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PREFACE

It must be acknowledged that the essays presented here do not constitute a systematic account of any sort but represent occasional forays. Some deal with matters that happened to evoke my interest, others grew out of a chance encounter with a text I deemed to be of particular value. Throughout, challenges of the work itself more than compensated the author's efforts.

Logic has always been of crucially important concern to philosophers. My own involvement with the history of logic goes back to my work on Leibniz in the 1950s (represented by Chapter 8 of the present book). Thereafter, during the 1960s, I devoted considerable effort to the contributions of the medieval logicians of the Arabic-using world (here represented in Chapters 2-6). Moreover, I have from time to time returned to the area to look at some aspects of the more recent scene, as Chapters 8-9 illustrate.

In some instances the present essays have been overtaken by subsequent events—events which in fact they helped to promote. This is true in particular in chapter 6's analysis of Arabic work regarding temporal modalities, which was instrumental in evoking the important contributions of Tony Street of Cambridge University.

I am very grateful to Estelle Burris for her patient and conscientious help in preparing the material for publication.

Nicholas Rescher
Pittsburgh PA
May 2006

Chapter 1

ON ARISTOTLE'S APODEICTIC SYLLOGISMS

1. INTRODUCTION

Virtually all modern modal logicians have been troubled by Aristotle's insistence that, given a valid *first figure* categorical syllogism (of the purely assertoric type, *XXX*, where *X* is to represent the actual and *L* is necessary) that take the format

Major Premiss (P_M)
Minor Premiss (P_m)
Conclusion

will have the corresponding modal syllogism (of type *LXL*)

Necessarily: P_M
P_m
Necessarily: C

also be valid. The correspondingly *LLL* syllogism must, of course, also be valid *a fortiori*, while the corresponding *XLL* syllogism will—so Aristotle has it—be invalid. Despite extensive discussions of the problem, a convincing rationale for Aristotle's theory has yet to be provided.[1] The aim of the present discussion is to propose a suggestion along these lines.

The leading idea of the present proposal is that, given syllogistic terms α and β, it is possible to define yet another term $[\alpha\beta]$ to represent the β-

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1 For an overview of the current position, together with references to the literature, see Storrs McCall, *Aristotle's Modal Syllogisms* (Amsterdam, 1963) and Nicholas Rescher, "Aristotle Theory of Modal Syllogisms and Its Interpretation," in *Essays in Philosophical Analysis* (Pittsburgh, 1969), pp. 33-60. For the general background of the Aristotelian syllogistic see Gunther Patzig, *Aristotle's Theory of the Syllogism* (Dordrecht, 1968).

species of α. As will be seen below, these bracketed terms represent a version of Aristotle's process of *ecthesis* ("selecting out" a part of the range of a syllogistic term). The [αβ]'s are specifically those α's which must be β relative to the hypothesis that they are α's (by conditional or relative necessitation). Thus they might, for example, be those humans (α) that must be female (β), as some certainty must be. The essential point regarding this special term, one that is central for our present purposes, is that it is such as to validate the inference:

$$\frac{A\alpha\beta}{LA\ \alpha[\alpha\beta]}$$

Intuitively, if all α's are β's, then all α's must be such that they are necessity [αβ]'s, where this is the α subspecies of the β's. (Thus if All mice are rodents, then All mice are necessarily members of the mouse subspecies of rodents.) Correspondingly, we would also have the inference:

$$\frac{I\alpha\beta}{LI\alpha[\alpha\beta]}$$

Thus if Some dogs are pomeranians, then Some dogs [viz. pomeranians] are necessarily members of the pomeranian subspecies of dogs.

Such "bracketed terms", as we shall call them provide the materials out of which our interpretation of Aristotle's apodeictic syllogisms will be constructed. Once terms of this type are introduced, it becomes an interesting and significant result that the apodeictic sector of the Aristotelian modal syllogistic follows *in toto* as a natural consequence.

2. THE TECHNICAL RESULT

The notation and terminology here used will be that of McCall's *Aristotle's Modal Syllogisms*, except for the additional primitive use of term-bracketing and replacing McCall's rule of substitution, p. 37, by:

(i″) Rule of Substitution of terms for variables, where this does not involve identifying terms.[2]

An axiomatization of the assertoric moods *XXX*—and correspondingly of the apodeictic moods *LLL*—in line with the above revisions will be assumed.

In order to extend this basis to include all the apodeictic moods, we adopt the following axiomatic rules with respect to bracketed terms:

Group 1: *Modal Inferences of Type X to L*

I. *C Aab LAa*[*ab*]
II. *C Iab LIa*[*ab*]

Group 2: *Modal Inferences of Type L to L*

I. *C LAab LA*[*ca*]*b*
II. *C LEab LE*[ca]*b*

These four rules together with the laws of conversion and of modal conversion suffice to yield all the apodeictic moods. To show that all the valid apodeictic moods are desirable on this basis, we shall prove Fitch-style all of those of the first figure:

Barbara *LXL*	1	*LAbc*	hyp
	2	*Aab*	hyp
	3	*LAa*[*ab*]	2, 1
	4	*LA*[*ab*]*c*	1, III
	5	*LAab*	2, 4, Barbara *LLL*

Celarent *LXL*	1	*LEbc*	hyp
	2	*Aab*	hyp
	3	*LAa*[*ab*]	2, 1
	4	*LE*[*ab*]*c*	1, IV
	5	*LEac*	3, 4, Celarent *LLL*

[2] We shall not attempt to formalize (i′) rigorously but the intent of (i″) is that (say) *l*[*ab*]*c*, *lbc*, *l*[*ab*][*cd*], and (even) *l*[*ab*][*ba*] or *I*[*ab*][*bb*] be regarded as substitution instances of *Iab*, but not *Iaa* or *I*[*ab*][*ab*].

Darii *LXL*	1	*LAbc*	hyp
	2	*Iab*	hyp
	3	*LIa*[*ab*]	2, II
	4	*LA*[*ab*]*c*	1, III
	5	*LIac*	3, 4, Darii *LLL*

Ferio *LXL*	1	*LEbc*	hyp
	2	*Iab*	hyp
	3	*LIa*[*ab*]	2, II
	4	*LE*[*ab*]*c*	1, IV
	5	*LOac*	3, 4, Ferio *LLL*

It should be noted that all the derivations follow a perfectly uniform plan, viz., (1) the use of bracketed terms to obtain (using I/II) a modalization from the assertoric minor premiss, in view of which (2) the bracketed term at issue in this minor can be subsumed as a special case under the apodeictic major (using III/IV).[3]

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[3] This substantiates the idea of Rescher *op. cit.* (pp. 53-55) that a leading intuition of Aristotle's apodeictic syllogistic is that of a special case falling under a necessary rule: In short, Aristotle espouses the validity of Barbara *LXL* not on grounds of abstract formal logic, but on grounds of *applied* logic, on *epistemological* grounds. What he has in mind is the application of modal syllogisms within the framework of a theory of scientific inference along the lines of his own conceptions. We must recognize that it is Aristotle's concept that in truly scientific reasoning the relationship of major to minor premiss is governed by the proposition:

major premiss: minor premiss:: general rule: special case

When we take note of this line of thought we see why Aristotle taught that the major premiss of a modal syllogism can strengthen the modality of the conclusion above that of the minor premiss. For a rule that is necessarily (say) applicable to all of a group will be necessarily applicable to any sub-group, pretty much regardless of how this sub-group is constituted. On this view, the necessary properties of a genus must necessarily characterize even a contingently differentiated species. If all elms are necessarily deciduous, and all trees in my yard are elms, then all trees in my yard are necessarily deciduous (even though it is not necessary that the trees in my yard be elms). The "special case" subsumption at issue here can be viewed as a mode of application of the *dictum de omni et nullo* (*ibid.*, pp. 54-55).

The adequacy of any formalization of Aristotle's theory of modal syllogisms depends not only on having the right theorems but also on lacking the wrong ones (which is where Lukasiewicz fails). An important test case is that the theory accepts Barbara *LXL* but omits Barbara *XLL*. We are safe on the first count; how do we fare on the second? Let us attempt to prove Barbara *XLL:*

Barbara *LXL*	1	*Abc*	hyp
2	L*Aab*	hyp	
	•		
	•		
	•		
n	*LAac*	?	

Clearly *LAac* is unavailable without the introduction of bracketed terms. Applying rule *I* to premiss *L* will yield *LAb*[*bc*]. This together with premiss 2 gives us *LAa*[*bc*]*—by* Barbara *LLL*. But now we are unable to proceed further; we simply cannot infer *LAac* from *LAa*[*bc*].[4] Since this is in fact our only method of attack, Barbara *XLL* cannot be proven.

The remaining first-figure syllogisms will also be blocked for the type *XLL*. Take Celarent first:

Celarent *XLL*	1	*Ebc*
2	L*Aab*	
	•	
	•	
	•	
n	*LEac*	

This is blocked because there is no way of obtaining an L-qualified proposition from an E-premiss (or any negative premiss).

Next consider Darii:

4 If all α's are necessarily β's-that-in-fact-are-γ's, it does not follow that all α's are necessarily γ's.

Darii *XLL*	1	*Abc*	
	2	L*Iab*	
	3	*Lab*[*bc*]	1, I
	4	*LIa*[*bc*]	2, 3, Darii *LLL*
		•	
		•	
		•	
	n	*LIac*	

But this inference cannot be accomplished because we cannot infer *LIac* from *LIa*[*bc*].[5]

Finally take Ferio:

Ferio *XLL*	1	*Ebc*
	2	L*Iab*
		•
		•
		•
	n	*LOac*

This inference *too* is blocked because there is no way of obtaining an *L*-qualified proposition from an E-premiss (or any negative premiss).

It might be noted that the four first figure *XLL* syllogisms are blocked by three principles:

(1) Disallowing the inference *of* any L-qualified proposition from a

[5] It deserves note that we cannot without serious consequences postulate the nonmodal counterpart of IV, viz., (IV) C *Eab E[ca]* b (together with the obvious moral principle that I- *CafJ* yields I- *CLaLfJ*). *For* IV entails C *l*[ca] *b lab*, whose modalized version is *CLl*[ca] *b LIab* or equivalently C *LIa*[*bc*] *LIac*. And just this principle must be excluded if Darii *XLL* is *to* be blocked. It is thus indicated that the assertoric counterparts *of* III and IV must be rejected, so that these represent specifically apodeictic modes *of* inference. In summary, by contrast with the acceptable theses I-IV, the following four theses should thus be rejected:

C LAa[*bc*] *LAac*
C LIa[*bc*] *LIac*
C Aab A[*ca*]*b*
C Eab E[*ca*] *b*

negative premiss.

(2) Disallowing the inference of *LAac* from *LAa*[*bc*].

(3) Disallowing the inference *of LIac* from *Lla*[*bc*].[6]

These last two principles amount *to: Disallowing the elimination of a bracketed term from an affirmative* premiss.

Thus if appropriate restrictions (of a rather plausible sort) are postulated for inferences involving bracketed terms, none of the apodeictic syllogisms Aristotle regards as illicit will be forthcoming.

If the machinery developed thus far is acceptable from an Aristotelian point of view, we can perhaps explain Aristotle's silence regarding the validity of $LA\alpha\alpha$. If we are to reject C $A\alpha\beta$ $LA\alpha\beta$ (which one must certainly reject), then given our machinery, we are committed to rejecting $LA\alpha\alpha$.[7] This may be seen as follows:

1	*Aab*	hypothesis
2	*LAa*[*ab*]	1, I
3	*LAbb*	by the thesis at issue
4	*LA*[*b*]*b*	3, III
5	*LAab*	2, 4, Barbara *LLL*

This serves to motivate omission of $LA\alpha\alpha$. We can only explain the lack of an explicit rejection by saying that if one must reject *LAaa*, one might well prefer doing so quietly. (Though if one is enough of an essentialist, it would seem not incongruous to take the view that among all the α's some should be α's of necessity but others merely by accident, so that $LA\alpha\alpha$ would not be acceptable.)[8] Although the Aristotelian modal syllogistic

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[6] Restrictions (2) and (3) are clearly plausible. If all or some α's are γ's, that does *not* mean they must necessarily be members of the γ-species of β'*s*.

[7] In consequence of this rejection it would no longer be necessary to introduce the above-mentioned restriction on McCall's rule of substitution.

[8] Previous attempts to formalize Aristotle's modal syllogic (specifically those of Lukasiewicz and McCall) also explicitly reject *LA/X/X*. See Jan Lukasiewicz, *Aristotle's Syllogistic*, 2nd edition (Oxford, 1957), p. 190, and Storrs McCall, *op. cit.*, p. 50.

must reject the thesis $LA\alpha\alpha$, the cognate thesis $LA[\alpha\alpha]$ is readily demonstrable:

1	Aaa	thesis
2	$LAa[aa]$	1 by ecthesis

Actually, although a strict proof does not seem available, it would appear that LA[αα][αα]—and indeed even LA[αβ][αβ]—could well be viewed as acceptable theses.

It is worthwhile to point out that the system, suggested here is consistent. We define a function h inductively as follows: (a) if α is a variable, $h(\alpha) = \alpha$, (b) $h([\alpha\beta]) = h(\beta)$, (c) $h(A\alpha\beta) = Ah(\alpha)\ h(\beta)$, (d) $h(I\ \alpha\beta) = Ih(\alpha)\ h(\beta)$, (e) $h(N\alpha) = Nh(\alpha)$, (f) $h(L\alpha) = h(\alpha)$ and (g) $h(C\alpha\beta) = Ch(\alpha)\ h(\beta)$. Clearly, if α is a theorem, $h(\alpha)$ is a theorem of the assertoric theory of the syllogism. So, our system is consistent if the assertoric theory is. But the latter is consistent.[9]

3. ECTHESIS

Aristotle does not give proofs for Baroco *LLL* and Bocardo *LLL* but merely outlines how they are to proceed *(An. pr.*, i. 8, 30a6). Both are to be proven by ecthesis.

We propose to construe this process—which Aristotle leaves somewhat mysterious—along the following lines:

(1) Nonmodal ecthesis

$$\frac{I\alpha\beta}{(\exists\gamma)\ [K\ a\ \gamma\ \alpha\ A\ \gamma\ \beta]} \qquad \frac{O\alpha\beta}{(\exists\gamma)\ [K\ a\ \gamma\ \alpha\ E\ \gamma\ \beta]}$$

(2) Modal ecthesis

$$\frac{LI\alpha\beta}{(\exists\gamma)\ [KLA[a\gamma]\alpha\ \ LA[\alpha\gamma]\beta]} \qquad \frac{LO\alpha\beta}{(\exists\gamma)\ [KLA[a\gamma]\alpha\ \ LE[\alpha\gamma]\beta]}$$

9 See, for example, J. C. Shepherdson's "On The Interpretation of Aristotelian Syllogistic," *Journal of Symbolic Logic*, vol. 21 (1956), pp. 137-147.

Ecthesis thus conceived, is a process for inferring universal propositions from particulars.[10] Its central feature in the modal case is its recourse to bracketed terms as introduced above. (It might be noted that the inferences in (1) and (2) are to be reversible into corresponding inverse forms.) Thus our construal of nonmodal ecthesis coincides with that of Patzig.[11] Aristotle's observations at *An. pr.*, i. 6, 28a 22-26, are simply a *statement* of the inverse form of the affirmative case of nonmodal ecthesis, rather than representing—as W. D. Ross complains—an attempt at "merely proving one third-figure syllogism by means of another which is no more obviously valid."[12]

Let us examine the argument for Baroco *LLL* as Ross[13] presents it. According to Ross (p. 317) the proof goes as follows: assume that all *B* is necessarily *A* and that some C is necessarily not *A*. Take some species of C (say D) which is necessarily not *A*. Then all *B* is necessarily *A*, all D is necessarily not *A*, therefore all D is necessarily not *B* (by Camestres *LLL*). Therefore some C is necessarily not *B*. The reasoning may be formulated as follows:

1	*LAba*	hyp
2	*LOca*	hyp
3	$(\exists d)$ $LE[cd]a$	ecthesis on 2
4	$(\exists d)$ $LE[cd]b$	1, 3, Camestres *LLL*
5	*LOcb*	4, inverse ecthesis

Next, consider the argument for Bocardo *LLL*. Ross *(ibid.)* construes the argument as follows: assume that some *C* is necessarily not *A* and that all *C* is necessarily *B*. Take a species of *C* (say *D*) which is necessarily not *A*.

10 The inverse inferences (closely akin to Darapti and Felapton) are, of course, also valid, so that we are, in effect, dealing with equivalences.

11 Cf. Gunther Patzig, *Aristotle's Theory of the Syllogism* (New York, 1968), pp. 156-168. In support of his interpretation of nonmodal ecthesis, Patzig cites *Anal. Pr.*, i.28, 43b43-Ha2 and Ha9-11, which appears to be a statement of the equivalence of the premisses and their respective conclusions in (1).

12 W. D. Ross, *Aristotle's Prior and Posterior Analyties* (Oxford, 1949), p. 32.

13 W. D. Ross, *ibid*.

Then all *D* is necessarily not *A*, all *D* is necessarily *B*, therefore some *B* is necessarily not *A* (by Felapton *LLL*). The reasoning also is readily formalized as follows:

1	$LOca$	hyp
2	$LAcb$	hyp
3	$(\exists d)\ LE[cd]a$	ecthesis on 1
4	$(\exists d)\ LE[dc]a$	3 (supposing $E[\alpha\beta]\gamma$ yields $E[\beta\alpha]\gamma$)
5	$(\forall d)\ LA[cd]b$	2, III
6	$LOba$	4, 5, Felapton *LLL*

The use of bracketed terms to explicate ecthesis along the lines outlined above thus provides a simple way to systematize the Aristotelian justification of certain apodeictic syllogisms.

4. CONCLUSION

The use of bracketed terms in connection with modal and ecthesis—involving reasonings—is analogous in one significant respect: In both cases their introduction allows us "to do the impossible" in Aristotelian logic—albeit in a perfectly legitimate way. In the one case we move from an assertoric to an apodeictic proposition:

$$\frac{A\alpha\beta}{LA\alpha[\alpha\beta]}$$

In the other case we move from a particular to a universal proposition:

$$\frac{LI\alpha\beta}{(\exists\gamma)LA[\alpha\gamma]\beta}$$

In both cases the bracketing operator enables us to "select" from among all the α's those which—given that a certain relationship holds between the α's and β's—bear a yet more stringent relation to the β's than the α's in general do.

The just indicated argument paradigm

$$\frac{LI\alpha\beta}{(\exists\gamma)LA[\alpha\gamma]\beta}$$

deserves further comment. It is crucial that the particularized relation the premiss lays down between *a* and *P* (their I-linkage) is necessary, otherwise the conclusion would clearly not be forthcoming. Thus perception—which can establish particular linkages *de facto* but not necessarily—cannot provide scientific knowledge.[14] Chance conjunctions in general cannot in the very nature of things be subject to demonstrations of necessity.[15]

That nonmodal ecthesis is a logically warranted (indeed virtually trivial) process can be seen along the following lines

1. Assume by way of hypothesis that: Some *a* is *b*.

2. Let X_1, X_2, . . . be specifically those *a's* that are b's and let us designate the group of these X_l, the "*a's* at issue", as *X*.

3. Then all these *X's* are *a's* (by definition of *X)* and moreover all *X's* are *b's*, and conversely (for the same reason).

Thus between the "*a's* at issue", viz., X_l, X_2, . . ., and *b* we have inserted a "middle term" (X) in such a way that (1) All the "a's at issue" are X's (and conversely) and (2) All X's are b's. No doubt here, in the assertoric (nonmodal) case, we have done this insertion in a logically trivial way.

But in the modal case when *Some a is necessarily* β the issue of inserting an intermediate X such that both *All the a's at issue are X* and *All X is necessarily* β is not trivial at all. For whereas the motivation of the first of the two inferences under consideration is essentially a matter of pure logic that of the second is at bottom not logical, but metaphysical. If *some* α's are necessarily α's, then—so the inference has it—there must be some α-delimitative species, the [αγ]'s, *all* of which are necessarily P's. If some metals are necessarily magnet-attracted then there must be a type of metal (e.g., iron) all of which is necessarily magnet attracted. The governing intuition operative here lies deep in the philosophy of nature: Whenever α's are such that some of them *must* be β's, then this fact is capable of *ration-*

[14] Cf. *Aal. Post.*, I 31.

[15] *Ibid.*

alization, i.e., there must in principle be a *natural kind* of α's that are necessarily (essentially, lawfully) β's.

A precursor version of the principle of causality is at work here: If some "men exposed to a certain virus" are in (the naturally necessitated course of things) "men who contract a certain disease", but some are not, then there must be some *characteristic* present within the former group in virtue of which those of its members exhibiting this characteristic *must all* contract the disease if exposed to it. To explain that some α's have to be β's we must find a naturally constituted species of the α's all the members of which are necessarily β's.[16] Thus given "Some α's are of necessity β's", it follows from the requisites of explanatory rationalization that for some species γ of the α's we have "All γ's are necessarily β's." We come here to what is essentially not a principle of logic but a metaphysical principle of rationalization. At this precise juncture, the logic of the matter is applied rather than pure—fusing with the theory of scientific explanation presented in *Posterior Analytics.*

From this standpoint, then, the principle of modal ecthesis

$$\frac{LI\alpha\beta}{(\exists\gamma)KLA[\alpha\gamma]\ \ LA[\alpha\gamma]\beta}$$

is based upon metaphysical rather than strictly logical considerations. This principle underwrites the equivalence:

$$LI\alpha\beta \text{ if and only if } \quad (\exists\gamma)LA[\alpha\gamma]\beta$$

This, in effect, is a "generalization principle for necessary connection". It stipulates that whenever a necessary connection exists between two particular groups α and β the matter cannot rest there. There must be-somehow, no matter how well concealed—a *universal* necessary relationship from which this particular case derives and in what it inheres. There can be no particular necessity as such: necessity, whenever encountered, is always a specific instance of a *universal* necessity. It is thus easy to see the

[16] The idea is closely analogous with the "generalization principle" in modern ethics, i.e., the thesis that if some certain men are obligated (or entitled) to do something, then this must be so because they belong to a group *all* of whose members are obligated (or entitled) to do so.

basis for Aristotle's policy (in *Posterior Analytics* and elsewhere) of assimilating necessity to universality. This perspective highlights Aristotle's fundamental position that science, since it deals with the necessary, cannot but deal with the universal as well. The irreducibly particular—the accidental—lies wholly outside the sphere of scientific rationalization.

Insofar as this view of the matter has merit, it stresses the conclusion that the fundamental motivation of Aristotle's modal syllogistic is heavily indebted to metaphysical rather than strictly logical considerations. Be this as it may, it is, in any case, significant that by introducing such an ecthesis-related specification of terms, the apodeictic sector of Aristotle's modal syllogistic is capable of complete and straightforward systematization.[17]

[17] This chapter was originally published in *The Review of Metaphysics*, vol. 24 (1971), pp. 178-84. It was written in collaboration with Zane Parks.

Chapter 2

AL-KINDĪ'S SKETCH OF ARISTOTLE'S ORGANON

1. INTRODUCTION[1]

Ya'qūb ibn Ishāq al-Kindī (c. 805-873), whose name was Latinized to Alkindus or Alkendus, was born in Basra, the descendent of a noble Arab tribe, the banū Kindah. The only notable Arabic philosopher of pure Arab descent, he was consequently dubbed "the philosopher of the Arabs".[2] Living at a time when, in the Arabic-speaking orbit, knowledge of Greek philosophy and science was almost wholly confined to the Syrian Christians, al-Kindī made an extensive study of Greek learning. A prolific writer, he composed numerous treatises—almost 300 titles are reported—mainly dealing with the natural sciences: mathematics (including music), physics (especially optics), geography, medicine, and others. In addition, al-Kindī made an oblique contribution to learning by acting as a patron and sponsor of Arabic translations of Greek works.

In the present discussion, our sole concern will be with al-Kindī as a logician, or more accurately as a student of logic. For he was, unlike his successor al-Fārābī, no specialist in logic. His encyclopedic interests embraced all of Greek science and philosophy, and his concern with logic was derivative in nature, resulting almost as a by-product from the fact that logic

1 This paper is part of a series of studies of Arabic contributions to logic supported by a National Science Foundation grant, which the author acknowledges gratefully.

2 The principal studies of al-Kindī, and in particular the important works of Flügel, Nagy, and Guidi-Walzer, are listed in the bibliography at the end of this paper. Reference may also be made to C. Brockelmann, *Geschichte der Arabischen Literatur* (Weimar: E. Felber, 1898) 1,209-210; and Supplement I, 372-374; *The Encyclopedia of Islam*, II, 1021 (L. Massignon); G. Sarton, *Introduction to the History of Science* (Baltimore: Pub. for the Carnegie Institution of Washington, by the Williams & Wilkins Company [1927-1948, reprinted 1962]) I, 559-560; and Ueberweg-Geyer, *Grundriss der Geschichte der Philosophie* (Berlin: E. S. Mittler & sohn, 1923-28) II, 303-304 and 720.

was not only an integral but even a fundamental branch of Greek philosophico-scientific knowledge.

From reports in the Arabic bio-bibliographical sources[3] we learn that al-Kindī wrote commentaries on, or more probably epitomes of, all parts of the Aristotelian Organon as well as the *Isagoge* of Porphyry, and that he also commented on the commentaries of Alexander of Aphrodisias on the *Rhetorica* and the *Poetica.* This makes al-Kindī the first *writer*, as opposed to translator, on logical subjects in Arabic, if we overlook the questionable case of Ibn al-Muqaffa'.[4] It is a matter which cannot but cause regret to students of the history of logic that none of these logical works of al-Kindī's have survived.

In view of these losses, it is a piece of good fortune that we possess, in Arabic, a treatise by al-Kindī bearing the title "On the quantity of the books of Aristotle and what is needful of them for the attainment of philosophy", the text of which was published by M. Guidi and R. Walzer in 1940.[5] This treatise contains a sketch, amounting to roughly a third of the whole, of Aristotle's Organon which qualifies as *the oldest extant Arabic logical text*.[6] Although this particular discussion of al-Kindī's has but little

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3 These data were already brought together in *Flügel* (1857). For reference of this form see the bibliography at the end of this paper.

4 The attribution in Arabic sources of logic-treatises to Abū 'Amr 'Abd-Allāh ibn al-Muqaffa' (d. 759 .A. D.), the famous translator of *Kalīlah wa-Dimnah*, the Persian "Fables of Bidpai", is for various reasons so implausible, that several authorities rejected such works as figments of the imagination of later bio-bibliographers. However, Paul Kraus convincingly argued in 1934 that the logician is the obscure son of this famous author, Muhammad ibn 'Abd-Allāh ibn al-Muqaffa' (d. c. 800 A. D.), and that he wrote (or more probably translated or even caused to be translated?) short epitomes of "the four books" of logic, based on Syriac sources. ("Zu ibn al-Muqaffa'") *Rivista degli Studi Orientali,* XIV (1934) 1-20.

5 *Guidi-Walzer* (1940) gives the *editio princips* of our text, together with an Italian translation. The Arabic text is also printed in *Abū Rīdah* (1950).

6 This statement requires slight qualification. In two instances the "old" Arabic translations of Aristotelian logical texts—antedating the work of Hunain ibn Ishāq and his associates—have survived: that of *Anal. Pr.* by one "Theodore" and that of *Soph. Elen.* by 'Abd-al-Mashīh ibn Nā'imah al-Himsī. But in these two cases the surviving versions were "modernized" in the school of Hunain. See R. Walzer, "New Light on the Arabic Translations of Aristotle," *Oriens,* vol. 6 (1953), pp. 91-142.

interest from the standpoint of its substantive logical contents, it is of significant value both for the historian of logic and for the student of intellectual tradition. This sort of combination of index and guide to Aristotle's works was seemingly a standard production of Arabic philosophers in the 850-950 period. We know, for example, that al-Fārābī (c. 873-950) composed a treatise "On the objectives (or: subject-materials) of Aristotle in each of his treatises" (*Kitāb fī aghād Aristūtālīs fī kull wāhid min kutubihi*).[7]

My aim here is to present an English translation of al-Kindī's Arabic text and to prefix to it some discussion both of the structure and substance of al-Kindī's remarks, and of their significance for the history of Arabic logic.

The first point of interest is al-Kindī's conception of the place of logic among the sciences. Following the tradition of the Hellenistic Aristotelianism of Alexandria, he arranges the sciences in the order: mathematics—and—logic, physics, metaphysics, and theology. Logic (and mathematics) are thus regarded as propaedeutic to all scientific inquiries, and the other disciplines being arranged in order of their decreasing involvement with matter. This ranking follows out ideas of Aristotle himself as laid down in the first chapter of book Eta of the *Metaphysics*. But al-Kindī and his Alexandrian predecessors go beyond Aristotle in regarding this ordering not only as the theoretical ranking of the sciences, but also as representing the didactic ordering of these disciplines for the program of philosophico-scientific studies. This concept of a complete parallelism between the systematic ranking of scientific subjects on the one hand and the didactic ordering of the program of studies on the other is applied by al-Kindī (and the Alexandrians) even to the individual books of the logical Organon.

Following Hellenistic models, al-Kindī regarded the division of the Aristotelian Organon into separate books as reflecting the organization of logic into distinct disciplines. This conception results in the standard Hellenistic-Syriac-Arabic division of logic into eight disciplines, each corresponding to an Aristotelian treatise as shown in Display 1.

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7 See the Fārābī bibliography of Ahmet Ates in Hilmi Ziya Ülken(ed.) *Fārābī Tetkikleri* (Istanbul: Üniversitesi Edebiyat Fakultesi Yayinlarindan, 1950), p. 113.

Display 1

BRANCHES OF LOGIC

Branch	Subject-Matter on the Standard Arabic View	Subject-Matter According to Al-Kindī	Basic Aristotelian Treatise
(1) Categories	categories (*al-maqūlāt*)	categories (*al-maqūlāt*)	*Categoriae*
(2) Hermeneutics	interpretation (*al-ībārah*)	interpretation (*al-tafsīr*)	*De Interpretatione*
(3) Analytics	syllogisms (*al-qiyās*)	conversion (*al-'aks*)	*Analytica Priora*[8]
(4) Apodictics	demonstration (*al-burhān*)	making-certain (*al-īdāh*)[9]	*Analytics Pisteriora*
(5) Topics	disputation (*al-jadal*) reasoning	dialectic (*Jadliyyah*)	*Topica*
(6) Sophistics	deception (*al-mughālitah*)	deception (*al-mughālitah*)	*Sophistici Elenchi*
(7) Rhetoic	rhetoric (*al-khitābah*)	persuasion (*al-balāghā*)	*Rhetorica*
(8) Poetics	poetry (*al-shīr*)	poetry (*al-shīr*)	*Poetica*

In grouping the *Rhetorica* and *Poetica* into the logical Organon, the Syriac and Arabic tradition follows a practice dating back at least to Simplicius (fl. c. 533 A. D.). It was also customary in Hellenistic-Syriac-Arabic practice to prefix to this listing as another branch of logic that of "Introduction" based upon Porphyry's *Isagōgē* as its basic text. Al-Kindī, being engaged in giving an inventory of Aristotle's treatises, of course, omits this work. His discussion makes it clear that, for al-Kindī, the principal objective of logic as a whole is the study of syllogistic arguments ("unions") in descending degrees of strength that decline from the *demonstrative* arguments of Analytics and Apodictics through the looser, but yet often reliable *dialectical* reasonings of Topics to the deceptive and fallacious arguments treated in Sophistics. How, or even whether, Al-Kindī would fit

8 The Syrians and the Arabs of al-Kindī's time confined the study of *Anal. Pr*. to the part ending with section seven of Book I., i.e., to the end of the discussion of categorical syllogisms. On this fact and its reasons see Max Meyerhof, "Von Alexandrien nach Baghdad," *Sitzungsberichte der Preußischen Akademie der Wissenschaften* (Philosophisch-Historische Klasse), XXIII (1930), pp. 389-429.

9 These terms were evidently closely linked in the 9th century usage. Thus we know that the mathematician Abū Sa'id Jābir ibn Ibrāhīm al-Sabī wrote a (surviving) work entitled *Idāh al-burhān* ("The making-certain of demonstration"). See H. Suter, *Die Mathematiker und Astronomen der Araber und ihre Werke* (Leipizig, 1900), p. 69, no. 162.

the *Rhetorica* and *Poetica* into this schematism is unclear. His treatment of these treatises is perfunctory at best.

It warrants note that al-Kindī, when characterizing the subject-matter of the various branches of logic, employs a terminology which occasionally (viz. in respect to item 2, 3, 4 and 7) reflects a more primitive Arabic practice than that which was to become standard in the wake of the translations of Hunain ibn Ishāq and his younger associates. It is significant, however, that in the main al-Kindī's approach and his terminology already correspond, almost everywhere, to the usual Arabic usage of the technical terminology of logic. Certain exceptions are noted in our footnotes.

One of the curious features of al-Kindī's discussion is his characterization of " Analytics" (i.e., of *Anal. Pr.* through I, 7, namely to the end of the treatment of categorical syllogisms) as being concerned, not with "the syllogism" as such, but with "the conversion of premisses".[10] His discussion brings out quite clearly the fact that al-Kindī thought of the main point of "Analytics" as being not as much the conception of the syllogism *per se*, but the reducibility of syllogistic arguments—in the main by conversion—to syllogisms of the first figure.

It is interesting to observe al-Kindī's attempts to put the technical terminology of logic to work in his discussion. One instance of this is the use of the technical term "quantity" (*kamiyyah*) in the title of his treatise. Another is his predilection for the technical term "species" (*naw*') over against some non-technical equivalent that would serve equally well.

The outstanding characteristic of al-Kindī's sketch is its very sketchiness. *Only* in the case of the first three works of the Aristotelian Organon—the *Categoriae*, *De Interpretatione*, and *Analytica Priora*—there is any attempt to go beyond an explanation of the meaning of the title of the treatise to an indication of its contents. Quite strikingly, more than half of al-Kindī's entire discussion of the Organon is devoted to its first three books. Everything else is given the most bare and sketchy treatment, but these are dealt with at some length, and some of their contents reported in outline. It seems to me not at all unlikely that, when writing the treatise here under discussion, the "four books of logic", i.e., *Oateg.*, *De Interp.* and *Anal. Pr.* (to I, 7), prefixed by Prophyry's *Isagoge*, were the only works of the Arabic logical Organon to which al-Kindī had access in translation or epitome.

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[10] Here also a terminological primitivism occurs—in that al-Kindī calls the pair of premisses of a syllogism its "head", *ra'*s.

Let us now bring our introduction to an end, and turn from the preliminaries to the presentation of al-Kindī's text.

2. AL-KINDĪ'S SKETCH OF ARISTOTLE'S ORGANON

[391: II][11] The books of Aristotle's are listed in the ordering which the student who seeks entry to them needs as an aid, as regards both their sequence and their arrangement, so that he might become a philosopher by their means. After the propaedeutic sciences there are four species of books:

(1) The "set of eight"[12] logical ones.

(2) The physical ones.

(3) Those which are not needed for physics, being by nature different from that which is in need of the material; for there exists alongside of the material that which is connected to it by one of the species of connection.

(4) Those which have no need for the material and have no connection with it in any way at all.[13]

Now as for the books of logic there are eight of them:

(I) The first of them is called *Categoriae* (*qātūghūriyās*) and deals with the categories, by which I mean the subject and the predicate. The subject is that which is called a substance; and the predicate is what is called an accident when it is predicated of a substance, but neither by what is attributed to it by its name nor by its definition.

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[11] It is so indicated the corresponding page of the text edition of *Guidi-Walzer* (1940).

[12] The text mistakenly reads "set of four".

[13] Cf. *Meta.*, Δ 1026a 13 and Λ 1069a 30. Essentially this same ordering of the sciences, viz. (i) propaedeutics (i.e., grammar and mathematics), (ii) logic, (iii) physics (including music), (iv) metaphysics and (v) theology, dominates the Arabic concept of the ordering of the sciences, and accounts for the tripartite division, logic-physics-metaphysics, of the Arabic philosophical encyclopedias.

What is called a "predicate" may be of two species:

Firstly, when the predicate is attributed to the subject by its nature and its definition—as, e.g., life is said to belong to man, for "man" is said of a living being, which is defined by the definition, "A living being is a substance which is sensible and mobile", in order to differentiate it from the things that are different from it. In this sense too is quality said to belong to [i.e., be predicated of white]. For quality is that which pertains to the white, and is said about it. This white thing is similar [in quality] to this white thing; and this white thing is not similar [in quality] to this white thing, [for] this shade is similar to this shade, but this shade is not similar to this shade. It is thus that quality gives the category according to species; the quality of a thing being the species which is predicated of it in virtue of its name and its definition.

Secondly, the other one of the two types of predicate is called a predicate through equivocation, and not by univocality. It gives neither a definition nor a name. Thus white is [in this equivocal sense] predicated of the white—I mean the body which is white. For the white—I mean the name of the white—is separated from the white; it is not a white particular concrete thing.[14] The white is a color which arrests the vision. But the white—I mean the body of the white—is not a color which arrests the vision. For the definition of the white cannot be applied [to a body], and the name *the white* does not apply to a particular concrete thing, but is split off (i.e., abstracted)[15] when white [i.e., the color] is split off (abstracted) from white things.

The categories which are predicated as accidents to the category which bears predicates, i.e., substance, are nine: quantity, quality, relation, place (lit.: where?), time (lit.: when?), action, passion, possession, and position, i.e., the situation of a thing.[16]

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[14] I read *'ayn* (particular-concrete-thing) with Abū Rīdah. This word was used by the earliest Arabic translators to render the Greek "substance" in the sense of *tode ti*. For this expression see the *Malfātīh al-'ulūm* of Muhammad ibn Ahmad al-Khwārizmī, p. 143, in the edition of the *Liber Mafātīh al-Olūm* by G. van Vloten (Lugduni-Batavorum, 1895).

[15] The Arabic root *shaqqa*, "to break off", "to tear off" is used in rendering Aristotle's *chōristos* and its cognates.

[16] Note that more space is devoted to the *Categoriae* in this outline than to the rest of the Organon combined.

(II) Now as for the second of the books of logic, it is called *Peri Hermēneias* (*Bāri Yārmāniyās: De Interpretatione*) which means "on interpretation"; meaning the interpretation of what is said in the *Oategoriae* and matters related to the existence of propositions (judgments) about an object and attribute—I mean [statements] composed of subject and predicate.

(III) As for the third of the books of logic, it is called *The First Analytica* (*Anālūtīqā* [*al-ūlā*]) which means "the conversion" of premisses.[17]

(IV) As for the fourth of the books of logic, it is called *The Second Analytica* (*Anālūtīqā al-thāniyyah*) and it is also specified by the name *Apodictica (Afūdiqtīqā),* which means "making certain".

(V) As for the fifth of the books of logic, it is called *Topica* (*Tūbīqā*), which means "places", meaning the places of discourse.

(VI) As for the sixth of the books of logic, it is called *Sophistica* (*Sūfistīqā*) which means "relating to the Sophists"; "Sophist" means one who is arbitrary.

(VII) As for the seventh of the books of logic, it is called *Rhetorica* (*Rītūrīqā*), which means "persuasive speaking".

(VIII) As for the eighth of the books of logic, it is called *Poetica* (*Buyītīqā*), which means "poetry".

This constitutes the quantity of the eight logical books.

[399: IX] Thus we say: As to the subject-matter of the book of Aristotle's which we delimit, the first of them, I mean the *Categoriae,* is a discourse about the ten single expressions (categories) which we have defined (above) by giving the description of everyone of them, (specifying) by what each of them is differentiated from all the others, and what each cov-

[17] I render *al-'aks min al-ra's* as "the conversion of premisses". The construction of Guidi-Walzer—namely that al-Kindī is playing on the Greek word *analyein* by resolving it into the elements *ana* (Arabic: *min al-ra*'s?) and *lyein* (Arabic: *'aks*)—seems to me fanciful and implausible.

ers, and what is general to the entire number of them, and what is special to each single one of them.

[The subject-materials of this book are three.] *The first* of them is the determination of the things which are the most basic in description and explanation. These are substance—as subject and substance—as-predicate. A substance-as-subject is a thing which does not have in it anything (else) as substance except an accident; for if an accident is [in] a subject, then an accident may be predicated of it—I mean said about it. [These points are made] to explain that a [primary] substance is sensible, and a secondary [substance] is not sensible, but is predicated [400] of the sensible; and that [primary] accidents are sensible and secondary accidents are not sensible, but are predicated of the sensible.

As to *the middle* (i.e., the second) [topic], it is explanation of the ten individuals (i.e., the categories), by describing them and [indicating] their general features and their special characteristics.

And as to *the last* (i.e., the third) [topic], it has to do with matters connected with these ten things (the categories) which exist in more than one of them; such as the "*prior*" [Greek: *to proteron*] and motion and "the together with" [i.e., simultaneity, Greek: *to hama*].

Now as to the subject-matter of the second book, called *De Interpretatione* (*Peri Hermēneias*)—it deals with interpretation. It discourses about the interpretation of propositions which serve as premisses of scientific syllogisms, i.e., "unions"[18] which have "reports"[19] that are affirmative or negative; and matters connected with that.

As to the first part [of this book], it explains about how a proposition comes to be [through the combination] of a name (noun) and a verb, and [it explains what] an inflected statement is, and the "reporting" of a statement.

And as for the next [part] it has to do with propositions composed of a name and a verb, as when we say "Sa'īd is writing"; there is no contingency (accident) in that.

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18 The word I translate as "union", namely *jāmi'ah*, is the Arabic equivalent of Greek *symplokē,* a term introduced by Alexander of Aphrodisias to represent the relationship of three categorical statements so linked by an appropriate overlap in their terms as to be capable of constituting the two premisses and the conclusion of a syllogism. This word became infrequent in the usage of Arabic logicians after the 9th century.

19 The word I translate as "report", namely *khabar*, is seemingly an obsolete Arabic equivalent of Greek *logos apophantikos,* i.e., proposition.

And as to the next [part], it deals with propositions composed of a name and a verb and a third (member), such as an increase of time when we say "Sa'īd is writing today"; there is no contingency in that.

And as to the next [part], it deals with propositions composed of a name and a verb and a third (member) and a fourth, as when we say "The sunlight is hot today and penetrating"; there is no contingency in that.

