Wilhelm Weber

On the Measurement of Electro-dynamic Forces

1848

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The work of Weber and Gauss linked electric phenomena to the dimensions of length, time and mass, uniting the electrostatic and electrodynamic. All subsequent conceptions of electric and magnetic units have been built upon Weber's foundation.

The first general publication of Weber's paper was
Electrodynamische Maassbestimmungen.
Annalen der Physik und Chemie, vol **73** no. 2, pages 193-240 (January 1848). (Often referred to as "Poggendorff's *Annalen*".)
Leipzig: J. A. Barth.

The article may be accessed in facsimile PDF pages on the web at
gallica.bnf.fr/ark:/12148/bpt6K15158z

A prefatory note in *Annalen* says the paper first appeared in:
Abhandlungen bei Begründung der Königl. Sächsischen Gesellschaft der Wissenschaften am Tage der 200jähr. Geburtstagfeier Leibnizens.
Leipzig: Fürstlich Jablonowskische Gesellschaft, 1846.

However, we have not seen this publication.

Here the paper is presented in a contemporaneous English translation, which appeared in
Richard Taylor, editor [and translator?].
Scientific Memoirs, selected from the Transactions of Foreign Academies of Science and Learned Societies.
Volume V, part 20, article 14.
London: Taylor and Francis, 1852.

The original English spelling and mathematical notation have been retained, but the decimal separator has been changed to a period on the baseline, and where a period on the line was used to indicate multiplication, it has been replaced by the current conventions.

A QUARTER of a century has elapsed since Ampère laid the foundation of electro-dynamics, a science which was to bring the laws of magnetism and electro-magnetism into their true connexion and refer them to a fundamental principle, as has been effected with Kepler's laws by Newton's theory of gravitation. But if we compare the further development which electrodynamics have received with that of Newton's theory of gravitation, we find a great difference in the fertility of these two fundamental principles. Newton's theory of gravitation has become the source of innumerable new researches in astronomy, by the splendid results of which all doubt and obscurity regarding the final principle of this science have been removed. Ampère's electro-dynamics have not led to any such result; it may rather be considered, that all the advances which have since been really made have been obtained independently of Ampère's theory,—as for instance the discovery of induction and its laws by Faraday. If the fundamental principle of electro-dynamics, like the law of gravitation, be a true law of nature, we might suppose that it would have proved serviceable as a guide to the discovery and investigation of the different classes of natural phenomena which are dependent upon or are connected with it; but if this principle is not a law of nature, we should expect that, considering its great interest and the manifold activity which during the space of the last twenty-five years that peculiar branch of natural philosophy has experienced, it would have long since been disproved. The reason why neither the one nor the other has been effected, depends upon the fact, that in the development of electro-dynamics no such combination of observation with theory has occurred as in that of the general theory of gravitation. Ampère, who was rather a theorist than an experimenter, very ingeniously applied the most trivial experimental results to his system, and refined this to such an extent, that the crude observations immediately concerned no longer appeared to have any direct relation to it. Electro-dynamics, whether for their more secure foundation and extension, or for their refutation, require a more perfect method of observing; and in the comparison of theory with experiment, demand that we should be able accurately to examine the more special points in question, so as to provide a proper organ for what might be termed the spirit of theory in the observations, without the development of which no unfolding of its powers is possible.

The following experiments will show that a more elaborate method of making electro-dynamic observations is not only of importance and consideration in proving the fundamental principle of electro-dynamics, but also because it becomes the source of new observations, which could not otherwise have been made.

Description of the Instrument

The instrument about to be described is adapted for delicate observations on, and measurements of, electro-dynamic forces; and its superiority over those formerly proposed by Ampère depends essentially upon the following arrangement.

The two galvanic conductors, the reciprocal action of which is to be observed, consist of two thin copper wires coated with silk, which, like multipliers, are coiled on the external part of the cavities of two cylindrical frames. One of these two coils incloses a space which is of sufficient size to allow the other coil to be placed within it and to have freedom of motion.

When a galvanic current passes through the wires of both coils, one of them exerts a rotatory action upon the other, which is of the greatest intensity when the centres of both coils correspond, and when the two planes to which the convolutions of the two coils are parallel form a right angle with each other. The common diameter of both coils is the axis of rotation. This respective position of the two coils constitutes the normal position, which they obtain in the instrument. Hence also the common diameter of the two coils, or their axis of rotation, has a vertical position, in order that the rotation may be performed in a horizontal plane.

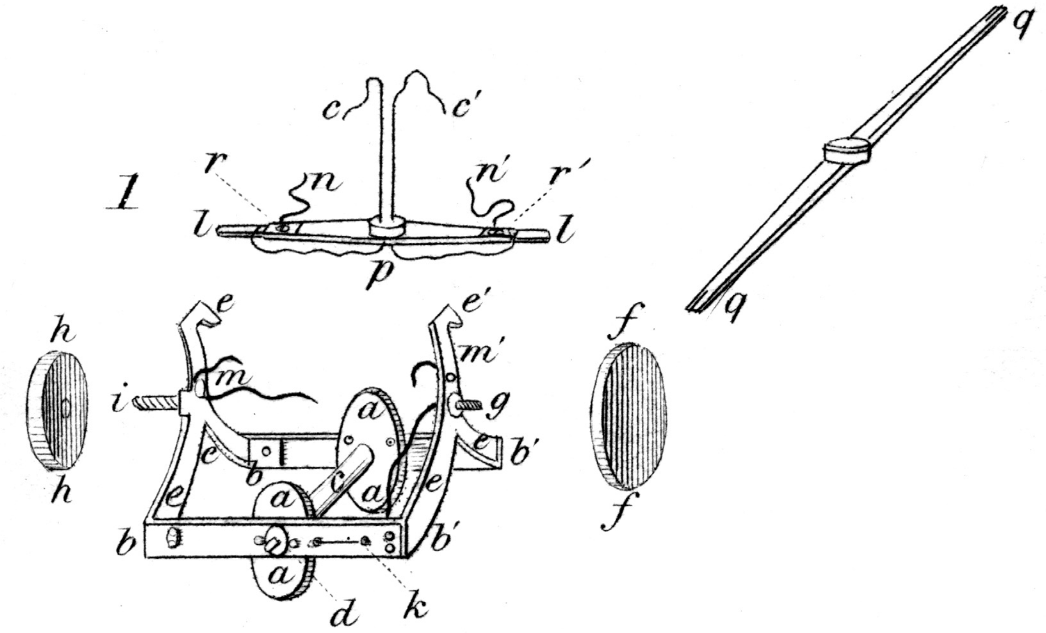
That coil which is to be rotated, to allow of the onward transmission and return of the current, must be brought into connexion with two immoveable conductors; and the main object of the instrument is to effect these combinations in such a manner that the rotation of the coil is not in the least interfered with even when the impulse is the least possible, as occurs when these connexions are effected by means of two points dipping into two metallic cups filled with mercury in which the two immoveable conductors terminate, as in Ampère's arrangement. Instead of these combinations, which on account of the unavoidable friction do not allow of the free rotation of the coil, in the present arrangement two long and thin connecting wires are used, which are fastened at their upper extremities to two fixed metallic hooks, in which the two immoveable conductors terminate, and at their lower extremities to the frame of the coil, and are there firmly united to the ends of the wires of the coil. The coil hangs freely suspended by these two connecting wires, and each wire supports half the weight of the coil, whereby both wires are rendered equally tense.

These two connecting wires thus effect the transmission of the galvanic current from one of the immoveable conductors to the coil, and back to the other immoveable conductor; and they effect this without the least friction interfering with the rotation of the coil.

These connecting wires are also of service, because each rotation of the coil through a certain angle corresponds to a definite rotatory momentum, which tends to diminish this angle, and is proportional to the sine of the angle of rotation; whence a standard is formed for all rotatory momenta, by the aid of which any other rotatory momentum acting upon the coil may be measured. This is effected according to those simple laws which Gauss has developed in the case of the bifilar magnetometer. Lastly, this measure may be made more or less delicate at pleasure, or as occasion may require, by the approximation or separation of the two connecting wires. This method of suspension not being accompanied with any friction, allows of increase in the weight of the suspended coil, which may be any amount provided it is not more than the connecting wires are capable of supporting. Hence a very long wire may be wound many times around the coil, and thus a very strong multiplication of the galvanic force be obtained. Moreover, this rotating coil may without injury be loaded with a speculum, which also rotates, and here, as in Gauss's magnetometer, may be used for the delicate measurement of angles; for provided friction be excluded, the application of delicate optical instruments in this case also does not form any impediment. Regarding the construction of the instrument in detail, as this has been described very perfectly by M. Leyser, the instrument-maker in Leipzic, I shall insert the explanation which he has given, and which refers to the figures sketched by him, fig. 1-10. The instrument is called an *Electro-dynamometer.*

Description of the Electro-dynamometer

Fig. 1 represents the little frame for supporting the reel which vibrates in the multiplier, seen diagonally. This frame consists of



two round ivory discs, *a a* and *a a,* which are riveted to two ivory plates, *b b′* and *b b′;* their distance apart is regulated by a small ivory roller, *c*. The latter is hollow, so that a metallic rod can be passed through it, and by means of a screw each of the discs with its plate can be fixed to the ends of the roller; and thus a reel is formed for the reception of the wire. One end of the wire to be coiled passes through the small hole *d*, and projects from it. When the wire is placed upon the reel and the end fixed by means of silk, the metallic supports, *e e e* and *e e e′*, of the reel are fixed to the ends of the plates above mentioned; thus, one support, *e e e′*, to which the speculum *f f* is screwed at *g*, is riveted at *b′ b′*; whilst the other support, *e e e*, to which the counterpoise *h h* is fixed by the screw *i*, is fastened by screws at *b b* ; so that this support, near the screws *b b*, may be thrown back in the direction *b b′*, in order that the entire reel may be conveniently placed in the multiplier. The commencement of the reel, which was left projecting through the hole at *d*, is now placed lengthwise along a portion of the plate *b b′* towards *b′*, until the circumference of the reel admits at *k* of its being again placed within the frame and then ascending to the support of the speculum, where by means of a small screw *m′* above the point at which the speculum is fixed, it comes into metallic contact with the support. The end of the reel is also brought into metallic contact with the other support by means of the screw *m;* this end must however be long enough not to stand in the way of the support when it is thrown back. When the speculum *f f* is now placed at *g,* and its counterpoise *h h* at *i*, the reel is prepared for suspension in the multiplier by the metallic threads. For this purpose both the supports of the reel terminate at *e* and *e′* in hooks or pieces in the form of ϒ, and the bifilar metallic threads are furnished below with a small ivory beam, *l l*, which towards each end terminates in a metallic plate, and this again in a small metallic cylinder; the latter fit into the above hooks or upsila of the support, and thus receive the reel. The bifilar metallic threads *n o* and *n′ o′* are united to the cross-beam *l l* in the following manner. The commencement *n* of the thread *n o* is fastened by means of a screw to the metallic plate *r*, proceeds a short distance towards *1,* and then returns through a small hole at the end of the plate beneath the beam *l l* to its centre *p*, where it runs through a small hole again above the beam, and can then be continued to *o* and further. The thread *n′ o′* is arranged in the same manner, its direction however being reversed; in the centre *p* of the beam *l l* each has a separate aperture, through which it passes; these lie very near each other, but are separated and kept isolated by the ivory. The index *q q* is placed upon the centre of the beam before the metallic threads *n o* and *n′ o′* are inserted.

Fig. 2 exhibits the lateral view of the vibrating reel, with the coil as placed upon the beam, and the mirror and counterpoise adapted and vibrating on the bifilar metallic threads. Only the very narrow anterior portion of the index is perceptible.

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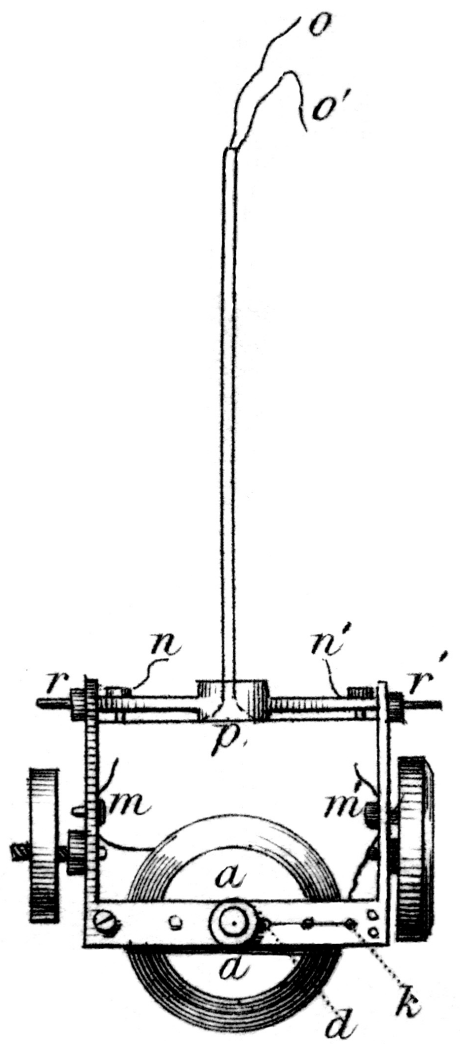

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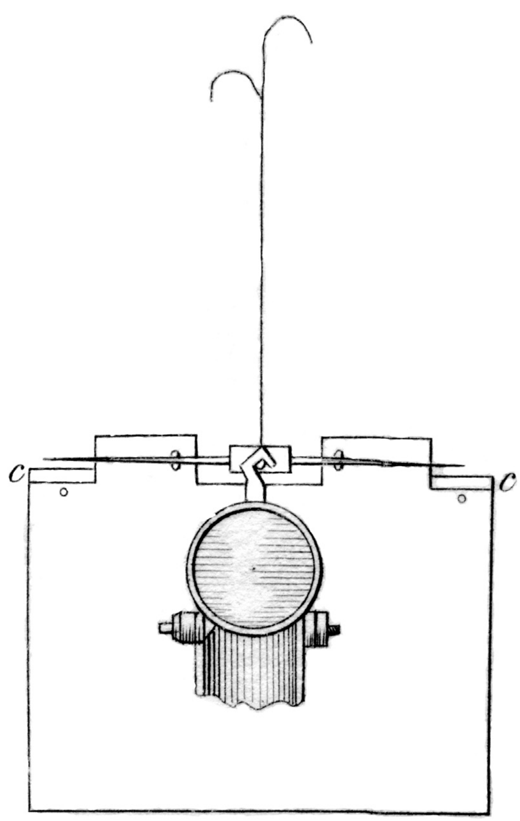
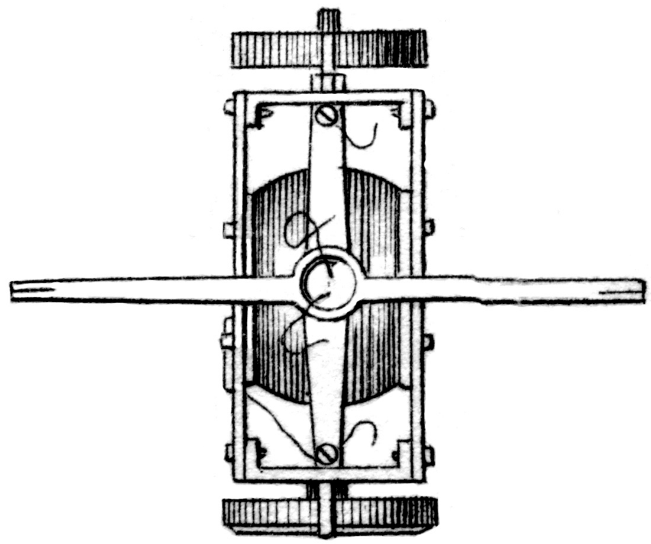


Fig. 3 represents the reel seen at right angles to the surface of the speculum; the hooks or upsila, as also the index vibrating above the scale-plate *c c*, are very distinctly seen.

