiftly than the other. If, then, *a* and *b* are combined, the combination will move more slowly than *a* alone. But the combination of *a* and *b* is larger than *a* is alone. Therefore, contrary to the assertion of our opponents, the larger body will move more slowly than the smaller. But this would be self-contradictory.

What clearer proof do we need of the error of Aristotle's opinion? And who, I ask, will not recognize the truth at once, if he looks at the matter simply and naturally? For if we suppose that bodies *a* and *b* are equal and are very close to each other, all will agree that they will move with equal speed. And if we imagine that they are joined together while moving, why, I ask, will they double the speed of their motion, as Aristotle held, or increase their speed at all? Let us then consider it sufficiently corroborated that there is no reason *per se* why bodies of the same material should move [in natural motion] with unequal velocities, but every reason why they should move with equal velocity. Of course, if there were some accidental reason, e.g., the shape of the bodies, this will not be considered among causes *per se*. Moreover, as we shall show in the proper place, the shape of the body helps or hinders its motion only to a small extent.

56 ON INTERSTITIAL VACUA

1. Marsilius of Inghen (ca. 1340–1396): Explanation of Condensation and Rarefaction, and Denial of Interstitial Vacua

Translated and annotated by Edward Grant[1]

In the fourteenth [question], we inquire whether it is possible that anything be rarefied and condensed.

And it is argued first that it is not possible. . . .

Aristotle argues the opposite of this at the end of the fourth book [of the *Physics*] in the treatise on the vacuum. There are many ways of responding to this question. One opinion assumes that condensation occurs by the entrance of the parts of a condensible body into certain separate vacuities within the parts of the condensible body; and rarefaction occurs by the generation of such vacuities. Against this opinion can be adduced all the arguments by which it was proved that it is impossible for a vacuum to exist.[2]

A second way assumes that condensation occurs by the departure of a more subtle body as appears with respect to a sponge, which is condensed by the

lated with Introduction and Notes by Stillman Drake (Madison, Wis.: University of Wisconsin Press, 1960), ch. 8, pp. 29–30. See also the end of section 3 above.

Although, in the argument reproduced here, the homogeneous but unequal bodies are falling in a uniform medium, there is no doubt of Galileo's conviction that the same argument could demonstrate that homogeneous bodies would also fall with equal speeds in a void. For he says later (Drabkin, p. 45) that a "body will move in a void in the same way as in a plenum. For in a plenum the speed of motion of a body depends on the difference between its weight and the weight of the medium through which it moves. And likewise in a void [the speed of] its motion will depend on the difference between its own weight and that of the medium. But since the latter is zero, the difference between the weight of the body and the weight of the void will be the whole weight of the body. And therefore the speed of its motion [in the void] will depend on its own total weight." Since, throughout this passage, "weight" signifies "specific weight," and since bodies move in the void in the same way as they do in a plenum, Galileo by the same method described here could easily have demonstrated equality of speed for homogeneous bodies falling in a void. Deeming it superfluous, he remained content with the declaration that "using the same method of proof in the case of the void as we used in the case of the plenum, we can show that bodies of the same material but of different size move with the same speed in a void" (Drabkin, pp. 48–49). In the First Day of the *Dialogues Concerning Two New Sciences (Discorsi)*, published in 1638, Galileo modified and corrected this opinion when he argued that *all* bodies of whatever composition and specific weight fall with equal uniformly accelerated speeds in the void, but not in a plenum. See the translation by Henry Crew and Alfonso de Salvio (New York: Macmillan, 1914), pp. 72, 74.

1. Translated from Book IV, Question 14, on folios 55 verso to 56 recto of Marsilius of Inghen's treatise cited in Selection 50.1, n. 1.

2. Marsilius argued this in the preceding, or thirteenth, question, from which an excerpt has been included in Selection 53.3.

explusion of air and rarefied by the entrance of air or water. I argue against this opinion, for if every condensation and rarefaction occurs in this way, then it follows that no body would be uniform [or homogeneous] except the most simple or subtle body. The consequent is false, because many stones or metals are uniform bodies. The consequence is proved,[3] because then such a [subtle and simple] body could not be condensed by the departure of a more subtle body, since there is no subtler [or rarer] body in it, for it is uniform throughout.

A third opinion is that condensation occurs by the penetration of parts of a body by the intrusion of the bodies [or parts] at its circumference [or surface] into the parts of the body situated around its center. And yet the body cannot be condensed to any extent whatever, because in every species there is a certain degree beyond which no individual of the species can be condensed. Therefore, it happens that bodies cannot mutually interpenetrate to any extent whatever but only to the extent that condensation can take place. And [in this opinion] rarefaction occurs by departure of one condensed body [or part] from another part. But one can argue against this opinion, because then it would follow that air, no part of which penetrates another part, could not be rarefied. The consequent is false, because fire can be made from air;[4] and the consequence is proved, because rarefaction occurs only by the departure of one part of a rarefiable body from another part, where these parts [had been assumed] previously to be mutually interpenetrable. A second argument is that if it happens that by condensation two bodies can mutually interpenetrate, it follows, by the same argument, that three, or four, or indeed an infinite number of bodies can mutually interpenetrate, as the Philosopher argues in this fourth book.[5]

After dismissing these opinions it must be noted that "condensation" is said to occur when any body is made smaller without the withdrawal of any part of it; and "rarefaction" is said to occur when a body is made larger without any other body entering from outside. In this respect, condensation and rarefaction differ from increase and decrease, for increase *(augmentatio)* occurs when some body is added, and decrease *(diminutio)* occurs when body is removed, as is clear in the first book of *On Generation*. Secondly, it must be noted that condensation is found in three ways. Sometimes it is violent, and this occurs only by compression of the condensed body—as when wax is compressed. In another way condensation occurs by means of a previous alteration. And in a third way it occurs from a previous generation.[6] Now these conclusions are posited.

The first is this: it is possible that something be condensed by compression alone without any other change having been made prior to the condensation. This is proved, for unless it were true, it would follow that no rectilinear local motion could be made. The consequent is contrary to experience,[7] but the consequence is proved, because the medium in which such a motion is made yields to the mobile, for otherwise there would be a penetration of bodies. Therefore, either the condensation occurs by the compression of the mobile—and the proposition is had—or another body yields to it, and a third body yields to that body, and so on up to the sky, which is impossible, because either the sky would yield to that body [which pressed against it] or that body would penetrate the sky. Each of these alternatives is absurd.

A second conclusion is this: It is possible that a body could be condensed by alteration without generation. For if a glass vessel were taken and heated in fire and the mouth of the vessel were then immediately placed in water with its bottom

3. The consequence declares that if condensation occurs by the departure of a more subtle body, then only the most subtle or simple bodies could be homogeneous. Since it follows that homogeneous bodies could not be condensed, the theory is absurd. The role of a "subtle matter" in increase and decrease is discussed by Nicholas of Autrecourt in section 2 (see also n. 40).

4. The consequent referred to is the statement in the preceding sentence that air could not be rarefied. This is simply false, Marsilius argues, presumably by appeal to the universally accepted opinions that fire can be made from air and that fire is rarer than air. Thus when air is converted to fire it is rarefied. In the very next sentence Marsilius goes on to show that although the consequent is false, it follows from the rejected opinion. For if rarefaction occurs only by the exit of one part from another part of previously interpenetrated parts, *and* the parts of air do not mutually interpenetrate, it follows that air will not rarefy.

5. In Physics IV.8.216b. 3–11, Aristotle argues that the volume of a cube placed in a void will occupy an equal volume of void, which will penetrate it. Thus the dimensions of cube and void will occupy the same place. "And if there can be two such things," declares Aristotle, "why cannot there by any number coinciding? This, then, is one absurd and impossible implication of the theory."

6. Marsilius illustrates each type further on.

7. That is, the consequent that "no rectilinear motion could be made" is contrary to our experience and hence is false. Therefore, it must be true that condensation could be made by compression alone, and Marsilius goes on to show this.

upward, it would appear obviously that the air is condensed from the cold by alteration;[8] and as the air is condensed so does the water rise, lest a vacuum be produced.

A third conclusion is that it is possible for a condensation to occur by means of a previous alteration *and* generation. This is proved, because it would otherwise follow that a denser element could not be made from a rarer element, which is false,[9] as is obvious in the second book of *On Generation*.

But there might be a doubt, first about the mode of condensation, i.e., how it happens, whether by the penetration of bodies or in some other way. Let the reply be that condensation occurs in this way: The parts [of the body] lie closer together than before. This is not true of the immediate parts,[10] but any of the mediate parts lie closer than before. Thus any two points that may be assigned in a condensed body are mutually closer than before condensation.

A second doubt might arise about whether or not something is lost by condensation; the same doubt may arise as to whether something is acquired in rarefaction. The reply to this could be made in different ways depending on the different opinions about quantity. If quantity is assumed to be a thing distinct from substance and quality, then quantity is a dimension by means of which something is extended, as whiteness *(albedo)* is a certain quality by means of which something is white *(album)*. In this event, it must be said that a new quantity is acquired in rarefaction and lost in condensation. But if every quantity were assumed to be a substance or quality, then it is not necessary that something be lost in condensation and acquired in rarefaction. . . .[11]

2. Nicholas of Autrecourt (b. ca. 1300; d. after 1350): The Existence of Interstitial Vacua Affirmed

Translated and annotated by Edward Grant[12]

ON VACUUM

We have probably destroyed many discussions. Now we want to take into consideration certain matters about which we have to say some unusual things, much of it in the previous manner. And [so] let us begin a discussion on the vacuum: whether this description that has been assumed reveals the "quid" of the word ["vacuum," namely] that by the word "vacuum" we understand that in which there is no body [but in which], nevertheless, a body can be [or exist]. And it seems that it does, because if not it would follow (1) that rectilinear

8. The alteration thus involves a change of quality from hot to cold.

9. The conversion of air to water, for example, would be conceived as a substantial change (or generation), since one element has been generated from another. It is also a qualitative change (or alteration), since air possesses the qualities hot and moist and water moist and cold. The change from air to water requires that hotness replace coldness.

10. Immediate parts are already in contact and cannot lie closer together.

11. Thus Marsilius allows that condensation and rarefaction can be properly interpreted on the basis of two different conceptions of quantity. If quantity is merely dimension or extension, then condensation, which is obviously a loss of extension, is necessarily a loss of quantity; and rarefaction will be a gain of extension and consequently of quantity. But if quantity is thought of as a substance or a quality, then nothing is lost or gained, presumably because the condensed or rarefied body will not possess more or less of whatever substance it is, a substance that will retain its identity; nor would it lose or gain with respect to quality, since its qualities would remain unaltered if its identity were preserved, regardless of the size it would attain as a consequence of rarefaction or condensation.

A minor objection has been omitted as well as very brief responses to the six principal reasons that were enunciated at the very beginning of the question and which were also omitted.

12. Translated from Autrecourt's *Exigit ordo executionis*, edited from a single known manuscript by J. Reginald O'Donnell, C.S.B., in "Nicholas of Autrecourt," *Mediaeval Studies*, I (1939), 179–280. The section on vacuum extends over pages 217–222.

In advocating the probable existence of indivisible atoms and interparticulate vacua Nicholas of Autrecourt either stands alone in the Middle Ages or belongs to a very small group whose members are not yet known. Believing that things and qualities do not undergo change (that is, are neither generated nor corrupted), Nicholas sought to account for apparent change by explaining generation as the coming together of indivisible, eternal, and unalterable atoms and corruption of bodies by the dissociation of these atoms. In the present selection he seeks not only to justify the existence of interstitial vacua as essential for a proper explanation of motion but attacks Aristotle's physics of motion based upon the existence of a plenum and the total denial of vacua of any kind. Unlike the ancient atomists Nicholas does not argue for an infinite universe and a plurality of worlds.

A thorough study and analysis of Nicholas' point of view appears in Julius R. Weinberg, *Nicolaus of Autrecourt: A Study in 14th Century Thought* (Princeton, N. J.: Princeton University Press, 1948).

local motion could not be; or (2) that two bodies could exist simultaneously [in the same place]; or (3) that with one thing moved, all things would necessarily be moved and change place. Thus in nothing can we persist so probably as in a vacuum; therefore vacuum must be assumed.

That all these positions are more absurd than the assumption *(positio)* of a vacuum seems to be sufficiently known and it should be conceded to us that they would follow. This is shown, because when this body changes place either (1) the place in which it was received was a vacuum and thus what has been proposed [is had], or (2) it is filled with a body and then either (a) the body remains and there will be two bodies [in the same place] simultaneously, which is another absurdity, or (b) it withdraws [from that place] and crosses to another place; and then I inquire in the same way as before.[13] By proceeding in this way, either you concede that the process always goes to infinity, or that two bodies will exist [in the same place] simultaneously; or you must concede a vacuum.

But here it might be replied that finally it [that is, the process of motion from one place to another] will terminate because a condensation will occur.[14] But we cannot relate *(dicere)* this discussion with the negation [or denial] of the vacuum, although, perhaps, Aristotle could according to his principles. For we do not assert that "dense" occurs by means of the generation of some new quality which did not exist before,[15] but [rather we declare] that "dense" occurs only by the retreat [or compacting] of bodies, as in wool; or because the parts come together, that is, because more [parts] are related more closely than before. And "rare" will not come to be unless the parts of this body are more separated than before. Thus "dense" or "rare" do not come to be except by the local motion of the parts. And so it is that another interpretation of rare and dense is of no help to us.[16]

There is another response which I heard from a certain worthy master which seems more probable with respect to Aristotle and our conclusion.[17] Let it be imagined that in a rectilinear local motion the situation is as follows: The motion continues [or endures] because it is conceived as if it were on a circle, so that a body located in *A* will be moved to place *B* and the body located in *B* will be moved to place *A* and so it will continue [in this way].[18] But this does not seem to suffice, [for] let us [now] imagine that around this space there are differences of position [or direction, namely] in front *(ante)*, behind *(retro)*, and so on. Now let this mobile be moved in a straight line. It seems that what is [directly] behind it, say air, could reach the place previously occupied by the mobile before what was in front of the mobile could reach that [very same] place; and what was behind it would enter that place; and so [it would happen that] the state

13. That is, if a body departs to make room for another, we must pose the same alternative about the new place which it will occupy.

14. If matter becomes more dense somewhere in this process, space will become available to accommodate all bodies, and the process will not be infinite.

15. That is, Aristotle could deny a vacuum and accept condensation or compression of matter because his explanation of the way bodies become dense and rare involves the generation of a new quality and does not, as Nicholas describes in the following sentences, involve compression and extension of particles in vacua (see Weinberg, pp. 160–161). For Aristotle, dense and rare are contrary qualities possessed at different times by one and the same matter. When a rare matter like air is generated from a dense matter like water, this is a qualitative change which one and the same quantity of matter undergoes (see *Physics* IV.9.217a.20—217b.26).

16. Perhaps Nicholas is making the point that Aristotle's interpretation of rare and dense is of no use to him because "rare" and "dense" involve local motion of parts or atoms. Hence even if a condensation occurred to terminate the process of motion from one place to another, the same problems described in the preceding paragraph would arise with respect to the parts of the body: Either the process goes to infinity or two bodies would exist in the same place simultaneously. If these alternatives are unpleasant, one ought to concede the existence of vacua.

17. What follows "will be more probable with respect to Aristotle and our conclusion" because it will show the absurdity of Aristotle's explanation and be favorable to Nicholas's position.

18. This explanation of rectilinear motion was frequently called "mutual replacement" or *antiperistasis*, the Greek term used by Aristotle in his brief mention of it in *Physics* IV.8.215a.15 and VIII.10.267a.15–21. It is clear that Aristotle did not utilize it to explain continuous violent rectilinear motion produced by a motive power in media such as air or water, because in "mutual replacement" "all the members of the series are moved and impart motion simultaneously, so that their motions also cease simultaneously: but our present problem concerns the appearance of continuous motion in a single thing, and therefore, since it cannot be moved throughout its motion by the same movent, the question is, What moves it?" (*Physics* VIII.10.267a.18–21). Thus while Aristotle may have accepted "mutual replacement" for nonrectilinear motion or, perhaps, for natural up or down rectilinear motion, he does not regard it as an appropriate explanation for violent rectilinear motion (his explanation of the latter appears in the section immediately preceding our quotation [VIII.10.266b.28–267a.15]). It seems reasonable to conclude that if Nicholas were criticizing Aristotle—and we note that his name is not explicitly associated with this argument—his

described above would never exist[19] unless a vacuum were conceded. Now, a body which is in front of a mobile does not appear to be moved unless it is pushed and yields. Now I ask whether in the same instant in which what is in *A* is [also] in *B*, that which is in *B* is [also] in *A*, and then it does not appear that this [that is, what is in front] yields more to this mobile than conversely [that is, than the mobile yields to what is directly in front of it]. Then how does it yield as if pushed?[20] Or, [if what is in *B* will be in *A*, or vice versa] in another [and different] instant and then, in this first instant, either two bodies were simultaneously [in the same place] or there was a vacuum. . . .[21] What I have said about the immediate instant you should understand in terms of the conclusion posited above, where it was said that a continuum is composed of indivisibles and time of instants.[22]

Let another argument be adduced with respect to new wine in a jar. When it becomes rarefied, there appears to be a greater quantity, not because of the generation of a new quality as above, nor because of the advent of new bodies, but only, it seems, because the parts, having [previously] come together as if there were a certain trampling down, now separate and are more distant from each other. What seems more intelligible in the description of rare and dense supports this argument. The usual description of dense is that "dense" is something whose parts lie closer together, so that water does not seem denser than air unless it is because its parts are mutually closer.[23] And so it is that the parts in a rare body are in some way further separated so that a vacuum intervenes—that is, the parts could be mutually closer [yet]. Hence the vacuum is something where there is no body [but] where, nevertheless, there could be a body. And this is how I expound this matter, for those who urge the opposite position seem to frighten people merely by the term ["vacuum"].

It is true that others use a description of "dense" in another signification, declaring that that is dense which yields less to the touch and "rare" is that which yields [readily to the touch]. But nothing of what the understanding seeks in this—namely *why* this body is denser than another—is determined by this approach. Moreover, they fall into a description of other things, namely of "the solid" *(solidi)* and "the light" *(levis)*. Whence it is that the concept of dense is grasped by the understanding by the sight of the body alone immediately upon seeing the body, because when we see it we say that it is dense. But "solid" is grasped by the understanding by means of touch, so that it could be said that a solid is that which yields to the touch with difficulty, and "light" is the opposite. And if we could say this, the investigation would be over, based on a solution of contraries. But I do not wish to determine this fully now.

And first, we should see how the assumption of a vacuum destroys the objection posited above, from which it was concluded that there is either no [rectilinear] local motion, or with one thing moved, all things are moved, and so on. It must be said that [on the assumption of a vacuum] this will be understood in local motion. For when a body is moved locally in air, the parts of the air are in some way compressed and come together, and thus it is not necessary that such a process continue to infinity or that two bodies exist simultaneously [in the same place]. And you might imagine that what was [just] stated could be explained in a manner similar to this: Assume that the subtle bodies

criticism could only be relevant to natural rectilinear motion. From the general tenor of the discussion it seems likely that Nicholas was rejecting the "mutual replacement" interpretation for both natural and violent rectilinear motion. At best, then, only that part which is relevant to natural motion might appropriately be used against Aristotle.

19. The situation just described requires that when body *a* in place *A* moves to place *B*, body *b* in place *B* must move to *A*. But, according to Nicholas, this will not occur. For if *b* is "in front" of *a* and *a* moves into *B*, a unit of air, say *c*, directly behind *A* will move into *A* before *b* itself can come around and occupy *A*. Thus *a* and *b* will be incapable of exchanging places. Only the assumption of a vacuum could circumvent this dilemma.

20. Another difficulty with "mutual replacement" arises in distinguishing between mover and moved. For if at the same instant, body *a* moves to *B* and body *b* moves to *A*, how can pusher and pushed be differentiated?

21. Now the bodies are assumed to exchange places not simultaneously but in different instants. Thus, if body *b* moved into place *A* in a first instant and body *a* moved into place *B* during the next, or second, instant, bodies *a* and *b* would be in place *A* simultaneously during the first instant. (A brief but difficult sentence, which plays no role in the argument, has been omitted.)

22. Discussed in a preceding section, "On Indivisibles."

23. Are we to assume that the only difference between air and water lies in the degree of separation of their constituent atoms? Or, do atoms of water differ from those of air? Considerations of this kind were not raised by Nicholas, perhaps because the main thrust of his discussion was to demonstrate the general plausibility and probability of the existence of atoms and void and to discredit traditional Aristotelian explanations. A detailed and comprehensive application of atomism to the physical world was apparently not attempted.

which are said to exist in wool are not interstitial[24] and that someone is moved in a room filled with wool; the parts [of the wool] would come together more and a greater proximity [of parts] would occur. If there are any good reasons against this conclusion, they would seem to be those of Aristotle in the fourth book of the *Physics*, for although they are few, they are by no means trivial. Against those who assume an internal [or interstitial] vacuum Aristotle adduces certain arguments in the fourth book of the *Physics*. But these arguments were directed more against the causes [or reasons] which led them to assume a vacuum than against the conclusion itself. But we do not use these causes [or reasons]. And the argument which follows seems very difficult and is thought to be the most subtle of his arguments, at least against one way of assuming a vacuum. [The argument is:] If a vacuum existed, it would follow that local motion would occur in an instant. The consequence is proved, because as is a ratio of medium to medium so is a ratio of motion to motion in speed and slowness, since it seems that a slower motion results from a greater resistance of a medium. But there is no ratio of a full medium to a void medium; therefore [there is] no [ratio] of motion to motion with respect to speed and slowness. Now if motion through a vacuum occurred in [some] time, there would be a certain ratio between the times; [but this does not happen]. Therefore, there will be motion in an instant.[25]

We ought to see first that this argument does not tell against the kind of vacuum which I have assumed, nor did Aristotle wish to adduce [it against this kind of vacuum][26] for we do not posit a pre-existing separate vacuum *(vacuum separatum praeexistens)* through which a motion could occur, but [rather we assume a vacuum only] between the parts of a body, and when the body must be moved locally those parts come together and are nearer each other. Now, if you wish to speak about the motion of this body whose parts come together [or compress] and which is first assumed to be moved through a medium–although it could [also] be received into a vacuum; nevertheless it was first assumed to be in a plenum—then, other things being equal, to the extent that these parts are compressed [or come together] faster so is the body moved faster, just as if we were to assume that it is moved through a pre-existing vacuum.[27] But although this [that is, Aristotle's] argument does not oppose us on this, it does oppose us in another way concerning the parts of the body which are compressed; for according to this argument, the parts come together within [the body] where there was a pre-existing vacuum [that is, pre-existing interstitial vacua], and thus it seems that the motion of these parts occurs in an instant.[28]

It can be shown from other ways that this argument does not conclude. As to whether the consequent is absurd, see above in the treatise [or section] on motion, for a mobile can be for an instant in any place in which it was not previously, and if by motion you understand something composed of several instantaneous changes *(ex pluribus mutatis)*, then motion would not occur in a single instant but in several. And whatever there might be about this consequent that we did not say is absolutely absurd above in the treatise [or section] on motion, I say that the consequent does not hold if one understands properly the manner in which the speed and slowness of one mobile is measured with respect to another as stated above in the treatise [or section] on the continuum when I replied to Aristotle's arguments.

Thus, leaving aside the agent, it [that is, a mobile] is not said to be slower unless in some way it rests.

24. The compression of wool would have been described by Aristotelians as the squeezing out of "subtle bodies" (presumably air) between the particles of wool. Nicholas suggests that if the existence of such "subtle bodies" between the particles of wool be denied, no good reasons could be suggested for supposing that the particles will not simply squeeze close together if someone walks into a room filled with wool. A similar explanation would be applicable to account for the compression of air.

25. A considerably altered representation of Aristotle's discussion in *Physics* IV.8.215b.20–216a.11.

26. The argument just described was intended by Aristotle for use against the existence of a separate vacuum and not against interparticulate vacua.

27. For Nicholas, the speed of a body is a direct function of the speed of its constituent atoms as they come together and make the body denser. How this is to be understood is not made clear. Is the body's speed greater because it becomes denser as a whole, or is the speed solely a function of the speeds of the individual particles? Since the speed of a body is made to depend primarily on the motion of its constituent particles, it follows, as Nicholas says, that a body could move in a hypothetically pre-existing separate vacuum (although he did earlier deny its existence) as well as in a material medium. Presumably, the external resistance of a material medium would act to reduce its speed.

28. Nicholas recognizes that although Aristotle's claim of instantaneous motion in a separate vacuum is inapplicable to his position, since he rejects the existence of such vacua, it might apply to atoms moving in the interstitial vacua. Thus it might be relevant on the microlevel of atomic motion.

One must therefore reflect that there is a certain essential [characteristic] *(essentialitas)* in motion, [namely] that just as one part of space is before another, so it is traversed before;[29] and this essential [characteristic] cannot be removed [or ignored]. And [now] consider slowness, which occurs because a mobile rests, so that to say that one mobile is slower than another is to say nothing other than that one mobile rests more than another.[30] Thus either rest belongs to [or inheres in] a mobile because of an intrinsic property *(ex determinatione)* of the mobile's nature, or [it occurs] from the resistance of a medium so that the parts of a medium do not come together easily. Now, therefore, should there be motion in a vacuum, why does it not follow that it should have a ratio to motion made in a vacuum in the sense that it had the essential [characteristic] of motion, [namely] that it traversed one part of the space before another, Hence you should not conclude that the motion made over the whole distance was made in an instant. Nor, again, [should you conclude that] there is no rest [for motions in the void]. For it is true to say [that there would be rest] if we do not assert that the total cause of rest in such a motion is the resistance of a medium but [rather assert that] the cause of rest is an intrinsic property *(determinatio)* of the moving thing.[31] And the understanding of succession which I assumed above, [namely] that it should be considered with respect to the order of parts in space, seems [also] to have been the understanding of Avempace.[32]

Other arguments are produced for denying a vacuum—as, [for example,] certain natural experiences *(experientiae)* which do not seem to have any rationale *(habere locum)* unless it be to show the impossibility of a vacuum. The first of these is the clepsydra, which is a vessel having an aperture below and above and filled with water. If a finger is held over the upper aperture, the water does not go forth even though by its [very] nature it should descend; should the finger be removed, the water descends. But that it does not descend in the first case does not seem explicable except by reason of an "abhorrence of a vacuum" *(fuga vacui)*, since a vacuum would exist if the water went forth, because another body could not enter through the top, since it is stopped up [by the finger]; nor could it enter through the bottom, because then two bodies would exist in the same place simultaneously, which is impossible.[33] Let it be said that this argument is not made against the conclusion which we hold, because we hardly suppose that such a separate vacuum *(vacuum separatum)*, which would be left in the vessel after the water had been removed, exists.

Now I have heard a subtle response [to this]: That the water does not flow forth is not because of any "abhorrence of a vacuum" *(fuga vacui)*, but it is imagined that the whole universe has its mode of fullness *(modum plenitudinis)*, so that to the extent that something exists it is able to enter into the universe of existent beings, which is considered to be good. Now, if this body should flow forth, another could not enter, and the whole universe would not be a total plenum and [yet] two bodies would [not] be [in the same place] simultaneously.[34] Now I choose the first explanation,[35]

29. This kinematic argument, used to justify motion in hypothetical separate vacua, was formulated by Thomas Aquinas (see Selection 55.1) and frequently repeated. As we see, Nicholas invokes it here in order to justify temporal motion of atoms through interstitial vacua and subsequently credits Avempace with having understood this argument. But in the form in which it is given here, it was probably derived ultimately from Thomas Aquinas (see Selection 55.1, n. 4).

30. An explanation which Nicholas probably adopted from the description of Arab atomism by Moses Maimonides (see *Guide of the Perplexed*, Bk. I, ch. 73). Earlier in the treatise (O'Donnell, p. 207), after assuming that space and time are composed of indivisible units, Nicholas argued that if a body moves through three units of space, say *a, b, c,* in three instants of time, and another body traverses them in six instants, the first is said to be moved with twice the speed of the second because it moves through each unit of space for only one instant and rests in none, whereas the second also rests for one instant in each unit. Obviously, all this is meant to apply to the motion of individual atoms. Thus not only will atoms move temporally through interstitial vacua but a ratio of the motions of any two atoms can be formed.

31. By assuming that rest is an intrinsic property of atoms and not dependent on the action of an external resistant medium, Nicholas can argue that motions in vacua are comparable. Why one atom should rest for one instant, another for two or three instants, and so on, is not explained.

32. See note 29.

33. The clepsydra illustration is also cited by Marsilius of Inghen (see Selection 53.1 and n. 25 thereto).

34. This strange argument assumes that matter enclosed in a vessel will seek to depart in order to join the totality of existent matter outside the vessel. Some form of the concept of a "community of matter" seems to be operative here.

35. Nicholas opts for nature's abhorrence of a separate vacuum as a better explanation than the one offered in this paragraph.

There is, [however,] doubt about the way in which we assume that a vacuum occurs. Suppose that this body is moved locally; you say that the parts of the body, say air, which were in the place to which the body moved, come together and occupy a smaller place than before.[36] But with regard to this, I inquire about the place from which the air withdraws, and [I ask] whether something could enter there. For if we deny it, then we shall hold a conception of [the term] vacuum that is held by others and we would be assuming a separate vacuum *(vacuum separatum)*; but if we do not deny that something could enter, and something should enter, say air, then, as before, we ask about the place from which this air withdraws. And so the reason which caused us to posit the abhorrence of a vacuum, does not allow us to avoid the other absurdity [of an infinite process].[37] Now, perhaps an appropriate response with respect to this should be to say that the parts of air which were moving behind the body are sometimes extended [or rarefied] so that fewer [parts] coming together fill the place up to a certain magnitude but not into infinity. Now, although we do not have to assume a vacuum on the side behind the body, nevertheless we must assume it for another [reason] lest we fall into an absurdity about the bodies which are in front of the body, [namely] that from one motion all things are moved. Thus, as we stated briefly, we do not approve of this, because [it would happen] that the parts of space would be pushed together in the first instant.[38] But perhaps we can consider this [problem] at another time.

Against what we say about vacuum there is the experience about the inflated bladder which cannot be compressed. Now this would not happen [those who oppose us say] if an interstitial void *(vacuum interceptum)* [actually existed,] enabling the parts to come together. With respect to this [inflated bladder] one of two explanations must be declared:[39] (1) Either [this is explained by the fact that] the parts of the body contained in the bladder cannot easily be compressed further, so that although the parts of air come together and motion occurs this way, they are not, however, so compressed but that between them any [separate] vacuum would remain. Therefore, the parts could be brought together [or compressed] up to a certain point *(ad aliquem modum)*, which does not [however, leave an empty] place in what is in the bladder. [Or] it might be explained in another way by saying (2) that this [lack of further compressibility of the bladder] happens because otherwise there would remain a wholly separate vacuum *(vacuum separatum)* in a part of the bladder, since another body could not enter [to occupy it].

The ancients produced one argument for the assumption of a vacuum, arguing that if there were no vacuum, local increase [or augmentation] could not occur. They used to prove this [by arguing that] since augmentation would not occur unless something were added to a body, either what is added is received in a plenum or in a vacuum. If in a plenum, two bodies would exist simultaneously [in the same place]; if in a vacuum, what has been proposed [is had]. [For] it is not valid to say that [the increase is received] into something that was previously filled by a certain subtle matter; in the first place because it seems to be nothing but [a type of explanation similar to] the abhorrence

36. Under these circumstances Nicholas seems to concede that a vacuum would not occur. For if there is a contraction of the air—whatever the reason—room will be made for the newly arrived body. But now he shifts ground and, in the next few lines, asks about the space which the moving body has vacated. Thus even if one mass of air, *a*, contracts in its place, allowing another mass of air, say *b*, to enter we must ask about the place which *b* has vacated. Either it is a separate vacuum, which Nicholas deems absurd, or since nature abhors a separate vacuum, as Nicholas believes, a body *c* will enter and occupy this place. But then the same question must be asked about the place vacated by *c*. We are therefore involved in an infinite regress.

37. That is, despite the fact that nature abhors a separate vacuum we cannot avoid the absurdity of an infinite process for every motion in a plenum. Only by assuming interstitial vacua can we avoid the dilemma, as Nicholas suggests in the next few lines.

38. If the world is a plenum and body *a* moved into place *B*, body *b* must vacate *B* and move to place *C*, from which body *c* must vacate, and so on. This infinite chain reaction must occur in an instant or no motion would ever cease. To avoid this absurdity, one should assume interstitial vacua.

39. These two explanations are offered by Nicholas as possible responses to his opponents. In each alternative he seeks to explain the absence of compressibility by retaining interstitial vacua and also denying the formation of a separate or continuous vacuum. The essence of such "solutions" lies in the assumption that particles of air will compress up to a certain point or not at all, stopping short at some hypothetical critical point at which a separate vacuum might form. In these ad hoc explanations, nature, acting through its atomic particles, seems to "know" the critical point. Unlike ancient atomism Nicholas' system falls short of a thoroughgoing mechanical determinism and relies on nature to "know" the critical point at which the normal behavior of atoms is suspended and no further compression would occur despite the fact that the interstitial vacua would allow further contraction.

[of a vacuum];[40] secondly because it [that is, the subtle matter] could not occupy a place greater than the body itself. With these [subtle] bodies interspersed, a whole arm was at one time but a magnitude of 1 foot, but it is now a magnitude of 2 feet.[41] Aristotle does not respond to this argument but says that these arguments refute themselves, for let there be a vacuum and then it follows that there will be no augmentation [or increase] because either it follows that any part of the increase will not be an increase or that augmentation does not occur by addition to some body, or that two bodies will exist simultaneously [in the same place], or that the whole body will be increased.[42] The proof of the consequent is this:[43] because if there are vacuities in an increased body, either these vacuities are everywhere—and then the whole will be a vacuum and nothing of it will be body, which is one of the absurdities—or a certain part of the increased body is void and a certain part full, and then either the full part lacking in void could be increased or not. If it be declared that it could be increased, then I ask whether this will occur by adding body or not. If not, the other absurdity holds;[44] if it is increased by adding body, then, with this part having been assumed absolutely full, two bodies, according to them, will then be in the same place [simultaneously].

But saving the reverence of Aristotle, this argument does not tell against them. First let us see what will be understood in augmentation. Assume a certain body that is composed of four indivisibles *A, B, C, D*. I say that there are vacuities between these indivisibles. Let us assume that one indivisible is placed between *A* and *B*. I say that with the advent of a suitable thing, they spread or extend themselves, as it were, so that they are separated; [for example,] say that *A* and *B* are separated by an extent of vacuum equal to two indivisibles and [then] they [namely the vacuities] receive within themselves what is appropriate to their nature.[45] And this is how an augmentation occurs. And the understanding of such a proposition as "Every part of an increased [or enlarged] body is increased" does not at all include the prime parts of the increased body, so that it be understood that the indivisible [part] is increased.[46] But the understanding [of all this] is that [only] after the augmentation of any composite part is obvious to the sense and has been compared to that which preceded the increase is it true to say that it is greater, as a hand is larger *(major)* after an increase than before; and so it is in other things.

Moreover, the ancients argued about a vessel filled with ashes which received as much water as if there were no ashes [whatever in the vessel].[47] Firstly, the Commentator, Averroes, who wished to solve this, says that he has not experienced it;[48] secondly, he says that this is because some parts of the water are corrupted and some parts of the ashes are corrupted. Now, he asserts this because if the ashes are afterward dried they do not appear in so great a quantity [as before]. With respect to the first [statement by Averroes], it seems that he could not be much of a lover of truth to neglect experience so readily;[49] [and concerning the second assertion] it seems dreadful *(fuga)* that he did not oppose what he said [with what the ancients proposed,] because the ancients said that if a smaller quantity appears, this is because the parts are nearer than before.

The ancients argued in [yet] another way[50]

40. By analogy with the explanation that "nature abhors a vacuum," where bodies rush in to fill spaces that might otherwise be left void, this explanation would have grosser matter rush in to replace "subtle matter," where the latter constituted no part of the bulk of the body and the former will now increase the quantity of matter. "Subtle matter" is also mentioned by Marsilius of Inghen (see the beginning of section 1 above).

41. If someone argues that augmentation is merely the replacement of subtle matter by gross matter, then how could a 1-foot magnitude interspersed with subtle matter become a 2-foot magnitude? That is, if a mere replacement is involved, the size of the body should remain constant. It would be as if the grosser matter were merely filling a vacuum. It is because of this that Nicholas likens this explanation to that of "nature abhors a vacuum."

42. See *Physics* IV.7.214b.3–10.

43. Here Nicholas shows how Aristotle's consequences follow from his assumptions. But in the next paragraph, he will reject them.

44. Namely that augmentation does not occur by the addition of matter to a body.

45. It would appear generally that the atomic interpretation of increase advocated here is such that whenever appropriate atoms enter the vacuities between the atoms of a body, immediately neighboring atoms will disperse sufficiently to allow for the subsequent intrusion of additional atoms, at which time another expansion will occur. In this way, a body is always potentially able to increase in size and quantity.

46. Individual particles or atoms do not increase; rather vacuities between atoms are occupied, producing an increase of body.

47. An illustration described by Aristotle in *Physics* IV.6.213b.20–22.

48. See Averroes, *Physics*, Book IV, Text 56 (folio 150 recto of the edition cited in Selection 44.1, n. 1).

49. The charge would seem to be that Averroes failed to test what could have been readily tested.

50. What follows is probably based on Aristotle, *Physics* IV.9.217a.10–19.

against those who denied a vacuum and assumed generation in things, for suppose that from the body of a lesser quantity a body of greater quantity is made, then either it is necessary that there be a vacuum or that a necessary condensation of bodies occur,[51] or that necessarily in another place there be generated [a body] of lesser quantity from a greater [body] in the same proportion as here [in this place] a body of greater quantity is generated from a smaller [body]. Now in order to avoid this, they concede [the existence of a vacuum]. The consequent follows, because the body that has been generated occupies a greater place than it did before.[52]

Again, when the argument concerns a body which is moved locally, it is either received in a plenum or in a vacuum. If in a plenum, two bodies will exist [in the same place] simultaneously; if in a vacuum, what has been proposed is had, [namely that a vacuum exists]. . . .[53]

3. Galileo Galilei (1564–1642): The Existence of Interstitial Vacua Affirmed

Translated by Henry Crew and Alfonso de Salvio[54]

Annotated by Edward Grant

SAGR. It still remains for you to tell us upon what depends the resistance to breaking, other than that of the vacuum; what is the gluey or viscous substance which cements together the parts of the solid?[55] For I cannot imagine a glue that will not burn up in a highly heated furnace in two or three months, or certainly within ten or a hundred. For if gold, silver and glass are kept for a long while in the molten state and are removed from the furnace, their parts, on cooling, immediately reunite and bind themselves together as before. Not only so, but whatever difficulty arises with respect to the cementation of the parts of the glass arises also with regard to the parts of the glue; in other words, what is that which holds these parts together so firmly?

[66]

SALV. A little while ago, I expressed the hope that your good angel might assist you. I now find myself in the same straits. Experiment leaves no doubt that the reason why two plates cannot be separated, except with violent effort, is that they are held together by the resistance of the vacuum; and the same can be said of two large pieces of a marble or bronze column. This being so, I do not see why this same cause may not explain the coherence of smaller parts and indeed of the very smallest particles of these materials. Now, since each effect must have one true and sufficient cause and since I find no other cement, am I not justified in trying to discover whether the vacuum is not a sufficient cause?

SIMP. But seeing that you have already proved that the resistance which the large vacuum offers to the separation of two large parts of a solid is really very small in comparison with that cohesive force which binds together the most minute parts, why do you hesitate to regard this latter as something very different from the former?

SALV. Sagredo has already answered this question when he remarked that each individual soldier was being paid from coin collected by a general tax of pennies and farthings, while even a million of gold would not suffice to pay the entire army.[56] And who knows but that there may be other extremely minute vacua which affect the smallest

51. That is, either space is made for the increased body by means of vacua or neighboring bodies must be compressed.

52. The implications and alternatives just described follow from the fact that the new body would occupy a greater place than before.

53. Approximately the last twelve lines of the section "On Vacuum" are omitted. They embody Nicholas' response to an objection raised by someone who might have been a contemporary (he is referred to as "the latest of the expounders of Aristotle") and are concerned with the relative or absolute nature of vacua.

54. Reprinted with the kind permission of the Macmillan Company from the First Day of *Dialogues Concerning Two New Sciences by Galileo Galilei*, translated from the Italian and Latin into English by Henry Crew and Alfonso de Salvio. . . (New York: Macmillan, 1914), pp. 18–19. Galileo's straightforward physical explanation should be compared to the preceding selection, where Nicholas of Autrecourt gives a complicated scholastic defense of interstitial vacua.

55. A reference to an earlier remark, quoted in Selection 53.4.

56. In connection with increasing the coherence of materials Sagredo had earlier declared (Crew and De Salvio, pp. 13–14): "I was wondering whether, if a million of gold each year from Spain were not sufficient to pay the army, it might not be necessary to make provision other than small coin for the pay of the soldiers." In terms of the present discussion, if a large single force were inadequate, perhaps a great number of minute forces acting together would be more effective.

particles so that that which binds together the contiguous parts is throughout of the same mintage? Let me tell you something which has just occurred to me and which I do not offer as an absolute fact, but rather as a passing thought, still immature and calling for more careful consideration. You may take of it what you like; and judge the rest as you see fit. Sometimes when I have observed how fire winds its way in between the most minute particles of this or that metal and, even though these are solidly cemented together, tears them apart and separates them, and when I have observed that, on removing the fire, these particles reunite with the same tenacity as at first, without any loss of quantity in the case of gold and with little loss in the case of other metals, even though these parts have been separated for a long while, I have thought that the explanation might lie in the fact that the extremely fine particles of fire, penetrating the slender pores of the metal (too small to admit even the finest particles of air or of many other fluids), would fill the small intervening vacua and would set free these small particles from the attraction which these same vacua exert upon them and which prevents their separation. Thus the particles are able to move freely so that the

[67]

mass *[massa]* becomes fluid and remains so as long as the particles of fire remain inside; but if they depart and leave the former vacua then the original attraction *[attrazzione]* returns and the parts are again cemented together.

In reply to the question raised by Simplicio, one may say that although each particular vacuum is exceedingly minute and therefore easily overcome, yet their number is so extraordinarily great that their combined resistance is, so to speak, multipled almost without limit. . . .

Measurement of Forces

57 ON MAXIMUM AND MINIMUM POWERS

John Buridan (ca. 1300–ca. 1358)

Translated and annotated by Edward Grant[1]

Questions on the *De caelo [On the Heavens]* of Aristotle

BOOK I, QUESTION 21

Next, we inquire whether a power [or force] ought to be defined by its maximum capability.[2]

This question is very difficult. And with respect to the intention of Aristotle and the Commentator, it must be noted that their words in opposition [to the arguments set forth above] should not be understood as pertaining to definition or description in a proper sense, because neither an active nor passive power is defined by a maximum or minimum but [rather] by a principle for transmuting another or [being transmuted] by another, just as the reasons given at the beginning of this question argue.[3] But coming now to Aristotle's intention, it must be understood that we do not know how strong a power is other than by its effect, and therefore we judge a motive power to be greater because it can move a greater mobile, [or] other things being equal, [it can move a mobile] quicker or through a greater distance or through a longer time; and so concerning other powers.

It must be said, therefore, that according to Aristotle an active power must be determined by its maximum capability, in the sense that we know how much strength an active power has by knowing its maximum capability or at least by knowing the maximum below which it is able to do everything. And I assume this distinction because of a great difficulty which will be investigated afterward. For some say that it is not a matter of giving the maximum of which a power is capable but of giving the minimum of which it is incapable; and that minimum could be the maximum, not of what it is able to do but below which it is able to do everything. This conclusion is asserted: When[4] we take the lifting power of Socrates or Plato, or of another, we see that that power is greater or stronger which can lift 100 pounds than that which can lift only 50. And we assume this. Then we also assume that any power that can lift 100 pounds can also lift 50 or 40, but not contrarily.

And then we argue as follows: We know by this [that is, the assumptions of the preceding paragraph] how strong a power is, and we can distinguish it from a stronger and weaker power [or force]. But this is by means of its maximum capability, or at least by its capacity to overcome everything below that point, but not otherwise. Therefore, and so on. The major [premise] is obvious. The minor [premise] is asserted, because to know a maximum capability includes two things, namely to know that it is able to do so much and to know that it is not able to do more. That we know it can do so much allows us to distinguish it from a lesser power [or force,] because that [lesser power] cannot do as much. And that we know it can do no more allows us to distinguish it from a greater power, since a greater power can do more. And thus we can know definitely how much it [that is, the maximum capability] is.

Nevertheless, I say, moreover, that not every active power can be determined in this way by its maximum capability or [the point] below which it can do everything, because the power of God, which according to the truth of faith is of infinite strength, cannot be determined in this way. For, indeed, no maximum can be given which God can move or do—neither a maximum velocity with which God can move the sky, nor an effect so perfect that God could not make it [even] more perfect. And this is because the infinite is not properly more in some degree, nor is it measurable or proportionable. Nevertheless, it must be noted that if we should want, in some way, to say how much is the motive or creative power of God, we should say that it is so much that in every mobile or

1. From *Iohannis Buridani Quaestiones super libris quattuor De caelo et mundo*, edited by Ernest A. Moody (Cambridge, Mass.: Mediaeval Academy of America, 1942), pp. 95, 96–100, 102–103, 109–112.

2. The expression "maximum capability" has been used to render the Latin phrase *maximum in quod ipsa potest*. Questions 21 and 22, large parts of which are translated here, are based upon a passage in Aristotle's *De caelo* (I.11.281a.7–27), which I now translate from a Latin translation possibly based on William of Moerbeke's medieval Latin translation from the Greek, made in 1271 and published in *Aristotelis opera cum Averrois commentariis* (Venice, 1562–1574; Frankfurt: Minerva, 1962), Vol. V, fol. 78 verso, col. 1, to 79 recto, col. 2. What follows is embedded in a lengthy discussion by Aristotle as to whether or not the world is eternal.

"Therefore if something can move through one hundred stades or lift a weight, we always speak of the most [or the maximum] that it can do, as to lift one hundred talents or to walk one hundred stades and it can accomplish any parts below these. And if there is a maximum *(excessum)* it would be necessary that the power be defined to the limit of the maximum [*excessus;* throughout this passage *excessus* has been used for 'maximum']. It is necessary, therefore, that what someone can do according to the maximum he can accomplish everything below that maximum, just as if someone who can raise one hundred talents can also lift two; and if someone walks through one hundred stades, he can walk through two stades. Moreover, a power is of the maximum, and what cannot do as much as the maximum cannot do more, just as what cannot walk a thousand stades obviously cannot walk through a thousand and one stades. Furthermore, nothing should trouble us [about this], for that which is properly possible is determined by the limit of the maximum. Now perhaps someone will insist that what has been said is not necessary, for someone seeing a stade will not necessarily see a smaller magnitude. On the contrary, he who can see a dot or hear a slight sound will [also] sense greater things. But this does not bear on the argument. For the maximum is determined either in the power or in the thing. What has been said is obvious, for vision which surpasses is of the smaller object, [whereas] a velocity which surpasses [another velocity] is of the greater body." The very last point should perhaps be understood in accordance with Aristotle's belief that a heavier body falls more quickly than a lighter body in natural motion.

The passage just quoted occasioned a considerable literature on maximum and minimum powers—indeed, the discussions were usually called *On Maximum and Minimum (De maximo et minimo)*—which had, as we shall see, both physical and mathematical aspects. In the present selection, which is intimately associated with the Aristotelian passage, it is the physical discussion that predominates, but, as Curtis Wilson explains in his treatment of this subject, for others, as, for example, Heytesbury, "The problem of the purely physical and qualitative character of different potencies recedes into the background and the discussion assumes. . .an extremely hypothetical, logicomathematical character. A set of rules is proposed as covering all imaginable cases of action-passion; the rules are couched in language borrowed from the *logica moderna*, and the center of interest appears to have shifted from that of answering strictly physical questions to the logicomathematical problem of setting extremes to classes or aggregates." (Curtis Wilson, *William Heytesbury: Medieval Logic and the Rise of Mathematical Physics* [Madison, Wis.: University of Wisconsin Press, 1956], ch. 3, "De maximo et minimo," p. 69).

At the outset, Buridan presents four arguments (omitted here) in defense of the opinion that a power ought not to be defined by its maximum capability. Then, after remarking that Aristotle and the Commentator, namely Averroes, say that "every power ought to be defined by its maximum capability," Buridan continues his discussion as translated here.

3. In the first of the four reasons, we read (*Iohannis Buridani . . .*, p. 95): "And it is argued in the negative [namely that a power ought not to be defined by its maximum capability] because in the fifth [V.12.1019a. 15–1019b.14] and ninth [IX.1.1064a.4–15] books of the *Metaphysics* powers are defined as active as well as passive, but not by a certain maximum. Indeed, it is said that an active power is a principle of transmuting another and a passive power is a principle of being transmuted by another."

4. Although the text has *quia*, I have used "when" rather than "since" or "because."

creatable thing it can move [that thing] greater [that is, with a greater velocity] or more perfectly or even to create [it more perfectly]. Indeed, it is by this that we believe Him to be infinite.

Next, concerning a passive power, it should also be understood that one must speak in a contrary way to [that in which one would speak about] an active power. For just as any active power that can act on a greater [thing] can also act on a lesser [thing], so also a passive power that can endure [or be acted upon by] a smaller active [power] can also be acted upon by a greater active power, but not the contrary—just as if Socrates can lift a great stone, so also can he lift a smaller stone; and if this stone can be lifted [or raised] by a strong force, it can also be lifted [or raised] by an even stronger force. And so, just as an active power should be determined by a maximum capability, so a passive power should be determined by the minimum which it can endure [or by which it can be stimulated], for by so much as the passive is said to be more passible [or receptive] by so much more does it suffer [or undergo change] from a lesser active power. Thus, if a passion should be [considered] good, a passive power is said to be better, stronger, and greater—not in acting or resisting, but in suffering and receiving—which receives more from a smaller object.

And by this [approach], the argument made about vision is solved.[5] For a visual power is passive and receptive of species from an object. Thus a visual power is called stronger, better, and more perfect, that receives from a smaller visible object a species sufficient to cause vision.

And so it appears that the [four] reasons given at the beginning of this question follow in their manner in accordance with the things said above, although a great doubt remains as to which part of the disjunctive [proposition] posited before is to be held;[6] but this will be discussed in another question. And it is obvious that it must be expressed in this manner.

BOOK I, QUESTION 22

Next, we inquire whether a power's maximum capability can be assigned.[7]

And it is argued "yes" on the authority of Aristotle and the Commentator, who say that a power or virtue must be determined by its maximum capability, which would not be true if that maximum could not be assigned.

The opposite [of this] is argued by the authority of many others, who say commonly that it is not [a question] of giving a maximum capability but the minimum of which it is incapable.

This question is very difficult because of the diversity of powers, both active and passive, and because of the diverse ways of acting or suffering and of moving or transmuting.

In the first place, therefore, I shall say something about the first power, namely the divine power. And, in accordance with faith, I posit this conclusion: No maximum mobile can be assigned which God cannot move, because, as stated before, I assume that an infinite mobile cannot be assigned; and yet, nevertheless, for every finite mobile that is assigned or assignable, God, by means of His infinite power, could make a greater finite mobile and move it. And so it may also be said that no minimum mobile can be assigned which God could not move, since God can move every mobile, great or small. Thus the power of God must not be determined by its maximum capability nor by the minimum of which it is incapable. I also say that there is no maximum velocity with which God is able to move [a mobile], since, indeed, whatever velocity may be assigned or assignable, He can move it [even] more quickly.

But now we must report what Aristotle would

5. Here Buridan refers to the fourth and final principal reason proclaimed at the beginning of the question (p. 96): "Again, Aristotle assigns the cause whereby a power must be determined by a maximum, namely because what is able to do a greater thing can do a lesser, but not conversely. Thus, if this cause should not be universally true, it must not be universally conceded that a power should be determined by its maximum capability. But this cause is not universally true, since it does not follow that if you can see a millet seed you can [also] see the thousandth part of it; for, indeed, many can distinguish and see greater things who cannot, however, distinguish smaller things." In the lines that follow, Buridan responds to this objection with the observation that vision is a passive—not active—power, so that any visual power capable of stimulation by a smaller object is a greater passive power than one whose threshold of visual stimulation can only be activated by a larger object.

6. A reference to the original question. At this juncture Buridan admits that he cannot decide the question "whether a power [or force] ought to be defined by its maximum capability." We are told that this will be considered in another question but apparently not in this treatise.

7. While admitting the difficulty in deciding whether a "maximum capability" can define a power, Buridan seeks to determine in this question whether a maximum power of any kind can be assigned. As will be seen, no maximum can be assigned to God's capacity for producing velocity, nor indeed to the action of an agent or power on a patient or resistance.

say about this. And it seems to me that Aristotle would say that a maximum velocity, or even a maximum slowness, can be assigned with which God can move the last [or outermost] sphere; and this maximum velocity, or even this maximum slowness, is that with which God does in fact move the last sphere. For Aristotle would say that God, because of His immutability and will and because of the unchangeability of the last sphere, cannot move this sphere quicker or slower than He now moves it.

However, all this notwithstanding, it must be said against Aristotle[8] that no maximum speed or maximum slowness can be assigned with which God can move, or even with which God does in fact move, [the last sphere]. On the contrary, whatever the velocity with which He moves something, He can move [it] with yet a greater velocity—I do not say greater extensively, but greater intensively. That is, in the last sphere what is moved cannot be assigned as quickest, for this whole sphere is not moved [with a] quickest [speed], since some parts of it, say near a pole, are moved more slowly than other parts.

Now if something were difformly [that is, nonuniformly] white or black so that from one end [apex? cone?; how the Latin *conus* ought to be rendered in this context is problematic] to the other end [apex? cone?] there would be a gradual diminution in whiteness part after part, this whole body would not, however, be denominated as a highest [or greatest] whiteness, even though the remission would begin from the end with the highest [or greatest degree of] whiteness. Indeed, by balancing one part against another, it ought more to be denominated by the mean degree between the greatest white part and the least white part.[9] Thus it could be said that if a stick were disposed in this way [that is, denominated by its mean degree], some part of the stick would be whiter than the [whole] stick and another would be less white than the [whole] stick. It is the same with respect to velocity in the issue proposed here. For the more remote parts of a sphere are moved more quickly and the parts nearer the poles are moved more slowly. It follows from this that the whole sphere is not moved most quickly or most slowly; on the contrary, a part remote from the pole is moved more quickly than the whole sphere and a part near the pole is moved more slowly than the whole sphere; and as a part is more remote from a pole—indeed from both poles—so much more quickly is it moved.

Now since this is so, it [that is, the whole sphere] ought not to be said to have a quickest motion. Indeed, by balancing one part against another it ought to be denominated more by a velocity mediate between the quickest and slowest moved parts.[10] And from this being so, it follows that no most intense velocity can be assigned in the sky unless there can be assigned a part most remotely distant from the poles. But such a velocity cannot be assigned unless you assume something without width, for [only] then would it be true that the circumferential line of the equinoctial circle would be most remote from the poles and would be moved most quickly. But we assume that such lines cannot be assigned.[11] Moreover, if between the poles you assign length, width, and depth to any part, it cannot be most distant from the poles. On the contrary, if it were divided into three parts according to latitude, the middle part of it would be more distant from the poles; therefore, no quickest motion can be assigned.

And the same appears [to be true] about slowness, because as a part around a pole is smaller so

8. As required by the sense of the argument, the Latin *secundum Aristotelem* has been rendered as "against Aristotle" rather than "according to Aristotle," its usual meaning.

9. The terminology and concepts embodied in this example are drawn from the subject matter of intension and remission of forms, which is described in Selection 43.

10. By analogy with the balancing of parts of whiteness in arriving at a mean degree of whiteness in the preceding paragraph, in this example concerning velocity Buridan implicitly assumes that the velocities of the points along the radius increase uniformly as they recede from the center. It is assumed intuitively that for every velocity less than the mean speed by whatever amount, there is a corresponding velocity greater than the mean speed by the same amount and that the points involved are equidistant from the point moving with the mean speed. All this is drawn from, and associated with, the mean speed theorem originating at Oxford in the 1330's (see Selections 42 and 43). It should be noted that in his *Treatise on Proportions (Tractatus de proportionibus)*, chapter 4, Part II, written in 1328, Thomas Bradwardine insisted, contrary to the position adopted by Buridan and most fourteenth-century scholastics, that the speed of a rotating radius be measured by the speed of its fastest moving point (see M. Clagett, *The Science of Mechanics in the Middle Ages* [Madison, Wis.: University of Wisconsin Press, 1959], p. 221).

11. For terminists or nominalists like Buridan, only physical—not mathematical—points, lines, and planes were conceded reality. Thus, only if the equinoctial circle were taken as mathematical—that is, lacking in width and depth—could a quickest point be admitted. Buridan, however, rejects such a move.

much the slower would it be moved; and thus a slowest motion cannot be assigned unless a minimum [velocity] can be assigned, which is impossible. . . .[12]

Having spoken of the powers of God and the intelligences, we must now consider natural and corporeal powers. It must be noted that some [natural powers] act without the resistance of a patient, as [for example,] light illuminating a medium, or a visible object multiplying its species in a diaphanous [or transparent] medium,[13] or into the eye; and so with many other [natural powers]. Other powers, however, act with the resistance of a patient, as [for example,] heating, or lifting a stone, or the downward motion of a heavy body, and so with many other [natural powers]. For it is a medium which resists. But first we shall speak about those things that act without resistance, [for it is here] that the major difficulty lies. In the first place we shall consider vision, but since illumination is required for vision, we shall speak first about illumination. . . .[14]

12. Although I have omitted the next section of the discussion, where Buridan considers the power of incorporeal intelligences (*Johannis Buridani*. . . , pp. 100–102), I include Curtis Wilson's summary of it (pp. 64–65; Wilson has provided a brief description of the entire question):

"SECOND: The powers of the celestial intelligences.—Here again it is probable that Aristotle would assign a maximum body which each intelligence can move, namely that sphere which it does in fact move, and also a maximum velocity with which each intelligence can move this body, namely that velocity with which it does in fact move its sphere. For the intelligences are finite and immutable in power, and the bodies which they move unalterable; hence the relation between mover and moved will remain constant, and each intelligence in moving its sphere will be acting according to its maximum power. But a doubt arises, supposing that God can augment the sphere of the moon, and further that the power of the mover must exceed the resistance of what is moved. For the power of the lunar motor will exceed the resistance of the lunar sphere by a divisible excess; let God then increase the resistance of the sphere by half this excess, and the lunar intelligence will still be able to move it. Hence the present lunar sphere is not the maximum which the lunar intelligence can move. Buridan answers that the reasoning is not demonstrative, because the celestial spheres offer no resistance to their movers, but have a pure inclination to their proper movement. In such a case, although the active power will exceed the passive in nobility and perfection, it is not necessary that its activity be greater than the passivity of the passive. The distinction between pure passivity and resistance is drawn more precisely in the next section."

13. For the concept of "multiplication of species," see Selections 62.1–4.

14. A lengthy discussion of illumination and vision (*Johannis Buridani*. . . , pp. 103–109) has been omitted. Wilson's summary of these sections is offered instead (pp. 65–67):

"THIRD: Light which illuminates a transparent medium.—To begin with, we must decide whether the medium offers resistance to the light which illuminates it. It would seem that it does insofar as it is opaque, for the less transparent and more opaque medium is illuminated less intensely and less far by the same light than the more transparent and less opaque. In opposition, however, it is objected that if opacity constituted a resistance, the illumination of the medium would become more intense and extend to a greater distance over a period of time, as we see in the case of heat-action. But this is false; the medium is illuminated instantaneously to its maximum potentiality. Buridan decides, therefore, that the medium is without resistance; the cause of its being more or less transparent is a greater or less passivity to light. Resistance is through a contrary or an inclination to a contrary; but light has no contrary.

"It may now be asked whether there is a maximum distance to which a given light can propagate its species in a given medium. If the light be of finite potency, and if neither the light nor the medium alter, then the light will illuminate part of the medium and part it will not; and between the two parts there will be only an indivisible or mathematical point. Hence the distance from the lightsource to this point will be the maximum distance to which the light propagates its species. It will also be the maximum distance to which the light *can* propagate its species; for the illumination does not improve with time, and the light cannot illuminate more than it in fact illuminates. A further consequence is that there is no minimum distance to which the light cannot propagate its species; for such a *minimum quod non* could not differ by a divisible magnitude from the *maximum quod sic* already assigned, the entire part of the medium beyond the dividing point being unilluminated.

"Again, as a corollary it follows that there is a maximum space through which a given visible body multiplies the species of color; since the action of light and that of color are entirely similar with respect to the multiplication of species. Thus it is possible for a given colored body to multiply its species to the surface of the eye and no further, or the point which is geometrically at the center of the eye and no further.

"FOURTH: A given medium to be illuminated by a light.—Buridan now turns to the case of the corresponding passive potency. Given a determinate medium, spherical in form, is there a minimum or maximum light which can illuminate the whole of it? Buridan answers that there is a *minimum quod sic;* and it will be precisely that light the maximum range of which extends throughout the given medium.

"FIFTH: Vision.—Here Buridan does not consider, as had Averroes, the case of different objects seen at a fixed distance, but rather keeps the object fixed and allows the distance to vary. Given a determinate power of vision *a*, a visible body *b*, and a uniform medium, is there a maximum distance through which *a* can see *b*? The common and probable answer, Buridan states, is in the negative. For suppose there were such a distance; through this distance *a* would see *b* with a certain intensity of vision (*visio aliquantae intensionis*). Now a form or quality which is naturally capable of remission or diminution in intensity is not corrupted all at once from a certain intensity to zero intensity, but remits in a continuous manner to its total corruption; hence if the distance between *a* and *b* is increased, *a* will continue to see *b*, albeit less and less intensely, until vision is finally reduced to zero intensity. Thus the original distance assigned was not a *maximum quod sic*, and no such can be assigned.

"And yet, although this reasoning appears demonstrative, a grand difficulty arises, which shows how the problem of assigning limits to potencies could become hopelessly involved with the problem of the physical nature of particular potencies. It was previously admitted (third section) that there is a maximum distance through which the given visible object *b* can multiply its visible species. Suppose this distance extends from *b* precisely to that point of the eye, say the center of the eye, at which vision becomes possible; will not this be the maximum distance through which the eye can see *b*? Thus we have two arguments: one based on the nature of vision as a remissible form, and leading to the assignment of a *minimum quod non;* the other based on the mode of propagation of the visible species of an object, as extending a definite maximum distance, and leading to the assignment of a *maximum quod sic* for vision as well. The two arguments cannot both stand.

"Buridan attempts to extricate himself from the difficulty by means of a distinction. There is a difference between 'the space in which *b* can be seen' and 'the space *through which b* can be seen'; *b* is in the first, but not in the second. Buridan then assigns a maximum space *in which* the body *b* can be seen; and a minimum space *through which b* cannot be seen, equal to the maximum *in which* it can be seen. Apparently—although Buridan does not say as much—this space is also equal to the maximum distance through which *b* can multiply its visible species. Thus if *b* is just outside the given space, it cannot be seen; if it approaches the eye by as little a distance as you please, it comes into the range of vision; and when it is altogether in the given space, it can be seen as a whole. However ingenious this distinction, it may be questioned whether it meets the central difficulty, which arises from the problem of the nature of light and of vision."

Now, finally, we must speak briefly about agents [acting] with [or against] the resistance of a patient. Concerning these, it is assumed as a principle that if a resistance is stronger than the active virtue or power, there is no action. But if the active power is stronger than the resistive power, there is an action and the action will be so much the greater or quicker as the excess of the active power to the resistance will be greater. These things are assumed. Then again we assume that the strength of the agent is not changed.

This first proposition is now posited: There cannot be assigned a maximum resistance by means of which an agent can act or with which it can act. For if *A* ought to move *B*, and you wish to assign the maximum resistance with which *A* moves *B*, let this maximum resistance be *CD*. Then I take a resistance equal to the active power and it [that is, the power] will be unable to move with this resistance. But it was able to move with resistance *CD*, therefore that equal resistance is greater than resistance *CD;* and let this be *CE*. But then the difference between *D* and *E* is divisible,[15] and let *I* represent the division. Then resistance *CI* is smaller than resistance *CE*, and consequently the active power exceeds it and so can move it.[16] But resistance *CI* is greater than resistance *CD*, so that resistance *CD* was not the maximum [resistance] with which *A* could move *B*, even though our adversary assumed it was a maximum.

From this, a second proposition that has been customarily assumed follows immediately, namely that a minimum resistance can be assigned with which *A* cannot move *B*, and this is the resistance that is equal to the active power. For it has been said that there is no action with such a resistance, but that however much smaller the resistance may be, the active power would exceed it and move it. Therefore, there is no smaller resistance that it could not move; but as there is nothing smaller [that it cannot move,] this is the minimum. Therefore, this is the minimum with which it was unable to move.[17]

15. That is, between *CD* and *CE*. The Latin reads incorrectly *inter D et C* ("between D and C") rather than *inter D et E*.

16. That is, $CE > CI > CD$.

17. The principles embedded in the two propositions of the last two paragraphs were used earlier in this very question, and although omitted from the translation were summarized in Wilson's account quoted above in note 12. Indeed, much the same proof appears also in an anonymous work entitled *Probationes conclusionum;* it is summarized by Wilson on page 58 of his volume on Heytesbury.

That no maximum resistance is possible Buridan demonstrated in the first proposition by utilizing one of the most fundamental axioms in medieval physics, namely that motion is impossible when the resistive force is equal to, or greater than, the motive force. Thus, if *F* is a force and *R* a resistive body, then if $F > R$, motion can occur. But it is obvious that *R* cannot represent *F*'s maximum capacity, since another resistance, say R_1, can be found such that $F > R_1 > R$; nor does R_1 represent *F*'s maximum capability, since R_2 can be found such that $F > R_2 > R_1$, and so on. Hence in the second proposition Buridan concludes that *the maximum capacity of any motive force must be measured by the least resistance which it is unable to move*, and this will equal the motive force (that is, $F = R$).

Of this proof as it appears in the *Probationes conclusionum*, Wilson remarks (pp. 58–59): "The proof is

And now more particularly, a third proposition is concluded, [which is] that one cannot assign [or specify] a maximum heavy body that Socrates can lift, but one can assign [or specify] a minimum heavy body that he cannot lift; and this is what is equal to his power.[18]

Similarly,[19] it must also be said if we posit a medium resisting the downward motion of a heavy body by its thickness [or density], we do not assign the thickest [or most dense] medium through which heavy body *A* can be moved downward but the least thickness [or denseness] through which it cannot be moved downward. Nor should a maximum weight be assigned which weight *A* can incline or raise in an [equal-arm] balance, but [rather] a minimum weight which cannot be inclined [or raised], and that is a weight equal to *A*. Therefore,

rigorous, if two assumptions are granted: (1) that the rule according to which no action or motion accrues from a ratio of equality between power and resistance is correct; and (2) that the sequence of all possible weights [where the anonymous *Probationes* speaks of weights, Buridan speaks of resistance in general—*Ed.*] forms a continuum. From this first illustration, indeed, it is possible to generalize, and to say that the medieval discussion of maxima and minima presents two major aspects of interest: (1) a physical or dynamical aspect, involving the relations between powers and resistances; and (2) a purely mathematical aspect, involving the concepts of continuity and limit.

"As physicists— or we might say 'speculative physicists,' since the theory here was seldom such as to yield conclusions admitting of empirical verification— the Schoolmen introduced numerous distinctions between types of powers and resistance, distinctions which necessitated a variation in the manner of assigning limits to potencies. For example, how is the power to be bounded if it is active or passive, debilitable (subject to weakening) or indebilitable, finite or infinite? If a power divides a resisting medium, how will the boundary be assigned when the medium is uniform; when it is difform (non-uniform)? What alterations in the form of the analysis must be introduced when the velocity and duration of the movement effected by a power are taken into account?

"The mathematical interest in the discussion lies, first of all, in the use of two kinds of boundaries for continuous sequences: one, in which the element serving as a boundary is itself a member of the sequence of elements which it bounds (*maximum quod sic* or *minimum quod sic*); and the other, in which the element serving as a boundary stands outside the range of elements which it bounds (*maximum quod non* or *minimum quod non*). In the preceding chapter we have noted (1) that this distinction is not capable of empirical verification; (2) but that, in recent times, it has proved of importance in the theory of functions and in the definition of irrational numbers; (3) that the medieval Schoolmen, on the contrary, were concerned with it only in connection with magnitudes which are, at least hypothetically, physical.

Once more, however, we find that the medieval discussion, particularly where it enters the context of logic, develops in extremely imaginative directions; becomes, in effect, a discussion of the ways in which aggregates may be bounded, and of the ways in which quantities may vary. The application of the two types of boundaries to finite and infinite aggregates and the incisive use of the notion of 'any' in connection with variable quantities permit us to speak of a logicomathematical interest in the Scholastic discussion."

In this physical discussion, and in the two examples following, Buridan has divided the continuum of resistances into two classes, one containing all resistances that can be moved, which includes all resistances smaller than the particular resistance that is equal to the motive force; the other class of resistances containing all those which the motive force cannot overcome and move, and embracing all resistances greater than the resistance which equals the motive force. Thus the *R* that equals *F* divides the continuum of resistances into two distinct classes, a division which also meets the requirements of Dedekind's postulate or cut enunciated in 1872 and which I quote in the translation of Wooster W. Beman:

"III. If *a* is any definite number, then all numbers of the system *R* fall into two classes, A_1 and A_2, each of which contains infinitely many individuals; the first class A_1 comprises all numbers a_1 that are $< a$, the second class A_2 comprises all numbers a_2 that are $> a$; the number *a* itself may be assigned at pleasure to the first or second class, being respectively the greatest number of the first class or the least of the second. In every case the separation of the system *R* into the two classes A_1, A_2 is such that every number of the first class A_1 is less than every number of the second class A_2." (Richard Dedekind, *Essays on the Theory of Numbers. I: Continuity and Irrational Numbers. II: The Nature and Meaning of Numbers* [Lasalle, Ill.: Open Court Publishing Co., 1948], p. 6.)

Where for Dedekind the assignment of the dividing point or number, *a*, was wholly arbitrary—that is, it could be assigned as the maximum term of the first class or the minimum term of the second class—it was hardly an arbitrary matter for Buridan and the scholastics who were concerned with real physical continua. Thus, for Buridan, the resistance that equals the motive force must necessarily be the minimum force which *F* cannot move, and hence the first and least term of the second class. As for the first class, it has no maximum resistance, since its successive resistances only approach the minimum resistance of the second class as a limit. Thus each physical case or example would have to be analyzed in order to determine to which of the two classes the dividing quantity is to be assigned. Despite this rather important difference between the Dedekind cut and its medieval counterpart, their similarity is nonetheless striking.

18. The results of the first two propositions are combined to form this third proposition.

19. Since the two Latin manuscripts used in establishing this edition diverge radically in their accounts of this paragraph, Moody included verbatim texts of both versions in parallel columns. Only the longer and seemingly more complete version in MS Bruges 477 has been translated here.

in any things whatever, [although] the greater mobile would resist more, no maximum can be assigned which *A* can move, but a minimum can be assigned which it cannot move.

But since some also speak about a maximum velocity, therefore I should speak of natural and not voluntary agents. Assuming that an active power is neither increased nor diminished, and that the resistance is not [increased or diminished], and [assuming further] that the resistance is such that *A* could move *B*, I could assign a maximum velocity and even a minimum velocity according to which *A* can move *B*. And it is that by which *A* does in fact move *B*, because in the case posited, *A* moves *B* with a certain velocity and cannot move *B* with a greater or smaller velocity. Indeed, from the aforesaid observations, it will always move it uniformly, since the same ratio of active force to resistance will always remain.

From[20] this one must conclude that when the medium remains uniform in the downward motion of a heavy body, the moving force is continually increased. This is due either to the attraction of the place or the impetus acquired by the motion,[21] or any other reason whatever about which I shall speak later. And this is obvious, because the natural motion of this heavy body downward is continually quicker and quicker, while the resistance remains the same, since we assumed a uniform medium. What I have said about local motions ["motions" renders *motibus*, which I have substituted for the textual reading *mobilibus*] must also be understood of alterations.

However, someone might object, because a drop of water thrown on a great fire acts [on it] and in a very short time extinguishes a glowing coal on which it falls. However, this great fire is of greater resistance than is the active power of the drop of water. Therefore, an agent acts even though the resistance is greater than its active power. Only it must be asserted that not the whole fire suffers from this drop but [only] a certain part of it whose resistance is exceeded at the beginning by the active power of the drop.

You ought to speak in this way about all other natural agents by considering that if an active power is increased in the downward motion of a heavy body, or if it is diminished, as in violent motions and in fatigable agents, then the velocity is increased or diminished. And so for all these cases it is necessary to reflect and respond according to the requirements [of each case].[22] And so it has been said.

Magnetism

58 AN ENCYCLOPEDIST'S DESCRIPTION OF THE MAGNET

Bartholomew the Englishman (fl. 1220–1250)

Translated and annotated by Edward Grant[1]

ON THE MAGNET

The magnet is an Indian stone of an iron-like color. It is found in India among the Troglodytes and attracts iron, for, as Isidore says, it makes a chain of rings so that the vulgar call it "living iron" *(ferrum vivum)*. It is believed that it also attracts clear glass, just as it does iron. As Augustine says, the force *(vis)* of it is so great that if anyone held the same magnet under a gold or bronze vessel and placed iron above it, the iron above will be moved by the motion of the [magnetic] stone underneath and follows [the magnet] in the same place. Whence, in a certain temple, a statue was made of

20. In the next two paragraphs I again translate from the longer version in MS Bruges 477, for the same reason given in the preceding note.

21. For the attraction of place and the role of impetus on bodies falling with accelerated motion, see Selection 49.

22. "The conclusion of Buridan's discussion is thus that a single rule cannot be given for assigning maxima and minima, but it is necessary to have regard for the particular exigencies of each case" (Wilson, pp. 68–69). For the most part, Buridan rejects Aristotle's position that a power is measured by its maximum capability (this is true of all the sections translated here). Only in vision does he partially accept Aristotle's viewpoint, but even here his attempt to reconcile it with the concept of a minimum which cannot be affected or altered failed (see n. 14).

1. Translated from *Bartholomaei Anglici De genuinis rerum coelestium terrestrium et inferarum proprietatibus libri XVIII.* (Frankfurt, 1601; Frankfurt, Minerva, 1964) pp. 746–747. This inferior, meager, and fantastic account should be compared to Peter Peregrinus' splendid exposition in the next selection.

iron which, by the power of the magnet, was seen to hang in air. Indeed, there is another species of magnet in Ethiopia which repels *(respuit)* iron and flees from itself. Furthermore, the magnet sometimes attracts iron from one angle and repels it from itself in another [angle]. Moreover, the darker [or bluer] a magnet is, the better it is. Isidore, a second *Dionysius*, and the *Lapidarius* add to this that this kind of stone restores husbands to wives and increases elegance and charm in speech. Moreover, along with honey, it cures dropsy *(hydropsim)*, spleen, fox mange *(alopecia)*,[2] and burns *(arsura)*. From the dizziness and confusion of the brain [caused by] the powder [of this magnet] when it is sprinkled over burning coals lying through the four angles of a house, it will seem that the house tumbles down immediately. And the magnet, just like the loadstone, when placed on the head of a chaste woman causes its poison *(virum suum)* to surround her immediately;[3] [but] if she is an adulteress she will instantly remove herself from bed for fear of an apparition. According to Plato, the Magi especially used this stone. A magnetic stone is hot and dry in the third degree, and has power to attract iron. There are mountains made of such stones and they attract and dissolve ships made of iron. Its dust, perfected with *apostolico*, an application for wounds, is especially valuable for wounds. Indeed the powder of a magnet in a quantity of two drams attracts iron [and] with the juice of fennel is valuable against dropsy, spleen, and fox mange, as Avicenna says, for it draws phlegm and black bile *(melancholia)*.

59 THE FIRST SYSTEMATIC DESCRIPTION IN EUROPE OF THE PROPERTIES OF THE LODESTONE

Peter Peregrinus (fl. 1269)

Translated by Brother Arnold (Joseph Charles Mertens)[1]

Annotated by Edward Grant

The Letter of Peregrinus

PART I

Chapter 1
Purpose of this Work

Dearest of Friends:[2]

At your earnest request, I will now make known to you, in an unpolished narrative, the undoubted though hidden virtue of the lodestone, concerning which philosophers up to the present time give us no information, because it is characteristic of good things to be hidden in darkness until they are brought to light by application to public utility. Out of affection for you, I will write in a simple style about things entirely unknown to the ordinary

2. For Isidore of Seville's description of fox mange, see Selection 89.

3. Presumably to ward off potential attackers.

1. *The Letter of Petrus Peregrinus On the Magnet, A.D. 1269*, translated by Brother Arnold (Joseph Charles Mertens), with Introductory Notice by Brother Potamian (M. F. O'Reilly) (New York: McGraw, 1904), pp. 3–34. The translation was made from a Latin edition published by Achilles Gasser in Augsburg, 1558. An earlier English translation was made by Silvanus P. Thompson. (*Epistle of Petrus Peregrinus of Maricourt, to Sygerus of Foucaucourt, soldier, concerning the magnet* [London: privately printed at the Chiswick Press, 1902] and a later one by H. D. Harradon ("Some Early Contributions to the History of Geomagnetism -I," *Journal of Terrestrial Magnetism and Atmospheric Electricity* [subsequently retitled *Journal of Geophysical Research*], 48 [1943], 6–17).

For descriptions of knowledge about magnetism in Greek and Roman antiquity, see M.R. Cohen and I.E. Drabkin, *A Source Book in Greek Science* (Cambridge, Mass.: Harvard University Press, 1948), pp. 310–314, and Duane H. D. Roller, *The "De Magnete" of William Gilbert* (Amsterdam: Menno Hertzberger, 1959), pp. 11–25 (for the Middle Ages, see pp. 25–42, which include a section on Peregrinus).

Although certain magnetic effects were known, for instance, chain-like attraction (magnetic induction), repulsion, and the action of magnets through certain media, no systematic description of the magnet's properties was given nor was magnetic polarity recorded. Indeed, no artificial magnets were made, and the application of magnetism to compasses was unknown. In this selection Peter Peregrinus reveals knowledge on all these matters. It would also seem that his letter was of some influence in the history of magnetism, for it is cited in William Gilbert's *De magnete* (1600) and in Athanasius Kircher's *De arte magnetica* (1641) and it was plagiarized by Jean Taisnier in the latter's *De natura magnetis* (1562). References to Gilbert's *De magnete* will be to the translation of Silvanus P. Thompson, made in 1900 and re-issued in 1958 in *The Collector's Series of Science*, published by Basic Books.

2. The letter was written to one Siger of Faucaucourt (or some variant of this place name), who was also from Picardy.

individual. Nevertheless I will speak only of the manifest properties of the lodestone, because this tract will form part of a work on the construction of philosophical instruments. The disclosing of the hidden properties of this stone is like the art of the sculptor by which he brings figures and seals into existence. Although I may call the matters about which you inquire[3] evident and of inestimable value, they are considered by common folk to be illusions and mere creations of the imagination. But the things that are hidden from the multitude will become clear to astrologers and students of nature, and will constitute their delight, as they will also be of great help to those that are old and more learned.

Chapter 2

Qualifications of the Experimenter[4]

You must know, my dear friend, that whoever wishes to experiment, should be acquainted with the nature of things, and should not be ignorant of the motion of the celestial bodies. He must also be skilful in manipulation in order that, by means of this stone, he may produce these marvelous effects. Through his own industry he can, to some extent, indeed, correct the errors that a mathematician would inevitably make if he were lacking in dexterity. Besides, in such occult experimentation, great skill is required, for very frequently without it the desired result cannot be obtained, because there are many things in the domain of reason which demand this manual dexterity.

Chapter 3

Characteristics of a Good Lodestone

The lodestone selected must be distinguished by four marks—its color, homogeneity, weight and strength. Its color should be iron-like, pale, slightly bluish or indigo, just as polished iron becomes when exposed to the corroding atmosphere. I have never yet seen a stone of such description which did not produce wonderful effects. Such stones are found most frequently in northern countries, as is attested by sailors who frequent places on the northern seas, notably in Normandy, Flanders and Picardy.[5] This stone should also be of homogeneous material; one having reddish spots and small holes in it should not be chosen; yet a lodestone is hardly ever found entirely free from such blemishes. On account of uniformity in its composition and the compactness of its innermost parts, such a stone is heavy and therefore more valuable. Its strength is known by its vigorous attraction for a large mass of iron; further on I will explain the nature of this attraction. If you chance to see a stone with all these characteristics, secure it if you can.

Chapter 4
How to Distinguish the Poles of a Lodestone

I wish to inform you that this stone bears in itself the likeness of the heavens, as I will now clearly demonstrate. There are in the heavens two points more important than all others, because on them, as on pivots, the celestial sphere revolves: these points are called, one the arctic or north pole, the other the antarctic or south pole. Similarly you must fully realize that in this stone there are two points styled respectively the north pole and the south pole. If you are very careful, you can discover these two points in a general way. One method for doing so is the following: With an instrument with which crystals and other stones are rounded let a lodestone be made into a globe and then polished.[6] A needle or an elongated piece of iron is then placed on top of the lodestone and a line is drawn in the direction of the needle or iron, thus dividing the stone into two equal parts. The needle is next placed on another part of the stone and a second median line drawn. If desired, this operation may be performed on many different parts, and undoubtedly all these lines will meet in two points just as all meridian or azimuth circles meet in the two opposite poles of the globe. One of these is the north pole, the other the south pole.[7]

3. Apparently Siger's inquiries on magnetism furnished the incentive for this important letter.

4. For Roger Bacon's praise of Peregrinus' talents as an experimenter, see the biographical sketch of Peregrinus, at the end of this source book.

5. S. P. Thompson remarks that Peregrinus' "reference to the navigation of the northern seas, . . . is of significance in the disputed question whether the use of the compass began with the Arab traders in the Levant or with the Northmen in the Baltic" ("Petrus Peregrinus De Maricourt and His Epistola de Magnete," *Proceedings of the British Academy*, II [1905–1906], 385). Actually, it may have been employed first by the Chinese (see n. 9).

6. Peregrinus may have been the first to actually shape a magnet, or, at the least, the first to speak of an artificially shaped magnet. It appears, then, that the first shaped magnets were spherical. In his *De magnete*, William Gilbert called the spherical magnet a *terrella*, a term which gained general acceptance. It was so called because the spherical *terrella* was conceived as a miniature spherical earth, for Gilbert considered the earth to be a great magnet.

7. This method of locating the poles is also described by William Gilbert in Book I, chapter, 3 of his *De magnete* (see p. 13 of Silvanus P. Thompson's translation of the *De magnete* cited in n. 1).

Proof of this will be found in a subsequent chapter of this tract.

A second method for determining these important points is this: Note the place on the above-mentioned spherical lodestone where the point of the needle clings most frequently and most strongly; for this will be one of the poles as discovered by the previous method. In order to determine this point exactly, break off a small piece of the needle or iron so as to obtain a fragment about the length of two fingernails; then put it on the spot which was found to be the pole by the former operation. If the fragment stands perpendicular to the stone, then that is, unquestionably, the pole sought;[8] if not, then move the iron fragment about until it becomes so; mark this point carefully; on the opposite end another point may be found in a similar manner. If all this has been done rightly, and if the stone is homogeneous throughout and a choice specimen, these two points will be diametrically opposite, like the poles of a sphere.

Chapter 5
How to Discover the Poles of a Lodestone and How to Tell Which is North and Which South

The poles of a lodestone having been located in a general way, you will determine which is north and which south in the following manner: Take a wooden vessel rounded like a platter or dish, and in it place the stone in such a way that the two poles will be equidistant from the edge of the vessel; then place the dish in another and larger vessel full of water, so that the stone in the first-mentioned dish may be like a sailor in a boat. The second vessel should be of considerable size so that the first may resemble a ship floating in a river or on the sea. I insist upon the larger size of the second vessel in order that the natural tendency of the lodestone may not be impeded by contact of one vessel against the sides of the other. When the stone has been thus placed, it will turn the dish round until the north pole lies in the direction of the north pole of the heavens, and the south pole of the stone points to the south pole of the heavens.[9] Even if the stone be moved a thousand times away from its position, it will return thereto a thousand times, as by natural instinct. Since the north and south parts of the heavens are known, these same points will then be easily recognized in the stone because each part of the lodestone will turn to the corresponding one of the heavens.

Chapter 6
How One Lodestone Attracts Another

When you have discovered the north and the south pole in your lodestone, mark them both carefully, so that by means of these indentations they may be distinguished whenever necessary. Should you wish to see how one lodestone attracts another, then, with two lodestones selected and prepared as mentioned in the preceding chapter, proceed as follows: Place one in its dish that it may float about as a sailor in a skiff, and let its poles which have already been determined be equidistant from the horizon, i.e., from the edge of the vessel. Taking the other stone in your hand, approach its north pole to the south pole of the lodestone floating in the vessel; the latter will follow the stone in your hand as if longing to cling to it. If, conversely, you bring the south end of the lodestone in your hand toward the north end of the floating lodestone, the same phenomenon will occur; namely, the floating lodestone will follow the one in your hand. Know then that this is the law: the north pole of one lodestone attracts the south pole of another, while the south pole attracts the north. Should you proceed otherwise and bring the north pole of one near the north pole of another, the one you hold in your hand will seem to put the floating one to flight. If the south pole of one is brought near the south pole of another, the same will happen.[10] This is because the north pole of one seeks the south pole of the other, and therefore repels the north pole. A proof of this is that finally the north pole becomes united with the south pole. Likewise if the south pole is stretched out towards the south pole of the floating lodestone, you will observe the latter to be repelled, which does not occur, as said before, when the

8. This method is also described by Gilbert in Book I, chapter 3, of the *De magnete* (Thompson, p. 14). Gilbert probably derived these two methods of locating poles from Peregrinus. He added a third using a *versorium*, "a piece of iron touched with a loadstone, and placed upon a needle or point firmly fixed on a foot so as to turn freely about. . . ." (Thompson, *De magnete*, p. 13).

9. In describing the magnetic compass—it had been in use long before Peregrinus wrote his letter—Alexander Neckham listed this property. Knowledge of the magnetic compass was had by the Chinese no later than 1088 (Joseph Needham, *Science and Civilisation in China* [Cambridge, Eng.: Cambridge University Press, 1954–1965], Vol. IV, Part I, p. 249), which is at least a century earlier than its mention by Neckham.

10. Repulsion between the like poles of a magnet is described by Roger Bacon in his *Opus Majus* (see Thompson, "Petrus Peregrinus. . . ," p. 378). The phenomenon of magnetic repulsion was known in Greek and Roman antiquity (for instance, Lucretius, *On the Nature of Things* VI) but was not associated with polarity.

north pole is extended towards the south. Hence the silliness of certain persons is manifest, who claim that just as scammony attracts jaundice on account of a similarity between them, so one lodestone attracts another even more strongly than it does iron, a fact which they suppose to be false although really true as shown by experiment.

Chapter 7
How Iron Touched by a Lodestone Turns Towards the Poles of the World

It is well known to all who have made the experiment, that when an elongated piece of iron has touched a lodestone and is then fastened to a light block of wood or to a straw and made float on water, one end will turn to the star which has been called the Sailor's star because it is near the pole; the truth is, however, that it does not point to the star but to the pole itself. A proof of this will be furnished in a following chapter. The other end of the iron will point in an opposite direction. But as to which end of the iron will turn towards the north and which to the south, you will observe that that part of the iron which has touched the south pole of the lodestone will point to the north and conversely, that part which had been in contact with the north pole will turn to the south. Though this appears marvelous to the uninitiated, yet it is known with certainty to those who have tried the experiment.

Chapter 8
How a Lodestone Attracts Iron

If you wish the stone, according to its natural desire, to attract iron, proceed as follows: Mark the north end of the iron and towards this end approach the south pole of the stone, when it will be found to follow the latter. Or, on the contrary, to the south part of the iron present the north pole of the stone and the latter will attract it without any difficulty. Should you, however, do the opposite, namely, if you bring the north end of the stone towards the north pole of the iron, you will notice the iron turn round until its south pole unites with the north end of the lodestone. The same thing will occur when the south end of the lodestone is brought near the south pole of the iron. Should force be exerted at either pole, so that when the south pole of the iron is made touch the south end of the stone, then the virtue in the iron will be easily altered in such a manner that what was before the south end will now become the north and conversely. The cause is that the last impression acts, confounds, or counteracts and alters the force of the original movement.

Chapter 9
Why the North Pole of One Lodestone Attracts the South Pole of Another and Vice Versa

As already stated, the north pole of one lodestone attracts the south pole of another and conversely; in this case the virtue of the stronger becomes active, whilst that of the weaker becomes obedient or passive. I consider the following to be the cause of this phenomenon: the active agent requires a passive subject, not merely to be joined to it, but also to be united with it, so that the two make but one by nature. In the case of this wonderful lodestone this may be shown in the following manner: Take a lodestone which you may call *AD*, in which *A* is the north pole and *D* the south; cut this stone into two parts, so that you may have two distinct stones; place the stone having the pole *A* so that it may float on water and you will observe that *A* turns towards the north as before; the breaking did not destroy the properties of the parts of the stone, since it is homogeneous; hence it follows that the part of the stone at the point of fracture, which may be marked *B*, must be a south pole; this broken part of which we are now speaking may be called *AB*. The other, which contains *D*, should then be placed so as to float on water, when you will see *D* point towards the south because it is a south pole; but the other end at the point of fracture, lettered *C*, will be a north pole; this stone may now be named *CD*. If we consider the first stone as the active agent, then the second, or *CD*, will be the passive subject. You will also notice that the ends of the two stones which before their separation were together, after breaking will become one a north pole and the other a south pole.[11] If now these same broken portions are brought near each other, one will attract the other, so that they will again be joined at the points *B* and *C*, where the fracture occurred. Thus, by natural instinct, one single stone will be formed as before. This may be demonstrated fully by cementing the parts together, when the same effects will be produced as before the stone was broken. As you will

11. Peregrinus observes here the fact that a magnet can be broken into pieces, each of which is a complete magnet with north and south poles. Its omission here is of little consequence, since the discussion is easily followed.

perceive from this experiment, the active agent desires to become one with the passive subject because of the similarity that exists between them. Hence *C*, being a north pole, must be brought close to *B*, so that the agent and its subject may form one and the same straight line in the order *AB*, *CD* and *B* and *C* being at the same point. In this union the identity of the extreme parts is retained and preserved just as they were at first; for *A* is the north pole in the entire line as it was in the divided one; so also *D* is the south pole as it was in the divided passive subject, but *B* and *C* have been made effectually into one.[12] In the same way it happens that if *A* be joined to *D* so as to make the two lines one, in virtue of this union due to attraction in the order *CDAB*, then *A* and *D* will constitute but one point, the identity of the extreme parts will remain unchanged just as they were before being brought together, for *C* is a north pole and *B* a south, as during their separation. If you proceed in a different fashion, this identity or similarity of parts will not be preserved; for you will perceive that if *C*, a north pole, be joined to *A*, a north pole, contrary to the demonstrated truth, and from these two lines a single one, *BACD*, is formed, as *D* was a south pole before the parts were united, it is then necessary that the other extremity should be a north pole, and as *B* is a south pole, the identity of the parts of the former similarity is destroyed. If you make *B* the south pole as it was before they united, then *D* must become north, though it was south in the original stone; in this way neither the identity nor similarity of parts is preserved. It is becoming that when the two are united into one, they should bear the same likeness as the agent, otherwise nature would be called upon to do what is impossible. The same incongruity would occur if you were to join *B* with *D* so as to make the line *ABDC*, as is plain to any person who reflects a moment. Nature, therefore, aims at being and also at acting in the best manner possible; it selects the former motion and order rather than the second because the identity is better preserved. From all this it is evident why the north pole attracts the south and conversely, and also why the south pole does not attract the south pole and the north pole does not attract the north.

Chapter 10

An Inquiry into the Cause of the Natural Virtue of the Lodestone

Certain persons who were but poor investigators of nature held the opinion that the force with which a lodestone draws iron, is found in the mineral veins themselves from which the stone is obtained; whence they claim that the iron turns towards the poles of the earth, only because of the numerous iron mines found there. But such persons are ignorant of the fact that in many different parts of the globe the lodestone is found; from which it would follow that the iron needle should turn in different directions according to the locality; but this is contrary to experience.[13] Secondly, these individuals do not seem to know that the places under the poles are uninhabitable because there one-half the year is day and the other half night. Hence it is most silly to imagine that the lodestone should come to us from such places. Since the lodestone points to the south as well as to the north, it is evident from the foregoing chapters that we must conclude that not only from the north pole but also from the south pole rather than from the veins of the mines virtue flows into the poles of the lodestone. This follows from the consideration that wherever a man may be, he finds the stone pointing to the heavens in accordance with the position of the meridian; but all meridians meet in the poles of the world; hence it is manifest that from the poles of the world, the poles of the lodestone receive their virtue. Another necessary consequence of this is that the needle does not point to the pole star, since the meridians do not intersect in that star but in the poles of the world. In every region, the pole star is always found outside the meridian except twice in each complete revolution of the heavens.[13a] From all these considerations, it

12. Peregrinus explains the attraction of unlike poles in terms of an agent, or active principle, and a patient, that which is acted upon (the agent–patient relationship was a popular type of medieval explanation invoked frequently in physical treatises). Thus a magnetized pole, whether north or south, which could cause another pole to move toward it would be labeled as active, its opposite as passive. Such a distinction is wholly arbitrary and of no value. See Cohen and Drabkin, p. 313, for Lucretius' atomistic explanation of magnetic attraction, which is devoid of any concept of polarity.

13. In dismissing this false theory that magnetic deposits in the earth attract the iron, Peregrinus was to turn away completely from the fruitful concept of associating magnetic properties with the earth. It remained for later authors, especially William Gilbert, to treat the earth as a large spherical magnet.

13a. Thus Peregrinus was aware that the pole star, Polaris, is not at true north but rotates around that point. Samuel Eliot Morison (*Admiral of the Ocean Sea, A Life of Christopher Columbus* [2 vols.; Boston: Little, Brown and Co., 1942], Vol. I. p. 271) remarks that Columbus discovered for himself the rotation of Polaris,

is clear that the poles of the lodestone derive their virtue from the poles of the heavens.[14] As regards the other parts of the stone, the right conclusion is, that they obtain their virtue from the other parts of the heavens, so that we may infer that not only the poles of the stone receive their virtue and influence from the poles of the world, but likewise also the other parts, of the entire stone from the entire heavens. You may test this in the following manner: A round lodestone on which the poles are marked is placed on two sharp styles as pivots having one pivot under each pole so that the lodestone may easily revolve on these pivots. Having done this, make sure that it is equally balanced and that it turns smoothly on the pivots. Repeat this several times at different hours of the day and always with the utmost care. Then place the stone with its axis in the meridian, the poles resting on the pivots. Let it be moved after the manner of bracelets so that the elevation and depression of the poles may equal the elevation and depressions of the poles of the heavens of the place in which you are experimenting. If now the stone be moved according to the motion of the heavens, you will be delighted in having discovered such a wonderful secret;[15] but if not, ascribe the failure to your own lack of skill rather than to a defect in nature.[16] Moreover, in this position I consider the strength of the lodestone to be best preserved. When it is placed differently, i.e., not in the meridian, I think its virtue is weakened or obscured rather than maintained. With such an instrument you will need no timepiece, for by it you can know the ascendant at any hour you please, as well as all other dispositions of the heavens which are sought for by astrologers.

PART II

Chapter 1

The Construction of an Instrument for Measuring the Azimuth of the Sun, the Moon, or any Star on the Horizon

Having fully examined all the properties of the lodestone and the phenomena connected therewith, let us now come to those instruments which depend for their operation on the knowledge of those facts. Take a rounded lodestone, and after determining its poles in the manner already mentioned, file its two sides so that it becomes elongated at its poles and occupies less space. The lodestone prepared in this wise is then enclosed within two capsules after the fashion of a mirror. Let these capsules be so joined together that they cannot be

"a fact which many late medieval and renaissance astronomers had denied. Practical seamen had assumed for centuries that the North Star marked true north.

14. Gilbert (*De magnete*, Bk. III, ch. I; Thompson, p. 116), scorns this and similar explanations of the causes of magnetism, declaring that "the crowd of philosophizers, in order to discover the reasons of the magnetical motions, called up causes lying remote and far away." Among those whose explanations he rejected are Peter Peregrinus (who thought that "the direction arises from the poles of the sky"), Cardano (who thought that "the turning of iron was caused by a star in the tail of the Great Bear"), and Marsilio Ficino (who argued that "the loadstone follows its own Arctick pole"). (See also Thompson, *De magnete*, p. 153).

Had Peregrinus been aware of magnetic declination and dip, it is not likely that he would have adopted the position that the poles of a magnet receive their virtue from the celestial poles. The phenomenon of magnetic declination is mentioned in a Chinese work by Shen Kua written around 1088. He observes that "magicians rub the point of a needle with the lodestone; then it is able to point to the south. But it always inclines slightly to the east, and does not point directly at the south" (Needham p. 249). Gilbert attributes the discovery of magnetic declination to Robert Norman, who described it in his book *Newe Attractive* in 1576.

15. Such a pivoted spherical magnet would be a perpetual motion machine exactly simulating the motion of the celestial sphere (see further on for Peregrinus' effort to construct a perpetual motion wheel) and would serve as a clock, as we are told a few lines further on. William Gilbert, taking cognizance of this claim, remarks (*De magnete*, Bk. VI, ch. 4; Thompson, pp. 223–224) that "Peter Peregrinus constantly affirms that a terrella suspended above its poles on a meridian moves circularly, making an entire revolution in 24 hours: which, however, it has not happened to ourselves as yet to see; and we even doubt this motion on account of the weight of the stone itself, as well as because the whole Earth, as she is moved of herself, so also is she propelled by other stars: and this does not happen in proportion (as it does in the terrella) in every part." In the Third Day of his *Dialogue Concerning the Two Chief World Systems*, Galileo criticized Gilbert for not vigorously rejecting the rotation of a spherical lodestone. For if the earth has a diurnal axial rotation, spherical lodestones already possess this motion "and to assign them a motion around their own centers would be to attribute to them a second movement quite different from the first. Thus they would have two motions; that is, a rotation in twenty-four hours about the center of the whole, and a revolution about their own centers. Now this second motion is arbitrary, and there is no reason whatever for introducing it" (trans. Stillman Drake [Berkeley, Calif.: University of California Press, 1962], pp. 413–414).

16. Convinced of the truth of his theory, Peregrinus anticipates the response of all who will try and fail by attributing their failure to lack of skill. There could be no experimental or experiential grounds for rejecting such a theory. In this it resembles the conviction of the alchemists that base metals are transmutable to gold. Every failure was attributable to faulty procedure, lack of knowledge, or misunderstanding.

separated and that water cannot enter; they should be made of light wood and fastened with cement suited to the purpose. Having done this, place them in a large vessel of water on the edges of which the two parts of the world, i.e., the north and south points, have been found and marked. These points may be united by a thread stretched across from north to south. Then float the capsules and place a smooth strip of wood over them in the manner of a diameter. Move the strip until it is equally distant from the meridian-line, previously determined and marked by a thread, or else until it coincides therewith. Then mark a line on the capsules according to the position of the strip, and this will indicate forever the meridian of that place. Let this line be divided at its middle by another cutting it at right angles, which will give the east and west line; thus the four cardinal points will be determined and indicated on the edge of the capsules. Each quarter is to be subdivided into 90 parts, making 360 in the circumference of the capsules. Engrave these divisions on them as usually done on the back of an astrolabe. On the top or edge of the capsules thus marked place a thin ruler like the pointer on the back of the astrolabe; instead of the sights attach two perpendicular pins, one at each end.[17] If, therefore, you desire to take the azimuth of the sun, place the capsules in water and let them move freely until they come to rest in their natural position. Hold them firmly in one hand, while with the other you move the ruler until the shadow of the pins falls along the length of the ruler; then the end of the ruler which is towards the sun will indicate the azimuth of the sun. Should it be windy, let the capsules be covered with a suitable vessel until they have taken their position north and south. The same method, namely, by sighting, may be followed at night for determining the azimuth of the moon and stars; move the ruler until the ends of the pins are in the same line with the moon or star; the end of the ruler will then indicate the azimuth just as in the case of the sun. By means of the azimuth may then be determined the hour of the day, the ascendant, and all those other things usually determined by the astrolabe.[18] A form of the instrument is shown in the following figure [Fig. 1].

Chapter 2
The Construction of a Better Instrument for the Same Purpose

In this chapter I will describe the construction of a better and more efficient instrument.[19] Select a

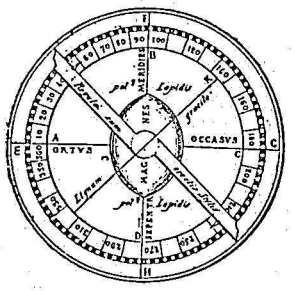


Fig. 1. Azimuth compass

vessel of wood, brass or any solid material you like, circular in shape, moderate in size, shallow but of sufficient width, with a cover of some transparent substance, such as glass or crystal; it would be even better to have both the vessel and the cover transparent. At the centre of this vessel fasten a thin axis of brass or silver, having its extremities in the cover above and the vessel below. At the middle of this axis let there be two apertures at right angles to each other; through one of them pass an iron stylus or needle, through the other a silver or brass needle crossing the iron one at right

17. S. P. Thompson calls this floating compass "the earliest compass with proper divisions" ("Petrus Peregrinus. . . ," p. 388).

18. Peregrinus conceives of this instrument as an astronomical one capable of performing the tasks of an astrolabe. That it was also intended to function as a mariner's compass is clear from the description of the second instrument, described in his next chapter, which was devised as an improvement over this one.

19. This instrument eliminates the need for water. It is a dry, pivoted compass placed in a circular box with a glass lid and provided with sights. Shortly after 1300—probably in southern Italy, perhaps in Amalfi—a light card with the compass points marked on it was affixed to the pivoting needle (see S. P. Thompson, "The Rose of the Winds; the Origin and Development of the Compass-Card," *Proceedings of the British Academy*, VI [1913], 181). This was the origin of the compass card, which was used effectively in the dry compass. The helmsman could now keep on course much more easily. The Chinese did not become aware of the dry, pivoted compass or the compass card until introduced to it by Dutch or Portuguese ships in the latter part of the sixteenth century (see Needham, p. 290). In truth, Peregrinus' pivoted compass of the cross-needle type with a graduated circle on which were marked the cardinal points was a major advance which he seems to have been the first to announce. Although the compass card was important, it is a mere refinement.

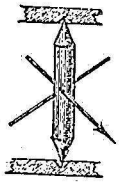


Fig. 2. Double-pivoted needle

angles.[20] Divide the cover first into four parts and subdivide these into 90 parts, as was mentioned in describing the former instrument. Mark the parts north, south, east and west. Add thereto a ruler of transparent material with pins at each end. After this bring either the north or the south pole of a lodestone near the cover so that the needle may be attracted and receive its virtue from the lodestone.

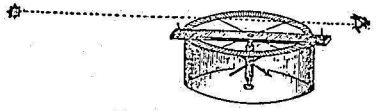


Fig. 3. Pivoted compass

Then turn the vessel until the needle stands in the north and south line already marked on the instrument; after which turn the ruler towards the sun if day-time, and towards the moon and stars at night, as described in the preceding chapter. By means of this instrument you can direct your course towards cities and islands and any other place wherever you may wish to go by land or sea, provided the latitude and longitude of the places are known to you. How iron remains suspended in air by virtue of the lodestone, I will explain in my book on the action of mirrors.[21] Such, then, is the description of the instrument.[22]

Chapter 3
The Art of Making a Wheel of Perpetual Motion

In this chapter I will make known to you the construction of a wheel which in a remarkable manner moves continuously. I have seen many persons vainly busy themselves and even becoming exhausted with much labor in their endeavors to invent such a wheel. But these invariably failed to notice that by means of the virtue or power of the lodestone all difficulty can be overcome.[23] For the construction of such a wheel, take a silver capsule like that of a concave mirror, and worked on the outside with fine carving and perforations, not only

20. When the magnetic needle lies north and south, the silver or brass needle will point west and east.

21. I am unaware of the existence of such a book on mirrors. Did he ever write one? Roger Bacon's statement about Peregrinus indicates that the latter was interested in optics (see the biographical sketch of Peregrinus at the end of this source book).

22. I have slightly truncated the last sentence, which Brother Arnold gives (p. 31) as "Such, then, is the description of the instrument illustrated below. (See Figs. 2 and 3.)" But an emended text by Timoteo Bertelli ("Sulla Epistola di Pietro Peregrino di Maricourt e sopra alcuni trovati e teorie magnetiche del secolo XIII. Memoria seconda" in *Bullettino di bibliografia e di storia delle scienze matematiche e fisiche*, I [Rome, 1868], 87) has only "Et hec est iam dicti Instrumenti descriptio." The two figures on p. 30 of Brother Arnold's translation appear neither in Gasser's edition nor in the manuscripts but were copied from diagrams supplied by Bertelli in a fold-out plate at the end of Volume I of the *Bullettino*. Since the text is sufficiently clear, Bertelli's figures have been omitted.

23. This account may represent the first genuine scientific attempt to construct a perpetual motion machine. Somewhat earlier in the thirteenth century Villard de Honnecourt (fl. 1225–1250) sketched a perpetual motion machine and supplied a brief legend which says: "Often have experts striven to make a wheel turn of its own accord. Here is a way to do it with an uneven number of mallets and with quicksilver" (quoted from Theodore Bowie, *The Sketchbook of Villard de Honnecourt* [Bloomington, Ind.: Indiana University Press, 1959], p. 134). Since no further elucidation is provided, Villard's concept cannot be readily compared with that of Peregrinus, who relied on the repulsive and attractive forces of magnetic poles and applied what he had learned about the behavior of magnets to construct a perpetual motion machine. In the fourteenth century scholastics generally rejected perpetual motion on theoretical grounds, arguing that inexhaustible forces *{vires infatigabiles}* did not exist in nature. Many would simply have supported Aristotle's contention in *De caelo* I.11.281a.26–27 that whatever had a beginning must eventually cease to be. Nicole Oresme also agreed that such perpetual motions did not occur in nature, but he added that there is no contradiction in conceiving a motion that has a beginning but no end (*Le Livre du ciel et du monde*, Bk. I, ch. 29; pp. 201–203 of A. D. Menut's translation [Madison, Wis.: University of Wisconsin Press, 1968]). After devising an illustration of a perpetual motion involving four wheels and incommensurable speeds, Oresme declares that "although such a series of events cannot occur in nature, nor be shown by material art or in destructible matter, nor endure so long [as forever], nevertheless, it contains or implies no contradiction whatever, nor is it within its own frame of reference incongruous to reason, . . ." Despite the healthy skepticism about perpetual motion prevailing in the fourteenth century, many believers are recorded through the centuries (see Henry Dircks, *Perpetuum mobile* [London, 1870]). Even so great a figure as Gottfried Leibniz sought to devise a perpetual motion machine (see A. Maier, *Metaphysische Hintergründe der spätscholastischen Naturphilosophie* [Rome: Edizioni di Storia e Letteratura, 1955], p. 269, n. 50).

for the sake of beauty, but also for the purpose of diminishing its weight. You should manage also that the eye of the unskilled may not perceive what is cunningly placed inside. Within let there be iron nails or teeth of equal weight fastened to the periphery of the wheel in a slanting direction, close to one another so that their distance apart may not be more than the thickness of a bean or a pea; the wheel itself must be of uniform weight throughout. Fasten the middle of the axis about which the wheel revolves so that the said axis may always remain immovable. Add thereto a silver bar, and at its extremity affix a lodestone placed between two capsules and prepared in the following way: When it has been rounded and its poles marked as said before, let it be shaped like an egg; leaving the poles untouched, file down the intervening parts so that thus flattened and occupying less space, it may not touch the sides of the capsules when the wheel revolves. Thus prepared, let it be attached to the silver rod just as a precious stone is placed in a ring; let the north pole be then turned towards the teeth or cogs of the wheel somewhat slantingly so that the virtue of the stone may not flow diametrically into the iron teeth, but at a certain angle; consquently when one of the teeth comes near the north pole and owing to the impetus of the wheel passes it, it then approaches the south pole from which it is rather driven away than attracted, as is evident from the law given in a preceding chapter. Therefore such a tooth would be constantly attracted and constantly repelled. In order that the wheel may do its work more speedily, place within the box a small rounded weight made of brass or silver of such a size that it may be caught between each pair of teeth; consequently as the movement of the wheel is continuous in one direction, so the fall of the weight will be continuous in the other. Being caught between the teeth of a wheel which is continuously revolving, it seeks the centre of the earth in virtue of its own weight, thereby aiding the motion of the teeth and preventing them from coming to rest in a direct line with the lodestone. Let the places between the teeth be suitably hollowed out so that they may easily catch the body in its fall, as shown in the diagram above. [Fig. 4.]

