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PREFACE

The INTERNATIONAL DICTIONARY OF APPLIED MATHEMATICS defines the terms and describes the methods in the applications of mathematics to thirty-one fields of physical science and engineering. These definitions and descriptions have been chosen to comprise those terms in general use and those methods which have proved most fruitful in practical calculations and analysis. Moreover, they are discussed in sufficient detail to permit the adaptation of methods from related fields. For example, electric circuits are treated not only from the standpoint of stability and topology, but also from that of dynamical analogies, because of the usefulness of this method in solving problems in acoustics and mechanics that are related to those in electric networks.

Furthermore, to broaden the usefulness of this book, the important terms which are used here in defining or describing a concept or method of applied mathematics are also defined in these pages, even though they belong in the fields of pure mathematics, science or engineering. Thus the definitions and methods of differential geometry, vector analysis, tensor analysis, mathematical statistics and other general mathematical fields are covered at considerable length. Moreover, since the primary objective of this book is to enable the reader to obtain the necessary mathematical results by the best available methods, the subject of numerical analysis is treated in some detail, and in certain cases where no analytical method is available, empirical relationships are cited.

To render available the fund of information which this book contains, a most comprehensive system of referencing has been developed. The key words occurring in a definition which are defined elsewhere in the book are printed

in boldface type. These references make available the necessary related information whether from applied mathematics, pure mathematics, science or engineering.

An important feature of this book is the group of four foreign language indices which list alphabetically the French, German, Russian and Spanish equivalents of the terms defined in this book, accompanied by their English equivalents. Thus the reader seeking a definition of a term in one of those languages need merely to find it with its English equivalent in the index for that language and then look up the English equivalent in the main part of this Dictionary. In cases where a foreign language has two or more equivalents of an English term, first consideration has been given to those which are essentially different from the corresponding English expression.

The editorial policy followed in the organization of this book has been directed to facilitate speedy and convenient reference. To that end, the methods and relationships which bear proper names such, for example, as those of Abel, Dyson, Euler, Maxwell, etc., have been stated as far as possible in uninflected form (e.g., without the use of the possessive). Furthermore, the entries beginning with adjectives have been listed generally under the shortest form of the adjective (that is, acoustic rather than acoustical, electric rather than electrical, etc.).

Necessarily, those questions of organization and arrangement have involved many compromises. All those who have worked on this book will welcome the views of the readers on the effectiveness of the arrangement, as well as the selection of entries or any other respect in which the plan and scope of this Dictionary can be improved.

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ABAMPERE. The cgs electromagnetic unit of current. It is that current which, when flowing in straight parallel wires 1 cm apart in free space, will produce a force of 2 dynes per cm length on each wire. One abampere is ten absolute amperes. (See **electromagnetic units**.)

ABBE-MAXWELL THEOREM. An axially symmetrical optical system is perfect if and only if it is plane-perfect for two different object planes perpendicular to the axis.

ABBE NUMBER. In correcting an optical system for first order chromatic aberration the Abbe number or reciprocal mean dispersion of a medium $\nu = (n_D - 1)/(n_F - n_C)$ is used as a discrete substitute for $dn/d\lambda$. The indices of refraction n_D, n_F, n_C are for light of the wavelength of the mean of two yellow sodium lines, D (5892.9Å), the blue hydrogen line, F (4861.327Å), and the mean of two red hydrogen lines, C (6562.8Å), respectively. For ordinary crown glass ν is approximately 60.

ABBE SINE CONDITION. An identity involving the second partial derivatives of the characteristic function of an optical system leads to an important relation that must be satisfied if a surface element containing a point source is to be imaged accurately into a corresponding surface element at the image point. This is Abbe's sine condition

$$ny \sin \theta = -n'y' \sin \theta'$$

where n, n' are indices of refraction, y, y' are distances from the optical axis, and θ, θ' are the angles light rays make with the optical axis in object, image space, respectively. If a system is corrected for spherical aberration and has an infinite exit pupil, then fulfillment of the sine condition is equivalent to freedom from coma.

The Abbe sine condition is equivalent to the extended sine relationship

$$ny \sin \theta + n'y' \sin \theta' = yy'\phi,$$

where ϕ is the power of the system, when the approximation $yy'\phi \cong 0$ is valid.

ABCOULOMB. The electromagnetic unit (emu) of charge. One abcoulomb is the amount of charge that passes when a steady current of one abampere flows for one second.

ABEL EQUATION. A mass point moves along a smooth curve in a vertical plane and under the influence of gravity alone. Given the time, t , required for the particle to fall from a point, x , to the lowest point on the curve as a function of x , what is the equation of the curve? The problem leads to a **Volterra integral equation** of the first kind

$$f(x) = \int_0^x \frac{\phi(t)dt}{\sqrt{2g(x-t)}},$$

which is a particular case of an Abel equation, where g is the acceleration of gravity.

A more general case of the Abel equation is

$$f(x) = \int_0^x (x-y)^{-\alpha} \phi(y)dy$$

where $f(x)$ is continuously differentiable for $x \geq 0$ and $0 < \alpha < 1$.

A first-order **differential equation**

$$y' = f_0(x) + f_1(x)y + f_2(x)y^2 + f_3(x)y^3$$

is also known as an Abel equation. When the $f_i(x)$ are given explicitly, the equation can often be converted into one of simpler type and solved in terms of elementary functions. In the general case the solution involves **elliptic functions**.

ABELIAN GROUP. A commutative group, namely such that $AB = BA$ where A, B are any two elements contained in it. A simple example is the **cyclic group** of order n .

ABEL IDENTITY. The sum of n terms of a sequence a_1, a_2, a_3, \dots , where $a_1 = b_1c_1, a_2 = b_2c_2$, etc., can obviously be written in the form

$$a_1 + a_2 + \dots + a_n = b_1s_1 + b_2(s_2 - s_1) + \dots + b_n(s_n - s_{n-1})$$

where $s_i = c_1 + c_2 + \dots + c_i$. Then this sum can be at once rearranged as

$$a_1 + a_2 + \cdots + a_n = s_1(b_1 - b_2) + s_2(b_2 - b_3) + \cdots + s_{n-1}(b_{n-1} - b_n) + s_n b_n,$$

which is the Abel identity. (Cf. **Abel inequality**.)

ABEL INEQUALITY. Let k_1, k_2, \cdots be a non-increasing sequence of positive numbers. Denote by A the greatest of the sums $|a_1|, |a_1 + a_2|, |a_1 + a_2 + a_3|, \cdots$, then the Abel inequality, which is easily deduced from the **Abel identity**, states that

$$|a_1 k_1 + a_2 k_2 + \cdots + a_n k_n| \leq A k_1.$$

This inequality is useful for proving certain tests of convergence.

ABEL TEST FOR CONVERGENCE. If $a_1 + a_2 + \cdots$ is convergent and u_1, u_2, \cdots is a bounded monotonic sequence, then $a_1 u_1 + a_2 u_2 + \cdots$ converges.

ABEL THEOREM ON POWER SERIES. If $a_0 + a_1 z + a_2 z^2 + \cdots$ is a power series with unity for its radius of convergence and with convergent sum of coefficients $a_0 + a_1 + a_2 + \cdots$, then the Abel theorem states that $\lim_{z \rightarrow 1} (a_0 + a_1 z + a_2 z^2 + \cdots)$ is equal to the sum $a_0 + a_1 + a_2 + \cdots$; that is, the power series is continuous up to and including the end point.

ABERRATIONAL CONSTANT. The aberrational constant is given by

$$k = \frac{2\pi a \operatorname{cosec} 1''}{cT\sqrt{1-e^2}}$$

in which a is the semi-major axis of the earth's orbit, c is the velocity of light, T is the sidereal year expressed in mean solar seconds, and e is the eccentricity of the earth's orbit. The value of the aberrational constant is about $20''.47$.

ABERRATION OF LIGHT, ASTRONOMICAL (CORRECTIONS FOR). The apparent change in the direction of a celestial object due to the component of the earth's motion perpendicular to the line of sight and the finite speed of light is known as the *aberrational correction*.

Calling θ the true direction and θ' the displaced direction (displacement due to aberration) there results

$$\sin(\theta - \theta') = \frac{v}{c} \sin \theta'$$

in which v is the velocity of the earth and c is the speed of light. Since v/c is a very small quantity one can write the above equation as

$$\theta - \theta' = \frac{v}{c} \sin \theta' \operatorname{cosec} 1'' \text{ which may be further simplified to } \theta - \theta' = k \sin \theta' \text{ in which } k = \frac{v}{c}$$

$\operatorname{cosec} 1''$ and k is known in astronomy as the **aberrational constant**. Since k does not exceed $20''.5$, then $\theta - \theta' = k \sin \theta$.

Since the aberrational displacements are in the plane of the ecliptic the aberrational corrections to celestial latitude B and celestial longitude λ are given by

$$\Delta\lambda = -k \sec B \cos(\lambda_0 - \lambda)$$

$$\Delta B = -k \sin B \sin(\lambda_0 - \lambda)$$

in which λ_0 is the longitude of the sun.

The aberrational corrections to the equatorial coordinates of right ascension and declination are much more complicated since the plane of the equator is inclined to the plane of the ecliptic in which the aberrational displacement takes place. Let

$$C = -k \cos \epsilon \cos \lambda_0$$

$$c = \frac{1}{15} \cos \alpha \sec \delta$$

$$d = \frac{1}{15} \sin \alpha \sec \delta$$

$$D = -k \sin \lambda_0$$

$$c' = \tan \epsilon \cos \delta - \sin \alpha \sin \delta$$

$$d' = \cos \alpha \sin \delta$$

in which ϵ is the obliquity of the ecliptic, α and δ are the true right ascension and declination, α' and δ' are the displaced right ascension and declination.

The values of $\log C$ and $\log D$ vary throughout the year due to the position of the sun. They are tabulated as the Besselian star numbers in almanacs for each day of the year.

Finally there results

$$\alpha' = \alpha + Cc + Dd$$

$$\delta' = \delta + Cc' + Dd'$$

ABERRATIONS OF AN OPTICAL SYSTEM. Let a point (x, y) in the object plane of an optical system be imaged by the point (x', y') in the image plane. In the approximation of Gaussian optics,

$$x' = mx, \quad y' = my$$

where m is the magnification. The aberrations of the system are the deviations,

$$x = x' - mx, \quad y = y' - my;$$

which are decomposed in several ways into constituent aberration functions (see **Seidel aberrations** and **Nijboer-Zernike aberration functions**) and various methods have been developed for the calculation of the constituent aberrations (see the **Schwarzschild-Kohlschütter formulas**).

ABERRATIONS OF LENSES, SEIDEL THEORY OF. See **Seidel aberrations**.

ABSCISSA. The horizontal coordinate of a point in a two-dimensional system, commonly rectangular Cartesian, and usually designated by x . Together with the **ordinate** it locates the position of the point in a plane.

ABSOLUTE ACCELERATION. The time rate of change of **velocity** with respect to axes fixed in space or to an **inertial frame** is a vector called absolute acceleration.

ABSOLUTE ACTIVITY. A quantity defined by the equation

$$\lambda = \exp\left(\frac{\mu}{kT}\right)$$

where k is the Boltzmann constant, T , the absolute temperature and μ , the **thermal potential**. In systems undergoing chemical reactions, this formula takes the following form:

$$\lambda_i = \exp\left(\frac{\mu_i}{kT}\right),$$

where λ_i is the absolute activity of component i , and μ_i is the chemical potential of component i .

ABSOLUTE ANGULAR MOMENTUM. The angular momentum as measured in an **absolute coordinate system**, hence the vector product of the position vector of a particle by the **absolute momentum** of the particle.

In the atmosphere the absolute angular momentum M per unit mass of air is equal to the sum of the angular momentum relative to the earth and the angular momentum due to the rotation of the earth:

$$M = ua \cos \phi + \Omega a^2 \cos^2 \phi,$$

where a is the radius of the earth, u the relative eastward speed, ϕ the latitude, and Ω the

angular speed of the earth. (See **angular momentum balance**, **conservation of angular momentum**.)

ABSOLUTE CONTINUITY. Let the function $f(x)$ be defined on the closed interval $[a,b]$ and let l denote the total length of a finite or countable number of nonoverlapping intervals (a_i, b_i) in $[a,b]$. If the limit, as l approaches zero, of the sum $\sum |f(b_i) - f(a_i)|$ is zero, then $f(x)$ is said to be absolutely continuous. An absolutely continuous function is *a fortiori* continuous.

ABSOLUTE CONVERGENCE. See **convergence**.

ABSOLUTE COORDINATE SYSTEM. (Or absolute reference frame.) In meteorology, that **inertial coordinate system** which has its origin on the axis of the earth and is fixed with respect to the stars. Thus, any mechanical quantities in meteorology defined with respect to this frame take into account the movement of the earth. (See **Coriolis force**, **absolute vorticity**.)

ABSOLUTE DERIVATIVE OF TENSOR. See **intrinsic derivative of tensor field**.

ABSOLUTE DIFFERENTIAL CALCULUS. Theory of the differentiation of **tensor fields**. Also called *tensor calculus*, *Ricci calculus*.

ABSOLUTE DISPLACEMENT. The change in **position** measured with respect to coordinate axes fixed in space or to an **inertial frame** is a vector called the absolute displacement.

ABSOLUTE HUMIDITY. (1) (Also called vapor concentration, vapor density.) In a system of moist air, the ratio of the mass of **water vapor** present to the volume occupied by the mixture; that is, the density of the water vapor component.

Absolute humidity is usually expressed in grams of water vapor per cubic meter or, in engineering practice, in grains per cubic foot. (Cf. **mixing ratio**, **specific humidity**, **relative humidity**, **dew point**.)

(2) As occasionally used in air conditioning practice, the number of grains of water vapor per pound of moist air, which is dimensionally identical with the **specific humidity**.

ABSOLUTE INVARIANT. See **invariant**.

ABSOLUTE ISOHYPSE. A line that has the properties of both constant pressure and con-

stant height above mean sea level. Therefore, it can be any **contour line** on a constant-pressure chart, or any **isobar** on a constant-height chart.

ABSOLUTE MAGNITUDE (STELLAR). The apparent stellar magnitude of a celestial object depends both upon the intrinsic brightness of the object and also upon the distance of the object from the earth. The absolute magnitude, either visual or photographic, is the brightness, expressed on the stellar magnitude scale, that it would have if the object were at a distance of 10 parsecs (stellar parallax $0''.1$). If we call M the absolute magnitude, m_g the apparent magnitude and ϕ'' the parallax in seconds of the object

$$M = m_g + 5 + 5 \log \phi''.$$

The sun has an apparent magnitude of -26.72 and a parallax of $206265''$. The absolute magnitude of the sun is 4.85 . Antares has an apparent magnitude of 1.22 and a parallax of $0''.009$. The absolute magnitude of Antares is found to be -4.0 . Hence it follows that actually Antares is nearly 4000 times as bright as the sun.

ABSOLUTE MOMENTUM. (Often called absolute linear momentum.) The (linear) **momentum** of a particle as measured in an **absolute coordinate system**; hence in meteorology, the sum of the (vector) momentum of the particle relative to the earth and the (vector) momentum of the particle due to the earth's rotation.

ABSOLUTE REACTION RATE THEORY. A reformulation of the **collision theory of chemical kinetics** due to Wigner, Eyring and others makes possible the expression of the rate constant in a more general form. Similar considerations may be applied to a great many of other rate processes (see **Eyring theory of transport processes**). Because the purpose of this theory is to calculate the rate in terms of molecular quantities alone, it is often called the absolute reaction rate theory.

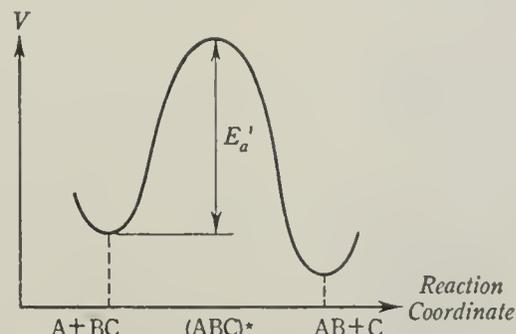
Consider the reaction



To simplify the discussion, assume that A, B and C always remain in a straight line. The course of the reaction may then be followed by noting the values of the two interatomic distances r_{AB} and r_{BC} . At the beginning of the

reaction r_{AB} is large and r_{BC} is small while at the end of the reaction r_{AB} is small and r_{BC} is large.

Let us introduce the potential energy surface. The representative point of the system moves on this surface along the so-called *reaction coordinate*. The potential energy along the reaction coordinate is represented schematically in the figure. The maximum of the



Potential energy along the *reaction coordinate*.

curve corresponds to a situation where three atoms are very close to one another. Moreover this point is a *maximum* along the reaction coordinate but a *minimum* for the direction normal to the reaction coordinate. Indeed the most probable path is the path involving the minimum potential energy in going from the initial to the final state.

Therefore the point considered corresponds to a *saddle point* of the energy surface. It is called the **activated complex**.

One may now assume that the reaction rate is the product of the following three factors: (1) the average number of activated complexes; (2) the characteristic frequency of the activated complex (that is, the inverse of its lifetime); (3) the *transmission coefficient*, K , which is the probability that a chemical reaction takes place after the system has reached the activated state.

Moreover the number of activated complexes is calculated by the equilibrium assumption. (See **equilibrium theory of chemical reactions**.)

Using this description of the reaction process one derives the following expression for the reaction constant

$$k = K \frac{\phi_r(T)}{\phi_A(T)\phi_{BC}(T)} \frac{kT}{h} \exp\left(-\frac{E^x}{kT}\right). \quad (2)$$

Here the ϕ terms are the **partition functions** $f(T, V)$, the volume factor being removed

$$f = V\phi. \quad (3)$$

ϕ_r corresponds to the activated complex, the degree of freedom associated with the reaction coordinate being removed; k is **Boltzmann's constant**, h is **Planck's constant**, E^x is the energy associated with the activated complex, or **activation energy** of the reaction.

This expression may also be written in the thermodynamic form

$$\begin{aligned} k &= K \frac{kT}{h} \exp\left(-\frac{\Delta G^\ddagger}{kT}\right) \\ &= K \frac{kT}{h} \exp[-(\Delta H^\ddagger - T\Delta S^\ddagger)/kT] \quad (4) \end{aligned}$$

where ΔG^\ddagger is a suitable **free energy** of activation and ΔH^\ddagger , ΔS^\ddagger the corresponding **enthalpy** and **entropy** of activation.

ABSOLUTE SCALAR. See **scalar, absolute**.

ABSOLUTE TEMPERATURE. See **temperature**.

ABSOLUTE TENSOR (TENSOR FIELD). Tensor (tensor field) of weight zero. Often called tensor (tensor field) when context admits no confusion.

ABSOLUTE VALUE. See **complex number**.

ABSOLUTE VELOCITY. The time rate of change of **position** measured with respect to **absolute coordinate system** (i.e., a system with coordinate axes fixed in space or to an **inertial frame**) is a vector called the absolute velocity. For example, (1) in a turbine, the average flow velocity measured with respect to the turbine casing. (See **velocity diagram**.) (2) In meteorology, the (vector) sum of the velocity of a fluid parcel relative to the earth and the velocity of the parcel due to the earth's rotation. The east-west component is the only one affected:

$$u_a = u + \Omega a \cos \phi;$$

where u and u_a are the relative and absolute eastward speeds, Ω the angular speed of the earth's rotation, a the radius of the earth, and ϕ the latitude of the parcel.

ABSOLUTE VISCOSITY. (Also called dynamic viscosity.) The property of a fluid which determines the shearing stresses which arise in it during motion. In pure shear flow, the shearing stress τ is proportional to the transverse velocity gradient du/dy ,

$$\tau = \mu \frac{du}{dy} \text{ (Newton's law of fluid friction)}$$

where μ is the absolute viscosity. In a general field of flow, u_1, u_2, u_3 , of a homogeneous, Newtonian, incompressible fluid, the shearing stresses are proportional to the respective rates of change of strain (Stokes' law). The symmetric stress tensor t_{ij} is assumed to be a linear function of the rate of strain tensor e_{ij} . Taking into account that in a fluid at rest the stress is an isotropic tensor, we put

$$t_{ij} = -p\delta_{ij} + \lambda\delta_{ij}e_{kk} + 2\mu e_{ij}$$

where δ_{ij} is the Kronecker delta. Since $t_{ij} = 0$ for $e_{ij} = 0$, we have $t_{ii} = -3p$ and $3\lambda + 2\mu = 0$. Consequently

$$t_{ij} = -p\delta_{ij} - \frac{2}{3}\mu\delta_{ij}e_{kk} + 2\mu e_{ij}$$

when p is now the hydrostatic pressure. The scalar μ is defined as the absolute viscosity. It is a function of the thermodynamic state of the fluid and is independent of the velocity field.

The ratio

$$\nu = \frac{\mu}{\rho}$$

of absolute viscosity to density is called *kinematic viscosity*.

ABSOLUTE VORTICITY. (1) The **vorticity** of a fluid particle determined with respect to an **absolute coordinate system**.

(2) The vertical component η of the absolute vorticity (as defined above) given by the sum of the vertical component of the vorticity with respect to the earth (the **relative vorticity**) ζ and the vorticity of the earth (equal to the **Coriolis parameter**) f :

$$\eta = \zeta + f.$$

ABSOLUTE ZERO. See **temperature**.

ABSOLUTE ZERO UNATTAINABILITY. See **thermodynamics, third law of**.

ABSORBANCE. The common logarithm of the **absorptance**. It may be applied to the total radiation, the visible radiation or to a particular part of the spectrum (spectral absorbance).

ABSORPTANCE. The ratio of the luminous flux absorbed by the body to the flux it receives. (See **spectral absorptance**; **internal**

absorptance; absorptivity; extinction coefficient.)

ABSORPTION. (1) The process whereby the total number of particles emerging from a body of matter is reduced relative to the number entering, as a result of interaction of the particles with the body. (2) The process whereby the kinetic energy of a particle is reduced while traversing a body of matter. This loss of kinetic energy of corpuscular radiation is also referred to as moderation, slowing, or stopping. (3) The process whereby some or all of the energy of sound waves or electromagnetic radiations is transferred to the substance on which they are incident or which they traverse. (4) The process of "attraction into the mass" of one substance by another so that the absorbed substance disappears physically.

ABSORPTION COEFFICIENT. (1) In the most general use of the term absorption coefficient, applied to electromagnetic radiation and atomic and sub-atomic particles, it is a measure of the rate of decrease in intensity of a beam of photons or particles in its passage through a particular substance. (See **absorption coefficient for light**.) (2) In the case of sound, the absorption coefficient (which is also called the acoustical absorptivity) is defined as the fraction of the incident sound energy absorbed by a surface or medium, the surface being considered part of an infinite area. (See also **acoustic attenuation coefficient**.) (3) For the absorption of one substance or phase in another, as in the absorption of a gas in a liquid, the absorption coefficient is the volume of gas dissolved by a specified volume of solvent; thus a widely-used coefficient is the quantity α in the expression $\alpha = V_0/Vp$, where V_0 is the volume of gas reduced to standard conditions, V is the volume of liquid and p is the partial pressure of the gas.

ABSORPTION COEFFICIENT, AMPLITUDE. See **acoustic attenuation coefficient**.

ABSORPTION COEFFICIENT FOR LIGHT. The absorption coefficient κ_ν measures the loss in intensity suffered by a beam of light of intensity I_ν when passing through an absorbing layer of thickness ds . κ_ν depends on the physical properties of the absorbing substance, on the frequency ν , and, some-

times, on the direction of flow of radiation. The decrease in intensity is therefore

$$dI_\nu/ds = -\kappa_\nu I_\nu. \quad (1)$$

It is sometimes useful to introduce the *mass absorption coefficient*

$$\kappa_{\nu,M} = \kappa_\nu/\rho \quad (2)$$

or the *atomic absorption coefficient*

$$\kappa_{\nu,at} = \kappa_\nu/n, \quad (3)$$

where ρ stands for the density of the absorbing substance and n for the number of absorbing atoms per cubic centimeter. The loss of intensity is then

$$dI_\nu/ds = -\kappa_{\nu,M}\rho I_\nu \quad (4)$$

or

$$dI_\nu/ds = -\kappa_{\nu,at}nI_\nu. \quad (5)$$

Another usage in optics is to express the absorption coefficient in terms of the wavelength rather than the frequency (as above). In this notation the absorption coefficient is written simply as $\alpha_\lambda = -dI/I$. Then by integration (see **absorption coefficient, integrated**) one arrives at the common form of **Bouguer's law of absorption**, $I = I_0 \exp(-\alpha x)$. In traversing perpendicularly a thin layer of absorbing material of thickness x , the amplitude of vibration of light is damped by the factor $\exp(-2\pi m x/\lambda)$ where m is the *absorption index* or *absorption constant*. In consequence the intensity is damped by $\exp(-4\pi m x/\lambda)$ and so the absorption coefficient is $\alpha = 4m/\lambda$ in terms of the absorption index. (For solutions, see **Beer's law**.)

ABSORPTION COEFFICIENT, INTEGRATED. The total intensity of an absorption feature (line or unresolved band) is given by

$$I_{abs} = \int (I_\nu^0 - I_\nu) d\nu = I_\nu^0 \Delta s \int \kappa_\nu d\nu.$$

Here I_ν^0 and I_ν stand for the initial and final intensities, κ_ν for the **absorption coefficient** at frequency ν , and Δs for the thickness of the absorbing layer. The integral

$$\int \kappa_\nu d\nu$$

of the absorption coefficient κ_ν over the frequency range is called the **integrated absorption coefficient**.

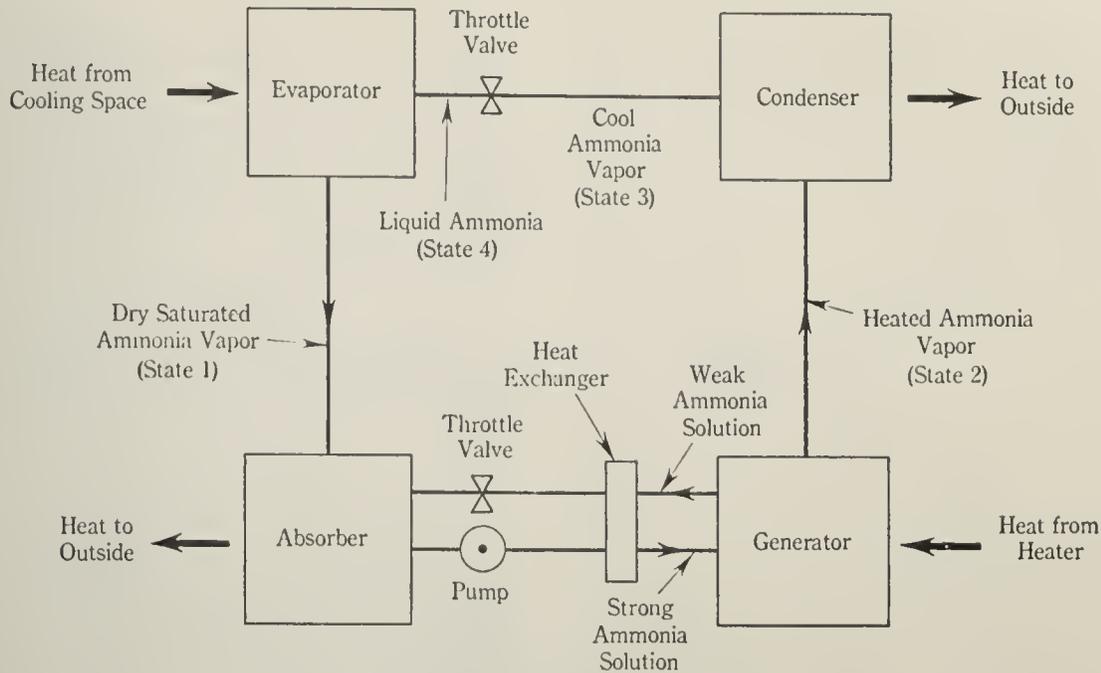


Fig. 1. Block diagram of absorption refrigerator.

ABSORPTION CONSTANT. A synonym for the optical absorption index. (See **absorption coefficient for light; index of refraction, complex.**)

ABSORPTION DISCONTINUITY. The discontinuities of the **absorption coefficient** κ_ν (in terms of frequency) or α_λ (using the wavelength notation), of a medium, correspond to spectral absorption lines and are often associated with anomalies in other frequency (wavelength) dependent properties of the medium, e.g., the refractive index.

ABSORPTION, EXPONENTIAL. The intensity of a beam of light of frequency ν passing through a homogeneous absorbing substance, decreases exponentially according to

$$I = I_0 e^{-\kappa_\nu s}$$

where I_0 and I are the initial and final intensity respectively, κ_ν the **absorption coefficient** for frequency ν , and s the thickness of the absorbing layer (see also **optical depth**).

Note that in wavelength notation, the coefficient κ_ν becomes $\alpha(\lambda)$, α_λ or simply α .

ABSORPTION FACTOR. Ratio of absorbed to unabsorbed radiation. (See also **optical depth**.)

ABSORPTION OF RADIANT ENERGY. The transformation of radiant energy to a different form of energy by the intervention of matter.

ABSORPTION REFRIGERATOR. A refrigerator in which the work of compression is considerably reduced by dissolving the refrigerant in a suitable liquid before compression. Thus the compression of a liquid is substituted for that of a gas. To compensate for it, it is necessary to introduce a source of heat at a temperature higher than that of the surroundings (i.e., of the cooling water available). (See Figures 1, 2 and 3.)

The mixture of gaseous and liquid ammonia is evaporated in the *evaporator* until it becomes dry saturated (state 1). After evaporation, the dry saturated vapor is passed into an *absorber* where it is absorbed in water, rejecting heat Q_s to the surroundings. The temperature of the water is only a little above atmospheric and the solubility of ammonia in water

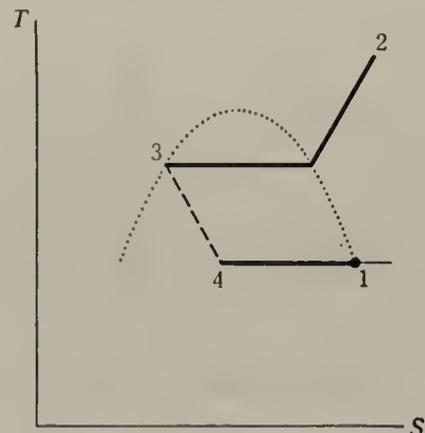


Fig. 2. Temperature-entropy diagram of absorption refrigerator.

is very high. The liquid solution is compressed in a pump, the process requiring a very small amount of work only ($W \simeq 0$). The liquid solution is then heated in the *generator*

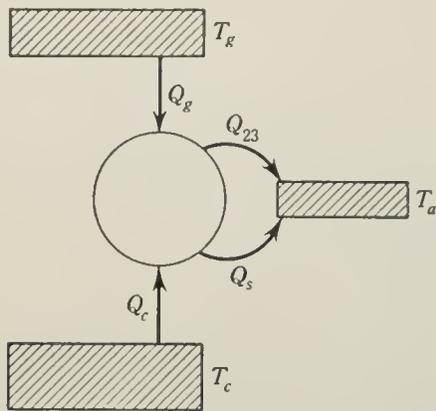


Fig. 3. Heat flow diagram of absorption refrigerator.

at a temperature T_g . This causes the ammonia to be driven off from the solution at the expense of heat Q_g absorbed. At this temperature the solubility of ammonia in water is considerably reduced. In order to maintain steady-state operation a weak solution is throttled back to the absorber, and the weak and strong solutions are made to exchange heat in a counter-flow heat exchanger (heat regeneration) for the sake of economy. Gaseous, usually superheated, ammonia at state 2 is used to complete a conventional refrigeration cycle: (a) cooling at constant pressure from state 2 to state 3 in the *condenser*, with the

possibility of undercooling below T_3 ; (b) throttling in a throttle valve to state 4, etc.

Energy is transferred at three levels: T_g in the generator, T_c in the evaporator (cold chamber), and T_a in the absorber and condenser. We have $T_g > T_a > T_c$. By the first law of thermodynamics

$$Q_g + Q_c = Q_{23} + Q_s.$$

In the ideal case there is no entropy increase due to the operation of the refrigerator and hence

$$\frac{Q}{T_g} - \frac{Q_c}{T_c} + \frac{Q_{23} + Q_s}{T_a} \geq 0.$$

The performance of the refrigerator can be judged by the ratio Q_c/Q_g of the refrigerating effect, Q_c , to the quantity of heat, Q_g , added at the highest temperature. It is seen that

$$\frac{Q_c}{Q_g} \leq \frac{T_c(T_g - T_a)}{T_g(T_c - T_a)}.$$

The so-called *Electrolux refrigerator* constitutes a special form of the absorption refrigerator in which the pump has been replaced by a syphon, and the throttling valve has been eliminated. Instead of throttling through a valve, ammonia is mixed with inert hydrogen so that its partial pressure is decreased, giving the same effect as throttling. Owing to its great simplicity, it is widely used in domestic refrigerators. (See Figure 4.)

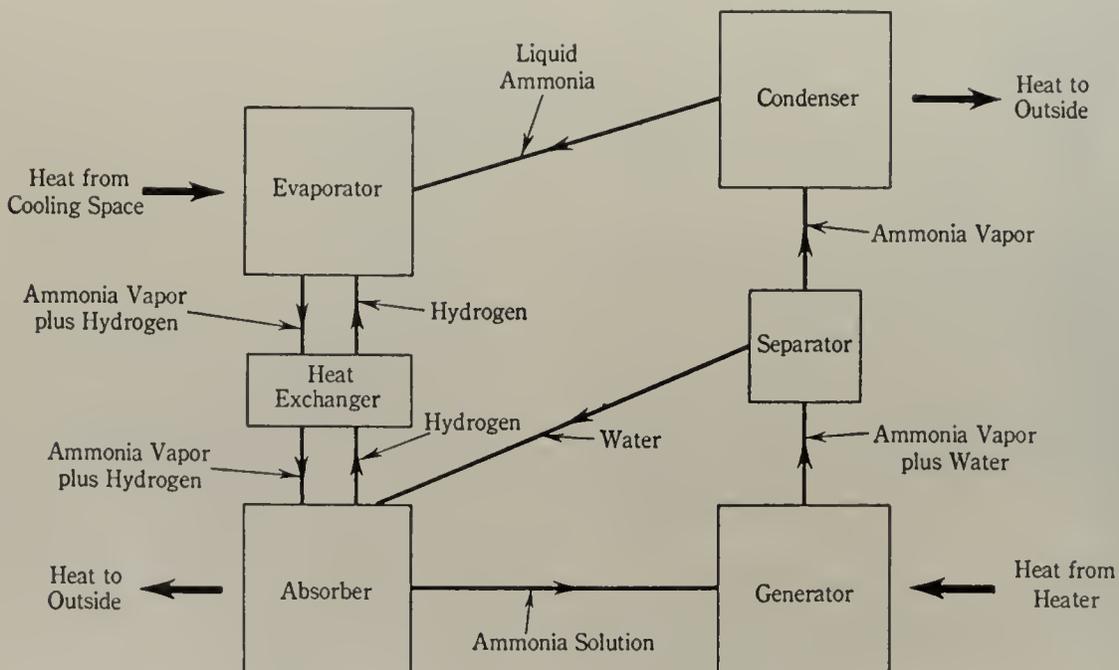


Fig. 4. Block diagram of Electrolux refrigerator.

The concentrated solution of ammonia in water is boiled in the *generator* and the syphon action through the connector tube makes it rise to the *separator*. The gaseous ammonia flows to the *condenser*, whereas the liquid water, while cooling, returns to the *absorber* through a liquid trap. The gaseous ammonia is cooled in the *condenser* shaped like a finned heat exchanger, and eventually condenses. The liquid ammonia flows through a trap to the *evaporator* filled with hydrogen so that its partial pressure decreases to that corresponding to the low temperature prevailing. In the process, the evaporator absorbs heat Q from the cooling space at that reduced temperature. The mixture of hydrogen and ammonia flows down through a heat exchanger to the absorber where the ammonia vapor enters into solution freeing the hydrogen in the process. The hydrogen rises to the evaporator, while the solution of ammonia in water flows to the generator to complete the cycle.

ABSORPTIVE POWER. That part of the radiation of a given wavelength which is absorbed by a body. When radiation falls on a surface, a portion A_λ of it is absorbed, a portion R_λ is reflected and a portion Z_λ is transmitted, so that

$$A_\lambda + R_\lambda + Z_\lambda = 1,$$

where A_λ is the absorptive power or absorptivity, R_λ is the reflecting power or reflectivity, Z_λ is the transmitting power or transmissivity at wavelength λ . Sometimes the adjective "monochromatic" is added to the above terms to emphasize that they relate to a definite wavelength.

ABSORPTIVITY. See *absorptive power*.

ABVOLT. See *electromagnetic units*.

ACCELERATING CONVERGENCE OF SERIES AND SEQUENCES. See *summation of series*.

ACCELERATION. The rate of change of the velocity with respect to the time is called acceleration. It is expressed mathematically by $\frac{d\mathbf{v}}{dt}$, the vector derivative of the velocity, \mathbf{v} with respect to the time, t . If the motion is in a straight line whose position is clearly understood, it is convenient to treat the velocity v and the acceleration $\frac{dv}{dt}$ as scalars with appro-

priate algebraic signs; otherwise they must be treated by vector methods.

Acceleration may be rectilinear or curvilinear, depending upon whether the path of motion is a straight line or a curved line. A body which moves along a curved path has acceleration components at every point. One component is in the direction of the tangent to the curve and is equal to the rate of change of the speed at the point. For uniform circular motion this component is zero. The second component is normal to the tangent and is equal to the square of the tangential speed divided by the radius of curvature at the point. This normal component which is directed toward the center of curvature also equals the square of the angular velocity multiplied by the radius of curvature. The acceleration due to gravity is equal to an increase in the velocity of about 32.2 ft per sec per sec at the earth's surface and is of prime importance since it is the ratio of the weight to the mass of a body. (For examples of acceleration in both curved and linear motion, see *kinematics*.)

ACCELERATION, ANGULAR. See *angular acceleration*.

ACCELERATION, CENTRIPETAL. See *centripetal acceleration*.

ACCELERATION, COMPATIBLE. Another name for *Coriolis acceleration*.

ACCELERATION, COMPLEMENTARY. Another name for *Coriolis acceleration*.

ACCELERATION, COMPOSITION LAW OF (RELATIVITY). Let a'_x, a'_y, a'_z be the components of the acceleration in Σ' and w'_x, w'_y, w'_z the velocity components in Σ' and a_x, a_y, a_z the components of the acceleration in Σ (v the velocity of Σ' with respect to Σ in the common x -direction) then

$$a_x = \left[\frac{c\sqrt{c^2 - v^2}}{c^2 + vw'_x} \right]^3 \cdot a'_x$$

$$a_y = \left[\frac{c\sqrt{c^2 - v^2}}{c^2 + vw'_x} \right]^2 \cdot \left[a'_y - \frac{vw'_y a'_x}{c^2 + vw'_x} \right]$$

$$a_z = \left[\frac{c\sqrt{c^2 - v^2}}{c^2 + vw'_x} \right]^2 \cdot \left[a'_z - \frac{vw'_z a'_x}{c^2 + vw'_x} \right].$$

ACCELERATION, CORIOLIS. See *Coriolis acceleration*.

ACCELERATION, NORMAL. See **centripetal acceleration**.

ACCELERATION OF GRAVITY. (1) The ratio of the weight of a material particle to its mass at any specific point in an approximately uniform gravitational field. This is the **acceleration** with which a body would fall in the absence of all other disturbing forces, such as those due to friction.

(2) Specifically, the acceleration with which a body falls *in vacuo* at a given point on or near a given point on the earth's surface. This acceleration, frequently denoted by g , varies by less than one percent over the entire surface of the earth. Its "average value" has been defined by the International Commission of Weights and Measures as 9.80665 M/S² or 32.174 ft/S². Its value at the poles is 9.8321 M/S² and at the equator 9.7799 M/S².

ACCELERATION POTENTIAL. The scalar function (if it exists) whose **gradient** is equal to the total acceleration of a fluid. In meteorology, this must include the **Coriolis acceleration**:

$$\frac{d\mathbf{V}}{dt} + 2\boldsymbol{\Omega} \times \mathbf{V} = -\nabla\chi$$

where \mathbf{V} is the velocity, $\boldsymbol{\Omega}$ the angular velocity of the earth, and χ the acceleration potential.

So defined, this potential exists if and only if the atmosphere is **barotropic**, in which case $\chi = \pi + gz$, where π is the **barotropic pressure function**, g , the acceleration of gravity, and z , the vertical coordinate.

However, if some thermodynamic variable σ be taken as vertical coordinate, the horizontal acceleration will have a potential

$$\frac{\partial \mathbf{V}_H}{\partial t} + \mathbf{V}_H \cdot \nabla_\sigma \mathbf{V}_H + 2\boldsymbol{\Omega} \times \mathbf{V}_H = -\nabla_\sigma \chi,$$

where \mathbf{V}_H is the horizontal velocity and ∇_σ the horizontal del operator in the σ -surface. In the case of pressure as vertical coordinate, $\sigma = p$, and $\chi = gz$. In the case of potential temperature θ as vertical coordinate, $\sigma = \theta$ and $\chi = c_p T + gz$, where c_p is the specific heat of air at constant pressure and T the absolute temperature. Note that the acceleration potential acts as the **stream function** in the case of **geostrophic equilibrium**. (See **velocity potential**; **potential**.)

ACCELERATION, TANGENTIAL. The component of acceleration along the path of motion is called the tangential acceleration. Its magnitude is dv/dt , the time rate of change of the **speed** along the path.

ACCELERATOR, PARTICLE. A device designed to give to charged particles, such as electrons or positive ions, the high energies necessary to penetrate the **Coulomb barrier** of the target nucleus.

ACCEPTOR BOND. See **donor bond**.

ACCESS TIME. (1) The time interval, characteristic of a memory or storage device, between the instant at which **information** is requested of the **memory** and the instant at which this information begins to be available in useful form. (2) The time interval between the instant at which information is available for storage and the instant at which it is effectively stored.

ACCOMMODATION COEFFICIENT. Let E_i be the average kinetic energy of the gaseous molecules which collide with a wall whose temperature corresponds to an equilibrium kinetic energy E_1 per molecule. If E_i and E_1 differ, the average kinetic energy of the reflected molecules will in general have a value E_r intermediate between E_i and E_1 . Knudsen has defined an *accommodation coefficient* α , which expresses how much of the maximum possible kinetic energy exchange does actually occur:

$$\alpha = \frac{E_i - E_r}{E_i - E_1}.$$

Similar accommodation coefficients can be introduced for the internal degrees of freedom, or even for chemical reactions.

ACCOMPANYING FLUID. The fluid in irrotational motion which travels with a moving vortex system. It surrounds the **substance of the vortex** and is itself surrounded by fluid in irrotational motion which does not travel with it but which passes by it as if it were a solid body. Hill's **spherical vortex** has no accompanying fluid.

ACCUMULATION POINT. One of a set such that any neighborhood of this point, no matter how small, contains a member of the set. All the points of a set which is everywhere dense are accumulation points.

ACCURACY. The degree of exactness actually possessed by an approximation, measurement, etc. It may be contrasted with precision, which is the degree of exactness with which the quantity is expressed. For example, as a value of π , the number 3.1428 is more precise than accurate.

ACHROMATIC. In the accepted colorimetric sense: (1) for primary light sources, the color of the **equi-energy spectrum** ($x = y = z = \frac{1}{3}$) is taken as achromatic; (2) for surface colors the light source serving as illuminant is taken as achromatic. On this basis, an ideal white surface is always defined as achromatic whatever may be the color of the light.

In optical design: (3) an optical system is achromatic if it is approximately corrected for chromatic aberration in the sense that the focal length is the same for two distinct wavelengths.

ACHROMATIC LIGHT, SPECIFIED. (1) Light of the same chromaticity as that having an **equi-energy spectrum**. (2) The colorimetric standard illuminants *A*, *B* and *C*, the spectral energy distributions of which were specified by the C.I.E. in 1931, with various scientific applications in view:

Standard A. Incandescent electric lamp of color temperature 2854°K.

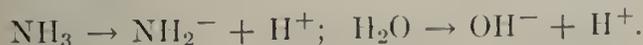
Standard B. Standard A combined with a specified liquid filter, to give a light of color temperature approximately 4800°K.

Standard C. Standard A combined with a specified liquid filter to give a light of color temperature approximately 6500°K.

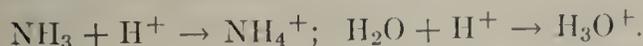
(3) Any other specified white light.

ACIDS AND BASES. DEFINITIONS OF BRØNSTED AND LEWIS. The two most common definitions of acids and bases are those of Brønsted and Lewis.

According to Brønsted, an acid is a proton donor; a base, a proton acceptor. For example NH_3 and H_2O are acids in the reactions:

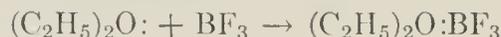


They act as bases in the reactions:



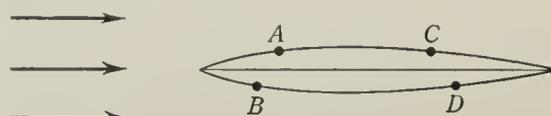
In a more general definition, Lewis calls a base any substance with a free doublet of electrons which it is capable of sharing with

an electron pair acceptor, which is called an acid. For example, in the reaction:



the ethyl ether molecule is a base, the boron trifluoride an acid. The resulting complex is a *Lewis salt*.

ACKERET THEORY. A theory of two-dimensional airfoils in supersonic flow. Since the theory is based on the linearized equation, it is valid only when the perturbation velocities are small, and for Mach numbers that are not either near 1 or large enough to make the hypersonic similarity parameter M_τ of order 1. The limitation to small perturbation velocities means that the theory can only be applied to thin airfoils, with sharp leading and trailing edges, at small angles of incidence.



Ackeret theory.

The theory gives the pressure coefficient at any point on an airfoil, where the surface is inclined at an angle θ to the stream, as

$$C_p = \frac{2\theta}{\sqrt{M^2 - 1}}, \quad (1)$$

where M is the Mach number of the stream. In Equation (1), the angle θ is considered to be positive if its sign corresponds to deflection of the stream at a concave corner. Thus, in the figure, θ and C_p are positive for the points *A* and *B*, and negative for *C* and *D*.

By integration of Equation (1) over the surface of an airfoil, a number of simple results relating to lift, drag and pitching moment may be obtained. Thus for a thin airfoil of any shape, at an incidence α ,

$$C_L = \frac{4\alpha}{\sqrt{M^2 - 1}}, \quad (2)$$

and for a symmetrical airfoil the center of pressure is at the half-chord point.

Drag coefficients calculated from Equation (1) do not include any boundary-layer effects and are, in fact, coefficients of wave drag. For a thin flat plate the wave drag coefficient is

$$C_D = \alpha C_L = \frac{4\alpha^2}{\sqrt{M^2 - 1}}. \quad (3)$$

Equation (3) expresses the simple condition that, in inviscid flow, the resultant force on a thin flat plate acts in a direction normal to the plate.

The total wave drag coefficient of an airfoil with thickness is the sum of two terms, the first given by Equation (3) and the second associated with the thickness of the airfoil. For a series of airfoils related by an **affine transformation**, the drag coefficient due to thickness at a given Mach number is proportional to the square of the thickness/chord ratio.

ACOUSTIC ABSORPTION COEFFICIENT. See **sound absorption coefficient of surfaces**.

ACOUSTIC ABSORPTIVITY. See **sound absorption coefficient of surfaces**.

ACOUSTIC AMPLITUDE EQUATIONS, FINITE. See **finite amplitude equations (acoustic)**.

ACOUSTIC ATTENUATION COEFFICIENT. When a plane harmonic sound wave is propagated through an attenuating medium, it is generally found that the amplitude of the wave decays exponentially with distance. The absolute value of the natural logarithm of the ratio of the peak sound pressures (or the peak particle velocities) at two points, unit distance apart, is called the acoustic attenuation coefficient or, more strictly, the acoustic amplitude attenuation coefficient. In the absence of scattering, this quantity can be related to other measures of internal friction (see **friction, internal**). The acoustic attenuation coefficient is generally measured in nepers per cm.

ACOUSTIC ATTENUATION CONSTANT. See **acoustic attenuation coefficient**.

ACOUSTIC CAPACITANCE. See **compliance, acoustic**.

ACOUSTIC CENTER, EFFECTIVE. The point from which spherically divergent sound waves, emitted by an acoustic generator, appear to diverge.

ACOUSTIC COMPLIANCE. See **compliance, acoustic**.

ACOUSTIC DAMPING. See **damping, acoustic**.

ACOUSTIC ENERGY EQUATION. See **Franklin equation**.

ACOUSTIC IMPEDANCE. See **impedance, acoustic**.

ACOUSTIC INERTANCE. See **inertance, acoustic**.

ACOUSTIC MASS. See **mass, acoustic**.

ACOUSTIC MEAN FREE PATH. See **mean free path, acoustic**.

ACOUSTIC OHM. See **acoustic units**.

ACOUSTIC PHASE CONSTANT. The acoustic phase constant is the imaginary part of the **acoustic propagation constant**. The commonly used unit is the radian per section or per unit distance. In the case of a symmetrical structure, the imaginary parts of both the transfer constant and the acoustic propagation constant are identical, and have been called the "wavelength constant."

ACOUSTIC POWER GENERATED. See **radiation, acoustic**.

ACOUSTIC PRINCIPLE OF SIMILARITY. For any acoustical system involving **diffraction** phenomena, it is possible to construct a new system on a different scale which will perform in similar fashion, provided that the wavelength of the sound is altered in the same ratio as the linear dimensions of the original system.

ACOUSTIC PROPAGATION CONSTANT. Of a uniform system or of a section of a system of recurrent structures, the natural logarithm of the complex ratio of the steady-state particle velocities (see **velocity, particle**), **volume velocities**, or pressures at two points separated by unit distance in the uniform system (assumed to be of infinite length), or at two successive corresponding points in the system of recurrent structures (assumed to be of infinite length). The ratio is determined by dividing the value at the point nearer the transmitting end by the corresponding value at the more remote point.

ACOUSTIC RADIATION. See **radiation, acoustic**.

ACOUSTIC REACTANCE. See **reactance, acoustic**.

ACOUSTIC RECIPROCITY THEOREM. See **reciprocity theorem, acoustical**.

ACOUSTIC REDUCTION FACTOR. The ratio of the sound energy density in a room containing a sound source to the sound energy in an adjoining receiving room:

$$R.F. = T.L. + 10 \log_{10} \frac{A}{S} \quad (1)$$

where $R.F.$ is the acoustic reduction factor, in decibels, A is the total absorption in the receiving room, in sabins, S is the area of the partition, in square feet and $T.L.$ is the transmission loss, in decibels.

Equation (1) shows that the reduction is due to both the loss introduced by the partition and the absorption in the receiving room.

The choice of a partition for insulating a room against sound involves a number of considerations. Some of the factors are the frequency distribution and intensity level of the components of the objectionable sound, the transmission frequency characteristics of the partition, the ambient noise or sound level in the receiving room which will mask the objectionable sound, and the response frequency characteristic of the ear.

ACOUSTIC RESISTANCE. See **resistance, acoustic.**

ACOUSTIC SYSTEM OF ONE DEGREE OF FREEDOM. An acoustic system of one degree of freedom is shown in Figure 1. In one degree of freedom the activity in every

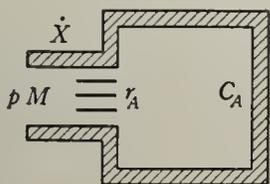


Fig. 1. Acoustic system of one degree of freedom.

element of the system may be expressed in terms of one variable. In the acoustic system an impinging sound wave of pressure p acts upon an inertance M and an acoustical resistance r_A comprising the air in the tubular opening which is connected to the volume or acoustical capacitance C_A . The acoustical resistance r_A is due to viscosity.

The principle of the conservation of energy states that the total store of energy of all forms remains constant if the system is isolated so that it neither receives nor gives out energy; in the case of transfer of energy the total gain or loss from the system is equal to the loss or

gain outside the system. The sum of the kinetic, potential, and heat energy during an interval of time is, by the principle of conservation of energy, equal to the energy delivered to the system during that interval. In the acoustical system of Figure 1 there are three forms of energy, namely, kinetic, potential, and heat energy.

Kinetic Energy. The kinetic energy T_{KA} stored in the inertance of the acoustical system is

$$T_{KA} = \frac{1}{2} M \dot{X}^2 \quad (1)$$

where M is m/S^2 , the inertance, in grams per (centimeter)⁴, m is the mass of air in the opening, in grams, S is the cross-sectional area of the opening, in square centimeters, $\dot{X} = S\dot{x}$ is the volume current, in cubic centimeters per second, \dot{x} is the velocity of the air particles in the opening, in centimeters per second.

It is assumed that all the air particles in the opening move with the same phase.

Potential Energy. The potential energy V_{PA} stored in the acoustical capacitance of the acoustical system is

$$V_{PA} = \frac{1}{2} \frac{X^2}{C_A} \quad (2)$$

where X is the volume displacement, in cubic centimeters, $C_A = V/\rho c^2$ is the acoustical capacitance, in (centimeters)⁵ per dyne, V is the volume of the cavity, in cubic centimeters, ρ is the density of air, in grams per cubic centimeter, and c is the velocity of sound, in centimeters per second.

Kinetic and Potential Energy. The total energy stored in the system is

$$W_A = T_{KA} + V_{PA} = \frac{1}{2} M \dot{X}^2 + \frac{1}{2} \frac{X^2}{C_A} \quad (3)$$

The rate of change of energy in the system is

$$\frac{dW_A}{dt} = M \dot{X} \ddot{X} + \frac{X \dot{X}}{C_A} \quad (4)$$

Dissipation of Energy. The acoustical energy is converted into heat by the dissipation due to viscosity as the fluid is forced through the narrow slits. The rate at which acoustical energy D_A is converted into heat is

$$D_A = r_A \dot{X}^2 \quad (5)$$

where r_A is the acoustical resistance, in acoustical ohms, and \dot{X} is the volume current in cubic centimeters per second.

Equation of Motion. The power delivered to the system must be equal to the rate of kinetic energy storage plus the rate of potential energy storage plus the power loss due to dissipation.

The rate at which work is done or power delivered to the acoustical system by the applied sound pressure is $\dot{X}Pe^{j\omega t} = p\dot{X}$.

The rate of increase of energy ($T_{KA} + V_{PA}$) of the system plus the rate at which work is done on the system or power delivered to the system by the external forces must be equal to the rate of dissipation of energy D_A . Writing this sentence mathematically yields the equation of motion for the acoustical system of Figure 1.

$$M\dot{X}\ddot{X} + r_A\dot{X}^2 + \frac{X\dot{X}}{C_A} = Pe^{j\omega t}\dot{X} \quad (6)$$

$$M\ddot{X} + r_A\dot{X} + \frac{X}{C_A} = Pe^{j\omega t}. \quad (7)$$

Solution of Equation of Motion. The steady-state solution of the differential Equation 7 is

$$\dot{X} = \frac{Pe^{j\omega t}}{r_A + j\omega M - \frac{j}{\omega C_A}} = \frac{p}{z_A}. \quad (8)$$

Acoustical Impedance. From Equation 8 the vector acoustical impedance z_A , in acoustical ohms, is

$$z_A = r_A + j\omega M - \frac{j}{\omega C_A}. \quad (9)$$

Acoustical Reactance. Acoustical reactance x_A is the imaginary part of the acoustical impedance z_A of Equation 10. The acoustical reactance x_A , in acoustical ohms, is

$$x_A = \omega M - \frac{1}{\omega C_A}. \quad (10)$$

Response. The response of the acoustical system of Figure 1 can be obtained from Equation 8. The response is the volume current \dot{X} , in cubic centimeters per second, for a constant

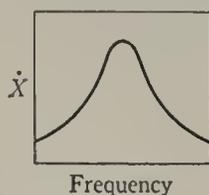


Fig. 2. Response-frequency characteristic of the acoustic system of Figure 1.

applied pressure p , in dynes per square centimeter. A typical response-frequency characteristic is shown in Figure 2.

Resonant Frequency. For a certain value of M and C_A there will be a certain frequency at which the imaginary component of the acoustical impedance is zero. This frequency is called the resonant frequency. At this frequency the ratio of the current to the applied voltage is a maximum. At the resonant frequency the current and voltage are in phase. The resonant frequency f_r , in cycles per second, is given by

$$f_r = \frac{1}{2\pi\sqrt{MC_A}}. \quad (11)$$

Power. The power P_A , in ergs per second, dissipated in the acoustical system of Figure 1 is given by

$$P_A = r_A\dot{X}^2. \quad (12)$$

ACOUSTIC UNITS. In acoustics, the centimeter-gram-second (cgs) system of units has been and is at present predominantly used; but some practical units such as English and metric system units of length are also being used; and the watt is commonly being employed for designating acoustic power. In recent years there has been a trend toward adoption of the rationalized meter-kilogram-second system of units in many fields of science and engineering. Therefore the MKS units are included in the table on page 15.

ACOUSTIC VELOCITY (SOUND VELOCITY). The velocity relative to the medium at rest at which infinitesimal disturbances are propagated through a fluid. It is given by

$$a^2 = (\partial p / \partial \rho)_s,$$

where s is entropy, in general. For **perfect gases** the expression is simplified to

$$a^2 = \gamma p / \rho = \gamma R T,$$

where γ need not be a constant and can depend on temperature. (For a perfect gas γ can be a function of temperature alone.)

The velocity of sound plays an important part in the study of the flow of compressible fluids because the character of the flow changes its nature according to whether the flow velocity is subsonic, sonic, or supersonic. (See **compressible fluid flow; nozzles; Mach number.**)

ACOUSTIC UNITS (COMPARISON OF CGS AND MKS UNITS)

Quantity	Dimensions	Cgs unit	MKS unit	Conversion Factor (Cgs units to MKS units)
Sound velocity (particle velocity)	LT^{-1}	cm per sec	meter per sec	10^{-2}
Volume velocity	L^3T^{-1}	cu cm per sec	cu meter per sec	10^{-6}
Sound energy	ML^3T^{-2}	erg	joule	10^{-7}
Force	MLT^{-2}	dyne	newton	10^{-5}
Sound pressure (sound energy density)	$ML^{-1}T^{-2}$	microbar (dyne per sq cm)	newton per sq meter	10^{-1}
Sound energy flux	ML^2T^{-3}	erg per sec	watt	10^{-7}
Sound intensity (specific sound energy flux)	MT^{-3}	erg per sec per sq cm	watt per sq meter	10^{-8}
Acoustic impedance (resistance, impedance)	$ML^{-4}T^{-1}$	acoustic ohm (dyne-sec per cm^6)	MKS acoustic ohm (newton-sec per meter)	10^5
Specific acoustic impedance (characteristic impedance)	$ML^{-2}T^{-1}$	rayl (acoustic ohm $\times cm^2$ or dyne-sec per cu cm)	MKS rayl (MKS acoustic ohm $\times sq meter$ or newton-sec per cu meter)	10

ACTION INTEGRAL. For a generalized coordinate q_i and its conjugate momentum p_i the phase integral,

$$J_i = \oint p_i dq_i,$$

is called the action integral (see **action variable**). The integral is taken over one complete cycle of a system and is therefore only defined for periodic, or multiply periodic, systems. The Bohr-Sommerfeld quantum condition states that the action integral of a system must be an integral multiple of Planck's constant ($J_i = nh$). (See **Bohr-Sommerfeld quantum theory**.)