Fig. 4 presents the view from above, in which the beam and the index form a right-angled cross.

4



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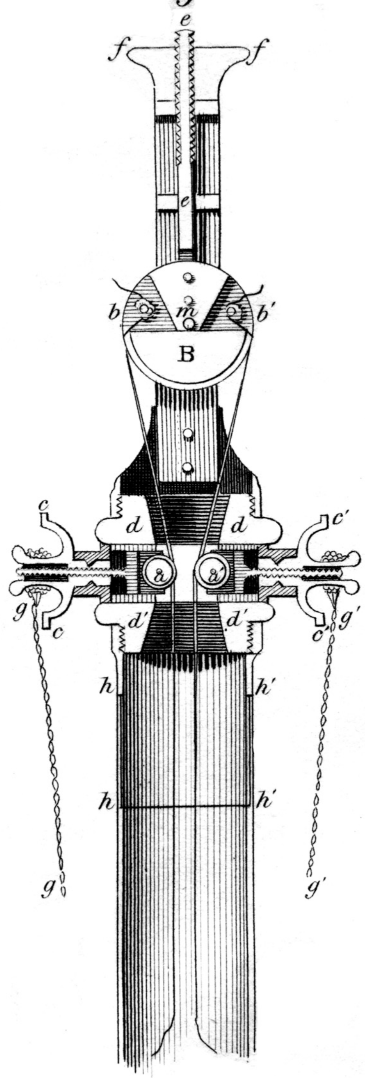


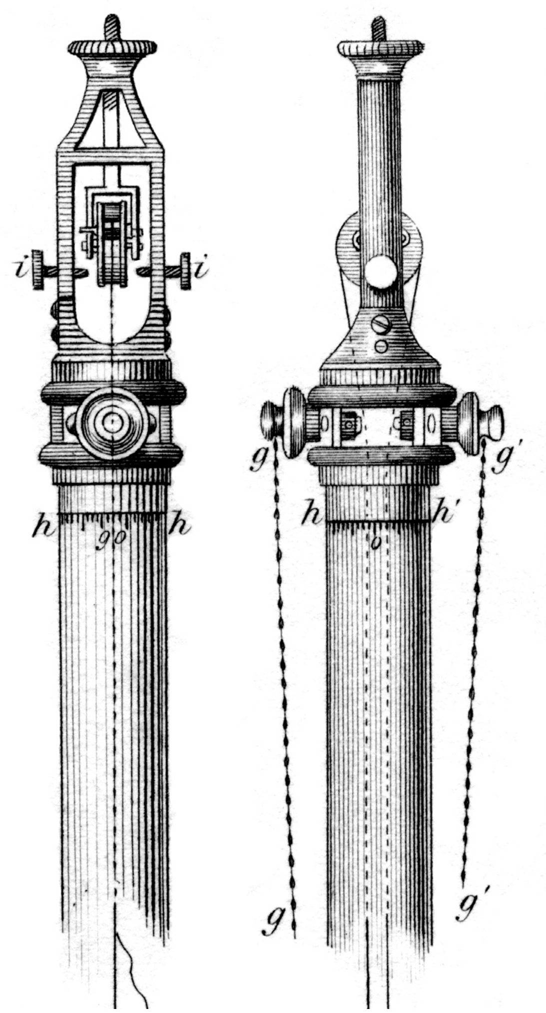
Fig. 5 serves to illustrate the further course of the bifilar metallic thread to its termination; for the sake of distinctness it is represented of twice the size of the other figures, and as seen in a vertical section. The bifilar metallic threads continue to ascend from *o* and *o′*, inclosed in a brass tube; they are wound round the moveable rollers *a* and *a′*, and are finally fixed to the ivory roller B at *b* and *b′* round rotating pegs. The threads can be wound up or unwound on these pegs or small rollers by means of a small key, according as the weight of the vibrating reel may render this requisite; the small rollers *a* and *a′* are also necessarily turned round at either of these operations. The ivory cylinder itself, B, with the prong and the screw *e e,* can also be screwed up or down by means of the nut, *f f;* and thus the vibrating reel may be arranged in the proper position as regards the multiplier, in the centre of which it should oscillate. At the same time the roller B, which is moveable in the prong *e e* around the peg *m,* assumes a state of equilibrium as soon as the vibrating reel is suspended freely from the bifilar metallic wires, since these wires act at band *b′* as it were at the ends of a lever, the centre of motion of which is at *m.* Thus the load of the vibrating cylinder is equally divided between the two threads.

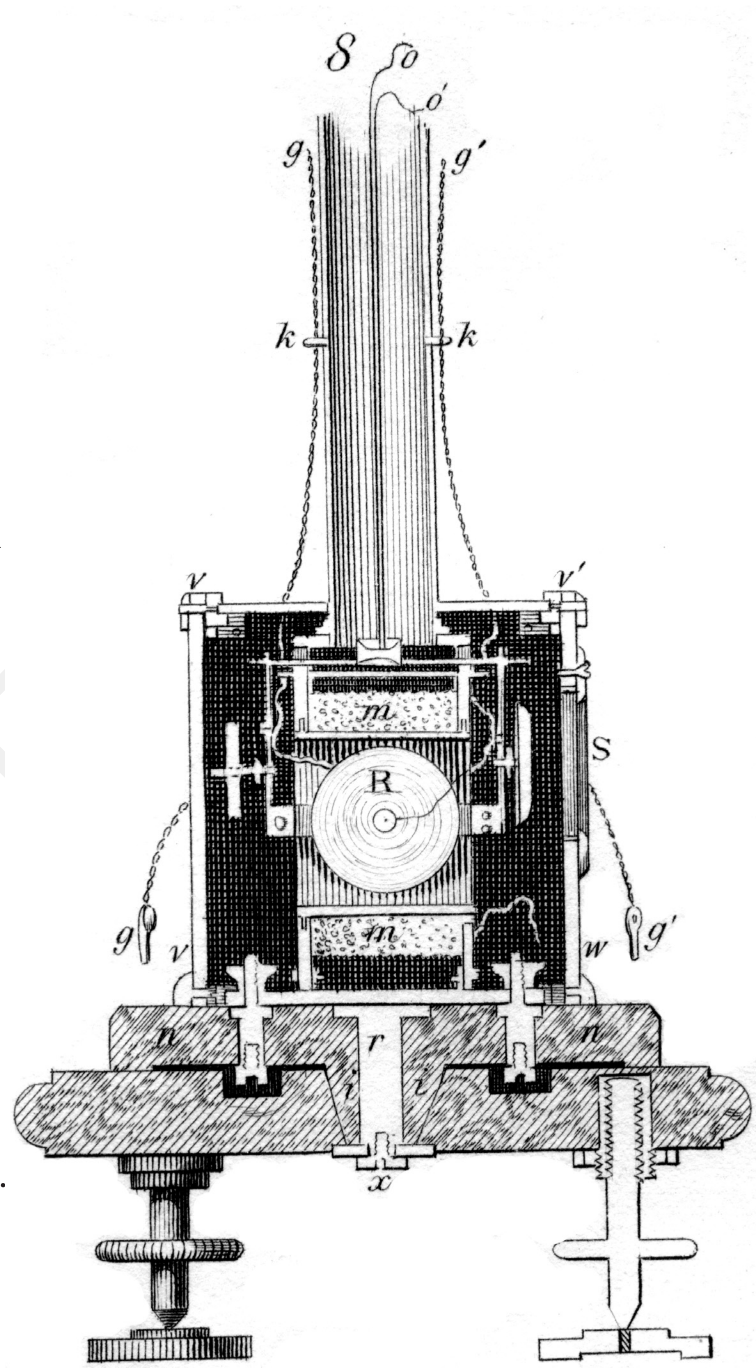
To allow of the approximation or separation of the two bifilar wires, the rollers *a* and *a′* are set in broad prongs, which, as seen in the figure, terminate in screws, by means of which they can be approximated or separated between two metallic plates (indicated by the lines engraved perpendicularly) with the nuts *c c* and *c′ c′*. The latter are fitted into a kind of case, indicated in the figure by lines drawn obliquely, in which they are fixed by a peg, but are not impeded as regards their rotation. The roller *a,* with its prong and screw, plate and nut *c c,* is isolated from the roller *a′*, with its prong and screw, plate and nut *c′ c′*, because the circular discs *d d* and *d′ d′*, which are perforated in the centre, and which connect them above and below, are made of ivory. To allow of the bifilar metallic wires being brought out conveniently, the nuts *c c* and *c′ c′* terminate in trumpet-shaped projections, as shown in the figure, from which hangs a wire *g g* and *g′ g′* thrice wound round. Hence a galvanic current takes the following course:- If it enters at *g,* it ascends to *g,* is communicated to the nut *c c,* and the roller *a* (it also ascends to *b,* but as *b* is isolated it returns), and runs down the threads to *o;* from *o* it proceeds (fig. 2) further down through the centre *p* of the transverse beam, then to its extremity *r*, where by the metallic contact with the support it runs down it, and at *m* enters the extremity of the reel itself, through the coils of which it continues, again making its exit at *d,* but again passing to the other support at *m′* through *k*, from *r′* along the transverse beam to its centre, and from this up to o'; from o' the current (fig. 5) again runs over the other roller a' into the nut $c'c'$, and finally arrives at the other conducting wire, $g'g'$. Thus the current, to arrive at one conducting wire $g'g'$ from the other $g\ g$, must necessarily run through the vibrating reel, inasmuch as the wire from g to g' is perfectly isolated. To do away with the torsion of the bifilar metallic wires, the whole of the upper portion of the instrument as far as $h\ h$ and $h'\ h'$ rotates horizontally, and is furnished with a torsion circle and an index, as is distinctly seen in figs. 6 and 7 at $h\ h'$.

Figs. 6 and 7 are not sectional, and fig. 6 belongs to fig. 2.

Fig. 7 exhibits the roller B with the prong and the screw $e\ e'$ of fig. 5 more distinctly; $i\ i$ here represent two screws, to fix the roller B on moving the instrument, without which precaution the bifilar threads would be easily injured.

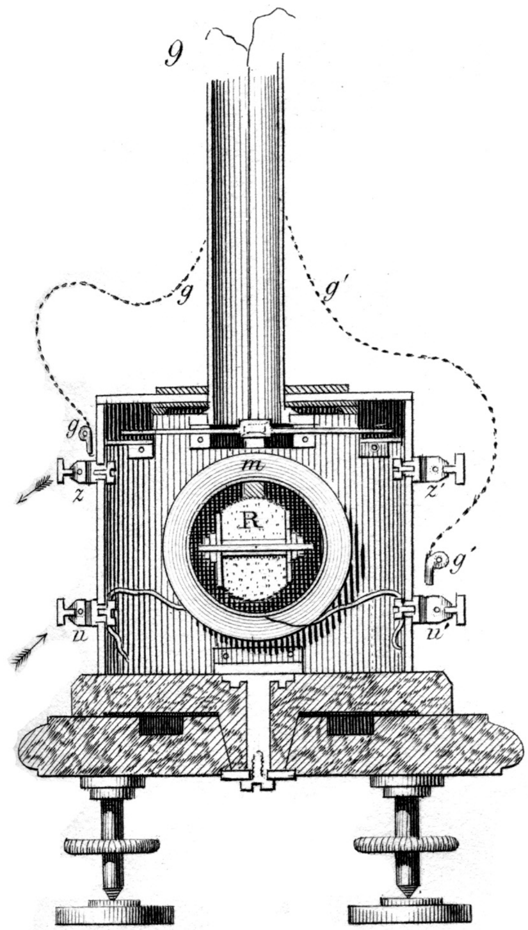
7 6

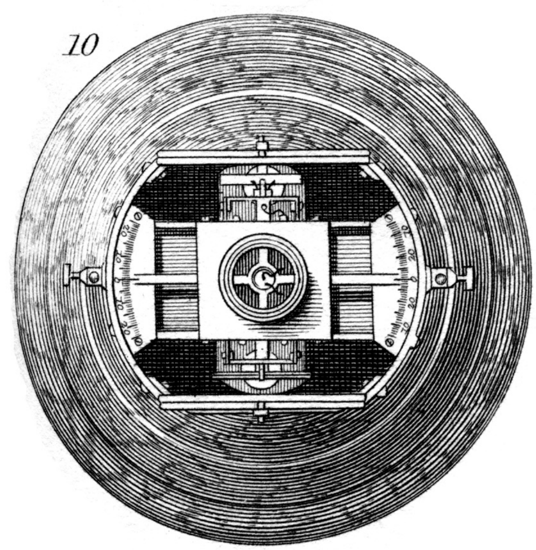




We now pass to fig. 8, which exhibits in a vertical section the lower part of the instrument, with the multiplier and the pedestal, which is constructed of serpentine. In it we first recognise fig. 2, suspended by the bifilar metallic wires *o* and *o*′, also as seen on a vertical section. The letters *m m* exhibit a section of the multiplier, wound round a brass drum furnished with wooden sides, in the interior of which the vibrating cylinder R is placed. These wooden sides support the tubes, within which the bifilar threads descend; the two scales for the index are also fixed to them.