And as to the next [part], it consists in an investigation about which [types of] proposition are the strongest in natural opposition; [whether] an affirmative to its negative, or an affirmative to another affirmative contradictory to it.

Now as to the subject-matter of the third book, called *Analytica (Priora)*, this is [devoted to] the clarification about "unions" of "premisses"[20], [explaining] what this is, and how it is, and why it is. A "union" of "premisses" is discourse in which various things are put forward [in such a way that] there becomes established through this another thing which was not evident in that (original) discourse, but yet is not a thing extraneous to that discourse. Now the very least [401] of which a "union" can be composed is a pair of two propositions which share one single term [in such a way that] there becomes established through them both a conclusion that was not evident in the two [premisses], but yet is not a thing extraneous to them both; i.e., is not a thing different from what joins the terms of the two [premiss] propositions.

A "union" of "premisses" can join its two premisses by three species of joining: (i) when the shared [i.e., middle] term occurs as subject in one of the two members [premisses] and as predicate in the other, (ii) when it [i.e., the middle term] occurs as predicate in both members together, and (iii) when it occurs as subject in both members together. And there are thus three species of "union": (I) those which unite truthfully and evidentially always—these are the *apodictic* ["unions"]; (II) those which unite truthfully in a connecting "union"[21] that may be either true or false, and these are the *dialectical* ["unions"]; and (III) those which unite falsely always,

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[20] The word I translate as "premiss" (always in quotes) is *mursilah*, apparently an obsolete Arabic equivalent of the later *muqaddimah* = premiss, which does occur (just once) in our text (p. 400, 1. 5).

[21] The word I render "connecting", from the Arabic root *qrm*, is a derivative from *qarīnah* = Greek *syzygia*, a technical innovation of Alexander of Aphrodisias to represent the relationship of two categorical statements so linked by a common term as to be capable of serving together as the premisses of a syllogism.

and these are the *sophistical* ["unions"].

The subject-matter of the *Analytica* is one of these three species of "union", namely the [apodictic] "union" of "premisses". Its object is to discourse about these "unions" of "premisses", primarily with a view to the discovery of apodictic unions, and secondarily with ancillary matters. Thus it discourses—firstly about wherein a "union" consists. Then [secondly] about how "unions" are linked together. Then [thirdly] about how many species (of "unions") there are which "make evident" [i.e., establish a conclusion], given their truth, by their very nature; and what can be established by a "motion"—I mean by a conversion or turning. Then [fourthly it discourses] about the introduction of premisses. And [fifthly] after that [it discourses] about the relationship of the second species and the third species of "union" towards the first species; on this ground this book is called the *Analytica* which means "breaking apart". Then [sixthly] it dwells [generally] upon "unions" and what is germane to them.

As to the subject-matter of the fourth book, called *Apodictica* (= *Anal. Post.*), i.e., "making certain", it discourses about conclusive "unions", I mean by those which give a demonstration what this is, and how it is, and how it functions, and what is needful for their composition. Then [it discourses about] the first principles of demonstration which are indispensable to a demonstration if it is to establish [a conclusion] which carries certainty for the intellect and perception.

As to the subject-matter of the fifth book, called *Topica*, it discourses about dialectical "unions" and the "places" of discourse which are necessary through a necessity external to themselves, and the fallacies that arise in this way and for these reasons. And [this book also gives] a clarification of "the five names"—to wit: genus, and species, and difference, and proprium, and accident—and of definition.

As to the subject-matter of the sixth book, called *Sophistica*—it discourses about fallacy in the make-up of "unions" whose construction does not satisfy the syllogistic conditions[22] upon premisses that compose a "union". The first [part of this book] discourses about how fallacy comes about; and the next [i.e., the second part] discourses about safeguards against the acceptance of such fallacies in this way.

As to the subject-matter of the seventh book, called *Rhetorica*, i.e., "oratory", it discourses about the three species of persuasion, i.e., persuasion in a tribunal, [402] and in an assembly, and about praise and blame as

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[22] I read *sharā'it* = conditions, with Abū Rīdah.

they go together in eulogy.

As to the subject-matter of the eight book, called *Poetica*, i. e., “poetics”, it discourses about the art of poetry [treating] of words and what metric is used in every species of poem, such as the poem-of-praise (= comedy) and the poem-of-mourning (= tragedy) and the poem-of-denunciation (= satire) and others.[23]

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[23] This chapter was originally published in *The New Scholasyticion*, vol. 37 (1965), pp. 44-58.

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Chapter 3

A NINTH-CENTURY ARABIC LOGICIAN ON: IS EXISTENCE A PREDICATE?

Now that the problem of whether or not "exists" is to be construed as a predicate is again the subject of active discussion in the philosophical literature,[1] it seems in order to reconsider significant stages in the history of the problem. The question at issue is frequently taken as arising from Kant's denial that existence is a predicate, a denial put forward in the interests of a refutation of the Ontological Proof. It may therefore be of interest to draw attention to a discussion of this question by the Arabic philosopher al-Fârâbî, which precedes the *Critique of Pure Reason* by well-nigh a millennium, and antedates St. Anselm himself by fully a century.

Abu Nasr al-Fârâbî was born in Farab, in Turkestan, not long after 870, and died at Damascus in 950, concluding a distinguished career as influential teacher and respected sage. Author of well over 70 philosophical treatises, al-Fârâbî devoted a large portion of his efforts to logic, writing extensive commentaries on Aristotle's logical work, and composing numerous shorter treaties devoted to special problems.[2] Of immediate interest here is

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1 See, for example, Rom Harré, "A Note on Existence Propositions," *The Philosophical Review,* 65 (1956), 548-549; G. Nakhnikian and W. C. Salmon, "'Exists' as a Predicate," *ibid.*, 66 (1957),535-542; H. S. Leonard, "The Logic of Existence," *Philosophical Studies,* 7 (1956), 49-64; N. Rescher, "On the Logic of Existence and Denotation," *The Philosophical Review,* 68 (1959), 157-180. A useful synthesis of earlier discussions is W. Kneale, "Is Existence a Predicate?" *Aristotelian Society Supplementary*, 15 (1936), reprinted in *Readings in Philosophical Analysis* ed. by H. Feigl and W. Sellars (N.Y., 1949), 29-43.

2 For a comprehensive listing of al-Fārābī's works see Max Horten, *Das Buch der Ringsteine Farabis* [Münster, 1906 (*Beitrage zur Geschichte der Philosophie des Mittelalters*), V, 3], XVIII-XXVIII. The fullest account of al-Fārābī's work is still that of Moritz Steinschneider, "Alfarabi," *Memoires de l'Académie Impériale des Sciences de Saint-Petersbourg,* serie 7, vol. 13 (1869).

a short collection entitled “Treatise on answers to questions asked of him” (*Risâltat fî jawâb masâ’il su’ila ‘anhâ*), which contains brief answers to some 40 miscellaneous questions, largely relating to logic.

Our present concern is with the sixteenth question, which I translate from the Arabic text edited by Friederich Dieterici:[3]

> *Question:* Does the proposition “Man exists” have a predicate, or not?
>
> *Answer:* This is a problem on which both the ancients and the moderns disagree; some say that this sentence has no predicate, and some say that it has a predicate.[4] To my mind, both of these judgments are in a way correct, each in its own way. This is so because when a *natural scientist* who investigates perishable things considers this sentence (and similar ones) it has no predicate, for the existence of a thing is nothing other than the thing itself, and [for the scientist] a predicate must furnish information about what exists and what is excluded from being.[5] Regarded from this point of view, this proposition does not have a predicate. But when a *logician* investigates this proposition, he will treat it as composed of two expressions, each forming part of it, and it [i.e., the composite proposition] is liable to truth and falsehood.[6] And so it does have a predicate from this point of view. Therefore the assertions are both together correct, but each of them only in a certain way.

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[3] *Al-Fârâbî’s Philosophische Abhandlungen* [Leiden (Brill), 1890], 90. A German translation of the eight treatises of al-Fârâbî published in this work was issued by Dieterici under the same title and imprint in 1892 (see 148-149 for our passage).

[4] By ‘ancients,’ the Islamic philosophers intend the Greek thinkers and their Hellenistic expositors, by ‘moderns’ the philosophers who used Arabic. Compare Averroes: “The ancient philosophers considered the First Principle ... as a simple existent. As to the later philosophers in Islam, they ... [also] accept a simple existent of this description.” *Tahâfut al-Tahâfut*, translated by S. Van den Bergh [Oxford, 1954], I, 237.

[5] That is to say, the predicate must give information regarding the nature (*mâhyaî*, what-ness, *quidditas*) of the thing in question. The existence of a thing (its *huwîya*, that-ness, *esse*) is not a part of its essence.

[6] Grammatically, “Man exists” is a complete sentence, with a grammatical subject, “man”, and a grammatical predicate, “exists”. Thus due to close parallelism between the logical and the grammatical relations (especially in Arabic) al-Fârâbî unhesitatingly classes “exists” as a legitimate grammatical (or logical) predicate. Even Kant agrees with this, affirming that: “zum logischen Prädicate kann alles dienen, was man will.”

Consideration of the question "Is 'exists' a predicate?" and of the logical issues involved in it thus goes back at least to the IXth century. Further, al-Fârâbî' insistence that the attribution of existence to an object adds nothing to its characterization, and provides no new information about it, effectively anticipates Kant's thesis that: "Sein ist offenbar kein reales Prädicat, d. i. ein Begriff von irgend etwas, was zum Begriffe eines Dinges hinzukommen könne."[7]

A word must be said as to the problems which occasioned al-Fârâbî's treatment of the matter. Al-Fârâbî, followed in this regard by Ibn Sînâ (Avicenna), wants clearly to distinguish the existence (*huwîya*) of a thing from its essence (*mâhîya*).[8] But if 'exists' is a predicate, then the existence of a thing would seem to become one of its properties, and could thus be held to be among the attributes constituting its essence. To preserve a clear distinction between essence and existence, al-Fârâbî denies that existence is a predicate (i.e., an *informative* predicate).[9]

The historical origin of the distinction between essence and existence has not yet wholly emerged from obscurity. In her masterly study of *La Distinction de l'essence et de l'existence d'aprés Ibn Sînâ* (Paris, 1937), Mlle. A.M. Goichon put the matter as follows:

> Ibn Sînâ la reçevait [i.e., la distinction de l'essence et de l'existence] de Fârâbî qui l'avait entrevue, mais sans lui donner tout son ampleur. Très probablement, tous deux l'ont considerée comme déduite des principes aristotéliciens, car ils n'en parlent jamais comme d'une découverte. Elle fait presque figure de lieu commun, et nulle référence ne permet d'affirmer quel texte la leur a inspirée. Peut-être les recherches dans les manuscrits, les traductions, les gloses anciennes, permettront-elles de déterminer la source. Pour le moment les matériaux nous manquent ... et nous ne pouvons remontrer avec certitude plus loin que Fârâbî. (op. cit., 131-132.)

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7 Compare also Averroes' view that "the word 'exists' does not indicate an entity added to its [i.e., a thing's] essence outside the soul, which is the case, when we say of a thing that it is white." *Tahâfut al-Tahâfut*, II, 118.

8 On their view, these coincide only in God.

9 Avicenna, however, held that existence is a predicate, and therefore, save with God, necessarily an accident (so that it would not be an essential property). Averroes, who denied the validity of the distinction between essence and existence, and argued against Avicenna on this ground, also condemned Avicenna's "mistake that the existence of a thing is one of its attributes." *Tahâfut al-Tahâfut*, I, 236; see also II, footnote 237.4.

There is no doubt, however, that the distinction was inspired by Aristotle, and took despite form in the hands of his commentators and expositors. There is nothing the Arabian distinction between *mâhîya* and *huwîya* that could not arise naturally out of explicative glosses on the following passage of the *Posterior Analytics*:

> He who knows what human or any other nature is, must know also that man exists; for no one knows the nature of what does not exist. ... But further, if definition can prove what is the essential nature of, a thing, can it also prove that it exists? And how will it prove them both by the same process, since ... what human nature is and the fact that man exists are not the same thing? Then too we hold that it is by *demonstration* that the being of everything must be proved—unless indeed to be were its essence; and since being is not a genus,[10] it is not the essence of anything. Hence the being of anything as fact is matter for demonstration; and this is the actual procedure of the sciences, for the geometer assumes the meaning of the word *triangle*, but that it is possessed of some attribute he proves. What is it, then, that we shall prove in defining essential nature? Triangle? In that case a man will know by definition what a thing's nature is—without knowing whether it exists. But that is impossible. (*An. Post*, 92b3-18 [Oxford translation]; cf. also 93a ff.)

For Arabic philosophy, then, the question "Is 'exists' a predicate?" arises, *not* from considerations relating to the Ontological Proof, but out of a desire to sharpen and clarify the Aristotelian distinction between the essence of things on the one hand, and their being or existence upon the other. Not the ontological argument, but the increasing systematization of the key concepts of Aristotle's logic occasioned al-Fârâbî to take up the problem of existence as a predicate.[11]

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[10] Cf. *Metaph.* 998bl4-24, 1045a34-68. Nor, on Aristotle's view, is being an attribute of thing§ *(An. Post.*, 9Oa2-4), This, in effect, amounts to al-Fârâbî's point that existence is not an *informative* predicate. Aristotle does, however, insist that being is a *predicat*e (*Metaph.* 1053b17-21), but his view and its grounds find accommodation in al-Fârâbî's assertion that existence is a logical predicate.

[11] This chapter was originally published in the *Journal of the History of Ideas*, vol. 2 (1960), pp. 428-30.

Chapter 4

AVICENNA ON THE LOGIC OF "CONDITIONAL" PROPOSITIONS

1. INTRODUCTION

Like most of the notable medieval Arabic philosophers working in the Aristotelian tradition, Abū ʻAlī al-Husain ibn ʻAbdallāh ibn Sīnā, better known under the Latinized name of Avicenna (980-1037), wrote extensively on logic. In their logical works, the Arabian philosophers invariably hewed to their Greek sources with painstaking care. It is consequently of some interest to find in Avicenna a discussion of the logic of hypothetical and disjunctive propositions which, beginning from a point of departure that is clearly Greek, and indeed Stoic in origin, goes beyond the discussion hitherto found in the accessible sources. The object of the present paper is to throw some light upon this chapter of Avicenna's logic.

2. "CONDITIONAL" PROPOSITIONS

Avicenna distinguishes between "attributive" (Arabic: *hamliyyah*) propositions, which ascribe a predicate to a subject, or deny it to the subject,[1] and "conditional" (*shartiyyah*) propositions, i.e., compound proposi-

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[1] *Livre des directives et remarques* (Kitāb al-Ishārāt wa-'l-Tanbīhāt), translated by A. M. Goichon (Paris and Beyrouth, 1951), p. 114. [This work is henceforth cited as "*I*".] *Le livre de science* (*Danesh—name*), pt. I (Logic and Metaphysics), translated by M. Achena and H. Massé (Paris, 1955), pp. 36-37. [This work is henceforth cited as "*D*".] Avicenna's fullest treatment of logic is to be found in his massive treatise *Al-Shifā'* whose logical sections are now appearing in print in Cairo under the auspices of the Egyptian Ministry of Education. The section of this work relevant to the present paper (No. IV on syllogistics, *al Qiyās*) has not yet appeared. Until it is available, the present discussion must be viewed as tentative.

In a work entitled *L'organon d'Aristote dans le monde arabe* (Paris, 1934), Ibrahim Madkour has made an extensive study of the Ishārāt. (The section of this work which will concern us here is treated on pp. 159-72.) Valuable though it is, Madkour's discussion is not always to be trusted on points of logic, and sometimes tions each of whose constituent propositions are displaced from their ordinary assertive function to play another role (I, 115). The paradigm examples of “attributive” propositions are “Man is an animal” and “Man is not a stone” (I, 116-117; D, 36). In the full light of his discussion, Avicenna’s “attributive” propositions are readily seen to correspond to *categorical* propositions. The paradigm examples of “conditional” propositions are “If the sun shines, it is day” and “Either this number is even, or it is odd” (I, 117-118; d. D, 36). Thus “conditional” propositions are *compounds* of “attributive” proposition, the compound statement being such as not to assert its components, but to relate them.

Avicenna considers two main types of “conditional” propositions: “conjunctive” (*muttasilah*) and “disjunctive” (*munfasilah*). The “conjunctive conditional” propositions correspond to *hypothetical* statements. The paradigm examples are “If the sun has risen, it is day”, and “If the sun has risen, it is not night” (I, 117-118; D, 41-42). The “disjunctive conditional” propositions correspond to *disjunctive* statements (in the sense of exclusive disjunction).[2] The paradigm examples are “Either this number is even, or it is odd” and “Either this number is even, or it is not-divisible into two even parts” (I, 118, D, 41-42).[3]

Avicenna’s distinctions correspond exactly with *those* found in Boethius’ treatise De *Syllogismo Hypothetico*,[4] which subsequently be-

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puts Avicenna into errors which he himself avoided.

2 The exclusive character of disjunction is quite clear throughout Avicenna’s discussion. For example: “The assertion of a disjunctive proposition consists in asserting an incompatibility—as when one says: ‘It is either thus, or it is so.’” (D. 44). Sometimes, however, Avicenna’s examples of disjunctions would be compatible with an inclusive construction of either . . . or”.

3 For fuller information regarding Avicenna’s classification of propositions, and for his terminology, see A. M.Goichon, *Lexique de la langue philosophique d’lbn Sīnā* (Paris, 1938), pp.305-318. That the distinctions just explained became part of the standard machinery of Arabic logic is shown by their inclusion in al-Abharī’s popular tract “Introduction to Logic” (Īsāghūjī fī-’l-mantiq). See E. E. Calverly’s translation in the *D. B. MacDonald Memorial Volume* (Princeton, 1933), pp. 15-85 (see pp. 80-81).

4 Migne, *Patrologia Series Latina*, vol. 64 (=*Boetii Opera Omnia*, v. II), pp. 831-876, see pp. 832-834. For two other points of agreement between Boethius and Avicenna regarding logical matters see S. M. Afnan, *Avicenna* (London, 1958), p. 84 and p. 97.

came established in Western logic.[5] (Since Latin writings were not available to the Arabs, this may be taken as further evidence in support of the general supposition that the pivotal ideas of Boethius’ work derive from Greek sources).[6] This correspondence is indicated in Display 1. Thus, for Avicenna, a “conditional” proposition may take either of the forms:

(1) “Conjunctive” case: If *A*, then *C*.

(2) “Disjunctive” case: Either *A* or *C*.

In both cases, a “conditional” proposition has two constituents, of which the former (i.e., *A*) is characterized as *antecedent* (*muqaddam*), and the latter (i.e., C) as *consequent* (tālī) [I, 117; D, 41]. Avicenna applies this terminology in the “disjunctive” as well as in the “conjunctive” case. When a “disjunctive conditional” proposition takes the form “Either *A*, or C_1, or C_2”, *both* C_1 and C_2 are characterized as consequents (D, 41-42). Avicenna also recognizes such complex “conditional propositions” as “*If A, then either* C_1 *or* C_2”, and “*Either if A then* C_1 *or it is not the case that if A then* C_2” (I, 129-130).

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5 See H. W. B. Joseph, *An Introduction to Logic* (2d. ed., Oxford, 1916), p. 348, n. 1. Cf. Sir William Hamilton’s *Lectures on Logic*, lecture, XIII.

Mlle. Goichon believes that Avicenna’s “conditional” propositions constitute “une sorte de proposition qui ne presente pas une correspondence exacte avec celle que l’on étudie en logique occidentale”, and conjectures that Avicenna derived this concept from Oriental sources (I, 115, footnote 1). But this view is unwarranted, because every detail of Avicenna’s characterization of “conditional” propositions corresponds precisely to Boethius’ treatment of the category of “hypothetical” propositions. In general, however, Miss Goichon clearly and rightly stresses Avicenna’s indebtedness in the analysis to Stoics sources (I, 57 and 67).

6 Regarding the occurrence of these distinctions in Chrysippus, see von Arnim, *Stoicorum Veterorum Fragmenta* (Leipzig, 1903), vol. II, p. 68; as cited by S. M. Afnan, *Avicenna* (London, 1958), p. 196, and also pp. 86-87. A discussion of the sources of Boethius is found in K. Dürr, *The Propositional Logic of Boethius* (Amsterdam, 1951), pp. 4-15. The distinctions in question apparently go back to the earlier peripatetics, Theophrastus and Eudemus in particular, and were subsequently taken up by the Stoics.

Display 1

PROPOSITIONAL TERMINOLOGY

"*Modern*" *Terminology*	*Boethius*' *Terminology*	*Avicenna*'*s* *Terminology*
I. Categorical Propositions	I. Categorical Propositions	I. Attributive Propositions
II. Non-Categorical Propositions	II. Hypothetical Propositions	II. Conditional Propositions
1) Hypothetical	1) Conjunctive	1) Conjunctive
2) Disjunctive	2) Disjunctive	2) Disjunctive

3. THE QUALITY OF "CONDITIONAL" PROPOSITIONS

According to Avicenna, "conditional" propositions can be either affirmative or negative. His paradigm examples of negative "conditionals" are: "*Not:* if the sun has risen, it is night", and "*Not:* either this number is even, or it is divisible into two equal parts" (I, 118; D, 43-44). He is explicit in emphasizing that the quality of a "conditional" proposition has nothing to do with the affirmativeness or negativity of its constituents, but depends solely upon whether the liaison or relationship between them is affirmed or denied (I, 118; d. D, 43).

With respect to the quality of "conditional" propositions, Avicenna thus presents the following classification:

Mode of "Conditional"	*Affirmative Form*	*Negative Form*
"conjunctive"	If *A*, then *C*	Not: if *A*, then *C*
"Disjunctive"	Either *A* or *C*	Not: either *A* or *C*

Avicenna apparently takes no account here of the fact that there is no way in which a proposition of the form "*Not: if A, then C*" can be transformed of the "conjunctive conditional" paradigm "*If X, then Y*".[7] Nor can "*Not:*

[7] In consequence of this, Western logicians did not divide the class of hypotheticals into the subdivisions of affirmative and negative. (See for example, J. Gredt, *Ele-*

either A or C" (in Avicenna's exclusive sense of "*either ... or*") be put into the form "*Either* X, *or Y*". Avicenna fails to note that in introducing the negative forms of "conditional" propositions in the way he does, he has, in effect, *broadened* the categories of "conjunctive" and "disjunctive" propositions beyond their original characterization.[8]

4. THE QUANTITY OF "CONJUNCTIVE CONDITIONAL" PROPOSITIONS

As a result of the work of Benson Mates, it is well-known that the Megarian logician Diodorus Cronus introduced a mode of implication characterized by the principle that "If *A*, then *C*" is to amount to:

At each and every time *t*: If *A*-at-*t*, then *C*-at-*t*.

Following Mates, we may symbolize this *Diodorean implication* in modem notation as: $(\forall t)(A_t \supset C_t)$.[9] Diodorus' paradigm example of a true implication statement is "If it is day, then it is light", and of a false one, "If it is day, then I am conversing".[10]

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menta Philosophiae Aristotelico-Thomisticae (Barcelona, 1946) I, pp. 37-40.)

8 Rather than taking this omission to represent a mere oversight on Avicenna's part, I believe it to be an (added) indication that Avicenna's logic draws upon sources in which the Stoic distinction between *denial* (*arnētikon*) and *negation* (*apophatikon*) is made. (See B. Mates, *Stoic Logic* [University of California Publications in Philosophy, vol. 26 (1953)], p. 31). If we start with discussions in which this distinction is presupposed, but assume it to be blurred in translation or exegisis, Avicenna's remarks are a natural consequence.

9 Benson Mates, "Diodorean Implication", *The Philosophical Review,* vol. 58 (1949), pp. 234-242; see especially p. 238. Cf. also Martha Hurst, "Implication in the Fourth Century B.C.", *Mind,* vol. 44 (1935), pp. 485-495; and Mates' *Stoic Logic*, Berkeley and Los Angeles (1953; University of California Publications in Philosophy, no. 26).

10 In the case of atemporal subject-matter, it would seem natural to substitute "case-in-which" for "time-at-which" phraseology, for example in a Diodorean-type rendering of the conditional "If a number is prime, it cannot be divided by four". Our very scanty sources regarding Diodorus however give no indication that he applied his analysis to atemporal cases.

The Diodorean conception of implication remained a living idea among the Stoic logicians.[11] It is well-known that the Arabic philosophers drew extensively on the work of the Stoics.[12] Thus it was that Avicenna found that Diodorean implication afforded a ready-made instrument for the quantification of "conditional" propositions.

Avicenna teaches that an affirmative "conjunctive conditional" proposition "*If A, then C*" may take the universal form,

(i) Always [i.e., "at all times"[13] or "in all cases"]: when *A*, then (also) *C*;

or the particular form,

(ii) Sometimes: when *A*, then (also) *C*.[14]

Correspondingly, the negative "conjunctive conditional" propositions can take the universal form,

(iii) Never: when *A*, then (also) *C*;

and the particular form,

(iv) Sometimes not: when *A*, then (also) *C*.[15]

Avicenna's discussion and his illustrative examples make it clear that what

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[11] See Mates' discussion, *op. cit.* p. 234. Sextus Empiricus quotes the remark of Callimachus that "Even the crows on the roof-tops are cawing about which conditionals are true" (*Adv. Math.* (Loeb), I, 309).

[12] See S. Horowitz's instructive study, "Ueber den Einfluss des Stoicismus auf die Entwicklung der Philosophie bei den Arabern", *Zeitschrift der Deutschen Morgenländischen Gesellschaft*, vol. 57 (1903), pp. 177 ff.

[13] Regarding Avicenna's emphasis upon this temporal construction see Miss Goichon's comment, I, p. 157, n.b.I.

[14] See I, 123; D, 43-44.

[15] See I, 123-124; D, 43-44.

he has in mind is most simply and accurately described in terms of the table:

Cases in which	*C holds*	*C does not hold*
A holds	I	II
A does not hold	III	IV

Here the universal affirmative (i) corresponds to the condition that compartment II is empty. (Note that this accounts for the terminology of “conjunctive” for hypotheticals—if II is empty, then *C* is always “conjoined” with *A*.) The particular affirmative (ii) corresponds to the circumstance that compartment I is non-empty (i.e., *A* and *C* are sometimes “conjoined”). Analogously, the universal negative (iii) corresponds to the circumstance that compartment I is empty, and the particular negative (iv) to the circumstance that compartment II is non-empty. The overall tenor of Avicenna’s discussion is summarized in Display 2 which shows treatment of “conditional conjunctive” propositions is in effect a generalization upon the Diodorean analysis of implication. The single universal affirmative mode of Diodorean implication is expanded into a full-scale treatment of this implication relationship, fully articulated with respect both to quantity and to quality.

5. THE QUANTITY OF “DISJUNCTIVE CONDITIONAL” PROPOSITIONS

In quantifying “conjunctive conditional” propositions, Avicenna, as we have seen, follows in the footsteps of the Stoics, carrying to their “logical conclusion” suggestions inherent in the Diodorean concept of implication. In the analogous quantification of “disjunctive conditional” propositions, Avicenna’s discussion takes yet another step beyond Stoic logic as we presently conceive it.

In the quantification of “disjunctive conditional” propositions of the form “*Either A, or C*”, Avicenna proceeds by close analogy with his Diodorean-style quantification of implication-statements of the form “*If A, then C*”. Thus Avicenna holds that an affirmative “disjunctive conditional” statement may take either the universal form,

(i) Always [i.e., “at all times” or “in all cases”]: either *A*, or C;

Display 2

AVICENNA'S CLASSIFICATION OF
CONJUNCTIVE CONDITIONAL PROPOSITIONS

From	Symbolic Rendition	Avicenna's Illustrative Paradigm
A (U.A.)	$(\forall t)\,(A_t \supset C_t)$ $(\forall t)\sim(A_t\ \&\ {\sim}C_t)$	"Always: when the sun has risen, it is day." (I, 123; D, 43-44)
E (U.N.)	$(\forall t)\sim(A_t\ \&\ C_t)$	"Never: when the sun has risen, it is night." (I, 123; D, 44)
I (P.A.)	$(\exists t)(A_t\ \&\ C_t)$	"Sometimes: when the sun has risen, it is cloudy." (I, 123; D, 44)
O (P. N.)[16]	$(\exists t)(A_t\ \&\ {\sim}C_t)$	"Sometimes not: when the sun has risen, it is cloudy." (I, 123-24; D, 44)

or the particular form,

(ii) Sometimes [i.e., "at certain times" or "in certain cases"]: either *A*, or *C*.[17]

Correspondingly, the negative "conjunctive conditional" propositions can take either the universal form,

(iii) Never [i.e., "at no times" or "in no cases"]: either *A*, or *C*;

or the particular form,

(iv) Sometimes [i.e., "at certain times" or "in certain cases"] not either *A*, or *C*.

[16] In Avicenna's discussion, following Aristotle (*Anal. Pr.*, 24a18-22), propositions of "indeterminate" quantity are also treated. A proposition is of indeterminate quantity when, like "Man is a writer" its quantity is indefinite, being wholly equivocal as between "All men are writers" and "Some men are writers" (I, 123-124; D, 44).

[17] See I, 123-124; D, 43-44.

Again, the exact construction Avicenna places upon these propositions is best described in terms of the table:

Cases in which	*C holds*	*C does not hold*
A holds	I	II
A does not hold	III	IV

The universal affirmative proposition (i) corresponds to the condition that compartments I and IV are both empty; and the particular affirmative (ii) corresponds to the circumstance that at least one of the compartments II and III is non-empty. Analogously, the universal negative (iii) corresponds to the circumstance that compartments II and III are both empty (i.e., *A* and C always either occur conjointly or are absent conjointly), while the particular negative (iv) corresponds to the circumstance in which at least one of the compartments I and IV are non-empty. The overall situation is surmised in Display 3.

We thus find that Avicenna’s discussion carries over to disjunctive propositions the Diodorean-style quantification which it provided for hypothetical propositions. It is possible that this might be found already in his Arabic predecessors,[18] or in some late Greek commentary on Aristotle’s logic written under Stoic influences.[19] But so far as I have been able to de

18 We know that al-Fārābī (c. 870-950) wrote on hypothetical propositions and inferences. (See C. Praml, *Geschichte der Logik im Abendlande*, vol. II, pp. 317-318). We know too that al-Fārābī’s teacher, Abū Bishr Mattā ibn Yūnus (c. 860-940) wrote a treatise on hypothetical syllogisms. (See M. Steinschneider, “Die Arabischen Uebersetzungen aus dem Griechischen”, *Zwölftes Beiheft zum Centralblatt für Bibliothekswesen* [Leipzig, 1893], p. 43.) Unfortunately, however, neither of these works has survived. However, al-Fārābī’s treatise on syllogistiés (*al-qiyās*), published by Mlle. M. Türker in 1958 (*Révue de la Faculté de Langues, d’Histoire, et de Géographie de l’Université d’Ankara*, vol. 16, 1958), does contain a short section on conditional syllogisms, giving a discussion which in large measure agrees, as far as it goes, with Avicenna’s treatment. Furthermore, al-Kindī (c. 800-873) is known to have been partial to hypothetical and disjunctive syllogisms. (See R. Walzer, “New Light on the Arabic Translations of Aristotle,” *Oriens*, vol. 6 (1953), p. 129.)

19 The concepts of Stoic logic penetrated into the other schools of Greek philosophy. See, for example, H. Matte in *Gnomon*, vol. 23 (1951), p.35.

Display 3

AVICENNA'S CLASSIFICATION OF "DISJUNCTIVE CONDITIONAL" PROPOSITIONS

From	Symbolic Rendition[20]	Avicenna's Illustrative Paradigm
A (U.A.)	$(\forall t)\ (A_t\ \mathrm{V}\ C_t)$	"Always: either a number is even, or it is odd." (I, 123 cf. D, 44)
E (U.N.)	$(\forall t)\ {\sim}(A_t\ \forall\ C_t)$	"Never: either the sun has risen, or it is day." (I. 123; Cf. D, 44)
I (P.A.)	$(\exists t)(A_t\ \mathrm{V}\ C_t)$	"Sometimes: either Zaid is in the house, or Amr is there." (I, 123; cf. D, 44)
O (P.N.)	$(\exists t){\sim}(A_t\ \mathrm{V}\ C_t)$	"Sometimes not: either a fever 'bilious', or it is 'sanguine'." (I, 123-24; cf. D, 44)

termine, Avicenna is the first writer in the history of logic to give an analysis of hypothetical and disjunctive propositions that is fully articulated with respect to quality and to quantity.

6. THE THEORY OF IMMEDIATE INFERENCE OF "CONDITIONAL" PROPOSITIONS

In the treatise under consideration, Avicenna dispatches the question of the theory of immediate inference for "conditional" propositions in one brief remark. He observes that, in the two cases of *contradiction* and of *conversion* the same rules apply which govern the "attributive", i.e. cate

[20] The upper case vee "V" is here used to symbolize exclusive disjunction, following Bochenski's usage in his discussion of Boethius in *Ancient Formal Logic* (Amsterdam, 1951), p. 107.

Display 4

CONTRADICTION AND CONVERSION

Categorical Inference[21]	Status of "Conjunctive Conditional" Analogue	Status of "Disjunctive Conditional" Analogue
Contradiction		
1) Of *A* and *O*	holds	holds
2) Of *E* and *I*	holds	holds
Conversion		
1) Of *A* (invalid)	fails	holds*
2) Of *E* (valid)	holds	holds
3) Of *I* (valid)	holds	holds
4) Of *O* (invalid)	fails	holds*

gorical, propositions, the antecedent playing the role of subject, and the consequent that of predicate (I, 131). The extent to which this remark is correct may be seen in the tabulation of Display 4. It is accordingly clear that Avicenna's statement is correct only with the exception of the two starred cases. But Avicenna is perfectly aware of this unorthodox feature of "disjunctive conditional" propositions, and himself comments upon it with admirable explicitness.[22] It seems necessary therefore to regard Avicenna's above-cited statement as an incautious formulation. What he should have said is that, with regard both to contradiction and conversion, all of the categorically valid inferences are also valid for "conditional" propositions, though the converse of this rule holds only in the case of "conjunctive conditional" propositions.

With regard to other kinds of immediate inference, it is clear that subalternation (*A* to *I*, E to 0), contrariety of (of *A* and E) and subcontrariety (of *I* and 0) also hold with respect both to "conjunctive conditional" and to "disjunctive conditional" propositions.

[21] It is assumed throughout that the requirement of existential import is satisfied.

[22] See D, 42-43, where Avicenna discusses the greater amenability to conversion of "disjunctive conditional" propositions vis-à-vis the disjunctive ones.

7. ANOTHER TREATMENT OF THE QUALITY AND QUANTITY OF HYPOTHETICAL AND DISJUNCTIVE PROPERTIES

To have a standard of comparison for assessing the treatment of the logic of "conditional" propositions to be found in Avicenna, it is useful briefly to examine the discussion of hypothetical and disjunctive propositions in a modern logic-manual written in the Western "Aristotelian" tradition. For this purpose, I have chosen J. Welton's comprehensive *Manual of Logic* (vol. I, 2d. ed., London 1896; cited henceforth as "ML").

The paradigm of a hypothetical proposition is taken as "If *M*, then *P*" (p. 181). Here *M* and *P* are understood to be strictly subject-predicate propositions, of the type "*S* is an *M*" and" *S* is a *P*", respectively. A hypothetical proposition is negative when its *consequent* is negated, so that the paradigm of a negative hypothetical is "*If M, then not P*". (It is thus recognized that the *denial* of a hypothetical is not itself of hypothetical form—a result that Avicenna apparently viewed with distaste.) The quantity of a hypothetical proposition is fixed by prefixing "always" for universals, and "sometimes" for particulars (p. 186). The four resulting modes are characterized as per Display 5:[23]

It is readily seen that, from a *strictly formal* standpoint, this analysis is entirely equivalent with that presented by Avicenna. A great difference, however, lies in the *semantical interpretation* of hypotheticals in the two treatments. For Avicenna, the U. A. proposition "*If A, then C*" is construed as: "In every *case in which A* holds true, so also does *C*". For Welton, on the other hand, "*If* M, *then P*" is to be construed as "For every *individual for which M* holds true, so also does *P*". Avicenna thus construes hypotheticals after the Stoic "case-in-which-true" manner, while Welton adheres to the "thing-for-which-true" construction of subject-predicate logic.

With respect to the theory of immediate inference for hypotheticals, Welton states that, on the analysis just given, "the whole doctrine of opposition is applicable" (ML, 244), and proceeds to show this in a derailed way.[24] In view of the formal equivalence just remarked upon, Avicenna can, of course, make the same claim.

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[23] ML, 244; see also p. 271. 24.

[24] ML, 244-246.

Display 5

Mode	Formulation	Interpretation
A (U.A.)	Always, if *M*, then *P*	$(s)\ (M_s \supset P_s)$
E (U.N)	Always, if *M* then not *P* Never, if *M* then *P*	$(s)\ (M_s \supset \sim P_s)$ $(s) \sim(M_s \ \&\ P_s)$
I (P. A.)	Sometimes, when *M*, then *P*	$(\exists s)\ (M_s \ \&\ P_s)$
O (P.N.)	Sometimes, when *M*, then not *P*	$(\exists s)\ (M_s \ \&\ \sim P_s)$

With regard to *disjunctive* propositions, one fundamental point of difference lies in the fact that Welton construes disjunction in terms of its *inclusive* applications (ML, 188-189). He proceeds to recognize four modes of disjunctive propositions as per Display 6: [25]

We may observe that, aside from the different (i.e., inclusive) construction of the disjunction relation "either ... or", there is a substantial formal analogy between the four modes of Welton's treatment and those of Avicenna's discussion. However, there is again a vast difference in the meaning which these two analyses accord to disjunction-statements. In Welton, the discussion is rigidly restricted to the confines of subject-predicate logic. In Avicenna we have the Stoic-Megaric notion of quantifying over "cases in which *X* holds". In Welton's analysis, on the other hand, we have only the orthodox "Aristotelian" notion of quantifying over "things to which *X* applies".

As regards the theory of immediate inference for disjunctive propositions, Welton explicitly recognizes that "the full doctrine of opposition cannot be applicable" (ML, 246). He is quite clear as to the modifications that are required.[26]

[25] ML, 192; see also p. 246.

[26] See ML, 274.

Display 6

Mode	Formulation	Interpretation
A (U.A.)	All *S*'s are either *P*'s of *Q*'s	$(s)\ (P_s \vee \mathrm{Q}_s)$
E (U.N)	No *S*'s are either *P*'s or *Q*'s	$(s) \sim(P_s \vee \mathrm{Q}_s)$
I (P. A.)	Some *S*'s are either *P*'s or *Q*'s	$(\exists s)\ (\sim P_s\ \&\ \sim \mathrm{Q}_s)$
O (P.N.)	Some *S*'s are neither *P*'s nor *Q*'s	$(\exists s) \sim(P_x \vee Q_s)$

8. CONCLUSION

The present deliberations could indicate that a fully articulated theory of the logic of hypothetical and disjunctive propositions is first to be found in the logical treatises of Avicenna. This theory may possibly be a product of late Greek rather than of originally Arabian logic, being a natural extension of ideas inherent in Stoic logic. At any rate, Avicenna is the earliest logician in whose writings this theory has thus far been identified.

As a comparison with the approach of "Aristotelian" logicians in the Latin West emphasizes, Avicenna's quantification of hypothetical and disjunctive propositions proceeds in truth-condition terms, rather than in the subject-predicate terms of the analysis given by European logicians. This difference of approach is clearly traceable to Stoic influences. Avicenna's treatment of "conditional" propositions thus affords a striking illustration of the fact that in Arabic logic, Stoic ideas were yet alive which did not figure in the more orthodox Aristotelianism which developed among the Latins.[27]

[27] This chapter was originally published in *The Notre Dame Journal of Formal Logic*, vol. 4 (1963), pp. 49-58.

Chapter 5

AVICENNA ON THE LOGIC OF QUESTIONS

In recent years the Logic of Questions has come into its own as a branch of logic theory which has generated widespread interest and has been extensively cultivated[1]. It is thus germane to call attention to the (relatively brief) treatment of the theory of questions by the famous Persian-Arabic philosopher Avicenna (980-1037).[2]

In several of his logical treatises, Avicenna seeks to provide an analysis and a systematic classification of questions.[3]

Avicenna's classification of questions is presented in Display 1.

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[1] A pioneer work of the recent discussions in M. and A. Prior, "Erotetic Logic," *The Philosophical Review*, vol. 64 (1955), pp. 43-59. Three important monographs are: D. Harrah, *Communication*: A *Logical Model* (Cambridge, Mass., 1963); N.D. Belnap, Jr., *An Analysis of Questions: Preliminary report* (Santa Monica, 1963); L. Aqvist, *A New Approach to the Logical Theory of Interrogatives*, pt. I (Uppsala, 1965). For a brief but synoptic discussion see the article "Questions" by C.L. Hamblin in P. Edwards (ed.), *The Encyclopedia of Philosophy*, vol. VII (New York, 1967), pp. 49-53.

[2] On Avicenna as a logician see N. Rescher, *The Development of Arabic Logic* (Pittsburgh, 1964), especially pp.149-155.