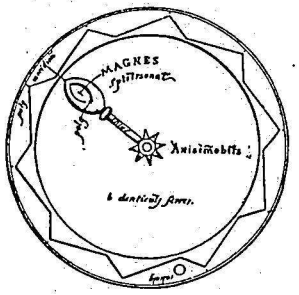


Fig. 4. Perpetual-motion wheel

Farewell: finished in camp at the siege of Lucera on the eighth day of August, Anno Domini MCCLXIX.

Optics

60 THE ENCYCLOPEDIC TRADITION IN OPTICS

1. Adelard of Bath (fl. 1116–1142): Natural Questions

Translated by Hermann Gollancz[1]

Annotated by David C. Lindberg

CHAPTER 23. WHAT THEORY IS TO BE HELD CONCERNING SIGHT?

NEPHEW: . . . We have heard what you have said about hearing; now let us see what you have to say about seeing.

ADELARD: Many different views have been expressed about the nature of sight, and it will be perhaps convenient first to set them forth, and then enquire which of them is the most reasonable.

NEPHEW: Then, if you approve, it shall be my task to state these various theories, and yours to state any objections there are to them when I have done so.

ADELARD: Your suggestion is a reasonable one.

NEPHEW: The theories I have been able to collect in various quarters about sight fall into four different groups. Some say that the mind, sitting in the brain as its chief seat, and looking forth upon outer things through open windows, viz., the eyes, gets knowledge of the shapes of things, and when it has got knowledge of them, judges them; it being always understood that nothing from the mind passes to the outside, and nothing from the shapes outside makes its way to the mind. Others, again, maintain that sight takes place through the approach of shapes, saying that the shapes of things give shape to the air that intervenes between themselves and the eyes, and that in this way the materials for judgment pass to the mind. Very many also assert that something is sent forth by the mind, i.e., visible breath, and that the shapes of the things that are to be seen meet it in mid-air: having taken shape from these, the breath returns to its seat, and presents the shape to the mind for it to exercise judgment upon. A fourth party maintains that no shapes of objects approach the eye, but that something which they call "fiery force," and which is produced in the brain by means of concave sinews, passes first through the eyes, and then to the objects to be seen, and by returning to its point of origin brings back to the mind, with the same quickness as it went, the shape impressed upon it as though by a potter.

Let us deal first with the first theory. The use of all the senses belongs, as Boethius declares in his "*De musica*," to all living creatures; but what is their strength, and what are the limits to their use, is not perfectly clear except to the intellect of the philosopher; for both the effects of things follow upon the antecedent causes according to a most subtle nexus, and causes along with their effects differ from one another by the most subtle differences, so that even philosophers themselves often fail in their understanding of nature. Thus in the present case, if they mean that the mind placed in the brain as a seat regards external objects through the windows of the eyes, then when they assert this they will either be attributing a sort of power to the windows of the eyes, or else assigning no means of contemplating external things to these windows. But by calling the eyes windows, they are attributing to them a certain faculty for seeing external objects: do they then mean that the mind is a corporeal thing? In that case it would require a free and unimpeded means of egress without risk of injuring itself by coming into contact with other things; or do they think that in that conception of shapes which we call sight, there is some corporeal power needed for the mind to prevent it itself from being checked by contact with any obstacle, and that hence windows are required? But neither the mind, nor its consideration of external things, are corporeal things, and therefore they need have no fear of contact with corporeal things, and consequently, so far as the things perceived by them are concerned, do not require any corporeal opening. The formula then of these people includes no force, nor do they assume any path for the mind through the windows of the eyes, since they do not require this. This being so, why do they say that it sees external things through the eyes, especially as the eyes too are not perforated, but are more solid than the other instruments of sense, so that if the mind required any corporeal power, the ears or the nostrils (which have passages through them) would supply it. Further if, as is asserted, the mind should see the shapes of things by looking at external things, how could the human mind, by regarding a mirror, see the shape of its own face? The mirror, being opposite to its windows, it would be able to see; but its own face, though opposite to it, could not be seen according to this idea. This theory therefore lacks consistency, and we must now discuss the second view previously stated.

ADELARD: By all means, nor do I see how the upholders of this first view are to get the better of your objections. It was on this account that a theory was propounded by the Stoics, that external forms make their way to the mind itself, and that it is imprinted by them as wax might be. This it was that made Boethius in his "*De Consolatione*" say, "in bye-gone days the Stoic school introduced us to a set of out-of-date old men who hold that perceptions and images are imprinted on our minds by external bodies, just as at times it is people's habit to make and leave marks on the smooth paper with a swift pen." Although Boethius in the same passage attacks this idea as being pointless, I should like to tell you my objections to it as follows:—The shapes of things to be seen approach the mind of the seer either in their own subject and along with it, or in another subject and along with that. Now it is impossible that it should do so in

1. The following selection is reprinted by kind permission of the heirs, Mr. Rodney Gollance and Mrs. Phyllis Simon, from *Dodi Venechdi (Uncle and Nephew), the Work of Berachya Hanakdan, now edited from MSS. at Munich and Oxford, with an English Translation, Introduction etc., to which is added the first English Translation from the Latin of Adelard of Bath's Quaestiones Naturales* by Hermann Gollancz (London: Oxford University Press, 1920), pp. 114–121. Adelard was both a translator of scientific works from Arabic to Latin and a writer of scientific treatises in his own right; the *Natural Questions* were written, by his own testimony, after he had received some exposure to Arabic science.

its own subject, and not along with it; nor does the body that is seen approach me: therefore it must be in another subject, and along with that.

NEPHEW: In air and with air. For the body, as they say, gives shape to air, and that air to other air, till it comes to that in the brain which acts upon the mind itself.

ADELARD: If approach to the mind therefore were to be by such a transmission of shapes how could this be accomplished should a glassy body be interposed? Would not the advance be prevented through such an obstacle? Would not the air in the neighbourhood of the glass be acted upon by it? Will it receive shape from that of the more distant and less splendid body? This is of course absurd, and absurd too is the theory we are discussing. However, let us, as Boethius says in the "*Topica*," admit it for the sake of argument, that we may see what follows. Let us grant that the progression of shapes as far as the eye, is made by the eye even to the brain, or the air of the brain; but let us note that passage which they call a sort of potter's impression. If the air of the brain imprints its shape on the mind, it gives either that shape actually and essentially, or another like it. But that it should be the same is impossible, for no individual shape can pass from one subject to another, as in the passing the shape would be without a subject, which is impossible. On the other hand, if it imprints another shape, it will at once require local parts in the mind just as it had in the air, and not finding them in it, it will not be able by moulding to give shape to it. Since then this theory is obviously untenable, let us pass to the next. This one—the third—is held by those who grant that a visible truth issues from the brain, but most certainly does not go so far as the body, which is relatively very far from it, for the reason that it finds the shapes it seeks in the intervening air.

Hence it has no need to do more than this, and consequently having been impressed returns, bringing back the shape to the mind by which it was sent forth. Those who say this do not realise what is the necessary consequence of their words; for if, as they assert, between the seer and the things to be seen, the air, by reason of the progression of similar shapes, offers to the visible breath that of which it was in search, then let us assume, first, a man looking from the east westwards, with the object of discerning the shape of a white body (and this, they say, he will find in the intervening air); next, at the same moment, assume another looking from the west eastwards, in order to behold a black body; and let him consequently by the same progression find the shape of the black body in the same intervening air: from this it follows, that the same or indivisible air is the subject alike of blackness and whiteness. Thus two contrary things, acting in opposition, are found in the same subject. But this is impossible, and we are therefore bound to reject the views of those who assert it. Finally, and beyond doubt, there is another consideration which, even if we put other objections on one side, is sufficient to upset all the views we have mentioned. It is this: we are familiar enough with the sight of our own shapes in a mirror; but this, though its reality is established by everyday life, does not agree with the theories we have recounted. It will be better, therefore, to deal with the view of which philosophy approves, and dismissing other theories as lacking strength, put our faith in this academic truth. This theory is as follows: In the brain there is generated a certain air of the most subtle nature, and made of fire, and consequently exceedingly light: this makes its way from the mind along the nerves, whenever it so pleases, and necessity arises, to see things outside. Hence it is called by physicists "visible spirit": being a body, it naturally requires a local exit, which exit it finds through the different concave[2] nerves, which the Greeks call "optic," extending from the brain to the eyes: then travelling to the body to be seen, it makes its way with wondrous speed, and being impressed with the shape of the body, it both receives and retains the impression, and then returning to its original position, it communicates the shape it has received. Now this spirit is called by philosophers "fiery force," and this force, when it finds a mirror opposite to it, or any other light-giving body, being reflected by it, returns as a result of the reflection to its own face, and still retaining the shape, when it enters, reveals it to the mind. You are not, however, to suppose that this fiery force found the shape of the face in the mirror; but, being reflected from the surface which is too smooth for it to abide there, received the shape while returning, and having received it, brought it back. This then is the divine theory which Plato has adopted, among other things, in his "*Timaeus*." "There are, in my opinion," says Plato, "two virtues in fire, one consuming and destructive, the other soothing and endowed with harmless light. With this one, therefore, in virtue

2. That is, hollow.

of which light bringing in the day unfolds itself, the divine powers are in harmony, for it has been their pleasure that the intimate fire of our bodies, own brother of the fire which is a passing bright, clear, and purged fluid, should flow through the eyes, and issue from them in order that through the eyes, slight, cramped, and affrighted, as it were, by the stouter substance, but yet offering a narrow medium, the more subtle clear fire might flow down through the same medium. Hence, when the light of day lends itself to the diffusion of sight, then no doubt the two like lights meeting in turn cohere into the appearance of a single body, in which the flashing brightness of the eyes meet, while the intimate brightness of the diffusion as it spreads is reflected by meeting with the image at close quarters. All this then goes through one and the same experience, and the result of that same experience, when either it touches something else, or is touched by it; moved by this contact, it spreads itself through the whole body, and making its way through that body to the mind and produces the sense which is called 'sight.' " . . .[3]

CHAPTER 24. WHETHER VISIBLE BREATH IS A SUBSTANCE OR AN ACCIDENT?

NEPHEW: I understand, and nothing could give me greater pleasure. For alike on philosophical grounds, from the physical effects of causes, and from ethical considerations, I am inclined to give my whole-hearted assent to Plato rather than to you.

I have now a task to impose both on Plato and on you, and I think it will annoy you both: for if, as has been previously explained, anything issues from the brain, whether with the physicists we call it visible breath, or—as Plato insists—the fire of our own bodies, it must necessarily be either substance or accident.

ADELARD: It is corporeal substance; for, as the philosopher says, fire is a most subtle substance, composed of the four elements.

CHAPTER 25. HOW CAN THIS SAME BREATH IN SO SHORT A TIME TRAVEL TO THE STARS AND RETURN FROM THEM?

NEPHEW: Certainly; for if it were an accident, it would not be able to pass to outer things without a subject. Since then you understand that it is a body, it is easy to understand what inconvenience would follow thence. The firmament, which by some is called *aplanos*,[4] and by others the further sphere, contains all body and place: and when I speak of stars, I mean those upper bodies which not seeming to move are fixed in the plane. Well then, this fiery breath with a single glance of the eye, so to speak, sees the stars; and we have to admit, that a body in so short a time traverses and returns through as great a density of air as there is between us and the moon and the infinite space that lies between us and the moon, and between the moon and the sun up to the very aplanos itself: and this is mere madness and quite impossible, since the actual breadth[5] of the whole earth bears no sort of proportion to the infinite diameter of the sky.

ADELARD: That is the sort of difficulty that a man gets into who knows nothing of nature: do you then pay careful attention and store up in your mind what I am going to say. Just as of spaces some are wide, some wider, and some widest of all, so of bodies some I hold are swift, others swifter, and others swiftest of all. This, however, not everyone can understand. Just as the extent of the sky and its shape as laid down by geometers is barely or not at all clear to the vulgar mind, so to it such a great speed in so short a time cannot be clear, though the eye in such people is swifter than the mind; for they measure, or rather mis-measure, everything according to the fallacious evidence of their senses in terrestrial matters. They think that the size of the sky exactly coincides with the earth, and that the bulk of the moon and sun and other things, which true reason would show them to exceed in size the earth, are not one whit greater than they seem to their bleary eyes. Those, however, who in matters of this sort are more apt to use reason, the incorporeal eye of the mind as their guide, just as they see clearly the magnitude of the external continent, or boundary, and the almost infinite extent of its diameter, so also see clearly the revolution of the heavens and the unutterably swift movement of the visible breath—and this in both cases, thanks to the use of reason. Just as the mind of the external sphere excels all created beings in the execution of virtues, so also this visible breath is more subtly perfected by the wonderful energy of creative virtue than all things compounded of elements. Just as the travelling forth of mind is the cause of that swift revolution, so the travelling forth of body is the reason of this swift going and returning, and it is not strange that

3. *Timaeus* 45B–45D.
4. That is, fixed.
5. Gollancz's edition has "breath," obviously a typographical error.

the man who is ignorant of this should also be ignorant of and wonder at its effects. One thing more I must tell you, and that is, that not all the physicists agree with you in saying that the upper stars are fixed in the aplanos, for they assert that they move inside the sky at a far lower altitude—this point, however, I will deal with later on, if I live long enough. For the present, now that impossibility and space are dwindling, let doubt dwindle also.

CHAPTER 26. HOW IS IT THAT WHILE THE EYE IS SHUT, THE VISIBLE BREATH IS NOT LEFT OUTSIDE?

NEPHEW: Nay, doubt grows from more to more, and I fancy that I myself shall dwindle away to utter nothingness before I give up doubting. Just imagine that while the visible breath is touching a star, the eye is shut. This is bound to happen, frequently, and the breath will then be left outside.

ADELARD: If only you would remember what I said previously, you would not raise this objection. The breath is very swift, and, as said above, it is established that it is sent by the mind, and through the mind. It is clear that the eye can be closed only by the mind, by which all voluntary motion is communicated to the body; let the consequence then of this also be clear, that that which sends also receives; and when it wills, shuts the door in such a way as to involve no injury to itself; and so that nothing of its own may remain outside, especially too when the thing itself is of such speed that it immediately receives it back again. Idle then would it be to grieve over a disaster of this sort when the thing sent obeys nature which sends it, and a door is no resistance to one who obeys.

CHAPTER 27. WHY THE BREATH DOES NOT HINDER ITSELF IN GOING AND RETURNING.

NEPHEW: I only hope that you, who while following Plato are tumbling into a pit, may have no causes for grief. I grant that in our discussion we should admit anything that has a possibility of truth, yet I think my next question will land you in difficulties. Assume that this breath, or if you will, fire, makes its way as far as to what is to be seen by it: is sight effected before it returns or afterwards?

ADELARD: It brings no message until its return.

NEPHEW: If therefore sight takes place immediately after its return, when we look at anything for a long while with fixed gaze and see it, it follows that it has both gone and returned at the same time, if we see the thing continuously and without any interval: hence both its going and returning will be simultaneous and without division, and therefore it hinders itself while going and returning.

ADELARD: Nay, it is you who hinder yourself, for as a result of your not understanding you make foolish objections; it does not follow that because we await anything with a continuous gaze that the breath when going is also returning, or hindering itself. When we say that it goes and returns so quickly, that when one return is accomplished, another starting takes place without a word, because there is no delay perceptible to any sense, since there is nothing that is manifest to sense, i.e., to the intellect, there is therefore no great need that it should go with the same speed after its return as it returned with after its going.[6] As, however, this interval is not discernible to sense, the journey wrongly seems continuous.

2. Alexander Neckam (1157–1217): Concerning the Natures of Things

Translated and annotated by David C. Lindberg and Greta J. Lindberg[7]

CHAPTER 153. CONCERNING SIGHT.

It is generally conceded that the more remote a thing, the smaller it appears. However, vapor can and commonly does prevent this general occurrence, for the body of the sun appears larger toward dawn on account of the remains of the nocturnal vapors than when it shines at midday. Moreover, a fish or anything placed in water seems larger in the water than out of it. Thus a dog swimming in water holding a piece of meat in its mouth is deceived by seeing a shadow and lets go of the meat that it was holding in its mouth, hoping to secure a larger piece for itself, but in vain. Let the waters represent tribulations; martyrs placed in tribulations were greater than in time of peace. The sun stands for power, which seems greater the more remote it is. Something worthy of admiration is found also in geometrical investigations: there is something that appears larger the more remote it is; for the closer the angle of tangency,[8] the smaller it appears to be. The cause of this is as follows: The larger the circle, the smaller is the angle of

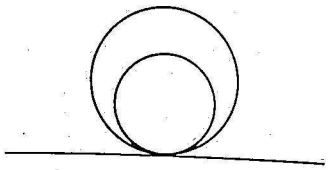


Fig. 1

tangency, and the smaller the circle, the larger is the angle of tangency, as can be seen in the adjacent figure [Fig. 1]. But the further away the circle, the smaller it appears to be; for the more remote it is, the smaller is the angle under which it is seen. However, the smaller the circle appears, the greater appears the angle of tangency; therefore, the more remote the angle of tangency, the larger it appears to be. Similarly, the further the acquaintance of a powerful man is from being achieved, the more worthy of praise is he considered to be. [However,] having become the friend of the powerful man, so much less desirable will his friendship appear to you.

Nevertheless, to some people it seems that this argument is invalid. The smaller the circumference, the larger the angle of tangency; therefore, the smaller the circumference appears to be, the larger the angle of tangency appears to be, since as the angle under which the circle appears decreases because of its elongation from the eye, so the angle under which the angle of tangency appears decreases as it is removed [from the eye]. But how? Under what angle is the angle of tangency viewed? Likewise, a straight rod appears bent in water, which is customarily attributed to reflection[9] of the rays from the surface of the water. [Now,] waters represent tribulations and the straight rod good works. Thus the works of the just, who are vexed by tribulations, are often regarded as bent, although they are [actually] straight. Furthermore, the man who is in a dark place sees a man standing in the light, but not vice versa; in the same way, unimportant people, whose fortune is dark, perceive the deeds of important people, but not vice versa.

A marvelous thing can be observed [even] by somebody poorly instructed in such things, since no parallel lines appear to be parallel.

A denarius placed in the bottom of a basin filled with water will be seen by one standing far away; when the water is drawn off, it will not be seen. Likewise, as long as there is tribulation in the soul, many things are observed and perceived which are not perceived when the tribulation has been removed. And as too much prosperity blinds the mind, so tribulation restores keener sight to the mind. Some one wishing to maintain that the denarius is not seen will contend that the water itself represents such a form to the eyes of the observer. To another, it will seem that a shadow appears because of the reflection made in the lucid body. But I think Aristotle is more to be believed than the masses.

To a person looking into water a tower not far removed from the water will appear to be inverted, although it is actually straight and erect; rejoice in the vanities of things, and they who do well will seem to you not to be standing upright, but inverted. An image is not formed in turbulent water but in quiet water; so also a disturbed mind is not attentive; anger hinders the soul, so that it cannot discern truth.

Although the body of the sun is round, it does not appear to have a convex surface, which occurs because of the impotence of sight and the excessive splendor of the sun. Similarly, the status of important men does not appear as it really is because of the splendor of the vainglory of conspicuous men; it is subject to the vicissitudes of life, although to the less prudent it appears otherwise. Furthermore, many are lifted in spirit who appear perfectly flat. Is it more necessary to ignore their thoughts than [those of] this powerful man? It is necessary that he fear many, who is feared by many.

Notice how certain animals see at night but not during the day on account of the whiteness of their eyes and the subtlety of their visual spirit.[10] For during the day the visual spirit (inasmuch as it is subtle and bright) is easily dispersed by the brightness of the air striking against the eyes, the whiteness of the eyes cooperating in this separation; and thus the power of sight of the spirits is

6. That is, the interval between return of the breath and its re-emission may be longer than the interval between the original emission of the breath and its return.

7. Translated from *Alexandri Neckam De naturis rerum libri duo*, edited by Thomas Wright (London, 1863), pp. 234–240. The two chapters translated here constitute all the optical material contained in Neckam's encyclopedia. They are valuable chiefly as illustrations of the low level of optical knowledge in the twelfth century.

8. On angles of tangency or contingent angles, see note 10 on Grosseteste's *Concerning Lines, Angles, and Figures*, Selection 61.1.

9. Although the cause of the rod's bent appearance is refraction, Neckam uses the term *reverberatio*.

10. On the visual spirit, see Adelard of Bath in the preceding section.

darkened by perdition. But at night, aided and strengthened by the darkness of the air, the visual spirit illuminates (by its own brightness) the air between the seeing thing and the thing to be seen, and thus it prepares the way for discerning the thing opposite; and this is evident in owls. In other animals, illuminated by the brightness of the day, the visual spirit produces sight during the day, because the eyes [of those animals] are black and the visual spirit is gross and dark; but at night, darkened by much greater obscurity, it does not perfect sight, but confuses it; and this is seen clearly in men. However, in bats, because their instruments [of perception] are varied and their visual spirit is midway between darkness and brightness, sight is achieved only at dawn or at dusk, because of the mediocrity [at those times] of the air; for during the day, because of the excessive brightness dividing the spirit, and at night, because of the excessive darkness obscuring the spirit, vision is as far removed from the bat as activity. So bright and subtle is the visual spirit in a lynx that by its irradiation, illuminating the dark air contained in the pores of a wall, the spirit is moved to see something placed on the other side of the wall. The eagle, however, although it has an excellent organ of vision (both because its visual spirit is strong and because its pupil is small, on which account very little of the spirit is scattered outside), exposes its eyes to the rays of the sun and [yet] directs its eyes without injury to seeing. But in us, because there is not so much visual spirit and the pupil of the eye is large, we are unable to look at the sun for very long on account of the [resulting] excessive dissolution of the spirits.

The basilisk[11] is said to kill man by sight alone, for it infects the air with the badness of its ray. The visual ray of a wolf is productive of stupor, and a man appearing unexpectedly and seen by a wolf is struck dumb for a time. Wherefore they say that bewitching is produced by the badness of the ray of the seeing thing; and this is why nurses lick the face of a bewitched little boy.

But if a ray issues forth, it is marvelous how it perceives the surface of the solar body, for the sun is 166 times and a fraction larger than the whole earth. This is why it appears to very inquisitive men that the sun is not [really] seen but that air adjacent to the earth and subject to the sun is seen. For how is the ray issuing forth turned back? How also can the ray be emitted [both] yesterday and today, or is a new ray emitted by the eye at every single instant? Moreover, when a certain man is sitting in a different place from others, how do those men see him if the rays emitted from their eyes [that is, his as well as theirs] encounter one another? Also, will one of the rays withdraw from the other, or will there be two rays [in the same place] at the same time?[12] Also, since the stars are much more distant from us than is the sun, how does a small ray extend itself to such a remote place without becoming thinner as it approaches the more remote obstacle? However, to him who is well instructed in physics, it is not doubtful that the visual ray is sent from the lining of the brain through the optic nerve to the window of the eye. And note how the two rays sent to the eyes intersect each other transversely in the form of a cross;[13] just as without faith in the cross the inner man does not see well, so neither is exterior vision achieved without the form of the cross. It should be noted also that a gross vapor sometimes unites itself to a visual ray, whence certain small black bodies sometimes appear to flit before human eyes. . . .[14]

CHAPTER 154. CONCERNING THE MIRROR.

When a mirror is whole, there being but one observer, only one image is produced; but should the glass be broken into several pieces, there appear as many images as there are breaks. And thus in Holy Scripture, as many understandings shine forth as there are expositions. But, marvelous thing!, remove the lead placed behind the glass; immediately no image is visible to the observer. [Likewise,] remove the foundation of the faith, and immediately you will no longer see yourself clearly in Holy Scripture. The lead can be understood to represent sin; thus in the mirror of Holy Scripture you will discern yourself less clearly unless you confess yourself to be a sinner. For if we say that we have not sinned, we deceive ourselves, and the truth is not in us.[15]

11. A fabulous lizard.

12. One of the rays is from the solitary man, the other from the group of men or from one member of the group. "Ray," as Neckam uses the term, denotes not a geometrical line or a narrow beam, but the entire radiation issuing from the eye or eyes of an observer or observers.

13. The optic nerves intersect in the lower forehead, most of the fibers proceeding to the opposite eye. In Neckam's view, rays pass from the brain to the eyes through the optic nerves, and thus they intersect in the form of a cross at this optic chiasma. On the chiasma, see Pecham's *Perspectiva communis*, I, Proposition 32 (Selection 62.7).

14. In the brief remaining portion of this chapter Neckam discusses the effects of a lunar ray on a horse's sore.

15. I John 1:8.

In a concave mirror, the image of the observer appears inverted; in a plane and convex mirror, erect. Who will provide a sufficient explanation of this?[16]

Also the pupil, which is a very small substance, is a mirror, in which the image of the observer shines back. However, for three days before the death of a man, the brightness of the pupil is already darkened, so that for [those] three days the image of an observer does not appear in it.

The image appearing in a mirror appears to conform itself to that of which it is the image. It laughs at a laughing person, and when the observer cries, the image seems to cry [also]. Thus the soul is the mirror of its Maker and ought to sympathize with the suffering Christ and rejoice with the risen and rejoicing Christ. Therefore, at different times different countenances are to be assumed, so long as we always have the countenance of one proceeding toward Jerusalem. . . .[17]

3. Bartholomew the Englishman (fl. 1220–1250): Concerning the Properties of Things

Translated and annotated by Bruce S. Eastwood[18]

BOOK VIII, CHAPTER 40. CONCERNING *lux* AND ITS PROPERTIES.

According to Basil, light *(lux)* is completely homogeneous. Authorities speak diversely on the question whether light is substance or accident. For Aristotle, light *(lumen)* is not corporeal, nor is it an emission by a body. Damascene likewise says that light *(lumen)* has no substance of its own. In *Super Genesim ad litteram*, Augustine maintains that light *(lux)* is a corporeal being; that of corporeal genera it is the simplest; that in efficacy it is the most diverse; that it is most mobile and penetrating while being least resistant; that it most highly unifies diverse and contrary things; that it is most convertible and is ultimately the origin and principle of natural motion. Light *(lux)*, he continues, is most perfectible, most pleasing, and most communicable. Whence there can be discovered in bodies nothing more useful, convenient, beautiful, swift, subtle, impassible, or virtuous than light.

Between *lux* and *lumen* there is a difference. *Lumen* is a certain emission, or irradiation, by the substance of *lux*. *Lux* is the original substance from which *lumen* arises. Now, if *lux* be considered in itself as an accident, as such it must be caused by the form of its subject. Thus, if *lux* in air be an accident, true *lux* is caused by the form of air—which cannot be. *Lux* changes its subject. This is manifest, because *lux* is naturally to the east at first, but subsequently it is in the west. The eastern light engenders more light immediately beside it, which new generation continues the process until light appears in the west. But accidents do not change their subject, nor do they act outside it, so that *lux* seems not to be an accident. Also, if it be an accident of the air, the air must move instantaneously *(subito)* from east to west in the same manner as light rays. However, such motion is incompatible with the air, as with any other element. Again, nothing is nobler than *lux*, but accident is less noble than substance, from which it seems that light is no accident, for air is far more ignoble than light.

If light be corporeal, it becomes difficult to understand how it may be in the air or any other transparent body, for instance, in crystals, since two bodies cannot simultaneously occupy the same place. Yet it is neither impossible nor difficult to posit light simultaneously as corporeal substance and as accompanying some body. We see water and ashes similarly mixed together, preserving the corporality and position of each component; the surface of each remains distinct from the surface of the other, and the parts of each are contained within their proper surface. . . . Light can likewise be in air or another body while retaining

16. Note the primitive understanding of mirrors and their properties: Neckam knew about erect and inverted images but regarded them as incomprehensible.

17. This chapter contains nothing further of optical interest.

18. This translation is based on a comparison of the Latin editions of Strassburg, 1485; Nürnberg, 1519; Frankfurt, 1601 [for the title of this edition, see Selection 58, n. 1—*Ed.*]; and the English version of London, 1582. The sections on vision and geometrical optics in this encyclopedia are extremely general and elementary—much less informative than Grosseteste's *De lineis*. However, Bartholomew's section on *lux*, excerpted here, nicely summarizes the main points of light metaphysics as found in the first half of Grosseteste's *De luce*. Bartholomew's encyclopedia represents a less technical but widely read version of this doctrine, which authors like Bacon and Witelo considered relevant to the science of optics. The authorities cited here by Bartholomew are Neoplatonic, Basil being the most important. While Grosseteste is not mentioned in this chapter, "Rupertus [Robertus] Linconiensis" is listed prior to Book I as an authority.

its own corporality and the continuity of its own substantial parts, . . . although the simplicity of light may prevent its being directly perceived in the medium. How much more wonderful is it that many lights may be brought together and united as one, still preserving unmixed the substantial form of each, by which each differs from the others. Now does any *lux* exist materially or formally by virtue of another. Dionysius teaches this openly in *De divinis nominibus*, as follows. When lamp lights *(lumina)* are in a house and gather together in common, each light remains discrete by one distinction and by distinct unity.[19] We observe many lights as one illumination and one clarity. Nor can anyone, I believe, separate one of these lights from the others, or from the air containing them all at once; nor can anyone discern one of these lights from another, as they exist confusedly among themselves. If one of the lamp lights is removed, it carries with it nothing from the others and relinquishes nothing to them. Indeed, there is a perfect unity, naturally commingled, and no part is confused, and the unity is such in corporeal air and in material light. Thus Dionysius teaches that while luminaries may be united, the substantial and accidental properties of each are preserved, either in increase or in decrease of illumination. . . .

Lux is diffused from the highest heaven to the center of the earth; this light is unitary and simple in its root and substance, but varied according to the diversity of its recipients, whether the sun or the other orbs, superior or inferior, having one substance, virtue, and operation of light, though without luminosity. This is the light created on the first day of Creation, according to Basil, and it preceded the sun and the stars made on the fourth day. The sun and stars are the vehicles of the primeval *lumen*, and because of the support of incorporeal *lux*, they are sufficient to shine perpetually without loss of their own substance. This *lux* reaches everywhere and perfects and influences all bodies, more or less. *Lux* therefore is the root and basis of every illumination; it is one in substance and is not contained accidentally by any lower body but rather contains all bodies. In comparison with them *lux* is quite formal, although it is in itself material and has position. So far as *lux* is material, it has part separate from part potentially. Because *lux* is in truth formal, it has part separate from part actually, for what matter has in potency, form has in actuality. Thus, when *lux* in a kind of bodies has a minimum of matter and a maximum of form, this light is closer to form than to matter. Therefore as matter it is at a minimum, that is, at a point; as form it is everywhere. . . . The form of *lux*, being the noblest of corporeal forms, extends its matter maximally. One point of *lumen* or of *lux* suffices to illuminate the whole world, because of the nobility of its matter and the highest actuality of its form, according to Algazel. Therefore *lux* is unitary and simple as it is uniform in essence, though various luminaries are distinct among themselves. . . .

Lux, as Basil says, is extremely mobile, incessantly moving and generating itself rectilinearly and circumferentially, diffusing itself in every direction, and moving instantaneously *(subito)*. *Lux* is most active and without resistance to internal penetration of anything. *Lux* engenders diversities and brings into unity and accord the contradictions in elements and mixed bodies. By the projection of its rays—direct, intersecting, refracting, and reflecting rays—*lux* brings things into being and conserves them, or it dissolves their existence, as Chalcidius says in commenting on the *Timaeus*. *Lux* rules and directs the life and permanence of everything. It naturally multiplies itself, generating *lux*, which likewise generates successive *lux*. This process occurs in an instant from a single point, filling the whole world.

19 In other words, as explained further on, the lamp lights remain quantitatively distinct as individuals ("by one distinction") and as integral entities ("by distinct unity").

61 ROBERT GROSSETESTE AND THE REVIVAL OF OPTICS IN THE WEST

Translations, introduction, and annotation by David C. Lindberg

Prior to the thirteenth century, Western optical knowledge was exceedingly limited. The best Greek and Islamic sources, including Ptolemy and Alhazen, were as yet unknown. Encyclopedists like Adelard of Bath and Alexander Neckam were thus unable to present more than a smattering of general knowledge gathered from traditional Western sources, such as Plato, Augustine, Boethius, Solinus, Chalcidius, and Isidore, and a few treatises of Greek and Islamic authorship that were just beginning to appear.

But Western optics was transformed in the first half of the thirteenth century through the efforts of Robert Grosseteste (d. 1253). Grosseteste had at his disposal several newly translated treatises on optics, including Euclid's *Optica* and *Catoptrica*,[1] Aristotle's *Meteorologica*, and Alkindi's *De aspectibus*. (It is possible, but unlikely, that he also knew Ptolemy's *Optica*.)[2] The profound influence of these treatises on the level of optical discussions is revealed by the two works of Grosseteste that follow. In another treatise, *De luce* (not included here, but available in English translation),[3] Grosseteste presents a metaphysics of light, according to which the world was created by the self-diffusion of a point of light into a spherical form. Optics thus became for him the fundamental science of nature, revealing not only the manner of creation of the material world but also the mode of all natural actions.

Grosseteste's influence was exercised not only through his written works but also through his association with Oxford University, first as lecturer in theology to the Franciscans and later as Bishop of Lincoln. His metaphysical conceptions influenced the next generation of writers on optics, and his stress on the value of optics and other scientific subjects served to justify a great deal of later scientific endeavor. Indeed, through his influence a scientific tradition was initiated among the Oxford Franciscans, which flowered later in the century in the works of Roger Bacon and John Pecham.[4]

1. Robert Grosseteste (ca. 1168–1253): Concerning Lines, Angles, and Figures[5]

The usefulness of considering lines, angles, and figures is very great, since it is impossible to understand natural philosophy without them. They are useful in relation to the universe as a whole and its individual parts. They are useful also in connection with related properties, such as rectilinear and circular motion. Indeed, they are useful in relation to activity and receptivity, whether of matter or sense; and if the latter, whether of the sense of vision, where activity and receptivity are apparent, or of the other senses, in the operation of which something must be added to those things that produce vision.

Since we have spoken elsewhere of those things that pertain to the whole universe and its individual parts and of those things that relate to rectilinear and circular motion,[6] we must now consider universal action insofar as it partakes of the nature of sublunary things *(inferiorum)*; this universal action is a subject receptive to diverse activities, insofar as it descends to operation in the matter of the world; and some things can be brought in as intermediaries, which are able to bring to perfection that which is advancing toward greater things *(maiora)*.

Now, all causes of natural effects must be expressed by means of lines, angles, and figures, for otherwise it is impossible to grasp their explanation. This is evident as follows. A natural agent multiplies its power from itself to the recipient, whether it acts on sense or on matter. This power is sometimes called species, sometimes a likeness, and it is the same thing whatever it may be called; and the agent sends the same power into sense and into matter, or into its own contrary, as heat sends the same thing into the sense of touch and into a cold body. For it does not act by deliberation and choice, and therefore it acts in a single

1. The latter is no longer attributed to Euclid. Both works have been translated by Paul Ver Eecke in *Euclide, L'Optique et la catoptrique* (Paris: Blanchard, 1959).

2. This is Albert Lejeune's position in his *L'Optique de Claude Ptolémée dans la version latine d'après l'arabe de l'émir Eugène de Sicile* (Louvain: Bibliothèque de l'Université, Bureaux du recueil, 1956), p. 31*. The alternative but, I believe, less easily defended position is adopted by A. C. Crombie, *Robert Grosseteste and the Origins of Experimental Science, 1100–1700* (Oxford: Clarendon Press, 1953), pp. 116, 120.

3. Robert Grosseteste, *On Light*, translated by Clare C. Riedl (Milwaukee: Marquette University Press, 1942).

4. On the Franciscan school, see A. G. Little, "The Franciscan School at Oxford in the Thirteenth Century," *Archivum Franciscanum Historicum*, XIX (1926), 803–874. However, note the cautions set forth by Stewart Easton in an appendix to his *Roger Bacon and His Search for a Universal Science* (Oxford: Blackwell, 1952), pp. 206–209.

5. I have used the text from *Die philosophischen Werke des Robert Grosseteste, Bischofs von Lincoln*, edited by Ludwig Baur in *Beiträge zur Geschichte der Philosophie des Mittelalters*, Vol. IX (Münster/Westfalen: Aschendorffsche Verlagsbuchhandlung, 1912), pp. 59–65. Frequent reference has been made to the prior, unpublished translation of this treatise by Bruce S. Eastwood in "The Geometrical Optics of Robert Grosseteste" (Unpublished Ph.D. dissertation, University of Wisconsin, 1964).

A glance at Baur's variant readings reveals that a more justifiable title would be "Concerning the Refraction and Reflection of Rays." Baur gives both titles, but the former has become so firmly established in the literature that a change seems inadvisable.

6. Baur, *Philosophischen Werke*, p. 79*, identifies the works of Grosseteste treating these other matters as *De sphera, De motu corporali et luce*, and *De motu supercelestium*.

manner whatever it encounters, whether sense or something insensitive, whether something animate or inanimate. But the effects are diversified by the diversity of the recipient, for when this power is received by the senses, it produces an effect that is somehow spiritual and noble; on the other hand, when it is received by matter, it produces a material effect. Thus the sun produces different effects in different recipients by the same power, for it cakes mud and melts ice.

The power from a natural agent reaches the recipient either along a shorter line—and then the power is more active, since the recipient is less distant from the agent—or along a longer line—and then it is less active, since the recipient is more distant. In any case, the power comes from the surface of the agent with or without mediation. If without mediation, the power comes either by a straight line or by a bent line. But if by a straight line, the action is stronger and better, as Aristotle proposes in Book V of the *Physics*,[7] since nature acts in the briefest possible manner; and a straight line is the shortest of all lines, as he says in the same place. Moreover, a straight line possesses equality because it has no angle; but the equal is superior to the unequal, as Boethius says in his *Arithmetic*.[8] However, nature acts in the briefest possible manner; consequently it acts best along a straight line. Similarly, every united power acts more strongly. But there is greater oneness and unity in a straight line than in a bent line, as is asserted in Book V of the *Metaphysics*;[9] therefore action in a straight line is stronger.

Now, a straight line falls either at equal angles (that is, perpendicularly) or at unequal angles. If at equal angles, the action [along it] is stronger for the three aforementioned reasons, since such a line is shorter and possesses equality, and the power comes uniformly along it to the parts of the recipient. But a line falling at equal angles on some body falls at right angles when it is incident on a plane, at acute angles on a concave body, and at obtuse angles on a sphere. This is illustrated as follows: If an incident line is drawn passing through the center of a sphere, it is perpendicular to the line of tangency, and between the line of tangency and the sphere, contingent angles are formed on both sides.[10] Therefore, such a line incident on a sphere forms two angles with the surface of the sphere, each of which is larger than a right angle, since it equals a right angle plus a contingent angle. Therefore, when power is incident at angles that are not merely equal but right angles, its action is seen to be the strongest, since complete equality and uniformity exist.

If there should be a line that is not straight (or one that is bent), then since it is not circular because a natural agent does not emanate its power along a circular path but along the diameter of a circle on account of the brevity of the latter, it is evident that such a line has angles. And this would not occur in the presence of a single medium or when a single body is encountered; but [in order for the line to be bent] there must be two media, since the power is multiplied along certain straight lines in the first medium and along others in the second. However, this can occur only in two ways, namely when the recipient body is dense so as to impede the passage of the power, chiefly in the case of our senses—and then it is called a reflected line, because the power returns—or when the opposing body is rare, thereby permitting passage of the power. If in the first way, then the power incident on a dense body falls either at equal angles (that is, perpendicularly) or at unequal angles. If the first of these, the power returns into itself along the same path by which it approached [the dense body]. The explanation of this is that lines incident on and reflected from a body form similar angles of the same size. Therefore, the power must be reflected at an angle equal to the angle of incidence and [thus] return in the same way [as it approached]. For if it should return at another angle or by another path, deviating to the right or to the left, it could not return at a [reflected] angle equal to the angle of incidence; rather, the angle would be larger or smaller. But if the power is incident nonperpendicularly, it returns along a path by which it is able to form an angle with the surface of the resisting body equal to the angle of incidence (that is, the angle formed by that body and the incident line) for the aforementioned reason. Thus the angles of incidence and reflection are always equal, which may now be assumed.

Since reflection occurs in these two modes, it is to be understood that a power reflected into itself is stronger than a power reflected along some other path, on account of the doubling of the power

7. Baur (p. 61) suggests that the statement may actually have been drawn from Aristotle's *Metaphysics* V.6.
8. I.32 and II.1, according to Baur, p. 61.
9. V.6.1016a ff., according to Baur, p. 61.
10. Medieval scientists regarded "contingent angles" (or angles of tangency) as having a finite magnitude. On the history of this view, see T. L. Heath, *The Thirteen Books of Euclid's Elements*, 2d ed. (New York: Dover, 1956), II, 39–42.

in the same place. Nevertheless, however much this may [seem to] follow from the theory of reflection, the action [of a power] is weaker when reflection is along the same path [as that of the approach,] because, since every reflection weakens a power, that reflection is weakened more which makes a power deviate completely [that is, 180°] from the direct progress that is proper to it if it is to pass through the body; and this is reflection along the path of incidence. And such a path is altogether contrary and opposite to the direct progress proper to the power.

When reflection occurs from a polished body possessing the nature of a mirror, then the reflection is best and the action strongest; but when reflection occurs from rough bodies, the species is dissipated and the action is weak. The Commentator assigns the cause of this in his treatise on sound,[11] saying that in the reflection of a species all parts of a polished body and a smooth surface concur in one action, on account of their evenness and uniformity; therefore the whole species is reflected from a polished body integrally, just as it approaches the body. But the parts of a rough body are uneven and the highest parts reflect the species first, and thus the parts do not unite in one action; for this reason the species is broken into parts, and therefore its action is not strong.

When reflection takes place from concave bodies, its action is stronger than when reflection takes place from plane or convex bodies, because rays reflected from a concave surface converge together; but this is not true of the other surfaces.

Now, if the body encountered by a power should not [entirely] prevent passage of the power, then a ray incident at equal angles (that is, perpendicularly) maintains rectilinear progress, and this is the strongest ray. But the ray incident at unequal angles deviates from the rectilinear path that it had in the first substance, which would be maintained if the medium [in which the power is propagated] were uniform. And this deviation is called refraction of the ray. This refraction is twofold: If the second substance is denser than the first, the ray is refracted to the right[12] and passes between the path of rectilinear progress[13] and the perpendicular drawn to the second body at the point of refraction; if, on the other hand, the second substance is subtler than the first, the ray is refracted toward the left, receding from the perpendicular on the other side of the path of rectilinear progress. And since these things are so, it should be understood that a power incident along a refracted line is stronger than a power incident along a reflected line, since a refracted line deviates slightly from a rectilinear course, which is strongest, while a reflected line deviates a great deal, [passing] in the opposite direction;[14] therefore reflection weakens a power more than refraction. Regarding the two modes of refraction, it can be asserted that a power refracted toward the right is stronger than that refracted toward the left, since the power refracted toward the right more nearly approaches the perpendicular course, whether we refer to the perpendicular drawn from the point of refraction or that drawn from the agent, from the same point of which issue both a perpendicular and a refracted line.[15]

Besides these three essential lines, there is a fourth accidental line, along which an accidental and weak power proceeds. However, this power does not come directly from the agent but from a power multiplied along any of the three aforementioned lines; for example, accidental light comes to all corners of a house from a ray falling through a window. This power is the weakest of all, since it does not issue directly from the agent but from a power of the agent directed along a straight, reflected, or refracted line. These things are asserted regarding lines and angles.

Regarding figures, there are two kinds that must be considered here. One of these, namely the sphere, is required by the multiplication of a power; for every agent multiplies its power spherically, since it does so in every direction and along all diameters—upward, downward, before, behind, to the right, and to the left. This is evident, since, for the same reason that a line extends in one direction from a centrally located agent, lines will extend likewise in every direction according to

11. Two manuscripts give the reading "chapter on sound in the second book of *De anima*." Crombie gives the reference as II.4.vi, fol. 143 recto of the Venice, 1551, edition of Averroes' commentary on Aristotle's *De anima*.
12. Grosseteste obviously had a figure in mind, although none appears in any of the manuscripts. Nevertheless, his intent is clear: in passing from a less dense to a more dense medium, the ray is refracted toward the perpendicular.
13. That is, the rectilinear extension of the incident ray.
14. It is, of course, not true that a reflected ray must deviate from rectilinearity more than a refracted ray.
15. This conclusion appears to contradict the earlier statement that bending weakens, for here Grosseteste seems to imply that a ray gains strength by deviating from its rectilinear course and approaching the perpendicular.

all differences of position—and thus necessarily in spherical fashion. This is asserted by the Commentator in the second book of *De anima*.[16] Moreover, wherever an organ of sense is placed, it is able to perceive such an agent at a suitable distance;[17] but this is possible only by means of a species or power coming from the agent. Therefore, that power is multiplied in all directions.

The other figure required for natural actions is the pyramid.[18] For if power should issue from one part of an agent and terminate at one part of the recipient, and if this should be true of all powers so that power always comes from one part of the agent to a single part of the recipient, no action will ever be strong or good. But action is complete when the power of the agent comes to every point of the recipient from all points of the agent or from its entire surface. But this is possible only by means of a pyramidal figure, since powers issuing from the single parts of the agent [which constitutes the base of the pyramid] converge and unite at the apex of the pyramid; and therefore all are able to act strongly on the part of the recipient encountered. Thus infinitely many pyramids are able to issue from one surface of an agent; as base of all these pyramids there is one thing, namely the surface of the agent, and there are as many apices as there are pyramids. The apices are located everywhere, at all the different points of the medium or recipient; and infinitely many pyramids issue in any one direction, some shorter and some longer. But there is no diversity [of power] in those pyramids of equal length or shortness, since, for their part, they all act identically, although diversity can originate with the recipient matter.

When one pyramid is shorter than another and both issue from the same agent, there is great difficulty [in determining] whether the apex of the shorter pyramid acts more strongly on the recipient. It must be asserted that a shorter pyramid acts more strongly, since its apex is less distant from its source, and consequently more power reaches the apex than in a longer pyramid; thus the recipient is more closely joined to the agent by the shorter pyramid, and therefore it is more strongly altered by the power [of the agent]. Moreover, if rays from the right side of the lateral surface of the shorter pyramid are extended beyond the apex in a continuous straight line, they form smaller angles[19] with rays from the left side of the lateral surface of the pyramid than do similar rays in a longer pyramid, as is evident from Proposition 21 of the first book of Euclid's *Geometry* and also to sense. In the same way, rays coming from the left side of the pyramid extended beyond the apex in a continuous straight line are more completely united with rays from the right side of the lateral surface of the pyramid than similar rays of a longer pyramid. Consequently, since every congregation and union is more active, the apex of a shorter pyramid acts more strongly and also alters the recipient more than the apex of a longer pyramid. Nevertheless, if it should be objected rationally that power comes to the apex of a longer pyramid from the whole surface of the agent,[20] that the power congregates there more completely because that apex is more acute than the apex of a shorter pyramid, that every united power acts more strongly, and moreover that the rays of a longer pyramid are closer to the perpendicular rays drawn from the extremities of the diameter of the agent, and therefore that longer pyramids are stronger because a perpendicular path is the strongest, it can be asserted that those reasons are quite conclusive insofar as they suffice; therefore, they prevail unless the stronger reasons mentioned above are to the contrary.

Here ends the treatise by [the Bishop] of Lincoln on the refraction and reflection of rays.

2. Robert Grosseteste: On the Rainbow[21]

Investigation of the rainbow is the concern of both the student of perspective and the physicist. It is for the physicist to know the fact and for the student of perspective to know the explanation. For this reason Aristotle, in his book on *Meteorology*, has not revealed the explanation, which concerns the student of perspective; but he has condensed the facts of the rainbow, which are the concern of the physicist, into a short discourse. Therefore, in the present treatise we have undertaken to provide the explanation, which concerns the student of perspective, in proportion to our limited capability and the available time.

In the first place, then, we state that perspective is the science based on visual figures and that this is subordinate to the science based on figures containing radiant lines and surfaces, whether that radiation is emitted by the sun, the stars, or some other radiant body. Nor is it to be thought that the emission of visual rays [from the eye] is only imagined and without reality, as those think who consider the part and not the whole. But it should be understood that the visual species [issuing from the eye] is a substance, shining and radiating like the sun, the radiation of which, when coupled with radiation from the exterior shining body, entirely completes vision.

Wherefore natural philosophers, treating that which is natural to vision (and passive), assert that vision is produced by intromission. However, mathematicians and physicists, whose concern is with those things that are above nature, treating that which is above the nature of vision (and active), maintain that vision is produced by extramission. Aristotle clearly expresses this part of vision that occurs by extramission in the last book of *De animalibus*, saying: "A deep-set eye sees from a distance; for its motion is neither divided nor destroyed, but a visual power leaves it and goes directly to the objects seen."[22] Again in the same book [Aristotle writes]: "The three senses referred to, namely vision, hearing, and smell, issue from the organs [of perception] as water issues from pipes, and for this reason long noses have a strong power of smell."[23] Therefore, true perspective is concerned with rays emitted [by the eye].

Perspective has three principal subdivisions, according to the triple mode in which rays are transmitted to the visible object. For either transmission of the ray to the visible object is in a straight line through a transparent medium of a single kind interposed between the observer and the object; or in a straight line to a body having a spiritual nature which is such as to make it a mirror, from which the ray is reflected to the object; or through several transparent media of different kinds, at the junctions of which the visual rays are refracted to form angles, so that the ray reaches the object not by direct approach but by a path of several straight lines joined at angles.

The first subdivision comprises the science called *"De visu,"* the second subdivision the science called *"De speculis."* The third part has remained untouched and unknown among us until the present time. Nevertheless, we know that Aristotle completed this third part, which surpasses the other parts in its exceeding subtlety and the greatly admired profundity of its natures.[24] This part of perspective, if perfectly understood, shows us how to make very distant objects appear close, how to make nearby objects appear very small, and how to make a small object placed at a distance appear as large as we wish, so that it would be possible to read minute letters from incredible distances or count sand, seeds, blades of grass, or any minute objects.[25] The way in which these astonishing things occur is made manifest as follows. The visual ray penetrating through several transparent substances of diverse natures is refracted at their junctions, and its parts, in the different transparent media existing at those junctions, are joined at an angle. This is revealed by that experiment which is considered fundamental in the book *De speculis*:[26] If an object is placed in a vessel and the observer stations himself at a position from which the object cannot be seen, the object will become visible when water is poured in. The same thing is also revealed by the fact that the subject of a continuous ray is a substance possessing a single [homogeneous] nature. Therefore, at the interface between two transparent media of different kinds, a visual ray must be interrupted. However, since the entire ray is generated by a single principle, and its continuity cannot be entirely destroyed unless its generation ceases, the discontinuity of the ray at the interface between the two transparent media must be incomplete. But the only mean between complete continuity and complete discontinuity is a point touching the two parts of one ray, [joined] not in a straight line but at an angle.

16. Averroes, as cited in note 11.
17. That is, neither too close nor too far.
18. Grosseteste uses the term "pyramid" to denote all figures having a plane or curved base and straight lines extending from every point on the base to a common apex; the base need not be a regular polygon.
19. That is, the angles supplementary to the angle of the apex.
20. Grosseteste evidently has in mind a convex surface, such as the surface of a sphere, so that a longer pyramid includes rays from a larger portion of the surface than do shorter pyramids.
21. I have translated this treatise from the Latin text edited by Ludwig Baur, pp. 72–78. Portions of this treatise have also been published in translation by Crombie, chapter 5. The majority of the manuscripts collated by Baur contain the title *De Iride;* however, the British Museum MS contains the title *De Irlde et speculo*, and Baur gives both titles.
22. *De generatione animalium*, V.1.781a.1–2.
23. *Ibid.*, V.1.781b.2–13.
24. Grosseteste apparently has in mind an ancient tradition to the effect that Aristotle wrote a book on optics; see Hero of Alexandria, *De speculis*, in *Heronis Alexandrini Opera quae supersunt omnia*, ed. L. Nix and W. Schmidt, Vol. 2, fasc.1 (Leipzig, 1900), p.318; compare Carl B. Boyer, "Aristotelian References to the Law of Reflection," *Isis*, XXXVI (1945–1946), 93, n. 14.
25. These views, long marveled at in Roger Bacon, were obviously borrowed by the latter from Grosseteste.
26. Pseudo-Euclid, *Catoptrica*, Definition 6.

The amount of divergence from rectilinearity of rays joined at an angle can be represented as follows. Imagine a ray from the eye incident through air on a second transparent medium, extended continuously and rectilinearly [into the second medium], and a line perpendicular to the interface drawn into the depth of the medium from the point at which the ray is incident on the [second] transparent medium. I say, then, that the path of the ray in the second transparent medium is along the line bisecting the angle enclosed by the ray which we have imagined to be extended continuously and rectilinearly and the perpendicular line drawn into the depth of the second transparent medium from the point of incidence of the ray on its surface.[27]

That the size of the angle of refraction of a ray may be thus determined is evident from an experiment similar to those by which we have learned that reflection of a ray by a mirror occurs at an angle equal to the angle of incidence. The same thing is revealed to us by the following principle of natural philosophy, namely that every operation of nature takes place in a manner as limited, well-ordered, brief, and good as possible.[28]

An object seen through several transparent media does not appear as it really is but appears to be situated at the intersection of the ray emitted by the eye, extended in a continuous straight line, and the line drawn perpendicularly from the visible object to the surface of the second transparent medium (that is, the one nearer the eye). This is revealed to us by similar reasoning and the same experiment as that by which we know that objects seen in mirrors appear at the intersection of the visual ray, extended in a straight line, and the perpendicular drawn [from the visible object] to the surface of the mirror.

Thus far we have considered the size of the angle of refraction at the interface between two transparent media and the place of appearance of an object seen through several transparent media. To these we add the following principles, which the optical theorist appropriates from the natural philosopher, namely that the size, position, and order of the visible object are determined from the size of the angle and the position and order of the rays under which the object is seen and that it is not great distance that makes an object invisible, except accidentally, but rather the smallness of the angle under which it is observed. From these principles, it is perfectly evident by geometrical reasoning how an object of known distance, size, and position will appear with respect to location, size, and position. From the same principles it is evident how one must shape transparent substances so that they will receive the rays emitted by the eye, according to the desired angle formed in the eye, and will draw the rays back [toward convergence] in any desired manner onto visible objects, whether these objects are large or small, far or near. Thus all visible objects are made to appear to the observer in the desired position and of the desired size; and very large objects, if so desired, are caused to appear very small; and conversely, very small objects placed far away are caused to appear large and very easily perceptible by sight.

The science of the rainbow is subordinate to this third part of perspective [that is, to the science of refraction]. Now, a rainbow cannot be formed by means of solar rays falling in a straight line from the sun into the concavity of a cloud,[29] for they would produce a continuous illumination of the cloud in the shape of the opening toward the sun through which the rays enter the concavity of the cloud rather than in the shape of a bow. Nor can a rainbow be formed by the reflection of solar rays from the convexity of the mist descending from a cloud as from a convex mirror, in such a way that the concavity of the cloud would receive the reflected rays and the rainbow would thus appear; for if that were so, the rainbow would not always be in the form of an arc, and as the sun rose the rainbow would become proportionately larger and higher, and as the sun set the rainbow would become smaller; the contrary, however, is evident to sense. Therefore, the rainbow must be formed

27. That is, the angle of refraction equals half the angle of incidence. Diagrams illustrating this conception appear in Crombie, page 121. This half-angle law of refraction is remarkably primitive by contrast with Ptolemy's elaborate attempts to find, by empirical means, the true mathematical law of refraction; see Albert Lejeune, *Recherches sur la catoptrique grecque* (Bruxelles: Palais des académies, 1957), pp. 152–166. It seems doubtful that Grosseteste would have formulated his half-angle law had he known Ptolemy's *Optica*.

28. This paragraph reveals that Grosseteste's half-angle law of refraction was determined not through measurement but rationally, on grounds of symmetry and brevity of action. See Bruce S. Eastwood, "Grosseteste's 'Quantitative' Law of Refraction: A Chapter in the History of Non-Experimental Science," *Journal of the History of Ideas*, XXVIII (1967), 403–414.

29. This is roughly the Aristotelian view, expressed in the *Meteorologica* III.2–5. On the history of the theory of the rainbow, see Carl B. Boyer, *The Rainbow: From Myth to Mathematics* (New York: Yoseloff, 1959), and Selection 63.2 from the *De iride* of Theodoric of Freiberg.

by the reflection of solar rays in the mist of a convex cloud. I maintain that the outside of a cloud is convex and the inside concave, as is evident from the nature of light and heavy. And that which we see of a cloud must be less than a hemisphere, although it may look like a hemisphere; and since the mist descends from the concavity of the cloud, it must be pyramidally convex at the top, descending to the earth, and therefore more condensed near the earth than in the higher part.[30]

Therefore, there are, in all, four transparent media through which a ray of the sun penetrates: [first,] pure air containing the cloud; second, the cloud itself; third, the higher and rarer mist coming from the cloud; and fourth, the lower and denser part of the same mist. Therefore, in accordance with what was said before about the refraction of rays and the size of the angle of refraction at the interface between two transparent media, solar rays must be refracted first at the interface between the air and the cloud and then at the interface between the cloud and the mist. By these refractions the rays converge in the density of the mist and, being refracted there once more as from the vertex of a pyramid, spread out not into a round pyramid but into a figure like the curved surface of a round pyramid[31] expanded opposite the sun. Therefore it assumes the shape of an arc, and in these [northern] regions the rainbow does not[32] appear in the south. And since the vertex of the aforementioned figure is near the earth and it is expanded opposite the sun, half the figure or more must fall on the surface of the earth and the remaining half or less onto a cloud opposite the sun. Therefore, near sunrise or sunset the rainbow appears semicircular and is larger, and when the sun is in other positions the rainbow appears as part of a semicircle; and the higher the sun, the smaller the [visible] part of the rainbow. For this reason, in regions where the sun closely approaches the zenith, the rainbow never appears at noon. Aristotle's claim that the variegated rainbow is of small measure at sunrise and sunset is not to be interpreted as smallness of size, but as smallness of luminosity, which occurs because the rays pass through a greater multitude of vapors at this time of day than at other times. Aristotle himself indicates this later, when he says that this [smallness] occurs because of the diminution of that which shines from the solar ray in the clouds.

However, since color is light mixed with a transparent medium—the transparent medium being diversified according to purity and impurity, while light is divided in a fourfold manner (according to brightness and darkness and according to multitude and paucity), and all colors being generated and diversified according to the combinations of these six differences—the variety of colors in different parts of one and the same rainbow occurs chiefly because of the multitude and paucity of solar rays. For where there is a greater multiplication of rays, the color appears clearer and more luminous, and where there is a smaller multiplication of rays, the color appears more bluish and obscure. And since the multiplication of light and the diminution determined by this multiplication result only from the splendor of the illumination on the mirror or from the transparent medium, which collects light in a certain place because of its shape [that is, the shape of the medium] and, after it has come together, diminishes it by separation, and since this disposition for the reception of light is not fixed, it is manifest that painters do not have the ability to represent the rainbow; nevertheless, its representation according to a fixed[33] disposition is possible.

Actually, the difference in color between one rainbow and another arises both from the purity and impurity of the recipient transparent medium and from the brightness and darkness of the light impressed on the medium. For if the transparent medium is pure and the light is bright, the color of the rainbow will be whitish and similar to light. But if the recipient transparent medium should contain a mixture of smoky vapors and the light is not very bright, as occurs near the rising or setting [of the sun], the color of the rainbow will be less brilliant and darker. Similarly, [the production of] all variations in color of the variegated bow is sufficiently apparent from other combinations of brightness and darkness of light and purity and impurity of the transparent medium.

Here ends the treatise on the rainbow by [the Bishop] of Lincoln.

30. For debate on the interpretation of the scheme described in this paragraph, see D. C. Lindberg, "Roger Bacon's Theory of the Rainbow: Progress or Regress?," *Isis*, LVII (1966), 240; Bruce S. Eastwood, "Robert Grosseteste's Theory of the Rainbow," *Archives internationales d'histoire des sciences*, XIX (1966), 323–324. Grosseteste's theory has also been analyzed by Crombie, chapter 5.

31. That is, a hollowed-out cone.

32. Baur's text does not include the negative, but Crombie (p. 126) argues for its inclusion and adduces manuscript support for his position.

33. According to Baur, the manuscripts collated for his text are unanimous in reading *non fixam*. However, omission of the negative seems to be required if sense is to be made of the sentence.

62 LATE THIRTEENTH-CENTURY SYNTHESIS IN OPTICS

Translations, introduction, and annotation by David C. Lindberg[1]

Although Robert Grosseteste greatly stimulated European interest in optics, his own investigations were hampered by the lack of many important sources. Ptolemy's *Optica* was just becoming known in the West, and it is doubtful that Grosseteste was familiar with it; Alhazen's *Perspectiva*, although translated late in the twelfth or early in the thirteenth century, had not come to Grosseteste's attention by the time he composed his optical works. Consequently, Grosseteste's knowledge of optics, though more complete than that possessed by any Western predecessor or contemporary, was primitive by comparison with the optical achievements of Islam; thus his works lack the scope and depth of such a work as Alhazen's great optical treatise, as even the most casual inspection will reveal.

However, by the second half of the century, the most advanced treatises of Greek antiquity and medieval Islam had been rendered into Latin. The most important of these for the development of optics were the *Perspectiva* of Alhazen, the *Optica* of Ptolemy, a number of works of Avicenna, and (toward the end of the period) the *Catoptrica* of Hero of Alexandria. These new riches, especially Alhazen's *Perspectiva*, occasioned a dramatic surge in the study of optics, which found its best expression in the works of Roger Bacon, Witelo, and John Pecham. Thus, whether in Alhazen's *Perspectiva* (which not only served as the principal source for Western writers but also circulated widely itself) or in the works of Bacon, Witelo, and Pecham which it inspired, the West possessed for the first time treatises in which almost the entire gamut of problems now classified as optical was systematically treated; moreover, in these treatises we find an enormous increase in the sophistication of mathematical techniques employed. Indeed, we find what must be considered a grand synthesis of optical knowledge, comprehensive in scope and incorporating the very best learning of Greece, Islam, and the Latin West. It speaks for the vigor and excellence of this synthesis, that the progress of optics throughout the remainder of the Middle Ages came about as scientists extended the application of principles set forth late in the thirteenth century[2] or as questions raised in the thirteenth century, but incompletely or unsatisfactorily answered, became the objects of inquiry and debate.

The selections appearing in this chapter are an attempt to convey the principal doctrines and techniques of this late thirteenth-century synthesis. They are drawn from the works of Roger Bacon, Witelo, and John Pecham, all of whom wrote on optics during the 1260's and 1270's. In addition, it has been deemed essential to include selections from the Latin text of Alhazen's *Perspectiva* to illustrate further the optical knowledge available in Europe after about 1250 and the origin of many of the ideas expressed by Bacon, Witelo, and Pecham.[3] Since Alhazen and his three thirteenth-century followers were in such complete agreement on most fundamental issues, it has been unnecessary to present the view of each individual on every question; rather, those passages have been selected that most clearly and concisely present the shared theory. In cases of major disagreement, as on the question of visual rays, alternative views are presented.

Finally, it should be noted that the magnitude of the thirteenth-century optical synthesis has made it both possible and necessary to omit all but two selections illustrating optical progress during the later Middle Ages: possible, because the thirteenth-century synthesis provided the framework within which later optics was pursued, so that the views presented here largely represent later medieval views as well; necessary, because late medieval discussions were of a highly particular sort impossible to treat adequately in a source book.[4]

1. Except for section 18, which was translated by Robert B. Burke.

2. Set forth, that is, by Alhazen's *Perspectiva* as well as by those thirteenth-century treatises based upon it. One could legitimately argue that this synthesis was actually Islamic and occurred in the eleventh century in the works of Alhazen and Avicenna. Nevertheless, their works did not become available to the West until the thirteenth century, whereupon they inspired further efforts by Bacon, Witelo, and Pecham; thus, so far as the optical knowledge of the West is concerned, the synthesis occurred in the late thirteenth century.

3. Unless otherwise noted, the selections have been translated or reprinted from the following editions: Alhazen, *Perspectiva*, from *Opticae thesaurus Alhazeni Arabis libri septem*, edited by Friedrich Risner (Basel, 1572); Witelo, *Perspectiva*, bound with Alhazen's *Perspectiva* in the *Opticae thesaurus*, but separately paginated; Roger Bacon, *The Opus Majus of Roger Bacon*, edited by J. H. Bridges (London, 1900); Roger Bacon, *De multiplicatione specierum*, included in Volume II of Bridges' edition of the *Opus maius*. The propositions from John Pecham's *Perspectiva communis* (both revised and unrevised versions) are reprinted by permission of the copyright owners, the Regents of the University of Wisconsin, from *John Pecham and the Science of Optics: Perspectiva communis* edited with an introdution, English translation, and critical notes by David C. Lindberg (Madison, Wis.: University of Wisconsin Press, 1970). The order in which the propositions appear in this source book and their page numbers in my Pecham volume are as follows: Part I, Proposition 27, revised version (p. 109); Propositions 29–34 (pp. 111–119); Propositions 43, 28, 37, 33, 38 (pp.127, 109–110, 121, 119, 121–123); Propositions 44–46 (pp. 127–131); Part II, Propositions 6, 20, 30 (pp.161–163, 171–173, 183–185); Part III, Proposition 4 (p.215); Proposition 16 (pp.229–231). Henceforth abbreviated citations will be used.

1. Roger Bacon (ca. 1219–1292): The Nature and Multiplication of Light or Species

OPUS MAIUS,[5] IV, DISTINCTION 2, CHAPTERS 1–2

Every efficient cause acts through its own power, which it exercises on the adjacent matter, as the light *(lux)* of the sun exercises its power on the air (which power is light [*lumen*] diffused through the whole world from the solar light [*lux*]). And this power is called "likeness," "image," and "species" and is designated by many other names, and it is produced both by substance and by accident, spiritual and corporeal, but more by substance than by accident and spiritual than corporeal. This species[6] produces every action in the world, for it acts on sense, on the intellect, and on all matter of the world for the generation of things, since one and the same thing is produced by a natural agent in whatever it acts upon because it does not have the power to deliberate; therefore it produces the same action in whatever it encounters. But if it acts on sense and the intellect, it produces species, as everybody knows; and if it acts on the contrary or on matter, it [also] produces species. And in things that have reason and intellect, although they do many things with deliberation and choice, nevertheless this operation (that is, the generation of species) is natural for them as for other things. Wherefore the substance of the mind multiplies its power inside the body and outside the body, and every body produces its power outside itself; angels move the world through powers of this kind. . . . Therefore, such powers of agents produce every action in this world. Now, there are two things to be considered with respect to these powers: one is the multiplication itself of the species and power from the place of its generation; the other is the varied action [of this species or power] in the world because of the generation and corruption of things. Since the second cannot be known without the first, the multiplication itself must first be described.

Every multiplication is according to lines, angles, or figures. As long as a species advances through a medium possessing a single rarity, as entirely in the heavens, entirely in fire, entirely in air, or entirely in water, it always maintains a rectilinear path, since Aristotle says in the fifth book of the *Metaphysics* that nature always acts in the shortest possible way, and a straight line is the shortest of all lines. . . .

But when there is a second body of different rarity and density, so that it is not altogether dense but alters the passage of the species in some way—like water, which is partly rare and partly dense, and similarly, crystal and glass and such substances through which we can see—then the species is incident on the second body either perpendicularly, and continues along the same straight line as before, or if it is not incident perpendicularly its rectilinear path is necessarily altered, and it forms an angle upon entering the second body. And this deviation from rectilinear progress is called refraction of the ray and species. The explanation of this is that the perpendicular is stronger and shorter and therefore nature acts in a better way along it, as geometrical demonstrations teach, of which further mention will be made below in the proper place. But this refraction is double, since if the second body is denser (as is the case in descending from the heaven into the lower regions), all stellar powers which are not perpendicularly incident on the sphere of the elements are refracted

4. On the history of late medieval optics, see Graziella Federici Vescovini, *Studi sulla prospettiva medievale* (Turin: Giappichelli, 1965).

5. Bridges, I, 111–117. Similar views on species appear in Bacon's *De multiplicatione specierum*, II, chapters 1–2 (Bridges, II, 457–465); however, as is frequently the case, Bacon expresses himself more concisely in his later and more refined *Opus maius*. Note the similarities between this passage and portions of Grosseteste's *Concerning Lines, Angles, and Figures*, included in the preceding selection.

6. Neither Witelo nor Alhazen employs the term "species," as do Bacon and Pecham. Nevertheless, as I have argued at length in my "Alhazen's Theory of Vision and Its Reception in the West," *Isis*, LVIII (1967), 321–341, Alhazen's and Witelo's "forms" are identical, in most respects, to Bacons and Pecham's "species."

between the direct path and the perpendicular erected at the place of refraction. And if the second body is subtler (as is the case in ascending from water to higher regions), then the direct path falls between the refracted ray and the perpendicular erected at the place of refraction.

When the second body is so dense that it in no way permits passage of a species—I speak of sensible passage, as concerns the judgment of human sight—then we say that the species is reflected. However, according to Aristotle and Boethius, the sight of a lynx penetrates walls. Therefore, in fact the species passes through, and this is true; however, human sight is unable to judge regarding this, but [only] regarding reflection, which necessarily takes place.[7] For because of the difficulty of passage through the dense body, since it has easy passage in the air through which it arrives, it multiplies itself more copiously in the direction from which it came [than straight ahead through the dense body]. . . .

But the fourth [kind of multiplication] is more necessary to the world, although it is called accidental multiplication. For light *(lumen)* is called accidental with respect to the principal light *(lux)*[8] coming from the thing, since the former does not come [directly] from the agent but from the principal multiplication, as the principal multiplication falls from the sun into a house through a window, but to a corner of the house comes accidental light from a ray passing through the window. The bodies of mortals could not always be exposed to the principal species without undergoing corruption, and therefore God has moderated all things through such accidental species.

The fifth [kind of multiplication] is different from the others, for it violates the common laws of nature and claims a special privilege for itself. And this multiplication occurs only in an animated medium as in the nerves of the senses; for a species follows the tortuous path of the nerve and does not concern itself with rectilinear passage.[9] And this takes place by the power of the mind directing the passage of the species, as the operations of an animated being require. . . .

OPUS MAIUS,[10] V.1, DISTINCTION 9, CHAPTER 4

But a species is not body, nor is it moved as a whole from one place to another; but that which is produced [by an object] in the first part of the air is not separated from that part, since form cannot be separated from the matter in which it is unless it should be mind; rather, it produces a likeness to itself in the second part of the air, and so on.[11] Therefore there is no change of place, but a generation multiplied through the different parts of the medium; nor is it body which is generated there, but a corporeal form that does not have dimensions of itself but is produced according to the dimensions of the air; and it is not produced by a flow from the luminous body but by a drawing forth out of the potentiality of the matter of the air. . . .

2. John Pecham (ca. 1230–1292): The Nature and Multiplication of Light or Species

PERSPECTIVA COMMUNIS, REVISED VERSION, I, PROPOSITION 27

Every natural body, visible or invisible, diffuses its power radiantly into other bodies. The proof of this is by a natural cause, for a natural body acts outside itself through the multiplication of its form. Therefore, the nobler it is, the more strongly it acts. And since action in a straight line is easier and stronger for nature, every natural body, whether visible or not, must multiply its species in a continuous straight line; and this is to radiate . . .

3. Roger Bacon: The Nature and Multiplication of Light or Species

DE MULTIPLICATIONE SPECIERUM,[12] II, CHAPTER 1

It is to be understood that lines along which multiplication [of species] occurs do not have length alone, extended between two points, but all of them [also] have width and depth, as the authors of books on sight determine. Alhazen demonstrates in his fourth book that every ray coming from a part of a body necessarily has width and depth as well as length.[13] Similarly, Jacob Alkindi says[14] that an impression is similar to that which produces it; now, the impressing body has three dimensions, the corporeal property of which [dimensions] is possessed by the ray. And he adds that a ray does not consist of straight lines between which are intervals, but multiplication is continuous, and therefore it will not lack width. And thirdly, he says that whatever lacks width, depth, and length is not perceived by sight; therefore a ray [if it lacks width and depth] is not seen, which is false. And we know that a ray must pass through some part of the medium, but every part of the medium has three dimensions.

7. That is, species pass through, but not in sufficient quantity to produce sight; however, they are reflected sufficiently to be observed.

8. Grosseteste was careful to distinguish between *lux* and *lumen* in his *De luce*. Here Bacon observes Grosseteste's distinction, but remarks elsewhere (*De multiplicatione specierum*, I, ch. 1; Bridges, II, 409) that the two terms are usually employed interchangeably. [Compare Bartholomew the Englishman's discussion in Selection 60.3.—*Ed.*]

9. As will be seen further on, Bacon and his contemporaries regarded the optic nerve as hollow and believed that the species of objects were propagated to the brain through the spirits filling the nerve.

10. Bridges, II, 71–72.

11. For a full discussion of the nature and propagation of species, see my "Alhazen's Theory of Vision and Its Reception in the West," *Isis*, LVIII (1968), 321–341.

12. Bridges, II, 459–460.

4. Witelo (fl. 1250–1275): The Nature and Multiplication of Light or Species

PERSPECTIVA,[15] II, THEOREM 3

Every line along which light passes from a luminous body to a facing body is a natural sensible line, having a certain width, within which may be supposed an imaginary mathematical line.

Light proceeds only from body, since it exists only in body. Hence it is evident that in the least light that can be supposed there is width, since we call "the least light" that which, if divided, no longer has the reality *(actus)* of light, since it would not be visible; but both parts [in case of division] would be destroyed, since neither of them would be light or would appear to sense. Therefore, in a radial line along which light is diffused, there is some width, and in the middle of that line is an imaginary mathematical line, parallel to which are all the other mathematical lines in that natural line. And since the least light proceeds toward the least part of a body that light can occupy, it must proceed along the mathematical line which is in the middle of the sensible line and along the extreme lines parallel to the middle line. And the least light is not incident on a mathematical point of the facing body but on a sensible point corresponding to all the indivisible mathematical points on which the mathematical lines of the sensible line can terminate. Therefore, we employ the fantasy of advancing mathematical lines to demonstrate the properties of light.

5. Witelo: The Speed of Propagation of Light

PERSPECTIVA,[16] II, THEOREM 2

Unimpeded light is necessarily moved in an instant through the whole of a medium proportioned to it.[17]

Let there be a line proportioned to the diffusion of a strong light, as the diameter of the world in the case of solar light. Let this line be *ABCD*, and let there be a strongly luminous body at point *A* [Fig. 1]. If it should be said that the light is moved through line *ABCD* in time and not in an instant, then in part of that time it is moved through line *AB*, and in the least sensible time through the least sensible line *AB*; for if it were moved through an

Fig. 1

13. Alhazen, *Perspectiva*, IV, sec. 16; Risner, p. 112.

14. Alkindi, *De aspectibus*, edited by A. A. Björnbo and Sebastian Vogl in *Abhandlungen zur Geschichte der mathematischen Wissenschaften*, Vol. XXVI, Part III (Leipzig: Teubner, 1912), ch. 11, p. 13.

15. Risner, pp. 63–64. Witelo wrote the *Perspectiva* no earlier than 1270.

16. Risner, p. 63.

17. Witelo's view that light is propagated instantaneously is opposed to the opinions of Alhazen and Roger Bacon. See Alhazen, *Perspectiva*, II, sec. 21; Risner, pp. 37–38. Also see Bacon, section 6 immediately following.

insensible space in a sensible time, it would follow that a sensible space is composed of insensible [parts], just as the time measured after that space [*AB*] is composed of partial times that are sensible.[18] Therefore, light is moved in the least sensible time through the least sensible space. But in the same time, the form of a luminous body weaker than that more strongly luminous body is moved through the same space, since there is no sensible space smaller than the least sensible space and no sensible time less than the least sensible time. Therefore, stronger and weaker lights are of equal strength,[19] which is impossible, since they involve contradictories. Therefore, it is impossible for light to be diffused in time through a medium proportioned to it, and thus it is necessary for this diffusion to take place in an instant, which is the proposition. . . .

6. Roger Bacon: The Speed of Propagation of Light or Species

OPUS MAIUS,[20] V.1, DISTINCTION 9, CHAPTER 3

But here arises a great doubt concerning the species of the eye and of the visible object, whether they are produced suddenly and in an instant or in time; and if in time, whether in a sensible and perceptible time or not. Alkindi endeavors to demonstrate in his book, *De aspectibus*,[21] that a ray traverses [some space] in an altogether indivisible instant, and he adduces quite a curious and probable argument when he says that if a certain species, such as the light of the sun when it rises, is produced in time in the first part of the air, then if that time is doubled in the second equal part and tripled in the third part, when it reaches the west it would be a time composed of many parts, great in proportion to the first time; and although the first time would be insensible, nevertheless the whole time will be sensible because of its magnitude, which is nearly incomparable with respect to the first time. And Aristotle says in the second book of *De anima* that although the multiplication of light over a small distance can be hidden from our sense, it cannot be hidden in the case of a distance as great as that between east and west.[22] Therefore, if a species were produced in a certain time, this would be perceptible to sense. But we do not perceive it; therefore it is not produced in time but in an instant. And Aristotle says in his book *De sensu et sensato* that the reasoning regarding light is different from that regarding the other sensibles; and concerning these others he teaches that their multiplications are in time.[23] Therefore, the multiplication of light takes place in an instant.

And all authors express the same view except Alhazen, who tries to refute this view in his second book, arguing as follows.[24] Consider the last instant in which light is at the *terminus a quo* and the first in which it is at the *terminus ad quem*. Since the instants are different, as he endeavors to prove by experiences, there will be a middle time between them. And he says that every alteration takes place in time, and the medium and the eye are altered by species. But these arguments of Alhazen have no efficacy. The first is refuted elsewhere; for it is not always necessary to specify the last instant of a thing's existence at the *terminus a quo*, as universally occurs in the generation of permanent things, but it is necessary to specify the first instant of the *terminus ad quem*, as Aristotle teaches in the eighth book of the *Physics*.[25] Thus, when Socrates, from nonwhite, becomes white, it cannot be said that now finally he is not white, taking now as an instant, but [it can be said that] now he is first white; for he is not white during the whole time that measures the change, and he becomes white at the end of that time, namely at the instant which is its *terminus*, as Aristotle teaches and as is certain, although it is too difficult to understand unless carefully explained; but this is inquired into elsewhere. Alhazen's second argument is of no value [either], for all those who suppose the opposite deny that the multiplication of light is successive and temporal change.

Nevertheless, an irrefutable argument for the opinion of Alhazen can be drawn from his statements in the seventh book. For there he teaches that from the same origin a perpendicular ray arrives sooner at the end of the space than a non-

18. That is, as the sum of the sensible partial times is equal to the whole time required to traverse space *AD*, so the sum of the insensible spaces corresponding to those partial times must equal the sensible space *AD;* but, as Witelo implies, it is absurd to hold that a sensible space is composed of insensible parts.

19. That is, if any light (or form) is moved through the least sensible space in the least sensible time (the view Witelo is attempting to refute), then strong and weak lights move with equal rapidity; but this, Witelo apparently thinks, violates the assumption of unequal strengths.

20. Bridges, II,68–71.

21. Björnbo and Vogl, chapter 15, pp. 25–27.

22. *De anima* II.7.418b.20.

23. *De sensu et sensato*, 6. 446b. 25ff.

24. Alhazen, *Perspectiva*, II, sec. 21; Risner, pp. 37–38.

25. *Physics* VIII. 8.

perpendicular ray. But sooner and later do not exist without time, as Aristotle maintains in the fourth and sixth books of the *Physics*.[26] And this is demonstrated without possibility of contradiction, for no finite power acts in an instant, as Aristotle says in the sixth book of the *Physics;* and he proves this, since if it did, a greater power would act in less than an instant, which is impossible. But the power of the eye and of its species and of all created things is finite; therefore it cannot act in an instant. And at the end of the eighth book of the *Physics*, he holds that a finite and an infinite power cannot act in the same or equal durations, since then they could have equal effects, and thus they would be equal.[27] But it is the property of an infinite power to act in an instant. Therefore, a finite power cannot do anything in an instant but only in time.

Moreover, as an instant is to time, so is a point to a line. Therefore, by permutation, as an instant is to a point, so is time to a line. But the traversal of a point occurs in an instant; therefore the traversal of every line occurs in time. Therefore, species traversing a linear space, however small, traverse it in time. Furthermore, prior and posterior in space are the cause of prior and posterior in traversal of the space and in duration, as Aristotle says in the fourth book of the *Physics*.[28] Therefore, since the space through which a species is moved has a prior and posterior [part], its traversal must have a prior and posterior part in itself and in its duration; but prior and posterior in duration do not exist without time, since they cannot exist in an instant. And if it should be declared that this is true of those things that are measured by parts of space but that this excludes species (as is supposed), still it is nothing, since this second statement is applicable only to spiritual existence. Since the species of a corporeal thing has a real corporeal existence in the medium and is a true corporeal thing, as we have shown above, it must necessarily be dimensional and therefore adapted to the dimensions of the medium.

On the other hand, if in a single instant the species were produced throughout the whole medium, then it would simultaneously be at the *terminus a quo*, in the intermediate space, and at the *terminus ad quem*. But this is impossible for many reasons. First, it follows from this that a created thing would be simultaneously in several places, and by the same reasoning in an infinite number of places, as has already been considered in the chapter on matter. Therefore, it would have infinite power and would be God or equal to God. Secondly, it is argued from this that when a thing is at its *terminus a quo*, it is altogether at rest and is not moved in any way; and when it is at its *terminus ad quem*, the motion has already occurred, and the motion occurs between these *termini*. Therefore, the species is simultaneously at rest before translation, has completed its motion, and is actually being moved through the whole space. Therefore, it is simultaneously moved and not moved, which is contradictory, as Aristotle argues (reducing the matter to impossibility) in another case in the sixth book of the *Physics*.[29]

The final explanation of this is that inasmuch as the multiplication of light does not depend on any other motion, we may assume that the heaven is at rest and not moved, for the multiplication of light can occur readily with the heaven at rest, and this will take place at the end of the world if the heaven stands still, as is believed. Therefore, if the multiplication of light is instantaneous and requires no time, there will be an instant without time, since there is no time without motion. But it is impossible for an instant to exist without time, as it is for a point to exist without a line. Therefore, the remaining possibility is that light is multiplied in time, and likewise all species of a visible thing and of the eye. Yet this time is not sensible, perceptible by sight, but insensible, since everybody knows by experience that he does not perceive the time required for light to travel from east to west.

7. John Pecham: The Structure of the Eye

PERSPECTIVA COMMUNIS,[30] I, PROPOSITIONS 29–34

Proposition 29. The eye would be unsuited for the perception of size if it were not round.[31]

26. *Physics* IV. 11.

27. *Physics* VIII. 10.

28. *Physics* IV. 11.

29. *Physics* VI. 6.

30. The anatomical scheme described by Pecham in these propositions is very close to that of Alhazen, Bacon, and Witelo. Compare Alhazen, *Perspectiva*, I, secs. 4–13 (Risner, pp. 3–7); Bacon, *Opus maius*, V, Distinctions 2–4 (Bridges, II, 12–30); Witelo, *Perspectiva*, III, Theorems 3–11 (Risner, pp. 85–90). For a more complete discussion of Pecham's views, see pp. 37 and 247–250 of my edition of the *Perspectiva communis*, cited in note 3. [See Mundinus' description of the eye, Selection 96.—*Ed.*]

31. The revised version of this proposition differs

Because of the many things that must be quickly perceived, the eye must be round so that it can easily move and turn. Furthermore, if the part through which the eye is acted upon were not spherical, the eye could see at a single glance only things equal [in size] to itself. This is evident, because vision occurs through straight lines incident perpendicularly on the eye[32] and intersecting at its center, as will be taught below. If the eye had a flat surface, only perpendiculars from a surface of equal size could reach it. For example, although impossible, let the eye have a plane surface *AB*, and let *CD* be the visible object [Fig. 2]. Then drop

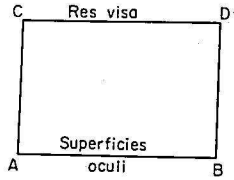


Fig. 2

a perpendicular from *B* to *D*. Likewise, draw another perpendicular from point *A* to point *C*. Since *AB* and *CD* are parallel (this may be assumed, since no absurdity follows from it), line *AC*, drawn perpendicularly by hypothesis, will be equal to line *BD;* consequently, line *BA* will equal line *DC*. That is, the thing seen cannot exceed the breadth of the eye. However, since this is false, it follows that the eye is not flat but more nearly spherical, and into its center rays can fall perpendicularly from a magnitude far greater [than the eye]. Moreover, roundness accords with interior capaciousness, because [the sphere] is the most capacious of all isoperimetric (that is, commensurable) figures.

Proposition 30. Some of the bodies constituting the eye must depart from complete sphericity.

For instance, if the consolidativa (that is, the white fat) that surrounds the eye were to surround it entirely, the eye would see nothing because the consolidativa lacks transparency. The uvea, too, has an opening in front, and the glacial humor[33] also departs from sphericity.

Proposition 31. Bodies of various dispositions are required for the constitution of the eye.

This is evident, for the part in which the power of sight resides[34] is very delicate and sensitive because it is transparent and watery and of the most delicate composition. If this were not so, it would be unsuited to the subtlety of the visual spirits coming from the brain; yet neither would the species in any way admit of being subject to the immaterial and purified. Furthermore, the influence of species can be perceived only by a most subtle body. This humor, however, would be easily corrupted if it were not surrounded by other stronger ones. Hence the eye is so disposed that there is an outermost tunic called the consolidativa,[35] strong and thick so as to maintain the entire eye in its proper arrangement. Within the consolidativa is a tunic called the cornea, because it is like horn. The cornea is hard, [as it must be] because of its exposure to the air, and transparent in order to be pervious to species.

Within the cornea is the tunic called the uvea. The uvea is black like a grape to darken the humor in which [the power of] sight resides, for unless that humor is darkened, the visible species will not appear in it. This is a strong tunic to prevent the enclosed humor from exuding, and so that species may pass through, it has a circular aperture in front,[36] the diameter of which is about the size of the side of the square that can be inscribed within the sphere of the uvea.

Within the uvea is the albugineous humor,[37] which is like the white of an egg. It is transparent, so that species may be conducted through it freely, and damp, to moisten the glacial humor so that the web surrounding it will not be corrupted by dryness.

The glacial humor,[38] which is like glass, is the innermost humor. It is moist in order to be sensitive to light—by virtue not only of transparency but also of the passibility of sense—and it is subtle so that it might be easily stimulated [by species]. It is also somewhat dense to permit the retention of species within it, for they would otherwise die away. This humor is divided into two parts: It has an anterior part[39] that is a portion of a larger sphere, and this part is concentric with the whole eye and parallel to the front of the eye; and it has a posterior part called the vitreous, more subtle than the anterior part. These two parts are surrounded by a certain fine web like a spider web, called the aranea;[40] the function of the aranea is to contain this moist humor. And so, according to the Physicist,[41] the eye has three humors and four tunics.

Others who investigate anatomy more attentively, however, consider (as it is observed in the book *De elementis*) that the uvea takes its origin from the pia mater and the cornea from the dura mater—the pia mater and dura mater being two webs that surround the cerebrum. This book adds that the eye is composed of three humors and seven tunics, the first of which is the conjunctiva or consolidativa; and it divides the cornea into two parts, calling the anterior part the cornea and the interior part the schrosim. It divides the uvea similarly, calling the anterior part the uvea and the posterior the secundina. Likewise the aranea is divided, the anterior part being termed the aranea and the interior part the retina. Thus to divide the eye, however, is not the concern of physics, which considers only what pertains to eccentricity or concentricity, refraction, and direction [of the rays].

Proposition 32. The duality of the eyes must be reduced to unity.

The benevolence of the Creator has provided that there should be two eyes so that if an injury befalls one, the other remains. However, their source is as follows. Two hollow nerves originate in the anterior part of the cerebrum and are directed toward the face. At first these are joined and form a single nerve, but then they branch into two [nerves] at the two hollow openings beneath the forehead; here they are spread apart, and on the ends of these nerves the eyes are formed. Accordingly, the species of visible things are received by both eyes, so that, if the species were not united, one thing would appear as two. This is evident also when a finger is placed under the eye and one eye is elevated from its [customary] position: one thing then looks like two because the species received through the two eyes are not joined in the common nerve. Therefore, [if vision is to be single,] species must be brought together in the common nerve and united there.

Proposition 33. It is necessary for some of the spheres constituting the eye to be mutually eccentric.

This is evident as follows. Since the species of the visible object is incident on the eye in the form of a pyramid, the vertex of which is to be imagined at the center of the eye, the rays converging at that center would intersect and proceed beyond if there were no change in transparency; and right would appear left, and left right. Hence nature has arranged for the interior glacial humor to have the same center as the cornea and albugineous humor, so that species traversing them should not be refracted before they reach the sensitive power residing in the glacial humor. . . .

Proposition 34. The centers of all the tunics and humors are contained on one line.

This is proved by an effect, for otherwise light could not enter all the tunics and humors in a regular manner, nor could any ray remain unrefracted. Consequently, there could be no certification[42] by the conveyance of the eye over the visible object from one end to the other, and that is false.

somewhat from the unrevised text presented here; however, since Pecham's revised version experienced so little circulation, it has seemed appropriate to print the text of the more popular unrevised version.

32. See I, Proposition 28, in section 13 of this selection.
33. That is, the crystalline lens.
34. The glacial humor, or crystalline lens; see note 38.
35. That is, the sclera. For a medieval drawing of the eye, see p. 115 of my edition of Pecham's *Perspectiva communis*, cited in note 3.
36. That is, the pupil.
37. That is, the aqueous humor.
38. The term "glacial humor" was employed before to denote the crystalline lens. Here it denotes the combined crystalline lens and vitreous humor.
39. The crystalline lens.

8. Alhazen (ca. 965–ca. 1039): The Lens as the Sensitive Organ of the Eye

PERSPECTIVA,[43] I, SECTION 16

And we would say in the first place that sight occurs only by means of the glacial humor, whether sight takes place through forms coming from the visible object to the eye or in some other way. Sight does not occur through one of the other tunics in front of it, since those tunics are merely instruments of the glacial humor; for if injury should befall the glacial humor, the other tunics remaining sound, sight is destroyed; if the other tunics should be corrupted, their transparency and the health of the glacial humor being retained, sight is not destroyed. And also, if there should be an obstruction in the opening of the uvea,[44] and the transparency of its humor were destroyed, sight would be destroyed along with the health of the cornea; and if the obstruction were removed, sight would return. Similarly, if a dense opaque fragment were placed within the albugineous humor in front of the glacial humor, between it and the opening of the uvea,

40. That is, the retina.
41. Alhazen.
42. On pecham's doctrine of certification, see I, Proposition 38, in section 13 below.
43. Risner, p. 8. Note that the headings of the sections (or enunciations of the propositions, if you will) in the Risner edition of Alhazen's *Perspectiva* did not originate with Alhazen but were added by Risner in the sixteenth century; therefore, they have not been translated.
44. That is, the pupil.

sight would be destroyed; and when that fragment is removed or is displaced in some direction from the direct line between the glacial humor and the opening of the uvea, sight will be restored. And medicine attests all these things. Therefore, the sense of sight is destroyed when the glacial humor is corrupted, the tunics in front of it remaining healthy. And this is the argument [to the effect that] the sense of sight is associated only with that [glacial] humor and not with the other tunics in front of it.[45]

9. Roger Bacon: The Lens as the Sensitive Organ of the Eye

OPUS MAIUS,[46] V.1, DISTINCTION 4, CHAPTER 2

The anterior glacial humor has many properties. The first and principal of these is that the visual power resides only in it, according to Alhazen and others. For all other humors anterior to it are its instruments and exist for its sake. For if the glacial humor should be injured, the others being preserved, sight is destroyed; and if the glacial humor is preserved, injury befalling the others (provided their transparency remains), sight is not destroyed but is still of service; for as long as the transparency within the glacial humor remains continuous with the transparency of the air, sight is not destroyed, provided the anterior glacial humor itself is preserved. And the anterior glacial humor is moist, so that it is more readily affected by the species of light and color, for very dry substances do not easily receive impressions. It is also subtle, since subtlety of body serves subtlety of feeling. And it is somewhat transparent, so that it can receive the forms of light and color, and they can pass through it to the common nerve. And it is [also] somewhat dense, so that it can retain species within itself for a long time, and they can appear to the visual power, and it can form a judgment. For if it were too transparent, species would pass through it [too readily] and would not remain so that a judgment could be formed. Therefore it must be somewhat dense, so that it may undergo a kind of pain on account of the species; for we see that strong lights and colors narrow the [pupil of the] eye and injure it and induce pain. But every action of light is of one nature (and the same is true of color), except that one [action] is stronger and another weaker; therefore, the eye always undergoes a kind of pain, although this is not always perceptible, as for example when the species are moderate. But painful effects cannot occur in a body unless it is quite dense, since if it is too rare, the species does not remain long enough to produce the effect of pain. . . .

10. Witelo: The Lens as the Sensitive Organ of the Eye

PERSPECTIVA,[47] III, THEOREM 4

And so it is evident from these things that the glacial humor is properly the organ of the visual power, for the transparency of only this humor is receptive of the forms of visible objects, and it is located in the middle of all the humors and tunics. And if any tunic or humor whatever, saving the glacial humor, should be injured, the eye always receives care by the aid of medicine and is healed, and sight is restored. However, when the glacial humor itself has been damaged, all of sight is destroyed without hope of restoration through the aid of medical treatment. And so principally the crystalline or glacial humor is the organ of the visual power, on which account it is carefully preserved.

11. Alhazen: The Act of Sight

PERSPECTIVA,[48] I, SECTIONS 25–26

Also it has been declared that this [glacial] humor is somewhat transparent and somewhat dense; and in this respect it is like glass. Now, since it possesses some transparency, it receives forms, and these pass through it because of its transparency. And since it [also] possesses some density, it prevents forms from passing through it because of its density; and [thus] forms are fixed in its surface and body, although weakly. Similarly, when light is incident on any transparent body in which there is some density, the light passes into it because of its transparency and is fixed in its surface because of its density.[49] And also, the glacial humor is suited for the reception of those forms and for perceiving them. Therefore, forms pass into the glacial humor on account of its sensible power of reception; and when the form reaches the surface of the glacial humor, it acts on it, and the glacial humor suffers because of the form, since it is a property of light to act on the eye and a property of the eye to suffer because of the light. And this effect, which light produces in the glacial humor, passes through the glacial body. . . . And when light passes through the glacial body, color passes through with it, for color is mixed with the light; and the glacial humor receives that action [of light and color], and the color passes through. And the glacial humor perceives on account of this action and suffering, because of the forms of visible things that are on its surface and pass through its whole body; and it perceives through the ordering of the parts of the form on its surface and throughout its body. . . .[50]

And this action produced in the glacial humor by light is a kind of pain; for certain pains are capable of being received, and [yet] the member is not injured because of them. Such pains are not manifest to sense, nor does the latter judge them a painful affliction. And the meaning of this is that light induces pain, since strong lights offend the eye and manifestly injure it (as the light of the sun when the observer looks at its body, or the light of the sun reflected to the eye from smooth bodies); for those lights induce manifest pain in the eye. And the action of all light on the eye is of the same kind and is diversified only according to more and less. Since all are of one kind, and the operation of strong lights is a kind of pain, all actions of light are a kind of pain. . . .

12 Witelo: The Act of Sight

PERSPECTIVA,[51] III, THEOREM 6

Sight occurs through the action of visible forms on the eye and the suffering of the eye on account of this form.

That visible forms act on the eye is evident from the second and third suppositions [of this book]; for the eye is injured by strong light, as in observation of the solar body or some other strong light (such as light reflected to the eye from a polished body or from some other very white body). In these cases, the eye is disabled in such a way that it fails in its operation until it is restored by its natural intrinsic power. But sight also suffers from sensible forms, since it sometimes retains in itself their strong impressions. For after the eye gazes for a long time at bright light or color, if it should then look at a dark or weakly illuminated place, it would [still] perceive very well the visible object that it observed directly before, along with its light, color, and shape.[52] And sometimes a strong color impressed on sight is mixed with the colors of things seen in [relative] darkness, and those things appear colored, mixed with the other color, as a bright green object that is observed makes white objects seen afterwards in a darker place appear mixedly green; and if the eye is closed, the form seen previously nevertheless appears to sight. Therefore, visible forms act on sight, and sight suffers on account of them. And since the visibles *per se* are light and color, and light is the hypostasis of colors—and light is always diffused spherically to every different position—it is plain that colors are diffused similarly. Thus, when the eye is opposite an illuminated or colored object, light is multiplied either by itself or with the color of the object,

45. Knowledge of the ancient technique of couching for the removal of cataracts should easily have refuted Alhazen's claim that the visual power resides in the lens. However, in the Middle Ages cataracts were not regarded as corruptions of the lens but as obstructions intervening between the lens and the pupil; indeed, Alhazen's remark about displacement of an opaque fragment from behind the uvea may be a reference to couching. Alhazen's argument, though based on a false premise, does appear to demonstrate that the visual power cannot reside in any humor anterior to the lens; but it does not prove (as Alhazen seems to have realized) that a humor or tunic behind the lens cannot possess the visual power; nevertheless, Alhazen does not claim simply that the visual power resides in the glacial humor or some humor posterior to it, but that it resides in the glacial humor. Probably because of this stress on the glacial humor as the sensitive organ, it was never (during the Middle Ages) regarded as a device for focusing rays on the retina.

46. Bridges, II, 27–28.

47. Risner, p. 87.

48. Risner, pp. 15–16.

49. A substance without any density would offer no resistance to the passage of light; however, all real transparent substances possess some density.

50. This point is crucial to Alhazen's theory of vision. Clear vision results when the form of the visible object is ordered on the surface and throughout the interior of the glacial humor exactly as on the object itself; or to express the idea more precisely, an object becomes visible when the species representing its individual points (or small parts) fall on the glacial humor in the same order as the points they represent and from which they issued. A one-to-one correspondence is thus established between points on the object and points on the surface of the glacial humor. Compare Pecham, *Perspectiva communis*, I, Proposition 37, in section 13 below.

51. Risner, pp. 87–88.

52. Witelo is referring to the after-image. Alhazen also stressed the after-image at the beginning of his *Perspectiva* and elsewhere.

and arriving at the surface of sight,[53] it acts on sight, and sight suffers from it. When light and color come simultaneously to the surface of sight and act on it, and sight suffers because of them, and the power of the soul achieves understanding because of the union of the visible forms and the soul's organ, then sight takes place on account of the presence of the visible forms acting on [the organ of] sight. . . .

13. John Pecham: The Act of Sight

PERSPECTIVA COMMUNIS, I, PROPOSITIONS 43, 28, 37, 33, 38

Proposition 43. The action of the visible object on the eye is painful.[54]

This is proved as follows. Because the action of the visible object on the eye is of a single kind and the action of bright lights on the eye is sensibly painful and injurious, it follows that all actions of light are painful and injurious, even if they do not seem to be. This is the argument of the Physicist[55] on the properties of sight, and it is seen to follow necessarily, since there is no visible object so delightful to the eye that when continuously observed it does not cause fatigue, the cause of which is clearly the preceding observation [of the object]. Now, the Physicist construes it this way, but philosophers who treat natural things say otherwise, [insisting that] since the sensible object is the perfection of sense, nothing induces pain in the act of perception unless it is excessive [in its action]. But the argument that if a superior sensible body induces pain, an ordinary one must do so also, is not compelling, for a violent action distresses while a moderate action delights and pleases. Therefore, what has been said here is limited to all prolonged observation and does not apply to every brief glance.[56]

Proposition 28. Sight occurs through lines of radiation perpendicularly incident on the eye.

This is obvious, for unless the species of the visible object were to make a distinct impression on the eye, the eye could not apprehend the parts of the object distinctly; and no distinction could be made among the partial species representing the parts of the object except by perpendicular lines, for otherwise the species would mingle together and would exhibit the object to sight confusedly. Besides, when perpendicular lines between the visible object and the eye have been intercepted, vision ceases. Accordingly, the opposite effect requires the opposite cause.[57]

Proposition 37. Vision takes place by the arrangement of the species on [the surface of] the glacial humor exactly as [the parts] of the object [are arranged] outside.

The possibility of this is obvious, notwithstanding the smallness of the glacial humor, because there are as many parts in the smallest of magnitudes as in the greatest. Species, however, are received without matter. Therefore, regardless of the size of the visible object, its species can be received distinctly and in [proper] order on [the surface of] the glacial humor. Unless this were so, the eye would not see the object distinctly; for if the species of two parts of the visible object were received in the same part of the glacial humor, the parts of the object would not be perceived distinctly by reason of the confusion of forms acting on the same part of the eye.[58]

53. *visus superficiem.* This could be translated "at the surface of the eye," but it appears to me from the context that Witelo has in mind the surface of the glacial humor, which contains the power of sight; hence, "the surface of sight."

54. This proposition is from the unrevised version of the *Perspectiva communis*; the revised version varies slightly.

55. That is, Alhazen. Pecham's reference is to the passage translated in section 11 above.

56. Here Pecham disagrees with Alhazen's conclusion that *every* action of light is painful.

57. In this proposition Pecham expresses one of the most original aspects of Alhazen's theory of sight (cf. Alhazen, *Perspectiva*, I, sec. 18; Risner, p. 9). A problem was created by the requirement that clear vision occurs only if species from the object fall on the glacial humor in the same order as the points from which they issued (see n. 50); for it was recognized that species issue in all directions from every point on the surface of the visible object, so that species or rays from every point on the object are incident on every point of the glacial humor. No one-to-one correspondence can thus be established. However, the problem is solved if all rays are ignored except those perpendicular to the surface of the eye and glacial humor: there is one such ray from each point on the visible object, and the complete set of such rays has the same arrangement on the surface of the glacial humor as on the visible object; moreover, these rays are rectilinear, since they are perpendicularly incident on the tunics of the eye and thus penetrate without refraction. Though not stated here, the principle which allows all nonperpendicular rays to be ignored is simply that refraction weakens, so that only perpendicular rays are powerful enough to influence the power of sight.

58. In this proposition Pecham deals with a further problem raised by Alhazen's intromission theory of

Proposition 33. . . . Upon meeting the interior glacial humor, which is eccentric to [the cornea and albugineous humor], or the vitreous humor, which is more subtle than the anterior glacial humor, the rays are separated and refracted away from the perpendicular. From there the species is carried to the place of interior judgment by the way of spirits.[59]

Proposition 38. Visible objects are perceived by means of the pyramid of radiation; perception is certified by the axis [of the pyramid] being conveyed over the visible object.

To be sure, the pyramid of radiation, impressed on the eye by the visible object, manifests the object to the eye; but the visible object is certified by a turning of the eye [all about] over the object, the latter being the base of the pyramid. For although the whole pyramid is perpendicular to the center of the eye, that is, the anterior glacial humor, it is not perpendicular to the whole eye. Therefore, only that perpendicular called the axis, which is not refracted,[60] manifests the object efficaciously, and other rays are correspondingly stronger and better able to manifest [the object] as they are closer to the axis. Therefore, the eye is turned about so that the object, which is perceived under the pyramid all at once, is discerned efficaciously by appearing along this perpendicular successively. . . .

14. Alhazen: The Debate about Visual Rays

PERSPECTIVA,[61] I, SECTION 23

It is permissible for us to assert that a transparent body receives something from sight and transmits it to the visible object and that perception occurs through the unbroken succession of that thing to the visible object, between the eye and the visible object. And this is the view of those who suppose that rays issue from the eye. Therefore, [for the sake of argument,] let it be supposed that this is true, that rays issue from the eye and pass through the transparent medium to the object of sight and that perception occurs by means of those rays. And since, [according to the view now under consideration,] perception occurs in this way, I inquire whether those rays return something to the eye. If perception occurs [only] through [such] rays, and they do not return anything to the eye, then the eye does not perceive. But the eye does perceive the object of sight, and [we have supposed that] it perceives only by the mediation of rays. Therefore, those rays that perceive the visible object [must] transmit something to the eye, by means of which the eye perceives the object. And since the rays transmit something to the eye, by means of which the eye perceives the object, the eye does not perceive the light and color in the visible object unless something comes to the eye from the light and color in the object; and this is delivered by the rays.

Therefore, according to all possibilities, sight does not occur unless something of the visible object comes from the object, whether or not rays issue from the eye. It has already been shown that sight is achieved only if the body intermediate between the eye and the visible object is transparent, and it is not achieved if the medium besight: If vision takes place through the action of perpendicular species from every point on the surface of the object converging towards an apex in the eye, how is it possible for these species to squeeze together as they approach the apex? Or, in mathematical rather than material terms, can there possibly be as many points (or small parts) on the tiny surface of the glacial humor as on the surface of a large visible object? The answers to these questions are, first, that species "are received without matter," so that squeezing is no problem, and second, that there are indeed as many points (or parts) in a small as in a large surface.

59. It is most important, in the theory of Alhazen and his thirteenth century followers, that the perpendicular species responsible for vision do not actually reach an apex and then reverse their order as they continue beyond the apex; for if such were the case, the visual impression would be inverted and the world would be seen upside down. (It should be noted that although the chief organ of vision is the glacial humor, vision is "completed" in the brain; thus it is necessary for the rays or species to maintain the proper order even after incidence on the surface of the glacial humor.) To explain why rays do not form an apex in the eye, Alhazen and his followers argued that the surface separating the glacial from the vitreous humor is situated anterior to the center of the eye, so that the species are refracted away from the center, thereby failing to reach an apex, and thus maintain their proper order as they progress through the vitreous humor and optic nerve to the "place of interior judgment" in the brain.