ACTION PRINCIPLE. In classical mechanics a variational principle (sometimes also called **Hamilton's Principle**) which states that the motion of a mechanical system described by coordinates $q_i = q_i(t)$ can be characterized by the requirement that the action integral $I(t_2, t_1)$

$$I(t_2, t_1) = \int_{t_2}^{t_1} L dt \quad (1)$$

where $L = L(q_i, \dot{q}_i, t)$ is the Lagrangian of the system, is stationary for the physically possible trajectories $q_i(t)$. More precisely, the action principle states that the variation $q_i(t) \rightarrow q_i(t) + \delta q_i(t)$ produces no change in I to first order

in $\delta q_i(t)$, if $\delta q_i(t)$ is an arbitrary variation vanishing at time t_1 and t_2 . The action principle is then equivalent to the Lagrangian equations of motion

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt} \left(\frac{\partial L}{\partial \dot{q}_i} \right) = 0 \quad (2)$$

(see **Lagrangian formalism**). Although the action principle states only that the action is stationary for variations vanishing at the end points t_1 and t_2 one can deduce the result of a variation which does not vanish at the end points. This is possible because each state of motion is defined by fixing as many coordinates as can be fixed independently (for the case of a single particle e.g., the three spatial coordinates at two different times, or three coordinates and three velocities at one time) and then the whole past and future is determined by the field equations. In the **quantum theory** the coordinates $q_i(t)$ of a particle are operators. In the **Heisenberg picture** they satisfy the same equations of motion (2) as in classical mechanics. This will be guaranteed if one assumes that the same action principle holds, namely $\delta I = 0$

$$I = \int_{t_2}^{t_1} L(q_i, \dot{q}_i) dt$$

for all variations of the q_i vanishing at t_1 and t_2 . In the quantum theoretic case by a variation is meant a change of q_i to $q_i + \delta q_i$ where

the δq_i are c-numbers and not operators. Because of the uncertainty principle (or equivalently due to lack of commutativity of $q(t)$ and $q(t')$ for $t \neq t'$) it is not possible in the quantum theory to assign values to the $q_i(t)$ throughout a physical motion. The motion is specified by giving values to the $q^i(t)$, or to the $\dot{q}_i(t)$, at a single time t . The future of the state cannot then be determined from the equations which are of second order in the time. The action principle, $\delta I = 0$, is therefore no longer sufficient to compute δI for variations which do not vanish at t_1 and t_2 . Additional statements must be made about the behavior of δI for variations δq which do not vanish at the end point. This additional information is contained in the Schwinger action principle which is the quantum mechanical generalization of the classical action principle. It can be stated as follows:

$$\delta \langle q_1 t_1 | q_2 t_2 \rangle = \frac{i}{\hbar} \langle q_1 t_1 | \delta I | q_2 t_2 \rangle$$

which action principle gives the changes in the transformation function $\langle q_1 t_1 | q_2 t_2 \rangle$ due to particular variations (including variations at the boundary), δI being the variation induced in I by such variations, where I , the action integral, is now an operator function.

The dynamical principle of the quantum theory of fields which governs the development of field systems between two space-like surfaces σ_1 and σ_2 , is called the Schwinger action principle. It states that

$$\delta \langle \sigma_1 | \sigma_2 \rangle = \left\langle \sigma_1 \left| \delta \left\{ i \int_{\sigma_2}^{\sigma_1} d^4x \mathcal{L}(\chi) \right\} \right| \sigma_2 \right\rangle$$

where the Lagrangian, which is a Hermitian operator function of the multicomponent Hermitian field $\chi(x)$, can in general be written in the form

$$\mathcal{L}(\chi) = \frac{1}{4} (\chi A^\mu \partial_\mu \chi - \partial_\mu \chi A^\mu \chi) - \mathcal{H}(\chi).$$

The A^μ are finite dimensional and skew Hermitian matrices. They can be reduced into two classes, one consisting of real antisymmetrical matrices and the other of imaginary symmetrical matrices. This expresses the separation of the total field χ into two fundamental classes of Bose-Einstein fields ϕ (integer spin fields) and Fermi-Dirac fields ψ (odd half integer spin field). Corresponding to the symmetry of the relevant parts of the matrices A^μ , the Bose-Einstein fields must essentially be commutative at space-like points) and the Fermi-Dirac fields

anticommutative in order that the first term in the Lagrangian be a non-trivial operator. Similarly the structure of \mathcal{H} must involve the ϕ and ψ in a symmetrical and antisymmetrical fashion respectively. It can be shown that \mathcal{H} , and hence \mathcal{L} , are even functions of the Fermi-Dirac field.

The finite multiplicity of the Hermitian field χ pertains in part to space time properties (spin) and in part to internal properties (charge, isotopic spin etc.).

ACTION, QUANTUM OF. Synonym for Planck constant, h . (See also action integral.)

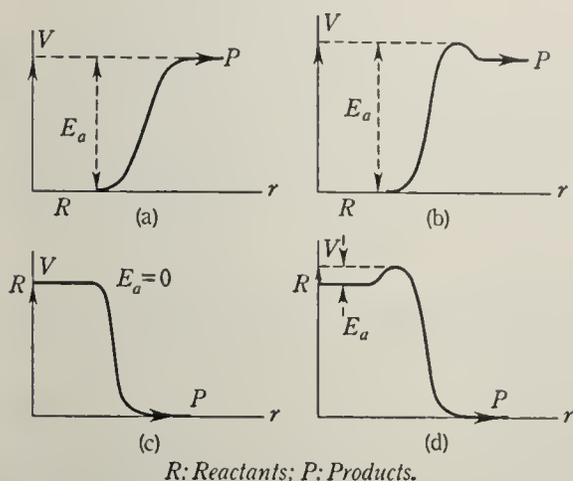
ACTION VARIABLE. If one of the momenta of a classical dynamical system yields a closed curve when graphed against the conjugate coordinate, the area contained within this curve is the action variable corresponding to this degree of freedom. Usually denoted by J .

ACTIVATED COMPLEX. A short-lived transition state which appears during chemical reactions, when the molecules can no longer be considered as reactants, and not yet as products. The activated complex is the point of maximum energy along the *reaction path* (see absolute reaction rate theory). The difference between the energy of the activated complex and that of the reactants is the **activation energy** of the reaction. The *activation entropy* is defined similarly as the difference between the **entropy** of the activated complex and that of the reactants.

ACTIVATION ENERGY. The excess energy over the ground state which must be acquired by an atomic system in order that a particular process may occur. Examples are the energy needed by a molecule to take part in a chemical reaction, by an electron to reach the conduction band in a semiconductor, and by a lattice defect to move to a neighboring site.

In the first example cited, the rate of an elementary chemical reaction can usually be expressed as a product of a function of the concentrations of the participants and of a rate constant. This latter can be written as $A \exp(-E_a/kT)$, where k is the Boltzmann constant, T , the absolute temperature, A , a frequency factor which varies slowly with the activation energy E_a which appears in the exponential. It is the minimum height of the potential barrier which must be crossed when one follows the reaction coordinate from the

reactants to the products. The figure illustrates four typical situations. It is to be noted that it is only a case (a) that E_a is equal to the energy of the reaction. (See **absolute reaction rate theory**; **equilibrium theory of reaction kinetics**; and **activated complex**.)



Activation energy. Variation of the energy along the reaction coordinate for two endothermic (a) and (b) and two exothermic (c) and (d) reactions. E_a is the activation energy.

ACTIVATION ENTROPY. See **absolute reaction rate theory**; and **activated complex**.

ACTIVATION RATE (NEUTRON REACTIONS). The activation rate for unit volume of a substance containing neutron flux is based upon the number of neutron-capture reactions taking place per unit time in a small energy range about a particular energy, E . Thus,

$$\text{overall activation rate} = \int_0^{\infty} dE \Sigma_{act}(E) \Phi(r, E)$$

where Σ_{act} is the macroscopic activation cross section and $\Phi(r, E)$ is the total neutron flux at energy E at point r .

ACTIVE FORCE. See **force**, **external** or **active**.

ACTIVE MOLECULES. See **unimolecular reactions**.

ACTIVE NETWORK. See **network**, **active**.

ACTIVITY. See **reference systems**.

ACTIVITY, ABSOLUTE. See **absolute activity**.

ACTIVITY COEFFICIENT. See **reference systems**.

ADAMS-BASHFORD METHOD. A method for solving an **ordinary differential equation**, $y' = f(x, y)$, expressed in terms of **difference operators**, which uses the **predictor**

$$y_{\nu+1} = y_{\nu} + h[f_{\nu} + \frac{1}{2}\nabla f_{\nu} + \frac{5}{12}\nabla^2 f_{\nu} + \frac{3}{8}\nabla^3 f_{\nu} + \dots],$$

and the **corrector**

$$y_{\nu+1} = y_{\nu} + h[f_{\nu+1} - \frac{1}{2}\nabla f_{\nu+1} - \frac{1}{12}\nabla^2 f_{\nu+1} - \frac{1}{24}\nabla^3 f_{\nu+1} - \dots].$$

The first amounts to approximating

$$\int_{x_{\nu}}^{x_{\nu+1}} f(x, y) dx$$

by means of a quadrature formula which utilizes values of f at x_{ν} and to the left only, the second utilizes also the value of f at $x_{\nu+1}$. The corrector is to be used with the best current approximation of $y_{\nu+1}$, and reapplied until the next approximation does not differ significantly from the previous. The formulas have significance when y and f are vectors. (See F. B. Hildebrand, *Introduction to Numerical Analysis*, McGraw-Hill Book Company, 1956; Lothar Collatz, *Numerische Behandlung von Differentialgleichungen*, 2nd edition, Springer, 1955).

ADAMS-STÖRMER METHOD. See **Störmer method**.

ADAPTATION. (1) The process taking place as the eye becomes accustomed to the luminance or the color of the field of view. (2) The final state of the process. In particular, the terms light adaptation and dark adaptation are used, according as the luminance is of the order of at least several **candela** per square meter, or less than some hundredths of a candela per square meter.

ADDITION OF TENSORS. The operation of forming the **sum** of (two) **tensors**.

ADDITIVE COMPLEMENTARY COLORS. Two lights are of complementary colors when it is possible to reproduce the color of a specified achromatic light by a suitable mixture of these two lights.

ADDITIVE MIXTURE OF COLORS. The mixture of light stimuli in such manner that they enter the eye simultaneously or in rapid succession and are incident on the same area

of the retina, or enter in the form of a mosaic which the eye cannot resolve.

ADDITIVE PROCESS. A **stochastic process** in which the increments over non-overlapping time intervals are independent; i.e., a process $\{x(t)\}$ is additive if, for $t_1 < t_2 < t_3 < \dots$, the increments $x(t_2) - x(t_1)$, $x(t_3) - x(t_2)$ \dots , are independent. The process is also known as a *random walk process*. An additive process may be defined in more than one dimension. It may also be defined for continuous t and is then sometimes known as a *differential process*.

ADDRESS. The symbol designating a particular cell in the **storage unit** of a **stored-program computer**.

ADDRESS PART. In an instruction, any part that is usually an address. (See also **code, instruction**.)

ADHESION, WORK OF. The work of adhesion W_{AB} between two liquids A and B is the increase in free surface energy (see **surface energy, free**) on separating 1 cm² of interface AB

$$W_{AB} = \gamma_A + \gamma_B - \gamma_{AB}$$

where γ_A and γ_B are the **surface tensions** of A and B respectively against their vapors, and γ_{AB} is the **interfacial tension**. For a solid-liquid interface the work of adhesion W_{SL} is defined as the work required to separate 1 cm² of interface in a vacuum to give a naked solid surface

$$W_{SL} = \gamma_S + \gamma_L - \gamma_{SL}$$

where γ_S and γ_L are the surface free energies measured in a vacuum. It may be shown that

$$W_{SL} = \gamma_S - \gamma_{SV_0} + \gamma_L(1 + \cos \theta_E)$$

where γ_{SV_0} is the surface tension of the solid covered by an absorbed film of liquid in equilibrium with the vapor, and θ_E the equilibrium **contact angle**.

ADIABATIC. (1) Occurring without gain or loss of heat by the system involved, for example adiabatic expansion. A completely adiabatic process is unrealizable, but it can be closely approximated in practice by providing good thermal insulation and by carrying out the process rapidly so that little time is allowed for heat flow in or out of the system. (2) If a thermally-isolated system moves through a series of equilibrium states, i.e., undergoes an

adiabatic and reversible process, the locus of the points representing these states on a graph is called an **adiabatic**. The present tendency is to avoid this usage and to replace adiabatic by the designations *isentrop*e or *isentrop*ic line. (See **adiabatic process**.)

ADIABATIC APPROXIMATION. The assumption that the electronic wave functions in a molecule or solid are distorted by the motion of the nuclei, but in such a way that their energy is a function only of the nuclear configuration at a given moment, and does not depend on the rate at which the nuclei are moving.

ADIABATIC BULK MODULUS. See **bulk modulus**.

ADIABATIC CHANGE. See **adiabatic process**.

ADIABATIC CHANGES OF RADIATION PRESSURE. Suppose radiation is confined in a container whose volume is altered. If this change is performed in such a way that the **entropy** remains constant, it is a reversible adiabatic change. For such a change the radiation pressure and the volume are related by the equation

$$pV^{4/3} = \text{const. (adiabatic).}$$

(See also **adiabatic equation of state**.)

ADIABATIC COOLING LIMIT. See **adiabatic saturation**.

ADIABATIC DEMAGNETIZATION. In a system whose state can be completely defined by the temperature and an external magnetic field **B**, the equation for a reversible adiabatic process is

$$S(T, \mathbf{B}) = \text{const. (adiabatic).}$$

For a paramagnetic substance at temperatures which are not too low (for example $T \geq 10^{-2} \text{°K}$) the entropy depends only on the ratio **B/T**. Therefore

$$\frac{\mathbf{B}}{T} = \text{const. (adiabatic).}$$

Thus when the field is reduced the temperature drops. This is the principle of cooling by adiabatic demagnetization.

ADIABATIC ENTHALPY DROP. See **enthalpy drop**.

ADIABATIC EQUATION OF STATE. The adiabatic equation of state expresses the relation between pressure and volume at constant **entropy** (instead of constant temperature). For a **perfect gas** of monoatomic molecules the adiabatic equation of state is of the form

$$pV^{\gamma} = \text{const.} \quad (1)$$

More generally, if γ is the ratio of the specific heats (see **thermal coefficients**) at constant pressure and at constant volume respectively, the adiabatic equation of state of a perfect gas is:

$$pV^{\gamma} = \text{const.}$$

ADIABATIC FLAME TEMPERATURE. See **combustion**.

ADIABATIC FLOW, ENERGY EQUATION FOR STEADY. See **energy equation for steady adiabatic flow**.

ADIABATIC HEAT DROP. See **enthalpy drop**.

ADIABATIC INVARIANT. A quantizable physical quantity which remains unchanged to some order of approximation under the action of a slowly varying spatial or temporal disturbance.

ADIABATIC IONIZATION POTENTIAL. See **ionization potential**.

ADIABATIC LAPSE RATE. The rate of decrease of temperature with height in an atmosphere in which a parcel of air displaced vertically assumes the same temperature as its surroundings if the changes are adiabatic. The potential temperature is then independent of height. For air of constant composition with gas constant R and ratio of specific heats γ , the adiabatic lapse rate Γ is given by

$$\Gamma = \frac{(\gamma - 1)g}{\gamma R} = \frac{g}{C_p}$$

which in the earth's atmosphere is 10°C per km with sufficient accuracy for practical purposes. The air is statically unstable or stable according to whether the lapse rate is greater or less than adiabatic, i.e., according to whether the lapse rate of **potential temperature** is positive or negative.

ADIABATIC LAW FOR QUANTIZED STATES (EHRENFEST). For a virtual, infinitely slow alteration of the **coupling** condi-

tions, the quantum numbers of the system do not change, and, in particular, the number of **terms** does not vary.

ADIABATIC PROCESS. Any thermodynamic process, reversible or irreversible, which takes place in a system without the exchange of heat with the surroundings. When the process is also reversible, it is called *isentropic*, because then the entropy of the system remains constant at every step of the process. (In older text-books isentropic processes were called simply adiabatic, or quasi-static adiabatic; the distinction between adiabatic and isentropic processes was not always sharply drawn.)

When a closed system undergoes an adiabatic process without performing work (*unresisted expansion*), its internal energy remains constant whenever the system is allowed to reach thermal equilibrium. Such a process is necessarily irreversible. At each successive state of equilibrium, the entropy of the system S_i , has a higher value than the initial entropy, S_0 . Example (see also **Joule's experiment**): When a gas at pressure p_0 , temperature T_0 , occupying a volume V_0 (see Figure 1) is allowed to expand

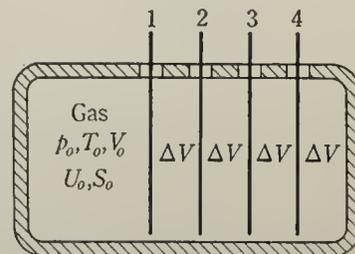


Fig. 1. Successive adiabatic expansions of gas by withdrawing slides.

progressively into volumes $V_1 = V_0 + \Delta V$, etc., by withdrawing slides 1, 2, etc., one after another, it undergoes such a process if it is enclosed in an adiabatic container. After each withdrawal of a slide, the irreversibility of the process causes the system to depart from equilibrium; equilibrium sets in after a sufficiently long waiting period. At each successive state of equilibrium $U_1 = U_2 = \dots = U_0$, but $S_0 < S_1 < S_2$, etc.

When an open system in steady flow undergoes an adiabatic process without performing external work, the enthalpy of the system regains its initial value at each equilibrium state, and the entropy increases as before. Example (see **Joule-Thomson experiment** and **throttling**): Successive, *slow* expansions through

porous plugs $P_1, P_2 \dots$ (Figure 2), when we have

$$H_1 = H_2 = \dots = H_0,$$

but

$$S_0 < S_1 < S_2, \text{ etc.}$$

This process is also necessarily irreversible (see **stagnation enthalpy, stagnation temperature**).

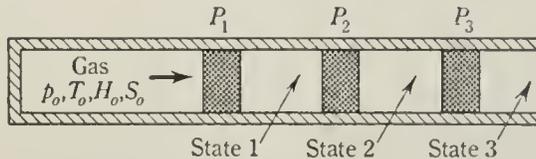


Fig. 2. Successive, slow adiabatic expansions of gas through porous plugs.

A closed system cannot perform an isentropic process without performing work. Example: (Figure 3). A quantity of gas enclosed by an ideal, frictionless, adiabatic piston in an adiabatic cylinder is maintained at a pressure p by

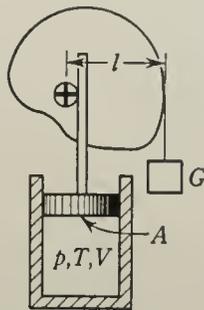


Fig. 3. Isentropic compression (or expansion) in cylinder.

a suitable ideal mechanism, so that $Gl = pA$ (A being the area of piston). When the weight G is increased (or decreased) by an infinitesimal amount dG , the gas will undergo an isentropic compression (or expansion). In this case

$$S = \text{const.}, \quad dS = 0, \quad (a)$$

at any stage of the process, but

$$U \neq \text{const.}, \quad H \neq \text{const.}$$

During an isentropic process of a closed system between states 1 and 2, the change in internal energy equals *minus* the work done between the two states, or

$$U_2 - U_1 = -W_{12}$$

work is done "at the expense" of the internal energy.

Equation (a) is the general equation of the process. In the case of a perfect gas, the differential Equation (a) can be written in three alternative forms:

$$\frac{dp}{p} + \gamma \frac{dv}{v} = 0$$

$$\frac{dT}{T} + (\gamma - 1) \frac{dv}{v} = 0 \quad (b)$$

$$\frac{dT}{T} - \frac{\gamma - 1}{\gamma} \frac{dp}{p} = 0$$

where γ can be a function of temperature. Since γ (or c_p or c_v) of a perfect gas is a tabulated function of temperature (temperature alone), it is necessary to evaluate certain integrals, in order to facilitate practical calculations. These are usually tabulated in the forms:

$$\left. \begin{aligned} \phi(T) &= \int_{T_0}^T c_p(T) \frac{dT}{T} \\ \phi_v(T) &= \int_{T_0}^T c_v(T) \frac{dT}{T} \\ p_r(T) &= p_0 \exp\left(\frac{\phi(T)}{R}\right) \\ v_r(T) &= v_0 \exp\left(-\frac{\phi_v(T)}{R}\right) \end{aligned} \right\} (c)$$

where T_0, p_0, v_0 denote an arbitrary *reference* state. (See Keenan and Kaye, *Gas Tables*, Wiley, 1958.) The integral equation of the isentropic process can then be written

$$\left. \begin{aligned} \phi(T) - \phi(T_1) &= R \ln\left(\frac{p}{p_1}\right) \\ \phi_v(T) - \phi_v(T_1) &= -R \ln\left(\frac{v}{v_1}\right) \\ \frac{p}{p_1} &= \frac{p_r(T)}{p_r(T_1)} \\ \frac{v}{v_1} &= \frac{v_r(T)}{v_r(T_1)} \end{aligned} \right\} (d)$$

where subscript 1 refers to the initial state of the system, and the symbols without subscripts refer to any other state. When the specific heats are constant, the above equations can be simplified to

$$\left. \begin{aligned} pv^\gamma &= \text{const.} \\ Tv^{\gamma-1} &= \text{const.} \\ \frac{T}{p^{\frac{\gamma-1}{\gamma}}} &= \text{const.} \end{aligned} \right\} (e)$$

The picture of an isentropic process in a **thermodynamic diagram** is called an isentrope or an isentropic. It is the locus of all states of constant entropy passing through the initial state 1. In a T, S diagram the isentrope is always a straight line as shown in Figure 4.

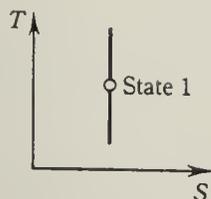


Fig. 4. Temperature-entropy diagram of isentropic process.

In any other system of coordinates, the shape of the curve depends on the thermodynamic properties of the system. The curve for a perfect gas with constant specific heats in p, V coordinates ($pV^\gamma = \text{const.}$) is shown in Figure 5.

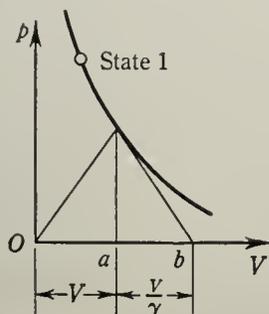


Fig. 5. Pressure-volume diagram for perfect gas at constant specific heat.

It is best obtained by cross-plotting from a log-log diagram. The subtangent ab at any state is equal to V/γ . For steam at moderate pressures and temperatures it is found empirically that it can be represented by the equation

$$Pv^n = \text{const.}$$

with $n = 1.3$, for superheated steam

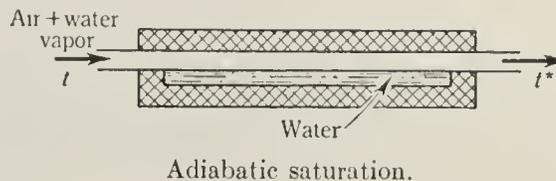
$n = 1.035 + 0.1x$, for wet steam of dry-ness fraction x

$n = 1.135$, for dry saturated steam.

(See also **Wilson line**.)

ADIABATIC SATURATION. When a mixture of air and water vapor is passed through an adiabatically insulated pipe over a large surface of water, the water will evaporate into the air until it becomes saturated. Owing to the extraction of the latent heat of vaporization, the temperature of the air will decrease, reaching a limit t^* at adiabatic saturation (*adiabatic cooling limit*). This temperature

is very close to that measured by a wet-bulb thermometer (thermodynamic wet-bulb temperature). By contrast, the actual temperature t is called the dry-bulb temperature.



The temperature t^* of adiabatic saturation is given approximately by Carrier's equation

$$p_w = p_s^* - \frac{p - p_s^*}{2,830 - 1.44t^*} (t - t^*)$$

where p_w is the partial pressure of water vapor in entering air, p_s^* is the saturation pressure at temperature t^* , p is the total pressure of entering moist air. (See figure.)

This formula is used to determine the relative humidity $\phi = p_w/p_s$ of the entering air from a measurement of t and t^* .

ADIABATIC WALL. (Perfect heat insulator.) Since in a rigorous development of the principles of thermodynamics it is necessary to introduce the concept of an adiabatic wall before the concept of heat, it is convenient to adopt the following alternative definition. If two closed systems are placed in contact through an adiabatic wall, their states can be varied independently of one another. Any state of one system can co-exist with any state of the other system through such a wall; the systems are not coupled in any way. (See **diathermal wall**.) The number of independent properties of the combined system is equal to the sum of the number of independent properties of the component systems.

ADIATHERMAL WALL. See **diathermal wall**.

ADJACENCY MATRIX. See **matrix, adjacency**.

ADJACENT VERTICES. See **vertices, adjacent**.

ADJOINT EQUATION. Consider the differential equation

$$L(u) = u'' + p(x)u' + q(x)u = 0.$$

The **operator adjoint** to $L(u)$ is

$$\bar{L}(v) = v'' - p(x)v' + [q(x) - p'(x)]v.$$

Then

$$vL(u) - u\bar{L}(v) = (d/dx)[P(u,v)]$$

is the **Lagrange identity**, and

$$P = uv[u'/u - v'/v + p(x)]$$

is the **bilinear concomitant**. If the adjoint equation $\bar{L}(v) = 0$ can be solved, the original equation $L(u) = 0$ is equivalent to the first order equation $P(u,v) = C$, where C is an arbitrary constant.

An equation identical with its adjoint is said to be **self-adjoint**. Any second-order differential equation may be put into self-adjoint form if the appropriate factor is introduced.

These procedures are not limited to equations of second order but may be generalized for those of any order. (See **Sturm-Liouville problem**.)

ADJOINT FLUX. See **flux, adjoint**.

ADJOINT OF A MATRIX. The adjoint of a square matrix **A** is the **transpose** of the matrix obtained from **A** by replacing each element of **A** by its **cofactor**.

ADJOINT OF AN OPERATOR. The adjoint A^* of an operator A is an operator such that, for all f and g in the domain of A , we have $(Af, g) = (f, A^*g)$. If $A^* = A$, then A is said to be self-adjoint. (Compare, e.g., **Sturm-Liouville problem**.)

ADJOINT SPINOR. See **Dirac equation**.

ADJOINT WAVE FUNCTION. In **Dirac electron theory** the row-vector $\bar{\psi} = \psi^* \gamma_4$ where ψ is the wave function, the asterisk denotes **Hermitean conjugate** and γ_4 is a **Dirac operator**.

ADMISSION. In a reciprocating steam engine, the point at which the inlet (admission) valve opens to feed steam into the cylinder.

ADMISSION, PARTIAL. In an impulse turbine, the device of admitting the working fluid (steam, combustion gases) over a portion of the inlet annulus only, in order to avoid the necessity of using inconveniently short blades.

ADMITTANCE. The ratio of current to voltage in an alternating current circuit. Therefore, admittance is the reciprocal of **impedance**. Thus if the impedance of a circuit element is

$$Z = R + jX, \quad |Z|^2 = R^2 + X^2,$$

its admittance is

$$Y = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2} = \frac{R}{|Z|^2} - j \frac{X}{|Z|^2} \\ \equiv G + jB.$$

The real part of admittance is called **conductance** (G); the imaginary part, **susceptance** (B). Note that positive reactance implies negative susceptance.

ADMITTANCE, DRIVING-POINT (between the j th Terminal and the Reference Terminal of an n -Terminal Network). The quotient of the complex alternating component I_j of the current flowing to the j th terminal from its external termination by the complex alternating component V_j of the voltage applied to the j th terminal with respect to the reference point when all other terminals have arbitrary external terminations. In specifying the driving-point admittance of a given pair of terminals of a network or transducer having two or more pairs of terminals, no two pairs of which contain a common terminal, all other pairs of terminals are connected to arbitrary admittances.

ADSORPTION. See **Gibbs division surface; adsorption isotherm**.

ADSORPTION, HEAT OF. When a gas or vapor is adsorbed on a solid, heat is liberated. There are two ways of expressing these heat effects: (1) Integral heat of adsorption, which is the total amount of heat, Q cal, given out when 1 g of the outgassed solid takes up m g of the gas or vapor. It is expressed as calories per g of adsorbent, for an adsorption of m g of adsorbate. (2) Differential heat of adsorption,

$$-\Delta H = M \frac{\partial \phi}{\partial m}$$

where M is the molecular weight of adsorbate. Heats of adsorption may be measured by means of a calorimeter. They may also be calculated from heats of immersion and condensation.

In the case of a liquid, the heat of adsorption of solute from solution may be determined from the thermodynamic relation:

$$\Delta H^\circ = \Delta G^\circ - T \frac{d(\Delta G^\circ)}{dT}$$

where ΔH° and ΔG° are the heat of and free energy (see **free energy** (1)) of adsorption. For dilute solutions:

$$\Delta G^\circ = -RT \ln \frac{C_{\text{solid}}}{C_{\text{solution}}} = -RT \ln \frac{\Gamma}{\gamma C_{\text{solution}}}$$

where C_{solution} is the concentration of the solute in solution, C_{solid} is the concentration of the adsorbed solute, Γ is the amount of adsorbed solute per unit area of surface, and γ is the thickness of the adsorbed film. Older values of the heat of adsorption are subject to sizable errors.

ADSORPTION ISOTHERM. The adsorption isotherm expresses the equilibrium condition between an adsorbed film, which is often a monolayer (see **localized monolayers**) and the vapor pressure.

If one considers a localized monolayer with negligible interactions between the adsorbed molecules and assumes, moreover, that all sites are equivalent, the adsorption isotherm is

$$\frac{\theta}{1 - \theta} = \frac{p}{p_o} \quad \text{or} \quad \theta = \frac{p}{p_o + p}$$

where θ is the fraction of sites occupied by molecules, p the vapor pressure and p_o a characteristic constant depending on temperature, the nature of the molecule and the energy of adsorption.

This formula was first derived by Langmuir and is known as the **Langmuir adsorption isotherm**.

Many other forms of the adsorption isotherm have been suggested or derived in the literature.

ADSORPTION ISOTHERM, LANGMUIR TREATMENT. See **Langmuir adsorption isotherm**.

ADVANCED POTENTIALS. The electromagnetic potentials at a point \mathbf{r} at time t due to sources at points $\mathbf{r} - \mathbf{R}$ and times $t + |\mathbf{R}|/c$. These sources lie on the future light cone through the point \mathbf{r} , t in space-time. Although these potentials arise in the mathematical formulation of classical field theory, they appear to have no physical significance. (See **retarded potentials**.)

ADVECTION. Transfer by the bulk motion of a fluid, e.g., horizontal transport by the wind. It is included in the more general term

convection which, however, more usually denotes transfer by turbulent or penetrative motions of a fluid.

Advection may be expressed in vector notation by

$$\mathbf{V} \cdot \nabla \phi$$

where \mathbf{V} is the wind vector, ϕ the atmospheric property, and ∇ the del operator. In three-dimensional **Cartesian coordinates**, it is

$$u \frac{\partial \phi}{\partial x} + v \frac{\partial \phi}{\partial y} + w \frac{\partial \phi}{\partial z}$$

If (u, v) is taken as the horizontal wind, and w , the vertical air velocity, the first two terms comprise the *horizontal advection* and the last term is the *vertical advection*. The property ϕ may itself be a vector field.

Often, particularly in synoptic meteorology, advection refers only to the horizontal or isobaric components of motion, that is, the wind field as shown on a synoptic chart.

AELOTROPIC MATERIAL. A material which is not isotropic. In elasticity theory, a material which is not isotropic with respect to its elastic properties. Also called *anisotropic material*.

AERODYNAMIC CENTER. If C_m is the pitching-moment coefficient on an airfoil at incidence α , the position of the pitching-moment axis on the chord line for which

$$\frac{dC_m}{d\alpha} = 0$$

is known as the aerodynamic center. For an unstalled airfoil, C_m varies almost linearly with α , so that the aerodynamic center is very nearly a fixed point, independent of α . For a symmetrical airfoil the aerodynamic center coincides with the center of pressure, since $C_m = C_L = 0$ when $\alpha = 0$.

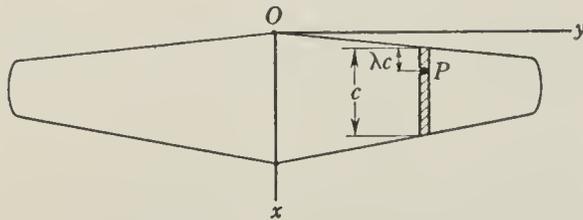
According to thin-airfoil theory, the aerodynamic center of a two-dimensional airfoil in subsonic flow is at the quarter-chord point. The linear theory of supersonic flow shows that the aerodynamic center of a two-dimensional airfoil is at the half-chord point.

AERODYNAMIC MEAN CHORD (OF WING). Referring to the figure on page 24 if c is the local chord of a wing at a distance y from the center line, the aerodynamic mean chord is defined as

$$\bar{c} = \frac{1}{S} \int_{-b/2}^{+b/2} c^2 dy \quad (1)$$

where S is the area and b is the span of the wing.

If a point $P(x,y)$ is at a distance λc from the leading edge, the point at a distance $\lambda \bar{c}$ from



Aerodynamic mean chord.

the leading edge of the aerodynamic mean chord is at a distance from the y -axis

$$\bar{x} = \frac{1}{S} \int_{-b/2}^{+b/2} x c dy. \quad (2)$$

Thus Equation (1) defines the length of the aerodynamic mean chord and Equation (2) defines its position along the x -axis.

The position of the mean quarter-chord point is found from Equation (2) by putting $\lambda = 1/4$.

AERODYNAMIC TIME UNIT. See **airplane stability**.

AEROFOIL. See **airfoil**.

AEROGRAM. (1) A thermodynamic diagram designed by Refsdal with logarithm of temperature as abscissa and the product of temperature and logarithm of pressure ($T \log p$) as ordinate, increasing downward. Isotherms are the only straight lines, but convenient spacing between dry adiabats, saturation adiabats, and vapor lines is obtained. The energy of a cyclic process is *not* proportional to the area enclosed by the curve representing the process. (2) The record traced by an aerograph.

AFFINE CONNECTION, FUNDAMENTAL. See **fundamental affine connection**.

AFFINE TENSORS AND FREE VECTORS. A quantity which behaves like a tensor under a linear (affine) coordinate transformation, but not under a general coordinate transformation is called an affine tensor. From an affine tensor it is possible to construct a *free vector*,

i.e., a vector not related to a given point (non-localized vector): Consider an affine tensor U_i^k and vector \mathbf{p}^k which is constant inside a "world-canal" (i.e., a region in the four dimensional world of tube shape, such that its lateral surface is space-like and the other two boundaries are time-like). If

$$\frac{\partial \sqrt{g} U_i^k \mathbf{p}^i}{\partial x^i} \equiv 0,$$

then the

$$J_k \equiv \iiint \sqrt{g} g_{ki} U_m^i \mathbf{p}^m dx^1 dx^2 dx^3$$

form the components of a free vector. The integral is extended over the lateral surface of the world canal.

AFFINE TRANSFORMATION. Using the **indicial notation** and **summation convention**, the transformation

$$\bar{x}^i = a_j^i x^j + b^i,$$

with a_j^i and b^i real constants, which transforms the point at x^i to \bar{x}^i . The transformation called non-singular or singular according as its determinant $|a_j^i|$ is non-zero or zero. If the constant terms b^i are zero, the transformation is *homogeneous affine*.

AFFINITY. See **chemical affinity**.

AFOCAL SYSTEM. An optical system with foci at infinity.

AFTER EFFECT, ELASTIC. Recovery of **strain** on unloading associated with delayed elasticity. (See **elasticity, delayed**.)

AGE APPROXIMATION. In the analysis of the energy-dependent transport equation in neutron transport theory, an approximation in which the collision density, appearing in the integrand of the collision integral, is replaced by the first two terms of its **Taylor expansion** about **lethargy** value u . The approximation, used in conjunction with a P_1 approximation for the angular dependence, leads directly to the Fermi Age Equation. (See **P_N Approximation**.)

AGE EQUATION (FERMI). In the theory of neutron slowing-down in matter, the age equation

$$\nabla^2 q(r, \tau') = \frac{\partial q}{\partial \tau'}(r, \tau')$$

describes the slowing-down density of neutrons as a function of space and generalized age, τ' . It is generally solved under particular boundary conditions and the initial condition $q(r, 0) = S(r)$, where $S(r)$ gives the space distribution of neutron sources. The generalized age,

$$\tau' = \int_E^{E_0} \frac{dE'}{E' \xi \Sigma_s + \gamma \Sigma_a} D(E')$$

reduces to the usual age, τ , when Σ_a , the macroscopic absorption cross section is zero. For definition of γ , see **Goertzel-Greuling approximation**.

AGE, NEUTRON. In the theory of neutron slowing-down in matter, the neutron age, $\tau(E, E_0)$ is defined by

$$\tau(E, E_0) = \int_E^{E_0} \frac{dE'}{E' \xi \Sigma_s(E')} D(E')$$

where E is the neutron energy, E_0 a reference energy, $D(E)$ is the diffusion coefficient appropriate to neutrons having energy E , ξ is the average logarithmic energy loss for a neutron in collision with an atom of the moderating material, and Σ_s is the microscopic cross-section for scattering. In the case of slowing-down in a mixture, average values of D and $\xi \Sigma_s$ are used. Note that the age has dimensions of length squared.

AGEOSTROPHIC WIND. The actual wind less the **geostrophic wind** (measured vectorially). If there is no frictional drag or eddy stress the ageostrophic wind \mathbf{v}_A is related to the acceleration in the same way as the geostrophic wind is related to the pressure gradient; thus

$$\mathbf{v}_A \times \mathbf{f} = \frac{D\mathbf{v}}{Dt}$$

where \mathbf{v} is the air velocity relative to the earth, assumed horizontal, and \mathbf{f} is twice the vertical component of the earth's rotation.

AIR, ATMOSPHERIC. Atmospheric air is a mixture of gases of remarkably constant composition, except for its *moisture content* (H_2O vapor). The composition of dry atmospheric air is as follows:

	Parts by Volume	Parts by Mass
N_2	0.7803	0.7547
O_2	0.2099	0.2320
A	0.00933	0.0128
CO_2	0.00030	0.00046
H_2	0.0001	0.00001
Ne	0.000018	0.000012
He	0.000005	0.0000007
Kr	0.000001	0.000003
Xe	0.0000009	0.0000004

For rapid calculation a simpler composition is often assumed:

	Parts by Volume	Parts by Mass
N_2	0.79	0.77
O_2	0.21	0.23

The gas constant for air is

$$R = 53.35 \text{ ft lbf/lbm } ^\circ\text{F} = 29.27 \text{ kp m/kg } ^\circ\text{C} \\ = 287.0 \text{ J/kg } ^\circ\text{C},$$

the molecular weight is

$$M = 28.964 \text{ lbm/lbmol} = 28.964 \text{ kg/kmol},$$

the density at 0°C and 760 mm Hg is

$$\rho_0 = 1.2928 \text{ kg/m}^3 = 0.08071 \text{ lbm/ft}^3.$$

When air is treated as a perfect gas with constant specific heats, it can be assumed that

$$c_p = 0.240 \text{ Btu/lbm } ^\circ\text{F} = 0.240 \text{ kcal/kg } ^\circ\text{C} \\ c_v = 0.171 \text{ Btu/lbm } ^\circ\text{F} = 0.171 \text{ kcal/kg } ^\circ\text{C} \\ \gamma = 1.40.$$

See also **atmosphere, standard**.

AIR COMPRESSOR. See **compressor**.

AIR EXCESS RATIO. Ratio of volume of excess air supplied for combustion to theoretical (stoichiometric) volume required for complete combustion. If λ denotes the **air ratio**, then the air excess ratio $\delta = \lambda - 1$.

AIRFOIL, ALSO AEROFOIL. A surface whose cross section is so shaped as to provide nearly **potential flow** around it; when placed in an air stream, an airfoil produces a large lift and a relatively small drag. Airfoils are used for airplane wings, turbine and turbo-compressor blades, etc.

AIRFOIL, LIFT AND MOMENT ON. See **lift and moment on an airfoil, general formulae**.

AIRFOIL STARTED FROM REST. It is supposed that an airfoil is started impulsively

from rest with velocity V , the angle of incidence α being kept constant. Immediately after starting, the circulation round the airfoil is zero and the streamlines in the vicinity of the trailing edge are as sketched in Figure 1. There is a rapidly rising pressure in the

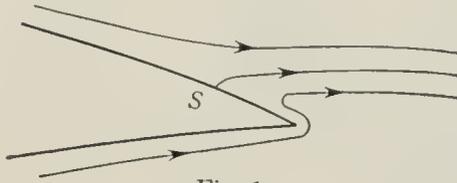


Fig. 1.

flow along the upper surface from the trailing edge towards the stagnation point S , and after a very short time a boundary layer develops in this region and separates, leading to the formation of a vortex as shown in Figure 2. This vortex, known as the *starting vortex*, does

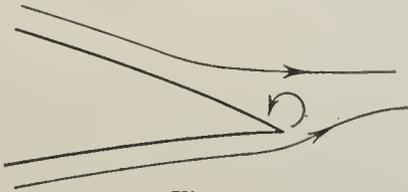


Fig. 2.

not remain in this position; it separates from the trailing edge and is left behind in the fluid as the airfoil moves on. The circulation round a path enclosing the airfoil and its wake remains zero, so that as the starting vortex is left behind in the wake the circulation round the airfoil alone becomes equal to the strength of the starting vortex, but of opposite sign.

The general problem of an airfoil in accelerated motion will now be considered in greater detail. Vorticity of the sign shown in Figure 2 is shed from the lower surface, and from the part of the upper surface between the stagnation point S and the trailing edge (Figure 1), while vorticity of opposite sign is shed from the part of the upper surface upstream of S . The former predominates, so that the net vorticity in the wake is of the sign shown in Figure 2. The shedding of net vorticity into the wake, and the associated growth of circulation around the airfoil, continue until

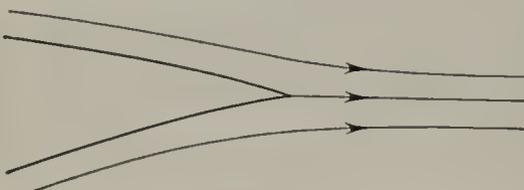


Fig. 3.

the fluid is flowing smoothly from the trailing edge as in Figure 3. This state is only approached asymptotically; the circulation and the lift then have their steady-flow values.

This discussion shows that, immediately after starting an airfoil from rest, the circulation is less than the final steady-flow value corresponding to the **Joukowski condition**. There is also a deficiency of lift, although the **Kutta-Joukowski law** relating the lift to the circulation for steady flow is not applicable in unsteady flow. The phenomenon of lift deficiency just after the start was studied theoretically by Wagner and is known as the *Wagner effect*. This term is also used with reference to the corresponding deficiency of lift immediately after an abrupt increase of incidence.

In the theoretical study of the problem by Wagner and others, it is assumed that the airfoil is thin, uncambered, and at a small angle of incidence. It is also assumed that the circulation at any instant is such that the fluid leaves the trailing edge smoothly, so that the velocity there is finite. It is found that the circulation and lift reach values within about 5% of the steady-flow values after the airfoil has travelled a distance of the order of 10 times the chord.

It is also found that immediately after an impulsive start, for an airfoil at constant incidence, or immediately after an abrupt increase of incidence from zero, for an airfoil at constant speed, the lift coefficient is exactly half the value in steady flow. (See A. Robinson and J. A. Laurmann, *Wing Theory*, Cambridge, 1956.)

AIRFOIL THEORY (TWO-DIMENSIONAL).

For calculating the properties of thin airfoils of small camber, at small angles of incidence, the methods of **thin-airfoil theory** are most useful. For thick airfoils, or for large angles of incidence, thin-airfoil theory becomes inaccurate and other methods must be used.

The family of airfoils known as **Joukowski airfoils** is obtained by conformal mapping of the flow past a circular cylinder, using the simple mapping function

$$\zeta = z + \frac{c^2}{z} \quad (1)$$

In the *Theodorsen method* Equation (1) is used to transform a given airfoil in the ζ -plane

into a contour in the z -plane which is not exactly a circle but does not differ greatly from a circle. This "pseudo-circle" in the z -plane is then related to a true circle in the z' -plane by a further transformation. The calculation involves a process of successive approximations, and although this converges rapidly the numerical work required is extensive. The method is exact and free from any restriction on thickness, camber or incidence, but it can only be used to calculate the pressure distribution on a given airfoil, and not for the inverse problem of designing an airfoil to have a given pressure distribution.

For this inverse problem, the *Lighthill method* may be used, if the thickness is so large that an exact theory is required. With this method it is unfortunate that the calculation has to be started with the velocity distribution prescribed, not on the airfoil itself, but on the circle into which the airfoil may be transformed. It is found in practice, however, that this does not restrict the usefulness of the method very seriously. The region outside the airfoil in the ζ -plane is transformed into the region outside the circle $|z| = a$ in the z -plane by means of a transformation function $\zeta(z)$ such that $\frac{d\zeta}{dz} \rightarrow 1$ as $z \rightarrow \infty$. The coordinates of the airfoil surface are obtained in terms of two integrals, involving the fluid speed and the direction of flow as functions of position on the circle in the z -plane. The fluid speed is prescribed at the start, and the direction of flow θ is found as the conjugate **Fourier series** of $\log q_0$, where q_0 is the fluid speed at zero incidence.

Lighthill's method has also been applied in designing cascades of airfoils, and contractions and bends in two-dimensional channels. For airfoils of small or moderate thickness, the approximate methods of Goldstein are more useful; these are considered briefly below. For very thick airfoils, especially those incorporating suction slots, the approximate methods cannot be used and Lighthill's method is unrivalled.

Goldstein has developed a series of systematic approximations to exact airfoil theory which lead to useful results with a comparatively small effort in computing. His first approximation corresponds to the linearized "thin-airfoil theory," while the second and third are higher order approximations allowing for greater thickness and camber. The

approximations are based on Theodorsen's method, and it is assumed that the differences between the pseudo-circle in the z -plane and the true circle in the z' -plane are small; this implies fairly small thickness and camber of the airfoil, but does not restrict the incidence. With this assumption, simplifications can be made in Theodorsen's method which reduce considerably the labor involved in computing the velocity distribution on a given airfoil. Moreover, Goldstein has been able to develop his approximate methods so that they can be used fairly easily for designing airfoils to have prescribed velocity distributions. (See A. Robinson and J. A. Laurmann, *Wing Theory*, Cambridge, 1956.)

AIR-FUEL RATIO. The ratio of the mass of air to the mass of fuel supplied in a combustion device.

AIR INJECTION. The process of injecting the fuel into a diesel engine with the aid of a blast of air from a compressor. In modern engines air injection has been replaced by *solid injection* when the fuel is compressed to several hundred atmospheres in an injection pump and injected into the cylinder through an *atomizer*. The object in either case is to inject the liquid fuel in the form of a fine spray.

AIR PARCEL. An imaginary body of air to which may be assigned any or all of the basic dynamic and thermodynamic properties of atmospheric air. A parcel is large enough to contain a very great number of molecules, but small enough so that the properties assigned to it are approximately uniform within it and so that its motions with respect to the surrounding atmosphere do not induce marked compensatory movements. It cannot be given precise numerical definition, but a cubic foot of air might fit well into most contexts where air parcels are discussed, particularly those related to **static stability**. Any fluid parcel may be defined similarly.

AIRPLANE ENERGY EQUATION. For an airplane of weight W , flying at height h with velocity V , the sum of the potential and kinetic energies is

$$E = W \left(h + \frac{V^2}{2g} \right).$$

Also, if D is the drag of the airplane and T is the propulsive thrust,

$$V(T - D) = \frac{dE}{dt} = W \left(\frac{dh}{dt} + \frac{V}{g} \frac{dV}{dt} \right).$$

Thus the rate of climb is

$$\frac{dh}{dt} = \frac{V}{W} (T - D) - \frac{V}{g} \frac{dV}{dt}.$$

AIRPLANE PERFORMANCE. For an airplane of weight W , flying at constant velocity and climbing at an angle θ , the required propulsive thrust T is given by

$$\frac{T}{W} = \gamma \cos \theta + \sin \theta, \quad (1)$$

where γ is the ratio of drag to lift. Thus the rate of climb, or vertical component of velocity, is equal to $V(T/W - \gamma)$, where V is the velocity of flight.

For horizontal flight at constant speed and incidence, the distance flown while using fuel of weight f is

$$\frac{V}{C\gamma} \log_e \frac{W}{W - f},$$

where W is the total weight of the airplane and fuel at the start and C is the ratio of the weight of fuel used per unit time to the propulsive thrust.

AIRPLANE STABILITY. In the theory of airplane stability it is usual to consider separately the *symmetric* disturbances, in which all the forces lie in the plane of symmetry, and the *asymmetric* disturbances in which there are rolling, yawing and side-slipping motions and associated forces.

The equations are expressed in terms of body axes, with origin at the center of gravity, chosen so that in undisturbed horizontal flight Ox is in the direction of flight and Oz is vertical and downward. The velocity components in undisturbed flight are $(U, 0, 0)$ and the components of the disturbance velocity are (u, v, w) . The angular velocities about the x, y and z -axes are p, q and r , while the moments of inertia about these axes are A, B and C . The components of the resultant aerodynamic force acting on the aircraft are (X, Y, Z) and the aerodynamic couples about the three axes are L, M and N . Then, if θ is the angular displacement about the y -axis from the undisturbed state and terms involving products of inertia are neglected, the equations for the

motion of the airplane when disturbed *symmetrically* are

$$\left. \begin{aligned} m\dot{u} &= X_u \cdot U + X_w \cdot W + X_q \cdot q \\ &\quad - mg\theta \\ m(\dot{w} - Uq) &= Z_u \cdot U + Z_w \cdot W + Z_q \cdot q \\ B\dot{q} &= M_u \cdot U + M_w \cdot W \\ &\quad + M_q \cdot q + M_{\dot{w}} \cdot \dot{w} + M_o, \end{aligned} \right\} (1)$$

where m is the mass of the aircraft.

In Equation (1), \dot{u} denotes $\frac{du}{dt}$ and X_u denotes $\frac{\partial X}{\partial u}$. M_o is the pitching moment due to displacement (if any) of the elevator control. With the exception of the term $M_{\dot{w}} \cdot \dot{w}$ in the third equation, terms involving aerodynamic forces or couples due to acceleration are negligible. In practice the terms $X_q \cdot q, Z_q \cdot q$ and $M_u \cdot U$, which are included here in the equations, are also negligible.

If ϕ and ψ are the angles of roll and yaw, respectively, the equations of motion for asymmetric disturbances are

$$\left. \begin{aligned} m(\dot{v} + Ur) &= Y_v \cdot v + Y_p \cdot p + Y_r \cdot r \\ &\quad + mg\phi \\ Ap &= L_v \cdot v + L_p \cdot p + L_r \cdot r \\ &\quad + L_o \\ C\dot{r} &= N_v \cdot v + N_p \cdot p + N_r \cdot r \\ &\quad + N_o \end{aligned} \right\} (2)$$

In Equation (2), L_o and N_o are the rolling and yawing moments due to displacement (if any) of the ailerons and rudder.

It is usual to convert Equations (1) and (2) into a dimensionless form. In principle, this is done by expressing all masses in terms of m , the mass of the airplane, all lengths in terms of a representative length l defining the scale of the airplane, and all times in terms of a quantity

$$\frac{m}{\rho US},$$

where ρ is the air density and S is the wing area.

The quantity $\frac{m}{\rho US}$ is known as the *unit of aerodynamic time*. The equations in this dimensionless form apply to any airplane of the specified shape, whatever its size, weight, or altitude of flight.

In solving Equations (1) and (2) it is often

assumed that the dimensionless forms of the derivatives X_u , Y_v , etc., are not affected by changes of airplane velocity. Then, expressing the solution for the disturbance velocities in the form $u = u_1 e^{\lambda t}$, $p = p_1 e^{\lambda t}$ and so on, a pair of quartic equations in λ is obtained, one for Equation (1) and another for Equation (2). The form of the solution of these quartic equations determines the stability or instability of the motion. (See W. J. Duncan, *Control and Stability of Aircraft*, Cambridge, 1952.)

AIR PREHEATING. In order to increase the overall efficiency of a steam plant, the combustion gases from the boiler can be used to preheat the air supplied for combustion. The air preheater is also known as an *economizer*.

AIR RATIO. Ratio of volume of air supplied for combustion to theoretical (stoichiometric) volume required for complete combustion.

AIRSCREW. An airscrew may be used either as a propeller, for converting shaft power into thrust, or as a windmill, for extracting energy from an air stream and converting this into shaft power. The lifting rotor of a helicopter in upward vertical flight may be regarded as a special form of propeller.

The usual theory of airscrews has been developed primarily for use in the design of propellers, but it is also applicable to windmills. Each element of a blade is regarded as an element of a two-dimensional airfoil, operating in a stream whose velocity is determined in magnitude and direction not only by the forward and rotational speeds, but also by the induced velocity due to the system of helical trailing vortices from the blades. It can be shown that the induced velocity at a given radius depends only on the thrust and torque loading at that radius.

In the usual method of calculating the induced velocities, the airscrew is replaced by an equivalent airscrew having an infinite number of blades and the same radial distribution of thrust and torque. This approximation gives very good results in many cases. For other cases, more refined theories are available in which allowance is made for the finite number of blades in calculating the induced velocities. (See H. Glauert, *The Elements of Aerofoil and Airscrew Theory*, Cambridge, 2nd Edition, 1947.)

AIR-STANDARD CYCLE. An ideal sequence of processes forming a cycle which serves as a standard of comparison for the evaluation of the performance of internal combustion engines. The essential idealization consists in replacing the process of combustion by the addition of heat from an external source and by assuming that the working fluid has the properties of air (perfect gas with constant, or variable, specific heats) throughout the cycle. The precise details of an air-standard cycle for a particular type of engine are to a certain extent arbitrary and must be fixed by convention.

AIR-STANDARD EFFICIENCY. The efficiency of an air-standard cycle which corresponds to a given internal-combustion engine.

AIR-STEAM MIXTURES. Mixtures of air and steam play an important part in the study of atmospheric conditions, drying and air conditioning. In evaluating their properties it is usually assumed that the air (or, generally, gas) is perfect and non-condensing, and that the steam (or, generally, the vapor) is also a perfect gas, except that the fact that the steam condenses under specified conditions is taken into account. Processes involving mixtures of gases and vapor are usually studied graphically. (See **psychrometric chart** and **Mollier diagram**.)

AIRY DIFFERENTIAL EQUATION. See **Airy functions**.

AIRY DISC. The diffraction pattern in the Gaussian image plane of an axially symmetrical optical system is given by

$$F(r) = \int_0^{\rho_0} J_0 \left(\frac{2\pi}{\lambda} \rho r \right) e^{i \frac{2\pi}{\lambda} W(\rho)} \rho d\rho$$

where $\rho_0 = n \sin \theta$ is the exit aperture, n is the index of refraction in image space, θ is the angle of projection, λ is the wavelength, $W(\rho)$ is the mixed characteristic function of the system, and r is radial distance in the image plane from the focus. (J_0 and J_1 are Bessel functions of orders 0 and 1.) If the system is free from spherical aberration so $W(\rho) = W(0)$ is constant, then

$$F(r) = \frac{\lambda \rho_0}{2\pi r} J_1 \left(\frac{2\pi}{\lambda} \rho_0 r \right) e^{i \frac{2\pi}{\lambda} W(0)}$$

The intensity of the diffraction patterns is

$$I(r) = I(0) \left\{ \frac{2J_1(\beta)}{\beta} \right\}^2, \quad \beta = \frac{2\pi\rho_0}{\lambda} r.$$

The first positive zero of $I(r)$, as determined from the first positive zero of $J_1(z)$ is approximately $r_1 = 0.61\lambda/\rho_0$. The Airy disc is then the illuminated circle in the image plane with center at the focus and radius r_1 .

AIRY FUNCTIONS. In an investigation of diffraction near a **caustic** surface Airy introduced what is now called the Airy integral

$$\begin{aligned} Ai(x) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \exp i(tx + \frac{1}{3}t^3) dt \\ &= \frac{1}{\pi} \int_0^{\infty} \cos (tx + \frac{1}{3}t^3) dt \end{aligned}$$

which satisfies Airy's differential equation

$$\frac{d^2y}{dx^2} = xy.$$

$x_0 - x$	$y_x(x_0)$	$y_x(x_0, x_1)$	$y_x(x_0, x_1, x_2)$	$y_x(x_0, x_1, x_2, x_3)$
$x_1 - x$	$y_x(x_1)$	$y_x(x_1, x_2)$	$y_x(x_1, x_2, x_3)$	
$x_2 - x$	$y_x(x_2)$	$y_x(x_2, x_3)$		
$x_3 - x$	$y_x(x_3)$			

The solutions of this equation are called Airy functions. A second solution is

$$Bi(x) = \frac{1}{\pi} \int_0^{\infty} \left\{ \exp (tx - \frac{1}{3}t^3) + \sin (tx + \frac{1}{3}t^3) \right\} dt.$$

AIRY INTEGRAL. See **Airy functions**.

AIRY STRESS FUNCTION. See **plane strain**.

AITKEN δ^2 PROCESS. See **delta-square process**.

AITKEN METHOD OF INTERPOLATION. The procedure is to form a table

$x_0 - x$	$y_x(x_0)$			
$x_1 - x$	$y_x(x_1)$	$y_x(x_0, x_1)$		
$x_2 - x$	$y_x(x_2)$	$y_x(x_0, x_2)$	$y_x(x_0, x_1, x_2)$	
$x_3 - x$	$y_x(x_3)$	$y_x(x_0, x_3)$	$y_x(x_0, x_1, x_3)$	$y_x(x_0, x_1, x_2, x_3)$
			

by means of the relations $y_x(x_i) = y_i$ and

$$\begin{aligned} y_x(x_i, x_j) &= \frac{\begin{vmatrix} y_x(x_i) & x_i - x \\ y_x(x_j) & x_j - x \end{vmatrix}}{(x_j - x_i)}, \\ y_x(x_i, x_j, x_k) &= \frac{\begin{vmatrix} y_x(x_i, x_j) & x_j - x \\ y_x(x_i, x_k) & x_k - x \end{vmatrix}}{(x_k - x_i)}, \\ &\dots \end{aligned}$$

Then $y_x(x_i, x_j)$ is the result of linear interpolation based upon the abscissae x_i and x_j ; $y_x(x_i, x_j, x_k)$ that of quadratic interpolation based upon the abscissae $x_i, x_j,$ and $x_k; \dots$. The process may be terminated whenever consecutive entries in a column agree to sufficiently many figures.