[3] Our principal sources are: (1) *Dânesh-nâme*, anonymously edited in Teheran in 1331 A.H. (=1912); tr. by M. Achena and H. Massé, *Avicenna: Le Livre de Science*, vol. I, Sections on logic and metaphysics (Paris, 1955), pp. 84-85; (2) *Kitâb al ishârât wa-'l-tanbîhât*, ed. J. Forget (Leiden, 1982); ed. with the commentary of Nâsir al-Dîn al-Tûsî (b.1201) by S. Dunyâ (Cario, 1960); tr. A.M. Goichon, *Livres des directives et remarques* (Paris, 1951); see pp. 85-86 of the Forget text and pp. 234-238 of the translation; (3) *Kitâb al-najât*, ed. M. Kurdî (Cario, 1938); the material on questions is extracted and translated in a series of footnotes on pp. 235-237 of A.M. Goichon, *op. cit.*

Display 1

AVICENNA'S CLASSIFICATION OF QUESTIONS

BASIC QUESTIONS (*mutâlib umhât*)

1. The *is-it* question (*hal al-shay*')

   i. Re existence *simply* (*mawjûd mutlaqan*)

   ii. Re existence *in-a-state* (*mawjûd bi-hal kadhâ*)

2. The *what-is-it* question (*mâ al-shay*')

   i. Re. essence of the thing (*dhât al-shay‹*)

      [a] definition (*hadd*)

      [b] description (*rasm*)

   ii. Re. meaning-of-the-world (*mafhûm al-ism*)

3. The *what-sort* question (*ayyu al-shay*')

   (Re. the genus, species, and difference of the thing)

4. The *why* question (*limâ al-shay*')

   i. Why *is*: the cause (the four causes: [a] material, [b] formal, [c] efficient, [d] final)

   ii. Why *said*: the reason

SUBSIDIARY QUESTIONS (*mutâlib juz'iyyah)*

5. The *how* question (*kayfa al-shay*')

6. The *where* question (*ayna al-shay*')

7. The *when* question (*matâ al-shay*')

8. The *how-much* question (*kammiyyat al-ashyâ*')[4]

[4] Listed in the *Dânesh-nâme,* but omitted in the *Ishârât*.

The principal distinctions involved in this classification are as follows:

1. *Basic questions vs. subsidiary questions.* The rationale here appears to be that a basic question is one regarding the *existence*, the *nature*, and the *causes* of a thing: and thus deal with (a) questions concerning *substance* (rather than "accidents", in the sense of Aristotelian categories other than substance), plus (b) questions concerning the *causes* (which are extra-categorial questions). By contrast, the subsidiary questions deal with accidental features. Apparently this is the reason why Avicenna designates[5] the four basic questions (1-4) as the *scientific* questions, dealing with matters of essence and existence, and he characterizes the subsidiary questions (5-8) as *non-scientific* precisely because they address accidental matters. For, of course, on the classical, Aristotelian view of the matter science deals with the essential features of thing and scientific knowledge of accidents is accordingly impossible.

2. *Questions of facts vs. questions of discourse*. Avicenna is clear and explicit in distinguishing considerations regarding the nature of things form those regarding the meanings of words (2i vs. 2ii), and in distinguishing considerations as to why things *are* as they are from those regarding why things *are spoken of* in certain ways (4i vs. 4ii).[6] It would seem that Avicenna's pointed formulation of the matter represents a substantial step towards the later distinction between nominal and real definition, a step indicated by but going beyond the work of the Stoics.

3. *The priority of questions*. The idea operative here is that it can prove infeasible to raise a question Q_1 (e.g., that regarding the *purpose* of a thing) if a suitable answer to an antecedently presupposed question Q_2 (e.g., that regarding the existence of the thing) is not forthcoming. Avicenna consequently maintains that, for example, the *why* question (4 i) is posterior to the *is-it* question (1 i). In such a case—when the *legitimacy of raising* the question Q_1 turns on the obtaining of an appropriate (affirmative or negative) answer to Q_2—

[5] *Le livre de science*, Vol. I, p. 84

[6] This distinction too comes from Aristotle. See *Metaphysics*, 1030a27-28. Cf. also *ibid*., 1029b13 and *Posterior Analytics* 92b6-8, 26.

question $Q2$ is said to have (logical) priority over $Q1$. In just sense, the question "Is X an *accomplished* flutist?" would be posterior to the question "Does *X* play the flute at all?": if the second is answered negatively, it would be pointless to raise the first.

It might seem at first blush that Avicenna's tabulation of questions was arrived at as a merely grammatical exercise, by simply compiling the interrogative particles of the Arabic language, the equivalents of *what*, *why*, *how*, *where*, *when* and the like. But this conception is mistaken, and loses sight of the venerable and august ancestry of the venture. For *Avicenna's tabulation of questions is in fact derived from Aristotle's categories*, duly augmented by the five predicable of Porphyry. There can be no doubt of this, in the face not only of the parallelism of the concepts at work here, but also the close correspondence of the Arabic terminology at issue in the discussion of categories.[7] The relevant data are assembled in Display 2. Two aspects of this tabulation should be noted especially:

1. It helps to bring out quite clearly the fact that Avicenna approaches questions from an *ontological* direction, viewing them all as questions asked about *an existing thing* (this was already implicit in the Arabic nomenclature), and that this thing is to be considered in isolation (hence the absence of the category of relation as well as its cognates posture and possession), and without regard to other things by which it may be affected or upon which it might be acting (hence the absence of the categories of action and passion).[8]

[7] Cf. D.M. Dunlop, "Al-Fârâbî's Paraphrase of the *Categories* of Aristotle," *The Islamic Quarterly*, vol. 4 (1958), pp. 168-197; vol. 5 (1959), pp. 21-54.

[8] In view of Avicenna's occasional dependence on Stoic sources, it is worth noting that his treatment of questions is clearly based on the Aristotelian doctrine of categories, in contrast to the simplified *Kategorienlehre* of the Stoics, for which see E. Zeller, *Die Philosophie der Griechen*, vol. III, pt. I, 5th ed. (*curavit* E. Wellman Leipzig, 1923; photoreprinted Hilesheim, 1963), pp. 93-105. Thus the question *is-it*? *what-is-it*? *why*? come straight out of *Posterior Analytics*, II i: while the Stoic doctrine is entirely different, with no place for *what*? *where*? *when*? *how much*?

Display 2

THE CORRESPONDENCE BETWEEN THE ARISTOTELIAN CATEGORIES: THE PORPHYREAN PREDICABLES AND AVICENNA'S QUESTIONS

Category/ Predicable	*Greek Names*	*Arabic Name*	*Question at Issue*	*Avicenna's Question*
1. substance	*όυσία*	*al-jawhar*	what thing?	(1i), (1ii)*
2. quantity	*ποσόν*	*al-kammiyah*	how much?	(8)
3. quality	*ποιόν*	*ayya*	what sort?	(3)
i. genus	*γένος*	*jins*		
ii. species	*'είδος*	*naw*[c]		
iii. difference	*διαφορά*	*fasl*		
iv. essential qualities	*ίδιον*	*dhât*	what nature?	(2i)
—definitive	*όρος*	*hadd*		
—descriptive		*rasm*		
v. accidents	*συμ βεβηκοτα*	*kayfa*	how functioning?	(5)
4. relation	*πρός τι*	*al-idâfah*	how related?	
7. posture	*κέίσθαι*	*wad*[c]	in what attitude?	
8. possession	*'εχειν*	*lahu*	with what ? accompaniments?	
9. action	*ποιέίν*	*an yaf*[c]*al*	what doing?	
10. passion	*πασχειν*	*an yuf*[c]*al*	what undergoing?	
5. place	*πόυ*	*ayna*	where?	(6)
6. time	*πότε*	*matâ*	when	(7)
(11. cause**	*'αιτία*	*al-sabab*	why?	(4))

Notes:

* Roughly, the two parts of this question ask regarding the primary and secondary substance at issue, respectively.

** Regarding this 11th entry see the discussion in the text.

2. It highlights the prominence of the rubic of causation as a category which the important role of *why*? questions endows with special significance.[9]

The introduction of *why*? questions has nothing to do with the elementary ontological analysis of the *Categories* but is required by the deeper analysis of *science*: to know a thing is to know its causes. Avicenna (in discussing demonstrative syllogism) *begins* with the analysis of questions in *Posterior Analytics*, II, *i* and fills it out by further use of the Categories. One must regard the *what-sort*? question as somehow derived from Aristotle's question *hoti*: "that it is the case."[10] For then the four Basic Questions coincide exactly with Aristotle's four "subjects for inquiry" [*loc. cit.*]. The process of their transmission can be traced through the Alexandrian Aristotelians to the Arabs in considerable detail.

Was the passage from categories (plus predicables) to question a transitional step which originated with the Arabs, rather than one which had already been taken earlier, with the Greeks? It is difficult to answer this question with absolute assurance, but there is circumstantial evidence to suspect that the answer is negative. For it is a well-established fact that most of the departures made by Avicenna in logic from orthodox Aristotelian positions trace back to ultimately Stoic sources. And it is clear that the Stoic logicians interested themselves in the logic of questions. For example Diogenes Laertius reports in his register of the logical works on Chrysippus (280-209 B. C.) that this important Stoic logician wrote an entire series of treatises on the logical theory of questions.[11] A meager modicum of in-

9 The theory of questions has an intimate relationship with the theory of demonstration, which deals with the establishment of answers. (Note that Avicenna's treatment of questions falls into the section on demonstration.) Regarding the kinship (particularly with respect to *why*? questions) see M. E. Marmura, "Ghazali and Demonstrative Science," *Journal of the History of Philosophy*, vol. 3 (1965), pp. 183-204 (see especially pp. 190-191).

10 Cf. A. M. Goichon, *op. cit.*, p. 236, n.2.

11 *Peri erôtêseôs* ("On Questions": 2 books), *Peri peuseôs* ("On Queries", 4 books), *Epitomê peri erôtêseôs kai peuseôs* ("Epitome on Questions and Queries"), *Peri apolriseôs* ("On Answers": 4 books), *Epitomê peri apokriseôs* ("Epitome on Answers"). See Diogenes Laertius, *Lives of the Eminent Philosophers*, VII: 191 (ed. D. H. Hicks in the Loeb seriaes, Vol. II, p. 300). The idea is at work here that a *question* (*erôtêma*) can be answered yes or no (e.g., "Is today Monday?"); a *query*

formation about this Stoic theory of questions is provided in sources available to us,[12] but this is unfortunately wholly insufficient to throw any light on the conjecture under discussion. None the less, it seems likely, all considered , that Avicenna's treatment of the logic of questions is (ultimately) indebted to the Stoic discussions on the subject. To be sure, the reference to "description" is the only point in Avicenna's classification of questions which, taken in isolation, is clearly Stoic and post-Aristotelian. But the tactic of realigning categorial ideas around the organizing theme of questions has the earmarks of a Stoic innovation.[13]

(*pysma*) is an interrogation that cannot be so answered (e.g., "What day is it?"). (*ibid*., VII: 66) Compare B. Mates, *Stoic Logic* (Berkeley, 1953), pp. 18-19.

[12] See C. Praml, *Geschichte der Logik im Abendlande*, vol. I (Leipzig, 1855; photoreprinted Graz, 1955), p. 441, n. 115.

[13] This chapter was originally published in the *Archiv für Geschichte der Philosophie*, vol. 49 (1967), pp. 1-6. I am grateful to Neil A. Gallagher and Charles H. Kahn for helpful suggestions and constructive criticisms.

Chapter 6

THE ARABIC THEORY OF TEMPORAL MODAL SYLLOGISTIC

1. BACKGROUND REGARDING THE TREATISE AND ITS AUTHOR

Arabic manuscript codex OR 12405 of the British Museum contains a logical treatise entitled *Sharh Al-takmīl fī ʻl-mantiq*, whose author is one Muhammad ibn Fayd Allā ibn Muhammad Amīn al-Sharwānī.[1] Nothing further is independently known about him,[2] apart from what can be gleaned from this manuscript itself. The codex contains two treatises by this scholar, written in the author's own hand, in a somewhat cramped naskhī of twenty-three lines per folio. In addition to the text at issue (in folios 72-104), it contains also (in folios 1-70) his contemporary on the well-known tract *Al-Hāshiyah* (or Al-Risālah) *al-sughrā fi ʻl-mantiq* or ʻAlī ibn Muhammad al-Jurjāni al-Sayyid al-Sharīf (A. D. 1340-1413).[3] Al-Sharwānī is thus a late medieval Persian scholar of presumably the early fifteenth century who must be considered obscure in view of his nearly total absence from the manuscript tradition. One item of biographical

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1 During the academic year 1967/68, the senior author of this paper [N .R.] spent a sabbatical term in England with the support of a grant-in-aid from the American Philosophical Society to examine the Arabic logical manuscripts of several libraries, the British Museum in particular. This occasioned contact with the manuscript now at issue, and the assistance of the American Philosophical Society is herewith gratefully acknowledged. This chapter was written with the assistance of Arnold vander Nat in checking various trabulations. Also thanks are due to Mr. Zakaria Bashier for his help in translating and interpreting several passages of Sharwānī's text.

2 He is nowhere mentioned in Brockelmann's *Geschichte der Arabischen Litteratur*.

3 For this logician see N. Rescher, *The Development of Arabic Logic* (Pittsburgh, 1964), pp. 222- 23.

information which can be gleaned from our text is that the author is the great-grandson of Al-Sadr al-Sharwānī Muhammad Sādi ibn Fayd Allāh ibn Muhammad Amīn, also otherwise unknown.

Al-Sharwānī’s treatise is of some interest because it enable us to confirm and extend in significant ways our information regarding the Arabic theory of temporal modal syllogistic as available from other sources.[4] This paper seeks to present the new light this source affords for our knowledge of Arabic logic. On the information now available it would seem that the theory of temporal modalities represents the most significant addition made by the medieval Arabic logicians to the body of logical material that they received from the Greeks. The entire subject has only begun to be studied in recent years, and until many more texts have been studied and analyzed our conclusions must remain provisional and tentative. Much work remains to be done before our feet can be set on firm ground. But already at this early juncture a claim can be entered with considerable assurance regarding the value and interest of such further work.

2. BASIC ELEMENTS OF THE ARABIC THEORY OF TEMPORAL MODALITIES: THE SIMPLE MODES

The theory of temporal modal syllogisms as presented in Arabic logical texts further qualifies the relation that the predicate bears to the subject in the four basic categorical propositions A (“All A is B”), E (“No A is B”), I (“Some A is B”), and 0 (“Some A is not B”) in certain characteristic ways.

For *simple* modal propositions two qualifications are added:

(1) a *modality* of one of the following four types:

i. (□): of necessity

ii. (◊): by a possibility

[4] N. Rescher, *Temporal Modalities in Arabic Logic, Foundations if Language,* Supplementary Series, no. 2 (Dordrecht, 1966) is the basic publication, although the data presumed there are extended and amplified in chapters 7-8 of *id., Studies in Arabic Philosophy* (Pittsburgh, 1967). The materials with which the present paper deals make it possible not only to extend but also in important ways to correct the presentation of the theory given in these earlier discussions.

iii. (∀): in perpetuity

iv. (∃): in (some) actuality, or, sometimes

(2) a *temporality* qualifying the modality of one of the following four types:

i. (E): when the subject at issue exists; that is, during times of the existence of the subject.

ii. (C): when the subject at issue exists and meets a certain condition as specified by the subject term of the proposition; that is, during times of the existence of the subject when it meets the condition stipulated by the subject term.

iii. (T): when the subject at issue exists during a definite, specifiable time; that is, during a certain *specified* and determinate period of the existence of the subject (e;g., its youth).

iv. (S): when the subject at issue exists during some indefinite, unspecifiable time; that is, during some *unspecified* and indeterminateperiod of the existence of the subject.

Note that the temporalities (T) and (S), being inherently time-restricted, do not allow of further qualification by the specifically temporal modalities ∀ and ∃.

In "order of strength" the four modalities are arranged as □, ∀, ∃, ◊, and may be termed necessity, perpetuity, actuality, and possibility, respectively. The order of strength of the temporalities E, C, T, S depends on their combination with modality. (Concerning these relative strengths see section VI below.) The four temporalities may be called the existential, the conditional, the temporal, and the spread temporality, respectively. Examples of categorical propositions displaying these temporalities are:

(E) All men are animals, as long as they exist.

(C) All writers move their fingers, as long as they write.

Table 1

STANDARD EXAMPLES OF THE SIMPLE MODES

(□E): All men are rational of necessity (as long as they exist).[5]

(□C): All writers move their fingers of necessity as long as they write.

(□T): The moon is eclipsed of necessity at the time when the earth is between it and the sun

(□S): All men breathe of necessity at some times.

(∀∃): All men are rational perpetually (as long as they exist).

(∀C): All writers move as long as they write.

(∃C): All writers move while they are writing.

(T): All writers move at the time they are writing.

(S): All men breathe at certain times.

(∃E): All men breathe (at some times).

(◊C): All writers move with a possibility while they are writing.

(◊T): The moon is eclipsed with a possibility at the time when the earth is between it and the sun.

(◊S): All men breathe with a possibility at all times.

(◊E): All writers move with a possibility (at some time).

(T) All moons are eclipsed at the time when the earth is between it and the sun.

[5] The existence condition is usually unstated.

(S) All men breathe at some times.

Table 2

SIMPLE MODES IN SHARWĀNĪ

TYPE	NAME
1. *Modes of Necessity*	
$\Box$E	absolute necessary
$\Box$C	general conditional
$\Box$T	absolute temporal (#)
$\Box$S	absolute spread (#)
2. *Modes of Perpetuality*	
$\forall$E	absolute perpetual
$\forall$C	general conventions
3. *Modes of Actuality*	
$\exists$E	general absolute
$\exists$C	absolute temporary or absolute continuing (#)
T	temporal absolute (*)
S	spread absolute)
4. *Modes of Possibility*	
$\Diamond$E	general possible
$\Diamond$C	possible continuing (#)
$\Diamond$T	spread possible (*)
$\Diamond$S	spread possible or perpetual possible (*)

NOTE: An asterisk (*) marks those modes missing in Qazwīnī and (#) marks those which are recognized by Qazwīnī but not listed or discussed by him.

In these examples modality has, for simplicity, been put aside.

The (simple) modal propositions arrived at by the full-scale use of this machinery are as shown in Table 1.

By ringing the changes on the two factors of modality and temporality, fourteen theoretical combinations arise. Six of these, (□E), (□C), (∀∃), (∀C), (∃E) and (◊E) are explicitly listed and discussed by al-Qazwīnī [ca. 1220-80], and he refers also to (□T), (□S), (3C), and (◊C), though not giving them an explicit place in his inventory.[6] Al-Sharwānī explicitly recognizes all fourteen, and his presentation of them is summarized in Table 2. The more detailed analysis of these modal propositions is deferred until section VI below.

In an earlier publication,[7] the analysis of the Arabic temporal modalities was based on the *Risālah al-shamsiyyah*, the *Sup Epistle* of al-Qazwīnī al-Kātibī.[8] AI-Qazwīnī's treatment differs from that of al-Sharwānī in that in Qazwīnī the temporalities (S) and (T) never occur with simple but *only* with compound propositions and in that Qazwīnī's analysis does not include the mode (∃C). Presumably, Qazwīnī assimilated these simple modes under the temporality condition (E). (So with Ibn al-Assāl [ca. 1190-12501 who appears to assimilate the weak modes (◊C) and (◊C) and (◊T) with (∃E) and (◊E), interchanging ◊ and ◊ on the one hand and ∀ and ∃ on the other).[9] Apart from this difference, there is complete agreement between Qazwīnī and Sharwānī regarding the nature and nomenclature of simple modes. Sharwānīs treatment in effect extends Qazwīnī s on the side of temporal conditionalization.[10]

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6 It would seem that the six modes are considered by al-Qazwīnī to be the *standard* modes—modes, as he says, "into which it is usual to inquire."

7 Rescher, *Temporal Modalities in Arabic Logic* (Dordrecht: Basil, 1972).

8 For this writer see *The Development of Arabic Logic*, pp. 203-204. Appendix I of Aloys Sprenger's *Dictionary if the Technical Terms Used in the Sciences of the Musulmans*, 2 (Calcutta. 1862), gives a text edition of this treatise, as well as an English translation of its nonmodal parts. (The latter are translated in *Temporal Modalities in Arabic Logic)*

9 For Assāl's account of modal propositions see Rescher, *Studies in Arabic Philosophy*.

10 Sharwānī seems to be following Qazwīnī's text quite closely. Besides the occurrence or amazing textual similarities between Sharwānī and Qazwīnī,

3. NEGATION AND CONVERSION FOR SIMPLE MODALITIES

The rule of negation for simple modal propositions is as follows. Let the initial proposition to be negated take the form

(modality/temporality) P

Then its contradictory takes the form

(O-modality/temporality) ~P

Here the O-modality is the *modal opposite* of the initial modality (formed by interchanging $\Box$ and $\Diamond$ on the one hand and $\forall$ and $\exists$ on the other). Moreover, the initial categorical proposition P is replaced by its *contradictory* ~P, and the temporality remains unchanged.

It must be noted at this juncture that in analyzing the modal propositions of Sharwānī we are dealing with *modes of predication* (modality *de re*) rather than with strictly propositional modes (modality *de dicta*) : the issue is one of qualifying the relation of the predicate to the subject rather than qualifying an entire categorical proposition. For example, the modal proposition

(1) ($\forall\exists$) (All men are animals)

is to be understood as

(2) All men are always animals

rather than as

(3) It is always true that all men are animals

The different between (2) and (3) becomes more striking when we consider the modal proposition

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Sharwānī explicitly refers to *The Sun Epistle* in his discussion of fourth figure moods.

(4) (~∀∃) (All men are animals)

Table 3

CONTRADICTORIES OF SIMPLE MODES

Original Proposition		Contradictory	
(□E)P	absolute necessary	(◊E)~P	general possible
(∀E)P	absolute perpetual	(∃E)~P	general possible
(∃E)P	general absolute	(∀E)~P	absolute perpetual
(◊E)P	general possible	(□E)~P	absolute necessary
(□C)P	general conditional	(◊C)~P	possible continuing
(∀C)P	general conventional	(∃C)~P	absolute continuing
(∃C)P	absolute continuing	(∀C)~P	general conventional
(◊C)P	possible continuing	(□C)~P	general conditional
(□T)P	absolute temporal	(◊T)~P	temporal possible
(T)P	temporal absolute	(T)~P	temporal absolute
(◊T)P	temporal possible	(□T)~P	absolute temporal
(□S)P	absolute spread	(◊S)~P	perpetual (spread) possible
(S)P	spread absolute	(∃S)~P	absolute perpetual[11]
(◊S)P	perpetual (spread) possible	(□S)~P	absolute spread

If we view the qualifying mode here as operating on the categorical proposition "All men are animals" then (4) becomes

(5) It is sometimes true that some men are not animals where as Sharwānī would have (4) be understood as

[11] Note that the *logical structure* of the spread absolute and the general absolute appears to be the same. The difference between these two modes seems .only to be that the spread absolute has connotations of *spreading* the attribution of the predicate, as in, for example, "all men breathe"—whereas the general absolute does not. On this matter see text section 6.

(6) All men are not always (i.e., sometimes not) animals.

Thus, for example, with regard to the *absolute necessary* (□∃) proposition and its contradictory, the *general possible* (◊∃), the following situation obtains:

Original Proposition	*Contradictory*
(□∃)(All A are B) = All A are (□∃)B	Some A are (◊∃) not B
(□∃)(No A are B) = All A are (□∃) not B	Some A are (◊∃) B
(□∃)(Some A are B) = Some A are (□∃) B	All A are (◊∃) not B
(□∃)(Some A are not B) = Some A are (□∃) not B	All A are (◊∃) B

The results of applying the negation principles are set out in Table 3.

The situation regarding conversion is more complex. The converse of a modal proposition (X)P is a modal proposition (Y)P° such that

(1) P° is a categorical converse (possibly by limitation) of P

(2) (Y)P° is the *strongest* modal proposition implied by (X)P.

The tables given below will indicate the relative strengths of modal propositions.

There seems to be no set procedure for obtaining converses other than the process of demonstration, either by *reductio* or by *supposition.* The results of such conversion demonstrations for the simple modal propositions are listed along with the results for compound modal propositions in Table 6 below. We illustrate the conversion procedure with the following examples.

1. (◊E) (All A is B) converts to (∃C) (Some B is A). Suppose not. Then, (∀*C*) (All B is not A), and this, together with the original, yields (∀*E*) (All A is not A). (For this first figure syllogism see Table 8 below.) But this conclusion is a contradiction.

2. (∃E) (All A is B) converts to (∃E) (Some B is A). Let us suppose that *x* is A, then *x* is B at some time, and hence some B is A at some time.

(For a more explicit analysis of this argument, cf. section VI below.)

4. THE COMPOUND MODES

Compound modes are formed from simple modes a *restriction.* This restriction can take only one of the two forms:

1. (~∀E): with non-perpetuity

2. (~□E): with non-necessity

However, the second form of restriction occurs only as a qualification of basic propositions whose temporality is existential, (E).

Taken by themselves, these restrictions qualify the relation of the predicate to the subject in exactly the same way as do the simple modes. For example:

(~∀E) (All A is B) ≡ All A is non-perpetually B

(~□E)(All A is B) ≡ All A is non-necessarily B

Moreover, letting P' be the *contrary* of P, we have the general equivalences:

(~∀E)*p* ≡ (∃E)P'

(~□E)P ≡ (◊E)P'

Note thus the difference between (~∀E)*P* and", (∀E)P, and between (~□E)P and ~(□E)P. (Our previous exegeses err in suggesting that it is the contradictory rather than the contrary that is at issue.)

The compound modes of categorical propositions are formed by qualifying the relation of the predicate to the subject by a simple mode-cum--restriction, so that in a compound mode there is a *twofold qualification* of the predication relation.

Thus, given a simple mode (X), we are to understand compound modal propositions as follows

(X & ~∀*E*) (All A is B) ≡ All A is (X)B, and they are not-perpetually B

≡ All A is (X)B, and they are sometimes not B
(X & ~□E)(All A is not B) ≡ All A is (X) not B, and they are not necessarily not B
≡ All A is (X) not B, and they are possibly B

The situation regarding the other categorical forms that are not displayed here is entirely analogous. Thus, for example, the general absolute (∃E) can be compounded into the non-perpetual existential (∃E & ~∀E)or the non-necessary existential (∃E & ~□E):

(∃E & ~∀E)(Some A is B) == Some A is sometimes B, and they are not always B
== Some A is sometimes B, and they are\ sometimes not B

(∃E & ~□E) (Some A is B)== Some A is sometimes B, and they are not necessarily B
== Some A is sometimes B, and they are possibly not B

Some further examples *of* compound modal propositions are:

(□C & ~∀E): all writers move of necessity as long as they write, but not perpetually.

(□T & ~ AE): All moons are of necessity not eclipsed at the time of the quarter mood, but not perpetually.

(◊E & ~□E): With a special possibility , all fires are cold.

Note here that an affirmative (negative) compound modal proposition is composed of an affirmative (negative) simple modal proposition and a negative (affirmative) general absolute or general possible.

The compound modes presented by Al-Sharwānī are set forth in Table 4. It does not appear that he considered this list to be exhaustive of all the obtainable compound modes. His considerations seem to have centered around those compound modes which were needed for conversion and-above all—*for* first syllogisms. (See Table 8 below.) For example, those compounds *of* possibility which are conspicuously absent in Table 4 are presumably missing because they *invariably* yield nonproductive syllogisms in the first figure.[12]

[12] In tact, when Sharwānī introduces the compound modes he specifically says that there are (only) *eight of them*, which he goes on to discuss, namely, (□C & ~ ∀E), (∀C & ~∀E), (□T & ~∀E), (□S & ~∀E), (∃C & ~∀E), (∃E & ~∀E), (∃E & ~□E), and (□E & ~□E). Later in the text, however, he mentions four additional modes.

Table 4

COMPOUND MODES IN SHARWĀNĪ

TYPE	NAME
1. Modes of Necessity	
$\Box E \mathbin{\&} {\sim}\forall\exists$	non-perpetual necessary (*) (impossible combination)
$\Box C \mathbin{\&} {\sim}\forall\exists$	special conditional
$\Box T \mathbin{\&} {\sim}\forall\exists$	temporal
$\Box S \mathbin{\&} {\sim}\forall\exists$	spread
2. Modes of Perpetuality	
$\forall E \mathbin{\&} {\sim}\forall\exists$	non-perpetual perpetual (*) (impossible combination)
$\forall C \mathbin{\&} {\sim}\forall\exists$	special conventions
3. Modes of Actuality	
$\exists E \mathbin{\&} {\sim}\forall\exists$	non-perpetual existential
$\exists C \mathbin{\&} {\sim}\Box\exists$	non-necessary existential
$\exists C \mathbin{\&} {\sim}\forall\exists$	non-perpetual continuing (*)
$T \mathbin{\&} {\sim}\forall\exists$	non-perpetual temporal absolute (*)
$S \mathbin{\&} {\sim}\forall\exists$	non-perpetual spread absolute (*)
4. Modes of Possibility	
$\Diamond E \mathbin{\&} {\sim}\Box\exists$	special possible[13]
$\Diamond E \mathbin{\&} {\sim}\forall\exists$	non-necessary existential[14]

(*) Missing in Qazwīnī

The matter seems to resolve itself if we consider the eight modes in question to be the *standard* modes "into which it is usual to inquire."

[13] Concerning this classification of modes, it should be mentioned that neither Qazwīnī nor Sharwānī presents a classification of modes as such. Rather, they refer to modes as follows. Propositions are either *actuals* or *possibles.* The actuals consist of the perpetuals, $\Box E$, $\forall E$; the conditionals, $\Box C$, $\Box C \mathbin{\&} {\sim}\forall E$; the conventionals, $\forall C$, $\forall C \mathbin{\&} {\sim}\forall E$; the continuing propositions, $\exists C$, $\exists C \mathbin{\&} {\sim}\forall E$; the temporals, $\Box T$, T, $\Box S$, S, $\Box T \mathbin{\&} {\sim}\forall E$, $\Box S \mathbin{\&} {\sim}\forall E$' the existentials, $\exists E \mathbin{\&} {\sim}\forall E$, $\exists E \mathbin{\&} {\sim}\Box E$; and the general absolute, $\exists E$. The possibles are $\Diamond C$, $\Diamond T$, .$\Diamond S$, $\Diamond E$, and $\Box E \mathbin{\&} {\sim}\Box E$.

[14] Note here that $(\Diamond E \mathbin{\&} {\sim}\Box E)P$-$(\Diamond E \mathbin{\&} {\sim}\Box E)\ P'$, where P' is the contrary of P. Thus it is described in the text as "composed of two general possibles, one negative, the other positive."

The negation of a compound mode follows the negation of each of its component modes in the following way:

> Given an affirmative (negative) compound modal proposition, which is an affirmative (negative) simple modal proposition *conjoined* with a negative (affirmative) general absolute, or a negative (affirmative) general possible-its negation is the negation of the simple modal proposition *disjoined* with an affirmative (negative) absolute perpetual, or an affirmative (negative) absolute necessary.

For example,

$\sim(\forall C \ \& \sim\forall E)$ all A is B) = some A is not B while they are A, or they are perpetually B

$\sim(\Box S \ \& \ \forall E)$ (All A is B) = some A is not Be possibly at all times, or they are perpetually B

Let us introduce some notation which will enable us to describe adequately the negation process for compound modes. Given a categorcal proposition P let us define P*, the pronominalization of P, as follows:

P	P*
All A is B	*they* (i.e., the A’s at issue) are B
All A is not B	*they* (i.e., the A’s at issue) are not B
Some A is B	*they* (i.e., the A’s at issue) are B
Some A is not B	*they* (i.e., the A’s at issue) are not B

We can now represent a compound mode $(X \ \& \sim\forall E)$, or $(X \ \& \sim\Box E)$ as follows:

$$(X \ \& \sim\forall E)P \equiv (X)P \ \& \ (\sim\forall E)P^*$$

$$(X \ \& \sim\Box E)P \equiv (X)P \ \& \ (\sim\Box E)P^*$$

Table 5

CONTRADICTORIES OF COMPOUND MODES

Compound Original	Contradictory
(□E) & ~∀E)P	(◊E) ~Pv(∀E)P*
(□C) & ~∀E)P	(◊C) ~Pv(∀E)P*
(□T) & ~∀E)P	(◊T) ~Pv(∀E)P*
(□S) & ~∀E)P	(◊S) ~Pv(∀E)P*
(∀E & ~∀E)P	(∃E) ~Pv(∀E)P*
(∀C & ~∀E)P	(∃C) ~Pv(∀E)P*
(∃C & ~∀E)P	(∀C) ~Pv(∀E)P*
(T & ~∀E)P	(T) ~Pv(∀E)P*
(S & ~∀E)P	(∀E) ~Pv(∀E)P*
(∃E & ~∀E)P	(∀C) ~Pv(∀E)P*
(∃E & ~□E)P	(∀E) ~Pv(□E)P*
(◊E) & ~∀E)P	(□E) ~Pv(∀E)P*
(◊E) & ~□E)P	(□E) ~Pv(□E)P*

and negation can be described as in Table 5.

The conversion process for compound modal propositions is essentially analogous to that for the simple modes. Given a compound mode (X & ~Y)P, its converse, if there is one, is a proposition (Z)P°, where (Z) can be either simple or compound, such that

1. P is the categorical converse of P

2. (Z)P° is the strongest modal proposition such that (X& ~Y)P implies (Z)P°

(For relative strengths of modal propositions, see section VI below.) Again, as for simple modes, the procedure for determining conversion is that of *demonstration.* Thus, for example, the special conventional converts to the non perpetual absolute continuing, in the universal affirmative case:

> (∀C & AE) (All A is B) converts to (∃C & ~∀E) (Some B is A). Otherwise, (∀C) (All B is not A) or (∀E) (they are A); so that (∀C) (All B is not A) or (∀E) (All B is A). But the first disjunct together with the original yields (∀C) (All A is not A), which is absurd, and the second disjunct together with the original simple mode yields (All B is B), which, with the original restriction (∃E) (All B is not B), in turn yields a pair of contradictories, (For these first figure syllogisms see Table 8 below.)

In Qazwīnī and Sharwānī there are three references to the "non-perpetual-about-some conventional": by Qazwīnī in *The Sun Epistle* †67/65 and †72/70, and by Sharwānī in the present text in Table 11 A (below) for the sixth mood (AEE) of the fourth figure.

In †67/65 Qazwīnī says that the universal negative general conditional and general conventional convert to the universal negative general conventional, and that the universal negative special conditional and special conventional convert to the *non-perpetual-about-some conventional.*

> The reason of this process in reference to the general conventional is that it is an *adherent* of both kinds of general propositions. The reason why the converted proposition is non-perpetual-about-some is because [if] it is not true that some B is with a general absolute C, [then] it is true by perpetuity that no B is C, and thus [this is] converted into perpetually no C is B. But the original proposition was [that no C is B as long as it is C but not perpetually, and so] that every C is B.

Now, the (partial) converse of (∀C & ~∀E) (No C is B) that *adheres* to the generals is (∀C) (No B is C). Also, (∀E) (No C is B), which is All C is sometimes B, converts to Some B is sometimes C, which is (~∀E) (Some B is not C). Thus the converse of (∀C & ~∀E) (All C is not B) is (∀C) (All B is not C) & (~∀E) (Some B is not C); and the latter is aptly called "the non-perpetual about-some conventional." We note, thus, that (1) *only a universal negative* special proposition converts to a non-perpetual-about-some conventional; and (2) the latter is a sort of halfway house between the universal negative specials and the particular negative specials—yet it is neither of them.

Table 6

CONVERSION OF MODAL PROPOSITIONS ACCORDING TO CATEGORICAL FORM

		A,I	E	O
	Original	Converse	Converse	Converse
	$(\Box E)P$	$(\exists C)P^\circ$	$(\forall E)P^\circ_d$	———
	$(\forall E)P$	$(\exists C)P^\circ$	$(\forall E)P^\circ_d$	———
	$(\Box C)P$	$(\exists C)P^\circ$	$(\forall E)P^\circ_d$	———
	$(\forall C)P$	$(\exists C)P^\circ$	$(\forall E)P^\circ_d$	———
(c)	$(\exists C)P$	$(\exists C)P^\circ$	———	———
(c)	$(\Box T)P$	$(\exists E)P^\circ$	———	———
(c)	$(\Box S)P$	$(\exists E)P^\circ$	———	———
(c)	$(T)P$	$(\exists E)P^\circ$	———	———
(c)	$(S)P$	$(\exists E)P^\circ$	———	———
(c)	$(\exists E)P$	$(\exists E)P^\circ$	———	———
	$(\Diamond X)P$	——(*)	———	———
	$(\Box E\ \&\ {\sim}\forall E)P$	$(\exists C\ \&\ {\sim}\forall E)P^\circ$	$(\forall E)P^\circ_1$(a)	———
	$(\forall E\ \&\ {\sim}\forall E)P$	$(\exists C\ \&\ {\sim}\forall E)P^\circ$	$(\forall E)P^\circ_1$	———
	$(\Box C\ \&\ {\sim}\forall E)P$	$(\exists C\ \&\ {\sim}\forall E)P^\circ$	$(\forall E)P^\circ_a$(#)	———
	$(\forall C\ \&\ {\sim}\forall E)P$	$(\exists C\ \&\ {\sim}\forall E)P^\circ$	$(\forall E)P^\circ_1$	———
(c)	$(\exists C\&\ {\sim}\forall E)P$	$(\exists E)P^\circ$	———	———
	$(\Box T\&\ {\sim}\forall E)P$	$(\exists E)P^\circ$	———	———
	$(\Box S\&\ {\sim}\forall E)P$	$(\exists E)P^\circ$	———	———
(c)	$(T\&\ {\sim}\forall E)P$	$(\exists E)P^\circ$	———	———
(c)	$(S\&\ {\sim}\forall E)P$	$(\exists E)P^\circ$	———	———
	$(\exists E\&\ {\sim}\forall E)P$	$(\exists E)P^\circ$	———	———
	$(\exists E\ \&\ {\sim}\Box E)P$	$(\exists E)P^\circ$	———	———
	$(\Diamond E\ \&\ {\sim}X)P$	——	———	———

This interpretation is further verified by the fact that Sharwānī has the non-perpetual-about-some conventional as a conclusion for the mood AEE-4, where the syllogism does not yield a particular special proposition.

The conversion results for both simple and compound modes is given in Table 6. These results are not given by Sharwānī, but are taken from

Qazwīnī, and Qazwīnī's account is supplemented by our own calculations. (In Table 6 we represent the *direct* converse of a universal negative proposition P, i.e., the converse *not* by limitation, as $P°_d$ and the converse of P by limitation as $P°_j$)

6. THE LOGICAL ANALYSIS OF MODAL PROPOSITIONS

We shall now attempt an analysis of the modal propositions considered thus far in terms of present-day symbolic notation. R_t is the basic operator for realization-at-time-t.[15] We shall make use of the following abbreviations:

$TQx = R_t(Qx)$	$\Box TQx = \Box R_t(Qx)$	$\Diamond TQx = \Diamond R_t(Qx)$
$SQx = R_s(Qx)$	$\Box SQx = \Box R_s(Qx)$	$\Diamond SQx = \Diamond R_s(Qx)$
$\exists Qx = (\exists t)R_t(Qx)$	$\exists\Box Qx = (\exists t)\Box R_t(Qx)$	$\exists\Diamond Qx = (\exists t)\Diamond R_t(Qx)$
$\forall Qx = (\forall t)R_t(Qx)$	$\forall\Box Qx = (\forall t)\Box R_t(Qx)$	$\forall\Diamond Qx = (\forall t)\Diamond R_t(Qx)$

Table 7 and 8 depict the overall results. In our symbolizations of modal propositions, *we shall systematically suppress the temporality condition* (E) relating to the existence of the subject. Concerning the symbolic rendition *of* modes, we take notice *of* only the following points. First, in adopting the symbolic machinery that we have, we assume here that all the usual quantificational and modal principles hold. Secondly, in the E-modes the existence condition has been suppressed; fully stated, ($\Box$E) (All A is B), for example, would be $(\forall x)[(\exists t)R_tAx \supset (\forall t)\Box R_t(Ex \supset Bx)]$. Thirdly, T and S modes are special time-instantiations, with regard to the existence *of* the subject, and accordingly, we use "T" and "S" as a time-constant.

Since the texts have very little to say about the implicational relations among modes, we must rely heavily on our symbolic interpretation *of* the modes to be able to say what relations hold. There is, in particular, a question concerning the relationship between the T-modes and the C-modes. As an example *of* the temporal absolute, (T), Sharwānī gives "All writers move at the time they are writing." Comparing this with the continuing absolute, ($\exists$C), "All writers move while they are writing," we would conclude that Sharwānī holds that $\exists C \rightarrow T$. Table 7, then, presents

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15 Note that $\Diamond E \;\&\; {\sim}\forall E$ is equivalent with $\exists E \;\&\; {\sim}\Box E$ and is thus the nonnecessary existential all over again.

Table 7

SIMPLE MODES OF THE A PROPOSITION (ALL A IS B)

Type	Example	Name
(□E)	(∀x)[∃Ax ⊃ ∀□Bx]	absolute necessary
(□C)	(∀x)[∃Ax ⊃ ∀□(Ax ⊃Bx]	general conditional
(□T)	(∀x)[∃Ax ⊃ □TBx]	absolute temporal
(□S)	(∀x)[∃Ax ⊃ □SBx]	absolute spread
(∀E)	(∀x)[∃Ax ⊃ ∀Bx]	absolute perpetual
(∀C)	(∀x)[∃Ax ⊃ ∀(Ax ⊃Bx]	general conventional
(∃C)	(∀x)[∃Ax ⊃ ∃(Ax &Bx]	absolute continuing
(T)	(∀x)[∃Ax ⊃ TBx]	temporal absolute
(S)	(∀x)[∃Ax ⊃ TBx]	spread absolute
(∃E)	(∀x)[∃Ax ⊃ ∃Bx]	general absolute
(◊C)	(∀x)[∃Ax ⊃ ∃◊(Ax &Bx]	possible continuing
(◊T)	(∀x)[∃Ax ⊃ ∃◊TBx	temporal possible
(◊E)	(∀x)[∃Ax ⊃ ∃◊Bx	general possible
(◊S)	(∀x)[∃Ax ⊃ ◊SBx]	perpetual possible

the implicational relations among modes as we have calculated them. Note that the compound modes (X & ~ AE) and (X & ~□E) both imply the simple mode (X). Table 9 depicts the situation here.