Fig. 10, a view of the instrument as seen from above, exhibits more accurately the scale and the metallic plates, to which the tube is fastened. The sides of this multiplier are in connexion with a strip of copper, which by means of two cap-screws can be connected with the upper part *n n* of the foot of serpentine. This portion, *n n,* with its cone *i i*, is capable of rotation in the lower part of the serpentine foot, and by means of the metallic bolt *r* is kept in connexion with it by the screw *x*. Since, as shown in fig. 8, both the speculum and the counterpoise project towards the wooden sides of the multiplier, the whole is protected from the influence of a current of air by a cylindrical wooden cover, which is fixed to the upper corners of the wooden sides of the multiplier. In the direction of the speculum to the counterpoise, however, this cylindrical cover is flattened, so as to allow of a free view through the cavity of the multiplier. The flat side





of the cover next the speculum can be opened or closed at pleasure by a wooden plate, which however, to enable us to use the mirror, is furnished with a flat parallel glass, S. The whole of the other flat side of the cover, which is turned towards the counterpoise, may be closed or opened by a glass plate. Thus the vibrating reel, when the sides of the cover are closed, can still be seen, and its free oscillation in the cavity of the multiplier be observed and regulated by means of the three screws in the serpentine pedestal. Moreover, from above downwards, above the graduation, the cover is closed by two glass plates, which are moveable towards each other in metallic grooves, and excavated in a semicircular form in the centre, to allow the tube in which the bifilar wires are suspended to pass through them. In fig. 8, $v\,v$ exhibits the glass plate at the side; $v'\,w$ is the wooden plate, with the flat parallel glass S at the other side; $v\,v'$ is one of the upper glass plates. The letters $k\,k$ are loops, through which the conducting wires $g\,g$ and $g'\,g'$ in fig. 6 descend; these wires are fixed in these loops to avoid their lying loosely throughout their entire length; they terminate in pegs, or small cylinders.

Fig. 9 also exhibits a vertical section, but at right angles to that of fig. 8; m is the multiplier, and R a section of the reel vibrating within it. At the side of the case we perceive four metallic knobs, marked $u\,u'\;z\,z'$. These are perforated crucially, and the perforation most distant from the case is furnished with a screw; on the inner side of the case it is fixed to it by another screw. Two of these knobs, u and u', are in metallic contact with the commencement and termination of the multiplier, so that a current from the knob u can run through the multiplier into the knob u', and *vice versa.* The other two knobs, z and z', are perfectly isolated; but all four of the knobs are very useful for reversing the current, and for effecting various combinations. In this figure also we see the index vibrating above the scale-plate, as also in fig. 3, where the case is supposed to be removed.

Let us now trace the course of a galvanic current, which enters the instrument at the knob *u;* it passes from *u* through the multiplier *m* and towards *u';* if the conducting wire *g' g'* with its metallic cylindrical extremity be now inserted into this knob, the current ascends in *g' g'*, and (fig. 5) towards the nut *c' c'* above the roller *a'*, then down within the tube to *o';* thence (fig. 2) from *o'* through the centre *p* of the transverse beam to *r' m' k d*, through the vibrating reel to *m r p o*, and (fig. 5) to *o,* ascending above the roller *a* in the nut *c c,* to the second conducting wire *g g* and (fig. 9) through *g g* down into the knob *z*, whence it runs into the other of the two exciting surfaces.

By means of the upper rotating part of the serpentine pedestal, the instrument may be arranged in any part of a hall or room as required. All the figures are drawn one-fourth of the linear magnitude of the electro-dynamometer, excepting fig. 5, which is one-half the real magnitude.

The wire on the vibrating reel is 200 metres in length, that of the multiplier 300; the first forms about 1200 coils, the latter about 900. The length of the bifilar wires, (which are very fine; composed of silver, and were heated to redness,) from the transverse beam to the small rollers *a a'*, was half a metre.

The price of the instrument is 10 guineas.

Observations Demonstrating the Fundamental Principle of Electro-dynamics.

The following observations were not made with the instrument which has just been described. However, it is unnecessary to describe separately the instrument made use of on this occasion, because it merely differs from the former in minor points of arrangement, which were less convenient than those in the latter. One important modification only requires to be mentioned, viz. that the multiplier, which in the above description assumes an invariable position, in which its centre coincides with the centre of the bifilarly-suspended reel, was left moveable, so that it could be placed in any position as regards the vibrating reel, for the purpose of extending the observations to all relative positions of the two galvanic conductors, which act upon each other. Now as these two conductors form two coils, one of which can enclose the other, and in the instrument described above the inner and smaller coil was suspended by two threads, to serve as it were as a galvanometer-needle, whilst the outer and larger coil, was fixed and formed the multiplier; it was requisite for the object in question to reverse the arrangement, and to suspend the outer and larger coil by two threads so as to use the inner and smaller coil as a multiplier, because it was only by this means that the position of the multiplier could be altered at pleasure without interfering with the bifilar suspension. It is at once seen that the external reel, on account of its size, has a greater momentum from inertia, which produces a longer duration of its vibration; this influence however may be easily compensated for when necessary by altering the arrangement of the bifilar suspension.

As regards the observations themselves, it remains to be remarked, that to render the results comparable, the intensity of the current transmitted by the two conductors of the dynamometer was, simultaneously with the observation on the dynamometer, accurately measured by a second observer with a galvanometer. This was requisite, because no reliance can be placed upon the constancy of the intensity of the current during a continued series of experiments, even when the so-called constant battery of Grove or Bunsen is used.

The first experiment was made by passing three currents of different intensity, *i.e.* from 3, 2 and 1 of Grove's elements, through the two conductors of the dynamometer, and observing the simultaneous deflections of the dynamometer and galvanometer. After making the necessary reductions, the following means were obtained as the deflections:—

Number of Grove's elements.	Deflections	
	Of the Dynamometer	Of the Galvanometer
3	440.038	108.426
2	192.255	72.398
1	50.195	36.332

If we denote the dynamometric observations by δ, and the galvanometric observations by γ, we obtain

$$\gamma = 5.15534\sqrt{\delta}$$

for if we calculate the values of γ from the values found by observation for δ according to this formula, we obtain in the order of the series,

108.144
72.589
36.786,

which exhibit less differences from the values of γ found by observation than could be anticipated, thus:

- 0.282
+ 0.191
+ 0.454.

The electro-dynamic force of the reciprocal action of two conducting wires, through which currents of equal intensity are transmitted, is therefore in proportion to the square of this intensity, which is exactly what is required by the fundamental principle of electro-dynamics.

A more extended series of experiments was then made for the purpose of ascertaining the dependence of the electro-dynamic force, with which the two conducting wires of the dynamometer react upon each other, upon the relative position and distance of these wires.

For this purpose the arrangement was effected in such a manner, that one conducting wire, *i.e.* the multiplier, could be placed in any position as regards the other, *i.e.* as regards the bifilarly-suspended coil, the latter forming the larger coil, which inclosed the former smaller one.

Both coils were always placed in such a position that their axes were in the same horizontal plane, and formed a right angle with each other.

The distance of the two coils was determined by the distance of their centres from each other, and was thus assumed as = 0 when the centres of the two coils coincided.

When the latter was not the case, in addition to the magnitude of the distance of the two centres, it was also requisite to measure the angle which the line uniting the two central points formed with the axis of the bifilarly-suspended coil, whereby the direction in which the centre of the multiplier was removed from the centre of the bifilarly-suspended coil was defined. For this purpose the four cardinal directions were selected at which the former angle had the value 0°, 90°, 180° and 270°, *i.e.* when the axis of the bifilarly-suspended coil, like the axis of the needle of a magnet, was arranged in the magnetic meridian, the centre of the multiplier was removed from the centre of the above coil, sometimes in the direction of the meridian, *north* or *south,* and sometimes in the direction at right angles to the magnetic meridian, *east* or *west.* In each of these different directions the multiplier was placed successively at different distances from the suspended coil.

This arrangement of different positions and distances of the two conducting wires of the dynamometer accurately corresponds, as is seen at a glance, to the arrangement of different positions and distances of the two magnets, upon which Gauss based his measurements, in demonstrating the fundamental principle of magnetism. The bifilarly-suspended coil here occupied the place of Gauss's magnetic needle and the multiplier the place of Gauss's deflection-rod. The only important difference is, that the mutual action of the magnets could only be observed from a distance; consequently in the magnetic observations that case was excluded in which the centres of the two magnets coincided; whilst in the electro-dynamic measurements of which we are now speaking, the system could moreover be rendered complete by the case, in which the centre of the two coils coincided.

Simultaneously with the observations made on the dynamometer, the intensity of the current which was transmitted through the two coils of the dynamometer was measured by another observer with a galvanometer. By these auxiliary observations I was enabled to reduce all the observations made on the dynamometer in accordance with the law shown above, (that the electro-dynamic force is in proportion to the square of the intensity of the current,) to an equal intensity of the current, and thus to render the results obtained comparable.

The following table gives the reduced mean values which were obtained in the different instances. The first vertical column shows the distance of the two coils of the dynamometer; above the other columns, the direction formed by the line uniting the two centres with the axis of the bifilarly-suspended coil directed towards the magnetic meridian is given:—

Distance in mm.	North, 0°	East, 90°	South, 180°	West, 290°
0	22960	22960	22960	22960
300	77.16	189.24	77.06	190.62
400	34.78	77.61	34.77	77.28
500	18.17	39.37	18.30	39.16
600	...	22.53	...	22.38

Moreover, the above table shows that the results obtained for an equal distance in opposite directions varying 180°, agree together as far as the observations could be depended upon.

These values, when reduced by taking their means, after converting the divisions of the scale into degrees, minutes and seconds, yield the following table:—

R.	v	v'
0.3	0° 49' 22''	0° 20' 3''
0.4	0° 20' 8''	0° 9' 2''
0.5	0° 10” 12''	0° 4' 44''
0.6	0° 5' 50''	

in which the same notation is adopted as used by Gauss in his *Intensitas Vis Magneticae*, *&c*. in the comparison of the magnetic observations.

According to the fundamental principle of electro-dynamics, we should be able to develope the tangents of the angle of deflection v and v' according to the diminishing odd powers of the distance R, and we should have

$$\tan v = aR^{-3} + bR^{-5}$$
$$\tan v' = \frac{1}{2}aR^{-3} + cR^{-5}$$

where *a, b* and *c* are constants to be determined from the observations. If now in the present instance we make

$$\tan v = 0.0003572R^{-3} + 0.000002755R^{-5}$$
$$\tan v' = 0.001786R^{-3} - 0.000001886R^{-5}$$

we obtain the following table of *calculated* deflections, and their difference from those *found by observation:*-

R.	v	Difference.	v'	Difference
0.3	0° 49' 22"	0"	0° 20' 4"	-1"
0.4	0° 20' 7"	+1"	0° 8' 58"	+4"
0.5	0° 10' 8"	+4"	0° 4' 42"	+2"
0.6	0° 5' 49"	+1"		

Thus in this agreement of the calculated values with those obtained by observation, we have a confirmation of one of the most universal and most important consequences of the fundamental princple of electro-dynamics, viz. *that the same laws apply to electro-dynamic forces exerted at a distance as to magnetic forces.*

In this application of the laws of magnetism to electro-dynamic observations, that case of the latter where the centres of the two coils of the dynamometer coincide must be excluded. Moreover, in this extension of the laws of magnetism to electro-dynamic observations, the values of three constants must be deduced from the observations themselves, which is unnecessary when we have recourse to the fundamental principle of electro-dynamics itself, and calculate directly from it the results which the observations should have yielded in accordance with it. Hence from the fundamental principle of electro-dynamics—

1. In that case in which the straight line uniting the centre of the two coils coincides with the axis of the bifilarly suspended coil,

when *m* designates the radius of the multiplying coil, *n* the radius of the bifilarly-suspended coil, and *a* the distance of the centres of the two coils, and for brevity we make

$$\frac{m\,m}{a\,a + n\,n} = v\,v,$$

$$\frac{n\,n}{a\,a + n\,n} = w\,w,$$

$$\frac{4\,a\,a + n\,n}{16\,(a\,a + n\,n)} = f,$$

$$\frac{8\,a^4 + 4\,a\,a\,n\,n + n^4}{64\,(a\,a + n\,n)^2} = g,$$

the electro-dynamic momentum of rotation which the multiplying coil exerts upon the bifilarly-suspended coil, when a current of the intensity i *passes through both coils*, is determined with sufficient accuracy to be

$$= -\frac{\pi\pi}{2} v^3 nniiS$$

S designating the following series :-

$$\begin{aligned} S = &+[\tfrac{1}{3} - ww] \\ &- \tfrac{3}{2}\left[\tfrac{3}{5} - ww - (3 - 7ww)f\right]vv \\ &+ \tfrac{15}{8}\left[\tfrac{5}{7} - ww - 2(5 - 9ww)f + 3(5 - 11ww)g\right]v^4 \\ &- \tfrac{35}{16}\left[\tfrac{7}{9} - ww - 3(7 - 11ww)f + 11(7 - 13ww)g\right]v^6 \\ &+ \tfrac{315}{128}\left[\tfrac{9}{11} - ww - 4(9 - 13ww)f + 26(9 - 15ww)g\right]v^8 \end{aligned}$$

et cetera

If in this equation we substitute the values known from direct measurement, in millimetres,

m = 44.4,

n = 55.8,

and successively

a = 300, 400, 500,

we obtain as the rotating momentum sought, the following three values to be multiplied by π π *i i* :—

- 1.4544
- 0.6547
- 0.3452,

Moreover,

2. In that case where the right line uniting the centres of both coils is at right angles to the axis of the bifilarly-suspended coil, *m*, *n* and *a* having the same signification, and

$$\frac{m\,m}{a\,a + n\,n} = v\,v,$$

$$\frac{a\,a}{a\,a + n\,n} = f,$$

$$\frac{n\,n}{a\,a + n\,n} = 4\,g\,v\,v,$$

the rotatory momentum required is

$$= +\pi v^3 nnii S'$$

S' expressing the following series :-

$$S' = +\tfrac{1}{3}$$
$$-\tfrac{3}{2}\left[\tfrac{1}{5} - \tfrac{10}{3} fg\right]vv$$
$$+\tfrac{15}{8}\left[\tfrac{1}{7} + \tfrac{2}{5}(1-14f)g + 42 ffgg\right]v^4$$
$$-\tfrac{35}{16}\left[\tfrac{1}{9} + \tfrac{3}{7}(2-18f)g - \tfrac{54}{5}(1-11f)fgg - 572 f^3 g^3\right]v^6$$
$$+\tfrac{315}{128}\left[\begin{array}{l}\tfrac{1}{11} + \tfrac{4}{9}(3-22f)g + \tfrac{12}{7}(1-22f+143ff)gg \\ +\tfrac{1144}{5}(1-10f)ffg^3 + \tfrac{24310}{3} f^4 g^4\end{array}\right]v^8$$

et cetera

If in this series we substitute for *m* and *n* the given values, and successively *a* = 300, 400, 500 and 600, we obtain as the rotating momentum required, the following values to be multiplied by π π *i i* :-

+ 3.5625
+ 1.4661
+ 0.7420
+ 0.4267

Lastly,

3. In that case where the centres of both coils coincide, when m designates the radius of the multiplier, and n' and n'' the least and greatest radius of the bifilarly-suspended coil, the rotatory momentum sought is

$$= \frac{\pi\pi m^3}{n''-n'} ii \left[\begin{array}{l} \frac{1}{3}\log\text{nat}\frac{n''}{n'} + \frac{9}{160}\left(\frac{1}{n''n''-n'n'}\right)mm - \frac{225}{14336}\left(\frac{1}{n''^4}-\frac{1}{n'^4}\right)m^4 + \\ \frac{6125}{884736}\left(\frac{1}{n''^6}-\frac{1}{n'^6}\right)m^6 + \frac{694575}{184549376}\left(\frac{1}{n''^8}-\frac{1}{n'^8}\right)m^8 + \ldots. \end{array} \right]$$

If in this formula we substitute the values known from direct measurement in millimetres,

$m = 44.4$
$n' = 50.25$
$n'' = 61.35$,

we obtain as the rotatory momentum the following value to be multiplied by $\pi\,\pi\,i\,i$:—

+442.714.