60. All rays perpendicular to the surface of the eye remain unrefracted until after they have passed through the surface of the glacial humor. However, only the central perpendicular ray (that is, the axis of the visual pyramid) is also perpendicular to the surface separating the glacial and vitreous humors, and this ray alone passes through the entire eye without refraction.

61. Risner, pp. 14–15. On the question of visual rays, see also Grosseteste's *On the Rainbow*, Selection 61.2, and Adelard of Bath's *Natural Questions*, Selection 60.1.

tween them is opaque. And it is evident that a transparent and an opaque body are distinguished only in the aforesaid way.[62] Since, as we have said and as has been demonstrated, the forms of the light and color in the visible object reach the eye (if they were [originally] opposite the eye), that which comes from the visible object to the eye (through which the eye perceives the light and color in the visible object no matter what the situation [with respect to visual rays]) is merely that form, whether or not rays issue [from the eye]. Furthermore, it has been shown that the forms of light and color are always generated in air and in all transparent bodies and are always extended to the opposite regions, whether or not the eye is present. Therefore, the egress of rays [from the eye] is superfluous and useless. Consequently, the eye does not perceive the light and color of the visible object unless form comes from the light and color [to the eye]. . . .

It remains only to consider the view of those who maintain that rays issue from the eye and to indicate what in this view is false and what true. Therefore, let us assert that if sight is due to something issuing from the eye to the visible object, that thing is either corporeal or incorporeal. If it is corporeal, it follows that when we look at the sky and see the stars in it, corporeal substance issues from our eye in that time and fills the whole space between heaven and earth, and [yet] the eye is in no way destroyed; and this is [obviously] false. Therefore, sight does not occur through the passage of corporeal substance from the eye to the visible object. But if that which issues from the eye is incorporeal, it will not perceive the object, since there is no perception except in corporeal things. Therefore, nothing issues from the eye to the visible object to perceive that object.[63] Now, it is evident that sight occurs through the eye; and since this is so, and the eye perceives the visible object only when something[64] issues from the eye to the visible object, and since that which issues forth does not perceive the object, therefore that which issues from the eye to the visible object does not return anything to the eye, by which the eye perceives the object. And [therefore,] that which issues from the eye is not sensible but conjectural, and nothing ought to be believed except through reason or by sight.[65]

However, those who assume that rays issue from the eye suppose this because they ascertain that the eye perceives the visible object and that between the eye and the object is space; and it is known[66] by mankind that there is no perception except by contact. Consequently, they have conjectured that sight does not occur unless something issues from the eye to the visible object, so that the thing issuing forth perceives the object in place of the eye, or rather receives something from the visible object and returns it to the eye; then the eye perceives that thing. And since corporeal substance cannot issue from the eye to perceive the object and nothing perceives a visible object unless it is corporeal, nothing remains except to conjecture that the thing issuing from the eye to the object of vision receives something from the object and returns it to the eye. And since it has been demonstrated that air and transparent bodies receive the form of the visible object and transmit it to the eye and to every facing body, that which they conjecture to return something from the visible object to the eye is nothing but air and the transparent bodies between the eye and the object of vision. Since air and transparent bodies transmit something from the visible object to the eye, regardless of the time and according to all arrangements (when the eye is opposite the visible object), without requiring that something issue from the eye, the reasoning that led from positing rays to maintaining their existence is unnecessary; for that which led them to say that rays exist is their supposition that sight cannot be completed except through something extended from the eye to the visible object, which returns something from the object to the eye. And since air and transparent bodies do this without requiring that something issue from the eye, and

62. That is, one allows the transmission of rays and thus the achievement of sight, while the other does not.

63. Note Alhazen's care in asserting that thus far he has proved only that visual rays (if they exist) are not responsible for visual perception, not that they are nonexistent. Alhazen, of course, was convinced that visual rays did not exist, as he makes clear later on, but the caution he displayed in this argument was to provide Western writers with room to maneuver. Thus Bacon, in an argument appearing in Selection 62.15, affirms the existence of visual rays and even assigns them a minor role in vision, while agreeing with Alhazen that the principal agents of vision are rays issuing from the observed object. This question is discussed in more detail in my "Alhazen's Theory of Vision," cited in note 6.

64. The Latin term *aliquid*, here translated, does not appear in the Risner text but is found in British Museum Royal MS. 12.G. VII, fol. 7 verso.

65. The last three words in this sentence *(vel a visu)* are included in the British Museum manuscript cited in note 64 but not in the Risner text.

66. The Risner text reads *magnum* but the British Museum MS reads *cognitum*.

moreover, since the air and transparent bodies are extended between the eye and the visible object without defect, it is useless to suppose that something else returns something from the object of vision to the eye. Therefore, it is useless to say that [visual] rays exist.

Moreover, all mathematicians who say there are rays use nothing in their demonstrations except imaginary lines, and they call them radial lines; . . . and the belief of those who consider radial lines to be imaginary is true, and the belief of those who suppose that something [really] issues from the eye is false. . . .[67]

15. Roger Bacon: The Debate about Visual Rays

OPUS MAIUS,[68] V. 1, DISTINCTION 7, CHAPTERS 3–4

And if Alhazen, Avicenna (in the third book of *De anima*), and Averroes (in his little book *De sensu et sensato*) are cited as opposed to this view [that species issue from the observer's eye], I reply that they are not opposed to the generation of the species of the eye or to its role in sight but [only] to those who have supposed that some body, as a visible species or something similar, extends from the eye to the visible object, by means of which the eye perceives the object, and that it seizes the species of the visible object and returns it to the eye . . . . Therefore, it must be maintained that the aforesaid philosophers, namely Alhazen, Avicenna, and Averroes, attack only this view, as is evident from their texts. . . .[69]

The explanation of this view [that species issue from the eye] is that every natural thing completes its action solely through its own power and species, as the sun and other celestial bodies produce the generation and corruption of things through their powers dispatched into the things of the world; inferior things do likewise, as fire dries and consumes and does many things by its power. And therefore the eye must perform the act of sight through its own power. But the act of sight is the perception of a visible object at a distance, and therefore the eye perceives the visible object through its own power multiplied to the object. Furthermore, the species of the things of the world are not immediately suited of themselves to bring to completion an action on the eye because of the nobility of the latter. Therefore, they must be assisted and excited by the species of the eye, which proceeds through the space occupied by the visual pyramid, altering and ennobling the medium and rendering it commensurate with sight. Thus the species of the eye prepares for the approach of the species of the visible object and, moreover, ennobles the species of the object so that it is wholly conformable to and commensurate with the nobility of the animate body (that is, the eye). But since this view is doubtful to many, I shall adduce, besides the verifications already presented, various true and certain experiences, according as they are relevant to other conclusions (which necessarily attend this view) in various places below.

Now, concerning the multiplication of this species, it must be recognized that it occurs in the same place, between the eye and the thing seen, as the species of the visible object; and it takes place along a pyramid, the apex of which is in the eye and the base of which is on the visible object. And just as the species of the object proceeds rectilinearly in a single [homogeneous] medium and is refracted in different ways when it encounters a medium of different transparency and is reflected when it meets the obstacle of a dense body, so the species of the eye proceeds along the same path as the species of the visible object. And although the species of the eye has the form of a pyramid, the apex of which is in the eye and the base of which is on all parts of the visible object, from the surface of the glacial humor [as base] proceed an infinite number of pyramids, all having the same base, and

67. There has been some confusion among historians about whether or not Alhazen really denied the existence of visual rays. The heading inserted by Risner (Alhazen's sixteenth-century editor) before the next section of Alhazen's text has caused most of the confusion: "Vision seems to occur through *συναύρεταν*, that is, rays simultaneously received and emitted." Directly beneath this heading, Alhazen's text reads: "It has been asserted on account of this that both schools of thought speak the truth and that both beliefs are correct and consistent; but one does not suffice without the other, and there can be no sight except through that which is maintained by both schools of thought" (I, sec. 24; Risner p. 15). However, what Alhazen means by this remark is simply that mathematicians, who are concerned to account for the phenomena mathematically, can legitimately use visual rays to represent the geometrical properties of sight. He does not compromise his denial of belief in the real existence of visual rays.

68. Bridges, II, 50–53.

69. With great insight Bacon observes that the attack on visual rays by the three Muslim philosophers mentioned actually overthrows only ancient visual ray theories like those of Plato and Ptolemy and that it does not apply to the theory he is about to propose.

their apices fall on individual points of the visible object,[70] so that all parts of the visible object appear as strongly as possible. Nevertheless, one pyramid is principal, namely the one whose axis is the line passing through the center of all parts of the eye (that is, the axis of the whole eye); for [the ray proceeding along] that [line] certifies everything, as has been said above and as will be explained more fully [below].

And although the species of visible objects, as the species of light and color, are mixed in a medium—that is, several lights are united and several colors are mixed, as has been said—and this species of the object and the species of the eye occupy the same common place, nevertheless there is no confusion of those species, nor mixing, nor do they unite; for they are not of the same kind nor of the same genus, since the pupil has no color, and color and light do not possess the power of soul. But the species of the eye is the species of an animate body, in which the power of soul rules, and therefore there is no comparison between it and the species of an inanimate thing so that they should be united. . . .

16. John Pecham: The Debate about Visual Rays

PERSPECTIVA COMMUNIS, I, PROPOSITIONS 44–46

Proposition 44. By assuming that sight occurs through rays issuing from the eye, mathematicians exert themselves unnecessarily.

For the manner in which vision occurs is adequately described above, by which [description] all the phenomena of vision can be saved. Therefore, it is superfluous to posit such rays. I say this following in the footsteps of the author of the *Perspectiva*.[71] Nevertheless, Alkindi teaches differently [in the treatise] *De aspectibus*, the Platonists have judged otherwise, and philosophers are seen in many places to understand differently. Augustine, who declares that the power of the soul has an effect on the light of the eye in a manner different from any that has hitherto been investigated, also teaches otherwise.

Proposition 45. Rays issuing from the eye and falling on a visible object cannot suffice for vision.

If it should be supposed that rays issue from the eye and fall on the visible object as if to seize it, either they return to the eye or they do not. If they do not return, vision is not achieved through them, since soul does not issue from body. If they do return, how do they do so? Are they animated? Are all visible objects mirrors by [virtue of the property of] reflecting rays? Furthermore, if the rays return to the eye with the form of the visible object, they go out in vain, since light itself (or the form of the visible object through the power of light) diffuses itself throughout the whole medium. Therefore, the visible object need not be sought out by rays as by messengers. Moreover, how would any power of the eye be extended all the way to the stars, even if the whole body were transformed into spirit?

Proposition 46. The natural light of the eye contributes to vision by its radiance.

For, as Aristotle says, the eye is not merely the recipient of action but acts itself, just as shining bodies do.[72] Therefore, the eye must have a natural light to alter visible species and make them commensurate with the visual power, for the species are emitted by the light of the sun but moderated with respect to the eye by mixing with the natural light of the eye. This is why Aristotle said that sight occurs when the outward action [of the eye] is strong, [for] when the action entering the eye is strong, as with a solar ray, it overpowers sight; nor, of itself, can this inward action be made commensurate with sight. It is evident, therefore, that there is some kind of emission of rays, but not of the Platonic type such that rays emitted by the eye are, as it were, immersed in the visible form and then returned to the eye as messengers. But rays [emitted by the eye] do have some effect on sight in the manner described above. This is evident as follows. Since vision is of the same kind in all animals, and certain animals are able to bestow a multiplicative power on colors by the light of their eyes so as to see them at night, it follows that the light of the eye has some effect on [external] light. Whether it goes beyond this, I do not determine, save only by following in the footsteps of the Author,[73] as I have said before.

70. These pyramids are reversed with respect to the principal visual pyramid, which has its apex in the observer's eye and its base on the visible object. That is, every point on the object serves as the apex of a pyramid of rays, the base of which is the surface of the glacial humor.

71. That is, Alhazen, in the passage translated in section 14 above.

72. *De generatione animalium* V.1.780a.5–15, 781a. 1–10.

73. Alhazen. But Pecham was also influenced by Bacon, as his reconciliation of the existence of visual rays with Alhazen's attack on visual rays reveals.

17. Witelo: The Debate about Visual Rays

PERSPECTIVA,[74] III, THEOREM 5

It is impossible for sight to be applied to the visible object by rays issuing from the eyes.[75]

If from the eyes should issue certain rays, by which the visual power is united with external objects, those rays are either corporeal or incorporeal. If corporeal, then when the eye sees stars and the sky, something corporeal issuing from the eye necessarily fills the entire space of the universe between the eye and the visible part of the sky, without diminution of the eye itself. But it is impossible that this should occur and also that it should occur so swiftly, the substance and size of the eye being preserved. If it should be said [instead] that the rays are incorporeal, then those rays do not perceive the visible object, since perception exists only in corporeal things. Therefore, the corporeal eye cannot perceive by the mediation of this insentient incorporeal ray. Nor do such incorporeal things return something to the eye, by which sight could perceive the visible object, since sight occurs only through contact between the eye and the visible form, because there is no action without contact. Therefore, if rays issuing from the eye return nothing to the eye, those rays do not produce sight. But if they return something to the eye, these are lights or colors, which appear by themselves [without the mediation of rays issuing from the eye] and which are multiplied to the eye among the [visual] rays. Therefore, [visual] rays do not cause sight to be applied to the things seen; but something else, which is multiplied to the eye, is by itself the cause of vision. It is therefore impossible for rays of themselves to cause vision, unless perhaps the lines drawn through the points of the forms multiplied from the surfaces of the visible object to the eye are called rays; for, as is evident by Theorem 2 of this book, in order that the object may actually appear, it must be possible to draw straight lines between any point on the surface of the visible object and a given point on the surface of sight. But such rays do not issue from the eyes.[76] Therefore, the proposition is evident.

18. Roger Bacon: Psychology of Visual Perception

Translated by Robert B. Burke[77]

OPUS MAIUS, V.1, DISTINCTION 1, CHAPTERS 2–4

Chapter 2. Concerning the internal faculties of the sensitive soul, which are imagination and the common sense (sensus communis).[78]

Since the optic, that is, the concave, nerves causing vision have their origin in the brain, and writers on optics ascribe, to a discriminative faculty through the medium of vision, the formation of judgments concerning twenty kinds[79] of visible things, which will be considered later, and since it is not known whether that discriminative faculty is among the powers of the soul, the organs of which are distinct in the brain, and since many other things to be treated later suppose a definition of the faculties of the sensitive soul, therefore we must begin with the parts of the brain and the faculties of the soul, in order that we may discover

74. Risner, p. 87.

75. In the thirteenth century only Witelo followed Alhazen in denying the existence of visual rays.

76. Witelo here distinguishes between rays actually issuing from the observer's eye and imaginary lines that can be drawn between points in the eye and points on the visible object. This echoes Alhazen's similar distinction; see note 67.

77. *The Opus Majus of Roger Bacon* (Philadelphia: University of Pennsylvania Press, 1928; reprinted, New York: Russell and Russell, 1962), II, 421–427. For this one long selection, it seemed preferable to reproduce Burke's translation of the *Opus maius* rather than to retranslate the Bridges text. However, a number of changes have been made in the interests of accuracy and clarity. First, the paragraphing and punctuation have been slightly modified. Second, a number of expressions have been given different translations; the most common changes are as follows: *virtus distinctiva* has been translated "discriminative faculty" instead of "distinct function"; *virtus* has been translated "power" or "faculty" instead of "function" or "impression"; *species* has been rendered as "species" instead of "form"; *sensibile*, which Burke renders as "sensation," "impression," "attribute," "property," or "quality," has in every case been rendered as "sensible"; and *collocantur sub* has been translated as "are subordinate to" instead of "are placed under" or "are under." Finally, Burke's translations of Bacon's citations of other works have been rephrased. Other modifications are noted as they appear.

78. On the internal faculties of the soul, see Harry A. Wolfson, "The Internal Senses in Latin, Arabic, and Hebrew Philosophic Texts," *Harvard Theological Review* XXVIII (1935), 69–133.

79. Instead of "kinds," Burke has "species," which is ambiguous because of its double meaning in optical contexts.

those things that are necessary for vision. Writers on optics give us a means to this end by showing how the visual nerves descend from the membranes of the brain and from the lining of the cranium, but no one explains all things necessary in this matter.

I say, then, as all writers on the subjects of nature, all physicians and authorities on optics agree, that the brain is enfolded in a double membrane, one of which is called the pia mater, which enfolds the brain by direct contact; and the other, the dura mater, which adheres to the concave side of the bone of the head called the cranium. For this latter membrane is harder, so that it may resist the bone, and the other is softer and tenderer owing to the softness of the brain, the substance of which is like marrow and ointment, with phlegm as the chief constituent, and with three distinctions, which are called chambers, cells, parts, and divisions. In the first cell there are two faculties: The one in the anterior part is the common sense, as Avicenna states in the first book of *De anima*, which is like a source[80] with respect to the particular senses, and like the center with respect to the lines extending from that same point to the circumference, according to Aristotle in the second book of *De anima*.[81] This common sense judges concerning each particular sensible. For the judgment is not completed in regard to what is seen before the species comes to the common sense, and the same is true in regard to what is heard and to the other senses, as is clear from the end of the work *De sensu et sensato*[82] and in the second book of *De anima*. This common sense forms a judgment concerning diversity in the sensibles[83] as, for example, that in milk whiteness is different from sweetness, a distinction which sight cannot make, nor taste, because they do not distinguish things in other categories, as Aristotle maintains in the second book of *De anima*. It judges concerning the operations of the particular senses, for vision does not perceive that it sees, nor hearing that it hears, but another faculty does, namely, the common sense, as Aristotle maintains in the second book of *De somno et vigilia*.[84] But the final action of this faculty is to receive the species coming from the particular senses and to complete a judgment concerning them. But it does not retain these impressions, owing to the excessive slipperiness of its own organ, according to the statement of Avicenna in the first book of *De anima*.

Therefore, there must be another faculty of the soul in the back part of the first cell, the function of which is to retain the species coming from the particular senses, owing to its tempered moistness and dryness, which is called imagination and is the coffer and repository of the common sense. Avicenna cites as an example a seal, the image of which water readily receives but does not retain, owing to its superabundant moistness; wax, however, retains the image very well, owing to its tempered moistness with dryness. Wherefore he says that it is one thing to receive and another to retain, as is clear from these examples. Such is the case in the organ of the common sense and of imagination.

The whole faculty, however, composed of these two, namely that which occupies the whole first cell, is called phantasia or the *virtus phantastica*. For according to the second book of *De anima* and *De somno et vigilia* and the book *De sensu et sensato*, it is evident that phantasia and the common sense are the same according to subject but differ according to being, as Aristotle says, and that phantasia and imagination are the same according to subject but differ according to being. Wherefore phantasia includes both faculties and does not differ from them except as the whole from the part. Therefore, since the common sense receives the species, and imagination retains it, a complete judgment follows regarding the thing, a judgment formed by phantasia.

Chapter 3. Concerning the sensibles that are apprehended by special senses, by the common sense, and by imagination.

We must note that imagination and the common sense and any particular sense form judgments by[85] themselves only concerning twenty-nine sensibles; as sight concerning light and color; touch concerning heat and cold, moistness and dryness; hearing concerning sound;[86] smell concerning odors; taste concerning savor. These are the nine proper sensibles[87] that belong to their own senses, as I have named them, of which no other particular sense can form a judgment. There are, moreover, twenty

80. The Latin term is *fons*, which Burke translates "fountain."

81. The proper reference, as indicated by Bridges, is *De anima* III.1–2.

82. See *De sensu et sensato*, 7.449a.

83. ". . .de diversitate sensibilium," which Burke translates "concerning differences of impressions on the senses."

84. *De somno et vigilia*, 2.455a.15–25.

85. Burke has "of," which is misleading.

86. Burke has failed to translate the words *auditus de sono*.

87. . . .*propria sensibilia*, translated "special properties" by Burke.

other sensibles, namely, distance, position, corporeity,[88] figure, magnitude, continuity, discreetness or separation, number, motion, rest, roughness, smoothness, transparency, thickness, shadow, darkness, beauty, ugliness, also similarity and difference in all these things and in all things composed of them. Besides these qualities there are some that are subordinate to one or more of these qualities, as order to position, and writing and painting to figure and order. Further examples are straightness and crookedness, and concavity and convexity, which are subordinate to figure; multitude and fewness, which are subordinate to number; equality, augmentation, and diminution, which are subordinate to similarity and difference; eagerness, laughter, and sadness, which are apprehended from the form of the face together with the shedding of tears; and moistness and dryness, which are subordinate to motion and rest. . . . The same is true concerning many other sensibles, which are reduced to the species and principal modes enumerated above belonging to visible things. All these matters are explained in the first book of Ptolemy's *Optica* and in the second book of Alhazen's *De aspectibus*,[89] and in other authors on optics.

There are, moreover, common sensibles, some of which Aristotle defines in the second book of *De anima* and in the beginning of his work *De sensu et sensato*, as, for example, magnitude, figure, motion, rest, and number.[90] These are not the only common sensibles but also all those mentioned before, although most writers on the subjects of nature do not consider this fact, because they are not expert in the science of optics. For common sensibles are not so called because they are perceived by the common sense but because they are determined by all the special senses or by several of them, and particularly by sight and touch, since Ptolemy states in the second book of his *Optica* that touch and sight participate in all these twenty. These twenty-nine, with those that are reduced to them, are apprehended by the special senses, and by the common sense, and by imagination, and these faculties of the soul cannot judge of themselves concerning other sensibles except by accident.

Chapter 4. Concerning the investigation of the estimative, memorative, and cogitative faculties.

But there are other sensibles *per se*, for animals use sense alone, since they do not possess intellect. The sheep, even if it has never seen a wolf, flees from it at once; and every animal experiences fear at the roaring of a lion, although it has never heard a lion before nor seen one. The same is true in regard to many things that are hurtful and contrary to the constitution or complexion of animals. The same principle holds good as regards what is useful and in conformity with their natures. For although a lamb may never have seen another lamb before, it runs to one and willingly remains with it, and the same is true concerning other animals. Brutes, therefore, have some perception in things advantageous and in things harmful. There is, then, something sensibly in them besides the twenty-nine sensibles mentioned above and besides those that are reduced to them. For there must be something more active and productive of change in the sentient body than light and color, because it not only causes comprehension[91] but also a state of fear or love or flight or delay. This is the property of the complexion belonging to each object by which it is assimilated to others in a nature special or general. Through this quality things agree, are strengthened and invigorated, or differ and oppose one another and are mutually harmful. Wherefore not only do light and color produce their species and powers, but to a far greater degree do the complexional qualities,[92] nay, the very natures of things as regards their substance, agreeing or disagreeing with one another, produce strong species, which change greatly the sensitive soul, so that it is moved to states of fear, horror, flight, or the opposites.

These species or powers coming from things, although they change and alter special senses and the common sense and imagination, just as they do the air through which they pass, yet no one of these faculties of the soul judges concerning these impressions, but of necessity a far nobler and more powerful faculty of the sensitive soul does, which is called estimation or the estimative faculty, as Avicenna states in the first book of *De anima*, a faculty which he says perceives the insensible forms connected with sensible matter. Sensible matter is spoken of here as that which is apprehended by the special senses and by the common sense, as are

88. Burke fails to translate *corporeitas*.

89. Ptolemy discusses these matters at the beginning of Book II of the *Optica*. See Alhazen, *Perspectiva*, II, sec. 15; Risner, p. 34.

90. *De anima* II.6.418a.15–20; *De sensu et sensato*, 1.437a.5–10.

91. Burke has "apprehension," which is ambiguous in the present context.

92. . . .*qualitates complexionales*, which Burke has rendered as "properties of complexions."

the aforesaid twenty-nine. We call insensible form that which is not taken cognizance of by those senses of themselves, since they are commonly called senses, although other faculties of the sensitive soul may equally well be called senses, should we wish so to name them, because they are parts of the sensitive soul. For every part of the sensitive soul can be called a sense, because it is in truth a sense, and a sensitive faculty. The statement, therefore, that qualities belonging to complexions are not apprehended by a sense must be understood as applying to a special sense and the common sense and imagination; but they can readily be apprehended by estimation, which although not called a sense is, however, a part of the sensitive soul.

But estimation does not retain a species, although it receives it like the common sense, and it therefore requires another faculty in the remotest part of the posterior cell to retain species coming to the estimative faculty and to be its storehouse and repository, just as imagination is the storehouse of the common sense. This is the memorative faculty, as Avicenna states in the first book of *De anima*.

Cogitation or the cogitative faculty is in the middle cell and is the mistress of the sensitive faculties. It takes the place of reason in brutes, and is therefore called the logical, that is, the rational faculty, not because it employs reason but because it is the ultimate perfection of brutes, just as reason is in man, and because the rational soul in man is united directly with it. By this faculty the spider weaves its geometrical web, and the bee makes its hexagonal house, choosing one of the figures that fill out space, and the swallow its nest. The same is true of all the works of brutes that are similar to human art. Man by means of this faculty sees wonderful things in dreams, and all the faculties both posterior and anterior of the sensitive soul serve and obey it, because they all exist on account of it. For the species[93] that are in the imagination multiply themselves into the cogitative faculty, although they exist in the imagination according to their nature primarily because of phantasia, which uses those species; but the cogitative faculty holds those species in a nobler way, and the species of the estimative and memorative faculties exist in the cogitative faculty in accordance with a nature nobler than that existing in those faculties, and therefore the cogitative faculty uses all the other faculties as its instruments.

In man there is in addition from without and from creation the rational soul, which is united with the cogitative faculty primarily and immediately, and uses this faculty chiefly as its own special instrument. Species are formed in the rational soul by this faculty. Wherefore, when this faculty is impaired, the judgment of reason is especially perverted, and when it is in a healthy condition the intellect functions in a sound and rational way.

19. John Pecham: The Geometry of Reflection

PERSPECTIVA COMMUNIS, II, PROPOSITIONS 6, 20, 30

Proposition 6. The angles of incidence and reflection are equal, and the incident and reflected rays are in the same plane as the line erected [perpendicularly] at the point of reflection.

The angle formed by the incident ray and either the surface of the mirror (on one side) or the imaginary line erected [perpendicularly] at the point of reflection (on the other side) is called the angle of incidence; the angle formed by these [that is, the surface of the mirror and the imaginary perpendicular] and the reflected ray is called the angle of reflection. Equality of the angles is gathered from experience and proved by reason in any of several ways; for if an incident ray could advance into the depth of a mirror, it would form (with the perpendicular extended into the depth [of the mirror] at the point of reflection) an angle equal to the angle of incidence because, according to Euclid, vertical angles are equal.[94] Therefore, the ray recoils in the same mode as that in which it would be transmitted [if it were not deflected by a reflecting surface], and consequently it must rebound at an equal angle [to the angle of incidence].[95] Accordingly, if it is incident on the mirror perpendicularly, it is reflected back on itself; if it is incident obliquely, it is reflected obliquely toward the other side. The same thing is evident in the motion of a body, since a heavy body descending vertically onto a solid body or projected perpendicularly along a line is driven back along the same line; if projected obliquely, it rebounds along a similar line on the other side. . . .[96]

93. Burke has "forms or species," but the Latin text simply reads *species*.

94. *Elements* I.15.

95. Note that although Pecham indicates that equality of angles can be gathered by experience, he argues from the principle of symmetry. Doubtless the experiences or experiments to which he referred were those described by Alhazen in the *Perspectiva*, IV, secs. 7–12; Risner pp. 104–109.

It is evident also that the three lines [that is, the incident ray, the reflected ray, and the perpendicular] are in the same plane, because a ray conforms as closely as possible to a rectilinear path, since straightness is natural to light. However, if a ray were to forsake that plane, it would be departing doubly from straightness, both by rebound and by deviation.

Proposition 20. In plane mirrors, and for the most part in others, the images appear at the intersection of the ray and the cathetus.

The cathetus is the line dropped perpendicularly from the visible object to the surface of the mirror, whether plane or spherical. Indeed, that which is seen in a mirror appears to be located at the imaginary intersection of the ray under which the object is seen and the perpendicular dropped from the object to the surface of the mirror. This can be explained by reference to Proposition 67 of Part I, since [by that proposition] the lengths of rays are perceived by the eye. But because the reflected part of the ray acts directly on sight and sight occupies itself with that part, through its mediation the part of the ray incident on the mirror is so perceived that the whole ray is apprehended by the eye as though proceeding without interruption in a straight line; for reflection cannot be discerned by the eye, which perceives nothing except the part of the ray that [actually] conveys the qualities [of the visible object] to sight. Therefore, an object seen in a mirror, if above the mirror, must appear beneath it at the imaginary intersection of the ray and the cathetus.[97]

For example, let *ABG* be the mirror, *CK* the visible object, and *D* the eye of the observer [Fig. 3]. Rays *KA* and *CB* fall from the visible object and are reflected to the eye along rays *AD* and *BD*. Consequently *KA* appears to be the [direct] continuation of *AD*, and *CB* of *BD;* it follows that *KA* and *CB* seem to be directed behind the mirror under the same angles at which they are reflected, because vertical angles are equal, and *DA* falls to *E* and *DB* to *F*. Furthermore, the object appears on the aforementioned perpendicular, that is, the cathetus, exactly as in its true location. . . .

Proposition 30. In convex spherical mirrors the image appears at the intersection of the ray and the cathetus, that is, the line dropped [from the object] to the center of the sphere.

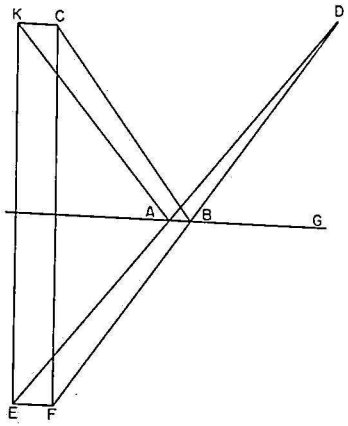


Fig. 3

This can be proved by experience and [can be shown to result] from natural causes, as it was for plane mirrors above. Nevertheless plane mirrors and convex spherical mirrors differ in that an object always appears as far below a plane mirror as it actually is above it, whereas in a convex spherical mirror the image appears sometimes on the surface of the mirror, sometimes inside the mirror, and sometimes outside it.[98] For example, let *E* be the visible point, *G* the eye, *N* the point of reflection, and *D* the center of the sphere [Fig. 4]. It is clear that the image is located at *K*. However, if the visible object should be placed at *B*, the image would appear at *O;* if the visible object should be placed still closer to the sphere, the image would

96. It must be asserted emphatically that Pecham is not implying that light is corpuscular; the mechanical example employed here is intended to elucidate only the equality of angles, not the nature of light.

97. The essence of Pecham's argument is that the eye knows the length of the ray by which the object is seen but is not aware of the bend; consequently, the object appears to be along the backward rectilinear extension of the reflected ray at its intersection with the cathetus. This makes ray *ED* in Fig. 3 equal to the sum of rays *KA* and *AD*, which is in accord with the eye's ability to determine the length of the entering ray. It should be noted, however, that this equality holds only for plane mirrors.

98. Although it is impossible for the image to be located anywhere except *behind* a convex mirror, Pecham

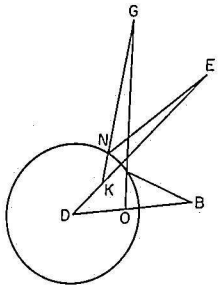


Fig. 4

appear outside the sphere, as will be evident by careful investigation.

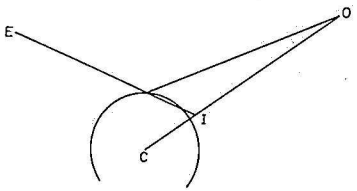
The point of reflection is easy to find in these [mirrors] when the eye and visible object are at exactly equal distances from the sphere. Otherwise the length [of time required] to find the point is greater than the difficulty or utility, as can be seen by examining the section [in Alhazen's *Perspectiva*] on images.[99] From that section also it is apparent that the image in convex spherical mirrors is nearer the mirror than is the visible object, [a relationship] which is in contrast to [that for] plane mirrors as shown above.

20. Witelo: A Problem of Image Formation by Reflection

PERSPECTIVA,[100] VIII, THEOREM 53

When the lines of incidence [drawn from the endpoints of some object] intersect each other in concave spherical mirrors, oblique lengths situated inside the point of intersection[101] appear as they [truly] are; but those that are outside the intersection of the lines appear as reversed images.

Let there be a concave spherical mirror *AG*, with center *M;* let *B* be the center of the eye, and let *DE* be a line situated obliquely above the surface of the mirror [Fig. 5]. Let the form of point *D* of this line be reflected to the eye *B* by point *A* of the mirror, and the form of point *E* by point *G*. Let the lines of incidence (that is, *DA* and *EG*) intersect one another at point *I;* and inside point *I*, situated obliquely with respect to the surface of the mirror, let there be line *KC*, point *K* of which is reflected from point *G* of the mirror and point *C* from point *A* of the mirror.[102] Thus, from point *D* to the center of the mirror, draw line *DM*, which, because of the obliquity of line *BA* with respect to the surface of the mirror (since line *DM* is perpendicular to the



is correct in asserting that the image may be inside, outside, or on the surface of the mirror. Consider the convex spherical mirror in the figure, with the visible object situated at *O* and the observer's eye at *E*. The image is located at the intersection of the cathetus and the rectilinear extension of the reflected ray, *I*, which is outside, though *behind*, the mirror.

99. V, sec. 39; Risner, pp. 150–151. This problem of determining the point of reflection, given the locations of the visible point and the observer's eye, is known as "Alhazen's problem." For a good discussion of "Alhazen's problem," see Paul Bode, "Die Alhazensche Spiegel-Aufgabe in ihrer historischen Entwicklung nebst einer analytischen Lösung des verallgemeinerten Problems," *Jahresbericht des physikalischen Vereins zu Frankfurt* (1891–92), pp. 63–107; cf. Sabetai Unguru, "Witelo as a Mathematician: A Study in XIIIth Century Mathematics, including a Critical Edition and English Translation of the Mathematical Book of Witelo's *Perspectiva*" (unpublished doctoral dissertation, University of Wisconsin, 1970), pp. 368–375. Much of the rest that has been written on the problem is misleading.

100. Risner, p. 355.

101. That is, between the point of intersection (*I* in Fig. 5) and the mirror. The collection of points such as *I* for all positions of the object and the eye constitutes the focal plane of the mirror, and although Witelo does not employ modern terminology in describing this plane, he is correct in asserting that objects situated outside it appear inverted or reversed, while those situated inside the focal plane appear erect or unreversed. This proposition well illustrates the advanced level to which geometrical optics was brought in the thirteenth century.

102. For ease of demonstration, Witelo has arranged lines *DE* and *KC* so that both are observed by means of the same incident and reflected rays (Fig. 5). But he need not have done so; any line situated above *I* (that is, outside the focal plane) will have an inverted image, and any line situated below *I* will have an erect image.

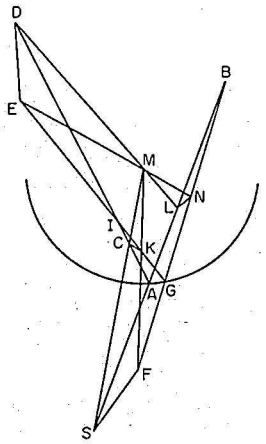


Fig. 5

same surface of the mirror by Book I, Theorem 72, of this treatise, because it passes through the center *M* of the mirror), intersects line *BA*, which is incident obliquely on the surface of the mirror, as Book I, Theorem 14, of this treatise is able to reveal; and let the point of intersection be *L*. Similarly, line *EM* intersects line *BG;* let the point of intersection be *N*. Thus it is evident, by Book V, Theorem 37, of this treatise, that the image of the form of point *D* is located at point *L*, and the image of the form of point *E* at point *N*.[103] Draw line *NL*, which will be the image of the whole line *DE*. Now image *NL* has reversed itself with respect to line *DE*, since point *N*, the image of the lower point *E*, is higher than point *L*, the image of the higher point *D*.

Also, let line *MK* be extended until it intersects the extension of line *BG*—they intersect because of the obliquity of line *BG* with respect to the surface of the mirror and the perpendicularity of line *MK*—and let the point of intersection be *F;* extend line *MC* until it intersects the extension of line *BA*, and let the point of intersection be *S*. Let line *FS* connect [these points of intersection]; then line *FS* will be the image of line *KC*, and as point *K* is higher than point *C*, so point *F* is higher than point *S*. Thus image *FS* has a position conforming to that of the visible object *KC*, situated in front of the mirror inside the point of intersection, *I*, of the incident lines. Therefore, the proposition is evident.

21. Alhazen: Paraboloidal Burning Mirrors

ON BURNING MIRRORS[104]

One of the most exquisite things invented by geometers, about which the ancients were concerned, in which the most excellent properties of geometrical figures are apparent, and which follows (according to them) from natural things, is the construction of mirrors that produce combustion through the reflection of a solar ray. Now, in inventing these mirrors, they employed various means. That is, they discovered that a ray is reflected from the surfaces of plane mirrors and also from the surfaces of spherical mirrors, and that the places to which the ray is reflected vary with the size of the various dimensions of the mirror. However, it became clear to them that a ray [proceeding from a given point on an object and] reflected from a plane mirror is reflected to one point from only one point [of the mirror] and that a ray reflected from a spherical mirror is reflected to one point from the circumference of only one of the circles lying in that sphere. And demonstrations of this are presented in their books. Thus some of them attempted to employ many plane mirrors joined together, from all of which rays would be reflected to one point, so that combustion would be strong. And those who invented these mirrors

103. Note that *DM* is the cathetus, since it is dropped from the visible point *D* perpendicular to the surface of the mirror, and *AB* is the reflected ray; the image of *D* is thus situated at intersection *L* of these two lines. The same principle is used to determine the location of the images of points *E*, *C*, and *K*.

104. Translated from the text edited by J. L. Heiberg and Eilhard Wiedemann, "Ibn al Haitams Schrift über parabolische Hohlspiegel," *Bibliotheca Mathematica*, Ser. 3, X (1910), 201–237. In the same article Heiberg and Wiedemann present a German translation based on both the Arabic and Latin texts, which texts, as their translation reveals, do not diverge significantly. In translating the excerpts that follow, I have consulted both this German translation and the English translation of an Arabic manuscript by H. J. J. Winter and W. 'Arafat, "Ibn al-Haitham on the Paraboloidal Focussing Mirror," *Journal of the Royal Asiatic Society of Bengal*, Ser. 3; Vol. XV, Nos. 1–2 (1949), pp. 25–40. This shorter work by Alhazen was translated into Latin probably by Gerard of Cremona or one of his school; in any case, it was known by the second half of the thirteenth century and became very popular during the later Middle Ages.

were famous men, such as Archimedes, Anthemius,[105] and others.

Then it occurred to them to consider the properties of the figures from which the ray is reflected. Thus they investigated the properties of conic sections[106] and discovered that the rays incident on the entire concave surface of a paraboloid are reflected to one and the same point. It thus became evident that the combustion produced by a mirror of this shape is stronger than the combustion produced by a mirror of any other shape. Nevertheless, they did not present an adequate demonstration of these facts, nor of the method by which they discovered them. Therefore, since in this matter there is great benefit and general utility, I have taken care to explain it and to reveal it, so that he who desires to know truth will have knowledge of it, and he who is concerned with the swiftness *(velocitatibus)*[107] of things will understand it. . . .

Premises agreed upon: A solar ray issues from the body of the sun to the surfaces of all kinds of mirrors along straight lines. All rays incident on plane mirrors are reflected from the surfaces[108] at equal angles. All rays incident on concave or convex mirrors are reflected from the surfaces at equal angles, formed with the plane surfaces tangent to those surfaces at the points of incidence. By the ray reflected at equal angles,[109] I mean that the reflected ray forms two equal angles with the straight line that is the common difference[110] between the surface of the two straight lines (that is, the incident and reflected rays) and the plane surface (that is, the surface of a [plane] mirror or the [plane] surface tangent to a concave or convex mirror). The rays that proceed along the straight lines reaching the surfaces of all mirrors and that are reflected at equal angles (whether from the surfaces of plane mirrors or from the [plane] surfaces tangent to the surfaces of concave and convex mirrors), that is, the lines that are reflected according to the manner of a reflected ray, are reflected in turn along those lines.[111] By the plane surface tangent to the concave surface, I mean that [surface] which has only one point [in common] with the concave surface. By the surface of the reflected line or ray, I mean the surface in which those two lines lie, that is, the line itself and the reflected line with which it forms an angle.[112]

[Consider] a section of a parabola, the axis of which is drawn, and [on which] a distance equal to one-fourth of the latus rectum is marked off from the end of the axis. In every [such] section, all lines drawn parallel to the axis, arriving at the section and variously reflected to the point that marks off the fourth part, form two equal angles with the line tangent to the section at the point [of reflection]. For example, let *ABG* be a section of a parabola, and let its axis be *AD* and its latus rectum *L* [Fig. 6]. From *AD*, I mark off line *AE* equal to one fourth of line *L*, and I draw line *TB* parallel to line *AD*. I connect *B* and *E* and draw tangent *KBH*. I say, then, that angle *TBK* equals angle *EBH*.

First, let angle *BEH* be acute. Then by the method of resolution, I suppose that angle *TBK* equals angle *EBH*.[113] And since line *TB* is parallel to line *DA*, angle *TBK* will equal angle *BHE*. But angle *TBK* equals angle *EBH* by assumption; therefore, angle *EBH* equals angle *BHE*. Therefore, line *BE* is equal to line *EH;* consequently, the square of *BE* equals the square of *EH*. Now I draw *BZ* perpendicular to the axis. Then the two squared quantities *EZ* and *BZ* [summed] are equal to the square of *EH*. But the square of *BZ* is equal

105. On Anthemius, see *Les opuscules mathématiques de Didyme, Diophane et Anthemius suivis du fragment mathématique de Bobbio*, translated by Paul Ver Eecke (Paris/Bruges: Desclée de Brouwer, 1940); Sir Thomas L. Heath, "The Fragment of Anthemius on Burning Mirrors and the 'Fragmentum mathematicum Bobiense,' *Bibliotheca Mathematica*, Ser. 3, VII (1907), 225–233. On Archimedes, see A. Rome, "Note sur les passages des catoptriques d'Archimède conservés par Théon d'Alexandrie," *Annales de la Société scientifique de Bruxelles*, Ser. A, LII (1932), 30–41.

106. The Latin text reads *sectionum pyramidum*, but cone is meant.

107. This makes absolutely no sense; the Arabic text reads "application."

108. . . . *ex superficiebus angulorum*. I have not translated *angulorum*, which makes no sense and is lacking in the Arabic text.

109. The text reads "right angles," but emendation is obviously required.

110. That is, the straight line formed by the intersection of the two planes.

111. Alhazen appears to mean that rays and the geometrical lines employed to represent them—rays have width and are thereby unlike geometrical lines—follow identical paths; that is, the rays move along the geometrical lines.

112. That is, the reflected line to which it is joined angularly.

113. Alhazen assumes that which is to be demonstrated; then, following the method of resolution (or analysis), he deduces from it some known mathematical relation. In the reverse process of composition (or synthesis) which follows, he begins with the known mathematical relation and deduces that which is to be demonstrated, namely the equality of angles.

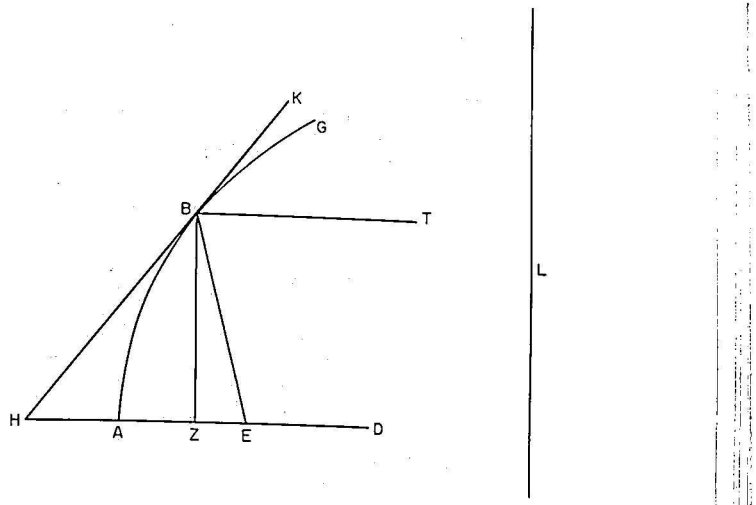


Fig. 6

to the product of *AZ* and *L*,[114] which is the latus rectum,[115] as the great Apollonius demonstrates in his book on conics.[116] Therefore, the square of *EZ* added to the product of *AZ* and *L* is equal to the square of *EH*. But *AE* is one fourth of *L;* thus four times the product of *AZ* and *AE* is equal to the product of *AZ* and *L*. Consequently, four times the product of *AZ* and *AE* added to the square of *EZ* is equal to the square of *EH*. Therefore, *AH* is equal to *AZ*.[117] But this is so because *BH* is tangent[118] and *BZ* is as arranged [that is, perpendicular to the axis].

And by the method of composition, I suppose everything as before, and I say that angle *TBK* equals angle *EBH*. The proof of this is as follows. I draw *BZ*, as before [perpendicular to the axis]. Since *BH* is tangent to the section [of the parabola] and *BZ* is as arranged, *AZ* will be equal to line *AH;* therefore four times the product of *AE* and *AZ* added to the square of *EZ* is equal to the square of *EH*. But *AE* is one fourth of *L;* consequently, four times the product of *AE* and *AZ* is equal to the product of *L* and *AZ*. Therefore, the product of *L* and *AZ* added to the square of *EZ* is equal to the square of *EH*. But the product of *L* and *AZ* is equal to the square of *BZ*, since *BZ* is as arranged [that is, perpendicular to the axis]; therefore, the square of *BZ* added to the square of *EZ* is equal to the square of *EH*. But the two squared quantities *BZ* and *EZ* [summed] are equal to the square of *BE;* thus the square of *BE* is equal to the square of *EH*, and *BE* is equal to *EH*. Therefore, angle *EBH* equals angle *BHE*. And again, *TB* is parallel to *DA;* therefore, angle *TBK* equals angle *BHE*. Therefore, angle *EBH* equals angle *TBK*. And similarly, all lines drawn parallel to the axis[119] are reflected variously to

114. By the defining equation of a parabola, expressed in Cartesian notation as $y^2 = Lx$, where L is the length of the latus rectum.

115. The Latin text reads "straight line," but latus rectum is obviously intended.

116. *De pyramidibus*.

117. A number of steps have been omitted, but it does indeed follow from $4AZ \cdot AE + EZ^2 = EH^2$ that $AH = AZ$.

118. Apollonius had demonstrated that for any parabola $AH = AZ$ if BH is tangent; see Heiberg and Wiedemann, p. 209, n. 1.

119. The text reads "diameter," but Alhazen obviously means the axis.

point E and form acute angles with AE. And that is what we wished to demonstrate. . . .[120]

When any concave reflecting surface having the same concavity as a paraboloid is placed opposite the body of the sun so that its axis is in a direct line with the body of the sun, rays issue from the body of the sun to its whole surface; and all of these rays are reflected to the point on the axis located a distance from the pole of the surface

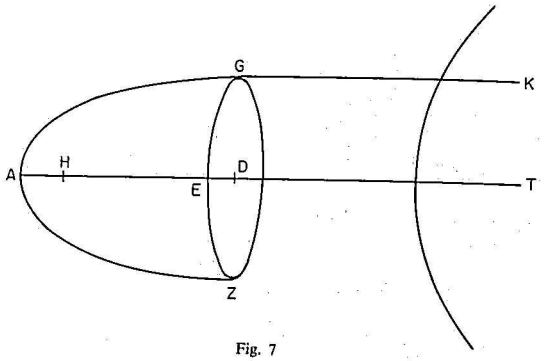


Fig. 7

equal to one fourth of the latus rectum of the section that generated the surface. For example, let there be a concave reflecting surface having the same concavity as a paraboloid, the pole of which is point A, its base the circle GEZ, its axis AD, and the distance of point H from point A equal to one fourth of the latus rectum of the section that generated the surface [Fig. 7]; and let the body of the sun (circle T) be placed opposite it, so that axis AD, when extended in a straight line, reaches point T within the body of the sun. I say, then, that rays issue from the whole body of the sun to this entire surface, and all are reflected to H.

This is demonstrated as follows. Since a ray proceeding from the body of the sun to the reflecting surface proceeds along straight lines, the ray proceeding from point T to point A proceeds along line AT. However, I choose a point at random on the circumference of the base of the reflecting surface, namely point G; and I imagine a line proceeding from point G parallel to line AT, such as line GK. Therefore, line GK, when extended rectilinearly, is incident on the body of the sun, since the separation between it and line AT is small, and insignificant by comparison with the body of the sun. Consequently, this line will always be incident near point T; but point T is within the body of the sun. Therefore, it is incident on the sun; thus let it be incident at point K.

Therefore, the ray that proceeds from point K to point G proceeds along line KG.[121] Similarly, for every line extended from a point on the reflecting surface parallel to the axis and reaching the body of the sun, the ray that issues from that point [on the body of the sun] to the point on the reflecting surface proceeds along that line. But it has already been declared that rays proceed from the body of the sun to the entire surface of the reflecting body along lines parallel to the axis. Therefore, I maintain that all of them are reflected to one point. And since surface $AGEZ$ is a concave surface having the same concavity as a paraboloid, when lines parallel to the axis encounter the surface and are reflected to point H, they form equal angles with the straight lines drawn in the same planes as the incident and reflected rays and tangent to the concave surface, as is evident in the figure above [Fig. 7]; and the rays that proceed along the straight lines drawn to the reflecting surfaces are reflected at equal angles with the lines [lying] in the same planes as the reflected lines and tangent to the reflecting surfaces. Therefore, the rays that proceed to the entire concave

120. The same demonstration is then repeated for the cases when angle BEH is right and obtuse.

121. Here Alhazen shifts from a discussion of geometrical lines to the rays propagated along them, in accordance with the distinction made in the fifth premise, in the third paragraph of this section.

surface along lines parallel to the axis are reflected along those lines that reach the point. And it has already been demonstrated that rays proceed from the body of the sun to the entire reflecting surface along lines parallel to the axis; therefore [these] rays, which are parallel to the axis, drawn to the entire concave reflecting surface having the same concavity as a paraboloid, are all reflected to point H. And this is the point located a distance from the pole of the surface equal to one fourth of the latus rectum. And this is what we wished to demonstrate. . . .[122]

22. Witelo: Paraboloidal Burning Mirrors

PERSPECTIVA,[123] IX, THEOREM 43

When a concave mirror, of the same concavity as a portion of a parabola, is placed opposite the sun so that its axis points directly toward the body of the sun, all rays parallel to the axis and incident on the mirror are reflected to one point of the axis, removed from the surface of the mirror a distance equal to one fourth of the latus rectum of the portion of the parabola generating the surface of the mirror. It is evident from this that it is possible to kindle fire by means of the surface of such mirrors.

Let there be a concave mirror having the same concavity as a portion of a parabola [Fig. 8]. Let

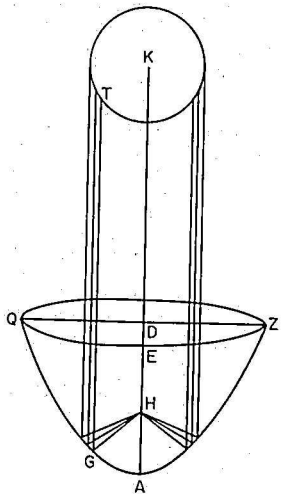


Fig. 8

its vertex be point A, its base circle QEZ, its axis AD; and let the distance of point H on the axis from the vertex of the mirror (point A) be equal to one fourth of line QZ, the latus rectum of the portion of the parabola AGQ, which generates the surface of the concave mirror by its motion about axis AD. Let the mirror be placed opposite the sun according to its axis AD; that is, let K be the center of the solar body, and let the mirror be situated so that its axis AD, if extended, reaches point K in the center of the sun. I say that all solar rays parallel to ray KA incident on the surface of the proposed mirror are reflected to point H of line AD, the axis of the mirror.

Since all rays proceeding from any point whatever of the solar body to any point of the surface of the mirror proceed along straight lines, as is evident by Book I, Theorem 2, of this treatise, it is clear that line KA is a straight line. Thus on the periphery of any portion of the parabola of the mirror—let this be $GAZQ$—mark point G at random; and from point G of the mirror to some point of the solar body—let this be T—draw line GT parallel to ray AK, which is incident on the surface of the mirror along axis AD, by Book I, Proposition 31 [of Euclid's *Elements*]. Now, it is necessary for the line from any point of the mirror extended parallel to ray AK to encounter the surface of the body of the sun, since the ratio between the surface of the mirror and the surface of the solar body is either zero or small; thus let there be point T, which is the end of line GT, on the surface of the solar body. Thus all lines that can be drawn from the surface of the mirror parallel to its axis AD encounter the solar body; and, like the ray incident along the axis, the incidence of all rays parallel to the axis takes place along these lines. This is true of all rays incident on any point whatsoever of the whole surface of the mirror, since (by Book I, Proposition 31 [of Euclid's *Elements*,]) from any given point, near or far, we know how to draw a line parallel to any given line, such as axis AD in the proposition.

Thus I say that all those rays are reflected from

122. The remainder of the treatise deals with the fabrication of paraboloidal mirrors.

123. Risner, pp. 401–402. This proposition abundantly reveals Witelo's close reliance on Alhazen.

the whole surface of the mirror to one point on the axis of the mirror, point *H*. Since all those rays are straight lines, it is evident from what has been asserted above that they form equal angles with the lines drawn from all their points of incidence to point *H;* therefore, by Book V, Theorem 20, of this treatise, all those rays are reflected along the lines passing through point *H*. From this it is clear that all rays incident on the periphery of a portion [of the paraboloid] and parallel to the ray incident along the axis[124] of that portion are reflected to the point on the axis that cuts a line segment equal to one fourth of the latus rectum of portion *GAZQ* [of the paraboloid] from the end of the axis at the periphery of that portion,[125] since every reflection from all regular polished bodies takes place according to equality of the angles formed between the incident and reflected lines and the line tangent to the surface of the mirror at the point where reflection occurs. And since all those lines intersect at point *H*, it is clear that the convergence of all those rays is in point *H;* therefore, every power of all the rays incident on the entire surface of the mirror is assembled at that point. And since every small beam brings with it some of the active power of the solar body, it is evident that the whole power is assembled at that point, namely the power of all rays parallel to axis *AD* incident on the surface of the mirror. Thus it is obvious that if combustible material should be placed at point *H*, fire can be kindled. And this is the best and strongest of all figures for assembling solar rays at one point, since solar rays are assembled at one point by its whole surface and its every point. Therefore, the proposition is evident.

23. Alhazen: Causal Analysis of Reflection

PERSPECTIVA,[126] IV, SECTION 18

The reason why light is reflected along a line having the same slope as the line by which the light approaches the mirror is that light is moved very swiftly, and when it falls on a mirror it is not allowed to penetrate but is denied entrance into that body. And since the original force and nature of motion still remain in it, the light is reflected in the direction from which it came, along a line having the same slope as the original ray. We can see the same thing in natural and accidental motion, for if we allow a heavy spherical body to descend perpendicularly onto a smooth body from a certain height, we will see it reflected along the same perpendicular by which it descended. In accidental motion,[127] if a mirror is raised to the height of a man and firmly fastened to a wall and a sphere is fixed to the tip of an arrow, and if the arrow is [then] projected by a bow at the mirror in such a way that the arrow and mirror have equal elevations and the arrow is horizontal, it is evident that the arrow approaches the mirror perpendicularly; and it will be seen to recede from the mirror along the same perpendicular. But if the motion of the arrow should be oblique relative to the mirror, the arrow will be reflected not along the line of incidence, but along a line horizontal (like the line of incidence) and of the same obliquity with respect to the mirror and the perpendicular. And it is clear that a motion of reflection is experienced by light because of the opposition of a polished body, since the stronger the repulsion or opposition, the stronger is the reflection of light.[128]

But the reason why the motions of approach and reflection are the same is as follows. When a heavy body descends along the perpendicular, the opposition of the polished body and the motion of descent of the heavy body are directly opposite: there is no motion except perpendicular motion, and opposition takes place along the perpendicular. Therefore, the body is opposed along the perpendicular, and thus it returns perpendicularly. But when the body descends along a sloping line, the line of descent falls between the perpendicular to the surface of the polished body (passing through that body) and the line in the surface orthogonal to this perpendicular. And if the motion were to penetrate beyond the point of its incidence, finding free passage, this line would fall between the perpendicular passing through the body and the line in the surface orthogonal to the perpendicular, and it would maintain the same measure of position with respect to the perpendicular passing [through the body] and the other line orthogonal to that perpendicular. For the measure of position of this motion is composed of a position along the perpendicular and a position along the orthogonal.[129] But since repulsion occurs along the perpendicular, because the body cannot repel motion according to the measure which it has along the perpendicular passing through the polished body and because a moderate thing would not enter, the body repels according to the measure of position along the perpendicular which it has along the orthogonal.[130] And when the regression of the motion has the same measure of position along the orthogonal which there was formerly along the orthogonal on the other side [of the point of reflection], it will also have the same measure of position along the perpendicular passing [through the body] as it had before.

But in the rebound of a heavy body, when the motion of repulsion ceases, the body descends because of its nature and tends toward the center [of the world]. However, light, which has the same nature of reflecting [as a heavy body], does not by nature ascend or descend; therefore, in reflection it is moved along its initial line until it meets an obstacle which terminates its motion.[131] And this is the cause of reflection.

124. The text reads "diameter," but Witelo obviously means the axis; see note 119.

125. The complicated phraseology at this point tends to obscure Witelo's meaning, which is simply that the point is situated on the axis at a distance from vertex *A* equal to one fourth of the latus rectum.

126. Risner, pp. 112–113.

127. That is, violent or forced motion.

128. Note the mechanical terms in which Alhazen discusses reflection. Nevertheless, it should not be thought that he regarded light as corpuscular; like his Western followers, he thought of light as a power or form propagated through a material medium. For a fuller analysis of Alhazen's explanation of reflection, see my introduction to the forthcoming facsimile reprint of the Risner edition.

24. Roger Bacon: Causal Analysis of Reflection

DE MULTIPLICATIONE SPECIERUM,[132] III, CHAPTER 1

If one speaks of the rebound of species from a body, it is evident from what has been said that this does not occur by violence; but when it is forbidden to pass through [the body] by the density opposing it, the species generates itself in a direction open to it. For if it were to be driven back like a ball from a wall, it would be necessary for it to be

25. Witelo: Causal Analysis of Reflection

PERSPECTIVA,[134] V, THEOREM 18

Everything seen by means of any mirror whatever is perceived by the eye under the shortest [possible] lines.

Let there be a mirror, in the surface of which is a straight or curved line *ACB*,[135] and let *D* be a point of the visible object and *F* the center of the eye; point *D* is seen to be reflected from point *C* of the mirror [Fig. 9]. I say that lines *FC* and *DC* [summed] are shorter than [the sum of] lines drawn from points *D* and *F* to any other points of the mirror. Let lines *ED* and *EF* be drawn to another point on the surface of the mirror; let this be *E*. These lines are neither shorter than, nor equal to,

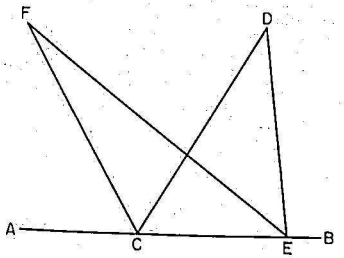


Fig. 9

129. Alhazen resolves the motion of an oblique ray into components perpendicular and parallel to the surface of the mirror.

130. This and the next sentence are most obscure; consequently, I have provided a very literal translation. It does seem clear, however, that Alhazen regards both the parallel and perpendicular components of motion as being preserved in reflection. This conclusion is consistent with the claim, earlier in the argument, that after the reflection of light "the original force and nature of motion still remain in it"; combined with the knowledge that the angles of incidence and reflection are equal, this implies that the perpendicular and parallel components of the reflected ray possess "the original force and nature of motion" of those components of the incident ray. Thus Alhazen has analyzed the reflection of light in a fully Cartesian manner.

131. Although the behavior of heavy bodies can elucidate the reflection of light, Alhazen here takes care to point out that light is not itself a heavy body.

132. Bridges, II, 505.

133. In this brief passage Bacon reasserts the point he has made elsewhere that light is not corpuscular and that reflection is not an instance of mechanical rebound; reflection is rather to be explained by the self-diffusive properties of species.

134. Risner, p. 198.

135. The meaning seems to be that line *ACB* is straight in a plane mirror, curved in a curved mirror.

lines *CD* and *CF*, but longer. Now, nature always acts along the shortest lines, as is evident from Theorem 5 of this book, and the multiplication of forms to the surfaces of mirrors is natural, since it, like every other diffusion of forms, takes place by the action of nature, as we have shown in [our] *Philosophia naturalis*,[136] in the chapter on natural action. Similarly, the reflection of forms from the surfaces of mirrors to the eye is purely natural, since it, like every other visual perception, takes place by the action of nature and is completed by the action of the soul, as is evident from the whole fourth book of this our book of science; and the soul is, as it were, the nature of animals. Therefore, it is evident that this diffusion, reflection, and comprehension of a form, which takes place of itself, is truly natural. Consequently it takes place along shorter lines, which was the proposition; for in vain would it take place along longer lines, since it could take place better and more surely along shorter lines.[137]

26. Alhazen: An Instrument for Investigating Refraction

PERSPECTIVA,[138] VII, SECTION 2

The construction of an instrument of refraction.[139]

That light indeed passes through air and is extended along straight lines was declared in the first book of this work; however, air is but one of the transparent substances. Light [also] passes through water, glass, and transparent stones, and [in them] it proceeds along straight lines. But this is learned by experience. If one would like to experience it, let him take a round plate of copper, not less than one cubit in diameter, thick enough to be rather strong. This plate should have a circular rim perpendicular to its surface, not less than two finger-breadths in height. In the middle of the back of the plate, perpendicular to its surface, there should be a small cylindrical body not less than three finger-breadths long. Now place this instrument in a lathe with which the turner fashions instruments of copper; place one point of the lathe in the middle of the plate and the other in the middle of the end of the body mounted on the back of the plate. Then scrape this instrument by revolution, with suitable abrasion, until the roundness of its rim, inside and out, is ascertained, and the interior and exterior surfaces are made even, and the two surfaces are made parallel. We also scrape the body on the back [of the plate] until it becomes round.

When this instrument has been completed by abrasion, we mark two diameters on its interior surface, crossing through its center and intersecting each other perpendicularly [Fig. 10]. Then we mark a point at the bottom of the rim of the instrument, one finger-breadth away from the end of one of the two intersecting diameters. From this point we draw a third diameter passing through the center of the plate and extended to [the limits of] its whole surface. From the two ends of this

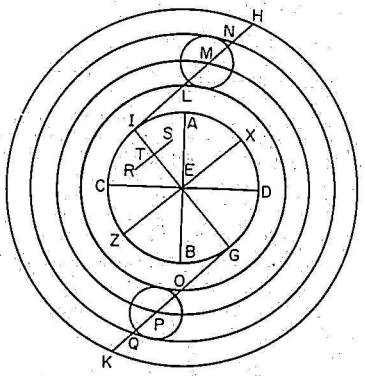


Fig. 10

136. This work has been lost.

137. Witelo's use of the principle of least action or minimum distance to explain the law of reflection is doubtless drawn from chapter 4 of Hero of Alexandria's *De speculis*, translated into Latin by William of Moerbeke at Witelo's request and completed on December 31, 1269. The date is given at the end of the holograph of this treatise, in Vat. Ottob. Lat. MS 1850, fol. 61v. Witelo's demonstration that lines forming equal angles with the "reflecting" surface satisfy the principle of least distance appears in Bk. I, Theorem 17 and Bk. V, Theorem 19.

138. Risner, pp. 231–233. Figure 11 has been corrected. Eilhard Wiedemann has published a German translation of portions of this passage in his "Ueber den Apparat zur Untersuchung und Brechung des Lichtes von Ibn al Haitam," *Annalen der Physik und Chemie*, n. s., XXI (1884), 541–544.

139. The same instrument is described by Witelo; however, there is no evidence that Witelo actually used it.

[third] diameter[140] we draw two lines[141] on the surface of the rim of the instrument, perpendicular to the surface of the plate. Then out of one of those lines we divide three small equal parts,[142] the first of which is next to the surface of the plate; and the length of each of these line segments is equal to the size of half a grain of barley. Thus, along this perpendicular line there are three points, which are the ends of those line segments. Then we return this instrument to the lathe and we mark on it three parallel circles,[143] passing through the three points on the perpendicular line at the end of the diameter. Thus the other perpendicular, which is perpendicular to the other end of this diameter, is also cut by those three circles, and three points are marked on it; [thus] there are, on each of the three circles, two opposite points, which are the ends of certain of their diameters. We then divide the middle of those three circles into 360 degrees and, if possible, into minutes. We form a round opening in the rim of the instrument, the center of which is the middle of the three points that are on one of the two lines perpendicular to the ends of the diameter of the plate, and its radius equal in size to the distance between circles. Thus the circumference of the opening extends between the two outermost parallel circles.[144]

After this, we take a thin rectangular plate of moderate thickness, the length of which is equal to the height of the rim of the instrument and the width of which is nearly that [Fig. 11]. Its

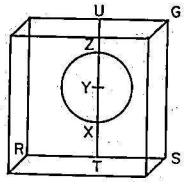


Fig. 11

[largest] surface is made as even as possible, and also [the surface of] its thickness, which falls along one of its ends, until the common difference between the surface of its face and the surface of its thickness is a straight line.[145] We divide this line into two equal parts, and from the middle we draw a straight line on the surface of its face, perpendicular to the straight line [*RS*] which is the common difference. Then we divide out of this perpendicular line, from the end that is above the common difference, three segments equal to each other and to one of the small lines marked along the perpendicular line on the rim of the plate; thus there are three points[146] on the perpendicular line on the face of the small plate. Then we pierce this small plate with a round hole, the center of which is the middle point of those that mark off the lines in it, and the radius of which is equal to any one of those small lines; consequently, this opening is equal to the opening in the rim of the instrument.

On the diameter of the [circular] plate, at the ends of which are the two perpendicular lines, we mark a point[147] in the middle of the line that connects the center of the plate and the end of the diameter that extends toward the opening; through this point we draw a line perpendicular to the diameter. Then we place the base of the small plate on this line so that the common difference of the small plate is superimposed on this line perpendicular to the diameter; and the point that divides the common difference of the small plate into two equal parts is placed over the point [*T*] marked on the diameter of the plate. When this has been done, the small plate is attached to the large plate by means of a complete joining and consolidation; the opening in the small plate will then be opposite the opening in the rim of the instrument.[148] And there is an imaginary line, which joins the centers of the two openings, in the plane of the intermediate of the three circles that are on the inside of the rim of the instrument; and it will be parallel to the diameter of the plate. The

140. Points *I* and *G* in Fig. 10.

141. *IH* and *GK*. Perspective has been distorted in Fig. 10 in order to make all parts of the apparatus visible. The concentric circles passing through points *L*, *M*, and *N* represent circular grooves or scratches made in the rim mounted perpendicular to the disk, which is represented by the innermost circle. Thus lines *IH* and *GK* are actually perpendicular to the disk and parallel to one another.

142. I have translated the passage rather literally. Alhazen means that the line is divided into four equal segments (*GO*, *OP*, *PQ*, and *QK* in the figure). Apparently segment *QK* does not count—hence three parts are "divided out."

143. *LO*, *MP*, and *NQ*.

144. There are two such circular openings, represented in Fig. 10 by the small circles centered on points *M* and *P*.

145. Line *RS* in Fig. 11.

146. *X*, *Y*, and *Z*. When Alhazen talks about "dividing out" three equal segments of the perpendicular line, he means dividing the line into four equal parts; see n. 142.

147. That is, point *T* in Fig. 10.

148. These two apertures, of the same size and carefully aligned, serve as a sighting device.

small plate, which is applied to the point, is like the rim of an astrolabe. When this has been completed, from the four distinct quarters between the first two diameters that intersect perpendicularly, we cut away that quarter adjacent to the quarter containing the opening;[149] and the place of this portion [of the rim] is evened out until it becomes one with the surface of the plate.

We then take a copper rod, not less, but more, than one cubit long and of square section, which is enclosed by four equal surfaces two finger-breadths wide; and its surfaces are made as even as possible, until they become equal and have right angles [Fig. 12]. Then we pierce one of its surfaces

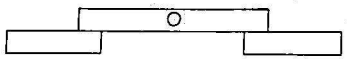


Fig. 12

with a round hole of such a size that it can receive the body on the back of the instrument and allow it to revolve, not easily, but with difficulty; and this hole should be perpendicular to the surface of the rod and should pass through to the other side. We place the instrument on the rod, and we put the body on the back of the instrument into the hole in the middle of the rod until the surface of the instrument rests on the surface of the rod. When this has been done, we cut from the ends of the rod those parts that extend beyond the diameter of the plate, for the rod is longer than the diameter of the plate, since we ordained it so. When we have cut the two excess lengths from the two ends of the rod, we replace them on the two ends of the rod in such a way that the two ends of the excess lengths are placed on the two ends of the remaining part of the rod.[150] And we place the surface of the excess lengths on the surface of the back of the instrument; and the amount of the two excess lengths that is placed upon the remaining part of the rod will equal one finger-breadth. This position having been noted, the two excess lengths will project over the two ends of the rod. Also, it will be better if that part of the body on the back of the instrument that sticks out [through the hole in the rod] is pierced and an iron peg inserted to keep the body from coming out. This having been done, the instrument is complete.

Then let the experimenter take a copper rule of small width (double the diameter of the opening in the rim of the instrument), of thickness equal to the diameter of this opening, and of length not less than half a cubit; this rule is perfected until it becomes very straight and true, and its surfaces become equal and parallel. We cut one of the sides of its width obliquely until the edge of its length and the edge of its width enclose an acute angle, so that one can thus easily turn and move it anywhere he wishes; and he makes its width at the other end perpendicular to the edge of its length. Then we divide this [latter] width into two equal parts, and from the place of division we draw a line on the surface of the face of the rule; this line is extended along the length of the rule and is perpendicular to its width. When this rule is placed on the surface of the plate, its upper surface will be in the plane of the intermediate of the three circles marked on the inside of the rim of the instrument; for the thickness of this rule is equal to the diameter of the opening, and the diameter of the opening is equal to the perpendicular extending from the center of the opening in the rim of the instrument to the surface of the plate, since the diameter of the opening is equal to two of the three small line segments marked off on the perpendicular line on the inside of the rim of the instrument. Thus, when this rule is mounted on the rim, and the surface of its width is on the surface of the plate, the line marked on its middle will be in the plane of the aforesaid middle circle, since the perpendicular that extends from any point on this line to the edge of the length of the rule is equal to the perpendicular extending from the center of the opening to the surface of the plate; for both of those perpendiculars are equal to the diameter of the opening.[151]

149. The purpose in removing this section of the rim was to provide the observer with unobstructed vision of the inside of the remainder of the rim: ". . . *inspiciat ad fundum aquae, ex quarta, cuius orae sunt abscisse* . . ." (*Perspectiva*, VII, sec. 3, p. 233).

150. See Fig. 12. The purpose of this rod is to support the disk in a vertical plane, thus allowing it to be inserted into a tank of water. The two "excess lengths," which extend beyond the circular disk, rest on the sides of the tank; since the hole in the rod is below these supports, it is possible to suspend the disk in such a way that its center is exactly at the surface of the water.

151. This copper rule is used to mark various positions of the light passing through the apertures in the course of actual experiments. For example, it can be used to determine the point on the surface of the water at which light passing through the upper pair of apertures is refracted or to position the disk with its center precisely at the surface of the water. Unfortunately, Alhazen's account of the use of the disk and associated equipment is far too verbose to be included here, but from his description of the apparatus, here presented, it is quite clear how one would proceed in general to measure angles of

27. Roger Bacon: The Geometry of Refraction

DE MULTIPLICATIONE SPECIERUM,[152] II, CHAPTER 3

It must be recognized, secondly, that multiplication [of species] is diversified along refracted lines; and refraction occurs in two ways. When the second medium is denser [than the first], refraction of the species at the surface of the second substance takes place between the direct path and the perpendicular extended into the second body from the point of refraction;[153] thus the species deviates from the direct path into the depth of the second body and divides the angle between the direct path and the perpendicular extended into the second body from the point of refraction. Nevertheless, it does not always divide that angle into two equal parts, though some think it does,[154] since greater deviation and less refraction from the direct path occur as a result of different densities of the second medium, according to the various differences in the angles of refraction determined by Ptolemy in Book V of his *De aspectibus* and Alhazen in his Book VII.[155] For the denser the second body, the more refraction deviates from the direct path, on account of the resistance of the denser medium, for density resists the ray, as Alhazen says.[156] But when the second substance is subtler [than the first], refraction of the species at the surface of the second body takes place beyond the direct path, by recession from the perpendicular drawn at the point of refraction; that is, the direct path falls between the refracted ray and the perpendicular. And the recession of the refracted ray and the size of the angle vary as the subtler [second] substance is more or less subtle.

Therefore, I will illustrate with diagrams the manner in which rays are understood to be refracted or unrefracted at plane and spherical surfaces. First, in plane surfaces, it is declared concerning a second substance of plane surface that the whole power [of refraction] is determined by its shape. Therefore, if the second substance should have a plane surface, a ray incident perpendicularly is not refracted, as has been said; but all nonperpendicular rays are refracted. And if the second body (which is plane) is denser, then, as has been said,

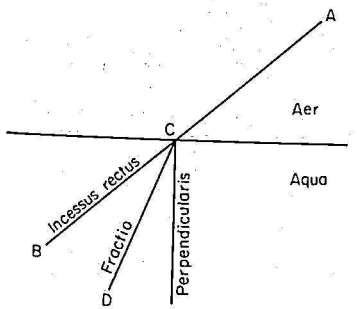


Fig. 13

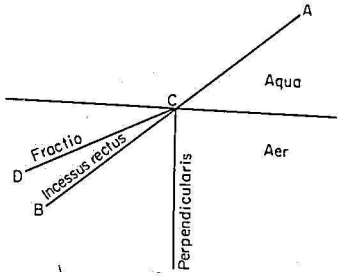


Fig. 14

incidence and refraction: If sunlight passes through the upper pair of apertures and is refracted at the surface of the water, a spot of light will appear on the submerged portion of the rim; the angles can then be determined from the markings on the middle of the three circles engraved on the inside of the rim. For additional detail, see A.C. Crombie's description of the use of Witelo's apparatus, which is identical to that of Alhazen, in *Robert Grosseteste and the Origins of Experimental Science, 1100–1700* (Oxford: Clarendon Press, 1953), pp. 220–223.

Alhazen adds a number of impressive refinements, not mentioned in the above account or by Crombie, which leave no doubt about his competence as an experimenter. Thus, for example, he suggests that a needle be placed over the middle of the aperture through which sunlight enters the apparatus; the resulting shadow in the middle of the spot of reflected light on the inside of the rim permits greater precision of measurement than the relatively large spot of light.

152. Bridges, II, 466–468.

153. That is, refraction is toward the perpendicular.

154. Chiefly Robert Grosseteste; cf. his treatise *On the Rainbow*, Selection 61.2.

155. Ptolemy's work on refraction has been analyzed by Albert Lejeune in *Recherches sur la catoptrique grecque* (Bruxelles: Palais des académies, 1957), pp. 153–166. On Alhazen, see *Perspectiva*, VII, secs. 10–12; Risner, pp. 243–247.

156. Alhazen, *Perspectiva*, VII, sec. 8; Risner, p. 240.

the ray is refracted between the direct path and the perpendicular extended into that second body from the point of refraction, in this manner [Fig. 13]. But if the second substance is subtler, direct ingress falls between the refracted ray and the perpendicular drawn from the point of refraction; for example: [see Fig. 14]. If, however, the second body is spherical and denser than the first, refraction occurs at its surface between the direct path and the perpendicular leading from its center to the point of refraction, in this manner [Fig. 15]. But if the second body should be subtler, the direct path falls between the perpendicular drawn from the point of refraction and the refracted ray itself, as here [Fig. 16]. And Ptolemy, in Book V of *De aspectibus*, and Alhazen, in his Book VII, agree regarding this double refraction; nor can it be otherwise. And they prove this by means of instruments, which they there show to be designed for this purpose, so that one can see with the senses the manner in which nonperpendicular rays are refracted. Likewise, they examine these refractions through causes and experiments.[157]

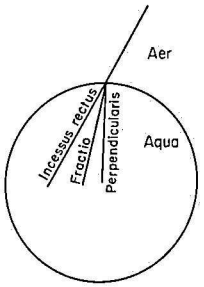


Fig. 15

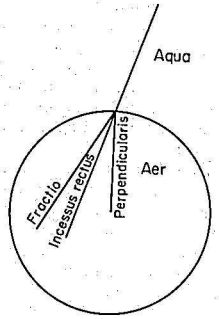


Fig. 16

28. Witelo: The Geometry of Refraction

PERSPECTIVA,[158] X, THEOREM 8

The angles of all refractions are revealed by means of tables.

Having observed by means of an instrument,[159] as closely as we were able, the angles of all refractions for all known transparent substances, by turns (as from air to water and glass, and from water to glass; and conversely, from water and glass to air, and from glass to water), we have discovered that the angles of refraction[160] are always the same [in passing] from any rare transparent substance to a denser transparent substance and [in passing] from that same dense substance [back] into the same rare substance. Accordingly, we have constructed these tables, of which this is the form [Table I]. Out in front, in the first column, we have placed the angles of incidence; then we have added the other angles according to the modes of their circles, which we have set forth at the heads of their columns. And so by means of these tables, experimentally formulated by means of the aforementioned instrument, the diligent investigator can learn all the angles of refraction for any and all media of diverse transparency.[161]

157. Note that Bacon makes no claim of having performed experiments himself, in contrast to Witelo in the following section.

158. Risner, pp. 412–413.

159. This is the instrument described by Alhazen, in section 26 above, and also by Witelo, II, Theorem 1 (Risner, pp. 61–63); X, Theorems 4–7 (Risner, pp. 407–412).

160. Witelo's "angle of refraction" is what is now usually referred to as the "angle of deviation," that is, the angle formed between the incident and refracted rays; Alhazen employs similar terminology. Hence this is a statement of the reciprocal law—the angle of deviation of a ray passing from substance *A* to substance *B* is equal to the angle of deviation of another ray traversing the same path from substance *B* to substance *A*.

161. Here and in the opening sentence of the theorem Witelo claims to have formulated these tables of refraction personally by experimental investigation. But there is ample evidence that this is untrue: In the first place, the values are not those given by experiment, but sets of numbers conforming to a regular progression—the

[Table I.]

Table of the quantity of the angles of incidence common to all the following	Refracted angles[a] from air to water		Angles of refraction of the same		Refracted angles from air to glass		Angles of refraction of the same		Refracted angles from water to glass		Angles of refraction of the same	
	deg.	min.	deg.	min.	deg.	min.	deg.	min.	deg.	min.	deg.	min.
10	7	55[b]	2	5	7	0	3	0	9	30	0	30
20	15	30	4	30	13	30	6	30	18	30	1	30
30	22	30	7	30	19	30	10	30	27	0	3	0
40	29	0	11	0	25	0	15	0	35	0	5	0
50	35	0	15	0	30	0	20	0	42	30	7	30
60	40	30	19	30	34	30	25	30	49	30	10	30
70	45	30	24	30	38	30	31	30	56	0	14	0
80	50	0	30	0	42	0	38	0	62	0	18	0

Table of the quantity of the angles of incidence common to all the following	Refracted angles from water to air		Angles of refraction of the same		Refracted angles from glass to air		Angles of refraction of the same		Refracted angles from glass to water		Angles of refraction of the same	
	deg.	min.	deg.	min.	deg.	min.	deg.	min.	deg.	min.	deg.	min.
10	12	5	2	5	13	0	3	0	10	30	0	30
20	24	30	4	30	26	30	6	30	21	30	1	30
30	37	30	7	30	40	30	10	30	33	0	3	0
40	51	0	11	0	55	0	15	0	45	0	5	0
50	65	0	15	0	70	0	20	0	57	30	7	30
60	79	30	19	30	85	30	25	30	70	30	10	30
70	94	30	24	30	101	30	31	30	84	0	14	0
80	110	0	30	0	118	0	38	0	98	0	18	0

[a]The expression "refracted angle" refers to the angle between the refracted ray and the perpendicular, that is, to what is now generally termed the "angle of refraction." For Witelo's "angle of refraction," see note 160.
[b]Printed editions of Witelo's *Perspectiva* give this angle as 7°45′. However, three early manuscripts that I have checked all give the value appearing here.

It is evident from these tables that the angles of incidence of the form of the same point [which successively assumes different positions] nearer the ray [that issues] from a point of the visible object incident perpendicularly on the surface of the transparent body, which [body] gives rise to refraction, are smaller; and those angles more remote from that ray are larger. For when a larger angle has been subtracted from the right angle of that [perpendicular] ray, the angle that remains is smaller than the remaining angle when a smaller angle is subtracted from the right angle. And in a transparent body denser than the first, the angle of refraction corresponding to the greater angle of incidence will be larger than the angle of refraction corresponding to the smaller angle of incidence.[162] Also, the excess of the greater angle of refraction over the smaller angle of refraction will be smaller than the excess of the greater angle of incidence over the smaller angle of incidence; and the ratio between the angle of refraction corresponding to the greater angle of incidence and that greater angle [of incidence] will be larger than the ratio between the angle of refraction corresponding to the smaller angle of incidence and that smaller angle [of incidence].

And the refracted angle by which the larger angle of incidence exceeds its angle of refraction is larger

differences between successive angles of refraction form an arithmetic progression with a common difference of one-half degree. Secondly, the upper half of the table, with the exception of the "angles of refraction" (which Witelo computed simply by subtracting the "refracted angles" from the angles of incidence) and the refracted angle of 7°55′, was taken from Ptolemy's *Optica*. Finally the lower half of the table was computed from the values in the upper half by erroneous application of the reciprocal law; consequently, it includes preposterous results, such as "refracted angles" (measured from the perpendicular) larger than 90° and no recognition at all of total internal reflection.

162. The reader is reminded that "angle of refraction" denotes the angle between the incident and refracted rays. The expression "angle of incidence," however, denotes the angle between the incident ray and the perpendicular.

than the refracted angle by which the smaller angle of incidence exceeds its angle of refraction. Thus the refracted angle, in a second transparent substance denser than the first, will always be smaller than the angle of incidence; and the ratio of those refracted angles to equal angles of incidence varies with the different densities of the [second] medium. For when refraction takes place in water and in glass, [the forms having come] through the same air and at equal angles of incidence, the refracted angles are more acute in the glass than in the water; thus the angles vary with the different transparencies. But if the second transparent medium is rarer [than the first], then the refracted angle is always larger than the angle of incidence; and the ratio of these angles to the other angles, since they are formed in the opposite manner to the aforesaid angles, will be as though the tables set forth above were arranged in the reverse manner. And the ratio of these refracted angles and angles of refraction to the same angle of incidence are varied according to greater and less rarity of the transparency of the second medium. For when refraction takes place from glass to water or to air, the angles formed in the air are larger than the angles formed in the water; accordingly, the ratio of the angles of refraction to the angles of incidence is varied.

These are the things that occur to lights and colors and universally to all forms in their diffusion through transparent bodies and in the refraction that occurs in all of them, both according to themselves and with respect to sight. And so what has been inquired into is evident.

29. John Pecham: Image Formation by Refraction

PERSPECTIVA COMMUNIS, III, PROPOSITION 4

Proposition 4. The image is located at the intersection of the pyramid under which the object is seen and the perpendiculars that one can imagine dropped from the visible object to the surface of the adjacent transparent medium.[163]

As was shown above,[164] everything that is viewed appears [as though] in a straight line; and by the apprehension of the ray through which the eye perceives the object, the object is judged to be at the end of the straight-line extension of the ray. Therefore, just as the appearance of the object at the intersection of the perpendicular and [the rectilinear extension of] the ray is considered basic to mirrors, so in the case at hand it [that is, the image] is at the intersection of the ray and the perpendicular dropped from the visible object. For example, let *A* be the eye, *B* the visible object, *BC* the bent ray (refracted at *C*, from which [point] proceeds [ray] *CA*) that presents the object to the eye, and *BLD* the perpendicular [Fig. 17]. I say that point *B* appears at *L*.

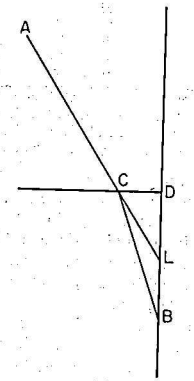


Fig. 17

30. Roger Bacon: Image Formation by Refraction

OPUS MAIUS,[165] V.3, DISTINCTION 2 CHAPTERS 2–3

Chapter 2. Concerning different places of appearance of the image by refraction at plane surfaces.

It should be understood that vision by refraction is at the intersection of the visual ray and the

163. As in the case of reflection, the image of any given point is located at the intersection of the *ray* under which the point is seen and the *perpendicular* dropped from the point to the refracting surface. However, an object is composed of many points, and the collection of the rays under which those points are seen constitutes a pyramid. Consequently, in the enunciation of this proposition, Pecham speaks of the intersection of the *pyramid* and the *perpendiculars*.

164. See *Perspectiva communis*, II, proposition 20, in section 19 above.

165. Bridges, II, 148–153.

cathetus, as has been stated for reflection. But this can occur in various and marvelous ways. However, in order that we may understand every diversity of such appearances, it is necessary to consider how such diversity occurs in plane, concave, and convex substances, when the eye is in the subtler or denser medium and the visible object conversely. Now, if the eye is in the subtler transparent medium, and between the eye and the visible object there is a denser medium, such as water or crystal or glass of plane surface or some other such transparent medium, then the object appears much larger than it is; for it is seen under a larger angle and is somewhat nearer than if there were no change in medium. The demonstration of this is evident in this figure [Fig. 18]. For the visible

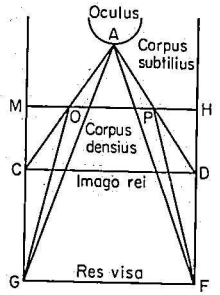


Fig. 18

point *F* will appear at *D*, where visual ray *AD* intersects the cathetus *FH;* similarly *G* will appear at *C*, where visual ray *AC* intersects cathetus *GM*. Therefore, the whole object *GF* will appear in position *CD* nearer to the eye and will be seen under a larger angle than if the substance [of the medium] were one; for it is seen under angle *OAP* through these two substances, but through one medium without refraction it would be seen under angle *GAF*.

But if the eye is in the denser medium and the visible object in the subtler medium, the contrary occurs; for the object will appear smaller both because it is seen under a smaller angle and because it appears more remote [Fig. 19]. Thus *O* is seen at *H*, and *F* at *K*, beyond the visible object, so that *OF* appears at *KH*, since visual ray *AB* intersects cathetus *HC* at *H*, and visual ray *AD* intersects cathetus *PFK* at *K*. And it is seen under a smaller angle than if it were seen through a

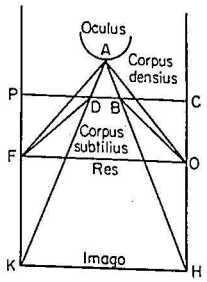


Fig. 19

single medium, since now the whole object is seen under angle *DAB* because of the refraction, whereas without refraction it is seen under the larger angle *FAO*.

Chapter 3. Concerning the different locations of the image in spherical media.

If the substances through which the eye sees are not plane, but spherical, there is great diversity. Now, either the concavity or the convexity of the substance is toward the eye. If the concavity, then there are four cases, two if the eye is in the subtler medium and two if the eye is in the denser medium. If the eye is in the subtler medium and the concavity of the medium is toward the eye, the eye can be between the center [of the concave surface] of the medium and the visible object, or the center can be between the eye and the visible object. And here there should be no notion of the center of the denser medium or of the center of the subtler medium, since the same thing is the center of both, and the concavity of each is toward the eye; for the same thing is the center of the containing sphere and the contained sphere. First, then, I shall state all the cases and then illustrate them with figures; it is necessary to proceed thus because of the brevity of the rules covering individual cases and the great number of figures. And all these things are revealed in the following figures, which are presented in the order of the eight aforementioned cases.[166]

166. I have had to correct all of Bridges' figures except the eighth; in each case, my correction has embodied principles clearly recognized by Bacon, and for most of the corrections it is possible to adduce either textual or manuscript support. For example, Bridges mistakenly interchanged point *C* and the eye in the first figure; my rearrangement is confirmed by the reference to angle

If the eye is in the subtler medium, and the concavity is toward the eye, and the eye is between the center and the visible object [Fig. 20], the

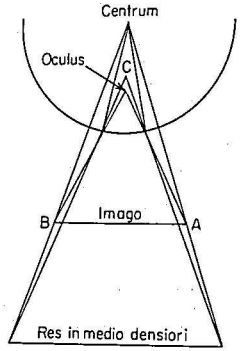


Fig. 20

object appears nearer than it [actually] is and [is seen] under a larger angle, for thus[167] the visual angle is larger than if straight lines are drawn from the eye to the ends of the object without refraction; and yet the image is smaller [in absolute terms] than the object itself. If the eye is in the subtler medium, and the concavity is toward the eye, and the center [of the concavity] of the denser substance is between the eye and the object [Fig. 21], the object still appears nearer; but the angle is smaller,

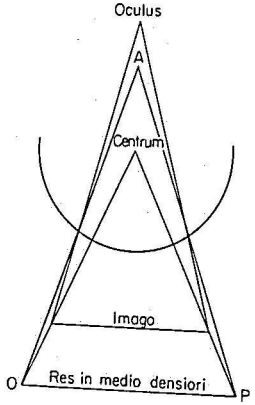


Fig. 21

and the image is smaller. But if the eye is in the denser medium, and the concavity is toward the eye, and the eye is between the center of the concave substance and the object [Fig. 22], then the object

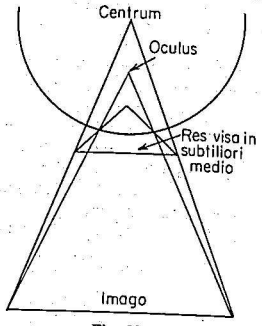


Fig. 22

appears beyond its [true] place, more remote [from the eye], and under a smaller angle; and the image will be larger [in absolute terms]. If the center of the concave substance is between the eye and the visible object, other conditions remaining the same [Fig. 23], the visible object still appears more

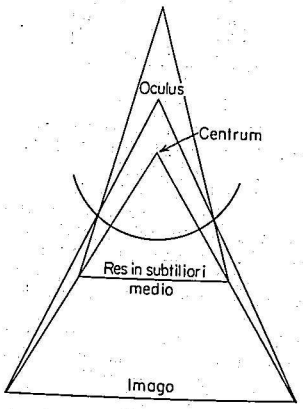


Fig. 23

BCA of this figure near the end of the chapter. Compare the figures in A. C. Crombie, *Augustine to Galileo*, 2nd ed., rev. (Cambridge, Mass.: Harvard University Press, 1961; reprinted, 1963, 1967, as *Medieval and Early Modern Science)*, I, 106–109.

167. That is, when the object appears nearer.

remote and under a larger angle; and the image is larger.

However, if the convexity of the substance is toward the eye, there are likewise four cases, two if the eye is in the subtler medium and two if the eye is in the denser medium. Now, if the eye is in the subtler medium, and the convexity of the medium in which the object is situated is toward the eye, then the visible object can be between the center and the eye, or the center can be between the eye and the visible object. If the object is between the eye and the center [Fig. 24], the image

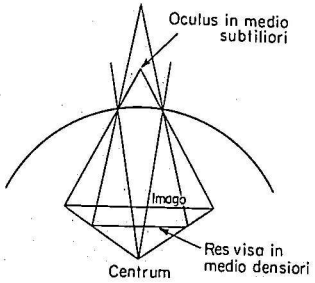


Fig. 24

is nearer and larger, and the angle is larger. If the center is between the eye and the visible object [Fig. 25], the image is still larger and the angle is

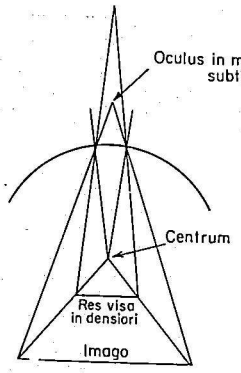


Fig. 25

larger; but the location of the image is more remote. However, if the eye is in the denser medium, and the visible object is between the eye and the center [Fig. 26], the image is more remote and

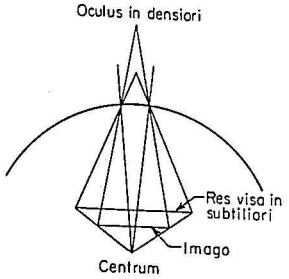


Fig. 26

smaller, and it is seen under a smaller angle. If the eye is in the denser medium, and the center is between the eye and the object [Fig. 27], the image

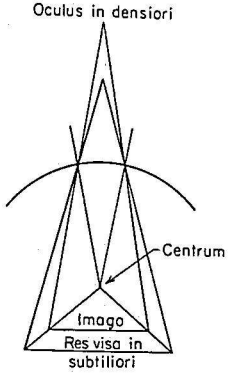


Fig. 27

is nearer and smaller and is seen under a smaller angle; and the size of the angle under which the object is seen is recognized to be smaller than it would be if the medium were one [substance]. And this is also the case whenever it [that is, the angle under which the object is seen] encloses the angle formed by the lines of direct progress, and they are terminated inside[168] it at another point; but [the

168. The Bridges text reads "outside," but the passage makes no sense unless emended.

angle under which the object is seen] is called greater, as is evident in the first figure [Fig. 20], when it converges at another point below [the angle formed by the lines of direct progress]; but then the angle formed by the lines of direct progress is smaller than the angle under which the object is seen, and therefore the angle under which the object is seen is larger than if there were one medium.[169] For then the object is seen under angle *BCA* enclosed under the direct lines. And in the following figure [Fig. 21] the object would be seen under angle *OAP*, enclosed by the direct lines, if the medium were one [substance], and thus under a larger angle than the angle formed by the refracted lines under which the object is seen through two media. And it is to be understood in this way in all the other figures that follow.

31. John Pecham: The Burning Glass

PERSPECTIVA COMMUNIS, III, PROPOSITION 16

Proposition 16. Fire can be kindled by the convergence of refracted rays.

It is evident from Proposition 17 of Part II, and the next to last proposition of the same part, that [fire can be kindled] by reflected rays in mirrors. The same thing happens when round transparent bodies are exposed to solar rays; but mirrors and transparent bodies are different in that fire is kindled between a mirror and the sun, whereas with transparent bodies, on the contrary, it is the transparent body that is between. For example, let there be a round crystal of diameter *YAZ*, and let rays *XC*, *XS*, *XR*, *XQ*, and *XP* fall on it from the sun [Fig. 28]. It is certain that only *XR* falls to the center *A* and proceeds unrefracted to *H*. Therefore, the others are refracted toward the perpendicular and fall from *C* to *B*, from *S* to *G*, from *Q*

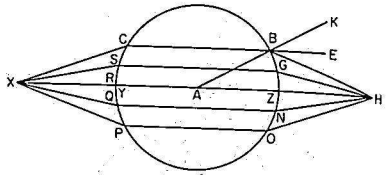


Fig. 28

to *N*, and from *P* to *O*. Accordingly, ray *CB* arriving at the concave surface of the air does not proceed directly to *E* but is refracted away from perpendicular *BK* to *H*, and similarly for the other rays. When these rays have been assembled and the air rarefied, fire is kindled [just] beyond the terminus of their species.[170]

32. Roger Bacon: Causal Analysis of Refraction

DE MULTIPLICATIONE SPECIERUM,[171] II, CHAPTER 3

However, they [that is, Ptolemy and Alhazen] assign the cause of refraction as follows. Since the descent of a perpendicular species is strong (as is evident in a falling stone, provided its descent is not diverted from the vertical), if something should impede perpendicular descent and make the stone deviate from a perpendicular course, it is manifest to sense that its ability to penetrate is weakened. Wherefore a man falling perpendicularly from a high place is killed by the fall. But if something should divert him from direct approach as he descends, he is spared insofar as he diverges from perpendicular approach. Similarly, if a sword or axe or some other instrument designed to cut is applied to a rod perpendicularly by the hand of the one wielding the instrument, it penetrates and divides the rod; [however,] if the instrument is applied obliquely, either it does not cut at all or it does not cut as much. But if the substance that receives an object falling perpendicularly should

169. Bacon is referring to the angles situated above the interface in each of the figures. In Fig. 20, for example, the lines of direct progress terminate at *C*, which is outside the angle under which the object is seen (terminating at the eye); consequently, the angle under which the subject is seen is larger than the angle formed by the lines of direct progress.

170. The closest approach by a medieval author to a geometrical analysis of lenses was discussion of the burning sphere (or cylinder). It is true that Alhazen and Witelo described a plano-convex (that is, semicylindrical) piece of glass or crystal, to be used in connection with their refraction apparatus, but this was so arranged that refraction occurred only at its plane surface. It was not employed as a lens, and the paths of rays through it were not traced.

171. Bridges, II, 468–470. For an analysis of medieval discussions of the cause of refraction, including Alhazen's, see my article, "The Cause of Refraction in Medieval Optics," *The British Journal for the History of Science*, Vol. IV, Part I, No. 13 (June 1968), 23–38, from which this selection (on pp. 36–38) is reprinted with the kind permission of the editors.

resist it altogether, the falling object will return along the same path by which it approached because of the strength of its approach, as a ball thrown against a wall or something that altogether resists [penetration]. And a ball falling obliquely deviates toward the other side of the perpendicular in accordance with [the direction of] its oblique fall, as is evident to sense; and it does not return along the path by which it approached [the wall] because of the weakness of its fall. Experiences of such things are without number.

But a transparent substance does not altogether resist [penetration by] species, causing them to return, for only a dense substance, which produces reflection, does that. Wherefore species incident on any transparent substance penetrate it, whether they are incident obliquely or perpendicularly. But since a perpendicular species is stronger and an oblique species weaker,[172] both cannot proceed in the same manner in a transparent substance. But there is no path except a rectilinear and a bent course. Now, since the rectilinear course is the stronger, it is suited to perpendicular species, allowing them to proceed along the same straight line in the second substance as in the first; and a bent path is suited to oblique species. Therefore, a perpendicular species does not bend and is not refracted, since bending is refraction; but an oblique species is refracted at the surface of the second substance, since it is more strongly resisted by the transparent substance than is a perpendicular species because of the weakness of the oblique species and the strength of the perpendicular species; for every transparent substance possesses some coarseness, by which it is able to resist somewhat and to impede the passage of species. As Alhazen says in the seventh [book of his *Perspectiva*],[173] transparency in natural things is limited so that it does not extend infinitely, and there is some coarseness even in celestial substance. Consequently, both perpendicular and oblique species are somewhat impeded, but oblique species more so. For, on account of its strength, the perpendicular is not diverted from a rectilinear path, though any coarseness whatever of the medium introduces a greater succession into its [that is, the perpendicular species'] traversal [of the medium]. Indeed, if there were a void medium without any natural coarseness, and if light or any body whatsoever could pass through it, there would be some succession caused by the medium simply as a result of the prior and posterior [character] of space itself,[174] quite apart from any resisting coarseness of the medium. But in a natural plenum, which has some natural coarseness, it is necessary that the coarseness (which neither altogether yields nor altogether impedes, but partly one and partly the other) should produce a new succession. Therefore, by its coarseness a transparent substance impedes both oblique and perpendicular species and introduces succession into both; and oblique and perpendicular species share this in common. But beyond this, a transparent substance impedes an oblique species more [than a perpendicular species] on account of the weakness [of the former] and does not allow it to continue in a straight line along the same path as it had formerly in the higher substance.[175]

Now, the reason why a species passing from a subtler into a coarser substance is refracted toward the perpendicular leading from the place of refraction (that is, between the rectilinear path and the perpendicular) is that just as the perpendicular course is strongest, so every course close to the perpendicular is stronger than any course more remote from the perpendicular, as reason declares and authors assert. Therefore, since the species [in the case under consideration] was moved much more swiftly in the more subtle substance than it could be in the second denser substance because of the great resistance offered by the coarseness of such a substance, the natural power generating the species desires the easier passage and chooses it;[176]

172. Supposedly, this has been demonstrated by the mechanical examples with which the discussion began: a sword cuts more deeply if applied to a rod perpendicularly, and a man's chances of survival increase as he is diverted from perpendicular descent. However, Bacon's claim is that perpendicular *traversal of a medium* is stronger than oblique traversal of the medium, whereas the examples demonstrate merely that perpendicular *incidence on a resisting surface* is stronger than oblique incidence on that surface.

173. VII, sec. 16; Risner, p. 252.

174. [The same justification for successive motion in a void was given by Thomas Aquinas; see Selection 55.1—*Ed.*]

175. Here Bacon argues that species incident obliquely on a transparent interface are refracted, while species incident perpendicularly are not, because oblique species are weaker and more easily deviated. The implication is that the medium resists equally in all directions and that variation is introduced only by the varying strength of the species. However, in the following paragraph, Bacon clearly asserts that the denser medium resists less in a direction perpendicular to its surface than in directions oblique with respect to its surface; a given species chooses a path closer to the perpendicular, because it finds easier passage in that direction.

176. Note Bacon's animistic terminology. Pecham,

and this is toward the perpendicular. Wherefore it must be refracted between the rectilinear path and the perpendicular leading from the place of refraction.

And since contraries are causes of contraries and the effects of contrary causes are contrary, it is necessary that when a species passes from a denser substance into a subtler substance, it bends away from the perpendicular, so that the rectilinear path falls between the line of refraction and the perpendicular drawn from the place of refraction. For since there is great resistance in the former dense substance (because its coarseness surpasses the coarseness of the second substance, which is very subtle so that species entering it act with ease), it is unnecessary for the species to seek an easier course (that is, toward the perpendicular), since every path is easy for it by comparison with the difficulty it encountered in the first substance. Therefore, the species is able to assume a direction away from the perpendicular, and it must necessarily choose this direction, since insofar as possible it acts uniformly and in one mode.[177] And therefore it passes reluctantly toward that which is violently contrary to its disposition. Accordingly, since in the first substance the species had great difficulty or [only] moderate ease of passage, on account of its uniformity (which it always maintains insofar as possible), it ought not choose the superior contrariety, according to which it would deviate toward the direction of easiest passage (that is, toward the perpendicular); but to a certain degree, as much as it can and ought, it will maintain [its] difficulty or moderate ease of passage; and this occurs as the species deviates from the rectilinear path in the direction away from the perpendicular, since such a course would suffice for it and be able to fulfill it. For example, when a species passes from a subtler into a coarser substance, it maintains its ease of traversal in the second substance, so that its passage in each substance is, insofar as possible, proportional and uniform; however, superfluous coarseness in the second substance [as when a species passes from a rare to a dense medium] excites the generative power of a species, so that it bends toward the direction of easier traversal.

33. Witelo: Causal Analysis of Refraction

PERSPECTIVA,[178] II, THEOREM 47

Although a perpendicular ray penetrates any transparent substance [without refraction], a ray obliquely incident on a second transparent medium of greater density [than the first medium] is refracted toward the perpendicular to the surface of the second transparent medium (drawn from the point of incidence); and [when incident] on a second transparent medium of greater rarity [than the first medium, the obliquely incident ray] is refracted away from the same perpendicular.

We [now] intend to support by a natural demonstration this [proposition], which has been proved previously by particular experiments employing an instrument. For all natural motions occurring along perpendicular lines are stronger [than others], since they are aided by the universal celestial power, which influences every body situated below according to the shortest straight line.[179] Moreover, the impulses of objects projected perpendicularly are stronger than of those that are projected obliquely. Similarly, perpendicular blows are stronger than all oblique blows, and among oblique blows, those which more closely approach the perpendicular are stronger.

And so, since the density of every body impedes the passage of light, it must be conceived that light is thrust back from passage by the resistance of a dense body, and even more by the resistance of a denser body. And we understand, by this resistance of a passive quality (that is, density) to an active quality (that is, light), a certain mode of the motion of light through the midst of resisting bodies, which are more or less susceptible to the impression of light; this is not to maintain that there would be some motion in the local transmutation of light itself (as is evident from Theorem 2 of this book), but that in the same instant light restrains or diffuses itself more, according to the diversity of the media.[180] And here we call this the motion of light itself. Thus every light passing through a transparent body traverses it with an exceedingly swift and insensible motion. And yet the motion occurs more swiftly through more transparent bodies than through less transparent bodies;[181] for every transparent body resists penetration by light more or less according as it participates more or less in transparency, since the coarseness of bodies is always resistant to penetration by light.

Therefore, when light passes obliquely through a transparent body and encounters a denser transparent body, the latter impedes the light more vigorously than the former rarer body impeded it; consequently, the resistance of the denser body must alter the motion of the light. And if the resistance should be strong, that motion would be refracted in the opposite direction;[182] and, in fact, since light does not strongly resist [deviation], it would not [be permitted to] proceed in its original direction. But if the resistance should be weak, in accordance with the greater rarity of a more transparent body,[183] the incident light would not be refracted in the opposite direction, nor would it be able to proceed in its original direction; rather, it would be altered in direction. (Nevertheless, it is not altered when incident perpendicularly on transparent bodies possessing any variety of transparencies, but penetrates all the bodies in a straight line, because the perpendicular [ray] is stronger than all others, and oblique [rays] nearer the perpendicular are stronger than those more remote from the perpendicular.) Therefore, when light is incident obliquely on a denser transparent body, it proceeds [after refraction] along a straight line approaching the perpendicular issuing from the point at which the light encounters the surface of the dense transparent body and extended above the surface of the denser body, since motion along the perpendicular line is easiest. Thus, if a ray of light is incident along a perpendicular line, it will continue in a straight line because of the strength of motion along the perpendicular. And if the ray is incident obliquely, it will not be able to continue directly because of the weakness of motion along oblique lines; consequently, it deviates toward some direction in which passage is easier than in the original direction.[184] But the easier motion and the one more aided by celestial influence is along the perpendicular, and that which is closer to the perpendicular has easier passage than that which is more remote from the perpendicular.

Accordingly, let many rays from point A of a luminous body be incident through medium AB, on the surface of another transparent body containing line $BCDE$; and let BF be the depth of the other transparent body and line AB the perpendicular to its surface [Fig. 29]. It is evident from the foregoing

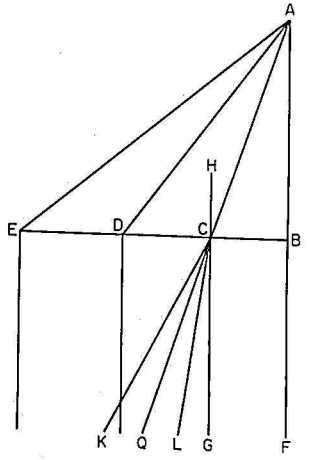


Fig. 29

explanations regarding the strength of perpendicular [rays] and from the experiments with instru-

who in other respects presented a similar explanation, was to rebuke him for this; see *Perspectiva communis*, I, Proposition 15.

177. Here is the fundamental principle employed by Bacon in his explanation of refraction. Species choose a direction in the second medium that will most nearly maintain uniformity of strength or action. Consequently, species passing from a less dense to a more dense medium deviate toward the perpendicular, where the greater ease of traversal (by comparison with traversal of more oblique paths) compensates for the greater resistance of the denser medium; species entering a less dense medium choose a more resistant path for the same reason.