Table 8

COMPOUND MODES

$(\Box E \,\&\, {\sim}\forall E)$	$(\forall x)\{\exists Ax \supset [\forall\Box Bx \,\&\, {\sim}\forall Bx]\}$	non-perpetual necessary
$(\Box C \,\&\, {\sim}\forall E)$	$(\forall x)\{\exists Ax \supset [\forall\Box(Ax \supset Bx) \,\&\, {\sim}\forall Bx]\}$	special condition
$(\Box T \,\&\, {\sim}\forall E)$	$(\forall x)\{\exists Ax \supset [\Box TBx \,\&\, {\sim}\forall Bx]\}$	temporal
$(\Box S \,\&\, {\sim}\forall E)$	$(\forall x)\{\exists Ax \supset [\Box SBx \,\&\, {\sim}\forall Bx]\}$	spread
$(\forall E \,\&\, {\sim}\forall E)$	$(\forall x)\{\exists Ax \supset [\forall Bx \,\&\, {\sim}\forall Bx]\}$	non-perpetual perpetual
$(\forall C \,\&\, {\sim}\forall E)$	$(\forall x)\{\exists Ax \supset [\forall(Ax \supset Bx) \,\&\, {\sim}\forall Bx]\}$	special conventional
$(\exists C \,\&\, {\sim}\forall E)$	$(\forall x)\{\exists Ax \supset [\exists(Ax \,\&\, Bx) \,\&\, {\sim}\forall Bx]\}$	non-perpetual continuing absolute
$(T \,\&\, {\sim}\forall E)$	$(\forall x)\{\exists Ax \supset [TBx \,\&\, {\sim}\forall Bx]\}$	non-perpetual temporal absolute
$(S \,\&\, {\sim}\forall E)$	$(\forall x)\{\exists Ax \supset [SBx \,\&\, {\sim}\forall Bx]\}$	non-perpetual spread absolute
$(\exists E \,\&\, {\sim}\forall E)$	$(\forall x)\{\exists Ax \supset [\exists Bx \,\&\, {\sim}\forall Bx]\}$	non-perpetual existential
$(\exists E \,\&\, {\sim}\Box E)$	$(\forall x)\{\exists Ax \supset [\exists Bx \,\&\, {\sim}\forall\Box Bx]\}$	non-necessary existential
$(\Diamond E \,\&\, {\sim}\Box E)$	$(\forall x)\{\exists Ax \supset [\exists\Diamond Bx \,\&\, {\sim}\forall\Box Bx]\}$	special possible

Table 9

RELATIVE STRENGTHS OF MODAL PROPOSITIONS

Simple Modes

$$
\begin{tikzcd}
\Box E \arrow[r] \arrow[d] & \Box C \arrow[r] \arrow[d] & \exists\Box C(*) \arrow[r, dotted] \arrow[d] & \Box T \arrow[r] \arrow[d] & \Box S \arrow[d] \\
\forall E \arrow[r] \arrow[d] & \forall C \arrow[r] \arrow[d] & \exists C \arrow[r, dotted] \arrow[d] & T \arrow[r] \arrow[d] & \exists E(\sim S) \arrow[d] \\
\Diamond S \arrow[r] & \forall\Diamond C(*) \arrow[r] & \Diamond C \arrow[r, dotted] & \Diamond T \arrow[r] & \Diamond E
\end{tikzcd}
$$

Compound Modes

$$
\begin{tikzcd}
\Box E \,\&\, {\sim}\forall E \arrow[r] \arrow[d] & \Box C \,\&\, {\sim}\forall E \arrow[rr, dotted] \arrow[d] & & \Box T \,\&\, {\sim}\forall E \arrow[r] \arrow[d] & \Box S \,\&\, {\sim}\forall E \arrow[d, "({\sim}S \,\&\, {\sim}\forall E)"] \\
\forall E \,\&\, {\sim}\forall E \arrow[r] & \forall C \,\&\, {\sim}\forall E \arrow[r] & \exists C \,\&\, {\sim}\forall E \arrow[r] & T \,\&\, {\sim}\forall E & \exists E \,\&\, {\sim}\forall E \arrow[d] \\
& & & & \exists E \,\&\, {\sim}\Box E \arrow[d] \\
& & & & E \,\&\, {\sim}\Box E
\end{tikzcd}
$$

When we view modes in the light of the symbolic apparatus just presented, it becomes clear that we may distinguish five additional modes. In section 8 below we will see that these five modes are necessary to describe adequately third figure syllogisms. We noted that the mode (T) is really a time-instantiation with respect to the temporality (E). In analogy we can also have a time-instantiation with respect to the temporality (C), thus giving rise to three new modes:

($\Box$TC) $(\forall x)[(\exists t)R_t AxC\Box R_T(Ax \,\&\, Bx)]$
continuing abolute temporal

(TC) $(\forall x)[(\exists t)R_t AxCR_T(Ax \,\&\, Bx)]$
continuing temporal absolute

Table 10

RELATIVE STRENGTHS OF MODAL PROPOSITIONS

Simple Modes

$$\begin{tikzcd}
\forall\Box E \arrow[r] \arrow[d] & \forall\Box C \arrow[r] \arrow[d] & \Box TC \arrow[r] \arrow[d] & \exists\Box C(*)\ (=SC) \arrow[r, dotted] \arrow[d] & \Box TE \arrow[r] \arrow[d] & \exists\Box E\ (=\Box S) \arrow[d] \\
\forall E \arrow[r] \arrow[d] & \forall C \arrow[r] \arrow[d] & TC \arrow[r] \arrow[d] & \exists C \arrow[r, dotted] \arrow[d] & TE \arrow[r] \arrow[d] & \exists E\ (=S) \arrow[d] \\
\forall\Diamond E\ (=S) \arrow[r] & \forall\Diamond C\ (=\Diamond SC) \arrow[r] & \Diamond TC \arrow[r] & \exists\Diamond C \arrow[r, dotted] & \Diamond TE \arrow[r] & \exists\Diamond E
\end{tikzcd}$$

($\Box$TC) $(\forall x)[(\exists t)R_tAxC\Box R_T(Ax \mathbin{\&} Bx)]$
continuing temporal possible

Also, since the temporality (S) is really the modality ($\exists$) combined with the temporality (E), we can, in analogy with the modes ($\Box$S) and ($\Diamond$S), distinguish the modes:

($\Box$SC) $(\forall x)[(\exists t)R_tAx \supset (Et)\Box R_t(Ax \mathbin{\&} Bx)]$
continuing absolute spread

($\Box$SC) $(\forall x)[(\exists t)R_tAx \supset (At)\Diamond R_t(Ax \mathbin{\&} Bx)]$
continuing perpetual possible

The compound modes that could be constructed out of the new modes are to be construed in analogy with the other compound, for example:

$$(\Box TC \mathbin{\&} \forall E)\ (\forall x)\{(\exists t)R_tAx \supset [\Box R_T(Ax \mathbin{\&} Bx) \mathbin{\&} {\sim}(\forall t)R_tBx]\}$$

To make the relation of the new modes to the other modes dearer. we present in Table 10 the relative strengths of the augmented number of simple modes. We shall in this table explicitly display the modalities ($\forall$)

and (∃) and the temporality (E) that are implicitly present in modes.

7. FIRST FIGURE SYLLOGISMS

In "The Sun Epistle" *Al-Risālah al-shamsiyyah* († 81), al-Qazwīnī lines the productive (i.e., valid) first figure modal syllogisms as follows:

> As to the first figure, its condition regarding modality is the actuality of the minor.[16] The conclusion here is the same as the major, if it [i.e., the major][17] is other than one of the two conditionals and the two conventionals; and otherwise [i.e., if the major *is* one of these four] it is like the minor when it is without the condition of the non-necessary or the non-perpetual, and the necessity which belongs specially [i.e., only] to the minor,[18] if the major is one of the two generals, and adding the non-perpetual to it if it is one of the two specials.

Sharwānī renders the last clause more clearly, and says:

> otherwise, it is like the minor, omitting the non-necessary, the non-perpetual, and the necessity special to it, if it was found in it, adding the nonperpetual of the major, if it was found in it.

Thus. the account for the first figure syllogisms is the following:

(1)The minor premise must he one of the seventeen actuals.[19]

---

[16] For details regarding this operator see N. Rescher and A. Urquhart, *Temporal Logic* (New York and Vienna, 1971).

[17] In the earlier translation of †81, in *Temporal Modalities in Arabic Logic,* the first sentence was erroneously translated as "as to the first figure, its condition [obtains] in relation to the modality operative for the minor."

[18] In the earlier translation of †81, in *Temporal Modalities in Arabic Logic,* the one of the two conditionals and the two conventionals" was interpreted as "if it (i.e., the minor) is other than . . . ." In the light of Sharwānī's account, however, the present interpretation is clearly the correct one.

[19] In that" earlier translation of †81, in *Temporal Modalities in Arabic Logic,* the phrase "and the necessity which belongs specially to the minor" was (erroneously) suppressed as a seeming corruption of the text.

(2) If the major is not one of ($\Box$C), ($\forall$C), ($\Box$C & ~$\forall$E), and ($\forall$C & ~$\forall$E), then the mode of the conclusion is that of the major.

(3) If the major is one of these four, then the major is one of these four, then the mode of the conclusion is like that of the minor except that

(a) the *restriction* of the conclusion is the same as the restriction of the major

(b) The conclusion is necessitated if and only if both the minor and the major are.

(4) All other moods are nonproductive.

Sharwānī supplements this account by a table. Table 11 gives Sharwānī's table as he himself presents it (using, of course. the mode names. rather than our symbolic abbreviations). Note that the table deals only with condition 3). Concerning the first figure Sharwānī says that there are four valid categorical moods (*Barbara*, *Celarent*, *Darii*, and *Ferio*) and that each of these four when mixed with modes gives rise to 374 productive moods[20] resulting from the seventeen actual minor modes times the twenty-two major modes the fourteen simples and the eight standard compounds ($\Box$C & ~$\forall$E), ($\forall$C & ~$\forall$E), ($\exists$C & ~$\forall$E), ($\Box$T & ~$\forall$E)($\Box$S & ~$\forall$E)($\exists$E & ~$\forall$E)($\exists$E & ~$\Box$E) and ($\Diamond$C & ~$\Box$E).[21]

[20] Note that there are twenty-two possible major, and minor, premises: fourteen simples and eight compounds, which divide into seventeen actuals and five possibles. Thus, for example, there are seventeen minor premises—the actuals-displayed in Table 8, since the possibles as minor are nonproductive.

[21] Note, thus, that when these 374 modal moods are combined with the four categorical moods there results a total of 1,496 productive syllogistic moods in the first figure alone.

Table 11#

	Minor \ Major	□C	∀C	□C & ~∀E	∀C & ~∀E
1	□E	□E	∀E	□E & ~∀E	∀E & ~∀E
2	∀E	∀E	∀E	∀E & ~∀E	∀E & ~∀E
3	□C	□C	∀C	□C & ~∀C	∀C & ~∀E
4	□T	□T	T	□T & ~∀E	T & ~∀E (*)
5	□S	□S	S	□S & ~∀E	S & ~∀E (*)
6	∀C	∀C	∀C	∀C & ~∀E	∀C & ~∀E
7	∃C	∃C	∃C	∃C & ~∀E	∃C & ~∀E
8	T	T	T	T & ~∀E	T & ~∀E
9	S	S	S	S & ~∀E	S & ~∀E
10	□C & ~∀E	□C	∀C	□C & ~∀E	∀C & ~∀E (*)
11	∀C & ~∀E	∀C	∀C	∀C & ~∀E	∀C & ~∀E
12	□T & ~∀E	□T	T	□T & ~∀E	T & ~∀E (*)
13	□S & ~∀E	□S	S	□S & ~∀E	S & ~∀E (*)
14	∃E & ~∀E	∃E	∃E	∃E & ~∀E	∃E & ~∀E
15	∃C & ~∀E	∃C	∃C	∃C & ~∀E	∃C & ~∀E
16	∃E & □E	∃E	∃E	∃E & ~∀E	∃E & ~∀E
17	∃E	∃E	∃E	∃E & ~∀E	∃E & ~∀E

(#) Display by Sharwānī.

(*) Proposed correction in accordance with condition (3b), removing the necessity of the simple component mode.

As far as we can determine the standard account given by both Qazwīnī and Sharwānī is correct except for moods containing the *continuing* modes (∃C)(∃C & ~∀E), and (◊C) in the major. In these first figure moods, as far as can be ascertained by independent calculation, the conclusion is (∃E)(∃E & ~∀E), and (◊E) respectively for each of the seventeen minors.

The account of modal syllogism in the first figure, duly corrected in the manner indicted, can be verified by means of the symbolic apparatus for modal propositions given above in section 6. It is intended to show by the following examples that the various claims regarding syllogistic results are in fact justified.

Example 1

Major: ($\Box$E) (All B are C) 1 $(\forall x)[\exists Bx \supset \forall\Box Cx]$
Minor: ($\exists$E & ~$\forall$E) (All A are B) 2 $(\forall x)[\exists Ax \supset [\exists Bx \,\&\, {\sim}\forall Bx]$

Conclusion: ($\Box$E) (All A are C) 3 $\exists Ax$

4	$\exists Bx \,\&\, {\sim}\forall Bx$	2, 3
5	$\exists Bx$	4
6	$\forall\Box Cx$	1, 5
7	$(\forall X)[\exists Ax \supset \forall\Box Cx]$	3-6

Example 2

Major: ($\Box$C & ~$\forall$E) (All B are C) 1 $(\forall x)[\exists Bx \supset(\forall\Box(Bx \supset CX) \,\&\, {\sim}\forall Cx)]$
Minor: ($\exists$E) (All A are B) 2 $(\forall x)[\exists Ax \supset \exists Bx]$

Conclusion: ($\exists$E & ~$\forall$E)) (All A are C) 3 $\exists Ax$

4	$\exists Bx$	2, 3
5	$\forall\Box(Bx \supset Cx) \,\&\, {\sim}\forall Cx)$	1, 4
6	$\forall\Box(Bx \supset Cx)$	5
7	$\exists Cx$	4, 6
8	$\exists Cx \,\&{\sim}\forall Cx$	5, 7
9	$(\forall x)[\exists Ax \supset(\exists Cx \,\&\, {\sim}\forall Cx)]$	3-8

Example 3

Major: ($\exists$C) (All B are C) 1 $(\forall x)[\exists Bx \supset \exists(Bx \,\&\, Cx)]$
Minor: ($\forall$C) (All A are B) 2 $(\forall x)[\exists Ax \supset (Ax \supset Bx)]$

Conclusion: ($\exists$E & ~$\forall$E)) (All A are C) 3 $\exists Ax$

4	$\forall(Ax \supset Bx)$	2, 3
5	$\exists Bx$	3, 4
6	$\exists(Bx \,\&\, Cx)$	1, 5
7	$\exists Cx$	6
8	$(\forall x)[\exists Ax \supset \exists Cx]$	3-7

Thus, for modal syllogisms in Sharwānī we have neither the Aristotelian rule of inference regarding modal syllogistics that the mode of the conclusion follows the mode of the major, since, as in Example 2, it sometimes follows the mode of the minor; nor the variant rule that it follows the minor, since, as in Example 1, it sometimes follows the major; nor the Theophrastean (*Peiorem*) rule that it follows the mode of the weaker premise, since, as in Example 1 it sometimes follows the mode of the stronger. Moreover, as in Example 3 the mode of the conclusion sometimes follows neither the mode of the major nor the minor. Note also that the restriction of the conclusion mode follows only the restriction of the major mode, as is illustrated in Examples 1 and 2. Note finally, as in Example 1, that when the major does not involve the temporality (*C*), the Aristotelian rule that the conclusion follows the major *does* obtain. In general, however, the logical situation in the theory of temporalized modal syllogistic of the Arab logicians is far more subtle than in the Aristotelian tradition of their Greek precursors.

8. SECOND, THIRD, AND FOURTH FIGURE SYLLOGISMS

The first figure syllogisms were held to be self-evident, and the other syllogisms were to be demonstrated by reduction to the first figure by converting one or both premises, by interchanging the premises and converting the conclusion, or by *reductio ad impossibile*

Concerning the second figure Sharwānī says that each of the four categorical moods (*Cesare*, *Camestres*, *Festino*, and *Baroco*), when combined with modes, gives rise to 144 productive moods.[22] Sharwānī's account, in perfect accord with Qazwīnī, [23] is as follows.

There are two conditions for valid syllogisms in the second figure: (1) truth by perpetuity must pertain to the minor (so that the minor is □E or ∀E), or the major must be one of the convertible negative propositions

---

[22] See note 12, section IV. Note also that the other four compounds never occur as premises in Sharwānī's account of the four figures, and that the eight standard compounds but for (∃C & ~∀E) are the only seven compounds discussed by Qazwīnī.

[23] The 2 perpetual minors times 17 actuals majors, plus the remaining 15 actual minors times the 6 negative convertible majors, plus the absolute necessary minor times the 5 possible majors, plus the absolute necessary major times the 5 possible minors, plus the 2 conditional majors times the 5 possible minors = 144 moods.

Table 12[#]

<table>
<tr><th colspan="2">Major / Minor</th><th>□C</th><th>□C & ~∀E</th><th>∀E</th><th>∀C & ~∀E</th></tr>
<tr><td>1</td><td>□C</td><td colspan="4" rowspan="4">∀C</td></tr>
<tr><td>2</td><td>∀C</td></tr>
<tr><td>3</td><td>□C & ∀C</td></tr>
<tr><td>4</td><td>∀C & ~∀E</td></tr>
<tr><td>5</td><td>∃E</td><td colspan="4">∃E</td></tr>
<tr><td>6</td><td>∃C</td><td colspan="4">∃C</td></tr>
<tr><td>7</td><td>T</td><td colspan="4">T</td></tr>
<tr><td>8</td><td>S</td><td colspan="4">S</td></tr>
<tr><td>9</td><td>∃E & ~∀E</td><td colspan="4" rowspan="2">∃E</td></tr>
<tr><td>10</td><td>∃E & ~□E</td></tr>
<tr><td>11</td><td>∃C& ~∀E</td><td colspan="4"></td></tr>
<tr><td>12</td><td>□T & ~∀E</td><td colspan="4" rowspan="2">T</td></tr>
<tr><td>13</td><td>□T</td></tr>
<tr><td>14</td><td>□S & ~∀E</td><td colspan="4" rowspan="2">S</td></tr>
<tr><td>15</td><td>□S</td></tr>
<tr><td>16</td><td>E</td><td colspan="3" rowspan="2">E</td><td rowspan="5">nonproductive</td></tr>
<tr><td>17</td><td>E & ~□E</td></tr>
<tr><td>18</td><td>◊C</td><td colspan="3">◊C</td></tr>
<tr><td>19</td><td>◊T</td><td colspan="3">◊T</td></tr>
<tr><td>20</td><td>◊S</td><td colspan="3">◊S</td></tr>
</table>

(*) As displayed by Sharwānī.

(i.e., one of □E, ∀E, □C, ∀C, DC & ~∀E, and ∀C & ~∀E); (2) a possibility proposition may be used only when the other premise is necessary (and is, thus, □E, □C, or □C & ~∀E). If both these conditions are met, then: if either premise is perpetually true, then the conclusion is ∀E; if the major is a conditional or a conventional proposition, then the mode of the conclusion is like that of the minor except without restriction and without necessity. All remaining moods are nonproductive.

In Table 12 we reproduce Sharwānī's table for the second figure. This table concerns only the case when the major is a conditional or a conventional proposition. The remaining cases are as just described.

As regards the third figure, there are six valid categorical moods (*Darapti*, *Felapton*, *Datisi*, *Ferison*, *Disamis*, and *Bocardo*), each producing 374 valid modal moods. The account given by Sharwānī is the following.

The condition for syllogisms in the third figure is that the minor premiss be one of the actual propositions. When the major is a conditional or a conventional proposition, the mode of the conclusion is like the mode of the converse of the minor, removing the nonperpetual from it or adding it to it, according as the major is general or special. Otherwise, the mode of the conclusion is like that of the major. All other moods are nonproductive.

Table 13 is the table that Sharwānī presents for the third figure. And this table is correct in its entirety. The undisplayed cases, however, present some difficulty, in that the given account seems not to describe them adequately.[24] As far as can be ascertained by independent calculation, the given account is correct except (1) when the major is a *continuing* mode, the conclusion is like the major with (C) weakened to (E) in certain places, and (2) when the minor is true by perpetuity, the conclusion is like the major with (E) *strengthened* to (C) in certain places. Specifically, the situation is as follows:

When the major is □C, ∀C, □C & ~∀E, ∀C & ~∀E, the mode of the conclusion is like the mode of the converted minor removing the non-perpetual from it or adding it to "it, according as the major is general or special. When the major is ∃C, ∃C & ~∀E the conclusion is like the major, with (C) weakened to (E) in all cases in which the modality (∀) does not pertain to the minor. When the major is ◊C, the conclusion is like the major with (C) weakened to (E) in all cases in which the modality (∀□) does not pertain to the minor.

Otherwise, when the major is not one of these seven, the following holds. When the minor is (□E), the conclusion is like the major, with (E) strengthened to (C) in all cases for which (∀) does not pertain to the major. When the minor is (∀E), the conclusion is like the major, with (E) strengthened to (C) in all cases for which (∀), (□), or (◊) do not pertain to the major. Otherwise (when the minor is neither (□E) nor (∀E),) the conclusion is like the major. (Note that we here have need of the new modes introduced in section 6 above.)

24 And so it is with all four figures: Sharwānī is in perfect accord with Qazwīnī.

Table 13*

<table>
<tr><th colspan="2">Major / Minor</th><th>□C</th><th>∀C</th><th>□C & ~∀E</th><th>∀C & ~∀E</th></tr>
<tr><td>1</td><td>□E</td><td colspan="2" rowspan="8">∃C</td><td colspan="2" rowspan="8">∃C & ~∀E</td></tr>
<tr><td>2</td><td>∀E</td></tr>
<tr><td>3</td><td>□C</td></tr>
<tr><td>4</td><td>□C & ~∀E</td></tr>
<tr><td>5</td><td>∀C</td></tr>
<tr><td>6</td><td>∀C& ~∀E</td></tr>
<tr><td>7</td><td>∃C & ~∀E</td></tr>
<tr><td>8</td><td>∃C</td></tr>
<tr><td>9</td><td>□T & ~∀E</td><td colspan="2">∃E(#)</td><td colspan="2">∃E & ~∀E(#)</td></tr>
<tr><td>10</td><td>□S & ~∀E</td><td colspan="2" rowspan="8">∃E</td><td colspan="2" rowspan="8">∃E & ~∀E</td></tr>
<tr><td>11</td><td>□T</td></tr>
<tr><td>12</td><td>□S</td></tr>
<tr><td>13</td><td>T</td></tr>
<tr><td>14</td><td>S</td></tr>
<tr><td>15</td><td>∃E & □E</td></tr>
<tr><td>16</td><td>∃E& ~∀E</td></tr>
<tr><td>17</td><td>∃E</td></tr>
</table>

(*) As Displayed by Sharwānī.

(#) Here Sharwānī has (∃C) and (∃C & ~∀E).

Finally, as to the fourth figure, Sharwānī says that there are eight valid moods, five of which are categorically valid (*Bramantip, Dimaris, Camenes*, (*Fesapo,* and *Fresison*), the other three (AOO, OAO, and IEO) being valid only when the negative premiss is one of the specials. Sharwānī, noting his departure from Qazwīnī, orders the moods as follows: (i) AD, (ii) IAI, (iii) EAO, (iv) OAO, (v) EIO, (vi) AEE, (vii) AOO, and (viii) IEO. The first, second, sixth, and eighth moods are reduced by interchanging the premises and converting the (resultant) conclusion; the third and the fifth by converting each of the premises; the seventh by converting the minor, resulting in a second figure syllogism; and the fourth by converting the major, resulting in a third figure syllogism.

The conditions for fourth figure syllogisms as given by Sharwānī are as follows. (1) Both premises must be actuals. (2) The negative propositions

Table 14*

<table>
<tr><th>Major / Minor</th><th>□E</th><th>∀E</th><th>□C</th><th>∀C</th><th>□C & ~∀E</th><th>∀C & ~∀E</th><th>Remaining Actuals</th></tr>
<tr><td>□E</td><td rowspan="6" colspan="4">∃C</td><td rowspan="2" colspan="2">(#)</td><td rowspan="9">∃E</td></tr>
<tr><td>∀E</td></tr>
<tr><td>□C</td><td rowspan="4" colspan="2">∃C& ~∀E</td></tr>
<tr><td>∀C</td></tr>
<tr><td>□C & ~∀E</td></tr>
<tr><td>∀C& ~∀E</td></tr>
<tr><td>∃C</td><td rowspan="3"></td><td rowspan="2" colspan="5">∃C(**)</td></tr>
<tr><td>∃C& ~∀E</td></tr>
<tr><td>Remaining Actuals</td><td colspan="5">∃E</td></tr>
</table>

(*) Not displayed, but only described by Sharwānī.
(#) Since the premises here are contradictory, the conclusion is problematic.
(**) Described by Sharwānī as (∃E).

in the syllogism must be convertible. (3) In the sixth mood (AEE) the minor must be true by perpetuity (or else the major mode must be one of the six negative convertibles).[25] (4) In the seventh mood (AOO), the major mode must be one of the six negative convertibles. (5) In the eighth mood (IEO) the minor must be one of the two specials, and the major one of the negative convertibles.

The productive combinations in both the first and second moods (AAI, IAI) are 289. Their condition is that the premises be actual propositions. The mode of the conclusion is the converse of the minor, if the minor is □E or ∀E, or if both premises are in the six negative convertibles. Otherwise, the conclusion mode is ~(∃E). All other cases are nonproductive. This

[25] For example, according to the text we are to have (∃E)P, (∀E)p’, therefore, (∃E)P”. Yet, it is clear that the following holds: (∀x) [∃Mx & ∃Px], (∃x) [∃Mx &. ∀Sx], therefore, (∃x) [∃Sx & ∃(Sx & Px)]. If our interpretation of modes is correct, the mood should thus be (∃E)P, (∀E)p’, therefore. (∃C)P”.

Table 15*

<table>
<tr><td colspan="2">Major
Minor</td><td>□E</td><td>∀E</td><td>□C</td><td>∀C</td><td>□C & ~∀E</td><td>∀C & ~∀E</td></tr>
<tr><td>1</td><td>□E</td><td colspan="2" rowspan="17">∀E

∀E</td><td colspan="4" rowspan="6">∃C</td></tr>
<tr><td>2</td><td>∀E 2</td></tr>
<tr><td>3</td><td>□C</td></tr>
<tr><td>4</td><td>∀C</td></tr>
<tr><td>5</td><td>∀C& ~∀E</td></tr>
<tr><td>6</td><td>□C& ~∀E</td></tr>
<tr><td>7</td><td>∃C & ~∀E</td><td colspan="2">∃C</td><td colspan="2" rowspan="2">∃C(#)</td></tr>
<tr><td>8</td><td>∃C</td><td colspan="2">∃C(#)</td></tr>
<tr><td>9</td><td>□T & ~∀E</td><td colspan="4" rowspan="9">∃E</td></tr>
<tr><td>10</td><td>□S & ~∀E</td></tr>
<tr><td>11</td><td>□T</td></tr>
<tr><td>12</td><td>□S</td></tr>
<tr><td>13</td><td>T</td></tr>
<tr><td>14</td><td>S</td></tr>
<tr><td>15</td><td>∃E & ~□E</td></tr>
<tr><td>16</td><td>∃E& ~∀E</td></tr>
<tr><td>17</td><td>∃E</td></tr>
</table>

(*) As displayed by Sharwānī.
(#) Sharwānī has (∀C) here.

situation is presented in Table 14.

The productive combinations in both the third and fifth moods (EAO, EIO) are 102. Their condition is the general condition that the premises be actual and that the negative premise be convertible. The moods are reduced to the first figure by converting each premiss. Thus, the conclusion is ∀E, if the major is ◊E or ∀E; otherwise, the mode *of* the conclusion is the same as .the mode *of* the converted minor after removing the non-perpetual from it. All other cases are nonproductive. The situation is presented in table 15.

The productive combinations in the fourth mood (OAO) are 34. The condition for the fourth mood is the general condition that the premisses be actual and that the negative premise be convertible and, therefore, that the major be one of the specials. The conclusion is the same as in the third

Table 16*

Minor \ Major		□C & ~∀E	∀C & ~∀E
1	□E		
2	∀E		
3	□C		
4	∀C	∃C & ~ ∀E	
5	□C& ~∀E		
6	∀C& ~∀E		
7	∃C & ~∀E		
8	∃C		
9	□T	∃E & ~∀E(#)	
10	□S		
11	□T & ~∀E		
12	□S & ~∀E		
13	T	∃E & ~∀E	
14	S		
15	∃E & ~□E		
16	∃E& ~∀E		
17	∃E		

(*) As displayed by Sharwānī.
(#) Sharwānī has (∃E) here.

figure after converting the major. Since the major is a special proposition, the conclusion is thus the same as the converse of the minor. All other cases are non-productive. The situation is as displayed in Table 16.

The productive combinations in the sixth mood (AEE) are 58. The condition for the sixth mood is the general condition that the negative premise (the minor) is one of the negative convertibles, and the particular condition that the minor is □E or ∀E, or that the major is a negative convertible. The conclusion mode is AE, if either premise is □E or ∀E; otherwise, the conclusion has the same mode as the converse of the minor. All other cases arc nonproductive. The situation is as displayed in Table 17.

Table 17*

<table>
<tr><th colspan="2">Major / Minor</th><th>□E</th><th>∀E</th><th>□C</th><th>AC</th><th>□C & ~∀E</th><th>∀C & ~∀E</th></tr>
<tr><td>1</td><td>□E</td><td colspan="2" rowspan="17">∀E</td><td colspan="2" rowspan="2">∀E(#)</td><td colspan="2" rowspan="2">∀E</td></tr>
<tr><td>2</td><td>∀E</td></tr>
<tr><td>3</td><td>□C</td><td colspan="2" rowspan="4">∀C</td><td colspan="2" rowspan="4">The non-perpetual about-some conventional</td></tr>
<tr><td>4</td><td>∀C</td></tr>
<tr><td>5</td><td>□C& ~∀E</td></tr>
<tr><td>6</td><td>∀C& ~∀E</td></tr>
<tr><td>7</td><td>□S& ~∀E</td><td colspan="4" rowspan="11">nonproductive</td></tr>
<tr><td>8</td><td>□T & ~∀E</td></tr>
<tr><td>9</td><td>□T</td></tr>
<tr><td>10</td><td>□S</td></tr>
<tr><td>11</td><td>T</td></tr>
<tr><td>12</td><td>S</td></tr>
<tr><td>13</td><td>∃C & ~∀E</td></tr>
<tr><td>14</td><td>∃C</td></tr>
<tr><td>15</td><td>∃C & ~∀E</td></tr>
<tr><td>16</td><td>∃E & ~□E</td></tr>
<tr><td>17</td><td>∃E</td></tr>
</table>

(*) As displayed by Sharwānī.
(#) Sharwānī has ∃C & ~∀E here.

Table 18*

<table>
<tr><th colspan="2">Major / Minor</th><th>□C & ~∀E</th><th>∀C & ~∀E</th></tr>
<tr><td>1</td><td>□E</td><td colspan="2" rowspan="2">∀E</td></tr>
<tr><td>2</td><td>∀E</td></tr>
<tr><td>3</td><td>□C</td><td colspan="2" rowspan="4">∀C</td></tr>
<tr><td>4</td><td>∀C</td></tr>
<tr><td>5</td><td>□C& ~∀E</td></tr>
<tr><td>6</td><td>∀C& ~∀E</td></tr>
</table>

(*) As displayed by Sharwānī.

The productive combinations in both the seventh and eighth moods (AOO, IEO) are 12. The condition for the seventh mood (AOO) is the particular condition that the major is one of the negative convertibles, and the general condition that the negative premise be convertible and, thus, that the minor premise is one of the specials. The mood is reduced to the second figure by converting the minor. All other cases are non-productive. The situation is displayed in Table 18.

The condition for the eighth mood (IEO) is the particular condition that the major be one of the negative convertibles and that the minor be one of the specials. The mood is reduced to the first figure by interchanging the premisses and converting the conclusion. All other cases are nonproductive. The situation is displayed in Table 19.

With the end of the fourth figure ends Sharwānī's 'account of temporal modal syllogisms, which, despite its occasional slips, reveals a detailed and sophisticated comprehension of temporal modal logic.

9. CONCLUSION

We have come to the end of a long and somewhat complicated account, and a word of retrospective appraisal is in order. Clearly, the Arabic logicians of the Middle Ages were in possession of a complex theory of temporal modal syllogisms, which they elaborated in great and sophisticated detail. When one considers that all reasoning was conducted purely verbally, largely on the basis of somewhat vague examples, without any symbolic apparatus, and even without abbreviative devices, one cannot but admire the level of complexity and accuracy. The logical acumen of these medieval scholars was of a very high order indeed. But their successors were not able to maintain this standard. Sprenger remarks in his translation of the *Shamsiyyah* of Qazwīnī:

> [The paragraphs dealing with modalized inferences] are omitted in the translation because they contain details on modals which are of no interest. The last named four paragraphs are also omitted in most Arabic text books on Logic, and are not studied in Mohammedan Schools.[26]

When the logical tradition of Islam passed from the hands of the scholars

[26] This clause is missing in the text, but Qazwīnī's otherwise identical discussion contains the clause. Cf. Table 15.

Table 19*

<table>
<tr><th colspan="2">Major / Minor</th><th>□C & ~∀E</th><th>∀C & ~∀E</th></tr>
<tr><td>1</td><td>□E</td><td colspan="2" rowspan="2">∀C & ~∀E(#)</td></tr>
<tr><td>2</td><td>∀E</td></tr>
<tr><td>3</td><td>□C</td><td colspan="2" rowspan="4">∀C & ~∀E</td></tr>
<tr><td>4</td><td>∀C</td></tr>
<tr><td>5</td><td>□C& ~∀E</td></tr>
<tr><td>6</td><td>∀C& ~∀E</td></tr>
</table>

(*) As displayed by Sharwānī.
(#) since the premises are contradictory here, the conclusion is problematic.

into that of the schoolmasters, the standard of work went into a not surprising decline. The medievals had a firmer grasp.[27]

[27] This chapter originally appeared under the same title in *Essays in Islamic Philosophy and Science* ed. by S. F. Hurani (Albany: State University of New York Press, 1975), pp. 189-221.

Chapter 7

CHOICE WITHOUT PREFERENCE: THE PROBLEM OF "BURIDAN'S ASS"

"A logical theory may be tested by its capacity for dealing with puzzles, and it is a wholesome plan, in thinking about logic, to stock the mind with as many puzzles as possible, since these serve much the same purpose as is served by experiments in physical science."

Bertrand Russell

"In things which are absolutely indifferent there can be no choice and consequently no option or will, since choice must have some reason or principle."

G.W. Leibniz

1. INTRODUCTION

The idea that the reasoned life, although rewarding, is not all that simple is already prominent in the earliest speculations on "wisdom" (*sophia*) out of which philosophy (*philo-sophia*), the love of wisdom, was to grow. Nor is this surprising. After all, a choice that is *reasoned* is more difficult to arrive at than a choice made haphazardly when, in the blithe manner of Mark Twain's dictum, "you pays your money and you takes your choice." But such reflections lead to the puzzle posed by the question: How is a reasoned choice among fully equivalent alternatives possible? We here confront the problem of *choice without preference*: a reasoned choice must proceed from a reasoned preference, but a reasoned preference among fully equivalent objects is patently impossible.

There are puzzles and puzzles—"idle" ones which can at best amuse a sated imagination, and "profound" ones which can lead the intellect into a deeper apprehension of the nature of things. The puzzle of equivalent choices is of the second kind, seeing that its analysis provides an occasion both for insight into the logic of reasoned choice, and for a better understanding of some important issues in the history of philosophy.

As is generally the case in matters of this sort, it is useful to consider the historical background. In elucidating the substantive philosophical contexts in which the problem of choice without preference has figured, and for which it has been viewed as fundamentally relevant, a historical survey brings to light primarily the three following issues: first, its context in Greek science, originally in cosmological discussions of the earth's place in the physical universe, and ultimately in more general considerations regarding physical symmetries (cf. Axiom 1 of Archimedes' treatise *On Plane Equilibriums*); second, its context in philosophico-theological discussion among the Arabs regarding the possibility of explaining God's actions in ways acceptable to reasoning men; and finally its medieval Scholastic context in ethico-theological discussions of man's freedom of the will.

So much for a preview of the historical aspects of our problem. With regard to the theoretical findings of the analysis, let it suffice here to note in a preliminary way that a study of choice without preference forces upon us a clear recognition of the difference between *reasons* on the one hand and inclining *motives* on the other. We shall see that an indifferent choice must be made (in effect) randomly. Now, when a random selection among indifferent objects is made by me, I do have a reason for my particular selection, namely the fact that it was indicated to me by a random selector. But I have no *preference* or psychological motivation of other sorts to incline me to choose this item instead of its (by hypothesis indifferent) alternatives. Such absence of psychological preference does not entail the impossibility of a rationally justifiable selection. A choice can therefore be vindicated as having been made reasonably even though it cannot be traced back to any psychological foundation. In short, we can have *reasons* for a choice even where there is no inclining *motive*. Thus, despite its seemingly abstruse and esoteric character of the issue, the puzzle of a reasoned choice among fully equivalent alternatives is not lacking in instructiveness from both the theoretical and the historical points of view.

2. THE PROBLEM

Can a reasonable agent choose a course of action, or an object, without a preference? It certainly appears on first view that this question has to be answered negatively. By the very concept of a "reasonable agent", it is requisite that such an individual have *reasons* for his actions. And when a reasonable choice among alternatives is made, this must, it would seem, have to be based upon a *preference* for the object actually chosen *vis-à-vis* its available alternatives. Where there is no *preference*, it would appear that no *reason* for a selection can exist, so that there apparently cannot be a *reasonable* way of making a choice. This line of reasoning seems to establish the precept: *No reasonable choice without a preference.*

However, despite the surface plausibility of this argument, it cannot be accepted as fully correct. For there is a well-known, indeed notorious counter-example: the dilemma or paradox of Buridan's Ass. This mythical creature is a hypothetical animal, hungry, and positioned midway between essentially identical bundles of hay. There is assumed to be no reason why the animal should have a preference for one of the bundles of hay over the other. Yet it must eat one or the other of them, or else starve. Under these circumstances, the creature will, being reasonable, prefer Having-one-bundle-of-hay to Having-no-bundle-of-hay. It therefore *must choose one* of the bundles. Yet there is, by hypothesis, simply no *reasons* for preferring either bundle. It appears to follow that reasonable choice must—somehow—be possible in the absence of preference.

It should at once be noted that the problem of the Identity of Indiscernibles, famous because of its prominent role in the philosophy of Leibniz, has no bearing upon the issue. For what is at stake in cases of choice without preference, such as the example of Buridan's Ass, is not there being *no difference* between the objects of choice (i.e., that they be strictly indiscernible), but merely that such differences as do admittedly exist are *either* entirely *irrelevant* to the desirability of these items (as the mint-markings of coins in current circulation have no bearing upon their value or worth), or *else* are simply *unknown* to the chooser. Thus indiscernability is not at issue here, but rather effective indistinguishability *qua* objects of choice—value-symmetry, in short—so that every identifiable reason for desiring one alternative is equally a reason for desiring the others. There is consequently no need for the issue of the identity of indiscernibles to concern us in the present context.

In the main, the problem of choice in the absence of preference is a theoretical, and not a practical problem. Real-life situations rarely confront us with strictly indifferent choices. Such situations do, however, appear to exist. For example, if a person were offered a choice between two fresh dollar bills, the only perceptible difference between which is that of their serial numbers, we would be greatly astonished if this selector could offer us a "reason" for choosing one of them rather than the other which could reasonably be regarded as cogent. While a difference between the bills does indeed exist, it simply does not constitute a valid difference as regards their preferability as objects of choice. And again, when purchasing a stamp at the post office, one is utterly indifferent as to which one on the sheet the agent gives one (for *him*, to be sure this indifference is eliminated by such factors as ease of access, etc., so that the *situation* is not one of indifference). However, though it is the case that indifferent choices are rare, the problem of choice without preference does, nevertheless, have the status of an interesting question in the theory of reasoned choice. And as such it also has—as we shall see—significant philosophical implications and consequences, and as well as a venerable history in philosophic thought.

3. THE HISTORY OF THE PROBLEM OF "BURIDAN'S ASS"

The problem of choice without preference has a long philosophical, and even literary, history. Its most noteworthy parts of which will be sketched in this section. The interest of this historical excursus lies both in the view that it provides of various formulations of our puzzle, and in its indication of the alternative philosophical problem contexts in which it has played a significant role.

Anaximander (ca. 610-ca. 545 BC)

According to a report of Origen, already certain of the early Greek cosmologists had held "that the earth is a celestial object (*meteôron*), supported [in the heavens] by nothing whatsoever, and remaining in its place on account of its equidistance from all."[1] From Aristotle we learn that just this was the position and the line of reasoning of the pre-Socratic philosopher, Anaximander of Miletus:

> There are some who name its [i.e., the earth's] indifference (*homoiotês*) as the cause of its remaining at rest, e.g., among the early philosophers Anaximander. These urge that that which is situated at the centre and is equably related to the extremes has no impulse to move in one direction—either upwards or downwards or sideways—rather than in another; and since it is impossible for it to accomplish movement in opposite directions at once, it necessarily remains at rest.[2]

[1] Origin, *Philosophoumena*, c. 6. My translation.

[2] Aristotle, *De caelo*, II 13, 295b10. Tr. by W.K.C. Guthrie in the Loeb Series. Regarding this passage and its bearing on Anaximander see E. Zeller, *Philosophie der Griechen*, vol. I, 7th edition, ed. by W. Nestle (Leipzig, 1923), p. 303, notes.

And this idea was endorsed by Socrates in Plato's *Phaedo*:

> "I am satisfied," he [Socrates] said, "in the first place that if [the earth] is spherical, and located in the middle of the universe, it has no need of air[3] or any other force of that sort to make it impossible for it to fall; it is sufficient by itself to maintain the symmetry of the universe and the equipoise of the earth itself. A thing which is in equipoise and placed in the midst of something symmetrical will not be able to incline more or less towards any particular direction; being in equilibrium, it will remain motionless."[4]

In the thought of Anaximander, then, that an object "placed in the midst of something symmetrical will not be able to incline more or less towards any particular direction" we have the conceptual origin, the germ as it were, of the problem of Buridan's Ass.[5] But this is only the start, and, a further step was required to reach our actual problem—the move to the concept of a psychological cancellation or balance among opposing motivations of equal strength, to a *psychological equilibrium of motives*, in short. This step was already taken by Aristotle.