This value suffers a reduction of about 1/29th when we take into consideration that all the turns of the two coils do not lie in one plane, which in this case exerts greater influence on account of their proximity than in the other cases. The above value thus becomes reduced to

$+427.5\pi\pi ii$

The numerical coefficients thus calculated should now be proportional to the observed values; and when multiplied by $\pi\pi ii$, the intensity of the current i being expressed according to the dimensions upon which the above measurements were based, should be equal.

In fact, when all the calculated numerical coefficients are multiplied by 53.06, and then arranged according to the analogy of the observed values, we obtain the following table of the calculated values, and their difference from those found by observation:-

Distance in millimeters	North or south, 0° or 180°	Difference	East or west, 90° or 270°	Difference
0	+22680.00	+280.00	+22680.00	+280.00
300	189.03	+0.90	77.17	- 0.06
400	77.79	-0.34	34.74	+ 0.03
500	39.37	-0.10	18.31	- 0.07
600	22.64	-0.18		

In this comparison of theory and experiment, the single factor 53.06 was deduced from observations; and this was merely done because this factor could not be determined with sufficient accuracy by direct measurements. The direct determination of this factor is based upon the ascertainment of the proportion of that measure of the intensity of the current, upon which the scale of the galvanometer used is based, to that absolute measure to which the theoretical expression refers. The measurements necessary for ascertaining this proportion could not all be effected with the requisite accuracy, because separate measures were not taken for this purpose. In fact, however, the above factor was provisionally, as well as circumstances permitted, determined by direct measurement, and found = 49.5. This result also exhibits an agreement with that previously deduced from the observations, which under the circumstances could not have been expected to be greater.

Observations Tending to Enlarge the Domain of Electro-dynamic Investigations

A. Observation of Voltaic Induction

If the bifilarly-suspended coil of the dynamometer be made to oscillate whilst a current is transmitted through it, or through the coil of the multiplier, or through both simultaneously, this motion is *inductive,* and excites a current in the conductor, through which no current was passing, or alters the current passing through this conductor. This mode of excitation of the current is called *voltaic induction.* The inducing motion, *i.e.* the velocity of the oscillating coil, is on each occasion diminished or *checked* by the antagonism of the currents excited by the voltaic induction and those conducted through the coil. This *check* to the vibrating coil *effected* by the voltaic induction may be accurately observed; and at the same time the motion of the oscillating coil itself, which *produces* the *voltaic induction,* may be accurately determined; and this twofold use of the dynamometer affords the data necessary for the more accurate investigation of the laws of voltaic induction.

The bifilarly-suspended coil closed in itself was made to oscillate to the greatest extent at which the scale permitted observations to be made, and its oscillations from 0 were counted until they became too minute for accurate observation. During the counting, the magnitude of the arc of oscillation was measured from time to time. These experiments were *first* made under the influence of voltaic induction, a current from three Grove's elements being conducted through the multiplying coil; the same experiments were *next* repeated, after the removal of the elements, without voltaic induction:—

With voltaic Induction.		Without voltaic Induction.	
Enumeration of the oscillations.	Arcs of oscillation.	Enumeration of the oscillations.	Arcs of oscillation.
0	764.10	0	650.80
9	679.14	14	601.43
18	604.05	25	564.90
35	484.15	52	485.28
47	414.60	82	409.62
57	365.50	109	353.08
74	292.27	134	306.70
85	253.30	163	261.08
103	200·80	189	226.33
118	165.56	212	198.68
130	141.37	232	178.26
143	119.33	254	157.98
157	100.49	284	134.17
179	75.59	309	116.30
196	60.58	328	105.25
210	50.08	369	83.68
		387	75.45

It is evident on comparison, that the diminution of the magnitude of the arc, which without the influence of induction from one oscillation to another amounted on an average to 1/180th, with the cooperation of the induction rose to 1/77th part.

When for the multiplying coil with the current transmitted through it, a magnet equivalent in an electro-magnetic point of view is substituted, the diminution of the arc is found to be equally great, *i.e.* the magnetic induction of this magnet is equal to the voltaic induction of the current in the multiplier.

The velocity which the inducing motion must possess for the intensity of the induced current to be equal to that of the inducing current, may also be deduced from these experiments.

B. Determination of the duration of Momentary Currents, as also its application to Physiological Experiments.

When the intensity of a *continued* constant current is to be determined, both the galvanometer (the sine- or tangent-compass) and the dynamometer may be used; but if the current, the intensity of which is to be determined, is merely of *momentary* duration, observation made with either of these instruments is not sufficient, because the deflection observed does not depend merely upon the intensity of the current, but also upon the duration itself. It is therefore requisite, in experimentally investigating the intensity of the current, also to determine its duration.

The two instruments, *i.e.* the galvanometer and the dynamometer, are complementary to each other, so that when the same momentary current is transmitted through both, and the deflection of both instruments thus produced is observed, both the duration and the intensity of the momentary current can be determined from these two observations. This reciprocity is based upon the circumstance that the observed deflection of both instruments depends in the same manner upon the duration of the momentary current, *i.e.* it is proportional to it, whilst it is not dependent in the same manner upon the intensity of the current, because the deflection of the galvanometer is in proportion to the intensity of the current.

Let s and ς indicate the duration of the oscillations of the galvanometer and dynamometer;

e' and ε the deflection at which both instruments remain when the same constant current of the intensity i' is transmitted through them;

Whilst e and ε indicate the extent of the deflection which both instruments attain in consequence of a momentary current of the duration Θ and of the intensity *i;* the following equation then gives the *duration* Θ:—

$$\Theta = \frac{1}{\pi} \times \frac{ss}{\varsigma} \times \frac{\varepsilon'}{e'e'} \times \frac{ee}{\varepsilon}$$

and the following that of the *intensity* of the current *i*

$$i = \frac{\varsigma}{s} \times \frac{e'}{\varepsilon'} \times i' \times \frac{\varepsilon}{e}$$

s, ς, e', ε', i', e and ε in these formulae are magnitudes which can be determined by observation.

This combination of the dynamometer with the galvanometer is of special importance in physiology, to investigate accurately the excitation of the nerves by galvanic currents. For it is found that nerves of sensation especially are quickly deadened by continued currents, and hence that for such experiments momentary currents are frequently required to be used. But the observed impressions of sense depend less upon the duration of the current than upon its intensity; and it is essential to be acquainted with both.

C. Repetition of Ampère's Fundamental Experiment with common Electricity and measurement of the duration of the Electric Spark on the discharge of a Leyden Jar.

It is evident from the preceding remarks, that the action of a current upon the dynamometer depends more upon the intensity of the current, to the square of which it is proportionate, than upon the duration of the current, to which it is simply proportional. Hence it follows that even a small quantity of electricity, when passed through the dynamometer within a very short period, so that it forms a current of very short duration but very great intensity, will produce a sensible effect. This is, in fact, the case when the small quantity of electricity which can be collected in a Leyden jar or battery is transmitted during its discharge through the dynamometer. By this means it was found that Ampere's fundamental experiment, which had previously been made only with powerful galvanic batteries, could also be made with common electricity.

When the same electricity, collected in Leyden jars, after having been transmitted through the dynamometer, was also conducted through a galvanometer and the deflection thus produced in both instruments was measured, in accordance with the above rules, the duration of the current, *i.e.* the duration of the electric spark on the discharge of the Leyden jar, and at the same time the intensity of the current could be determined, admitting that the current might be considered as uniform during its brief duration.

It is well known that in experiments of this kind the discharge of the Leyden jar is effected by means of a wet string, to prevent its taking place through the air instead of through the fine wires of the two instruments. In this manner a series of experiments was made: a battery of eight jars being discharged through a wet hempen string, 7 millimetres in thickness and of different lengths, the following results were obtained:-

Length of the string. Millimetres.	Duration of the spark. Seconds
2000	0.0851
1000	0.0345
500	0.0187
250	0.0095

Hence the duration of the spark was nearly in proportion to the length of the string; for the observed duration of the spark is:—

Seconds.
0.0816 + 0.0035
0.0408 - 0.0063
0.0204 - 0.0017
0.0102 - 0.0007

The first part of the duration of the spark is thus exactly in proportion to the length of the string; but the second part is so small that it may be considered as arising from error of observation, which was unavoidable.

It is thus evident that the result obtained by Prof. Wheatstone, according to which the duration of the spark on discharge by simple metallic conductors is infinitely short in comparison with that ascertained in the present case, is in direct accordance with this result.

When a rapid alternation of positive and negative currents ensues in a conducting wire, the continued motion of the electricity becomes converted into an *oscillation.* An oscillation of this kind cannot however be observed by means of a galvanometer (for instance, a sine- or tangent-compass), because in this case the effects of the successive opposite oscillations destroy each other.

But the case is different with the dynamometer, in the two coils of which the direction of the vibration always changes simultaneously, and in which the deflection observed is in proportion to the square of the intensity of the current; for it is self-evident that the simultaneous change of the direction in both coils can exert no influence upon the action, because in the dynamometer a negative current transmitted through both coils produces a deflection towards the same side as a positive current transmitted through both coils. The occurrence of the deflection of the dynamometer to one side or the other does not, as in the galvanometer, depend upon the direction of the transmitted current, but merely upon the mode of connexion of the extremities of the wires of both coils.

But an electric vibration may be readily produced in a conducting wire by a magnetized steel bar vibrating so as to produce a musical sound, when one portion of the conducting wire, forming as it were the inducing coil, surrounds the free vibrating end of the bar, so that the direction of the vibration is at right angles to the plane of the coils of the wire. All vibrations of the bar on one side then produce positive currents in the wire, and all the vibrations on the other side produce negative currents, which follow each other as rapidly as the sonorous vibrations themselves.

When the ends of the wire of the inducing coil are united to the ends of that of the dynamometer, a deflection of the latter during the vibration of the bar is observed, which can be accurately measured. This deflection remains unaltered so long as the intensity of the sonorous vibrations remains unaltered, but speedily diminishes when the intensity of the sonorous vibrations diminishes; and when the amplitude of the sonorous vibrations has fallen to a half, it then amounts to the fourth part only.

The dynamometer thus presents a means of estimating the intensity of sonorous vibrations, which is of importance, because methods adapted to these measurements are still much required.

In addition to the investigations which we have hitherto considered, and which are based on the use of the dynamometer, there are others which will be subsequently treated of, when some modifications in the construction of this instrument for special objects will also be more accurately detailed.

On the Connexion of the Fundamental Principle of Electro-dynamics with that of Electro-statics

The fundamental principle of electro-statics is, that when two electric (positive or negative) masses, denoted by e and e, are at a distance r from each other, the amount of the force with which the two masses act reciprocally upon each other is expressed by

$$\frac{ee'}{rr},$$

and that repulsion or attraction occurs accordingly as this expression has a positive or negative value.

On the other hand, the fundamental principle of electro-dynamics is as follows:— When two elements of a current, the lengths of which are α and α' and the intensities i and i', and which are at the distance r from each other, so that the directions in which the positive electricity in both elements moves, form with each other the angle ε, and with the connecting right line the angles Θ and Θ', the magnitude of the force with which the elements of the current reciprocally act upon each other is determined by the expression

$$-\frac{\alpha\alpha' ii'}{rr}(\cos\varepsilon - \tfrac{3}{2}\cos\Theta\cos\Theta')$$

and repulsion or attraction occurs according as this expression has a positive or negative value. The expressions of the rotatory momentum exerted by one coil of the dynamometer upon the other, developed at pages 21 - 23, are all deduced from this fundamental principle.

The *former* of the two fundamental principles mentioned refers to two electric masses and their antagonism, the *latter* to two elements of a current and their antagonism. A more intimate connexion between the two can only be attained by recurring, likewise in the case of the elements of the current, to the consideration of the electric magnitudes existing in the elements of the current, and their antagonism.

Thus the next question is, what electric magnitudes are contained in the two elements of a current, and upon what mutual relations of these masses their reciprocal actions may depend.

If the mass of positive electricity in a portion of the conducting wire equal to a unit of length be represented by e, and consequently the mass of the positive electricity contained in the elements of the current, the length of which is $= \alpha$, by αe, and if u indicates the velocity with which the mass moves, the product $e\,u$ expresses that mass of positive electricity which in a unit of time passes through each section of the conducting wire, to which the intensity of the current i must be considered as proportional; hence, when a expresses a constant factor,

$$aeu = i$$

If now αe represent the mass of positive electricity in the element of the current a, and u its velocity, $-\alpha e$ represents the mass of negative electricity in the same element of the current, and $-u$ its velocity.