It is not necessary that the abscissae be equally spaced, or even ordered, but only that they be distinct. The method of Neville is based upon the same principle but utilizes the table

AITKEN NUCLEI. The microscopic particles in the atmosphere which serve as condensation nuclei for droplet growth during the rapid adiabatic expansion produced by an Aitken dust-counter. These nuclei, often numbering many tens of thousands per cubic centimeter in city air, are both solid and liquid particles whose diameters are of the order of tenths of microns or even smaller.

ALBEDO. (1) The **reflection factor** of a diffusely reflecting surface, e.g., a planet. (2) In particle physics, the albedo is the ratio of the current of particles (neutrons, photons, . . .) reflected from a surface, to the current incident upon the surface. (3) In cosmic ray

physics, albedo refers to those energetic charged secondary cosmic rays which move generally upward. Since some of these particles go far into the upper atmosphere and return to the earth's surface (because of the effect of the earth's magnetic field), measurements of primary cosmic radiation must be corrected for the contribution of the albedo particles. (4) In determining the albedo of a planet, it is evident that observations must be taken at all phases of the object as seen from the earth and an average value obtained. The average values of the albedo for various members of the solar system are:

Mercury... 7%	Saturn.... 42%
Venus.... 59	Uranus... 45
Earth.... 29	Neptune.. 52
Ceres.... 6	Pluto.... 3(?)
Jupiter... 44	

ALFÉN WAVE. (Also called magnetohydrodynamic wave.) A transverse wave in a **magnetohydrodynamic** field in which the driving force is the tension introduced by the magnetic field along the lines of force. The dynamics of such waves are analogous to those in a vibrating string, the phase speed being given by

$$C^2 = \mu H^2 / 4\pi\rho,$$

where μ is the permeability, H the magnitude of the magnetic field, and ρ the fluid density. Dissipative effects due to fluid viscosity and electrical resistance may also be present. (For further details see T. G. Cowling, *Magneto-hydrodynamics*, Interscience, 1957.)

ALGEBRA. In general, algebra is the theory of multiplication and addition; see **field**, etc. Specifically, an algebra (the concept is important, e.g., in quantum mechanics, and in crystallography) is defined as follows:

Let K be any field (in physical practice, usually a complex field) whose elements we shall call scalars. Let γ_{ijk} , with $i, j, k = 1, 2, \dots, n$, be any n^3 elements of K . Then the set of all ordered n -tuples (x_1, x_2, \dots, x_n) of element of K is called an algebra over K if addition, multiplication and scalar multiplication are defined thus:

$$\begin{aligned} (x_1, x_2, \dots, x_n) + (y_1, y_2, \dots, y_n) \\ = (x_1 + y_1, x_2 + y_2, \dots, x_n + y_n) \end{aligned}$$

$$\begin{aligned} (x_1, x_2, \dots, x_n)(y_1, y_2, \dots, y_n) \\ = (\gamma_{ij1}x_i y_j, \gamma_{ij2}x_i y_j, \dots, \gamma_{ijn}x_i y_j), \\ \lambda(x_1, x_2, \dots, x_n) = (\lambda x_1, \lambda x_2, \dots, \lambda x_n), \end{aligned}$$

where λ is a scalar and we have used the summation convention. Setting

$$\begin{aligned} l_1 = (1, 0, \dots, 0), l_2 = (0, 1, \dots, 0), \dots, l_n \\ = (0, 0, \dots, 1) \end{aligned}$$

we have $(x_1, x_2, \dots, x_n) = x_i l_i$, and $l_i l_j = \gamma_{ijk} l_k$. The l_i form a basis of the algebra and the γ_{ijk} are its structure constants.

ALGEBRAIC EQUATIONS. An equation, or set of **simultaneous equations**, in which the unknowns occur as rational functions only. Hence the equations are expressible by equating polynomials to zero. Here the case of a single equation in one unknown is considered:

$$\begin{aligned} a_0 P(x) \equiv a_0 x^n + a_1 x^{n-1} + \dots + a_n = 0, \\ a_0 \neq 0, \end{aligned}$$

where the a_i do not depend upon x , and are called the coefficients of the equation, while n is the degree. This equation is equivalent to

$$\begin{aligned} P(x) \equiv x^n - c_1 x^{n-1} + \dots + (-1)^n c_n = 0, \\ c_i = (-1)^i a_i / a_0, \end{aligned}$$

where the c_i are the elementary **symmetric functions** of the roots x_i .

The *remainder theorem* states that if $a_0 P(x)$ is divided by $x - r$, the remainder is $a_0 P(r)$:

$$a_0 P(x) \equiv (x - r)Q(x) + a_0 P(r);$$

the *factor theorem* is a corollary and states that if r is a root of $P(x) = 0$, then $x - r$ divides $P(x)$. The "fundamental theorem of algebra" states that every algebraic equation has a root, real or complex. These theorems imply that $P(x)$ can be factored completely:

$$P(x) = (x - x_1)(x - x_2) \dots (x - x_n),$$

each x_i being a root. If $x_i = x_j$ for some $i \neq j$, then $x_i = x_j$ is a double root; if $x_i = x_j = x_k$, a triple root, \dots . Counting a root x_i of multiplicity m as being m coincident roots, one says that an algebraic equation of degree n has exactly n roots (neither more nor less). A **Taylor series** expansion gives

$$\begin{aligned} P(x - r) = P(r) + (x - r)P'(r) + \dots \\ + (x - r)^n P^{(n)}(r) / n! \end{aligned}$$

Hence r is a root of multiplicity m if and only if $0 = P(r) = P'(r) = \dots = P^{(m-1)}(r) \neq P^{(m)}(r)$, hence r satisfies the derived equations

$$P^{(i)}(x) = 0, \quad i = 0, 1, \dots, m - 1.$$

In general, setting $y = x - r$, if

$$b_i = P^{(n-i)}(r)/(n - i)!,$$

then any root of

$$y^n + b_1y^{n-1} + \dots + b_n = 0$$

is r less than a root x_i , hence the roots are said to have been reduced by r . Repeated **synthetic division** can be applied to the original equation to obtain the b_i , since $b_n = P(r)$ is the remainder after dividing $P(x)$ by $x - r$; b_{n-1} that after dividing the quotient by $x - r$; \dots .

Other useful transformations are the following:

The roots of

$$a_0y^n - a_1y^{n-1} + \dots + (-1)^na_n = 0$$

are the negatives of the roots of the original; those of

$$a_0y^n + a_1\alpha y^{n-1} + \dots + a_n\alpha^n = 0$$

are α times the roots of the original; those of

$$a_ny^n + a_{n-1}y^{n-1} + \dots + a_0 = 0$$

are the reciprocals of those of the original. These can be derived by setting $x = -y$, $x = y/\alpha$, and $x = 1/y$.

When the coefficients a_i are integers, then any rational root when expressed as a fraction p/q in lowest terms is such that p divides a_n and q divides a_0 without remainder. In particular, any integral root must be a divisor of a_n . In principle all rational roots can be found exactly, and once any root r is known, all other roots must satisfy $Q(x) = 0$ where $Q = P/(x - r)$. For irrational roots see **Budan theorem**, **Sturm theorem**, **Horner method**, **Newton method**, **Bernoulli method**, **Graeffe method**, **iterative methods**.

While these methods, except for Horner's, apply as well to complex roots as to real, it may be convenient to evaluate $P(z)$ with $z = x + iy$, and write

$$P(z) = R(x, y) + iJ(x, y),$$

after collecting real and pure imaginary terms.

Then

$$R(x, y) = J(x, y) = 0$$

are two **simultaneous equations** in x and y , and any real solution (x, y) determines a complex solution $z = x + iy$ of $P(z) = 0$. More to be recommended, however, is the **Bairstow method** for complex roots, which is, in fact, an adaptation of Newton's method to complex roots.

ALGEBRA OF A GROUP. The algebra of polynomials, with coefficients in a field K , of the elements a_1, a_2, \dots, a_n of a finite group G , where if

$$x = k_1a_1 + k_2a_2 + \dots + k_na_n$$

$$y = k'_1a_1 + k'_2a_2 + \dots + k'_na_n,$$

then

$$x + y = (k_1 + k'_1)a_1 + \dots + (k_n + k'_n)a_n$$

and $xy = l_1a_1 + l_2a_2 + \dots + l_na_n$,

where x and y have been multiplied together as polynomials, the product a_ia_j being determined by the law of multiplication of the group.

ALGORITHM. A term derived from the word algorism, which meant the art of computing with Arabic numerals. The term algorithm is now used (1) to denote any method of computation, whether algebraic or numerical, or (2) any method of computation consisting of a comparatively small number of steps; the steps to be taken in a preassigned order and usually involving iteration, which are specifically adapted to the solution of a problem of some particular type. The best known example is Euclid's algorithm for finding the highest common factor of two given numbers. Algorithms play an important role in the theory of computing machines.

ALIASING. The introduction of error in the **Fourier analysis** of a discrete sampling from continuous data, by which frequencies too high to be analyzed with the given sampling interval contribute to the amplitude of lower frequencies. (See also **Nyquist frequency**.)

ALIENATION. A term used, mainly in psychology, to denote the departure of random variables from dependence on one another. If r is the coefficient of **correlation** between two variables x and y , their *coefficient of alienation* is defined as $\sqrt{(1 - r^2)}$. Likewise, if x and y

are p -way and q -way vectors with dispersion matrices v_1 and v_2 and if v is the dispersion matrix of all $(p + q)$ variables together, the vector alienation coefficient is defined as $\sqrt{\{|v|/|v_1||v_2|\}}$.

ALLOWED TRANSITION. See transition, allowed; selection rules; transition, forbidden.

ALPHA DISINTEGRATION (ALPHA DECAY). The process whereby a nucleus disintegrates spontaneously with the emission of an alpha particle (see radioactivity). The alpha particle is assumed to be formed at the instant of emission at the surface of the nucleus.

ALPHA DISINTEGRATION ENERGY. (1) The energy of disintegration of an α -disintegration process, equal to the value $Q_\alpha = E_\alpha + E_R$ where Q_α is the α -disintegration energy, E_α is the kinetic energy of the α -particle, and E_R is the recoil kinetic energy of the product atom. If M_α and M_R are the masses of the α -particle and the recoil atom, respectively, then

$$E_\alpha = \frac{M_R}{M_R + M_\alpha} Q_\alpha$$

and

$$E_R = \frac{M_\alpha}{M_R + M_\alpha} Q_\alpha.$$

(2) Often, the ground state α -disintegration energy, which is the total energy evolved, including the energy of gamma and associated radiations, when the disintegrating and product nuclei are in their ground states: Q_α .

ALPHA-PARTICLE MODEL OF NUCLEI.

The nuclei of mass $A = 4n$ and charge $Z = 2n$, where n is an integer, are known to have binding energies somewhat in excess of their neighbors, and approximately equal to the total binding energy of n free alpha-particles, or helium nuclei. For example, the binding energy of O^{16} is 127 Mev, whereas that of 4 free α -particles is 113 Mev. Consequently a model has been proposed whereby these nuclei are supposed to consist of more or less stable α -particles as sub-units bound together by weak residual attraction forces similar to the van de Waals forces in molecules. The theory does not stand up to detailed investigation.

ALTERNATING CURRENT (A-C). Current in which the charge-flow periodically reverses, as opposed to direct current, and whose average value is zero.

ALTERNATING CURRENT CIRCUIT. Suppose that any element of a circuit, such as a straight wire, a coil, or a capacitor, carries a current that varies periodically with the time t . A simple example occurs when the current I can be expressed as

$$I = I_o \cos \omega t = I_o \cos 2\pi ft,$$

where I_o is a constant amplitude, f is the frequency, and ω is the angular frequency. The potential difference required to maintain this current is then also periodic and can be shown to be

$$V = V_o \cos (\omega t + \phi),$$

where V_o is the voltage amplitude and ϕ is the phase angle. Further,

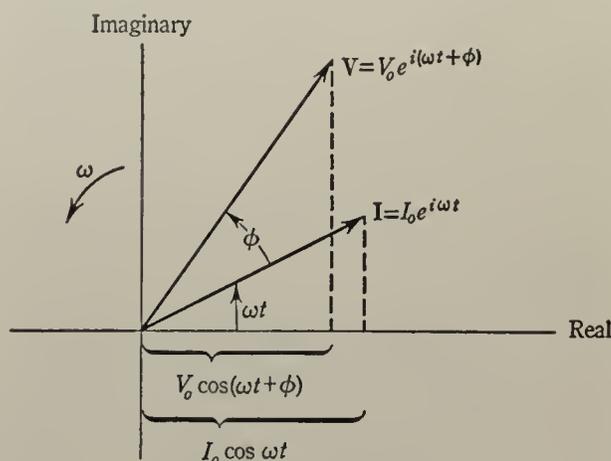
$$V_o = I_o Z,$$

where Z is the magnitude of a quantity known as the impedance, and

$$\tan \phi = X/R$$

where X is the reactance and R is the effective resistance.

It is often convenient to express the current through and voltage across the circuit element by a **complex number**, $\mathbf{I} = I_o e^{i\omega t}$ or $\mathbf{V} = V_o e^{i(\omega t + \phi)}$ and to consider the instantaneous current or voltage to the real part of the number. The situation can then be represented on an Argand diagram as shown in the figure. The



Argand diagram for an alternating current circuit.

vectors \mathbf{I} and \mathbf{V} are then each of constant length, I_o and V_o respectively, and both rotate about the origin with an angular velocity ω , so that the phase angle ϕ between them remains constant. Then one can write

$$\mathbf{V} = \mathbf{Z}\mathbf{I} = (R + iX)\mathbf{I},$$

where \mathbf{Z} is the (complex) impedance and $i = \sqrt{-1}$.

The magnitude of \mathbf{Z} is then

$$Z = (\mathbf{Z}\mathbf{Z}^*)^{1/2} = (R^2 + X^2)^{1/2},$$

\mathbf{Z}^* being the **complex conjugate** of \mathbf{Z} .

For a pure **resistance** R :

$$\mathbf{Z}_R = Z_R = R; \quad \phi_R = 0;$$

for a pure **inductance** L :

$$\mathbf{Z}_L = iZ_L = i\omega L; \quad \phi_L = 90^\circ;$$

and for a pure **capacitance** C :

$$\mathbf{Z}_C = iZ_C = -i/\omega C; \quad \phi_C = -90^\circ.$$

Any real circuit element can either be approximated (at a particular frequency) by one of the ideal elements mentioned above or can be considered to be built up of a combination of such elements. Thus a coil of wire, such as a solenoid, can often be considered to consist of a resistance R in series with an inductance L . When it carries an alternating current \mathbf{I} , the potential difference across its terminals is

$$\mathbf{V} = \mathbf{I}R + i\mathbf{I}X_L = \mathbf{I}Z,$$

so that

$$\mathbf{Z} = R + i\omega L \quad \text{and} \quad Z = (R^2 + \omega^2 L^2)^{1/2}$$

and

$$\tan \phi = L/R.$$

Similarly, a capacitor or condenser can be approximated as a pure capacitance C in parallel with a leakage resistance R . Then, if \mathbf{I}_R is the current through the resistance and \mathbf{I}_C that through the capacitance,

$$\mathbf{V} = \mathbf{I}_R R = i\mathbf{I}_C X_C = -i\mathbf{I}_0/\omega C.$$

Taking the total current $\mathbf{I} = \mathbf{I}_R + \mathbf{I}_C$, it is found that

$$\mathbf{Z} = \mathbf{V}/\mathbf{I} = R(1 - i\omega CR)/(1 + \omega^2 C^2 R^2),$$

$$Z = R/(1 + \omega^2 C^2 R^2)^{1/2}$$

and

$$\tan \phi = -\omega CR.$$

Complex circuits may be built up of combinations of elements of the type discussed and the impedance of the combination found by the applications of the **Kirchhoff laws** of networks to the instantaneous currents through and potential drops across the elements. When the current variation is more complicated than

that expressed by the simple sinusoidal function, e.g., if it includes a third harmonic:

$$I = I_1 e^{i\omega t} + I_3 e^{3i\omega t},$$

a separate solution may be carried out for each value of ω and the real parts of resulting instantaneous currents and voltages may be added.

At any instant, energy is supplied to a circuit element at the rate

$$\begin{aligned} P_i &= IV = [I_o \cos \omega t][V_o \cos (\omega t + \phi)] \\ &= I_o V_o (\cos^2 \omega t \cos \phi - \sin \omega t \cos \omega t \sin \phi). \end{aligned}$$

This instantaneous power may be either positive or negative; when $\phi \neq 0$ power is delivered to the circuit during part of the cycle and returned to the source during other parts. The net energy delivered during any single cycle is

$$\begin{aligned} E &= \int_0^{2\pi/\omega} P_i dt = I_o V_o \left[\cos \phi \int_0^{2\pi/\omega} \cos^2 \omega t dt \right. \\ &\quad \left. + \sin \phi \int_0^{2\pi/\omega} \sin \omega t \cos \omega t dt \right]. \end{aligned}$$

The average power is $P = Ef + E\omega/2\pi$. As the second integral above is zero, the only contribution comes from the first integral, which has the value π/ω . Hence

$$P = \frac{1}{2} I_o V_o \cos \phi = I_{rms} V_{rms} \cos \phi,$$

where I_{rms} and V_{rms} are respectively the root-mean-square (or effective) current and the root-mean-square voltage. The factor $\cos \phi$, giving the ratio of the power to the product of the effective current and effective voltage, is called the *power factor*. (See also **multiphase circuits**.)

ALTERNATING GRADIENT FOCUSING.

The qualitative concept involved in alternating gradient focusing can best be seen by considering a charged particle moving in the z -direction of a Cartesian coordinate system. If from $z = 0$ to $z = A$ there is a magnetic flux density \mathbf{B} , whose x and y -components are respectively $B_o x$ and $B_o y$ where B_o is a constant, and if from $z = A$ to $z = 2A$ there is a flux density with the x and y -components respectively, $-B_o x$ and $-B_o y$, then the particle which is moving along the z -direction will obtain a velocity directed towards the z -axis by the time it reaches $z = 2A$. In other words, a sequence of alternating gradient fields will tend to focus the particle towards the axis.

This principle is used in high energy accelerators to prevent spreading of the beam.

ALTERNATING GROUP. The group of even permutations of n objects.

ALTERNATING SERIES. A series whose terms are alternately positive and negative. The alternating series test for convergence states that if the terms of an alternating series decrease monotonically in absolute value to zero, then the series converges.

ALTERNATING TENSOR. See **tensor, alternating**.

ALTERNATION OF MULTIPLICITIES. See **multiplicities, alternation of**.

ALTERNATIVE (THEOREM OF THE, FOR LINEAR EQUATIONS). The system

of n linear equations in n unknowns $\sum_{k=1}^n a_{ik}x_k =$

y_i , ($i = 1, 2, \dots, n$) with fixed a_{ij} , either has a unique solution for every set of constants y_i or else the corresponding homogeneous equations (with $y_i \equiv 0$) have a positive number r , with $0 < r \leq n$, of linearly **independent** solutions $x_{11}, x_{12}, \dots, x_{1n}, \dots, x_{r1}, x_{r2}, \dots, x_{rn}$, in which

case the transposed equations $\sum_{k=1}^n a_{ki}z_k = 0$,

also have r linearly independent solutions $z_{11}, z_{12}, \dots, z_{1n}, \dots, z_{r1}, z_{r2}, \dots, z_{rn}$ and the original (non-homogeneous) equations are solvable only if y_1, y_2, \dots, y_n is orthogonal to every

$z_{i1} \dots z_{in}$, that is only if $\sum_{j=1}^n y_j z_{ij} = 0$, $i =$

$1, 2, \dots, r$ the solution then being uniquely determined, except that any solution of the homogeneous equations may be added to it. (Compare **Fredholm theorem for integral equations**.)

ALTITUDE (ASTRONOMIC). The coordinate of a celestial object, on the horizontal system of coordinates, that is measured from the horizon along the vertical circle through the object through 90° . The determination of altitude is the fundamental observation for the determination of a line of position on which the ship (either air or sea borne) is located. The measurement of altitude of the pole star by sailors, for determination of the latitude of their ship, has been referred to in literature for at least 2500 years. The altitude of an object is determined by navigators by the use

of a sextant, either of the mariner's or the bubble type. When the mariner's sextant is used, the altitude of the lower limb of an astronomic object is measured from the apparent horizon. To obtain the astronomic altitude the observed altitude must be corrected for instrumental errors, dip of the horizon, refraction, and semi-diameter (if the altitude of the lower limb of the sun or moon was measured). If the bubble type of sextant is used, corrections must be applied for instrumental errors, refraction, and **Coriolis force**.

AMAGAT COORDINATES. Representation of the equation of state of a substance involving a family of isotherms plotted in a system of coordinates PV versus P . In the case of a perfect gas ($PV = RT$) this gives a family of straight horizontal lines; hence, in the case of a real gas, the diagram shows up clearly the deviations from perfect-gas behavior.

AMAGAT LAW. (Also known as the Leduc law.) The volume of a mixture of (perfect) gases is equal to the sum of the volumes of the components, each taken at the (total) pressure and temperature of the mixture.

AMAGAT UNIT. A unit of volume often used in the study of the equation of state for gases. It is by definition the molar volume of the gas at 0°C and 1 atmosphere. The exact value of this unit depends upon the gas considered, but is approximately equal to 2.24×10^4 cm^3/mole .

AMPERE. A unit of electrical current, abbreviated as A or amp. (1) The absolute ampere is exactly one-tenth of an **abampere**, the electromagnetic unit of current. The absolute ampere has been the legal standard of current since 1950. (2) The International ampere, the legal standard before 1950, is the steady current which must flow across a surface in order that one International **coulomb** of charge shall pass the surface during each second. 1 Int. amp = 0.999835 abs. amp.

AMPERE LAW. The term is used loosely for any one of several expressions that connect the magnetic field produced by a steady current with the magnitude of the current and the geometry of the system. In the following discussion rationalized MKSA units are used; to adapt to other systems see **electromagnetic units**.

The differential form, sometimes called the *Laplace law*, gives the field $d\mathbf{H}$ produced by a current I flowing in a differential length $d\mathbf{l}$ of a filamentary conductor as $d\mathbf{H} = I (d\mathbf{l} \times \mathbf{r}/r^3$, where \mathbf{r} is the (vector) displacement of the point at which the field is measured from the element $d\mathbf{l}$. The differential vector $d\mathbf{l}$ has the same sense as the (positive) current flow. (For meaning of $d\mathbf{l} \times \mathbf{r}$ see **vector product**.) After integration over an infinitely long straight filament, this form of the law yields the **Biot-Savart law**.

The integral, or circuital form of the law states that

$$\oint \mathbf{H} \cdot d\mathbf{s} = I,$$

where the line integral is taken around any closed path that encloses the current I . (For the meaning of $\mathbf{H} \cdot d\mathbf{s}$ see **scalar product**.) The value of this integral is often called the *magnetomotive force*. When this form of the law is combined with a **symmetry argument** it, like the differential form, leads to the Biot-Savart law.

In the course of the development of his electromagnetic theory, Maxwell modified the Ampere law to allow for the effect of **displacement currents**, as well as conductive currents.

The Biot-Savart law is sometimes called the Ampere law.

AMPERE RULE. The magnetic flux generated by a current in a wire encircles the current in the counterclockwise direction, if the current is approaching the observer. This is also referred to as the "right-hand rule," since if the thumb of the right hand is directed along the current, the fingers will curl in the direction of the magnetic flux.

AMPERE THEOREM. The magnetic field due to an electric current flowing in any circuit is equivalent at external points to that due to a simple magnetic shell, the bounding edge of which coincides with the conductor, and the strength of which is equal to the strength of the current. (See **magnetic shell, equivalent**.)

AMPERE TURN. The unit of magnetomotive force in MKSA units. The integral form of Ampere's law gives the magnetomotive force as

$$\mathcal{H} = \oint \mathbf{H} \cdot d\mathbf{s} = I = Ni,$$

where I is the total current enclosed by the line integral. This total current may result from N turns of wire each carrying a current i . Thus a magnetomotive force of 1 ampere turn may result from 1 ampere flowing in 1 turn of wire, from 0.1 ampere flowing in 10 turns of wire, etc.

AMPERE TURN PER METER. The unit of magnetic field strength, \mathbf{H} , in the MKSA system. A magnetic field strength of magnitude equal to that inside a long solenoid having N turns in a length L , where $N/L =$ one meter. (See **ampere turn**.)

AMPERIAN CURRENTS. The complete equivalence between magnetic dipoles of magnetized matter and of circulating electric currents enables one to postulate that the magnetization of matter is due to circulating currents, known as Amperian currents (or *magnetization currents*). Unlike ordinary currents, the flow of Amperian currents is persistent and involves no dissipation. Amperian currents are due to the orbital and spin magnetic moments of electrons.

AMPLIFICATION FACTOR. A measure of the relative effectiveness of voltages on the plate and grid of a vacuum tube, defined as the negative of the ratio of the infinitesimal plate voltage change necessary to counteract a given infinitesimal change in grid voltage in order to keep the plate current constant.

$$\mu \equiv \left(- \frac{\partial V_p}{\partial V_g} \right)_{i_p \text{ const.}}$$

AMPLITUDE. (1) If a complex number is represented in polar coordinates it becomes $r(\cos \theta + i \sin \theta)$ and the angle θ is the amplitude, argument, or phase of the number. (See **de Moivre theorem**.) The term is also used to designate a parameter occurring in **elliptic functions** and **integrals**. (2) The crest or maximum value of a periodic (or specifically a simple harmonic) function of space or time or, more generally, any parameter that when changed, merely represents a change in scale factor. In amplitude-modulation systems, this quantity becomes a function of time, and its instantaneous value is of importance, but it is still referred to as the amplitude.

AMPLITUDE (INITIAL). The maximum displacement from a reference point of a pe-

riodic function. The periodic function can describe the motion of a pendulum point, the motion of a point on a spring or of a point on a mechanical system which is performing oscillatory motion or it may represent the motion of a wave in a medium.

AMPLITUDE EQUATIONS, FINITE ACOUSTIC. See **finite amplitude equations (acoustic)**.

AMPLITUDE OF NORMAL CURVATURE. See **curvature, amplitude of normal**.

AMPLITUDE OF OSCILLATION. The maximum excursion from the mean position is called the amplitude, e.g., A in the **simple harmonic** oscillation $x = A \sin(\omega t + \phi)$. Also used less precisely to denote the excursion, x at any time t .

A.M.U. See **atomic mass unit**.

ANABATIC WIND. An up-slope wind, particularly noticeable on hillsides warmed by sunshine when the air in a valley is stably-stratified or when large stationary convection clouds (cumulus) are situated over the hill-tops.

ANALOG COMPUTER. A computing device in which quantities are directly represented by physical magnitudes: distances, voltages, currents, rotations, etc. Hence the mathematical operations are direct operations upon these magnitudes. Examples are slide rules, differential analyzers, and most special purpose computers, as well as wind tunnels and model basins. Analog computers are to be contrasted with **digital computers**, although hybrid forms exist. Most often, as in digital differential analyzers, in overall use the machine appears as an analog machine but contains components that operate digitally.

ANALOGOUS SYSTEMS (NETWORKS). Systems whose behavior is described by the same type of equations.

ANALOG-TO-DIGITAL CONVERTER. Any device for providing a digital representation of quantities initially represented in analog form, i.e., by physical magnitudes.

ANALYSIS. That branch of mathematics which deals with limit processes. Analysis may thus be regarded as a branch of topology.

Traditionally, the name is given to the study of functions of real or complex variables arising naturally from the differential and integral calculus.

ANALYSIS OF COVARIANCE. An extension of the **analysis of variance**. In the latter the analysis is concerned with sources of variability affecting a set of observations of a single variable y . It may be that there exist variables x (say one for simplicity) which are correlated with y but unaffected by sources of variability; and we wish to abstract the effect of x from y before analyzing the latter. For example, in an agricultural field-trial to test different fertilizers on certain plots, there may be prior differences in fertility in the plots (x) which can be measured and affect the yields (y), whereas our object is to test the effect of treatments on y irrespective of x . The object is attained by finding the **regression** of y on x , removing the effect of x and testing the residual values of y in a variance analysis. The process involves the calculation of variances of x and y and their covariance—hence the name. The procedure can be extended to many variables and exact tests of significance are available.

ANALYSIS OF VARIANCE. Basically an arithmetical technique for segregating the sources of variability affecting a set of observations. Two main types of analysis may be distinguished, and the simplest cases of each are described below.

(1) Suppose we have n observations in each of m classes and assume that the j th observation in the i th class can be written in the form

$$y_{ij} = a + b_i + c_{ij} \\ i = 1 \cdots m; \quad j = 1 \cdots n,$$

where b_i, c_{ij} are variates distributed about zero with variances σ_b^2, σ_c^2 . We can calculate a sum of squares about the overall mean, $\sum_{i,j} (y_{ij} - \bar{y})^2$, with $(mn - 1)$ **degrees of freedom** from the whole set of observations. From each class we can calculate a sum of squares about the class mean, $\sum_{i,j} (y_{ij} - \bar{y}_i)^2$, with $(n - 1)$ d.f.; the corresponding mean square will estimate σ_c^2 , and the best estimate is obtained by pooling these sums of squares and degrees of freedom within classes. In the same way we can calculate a sum of squares between

classes, with $(m - 1)$ d.f., as $n \sum_i (\bar{y}_i - \bar{y})^2$ (in practice, class totals rather than class means would be used in the computation); the **expected value** of the corresponding mean square is $n\sigma_b^2 + \sigma_c^2$. The degrees of freedom and the sums of squares between and within classes should total exactly to the total degrees of freedom and sum of squares. The computations can be conveniently set out in an analysis of variance table as shown:

Source of Variation	D.F.	S.S.	Expected M.S.	M.S.
Between classes...	$m - 1$	$n \sum_i (\bar{y}_i - \bar{y})^2$	$n\sigma_b^2 + \sigma_c^2$	
Within classes...	$m(n - 1)$	$\sum_i \sum_j (y_{ij} - \bar{y}_i)^2$	σ_c^2	
Total...	$(mn - 1)$	$\sum_{i,j} (y_{ij} - \bar{y})^2$		

If $\sigma_b^2 = 0$, the two mean squares form independent estimates of σ_c^2 and their ratio therefore follows the **F-distribution** if we assume normal distributions. An estimate of σ_b^2 is given by

$$\left(\frac{\text{M.S. between classes} - \text{M.S. within classes}}{n} \right)$$

The quantity $\sqrt{(1 - \sigma_c^2 / (\sigma_b^2 + \sigma_c^2))}$ is called the intra-class correlation; it measures the resemblance of two members of the same class as compared with two unrelated individuals.

(2) Suppose that the observations can be classified in two ways and that we can write

$$y_{ij} = a + b_i + c_j + \epsilon_{ij}$$

$$i = 1 \cdots m; \quad j = 1 \cdots n,$$

$$\sum b_i = 0, \quad \sum c_j = 0$$

where a, b_i, c_j are constants and ϵ_{ij} is a random variate with mean zero and variance σ^2 . The observations can now be set out in a two-way table, and if we denote the row and column means by \bar{y}_i, \bar{y}_j , we can calculate two sums of squares

$$n \sum_i (\bar{y}_i - \bar{y})^2, \quad m \sum_j (\bar{y}_j - \bar{y})^2$$

with $(m - 1)$ and $(n - 1)$ d.f. which measure the amount of variation between these means. Subtracting these from the total sum of squares, $\sum_{i,j} (y_{ij} - \bar{y})^2$ with $(mn - 1)$ d.f., we are left with a residual sum of squares with $(m - 1)$

$(n - 1)$ d.f.; the corresponding mean square provides an estimate of σ^2 . The analysis of variance table now takes the form

Source of Variation	D.F.	S.S.	M.S.
Difference between b 's...	$m - 1$	$n \sum_i (\bar{y}_i - \bar{y})^2$	
Difference between c 's...	$n - 1$	$m \sum_j (\bar{y}_j - \bar{y})^2$	
Residual.....	$(m - 1)(n - 1)$	by subtraction	s^2
Total.....	$(mn - 1)$	$\sum_{i,j} (y_{ij} - \bar{y})^2$	

If all the b 's are zero, the first mean square in the table provides an independent estimate of σ^2 , and an overall test of significance for differences between the b 's can thus be derived from **Fisher's distribution**; the c 's can similarly be tested by means of the second mean square. More usefully, the variance of a row mean is estimated as s^2/n and that of a column mean as s^2/m . Fiducial or confidence limits for single mean or for a **contrast** between the means can therefore be derived using the **t-distribution** with $(m - 1)(n - 1)$ d.f.

(3) The analysis of variance can usefully be applied to **regression** problems, of which (2) above is a special case. Suppose that the **normal equations** arising in the regression calculations are

$$a_{11}b_1 + a_{12}b_2 + \cdots + a_{1p}b_p = h_1$$

$$a_{21}b_1 + a_{22}b_2 + \cdots + a_{2p}b_p = h_2$$

$$a_{p1}b_1 + a_{p2}b_2 + \cdots + a_{pp}b_p = h_p$$

Then we can set up an analysis of variance in the form

Source of Variation	D.F.	S.S.	M.S.
Regression....	p	$\sum_i b_i h_i$	
Residual.....	$n - p - 1$	by subtraction	s^2
Total.....	$n - 1$	$\sum (y - \bar{y})^2$	

and the mean square in the second line estimates the variance of the deviations of the observations from their expected values. (See also **analysis of covariance**.)

ANALYTIC CONTINUATION. If the function $f_1(z)$ is analytic in a domain D_1 of the complex plane and $f_2(z)$ in a domain D_2 partly overlapping D_1 , and if $f_1(z) = f_2(z)$ in the overlapping part, then $f_2(z)$ is called the analytic continuation of $f_1(z)$. The method of obtaining $f_2(z)$ from $f_1(z)$ consists of ex-

pressing $f_1(z)$ as a power series with a certain circle of convergence; then re-arranging the series about a different center so that its new circle of convergence extends beyond the old one and so on successively.

ANALYTIC CURVE. A curve whose parametric equations, in three dimensions for example, are of the form,

$$x = x(t), \quad y = y(t), \quad z = z(t),$$

where $x(t)$, $y(t)$, $z(t)$ are analytic functions of the real variable t .

ANALYTIC FUNCTION. A function of a real or complex variable is analytic at a point if it can be represented in a neighborhood of this point by its **Taylor series** about it. A function of a complex variable z , say $w(z)$, is analytic at any point in the z -plane if it is regular there, i.e., if $w(z)$ and $\frac{d}{dz} \{w(z)\}$ are both finite and single-valued at the point. If this is true for all points on a contour or in a certain region, $w(z)$ is analytic on that contour or within that region.

If $z = x + iy$ and $w = u + iv$, where x, y, u, v are real, then $w(z)$ is analytic at any point where the **Cauchy-Riemann relations**

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} \quad \text{and} \quad \frac{\partial v}{\partial x} = -\frac{\partial u}{\partial y}$$

hold and where the partial derivatives involved are continuous.

ANAMORPHIC SYSTEM. An optical system in which the magnification is a function of direction in the image plane.

ANASTIGMAT. An optical instrument that is corrected for astigmatism and curvature of field, i.e., which satisfies **Petzval's condition**.

ANCILLARY STATISTIC. In the theory of statistical estimation, a function of the observations which adds information additional to that furnished by the statistic used as an estimate. The term was introduced by R. A. Fisher who appears, however, to have altered the meaning so that estimation with the use of an ancillary statistic is now equivalent to **conditional estimation**.

ANELASTIC MATERIAL. A material having a constitutive equation which includes non-elastic effects, such as viscoelasticity or plasticity.

ANEMOMETER LEVEL. (1) A level above the surface of the earth which is taken as the initial point of the **Ekman spiral**. Below this level is the **surface boundary layer**, above it the **Ekman layer**. (2) The height above the ground at which an anemometer is exposed; or, vaguely, the ideal height of exposure.

ANGLE CHANGE, BALANCING. A complementary procedure to **moment distribution** for solving **rigid frame** problems by successive approximations. Angle changes instead of moments are determined. Clockwise rotation is taken as positive (see **slope-deflection**).

ANGLE CHARACTERISTIC FUNCTION. If F, F' are the feet of perpendiculars from the origin onto a ray in object, image space, respectively, then the angle characteristic function T is the optical path length along the ray from F to F' . If (p, q, r) and (p', q', r') are the optical direction cosines of the ray in object and image spaces, then

$$T = T(p, q, r, p', q', r')$$

and T is simply related to the point characteristic function (see **Hamilton's characteristic**.)

$$T = V(x, y, z, x', y', z') + (px + qy + rz) - (p'x' + q'y' + r'z')$$

where (x, y, z) and (x', y', z') are the object and image points on the ray. The properties of T are thus simply obtained from the properties of V , but for many applications T is the most convenient characteristic function. In particular, there is the elimination rule for calculating the angle characteristic of a stratified medium:

Find the characteristic T_i , $i = 1, 2, \dots, n$, for each layer of the stratified medium. The angle characteristic for a pair of consecutive layers is then

$$T_{i, i+1} = T_i + T_{i+1}$$

where the intermediate variables are eliminated by means of equations expressing the fact that the sum so obtained has a stationary value with respect to arbitrary variations of the intermediate variables. Iterating this process leads to the characteristic of the stratified medium.

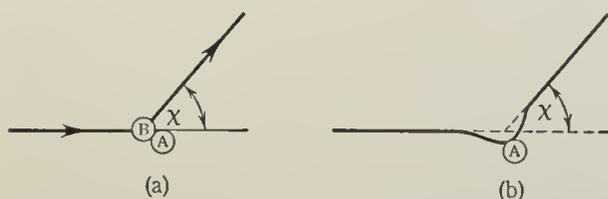
$$T_{1, 2 \dots, n} = T_1 + T_2 + \dots + T_n.$$

ANGLE OF ATTACK. See **angle of incidence**.

ANGLE OF CONTACT. See *capillarity*.

ANGLE OF CONVERGENCE. When the gaze is fixed on a point P in the binocular field of view then the angle between the rays through P and the nodal points of the eyes is the angle of convergence. (See *Vieth-Müller torus*.)

ANGLE OF DEFLECTION. The angle χ between the relative velocity vectors before and after the collision of two molecules. For hard spheres (see *hard sphere model*), χ depends on the *impact parameter* b only. For



Angle of deflection χ in the case of a collision between two hard spheres (a) and in that of a more realistic (attractive + repulsive) potential (b).

more realistic potentials, where the intermolecular forces vary continuously with the distance, it depends on both b and the relative velocity. (See figure.)

ANGLE OF DEVIATION. The angular change in the direction of a ray of light, or other electromagnetic radiation, upon entering another medium. (See *angle of incidence*.)

ANGLE OF DIFFRACTION. The angle between the direction of an incident beam of light and any resulting diffracted beam.

ANGLE OF INCIDENCE. See *incidence, angle of*.

ANGLE OF PRINCIPAL AZIMUTH. Circularly-polarized light incident on a metallic surface at the *angle of principal incidence* will be reflected as plane-polarized light. The angle between the plane of vibration of the reflected light and the plane of incidence is the angle of principal azimuth.

ANGLE OF PRINCIPAL INCIDENCE. Plane-polarized light incident at an angle upon a metallic surface becomes in general elliptically-polarized. There is one particular angle of incidence, called the principal angle of incidence, for which one axis of the ellipse lies in the plane of incidence.

ANGLE OF REFLECTION. The angle between a ray of light, after a reflection at a surface, and the normal to the surface. This angle and the angle of *incidence* are in the same plane, and are equal angles.

ANGLE OF REFRACTION. The angle between a ray of light after *refraction* at an interface between two media and the normal to the interface. This angle and the angle of *incidence* are in the same plane. (See the *Snell law*.)

ANGLE OF SHEAR. See *simple shear*.

ANGLE VARIABLE. In classical mechanics, the dynamical variable w conjugate to the *action variable* J , and thus defined only for periodic motion. For such motion of frequency ν , $w = \nu t + \text{const.}$

ANGSTROM (A). The ångström, or ångström unit, named for the pioneer spectroscopist Ångström, is equal to 0.00000001 cm or 10^{-10} meter, and is therefore sometimes called the "tenth-meter." The unit so defined is the *absolute ångström*. To obviate inconvenience due to variation of spectroscopic wavelengths with air pressure, another unit, the *international ångström*, has been adopted, such that the wavelength of the red line of cadmium is exactly 6438.4696 of these units. The two units are equal only in a vacuum.

ANGSTROM COEFFICIENT. The coefficient A in Ångström's formula for the scattering coefficient for dust in the atmosphere.

$$S = A\lambda^{-B}$$

in which λ is wavelength and B depends on the particle size.

ANGSTROM, INTERNATIONAL. See *ångström*.

ANGULAR ACCELERATION. The time rate of change of *angular velocity* is a vector called angular acceleration.

ANGULAR CORRELATION IN NUCLEAR PHYSICS. Successive radiations emitted by atomic nuclei during radioactive decay or nuclear reactions are usually correlated in angle of emission. A measurement of the correlation gives information on the spins of the nuclear energy levels involved in the process.

ANGULAR DIAMETER. See *apparent diameter*.

ANGULAR DISPLACEMENT. Change in angular position. Finite angular displacement is not a vector quantity, although it has both magnitude and direction.

ANGULAR DISTRIBUTION IN NUCLEAR PHYSICS. In nuclear physics, the distribution in angle, relative to an experimentally specified direction, of the intensity of particles or photons resulting from a nuclear or an extra-nuclear process. Commonly, the specified direction is that of an incident beam, and the angular distribution is that of particles which are scattered or are the products of nuclear reactions. In such cases, the angular distribution in the **laboratory system** will depend upon the momenta and angular momenta of the incident and ejected particles. Alternatively, the specified direction might be that of an applied field, or a direction of polarization, or the direction of emission of an associated radiation. The measurement of the angular distribution of radiations emitted in a nuclear process is a common tool for investigating the spins of **nuclear energy levels**.

ANGULAR IMPULSE. The quantity

$$\sum_i \int (\mathbf{r}_i \times \mathbf{F}_i) dt,$$

the time integral of the moment about a given point. Angular momentum is conserved when the sum of the angular impulses of all forces about a fixed point is zero. In general angular impulse about a fixed point equals change in **angular momentum** about the same point.

ANGULAR MAGNIFICATION. In an axially symmetric optical system, the ratio of the tangent of the angle with the optical axis made by a ray upon emergence from the system to the tangent of the angle for the conjugate incident ray.

ANGULAR MOMENTUM. A **conserved quantity** which remains constant in an isolated system. For a single particle of mass m , rotating in a circle of radius r with an angular velocity ω (in radians/second) the angular momentum is $mr^2\omega$. For a symmetric body rotating about a fixed axis the angular momentum is

$$H = (\sum m_i r_i^2) \omega = I \omega,$$

or in vector notation, $\mathbf{H} = I\boldsymbol{\omega}$, where $\boldsymbol{\omega}$ is a vector whose direction is given by the direction

of progression of a right-handed screw. $I = \sum m_i r_i^2$ is the moment of inertia with r_i being the perpendicular distance of the i^{th} mass m_i to the axis of rotation.

For a body not rotating about a fixed axis, the situation is more complicated. The angular momentum of a single particle about a point, whether or not it be moving about this point, is in vector notation $m\mathbf{r} \times \mathbf{v} = m\mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r})$ where \mathbf{r} is its vector distance to the point, \mathbf{v} its instantaneous velocity, and $\boldsymbol{\omega}$ its instantaneous angular velocity about the point. For a rigid body, the angular momentum is then given by $H = \sum m_i r_i \times (\boldsymbol{\omega} \times r_i) = \sum m_i [r_i^2 \boldsymbol{\omega} - (r_i \cdot \boldsymbol{\omega}) r_i]$. This can be written as:

$$H_x = I_{xx}\omega_x - I_{xy}\omega_y - I_{xz}\omega_z,$$

$$H_y = -I_{yx}\omega_x + I_{yy}\omega_y - I_{yz}\omega_z,$$

$$H_z = -I_{zx}\omega_x - I_{zy}\omega_y + I_{zz}\omega_z,$$

where the I_{xx} , I_{xy} , etc., are the **moments and products of inertia**. In this case we do not in general have $\mathbf{H} = I\boldsymbol{\omega}$. The angular momentum vector thus does not have the same direction as the angular velocity vector; rather angular momentum vector is related to the angular velocity vector by a linear transformation through the moment of inertia tensor, which is defined by the nine components I_{xx} , etc.

If the principal axes are used, the products of inertia vanish and the angular momentum is given by $H_1 = I_{11}\omega_1$, $H_2 = I_{22}\omega_2$, $H_3 = I_{33}\omega_3$, where ω_1 is the component of $\boldsymbol{\omega}$ along principal axis number 1, etc.; and where I_{11} is the moment of inertia computed about axis number 1, etc. For the special case of a body rotating about a principal axis we again have $\mathbf{H} = I\boldsymbol{\omega}$.

ANGULAR MOMENTUM (ATOMIC AND NUCLEAR PHYSICS). The angular momentum, or moment of momentum, of an elementary particle or system of such particles that either (1) spins about an axis or behaves as though it spins about an axis, or (2) revolves in an orbit or behaves as though it revolves in an orbit. For an elementary particle, the angular momentum associated with axial rotation is called the *intrinsic angular momentum*, or **spin**. When a nucleus is considered as a single particle, its total angular momentum is also referred to as its spin. For an elementary particle, the component in a particular direction of both kinds of angular momentum is quantized; the quantum of spin

angular momentum is $\frac{1}{2}\hbar$, and the quantum of orbital angular momentum is \hbar .

The three Cartesian components of angular momentum are defined in quantum mechanics by the commutator relations

$$[J_x, J_y] = i\hbar J_z \quad (\text{and cyclic permutations})$$

where $2\pi\hbar$ is Planck's constant. It then follows that the square of the total angular momentum

$$\mathbf{J}^2 = J_x^2 + J_y^2 + J_z^2$$

commutes with each of the three components. Simultaneous eigenstates of \mathbf{J}^2 and any one component, say J_z , may then be constructed. They satisfy the eigenvalue equations

$$\mathbf{J}^2\phi(jm) = j(j+1)\hbar^2\phi(jm)$$

$$J_z\phi(jm) = m\hbar\phi(jm)$$

where j is integral or half-integral, and m runs in integer steps from $-j$ to $+j$. The orbital angular momentum $\mathbf{L} = \mathbf{r} \times \mathbf{p}$ satisfies these relations, provided the quantum numbers j and m are in this case integral. The half-integral eigenvalues arise from the spin motion which has to be defined directly through the commutation relations.

ANGULAR MOMENTUM BALANCE. An integral requirement for a system that is characterized by **conservation of angular momentum**. A description of the angular-momentum balance of a physical system includes: (1) a description of the partition of **absolute angular momentum** within the system; and (2) a description of the mechanism by which absolute angular momentum is transferred from one portion of the system to another and also between the system and its surroundings.

ANGULAR MOMENTUM DENSITY (ELECTROMAGNETIC FIELD). For a region free of charge and current, the angular momentum density of the electromagnetic field with respect to the origin of the radius vector \mathbf{r} is given by $\mathbf{M} = \mathbf{r} \times \mathbf{P}$, where \mathbf{P} is the electromagnetic momentum density. (See **energy-momentum tensor**.)

ANGULAR MOMENTUM, ORBITAL. The angular momentum, \mathbf{H} , associated with the motion of a particle moving in an **orbit**.

$$\mathbf{H} = m\mathbf{r} \times \mathbf{v},$$

where m is the mass of the particle, \mathbf{r} its vector displacement from the origin (fixed in an **inertial**

system) and \mathbf{v} is the velocity. (See **angular momentum**; **angular momentum (atomic and nuclear physics)**.)

ANGULAR MOMENTUM VECTORS, QUANTUM THEORETICAL ADDITION OF. Let \mathbf{A} and \mathbf{B} be two angular momentum vectors. Their magnitudes, according to quantum theory are given by $\sqrt{A(A+1)}(h/2\pi)$ and $\sqrt{B(B+1)}(h/2\pi)$, respectively [or approximately by $A(h/2\pi)$, and $B(h/2\pi)$]. Here A and B are the quantum numbers which can be integral, 0, 1, 2, \dots , or half integral, $\frac{1}{2}$, $\frac{3}{2}$, $\frac{5}{2}$, \dots . The resultant \mathbf{C} , of \mathbf{A} and \mathbf{B} , is obtained in the usual way from the parallelogram of vectors. However, \mathbf{C} is again quantized, that is to say, can only take the values $\sqrt{C(C+1)}(h/2\pi)$ [or approximately $C(h/2\pi)$], where C is integral when A and B are both integral or both half integral, but is half integral when only A or only B is half integral. Therefore, in order to obtain the quantized values of \mathbf{C} , \mathbf{A} and \mathbf{B} can be added only in certain directions relative to one another. The possible values of C are

$$C = (A + B), (A + B - 1), \\ (A + B - 2), \dots, (A - B).$$

ANGULAR NEUTRON FLUX. See **flux, neutron, angular**.

ANGULAR RESOLVING POWER. See **resolving power**.

ANGULAR VELOCITY. A quantity relating to rotational motion. Its instantaneous value is defined as the vector, whose magnitude is the time rate of change of the angle θ rotated through, for example, $d\theta/dt$, and whose direction is arbitrarily defined as that direction of the rotation axis for which the rotation is clockwise. The usual symbol is ω or Ω .

The concept of angular velocity is most useful in the case of rigid body motion. If a rigid body rotates about a fixed axis and the position vector of any point P with respect to any point on the axis as origin is \mathbf{r} , the velocity $\dot{\mathbf{r}}$ of P relative to this origin is $\dot{\mathbf{r}} = \boldsymbol{\omega} \times \mathbf{r}$, where $\boldsymbol{\omega}$ is the instantaneous vector angular velocity. This indeed may serve as a definition of $\boldsymbol{\omega}$.

The average angular velocity may be defined as the ratio of the angular displacement divided by the time. In general, however, this is not a vector, since a finite angular dis-

placement is not a vector. The instantaneous angular velocity is more widely used.

Angular velocities, like linear velocities, are vectorially added, for example, if a top is spinning about an axis which is simultaneously being tipped over toward the table, the resultant angular velocity is the vector sum of the angular velocities of spin and of tipping. (This enters into the theory of **precession**.) The derivatives of the **Eulerian angles** are sometimes very useful in describing the angular motion of a rigid body which has components of angular velocity about all its principal axes.

ANGULAR VELOCITY, EFFECTIVE. The effective angular velocity at a point is the root mean square value of the instantaneous **angular velocity** over a complete cycle at the point. The unit is the radian per second.

ANHARMONICITY. (1) The term mechanical anharmonicity refers to a mechanical vibration in which the **restoring force** acting on the system does not vary linearly with the displacement of the system from its **equilibrium position**. (2) A phenomenon to which the term electrical anharmonicity is applied occurs in the calculation of the intensities of infrared bands. This phenomenon is a departure of the variation of **dipole moment** with internuclear distance from a strictly linear relationship.

ANHARMONIC OSCILLATOR. See **oscillator, anharmonic**.

ANHARMONIC TERMS. Terms in the expressions for the potential energy of a molecule or a crystal which depend on the third and higher powers of the displacements of the nuclei.

ANIONS. See **electroneutrality**.

ANISOTROPIC MATERIAL. See **aeotropic material**.

ANISOTROPY ENERGY. The anisotropy energy, or the magneto-crystalline energy, as it is sometimes called, of a ferromagnetic crystal, is the excess energy required to magnetize a crystal along the *hard* directions of magnetization over the energy required to magnetize it along the *easy* directions. In a cubic crystal such as iron, it is a function of the direction cosines α_1 , α_2 , α_3 , which the direction of magnetization makes with the cubic axes. Sym-

metry considerations show that only even powers of each α are involved, and the anisotropy energy f_k can be expressed by the series

$$f_k = K_1(\alpha_1^2\alpha_2^2 + \alpha_2^2\alpha_3^2 + \alpha_3^2\alpha_1^2) \\ + K_2\alpha_1^2\alpha_2^2\alpha_3^2 + \text{terms of higher degree}$$

It is found that the sum of the first two terms gives an adequate description of the behavior in most cases, and that K_1 and K_2 are material properties which depend on temperature.

ANISOTROPY, MAGNETIC. See **magnetic anisotropy**.

ANNIHILATION. See **creation and annihilation operators**.

ANNIHILATION OPERATOR. See **creation and annihilation operators**.

ANNULUS. The part of the plane lying between two concentric circles.

ANOMALISTIC YEAR. See **year**.

ANOMALOUS DISPERSION. An abrupt change in the dispersion $dn/d\lambda$ of the index of refraction n as a function of wavelength λ . Such changes occur in the neighborhood of an absorption band of the spectrum.

ANOMALOUS MAGNETIC MOMENT FOR A SPIN $\frac{1}{2}$ PARTICLE. The **Dirac theory** for a spin $\frac{1}{2}$ charged particle predicts that such a particle should have a magnetic moment equal to

$$\mu_0 = e\hbar/2mc$$

where m is the mass and e the charge of the particle. The difference $\Delta\mu$ between the observed magnetic moment and the value $e\hbar/2mc$ is called the anomalous magnetic moment of the particle. The anomalous moment arises from the interaction of the particle with the vacuum fluctuations of the (quantized) electromagnetic field and with those of any other field with which the particle can interact.

For the case of the electron where only the interaction with the quantized electromagnetic field need be considered, one can treat the modifications of the value of the magnetic moment from the Dirac value by the application of **covariant perturbation theory**. Up to the present the magnetic moment of the electron has been computed to fourth order in the coupling constant,

$$\alpha = \frac{e^2}{4\pi\hbar c} = \frac{1}{137}$$

with the result that

$$\begin{aligned} \Delta\mu &= \left(\frac{\alpha}{2\pi} - 0.328 \frac{\alpha^2}{\pi^2} \right) \frac{e\hbar}{2mc} \\ &= 0.0011596 \frac{e\hbar}{2mc}. \end{aligned}$$

The best experimental value of $\Delta\mu$, as obtained from a measurement of the g factor of free electrons is

$$\Delta\mu_{exp} = (0.001162 \pm 0.000004) \frac{e\hbar}{2mc}$$

in good agreement with the theoretical value.

For the μ -meson there is a contribution to the theoretical value of the anomalous magnetic moment due to its interaction with vacuum fluctuations of the electron-positron field (vacuum polarization effect due to virtual electron-positron creation and annihilation).

The magnitude of this contribution is $\frac{\alpha^2}{\pi^2} \times 1.08\mu_0$ so that the theoretically predicted μ -meson anomalous moment is

$$\begin{aligned} \Delta\mu_\mu &= \left(0.50 \frac{\alpha}{\pi} + 0.75 \frac{\alpha^2}{\pi^2} \right) \mu_0 \\ &= 0.0011654\mu_0. \end{aligned}$$

The best experimental value of the anomalous magnetic moment of the μ -meson at present is 0.0015 ± 0.0006 . The values of the magnetic moment of the neutron and proton are

$$\begin{aligned} \mu_p &= 2.7896 \frac{e\hbar}{2Mc} \\ \mu_n &= 1.9103 \frac{e\hbar}{2Mc} \end{aligned}$$

where M is the proton mass. Their anomalous moment is thus

$$\begin{aligned} \Delta\mu_p &= 1.7896 \frac{e\hbar}{2Mc} \\ \Delta\mu_n &= -1.9103 \frac{e\hbar}{2Mc}. \end{aligned}$$

Due to the largeness of the coupling constant characterizing the interaction of nucleons with mesons, valid and reliable theoretical

computations of the expected anomalous moments of the nucleons based on pseudoscalar meson theory are very difficult to perform and to date no satisfactory calculation has been performed.

No data is as yet available on the magnetic moments of the hyperons.

We next briefly illustrate the type of Feynman diagrams which contribute to the anomalous magnetic moment. To second order in e , the electron's anomalous magnetic moment can be computed from the three Feynman diagrams in Figure 1,

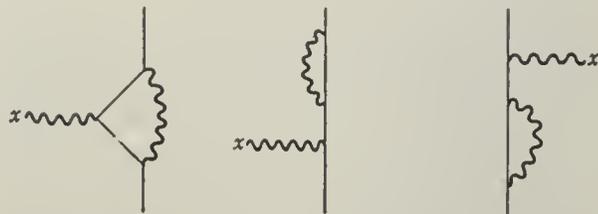


Fig. 1.

where the wavy line with an x denotes an interaction with an external electromagnetic field supposed weak enough to allow a perturbation treatment of its effects. To fourth order in e , but still only considering one interaction with the external field, the diagrams in Figure 2 contribute

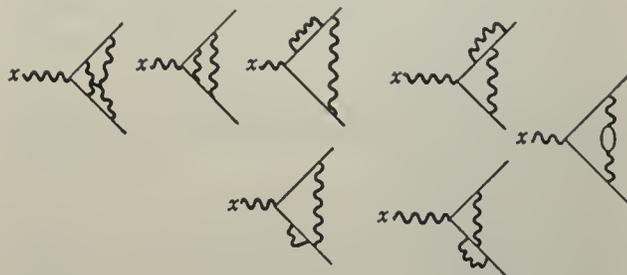


Fig. 2.