178. Risner, pp. 81–83.

179. Witelo introduces a novel element into discussions of refraction: motion along the vertical (which Witelo takes also to be the perpendicular to the refracting surface) is stronger because a celestial power aids it.

180. In this theorem, translated in section 5, Witelo argues that light is propagated instantaneously; consequently, one cannot speak of its local motion or transmutation.

181. This appears to contradict previous arguments on the instantaneous propagation of light. While it might be maintained that Witelo's earlier arguments *succeed* in demonstrating merely that the time required for the propagation of light is insensibly small, it appears that his *intention* was to demonstrate that the propagation of light over a finite space does not require any time at all.

182. That is, the motion would be turned back on itself, as in reflection. However, here Witelo prefers to continue to speak of the process as refraction. This is legitimate, since (as the present argument suggests) reflection is simply a more intense degree of refraction.

183. Rarity and transparency were commonly equated in medieval optics.

184. It is apparent that Witelo's explanation of the direction of refraction is essentially the same as Bacon's.

ments described in Theorems 42 and 44 of this book that a perpendicular ray incident along line *AB* penetrates the entire body *BEF*. However, if the ray incident along line *AC* were to traverse body *BEF* without deviation, [this would imply that] there is no difference in transparency between bodies *ABE* and *BEF*, contrary to hypothesis; accordingly, because of the difference in resistance [of the two substances], line *AC* will not be extended directly. But if a ray has been moved freely along line *AC* through a body offering little resistance, it cannot be moved along the same line in a body resisting more or less. Therefore, if body *BEF* is denser than body *ABE*, it is evident from the foregoing that passage through the former is more difficult. If, therefore, line *AC* is refracted away from the perpendicular drawn to surface *BCDE* of the body from point *C*—let this perpendicular be *CG*—it will be weakened and will fail to achieve its full effect; therefore, it has been thwarted. But it was accepted at the outset that nature is not thwarted;[185] therefore, as was demonstrated experimentally in Theorem 43 of this book, line *AC* must be refracted toward perpendicular *CG*, so that its action may be strengthened. The same is true of rays incident along lines *AD* and *AE*.

But if the body containing line *BCDE* in its surface should be of rarer transparency (like body *ABE* [in the foregoing case]), perpendicular ray *AB* would still penetrate without refraction, because of the strength of action [along the perpendicular]; but the ray passing through the denser body along line *AC* and incident on the surface of the rarer body at point *C* would not encounter resistance as before. And since forms have the property of always diffusing themselves according to the amplitude of every material suited to [receiving] them, it is evident that ray *AC* does not continue along line *AC*, for this would be the case only if the transparent bodies were equivalent in their resistance to the reception of light, contrary to hypothesis. Therefore, ray *AC* is refracted, but not toward perpendicular *CG*, since refraction in such a direction would not result from the resistance of the material, but from victory of the acting form over matter more suited [to receiving the form] than the first [denser matter]. Therefore, by its own power, the form diffuses itself away from its initial advance along line *AC* and in the direction away from perpendicular *CG* and its parallel *BF*. The same is true of all other oblique rays, such as *AD* and *AE*.

And so the motion of rays incident obliquely on a body of denser transparency, such as *BEF*, is composed of a motion in the direction of perpendicular *AB* (crossing through body *BEF* in which the motion takes place) and of a motion along line *CB*, which is perpendicular to line *CG*. Now, since perpendicular passage is the strongest and easiest of motions and the density of the body opposes [the reaching of] the goal toward which the motion was directed, line *AC* is necessarily moved toward perpendicular *CG* issuing from point *C*, where ray *AC* encounters the surface of the denser body. And since it is prohibited from that [initial] motion because of the grossness of the medium and the nature of the other motion (that is, along line *CB*), which is not altogether destroyed but only impeded by the resistance of the medium,[186] the light will deviate toward point *B*, always approaching perpendicular *ABF*. Thus, in a second medium of grosser transparency than the first, refraction of ray *AC* occurs along line *CL*, which is closer to perpendicular *CG* issuing from point *C* (where ray *AC* encounters the denser body) than is line *AC* (along which the ray was incident on the surface of the denser body) extended beyond point *C* to point *Q*, to the same perpendicular extended beyond point *C* to point *H*. Consequently, angle *ACH* is greater than angle *LCG*. Nevertheless, [refracted ray *CL*] does not intersect perpendicular *BF* in the direction of point *F*, but in the direction of point *A*, by Book I, Theorem 2, of this treatise, since *CL* intersects line *CG* (which is parallel to *BF*) at point *C*.

But if ray *AC* issues from the grosser body to the more subtle, then since the latter has less resistance, motion of the ray will be swifter and more diffusive in it than in the former. And since the resistance of a denser medium always occasions an oblique light, so that it unites with the perpendicular line drawn to the surface of that body at the point of incidence (that is, line *CG*), and in a medium of rarer transparency the resistance is less than [the resistance of] the first [denser medium], it is evident that the light is moved toward the region from which the greater motion was prohibited by the resistance.

185. This is much like Bacon's principle of uniform action.

186. Here Witelo attempts an explanation of refraction in terms of the perpendicular components of the incident and refracted rays. Apparently, the ray assumes a direction closer to the perpendicular in the denser medium because (or partly because) its component parallel to the interface is weakened (or slowed?). It is not clear, however, what is happening to the perpendicular component at the same time. Nevertheless, the resemblance to Descartes' later analysis is noteworthy and suggests some reservations regarding Descartes' originality.

Thus, in a rarer transparent body, light is moved away from the perpendicular, so that angle *GCK* is larger than angle *ACH*. However, when light *AC* is refracted by a second body of rarer transparency than that of the first body, motion of the light always takes place between lines *CG* and *CE*, since when angle *GCE* is right, angle *GCK* cannot be right. Therefore, the proposition is evident.

63 LATE MEDIEVAL OPTICS

Introduction by David C. Lindberg

Fourteenth- and fifteenth-century optics developed within the framework (broadly defined) established in the thirteenth century. Indeed, the most notable optical treatises of the fourteenth century were several long technical commentaries on the *Perspectiva communis* of John Pecham. The extent to which this late medieval tradition continued to debate questions raised in the thirteenth century is illustrated by the first section, which reproduces the enunciations of the fifteen questions discussed by Henry of Hesse in his *Questions* on Pecham's *Perspectiva communis*.[1]

Few problems were debated more urgently in the thirteenth century than the cause of the rainbow, treated unsuccessfully by every major thirteenth-century author. Perhaps the most dramatic development of fourteenth-and fifteenth-century optics was the first correct geometrical analysis of this problem, by Theodoric of Freiberg early in the fourteenth century. The second section, from Theodoric's *De iride*, embodies all the essential geometrical elements of his theory.

1. Henry of Hesse (1325?–1397): Questions Concerning Perspective

Translated and Annotated by David C. Lindberg[2]

1. Whether light is multiplied by means of rays.
2. Whether a luminous body terminates a pyramid of its light in every point of the medium.
3. Whether light incident through a triangular aperture is reduced to roundness by nature.
4. Whether a luminous body appears larger from afar than from nearby.
5. Whether every ray incident or issuing forth under an obtuse angle is oblique.[3]
6. Whether a perpendicular ray is strongest.
7. Whether an opaque body smaller than the luminous body always casts a pyramidal shadow.
8. Whether the moon increases in light [in passing] from conjunction to opposition.
9. Whether everything seen is viewed by means of straight lines.
10. Whether every visual perception takes place under an angle.
11. Whether all visible intentions are apprehended or revealed through the species of colors.
12. Whether reflection takes place from the surface of every denser body.
13. Whether everything seen by reflection appears at the intersection of the reflected ray and the cathetus.
14. Whether rays are refracted in a difform medium.
15. Whether the rainbow appears by means of circular reflection from a cloud.

2. Theodoric of Freiberg (b. ca. 1250; d. shortly after 1310): On the Rainbow

Translated and annotated by William A. Wallace, O.P.[4]

PART II.

Chapter 25

In general, concerning the matter or subject [body]

1. All but one or two of these questions had been explicitly raised by Pecham or his contemporaries, Bacon and Witelo. It has been impossible, in this *Source Book*, to include more than the enunciations of Henry's *Questions*.

2. Translated from an unpublished edition by H. L. L. Busard.

3. The meaning of this question is clarified by the contrasting case of a perpendicular ray, which is incident or issues forth under two right angles.

4. From *Theodoricus Teutonicus de Vriberg: De Iride et Radialibus Impressionibus* (Dietrich von Freiberg: Über den Regenbogen und die durch strahlen erzeugten Eindrücke), edited by Joseph Würschmidt, *Beiträge zur Geschichte der Philosophie des Mittelalters*, Vol. XII, pts. 5–6 (Münster/Westfalen: Aschendorffsche Velagsbuchhandlung, 1914), pp. 113–114, 119–120, 133–138, 151, 153–154, 156–159. I have corrected Würschmidt's reading, as noted below, by reference to the Leipzig manuscript, Cod. Lips. 512, fols. 47 recto to 72 verso.

in which the rainbow is generated, its efficient cause, and its mode of radiation.[5]

The [radiant] impression that we call the rainbow extends above the horizon to the upper part of the airy [that is, atmospheric] region in the form of an arc, with the arch on top and with both extremities touching the plane of the horizon when it is fully formed.[6]

The matter or subject body in which this impression comes to exist is twofold. One is a dewy cloud, that is, a cloud resolved into drops or watery droplets of dew shaped into spherical form, located in or around the place of their generation, and detained and suspended there partly by the power of the generator and alterator, which has not yet been separated from it, and partly from the nature of a certain levity, from the fact that its substance shares markedly in the concrete nature of an airy property, as has been noted in Part I.[7] The other subject body or matter of the aforementioned impression or rainbow is a collection of falling drops released from a cloud as rain. Although these drops, which are like watery spherulets, continually move downward in their fall, the fact that some drops succeed others in the same places causes this impression to appear to sight as being in the same place.[8]

These [notes], therefore, have been made with respect to the matter or subject body in which the aforementioned radial impression that we call the rainbow comes to exist. Its efficient cause is radiation from a star that is shining brightly, such as the sun and the area immediately surrounding it—for it is not the radiation of a single form that causes the described four colors, as has been noted above.[9] This radiation is effected after the third mode of radiating among those that have been enumerated in the preceding Part I,[10] a radiation, I say, incident on some one of the described bodies at a place on the spherical particles fixed by the configuration between this place, the star that is radiating, and the sight to which such radiation is reflected. This incidence of radiation and its reflection to sight occur only in an area that is fixed geometrically by nature.[11]

Chapter 29

Concerning the incidence and reflection of solar radiation whereby the colors of the [lower] rainbow are generated.

Another mode of solar radiation takes place in some of the described spherical droplets, which the foregoing altitude circle *ADB* intersects at right angles [Fig. 1].[12] This mode, I say, is different from

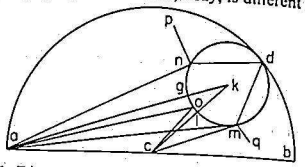


Fig. 1. Diagram from Theodoric of Freiberg's *On the Rainbow*, showing the path of solar radiation through an individual drop when generating the colors of the lower rainbow. The sun is at *a*, the observer at *c*, and the raindrop at *d*.

5. The title of this introductory question reveals Theodoric's basic Aristotelian methodology, wherein he is seeking the various causes (material, efficient, and so on) of the rainbow. For a full discussion of this method, see W. A. Wallace, O. P., *The Scientific Methodology of Theodoric of Freiberg*. Studia Friburgensia, New Ser., No. 26 (Fribourg: The University Press, 1959).

6. In Part I, chapter 6, of this treatise Theodoric classified the rainbow as one of fifteen "radiant impressions" produced in the upper atmosphere by light from the sun or other heavenly body. Part II of the work is concerned with the lower rainbow; Part III, with the upper rainbow; and Part IV, with the remaining radiant impressions.

7. Theodoric's views of gravity and levity, while Aristotelian, differed in several respects from those of his contemporaries. For details, see W. A. Wallace, O.P., "Gravitational Motion according to Theodoric of Freiberg," *The Thomist* XXIV (1961), 327–352; reprinted in *The Dignity of Science*, edited by J. A. Weisheipl, O.P., (Washington, D.C.: The Thomist Press, 1961), pp. 191–216.

8. Albert the Great (d. 1280), like Theodoric a German Dominican and a Prior Provincial of Teutonia, is usually credited with being the first to have recognized the role of the individual drops in the production of the rainbow. See Carl B. Boyer, *The Rainbow: From Myth to Mathematics* (New York: Thomas Yoseloff, 1959), pp. 94–99, especially p. 96.

9. Theodoric had worked out his own peculiar theory of color formation, whereby two forms (more and less bright) are required to generate the four colors. For details, see *The Scientific Methodology of Theodoric of Freiberg*, pp. 188–205, 152–173.

10. Five different modes of radiation are described by Theodoric in chapters 7–11 of Part I; these are summarized in *The Scientific Methodology of Theodoric of Freiberg*, pp. 181–182.

11. This conclusion is the result of extensive experimentation by Theodoric that has been well described by A. C. Crombie, *Robert Grosseteste and the Origins of Experimental Science, 1100–1700* (Oxford: Clarendon Press, 1953), pp. 233–259; also by Boyer, pp. 110–128. See note 24.

12. The diagram of Fig. 1 is based on the faulty Aristotelian geometry of a "meteorological sphere" and its "altitude circle." See the detailed discussion in Boyer, pp. 119–122.

the foregoing [mode] both with respect to the place and with respect to the mode of incidence and reflection. For in the first mode the incidence of the radiation and its reflection take place on the surface of the spherical droplet that is on the convex side in the direction of the sun and the [person] viewing. In the mode of which we now speak, however, the incidence and reflection to sight take place from the interior of the droplet, or from the concave spherical surface in the direction away from the sun and the [person] viewing, and this according to the third mode of radiation explained above in Part I. For the solar ray is incident on the surface of the sphere facing the sun; it is incident, I say, at a place higher than the point where the perpendicular *ag* falls [on the surface]; and let this place of incidence be *n* and the incident ray *an*. For in this place the described ray, entering the body of the drop, is refracted toward the perpendicular of its incidence, because such a drop is a translucent body of greater termination or density and, as a consequence, of lesser transparency than the air through which the described ray passes on its way to the drops.[13]

According to this mode of refraction and of penetration, therefore, the ray passing through the body of the drop arrives at its concave [surface] at point *d* over line *nd*, and the described ray is again reflected from this place of incidence, or *d*—as by a concave mirror, since it has the nature of a mirror—and is reflected toward the surface of the sphere on which it was first incident. Passing through the body of the sphere it arrives at point *m*, at a place below the perpendicular *ag* coming from the sun; moreover, the same place of incidence will be below the perpendicular *cl*, coming from the [person] viewing to the indicated drop. The ray so reflected from the concavity of the foregoing surface at the place already described is *dm*, and the radiation leaving this place on the body of the sphere is refracted, on leaving, away from the perpendicular of its incidence, because the air which it enters on leaving the body of this sphere is of more subtle substance and, as a consequence, of greater transparency and translucency. Thus the radiation passing from the point of such refraction arrives at the [person] viewing over line *mc*, and in a place and at a configuration fixed by the nature of the star, of the drop, and the [position of the] viewer. Again, the colors of the rainbow appear at the places of incidence, refraction, and reflection, as will now be explained; this is apparent in the description of the foregoing figure.

Chapter 38

Concerning the mode whereby the colors of the lower rainbow are transmitted to the viewer.

Now we must consider the mode whereby the colors that appear in the rainbow are transmitted to the viewer, and this for the case of the lower rainbow. To do so we must review some knowledge from physics that is required for this purpose, and first this [fact], which has been demonstrated above, that radiation of a luminous object, precisely as luminous, when obliquely incident and traveling through a watery or crystalline drop—especially if the drop is of very small size—such radiation, I say, projects the color red from its part that is incident around the sides, angles, and angular sides of such a drop, then [the color] yellow, then green, and finally blue in the portion that on entrance is farther from the angles and the angular sides of the sphere and stays more toward the middle of the drop.[14] Also supposed is this [fact], already indicated, that the place of elevation of the drops on the altitude circle from which the rainbow appears is not a point, but occupies a distance on the altitude circle that corresponds to the width of the entire rainbow and also of each of its component colors.

Another [factor] to be similarly noted is that the place of incidence of the radiation on any drop from which the colors of the rainbow shine on the surface of the drop is not a point but has a width, the different parts of which reflect different colors of the rainbow in such a way that each width of these parts contains a proportionate amount of each of the said colors.

Likewise the place on the opposite surface in the interior of the drop on which such [radiation] is incident has a width that is proportioned to the aforementioned [partial] widths, as has been noted. As a consequence, moreover, the place on the drop from which the foregoing colors depart and are transmitted to the viewer has a width that is proportioned to the width of incidence already mentioned.

13. Theodoric obtained his knowledge of the law of refraction from Alhazen (ibn al-Haitham), whom he cites fifteen times in *De iride* as the "auctor *Perspectivae*."

14. According to Theodoric's theory of color formation, the color red would be generated in the part of the drop where there is greater clarity and boundedness, that is, by rays passing closest to the drop's surface, whereas the other colors would be generated by rays passing more toward the middle of the drop; see *The Scientific Methodology of Theodoric of Freiberg*, pp. 188–196.

Again, this too is to be remembered, that the more obliquely radiation is incident on one side at a position more remote from the perpendicular, the more obliquely it departs toward the viewer on the other side and at a position more remote from the same perpendicular.

Moreover, with the foregoing, [the point] that has been mentioned above is not to be omitted, namely that with one drop receiving such radiation in this manner and transmitting it to the viewer, all of the mentioned colors seen at the drop do not appear be *defgh;* moreover, let it suffice that each aggregate or collection of drops so located as to [produce] each of the mentioned colors be represented by one drop. Then, let there be marked off on each of the mentioned drops a place of incidence having a width, as has been said, and let this be the arc *ln*, and similarly a place at which the said radiation is incident in the interior of the drop, since it has a width, *os*, in the foregoing manner; [then,] the place of exit toward the viewer *tz*,[10] the partial positions within the same width *tvxyz*, the

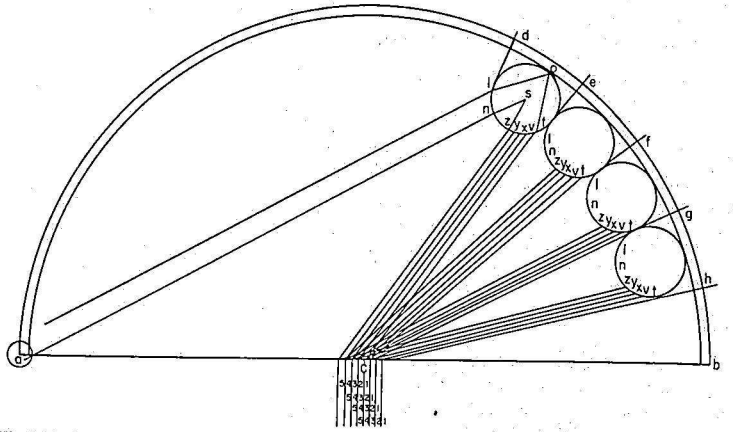


Fig. 2. The formation of the lower rainbow, showing the four drops (or collections of drops) that produce the four colors Theodoric held were present in the bow. The sun is at *a*, the observer at *c*, and the four drops at *de*, *ef*, *fg*, and *gh*.

come to the viewer in one and the same spot, but, depending on the various positions of the viewer with respect to this drop, different colors come to view. Hence, if the viewer sees all the colors simultaneously, as happens in the rainbow, this must come from different drops that are differently situated with respect to the viewer, and the viewer with respect to them.

I will therefore draw a horizon represented by the line *ab*, which also will represent the diameter of a sphere whose center is *c* [Fig. 2].[15] Let the altitude circle cutting the horizon at right angles be *adb*. Let the position on the altitude circle to which the spherical drops are elevated, and in which the rainbow appears, be the arc *dh*. Since this is not a point, but has a width, as has been said, let the parts of the position in which the individual colors place where the total radiation carrying the colors of the rainbow falls on the diameter of the sphere on the horizon, *51*, [and] the partial places at which the individual colors fall, *54321*. And let the sun be on the surface of the horizon so as to provide an example that is easily seen.

With the foregoing [points] that are here called to mind, this too should not be forgotten, as has been shown above, that the arc described on the drop that is included between the place of inci-

15. This diagram is based on the same faulty geometry as Fig. 1; see note 12.

16. Würschmidt's text gives the letters *SZ* here; I have emended his reading on the basis of Cod. Lips. 512, fol. 65 recto. Crombie (p. 253, n. 1) notes a similar change on the basis of the Basel manuscript, Cod. Basil. F. IV. 30, fol. 33 verso.

dence, which is *ln*, and the place of departure toward the viewer, *zyxvt*, is larger or smaller according as the spherulet is more or less elevated above the horizon on the altitude circle. Therefore, [considering] the spherical droplet at the highest place on the altitude circle from which radiation incident from the sun can be transmitted to the viewer, namely, at position *de*, the [incident] radiation falls on the surface of the drop at arc *ln* and, entering the body of the drop there and being refracted toward the perpendicular, falls on the opposite internal surface of the drop at arc *os;* then it returns, as if reflected by a mirror, to the first surface and falls on the arc *tz*. Then, in leaving, it is refracted away from the perpendicular and, arriving at the viewer, represents the color red in the portion that when passing through the body of the drop is incident around the angles and sides and confinement of the boundaries of the drop.[17] Thus the portion *vt* of the aforementioned radiation comes to the viewer in the center of the [meteorological] sphere with the color red at the farther limit *21* on the plane of the horizon. The other three colors, yellow, green, and blue, fall on the diameter of the sphere[18] *acb* between the viewer and the sun, namely yellow at position *32*, green at position *43*, and blue at position *54* at the other limit of these markings. These do not come to the viewer, but the red color alone comes to the viewer from this drop and, in fact, from the entire collection of spherical droplets that are located on the altitude circle at position *de*.

The radiation that is incident on drops that are lower on the altitude circle, namely at position *ef*, so arrives at the viewer that the color yellow from the portion *xv* of the arc *zyxvt* comes to the viewer at the second position on the horizon toward the lower limit, *32*, from each of the drops located at the position *ef* on the altitude circle. The color red, on the other hand, falls on the diameter of the sphere[19] *[acb]* in front of the viewer, namely between the viewer and the place where the rainbow appears. The other two [colors], namely green and blue, fall on the diameter of the sphere *[acb]* behind the viewer, namely between the viewer and the sun, and none of these come to the viewer except the yellow.

Considering the drops located at the third position lower on the altitude circle already described, namely *fg*, from each of these the color green comes to the viewer in the portion *yx* of the arc *zyxvt*. The red and the yellow, on the other hand, fall on the diameter of the sphere[20] *[acb]* in front of the viewer, namely between the viewer and the place where the rainbow appears. The fourth [color], namely blue, falls on the diameter of the sphere behind the viewer, [that is,] between the viewer and the sun, at the positions noted in the preceding [account].

From the drops at the fourth and lowest of the described positions on the altitude circle, namely at *hg*, the fourth color, blue, comes to the viewer at the portion of *zy* of the arc *zyx;* the other three colors, red, yellow, and green, fall on the diameter of the sphere in front of the viewer or between the viewer and the rainbow,[21] and none of these colors come to the viewer except the color blue.

Thus all the colors of the rainbow [are seen] at the same time, and the entire rainbow appears on the altitude circle in different spherical droplets according as they are more or less elevated along the arc *hd* at various positions, from each of which the individual colors come to the viewer in the manner described above.

On the other hand, no incident radiation comes to the viewer from the drops that are located on the altitude circle above the mentioned arc *hd*. Those [drops] that are located below the mentioned arc, however, transmit some incident radiation to the viewer, but not with the colors of the rainbow; rather [they transmit] a kind of white light, unmixed with colors, as has been treated above and will be explained more fully below.[22]

Therefore, according to what has already been said, since the color red is radiated from the highest position on the altitude circle, yellow from the [position] next to this, then green from the third [position], and blue from the lowest [position],

17. This passage again presupposes Theodoric's theory of color formation; see note 14.

18. Würschmidt's text gives the reading *sphaerulae*, meaning spherulet or drop; I have emended his reading to *sphaerae* on the basis of Cod. Lips. 512, fol. 65 recto. The same error occurs with some frequency, as noted below. Crombie (p. 254, nn. 3–5, and so on) has made similar corrections on the basis of the Basel manuscript, Cod. Basil. F. IV. 30, fol. 33 verso ff.

19. Cod. Lips. 512, fol. 65 verso.

20. *Ibid.*

21. I have corrected Würschmidt's text to read *ante visum sive inter visum et iridem* in place of *ante visum et iridem* on the basis of Cod. Lips. 512, fol. 65 verso; see Crombie, p. 254, n. 6, for the same emended reading based on the Basel Manuscript.

22. This paragraph provides Theodoric's explanation of "Alexander's band" and the "Aphrodisian paradox" unaccounted for by previous Aristotelian explanations of the rainbow; for a detailed explanation, see Boyer, pp. 64, 119–123, 215.

as a result the highest and outermost periphery [of the rainbow] is red, the next under [it is] yellow, then follows green, and the lowest and innermost [periphery is] blue.

PART III.

Chapter 3

In general, concerning the difference between the [modes of] radiation whereby the two rainbows are variously generated.

The mode of radiation by which the upper rainbow is generated, to speak only in general, is this [Fig. 3]. Just as in the lower rainbow solar

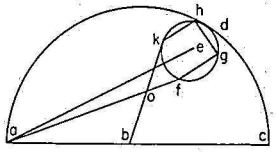


Fig. 3. The path of solar radiation through an individual drop when generating the colors of the upper rainbow. The sun is at *a*, the observer at *b*, the raindrop at *d*.

radiation falls on any drop on the part that is higher or more remote from the diameter of the sphere[23] of the world, and this either higher or more to the right or to the left, as has been explained above, and is reflected to the viewer from the lower part of the drop, or that closer to the diameter of the sphere of the world, so correspondingly in the present case, or in the generation of the upper rainbow, solar radiation falls on the drop at the part that is lower or closer [for instance, point *f*] to the diameter of the sphere of the world [that is, *ac*], whether the drop is elevated above the horizon on the altitude circle at the summit of the rainbow or is located to the right or to the left of the viewer; and thus this radiation, entering the body of the drop and penetrating to the opposite concave surface of the same drop [for instance, at points *g* and *h*], is reflected from it to the viewer [at point *b*] from the part of the drop that is higher or more remote [for instance, point *k*] from the diameter of the sphere of the world [that is, *ae*].

The mode of this type of radiation can be verified by experiment, if one uses a translucent crystalline stone, which is called a beryl, or any clear spherical drop that is so situated with respect to the sun and the viewer, namely with the viewer located between the sun and such a drop located off to one side of the straight line that goes from the sun to the viewer.[24] But this suffices in general.

Chapter 4

In particular, concerning the place or quantity of elevation on the altitude circle. . . of the upper rainbow.

Now to be considered more in particular is [the fact] that the place of generation of this rainbow determined on the altitude circle is eleven [degrees] or approximately this degree of elevation higher on the altitude circle than the place of elevation of the lower [rainbow]. . . .[25]

Chapter 6

Concerning the four conditions or modes of this upper rainbow, which are deduced from the foregoing, with certain introductory notes that are inserted for considering them in particular.

From the foregoing mode of radiation whereby this rainbow is generated, therefore, we may deduce its four modes or conditions: the first, namely that the situal arrangement of these four colors in the upper rainbow is just the opposite of what it is in the lower rainbow.[26] The second also we deduce from physics, namely its shape; the third also, namely why this rainbow is more obscure than the lower [rainbow]; [and] the fourth, why it appears with less frequency than the lower [rainbow]. . . .[27]

Chapter 7

In detail, concerning the cause of the arrangement of the colors of the upper rainbow as these appear to sight.

23. See note 18; Cod. Lips. 512, fol. 68 recto; Crombie p. 256, n. 2.

24. This passage only hints at the experimental methodology used by Theodoric to verify his theory. His ingenious coupling of theory and experiment, in Boyer's expression, "represents one of the greatest scientific triumphs of the Middle Ages. It is difficult to overestimate the significance of his step in immobilizing the raindrops. For the first time the rainbow in the laboratory could be studied more thoroughly than the astronomer in the observatory scans the heavens" (Boyer, p. 124).

25. By his own account Theodoric used the astrolabe to measure such angles; see Part II, chapter 34 (Würschmidt, p. 127, ll. 5–6). The angle he reports in this passage is correct, whereas the angle he measured for the radius of the lower bow is not; for details, see Boyer, pp. 120–121.

26. Theodoric was the first, it may be noted, to provide an essentially correct explanation for the phenomenon of color reversal in the secondary rainbow. See the translation from Part III, chapter 7, following.

27. The explanations for these remaining properties of the upper rainbow are given in *The Scientific Methodology of Theodoric of Freiberg*, pp. 218–223; compare Boyer, pp. 117–118.

Therefore, considering the spherical drop or, in fact, the entire collection of drops elevated on the viewer and the center of the rainbow *k*. . . .

Therefore, [after having discussed the drops at

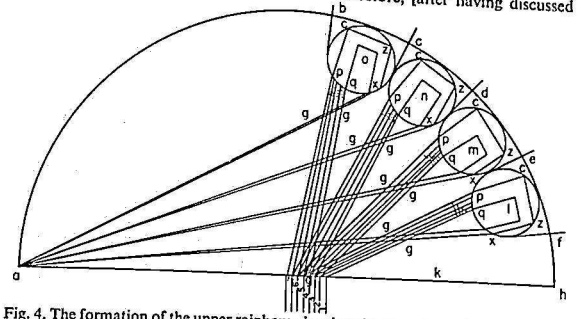


Fig. 4. The formation of the upper rainbow, showing the four drops (or collections of drops) that produce the four colors that Theodoric held were present in the bow. The sun is at *a*, the observer at *g*, and the four drops at *bc*, *cd*, *de*, and *ef*.

altitude circle at arc *fb*, [let us examine] the lowest portion of the arc, *fe* [Fig. 4]. So as to have a very clear example, let the sun be at the horizon [for instance, at *a*]; then solar radiation falls on each one of the mentioned drops at its lower side, namely at arc *x*, and there entering the body of the drop rotates, through its incidences and reflections, around the opposite concave surface of the drop until it arrives at its place of exit, namely at arc *pq*. On emerging from this position it is refracted away from the perpendicular because it meets a body of greater transparency, namely air; thus, by considerable circumgyration it arrives at the line of original incidence, *ax*, and, intersecting this at *gg*,[28] comes to the viewer [at] *g*. Therefore, since for the particular portion or part of the radiation that emerges near *p* the path of rotation comes closest and nearest the angles, arcs, and angular sides of the drop, it is certain,[29] from the foregoing, that this same part of the radiation, which is designated in the diagram by one mark [|], will present the color red to the viewer. The other three colors, namely yellow, green, and blue, which are designated by the remaining marks [|| , ||| , ||||], fall on the diameter of the sphere[30] between the *ed*, *dc*, and *cb*], since in the generation of this [upper] rainbow the color red comes to the viewer from the lowest position within the arc *fb*, which is determined by nature to produce this type of rainbow in view of the elevation of the spherical drops within it, and since yellow is reflected to the viewer next to the red from the next higher portion of the mentioned arc, while the color green radiates from the third portion of the said arc, two [places] above [the red], and finally blue comes from the topmost [part], it is clear that the colors of this upper rainbow are arranged in an order that is the reverse of that seen in the lower rainbow. . . .

28. Cod. Lips. 512, fol. 69 verso, gives a single *g* here, although there are two *g*'s on the diagram in the manuscript corresponding to Fig. 4. I have inserted *gg* so as to avoid the ambiguity otherwise introduced by Theodoric's using *g* also to designate the viewer at the center of the meteorological sphere. Würschmidt gives the reading *f*, following the Basel manuscript; compare Crombie, p. 258, n. 1.

29. I have replaced Würschmidt's *tertium*, meaning third, by *certum*, following Cod. Lips. 512, fol. 69 verso, where the "c" is written like a "t," as in many fourteenth-century manuscripts. The conclusion again presupposes Theodoric's theory of color formation; see note 14.

30. See note 18. Cod. Lips. 512, fol. 69 verso.

Astronomy, Astrology, and Cosmology

Astronomy

64 THE TWO MOST POPULAR MEDIEVAL HANDBOOKS OF THE ELEMENTS OF ASTRONOMY

1. John of Sacrobosco (d. ca. 1244–1256): On The Sphere

Translated by Lynn Thorndike[1]

Annotated by Edward Grant

PROEMIUM

Contents of the Four Chapters[2]

The treatise on the sphere we divide into four chapters, telling, first, what a sphere is, what its center is, what the axis of a sphere is, what the pole of the world is, how many spheres there are, and what the shape of the world is. In the second we give information concerning the circles of which this material sphere is composed and that supercelestial one, of which this is the image, is understood to be composed. In the third we talk about the rising and setting of the signs, and the diversity of days and nights which happens to those inhabiting diverse localities, and the division into climes. In the fourth the matter concerns the circles and motions of the planets, and the causes of eclipses.

CHAPTER 1

Sphere Defined.

A sphere is thus described by Euclid:[3] A sphere is the transit of the circumference of a half-circle upon a fixed diameter until it revolves back to its original position. That is, a sphere is such a round and solid body as is described by the revolution of a semicircular arc.

By Theodosius[4] a sphere is described thus: A sphere is a solid body contained within a single surface, in the middle of which there is a point from which all straight lines drawn to the circumference are equal, and that point is called the "center of the sphere." Moreover, a straight line passing through the center of the sphere, with its ends touching the circumference in opposite directions, is called the "axis of the sphere." And the two ends of the axis are called the "poles of the world."

Sphere Divided.

The sphere is divided in two ways, by substance and by accident. By substance it is divided into the ninth sphere, which is called the "first moved" or the *primum mobile;* and the sphere of the fixed stars, which is named the "firmament"; and the seven spheres of the seven planets, of which some are larger, some smaller, according as they the more approach, or recede from, the firmament. Wherefore, among them the sphere of Saturn is the largest, the sphere of the moon the smallest, as is shown in the accompanying figure.[5]

1. This selection, which includes all of chapters 1, 2, and 4 of John of Sacrobosco's *On the Sphere (De sphaera)* as translated by Lynn Thorndike, is reprinted here by permission of the University of Chicago Press from Lynn Thorndike, *The Sphere of Sacrobosco and its Commentators* (Chicago: University of Chicago Press, 1949), pp. 118–129, 140–142.

This nontechnical account, written primarily as a textbook for use in the universities, was probably the most popular astronomical and cosmological treatise of the Middle Ages and served as the basis of many later commentaries. For instances of its considerable influence on Pierre d'Ailly's *Ymago Mundi*, see Selection 81.

2. This caption, and the many which follow, have no basis in the Latin text and were supplied by Thorndike, presumably for the convenience of the reader.

3. In Book XI, Definition 14, of the modern Greek edition (see Thomas Heath: *The Thirteen Books of Euclids Elements*, 2d. ed. [New York: Dover, 1956], III, 261). Sacrobosco probably derived it from one of the versions of Adelard of Bath's translation of Euclid's *Elements*, perhaps from Campanus of Novara's popular edition (see Selection 27, n. 1.).

4. From Theodosius of Bithynia's (b. ca. 180 B.C.) *On Spheres (De spheris)*, a work originally written in Greek but translated into Latin in the twelfth century by Gerard of Cremona (see Selection 7, no. 5).

5. Although Sacrobosco mentions an accompanying figure, none appears here or elsewhere in Thorndike's edition and translation. Perhaps Sacrobosco never supplied diagrams, or, if he did, they may have been omitted in the subsequent manuscript copies. Whatever the explanation, the figure intended here was probably one that contained all the celestial spheres and their associated planets arranged concentrically.

By accident the sphere is divided into the sphere right and the sphere oblique. For those are said to have the sphere right who dwell at the equator, if anyone can live there. And it is called "right" because neither pole is elevated more for them than the other, or because their horizon intersects the equinoctial circle and is intersected by it at spherical right angles. Those are said to have the sphere oblique who live this side of the equator or beyond it. For to them one pole is always raised above the horizon, and the other is always depressed below it. Or it is because their artificial horizon intersects the equinoctial at oblique and unequal angles.

The Four Elements.

The machine of the universe is divided into two, the ethereal and the elementary region. The elementary region, existing subject to continual alteration, is divided into four. For there is earth, placed, as it were, as the center in the middle of all, about which is water, about water air, about air fire, which is pure and not turbid there and reaches to the sphere of the moon, as Aristotle says in his book of *Meteorology*. For so God, the glorious and sublime, disposed. And these are called the "four elements" which are in turn by themselves altered, corrupted and regenerated. The elements are also simple bodies which cannot be subdivided into parts of diverse forms and from whose commixture are produced various species of generated things. Three of them, in turn, surround the earth on all sides spherically, except in so far as the dry land stays the sea's tide to protect the life of animate beings. All, too, are mobile except earth, which, as the center of the world, by its weight in every direction equally avoiding the great motion of the extremes, as a round body occupies the middle of the sphere.

The Heavens.

Around the elementary region revolves with continuous circular motion the ethereal, which is lucid and immune from all variation in its immutable essence. And it is called "Fifth Essence" by the philosophers. Of which there are nine spheres, as we have just said: namely, of the moon, Mercury, Venus, the sun, Mars, Jupiter, Saturn, the fixed stars, and the last heaven. Each of these spheres incloses its inferior spherically.

Their Movements.

And of these there are two movements. One is of the last heaven on the two extremities of its axis, the Arctic and Antarctic poles, from east through west to east again, which the equinoctial circle divides through the middle. Then there is another movement, oblique to this and in the opposite direction, of the inferior spheres on their axes, distant from the former by 23 degrees. But the first movement carries all the others with it in its rush about the earth once within a day and night although they strive against it, as in the case of the eighth sphere one degree in a hundred years.[6] This second movement is divided through the middle by the zodiac, under which each of the seven planets has its own sphere, in which it is borne by its own motion, contrary to the movement of the sky, and completes it in varying spaces of time—in the case of Saturn in thirty years, Jupiter in twelve years, Mars in two, the sun in three hundred and sixty-five days and six hours, Venus and Mercury about the same, the moon in twenty-seven days and eight hours.

Revolution of the Heavens from East to West.

That the sky revolves from east to west is signified by the fact that the stars, which rise in the east, mount gradually and successively until they reach mid-sky and are always at the same distance apart, and, thus maintaining their relative positions, they move toward their setting continuously and uniformly. Another indication is that the stars near the North Pole, which never set for us, move continuously and uniformly, describing their circles about the pole, and are always equally near or far from one another. Wherefore, from those two continuous movements of the stars, both those that set and those which do not, it is clear that the firmament is moved from east to west.

The Heavens Spherical.

There are three reasons why the sky is round: likeness, convenience, and necessity. Likeness, because the sensible world is made in the likeness of the archetype, in which there is neither end nor beginning; wherefore, in likeness to it the sensible world has a round shape, in which beginning or end cannot be distinguished. Convenience, because of all isoperimetric bodies the sphere is the largest and of all shapes the round is most capacious. Since largest and round, therefore the most capacious. Wherefore, since the world is all-containing, this

6. The value of the precession of the equinoxes as given in Ptolemy's *Almagest* and subsequently cited in other astronomical treatises, for example, in Alfraganus' popular astronomical work (see n. 8).

shape was useful and convenient for it. Necessity, because if the world were of other form than round—say, trilateral, quadrilateral, or many-sided—it would follow that some space would be vacant and some body without a place, both of which are false, as is clear in the case of angles projecting and revolved.[7]

A Further Proof.

Also, as Alfraganus says,[8] if the sky were flat, one part of it would be nearer to us than another, namely, that which is directly overhead. So when a star was there, it would be closer to us than when rising or setting. But those things which are closer to us seem larger. So the sun when in mid-sky should look larger than when rising or setting, whereas the opposite is the case; for the sun or another star looks bigger in the east or west than in mid-sky. But, since this is not really so, the reason for its seeming so[9] is that in winter and the rainy season vapors rise between us and the sun or other star. And, since those vapors are diaphanous, they scatter our visual rays so that they do not apprehend the object in its true size, just as is the case with a penny dropped into a depth of limpid water, which appears larger than it actually is because of a like diffusion of rays.

The Earth a Sphere.

That the earth, too, is round is shown thus.[10] The signs and stars do not rise and set the same for all men everywhere but rise and set sooner for those in the east than for those in the west; and of this there is no other cause than the bulge of the earth. Moreover, celestial phenomena evidence that they rise sooner for orientals than for westerners. For one and the same eclipse of the moon which appears to us in the first hour of the night appears to orientals about the third hour of the night, which proves that they had night and sunset before we did, of which setting the bulge of the earth is the cause.

Further Proofs of This.

That the earth also has a bulge from north to south and vice versa is shown thus: To those living toward the north, certain stars are always visible, namely, those near the North Pole, while others which are near the South Pole are always concealed from them. If, then, anyone should proceed from the north southward, he might go so far that the stars which formerly were always visible to him now would tend toward their setting. And the farther south he went, the more they would be moved toward their setting. Again, that same man now could see stars which formerly had always been hidden from him. And the reverse would happen to anyone going from the south northward. The cause of this is simply the bulge of the earth. Again, if the earth were flat from east to west, the stars would rise as soon for westerners as for orientals, which is false. Also, if the earth were flat from north to south and vice versa, the stars which were always visible to anyone would continue to be so wherever he went, which is false. But it seems flat to human sight because it is so extensive.

Surface of the Sea Spherical.

That the water has a bulge and is approximately round is shown thus: Let a signal be set up on the seacoast and a ship leave port and sail away so far that the eye of a person standing at the foot of the mast can no longer discern the signal. Yet if the ship is stopped, the eye of the same person, if he has climbed to the top of the mast, will see the signal clearly. Yet the eye of a person at the bottom of the mast ought to see the signal better than he who is at the top, as is shown by drawing straight lines from both to the signal. And there is no other explanation of this thing than the bulge of the water. For all other impediments are excluded, such as clouds and rising vapors.[11]

7. Aristotle's *De caelo* (II.4.278a.12–23) is the source of the argument for the *necessary* sphericity of the universe.

8. A reference to the *Differentie scientie astrorum* (or *Liber de aggregationibus*, as it was also known) of Al-Farghani (known as Alfraganus in Latin) (fl. 813–833), an Arab astronomer. Translated twice into Latin (by John of Seville and Gerard of Cremona), it was influential in the history of medieval astronomy. For a modern Latin edition, see Francis J. Carmody, *Al Farghani, Differentie scientie astrorum* (Berkeley, Calif., 1943).

9. The explanation which follows is ultimately—if not directly—derived from Ptolemy's *Almagest*, Book I, chapter 3. There, Ptolemy explains that "what makes the apparent size of a heavenly body greater when it is near the horizon is not its smaller distance but the vaporous moisture surrounding the earth between our eye and the heavenly body. It is the same as when objects immersed in water appear larger, and in fact, the more deeply immersed, the larger" (trans. Cohen and Drabkin, *A Source Book in Greek Science* [Cambridge, Mass.: Harvard University Press, 1948], p. 124, and repeated on p. 283, where, in a note, they explain that "the apparent increase of size of sun and moon at the horizon is largely due to an optical illusion, not to refraction").

10. In the next two paragraphs the substance of the arguments for the earth's sphericity are derived ultimately from Ptolemy's *Almagest*, Book I, chapter 4.

11. In a similar manner, but using a different specific

Also, since water is a homogeneous body, the whole will act the same as its parts. But parts of water, as happens in the case of little drops and dew on herbs, naturally seek a round shape. Therefore, the whole, of which they are parts, will do so.

The Earth Central.

That the earth is in the middle of the firmament is shown thus. To persons on the earth's surface the stars appear of the same size whether they are in mid-sky or just rising or about to set,[12] and this is because the earth is equally distant from them. For if the earth were nearer to the firmament in one direction than in another, a person at that point of the earth's surface which was nearer to the firmament would not see half of the heavens. But this is contrary to Ptolemy[13] and all the philosophers, who say that, wherever man lives, six signs rise and six signs set, and half of the heavens is always visible and half hid from him.

And a Mere Point in the Universe.

That same consideration is a sign that the earth is as a center and point with respect to the firmament, since, if the earth were of any size compared with the firmament, it would not be possible to see half the heavens. Also, suppose a plane passed through the center of the earth, dividing it and the firmament into equal halves. An eye at the earth's center would see half the sky, and one on the earth's surface would see the same half. From which it is inferred that the magnitude of the earth from surface to center is inappreciable and, consequently, that the magnitude of the entire earth is inappreciable compared to the firmament.[14] Also Alfraganus says that the least of the fixed stars which we can see is larger than the whole earth. But that star, compared with the firmament, is a mere point. Much more so is the earth, which is smaller than it.

The Earth Immobile.

That the earth is held immobile in the midst of all, although[15] it is the heaviest, seems explicable thus. Every heavy thing tends toward the center. Now the center is a point in the middle of the firmament. Therefore, the earth, since it is heaviest, naturally tends toward that point.[16] Also, whatever is moved from the middle toward the circumference ascends. Therefore, if the earth were moved from the middle toward the circumference, it would be ascending, which is impossible.

Measuring the Earth's Circumference.

The total girth of the earth by the authority of the philosophers Ambrose, Theodosius, and Eratosthenes is defined as comprising 252,000 stades, which is allowing 700 stades for each of the 360 parts of the zodiac (*sic*).[17] For let one take an astrolabe on a clear starry night and, sighting the pole through both apertures in the indicator, note the number of degrees where it is. Then let our measurer of the cosmos proceed directly north until on another clear night, observing the pole as before, the indicator stands a degree higher. After this let the extent of his travel be measured, and it will be found to be 700 stades. Then, allowing this many stades for each of 360 degrees, the girth of the earth is found.

And Diameter.

From these data the diameter of the earth can be found thus by the rule for the circle and diameter. Subtract the twenty-second part from the circuit of the whole earth, and a third of the remainder—that is, 80,181 stades and a half and third part of one stade—will be the diameter or thickness of the terrestrial ball.[18]

CHAPTER 2

OF THE CIRCLES AND THEIR NAMES

Celestial Circles.

Of these circles some are larger, some smaller,

illustration, Ptolemy, in Book I, chapter 4, of the *Almagest*, also asserts the sphericity of the surface of the earth's waters when he declares that "whenever we sail towards mountains or any high places from whatever angle and in whatever direction, we see their bulk little by little increasing as if they were arising from the sea, whereas before they seemed submerged because of the curvature of the water's surface" (trans. R. C. Taliaferro in *Great Books of the Western World* [Chicago: Encyclopaedia Britannica, 1952], XVI, 9).

12. Earlier Sacrobosco had observed that the celestial bodies *appear* larger (though they are not actually so) when rising or setting and attributed this to refraction (see the previous paragraph "A Further Proof" and n. 9). Here, in apparent contradiction, he declares flatly that the stars *appear (stelle apparent)* exactly the same size in every part of the sky.

13. See *Almagest*, Book I, chapter 5 ("That the Earth is in the Middle of the Heavens"; Taliaferro, p. 10).

14. Ptolemy is again the ultimate source of this argument in *Almagest*, Book I, chapter 6 ("That the Earth Has the Ratio of a Point to the Heavens").

15. In his lengthy review of Thorndike's volume on Sacrobosco, Edward Rosen (*Isis*, XL [1949], 260, col. 1) rightly insists that in this context *cum* be translated as "because" rather than "although."

16. This Aristotelian argument could have been derived from Ptolemy's *Almagest*, Book I, chapter 7 (see Selection 67.1).

17. See Selection 81, chapter 5 and n. 19.

18. See Selection 81, chapter 5 and n. 22.

as sense shows. For a great circle in the sphere is one which, described on the surface of the sphere about its center, divides the sphere into two equal parts, while a small circle is one which, described on the surface of the sphere, divides it not into two equal but into two unequal portions.

The Equinoctial.

Of the great circles we must first mention the equinoctial. The equinoctial[19] is a circle dividing the sphere into two equal parts and equidistant at its every point from either pole. And it is called "equinoctial" because, when the sun crosses it, which happens twice a year, namely, in the beginning of Aries and in the beginning of Libra, there is equinox the world over. Wherefore it is termed the "equator of day and night," because it makes the artificial day[20] equal to the night. And 'tis called the "belt of the first movement."

The Two Movements Again.

Be it understood that the "first movement" means the movement of the *primum mobile,* that is, of the ninth sphere or last heaven, which movement is from east through west back to east again, which also is called "rational motion" from resemblance to the rational motion in the microcosm, that is, in man, when thought goes from the Creator through creatures to the Creator and there rests.

The second movement is of the firmament and planets contrary to this, from west through east back to west again, which movement is called "irrational" or "sensual" from resemblance to the movement of the microcosm from things corruptible to the Creator and back again to things corruptible.

The North and South Poles.

'Tis called the "belt of the first movement" because it divides the *primum mobile* or ninth sphere into two equal parts and is itself equally distant from the poles of the world. It is to be noted that the pole which always is visible to us is called "septentrional," "arctic," or "boreal." Septentrional" is from *septentrio,* that is, from Ursa Minor, which is derived from *septem* and *trion,* meaning "ox," because the seven stars in Ursa move slowly, since they are near the pole. Or those seven stars are called *septentriones* as if *septem teriones,* because they tread the parts about the pole. "Arctic" is derived from *arthos,* which is Ursa Maior, for 'tis near Ursa Maior. It is called "boreal" because it is where the wind Boreas comes from. The opposite pole is called "Antarctic" as opposed to "Arctic." It also is called "meridional" because it is to the south, and it is called "austral" because it is where the wind Auster comes from. The two fixed points in the firmament are called the "poles of the world" because they terminate the axis of the sphere and the world revolves on them. One of these poles is always visible to us, the other always hidden. Whence Virgil:

This vertex is ever above us, but that
Dark Styx and deep Manes hold beneath our feet,[21]

The Zodiac.

There is another circle in the sphere which intersects the equinoctial and is intersected by it into two equal parts. One half of it tips toward the north, the other toward the south. That circle is called "zodiac" from *zoe,* meaning "life," because all life in inferior things depends on the movement of the planets beneath it. Or it is derived from *zodias,* which means "animal," because, since it is divided into twelve equal parts, each part is called a sign and has its particular name from the name of some animal, because of some property characteristic of it and of the animal, or because of the arrangement of the fixed stars there in the outline of that kind of animal. That circle in Latin is called *signifer* because it bears the "signs" or because it is divided into them. By Aristotle in *On Generation and Corruption* it is called the "oblique circle," where he says that, according to the access and recess of the sun in the oblique circle, are produced generations and corruptions in things below.[22]

The Twelve Signs.

The names, order, and number of the signs are set forth in these lines:

There are Aries, Taurus, Gemini, Cancer, Leo, Virgo,

19. That is, the celestial equator.

20. In his review (p. 259, cols. 1–2) Rosen observes that "the 'natural day' had traditionally been defined as the period from sunrise to sunset, in contradistinction to the full period of twenty-four hours from sunrise to sunrise. Sacrobosco reversed the terminology, applying the term 'natural day' to the twenty-four hour period, and 'artificial day' to the interval from sunup to sundown."

21. *Georgics* I. 242–243.

22. *On Generation and Corruption* I. 10.336a.32—336b.24.

Libra and Scorpio, Architenens, Caper, Amphora, Pisces.

Morevoer, each sign is divided into 30 degrees, whence it is clear that in the entire zodiac there are 360 degrees. Also, according to astronomers, each degree is divided into minutes, each minute into 60 seconds, each second into 60 thirds, and so on. And as the zodiac is divided by astronomers, so each circle in the sphere, whether great or small, is divided into similar parts.

While every circle in the sphere except the zodiac is understood to be a line or circumference, the zodiac alone is understood to be a surface, 12 degrees wide of degrees such as we have just mentioned. Wherefore, it is clear that certain persons in astrology lie who say that the signs are squares, unless they misuse this term and consider square and quadrangle the same. For each sign is 30 degrees in longitude, 12 in latitude.

The Ecliptic.

The line dividing the zodiac in its circuit, so that on one side it leaves 6 degrees and on the other side another 6, is called the "ecliptic," since when sun and moon are on that line there occurs an eclipse of sun or moon. The sun always moves beneath[23] the ecliptic, but all the other planets decline toward north or south; sometimes, however, they are beneath the ecliptic.[24] The part of the zodiac which slants away from the equinoctial to the north is called "northern" or "boreal" or "Arctic," and those six signs which extend from the beginning of Aries to the end of Virgo are called "northern." The other part of the zodiac which tips from the equinoctial toward the south is called "meridional" or "austral," and the six signs from the beginning of Libra to the end of Pisces are called "meridional" or "austral."

Extended Uses of "Sign."

When it is said that the sun is in Aries or in another sign, it should be understood that *in* is taken for *beneath* according as we now accept sign. In another meaning a sign is called a "pyramid," whose quadrilateral base is that surface which we call a "sign," while its apex is at the center of the earth. And in this sense we may properly say that the planets are *in* signs. "Sign" may be used in a third way as produced by six circles passing through the poles of the zodiac and through the beginnings of the twelve signs. Those six circles divide the entire surface of the sphere into twelve parts, wide in the middle but narrower toward the poles, and each such part is called a "sign" and has a particular name from the name of that sign which is intercepted between its two lines. And according to this usage stars which are near the poles are said to be "in signs." Also think of a body whose base is a sign in this last sense which we have accepted but whose edge is on the axis of the zodiac. Such a body is called a "sign" in a fourth sense, according to which usage the whole world is divided into twelve equal parts, which are called "signs," and so whatever is in the world is in some sign.

Colures.

There are two other great circles in the sphere which are called "colures," whose function is to distinguish solstices and equinoxes.[25] "Colure" is derived from *colon,* which is a member, and *uros,* which is a wild ox, because, just as the lifted tail of the wild ox, which is its member, describes a semicircle and not a complete circle, so a colure always appears to us imperfect because only one half of it is seen.

The colure distinguishing the solstices passes through the poles of the universe and through the poles of the zodiac and through the greatest declinations of the sun, that is, through the first degrees of Cancer and Capricorn. Wherefore, the first point of Cancer, where that colure intersects the zodiac, is called the "point of the summer solstice," because, when the sun is in it, the summer solstice occurs and the sun cannot approach further toward our zenith. The zenith is a point in the firmament directly above our heads. The arc of the colure which is intercepted between the point of the summer solstice and the equinoctial point[26] is called the "sun's greatest declination" and is according to Ptolemy, 23 degrees and 51 minutes, according to Almeon,[27] 23 degrees and 33 minutes. Similarly, the first point of Capricorn is called the "point of the winter solstice," and the arc of the colure intercepted between that point and the

23. Although the expression is *sub ecliptica,* Sacrobosco means that the sun moves along the ecliptic line as its immediate background. It is not inclined (or declined) to the ecliptic as are the other planets.

24. Presumably, when they cross the ecliptic.

25. See Selection 81, chapter 6 and n. 32.

26. Rather than "equinoctial point," *equinoctialem* should be translated here as "celestial equator" (see Rosen, p. 259, col. 1).

27. Thorndike notes that Almeon probably represents the Abbasid caliph al-Ma'mun, who ruled from 813 to 833. It was during his reign that a value of 23 degrees 33 minutes was determined for the inclination of the ecliptic.

equinoctial is called the "sun's greatest declination" and is equal to the former.

The other colure passes through the poles of the universe and through the points of Aries and Libra where are the two equinoxes, whence it is called the "colure distinguishing the equinoxes." Those two colures intersect at the poles of the world at spherical right angles. The signs of the solstices and equinoxes are stated in these verses:

These two solstices make, Cancer and Capricorn
But Aries and Libra equal the nights to days.[28]

The Meridian.

There are yet two other great circles in the sphere, namely, the meridian and the horizon. The meridian is a circle passing through the poles of the world and through our zenith, and it is called "meridian" because, wherever a man may be and at whatever time of year, when the sun with the movement of the firmament reaches his meridian, it is noon for him. For like reason it is called the "circle of midday." And it is to be noted that cities of which one is farther east than the other have different meridians. The arc of the equinoctial intercepted between two meridians is called the "longitude" of the city.[29] If two cities have the same meridian, then they are equally distant from east and from west.

The Horizon.

The horizon is a circle dividing the lower hemisphere from the upper, whence it is called "horizon," that is, "limiter of vision." It is also called the "circle of the hemisphere." Moreover, the horizon is twofold—that is, right, and oblique or slanting. Those have a right horizon and right sphere whose zenith is on the equinoctial, since their horizon is a circle passing through the poles of the world cutting the equinoctial at right angles, wherefore it is called "right horizon" and "right sphere." But those to whom the pole of the world is raised above the horizon have an oblique or slanting horizon, since their horizon intersects the equinoctial at unequal and oblique angles and is called "oblique horizon" and the sphere "oblique" or "slanting." Moreover, the zenith over our heads is always the pole of the horizon.

Elevation of the Pole.

From these things it is evident that the elevation of the pole of the world above the horizon is as great as the distance of the zenith from the equator, which is shown in this way. Since in every natural day either colure twice joins or becomes identical with the meridian, whatever is true of one holds for the other. Take, then, a fourth part of the colure distinguishing the solstices, which is from the equinoctial to the pole. Take another fourth part of the same colure, which is from zenith to horizon. Since the zenith is the pole of the horizon, those two quarters, since they are quarters of one and the same circle, are equal. But if equals are subtracted from equals, or the same thing common to both is subtracted, the remainders will be equal. Therefore, if we subtract the common arc, namely, that between the zenith and the pole, the remainders will be equal, namely, the elevation of the pole above the horizon and the distance of the zenith from the equinoctial.

Tropics of Cancer and Capricorn.

Having told of the six great circles, we must speak of the four smaller circles. Be it noted, then, that the sun, when in the first point of Cancer of the summer solstice, as it is carried by the firmament describes a circle, which is the one last described by the sun in the direction of the Arctic pole. Wherefore it is called the "circle of the summer solstice" for the reason aforesaid, or the "summer tropic" from *tropos*, which is "turning," because then the sun begins to turn toward the lower hemisphere and to recede from us. The sun again, when in the first point of Capricorn or winter solstice, as it is carried by the firmament describes another circle which is the one last described by the sun in the direction of the Antarctic pole, whence 'tis called the "circle of the winter solstice" or the "winter tropic," because then the sun turns toward us.

Arctic and Antarctic Circles.

Since the zodiac slants from the equinoctial, the pole of the zodiac will decline from the pole of the world. Therefore, since the eighth sphere and the zodiac, which is a part of it, are moved about the axis of the world, the pole of the zodiac, too, will move about the pole of the world. And that circle which the pole of the zodiac describes about the Arctic pole of the world is called the "Arctic circle." And that circle which the other pole of the zodiac describes about the Antarctic pole is called the "Antarctic circle."

28. The author of these lines is unidentified.

29. Since the Latin term is *civitatum*, "of the city" should be translated "of the cities," that is, the intercepted arc provides a measure of arcal distance between the cities.

As great as is the maximum declination of the sun, so great is the distance of the pole of the world from the pole of the zodiac, which is shown in this way. Take the colure distinguishing the solstices which passes through the poles of the world and the poles of the zodiac. Since all quarters of one and the same circle are equal, the quarter of this colure between equator and pole is equal to the quarter of the same colure from the first point of Cancer to the pole of the zodiac. Then, if we subtract from those equals the common arc from the first point of Cancer to the pole of the world, the remainders will be equal, namely, the maximum declination of the sun and the distance from the pole of the world to the pole of the zodiac. Moreover, since the Arctic circle at every point is equidistant from the pole of the world, it is evident that that part of the colure which lies between the first point of Cancer and the Arctic circle is almost double the maximum declination of the sun or the arc of the same colure intercepted between the Arctic circle and the Arctic pole, which is equal to the maximum declination of the sun. Since that colure, like other circles in the sphere, has 360 degrees, a quarter of it will be 90 degrees. Then, since the maximum declination of the sun according to Ptolemy is 23 degrees and 51 minutes and of as many degrees is the arc which is between the Arctic circle and the Arctic pole, if those two combined, which make about 48 degrees, are subtracted from 90, the remainder will be 42 degrees, as is the arc of the colure which lies between the first point of Cancer and the Arctic circle. So it is clear that that arc is almost double the maximum declination of the sun.[30]

It is also to be noted that the equinoctial with the four small circles are called "parallels," as it were equidistant, not that the first is as far from the second as the second is from the third, because this is false, as has already been shown, but because any two taken together are equidistant at every point. They are called the "equinoctial parallel," the "parallel of the summer solstice," the "parallel of the winter solstice," the "Arctic parallel", and the "Antarctic parallel." It is further to be noted that the four minor parallels, namely, the two tropics and the Arctic parallel and Antarctic parallel, distinguish five zones or five regions in the heaven. Wherefore, Virgil:

Five zones possess the sky, of which one is ever
Red from blazing sun and ever burnt by fire.[31]

Also a like number of zones is distinguished on earth directly beneath the said zones. Wherefore, Ovid:

. . . and just as many zones are marked on earth.[32]

The Five Zones.

That zone which lies between the tropics is said to be uninhabitable[33] because of the heat of the sun, which ever courses between the tropics. Similarly, the zone of earth directly beneath it is said to be uninhabitable because of the fervor of the sun, which ever courses above it. But those two zones which are described by the Arctic circle and the Antarctic circle about the poles of the world are uninhabitable because of too great cold, since the sun is far removed from them. The same is to be understood of the zones of earth directly beneath them. But those two zones of which one is between the summer tropic and the Arctic circle and the other between the winter tropic and the Antarctic circle are habitable and tempered from the heat of the torrid zone between the tropics and from the cold of the extreme zones which lie about the poles. The same is to be understood of the stretches of earth directly beneath them.

CHAPTER 4[34]

Movement of the Sun.

It should be noted that the sun has a single circle in which it is moved in the plane of the ecliptic, and it is eccentric. Any circle is called "eccentric"[35] which, like that of the sun, dividing the earth into equal parts, does not have the same center as the earth but one outside it. Moreover, the point in the eccentric which approaches closest to the firmament is called *aux* or *augis*, meaning

30. Compare to Pierre d'Ailly's discussion in Selection 81, chapter 6. Oresme, on whose *Traitié de l'espere* d'Ailly depended directly, probably drew upon Sacrobosco's account of the four smaller circles and the sun's maximum declination.

31. *Georgics* I. 233–34.

32. *Metamorphoses* I. 48.

33. Compare d'Ailly's account in Selection 81, chapter 6, and see also notes 35 and 38. In the latter note it will be seen that Columbus disagreed with Sacrobosco and d'Ailly by insisting upon the habitability of the torrid zone.

34. In chapter 3, which has been omitted here, Sacrobosco discusses risings and settings, ascensions, inequalities of days, the number of times the sun passes directly overhead during a year for those whose zeniths are at the four smaller circles and the zones between them, and the seven climes.

35. Sacrobosco's definitions of "eccentric," "aux" and other technical astronomical terms in this fourth chapter should be compared to those given in the *Theorica planetarum* in the next section.

"elevation." The opposite point, which is farthest removed from the firmament, is called the "opposition" of the *aux*.

Moreover, there are two movements of the sun from west to east, one of which is its own in its eccentric, by which it is moved every day and night about 60 minutes. The other is the slower movement of the sphere itself on the poles of the axis of the circle of the signs, and it is equal to the movement of the sphere of the fixed stars, namely, 1 degree in a hundred years. From these two movements, then, is reckoned the sun's course in the circle of the signs from west to east, by which it cleaves the circle of the signs in 365 days and a fourth of one day, except for a small fraction which is imperceptible.

Of the Other Planets: Equant, Deferent, and Epicycle.

Every planet except the sun has three circles, namely, equant, deferent,[36] and epicycle. The equant[37] of the moon is a circle concentric with the earth and in the plane of the ecliptic. Its deferent is an eccentric circle not in the plane of the ecliptic —nay, one half of it slants toward the north and the other toward the south—and the deferent intersects the equant in two places, and the figure of that intersection is called the "dragon" because it is wide in the middle and narrow toward the ends. That intersection, then, through which the moon is moved from south to north is called the "head of the dragon," while the other intersection through which it is moved from north to south is called the "tail of the dragon."[38] Deferent and equant of each planet are equal, and know that both deferent and equant of Saturn, Jupiter, Mars, Venus, and Mercury are eccentric and outside the plane of the ecliptic, and yet those two are in the same plane. Also every planet except the sun has an epicycle. An epicycle is a small circle on whose circumference is carried the body of the planet, and the center of the epicycle is always carried along the circumference of the deferent.

Stationary, Direct, and Retrograde.

If, then, two lines are drawn from the center of the earth to include an epicycle, one on the east and the other on the west, the point of contact on the east is called the "first station," while the point of contact to the west is called the "second station." And when a planet is in either of those stations it is called "stationary." The upper arc of the epicycle intercepted between those two stations is called "direction," and when the planet is there it is called "direct." But the lower arc of the epicycle between the two stations is called "retrogradation," and a planet existing there is called "retrograde." But the moon is not stationary, direct, or retrograde because of the swiftness of its motion in its epicycle.[39]

Cause of Lunar Eclipse.

Since the sun is larger than the earth, it is necessary that half the sphere of earth be always illuminated by the sun and that the shadow of the earth, extended into the air like a cone, diminish in circumference[40] until it ends in the plane of the circle of the signs inseparable from the nadir of the sun. The nadir is a point in the firmament directly opposite to the sun. Hence, when the moon at full is in the head or tail of the dragon beneath the nadir of the sun, then the earth is interposed between sun and moon, and the cone of the earth's shadow falls on the body of the moon. Wherefore, since the moon has no light except from the sun, it actually is deprived of light and there is a general[41] eclipse, if it is in the head or tail of the dragon directly but partial if it is almost within the bounds determined for eclipse. And it always happens at full moon or thereabouts. But, since in every opposition—that is, at full moon—the moon is not in the head or tail of the dragon or beneath the nadir of the sun, it is not necessary that the moon suffer eclipse at every full moon.

36. In the *Theorica planetarum* these circles are called "eccentric equant" and "eccentric deferent" (see next section, par. 32).

37. Sacrobosco's use of the term "equant" here is at the least unfortunate, and probably confused. For the relation of the lunar circles, see paragraphs 10–16 and 29–30 of the *Theorica planetarum*, in the next section.

38. It is not clear whether the two intersecting circles described here, namely the "equant" and deferent circles of the moon, are identical with the two intersecting circles described in the *Theorica planetarum*, paragraph 29, namely the eccentric circle of the moon and the eccentric circle of the sun, or ecliptic. Perhaps Sacrobosco's "equant" corresponds to the eccentric circle carrying the nodes 3 minutes daily from east to west, described in paragraph 30 of the *Theorica*.

39. A more faithful translation of this last sentence would be: "However, the moon is not assigned a station, direction, or retrogradation and therefore is not said to be stationary, direct, or retrograde because of the swiftness of its motion in its epicycle."

40. "Roundness" would be a better rendering of *rotunditate* than "circumference."

41. In this context *generalis* should be translated as "total" rather than "general."

Cause of Solar Eclipse.

When the moon is in the head or tail of the dragon or nearly within the limits and in conjunction with the sun, then the body of the moon is interposed between our sight and the body of the sun. Hence it will obscure the brightness of the sun for us, and so the sun will suffer eclipse—not that it ceases to shine but that it fails us because of the interposition of the moon between our sight and the sun. From these it is clear that a solar eclipse should always occur at the time of conjunction or new moon. And it is to be noted that when there is an eclipse of the moon, it is visible everywhere on earth. But when there is an eclipse of the sun, that is by no means so. Nay, it may be visible in one clime and not in another, which happens because of the different point of view in different climes. Whence Virgil most aptly and concisely expresses the nature of either eclipse:

Varied defects of the moon, and of the sun travails.[42]

Eclipse during the Passion Miraculous.

From the aforesaid it is also evident that, when the sun was eclipsed during the Passion and the same Passion occurred at full moon, that eclipse was not natural—nay, it was miraculous and contrary to nature, since a solar eclipse ought to occur at new moon or thereabouts. On which account Dionysius the Areopagite is reported to have said during the same Passion, "Either the God of nature suffers, or the mechanism of the universe is dissolved."

2. Anonymous: The Theory of the Planets

Translation and introduction by Olaf Pedersen

Annotated by Edward Grant

From the middle of the thirteenth century the primary source of theoretical astronomy was Ptolemy's *Almagest*, which was accessible in various Latin translations. This text was, however, much too advanced and difficult to be of any use to students at an elementary level, and comparatively few manuscripts of these translations are known. For teaching purposes in the Liberal Arts course in the universities it was therefore usually replaced by one of a number of elementary manuals, the most popular of which was the *Theorica Planetarum* which is still extant in hundreds of manuscripts dating from the thirteenth to the sixteenth centuries. In astronomical codices from the Middle Ages it was often placed immediately after the equally popular works of John of Sacrobosco, the *Tractatus de Sphaera*, the *Compotus*, and the *Algorithmus*. It was usually followed by a set of astronomical tables and one or two treatises on instruments such as the quadrant or the astrolabe, the whole forming a kind of astronomical compendium which, with some additions, remained essentially unchanged throughout the centuries.

Unfortunately neither the author nor the date of the *Theorica Planetarum* can be determined with certainty. The great majority of the manuscripts are anonymous, from which it follows that the author was unknown to most medieval scholars. A few among them ascribed the text to one of a dozen authors, among whom was the great translator Gerard of Cremona (d. 1187), and in the many early printed editions it was commonly listed among his works. In the nineteenth century Boncompagni [*Atti dell'Accad. Pont. de'Nuovi Lincei* IV (1852), 449 ff] rejected this ascription and tried to establish a case for the Italian astrologer Gerard of Sabbioneta (about 1250) as the author, whereas Duhem [*Le Système du Monde* (Paris: Hermann, 1913–1959), III, 219 ff.] sought to give it back to Gerard of Cremona. However, in the light of modern scholarship none of these assumptions can be maintained and the question remains unsolved. The most probable hypothesis is that the *Theorica Planetarum* was composed by an unknown teacher of astronomy as a manual destined to supplement the meager treatment of planetary theory in the fourth book of the *Sphaera* of Sacrobosco. There are some reasons to suppose that this happened in Paris sometime during the period 1260 to 1280.

The translation given below is based upon one of the oldest and best preserved manuscripts in the Royal Library in Copenhagen as MS. Latin Add. 447, 2°, 49r–56r, dating from perhaps a little before 1300. The division into eight chapters is found in most manuscripts, and also the chapter headings given here. For the sake of easy reference the translator has found it convenient to divide the

42. *Georgics* II.478.

text into short paragraphs numbered consecutively from 1 to 121. As for the translation, some of the technical terms, for instance, *aux* or *motus*, have to be used in their Latin form because in their Ptolemaic sense they have no precise equivalents in modern English. The reader should consult Professor D. J. de Solla Price's *The Equatorie of the Planetis* (Cambridge, Eng.: The University Press, 1955) for a more detailed discussion of such problems.

Here begins the theory of the seven planets.

I. THEORY OF THE SUN'S MOTION

1. A circle whose center is not at the center of the world is called an eccentric circle, or a circle with a displaced cusp, or an outgoing center.

The part of the eccentric which has the greatest distance from the center of the world is called aux, or the greatest distance. The part nearest to the center is called the opposite aux, or the least distance. Therefore, the two parts of the circle situated in the middle between the aux and the opposite aux are called the mean distances.

2. The Sun has its own motion in its eccentric circle, where it every day moves 59 minutes 8 seconds with uniform speed, so that it must perform a nonuniform motion through the zodiac from west to east; but the universe rotates from east to west with a uniform motion.

3. The mean motus of the Sun is an arc of the zodiac cut off between two lines, one from the center of the Earth to the place of Aries, and another from the same center to the firmament, parallel to a line from the center of the eccentric through the center of the Sun.

4. The true motus of the Sun is defined as an arc of the zodiac lying between the Head of Aries and a line from the center of the Earth through the center of the Sun.

5. The equation of the Sun is defined as an arc of the zodiac lying between the mean motus and the true motus of the Sun. The equation of the Sun is zero when the Sun is at the aux or the opposite aux, but when it is at the mean distances the equation is maximum.

6. The argument of the Sun is defined as an arc of the zodiac lying between the aux and the line defining the mean motus.

7. The aux of the Sun in the second sense is defined as an arc of the zodiac lying between the Head of Aries and the line through the aux of the eccentric circle.

8. Throughout one half of the heavens the mean motus is greater than the true motus so that the equation must here be subtracted; throughout the other half it is smaller, so that the equation must be added.

9. To find the mean motus of the Sun means to find a certain arc of the zodiac which has the same ratio to the whole zodiac as the arc of the eccentric circle described by the Sun has to the whole eccentric circle. This is found by the parallel line as seen in the following figure [Fig. 1].

II. THEORY OF THE VARIOUS MOTIONS OF THE MOON

10. The Moon's epicycle, or orbit of revolution, or small orbit, is defined as the small circle whose center moves from west to east upon the circumference of the eccentric.

11. The eccentric circle of the Sun is immovable apart from the motion of the eighth sphere, but the eccentric circle of the Moon moves every day about 11 degrees from east to west. The center of the eccentric describes a certain small circle around the center of the world. The center of the epicycle of the Moon moves about 13 degrees per day from west to east, and the aux of the eccentric moves 11 degrees per day from east to west. The center of the Sun moves almost 1 degree per day in the opposite direction to the firmament.

12. From this it appears that if the center of the Sun, and the aux of the eccentric of the Moon, and the center of the epicycle of the Moon were at a certain place of the heavens at a certain hour, then the following day the Sun would have moved 1 degree towards the east from the said place. The aux of the eccentric would have moved 11 degrees towards the west and the center of the lunar epicycle 13 degrees towards the east, wherefore the Sun will then be in the middle between the aux and the epicycle center. If, therefore, the distance between the Sun and the center of the lunar epicycle is doubled, we get the distance between the aux and the epicycle. This distance is called the Moon's centrum, or the double interval.

13. This makes it clear that these three points are either at the same place or that the Sun is in the middle between the two others, or at their opposite point.

14. It is also seen that the center of the lunar epicycle goes round the eccentric circle twice in a month, and that it is at the aux when in conjunction with, or in opposition to, the Sun. In quadrature it is at the least distance.

15. When the Moon is in the upper part of the

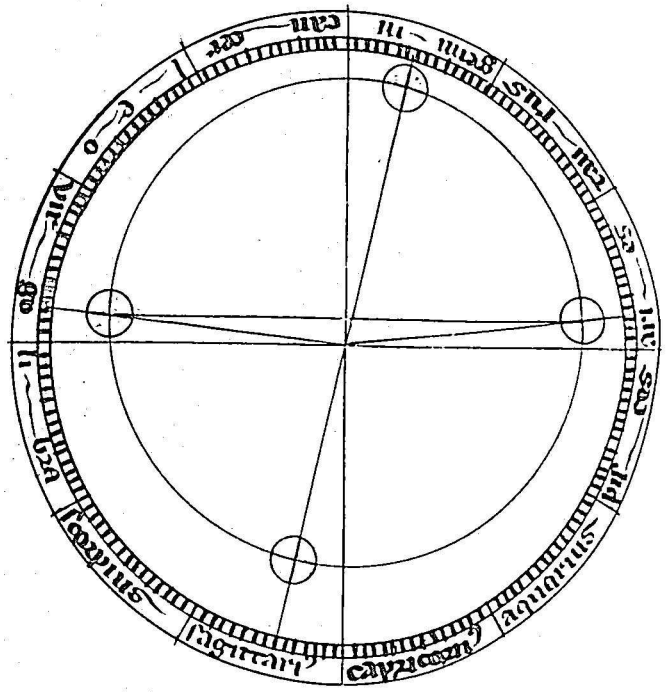


Fig. 1. Motions of the Sun.

circumference of the epicycle it moves from east to west and therefore has a slow motion. In the lower part the motion is in the opposite direction and therefore fast. The other planets behave in the opposite manner.

16. Just as the Sun moves uniformly around the center of its eccentric, the center of the lunar epicycle moves uniformly around the center of the world.

17. The mean motus of the Moon is an arc of the zodiac beginning at Aries and continuing according to the order of the signs, that is, Aries, Taurus, and so on, and ending at a line from the center of the Earth through the center of the epicycle.

18. The true place of the Moon is cut off by a line from the center of the world through the center of the Moon's body.

19. The mean aux of the epicycle is defined as the point of intersection between the upper part of the epicycle and a line passing through the center of the epicycle from a certain point opposite to the center of the Moon's eccentric, and with the same distance from the center of the Earth as this center of the eccentric.

20. The true aux is defined as the point of the epicycle where it is cut by a line from the center of the world through the center of the epicycle to the upper part of the latter.

21. The equation of center is a small arc of the epicycle lying between the mean and the true auges.

22. The mean argument of the Moon is defined as an arc lying between the mean aux and the center of the Moon's body and reckoned in accordance with the motion of the Moon upon the epicycle.

23. The true argument is defined as an arc between the true aux and the center of the Moon's body.

24. The equation of center is zero, that is, the true aux and the mean aux are the same, when the center of the epicycle is at the aux of the eccentric (that is, in mean conjunction and mean opposition) and also when it is at the opposite aux (that is, in quadratures between the Moon and the Sun).

25. The lunar equation of argument is defined as an arc of the zodiac lying between the mean motus and the true motus. This equation is zero when the epicycle center is at the aux or the opposite aux of the eccentric, the Moon being at the same time at the greatest or least distance of the epicycle. The equation of argument is maximum when the epicycle center is at the mean distances of the eccentric and the Moon at the mean distances of its epicycle.

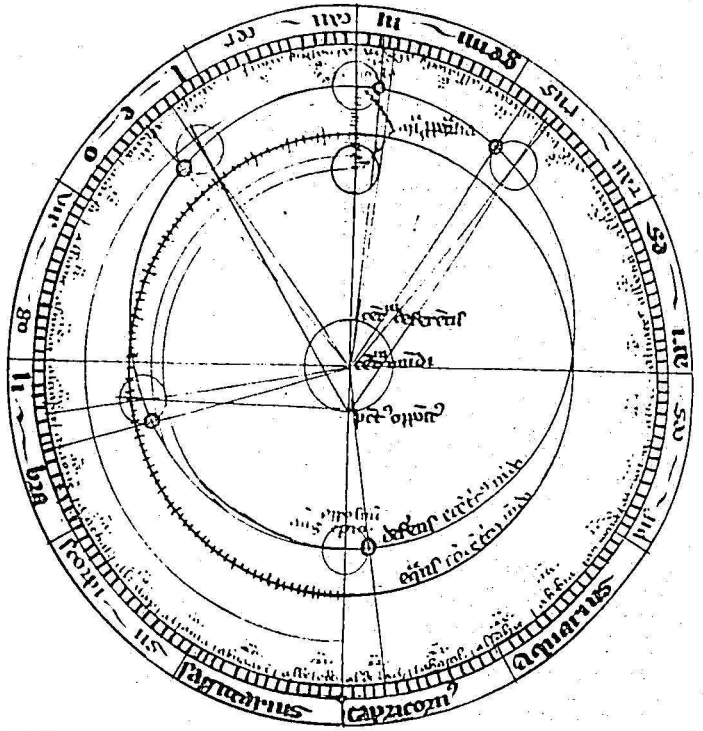


Fig. 2. Motions of the Moon. [(1) is the center of the world (*centrum mundi*), namely the Earth (concentric with it is the celestial sphere divided into its 12 familiar zodiacal signs; also shown are the 28 numbered houses of the Moon, whose names were transliterated from Arabic); (2) is the center of the deferent (*centrum deferentis*), that is, the center of the circle "bearing" the epicycle; (3) is the equant point (*punctus oppositus*) or center of mean motion; and (4) represents proportional minutes (*minuta proportionalia*). The abbreviated forms of the Latin terms used in the figure have been expanded in this caption, which was supplied by V. E. Thoren.—*Ed.*]

When the Moon is on the western half of the epicycle, the mean motus is greater than the true motus; here, therefore, the equation of argument must be subtracted. Upon the eastern half the opposite is true.

26. You must know that when the epicycle center is at the opposite aux of the eccentric, the equations of argument are greater than when it is at the aux. The difference between these equations of argument, with the epicycle center at the aux and the opposite aux, is called the equation of the diameter variation of the small circle [that is, of the epicycle].

27. The equations of argument which are recorded in the tables presuppose that the epicycle center is always at the aux of the eccentric; but when the epicycle center is found at other places, the equations of argument increase as the epicycle approaches the opposite aux, since the epicycle center comes nearer to the center of the Earth.

28. The equations of argument are found by means of proportional parts. These proportional parts are defined as sixty small parts of a line twice as long as the line between the center of the Earth and the center of the eccentric, and divided into sixty parts. The line from the center of the Earth to the opposite aux does not contain any of these parts, but the line to the aux contains them all. Other lines to other points contain a number of those parts, corresponding to their nearness or remoteness from the aux and the opposite aux, as seen from the figure [Fig. 2].

III. THEORY OF THE MOTION OF THE DRAGON'S HEAD

29. In the following we consider the Head and Tail of the Dragon. The eccentric circle of the Moon deviates from the path of the Sun in two directions, namely to the North and to the South, intersecting the eccentric circle of the Sun at two places which are always opposite. These intersections are called the Head and Tail of the Dragon. The intersection where the Moon begins to turn to the north is called the Head, and the other is the Tail.

30. These points of intersection move daily about 3 minutes from east to west. They are carried by a certain eccentric circle existing in the heaven of the Moon and equal to the eccentric in magnitude. It lies in the plane of the zodiac, that is, in the path of the Sun, and this motion is unlike the motion of the planets, which is from west to east.

31. In order to obtain a similarity in motions, the Head of Genzahar is said to have a mean motion contrary to the firmament and equal to that which it actually has along with the firmament. It follows that the mean motus of the Head subtracted from twelve signs is equal to the true place of the Dragon's Head, reckoned according to the order of the signs, which appears from the following figure [Fig. 3].

IV. THEORY OF THE MOTIONS OF THE THREE SUPERIOR PLANETS

32. We shall now proceed with the three superior planets. You must therefore note that any of the three superior planets has two eccentric circles lying in the same plane and immovable, apart from the motion of the eighth sphere and the diurnal motion around the Earth from east to west. One of them is called the eccentric deferent, along the circumference of which the epicycle center is carried from west to east. The other is called the eccentric equant, about the center of which the center of the epicycle moves uniformly, describing equal angles in equal times.

33. These two eccentrics are equally elevated towards the same place in the heavens, and the one whose center is nearest to the Earth is the deferent. The other one is the equant. The deferent center has the same distance from the center of the Earth as from the center of the equant because it is in the middle. And these three centers are upon one line, and both eccentric circles have the same magnitude.

34. In the lunar theory the diameter of the epicycle points towards the center of the Earth when the epicycle center is at the aux or the opposite aux, and later, at other places of the eccentric, it inclines towards a certain point opposite to the center of the eccentric and at the same distance from the center of the world as the center of the eccentric. In the three superior planets the diameter of the epicycle likewise points to the center of the world when the epicycle center is at the aux or the opposite aux of the eccentric, but when the epicycle center is at other points of the eccentric this diameter points towards the center of the equant. We call this "reflection".[43]

35. On the superior part of the epicycle the Moon moves from east to west, but the opposite way on the inferior part. However, these three planets

43. The term "reflection" *(reflexio)* has a quite different application here than it does in the section translated in note 47, where it signifies terrestrial parallax.

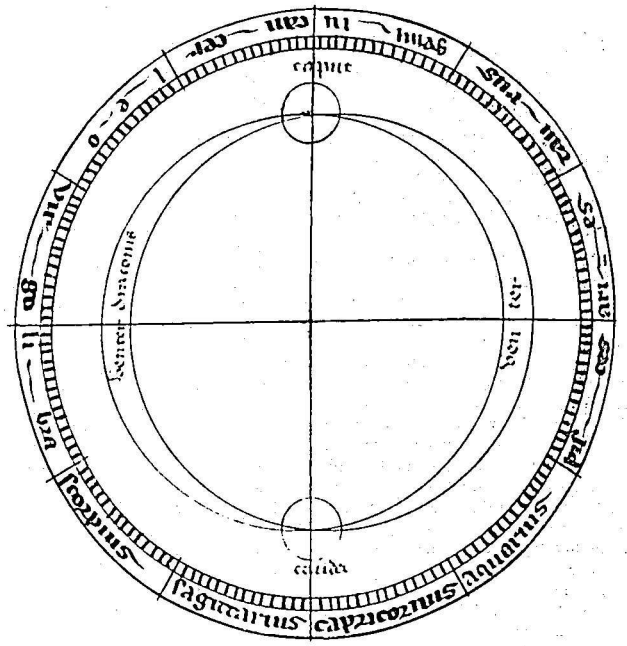


Fig. 3. Motion of the Dragon's Head.

move from west to east on the superior and the opposite way on the inferior part.

36. When any of these planets is in conjunction with the Sun with respect to the mean motus, it will always be upon the superior part of its epicycle, that is, at the mean aux. But in mean opposition to the Sun it will be at the lowest point of its epicycle, and in quadrature with the Sun it is at the mean distances of the epicycle. Therefore, it will go around the epicycle in the same time as the Sun needs for returning to a conjunction.

37. The mean aux of the epicycle is defined as a point upon its superior part where a line from the center of the equant through the center of the epicycle intersects the latter. This aux does not change.

38. The true aux is a point defined by a line from the center of the Earth through the center of the epicycle, and this aux changes with the increase or decrease of the equation of center of the epicycle.

39. The equation of center of the epicycle is an arc of the latter lying between the mean and true auges.

40. The equation of center in the zodiac is an arc of the latter lying between the mean motus and true motus of the epicycle, and the ratio between the equation and its corresponding circle is the same in the two cases. This can be proved by a line lying between the two parallels. Consequently, if one equation is found in a table, we also have the other.

41. The mean motus of any of these planets, that is, of their epicycles, is an arc of the zodiac lying between Aries and a line from the center of the Earth parallel to the line passing from the center of the equant through the center of the epicycle.

42. The true place of the epicycle is an arc of the zodiac between Aries and a line from the center of the Earth through the center of the epicycle.

43. The true place of the planet is determined by a line from the center of the Earth through the center of the planet.

44. The aux of the planets in the second sense is defined, as in the solar theory, as an arc of the zodiac beginning at Aries and ending at a line drawn from a certain point of the Earth just beneath the auges of the eccentric circle.

45. The mean centrum of the planet is defined as an arc of the zodiac between the aux of the eccentric and the mean motus of the epicycle. It is this arc which is called "argument" in the solar theory and centrum, double interval, or double distance in the Moon.

46. The true centrum is defined as an arc of the zodiac lying between the aux of the eccentric and the true place of the epicycle.

47. The mean argument is defined as an arc of the epicycle lying between the mean aux and the center of the planet.

48. The true argument is defined as an arc of the epicycle lying between the true aux and the center of the planet.

49. In one half of the heavens the equation of center in the zodiac must be subtracted from the mean centrum, whereas the equation of center in the epicycle must be added to the mean argument in order to obtain the true centrum and the true argument. In the other half the opposite is true, which is easily seen from a figure [Fig. 4]. When

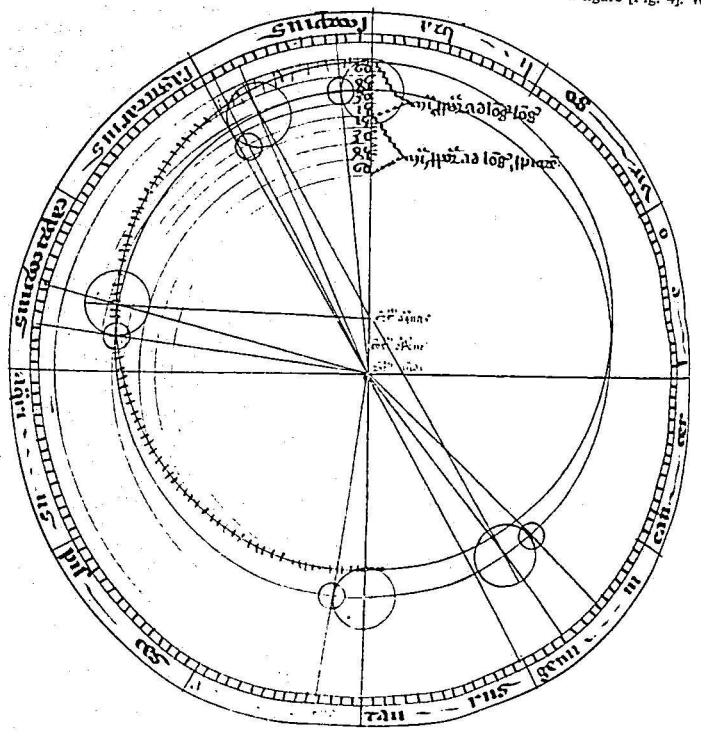


Fig. 4. Motions of the three superior planets.

the epicycle center is at the aux or the opposite aux, the said equations are zero.

50. The planet's equation of argument is defined as an arc of the zodiac between the true place of the planet and the true place of the epicycle.

51. If the true argument of the planet has the same value, it is clear that the more the epicycle center approaches the center of the Earth, the more the equation of argument will increase. The reason is that the equations of argument are greater when the epicycle center is at the least distance of the eccentric circle than when it is at the mean distances, and that the equations are greater at these mean distances than at the aux.

52. In tables the equations of argument are written as if the epicycle center were always at the mean distances of the equant, not because it moves upon the equant but because it has a uniform motion about the center of the equant. The epicycle center is said to be at the mean distance of the equant when the diameter of the epicycle is perpendicular to the diameter of the world passing through the center of the eccentric circles.

53. The differences between the equations at the mean distance and at the aux are called the diameter variations of the small circle [that is, the epicycle] corresponding to the greatest distance. The differences between the equations at the mean distance and at the opposite aux are called the diameter variations of the small circle corresponding to the least distance, and these variations are recorded in the tables.

54. The excess of a line drawn from the center of the Earth to the aux of the equant over another line from the same center to its mean distance is divided into sixty parts, called proportional parts corresponding to the greatest distance. The excess of a line drawn to the mean distance over a line drawn to the least distance is divided into sixty parts, called proportional parts corresponding to the least distance. Therefore, the diameter variations corresponding to the greatest distance must be subtracted from, or those corresponding to the least distance added to, the equation of argument, which is clearly seen from a figure [Fig. 4] by the attentive reader.

55. As mentioned above, the more the epicycle center approaches the center of the Earth, the more the equation of argument will increase, as seen from the following figure [Fig. 4].

V. THEORY OF THE MOTIONS AND CIRCLES OF VENUS AND MERCURY

56. We continue with Venus and Mercury. Mercury has two eccentric circles of the same magnitude and lying in the same plane, of which the equant is nearest to the center of the Earth. For the deferent center is twice as far away from the equant center as the latter is from the center of the Earth, because a certain small circle must pass through these two centers of the eccentrics.

57. The deferent center of Mercury moves upon the circumference of this small circle from east to west with a daily motion equal to that with which the Sun moves contrary to the firmament, carrying the aux with it. For in the same time in which the Sun goes round the firmament, the aux of the eccentric deferent and also any other point of it will go round the equant, and the center of the eccentric deferent will go round its small circle. Sometimes it therefore happens that the equant and deferent centers are at the same place, so that the two circles here are one and the same. Except for this moment, the deferent will be nearer to the firmament than the equant. The deferent moves so that it describes equal angles in equal times around the center of the equant.

58. The equant is immovable apart from the motion of the eighth sphere.

59. The epicycle of Mercury moves upon the circumference of its eccentric from west to east with the same speed with which the Sun moves contrary to the firmament. It therefore appears that the epicycle center of Mercury goes twice around its eccentric in one year, once because of its own motion and once because of the motion of the aux, just as the epicycle of the Moon goes twice around the circumference of its eccentric in one month.

60. Similar to the epicycles of the other planets, this epicycle also has two motions, that is a mean and a true one. The mean motus is determined by a line from the center of the Earth parallel to the line from the center of the equant through the center of the epicycle, but this mean motus is the same as the mean motus of the Sun.

61. Sometimes these three lines are parallel, that is, the line from the center of the Sun's eccentric through the center of the Sun, and the other line from the center of the eccentric equant of Mercury through the center of its epicycle, and the line from the center of the Earth parallel to the two others. And since these three lines move equally, they will always be parallel, or they will, from time to time, be two lines only, or even one and the same line. From this it does not follow that

the epicycle center of Mercury and the center of the Sun are at the same place, or that the line from the center of the Sun's eccentric through the center of the Sun is the same as the line from the center of the equant through the center of the epicycle.

62. The true place or motus of the epicycle is determined by a line from the center of the Earth through the center of the epicycle.

63. As already mentioned, the center of the epicycle and the aux of the eccentric move in opposite directions at the same rate, equal to that of the mean motus of the Sun. In equal times the center of the epicycle and the aux and any other point of the eccentric deferent always describe equal angles about the center of the equant. The center of the deferent also moves at the same rate upon the small circumference in such a way that it describes equal arcs upon the equant in equal times, thus describing unequal arcs upon the small circumference. Thus, when it has moved through a quarter of its small circle, it will not have moved through a quarter of the equant.

64. From what is said above, it can also be shown that the aux of the deferent cannot be at every point of the equant, since the lines passing from the center of the Earth through the center of the deferent and pointing to the aux of the latter will always lie inside an arc of the equant, confined by two lines tangential to the small circle upon which the deferent center moves and passing through the center of the Earth.

65. From this it follows that the aux of the deferent is always confined to this arc, that is, that it now approaches and now recedes from the aux of the equant and this to both sides of the immovable aux of the equant; and although the aux of the deferent cannot be outside the said arc, nevertheless the point of the deferent, which in a way is the aux point, can be at any point of the equant, that is, in line with it. The reason is that the aux is continually changing.

66. When the aux of the deferent is moving away from the aux of the equant towards the west, or inward, then the intersection between the deferent and the equant following after the aux of the latter according to the order of the signs will approach the aux of the equant, and the other intersection will recede from it. The opposite is true when the aux of the deferent moves away from the aux of the equant towards the east.

67. Each time the epicycle center of Mercury coincides with the aux of the deferent, that is, falls upon it, then the aux of the deferent will be at the aux of the equant. It then follows that the centers of the Earth, the equant, and the deferent, and the auges and the opposite auges, and the center of the epicycle will lie upon the same diameter of the world. And when the epicycle center is at the opposite aux of the deferent, then the points mentioned above are upon the same diameter and the two eccentric circles become one.

68. It also follows from what is said above that although the epicycle center goes round its deferent twice in a year, it will nevertheless be at the aux only once, for when the epicycle center is upon one half of its equant the center of the eccentric will move upon the opposing half of its small circle. And when the epicycle center is at the aux it is at the point of the deferent which can be farthest away from the Earth. Thus it can be at the point (of the deferent) farthest away from the Earth, but it cannot be at the point nearest to the Earth, for when it is at the opposite aux, then the two circles are one and the same and the epicycle center is at the opposite aux of the equant.

69. But there are two places in which the epicycle center comes as near to the center of the Earth as possible, whereas it cannot come so near at other places. These two places are points opposite to the aux of the equant and endpoints of the lines tangential to the small circle carrying the center of the deferent and passing through the center of the Earth.

70. Therefore, when the epicycle center is at the aux, it immediately begins to move towards the east upon its deferent, and likewise the center of the deferent begins to move towards the west upon its small circle. And when the epicycle center is at the head of the tangents in the neighborhood of the opposite aux of the equant, then the deferent center will be at the point of contact between this line and the small circle. The aux of the deferent is then as far as possible from the aux of the equant, and the epicycle center is at the opposite aux of the deferent.

71. At this moment, therefore, it (that is, the epicycle center) is as near as possible to the Earth, because the deferent center now descends more on its small circle and the opposite aux is more removed from the center of the Earth, which is easily seen by anyone who regards the figure [Fig. 5] and understands the motion. And as long as the epicycle center is upon the arc of the deferent lying between the points or heads of the tangents, the heads being near to the opposite aux of the equant, it will always be at the opposite aux of the

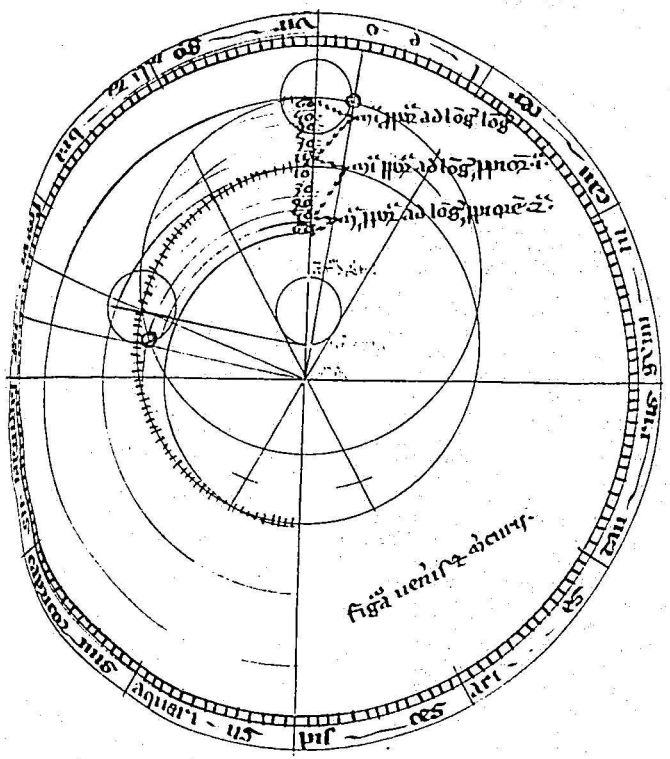


Fig. 5. Motions of Venus and Mercury.

deferent. This follows from the fact that as the epicycle moves in one direction, the deferent center will move as much in the other. Therefore, they will always be upon the same line, passing through the center of the Earth and the center of the deferent. And yet the epicycle center never comes so near to the center of the Earth as it does at the heads of the tangents.

72. The mean motus of Venus and Mercury, the mean and true centrums, the mean and true arguments, the equations of center in the zodiac and in the epicycle, the mean and true auges, and the equation of argument—all these things are defined in Venus and Mercury as in the three superior planets.

73. As in the three superior planets, the equations of argument of Mercury increase as the epicycle center approaches the Earth. The equations which are written in the tables presuppose that the epicycle center is at the point of intersection between the equant and the deferent.

74. For when the epicycle center is at the aux, it immediately begins to move towards the east, and the point of intersection which is before it moves towards it, so that they will be conjoined. And the equations of argument corresponding to this

position are written one after the other in the tables. The same thing takes place at the other point of intersection. When the epicycle center is at other places of the deferent, the equations of argument are found by proportional parts.

75. There must therefore be three sets of proportional parts. The first are the proportional parts corresponding to the greatest distance and defined as sixtieth parts of the difference between a line from the center of the Earth to the center of the epicycle, the latter being at the aux of the deferent, and another line to the intersection of the circles. The proportional parts corresponding to the least distance are sixtieth parts of the difference between a line from the Earth to the intersection, and the tangents [from the Earth] to the points where the epicycle center comes nearest to the Earth. Or again, from the same place to the opposite aux of the equant. As this is a moving line, the proportional parts will vary.

76. The diameter variation of the small circle can be described in two ways, as in the three superior planets.

77. Venus has a deferent and an equant situated as in the three superior planets, and with their highest points at the same place as that of the eccentric circle of the Sun. Its epicycle center moves with the same speed as the Sun, so that the mean motus of the Sun is also the mean motus of Venus, because there are two lines parallel to the line from the center of the Earth, one of which goes from the center of the Sun's eccentric circle through the center of the Sun, and the other from the center of the equant through the center of the epicycle.

78. The deferent and the equant are immovable, apart from the motion of the eighth sphere, and apart from the fact that the deferent moves in latitude to the south and to the north, so that when the deferent and equant sometimes lie in the plane of the ecliptic, it will soon move away from this line towards each pole, and the deferent will incline relative to the ecliptic. We shall treat of this motion in the chapter on latitudes.

79. All other things concerning Venus are similar to the three superior planets. And this will be sufficient for Venus and Mercury.

VI. ON THE RETROGRADE MOVEMENTS OF THE PLANETS

80. In the following we deal with the retrograde movements of the planets. A planet is called direct when its motion is supported by the motion of the epicycle against the firmament. It is called retrograde when its motion is not supported by the motion of the epicycle.

81. The first station is defined as the point of the epicycle where the planet begins its retrograde motion. The second station is the point where it begins to be direct.

82. The Moon cannot be said to exhibit any of these three phenomena, although it has an epicycle, because this epicycle always moves faster than the Moon itself. Yet in the superior part of its epicycle the Moon is called slow and in the inferior part fast in its motion.

83. The first station in the second sense is defined as the arc of the epicycle between the true aux of the latter and the first station in the first sense.

84. The second station in the second sense is defined as the arc of the epicycle lying between the true aux and the second station in the first sense, and stretching through the first to the second station.

85. The arc of retrogradation is an arc of the epicycle lying between the first and the second station. This arc decreases as the epicycle center approaches the center of the Earth, so that the stationary points are changing.

86. If the arc of the first station is subtracted from the second station, the arc of retrogradation will remain, and if the first station is subtracted from the whole circle, the second station remains, because the arc *ABC* is equal to the arc *ACB*.

87. The mean motus of any planet corresponding to a given time not appearing in the tables is found in this manner: Take the radix[44] corresponding to a collected year. Then take the mean motus from the expanded years corresponding to the intermediate years between the first of the collected years and the year you seek. Subtract this motus from the radix if it be possible. If not, add one full revolution of twelve signs to the radix. The remainder is the desired mean motus of the planet.

88. A mean conjunction or a mean opposition is defined with reference to the mean motus, a true conjunction with reference to the true motus, and a visible one with reference to the visible [planets] and this with attention to sign, degree, and minute.

89. An eclipse digit is defined as the twelfth part of the diameter of the lunar or solar body.

90. The minutes of half duration are the minutes (of arc) through which the Moon moves from the beginning of an eclipse until the middle of it, if it is

44. For the concept of *radix*, see Selection 65.

not totally obscured, or until the beginning of the total eclipse, if it is totally obscured. In the Sun, too, the minutes of half duration are the minutes through which the Moon moves from the beginning to the middle of a solar eclipse.

91. The minutes of half the time of total obscuration are the minutes through which the Moon proceeds from the beginning of a [total] eclipse until the middle. If, therefore, these minutes are divided by the average motion of the Moon in one hour the result is the time used for the motion between these points.

92. The residual minutes or points are defined as the part of the diameter of the epicycle remaining between the aux and the direction of the Moon.

93. The numbers written in the columns of the eclipse tables are arguments of equivalent latitudes, or equivalent latitudes.

94. The great years are computed to the number of terms and the least years to the smallest revolutions. Some are intermediate between the great and least years of the Sun and the Moon. The great years belong to their great revolutions, the least years to their least revolutions, and the mean years to their mean revolutions.

95. The planets are said to be slow and of least speed when they are retrograde, but fast, that is, with increased speed, when they are direct. The planets are said to be numerically increased when the equation of argument is added to the mean motus, and to be numerically decreased when the opposite is true. They are called increased as to their light when they recede from the Sun or the Sun from them, but decreased as to their light when they come nearer to the Sun or the Sun to them.

96. The natures, properties, and operations of the planets and the signs are determined in such a way that one always begins with the Sun, because it is the most noble of the planets, and with Aries as the noblest of the signs, and from noon as the nobler part of the day, and from the site of the equator, which is in the middle of the world.

VII. ON THE LATITUDES OF THE PLANETS

97. The latitude of a planet is defined as its distance from the path of the Sun. The declination of a planet is its distance from the equinoctial line. It follows that the Sun has no latitude but only a declination. This declination of the Sun is found by means of the distance of the Sun's center from the first intersection between the circle of the Sun and the equinoctial line, that is, from the beginning of Aries.

98. The latitude of the Moon is found by means of the distance of the center of the Moon's body from the first intersection between the circles of the Moon and the Sun, that is, from the Dragon's Head, and thus we also find the declination of that degree of the zodiac where the Moon is, and for the same time we find the latitude of the Moon from the zodiac, that is, from the path of the Sun. And if both of them, namely the latitude and the declination, are to the north or to the south, we combine them, and the result is the Moon's declination from the equinoctial; but if they are different, we must subtract the smaller from the greater. In this way the declination is found in the other planets too.

99. It must be noted that the eccentric circle of the Moon always deviates from the path of the Sun in the same way and that its epicycle always lies in the plane of the eccentric, so that the Moon has only one latitude. The other five planets have a double latitude, one by which the epicycle deviates from the eccentric and another due to the eccentric itself, which deviates from the path of the Sun.

100. The latitude corresponding to the epicycle is found by means of a binary table, and the latitude corresponding to the eccentric by means of a quaternary table. A table is called binary[45] because it has two entrances, and quaternary because it has four. A binary table is computed for six signs, that is, half a circle, and a quaternary for a quarter circle. This means that a binary table is computed for six signs, namely a particular table for each; but a quaternary table for three signs, that is, a particular table for one sign, but serving for four signs by equivalence.

101. Because a binary table corresponds to the epicycle, it is entered by the argument, and because a quaternary table corresponds to the eccentric, it is entered by the distance from the ascending node (the Dragon's Head). The latitude recorded in a binary table is the distance of the various parts of the circumference of the epicycle from the circumference of the eccentric, this distance being computed towards the path of the Sun, which is called ecliptica because eclipses of the Sun and the Moon occur upon this line or in its neighborhood.

102. The epicycle is inclined to the eccentric in such a way that the planet is always between the ecliptic and the center of the epicycle, except when the latter is at the Head or the Tail of the Dragon. For then the epicycle lies in the plane of the eccentric. But when the planet is at the aux of the epicycle it has its maximum distance from the path of the Sun, so that we here find the maximum latitude in the table. At places in between, a smaller latitude is found, because the planet has a smaller distance.

45. Binary tables appear in Selection 65.

103. The planetary latitude recorded in the quaternary table is the distance of the circumference of the eccentric from the path of the Sun. This distance is very small in the neighborhood of the nodes, but greatest at places three signs from the latter. In a quaternary table we thus find a small latitude at the beginning and the maximum latitude at the end, with intermediate latitudes in between.

104. Thus it is clear that when these two latitudes have been found, one of them must always be subtracted from the other.

105. In order to prove his mastery, the computer of the tables has refrained from recording the true values of the said latitudes, one of which is always subtracted from the other. Instead he gives equivalent values by means of which you get the same result by dividing one by another, as that resulting from the subtraction of one of the true values from another. For to every subtraction corresponds a certain division, and conversely. For the same thing results if you divide 60 by 30 as if you subtract 2 from 4, and consequently one latitude is divided by another.[46]

106. When the epicycle center is at the nodes the latitudes are zero. Then the epicycle lies in the plane of the eccentric and its center on the path of the Sun. That the values quoted in the tables are not the true ones is seen from the fact that the table of Mercury contains a latitude greater than 6 degrees, so that it would sometimes be outside the zodiac. But its true latitude appears by dividing one of these numbers by the other.

107. What I have said is corroborated by the tables of the real latitudes, if such tables are at hand, in which subtraction or addition takes the place of division. And it is stated that if you divide the second latitude by the first, you are reckoning the latitude from the path of the Sun, but if you divide the first by the second, you are reckoning from the limb of the zodiac, for a latitude of 2 degrees according to one way of reckoning is the same as the latitude of 4 degrees according to the other.

108. The Head and the Tail of the three superior planets are immovable, but the Head and Tail of Venus and Mercury are moving, so that the argument of latitude of the latter is found in a different way from that of the former.