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[3] According to Aristotle (*De caelo*, II 13, 294b14), Anaximenes, Anaxgoras and Democritus held that the earth stays in place "owing to the air beneath, like the water in *klepsydrae*."

[4] Plato, *Phaedo*, 108 E. Tr. by R.S. Bluck (London, 1955). This reasoning is endorsed also by Parmenides and by Democritus (see Aetios III, 15, 7), who are also reported to have characterized the state resulting from the earth's equidistance from the cosmic extremities as one of *isorropia* (equilibrium: the term used by Pre-Socratics and by Plato in the citation). Again, according to a report of Achilles (*Isagogê*, 4; ed. by V. Arnim, vol. II, p. 555), "The Stoics . . . [hold that] the earth will remain in the center, being kept in equilibrium by the pressure of air from all sides. And again, if one takes a body and ties it from all sides with cords and pulls them with precisely equal force, the body will stay and remain in its place, because it is dragged equally from all sides." (I take the reference and the translation from S. Sambursky, *Isis*, vol. 49 [1958], pp. 331-335.) Cp. the "explanation" given in medieval times by eager Christians anxious to refute the supposed miracle that Mohammed's coffin had floated unsupported in mid-air, by claiming that it was made of iron and was supported just midway between two precisely equal magnets.

[5] Compare Archimedes' axiom: "I postulate that equal weights at equal distances balance, and equal weights at unequal distances do not balance, but incline towards the weight which is at the greater distance" (*On Plane Equilibriums*, tr. by I. Thomas in *Greek Mathematics* [Loeb], vol. ii, p. 207, Axiom I).

Aristotle (384-322 BC)

In criticizing as inadequate the very view we have just considered that the earth is sustained in space through the equipoise of the surrounding heavens, Aristotle contrasts this view with his own theory of *natural place*, to the distinct advantage of this latter theory:

> The reason [for the earth's position] is not its impartial relation to the extremes: that could be shared by any other element, but motion towards the center is peculiar to earth . . If . . .the place where the earth rests is not its natural place, but the cause of its remaining there is the constraint of its "indifference" (on the analogy of the hair which, stretched strongly but evenly at every point, will not break, or the man who is violently but equally hungry and thirsty, and stands at an equal distance from food and drink, and who therefore must remain where he is), then they [i.e., Anaximander and the other supporters of this view] ought to have inquired into the presence of fire at the extremes . . .Fire when placed at the centre is under as much necessity to remain there as earth, for it will be related in the same way to any one of the points on the extremity; but in fact it will leave the centre, and move as we observe it to do, if nothing prevents it, towards the extremity. . . [6]

Here, in Aristotle's extension of the mechanical equilibrium cases into his example of the man torn between equal attraction to food and drink, the physical theme of an equilibrium of forces was first transformed into a psychological balance of motives.

The sixth century Aristotelian commentator Simplicius offers the following discussion on this passage:

> The sophists say that if a hair composed of similar parts is strongly stretched and the tension is identical throughout the whole, it would not break. For why would it break in this part rather than that, since the hair is identical in all its parts and the tension is identical? Analogously also in the case of a man who is exceedingly hungry and thirsty, and identically so in both, and identically lacking in food and drink, and for this reason identically motivated. Necessarily, they say, this man remains at rest, being moved to neither alternative. For why should he move to this one first, but not that, in a smuch as his need, and thus his motivation, is identical [on each side] . . . The solution of such examples of identity is hardly surprising. For it is clear that the hair breaks. Even hypothesizing a fictitious thing with parts thus identical, plainly an identical tension at the ends and the middle is impossible. As to the other example, even if the man were equally distant, thirst would press him more. And if neither this nor that presses more, he will choose whatever he first happens on, as when two pleasant sights lie equally in our view. Whatever happens first we choose first. For identity does not completely obviate the choice,

6 Aristotle, *De caelo*, II 13, 295b24. Tr. by W.K.C. Guthrie in Loeb series.

> but simply makes the drive [towards one alternative] slower by the diversion of the other.[7]

In his discussion of the choice problem, Simplicius rigidly preserves the psychological character of the example as instancing a psychological equilibrium of motives. Simplicius' proposed solution to the problem does, however, offer, an interesting and original suggestion, viz., that indifferent choices can be resolved on grounds of *convenience*, and in particular, that this can be accomplished by selecting the alternative upon which "we happen first". We shall have occasion to revert to this suggestion below.

Before the definition of the philosophic problem of choice without preference was to attain its ultimate logical sharpness of formulation, it was necessary that the mode of indifference at issue should become transformed from a *psychological* balance among diverse motivations into a strict *logical* indifference: a choice in the face of essentially identical alternatives. This was the step taken by al-Ghazâlî, the Algazel of the Schoolmen, and taken first, it would seem, by him.

Ghazâlî (1058-1111)

In his great work on the *Incoherence of the Philosophers*, the Arabic philosopher-theologian Ghazâlî is concerned, *inter alia*, to defend the orthodox Moslem theological thesis of the createdness of the world against the view maintained by the Arabic Aristotelians that the universe is eternal. One of the reasonings which Ghazâlî is concerned to refute is an argument against the createdness of the world based on a concept of sufficient reason: Why, if the world is the creation of God, did he elect to create it when he did, rather than earlier or later?[8] Speaking, for the moment, on behalf of the (Aristotelian) philosophers, Ghazâlî presses this question home against the supporters of the createdness of the world:

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7 *Commentaria in Aristotelem Graeca* (Royal Prussian Academy), vol. VII, *Simplicii in Aristotelis de Caelo Commentaria*, ed. by I.L. Heilberg (Berlin, 1894), pp. 533-534. My translation.

8 "How will you defend yourselves, theologians, against the philosophers, when they . . . [say] that times are equivalent so far as the possibility that the Divine Will should attach itself to them is concerned . . .?" Averroes' *Tahâfut al-Tahâfut*, tr. by S. van den Bergh (London, 1954), vol. I, p. 18. (All questions from this work are drawn from this edition.) Ghazâli's work is quoted *in extenso* in Averroes' commentary thereon, *The Incoherence of the Incoherence*.

> But we philosophers know by the necessity of thought that one thing does not distinguish itself from a similar except by a differentiating principle, for if not, it would be possible that the world should come into existence, having the possibility both of existing and of not existing, and that the side of existence, although it has the same possibility as the side of non-existence, should be differentiated without a differentiating principle. If you answer that the Will of God is the differentiating principle, the one has to inquire what differentiates the will, i.e., the reason why it has been differentiated in such or such way. And if you answer: One does not inquire after the motives of the Eternal, well, let the world then be eternal, and let us not inquire after its Creator and its cause, since one does not inquire after the motives of the Eternal![9]

In opposing this argument, Ghazâlî proceeds by a closer examination of the concept of *will*, seeking to establish the drastic-seeming remedy of a denial that the concept of a sufficient reason for action is applicable to the supreme being,[10] whose will can of itself constitute a differentiating princi-

9 Averroes,*Tahâfut al-Tahâfut*, vol. I. p. 18. Compare R.G. Collingwood's discussion in *The Idea of Nature* (Oxford, 1945): "Unless God had a reason for His choice [to create the world as He did], it was no choice: it was something of which we have no conception whatever, and calling it a choice is merely throwing dust in our own eyes by pretending to equate it with a familiar human activity, the activity of choosing, which we do not in fact conceive it to have resembled. Choice is choice between alternatives, and these alternatives must be distinguishable, or they are not alternatives; moreover one must in some way present itself as more attractive than the other, or it cannot be chosen. [Cp. Averroes and Leibniz below—N.R.] . . . To speak of Him as choosing implies either that He chooses for a reason . . . or else He chooses for no reason, in which case he does not choose. And the dilemma cannot be evaded by a profession of reverent ignorance. You cannot wriggle out of it by saying that there are mysteries into which you will not pry: that God's ways are past finding out, or (if you prefer one kind of humbug to another) that these are ultimate problems. . . . Humbug of that kind arises from a kind of pseudo-religiosity. . . . It is humbug, because it was yourself that began prying into these mysteries. You dragged the name of God into your cosmology because you thought you could conjure with it. You now find you cannot; which proves, not that God is great, but that you are a bad conjurer" (pp. 40-41). Compare Spinoza, who flatly characterizes the "will of God" as "the refuge for ignorance" (*Ethics*, Bk. I, Appendix).

10 In Christian theology, this was the position of Duns Scotus: "If it be asked why the divine will is determined rather to one of two incompatables than to the other, I reply: it is foolish (*indisciplinatus*) to seek causes and demonstrations for all things . . . there is no cause on account of which the will wills, just as there is no willing to will" (*Opus oxoniensis*, I vii 5, 23-24). My translation is from the Latin cited by C.R.S. Harris in *Duns Scotus* (Oxford, 1927), vol. I, p. 181.

ple.[11] We must accept the idea of a "mere" will—of a choice made not conditionally because it subserves some other willed purpose, but categorically—simply and solely because its willer would have it so.[12] It is of the essence of will, Ghazâlî argues, that choice without reason be possible. Here the will can provide a substitute for reason out of its own resources: *stet pro ratione voluntas*.[13]

> We answer: The world exists, in the way it exists, in its time, with its qualities, and in its space, by the Divine Will and will is a quality which has the faculty of differentiating one thing from another, and if it had not this quality, power in itself would suffice. But, since power is equally related to two contraries and a differentiating principle is needed to differentiate one thing from a similar, it is said that the Eternal possesses besides His power a quality which can differentiate between two similars. And to ask why will differentiates one of two similars is like asking why knowledge must comprehend the knowable, and the answer is that "knowledge" is the term for a quality which has just this nature. And in the same way, "will" is the term for a quality the nature or rather the essence of which is to differentiate one things from another.[14]

In Jewish theology, this view is espoused by Moses Maimonides: "We remain firm in our belief that the whole Universe was created in accordance with the will of God, and we do not inquire for any other cause or object. Just as we do ask what is the purpose of God's existence so we do not ask what was the object of His will, which is the cause of the existence of all things with their present properties, both those that have been created and those that will be created" (*Guide for the Perplexed*, vol. III, p. 13, tr. by M. Friedländer [American edition, 1946], p. 276).

11 In his controversy with Leibniz, Samuel Clarke maintained just this thesis, "Tis very true, that nothing *is*, without a sufficient *reason* it is, and why it is *thus* rather than *otherwise*. . . . But *sufficient reason* is of times no other, than the *mere Will of God*." (Second reply, §1.)

12 This idea is not unfamiliar to readers of the *Arabian Nights* as a characteristic feature of the type of medieval oriental despotism there depicted. When one in authority gives as his "reason" for wanting a thing done that "It must needs be so, there is no help for it," this is to be accepted as constituting a very convincing reason indeed.

13 See note 18 below.

14 Averroes, *Tahâfut al-Tahâfut*, vol. I, p. 19.

Ghazâlî proceeds to illustrate by means of an example that this capacity of differentiating where there is no difference is an essential characteristic power of all will, human as well as divine. This example is the focus of our present interest, and merits quotation in full:

> How, then, will you refute those who say that rational proof has led to establishing in God a quality the nature of which is to differentiate between two similar things? And, if the word "will" does not apply, call it by another name, for let us not quibble about words! . . . Besides, we do not even with respect to our human will concede that this cannot be imagined. Suppose two similar dates in front of a man who has a strong desire for them, but who is unable to take them both. Surely he will take one of them through a quality in human nature of which is to differentiate between two similar things. All the distinguishing qualities you have mentioned, like beauty or nearness or facility in taking, we can assume to be absent, but still the possibility of the taking remains. You can choose between two answers: either you merely say that an equivalence in respect to his desire cannot be imagined—but this is a silly answer, for to assume it is indeed possible—or you say that if an equivalence is assumed, the man will remain for every hungry and perplexed, looking at the dates without taking one of them, and without a power to choose or to will, distinct from his desire. And this again is one of those absurdities which are recognized by the necessity of thought. Everyone, therefore, who studies, in the human and the divine, the real working of the act of choice, must necessarily admit a quality the nature of which is to differentiate between two similar things.[15]

Here for the first time the problem of choice without preference is given its ultimate logical formulation. The examples in explanation of Anaximander's views involve a physical balance through the equilibrium of forces; and in Aristotle's example we have the psychological balance of contrary drives or motivations of equal intensity. Ghazâlî's formulation, however, sharpens the dilemma to its logical edge: it poses the problem of *the possibility of rational choice in the face of essentially identical alternatives*.

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[15] Averroes, *Tahâfut al-Tahâfut*, vol. I, p. 21. This is Ghazâli's reply to a hypothetical philosopher-opponent who said, "The assumption of a quality the nature of which is to differentiate one things from a similar one is something incomprehensible, say even contradictory, for 'similar' means not to be differentiated, and 'differentiated' means not similar. . . . If someone who is thirsty has before him two cups of water, similar in everything in respect to his aim, it will not be possible for him to take either of them. No, he can only take the one he thinks more beautiful or lighter or nearer to his right hand, if he is right-handed, or act for some such reason, hidden or known. Without this the differentiation of the one from the other cannot be imagined" (*Ibid*., p. 19).

By right of historical precedence, then, the problem of Buridan's Ass ought perhaps more appropriately be denominated as that of *Ghazâlî's Dates*. However, it seems likely—in view of the manner in which Ghazâlî introduces the problem into his discussion—that he found it already in current consideration.[16] He employs it as an example admirably suited to support the concept of a "mere" will—inscrutable from the standpoint of reasons and reasonings, capable of effecting differentiation where there is no difference.[17]

Ghazâlî associates himself with the school of Moslem theologians called Ash'arites, after its founder al-Ash'ari. Opposing the rationalistic Mu'tazilites, the Ash'arites make room for a certain irrationality, or better, non-rationality in matters theological, denying that reason alone is capable of attaining religious truths:

> The difference between the Ash'arite and Mu'tazilite conceptions of God cannot be better expressed than by the following passage which is found twice in Ghazâlî . . . and to which by tradition is ascribed the breach between al-Ash'ari and the Mu'tazilites.

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[16] He may well have owed it to a Syriac or Arabic commentator on Aristotle, presumably in a gloss on *De Caelo*, 295b10-35, although the Greek commentators do not seem to have modified Aristotle's formulation of the example (cf. the quotation for Simplicius given above and also see C.A. Brandis' edition of the *Scholia in Aristotelem*, published by the Royal Prussian Academy, Vol. 4 [1836], p. 507). Thus Léon Gauthier argues that Ghazâli must have found the example already present in al-Fārābī or in Avicenna "because he explicitly states at the end of the first Preamble of the *Tahâfut* that throughout this work, in refuting the doctrines of the Greek philosophers, especially Aristotle and his commentators, he limits his considerations to those ideas taken up and endorsed by their two great Moslem disciples, al-Fārābī and Avicenna" ("L'Argument de l'Ane de Buridan et les Philosophes Arabes," *Mélanges René Basset [Publications de l'Institut des Hautes-Études Marocaines,* Vol. X], Paris, 1923, pp. 209-233; see p. 224).

[17] This position was adopted by many (Western) scholastics. Johannes Gerson, for example, says that the will *est sibi frequenter sufficiens causa vel ratio* and that it can choose one thing and reject another in such a manner that *nec exterior alia ratio quarenda est: sic voleo, sic jubeo; stat pro ratione voluntas (Opera Omnia,* ed. by M.L.E. Du Pin [Antwerp-Amsterdam, 1706], vol. III, pp. 443-444.) On Gerson's theory of the will see H. Siebeck, "Die Willenslehre bei Duns Scotus und seinen Nachfolgern," *Zeitschrift für Philosophie und Philosophische Kritik*, vol. 112 (1898), pp. 179-216.

> Let us imagine a child and a grown-up in Heaven who both died in the True Faith, but the grown-up has a higher place than the child. And the child will ask God, "Why did you give that man a higher place?" And God will answer, "He has done many good works." Then the child will say, "Why did you let me die so soon that I was prevented from doing good?" God will answer, "I knew that you would grow up a sinner, therefore it was better that you should die a child." Then a cry goes up from the damned in the depths of Hell, "Why, O Lord, did you not let us die before we became sinners?"

Ghazâlî adds to this: "The imponderable decisions of God cannot be weighed by the scales of reason and Mu'tazilism."[18]

Averroes (1126-1198)

In his book on the *Incoherence of the Incoherence*, a detailed critical commentary on Ghazâlî's *Incoherence of the Philosophers*, Averroes undertook to defend the Arabic Aristotelians against Ghazâlî's onslaught. It is worth quoting in full his criticism of Ghazâlî's example of the dates:

> It is assumed that in front of a man there are two dates, similar in every way, and it is supposed that he cannot take them both at the same time. It is supposed that no special attraction need be imagined for him in either of them, and that nevertheless he will of necessity distinguish one of them by taking it. But this is an error. For, when one supposes such a thing, and a willer whom necessity prompts to eat or to take the date, then it is by no means a matter of distinguishing between two similar things when, in this condition, he takes one of the two dates . . . whichever of the two dates he may take, his aim will be attained and his desire satisfied. His will attaches itself therefore merely to the distinction between the fact of taking one of them and the fact of leaving them altogether; it attaches itself by no means to the act of taking one definite date and distinguishing this act from leaving the other (that is to say, when it is assumed that the desires for the two are equal); he does not prefer the act of taking the one to the act of taking the other, but he prefers the act of taking one of the two, whichever it may be, and he gives a preference to the act of taking over the act of leaving. This is self evident. For distinguishing one from the other means giving a preference to the one over the other, and one cannot give a preponderance to one of two similar things in so far as it is similar to the other—although in their existence as individuals they are not similar since each of two individuals is different from the other by reason of a quality exclusive to it. If,

[18] S. van den Bergh, p. x of his Introduction to the *Tahâfut al-Tahâfut*. Compare St. Paul: "Nay but, O man, who art thou that repliest against God? Shall the thing formed say to him that formed it, Why hast thou made me thus? Hath not the potter power over the clay, of the same lump to make one vessel unto honour and another unto dishonour?" (*Romans* 9:20-21). Cp. also Omar Khayyam's *Rubaiyât*.

> therefore, we assume that the will attaches itself to that special character of one of them, then it can be imagined that the will attaches to the one rather than the other because of the element of difference existing in both. But then the will does not attach to two similar objects, in so far as they are similar.[19]

Essentially, then, Averroes’ position was that: (1) it is necessary to grant the preferability of taking-one-date over against taking-neither-date, but (2) there would be no reasonable way of choosing one particular date were it actually to follow from the hypothesis of the problem that there is no reason for preferring one over the other, however, (3) since there are two distinct dates, they must be *distinguishable* so that there must be some element of difference—at least a difference in *identity*—between them, and the will can and must therefore fix upon such an element of difference as a “reason” for preference. Thus Averroes simply reasserts—in the teeth of Ghazâlî’s example—the impossibility of choice without preference. And he resolves the impasse by having a difference of the sort that must inevitably exist provide the “reason” for a choice.[20]

The obvious criticism of Averroes’ solution is implicit in the quotation marks that have been put about the word *reason*. For it is assumed in the

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[19] *Tahâfut al-Tahâfut*, vol. I, pp. 22-23.

[20] In his very valuable footnotes on Averroes’ text, S. van den Bergh, the learned translator of the *Tahâfut al-Tahâfut* into English, writes: “Averroes misses the point here completely. Certainly the donkey will take one or the other of the two bundles rather than die, but the question is what determines its taking the one rather than the other. Obviously it will take the one that comes first to hand; only, when there is a complete equivalence of all conditions, this is impossible, and Spinoza says bluntly that the donkey will have to die. As a matter of fact, in such cases a complete equivalence of psychological and physical conditions is never reached; no living body even is strictly symmetrical, and if *per impossible* such an equivalence could be momentarily reached, the world is changing, not static, and the donkey will move and not die” (*Tahâfut al-Tahâfut*, vol. II, p. 20). The point here is twofold: (1) that a complete equilibrium of opposing motivations can never actually be reached, and (2) that even if such an equilibrium, albeit impossible, were to be reached, such a condition would necessarily pass due to an inherent instability. The first of these has been asserted in the present context by numerous writers—Montaigne, Bayle, and Leibniz among others—and we shall return to it below. However, van den Bergh is the first to urge the second thesis: that a psychological equilibrium would be intrinsically unstable, and would become resolved because “the world is changing, not static”. However, since physical equilibrium is in theory possible in a changing world, it would seem that a better case must be made out for this thesis.

defining statement of the problem that the differences among the objects are such as to have no rationally valid bearing on the matter of their relative preferability. There therefore is, *by hypothesis*, no legitimacy or validity from the standpoint of reasonableness, in any attempt to base a reasoned preference upon these differences.

Khôdja Zâdeh (1415-1488)

The Turkish philosopher Khôdja Zâdeh, in his reply to Averroes' criticism of the theologians, written in Arabic, again under title of Ghazâlî's work *Tahâfut al-Falâsifa*, takes up the problem of choice without preference just where Averroes had left it (of course without knowledge of the intervening Western discussions). Taking the part of Ghazâlî against Averroes, Khôdja Zâdeh argues that a genuinely "free" agent can *ipso facto* resolve the paradox of choice without preference:

> If one puts a loaf of bread before a hungry man, he will begin to eat a certain part to the exclusion of all others, without determination of a volition favoring this part to the rest. You object: "I do not grant that he will begin to eat a part without determination of a volition in its favour; for why would not this volition have for its deciding reason that one part is closer to him, or more appealing, or better baked?" I reply: "By hypothesis it has been assumed that all is, without exception, alike in each part of the loaf. And so the man either cannot start to eat some one particular part and will therefore starve (which is manifestly absurd), or else he will start somewhere to satisfy his desire."[21]

The force of the example is thus presumably to militate for acknowledging the difference between the realm of a rational nature where a balance of forces creates equilibrium and the rational realm where a balance of reasons leaves room for free will.

Khôdja Zâdeh then goes on to give a more sophisticated example of choice without preference, which is cited here in Léon Gauthier's epitome:

> If one can demonstrate . . . that in some instance God [in creating the world] must choose among two or more strictly equivalent alternatives, one will have upset in one decisive stroke . . . the premiss on which the argument of the Hellenizing philosophers is founded, the principle of sufficient reason. But there are numerous such instances. . . . Thus with any of the celestial spheres (of Greco-Arabian astronomy) God has an arbitrary choice among an infinity of strictly identical alter-

[21] Rendered from the French of Léon Gauthier's translation in "L'Argument de l'Ane de Buridan et les Philosophes Arabes," *Mélanges René Basset, op. cit.*, pp, 209-233; see pp. 227-228.

> natives in selecting the two points which serve as poles or the circle which serves as equator or the line which serves as axis. And again, with respect to the motion of each sphere, a direction of rotation and a particular speed must be chosen arbitrarily; similarly on each eccentric there is an arbitrary choice of a center for the epicyclic sphere and on this sphere itself the place of the planet which it carries must be selected, and so on.[22]

In this astronomical formulation, the example is transformed from a choice between two indifferent alternatives into one among infinitely many, a complication which induces no fundamental change in our problem. Like some Western writers, Khôdja Zâdeh is satisfied with insisting *that* "free will" can resolve a situation of choice among indifferent objects, without explaining *how* this is possible.

St. Thomas Aquinas (c. 1227-1274)

In discussing "Whether Man Chooses of Necessity or Freely" Aquinas employed the example of choice without preference as a means of formulating a possible objection to the thesis of freedom of the will (an objection which he subsequently endeavors to refute).

> If two things are absolutely equal, man is not moved to one more than to the other; thus if a hungry man, as Plato says,[23] be confronted on either side with two portions of food equally appetizing and at an equal distance, he is not moved towards one more than to the other; and he finds this the reason of the immobility of the earth in the middle of the world.[24] Now if that which is equally (eligible) with something else cannot be chosen, much less can that be chosen which appears as less (eligible). Therefore if two or more things are available, of which one appears to be more (eligible), it is impossible to choose any of the others. Therefore that which appears to hold the first place is chosen of necessity. But every act of choos-

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[22] *Ibid*., pp. 229-230. Gauthier thinks that Khôdja Zâdeh derived the basic idea of his example from the Muslim theologians (*ibid*., pp. 230-231). See also Léon Gauthier, *Ibn Rochd* (*Averroes*), (Paris, 1948), pp. 221-222. These astronomical examples are only variations on a theme of al-Ghazâlî. (See *Tahâfut al-Tahâfut, op. cit*., pp. 124, 144.)

[23] Aristotle is apparently meant here, though there is a transition to Plato toward the end of this sentence.

[24] The text I am quoting reads "and he finds the reason of this in the immobility of the earth in the middle of the world"—which simply does not make sense. The original reads: " . . ., *ut Plato dicit, assignans rationem quietis terrae in medio*."

> ing is in regard to something that seems in some way better. Therefore every choice is made necessarily.[25]

An obvious weakness in this argument obtains with regard to its question-begging presupposition that "every act of choosing is in regard to something that seems in some way better." For the problem of choice without preference arises when just this condition is falsified, and it is here that our determinist's case is weakest (cf. the discussion of Spinoza below). However, Aquinas does not capitalize on this weakness of the objection. After a general critique of determinism, he returns briefly to the example of choice among equals, in effect dismissing it summarily:

> If two things be proposed as equal under one aspect, nothing hinders us from considering in one of them some [other] particular point of superiority, so that the will has a bent towards the one rather than towards the other.[26]

Thus Aquinas does not view the problem of choice among equals as a hopeless paradox which condemns its victim to utter inaction, since—so he insists—the will has the capacity of viewing them under some aspect under which one of them is accorded "some particular point of superiority". Aquinas' position is not far removed from Ghazâlî's mere will, capable even in the absence of difference of itself of providing a differentiation to facilitate a rational choice. But this too begs the pivotal question. For what happens when that "point of superiority" is absent? (Clearly we cannot synthesize it *ex nihilo*.)

Dante (1265-1321)

Our problem now for the first time steps forth upon the stage of world literature. The events of the *Divine Comedy*, Paradiso III, bring to Dante's mind two perplexing moral problems which—in Canto IV—he wishes Beatrice to clarify for him: Does an evil action performed under duress detract from the moral merit of the agent? Can a good action done in atonement lessen the moral onus of a wrongful deed?

> Between two foods alike to appetite, and like afar, a free man, I suppose, would starve before of either he would bite;

[25] *Summa Theologica*, II, i, 13.6. Cited from the translation of the Fathers of the English Dominican Province (2nd edition, London, 1927).

[26] *Loc. cit.*

> So would a lamb, between the hungry throes of two fierce wolves, feel equipoise of dread, so hesitate a hound between two does.
>
> Whence by my doubts alike solicited inevitably, censure can be none, nor commendation if I nothing said.
>
> And I said nothing; but desire upon my face was pictured, questioning as well set forth more fervently than words had done.
>
> * * *
>
> So Beatrice did, and said: "I see one yearning and the other draw thee so, that eagerness ties up thy tongue to breathe no dear concerning."
>
> * * *
>
> These questions balance, equally the beam of thy desire.

In his notes on this passage[27] the translator aptly remarks that, "It is in artistic keeping that a Canto dealing so largely with the dilemma of the broken vow should begin with this ancient paradox." Furthermore, it is noteworthy that this problem context of punishment and reward in the world to come in which the example occurs in Dante, is essentially the same as in discussions by Arabic philosophers and theologians (see the conclusion of the foregoing discussion of Ghazâlî). However, the problem as presented by Dante is that of conflict among equal desires (or fears—here for the first time), as in Aristotle's formulation, and not that of choice between essentially identical objects, as with Ghazâlî and the Arabs.

[27] Dante, *Divine Comedy*, II, 1-26 (with deletions). I quote from the translation prepared by Melville B. Anderson for the Oxford University Press edition of the *Divine Comedy* in "The World's Classics" series. Many of the great philosophical problems and controversies of the age are discussed in the *Divine Comedy*. [NR—check the reference]

Buridan (c. 1295-1356)

It has long occasioned astonishment that Buridan's Ass is nowhere to be met with Jean in Jean Buridan's writings.[28] Among others, Bayle, Schopenhauer, and Sir William Hamilton attest to long hours of fruitless search.[29] Bayle has even conjectured that the phrase "Ass of Buridan" may first have gained currency in connection with an entirely different point of logical difficulty or complexity discussed by Buridan as a *pons asinorum* in logic,[30] and subsequently the phrase came to be shifted in its application to the well-known ambivalence example.[31]

There is no question, however, but that Buridan was familiar, in essence with the example to which he lent his name. In his unpublished commen-

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28 See B. Geyer in the 11th (last) edition of vol. II, *Die Patristische und Scholastische Philosophie* of F. Ueberweg's *Grundriss der Geschichte der Philosophie* (Berlin, 1929), p. 597.

29 Bayle writes: "The Ass of Buridan was a kind of proverb or example which was long used in the schools. I do not know if I have determined with precision just what it was, for I have found no one able to explain it to me, nor any book that enters into detail on this matter" (*Dictionnaire*, art. "Buridan"). Schopenhauer writes that, "one has now been vainly searching his writings for some hundred years" for the ass of Buridan, and that, "I myself own an edition of his *Sophismata*, apparently printed already in the fifteenth century . . . in which I have repeatedly searched for it in vain, although asses occur as examples on virtually every page" (*Prize Essay on the Freedom of the Will*, p. 58 of the original edition). Sir William Hamilton states that, "the supposition of the ass, etc., is not, however, as I have ascertained, to be found in his [i.e., Buridan's] writings" (*Reid's Works*, ed. by W. Hamilton, vol. I, p. 38 in the seventh, eighth, and possibly other editions). Pierre Duhem writes, "I have searched in vain for the argument of the ass in all of the writings attributed to Buridan; in those places where it might reasonably occur, we encounter instead wholly different examples" (*Études sur Léonard de Vinci*, Vol. III [reprint, Paris, 1955], p. 16).

30 This would probably have been the set of rules for determining suitable middle terms in the construction of syllogistic arguments in support of a given conclusion, which have long been ascribed (incorrectly) to Buridan. See B. Geyer's revision of Vol. II of Ueberweg's *Grundriss*, *op. at.*, p. 597.

31 Bayle, *Dictionnaire*, art. "Buridan." Bayle also suggests (*ibid*.) that the phrase may originally have referred to the "*an*" (Latin) of Buridan—along the lines of *utrum* as common in Scholastic usage—and subsequently metamorphosed into the "asne" (French) of Buridan. This "explanation" is rather far-fetched.

tary on Aristotle's *De Caelo*,[32] in a gloss on the very passage of II, 13 which we had occasion to examine above, Buridan gives the example of a dog—not an ass!—dying of hunger between two equal portions of food.[33] It is clear, however, that this transposed example in an obscure manuscript could scarcely have been the direct origin of the notorious paradox, and that it must have been associated with Buridan in some more immediate and prominent way. It is highly probable that the example was given by Buridan (in its henceforth traditional description of an ass placed between equally appetizing heaps of hay) in some more memorable manner, possibly in one of his several yet unpublished commentaries on Aristotle, or perhaps it arose in a verbal context, either in his widely reputed lectures, or in oral disputation or discussion.[34]

At any rate, the example of the ass fits in a very natural and congenial way into the problem context of Buridan's theory of the will. In his *Quaestiones* on the *Nicomachean Ethics*, Buridan treats of the problem of the

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[32] I am here referring to Buridan's *Expositio textus* of the *De caelo*, and not the *Quaestiones* which he also devoted to that work. The former is unpublished, and exists in only two MS versions: Bruges 477 (210 v-238v), and Vat. lat. 2162 (57r-79r). (See Anneliese Maier, *Zwei Grundprobleme der Scholastischen Naturphilosophie* [Rome, 1951], p. 205.) The *Quaestiones super libris quattuor de Caelo et mundo* have been published by E.A. Moody (Cambridge, Mass., 1942). In this work there is, however, no mention of our example.

[33] See the article "Buridan" by L. Minio-Paluello in the *Encyclopedia Britannica* (1956 edition). This almost, though not quite, bears out Schopenhauer's conjecture that Buridan's example was adopted form that of Aristotle's man perplexed by a choice between food and drink, but that Buridan, "changed the man to an ass, solely because it was the custom of this parsimonious Scholastic to take for his example either Socrates and Plato, or *asinum*" (*Freedom of the Will*, p.59). It would clearly be unseemly to present the greats in perplexity.

[34] This latter possibility would accord will with the oft-voiced conjecture that the example of Buridan's Ass actually derives from an *objection* to Buridan's views. (See, for example, B. Geyer's revision of Ueberweg's *Grundriss*, p. 597.) Correspondingly, Sir William Hamilton has plausibly conjectured that "perhaps it [i.e., the example of Buridan's Ass] was orally advanced in disputation, or in lecturing, as an example in illustration of his Determinism; perhaps it was employed by his opponents as an instance to reduce that doctrine to absurdity" (*Reid's Works*, ed. by Hamilton, vol. II, p. 690). We know that for many years Buridan *professa dans l'université de Paris avec une extrème reputation* (Bayle, *Dictionnaire*, art. "Buridan").

human freedom.[35] He asks: "Would the will, having been put between two opposites, with all being wholly alike on both sides, be able to determine itself rather to one opposed alternative than to the other?"[36] As an illustration of a problem of this type, Buridan addresses the situation of two alternative routes leading to the same destination, though not, alas, our ass example.[37]

Buridan's answer to the problem of indifferent choice is given in terms of his theory of the will. The will, he holds, does not decide spontaneously from within its own resources, but is subject to the commands of reason. As reason judges, so rules the will. When reason deems one object a greater good than another, the will can only opt—other things being equal—for the higher good. Should two of its objects be adjudged by reason as wholly equivalent, the will will be unable to act by breaking the deadlock of itself.[38] Buridan supports this intellectual determinism of the will by saying that those who claim free will for man but deny it to animals find themselves in difficult straits:

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[35] *Quaestiones super decem libros Aristotelis ad Nicomachum*, III, I.

[36] Translated from the quotation given by P. Duhem, *Études sur Léonard de Vinci*, vol. III, pp. 17-18 (reprint, Paris, 1955). This formulation of the problem of freedom derives from Buridan's master. William of Ockham, who characterizes freedom as: *potestas qua possum indifferenter et contingenter effectum ponere, ita quod possum eundem effectum causare nulla diversitate circa illam pontentiam facta (Quodlibeta*, vol. I, p. 16).

[37] The instance he gives is the following "I could go from Paris to Avignon either *via* Lyon or Dun-le-Roy." Ludovico Molina says that "Cyril of Alexandria wrote in the third chapter of his four-book commentary on St. John's Gospel: 'Man is an animal that has freedom, and can choose to elect either the right or the left road (i.e., either virtue or vice)'" (*Concordia liberi arbitrii*, XIV, xiii, 23 §4). Buridan also discusses the problem of choice confronting the mariner in a stormy sea, agonized whether to jettison his cargo or risk his life. See P. Duhem, *Études sur Léonard de Vinci*, vol. III, p. 18.

[38] See Albert Stöckl, *Geschichte der Philsophie*, 2nd edition (Mainz, 1875), p. 531. Buridan does, however, grant to the will the status of a *facultus suspensiva* which, while it cannot go *counter* to reason, can suspend option so that, should the position of reason change upon further examination of the matter of its initial judgment, it would be possible to choose the object *now* deemed the greater good, though formerly deemed the lesser (*ibid*).

> It seems to me that, to show the difference between the freedom of our will and the lack of freedom to which the actuating faculty of a dog is subject, it would be better to trust to faith than to natural reason.[39] For it would be difficult indeed to show that when our will is wholly indifferent between two opposed acts, it [in contradistinction to the actuating faculty of a dog] could decide for one or the other alternative without being so determined by some external factor.[40]

It is therefore easy to see how, in the context of Buridan's theory of will, the ass example might with its characteristic double-edgedness serve either (1) as a somewhat drastic example in illustration of Buridan's intellectual determinism of the will, or (2) as an example adduced by Buridan's opponents on an attempt to render this doctrine absurd.

In any case, the story of "Buridan's Ass" passed (in various guises) even into the popular lore of all the European peoples. I cite as one instance the Spanish folktale, apparently of late medieval vintage:

> EL BURRO DE BURIDÁN: Una día el burro de un filósofo llamado Juan Buridán—y por eso llamado el burro de Buridán—perece de hambre y sed. Teniendo a un lado una gran cantidad de avena y otro un cubo de agua, el burro nunca puede saber si tiene sed o hambre. El burro no sabe que decidir: si comer o beber. En esta horrible vacilación le sorprende la muerte.[41]

Rabelais (ca. 1490-1553)

In Francisco Rabelais' *Gargantua and Pantagruel* our example once again receives a literary treatment:

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39 This is Buridan's final and considered position on the subject of the freedom of the will: he holds this *not* to be subject to philosophical demonstration or refutation, but a matter of *faith*. (See pp. 84-85 of K. Michalski, "Les courants philosophiques à Oxford et à Paris pendant le XIV^{e} Siècle," *Bulletin Internationale de l'Académie Polonaise des Sciènces et des Lettres* [Classe d'Histoire et de Philosophie, 1919-1920], pp. 59-88.)

40 Buridan, *In Metaphysicam Aristotelis Quaestiones*, quoted by P. Duhem in *Études sur Léonard de Vinci*, vol. III, pp. 20-21. Duhem in this work attributes these *Quaestiones* on the *Metaphysics* to *another* John Buridan, but in the face of manuscript evidence discovered by himself, he subsequently reverses himself (*Le système du monde*, vol. IV,p. 126).

41 Angel Flores, *First Spanish Reader* (New York, 1964), p. 2. This balancing of hunger and thirst carries us back to Aristotle.

> At Pantagruel's birth, none was more amazed and perplexed than his father Gargantua. On the one hand, he saw his wife Badebec dead, on the other, his son Pantagruel, large as life and much noisier. He was at a complete loss what to say or do. A terrible doubt racked his brain: should he weep over the death of his wife or rejoice over the birth of his son? On either hand, sophistical arguments arose to choke him. He could frame them capitally *in modo et figura*, according to the modes and figures of the syllogism in formal logic. But he could not resolve them. So there he was, fretting like a mouse caught in a trap, or a kite snared in a gin.[42]

Giving full play to his provocative genius, Rabelais devised a highly dramatic and characteristically tragicomic setting for this ancient problem.

Montaigne (1533-1592)

In his *Essais*, Michel de Montaigne discusses the problem of choice without preference—again in Aristotle's formulation—as an intellectual curiosity, a difficulty of the sort that give spice and stimulus to the cultivation of philosophical speculations, that curious pursuit of the paradoxical creature *homo sapiens*:

> It is a pleasant imagination, to conceive a spirit justly ballanced betweene two equall desires. For, it is not to be doubted, that he shall never be resolved upon any match: Forsomuch as the application and choise brings an inequality of prise: And who should place us betweene a Bottle of Wine and a Gammon of Bacon, with a equall appetite to eat and drinke, doubtlesse there were noe remedy but to die of thirst and hunger.[43]

However, in condemning Buridan's ass to death, Montaigne proposes to draw the venom of the paradox, by reducing it to the status of a strictly abstract and purely fanciful *hypothetical* difficulty that could not possibly arise in a real or *practical* context.

> In my opinion, it might . . . be said, that nothing is presented unto us, wherein there is not some difference, how light so ever it bee: and that either to the sight, or to the feeling, there is ever some choise, which tempteth and drawes us to it, though imperceptible and not to bee distinguished.[44] In like manner, hee that shall

[42] Rabelais, *Gargantua and Pantagruel*, Bk. II, p. 3, tr. by Jacques LeClerq (Modern Library edition).

[43] *Essais*, Bk. II, chap. 14. Cited from John Florio's translation (Everyman's Library edition, Bk. II, p. 333).

> presuppose a twine-thrid equally strong all-through, it is impossible by all impossibilitie that it breake, for, where could you have the flaw or breaking to beginne? And at once to breake in all places together, it is not in nature.[45]

Montaigne’s resolution of the problem flatly maintains that strict identity among objects “is not in nature,” so that choice among identicals becomes a purely imaginary complication.

Gataker (1574-1654)

A most interesting discussion of the uses of random selection occurs in the study *Of the Nature and Use of Lots: A treatise Historical and Theological* by Thomas Gataker a sixteenth century English scholar and divine, first published in London in 1616 (second edition, here cited, published in 1627), Gataker considered a great number of historical examples of the use of lots in the Old and New Testament (e.g., the selection of a successor to the apostle Judas, *Acts*, 1:23-26); in the assignment of priesthoods and public offices in Greece; in Hebrew, Greek, Roman, and other legal practice; and the like. He defined a “lot” as an “event merely casual purposely applied to the deciding of some doubt” (p. 9), “casual events” being “such as might all out in like sort diversely, and are not determined by any art, foresight, forecast, counsell, or skill of those that either act them, or make use of them” (p. 14). Quoting with approval the dictum that “chance is founded, and dependeth upon Man’s ignorance (*fortuna in ignorantia nostra fundatur*)” (p. 37), Gataker criticized the view that “a Lot discovereth to men God’s hidden will” (p. 25), and argued that “Lots are not to be used in [a] question of Fact past and gone . . . for that is no ordinarie Lot able to decide; but where some question is who has the right to a thing; in which case, notwithstanding the Lot is not used to determine who in truth hath right to it, but who for peace and quietnesse sake shall enjoy it” (p. 148). Gataker insisted that, “concerning the matter or businesses wherein Lots may lawfully be used, the rule of Caution in general is this, that Lots are to be used in things indifferent onely” (p. 125), for:

> . . . many good things there are that may at sometime be done, where of a man may make chose whether of them hee will doe, being not necessarily tied unto, or enjoyned any one of them: As for a student having divers bookes about him in his study, it is indifferent to choose one, this or that, refusing the rest, for present em-

44 Compare Leibniz’ “Petites perceptions.”

45 *Essais*, Bk. II, chap. 14.

> ployment, there being no speciall occasion to urge the use of one more than another: Or for a man that carrieth a pair of knives about him, it is indifferent to draw and use either when occasion requireth (as Plutarch says, *de Stoic, contradict.*). (P. 128.)

Gataker's distinguished clerical career was brought into jeopardy by accusations of favoring games of chance, growing out of his defense of the use of lots. He has the distinction of being the first to suggest the employment of random-selection devices as a means of resolving the problem of indifferent choice in public policy situations where some preferential selection is desirable "for peace and quietnesse sake".