For the μ -mesonic calculation all the above diagrams will occur. There is, moreover, the diagram (Figure 3) indicated below (we denote μ -meson lines by double lines) wherein the vacuum polarization effect is due to electrons. This term involves the ratio of the



Fig. 3.

electron mass to that of the μ -meson logarithmically and gives rise to the following contribution to the μ -meson anomalous moment:

$$\delta\mu_{EVP} = \frac{\alpha^2}{\pi^2} \left[\frac{1}{3} \ln \frac{m_\mu}{2m_e} + \frac{1}{3} \log 2 - \frac{2.5}{3.6} \right]$$

$$+ \text{terms of order } \frac{\alpha^2 m_e}{\pi^2 m_\mu}$$

$$\approx \frac{\alpha^2}{\pi^2} 1.08.$$

ANOMALY, TRUE AND ECCENTRIC. In theory Kepler's second law should predict the position of an object moving about the sun in an orbit, provided that one knows the semi-major axis, a , the eccentricity, e , the time that the object was at perihelion, τ , and the orbital period T . Let P represent the position of the object at the time t . The radius vector sweeps out the area ASP in the time $(t - \tau)$ and, by the second law of Kepler, the proportionality relation is

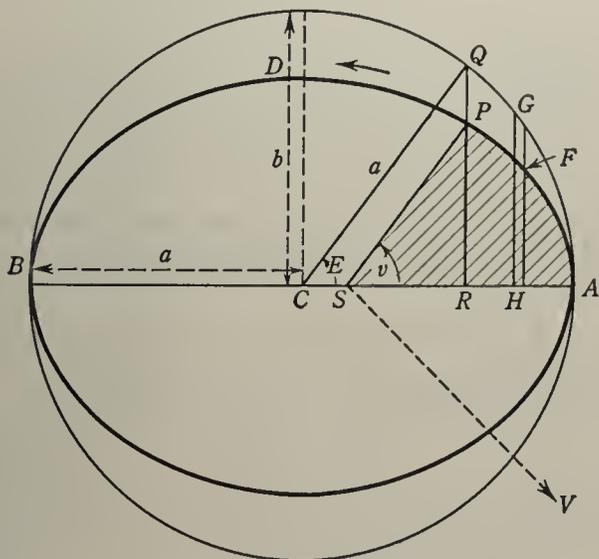
Area ASP :Area of ellipse :: $(t - \tau)$: T
or

$$\text{Area } ASP = \frac{\pi ab}{T} (t - \tau)$$

Introducing the angular motion $n = \frac{2\pi}{T}$

$$\text{Area } ASP = \frac{1}{2} nab(t - \tau)$$

in which $b^2 = a^2(1 - e^2)$. This equation apparently gives the desired position, but in practice it is very inconvenient to use.



Anomaly, true and eccentric.

From the figure: The radius vector makes an angle v with the major axis that is known as the true anomaly. It is linked to the orbital elements since $v = \theta - \omega$.

Now draw a circle of radius, a , centered on C . The line RP , perpendicular to the major axis, is extended to the circle of construction and meets it in Q . The angle E at the center, is known as the eccentric anomaly, E .

Since the orbit is an ellipse, $PR:QR::b:a$ and it may be shown that $r \cos v = a (\cos E - e)$ and

$$r = a(1 - \cos E)$$

and this equation gives the radius vector in terms of the eccentric anomaly E .

Further trigonometric analysis will result in an equation

$$\tan \frac{v}{2} = \sqrt{\frac{1+e}{1-e}} \tan \frac{E}{2}$$

which expresses the true anomaly, v , in terms of the eccentric anomaly E .

Kepler's Equation. From the definition of mean angular motion, $n = \frac{2\pi}{T}$, one defines the mean anomaly M as $M = n(t - \tau)$. Also, the area $SPA = \frac{1}{2}(abM)$. We wish to express this area in terms of the eccentric anomaly, E . The shaded area (in figure) is equal to area PSR plus the area RPA . The area PSR may be shown to be equal to $\frac{1}{2}ab \sin E (\cos E - e)$. The area of RPA is computed as follows: It is bounded by the elliptic arc PFA . Divide it into strips perpendicular to AB , and extend the strips to the circle on AB as a diameter.

Consider the strip FH . Since $FH = \frac{b}{a} GH$, the sum of all of the strips forming the area RPA is equal to $\frac{b}{a}$ times the sum of all the strips forming the area QRA and the area $RPA = \frac{b}{a} QRA$.

The area of QRA is the area of CQA minus the area of QCR . The angle $QCA = \frac{1}{2}a^2E$ and the area of the triangle QCR is $\frac{1}{2}a^2 \sin E \cos E$. One may now determine the area $RPA = \frac{1}{2}ab(E - \sin E \cos E)$. Add RPA to the area of PSR , above, and the result is

$$SPA = \frac{1}{2}ab(E - e \sin E)$$

whence

$$E - e \sin E = M \equiv n(t - \tau)$$

which is Kepler's equation.

Kepler's equation is thus a relation between the eccentric anomaly, E , and the

mean anomaly, M . It should be noted that both E and M are expressed in circular measure. Solutions of this equation for orbit computers are found in many computer tables.

ANTENNA. Any structure used to couple a transmitter or receiver to a medium in which electromagnetic waves can propagate. These structures range in form from simple straight wires, used either singly or in arrays, to quasi-optical devices employing metallic mirrors or dielectric lenses.

ANTENNA RESISTANCE. The quotient of the power supplied to the entire antenna circuit by the square of the effective antenna current referred to a specific point. Contributions to the antenna resistance come from such sources as **radiation resistance**, ground resistance, radio frequency resistance of conductors in the antenna circuit, and equivalent resistance due to corona, eddy currents, insulator leakage and dielectric power loss.

ANTENNA RESONANT FREQUENCY. That frequency or frequencies at which the antenna appears as a pure resistance.

ANTIBONDING ELECTRONS. See **molecular orbital theory of valence**.

ANTIBONDING ORBITAL. See **orbital, bonding**.

ANTICLASTIC CURVATURE. See **curvature, anticlastic**.

ANTICLASTIC SURFACE. A surface or portion of a surface on which the two principal radii of **curvature** at each point have opposite signs. Also called *surface of negative total curvature*.

ANTICOMMUTATION. The anticommutator of two linear operators A and B is the quantity $AB + BA$. Two operators anticommute, if $AB = -BA$. In quantum mechanics, **anticommutation rules** which give the expression for the anticommutator of two operators are sometimes of importance. The **creation** and **annihilation operators** related to the description of a system of **fermions** satisfy simple anticommutation rules.

ANTICOMMUTATION RULES. The commutation rules satisfied by the **creation** and **destruction** operators for particles obeying Fermi-Dirac statistics. Let b_n and b_n^* be the

destruction and creation operator for a (Fermi-Dirac) particle in the (one-particle) state characterized by the observables n . The Jordan-Wigner anticommutation rules then state that

$$[b_n, b_m]_+ = [b_n^*, b_m^*]_+ = 0 \quad (1)$$

$$[b_n, b_m^*]_+ = \delta_{nm} \quad (2)$$

where $[A, B]_+ = AB + BA$. (3)

The anticommutator $[A, B]_+$ is sometimes also denoted by $\{A, B\}$.

Consider the case of the spin $\frac{1}{2}$ particles. In a quantum-field theoretic description, a system of free spin $\frac{1}{2}$ particles of mass m is described by a field operator $\psi(x)$ which is a four component spinor operator satisfying

$$(-i\gamma^\mu \partial_\mu + m)\psi(x) = 0 \quad (4)$$

where $\gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\mu = 2g^{\mu\nu}$, γ^0 is hermitian and γ^j ($j = 1, 2, 3$) anti-hermitian. Let f_n , $n = 1, 2 \dots$, be any infinite set of positive energy solutions of the Dirac equation $(-i\gamma^\mu \partial_\mu + m)f_n(x) = 0$ normalized and orthogonal in the scalar product

$$(f, g) = \int \tilde{f}(x) \gamma^\mu g(x) d\sigma_\mu(x) \quad (5)$$

($\tilde{f} = f^* \gamma^0$). One then can define the destruction operator as follows:

$$a_j = \int \tilde{f}_j(x) \gamma_\mu \psi(x) d\sigma^\mu(x). \quad (6)$$

If the field is neutral (self-charge conjugate, i.e., $\psi^C(x) = \psi(x)$ where $\psi^C(x) = C\psi^*(x)$ and C such that $C^{-1}\gamma^\mu C = -\gamma^{\mu T}$, $C^*C = 1$, $C^T = -C$) then (6) defines all the destruction operators that have to be defined. If the field is charged, the destruction operators for the **anti-particles** are defined by the equation

$$b_j = \int \tilde{f}_j(x) \gamma_\mu \psi^C(x) d\sigma^\mu(x), \quad j = 0, 1, 2 \dots \quad (7)$$

The commutation rules of a_i and b_i are

$$a_j a_k + a_k a_j = 0 \quad (8)$$

$$a_j a_k^* + a_k^* a_j = \delta_{jk} \quad j, k = 1, 2 \dots \quad (9)$$

$$a_j b_k + b_k a_j = a_j^* b_k^* + b_k^* a_j^* = 0 \quad (10)$$

$$b_j b_k + b_k b_j = 0 = b_j^* b_k^* + b_k^* b_j^* \quad (11)$$

$$a_j b_k^* + b_k^* a_j = a_j^* b_k + b_k a_j^* = 0. \quad (12)$$

One notes that the b_j behave like additional a 's associated with the charge conjugate wave

functions. There is therefore no loss of generality in considering the set $a_j, j = 1, 2, \dots$, satisfying (10) and (11). Now define the operator of "the number of particles in state j " by the equation

$$N_j = a_j^* a_j \quad j = 1, 2.$$

It follows immediately from the anticommutation rules that

$$N_j^2 = N_j \quad \text{and} \quad [N_j, N_i] = 0$$

so that the possible occupation numbers for the states can only be one or zero, in accordance with the Pauli principle. It can be demonstrated that every discrete representation of the commutation rules (10), (11) by bounded operators is a direct sum of irreducible representations. The unitarily inequivalent irreducible representations are \aleph_1 in number. For only one of these representations does a nonparticle state (i.e., a state $|0\rangle$ for which $a_j|0\rangle = 0$ for all j) and an operator of the total number of particles, $\sum_{j=1}^{\infty} N_j$, exist.

ANTICYCLONIC MOTION. Circulatory motion relative to the earth in the direction opposite to the vertical component of the earth's rotation; clockwise when viewed from above in the northern hemisphere. Such motion is produced in the atmosphere by horizontal divergence of air at rest, the vortex lines then being compressed vertically.

ANTICYCLONIC VORTICITY. Vorticity relative to the earth in an anticyclonic direction, i.e., in a direction opposite to the vertical component of the earth's rotation.

ANTIDERIVATIVE (OF A FUNCTION). A function whose derivative is the given function.

ANTIFERROELECTRICITY. A dielectric phenomenon in which neighboring lines of spontaneously polarized ions are aligned in anti-parallel directions.

ANTIFERROMAGNETISM. In the Heisenberg theory of ferromagnetism, the spins of electrons on neighboring atoms depends on the sign of the exchange integral. When this integral is positive, ferromagnetism is observed; when it is negative and sufficiently great in magnitude, an anti-parallel arrangement of neighboring spins occurs, and this is

interpreted as leading to the phenomenon of antiferromagnetism. The temperature T_c , at which the susceptibility is a maximum, is referred to as the Curie temperature, or sometimes as the Néel temperature. Simple models of antiferromagnetism lead to the relation

$$\chi_0 = \frac{2}{3}\chi_c$$

where χ_0 is the susceptibility at $T = 0$, and χ_c is the susceptibility at T_c . The relation derived for the susceptibility χ at temperature $T > T_c$ is

$$\chi = \frac{c}{T + aT_c}$$

where c is a constant and, if only coupling between nearest neighbors is considered, a is unity.

ANTI-LINEAR OPERATOR. An operator, T , mapping any linear manifold \mathfrak{M} of a Hilbert space \mathfrak{H} onto another linear manifold $\mathfrak{N} \subseteq \mathfrak{H}$ is called anti-linear if the following relation holds for any vectors f, g , of \mathfrak{M} and complex numbers λ, μ

$$T(\lambda f + \mu g) = \bar{\lambda} T f + \bar{\mu} T g$$

where $\bar{\mu}$ denotes the complex conjugate of μ . Multiplication of an anti-linear operator by a linear operator yields an anti-linear operator, whereas the product of two anti-linear operators is linear. If \mathfrak{M} is dense in \mathfrak{H} , one can define the antihermitian conjugate operator T^\dagger by

$$\begin{aligned} (T f, g) &= (T^\dagger g, f) \\ &= \overline{(f, T^\dagger g)} \end{aligned}$$

ANTILOGARITHM (OF A NUMBER). A number whose logarithm is the given number.

ANTI-NODAL POINTS. In Gaussian optics the anti-nodal points, or negative nodal points, are conjugate points for which the lateral magnification is $(-1)^{\nu} n/n'$ where n, n' are the refractive indices of object and image space, and ν is the number of reflections in the optical system.

ANTIPARTICLES. Every fermion particle which has been discovered thus far seems to have the property that it is a member of a family: together with this particle of mass m , spin $1/2$, charge e , and magnetic moment μ , there exists an "anti-particle" also having a mass m , spin $1/2$, but a charge $-e$, and mag-

netic moment $-\mu$. (This statement has been verified for the case of neutrinos, electrons, μ -mesons (the leptons) and of the protons, neutrons and Λ -particles. To each such fermion family there corresponds in the present formulation of quantum field theory, a **spinor** field operator. This spinor field operator describes simultaneously the two kinds of particles with electric charge and magnetic moment of opposite sign. This is displayed by the decomposition of the (free) field operator in terms of the creation and annihilation operator for the particle and antiparticle

$$\psi(x) = \int \frac{d^3p}{p_0} \left(\frac{m}{E_p}\right)^{1/2} \sum_r \{b_r(\mathbf{p})u_+^r(\mathbf{p})e^{-ipx} + d_r^*(\mathbf{p})v_-^r(\mathbf{p})e^{ipx}\}$$

where $b_r(\mathbf{p})$ is a destruction operator for a particle of momentum \mathbf{p} and spin r and $d^*(\mathbf{p})$ a creation operator for an antiparticle of momentum \mathbf{p} and spin r . (u_+ and v_- are positive and negative energy solutions of the **Dirac equation** for momenta \mathbf{p} and $-\mathbf{p}$ respectively.) (See also **hole theory**.)

ANTI-PRINCIPAL POINTS. In **Gaussian optics** the anti-principal points, or negative principal points, are conjugate points for which the **lateral magnification** is -1 .

ANTIRESONANCE (PARALLEL IMPEDANCE). In general, a condition of maximum impedance, as results when two or more impedors are connected in parallel, under such conditions that (for resistanceless impedors), Z approaches infinity when $\omega_1^2 = 1/LC$. The term antiresonance is commonly used with suitable modifiers.

ANTI-STOKES LINES. Raman lines (see **Raman effect**) displaced toward shorter wavelengths with respect to the incident line.

ANTISYMMETRIC FUNCTION. A function which is transformed into its negative when a pair of the independent variables is interchanged.

ANTISYMMETRIC RELATION. A binary relation R is called antisymmetric (see, e.g., **lattice**) if aRb and bRa imply $a = b$.

ANTISYMMETRIC TENSOR. See **tensor, skew-symmetric**.

ANTISYMMETRIC WAVE FUNCTION. A wave function is antisymmetric if it changes sign in interchanging the variables of two particles: $\psi(x_1, x_2, \dots, x_N) = -\psi(x_2, x_1, \dots, x_N)$, where x_i stands for all the variables of the i^{th} particles. The states of a system of N fermions is described by such an antisymmetric wave function. (See also **exchange energy**.)

ANTITRIPTIC WIND. A wind in which the pressure gradient force is almost balanced by the drag at the ground, other forces and accelerations being small by comparison.

ANTI-UNITARY OPERATOR. An operator W is said to be anti-unitary if it maps the Hilbert space \mathcal{H} onto itself and obeys

$$\begin{aligned} (Wg, Wf) &= (f, g) \\ &= \overline{(Wf, Wg)} \end{aligned} \quad (\text{a})$$

for every f and g in \mathcal{H} . It follows from the definition that W is an anti-linear operator, and that it has an inverse W^{-1} , which is identical with the antihermitian conjugate operator W^\dagger . (See **time inversion operation**.)

APERTURE. (1) In general, an opening allowing the passage of radiant energy. (2) In particular for axially symmetrical optical systems, the aperture, or the aperture stop, is a circle in a plane perpendicular to the axis that just allows the passage of a minimal axial bundle of rays. The size of the aperture is specified by either the angular aperture (aperture angle) or the angle of projection. (See **aperture angle**.)

APERTURE ANGLE. The angle subtended by the radius of the **entrance pupil** of an optical instrument at an axial object point. The **projection angle** is similarly associated with the **exit pupil** and an image point.

APERTURE, EXIT. See **exit aperture**.

APLANAT. An optical system, in which the first two **Seidel aberrations** (spherical aberration and coma) are reduced to very small values and the largest remaining aberration is the off-axis astigmatism, used to flatten the field.

APLANATIC POINTS OF A SPHERE. Consider a sphere of radius R separating two

media of refractive indices n_0 and n_1 . There exist two points at distances

$$r_0 = \frac{n_0}{n_1} R, r_1 = \frac{n_1}{n_0} R,$$

from the center of the sphere so that a spherical wave diverging from either point is refracted at the surface of the sphere into a spherical wave apparently diverging from the second point. The two points are virtual **conjugate points**. (See also the definition following.)

APLANATIC SURFACE. A surface separating two media is an aplanatic surface under refraction (or reflection) if there exist two **conjugate points** (either real or virtual). The aplanatic surfaces of revolution are the **Cartesian surfaces**.

APOCHROMATIC SYSTEM. An optical system having the same focal length for at least three wavelengths. An apochromatic system is thus more highly corrected for chromatic aberration than an **achromatic system**.

APODIZATION. A modification of the amplitude transmittance of the **aperture** of an optical system for the purpose of reducing or suppressing the energy content of the diffraction rings relative to that of the central maximum or **Airy disc**.

APPARENT DIAMETER. The angle subtended, at the eye of an observer, by a diameter of an object is the apparent diameter or *angular diameter* of the object.

APPARENT MAGNITUDE. See **stellar luminosities**.

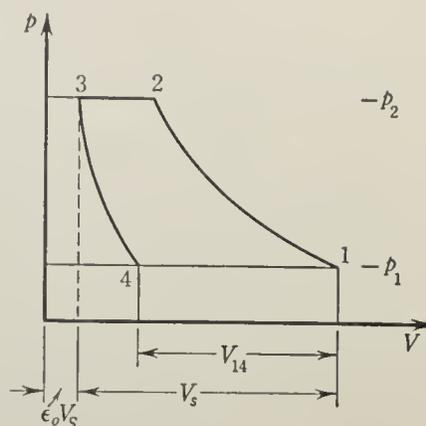
APPARENT VOLUMETRIC EFFICIENCY. Ratio of effective swept volume to total swept volume of a compressor or a four-stroke internal combustion engine. The effective swept volume V_{14} is smaller than the swept volume V_s owing to the effect of the clearance volume $\epsilon_0 V_s$. Hence the apparent volumetric efficiency

$$\mu = V_{14}/V_s$$

is a measure of the influence of clearance on delivery. Assuming that compression 12 and expansion 34 are polytropic ($pV^n = \text{const}$) it is found that the apparent volumetric efficiency is reduced to zero when the pressure ratio

$$\frac{p_2}{p_1} = \left(\frac{1}{\epsilon_0} + 1 \right)^n.$$

This puts an upper limit on the pressure ratio p_2/p_1 which can be used with a given **clearance ratio** ϵ_0 . In practice, the pressure ratio p_2/p_1 is kept well below this limit for other



reasons (lubrication, thermal stresses, saving in work through interstage cooling), and when high values of p_2/p_1 are required it is necessary to employ multi-stage compressors.

APPLICABLE SURFACES. Two surfaces are said to be applicable to each other, if there exists a one-to-one correspondence between the points on the two surfaces such that the lengths of corresponding linear elements on the two surfaces are equal.

APPLIED FORCE. See **force, applied**.

APPROXIMATION IN THE MEAN. A function $f(x)$ is approximated in the mean by the sum

$$\sum_{i=1}^n a_i u_i \quad \text{if} \quad \lim_{n \rightarrow \infty} \int (f - \sum a_i u_i)^2 w dx = 0$$

where w is the appropriate **weighting function**.

APSE LINE. The line joining the centers of two molecules at the point of closest approach during a collision. A unit vector \mathbf{k} along the apse line is frequently used, together with the initial relative velocity, to characterize the collision.

ARAGO SPOT. If an object casts a circular shadow on a plane when illuminated by a point source, then a bright region, called the Arago spot, appears in the diffraction pattern at the center of the shadow. (See **diffraction cvolute**.)

ARCH. A curved bar or truss subjected to combined compression and bending produced by forces in its plane. In a masonry or **Voussoir arch** which is unable to carry tensile stress, the **pressure line** or funicular polygon must be inside the arch from the crown or highest point to the springing or juncture with the foundation. When the moving or live load is large compared with the dead weight, appreciable binding occurs and reinforced concrete or steel arches are required.

Arches may be classified as fixed or hinged. A fixed support resists rotation and translation. It exerts thrust, transverse force and moment. A hinged support permits rotation and exerts thrust and transverse force only. A fixed arch is fixed at both supports. A two-hinged arch is hinged at both supports. A three-hinged arch has a third hinge, usually at the crown and is a statically determinate structure. When foundation conditions are unfavorable, the outward thrust of the arch may be carried by a tension member between supports. Such a structure is called a **tied arch**.

ARCH, HINGED. See **arch**.

ARCHIMEDES PRINCIPLE. The resultant upward force due to fluid pressure, on a body wholly or partially submerged in a stationary fluid, is equal to the weight of the fluid displaced by the body. (See also **stability of a floating body**.)

ARCH, TIED. See **tied arch**.

ARC-SINE DISTRIBUTION. A probability distribution which arises in the theory of recurrent events. It is of the form

$$p(x \leq x_0) = \frac{2}{\pi} \arcsin \sqrt{x_0}, \quad 0 \leq x \leq 1.$$

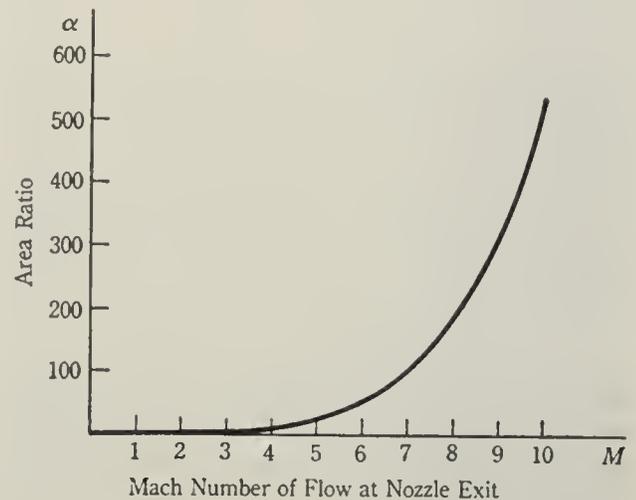
ARC SPECTRUM. The spectrum of the neutral atom of an element. (See also **spark spectrum**.)

AREA, CENTER OF. See **centroid (of a geometrical figure)**.

AREAL VELOCITY. See **velocity, areal**.

AREA RATIO. In a convergent-divergent nozzle, the ratio of the exit area to the throat area. The area ratio determines the **Mach number** of the flow at exit on condition that the flow in the throat is sonic. The diagram

gives the relation between the exit Mach number and the area ratio α for *isentropic flow* and for a perfect gas with constant specific



heats and a value of the specific heat ratio (γ) of 1.4.

AREA RATIO, NOZZLE. See **area ratio; nozzle area ratio**.

ARGAND (OR GAUSS) PLANE. A mapping of the complex numbers onto a Cartesian plane (a **Euclidean plane** with **Cartesian coordinates**) is called the Argand plane (or the Gauss plane) if the complex number $z = x + iy$ corresponds to the vector (or alternatively to its end-point) issuing from the origin with end-point x, y . The x -axis is the *real axis* and the y -axis is the *imaginary axis*. The length $r = (a^2 + b^2)^{1/2}$ of the vector gives the *modulus* or *absolute value* of z , and the angle θ between the real axis and the vector gives the *argument* of z . The resulting diagram is of great aid to visualization because it allows a simple geometric interpretation of the multiplication of complex numbers; for then, in polar form, $z = r(\cos \theta + i \sin \theta) = re^{i\theta}$, and if $\zeta = \rho(\cos \phi + i \sin \phi) = \rho e^{i\phi}$ then $z\zeta = r\rho e^{i(\theta+\phi)}$ which means that the moduli have been multiplied and the arguments have been added.

ARITHMETIC MEAN. The arithmetic mean of a number of observations x_i ($i = 1, \dots, n$) is the simple average $\Sigma x_i/n$. Thus if a and b are the first and last terms respectively of an **arithmetic progression**, then the intermediate terms are said to be arithmetic means between a and b . In particular $\frac{a+b}{2}$ is the arithmetic

mean between a and b . The arithmetic mean is commonly referred to simply as the mean.

The arithmetic mean is the most generally used **measure of location**. In samples from a normal distribution, the mean is an efficient, and indeed sufficient, estimate of the location parameter, though in other distributions other estimators may be preferable. If \bar{x} is the mean of a sample drawn from a finite population of N members, $N\bar{x}$ is an unbiased estimate of the population total.

A useful generalization of the arithmetic mean is the weighted mean. If each observation x_i has attached to it a weight w_i the weighted mean is given by $\Sigma w_i x_i / \Sigma w_i$, observations with large weights being given greater influence. If we have a set of estimates x_i of a quantity ξ , and if the variance of x_i is σ^2/w_i with w_i known, the estimate of ξ with least variance is the weighted mean of the x_i , and the variance of this estimate is $\sigma^2/\Sigma w_i$.

ARITHMETIC PROGRESSION. A sequence of numbers $a, a + d, a + 2d, \dots, a + (n - 1)d$ in which each term is greater than its predecessor by a constant d , called the common difference.

ARITHMETIC UNIT. The portion of a stored-program computer devoted to the actual execution of the commands.

ARRHENIUS EQUATION. See order of chemical reactions.

ARTICULATION POINT. See vertex, cut.

ARZELA THEOREM. Same as Ascoli theorem.

ASCOLI THEOREM. If $f_1(x), f_2(x), \dots$ is any set of functions defined and equicontinuous on a closed interval $[a, b]$, then there exists a subsequence $f_{i_1}(x), f_{i_2}(x), \dots$ which converges uniformly on $[a, b]$ to a limit function $f(x)$. The theorem can be extended directly to any number of independent variables.

ASPECT RATIO (OF WING). For a wing of span b and area S the aspect ratio is equal to b^2/S . For a wing of rectangular plan form this is simply the ratio span/chord.

ASPIRATION PSYCHROMETER. An instrument used to measure the humidity of air. It consists of two thermometers, wet-bulb and dry-bulb, over which a stream of the air whose humidity is to be measured is maintained by

a small blower. The wet-bulb thermometer has its bulb covered with an envelope which is kept soaked with moisture. The knowledge of the two different temperatures is sufficient to determine the humidity. (See psychrometry, psychrometric chart, wet-bulb temperature, Mollier H, S diagram, adiabatic cooling limit.)

ASSEMBLY. (1) A thermodynamic macroscopic system. (2) An ensemble.

ASSEMBLY STRESSES. Stresses set up during fabrication by glueing, bolting, riveting, or welding due to initial mismatch, permanent deformation or differential cooling. Stresses over large areas may be termed **reaction stresses** to distinguish them from **residual stresses** which are localized.

ASSOCIATE BERTRAND CURVES. See Bertrand curves.

ASSOCIATED POLYNOMIALS (OR FUNCTIONS). See under the respective names; e.g., for associated Legendre polynomials, see Legendre polynomials, similarly for Laguerre, etc.

ASSOCIATED TENSORS. See tensors, associated.

ASSOCIATED VECTORS. See vectors, associated.

ASSOCIATE MATRIX. See hermitian conjugate.

ASSOCIATION. In statistics this expression has a precise meaning, namely, the relationship between variables which are simply dichotomized. In a 2×2 table of the following kind

	Attribute		Totals
	A	not A	
Attribute B	a	b	$a + b$
not B	c	d	$c + d$
Totals	$a + c$	$b + d$	N

where, for example, a is the number of individuals bearing attributes A and B , there is said to be positive or negative association according as a is greater than or less than $(a + b)(c + d)/N$. Various coefficients can be constructed to measure the intensity of associa-

tion, the commonest being the *coefficient of association*

$$Q = \frac{ad - bc}{ad + bc}$$

ASSOCIATIVE LAW. An operation, denoted, for example, by \circ , obeys the associative law if $(a \circ b) \circ c = a \circ (b \circ c)$ for all a, b, c in its domain. Thus, in elementary arithmetic:

$$a + (b + c) = (a + b) + c;$$

$$(ab)c = a(bc) = abc.$$

ASTIGMATIC DIFFERENCE. The distance between the **tangential** and **sagittal foci** on a ray.

ASTIGMATISM. (1) Qualitatively, astigmatism is the image error due to the lack of coincidence of the **sagittal** and the **tangential foci** on a ray. (2) Quantitatively, the third of the five **Seidel aberration functions** is the astigmatism function of a system. The astigmatism is also measured by the **astigmatic difference**. (See also the astigmatism of a surface.)

ASTIGMATISM OF A SURFACE. The astigmatism of an optical surface separating two media with relative index of refraction n is $(n - 1)|\kappa_1 - \kappa_2|$ where κ_1, κ_2 are the principal curvatures of the surface. For a surface given explicitly, $z = z(x,y)$, the astigmatism is approximately

$$(n - 1)\sqrt{(z_{xx} - z_{yy})^2 + (2z_{xy})^2}$$

if the surface is nearly flat in the sense that $(z_x)^2, (z_y)^2, (z_x z_y)$ are negligible.

ASTRONOMICAL COORDINATE SYSTEMS. The table at the bottom of this page presents the spherical coordinate systems in common use in astronomy and navigation. The corresponding terrestrial coordinates are shown for the purpose of clarity.

ASTRONOMICAL DISTANCE UNITS. (1) *The Astronomical Unit.* The mean distance of the earth from the sun, 92,900,000 miles, is known as the astronomical unit. It is used for distances within the solar system and, in a few cases, for stellar distances.

(2) *Stellar Parallax.* The angle subtended at a star by the mean distance of the earth from the sun, i.e., one astronomical unit, is the stellar parallax of the star and is symbolized by p'' .

$$\text{Distance (in astronomical units)} = 206265/p''$$

(3) *The Parsec.* The distance at which one astronomical unit would subtend one second of arc is known as the parsec.

$$1 \text{ parsec} = 3.08 \times 10^{13} \text{ km}$$

$$= 1.92 \times 10^{13} \text{ miles.}$$

(4) *The Light Year.* The distance that would be covered by an object moving with the velocity of light in one year is defined as the light year.

$$1 \text{ light year} = 9.46 \times 10^{12} \text{ km}$$

$$= 5.88 \times 10^{12} \text{ miles.}$$

$$\text{Distance (in parsecs)} = 1''/p''$$

$$\text{Distance (in light years)} = 3''.26/p''$$

ASTRONOMICAL COORDINATE SYSTEMS

EARTH	CELESTIAL EQUATOR	HORIZON	ECLIPTIC	GALACTIC
Equator	Celestial equator	Horizon	Ecliptic	Galactic equator
Poles	Celestial poles	Zenith, nadir	Ecliptic poles	Galactic poles
Meridians of longitude	Hour circles, celestial meridians	Vertical circles	Circles of latitude	
Prime meridian, meridian of Greenwich	Hour circle of Greenwich meridian, local celestial meridian	Principal vertical circle, prime vertical circle	Circle of latitude through the Vernal equinox.	Great circle through galactic poles and intersection of galactic equator and celestial equator at about <i>SHA</i> 80°
Parallels of latitude	Parallels of declination	Parallels of altitude	Parallels of latitude	
Latitude	Declination	Altitude	Celestial latitude	Galactic latitude
Colatitude	Polar distance	Zenith distance	Celestial colatitude	Galactic colatitude
Longitude	<i>SHA, R.A., GHA, LHA, t.</i>	Azimuth, azimuth angle, amplitude	Celestial longitude	Galactic longitude

ASTRONOMICAL REFRACTION. In any type of astronomical observing the light from the external object must pass through the atmosphere of the earth. The index of refraction of the atmosphere increases from zero at the point of entry to the value at the observer. The rate of increase of this index of refraction depends upon a number of variables that are very imperfectly known at the present time.

The major effect of this atmospheric refraction is to cause objects to appear to be at a greater altitude than they would be if there were no atmosphere. The amount of this refraction is zero for an object in the zenith, and it increases with diminishing altitude. Below altitude 15° the value of the refraction is uncertain, although some tables of refraction include values for less than 15° .

As a result of a number of long series of observations at various observatories all over the earth, tables of "mean refraction" have been published. Corrections to the mean refraction for the air temperature and barometric pressure of the observer's station are published with the tables.

A fair approximation to mean refraction, corrected for the observer's air temperature and barometric pressure, may be obtained from the relationship

$$R = \frac{983B}{460 + T} \cot h_o$$

in which B is the barometric pressure in inches, T is the air temperature on the Fahrenheit scale, h_o is the observed altitude of the object, and R is the refraction in seconds of arc. The corrected altitude would be $h = h_o + R$.

Since refraction is along a vertical circle, the azimuth is not affected by refraction. To determine the effects of refraction on the celestial coordinates other than those in the altazimuth system, the astronomical spherical triangles must be solved.

ASTRONOMICAL UNIT. See astronomical distance units.

ASYMMETRIC TOP MOLECULE. See polyatomic molecules, types of.

ASYMPTOTE. The limiting position of a tangent to a curve, where the point of contact is at an infinite distance from the origin. Where there are no infinite branches, as in the

cases of the circle and the ellipse, there is no real asymptote.

Suppose the equation of a given curve can be expanded in a power series

$$y = f(x) = \sum_{k=0}^n a_k x^k + \sum_{k=1}^{\infty} b_k/x^k = S_1 + S_2.$$

Then, if $\lim_{x \rightarrow \infty} S_2 = 0$, the equation of the asymptote is $y = S_1$. If this equation is linear, the asymptote is a straight line; otherwise, it is a more complicated curve. In the linear case, the equation of the asymptote may be written as

$$y = mx + b; \quad m = \lim_{x \rightarrow \infty} f'(x);$$

$$b = \lim_{x \rightarrow \infty} [f(x) - xf'(x)].$$

For example, for the hyperbola $x^2/a^2 - y^2/b^2 = 1$, we have

$$y = \pm \frac{b}{a} \sqrt{x^2 - a^2} = \frac{b}{a} (x + \frac{1}{2}x^{-1} + \dots),$$

so that the equations of the asymptotes are

$$y = \pm \frac{b}{a} x.$$

ASYMPTOTIC CONDITION. The requirement that a field theory (see relativistic quantum field theory) have an interpretation in terms of asymptotic particle observables. The asymptotic particle observables are a set of operators which represents the observables of stationary incoming and outgoing beams of particles having a sharp mass, and the same transformation, statistical, and orthogonality properties as free particles.

ASYMPTOTIC DIRECTION. See conjugate directions.

ASYMPTOTIC LINE (ON A SURFACE). A curve whose tangent at each point lies in an asymptotic direction at that point.

ASYMPTOTIC REACTOR THEORY. The mathematical formulation of neutron transport theory in which the behavior of the neutron flux, which is generally agreed to hold far from boundaries and sources, is assured to hold throughout the system. In particular, it is assured that the flux is separable into space and energy factors, and that the space factor follows the equation $(\nabla^2 + B^2)\psi(\mathbf{r}) = 0$. (See buckling; first fundamental theorem.)

ASYMPTOTIC SERIES. A divergent series of the form

$$A_0 + A_1/x + A_2/x^2 + \cdots + A_n/x^n + \cdots$$

is an asymptotic representation of a function $f(x)$ if

$$\lim_{x \rightarrow \infty} x^n [f(x) - S_n(x)] = 0$$

for any value of n , where S_n is the sum of the first $(n + 1)$ terms of the series. For any fixed value of x , there is a corresponding value of n for which the "error" $|f(x) - S_n(x)|$ is a minimum, and, by taking x sufficiently large, we may make this minimum error as small as desired.

A familiar example of an asymptotic series is the **Euler-Maclaurin formula**.

Asymptotic expansions are unique; that is, a given function can be represented by only one such series. They may be integrated, two or more of them may be multiplied together, but in general they may not be differentiated.

ATHERMAL SOLUTIONS. A class of non-ideal solutions for which the excess Gibbs free energy (see **excess functions**) is mainly due to the excess entropy

$$g^E \approx -T_s^E.$$

Such a behavior is found in mixtures of macromolecules for which the deviations from ideality are mainly due to the difference in size between solvent and solute. (See **polymer mixtures**.)

ATHERMANOUS. Opaque to infrared radiation. (See **diathermanous**.)

ATMOSPHERE. (1) The layer of air which surrounds the earth. (2) A gaseous envelope surrounding a body, or a mass of gas occupying a region. Hence (3), the gas filling or circulating through a furnace to provide desired conditions during heating or for metallurgical purposes. (4) A standard unit of pressure (see **atmosphere, standard**). (5) In Germany, the pressure of 1 kp/cm², i.e., of one kilopond (kilogram force) per one square meter.

ATMOSPHERE, STANDARD. (1) A unit of pressure, defined as the pressure exerted by a column of mercury 760 mm high, having a density of 13.5951 gm cm⁻³ and subject to

gravitational attraction of 980.665 dyne gm⁻¹, which is equal to 1.013250×10^6 dyne cm⁻². (2) In meteorology, an atmosphere having a sea-level pressure of 1.013250×10^6 dyne cm⁻² (1,013.25 millibars); a sea-level temperature of 15°C; and a lapse-rate of 6.5°C up to 11 km. 1 atm = 1.013250 bar = 101325 N/m² = 1013250 dyne/cm² = 1.03323 kp/cm² = 760 Torr = 14.6960 lbf/in².

ATMOSPHERIC PRESSURE-ALTITUDE RELATIONS. Pressure-altitude relations in the atmosphere are mathematically precise and can be determined from the hydrostatic equation

$$\frac{\partial p}{\partial h} = -\rho g,$$

and the equation of state for air

$$p = \rho RT,$$

which leads to the relation

$$p = p_0 \left[\frac{T_0 - \lambda h^{gm/R\lambda'}}{T_0} \right].$$

where λ is the lapse rate, m is the molecular weight of the air sample, and the zero subscripts denote conditions at height zero. In terms of γ , the ratio of specific heat at constant pressure to that at constant volume, the derivation is

$$dp = -\rho g dh \quad (1)$$

where ρ is density at height h .

Since for adiabatic expansion

$$p\rho^{-\gamma} = p_0\rho_0^{-\gamma} \quad (2)$$

where the subscript ₀ denotes sea-level conditions, then (1) becomes,

$$dp = \left(\frac{p}{p_0} \right)^{1/\gamma} \rho_0 g dh \quad (3)$$

which yields on integration

$$\left(\frac{p}{p_0} \right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{\gamma-1}{\gamma} \frac{\rho_0 g h}{p_0}, \quad (4)$$

For use as a convenient reference, various standard atmospheres have been established. That of the NACA is given on the next page.

NACA STANDARD ATMOSPHERE

An arbitrary and fictitious atmosphere established for comparison purposes. At sea level, a pressure of 29.92 inches (760 mm) of mercury, a temperature of 50°F (15°C), with the air perfectly dry is the beginning assumption. This state is considered to be the average condition prevailing at 40° North latitude. The temperature in the *standard atmosphere* decreases with altitude. At 40°N latitude it drops approximately 1.98°C for each 1000 feet increase in altitude until 35,332 feet is reached. After this the temperature remains at -55°C. The *meteorological average* temperature decrease is calculated at 0.5°C per 100 meters increase in altitude (or about 1°F per 300 feet). This amounts to 1.52°C for each 1000 feet increase in altitude. The following table gives the values of various parameters as established for the Standard Atmosphere:

Altitude (Feet)	Temperature (°F)	Pressure				Density (Slugs/ft ³)
		Psia	Inches mercury	Millibars	Atmospheres	
0	59.0	14.7	29.92	1013	1.0	0.002378
2,500	50	13.4	27.3	930	.910	
5,000	41.2	12.2	24.89	850	.850	0.002047
7,500	32	11.11	22.4	750	.750	
10,000	23.3	10.11	20.58	699	.688	0.001755
15,000	5.5	8.29	16.88	571.62	.564	0.001496
20,000	-12.3	6.75	13.75	465.63	.459	0.001268
25,000	-30	5.45	11.1	375.89	.371	0.001068
30,000	-48	4.36	8.88	300.0	.295	0.000893
35,000	-65.8	3.46	7.00	238.4	.235	0.000740
40,000	-67	2.72	5.54	187.61	.185	0.000585
45,000	-67	2.14	4.36	147.65	.146	0.000462
50,000	-67	1.68	3.44	116.15	.115	0.000364
55,000	-67	1.32	2.71	91.43	.091	0.000287
60,000	-67	1.04	2.13	72.13	.071	0.000227
65,000	-67	.83	1.68	56.89	.056	0.000179
70,000	-67	.65				0.000141
80,000	-67	.41				0.000088
90,000	-67	.26				0.000055
100,000	-67	.16				0.000035

ATOM, BOHR. A model of the hydrogen atom (and of hydrogen-like ions such as He⁺ and Li⁺⁺) based on the nuclear atom of Rutherford, and on classical mechanics except for the superposition of non-classical, quantum, conditions. An electron of charge $-e$ and mass m experiences a **Coulomb force** of attraction to a nucleus of positive charge $+Ze$ (where Z is 1 for hydrogen, 2 for ionized helium (He⁺), etc.) given (in mksa units) by $F = Ze^2/4\pi\epsilon_0 r^2$. This attractive force provides the centripetal force necessary for the electron to move in a circle of radius r , i.e., $Ze^2/4\pi\epsilon_0 r^2 = mv^2/r$. Classically, an electron moving in a circular orbit would radiate electromagnetic waves and hence would spiral into the nucleus. Bohr assumed as his first non-classical condition that this classical radiation law did not hold if the electron were in one of a discrete set of "stationary" orbits determined by a quantum condition on its angular momentum,

i.e., if $mvr = nh/2\pi$, where h is the Planck constant. This condition, together with the force equation, leads to the expression for the radii of the allowed orbits: $r_n = n^2 h^2 \epsilon_0 / \pi e^2 n$ ($n = 1, 2, \dots$) and to the expression for the energy of the electron when it is in one of the allowed orbits: $E_n = -me^4/8n^2 h^2$ ($n = 1, 2, \dots$) (in mksa units). The negative sign results from the fact that the zero of potential energy has been taken at infinity.

The second non-classical condition imposed by Bohr was that a photon of radiation is emitted in the transition from a higher to a lower stationary state, with the frequency, f , of the radiation being given by the Planck law, i.e., $f = \Delta E_m - n/h = (E_m - E_n)/h$.

In a modified form the Bohr atom can include elliptic as well as circular orbits accounted quite well for the spectra of hydrogen and hydrogen-like ions, but was unable to account for many of the features of more com-

plex atoms. It has been superseded by the quantum mechanical theory of the atom. (See also *wave mechanics*; *Schrödinger equation*.)

ATOM, BOHR-SOMMERFELD. A modification of the Bohr model of the atom (see *atom, Bohr*), in which quantized elliptic orbits, as well as circular orbits, are allowed. The quantum conditions of the Bohr-Sommerfeld quantum theory were applied to the radial and angular components of the motion, leading to two quantum numbers instead of one, the radial quantum number (n') and the azimuthal quantum number (k). The principal quantum number $n = n' + k$ largely determines the energy of the stationary states as in the Bohr atom. However, there is a small relativistic effect on the energy expressed in terms of the *fine structure* constant and of both n and k . The quantum number n can take on values 1, 2, \dots , and the quantum number k can take on the values 1, 2, \dots , n .

In addition to the principal and azimuthal (angular momentum) quantum numbers, n and k , a third, the magnetic quantum number m appeared as a result of space quantization. Thus, the component of the angular momentum in some given direction can only take on discrete values, $+mh/2\pi$, where $m = 0, 1, 2, \dots, k$.

The Bohr-Sommerfeld model of the atom has been replaced by the wave-mechanical or quantum mechanical atom, yet as a modelistic visual aid it is still used (with caution), e.g., in the vector model of the atom. Its major fault lies in the fact that it assigns precise orbits to the electrons and is therefore in violation of the Heisenberg *uncertainty principle*. (See also *atom, de Broglie*; *atom, vector model of*; *quantum numbers of electrons in atom*.)

ATOM, DE BROGLIE. Similar in all respects to the Bohr atom (see *atom, Bohr*) except that the quantum condition on the angular momentum electron, $mvr = nh$, results not as an *ad hoc* assumption but as a result of imposing a condition on the wavelength of the de Broglie wave associated with the electron. This wavelength, λ , should fit an integral number of times into the circumference of the circular orbit, thus $2\pi r = n\lambda$. If this condition is combined with the de Broglie relation between wavelength and momentum, $mv = h/\lambda$, the Bohr condition results.

ATOM, ELECTRON SHELLS IN. See *electron shells in atom*.

ATOM, EVEN TERM OF. See *even term of an atom*.

ATOMIC ABSORPTION COEFFICIENT. See *absorption coefficient*.

ATOMIC BOND (ALSO CALLED HOMOPOLAR BOND, OR COVALENT BOND). The chemical bond between two neutral atoms. While ionic binding of atoms in a molecule can be explained on a classical basis, the fact that neutral atoms can attract one another strongly and form very stable (homopolar) molecules such as, for example, H_2 , N_2 and CO , can be explained only on the basis of quantum mechanics. Rigorous theoretical treatment of homopolar binding is possible only in the simplest cases (H_2^+ and H_2). For the discussion of more complicated cases various approximation methods have been developed. (See *atomic bond energy*; *hydrogen molecule, Heitler-London theory for*; *Heitler-London theory of valence*; *orbital theory of valence*.)

ATOMIC BOND ENERGY. The molecular wave function is approximated in the *Heitler-London* method (see also *valence bond method*) by a superposition of possible "*atomic*" and "*ionic*" configurations of the molecule. Each of these corresponds to a possible situation of the system electrons + nuclei at infinite separation of the nuclei. In the case of the H_2 molecule, for example, if A and B refer to the nuclei, and 1 and 2 to the electrons, the symmetrical wave function is:

$$\psi = a_1\psi_A(1)\psi_B(2) + a_2\psi_A(2)\psi_B(1) \\ + a_3\psi_A(1)\psi_A(2) + a_4\psi_B(1)\psi_B(2).$$

The two first terms of the right hand side of this equation correspond to atomic configurations at infinite separation, since each electron is linked to a nucleus. Their contribution to the bond energy is called the *atomic, homopolar, or covalent bond energy*. (For further treatment, see *hydrogen molecule, Heitler-London theory for*.)

The other two terms represent ionic configurations ($H^+ + H^-$) at infinite separations of the nuclei, and their contribution to the bond energy is called the *ionic bond energy*. The ratio of the atomic to ionic bond energies

varies from one molecule to another. (See **ionic bond character**.) Note that the ionic configurations are responsible for the **electric dipole moment** of heteropolar molecules.

ATOMIC CONFIGURATION. See **configuration, atomic**; **atomic bond energy**.

ATOMIC ENERGY. (1) The constitutive internal energy of the atom, which would be absorbed when the atom is formed from its constituent particles, and released when it is broken up into them. This is identical with the binding energy and is proportional to the mass defect. (2) Energy released as the result of the disintegration of atomic nuclei, particularly in large scale processes.

ATOMIC ENERGY LEVELS. (1) The values of the energy corresponding to the stationary states of an isolated atom. (2) The set of stationary states in which an atom of a particular species may be found, including the ground state, or normal state, and the excited states.

ATOMIC ENERGY LEVELS, STATISTICAL WEIGHT. See **statistical weight of atomic energy levels**.

ATOMIC EXCITATION FUNCTION. See **excitation function, atomic**.

ATOMIC FREQUENCY. Each atom in a solid vibrates about its equilibrium position with a frequency ν characteristic for that solid.

Estimations of atomic frequencies may be made in several ways including:

(1) Measurement of the elastic constants of the solid, which are related to the **Debye temperature** Θ (and hence ν) by

$$\Theta = h\nu/k = 35.74 \times 10^{-4} M^{-1/2} \kappa^{-1/2} \rho^{-1/6} [f(\sigma)]^{-1/2}$$

where

$$f(\sigma) = 2[2(1 + \sigma)/3(1 - 2\sigma)]^{3/2} + [(1 + \sigma)/3(1 - \sigma)]^{3/2}$$

where M is atomic weight, κ is compressibility, ρ is density, σ is Poisson's ratio, k is Boltzmann's constant, and h is Planck's constant.

(2) Measurement of the compressibility κ of the solid, when

$$\nu^2 = AN^{2/3} V_a^{1/3} M^{-1} \kappa^{-1}$$

where V_a is atomic volume, N is Avogadro's number, and A is **Madelung's constant**.

(3) Measurement of the coefficient of expansion α of the solid, when

$$\nu = 2.91 \times 10^{11} (C_v/V_a^{2/3} \alpha)^{1/2}$$

where C_v is the molar specific heat at constant volume.

(4) Determination of the melting point, T_m , of the solid, when

$$\nu = K(T_m/MV_a^{2/3})^{1/2}$$

where K is a constant which must be determined experimentally.

(5) From measurement of the wavelengths of the "residual rays" remaining after several reflections of infrared radiation from the solid. The rays are caused by resonance of the atomic ions so that their wavelength is directly related to the atomic frequency.

ATOMIC (SPECIFIC) HEAT. See **Dulong-Petit rule**.

ATOMIC HEAT OF FORMATION. Of a substance, the difference between the enthalpy of one mole of that substance and the sum of the enthalpies of its constituent atoms at the same temperature; the reference state for the atoms is chosen as the gaseous state. The atomic heat of formation at 0°K is obviously equal to the sum of all the bond energies of the molecule, or to the sum of all the dissociation energies involved in any scheme of step-by-step complete dissociation of the molecule. (See also **bond energy**; **dissociation energy**.)

ATOMIC LINE STRENGTH. See **line strength, atomic**.

ATOMIC MASS. The mass of an atom or nuclide expressed in **atomic mass units**, usually on the physical scale (a.m.u.). Examples (E. R. Cohen and W. M. DuMond, 1955, *Encycl. Phys.*, Springer-Verlag, Berlin):

Atomic mass of neutron:

$$M_n = 1.008982 \pm 0.000003$$

Atomic mass of hydrogen:

$$M_H = 1.008142 \pm 0.000003$$

Atomic mass of deuterium:

$$M_D = 2.014735 \pm 0.000006$$

Atomic mass of proton:

$$M_p = 1.007593 \pm 0.000003$$

Atomic mass of electron:

$$Nm = (5.48763 \pm 0.00006) \times 10^{-4}$$

(N is Avogadro's constant, m is electronic rest mass.)

ATOMIC MASS CONVERSION FACTOR.

The ratio r of the values for the atomic mass in terms of the chemical scale and in terms of the physical scale, respectively. The chemical scale is based on an atomic weight 16 for oxygen in its natural isotopic composition, the physical scale on an atomic weight 16 for the oxygen isotope of mass 16. Since the isotopic composition of oxygen varies slightly with its source, experimentally determined values of r vary somewhat ($r = 1.000278$ for air and carbonates, $r = 1.000268$ for water and rocks). R. T. Birge (1942) gives $r = 1.000272$ on the basis of the abundance ratio $O^{16}:O^{18}:O^{17} = 506:1:204$. Adoption of a definite value for r by international agreement is pending at the time of writing (1959).

ATOMIC MASS UNIT (a.m.u.). The unit of atomic mass in terms of the physical scale, i.e., the weight, in grams, of an atom of atomic weight 1, based on an atomic weight 16.0000 for the oxygen isotope O^{16} .

$$1 \text{ a.m.u.} = 1.67330 \times 10^{-24} \text{ g.}$$

(See also **atomic mass conversion factor**.)

ATOMIC MULTIPLETS, HUND RULES FOR. See **Hund rules for atomic multiplets**.

ATOMIC NUMBER. The atomic number Z , the ordinal number of an element in the periodic system, is equal to the electric charge of the nucleus in terms of the electronic charge, and gives the number of electrons surrounding the nucleus in a neutral atom.

ATOMIC NUMBER, EFFECTIVE. A number calculated from the atomic numbers of a compound or mixture, giving the latter's characteristics as an x-ray absorbing medium. An element of this atomic number would interact with photons in the same way as the compound or mixture. Various formulae for this number have been developed, of which the following are perhaps best known:

Spier: Based on theoretical considerations involving absorption and scattering coefficients (*Brit. J. Radiol.* **19**, 52, 1946):

$$\begin{aligned} Z &= \text{effective atomic number} \\ &= (a_1 Z_1^{2.94} + a_2 Z_2^{2.94} + \dots)^{1/2.94} \end{aligned}$$

where Z_1, Z_2 , etc., are atomic numbers of individual constituents, and a_1, a_2 , etc., the fractional electron contents of elements Z_1, Z_2 , etc., in the compound.

Fricke and Glasser: Based on theoretical considerations of photoelectron production (*Fortsch. a. d. Gebiete der Roentgenstrahlen*, **33**, 243, 1925):

$$Z = \sqrt[3]{\frac{a_1 Z_1^4 + a_2 Z_2^4 \dots}{a_1 Z_1 + a_2 Z_2 \dots}}$$

where Z_1, Z_2 , etc., are the atomic numbers of the constituents, and a_1, a_2 , etc., their fractions by weight.

ATOMIC ORBITAL. See **orbital**.

ATOMIC RADIUS. The length of a chemical bond can often be evaluated as a sum of two radii characteristic of the atoms involved in the bond, and of the type of bond (single, double or triple). Tables of such radii and an additivity rule allow one to predict the size of the skeleton of many molecules with reasonable accuracy, except when appreciable resonance with neighboring bonds takes place. (See also **bond length; bond character**.)

ATOMIC REFRACTION. The product of the **specific refraction** of an element by its atomic weight.

ATOMIC SCATTERING FACTOR (ATOMIC SCATTERING POWER). A quantity defined as the ratio of the amplitude of the radiation scattered by an atom to that scattered by a single electron. For radiation of wavelength λ , scattered at an angle θ , the scattering factor is given by the relation

$$f = \int_0^\infty U(r) \frac{\sin kr}{kr} dr$$

where $k = 4\pi \sin \theta / \lambda$. $U(r)$ is a radial distribution function, such that $U(r)dr$ represents the number of electrons between spherical surfaces of radii r and $r + dr$, with their centers at the nucleus of the atom. It may be noted that only at $\theta = 0$ is f equal to Z , the number of electrons in the atom; for all other angles of scattering $f < Z$.

ATOMIC SPECTRA. The radiation absorbed or emitted by (free) atoms and ions, corresponding to transitions between two electronic states (see **combination principle**). If both electronic states involved are stable atomic states the corresponding radiation consists of discrete lines; if the transition involves the removal of an electron (ionization) the emitted

or absorbed radiation is continuous. Atomic spectra in the usual sense of the term are the spectra in the optical and x-ray regions, corresponding to electronic transitions between atomic energy levels of considerably different energy (i.e., different electronic orbitals). For atoms and ions with comparatively simple electronic structure, especially hydrogen, helium, the alkali metals, the alkaline earths, and the corresponding ions (see **Sommerfeld-Kossel displacement law**) these spectra form characteristic series of lines whose separation and intensity decreases in regular manner towards shorter wavelengths. At the point of convergence, the series limit, the series merge into regions of continuous absorption or emission. (See also **Rydberg equation**, **Ritz formula**, **Hicks formula**.) In a wider sense the term atomic spectra includes radiation in the millimeter and centimeter region, corresponding to transitions between component levels of the same multiplet or of the same hypermultiplet.

ATOMIC SPECTRA, BURGER-DORGELO-ORNSTEIN SUM RULE FOR. See **Burger-Dorgelo-Ornstein sum rule for atomic spectra**.

ATOMIC SPECTRA, FINE STRUCTURE OF. See **fine structure**.

ATOMIC STATE, MEAN LIFETIME OF. See **mean lifetime of an atomic state**.

ATOMIC STRUCTURE, COUPLING IN. See **coupling in atomic and molecular structure**.

ATOMIC TERM SYMBOLS. The different energy levels of an atom are designated S , P , D , F , \dots terms according as the quantum number of the resultant electronic orbital angular momentum has the values $L = 0, 1, 2, 3, \dots$. The value of the multiplicity $2S + 1$ (S , quantum number of the resultant electron spin) is added to the symbol for L as a left superscript, the value of J , the quantum number of the resultant total electronic angular momentum, as right subscript.

ATOMIC UNITS (HARTREE). In atomic problems it is often convenient to use the set of units; \hbar , the quantum of angular momentum (Dirac \hbar); m , the mass of the electron; and e , the charge on the electron, rather than the esu, emu, or mksa systems. In the atomic system of units, the unit of length is the radius of the

first Bohr orbit $a = \hbar^2/mc^2$, and the unit of time is $t = \hbar^3/mc^4$, which is the time for the electron to travel one radian in the first Bohr orbit.

ATOMIC WEIGHT. The mass of an atom of any element, the mass of the oxygen atom being taken at 16 atomic mass units. The atomic weight is also called the equivalent weight or the relative weight. Since many elements, as they commonly occur in nature, are mixtures of isotopes, the accepted values of their atomic weights are in reality mean values of the isotopic atomic weights of the various isotopes present. The atomic weight, as defined above, is often called the "chemical atomic weight," taking as its basis a value of 16, for ordinary atmospheric oxygen. Since atmospheric oxygen consists of a mixture of 3 different isotopes, a "physical atomic weight" scale has been established which assigns to the lowest mass isotope the value 16. (See next entry.)

ATOMIC WEIGHT UNIT (a.w.u.). The unit of atomic mass in terms of the chemical scale, i.e., the weight, in grams, of an atom of atomic weight 1, based on the atomic weight 16 for oxygen of the isotopic composition found in natural substances (e.g., fresh lake or rain water).

$$1 \text{ a.w.u.} = 1.6738 \times 10^{-24} \text{ g.}$$

(See also **atomic mass conversion factor**.)

ATOMIZER. See **air injection**.

ATOM, ODD TERM OF. See **odd term of an atom**.

ATOM, RUTHERFORD. A theory of the structure of the atom in which a small nucleus concentrates practically all the mass, and has a positive charge equal to the atomic number. An equal number of electrons, surrounding the nucleus, fill the major part of the atomic volume. This idea is the basis of the **Bohr-Sommerfeld atom**, and of all subsequent theories.

ATOM, THOMSON. An early model of an atom in which the positive charge was considered to be distributed continuously throughout the volume of a sphere, and in which negative electrons (of sufficient number to give electric neutrality) were imbedded. The electrons normally occupied positions of equilibrium, but were capable of executing harmonic

motions about their equilibrium positions when disturbed (e.g., by an incident electromagnetic wave).

ATOM, VECTOR MODEL OF. In quantum mechanics, as in classical physics, **angular momentum** is a vector quantity whose direction, for a particle moving in a plane orbit, is perpendicular to the plane of the orbit and with a sense given by the progression of a right-handed screw. In quantum mechanics, however, angular momenta (both orbital and spin) are quantized in magnitude and direction and special rules have to be set up for their vector addition (see **angular momentum, coupling of**). The anomalous **Zee-man effect** spectral doublets, **hyperfine structure**, and many other spectral phenomena are explained in terms of the vector addition of the various vector contributions to the total angular momentum of atoms, nuclei, etc. (See also Semat, *Introduction to Atomic and Nuclear Physics*, Rinehart and Co., 1954.)

ATTACK, ANGLE OF. See **incidence, angle of**.