109. The Head of Venus (and Mercury) moves at such a rate that its true place always has the same distance from it [that is, the planet] as the true place of the Head quoted in the canon to the table deviates from the place determined by the mean motus of the Sun and their equated argument. Consequently, we must add their equated arguments to the mean motus of the Sun.

110. The true places of the Heads are by definition computed from Aries according to the order of the signs, that is, Aries, Taurus, Gemini, and so on.

111. The mean places of the Heads are by definition computed in the other direction, that is, Aries, Pisces, Aquarius, and so on, so that the mean motus of the Head, together with its true motus, makes twelve signs. If the mean motus of the Head is subtracted from twelve signs, the true motus remains.

112. The computers of the tables based upon Arim[47] are said to have been Nembroth, Hermes, Ycominus, Ptholomeus, Albategni, Albumazar, Algorismus. Arim has the same distance from the gates of Alexander and Hercules projected upon the equator. It has, namely, a distance of 90 degrees from the gates of Alexander in the east and also of 90 degrees from the gates of Hercules in the west and a distance of 90 degrees from each pole.

113. If you wish to adjust the tables to other localities, you will have to subtract the mean motus of the stars in as many hours as there are between these localities and Arim, that is, subtract or add it in the collected years only.

114. When the planets are equated before or after noon, you can find the ascendant and the hour for the said time. If you wish to do so, you must place the degree of the Sun upon the meridional line [of an astrolabe] and note the place of the almuri upon the degrees of the limb. Let it go 15 degrees forwards or backwards for each of the hours you had before or after noon, and you will find the ascendant for the present hour.

115. If the Moon is in the middle of the heavens and you equate it by means of a table for a certain region, you will know the longitude between the two regions by the difference between the places of the Moon without having to wait for an eclipse.

116. If you take the altitude of the lower limb of

46. The description in this paragraph is of an Indian method foreign to Ptolemaic astronomy. See A. S. Kennedy and W. Ukashah, "Al-Khwārizmī's Planetary Latitude Tables," *Centaurus*, Vol. 14 (1969), 86–96.

47. Arim was the Indian city, Ujjain, which served as the standard meridian for earlier astronomical tables.

the Sun and Moon with the dorsum of an astrolabe, and then the altitude of the upper limb, and notice the displacement of the alidade, you will know the magnitudes of the diameters of the Sun and the Moon, but you must look at the Sun through a cloth.

117. Note that Thabit says that the auges are said to move 7 degrees towards the east during nine hundred years, and the same number of degrees to the west during another nine hundred years.[48] Moreover, they are, according to Albategni, said to move 1 degree in sixty years and four months, and always towards the east. But Alfraganus states that they move 1 degree in one hundred years, and also always towards the east.

118. Note also that as long as the Sun is on that half of its eccentric which is farthest away from the Earth, that is, at the greatest distance, the alidade on the dorsum of the astrolabe is higher than the degree of the Sun upon the rete placed upon the almucantarat at noon. The opposite is true upon the other half of the eccentric. And upon that day when the distance between these two altitudes is greatest, the Sun will be at the aux upon that half of its eccentric first mentioned above. The value of this distance is the same as the eccentricity of the Sun, that is, about 2.5 degrees.

119. If the degree of the Sun is placed among the almucantarat upon the number corresponding to that by which the nadir of the Sun falls below the place beyond which it rests, the hour determined here will be wrong by so much. If, in this way, we have considered the day when the Sun at noon is at the aux in one year and have measured its altitude by the alidade, and if we do the same the next year at the same hour and measure how much the altitude is increased, we will know how much the aux has moved in one year.

120. In this way Albategni is said to have found how much the planetary auges move per year, per month, and per day, and he computed tables of this motion, and he had a great astrolabe of 3 cubits or more.[49]

VIII. THE DETERMINATION OF THE PLANETARY ASPECTS

121. The aspect of the planets can be found in this way. Enter the table of right ascensions of the signs which begins from Aries at the equated length of any planet. The number of degrees found under the sign containing this planet must be retained. Thereupon enter at the equated length of any other planet, and the number of degrees found under its sign must be taken and retained. Of these two numbers taken in this way, the smaller deviates from the greater. If it amounts to the

48. This is probably a reflection of Thabit ibn Qurra's theory of trepidation, as given in his work *On the Motion of the Eighth Sphere*. See J. L. E. Dreyer, *A History of Astronomy from Thales to Kepler*, 2d. ed. (New York: Dover, 1953), pp. 276–277.

49. Some manuscripts of the *Theorica planetarum* contain an additional passage that is lacking in the Copenhagen manuscript from which the present translation was made. Because Professor Pedersen considers this passage to be a later interpolation, I have relegated to this note my translation of it (made from MS. Florence, Ashburnham 208, p. 51, col. 2, to p. 52, col. 2, and an edition of the *Theorica* published in 1531):

"A line drawn from the center of the earth through the center of the planet's body [and on] to the sky reveals the true place of the planet, as is obvious from the radiation of the sun.

"A line drawn from our position through the center of the planet's body [and on] to the sky shows the apparent place of the planet; but it is there only when the planet is in our zenith. For then [that is, when the planet is in our zenith] the line drawn from the center of the earth to the center of the planet's body and the line drawn from our position are the same, and the place in which this line is terminated is the same; at other places, [however,] it is not the same.

"Therefore, the arc cut off between the true place and the apparent place of the planet is called the reflection *(reflexio)* [that is, terrestrial parallax] or difference of aspect *(diversitas aspectus)*. The said reflection occurs sometimes in latitude, sometimes in longitude, sometimes diverse [that is, in both latitude and longitude; and] sometimes it is greater, sometimes smaller. It must be understood that in the planets above the sun, namely Saturn, Jupiter, and Mars, there is no reflection perceptible to the sense. It occurs only in the sun, Mercury, Venus, and the moon. In the sun, a reflection of about 3 minutes is found; and when the sun is near the horizon it [reflection] is a maximum; further this [maximum is also] perceived in the moon [that is, when the moon is at the horizon]. Furthermore, the reflection is wholly in longitude when a planet is on the meridian of the observer; and it is wholly in latitude when a planet is on the circle of altitude; and it is diverse, that is, partly in longitude, partly in latitude, when a planet is neither on the meridian nor on the circle of altitude. And it is sometimes greatest because of one, or both, of two causes. For there are two causes which produce a greater reflection as well as a smaller reflection. One of these is when a planet is more distant from our zenith; therefore, reflection would be greatest near the horizon. The other cause of reflection is when a planet is nearer the earth. Hence, when the moon is at the least distance [from the earth] and near the horizon, the total reflection is 1 degree 44 minutes, which is the maximum of all reflections. When, however, it is at the greatest distance [from the earth], the total reflection is 54 minutes. Moreover, during hours of eclipses, the greatest reflection is 1 degree 4 minutes. . . ."

Approximately three lines of garbled text are omitted.

sixth part of a circle, those two planets have the sixth aspect, because there is a sixth part of a circle between them. If there remains the fourth part of a circle, it will be the fourth aspect; [if there remains half a circle, it will be the second aspect.] And if there remain so many degrees as to make up the third part of a circle, it will be the trine aspect. But if there remains half of the circle, it will be the aspect of opposition. If there remain more or less degrees, the planets have no aspect. If nothing remains, then these planets will be in corporal conjunction. If there remain so many degrees as to make up another splendor, then these planets will be conjoined as to their light and not as to their body.

Explicit.

65 EXTRACTS FROM THE ALFONSINE TABLES AND RULES FOR THEIR USE

John of Saxony (fl. 1327–1355)

Translated by Victor E. Thoren and Edward Grant

Introduction and annotation by Victor E. Thoren

The *Alfonsine Tables* were prepared during the third quarter of the thirteenth century as part of an extensive scientific and literary program sponsored by Alfonso X of Castile. From Spain they passed to Paris, where, in 1327, they were drawn up in an essentially new form and provided with a set of Latin precepts by John of Saxony. In that form they soon spread throughout Europe, to enjoy—first in manuscript, then in numerous printed editions—a reign of three hundred years as the Latin West's authoritative astronomical work.

From a purely technical standpoint the *Alfonsine Tables* are of little interest. They are essentially the tables of the *Almagest*, shorn of their explanatory and justificatory apparatus and recomputed in accordance with minor adjustments in a few of the constants. There is one respect, however, in which the *Tables* are quite distinctive; namely their peculiarly complete utilization of the sexagesimal system. Instead of the usual grouping of days into 365's, they employ powers of 60; and instead of the traditional grouping of degrees into twelve signs of 30 degrees each, they adopt six "physical" signs of 60 degrees each. How truly remarkable these innovations are has emerged only comparatively recently, through the discovery that they are not really Alfonsine at all—that they constitute a departure from the form of the original tables themselves! By whom the modifications were introduced has yet to be ascertained. All that is known is that the Spanish introduction to the original *Tables* (which have not survived in Spain) describes something quite different from what left Paris fifty years later to circulate Europe under Alfonso's name; and that the author of that version, John of Saxony, credits the difference neither to himself nor to his teacher John of Linières. As for the scheme itself, in addition to providing a very tidy symmetry, it allows a most ingenious abbreviation of the task of tabulating the various mean motions—a crucial consideration in an era when all tables had to be hand-copied.

The immediate source of the material presented here is the first printed edition of the *Tables*, published at Venice in 1483.[1] The portion of the work selected is the minimum required for the prediction of a lunar eclipse: about 30 percent of the introductory *canones* and about 20 percent of the complete set of tables. To facilitate comprehension of the steps involved in an eclipse prediction, a sample prediction of eclipses has been included between the translated precepts and the appended tables. Boldface numbers in parentheses inserted within the introductory *canones* refer to illustrative operations in the sample calculation. Obvious misprints have been corrected without recourse to the usual scholarly apparatus.

VIII. *To set out what is to be understood by the term radix,[2] when the radix of this or that motion is mentioned.*

It is to be known that the radix of any *motus*[3] is

1. *Alfontij regis castelle illustrissimi celestium motuum tabule necnon stellarum fixarum longitudines ac latitudines Alfontij tempore ad motus veritatem mira diligentia reducte. Ac primo Joannis Saxoniensis in tabulas Alfontij canones ordinati incipiunt faustissime* (Venice: Erhard Ratdolt, 1483).

2. This is the first of several technical terms which seem to be better left untranslated. As with most of them, it is defined in the text, according to which it would have to be rendered as "epoch value" if it were to be translated. It has been pluralized in the Latin form, *radices*.

3. The Latin term *motus* is used to signify both "movement" and the position resulting from a particular movement. Its most common appearance in the latter sense is as a technical term indicating angular distance from the first point of Aries (see pars. 3, 17, and 41 of

nothing other than the place in the circle of the signs in which that motus stood at the beginning of the era of which it is the radix. For example, in the table of radices[4] of the sun, the radix of the Incarnation of Christ is 4 signs 38 degrees 21 minutes. This means that where this number falls in the zodiac (beginning the reckoning from Aries), there the line of mean motus of the sun was situated at noon of the last day of December[5] (or the beginning of January) at the time of Christ. This should also be understood for all [the rest of] the mean motuses. [On the other hand,] the radix of any argument (when I speak of "argument," I always refer to motion in an epicycle) is the distance of the body of the planet from the mean *aux*[6] of the epicycle at the beginning of the era of which it is the radix. For example, in the table of radices for the argument of the moon, the radix at the Incarnation of Christ is 3 signs 19 degrees, and so on. This means that in the time of Christ at the beginning of January, as I said, the distance of the body of the moon from the mean aux was this much; for just as the mean motuses were reckoned from Aries in the circle of the signs, so the mean arguments are reckoned from the mean aux in the epicycle. . . .

XI. *To find by means of tables the mean motuses of the sun, moon, and the rest of the planets; the mean arguments of the moon, Venus, and Mercury; and the motion of access and recess of the eighth sphere; the motus of the auges and the fixed stars; the argument of latitude of the moon; and the motus of the head of the dragon.*

From the beginning of whatever era you use as reference, up to the hour you wish to consider, reduce the total time to fourths, thirds, seconds, and units of days (1); likewise, reduce the hours and minutes of hours, if you have them, to minutes and seconds of days. This done, write first the radix (2) of that motion which you wish to know for the time of the era in terms of which you are working. For example, if you are seeking the mean motus of the sun, write the radix of the sun; and if you are working in years of Christ, write the radix of the Incarnation. If you want to work to some other years, write the radix corresponding to them. With the radix written aside, enter with the number of fourths into the table of that motion you seek, and what you find in the fourth line after the line of the number (namely in the line entitled "4ths") are signs, then degrees, next minutes, and so forth, according to the order of the fractions (2). Write them down under the chosen radix, each under its own kind: signs under signs, degrees under degrees, and so on. Then enter the same table with the number of thirds, and what you find in the line entitled "3ds" are the signs, next degrees, and so on in order: Write them down under their own kinds in the usual order. Then enter with the number of seconds, and what you find in the line entitled "2ds" are the signs, then degrees, and so on. Then enter the same table with the number of units, and what you find in the line entitled "units" are signs, then degrees, and so on: Write them under the others already written. This done, if you have minutes of the day, enter into the same table with them, and what you find in the first line are degrees, then minutes, and so forth: Write them down under the others, each under its kind. Then enter the same table with the seconds of days, if you have them, and what you find in the first line are minutes of a degree, and so forth: Write them under the others already written. Continue in the same fashion if you have more fractions of time. Now, that these fractions of degrees correspond to any fraction of time is obvious from [the introductory paragraph?][7] of this table.

This done, add everything together column by column, beginning from the smallest fraction. As often as the sum surpasses 60, one unit is to be substituted for them in the place of the next larger fraction. This procedure is to be followed up to

the *Theorica planetarum* in Selection 64.2). Where it is being used in this way, it has been left untranslated and given an anglicized plural, "motuses." Where it is intended in its more general sense, it has been rendered as "motion" or "movement."

4. In the table of radices cited (as well as for each of the other sixteen motions he tabulates), Alfonso provides radices for ten different eras; for instance, those of the Flood, Alexander, Julius Caesar, and the Hegira. For the purpose of the present translation, the radix of the Incarnation of Christ has been extracted from each of the tables relevant to the sun or moon, and all have been combined in the table of radices (Table [A]).

5. Note that Alfonso reckons his year from half a day *before* the beginning of the civil year rather than half a day *after*, as was the usual astronomical custom.

6. In general terms, the *aux* is the point farthest from the earth on any circle—apogee, in modern parlance (see pars. 1, 19, and 20 of the *Theorica planetarum* in Selection 64.2). Adapted from the Arabic, it has been pluralized in the Latin form, *auges*.

7. The text reads *per titulum inferiorem ipsius tabulae*. The treatise opens with "Time is the measure of the motion of the prime mover, as Aristotle puts it in the fourth [book] of the *Physics*. Therefore, when we wish to know a motion, it is necessary that we know the times beforehand; so that with the quantity of time known, we can know the motion corresponding to it."

signs, since 60 degrees equal 1 sign. 6 signs make one revolution or one circle, so if by adding the signs together, more than 6 signs emerge, subtract 6 as often as you can. Set the remainder aside,[8] for it is what you have been seeking.

It should be understood that the radices taken in these tables are for the meridian of Toledo (53), so that by this procedure you have motions only for the meridian of Toledo. Now, if you wish to find the radices of motion for your meridian or any others, you need to know whether your city is farther east or west. And for however much time, seek the motion you want,[9] and subtract it from the motion that has been found if the city is farther east, but add it if the city is farther west. What emerges after the addition or subtraction is the motion you seek for the meridian of your city. Note that you could abbreviate the work considerably by reducing all the radices to your meridian at one time, and it would serve you as long as you live. Note also that you can seek all the mean motuses and mean arguments in the beginning of any year and place them on one list and take that year as your radix. This will greatly reduce your work, because for the whole of this year it will be necessary to enter only with seconds and units (and perhaps, minutes and seconds of days, if you should have minutes and seconds of days).

XII. *To find the time of mean conjunction and opposition of the sun and moon by the tables made for this purpose.*

For the day during which you estimate that conjunction or opposition will occur, reduce the appropriate time interval to fourths, thirds, seconds, and units according to the method already given. This done, write first the radix of the mean elongation (2) of the moon, working in the same way as was taught concerning the mean motions. When you have made all the entries and added the numbers, if exactly 6 signs result for that time, then there is a mean conjunction; if 3, there is a mean opposition. For mean conjunction is defined as the situation of the lines of mean motus in the same place of the zodiac, while mean opposition is when they are in opposite places.

If neither precisely 6 nor precisely 3 signs result, then, if you wish to find conjunction, subtract your result from 6 signs and set the remainder aside; if you wish to find opposition, subtract your result from 3 signs and set the remainder aside. This done, seek the remainder in the table of mean elongations. If there are any signs, seek them in the first line: they are days (3). If, however, you do not find a whole number of signs or degrees exactly, take the nearest smaller number, find the number standing opposite in the line of numbers, and write it aside; for these are days. Then subtract what you took in the table (namely the signs, degrees, and so on) from the number which you had (namely from the signs, degrees, and so on), and with the remainder enter again into the same table. If there are degrees in the remainder, seek them in the first line after the line of numbers; and if you find them there, what was written in the line of numbers are minutes of days: Write them down after the days you have already noted. If, however, there are so many degrees that you cannot find them in the first line after the line of the numbers, seek them (or the nearer lesser number) in the second line after the line of the numbers, in that part where there is zero or a cipher in the first line. Then what is written in the line of the numbers are days: Write them aside under the others already written. This done, if after subtracting that with which you entered the table from your number, there should be a remainder, seek it again in the same table. If there are minutes in the remainder and you can find them in the first line after the line of the numbers, then what is written in the line of the numbers are seconds of days. But if there are so many minutes as cannot be found in the first line after the line of the numbers, then seek them in the second line (namely, in that part where the zero is); and what is written in the line of numbers are minutes of days. The small table written below shows these denominations. Enter in the same manner until the number is used up: It is very much the same operation as where you sought the years in fourths, thirds, seconds, and units from the table.

This done, add the days, minutes of days, and the rest of the fractions that you found by the operations to the time with which you [first] sought the mean elongation. What results from the addition will be the time of mean conjunction or opposition.

If, now, having conjunction, you wish to get opposition in an easier way, or, having opposition, you want to get the following conjunction, then, since both intervals are equal, add 14 days, 45 minutes of a day, 55 seconds, 3 thirds, and 48 fourths of a day to the time of conjunction if you want the

8. The imperative *serva* occurs frequently. It is rendered throughout as "set aside."

9. That is, for the time equivalent to the difference in longitude, find the distance traveled by the planet involved.

following opposition, or to the time of opposition if you want the following conjunction. From this it is clear that, having the first conjunction of the year, all the conjunctions and oppositions of that year can be found by the addition of numbers alone. If you wish to work with this method, seek the first conjunction of the year in this way: Reduce to fourths, thirds, seconds, and units the total time from the beginning of your era up to the beginning of the year whose first conjunction you seek. With them, enter the table in the way already described in this chapter. Subtract what results from 6 signs, and with what remains, enter in the way already given, taking days, minutes of days, and so forth: Set these aside, for these are the days and minutes of days, and so on, from the beginning of the year to the conjunction immediately following. Now add 29 days, 31 minutes of a day, 50 seconds, 7 thirds, and 36 fourths of a day (for that is the time from one mean conjunction to another) to the [time of] conjunction just set aside (5). And if you add this interval to the second conjunction, there results a third; and in the same way, others. If you wish to know oppositions, add to the time of any conjunction the time previously given, namely 14 days, 45 minutes of a day, 55 seconds, 3 thirds, and 48 fourths of a day, since this is the time from conjunction to opposition and from opposition to mean conjunction. If you also want to find quadratures, that is, the distance of a quarter circle between the mean motus of the moon and the mean motus of the sun, add 7 days, 22 minutes of a day, 57 seconds, 31 thirds, and 54 fourths of a day to the time of [mean] conjunction and you will have first quadrature; add this same amount to the time of opposition and you will have second quadrature. You can write these numbers in the margins of the tables so you can find them quickly.

XIII. *To determine whether an eclipse is possible during any conjunction or opposition.*

Seek the argument of latitude of the moon for the time of mean conjunction, according to the rules of Alfonso and John of Linières (as appears in the thirty-third proposition of his work), if you are inquiring as to the possibility of a solar eclipse, or for the time of opposition, if you are checking for a lunar eclipse (4). If the argument of latitude of the moon turns out to be no signs and less than 12 degrees, or 5 signs and more than 48 degrees, or 2 signs and more than 48 degrees, or 3 signs and less than 12 degrees, say that an eclipse would be possible (6). But if you get a result outside these limits, an eclipse is impossible.

XIV. *To master the means of correcting the tables of uniform motion and even constructing them anew when the motion for any day whatever is known.*

You know the motion of one day of the table which you wish to correct, for it is the motion of any given day which is written in the first line of the table. So when you doubt whether some line in the table is correctly copied, look at the line immediately preceding the one whose truth you doubt. Write it down, then write the motion of one day (namely the first line of the table) and add the two together: What results will be the value of the line you doubt. But suppose it is the rectitude of the first line itself that is in doubt. Then take any line whatever in the table, write it down, and take the immediately preceding line and write it under the one previously written. Subtract it from the upper, and what remains is the motion of one day, namely the first line of the table. If they agree, fine; if they do not agree, however, take two successive lines in another section, subtract one from the other, and check agreement until you have certified the motion of one day, since it is not likely that the table will be erroneous in many places. And when you have certified it, you will be able to correct the whole table by the said motion of one day; if it were all false, you could even make a whole table from the beginning, in the following manner. Write the motion of a day first, then double it: the doubled value is the motion of two days. To this, add the motion of one day, and there will result the motion of three days; add again the motion of one day, and there will result the motion of four days. Do this until you complete sixty lines and you will have finished the whole table.

XV. *To find the true place of the aux of any planet by the tables.*

Seek first the motus of the auges and fixed stars according to the method given for the mean motuses, but do not take any radix: Set it aside (8). Then seek the motion of access and recess of the eighth sphere and add it to the radix of the eighth sphere. This done, if there are any signs here, resolve them into degrees and add the degrees to the others, if there are degrees in addition to the signs. Then enter the table of equations of access and recess, and find the given number of degrees in the line of the numbers (9), if there are less than 90 degrees. If there are more than 90, however, reckon in backward or contrary order; that is, if you have 91, take 89, and if 92, take 88, if 93, 87, and so forth, back to the beginning of the table, where the

equation is null for 180 degrees. If there are more than 180 degrees, begin at the beginning of the table; thus 181 will be 1, 182 will be 2, and 183, 3, and so forth up to 90 again, which will be 270 degrees. But if there are more than 270 degrees, work backward again from the end of the table towards the beginning; that is, for 271 take 89, and for 272 take 88, and so forth right to the beginning of the table, which will be 360, where the equation is again null. But you will not need this retrograde computation in our lifetime, because it will be many years before 90 degrees accumulate.

Seek, then, the number of degrees in the lines of the numbers, and see how many degrees, minutes, and seconds stand opposite it. Write them down in two places. This done, if you have any minutes beyond the degrees, enter in the same way again with one more than the number of degrees with which you entered. That is to say, if you entered first with 67 degrees, and if there are some minutes beyond 67 degrees, enter with 68 degrees. See what equation stands opposite, and write it down. This done, see whether the first equation (namely, that which you took first) is greater than the second, or the contrary. Subtract the smaller from the greater, and what remains is called the difference between the two equations. Of this difference, take the proportional parts according to the proportion of the minutes beyond degrees to 60.

The proportional parts are found in this fashion. If there are minutes and seconds in the difference between the two equations, reduce them to the same denomination (namely to seconds): This is the first number. Then, in the same way, reduce those minutes and seconds beyond the degrees, with which you did not enter the tables, to seconds: This is the second number. Then multiply the second number by the first, divide what results by 60, and the quotient will be the proportional parts. Add these proportional parts to the first equation taken if the second was greater; but subtract them from the first if the second was smaller. What emerges after the addition or subtraction is the equation of the motion of the eighth sphere. This done, add this equation of the eighth sphere to the motion of the auges and fixed stars (which I directed to be set aside) if the degrees with which you sought the equation were fewer than 180, but subtract if they were more. Add what results after the addition or subtraction to whatever radix of the aux you choose, and you will have its true place (10). Note that this operation is necessary for determining the motion of the planets, because for no planet except the moon can the true place be found without the motion of the aux, as will appear from what follows.

XVI. *To find the true place of the sun by the tables.*

Seek first the mean motus of the sun and set it aside (11). You know the aux of the sun from the preceding chapter. Subtract it from the mean motus of the sun if you can (that is, if the mean motus is greater than the aux). But if you cannot subtract it (that is, if the aux is greater than the mean motus), add one revolution, or 6 signs, and then subtract the aux. What emerges after the subtraction is the argument of the sun (12). Note here that in this chapter and in all succeeding ones, whenever the rules dictate the subtraction of such a number from such a number, it will always be necessary to subtract in that way, whether the number from which the subtraction is to be made is greater or smaller than that which is to be subtracted. If that from which the subtraction is supposed to be made is smaller, then one revolution must be added and the subtraction made. This should be noted very carefully, since the situation will often arise in your work.

Having the argument of the sun, enter with it into the table of equations of the sun, and seek the same number in the lines of numbers (13). If the whole number of the argument can be found exactly (that is, if there are only degrees and signs, without minutes and so forth), you will find the equation of the sun opposite. Subtract the equation from the mean motus if the argument of the sun (by which you found the equation) is less than 3 signs, but add it to the mean motus if the argument is more than 3 signs. What emerges after the addition or subtraction is the true place of the sun in the ninth sphere. If great precision is not necessary, you can proceed in this fashion. See whether there are any minutes and seconds beyond the signs and degrees in the argument of the sun. If there are more than 30, add 1 degree in place of them to the degrees of the argument: if there are less than 30, do not worry about them. Thus, with one entry, you have the equation, without the necessity of computing proportional parts. The method is sufficiently precise for knowing the degree of the sun's position and can be used in the same way for the rest of the planets. But since this does not suffice if you wish to work precisely, it is necessary to give the complete method. . . . [Details of interpolation procedure follow.]

XVII. *To find the true place of the moon by the tables.*

Seek first the mean motus of the moon (26) and set it aside. Seek also the mean motus of the sun and subtract it from the mean motus of the moon. Double what remains, and what results will be the mean center of the moon (27); for the *Theorica* says that if the distance between the sun and moon is doubled, the distance between the aux of the eccentric of the moon and the mean motus of the moon is obtained, and this distance is called the mean center of the moon. Set it aside with the mean motus. Then seek the mean argument of the moon (30) and set it aside also. These things found and set aside, enter into the table of equations of the moon with the mean center, and seek the equation of center (28), with a double entry [by interpolation] if there are any minutes in the [mean] center beyond the signs and degrees, as set out in the directions regarding the sun. In the same place (namely where you entered with the mean center), look up the proportional minutes (29) which stand opposite, and set them aside. Then see whether the mean center is less than 3 signs, or more; if less, add the equation of the center to the mean argument, but if more than 3 signs, subtract the same equation from the mean argument. What results from the addition or subtraction will be the corrected[10] or true argument of the moon (31), which is the distance of the body of the moon from the true aux of the epicycle.

This had, enter with it into the same tables, seeking the corresponding number in the lines of numbers, and you will find the equation of the argument standing opposite. Set it aside: it is called the first equation. Then enter with one degree more, and again take the equation standing opposite, which is called the second equation. Subtract the lesser from the greater, and the difference of the two equations will remain, of which you take the proportional parts (namely, the proportion of the minutes beyond the degrees to 60). Add the result to the first equation if the second is greater, but subtract from the first if the second is smaller. What remains after the addition or subtraction is the first examined[11] equation of the argument (33).

In the same place extract also the variation of the diameter of the small circle (32), making a double entry if it is necessary (that is, if there are any minutes, and so forth, beyond the degrees in the corrected argument). Then take the proportional part of the variation of the diameter according to the proportion of the proportional minutes (which I earlier told you to set aside) to 60 (34). You know how to do this from the rules for the sun. Add the proportional parts to the first examined equation of the argument, to get the second examined equation of the argument, which you add to the mean motus of the moon if the corrected argument is more than 3 signs, and subtract if less. For the *Theorica* says that if the moon is in the half of the epicycle which one regards as west on the right-hand side, the mean motus is greater than the true, so that the equation of the argument is to be subtracted; in the other half the contrary obtains, so that it is to be added. Add or subtract as necessary, then, and what results will be the true place of the moon (35) in the ninth sphere.

XVIII. *To find the true place of the head of the dragon by the tables.*

It should be known that the motion of the intersections of the lunar eccentric with the ecliptic (which is called head or tail of the dragon) is not like the motion of the planets but is in the opposite direction, as the *Theorica* says. The head of Genzahar is said to move as much in mean motus against the firmament as it (in reality) goes with the firmament,[12] so that by subtracting the mean position of the head of the dragon from 6 signs, there remains its true place computed according to the succession of the signs. Therefore, when you want the true place of the head of the dragon, find its mean motus in its table, subtract it from 6 signs, and what results from the subtraction is the true place of the head of the dragon reckoned according to the succession of the signs from Aries. Having, then, the true place of the moon by a preceding rule, and the true place of the head of the dragon by this rule, you can determine the true

10. The phrase is *argumentum equatum sive verum.* "Corrected" seems to be the sense of *equatum*, which appears frequently in astronomical literature up through the seventeenth century.

11. The phrase is *equatio argumenti primo examinata.* "Corrected" seems to be the sense of *examinata*, but if that were meant, *equata* might have been used. Perhaps "approximated" is what is intended.

12. Thus, the ascending node (☊) actually slides *backward* (in the direction of the nightly movement of the stars) toward the vernal equinox (♈). This means that the longitude of the node (☊ − ♈) is a *decreasing* quantity. The straightforward way of handling it would be to establish an epoch value for (☊ − ♈), sum up the regression during the time interval involved, and *subtract* the (backward) movement of the node (△ ☊) from the epoch value to get the current value (☊ − ♈ − △☊). John does it by using 360° − (☊ − ♈) as the epoch value, *adding* △☊, then subtracting the whole [360° − (☊ − ♈) + △ ☊] from 360°, to get (☊ − ♈ − △ ☊).

place of the moon's latitude in this way: Subtract the true place of the head of the dragon from the true place of the moon, and the true argument of latitude of the moon will remain.[13] The true argument of latitude of the moon is defined as the distance of the body of the moon from the head of the dragon. It is called the moon's argument of latitude because by means of it the latitude of the moon from the ecliptic is found. . . .

XXVI. *To find the extent and duration of a solar eclipse from the tables.*

According to the teaching of the rules established for the purpose, seek a mean conjunction of the sun and moon for which the possibility of an eclipse exists (6). This found, seek the mean motus of the sun (11), the mean motus of the moon, and the mean argument of the moon (14) for the time of mean conjunction. You do not need to worry about the argument of latitude yet, whatever the other rules say. Seek also the mean motion of the eighth sphere, so that you can get the aux of the sun (10) according to the rules established for this purpose. Upon finding these things, subtract the aux of the sun from the mean motus, so that the argument of the sun remains (12). With it, enter the table of solar equations, seeking the number matching the argument in the lines of numbers, and finding the equation opposite it (13). Subtract [it] from the mean motus of the sun if the argument of the sun is less than 3 signs, but add if it is more. What emerges from the addition or subtraction will be the true place of the sun for the time of mean conjunction: Set it aside. Then enter the table of lunar equations with the mean argument of the moon, seeking its like in the lines of numbers (15). After finding the equation of the argument standing opposite, subtract [it] from the mean motus if the argument is less than 3 signs, but add if it is more than 3 signs; and what results is the true place of the moon.[14]

Examine the true places of the sun and moon. If they agree in signs, degrees, and minutes, true and mean conjunction occur at the same time. But if they do not agree, subtract the lesser from the greater and set [what is termed] the distance [*longitudo*] aside (16). See of what it is the distance: It is the distance of that [body] which precedes the other[15] in the order of the signs (16). To this distance add a twelfth part of itself, and divide the whole in half. Add the half to the mean argument of the moon if the distance is "of the sun", but subtract from it if the distance is "of the moon." What results after the addition or subtraction is the argument of the moon,[16] corrected for the purpose of finding the motion of the moon in 1 minute of the day (17): Set it aside. Then enter with the argument of the sun into the table of the motion of the sun in 1 minute[17] of a day, and what you find standing opposite is the motion of the sun in 1 minute of a day (19). Likewise, enter with the argument of the moon (corrected, as I said) into the table of the motion of the moon in 1 minute of a day, and what you find standing opposite is the amount the moon moves in 1 minute of a day (18). Then subtract the motion of the sun in 1 minute of a day from the motion of the moon in 1 minute of a day, and there will remain the fraction or "excess" per minute of a day (20). For every degree of distance between the sun and moon, add to this excess 1 minute of motion in 1 degree, and for

13. That is, subtract ☊ − ♈ from ☽ − ♈ to get ☽ − ☊.

14. The involved computation of the moon's true place (related in sec. XVII and shown in the sample calculation) is here legitimately abbreviated by the fact that the computation is for syzygy. Since twice the elongation is 0 degrees, steps (27) to (29) and (31) are eliminated.

15. At another place John elaborates: "Thus the distance is *of the sun* if the moon has not yet reached the sun in conjunction or [if it has not yet reached] the nadir of the sun in opposition. It is *of the moon* if the moon precedes the sun in conjunction or [precedes] the nadir of the sun in opposition."

16. The ultimate object of this paragraph is to estimate the time from mean to true conjunction (or opposition). During that time t, the moon, traveling at a velocity V, will move not only through the "distance" D, but also through an extra angle d resulting from the velocity of the sun v during t.

Thus, $Vt = D + d$, where $d = vt$.

Then, $D = Vt - vt = t(V - v)$; so $t = \frac{D}{V - v}$.

With D known, all that is necessary to get t is a knowledge of the velocities of the sun and moon at the time in question. Both vary with position in the epicycle; and the moon moves sufficiently fast in the epicycle to necessitate an attempt at determining its *average* position (and, hence, its average velocity) through the interval involved. The *longitude* through which the moon moves is $D + d$; in the short term it can be used as an approximation of the *argument* through which the moon moves. From the equations above, $d = vt = v[D/(V - v)]$. With V and v equaling roughly 13 degrees and 1 degree, it is clear that the approximation $d = D/12$ is a very reasonable one. Then half of $d = D/12$ added to (or subtracted from) the mean argument of the moon will approximate the position of the moon in the epicycle halfway through the overtaking period.

17. Omitted for the sake of abbreviating the tables. The *hourly* velocities (that is, motions in one twenty-fourth rather than one sixtieth of a day) given in Table [P] are used in the sample calculation.

every second, add a third[18] if the moon is in the lower part of its epicycle; but subtract if it is in the upper part of its epicycle. What remains will be the corrected excess, which is to be divided into the distance between the sun and moon (21): The quotient will be the number of minutes of the day between mean and true conjunction. (If the distance is smaller than the excess, multiply the distance by 60, then divide by the excess, and what results will be seconds of the day. If anything is left over, multiply by 60 and divide by the excess, and what results will be [units of] the next smaller fraction. You can do this as many times as you wish.) The minutes and seconds are to be added to the time of mean conjunction if the distance [you found] was of the sun, but subtracted if it was of the moon. What results will be the time of true conjunction, very closely approximated indeed, if you have done the work well.

For this time, seek the mean motus of the sun (24), and the mean motus (26), mean center (27), and mean argument (30) of the moon. Then correct the sun and moon with all the precision you can; and if they agree in signs, degrees, and minutes, that will suffice. If they do not agree, however, subtract the lesser from the greater, leaving the distance (35), which you set aside. Then, to the mean motus of the sun that you just now found, add the movement of the sun in 1 second of a day;[19] do the same to the argument and correct the sun as before. This done, subtract the place [motus] of the sun found previously from the place found just now, and what results will be the motion of the sun in 1 second of a day. Do the same thing with the moon: That is, correct it by 1 second of a day after the time for which you first figured it, then subtract the first value from the second to get its motion in 1 second of a day. This done, subtract the motion of the sun in 1 second of a day from the motion of the moon in 1 second of a day to get the excess (37), by which you divide the distance to get seconds of a day (38). If anything remains, multiply by 60 and divide again by the same number as before to get thirds of a day, and so on as far as you wish. When this is finished, add the time of this division to the time of true conjunction already found if the last distance was of the sun; but subtract it if it was of the moon. What results from the addition or subtraction will be the time of true conjunction uncorrected for the equation of days.

For this [time], then, seek the places of the sun and moon precisely corrected (39); and seek also the mean motus of the head of the dragon (42) for this time, which you add[20] to the true place of the moon (41). What results will be the second corrected argument of latitude of the moon (42): Set it aside. Then reduce the fractions of a day that you have for the time of truest conjunction to hours and fractions of an hour (7), and add them (46) to the equation of days, which is found in this manner. Enter the table of the equation of days with the degrees of the sun, using a double entry if it is necessary (that is, if there are any minutes beyond complete degrees in the place of the sun). And for every degree found here, take 4 minutes of an hour (40) and for every minute, 4 seconds; and for every second, 4 thirds. Add[21] them to the hours, minutes, and so on, of truest conjunction, and you will have the time of truest conjunction corrected for the equation of days. . . . [The rest of the details of solar eclipse prediction follow. Only what has already been presented is necessary for understanding the procedure for computing lunar eclipses.]

XXVIII. *To find the extent and duration of lunar eclipses.*

Seek first mean opposition (namely the one for which you found the possibility of a lunar eclipse) and then true opposition, working in exactly the same way as was said for conjunction, except that

18. The rule of thumb given here is not only garbled in the text, but is also somewhat different from that given in an earlier (omitted) discussion. What is intended is a correction of 1 part in 3600, less 1; for example, 4 seconds for a distance of 5°. It is copied from the Toledan Tables, in which the original rule of al-Battani (simply 1 part in 3600) has been slightly corrupted. Obviously, the correction is so small as to be scarcely worth considering, but it is theoretically valid. Because of the eccentricity of Ptolemy's lunar orbit, the moon is very slightly closer to the earth (and hence moves somewhat faster) when just outside of syzygy than in syzygy itself.

19. This operation is a purely academic exercise. It is unlikely to produce a prediction differing by as much as 1 second from one derived from use of the table of the sun's motion per minute of the day. The value used in arriving at (37) is one sixtieth of the hourly velocity tabulated in [P].

20. In section XVIII (and see n. 12) the mean motus of the head of the dragon is defined as the *retrograde* arc from the first point of Aries to the ascending node: ♈ − ☊ instead of the usual ☊ − ♈. This means that it must be added to, rather than subtracted from, the place of the moon (☽ − ♈) to get the argument of latitude (☽ − ☊).

21. In order to render all the corrections additive, the table has been augmented by a small portion of what would rightfully be the radix of the sun.

in finding conjunction it is the distance between the sun and moon that is sought, while for opposition it is the distance between the nadir of the sun and the body of the moon that is sought. The work differs in no other way. If I described here the method of working, I would do nothing but repeat what was said.

Having the time of truest opposition, then, seek for that time the mean motus of the head of the dragon, and add it to the true place of the moon found for the time of truest opposition. What results is the second corrected argument of latitude. Then seek the equation of days by entering with the position of the sun into the table made for this purpose. Reduce the equation to fractions of hours, as was said concerning conjunction, and add these fractions of hours to the time of true opposition. What results is the time of true opposition corrected by the equation of days. This you take as the time of mid-eclipse (46).

With the second corrected argument of latitude (42), enter the table for lunar eclipses at greatest distance (43) and extract the points of eclipse which stand opposite: Use a double entry if it is necessary, following the same method given for solar eclipses. At the same time, take the minutes of *casus* and the minutes of *mora*,[22] if there is any mora; or only the minutes of casus if there are no minutes of mora (for if 00 appears in the line entitled "mora," it has no mora). Now, it is to be understood that mora is defined as the time from the beginning of total obscuration (namely when the whole moon has entered the shadow) up to the beginning of the return of light. And what is called the minutes of mora is the space of the heavens traversed by the moon's own motion from the beginning of mora up to mid-eclipse. The minutes of casus are defined as the space which the moon traverses from the beginning of eclipse up to the beginning of mora. Having the points of eclipse, the minutes of casus, and the minutes of mora for the greatest distance, then set them aside as what was to have been found if the moon was in the true aux of its epicycle. But if the moon was not in the aux of its epicycle, enter with the same argument of latitude into the table for lunar eclipses at the least distance (44), and take here the points of eclipse, minutes of casus, and minutes of mora, by working in the same manner as you did with the greatest distance. Set them aside as the things to have been found if the moon was in opposition to the true aux of its epicycle. In short, if the moon was in the true aux of its epicycle, enter only at the greatest distance; while if it was in opposition to the aux of the epicycle, enter only at the least distance. But if it was neither in the aux nor in opposition to the aux, enter both distances in the way already said. Then subtract the points of eclipse found for the greatest distance from the points found for the least distance, and the minutes of casus from minutes of casus, and minutes of mora from minutes of mora, and set the differences aside, each by itself. Then, with the corrected argument of the moon for the hour of true opposition, enter the table of proportions tabulated in increments of 2 degrees, and (using a double entry if necessary) take the proportional minutes standing opposite (45). Multiply these proportional minutes by each difference previously set aside, and divide by 60 to get the proportional part of each of the quantities. Add these to the points of eclipse, minutes of casus, and minutes of mora found for the greatest distance, each to its own kind (namely to the proportional parts of the difference of points, add the points; and to the proportional parts of the minutes of casus, add the minutes of casus; and to the proportional parts of the difference of the minutes of mora, add the minutes of mora). You will then have all these quantities corrected according to the distance of the moon from the earth.

Now, divide the minutes of casus (as just corrected) by the excess of the moon in 1 hour, and the result will be hours. And if there is any remainder, multiply by 60 and divide by the same thing as before to get minutes of an hour: and if something still remains, multiply again by 60 and divide as before to get seconds of an hour. This done, you will have the hours, minutes, and seconds from the beginning of eclipse up to the beginning of mora (47), if there is going to be any mora; or from the beginning of eclipse up to the middle, if there will be no mora. In the same way, divide the minutes of mora by the excess of the moon in 1 hour, and there will result the hours, minutes, and seconds (or minutes and seconds, alone, if there are no hours) from the beginning of mora up to mid-eclipse (48). This done, if you do not have any mora, subtract the hours from the beginning of eclipse up to the middle from the time of truest opposition corrected by the equation of days. What remains will be the time of the beginning of the eclipse, and if you add these same hours to the time of truest opposition you will get the time of the end

22. As will appear from the definitions accompanying them, *casus* and *mora* refer to the periods of partiality and totality of the eclipse.

of the eclipse. However, if the eclipse will have mora, add the hours, minutes, and seconds from the beginning of eclipse to the beginning of mora to the hours, minutes, and seconds from the beginning of mora to the middle of the eclipse. Subtract the whole from the time of truest opposition to get the time of the beginning of the eclipse (49); and if you add the same thing, there will result the time of the end of the eclipse (50). Then subtract the time from the beginning of mora to mid-eclipse from the time of truest opposition, and there will remain the beginning of mora; and if you add the same, that will be the end of mora.

If you double the time from the beginning of eclipse to mid-eclipse, the result will be the total duration of the eclipse from beginning to end. If you double the time from the beginning of mora to mid-eclipse, the result will be the total mora (that is, the length of time the moon stays completely in the shadow). Then, if you wish to make a diagram, multiply the hours from the beginning of eclipse up to mid-eclipse by the motion of the moon in 1 hour (51), and subtract what results from the true place of the moon found for the time of truest opposition, and also from the argument of latitude (52), so that the true place of the moon for the time of the beginning of the eclipse will emerge. From the argument of latitude, subtract the motus of the head of the dragon[23] to get the argument of latitude corrected for the time of the beginning of the eclipse; and if you add the same thing that you just now subtracted, you will get these things for the time of the end of the eclipse. Moreover, you find the latitude of the moon for these three times (52) by entering the table of latitudes of the moon with these three arguments of latitude (namely for the beginning, middle, and end of the eclipse). Then you can make a diagram according to the teaching of Master John of Linières. . . .[24]

Prior to the seventeenth century, *canones* such as the foregoing constituted the ultimate form of help for the would-be calculator. The greatest concession found in such theoretical works as the *Almagest* or *De Revolutionibus* was an occasional set of precepts for carrying out an isolated operation. The following sample exercise is thus an anachronism. As seventeenth-century astronomers were to admit, however, even the experienced calculator can benefit from an explicit display of the various conventional usages adopted by a particular tabulator[25] and for the uninitiated such a picture is, indeed, worth a thousand words. In order to maximize its value while minimizing the commentary involved, the sample calculation (see Table 2) has been harnessed with a rather complicated reference system. The first element is a bold face marginal number labeling the various operations and results, by means of which the reader can move from the textual description of a given procedure to the illustration of it. The second element enables the reader to move in the opposite direction: If the short comments in the lefthand column are not sufficient to convey the drift of the computation, the last previous roman numeral in the same column will direct him back to the appropriate section in the text. The third element indicates the source of the numbers actually involved in the calculation: A letter in brackets identifies the table from which the corresponding number in the fourth (computation) column derives, while a parenthetical boldface number signifies that the number opposite it in the fourth column has simply been carried forward from some previous operation. In the computation column the prevalent operation is addition. Where a subtraction is involved, a minus sign has been entered for the appropriate number. Other operations (except rounding and interpolating) have been indicated in the explanatory columns.

23. While strictly accurate, this correction is so small that it has been omitted from the sample calculation.

24. The printed edition omits the concluding tribute "*a quo habeo scientiam meam*" found in most manuscripts.

25. See, for example, *Histoire de l'Astronomie du Moyen Age* (Paris, 1819), pp. 251–252, where Delambre, intending to provide illustrative computations for September 20, 1476, actually obtains positions for the following year.

Table 2

Sample Investigation of Eclipses for the Year 1511

	XI	[B]	1000^{y}	1	41	27	30
			500		50	43	45
			10		1	0	52
(1)	days: epoch to beginning 1511			2	33	12	7^{d}
		[A]	Radix	3	24°	25	50
		[J]	2 4ths	53	23	15	44
			33 3rds	17	40	53	49
			12 2nds	26	17	20	20
			7 units	1	25	20	7
(2)	elongation of moon at new year			0	11°	15	50
	XII		3^{ss}−(2)	2	48°	44	10
		[J]		2	38	28	47
					10	15	23
					10	9	32
(3)	time to first mean opposition					5	51
						5	53
	XIII		Rx	3	34°	28	43
		[K]	2	31	18	44	52
			33	34	6	39	20
			12	38	45	7	52
			20^{d}	4	24	35	13
			50		11	1	28
			29			6	24
(4)	argument of latitude at opposition			4	20°	43	52
		[K]	29^{d}	6	23°	39	4
			31		6	50	7
			50			11	1
(5)	monthly increase arg. lat.			0	30°	40	12

13^{d} 50 29

In three months the argument of latitude will fall within the eclipse limits. As it happens, however, the resulting eclipse occurs during daylight hours in Europe. Six months later, the moon approaches the other node: that is,

(6) (4) + 9 × (5) = 2^{ss}57°.

			(3)			13^{d}	50	29
			(29^{d} 31 50) times 9 mos.	4	25	46	30
(7)	since 4 33^{d} = end of Sept.			4	39^{d}	37	or
		[C$_2$]	14^{h} 48^{m}	of	7	Oct.
	time of mean opposition		2 33 16	46^{d}	37	
	XV	[F]	2	0	8°	41	23
			33		2	23	23
			16			1	10
			46				3
(8)	auges and fixed stars				11°	5	59
			Rx	5	59°	12	34
		[E]	2	1	0	49	38
			33		16	43	39
			16			8	7
			46				23
	eighth sphere			1	16°	54	21
(9)	equation	[D]			+ 8°	45	49
			(8)		11	5	59
			Rx	1	11	25	23
(10)	aux ⊙			1	31	17	11

Table 2 (continued)

XVI

	Rx	4	38°	21	1
[G]	2	16	39	14	38
	33	31	34	47	32
	16	15	46	13	14
	46^d		45	20	23
	37			36	28
		3	24°	33	16
(10)		− 1	31	17	11
		1	53°	16	5
[M]			− 2°	1	9
	Rx	3	19°	0	15
[I]	2	47	55	0	42
	33	8	40	37	42
	16	29	2	23	20
	46^d	10	0	59	22
	37		8	3	24
		3	6°	4	45
[N]			+ 0°	34	15
(13)			− 2°	1	9
			2°	35	24
(16) ÷ 12				+ 12	57
			2°	48	21
			− 1	24	10
(14)		3	6°	4	45
		3	4°	40	35
[P]				36	20
[P]				2	29
				33	51
			− 4^h	$35\frac{1}{2}^m$	
[C_1]			0^d	11	29
[G]	11′			0°	10	51
	29″					29
				− 0°	11	20
(12)		1	53	16	5
		1	53°	4	45
[M]				− 2°	1	13
(11)		3	24°	33	16
(22)				− 0°	11	20
		3	24°	21	56
(23)				− 2°	1	13
		3	22°	20	43
	Rx	2	2°	46	50
[H]	2	10	2	30	22
	33	49	15	41	21
	16	30	49	20	20
	46^d	10	6	6	51
	37		8	7	32
		0	24°	33	16
[H]	11′		− 2	24	56
	29		−	6	22
		0	22°	1	58
(24)		− 3	24°	21	56
		2	57°	40	02
		5	55°	20	04
[N]				− 0°	42

(11) motus ☉
(12) arg. ☉
(13) eq. ☉

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(14) arg. ☽
(15) eq. ☽
(16) "distance," "of the moon"
(17) corrected arg. ☽
(18) vel. ☽
(19) vel. ☉
(20) excess
(21) correction (16) ÷ (20)

(22) corrected arg. ☉
(23) eq. ☉
(24) mean ☉
(25) true ☉

XVII

(26) mean ☽
mean ☉
(27) 2 (☽ − ☉)
(28) eq. center

Table 2 (continued)

(29) prop. parts
(30) mean arg. ☽
(31) true arg. ☽
(32) div. diam.
(33) eq. arg.
(34)

true ☽

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(35) "distance"
(36) vel. ☽
vel. ☉
(37) excess
(38) correction

[N]					00
[I]	11′	0	2°	23	43
	29			6	19
			− 2°	30	2
(14)		3	6	4	45
		3	3°	34	43
(28)			− 0°	42	
		3	2°	53	
[N]				0°	11
[N]				+ 16′	17″
(29) × (32)				00	00
(26)		0	22°	1	58
		0	22	18	15
(25)		− 3	22	20	43
			− 0°	2	28
[P]					36″22 per^m
[P]					2″29 per^m
					33″53 per^m

(35) ÷ (37) + 4^m or 0^d 0 10

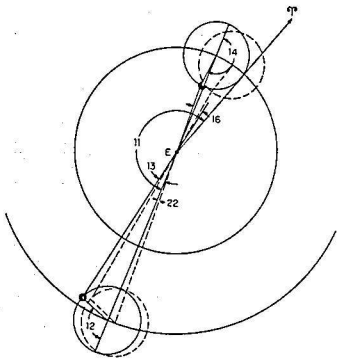

Table 2 (continued)

	movement ⊙	(25)		+ 0°	0	10
				3	22	20	43
(39)	longitude			3	22°	20	53
	mid-eclipse						
(40)	eq. of days	[Q]		7°	36′	or	30½m
(41)	longitude ☽	(39)		0	22°	21	
	mid-eclipse	Rx		1	31	56	
	XXVIII	[L]	2	21	16	15	
	mean motus		33	44	50	58	
	of the head		16		50	50	
	of the dragon		46d		2	26	
			26			1	
(42)	arg. lat.			2	54°	47	

	distance in epicycle	points		casus		mora	
(43)	greatest [S]	10	50	44	19	0	0
(44 a, b, c)	least [T]	13	04	45	20	11	57

(45) From (31) the table of proportions (R) indicates (59p 57′) that no further interpolation is necessary. The ☽ is essentially at its least distance, so the values of (44) obtain.

	mean opp.	(7)	7 Oct.	14h	48m
		(21)		— 4h	35½m
		(38)		+	4m
		(40)		+	30½m
(46)	MID-ECLIPSE		7 Oct.	10h	47m
(47)	half-duration of partiality	(44b) ÷ (37)		1h	20m
(48)	half-duration of totality	(44c) ÷ (37)		0h	21m
(49)	BEGIN ECLIPSE		7 Oct.	9h	6m
	BEGIN TOTALITY	(47)		1	20
				10h	26m
	END TOTALITY	2 × (48)		0	42
				11h	8m
(50)	END ECLIPSE	(47)		1	20
				12h	28m
(51)	half-distance of eclipse	(47 + 48) × (36)		1°	1′

		BEGINNING	MIDDLE	END
	longitudes of eclipse	0 21° 20	0 22° 21	0 23° 22
(52)	args. of latitude	2 53° 46	2 54° 47	2 55° 48
	latitudes [0]	32′ 32″	27′ 15″	21′ 56″

Copernicus (*De revolutionibus orbium coelestium* IV.5) "painstakingly observed" this eclipse and found that it began 1½ hours before midnight and ended 2½ hours after midnight.

(53) When corrected by 1h36m, to account for the 24° difference in the longitudes of Frauenburg and Toledo, these times become 9h16m and 12h44m.

Table [A] of radices for the incarnation of Christ.

Radix	S	G	m	s
Movement of the eighth sphere	5	59	12	34
Movement of the aux of the sun	1	11	25	23
Motus of the sun	4	38	21	1
Motus of the moon	2	2	46	50
Argument of the moon	3	19	0	15
Argument of latitude of the moon	3	34	28	43
Elongation of the moon	3	24	25	50
Movement of the head of the dragon	1	31	55	53

Table [B] for converting years and months into days.

Years	Days 4ths	3ds	2ds	Units	*	Years	Days 3ds	2ds	Units	*		Normal Years	Leap Years
40	0	4	3	10		1	0	0	5	15	January	0 31	0 31
60	0	6	5	15		2	0	1	10	30	February	0 59	1 0
80	0	8	7	0		3	0	1	16	15	March	1 30	1 31
100	0	10	8	45		4D	0	1	21	0	April	2 0	2 1
200	0	20	17	30		5	0	2	26	15	May	2 31	2 32
300	0	30	26	15		6	0	3	31	30	June	3 1	3 2
400	0	40	35	0		7	0	4	37	45	July	3 31	3 32
500	0	50	43	45		8D	0	4	42	0	August	4 32	4 33
600	1	0	52	30		9	0	5	47	15	September	4 63	4 64
700	1	11	1	15		10	1	0	53	30	October	5 4	5 5
800	1	21	10	0		11	1	1	58	45	November	5 34	5 35
900	1	31	18	45		11D	1	2	3	0	December	6 5	6 6
1000	2	41	27	30		12	1	2	9	15			
2000	5	23	55	0		24	1	5	13	30			
3000	8	4	22	30		35	1	8	17	45			
4000	10	45	50	0		36D	1	10	24	0			
5000	13	27	17	30		37	1	13	28	15			
6000	16	8	45	0		48	1	16	34	30			
7000	18	50	12	30		49	1	19	39	45			
8000	21	31	40	0		50D	2	1	45	0			

*This column for tabular symmetry only; not to be used in computation

Table [C1] for finding minutes of a day and their fractions from minutes of an hour and their fractions.

m	M	S	m	M	S	m	M	S
1	0	2 30	31	1	17 30	1	0	24
2	0	5 0	32	1	20 0	2	0	48
3	0	7 30	33	1	22 30	3	1	12
4	0	10 0	34	1	25 0	4	1	36
5	0	12 30	35	1	27 30	5	2	0
6	0	15 0	36	1	30 0	6	2	24
7	0	17 30	37	1	32 30	7	2	48
8	0	20 0	38	1	35 0	8	3	12
9	0	22 30	39	1	37 30	9	3	36
10	0	25 0	40	1	40 0	10	4	0
11	0	27 30	41	1	42 30	11	4	24
12	0	30 0	42	1	45 0	12	4	48
13	0	32 30	43	1	47 30	13	5	12
14	0	35 0	44	1	50 0	14	5	36
15	0	37 30	45	1	52 30	15	6	0
16	0	40 0	46	1	55 0	16	6	24
17	0	42 30	47	1	57 30	17	6	48
18	0	45 0	48	2	0 0	18	7	12
19	0	47 30	49	2	2 30	19	7	36
20	0	50 0	50	2	5 0	20	8	0
21	0	52 30	51	2	7 30	21	8	24
22	0	55 0	52	2	10 0	22	8	48
23	0	57 30	53	2	12 30	23	9	12
24	1	0 0	54	2	15 0	24	9	36
25	1	2 30	55	2	17 30	25	10	0
26	1	5 0	56	2	20 0	26	10	24
27	1	7 30	57	2	22 30	27	10	48
28	1	10 0	58	2	25 0	28	11	12
29	1	12 30	59	2	27 30	29	11	36
30	1	15 0	60	2	30 0	30	12	0

Table [C2] for finding hours and fractions of hours from minutes of a day and their fractions.

M	H	m	M	H	m	M	H	m
1	0	24	31	12	24	61	24	0
2	0	48	32	12	48	62	24	24
3	1	12	33	13	12	63	24	48
4	1	36	34	13	36	64	25	12
5	2	0	35	14	0	65	25	36
6	2	24	36	14	24	66	26	0
7	2	48	37	14	48	67	26	24
8	3	12	38	15	12	68	26	48
9	3	36	39	15	36	69	27	12
10	4	0	40	16	0	70	27	36
11	4	24	41	16	24	71	28	0
12	4	48	42	16	48	72	28	24
13	5	12	43	17	12	73	28	48
14	5	36	44	17	36	74	29	12
15	6	0	45	18	0	75	29	36
16	6	24	46	18	24	76	30	0
17	6	48	47	18	48	77	30	24
18	7	12	48	19	12	78	30	48
19	7	36	49	19	36	79	31	12
20	8	0	50	20	0	80	31	36
21	8	24	51	20	24	81	32	0
22	8	48	52	20	48	82	32	24
23	9	12	53	21	12	83	32	48
24	9	36	54	21	36	84	33	12
25	10	0	55	22	0	85	33	36
26	10	24	56	22	24	86	34	0
27	10	48	57	22	48	87	34	24
28	11	12	58	23	12	88	34	48
29	11	36	59	23	36	89	35	12
30	12	0	60	24	0	90	36	0

Table [D] of equations for the access and recess of the eighth sphere.

G	m	s	G	m	s	G	m	s
1	0	9 37	31	4	37 17	61	7	11 49
2	0	18 49	32	4	45 15	62	7	20 19
3	0	28 11	33	4	53 14	63	8	0 41
4	0	37 23	34	5	1 13	64	8	41 16
5	0	46 52	35	5	9 11	65	8	51 52
6	0	56 12	36	5	17 10	66	9	2 28
7	1	5 31	37	5	25 8	67	9	13 4
8	1	14 48	38	5	33 7	68	9	23 41
9	1	24 4	39	5	41 5	69	9	34 17
10	1	33 20	40	5	49 4	70	9	44 53
11	1	42 34	41	5	57 2	71	9	55 30
12	1	51 46	42	6	5 1	72	10	6 6
13	2	0 57	43	6	12 59	73	10	16 43
14	2	10 6	44	6	20 58	74	10	27 19
15	2	19 13	45	6	28 56	75	10	37 56
16	2	28 17	46	6	36 55	76	10	48 32
17	2	37 16	47	6	44 53	77	10	59 9
18	2	46 11	48	6	52 52	78	11	9 45
19	2	55 2	49	7	0 50	79	11	20 22
20	3	3 49	50	7	8 49	80	11	30 58
21	3	12 32	51	7	16 47	81	11	41 35
22	3	21 10	52	7	24 46	82	11	52 12
23	3	29 45	53	7	32 44	83	12	2 48
24	3	38 17	54	7	40 43	84	12	13 25
25	3	46 47	55	7	48 41	85	12	24 1
26	3	55 14	56	7	56 40	86	12	34 38
27	4	3 37	57	8	4 38	87	12	45 14
28	4	12 58	58	8	12 37	88	12	55 51
29	4	20 55	59	8	20 35	89	13	6 27
30	4	29 10	60	8	28 34	90	13	17 4

Table [E] of the mean motion of access and recess of the eighth sphere.

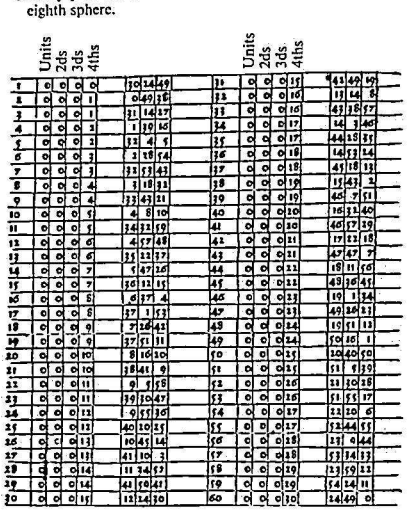

Table [F] of the mean movement of the auges and the fixed stars.

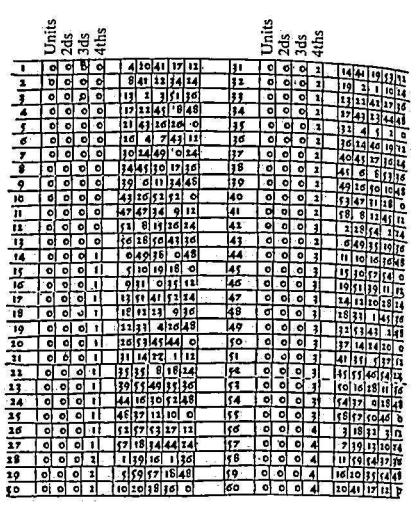

Table [I] of the mean argument of the moon.

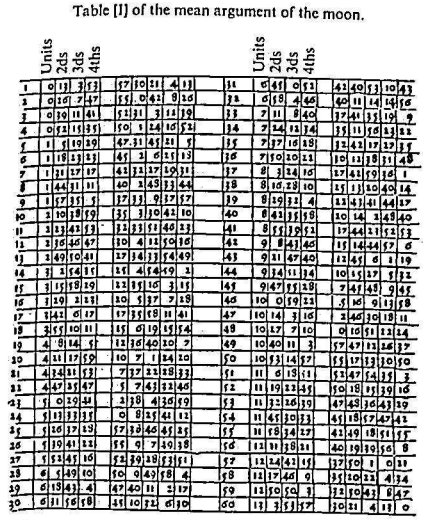

Table [J] of the mean elongation of the moon.

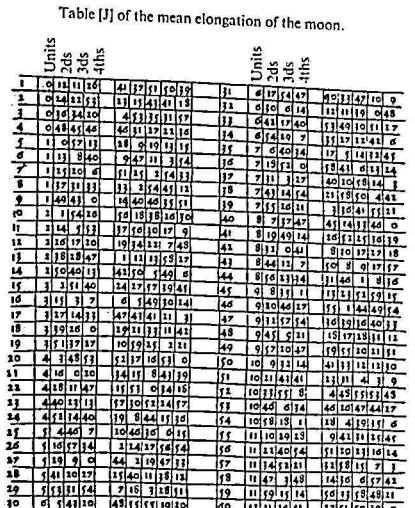

Table [G] of the mean motus of the sun.

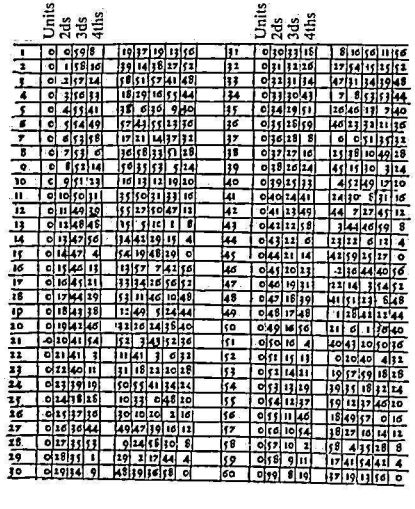

Table [H] of the mean motus of the moon.

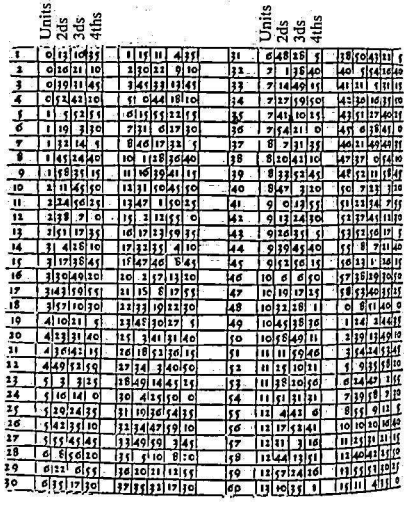

Table [K] of the argument of latitude of the moon.

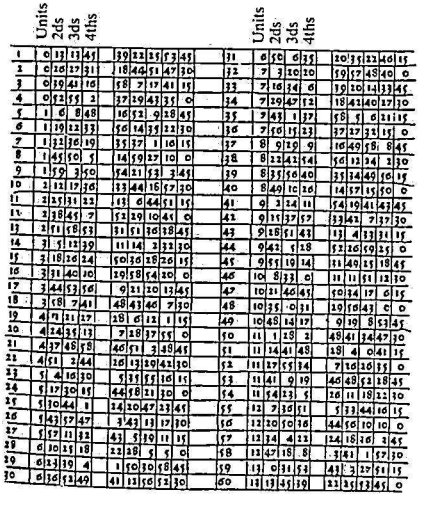

Table [L] of the mean motion of the head of the dragon.

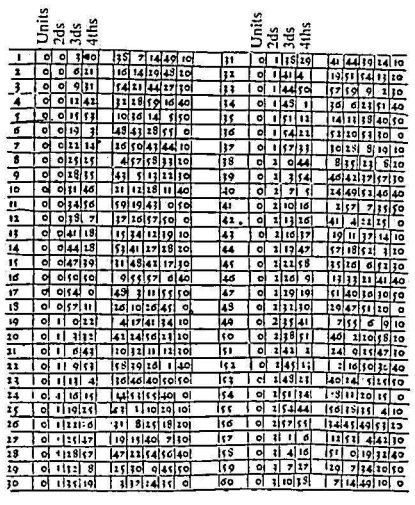

Table [M] of the equation of the sun.

Common numbers S G S G	Equations G m s	Common numbers S G S G	Equations G m s	Common numbers S G S G	Equations G m s
0 1 5 59	0 2 10	0 31 5 29	1 4 46	1 1 4 59	1 5 58
0 2 5 58	0 4 19	0 32 5 28	1 6 37	1 2 4 58	1 10 0
0 3 5 57	0 6 27	0 33 5 27	1 8 28	1 3 4 57	1 14 1
0 4 5 56	0 8 36	0 34 5 26	1 10 19	1 4 4 56	1 18 0
0 5 5 55	0 10 44	0 35 5 25	1 12 9	1 5 4 55	1 21 58
0 6 5 54	0 12 53	0 36 5 24	1 13 58	1 6 4 54	1 25 55
0 7 5 53	0 15 2	0 37 5 23	1 15 48	1 7 4 53	1 29 51
0 8 5 52	0 17 10	0 38 5 22	1 17 34	1 8 4 52	1 33 46
0 9 5 51	0 19 19	0 39 5 21	1 19 6	1 9 4 51	1 37 2
0 10 5 50	0 21 28	0 40 5 20	1 20 48	1 10 4 50	1 40 36
0 11 5 49	0 23 36	0 41 5 19	1 22 29	1 11 4 49	1 44 10
0 12 5 48	0 25 45	0 42 5 18	1 24 10	1 12 4 48	1 47 43
0 13 5 47	0 27 53	0 43 5 17	1 25 50	1 13 4 47	1 51 16
0 14 5 46	0 30 1	0 44 5 16	1 27 29	1 14 4 46	1 54 48
0 15 5 45	0 32 8	0 45 5 15	1 29 8	1 15 4 45	1 58 20
0 16 5 44	0 34 16	0 46 5 14	1 30 46	1 16 4 44	2 1 51
0 17 5 43	0 36 23	0 47 5 13	1 32 23	1 17 4 43	2 5 22
0 18 5 42	0 38 30	0 48 5 12	1 33 59	1 18 4 42	2 8 52
0 19 5 41	0 40 37	0 49 5 11	1 35 30	1 19 4 41	2 12 22
0 20 5 40	0 42 43	0 50 5 10	1 37 0	1 20 4 40	2 15 51
0 21 5 39	0 44 49	0 51 5 9	1 38 30	1 21 4 39	2 19 20
0 22 5 38	0 46 55	0 52 5 8	1 39 58	1 22 4 38	2 22 48
0 23 5 37	0 48 59	0 53 5 7	1 41 27	1 23 4 37	2 26 15
0 24 5 36	0 51 4	0 54 5 6	1 42 54	1 24 4 36	2 29 42
0 25 5 35	0 53 4	0 55 5 5	1 44 14	1 25 4 35	2 33 8
0 26 5 34	0 55 3	0 56 5 4	1 45 34	1 26 4 34	2 36 34
0 27 5 33	0 57 1	0 57 5 3	1 46 53	1 27 4 33	2 39 57
0 28 5 32	0 58 59	0 58 5 2	1 48 10	1 28 4 32	2 43 20
0 29 5 31	1 0 57	0 59 5 1	1 49 28	1 29 4 31	2 46 42
0 30 5 30	1 2 54	1 0 5 0	1 50 44	1 30 4 30	2 50 3

Table [N] of the equations of the moon.

Common numbers S G S G	Equation of center G m	Prop. parts m	Variation of the diameter G m	Equation of the argument G m s
0 1 5 59	0 9	0	0 3	0 4 46
0 2 5 58	0 18	0	0 5	0 9 31
0 3 5 57	0 27	0	0 7	0 14 15
0 4 5 56	0 36	0	0 10	0 19 0
0 5 5 55	0 45	0	0 12	0 23 44
0 6 5 54	0 53	0	0 14	0 28 28
0 7 5 53	1 2	0	0 17	0 33 11
0 8 5 52	1 11	0	0 19	0 37 54
0 9 5 51	1 20	0	0 21	0 42 37
0 10 5 50	1 29	0	0 24	0 47 19
0 11 5 49	1 38	0	0 26	0 52 0
0 12 5 48	1 46	1	0 28	0 56 41
0 13 5 47	1 55	1	0 31	1 1 20
0 14 5 46	2 4	1	0 33	1 5 59
0 15 5 45	2 13	1	0 35	1 10 38
0 16 5 44	2 22	1	0 38	1 15 15
0 17 5 43	2 31	1	0 40	1 19 51
0 18 5 42	2 39	1	0 42	1 24 27
0 19 5 41	2 48	1	0 45	1 29 0
0 20 5 40	2 57	2	0 47	1 33 32
0 21 5 39	3 5	2	0 49	1 38 3
0 22 5 38	3 14	2	0 52	1 42 33
0 23 5 37	3 23	2	0 54	1 47 1
0 24 5 36	3 31	2	0 57	1 51 27
0 25 5 35	3 40	2	0 59	1 55 52
0 26 5 34	3 49	2	1 1	2 0 15
0 27 5 33	3 57	3	1 3	2 4 37
0 28 5 32	4 6	3	1 6	2 8 57
0 29 5 31	4 15	3	1 8	2 13 16
0 30 5 30	4 23	3	1 10	2 17 29

Common numbers S G S G	Equations G m s	Common numbers S G S G	Equations G m s
2 1 3 59	1 53 46	2 31 3 29	1 5 1
2 2 3 58	1 52 35	2 32 3 28	1 2 54
2 3 3 57	1 51 24	2 33 3 27	1 0 47
2 4 3 56	1 50 12	2 34 3 26	0 58 40
2 5 3 55	1 48 59	2 35 3 25	0 56 33
2 6 3 54	1 47 46	2 36 3 24	0 54 25
2 7 3 53	1 46 20	2 37 3 23	0 52 17
2 8 3 52	1 44 53	2 38 3 22	0 50 9
2 9 3 51	1 43 26	2 39 3 21	0 48 1
2 10 3 50	1 41 57	2 40 3 20	0 45 53
2 11 3 49	1 40 27	2 41 3 19	0 43 44
2 12 3 48	1 38 57	2 42 3 18	0 41 35
2 13 3 47	1 37 25	2 43 3 17	0 39 26
2 14 3 46	1 35 53	2 44 3 16	0 37 16
2 15 3 45	1 34 20	2 45 3 15	0 35 6
2 16 3 44	1 32 46	2 46 3 14	0 32 56
2 17 3 43	1 31 12	2 47 3 13	0 30 35
2 18 3 42	1 29 37	2 48 3 12	0 28 19
2 19 3 41	1 27 50	2 49 3 11	0 26 1
2 20 3 40	1 26 3	2 50 3 10	0 23 42
2 21 3 39	1 24 16	2 51 3 9	0 21 22
2 22 3 38	1 22 28	2 52 3 8	0 19 1
2 23 3 37	1 20 40	2 53 3 7	0 16 40
2 24 3 36	1 18 51	2 54 3 6	0 14 19
2 25 3 35	1 17 0	2 55 3 5	0 11 58
2 26 3 34	1 15 8	2 56 3 4	0 9 36
2 27 3 33	1 13 13	2 57 3 3	0 7 12
2 28 3 32	1 11 16	2 58 3 2	0 4 48
2 29 3 31	1 9 10	2 59 3 1	0 2 24
2 30 3 30	1 7 7	3 0 3 0	0 0 0

Common numbers S G S G	Equation of center G m	Prop. parts m	Variation of the diameter G m	Equation of the argument G m s
1 1 4 59	5 44	1	2 5	4 4 7 47
1 2 4 58	5 32	1	2 6	4 10 17
1 3 4 57	5 19	1	2 7	4 15 3
1 4 4 56	5 7	1	2 9	4 15 51
1 5 4 55	4 55	1	2 10	4 19 3
1 6 4 54	4 22	1	2 12	4 20 27
1 7 4 53	4 10	1	2 13	4 22 47
1 8 4 52	3 57	1	2 14	4 25 1
1 9 4 51	3 44	1	2 15	4 27 11
1 10 4 50	3 32	1	2 16	4 29 18
1 11 4 49	3 19	1	2 17	4 31 20
1 12 4 48	3 6	1	2 18	4 33 18
1 13 4 47	2 53	1	2 19	4 35 11
1 14 4 46	2 40	1	2 20	4 36 59
1 15 4 45	2 27	1	2 21	4 38 43
1 16 4 44	2 14	1	2 22	4 40 23
1 17 4 43	2 1	1	2 23	4 41 59
1 18 4 42	1 48	1	2 24	4 43 29
1 19 4 41	1 35	1	2 25	4 44 55
1 20 4 40	1 22	1	2 26	4 46 17
1 21 4 39	1 9	1	2 27	4 47 35
1 22 4 38	0 56	1	2 28	4 48 47
1 23 4 37	0 43	1	2 29	4 49 55
1 24 4 36	0 30	1	2 30	4 50 58
1 25 4 35	0 17	1	2 31	4 51 56
1 26 4 34	0 4	1	2 32	4 52 49
1 27 4 33	0 9	2	2 33	4 53 37
1 28 4 32	0 22	2	2 34	4 54 20
1 29 4 31	0 35	2	2 35	4 54 58
1 30 4 30	0 47	2	2 36	4 55 31

Common numbers S G S G	Equation of center G m	Prop. parts m	Variation of the diameter G m	Equation of the argument G m s
1 31 4 29	1 0	2	2 37	4 55 59
1 32 4 28	1 13	2	2 37	4 55 37
1 33 4 27	1 25	2	2 38	4 55 49
1 34 4 26	1 38	2	2 38	4 55 55
1 35 4 25	1 50	2	2 38	4 56 0
1 36 4 24	2 2	2	2 38	4 55 56
1 37 4 23	2 14	2	2 38	4 55 43
1 38 4 22	2 26	2	2 39	4 55 25
1 39 4 21	2 38	2	2 39	4 55 1
1 40 4 20	2 50	2	2 39	4 54 32
1 41 4 19	3 2	2	2 39	4 53 58
1 42 4 18	3 14	2	2 39	4 53 18
1 43 4 17	3 25	2	2 40	4 52 33
1 44 4 16	3 37	2	2 40	4 51 42
1 45 4 15	3 48	2	2 40	4 50 46
1 46 4 14	4 0	2	2 40	4 49 45
1 47 4 13	4 11	2	2 40	4 48 39
1 48 4 12	4 22	2	2 40	4 47 27
1 49 4 11	4 33	2	2 40	4 46 10
1 50 4 10	4 44	2	2 39	4 44 47
1 51 4 9	4 55	2	2 39	4 43 17
1 52 4 8	5 6	2	2 39	4 41 42
1 53 4 7	5 16	2	2 38	4 40 1
1 54 4 6	5 27	2	2 38	4 38 15
1 55 4 5	5 37	2	2 37	4 36 23
1 56 4 4	5 47	2	2 37	4 34 25
1 57 4 3	5 57	2	2 36	4 32 22
1 58 4 2	6 7	2	2 35	4 30 13
1 59 4 1	6 16	2	2 34	4 27 59
2 0 4 0	6 26	2	2 33	4 25 40

Table [N] of the equations of the moon.

Common numbers	Equation of center	Prop. parts	Variation of the diameter	Equation of the argument
S G S G	G m	m	G m	G m s
2 1 3 59	13 3	43	3 30	4 24 38
2 2 3 58	13 1	44	2 29	4 23 11
2 3 3 57	12 59	44	3 27	4 19 35
2 4 3 56	12 56	45	2 26	4 16 58
2 5 3 55	12 53	45	2 25	4 14 13
2 6 3 54	12 50	45	2 23	4 11 23
2 7 3 53	12 46	46	2 22	4 8 28
2 8 3 52	12 41	46	2 21	4 5 31
2 9 3 51	12 36	47	2 19	4 2 30
2 10 3 50	12 30	47	2 18	3 59 20
2 11 3 49	12 23	47	2 17	3 56 5
2 12 3 48	12 16	48	2 15	3 52 47
2 13 3 47	12 9	48	2 14	3 49 23
2 14 3 46	12 2	49	2 12	3 45 52
2 15 3 45	11 54	49	2 10	3 42 17
2 16 3 44	11 46	50	2 9	3 38 37
2 17 3 43	11 38	50	2 7	3 34 53
2 18 3 42	11 29	51	2 5	3 31 3
2 19 3 41	11 20	51	2 3	3 27 10
2 20 3 40	11 11	52	2 1	3 23 12
2 21 3 39	11 2	52	1 58	3 19 9
2 22 3 38	10 53	53	1 56	3 15 2
2 23 3 37	10 43	53	1 54	3 10 50
2 24 3 36	10 33	53	1 51	3 6 33
2 25 3 35	10 23	54	1 49	3 2 15
2 26 3 34	10 11	54	1 46	2 57 51
2 27 3 33	10 0	54	1 43	2 53 23
2 28 3 32	9 48	55	1 41	2 48 51
2 29 3 31	9 35	55	1 38	2 44 15
2 30 3 30	9 22	55	1 35	2 39 35

Common numbers	Equation of center	Prop. parts	Variation of the diameter	Equation of the argument
S G S G	G m	m	G m	G m s
2 31 3 29	9 8	56	1 32	2 34 52
2 32 3 28	8 53	56	1 29	2 30 6
2 33 3 27	8 38	56	1 26	2 25 16
2 34 3 26	8 22	56	1 24	2 20 23
2 35 3 25	8 5	57	1 21	2 15 26
2 36 3 24	7 48	57	1 18	2 10 26
2 37 3 23	7 31	57	1 16	2 5 22
2 38 3 22	7 14	57	1 13	2 0 17
2 39 3 21	6 56	57	1 10	1 55 9
2 40 3 20	6 39	58	1 8	1 49 58
2 41 3 19	6 21	58	1 5	1 44 44
2 42 3 18	6 3	58	1 2	1 39 27
2 43 3 17	5 45	58	0 59	1 34 9
2 44 3 16	5 27	58	0 56	1 28 47
2 45 3 15	5 8	59	0 53	1 23 26
2 46 3 14	4 49	59	0 49	1 18 1
2 47 3 13	4 30	59	0 46	1 12 36
2 48 3 12	4 11	59	0 43	1 7 6
2 49 3 11	3 52	59	0 39	1 1 36
2 50 3 10	3 32	59	0 36	0 56 5
2 51 3 9	3 13	59	0 32	0 50 32
2 52 3 8	2 53	60	0 29	0 44 58
2 53 3 7	2 32	60	0 25	0 39 23
2 54 3 6	2 11	60	0 21	0 33 47
2 55 3 5	1 50	60	0 18	0 28 10
2 56 3 4	1 29	60	0 15	0 22 33
2 57 3 3	1 7	60	0 11	0 16 56
2 58 3 2	0 45	60	0 8	0 11 18
2 59 3 1	0 23	60	0 4	0 5 40
3 0 3 0	0 0	60	0 0	0 0 0

Table [O] of the latitude of the moon.

Common numbers	Ascending latitudes 0	1	2	Descending latitudes 3	4	5
1 29	0 5 13	2 34 24	4 22 22	4 59 58	4 17 7	3 33 17
2 28	0 10 27	2 18 52	4 24 51	4 59 50	4 14 22	3 30 40
3 27	0 15 40	2 43 17	4 27 14	4 59 35	4 11 34	3 10 2
4 26	0 20 53	2 47 39	4 29 34	4 59 15	4 8 37	2 11 22
5 25	0 26 7	2 52 0	4 31 49	4 58 51	4 5 35	2 6 40
6 24	0 31 19	2 56 10	4 33 59	4 58 22	4 2 27	2 1 56
7 23	0 36 31	3 0 21	4 36 4	4 57 45	3 59 28	1 57 8
8 22	0 41 42	3 4 29	4 38 4	4 57 4	3 56 16	1 52 17
9 21	0 46 52	3 8 35	4 40 0	4 56 17	3 53 0	1 47 23
10 20	0 52 2	3 12 39	4 41 52	4 55 25	3 49 40	1 42 27
11 19	0 57 9	3 16 39	4 43 38	4 54 26	3 46 17	1 37 29
12 15	1 2 16	3 20 35	4 45 18	4 53 25	3 42 49	1 32 31
13 17	1 7 23	3 24 26	4 46 52	4 52 17	3 39 17	1 27 33
14 16	1 12 30	3 28 13	4 48 20	4 51 3	3 35 41	1 22 35
15 15	1 17 36	3 32 0	4 49 44	4 49 44	3 32 0	1 17 36
16 14	1 22 35	3 35 41	4 51 3	4 48 20	3 28 15	1 12 30
17 13	1 27 33	3 39 17	4 52 17	4 46 52	3 24 26	1 7 23
18 12	1 32 31	3 42 49	4 53 25	4 45 18	3 20 35	1 2 16
19 11	1 37 29	3 46 17	4 54 28	4 43 36	3 16 39	0 57 9
20 10	1 42 17	3 49 40	4 55 25	4 41 52	3 12 39	0 52 1
21 9	1 47 13	3 53 0	4 56 17	4 40 0	3 8 35	0 46 52
22 8	1 52 17	3 56 16	4 57 4	4 38 4	3 4 29	0 41 42
23 7	1 57 8	3 59 28	4 57 45	4 36 4	3 0 21	0 36 31
24 6	2 1 56	4 2 37	4 58 21	4 33 59	2 56 10	0 31 19
25 5	2 6 40	4 5 38	4 58 51	4 31 49	2 51 57	0 26 7
26 4	2 11 22	4 8 37	4 59 15	4 29 34	2 47 39	0 20 53
27 3	2 16 2	4 11 34	4 59 34	4 27 14	2 43 17	0 15 40
28 2	2 20 40	4 14 22	4 59 50	4 24 51	2 38 51	0 10 27
29 1	2 25 17	4 17 7	4 59 58	4 22 22	2 34 24	0 5 13
30 0	2 29 52	4 19 47	5 0 0	4 19 47	2 29 52	0 0 0
	11	10	9	8	7	6 b

Table [P] of hourly movements of the sun and moon.

Common numbers	Movements Moon	Sun
S G S G	m s	m s
0 1 5 59	30 38	2 24
0 2 5 58	30 37	2 24
0 3 5 57	30 35	2 24
0 4 5 56	30 34	2 24
0 5 5 55	30 32	2 24
0 6 5 54	30 31	2 24
0 7 5 53	30 29	2 24
0 8 5 52	30 28	2 24
0 9 5 51	30 26	2 24
0 10 5 50	30 25	2 24
0 11 5 49	30 23	2 24
0 12 5 48	30 22	2 24
0 13 5 47	30 20	2 24
0 14 5 46	30 19	2 24
0 15 5 45	30 17	2 24
0 16 5 44	30 16	2 24
0 17 5 43	30 14	2 24
0 18 5 42	30 13	2 24
0 19 5 41	30 11	2 24
0 20 5 40	30 10	2 24
0 21 5 39	30 8	2 24
0 22 5 38	30 7	2 24
0 23 5 37	30 5	2 24
0 24 5 36	30 4	2 24
0 25 5 35	30 2	2 24
0 26 5 34	30 1	2 24
0 27 5 33	29 59	2 24
0 28 5 32	29 58	2 24
0 29 5 31	29 56	2 24
0 30 5 30	29 55	2 24

Common numbers	Movements Moon	Sun
S G S G	m s	m s
0 31 5 29	29 53	2 24
0 32 5 28	29 52	2 24
0 33 5 27	29 50	2 24
0 34 5 26	29 49	2 24
0 35 5 25	29 47	2 24
0 36 5 24	29 46	2 24
0 37 5 23	29 44	2 24
0 38 5 22	29 43	2 24
0 39 5 21	29 41	2 24
0 40 5 20	29 40	2 24
0 41 5 19	29 38	2 24
0 42 5 18	29 37	2 24
0 43 5 17	29 35	2 24
0 44 5 16	29 34	2 24
0 45 5 15	29 32	2 24
0 46 5 14	29 31	2 24
0 47 5 13	29 29	2 24
0 48 5 12	29 28	2 24
0 49 5 11	29 26	2 24
0 50 5 10	29 25	2 24
0 51 5 9	29 23	2 24
0 52 5 8	29 22	2 24
0 53 5 7	29 20	2 24
0 54 5 6	29 19	2 24
0 55 5 5	29 17	2 24
0 56 5 4	29 16	2 24
0 57 5 3	29 14	2 24
0 58 5 2	29 13	2 24
0 59 5 1	29 11	2 24
1 0 5 0	29 10	2 24

Common numbers	Movements Moon	Sun
S G S G	m s	m s
1 1 4 59	29 8	2 24
1 2 4 58	29 7	2 24
1 3 4 57	29 5	2 24
1 4 4 56	29 4	2 24
1 5 4 55	29 2	2 24
1 6 4 54	29 1	2 24
1 7 4 53	28 59	2 24
1 8 4 52	28 58	2 24
1 9 4 51	28 56	2 24
1 10 4 50	28 55	2 24
1 11 4 49	28 53	2 24
1 12 4 48	28 52	2 24
1 13 4 47	28 50	2 24
1 14 4 46	28 49	2 24
1 15 4 45	28 47	2 24
1 16 4 44	28 46	2 24
1 17 4 43	28 44	2 24
1 18 4 42	28 43	2 24
1 19 4 41	28 41	2 24
1 20 4 40	28 40	2 24
1 21 4 39	28 38	2 24
1 22 4 38	28 37	2 24
1 23 4 37	28 35	2 24
1 24 4 36	28 34	2 24
1 25 4 35	28 32	2 24
1 26 4 34	28 31	2 24
1 27 4 33	28 29	2 24
1 28 4 32	28 28	2 24
1 29 4 31	28 26	2 24
1 30 4 30	28 25	2 24

Common numbers	Movements Moon	Sun
S G S G	m s	m s
1 31 4 29	28 23	2 24
1 32 4 28	28 22	2 24
1 33 4 27	28 20	2 24
1 34 4 26	28 19	2 24
1 35 4 25	28 17	2 24
1 36 4 24	28 16	2 24
1 37 4 23	28 14	2 24
1 38 4 22	28 13	2 24
1 39 4 21	28 11	2 24
1 40 4 20	28 10	2 24
1 41 4 19	28 8	2 24
1 42 4 18	28 7	2 24
1 43 4 17	28 5	2 24
1 44 4 16	28 4	2 24
1 45 4 15	28 2	2 24
1 46 4 14	28 1	2 24
1 47 4 13	27 59	2 24
1 48 4 12	27 58	2 24
1 49 4 11	27 56	2 24
1 50 4 10	27 55	2 24
1 51 4 9	27 53	2 24
1 52 4 8	27 52	2 24
1 53 4 7	27 50	2 24
1 54 4 6	27 49	2 24
1 55 4 5	27 47	2 24
1 56 4 4	27 46	2 24
1 57 4 3	27 44	2 24
1 58 4 2	27 43	2 24
1 59 4 1	27 41	2 24
2 0 4 0	27 40	2 24

Common numbers	Movements Moon	Sun
S G S G	m s	m s
2 1 3 59	14 17	2 29
2 2 3 58	14 20	2 29
2 3 3 57	14 23	2 29
2 4 3 56	14 26	2 30
2 5 3 55	14 29	2 30
2 6 3 54	14 32	2 30
2 7 3 53	14 35	2 30
2 8 3 52	14 38	2 30
2 9 3 51	14 40	2 30
2 10 3 50	14 43	2 30
2 11 3 49	14 44	2 30
2 12 3 48	14 46	2 30
2 13 3 47	14 48	2 30
2 14 3 46	14 50	2 30
2 15 3 45	14 52	2 30
2 16 3 44	14 53	2 30
2 17 3 43	14 58	2 30
2 18 3 42	15 0	2 30
2 19 3 41	15 3	2 30
2 20 3 40	15 5	2 30
2 21 3 39	15 7	2 30
2 22 3 38	15 9	2 30
2 23 3 37	15 11	2 31
2 24 3 36	15 13	2 31
2 25 3 35	15 15	2 31
2 26 3 34	15 18	2 31
2 27 3 33	15 20	2 31
2 28 3 32	15 22	2 31
2 29 3 31	15 24	2 31
2 30 3 30	15 26	2 31

Common numbers	Movements Moon	Sun
S G S G	m s	m s
2 31 3 29	15 29	2 31
2 32 3 28	15 32	2 31
2 33 3 27	15 34	2 31
2 34 3 26	15 36	2 32
2 35 3 25	15 38	2 32
2 36 3 24	15 40	2 32
2 37 3 23	15 43	2 32
2 38 3 22	15 45	2 32
2 39 3 21	15 47	2 32
2 40 3 20	15 49	2 32
2 41 3 19	15 50	2 32
2 42 3 18	15 52	2 32
2 43 3 17	15 54	2 32
2 44 3 16	15 57	2 32
2 45 3 15	15 59	2 33
2 46 3 14	16 1	2 33
2 47 3 13	16 3	2 33
2 48 3 12	16 5	2 33
2 49 3 11	16 7	2 33
2 50 3 10	16 9	2 33
2 51 3 9	16 11	2 33
2 52 3 8	16 13	2 33
2 53 3 7	16 15	2 33
2 54 3 6	16 17	2 33
2 55 3 5	16 19	2 34
2 56 3 4	16 21	2 34
2 57 3 3	16 22	2 34
2 58 3 2	16 23	2 34
2 59 3 1	16 24	2 34
3 0 3 0	16 25	2 34

Table [Q] of the equation of days

G	Capricorn m	s	Aquarius m	s	Pisces m	s	Aries m	s	Taurus m	s	Gemini m	s
1	3	41	0	37	0	12	2	15	4	41	5	33
2	3	33	0	33	0	15	2	20	4	44	5	33
3	3	25	0	30	0	17	2	25	4	48	5	33
4	3	18	0	26	0	20	2	30	4	52	5	33
5	3	11	0	23	0	22	2	35	4	55	5	33
6	3	4	0	20	0	25	2	41	4	59	5	32
7	2	57	0	18	0	28	2	46	5	2	5	29
8	2	50	0	15	0	31	2	52	5	5	5	26
9	2	43	0	13	0	35	2	57	5	8	5	23
10	2	37	0	10	0	38	3	3	5	10	5	20
11	2	31	0	8	0	42	3	8	5	13	5	17
12	2	24	0	6	0	45	3	14	5	16	5	15
13	2	17	0	5	0	49	3	19	5	18	5	12
14	2	10	0	4	0	54	3	24	5	20	5	10
15	2	3	0	3	0	58	3	29	5	22	5	8
16	1	56	0	2	1	3	3	34	5	23	5	5
17	1	49	0	1	1	7	3	39	5	25	5	3
18	1	43	0	0	1	12	3	44	5	27	5	0
19	1	37	0	0	1	16	3	49	5	28	4	56
20	1	31	0	1	1	21	3	54	5	29	4	53
21	1	25	0	1	1	25	4	0	5	30	4	49
22	1	19	0	1	1	30	4	5	5	31	4	46
23	1	14	0	2	1	34	4	11	5	31	4	43
24	1	9	0	2	1	39	4	16	5	32	4	40
25	1	4	0	3	1	44	4	20	5	32	4	36
26	0	59	0	4	1	49	4	23	5	32	4	33
27	0	54	0	6	1	54	4	27	5	32	4	30
28	0	49	0	7	2	0	4	31	5	33	4	27
29	0	44	0	8	2	5	4	34	5	33	4	24
30	0	41	0	10	2	10	4	39	5	33	4	20

Table (Q) of the equation of days

G	Cancer m	s	Leo m	s	Virgo m	s	Libra m	s	Scorpio m	s	Sagittarius m	s
1	4	16	3	4	3	48	6	9	7	49	6	59
2	4	14	3	4	3	51	6	14	7	50	6	55
3	4	10	3	4	3	54	6	18	7	51	6	50
4	4	7	3	4	3	57	6	22	7	52	6	44
5	4	3	3	4	4	1	6	27	7	52	6	38
6	4	0	3	4	4	6	6	32	7	53	6	32
7	3	57	3	4	4	12	6	36	7	53	6	25
8	3	54	3	4	4	17	6	41	7	54	6	19
9	3	51	3	4	4	21	6	45	7	54	6	12
10	3	48	3	5	4	26	6	49	7	53	6	6
11	3	45	3	6	4	31	6	53	7	52	5	59
12	3	42	3	7	4	36	6	57	7	51	5	53
13	3	39	3	8	4	41	7	1	7	50	5	48
14	3	36	3	9	4	46	7	6	7	49	5	44
15	3	33	3	10	4	51	7	10	7	47	5	39
16	3	30	3	11	4	56	7	15	7	45	5	32
17	3	27	3	13	5	1	7	19	7	43	5	25
18	3	25	3	14	5	6	7	23	7	41	5	18
19	3	23	3	16	5	11	7	27	7	39	5	11
20	3	21	3	17	5	16	7	31	7	37	5	4
21	3	19	3	19	5	21	7	34	7	34	4	57
22	3	17	3	21	5	26	7	36	7	31	4	49
23	3	15	3	24	5	31	7	38	7	27	4	42
24	3	13	3	27	5	36	7	39	7	23	4	34
25	3	11	3	30	5	41	7	41	7	20	4	27
26	3	9	3	33	5	45	7	42	7	16	4	19
27	3	8	3	36	5	50	7	44	7	12	4	12
28	3	7	3	39	5	55	7	45	7	9	4	4
29	3	6	3	42	5	59	7	47	7	5	3	56
30	3	5	3	45	6	4	7	48	7	3	3	49

Table [R] of proportions.

Deg.	Prop. min.	Deg.	Prop. min.	Deg.	Prop. min.
0 2	0 2	1 2	14 51	2 2	45 0
0 4	0 6	1 4	15 45	2 4	46 0
0 6	0 12	1 6	16 41	2 6	47 7
0 8	0 20	1 8	17 35	2 8	47 46
0 10	0 30	1 10	18 30	2 10	48 37
0 12	0 42	1 12	19 26	2 12	49 30
0 14	0 57	1 14	20 30	2 14	50 19
0 16	1 11	1 16	21 26	2 16	51 6
0 18	1 14	1 18	22 26	2 18	51 50
0 20	1 55	1 20	23 26	2 20	52 33
0 22	2 18	1 22	24 26	2 22	53 11
0 24	2 43	1 24	25 26	2 24	53 48
0 26	3 5	1 26	26 28	2 26	54 14
0 28	3 25	1 28	27 40	2 28	54 59
0 30	3 54	1 30	28 33	2 30	55 14
0 32	4 21	1 32	29 44	2 32	56 8
0 34	4 50	1 34	30 46	2 34	56 42
0 36	5 21	1 36	31 48	2 36	57 15
0 38	5 57	1 38	32 50	2 38	57 43
0 40	6 34	1 40	33 53	2 40	58 8
0 42	7 13	1 42	34 54	2 42	58 31
0 44	7 52	1 44	35 50	2 44	58 50
0 46	8 32	1 46	36 58	2 46	59 7
0 48	9 15	1 48	38 0	2 48	59 21
0 50	10 0	1 50	39 0	2 50	59 33
0 52	10 46	1 52	40 0	2 52	59 43
0 54	11 33	1 54	41 0	2 54	59 51
0 56	12 21	1 56	42 0	2 56	59 56
0 58	13 10	1 58	43 0	2 58	59 58
1 0	14 0	2 0	44 0	3 0	60 0 m

Table [S] for lunar eclipses at the greatest distance in the epicycle.

Arguments of latitude Northerly						Arguments of latitude Southerly						Digits		Minutes Casus		Minutes Mora	
0	11	0	3	49	0	3	11	0	5	49	0	0	0	0	0	0	0
0	10	30	3	49	30	3	10	30	5	49	30	0	40	12	10	0	0
0	10	0	3	50	0	3	10	0	5	50	0	1	20	19	30	0	0
0	9	30	3	50	30	3	9	30	5	50	30	2	0	14	11	0	0
0	9	0	3	51	0	3	9	0	5	51	0	2	35	18	7	0	0
0	8	30	3	51	30	3	8	30	5	51	30	4	32	31	13	0	0
0	8	0	3	52	0	3	8	0	5	52	0	5	30	34	10	0	0
0	7	30	3	52	30	3	7	30	5	52	30	6	35	36	27	0	0
0	7	0	3	53	0	3	7	0	5	53	0	7	53	38	42	0	0
0	6	30	3	53	30	3	6	30	5	53	30	8	11	40	18	0	0
0	6	0	3	54	0	3	6	0	5	54	0	9	20	42	11	0	0
0	5	30	3	54	30	3	5	30	5	54	30	10	17	43	56	0	0
0	5	0	3	55	0	3	5	0	5	55	0	11	14	44	53	0	0
0	4	30	3	55	30	3	4	30	5	55	30	12	11	45	4	0	0
0	4	0	3	56	0	3	4	0	5	56	0	13	9	45	42	10	21
0	3	30	3	56	30	3	3	30	5	56	30	14	7	46	1	11	47
0	3	0	3	57	0	3	3	0	5	57	0	15	4	46	46	13	48
0	2	30	3	57	30	3	2	30	5	57	30	16	2	47	38	17	28
0	2	0	3	58	0	3	2	0	5	58	0	17	0	48	31	19	14
0	1	30	3	58	30	3	1	30	5	58	30	17	57	49	3	20	11
0	1	0	3	59	0	3	1	0	5	59	0	18	53	49	52	20	51
0	0	30	3	59	30	3	0	30	5	59	30	19	50	50	19	21	26
0	0	0	3	0	0	3	0	0	6	0	0	20	46	50	46	21	52

Table [T] for lunar eclipses at least distance in the epicycle.

Arguments of latitude Northerly						Arguments of latitude Southerly						Digits		Minutes Casus		Minutes Mora	
0	13	0	3	47	0	3	13	0	5	47	0	0	26	12	25	0	0
0	12	30	3	47	30	3	12	30	5	47	30	1	13	10	52	0	0
0	12	0	3	48	0	3	12	0	5	48	0	2	2	10	7	0	0
0	11	30	3	48	30	3	11	30	5	48	30	2	50	10	23	0	0
0	11	0	3	49	0	3	11	0	5	49	0	3	36	14	27	0	0
0	10	30	3	49	30	3	10	30	5	49	30	4	24	17	0	0	0
0	10	0	3	50	0	3	10	0	5	50	0	5	29	41	27	1	0
0	9	30	3	50	30	3	9	30	5	50	30	6	30	43	26	0	0
0	9	0	3	51	0	3	9	0	5	51	0	6	54	45	21	0	0
0	8	30	3	51	30	3	8	30	5	51	30	7	41	47	25	0	0
0	8	0	3	52	0	3	8	0	5	52	0	8	32	49	28	1	0
0	7	30	3	52	30	3	7	30	5	52	30	9	26	51	0	0	0
0	7	0	3	53	0	3	7	0	5	53	0	10	21	52	44	0	0
0	6	30	3	53	30	3	6	30	5	53	30	11	16	54	9	0	0
0	6	0	3	54	0	3	6	0	5	54	0	12	13	55	20	0	0
0	5	30	3	54	30	3	5	30	5	54	30	13	11	56	19	0	0
0	5	0	3	55	0	3	5	0	5	55	0	13	27	43	53	14	0
0	4	30	3	55	30	3	4	30	5	55	30	14	25	40	56	17	51
0	4	0	3	56	0	3	4	0	5	56	0	15	0	38	0	19	47
0	3	30	3	56	30	3	3	30	5	56	30	15	50	36	51	22	32
0	3	0	3	57	0	3	3	0	5	57	0	16	55	35	17	25	0
0	2	30	3	57	30	3	2	30	5	57	30	17	17	33	15	27	0
0	2	0	3	58	0	3	2	0	5	58	0	17	55	31	15	33	31
0	1	30	3	58	30	3	1	30	5	58	30	18	30	29	5	35	5
0	1	0	3	59	0	3	1	0	5	59	0	19	14	26	49	37	2
0	0	30	3	59	30	3	0	30	5	59	30	20	43	24	26	37	56
0	0	0	3	0	0	3	0	0	6	0	0	21	31	24	25	37	37

66 AN ATTACK UPON ASTROLOGY

Nicole Oresme (ca. 1325–1382)

Translated and annotated by G. W. Coopland[1]

Here begins Master Nicole Oresme's Book against Divination.

It is my aim, with God's help, to show in this little book, from experience, from human reason, and from authority, that it is foolish, wicked, and dangerous even in this life, to set one's mind to know or search out hidden matters or the hazards and fortunes of the future, whether by astrology, geomancy, nigromancy, or any other such arts, if they can correctly be called arts; and, further, that such things are most dangerous to those of high estate, such as princes and lords to whom appertains the government of the commonwealth.

Hence I have written this little book in French so that laymen may understand it for I have heard that many of them are overmuch given to such stupidities. At a former time I wrote in Latin on this matter and if any man wishes to attack what I shall say let him do it openly and with set argument, not in mere slander, and let him write against it and I shall reply so far as I am able, for in such wise we may arrive at truth. In any case what I say is submitted to the correction of those whom it concerns. And I beg that I may be excused for my rough manner of expression because I have never learned or been used to set forth or write anything in French.

Here follow the chapters of the book.

The first; of the arts by which men enquire into hidden and secret things.

The second; as to how much truth there is in the various branches of astrology.

The third; as to what truth there is in the other arts aforesaid.

The fourth; a reply to an objection.

The fifth; on the arguments in favour of princes applying themselves to these sciences.

The sixth; the arguments in favour of the possibility of knowing the future by these methods.

The seventh; the arguments to show that divination is a possible and profitable thing.

The eighth; true proof of the contrary from experience.

The ninth; on my proposition from authority.

The tenth; proof of my proposition by reason.

The eleventh; that there is no certainty in such arts.

The twelfth; on how men are deceived by such arts.

The thirteenth; on the fitting attitude of princes to such sciences.

The fourteenth; replies to the arguments of the fourth chapter.

The fifteenth; replies to the arguments of the fifth chapter.

The sixteenth; replies to the arguments of the sixth chapter.

The seventeenth; recapitulation and conclusion of the whole work.

Here ends the Prologue.

CHAPTER 1

There are many arts or sciences by means of which men are accustomed to enquire into the future or into things occult, secret, and hidden, or which can be applied to such uses. One of these is astrology which appears to me to have six principal parts. The first of these has to do chiefly with the movements, the signs, and the measurements of the heavenly bodies, so that by means of tables, constellations, eclipses and suchlike things in the future can be known.[2] The second is concerned with the qualities, the influences, and physical powers of the stars, with the signs of the zodiac, with degrees, with the heavenly signs, and so on; as, for instance, that a star in one quarter of the sky signifies or has power to cause heat or cold, dryness or moisture, and similarly with other physical effects. This part of astrology is introductory to the making of predictions. The third part deals with the revolutions of the stars and with the conjunctions of the planets, and is applied to three kinds of predictions; first, that we may know from the major conjunctions the great events of the world, as plagues, mortalities, famine, floods, great wars, the rise and fall of kingdoms, the appearance of prophets, new religions, and similar changes; next, that we may know the state of the atmosphere, the changes in the weather, from hot to cold, from dry to wet, winds, storms, and such movements in nature; third, that we may judge as to the humours of the body and as to taking medicine and so on.

The fourth part has to do with nativities, and especially with decisions as to a man's fortune, from the constellation which is in the heavens at the moment of his birth.

The fifth deals with interrogations, that is, decides and answers a question according to the constellation which is in the heavens at the time when the question is asked.

The sixth is of elections by which the time to start a journey or to undertake a task is ascertained and in this part is included the branch which teaches how to make images, carettes, rings, and such things.

The other sciences are geomancy, hydromancy, and similar devices, palmistry, experiments, and such auspices as sneezing, encounters, arguments drawn from the song or flight of birds, from the members of dead animals, magic art, nigromancy, interpretation of dreams, and many other vanities which are not sciences properly speaking.

CHAPTER 2

The first part of astrology is speculative and mathematical, a very noble and excellent science and set forth in the books very subtly, and this part can be adequately known but it cannot be known precisely and with punctual exactness, as I have shown in my treatise on the Measurement of the Movements of the Heavens and have proved by reason founded on mathematical demonstration.[3]

The second part is a part of natural science and is a great science and it too can be known so far as its nature is concerned but we know too little about it and in particular the rules in the books are false, as Averroes says, and have either slight proof or none. And some of them which were fulfilled in the place or at the time when they were laid down, are false in other places or at the present time: for the fixed stars which according to the ancients have great influence are not now in the position they were in then and these same positions are used in making predictions.

And of the three subdivisions of the third part of astrology the first, which is concerned with the great events of the world, can be and is sufficiently well known but only in general terms. Especially we cannot know in what country, in what month, through what persons, or under what conditions, such things will happen, or the other particular circumstances. Secondly, as regards change in the weather, this part by its nature permits of knowledge being acquired therein but it is very difficult and is not now, nor has it ever been to any one who has studied it, more than worthless, for the rules of the second part are mostly false as I have said, and are assumed in this branch. And, similarly, the detailed rules bearing on this part are false, so that we see every day that sailors and husbandmen can prophesy changes in the weather better than the astronomers. In the third place, so far as medicine is concerned, we can know a certain amount as regards the effects which ensue from the course of the sun and moon but beyond this little or nothing. All this third part of astrology has to do chiefly with physical effects; the parts which follow, with the effects of fortune.

The fourth part, of nativities, is not in itself beyond knowledge, so far as the complexion and inclination of the person born at a given time are in question, but cannot be known when it comes to fortune and things which can be hindered by the human will, and this section has to do with those things rather than with physical effects. And one often sees in practice that two people are born at intervals of time so minute as cannot be recorded and yet their fortunes are quite different, so I say that this part of astrology cannot be known and the rules written down on it are not true.