Spinoza (1632-1677)

The problem of choice without preference was taken up by Benedict de Spinoza as a source of possible objection to determinism. If two objects of choice are essentially identical (so that there is no difference in the relevant causal factors militating for selection of one *vis-à-vis* the other), and it is granted that a selection of one of them is possible, would this not reveal a rift in the framework of causal determinism? If choice in situations of indifference were accepted as possible, would this not concede the operation of a free will capable of supplementing causal determinations in such cases, and thus possibly even supplanting them in others?[46]

> It may be objected that if a man does not act from freedom of the will, what would he do if he were in a state of equilibrium, like the ass of Buridanus? Would he not perish from hunger and thirst? and if this be granted, do we not conceive him as a statue of a man or an ass [i.e., rather than as a real human being or animal]? If I deny that he would thus perish, he will consequently determine himself and possess the power of going where he likes and doing what he likes.[47]

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[46] This argument underlies use of the phrase "liberty of indifference" regarding which Dugald Stewart writes: "The phrase *Liberty of Indifference,* . . . has been so frequently substituted . . . for the older, simpler, and much more intelligible phrase of *Free-will*. . . . It certainly conveys but a very inadequate notion of the thing meant;—the power, to wit, of choice or *election*; and that not only among things indifferent, but (*a fortiori*) between right and wrong, good and evil" (*Active and Moral Powers*, Appendix on Free Agency, iii). Insistence on the important of *indifference* of will for free (and thus morally responsible) action goes back to Duns Scotus. (See C.R.S. Harris, *Duns Scotus* [Oxford, 1927], vol. II, p. 309.)

[47] Spinoza, *Ethics*, vol. II, final Scholion, quoted from the translation of W.H. White and A.H. Stirling (Oxford, 1927). This passage does not show Spinoza at his best, since it naively depicts determinism as incompatible with "the power of going

Spinoza's imagined opponent here presses the determinist with the objection that surely a *real* agent would not rest inactive in a case of choice under conditions of equilibrium or stalemate among opposing determinations. Spinoza, undaunted by the objection, maintained that—however unreasonable such inactivity might seem—it is just precisely what *would* actually *have* to happen:

> With regard to the objection, I say that I entirely grant that if a man were placed in such a state of equilibrium he would perish of hunger and thirst, supposing he perceived nothing but hunger and thirst, and the food and drink were equidistant from him. If you ask me whether such a man would not be thought as ass rather than a man, I reply that I do not know; nor do I know what ought to be thought of a man who hangs himself, or of children, fools and madmen.[48]

In Spinoza's discussion, then, the problem of Buridan's Ass recurs in its Thomistic setting, in the context of the free-will issue. With Aquinas, however, the example served as part of an (ultimately rejected) argument *against freedom of the will*, while with Spinoza it becomes part of an (ultimately rejected) *objection to a thorough-going determinism* with respect to the choices of responsible agents.[49] For Spinoza makes short shrift of the objection, by insisting that where opposing motivations are actually in strict equilibrium, inaction is the only arguable result. Like Leibniz after

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where [one] likes and doing what [one] likes". Leibniz agrees with Spinoza in opposition to those who hold that the locus of human liberty of will is to be sought in situations of indifference of choice: "We [can] become as it were masters of ourselves, and make ourselves think and do at the time as we should *wish* to will and as reason commands. But it is always through determined paths, and never *without a reason*, or by means of the imaginary principle of perfect indifference or equilibrium. . . . I here say *without a reason* to mean without the opposition of other inclinations, or without being in advance disposed to turn aside the mind, or without any other means equally explicable. To assert otherwise is to revert to the chimerical, as in the empty faculties or occult qualities of the scholastics, in which there is neither rhyme nor reason" (*New Essays*, II, xxi, 47; my translation largely follows that of A.G. Langley [LaSalle, various dates]).

48 Spinoza, *Ethics*, vol. II, final Scholion. "There are more things in heaven and earth, Horatio, than are dreamt of in your philosophy." Spinoza seemingly has the rare candor to admit this with respect to his own system.

49 Note that in both instances the example is used in support of positions which the authors are endeavoring to rebut (though contrary positions to be sure). It is interesting that the example so often occurs in this manner.

him, Spinoza was willing to push the principle of sufficient reason to its logical conclusion.

Bayle (1647-1724)

Pierre Bayle provided what may be seen as perhaps the most comprehensive and the most competent *substantive* discussion of the problem of Buridan's Ass.[50] His discussion of the problem in his *Dictionnaire* shows that Bayle was clearly aware of the role of the example in relation to the issue of freedom of the will; he perceived its bearing on the question of the amenability of God's choices to human rationalization; and lastly, and very importantly, Bayle recognized its relevance to actually occurring situations of strict logical indifference of choice, and comments on the procedures of resolution actually used in such cases.

Bayles views the example of Buridan's Ass was applicable—in the first analysis—to the issue of man's freedom of will:

> I had long thought that it [i.e., Buridan's Ass] was an example that Buridan gave to illustrate the state of dependence in which animals stand with respect to their objects of sensation. Those who maintain the doctrine of Free Will in its correct sense assert that man has the capacity to determine himself to the left or to the right in cases in which the motivating forces are equally strong in the opposed objects. For they claim that the human soul is able to say—without having any reasons other than the use of its freedom—*I prefer this to that, although I see nothing in it which makes it more worthy of my selection than the other*. But they do not grant this capacity to dumb animals, holding that these are not able to determine themselves in the presence of two objects which draw them equally to either side. For example, a famished ass would die of starvation between two bushels of oats acting equally upon its faculties; for, having no reason to prefer one to the other, it would remain motionless just like a piece of iron between two magnets of equal force. The same would happen if hunger and thirst pressed equally and before it [i.e., the ass] were a bushel of oats and a bucket of water which acted with equal force upon its faculties. It would not know which way to turn—if it ate before drinking, it would have to be because its hunger was greater than its thirst, or else the appeal of the water feebler than that of the oats, which is contrary to the hypothesis. Buridan would thus have made use of this example to show that unless external forces determine

[50] This is so despite Bayle's disclaimer, "I do not know if I have properly grasped [the Problem of Buridan's Ass] for I have been unable to find anyone who could explain it to me, not any book that enters into detail on this subject" (*Dictionnaire*, art. "Buridan."). There is a vitiating flaw in Bayle's discussion of the *history* of the problem: he does not realize that it antedates Buridan.

> an animal, its own soul has not the capacity to effect a choice between equal objects.[51]

However, Bayle is troubled by the element of sophistry inherent in the double-edged character of the example in its application to the problem of freedom. He therefore thinks it proper to moot the possibility that the example of Buridan's Ass was introduced as a mere debating trick, a paradox to confound opponents in scholastic debate.

> I have since had another thought, to wit that Buridan's Ass may have been a Sophism propounded by that philosopher as a kind of dilemma, so that whatever response to it one gave him, he would draw embarrassing conclusions from it. He would hypothesize either a famished ass, placed between two bushels of oats and attracted to them equally, or an ass pressed equally by hunger and by thirst, placed between a bushel of oats and a pail of water which act with equal attraction upon its faculties. Having made this supposition, he queried: "What would the ass do?" If the response is given that it rests immobile, he counters: "What! The ass would starve between two bushels of oats, or die of thirst and hunger, though in the proximity of food and drink!" This would seem absurd and would enable him to win all of the humorous to his cause against him who had given the reply. If, on the other hand, one replied to him that the ass would not be so foolish as to die of hunger or thirst in such a situation, he replies: "What! It would go to one side rather than to the other, although nothing whatever urges more strongly in that direction than its opposite. So then it is either endowed with free will, or else you are saying that given two weights in balance or equilibrium, one would be able to move the other." These two consequences being absurd, there would be no way out but to concede that the ass would find itself more strongly attracted by one of the objects—but this is contrary to the hypothesis. Thus Buridan would have the upper hand no matter what reply would be given to his query.[52]

Scholasticism is, in Bayle's opinion, particularly susceptible to use of a debating trick of this kind, due to its insistence on the determinability of reasoned causes for all things.

> It is small wonder that the Ass of Buridan became renowned in the Schools. . . . The Scholastics tormented themselves in such manner to assign a cause to every effect, that they sought the reason, for example, why any one particular degree of heat is produced rather than another. Heat according to them is a type of quality

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[51] Pierre Bayle, *Dictionnaire,* art. "Buridan." The translations given from this source are my own.

[52] *Ibid*.

> that comprehends within itself an infinity of particular possibilities. Fire, for example, realizes one of these particular possibilities every time it warms water—but why rather one [resulting temperature] than the other? Consider this from every standpoint, and you will find no one constant fixed factor excepting in the will of God alone. It becomes necessary here to violate the Scholastic Axiom, *Non est Philosophi recurrere ad Deum*. . . . If you try to ascend above this point, and ask why God chose rather one particular degree of heat than another, one must answer you: His supreme independence gives him the right and power of choice without determination by any superiority in the object of choice.[53]

Thus Bayle also appreciated the bearing of the example on the problem of the amenability of God's choices to human rational explanation. (He was apparently the first writer in the Christian tradition to do so.)

Bayle's own view on the issue of choice without preference is that the sorts of examples which have traditionally been used to illustrate the problem have placed it in an unrealistic light, depicting it as a purely ideal or theoretical difficulty which could not arise in fact. He says that, "my own belief [is that the case of] an ass, famished and attracted equally towards two bushels of oats, and remaining immobile because of this equal attraction would be a physical impossibility (*un cas physiquement impossible*)."[54] Such examples do not do justice to the problem, Bayle feels, because situations of indifference can and do arise in real-life contexts, and when they do so, they do not end in an impasse, since means of extrication exist:

> There are at least two ways in which a man can free himself from the snare of this equilibrium. The one is that which I have already mentioned—to flatter himself with the pleasing notion that he is master in his own house, not dependent upon his objects, and perform one of the acts, saying, "I choose to give this one preference over that, simply because it suits me to do so." And here what determines him is not bound up with the object; the determining motive derives solely fro the sphere of ideas that men have regarding their perfections, or rather their natural endowments. The other mode of resolution is that of fate or chance. A man is assigned the task of deciding the precedence of two ladies at court. If he finds nothing about them to support a determination, and it is quite necessary that he must give one precedence over the other, he would not be brought to a standstill, for he would simply have them draw straws. The same would be done in case a man has engaged himself to play at cards with each of two ladies, and wishes to avoid giving

[53] *Ibid*.

[54] *Ibid*.

either of them any shade whatever of preference. The short straw would decide with whom he would play first. Thus the equilibrium need not render its victim immobile, as Spinoza would have it. One would be able to find the remedy.[55]

Bayle may thus be regarded as being, along with Gataker, among the first explicitly to recognize in the paradox of Buridan’s Ass a generic problem of choice without preference that does in fact arise in actual situations. And this recognition enables him to discuss in a fresh and realistic way the resolution of this problem, and to indicate the devices by which in fact and practice people can extricate themselves from perplexities of this kind, to wit, the instrumentalities of randomness.

Leibniz (1646-1717)

The ass of Buridan is immobilized in the setting of G. W. Leibniz’s philosophy because on Leibnizian principles there must be a *sufficient reason* for all occurrences, and this condition would be violated in the example. Thus in his third letter to Clarke, Leibniz writes:

> My axiom has not been fully understood, and . . . the author [i.e., Clarke] while seeming to grant it, has really denied it. “It is true,” he says, “that nothing exists without a sufficient reason why it is, and why it is thus rather than otherwise,” but he adds that this sufficient reason is often the simple or *mere will* of God. . . . But this is simply maintaining that god wills something without there being a sufficient reason for his will, contrary to the axiom. . . . This is to relapse into the loose indifference which I have amply refuted [in the *Theodicy*] and which I have shown to be absolutely chimerical, even in created beings . . .[56]

However, Leibniz’s Principle of Sufficient Reason would not sentence Buridan’s poor animal to death, for a way out of the impasse is made possible by his concept of *petites perceptions*, infinitesimal psychic occurrences beneath the threshold of any conscious awareness, which can act as imperceptible motivations in effecting a choice.

> All our unpremeditated actions are the result of a concurrence of *petites perceptions*, and even our habits and our passions, which so much influence our [conscious] deliberations, come therefrom. . . . I have already remarked that he who would deny thee effects in the sphere of morals would imitate those ill taught per-

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[55] *Ibid*.

[56] §7 of the third letter to Clarke.

> sons who deny insensible corpuscles in physics. And yet, I see that among those who discuss freedom of the will there are some who, taking no notice of these unperceived impressions which are capable of inclining the balance, imagine an entire indifference in moral actions, like that of the ass of Buridan equally torn between two meadows.[57]

And again, in the *Theodicy*, Leibniz writes:

> There is never any *indifference of equipoise*, that is [situations of choice] where all is completely even on both sides, without any inclination towards either . . . By this false idea of an indifference of equipoise the Molinists were much embarrassed. They were asked not only how it was possible to know in what direction a cause absolutely indeterminate would be determined, but also how it was possible that there should finally result there from a determination for which there is no source. To say with it that there is the privilege of the free cause is to say nothing but only to grant that cause the privilege of being chemical. In consequence of this, the case also of Buridan's ass between two meadows, impelled equally towards each of them, is a fiction that cannot occur in the universe, in the order of Nature, although M. Bayle may be of another opinion . . . For the universe cannot be halved by a plane through the middle of the ass, which is cut vertically through its length, so that all is equal and alike on both sides . . . Neither the parts of the universe nor the viscera of the animal are alike nor are they evenly placed on both sides of this vertical plane. There will therefore always be many things in the ass and outside the ass, although they may not be apparent to us, which will determine him to go to one side rather than the other. And although man is free, and the ass is not, nevertheless for the same reason it must be true that in man likewise the case of a perfect equipoise between two courses is impossible.[58]

Thus Leibniz' position is neatly summarized in his correspondence with Clarke:

> In things which are absolutely indifferent there can be no choice and consequently no election or will, since choice must have some reason or principle.[59] To say that the mind may have good reasons for acting when it has no motives, and when

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[57] G. W. Leibniz, *New Essays*, II, i, 15. My translation follows that of A.G. Langley (La Salle, various dates).

[58] G. W. Leibniz, *Theodicy*, §§46-49. (I quote the translation by Austin Farrer [New Haven, 1932].) Cf. also §§302 ff. for Leibniz' critique of Bayle's discussion of indifference of choice.

[59] §1 of Leibniz' fourth letter.

> things are absolutely indifferent . . . is a manifest contradiction. For if there are good reasons for the course it adopts, the things are not indifferent to it.[60]

Leibniz' solution can be viewed as acceptable only if it is conceded that there are always bound to be actually present factors (possibly unnoticed) which "incline the balance" between the objects of choice. But what if this is not conceded, and the hypothesis of a thoroughgoing similarity of these objects is strictly insisted on? To avert this line of attack, Leibniz would fall back on his Principle of the Identity of Indiscernibles, according to which no two distinct objects can be strictly comparable in the requisite manner.[61] But this is another topic, and a large one, lying beyond the realm of present discussion.

Wolff (1679-1754)

In his *Psychologia Empirica*, Christian Wolff gives as a concrete illustration of choice without preference the example of selection of an individual from a species to provide a specimen to serve as a typical representative for scientific study:

> If we neither desire nor are repelled by an object, we are said to be *indifferent* to it, and the state of mind towards an object thus indifferently considered is called a *state of indifference*. The existence of such states of indifference is attested by experience. For example, some one sees many small stones by a riverside, and regards them without either desire or dislike. He takes some of them up in his hand, to study them more closely, and then throws them away, having wanted them solely for the purposes of examination, without singling out these as preferable to the others, and returning to a state of indifference among them from his examination of the particular specimen.[62]

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[60] §16 of Leibniz' fifth letter. Compare also the following passage: "There is indifference, when there is no more reason for one than for the other. The opposite is determination. . . . All actions are determined, and never indifferent. For there is always a reason inclining us to one rather than the other, since nothing happens without a reason. . . . A liberty of indifference is impossible. So it cannot be found anywhere, not even in God. For God is selfdetermined to do always the best. And creatures are always determined by internal or external reasons." G.W. Leibniz, *Philosophische Schriften*, ed. by C.I. Gerhardt, vol. VII (Leipzig, 1885), p. 109. Quoted by B. Russell in *A Critical Exposition of the Philosophy of Leibniz* (Cambridge, 1900), pp. 193-194.

[61] Cf. §§49, 304, and 307 of the *Theodicy*.

Wolff rightly appreciates that feature-indistinguishability is not a prerequisite for evaluative equivalency.

Kant (1724-1804)

Immanual Kant nowhere discusses the problem of Buridan's Ass *explicitly*, but it is implicitly present in a behind-the-scenes way in this discussion of will and of the freedom of the will. Kant characterized *will* as of itself a causal agency:

> The *will* is a kind of causality belonging to living things in so far as they are rational, and freedom would be this property of such causality that it can be efficient, independently of foreign causes *determining* it; just as *physical necessity* is the property that the causality of all irrational beings has of being determined to activity by the influence of foreign causes.[63]

Kant distinguished between (1) the Rational Will which urges those principles of duty that *reasonableness* lays upon men, stipulating objectively and unconditionally how they *ought* to act, and (2) the Elective Will that is operative in making our strictly subjective day-to-day choices. Regarding these modes of will, Kant wrote:

> Laws proceed from the Rational Will; maxims from the Elective Will. The latter is in man a free elective will, the Rational Will, which is directed to nothing but the [moral] law alone, cannot be called either free or unfree, because it is not directed to actions, but immediately to the legislation for the maxims of actions . . . Consequently it [the Rational Will] is absolutely necessary, and is even *incapable* of constraint. It is therefore only the Elective Will that can be called *free*.[64]

The "freedom" of the Elective Will resides in that its choice is only conditioned, and not wholly determined for an individual by sensuous presentations relating to the objects of choice: "[The free Elective Will] is one which is *affected*, but not *determined* by impulses . . . The *freedom* of

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62 Christian Wolff, *Psychologia empirica*. §585, my translation.

63 Immanuel Kant, *Fundamental Principles of the Metaphysic of Morals*, III, tr. by T.K. Abbott in *Kant's Theory of Ethics* (London, 1873), p. 65 of third (1883) and subsequent editions.

64 Immanuel Kant, *Introduction to the Metaphysic of Morals*, IV; Francis Abbott, *Kant's Theory of Ethics*, p. 282.

the elective will just is that independence of its *determination* on sensible impulses."[65]

This concept of the will as a spontaneous causative agency forms the background of Kant's presentation of the paradox of freedom in the third antinomy of pure reason:

> THESIS: To explain . . . appearances it is necessary to assume that there is also another causality, that of freedom. Proof: . . . We must . . . assume a causality through which something takes place, the cause of which is not itself determined, in accordance with necessary laws, by another cause antecedent to it, that is to say, an *absolute spontaneity* of the cause, whereby a series of appearances, which proceeds in accordance with laws of nature, begins of *itself* . . .
>
> ANTITHESIS: There is no freedom; everything in the world takes place solely in accordance with laws of nature. Proof: Assume that there is freedom in the transcendental sense, as a special kind of causality in accordance with which the events in the world can have come about, . . . it then follows that not only will a series have its absolute beginning in this spontaneity, but that the very determination of this spontaneity to originate the series, that is to say, the causality itself, will have an absolute beginning; there will be no antecedent through which this act, in taking place, is determined in accordance with fixed laws . . . Transcendental freedom thus stands opposed to the law of causality; and the kind of connection which it assumes as holding between the successive states of the active causes renders all unity of experience impossible. It is not to be met with in any experience, and is therefore an empty thought-entity.[66]

Since freedom of the will involves such an antinomy, its status on Kantian principles must be that of a postulate of practical reason.

The freedom of the Elective Will from *complete* determination by its sensuous materials is the distinguishing characteristic of *human* as opposed to *animal* will:

> Freedom in the practical sense is the will's independence of coercion through sensible impulses, for a will is sensuous insofar as it is *pathologically affected*, i.e., by sensuous motives; it is *animal (arbitrium brutum*) if it can be pathologically *necessitated*. The human will is certainly an *arbitrium sensitivum*, not, however, *brutum*,

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[65] *Ibid*., I, p. 268.

[66] Immanuel Kant, *The Critique of Pure Reason*, tr. by N.K. Smith (London, 1929), pp. 409-411.

> but *liberum*. For sensibility does not necessitate its action. There is in man a power of self-determination, independently of any coercion through sensuous impulses.[67]

Thus Kant returns to Buridan's own position that humans, unlike donkeys, would be able to resolve Buridan-type choice situations.

Kant's discussion of the will and its nature thus derives all of its key constituent elements from the historical contexts the problem of choice without preference:

(1) The nature of *will* as a faculty capable of playing a causal role in situations of choice (Ghazâlî, etc.).

(2) The concept of *freedom* of the will as involving a lack of complete dependence on the nature of its objects (Ghazâlî, Aquinas, and the *mere will* tradition).

(3) Absence of determination by its objects as the essential difference between animal and human will (Buridan).

Reid (1710-1796)

One of the principal interpretations which has been placed upon the example of Buridan's Ass is that an equilibrium of contrary determining motives must, if equal in strength, result in inaction. Such reasoning, according to Thomas Reid, rests upon a spurious analogy:

> Some philosophers . . . say, that, as the balance cannot incline to one side more than the other when the opposite weights are equal, so a man cannot possibly determine himself if the motives on both hands are equal: and, as the balance must necessarily turn to that side which has the most weight, so the man must necessarily be determined to that hand where the motive is strongest. And on this foundation some of the schoolmen maintained that, if a hungry ass were placed between two bundles of hay equally inviting, the beast must stand still and starve to death, bring unable to turn to either, because there are equal motives to both. This is an instance of that analogical reasoning which I conceive ought never to be trusted . . . The argument is no better than this—that, because a dead animal moves only as it is pushed, and, if pushed with equal force in contrary directions, must remain at rest; therefore the same thing must happen to a living animal; for, surely, the si-

[67] *Ibid*. (The Antinomy of Pure Reason, Appendix III), p. 465.

> militude between a dead animal and a living, is as great as that between a balance and a man.[68]

Reid rightly perceived the conceptual origin of the problem in a physical analogy—and analogy that he regarded as altogether invalid.

In Reid’s discussion, the example of Buridan’s Ass is treated as a pillar of support for the thesis, denied by Reid, that *action* must take place in a manner proportionate with *motivation*. And he treated the putative invalidity of the supporting scale/will analogy as destroying the case for this thesis:

> Cases frequently occur, in which an end that is of some importance, may be answered equally well by any one of several different means. In such cases, a man who intends the end finds not the least difficulty in taking one of these means, though he be firmly persuaded that it has no title to be preferred to any of the others. To say that this is a case that cannot happen, is to contradict the experience of mankind; for surely a man who has occasion to lay out a shilling or a guinea, may have two hundred that are of equal value, both to the giver and to the receiver, any one of which will answer his purpose equally well. To say, that, if such a case should happen the man could not execute his purpose, is still more ridiculous, though it may have the authority of some of the schoolmen, who determined that the ass, between two equal bundles of hay would stand still till it died of hunger.[69]

In a way reminiscent of Wolff’s stones, Reid proposes coinage as a realistic, rather then makeshift, instance of a situation in common life involving objects among which choice is—for practical purposes—thoroughly indifferent.[70]

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[68] Thomas Reid, *On the Intellectual Powers*, vol. I, p. 4; *Reid’s Works*, ed. by Hamilton, *op. cit.*, p. 238 of the seventh, eights, and possibly other editions.

[69] Thomas Reid, *On the Active Powers*, Bk. IV, p. 4; *Reid’s Works, op. cit.*, Bk. II, p. 609.

[70] The sceptics of antiquity were fond of dwelling on the limitations of man’s sensory discriminations by adducing the difficulty in distinguishing between observationally identical items, such as similar eggs, and impressions of the same seal. See Augustine, *Soliloquies*, Bk. I, p. 6.

Schopenhauer (1788-1860)

The problem of Buridan's Ass was adduced by Arthur Schopenhauer as providing conclusive demonstration of the absurdity of the free-will doctrine.

> The really profound philosophers of all ages—however diverse their views in other respects—have agreed in asserting the necessity of acts of will in accordance with their motives, and have united in rejecting the *liberum arbitrium*. The incalculably preponderant majority of men, incapable of real thought and ruled by appearances and by prejudice, has at all times stubbornly resisted this truth. Philosophers have therefore been at pains to express it in the most pointed and even exaggerated terms. The most familiar of these devices is the famous Ass of Buridan, for which one has not been vainly searching in his writing for some hundred years . . . [71]

Schopenhauer thus maintains that the problem of Buridan's Ass reveals the untenability of the thesis of freedom of the will in showing that a selection unconditioned by determining factors is indefensible, indeed, inconceivable.

While the problem context of the puzzle of choice without preference is here again provided by its ancient setting of determinism *vs*. free will, nevertheless, Schopenhauer's particular way of using the problem—as though it gave a plain and incontestable proof of the absurdity of free will—seems to have originated with himself (his claim to the contrary notwithstanding).

de Morgan (1806-1871)

It is fitting that our ancient paradox is the starting-point of Augustus de Morgan's *Budget of Paradoxes*. Here we read:

> Buridan was for free-will—that is, will which determines conduct, let motives be ever so evenly balanced. An ass is *equally* pressed by hunger and by thirst; a bundle of hay is on one side, a pail of water on the other. Surely, you will say, he will not be ass enough to die for want of food or drink; he will then make a choice—that is, will choose between alternatives of equal force. The problem became famous in the schools; some allowed the poor donkey to die of indecision; some denied the possibility of the balance, which was no answer at all. The following question is more difficult and involves free-will to all who answer—"Which you please." If the northern hemisphere were land, and all the southern hemisphere wa-

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[71] Arthur Schopenhauer, *Prize Essay on the Freedom of the Will*, *op. cit.*, p. 58 of the original edition. My translation.

> ter, ought we to call the northern hemisphere an island, or the southern hemisphere a lake? Both the questions would be good exercises for paradoxers . . .

What we have in this somewhat different but hindered case is also a balance of reasons via a symmetry of arguments.

Lewis Carroll (1832-1898)

In view of the venerable history of our problem, it is not at all surprising that it also has received the (perhaps dubious) honor of humorous treatment. In *Alice in Wonderland* and in *Through the Looking Glass*, Lewis Carroll delights to poke fun at various old and respected pieces of equipment in the logician's arsenal. The problem of Buridan's Ass is up for consideration in the episode of Tweedledum and Tweedledee.

> They were standing under a tree, each with an arm around the other's neck, and Alice knew which was which in a moment, because one of them had "DUM" embroidered on his collar, and the other "DEE." "I suppose they've each got 'TWEEDLE' round at the back of the collar," she said to herself . . . "I know what you're thinking about," said Tweedledum; "but it isn't so, nohow." "Contrariwise," continued Tweedledee, "if it was so, it might be: and if it were so, it would be; but as it isn't, it ain't. That's logic." . . . Alice did not like shaking hands with either of them first, for fear of hurting the other one's feelings; so, as the best way out of the difficulty, she took hold of both hands at once: the next moment they were dancing round in a ring. This seemed quite natural (she remembered afterwards) . . .[72]

With Rabelais, it is the ludicrous side of the puzzle that appealed to Lewis Carrol.

Frank R. Stockton (1854-1902)

Yet another literary employment of the idea of choice without preference is its role in providing the basis for the plot of Frank R. Stockton's intriguing short story "The Lady, or the Tiger?"

> When a subject [of this mythical monarch] was accused of a crime of sufficient importance to interest the King, public notice was given that on an appointed day the fate of the accused person would be decided in the king's arena . . . when all

[72] Lewis Carroll, *Through the Looking Glass*, chap. IV. In an episode of the television science-fiction series *Star-Trek* broadcast in January 1969, the villain transforms himself into a duplicate of the hero, confronting the latter's collaborators with a vexatious puzzle.

> the people had assembled in the galleries, and the King, surrounded by his court, sat high up on his throne of royal state on one side of the arena he gave a signal, a door beneath him opened, and the accused subject stepped out into the amphitheatre. Directly opposite him, on the other side of the enclosed space, were two doors, exactly alike and side by side. It was the duty and the privilege of the person on trial, to walk directly towards these doors and open one of them. He could open either door he pleased: he was subject to no guidance or influence . . . If he opened the one, there came out of it a hungry tiger, the fiercest and most cruel that could be procured, which immediately sprang upon him and tore him to pieces as a punishment for all his guilt . . . But, if the accused person opened the other door, there came forth from it a lady, the most suitable for his years and station that his majesty could select among his fair subjects; and to this lady he was immediately married, as a reward of his innocence . . This was the king's semi-barbaric method of administering justice. Its perfect fairness is obvious. The criminal could not know out of which door would come the lady: he opened either he pleased, without having the slightest idea whether, in the next instant, he was to be devoured or married.[73]

The choice of the accused, however conditioned by a preference as to ultimate results, is most clearly made without any preference between the doors that are the immediate objects of choice. Here it is in fact *ignorance* that creates a symmetry of arguments; the objects are different enough but what we lack is differentiating information.

Perhaps this light note marks a good point for concluding the historical survey of the problem of choice without preference. Let us turn now, with all due seriousness, to a reasoned analysis of the problem and its resolution.

4. CHOICE IN THE ABSENCE OF PREFERENCE

The leading idea which underlies the sensible resolution of the Buridan's choice perplex inheres in the similarity of logical structure between the problems (1) of choice in the case of symmetry of *knowledge*, and (2) of choice in the case of symmetry of *preference*. To establish the kinship which obtains here, let us first examine the problem of choice with symmetric knowledge.

Consider the following example, a simple variant of Frank Stockton's problem of the lady and the tiger: A person is offered a choice between two similar boxes. He is told only that one box contains some prize, and that

[73] Frank R. Stockton, *The Lady, Or the Tiger? and Other Stories* (New York, 1884).

the other is empty. He is not told which is which. Here there is no problem of absence of preference: the person has a clear preference for the treasure-box. The only lack is one of *information*—the choice is to be made in the face of absence of any clue as to the *identity* of this treasure-box. While they may differ in other ways (color, for example), with regard to the crucial question—"Which box is empty and which one holds the prize?"—the available information about the boxes is completely *symmetric*.

This example, then, is an instance of the problem of choice under conditions of symmetric information with respect to a particular preferential issue. How, in such cases, can a *reasonable* person go about making a *rationally defensible* choice?

The sensible answer to this question is in fact simple, well-known, and uncontroversial. For consider the example of the boxes. By the hypothesis which defines the problem, there is no item of information at the disposal of the chooser which could be embraced by him as a *reason* for selecting one box rather than the other. This person therefore simply cannot *reasonably* incline toward one box *vis-à-vis* the other. And this fact of itself must accordingly characterize the manner of his choice. In short, if rational, he must make his selection in a manner which does not favor one box over against the other: he must make his selection in a *random* manner.

This is a matter susceptible of reasoned demonstration. Assume that the boxes are labeled *A* and *B*. Given that, (by hypothesis) the choice of one box produces a result preferable to the rejection of both, the following three courses of action remain available and are mutually exclusive and exhaustive:

(1) To make the choice in some manner that favors selection of Box *A* rather than Box *B*.

(2) To make the choice in some manner that favors selection of Box *B* rather than Box *A*.

(3) To make the choice by means of a selection process that is wholly impartial as between Box *A* and Box *B*, i.e., to choose *randomly*.

Observe, to begin with, that probabilistic considerations as to expected gain do not enter in at all—on the basis of the available information it is *equally probable* that Box *A* holds the treasure as Box *B*, so that the expected gain with *any* of the three procedures (1)—(3) is precisely the same, viz., one-half the value of the treasure. Thus on the sole grounds of expected gain there is no difference among these alternatives. But from the standpoint of *reasonableness* there is a very significant difference among the selection procedures. For by the defining hypothesis of the problem, there is no known reason for favoring Box *A* as against Box *B*, or conversely. This very fact renders it rationally indefensible to adopt (1) or (2). *Per contra*, this symmetry of knowledge *of itself constitutes an entirely valid reason for adopting* (3). This line of reasoning establishes the thesis—pivotal for present purposes—that: *In the case of symmetric knowledge, random choice is the reasonable policy*.

It is be useful to note a corollary of this thesis. When such a problem of choice with symmetric information arises, there is no reason (by the very nature of the problem) why we ought not to regard the *arrival order* in which the choices are given in the formulation or situation of the problem as being purely adventitious, i.e., as a random ordering. The following policy would thus be entirely reasonable and justified: whenever confronted with a choice in the face of symmetric knowledge, to select that alternative which is the first[74] to come to view. (Compare the discussion of Simplicius given above.) Such a policy is defensible as entirely reasonable, since under usual circumstances the arrival order can be taken, by the defining hypothesis of the problem, to be a random ordering.

It is important to note that the matter of a *policy* of choice is very important in this context. When I make a choice among symmetrically characterized alternatives, I *can* defend it, reasonably, by saying, "I chose the first mentioned (or the like) alternative, because I *always* choose the first-mentioned (etc.) in these cases."[75] But I cannot (reasonably) defend the choice by saying, "I chose the first mentioned alternative because this seemed to me to be the thing to do *in this case*, though heaven knows what I would do on other occasions."

The adequacy of such selection policies in the face of indifference based on "convenience" is of fundamental importance because this alone averts an infinite regress of random selections in cases of indifferent

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[74] Or "the last" or "the second" or "the penultimate" etc., etc.

[75] "Why?" "Because this amounts to a random choice." "Why do you choose randomly?" "Because random choice is the only rationally defensible policy in such cases." (Why?—Re-read the foregoing!).

choice. For if such choice had always to be made by a random device, the following regress would at once ensue: We are to choose between the indifferent alternatives *A* and *B*. We take a random instrument, say a coin, as means of resolution (since, by hypothesis, we must have actual recourse to a randomizing instrument). We must now, however, choose between the alternatives:

(I) To associate *heads* with alternative *A* and *tails* with alternative *B*.

(II) To associate *tails* with alternative *A* and *heads* with alternative *B*.

It is at once, obvious that this is itself an indifferent choice. Thus if the resolution of our initial indifferent choice between *A* and *B* requires use of a random device, we must, first of all resolve another indifferent choice, that between the alternatives (I) and (II), or their analogues. But now if this choice too must be effected by a random device, it is clear that we shall be faced with another, analogous situation of indifferent choice, and so on *ad infinitum*. Only if we recognize that selections in the face of choice without preference can be effected on the basis of selection policies based on "convenience," and do not invariably necessitate actual employment of actual random devices, can this infinite regress of random selections be circumvented.

It should also be noted, however, that a systematic policy of choice such as, for example, invariable selection of the first-occurring alternative is not a *universally* appropriate substitute for selection by actual outright use of a random device or process. Consider, for example, the following situation of choice. A (fair) coin is tossed. *A* tries to guess the outcome: heads or tails. *B* tries to guess *A*'s selection. If *B* guesses correctly, he wins a penny from *A*, if correctly he pays a penny to him. How is *A* to chose his guesses? Clearly, it would be a poor proposition for *A* always to guess heads, even though he is in a position of total ignorance and indifference with regard to the outcome of heads or tails. And the same holds true of any other program of choice, such as always guessing tails, or alternating, or the like. All of these run the risk that *B* can discern the guessing pattern involved, and then capitalize on this information. The only defensible course, in a situation such as this, is to have outright recourse to a random process or device. (This randomizing instrument may, however, be the human mind, since people are presumably capable of making arbitrary selections with respect to which they can be adequately certain in their own mind that the choice was made haphazardly, and without any "reasons" whatsoever. To be sure, this process is open to the possible intrusion of unrecognized biases, but then so are physical randomizers such as coins. The randomness of any selection process is a matter which, in cases of importance, shall be checked by empirical means.)

Let us now turn from this discussion of choice in the face of symmetric *knowledge* to the problem of symmetric *preference*. It is clear upon careful consideration that the matter of choice without preference—i.e., under conditions of symmetric preference—can actually be subsumed under the topic of symmetric knowledge as a special case. For in a case of strictly symmetric preference (two essentially similar dates, glasses of water, bales of hay, corn, etc.), the knowledge or information at our disposal constrains us to regard the objects of choice as equally desirable, because in the circumstances every possible reason for valuing one applies, *mutatis nomine*, to the other(s). So far as the factor of their value or desirability for us is concerned, our knowledge regarding each object is precisely the same.[76] Problems of choice with symmetric valuation can therefore be regarded as simply a species within the symmetric knowledge genus owing to the equivalence of our relevant information in the symmetric value case.

It thus follows that the device of random selection will also provide the means of resolution appropriate for symmetric preference choices. To test the correctness of this resolution, consider Ghazâlî's example of a man who had the choice between two ostensibly identical dates. Logically, there are three courses of action open to him, with the ensuing reward as indicated.

	Course of Action	Reward
(1)	To select *neither* date, for lack of a preference	Nothing
(2)	To fix upon one of the dates by means of some selection procedure which favors one over the other	One date

[76] It is clear, then, that this analysis does not apply in the case of the *psychological* dilemma of a balance among diverse motivations of equal force (e.g., hunger and thirst), but obtains solely with respect to the *logical* dilemma of choice among strictly comparable alternatives.

(3) To select one of the dates at *random* One date

It is clear that these three courses of action are mutually exclusive and exhaustive. But a reasonable person cannot opt for (1), because its associated reward is of lesser value than that of its alternatives. Further, the defining hypothesis of the example—viz., that there is no known reason for preferring one date to the other—of itself constitutes a reason for rejecting (2). Random selection is the only means of avoiding favoring one alternative over the other. And just this constitutes a valid reason for adoption of (3).

These considerations, then, serve to establish the proposition that: *Random selection is the rationally appropriate procedure for making choices in the face of symmetric preference.* The concept of random selection provides an answer to the problem of choice without preference which, is demonstrably, its only *reasonable* (i.e., rationally defensible) resolution.

This proposed resolution of the problem of choice without preference is in fact substantially that which was first proposed by Gataker and Bayle as a general means of solution—though in their case without any justifying discussion of the rationale establishing the validity of this solution. Bayle based his suggestion on the fact that when the problem of choice without preference actually arises in real situations—in particular in the instance of court-precedence cases—resolution by change selection is generally regarded as acceptable, and indeed has acquired the status of customary, official mode of resolution.[77] But of course custom-conformity does not of itself constitute validation but at best supplies some empirical evidence in support of the reasonableness of the proposed resolution.

5. A POSTSCRIPT ON PHILOSOPHICAL ISSUES

In examining the substantive philosophical contexts in which the problem of choice without preference has figured, and for which it has been

[77] According to a *New York Times* report (Monday, January 12, 1959, p. 6), "chance is the arbiter prescribed by Swedish law for breaking tie votes in Parliament," The report states that "a drawing of lots may decide the fate of a controversial pension plan," but goes on to observe that "legislation by lottery has never yet been necessary on any major issue." Again, when Hawaii was admitted as the fiftieth state of the United States, and two new senators were elected, random devices were used by the Senate to decide which of the two new Hawaiian senators would have seniority (decision by a coin-toss), and which would serve the longer term (decision by card-drawing). See the *New York Times* front-page report of August 25, 1959.

viewed as fundamentally relevant, our historical survey has brought the three following issues to the fore:

1. Its Greek context in cosmological discussion of the earth's place in the physical universe (Anaximander, Plato, Aristotle).

2. Its Scholastic context in ethico-theological discussion of man's freedom of will (Aquinas, probably Buridan, and others).

3. Its Arabic context in epistemologico-theological discussion of the amenability of God's choices to reason and to human rationalization, i.e., the possibility of explaining God's actions in ways acceptable to reasoning men (Ghazâlî, Averroes).

The entire problem of a choice balanced among indifferent objects originates, historically and conceptually, in an analogy with physical equilibria, such as a body immobile under the pull of opposing forces (see Plato's *Phaedo* 108 E, *vice* Anaximander), or a balance-bar at rest under the pressure of opposed, but equal weights (embodied in Axiom 1 of Archimedes' *On Plane Equilibriums*). Here, the issues involved are not properly philosophical, and the definition of the example is still in its embryonic form, dealing either with mechanical equilibrium, or with psychological balance among conflicting motivations of comparable strength. The problem has not yet reached its philosophically pertinent definition as one of selection among *logically* indifferent alternatives, which it achieved only in the middle ages.

It is a common occurrence in the history of philosophical concepts that a purely scientific discovery or idea metamorphoses—through application to a novel setting in a more far-reaching context—into a matter of philosophical concern and significance. And just this happened with Aristotle's psychological analogy in the present case. Only with the Aristotelian commentators (in Islam, Ghazâlî, and Averroes) did the philosophical problem of choice among strictly indifferent objects reach its ultimate *logical* formulation. Genetically, as correctly noted by Reid, the philosophic problem of choice without preference descends from a physical problem-setting, deriving ultimately from analogy with mechanical equilibrium.

With respect to the free-will context, it must be recognized that use of the Buridan example rests upon, and is inextricably embedded in, the scholastic identification of *cause* and *reason*.[78] Once we reject this identification, as indeed we must, the bearing of the example changes. For a situation of choice in which a preferring *reason* for a selection is absent need not now be one in which no *cause* (other than the agency of a "free will") is operative in leading to choice. Thus outside the context of scholastic presuppositions, the example becomes incapable of establishing the immobilization that it claims.

It deserves stress that our problem serves also to highlight the difference between *reasons* and *motives*. When a random selection among indifferent objects is made by me, I do have a *reason* for my particular selection, namely the fact that it was indicated to me by a random selector. But I have no *preference* or psychological motivation of other sorts to incline me to choose this item instead of its (by hypothesis indifferent) alternatives. Such absence of psychological preference does not entail the impossibility of a logically justifiable selection. A choice can, therefore, be logically vindicated as having been made reasonably even though it cannot be traced back to any psychological foundation. In short, we can have *reasons* for a choice even where there is no *motive*.

We come down to the remaining context, the rationalizability of divine choice. Before entering upon a closer consideration of this matter, it is desirable first to take up some other, preliminary observations.