We have also, when

$$ae'u' = i',$$

$\alpha' e'$ as the mass of positive electricity in the second element of the current α', and u' its velocity, and lastly, $-\alpha' e$ as the mass of negative electricity, and $-u'$ as its velocity. If now for i and i', in the expression of the force which one element of a current exerts upon another, their values $i = a\,e\,u$, and $i' = ae'u'$ are substituted, we then obtain for them

$$-\frac{\alpha e \alpha' e'}{rr} aauu' \left(\cos\varepsilon - \tfrac{3}{2}\cos\Theta\cos\Theta'\right).$$

If now we *first* consider in this expression $\alpha e \alpha' e'$ as the product of the *positive* electric masses αe and $\alpha' e'$ in the two elements of the current, and uu' as the product of their velocities u and u', and if we denote by r the variable distance of these two masses in motion; and lastly, by s and s', the length or a portion of each of the two conducting wires, to which the elements of the current α and α' just considered belong, estimated from a definite point of origin and proceeding in the direction of the *positive* electricity, as far as the element of the current under consideration, we then know that the cosines of the two angles Θ and Θ', which the two conducting wires in the situation of the elements of the current mentioned form with the connecting right line $r_{/}$, may be represented by the partial differential coefficients of $r_{/}$ with respect to $s_{/}$ and $s_{/}'$; thus

$$\cos\Theta = \frac{dr_{/}}{ds_{/}},\ \cos\Theta' = -\frac{dr_{/}}{ds_{/}'},$$

we then have

$$\cos\varepsilon = -r_{/}\frac{ddr_{/}}{ds_{/}\,ds_{/}'} - \frac{dr_{/}\,dr_{/}}{ds_{/}\,ds_{/}'}$$

as the cosine of the angle ε which the directions of the two conducting wires form with each other. Moreover, if the differential coefficients above mentioned be substituted for the cosines of the three angles ε, Θ and Θ', we have

$$-\frac{\alpha e\alpha' e'}{r_{/}r_{/}}a\,a\,u\,u'\left(\frac{1}{2}\frac{dr_{/}\,dr_{/}}{ds_{/}\,ds_{/}'} - r_{/}\frac{ddr_{/}}{ds_{/}\,ds_{/}'}\right)$$

as the expression of the force with which one element of the current acts upon the other.

Secondly, if in the above expression, $-\alpha e\,\alpha' e'$ be considered as the product of the *positive* electric mass αe of one element of the current α into the *negative* electric mass $-\alpha' e'$ of the other element of the current α', and $-uu'$ as the product of their velocities u and $-u'$; moreover, if the variable distance of these two moving masses be denoted by $r_{//}$ and by $s_{/}$ and $s'_{//}$ the length of a portion of each of the two conducting wires, to which the elements of the current under consideration belong, taken from a definite point of origin, and proceeding in that direction in which, in the first the *positive;* in the second the *negative* electricity runs, as far as the element of the current mentioned, we obtain in the same manner

$$\cos\Theta = \frac{dr_{\prime\prime}}{ds_{\prime}}, \quad \cos\Theta' = \frac{dr_{\prime\prime}}{ds_{\prime\prime}'}$$
$$\cos\varepsilon = r_{\prime\prime}\frac{ddr_{\prime\prime}}{ds_{\prime}ds_{\prime\prime}'} + \frac{dr_{\prime\prime}dr_{\prime\prime}}{ds_{\prime}ds_{\prime\prime}'}.$$

On substituting these values, we have the following expression for the force with which one element of the current acts upon the other:-

$$+\frac{\alpha e\alpha' e'}{r_{\prime\prime}r_{\prime\prime}}aauu'\left(\frac{1}{2}\frac{dr_{\prime\prime}dr_{\prime\prime}}{ds_{\prime}ds_{\prime\prime}'} - r_{\prime\prime}\frac{ddr_{\prime\prime}}{ds_{\prime}ds_{\prime\prime}'}\right).$$

If, *thirdly*, we consider in the original expression $-\alpha e\,\alpha' e'$ as the product of the *negative* electrical masses $-\alpha e$ and $-\alpha' e'$ into the two elements of the current, and $u\,u'$ as the product of their velocities $-u$ and $-u'$ and $r_{\prime\prime\prime}$ denote the variable distance of these two moving masses, and lastly, $s_{\prime\prime}$ and $s_{\prime\prime}'$ denote the length of a portion of each of the two conducting wires to which the elements of the current under consideration belong, calculated from *u*, definite point of origin, and proceeding in that direction in which the *negative* electricity runs, as far as the element of the current under consideration; we have

$$\cos\Theta = -\frac{dr_{\prime\prime\prime}}{ds_{\prime\prime}}, \quad \cos\Theta' = \frac{dr_{\prime\prime\prime}}{ds_{\prime\prime}'}$$
$$\cos\varepsilon = -r_{\prime\prime\prime}\frac{ddr_{\prime\prime\prime}}{ds_{\prime\prime}ds_{\prime\prime}'} - \frac{dr_{\prime\prime\prime}dr_{\prime\prime\prime}}{ds_{\prime\prime}ds_{\prime\prime}'}.$$

On substituting these values, we have a third expression for the force with which one element of the current acts upon the other, namely,

$$-\frac{\alpha e\alpha' e'}{r_{\prime\prime\prime}r_{\prime\prime\prime}}aauu'\left(\frac{1}{2}\frac{dr_{\prime\prime\prime}dr_{\prime\prime\prime}}{ds_{\prime\prime}ds_{\prime\prime}'} - r_{\prime\prime\prime}\frac{ddr_{\prime\prime\prime}}{ds_{\prime\prime}ds_{\prime\prime}'}\right).$$

In fine, if, *fourthly,* in the original expression we consider $-\alpha e\ \alpha' e'$ as the product of the *negative* electric mass $-\alpha e$ of the element of the current α into the *positive* electric mass $\alpha' e'$ of the element of the current α', and $-u\,u'$ as the product of their velocities $-u$ and u'; if, moreover, $r_{////}$ designate the variable distance of these two moving masses, and $s_{//}$ and $r_{/}$, $r_{//}$, $r_{///}$, $r_{////}$, the length of a portion of each of the two conducting wires to which the elements of the current under consideration belong, calculated from a defined point of origin, proceeding in that direction in which in the first the *negative,* in the second the *positive* electricity runs, we have

$$\cos\Theta = -\frac{dr_{////}}{ds_{//}},\ \cos\Theta' = -\frac{dr_{////}}{ds'_{/}}$$

$$\cos\varepsilon = r_{////}\frac{ddr_{////}}{ds_{//}ds'_{/}} + \frac{dr_{////}dr_{////}}{ds_{//}ds'_{/}}.$$

If now these values be substituted, we have the fourth expression of the force with which one element of the current acts upon the other, viz.

$$+\frac{\alpha e\alpha' e'}{r_{////}r_{////}}aauu'\left(\frac{1}{2}\frac{dr_{////}dr_{////}}{ds_{//}ds'_{/}} - r_{////}\frac{ddr_{////}}{ds_{//}ds'_{/}}\right).$$

Now at that moment in which the electric masses alluded to occur in the two elements α and α', the distances $r_{/}$, $r_{//}$, $r_{///}$, $r_{////}$, have all the same value, which is expressed by r. Hence the four expressions of the electro-dynamic force of the two elements of the current α and α' become converted into the following:-

$$-\frac{\alpha e\alpha' e'}{rr}aauu'\left(\frac{1}{2}\frac{dr_{/}dr_{/}}{ds_{/}ds'_{/}} - r\frac{ddr_{/}}{ds_{/}ds'_{/}}\right), \ldots . (1.)$$

$$+\frac{\alpha e\alpha' e'}{rr}aauu'\left(\frac{1}{2}\frac{dr_{//}dr_{//}}{ds_{/}ds'_{//}} - r\frac{ddr_{//}}{ds_{/}ds'_{//}}\right), \ldots . (2.)$$

$$-\frac{\alpha e\alpha' e'}{rr}aauu'\left(\frac{1}{2}\frac{dr_{///}dr_{///}}{ds_{//}ds'_{//}} - r\frac{ddr_{///}}{ds_{//}ds'_{//}}\right), \ldots . (3.)$$

$$+\frac{\alpha e\alpha' e'}{rr}aauu'\left(\frac{1}{2}\frac{dr_{////}dr_{////}}{ds_{//}ds'_{/}} - r\frac{ddr_{////}}{ds_{//}d'_{/}}\right), \ldots . (4.)$$

from which we can construct the fifth expression, viz. (5.) :—

$$-\frac{\alpha e\alpha' e'}{rr}\frac{aa}{4}uu'\Big[\frac{1}{2}\Big(\frac{dr_I dr_I}{ds_I ds_I'} - \frac{dr_{II}dr_{II}}{ds' ds_{II}'} + \frac{dr_{III}dr_{III}}{ds_{II}ds_{II}'} - \frac{dr_{IIII}dr_{IIII}}{ds_{II}ds_I'}\Big)$$
$$- r\Big(\frac{d\,dr_I}{ds_I ds_I'} - \frac{d\,dr_{II}}{ds_I ds_{II}'} + \frac{d\,dr_{III}}{ds_{II}ds_{II}'} - \frac{d\,dr_{IIII}}{ds_{II}ds_I'}\Big)\Big].$$

The four variable distances r_I, r_{II}, r_{III}, r_{IIII}, are now respectively dependent upon the variable magnitudes of the paths s_I and s_I', s_I and s'_{II}, s_{II} and s'_{II}, s_{II} and s_I', through which the moveable masses to which they refer have passed in the two given conducting wires, and which consequently are again functions of the time *t*. On developing their complete differentials, we have

$$dr_I = \frac{dr_I}{ds_I}ds_I + \frac{dr_I}{ds_I'}ds_I',$$

$$dr_{II} = \frac{dr_{II}}{ds_I}ds_I + \frac{dr_{II}}{ds_{II}'}ds_{II}',$$

$$dr_{III} = \frac{dr_{III}}{ds_{II}}ds_{II} + \frac{dr_{III}}{ds_{II}'}ds_{II}',$$

$$dr_{IIII} = \frac{dr_{IIII}}{ds_{II}}ds_{II} + \frac{dr_{IIII}}{ds_I'}ds_I';$$

If these differentials are respectively divided by the elements of the time *dt*, and their squares dt^2, and admitting that

$$\frac{ds_I}{dt} = \frac{ds_{II}}{dt} = u, \quad \frac{ds_I'}{dt} = \frac{ds_{II}'}{dt} = u',$$

we have

$$\frac{dr_I}{dt} = u\frac{dr_I}{ds_I} + u'\frac{dr_I}{ds_I'},$$

$$\frac{dr_{II}}{dt} = u\frac{dr_{II}}{ds_I} + u'\frac{dr_{II}}{ds_{II}'},$$

$$\frac{dr_{III}}{dt} = u\frac{dr_{III}}{ds_{II}} + u'\frac{dr_{III}}{ds_{II}'},$$

$$\frac{dr_{IIII}}{dt} = u\frac{dr_{IIII}}{ds_{II}} + u'\frac{dr_{IIII}}{ds_I'};$$

moreover,

$$\frac{d\,d\,r_{I}}{d\,t^2} = u\,u\,\frac{d\,d\,r_{I}}{d\,s_{I}^2} + 2\,u\,u'\,\frac{d\,d\,r_{I}}{d\,s_{I}\,d\,s_{I}'} + u'\,u'\,\frac{d\,d\,r_{I}}{d\,s_{I}'^2},$$

$$\frac{d\,d\,r_{II}}{d\,t^2} = u\,u\,\frac{d\,d\,r_{II}}{d\,s_{I}^2} + 2\,u\,u'\,\frac{d\,d\,r_{II}}{d\,s_{I}\,d\,s_{II}'} + u'\,u'\,\frac{d\,d\,r_{II}}{d\,s_{II}'^2},$$

$$\frac{d\,d\,r_{III}}{d\,t^2} = u\,u\,\frac{d\,d\,r_{III}}{d\,s_{II}^2} + 2\,u\,u'\,\frac{d\,d\,r_{III}}{d\,s_{II}\,d\,s_{II}'} + u'\,u'\,\frac{d\,d\,r_{III}}{d\,s_{II}'^2},$$

$$\frac{d\,d\,r_{IIII}}{d\,t^2} = u\,u\,\frac{d\,d\,r_{IIII}}{d\,s_{II}^2} + 2\,u\,u'\,\frac{d\,d\,r_{IIII}}{d\,s_{II}\,d\,s_{I}'} + u'\,u'\,\frac{d\,d\,r_{IIII}}{d\,s_{I}'^2}.$$

From the four last equations we get immediately—

$$2\,u\,u'\,\frac{d\,d\,r_{I}}{d\,s_{I}\,d\,s_{I}'} = +\frac{d\,d\,r_{I}}{d\,t^2} - u\,u\,\frac{d\,d\,r_{I}}{d\,s_{I}^2} - u'\,u'\,\frac{d\,d\,r_{I}}{d\,s_{I}'^2},$$

$$-2\,u\,u'\,\frac{d\,d\,r_{II}}{d\,s_{I}\,d\,s_{II}'} = -\frac{d\,d\,r_{II}}{d\,t^2} + u\,u\,\frac{d\,d\,r_{II}}{d\,s_{I}^2} + u'\,u'\,\frac{d\,d\,r_{II}}{d\,s_{II}'^2},$$

$$2\,u\,u'\,\frac{d\,d\,r_{III}}{d\,s_{I}\,d\,s_{II}'} = +\frac{d\,d\,r_{III}}{d\,t^2} - u\,u\,\frac{d\,d\,r_{III}}{d\,s_{II}^2} - u'\,u'\,\frac{d\,d\,r_{III}}{d\,s_{II}'^2},$$

$$-2\,u\,u'\,\frac{d\,d\,r_{IIII}}{d\,s_{II}\,d\,s_{I}'} = -\frac{d\,d\,r_{IIII}}{d\,t^2} + u\,u\,\frac{d\,d\,r_{IIII}}{d\,s_{II}^2} + u'\,u'\,\frac{d\,d\,r_{IIII}}{d\,s_{I}'^2}.$$

Now the differential coefficients

$$\frac{d\,d\,r_{I}}{d\,s_{I}^2},\ \frac{d\,d\,r_{II}}{d\,s_{I}^2},\ \frac{d\,d\,r_{III}}{d\,s_{II}^2},\ \frac{d\,d\,r_{IIII}}{d\,s_{II}^2}$$

have the same value, which is dependent merely upon the position and form of the *first* conducting wire, and which we shall denote by $\frac{d\,d\,r}{d\,s^2}$. This applies also to the differential coefficients

$$\frac{d\,d\,r_{I}}{d\,s_{I}'^2},\ \frac{d\,d\,r_{II}}{d\,s_{II}'^2},\ \frac{d\,d\,r_{III}}{d\,s_{II}'^2},\ \frac{d\,d\,r_{IIII}}{d\,s_{I}'^2},$$

all of which denote the same magnitudes, which are dependent merely upon the position and form of the *second* conducting wire, and which for brevity we shall denote by $\frac{d\,d\,r}{d\,s'^2}$.