The fifth part, of interrogations, and the sixth, of elections, have no reasonable foundation and there is no truth in them and Averroes says in his commentary on the XIIth book of Metaphysics that the part about images sprang from the corruption of philosophy and the fables of the pagans; and any one who has read the works of Hyginus[4] and Aratus,[5] who deal with this matter, can un-

1. [Reprinted with the permission of the Harvard University Press from G. W. Coopland, *Nicole Oresme and the Astrologers: A Study of His Livre de Divinacions* (Cambridge, Mass.: Harvard University Press, 1952), pp. 51, 53, 55, 57, 61, 71, 75, 87, 89, 91, 93, 109, 111, 113. The present selection was translated from Oresme's *Livre de Divinacions*.

As will be seen, although Oresme was tolerant of certain features of astrology, he was utterly opposed to its determinism and wholly sceptical of its ability to predict future events. Indeed, in his *On Ratios of Ratios* and *Treatise on the Commensurability or Incommensurability of Celestial Motions*, he offered mathematical reasons for denying astrology's predictive claims (see Selection 69, Part III).—*Ed.*]

2. [Here we have astronomy proper.—*Ed.*]

3. [At least three candidates for this reference come to mind: his *Treatise on the Commensurability or Incommensurability of the Celestial Motions*, Part III (Selection 69); *On Ratios of Ratios*, where the mathematical theory is formulated in favor of the probable incommensurability of celestial motions (see Selection 51.2 and notes 80 and 84 thereto); and the *Ad pauca respicientes* (see Selection 69).—*Ed.*]

derstand that such images can have no effect unless it be by magic art or by nigromancy.

CHAPTER 4

Some may say that the aforesaid sciences, and especially astrology, are said to be of very ancient authority and are set forth by prophets, by reasonable authors, and in authentic books, and yet I attack some portions and some rules of these sciences and the arts that have been named without giving reasons, like a man who hates them and yet knows nothing about them. In reply I say, first, that it would be too long a business to demonstrate in detail the falsity and weakness of the principles on which the sciences that I call worthless are based. Again, it is not easy to deal with the matter in French or in such a way as laymen could easily understand. Further, I am prepared, if necessary, to defend what I shall say by set argument in Latin against those who wish to maintain the contrary, in a fitting way. And it is a matter of every-day experience that such divinations fail to speak truth. And, indeed, the real reply is that my main aim in this work is not to prove the falsity of such pursuits but, leaving on one side that question, to demonstrate that it is foolish and perilous to make use of such arts so far as enquiring into particular fortunes is concerned. And to prove this does not require that one should know the said arts perfectly, just as a man who condemns tables or dice need not be a master of those games.

CHAPTER 8

Notwithstanding all the above arguments, I wish to demonstrate with certainty my main proposition; and first by experience and by induction from past history. And I say that princes and others who have set themselves to such matters have come to a bad end and have had evil fortune in this world. And consider Zoroaster, King of the Bactrians, who was such a great astronomer, and who was the first to invent the arts of magic; he was slain by Ninus who deprived him of his life and of his kingdom. . . .[6] Hence it is clear that the conclusion may be drawn from this great induction, from so many experiences, observations, instances, that divination was never at any time propitious.

CHAPTER 9

Now I will prove my proposition by argument from authority. First, Moses in the 18th chapter of Deuteronomy teaches and commands the people of Israel, when they come into the promised land, to beware lest they follow the abominations of the poeple of that land and forbids them to have among them any manner of diviners, or to put questions to them, for our Lord abominates such things, and for such manner of iniquity would take away their land and would give it to the people of Israel. For he had said previously, in the 9th chapter, that they should not possess that land because of their righteousness, but that the inhabitants would be destroyed for their iniquities. Surely, any man who considers this text of the Scriptures, so ancient, so authentic, so approved, received, and commended, by the whole world, whether Christian, Jew, or Saracen, must be in great fear and horror of listening to such divinations; for the Scripture gives no reason but this for the destruction of those peoples and for the ruin of those kingdoms. . . .[7]

CHAPTER 11

Although sometimes a person may, by nature, be able to see what is absent or to come, in dreams, or sickness, or when he has been put out of his senses by magic art, as I have shown elsewhere, yet such visions are often false or occult, without certainty and dangerous of acceptance, and there is still less certainty in the other arts aforesaid. For, in the case of astronomy, which has the most appearance of truth, a philosopher named Baren says that a diligent interpreter or expositor of dreams will investigate and will find the truth more easily than an astronomer.[8] "For," says he, "the institutes of astronomy are too defective," and he goes on to say that he saw in his own day many who knew astrology more than excellently yet in their predictions they were variable and discordant. Also Avicenna, in his Metaphysics, expresses the opinion that the disposition of the heavens and of the stars cannot be completely known, and even supposing we did know it we could know nothing

4. See *C. Julii Augusti Liberti Fabularum ejusdem Astronomicon libri quattuor*, etc., Basle, 1535, for illustrations of this.

5. Aratus (c. 270 B.C.) the author of the *Phoenomena et Prognostica*. Oresme is probably indebted to Haly's *Commentary* [that is, Haly Eben Rodan's Commentary on Ptolemy's *Tetrabiblos* or *Quadripartitum*, as it was known in Latin—*Ed.*] for the reference.

6. [Many similar "historical" instances are cited.—*Ed.*]

7. [References to other "authorities" fill out the remainder of the chapter.—*Ed.*]

8. Long search and enquiry in many quarters have revealed no clue to this reference.

of absent things.[9] And he says further that many of the rules of astrology are based on poetry and on rhetoric, that is to say on fables and imaginings that cannot be accepted in natural science. Hence he says that we must not believe the predictions of the astrologers although of our good pleasure we may allow their writings to be true, out of respect for their science. Again, according to Ptolemy in the Quadripartitum, those who live towards the south are more apt for the knowledge of astrology than those who live towards the north,[10] and to this Haly adds that those who live towards the east are similarly more fitted for it than those who live further west. And this is true, and more especially so in the case of judicial astrology. And, according to the opinion expressed by Ptolemy in his Centiloquium[11] and also according to human reason, those men can profit little in astrology who are not inclined thereto by nature; hence it follows that Frenchmen, and still more Englishmen and those further away yet, who live near the confines of the habitable north or of the west, can commonly have little profit from judicial astrology. And, in particular, as I indicated briefly in the second chapter, the part of astrology which deals with the shapes of the constellations is nearly all founded on the stories of the poets who pretended that good princes and certain others were changed into stars. And just as old women to-day speak of St. Martin's Staff,[12] or the True Cross, or St. James' Road,[13] so the poets gave to the stars the names of persons or things that in ancient times were held in common memory or reverence. Thus Andromeda, Perseus and Hercules were named and the astronomers accepted those names because it was convenient to have separate names to distinguish the stars, and every star or constellation did not possess a shape or outline which suggested a name like the Crown, the Wain and others. This one may know from the old histories and books which deal with the origin of these names, as, for instance, Aratus, Ovid,[14] Fulgeratus,[15] and many others. And certain of the rules of judicial astrology are based on the names of the heavenly shapes, and from this it is clear that there is no certainty in these matters. And St. Ambrose stresses this and relates how they say that if a man is born under the sign of the Ram he will be generous, because the sheep parts willingly with its fleece, and if he is born under the sign of the Bull he will be a tiller of the ground, and such-like nonsense.[16] Also we have books of "experiments of images", such as that of Olbrizabalis,[17] which, similarly, are based on the manners or deeds or doings of the personages after whom these shapes are named, just as if one were to construct an "image" to cure fevers when St. Martin's Staff was in the ascendant. And some wish to connect the images in geomancy with these shapes, but without justification, for they have no resemblance to them, either in number of stars, or outline or shapes of the figures. And, even if they were alike, it would not be possible to draw any conclusion as to their predictions so we can see plainly that the whole thing is worthless. But it would be waste of time to delay to attack it, for the man who is willing to listen to such things is lacking in

9. See *Liber Avicennae de philosophia prima sive scientia divina*, ed. 1492 (Bodleian Lib.), especially the first chapter of the 10th tractatus, f. 108 recto, col. 2 [the same folio reference applies to a 1508 edition of Avicenna's works reprinted in Frankfurt am Main by Minerva G. m. b. H. in 1961—*Ed.*]. For Avicenna's views on astrology see also A. F. Mehren, *Vues d'Avicenne sur l'Astrologie et sur le rapport de la responsabilité humaine avec le destin*, in *Homenaje a D. Francisco Codera*, Zaragoza, 1904, pp. 235–250. The article is based on examination of Avicenna's *Réfutations des Astrologues* and *Traité sur le destin* as existing in Arabic MSS. at Leyden. Avicenna compared judicial astrology with the quack side of medicine, and criticised the fact that the astrologers confined influence to the planets and neglected the millions of stars of the Milky Way; he dealt, too, with many of the traditional dilemmas and difficulties.

10. Ptolemy, *Quadripartitum*, II, cap. 2.

11. ["The astrological *Centiloquium* or *Karpos*, and other treatises on divination and astrological images ascribed to Ptolemy in medieval Latin manuscripts are probably spurious, but there is no doubt of his belief in astrology" (L. Thorndike, *A History of Magic and Experimental Science*, [New York: Macmillan, 1929], I, 111). The *Tetrabiblos*, or *Quadripartitum*, is believed to be Ptolemy's.—*Ed.*]

12. This is no longer well known in France, though it may still be used in certain districts.

13. The Milky Way is still so called in France. In England it was once called Watling Street and Walsingham Way; see on this Skeat's edition of Chaucer (1894), the notes to the *Hous of Fame*, II, 1.939.

14. e.g. *Fasti*, II and V.

15. [Coopland mentions a *Fulgentius*, cited by Pierre d'Ailly (in his *Principium in Cursum Bibliae*) as the author of *Mythologies;* and also cites a work *Fabii Planciadis Fulgentii Mythologicon*, Amsterdam, 1681.—*Ed.*]

16. [St. Ambrose, *Hexameron*, Bk. IV.—*Ed.*] The work of St. Ambrose here used by Oresme is one of the fullest and most interesting of orthodox condemnations of divination.

17. [Identified by Coopland as Albohaly (Yahyā ibn Ghālib, Abu Ali Al-Khaiyat). Thorndike (*History of Magic*, II, 75) mentions that in 1153, John of Spain, or John of Toledo, translated the *Nativities* of Albohali.—*Ed.*]

reason, so that it would be trouble wasted, and, if he listens to reason, he need only consider the matter for himself and he will soon find that so far as predictions are concerned, they are mere foolishness without base or reason. Further, it appears to me to be ridiculous to believe that an astrologer or geomancer or other of the sort can make sure predictions as to things in the future, which are fortuitous and under control of the changeable human will, when he can't say what sort of a day it will be to-morrow or prophesy a change in the weather or in the wind. Yet these latter things happen independently of fortune and result from the influence of the heavens and cannot be impeded except by divine miracle. Also, supposing a fortuitious result were known to the man to whom it is to happen, the which event depends on free will, I pose this question. Either it can be prevented or it cannot. If it cannot then it follows that all things happen of necessity and it is not worth while to ascertain them beforehand. And if it can be prevented then it cannot be said to be known. So that it seems to be impossible to know the future, precisely and certainly, by any human art. And that is what the Sage says in Ecclesiastes, namely, that man has many afflictions and cannot know by any message what is to come.[18] And, on this, it is written in a letter of King Alexander that when he asked his fate of the sacred trees the reply was that he would die poisoned. And when he asked at whose hand, he was told that he could not know this, for if he did he might impede the decrees of the fates.[19] And to any man who studies attentively the books of predictions it is plain, whatever they may say to the contrary, that the writers believe everything happens of necessity. And Julius Fermacus in his book, which includes almost the whole of judicial astrology, states this explicitly,[20] but it is false and contrary to the Faith and is condemned and attacked by every law. And Tully wrote a book on Divinations in which he gives plenty of stories and examples to illustrate the fact that there is no certainty in these matters;[21] and following his example I have been pleased to name this treatise, "On Divination". Further, besides the arts I named in the first chapter, there are others which are more or less promising and yet entirely uncertain, such as Saturn's head,[22] the notary art, the contrenotary art, pyromancy, spatulomancy, the art of sparks, divination by means of metals, wax, bread, sneezes, thunder, and similar sorceries. But I won't stop to reprove them for it is not my main purpose. At the same time, I do not say that some things that are absent, or still to come, cannot be known by prophecy, or by divine revelation, or by the use of human reason, and without the aid of the sciences in question; but such visions come to men of sober and peaceful life, whose souls are like clear and shining mirrors, clean from all worldly thoughts. So that Rabbi Moses the Egyptian says that concupiscence and melancholy prevent prophecy[23] and Isaiah speaks to the same effect.[24] And sometimes, wars, plagues, and death can be conjectured from the stars, from comets, from monstrosities, from extraordinary appearances in the atmosphere, as we see in the second book of Machabees,[25] and in the history of the downfall of Jerusalem, and in many other places, especially in the history of Rome, but these conjectures can be only general and without certainty. With regard to that, Seneca says that a comet appeared in the reign of Octavian which was followed by good and not evil and he says this removed the ill repute of comets, for previously all the authorities attributed an evil significance to them.[26]

CHAPTER 15.

The aim of the sixth chapter was to show that it is possible to know the future by means of art. My reply is that it is possible only by the means set

18. *Ecclesiastes* VIII, 6, 7.

19. [Coopland's reference is to a work by Valerius on the deeds of Alexander the Great of Macedon (*Valerii Alexandri Polemi Res Gestae Alexandri Macedonis*, p. 190; no date or place of publication is supplied).—*Ed.*]

20. [That is, Julius Firmicus.—*Ed.*] See *Julii Firmici Materni Matheseos Libri VIII*, ed. Kroll and Skutch, 1897, 1913. . . p. 17.

21. *De Divinatione*, II, generally.

22. I have found no mention of this in any authority; the only famous head in this connection seems to be that credited to Roger Bacon.

23. Moses Maimonides, called by the Latins Rabbi Moyses of Cordova. [A selection from Maimonides' *Guide of the Perplexed* is printed in Selection 68.1.—*Ed.*] I have read only his *Director Dubitantium*, and that only as translated by Friendlander, New York, 1904. . .

In the *Director Dubitantium* there are many passages justifying Oresme's reference, e.g. p. 228 (*ed. cit.*), "You will find therefore that prophets are deprived of the faculty of prophesying when they mourn or are angry, or are similarly affected. Inspiration does not come upon a man when he is sad or languid and prevalence of sadness and dullness was undoubtedly the direct cause of the interruption of prophecy during the exile."

24. Possibly, though the passage is not conspicuous.

25. II Mach. III, 24 *et seq.*; X, 29, 30, *etc.*

26. *Nat. Quaes*, VII, Sect. 17. [Coopland observes that Oresme expands Seneca's passage.—*Ed.*]

forth in the eleventh chapter. Now, as regards the argument drawn from the dumb animals who have this foreknowledge, I say that Pliny, in the seventh book of his Natural History,[27] describes the miseries of human nature in comparison with, and in regard to, the other animals, and says that man is born dumb, helpless, impotent, able to do nothing without teaching except weep, and that this is not true of the other animals; does it not ensue, then, that if nature has given a gift to another animal that she has given it to man? But God and nature have given to man something better, namely, reason and memory, by use of which he can acquire human prudence by consideration of things past, present, and possible in the future, according to reasonable conjecture, by good counsel, and by legitimate industry, better than by listening to such divinations; and this is Seneca's opinion, as given in chapter ten. The second argument on human prudence may be met similarly by what I say here. And in this matter of provision for what may happen in the future, great assistance may be obtained from consideration of the history of times past. This is the view of Sallust,[28] and lords who have the task of government should sometimes read such histories, or hear them read, instead of the impossible fables to be found in many romances. And because the most reliable histories that we possess are in Latin, it is very desirable that princes should know Latin. As to a further argument, namely, that Ptolemy laid down that some men have a natural tendency towards power to foretell the future and can assist that power by art, my reply is that, if the tendency is towards prophesying changes in the atmosphere or weather, it may be assisted by astronomy or still better by observation of changes in the atmosphere, as Ptolemy says, and he calls such changes the secondary stars.[29] But, if the tendency is towards prophesying by the help of visions, one can help this by leading a sober and peaceful life, as I said in the eleventh chapter, and in no other way unless it be by magic art or divine miracle, and, even here, not in regard to fortunes but in the respects treated of in the eleventh chapter. As for the remaining arguments, there are two points and not more that call for an answer. The first is Bernard's when he says that the sky is a book wherein are written the fortunes of kings. I reply that what is to happen in the future is not written in the sky, except in so far as from congruent movements we may know future constellations which are, or will be, cause or sign of various inclinations and diverse fortunes, saving always the freedom of the human will. The second point is that astrology would be worthless and God would have displayed this book, the sky, to us, without significance, if we cannot derive such knowledge from it. My reply is that this is not so, and that astrology has three very noble ends. The first is to have knowledge of such great matters, for to this, according to the philosophers, is human nature naturally inclined. And, as Aristotle says in a work that we no longer possess,[30] if it is a delectable and gracious thing to hear or know the ordinance and description of a noble royal palace, or a beautiful mountain, how much more excellent is it and more desirable to have knowledge of the form of the heavens, which exceeds and surpasses in beauty all things visible; and Tully says this in his book on the Nature of the Gods.[31] And Aristotle on this point says in the book of the Heavens and the Earth that it is better to know imperfectly a great matter than much more completely a matter of less moment, for, according to the philosophers, the natural perfection of the human understanding is attained in the study of such things. The second end, and the chief, is that it gives great aid in the knowledge of God the Creator. And Isidore says that it was ordained last of secular sciences so that by means of it the thought of man should be withdrawn from earthly, though praiseworthy, things to contemplation of the divine,[32] on which Tully says, in the work named above, that the argument that had most weight in bringing ancient philosophers to a knowledge of God was the marvellous quantity and most ordered movement of the heavens and the stars.[33] And the Scriptures speak similarly in the Book of Wisdom[34] and the Apostle in the Epistle to the Romans;[35] and Saint Denis, in an Epistle that he wrote, confesses that what he knew of astrology was the occasion of his being

27. The famous passage which introduces the book.

28. *De Bello Jugurthino* IV; the sentiment is there expressed only by implication, as also in the opening of the *De conjuratione Catilinae*.

29. [Apparently drawn from Haly's commentary on No. XIII of the *Centiloquium* attributed to Ptolemy. See n. 11.—*Ed.*]

30. [Perhaps a reference to some pseudo-Aristotelian treatise which was lost, or unavailable, in Oresme's day.—*Ed.*]

31. [*De natura deorum*, Bk. II, 40, 104.—*Ed.*]

32. Isidore, *Etymologies*, III, 71.

33. The thought is not expressed in this form in Cicero, but *De natura deorum*, II, 61, has the substance of it. . . .

34. XIII, 2, 4, 5.

35. I. 20.

converted to the faith of Jesus Christ.[36] The third end of astrology and the least important is to ascertain certain dispositions of this lower and corruptible nature, whether present or to come, and nothing beyond that, as I have laid down in several passages and especially in the second chapter. I say then that astrology is not worthless and it is not without purpose that God and Nature have displayed the heavens to us.

Cosmology

67 ON THE POSSIBLE DIURNAL ROTATION OF THE EARTH

Introduction by Edward Grant

Prior to Copernicus very few had accepted the daily rotation of the earth as a part of their astronomy and cosmology. The most notable proponents of its rotation were the Greeks Heraclides of Pontus and Aristarchus of Samos, whose influence in subsequent centuries was extremely minimal, as the physical and cosmological opinions of Aristotle and Ptolemy dominated Greek, Arabic, and medieval thought. But reports by Ptolemy, Simplicius, and even Aristotle (whose report on Plato seemed to classify Plato as an advocate of the earth's motion) preserved knowledge of these earlier views about the earth's rotation and gave rise in the Middle Ages to important discussions by John Buridan and Nicole Oresme, who, after formulating arguments in behalf of the earth's diurnal rotation, accepted the traditional position for quite different reasons. Although it has never been determined whether either one influenced Copernicus, it will be evident that a number of their arguments in favor of the earth's rotation appear in strikingly similar fashion in Book I of the *De revolutionibus*.

1. Ptolemy (fl. 127–151): The Immobility of the Earth in the Center of the World

Translated by R. Catesby Taliaferro[1]

Annotated by Edward Grant

7. THAT THE EARTH DOES NOT IN ANY WAY MOVE LOCALLY

By the same arguments as the preceding it can be shown that the earth can neither move in any one of the aforesaid oblique directions, nor ever change at all from its place at the centre. For the same things would result as if it had another position than at the centre. And so it also seems to me superfluous to look for the causes of the motion to the centre when it is once for all clear from the very appearances that the earth is in the middle of the world and all weights move towards it. And the easiest and only way to understand this is to see that, once the earth has been proved spherical considered as a whole and in the middle of the universe as we have said, then the tendencies and movements of heavy bodies (I mean their proper movements) are everywhere and always at right angles to the tangent plane drawn through the falling body's point of contact with the earth's surface. For because of this it is clear that, if they were not stopped by the earth's surface, they too would go all the way to the centre itself, since the straight line drawn to the centre of a sphere is always perpendicular to the plane tangent to the sphere's surface at the intersection of that line.

All those who think it paradoxical that so great a weight as the earth should not waver or move anywhere seem to me to go astray by making their judgment with an eye to their own affects and not to the property of the whole. For it would not still appear so extraordinary to them, I believe, if they stopped to think that the earth's magnitude compared to the whole body surrounding it is in the ratio of a point to it. For thus it seems possible for that which is relatively least to be supported and pressed against from all sides equally and at the same angle by that which is absolutely greatest and homogeneous. For there is no "above" and

36. See *Acta Sanctorum* [*Acts of the Saints—Ed.*], IV, Oct. IX, cols. 774–5, for the wonders of the heavens which influenced St. Denis.

1. Reprinted with the permission of the *Encyclopaedia Britannica* from Ptolemy's *Almagest*, Book I, chapter 7, as translated by R. Catesby Taliaferro in *Great Books of the Western World* (Chicago: Encyclopaedia Britannica, 1952), XVI, 10–12.

"below" in the universe with respect to the earth, just as none could be conceived of in a sphere. And of the compound bodies in the universe, to the extent of their proper and natural motion, the light and subtle ones are scattered in flames to the outside and to the circumference, and they seem to rush in the upward direction relative to each one because we too call "up" from above our heads to the enveloping surface of the universe; but the heavy and coarse bodies move to the middle and centre and they seem to fall downwards because again we all call "down" the direction from our feet to the earth's centre.[2] And they properly subside about the middle under the everywhere-equal and like resistance and impact against each other. Therefore the solid body of the earth is reasonably considered as being the largest relative to those moving against it and as remaining unmoved in any direction by the force of the very small weights, and as it were absorbing their fall. And if it had some one common movement, the same as that of the other weights, it would clearly leave them all behind because of its much greater magnitude. And the animals and other weights would be left hanging in the air, and the earth would very quickly fall out of the heavens. Merely to conceive such things makes them appear ridiculous.

Now some people, although they have nothing to oppose to these arguments, agree on something, as they think, more plausible. And it seems to them there is nothing against their supposing, for instance, the heavens immobile and the earth as turning on the same axis from west to east very nearly one revolution a day,[3] or that they both should move to some extent, but only on the same axis as we said, and conformably to the overtaking of the one by the other.

But it has escaped their notice that, indeed, as far as the appearances of the stars are concerned, nothing would perhaps keep things from being in accordance with this simpler conjecture, but that in the light of what happens around us in the air such a notion would seem altogether absurd.[4] For in order for us to grant them what is unnatural in itself, that the lightest and subtlest bodies either do not move at all or no differently from those of contrary nature, while those less light and less subtle bodies in the air are clearly more rapid than all the more terrestrial ones; and to grant that the heaviest and most compact bodies have their proper swift and regular motion, while again these terrestrial bodies are certainly at times not easily moved by anything else—for us to grant these things, they would have to admit that the earth's turning is the swiftest of absolutely all the movements about it because of its making so great a revolution in a short time, so that all those things that were not at rest on the earth would seem to have a movement contrary to it, and never would a cloud be seen to move toward the east nor anything else that flew or was thrown into the air. For the earth would always outstrip them in its eastward motion, so that all other bodies would seem to be left behind and to move towards the west.

For if they should say that the air is also carried around with the earth in the same direction and at the same speed, none the less the bodies contained in it would always seem to be outstripped by the movement of both.[5] Or if they should be carried around as if one with the air, neither the one nor the other would appear as outstripping, or being outstripped by, the other. But these bodies would always remain in the same relative position and there would be no movement or change either in

2. Unlike Aristotle, Ptolemy seems here to deny an absolute "up" and "down" with respect to the earth. These directions are only relative.

3. This interpretation was held by Heraclides of Pontus and Aristarchus of Samos (see sections 3 and 5 of this selection and n. 22). By reporting this important minority opinion, Ptolemy joins Simplicius *(Commentary on Aristotle's De caelo)* as one of three primary sources introducing this conception to medieval authors (the third is Aristotle's *De caelo*, where Aristotle seems to attribute to Plato a belief in the axial rotation of the earth at II.13.293b.30–32 and to mention it again, rather vaguely, in II.14.296.b.1–4). Aquinas probably derived it from Simplicius' commentary in the translation made available by William Moerbeke (see number 47 of Moerbeke's list in Selection 8); Oresme probably knew all three sources (see section 5 of this selection).

4. It is of interest to note that Ptolemy concedes that the earth's diurnal rotation is compatible with the purely astronomical phenomena, while holding it absurd in terms of physics.

5. Buridan was unquestionably familiar with this argument, for he reports that adherents of diurnal rotation say that "the earth, water, and the air in the lower region are moved simultaneously with diurnal motion (see pars. 7 and 8 of section 4 of this selection). While Buridan appears to accept this, he rejects diurnal rotation by arguing that bodies moved violently ought to resist the motion of the air as it accompanies a moving earth. Experience shows that they do not (see par. 9). Copernicus explicitly reports and rejects Ptolemy's opinion on the disruptive effects of a terrestrial motion (see Bk. I, ch. 7 and ch. 8, of Copernicus' *De revolutionibus*, in section 6 of this selection).

the case of flying bodies or projectiles. And yet we shall clearly see all such things taking place as if their slowness or swiftness did not follow at all from the earth's movement.[6]

2. Thomas Aquinas (ca. 1225–1274): The Immobility of the Earth in the Center of the World

Translated and annotated by Edward Grant[7]

1. Inasmuch as the Philosopher [that is, Aristotle] has described the opinions of others concerning the earth, he here considers it according to the truth [of the matter]. First he solves the problem of the earth's place and rest; and secondly, where he says "Its shape must necessarily be spherical"[8] (297a.9), he considers its shape. He makes two points concerning the first part: First he determines the truth by means of natural reasons; and secondly, where he says "This view is further supported. . . " (297a.3), he resorts to astronomical appearances. With regard to the first of these [two parts], he makes two points: First he shows that it is impossible for the earth to be moved; secondly, where he says ["From what we have said the explanation of the earth's immobility] is also apparent" (296b.28), he gives the true explanation for the earth's immobility from premises.

Concerning the first part,[9] he makes three points. First he directs attention to what must be discussed first, namely whether the earth moves or rests. If it moves, then we must move on to other things that have to be considered about the earth. And thus he posits this first[10] in order to assume it as a beginning for what is to follow.

Secondly, where he says "For, as we said, [there are some who make it one of the stars]. . . " (296a.25), he shows the necessity for the aforesaid inquiry.[11] For, as stated above,[12] some, namely the Pythagoreans, assumed that the earth was moved around the center of the world—as if it were one of the planets;[13] others, indeed, assuming that the earth is in the center, as it is written in the *Timaeus*, say that it is rotated around the middle of the pole—that is, around the axis dividing the sky down the middle.[14]

2. Thirdly, where he says "That both views are untenable. . ." (296a. 27), he shows by means of four reasons that it is impossible for the earth to be moved in this way. In the first of these [reasons] he takes it as a starting point that if the earth were moved circularly—whether in the center of the world or outside of it—such a motion would be necessarily violent. Now, it is obvious that circular motion is not a proper and natural motion for the earth; for if it were, every particle of earth should also have this motion, since, as stated above, a natural motion is the same for whole and part. But we see that this is false, because all parts of the earth are moved with a rectilinear motion toward the center of the world. If, however, the motion of the earth were circular, it would be violent and contrary to nature, and [therefore] could not be eternal, since, as was said earlier, nothing violent is eternal. But if the earth were moved circularly it would be necessary that such a motion be eternal—assuming, [of course,] that the world is eternal. In his [that is, Aristotle's] opinion this is so, because it is necessary that the universal order be eternal, and the motion or rest of the principal parts of the world [that is, heavens and earth] are part of this order. It follows, therefore, that the earth is not moved with a circular motion.

3. Where he says "Again, everything [that moves with the circular movement] . . ." (296a.34), he posits the second reason, which is this. All bodies that are moved circularly seem to drop back, that is, do not maintain a constant position, because every one of them—except the first sphere, which according to Aristotle is the sphere of the fixed stars—is moved with several motions and not by one only. Therefore, if the earth—whether it is in the center or outside the center—has a circular motion, it would necessarily be moved with several motions, namely with the motion of the first sphere around the equinoctial poles [that is, the poles of the equator], and with some other proper motion around the poles of the zodiac. But this cannot be, for if it were so, the fixed stars would exhibit changes and turnings with respect to the earth, which, in virtue of its proper motion, would lag behind and not return to the same point together with a fixed star—just as happens with the planets. This applies either to the whole earth or any part that may be designated. And so it would follow that the fixed stars would not always be seen to rise and set at the same part of the earth. But this does not happen, because the fixed stars rise and set at the same designated places. Therefore, the earth is not moved circularly.[15]

4. Where he says "Further, the [natural] movement [of the earth] . . ." (296b. 7), he asserts the

6. This last argument, that there would be no detectable motion if the air and everything in it rotated with the earth, seems to have been answered indirectly by Oresme and Copernicus. Thus Oresme says (in section 5) that "according to this opinion, not only the earth moves, but also with it the water, and the air, as we stated above, although the water and air here below may be moved in addition by the winds or other forces." Perhaps influenced by Oresme, Copernicus conjectures that despite the rotation of the earth, "the air nearest Earth, with the objects suspended in it, will be stationary unless disturbed by wind or other impulse which moves them this way or that—for a wind in the air is as a current in the sea" (Bk. I, ch. 8, in section 6).

7. Translated from Aquinas' *Commentaria in libros Aristotelis De caelo et mundo*, Lib. II, cap. XIV, Lectio 26, in *Sancti Thomae Aquinatis Opera Omnia iussu impensaque Leonis XIII* (Rome, 1886), Vol. III, p. 218, col. 1, to p. 221, col. 2. Wherever Thomas quotes the words of Aristotle, I have consulted the Oxford English translation, *The Works of Aristotle Translated into English under the Editorship of W. D. Ross*, Vol. II (1930; reprinted 1953).

8. Thomas comments upon this in Lectio 27.

9. That is, Aristotle's intention to determine the truth by natural reasons.

10. The motion of the earth.

11. The inquiry into the motion of the earth is necessary, because those who assume that the earth moves disagree as to the kind of motion it has.

12. Lectio 20, note 3.

13. Since that aspect of Aristarchus' view which assumed the earth's revolution around the sun was apparently unknown in the Middle Ages, Aristotle's account (293a.20—293b.6) of the Pythagorean view (to which Thomas alludes here)—that the earth is like a planet moving round a central fire—was the only other opinion available in which the earth was given an orbital motion in contradistinction to an axial rotation. This Pythagorean view was mentioned by Copernicus (see ch. 5 of section 6 of this selection).

14. It has been argued whether Aristotle meant to attribute to Plato (in the *Timaeus*) the view that the earth had an axial rotation. This was discussed earlier in Lectio 21, par. 5 (p. 204, col. 2, to p. 205, col. 2), where Thomas comments upon Book II, chapter 13 (293b.30–32). Thomas is quite explicit that Plato believed in the absolute immobility of the earth at the center of the universe and that Aristotle did not mean to attribute such a view to Plato. All Aristotle meant to say, according to Thomas, is that in the *Timaeus* it is assumed that the earth is packed, so to speak, around the axis of the world. Numerous scholars, ignorant of Thomas' opinions, adopted the same interpretation. It was J. L. E. Dreyer who first claimed that "this simple explanation [that is, that the expression "as it is written in the *Timaeus*" in 293b.32 refers only to the earth lying in the center about the axis of the universe and not to any motion of the earth—*Ed.*] was brought forward already six hundred years ago by good old Thomas Aquinas, a fact which neither Zeller nor Martin nor any other modern writer seems to have noticed" (*A History of Astronomy from Thales to Kepler*, 2d. ed. [New York: Dover, 1953] p. 78). Sir Thomas Heath, it should be noted, rejects Thomas' interpretation (*Aristarchus of Samos* [Oxford: Clarendon Press, 1913], p. 177).

Rather than Plato, Thomas believed that Aristotle had in mind Heraclides of Pontus ("Moreover we can say briefly that a certain Heraclitus Ponticus—whose opinion Aristotle presents here—assumed that the earth was moved in the center [of the world] and that the sky rested." Lectio 21, par. 5; p. 205, col. 2.). Heath (*Aristarchus of Samos*, p. 240) believes it implausible that Aristotle was representing the view of Heraclides, who assumed a daily axial rotation:

"It seems likely, as Dreyer suggests, that in speaking of a motion of the earth 'at the centre itself,' Aristotle is not thinking of a rotation of the earth in *twenty-four hours*, i.e. a rotation replacing the apparent revolution of the fixed stars, as Heraclides assumed that it did; for he does not mention the latter feature or give any arguments against it; on the contrary, he only deals with the general notion of a rotation of the earth, and moreover mixes up his arguments against this with his arguments against a translation of the earth in space. He uses against both hypotheses his fixed principle that parts of the earth, and therefore the earth itself, move naturally towards the centre. Whether, he says, the earth moves away from the centre or *at* the centre, such movement could only be given to it by *force;* it could not be a natural movement on the part of the earth because, if it were, the same movement would also be natural to all its parts, whereas we see them all tend to move in straight lines towards the centre; the assumed movements, therefore, being due to force and against nature, could not be everlasting, as the structure of the universe requires."

15. This unclear and quite confused passage must be examined against the background of the specific Aristotelian passage (296a.34—296b.6) which it purports to explain. Thomas Heath translates it as follows (pp. 240–241):

"Further, all things which move in a circle, except the first (outermost) sphere, appear to be left behind and to have more than one movement; hence the earth, too, whether it moves about the centre or in its position at the centre, must have two movements. Now, if this occurred, it would follow that the fixed stars would exhibit passings and turnings. This, however, does not appear to be the case, but the same stars always rise and set at the same places on the earth."

Heath supplies an important commentary to this passage (p. 241), which is worthy of full citation:

"The bodies which appear to be 'left behind and to have more movements than one' are of course the planets. The argument that, if the earth has one movement, it must have two, is based upon nothing more than analogy with the planets. Aristotle clearly inferred as a corollary that, if the earth has two motions, one must be oblique to the other, for it would be the obliquity to the equator in at least one of the motions which would produce what he regards as the necessary consequence of his assumption, namely that the fixed stars would not always rise and set at the same places. As already stated, Aristotle can hardly have had clearly in his mind the possibility of one single rotation about the axis in *twenty-four hours* replacing exactly the apparent daily rotation; for he would have seen that this would satisfy his necessary condition that the fixed stars shall always rise and set at the same places, and therefore that

third reason, which depends upon the motion of parts of the earth and of the whole. He makes three points concerning this. In the first place, he proposes what the natural motion of the earth and its parts must be; secondly, where he says "but it might be [questioned] . . ." (296b. 9), he raises a certain doubt concerning this; and thirdly, he proves what he proposed.

In the first place, then, he says that motion of parts of the earth—in accordance with the nature of earth—is toward the center of the whole world; and, similarly, if the whole earth were outside the center of the world, it would, by its very nature, be moved to the center of the world, because the natural motion of the whole is the same as that of a part.

5. Then, when he says "but it might be [questioned]. . . " (296b.9), he raises a certain doubt. First, he proposes the doubt, saying that if the earth were assumed in the middle or center of the world—he means by this that the center of the whole world and of this earth would be the same—one can ask to which of these are heavy bodies—especially parts of the earth—moved naturally. That is, are they moved to the center because it is in the center of the world, or because it is the center of the earth?

Secondly, where he says "The goal, surely. . . " (296b.13), he resolves the doubt, saying that heavy bodies are moved to the center because it is the center of the whole world. For the motion of heavy bodies is contrary to the motion of light bodies; but light bodies, especially fire, are moved to the extremity of the [innermost] celestial body.[16] Therefore, heavy bodies, especially earth, are moved to the center of the earth, not, however, per se but accidentally, because the center of the earth and the center of the world coincide. . . .

6. Thirdly, where he says "That the center of the earth is the goal of their movement. . ." (296b.18), he proves what he assumed, namely that heavy bodies and parts of the earth are moved to the center. . . .[17]

7. Where he says "It is clear, then. . ." (296b. 21), he concludes what he has proposed. But he infers two propositions. The first of these is that the earth is in the center of the world, which is deduced from premises in the following way: [1] All heavy bodies are self-moved toward the center of the world; [2] furthermore, as was [just] demonstrated, they are also moved toward the center of the earth; therefore, the center of the earth is the center of the world. And so the earth is in the center of the world.

The second proposition is that the earth is immobile. This is deduced from premises in the following way: [1] Nothing is moved [when it is] in a place to which it is moved naturally, since it is natural for it to rest there; [2] but earth is moved naturally toward the center of the world; therefore, it is not moved in the center. Moreover, as has been shown, the earth cannot but be in the center of the world; therefore, the earth is in no way moved.

8. He posits the fourth reason where he says "because heavy bodies forcibly thrown. . . " (296b.23). We see that if a stone were suspended above some table, it would fall down in a straight line; and as long as its motion is downward it will fall along the same straight line. Now, if the table were not moved, the stone would fall to the [very] same place where it fell before; but if the table were moved, the stone would fall to another place that would be as far removed [from the previous place] in proportion as the height from which the stone is dropped increases, since a greater time will elapse between the beginning and the end of the fall. However, we see that heavy bodies fall downward in a regular manner—that is, in a straight line—and that they fall to the same place on earth when dropped repeatedly from a particular place. Now, someone might say that because of the slowness of the earth's motion, there is an imperceptible distance between each of these places. But this overlooks the fact that if someone were to cause the stone to fall downward time after time for an infinite number of times, the same thing would occur, so that the extent of the time [involved] should make the distance between the places perceptible. And so it is obvious that the earth is not moved.

Then, in conclusion, he deduces from premises that the earth can neither be moved nor located outside the center of the world. . .

11. That the earth is not movable from place to place with a translatory motion results from the fact that it is always in the center. But should it be moved with any motion whatever, it would follow, again, that in virtue of its velocity all other motions—whether of clouds or animals—would be concealed from us. For that does not appear to be moved which is moved more slowly near a body that is moved more quickly.[18]

Therefore, in closing, the Philosopher concludes that enough has been said on the relation between the position, motion, and rest of the earth.

he would have to get some further support from elsewhere to his assumption that the earth must have *two* motions. Still less could he have dreamt of the possibility of Aristarchus's later hypothesis that the earth has an annual revolution as well as a daily rotation about its axis, which hypothesis satisfies, as a matter of fact, both the condition as to two motions and the condition as regards the fixed stars."

It is immediately evident that Thomas has made explicit the two motions left unspecified by Aristotle. We have both a motion around the poles of the earth (equinoctial poles) and the poles of the zodiac. Thomas, following Aristotle, tells us that if the earth moved at all it would necessarily possess these two motions—whether or not it is at the center of the world. If it were at the center of the world, it would have an axial rotation—but no translatory motion—and simultaneously rotate around the poles of the zodiac—that is, ecliptic—which is inclined to the earth's axis by approximately 23½ degrees. However, despite the obliquity of the second motion to the first, and assuming uniformity of motion, the risings and settings would, it seems, follow regular patterns. And if we assume that the earth is not in the center of the universe, the two motions attributed to the earth would be those applicable to any planet—that is, a daily motion around the equinoctial poles as it is carried along with the other planets by the daily rotation of the sphere of the fixed stars (hence no axial rotation peculiar to the earth is involved), and a proper motion, that is, an annual periodic motion stemming from the motion of the sphere in which the earth—here treated as any other planet—would be embedded. On this interpretation, then, the earth would be assumed to have two translatory motions but no daily axial rotation—and this would be consistent with Thomas' comparison of the earth's motion to that of a planet. If Thomas had this scheme in mind, and the two motions are assumed uniform and commensurable, his consequence that "the fixed stars would not always be seen to rise and set at the same part of the earth" would again not follow. The phenomena of risings and settings would proceed in regular patterns.

Finally, it might be asked whether we do not have here, perhaps, a reflection of the heliocentric system enunciated by Aristarchus, where one motion of the earth would be a daily axial rotation and the other a revolutionary translatory motion around the sun in the course of a year. Such an interpretation seems unwarranted, since Thomas makes no mention of the sun as center. From his all too brief description, one is only justified in assuming that the two motions are of the same kind for each of the two possibilities. Thus, as we have seen, if the earth is at rest in the center of the universe, the two motions are axial; if it is not in the center, both are translatory. There is no justification for taking one motion as axial and the other as rotational. But just this combination of motions would be required if we are to see in this discussion the principal assumptions enunciated by Aristarchus. However, on the remote possibility that this is what Thomas intended, his consequence that such motions of the earth could not account for stellar risings and settings would be false.

Perhaps the underlying difficulty in all this lies in an implicit, but rash, inference. For Aristotle and Aquinas seem to have inferred that because a planet appears to move irregularly against the background of fixed stars, an observer on that planet would necessarily see only irregular risings and settings of these stars. Then, by analogy, they concluded that if the earth had the two motions of a planet, observers on earth would also see variations in the risings and settings of stars.

16. Fire would rise no higher than the concave surface of the moon, beyond which lay the celestial region that was filled with a fifth element, aether, extending all the way to the outermost sphere (that is, the sphere of the fixed stars).

17. The "proof" according to Aristotle is "indicated by the fact that heavy bodies moving toward the earth do not move parallel but so as to make equal angles, and thus to a single centre, that of the earth" (296b.19–21). That is, wherever heavy bodies fall toward a point on the surface of the earth, they fall at right angles to a tangent drawn at that point.

18. Brevity and lack of clarity make this passage difficult to comprehend. Thomas seems to argue that if the earth moved (translatory or rotatory), its speed would be greater than that of any other body on, or near, its surface. In this event, bodies on the earth would appear to be unmoved on the strange principle that a body will appear stationary when it is "moved more slowly near a body that is moved more quickly." It is possible that Thomas has garbled one of Ptolemy's arguments against the earth's axial rotation which we have quoted elsewhere (see section 1 of this selection).

19. Translated from Aquinas' *Commentaria in libros Aristotelis De caelo et mundo*, Lib. II, cap. VIII, Lectio 11, in *Sancti Thomae Aquinatis Opera Omnia*, Vol. III, p. 162, col. 1. Thomas is here commenting upon the first part of II.8 (289b.1—290a.7) of Aristotle's *On the Heavens*. For the translation of Thomas' quotations of Aristotle, I have consulted the Oxford English translation.

20. This is a reference to the opening words of II.9 of Aristotle's *On the Heavens*.

3. Thomas Aquinas: Heraclides of Pontus, and Aristarchus Mentioned as Proponents of the Earth's Diurnal Rotation

Translated and annotated by Edward Grant[19]

1. After the Philosopher [that is, Aristotle] shows what sort of nature the stars have, he takes up their motion here. First he shows how the stars are moved; secondly, where he says "From all this it is clear. . . " (290b. 12),[20] he shows whether sounds could be produced by their motion.

On the first point he offers three reasons to show that the stars are not self-moved, but [rather] that

they are carried by the motion of the orbs [that is, spheres]. The first of these reasons is put forth by way of a comparison of stars to orbs. In this reason, indeed, he presupposes one thing that is evident to the senses: We see that the stars and the entire sky are moved. Now, of necessity, this could happen in [only] three ways:[21] in one way, each—namely both star and sphere—is at rest; in another way, each of them is moved; in the third way, one of them rests and the other is moved. This division being assumed, he takes up the three parts just mentioned.

2. He first takes up the first way, where he says "That both should be at rest. . . " (289b.4). On this point he says that it is impossible to say that each—namely star and sphere—rests if it is also assumed that the earth rests. For the apparent motion of the stars could not be saved if the stars, which appear to be moved, and the men who view the stars should both be at rest. Indeed, the appearance of motion is caused either by the motion of the thing seen or by the motion of the one who sees it. For this reason some people, assuming that the stars and the whole sky rest, have posited that the earth on which we dwell is moved once daily from west to east around the equinoctial poles. Thus by our motion, it seems to us that the stars are moved in a contrary direction, which is what Heraclides of Pontus and Aristarchus are said to have posited.[22] However, in the present discussion, Aristotle assumes that the earth rests; but later he will prove it.

4. John Buridan (ca. 1300—1358): The Compatibility of the Earth's Diurnal Rotation with Astronomical Phenomena

Translation and commentary by Marshall Clagett[23]

Annotated by Edward Grant

BOOK II, QUESTION 22

1. It is sought consequently whether the earth always is at rest in the center of the universe. . . .

This question is difficult. For in the first place there is a significant doubt as to whether the earth is directly in the middle of the universe so that its center coincides with the center of the universe. Furthermore, there is a strong doubt as to whether it is not sometimes moved rectilinearly as a whole, since we do not doubt that often many of its parts are moved, for this is appearent to us through our senses. There is also another difficult doubt as to whether the following conclusion of Aristotle is sound, namely, if the heaven is by necessity to be moved circularly forever, then it is necessary that

21. After some discussion Aristotle concludes that "since . . . we cannot reasonably suppose either that both are in motion or that the star alone moves, the remaining alternative is that the circles should move, while the stars are at rest and move with the circles to which they are attached" (289b.31–34). By circles Aristotle means orbs or spheres. Almost without exception in the Middle Ages it was held that the stars and planets are not self-moved but are carried around by the spheres in which they were thought to be embedded.

22. Thomas almost certainly gets this from Simplicius' commentary on Aristotle's *On the Heavens*, which was translated from Greek into Latin by William of Moerbeke in 1271. Indeed, Simplicius mentions Heraclides and Aristarchus as proponents of the earth's diurnal rotation in commenting on the very same passage as St. Thomas (*De caelo* II.8.289b.5–7). Cohen and Drabkin translate the relevant passage in Simplicius' commentary as follows: "There have been some, among them Heraclides of Pontus and Aristarchus, who thought that the phenomena could be accounted for by supposing the heaven and stars to be at rest, and the earth to be in motion about the poles of the equator from west [to east] making approximately one complete rotation each" (*A Source Book in Greek Science* [Cambridge, Mass.: Harvard University Press, 1948], pp. 106–107). Heraclides (Thomas spells it Heraclitus) lived ca. 388 B.C. to ca. 310 B.C.; Aristarchus of Samos ca. 310 B.C. to ca. 230 B.C. This translation also serves as a very accurate rendition of Moerbeke's Latin translation on folio 71 verso, col. 1, of the Venice edition of 1540. Also in this same translation of Simplicius' commentary are two other references to Heraclides of Pontus as a proponent of the earth's diurnal rotation (see fols. 83 recto, col. 1, and 87 verso, col. 1). Medieval discussions of the earth's daily axial rotation were, for the most part, probably derived ultimately from Simplicius via Moerbeke's translation rather than from Ptolemy's *Almagest*, where it is also mentioned (see section 1 of this selection). This is likely not only because few would have read so formidable a work as the *Almagest* but also because Ptolemy makes no mention of Heraclides or Aristarchus, thus indicating that where one or both of these names are associated with a discussion of the earth's diurnal rotation, Simplicius is the probable source (see n. 3).

23. Reprinted with permission of the Regents of the University of Wisconsin from Marshall Clagett, *The Science of Mechanics in the Middle Ages* (Madison, Wis.: University of Wisconsin Press, 1959), pp. 594–599. Clagett's translation was made from E. A. Moody's Latin edition of Buridan's *Quaestiones super libris quattuor De caelo et mundo* (Cambridge, Mass.: Mediaeval Academy of America, 1942), pp. 226–232. The omissions, as indicated by ellipses, are Clagett's.

the earth be at rest forever in the middle. There is also a fourth doubt whether, in positing that the earth is moved circularly around its own center and about its own poles, all the phenomena that are apparent to us can be saved (*possent salvari omnia nobis apparentia*). Concerning this last doubt let us now speak.

2. It should be known that many people have held as probable that it is not contradictory to appearances for the earth to be moved circularly in the aforesaid manner, and that on any given natural day it makes a complete rotation from west to east by returning again to the west—that is, if some part of the earth were designated [as the part to observe]. Then it is necessary to posit that the stellar sphere would be at rest, and then night and day would take place through such a motion of the earth, so that that motion of the earth would be a diurnal motion (*motus diurnus*). The following is an example of this [kind of thing]: If anyone is moved in a ship and he imagines that he is at rest, then, should he see another ship which is truly at rest, it will appear to him that the other ship is moved. This is so because his eye would be completely in the same relationship to the other ship regardless of whether his own ship is at rest and the other moved, or the contrary situation prevailed.[24] And so we also posit that the sphere of the sun is everywhere at rest and the earth in carrying us would be rotated. Since, however, we imagine that we are at rest, just as the man located on the ship which is moving swiftly does not perceive his own motion nor the motion of the ship, then it is certain that the sun would appear to us to rise and then to set, just as it does when it is moved and we are at rest.

3. It is true, however, that if the stellar sphere is at rest, it is necessary to concede generally that the spheres of the planets are moving, since otherwise the planets would not change their positions relative to each other and to the fixed stars. And, therefore, this opinion imagines that any of the spheres of the planets moved evidently like the earth from west to east, but since the earth has a lesser circle, hence it makes its rotation (*circulatio*) in less time. Consequently, the moon makes its rotation in less time than the sun. And this is universally true, so that the earth completes its rotation in a natural day, the moon in a month, and the sun in a year, etc.

4. It is undoubtedly true that, if the situation were just as this position posits, all the celestial phenomena would appear to us just as they now appear. We should know likewise that those persons wishing to sustain this opinion, perhaps for reason of disputation, posit for it certain persuasions. . . . The third persuasion is this: To celestial bodies ought to be attributed the nobler conditions, and to the highest sphere, the noblest. But it is nobler and more perfect to be at rest than to be moved. Therefore, the highest sphere ought to be at rest. . . .

The last persuasion is this: Just as it is better to save the appearances through fewer causes then through many, if this is possible, so it is better to save [them] by an easier way than by one more difficult. Now it is easier to move a small thing than a large one. Hence it is better to say that the earth, which is very small, is moved most swiftly and the highest sphere is at rest than to say the opposite.[25]

5. But still this opinion is not to be followed. In the first place because it is against the authority of Aristotle and of all the astronomers (*astrologi*). But these people respond that *authority does not demonstrate*, and that it suffices astronomers that they posit a method by which appearances are saved, whether or not it is so in actuality. Appearances can be saved in either way; hence they posit the method which is more pleasing to them.

6. Others argue [against the theory of the earth's diurnal rotation] by many appearances (*apparentiis*). One of these is that the stars sensibly appear to us to be moved from the east to the west. But they solve this [by saying] that it would appear the same if the stars were at rest and the earth were moved from west to east.

7. Another appearance is this: If aynone were moving very swiftly on horseback, he would feel the air resisting him. Therefore, similarly, with the very swift motion of the earth in motion, we ought to feel the air noticeably resisting us. But these [supporters of the opinion] respond that the earth, the water, and the air in the lower region are moved simultaneously with diurnal motion. Consequently there is no air resisting us.[26]

24. See the similar discussions of relative motion in a ship by Oresme (section 5) and Copernicus (section 6).

25. Oresme presents the same argument (section 5) as did Copernicus later in rebutting Ptolemy near the beginning of Book I, chapter 8, of *De revolutionibus* (see section 6). Galileo, in turn, would repeat Copernicus' arguments in the *Dialogue Concerning the Two Chief World Systems* (see p. 115 of Stillman Drake's translation [Berkeley, Calif.: University of California Press, 1962]).

26. Ptolemy had already formulated this counterargument against his own position. However, Buridan makes no mention of Ptolemy's response to it (see section 1 and n. 6).

8. Another appearance is this: Since local motion heats, and therefore since we and the earth are moved so swiftly, we should be made hot. But these [supporters] respond that motion does not produce heat except by the friction *(confricatio)*, rubbing, or separation of bodies. These [causes] would not be applicable there, since the air, water, and earth would be moved together.

9. But the last appearance which Aristotle notes is more demonstrative in the question at hand. This is that an arrow projected form a bow directly upward falls again in the same spot of the earth from which it was projected. This would not be so if the earth were moved with such velocity. Rather before the arrow falls, the part of the earth from which the arrow was projected would be a league's distance away. But still the supporters would respond that it happens so because the air, moved with the earth, carries the arrow, although the arrow appears to us to be moved simply in a straight line motion because it is being carried along with us. Therefore, we do not perceive that motion by which it is carried with the air. But this evasion is not sufficient because the violent impetus of the arrow in ascending would resist the lateral motion of the air so that it would not be moved as much as the air. This is similar to the occasion when the air is moved by a high wind. For then an arrow projected upward is not moved as much laterally as the wind is moved, although it would be moved somewhat. . . .[27]

10. Then I come to the other doubts. One would be whether the earth is situated directly in the middle of the universe. It should be answered in the affirmative. For we suppose that the place [designated] absolutely *(simpliciter)* as "upward," insofar as one looks at this lower world, is the concave [surface] of the orb of the moon. This is so because something absolutely light, i.e., fire, is moved toward it. For since fire appears to ascend in the air, it follows that fire naturally seeks a place above the air, and this place above the air is at the concave [surface] of the orb of the moon; because no other element appears to be so swiftly moved upward as fire. Now the place downward ought to be the maximum distance from the place upward, since they are contrary places. Now that which is the maximum distance from the heaven is the middle of the universe. Therefore the middle of the universe is absolutely downward. But that which is absolutely heavy—and earth is of this sort—ought to be situated absolutely downward. Therefore, the earth naturally ought to be *in* the middle of the universe or *be* the middle of the universe.

11. But it is a significant difficulty as to whether the center of magnitude in the earth is the same as the center of gravity *(medium gravitatis)*. It seems according to some statements that it is not. This is because if a large region of the earth is not covered with waters due to the habitation of animals and plants, and the opposite part is covered with waters, it is clear that the air which is naturally hot, and the sun, make hot the noncovered part, and thus they make it to some degree more subtle, rarer, and lighter. The covered part remains more compact and heavier. Now if one body in one part is lighter and in an opposite part heavier, the center of gravity will not be the center of magnitude. Rather with the center of gravity given, the greater magnitude will be in the lighter part, just as in the case of balances if on one side a stone is placed and on the other side wool [and they balance], the wool will be of a much greater magnitude.

12. With this understood, it ought to be seen which of those centers is the center of the universe. It should be answered immediately that the center of the universe is the center of gravity of the earth. This is because, as Aristotle says, all parts tend toward the center of the universe through their gravity,[28] and a part which is heavier would displace another, and thus finally it is necessary that the center of the universe coincide with the center of gravity. From these arguments it follows that the earth is nearer to the heaven in the part not covered with waters than in the covered part, and thus at the covered part there is greater declivity, and so the waters flow to that part. So, therefore, the earth, with respect to its magnitude, is not directly in the center of the universe. We commonly say, however, that it is in the center of the universe, because its center of gravity is the center of the universe.

13. By this another doubt is solved, evidently, whether the earth is sometimes moved according to its whole in a straight line. We can answer in the affirmative because from this higher [part of] the earth many parts of the earth (i.e., debris) continually flow along with the rivers to the bottom of the sea, and thus the earth is augmented in the covered part and is diminished in the uncovered part. Consequently, the center of gravity does not remain the same as it was before. Now, therefore, with the center of gravity changed, that which has newly become the center of gravity is moved so that it will coincide with the center of the universe, and that point which was the center of gravity before ascends and recedes, and thus the whole earth is elevated toward the uncovered part so that the center of gravity might always become the center of the universe. And just as I have said elsewhere, it is not apparent how it could be saved unless the mountains were consumed and destroyed sometimes, nay infinite times, if time were eternal. Nor is any other way apparent by which such mountains could be generated. This was spoken of elsewhere, so I shall now desist. . . .

27. See Clagett's Commentary below.
28. *On the Heavens* II.14, 296b.7–25.

COMMENTARY

The first passage outlines the various "doubts." In passage 2 the relative nature of the perception of motion is introduced: "If anyone is moved in a ship and he imagines that he is at rest, then, should he see another ship which truly is at rest, it will appear to him that the other ship is moved." In the same way, an observer on a moving earth would think he was at rest and the heaven, truly at rest, he would believe to be in motion. On the other hand (passage 3) the planets change their positions relative to each other and the fixed stars. Hence they are in movement. But all celestial phenomena (passage 4) would appear to us as they do regardless of whether the heavens or the earth rotate.

Furthermore, according to Buridan, certain persuasions can be introduced to support the diurnal rotation of the earth. After presenting some that involve the natural superiority of the heavens over the earth, Buridan in the last persuasion gives the principle of economy or simplicity; saying that it is better to save appearances by the simpler and easier movement, i.e., the rotation of the earth.

In passage 5 Buridan notes that the authority of Aristotle is against the diurnal rotation of the earth. But the holders of the theory of the earth's rotation would however answer rightfully, "Authority does not demonstrate." Furthermore, astronomers are only interested in finding the simplest way to save appearances. They are not concerned with whether the theory is so in actuality. Here Buridan is assigning the same sort of role to astronomers as did the Greek astronomer Geminus, whom Simplicius quotes as saying that the astronomer merely seeks hypotheses by the assumption of which the phenomena will be saved; it is not his business, but rather that of the physicist, to determine which hypothesis is actually more suitable to nature.[29]

In passage 7 Burdian presents the argument that, as the result of the diurnal rotation of the earth, one would expect a continually resisting wind. But the answer given is that the earth, water, and air are moved together. This would also explain why there is no heat generated by the friction between the air and the moving earth (passage 8). Finally, Buridan presents as an answer to the apparent rectilinear ascent and descent of arrows or projectiles by saying that the arrows are carried along with the air in the same diurnal motion as the earth. Buridan says this is not true, for the vertical motion of the arrow would resist the air. Perhaps Oresme, later, realized that it is not the air that causes the arrow to move laterally but rather the fact it is a part of the mechanical system of the rotating earth, just as a man moving his hand vertically up and down a mast is part of the mechanical system of the boat.[30]

In passage 11–13, Buridan argues that geological processes are always causing a redistribution of the matter of the earth; hence they are continually changing its center of gravity. But the center of gravity always strives to be at the center of the universe. Hence the earth is constantly shifting about near the center of the universe.

5. Nicole Oresme (ca. 1325–1382): The Compatibility of the Earth's Diurnal Rotation With Astronomical Phenomena and Terrestrial Physics

Translated by Albert D. Menut[31]

Annotated by Edward Grant

7. There are others who hold that the earth is at the center of the world and that it revolves and

29. This explicit distinction reported by Simplicius (the passage is translated in Cohen and Drabkin, pp. 90–91) appears in his commentary on Aristotle's *Physics*, which was not available in Latin translation during the Middle Ages. However, a similar sentiment was expressed by Simplicius in Book III, chapter 1, of his commentary on Aristotle's *De caelo* (the passage is translated in E. Grant, "Late Medieval Thought, Copernicus, and the Scientific Revolution," *Journal of the History of Ideas*, XXIII [1962], p. 198.)
30. See Oresme's discussion in the next section.
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moves in a circuit around the pole established for this purpose, as is written in Plato's book called *Timaeus*.[32]

G. This was the opinion of a philosopher named Heraclides Ponticus,[33] who maintained that the earth moves circularly and that the heavens remain at rest. Here Aristotle does not refute these theories, possibly because they seemed to him of slight probability and were, moreover, sufficiently criticized in philosophical and astrological writings.

However, subject, of course, to correction, it seems to me that it is possible to embrace the argument and consider with favor the conclusions set forth in the above opinion that the earth rather than the heavens has a diurnal or daily rotation. At the outset, I wish to state that it is impossible to demonstrate from any experience at all that the contrary is true; second, that no argument is conclusive; and third, I shall demonstrate why this is so. As to the first point, let us examine one experience: we can see with our eyes the rising and setting of the sun, the moon, and several stars, while other stars turn around the arctic pole. Such a thing is due only to the motion of the heavens, as was shown in Chapter Sixteen, and, therefore, the heavens move with daily motion. Another experience is this one: if the earth is so moved, it makes its complete course in a natural day with the result that we and the trees and the houses are moved very fast toward the east; thus, it should seem to us that the air and wind are always coming very strong from the east and that it should make a noise such as it makes against the arrow shot from a crossbow or an even louder one, but the contrary is evident from experience.[34] The third argument is Ptolemy's—namely, that, if someone were in a boat moving rapidly toward the east and shot an arrow straight upward, it would not fall in the boat but far behind it toward the west. Likewise, if the earth moves so very fast turning from west to east and if someone threw a stone straight upward, it would not fall back to the place from which it was thrown, but far to the west; and the contrary appears to be the case.[35] It seems to me that what I shall say below about these experiences could apply to all other theories which might be brought forward in this connection. Therefore, I state, in the first place, that the whole corporeal machine or the entire mass of all the bodies in the universe is divided into two parts: one is the heavens with the sphere of fire and the higher region of the air; all this part, according to Aristotle in Book I of *Meteors*,[36] moves in a circle or revolves each day. The other part of the universe is all the rest—that is, the middle and lower regions of the air, the water, the earth, and the mixed bodies—and, according to Aristotle, all this part is immobile and has no daily motion. Now, I take as a fact that local motion can be perceived only if we can see that one body assumes a different position relative to another body. For example, if a man is in a boat *a*, which is moving very smoothly either at rapid or slow speed, and if this man sees nothing except another boat *b*, which moves precisely like boat *a*, the one in which he is standing, I maintain that to this man it will appear that neither boat is moving.[37] If *a* rests while *b* moves, he will be aware that *b* is moving; if *a* moves and *b* rests, it will seem to the man in *a* that *a* is resting and *b* is moving, just as before.[38] Thus, if *a* rested an hour and *b* moved, and during the next hour it happened conversely that *a* moved and *b* rested, this man would not be able to sense this change or variation; it would seem to him that all this time *b* was moving. This fact is evident from experience, and the reason is that the two bodies *a* and *b* have

Nicole Oresme, Le Livre du ciel et du monde, edited by Albert D. Menut and Alexander J. Denomy, C.S.B. †, translated with an Introduction by Albert D. Menut (Madison, Wis.: University of Wisconsin Press, 1968), pp. 519–539.

The present selection constitutes virtually the whole of Book II, chapter 25, of Oresme's translation from Latin into French of Aristotle's treatise *On the Heavens (De caelo)*, along with Oreme's accompanying commentary (see Selection 71, n. 1). The letters *T* and *G* represent Aristotle's text and Oresme's commentary (or gloss), respectively.

32. These words of Aristotle's are found in *On the Heavens* II.13.293b.30–32. The chapter which follows consists of Oresme's lengthy commentary on these few words relating to the possible diurnal axial rotation of the earth.

33. See note 3.

34. The same point was mentioned and refuted by Buridan (see par. 7 of section 4).

35. Although Ptolemy mentions neither boat nor stone, these arguments reproduce faithfully the substance of his counterargument in the penultimate paragraph of Book I, chapter 7, of the *Almagest* (see section 1).

36. Probably *Meteorologica* I.3.340b.10–12, where, however, there is some dispute as to whether Aristotle intended to subdivide it quite as Oresme presents it.

37. This particular illustration of the relativity of motion was not given by Buridan in paragraph 2 of section 4, but was included by Copernicus near the beginning of Book I, chapter 5 of *De revolutionibus* (see section 6).

38. This is virtually the same point made by Buridan in paragraph 2 of section 4.

a continual relationship to each other so that, when *a* moves, *b* rests and, conversely, when *b* moves, *a* rests. It is stated in Book Four of *The Perspective* by Witelo that we do not perceive motion unless we notice that one body is in the process of assuming a different position relative to another.[39] I say, therefore, that, if the higher of the two parts of the world mentioned above were moved today in daily motion—as it is—and the lower part remained motionless and if tomorrow the contrary were to happen so that the lower part moved in daily motion and the higher—that is, the heavens, etc.—remained at rest, we should not be able to sense or perceive this change, and everything would appear exactly the same both today and tomorrow with respect to this mutation. We should keep right on assuming that the part where we are was at rest while the other part was moving continually, exactly as it seems to a man in a moving boat that the trees on shore move. In the same way, if a man in the heavens, moved and carried along by their daily motion, could see the earth distinctly and its mountains, valleys, rivers, cities, and castles, it would appear to him that the earth was moving in daily motion, just as to us on earth it seems as though the heavens are moving. Likewise, if the earth moved with daily motion and the heavens were motionless, it would seem to us that the earth was immobile and that the heavens appeared to move; and this can be easily imagined by anyone with clear understanding. This obviously answers the first experience, for we could say that the sun and stars appear to rise and set as they do and that the heavens seem to revolve on account of the motion of the earth in which we live together with the elements. To the second experience, the reply seems to be that, according to this opinion, not only the earth moves, but also with it the water and the air, as we stated above, although the water and air here below may be moved in addition by the winds or other forces.[40] In a similar manner, if the air were closed in on a moving boat, it would seem to a person in that air that it was not moving. Concerning the third experience, which seems more complicated and which deals with the case of an arrow or stone thrown up into the air, etc., one might say that the arrow shot upward is moved toward the east very rapidly with the air through which it passes, along with all the lower portion of the world which we have already defined and which moves with daily motion; for this reason the arrow falls back to the place from which it was shot into the air. Such a thing could be possible in this way, for, if a man were in a ship moving rapidly eastward without his being aware of the movement and if he drew his hand in a straight line down along the ship's mast, it would seem to him that his hand were moving with a rectilinear motion; so, according to this theory it seems to us that the same thing happens with the arrow which is shot straight down or straight up.[41] Inside the boat moved rapidly eastward, there can be all kinds of movements—horizontal, criss-cross, upward, downward, in all directions—and they seem to be exactly the same as those when the ship is at rest.[42] Thus, if a man in this boat walked toward the west less rapidly than the boat was moving toward the east, it would seem to the man that he was approaching the west when actually he was going east; and similarly as in the preceding case, all the motions here below would seem to be the same as though the earth rested. Now, in order to explain the reply to the third experience in which this artificial illustration was used, I should like to present an example taken from nature, which, according to Aristotle, is true. He supposes that there is a portion of pure fire called *a* in the higher region of the air; this fire, being very light, rises as high as

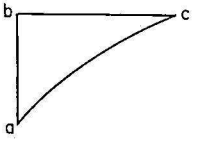


Fig. 1

39. *Vitellonis Thuringopoloni Opticae libri decem*, Bk. IV, par. 110, p. 167, of the edition published by Friedrich Risner (Basel, 1572). This declaration of the relativity of motion was taken by Witelo from Book II, paragraph 49, of the *Optics* of Alhazen, which, although separately paginated, was published by Risner in the same volume that included Witelo's *Optics;* Alhazen's statement appears on p. 60.

40. See note 6.

41. Oresme is here proposing a very different explanation than Buridan for the phenomenon of the arrow shot vertically into the air and falling vertically down to approximately the place from which it was shot. Where Buridan invoked the impetus theory (see par. 9 of section 4) to explain the arrow's return and inferred from it that the earth and its ambient air did not rotate, Oresme argues that the arrow might share the rotational motion of earth and air and hence would naturally fall to the same place from which it was shot.

42. In *De revolutionibus*, Book I, chapter 8, Copernicus says the very same thing (see section 6), as does Galileo in the *Dialogue Concerning the Two Chief World Systems* (Drake, p. 142).

possible to a place called *b* near the concave surface of the heavens [see Fig. 1]. I maintain that, just as with the arrow above, the motion of *a* in this case also must be compounded of rectilinear and, in part, of circular motion, because the region of the air and the sphere of fire through which *a* passed have, in Aristotle's opinion, circular motion. If they were not thus moved, *a* would go straight upward along the line *ab;* but because *b* is meanwhile drawn toward *c* by circular and daily motion, it appears that *a* describes the line *ac* as it ascends and that, therefore, the movement of *a* is compounded of rectilinear and of circular motion, and the movement of the arrow would be of this kind of mixed or compound motion[43] that we spoke of in Chapter Three of Book I. I conclude, then, that it is impossible to demonstrate by any experience that the heavens have daily motion and that the earth does not have the same.

With regard to the second point, if it could be demonstrated by rational arguments, in my opinion they would be the following, to which I shall reply in a manner that could be employed to refute all other pertinent argument. First, every simple body has a single simple motion, and the earth is a simple element which has, according to its various parts, natural rectilinear movement downward. So, it can have no other motion, a fact fully explained in Chapter Four of Book I. Circular motion is not natural to the earth for it has another motion, as already noted; if circular motion is violent to it, the earth could not be perpetual, as appears in several passages of Book I. All local motion is relative to some body at rest, as Averroes states in Chapter Eight,[44] from which he concludes that the earth must be at rest in the center of the heavens. Now, all motion is produced by some motive power or force, as shown in Books Seven and Eight of the *Physics*,[45] and the earth cannot move circularly because of its weight; if it is so moved by an external force, this movement would be violent and not perpetual. If, in reality, the heavens did not have diurnal motion, all astronomy would be false as well as a large part of natural philosophy throughout which such motion is taken for granted. It would, moreover, contradict Holy Scripture which states: The sun riseth and goeth down and returneth to his place; and there rising again, maketh his round by the south and turneth again to the north; the spirit goeth forward surveying all places round about and returneth to his circuits.[46] And it is also written of the earth that God made it motionless: Etenim firmavit orbem terre, qui non commovebitur.[47] The Scripture states that the sun stopped its course in Joshua's time and returned in King Hezekiah's;[48] if, as is posited in this theory, it is the earth that moves and the heavens that remain motionless, then this stopping would have been a turning backward, which would have been more than a stoppage. And this is contrary to the statement in the Scriptures. As for the first argument where it is stated that every simple body has a single simple motion, I say that the earth, which as a whole is a simple body, has no movement, according to Aristotle in Chapter Twenty-two. Against the interpretation of anyone who maintained that Aristotle means that this body has a single simple motion not proper to itself as a whole, but applying only to its parts when they are out of their proper place, we can cite the case of air which moves downward when it is in the region of fire and upward when it is in the region of water, both being simple movements. Therefore, we can say with an ever greater show of reason that each simple body or element of the universe, with the possible exception of the sovereign heaven, moves in its proper place with circular motion. If any part of such a body is out of its place or outside the main body, it returns to it as directly as it can, once the hindrance is removed; this would surely happen if some part of the heavens were to get outside. It is not necessary that a simple body have its own simple motion in its proper place and another motion in its parts when they return to their proper place, and, according to Aristotle, we shall have to grant this assumption, as I shall do a little later. To the second

43. Like Oresme, Copernicus would also argue (Bk. I, ch. 8, of *De revolutionibus;* see section 6) that on the assumption of the earth's rotation, rising and falling objects would have a motion that is the resultant of two motions, rectilinear and circular.

44. Averroes, Commentary on *De caelo*, Book II, Comment 18, on folio 107 verso, paragraph 1, in Volume V of the edition cited in Selection 44, n. 1.

45. That is, Aristotle's *Physics*.

46. Ecclesiastes 1:5–6.

47. Psalms 92:1.

48. Joshua 10:12–14. This Scriptural account was also used against the Copernican theory, as Galileo reports in the Third Day of the *Dialogue Concerning the Two Chief World Systems* (Drake, p. 357), and even long before, in his 1615 *Letter to Madame Christina of Lorraine Grand Duchess of Tuscany Concerning the Use of Biblical Quotations in Matters of Science* (translated into English by Stillman Drake in *Discoveries and Opinions of Galileo* [New York: Doubleday, 1957], pp. 211–215), where Galileo attempts to reconcile the Joshua passage with the Copernican theory (see n. 51).

argument, I say the motion is natural to the earth as a whole and in its place; however, its parts have a different natural motion, rectilinear upward and downward, when they are out of their natural place. According to Aristotle, we must admit the same with respect to fire, parts of which move naturally upward when out of their proper place, and besides, also according to him, the entire element of fire in its sphere and in its place moves perpetually with diurnal motion, which could not be a true statement if its movement were violent. Now, in the theory we are discussing, it is not the element of fire, but the earth that moves in this manner. I say no to the third argument which states that all motion requires some body to be at rest, unless the motion must be perceptible to the senses; to make such motion apparent, it would suffice that the first body be moved in a different manner. But it is not required that there be a second body in order that this motion should exist, as was explained in Chapter Eight. Assuming that the heavens have diurnal movement and that the earth is moving in the opposite direction or imagining even that the earth were annihilated, we would note that the heavens had not stopped moving on this account; nor would they move faster or more slowly because neither the intelligence which moves the heavens nor the moving body of the heavens as a whole would be disposed to do otherwise. Besides, if it is assumed that circular motion did require another body at rest, such a body would not be situated in the middle of the one moving; in the middle of a millstone of a flour mill or of any similar moving body, nothing is at rest save a single mathematical point which is not a body, and the same is true at the center of the movement of the polar star. Thus, it could be said that the sovereign heaven rests or moves differently from the motion of the other bodies because its movement requires the existence of the other motions or requires that they be perceptible to the senses. To the fourth argument, we can say that the force causing this lower region of the world to move in a circle is its nature or form; and this same force—similar in nature to that which draws iron to the magnet—moves the earth to its proper place when it gets outside. Besides, I ask Aristotle what force it is that moves fire in the diurnal movement of its sphere, for we cannot say that the heavens pull it thus or seize it violently not only because this motion is perpetual, but also because the concave surface of the heavens is so highly polished, as noted in Chapter Eleven, that it passes over the sphere of fire without rubbing, pulling, or pushing, as stated in Chapter Eighteen. So, we must say that fire is moved circularly by its own nature and form or by some intelligence or celestial influence. Exactly the same could be said by one who maintains that the earth has diurnal rotation and that the sphere of fire remains at rest. I say to the fifth argument, where it is held that, if the heavens did not make a rotation from day to day, the whole of astronomy would be false, that such a statement is not true, because all heavenly aspects, conjunctions, oppositions, constellations, figures, and influences would be exactly as they are in every respect, as is apparent from what was stated in reply to the first experience; and the astronomical tables of the heavenly motions and all other books would remain as true as they are at present, save that, with respect to diurnal motion, one would say that it is *apparently* in the heavens, but *actually* in the earth; no other effect would follow or result from one theory more than from the other. Aristotle's statement in Chapter Sixteen is pertinent in this connection, namely, that the sun seems to us to turn and twist and the stars to flicker and twinkle and that whether the thing we see moves or whether our vision moves makes no difference; and in the present case one could say that our vision is affected by diurnal movement. One could answer the sixth argument, which concerns the reference in Holy Scripture about the sun's turning, etc., by saying that this passage conforms to the customary usage of popular speech just as it does in many other places, for instance, in those where it is written that God repented, and He became angry and became pacified, and other such expressions which are not to be taken literally.[49] And more pertinent to our present subject, we read that God covers the heavens with clouds: Qui operit celum nubibus,[50] while the fact is that the heavens cover the clouds. Thus, we could say that the heavens, rather than the earth, appear to move with diurnal motion, while the truth is the exact opposite. And we could say that, in reality, the earth does not move from its place, nor apparently within its place, but it does actually move within its place. To the seventh argument, we could reply in much the same manner that in the time of Joshua the sun stopped and that in the time of

49. To illustrate Oresme's point that Scripture is not to be taken literally when it speaks of God repenting, and so on, Menut cites Genesis 6:6, Isaiah 47:6, Psalms 59:3, I Par. 13:10, Psalms 105:40.

50. Psalms 147:8.

Hezekiah it returned, but only apparently so; for, in fact, it was the earth which stopped moving in Joshua's time and which later in Hezekiah's time advanced or speeded up its movement; whichever occurrence we prefer to believe, the effect would be the same.[51] The latter opinion seems more reasonable than the former, as we shall make clear later.

Regarding the third point of this discussion, I want to present several opinions or reasons favorable to the theory that the earth moves as we have stated. In the first place, everything that requires another thing for its natural existence must aim at receiving the good it derives from the other through the motion or action natural to it. In this way we can see that each element moves to its natural place where it is conserved and that it goes to its place rather than its place coming to it. Thus the earth and the elements here below which require the heat and influence of the heavens round about them must needs be disposed by their movements to receive these benefits in due degree, just as, to speak familiarly, the meat being roasted before the fire receives around it the heat of the fire by being turned and not by the turning of the fire around the meat. If neither experience nor reason indicates the contrary, it is much more reasonable, as stated above, that all the principal movements of the simple bodies in the world should go or proceed in one direction or manner. Now, according to the philosophers and astronomers, it cannot be that all bodies move from east to west; but, if the earth moves as we have indicated, then all proceed alike from west to east—that is, the earth by rotating once around the poles from west to east in one natural day and the heavenly bodies around the zodiacal poles: the moon in one month, the sun in one year, Mars in approximately two years, and so on with the other bodies.[52] It is unnecessary to posit in the heavens other primary poles or two kinds of motion, one from the east to the west and the other on different poles in the opposite direction, but such an assumption is definitely necessary if the heavens move with diurnal motion. . . . Although Averroes says in Chapter Twenty-two that motion is nobler than rest,[53] the contrary seems true, because, again on Aristotle's authority in Chapter Twenty-two, the noblest thing possible achieves its perfection without movement, and this is God Himself. Rest is the end purpose of motion, and so Aristotle holds that the bodies here below move to their natural places in order to rest there.[54] A further sign that rest is best is that we pray for the dead that God may give them rest: Requiem eternam, etc. Therefore, to rest or to be moved less is a better and nobler condition than to be moved or to be moved farther and farther from rest.[55] From this, it seems that the position we have taken above is very reasonable, for it could be said that the earth, the vilest element, along with the other elements here below make their rotation very fast, that the sovereign air and fire move less fast—as can be observed in the case of the comets—and that the moon and lunar sphere move still more slowly, for it moves in a month only the distance the earth travels in a natural day. Proceeding in this manner, the higher heavens make their revolution more slowly yet, although there is some variation, and this process continues up to the heaven of the fixed stars, which is motionless or makes its revolution very slowly, according to some in thirty-six thousand years or one degree in one hundred years.[56] In this way and no other can we solve the question proposed by Aristotle in Chapter Twenty-one, with only slight additions. It is not necessary to assume so many degrees of things nor such obscure difficulties as Aristotle introduces in his reply in Chapter Twenty-two. It is indeed very reasonable that the bodies that are larger or farther from the center should make their circuit or revolution in longer

51. Oresme's solution to the "Joshua problem" is more daring than Galileo's, since it was more at variance with the literal sense of the text. Galileo argued that the sun controls all celestial motions, which would cease immediately if the sun's motion ceased. "The sun, then," says Galileo, "being the font of light and the source of motion, when God willed that at Joshua's command the whole system of the world should rest and should remain for many hours in the same state, it sufficed to make the sun stand still. Upon its stopping, all the other revolutions ceased; the earth, the moon, and the sun remained in the same arrangement as before, as did all the planets; nor in all that time did day decline towards night, for day was miraculously prolonged. And in this manner, by the stopping of the sun, without altering or in the least disturbing the other aspects and mutual positions of the stars, the day could be lengthened on earth—which agrees exquisitely with the literal sense of the sacred text" (from Galileo's *Letter to Madame Christina*, pp. 213–214 of Drake's translation).

52. Buridan says virtually the same thing in paragraph 3 of section 4.

53. Averroes, *Commentary on De caelo*, Book II, Comment 61, on folio 140 recto, paragraph A, in Vol. V of the edition cited in Selection 44, n.1.

54. *Physics* V.6.230b.25–27.

55. Buridan offers much the same argument in favor of the immobility of the heavens and the motion of the earth (see the third persuasion in par. 4 of section 4).

56. Ptolemy's value for the precession of the equinoxes.

time than those nearer the center, because, if they made their circuit in the same or equal time, their movements would have to be excessively fast. So we could say that nature compensates by ordaining that the rotations of the bodies farther from the center shall be accomplished in much longer time. Accordingly, because of its great size, the sovereign or primary heaven takes a very long time to make its circuit or rotation although it moves very fast. But the earth, which has a very small circuit, can cover the distance in one diurnal movement, while the other bodies intermediate between the highest and lowest heavens accomplish their circuits in time periods midway between the extremes, although these periods are not proportionate. In this way, a constellation near the north, i.e., the Great Bear which we call the Chariot, does not move backward, the chariot in front of the oxen, as it would if moved with diurnal motion; but it actually goes forward in the right direction. All philosophers say that an action accomplished by several or by large-scale operations which can be accomplished by fewer or smaller operations is done for naught.[57] And Aristotle says in Chapter Eight of Book I that God and nature do nothing without some purpose.[58] Now, if it is true that the heavens move with diurnal motion, it becomes necessary to posit in the major bodies of the universe and in the heavens two contrary kinds of movement: one from east to west and the other from the opposite direction, as we have often stated. And with regard to diurnal motion, we must assume an excessively great speed; for, if we consider thoughtfully the height or distance of the heavens, their magnitude, and the immensity of their circuit, mindful that this circuit is traveled in but one day's time, no man could imagine or conceive how marvelously swift and excessively great, how far beyond belief and estimation their speed must be.[59] Since, then, all the effects we see could be produced and all appearances saved by substituting for the diurnal movement of the heavens a smaller operation, namely, the diurnal motion of the earth, a very small body as compared with the heavens, and by so doing avoid the multiplication of operations so diverse and so outrageously great, then it follows that God and nature must have created and arranged them for naught; and this is an inadmissible conclusion, as we have often said. Assuming the entire heavens to move with daily motion and, in addition, assuming the eighth sphere to have a different motion, as the astronomers believe, then we must necessarily posit a ninth sphere moving only with diurnal motion. However, if we assume that the earth moves as stated above, then the eighth heaven moves with a single slow motion and it is consequently unnecessary to imagine a ninth natural sphere invisible and starless; for God and nature would have made this ninth sphere for naught since by another method, i.e., assuming the earth to move, everything can remain exactly as it is. Also, when God performs a miracle, we must assume and maintain that He does so without altering the common course of nature, in so far as possible. Therefore, if we can save appearances by taking for granted that God lengthened the day in Joshua's time by stopping the movement of the earth or merely that of the region here below—which is so very small and like a mere dot compared to the heavens—and by maintaining that nothing in the whole universe—and especially the huge heavenly bodies—except this little point was put off its ordinary course and regular schedule, then this would be a much more reasonable assumption. And appearances can be saved in this way, as is evident from the reply to the seventh argument, presented against this opinion. As much could be said with regard to the return of the sun in Hezekiah's time. Thus, it is apparent that one cannot demonstrate by any experience whatever that the heavens move with diurnal motion; whatever the fact may be, assuming that the heavens move and the earth does not or that the earth moves and the heavens do not, to an eye in the heavens which could see the earth clearly, it would appear to move; if the eye were on the earth, the heavens would appear to move. Nor would the vision of this eye be deceived, for it can sense or see nothing but the process of the movement itself. But if the motion is relative to some particular body or object, this judgment is made by the senses from within that particular body, as Witelo explains in *The Perspective*;[60] and the senses are often deceived in such cases, as was related above in the example of the man on the moving ship. Afterward, it was demonstrated how it cannot be proved conclusively by argu-

57. This principle was expressed by Aristotle as nature doing nothing in vain (see n. 58, and compare to Ockham's "razor," Selection 40, n. 13).

58. Aristotle, *De caelo* I.4.271a.33; see also II.8.290a.31.

59. The same substantive argument was made by Buridan (par. 4 of section 4) and Copernicus (ch. 7 of section 6). See note 25.

60. See note 39.

ment that the heavens move. In the third place, we offered arguments opposing their diurnal motion. However, everyone maintains, and I think myself, that the heavens do move and not the earth: For God hath established the world which shall not be moved, in spite of contrary reasons because they are clearly not conclusive persuasions. However, after considering all that has been said, one could then believe that the earth moves and not the heavens, for the opposite is not clearly evident. Nevertheless, at first sight, this seems as much against natural reason as, or more against natural reason than, all or many of the articles of our faith.[61] What I have said by way of diversion or intellectual exercise can in this manner serve as a valuable means of refuting and checking those who would like to impugn our faith by argument.

6. Nicolaus Copernicus (1473–1543): The Compatibility of the Earth's Diurnal Rotation with Astronomical Phenomena and Terrestrial Physics

Translated by Charles Glenn Wallis[62]

Annotated by Edward Grant

PREFACE AND DEDICATION TO POPE PAUL III

. . . . . .

Accordingly, when I had meditated upon this lack of certitude in the traditional mathematics concerning the composition of movements of the spheres of the world, I began to be annoyed that the philosophers, who in other respects had made a very careful scrutiny of the least details of the world, had discovered no sure scheme for the movements of the machinery of the world, which has been built for us by the Best and Most Orderly Workman of all. Wherefore I took the trouble to reread all the books by philosophers which I could get hold of, to see if any of them even supposed that the movements of the spheres of the world were different from those laid down by those who taught mathematics in the schools. And as a matter of fact, I found first in Cicero that Nicetas[63] thought that the Earth moved. And afterwards I found in Plutarch that there were some others of the same opinion: I shall copy out his words here, so that they may be known to all:[64]

> Some think that the Earth is at rest; but Philolaus the Pythagorean says that it moves around the fire with an obliquely circular motion, like the sun and moon. Herakleides of Pontus and Ekphantus the Pythagorean do not give the Earth any movement of locomotion, but rather a limited movement of rising and setting around its centre, like a wheel.

Therefore I also, having found occasion, began to meditate upon the mobility of the Earth. And although the opinion seemed absurd, nevertheless because I knew that others before me had been granted the liberty of constructing whatever circles they pleased in order to demonstrate astral phenomena, I thought that I too would be readily permitted to test whether or not, by the laying down that the Earth had some movement, demonstrations

61. In the final analysis, then, Oresme denies the diurnal rotation of the earth. He argues that although one might plausibly believe in the earth's rotation—for there are no persuasive arguments to deny it conclusively—it seems contrary to natural reason, even more so than do some articles of the faith. Thus Oresme acquiesces in tradition, custom, and "natural reason," to conclude in favor of the earth's immobility. He sought to humble reason and show that physical arguments could not establish a relatively simple physical problem. If reason is impotent in the solution of a physical problem, how much more impotent would it be in coping with articles of the faith, which some men had tried to demonstrate by reason. Thus Oresme's elaborate and brilliant arguments were designed to protect the faith from demonstrations based on applications of reason and science, both of which seem incapable of deciding straightforward physical problems. By showing that it was impossible to know which alternative is *really* true, Oresme, the theologian, succeeded in using reason to confound reason.

62. Reprinted by permission of Encyclopaedia Britannica from *Nicolaus Copernicus on the Revolutions of the Heavenly Spheres*, translated by Charles Glenn Wallis in Great Books of the Western World (Chicago: Encyclopaedia Britannica, 1952), XVI, pp. 508, 514–521. This selection contains an excerpt from Copernicus' Preface and chapters 5 to 9 of Book I of the *De revolutionibus orbium coelestium*. Another, but inferior, translation of Preface and Book I has been made by John F. Dobson, assisted by Selig Brodetsky, in *Occasional Notes of the Royal Astronomical Society*, Vol. II, No. 10 (May 1947).

63. Cicero's *Academica*, Book II, chapter 29 (the quotation is given in n. 67). Copernicus uses the name Nicetus (not Nicetas, as given here, or Hicetas, as used by Wallis, in Bk. I, ch. 5) for Hicetas, the form employed by Cicero.

64. *On the Opinions of the Philosophers (De placitis Philosophorum)*, Book III, chapter 13. Copernicus actually quoted the Greek text.

less shaky than those of my predecessors could be found for the revolutions of the celestial spheres.

. . . . . .

BOOK I

5. Does the Earth Have a Circular Movement? And of Its Place

Now that it has been shown that the Earth too has the form of a globe, I think we must see whether or not a movement follows upon its form and what the place of the Earth is in the universe. For without doing that it will not be possible to find a sure reason for the movements appearing in the heavens. Although there are so many authorities for saying that the Earth rests in the centre of the world that people think the contrary supposition inopinable and even ridiculous; if however we consider the thing attentively, we will see that the question has not yet been decided and accordingly is by no means to be scorned. For every apparent change in place occurs on account of the movement either of the thing seen or of the spectator, or on account of the necessarily unequal movement of both. For no movement is perceptible relatively to things moved equally in the same directions—I mean relatively to the thing seen and the spectator.[65] Now it is from the Earth that the celestial circuit is beheld and presented to our sight. Therefore, if some movement should belong to the Earth it will appear, in the parts of the universe which are outside, as the same movement but in the opposite direction, as though the things outside were passing over. And the daily revolution in especial is such a movement. For the daily revolution appears to carry the whole universe along, with the exception of the Earth and the things around it. And if you admit that the heavens possess none of this movement but that the Earth turns from west to east, you will find—If you make a serious examination—that as regards the apparent rising and setting of the sun, moon, and stars the case is so. And since it is the heavens which contain and embrace all things as the place common to the universe, it will not be clear at once why movement should not be assigned to the contained rather than to the container, to the thing placed rather than to the thing providing the place.

As a matter of fact, the Pythagoreans Herakleides and Ekphantus were of this opinion[66] and so was Hicetas[67] the Syracusan in Cicero; they made the Earth to revolve at the centre of the world. For they believed that the stars set by reason of the interposition of the Earth and that with cessation of that they rose again. Now upon this assumption there follow other things, and a no smaller problem concerning the place of the Earth, though it is taken for granted and believed by nearly all that the Earth is the centre of the world. For if anyone denies that the Earth occupies the midpoint or centre of the world yet does not admit that the distance [between the two] is great enough to be compared with [the distance to] the sphere of the fixed stars but is considerable and quite apparent in relation to the orbital circles of the sun and the planets; and if for that reason he thought that their movements appeared irregular because they are organized around a different centre from the centre of the Earth, he might perhaps be able to bring forward a perfectly sound reason for movement which appears irregular. For the fact that the wandering stars are seen to be sometimes nearer the Earth and at other times farther away necessarily argues that the centre of the Earth is not the centre of their circles. It is not yet clear whether the Earth draws near to them and moves away or they draw near to the Earth and move away.

And so it would not be very surprising if someone attributed some other movement to the earth in addition to the daily revolution. As a matter of fact, Philolaus the Pythagorean—no ordinary mathematician, whom Plato's biographers say Plato went to Italy for the sake of seeing[68]—is supposed to have held that the Earth moved in a circle and wandered in some other movements and was one of the planets.

Many however have believed that they could show by geometrical reasoning that the Earth is in the middle of the world; that it has the proportionality of a point in relation to the immensity of the heavens, occupies the central position, and for this reason is immovable, because, when the universe

65. The same statement was made earlier by Oresme (see section 5 and note 37).

66. See the quotation from Plutarch in the Preface.

67. See the Preface and note 2. Here is the quotation from Cicero's *Academica*, Book II, chapter 29 (trans. H. Rackham in the Loeb Classical Library, p. 627):

> "The Syracusan Hicetas, as Theophrastus asserts, holds the view that the heaven, sun, moon, stars, and in short all of the things on high are stationary, and that nothing in the world is in motion except the earth, which by revolving and twisting round its axis with extreme velocity produces all the same results as would be produced if the earth were stationary and the heaven in motion; and this is also in some people's opinion the doctrine stated by Plato in *Timaeus*, but a little more obscurely."

68. See Plutarch, *De placitis philosophorum*, III.13.

moves, the centre remains unmoved and the things which are closest to the centre are moved the most slowly.

6. On the Immensity of the Heavens in Relation to the Magnitude of the Earth[69]

It can be understood that this great mass which is the Earth is not comparable with the magnitude of the heavens, from the fact that the boundary circles—for that is the translation of the Greek ὁρίζοντες—cut the whole celestial sphere into two halves; for that could not take place if the magnitude of the Earth in comparison with the heavens, or its distance from the centre of the world, were considerable. For the circle bisecting a sphere goes through the centre of the sphere, and is the greatest circle which it is possible to circumscribe.

Now let the horizon be the circle *ABCD* [Fig.2],

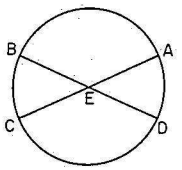


Fig. 2

and let the Earth, where our point of view is, be *E*, the centre of the horizon by which the visible stars are separated from those which are not visible. Now with a dioptra or horoscope or level placed at *E*, the beginning of Cancer is seen to rise at point *C*; and at the same moment the beginning of Capricorn appears to set at *A*. Therefore, since *AEC* is in a straight line with the dioptra, it is clear that this line is a diameter of the ecliptic, because the six signs bound a semicircle, whose centre *E* is the same as that of the horizon. But when a revolution has taken place and the beginning of Capricorn arises at *B*, then the setting of Cancer will be visible at *D*, and *BED* will be a straight line and a diameter of the ecliptic. But it has already been seen that the line AEC is a diameter of the same circle; therefore, at their common section, point *E* will be their center. So in this way the horizon always bisects the ecliptic, which is a great circle of the sphere. But on a sphere, if a circle bisects one of the great circles, then the circle bisecting is a great circle. Therefore the horizon is a great circle; and its centre is the same as that of the ecliptic, as far as appearance goes; although nevertheless the line passing through the centre of the Earth and the line touching to the surface are necessarily different; but on account of their immensity in comparison with the Earth they are like parallel lines, which on account of the great distance between the termini appear to be one line, when the space contained between them is in no perceptible ratio to their length, as has been shown in optics.

From this argument it is certainly clear enough that the heavens are immense in comparison with the Earth and present the aspect of an infinite magnitude, and that in the judgment of sense-perception the Earth is to the heavens as a point to a body and as a finite to an infinite magnitude. But we see that nothing more than that has been shown, and it does not follow that the Earth must rest at the center of the world. And we should be even more surprised if such a vast world should wheel completely around during the space of twenty-four hours rather than that its least part, the Earth, should. For saying that the centre is immovable and that those things which are closest to the centre are moved least does not argue that the Earth rests at the centre of the world. That is no different from saying that the heavens revolve but the poles are at rest and those things which are closest to the poles are moved least. In this way Cynosura [the pole star] is seen to move much more slowly than Aquila or Canicula because, being very near to the pole, it describes a smaller circle, since they are all on a single sphere, the movement of which stops at its axis and which does not allow any of its parts to have movements which are equal to one another. And nevertheless the revolution of the whole brings them round in equal times but not over equal spaces.

The argument which maintains that the Earth, as a part of the celestial sphere and as sharing in the same form and movement, moves very little because very near to its centre advances to the following position: therefore the Earth will move, as being a body and not a centre, and will describe in the same time arcs similar to, but smaller than, the arcs of the celestial circle. It is clearer than daylight how false that is; for there would necessarily always be noon at one place and midnight at another, and so the daily risings and settings could not take place, since the movement of the whole and the part would be one and inseparable.

69. Virtually the same chapter title was used by Ptolemy in Book I, chapter 6, of the *Almagest*.

But the ratio between things separated by diversity of nature is so entirely different that those which describe a smaller circle turn more quickly than those which describe a greater circle. In this way Saturn, the highest of the wandering stars, completes its revolution in thirty years, and the moon which is without doubt the closest to the Earth completes its circuit in a month, and finally the Earth itself will be considered to complete a circular movement in the space of a day and a night. So this same problem concerning the daily revolution comes up again. And also the question about the place of the Earth becomes even less certain on account of what was just said. For that demonstration proves nothing except that the heavens are of an indefinite magnitude with respect to the Earth. But it is not at all clear how far this immensity stretches out. On the contrary, since the minimal and indivisible corpuscles, which are called atoms, are not perceptible to sense, they do not, when taken in twos or in some small number, constitute a visible body; but they can be taken in such a large quantity that there will at last be enough to form a visible magnitude. So it is as regards the place of the earth; for although it is not at the center of the world, nevertheless the distance is as nothing, particularly in comparison with the sphere of the fixed stars.

7. Why the Ancients Thought the Earth Was at Rest at the Middle of the World as Its Centre[70]

Wherefore for other reasons the ancient philosophers have tried to affirm that the Earth is at rest at the middle of the world, and as principal cause they put forward heaviness and lightness. For Earth is the heaviest element; and all things of any weight are borne towards it and strive to move towards the very centre of it.

For since the Earth is a globe towards which from every direction heavy things by their own nature are borne at right angles to its surface, the heavy things would fall on one another at the centre if they were not held back at the surface; since a straight line making right angles with a plane surface where it touches a sphere leads to the centre. And those things which are borne toward the centre seem to follow along in order to be at rest at the centre. All the more then will the Earth be at rest at the centre; and, as being the receptacle for falling bodies, it will remain immovable because of its weight.[71]

They strive similarly to prove this by reason of movement and its nature. For Aristotle says that the movement of a body which is one and simple is simple, and the simple movements are the rectilinear and the circular. And of rectilinear movements, one is upward, and the other is downward. As a consequence, every simple movement is either toward the centre, *i.e.*, downward, or away from the centre, *i.e.*, upward, or around the centre, *i.e.*, circular. Now it belongs to earth and water, which are considered heavy, to be borne downward, *i.e.*, to seek the centre: for air and fire, which are endowed with lightness, move upward, *i.e.*, away from the centre. It seems fitting to grant rectilinear movement to these four elements and to give the heavenly bodies a circular movement around the centre. So Aristotle.[72] Therefore, said Ptolemy of Alexandria, if the Earth moved, even if only by its daily rotation, the contrary of what was said above would necessarily take place. For this movement which would traverse the total circuit of the Earth in twenty-four hours would necessarily be very headlong and of an unsurpassable velocity. Now things which are suddenly and violently whirled around are seen to be utterly unfitted for reuniting, and the more unified are seen to become dispersed, unless some constant force constrains them to stick together. And a long time ago, he says, the scattered Earth would have passed beyond the heavens, as is certainly ridiculous; and *a fortiori* so would all the living creatures and all the other separate masses which could by no means remain unshaken. Moreover, freely falling bodies would not arrive at the places appointed them, and certainly not along the perpendicular line which they assume so quickly. And we would see clouds and other things floating in the air always borne toward the west.[73]

8. Answer to the Aforesaid Reasons and Their Inadequacy

For these and similar reasons they say that the Earth remains at rest at the middle of the world and that there is no doubt about this. But if someone opines that the Earth revolves, he will also say that the movement is natural and not violent. Now things which are according to nature produce effects contrary to those which are violent. For things to which force or violence is applied get

70. The immobility of the earth was also the subject of Book I, chapter 7, of the *Almagest* (see section 1).
71. Based on Aristotle, *De caelo* II.14.296b.
72. Aristotle, *De caelo* I.2–3; III.3–5.
73. Most of this paragraph is drawn from the *Almagest*, Book I, chapter 7 (see section 1).

broken up and are unable to subsist for a long time. But things which are caused by nature are in a right condition and are kept in their best organization. Therefore Ptolemy had no reason to fear that the Earth and all things on the Earth would be scattered in a revolution caused by the efficacy of nature, which is greatly different from that of art or from that which can result from the genius of man. But why didn't he feel anxiety about the world instead, whose movement must necessarily be of greater velocity, the greater the heavens are than the Earth? Or have the heavens become so immense, because an unspeakably vehement motion has pulled them away from the centre, and because the heavens would fall if they came to rest anywhere else?

Surely if this reasoning were tenable, the magnitude of the heavens would extend infinitely. For the farther the movement is borne upward by the vehement force, the faster will the movement be, on account of the ever-increasing circumference which must be traversed every twenty-four hours: and conversely, the immensity of the sky would increase with the increase in movement. In this way, the velocity would make the magnitude increase infinitely, and the magnitude the velocity. And in accordance with the axiom of physics that *that which is infinite cannot be traversed or moved in any way*, then the heavens will necessarily come to rest.[74]

But they say that beyond the heavens there isn't any body or place or void or anything at all; and accordingly it is not possible for the heavens to move outward:[75] in that case it is rather surprising that something can be held together by nothing. But if the heavens were infinite and were finite only with respect to a hollow space inside, then it will be said with more truth that there is nothing outside the heavens, since anything which occupied any space would be in them; but the heavens will remain immobile. For movement is the most powerful reason wherewith they try to conclude that the universe is finite.

But let us leave to the philosophers of nature[76] the dispute as to whether the world is finite or infinite, and let us hold as certain that the Earth is held together between its two poles and terminates in a spherical surface. Why therefore should we hesitate any longer to grant to it the movement which accords naturally with its form, rather than put the whole world in a commotion—the world whose limits we do not and cannot know? And why not admit that the appearance of daily revolution belongs to the heavens but the reality belongs to the Earth? And things are as when Aeneas said in Virgil: "We sail out of the harbor, and the land and the cities move away."[77] As a matter of fact, when a ship floats on over a tranquil sea, all the things outside seem to the voyagers to be moving in a movement which is the image of their own, and they think on the contrary that they themselves and all the things with them are at rest.[78] So it can easily happen in the case of the movement of the Earth that the whole world should be believed to be moving in a circle. Then what would we say about the clouds and the other things floating in the air or falling or rising up, except that not only the Earth and the watery element with which it is conjoined are moved in this way but also no small part of the air and whatever other things have a similar kinship with the Earth? whether because the neighbouring air, which is mixed with earthly and watery matter, obeys the same nature as the Earth or because the movement of the air is an acquired one, in which it participates without resistance on account of the contiguity and perpetual rotation of the Earth. Conversely, it is no less astonishing for them to say that the highest region of the air follows the celestial movement, as is shown by those stars which appear suddenly—I mean those called "comets" or "bearded stars" by the Greeks. For that place is assigned for their generation; and like all the other stars they rise and set. We can say that that part of the air is deprived of terrestrial motion on account of its great distance from the Earth. Hence the air which is nearest to the Earth and the things floating in it will appear tranquil, unless they are driven to and fro by the wind or some other force, as happens. For how is the wind in the air different from a current in the sea?[79]

But we must confess that in comparison with the world the movement of falling and of rising bodies is twofold and is in general compounded of the

74. Although both Buridan and Oresme used the argument that it is more appropriate for the relatively small earth to rotate daily than for the vast heavens to do so (see section 4, par. 4; n. 25; and section 5), neither includes mention of a vehement motion *(motus vehementia)* or force expanding the heavens to infinity.

75. Aristotle, *De caelo* I.9. For those who maintained the existence of an infinite void beyond the cosmos, see Selection 73.

76. "Philosophers of nature" translates *physiologi*.

77. *Aeneid* III.72.

78. Oresme had already made this point (see section 5 and n. 42).

79. See note 6.

rectilinear and the circular.[80] As regards things which move downward on account of their weight because they have very much earth in them, doubtless their parts possess the same nature as the whole, and it is for the same reason that fiery bodies are drawn upward with force. For even this earthly fire feeds principally on earthly matter; and they define flame as glowing smoke. Now it is a property of fire to make that which it invades to expand; and it does this with such force that it can be stopped by no means or contrivance from breaking prison and completing its job. Now expanding movement moves away from the centre to the circumference; and so if some part of the Earth caught on fire, it would be borne away from the centre and upward. Accordingly, as they say, a simple body possesses a simple movement—this is first verified in the case of circular movement—as long as the simple body remains in its unity in its natural place. In this place, in fact, its movement is none other than the circular, which remains entirely in itself, as though at rest. Rectilinear movement, however, is added to those bodies which journey away from their natural place or are shoved out of it or are outside it somehow. But nothing is more repugnant to the order of the whole and to the form of the world than for anything to be outside of its place. Therefore rectilinear movement belongs only to bodies which are not in the right condition and are not perfectly conformed to their nature—when they are separated from their whole and abandon its unity. Furthermore, bodies which are moved upward or downward do not possess a simple, uniform, and regular movement—even without taking into account circular movement. For they cannot be in equilibrium with their lightness or their force of weight. And those which fall downward possess a slow movement at the beginning but increase their velocity as they fall. And conversely we note that this earthly fire—and we have experience of no other—when carried high up immediately dies down, as if through the acknowledged agency of the violence of earthly matter.

Now circular movement always goes on regularly, for it has an unfailing cause; but [in rectilinear movement] the acceleration stops, because, when the bodies have reached their own place, they are no longer heavy or light, and so the movement ends. Therefore, since circular movement belongs to wholes and rectilinear to parts, we can say that the circular movement stands with the rectilinear, as does animal with sick. And the fact that Aristotle divided simple movement into three genera: away from the centre, toward the centre, and around the centre, will be considered merely as an act of reason, just as we distinguish between line, point, and surface, though none of them can subsist without the others or without body.

In addition, there is the fact that the state of immobility is regarded as more noble and godlike than that of change and instability, which for that reason should belong to the Earth rather than to the world[81] I add that it seems rather absurd to ascribe movement to the container or to that which provides the place and not rather to that which is contained and has a place, *i.e.*, the Earth. And lastly, since it is clear that the wandering stars are sometimes nearer and sometimes farther away from the Earth, then the movement of one and the same body around the centre—and they mean the centre of the Earth—will be both away from the centre and toward the centre. Therefore it is necessary that movement around the centre should be taken more generally; and it should be enough if each movement is in accord with its own centre. You see therefore that for all these reasons it is more probable that the Earth moves than that it is at rest—especially in the case of the daily revolution, as it is the Earth's very own. And I think that is enough as regards the first part of the question.

9. Whether Many Movements Can be Attributed to the Earth, and Concerning the Centre of the World

Therefore, since nothing hinders the mobility of the Earth, I think we should now see whether more than one movement belongs to it, so that it can be regarded as one of the wandering stars. For the apparent irregular movement of the planets and their variable distances from the Earth—which cannot be understood as occurring in circles homocentric with the Earth—make it clear that the Earth is not the centre of their circular movements. Therefore, since there are many centres, it is not foolhardy to doubt whether the centre of gravity of the Earth rather than some other is the centre of the world. I myself think that gravity or heaviness is nothing except a certain natural appetency implanted in the parts by the divine providence of the universal Artisan, in order that they should unite with one another in their oneness and wholeness and come together in the form of a globe. It is believable that this affect is present in the sun,

80. See note 43.

81. See section 4, paragraph 4.

moon, and the other bright planets and that through its efficacy they remain in the spherical figure in which they are visible, though they nevertheless accomplish their circular movements in many different ways. Therefore if the Earth too possesses movements different from the one around its centre, then they will necessarily be movements which similarly appear on the outside in the many bodies; and we find the yearly revolution among these movements. For if the annual revolution were changed from being solar to being terrestrial, and immobility were granted to the sun, the risings and settings of the signs and of the fixed stars—whereby they become morning or evening stars—will appear in the same way; and it will be seen that the stoppings, retrogressions, and progressions of the wandering stars are not their own, but are a movement of the Earth and that they borrow the appearances of this movement. Lastly, the sun will be regarded as occupying the centre of the world. And the ratio of order in which these bodies succeed one another and the harmony of the whole world teaches us their truth, if only—as they say—we would look at the thing with both eyes.