The solution presented in the foregoing section establishes the central role played by *randomness* in the theory of rational choice and decision. A *rational* person must, by the very meaning of the term, fashion his belief and his action in tune with the *evidence* at his disposal. In symmetric choice situations, therefore, in which the manifold of reasons—the available ramification—bears identically on every side, he must choose—as has been seen—in a random manner. In such cases, the "reasons" for his choice are independent of any distinguishing characteristic of the object of choice. Here, reasonable choice comes to be possible in the absence of preference only by essentially abdicating the right of choice, in delegating the selection to a random process. Seeing that there simply is *no reasonably defensible way of actually "choosing"* among alternatives in the face of symmetric knowledge. We have either to hand the task of fixing upon a selection over to some random mechanism, by making it contingent upon the

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[78] Schopenhauer's monograph on *The Fourfold Root of the Principle of Sufficient Reason* provides an extended critique of this confusion of *logical reason* with real cause.

outcome of such a device, or else we have to make our selection in accordance with the prescript of some predetermined policy which we can defensibly construe as constituting a random selection process. In either event, we can be said to have "made a *choice*" purely by courtesy. It would be more rigorously correct to say that we have *effected a selection*. In situations of choice without preference, a reasonable person is not condemned to paralysis and inaction. He can and does select, but does so in a random manner, and thus at the price that "his choice" is "his" in only a Pickwickian sense.[79]

Thus in a world in which all things are indifferent, all choices are random, and wisdom and morality will alike come to naught. Just this is the criticism advanced by Cicero against the Stoic teaching that all things of this world should be "indifferent" to the wise man. Cicero writes:

> If we maintained that all things were absolutely indifferent, the whole of life would be thrown into confusion . . . and no function or task could be found for wisdom, since there would be absolutely no distinction between the things that pertain to the conduct of life, and no choice need be exercised among them.[80]

Consequently the idea of randomness must play a key part in the theory of rational choice. The concept of randomness which is at issue here is not that of mathematics, as characterized by the criteria which govern the construction of random number of tables. Rather, it is its logical cognate: an alternative is *randomly selected* (in this logical sense) if the selection situation is such that the sum total of the weight of evidence for the selection of the chosen alternative is equal to the weight of evidence for selection of its competing alternatives. (Symmetric information or evidence is a special case of evidence of equal weight.) This concept of randomness as based on evidence is a wholly logical or epistemological concept, which relativizes randomness to knowledge and ignorance.[81]

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[79] It should be noted that in games of chance, situations in which *rational* choices of courses of action must be made probabilistically can also arise (when the *optimal* strategy is one which is *mixed*). See any text or treatise on the mathematical Theory of Games.

[80] Cirero, *De finibus*, Bk. III, § 50. I quote H. Rackham's translation in the Loeb series.

Another line of consideration is worth noting in this connection. Already Pierre Bayle quite correctly perceived that the problem of choice without preference can take on two forms: (1) selection of one among several (exclusive) alternatives that are essentially identical as regards their desirability-status as objects of possession or realization, i.e., choice without preference among the *objects* involved (the problem of Buridan's Ass), and (2) selection of one among several alternative claimants, whose claims are indivisible and uncompromisable, and whose claims are essentially identical in strength, and must therefore in fairness be treated alike, i.e., choice without preference among the *subjects* involved. Bayle properly recognizes that the device of random selection provides a means of resolution that is entirely appropriate for both cases alike. Random selection, it is clear, constitutes the sole wholly satisfactory manner of resolving exclusive choice between equivalent claims in a wholly fair and unobjectionable manner. Only random, and thus strictly "unreasoned" choice provides an airtight guarantee that there is no answer forthcoming to the question: "Why was this alternative, rather than another, selected?" Random choice thus guarantees that the other alternatives *might just as well* (in the strictest of senses) have designated. Where there is no way of *predicting* the outcome in advance no charge of preferential treatment can possibly be substantiated. Thus random choice affords the appropriate avenue of resolution for selection-situations in which considerations of fairness leave no other courses of immediate resolution open as acceptable or as defensible.

This consideration has further implications of philosophical import. For one thing, it is surely a *contingent* fact that random processes and devices exist in the world: it is logically feasible to conceive of a possible universe without them. Now the abstract problem of choice without preference is, in its abstract essentials, a theoretical and not a practical problem. It seems curious that the solution of this *theoretical* problem hinges upon the availability of an instrumentality (viz., random choice) whose existence is *contingent*. Surprisingly, it is thus possible to conceive of circumstances (specifically, symmetric choice situations) in which the possibility of rational

[81] Cf. Hume's thesis that "though there be no such thing as *chance* in the world; our ignorance of the real causes of any event has the same influence on the understanding, and begets a like species of belief or opinion" (*Enquiry*, Bk. VI, first sentence). For further explanation of the concept of evidence and of measures of evidential weight, the reader is referred to the writer's paper on "A Theory of Evidence," *Philosophy of Science*, vol. 25 (1958), pp. 83-94.

action depends upon an otherwise wholly extraneous matter of contingent fact (the availability to rational agents of random selection methods). (The availability of random selection policies does however blunt the concept of this consideration.)

Now let us finally return to examining the bearing of the foregoing discussion upon the question—much disputed, alike in medieval Islam, Judaism, and Christianity—as to the reasonableness of God's choices.[82] Here it is—or should be—perfectly clear that as a means of resolving the problem of choice without preference the proposed solution is entirely inapplicable. Orthodox Islamic theology, no less than Christian or Judaic, cannot grant that the concept of random selection has any applicability to the divinity. There can be no chance mechanisms or processes whose outcome is not known to God, nor need He trouble with weights of evidence: in postulating divine omniscience, no possibility is left open for random choice.[83] God's knowledge being complete and timeless, selection cannot be delegated by Him to some contingently future outcome or to some element of serial ordering, such as "the first" (or "the last") alternative.

If follows that the proposed resolution of the problem of choice without preference must be held to apply to the human sphere alone, and not the divine. Only man's ignorance permits him to resolve questions of choice without preference behind the veil of chance.

Once we allow (against Leibniz) the possibility that strictly indifferent choices can arise for the supreme being, we must, I think, be prepared to grant the right to Ghazâlî, against the Arabic Aristotelians. For here a solution is possible only in terms of an inscrutable will, capable of effecting out of its own resources differentiations in the absence of any relevant difference.[84] In this regard, it is clear, we must consequently renounce the possi-

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82 The significance of this discussion does not hinge on the issue of God's existence. Its bearing is entirely hypothetical: if there were a God along the traditional lines how would he function?

83 In the "Introduction" to the *Analogy*, Bishop Butler writes: "Probable evidence, in its very nature, affords but an imperfect kind of information; and is to be considered as relative only to beings of limited capacities. For nothing which is the possible object of knowledge, whether past, present, or future, can be probable to an infinite intelligence; since it cannot be discerned absolutely as it is in itself, certainly true or certainly false. But to us, probability is the very guide of life."

84 Ghazâli makes the point (*Tahâfut al-Tahâfut*, vol. I, pp. 18-19) that it is senseless to speak of God making choices by chance, for instead of saying, "God chose to do

bility of human rationalization of divine acts. The problem of choice without preference was a shrewdly selected example in support of the position maintained by the Islamic theologians in their dispute with the philosophers: this problem does illustrate effectively the thesis of Arabic scholasticism that choices made by the divine intellect may ultimately prove inscrutable in human terms of reference.[85, 86]

so-and-so in a chance manner" one might instead just a well say, "So-and-so happened by chance."

[85] Jonathan Edwards offers the following remarks: "If, in the instance of the two spheres, perfectly alike, it be supposed possible that God might have made them in a contrary position: that which is made at the right hand, being made at the left: then I ask, whether it is not evidently equally possible, if God had made but one of them, and that in the place of the right-hand globe, that he might have made that numerically different from what it is, and numerically different from what he did make it; though perfectly alike, and in the same place . . .? Namely, whether he might not have made it numerically the same with that which he has now made at the left hand, and so have left that which is now crated at the right hand, in a state of non-existence? And if so, whether it would not have been possible to have made one in that place, perfectly like these, and yet numerically different from both? And let it be considered, whether from this notion of a numerical difference in bodies, perfectly alike . . . it will not follow, that there is an infinite number of numerically different possible bodies, perfectly alike, among which God chooses, by a self-determining power, when he sets about to make bodies." (*A Careful and Strict Enquiry into the Modern Prevailing Notions of that Freedom of Will which is supposed to be Essential to Moral Agency, Virtue and Vice, Reward and Punishment, Praise and Blame* [Boston, 1754], pt. II, sect. xii, subsect. i. Quoted from A. N. Prior, *Past, Present and Future* [Oxford, 1967], pp. 141-142.)

[86] This chapter is a somewhat expanded version of an article originally published in *Kantstudien*, vol. 51 (1959/1960), pp. 142-175.

Chapter 8

LEIBNIZ'S INTERPRETATION OF HIS LOGICAL CALCULI

It is scarcely possible to overestimate the debt which the contemporary student of Leibniz's logic owes to Louis Couturat. His historical researches rescued the logical work of Leibniz from the oblivion of neglect and forgetfulness.[1] They revealed that Leibniz developed in succession several versions of a "logical calculus" (*calculus ratiocinator* or *calculus universalis*). In consequence of Couturat's investigations it has become well known that Leibniz's development of these logical calculi adumbrated the notion of a logistic system[2]; and for these foreshadowings of the logistic treatment of formal logic Leibniz is rightly regarded as the father of symbolic logic.

Our gratitude to Couturat must, however, be accompanied by the realization that his own theory of logic is gravely defective. Couturat was persuaded that the extensional point of view in logic is the only one which is correct, an opinion now quite antiquated, and shared by no one.[3] This prejudice of Couturat's marred his exposition of Leibniz's logic. It led him to battle with windmills: he viewed the logic of Leibniz as rife with shortcomings stemming from an intensional approach.

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1 Couturat's exposition of Leibniz's logical work is contained in his *La Logique* de *Leibniz* (Paris, 1901), and the previously unpublished writings on which this is based are given in his *Opuscules et fragments inedits de Leibniz* (Paris, 1903). Couturat discusses Leibniz's logical calculi in the eighth chapter, "Le Calcul Logique," of *Logique.*

2 See Alonzo Church's definition in *The Dictionary of Philosophy*, edited by D. Runes (New York, n. d.). With this compare Leibniz's discussion on pages 204-207 of volume seven of *Die philosophischen Schriften von G. W. Leibniz* edited by C. I. Gerhardt (Berlin, 1890).

3 The extensional interpretation of logic is, he claims, "la seule qui permette de soumettre la logique au traitement mathématique" (*Logique,* p. 32).

The task of this paper is a re-examination of Leibniz's logic.[4] It will consider without prejudgment how Leibniz conceived of the major formal systems he developed as *logical* calculi—that is, these systems will be studied with a view to the interpretation or interpretations which Leibniz himself intends for them. The aim is to undo some of the damage which Couturat's preconception has done to the just understanding of Leibniz's logic and to the proper evaluation of his contribution.

A remark concerning the mode of presentation adopted in this paper is in order. In describing the logical calculi devised by Leibniz we employ the schematism provided by the concept of a *logistic system*. Admittedly Leibniz did not possess a full and complete grasp of the logistic method, a fact evidenced by several misgivings which we shall have to express in the course of our exposition. However, even a cursory perusal of the writings we shall have occasion to cite suffices to justify our course. Leibniz is sufficiently close to a logistic treatment of logic to make a cautious and careful extrapolation to explicitly logistic formulation historically legitimate, as well as highly helpful towards securing understanding of what he has in mind.

From this standpoint, then, Leibniz's most fully developed efforts at a symbolic treatment of logic have a common basis comprising the following five features:

1. Variables whose range is a set of otherwise unspecified objects called "terms" (*termini*).

2. Singulary and binary operators on "terms" yielding "terms".

3. Binary relations between "terms", including equality.

4. The following three rules of inference[5]:

[4] A limitation must be mentioned. We deal only with the mature portion of Leibniz's logical work, not with his earlier efforts, prior to 1679. Regarding these, reference should be made to Karl Dürr's article, *Leibniz' Forschungen im Gebiet der Syllogistik*, in *Leibniz zu seinem 300. Geburtstag* (Berlin, 1949), to the exposition of Leibniz's arithmetic treatment of logic on pages 126-129 of Jan Lukasiewicz's *Aristotle's syllogistic* (Oxford, 1951), and, of course, to Couturat's *Logique.*

[5] Leibniz possessed to an insufficient extent the distinction—basic to the concept of a rule of inference—between a statement of the system and a statement about the system in a meta-language. However, he is often sufficiently close to an apprecia-

i. Equality obtains between "term"-denoting complexes iff (if and only if) they are inter-substitutable in asserted statements.[6]

ii. In any asserted statement involving some "term"-variable this may be replaced throughout by another "term"-variable, or by some other "term"-denoting complex, and the result will again be an asserted statement.[7]

iii. The *modus ponens* rule.[8]

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tion of the distinction in question to justify the explicit formulation of rules of inference. Thus, for example, in the marginalia given on pages 223-227 of vol. 7 of *Phil. Schr.* (Gerhardt), Leibniz distinguishes between *verae propositiones,* Le., assertions of the system, and *principiae calculi,* i.e., rules for obtaining further assertions from given ones.

6 Leibniz's classic definition of equality, "Eadem sunt quorum unum in alterius locum substitui potest, salva veritate" (*Phil. Schr.* (Gerhardt), vol. 7, p. 219), is defective both as regards the distinction of use and mention, and that between object and meta-language. Our statement has repaired these defects.

7 Quicquid conclusum est in literis quibusquam indefinitis, idem intelligi debit conclusum in aliis quibuscunque easdem conditionibus habentibus, ut quia verum est *ab* est *a,* etiam verum erit *bc* est *b,* imo et *bcd* est *bc* (*Phil. Schr.* (Gerhardt), vol. 7, p.224).

8 Leibniz cannot, in view of footnote 5, give a wholly adequate statement of this rule. However, he does come quite close. First some usages must be explained. A *proposition vera* is what we should term an *asserted statement* (of the system) (*Phil. Schr.* (Gerhardt), vol. 7, pp. 218 if.). Statements play an axiomatic role if they are either self-evident truths (*Propositiones per se verae*) or are arbitrarily assumed and asserted without proof (*propositiones positvae*) (loc. cit.). If a proposition is an implication of the form *Si . . ., ergo* - - -, it is a *consequentia.* (Loc. cit. Compare the scholastic usage as discussed in the third chapter of P. Boehner's *Medieval Logic* (Manchester, 1952).) Now Leibniz states, "Proposition vera est, quae ex positis et per se veris per consequentias oritur," i.e., "A statement is asserted which is obtained from posited statements and statements true-in-themselves (i.e., from the axioms) by utilizing implications" (*Phil. Schr.* (Gerhardt), vol. 7, p. 219). No matter how this is taken, it would appear that the *modus ponens* rule is implied.

5. A group of asserted statements which provides the axiomatic basis for the system.

Leibniz's efforts at a symbolic treatment of formal logic led him to construct three main versions of a logical calculus.[9] We will study these systems, proceeding chronologically.

The first of these systems, dating from around 1679, grew out of Leibniz's attempt to devise an arithmetic treatment of logic.[10] An exposition is given in the essays *Specimen calculi universalis*[11] and *Ad specimen calculi universalis addenda*[12]. Leibniz employs lower-case Roman letters for "term"-variables. Operators on "terms" are 'non' (singulary), and juxtaposition (binary). Relations between "terms" are 'est', its negation 'non est', equality (for which Leibniz used "eadem sunt" or other forms, such as "sunt idem", but which we represent by the conventional =), and inequality ("diversa sunt"). The following are asserted statements of this system:

1. *a* est *a*

2. *a* = non-non-*a*[13]

3. *a* est *b* iff non-*b* est non-*a*

4. If *a* est *b* and *b* est *c*, then *a* est *c*

5. *a* est *b* and *b* est *a* iff *a* = *b*[14]

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[9] Besides the treatment in Couturat's *Logique.* there is an illuminating discussion of all three of these systems in Karl Dürr's article, "Die mathematische Logik von Leibniz," *Studia philosophica* (Basel), vol. 7 (1947), pp.87-102.

[10] Cf. footnote 4.

[11] *Phil. Schr.* (Gerhardt), vol. 7, pp.219-221.

[12] *Phil. Schr.* (Gerhardt), vol. 7, pp. 221-227. Couturat's discussion of this system is given on pages 336-343 of *Logique*.

[13] In Leibniz's own notation, this would be written: "Eadem sunt *a* et non-non-a." We will content ourselves with this one example.

[14] On pages 219-221 of volume 7 of *Phil. Schr.* (Gerhardt), Leibniz offers an ingenious proof of 5, based on the fact that well-formed formulas of the calculus of this

6. $a \neq b$ iff not $a = b$

7. *a* non est *b* iff not *a* est *b*

8. If $a = b$, then $b = a$

9. If $a = b$ and $b = c$, then $a = c$

10. $a = aa$

11. $ab = ba$

12. *a* est *bc* iff *a* est *b* and *a* est *c*

13. If *a* est *b*, then *ca* est *cb*

14. If *b* est *a* an *c* est *a*, then *bc* est *a*[15]

15. If *a* est *b* and *c* est *d*, then *ac* est *bd*

16. *a est a*

17. *a est b*

18. If *a* is proper: *a* non est non-*a*[16]

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system must have one of a limited number of forms. This is, perhaps, the earliest example of what has come to be known as a *syntactic metatheorem.*

[15] Si *b* est *a* et *c* est *a*, etiam *be* erit *a* (*Phil. Schr.* (Gerhardt), vol. 7, p. 222). Couturat renders this as, "Si *a* est *c*, ou *est c*, on peut affirmer que *ab* est *c*." explaining in a footnote that Leibniz erroneously says "et" (*Logique*, p.340). But Leibniz is not wrong, and 14 follows from 4, 10, and 13, all of which Couturat renders correctly.

[16] A "term" *a* is *proper* if there is no "term" *b* such that *a* est *b non-b.* The propriety condition is essential to the consistency of the system, as is shown by the following refutation of an unqualified 18:

(1) *b* non-*b* est non-*b* by 17

(2) *b* non-*b* est *b* by 16

19. In *a* is proper: If *a* est non=*b*, then *a* non est *b*.[17]

In listing the assertions of this system no effort has been made to distinguish between axiomatic and derived assertions, though Leibniz's own expositions do distinguish between axioms (*Propositiones per se verae*) and propositions established from them (*verae propositiones*).[18] The principal reason for abandoning this distinction here is that several brief sketches were drawn up by Leibniz for the system we are considering (as well as for those we have yet to consider), and the set of assertions used as axioms varies from one expository sketch to another. The very scale of the logical sector of the Leibnizian corpus makes it impractical to attempt a complete listing of places where the various assertions are to be encountered.[19]

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(3) non-*b* est non (*b* non=*b*) by 3, (2)

(4) *b* non-*b*) est non (*b* non-*b*) by 4, (1), (3).

Leibniz does not always state the propriety condition explicitly, when required. However, in one of his writings he indicates that he understands the traditional categorical propositions always to contain the tacit assumption that the terms which enter are proper: "In omnibus tamen tacite assumitur terminum ingredientem esse Ens" (*Phil. Schr.* (Gerhardt), vol. 7, p. 214). The formulation is in the terminology of Leibniz's second version of logical calculus.

17 In view of 2, this can be put in the form

19*. If *a* is proper: If *a* est *b*, then *a* non est non-*b*

by substituting "*non-b*" for "*b*" in 19. This requires mention because, as we shall see, 19* plays an important role in the interpretations.

18 See footnote 8.

19 I will therefore confine myself here to referring each of the forty assertions we shall encounter to one occurrence in Leibniz's writings. I have selected in each case the occurrence which, to my knowledge, may be presumed to be the earliest chronologically. Regarding assertions 5, 14, 21, and 27, see the footnotes *ad hoc*. Reference may be made to vol. 7 of *Phil. Schr.* (Gerhardt) for assertions 1 (p. 218), 4 (p. 218), 6 (p. 225),7 (p. 218), 10-13 (p. 222), 15 (p. 223), 16-17 (p. 218), 18 (p. 224),23 (p. 212), 25 (p.230), 26 (p.237), 28 (p.232), 29 (p.239), 30-31 (p.232), 32-36 (p.229), 37 (p.234), 38 (p.230), and 39-40 (p.233). Finally, refer to Couturat's *Opuscules* for assertions 2-3 (p.379), 8-9 (p.365), 19-20 (p.378), 22 (p.233), and 24 (p.261).

The consistency of this system is immediately apparent from the existence of the following interpretation: let the "terms" be sets, 'non' complementation, let juxtaposition represent intersection, 'est' inclusion, and let propriety be non-nullity. This interpretation also serves the purpose of exhibiting the relation of Leibniz's first logical calculus to our modern class-logic, which will become plainer yet in the later discussion of Leibniz's own interpretations of this system.

Our presentation of Leibniz's interpretations of his systems of logical calculus involves a departure from Leibniz's own mode of presentation. Although he was clearly aware of the distinction between an abstract axiomatic system and the interpreted system obtained from it by specifying meanings for the symbolism, Leibniz's own expositions commonly develop a system and one-or-more of its intended interpretations side by side. As an expository asset, and particularly to gain added clarity, this paper draws the line sharply, and lays down each interpretation in a separate, explicit fashion. Leibniz's relevant writings show this to be well warranted by his own treatment.[20]

To lay down an interpretation for Leibniz's first logical calculus it suffices to specify (1) the set of "terms", (2) the effects of the *non* operator, and of the operation of juxtaposition, (3) the meaning of the relation *est*, and (4) the meaning of propriety; provided that this is done so as to satisfy the assertions. This is so because the meanings of "non est," "=", and "≠" will then be determined by 7, 5, and 6, respectively.

One interpretation given by Leibniz for this system is obtained by letting "terms" be predicates taken in intension, i.e., properties. The result of operating on a "term" (property) by *non* is the property of not having the property in question—*non* represents the *negatio* or negation of properties. The result of juxtaposing two "terms" is the property of possessing both of the properties in question—juxtaposition represents the *additio* or *conjunctio*, the joining of properties. A "term" (property) is proper if it is not of universal comprehension (i.e., null extension). Finally, the result of linking two "term" names by 'est' is the statement that the former property contains the latter in its intension or comprehension. Thus '*a* est *b*' symbolizes the universal affirmative proposition that whatever is characterized by the property *a* is also characterized by the property *b*, i.e., that all *a*'*s* are *b*'*s*.

[20] See especially the *Generales inquisitiones de analysi notionum et veritatum* (*Opuscules* (Couturat), pp. 356-399), and the essays cited in footnote 49.

A second interpretation is given by Leibniz for this system. This is obtained by letting "terms" be predicates taken in extension, i.e., classes. The result of operating on a "term" (class) by *non* is the class of all objects[21] not belonging to the class in question. The result of juxtaposing "terms" is the class of all objects belonging to both of the classes in question. A "term" (property) is proper if it is not of null extension (universal comprehension). Finally, the result of linking two "term" names by 'est' is the statement that the former class is contained in the latter in extension, i.e., '*a* est *b*' symbolizes the universal affirmative proposition that all *a*'*s* are *b*'*s*.

It is plain that in these two interpretations of his first system Leibniz treats adjectives (property-names) and substantives (class-names) in an entirely parallel fashion. He justifies this by remarking that to any adjectival property, such as (*is an*) *animal*, there is a corresponding substantival class, in this case *the animals*. And he asserts that as regards symbolic treatment the distinction between these is irrelevant.[22] By exploiting this duality of property intension and class extension Leibniz is able to provide a twin-interpretation for his first system of logical calculus, and so to develop its possibilities of interpretation. This important point is entirely missed by Couturat, who views Leibniz's discussion of this matter as needless verbiage, calculated to accommodate the scholastics.[23] Indeed, Couturat is kept from a proper understanding of the first of Leibniz's interpretations of this system by his conviction that any but the extensional point of view is in-

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[21] As objects (*entia*), Leibniz holds, one can take either all actually existing things, or else all which are (logically) possible. The *dictum de omni et nullo* must then be taken in the appropriate sense (*Phil. Schr.* (Gerhardt). vol. 7, p. 214).

[22] *Substantivum* [n. b.] est quod includit nomen Ens vel res; *Adjectivum* quod non includit. Ita animal est substantivum, seu idem quod ens animale. Rationale est adjectivum, fit enim demum substantivum, si adjicias Ens, dicendo Ens rationale, vel per compendium una voce (si jocari licet) Rational. Ut ex termino Ens animale: animal. (n. b. Rae definitiones usui scholae sunt accommodatae, sed in characteristibus [i.e., in the symbolism] necesse non est differentiam nominis substantivi atque adjectivi apparare, neque illa vero usum habet ullum (*Phil. Schr.* (Gerhardt), vol. 7, p.227).

[23] Witness Couturat's comment on the passage cited in the previous footnote: "Cette influence scolastique se revele par les définitions des termes de la logiqué traditionelle (grammaticale) dont Leibniz reconnait lui-meme l'inutilite', (*Logique,* p.337, notes).

adequate for logic. [24]

In this first system Leibniz is able to treat the entire classical doctrine of categorical propositions. Thus, S *a P*, *Se P*, *S i P,* and *S o P* are rendered (in *either* interpretation) as *S* est *P*, *S* est *non-P*, *S* non est *non-P*, and *S* non est *P*, respectively. The symbolic version of the entire classical theory of immediate inference and the syllogism is available in the assertions of the system. It is at this point that the propriety condition acquires significance. The assertions (particularly 19*) which guarantee the validity of two of the classical modes of inference, subalternation and partial conversion, are dependent on the propriety of the terms involved. And Leibniz is, throughout *all* of his logical work concerned to preserve the validity of the entire classical theory of immediate inference and of the syllogism.[25] With him symbolic logic was the symbolic treatment of the classical, traditional logic.

Leibniz's second system was developed in the 1685-6 period during which his metaphysical system assumed its final and completed form in virtually every detail. This is no mere coincidence. This system played a central role in Leibniz's solution of a logical problem on which he felt the progress of his metaphysics to depend: the problem of reconciling his belief that there are true, contingent propositions with his conviction that all true propositions are analytic.[26] This accounts for Leibniz's subsequent marginal comment, "Hie egregie progressus sum", on the manuscript *Generales inquisitiones de analysi notionum et veritatum*[27], which is the main vehicle for the presentation of this system. The essays *Principia calculi rationalis*[28] and *Difficultates logicae*[29] also throw light on this system, as do

[24] Couturat holds that the extensional view of logic is "la seule qui permette de soumettre la Logique au traitement mathematique" (*Logique*, p. 32). This prejudice on his part leads Couturat to hold Leibniz's intensional point of view responsible for the shortcomings—generally rather imagined than actual—of his logical work (*Logique*, pp. 30-32, 353-54, 359-62, 373-77, and elsewhere).

[25] One of the clearest expressions of this concern is the essay *Difficultates logicae* (*Phil. Schr.* (Gerhardt), vol. 7, pp. 211-217).

[26] Regarding Leibniz's solution see the writer's article, *Contingence in the philosophy of Leibniz, Philosophical review,* vol. 61 (1952), pp. 26-39.

[27] *Opuscules* (Couturat), pp. 356-399, and d. also *ibid.*, pp. 261-264.

[28] *Opuscules* (Couturat), pp. 229-231.

[29] *Phil. Schr.* (Gerhardt), vol. 7, pp. 211-217.

the later sketches (1690) *Primaria calculi logici fundamenta*[30] and *Fundamenta calculi logici*[31].

This second of Leibniz's systems of logical calculus is an extension of the first. It includes as assertions all those of the first system, though there are some changes in the notation. Upper case Roman letters are used for variables. "Continet" occasionally replaces "est", and the equality of *A* and *B* is rendered: "$A \infty B$" or "$A \propto B$" or "coincidunt *A* et *B*" or sometimes "aequivalent *A* et *B*". Also, there is one fundamentally new element, the "term"-constant *Ens* or *Res.* This is introduced in connection with the propriety condition; propriety is given by the definition: *A* is proper iff *A* non est non-Ens.[32] As assertions this system adds the following five to the nineteen we have listed for the first system:

20. If *A* is proper: *A* est *B* iff $AB = A$

21. If *A* non est non-Ens, then *A* est Ens[33]

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[30] *0puscules* (Couturat), pp.232-237.

[31] *Opuscules* (Couturat), pp. 421-423. Couturat's discussion of this system is given on pages 344-362 of *Logique.*

[32] *Opuscules* (Couturat), p. 233. Cf. footnote 16.

[33] *Opuscules* (Couturat), p. 233. By 3, an equivalent formulation of 21 is

21 *. non-Ens est non-A, unless *A* est non-Ens.

Thus Leibniz states, "Non Ens est mere privativum, sive non-*Y*, id est non-A, non-B, non-C, etc., idque quod vulgo dicunt nihili nulla esse proprietates" (Ibid., p. 356). Again, another formulation, in virtue of 19, is

21 * *. non-Ens non est *A*, unless *A* est non-Ens.

Leibniz gives this also: "Esto *N* non est *A*, *N* non est *B,* item *N* non est C, et ita porro, tunc did potest *N* est Nihil [i.e., non-Ens]. Hue pertinet quod vulgo dicunt, non Entis nulla esse Attributa" (Couturat, *Logique*, p.349, notes).

Couturat is patently misguided when he remarks in discussing this last passage (*Ibid.*, and d. p. 353, notes) that, "cette définition, inspirée, comme on voit, de la tradition scholastique, n'a aucune valeur. Tout au contraire, on définit a présent le zero logique comme le terme qui est contenu dans tous les autres (en extension), comme le sujet de tous les prédicats possibles."

22. *A* est non-Ens iff *A* non est Ens

23. If *A* is proper: *A* est *B* iff *Anon-B* est non-Ens

24. If *A* est *Bnon-B,* then *A* est non-Ens

The nature of this augmented system is perhaps best apprehended by considering an interpretation in terms of modern class-logic, which also shows the consistency of the system. Let the "terms" be sets, 'non' complementation, let juxtaposition represent intersection, *Ens* the universal class, and propriety non-nullity. Finally, let 'est' represent inclusion among non-null sets, in accord with the rule: "*A* est *B*" is "*A=A*" or "*A* C *B* & *AB=FA*" according as *B* is empty (null) or not. All assertions of the second system are readily verified for this interpretation.[34]

Let us now turn to Leibniz's own interpretations for this system. Since it is an extension of the first system, the only additional step in laying down an interpretation for this second system is specification of the meaning of the "term" Ens.

One interpretation given by Leibniz for this system is virtually the same as the intensional interpretation of his first system. The "terms" are predicates in intension (i.e., properties), and the *non-operator* and juxtaposition are defined as in the analogous case of the previous system. The result of linking "term" names by 'est' is again the statement that the former property contains—"continet" is occasionally used in place of "est"—the latter in intension: if *A* est *B*, all *A*'*s* are *B*'*s*. The "term" (property) *Ens* is of null comprehension (universal extension); it represents the property containing no (proper) property in its intension or comprehension.[35]

Leibniz also provides an extensional interpretation for this system which is, essentially, the same as that of the first system. The "terms" are predicates in extension (classes), and the *non-operator* and juxtaposition are respectively complementation and intersection, as with the first system. The "term" *Ens* is the class of universal extension,[36] and propriety is, there-

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[34] The method of proof by cases facilitates the check.

[35] If *non-A* is proper, *non-A* est Ens, whence non-Ens est *A*. Thus non-Ens is of (virtually) universal intension, and so the intension of Ens is null.

[36] Ens is the class of all things (*entia*). See footnote 21.

fore, non-nullity. Finally, "est" represents the containment of (proper) classes: "*A* est *B*" signifies that the class *A* is contained in the class *B*, i.e., that all *A*'*s* are *B*'*s*.

In both of these interpretations the classical theory of immediate inference and of the syllogism can be accommodated. Leibniz offers several sets of renditions of the categorical propositions. Among these are the three indicated in Display 1:[37]

Display 1

CATEGORICAL PROPOSITIONS

	(1)	(2)	(3)
a:	*S* non-*P* est non-Ens	*S est P*	$SP = S$
e:	*SP* est non-Ens	*S* est non-*P*	$SP \neq SP$Ens
i:	*SP* est Ens	*S* non est non-*P*	$SP = SP$Ens
o:	*S* non-*P* est Ens	*S* non est *P*	$SP \neq S$.

One point regarding this system has led to some misunderstanding. This is Leibniz's occasional use of "continet" for "est". He employs this usage only when dealing with the intensional interpretation, which is quite proper, since the fact that *A* contains *B* in its intension or comprehension—i.e., that all *A*'*s* are *B*'*s*—is represented by "*A* est *B*". On the other hand, if "est" is to be interpreted in terms of containment in the extensional interpretation, then "*A* est *B*" must be read "*A* is contained in *B*", or else, if "est" is still to be read as "contains", then *A* and *B* must be interchanged. This is explicitly stated by Leibniz in several places.[38] It has been misconstrued as being a statement on Leibniz's part to the effect that "*A* est *B*"

[37] Such sets of symbolic versions of categorical propositions are given in many places, including pp. 211-217 of vol. 7 of *Phil. Schr.* (Gerhardt), and pp. 232-33 of *Opuscules et fragments* (Couturat). Couturat's apparent denial (*Logique*, p. 30) notwithstanding, the intensional interpretation of this second system is adequate to classical syllogistic logic.

[38] *Opuscules* (Couturat) , pp.300 (top) and 384-385.

may be taken as symbolizing "*A* contains *B* (in extension)", and thus as stating in an extensional interpretation of the system that all *A*'*s* are *B*'*s*. This does not yield a valid interpretation of the system, or rather more accurately, it could be correct only if juxtaposition were to represent alternation (i.e., set union or addition),[39] whereas it is uniformly and consistently used by Leibniz to represent conjunction (i.e., set intersection or multiplication). Couturat is guilty of this misconstruction,[40] and on this basis he accuses Leibniz of falling into error by misguided adherence to an intensional point of view. (The purported error in question is that Leibniz fails, because of intensional prejudice, to take juxtaposition as extensional—rather than intensional—union or addition.[41])

Leibniz offers still another interpretation of this second system, one which makes it the forerunner of C. I. Lewis's systems of strict implication. In this interpretation "terms" are propositions, *non* represents negation, juxtaposition represents conjunction, and *est* stands for the relation of entailment.[42] *Ens* represents logical necessity or logical truth, and so propriety is logical consistency.[43] Leibniz rightly views this system, thus interpreted, as a modal logic[44], and thus merits Lewis's estimation of him as a precursor.

In this interpretation, and in it alone, the result of linking "terms" (propositions) by "est" is again a "term". Thus formulas such as "(*A* est *B*) est (C est *D*)" are meaningful in this interpretation.[45] It is also of interest to

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[39] See assertions 16 and 17.

[40] *Logique*, pp. 30-32.

[41] See footnote 24.

[42] Cum dico *A* est *B*, et *A* et *B* sunt propositiones, intellego ex *A* sequi *B*. "*A* est *B*" is held to be the symbolic version of "*A* infert *B*" or "*B* sequitur ex *A*" (Couturat, *Logique*, p.355, notes).

[43] *A* is necessary iff *A* = Ens. *A* is impossible if non-A is necessary, and it readily follows that, "Quod continet *B non-B*, idem est quod impossibile" (*Opuscules* (Couturat), p.368).

[44] Regarding Leibniz's conception of this interpretation see especially the *Generales inquisitiones*.

[45] See Couturat, *Logique*, p. 355.

observe that Leibniz exploited the opportunity, afforded by assertion 23, of defining entailment in terms of negation, conjunction, and the notion of possibility.[46]

We now turn to Leibniz's third and final system of logical calculus. This system was developed in 1690. The writer conjectures that the motivating force underlying its development was Leibniz's growing conviction that the notions of part, whole, and containment are the fundamental concepts of logic.[47] For this system may justly be characterized, as will appear below, as an axiomatic theory of containment. Its principal expositions are the tract *De formae logicae comprobatione per linearum ductus*,[48] as well as several brief, untitled essays reproduced in the seventh volume of Gerhardt's edition of Leibniz's philosophical works.[49]

This system can be viewed as an improved extension of the first system. Upper-case Roman letters, *A*, *B*, C, ..., are "term"-variables. N (sometimes *Nihil*) is a "term"-constant. *non* is a singulary "term"-operator, + (sometimes ⊕) and - (sometimes ∴) are binary "term"-operators. *I nest* is a binary relation between "terms"; but "*A* inest *B*" is also occasionally written by Leibniz in one of the alternative forms: "*A* est in *B*" or "*B* continet *A*". The notation for' =' is '∞ or '∝', and for" $A \neq B$" Leibniz uses "non $A \infty B$" or "non $A \propto B$". Finally, there are two further binary "term"-relations, $\underline{\wedge}$, and its negate, A, where "$A \wedge B$" is written "communicant *A* et *B*" or "communicantia sunt *Aet B*" or "compatibilia suilt *A* et *B*", and "$A \underline{\wedge} B$" is symbolized similarly, but with an appropriate insertion of "non".

The assertions of this system are:

Group I. Assertions 1 through 9 of the first system, with "inest" in place of "est".

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[46] Couturat, *Logique*, p. 355.

[47] The concept of containment provided the central idea underlying Leibniz's interpretations of his first two systems. Already in the second system the notation "continet" was occasionally used in place of "est".

[48] *0puscules* (Couturat), pp.292-321, and d. pp.267-270. Couturat's discussion of this system is given on pages 362-385 of *Logique*.

[49] Number XVI, pp. 208-210, number XIX, pp. 228-235, and number XX, pp. 236 - 247. The last two are available in an English translation in the appendix of C.I. Lewis's *Survey of Symbolic Logic* (Berkeley, 1918).

Group II. 25. $A = A + A$

26. $A + B = B + A$

27. $(A + B) + C = A + (B + C)$[50]

28. If A inest B and C inest B, then $A + C$ inest B

29. If A inest B, then $C + A$ inest $C + B$

30. If A inest B and C inest D, then $A + C$ inest $B + D$

31. A inest B iff $B + A = B$

32. $A \wedge B$ iff not $A \underline{\triangle} B$

33. $A \wedge B$ if $B \wedge A$

34. $A \wedge B$ iff A inest non-B

35. $A \underline{\triangle} B$ iff there is a C, $C \neq N$, such that C inest A and C

36. $A - B = C$ iff $A = B + C$ and $C \wedge B$

37. $A - B \wedge B$

38. $A - A = N$

39. $A + N = A$

40. N inest A.

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[50] Leibniz nowhere explicitly states this associative law. However he uses it in proofs, and he writes sums without parentheses (*Phil. Schr.* (Gerhardt), pp. 228 ff.). [Note that Leibniz is elsewhere scrupulous in their use, e.g., (*Opuscules* (Couturat) pp. 356 ff)]. K. Dürr also adds this associative law in his exposition of Leibniz's logic, rightly saying that, "Diese Ergänzung dient lediglich dazu, das Verständnis des Systems von Leibniz zu erleichtern; es wird dadurch an dem System nichts Wesentliches verändert" (*Die mathematische Logik von Leibniz*, p. 100).

The consistency of this system follows from the existence of the following interpretation: Let "terms" be classes, N the null class, *non* complementation, + class union, *inest* class inclusion, and let $\underline{\wedge}$, $\wedge$, and - be defined by 35, 32, and 36, respectively. This interpretation is also of interest in connection with the following, more general considerations.

Leibniz explicitly intends this system to provide an abstract theory of containment. Given a sound application of the concepts of whole and of part, an interpretation of this third system is, Leibniz claims, available.[51] For if such a notion of containing is given, then "*A* inest *B*" can be taken to represent "*B* contains *A* (in the sense in question)", "*non-A*" represents that containing everything not contained in *A*, N is that which contains nothing, *A* + *B* contains everything contained in *A* or in *B* or both, and all else may be interpreted correspondingly.

In the light of our discussion of the previous systems, it is clear how Leibniz constructs interpretations of this system as a logic of predicates in intension and also as a logic of predicates in extension. Thus the four categorical propositions, *a*, *e*, i, and *0*, are rendered *S* inest *P*, $S \wedge P$, $S \underline{\wedge} P$, and *S* non inest *P* extensionally, and in intension as *P* inest *S*, $P \wedge S$, $P \underline{\wedge} S$, and *P* non inest *S*. Again, the classical theory of immediate inference and the syllogism is available in the assertions of the system. However, assertion 35 is required both for subalternation and partial conversion, and so both of these inferences must be conjoined with an explicit statement of the non-nullity of the terms involved. In this feature of explicitness, together with its more abstract and general nature, resides the superiority of Leibniz's third system over the first two.

Here our survey of Leibniz's interpretations of his three principal logical calculi reaches its end. We have seen that these interpretations are of three types: a logic of predicates in intension, a logic of predicates in extension, and a modal logic of propositions. In each case our investigation has revealed the soundness of the interpretation. We have found nothing to support Couturat's contention that Leibniz's favoritism toward an intensional point of view had dire consequences for his logic.[52] If Leibniz's

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[51] That is why this system is presented as *a Non inelegans specimen demonstrandi in abstractis* (*Phil. Schr.* (Gerhardt), vol. 7, pp. 228 ff.; d. Lewis, *Survey of Symbolic Logic*, pp. 373-379). This essay—especially the third definition and the various scholia—shed much light on Leibniz's conception of this third system of logical calculus.

[52] Lewis's evaluation of Leibniz's logic is of interest: "It is a frequent remark upon

logical calculi do not possess the symmetry and elegance of later algebras of logic it is not because of his intensional conception of logic, but because his greater commitment to traditional logic inclines him to weigh more heavily the semantically logical, rather than the systemically algebraic, considerations.[53]

Leibniz' contributions to logic that he failed to accomplish this or that, or erred in some respect, because he chose the point of view of intension instead of that of extension. The facts are these: ... He preferred the point of view of intension, or connotation, partly from habit and partly from rationalistic inclination ... This led him into some difficulties which he might have avoided by an opposite inclination or choice of example, but it also led him to make some distinctions the importance of which has since been overlooked and to avoid certain difficulties into which his commentators have fallen." (*Survey of Symbolic Logic*, p. 14.)

53 This chapter was originally published in *The Journal of Symbolic Logic*, vol. 19 (1954), pp. 1-13.

Chapter 9

RUSSELL AND MODAL LOGIC

1. INTRODUCTION

Bertrand Russell's repute as one of the founding fathers of modern symbolic logic is secure for all time, and his claims to greatness as a logician are established to an extent beyond my meagre capacity to alter for better or for worse. Accordingly, it is no real unkindness to Russell's memory to observe in the interests of historical justice that he, too, once more illustrates the rather trite precept that even scholars of deservedly great stature can exhibit a bias of intellect that produces unfortunate side effects. At any rate, the aim of this present discussion is to note the substantially negative import of Russell's work for the evolution of modal logic, whose rapid growth since the late 1940s is unquestionably one of the most exciting developments in contemporary logical research.