ATTENUATION. (1) The decrease of flux density (as, for example, light in an absorbing medium) with the distance propagated. (2) In statistics, the weakening (lowering in absolute value) of a **correlation** coefficient due to errors of observation in the variables. (3) For attenuation of sound, see **acoustic attenuation coefficient**.

ATTENUATION CONSTANT. See **propagation constant**.

ATTENUATION FACTOR. (1) In traversing a distance x in a medium with absorption coefficient α the amplitude of vibration of light is damped by the factor $\exp(-\alpha x)$, called the attenuation factor. (2) A measure of the opacity of a layer of material to sound waves traveling through it. It is equal to I_0/I where I_0 and I are the sound intensities of the wave entering and leaving the layer, respectively.

AUDIBILITY, THRESHOLD OF (THRESHOLD OF DETECTABILITY). For a specified signal, the minimum effective sound pressure of the signal that is capable of evoking an auditory sensation in a specified fraction of the trials. The characteristics of the signal, the manner in which it is presented to the

listener, and the point at which the sound pressure is measured should be specified.

Unless otherwise indicated, the ambient noise reaching the ears is assumed to be negligible. The threshold may be expressed in **decibels** relative to 0.0002 **microbar** or to 1 **microbar**. Instead of the method of constant stimuli, which is implied by the phrase "in a specified fraction of the trials," another psychophysical method (which should be specified) may be employed.

AUDIOGRAM. A graph showing **hearing loss**, percent hearing loss, or percent hearing (see **hearing, percent**) as a function of frequency.

AUGER COEFFICIENT. The ratio of Auger yield to fluorescence yield, which is also expressed quantitatively as the ratio of the number of Auger electrons to the number of x-ray photons ejected. (See **Auger effect**.)

AUGER EFFECT. (Also called auto-ionization or pre-ionization.) The radiationless transition in an atom from a discrete electronic level to a continuous (ionized) level of the same energy. Levels for which the probability of radiationless transition is high are broadened, with consequent broadening of spectral lines corresponding to radiative transitions involving this level. The selection rules for radiationless transitions are

$$\Delta J = 0; \quad \Delta S = 0; \quad \Delta L = 0; \quad \text{even} \leftarrow | \rightarrow \text{odd.}$$

AUGER YIELD. The Auger yield for a given excited state of an atom of a particular element is the probability of deexcitation by the **Auger effect** rather than by x-ray emission; it is the difference between unity and the **fluorescence yield** for that state, and also equals the sum of the Auger yields for the various possible Auger transitions from that state.

AUGMENTED THRUST RATIO. The ratio of thrust obtained by a turbojet with its afterburner operating to that without. It is given by the relationship:

$$F_a/F = \frac{V_{ja} - V}{V_j - V} = \frac{\sqrt{T_{ea}/T_e} - \nu}{1 - \nu},$$

where F_a is the thrust with afterburner operating, F is the thrust without, V_{ja} is the exhaust jet velocity with afterburner operating, V_j is

the exhaust jet velocity without, V is the inlet velocity (velocity of vehicle), T_{ea} is the exit temperature ($^{\circ}\text{R}$) with afterburner operating, T_e is the exit temperature without, and ν is the ratio V/V_j .

AUTOBAROTROPY. The state of a fluid which is characterized by both **barotropy** and **piezotropy** when the coefficients of barotropy and piezotropy are equal. This condition indicates that the fluid will remain barotropic.

AUTOCORRELATION. The **correlation** of the members of a time-series among themselves. More precisely, if $\{x_t\}$ is the **realization** of a stochastic process with mean m and variance σ^2 , the autocorrelation coefficient of order k is ρ_k , the limiting value, as $n \rightarrow \infty$, of

$$\frac{1}{n\sigma^2} \sum_{t=1}^n (x_{t+k} - m)(x_t - m).$$

The expression is also used for a sample value of ρ_k derived from a realization of finite length n , the quantities m and σ^2 then being replaced by estimators calculated from the realization. An alternative name for the sample value is *serial correlation*.

AUTOCORRELATION COEFFICIENT IN TURBULENCE. A coefficient relating a fluctuating quantity with its value at another point separated by a fixed interval of time or space, used in the statistical theory of **turbulence**. Assuming that mean values can be properly defined, i.e., that

$$\bar{Q} = \frac{1}{2\Theta} \int_{-\Theta}^{\Theta} Q(\theta) d\theta$$

is independent of Θ for a wide range of values of Θ , Taylor's (Lagrangian) autocorrelation coefficient R_ξ for a single particle moving in a field of turbulent motion is definable at time t by

$$R_\xi = \overline{u'(t)u'(t + \xi)/u'^2}$$

where u' is the fluctuating velocity component being analyzed, and ξ is a time interval. This coefficient, being a characteristic of the turbulence, must yield a **mixing length** which is also a measure of the turbulence. Since $R_\xi \rightarrow 0$ as $\xi \rightarrow \infty$ it is reasonable to suppose that $\int_0^t R_\xi d\xi$ tends to a definite limit as $t \rightarrow \infty$, in which case, together with the turbulent intensity $(\overline{u'^2})^{1/2}$, it yields the length

$$l = (\overline{u'^2})^{1/2} \int_0^\infty R_\xi d\xi.$$

The measurement of the Lagrangian autocorrelation coefficient presents serious practical difficulties; it is therefore more convenient to use the Eulerian coefficient defined by a space interval x ; thus

$$r_x = \overline{u'(X)u'(X + x)/u'^2},$$

the fluctuations u' at X and $X + x$ being measured simultaneously.

Still easier to measure is the correlation with the same quantity observed at the same point after an interval τ , which we may define as

$$r_\tau = \overline{u'(t)u'(t + \tau)/u'^2},$$

but its interpretation requires that the frequency analysis of the local velocity fluctuations be approximately identical with that produced when the wave-number analysis of the instantaneous variation of velocity along a line in the direction of the mean motion is advected with the mean velocity. This is only true if the fluctuations are small compared with the mean flow. In that case x is assumed to be the same as $\bar{u}\tau$ where \bar{u} is the mean velocity.

The above definitions are strictly applicable only to homogeneous isotropic turbulence.

AUTOCORRELATION FUNCTION. A function used, for example, in statistics and in information theory of the form

$$\phi_{11}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(t)f(t - \tau) dt$$

where τ is a time-delay parameter. The function $f(t)$ is considered to be continuous, and the function rejects phase information in regard to $f(t)$.

The autocorrelation function is related to the power spectrum of $f(t)$ by the Fourier-transform pair

$$\phi_{11}(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \Phi_{11}(\omega) e^{j\omega\tau} d\omega$$

$$\Phi_{11}(\omega) = \int_{-\infty}^{\infty} \phi_{11}(\tau) e^{-j\omega\tau} d\tau$$

where $\Phi_{11}(\omega)$ represents the power density spectrum of $f(t)$.

AUTOFRETTAGE. The process of inducing a favorable distribution of initial or residual

stress in a tube. Compressive hoop stresses which reduce the tension caused by interior pressure may be produced at the inside diameter by winding the tube with wire under tension, by shrinking one or more tubes on the outside of the inner one, or by applying interior pressure above the elastic limit but below the flow limit or collapse load.

The plastic **limit load** or pressure is not altered by autofrettage but the **fatigue** life under lower pressure is increased.

AUTO-IONIZATION. See **Auger effect**.

AUTOMORPHISM. See **group (or homomorphism)**.

AUTOMORPHISM GRAPH. See **graph, automorphism**.

AUTOREGRESSION. A **regression** relationship connecting the members of a time-series whereby the value at one point, say u_t , is expressible in terms of values at previous points, u_{t-1}, u_{t-2}, \dots , plus a **stochastic** term. The commonest case is that of *linear autoregression* exemplified by

$$u_t = \alpha_1 u_{t-1} + \alpha_2 u_{t-2} + \dots + \alpha_k u_{t-k} + \epsilon_t$$

where the α 's are parameters and ϵ is a **random variable**. A series obeying such a relation may be regarded as the realization of a particular kind of stochastic process, known as an autoregressive process. The simple case when $k = 1$ is usually known as a *Markov scheme*; that for $k = 2$ is called a *Yule scheme*.

AUXILIARY EQUATION. Using the symbol $D = d/dx$, a given linear **differential equation** with constant coefficients may be written in the form

$$D^n y + a_1 D^{n-1} y + \dots + a_n y = 0.$$

The auxiliary equation is

$$r^n + a_1 r^{n-1} + a_2 r^{n-2} + \dots + a_n = 0,$$

regarded as an algebraic equation in the unknown r . If the roots of this auxiliary equation are r_1, r_2, \dots, r_n , then the general solution of the original differential equation is

$$y = \sum_i c_i e^{r_i x}$$

the c_i 's being arbitrary constants, provided the roots are all unequal. If one of them, r_g is repeated g times the corresponding terms in the general solution of the differential equation are

replaced by $c_g(1 + a_1 x + a_2 x^2 + \dots + a_{g-1} x^{g-1})e^{r_g x}$. Imaginary roots, if they occur, have the form $r_{\pm} = A \pm iB$ and the term in the solution becomes $e^{Ax}(c_+ \cos Bx + c_- \sin Bx)$.

AVAILABILITY. See **available energy**.

AVAILABLE ENERGY. The maximum quantity of work which can be performed by a closed system in a given state (energy E , volume V , etc.) as it undergoes a transformation at the end of which it is in thermodynamic and mechanical equilibrium with the surroundings, usually the terrestrial atmosphere. The surroundings is imagined to be in thermal equilibrium, and infinite in extent, its pressure and temperature being p_o and T_o , respectively. Denoting the properties of the system by subscript o when it has reached equilibrium with the surroundings, it can be shown that the available energy

$$\alpha = W_{max} = (E + p_o V - T_o S) - (E_o + p_o V_o - T_o S_o).$$

The preceding relation is derived from the second law of thermodynamics, and hence is a consequence of it under the specified conditions, that is, in particular when the system interacts with one source of heat at temperature T_o , that of the surroundings. It is implied that the process from state (E, V, S) to state (E_o, V_o, S_o) is reversible and that the change in the sum of the entropies of the system and its surroundings is zero.

AVAILABLE POTENTIAL ENERGY. That portion of the total **potential energy** which may be converted to **kinetic energy** in a **closed system**.

AVERAGE DEVIATION. The average deviation (A.D.) of a number of like quantities X_j is the average of the absolute departures of their individual values from their mean

$$\text{A.D.} = \frac{\sum_{j=1}^n |X_j - \bar{X}|}{n},$$

where the mean

$$\bar{X} = \frac{\sum_{j=1}^n X_j}{n}.$$

The A.D. is a measure of the expected departure of any single observation of a series from the "true" value which would be obtained by the averaging of an infinite series of observations. The average deviation of the mean (a.d.) is

$$\text{a.d.} = \text{A.D.} \sqrt{n - 1}.$$

A.D. and a.d. are related to other measures of uncertainty if the individual values are distributed according to a particular rule, such as the Gaussian error function.

AVERAGE POTENTIAL MODEL FOR MIXTURES. Statistical treatments of mixtures (Prigogine, Bellemans, Scott) are usually based on the statistical theorem of corresponding states and on the introduction of *average-composition-dependent interaction constants*. If limited to first-order terms, they reduce to the theory of conformal solutions but they give also an approximate estimate of higher order terms in the difference of intermolecular forces.

AVOGADRO HYPOTHESIS. Equal volumes of gases under the same conditions of pressure and temperature contain the same number of molecules. (See Avogadro number.)

AVOGADRO NUMBER. The number of molecules in a mole ($N = 6.0248 \times 10^{23}$ mole⁻¹). It is equal to the ratio of the gas constant R to the Boltzmann constant k .

A.W.U. See atomic weight unit.

AXES, MOVING. See moving axes.

AXES OF MOLECULES, PRINCIPAL. See moments of inertia of molecules.

AXIAL BUNDLE. A bundle of rays emanating from an object point on the axis of an axially symmetric optical system.

AXIAL-FLOW COMPRESSOR. A rotary compressor in which the fluid flows mainly in the axial direction, in contrast to a centrifugal compressor in which the flow is mainly radial.

AXIAL-FLOW TURBINE. A turbine in which the fluid flows mainly in the axial direction in contrast with a *Ljungström turbine* in which the flow is mainly radial.

AXIAL LOAD. A tensile or compressive force applied along the effective centerline of a straight member. The term may also be employed for an eccentric load parallel to the

axis, with the term central axial load used to indicate the absence of bending.

AXIAL MAGNIFICATION. The ratio of the interval between two adjacent image points on the axis of an optical instrument to the interval between the conjugate object points.

AXIAL RAY. (1) Two linearly independent optical paths, one from an axial object point and one from an off-axis object point, determine all paraxial optical paths in the Gaussian optics of a system. These are referred to as the axial and field rays, respectively. (2) In general, a meridional ray of an optical system intersecting the axis in the object plane is an axial ray.

AXIAL THRUST. The axial component of force exerted on a nozzle, turbine or turbo-compressor by the fluid passing through it. It consists of a term due to change of momentum and a term due to a difference in pressures at inlet and exit.

AXIAL VECTOR. See pseudovector.

AXIS, CENTROIDAL. See centroidal axis.

AXIS, NEUTRAL. See neutral axis.

AXIS, OPTIC(AL). (1) A direction through a doubly-refracting crystal along which no double refraction occurs. A uniaxial crystal has one such direction, a biaxial crystal has two such directions. (2) The surface of a mirror and all surfaces of a lens or lens system are usually figures of revolution about a common axis, called the optical axis.

AXIS, PRINCIPAL. A line for which some quantity or property has a maximum, minimum or stationary value. *Principal axes of stress* for a point of a body are directions in which the principal stresses act. *Principal axes of inertia* for a point of a body are axes about which the moment of inertia is stationary; they are the axes of the inertia ellipsoid associated with the point. (See Mohr circle.)

AXIS, STRONG. The cross section of any beam in bending will have one line through its centroid about which the area moment of inertia

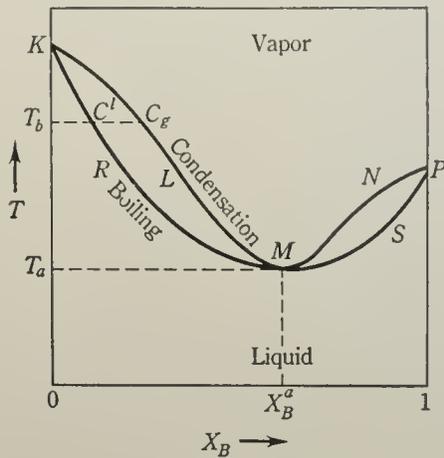
$$I_x = \int_A y^2 dA$$

is maximum. This line is known as the strong axis. In an I-beam it is the line in the cross

section perpendicular to the web at the mid-height of the beam and often called the 1-1 or $x-x$ axis.

AXIS, WEAK. The line in the cross section about which the area **moment of inertia** is least. In an I-beam cross section it is the bisector of the web perpendicular to the flanges and often is called the 2-2 or $y-y$ axis.

AZEOTROPIC SYSTEMS. Consider a mixture of water and alcohol in the presence of the vapor. This system of two phases and two components is *divariant* (see **phase rule**). Now choose some fixed pressure and study the composition of the system at equilibrium as a function of temperature. The experimental results are shown schematically on the figure.



Azeotropic system.

The vapor curve $KLMNP$ gives the composition of the vapor as a function of the temperature T , and the liquid curve $KRMSP$ gives the composition of the liquid as a function of the temperature. These two curves have a common point M . The state represented by M is that in which the two states, vapor and liquid, have the same composition x_B^A on the mole fraction scale. Because of the special

properties associated with systems in this state, the point M is called an *azeotropic point* and the system is said to form an *azeotrope*. In an azeotropic system, one phase may be transformed to the other at constant temperature, pressure and composition without affecting the equilibrium state. This property justifies the name *azeotropy*, which means a system which boils unchanged.

AZEOTROPY. See **azeotropic systems**.

AZIMUTH (ASTRONOMIC). The coordinate in the horizontal system of coordinates that is measured westward in the plane of the horizon from the prime vertical (prime meridian or south point of the horizon) to the intersection of the vertical circle through the object, with the horizon.

The *azimuth angle* is measured at the center of the celestial sphere in the plane of the horizon from the direction of the elevated pole (north or south to agree with the latitude) east or west through 180° .

The *altitude of an object* is the arc of the horizon either east or west to the intersection of the vertical circle through the object with the horizon.

Any of these quantities may be expressed in terms of arcs of the *almucantor* (parallel plane to the equator) through the object.

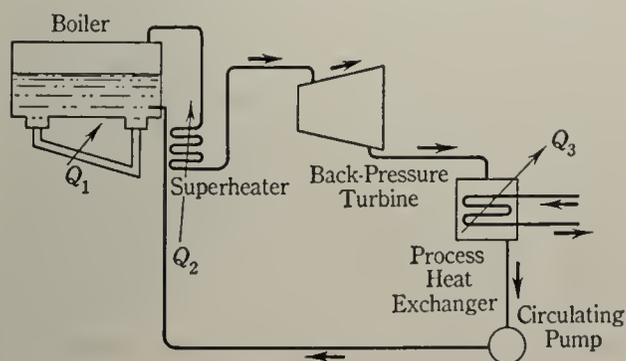
AZIMUTH ANGLE. (1) The angle ϕ in the **spherical coordinate system**. (2) For **plane-polarized light** incident on the surface of a dielectric, the angle between the plane of vibration and the normal to the plane of incidence. This same word applies to incident, reflected and refracted light. (3) When a celestial object is located with respect to the horizon and a meridian plane, the azimuthal angle is that between the meridian and the vertical plane through the body and the zenith.

B

BABINET PRINCIPLE. Two diffraction screens are complementary if the clear regions of the first are opaque regions of the second and *vice versa*. J. Babinet proved that the **Fresnel diffraction** patterns of a point source, due to complementary screens, are identical except for a region near the central image.

BACK FOCAL LENGTH. The axial distance between the last optical surface in an axially symmetrical optical system and the second (or image space) focal point.

BACK-PRESSURE TURBINE. In industrial processes and in building heating, it is necessary to supply heat at a moderate temperature only, namely at a temperature far below that which can be maintained in a furnace. Consequently, supplying heat directly constitutes a loss from the point of view of the second law of **thermodynamics**, because the temperature difference could be made use of for the production of work. It is more economical to produce steam at the highest temperature possible and to expand it in a back-pressure turbine to the temperature required for heating or processing. The latter temperature determines the back pressure for the turbine.



Back-pressure turbine.

(See also **extraction turbine**.)

BACK-SCATTERING (BACKWARD SCATTERER). The scattering of radiant energy into the hemisphere of space bounded by a plane normal to the direction of the incident radiation and lying on the same side as the incident ray; the opposite of forward scatter.

tion and lying on the same side as the incident ray; the opposite of forward scatter.

BACK-SCATTERING COEFFICIENT B (ECHOING AREA). For an incident plane wave, the back-scattering coefficient B is 4π times the ratio of the reflected power per unit solid angle (Φ_r) in the direction of the source divided by the power per unit area (W_i) in the incident wave:

$$B = 4\pi \frac{\Phi_r}{W_i} = 4\pi r^2 \frac{W_r}{W_i},$$

where W_r is the power per unit area at distance r . For large objects, the back-scattering coefficient of an object is approximately the product of its interception area by its scattering gain in the direction of the source, where the interception area is the projected geometrical area and the scattering gain is the reradiated power gain relative to an isotropic radiator.

BACKWARD DIFFERENCE OPERATOR. See **difference operators**.

BACKWARD DIFFERENCES. See **difference operators**.

BAIRSTOW METHOD. A method for finding complex roots of an algebraic equation.

Let $z^2 + az + b$ be a trial divisor of $f(z)$ and form

$$f(z) = (z^2 + az + b)^2 Q(z) + (z^2 + az + b)q(z) + r(z),$$

where

$$r(z) = r_1z + r_0, \quad q(z) = q_1z + q_0.$$

This means that r is the remainder after dividing f by $z^2 + az + b$, and q the remainder after dividing the quotient. Solve

$$(aq_1 - q_0)\delta a - q_1\delta b = -r_1,$$

$$bq_1\delta a - q_0\delta b = -r_0$$

for δa and δb ; then $z^2 + (a + \delta a)z + b + \delta b$ will be, in general, closer to a true divisor. The

method is an adaptation of the Newton method to finding complex roots and was originally described by Bairstow; later by Hitchcock.

BALANCE. A horizontal shaft with attached masses (as, for example, the armature of a motor or the rotor of a turbine) is said to be in *static balance* if at rest there is no turning moment about the axis of the shaft in any angular position. A rotor in static balance may produce oscillating bearing reactions when the shaft rotates. The reduction of these oscillatory forces to an acceptable value is called *dynamic balancing*. Balance is achieved when a **principal axis of inertia** of the rotor coincides exactly with the axis of the shaft.

BALANCED DESIGN. See **incomplete block**.

BALANCING, DETAILED. See **detailed balancing**.

BALMER SERIES. Series of lines in the spectra of atomic hydrogen and the hydrogen-like ions He^+ , Li^{++} , \dots . The wave numbers $\nu_n = \frac{1}{\lambda_n}$ are given by the formula

$$\nu_n = RZ^2 \left[\frac{1}{2^2} - \frac{1}{n^2} \right], \quad (n = 3, 4, \dots),$$

where R stands for the **Rydberg constant** and Z for the **atomic number**.

The first lines of the Balmer series of hydrogen ("Balmer lines") were discovered early as strong absorption lines in the visible region of the solar spectrum, and are traditionally denoted as $\text{H}_\alpha(\lambda, 6563 \text{ \AA})$, $\text{H}_\beta(\lambda, 4861 \text{ \AA})$, $\text{H}_\gamma(\lambda, 4340 \text{ \AA})$, etc.

BALMER TERMS. The energy levels of the hydrogen atom and of the hydrogen-like ions, without consideration of fine structure, i.e., the energy of the **Bohr orbits**

$$E_n = - \frac{2\pi^2 \mu e^2 Z^2}{h^2 n^2}.$$

Here h is Planck's constant, e is the electronic charge, Z is the atomic number, n is the principal quantum number

$$n = 1, 2, 3, \dots,$$

and μ is the reduced mass of the electron

$$\mu = \frac{mM}{m + M},$$

(m and M are electronic and nuclear mass respectively).

BANACH SPACE. A complete, **normed vector space**. The most important examples of Banach spaces have functions for their elements, e.g., the space C of functions continuous on a closed interval with norm equal to maximum of absolute value. A **Hilbert space** is a particular example of a Banach space.

BAND. In the spectrum of a molecule, a group of closely spaced lines, corresponding to the possible transitions from the rotational energy levels belonging to one vibrational level to the rotational levels of another vibrational level (see **molecular energy levels**, **vibrational energy levels of a molecule**, **rotational energy levels of a molecule**). The rotational fine structure of a band depends on the **selection rules** which are valid for the transition concerned, and varies widely between the bands of different molecules, and between bands corresponding to different types of transitions in the same molecule. (For more detail see **rotation-vibration spectra of molecules**.) Lines having the same difference $\Delta J = J' - J''$ between the rotational quantum numbers of the upper and lower state form series of regularly spaced lines, called branches (Q -branch for $\Delta J = 0$, R - and P -branch for $\Delta J = +1$ and -1 , respectively, S - and O -branch for $\Delta J = +2$ and -2 , respectively, \dots).

BAND, CONDUCTION. See **conduction band**.

BAND EDGE ENERGY. The energy of the edge of the **conduction band** or **valence band** in a solid; that is, the minimum energy required by an electron in order that it may be free to move in a semiconductor or the maximum energy it may have as a valence electron.

BAND, FUNDAMENTAL. See **fundamental band**.

BAND ORIGIN. See **zero line of a band**.

BAND PRESSURE LEVEL. The effective sound pressure level (see **sound pressure, effective**) for the sound energy contained within a specified frequency band of a given sound. Both the width of the band and the reference pressure should be stated.

BAND SEQUENCES. See **band system**.

BANDS, OVERTONE. See *overtone bands*.

BAND SPECTRA. See *band system; molecular spectra*.

BAND SYSTEM. In the spectrum of a molecule, a band system consists of the bands corresponding to all the transitions possible between the **vibrational energy levels** of one electronic state to the vibrational levels of another electronic state. (For formulas representing the wave numbers of the **zero lines** of a band system, see **electronic spectra of molecules**.) In the case of a diatomic molecule the wave numbers of the zero lines (or of the band heads) of the bands belonging to a band system are conveniently arranged in a table (Deslandres table) such that bands with the same lower vibrational quantum number v'' and increasing upper vibrational quantum number v' (v' progressions) form vertical columns, and bands with the same upper vibrational quantum number v' and increasing lower vibrational quantum number v'' (v'' progressions) form successive horizontal rows. The differences between successive columns or successive rows of wave numbers correspond to the differences between successive vibrational energy levels of the lower and upper electronic states, respectively. The bands forming diagonals in the Deslandres table correspond to transitions with $v' - v'' = \text{const.}$ (*Band sequences*.)

BAND THEORY OF SOLIDS. Electrons in a periodic lattice of ions occupy energy levels which fall into groups or bands. In the *allowed bands*, the energy levels are closely spaced, and form effectively a continuum. There are, however, ranges of energy in which there are no levels at all, and these are called *forbidden bands* or *energy gaps*. The electrical properties of the material depend on how the electrons are distributed in the various bands. Thus, when only the lower bands are filled, and there is a wide energy gap above them, the material behaves as an *insulator*. When the gap is narrower, or when impurity centers are present, which create extra levels, some electrons may be raised to the **conduction band**, where they can carry current. The material is then a *semiconductor*. In metals, there is a partially-filled band in which the electrons behave as if they were free.

BANDWIDTH. (1) The number of cycles per second expressing the difference between the limiting frequencies of a frequency band. (2) For an amplifier or other device, the range of frequencies for which some characteristic (such as gain) falls within specified limits (e.g., not less than one half of its maximum value). (3) In radio communication usage the number of cycles per second required to convey the information being transmitted, either visual or aural.

BANDWIDTH, EFFECTIVE. (1) For a specified **transmission system**, the bandwidth of an ideal system which (a) has uniform transmission in its **pass band** equal to the maximum transmission of the specified system, and (b) transmits the same power as the specified system when the two systems are receiving equal input signals having a uniform distribution of energy at all frequencies. This may be expressed as follows:

$$\text{Effective bandwidth} = \int_0^{\infty} Gdf$$

where f is the frequency in cycles per second and G is the ratio of the power transmission at the frequency f , to the transmission at the frequency of maximum transmission.

(2) For a band-pass filter (see **filter, band-pass**) the width of an assumed rectangular band-pass filter having the same **transfer ratio** at a reference frequency, and passing the same mean-square value of a hypothetical current and voltage, having even distribution of energy over all frequencies.

BANDWIDTH OF A WAVE. The least frequency interval outside of which the **power spectrum** of a time-varying quantity is everywhere less than some specified fraction of its value at a reference frequency. This definition permits the spectrum to be less than the specified fraction with the interval. Unless otherwise stated, the reference frequency is that at which the spectrum has its maximum value.

BAND, ZERO LINE OF. See *zero line of a band*.

BAR. The general term for a structural member whose length is much greater than its cross-sectional dimensions. A bar under compression may be called a **column** or a **strut**. A

bar in bending is a **beam**, a bar in torsion is a **shaft**, a bar in tension may be called a **tie**.

BARBA LAW. The statement that geometrically similar bars of the same material but of different size deform similarly. Deviations from similarity are labeled as **size effects**.

BARDEEN-COOPER-SCHRIEFFER THEORY OF SUPERCONDUCTIVITY (1957).

An extension of the ideas of the Fröhlich-Bardeen theory in which the exchange of virtual phonons and the short range Coulomb repulsions between the members of a pair of electrons having spins in opposite directions lead to a correlation of the electrons in a superconductor. The correlation leads to stable electron pairs having energies near the **Fermi level** and seems to account satisfactorily for the major properties of superconductivity.

BARN. The unit of nuclear cross section, equal to 10^{-24} square centimeter. The most common sub-unit is the millibarn, equal to 10^{-27} cm².

BAROCLINIC. Possessing density gradients; when applied to the atmosphere implying the presence of horizontal temperature gradients and therefore of vertical gradients of horizontal wind. Potential energy is stored in a baroclinic airstream, and a measure of the availability of this for the production of wind systems is the **Richardson number**, $Ri = g\beta/U'^2$. Here

$$\beta = \frac{1}{\theta} \frac{\partial \theta}{\partial z}$$

is the static stability represented by the vertical gradient of potential temperature θ , and U' is the wind shear representing, through the thermal wind equation, the horizontal gradient of temperature. Thus for geostrophic winds

$$Ri = f^2 \theta \frac{\partial \theta}{\partial z} / g \left(\frac{\partial \theta}{\partial x} \right)^2,$$

where z is measured vertically, and x horizontally normal to the isotherms. f is twice the vertical component of the earth's rotation. The instability of a baroclinic airstream increases with decreasing Ri and disturbances like extra-tropical depressions may arise in baroclinic airstreams whose **Rossby number** is of the order of unity.

BAROCLINITY. The state of being **baroclinic**.

BAROMETER FORMULA. Equation for the density $\rho(z)$ of an isothermal gas at a height z in the earth's gravitational field (constant gravitational acceleration g),

$$\rho = Ce^{-\beta mgz},$$

where $\beta = 1/kT$ (k is Boltzmann's constant, T , absolute temperature), m , the mass of one atom of the gas, and C , a normalizing constant.

BAROTROPIC. Of uniform density; when applied to the atmosphere implying the absence of horizontal temperature gradients and therefore of vertical gradients of horizontal wind because of the absence of a **thermal wind**. A barotropic atmosphere moves horizontally with a velocity independent of height.

BAROTROPIC INSTABILITY. The hydrodynamic **instability** arising from certain distributions of **vorticity** in a two-dimensional non-divergent flow. This is an **inertial instability** in that kinetic energy is the only form of energy transferred between current and perturbation. The variation of vorticity (i.e., shear) in the basic current may be concentrated in discontinuities of the horizontal wind shear (to be distinguished from **Helmholtz instability**, where the velocity itself is discontinuous) or may be continuously distributed in a curved velocity profile. A well known necessary condition for barotropic instability is that the vorticity must change sign (i.e., vanish) at a point of maximum shear.

In barotropically unstable flow, certain isolated characteristic wavelengths are associated with **neutral waves**, separating the damped from the amplified waves. Applications to atmospheric jet-stream wind profiles have calculated neutral wavelengths to be of the order of 9000 km, with shorter waves damped and longer waves amplified. It has been suggested therefore that the shorter cyclone waves provide a source of kinetic energy to the jet stream.

BAROTROPIC MODEL. Any of a number of model atmospheres in which some of the following conditions exist throughout the motion: coincidence of pressure and temperature surfaces; absence of vertical wind shear; absence of vertical motions; absence of horizontal velocity **divergence**; and conservation of the vertical component of **absolute vorticity**.

Certain of these models would be more prop-

erly called **autobarotropic** models; often the barotropic model is the same as the **equivalent-barotropic model** used in numerical forecasting.

BAROTROPIC PRESSURE FUNCTION.

The scalar function in a **barotropic** fluid, the gradient of which is equal to the specific pressure force:

$$\nabla\pi = \alpha\nabla p,$$

where π is the barotropic pressure function, α , the specific volume, and p , the pressure.

BAROTROPIC VORTICITY EQUATION.

The **vorticity equation** in the absence of horizontal divergence and vertical motion, so that the absolute vorticity of a parcel is conserved,

$$\frac{d}{dt}(\zeta + f) = 0,$$

where ζ is the **relative vorticity** and f the **Coriolis parameter**. This equation may also be interpreted as governing vertically-averaged flow in which divergence is present but wind direction is constant with height. (See also **equivalent-barotropic model**.)

BAROTROPY. The state of being **barotropic**.

BARREL DISTORTION. See **geometric distortion**.

BARRIERS, NOISE LEVEL REDUCTION OF. See **noise level reduction of barriers**.

BARTLETT FORCE. A central force between two nucleons which is derivable from a potential containing an operator which exchanges the **spins** of the two particles but not their positions. Nuclear forces are known to be spin-dependent, and some of their properties can be described in terms of the Bartlett force.

BARYCENTRIC PARALLAX. A small oscillatory motion of the earth around its orbit about the sun due to the fact the center of mass of the earth-moon system does not coincide with the center of the earth, but moves about as the moon rotates around the earth. The period of this motion is one synodic month. Data on the barycentric parallax are chiefly of use in calculating the mass ratio of earth and moon.

BARYON. The generic name of the "elementary" particles of mass in the interval from the nucleonic mass up to (but not including) the deuteron mass. The word "elementary particle" is here used in the sense of a system having a definite mass and spin, a lifetime long compared to **strong interaction** times (which are of the order $\sim 10^{-23}$ sec) and having a well defined charge and other electromagnetic properties. At present (1960) eight such particles are known. They are listed in Table 1 below.

The six particles listed in Table 1 which are heavier than the neutron or proton are called *hyperons*.

BASE OF A SYSTEM OF LOGARITHMS. See **logarithm**.

BASE OF A SYSTEM OF NUMBERS. See **radix**.

BASES. See **acids and bases, definitions of Brønsted and Lewis**.

BASE VECTORS (FOR A COORDINATE SYSTEM). If \mathbf{r} denotes the vector position relative to a fixed origin, of a point P in space and x^i are the contravariant coordinates of P in a curvilinear coordinate system x , then for each value of i , $\partial\mathbf{r}/\partial x^i$ is a vector tangential to the coordinate line x^i through P . The vectors $\partial\mathbf{r}/\partial x^i$ are called the base vectors for the curvilinear coordinate system x .

TABLE I

Particte	Mass (in electron masses)	Mean life (in seconds)	Charge (in multiple of e)
p (proton)	1836.12 ± 0.04	stable $> 4 \times 10^{23}$ years	+1
n (neutron)	1838.65 ± 0.04	1050 ± 200	0
Λ^0	2182.5 ± 0.4	$2.9 \pm 0.2 \times 10^{-10}$	0
Σ^0	2329.9 ± 0.3	$< 2 \times 10^{-12}$	0
Σ^+	2327.9 ± 0.6	$0.7 \pm 0.1 \times 10^{-10}$	+1
Σ^-	2341.6 ± 0.8	$1.6 \pm 0.2 \times 10^{-10}$	-1
Ξ^-	2585 ± 6	$\sim 3 \times 10^{-10}$	-1
Ξ^0	$\sim 2587 \pm 10$?	0

BASE VECTORS, RECIPROCAL (FOR A COORDINATE SYSTEM IN THREE-DIMENSIONAL SPACE). The vectors \mathbf{a}^i given by

$$\mathbf{a}^i = \frac{\mathbf{a}_2 \times \mathbf{a}_3}{[\mathbf{a}_1 \mathbf{a}_2 \mathbf{a}_3]}, \text{ etc.,}$$

where \mathbf{a}_i are the base vectors, $[\mathbf{a}_1 \mathbf{a}_2 \mathbf{a}_3]$ is the scalar triple product of these and $\mathbf{a}_2 \times \mathbf{a}_3$ is the vector product of \mathbf{a}_2 and \mathbf{a}_3 .

BASIC STIMULUS. The standard stimulus (usually **achromatic**) used to determine the units of the reference stimuli of any **trichromatic system**.

BATHO FORMULA. The average shear stress in a thin-walled tube subject to torsion without appreciable distortion of the tube is $\tau = 2G\theta A/s$ where s is the mean perimeter, θ , the angle of twist per unit length, A the cross-sectional area and G the **shear modulus**.

BAUER-WACH TURBINE. A turbine driven by the exhaust steam from a reciprocating steam engine. (See **exhaust-gas turbine**.)

BAUSCHINGER EFFECT. Reduction of the yield stress for reversed loading after plastic flow. More generally, the reduction of the **yield limit**, when the yield stress system changes, or changes direction from that of first loading.

BAY. The portion of a structure between two adjacent **bents** or **frames**.

BAYES' SOLUTION. In the theory of statistical decision-functions, a Bayes' solution is one which minimizes the average risk involved, the average taking place over some prior probability distribution.

BAYES' THEOREM. Let $c_1, c_2, c_3 \dots c_s$ be some s mutually exclusive random events such that one of them must happen and let $P(c_i)$ be the **probability** of c_i . These events we shall describe as causes. Let E be some event which can be directly observed and let $P(E|c_i)$ be the probability that E will occur on the assumption that c_i has already happened. Let $P(c_i|E)$ be the probability of c_i after E has occurred. The probabilities $P(c_i)$ are called "a priori" probabilities, i.e., probabilities not at all dependent on the event E , whereas $P(c_i|E)$ are called "a posteriori" probabilities

since these depend on the event E having occurred. Bayes' theorem states

$$P(c_i|E) = \frac{P(c_i) \cdot P(E|c_i)}{\sum_{j=1}^s P(c_j)P(E|c_j)}$$

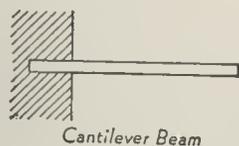
The theorem states exactly how the probability of a certain "cause" changes as different events actually occur. The theorem was published posthumously in 1764. When all conditions of the theorem are fulfilled there is no objection to Bayes' theorem. The difficulties in applying the theorem depend upon the fact that $P(c_i)$, the "a priori" probabilities, are not known and are assumed often to be equal in the absence of other knowledge. This is known as *Bayes' postulate*, or the "equidistribution of ignorance." Many statisticians find it unacceptable and most modern theories of inference attempt to avoid it.

BEAM. A body bounded by a right cylindrical (or prismatic) surface and by two parallel planes which are normal to the generators of the cylindrical surface, the dimension of the body parallel to the generators being large compared with the characteristic dimensions perpendicular to them. Also called *rod*, *shaft* or *bar*. By an extension of meaning, the cross-sectional form may vary, if explicitly stated, as when the term is used to designate certain rolled or extruded sections, such as I-beams, WF-beams, T-beams, etc. Frequently, the term beam is reserved for a **bar** subjected primarily to bending so that its curvature changes. For pure bending about a principal axis in the elastic range $1/\rho = M/EI$ for an initially straight beam where ρ is the radius of curvature, M is the **bending moment**, EI is the **flexural rigidity** (E is **Young's modulus** and I is **area moment of inertia**.) The curvature $1/\rho$ is approximated by d^2y/dx^2 in most problems. Maximum bending stress in an elastic beam is given by Mc/I where c is the distance from the **neutral axis** to the extreme fiber.

BEAM, BENDING OF. See **flexure, simple (of a beam)**.

BEAM, CANTILEVER. A beam which is rigidly connected at one end to a fixed support and free to move at the other end. This theoretically fixed condition rarely occurs because of deformation of the supporting ma-

material. The maximum **bending moment** and maximum **shear** occur simultaneously at the face of the support. The usefulness of this



type of beam is demonstrated in structures such as canopies, unbraced airplane wings and cantilever retaining walls.

BEAM-COLUMN. A beam-column is a **bar** subject to appreciable axial compression P , as well as bending. Non-linear problems arise because the transverse deflections y which occur increase the bending moment significantly by the addition of a Py term. The elastic deflections which would be found in the absence of P are magnified by $1/(1 - P/P_E)$ approximately, where P_E is the **Euler buckling load**. (See **critical load—Southwell's method**.)

BEAM, COMPOSITE. A beam which is composed of two materials properly bonded together and having different properties. **Reinforced concrete** beams, steel beams mechanically bonded to a concrete deck, and wood beams reinforced with steel plates are typical examples. It has become a rather prevalent practice in recent years to provide for composite action between the steel stringers and concrete floor of highway bridges. This permits the use of lighter beams than would otherwise be possible.

The analysis of composite beams depends on the assumption that a plane section before bending remains plane after the **load** is applied. Therefore the two materials must be connected in such a way that they will act as a unit. This condition is realized in the reinforced concrete beam by means of the bond (see **bond stress**) between the reinforcing rods and the concrete. In the case of reinforced wood beams the parts are connected by bolts properly spaced to resist the shearing forces (see **shear**) between the plates and the beam. Steel beams are bonded to the concrete flange by means of lugs or spirals welded into the top flange of the steel beam.

The flexure formula is applicable to composite elastic beams if the beam is transformed into an equivalent homogeneous section by means of the transformed area method

which is found in texts on strength of materials and reinforced concrete design.

BEAM, CONJUGATE. The equations for small deflections of **beams**

$$\frac{d^2y}{dx^2} = \frac{d\theta}{dx} = \frac{M}{EI}$$

and the relation between moment, M , shear, V , and transverse load per unit length

$$\frac{d^2M}{dx^2} = \frac{dV}{dx} = q$$

show that the shear and bending moment in a conjugate beam loaded by M/EI is the slope and deflection of the actual beam, respectively. Boundary conditions for the conjugate beam must be chosen appropriately. An actual hinged end transforms to a hinged end, a fixed end becomes free, and conversely.

BEAM, CONTINUOUS. A **beam** which has more than two supports and therefore is statically **indeterminate** unless hinges are inserted. The continuity over the supports increases the load-carrying capacity. The junction conditions at a support are continuity of moment, except when an external moment is applied, and continuity of slope in the elastic range. (See **Clayton theorem of three moments**.)

BEAM CURVATURE. See **curvature of beam**.

BEAM, CURVED. A **beam** curved in the plane of bending. Although displacement may vary linearly, the strain distribution is non-linear because of this initial curvature. (See **Winkler-Bach formula**.) When the depth of the beam is small compared to its radius of curvature, the non-linearity is negligible and the equations for straight beams apply with change in curvature replacing curvature.

BEAM, DEEP. A beam whose length is a small multiple of its depth, say less than five, and which carries transverse load by a combination of beam and arch action. For such a short or deep beam, the assumption of plane cross sections remaining plane does not give good answers for the maximum stresses induced.

BEAM EQUATIONS OF MOTION. The differential equation of small motion of a beam, ignoring rotary inertia and shear, is

$$\frac{\partial^2}{\partial x^2} \left(EI \frac{\partial^2 y}{\partial x^2} \right) + \rho A \left(\frac{\partial^2 y}{\partial t^2} \right) = f(x, t)$$

where EI is the flexural rigidity, ρ is the mass density, A is the cross-sectional area and f is the applied force per unit length of beam.

BEAM, FIXED-ENDED OR ENCASTRÉ.

A beam with ends so firmly connected to the supports that in the elastic range the end



Fixed-ended beam.

tangents remain fixed in direction. At loads sufficient to produce plastic hinges at the ends of the beam, the supports are presumed capable of carrying the full plastic moment.

BEAM, HAUNCHED. A beam whose thickness increases toward its supports. Tables and graphs are available for both reinforced concrete and metal beams to permit the designer to employ the same techniques as for the simpler problems of uniform cross section.

BEAM ON ELASTIC FOUNDATION. The term elastic foundation usually refers to the simplest type in which the local foundation reaction ky is proportional to the local deflection y . The governing differential equation for the bending of an elastic beam of flexural rigidity EI then is

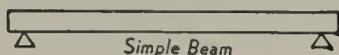
$$\frac{\partial^2}{\partial x^2} \left(EI \frac{\partial^2 y}{\partial x^2} \right) + ky = q(x)$$

where q is the applied load per unit length.

BEAM, OVERHANGING. A beam which extends beyond its support at one or both ends. The overhang is a cantilever whose support end is not fully fixed. (See **bending moment**.)

BEAM, SHORT. A deep beam.

BEAM, SIMPLE. A beam which rests on two end supports in such a manner that its ends are free to rotate on the supports. Since riveted



connections are elastic, the end restraint offered by this type of connection is not very reliable. Therefore, beams and girders which

are riveted to supports are generally assumed to be simply supported unless a special type of moment connection is used.

BEARING. (1) Any member used to support, guide, or restrain a moving part of a mechanism. (2) That part of a structure that transmits load to the supports, such as a pedestal under a bridge. A compressive force or stress is often referred to as a *bearing stress*. Also the load-carrying capacity under thrust as for a footing or a pile foundation is called *bearing capacity*. (3) A term used in navigation, surveying, and a variety of other activities to describe direction. Various methods are used to define bearing. Frequently *true bearing* is defined as the direction measured from the north to the east through 360° . However, one might well find a southeast direction described as bearing 135° , 135° true, southeast, south 45° east, north 135° east, etc., etc.

Bearing may be determined by compass, in which case it must be corrected for deviation and variation to obtain a true bearing.

Bearing may be *relative*, in which case other information must be furnished before the actual direction can be found. For example, a look-out on a ship on heading 045° true and an object is seen 65° to the right of the bow. In this case he might report as 065° right, in which the deck officer would interpret this direction as 110° true.

The various professions dealing with directions have their own schemes for naming a direction. Astronomers seldom use the term bearing, preferring azimuth, measured from the north to the right (i.e., to the east) through 360° .

BEAT(S). A series of alternate maxima and minima in **vibration** amplitude, produced by the **interference** of two wave trains of different frequency.

BEAT FREQUENCY. The difference frequency resulting from the combination of two applied frequencies.

BEATTIE AND BRIDGEMAN EQUATION.

A form of the **equation of state**, relating the pressure, volume, and temperature of a gas, and the **gas constant**. The Beattie and Bridgeman equation applies a correction for reduction of the effective number of molecules by molecular aggregation, due to various causes.

It has been stated in more than one form, an example of which is:

$$P = \frac{RT(1 - \epsilon)}{V^2} (V + B) - \frac{A}{V^2}$$

in which P is the pressure, T is the absolute temperature, V is the volume, R is the gas constant and A , B and ϵ are quantities defined in terms of five empirical constants, A_0 , B_0 , a , b , and c by the following relationships:

$$A = A_0 \left(1 - \frac{a}{V}\right)$$

$$B = B_0 \left(1 - \frac{b}{V}\right)$$

$$\epsilon = \frac{c}{VT^3}$$

BEER LAW. The law discovered by Beer in 1852, that the absorption coefficient for light passing through a solution of a given salt in a non-absorbing solvent is proportional to the concentration. The absorption coefficient a of the Bouguer law can therefore be expressed as αc , where α is the molar absorption coefficient and c is the concentration in moles per liter. Sometimes written

$$I = I_0 e^{-\alpha c x}$$

where I is the intensity of light transmitted, I_0 is the intensity of the incident light, e is the natural logarithmic base, c is the concentration of the solution in moles per liter, x is the thickness of the transmitting layer, and α is the molar absorption coefficient.

In general Beer's law is used for light of a particular narrow wavelength band and then α is the specular molar absorption coefficient.

The Beer law is also written

$$I = I_0 10^{\epsilon c x}$$

where ϵ is known as the molar extinction coefficient. Not all solutions obey the Beer law. (See also the **Bouguer law**.)

BEHRENS-FISHER TEST. A statistical test of significance for the difference between the means of two samples which emanate from normal (Gaussian) populations with different variances. The test is based on fiducial probability (see **fiducial inference**) and is not universally accepted as valid.

BEI. The imaginary part of the **Bessel function** $J_n(xe^{\pm 3\pi i/4}) = \text{ber}_n(x) \pm i \text{bei}_n(x)$, where the real part is denoted by **ber**.

BEL. See **gain**, **gain ratio**.

BELLEVILLE SPRING. A sprung flat disc or washer in the form of a conical shell or an assemblage of shells. (See **springs**.)

BELTRAMI DIFFERENTIAL PARAMETERS. If ϕ and ψ are invariant functions of position on a surface for which the contravariant **metric tensor** is $a^{\alpha\beta}$, then $a^{\alpha\beta}\phi_{,\alpha}\phi_{,\beta}$ is a surface invariant called Beltrami's first differential parameter of ϕ . $a^{\alpha\beta}\phi_{,\alpha\beta}$ is a surface invariant called Beltrami's second differential parameter of ϕ . $a^{\alpha\beta}\phi_{,\alpha}\psi_{,\beta}$ is a surface invariant called Beltrami's mixed differential parameter of ϕ and ψ . In a surface coordinate system u^α , $\phi_{,\alpha}$ denotes $\partial\phi/\partial u^\alpha$ and $\psi_{,\beta}$ denotes $\partial\psi/\partial u^\beta$.

BELTRAMI-ENNEPER THEOREM. The torsions of the two **asymptotic lines** through a point on a surface are the positive and negative square roots of the negative of the Gauss curvature. (See **curvature of a surface**, **second**.)

BELTRAMI FLOW. A fluid motion in which the **vorticity** vector is parallel to the velocity vector at every point of the fluid.

BELTRAMI-MICHELL COMPATIBILITY EQUATIONS. Differential equations which must be satisfied by a static stress field in a body of isotropic elastic material satisfying the **generalized Hooke law**. In a rectangular Cartesian coordinate system x , they may be written, using the **indicial notation** and **summation convention**, as

$$\begin{aligned} \frac{\partial^2 \sigma_{ij}}{\partial x_k \partial x_k} + \frac{1}{1 + \nu} \frac{\partial^2 \sigma_{kk}}{\partial x_i \partial x_j} \\ = - \frac{\nu}{1 - \nu} \delta_{ij} \frac{\partial F_k}{\partial x_k} - \left(\frac{\partial F_i}{\partial x_j} + \frac{\partial F_j}{\partial x_i} \right), \end{aligned}$$

where σ_{ij} denotes the stress components and F_i the components of the body forces per unit volume in the system x ; ν is **Poisson's ratio** and δ_{ij} is the **Kronecker delta**. The equation represents six independent equations. It is derived from the equation of compatibility of strain for infinitesimal deformations by using the **generalized Hooke law** and the equations

of equilibrium to effect substitutions. Also called *equations of compatibility of stress*

BÉNARD CELLS. The configuration assumed when slow overturning motion is induced in a viscous fluid by heating it below. The cells are usually square or hexagonal when the fluid is confined between horizontal planes, the hexagonal form requiring a slightly smaller **Rayleigh number** to produce the motion.

In a fluid in which the viscosity decreases as temperature increases, the motion is upwards in the center of the cells and downwards at the edges. In a gas it is usually downwards in the center.

The motion is slow, so that the inertia forces are negligible and the viscous stresses are balanced by the pressure gradients produced by the density variations. The heat is transported from the hot to the cold surface partly by convection and partly by conduction.

The cells are about 2½ times as wide as they are deep and the precise configuration depends on the conditions imposed on the velocity and temperature at the bounding surfaces.

If both upper and lower surfaces are perfectly conducting and the cells of square plan, the ratio, *a*, of cell width to height and the Rayleigh number, required just to produce motion, are as follows:

Upper Boundary	Lower Boundary	<i>a</i>	Rayleigh Number
free	free	$\pi/\sqrt{2} = 2.22$	$27\pi^2/4 = 651$
free	rigid	2.67	1100
rigid	rigid	3.14	1708

where at a rigid boundary the velocity is zero and at a free boundary there is no shear stress, while the temperature is uniform in both cases.

In this case the Rayleigh number is defined as $g\beta\alpha h^4/\kappa\nu$, α being the coefficient of cubical expansion, β the vertical temperature gradient, h the depth, and κ and ν the thermometric conductivity and dynamic viscosity of the fluid.

BENDING. Change of **curvature**. When produced by a transverse load, bending is accompanied by shearing deformation, which is not generally of great importance except in short or deep **beams** or in beams whose web is designed explicitly to carry the shear with minimum material.

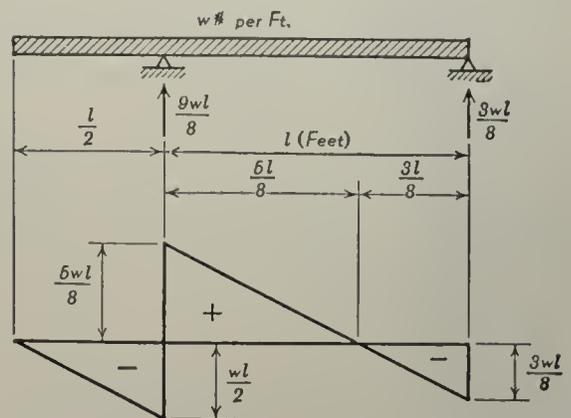
BENDING A LENS. In the approximation of thin lens theory, i.e., negligible lens thickness,

the power of a lens with spherical surfaces of radii r_1, r_2 is $\phi = (n - 1) (r_1^{-1} - r_2^{-1})$ where n is the index of refraction. By bending the lens of an axially symmetric lens system is meant varying the curvatures of the spherical surfaces so that $(r_1^{-1} - r_2^{-1})$ is an invariant for each component lens of the system. Then the **power**, the **Petzval sum**, and the longitudinal and lateral chromatic variations of focus remain constant, while in general the **spherical aberration**, **coma**, **astigmatism**, and **distortion** of the system vary. As a result, bending the component lens of a system is a basic perturbation method in lens design.

BENDING, MEMBRANE ANALOGY. See **membrane analogy bending**.

BENDING MOMENT. The bending moment at any cross section of a beam is the component in the cross section of the vector sum of the moments of all the forces acting on the beam to one side of the section about the **centroid** of the cross section. An internal resisting moment at any section is the sum of the moments of the normal or bending stresses about an axis in the cross section passing through the centroid.

A graphical representation of the variation of bending moment on a beam is called a *bending moment diagram*. An illustration of a bending moment diagram is given below for an overhanging beam with a uniformly dis-



Bending moment diagram.

tributed load covering the entire length of the beam. The maximum bending moments which are indicated on the diagram occur where the **shear** changes sign. Since the product of a force and distance is a moment, bending moments are expressed as foot-pounds, inch-pounds, etc.

BENDING OF A BEAM. See flexure, simple (of a beam).

BENDING, PURE. Bending unaccompanied by shear or normal force. The bending moment is constant so that plane cross sections must remain plane because of symmetry.

BENDING, UNSYMMETRICAL. See unsymmetrical bending.

BENEDICT-WEBB-RUBIN EQUATION. An empirical equation of state for a gas of the form

$$p = R\rho T - \left(B_o RT - A_o - \frac{C_o}{T^2} \right) \rho^2 + (bRT - a)\rho^3 + a\alpha\rho^6 + [(1 + \gamma\rho^2)c\rho^3/l^3] \exp(-\gamma\rho^2),$$

where R , B_o , A_o , C_o , b , a , c , α , γ are empirical constants, different for every gas. The equation is nearly of the virial form (except for the last term) and covers a wide range of pressures and temperatures. It is particularly useful for mixtures of inert gases, when the following empirical combination rules apply:

If X denotes any of the constants A_o , C_o , γ , then for the mixture (no subscript) in terms of two components (subscripts 1 and 2), we can write

$$X = (x_1 X_1^{1/2} + x_2 X_2^{1/2})^2.$$

Here x_1 and x_2 are the respective mole fractions. If Y denotes any one of b , a , c , α then

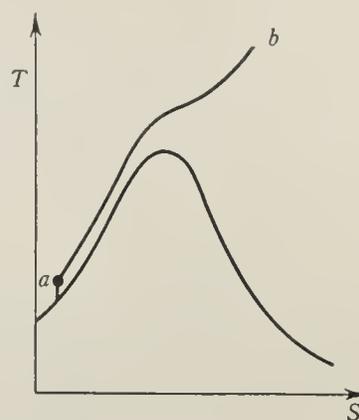
$$Y = (x_1 Y_1^{1/3} + x_2 Y_2^{1/3})^3,$$

etc., for n components. The constant B_o can be calculated from

$$B_o = x_1 B_{o1} + x_2 B_{o2}.$$

BENSON BOILER. A boiler in which steam is generated at a pressure higher than the critical ($= 3206.2 \text{ lbf/in}^2 = 225.65 \text{ kp/cm}^2 = 218.39/\text{atm}$). In this manner the transition from the liquid to the superheated vapor phase is continuous and no bulky boiler drum is required. The sequence of states traversed by the working liquid in a Benson boiler is shown in the figure. The boiler can be operated without trouble also at pressures somewhat lower than the critical, because the change in volume on evaporation is still quite small. The boiler can be regulated by regulating its pressure.

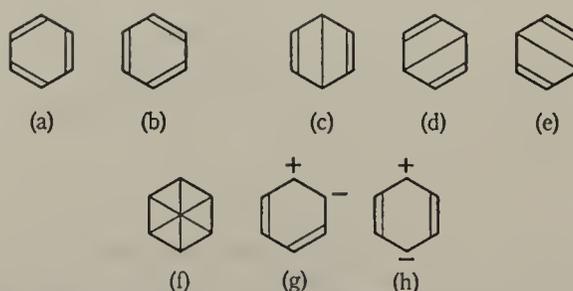
The power consumption of the high-pressure pump is considerable.



T,S diagram for Benson boiler. (Point a , compressed water; curve b , superheated steam.)

BENT. A transverse frame which forms an integral part of a structural unit or supports another structural unit. Bents are designed to carry lateral as well as vertical loads, and are made of structural steel, reinforced concrete, prestressed concrete, or wood. They are used extensively in connection with viaducts and industrial buildings. Viaduct bents consist of columns held firmly together by bracing in horizontal and vertical planes. Bents in buildings are composed of a roof truss and the supporting columns or of a roof girder rigidly attached to the columns so as to form a rigid frame. The bents support longitudinal beams, commonly called purlins, on which the roof is laid.

BENZENE, STRUCTURES OF. In order to explain the observed stability of the benzene molecule (and of other aromatic molecules), different structures have been suggested in the pioneer days of organic chemistry.



Structures proposed for the benzene molecules. [(a and b), Kekulé structures; (c, d and e), Dewar structures; (f), Claus structure; (g and h), Ionogenic structures.]

The valence bond method calculates the energy of these molecules by means of a

weighted average of such configurations. The better known of these are the two *Kekule* and the three *Dewar structures* (a, b, and c, d, e of the figure, respectively). The structure (f) proposed by *Claus* must not be considered in the valence bond method, since it can be expressed as a linear combination of the other five configurations. *Ionogenic structures* (g, h) have also been considered.

BERGMANN SERIES. A series of lines in the spectra of the simpler atoms, the alkalis, alkaline earths, helium, etc., corresponding to transitions from an upper *F* state to a lower *D* state. (Also called **fundamental series**.)

BERNOULLI DISTRIBUTION. See **binomial distribution**.

BERNOULLI EQUATION. A first order non-linear differential equation

$$\frac{dy}{dx} + f(x)y = g(x)y^n.$$

It may be made **linear** by the substitution $y = u^{n-1}$, giving

$$\frac{du}{dx} + (1 - n)f(x)u = (1 - n)g(x).$$

BERNOULLI EQUATION APPLIED TO FLUIDS. An integrated form of the equation of steady motion, viz.

$$\int \frac{dp}{\rho} + \frac{1}{2}v^2 + \Omega = \text{const.}$$

along a streamline or vortex line, where Ω is the potential of the body forces. If the motion is irrotational there are no vortex lines and the expression is constant throughout the fluid. In general it is constant on a surface composed of all the vortex lines through a streamline, called a *Bernoulli surface*.

In a liquid of constant density moving irrotationally under gravity

$$p + \frac{1}{2}\rho v^2 + g\rho z = p_0$$

where p_0 is the **stagnation pressure** at the height $z = 0$. The effect of gravity may be removed by writing $p' = p + g\rho z$. p' is then the departure of the pressure from the hydrostatic value, and the Bernoulli equation is

$$p' + \frac{1}{2}\rho v^2 = p'_0$$

and p'_0 is then called total pressure.