On summation, bearing this in mind, we have

$$2uu'\left(\frac{ddr_{\prime}}{ds_{\prime}ds'_{\prime}}-\frac{ddr_{\prime\prime}}{ds_{\prime}ds'_{\prime\prime}}+\frac{ddr_{\prime\prime\prime}}{ds_{\prime\prime}ds'_{\prime\prime}}-\frac{ddr_{\prime\prime\prime\prime}}{ds_{\prime\prime}ds'_{\prime}}\right)$$
$$=\frac{ddr_{\prime}}{dt^2}-\frac{ddr_{\prime\prime}}{dt^2}+\frac{ddr_{\prime\prime\prime}}{dt^2}-\frac{ddr_{\prime\prime\prime\prime}}{dt^2}.$$

But from the first four equations, after they have been squared, we have

$$2uu'\frac{dr_{\prime}dr_{\prime}}{ds_{\prime}ds'_{\prime}}=+\frac{dr_{\prime}^2}{dt^2}-uu\frac{dr_{\prime}^2}{ds_{\prime}^2}-u'u'$$
$$-2uu'\frac{dr_{\prime\prime}dr_{\prime\prime}}{ds_{\prime}ds'_{\prime\prime}}=-\frac{dr_{\prime\prime}^2}{dt^2}+uu\frac{dr_{\prime\prime}^2}{ds_{\prime}^2}+u'u'$$
$$2uu'\frac{dr_{\prime\prime\prime}dr_{\prime\prime\prime}}{ds_{\prime\prime}ds'_{\prime\prime}}=+\frac{dr_{\prime\prime\prime}^2}{dt^2}-uu\frac{dr_{\prime\prime\prime}^2}{ds_{\prime\prime}^2}-u'u'$$
$$-2uu'\frac{dr_{\prime\prime\prime\prime}dr_{\prime\prime\prime\prime}}{ds_{\prime\prime}ds'_{\prime}}=-\frac{dr_{\prime\prime\prime\prime}^2}{dt^2}+uu\frac{dr_{\prime\prime\prime\prime}^2}{ds_{\prime\prime}^2}+u'u'$$

Now the differential coefficients $\frac{dr_{\prime}^2}{ds_{\prime}^2}, \frac{dr_{\prime\prime}^2}{ds_{\prime}^2}, \frac{dr_{\prime\prime\prime}^2}{ds_{\prime\prime}^2}, \frac{dr_{\prime\prime\prime\prime}^2}{ds_{\prime\prime}^2}$

have also the same value, which shall be denoted by $\frac{dr^2}{ds^2}$, as have likewise $\frac{dr_{\prime}^2}{ds_{\prime}'^2}, \frac{dr_{\prime\prime}^2}{ds_{\prime\prime}'^2}, \frac{dr_{\prime\prime\prime}^2}{ds_{\prime\prime}'^2}, \frac{dr_{\prime\prime\prime\prime}^2}{ds_{\prime}'^2}$,

which we shall denote by $\frac{dr^2}{ds'^2}$.

On summation, keeping this in view, we have

$$2uu'\left(\frac{dr_{\prime}dr_{\prime}}{ds_{\prime}ds'_{\prime}}-\frac{dr_{\prime\prime}dr_{\prime\prime}}{ds_{\prime}ds'_{\prime\prime}}+\frac{dr_{\prime\prime\prime}dr_{\prime\prime\prime}}{ds_{\prime\prime}ds'_{\prime\prime}}-\frac{dr_{\prime\prime\prime\prime}dr_{\prime\prime\prime\prime}}{ds_{\prime\prime}ds'_{\prime}}\right)$$
$$=\frac{dr_{\prime}^2}{dt^2}-\frac{dr_{\prime\prime}^2}{dt^2}+\frac{dr_{\prime\prime\prime}^2}{dt^2}-\frac{dr_{\prime\prime\prime\prime}^2}{dt^2}.$$

On substituting these values in the fifth expression found for the electro-dynamic force, it becomes

$$-\frac{\alpha e\alpha' e'}{rr}\frac{aa}{16}\left[\left(\frac{dr_{\prime}^2}{dt^2}-\frac{dr_{\prime\prime}^2}{dt^2}+\frac{dr_{\prime\prime\prime}^2}{dt^2}-\frac{dr_{\prime\prime\prime\prime}^2}{dt^2}\right)\right.$$
$$\left.-2r\left(\frac{ddr_{\prime}}{dt^2}-\frac{ddr_{\prime\prime}}{dt^2}+\frac{ddr_{\prime\prime\prime}}{dt^2}-\frac{ddr_{\prime\prime\prime\prime}}{dt^2}\right)\right],$$

an expression which may be resolved into the four following members:—

$$-\frac{\alpha e \alpha' e'}{r_{I} r_{I}} \frac{aa}{16}\left(\frac{dr_{I}^2}{dt^2} - 2 r_{I} \frac{ddr_{I}}{dt^2}\right),$$
$$+\frac{\alpha e \alpha' e'}{r_{II} r_{II}} \frac{aa}{16}\left(\frac{dr_{II}^2}{dt^2} - 2 r_{II} \frac{ddr_{II}}{dt^2}\right),$$
$$-\frac{\alpha e \alpha' e'}{r_{III} r_{III}} \frac{aa}{16}\left(\frac{dr_{III}^2}{dt^2} - 2 r_{III} \frac{ddr_{III}}{dt^2}\right),$$
$$+\frac{\alpha e \alpha' e'}{r_{IIII} r_{IIII}} \frac{aa}{16}\left(\frac{dr_{IIII}^2}{dt^2} - 2 r_{IIII} \frac{ddr_{IIII}}{dt^2}\right).$$

Each of these four members refers exclusively to *two* of the four electric masses distinguished in the two elements of the current, viz. the *first* member to the two positive masses αe and $\alpha' e'$, the relative distance of which is r_{I}, velocity $\frac{dr}{dt}$, and acceleration $\frac{ddr_{I}}{dt^2}$; the *second* to the positive mass αe in the first, and to the negative mass $-\alpha' e'$, in the second element the relative distance of which is r_{II}, velocity $\frac{dr_{II}}{dt}$, and acceleration $\frac{ddr_{II}}{dt^2}$, and so on; and in fact all four are members of the masses to which they refer, the distance, velocity and acceleration of which are composed *in exactly the same manner.*

Hence it is evident that if the entire expression of the electrodynamic force of two elements of a current be considered as the sum of the forces, which each two of the four electric masses they contain exert upon each other, this sum would, be decomposed into its *original constituents,* the four above members representing individually the four forces which the four electric masses in the two elements exert *in pairs* upon each other.

Hence also the force with which any positive or negative mass E acts upon any other positive or negative mass E', at the distance R, with a relative velocity of $\frac{d\mathbf{R}}{dt}$, and acceleration $\frac{dd\mathbf{R}}{dt^2}$, may be expressed by

$$-\frac{aa}{16}\times\frac{\mathrm{EE}'}{\mathrm{RR}'}\left(\frac{d\mathrm{R}^2}{dt^2}-2R\frac{dd\mathrm{R}}{dt^2}\right);$$

for this fundamental principle is necessary and at the same time sufficient to allow of the deduction of Ampère's electro-dynamic laws, which are confirmed by the above measurements.

However, this new fundamental principle of electro-dynamics is in its nature more *general* than that formerly laid down by Ampère; for the latter refers merely to the special case, in which four electric magnitudes are given at the same time, subject to the conditions premised for invariable and undisturbed elements of the current, whilst such a limitation to the above conditions does not occur in the former. This fundamental principle, consequently, admits of application in those cases where the former is inapplicable; hence its greater utility.

If, lastly, the newly-discovered fundamental principle of electro-dynamics be compared with the fundamental principle of electro-statics mentioned at the commencement, we see that each estimates a force which *two electric masses* exert upon each other; but that in the cases hitherto considered, one of the two forces disappears each time, whence the other only requires consideration. This occurs *first* in all cases which belong to electrostatics, because here the force determined by the new principle of electro-dynamics always disappears; but it also occurs, *secondly,* in all cases belonging to electro-dynamics which have yet come under consideration, where relations are constantly pre-supposed to exist, in which all forces estimated by the principle of electro-statics are mutually checked.

Thus the two principles are complementary to each other, and hence they may be combined to form a general fundamental principle *for the whole theory of electricity,* which comprises both electro-statics and electro-dynamics.

By the fundamental principle of electro-statics, a force

$$= \frac{EE'}{RR}$$

was found for two electric masses E and E' at the distance R; if this force be then added to that yielded by the new principle of electrodynamics,

$$= -\frac{aa}{16} \times \frac{EE'}{RR'} \left(\frac{dR^2}{dt^2} - 2R \frac{ddR}{dt^2} \right)$$

we obtain, as the general expression for the complete determination of the force which any electric mass E exerts upon another E', whether at rest or in motion,

$$\frac{EE'}{RR'} \left(1 - \frac{aa}{16} \times \frac{dR^2}{dt^2} + \frac{aa}{8} \times R \frac{ddR}{dt^2} \right),$$

For a definite magnitude assumed for the purpose of measuring the time, in which $a = 4$, this expression becomes

$$\frac{EE'}{RR'} \left(1 - \frac{dR^2}{dt^2} + 2R \frac{ddR}{dt^2} \right).$$

Moreover, supposing that both R and $\frac{dR}{dt}$, are functions of *t*, consequently that $\frac{dR}{dt}$, is to be regarded as a function of R, which we shall denote by [R], we may also say that the *potential* of the mass E, in regard to the situation of the mass E', is

$$= \frac{E}{R} (1 - [R]^2);$$

for the partial differential coefficients of this expression, with respect to the three coordinates *x, y, z,* yield the components of the decomposed accelerating force in the directions of the three coordinate axes.

Lastly, if by the *reduced relative velocity* of the masses E and E', we understand that relative velocity which these magnitudes,—the distance of which apart at the moment supposed was R, the distance of which apart at the moment supposed was R, the relative velocity $\frac{dR}{dt}$, and the acceleration $\frac{ddR}{dt^2}$, *if the latter were constant*,—would possess at that instant in which both, in accordance with this supposition, met at one point, and if V denoted this *reduced relative velocity*, the above expression,

$$\frac{EE'}{RR'}\left(1-\frac{dR^2}{dt^2}+2R\frac{ddR}{dt^2}\right).$$

becomes converted into the following,

$$\frac{EE'}{RR}(1-VV),$$

which may be verbally expressed as follows:— *The diminution arising from motion of the force with which two electric masses would act upon each other when they are at rest, is in proportion to the square of their reduced relative velocity*.

Thus the expressions given for the determination of the force which two electric masses exert upon one another are now confirmed—

1st. As regards the entire domain of electro-statics;

2nd. As regards that domain of electro-dynamics the object of which is the consideration of the forces of the elements of the current when invariable and undisturbed; hence

3rdly. Its confirmation, as regards all that domain of electro-dynamics which is not limited to the invariable and undisturbed state of the elements of the current, is all that remains to be desired.

Theory of Voltaic Induction

It has already been mentioned that the principle of electro-dynamics laid down by Ampère refers merely to the special case, where four electric masses occur under the conditions premised to exist where two invariable and undisturbed elements of a current are concerned. Under conditions where these premises do not exist, the new fundamental principle only can be applied for the *à priori* determination of the forces and phenomena; and it is exactly in this way that the greater advantage of the new principle, arising from its more general application, will be exhibited.

The case in which the principle of electro-dynamics laid down by Ampère is inapplicable, thus occurs even when one element of a current is disturbed or its intensity varies; in addition to which it may also happen, that instead of the other element of the current, one element only of the conductor of a current may be present, without however any current being present in it. In fact, we know from experience that currents are then excited or *induced,* and the phenomena of these induced currents are comprised under the name of *voltaic induction;* but none of these phenomena could be predicted or estimated *à priori* either from the principle of electro-statics or the principle of electro-dynamics laid down by Ampère. It will now however be shown, that by means of the new fundamental principle as laid down here, the laws for the *à priori* determination of all the phaenomena of voltaic induction may be deduced. It is evident that the laws of voltaic induction deduced in this manner are correct, so far only as we are in possession of definite observations.

For the purpose of this deduction the magnitudes concerned may be denoted as follows:— α and α' denote the length of two elements, the former of which, α, is supposed to be *at rest.* This supposition does not limit the generality of the consideration, because every movement of the element α may be transferred to α', by attributing the opposite direction to it in α'. The four following electric masses are distinguished in these two elements, viz.-

$+\alpha e$, $-\alpha e$, $+\alpha' e'$, $-\alpha' e'$

The *first* of these masses $+\alpha e$ would move with the velocity $+u$ in the direction of the quiescent element α, which forms the angle Θ with the right line drawn from α to α'. This velocity during the element of time *dt* would alter by $+du$.

The *second* mass $-\alpha e$ would move, in accordance with the determinations relating to a galvanic current, in the same direction as the velocity $-u$, *i.e.* backwards, and this velocity during the element of time *dt* would alter by $-du$.

The *third* mass $+\alpha' e'$ would move with the velocity $+u'$ in the direction of the element α', which with the right lines drawn from α to α', and produced, forms the angle Θ. This velocity in the element of time *dt* would alter by $+du'$. Moreover, this electric mass would itself share the motion of the element α', which takes place with the velocity v in a direction which forms the angle η with the prolonged right line drawn from α to α', and is contained in a plane lying in this right line, which with the plane running parallel with the element α through the same right line, encloses the angle γ. The velocity v would alter during the element of time *dt* by dv.

The *fourth* mass $-\alpha' e'$ would move, in accordance with the determinations for a galvanic current, in the direction of the element α', with the velocity $-u'$ which during the element of time *dt* alters by $-du'$; but, moreover, like the previous mass, would itself acquire the velocity v of the element $-\alpha'$ in the direction already indicated.

The distances of the two former masses from the two latter, at the moment under consideration, are equal to the distance r of the two elements themselves; but since they do not remain the same, they may be denoted by r_1, r_2, r_3, r_4.

Lastly, if two planes pass through the right line drawn from α to α', the one parallel to α, the other to α', ω would denote the angle enclosed by these two planes.