2. PHILOSOPHICAL BACKGROUND

From the very first, Russell was on philosophical grounds reluctant, nay unwilling, to recognise the merely possible (i.e. the *contingently* possible) as a distinct category. His *Critical Exposition of the Philosophy of Leibniz* (Cambridge University Press, 1900) exemplifies this attitude. It was, Russell held, improper for Leibniz, given his own commitments, to espouse the category of mere possibility, and to maintain the contingency of actual truth: he should have held that all truths about the world are necessary. Thus, Russell reproached Leibniz with not reaching more Spinozistic conclusions: had Leibniz traced out his own lines of thought more rigorously he would have arrived at the position of Spinoza.

Russell's criticism of Kant in the *Principles of Mathematics* (Cambridge University Press, 1903) gives another revealing insight into his position. According to Russell, the Kantian analysis of the foundations of necessity is drastically insufficient. In tracing the source of necessity to the categories and forms of the human understanding, Kant—so Russell holds—merely provides a contingently factual basis that cannot provide an appropriate foothold for necessity proper:

> [On the Kantian theory of necessity] we only push one stage further back the region of “mere fact”, for the constitution of our minds remains still a mere fact. The theory of necessity urged by Kant, and adopted . . . by Lotze, appears radically vicious. Everything is in a sense a mere fact. (section 430)

The philosopher, Russell seems to imply, is engaged on a quest for the necessity of things that does not permit him to rest content, at any stage, with anything that is a matter of mere fact. Just this attitude lay behind Russell’s rejection at this stage of the empiricist philosophy of mathematics of John Stuart Mill, which would not provide a suitable account of the necessity of mathematical truth.

The philosophical roots of the early Russell’s discontent with merely factual truth are to be found in his prolonged flirtation with the philosophy of Spinoza, a marked feature of *Mysticism and Logic* and vividly at work in the splendid essay on ‘A Free Man’s Worship’. Drawn to Spinozistic necessitarianism on powerful ideological grounds, Russell shied away from all traces of Leibnizian possibilism.

Himself a determinist of more or less classical proportions, Russell was committed to a necessitarianism that left him disinclined on philosophical grounds to allocate a logically useful role to the modal distinctions between the possible, the actual and the necessary. Like his hero, Spinoza, he was prepared to maintain that there will, in the final analysis, be a *collapse* of modality: that the actual itself is more or less necessary, so that the possible vanishes as a distinct category. This philosophical stance was, I believe, significantly operative in Russell’s negative view *as a logician* regarding the utility and prospects of modal logic.

3. MATHEMATICAL BACKGROUND

Russell’s philosophical perspectives were, of course, substantially influenced by his preoccupation with mathematics. The early Russell pioneered the tendency, destined to become predominant in his later years, of approaching logico-philosophical problems from the mathematical point of view. Now mathematics has, of course, no place for modal distinctions: in mathematics it is altogether otiose to differentiate between the actual and the necessary, and there is no room at all for the contingently possible. Throughout the mathematical domain the drawing of modal distinctions is effectively beside the point.

Moreover, it seems particularly pointless to apply the concept of necessity to the theses of a mathematical system like Riemannian geometry; what is necessary—and also what is mathematically interesting—are the relationships of deductive consequence by which theorems follow from axioms. Accordingly, we find Russell maintaining in *The Principles of Mathematics* (1903) that:

> Thus any ultimate premiss is, in a certain sense, a mere fact. . . . The only logical meaning of necessity seems to be derived from implication. A proposition is more or less necessary according as the class of propositions for which it is a premiss is greater or smaller. In this sense the propositions of logic have the greatest necessity, and those of geometry have a high degree of necessity. (section 430)

The *relative* necessity of mathematical propositions is to be defined in terms of implicative relationships, according as the body of the propositions that are needed as premisses for it is the less or as that of propositions for which it can serve as premiss is the greater. And the necessity of deductive consequence is the basic mode of necessity as well as the only ultimately genuine *form* thereof. But now—once one follows Russell in defining pure mathematics as 'the theory of propositions of the if-then form'—one arrives at a view of pure mathematics that sees all its propositions as having this necessity of consequence. This strictly relativised necessity of deductive consequence is all one needs in the philosophy of mathematics, and so there is—for example—little point of speaking of the theorems of a mathematical system as necessary in ways other than as shorthand for 'necessary relative to the axioms'. Absolute and unrelativised necessity is not only otiose, but obscurantist as well: the 'necessity of consequence' is the ultimately basic form of necessity.

At this point Russell's logicism intervenes decisively in the dialectic of thought. If the basis of our concern with logic is its role in the rational articulation of mathematics; nay, if there is at bottom a fundamental *identity* of logic with mathematics, then the handwriting is on the wall. For if mathematics has no real need for modal distinctions and no room for contingent possibilities, a modal *logic* becomes almost a contradiction in terms. This standpoint blocks any concern on the logician's part with mere possibilities and alternative possible worlds. Such concerns of traditional philosophy come to be seen as metaphysical sophistries upon which the logician must simply turn his back.

Thus, both from the point of departure of his philosophical determinism and from that of his mathematical logicism, Russell was powerfully predisposed against the maintenance of modal distinctions which could only secure their validation in a rationale that recognises the prospect of contingent possibilities. These considerations provide the background for understanding Russell's relationship to those logicians—preeminently Hugh MacColl, C. I. Lewis and Jan Lukasiewicz—who pressed for the recognition of modal distinctions during the period (roughly 1895-1925) when Russell was actively preoccupied with logic. Let us examine this phenomenon in some detail.

4. INTERACTIONS: RUSSELL AND MACCOLL

In a series of articles published over a period of some thirty years beginning in 1880,[1] Hugh MacColl argued a number of points which any modern modal logician will recognise as foundational for his entire subject:

a. that there is a crucial difference between propositions that obtain merely *de facto* and those that obtain of necessity; between those which *must* hold and those which *mayor may not* hold (even if they actually do so). (The former type of truths MacColl characterised as *certain*, the latter as *variable*.)

b. that there is a crucial difference between a *material* implication and genuine implication. 'For nearly thirty years', he complained in 1908, 'I have been vainly trying to convince them [i.e. logicians] that this supposed invariable equivalence between a conditional (or implication) and a disjunction is an error.'[2]

c. that a satisfactory logic of modality must distinguish between actually existing individuals and merely possible ones; and that, accordingly, in constructing quantificational logic we should *not* simply and automatically presuppose that we are dealing with actually existing individuals.

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[1] MacColl published some forty books and papers during the years 1877-1910. For details see the bibliography by A. Church in *Journal of Symbolic Logic*, vol. I (1936: 132-3).

[2] *Mind*, vol. XVII (1908: 151-2); see p. 152.

Russell, of course, would have none of this. As far as he was concerned, all of MacColl's doctrines were the results of rather elementary errors. His distinction between certain and variable statements, for example, results from not distinguishing between propositions and propositional functions, and is simply a misguided and misleading way of dealing with the difference between them. There is no need to go beyond the twofold categorisation of propositions proper as true and false.[3]

In sum, Russell's philosophical positions and allegiances led him to dismiss all of MacColl's doctrines as so much old-fashioned fairy tale nonsense.[4]

5. INTERACTIONS: RUSSELL AND MEINONG

One idea operative in MacColl—and even more prominently in the work of Alexius Meinong—came to arouse Russell's particular ire: the idea of unrealised or non-actual particulars. The conception that there are non-existent individuals—i.e. particulars which don't exist in this, the actual world but could exist in some alternative dispensation—represents an idea of longstanding credentials in philosophy, figuring in the Presocratics; in medieval scholasticism, and in Leibniz, in addition to its prominent role in the philosophy of Brentano.[5]

According to Russell, MacColl's distinction between actual and merely possible individuals is the result of an incorrect theory of naming accord-

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[3] "'If' and 'Implication': a Reply to Mr. MacColl", *Mind*, vol. XVIII (1908: 300-1; cf. *Introduction to Mathematical Philosophy* (London, George Allen & Unwin, 1919: 165).

[4] For Russell the uncongeniality of MacColl's ideas was compounded *by* that of his somewhat idiosyncratic logical symbolism. The Russell Archives at McMaster University contain some twenty-five letters and postcards sent by MacColl to Russell over the years 1901-9. In one of these MacColl complains that he and Russell have as much difficulty understanding one another as would an Englishman who knows little or no French and a Frenchman who knows little or no English. In a later handwritten annotation of a typed transcription of a letter of MacColl's, dated 28 May 1905, Russell writes: 'MacColl was a symbolic logician of some eminence. I have a very large number of letters from him, but I have not included them in this [typed] selection because they are in his difficult symbolism.'

[5] For the history see the chapter on 'The Conception of Nonexistent Possibles' in N. Rescher, *Essays in Philosophical Analysis* (University of Pittsburgh Press, 1969).

ing to which any combination of letters which functions grammatically as a name must actually name something in virtue of this function. When in fact the name names nothing (e.g. 'Pegasus')—and is thereby in Russell's opinion not properly speaking a name at all—MacColl, under the spell of linguistic usage, provides a referent for the name in the guise of a merely possible individual. For Russell those theoreticians who, like MacColl and Meinong, accept an ontology of 'merely possible objects' have fallen victim to the logically deceptive distortions inherent in our ordinary use of language.[6]

This conception of merely possible individuals is altogether anathema to Russell, who indeed flatly dismisses the very meaningfulness of ascribing existence to individuals. He writes:[7]

> For the present let us merely note the fact that, though it is correct to say 'men exist', it is incorrect, or rather meaningless, to ascribe existence to a given particular *x* who happens to be a man. Generally, 'terms satisfying ϕx exist' means ϕx is sometimes true'; but '*a* exists' (where *a* is a term satisfying ϕx) is a mere noise or shape—devoid of significance.

Rather than run the risk of having to put up with non-existent possible individuals, Russell is willing to dispense altogether with the whole process of attributing existence to things.

6. INTERACTIONS: RUSSELL AND C. I. LEWIS AND J. LUKASIEWICZ

It is also illuminating to consider Russell's reactions to the work of two other pioneers of modal logic in its contemporary guise as a branch of symbolic logic. I think here primarily of C. I. Lewis whose important *Survey of Symbolic Logic* appeared in 1918 and Jan Lukasiewicz whose important historical and systematic inquiries came into increasing prominence after the early 1920s. It is a perhaps surprising, but, I think, interesting and not insignificant fact that one can search Russell's pages in vain for any

6 These considerations in the backwash of Russell's classic paper: 'On Denoting', *Mind*, vol. XIV (1905): 479-93)—that 'paradigm of philosophy' as F. P. Ramsey and G. E. Moore called it, and as indeed it was for much of English philosophy during the interwar era—are doubtless too familiar to need detailed documentation.

7 *Introduction to Mathematical Philosophy*, op. cit.

recognition of the work of these men. (Both in the second edition of the *Principia* (1925-7) and in the second edition of the *Principles* (1937) Russell preserves total silence with respect to all these developments.) And this seems especially inexplicable in view of the fact that the issues that provided these writers with their entry-point into the realm of modal ideas were topics of very special interest to Russell. (In Lewis's case the motivating issue was the philosophy of Leibniz, in Lukasiewicz's it was that of determinism and problems of prediction and future contingency in the context of the philosophy of Aristotle.) Again, it is also startling that Russell also ignores totally the development of mathematical intuitionism, especially the writings of L. E. J. Brouwer, whose work provides a possible bridge to the modal realm from points of departure in the philosophy of mathematics.

A clear picture emerges: in *pre-Principia* days Russell sharply opposed philosophers like MacColl and Meinong who sought to promote concern with the logic of modalities; in *post-Principia* days, secure on his own logico-mathematical ground, Russell simply ignored writers like Lewis and Lukasiewicz and the intuitionists whose work could provide a basis for the introduction of modalities into the framework of symbolic logic. Where modal logic was concerned, Russell adopted Lord Nelson's precedent, and stolidly put his telescope to the blind eye.

7. THE FASCINATION WITH TRUTH-FUNCTIONALITY: LOGICAL ATOMISM AND LOGICAL POSITIVISM

The line of thought operative here is, of course, intimately linked to Russell's theory of logical constructions, and to the methodological precept of logical constructionism, which he articulates as follows.[8]

> The supreme maxim in scientific philosophising is this: Whenever possible, logical constructions are to be substituted for inferred entities.

Clearly the dismissal of all inferred entities and processes points towards a demise of potentialities, powers and causal efficacy that pulls the rug out from the main motivation for recognising possibility and contingency. The logical construction of something real will, quite evidently, be a construc-

8 Quoted in Rudolf Carnap, *Der logische Aufbau der Welt: Scheinprobleme in der Philosophie* (Berlin-Schlachtensee, Weltkreis-Verlag, 1928: I).

tion from elements that are themselves altogether actual (real). This facet of Russell's philosophy provides yet another facet of his rejection of modality.

The generally reductivist penchant of the theory of logical constructions found its clearest expression in various aspects of Russell's reductivistic programme in logic and the extraction of all logical operations from atomic elements by truth-functional modes of combination.

It is important to recognise that Russell was himself deeply caught up in the ideology of two-valued truth-functionality that was part of the heritage of Frege and received its canonical formulation in Ludwig Wittgenstein's *Tractatus Logico-Philosophicus*. The Russellian programme of 'The Philosophy of Logical Atomism'—as well as Wittgenstein's Tractarian theory correlated with it—envisaged the definitional reduction of all concepts of interest and utility in the precise sector of philosophy to truth-functional conjoinings or combinations of basic propositions which (since meaningfully definite) will themselves be either true or false.[9] Being truth-functional, these modes of combination have the feature that the truth-status of a compound can always be determined in terms of the respective status of its several constitutive components.[10]

Thus significant weight came to be borne by not strictly logical, but essentially methodological (perhaps even metaphysical) considerations, as built into the philosophy of logical atomism. We do not have a rationally adequate grasp of theses that have not been analysed into their components. Such analysis calls for indicating the component elements and composing structures through which the logical character of complexes is determined (*truth-functionally* determined) in terms of the status of the component elements. If a thesis that is internally complex in its conceptual

9 But note that Wittgenstein gave hints from which Carnap developed his rationalisation of modal logic.

10 The strengths and limitations of this programme were brought into clearest relief in A. Tarski's classic essay on the concept of truth in formalised languages, 'Der Wahrheitsbegriff in den formalisierten Sprachen', *Studia Philosophica, Warsaw*, vol. I (1935-6: 261-405); German trans., original in Polish (1930). It is of interest in illustrating the pervasiveness of the truth-functional tenor of thought that Tarski in the early 1930s rejected the proposal (of Z. Zawirsky and H. Reichenbach) to consider the probability calculus as a form of many-valued logic on the grounds that probabilities do not behave in a truth-functional manner; cf. the discussion in N. Rescher, *Many-valued Logic* (New York, McGraw-Hill, 1969: 184-8).

structure does not submit to truth-functional analysis, this is a mark of an internal imprecision whose toleration is a concession to obscurantism.

The stress upon logical reducibility was, of course, vastly congenial to the ethos of logical positivism. This took increasingly definite form during the first decade after the First World War, whose programmatic menu offered a Hobson's choice between the *reduction* and the *abandonment* of philosophically problematic concepts. We are brought back to the influence of that 'paradigm of philosophy' (according to F. P. Ramsey and G. E. Moore), the Theory of Descriptions, according to which some conception that is standardly operative in our ordinary scheme of thought about things is reductively annihilated as the mere product of linguistic illusion.

Now the critical fact which all concerned recognised as a feature of modal concepts is that none of them—be they absolute (like possibility or necessity) or relative (like entailment or strict implication)—will be truth-functional. It was throughout recognised by all concerned as a vain enterprise to analyse modal concepts in two-valuedly truth-functional terms. And for Russell and the bulk of the positivistically-inclined logical tradition that followed him down to the days of Goodman and Quine, this very fact provided the basis for a rejection of modality.

But from the first there were dissentients. The logical structure of the basic conceptual situation was created by an inconsistent triad:

a. the insistence upon propositional two-valuedness;

b. the insistence upon the truth-functionality of all proper propositional operators and connectives;[11]

c. the legitimacy of modal distinctions.

As indicated, Russell and his positivist congeners abandoned (c), but others took a different route. In his single-minded pursuit of a concept of relative necessity and a really viable analysis of if-then, C. I. Lewis gave up the truth functionality of (b) and developed the theory of strict implication. Jan Lukasiewicz in his pursuit of an Aristotelian theory of future contingency gave up (a), and developed many-valued logic. (Brouwer and his Intuitionist followers gave up the entire concept of mathematico-logical analysis

11 There is no essential link between two-valuedness and truth-functionality. Connectives in a many-valued logic can, of course, be truth-functional.

upon which the concept of propriety operative in (b) is based.)

The ideological penchants and predilections of logical positivism involved: (i) a commitment to a sharp-edged criterion of truth that was unwilling to tolerate the pluralism of a theory of degrees of truth or to acknowledge—as apart from the altogether meaningless—any shades and gradations as between the true and the false, and (ii) a commitment to a criterion of meaning unwilling to recognise as meaningful conceptions not definition ally reducible to the clear conceptions of a canonical basis. Accordingly, while the appropriateness of efforts at a corresponding reduction of conceptions like absolute and relative modalities (or of counterfactual conditionals, to take another example) might be recognised, the failure of such a quest for two-valued reduction was taken as to be construed to spell not the inadequacy of the reductive programme, but the illegitimacy of the putatively irreducible concepts. In this positivistic atmosphere, the Russellian distaste for modal concepts hardened into an attitude of virtually dogmatic rejection of modal logic.[12]

8. EFFECTS

Orthodox two-valued and truth-functional logic—'classical' logic as it is now frequently called—in the form given it by Russell, his associates, and their followers, enjoyed enormous successes. The mainstream of development in the tradition of logicians like Hilbert, Gödel, Tarski, Church, Rosser, et al. developed logic into a powerful instrument for exploring the foundations of mathematics and more than justified Russell and Whitehead's selection of that proud title of their monumental work.

Modal logic remained in the shadows for a long time. It did not really begin to come into its own until the development of modern 'modal semantics, largely under the impetus of Rudolf Carnap[13] (erecting a structure of his own on foundations laid by Wittgenstein and Tarski). It was Carnap who first successfully elaborated the possible-world semantics which those who followed in his wake were to, build up into the grandiose structure we

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[12] The reluctance or inability of logical positivism to come to serious and effective grips with the logic of modality proved a serious stumbling-block to the success of the movement. See Hans Poser, 'Das Scheitern des logischen Positivismus an modaltheoretischen Problemen', *Studium Generale*, vol. 24 (1971: 1522-35).

[13] *Introduction to Semantics* (Cambridge, Mass.: Harvard University Press, 1942); and especially *Meaning and Necessity* (University of Chicago Press, 1947).

know today. Until the late 1940s it remained to all intents and purposes the concern of a few eccentric philosophical guerrillas concerned to snipe from the sidelines as the main column of modern mathematical logic marched by *en route* from victory to victory in directions appointed for it by the orientation of Russell's work. Thus, during the period from the early 1920s to the late 1940s, the great bulk of logicians and logically-concerned 'philosophers—indeed virtually everyone outside the range of the personal influence of Lewis and Lukasiewicz[14]—adhered to Russell's negative stance towards modal concepts. The great successes of the Russellian vision of logic in the mathematical sphere gave a massive impetus to his negative view of modality. The upshot was, I think it not unfair to say, that the development of modal logic was set back by a full generation.

There is no fundamental historical reason why modern symbolic modal logic could not have developed substantially sooner. The basic tools forged by MacColl and Lewis lay to hand by 1920, as did those hints of Wittgenstein's *Tractatus* (relating to probability) from which Carnap first systematised the possible-worlds interpretation of modal logic. There is no reason of historical principle why the logic of modality which surged up shortly after Carnap's *Meaning and Necessity* (University of Chicago Press, 1947) could not have begun soon after 1920. This development was certainly delayed by a full generation during the period between the two world wars. This delay can be attributed in no small part to views and attitudes held by Russell and promulgated under the influence of his massive authority.

It would be just plain wrong to say that the time was not ripe for the development of modal logic in the period between the two world wars. The ideas were there, the pioneering work was being done, the relevant publications were part of the public domain. But this work simply did not have the reception it deserved—far too little attention was paid to it. And this was due not to any lack of intrinsic interest or importance or to any *logical* disqualifications, but principally to *ideological* factors. Put bluntly, the development of modal logic was retarded primarily because Russell and his positivist followers found modal conceptions *philosophically* uncongenial. And the influence of Russell was a crucially operative factor here. There is no question in my mind that if Russell had possessed a more urbane, tolerant and receptive interest in logical work that did not resonate to his own immediate philosophical predilections, it would have done a great deal of good.

[14] Brouwer and the non-classicists in the foundations of mathematics, of course, had no interest in modal logic, since modal concepts play no role in mathematics.

9. ASSESSMENT

Russell's work and the stimulus it exerted upon others was responsible for a massive forward step in the development of modern symbolic logic in its 'classical' articulation, in a form eminently suited to mathematical developments and applications. The massive proportions of his contribution cannot be questioned. Nevertheless, in so far as the line of thought presented here is at all correct, it appears that baneful consequences ensued from Russell's work and its influence for the development of modal logic. But the question remains: Was this just an unfortunate historical accident or was it something for which Russell himself deserves a certain measure of responsibility?

This question is certainly not otiose or irrelevant. We know full well that the blame for abuse by later followers of the contributions of a master must not inevitably be laid on his own doorstep. We cannot reproach the humane Dr. Guillotin—concerned only to minimise the agonies of criminals condemned to execution—with the excesses of the abuse of his favoured implement during the Terror phase of the French Revolution. Nor can we reproach that devoted and conscientious scholar Darwin for the callous application of his ideas by some among the Social Darwinists. A master innovator can fall blameless victim to the rationally unbridled zeal and unrestrained excesses with which his followers exploit his ideas. He can certainly fall the unhappy hostage of an unforeseen and to him almost certainly unwelcome abuse of his ideas.

But is this defensive line available in Russell's case to blunt the charge of responsibility for impeding the development of modal logic? I think not, because in this case the central factor is a question not of the unforeseen and presumably undesired consequences of certain innovations, but of Russell's own views and positions. His own deliberately held negative views towards modal conceptions—opinions espoused on conscious and philosophically reasoned grounds—were themselves operative forces behind the impact of Russell's position in this sphere.

The distaste for modal logic in Anglo-American philosophy during the period between the two world wars was virtually initiated by Russell and largely propagated by his great influence. The development of modal logic was impeded neither by accidental factors nor because this branch of logic is itself lacking in substantive interest from a logical point of view, but because many logicians were led under Russell's influence to regard it as philosophically distasteful. It seems to me by no means unjust to place squarely at Russell's door a substantial part of the responsibility for the stunted development of modal logic during the two generations succeeding the pioneering days of Hugh MacColl. Russell's philosophically inspired attitudes propagated a negative view of modal logic and helped to produce that disinclination to take modality seriously which can still be seen at work among our own contemporaries of the older generation (e.g. W. V. Quine and N. Goodman). The very success of Russell's work in the more mathematically oriented sectors of logic gave authority and impact to his antagonistic stance towards the logic of modality. For the development of *this* area of logic, at any rate, Russell's work represented a distinctly baneful influence.

Please do not misunderstand my intentions. It is not really my aim to accuse Russell of any moral dereliction in regard to modal logic. After all, every philosopher is entitled to his full share of human failing, myopia and even prejudice. My concern is not so much with moral as with causal responsibility. It is my prime aim to establish the *causal* fact that Russell's disinclination towards modal conceptions substantially retarded the development of modal logic. As regards issue of praise or blame I leave it to the reader to draw his own conclusions.

Certainly nothing could be more wise and urbane than the pious sentiments of the concluding paragraph of Russell's review in *Mind* of MacColl's *Symbolic Logic and Its Applications*:[15]

> The present work . . . serves in any case to prevent the subject from getting into a groove. And since one never knows what will be the line of advance, it is always most rash to condemn what is not quite in the fashion of the moment.

Anyone concerned for the health and welfare of modal logic as an intellectual discipline cannot but wish that Russell himself-and especially that majority among his followers who were perhaps even more royalist than their king—had seen fit to heed this eminently sound advice.[16]

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[15] *Mind*, vol. XV (1906), pp. 255-60; see p. 26.

[16] This chapter was originally published in G. W. Robert (ed.), *Bertrand Russell Memorial Volume* (London: George Allen & Unwyn, 1979), pp. 139-49.

Chapter 10

DEFAULT REASONING

Default reasoning is a new branch of inductive logic which emerged to prominence in the last decades of the twentieth century. Branches of knowledge sometimes originate in the work of single individuals like a mighty river such as the Nile they spring from a single source, much as Aristotle originated syllogistic logic. In other cases, a discipline originates in the unification of a myriad individual findings like a river formed from the gradual confluence of myriad rivulets. The development of default logic illustrates this phenomenon. In fact, the discipline was not so much originated as grown—assembled from parts and pieces of material developed in different problem areas.[1]

1. DEFAULT INFERENCE

A *default* in logic is a fall-back position in point of conclusion-drawing—one to which we can appropriately take resort when things go wrong. But of course things ought not to go wrong in logic. So what is going on here?

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1 Regarding default reasoning and its ramifications see "Common-Sense Reasoning" in *The Routledge Encyclopedia of Philosophy* (London: Routledge, 2000); William L. Harper, "A Sketch of Some Recent Developments in the Theory of Conditionals," in W. L. Harper, L. G. Pearson, and Robert Stalnaker (eds.) *IFS: Conditionals, Belief, Decision, Chance and Time* (Dordrecht, D. Reidel, 1981); Henry E. Kyburg, Jr. and Chou Man Teng, *Uncertain Inference* (Cambridge: Cambridge University Press, 2001); Hans Rott, *Choice and Inference: A Study of Belief Revision and Nonmonotonic Reasoning* (Oxford: Clarendon Press, 2001); Alexander Bochman, *A Logical Theory of Nonmonotonic Inference and Belief Change (Berlin: Springer, 2001);* J. L. Pollock, "A Theory of Defeasible Reasoning," *International Journal of Intelligent Systems*, vol. 6 (1991), pp. 33-54; R. B. Reiter, "Nonmonotonic Reasoning," *Annual Review of Cognitive Science*, vol. 2 (1987), pp. 147-86; as well as Nicholas Rescher, *Induction* (Oxford: Basil Blackwell, 1980) and *Presumption* (Cambridge: Cambridge University Press, 2006).

Orthodox inferential reasoning proceeds via *logically valid* inference processes which, as such, do—and must—lead to true conclusions when the premisses are true. By contrast, default reasoning—which involves an information-gap between premisses and conclusion—is fundamentally inductive in that here premisses that are true will lead to plausible (though possibly false) conclusions.

The logical validity of inference rules in standard (truth-functional) logic is determined on an input-output basis, a valid rule being one that will invariably yield true outputs (conclusions) from true premisses. All such inference rules will faithfully and unfailingly transmit the truth of premisses to the conclusions. By contrast, the inference processes of default logic are such that the truth of the premisses does not assure that of the conclusion but will at most establish that conclusion as plausible. Such inferences are *ampliative*: the conclusion can go beyond what the premisses provide, thanks to a shortfall of information. And this means that such reasonings are fallible and can—and occasionally will—lead from true premisses to false conclusions.

We shall represent *logically valid deducibility* (in its classical construction) by $\vdash$, and by contrast use $\Vdash$ to represent the *plausible inferability* at issue with default reasoning.

Some examples of inference-processes in default logic are as follows:

(1) p is highly likely $\Vdash p$

(2) p is very likely, q is very likely $\Vdash p$ & q is very likely

(3) there is strong evidence in favor of p and no more than weak evidence against it $\Vdash p$

(4) p has obtained in all past instances $\Vdash p$ will obtain in the next instance

If all we are told of some number is that it is a prime, we would, plausibly enough, conclude that it is not an even integer—even though we are aware that this conclusion will prove false once out of an infinity of cases (viz. that of the number two).

As these examples indicate, the inference processes of default logic can all be assimilated to a deductive pattern of the following structure (which does clearly obtain as valid in traditional logic):

- In all ordinary (normal, standard, commonplace) cases, whenever *P*, then *Q*.
- *P* obtains in the case presently at hand
- <The present case is an ordinary (normal, standard, etc.) one.>

∴ *Q* obtains in the present case

Here that third, usually tacit and thereby enthymematic, premiss plays a pivotal role. And it is, in general, able to do so not because we have secured it as a certified truth, but simply because it is a plausible (albeit defeasible) presumption that is strongly supported by the available evidence though not, of course, guaranteed. Default reasoning accordingly rests on arguments which would be valid if all of their premisses—explicit and tacit alike—were authentic truths, which they are not since at least one of the critical premisses of the argument is no more than a mere presumption.

Such a *defeasible presumption* is emphatically not to be regarded as an established truth but merely something that holds only provisionally, as long as is no counter-indicatively conflicting information comes to light. Against this background the procedure that is definitively characteristic of default reasoning is:

> To treat what is generally (normally, standardly, generally, usually, etc.) the case *as if* it were the case always and everywhere, and therefore as applicable in the present instance.

Here, in effect, ignorance is bliss: where there is no good reason to see the case at hand as being out of the ordinary, we simply presume it to be an ordinary one in the absence of visible counter-indications. Such reliance on a *principle of presumption* to the effect that what generally holds also holds here in the case presently at hand that defines the modus operandi of default reasoning.

2. FACING THE PROSPECT OF ERROR

Of course such plausible presumption can go awry. For it may well happen that the situation at hand fails to be standard and representative as the enthymematic comportment of the argument requires. This is brought out vividly in John Godfry Saxe's poem "The Blind Men and the Elephant" which tells the story of certain blind sages, those "six men of Indostan/To learning much inclined/Who went to see the elephant/(Though all of them were blind).” One sage touched the elephant’s “broad and sturdy side” and declared the beast to be “very like a wall”. The second, who had felt its tusk, announced the elephant to resemble a spear. The third, who took the elephant’s squirming trunk in his hands, compared it to a snake; while the fourth, who put his arm around the elephant’s knee, was sure that the animal resembled a tree. A flapping ear convinced another that the elephant had the form of a fan; while the sixth blind man thought that is had the form of a rope, since he had taken hold of the tail.

> And so these men of Indostan,
> Disputed loud and long;
> Each in his own opinion
> Exceeding stiff and strong:
> Though each was partly in the right,
> And all were in the wrong.

None of those blind sages was altogether in error, it is just that the facts at their disposal were nontypical and unrepresentative in a way that gave them a biased and misleading picture of reality. It is not that they did not know truth, but rather that an altogether plausible inference from the truth they knew propelled them into error.

But since such a policy of typicality presumption may well lead us down the primrose path into error, how is it ever to be justified? The answer here lies precisely in the consideration that what is at issue is not a truth-claim but a policy or procedure. And such policies of procedure are not justified in the theoretical (i.e., factual) order but in the practical or pragmatic order of deliberation. The validation at issue runs roughly as follows:

1. We have questions to which we need a (satisfactory) answer, and in the face of this we take the stance that—

2. We are rationally entitled to use a premiss that holds good promise of finding one (i.e., is effective or more effective than the other available alternatives) even though it may occasionally fail.

On this basis we proceed subject to the idea that if and when things go wrong, this is a bridge we can cross when we get there, invoking “explanations" and excuses to indicate the unusual (anormal, extraordinary) circumstances of the case.

Even as in real life we cannot manage our affairs sensibly without running risks, so in the cognitive life one must, on occasion, take the risk of error in stride, since the inevitable result of a radical nothing-risk policy is the nothing-have of radical skepticism. And this situation is particularly prominent in inductive contexts.

3. INDUCTION AS DEFAULT REASONING

The term "induction" is derived from the Latin rendering of Aristotle's *epagôgê*—the process for moving to a generalization from its specific instances.[2] Gradually extended over an increasingly wide range, induction can be seen as a question-answering device encompassing virtually the whole range of non-deductive reasoning. Thus consider a typical inductive argument—that from "All the magnets we have examined attract iron filings" to "All magnets attract iron filings". It would be deeply problematic to regard this as a deductive argument that rests on the (obviously false) premiss: "What is the case in all examined instances is universally the case." Rather, what we have here is a plausible presumption that takes the cases in hand to be typical and generally representative in the absence of concrete counterindications—that is, an instance of default reasoning.

Induction, so regarded, is accordingly not so much a process of *inference* as one of presumption-based *truth-estimation*. We clearly want to accomplish our explanatory gap-filling in the least risky, the minimally problematic way, as determined by plausibilistic considerations. This is illustrated by such examples as:

- There is smoke yonder
- Usually, where(ever) there's smoke, there's fire
- <The present situation fits the usual run>

∴ There is fire yonder

or again

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[2] See W. D. Ross, *Aristotle's Prime and Posterior Analytics* (Oxford: Clarendon Press, 1949), pp. 47-51.

- Two thirds of the items in the sample are defective
- <The sample is representative of the whole>

$\therefore$ Two thirds of the items in the whole population are defective

(Here the enthymematically tacit premisses needed to make the argument deductively cogent have been indicated.)

Its reliance on a presumption of typicality, normalcy, or the like, means that any inductive process is inherently chancy. Induction rests on presumption-geared default reasoning and its conclusions are thus always at risk to further or better data since what looks to be typical or representative may in due course turn out not to be so. And as such considerations indicate, presumption is a significant—perhaps even the most important—tool in default reasoning.[3]

4. DEFAULT REASONING AS NONMONOTONIC

In virtue of the fact that default reasonings rest on a presumption of normality, typicality, or the like, it may well transpire that while a premisses $\Vdash$ implies a certain conclusion nevertheless the conjunction of this premisses with some further propositions may fail to do so. Such implications are called non-monotonic because while "If *p* then *q*" obtains, nevertheless it can happen that *q* sometimes fails to obtain in certain circumstances, where *p* holds, so that:

$p \Rightarrow q$ need not yield $(p \;\&\; r) \Rightarrow q$

Additional information can destabilize default implications.

Clearly, the reason why the monotonicity-characterizing principle

- Whenever $p \vdash q$, then $(p \;\&\; r) \vdash q$

works in deductive context, is that here there is then no normality linkage between *p* and *q* which requires the addition of further material that may or may not be forthcoming—as per a stipulation of normalcy or of "all things equal" in the case of inductive reasonings. The reliance of default reason-

[3] On this theses see the author's *Presumption* (Cambridge: Cambridge University Press, 2006).

ing on a presumption of normality, typicality, or the like, means that throughout this domain new information can undo earlier findings.

Thus consider the claim

—If you are in America, then you might be in New York.

This is, of course, perfectly correct. But it will not do to “strengthen” the antecedent as per

—If you are in America and you are in Texas, then you might be in New York.

The conclusions we arrive at with nonmonotonic implication relations are no more than presumption. For in making the inference we have to presume that the situation is not one where some yet unseen conclusion-averting circumstance comes into operation.

This state of affairs also means that with nonmonotonic implications *modus ponenes* fails: the combination of p and $p \Rightarrow q$ need not *demonstrate* that q obtains but may do no more than to establish a *presumption* to that effect.

Nonmonotonicity is thus a standard feature of default inference as is illustrated by contrasting

—If I had put sugar in the tea then it would have tasted fine

with

—If I had put sugar and cayenne pepper in the tea, then it would have tasted fine.

Or again, contrast:

—If you greet him, he will answer politely.

with

—If you greet him with an insult, he will answer politely.

After all, that first implication effectively (but tacitly) comes to

—If you greet him *in the usual and ordinary way*, he will answer politely

and the antecedent of the second implication violates that initial condition.

With default inferences we have to do with what is, from the standpoint of standard logic, a decidedly eccentric mode of reasoning. For no qualification additional to the antecedent as such can abrogate what a valid monotonic implication implies: the antecedent will, in and of itself, suffice to guarantee the consequent. But whenever that “inevitably (invariably unavoidably, etc.)” becomes weakened to “generally, usually, probably, possibly, etc.)”, the monotonicity that is requisite for authentic implication is lost. To obtain a conclusion we must now suppose that nothing untoward is hidden from our sight—that nothing unmentioned intervenes. And this always brings the factor of presumption upon the scene.

5. SOME COMFORTING CONSIDERATIONS

But what if those normality presumptions should prove unjustified? How are we to proceed in the context of conclusions arrived at by reasoning that we see as potentially misleading? The short answer is: Cautiously! But a somewhat more informative response lies in the important prospect of *blurring* that conclusion—making it less specified and detailed. As stated at the outset default reasoning calls for the possibility of resort to a fall-back position. And in managing our cognitive risk’s we can always fall back upon *vagueness* and its inherent qualifications.

With default reasoning in general and induction in particular we run the risk that our conclusions may run awry thanks to out reliance on (generally tacit) suppositions of normality or typicality that may fail in the circumstances at hand. To offset the risk error we can resort to the introduction of decreasing definiteness for the sake of increasing security. Thus instead of reasoning

- *q* is highly likely wherever *p*
- In he present case *p* obtains
- The present case looks to be a typical, majority-conforming one

<Looks are not deceiving here>

∴ In the present case *q* obtains

we would instead reason to

- In the present case *q* probably obtains

Thereby taking a sensible step in the direction of safety. But of course likelihoods do not answer yes/no question, and where such question confront us we have little choice but to resort (circumstances permitting) to chance the risks of the presumption of typicality/normality that characterizes default reasoning. There are, however, some promising precautions here.

After all, a fundamental feature of inquiry is represented by the following observation:

THESIS 1: *Insofar as our thinking is vague, truth is accessible even in the face of error.*

Consider the situation where you correctly accept *P*-or-*Q*. But—so let it be supposed—the truth of this disjunction roots entirely in that of *P* while *Q* is quite false. However, you accept *P*-or-*Q* only because you are convinced of the truth of *Q*; it so happens that *P* is something you actually disbelieve. Yet despite your error, your belief is entirely true.[4] Consider a concrete instance. You believe that Mr. Kim Ho is Korean because you believe him to be a North Korean. However he is, in fact a South Korean, something you would flatly reject. Nevertheless your belief that he is Korean is unquestionably correct. Thanks to the indefiniteness of that disjunctive belief at issue, the error in which you are involved, although real, is not so grave as to destabilize the truth of your belief.

This example illustrates a more far-reaching point.

THESIS 2: *There is, in general, an inverse relationship between the precision or definiteness of a judgment and its security: detail and probability stand in a competing relationship.*

It is a basic principle of epistemology that increased confidence in the correctness of our estimates can always be purchased at the price of decreased accuracy. We *estimate* the height of the tree at around 25 feet. We

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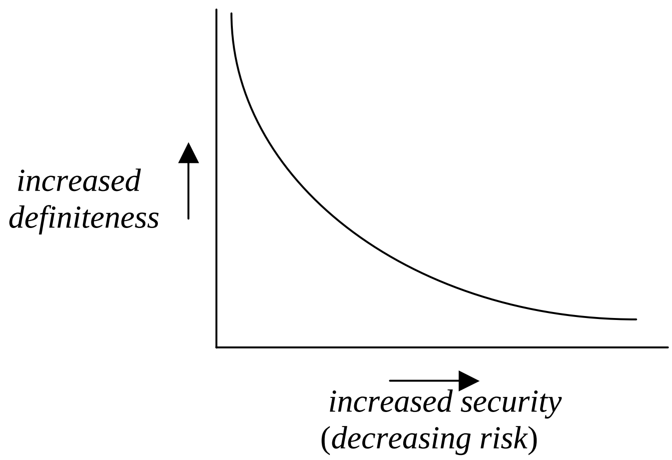
[4] Examples of this sort indicate why philosophers are unwilling to identify *knowledge* with *true belief*.

are *quite sure* that the tree is 25±5 feet. We are *virtually certain* that its height is 25±10 feet. But we are *completely and absolutely sure* that its height is between 1 inch and 100 yards. Of this we are completely sure, in the sense that we deem it absolutely certain, secure beyond the shadow of a doubt, as certain as we can be of anything in the world, so sure that we would be willing to stake our life on it, and the like. With any sort of estimate, there is always a characteristic trade-off relationship between the evidential *security* of the estimate on the one hand (as determinable on the basis of its probability or degree of acceptability), and the informative *definiteness* (exactness, detail, precision, etc.) of its asserted content on the other. Vaguer and looser statements are for that very reason more secure because they embody larger margins of error. This relationship between security and definiteness is graphically characterized by a curve of the general form of an equilateral hyperbola (see Display 1). And this sort of relationship holds just a well for our *truth* estimates as of others.

Display 1

THE SECURITY/DEFINITENESS RELATIONSHIP



Note: The overall quality of the information provided by a claim hinges on combining its security with its definiteness. Given suitable ways of measuring security (*s*) and definiteness (*d*) the curve at issue can be supposed to be an equilateral hyperbola obtained with *s* x *d* as constant.

This state of affairs has far-reaching consequences. It means, in particular, that no secure statement about objective reality can say exactly and in complete detail how matters stand universally always and everywhere. To capture the full complexity of the truth of the matter of things by means of language we must often proceed by way of “warranted approximation”. In general we can be sure of how things “usually” are and how they “roughly” are, but not how they always and exactly are. And this impels our reasoning in the direction of presuppositions of normalcy, typicality, and the like, which are characteristic of default argumentation.

But be this as it may, the present considerations indicate that “inductive inference” as traditionally conceived affords a paradigm instance of default reasoning, which itself emerges in their light as an exercise in standard deductive inference subject to a recourse to the potentially defeasible presumption of typicality.

Yet how is the adoption of a potentially defeasible thesis to qualify as rationally appropriate? The answer, as noted above, lies in the general principle of risk management. For what is at issue with presumption is at bottom less an endorsement of the truth than the implementation of a policy. And rationality here—as elsewhere in matters of practical procedure—pivots on the principle of a favorable balance of potential benefit over potential loss. In many situations default reasoning affords our best-available pathway to our ultimately very practical need for information—for answering in a cogent and epistemically responsible way a question that we need to resolve. For in truth-estimation as in so much of life one must rest content with doing the best one can actually manage to achieve in the circumstances. The theory of inference under the sub-ideal conditions of imperfect information is a comparative newcomer to the logico-philosophical scene and entry into this domain is one of the salient innovations of contemporary logic.

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