68 ON SAVING THE PHENOMENA AND THE REALITY OR UNREALITY OF EPICYCLES AND ECCENTRICS

Introduction by Edward Grant

The following selections from Moses Maimonides, Bernard of Verdun, and John Buridan reflect a controversy about the role of epicycles and eccentrics in astronomy and cosmology that is traceable to the Arabs and beyond them to the Greeks, especially Posidonius, Geminus, and Ptolemy. Here is Duhem's characterization of the problem:[1]

> One could conceive the hypotheses of astronomy as simple mathematical fictions which the geometer combines in order to render the celestial movements accessible to his calculations. Or one could view them as the description of concrete bodies, of movements actually realized. In the first case, a single condition is imposed on these hypotheses, that of *saving the appearances;* in the second case, the freedom of the one who imagines these hypotheses is very narrowly limited. If he is one who is versed in a philosophy that pretends to know something of the celestial essence, he will necessarily express his hypotheses in accordance with the precepts of that philosophy.
>
> On the subject of astronomical hypotheses, the author of the *Almagest*, and the Hellenic thinkers who came after him, adopted the first of these two opinions. They could, therefore, construct their geometric theories without concerning themselves about the different physics about which they disputed; they could choose their suppositions without troubling themselves if agreement was lacking between the results of their calculations and the data of observation.
>
> On the contrary, however, after the author of the *Hypotheses [of the Planets]*,[2] with Thabit ibn Qurrah and Ibn al Haitham [Alhazen], Arab astronomers desired that the hypotheses imagined by them should correspond to the true movements of really existing solid or fluid bodies. Hence they rendered these hypotheses amenable to the laws assumed by physics.
>
> But the physics professed by most of the philosophers of Islam was peripatetic [that is, Aristotelian] physics, the physics that Xenarchus and Sosigenes had long ago opposed to the astronomy of eccentrics and epicycles showing that the reality of the latter could not be reconciled with the truth of the former. The realism of Arab astronomers would necessarily provoke the Peripatetics of Islam into an ardent and merciless quarrel against the doctrines of the *Almagest*.
>
> In the twelfth century, we see this quarrel—led vigorously by the most distinguished Arab thinkers, Ibn Bâdja (Avempace), Ibn Tofail (Aboubekr), Ibn Roshd (Averroes), Moses ben Maimon (Maimonides)—produce an astronomical system, the system of al-Bitruji (Alpetragius), which, until the sixteenth century, will attempt unceasingly to be substituted for the system of Ptolemy.

1. My translation from *Le Système du monde* (Paris: Hermann, 1913–1959), II, 130–131.
2. Ptolemy himself was author of the *Almagest* and *Hypotheses of the Planets*. Thus in the former treatise he was primarily saving the astronomical phenomena by use of eccentrics and epicycles in a purely geometric manner without concern for the mechanical relations between these geometric devices. In the *Hypotheses*, however, Ptolemy is concerned exclusively with actual physical epicyclic and eccentric spheres and the manner in which they are physically located one within the other.

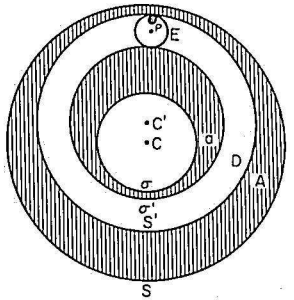


Fig. 1

Figure 1[3] conveys something of Ptolemy's effort to conceive physical eccentrics and epicycles. Almost all subsequent efforts of a similar kind were based on Ptolemy's account. The two spherical surfaces S and σ have as their center C, the center of the world. These constitute the *sphere of the planet*. The two spherical surfaces S' and σ' have as their common center C', the center of the eccentric. What lies between S' and σ', namely D, comprises the *deferent orb*. The epicyclic sphere, E, is located in D between S' and σ'. Thus the epicyclic sphere has a diameter equal to the thickness of the deferent orb. Set in the sphere of the epicycle is the planet P. The solid masses A and a, which are enclosed by the surface S, S' and σ', σ, respectively, are each moved with two rotations around axes passing through C, the center of the world. One of these rotations, the daily motion, is effected from east to west around the poles of the center of the world; the other rotation is very slowly from west to east around the poles of the ecliptic. Thus double rotation is communicated by A and a to D, the deferent orb, which is itself carried in a third motion around C', the center of the eccentric planet. All these motions are conceived as communicated to E, the epicyclic sphere, which, in turn, rotates around an axis passing through its center. The great circles, epicycles, and eccentrics that are treated in Ptolemy's *Almagest* are derived from this three dimensional cosmological description as follows: (1) By the rotation of the epicyclic sphere, the planet would describe a great circle on it which is the epicycle circle; (2) an eccentric circle, or deferent, is described as the center of the epicyclic sphere carried round by the deferent orb; and (3) the epicyclic sphere is subject to an oscillatory movement by means of which one can explain the inclination of the plane of the epicycle on the plane of the eccentric circle.

In the following selections it will be seen that Maimonides presents a thoroughly Aristotelian viewpoint, Bernard of Verdun argues for the existence of epicycles and eccentrics but partially justifies them by use of Aristotelian principles, whereas Buridan, in a more qualified position, emphatically denies the existence of epicycles but seems to argue for the physical existence of eccentrics. It will be evident that in their reports of the arguments of others and in their attacks against epicycles and eccentrics, Maimonides and Averroes strongly influenced the history of this problem and the responses to it. Helping to prolong the dispute was the fact that epicycles and eccentrics had not yet been devised in Aristotle's lifetime, thereby allowing the disputants to justify their positions by direct appeal to Aristotelian principles.

1. Moses Maimonides (1135–1204): The Reality of Epicycles and Eccentrics Denied

Translated by Shlomo Pines[4]

Annotated by Edward Grant

You know of astronomical matters what you have read under my guidance and understood from the contents of the "Almagest." But there was not enough time to begin another speculative study with you. What you know already is that as far as the action of ordering the motions and making the

3. Reproduced from Duhem, *Le Système du monde*, II, 91. The description follows Duhem.
4. Reprinted with the kind permission of the University of Chicago Press from *The Guide of the Perplexed, Moses Maimonides*, translated with an Introduction and Notes by Shlomo Pines with an introductory essay by Leo Strauss (Chicago: University of Chicago Press, 1963), pp. 322–327. *The Guide of the Perplexed*, of which the present selection constitutes the whole of Book II, chapter 24, was written originally in Arabic between 1187 and 1190 and translated into Hebrew by Samuel ibn Tibbon at Arles (France) in 1204. From the latter translation, a Latin translation was made during the first half of the thirteenth century. The *Guide* was well known and influential in the Latin West, being cited by Thomas Aquinas and very likely conveying some of the arguments and opinions to which Bernard of Verdun and John Buridan reacted.

course of the stars conform to what is seen is concerned, everything depends on two principles: either that of the epicycles or that of the eccentric spheres or on both of them. Now I shall draw your attention to the fact that both those principles are entirely outside the bounds of reasoning and opposed to all that has been made clear in natural science. In the first place, if one affirms as true the existence of an epicycle revolving round a certain sphere, positing at the same time that that revolution is not around the center of the sphere carrying the epicycles—and this has been supposed with regard to the moon and to the five planets—it follows necessarily that there is rolling, that is, that the epicycle rolls and changes its place completely. Now this is the impossibility that was to be avoided, namely, the assumption that there should be something in the heavens that changes its place. For this reason Abū Bakr Ibn al-Sā'igh[5] states in his extant discourse on astronomy that the existence of epicycles is impossible. He points out the necessary inference already mentioned. In addition to this impossibility necessarily following from the assumption of the existence of epicycles, he sets forth there other impossibilities that also follow from that assumption. I shall explain them to you now.

The revolution of the epicycles is not around the center of the world. Now it is a fundamental principle of this world that there are three motions: a motion from the midmost point of the world, a motion toward that point, and a motion around that point. But if an epicycle existed, its motion would be neither from that point nor toward it nor around it.

Furthermore, it is one of the preliminary assumptions of Aristotle in natural science that there must necessarily be some immobile thing around which circular motion takes place. Hence it is necessary that the earth should be immobile. Now if epicycles exist, theirs would be a circular motion that would not revolve round an immobile thing. I have heard that Abū Bakr has stated that he had invented an astronomical system in which no epicycles figured, but only eccentric circles.[6] However, I have not heard this from his pupils. And even if this were truly accomplished by him, he would not gain much thereby. For eccentricity also necessitates going outside the limits posed by the principles established by Aristotle, those principles to which nothing can be added. It was by me that attention was drawn to this point. In the case of eccentricity, we likewise find that the circular motion of the spheres does not take place around the midmost point of the world, but around an imaginary point that is other than the center of the world. Accordingly, that motion is likewise not a motion taking place around an immobile thing. If, however, someone having no knowledge of astronomy thinks that eccentricity with respect to these imaginary points may be considered—when these points are situated inside[7] the sphere of the moon, as they appear to be at the outset—as equivalent to motion round the midmost point of the world, we would agree to concede this to him if that motion took place round a point in the zone of fire or of air, though in that case that motion would not be around an immobile thing. We will, however, make it clear to him that the measures of eccentricity have been demonstrated in the "Almagest" according to what is assumed there. And the latter-day scientists have given a correct demonstration, regarding which there is no doubt, of how great the measure of these eccentricities is compared with half the diameter of the earth, just as they have set forth all the other distances and dimensions. It has consequently become clear that the eccentric point around which the sun revolves must of necessity be outside the concavity of the sphere of the moon and beneath the convexity of the sphere of Mercury. Similarly the point around which Mars revolves, I mean to say the center of its eccentric sphere, is outside the concavity of the sphere of Mercury and beneath the convexity of the sphere of Venus. Again the center of the eccentric sphere of Jupiter is at the same distance—I mean between the sphere of Mercury and Venus. As for Saturn, the center of its eccentric sphere is between the spheres of Mars and Jupiter. See now how all these things are remote from natural speculation! All this will become clear to you if you consider the distances and dimensions, known to you, of every sphere and star, as well as the evaluation of all of them by means of half the diameter of the earth so that everything is calculated according to one and the same proportion and the eccentricity of every sphere is not evaluated in relation to the sphere itself.

Even more incongruous and dubious is the fact that in all cases in which one of two spheres is inside the other and adheres to it on every side, while the centers of the two are different, the smaller sphere can move inside the bigger one without

5. That is, Ibn Bajja, or Avempace (see Selection 44).
6. This view is discussed by Buridan (see section 3 below).
7. That is, beneath.

the latter being in motion, whereas the bigger sphere cannot move upon any axis whatever without the smaller one being in motion. For whenever the bigger sphere moves, it necessarily, by means of its movement, sets the smaller one in motion, except in the case in which its motion is on an axis passing through the two centers. From this demonstrative premise and from the demonstrated fact that vacuum does not exist and from the assumptions regarding eccentricity, it follows necessarily that when the higher sphere is in motion it must move the sphere beneath it with the same motion and around its own center. Now we do not find that this is so. We find rather that neither of the two spheres, the containing and the contained, is set in motion by the movement of the other nor does it move around the other's center or poles, but that each of them has its own particular motion. Hence necessity obliges the belief that between every two spheres there are bodies other than those of the spheres. Now if this be so, how many obscure points remain? Where will you suppose the centers of those bodies existing between every two spheres to be? And those bodies should likewise have their own particular motion. Thābit[8] has explained this in a treatise of his and has demonstrated what we have said, namely, that there must be the body of a sphere between every two spheres. All this I did not explain to you when you read under my guidance, for fear of confusing you with regard to that which it was my purpose to make you understand.

As for the inclination and deviation that are spoken of regarding the latitude of Venus and Mercury, I have explained to you by word of mouth and I have shown you that it is impossible to conceive their existence in those bodies. For the rest Ptolemy has said explicitly, as you have seen, that one was unable to do this, stating literally: No one should think that these principles and those similar to them may only be put into effect with difficulty, if his reason for doing this be that he regards that which we have set forth as he would regard things obtained by artifice and the subtlety of art and which may only be realized with difficulty. For human matters should not be compared to those that are divine.[9] This is, as you know, the text of his statement. I have indicated to you the passages from which the true reality of everything I have mentioned to you becomes manifest, except for what I have told you regarding the examination of where the points lie that are the centers of the eccentric circles. For I have never come across anybody who has paid attention to this. However this shall become clear to you through the knowledge of the measure of the diameter of every sphere and what the distance is between the two centers as compared with half the diameter of the earth, according to what has been demonstrated by al-Qabīsī[10] in the "Epistle Concerning the Distances." If you examine those distances, the truth of the point to which I have drawn your attention will become clear to you.

Consider now how great these difficulties are. If what Aristotle has stated with regard to natural science is true, there are no epicycles or eccentric circles and everything revolves round the center of the earth. But in that case how can the various motions of the stars come about? Is it in any way possible that motion should be on the one hand circular, uniform, and perfect, and that on the other hand the things that are observable should be observed in consequence of it, unless this be accounted for by making use of one of the two principles[11] or of both of them? This consideration is all the stronger because of the fact that if one accepts everything stated by Ptolemy concerning the epicycle of the moon and its deviation toward a point outside the center of the world and also outside the center of the eccentric circle, it will be found that what is calculated on the hypothesis of the two principles is not at fault by even a minute. The truth of this is attested by the correctness of the calculations—always made on the basis of these principles—concerning the eclipses and the exact determination of their times as well as of the moment when it begins to be dark and of the length of time of the darkness. Furthermore, how can one conceive the retrogradation of a star, together with its other motions, without assuming the existence of an epicycle? On the other hand, how can one imagine a rolling motion in the heavens or a motion around a center that is not immobile? This is the true perplexity.

However, I have already explained to you by word of mouth that all this does not affect the astronomer. For his purpose is not to tell us in which way the spheres truly are, but to posit an astronomical system in which it would be possible for the motions to be circular and uniform and to

8. Thabit ibn Qurra, a pagan Arab astronomer and mathematician who died in 900.
9. *Almagest* XIII.2.
10. A tenth-century astronomer.
11. Epicycles and eccentric circles are the two principles.

correspond to what is apprehended through sight, regardless of whether or not things are thus in fact.[12] You know already that in speaking of natural science, Abū Bakr Ibn al-Sā'igh expresses a doubt whether Aristotle knew about the eccentricity of the sun and passed over it in silence—treating of what necessarily follows from the sun's inclination, inasmuch as the effect of eccentricity is not distinguishable from that of inclination—or whether he was not aware of eccentricity. Now the truth is that he was not aware of it and had never heard about it, for in his time mathematics had not been brought to perfection. If, however, he had heard about it, he would have violently rejected it; and if it were to his mind established as true, he would have become most perplexed about all his assumptions on the subject. I shall repeat here what I have said before.[13] All that Aristotle states about that which is beneath the sphere of the moon is in accordance with reasoning; these are things that have a known cause, that follow one upon the other, and concerning which it is clear and manifest at what points wisdom and natural providence are effective. However, regarding all that is in the heavens, man grasps nothing but a small measure of what is mathematical; and you know what is in it. I shall accordingly say in the manner of poetical preciousness: *The heavens are the heavens of the Lord, but the earth hath He given to the sons of man.*[14] I mean thereby that the deity alone fully knows the true reality, the nature, the substance, the form, the motions, and the causes of the heavens. But He has enabled man to have knowledge of what is beneath the heavens, for that is his world and his dwelling-place in which he has been placed and of which he himself is a part. This is the truth. For it is impossible for us to accede to the points starting from which conclusions may be drawn about the heavens; for the latter are too far away from us and too high in place and in rank. And even the general conclusion that may be drawn from them, namely, that they prove the existence of their Mover, is a matter the knowledge of which cannot be reached by human intellects.[15] And to fatigue the minds with notions that cannot be grasped by them and for the grasp of which they have no instrument, is a defect in one's inborn disposition or some sort of temptation. Let us then stop at a point that is within our capacity, and let us give over the things that cannot be grasped by reasoning to him who was reached by the mighty divine overflow so that it could be fittingly said of him: *With him do I speak mouth to mouth.*[16] That is the end of what I have to say about this question. It is possible that someone else may find a demonstration by means of which the true reality of what is obscure for me will become clear to him. The extreme predilection that I have for investigating the truth is evidenced by the fact that I have explicitly stated and reported my perplexity regarding these matters as well as by the fact that I have not heard nor do I know a demonstration as to anything concerning them.

2. Bernard of Verdun (fl. late thirteenth century): The Reality of Epicycles and Eccentrics Affirmed

Translated and annotated by Edward Grant[17]

Distinction III
On Explaining the Causes of the Apparent Irregularities in the Sun's Motion

CHAPTER 2

The second chapter will set forth some things that are necessary for showing that in many planets *(stellis)* and the sun it is necessary to assume an eccentric orb or a concentric [orb] with an epicycle whose radius is equal to the aforementioned eccentricity [Fig. 2]. According to *On the Heavens*, Book II,[18] the irregularity of motion is not in the celestial bodies [themselves], for if an irregularity

12. An attitude that was widely prevalent in the Middle Ages, perhaps wholly, or in part, the result of this statement, but which goes back ultimately to Simplicius' *Commentary on Aristotle's Physics*, and even beyond, to Geminus and Posidonius (see Cohen and Drabkin, *A Source Book in Greek Science* [Cambridge, Mass.: Harvard University Press, 1948], pp. 90–91). Buridan repeats it (see near the end of Question 10 in section 3.)

13. Compare to Book II, chapter 22.

14. Psalms 115:16.

15. In a note Pines explains (p. 327, n. 12), "In Ibn Tibbon's translation the passage has a different meaning: 'The general proof from them is that they indicate the existence of their Mover, but the knowledge of other matters concerning them cannot be reached by human intellects.' "

16. Numbers 12:8.

17. Translated from Bernard of Verdun's *Tractatus super totam astrologiam* edited by P. Polykarp Hartmann, O.F.M. (Werl, Westphalia: Dietrich-Coelde Verlag, 1961), pp. 67–72.

18. Aristotle, *De caelo* II.6 (288a. 13—289a.10). In this chapter Aristotle shows that there are no irregularities in celestial motions.

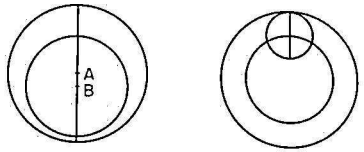


Fig. 2

were in them, there would be an intension and remission [of motion][19] in the beginning and end; indeed a natural motion is intensified at the end. But this is repugnant [when applied] to celestial circulation. Or such irregularity will result from a variation in the motive power, which is not true, because such movers are separate and do not undergo any change of power, nor does the substance of a [celestial] mobile change, since it is wholly immune from change. Furthermore, an intension and remission [of motion] occurs because of an impotence. Now, an impotence is contrary to nature, for nature intends what is best. Nothing is contrary to nature there [in the celestial regions], since celestial bodies are simple and in their proper region and nothing is contrary to them.

A second thing that is set forth is that in the planets, and especially in the moon, there seems to occur an advance *(accessus)* toward the center of the earth and a withdrawal *(recessus)* from the center by the same [celestial body] according to a greater or lesser amount. . . .

A third thing that is set forth is that in the motion of the sun and all the planets there appears an irregularity in speed and slowness, and all admit this, so that it is unnecessary to delay on this point.

CHAPTER[20] 4

In this fourth chapter a twofold way of saving the previously mentioned appearances is posited; the first is excluded and the second is inferred to be necessary.[21] All who speak reasonably on this subject say there is only a twofold way of saving the aforesaid phenomena. One way is touched upon by Averroes in Book XI of the *Metaphysics*,[22] where he says that true astronomy is based upon natural principles that destroy eccentrics and epicycles, which can happen if one orb is moved on many poles so that these motions are composed of many motions. Yet he confesses that he cannot explain this astronomy. Thus it seems to some that Alpetragius (al-Bitruji), following him and perhaps aroused by these things, developed the roots of Averroes into beautiful branches, flowers, and fruits.[23] For he assumes that the planets are moved with particular motions on poles other than the poles of the first mobile, namely on the poles of the zodiac. He assumes that they are moved on inclined circles whose poles are moved on the circumference of a small circle around or near the poles of the zodiac. Thus the planets are moved against the order of the signs through one half [of the circle] and with the order of the signs in the other half. And when the arc that runs counter to the order of the signs has been traversed and it corresponds to more of the zodiac than the orb of the planet will have traversed in revolving around the poles of the zodiac, a planet will appear retrograde; when these two are equal, the planet will appear stationary; and when the arc that has been traversed in the contrary order of the signs is less than the part traversed by the planet along the zodiac, the planet will have a direct movement but at a slow speed. However, when it moves in the order of the signs it appears to move quickly. And for every planet each of these events occurs in the very same point of the small circle.[24]

19. That is, the velocity of celestial bodies would increase and decrease. This is denied by Aristotle.

20. This is actually chapter 3, but an error lists it as chapter 4. The next will be listed as 5, but is actually 4.

21. Rashly assuming that his two alternatives are exhaustive, Bernard believes that he has arrived at a necessary truth by elimination. This mode of reasoning is frequently encountered in medieval treatises (see Selection 51, n. 30; after eliminating four theories, Bradwardine considers a fifth as necessarily true).

22. Actually Book XII, Comment 45, of the *Metaphysics*. These arguments are mentioned in the next section (see n. 49).

23. Duhem noted (*Le Système du monde*, III, 447) that all of chapter 4 to this point was taken substantially, though not verbatim, from Roger Bacon, *Epistola ad Clementem IV in Opus maius*, Part II, ch. 2.

24. The *De motibus celorum (On the Celestial Motions)* of al-Bitruji, in which this interpretation is found, was translated from Arabic into Latin by Michael Scot in 1217. Apparently following the path of Averroes, al-Bitruji opposed Ptolemy and denied epicycles and eccentrics. It will be useful to quote part of J. L. E. Dreyer's description of this system (*A History of Astronomy from Thales to Kepler*, 2d ed. [New York: Dover, 1953], pp. 265–266):

"The object of this system was to explain the constitution of the universe as it really is, and not merely to represent the motions of the planets geometrically, so as to be able to foretell their places in the heavens at any time; and the author (be he Ibn Tofeil or Al Betrugi *alias* Alpetragius) specially disclaims any intention of testing the theory by comparing it with observations or of accounting for minor details of the motions. The

This position is seen to be impossible in many ways. For when any point of the sphere has been moved, the whole sphere and any point of it have been moved except the two points which are the poles of motion. Therefore, when the poles will have traversed a fourth part of the small circle on which they are revolved, according to Alpetragius, the whole sphere will have traversed a fourth of the distance of its revolution; similarly when the whole [small] circle is traversed by the poles, the whole sphere will traverse its whole distance. For when the line coming from the center of the said small circle through the pole of the sphere, with whose motion the planet is moved, has terminated at the planet, not only when it has traversed the small circle does it also traverse the arc of the zodiac intercepted between the lines touching the small circle and coming from the pole of the world and terminating in the zodiac, but it traverses the whole zodiac. From all this, the great laughability of this position is obvious.[25]

Again, in this theory, direct and retrograde arcs would always be equal, and stationary points always the same. All these things are wholly false.

Again, direct and retrograde motions would be quickest when a planet would be at its maximum latitude; the opposite of this is certain and is shown below. Again, the sun would have a latitude, which is wholly false.

Again, if this way were true, the things we premised in the preceding chapter, which are very obvious to any one capable of reasoning, would be destroyed. Although many arguments could be brought against this position, these should suffice.

From all this, the first way is impossible and insufficient for saving the things shown before, which anyone capable of reasoning necessarily has to assume. The second way is left and therefore is necessary, namely the way which assumes eccentric and epicycle at the same time and a plurality of orbs around different poles of revolution in the planets other than the sun. In the sun [there is needed] only an eccentric, which is more likely, or a concentric with an epicycle. By these means, all the aforementioned absurdities are avoided and all the things mentioned in the preceding chapter are saved. Maintaining this from the beginning [of astronomical investigation], all the things which we happen to know about celestial motions and the distances and magnitudes of celestial bodies were investigated truly up to this day. If the basis of all this was false, this could not have happened. For, in any thing whatever, a small error at the beginning is great at the end.

Moreover, all the celestial phenomena are consonant with this way and in contradiction with the

leading idea is that of the homocentric spheres, each star being attached to a sphere, and the motive power is the ninth sphere, the sphere outside that of the fixed stars. The Spanish philosopher ought therefore to have been content with the system of Eudoxus or its modification by Aristotle (whom he never mentions by name, but only as "the sage"), but unfortunately he became possessed with the notion that the prime mover must everywhere produce only a motion from east to west, and he had therefore to reject the independent motion of the planets from west to east, and revert to the old Ionian idea that the seven planets merely perform the daily revolution with a speed slightly slower than that of the fixed stars. The true speed of the *primum mobile* is a little faster than this; the eighth sphere performs a revolution in a slightly longer period (24 hours), and the effect of the prime mover is gradually weakened more and more, with increasing distance, until we find the sphere of the moon, being furthest from the prime mover, taking nearly twenty-five hours to complete a revolution. This was the old primitive Ionian idea, but Al Betrugi (or his teacher) saw that this was not sufficient, as not only is the pole of the ecliptic different from that of the equator, which prevents the planets from moving in closed orbits, but the planets do not even keep at the same distance from the pole of the ecliptic but have each their motion in latitude, as well as a variable velocity in longitude; and all this had yet to be accounted for. The ninth sphere has but one motion, but the eighth has two, that in longitude (precession) and another which is caused by the pole of the ecliptic describing a small circle round a mean position thereby producing the supposed oscillation or trepidation of the equinoxes. Similarly, the pole of each planet describes a small circle round a mean position (i.e. the pole of the ecliptic), thereby producing inequalities in longitude and motion in latitude. Whenever the actual orbit-pole of a planet is on the parallel of the mean pole, it is obvious that the planet will perform its daily revolution with its mean velocity, while the velocity is increased or lessened when the actual pole is respectively at its minimum or maximum distance from the pole of the heavens (the motion of the pole of the orbit being added to or subtracted from the motion of the planet), so that the epicycle is hereby rendered superfluous."

Dreyer calls this a retrograde step in the history of astronomy. Duhem believed that Bernard of Verdun did not fully understand al-Bitruji's system and made unjust criticisms (see *Le Système du monde*, III, 449,450). No evaluation of al-Bitruji's system or Bernard's criticism will be attempted here. The Latin text has been edited by F. J. Carmody, *Al-Bitruji De motibus celorum* (Berkeley: University of California Press, 1952).

It is of considerable interest to realize that the only other contending theory is not the vague, qualitative Aristotelian system of homocentric spheres described in the *Metaphysics*, but al-Bitruji's mathematically formulated alternative.

25. The substance and meaning of this criticism elude me.

first way. Therefore, it is necessary that the premises stand and be conceded with the same necessity as the celestial motions and the whole of nature, and so on. To deny the things that are very certain in every argument *(omni ratione)* because of certain sophistic arguments is as foolish as are the follies of the ancients who falsely denied motion, all transmutations, and a plurality of beings, the contrary of which is apparent to sense alone. For these things cannot be demonstrated, just as one cannot demonstrate that fire is hot or that substance and accident are in existent things; but we accept this by the senses. Thus the Philosopher [Aristotle] says that we do not have to dignify every argument, and he says it is absurd to seek a reason for these things. All our reasoning always presupposes the senses.[26]

CHAPTER 5

In this fifth chapter are presented the arguments of those who attack the way stated above and the response to them. For they argue that if there is an eccentric or epicyclic orb then either the body of the sky will be divisible or rarefiable or there will be a vacuum, or two bodies in the same place. Indeed the part of the eccentric farthest from the earth will take the place of the part of the eccentric nearest the earth, and conversely. Similarly an epicycle that is revolved through an orb will cut it.

But it should be replied that the first difficulty is avoided because an eccentric is not assumed to be revolved around the center of the world, except with an accidental motion, but around its own center, so that its different parts succeed themselves continually in the points or places of the farther and nearer longitude that are imagined in the convexity of the orb surrounding the eccentric.

Similarly, the second difficulty is avoided because we do not assume that the center of the epicycle is moved on the circumference of the deferent in reality *(secundum veritatem)*, but we assume that the epicycle is a small sphere located between the thickness of the [eccentric] orb and is moved with the motion of the orb in which the planet is immediately located.[27] And, as will be seen, this sphere embraces several spheres required for saving the different appearances of such a motion. Nor does it seem absurd if the motion of superior [planets] is distributed to the lower spheres. I say this about the motion of the center of the epicycle inclined in the orb of the eccentric, [that is,] the motion of such lower eccentrics is the aggregate of their own proper motions and the motions of all the higher [or upper] eccentrics; and the inclination *(declivitas)* of their motions [is the aggregate of] their own proper inclinations and that of the higher [spheres]. For example, if in the sun, in which no inclination or latitude appears, a latitude or inclination were to appear, an epicycle carrying the sun should be assumed revolving in latitude in order to compensate for the latitude that is to be offset. Or a multitude of great spheres could be assumed, making the sun accord with its appearances; or if this does not seem suitable, immobile spheres could be posited [between the different orbs] that are not of the same thickness in their parts, so that it would not be necessary to assume that the motion of the lower spheres is composed of the motions of all the higher spheres[28] and it would not be necessary to assume a division of the spheres and a penetration or vacuity. Nor are these things assumed superfluously, since the motions of the other [spheres] makes them necessary. Thus we can understand that in all natural things, some things are in them for their own sake, and some because of other things only. And it is the same way in artificial things; for all the parts of a cither do not produce sound, and yet if the parts that produce no sound were not in the cither, the others would not produce sound. Also in animals there are bones that are immobile per se, in which other bones are revolved.

Furthermore, even the beauty and perfection of creatures consists more in variety, numerousness, and proportionality than in complete uniformity; and just as the beauty *(formositas)* of the starry orb consists in the variety and numerousness of the stars, so [does the beauty] of the lower [celestial bodies or planets consist] in the variety and numerosity of their motions and orbs, as is declared in Book II of *On the Heavens*.[29] In the second place, one may argue this way: If the moon and any planet are rotated by means of the motion of an epicycle, then the same part of the body of that planet does not always face the earth, the contrary of which is shown by Aristotle regarding the moon, whose spot always appears to us in the same shape [or form].[30]

26. The dependence of induction on the senses is emphasized in *Posterior Analytics* I.18.81a.38—81b.9.

27. See the figure in the Introduction to this selection.

28. The immobile spheres between the orbs would not transmit motions from the outer orbs to the inner orbs.

29. 292b.25—293a.14.

30. Aristotle argues (*On the Heavens* II.290a.25–27) that since the moon always shows its same face to us, it

I reply that if this were so, it [that is, the moon or any planet] is countered [or brought back *(reducitur)*] by the motion of some of the orbs containing the epicycle, in whatever way this would be required.[31] Moreover, granting that the same part of the body of a planet does not always face the earth, yet it could, nevertheless, always appear to us in the same shape [or form] because we assume that we see every part of the moon's body, since it is like a solid spherical body of great transparency which extends to every part of its depth and in the interior of it there is a spot of spherical shape. Therefore, however this body may be revolved, the spot always appears to us in the same shape [or form].[32]

Furthermore, every natural motion is away from a center, toward a center, or around a center. It must be said that since the motion of eccentrics and epicycles are circular, they move around a center, since they move around their own centers. Thus the Philosopher, at the beginning of *On the Heavens*[33] divides local motion in two ways, namely into rectilinear and circular motion, since every motion is straight or circular or a mixture of these. For these are the only two simple motions, since these magnitudes[34] are the only simple magnitudes. And this includes the circle, which is around a center; and thus he calls circular motion and motions around a center the same. Hence the center *(medium)* around which the prime motion [of the outermost sphere] occurs, and takes with it all the orbs, is the absolute center [of the world]. And the centers *(media)* around which the lower orbs are moved are centers in a relative sense *(secundum quid)*. At present, what we have said about these matters is sufficient.

3. John Buridan (ca. 1300—ca. 1358): An Intermediate Position: Epicycles Denied, Eccentrics Affirmed

Translated and annotated by Edward Grant[35]

Book XII

QUESTION 10

In the tenth question, we inquire whether epicycles are to be assumed in celestial bodies.

First it is argued that they are not to be assumed,[36] by the authority of the Commentator [Averroes], who expressly tries to disprove them. Next by the authority of Aristotle, who, in enumerating the celestial motions and spheres in this twelfth book, did not enumerate epicycles. Indeed even in the second book of *On the Heavens and World*, in saving the irregularities *(difformitates)* of the planetary motions that are apparent to us, he says that these irregularities arise because several motions are brought together in the same mobile. Thus he does not assume that they arise by eccentrics or epicycles.

Likewise, the Commentator argues that every celestial motion ought to be a simple motion and every simple motion is (1) away from the center of the world, or (2) toward the center of the world, or (3) around the center of the world, as is taught in the first book of [Aristotle's] *On the Heavens*.[37] Therefore, every celestial motion must be one of these [three possibilities]. But if the motion of an epicycle were assumed, it would be none of these. For it would not be away from the center, because

cannot be said to roll or revolve. This argument is extended to all celestial bodies. This second objection against epicycles may have been drawn from Roger Bacon's *Opus Tertium* or the *Communia naturalium*, Book II.

31. Bernard argues that the planets could be carried on epicycles and yet always present the same face to us if only we understand that in the total machinery of spheres and orbs for each planet there are spheres that would counter the epicycle's motion and allow us to see only the same face. Bernard offers no technical details for achieving this.

32. In this interesting response, Bernard assumes that the moon is a transparent sphere with a spherical opaque spot lying at its center. Thus however the moon may turn or rotate, we will see the spot embedded in the transparent moon and, to a terrestrial observer, always possessed of the same shape.

33. I.2.268b.17f.

34. That is, straight line and circle.

35. From Buridan's *Questions on the Metaphysics of Aristotle* as published in *In Metaphysicen Aristotelis Questiones argutissimae Magistri Ioannis Buridani in ultima praelectione ab ipso recognitae et emissae. . .* (Paris, 1518; Frankfurt: Minerva G.M.B.H., 1964), fols. 73 recto to 74 recto.

36. By first stating the arguments for the negative position and subsequently adopting it, Buridan diverges from the usual pattern of a *questio* (for a similar instance, see Selection 31, n. 7). Despite a rejection of epicycles in this question, Buridan allows for a qualified acceptance of them at the conclusion of the next question (see n.54).

37. This threefold Aristotelian analysis of motion is also repeated by Maimonides (see the beginning of section 1) and Bernard of Verdun (near the end of section 2).

that motion would be the motion of light bodies, namely an upward motion; nor would they move toward the center, because that motion is the motion of heavy bodies. Nor would it move around the center of the world, because this epicycle is not around the earth [that is, does not have the earth as its center]. Indeed it is above [the earth] in an orb and in this orb it has both its own center and circumference. Therefore, such an epicycle must not be assumed. Moreover, every celestial motion ought to be circular and, consequently, must be around something [that is, around a physical center]. But the motion of this epicycle would not be around anything, since it does not have a center—except an indivisible [center or point] around which it is moved. Such an indivisible [entity] is not a natural center. Indeed it is nothing. Hence this motion is not around anything.

Nevertheless, Ptolemy and all modern astronomers *(astrologi)* assume the opposite. For otherwise the appearances of the planets could not be saved, especially their approaches *(approximationes)* and recedings *(elongationes)* from the earth which we obviously and notably experience. Thus it is that sometimes the moon is so high that it cannot be totally eclipsed, but the cone of the earth's shadow falls onto the middle of it, so that while it is seen to have been eclipsed in the middle, the circumference remains in light. And sometimes the moon is so near to the earth that it falls deeply into the earth's shadow and remains totally eclipsed.

It must be understood that in the world the natural center is this earth. Therefore, an indivisible center is not assumed except in the imagination. But nevertheless a point can be imagined in the center of the earth as if it were the center of the world, and then all spheres having their centers in the center of the earth are called concentric. But in all the spheres of the planets, the moderns assume a sphere which they call eccentric and which is wholly around the earth; it does not have its center in the center of the world, but outside it. And for this reason, it is called eccentric. Furthermore, they assume little spheres *(spherulas)*, which they call epicycles, that are fixed in the width of the eccentric. The planet is fastened on these epicycles. But then it is necessary that the whole sphere of the planet be concentric, for otherwise there would necessarily be a vacuum between the total spheres of two planets. Indeed it is necessary to imagine two spheres of very nonuniform magnitude, [one] above the eccentric, the other below the eccentric, and each of them is very wide on one side and on the other side of virtually no magnitude.[38] Then it is necessary to imagine that this whole sphere composed of all the things just mentioned is moved simultaneously with a daily motion from east to west, carrying the eccentric with it; but the eccentric is also moved on the poles of the zodiac between the two parts [or sides of the whole sphere] in a motion contrary to the daily motion, namely from west to east. Then, thirdly, the epicycle, carrying the planet, is moved on its [own] proper center in this eccentric.

After saying all this, we must let it be known how this appears to us. It seems to me very probable that epicycles should not be assumed, because if an epicycle is not posited in the orb of the moon it ought not to be posited in the orb of the other planets, since all the reasons which apply to the other planets should also apply to the moon; and should anyone assume epicycles on the spheres of the other planets, he should similarly assume an epicycle on the sphere of the moon. Therefore, if it were shown that an epicycle should not be assumed in the [orb of the] moon, it could be concluded that no such epicycles should be assumed.

Then I argue that it ought not to be assumed for the moon, because then it would follow that in that spot *(macula)* of the moon which appears as if it were an image of a man whose feet always appear to be below [or toward the bottom], the feet would sometimes appear above [in the upper part of the moon]. But we experience that this consequent is false, since this image always appears situated in the same way with respect to us, on the assumption that it appears to us in the same part of the sky, for example, on the meridian. But I prove the principal consequent,[39] because if the feet appear to us below when the moon is in the *aux* [that is, apogee] of the epicycle, it should follow that when it reaches the opposite of of the *aux* [that is, perigee] the feet should appear above, since by the motion of the epicycle the moon has traversed [a distance such that] the part of the moon which previously was above is now below.

It must be understood that there is no escape in

38. The figure in the introduction to this selection illustrates Buridan's description.

39. Buridan wishes to prove that if a lunar epicycle were assumed, it must follow that the feet of the man in the moon (or the spot on the moon) will sometimes appear in the upper part and sometimes in the lower part of the moon. For Bernard of Verdun's denial of this argument, see near the end of section 2, and note 32.

resolving this argument except in one way only: namely by saying that just as this epicycle is moved around its proper center, so also is the body of the moon moved around its proper center, in a motion contrary to that of the epicycle and with an equal speed. Thus the epicycle completes its circulation in the same time as the moon completes its circulation. It is certain that if this conception *(imaginationem)* were true, the argument would be solved, since the part of the moon which was previously above will always remain above. But there is an objection against this conception, because if the body of the moon had a special motion, by a parity of reasoning the same would seem to apply to the other planets and stars, since any star *(stella)* [or planet][40] is a spherical body like the moon. No reason can be offered for the moon's motion which ought not to apply to any other star [or planet]. But then, according to Aristotle, it would be necessary to assume as many intelligences as there are stars in the sky because each star would require a special *(proprium)* mover for its special motion. Aristotle did not assign [or concede] such a multitude [of motions and intelligences].

Moreover, according to Aristotle, these different motions of the stars and planets are required for explaining the difference of species and transmutations in lower [or inferior] things. Therefore, motions must not be posited in the sky that are not associated [directly] with a star. Otherwise there would not follow any diverse action in lower things, but by such a motion of the sun or Jupiter no diversification would appear to us, because it is probable that the sun is uniform and that the upper part of it is just like the lower part. And thus there is no force *(vis)* which a part of the sun would reveal to us. And so it is that if the motion of the sun were around a special center [other than the earth], it would be in vain, conferring nothing on this lower world. But if the sun does not have such a motion, it does not seem reasonable that the moon should have it, since the sun is much nobler than the moon.

To the authority of the astronomers, the Commentator replies that this way of assuming or imagining epicycles and eccentrics is quite valid for computation and for knowing the positions and dispositions of the planets with respect to each other and with respect to us; and astronomers seek nothing more than this. Thus, although such conceptions are useful for them, they do not exist in the thing [that is, in the celestial spheres].[41] And when it is also said that the appearances [of the planets] cannot be saved, I readily concede that they cannot be saved without the assumption of epicycles or eccentrics. But all the appearances can be saved by eccentrics [alone] without epicycles. And this will be seen in another question.

QUESTION 11

In the eleventh question, we inquire whether eccentric orbs are to be assumed in the sky.

By the authority of the Commentator, who attempts to reject them, it is argued that [eccentric orbs are] not [to be assumed in the sky]. Furthermore, it follows that the motions of these eccentrics would not be simple; and yet all celestial motions ought to be simple, since they arise from simple movers. But the first consequence is proved,[42] because that eccentric motion which participates in circular motion would also participate in somewhat of an upward motion and somewhat of a downward motion, so that sometimes a planet would appear nearer the earth—and therefore [participate in] more of a downward motion—and sometimes farther from the earth—and thus [participate in] more of an upward motion.[43] Now, such motions, which participate in circular, upward, and downward movements, are not simple, and so on.

Furthermore, in the second book of *On the Heavens,* Aristotle does not save the motions and appearances by such eccentrics but [rather] by combining several motions in the same orb. It was not Aristotle's intention that there be such eccentrics.

Again, the world ought to be one and as uniform as possible, since it depends on a single and simple principle. But it would detract from its

40. In this context it seems best to understand *stella* as "planet" rather than "star."

41. An opinion also held by Maimonides. However, in at least one instance, Averroes seems to cast slight doubt on the appropriateness of epicycles and eccentrics for computation. In *Metaphysics* XII.8, Comment 45, he says with respect to epicycles and eccentrics that "present-day astronomy does not deal with realities, and is suitable merely for computing unrealities" (translated by Francis Carmody, "The Planetary Theory of Ibn Rushd," *Osiris,* X [1952], 572).

42. That is, Buridan now demonstrates that the motion of eccentrics is not simple, but composite. He argues (near the end of section 3), however, that it is sufficient "that an eccentric motion is simple, uniform, and regular with respect to its center, although not with respect to the earth."

43. Since the center of the eccentric circle differs from the earth's center, the eccentric will allow for variation in the distance of a planet from the earth.

unity if it did not have the same circumference and center for all the spheres composing it: for example, that the natural circumference would be the highest sphere and the natural center would be the earth. Now, assuming eccentrics would destroy the unity of the world, since the earth would not be the center of all the celestial spheres.

Again, it would follow that only in the sky would there be bodies whose function it is to fill a vacuum, which seems absurd and against Aristotle, who says that every celestial sphere is disposed for carrying a [single] planet. . . .[44]

Again, not only is an intelligence the cause of celestial motion but it is also the cause of the orb, since it should not be said that the sky does not have a cause. Now, as Aristotle says, it is imagined that from a simple intelligence there ought to arise only one simple and regular motion. Thus it is probable that an orb caused by a simple intelligence ought to be one and therefore regular. But those who posit eccentrics assume very irregular orbs, as it seems, for those planets that are said to have an eccentric. Therefore, eccentrics should not be assumed.

These are the arguments of the Commentator against eccentrics in the twelfth book of the *Metaphysics* and in the second book of *On the Heavens.*[45]

Ptolemy and all modern astronomers argue for the opposite position, and the argument is based on calculations *(rationes),* because no way has yet been found, nor does it appear that it could be found, for saving the appearances—especially concerning the receding and approach of the planets [away from and] toward the earth—except by assuming eccentrics or epicycles.

It must be noted that we observe many apparent irregularities in the planets and stars. First there are the different retardations of the planets from the daily motion; and in this they are assumed to be moved with a proper motion contrary to the daily motion, so that with its proper motion the sun completes one circulation in one year and similarly Venus and Mercury; and the moon in one month, Mars in two years, Jupiter in twelve years, and Saturn in thirty-two years.[46] The retardation is such that if the sun and a fixed star rise simultaneously today, they will not rise simultaneously tomorrow, but the fixed star would rise before. Thus in one year a fixed star rises more often than the sun, namely more often in one period.

A second irregularity is the common withdrawal from our tropic to the other tropic and back again. . . ,[47] and because of this nonuniformity an oblique circle is assumed, namely the zodiac, under which the planets move about on poles other than the poles of the daily motion.

A third irregularity is the inclination *(declinatio)* of a planet from the middle line of the zodiac, which is usually called the ecliptic. It is an inclination toward the south or north, depending on the width of the zodiac.

A fourth irregularity is speed and slowness. For it is noticeably appearent that a planet moves sometimes quicker and sometimes slower. Thus it is that sometimes the moon passes through a sign [that is, 30 degrees] in less time and sometimes in a considerably greater time.

A fifth irregularity is retrogradation. For some planets in moving from west to east contrary to the daily motion not only appear to be moved sometimes quicker and sometimes slower but even appear to regress, so that they return from west to east more quickly than the fixed stars. And you ought to know that by the association of several different motions in the same sphere—as Aristotle says in the second book of *On the Heavens*—all these irregularities are easily saved without imagining epicycles or eccentrics.[48]

44. A few lines are omitted here.

45. The arguments of the Commentator, Averroes, in both the *Metaphysics* and *On the Heavens* have been translated by F. Carmody ("The Planetary Theory of Ibn Rushd," *Osiris,* X [1952], 556–586). Some of the relevant sections will now be quoted:

"We claim that eccentrics and epicycles are unnatural, and that epicycles are impossible. A body moved in a circle can only be moved about the center of the universe; if there were circular motion not associated with this center, we would have to suppose another and different center, hence another and different earth, which is impossible, as stated in the Physics.

"The same perhaps holds for Ptolemy's eccentrics: if there were many centers, there would be many solid bodies different from the earth, and the center would not be unique, for it would have breadth and be divided; all this is impossible. Furthermore, accepting eccentrics, we would have to find among heavenly bodies some superfluous ones, useful merely to fill spaces, as in the bodies of animals. Nothing in planetary motion leads us to suppose epicycles or eccentrics" (p. 568; from Averroes, *Metaphysics* XII.8, Comment 45).

"A body moving in a circle can only move about a center; hence eccentrics are impossible unless between celestial bodies there is either a vacuum or filling-in bodies which are neither naturally round nor naturally moved" (p. 570; *On the Heavens* II.6, Comment 35).

46. Thirty years would be more nearly correct.

47. Two lines are omitted here.

48. According to Buridan, the first five irregularities

A sixth irregularity is the nearness of a planet to the earth and at times its remoteness or distance from it. This irregularity is well experienced and demonstrated. It seems to me that this irregularity cannot be saved without eccentrics or epicycles, since any concentric sphere in the world would be everywhere equidistant from the earth. By virtue of the motion of such a sphere a planet could never be nearer or farther. The Commentator tried briefly to save this appearance in another way, but he could not, as he confessed.[49] In the second place, it seems to me that these appearances could be saved by eccentrics without epicycles, or even by epicycles without eccentrics. An eccentric has one of its sides nearer the earth and the other farther away, so that when the planet is on the side of the eccentric more distant from the earth it would be farther away from us; but on the other side it would be nearer to us, although no epicycle was assumed. And similarly, even if there were no eccentric, yet an epicycle could have one side nearer to us and the other more distant from us.[50]

But some argue against the previously stated conclusion, because when the planet approaches or recedes there appears a very great irregularity, since sometimes it approaches very close before it begins to withdraw and sometimes it approaches less than half as much before it begins to withdraw.[51] But this could not be saved by assuming an eccentric alone or an epicycle alone. To this argument it could be replied that if anyone wished to assume epicycles without eccentrics it would be necessary, in order to save this appearance [or phenomenon], to posit an epicycle on an epicycle, and then every irregularity which is saved by an eccentric and an epicycle could be saved by these two epicycles.[52] Also, whoever wished to assume eccentrics without epicycles would have to assume two eccentrics, one on the other, just as we assumed an eccentric on a concentric. Then everything that was saved by an eccentric and an epicycle could be saved by these two eccentrics. But, to put it briefly, it does not seem to me that any of these ways is really demonstrable or disprovable.

To the arguments put forth by the Commentator one can say this: that an eccentric motion is simple, uniform, and regular with respect to its center, although not with respect to the earth. And this suffices. It is not necessary to assume the world as one being but as a union of many things having a mutual order and one principle; nor is it necessary to assume that these [celestial] bodies would serve only to fill a vacuum. Rather it should be assumed that the whole [world] is made up of these things and is moved on an eccentric simultaneously with the daily motion and that the different parts of the eccentric are related to the earth in different ways. For God and the intelligences intend such irregularities for the purpose of better explaining all the different material species in this world, and it is not necessary that the sphere moved by one intelligence be itself one [that is, undivided], because there is no discontinuity there, since the whole world contains many spheres and it [the world] is moved all at once by the simplest prime mover. Nor [indeed] is there any irregularity of magnitude with respect to the moving intelligence, since the whole sphere [of the world] is concentric and of uniform magnitude and it is moved all at once by its proper intelligence. But, again, an eccentric has a special intelligence by which it is moved with a certain other motion; and this eccentric is also of a uniform magnitude and

can be explained without epicycles or eccentrics, but not the next, or sixth, irregularity.

49. Probably based on Averroes' remarks in *Metaphysics* XII.8, Comment 45 (Carmody, p. 572): "But Ptolemy did not understand that the ancients postulated spiral motion because epicycles and eccentrics are impossible; wherefore we must return to a study of the true theories based on rational precepts. I have set up a system of motion on a single sphere but about two or more different poles, as the appearances required; by this motion one can, for the asters, explain speed, slowness, advance and recession, and other motions for which Ptolemy was able to set up no theory; and from this, also, lesser and greater distance, as for the moon. In my youth I hoped to complete this study, but in my old age I despair of this; yet perhaps these statements will lead someone to further study." Perhaps al-Bitruji was motivated by these early efforts of Averroes (see n. 24).

50. Underlying Buridan's statement is the demonstrated equivalence in representing the apparent irregular motion of a planet from west to east (especially the sun's annual motion) either by (1) an eccentric circle whose center is not the earth, or (2) a concentric circle with the earth at its center but on which an epicycle is assumed whose radius equals the eccentricity of the eccentric circle described in (1). A demonstration of this equivalence appears in Ptolemy's *Almagest*, Book III, chapter 3 (see pp. 86–91 of R. Catesby Taliaferro's translation of the *Almagest* in Vol. XVI of the *Great Books of the Western World* [Chicago: Encyclopaedia Britannica, 1952]).

51. Buridan observes that the perigee of a planet varies, a phenomenon that was not representable by an epicycle or a concentric or an eccentric circle alone.

52. That is, a concentric circle with an epicycle on an epicycle can be made equivalent to an eccentric with an epicycle.

moved uniformly around its center.[53] And so, by assuming eccentrics, these arguments are avoided. As for those who wish to assume epicycles *(epicyclos)*, there is no strong argument against them except for the spot *(macula)* on the moon.[54] And so Question 11 is terminated.

69 ON THE COMMENSURABILITY OR INCOMMENSURABILITY OF CELESTIAL MOTIONS

Nicole Oresme (ca. 1325–1382)

Translation, introduction, and annotation by Edward Grant[1]

Nicole Oresme's fascination with the subject of commensurability and incommensurability in mathematics, physics, and cosmology is evidenced by a number of treatises in which he saw fit to discuss, or at least mention, it. Sometime after 1351, and perhaps between 1351 and 1360, he composed *On Ratios of Ratios (De proportionibus proportionum)*, in which the propositions of the first three chapters were concerned with commensurable and incommensurable relations between various types of ratios, and the fourth chapter developed the application of these propositions to terrestrial and celestial motions.[2] Even prior to this, however, Oresme seems already to have written *Ad pauca respicientes* (these are the opening words of the treatise),[3] in which he concerned himself almost exclusively with the kinematics of circular motion.

[His] purpose was to determine the consequences deriving from the motions of points or bodies moving with different uniform circular velocities. These bodies, usually two or three, are, in some propositions, assumed to move with mutually commensurable velocities and in others with incommensurable velocities.

In contrast to astronomers, and especially in Part I, Oresme is generally interested in precise punctual relations between the moving bodies. That is, he wishes to specify the exact point or points at which the bodies will meet or oppose one another, and if the bodies move commensurably, to determine the precise time intervals between such occurrences. On the assumption of incommensurable speeds, such precision is impossible.

In Part 2 Oresme shifts his point of emphasis and concerns himself with general dispositions and relative positions of two or more mobiles before and after they enter into actual conjunction or opposition. The particular points in which these events occur are now of little significance.

Of great interest is Supposition II of Part 1 (p. 385), where Oresme assumes that any two quantities are probably incommensurable. Although this supposition is not utilized until the very end of the treatise, its application is the highlight of the *Ad pauca respicientes*. Having formulated numerous abstract propositions involving points or bodies moving with commensurable or incommensurable speeds, Oresme, in Proposition XVII of Part 2 (p. 423), invokes Supposition II and applies it to celestial motions in order to demonstrate that no configuration or relationship of celestial bodies can ever repeat. This constitutes the basis of his repudiation of astrological prediction in Proposition XIX (pp. 425, 427). If every celestial configuration is unique—as it must be on the assumption of incommensurable speeds, which by Supposition II are probable—astrological prediction is hopeless for it must rely on cumulative observations of precisely recurring events.[4]

53. It seems obvious from the preceding that Buridan argues here for a physically real eccentric sphere which is moved around *a center other than the earth* by its own special intelligence. This represents a total break with a central theme in Aristotelian cosmology.

54. Probably a reference to Question 10, where Buridan argued that if epicycles existed, the spot or man in the moon should appear sometimes in the upper part, sometimes in the lower part, of the moon. Since such a phenomenon was not experienced, this was taken as a weakness of the epicycle theory. Now Buridan says that if one were ready to accept this imperfection as an inherent defect of epicyclic theory, the latter would be serviceable in all other respects. See also note 39.

1. Reprinted with permission of the copyright owners, the Regents of the University of Wisconsin, from *Nicole Oresme and the Kinematics of Circular Motion: "Tractatus de commensurabilitate vel incommensurabilitate motuum celi,"* edited with an introduction, English translation, and commentary by Edward Grant (Madison, Milwaukee, and London: University of Wisconsin Press, 1971), pp. 179, 193, 201, 215–217, 237–243, 249–253, 261–265, 273–277, 279, 319–321. Where it seemed relevant, I have incorporated into the notes material from the introduction and commentary. Occasionally these have been altered.

2. Excerpts from this treatise appear in Selections 25 and 51.2.

3. See pages 76–78 of my edition of Oresme's *De proportionibus proportionum* (for full title, see Selection 25, n. 1).

4. *Ibid.*, pp. 121–122; the page numbers given in parentheses are references to my edition.

There is reason to believe that Oresme intended to add a revised version of the *Ad pauca respicientes* to the *De proportionibus* as an illustration of one important application of incommensurability. In the process of revision, however, Oresme probably became dissatisfied with its organization and content and subsequently reworked and enlarged the propositions and format so thoroughly that a wholly new treatise emerged, which he called *Treatise on the Commensurability or Incommensurability of the Celestial Motions (De commensurabilitate vel incommensurabilitate motuum celi)*.[5] Now unsuitable as an addition to the *De proportionibus proportionum*, it was issued as a separate and rather lengthy treatise.

The *De commensurabilitate vel incommensurabilitate motuum celi* consists of a prologue and three parts. After the presentation of a few preliminary definitions and assumptions, the remainder of Part I is devoted to 25 propositions in which bodies, or mobiles, are assumed to move with commensurable speeds on concentric circles (in Proposition 20 the circles are eccentric). Part II contains 12 propositions in each of which at least two of the motions are incommensurable. In these two parts, Oresme's objective is to derive various consequences from the motions of two or more bodies whose speeds are first assumed to be commensurable and then incommensurable. In most propositions, these kinematic consequences are applied arbitrarily to astronomical aspects—conjunction is used as the paradigm case—to determine the times in which, and places where, these can occur in terms of the specific data and assumptions. Oresme concludes the treatise by presenting Part III in the form of a debate presided over by Apollo and involving, as antagonists, personifications of Arithmetic and Geometry, who argue whether the celestial motions are commensurable (Arithmetic) or incommensurable (Geometry).[6]

The setting of the debate is a dream in which Apollo and his Muses appear before Oresme. Following the debate, and just as Apollo is on the verge of rendering a judgment as to whether the celestial motions are commensurable or not, Oresme awakens and the conclusion is left in doubt. The attentive reader, however, would have had good reason to suppose that Oresme really considered that the celestial motions were probably incommensurable, for it was only this position that enabled him to discredit exact astrological prediction with mathematical certainty (see Part III of this selection and notes 80 and 84).

Although Oresme had predecessors who discussed the incommensurability of celestial motions and some of its consequences, no one is yet known who gave it such full and subtle mathematical development, nor who exerted as great an influence on subsequent discussants (for instance, Henry of Hesse, Marsilius of Inghen, Pierre d'Ailly, and probably Jerome Cardano).

PART I

. . . . . .

I take the commensurability and incommensurability of circular motions in terms of the magnitude of the angles described around the center or centers, or in terms of the circulations, which is the same thing. Thus, things are moved commensurably when, in equal times, they describe commensurable angles around the center, or when they complete their circulations in commensurable times. Circulations are incommensurable when they are completed in incommensurable times, and when, in equal times, incommensurable angles are described around the center. Accordingly, conjunctions, oppositions, aspects, and all the motions ascribed to the heavens by astronomers are to be measured in this way, since a ratio of velocities varies as a ratio of the circular lines described by the mobiles. Whether or not it ought to be taken in this way is not relevant to what is proposed here. Moreover, incommensurability can be found in every kind of continuous thing, and in all instances in which continuity is imaginable, either extensively or intensively. For a magnitude can be incommensurable to a magnitude, an angle to an angle, a motion to a motion, a speed to a speed, a time to a time, a ratio to a ratio, a degree to a degree, and a voice to a voice, and so on for any similar things.

The return of *one mobile* along a circular path from any point to that same point, I call a *circulation*. The return of *several mobiles* from any state to a wholly similar state, or aspect, I call a *revolution*.[7]

5. *Ibid.*, pp. 79–80.

6. *Nicole Oresme and the Kinematics of Circular Motion*, p. 6 ("Summary and Analysis of the *De Commensurabilitate*") of my edition of Oresme's *De commensurabilitate*.

7. A *circulation* of a single body is analogous to a sidereal period in astronomy, but a *revolution* has no direct counterpart in astronomy, since it requires that the bodies in motion, or mobiles, return to the very same point or configuration taken as absolutely fixed in space. Thus a revolution is more like a Great or Perfect Year in ancient and medieval astrology, a concept in which all planetary bodies return simultaneously to the same celestial configuration from which they started, an event which was thought to occur at regular intervals. In the *De commensurabilitate*, Oresme even includes as instances of a Great Year the return of any two planets to the same point, and even includes the case of a single celestial body with two or more simultaneous motions, if, in equal time intervals, these motions always brought it back to the very same point from which it started (see Part I, Proposition 22). The Great Year was mentioned by Plato (*Timaeus* 39D) and other ancient authors. See also selection 51.2, n. 86.

In this little book, my purpose is to consider *exact* and *punctual aspects* of mobiles that are moved circularly. I do not, however, propose to deal with aspects near a point which is usually the intention of astronomers, who care only that there be no sensible discrepancy—even though a minute, undetectable error would produce a perceptible discrepancy when multiplied over a long [period of] time.

. . . . . .

Proposition 4. If two mobiles are now in conjunction, it is necessary that they conjunct in the same point at other times.

In this part of the book the discussion is always about things moved commensurably. And so let A and B be in conjunction in point g, since, as far as what is considered here, a conjunction in a point is the same as on a line or surface. Since the motions are commensurable, it follows by the fifth [proposition] of the tenth [book] of Euclid[8] that a ratio of these two motions is as a ratio of two numbers. Therefore, let A's motion be as number C, and B's motion as number D. Then, in the same time in which A makes C circulations, B makes D circulations, so that at the end of this time A and B will have completed an exact number of circulations and each of them will be where it is now. Consequently, they will be in conjunction in the point in which they are now.

For example, let A's velocity be as 5 and B's as 3. Then, when A will have made 5 circulations, B will have made 3 and they will be where they are now. A similar argument could be made for past time, namely, that they had conjuncted at other times.

Proposition 8. [How] to determine the place of the first conjunction following the present conjunction of these two mobiles.

Since, by the sixth proposition,[9] it has already been shown [how to find] the time required for such a conjunction, there only remains to be seen how great a distance either of these [two] mobiles traverses in that [very same] time, and then to subtract the whole circle from that distance as many times as possible. This is the same as dividing the whole circle by that distance so that what has been proposed will be found.

For example, from the velocity given in the case presented before,[10] it is obvious that in one day B traverses $\frac{3}{5}$ of its circle. But since by the sixth proposition, $7\frac{1}{2}$ days represents the time elapsed before the first conjunction following [the present conjunction], it is evident that during that time B traverses $\frac{3}{5}$ of a circle plus $\frac{1}{2}$ of $\frac{3}{5}$ of a circle[11]—that is, the whole circle plus half of it. Consequently, the [very next] conjunction following will occur in a point opposite the point in which A and B are now in conjunction. The same result will be obtained by operating with the motion of A.

Proposition 12. If there were more than two mobiles, it is possible that no more than two will ever be in conjunction at the same time.

Let A, B, and C be three mobiles. By the preceding propositions,[12] A and B can conjunct in only a fixed number of points, or places, and nowhere else; let any such point be called d. Similarly, B and C conjunct in a certain number of fixed points, and nowhere else; let any such point be called e. Then, if no d is an e, which is possible, these three mobiles will never be in conjunction at the same time.

8. "Any two commensurable magnitudes have a ratio of a number to a number" *(Omnium duarum quantitatum communicantium est proportio tanquam numeri ad numerum)*. See p. 247 of the edition by Campanus of Novara cited in Selection 27, n. 1.

9. In Part I, Proposition 6, Oresme shows "[How] to find the time of the first conjunction that will follow a [present] conjunction of two mobiles whose velocities have been given." In this proposition, two bodies are assumed to be in conjunction at the outset and the time that will elapse before their very next point of conjunction is then determined.

10. The data are drawn from Part I, Proposition 6, where body B traverses 1/5 of its circle per day and body A traverses 1/3 of its circle per day. To determine the place of their very next conjunction, one can work with the daily distance traversed by either B or A; Oresme chooses the former.

11. That is, in 7 days B will traverse 7/5 of its circle and in the remaining 1/2 day it will traverse $(1/2) \cdot (1/5) = 1/10$ of its circle. Adding, $(7/5) + (1/10) = 3/2$. Thus in $7\frac{1}{2}$ days, B will have traversed its circle $1\frac{1}{2}$ times and meet A in conjunction in a point diametrically opposite their first conjunction. That A will also be in the same point in $7\frac{1}{2}$ days is shown by multiplying $7\frac{1}{2} \cdot \frac{1}{3} = 2\frac{1}{2}$. Thus, while B has traversed its circle $1\frac{1}{2}$ times, A has traversed its circle $2\frac{1}{2}$ times, and they will conjunct.

12. Propositions 7, 8, 10, and 11 are relevant here.

In a like manner, if there were four or five, or any number of mobiles, it can happen that only two will ever conjunct at the same time since it is possible that when the mobiles are taken two at a time, they do not share any common places of conjunction; but it is also possible that they could share some common points, and that [for example], of six mobiles there might be simultaneous conjunctions of three, or four and no more, or five, or all; and the same reasoning applies to other sets of mobiles.

For example, assume that A and B are in conjunction in point d [Fig. 1], and mobile C precedes

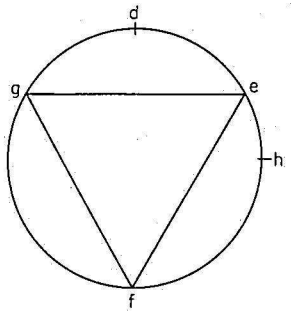


Fig. 1

them by $\frac{1}{8}$ part of the circle—i.e., by $1\frac{1}{2}$ [zodiacal] signs. And let the velocity of A be as 4, that of B as 2, and C as 1; let point e be separated from point d by $\frac{1}{4}$ part of the circle—i.e., by two signs—and let f and g be two other points such that e, f, and g divide the circle into three equal parts. [Finally] assume that h is another point separated from point d by $\frac{1}{4}$ part of the circle—i.e., by three signs.

Having now set out the data, we shall demonstrate, by [means of] the ninth proposition,[13] that A and C will conjunct first in point e;[14] and by the same [ninth] proposition it will be obvious that B and C will conjunct first in point h;[15] and by the tenth, or even eleventh, proposition it will be shown that A and B will conjunct only in point d,[16] and, by the same proposition, that B and C will conjunct only in point h. It follows immediately from this that A, B, and C will never conjunct simultaneously. Indeed, by the same [tenth] proposition it will be seen that A and C conjunct only in the other three points e, f, and g, from which it could again be argued that A, B, and C will never conjunct simultaneously. When more mobiles are involved it can happen that 3, or 4, or any number of them can be in conjunction simultaneously. How this happens will be considered later.

Proposition 22. [How] propositions [that are] similar [to those demonstrated previously for two or more distinct mobiles] can be applied to one and the same mobile moving with several [simultaneous] motions.

For example, let it be assumed that the sun has only two motions, namely, proper and diurnal; and on the ninth sphere, let A be the first point of Cancer, which describes the same circle every day, namely, the summer tropic; and let B be the center of the sun describing the ecliptic in one year with a proper motion [Fig. 2]. [Finally] assume that

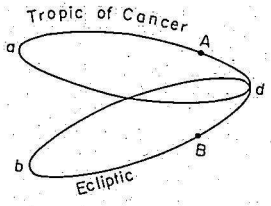


Fig. 2

A and B are now in point d, which is imagined as motionless. Now on the assumption that these motions are commensurable, the ratio of the motions will be rational. If this ratio were multiple, it follows that A and B will never be together simultaneously except in point d. This can be made clear at once by an example, for if A should make 100 circulations in the same time that it takes B to make one circulation, then at the termination of

13. Part I, Proposition 9, reads: "[How] to determine the place and time of the first conjunction following [a given conjunction] when a distance between the two mobiles has been assigned."

14. From the data, $V_a/V_c = 4/1$, where V is a scalar velocity. Initially A and B are in conjunction in d, and C precedes them by 1/8 part of a circle or $1\frac{1}{2}$ zodiacal signs. Now, since $V_a/V_c = 4/1$, as A moves 2 signs from d to e, C, which precedes A by $1\frac{1}{2}$ signs, will move $\frac{1}{4}$ sign and conjunct with A in e. Indeed, it is evident that A and C can conjunct only in points e, f, and g.

15. Since C precedes B by $1\frac{1}{2}$ signs and $V_B/V_C = 2/1$, it follows that C and B will conjunct in h, which is separated from d, and hence from B, by 3 signs and from C by $1\frac{1}{2}$ signs. Thereafter, B and C will conjunct only in h.

16. Since $V_A/V_B = 2/1$ and A and B are initially in conjunction in d, it follows that A and B will conjunct only in d.

these 100 circulations—and not before—A and B will be in point d; and this will always occur every 100 days.

But if the ratio of the motions is not multiple, a fraction must be assumed in its denomination. Furthermore, the denominator of the fraction indicates the number of points in which A and B can meet. Thus, if A makes $100\frac{1}{3}$ circulations in the same time as B makes one circulation, it follows that they would be found in a point opposite d, the first point; and after the next sequence [of circulations], they would be found in d [itself]. [In this example] there are only two fixed places where A and B could meet. But B could meet A in exactly three fixed points, if B makes one circulation when A makes $100\frac{1}{3}$; and if [A makes] $100\frac{1}{3}$ [circulations], there would be four [points of contact], and so on. Also, if A should complete $100\frac{1}{3}$ circulations while B makes one circulation, there would be five places where they could meet, since in that time A would traverse its circle a certain number of times plus $\frac{1}{3}$ of it, and during another [equal] time [period] would again make just as many circulations plus $\frac{1}{3}$ [of one]; and this applies to all [such cases].[17]

The number and arrangement of the celestial points on which the sun could enter the first point of Cancer—as well as other degrees of the zodiac—become apparent from all this. Thus if the sun were moved with only two motions and the solar year were measured by an exact number of integral days, it is obvious that the sun could enter [the first point of] Cancer only when it coincided with one [particular] meridian; and after another [equal interval of] time it would again [enter Cancer] on the same meridian, and never on another. But if the year contained a certain number of days exactly plus some $\frac{1}{3}$ part, or parts, of a day—as, for example, $365\frac{1}{3}$ days—there would be only four equidistant points on this tropical circle on which the sun could enter the first point of Cancer; and the same applies to other degrees [of the zodiac].

If, however, three motions were involved, then, by the method already outlined, they will be taken two at a time and their combinations mutually compared in quite the same way as in our previous discussion of a conjunction of three mobiles;[18] the same procedure would be followed for four or more motions.

From all this, it follows that whatever the number of motions involved—provided they are all mutually commensurable—there is always, for any given disposition, a certain number of fixed places—and no more—in which that mobile can be related in the same way. And when it has proceeded through all these places maintaining this same relationship, it will begin, once again, to run through them exactly as before.

17. To explain all this refer to Fig. 2, where circle a is the tropic of Cancer (summer tropic) and A is the first point of Cancer. Let A_n, where n can be any integer, represent any point on circle a. Circle b is the ecliptic on which the center of the sun, B, moves with a uniform motion and completes one circulation in a year. Finally, d is the only point of contact between circles a and b. Therefore, any point, A_n, will become the first point of Cancer when it is in d simultaneously with B, the center of the sun.

Initially, assume that B and some point on the tropic of Cancer, say A_1, are in d simultaneously. Let V_a and V_b represent the commensurable velocities of circles a and b, respectively. Now if $V_a/V_b = n/1$, where n is an integer, then, since A_1 is carried around by circle a, and B by circle b, it follows that when a makes n circulations and b one circulation, A_1 and B will meet again in d. Thus, in Oresme's first example, whenever circle a completes 100 circulations while b completes one circulation, A_1 and B will meet in d.

But if the ratio of velocities of the circles is not exactly a multiple ratio, but includes some fractional part of a circulation, the denominator of the fraction will indicate the exact number of points of circle a that B could meet in d. Thus, if $V_a/V_b = P\frac{m}{n}/1$, n will represent the total number of points on circle a that will meet B in d. In the mixed fraction, P is the integral number of circulations that circle a will complete in some given time and m/n represents an additional fractional part of a circulation. Since the n points on circle a will divide it into equal parts, it is only necessary to determine the distance that separates two successive meetings in d in order to arrange the sequence of the points on circle a through which the sun can enter the first point of Cancer. For example, should circle a complete $100\frac{1}{3}$ circulations to one for b, there would be four equidistant points on a that would meet B in d at equal intervals. If A_1 and B were initially in d, then, after $100\frac{1}{3}$ circulations, A_2 will meet B in d; after the next $100\frac{1}{3}$ circulations of a, A_3 will meet B in d; and finally, A_4 will arrive in d simultaneously with B after the next $100\frac{1}{3}$ circulations of circle a. This will be followed, once again, by a meeting of A_1 and B in d, after which the cycle will repeat ad infinitum.

If, however, circle a completes $100\frac{1}{3}$ circulations to every one for circle b, there will be five equidistant points on a that will meet B in d, but the order of the points meeting B will not be A_1, A_2, A_3, A_4, and A_5, where each point is separated from its immediate neighbor by 1/5 of the circle. For if at the beginning, A_1 and B are together in d, after the next $100\frac{1}{3}$ circulations of circle a, point A_3—not A_2—will meet B in d; and after the next series of circulations A_5 will meet B; then A_2, and finally, A_4. The order of points which meet B in d is, therefore, A_1, A_3, A_5, A_2, and A_4, after which the cycle is repeated ad infinitum.

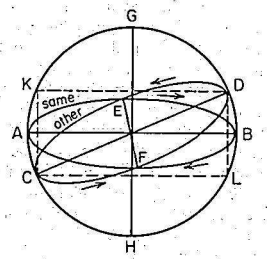
18. As, for example, in Part I, Proposition 12.

For example, let *d* be one of only four fixed points on which the sun can enter [the first point of] Cancer. I say that if the sun begins from point *d* and moves continuously for four years describing daily a new spiral until it returns to point *d* at the end of the fourth year, it would then resume its course from the beginning; and so on, endlessly. In the same manner, this would also happen if there were more points of this kind, but then such a revolution would be completed in a greater time. The same thing could be said about the moon and any of the other planets.

Therefore, if all the motions by which the sun is moved were commensurable, the sun's center, moving within a space imagined as immobile, would describe a finite path, or line, that is like a circular line without terminating points. And yet it does not describe a circular path. Indeed, its path is composed of many circular-like or spiral lines that are [interlaced or] interwoven in a certain way. Some of these lines begin toward the tropic of Cancer and proceed in the direction of the tropic of Capricorn, which is the path the sun follows if beginning its motion there; and at other times the sun returns in the opposite direction describing lines that intersect those made previously—as if weaving a pattern.[19] Also the number of circular-like lines, or spirals, between the two tropics equals the number of days that it takes the sun to depart from and return to the point which is imagined as fixed in the heavens. The [end] product of such an interweaving between the two tropics is similar to a quadrangle, oblong, or rhombus.[20] But throughout all eternity, the center of the sun never was, nor will ever be, inside the area or inner space of these [figures]. Furthermore, once this path has been traversed by the sun in its period, it begins again to move over the same path as before. The same thing could be said of the moon and the other planets.

Thus, any mobile taken independently, but having several motions, has a fixed period upon the completion of which it begins again; and this goes on an infinite number of times. This period can be called the Great Year of that mobile,[21] just as, in a similar way, any two celestial mobiles that might be taken complete their courses simultaneously in a fixed period of time, after which they begin over again as before; and this can be said about three mobiles, or [indeed] any number of them. These mobiles can also be said to have a Great Year, just as some say that 36,000 solar years constitute a Great Year of the sun and the eighth sphere. But a Great Year of all the planets and the eighth sphere is very much longer.[22] In brief, if all the celestial motions are mutually commensurable,

19. As the sun moves between the two tropics or solstices with two simultaneous motions, diurnal and annual, it traces out a spiral line which results from the fact that its two motions are oppositely directed. For each day of the year there is a corresponding spiral line, and at the end of every year a pattern is traced which is similar to a fabric woven with criss-crossing lines. Oresme may have read about this pattern of spiral lines in Chalcidius' Latin translation of Plato's *Timaeus*, where in 39A, B, Plato discusses the simultaneous motions of the circle of the Same (that is, the celestial equator which rotates daily from east to west carrying all the planets with it) and the circle of the Different or Other (that is, the ecliptic, or more properly, the zodiacal band, on which each planet moves in its period of revolution from west to east). The generation of Plato's spiral is summarized by Sir Thomas Heath as follows (*Aristarchus of Samos* [Oxford, Clarendon Press, 1913], p. 169; the figure appears on p.160.):



Suppose a planet to be at a certain moment at the point F. It is carried by the motion of the Same about the axis *GH*, round the circle *FAEB*. At the same time it has its own motion along the circle *FDEC* [i.e. the ecliptic]. After 24 hours accordingly it is not at the point *F* on the latter circle, but at a point some way from *F* on the arc *FD*. Similarly after the next 24 hours, it is at a point on *FD* further from *F*; and so on. Hence its complete motion is not in a circle on the sphere about *GH* as diameter but in a spiral described on it. After the planet has reached the point on the zodiac (as *D*) furthest from the equator it begins to approach the equator again, then crosses it, and then gets further away from it on the other side, until it reaches the point on the zodiac furthest from the equator on that side (as *C*). Consequently the spiral is included between the two small circles of the sphere which have *KD*, *CL* as diameters."

20. In the figure in note19, we can produce a right angled quadrilateral figure if we join lines *KD* (the diameter of the tropic of Cancer) and *CL* (the diameter of the tropic of Capricorn) by joining the points *K* and *C* and *D* and *L* by straight lines.

21. See note 7.

22. See note 7.

it is necessary that there be one maximum [or perfect] period for all of them at the same time, [a period] which is repeated after its termination. But if the world were eternal, the perfect period [or Great Year], although it would always be similar, would not be repeated in the same place, but, rather, in infinite [different] places.[23]

PART II

In the second part of this work, it is assumed that some of the celestial motions are incommensurable. We must now show what follows from that assumption.

Proposition 1. If two mobiles have moved with incommensurable velocities, and are now in conjunction, they will never conjunct in that same point at other times.

Assume that *A* and *B* are in point *d*. If, at some later time, they should meet in point *d*, it follows that during that time each will have made a certain number of exact circulations. Let *E* be the number of circulations that will have been made by *A*, and *G* the number that will have been made by *B*. Since a ratio of velocities varies as the ratio of the distances traversed [by these velocities] during the same time, it follows directly that the speeds of these mobiles are related as the two numbers *E* and *G*. Therefore, the velocities are commensurable, which is contrary to what was assumed.

For example, if *A* and *B* are now in point *d*, and at some later time they are again in conjunction in the same point, assume that *A* will have made 5 circulations and *B* will have made 3. It is obvious, then, that their velocities are related as the numbers 5 and 3, and that *A* is moved more quickly than *B* by a superpartient two-thirds ratio.

And one can show in the same manner that they were never before in conjunction in point *d*. All this is applicable, in just the same way, to any other aspect. Thus if *A* is in point *d*, and B is in the point opposite [to *d*], they will never be in opposition at other times in these very same points. This could be demonstrated in the same manner as before. Hence, the [period of] revolution of these mobiles is infinite; indeed, it might be truer to say that there is no period of revolution at all.

Proposition 2. If two mobiles are now in conjunction, they will never conjunct at other times in any point separated from their present point of conjunction by a part of the circle commensurable to the whole circle.

From the eighth and ninth [propositions] of the tenth [book] of Euclid, it follows at once that if something is added to each of two commensurable quantities, and is commensurable to each of them, the wholes will be commensurable. Therefore, if to each of two sets of circulations—there may be any number of circulations in each set—there is added a part of a circulation commensurable to a whole circulation, the wholes will be commensurable. Now if *A* and *B* are in conjunction in some point separated from point *d*, then, in the time that has intervened [since their conjunction in *d*], each will have completed an exact number of circulations plus some part of a circulation commensurable to a whole circulation.[24] Their motions must, therefore, be commensurable, since the distances traversed in the same period of time would be commensurable.

For example, if *A* and *B* are in conjunction in some point that is separated from point *d* by $\frac{1}{4}$ part of the circle, and if *A* had by then completed $5\frac{1}{4}$ circulations and *B*, $3\frac{1}{4}$ circulations, the ratio of the motions of *A* and *B* would be as $5\frac{1}{4}$ to $3\frac{1}{4}$—i.e., as 21 to 13, which is a ratio of $1\frac{8}{13}$. The same may be said for any other aspect. Thus if *A* is presently in point *d* and *B* is in the point opposite *d*, then, by taking a part of the circle commensurable to the whole circle, one can demonstrate by the same method that they will never again be in opposition in points that are separated from their present points of opposition by distances that are commensurable to the whole circle. Any other disposition can be treated in this manner. And the same argument applies to the past as well as the future. One may, therefore, conclude that the distance, or sector of a circle, that separates two successive conjunctions is incommensurable to the whole circle; and this may be said about any other aspect. And similarly can the time between

23. If the world were eternal without beginning, we would be ignorant of the unique point or configuration from which all the motions commenced and to which a Great Year could be uniquely referred. Without such knowledge, an infinite number of points or configurations would be equally privileged to serve as reference points.

24. That is, each of the bodies, *A* and *B*, will have completed an integral number of circulations with respect to *d* plus the same angular part of the circle. If that angular part of the circle is commensurable to the whole circle, it follows that the distances traversed by *A* and *B*, as expressed in whole numbers of circulations plus the commensurable fractional part, will be commensurable. Since the speeds are proportional to the distances traversed, the velocities will be commensurable, which is contrary to the assumption that they are incommensurable.

two such conjunctions be related to the time it takes each mobile to complete one circulation; and these times of circulation are likewise mutually incommensurable.

Proposition 7. It is possible that three or more mobiles whose motions are mutually incommensurable are now in conjunction but can never conjunct in another place. Therefore, in one case of incommensurability it follows that the mobiles can be in conjunction an infinite number of times, but in another case this does not follow.[25]

As before, let *A*, *B*, and *C* be in point *d*. Let us now reconsider something that was demonstrated in the second proposition of the first part—namely, that if a continuum were divided into infinity by one rational [geometric] proportionality, it could also be imagined as divisible by [yet] another rational [geometric] proportionality; and these proportionalities are incommunicant [or prime to each other].[26] If this continuum were a circle, then, except for one point only, no point in one division is a point in the other division. However, if the proportionalities were communicant, there would be points in common between the two divisions.

The same distinctions apply to divisions made by proportionalities consisting of irrational ratios. Thus if one proportionality is based on half of a double ratio [i.e., $(2/1)^{1/2}$], and the other is based on half of an octuple ratio [i.e., $(8/1)^{1/2}$], these are communicant; but if the proportionalities are based on half of a double ratio and half of a triple ratio [i.e., $(3/1)^{1/2}$], they are incommunicant [or prime to each other].[27] Hence it is possible for the continuum to be divided by points that are separated by incommensurable distances, so that no point in one division is also a point in the other division—except, perhaps, one point. If the continuum is circular, two such divisions are possible in a circle. Now since these two series of infinite numbers will begin in sequence from point *d* and they are separated from each other by incommensurable distances, the two series will not meet in [or share] any point. It is possible, then, that [beginning] from point *d*, all the other infinite number of points in which *A* and *B* will conjunct are incommunicant with those in which *B* and *C* will conjunct, so that *B* and *C* will never conjunct in the same places where *A* and *B* conjunct. Therefore, *A*, *B*, and *C* will never conjunct at other times.

For example, assume that *A* and *B*, which are now in *d*, are afterward in conjunction in *e*, then in *f*, and so on [Fig. 3]. Then, because of the

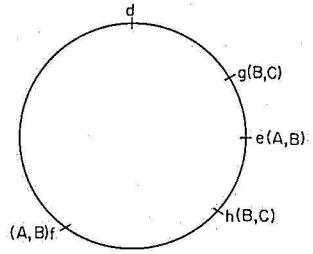


Fig. 3

25. In Part II, Proposition 6, Oresme had shown that three mobiles whose motions were respectively uniform but mutually incommensurable could conjunct again in other points. Indeed they could conjunct an infinite number of times but never twice in the same place. In Proposition 7, however, Oresme establishes conditions which would deny the results of Proposition 6. For now, three mobiles in conjunction will never again conjunct elsewhere.

26. In Part I, Proposition 2, it was demonstrated that if, for example, a continuum were divided into 2^n successive parts (where n is 1, 2, 3, 4, . . .) and then into 3^n successive parts, these two geometric proportionalities would not divide the continuum into parts which had common points of division (except for 1, which would represent the whole continuum before any divisions were made). This follows from the fact that the numbers following unity in each proportionality are mutually prime to each other (that is, 2 and 3). Where this is not the case, there will be points in common. Oresme asserts that statements true of rectilinear continua apply also to circular continua, the only difference being that where a single point can divide a rectilinear continuum, two points are required for division of a circle. In Part II, Proposition I, Oresme considered only rational proportionalities, but he now concentrates on irrational proportionalities.

27. They are "communicant" or commensurable because $(2/1)^{1/2}$ and $(8/1)^{1/2}$ can be related by a rational exponent—that is, $(8/1)^{1/2}=[(2/1)^{1/2}]^{3/1}$. Hence these two irrational ratios constitute a "rational ratio of ratios." But the irrational ratios $(2/1)^{1/2}$ and $(3/1)^{1/2}$ form an "irrational ratio of ratios" since no rational exponent can relate them—that is, $(3/1)^{1/2}\neq[(2/1)^{1/2}]^{p/q}$, where p/q is a rational ratio. These distinctions were elaborated by Oresme in his earlier work, *On Ratios of Ratios (De proportionibus proportionum)*, especially Part I, Proposition 9 (see pp. 200–204 of my edition; also see Selection 51.2, n. 51).

If a circle were divided from the same point by the "commensurable" proportionalities $(2/1)^{1/2}$ and $(8/1)^{1/2}$, it is clear that all points represented by $[(2/1)^{1/2})]^{3n}$ and $[(8/1)^{1/2}]^n$, where $n = 1, 2, 3, 4, \ldots$, are common to both proportionalities, since for any value of n both ratios would be equal and represent the very same point. Thus when $n = 2$, both ratios equal 8/1.

regularity of the motions, points *d*, *e*, *f*, and so on, are separated by equal distances. Let the whole circle be related to arc *de* as half of a sesquialterate ratio.[28] Also, assume that *B* and *C*, which are in point *d*, are afterward in conjunction in *g*, then in *h*, and so on. [Finally] let the whole circle be related to arc *dg* as half of a sesquitertian ratio [namely, as $(4/3)^{1/2}$].[29]

Now since half of a sesquialterate and half of a sesquitertian are incommensurable ratios, as is shown in the *De proportionibus proportionum*, it follows that the aforementioned divisions are incommunicant. Thus, *A*, *B*, and *C* will never conjunct at other times; this applies also to the past and to more [than three] mobiles.[30]

It should be noted, however, that the circle ought not to be divided by such points in accordance with a ratio of velocities, or any ratio commensurable to it. This is immediately obvious in rational ratios, for if *A* were moved four times quicker than *B*, the points of conjunction would divide the circle into three, not four, parts; and the whole circle would be triple any one of these parts. This is shown by the eleventh proposition of the first part.

Proposition 11. . . . On the assumption of the incommensurability and eternity of motions, it is truly beautiful to contemplate how such a configuration as an exact conjunction occurs only once through all of infinite time, and how it was necessary through an eternal future that it occur in this [very] instant with no conjunction like it preceding or following. One cannot even find a reason as to why it happens at that time [rather] than at another time,[31] unless it be because the velocities of motions and the unalterable inclinations of moving bodies are [simply that way].

And if [celestial] configurations are the continuous causes of effects in the lower regions, then, whenever extraordinary aspects concern a whole species, there would occur such a disposition that never again will there be one like it in this world. Speaking naturally, it does not seem inconceivable that a great conjunction of the planets, different from anything that happened before, could produce some individual unlike any other which would begin as a new and previously unseen species in either substance or accident, just as Pliny, in the twenty-sixth book [of his *Natural History*], says that with regard to sickness "the face of man has been afflicted with new and unknown diseases in every age."[32] And, if the world were eternal, perhaps it is even possible that once such a species has come into being, it would never cease to exist; or it might at some time cease to exist because of the power of another configuration.[33] Similar things may be said about similar corollaries that are deducible from what has been said.

Proposition 12. Propositions enunciated previously [for two or more distinct mobiles] will now be applied to [one and] the same mobile moving with several [simultaneous] motions.

Now just as in the twenty-second proposition of the first part of this work, the motions discussed were commensurable, so in this proposition they are to be taken as incommensurable. For example, assuming that the sun is moved with only two incommensurable and perpetual motions—namely, proper and diurnal—let *A* be the first point of Cancer which daily describes one and the same circle, namely the summer tropic; and let *B* be the center of the sun's body which describes the ecliptic in one year.[34] And [finally] let *A* and *B* be in point *d* imagined motionless in space—i.e., *A* and *B* are in conjunction there.

I say, then, that *A* and *B* will never be simultaneously in point *d* at other times, and this can be demonstrated exactly as was the first proposition [of this part]. Thus, if the sun should enter the first point of Cancer on some [particular] meridian, it will never enter, nor has it ever entered, this sign

28. That is, circumference/arc $de = (3/2)^{1/2}/1$.

29. That is, circumference/arc $dg = (4/3)^{1/2}/1$.

30. That *A*, *B*, and *C* will never conjunct anywhere else except *d* follows from the fact that irrational ratios $(3/2)^{1/2}$ and $(4/3)^{1/2}$ would not share any common points on the circle except *d*, which they share as a common starting point. The motions of *A* and *B* are such that after leaving *d* their points of conjunction will always differ from those of *B* and *C*. For the circumference of the circle is related to the arcs separating the successive conjunctions of *A* and *B* in the ratio of $(3/2)^{1/2}/1$; and the circumference of the circle is related to the arcs separating the successive conjunctions of *B* and *C* as $(4/3)^{1/2}/1$. In effect, arcs *de* and *dg* are incommensurable, so that when *B* conjuncts with *A* it cannot simultaneously conjunct with *C*.

31. Oresme raised this same question in his *Ad pauca respicientes*, Part II, Proposition 18 (see p. 425 of my edition cited in Selection 25, n. 1); *Questiones super De celo*, Book I, Question 24 (see pp. 419–420 of Claudia Kren's Ph.D. dissertation "The 'Questiones super De celo' of Nicole Oresme," University of Wisconsin, 1965); and his *Le Livre du ciel et du monde*, Book I, chapter 34, 57a–57c (on p. 243 of Albert Menut's translation [Madison, Wis.: University of Wisconsin Press, 1968]).

32. Pliny, *Natural History* XXVI, 1, 1 in the translation of W. H. S. Jones (Loeb Classical Library).

33. This is also discussed in the three other Oresme treatises cited in note 31.

34. Figure 2 of Part I, Proposition 22, illustrates this.

at other times when on this same meridian; nor, indeed, as shown in the second proposition of this part, will it enter when it is on any meridian separated from this one by a distance that is commensurable [to the whole circle of the summer tropic]. There is also an infinite set of points scattered everywhere on circle *a* in each of which *B* has entered Cancer; and there is [also] another infinite set of points in each of which *B* will enter this same sign in the future. Then, as was the case for conjunctions in the fourth proposition of this part, no part [or arc] of this circle is so small that it does not contain some meridian on which *B* appeared [in the past] when it was in the first point of Cancer; nor can it be so small that it does not contain some meridian on which *B* will appear in the first point of Cancer sometime [in the future].[35] One should understand that what has been said about the first point of Cancer applies also to any point of the zodiac.

It follows from all this that *B* describes daily a new spiral—imagined as motionless in space—which it has never before described; and it traverses a path which it has never before traversed. And so, by its track, or imagined flow, *B* seems to extend a spiral line that has already become infinite [in length] from the infinite spirals that were described in the past. This infinite spiral line is constituted from the spirals that were described between the two tropics, and from the intersections between the earlier and later spirals. Consequently, through an eternal time, *B* appears twice in any point of intersection [between these spiral lines][36]—and only once, or never, in any of the other points.[37]

In accordance with what has been imagined here, the whole celestial space between the two tropics is traced by *B*, leaving behind a web-or net-like figure expanded through the whole of this space. The structure of this figure was already infinitely dense through the course of an infinite past time, and yet, nonetheless, it will be made continually more dense, since it produces a new spiral every day. Throughout the extent of such a space, *B* has appeared between any two assignable points and yet, despite this, in the very same space [between any two such points] there are an infinite number of points in which *B* never was, nor ever will be. There is also another infinite [set of points] in each point of which *B* enters only once through all eternity; and there is yet another infinite set of points through each of which *B* passes twice during the course of an infinite time; but there is no point through which it could pass more than twice.

. . . . . .

Furthermore, from the incommensurability mentioned above, it might be [the case] that the mean solar year contained a certain number of days and a part of a day incommensurable to a whole day. If this were assumed [to be true], the precise length of the year could not possibly be expressed in numbers, nor could a perpetual almanac be established, or a true calendar found.[38]

35. The enunciation of Part II, Proposition 4, reads: "No sector of a circle is so small that two such mobiles could not conjunct in it at some future time, and could not have conjuncted in it sometime [in the past]." This proposition, which was concerned with two bodies moving with incommensurable speeds, is now applied to the motion of a single body, the sun, moving with two simultaneous, but incommensurable, motions.

36. In this paragraph Oresme turns, once again, to the "Platonic spiral." In Part I, Proposition 22, the simultaneous motions of the sun were assumed commensurable and the spiral line traced by its path was of finite length (though not terminating in any one point), which the sun perpetually retraced as it moved between the tropics (see n. 19). But with its simultaneous motions assumed incommensurable, the sun's path would be an infinite spiral having neither beginning nor end, for it already has described a new spiral every day through an infinite past and will describe a new daily spiral through an infinite future. Although Oresme does not elaborate, it is clear that the spiral must be infinite, since the sun has always entered the tropic of Cancer at a different point and then spiraled down toward the tropic of Capricorn (the formation of the spiral is explained in n. 19), which it always entered, and always will enter, at a different point. It then spirals upward once again toward the tropic of Cancer entering at a new point. Under these circumstances and assuming an infinite past, as Oresme does (of course, as a Christian, he believed that the world had a temporal beginning), the spiral will be of infinite length without beginning or end. Furthermore, since the sun never retraces a previously described spiral, its path can intersect any other spiral line at only one point, and hence through all eternity the sun will have been in every one of these points of intersection twice, but no more.

37. The class of points through which *B*, the sun, passes only once includes all points on the infinite spiral line which it has traced, exclusive of points of intersection. Points through which it never passes are presumably points on either tropical circle that cannot serve as the first point of Cancer or Capricorn. For example, by Part II, Proposition 2, the sun will never enter Cancer on any point that is separated from one which it previously entered by an angular distance that is commensurable to the whole circle.

38. The same opinions were expressed by Oresme in his *De proportionibus proportionum* (see Selection 51.2), *Ad pauca respicientes*, Part II, Proposition 16 (see pp. 420–422, ll. 192–197, of my edition cited in full in Selection 25, n. 1), and his *Questions on the Geometry of Euclid (Quaestiones super geometriam Euclidis)* (edited by H. L. L. Busard [Leiden: Brill, 1961], p. 25, ll. 5–10).

PART III

. . . . . .

[*The Oration of Geometry*][39]

. . . . . .

Furthermore, those who object that man would be ignorant [of celestial events if these motions were incommensurable] are unconvincing. For it is enough to know beforehand that a future conjunction, or eclipse, of this mobile falls below a certain degree, minute, second, or third; nor is it necessary to predict the exact point or instant of time [in which these will occur], since, as Pliny says, "the measure of the heavens is not reducible to inches."[40] And according to Ptolemy, we are unable to determine the exact truth in such matters.[41] In these matters, then, anyone who announces results that are free of noticeable error would seem to have determined things adequately and beautifully. But if men knew all motions exactly, it would not be necessary to make further observations, or to record the celestial revolutions with attentive care. As far as such excellent things are concerned, it would be better that something should always be known about them, while, at the same time, something should always remain unknown, so that it may be investigated further. For such an inquiry, acquired with a sweet taste, would divert noble minds from terrestrial things and, with their desire continually aroused, totally engage and engross those [already] occupied in so respectable an exercise of high-minded endeavor. If, however, these motions were known punctually, and this Great [or Perfect] year were possible, then, surely, all things to come, and the whole order of future events, could be foressen by men, who could then construct a perpetual almanac based on all the effects of the world. They would become like the immortal gods. But it is the gods, not men, who know future times and moments, which are subject to the divine power alone. Indeed, it would be very repugnant that men should come to know about future events beforehand. It seems arrogant of them to believe that they can acquire a foreknowledge of future contingents, only some of which are subject to celestial powers.

It seems better, therefore, to assume the incommensurability of the celestial motions, since these difficulties do not follow from that [supposition]. Indeed, incommensurability is shown in yet another way, for, as demonstrated elsewhere,[42] when any two unknown magnitudes have been designated, it is more probable that they are incommensurable than commensurable, just as it is more probable that any unknown [number] proposed from a multitude of numbers would be nonperfect rather than perfect. Consequently, with regard to any two motions whose ratio is unknown to us, it is more probable that that ratio is irrational than rational—provided that no other consideration intervenes that was not taken into account in what has already been discussed.

70 ON COMETS

Albertus Magnus (ca.1193–1280)

Translated by Lynn Thorndike[1]

Introduction and annotation by Edward Grant

In this selection Albertus Magnus not only reports a number of explanations of comets which Aristotle had refuted but adds a few others proposed subsequently.[2] A basic issue was whether comets are celestial or sublunar phenomena. For

39. Part III consists of an introduction in which Oresme dreams he is conversing with Apollo, of whom he inquires whether the celestial motions are commensurable or incommensurable. Arithmetic orates first on behalf of commensurability, followed by Geometry, who argues that the celestial motions must be incommensurable. The excerpt which follows is but a part of Geometry's lengthy discourse, which represents Oresme's true opinion.

40. Pliny, *Natural History* II, 21, 87.

41. This purported remark of Ptolemy was drawn by Oresme from Al-Battani, *De scientia stellarum (On the Science of the Stars)*. See chapter 52, folio 81 recto, of the edition published in 1537.

42. Undoubtedly a reference to Oresme's *De proportionibus proportionum*, Chapter III, Proposition 10, which is included in Selection 25.

1. Reprinted with permission of the University of Chicago Press from *Latin Treatises on Comets Between 1238 and 1368* A.D., edited by Lynn Thorndike (Chicago: University of Chicago Press, 1950), pp. 62–76. Thorndike's translation was made from Book I, Tractate III, of Albert's commentary on Aristotle's *Meteorologica* as printed in the Borgnet edition, IV, 499–508.

2. A good summary account of cometary theory to the end of the fourteenth century is provided by C. Doris Hellman, *The Comet of 1577: Its Place in the History of Astronomy* (New York: Columbia University Press, 1944), ch. 1, pp. 13–65.

Albert, following Aristotle, they are sublunar, occurring in the uppermost regions of the terrestrial world. Since Aristotle's explanation in Book I of the *Meteorologica* became the standard account of comets in the Middle Ages, it will be useful to quote the most relevant and substantive portions of it.

When the sun warms the earth the evaporation which takes place is necessarily of two kinds, not of one only as some think. One kind is rather of the nature of vapour, the other of the nature of a windy exhalation. That which rises from the moisture contained in the earth and on its surface is vapour, while that rising from the earth itself, which is dry, is like smoke. Of these the windy exhalation, being warm, rises above the moister vapour, which is heavy and sinks below the other. Hence the world surrounding the earth is ordered as follows. First below the circular motion comes the warm and dry element, which we call fire, for there is no word fully adequate to every state of the fumid evaporation: but we must use this terminology since this element is the most inflammable of all bodies. Below this comes air. We must think of what we just called fire as being spread round the terrestrial sphere on the outside like a kind of fuel, so that a little motion often makes it burst into flame just as smoke does: for flame is the ebullition of a dry exhalation. So whenever the circular motion stirs this stuff up in any way, it catches fire at the point at which it is most inflammable. The result differs according to the disposition and quantity of the combustible material. If this is broad and long, we often see a flame burning as in a field of stubble: if it burns lengthwise only, we see what are called "torches" and "goats" and shooting-stars. Now when the inflammable material is longer than it is broad sometimes it seems to throw off sparks as it burns. (This happens because matter catches fire at the sides in small portions but continuously with the main body.) Then it is called a "goat." When this does not happen it is a "torch." But if the whole length of the exhalation is scattered in small parts and in many directions and in breadth and depth alike, we get what are called shooting-stars. (*Meteorologica* I.4.341b.7–35 in the Oxford translation of E. W. Webster)

In chapter 7 (344a.9—344b.17) Aristotle pursues the matter further with reference to comets:

We know that the dry and warm exhalation is the outermost part of the terrestrial world which falls below the circular motion. It, and a great part of the air that is continuous with it below, is carried round the earth by the motion of the circular revolution. In the course of this motion it often ignites wherever it may happen to be of the right consistency, and this we maintain to be the cause of the "shooting" of scattered "stars." We may say, then, that a comet is formed when the upper motion introduces into a gathering of this kind a fiery principle not of such excessive strength as to burn up much of the material quickly, nor so weak as soon to be extinguished, but stronger and capable of burning up much material, and when exhalation of the right consistency rises from below and meets it. The kind of comet varies according to the shape which the exhalation happens to take. If it is diffused equally on every side the star is said to be fringed, if it stretches out in one direction it is called bearded. We have seen that when a fiery principle of this kind moves we seem to have a shooting-star: similarly when it stands still we seem to have a star standing still. We may compare these phenomena to a heap or mass of chaff into which a torch is thrust, or a spark thrown. That is what a shooting-star is like. The fuel is so inflammable that the fire runs through it quickly in a line. Now if this fire were to persist instead of running through the fuel and perishing away, its course through the fuel would stop at the point where the latter was densest, and then the whole might begin to move. Such is a comet—like a shooting-star that contains its beginning and end in itself.

When the matter begins to gather in the lower region independently the comet appears by itself. But when the exhalation is constituted by one of the fixed stars or the planets, owing to their motion, one of them becomes a comet. The fringe is not close to the stars themselves. Just as haloes appear to follow the sun and the moon as they move, and encircle them, when the air is dense enough for them to form along under the sun's course, so too the fringe. It stands in the relation of a halo to the stars, except that the colour of the halo is due to reflection, whereas in the case of comets the colour is something that appears actually on them.

Now when this matter gathers in relation to a star the comet necessarily appears to follow the same course as the star. But when the comet is formed independently it falls behind the motion of the universe, like the rest of the terrestrial

world. It is this fact, that a comet often forms independently, indeed oftener than round one of the regular stars, that makes it impossible to maintain that a comet is a sort of reflection, not indeed, as Hippocrates and his school say, to the sun, but to the very star it is alleged to accompany—in fact, a kind of halo in the pure fuel of fire.

The Aristotelian interpretation of comets as sublunar phenomena was exceptionally long-lived. Not until observation of the comet of 1577 by Tycho Brahe was their supralunar and celestial locale determined by parallax. Indeed, "no claim can be made that immediately after the appearance of that comet all who wrote on astronomy completely dropped Aristotelian and similar theories and immediately recognized comets as celestial bodies moving according to the general laws governing the motion of such bodies. These laws, indeed, as applied to the solar system, were first stated in their present generalized form by Kepler in the beginning of the seventeenth century" (C. D. Hellman, *The Comet of 1577*, p. 307). But it was not until the 1680's that Dörfel determined a parabolic path for some comets and Halley an elliptical path for others.

CHAPTER 1
OF THE ERROR OF THOSE WHO SAID THAT A COMET IS A CONJUNCTION OF SEVERAL STARS

Now let's speak of comets or stars having tails, and of what sort they are with respect to efficient form, and how they vary in kind and figure. Some physicists who deviated from nature have said —and, as Seneca says in the book of *Natural Questions*, Apollonius, too, was of that opinion—that stars having tails are many stars gathered together in the lower spheres, which are mobile like the planets and not fixed like the stars of the eighth sphere.[3] Every planet may be in conjunction with every other planet in two ways, truly and not truly. Truly indeed when one eclipses another, and then the lower one is seen and not the upper. Not truly, moreover, again in two ways, namely, that they are in the same part of the meridian, although they do not touch or eclipse; and this is called being on the same meridian but not on the same circle of altitude. Moreover, the meridian is that arc which starts from the North Pole, goes straight overhead through the zenith, crosses the equator, and passes through the South Pole. The circle of altitude is that which comes from the east through the star under observation and goes on to the west. So it often happens that two planets lie on the same meridian but not on the same arc of the circle of altitude, because one planet may be farther north and the other farther south, and then they are said to be in conjunction so far as the meridian is concerned, but not truly, because they're not in the line of the same circle of altitude. Sometimes, moreover, by their motion they are so close on the meridian that their bodies touch or so nearly touch that the eye cannot discern any of the arc of the meridian between the body of one and the body of the other, and then it may be that one is below the other, yet the eye sees them as a single star made from two or three. And this they said was a comet, and the reason for the deception was that it undoubtedly is like a comet, for, when two or more planets are thus gathered, a person looking at them sees a long continuous light like a tail. This, then, is the opinion of the said physicists as to what a comet is.

CHAPTER 2
OF THE OPINION OF THOSE WHO SAID THAT A COMET IS VAPOR CLINGING TO A PLANET, JUST AS THE MORNING SUN IS SEEN WITH THE COLOR OF VAPOR

Certain Pythagoreans among the Italian philosophers said that a tailed star is only one star, and not many gathered together, although by one star they did not mean just one in number, which is always tailed, because many in turn might be tailed according to them; for this tailed star is one of those that move, that is, one of the planets which sometimes seems to have a tail.[4] For when it rises, it looks small, since the eye, surveying the horizon of our habitable hemisphere, sees only a little of it because of the thick vapor close to the ground between our vision and the rising star. But when it has risen some space above the horizon, the vapor is thinner, and then the star is seen in the vapor, and sometimes the vapor is spread out like long hairs and the planet shines in it, and that diffusion of vapor seems to be its tail, and the star changes color as the vapor changes. For in pure vapor the tail will seem white, and in moist it will appear red; and if what is between us and the star is very watery, the tail will seem golden-red or green; and if

3. Probably based on Aristotle's remark that "Anaxagoras and Democritus declare that comets are a conjunction of the planets approaching one another and so appearing to touch one another" (*Meteorologica* I.6. 342b.25–29).

4. Compare *Meteorologica* I.6.342b.30–343a.20.

there are dark clouds, the tail will appear black. This, then, is the reason, as the Pythagoreans say, why sometimes a tailed star appears.

CHAPTER 3

OF THE OPINION OF THOSE WHO SAID THAT A TAILED STAR IS THE IMPRESSION OF A STAR ON HUMID AIR AS IN A MIRROR

Certain others talked similarly concerning a comet so far as matter is concerned, since they said it was an impression on moist vapor, although they differ from these as to the location of that vapor. And they are imitators of Hippocrates of Cos and were his disciples, Nichius and Paul. For they said that, although it was one of the planets which seemed tailed, yet the tail was not an essential part of it.[5] But rather, when one of the five planets moved in its varied course, now rising from the east, now descending to a western sign, now in latitude north of the equator, now carried southward in the opposite direction beyond the equator, then our vision would sometimes see it as if it had a shining tail. This came about because this star impressed its light as in a mirror on the humid and pure air in the upper region of the air, and the air which first received the light reflected it in the humid air adjoining, just as a mirror reflects the form which it receives; and so the air kept reflecting it until a long light appeared about the star, and this is called the tail of the star. Moreover, they call humid air a moist vapor of subtle substance and clear, which they say is drawn to the upper air by the force of the sun, which, according to the Egyptian physicists, draws that humor to nourish itself and the other planets even beyond the sphere of fire. Moreover, such a star, according to them, is [not] seen in the north, where every star has more of its orbit above than below the earth because of the obliquity of the horizon; and so the sun, when in the northern signs, makes the days longer than the nights; and so they say that that star stays above earth longer than another.

Some give a different explanation,[6] that that moist vapor which is attracted for nourishment, although diminished, remains standing about the star until consumed and so is seen to rotate with it for some time. Those supporting this view give the reason that in the west, where is completed the revolution of the star above the hemisphere, there is abundant humor from cold and vapors and that there it frequently appears tailed. They add that in the north, where humor abounds because of the cold, and in the south, where humor abounds which the planets attract as they cross those regions, a tailed star is more often seen; and when a star is in the air where there is no humor, north and south, it is not seen to have a tail. But, although this sometimes happens in the circle farthest south, as in the small circle which the movement of the pole of the zodiac describes to the south, yet it does not appear to us there because of the too great remoteness from our vision. But when it is in the north over our habitable world, then its tail is visible to us, since then our sight can comprehend it because it is near and overhead. These, then, are the views which Aristotle found had been expressed as to comets.

CHAPTER 4

A DIGRESSION TO GIVE THE OPINIONS OF SENECA AND JOHN OF DAMASCUS AND OF CERTAIN MODERNS AS TO COMETS

There are still some false views which Aristotle did not bother to note which we give here in order to overthrow them at the same time. For John of Damascus says (II, 7):

> Often comets are formed, signs signifying the deaths of kings, which are not of those stars which were generated from the beginning, but by divine command are formed to suit the time being and again are dissolved.