Then, on applying the new principle, we obtain as the *sum* of the forces which act upon the *positive* and *negative* electricity in the element α', *i.e.* as the force which moves the element α' itself, the following expression:—

$$-\frac{aa}{16}\times\frac{\alpha e\times\alpha' e'}{rr}\times\left\{\left(\frac{dr_1^2}{dt^2}+\frac{dr_2^2}{dt^2}-\frac{dr_3^2}{dt^2}-\frac{dr_4^2}{dt^2}\right)-2r\left(\frac{ddr_1}{dt^2}+\frac{ddr_2}{dt^2}-\frac{ddr_3}{dt^2}-\frac{ddr_4}{dt^2}\right)\right\}$$

But for the *difference* of these forces, upon which the *induction* depends, we have the following expression:-

$$-\frac{aa}{16}\times\frac{\alpha e\times\alpha' e'}{rr}\times\left\{\left(\frac{dr_1^2}{dt^2}-\frac{dr_2^2}{dt^2}+\frac{dr_3^2}{dt^2}-\frac{dr_4^2}{dt^2}\right)-2r\left(\frac{ddr_1}{dt^2}-\frac{ddr_2}{dt^2}+\frac{ddr_3}{dt^2}-\frac{ddr_4}{dt^2}\right)\right\}$$

Moreover, when, in addition to the motions of the electric masses in their conductors, the motion common to them and their conductors is taken into account, we have the following expression for the first differential coefficients:—

$$\frac{dr_1}{dt} = -u\cos\Theta + u'\cos\Theta' + v\cos\eta,$$

$$\frac{dr_2}{dt} = +u\cos\Theta - u'\cos\Theta' + v\cos\eta,$$

$$\frac{dr_3}{dt} = -u\cos\Theta - u'\cos\Theta' + v\cos\eta,$$

$$\frac{dr_4}{dt} = +u\cos\Theta + u'\cos\Theta' + v\cos\eta.$$

Hence

$$\left(\frac{dr_1^2}{dt^2}+\frac{dr_2^2}{dt^2}-\frac{dr_3^2}{dt^2}-\frac{dr_4^2}{dt^2}\right)=-8uu'\cos\Theta\cos\Theta'$$

$$\left(\frac{dr_1^2}{dt^2}-\frac{dr_2^2}{dt^2}+\frac{dr_3^2}{dt^2}-\frac{dr_4^2}{dt^2}\right)=-8uu'\cos\Theta\cos\eta$$

We obtain the second differential coefficients when the variability of the velocity u, u', and v is also taken into account:—

$$\frac{d\,d\,r}{d\,t^2} = +\,u \sin\Theta\,\frac{d\Theta_1}{d\,t} - u' \sin\Theta'\,\frac{d\Theta'_1}{d\,t} - v \sin\eta\,\frac{d\eta_1}{d\,t}$$
$$- \cos\Theta\,\frac{d\,u}{d\,t} + \cos\Theta'\,\frac{d\,u'}{d\,t} + \cos\eta\,\frac{d\,v}{d\,t},$$

$$\frac{d\,d\,r_2}{d\,t^2} = -\,u \sin\Theta\,\frac{d\Theta_2}{d\,t} + u' \sin\Theta'\,\frac{d\Theta'_2}{d\,t} - v \sin\eta\,\frac{d\eta_2}{d\,t}$$
$$+ \cos\Theta\,\frac{d\,u}{d\,t} - \cos\Theta'\,\frac{d\,u'}{d\,t} + \cos\eta\,\frac{d\,v}{d\,t},$$

$$\frac{d\,d\,r_3}{d\,t_2} = +\,u \sin\Theta\,\frac{d\Theta_3}{d\,t} + u' \sin\Theta'\,\frac{d\Theta'_3}{d\,t} - v \sin\eta\,\frac{d\eta_3}{d\,t}$$
$$- \cos\Theta\,\frac{d\,u}{d\,t} - \cos\Theta'\,\frac{d\,u'}{d\,t} + \cos\eta\,\frac{d\,v}{d\,t},$$

$$\frac{d\,d\,r_4}{d\,t^2} = -\,u \sin\Theta\,\frac{d\Theta_4}{d\,t} - u' \sin\Theta'\,\frac{d\Theta'_4}{d\,t} - v \sin\eta\,\frac{d\eta_4}{d\,t}$$
$$+ \cos\Theta\,\frac{d\,u}{d\,t} + \cos\Theta'\,\frac{d\,u'}{d\,t} + \cos\eta\,\frac{d\,v}{d\,t}.$$

Consequently it becomes

$$\left(\frac{ddr_1}{dt^2}+\frac{ddr_2}{dt^2}-\frac{ddr_3}{dt^2}-\frac{ddr_4}{dt^2}\right) = +u\sin\Theta\left(\frac{d\Theta_1}{dt}-\frac{d\Theta_2}{dt}-\frac{d\Theta_3}{dt}-\frac{d\Theta_4}{dt}\right)$$
$$-u'\sin\Theta'\left(\frac{d\Theta'_1}{dt}-\frac{d\Theta'_2}{dt}+\frac{d\Theta'_3}{dt}-\frac{d\Theta'_4}{dt}\right)$$
$$-v\sin\eta\left(\frac{d\eta_1}{dt}+\frac{d\eta_2}{dt}-\frac{d\eta_3}{dt}-\frac{d\eta_4}{dt}\right)$$

and

$$\left(\frac{ddr_1}{dt^2}-\frac{ddr_2}{dt^2}+\frac{ddr_3}{dt^2}-\frac{ddr_4}{dt^2}\right) = +u\sin\Theta\left(\frac{d\Theta_1}{dt}+\frac{d\Theta_2}{dt}+\frac{d\Theta_3}{dt}+\frac{d\Theta_4}{dt}\right)$$
$$-u'\sin\Theta'\left(\frac{d\Theta'_1}{dt}+\frac{d\Theta'_2}{dt}-\frac{d\Theta'_3}{dt}-\frac{d\Theta'_4}{dt}\right)$$
$$-v\sin\eta\left(\frac{d\eta_1}{dt}-\frac{d\eta_2}{dt}+\frac{d\eta_3}{dt}-\frac{d\eta_4}{dt}\right)$$
$$-4\cos\Theta\,.\,\frac{du}{dt}.$$

The differential coefficients $\frac{d\Theta_1}{d\,t}, \frac{d\Theta'_1}{d\,t}, \frac{d\eta_1}{d\,t}$, &c. are easily developed according to the well-known laws of trigonometry; and we thus obtain the following expressions, viz.-

$$r_1 \frac{d\Theta_1}{dt} = + u \sin\Theta - u' \sin\Theta' \cos\omega - v \sin\eta \cos\gamma,$$

$$r_1 \frac{d\Theta'_1}{dt} = - u' \sin\Theta' + u \sin\Theta \cos\omega - v \sin\eta \cos(\omega + \gamma),$$

$$r_1 \frac{d\eta_1}{dt} = - v \sin\eta + u \sin\Theta \cos\gamma - u' \sin\Theta' \cos(\omega + \gamma),$$

$$r_2 \frac{d\Theta_2}{dt} = - u \sin\Theta + u' \sin\Theta' \cos\omega - v \sin\eta \cos\gamma,$$

$$r_2 \frac{d\Theta'_2}{dt} = + u' \sin\Theta' - u \sin\Theta \cos\omega - v \sin\eta \cos(\omega + \gamma),$$

$$r_2 \frac{d\eta_2}{dt} = - v \sin\eta - u \sin\Theta \cos\gamma + u' \sin\Theta' \cos(\omega + \gamma),$$

$$r_3 \frac{d\Theta_3}{dt} = + u \sin\Theta + u' \sin\Theta' \cos\omega - v \sin\eta \cos\gamma,$$

$$r_3 \frac{d\Theta'_3}{dt} = + u' \sin\Theta' + u \sin\Theta \cos\omega - v \sin\eta \cos(\omega + \gamma),$$

$$r_3 \frac{d\eta_3}{dt} = - v \sin\eta + u \sin\Theta \cos\gamma + u' \sin\Theta' \cos(\omega + \gamma),$$

$$r_4 \frac{d\Theta_4}{dt} = - u \sin\Theta - u' \sin\Theta' \cos\omega - v \sin\eta \cos\gamma,$$

$$r_4 \frac{d\Theta'_4}{dt} = - u' \sin\Theta' - u \sin\Theta \cos\omega - v \sin\eta \cos(\omega + \gamma),$$

$$r_4 \frac{d\eta_4}{dt} = - v \sin\eta - u \sin\Theta \cos\gamma - u' \sin\Theta' \cos(\omega + \gamma).$$

Now since for the moment under consideration $r_1=r_2=r_3=r_4=r$, we thus get

$$r\left(\frac{d\Theta_1}{dt} - \frac{d\Theta_2}{dt} - \frac{d\Theta_3}{dt} + \frac{d\Theta_4}{dt}\right) = -4u' \sin\Theta' \cos\omega,$$

$$r\left(\frac{d\Theta_1}{dt} + \frac{d\Theta_2}{dt} + \frac{d\Theta_3}{dt} + \frac{d\Theta_4}{dt}\right) = -4v \sin\eta \cos\gamma;$$

again:

$$r\left(\frac{d\Theta'_1}{dt} - \frac{d\Theta'_2}{dt} + \frac{d\Theta'_3}{dt} - \frac{d\Theta'_4}{dt}\right) = +4u \sin\Theta \cos\omega,$$

$$r\left(\frac{d\Theta'_1}{dt} + \frac{d\Theta'_2}{dt} - \frac{d\Theta'_3}{dt} - \frac{d\Theta'_4}{dt}\right) = 0;$$

lastly:

$$r\left(\frac{d\eta_1}{dt}+\frac{d\eta_2}{dt}-\frac{d\eta_3}{dt}-\frac{d\eta_4}{dt}\right)=0,$$

$$r\left(\frac{d\eta_1}{dt}-\frac{d\eta_2}{dt}+\frac{d\eta_3}{dt}-\frac{d\eta_4}{dt}\right)=+4u\sin\Theta\cos\gamma$$

These values by substitution become

$$r\left(\frac{ddr_1}{dt^2}+\frac{ddr_2}{dt^2}-\frac{ddr_3}{dt^2}-\frac{ddr_4}{dt^2}\right)=-8uu'\sin\Theta\ \sin\Theta'\cos\omega,$$

$$r\left(\frac{ddr_1}{dt^2}-\frac{ddr_2}{dt^2}+\frac{ddr_3}{dt^2}-\frac{ddr_4}{dt^2}\right)=-8uv\sin\Theta\sin\eta\cos\gamma,$$

With these values, the *sum* of the forces which act upon the positive and negative electricity in the element α' is

$$=-\frac{\alpha\alpha'}{rr}\times aeu\times a'e'u'(\sin\Theta\sin\Theta'\cos\omega-\tfrac{1}{2}\cos\Theta\cos\Theta'\eta)$$

If in this equation the angle which the directions of the two elements α and α' form with each other be denoted by ε, and, as in page 33, i and i' be substituted for *a e u* and *a'e'u',* the above sum, with slight transposition, becomes

$$=-\frac{\alpha\alpha' ii'}{rr}(\cos\varepsilon-\tfrac{3}{2}\cos\Theta\cos\Theta'),$$

the same expression at which Ampère arrived where the elements of the current are invariable and undisturbed, *i.e.* the electrodynamic force acting upon the entire element α' is determined in the same manner when the conductors are in motion and the intensities of the current variable, as when the intensities of the current remain invariable and the conductors undisturbed. Hence Ampère's law is of general application in the determination of the forces which act upon the entire element of the current when the position of the elements of the current and the intensities of the current are given. The application of this law merely requires that the intensities of the current when variable, as also the position when variable, be given *for each individual moment,* and further, the intensities of the currents, including that part added at each moment in consequence of induction.

But as regards the *difference* of the forces which act upon the *positive* and *negative* electricity in the element α', by which these two electricities are separated from each other, and move in the conductor in opposite directions, this now becomes

$$= -\frac{\alpha\alpha'}{rr} \times aeu \times ae'v\,(\sin\Theta\,\sin\eta\,\cos\gamma - \tfrac{1}{2}\cos\Theta\,\cos\eta)$$

$$- \tfrac{1}{2}\frac{\alpha\alpha'}{r}\,aa'ee'\cos\Theta \times \frac{du}{dt},$$

The force thus determined then tends to separate the *positive* and *negative* electricity in the induced element α' in the direction of the right line *r.* When the conductor is linear, however, separation cannot occur in this direction, but only in the direction of the induced linear element α' itself, which forms the angle Θ' with the produced right line *r.* By thus decomposing the whole of the above separating force in this direction, *i.e.* by multiplying the above value by cos Θ', we find the force, which effects the true separation,

$$= -\frac{\alpha\alpha'}{rr}\,i\,(\sin\Theta\,\sin\eta\,\cos\gamma - \tfrac{1}{2}\cos\Theta\,\cos\eta) \times ae'v\cos\Theta' - \tfrac{1}{2}\frac{\alpha\alpha'}{r}\iota$$

$$\times \cos\Theta\,\cos\Theta' \times \frac{di}{dt}$$

This expression, divided by *e',* gives *the electromotive* force exerted by the inducing element α*,* upon the induced element α', in the ordinary direction,

$$= -\frac{\alpha\alpha'}{rr} i (\sin\Theta \sin\eta \cos\gamma - \tfrac{1}{2}\cos\Theta \cos\eta) \times av\cos\Theta' - \tfrac{1}{2}\frac{\alpha\alpha'}{r} a \times \cos\Theta \cos\Theta' \times \frac{di}{dt}$$

This is therefore *the general law of voltaic induction, as found by deduction from the newly laid down fundamental principle of the theory of electricity.*

If we now, *first,* take the case in which no alteration occurs, in the intensity of the current, thus

$$\frac{di}{dt} = 0,$$

we have *the law of the induction exerted by a constant element of a current upon the element of a conductor moved against it*, *i.e*. the *electromotive* force becomes

$$= -\frac{\alpha\alpha'}{rr} i (\sin\Theta \sin\eta \cos\gamma - \tfrac{1}{2}\cos\Theta \cos\eta) \times av\cos\Theta'$$

or, when ε denotes the angle which the direction of the inducing element of the current forms with the direction in which the induced element itself is moved, by a transformation which is readily made it becomes

$$= -\frac{\alpha\alpha'}{rr} i (\cos\varepsilon - \tfrac{3}{2}\cos\Theta \cos\eta) \times av\cos\Theta'$$

The induced current is positive or negative according as this expression has a positive or negative value; by a positive current being understood one, the positive electricity of which moves in that direction of the element α', which with the produced right line *r* forms the angle Θ'.

Now if *e.g.* the elements α and α' are parallel to each other, and if the direction in which the latter is moved with the velocity v is contained within the plane of these two parallels, and at right angles to their direction, we have, when α' by its motion recedes from α,

$$\Theta = \Theta', \quad \cos\eta = \sin\Theta, \quad \cos\varepsilon = 0;$$

consequently the *electromotive* force is

$$= +\tfrac{3}{2}\frac{\alpha\alpha}{rr} i \sin\Theta \cos^2\Theta \times av.$$

This value is always positive, because we must consider $\Theta < 180^\circ$; and this *positive* value here denotes an induced current of the same direction as the inducing current, in conformity with that which has been found by experiment for this case.

Under the same conditions, with the difference merely that the element α' by its motion becomes approximated to the element α, we have

$$\Theta = \Theta', \quad \cos\eta = -\sin\Theta, \quad \cos\varepsilon = 0;$$

consequently the *electromotive* force becomes

$$= -\tfrac{3}{2}\frac{\alpha\alpha'}{rr} i \sin\Theta \cos^2\Theta \; av$$

The negative value of this force denotes an induced current, in the opposite direction to that of the inducing current, also in conformity with that found by experiment for this case.

As is well known, voltaic induction may be produced in two essentially different ways; for currents may be induced by *constant* and by *variable* currents. It is produced by *constant* currents either when the conductor through which the current passes is moved towards that conductor in which a current is about to be induced, or *vice versa*. It may be induced by *variable* currents even when the conductor through which the variable current passes remains undisturbed as regards that conductor in which a current is about to be induced.