BERNOULLI METHOD. Given the algebraic equation

$$(1) \quad x^n + a_1x^{n-1} + \dots + a_n = 0,$$

let h_0, h_1, \dots, h_{n-1} be arbitrary numbers, not all zero, and form h_n, h_{n+1}, \dots , by

$$h_{n+\nu} + a_1h_{n+\nu-1} + \dots + a_nh_\nu = 0.$$

If (1) has a unique root of largest modulus, then in general the quotients $h_{\mu+1}/h_\mu$ approach that root. The method can be extended to transcendental equations. Let

$$(2) \quad f(z) \equiv 1 + c_1z + c_2z^2 + \dots$$

converge in some circle about the origin in the complex plane, and let

$$(3) \quad g(z) \equiv g_0 + g_1z + g_2z^2 + \dots$$

represent any function analytic in the same circle, and having no zero in common with $f(z)$. Let

$$h_0 = g_0,$$

$$c_1h_0 + h_1 = g_1,$$

$$c_2h_0 + c_1h_1 + h_2 = g_2$$

$$\dots \dots \dots$$

Then if $f(z)$ has a unique zero of smallest modulus lying within that circle, then $h_\mu/h_{\mu+1}$ approaches that zero. If there are two zeros whose moduli are less than those of all others, then the roots of

$$\begin{vmatrix} z^2 & h_\mu & h_{\mu+1} \\ z & h_{\mu+1} & h_{\mu+2} \\ 1 & h_{\mu+2} & h_{\mu+3} \end{vmatrix} = 0$$

approach those zeros of $f(z)$. Likewise one can form cubics whose roots approach the three smallest roots. (See Alston S. Householder, *Principles of Numerical Analysis*, McGraw-Hill Book Company, 1953.) The QD algorithm improves upon this principle. (See National Bureau of Standards, *Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues*, NBS Appl. Math. Series 49, 1958.)

BERNOULLI NUMBERS. Coefficients B_ν of the **Bernoulli polynomials**. Except for B_1 , those with odd subscripts vanish. The values of the first several are

$$B_0 = 1, \quad B_1 = -\frac{1}{2}, \quad B_2 = \frac{1}{6}, \quad B_4 = -\frac{1}{30},$$

$$B_6 = \frac{1}{42}, \quad B_8 = -\frac{1}{30}, \quad B_{10} = \frac{5}{66}.$$

They are the coefficients in the expansion

$$u/(e^u - 1) = \sum_0^{\infty} B_\nu u^\nu / \nu!$$

They occur in the **Euler-Maclaurin formula**. (See Niels Erik Nörlund, *Vorlesungen über Differenzenrechnung*, Berlin, Springer, 1924.)

BERNOULLI POLYNOMIALS. These polynomials $B_\nu(x)$ of degree ν occur as coefficients in the expansion

$$ue^{xu}/(e^u - 1) = \sum_0^{\infty} u^\nu B_\nu(x) / \nu!$$

They satisfy the differential equations with boundary conditions

$$B'_\nu(x) = \nu B_{\nu-1}(x), \quad B_\nu(1) = (-1)^\nu B_\nu(0),$$

and can be written symbolically in the form $B_\nu(x) = (B + x)_\nu$, where the right member is to be expanded as though ν were an exponent:

$$(B + x)_\nu = B_\nu + \nu x B_{\nu-1} + \frac{\nu(\nu-1)}{2} x^2 B_{\nu-2} + \dots$$

and the coefficients are the **Bernoulli numbers**. In the same fashion a recursion for the Bernoulli numbers is

$$B'_\nu = (B + 1)_\nu, \quad \nu > 1.$$

Both the **Bernoulli numbers** and the Bernoulli polynomials are defined in other ways. They are used in numerical integration formulas and in the calculus of finite differences. (See Niels Erik Nörlund, *Vorlesungen über Differenzenrechnung*, Berlin, Springer, 1924.)

BERNOULLI PROBABILITY FUNCTION. See **binomial distribution**.

BERNOULLI THEOREM. This theorem was published posthumously in 1713. Let the **probability** of an event p be constant from trial to trial, and the probability of failure be denoted by q . Let the **relative frequency** be denoted by $\frac{x}{s}$ where x is the number of successes in s trials. Let $P\left(\left|\frac{x}{s} - p\right|\right)$ denote the probability of obtaining the absolute value of the deviation $\frac{x}{s} - p$. Usually this deviation will be small. Bernoulli's theorem states that as the number of trials s increases, the probability of a

difference in absolute value more than a stated positive amount ϵ approaches zero. In symbols

$$\lim_{s \rightarrow \infty} P\left(\left|\frac{x}{s} - p\right| > \epsilon\right) = 0.$$

It should be clear that the theorem does not state

$$\lim_{s \rightarrow \infty} \frac{x}{s} = p.$$

The theorem states roughly that if s is sufficiently large usually $\frac{x}{s}$ will not differ much from p . However, cases may occur though quite infrequently when $\frac{x}{s}$ will differ considerably from p . Bernoulli's theorem is one of the "laws of large numbers."

BERNOULLI TRIAL SOLUTION. See **separation of variables**.

BERNSTEIN POLYNOMIAL. Given a function $f(x)$ defined on the closed interval $[0,1]$ the polynomial

$$B_n(x) = \sum_{i=0}^n f(i/n) \binom{n}{i} x^i (1-x)^{n-i},$$

where $\binom{n}{i}$, is the binomial coefficient, is the Bernstein polynomial of order n for the function $f(x)$. These polynomials are important in the theory of probability in the summation of divergent series, and in the theory of approximation.

BERTHELOT'S EQUATION OF STATE. A form of the **equation of state**, relating the pressure, volume, and temperature of a gas, and the gas constant R . The Berthelot equation is derived from the **Clausius equation** and is of the form

$$PV = RT \left[1 + \frac{9PT_c}{128P_c T} \left(1 - 6 \frac{T_c^2}{T^2} \right) \right]$$

in which P is the pressure, V is the volume, T is the absolute temperature, R is the gas constant, T_c is the critical temperature, and P_c the critical pressure.

BERTRAND CURVES. A pair of curves having their principal normals in common are called *associate*, or *conjugate*, *Bertrand curves*.

BESSEL FUNCTION. The differential equation

$$x^2y'' + xy' + (x^2 - n^2)y = 0, \quad n = \text{const.}$$

is called Bessel's equation of order n . Certain of its solutions (see below) are called Bessel functions. The general solution is

$$y = AJ_n(x) + BY_n(x)$$

where

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{\Gamma(k+1)\Gamma(k+n+1)} \left(\frac{x}{2}\right)^{n+2k}$$

and $Y_n(x) = J_{-n}(x)$ if n is not an integer. These functions are called Bessel functions of the first kind. If n is an integer, then $J_{-n}(x) = (-1)^n J_n(x)$, so that $Y_n(x)$ is defined as

$$Y_n(x) = \lim_{k \rightarrow n} \frac{J_k(x) \cos kx - J_{-k}(x)}{\sin k\pi}$$

which is called a Bessel function of the second kind. The functions, much used in physics,

$$H_n^{(1)}(x) = J_n(x) + iY_n(x) \quad i = \sqrt{-1}$$

$$H_n^{(2)}(x) = J_n(x) - iY_n(x)$$

are Bessel functions of the third kind; they are also called Hankel functions of the first and second kind, respectively. Other combinations of Bessel functions are also given names. These functions have certain standard properties of recurrence, orthogonality, etc. (See **special functions**.)

BESSEL FUNCTION, MODIFIED. The modified Bessel function of the first kind is

$$I_n(z) = i^{-n} J_n(iz),$$

and that of the second kind is

$$K_n(z) = \frac{1}{2}\pi (\sin n\pi)^{-1} [I_{-n}(z) - I_n(z)].$$

BESSEL FUNCTION, SPHERICAL. The function,

$$j_n(z) = \sqrt{\frac{\pi}{2z}} J_{n+\frac{1}{2}}(z),$$

where $J_{n+\frac{1}{2}}(z)$ is a **Bessel function** of half integral order.

BESSEL INEQUALITY. Let u be a vector in Euclidean n -space with length $\|u\|$ and let $u_1, u_2 \dots u_r$ be an orthonormal set of vectors.

Then Bessel's inequality states that

$$(u, u_1)^2 + \dots + (u, u_r)^2 \leq \|u\|^2,$$

where the parentheses denote scalar product.

The particular case $r = n$, for which equality holds, is called the Pythagorean theorem or Parseval's equation. Similarly, if u_1, u_2, \dots is a complete orthonormal set in **Hilbert space**, then Bessel's inequality states that $(u, u_1)^2 + (u, u_2)^2 + \dots \leq \|u\|^2$.

For example, if scalar product is defined by integration as in the case of a standard Fourier series, then $(u, u_i) = \int f u_i dx$ and Bessel's inequality states that $\sum_{i=1}^{\infty} \left(\int f u_i dx\right)^2 \leq \int f^2 dx$.

BESSEL INTERPOLATION FORMULA.

$$E^u = E^{1/2} \left[\mu + (u - \frac{1}{2})\delta + \binom{u}{2} \mu \delta^2 + \frac{u - \frac{1}{2}}{3} \binom{u}{2} \delta^3 + \binom{u+1}{4} \mu \delta^4 + \frac{u - \frac{1}{2}}{5} \binom{u+1}{4} \delta^5 + \dots \right].$$

(For notation see **difference operators**.)

The formula is most useful for $0 < u < 1$. Taking $v = u - \frac{1}{2}$, an alternative, and more symmetric, expression is

$$E^u = E^{1/2} \left[\mu + v\delta + \frac{1}{2!} (v^2 - \frac{1}{4}) \mu \delta^2 + \frac{1}{3!} v(v^2 - \frac{1}{4}) \delta^3 + \frac{1}{4!} (v^2 - \frac{1}{4})(v^2 - \frac{9}{4}) \mu \delta^4 + \frac{1}{5!} v(v^2 - \frac{1}{4})(v^2 - \frac{9}{4}) \delta^5 + \dots \right].$$

BETA DISINTEGRATION (BETA-DECAY).

The process whereby a nucleus disintegrates spontaneously with the emission of β -particles (i.e., electrons or positrons) and neutrinos (see **radioactivity**). Since electrons and neutrinos cannot exist within the nucleus, they are assumed to be emitted by means of a weak coupling between the electron and neutrino fields on the one hand, and the neutron and proton fields on the other, in much the same way as electromagnetic radiation can be emitted when a charged particle changes its state of motion.

Thus, the process of β -decay involves the simultaneous interaction of four fields. The strength of the coupling is far weaker than the coupling between the electron and photon fields. The exact nature of the coupling is not yet certain; but recently it has been found that parity is violated in the process of β -decay, and this discovery is likely to lead to a better understanding of the process itself.

BETA DISINTEGRATION ENERGY. (1)

The disintegration energy of a beta-decay process; symbol Q_β . For negatron emission it is equal to the sum of the kinetic energies of the β -particle, the neutrino, and the recoil atom (usually negligible), and is obtained experimentally from the maximum energy of the β -particle spectrum. For positron emission, the energy equivalence of two electron rest-masses must be added to the aforementioned sum, since the products include the positron and one negative electron in addition to the neutral product atom, the energy equivalent to their masses ultimately appears as radiant energy of annihilation radiation. For electron capture, the disintegration energy is equal to the sum of the kinetic energy of the neutrino and the electronic excitation energy of the product atom (usually, the binding energy, in the neutral product atom, of an electron equivalent to that which was captured).

(2) Often, implicitly, the ground-state β -disintegration energy, which is the total energy released in a β -transition between isobars in their ground states; symbol Q_{β_0} . It includes the energies of any γ -radiation and associated radiations following the β -process itself.

BETA DISTRIBUTION. A statistical distribution of the form

$$dF = \frac{1}{B(\alpha, \beta)} x^{\alpha-1} (1-x)^{\beta-1} dx, \quad 0 \leq x \leq 1.$$

Another form of the type

$$dF = \frac{y^{\alpha-1}}{B(\alpha, \beta)(1+y)^{\alpha+\beta}} dy, \quad 0 \leq y \leq \infty,$$

is sometimes known as a beta distribution of the second kind. The first form is also known as a Pearson Distribution of Type I and the second form as a Pearson Distribution of Type VI.

BETA FUNCTION. An improper integral

$$B(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx; \quad m > 0, n > 0.$$

It can be expressed in terms of the gamma function by the equation

$$B(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$

BETA-RAY SPECTRUM. See spectrum, beta-ray.

BETHE METHOD. In the theory of the slowing down of neutrons at large distances from a source, one of the effective analytical methods. The starting point of the analysis of the transport equation is the integral equation form and its goal, the elucidation of the constants in an expression such as,

$$N(x, E) \cong Ax^{-1/2} \exp \left(B \ln \left(\frac{E_0}{E} \right) - C |x| \right)$$

for the neutron density at large distance x from a plane isotropic source. Bethe's method is particularly effective at distances such that

$$\frac{\lambda}{3M^2} |x| \gtrsim 1,$$

where λ is the mean free path (assumed) independent of energy, and M^2 is the slowing down area.

BETHE-SALPETER EQUATION. The relativistic equation describing the evolution and properties of a two-particle system. Consider for example the case of two fermions, described by quantized fields $\psi^a(x)$ and $\psi^b(y)$, respectively, which interact through the medium of a boson field ϕ .

Let the equation of motions of these operators be

$$\left. \begin{aligned} (-i\gamma^\mu \partial_\mu + m_a)\psi^a(x) &= J^a(x) \\ (-i\gamma^\mu \partial_\mu + m_b)\psi^b(y) &= J^b(x) \\ (\square + \mu^2)\phi(x) &= j(x) \end{aligned} \right\} \quad (1)$$

where J^a , J^b and j are the sources of the ψ^a , ψ^b and ϕ fields, respectively, and are operator functions of ψ^a , ψ^b , ϕ . The Bethe-Salpeter amplitude for the two particle state $|\Psi\rangle$ is then defined as

$$\chi(x, y) = (\Psi_0, T(\psi^a(x)\psi^b(y))\Psi). \quad (2)$$

where T denotes the **chronological operator**, and $|\Psi_0\rangle$ is the vacuum state vector. The Bethe-Salpeter equation is the equation obeyed by this amplitude and is of the form

$$(-i\gamma^\mu\partial_{x\mu} + m_a)(-i\gamma^\mu\partial_{y\mu} + m_b)\chi(x,y) = -i\int d^4x' \int d^4y' K(x,y; x',y')\chi(x',y') \quad (3)$$

where K is a kernel determined from the right-hand side of Equation (1).

BETTI NUMBER. See **graph**, **nullity**.

BETTIS RECIPROCAL THEOREM. See **reciprocal theorem**.

BEV. Symbol for one thousand million (i.e., one billion) electron volts, a unit of energy equal to 1.602×10^{-3} erg. Sometimes denoted by **Gev**.

BIANCHI IDENTITY. The relation

$$B^i_{ajk,l} + B^i_{akl,j} + B^i_{alj,k} = 0$$

between covariant derivatives of the **curvature tensor** B^i_{jkl} .

BIAXIAL CRYSTAL. A crystal which has two distinct **optic axes**.

BIAXIAL STRESS. See **stress**, **combined**.

BIEDENHARN IDENTITY. The relation

$$\begin{aligned} \left\{ \begin{matrix} abc \\ def \end{matrix} \right\} \left\{ \begin{matrix} abc \\ d'e'f' \end{matrix} \right\} &= \sum_k (-1)^{a+b+c+d+e+f+d'+e'+f'} \\ &\cdot (2k+1) \left\{ \begin{matrix} gee' \\ aff' \end{matrix} \right\} \left\{ \begin{matrix} gff' \\ bdd' \end{matrix} \right\} \left\{ \begin{matrix} gdd' \\ cee' \end{matrix} \right\} \end{aligned}$$

between the six-j symbols of Wigner, which was derived by Biedenharn (1953) and also by Elliott (1953). It should be noted that the six-j symbol is given by Racah's symmetrized W coefficient (1957):

$$\mathbb{W} \left\{ \begin{matrix} abc \\ def \end{matrix} \right\} \equiv \left\{ \begin{matrix} abc \\ def \end{matrix} \right\}.$$

BIENAYMÉ-CHEBYSHEV INEQUALITY. If x is a random variable with mean m and variance σ^2 , the inequality states that

$$P\{|x - m| > t\sigma\} \leq 1/t^2, \quad \text{all } t > 1.$$

It is a particular case of **Chebyshev inequality**.

BIGIT. A binary digit. This term has been proposed in order to reserve **bit** for use only as a measure of information.

BIHARMONIC EQUATION. The partial differential equation

$$\partial^4\phi/\partial x^4 + 2\partial^4\phi/\partial x^2\partial y^2 + \partial^4\phi/\partial y^4 = 0$$

satisfied, for instance, by small deflections of a plate. The usual coefficients for a square mesh are in the form (see **star**)

$$\begin{array}{ccccc} & & 1 & & \\ & 2 & -8 & 2 & \\ 1 & -8 & 20 & -8 & 1 \\ & 2 & -8 & 2 & \\ & & 1 & & \end{array}$$

BILATERAL. This term has rather wide usage in the same sense as the term **reciprocal**, meaning that the reciprocity principle applies, and that the network to which the term refers is capable of transmission equally in two directions between a specified pair of ports.

BILINEAR CONCOMITANT. See **adjoint equation**; **Sturm-Liouville problem**.

BINARY. In the binary representation of numbers, the **radix** is 2, this being used in many digital computers.

BINARY MIXTURE CYCLES. Engine or refrigeration cycles which involve the use of solutions. The saturation temperature of a solution (binary mixture) is higher, at a given pressure, than that of the liquid and in this manner superheated vapor can be obtained at a lower operating pressure. For example a 84.6% KOH solution in water boils at 900°F under a pressure of 215 lbf/in², giving off steam at that temperature. Pure water boiling at that pressure could yield steam at 384°F only. The steam obtained in this way could be made to perform work in a suitable engine, and the condensate could be re-absorbed in a concentrated solution. In the process a considerable quantity of heat of absorption would be liberated, and that heat could be utilized for the evaporation of water. Binary mixture cycles have been proposed, but not actually used for the generation of power; the reverse cycle is realized in an **absorption refrigerator**.

BINARY POINT. The radix point in the representation of a number with the **radix** 2.

BINARY STARS. The term binary star was apparently first introduced by Sir William Herschel in 1802 to designate "a real double

star—the union of two stars that are formed together in one system by the laws of attraction.” When the relative motion of the components of a double star is established, systematic observations are begun. Usually one of the components is brighter than the other and this is considered as stationary and known as the *primary*. The angular distance of the secondary from the primary is called the *distance*. The direction of the secondary from the primary is known as the *position angle*. Position angle is measured from the direction of the north celestial pole toward the east through 360° .

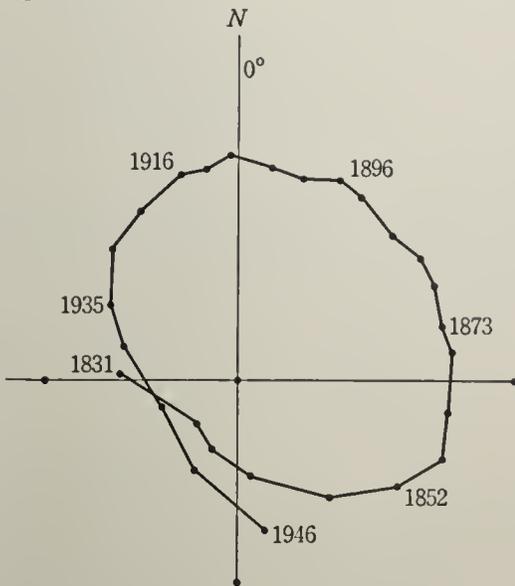


Fig. 1. Binary star—observed position.

Figure 1 shows the observations of a binary star extending from 1831 to 1946. Proper “smoothing” of the data will provide what is called the apparent orbit. This orbit will be an ellipse, and the Keplerian law of areas will be maintained by the line joining the stars in direction, length, and time. However, it is very unlikely that the primary star will be at the focus of the apparent ellipse as determined by the measurements.

To determine the apparent orbit of a binary star, i.e., the ellipse that will best fit the observations, one must first fit an ellipse to the plotted points by estimation. Figure 2 shows such an ellipse. In the fitting of the ellipse to the observations the position of the primary is ignored.

Letting the direction defining the position angle 0° be considered as the x -axis of a coordinate system and position angle 90° be the direction of the y -axis, a plane is defined that is perpendicular to the line of sight. The origin

of this system is the primary star, which is assumed to be fixed. Points on the ellipse that best fit the observations can be referred to the Cartesian system. If the length of SC is ρ and KSC is the angle θ then for C : $x = \rho \cos \theta$ and $y = \rho \sin \theta$. Five such measurements spread

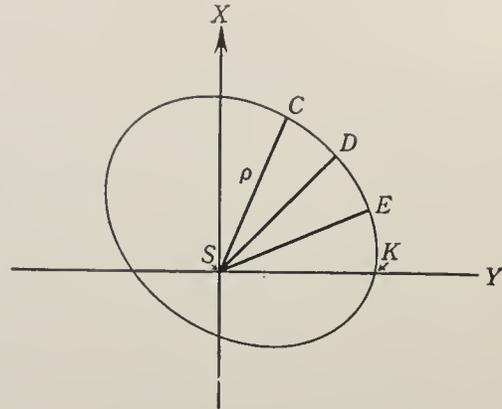


Fig. 2. Binary star—orbital ellipse fitted to observed position.

out over the ellipse are enough to determine the equation of the ellipse as $Ax^2 + 2Hxy + By^2 + 2Gx + 2Fy + 1 = 0$. In actual practice there are a great many observations and each one will yield xy coordinates to be represented. Many methods have been used by different computers. There is a further consideration to use in testing the apparent ellipse. Kepler's second law provides that equal areas are swept out in equal times by the radius vector of an ellipse. In projection the ratio of areas will be maintained. Hence, if t_1, t_2, t_3 are the times at which the companion star is at points C, D, E then the ratio of the area CSD to the area DSE must be equal to the ratio of $(t_2 - t_1)$ to $(t_3 - t_2)$. This can be carried on for points all around the ellipse, determining the areas with a planimeter. This is, of course, only an apparent orbit.

There are many methods for the determination of the true orbit of a binary star. However, they are essentially exercises in geometry and trigonometry. (C.f. **spectroscopic binary stars**; **eclipsing binary stars**.)

BINARY STARS, DYNAMIC PARALLAX OF. See **dynamic parallax (of binary stars)**.

BINARY STARS, ECLIPSING. See **eclipsing binary stars**.

BINARY STARS, SPECTROSCOPIC. See **spectroscopic binary stars**.

BINAURAL PHASE EFFECT. If the sound intensity at the ears is maintained alike and differences in phase are introduced, there is an angular displacement θ of the apparent sound source from the median plane. The relation between this angle and the phase difference ϕ is given by

$$\theta = K\phi,$$

where K depends upon frequency.

BINDING ENERGY. This term is used in atomic physics with two closely related meanings: the binding energy of a particle (or other entity) is the energy required to remove the particle from a system; the binding energy of a system is the energy required to disperse the system into its constituent entities. Explicit definitions are obviously necessary.

Some explicit definitions for the binding energies of particles are the following:

(1) The **electron** binding energy is the energy necessary to remove an electron from an atom. It is identical with the **ionization potential**.

(1a) The total electron binding energy is the energy necessary to remove all the electrons from an atom to infinite distances, so that only the nucleus remains. It is equal to the sum of the successive **ionization potentials** of that atom.

(2) The **proton** binding energy is the energy necessary to remove a single proton from a nucleus. Most known proton binding energies are in the range 5–12 mev, although that for H^2 is 2.23 mev, that for He^4 is 19.81 mev, and those for Li^5 and Be^9 are negligible.

(3) The **neutron** binding energy is the energy required to remove a single neutron from a nucleus. Most known neutron binding energies are in the range 5–8 mev, though that for H^2 is 2.23 mev, that for Be^9 is 1.67 mev, and that for C^{12} is 18.7 mev.

(4) The **alpha-particle** binding energy is the energy required to remove an alpha-particle from a nucleus. For most light nuclides the alpha-particle binding energy is positive and is equal to several mev. For nuclides of mass number about 125, it is approximately zero. For nuclides of mass number about 150 to 200, it is negative by about 1 to 3 mev, but the lifetimes for alpha-disintegration are generally too long for detection of alpha-activity. For most nuclides of mass number exceeding 200, the alpha-particle binding energy is negative

by about 4 to 8 mev, leading to observable alpha-activity.

Some explicit definitions for the binding energies of systems are:

(1) The nuclear binding energy is the energy that would be necessary to separate an atom of **atomic number** Z and **mass number** A into Z hydrogen atoms and $A-Z$ neutrons. This energy is the energy equivalent of the difference between the sum of the masses of the product hydrogen atoms and neutrons, and the mass of the atom; it includes the effect of electronic binding. (See total electron binding energy above.)

(2) The binding energy of a solid is the energy required to disperse a solid into its constituent atoms, against the forces of cohesion. In the case of ionic crystals, it is given by the **Born-Mayer equation**.

BINGHAM MATERIAL. A material exhibiting a **yield limit** σ_0 and viscous resistance to flow for stresses above this limit:

$$\begin{aligned} \dot{\epsilon} &= \frac{1}{\nu}(\sigma - \sigma_0), & \sigma > \sigma_0 \\ \dot{\epsilon} &= 0, & \sigma < \sigma_0. \end{aligned}$$

BINODALS. Consider the volume-composition diagram of a binary mixture for states corresponding to the coexistence of a liquid and of a gaseous phase. If the temperature is below the critical region (see **critical point; critical phenomena in the vaporization of mixtures**) one obtains a diagram of the form represented in Figure 1.

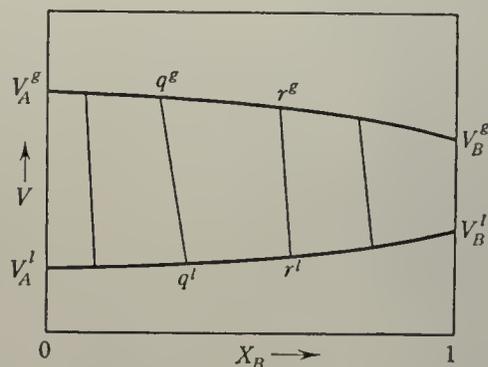


Fig. 1. Volume-composition diagram of a binary mixture below the critical region.

The line $V_A^g V_B^g$ corresponds to the molar volumes of the vapor phase, while $V_A^l V_B^l$ relates to the liquid phase. The lines such as r^l-r^g , q^l-q^g , etc., joining two phases in equilibrium are called binodals by the Dutch School

(van der Waals, Kohnstamm). In the critical region the diagram takes the form indicated in Figure 2. K is the critical point. The curve $V_A^g-K-V_A^l$ is the *saturation curve*. (See also *spinodal curve*.)

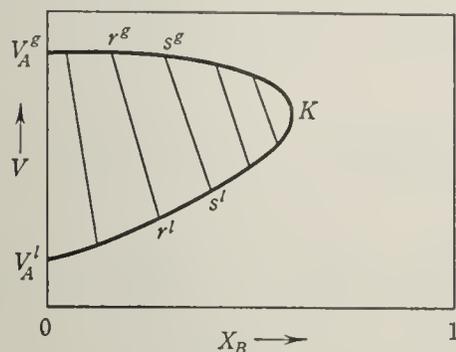


Fig. 2. Volume-composition diagram of a binary mixture in the critical region.

BINOMIAL COEFFICIENT. The coefficient of the variable in a **binomial series** $(a + x)^n$. The $(k + 1)$ th coefficient of order n is $\frac{n!}{k!(n - k)!}$,

usually designated by the symbol $\binom{n}{k}$. It equals the number of **combinations** of n things taken k at a time. Properties of the coefficients include

$$\binom{n}{0} = \binom{n}{n} = 1; \quad \binom{n}{k} + \binom{n}{k+1} = \binom{n+1}{k+1}; \quad \binom{n}{n-k} = \binom{n}{k}.$$

BINOMIAL DISTRIBUTION. If a trial can have one of two mutually exclusive results (say "success" and "failure") and if the probability p of a "success" is constant over a series of n independent trials, then the probability of obtaining r "successes" is

$$P_r = \frac{n!}{r!(n - r)!} p^r q^{n-r}$$

where $q = 1 - p$. This function is sometimes called the *Bernoulli probability function*, and the probability distribution of r is called the *binomial distribution* since the probabilities can be obtained by expanding $(q + p)^n$ by the binomial formula. The mean of the distribution is np , the variance npq . It is sometimes known as the *Bernoulli distribution*.

The *negative binomial distribution* has formally the same probability function but p

and n are negative and n is not restricted to integral values.

BINOMIAL THEOREM. The theorem that

$$(a + x)^n = \sum_{i=0}^n \binom{n}{i} a^{n-i} x^i,$$

where $\binom{n}{i}$ is a binomial coefficient.

BINORMAL. The **normal to a space curve at a point**, which is perpendicular to the osculating plane at that point. The *unit binormal* to the curve at P is a unit vector in the direction of the binormal, usually chosen so that the unit tangent, unit normal and unit binormal form a right-handed system.

BIOT NUMBER. In solid propellant rocket propulsion, a parameter describing the rate of transfer of heat from the center of a burning grain to the outer cylindrical surface. It is given by:

$$\mathfrak{B} = \frac{hR_e}{\lambda}$$

where \mathfrak{B} is the Biot number, h is the heat transfer coefficient, R_e is the external radius of grain, and λ is the thermal conductivity.

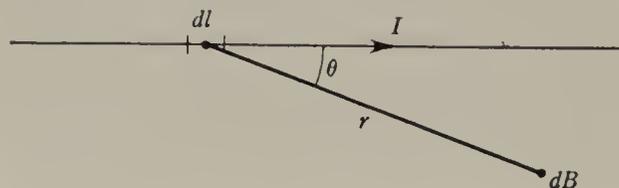
BIOT-SAVART LAW. A law giving the magnetic field in the neighborhood of a long, straight conductor carrying a steady current I . This law, discovered experimentally, may be written

$$B = \frac{\mu_0 I}{2\pi r}$$

where r is the normal distance from the conductor and I is the magnitude of the current flowing in the conductor. The above can be derived from the relationship

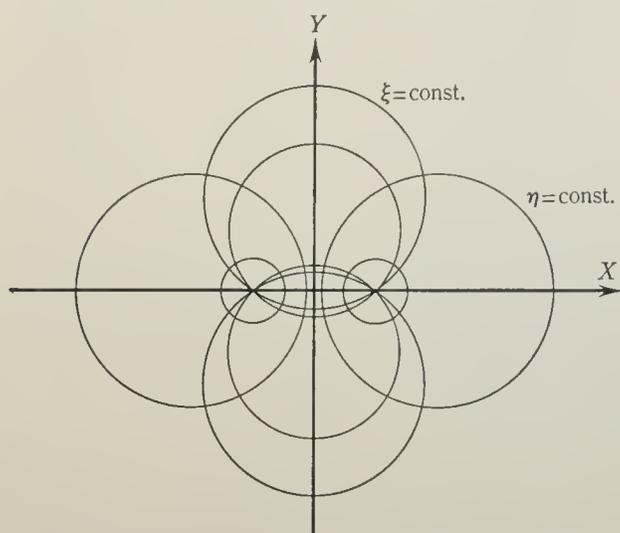
$$dB = \frac{\mu_0 I dl \sin \theta}{4\pi r^2}$$

which gives the incremental magnetic field due to a current element of length dl and strength I in terms of distance r and angle θ in the figure (dB is directed into the page).



This incremental law was first proposed by Biot, but it is often referred to as the *Ampere law* or *Ampere formula*.

BIPOLAR COORDINATES. Choose two points $\pm a$ on the X -axis of a rectangular coordinate system. Then any point in the XY -plane could be measured in either of two polar coordinate systems where the poles are at $x = \pm a$; the polar axis in both systems is the X -axis; the two polar angles are θ_1, θ_2 ; the two



Bipolar coordinate system.

radius vectors are r_1, r_2 . Define the parameters $\xi = \theta_1 - \theta_2$; $\eta = \ln r_2/r_1$ and, in terms of these parameters,

$$x = \frac{a \sinh \eta}{\cosh \eta - \cos \xi}; \quad y = \frac{a \sin \xi}{\cosh \eta - \cos \xi}$$

which are families of circles along the X - and Y -axes, respectively. Translation of these circles along the Z -axis produces the curvilinear coordinate system known as bipolar coordinates. They are families of right circular cylindrical surfaces with axes parallel to the Z -axis and centers at $y = 0, x = 0$, respectively ($\xi, \eta = \text{constant}$), where

$$0 \leq \xi \leq 2\pi; \quad -\infty \leq \eta \leq \infty.$$

The third surface is a plane perpendicular to the Z -axis ($z = \text{constant}$).

BIPRISM. An optical prism with a vertex angle only slightly less than 180° , devised by Fresnel to produce a double virtual image of a point source.

BIQADRATIC EQUATION. An algebraic equation of the fourth degree in one or more

variables, also called a quartic equation. If, as is usually the case, only one variable occurs, its general form is $a_0x^4 + a_1x^3 + a_2x^2 + a_3x + a_4 = 0$. The transformation $x = y - a_1/4a_0$ removes the term in x^3 , so that the equation becomes $a_0y^4 + b_2y^2 + b_3y + b_4 = 0$. The standard form is $y^4 + py^2 + qy + r = 0$, where $p = b_2/a_0, q = b_3/a_0, r = b_4/a_0$.

BIQUINARY. A form of number representation combining the radices 2 and 5. (See radix.)

BIREFRINGENCE. A synonym for double refraction.

BIREFRINGENCE, FLOW. In the flow of viscous liquids, composed of anisotropic molecules, the forces of shear tend to orient the molecules, leading to birefringence.

BIRTH-AND-DEATH PROCESS. A stochastic process, usually Markovian, which describes a "population" the members of which may "die" or "give birth" to new individuals. A great many processes are of this type, e.g., epidemics, sub-atomic showers and certain kinds of factory production. The limiting case where no "deaths" occur is known as a birth process.

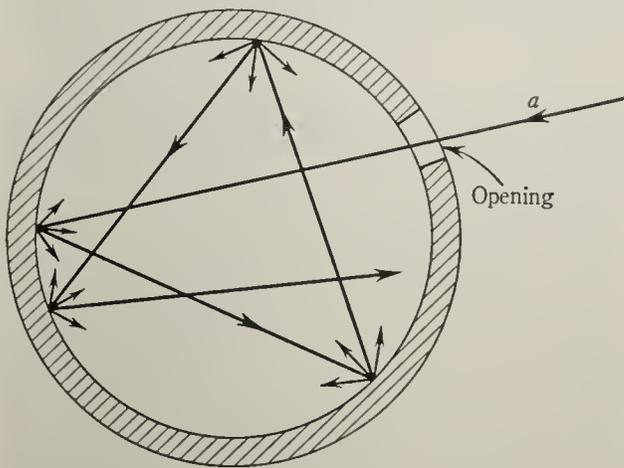
Its mathematical description is as follows: The system described by a birth-and-death process changes only through transitions from states E_n to their next neighbors E_{n+1} or E_{n-1} . If at any time t the system is in state E_n , the probability that during $(t, t+h)$ the transition $E_n \rightarrow E_{n+1}$ occurs equals $\lambda_n h + o(h)$, and the probability of $E_n \rightarrow E_{n-1}$ equals $\mu_n h + o(h)$. The probability that during $(t, t+h)$ more than one change occurs is $o(h)$. Here, the term $o(h)$ denotes a quantity which is of smaller order of magnitude than h . The definition of a birth process follows analogously.

BISERIAL CORRELATION. The correlation between two variables when one of them can take only one of two values or is a simple dichotomy. Earlier forms of biserial correlation coefficients depended on the assumption that the dichotomy was based on an underlying normal (Gaussian) distribution. The so-called point biserial coefficient assigns the values 0 and 1 to the dichotomic classification and proceeds to calculate a product-moment correlation coefficient on that basis.

BIT. (1) A unit of measure of information; viz., that amount represented by one of two alternatives. A binary digit, i.e., 0 or 1, in number representation with the **radix 2**. (2) In digital computer terminology, the term bit has been extended to denote a character (in a two-character language), an abbreviation for "binary digit," or a unit of storage capacity.

BIVARIANT SYSTEM. See **phase rule**.

BLACK BODY. A body whose surface absorbs all radiation and does not reflect any. Ideal black bodies do not exist in nature, but a very good approximation is obtained at a small opening of a hollow box, like the one shown in the figure. The interior of the hollow



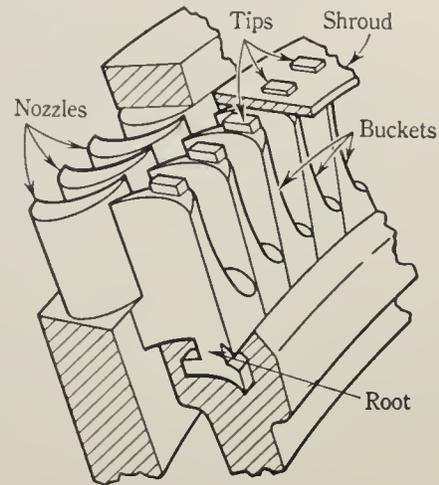
Black body.

box is made of an irregular surface of reasonably high absorptive power, and its walls are kept at a constant temperature. Any ray, say a in the figure, entering through the opening undergoes a multiple reflection before it can emerge again. At each reflection a proportion of the energy is absorbed, and consequently, the amount of energy leaving through the opening is extremely small. Hence the surface of the opening acts very nearly like that of an ideal black body. A hollow box of this type is sometimes called a "hohlraum" (German). Since the absorptivity of a black body is equal to unity, its emissivity, by the **Kirchhoff law**, is also unity, and it emits the maximum possible amount of radiation.

BLACK BODY RADIATION LAWS. See **Planck radiation formula**; **Wien displacement law**; **Stefan-Boltzmann law**; **Kirchhoff law**.

BLADE. An element of a turbine or turbo-compressor stage which effects a change in

the flow direction of the working fluid and which forms the channel for it. The part of the blade which is nearest the section at which it is fixed to the wheel is called its root, the other end is called its tip. Sometimes the tips of the blades are fixed in a shroud (see figure).



Blade details.

A two-dimensional arrangement of blade sections is called a *cascade*. The blade cross sections are made in the shape of airfoils.

BLADE EFFICIENCY. (Also known as diagram efficiency.) In a turbine or turbo-compressor stage, the ratio of the useful work per unit mass flow and time to the real **enthalpy drop** across the stage. (See **stage efficiency**.)

BLADE-SPEED RATIO. In a turbine or compressor **velocity diagram** the ratio u/V of peripheral velocity to absolute velocity at entry.

BLADE, TWISTED. In a turbine or turbo-compressor stage, the velocity diagram changes along the (finite) blade length. The angles of the flow can only coincide with the entry and exit angles of the blade at one section (usually at mid-length), at the *design diameter*. At all other sections, entry is oblique, and the degree of reaction changes along the blade length. These deviations from ideal flow are the source of losses in efficiency. In order to avoid them, the blade angles (see **velocity diagram**) may be continuously varied, and this causes the blades to be *twisted*.

The amount of twist can be calculated in two alternative ways: the *free vortex type* and the *constant reaction type*. In the free vortex type, the flow velocity in front of and behind the blade is made to vary inversely with the

radius (in an axial arrangement), in the same way as in a free vortex. In the constant reaction type the inlet and outlet blade angles are the same for the rotor and stator. The free vortex type has the advantage that there exists radial equilibrium between the pressure forces and the centrifugal forces. However, the amount of twist required is rather high, being somewhat smaller for the stator blades. The constant reaction type gives a radial increase in velocity, which is incompatible with a constant axial velocity. Equilibrium can only be established by increasing the axial velocity with decreasing radius, but the necessary twist is smaller and the same for rotor and stator blades.

BLASIUS EQUATION. (1) The differential equation

$$2f''' + ff'' = 0 \quad (a)$$

which describes the stream function

$$\psi = (\nu x U_\infty)^{1/2} f(\eta) \quad (b)$$

where

$$\eta = (U_\infty/\nu x)^{1/2}$$

in boundary layer flow past a flat plate at zero incidence (and, hence, zero pressure gradient along the flow). The function $f(\eta)$ determined by the above equation and the boundary conditions $f(0) = 0, f(\infty) = 1$, is called the *Blasius function*. Equation (a) must be integrated numerically and the Blasius function, together with its derivatives, is available in tabular form (see Schlichting, transl. Kestin, *Boundary Layer Theory*, McGraw-Hill, 1960).

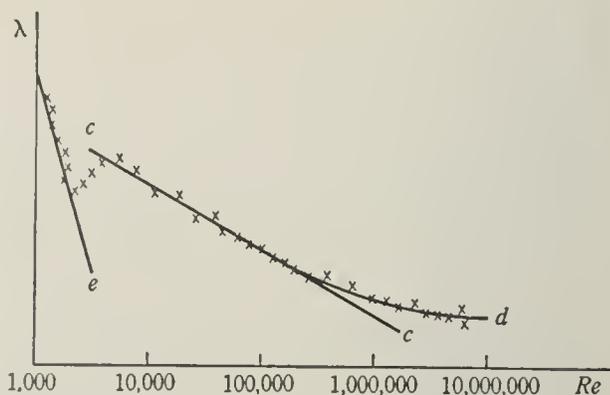
(2) The empirical formula

$$\lambda = 0.3164 Re^{-0.25} \quad (c)$$

which describes the relation between the pressure loss coefficient

$$\lambda = \frac{\Delta p}{\left(\frac{1}{2}\rho\bar{u}^2\right) \frac{l}{d}}$$

for fully developed turbulent flow in a pipe of diameter d and the Reynolds number $Re = \rho\bar{u}d/\mu$ (where Δp is the pressure loss over length l ; \bar{u} is the mean velocity; ρ is the (constant) density; μ is viscosity. The Blasius equation can be used in the range $2300 < Re < 100,000$. A comparison between the Blasius formula and experiment is shown in the figure.



Logarithmic graph of pressure loss coefficient (λ) of fluid flow in smooth pipes plotted against Reynolds number. The experimental results are denoted by the small crosses, the graph of the Blasius formula (c in accompanying article) is c, the graph of the Prandtl formula (d in accompanying article) is d, and the graph of the laminar flow formula (e in accompanying article) is e.

At higher Reynolds numbers, the loss coefficient is given by Prandtl's semi-theoretical formula

$$\frac{1}{\sqrt{\lambda}} = 2.0 \log_{10} (Re\sqrt{\lambda}) - 0.8 \quad (d)$$

and in laminar flow, the theoretical solution gives

$$\lambda = \frac{64}{Re} \quad (e)$$

All formulae are valid for smooth pipes only. (See **lift and moment on an airfoil, general formulae**.)

BLASIUS FORMULA. See Blasius equation.

BLAST LOADING. The forces applied to a structure by the pressure waves emanating from an explosion are called the blast loading. Both reflection and diffraction must be taken into account. The peak pressures may be several times those of the incident shock, but last only a short time. Drag or dynamic pressure loading behind the shock front may be more damaging.

BLEEDING. The action of extracting steam between the stages of a turbine, in particular for the purpose of feed-water heating. (See **carnotization**.)

BLOCH FUNCTION. It can be shown that the wave function of an electron in a periodic lattice has the form

$$\psi = u(\mathbf{r})e^{i\mathbf{k}\cdot\mathbf{r}}$$

where $u(\mathbf{r})$ has the periodicity of the lattice (i.e., is the same in every unit cell) and \mathbf{k} is the **wave vector** of the electron.

BLOCH THEOREM. (1) The lowest state of a quantum-mechanical system in the absence of a magnetic field can carry no current; an important result for the theory of **superconductivity**. (2) The statement that every electronic wave function in a periodic structure can be represented by a **Bloch function**. (Bloch's proof was general for solutions of the Schrödinger equation with a periodic potential.)

BLOCK. In computer practice, a group of words considered as a unit.

BLOCKAGE CORRECTION. See **wind tunnel interference**.

BLOCK RELAXATION. A variant of the method of **relaxation** for solving linear systems in which one solves two or more equations of the set simultaneously for the corresponding unknowns in terms of current approximations to the others. This generally speeds convergence if convergence occurs.

BLOW-DOWN. The sudden reduction in pressure inside a reciprocating engine cylinder following the opening of the exhaust valve or ports.

BLOWER. A device for supplying a stream of air at relatively low pressure in which compression is performed by a rotating member. Compression may be performed either by dynamic action, as in a **turbo-compressor**, or by positive displacement, as in a **Roots blower**.

BLUFF BODY, FLOW PAST. A bluff body may be defined, rather loosely, as a body on which the positive pressure gradient is large enough to cause separation of the boundary layer. This definition is a loose one, because in general the conditions for separation depend on the Reynolds number, and in particular on whether the boundary layer is laminar or turbulent. Nevertheless, the definition is convenient, serving to distinguish broadly between this class of body and the class of *streamline bodies* on which separation does not occur upstream of the trailing edge or tail.

The drag of a bluff body is relatively large, because the rise of pressure towards the rear of the body, which would occur in potential

flow, does not occur in a real fluid downstream of separation. Thus there is a relatively low pressure at the rear of the body and a large drag due to the distribution of normal pressure. The surface-friction drag is of the same order as for a streamline body and is usually almost negligible in comparison with the normal-pressure drag.

Because a turbulent boundary layer separates less readily than a laminar one, the position of separation on a bluff body is further downstream when the boundary layer is turbulent at separation than when it is laminar. Thus with a turbulent boundary layer the pressure at the rear of the body is higher, and the drag coefficient is lower, than with a laminar boundary layer. This leads to a considerable reduction of drag coefficient as the Reynolds number is increased through the *critical Reynolds number*, at which the boundary layer at separation changes from the laminar to the turbulent state. At Reynolds numbers a little below this critical value, slight roughening of the surface may cause the boundary layer to become turbulent before separation and hence lead to a large reduction of drag.

(See also **drag; separation of boundary layer**.)

BLUNDER. Any malfunction, human or otherwise, such as faulty arithmetic, use of an incorrect formula, etc., in a computation. The term blunder is also applied to the result of such a malfunction. (See **error**.)

BLUR CIRCLE. See **circle of least confusion**.

BODE DIAGRAM. See **frequency response representation**.

BODE LAW. In the latter part of the 18th century an empirical relationship was noticed between the mean distance of the various **planets** from the sun. This relationship was first published by Bode in 1772 and has since become known as Bode's law, in spite of the fact that there is certain evidence that it was known and used by Titius a number of years previous to the time of its announcement.

Bode's Law may be stated as follows: write down a series of 4's; to the first one add 0, to the second one add 3, to the third one add $6 = 3 \times 2$, to the fourth one, $12 = 6 \times 2$, to the fifth, $24 = 12 \times 2$, etc.; the resulting numbers divided by 10 will give the approximate

mean distances of the planets from the sun in **astronomical units**. The sequence is as follows:

Planet	Bode Distance	Mean Distance
Mercury.....	4 + 0 = 4	0.39
Venus.....	4 + 3 = 7	0.72
Earth.....	4 + 6 = 10	1.00
Mars.....	4 + 12 = 16	1.52
	4 + 24 = 28	
Jupiter.....	4 + 48 = 52	5.20
Saturn.....	4 + 96 = 100	9.53
Uranus.....	4 + 192 = 196	19.19
Neptune.....	4 + 384 = 388	30.07
Pluto.....	4 + 768 = 772	39.5

The value in the last column is the actual mean distance of the planet from the sun in astronomical units.

BODE RELATIONSHIP. See frequency response representation.

BODY FORCES. Forces on matter not resulting from inertia or contact with adjacent matter, e.g., gravitational or electromagnetic forces. They are usually assumed to be proportional to the mass to which they are applied.

BÖHMER INTEGRALS. See Cornu spiral.

BOHR FREQUENCY CONDITION. The assumption made by Bohr, in extension of Planck's quantum theory, that the frequency ν' (sec^{-1}) of radiation absorbed or emitted by an atom is proportional to the energy difference $E_1 - E_2$ between two quantum states, according to the relation

$$h\nu' = E_1 - E_2$$

where h stands for Planck's constant.

BOHR MAGNETON. The Bohr magneton (sometimes referred to as electronic Bohr magneton) is the unit used to measure the magnetic moment due to the orbital motion and the spin of the electron. The Bohr magneton is defined (in emu) as:

$$\mu_o = \frac{he}{4\pi mc} = 0.92731 \times 10^{-20} \text{erg gauss}^{-1}.$$

It is equal to the magnetic moment of an electric point charge moving in an orbit with angular momentum $h/2\pi$. Here e and m stand, respectively, for the electronic charge and mass, c for the velocity of light, and h for Planck's constant.

BOHR ORBITS. The orbits of the electron about the nucleus in Bohr's original model of the hydrogen atom. (See **electron orbits in an atom**.)

BOHR RADIUS. The radius of the innermost circular electron orbit (in non-rationalized c.g.s. units)

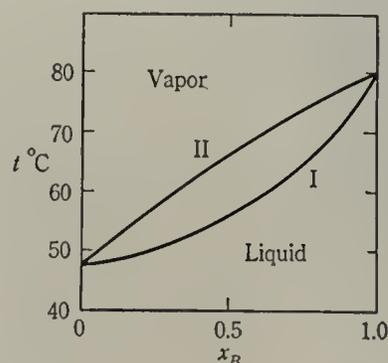
$$a_0 = \frac{h^2}{4\pi^2 m e^2} = (5.29172 \pm 0.00002) \times 10^{-9} \text{ cm}$$

in Bohr's model of the hydrogen atom, sometimes also referred to as first Bohr radius. Here h stands for Planck's constant, e and m respectively for charge and mass of the electron. The Bohr radius is often used as a unit of length in atomic physics.

BOILER. A device for the production of a flow of steam at high pressure and temperature for use in heating, in the production of power with the aid of a turbine, etc.

BOILING. The process of change of phase from liquid to gaseous states, also referred to as *evaporation*. The opposite process is called *liquefaction*. Sometimes a distinction is made between boiling, which involves the vigorous production of bubbles in the liquid, and evaporation which occurs mainly at the surface of the liquid and proceeds much more calmly.

BOILING CURVE AND CONDENSATION CURVE. Consider the phase diagram of a binary system forming a liquid and a vapor phase at constant pressure. Curve I is the boiling curve, which gives the coexistence temperature as a function of liquid composition; and curve II is the condensation curve, which gives the coexistent temperature as a function of the composition of the vapor phase. If the



Temperature-composition of a liquid-vapor system at constant pressure.

temperature is increased, vaporization begins when the boiling curve is crossed. Inversely, condensation begins when the temperature is decreased below the condensation curve.

BOILING POINT. The temperature at which boiling occurs for a given pressure. Sometimes the pressure is specified as that of 1 atmosphere. (See *phase change*.)

BOILING POINT CONSTANT. Consider a dilute solution in which all solute species may be regarded as non-volatile. The vapor in equilibrium with the solution is then formed from the solvent only. Call T° the boiling point of the pure solvent at the pressure concerned, and T the boiling point of the solution. For a dilute solution the difference

$$\theta = T - T^\circ \quad (1)$$

will be small compared with T° .

If the solution is also ideal one has

$$\theta = \frac{R(T^\circ)^2}{\Delta_e h^\circ} \frac{M_1}{1000} \sum_s m_s = \theta_e \sum_s m_s. \quad (2)$$

M_1 is the molar mass of the solvent, $\Delta_e h^\circ$ its latent heat of vaporization in kcal per mole at temperature T° , m_s the **molality** of solute s ; θ_e is called the *boiling point constant*, or *ebullioscopic constant*. It depends only on the properties of the solvent. For water

$$\theta_e = 0.51^\circ\text{C}.$$

BOLTZMANN CONSTANT. The multiplying constant, k , in the relation between **entropy** of a state and the logarithm of the probability of that state, or in the relation between the average kinetic energy and the absolute temperature. The Boltzmann constant is also equal to the ratio of the **gas constant** to the **Avogadro constant**. Its value is 1.3803×10^{-16} g cm² sec⁻² degree C⁻¹.

BOLTZMANN DISTRIBUTION. See *Maxwell-Boltzmann distribution*; *Boltzmann principle*, *distribution*.

BOLTZMANN EQUATION. See *transport equation*.

BOLTZMANN FACTOR. (1) The temperature-dependent **statistical weight** factor $e^{-\beta E}$ for a particle of energy E in a system at a temperature $1/k\beta$ (k is the Boltzmann constant). (See also *Maxwell-Boltzmann distribution*.) (2) A correction factor applied to

calculated line intensities in spectra due to thermal excitation.

BOLTZMANN FORMULA. See *Boltzmann principle*.

BOLTZMANN H-THEOREM. See *H-theorem*.

BOLTZMANN-PLANCK EQUATION. The relation between **entropy** S and number of **complexions** W ,

$$S = k \ln W,$$

where k is the **Boltzmann constant**.

BOLTZMANN PRINCIPLE (DISTRIBUTION). A somewhat general law relating to the statistical distribution of large numbers of minute particles subject to thermal agitation and acted upon by a magnetic, and electric, or a gravitational field, or by inertia. The number of particles, N , per unit volume in any region of the field, when the system is in statistical equilibrium, is given by the equation

$$N = N_0 e^{-\beta E}.$$

Here N_0 is the number of particles per unit volume in a region of the field where E , the potential energy of the particle, is zero, and where $e^{-\beta E}$ is the **Boltzmann factor**, which is temperature-dependent. (See *Maxwell-Boltzmann distribution*.)

BOLTZMANN STATISTICS. In the case of a system of independent particles, there will be $N!/n_1!n_2! \cdots$ different **microstates** corresponding to the same **macrostate** characterized by the numbers n_i of particles in the i -th single particle level, provided no symmetry considerations are taken into account. The macrostate characterized by a set of n_i will then have a weight proportional to $\Pi_j (n_j!)^{-1}$. This corresponds to Boltzmann statistics.

BOLTZMANN TRANSPORT EQUATION. See *transport equation*, *Boltzmann*.

BOLTZMANN TRANSPORT EQUATION FOR NEUTRONS. See *neutron transport theory*.

BOLTZMANN-VLASOV EQUATIONS. The basic equations which govern a high temperature plasma in which the collisional mean free path is much greater than all the characteristic lengths of the system, first used by Vlasov. They are

$$\frac{\partial f_j}{\partial t} + \mathbf{v} \cdot \nabla f_j + \frac{e_j}{m_j} \left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} \right] \cdot \nabla \mathbf{v} f_j = 0 \quad (1)$$

$$\nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t} \quad (2)$$

$$\nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} \quad (3)$$

$$\nabla \cdot \mathbf{B} = 0 \quad (4)$$

$$\nabla \cdot \mathbf{E} = 4\pi\sigma \quad (5)$$

$$\mathbf{j} = \sum_j e_j \int f_j \mathbf{v} d^3\mathbf{v} \quad (6)$$

$$\sigma = \sum_j e_j \int f_j d^3\mathbf{v}. \quad (7)$$

Equation (1) is the usual Boltzmann equation for the one-particle distribution functions f_j of charge and mass e_j and m_j respectively. The electromagnetic fields \mathbf{E} and \mathbf{B} are determined by the usual Maxwell Equations (2)–(5). The current density \mathbf{j} and the charge density σ are given by Equations (6) and (7). The summations in (6) and (7) are over all particle species.

BOLZANO-WEIERSTRASS THEOREM. If A is a bounded set containing infinitely many points in a metric space S , then A has at least one limit point. This theorem is of fundamental importance in analysis, in the case, for example, that S is the set of real numbers.

BOND ANGLE. In a polyatomic molecule, the angle formed by the lines connecting one nucleus with two others, that is, the angle between two chemical bonds.

BOND CHARACTER. Empirical additive rules and tables of atomic radii have been used to predict the skeleton size of various molecules (see **atomic radius; bond length**). For the atoms which can form single, double or triple bonds, a different radius must be used for each kind of bond.

In some cases, the observed deviation from this additive rule can be attributed to a *change in bond character* caused by **resonance** with neighboring bonds. (For instance, the C—Me distance in toluene is smaller than the C—C distance in diamond.) The bond character can be estimated from the observed interatomic distance, and from a plot of bond length versus bond character for substances

whose bond character is unambiguous; examples are diamond, graphite, benzene, and ethylene, with respective double bond characters of 0, $\frac{1}{3}$, $\frac{1}{2}$, and 1.

BOND, CONJUGATED DOUBLE. The alternation of single and double bonds. By electron transfer the positions of the single and double bonds can be exchanged, leading to an exchange energy.

BOND, COORDINATE. See **coordinate bond**.

BOND, COVALENT. See **homopolar bond**.

BOND, ELECTROSTATIC. See **heteropolar bond**.

BOND, ELECTROVALENT. See **heteropolar bond**.

BOND ENERGIES. It has been suggested that ${}_a Q_f^\circ$, the heat of formation of a molecule from its constituent atoms (see **atomic heat of formation**), could be computed from a table of average bond energies, and the assumption of additivity:

$${}_a Q_f^\circ = \sum_{\substack{\text{all types} \\ \text{of bonds}}} n_{X_i-X_j} \cdot E_{X_i-X_j}$$

$n_{X_i-X_j}$ is the number of bonds between the two atomic species, X_i and X_j , in the molecule. $E_{X_i-X_j}$ is the average bond energy associated with each of these bonds. Fairly accurate predictions of the heats of formations of organic molecules can be made in this way, particularly for the *larger* hydrocarbons, alcohols and other aliphatic derivatives.

The differences between the observed heats of formation of *cis-trans* isomers and of branched and unbranched hydrocarbon chains show that the additivity rule is not strictly rigorous. Various improvements have been suggested.

It is only in the case of diatomic molecules that bond energies and dissociation energies (see **dissociation energies**) are equivalent. In the simple case of the H_2O molecule, the dissociation energies of the two successive steps $\text{H}_2\text{O} \rightarrow \text{H} + \text{OH} \rightarrow \text{H} + \text{H} + \text{O}$, differ by about 10%.

BOND ENERGY, ATOMIC. See **atomic bond energy**.

BOND, HETEROPOLAR. See **heteropolar bond**.

BOND, HOMOPOLAR. See **homopolar bond**.

BOND, HYBRID. See **hybridization of eigenfunctions**.

BONDING ORBITALS. In the molecular orbital theory (see **molecular orbital method**) of the chemical bond, when two atoms are brought together their electrons move into characteristic orbitals of the molecule. These electrons and orbitals are called either bonding or anti-bonding depending on whether they strengthen or weaken the resulting chemical bond. If their effect on the bond energy can be neglected, they are non-bonding orbitals or non-bonding electrons.

For example, the two electrons in the fundamental state of the H_2 molecule are bonding electrons. But when two H atoms with electrons of parallel spin are brought together, one electron only goes into a bonding orbital. The other moves into a strongly anti-bonding orbital, leading to an unstable molecule. The inner K orbitals in the Li_2 or Na_2 molecules are non-bonding. (See **bond types**.)