Seneca, moreover, in the book of *Natural Questions* thus expresses his opinion concerning comets, saying:

> A comet is a star created together with the works of nature, whose nature it may be is ignored, yet that it is not transient fire is proved thus. Whatever things air produces are short-lived. How then can a comet last long in air, when air itself is not long permanent? For they are born in what is flighty and mobile, and it cannot be that fixed fire should have its seat and so pertinaciously adhere in a wandering body.[7]

And later he adds in the same place:

> A comet has its seat and so is not quickly expelled but traces its measured course, nor is extinguished but passes on.[8]

There are also certain learned moderns who say that a comet is the impression of one of the five

5. Based on *Meteorologica* I.6.343a.23–30.
6. The following explanation is drawn from *Meteorologica* I.6.343a.1–20.
7. Seneca, *Natural Questions*, VII.22. Thorndike notes that Albert's version of this passage differs considerably from the modern edition.
8. *Natural Questions*, VII.23.

planets, not merely in fire, nor merely in air, but rather at the meeting line of fire and air, where the convex surface of the air mingles with the concave of fire, for there there are certain obscure lines of air and certain luminous, as they say, lines of fire which are further illuminated by the light descending from one of the five planets, and so the tail seems to them to be made up of obscure lines and luminous lines intermixed. These have four arguments for themselves, which I will briefly touch upon mediately rather than argumentatively. For they say that it seems unfitting, if the fixed stars are the causes of some impression and the planets of none, since it is evident that the planets are much stronger in moving inferiors than the fixed stars are. If, therefore, they are causes of some impression, it is proved inductively that no impression suits them except tailed stars. The second line of argument is that comets are generated either in the way aforesaid or from dry terrestrial vapor. But if from dry terrestrial vapor, either that vapor is uniform or not uniform. If it is uniform, either it is raised all at once or successively. Not all at once, because, if all at once, then it would also be inflamed and illuminated all at once, and so would appear in a round form like *assub*, as will be shown below and so the comet would not be seen in elongated form. But if not raised all together, then it will be long and inflamed, standing erect in air; therefore, it would look like a lance or perpendicular fire. But if it is not uniform, either it is raised all at once, or not all at once. If it is raised simultaneously, then the first subtle part will be raised and set on fire, and the lower because of its coarseness will be spread out, and thus it will seem perpendicular and broad below; and so again it will not be in the figure of a comet. But if it is not raised simultaneously, then the subtle part will rise without the coarse; therefore, it will also be set afire without it; therefore, the coarse will never follow the subtle; therefore, the figure of that fire will never be elongated by the coarse part set afire and joined to the subtle; and so again it will not have the figure of a comet. The third argument is that if it is of terrestrial vapor set afire, it seems that the same should happen in space as happens in the case of *assub*, namely, that it descend that it may straightway ascend, and this we do not see, since often it long moves in a circle. Therefore, it is not of inflamed terrestrial vapor. The fourth argument is that no inflamed terrestrial vapor, which at the same time rises to the hot upper region of the air and secondarily descends to earth, appears to rise in the air, since either it is consumed in that upper region in which fire triumphs, as said above, or it is weighed down with cold and depressed to earth. But a comet often appears to rise in the air; therefore, it is not from terrestrial vapor inflamed in or below the upper region of air. These, then, are the views about comets which Aristotle did not touch upon.

CHAPTER 5

A DIGRESSION GIVING THE CORRECT VIEWS OF PHYSICISTS, AVICENNA, ALGAZEL, PTOLEMY, AND MANY OTHERS, AS TO COMETS, WHO ALL AGREE ON THE SAME POINT.

Having excluded all these erroneous views, as our rebuttal of them below will show, we will give the correct view as to comets, and we will confirm it by the authority of many physicists and even by the arguments which those philosophers advance for themselves. I say, then, that a comet is nothing else than coarse terrestrial vapor, whose parts lie very close together, gradually rising from the lower part of the upper region of the air to its upper part, where it touches the concave surface of the sphere of fire and there is diffused and inflamed and so often seems long and diffuse. I say "terrestrial vapor" to denote the material of the vapor. And I say "coarse" because, if it were subtle, it would quickly evaporate and be dissipated. And I say "whose parts are close together," since it is well mixed and viscous in so far as such vapor which is not actually humid and viscous can be. And it is said "ascend gradually," since in rain-producing vapors are intermingled some ignited terrestrial parts which do not all descend with the rain, and those which do descend for the most part reascend with evaporation. And they escape beyond the middle cold region of the air because of their state and stay there and multiply. And from that multiplying, as from some reservoir, they gradually ascend because of the heat of the upper region of the air, because they have much constancy in many respects. At first, the mass is diffused by the heat of the fire and afterward inflamed; yet in the middle it always remains dense, since it is supplied by its reservoir below it, and so its flame is very white and thick. But that which is diffused from it at a distance to the sides is tenuous and has a thin flame like a white cloud, and this is called the "tail." Moreover, it lasts through all the time until its supply is exhausted. This is the view of the philosopher Constantinus, who is said to have comment-

ed in Greek on the *Liber meteororum* of Aristotle, where he speaks of comets.[9]

Moreover, that it is so, testify the illustrious philosophers Avicenna and Algazel. For Avicenna speaks thus: "A star which is called tailed is made of thick ignited fumes, which, since it is quickly converted, sometimes revolves circularly with the sphere of fire." Algazel in his *Physics* says:

> If fire prevails over elevated fumes, it will purge them of darkness and then all the fumes will be converted into fire. And only one of two things can take place there: either it will be ignited, or it will become pure fire, because there is no frigidity there. But if it is thick, it will indeed ignite; but because it is not such as is readily altered, it will so remain for some time and will seem a tailed star. Which revolves with the sky, because the parts of fire are continuous with the concave surface of the heavens; so it revolves because of its relationship.

From these words is shown concerning comets what we have said before, and so Algazel says that the matter of the coarse vapor revolved with the sky seems threefold. For sometimes it is flame, as was just said. But sometimes coarse and black, and then it is ignited like coal, and that comet looks red. Sometimes, moreover, the fire with it is extinguished, because the matter is too coarse, and it remains smoky, and then it looks like coal that is black and extinct; and so it is that dark apertures are seen in the sky, which phenomenon is called by the people "perforation of the sky."

Alfraganus, moreover, in his *Astronomy* proves that fire is moved in a circle from the fact that the vapor set on fire adhering to it rotates in a circle beneath it, saying:

> Circular motion is derived from the sky apart from natural motions; and this is so, since we see in the element fire a circular motion like the motion of the sky, as in the case of scintillations of stars apart from natural motions, of flaming vapors in the upper air, in evening twilight, so that the spectator thinks them stars and sees them moved with the motion of the stars and following these until they are hidden.

From this it is inferred that they are like stars and not stars and are vapors ignited in the air. Ptolemy implies the same in his *Centiloquium*, third proposition, where he says that third judgment is from second stars, on which three commentators, namely, Haly and Abraham and Bugaforus, remark that second stars are the effects of the stars upon vapor elevated and ignited, as in comets and *assub* and the like. Albumasar also implies this in the book on *Conjunctions of the Planets*, where he discusses the quality of the science of the generation of superior individuals apart from inferior accidents and says that one part of that signification is from superior meteors, such as fire and *assub* and comets. And he adds that they are generated from one matter by the power of Mars alone, without any other star. So it is clear that this view, which we have stated, is that of all the better philosophers.

Reason, too, rallies to the support of this opinion, since it is evident that flame is nothing but kindled fumes; now a comet is a sort of flame, as is apparent to the sight; therefore, it is kindled fumes. Further, every other impression which is luminous, having a distinct light by itself, is from kindled vapor; therefore, this too. Besides, let us ask our modern doctors, since a comet is light kindled by one of the five planets at the boundary of air and fire and since the planets always are moved above this kind of boundary where air is mixed with fire, why don't comets always appear? Also why don't several comets appear at the same time, since several planets may rise simultaneously above this mixture of air and fire? Also why is diverse form seen in comets? Since, as Seneca says, sometimes it revolves, pointing its tail upward, sometimes casting it downward, and sometimes sideways. It seems that they would be unable to make any reasonable reply. Furthermore, if a comet is always produced by one of the five planets, then it should never be seen outside the path of the planets. But the path of the planets is in the zodiac or very little outside; therefore, a comet should always be seen in the zodiac or near it; and yet this is false, since Aristotle says that we see comets to the north and south and in every part of the sky. Moreover, I with many others in Saxony in the year 1240 from the Incarnation of the Lord saw a comet close to the North Pole, and it projected its rays between east and south, but more toward the east, and it is evident that there was not the path of any planet. But, since also the other three opinions which Aristotle mentioned say that a comet is generated by the five planets, therefore this opinion together with them in this respect will receive disproof.

9. Thorndike observes that "the allusion seems to be to Konstantinos Psellos, usually known as Michael."

CHAPTER 6
A DIGRESSION DISPROVING THE OPINION OF SENECA AND OF JOHN OF DAMASCUS

That the statement of Seneca is an error is patent, since what vanishes without setting but within the hemisphere, that is corrupted not completing its course through the hemisphere. Moreover, a comet thus vanishes before the eyes of men; therefore, it is extinguished and disappears. Moreover, that it vanishes before the eyes of men is proved by two experiences which Seneca himself records in the book of *Questions*,[10] saying:

> After the death of Demetrius, king of Syria, shortly before the Achaean War, a comet shone forth not smaller than the sun, at first fiery and rubicund and emitting a clear light; then gradually its size was restricted and its clarity vanished; finally it totally died out.

From this, then, is evident what has been said. He recorded the second in the same place, saying:

> While Attalus was ruling in Greece a moderate-sized comet appeared; then it spread out and reached the equator so that it equaled in extent that part of the sky called the Milky Way, and there within an instant it ceased to be.[11]

The opinion of John of Damascus, that God makes a new star, is not probable, since there is no reason why He should make it in different figures and colors. But a comet is seen in different figures and colors; therefore, it is not a work of God alone. That it has different figures is evident from the words of Seneca, who says:

> The form of a comet is not single, for there were many comets from which flame hung like a beard, and those which spread their hairs in every direction about them, and those in which fire was diffuse but tending to a point: yet all are of the same nature and are rightly called comets. Whose forms, since they appear at long intervals, are difficult to compare. Indeed, at the very time they appear, observers do not agree as to their appearance, but as each is sharp or dull sighted, so he says they are more brilliant or rubicund or obscure.[12]

Since, therefore, this variety of form is because of some pre-existing matter from which nature works, it is evident that a comet is not then produced by divine work only. So then it is evident, with regard to the three opinions of Seneca, John of Damascus, and certain doctors of our time, that they are false, and that that which is true is to be held.

CHAPTER 7
A DIGRESSION TO DISPROVE THE FIRST OPINION WHICH ARISTOTLE DISPROVES

What Anaxagoras and Democritus said,[13] and Apollonius after them, that a comet is many planets congregated, Seneca disproves by four reasons, of which the first two are taken from experiences. For if a comet appeared before the Achaean War as large as the sun, whereas no other stars, albeit gathered together, ever equal the sun, it is evident that a comet is not generally a congregation of stars. The second case is that the comet which appeared while Attalus was ruling in Greece so extended itself that it equaled in immense extent that region of the sky known as the Milky Way. For the planets cannot come together so that they occupy continuously so long a tract of the sky. Two other arguments are based on reason, one from the position of the planets, which he states in these words: "Apollonius says that not one comet is made up of planets but many planets are comets." But if this were so, comets would move within the limits of the zodiac. The second argument is from the nature and property of the stars, which he states thus: "A star never shines through a star, since otherwise one would not eclipse another; through a comet sometimes, as through a cloud, objects beyond are seen, whence it appears that a comet is not a star but a light and tumultuous fire." On the same point Seneca further remarks: "It is certain that comets are not seen in one part of the sky only but both in the east and west, even more frequently to the north."

CHAPTER 8
DISPROOF OF THE SECOND AND THIRD OPINION BY REASONING OF ARISTOTLE

This being so, let us overthrow the other two opinions by the arguments of Aristotle. Since we have already repeated their utterances concerning comets, we ought now to ask whether these are true or false. I say, then, that often we see comets outside the eighth sphere of the fixed stars, and so it cannot be that a comet is one or more of the fixed stars, since then it would be in the first belt, which is the eighth sphere which encloses all the starry sphere, which is false because we see comets

10. *Natural Questions*, VII.15.
11. *Ibid.*
12. *Natural Questions*, VII.11.
13. See note 3.

outside that sphere far below it. But if it were one of the planets, then it would never be seen except in the spheres of the planets, which is false because it is seen outside these, beneath them, and outside the circle of the signs. But if comets were stars seen through humid air, as those of the second opinion said, then when the star comes to mid-sky, where our vision observes it without the interposition of humid air, it ought to appear without a tail. And this again is false, since a comet sometimes appears anywhere in the revolving sky and sometimes for many days. Furthermore, if the tail were caused by humid air, then every star seen through humid air would be a comet; so comets would be as numerous as the stars and would not appear only in one of them but in many at once. And this is false, since we do not find stars having tails except five, and and these not at once but successively, as we shall state below. And their assertion that a comet is not found except in the north is false, since one is often seen elsewhere than in the north, for a comet is sometimes seen with the sun preceding it as it rises in summer. Moreover, the summer rising of the sun is the sign in which the sun rises in summer, such as the northern signs in which it moves from the beginning of Aries to the beginning of Libra, because this summer rising is interpreted broadly for any rising in which the day exceeds the night, since then the sun regards us more directly and warms our habitation. And likewise tailed stars are in the south as time varies. And this happens especially in the city of Arim, which is on the equator at the center of the world, ninety degrees from the Arctic pole and ninety from the Antarctic, and ninety to the east and ninety to the west of the habitable world. For the region at the equator is more temperate than any other spot in this world, as Ptolemy says and Avicenna and many other philosophers. Comets are also seen in the north in the morning, rising before the sun in its winter rising, and this cannot be otherwise than when the sun rises in winter in southern signs, since then the comet was rising in the north in northern signs. All this that has been said shows us that tailed stars are other than the fixed stars and planets which they have said, and are seen in summer and in winter in various parts of the sky and world. So then is shown the error of their statements.

CHAPTER 9

ARISTOTLE'S OPINION CONCERNING COMETS

I say, then, that stars having tails effectively and by the place of their generation are air inflamed by the vicinity of fire and which contains the matter from afar and the comet itself. I say air close to fire which contains in itself the heat of fire, since, when dry coarse vapor is inflamed in it, then a continuation is made with the light of the flame of the comet from the lower air where stands vapor of the same nature, and so it is elongated from this extension continuous with it; and such is a comet, as is clear from the aforesaid. Some expound otherwise this opinion of Aristotle, but I do no violence to their exposition, since almost all philosophers who have followed Aristotle agree with our interpretation.

CHAPTER 10

A DIGRESSION EXPLAINING WHY THERE ARE SAID TO BE FIVE COMETS AND NO MORE

Someone may ask according to that interpretation of Aristotle why there are said to be only five comets. For if the number is not taken from the planets, it will seem to depend merely on difference in material, but differences in material are infinite and so should not be stated as five. Moreover, those stars are called "secondary" by the philosophers; therefore, there should be some primary stars which are their causes; it is evident that these are not fixed stars, it seems therefore that they are planets; they are not the two luminaries, so they should be the five planets. But, according to this, they should be only in or near the zodiac, whereas it was held above that they are seen in every part of the sky.

To this without prejudice I think it should be answered that in truth, according to astronomers who base judgments upon them, five comets are not reckoned as five planets, as we will state below, but either according to five differences of the vapor composing them or according to five forms of stars having tails. The differences of vapors are that all cometary vapor, though of a coarse and sticky variety, yet may be relatively coarse or subtle or halfway between. Moreover, where there is a midway mean, there is also a mean near either extreme, and so there will be three means, namely, halfway, and between it and subtle, and between it and coarse. Nor can there be greater diversity of vapor in variety, and so comets number five. Also they are five by figure, for either the vapor surrounds it in a circuit, or from lightness is above the comet, or hangs downward, or to one side, in which case it is evidently either from both sides or from one side: and so they are again five. It may be, however, that the primary difference is one of matter, since

it causes difference in form, since thin vapor takes on one shape, and thick another. That they are called "secondary stars" is because they signify the force of the planets in diverse places, as we shall soon state, yet not on this account are they numbered according to the planets. It may be, too, that difference in vapor as coarse or subtle is a difference in matter; yet it is not purely material but rather ordained for form, since by intrinsic and extrinsic heat such matter takes on such formality, and this question of diversity is one which properly belongs to the physicist.

CHAPTER 11

A DIGRESSION WHY COMETS SIGNIFY THE DEATH OF POTENTATES AND WARS

Now it must be asked if we can comprehend why comets signify the death of magnates and coming wars, for writers of philosophy say so. The reason is not apparent, since vapor no more rises in a land where a pauper lives than where a rich man resides, whether he be king or someone else. Furthermore, it is evident that a comet has a natural cause not dependent on anything else; so it seems that it has no relation to someone's death or to war. For if it be said that it does relate to war or someone's death, either it does so as cause or effect or sign. Evidently it does not have sufficient relation to anything as cause, since it is neither efficient cause, nor formal, nor final, nor material. Similarly it is proved not to be an effect, since neither necessarily follows the other, but an effect necessarily follows its cause. Nor does it relate even as a sign, since it has not conformity, but every sign has conformity with what it signifies. Furthermore, as some say, the distinction of five comets depends on the five planets, and, since not all the planets have such bad effects, since Jupiter signifies fortune and similarly Venus and Mercury do not signify misfortune, it seems that comets should not as a rule be signs of evil.

Consider, then, things like them, for as Albumasar says in the seventh tractate *On the Conjunctions of the Planets*, the event of fires and *assub* and comets is not inferred from any planet except in the air, especially when its rays were in terrestrial or aerial signs, the moon not impeding, since it stirs up humid, aqueous vapor which might prevent such fiery phenomena. Hence it is clear that it is not stated that comets are interpreted according to the five planets, since such phenomena are only of the character of Mars and so are from it as prime mover, except perhaps occasionally, as in the conjunction of Jupiter and Mars, since from that conjunction scintillations and coruscations and running fires are also moved through the air.

In answer to the objection as to land of poor and rich, it should be said that it indicates the destruction of both according to the diversity of period of which we have spoken at the end of the second book *On Generation and Corruption*. But the death of kings is noticed more because of their fame, since their periods have more planetary dignity, and so greater significations are referred to them. In answer to the assertion that it has no natural cause related to anyone's death, it should be said that it is not the immediate cause of the death of kings; yet its first moving cause denotes the force of Mars, which is a sign of death especially by violence, and then Mars dominates the commotion of the elements; also it is lord in the period and shows the cause of death, so far as the prime mover may be called the cause, which is not necessary but rather inclinatory. As to the question whether it be sign or cause, it should be said that it is properly sign, since, as Aristotle says in the book *On Sleep and Waking*, such are like a counselor whose advice can be changed by finding better. So the rule of Mars signifies wars and death of inhabitants and disturbances of rulers, as inclining to this by wrath and heat and drought, from which comes animosity or exciting of peoples against one another. And so the significance of comets is primarily connected with Mars, which is the cause of war and destruction of peoples; and so a comet is said to signify these, although its immediate significance is not connected with them.

71 THE POSSIBILITY OF A PLURALITY OF WORLDS

Nicole Oresme (ca. 1325–1382)

Translated by Albert D. Menut[1]

Introduction and annotation by Edward Grant

Oresme's analysis of the possible existence of other worlds was perhaps the most cogent and

1. *Nicole Oresme: Le Livre du ciel et du monde*, edited by Albert D. Menut and Alexander J. Denomy, C.S.B.†;

penetrating discussion of this question during the Middle Ages. Not only did he incorporate many of the standard arguments, but he went considerably beyond, adding new insights and subtleties. In his response, he followed a somewhat usual pattern by distinguishing different possible interpretations of a "plurality of worlds," taking up each in turn. Of special interest is the third interpretation involving the simultaneous existence of many worlds. Here Oresme's objective is to demonstrate the inconclusiveness of Aristotle's arguments that only one world is possible. Although writing one hundred years later, Oresme's attitude reflects the influence of Article 34 of the Condemnation of 1277, which denounced those who would deny that God could create as many worlds as He pleased (see Selection 13). Not only does Oresme advocate significant cosmological ideas, but he attempts to cope with some of the physical problems that would have immediately arisen for Aristotelians. Indeed, he seeks to apply and extend Aristotelian physical principles to these other possible worlds, whose existence Aristotle had denied categorically. In the end, however, Oresme upholds the traditional Aristotelian and Christian position that there is only one world. It was never his intention to overturn tradition, but merely to demonstrate the possibility—and no more—that other worlds could exist quite readily if God but chose to create them.

Of the three interpretations discussed here, the first and third were of ancient vintage—namely the concept of a succession of single worlds and a plurality of simultaneous, separate, existing worlds scattered about in space. The second interpretation—where the plurality is simultaneous but the worlds are contained within each other—appears to have been a medieval invention. Albert of Saxony—and Oresme himself in his earlier *Questiones super de celo (Questions on De caelo)*, Bk. I, Question 18—even conceived of these worlds as arranged concentrically or eccentrically. As Albert explains it (*Questions on De caelo*, Bk. I, Question 11, fol. 95r, col. 1, of the edition cited in n. 1 to Selection 35):

> Several worlds can be imagined simultaneously which are either concentric or eccentric. Indeed, they would be concentric if beyond the last heaven of our world we should imagine an earthy orb, and beyond that an aqueous orb, and beyond that an airy orb, and beyond that a fiery orb, and beyond that celestial orbs like the celestial orbs of our world; or, if inside our earth, it could be imagined that the aggregate of celestial orbs was contiguous with the concavity of the orb of our earth and within this aggregate of supercelestial orbs, four elements could be imagined, just as in our world. And so, by ascending and descending, an infinite number of concentric worlds can be imagined.
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> Similarly, we can imagine several eccentric worlds: either (1) one lies wholly outside the other, and this could be [imagined] in the way several globes are placed in a sack, or [it might be imagined that they are inside but] do not touch; or (2) that one does not lie wholly outside the other, but that there is another world in some part of our world, as if, [for example,] another world were imagined in the moon, or in the sun, and in the other planets.

Now we have finished the chapters in which Aristotle undertook to prove that a plurality of worlds is impossible, and it is good to consider the truth of this matter without considering the authority of any human but only that of pure reason. I say that, for the present, it seems to me that one can imagine the existence of several worlds in three ways. One way is that one world would follow another in succession of time, as certain ancient thinkers held that this world had a beginning because previous to this all was a confused mass without order, form, or shape. Thereafter, by love or concord, this mass was disentangled, formed, and ordered, and thus was the world created. And finally after a long time this world will be destroyed by discord and will return to the same confused mass, and again, through concord, another world will then be made. Such a process will take place in the future an infinite number of times, and it has been thus in the past. But this opinion is not touched upon here and was reproved by Aristotle in several places in his philosophical

translated by Albert D. Menut (Madison, Wis.: University of Wisconsin Press, 1968), pp. 167–179. Reprinted with permission of the copyright owners, The Regents of the University of Wisconsin. Of Oresme's *Le Livre du ciel et du monde (Book on the Heavens and the World)*, which is a French commentary on Oresme's own French translation of Aristotle's *De caelo (On the Heavens)*, the present selection includes most of Book I, chapter 24, and is an actual commentary on I.9.279a.12–279b.4 of Aristotle's *De caelo*. Completed in 1377, this represents Oresme's last extant work before his death in 1382. It was one of his best. Apart from a few minor changes and the omission of a few cross-references and folio numbers to one of the French manuscripts, the translation is faithfully reproduced.

works.[2] It cannot happen in this way naturally, although God could do it and could have done it in the past by His own omnipotence, or He could annihilate this world and create another thereafter. And, according to St. Jerome,[3] Origen used to say that God will do this innumerable times.

Another speculation can be offered which I should like to toy with as a mental exercise. This is the assumption that at one and the same time one world is inside another so that inside and beneath the circumference of this world there was another world similar but smaller. Although this is not in fact the case, nor is it at all likely, nevertheless, it seems to me that it would not be possible to establish the contrary by logical argument; for the strongest arguments against it would, it seems to me, be the following or similar ones. First, if there were another world inside our world, it would follow that our earth is where it is by constraint, because inside this earth and beneath its circumference toward its center would be another heaven and other elements,[4] etc. Also, the earth of the second world would be absolutely massive and at the center of both worlds; and the earth of our world would be empty and concave and neither the whole earth nor any part of it would be at the center. Thus, since their natural places are different, it follows from what is said in Chapter Seventeen that these two worlds are of different form so that the world beneath us and this our world would be dissimilar, etc. Also, all natural bodies are limited in bigness and smallness, for the size of a man could diminish or grow so much that he would no longer be a man, and the same with all bodies. So, the world we have imagined inside our own world and beneath its circumference would be so small that it would not be a world at all, for our sun would be more than 2,000 times the size of the other and each of our stars would be larger than this imaginary world. To pursue our thought, one could dig in the ground deep enough to reach the earth of the other world beneath ours. This is an untenable absurdity. Also, we should have to posit two Gods, one for each world, etc. Likewise, we might assume another world like our own to exist in the moon or some other star, etc. Or we could imagine another world above and another beneath the one which is our world, etc. To show that these and similar speculations do not preclude the possibility of such a thing, I will posit, first of all, that every body is divisible into parts themselves endlessly divisible, as appears in Chapter One; and I point out that *large* and *small* are relative, and not absolute terms, used in comparisons. For each body, however small, is large with respect to the thousandth part of itself, and any body whatsoever, however large, would be small with respect to a larger body. Nor does the larger body have more parts than the smaller, for the parts of each are infinite in number.[5] Also from this it follows that, were the world to be made between now and tomorrow 100 or 1,000 times larger or smaller than it is at present, all its parts being enlarged or diminished proportionally, everything would appear tomorrow exactly as now, just as though nothing had been changed. And, if a stone in a quarry had a small opening in it or a concavity full of air, it is not necessary to say that this stone is outside its natural place. Likewise, if there were a concavity the size of an apple full of air at the earth's center, it would not follow that the earth was out of its natural place nor that it was there by violence. Also, if such concavity were to become a bit larger and then still larger until it became very large, we could not place a limit upon this growth at which point one could say the earth would be out of its natural place, precisely because large and small are relative terms, as we have already said. Therefore, for the earth to be in its natural place, it is enough that the center of its weight should be the center of the world, regardless of the concavity inside the earth, provided that it be held firmly together. And this is the answer to the first argument; for, if a world were enclosed within a concavity inside our earth, nevertheless our earth would be in its natural place since the center of the world would be the middle or center of its weight.[6] A propos, I say further that, according to

2. This is the view of Empedocles (fifth century B.C.). Among other places, see Aristotle's *Physics* VIII.1.252a.7–10; VIII.9.265b.19–21; *On the Heavens* I.10.279b.15–17; II.13.295a.30—295b.1; III.2.301a.16–20; *Metaphysics* I.4.985a.22–30.

3. In a letter to Avitus (see letter 124 in Vol. 3 of I. Hilberg's edition of St. Jerome's letters, *Sancti Eusebii Hieronymi epistulae* [Vienna: F. Tempsky, 1918]).

4. By extension of Aristotelian principles to this hypothetical situation, it could only be concluded that the earth above the heaven would be out of its natural place and could be maintained there only by force.

5. We have here the assertion of a one-to-one correspondence between an infinite set and its subset. This important concept was expressed at least as early as the Greek Stoics and repeated in the Middle Ages by many scholastics, including Roger Bacon. Galileo later applied this notion to numbers, showing a one-to-one correspondence between the infinite set of natural numbers and its subset of square numbers.

6. That is, the center of the world would correspond to

Scripture, water is above the heavens or the firmament; whence the psalm says:[7] Who stretchest out the heavens, etc., Who coverest these heavens with water. And, elsewhere: Bless the Lord, ye waters that are above the heavens.[8] And if this water were not heavy in substance if not in fact, then it would not be water. For this reason it is said to be solid and as though frozen or solidified and is called the glacial or the crystalline heavens. Accordingly, this heaven or this water is in its natural place, in spite of the fact that all the other heavenly spheres and elements are enclosed within the concavity of this sphere, for it is solid and the center of its weight is the center of the world. To the second argument I reply that, even if this earth were hollow and concave, nevertheless it would be in the center of the world or worlds, just as though this were its proper place, taking *place* in the sense of the second member of the distinction made in reply to an argument in Chapter Seventeen.[9] From this it appears that our earth and the earth of the other world within it would be in the same place. To the third argument, which stated that all natural bodies are limited in quantity, I say that in this world they are limited to one quantity or size and that in another world they would be fixed at other limits, for large and small, as we have said, are relative terms which do not mean variation or difference in form. Accordingly, we see men—all of the same form—larger in one region and smaller in another. To the fourth argument, where it was stated that one could dig deep enough into the earth, etc., I answer that nature would not permit this, any more than one could naturally approach the sky close enough to touch it. To the fifth argument, regarding the possibility of two Gods, it does not follow; for one sovereign God would govern all such worlds, but it is possible that additional intelligences would move the heavenly bodies of one world and other intelligences the heavens of the other world. To the sixth argument, where it was said that by analogy one could say there is another world inside the moon, and to the seventh, where it was posited that there are several worlds within our own and several outside or beyond which contain it, etc., I say that the contrary cannot be proved by reason nor by evidence from experience, but also I submit that there is no proof from reason or experience or otherwise that such worlds do exist. Therefore, we should not guess nor make a statement that something is thus and so for no reason or cause whatsoever against all appearances; nor should we support an opinion whose contrary is probable; however, it is good to have considered whether such opinion is impossible.[10]

The third manner of speculating about the possibility of several worlds is that one world could be entirely outside the other in an imagined space,[11] as Anaxagoras held. This solitary type of other world is refuted here by Aristotle as impossible. But it seems to me that his arguments are not clearly conclusive, for his first and principal argument states that, if several worlds existed, it would follow that the earth in the other world would tend to be moved to the center of our world and conversely, etc., as he has loosely explained in Chapters Sixteen and Seventeen.[12] To show that this consequence is not necessary, I say in the first place that, although *up* and *down* are said with several meanings, as will be stated in Book II [see ch. 4], with respect to the present subject, however, they are used with regard to us, as when we say that one-half or part of the heavens is up above us and the other half is down beneath us. But up and down are used otherwise with respect to heavy and light objects, as when we say the heavy bodies tend downward and the light tend upward. Therefore, I say that up and down in this second usage indicate nothing more than the natural law concerning heavy and light bodies, which is that all the heavy bodies so far as possible are located in the middle of the light bodies without setting up for them any

the center of gravity of the earth, but not to the actual center of the earth.

7. Psalms 103:2–3.

8. See Daniel 3:60.

9. In Book I, chapter 17 (Menut translation, p. 139), Oresme distinguishes between (1) place as the immediate container of a thing—as, for example, the concave surface of a barrel of wine is the container of the wine—and (2) place "as that which defines the proper state of a body seated in its unique natural location; in this sense the center of the world is the place natural and proper to the earth and to all the multitude of objects possessing weight, . . ." Thus, even if the earth were hollow and concave, its center would still coincide with the geometric center of the world. With this approach, Oresme is satisfied that the earths of every world are in their natural places, for every one of their centers would coincide with the geometric center of the universe.

10. And so Oresme shows that a series of worlds arranged one inside the other is not a contradictory concept and is therefore logically possible. For this particular interpretation of a plurality of worlds, Albert of Saxony adopted much the same attitude as Oresme.

11. In this sentence I have altered slightly Menut's translation.

12. Aristotle makes this point in *On the Heavens* I.8.276b.10–18.

other motionless or natural place. This can be understood from a later statement and from an explanation in the fourth chapter, where it was shown how a portion of air could rise up naturally from the center of the earth to the heavens and could descend naturally from the heavens to the center of the earth.[13] Therefore, I say that a heavy body to which no light body is attached would not move of itself;[14] for in such a place as that in which this heavy body is resting, there would be neither up nor down because, in this case, the natural law stated above would not operate and, consequently, there would not be any up or down in that place. This can be clarified by what Aristotle says in Book Four of the *Physics*, namely, that in a void there is no difference of place with respect to up or down.[15] Therefore, Aristotle says that a body in a vacuum would not move of itself.[16] In the eleventh chapter of this first book it appears, according to Aristotle, that, since nothing is lower than the center of the earth, nothing is or can be higher than the circumference or the concavity of the lunar sphere,[17] the place proper to fire, as we have often said. Thus, taking up in the second sense above, beyond or outside of this circumference or heaven there is no up nor down. From this it follows clearly that, if God in His infinite power created a portion of earth and set it in the heavens where the stars are or beyond the heavens, this earth would have no tendency whatsoever to be moved toward the center of our world. So it appears that the consequence stated above by Aristotle is not necessary. I say, rather, that, if God created another world like our own, the earth and the other elements of this other world would be present there just as they are in our own world. But Aristotle confirms his conclusion by another argument in Chapter Seventeen and it is briefly this: all parts of the earth tend toward a single natural place, one in number; therefore, the earth of the other world would tend toward the center of this world. I answer that his argument has little appearance of truth, considering what is now said and what was said in Chapter Seventeen. For the truth is that in this world a part of the earth does not tend toward one center and another part toward another center, but all heavy bodies in this world tend to be united in one mass such that the center of the weight of this mass is at the center of this world, and all the parts constitute one body, numerically speaking. Therefore, they have one single place. And if some part of the earth in the other world were in this world, it would tend toward the center of this world and become united with the mass, and conversely. But it does not have to follow that the portions of earth or of the heavy bodies of the other world, if it existed, would tend to the center of this world because in their world they would form a single mass possessed of a single place and would be arranged in up and down order, as we have indicated, just like the mass of heavy bodies in this world. And these two bodies or masses would be of one kind, their natural places would be formally identical, and likewise the two worlds.[18]

13. In Book I, chapter 4, Oresme set up certain hypothetical conditions to show how one might disagree with Aristotle's contention that a simple or elemental body has only one simple motion—that air, for example, rises in the region of water and always falls when in the region of fire, in both instances seeking its natural region of air. Oresme counters this with an illustration: "I imagine the case of a tile or copper pipe or other material so long that it reaches from the center of the earth to the upper limit of the region of the elements, that is, up to the very heavens. I say that, if this tile were filled with fire except for a small amount of air at the very top, this air would drop down to the center of the earth for the reason that the less light body always descends beneath the lighter body. And if this tile were full of water save for a small quantity of air near the center of the earth, this air would mount up to the heavens, because by nature air always moves upward in water. From these it appears that air can, by reason of its nature, descend and move upward to the distance of the semidiameter of the sphere of the elements. Now, these two motions are both simple and contrary, and thus a simple body is by its nature capable of moving in two simply contrary motions." Oresme immediately qualifies this example, but it is the object of the reference above.

14. From the discussion that follows, Oresme illustrates this situation by assuming a heavy body in void space where presumably "no light body is attached" or in contact with the heavy body. Under these circumstances, the heavy body would remain at rest since there is no up or down in void and no lighter medium to fall through, as happened in the example to which he referred in note 13.

15. *Physics* IV.8.215a.6–13. Aristotle's fundamental point is that in void one part of emptiness is like every other, so that no absolute directions such as up or down can be identified or isolated.

16. Lacking directional places like "up" or "down", Aristotle denied that natural motion could occur in a vacuum, since, for him, all natural motion was necessarily directed toward a place.

17. Aristotle does not actually mention the center of the earth and the concavity of the lunar sphere in the chapter referred to by Oresme. But he does discuss the upper and lower contrary directions associated with the contrary motions of up and down. See *De caelo* I.6.273a.7–18.

18. Oresme considers each world a closed system where every bit of matter acts in accordance with Aristotle's principles of natural motion and remains in its own

In Chapter Twenty Aristotle mentions another argument from what was said in the *Metaphysics*—namely, that there cannot be more than one God and, therefore, it seems there can be only one world. I reply that God is infinite in His immensity, and, if several worlds existed, no one of them would be outside Him nor outside His power; but surely other intelligences would exist in one world and others in the other world, as already stated. And my reply to this argument is given more fully in Chapter Twenty. He argues again in Chapters Twenty-two and Twenty-three of which the purport is briefly this: this world is composed of all the matter avilable for the constitution of a world, and outside this world there can be no body or matter whatsoever. So it is impossible that another world exists.[19] In reply, I say in the first place, that, assuming that all the matter now existing or that has ever existed is comprised in our world, nevertheless, in truth, God could create *ex nihilo* new matter and make another world. But Aristotle would not admit this. Thus, I say, secondly, that, assuming that nothing could be made save from matter already existing and considering the replies we have given to Aristotle's first arguments regarding this problem—arguments whose substance he repeats and employs here in the present case—nonetheless he does not prove that another or more than one world besides our own could not now exist or may not always have existed, just as he states this world of ours to exist without beginning or end. He argues again in Chapter Twenty-four that outside this world there is no place or plenum, no void, and no time;[20] but he proves this statement by saying that outside this world there can be no body, as he has shown by the reasoning above to which I have replied; so it is unnecessary to answer this argument again. But my position could be strengthened or restated otherwise; for, if two worlds existed, one outside the other, there would have to be a vacuum between them for they would be spherical in shape; and it is impossible that anything be void, as Aristotle proves in the fourth book of the *Physics*.[21] It seems to me and I reply that, in the first place, the human mind consents naturally, as it were, to the idea that beyond the heavens and outside the world, which is not infinite, there exists some space whatever it may be, and we cannot easily conceive the contrary. It seems that this is a reasonable opinion, first of all, because, if the farthest heaven on the outer limits of our world were other than spherical in shape and possessed some high elevation on its outer surface similar to an angle or a hump and if it were moved circularly, as it is, this hump would have to pass through space which would be empty—a void—when the hump moved out of it.[22] Now, if we assumed that the outermost heaven was not thus shaped or that nature could not make it thus, nevertheless, it is certainly possible to imagine this and certain that God could bring it about.[23] From the assumption that the sphere of the elements or of all bodies subject to change contained within the arch of the heavens or within the sphere of the moon were destroyed while the heavens remained as they are, it would necessarily follow that in this concavity there would be a great expanse and empty space. Such a situation can surely be imagined and is definitely possible although it could not arise from purely natural causes,[24] as Aristotle

world. Should it be removed to another world, it would become a part of that world and move with respect to the absolute places of that world. John Buridan devoted a special question to this problem and came to the same conclusion as did Oresme—*Questions on De caelo*, Book I, Question 18: "Whether, if there were several worlds, the earth of one would be moved naturally to the middle of another world" (edition of E. A. Moody [Cambridge, Mass.: Mediaeval Academy of America, 1942], p. 83). Others, including Albert of Saxony, also adopted this position. Although none believed in the actual existence of more than one world, they were compelled to acknowledge that God could create more if He wished. On the assumption that he did create more and they existed simultaneously, many scholastic natural philosophers seemed interested in pursuing the seeming cosmological and physical consequences that would follow from a hypothetical plurality of worlds. Most concluded that no physical absurdities would follow and seemed satisfied that they could save the hypothetical phenomena which they had imagined.

19. *On the Heavens* I. 9.279a.7–11.

20. *On the Heavens*, I.9.279a.12–19.

21. *Physics* IV.6–9. For much of the remainder of this section Oresme considers the existence of an infinite void space lying beyond our finite cosmos. This topic is discussed in the next selection.

22. With only slight alterations, Oresme is here utilizing an illustration which Aristotle employs to demonstrate the sphericity of the universe in *On the Heavens* II.4.278a.12–23.

23. Appeals to God's absolute power to do a multiplicity of things deemed absurd by Aristotle were commonplace as a consequence of the Condemnation of 1277 (Selection 13).

24. Imagining that God annihilates all or part of the elemental spheres, thereby leaving behind a void space, was commonplace in the fourteenth century. The various consequences relevant to problems of motion that would have followed upon such an action were frequently described and discussed (for an illustration, see Selection 53.1).

shows in his arguments in the fourth book of the *Physics*, which do not settle the matter conclusively, as we can easily see by what is said here. Thus, outside the heavens, then, is an empty incorporeal space quite different from any other plenum or corporeal space,[25] just as the extent of this time called eternity is of a different sort than temporal duration, even if the latter were perpetual, as has been stated earlier in this chapter. Now this space of which we are talking is infinite and indivisible, and is the immensity of God and God Himself,[26] just as the duration of God called eternity is infinite, indivisible, and God Himself, as already stated above. Also, we have already declared in this chapter that, since our thinking cannot exist without the concept of transmutation, we cannot properly comprehend what eternity implies; but, nevertheless, natural reason teaches us that it does exist. In this way the Scriptural passage, Job 26: [7], which speaks about God can be understood: Who stretchest out the north over the empty place. Likewise, since apperception of our understanding depends upon our corporeal senses, we cannot comprehend nor conceive this incorporeal space which exists beyond the heavens. Reason and truth, however, inform us that it exists. Therefore, I conclude that God can and could in His omnipotence make another world besides this one or several like or

25. On the basis of other statements made in *Le Livre du ciel et du monde*, Oresme conceived of this void space as infinite and extensionless, because God, who is identified with it (see the next few lines of this selection) is infinite and extensionless (though, paradoxically, omnipresent). Not only did Oresme view this infinite, incorporeal void space as the container of many possible worlds but in Book II, chapter 8, also envisioned it as the spatial backdrop against which one could conceive of an absolute motion arising if God chose to move our whole spherical finite cosmos in a straight line. In the quotation which follows, we find an appeal to God's absolute power as well as an actual reference to Article 49 of those condemned in 1277:

"But perhaps someone will say that to move with respect to place is to change one's position in relation to some other body which may, or may not, be in motion itself. Yet I say that this is not valid primarily because there is an imagined infinite and immobile space outside the world, as was stated at the end of the twenty-fourth chapter of the first book, and it is possible without contradiction that the whole world could be moved in that space with rectilinear motion. To say the contrary is an article condemned at Paris. Now assuming such a motion, there would be no other body to which the world could be related with respect to place, and the description given above would be invalid." (The translation is my own from Menut's French text on pp. 368, 370.)

In the famous Clarke-Leibniz correspondence of 1715–1716, Samuel Clarke found this unusual illustration useful in defending Newton's absolute space against Leibniz's relational concept of space. Clarke argued that "if space was nothing but the order of things coexisting [as Leibniz maintained], it would follow that if God should remove the whole material world entire, with any swiftness whatever, yet it would still always continue in the same place" (Par. 4 of Clarke's Third Reply to Leibniz in 1716, quoted, with altered punctuation, from H. G. Alexander, ed., *The Leibniz-Clarke Correspondence* [New York: Philosophical Library, 1956], p. 32). The point of Clarke's argument is to show that if space is but a relation between coexistent things—as Leibniz held—and God moved the world taken as a single thing, the world could not be said to have undergone any motion, since there would be no other existent thing to which it could be related. For Clarke, as for Oresme, the hypothetical conditions under which God moves the world would indeed produce a motion, one that is absolute.

26. Oresme here identifies the infinite extramundane void space with God Himself (in this he differs from Bradwardine; see Selection 73.1, n. 25). Despite a lack of explicitness, and even some ambiguity, it seems that Oresme would have denied physical extension to the extramundane void. He does explain that "God in His infinite grandeur without any quantity and absolutely indivisible, which we call immensity, is necessarily all in every extension or space or place which exists or can be imagined. This explains why we say God is always and everywhere . . ." (Bk. II, ch. 2; Menut translation, p. 279). Thus God, though dimensionless, would be in every physically extended space. Is it not plausible, then, to assume that the infinite void beyond the cosmos is also physically extended and occupied wholly by God, just as He would occupy any finite physical space? This seems unlikely, however, for as the passage above indicates, Oresme identified the extramundane void with God Himself, who is dimensionless; hence this particular space would also be dimensionless. But nowhere does Oresme identify "God Himself" with actual physical or dimensional spaces; he rests content to say only that God is in them because He is everywhere.

It would seem, then, that Oresme's identification of God with an infinite void beyond the cosmos conferred upon the latter a transcendent and nondimensional character (this is not to say, however, that Oresme did not conceive of bodies moving through this "transcendent" infinite space, or that it might not serve as the space of other worlds; see n. 25). Many medieval authors would have construed it in much the same manner (not, however, Johannes de Ripa; see Selection 73, introduction). Some authors in the seventeenth and eighteenth centuries—for example, Henry More, Joseph Raphson, and Isaac Newton—would come to associate God with a physically extended infinite void and conceive of God as actually extended in physical space. Indeed, Raphson, Newton, and Samuel Clarke conceived of physical space as an attribute of God (see my article "Medieval and Seventeenth-Century Conceptions of an Infinite Void Space Beyond the Cosmos," *Isis*, LX [1969], 57–59). Thus, as Leibniz was to note in his controversy with Samuel Clarke, God was in danger of becoming a physically extended being.

unlike it. Nor will Aristotle or anyone else be able to prove completely the contrary. But, of course, there has never been nor will there be more than one corporeal world, as was stated above.

72 ON THE EXISTENCE OF AN IMAGINARY INFINITE VOID SPACE BEYOND THE FINITE COSMOS

Hermes Trismegistus to Asclepius: Void beyond the Cosmos Lacks Matter but not Spirit

Translated by Walter Scott[1]

Introduction and annotation by Edward Grant

The Latin dialogue between Hermes Trismegistus and Asclepius was translated from a Greek original now lost. The text, which for a variety of reasons is faulty, has required considerable emendation based on careful study. Scott explains in his translation, "I have aimed at expressing what I suppose to have been the meaning of the original Greek, rather than the meaning—or, too frequently, the absence of meaning—of the Latin" (*Hermetica*, I, 51). *Asclepius* was subdivided into three parts by Scott because it seemed to consist of that many distinct and unrelated parts. The following selection is from *Asclepius* III, which Scott believes to have been composed in the original Greek in Egypt by an Egyptian sometime between 260 and 310, indeed perhaps between 268 and 273. Its translator into Latin is unknown, but Scott suggests C. Marius Victorinus, who died soon after 362. The treatise was reasonably well known in the Latin Middle Ages, and a number of manuscripts of it exist.

This selection on void may have influenced Bradwardine (see Selection 73.1 and n. 7 thereto) and other Christians to suppose that beyond the cosmos there existed a space void of matter but not God. Christians could easily have placed such an interpretation on Hermes Trismegistus' declaration that "I hold that not even the region outside the Kosmos is void, seeing that it is filled with things apprehensible by thought alone, that is, with things of like nature with its own divine being." Since all space is occupied by matter or spirit, Hermes Trismegistus denies the existence of void. But void in the usual sense of a place devoid of matter may indeed exist.

But as to Void, which most people think to be a thing of great importance, I hold that no such thing as void exists, or can have existed in the past, or ever will exist. For all the several parts of the Kosmos are wholly filled with bodies of various qualities and forms, each having its own shape and magnitude; and thus the Kosmos as a whole is full and complete. Of these bodies, some are larger, some are smaller; and they differ in the greater or lesser firmness of their substance. Those of them which are of firmer substance are more easily seen, as are also those which are larger; whereas smaller bodies, and those which are of less firm substance, are almost or quite invisible, and it is only by the sense of touch that we are made aware of their existence. Hence many people have come to think that these bodies do not exist, and that there are void spaces; but that is impossible. And the like holds good of what is called the "extramundane," if indeed any such thing exists; for I hold that not even the region outside the Kosmos is void,[2] seeing that it is filled with things apprehensible by thought alone, that is, with things of like nature with its own divine being. And so our Kosmos also—the sensible universe, as it is called—is wholly filled with bodies, and living bodies, suited to its character. The shapes presented by these bodies to our sight differ in magnitude; some of these shapes are very large; others are very small, when the distance of the objects makes them appear small to us; and some things, on account of their extreme minuteness of tenuity, are wholly invisible to us, and are consequently supposed by many people to be nonexistent.

And so, Asclepius, you must not call anything void, without saying what the thing in question is void of, as when you say that a thing is void of fire or water or the like. For it is possible for a thing to be void of such things as these, and it may consequently come to *seem* void; but the thing that

1. In *Hermetica, the Ancient Greek and Latin Writings Which Contain Religious or Philosophic Teachings Ascribed to Hermes Trismegistus*, edited with English Translation and Notes by Walter Scott (Oxford: Clarendon Press, 1924), I, 317–321.

2. Certain Stoics (notably Posidonius) believed in the existence of an infinite void beyond the visible and finite cosmos (but denied the existence of void within the cosmos). The author of *Asclepius* III, however, speaks of an extramundane region in hypothetical terms only.

seems void, however small it be, cannot possibly be empty of spirit and of air.

And the like must be said of Space.[3] The word "space" is unmeaning when it stands alone; for it is only by regarding something which is in space, that we come to see what space is; and apart from the thing to which it belongs, the meaning of the term "space" is incomplete. Thus we may rightly speak of the space occupied by water, and fire, and so on (but not of space alone). For as there cannot be a void, so it is impossible to determine what space is, if you regard it by itself. For if you assume a space apart from something which is in it, it will follow that there is a void space; and I hold that there is no such thing as that in the universe. If void has no existence, then it is impossible to find any real thing answering to the word "space" taken by itself.

73 ON A GOD-FILLED EXTRAMUNDANE INFINITE VOID SPACE

Introduction, translations, and annotation by Edward Grant

In these selections on the existence of an infinite extramundane void space, we shall see an essentially Christian and theological reaction to Aristotle's claim that there can be "neither place, nor void, nor time outside the heaven,"[1] a conclusion reached in the process of rejecting the existence of other worlds. Despite Aristotle's position, subsequent authors could not refrain from inquiring about the hypothetical disposition of objects, say a lance or an arm, pushed through the outermost celestial sphere. Where Aristotle would have deemed such questions absurd, arguing nothing whatever existed beyond the world, Stoics and some others who subscribed to a finite spherical universe came to hold that an infinite void lay beyond (see the previous selection, n. 2). Thus a lance or an arm pushed beyond the universe would emerge into an infinite void. This type of example and Stoic advocacy of an extramundane infinite void came to be widely known in the Middle Ages through William of Moerbeke's Latin translation of Simplicius' *Commentary on Aristotle's De caelo* (see Selection 8, no. 47), where a brief summary of Stoic arguments was presented.[2] This, along with the general availability of *Asclepius III*, where, as we have seen, it was emphasized that if extramundane void existed it would be filled with spirit,[3] came to constitute one of the two major sources for medieval discussions of extramundane void. When we add to all this the theological concept of God's absolute power which emerged from the Condemnation of 1277 (see Selection 13) we have the major historical influences that shaped Bradwardine's interesting arguments. And the other authors from whom excerpts have been included in this selection[4] would argue, as did Bradwardine, for the existence of an infinite extramundane void filled with an omnipresent God, a void that was very different indeed from the spiritless and truly empty void proposed by the Stoics. This medieval conception seems ultimately to have influenced the Cambridge Platonists, (for instance, Henry More), Otto von Guericke, Isaac Barrow, and Sir Isaac Newton, whose absolute space is ontologically strikingly similar to the medieval God-filled extramundane void space.

In the selections from Bradwardine, Oresme, and the Coimbra Jesuits, we appear to have a medieval

3. Throughout this paragraph Scott used the term "space" (instead of "place") to translate *locus*, not *spatium*. However, "place"—in the restricted sense of the container of a body—would have been preferable. Although *locus* and *spatium* appear sometimes to be used interchangeably in the sense of a container, the term *spatium* is more usually introduced when distance or extent is intended.

1. *De caelo* I.9.279a.12–13, 17–18.

2. Simplicius tells us (fol. 44 verso, col. 2, of the edition published in Venice, 1540): "The Stoics, however, thinking that there is a vacuum beyond the sky, prove it by this kind of assumption: Let it be assumed that someone standing motionless at the extremity [of the world] extends his hand upward. Now, if his hand does extend, they take it that there is something beyond the sky to which the hand extends. But if the arm could not be extended, then something will exist outside that prevents the extension of the hand; but if he then stands at the extremity of this [obstacle that prevents the extension of his hand] and extends his hand, the same question as before [is asked], since something could be shown to exist beyond that being." I have quoted this translation from my article "Medieval and Seventeenth-Century Conceptions of an Infinite Void Space beyond the Cosmos," *Isis*, LX (1969), 41. Thomas Aquinas discussed these arguments in his *Expositio in De caelo*. They seem also to have been known to John Buridan and Nicole Oresme.

3. See Selection 72, introduction. In Bk. XI, ch. 5 of the *City of God*, St. Augustine says much the same thing, and was probably familiar with the Latin version of *Asclepius* III.

4. Summaries and analyses of their discussions are given in my article in *Isis* cited in note 2. Some of the notes in this selection are based upon my article.

tradition in which an extramundane infinite void was conceived as either dimensionless (Bradwardine and Oresme are hardly clear on this point; for an explicit statement from the Coimbra Jesuits see Article 4, section 3 below), or dimensional in a special transcendent sense associated with the "dimensions" of a Divine Being who is Himself extensionless. Bradwardine and the Coimbra Jesuits, and probably Oresme, are quite emphatic in characterizing this infinite extramundane void as an existent entity—for it is the immensity of God Himself, who cannot be confined to a finite world—that has no real and positive being of its own. These authors seem to have conceived of infinite extramundane void space—it is also characterized as imaginary—as coextensive (in a transcendental sense) with God's total immensity (or "extent") and therefore eternal, because it is God Himself.

But another important view (not represented in this selection) was proposed around 1350 by Johannes de Ripa. He conceived an imaginary infinite extramundane void space as being less "extensive" than the immensity of God Himself. Indeed, God's infinite immensity was said to circumscribe and infinitely exceed the immensity of the imaginary infinite void beyond the world (see "Jean de Ripa, I Sent. Dist. XXXVII: De Modo Inexistendi Divine Essentie in Omnibus Creaturis. Édition critique par André Combes et Francis Ruello, présentation de Paul Vignaux. Immensité Divine et Infinité Spatiale," *Traditio*, XXIII (1967), 235, 238 (text), 199 (Vignaux's discussion). Perhaps because he radically distinguished between the dimension of the infinite void space and God's infinitely more extensive immensity, Johannes conceived of the infinite void as a possible three dimensional enity. Of great interest, however, is the fact that he too denied that this imaginary infinite extramundane space was a positive and real thing (Vignaux, pp. 232, 233, 234).

Whether Otto von Guericke conceived of imaginary extramundane infinite void as coextensive with God Himself or inclined rather to a position more in accord with the view of Johannes de Ripa is difficult to determine. In one instance he declares that God can be contained by no place, since "He Himself is the place for Himself, or the thing that is empty *(vacuum)* of every creature, or the space, or universal container of all things." Elsewhere, however, he asserts that "the space and vacuum of the whole creation is in Him, by Him, and for Him" (see section 4). On a conjectural basis, it would seem appropriate to regard von Guericke as closer to Bradwardine and the Coimbra Jesuits than to de Ripa in believing that total infinite void space is coextensive with God Himself.

On one significant point von Guericke breaks completely with his medieval predecessors just mentioned: He insists that the imaginary space beyond the world is indeed a *real and positive* thing (see near end of Bk. II, ch. 6, in section 4). Perhaps this judgment was generated in part by seventeenth-century experiments and discoveries concerning atmospheric pressure, achievements in which von Guericke had himself played a leading role. Not only had it been demonstrated that nature did not abhor a vacuum (see Selection 54) but production of three-dimensional artificial vacua might have encouraged acceptance of the vacuum as in some sense a real three-dimensional entity. Moreover, ancient atomism, with its three-dimensional void extending to infinity, and Stoic cosmology, with its sealed finite world surrounded by a real three-dimensional infinite void, were well known, and each had its partisans. For some, or all, of these reasons, it is perhaps not surprising that Otto von Guericke and others in the seventeenth century came to regard three-dimensional void space as positive and real.

1. Thomas Bradwardine (ca. 1290–1349)[5]

Bradwardine employed a quasi-mathematical form, announcing propositions and deriving corollaries from them. A French translation of many of the passages relevant to infinite void space was made by Alexandre Koyré: "Le vide et l'espace infini au XIV[e] siècle," *Archives d'Histoire Doctrinale et Littéraire du Moyen Age*, 24th Year (1949), pp. 45–91. My translations and annotations in this and the following selections on extramundane void space were an outgrowth of a research project on the concept and role of void space in the history of medieval physics supported by the Division of Social Sciences of the National Science Foundation.

[COROLLARIES]

1. First, that essentially and in presence, God is necessarily everywhere in the world and all its parts;

5. This selection is from Thomas Bradwardine, *De causa Dei contra Pelagium et De virtute causarum. . .* (London: Henry Savile, 1618), Bk. I, ch. 5, pp. 177–180. This theological treatise was apparently written in 1344. In a manner similar to Spinoza in the *Ethics*,

2. And also beyond the real world in a place, or in an imaginary infinite void.[6]

3. And so truly can He be called immense and unlimited.

4. And so a reply seems to emerge to the old questions of the gentiles and heretics—"Where is your God?" And, "Where was God before the [creation of the] world?"

5. And it also seems obvious that a void can exist without body, but in no manner can it exist without God.[7]

. . . . . .

That God is necessarily everywhere in the world follows obviously.[8] For if He were not in any [particular] place in the world, but could be in it—since, by the seventh part of the [corollary] of the first [chapter], He is omnipotent—He cannot do this by His motion, nor through the motion of any creature, since there is a creature there now, and there was always one there previously. And the same reasoning applies to any other creature. This suffices to show the existence of God there, but not by his motion, as the chapter has shown. The same thing is plainly shown by a corollary of the second [chapter] of this book and by its demonstration.

In order to demonstrate the second part,[9] I assume that *A* is the fixed, imaginary place of this world, and *B* is simultaneously an imaginary place beyond the world that is separated from *A*; and I assume that God moves the world from *A* to *B*, placing it in *B*. Then, by the first part [or corollary] of this [fifth] chapter and by a corollary of the second [chapter] and its demonstration, God is now in *B*.[10] Therefore, either He was there [in *B*] before, or not. If He was there, then, by the same reasoning, He was there before and can now be imagined as everywhere outside the world.[11] If He was not there before, then, by the same reasoning, He is not now in *A*, but [must be] in *B*. Therefore, God was in *A* before, but not in *B;* and He is not now in *A* but in *B*, which is completely separate. Consequently, He withdrew from *A* and came to *B*, so that He was moved locally or with respect to position, just as our soul is moved with the motion of our body.

Some of those who follow the Philosopher [Aristotle] in Book I of *On the Heavens* reply to this. They assume that every local motion is necessarily upward, downward, or circular—that is, away from the center [of the world], toward the center, or around the center.[12] But they say that if this motion [from *A* to *B*] were assumed, it could not be any one of the ways just mentioned. For this reason, they say that it is impossible for the world to be moved. But these [followers of Aristotle] seriously diminish and mutilate the divine—indeed, omnipotent—power. For, in the beginning, God could have created this world in *B*. Why, then, is He unable to put it in *B* now? Furthermore, He can now create another world in *B*. Why, then, is He unable to put this world in *B*? This reply [that the world cannot be moved] is condemned by Stephen, Bishop of Paris, in these words: "That God could not move the heavens [that is, the world] with rectilinear motion; and the reason is that a vacuum would remain."[13] But this response does not avoid the difficulty. For it could be assumed that without [resort to] local motion, God could create another world in *B* and annihilate the world in *A*. [On this assumption,] the difficulty returns.[14]

6. Bradwardine nowhere explains what he means by "imaginary infinite void." Perhaps it was "imaginary" because of its incomprehensibility, which arose from its necessary lack of dimensionality stemming from the fact that God, who filled it, is dimensionless. Or, perhaps he rather inclined toward Oresme's indirect suggestion (see Selection 71, at the end) that infinite incorporeal space is "imaginary" because it is imperceptible to the senses and comprehensible—if at all—by the reason alone. A few authors discuss this term explicitly (see Question 2, Article IV, of section 3 and Bk. I, ch. 35 of section 4).

7. The influence of *Asclepius* III is detectable here, for, as we saw in the preceding selection, its author insisted that if void existed, it would of necessity be filled with spirit. This treatise was known in the Middle Ages as *De aeterno verbo (On the Eternal Word)*. Although Bradwardine does not mention it in the section translated here, he cites it elsewhere in the *De causa Dei*. Indeed, he calls Hermes Trismegistus "a brilliant prophet and glorious philosopher" ("Hermes Mercurius Trismegistus, clarus propheta et philosophus gloriosus"; *De causa Dei*, p. 98). A quote from *Asclepius* appears in Question 2, Article II, of section 3.

8. The first corollary is now demonstrated. Bradwardine argues that without undergoing movement, God is omnipresent in the world.

9. That is, the second corollary, namely that God exists beyond the world in an imaginary infinite void.

10. After moving the world from *A* to *B*, God must be in *B*, the place of the world, since by the first corollary God is everywhere inside the cosmos.

11. Obviously, place *B* can be rightly conceived as representing each and every place outside, or beyond, the world. Hence God is everywhere.

12. Aristotle, *De caelo* I.2.268b.16–24.

13. This is Article 49 of the Condemnation of 1277 (see Selection 13 and n. 15 thereto). Reliance on God's absolute power to perform acts contrary to Aristotelian physics and metaphysics is a vital component of Bradwardine's over-all approach.

14. Thus, whether the world is transferred from place

Other followers of the Philosopher would, it seems, respond to this by saying that there is no place or void outside of [or beyond] the world;[15] therefore God is not there and the world cannot move there. But as a consequence these [followers of the Philosopher] must then say that God, for all His omnipotence, could not have made the world greater or smaller in any thing, which would very much restrict and confine His omnipotence. For they must say that God necessarily makes the world in place *A*, and that place *A*, and no other, existed before there was a world.[16] But why this place, and no other? Why was the world just this size and no greater or smaller? For, indeed, either this [that is, the size of the world] was fixed by God, or by itself and not by God. If by God, then, in virtue of his infinite power, He could make a place greater and greater; and [He could] make another place, and yet another place, without end. If, however, the size is determined by the world itself —and not by God—what power determines this? What is the reason for it? What nature has fixed the world at this [particular size], and inviolately determined the limit beyond which it cannot pass?[17]

Moreover, as no one can ignore, this imaginary place *(situs imaginarius)* could have no positive nature, for otherwise there would be a certain positive nature which is not God, nor from God, the opposite of which was shown in the second and third [chapters] of this [book]; such a nature would be coeternal with God, something no Christian can accept. It was for this reason that Bishop Stephen of Paris condemned an article asserting that "many things are eternal,"[18] and that even if He wished, God could not destroy them; therefore, He is deprived of omnipotence. The argument of these [Aristotelians] tells against them. For if, according to the assumption of the Philosopher and his followers, there could be no void, nor any imaginary place not filled with body, [then] the world is eternal, which is heretical, and which these people [themselves] deny;[19] or, prior to the creation of the world, there was an imaginary void place, unoccupied by any body. The previously cited article [of Bishop Stephen of Paris] condemned this reply and its irrational argument.[20] Since these arguments do not serve to hinder us in any way, it appears that the reasoned premise stands unscathed.[21]

On the contrary, creating the world and every part of it in the beginning, God was simultaneously with the world and every part of it. For . . . in every motion and action, the mover and agent exists simultaneously with what is moved and suffers the action. Thus Augustine in his second Homily on John, 1, [where John says] "He was in the world and the world was made by Him,"[22] asks, "How was He in the world?" And he replies, "As the artist, ruling what he makes." For He does not make things as does a carpenter who is outside the box that he makes, a box which, during its construction, is also located in another place [than the carpenter]. And though he is very near to it, yet he who makes it is located in another place, and is external to what he makes. God, however, is infused into the world He makes, which is placed wherever He makes it. And He does not withdraw from anything and is not external . . . [to what He

A to *B* by local motion, a possibility denied by Aristotelians, or by the divine power annihilating the world in *A* and recreating it in B, the same problem arises: Was God in place *B* before the creation of the world?

15. Aristotle, *De caelo* I.9.279a.12–13, 17–18.

16. No proper Aristotelian could have subscribed to the consequence drawn by Bradwardine, since it was a fundamental tenet of Aristotelian cosmology and physics that the universe was not itself in a place (*Physics* IV.5.212b.7–11). Perhaps Bradwardine thought that by restricting God's omnipotence with the claim that He "could not have made the world greater or smaller," the Aristotelians were committed to the consequence that "place *A*, and no other, existed before there was a world." That is, *A* was a pre-existent place into which God had to fit the world exactly; therefore, He could not tinker with its size.

17. Apparently deeming the alternative opinion absurd, Bradwardine adopts, without further comment, the position that God determined the size of the world and the size of its place and could have made it any size whatever.

18. Probably a reference to Article 52 of the Condemnation of 1277.

19. The sense of this argument would appear to be that if no void space or place ever existed, then it follows that a plenum has always existed. Hence matter—and consequently, the world—is eternal. As Christians, the Aristotelians, whom Bradwardine here opposes, would have been committed to the repudiation of such a position.

20. The alternative argument, that "prior to the creation of the world, there was an imaginary void place, unoccupied by any body," is also rejected by appeal to Article 52 of those condemned in 1277. Such a pre-creation void place conceived as mere emptiness devoid of matter *and God* would be a thing coeternal with God. As Bradwardine will argue in what follows, God himself must necessarily exist in such a void place—indeed in all such void places, for He is omnipresent.

21. The "reasoned premise" is that "this imaginary place could have no positive nature"; therefore, no positive thing is coeternal with God.

22. John 1:10.

makes]. The presence of [His] majesty makes what He makes; [and] His presence governs what He has made. Therefore, He was in the world even as the world was made by Him. . . .

God is a foundation without a foundation—that is, He is based in no prior thing but is rather the first and original foundation of all other things, on which all other things rest, since they are unstable themselves, while the foundation by which they are always maintained and continuously supported is itself fixed, stable, and immobile, as was shown in many ways in the second [chapter]. However, the contrary of this is not true. God can be in any place He wishes without the need of a creature; and He is not there anew, since He does not get there by His motion, as the fifth [chapter] has shown;[23] therefore He is there in eternal rest. Hence God persists essentially by himself in every place, eternally and immoveably everywhere. Furthermore, it is more perfect to be everywhere in some place, and simultaneously in many places, than in a unique place only; and a spirit that can do this is more perfect than a body unable to do this. But God is an infinitely perfect spirit, as the first assumption and its demonstration[24] reveal. Hence this is infinitely suitable for Him, and [it is so] without the need of any creature, as shown above, and without any mutability whatever. God is, therefore, necessarily, eternally, infinitely, everywhere in an imaginary infinite place, and so truly omnipresent, just as He can be said to be omnipotent. In a certain sense, and for a similar reason, He can also be called infinite, [as for example], infinitely great, or of infinite magnitude, even though extensive [is taken in] a metaphysical and improper sense. For He is infinitely extended without extension and dimension.[25] For truly, the whole of an infinite magnitude and imaginary extension, and any part of it, coexist simultaneously, for which reason He can be called immense, since He is unmeasured, nor is He measurable by any measure; and He is unlimited, because nothing surrounds Him fully as a limit; nor, indeed, can He be limited by anything, but [rather] He limits, contains, and surrounds all things. Thus Sixtus[26] the Pythagorean says in his aphorisms (*in sententiolis suis*) that "you could not discover the magnitude of God even if you could fly with wings." This opinion is attested to by the second of the twenty-four definitions of God posited by the twenty-four Philosophers,[27] which says, "God is an infinite sphere whose center is everywhere and circumference nowhere";[28] and the eighteenth definition, which says, "God is a sphere that has as many circumferences as points"; and the tenth definition, which says, "God is that whose power is not numbered, whose being is not enclosed, [and] whose goodness is not limited. . . ."

. . . .

. . .Indeed, if against the possibility of the rectilinear motion of the whole world,[29] which was touched on above, anyone should oppose the opinion of the Philosopher mentioned before, namely that every motion is from the center, or around the center of this world, it must be said that he speaks of species of natural motions of natural bodies and parts of this world. But if anyone objects because of the vacuum [that would be left behind], it must be said that perhaps he under-

23. See the discussion immediately after the Corollaries at the beginning of this section, and note 8.

24. A reference to the first supposition of the first chapter, which declares that "God is the highest perfection and good because there can be nothing more perfect or better." *De causa Dei*, p. 1; this is demonstrated on pages 1–2.

25. Since the infinite void prior to the creation of the world was occupied by an omnipresent God who is "infinitely extended without extension and dimension," it seems plausible to conclude that the infinite extra cosmic void was thought to be dimensionless—especially since Bradwardine had earlier argued that the void possessed no independent existence, for then it would be coeternal with God. Although Bradwardine did not identify infinite void space with God or call it an attribute of God, Oresme did make such an identification but, like Bradwardine, seems to have denied dimensionality to this void. Others would not only confer dimensionality upon it (see Introduction; near end of Book II, ch. 6, of section 4; and n. 71), but would also conceive of God as in some sense physically extended within this space, which was even described as His attribute (see Selection 71 and note 26 thereto).

26. Is this Sextus, or Sextius, the Pythagorean, who lived in the reign of Augustus or Tiberius, sometime between 29 B.C. and A.D. 37? I have not located the quotation.

27. The three definitions following are drawn from the *Book of the XXIV Philosophers*, a popular Pseudo-Hermetic treatise. The Latin text was edited by Clemens Baeumker, "Das pseudo-hermetische 'Buch der vierundzwanzig Meister' (Liber XXIV philosophorum)" in *Beiträge zur Geschichte der Philosophie des Mittelalters*, Band XXV, Heft 1–2 (Münster, 1927), pp. 194–214. The three definitions appear on pages 208, 210, 212.

28. This definition became widely known in the Renaissance and influenced, among others, Nicholas of Cusa, Marsilio Ficino, Giordano Bruno, and Robert Fludd, as well as the anonymous Jesuit commentator at Coimbra, from whose treatise we have included a selection on infinite void (see Question 2, Article II, of section 3, and n. 43).

29. See note 13.

stands that vacuum must not be posited necessarily and naturally in the world nor even outside the world, because of the effects which the Pythagoreans, Democritus and Leucippus, and many ancient philosophers, assumed from it, namely because of local motion, motion of augmentation and diminution, condensation and rarefaction, motion of alteration, and for distinguishing beings lest they be one, or because of respiration of the heavens from an infinite void beyond it, as is obvious in [Book] 4 of the *Physics*, [Books] 3 and 4 of *On the Heavens and the World*, and [Book] 2 of *On the Soul*, and also in other places.[30]

And it seems that vacuum can be assumed in two ways: in one way only privatively, namely as pure privation itself or the pure lack of a plenum; in another way positively, and this can be assumed in two ways. In one way, it would only be a corporeal dimension separated from natural forms, as if it were a mathematical body, or a quantity by itself separated from other natural things; it is sometimes taken in this way in the fourth [book] of the *Physics*. In another way it would also have an active power and be moved according to place.[31] Vacuum is also defined by him [Aristotle] as a place that is not filled, but is capable of being filled. Moreover, according to him [Aristotle], place is an ultimate [surface], namely the last surface of the containing body.[32] Furthermore, such a void is not outside the heavens; nor is there a place [beyond the heavens] that could be filled by natures formed [or constructed] from natural things. Nevertheless, these arguments do not at all convince with an irrefutable necessity. Indeed, by means of His absolute power, God could make a void anywhere that he wishes, inside or outside of the world. Truly, even now, there is in fact an imaginary void place outside of the world, which I say is void of any body and of everything other than God, as shown by what has preceded. . . .

However, one might argue against [my position by saying] that if such a void place existed outside this world, another world would be possible there. This is contrary to the same Philosopher in his *On the Heavens*, [Text] 76,[33] and afterward where he rejects this in many places. But who would deny or doubt this conclusion, except, perhaps, one who denies the omnipotence of God[34]. . . .

2. Nicole Oresme (ca. 1325–1382)

See Selection 71.[35]

3. Jesuit Commentators at the College of Coimbra, Portugal (second half of sixteenth century)[36]

Question 2

Whether or Not God Exists Beyond the Sky [or Celestial Heavens]

ARTICLE I

The Argument On the Negative Side of the Controversy

Those who have followed the negative part of this question are Scotus[37] in a unique question of

30. Here Bradwardine has brought together from a number of Aristotelian works phenomena which the atomists sought to explain by the motion of atoms in void space. He argues that one ought not to posit the existence of vacuum for the reasons which they offered, namely that it could explain the above-mentioned phenomena.

31. Aristotle, *De caelo* IV.2.309b.15–19.

32. Both definitions appear in Book IV of the *Physics*.

33. Aristotle's preliminary statement against a plurality of worlds appears in Book I, Text 76, of the medieval text of Aristotle's *De caelo*; the argument does not conclude until Book I, Text 85. In the modern editions and translations of *De caelo*, see I.8.276a.18—277a.12.

34. Here, again, Bradwardine rejects Aristotle's restriction on God's absolute power to create more worlds if He pleased. In so doing, Bradwardine abides by the spirit which motivated the Condemnation of 1277, probably with Article 34 in mind.

35. This selection properly begins on page 552 with the words: "He argues again in Chapter Twenty-four that outside this world there is no place or plenum, no void, and no time; . . ."

36. This selection is translated from the Commentary on Aristotle's *Physics* published in Cologne, 1602, under the title *Commentariorum Collegii Conimbricensis Societatis Iesu: In octo libros Physicorum Aristotelis Stagiritae* . . . . It is drawn from Book VIII, chapter 10, Question 2, Articles I, II (part), and IV, on cols. 514, 518–519.

37. In this question John Duns Scotus (ca. 1265–1308) denied the necessity for God's omnipresence in an infinite extramundane void on grounds that God's presence in a place was not a necessary precondition for His acting in that place. God's will, not His omnipresence, was taken as the basis of His actions. Hence, He could act in a place remote from His presence. With "action at a distance" as God's *modus operandi*, Scotus denied the necessity of God's presence in the empty place where He came to create the world and, a fortiori, rejected the necessity for God's omnipresence in an infinite void

[Book] I, Distinction 37 [of his *Sentence Commentary*]; St. Bonaventure in Article II, Question 1, Number 22 [of his *Sentence Commentary*, and] Capreolus,[38] in a unique question, Article III, and others.

In behalf of this position, some have put forth this most powerful argument: (1) There is nothing beyond the sky; but God cannot be in nothing; therefore He is not beyond the sky.

(2) Again, if God were in a space, which we can imagine beyond the encompassing sky, [and] since this space is, in truth, nothing positive or real, then by a parity of reasoning *(pari ratione)* it would be possible to say that God is in darkness, and in other privations, which is absurd.

(3) Next, just as we conceive such a space only in the mind, so it is a consequence that God is not in the thing itself but is said to exist in it only in our conception. Therefore, God does not really exist beyond the sky [or heaven].

(4) It happens that substances devoid of matter are not in place except by operation. Indeed, God operates nothing beyond [or outside] the world, just as He did not operate anything before the world was founded, which St. Augustine says in Confessions, Book XI, chapter 12: "[How, then, shall I respond] to him who asks, 'What was God doing before he made heaven and earth?' I do not answer, as a certain one is reported to have done facetiously (shrugging off the force of the question), 'He was preparing hell.' " And a bit later on [Augustine says,] "Before God made heaven and earth, he did not make anything at all. For if he did, what did he make unless it were a creature?"[39] Finally, St. Augustine holds the same opinion on this in his book *Against Maximinus*, chapter 21, where to those wishing to know where God was before He made the world, he [Augustine] replied that He was in Himself.[40] This is repeated in Psalm 120[41] in these words: "I have lifted up my eyes to the mountains." There is no mention here of an imaginary space in which God existed.

ARTICLE II

AN AFFIRMATIVE PART CONSISTS OF THE TESTIMONY OF PHILOSOPHERS

The opposite part of the question, however, is true, especially what Trismegistus teaches in *Asclepius*, where he says: "God who dwells above the summit of the highest heaven, is present everywhere, and from all around He watches all things. For there is beyond the sky a space *(spatium)* without stars removed from all things corporeal."[42] To these statements, he adds this definition of God, which is said to have been furnished by the same person: "God is an intelligible[43] sphere whose center is everywhere and circumference nowhere." In this description, the greatest perfection of the divine nature is signified by the sphere, the most absolute *(absolutissima)* [or perfect] of all figures. . . .

Moreover, that the same opinion concerning the existence of God beyond the world was followed by certain other ancient philosophers, as well as by Plato and Aristotle,[44] is testified to by Eugubinus in Book IV of *De perenni philosophia (On Perennial Philosophy)*, chapters 1 and 2. . . .

ARTICLE IV

WHAT IS IMAGINARY SPACE? WHEN THIS HAS BEEN DETERMINED [WE SHALL INQUIRE WHETHER] GOD EXISTS IN IT.

The solution to the arguments of the first article. For the most thorough explication both of our opinion and the arguments, certain things must be considered about space, and we shall do this with respect to space that is within and beyond the sky.

space. See Scotus' *Questio unica* in Book I, Distinction 37, of his *Questions on the Sentences* in *Joannis Duns Scoti . . . opera omnia editio nova juxta editionem Waddingi XII tomos continentem* . . . (Paris: Vivès edition, 1893), Vol. X, p. 597, col. 2.

38. That is, Johannes Capreolus (d. 1444), sometimes called "The Prince of Thomists" *(Thomistarum princeps)*.

39. These two passages from Augustine's *Confessions* are quoted here from the translation of Albert C. Outler, *Augustine: Confessions and Enchiridion* (Philadelphia: Westminster Press, 1955), p. 253.

40. I was unable to locate this statement in the work and chapter cited.

41. The text has 122.

42. Since the text of *Asclepius* III possessed by our author at Coimbra differs somewhat from that edited by Walter Scott in *Hermetica* . . . (Oxford: Clarendon Press, 1924), I, 324, I have altered the latter's translation accordingly. As the most significant variation, it should be noted that where Scott's text has *"huic est enim ultra caelum locus"* (translated as "his abode is beyond heaven"), the Coimbra text has *"Est enim ultra caelum spatium,"* which I have rendered as "For there is beyond the sky a space." See also note 7.

43. Although the text has *intelligibilis*, it should read *infinita*—that is, "God is an infinite sphere . . . ." This definition, which is not from *Asclepius* but is found in the pseudo-Hermetic *Book of the XXIV Philosophers*, was also quoted by Bradwardine and others (see Selection 73.1 and n. 28 thereto).

44. This is not true for Aristotle, unless one is prepared to argue that the prime mover was located beyond the world.

The first is that this [imaginary] space is not a true quantity possessed of three dimensions;[45] moreover, bodies cannot be received in it, since several such dimensions cannot be in the same position simultaneously by [any actions of] the forces of nature *(naturae viribus)*. Furthermore, it cannot be another real and positive being, since, beside God, no such thing could exist from eternity.[46] But this space always existed and always ought to be.

A second thing about it is that this space is not a being [or entity] of the reason,[47] since by means of this thing itself bodies are received within the world without the action of the intellect; and they can [also] be received outside [or beyond] the world if they were created there by God. Therefore, the dimensions of this space are not usually said to be imaginary because they are fictions *(fictitiae)* or depend solely on a mental conception or that they are thought to be beyond the understanding *(nec extra intellectum dentur)*, but because we imagine them in space, in a certain relation corresponding to the real and positive dimensions of bodies.[48]

We assert, therefore, that God is actually in this imaginary space, not as in some real being but through his immensity, which, because the whole universality of the world cannot [accommodate it], must of necessity also exist in infinite spaces beyond the sky.[49] But God Himself exists in one way in Himself, in another way in created things, and still another way outside the world. For, indeed, He is in Himself because He needs the support of no one; He is in created things by essence, presence, and power; He is beyond the sky because He cannot be excluded from any place, whether true or imaginary. And yet, He is not present in the imagination by coexisting with any creature, but [rather He is there] according to the real and infinite existence of His being.

Now we must explain the arguments which we set forth at the beginning. To the first [see beginning of Article I], we deny that God cannot be in nothing—that is, in a space which is not a real and positive being; otherwise no stone could exist beyond the world by [an act of] the divine power.[50]

To the second [argument], we must deny that the same reasoning applies to space as to darkness and other privations. For just as space is appropriately disposed to receive bodies, this is not true of other negations or privations. And so it is also fitting that God should be said to be in this space, but that He should not be said to be in other negations or privations. To the third [argument we say] that that space beyond the world should be conceived by us as if it were truly there, but not as a real positive being. To the fourth [argument we say] that God is not properly said to exist in a place, since He is not enclosed by any spaces. But created substances devoid of matter do not exist in a place by operation but by their substances considered in a certain way;[51] however, this cannot be explained more concisely in this place. To the last [and fifth argument we say that] although St. Augustine makes no mention of imaginary space in his book, he, nevertheless, made [allusion to it] with clear and lucid words, in a place cited by us in *The City of God*, Book II.[52] Since these things are so, it is clear that Aristotle,[53] in Book I of *Metaphysics*, Explanation [that is, Text] 14, was not right to label as an error and find fault with this opinion, which

45. Insofar as this discussion applies to space beyond the world, our Jesuit author agrees with Bradwardine and Oresme that imaginary space lacks dimensions (see n. 25 above and Selection 71, n. 26).

46. Both Bradwardine and Johannes de Ripa held this opinion (see Selection 72, n. 20, and the introduction to this selection).

47. That is, it is not created by the reason.

48. Otto von Guericke argued (near end of Bk. II, ch. 6, in section 4) that imaginary space is not just *like* a positive and real thing but actually *is* a positive and real thing.

49. After our author ascribes an essentially unreal but distinct status to imaginary space—that is, its dimensions merely *correspond* to the positive and real dimensions of bodies, for in itself it is nothing—he insists that God is actually in this imaginary space, a space which always existed and in which the world was created. Here we ought, presumably, to understand that since this infinite space is eternal (see preceding part of Question 2, Article IV in section 3) it is assumed to be God himself, for otherwise it would be an eternal thing coexistent with, but separate from, God. In this, the Coimbra Jesuits agree with Bradwardine and Oresme that infinite space is God Himself and, therefore, coextensive and eternal.

50. Since no one would deny God's power to create a stone in the unreal space beyond the cosmos, there is no good reason for supposing that God Himself could not be in this space.

51. A spiritual substance is not properly locatable. Thus God exists in this unreal space only in a transcendent sense.

52. Earlier, our author cited *The City of God*, Bk. II, ch. 5, presumably the reference intended here. In this chapter Augustine actually expresses disbelief in the existence of infinite places or spaces beyond the world. He is convinced that there is no place beyond this one world. But he insists that if infinite spaces existed beyond the world, God would be omnipresent in them, since no reason could be found for confining God to our relatively small finite world.

53. The text reads *Aquarium* instead of *Aristotelem*.

we follow, concerning the existence of God beyond the sky.

4. Otto von Guericke (1602–1686)[54]

BOOK I

Chapter 35

Because according to Aristotle, as the prince of philosophers, and as the more common opinion of both philosophers and theologians, there are no more worlds beyond this world, this provides a good reason that the world must ultimately have a limit, since it is also against God and nature that something (with the exception of God) be infinitely extended. And so finally, the human mind asks, What is outside [or beyond] the world (or its limits: either the so-called prime mobile, or the empyrean heaven, or those waters believed to be beyond the heavens, or something else)? Or what contains or circumscribes the world? Aristotle, especially in Book III, chapters 8 and 9, of *De caelo*, expressly declares that beyond the last [or outermost] sky [or heaven,] (which, for him, is the starry sky or natural body or substance of the last revolving thing *[conversionis]*) there is neither place *(locum)*, nor time *(tempus)*, nor vacuum *(vacuum)*, but absolutely nothing *(nihil)*;[55] indeed, by his arguments, he zealously seeks to confirm that no natural body could be there, and so on.

Now, some state that there is a certain imaginary space which they call "possible locations" *(ubicationes possibiles)*. They offer an occasion for imagining or conceiving the different ways [of thinking about] these various interpretations.

The Coimbra group *[Conimbricenses]*, in commenting on Aristotle's *Physics*, Book VIII, chapter 10, Question 2, Article IV, declares "concerning the space which is found within and beyond the sky. . . ."[56] And in their explication of Book I, chapter 9, of *De caelo*, they confirm this when they write: "On the other hand, although there is no body beyond the sky, there is, nevertheless, a space *(spatium)* or receptacle *(capedinem)* for receiving bodies."[57]

Others say that imaginary space is nothing other than nothing.

Others hold that it is empty of all reality.

Others proclaim that it is the negation of all being.

Others respond [to these last three descriptions as follows]: (1) As for its being "nothing," if it were nothing, space could not be conceived, for "nothing" could have neither extent, nor width, nor length, nor depth. With respect to [description] (2), they infer that if imaginary space is space that is empty of all reality and being, and since the same thing is said of vacuum, therefore vacuum or imaginary space are one and the same thing. As for [description] (3), they reply that if imaginary space is the negation of every being, then it cannot also be called "space."[58]

Furthermore, some desire that imaginary space be merely something fictitious. From this they conclude that before the creation of the world there was no thinking [or conceiving] intellect [and] therefore no imaginary space. And since God cannot be in that which does not exist (as in chimeras), but only in that which does exist,

54. This selection is from *Ottonis de Guericke Experimenta Nova (ut vocantur) Magdeburgica De Vacuo Spatio primum a R. P. Gaspare Schotto, e Societate Jesu, et Herbipolitanae Academiae Matheseos Professore. Nunc vero ab ipso Auctore Perfectius edita, variisque aliis Experimentis aucta . . .* (Amsterdam, 1672; Aalen: O. Zeller, 1962), pp. 51, col. 1, to 52, col. 2; pp. 61, col. 1, to 62, col. 2; pp. 63, col. 1, to 65, col. 2.

Probably through the work of Gaspar Schott, Otto von Guericke was thoroughly familiar with the different viewpoints on imaginary void space, and was especially influenced by the opinions of the Coimbra Jesuits (see section 3), who transmitted a variety of medieval opinions. In this way von Guericke probably played a significant role in disseminating the medieval position represented by Bradwardine and Oresme (indeed, he may have even read Bradwardine's *De causa Dei* in the 1618 edition).

In the history of science von Guericke's treatise, *New Magdeburg Experiments on Void Space*, is most famous for its description of pneumatic experiments, especially the celebrated "Magdeburg Hemispheres," in which two hollow bronze hemispheres were fitted together and subsequently evacuated. Despite the efforts of two teams of eight horses harnessed to each hemisphere and pulling in opposite directions, the evacuated hemispheres could not be pulled apart. See Selection 53.2 for Buridan's similar experiment.

55. See introduction to this selection. Like his medieval predecessors, von Guericke's point of departure is Aristotle's denial of void or any kind of existence outside the finite world.

56. Since von Guericke now quotes much of the substance of the first four paragraphs of Article IV, translated in section 3 above, I have omitted that section here. The edition from which he quoted differs slightly from the one I used.

57. The Coimbra Jesuits also commented upon Aristotle's *De caelo* (and virtually all the physical treatises).

58. Presumably, for these critics, space is a positive thing.

imaginary space does not exist, according to their view. Therefore, God is not in imaginary spaces.

Others understand imaginary space as some merely possible, immense, corporeal mass that is diffused everywhere into infinity; or [imaginary space is] a possible location of such a corporeal mass [or quantity].

Some say (as does Lessius in Book 2 of *De perfectione divino*): "Imaginary space is God Himself, who, in accordance with His immensity is necessarily everywhere, or infinitely, diffused."[59] For if someone could transcend all the [celestial] heavens and seek what is in the space which he forms in his imagination, he would surely find God.

Some wholly reject imaginary space for this reason: because what is conceived *only* by our imagination or mind is not something about a real thing. For example, someone could conceive or imagine that he has 1000 gold pieces, when, indeed, he does not have a single one. Therefore, there is no imaginary space.

A certain more recent philosopher, René Descartes, in his *Principles of Philosophy*, Part II, Number 21, observes that this imaginary space is something truly real when he says:[60] "We likewise recognise that this world, or the totality of corporeal substance, is extended without limit, because wherever we imagine a limit we are not only still able to imagine beyond that limit spaces indefinitely extended but we perceive these to be in reality such as we imagine them, that is to say, that they contain in them corporeal substance indefinitely extended." "For," as has already been shown in his [Descartes's] opinion "the idea of extension that we perceive in any space whatever is quite evidently the same as the idea of corporeal substance.[61] It is thus not difficult to infer from all this that the earth and heavens are formed of the same matter, and that even were there an infinitude of worlds, they would all be formed of this nature *(natura)*,[62] from which it follows that there cannot be a plurality of worlds, because we clearly perceive that the matter whose nature consists in its being an extended substance only, now occupies all the imaginable spaces where these other worlds could alone be, and we cannot find in ourselves the idea of any other matter.[63] There is, therefore, but one matter in the whole universe, and we know this by the simple fact of its being extended," and so on. And this is the understanding or observations of the said René [Descartes].[64]

Finally, if imaginary space is conceded, it follows that it is infinite and immense or infinitely expanded in all its parts, that is, in length, width, and depth;[65] it also follows that it is incorruptible, sempiternal, immobile, fixed, and permanent, so that by no force or reason could it be destroyed, so that it could be the receptacle [or container] of any body whatever, whether great or small. On this matter, see several places in Book II, following.

. . . . . .

BOOK II

Chapter 6

Whether Space, or the Universal Container of All Things, Is Finite or Infinite

[After reviewing, in the first half of this chapter, the different opinions on whether an imaginary space exists beyond the world, von Guericke rejects Descartes's opinion (quoted in the preceding book), declaring that he will demonstrate its falsity throughout the treatise, for it is "against God to assert something infinite and immense; for only God could know no limits to his extension." At this point, von Guericke sets forth his own opinion.—*Ed.*].

Now let us proceed to what has been proposed,

59. A conception of imaginary space that is almost identical with Oresme's (see Selection 71). The Lessius referred to here may be Leonard Lessius (1554–1623), a Jesuit professor in theology at the University of Louvain.

60. Von Guericke's quotations from Descartes are given here in the translation by Elizabeth S. Haldane and G. R. T. Ross, *The Philosophical Works of Descartes* (Cambridge, England: Cambridge University Press, 1911; reprinted, with corrections, 1931), I, 264–265.

61. Part II, Principle 21, terminates here, but von Guericke continues and also quotes Principles 22 and 23.

62. Descartes has *materia*, not *natura*. Except for this, and *constitit* for *consistit* (a few words after), von Guericke's quotation agrees with the corresponding passage in the edition by C. Adam and P. Tannery, *Oeuvres de Descartes* (Paris: L. Cerf, 1905), VIII, 52.

63. Principle 22 ends here, followed by the first few lines of Principle 23.

64. Descartes' "imaginary space" is an indefinitely extended plenum, not a void.

65. In light of later remarks, this appears to represent von Guericke's own interpretation of extramundane infinite void space. For him, then, it is three-dimensional, despite the fact that earlier, in Book II, chapter 4 (p. 57 of the 1672 ed.), he declared that he is not considering space three-dimensionally, as is commonly done, but rather as a universal container of all things "which is not to be conceived according to quantity, or length, width, and depth." For evidence that he conceived extramundane space as three-dimensional, see page 565 and notes 69 and 71; and page 567 and note 84.

namely whether there is a space beyond the world and, [if so, whether] it is finite or infinite.

Since it is certain that the world has a limit,[66] the human mind inquires especially about the last terminus of the world. What indeed is it (whether it ends, or the first mover is a fiction, or [whether] one ought to believe there are waters beyond the heavens, or some other thing)? What contains or circumscribes all these things?

Assuming, according to the more common opinion, that there is "nothing" *(nihil)* beyond the world (taking "nothing" in the Aristotelian sense)[67] then if someone should reach the last confines of the world, he either could, or could not, extend his arm beyond the last surface of the last heaven or hurl a spear and a stone into this nothingness.[68]

In the first case, if one could [do this], there would be nothing to obstruct it and so it would be true that "nothing" is there; but nevertheless *space* is there (for if no space were there, a stone could not be projected beyond, nor an arm be extended because it cannot be in something into which it cannot be sent; nor could anything be in it, because what is not has neither extent nor width but is absolutely incapable of receiving anything), and consequently, there would be something [out there].

In the second case, if one could not [do this], something must necessarily impede [or obstruct] it, [since] one [body] could not occupy the place of another. Now, what obstructs it will be, necessarily, something hard or corporeal. Therefore, something will exist outside the last boundary [or terminus] of the world, since [mere] "nothing" cannot obstruct. Nor can what obstructs be called "nonbeing." [But] you say: *It will not be a body or some other [thing] that resists [or opposes] but this nothing (nihil), or lack of space and place, resists and thus opposes nature as if the spear or stone were repelled by a hard body.* But who will be able to understand this? That what is not could obstruct and have a certain nature of resisting. For appearances teach us that lack of place or space is nothing other than one body impeding another, not that one body occupies the space of another [simultaneously]; however, with the one body removed, another can occupy that space. Indeed, since it cannot be denied that the Divine Essence, infinite [and] immense, is beyond the world, then all the more reason it is appropriate that it [that is, the space beyond the world] is immense in extent, width, depth, that is, according to space or expanse;[69] but it is not to be considered an absolutely infinite nothing (indeed, according to what will be said in the following chapters, we know that it is not [an infinite nothing]).

It follows, moreover, than unless the spear or stone were impeded by another body, it could be projected beyond and such a projection *(jactus)* could be continued into infinity.

Furthermore, since the projection of a spear or stone could be continued into infinity beyond the world and there is nothing to stop it, while, nevertheless, such a projection could not be accomplished except in space, it follows:

1. That the *nothing (nihil)* beyond the world and *space (spatium)* are one and the same; and so-called *imaginary space* is true space, for imaginary space (in the common opinion of philosophers) is nothing and nothing is space, and the space which they call imaginary is true space *(spatium verum.)* Nor can it be objected that whatever is imaginary is not a positive and real thing[70] even if it is not always efficacious. For it is necessary that at least we imagine everything we never see and which is beyond our grasp, as, for example, it is necessary that one who never sees Rome, or a spirit, or an exotic animal, or some other thing, must imagine them; and because no one can comprehend the infinite, it is grasped at least in some way, by the imagination. Meanwhile, it does not follow that Rome, or spirit, or the infinite, are not real *(verum)* or positive things.

2. If the projection of a spear or stone could be continued into infinity and no terminus for resting is revealed, it follows that this space beyond the world is extended or expanded without end toward all parts and thus is infinite and immense.[71] Moreover, if it is "infinite" and "immense" what do we mean by these? Before we can respond to this question, it will be necessary to inquire about *those things that exist (ea quae sunt)*.

66. Von Guericke accepted and defended the truth of the Copernican system.

67. That is, as absolute privation.

68. This statement, and the two cases that follow, derive ultimately from Stoic arguments, probably from Simplicius' version, which is quoted in note 2.

69. In this statement von Guericke indicates that extramundane space is three-dimensional.

70. Here von Guericke disagrees with the Jesuit authors at Coimbra, who, as we have seen (end of section 3), argued that imaginary space is not a real and positive being. See also the introduction to this selection.

71. Three-dimensionality seems to be implied here. This is a description of an inertial motion which had already been given earlier on this page.

Chapter 7
On that which is and that which is said not to be.

. . . . . .

Thus, if it were asked, "What is to be understood [as existing] before the world was established," and someone responds, "What is Uncreated," and another responds, "Nothing," each would be a proper response. For the one who says "Uncreated" answers as rightly as the one who says "Nothing." Indeed, he [that is, the one who answers "Uncreated"] is thinking of what was created, because it [the "Uncreated"] was surely the "Nothing" of it [that is, of the Created]. Nevertheless, it was and yet is Uncreated.

In the same way, we say that the heaven and earth were made from Nothing, that is, from the Nothing [or negation] of what was created, that is, from the Uncreated.[72] And just as all things have been created from Nothing, so have all things been received and fixed in Nothing—that is, in the Uncreated. For where created things subsist and are now, Nothing was there before and they were received in Nothing, that is, in the Uncreated, since they could not be received in any created thing, both because it was not, and because if it was it was not a Nothing but a created something. Moreover, if it was [or did exist], something could not be received in something. Therefore, it was Nothing in which all things were received, that is, in the Uncreated.

Therefore, everything is in Nothing *(nihilo)* and if God should reduce the fabric of the world *(machinam mundi)*, which he created, into Nothing *(nihilum)*, nothing would remain of its place other than Nothing *(nihil)*, that is, the Uncreated *(increatum)*. For the "Uncreated"[73] is that whose beginning does not pre-exist; and Nothing, we say, is that whose beginning does not pre-exist. Nothing contains all things. It is more precious than gold, free of origin and distinction, more joyous than the appearance of beautiful light, more noble than the blood of kings, comparable to the heavens, higher than the stars, more powerful than a stroke of lightning, perfect and blessed in every part. Nothing always inspires. Where Nothing is, there ceases the jurisdiction of all kings. Nothing is without any mischief. According to Job[74] *[Hiob]*, the earth is suspended over Nothing. Nothing is outside the world. Nothing is everywhere. They say the vacuum is Nothing; and they say that imaginary space and space itself is Nothing.

Chapter 8
Whether Space, or the Universal Container of All Things, Is a Created or an Uncreated Something?

Since vacuum, and imaginary space, or space itself, is Nothing, [and] Nothing is the negation of one thing and the affirmation of another (as was demonstrated in the preceding chapter),[75] it follows that vacuum, or imaginary space, or space itself is one of these two things that are—it is either created or uncreated, for no third thing is given. But that vacuum, or imaginary space, or space itself, is the Nothing [or negation] of the created is acknowledged by Jacques du Bois, an ecclesiastic of Leyden, in his *Dialogus Theologico-Astronomicus*[76] (written against Galileo to the Galileans Philip of Lansberg[77] and others, who declare that the sun is in the middle of the world and that the earth revolves) when he writes on page 39: "Who denies the vacuum beyond the extreme heavens contradicts *(contradicit)* the infinity of the divine essence, for they do not take the God of the Heaven of Heavens as Solomon witnesses in I Kings, 8, verse 27.[78] Therefore, beyond these heavens there is some place *(ubi)* in which there also is this divine essence, which cannot be contained by any limits. We are accustomed to call this place *(ubi)* "vacuum" because it contains no body. Accordingly, before the creation of the universe, God was somewhere; indeed He was everywhere infinitely in this infinite abyss which could not be filled by a finite body, however large, which is what the supreme [or outermost] heaven is (note: here the author [Jacques du Bois] declares with Aristotle that the heaven [or sky] is a body).[79] For to declare with René Descartes that the whole of corporeal substance has no limits to its extension is, in reality, to declare an infinite creation, which God and nature abhor, and so on"[80]

It is clearly obvious from these words that Jacques du Bois states that God is infinitely everywhere in some void place *(in aliquo ubi, vacuo)*. But this statement was not well put: "Therefore, there is some place in which there also is this divine essence." and so on. But [instead] he ought to say: "Therefore, there is a place or space, not *in which* the divine essence is, but which is itself the divine essence." For God can be contained by no place *(ubi)* or vacuum or space, because He Himself is the place *(ubi)* for Himself, or the thing that is empty *(vacuum)* of every creature, or the space, or universal container of all things.

On the other hand, Father Athanasius Kircher[81] of the Society of Jesus, a most famous mathematician, although he attacks the world system of Copernicus and concedes no void space in nature, nevertheless, where he takes up imaginary space in his *Itinerarium Ecstaticum*,[82] Dialogue 2, chapter 9, Number 4, he uses the following words:

"Since God is infinite in actutality, he also necessarily fills every void and empty space (which everyone conceives as immense and infinite) with his substance and presence, eliminating every nothing, every vacuum, emptiness, and nonbeing. And so, outside of God, it is necessary that neither Nothing, nor emptiness, nor vacuum be left. For if there is a nothing outside of God, you must acknowledge that God is not infinite in the least, inasmuch as the divine substance would be confined and contained by this nothing, which is impossible. Hence, it follows again that the world does not subsist in Nothing, but is received in God. Therefore, when you imagine this imaginary space beyond the world, do not imagine it as nothing, but conceive it as a fullness of the Divine Substance extended into infinity. For He who has perfected and created all things by Himself and His fullness receives, confers, carries, encompasses, contains, preserves, and so on."

From these words, nothing other follows than that *space (spatium)*, vacuum *(vacuum)*, and emptiness *(inane)*, which we conceive as immense outside the world, is not Nothing or not-being as understood in the opinion of Aristotle, but it is the Nothing [or negation] of the created, that is, the uncreated. But Father Kircher speaks improperly when he says "that God fills all imaginary space, vacuum, or emptiness, by His substance and presence," and so on, because he himself [Kircher] denies their existence in the nature of things. How can God fill what is not? He ought to have said with Lessius (who is quoted above) that "imaginary space, vacuum, or the Nothing beyond the world, is God Himself," as he finally does when he announces that the space beyond the world is not Nothing, but is the fullness of the Divine Substance.

Now, we ought to consider that the infinite essence of God is not contained in space, or vacuum, since God, who is present everywhere, is not contained in space or vacuum, but the space and vacuum of the whole creation is in Him, by Him, and for Him. For He is an essence other than all created things [and] contained by the periphery of none of them nor (as I would say) comprehended by any enclosure, but infinite and therefore as much outside all things as within all things. Accordingly, he is also present in all our actions and brings us to judgment for them. "Am I a God at hand, saith the Lord and not a God far off? Can any hide himself in secret places that I shall not see him? saith the Lord. Do not I fill heaven and earth? saith the Lord" (Jeremiah, chapter 23, verse 23).[83]

Thus in the same manner as the said authors consider the space beyond the world as the Divine Essence Himself, so also is it necessary that they consider the world within in the same way or quality. For while they assert that the space beyond is infinite, immense, and full of the divine essence, it follows that within the world it must be the same way, because this world is contained in this infinite space (indeed, in relation to the immensity of this space, it does not have the likeness of a point or atom)[84] and cannot be excluded from it so

72. That is, the Nothing from which the world was created is the Uncreated.

73. What follows can only be described as an *Ode to Nothing*. The contrast with the Stoic conception of infinite void is striking. No longer is the infinitely extended void an inefficacious, impotent, and completely empty three-dimensional space. It has become a positive, real, all-powerful divine force—God Himself.

74. Job 26:7.

75. Omitted here.

76. A reference to Du Bois' *Dialogus Theologicus-Astronomicus in quo ventilatur quaestio An Terra in centro universi quiescat, . . . an vero, sole quiescente, terra circa eam feratur (Theological-Astronomical Dialogue in which is aired the question whether the earth rests in the center of the universe, . . . or whether, with the sun at rest, the earth is carried around it)*, published in Leyden, 1653.

77. 1561–1632.

78. "But will God indeed dwell on the earth? behold, the heaven and heaven of heavens cannot contain thee; how much less this house that I have builded?" (King James Version).

79. This parenthetical note within the quotation from Du Bois was supplied by von Guericke.

80. The quotation from Du Bois terminates here.

81. 1601–1690. After teaching mathematics and philosophy at the Jesuit college in Würzburg, Kircher settled in Rome, where he taught mathematics, physics, and oriental languages.

82. Published in 1656.

83. This quotation embraces verses 23 and 24 of the King James Version, the translation used here.

84. A comparison between two physical magnitudes seems implied here, once again suggesting that von Guericke thought of extramundane infinite void space as three-dimensional.

that space withdraws [or diminishes] to the extent that this world system has been received in it; but as the space also contains, it penetrates and thus receives, confers, carries, encompasses, contains, preserves, and so on, all the bodies existing in the world system. Nor does it matter whether the world is located here or elsewhere by an infinite number of parasangs,[85] as we shall see in a following chapter. The Coimbra *[Conimbricenses]* discussions confirm all these things, as can be seen [above] in [Book I], chapter 35.[86]

85. A parasang is a Persian league containing 30 stades. Its exact measure is irrelevant, since von Guericke speaks of an infinite number of them.

86. For the references, see page 563.

Alchemy and Chemistry

74 ON THE FORMATION OF MINERALS AND METALS AND THE IMPOSSIBILITY OF ALCHEMY

Avicenna (980–1037)

Translated and annotated by E. J. Holmyard and D. C. Mandeville[1]

The time has now arrived for us to give an account of the properties of mineral substances. We say, therefore, that mineral bodies may be roughly divided into four groups, *viz.* stones, fusible substances, sulphurs and salts.[2] This is for the following reason: some of the mineral bodies are weak in substance and feeble in composition and union, while others are strong in substance. Of the latter, some are malleable and some are not malleable. Of [the former, *i. e.*] those which are feeble in substance, some have the nature of salt and are easily dissolved by moisture, such as alum, vitriol, sal-ammoniac and *qalqand*,[3] while others are oily in nature and are not easily dissolved by moisture alone, such as sulphur and arsenic [sulphides].[4]

Mercury is included in the second group, inasmuch as it is the essential constituent element of malleable bodies or at least is similar to it.[5]

All malleable bodies are fusible, though sometimes only indirectly, whereas most non-malleable substances cannot be fused in the orthodox way or even softened except with difficulty.

The material of malleable bodies is an aqueous substance united so firmly with an earthy substance that the two cannot be separated from one another. This aqueous substance has been congealed by cold after heat has acted upon it and matured it. Included in the group [of malleable bodies], however, are some which are still quick[6] and have not congealed on account of their oily nature; for this reason, too, they are malleable.[7]

1. [This is the third part ("On the Four Species of Mineral Bodies") of Avicenna's *"De congelatione et conglutinatione lapidum" being sections of the Kitâb al-Shifâ'*, edited . . . by E. J. Holmyard and D. C. Mandeville. For a full description and discussion of this treatise see Selection 78, introduction and note 1.

2. [Holmyard and Mandeville cite a classification in the *Kitâb al-Asrâr* of al-Râzî (d. 924) in which terrestrial elements are divided into the following six classes: (1) spirits (four kinds); (2) bodies (seven kinds); (3) stones (thirteen kinds); (4) vitriols (five); (5) niters (six); (6) salts (eleven). Other schemes of classification were also proposed. On al-Râzî, see Selection 76, n. 129—*Ed.*]

3. qalqand, Χάλκανθος, green vitriol, $FeSO_4$. See later in this selection.

4. zarnîkh, realgar, orpiment, As_2S_2 and As_2S_3.

5. This is not merely an expression of Ibn Sînâ's natural caution but a reference to general alchemical opinion, *viz. Our mercury is not the mercury of the vulgar.*

6. In the sense of *alive*, or as in *quicksilver*. This passage proved very troublesome to the Latin translator, or at least to the copyists,the general rendering being: *Et erit exemplum a vino quod nondum gelavit propter suam unctuositatem: et ideo non est ductile.* ["And an example is wine, which scarcely congeals because of its oiliness; and so it is not malleable."—*Ed.*] The Trinity MS. 1122 [see Selection 78, n. 3.—*Ed.*], however, seems to read *a vivo* for *a vino*, so that it is possible that Alfred the Englishman was not far out. [That is *a vivo* would give "And an example is the living. . . . " which would bring it quite near the sense given the passage by Holmyard and Mandeville.—*Ed.*]

7. On the theory that metals are composed of an aqueous substance and an earthy substance, *cf.* Aristotle, following Heraclitus (W. D. Ross, *Aristotle*, London, 1923, p. 109):—"There are two 'exhalations' produced by the sun's rays acting on the surface of the earth. When the sun's rays fall on dry land, they draw up from it an exhalation which is hot and dry, and which Aristotle likens for the most part to smoke but also to fire and wind. When they fall upon water, they draw up an exhalation which like water is moist and cold, and is called the vaporous in opposition to the smoky exhalation. The dry exhalation consists of minute particles of earth on the way to being fire, and exhibiting already, though in a weaker degree, the properties of fire—heat and dryness. The moist exhalation consists of minute particles of water on the way to becoming air, but exhibiting in the main the qualities of water—coldness and moisture. . . . Aristotle turns next to the effects produced by the exhalations when 'imprisoned' in the earth, *i.e.* the minerals. These are divided into the metals, which are formed by the moist exhalation, and the 'fossiles', formed of the dry; most of the latter are said to be either 'coloured powders' or stones formed out of such."

According to Jâbir ibn Hayyân there are two constituents of metals, an earthy smoke and a watery steam. Condensation of these exhalations in the bowels of the earth gives rise to sulphur and mercury; combination of sulphur and mercury results in the formation of metals. [Little is known of Jâbir ibn Hayyân, an Islamic alchemist of the eighth century who acquired so great a reputation that long after his death many alchemical treatises written in the tenth century were ascribed to him by their authors. These treatises, many of which were written by members of a sect known as the Brethren