Just as the particular law of the first kind of voltaic induction was at once found from the *general laws of voltaic induction* deduced above by the conditional equation

$$\frac{di}{dt} = 0,$$

so we also find the peculiar law of the latter kind of voltaic induction by the conditional equation

$$v = 0.$$

Thus if we take, *secondly,* the case in which *no motion of the conductors as regards each other takes place*, or where $v = 0$, the *law of the induction of a variable current upon that element of a current which is not moved as regards it*, or the value of the electromotive force becomes

$$= -\tfrac{1}{2}\frac{\alpha\alpha'}{r} a \cos\Theta \cos\Theta' \times \frac{di}{dt}$$

Hence the induction, during the element of time *dt*, *i.e.* the product of this element of time into the acting *electromotive force,* becomes

$$= -\frac{a}{2} \times \frac{\alpha\alpha'}{r} \cos\Theta \cos\Theta' \times di$$

consequently the induction for any period of time in which the intensity of the induced current increases by i, whilst r, Θ and Θ' remain unchanged, is

$$= -\frac{a}{2} \times \frac{\alpha\alpha'}{r}\, i \cos\Theta \cos\Theta'$$

The *positive* value of this expression denotes a current induced in the element α' in the direction of α', which with the produced right line *r* forms the angle Θ'; the *negative* value denotes an induced current in the opposite direction.

When the two elements α and α' are parallel, and Θ = Θ', the above expression, when the intensity of the current is *increasing*, or where the value of *i* is positive, has a *negative* value, *i.e.* when the intensity of the current is on the increase in α' a current is excited in α' in an opposite direction to that of the inducing current. The reverse applies when the intensity of the current, diminishes. Both results agree with well-known facts. The proportionality of the induction to the variation of the intensity *i* of the inducing current is also in accordance with experiment.

Lastly, if we return from the consideration of these two distinct kinds of *voltaic induction* to the general case, whereat the same time the intensity of the inducing current is variable and the two conductors are in motion as regards each other, the *electromotive force* exerted by the variable element of a current upon the *moved* element of a conductor is found to be simply as the *sum of the electromotive forces* which would occur—

1. If the element of the conductor *were not in motion* at the moment under consideration;

2. If the element of the conductor were in motion, but the *intensity of the current* of the induced element did *not* alter at the moment under consideration.

Appendices

1. Grove's battery

The description below of the "galvanic cell" used by Weber is taken from the most popular physics text of the day, *Ganot's Physics*[1]. The distinctive feature of this type of cell is that it is capable of supplying 12 amperes at 1.8 volts.

The cell was invented in 1839 by Sir Robert Grove, who also invented the first fuel cell. In addition to his work in electrochemistry, Grove was a successful lawyer and ultimately a judge.

The chief disadvantage of these cells is described in a text of 1876:

"The prevailing and permanent objection to the use of this arrangement for manufacturing purposes, and also for many experimental purposes, is that it emits nitrous fumes which corrode everything within their reach, and prove very disagreeable and hurtful to every person breathing them."[2]

Up to about the time of the Civil War, the telegraphy system in the United States largely depended upon batteries of Grove's cells. In this service the platinum was a modest strip instead of the substantial S-shaped plate described below, and the nitric acid was removed and replaced each night. Eventually the nitric dioxide problem and the need for a more constant voltage forced a change to a different sort of cell.

809. **Grove's battery**.— In this battery the copper sulphate solution is replaced by nitric acid, and the copper by platinum, by which

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1 *Elementary Treatise on Physics Experimental and Applied for the use of colleges and schools.* Translated and edited from Ganot's *Éléments de Physique* (with the Author's sanction) by E. Atkinson. 12th edition. London:Longmans, Green, and Co., 1886.

2 James Napier. *A Manual of Electro-Metallurgy: including the Applications of the Art to Manufacturing Processes.* 5th edition. London: Charles Griffin and Company, 1876.. Page 49.

greater electromotive force is obtained. Fig. 710 represents one of the forms of a couple of this battery. It consists of a glass vessel, A, partially filled with dilute sulphuric acid (1:8); of a cylinder of zinc, Z, open at both ends; of a vessel, V, made of porous earthenware, and containing ordinary nitric acid; of a plate of platinum, P (fig. 711), bent in the form of an S, and fixed to a cover, *c*, which rests on the porous vessel. The platinum is connected with a binding screw, *b*, and there is a similar binding screw on the zinc. In this battery the hydrogen, which would be disengaged on the platinum meeting the nitric acid, decomposes it, forming hyponitrous acid, which dissolves, or is disengaged, as nitrous fumes. Grove's battery is the most convenient, and one of the most powerful of the two fluid batteries. It is, however, expensive, owing to the high price of platinum; besides which the platinum is liable, after some time, to become brittle and break very easily. But as the platinum is not consumed, it retains most of its value, and when the plates which have been used in a battery are heated to redness, they regain their elasticity.

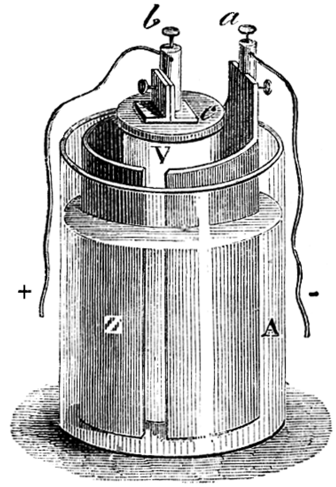

Fig. 710.

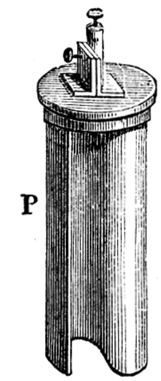

Fig. 711.

810. **Bunsen's battery**.-Bunsen's, also known as the zinc carbon battery, was invented in 1843; it is in effect a Grove's battery, where the plate of platinum is replaced by a cylinder of carbon. This is made either of the graphitoidal carbon deposited in gas retorts, or by calcining in an iron mould an intimate mixture of coke and bituminous coal, finely powdered and strongly compressed. Both those modifications of carbon are good conductors. Each element consists of the following parts: I. a vessel,

F (fig. 712), either of stoneware or of glass, containing dilute sulphuric acid; 2. a hollow cylinder, Z, of amalgamed zinc; 3. a porous vessel, V, in which is ordinary nitric acid; 4. a rod of carbon,C, prepared in the above manner. In the vessel F the zinc is first placed, and in it the carbon C in the porous vessel V as seen in P. To the carbon is fixed a binding screw, *m*, to which a copper wire is attached, forming the positive pole. The zinc is provided with a similar binding screw, *n*, and wire, which is thus a negative pole.

A single cell of the ordinary dimensions, 20 cm. in height and 9 in diameter, gives a current of 12 to 13 amperes when on short circuit, that is when it is closed without measurable resistance.

The elements are arranged to form a battery (fig. 713) by connecting each carbon to the zinc of the following one by means of the clamps *mn*, and a strip of copper, *c*, represented in the top of the figure. The copper is pressed at one end between the carbon and the clamp, and at the other it is soldered to the clamp *n*, which is fitted on the zinc of the following element, and so forth. The clamp of the first carbon and that of the last zinc are alone provided with binding screws, to which are attached the wires.

The chemical action of Bunsen's battery is the same as that of Grove's, and being equally powerful, while less costly, is very generally used on the Continent. But though its first cost is less than that of Grove's battery, it is more expensive to work, and is not so convenient to manipulate.

2. Multiplier

821. Galvanometer or multlplier.-The name *galvanometer*, or sometimes *multiplier* or *rheometer*, is given to a very delicate apparatus by which the existence, direction, and intensity of currents may be determined. It was invented by Schweigger a short time after Oersted's discovery.

In order to understand its principle, let us suppose a magnetic needle suspended by a filament of silk (fig. 720), and surrounded in the plane of the magnetic meridian by a copper wire, *mnopq*, forming a complete circuit round the needle in the direction of its length.

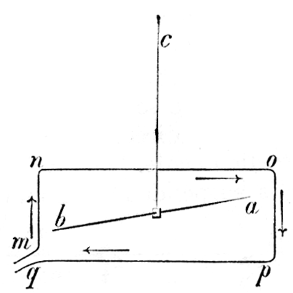

Fig. 720.

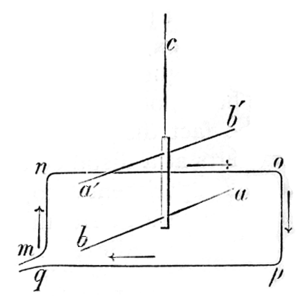

Fig. 721.

When this wire is traversed by a current, it follows, from what has been said in the previous paragraph, that in every part of the circuit an observer lying in the wire in the direction of the arrows, and looking at the needle *ab*, would have his left always turned towards the same point of the horizon, and consequently, that the action of the current in every part would tend to turn the north pole in the same direction; that is to say, that the actions of the four branches of the circuit concur to give the north pole the same direction. By coiling the copper wire in the direction of the needle, as represented in the figure, the action of the current has been *multiplied*. If, instead of a single one, there are several circuits, provided they are insulated, the action becomes still more multiplied, and the deflection of the needle increases. Nevertheless, the action of the current cannot be multiplied indefinitely by increasing the number of windings, for, as we shall presently see, the strength of a current diminishes as the length of the circuit is increased.

As the directive action of the earth continually tends to keep the needle in the magnetic meridian, and thus opposes the action of the current, the effect of the latter is increased by using an astatic system of two needles, as shown in fig. 721. The action of the earth on the needle is then very feeble, and, further, the actions of the current on the two needles become accumulated. In fact, the action of the circuit, from the direction of the current indicated by the arrows, tends to deflect the north pole of the lower needle towards the west. The upper needle *a'b'*, is subjected to the action of two contrary currents, *no* and *qp*, but as the first is nearer, its action preponderates. Now this current passing below the needle, evidently tends to tum the pole *a'* towards the east, and, consequently, the pole *b'* towards the west; that is to say, in the same direction as the pole *a* of the other needle.

From these principles it will be easy to understand the action of the *multiplier*. The apparatus represented in fig. 722 consists of a thick brass plate, D, resting on levelling screws; on this is a rotating plate, P, of the same metal, to which is fixed a copper frame, the breadth of which is almost equal to the length of the needles. On this is coiled a great number of turns of wire covered with

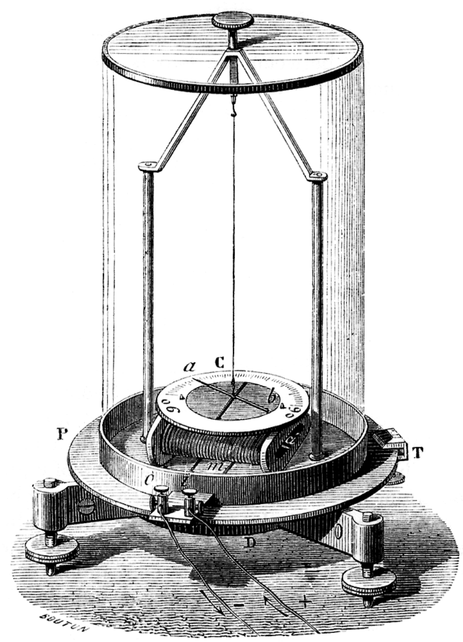

Fig. 722.

silk. The two ends terminate in binding screws, *i* and *o*. Above the frame is a graduated circle, C, with a central slit parallel to the direction in which the wire is coiled. The zero corresponds to the position of this slit, and there are two graduations on the scale, the one on the right and the other on the left of zero, but they only extend to 90°. By means of a very fine filament of silk, an astatic system is suspended; it consists of two needles *ab* and *a'b',* one above the scale, and the other within the circuit itself. These needles, which are joined together by a copper wire, like those in fig. 608 and fig. 721, and cannot move separately, must not have exactly the same magnetic intensity; for if they are exactly equal, every current, strong or weak, would always put them at right angles with itself.

In using this instrument the diameter, to which corresponds the zero of the graduation, is brought into the magnetic meridian by turning the plate P until the end of the needle *ab* corresponds to zero. The instrument is fixed in this position by means of the screw-clamp T.

The length and diameter of the wire vary with the purpose for which the galvanometer is intended. For one which is to be used in observing the currents due to chemical actions, a wire about 1/6 millimetre in diameter, and making about 800 turns, is well adapted. Those for thermo-electric currents, which have low intensity, require a thicker and shorter wire; for example, thirty turns of a wire 2/3 millimetre in diameter. For very delicate experiments, as in physiological investigations, galvanometers with as many as 30,000 turns have been used.

By means of a delicate galvanometer consisting of 2,000 or 3,000 turns of fine wire, the coils of which are carefully insulated by means of silk and shellac, currents of high potential, as those of the electrical machine (791) may be shown. One end of the galvanometer is connected with the conductor, and the other with the ground, and on working the machine the needle is deflected, affording thus an illustration of the identity of statical with dynamical electricity.

The deflection of the needle increases with the strength of the current; the relation between the two is, however, so complex, that it cannot well be deduced from theoretical considerations, but requires to be determined experimentally for each instrument. And in the majority of cases the instrument is used as a galvanoscope or rheoscope-that is, to ascertain rather the presence and direction of currents-than as a galvanometer or rheometer in the strict sense; that is, as a measurer of their intensity. The term galvanometer is, however, commonly used.

The *differential galvanometer* consists of a needle, as in an ordinary galvanometer, but round the frame of which are coiled two wires of the same kind and dimensions, carefully insulated from each other, and provided with suitable binding screws, so that separate currents can be passed through each of them. If the currents are of the same strength but in different directions, no deflection is produced; where the needle is deflected one of the currents differs from the other. Hence the apparatus is used to ascertain a difference in strength of two currents, and to this it owes its name.

When a current is passed through a galvanometer, the needle does not usually at once attain its final position of equilibrium, but oscillates about this position, which in observations causes much loss of time. These oscillations are damped partly by surrounding the needle by thick masses of copper, the effect of which will be afterwards explained (905), and partly by increasing the magnetisation of the needle. Galvanometers in which the needle acquires at once this final deflection are known as *aperiodic*, or *dead-beat galvanometers*.

When a current of very small duration is passed through a galvanometer, a momentary deflection or swing of the needle will be produced. The product of a constant into the sine of half the angle of the first swing is then a measure of the strength of the current, so that if momentary currents of different strengths are passed through one and the same galvanometer they will be measured by the sines of the corresponding angles of deflection or swings, or by the angles themselves where these are small. This is known as the *ballistic method* of measuring currents, and the galvanometers adapted for the purpose are known as *ballistic galvanometers*.