BOND, IONIC. See **heteropolar bond**.

BOND LENGTH. Distance between the nuclei of the two atoms participating in a bond. (See **atomic radius**; **bond character**; **mobile bond order**.)

BOND ORDER. See **mobile bond order**.

BOND, π - π . A covalent bond resulting from the interaction of two electrons having π -wave functions. The resulting wave function must be symmetrical with respect to an interchange of the two electrons, i.e., the wave functions of the two electrons must point in the same direction. This leads to a rigidity of the bond: the terminal groups of atoms are not free to rotate with respect to one another. A good example of this rigidity is the *cis-trans* isomerism for C=C double bonds.

BOND, σ - σ . A covalent bond resulting from the interaction of two electrons having σ -wave functions. In view of the symmetry character of these wave functions, the terminal groups of atoms should be free to rotate with respect to one another around the bond axis. (See, however, **hindered rotation**.)

BOND STRESS. The effectiveness of reinforcement, such as that of steel in concrete or

fibers in reinforced plastic, depends upon the transmission of load from the weaker material to the reinforcement. If this transfer takes place inside the member (as contrasted with post-stressing technique) it must be by shear on the surface of the reinforcement. This shear stress is called bond stress and often is the limiting feature. The smaller the diameter of the reinforcement for a given cross-sectional area of rod, wire or fiber, the lower the bond stress.

BOND TYPES. One may classify the various types of interatomic (and intermolecular) bonds occurring in nature into 4 broad categories:

(1) *Electrostatic bonds* resulting from Coulomb forces between positive and negative ions (ionic bonds) or between dipoles or multipoles. The forces are non-directional; the **coordination number** is high. (See **heteropolar bond**.)

(2) *Covalent bonds* in which the atoms share one or more electrons; in some cases, each atom provides valency electrons (*pure covalent bond*); in other cases, the valency electrons are provided by one of the atoms only (*coordinate bond*). The forces are directional and the coordination number usually low. (See **homopolar bond**.)

(3) *Metallic bonds*. The attractive forces result from the exchange interaction of the electron gas with the ionic lattice. The forces are non-directional, and the coordination number high.

(4) *Van der Waals bonds*. Van der Waals forces are important in molecular crystals. They result from London **dispersion forces**. They are non-directional, and have a high coordination number.

Perfect cases of the three first types are seldom encountered in nature, but bonds exhibiting virtually all degrees of transition between them exist. In most cases, however, one or the other of these limiting cases is a better first approximation for the calculation of the bond energy. The bond is said to be primarily electrostatic, covalent, or metallic in character.

(See also **atomic bond energy**; **valence forces**, **subsidiary**; **valency**, **directed**.)

BOOLEAN ALGEBRA. A distributive lattice which has universal bounds and has complements. (A lattice is distributive if $a(b + c)$

$= ab + ac$ and $a \times bc = (a \times b)(a \times c)$, has universal bounds if it contains elements O and I with $0 \leq a \leq I$ for all a , and has complements if for every a there exists an a' such that $aa' = O$ and $a + a' = I$.) Boolean algebra can be characterized in many other, equivalent ways. The subsets a, b, \dots of a set of objects S form a Boolean algebra if ab denotes the intersection and $a + b$ the union of a and b . The algebra of statements $a, b, c \dots$ with connectives "and," "or," "not" form a Boolean algebra if $a + b$ means a or b (including the possibility of both), while ab means a and b . Boolean algebras are important in the design of computing machines.

BORE. See hydraulic jump.

BORN AND VON KÁRMÁN THEORY. This theory followed the Debye theory of specific heats, in which a solid is treated as a continuous medium with a characteristic maximum frequency ν_{max} . In the Born and von Kármán theory, the solid is treated as an infinitely extended vibrating atomic lattice in which the vibrations of any given particle influence only immediately adjacent particles. This results in an energy equation of the form

$$E = \sum_p^{1,2,3} \iiint_0^{2\pi} \frac{N}{2\pi^3} f(\nu_p) d\phi d\psi d\chi$$

where

$$f(\nu_p) = h\nu / [\exp(h\nu/kT) - 1]$$

and ϕ, ψ, χ , are phase differences in the vibrations of neighboring points at a given instant, N is the number of atoms in the lattice, and k is the Boltzmann constant. There are three sums for the three types of vibration (one longitudinal and two transverse).

BORN APPROXIMATION. A quantum mechanical approximation method used particularly for the computation of cross sections in scattering problems. This approximation can be used when the relative kinetic energy of two particles is large compared to their interaction energy. The approximation consists of the assumption that the wave function of the system is made up of an incident plane wave, e^{ikz} , plus a spherical scattered wave, $v(\mathbf{r})$, with $|v(r)| \ll 1$. Qualitatively, this approximation is based on the idea that the large size of the relative kinetic energy compared to the interaction energy insures that the distortion of the incoming plane wave by the potential

can be neglected and that the weak potential will scatter a very small part of the incident wave.

BORN-HABER CYCLE. A thermodynamic cycle of operations used to obtain the electron affinities of certain atoms. It may be illustrated by consideration of the metal halides, when the cycle includes:

(1) A break-up of the crystal into ions, with attendant energy change U , the lattice energy. (See **Born-Mayer equation**.)

(2) Change of the ions formed into atoms, with energy changes of I , the ionization energy for the positive ion; and E , the electron affinity, for the negative ion.

(3) Condensation of the gaseous atoms into (a) a metallic solid, involving an energy change S , where S is the heat of sublimation per gram-mole, and (b) a halogen molecule, with energy change $\frac{1}{2}D$, where D is the dissociation energy.

(4) Reaction of metal and halogen to form the metal halide, the heat of reaction being Q . Then

$$E = Q + S + \frac{1}{2}D - U.$$

BORN-INFELD THEORY. A non-linear modification of Maxwell's equations. The Maxwell equations themselves remain unchanged, but ϵ and μ are related by the equations

$$\mu = \frac{1}{\epsilon} = \sqrt{1 + \frac{\mathbf{B}^2 - \mathbf{E}^2}{b^2}}.$$

(In a later modification

$$\mu = \frac{1}{\epsilon} = \sqrt{1 + \frac{\mathbf{B}^2 - \mathbf{E}^2}{b^2} + \frac{(\mathbf{E} \cdot \mathbf{B})^2}{b^4}})$$

b is a new universal constant, viz., the absolute electrostatic field at the center of a point electron. The electrostatic field of a point electron of charge e at a distance r is

$$\mathbf{E} = \frac{e}{\sqrt{r^4 + \frac{e^2}{b^2}}}.$$

The total electrostatic energy W is finite, viz.,

$$W = 1.2361 \left(\frac{e^2}{a} \right)$$

where

$$a = \sqrt{\frac{e}{b}} \text{ ("electronic radius").}$$

The theory is not unique and does not lead to new results which may be compared with those of experiment.

BORN-MAYER EQUATION. An equation which expresses the lattice energies of crystals in the form of the equation

$$U = \frac{Z^2 e^2 N A}{r} - b \exp(-rp) + \frac{c}{r^6} + \epsilon_0$$

where r is the interatomic distance. The first term represents the **Coulomb energy**, where Ze is the ionic charge, $2N$ is the number of ions in the crystal, and A , the **Madelung constant**, which varies with crystal structure. The second term allows for repulsive forces, the factor p being calculated from results of measurement of the compressibility of the material. The third term accounts for the **van der Waals forces**, while the final term ϵ_0 represents the **zero-point energy**.

BORN-OPPENHEIMER METHOD. An argument for calculating the **force constants** between atoms in a molecule or solid, based on the observation that the motion of the electrons is so rapid compared with that of the heavier nuclei that it can be assumed that the electrons follow the motion of the nuclei adiabatically. That is, one calculates the eigenvalues of energy for the electrons with the nuclei in fixed positions; the variation of this electronic energy with the configuration of the nuclei may then be treated as a contribution to the potential energy of the interatomic forces.

BORN REPULSION. The interaction of the electron shells of two atoms leads to a strong repulsion at sufficiently short range (**Pauli exclusion principle**). This effect, which is difficult to calculate, was represented empirically by Born by an additive term c/r^n in the potential energy of interaction for his ionic lattice model. (See **Born-Mayer equation**.) A similar term was used by Lennard-Jones to represent the intermolecular repulsion at close range. (See **Lennard-Jones potential**; **index of repulsion**.)

BOSANQUET LAW. A relationship in magnetism analogous to the **Ohm law** in electricity. It is of the form

$$\text{Flux} = \frac{\text{magnetomotive force}}{\text{reluctance}}; \quad \phi = \mathfrak{F}/\mathcal{R}.$$

Ignoring frequency, the reluctance of an air gap of length l and cross-sectional area A is

$$\mathcal{R}_g = \frac{l}{\mu_v A}$$

while that of the core of length L is

$$\mathcal{R}_c = \frac{L}{\mu_s \mu_v A}$$

making the total: $\mathcal{R} = \mathcal{R}_g + \mathcal{R}_c$. The magnetomotive force, in rationalized units, is $\mathfrak{F} = NI$. Using mksa units, l is in meters, A in square meters, I in amperes, \mathfrak{F} in ampere-turns, and the flux (ϕ) in webers.

BOSE-EINSTEIN DISTRIBUTION. (1) In a gas composed of a fixed number of particles that obey **Bose-Einstein quantum statistics**, the number of particles, n_i , in a state of energy, E_i , is given by

$$n_i = 1/[Ae^{E_i/kT} - 1],$$

where A is a constant that depends on the total number of particles present. Gases composed of particles with integral spin, such as helium of mass number 4, obey this distribution law.

(2) In applying the Bose-Einstein statistics to photons, the assumption of a fixed number of photons cannot be made. This fact leads to a slightly different distribution function than the one above, namely the constant A is unity, i.e.,

$$n_i = 1/[e^{E_i/kT} - 1].$$

This distribution law leads to the **Planck radiation formula** for black bodies.

BOSE-EINSTEIN GAS. Systems of non-interacting or only weakly interacting **bosons**. A perfect Bose-Einstein gas shows the phenomenon of **Einstein condensation**. A system of weakly interacting bosons can represent **helium gas**. Of special interest in that connection is the so-called **hard sphere boson gas** which shows a transition similar to the **lambda transition** of helium.

BOSE-EINSTEIN LIQUID. Liquid whose constituent particles are **bosons**. The best known example is ordinary **liquid helium**.

BOSE-EINSTEIN STATISTICS. In the case where the particles in a system of independent particles are described by a symmetrical wave function, there will be only one possible **macrostate** characterized by a set of numbers n_i of particles in the i -th single particle level, namely, the one corresponding to a symmetric superposition of all possible **microstates**. This means that in Bose-Einstein statistics every macrostate will have the same weight. (See also **quantum statistics**.)

BOSON. The generic name for the elementary particles obeying **Bose-Einstein statistics**. The connection of spin with statistics establishes that all bosons have integral spin or zero spin.

BOSONS, COMMUTATIVE RULES FOR. See **commutative rules for bosons**.

BOUGUER LAW. The law discovered by Bouguer in 1729, that the absorption of light in homogeneous materials depends on thickness according to the exponential expression given under **absorption coefficient**. The law was rediscovered by Lambert and is often referred to as the Lambert law of absorption. It is also related to the Beer law, which is obtained by setting the absorption coefficient $a = ac$. When the law is expressed in terms of common logarithms:

$$I = I_0 10^{-kx},$$

k is known as the *extinction coefficient*.

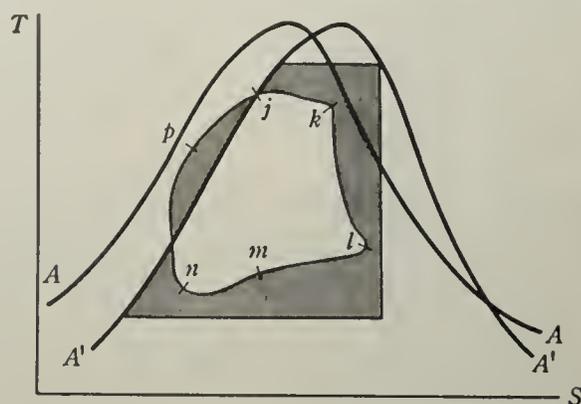
These laws may be applied to a narrow band of wavelengths, giving the *specular coefficient of absorption* or the *specular extinction coefficient* according to the form of the equation used. They do not hold for broad spectral bands if the coefficients depend appreciably on the wavelength.

BOULVIN DIAGRAM. The sequence of processes in a reciprocating steam engine cannot be drawn on a T,S diagram because the mass of steam contained in the engine cylinder varies during one cycle. It is much smaller during compression than during expansion, and varies continuously during admission and exhaust. The difficulty can be circumvented if certain more or less arbitrary assumptions, suggested by Boulvin, are made. It is assumed

that the cylinder is filled with a mass $m + m_r$ (m is the mass of steam admitted; m_r is the mass of residual steam) during the whole of the cycle, and that the processes in the boiler and condenser take place in the cylinder. It is imagined that there is no exhaust, but that the steam condenses in the cylinder, the heat of condensation being rejected through the walls, and the volume of steam being reduced practically to zero. Similarly, it is imagined that there is no admission but that the condensate evaporates in the cylinder. In this way an imaginary cycle with constant mass $m + m_r$ is assumed to be performed.

The mass of steam m admitted is calculated from the measured steam consumption, and the mass of residual steam, m_r , is calculated under the assumption that it is dry saturated at the beginning of compression.

A Boulvin diagram is shown in the figure. In it, $jklmnp$ represents the indicator diagram



Boulvin diagram.

cross-plotted from a p,V diagram. Line A is the boundary curve for mass m , and line A' is the boundary curve for mass $m + m_r$. The shaded areas represent the quantities of heat exchanged with the cylinder walls.

The Boulvin diagram illustrates the nature of the exchanges of heat which takes place in the engine cylinder. Its importance is primarily historical, owing to the fact that reciprocating steam engines are at present confined to marginal applications only.

BOUND. Let S be a set of real numbers, e.g., a sequence, the values of a function, etc. A number M , if it exists, which is greater than or equal to any member of S , is an upper bound for S , and the least such upper bound if it exists is called the *least upper bound* (l.u.b.) of S , and similarly for lower bounds.

BOUNDARY CIRCLES. In impedance matching, circles of constant standing-wave ratio which represent the boundaries of all transformation circles which may be transformed to the definition circle.

BOUNDARY CONDITION. A specified requirement for the solution of a given differential equation or set of equations at a given set of values of the independent variables.

BOUNDARY CONDITION, KINEMATIC. See kinematic boundary condition.

BOUNDARY CONDITIONS, MARK. In the analysis of the Milne problem in transport theory by means of the spherical harmonics method, the (vacuum) boundary condition—in the n^{th} order approximation—which states that the angular flux vanishes at $\frac{1}{2}(n+1)$ positive values μ_i . The μ_i are given by the positive roots of $P_{n+1}(\mu) = 0$, where $P_n(\mu)$ is the n^{th} Legendre polynomial. These conditions were first propounded by C. Mark.

BOUNDARY CONDITIONS, MARSHAK. In the analysis of the Milne problem in transport theory by means of the spherical harmonics method, the (vacuum) boundary condition—in the n^{th} order approximation—which states that the first $\frac{1}{2}(n+1)$ odd moments of the returning flux vanish. These conditions were first propounded by R. E. Marshak.

BOUNDARY CONDITIONS, NEUTRON DIFFUSION THEORY. Across an interface between two media, the neutron current, as given by Fick's Law, and the neutron flux are assumed continuous. At an outer boundary the neutron flux does not vanish; it is assumed to vanish at a point, the extrapolated end-point (z_0), lying outside the boundary. Alternatively, one may say that the magnitude of the logarithmic derivative of the flux, evaluated at the boundary, is equal to the reciprocal of a length, the linear extrapolation length, l . In general $l \neq z_0$.

BOUNDARY CONDITIONS, NEUTRON TRANSPORT THEORY. In moving across an interface between two media in direct contact, a packet of neutrons suffers no change. More precisely $N(\mathbf{r} + R\hat{\Omega}, E, \hat{\Omega})$ is a continuous function of R for $\mathbf{r} + R\hat{\Omega}$ at the interface. At an outer boundary of a geometrically simple medium $N(\mathbf{r}, E, \hat{\Omega}) = 0$ for all $\hat{\Omega}$ directed into the medium, and r on the surface.

BOUNDARY CONDITIONS ON THE ELECTROMAGNETIC FIELD VECTORS.

(1) At an uncharged boundary between two media the normal components of the magnetic flux density \mathbf{B} , and of the electric displacement \mathbf{D} are continuous. If the boundary carries a surface charge, then the difference in the normal components of \mathbf{D} is equal to the surface charge density. (2) Also the tangential component of the electric field intensity \mathbf{E} is continuous at a boundary, and the tangential component of the magnetic field intensity \mathbf{H} is continuous if the boundary carries no surface current. If there is a surface current, the difference in the tangential components of \mathbf{H} on the two sides of the boundary is the surface current per unit length flowing along the boundary.

BOUNDARY CONDITION, STANDARD. When applied to the Schrödinger equation of quantum mechanics, this condition requires that the wave function and its derivative vanish at infinity.

BOUNDARY CURVE. See saturation curve.

BOUNDARY LAYER. The Navier-Stokes equations for flow of a viscous fluid are non-linear, and solutions can only be obtained for a few special cases. For this reason, much theoretical work in fluid dynamics has been concerned with a hypothetical *ideal fluid*, which differs from real fluids in two important respects. The ideal fluid is assumed to have zero viscosity, so that there can be no shear stresses, and in addition it is assumed that it can have a non-zero velocity at a solid boundary, i.e., that it can slip at the boundary. Real fluids have viscosity, so that shear stresses are developed, and they must have zero velocity of slip at a solid boundary (except for gases at very low densities).

With a fluid of zero viscosity the condition of no slip at a boundary can only be satisfied by introducing a vortex sheet at the boundary. In a real fluid, with viscosity, there is an outward diffusion of vorticity from the boundary into the fluid. At high Reynolds numbers, however, the rate of outward diffusion of vorticity is small compared with the rate of convection with the stream, so that the vorticity is confined to a layer, the boundary layer, whose thickness is small compared with the length of the body. Thus, in the important

case of a body in a uniform stream, the flow outside the boundary layer is irrotational and may be calculated by neglecting viscosity. Effects of viscosity are confined to the boundary layer, and *if this does not separate* its only effect on the irrotational flow is equivalent to a small change in the shape of the body. If the boundary layer separates there are important effects on the irrotational flow which will be discussed later.

A boundary layer may be either *laminar* or *turbulent*, or in a state of transition between these two conditions. In a laminar boundary layer the shear stresses are caused directly by viscosity, while over the greater part of a turbulent layer the shear stresses are due mainly to the *Reynolds stresses* associated with the turbulence, the viscous stresses being comparatively small.

At low Reynolds numbers, and when the pressure gradient in the stream direction is strongly negative, laminar boundary layers are stable. Instability of a laminar boundary layer, followed by transition to turbulent flow, occurs more readily as the Reynolds number or the positive pressure gradient is increased.

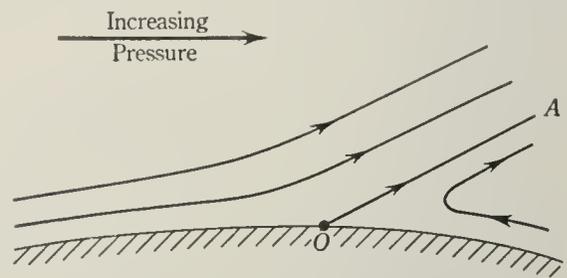
For a laminar boundary layer it can be shown that δ/l , v/u and $\tau_o/\rho U^2$ are all $O(R^{-1/2})$, where δ is the thickness of the layer, l is the length of the body, v and u are velocity components normal and tangential to the surface, τ_o is the shear stress at the boundary, and $R = \frac{\rho U l}{\mu}$ is

the Reynolds number based on the stream velocity U . Thus at high Reynolds numbers $\delta \ll l$ and $v \ll u$. As a consequence the Navier-Stokes equations can be simplified by omitting certain terms; in particular the variation of pressure across the boundary layer can be neglected. Approximate solutions of the resulting *boundary-layer equations* can be obtained by various methods.

The flow in a turbulent boundary layer is inherently much more complicated than that in a laminar layer, because in the former case there is no simple relation between the shear stress and the distribution of mean velocity. The methods available at present for calculating turbulent boundary layers rely on empirical data to a considerable extent.

Separation of a boundary layer can only occur when the pressure gradient in the stream direction is positive. Because the kinetic energy of the fluid in the boundary layer is

reduced by the action of viscosity, the pressure gradient may cause the fluid near the boundary to be brought to rest. In the figure



Separation of boundary layer.

this occurs at the point O , and the flow further downstream is reversed in direction. The limiting streamline at the boundary (of zero velocity) diverges from the boundary along OA , and the other streamlines are also deflected outward. There is a considerable effect on the irrotational flow outside the boundary layer, leading to a change of pressure distribution which in turn affects the flow in the boundary layer. Thus the final flow pattern, and in particular the position of the separation point O , depend on an interaction between the flow inside the boundary layer and the external flow.

When separation does not occur, the observed pressure distribution at high Reynolds numbers usually agrees well with the results of ideal fluid theory, because the boundary layer is thin and has only a small effect on the irrotational flow. When separation occurs, however, the separated boundary layer may cause large changes in the boundary conditions for the irrotational flow, so that the pressure distribution on the body differs considerably from that calculated for an ideal fluid. (See also **laminar boundary-layer equations; separation of boundary layer; turbulent boundary layer.**)

BOUNDARY LAYER, CALCULATION OF DEVELOPMENT. See **transition of turbulent flow in a boundary layer.**

BOUNDARY-LAYER CONTROL. Boundary-layer control is applied in two forms. In one of these, fluid is sucked into a porous or perforated wall to increase the stability of a laminar **boundary layer**, thus preventing or delaying transition to turbulent flow and reducing the drag due to surface friction. The volume flow required per unit area of surface

is of order $U \times 10^{-4}$ at Reynolds numbers of 10^6 to 10^7 , where U is the free-stream velocity.

The other form of boundary-layer control is applied to prevent separation of a boundary layer, either laminar or turbulent. This form of control may be applied by sucking fluid into the surface, by blowing fluid tangentially along the surface in the direction of the stream or, for a turbulent boundary layer, by various devices such as vortex generators which promote more rapid mixing in the layer.

BOUNDARY LAYER DISPLACEMENT THICKNESS. See displacement thickness of boundary layer.

BOUNDARY LAYER ENERGY EQUATION. See energy equation for boundary layer.

BOUNDARY LAYER, ENERGY THICKNESS OF. See energy thickness of boundary layer.

BOUNDARY LAYER, FORM PARAMETER FOR TURBULENT. See form parameter for turbulent boundary layer.

BOUNDARY LAYER IN A COMPRESSIBLE FLUID. When a compressible fluid flows past an insulated body at a high Mach number, the temperature at the body is greater than that in the free stream. At high Reynolds numbers the change of temperature occurs across a thin layer adjacent to the body, the *thermal boundary layer*. For all gases the Prandtl number is of order 1, and as a consequence the thickness of the thermal boundary layer is of the same order as that of the ordinary (velocity) boundary layer.

In general, calculation of boundary-layer flow in a compressible fluid is difficult, because of the interaction between the thermal and velocity boundary layers and because of the variation of viscosity with temperature. Non-dimensional quantities such as the skin-friction coefficient depend not only on the Reynolds number, external pressure distribution and surface roughness (as for an incompressible fluid), but also on (i) the Mach number, (ii) the Prandtl number, (iii) the functional relation between viscosity and temperature, (iv) the conditions of heat transfer at the boundary.

For either a laminar or a turbulent boundary layer at a given Reynolds number, the skin friction coefficient decreases with increas-

ing Mach number. With zero heat transfer, increasing Mach number reduces the stability of a laminar **boundary layer**, but heat transfer from the fluid to the body leads to a considerable increase of stability.

BOUNDARY LAYER IN UNSTEADY FLOW. When the flow in a boundary layer

is unsteady, an additional term $\frac{\partial u}{\partial t}$ appears in the equations. Thus for a laminar **boundary layer** the equations are

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad (1)$$

and the **equation of continuity**. The boundary conditions are the same as for steady flow, except that the velocity in the potential flow is now $U(x,t)$. The pressure gradient $\frac{\partial p}{\partial x}$ is determined by the potential flow, in accordance with the equation

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} \quad (2)$$

The equations can be solved by a process of successive approximation. Neglecting the terms in $\frac{\partial u}{\partial x}$, $\frac{\partial u}{\partial y}$ and $\frac{\partial U}{\partial x}$ in Equations (1) and (2), a first approximation to $u(x,y,t)$ is obtained from the equation

$$\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial y^2} = \frac{\partial U}{\partial t} \quad (3)$$

This first approximation is then used to calculate approximately the neglected terms and hence obtain a second approximation to $u(x,y,t)$, and so on.

The method outlined above can be used for a laminar boundary layer that is either started from rest or in periodic motion. For a **turbulent boundary layer** in unsteady flow there is no satisfactory theoretical approach and very few experiments have been made. (See also **laminar boundary-layer equations**.)

BOUNDARY LAYER, LAMINAR. See laminar boundary layer equations.

BOUNDARY LAYER, MOMENTUM EQUATION FOR. See momentum equation for boundary layer.

BOUNDARY LAYER, MOMENTUM THICKNESS OF. See momentum thickness of boundary layer.

BOUNDARY LAYER ON A FLAT PLATE, LAMINAR. For a thin flat plate at zero incidence, in a steady uniform stream of incompressible fluid, the pressure is uniform and the laminar boundary-layer equations reduce to

$$u \frac{\partial u}{\partial x} + \nu \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (1)$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (2)$$

Taking the x -axis along the plate, with the origin at the leading edge, the boundary conditions are $u = v = 0$ at $y = 0$ and $u = U$ at $y = \infty$, where U is the velocity of the free stream.

The velocity profiles at different values of x are all similar, so that if u and y are expressed in suitable nondimensional terms all the velocity profiles are given by a single function. A stream function ψ is introduced, so that

$$u = \frac{\partial \psi}{\partial y} \quad \text{and} \quad v = \frac{\partial \psi}{\partial x}.$$

Then, putting

$$\eta = y \left(\frac{U}{\nu x} \right)^{1/2} \quad \text{and} \quad \psi = (\nu U x)^{1/2} f(\eta),$$

Equation (1) reduces to

$$ff'' + 2f''' = 0, \quad (3)$$

where the primes denote differentiations with respect to η . Equation (3) has been solved numerically, with the appropriate boundary conditions, by expressing f as a series in powers of η . A few points from the numerical solution are given in the following table.

η	0	1	2	3	4	5	6
f	0	0.1656	0.6500	1.3968	2.3058	3.2833	4.2796
$f' = \frac{u}{U}$	0	0.3298	0.6298	0.8460	0.9555	0.9915	0.9990

The shear stress at the wall is $\tau_0 = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}$.

Hence it can be shown that the skin friction coefficient is

$$C_f = \frac{\tau_0}{\frac{1}{2}\rho U^2} = 0.664 \left(\frac{\nu}{Ux} \right)^{1/2}.$$

If D is the drag per unit breadth of a plate of length l , wetted on both sides, integration of the expression for C_f shows that the drag coefficient is

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 l} = 2.656 \left(\frac{\nu}{Ul} \right)^{1/2}.$$

Experiments with laminar boundary layers on flat plates agree well with these results, provided the Reynolds number $\frac{Ux}{\nu}$ is not less than about 10^4 . In a stream of very low turbulence the boundary layer may remain laminar for values of $\frac{Ux}{\nu}$ up to about 3×10^6 .

BOUNDARY LAYER, REATTACHMENT AFTER SEPARATION. See reattachment of boundary layer after separation.

BOUNDARY LAYER, SEPARATION OF. See separation of the boundary layer.

BOUNDARY-LAYER SHOCK-WAVE INTERACTION. See shock-wave boundary-layer interaction.

BOUNDARY LAYER, TRANSITION OF TURBULENT FLOW IN. See transition of turbulent flow in a boundary layer.

BOUNDARY LAYER, TURBULENT. See turbulent boundary layer.

BOUNDARY VALUE PROBLEM. A type of problem encountered in the solution of a differential equation or a system of differential equations which must meet given requirements for the value(s) of the independent variable, called the *boundary points*. For **ordinary differential equations** the usual numerical method of solution is to approximate the derivatives by finite differences and solve the resulting difference equations. This leads to **matrix inversion**, and the solution of linear equations, and possibly to the need for finding **eigenvalues and eigenvectors**. (See also **partial differential equations**.)

The *Dirichlet problem*, often called the *first*

boundary value problem, can be stated in the form: Assume a function of f continuous over a boundary surface S , with R the region bounded. The boundary problem is to find a solution of the **Laplace equation** $\nabla^2\phi = 0$ that is regular in R , continuous in $R + S$ and satisfies the equation $\phi = f$ at the boundary. This solution can be expressed in terms of **Green's function**.

The *Neumann problem*, often called the *second boundary value problem*, can be stated in the form: Assume a function of f continuous over a boundary S , with R the region bounded. The boundary value problem here is also (as in the Dirichlet problem above) to find the solution of the Laplace equation, with the conditions specified above, but with the added

conditions that the surface integral $\iint_S f dS$ vanishes, and also that the normal derivative is equal to f on the boundary surface. The solution can be expressed in terms of **Neumann's function**.

The *Robin problem*, often called the *third boundary value problem*, is subject to conditions specified above for the Neumann problem, but the equation it must satisfy is

$$g\partial\phi/\partial n + \psi\phi = f,$$

where g and ψ , as well as f , are functions that are continuous on S . The solution can be expressed in terms of **Robin's function**.

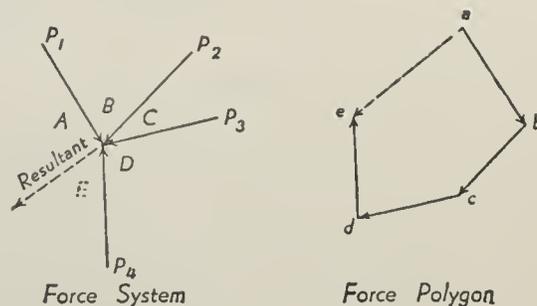
BOUNDED. A function $f(z)$ is said to be bounded if there exists a constant M such that $|f(z)| < M$ for all z in the domain of definition of f . A sequence of numbers a_1, a_2, \dots is bounded if $|a_n| < M$ for all n . A set of points is bounded if their distances from any one of them is bounded. (See also **variation**, **bounded**.)

BOUND PARTICLE. A particle surrounded by a potential barrier, and having insufficient kinetic energy to pass over the top of the barrier. In quantum mechanics, a particle which is classically bound may penetrate through a barrier. (See **penetration probability**.)

BOUSSINESQ PROBLEM. See **problem of Boussinesq and Cerruti**.

BOW NOTATION. Bow's notation is a standard method of representing, by letters of the alphabet, forces and stresses in graph-

ical analysis. This analysis may consist of such problems as the graphical solution of stresses in simple framed structures or the determination of the **resultant** of an independent system of unbalanced forces lying in the same plane and having a common point of application. The accompanying figure illustrates the method of applying Bow's nota-



tion to the latter system. Let P_1, P_2, P_3 and P_4 be a system of unbalanced forces lying in the same plane and having a common point of application. Denote the space between the line of action of each force by the letters A, B, C and D . Next construct a figure called a force polygon. This is accomplished by drawing a line parallel to P_1 and laying off its magnitude to a definite scale denoting the ends of the line by the letters a and b . From point b lay off bc equal in magnitude and parallel to P_2 . Repeat the operation for the other forces. Upon completion of this graphical figure it will be found, in general, that the line representing P_4 will not pass through point a . The distance from point a to end of this line, which will be lettered e , represents the value of the resultant of P_1, P_2, P_3 and P_4 according to the scale used. The direction of ae determines the line of action of the resultant. Thus, in Bow's notation a force in space is designated by the space letters on either side of it, whereas the forces as part of the force polygon are named by the letters at their extremities. This notation is further illustrated in the accompanying figure.

BOYLE-CHARLES LAW. The **Boyle law** expresses the variation of pressure and volume of a body of ideal gas at constant temperature; the **Charles law** expresses the proportionality of pressure to absolute temperature (at constant volume), while the **Gay-Lussac law** states the proportionality of volume to absolute temperature (at constant pressure).

A single statement covering all these relationships is the Boyle-Charles law, which leads

to, or is a form of, the ideal gas law. Its mathematical expression may be written:

$$pv = p_0v_0(1 + at),$$

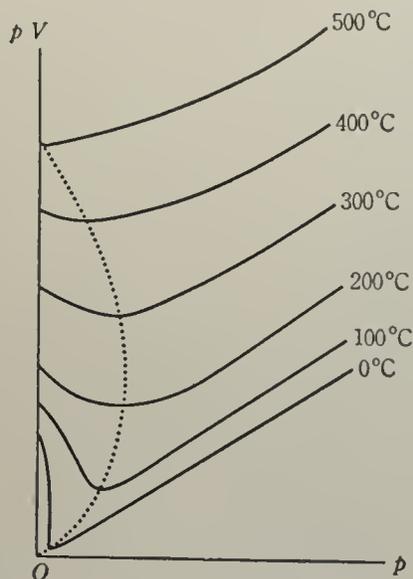
in which p_0v_0 is the value of pv at temperature $t = 0$, and a is the volume coefficient of expansion for the gas in question. If the centigrade scale is used, a is for all gases approximately equal to $\frac{1}{273}$ or 0.003663 per centigrade degree.

BOYLE CURVE. See Boyle temperature.

BOYLE LAW. (Mariotte law, law of Boyle-Mariotte.) At constant temperature the volume of a gas varies inversely as the pressure, and the pressure varies inversely as the volume. In other words, the product of the pressure and volume of a gas is constant at a given temperature. This law holds only for the ideal or **perfect gas**; all real gases depart from it to a greater or lesser extent.

BOYLE-MARIOTTE, LAW OF. See Boyle law.

BOYLE TEMPERATURE. The temperature, different for each gas, at which the **perfect gas** law is obeyed for a wide range of pressures from zero upwards. The figure represents an **Amagat diagram** for carbon dioxide. It is seen



Amagat diagram for carbon dioxide. The Boyle curve is the dotted line, while the Boyle temperature is the intersection of that line with the pV -axis.

that the isotherms in the gaseous phase pass through flat horizontal minima. If carbon dioxide obeyed the perfect gas law $pV = RT$, its isotherms would be straight horizontal lines.

The real gas behaves most nearly like a perfect gas in the neighborhood of the above minima. Their locus, shown by a dotted line, is called the *Boyle curve*. The isotherm (500°C for carbon dioxide) which corresponds to the intersection of the Boyle curve with the axis $p = 0$, possesses a large range of pressures from zero upwards in which the gas behaves very nearly like a perfect one. Hence the temperature which corresponds to the isotherm passing through the intersection of the Boyle curve with the axis $p = 0$ is called the Boyle temperature of a gas.

For a gas obeying the van der Waals equation, the equation of the Boyle curve in reduced coordinates is the parabola

$$(p_r V_r)^2 - 9(p_r V_r) + 6p_r = 0.$$

Hence the reduced Boyle temperature corresponds to

$$p_r V_r = 9 \quad \text{or} \quad T_r = \frac{27}{8} = 3.375.$$

The table below gives Boyle temperatures for several gases:

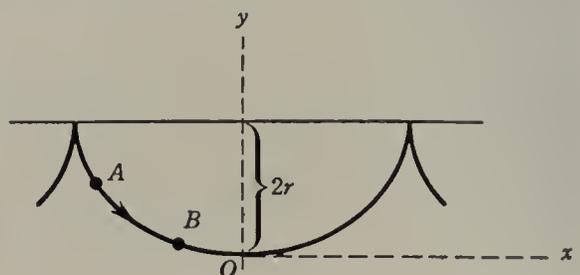
Gas	Boyle Temperature
He	-249°C
H_2	-166°C
N_2	$+51^\circ\text{C}$

If the **virial equation** is used, it is seen that the Boyle temperature corresponds to that temperature for which the second virial coefficient vanishes, or $B = 0$.

The general equation of the Boyle curve is

$$\left\{ \frac{\partial(pV)}{\partial p} \right\}_T = V + p \left(\frac{\partial V}{\partial p} \right)_T = 0.$$

BRACHISTOCHRONE (CURVE OF SHORTEST DESCENT). The curve along which a particle will fall from one point to another in the shortest time, under the in-



fluence of gravity only. (See **calculus of variations**.) In the illustration, a brachistochrone is the curve connecting two points *A* and *B* in the same vertical plane such that a mass sliding frictionless under the sole influence of gravitation will reach *B* in the shortest possible time after leaving *A*. The curve is a cycloid. Its equation is

$$x = r(\theta + \sin \theta)$$

$$y = r(1 - \cos \theta).$$

(See also **tautochrone** and **Huygen's pendulum**.) The line of descent is

$$t = \sqrt{\frac{r}{g}} \Delta\theta.$$

BRACKETT SERIES. Line series in the spectra of the hydrogen and the hydrogen-like ions He^+ , Li^{++} , etc. The wave numbers

$$\nu_n = \frac{1}{\lambda_n}$$

are given by the formula

$$\nu_n = RZ^2 \left[\frac{1}{4^2} - \frac{1}{n^2} \right], \quad (n = 5, 6, \dots),$$

where *R* stands for the **Rydberg constant** and *Z* for the **atomic number**.

BRAGG LAW. The law expressing the condition under which a crystal will reflect a beam of x-rays with maximum distinctness, at the same time giving the angle at which the reflection takes place. This law is also valid for the reflection of de Broglie waves associated with electrons, protons, neutrons, etc. For x-ray reflection it is customary to use the complement of the angle of incidence and reflection, that is, the angle which the incident or the reflected beam makes with the crystal planes, rather than with the normal. Let this "Bragg angle" be θ . If the planes or layers of atoms are spaced at a distance *d* apart, and if λ is the wavelength of the x-rays, Bragg's law is expressed by the equation

$$\sin \theta = \frac{n\lambda}{2d}.$$

The condition for an intensity maximum is that *n* must be a whole number. For example if the planes of rock salt parallel to the natural cubical

faces are spaced at $d = 2.814 \times 10^{-8}$ cm or 2814 x-units, and if the incident rays have a component of wavelength $\lambda = 714$ x-units, the above equation gives $\sin \theta = 0.1269n$. Then if the crystal is rotated slowly, there will be a distinct reflection when θ reaches $7^\circ 17'$ ($n = 1$), again at $14^\circ 42'$ ($n = 2$), also at $22^\circ 23'$ ($n = 3$), etc.

BRAGG RULE. An empirical relationship whereby the mass **stopping power** of an element for α -particles is inversely proportional to the one-half power of the atomic weight. This relationship is also stated in the form that the atomic stopping power is directly proportional to the one-half power of the atomic weight. The wide usefulness of the Bragg rule is due to the fact that it leads to relations between the stopping powers of different elements for α -particles. It also applies to other heavy charged particles as well as α -particles, and to the same degree of approximation.

BRAKE HORSEPOWER. The power output measured at the driving shaft of an engine. In the case of a prime mover it represents the net power delivered.

BRAKE MEAN EFFECTIVE PRESSURE. The ratio of the (indicated) **mean effective pressure** *P* to the mechanical efficiency η_m :

$$P_b = \frac{P}{\eta_m}.$$

Hence the **brake horsepower** of a reciprocating engine is given by

$$\text{IP}_b = \frac{P_b n V_s}{k}.$$

(For explanation of symbols see **mean effective pressure**.)

BRAKE THERMAL EFFICIENCY. (Also known as overall efficiency.) Ratio of brake horsepower of a prime mover to the energy supplied in producing it. The energy is usually supplied in the fuel, and in each particular case it is necessary to specify whether the brake thermal efficiency is based on its higher or lower calorific value. Standards of different countries differ on this point. The specific fuel consumption or the specific steam consumption are, in effect, reciprocals of the brake thermal efficiency.

BRANCH. (1) See **branch point**. (2) In topology, an **element** of a **tree**.

BRANCH CUT. See **branch point**.

BRANCHING FRACTION. In a radio-nuclide undergoing disintegration by more than one mode, the ratio of the number of atoms disintegrating by a particular mode to the total number of atoms disintegrating (per unit time), is the branching fraction for the particular mode of disintegration.

BRANCH POINT. If $f(z)$ is a multivalued function of the complex variable z and there exists a single-valued analytic function $g(z)$ such that at each z for which $f(z)$ is defined, the value of $g(z)$ coincides with one of the values of $f(z)$, then $g(z)$ is called a *branch* of $f(z)$. A curve in the domain of definition of $f(z)$ such that, if the points on this curve are removed, the remainder of the domain of definition is an open set for which there exists a branch of $f(z)$, is called a *branch cut*. A point at which a branch cut originates is called a branch point. Thus, if $f(z) = z^{1/2}$, the negative real axis, including the origin, is an example of a branch cut, while the origin itself is a branch point. (See also **Riemann surface**.)

BRAVAIS-MILLER INDICES. A modification of the **Miller indices** suitable for describing hexagonal crystals. In this system, three axes are taken, perpendicular to the **hexagonal axis** and at angles of 120° to one another. The symbols then consist of the reciprocal intercepts on these axes, followed by the reciprocal intercept on the hexagonal axis, all reduced to integers, e.g., (0001). The first three indices are not independent but must add to zero.

BRAVAIS POINTS. In **Gaussian optics**, the self-conjugate points of an optical system. A system thus has either zero, one, or two Bravais points.

BRA VECTOR. A vector in **Hilbert space**, contragradient to a **ket vector**, denoted by the symbol $\langle B|$. The scalar product of a bra and ket vector forms a scalar, the **Dirac bracket** expression $\langle B|A\rangle$. On the other hand, $|A\rangle\langle B|$ is an operator.

BRAYTON CYCLE. See **Joule cycle**.

BREAK-FREQUENCY. See **frequency response representation**.

BRETT FORMULAS. In the analysis of the torsion of thin-walled closed sections, it may be assumed that the shear stress is uniform across the thickness of the tube wall. Brett formula is a name given to a specific result obtained, as for example,

$$\tau = T/2At$$

where τ is the shear stress, T , the twisting moment and A , the area enclosed by the midline of the closed tube of local thickness t . (See **Batho's formula**.)

BREEDING GAIN. In nuclear reactor theory, the **breeding ratio** minus one.

BREEDING RATIO. In nuclear reactor theory, the number of fissionable nuclei produced, via neutron capture, per fissionable nucleus destroyed.

BREIT-WIGNER FORMULA. A formula relating the cross section σ of a particular nuclear reaction to the energy E of the incident particle, when E lies close to the energy E_0 required to form a discrete resonance level of the compound nucleus. If the spin of the incident particle is i and of the target nucleus I , and if J is the spin of the compound state, then the cross section for a reaction in which particle a is absorbed and particle b emitted is given by

$$\sigma_{(a,b)} = \frac{(2J + 1)\pi\lambda^2}{(2i + 1)(2I + 1)} \frac{\Gamma_a\Gamma_b}{(E - E_0)^2 + \frac{1}{4}\Gamma^2}$$

Here Γ_a/Γ is the probability that the compound state, once formed, will disintegrate by the re-emission of the incident particle without loss of energy, while Γ_b/Γ is the probability that particle b will be emitted from the compound state; Γ is the width of the level; and $2\pi\lambda$ is the de Broglie wavelength of the incident particle. In the case of the radiative capture of slow neutrons, for example, Γ_n is proportional to the incident neutron velocity, while Γ_γ is approximately independent of neutron energy, but much larger than Γ_n . If there is no competing process other than elastic scattering of the neutrons, we then have $\Gamma = \Gamma_n + \Gamma_\gamma \approx \Gamma_\gamma = \text{constant}$. The term λ^2 is inversely proportional to the square of the incident neutron velocity, hence $\sigma(n,\gamma) \propto E^{-1/2}/\{(E - E_0)^2 + \frac{1}{4}\Gamma^2\}$ approximately. The energy dependence is in general more complicated than this, but the main effect in the immediate neighborhood

of the resonance is produced by the Lorentz denominator.

BREWSTER ANGLE. The Brewster angle or polarizing angle (see **polarized light**) of a dielectric is that angle of incidence i for which a wave polarized parallel to the plane of incidence is wholly transmitted (no reflection). Thus $\tan i = n/n'$ where n and n' are the indices of refraction in the media before and after refraction. An unpolarized wave of intensity I_0 incident at this angle is resolved into a transmitted partly polarized component and a reflected component of intensity I which is now 100% polarized with its electric vector perpendicular to the plane of incidence and where

$$I = \frac{1}{2}I_0 \sin^2 (i - r) = \frac{1}{2}I_0 \left(\frac{n'^2 - n^2}{n'^2 + n^2} \right)^2.$$

If, on the other hand, the incident light were already 100% polarized at Brewster's angle,

$$I = I_0 \left(\frac{n'^2 - n^2}{n'^2 + n^2} \right)^2.$$

(See **Brewster law**.)

BREWSTER LAW. A simple relationship, for any dielectric reflector, between the polarizing angle (see **polarized light**) for the reflected light of a particular wavelength and the relative index of refraction of the substance for the same wavelength. The relationship is that the tangent of the polarizing angle is equal to the relative refractive index.

BRIGGS LOGARITHMS. See **logarithm**.

BRILLOUIN EFFECT. Upon the scattering of monochromatic radiation by certain liquids, a doublet is produced, in which the frequency of each of the two lines differs from the frequency of the original line by the same amount, one line having a higher frequency, and the other a lower frequency.

BRILLOUIN FUNCTION. In the quantum theory of paramagnetism, a quantum-mechanical analog of the Langevin equation (see **Weiss theory; Langevin function**) was developed by Brillouin, with the following result:

$$\frac{I_s}{I_0} = \frac{2J + 1}{2J} \operatorname{ctnh} \frac{(2J + 1)a}{2J} - \frac{1}{2J} \operatorname{ctnh} a/2J$$

where $a = Jg\beta(H + NI_s)/kT$, β is the Bohr magneton, $eh/4\pi mc$, J is the momentum quantum number, and

$$g = \frac{2mc}{e} = \frac{\text{magnetic moment}}{\text{angular momentum}};$$

(g is 1 for orbital motion alone; 2, when the moment is due entirely to spin).

BRILLOUIN ZONE. An electron moving within an ionic crystal moves in a potential field which may be approximated to as that of a constant potential within the crystal (as in the elementary Drude-Lorentz theory), modified by a varying potential which varies as the periodicity of the lattice. The allowed solutions of the wave equation for such a system are those for which the energy lies in a series of bands, the wave vector \mathbf{k} of the electron being imaginary at other values. The values of \mathbf{k} at which discontinuities occur lie at the surfaces of polyhedra in \mathbf{k} -space called Brillouin zones. The Brillouin zones may be calculated for a given lattice structure.

BRITISH THERMAL UNIT (ABBREVIATED B.T.H.U. OR BTU). The unit of heat in the English system of units. Originally defined as $\frac{1}{150}$ part of the quantity of heat required to raise 1 lbm of water from its freezing point to its boiling point at a pressure of one atmosphere. Nowadays the unit is defined by the equation

$$1 \text{ kcal}_{\text{IT}}/\text{kg} = 1.8 \text{ Btu}/\text{lb}$$

exactly. Here 1 kcal_{IT} denotes the international steam tables kilocalorie. (See **thermal units**.)

$$\begin{aligned} 1 \text{ BTU} &= 1055.056 \text{ J} = 1075.857 \text{ kpm} \\ &= 0.252074 \text{ kcal}_{15^\circ} = 0.251996 \text{ kcal}_{\text{IT}} \\ &= 2.93071 \times 10^{-4} \text{ kw.hr}_{\text{int}}. \end{aligned}$$

BRITTLE FRACTURE. Some structural materials at normal operating temperature fail in a brittle manner. Cast iron, unreinforced concrete, glass and some plastics are common examples. Far more troublesome is the brittle fracture of metals, especially structural steel, which normally exhibit great ductility. For each type of test for brittle behavior (notched bar impact, notched bar tension or slow bend, etc.), there is a transition temperature or range below which the material behaves in a brittle

manner and above which failure is truly ductile. Evidence is accumulating that the prior strain history of the material is of great importance in the effective exhaustion of ductility.

BROADENING OF SPECTRAL LINES. A spectral line emitted by an atomic or nuclear system does not consist of a single frequency, but rather of a continuous group of frequencies, which may be very narrow in its extent. The inherent width of a line is known as its **natural width**. A spectral line may be additionally broadened by **Doppler broadening** and by collision, or **pressure broadening**. In the latter case, the lifetime τ of an excited state may be reduced during a collision, which in turn increases the energy level width, Γ , of the excited state through the relation, $\Gamma = \hbar/\tau$. Since the change in energy ΔE in the transition and the frequency of emitted radiation are related by $\Delta E = h\nu$, any broadening of the energy change results in a broadening of the frequency spectrum.

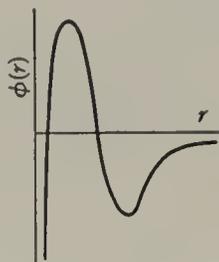
BRØNSTED AND LEWIS TREATMENT OF ACIDS. See acids and bases, definitions of Brønsted and Lewis.

BROUWER FIXED POINT THEOREM. The theorem that any continuous transformation of a circle (circumference and interior) into itself must have at least one fixed point. The analogue in **Hilbert space** is often useful for proving the existence of a solution of a differential problem. There are numerous generalizations.

BUCKINGHAM POTENTIAL. The intermolecular potential

$$\phi(r) = b \exp[-ar] - cr^{-6} - dr^{-8} \quad (1)$$

has an exponential repulsive part and an attractive part which is the sum of two power laws. It contains four parameters a , b , c , d . The choice of the exponents -6 and -8 in



Buckingham potential.

the attractive part is made for theoretical reasons (see **intermolecular forces**); the exponential form of the repulsive part is somewhat arbitrary. This potential becomes unrealistic for very small distances (see figure).

BUCKLING. (1) The abrupt change from one equilibrium geometric configuration to a neighboring or a distant configuration. The simplest example is provided by a perfectly straight column under perfectly axial compression. As the axial load increases the straight configuration is always permissible from the point of view of equilibrium. Above the **Euler buckling load** for an elastic column or the **tangent modulus load** for an elastic-plastic one, a bent configuration also is permissible. The actual behavior depends upon the unavoidable initial imperfections of geometry or loading. In practice, therefore, if a neighboring buckled configuration exists the structure will buckle. If a distant one exists, as in many problems of shells and most problems of buckling in the plastic range, the eigenvalue problem associated with the buckling of perfect structures may have little practical relevance. It is necessary to follow the detailed history of the deflection and stress in an initially imperfect structure as the load is increased from zero. (2) In nuclear reactor theory, the distribution of thermal neutron flux far from boundaries and sources is assumed to satisfy the equation

$$(\nabla^2 + B^2)\Phi(\mathbf{r}) = 0.$$

B^2 , the buckling, when expressed in terms of material properties is an intensive quantity, called the *material buckling*. When the differential equation is assumed to hold throughout an unreflected system (see **first fundamental theorem**) and to describe a critical chain reaction, the lowest **eigenvalue**, B^2 , dictated now by the shape of the system, is called the *geometrical buckling*.

BUCKLING, IMPACT. At high rates of loading, mass acceleration terms become important and higher transient axial loads can be carried than the axial loads which would cause static **buckling**.

BUCKLING LAG. The difference in load carrying capacity between a buckled sheet and a fully effective sheet may be described as due to buckling lag.

BUCKLING, LATERAL. See lateral buckling.

BUCKLING, LOCAL. The load carrying capacity of a structure is often limited by local buckling or crippling. A column or frame may have sufficient strength if it acts as a unit but the flange or some other structural element may buckle over a short length, precipitating collapse of the entire structure.

BUCKLING, TORSIONAL OR TWIST. A column of open or profile section is relatively flexible and weak in torsion. If buckling about the weak axis is prevented, the column will tend to fail by twisting or torsional buckling. The lateral buckling of beams is an allied phenomenon.

BUDAN THEOREM. Given any polynomial $P(x)$ in the single variable x , let P and its derivatives be evaluated at $x = r$, for any r . Ignoring vanishing derivatives, let V_r represent the number of variations in sign in the sequence P, P', P'', \dots . Then the number of roots of $P(x) = 0$ exceeding r is $V_r + 2r$, where r is an integer ≥ 0 . **Descartes's rule of signs** is a particular case with $r = 0$, where V_0 is the same as the variations in sign in the coefficients. Any r for which $V_r = 0$, all $P^{(i)}$ having the same sign, is an upper bound for the roots of $P(x) = 0$. In case $V_r - V_s = 1$, there must be exactly one root on the interval from r to s . All roots are counted according to multiplicity, e.g., a double root is counted twice.

BUILDING-UP PRINCIPLE. The method of deriving the different electronic terms of an atom or molecule by the hypothetical bringing together of their components. In the case of an atom this means successive addition to the nucleus of electrons in quantum states permitted by the **Pauli principle**. In the case of a molecule there are various alternatives. One is a procedure analogous to that used in the case of an atom, that is, successive addition of electrons to the nuclei with fixed internuclear distances, corresponding to those in the completed molecule. Another is the bringing together of the whole component atoms, a third the drawing apart of the component atoms from an assumed position of zero separation ("splitting the united atom").

BUILD-UP FACTOR. In the study of the penetration of radiation through matter, the

point kernel (see **kernel, point**) representing the flux at one point due to unit source at another, will deviate from the first-flight or transport kernel, $(4\pi R^2)^{-1} \exp(-\Sigma R)$. The deviation, due to multiple scattering, is expressed by multiplying the transport kernel (see **kernel, transport**) by the factor $B(R)$, called the build-up factor.

BULK COMPLIANCE. See **compliance, bulk**.

BULK MODULUS (MODULUS OF VOLUME ELASTICITY). The isothermal or isentropic bulk moduli are reciprocals of the isothermal or isentropic **coefficients of compressibility**. Hence

$$B_T = -V \left(\frac{\partial p}{\partial V} \right)_T$$

$$B_S = -V \left(\frac{\partial p}{\partial V} \right)_S$$

In older publications the isentropic bulk modulus is called the adiabatic bulk modulus.

BULK TEMPERATURE. In heat transfer, the temperature of the bulk of the fluid. More precisely, in internal flow, the weighted mean temperature of the fluid in a cross section; in external flow, the temperature of the fluid at a large distance from the body, or the so-called free-stream temperature.

BURGER-DORGELO-ORNSTEIN SUM RULE FOR ATOMIC SPECTRA. See **multiplet intensity rules**.

BURGERS MATERIAL. Material represented by four-element model. (See **viscoelastic models**.)

BURGERS VECTOR. A vector which defines the direction and magnitude of the slip produced by a **dislocation**.

BURNETT EQUATIONS. The Enskog-Chapman method for the solution of the Boltzmann equation presents the distribution function as a series of successive approximations (see **Enskog series**). The first order approximation leads to the **Navier Stokes equations** of hydrodynamics, in which the **flux vectors** are linear functions of the first order in the gradients of the macroscopic quantities (density, velocity, temperature). It is only valid when these gradients are small.

The second approximation leads to the

Burnett equations, where the flux vectors then depend on the *second derivative* of these macroscopic quantities, and on the *square of the gradients*. There is some doubt as to the rapidity of the convergence of the Enskog series, and attempts to apply the Burnett equations to such phenomena as shock waves and the transmission of sound waves in rarefied gases have led to disappointing results.

BURNING RATE. For solid propellant fuels, the rate of burning (r) is the linear measure (normal to the burning surface) of the propellant grain which is consumed in unit time, usually expressed in inches per second.

BURNING RATE, EROSIVE. The increase in burning rate over normal conditions, which is incident to the sweep of hot gases over a burning propellant surface.

BURNING RATE, LINEAR. The rate at which a solid propellant burns normal to its surface under design combustion chamber pressure. The burning rate is in accordance with the Muraour relationship. It is given by the equation:

$$r_0 = a_2 p_c^n = \frac{dw/dt}{A_p \gamma_p},$$

where r_0 is the linear burning rate, a_2 is an experimental constant, p_c is the chamber pressure, n is a variable exponent, dw/dt is the rate of flow of propellant by weight, A_p is the burning surface and γ_p is the specific weight of propellant.

BURNT CASES. Gases resulting from the combustion of a fuel with air in an internal combustion engine or other appliance (e.g., a boiler). The composition of the burnt gases is determined by the fuel composition, the air

ratio, and the completeness of the combustion process.

BUSEMANN SECOND-ORDER THEORY. A second-order theory, applicable to two-dimensional airfoils in supersonic flow. At a point on an airfoil where the surface is inclined at an angle θ to the stream, the pressure coefficient is

$$C'_p = \frac{2\theta}{\sqrt{M^2 - 1}} + \left[\frac{(M^2 - 2)^2 + \gamma M^4}{2(M^2 - 1)^2} \right] \theta^2, \quad (1)$$

where M is the Mach number of the stream. The sign of θ is the same as for the **Ackeret theory**, and if the term in θ^2 in Equation (1) is neglected the Ackeret formula is obtained.

When Equation (1) was first derived by Busemann, a term in θ^3 was also obtained, but this third-order term is affected by entropy changes in shock waves and is not included in the usual form of the theory.

In the expression for the lift coefficient of an airfoil the second-order terms cancel, so that the lift given by Equation (1) is the same as that given by the Ackeret theory. Also, for an airfoil which is symmetrical about the chord line and about the normal to the chord line through its center, the second-order terms in the expression for the drag cancel, so that again Equation (1) gives the same result as the Ackeret theory. In general, however, the second-order term in Equation (1) does affect the drag. It also affects the pitching moment, usually moving the center of pressure of a symmetrical airfoil *ahead* of the half-chord position given by the linear theory of Ackeret.

BY-PASS ENGINE. A type of **jet-propulsion** engine in which a fan driven by the gas-turbine engine provides an additional flow of air in the jet.

C

C. In **neutron scattering** and **transport theory**, the average number of neutrons appearing as the result of the collision of a neutron with an atomic nucleus.

CABLE. The word cable often implies the idealization of a tension member which has no flexural strength or rigidity.

CALCULATOR. In current usage the term is used interchangeably with **computer** as either one who or that which calculates or computes. Since the term "computer" more often designates a machine in current literature, it is suggested here that "calculator" be applied only to the human being, and "computer" only to the machine.

CALCULUS. The word comes from the Latin *calculus*, a stone or pebble used in reckoning. The unqualified word is usually taken to mean differential and integral calculus. For other kinds of calculus (e.g., calculus of **finite differences**, calculus of **variations**) see the respective headings.

CALCULUS OF VARIATIONS. See **variations**, **calculus of**.

CALCULUS, SPINOR. See **spinor calculus**.

CALENDAR EQUATION. An empirical (thermal) equation of state for steam:

$$v = 47 \frac{T}{p} + 0.001 - 0.075 \left(\frac{273}{T} \right)^{1/3}$$

where v , the specific volume, is expressed in m^3/kg , T in $^\circ\text{K}$, and p in kp/cm^2 . It is sufficiently accurate for pressures up to 20 kp/cm^2 (about 300 lb/in^2) and for temperatures below 500°C (932°F). The equation is not dimensionally correct; it is a modified form of the Clausius equation. (See also **steam tables**.)

CALLIER COEFFICIENT. The density of photographic negatives measured by parallel light is greater than if measured by diffused

light. This effect is caused by scattering. Callier defined a coefficient as

$$Q = \frac{\text{density measured by parallel light}}{\text{density measured by diffused light}}$$

The average value of Q is about 1.4 ± 0.2 .

CALORESCENCE. The transformation of infrared radiant energy into visible radiation.

CALORIC EQUATION OF STATE. Often a distinction is made between the various types of equations of state. The relation between pressure, volume and temperature, the so-called p, V, T , relation, is then called the *thermal equation of state*. Equations involving a caloric parameter, such as **enthalpy**, **internal energy**, **entropy**, etc., are then called caloric equations of state.

CALORIE. See **thermal units**.

CALORIFIC VALUE. The number of units of heat obtained by the complete combustion of a unit mass of a substance, usually a fuel. In measuring the calorific value, it is stipulated that the products of combustion are cooled to the initial temperature of the fuel, and it is necessary to specify whether combustion has taken place at constant volume or constant pressure. (See also **combustion**.)

CALORIMETRIC COEFFICIENT. One of the following six coefficients, used to express the rate of absorption of heat during reversible changes of pressure, volume, and temperature:

(1) Heat of compression at constant volume, $(\partial q/\partial p)_v$.

(2) Heat of expansion at constant pressure, $(\partial q/\partial v)_p$.

(3) Specific heat at constant volume, $(\partial q/\partial T)_v$.

(4) Specific heat at constant pressure, $(\partial q/\partial T)_p$.

(5) Latent heat of change of pressure, $(\partial q/\partial p)_T$.

(6) Latent heat of change of volume, $(\partial q/\partial v)_T$.

In these expressions, q is the quantity of heat, p , the pressure, v , the volume and T , the absolute temperature.

CANDELA. The unit of luminous intensity. The magnitude of the candela is such that the luminance of a full radiator at the temperature of solidification of platinum is 60 candelas per square centimeter. Definition adopted by the 9th General Conference on Weights and Measures (1948).

CANDELA PER SQUARE METER. A metric unit of luminance, recommended by the C.I.E.

CANDLE. A common synonym for the unit of luminous intensity, the **candela**.

CANONICAL. Of standard form, especially if the form is simple. For example, a canonical matrix of a certain kind has non-zero elements only on the main diagonal.

CANONICAL COORDINATES. The canonical coordinates of a particle are its coordinates q_i and its conjugate momenta $p_i = \frac{\partial L}{\partial \dot{q}_i}$; $\left(\dot{q}_i = \frac{dq_i}{dt}\right)$. L is the **Lagrangian** of the system. The definition of L is $L(q_i, \dot{q}_i, t) = T(q_i, \dot{q}_i, t) - V(q_i, t)$. T is the kinetic energy and V the potential energy of the system.

By means of the defining equation for p_i , the velocities \dot{q}_i can be replaced in terms of the p_i in the Lagrangian, and the Lagrangian $L^*(q_i, p_i, t)$ results.

$$L^*(q_i, p_i, t) = L(q_i, \dot{q}_i, t)$$

$$\text{i.e., } T^*(q_i, p_i, t) - V^*(q_i, t) = T(q_i, \dot{q}_i, t) - V(q_i, t)$$

$$\text{But } V^*(q_i, t) \equiv V(q_i, t)$$

Therefore

$$L^*(q_i, p_i, t) = T^*(q_i, p_i, t) - V(q_i, t)$$

$$\text{and } T^*(q_i, p_i, t) = T(q_i, \dot{q}_i, t).$$

where the dots denote the derivative with respect to time.

CANONICAL CORRELATION. Suppose there is a p -variate population with variates x_1, x_2, \dots, x_p , and a q -variate population with variates y_1, y_2, \dots, y_q , and suppose for definite-

ness that $p \geq q$. Then it is possible to find p linear combinations of the x 's, u_1, u_2, \dots, u_p and q linear combinations of the y 's, v_1, v_2, \dots, v_q all with zero mean and unit variance, such that

$$i \quad \text{cov}(u_i u_j) = 0 \quad i \neq j$$

$$ii \quad \text{cov}(v_i v_j) = 0 \quad i \neq j$$

$$iii \quad \text{cov}(u_i v_j) = 0 \quad i \neq j$$

where $\text{cov}()$ denotes **covariance**. The correlation between u_i and v_i is called a canonical correlation; at most q of these correlations are non-zero. If u and v are the linear combinations corresponding to the largest canonical correlation, then v is the linear combination of the y 's which can be predicted from the x 's by linear regression with least residual variance (the most predictable criterion), and u is the appropriate predictor.

CANONICAL ENSEMBLE. An ensemble with a density ρ given by the equation

$$\rho = e^{\beta(\psi - \epsilon)}$$

where $\beta = 1/kT$ (k , Boltzmann's constant; T , absolute temperature), ψ , a normalizing constant and ϵ , the energy of the system.

CANONICAL EQUATIONS OF MOTION. For the k th set of generalized coordinates and conjugate momenta in a conservative dynamical system, there can be written a pair of first-order partial differential equations:

$$\frac{\partial H}{\partial p_k} = \frac{dq_k}{dt}$$

$$\frac{\partial H}{\partial q_k} = -\frac{dp_k}{dt}$$

where H is the **Hamiltonian function** for the system and p_k and q_k are generalized momenta and coordinates, respectively. They are said to be *canonically conjugate variables*. These equations are called the canonical equations of motion. (See also **canonical coordinates; coordinates and momenta (generalized)**.)

CANONICAL EQUATIONS OF OPTICS. The optical paths $x = x(z)$, $y = y(z)$ in a medium of refractive index $n = n(x, y, z)$ are the solutions of the canonical equations.

$$\frac{dx}{dz} = \frac{\partial H}{\partial p}, \quad \frac{dy}{dz} = \frac{\partial H}{\partial q},$$

$$\frac{dp}{dz} = -\frac{\partial H}{\partial x}, \quad \frac{dq}{dz} = -\frac{\partial H}{\partial y},$$

where

$$H = -r = -\sqrt{n^2 - p^2 - q^2}$$

is the **Hamiltonian function** of optics and (p, q, r) is a tangent vector, to the optical path, of length n . The canonical equations are the **Euler equations** of the **canonical form of the Fermat principle**.

CANONICAL FORM OF THE FERMAT PRINCIPLE. The optical paths joining the points P_0, P_1 in a medium with refractive index $n = n(x, y, z)$ are the extremum of

$$V = \int_{P_0}^{P_1} (pdx + qdy + rdz)$$

where (p, q, r) is a tangent vector, to the optical path, of length n . The **Euler equations** of this variational problem are the **canonical equations of optics**.

CANONICAL TRANSFORMATION. A transformation from one set of generalized coordinates and momenta to a new set such that the form of the **canonical equations of motion** is preserved. This usually involves finding a transformation function S which is a continuous and differentiable function of the old and new generalized coordinates and the time. The transformation can be defined by

$$L(q, \dot{q}) = L'(Q, \dot{Q}) + \frac{dS}{dt}$$

$$S = S(q, Q, t)$$

$$p_k = \frac{\partial S}{\partial q_k}; \quad P_k = -\frac{\partial S}{\partial Q_k};$$

$$H = \bar{H} - \frac{\partial S}{\partial t}$$

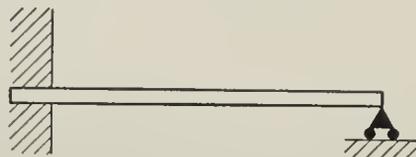
where L is the **Lagrangian function** in the original set of coordinates, and L' is the Lagrangian function in the transformed set of coordinates. H is the **Hamiltonian function** in terms of p, q, t , and \bar{H} is the Hamiltonian function in terms of P, Q, t .

CANONICAL VARIABLES. See **Lagrangian formalism**.

CANTILEVER. Pertaining to overhang or projection beyond supports. (See **beam, cantilever** and **beam, overhanging**.)

CANTILEVER BEAM. See **beam, cantilever**.

CANTILEVER, PROPPED. A beam fixed at one end and propped or simply supported at the other.



Propped cantilever.

CAPACITANCE. (1) Electrical capacitance is the ratio of the **electric charge** given a body to the resultant change of **potential**. It is usually expressed in **coulombs** of charge per volt of potential change, that is, in terms of the farad, or its submultiples; the cgs electromagnetic unit is the abfarad. If a conductor is completely isolated, that is, far removed from other conductors, including the earth, and is surrounded by a homogeneous, perfectly insulating dielectric, its capacitance depends only upon the size and shape of its external surface, and upon the **dielectric constant** of the surrounding medium. The capacitance of a circuit, whether "distributed" or intentionally introduced by means of capacitors, acts as a capacitor in parallel with the conductor, and may have marked effect upon alternating or variable currents traversing it.

(2) For acoustical capacitance, see **compliance, acoustic**.

CAPACITANCE, ACOUSTIC(AL). See **compliance, acoustic**.

CAPACITANCE, GEOMETRIC. The **capacitance** of an isolated conductor *in vacuo*. For a sphere of radius a ,

$$V = \frac{Q}{a}, \quad \text{hence } C = Q/V = a$$

where electrostatic units are used. For a hemisphere of radius a , $C = 0.845a$. For many other bodies C is approximated by $\sqrt{S/4\pi}$, where S is the surface area.

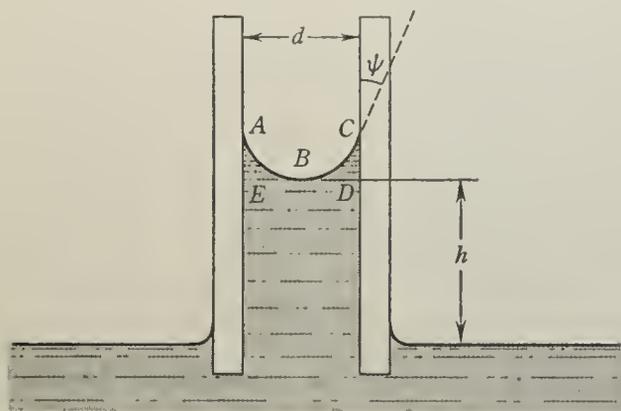
CAPACITANCE, SPECIFIC ACOUSTIC.

That coefficient which, when multiplied by 2π times the frequency, is the reciprocal negative imaginary part of the specific acoustic impedance. (See **impedance, specific acoustic**.) The dimensions of specific acoustic capacitance are cm^5/dyne .

CAPACITOR. A device whose primary purpose is to introduce **capacitance** into an electric circuit or network.

CAPACITY, CARRYING. See **carrying capacity**.

CAPILLARITY. For convenience in analyzing the phenomena of capillarity it is usual to assume that there is a **surface tension** at every surface separating a liquid from a gas,



Capillarity.

or from a solid, or from another liquid. The surface tension T is defined as the tensile force acting across any line in the surface, per unit length of line. The potential energy per unit area of surface, due to the surface tension, is also equal to T .

The surface tension of any liquid in contact with a gas *decreases* with increasing temperature. For an unassociated liquid the surface tension is zero at the critical temperature.

As a consequence of surface tension there is a difference of pressure across a curved surface, the pressure being greater on the concave side. For a spherical surface of radius R the pressure difference is $2T/R$.

It can be shown that surface tension causes a cylindrical liquid surface to become unstable when the wavelength of a disturbance imposed on it exceeds the circumference. This is one of the principal causes of the breaking-up of liquid jets into small drops.

When a liquid surface meets a solid, the angle between the surface of the liquid and

that of the solid, at any point on the line of contact, is known as the *angle of contact* of that particular liquid and solid.

If a cylindrical tube of small internal diameter, open at both ends, is held vertically with one end immersed in a liquid, surface tension will cause the liquid to rise in the tube as in the figure, provided the angle of contact for the liquid and the tube is less than 90° . If the diameter of the tube d is very small, so that the weight of liquid in the region $ABCDE$ is negligible, the height h to which the liquid rises in the tube is

$$\frac{4T \cos \psi}{\rho g d},$$

where ρ is the density of the liquid and ψ is the angle of contact at the wall of the tube. In many cases the liquid "wets" the tube, so that ψ is small and $\cos \psi = 1$. For a few cases, such as mercury in glass, ψ is greater than 90° so that h is negative.

CAPILLARY WAVES. Waves on a liquid surface in which the surface tension T provides an appreciable force. The pressure under the surface is increased by amount $T \nabla^2 \zeta$, where ∇^2 is the two-dimensional Laplacian operator and ζ is the displacement from a plane, assumed small. In such cases gravity, which produces a pressure increment $g\rho\zeta$ is, in effect, replaced by $[g + \frac{T}{\rho} \nabla^2]$.

Two-dimensional waves on deep water, of length $2\pi/k$, then have speed c given by

$$c^2 = \frac{g}{k} + \frac{Tk}{\rho}$$

which has a minimum value when $k^2 = g\rho/T$. For waves with shorter length the velocity increases as wavelength decreases, so that the group velocity exceeds the phase velocity. These particular waves are called *ripples*. On water ripples have a wavelength less than about 1–2 cm (the value depending largely on T , which varies with temperature).

CARATHÉODORY PRINCIPLE. An independent formulation of the second law of **thermodynamics**. It is best stated in two parts:

Part 1. In the neighborhood of any thermodynamic state of any system there exist states which are inaccessible from it along any *reversible adiabatic* (i.e., isentropic) path.

Part 2. In the neighborhood of any thermodynamic state of any system there exist states which are inaccessible from it as a result of any (reversible or irreversible) **adiabatic process**.

Carathéodory's principle constitutes a generalization of known experimental facts. The two parts of it are sufficient for the formulation of the second law and for derivation of its most essential corollaries. (See **thermodynamics, second law of**.)

CARATHÉODORY (MATHEMATICAL) THEOREM. A theorem which relates the property of integrability of **Pfaffian expressions** (linear differential forms) to the existence of inaccessible points.

Given a Pfaffian

$$dy = \sum X_i dx_i \quad (a)$$

where the X_i s are functions of the x_i in an n -dimensional space \mathfrak{X}_i , two neighboring points x_i, x'_i are called *accessible* if they lie on an element of a curve satisfying the Pfaffian differential equation

$$\sum X_i dx_i = 0. \quad (b)$$

Otherwise they are called *inaccessible*.

The Pfaffian Equation (a) is called integrable if a function $\lambda(x_i)$ of the x_i s exists, such that

$$dy = \lambda(x_i) d\phi(x_i) \quad (c)$$

where $\phi(x_i)$ is also a function of the n independent variables x_i . The function λ is then called an *integrating factor* of the Pfaffian.

It may be noted that: (1) a Pfaffian in two independent variables x_1, x_2 always possesses an integrating factor, but that for $n \geq 3$ this is no longer the case; (2) if one integrating factor exists, there are infinitely many, because $\lambda' = \lambda d\psi/d\phi$ is also an integrating factor if $\psi(\phi)$ is any given function of ϕ ; (3) the curves which satisfy Equation (b) in the case of an integrable Pfaffian, also satisfy the condition $\phi(x_i) = \text{const}$. Here, and above, the symbols $X_i(x_i), \phi(x_i)$, etc., denote functions of all the n variables x_i .

Carathéodory's mathematical theorem asserts that if the Pfaffian Equation (a) is integrable, then in the neighborhood of any given point $A(x_i)$, however small, there exist points which are inaccessible from it.

The converse theorem is also true. In other words, if the Pfaffian Equation (a) possesses the property that in every arbitrarily-close neighborhood of a given point $A(x_i)$ there exist

inaccessible points, then the Pfaffian is integrable.

Both theorems can be summarized in the statement that the existence of inaccessible points in the neighborhood of a given point is both necessary and sufficient for the integrability of a Pfaffian expression.

The converse theorem plays an important part in Carathéodory's formulation of the second law of **thermodynamics**.

CARATHÉODORY THEOREM (IN OPTICS). Two surface elements can be imaged by a continuous manifold of rays tangential to both elements only if the optical magnitudes of the elements are equal, i.e., $|ndS| = |n'dS'|$.

CARDINAL NUMBER. It is difficult to give a satisfactory definition of a cardinal number, but this difficulty is not of great importance, since it is clear under what condition two sets contain the same cardinal number of objects, namely that they can be put into one-to-one correspondence with each other. This cardinal number is called the *power* of the set. An **infinite set** that can be put into one-to-one correspondence with the set of positive integers is said to be countable (or denumerable, or enumerable). A cardinal number which is not finite is called transfinite.

CARDINAL POINTS OF AN OPTICAL SYSTEM. The **focal points**, the **principal points** and the **nodal points** constitute the cardinal points of an optical system.

CARDINAL STIMULI. Four standard stimuli by means of which the three **reference stimuli** and the basic stimulus of any **trichromatic system** may be defined. Light of wavelengths 700, 546.1 and 435.8 millimicrons and illuminant B have been adopted by the C.I.E.

CARLSON METHOD. See S_N method in neutron transport theory.

CARNOT CYCLE. The hypothetical cycle in the diagram of Figure 1. The cycle consists of the following *reversible* processes: 1–2, iso-

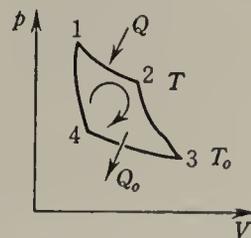


Fig. 1. Carnot cycle— p, V diagram.

thermal expansion during which a quantity of heat Q is transferred to the working fluid; 2-3, isentropic expansion; 3-4, isothermal compression during which a quantity of heat Q_o is rejected by the working fluid; 4-1, isentropic compression to the initial state 1. If the working fluid is a perfect gas, then

$$V_1V_3 = V_2V_4. \quad (a)$$

The efficiency of a Carnot cycle

$$\eta = \frac{\oint dW}{Q} = \frac{Q - |Q_o|}{Q} = \frac{T - T_o}{T}. \quad (b)$$

Also

$$\frac{Q}{T} = \frac{|Q_o|}{T_o}$$

so that

$$\frac{Q}{T} + \frac{Q_o}{T_o} = 0.$$

(Here Q_o is negative according to the sign convention: see **heat**.)

The efficiency of a Carnot cycle is independent of the physical properties of the working fluid, and, as shown by Equation (b), is a function of the two thermodynamic temperatures T and T_o only. If the two constant temperatures T and T_o are not measured on the thermodynamic temperature scale, but on an arbitrary empirical temperature scale, so that

T corresponds to θ and

T_o corresponds to θ_o ,

we have

$$\eta = 1 - \frac{f(\theta)}{f(\theta_o)} \quad (c)$$

where $f(\theta)$ is a universal function depending only on the properties of the empirical temperature scale. Its representation, Figure 2, in a

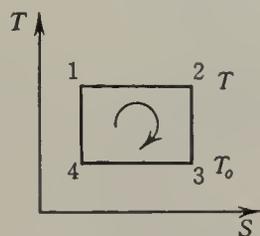


Fig. 2. Carnot cycle— T,S diagram.

T,S diagram is particularly simple. Since a reversible Carnot cycle has the highest efficiency of all cycles, reversible or irreversible, operating between a highest temperature T and a lowest temperature T_o , it can be used as a

standard of comparison for real cycles. It also represents the ideal arrangement for the conversion of heat into work.

The properties of Carnot cycles can be used for an elementary mathematical formulation of the second law of **thermodynamics** and the concept of **entropy**.

The forward Carnot cycle (as discussed above), can, in principle, be used for the production of power (conversion of heat into work). When the cycle is operated in *reverse*, it can be used in principle as a refrigerator or heat pump. When the temperature, T , is equal to that of an available cooling source (atmosphere), the cycle rejects heat Q to it and absorbs heat Q_o at a lower temperature T_o . It can thus serve as a *refrigerator* maintaining the temperature of a body at $T_o < T$ by removing a heat leak Q_o into it at the expense of the work,

$$W = Q - |Q_o|.$$

When the temperature T_o is equal to that of an available source (surrounding atmosphere), the cycle can extract heat Q_o from it and maintain a body at a temperature $T > T_o$ by supplying heat Q to it to cover its heat leaks at the expense of work W (*heat pump*). In either case, the Carnot cycle represents the minimum amount of work W required for the purpose under given conditions (determined by T and T_o).

The performance of a refrigerator is expressed by the coefficient of performance

$$e = \frac{|Q_o|}{W} = \frac{|Q_o|}{|Q| - |Q_o|} = \frac{T_o}{T - T_o}$$

and the quantity of heat $|Q_o|$ extracted from the cooling chamber is called the *refrigerating effect*.

There exists no agreed coefficient to express the operation of a heat pump, but the ratio

$$\frac{Q}{W} = \frac{T}{T - T_o}$$

can serve the purpose.

It should be noted that the efficiency of the forward cycle is highest when T is as high as possible. Since, in practice, T_o will always be fixed by the temperature of the surrounding atmosphere, a high efficiency corresponds to a large difference $T - T_o$. In contrast, a high coefficient of performance, or a high effectiveness of a heat pump corresponds to a small difference $T - T_o$.

It would appear that decreasing T_o for a

power cycle below that of the surrounding atmosphere is advantageous in that the efficiency η is increased. However, it must be realized that this can only be achieved at the expense of work in operating a refrigerator, and no advantage is gained.

CARNOTIZATION. A process of feed-water heating by bleeding steam in a steam plant, usually steam-turbine plant, in order to increase its thermal efficiency. Since the ideal limit of efficiency is that of a **Carnot cycle**, the process of increasing the efficiency of the cycle is called "carnotization."

The plant is drawn schematically in Figure 1. The feed-water from the condenser is supplied by the feed pump, and pre-heated in the 3 feedwater heaters with the aid of steam from the multi-stage turbine. The condensate passes through throttling valves from each pre-heater to the next one with a lower pressure and partly evaporates, the steam being utilized for pre-heating in the next stage. From the last pre-heater the condensate passes through the counter-flow heat exchanger in which it transfers its sensible heat, and finally reaches the inlet duct of the feed pump through the throttling valve.

The process is illustrated in the T,S diagram of Figure 2. There it has been assumed that steam at three different pressures is bled from the plant and is used to heat the feed-water in stages. The quantity of steam bled from the turbine is so adjusted that, for example at pressure p_3 , its heat of condensation (the rectangle bounded by the line cd , the abscissas

from c and d to the S -axis and the S -axis itself) is equal to the sensible heat of the feed water between the temperatures corresponding to pressures p_3 and p_2 (the figure bounded by the line ab , the abscissas from a and b to the

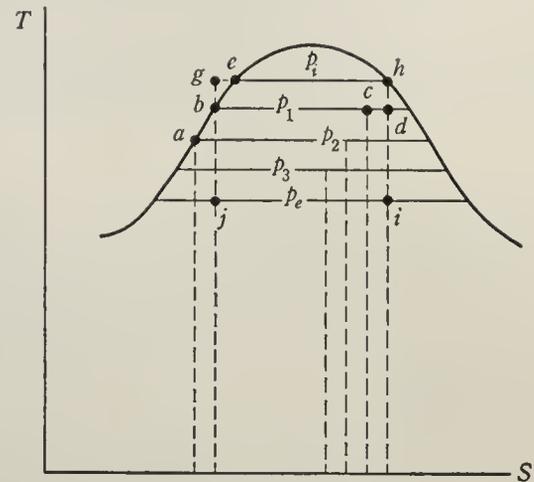


Fig. 2. T,S diagram of Carnotization process.

S -axis and the S -axis itself). By relating similarly the heat of condensation of the steam for the other pressure intervals to the sensible heat of the feed water for the corresponding temperature differences, and summing heats of condensation and sensible heat, it is found that the ideal work of the modified cycle differs from that of an equivalent Carnot cycle $ghij$ only by the small triangle geb ; hence the efficiency is almost equal to that of a Carnot cycle. As the number of stages is increased, the approximation to a Carnot cycle becomes closer. The above cycle is also known as the regenerative (steam) cycle, and it is said that

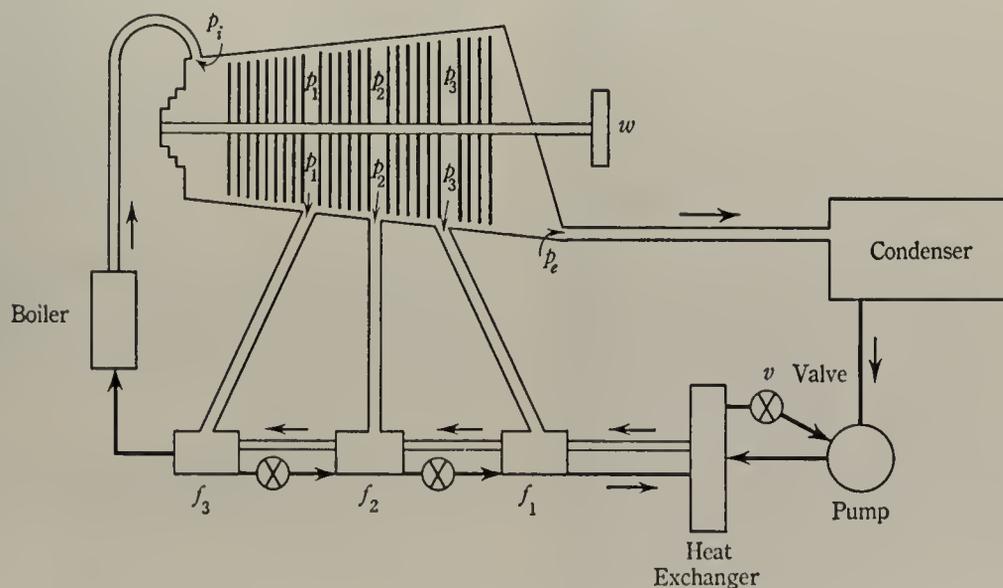


Fig. 1. Steam power plant with interstage bleeding, illustrating Carnotization. (Certain auxiliary equipment, e.g., superheater, economizer, etc., not shown.)

heat is being "regenerated" in the heat exchangers.

CARNOT THEOREM. The most essential corollary of the second law of **thermodynamics**. It can be stated in the following way: For any system there exist two properties, its thermodynamic temperature T and its entropy S such that in any infinitesimal reversible process, the quantity of heat exchanged is given by

$$dQ = TdS.$$

Furthermore, the thermodynamic temperature T is a positive function of the empirical temperature θ measured on an arbitrary empirical temperature scale. The Carnot theorem is a direct consequence of Part 1 of the **Carathéodory principle**.

Some authors prefer to reserve the designation of Carnot's theorem to the following corollary of the second law of thermodynamics: No engine operating between two reservoirs of fixed temperatures T and T_o ($T > T_o$) can have a higher efficiency η than that of a reversible Carnot cycle operating between the same sources, thus,

$$\eta < \eta_c$$

$$\text{where } \eta_c = 1 - \frac{T_o}{T}.$$

CARRIER DENSITY. In thermal equilibrium, the density, n , of electrons, and p , of holes in a non-degenerate semiconductor are related by the equation

$$np = N_c N_v \exp(-E_G/kT)$$

where E_G is the width of the **energy gap** between the **valence band** and the **conduction band**, and N_v , N_c are the effective densities of states in these bands, i.e.,

$$N_c = N_v = 2(2\pi mkT/h^2)^{3/2}.$$

CARRIER EQUATION. See **adiabatic saturation**.

CARRY. (1) A signal, or expression, produced as a result of an arithmetic operation on one digit place of two or more numbers expressed in positional notation and transferred to the next higher place for processing there. (2) Usually a signal or expression as defined in (1) above which arises in adding, when the sum of two digits in the same digit place equals or exceeds the base of the number system in use. If a carry into a digit place will result

in a carry out of the same digit place, and if the normal adding circuit is bypassed when generating this new carry, it is called a *high-speed carry*, or *standing-on-nines carry*. If the normal adding circuit is used in such a case, the carry is called a *cascaded carry*. If a carry resulting from the addition of carries is not allowed to propagate (e.g., when forming the partial product in one step of a multiplication process), the process is called a *partial carry*. If it is allowed to propagate, the process is called a *complete carry*. If a carry generated in the most significant digit place is sent directly to the least significant place (e.g., when adding two negative numbers using nines complements) that carry is called an *end-around carry*. (3) In direct subtraction, a signal or expression as defined in (1) above which arises when the difference between the digits is less than zero. Such a carry is frequently called a *borrow*. (4) The action of forwarding a carry. (5) The command directing a carry to be forwarded.

CARRYING CAPACITY (LOAD). The load-carrying capacity of a structure is the maximum load which can be applied prior to complete collapse or is the maximum load which can be applied without causing clearly excessive deflection or danger of **brittle fracture**. (See **limit load**.)

CARRYOVER FACTOR. When elastic continuous (rigid) frames are analyzed by the **moment distribution** technique, moments are balanced or distributed at each joint and part of the distributed moments are carried over to neighboring joints. The fraction carried over is called the carryover factor. When individual beams and columns are of uniform cross section the carryover factor is one half. No moment is carried over to a hinged end, however.

CARRY, SELF-INSTRUCTED. A system of executing the **carry** process in which **information** is allowed to propagate to succeeding places as it is generated and without receipt of a specific signal.

CARRY, SEPARATELY-INSTRUCTED. A system of executing the **carry** process in which **carry information** is allowed to propagate to succeeding places only on receipt of a specific signal.

CARTESIAN COORDINATES. See *coordinate*.

CARTESIAN OVAL. See *Cartesian surfaces*.

CARTESIAN SURFACES. The aplanatic surfaces of revolution are the Cartesian surfaces generated by revolving the *Cartesian oval*

$$n_0 \sqrt{x^2 + y^2} \pm n_1 \sqrt{(x - a)^2 + y^2} = c$$

about the x -axis. The Cartesian oval is a fourth degree algebraic curve. In certain cases, when the surface is aplanatic under reflection, or in the case of refraction if one of the conjugate points is at infinity, then the Cartesian oval degenerates to a conic section. When both conjugate points are at infinity, the Cartesian oval degenerates to a line.

Descartes' investigation of the aplanatic properties of the conics is evidence that he anticipated Snell's discovery of the laws of refraction.

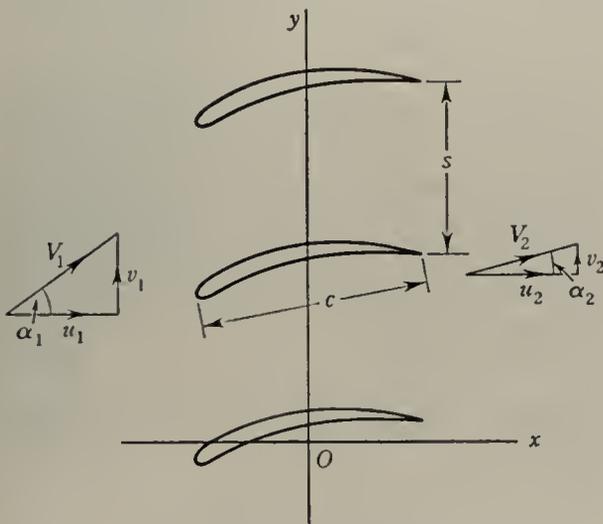
CARTESIAN TENSOR. See *tensor field, Cartesian*.

CARTESIAN TENSOR FIELD. See *tensor field, Cartesian*.

CARTESIAN VECTOR FIELD. See *vector field, Cartesian*.

CASCADED CARRY. See *carry*.

CASCADE OF AIRFOILS. An infinite set of similar airfoils, all placed at the same incidence and equally spaced. In the figure, only three of the airfoils are shown, but the set is assumed to extend to $y = \pm \infty$. The velocities



Cascade of airfoils.

at $x = -\infty$ and $x = +\infty$ are assumed to be uniform and equal to V_1 and V_2 respectively, with components u_1, v_1, u_2 and v_2 as shown. The equation of continuity then shows that $u_1 = u_2$. From the equation of momentum it can be shown that the **Kutta-Joukowski law**, relating lift to circulation, applies to a cascade if the stream velocity is taken to be the vector mean velocity

$$V = \frac{1}{2}(V_1 + V_2).$$

Thus for each airfoil, if Γ is the circulation, the lift is $\rho V \Gamma$ acting in a direction normal to V . The lift coefficient C_L , defined in terms of the vector mean velocity V , is

$$C_L = 2 \frac{s}{c} (\tan \alpha_1 - \tan \alpha_2) \cos \alpha_m,$$

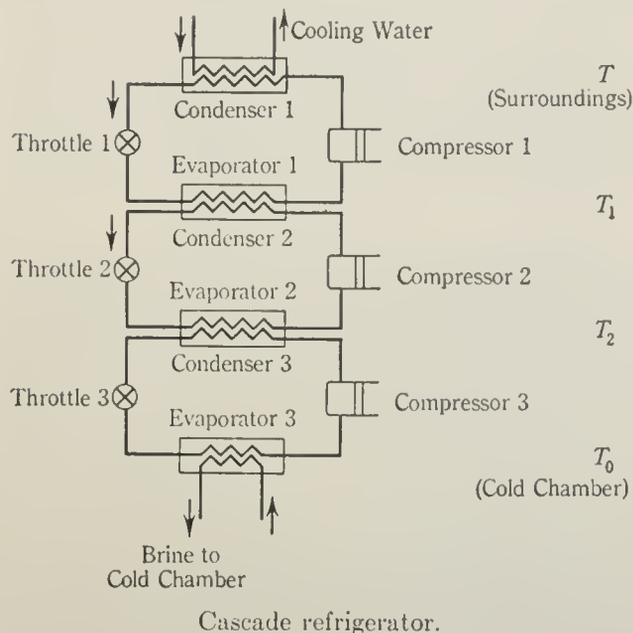
where $\tan \alpha_m = \frac{1}{2}(\tan \alpha_1 + \tan \alpha_2)$.

The flow through a cascade of airfoils can be calculated in various ways by the use of conformal transformations. For the simple case of a cascade of thin flat plates, the cascade in the ζ -plane can be transformed into a circle in the z -plane. In the general case, when the airfoils have thickness and camber, the cascade in the ζ -plane can be transformed into an oval in the z -plane. If the airfoils are thin and of small camber, this oval in the z -plane approximates to a circle and can be transformed into a true circle in the z' -plane by a method analogous to Theodorsen's method (see **airfoil theory**) for a single airfoil.

The cascades that are of practical importance usually consist of airfoils of large camber. The solution by conformal transformation is then very lengthy and complicated, and an alternative method is often used, in which each airfoil of the cascade is represented by a distribution of vorticity along the center line. This implies that the thickness of the airfoil is neglected, i.e., the method is analogous to thin-airfoil theory as applied to a single airfoil. (See A. Robinson and J. A. Laurmann, *Wing Theory*, Cambridge, 1956.)

CASCADE REFRIGERATOR. An arrangement of refrigerators, each filled with a different working fluid, operating in series, the evaporator of one serving as the condenser for the next. In this manner low refrigeration temperatures can be achieved with reasonable coefficients of performance. In addition, the properties of the refrigerants can be suited to

the respective temperature ranges. Cascade refrigerators have been used to liquefy gases in the last stage.



CASTIGLIANO FIRST THEOREM. Within the framework of classical elasticity theory, we consider a body to be subjected to concentrated forces with components $P_i^{(\alpha)}$ ($i = 1, 2, 3$; $\alpha = 1, 2, \dots$) in a rectangular Cartesian coordinate system x . The components $u_i^{(\alpha)}$ in the system x of the displacement undergone in the deformation, by the particle of the body at which the force $P_i^{(\alpha)}$ is applied, are given by $u_i^{(\alpha)} = \frac{\partial U}{\partial P_i^{(\alpha)}}$, where U is the strain energy for the deformations produced by the system of forces $P_i^{(\alpha)}$ ($\alpha = 1, 2, \dots$).

When the boundary displacements are zero wherever they are specified and the stress-strain relation is elastic, the complement of the strain energy is a minimum. For a linearly elastic system, the complement of the strain energy equals the strain energy, so that the strain energy is also a minimum. The derivative of the strain energy with respect to any applied force P_i is then the displacement of the point of application of the force in the direction of the force:

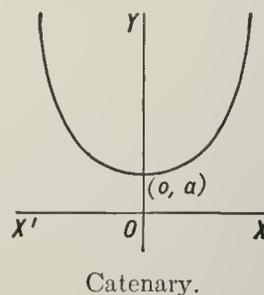
$$\frac{\partial U}{\partial P_i} = \delta_i.$$

CATACAUSTIC. A caustic produced by reflection.

CATENARY. The locus of the transcendental equation

$$y = \frac{a}{2} (e^{x/a} + e^{-x/a}) = a \cosh \frac{x}{a}$$

The curve can be generated by the focus of a parabola rolling along a straight line and its shape is that taken by a uniform, heavy flexible cable freely suspended from its ends. The involute of the catenary is called the *tractrix*.



CATENARY CURVE OR CABLE. A homogeneous flexible cable, when suspended from two fixed points and allowed to hang freely, assumes the form of a **catenary** curve. The equation for a catenary cable with a linear mass density ρ is

$$y = \frac{A}{\rho g} \cosh \frac{\rho g x}{A}$$

where g is the acceleration of gravity and A is a constant equal to ρg times the perpendicular distance from the x -axis to the lowest point of the catenary.

CATION. See **electroncutrality**.

CATOPTICS. That part of optics dealing with reflection, especially spectral reflection. Formally called *anacamptics*, though "*Catoptics*" was the title of Euclid's book on reflection. (See **dioptrics**.)

CATOPTRIC SYSTEM. (1) If an optical system is convergent, it is called catoptric if the object space focal length is positive and the image space focal length is negative. If a lens system is divergent, it is called catoptric if the object space focal length is negative and the image space focal length is positive. (2) Any mirror system.

CAUCHY (INTEGRAL) CONVERGENCE TEST. If

$$\lim_{n \rightarrow \infty} |a_n|^{1/n} < 1,$$

then the infinite series

$$\sum_{n=1}^{\infty} a_n$$

converges absolutely. A corollary, known as the *Cauchy ratio test*, or sometimes as *d'Alembert's test*, involves

$$\lim_{n \rightarrow \infty} |a_{n+1}/a_n| = r.$$

If $r > 1$, the series converges; if $r < 1$, it diverges; if $r = 1$, the test fails and the series may either converge or diverge.

CAUCHY DISTRIBUTION. A statistical distribution of the form

$$dF = \frac{dx}{\pi(1+x^2)}, \quad -\infty \leq x \leq \infty.$$

CAUCHY-EULER EQUATION. Synonym of Euler-Lagrange equation; see **variations, calculus of**.

CAUCHY FORMULA FOR REFRACTIVE INDEX. There is no simple formula expressing the index of refraction of a medium as a function of wavelength. A number of semi-empirical formulas have been used. One of these is *Sellmeier's equation*.

$$n^2 = 1 + \frac{A_1 \lambda^2}{\lambda^2 - \lambda_1^2}$$

where A_1 is a constant characteristic of the material and λ_1 is an idealized absorption wavelength of the medium. When $\lambda_1 \ll \lambda$ this reduces to

$$n^2 = 1 + A_1 + \frac{A_1 \lambda_1^2}{\lambda^2}.$$

Cauchy suggested that the Laurent expansion

$$n = \sum a_n \lambda^{-2n}$$

would be more satisfactory. **Herzberger's dispersion formula** is the best currently available for most optical material.

CAUCHY INTEGRAL FORMULA. The formula

$$f(z) = \frac{1}{2\pi i} \int_C \frac{f(\zeta)}{\zeta - z} d\zeta,$$

where the path of integration C consists of a finite number of simple closed rectifiable

curves in the complex plane bounding a region D in which $f(z)$ is analytic.

CAUCHY INTEGRAL THEOREM. If $f(z)$ is an analytic function of a complex variable z which has no singular point within or on a given closed curve C , then

$$\int_C f(z) dz = 0,$$

where the **integral** is extended over the entire contour C . (See also **Morera theorem**.)

CAUCHY NUMBER. A non-dimensional number arising in the study of the elastic properties of a fluid. It may be written $U^2 \rho / e$, where U is a characteristic velocity, ρ the density, and e the modulus of elasticity of the fluid. It is the square of the **Mach number**.

CAUCHY PROBLEM FOR PARTIAL DIFFERENTIAL EQUATIONS. The Cauchy problem for a set of partial differential equations can be stated, in general form, as follows:

Let t, x_1, x_2, \dots, x_n denote independent variables, while u_1, u_2, \dots, u_m are the dependent variables. Let there be given m equations of the form

$$\frac{\partial^{k_i} u_i}{\partial t^{k_i}} = F_i, \quad i = 1, 2, \dots, m$$

where F_i is a function of the independent and dependent variables and of partial derivatives of the latter of order not higher than k_i . In physical applications, t is usually the time and x_1, x_2, \dots, x_n are space coordinates.

For a fixed $t = t_0$, let the following conditions (the initial or Cauchy data) be prescribed

$$\frac{\partial^k u_i}{\partial t^k} = \phi_{ik}(x_1, x_2, \dots, x_n), \quad k = 0, 1, \dots, k_i - 1,$$

where the functions ϕ_{ik} are given over some region of the space (x_1, x_2, \dots, x_n) .

For example, for a vibrating membrane ($n = 2, m = 1, k_1 = 2$) there is one hyperbolic partial differential equation

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2}$$

and the Cauchy conditions are

$$u(t_0, x_1, x_2) = \phi(x_1, x_2),$$

and

$$\dot{u}(t_0, x_1, x_2) = \psi(x_1, x_2,$$

representing the initial position and velocity, respectively. (See also **Riemann method**.)

CAUCHY RELATIONS (IN CRYSTAL STRUCTURE). If it is assumed that the forces between the atoms of a crystal lattice are central forces (i.e., that they act along the lines joining the centers of the atoms), and if it is also assumed that each atom is at a center of symmetry (i.e., that if \mathbf{r}_{jk} is the vector joining two atoms, replacing \mathbf{r}_{jk} by \mathbf{r}_{kj} does not change the structure), certain relations can be established between the elastic constants of the unstrained crystal.

Thus, if the relations between the six components of the stress tensor p_{ij} and the six components of the strain tensor s_{ij} are written in the usual way, we have

$$p_i = \Sigma c_{ij}s_j.$$

There are thus 36 elastic constants c_{ij} but the relation $c_{ij} = c_{ji}$ necessary for the existence of a strain energy function reduces this number to 21. The Cauchy relations state that in addition

$$c_{12} = c_{66}, \quad c_{23} = c_{44}, \quad c_{31} = c_{55}$$

$$c_{14} = c_{65}, \quad c_{25} = c_{46}, \quad c_{36} = c_{54}.$$

Thus, if these relations were true, there could be no more than 15 independent elastic constants for a crystal.

Ionic crystals are found to satisfy the Cauchy relations reasonably well, but metals do not, because the interatomic forces are not central.

Poisson showed that, for an isotropic solid, the assumption of central forces led to a single elastic constant, the value of **Poisson's ratio** being always 0.25.

CAUCHY RELATIONS IN ELASTICITY THEORY. Relations between the elastic constants in the **generalized Hooke's law**, which result from the assumption that the material consists of particles, between each pair of which a force acts along the line joining them, the magnitude of this force depending on the distance between the pair of particles.

CAUCHY-RIEMANN EQUATIONS. For functions $u = u(x,y)$ and $v = v(x,y)$ the Cauchy-Riemann equations are

$$\frac{\partial u}{\partial v} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}.$$

These equations are satisfied by the real and imaginary parts of an analytic function $f(z) = u(x,y) + i(v,y)$ of the complex variable z .

CAUCHY SEQUENCE. A sequence a_1, a_2, \dots of elements of a metric space is called a Cauchy sequence if, for every $\epsilon > 0$, there exists an integer N such that $|a_m - a_n| < \epsilon$ for all $m, n > N$.

CAUSALITY, MICROSCOPIC. See **causality principle (2)**.

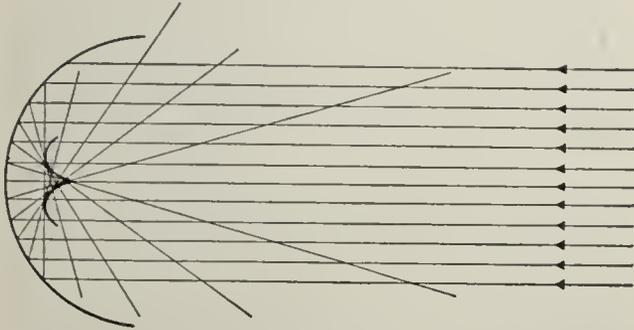
CAUSALITY PRINCIPLE. (1) The hypothesis that a precisely determined set of conditions will always produce precisely the same effects at a later time. Classical physics was based on the idea that the precise determination of the initial conditions was possible in principle. Quantum mechanics is based rather on the **Heisenberg uncertainty principle**, which indicates that such precise determination is impossible. (2) The principle of microscopic causality is the statement that signals never propagate with a velocity faster than that of light, no matter how short the distance involved. In the language of local quantum field theory, it is expressed by the statement that the commutator of two observable (Hermitian) Heisenberg field operators, taken at different space-time points, vanishes if the separation between these points is space-like. If $A(x)$ and $B(y)$ are two such Heisenberg operators, causality asserts that

$$[A(x), B(y)] = 0 \quad \text{for } (x - y)^2 < 0.$$

The arguments which lead to this formulation are as follows: in quantum theory the lack of commutativity of two observable operators implies that these cannot be measured simultaneously with arbitrary accuracy. However, measurements at points which are separated by space-like distances can never perturb one another, since relativity demands that signals (energy) cannot be propagated with a velocity greater than that of light. Hence the commutator of observables with space-like connections must vanish.

CAUSTIC. One of the earliest discoveries in optics (F. Maurolycus, 1575) was that the rays of a normal congruence are tangent to a surface, called the *caustic surface*, and that the

general ray is tangent to the caustic surface in two points. For example, the caustic of a parabolic mirror due to an incident beam of rays parallel to the axis of symmetry is a semi-cubic paraboloid of revolution. Gaviola's caustic test for parabolic mirrors is based on the caustic being the locus of centers of curvature of the parabolic sections.



Caustic of a parabolic mirror.

The most instructive definition characterizes the caustic surface as a set of caustic curves. A *caustic curve* is the edge of regression of the polar developable of a line of curvature on a **wave front**. This means that the set of normal planes of a line of curvature of a wave front are the tangent planes of a **developable surface**, the *polar developable*. The characteristic lines, or rulings, of the polar developable are the tangent lines of the edge of regression or caustic curve. This shows that the caustic surface consists of two sheets, whose points are the centers of curvature of the wave front, or the **tangential** and **sagittal foci** of the rays.

CAVITATION. In a flowing liquid, if the motion causes the absolute pressure at any point to fall below the vapor pressure p_v , bubbles of vapor are usually formed. Usually, at some point further downstream, the pressure rises again to some value above p_v ; the bubbles of vapor then collapse rapidly and often a loud noise is emitted and the high pressures generated locally cause pitting and corrosion of solid objects.

Since any pressure variations in the vapor cavity are very small compared with those in the liquid, the boundary of the vapor cavity may be considered to be one of constant pressure. This has sometimes been applied as a method of determining the shape of a solid boundary to give constant pressure on the boundary.

The *cavitation number* of a stream of liquid pressure p_1 and velocity v is defined as

$$\lambda = \frac{p_1 - p_v}{1/2\rho v^2}.$$

This number is a measure of the tendency for cavitation to occur when the stream is disturbed by the introduction of a solid body. If C_p is the pressure coefficient at any point on the body, cavitation may be expected to occur wherever $-C_p > \lambda$. Thus a small value of λ means that the body must introduce only small disturbances if cavitation is to be avoided.

Cavitation is of great practical importance in connection with water turbines and ships' propellers. In both these cases it is desirable to use the highest possible blade speed, but the speed that can be used is restricted by the necessity for avoiding cavitation, with its associated loss of efficiency and corrosive damage to the solid surface.

It has been shown that in water from which all dissolved air has been carefully removed it is possible for the pressure to fall considerably below p_v without the formation of any bubbles of vapor. It is even possible for the absolute pressure in the water to become negative.

CAVITATION NUMBER. See **cavitation**.

CAVITY FIELD IN A DIELECTRIC. The cavity field is the electrical field inside a spherical cavity of dielectric constant ϵ_2 , which is included in a medium of dielectric constant ϵ_1 , when the dipole moment of the sphere is 0 (i.e., when the spherical cavity is not polarized, see **polarization**). If the static field inside the dielectric ϵ_1 has, at a sufficiently large distance from the cavity, the value E_∞ , the cavity field G has the value

$$G = \frac{3\epsilon_1}{2\epsilon_1 + \epsilon_2} E_\infty.$$

For an empty sphere $\epsilon_2 = 1$, and

$$G = \frac{3\epsilon_1}{2\epsilon_1 + 1}.$$

(See also **reaction field**.)

CAYLEY-KLEIN PARAMETER. One of a set of four quantities used to describe the orientation of a rigid body. They are not linearly independent but may be expressed in terms of the **Euler angles**. When combined into matrix

equations they are related to the Pauli spin matrices which occur in quantum mechanics. (See also **Euler-Rodrigues parameters**.)

C-CIRCUIT. An alternate term for **fundamental circuit**.

ČEBYŠEV. See **Chebyshev**.

CELESTIAL SPHERE. An imaginary sphere of infinite radius upon which the celestial bodies are assumed to be fixed. The celestial sphere is used as a convenience in solving problems in celestial mechanics, navigation, and related fields. Points on the celestial sphere are determined by their celestial altitude and celestial azimuth. Since the celestial sphere is infinite in size, the earth's radius is not significant in comparison, and calculations are based upon the assumption that terrestrial observations are made from the center of the earth rather than the surface.

CELL, BINARY. In computers, an elementary unit of storage which can be placed in either of two stable states.

CELL EMF (FORMULA FOR). Since the electrical work done by a reversible cell equals the decrease of Gibbs free energy (**Gibbs function**) in the cell reaction

$$-\Delta G = W = EIt = Eq = EnF.$$

It follows that

$$E = -\frac{\Delta G}{nF}$$

where ΔG is the free energy change, n , the number of faradays passing through the cell, and F is the faraday (96,500 coulombs).

CELL MODEL OF THE LIQUID STATE. For densities as high as those in the liquid state far below the critical temperature, one may expect a certain order in the mutual distribution of the molecules. Intermolecular distances between first neighbors smaller than the molecular diameter are prohibited by the strong repulsive forces (see **intermolecular forces**) while distances much larger than the mean intermolecular distances are statistically very unlikely.

This more or less regular structure forms the basis of the cell models. In its simplest form, it assumes that each molecule is confined to its own cell (Eyring, Eyring and Hirschfelder, Lennard-Jones and Devonshire).

The field acting on each molecule in its cell is rapidly fluctuating, and may be replaced by an *average field* of spherical symmetry. If the lifetime of a molecule in its cell is sufficiently long, the exchange of places may be neglected in the evaluation of the thermodynamic properties (but of course not in the evaluation of the transport properties).

Denote by Ψ the **partition function** corresponding to a molecule in its cell referred to the energy of the particle at the center of the cell, and denote by $\omega(r)$, the mean energy of interaction. Then the *cell partition function* is

$$\Psi = 4\pi \int_{cell} \exp \left\{ \frac{1}{kT} [-(\omega(r) - \omega(0))] \right\} \cdot r^2 dr \quad (1)$$

where $\omega(0)$ is the value of $\omega(r)$ at the center of the cell. The configurational partition function of the liquid may be written

$$Q = \Psi^N \exp \left[-\frac{1}{2kT} N\omega(0) \right] \quad (2)$$

where $\frac{1}{2} N\omega(0)$ is the energy of the system when all particles are at the centers of their cells.

Using an explicit expression for $\omega(r)$, all thermodynamic properties may be deduced from (2).

The cell model gives a reasonable semi-quantitative description of the thermodynamic properties of a liquid. It suffers, however, from the basic weakness of a one-particle model. **Correlations** between molecules are not properly taken into account and the **entropy** is underestimated. The cell model has also been used for solutions (Prigogine).

CELSIUS TEMPERATURE SCALE. See **temperature**.

CENSORING. In the theory of sampling, a sample is said to be censored when the values of some of the members are unknown or deliberately ignored. If values exceeding (or falling short of) a fixed variate-value are unknown or discarded the sample is sometimes said to be *truncated*. The distinction between censoring and truncation is sometimes blurred and usage is not uniform.

CENTER, INSTANTANEOUS. The point about which a body having general motion may be considered to be in pure rotation (i.e., without **translation**) for any instant. The in-

stantaneous center is not necessarily on the body; in fact, it can be, in the case of rectilinear motion, infinitely distant.

CENTER OF A GROUP. The set (also called the central) of those elements of the **group** which commute with every element of the group. The center always includes the unit element.

CENTER OF AREA. See **centroid** (of a geometrical figure).

CENTER OF A SET OF VALUES. In statistics, the center of a set of values is the point midway between the two extreme values.

CENTER OF COLLINEATION. In **Gaussian optics**, the center of collineation is the point common to all rays joining **conjugate points** of (1) a spherical refracting surface, or (2) a thin lens. In case (1) it is the center of the sphere, in (2) it is the **optical center**.

CENTER OF CURVATURE. See **curvature**.

CENTER OF CURVE. See **centroid** (of a geometrical figure).

CENTER OF GRAVITY. The point through which the resultant of the gravitational forces on a body acts for every position of the body. This point exists when the dimensions of the body are negligible compared with its distance from the center of the earth. Designating the total weight of the body by W , the weight per unit of volume V , by ρ , the position vector of any point by \mathbf{r} , and the position vector of the center of gravity by \mathbf{r}_g .

$$W\mathbf{r}_g = \int_V \rho \mathbf{r} dV.$$

CENTER OF INVERSION. (1) For geometrical meaning, see **inverse surfaces**. (2) A **symmetry element** possessed by certain crystals, whereby the crystal can be brought into self-coincidence by the operation $\mathbf{r} \rightarrow -\mathbf{r}$, where \mathbf{r} is the vector position of a point in the crystal referred to the center of inversion.

CENTER OF MASS. That point in a body about which the sum of the moments of all the individual masses constituting the body is zero. If the individual (point) masses be called m_j and the vector position of m_j from some fixed origin be called \mathbf{r}_j , the center of mass is located \mathbf{r} from the origin, where

$$\mathbf{r}\Sigma m_j = \Sigma m_j \mathbf{r}_j$$

the summation being extended over all of the masses constituting the body. For a body whose mass is continuously distributed over its volume, the coordinates of the center of mass are:

$$x_c = \frac{\int x\rho dx}{M}, \quad y_c = \frac{\int y\rho dy}{M}, \quad z_c = \frac{\int z\rho dz}{M},$$

where M is the total mass of the body and $\rho = f(x, y, z)$ is the density as a function of position. If the density is constant throughout, the center of mass coincides with the center of volume. (See **centroid of a geometrical figure**.)

CENTER-OF-MASS (COORDINATE) SYSTEM. In general, any frame of reference moving with the center of mass of a system. Calculations in the center of mass system are often simpler than in other coordinate systems because the total **momentum** in the former is always zero. (See **L-system** and **C-system**.)

CENTER OF MOMENTS. The point about which moments are computed. Symbolizing moment as $\mathbf{r} \times \mathbf{F}$, the center of moments is the origin from which the position vector \mathbf{r} to any point on the line of action of the force \mathbf{F} is measured.

CENTER OF OSCILLATION. The frequency of oscillation of a physical pendulum is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{gl}{k^2 + l^2}},$$

where l is the distance from the point of suspension to the center of mass, and k is the **radius of gyration**. A simple pendulum having the same frequency will be of length

$$l_1 = \frac{k^2 + l^2}{l}.$$

The point which lies at a distance l_1 from the point of suspension on the line through the point of suspension and center of mass is called the center of oscillation. If the physical pendulum were to be suspended from this point, its frequency would be the same as for the original point of suspension. (See also **pendulum, physical**; **pendulum, simple**.)

CENTER OF PERCUSSION. See *percussion, center of*.

CENTER OF PRESSURE. The point on an area of a submerged body at which the resultant of the pressure force on the area acts.

CENTER OF SHEAR. See *shear center*.

CENTER OF SUSPENSION. See *center of oscillation*.

CENTER OF SYMMETRY. See *symmetry, center of*.

CENTER OF TWIST. See *shear center*.

CENTER OF VOLUME. See *centroid (of a geometrical figure)*.

CENTIGRADE HEAT UNIT (ABBREVIATED C.H.U. OR CHU). Originally defined as $\frac{1}{100}$ part of the quantity of heat required to raise 1 lbm of water from its freezing to its boiling point at atmospheric pressure. This unit was used in engineering calculations based on the British system of units together with the Celsius (Centigrade) temperature scale. Now largely obsolete. (See *thermal units*.)

CENTIGRADE TEMPERATURE SCALE. See *temperature*.

CENTRAL DIFFERENCE OPERATOR. See *difference operators*.

CENTRAL FORCE. See *force, central*.

CENTRAL LIMIT THEOREM. An important theorem in probability to the general effect that the sum of n independent random variables with finite *variances*, suitably standardized, tends to the *normal (Gaussian) distribution* as n tends to infinity. There are various forms of the theorem; a necessary and sufficient condition is that no variate dominates the others. More general theorems concerning random variables which do not possess finite variances, or are not independent, are possible and are also known as central limit theorems.

CENTRAL MEAN OPERATOR. See *difference operators*.

CENTRAL POINT OF A GENERATOR. See *ruled surface*.

CENTRAL QUADRIC. A quadric which has a center, e.g., an ellipsoid or hyperboloid. By

a suitable choice of the rectangular Cartesian reference system x, y, z , the equation of any central quadric may be written in the form

$$\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 1,$$

where a, b and c are real constants. The origin of the coordinate system is then the center of the quadric and the directions of the axes x, y, z are the *principal axes of the quadric*. If a, b and c are all positive, the quadric is an *ellipsoid*. If $a > b > c$ and c is negative, while a and b are positive, the quadric is a *hyperboloid of one sheet*. If a is positive and b and c are both negative, it is a *hyperboloid of two sheets*.

CENTRIFUGAL. Moving outward or directed outward, in the sense of away from a center.

CENTRIFUGAL COMPRESSOR. See *axial-flow compressor*.

CENTRIFUGAL FORCE. A radially outward force experienced by an observer in a reference frame which is rotating at an *angular velocity* ω with respect to an *inertial frame*. The centrifugal force is an *inertial force* and may be considered as the reaction to the *centripetal force* necessary to hold the observer at a fixed point in the rotating frame, and thus has a magnitude equal to and a direction opposite to the centripetal force.

CENTRIPETAL ACCELERATION. The acceleration toward the center of curvature to which any particle moving in a curved path is subject. It is equal to v^2/r where v is the speed of the particle and r is the radius of curvature of the path (see *centripetal force*). An equivalent term is *normal acceleration*.

CENTRIPETAL FORCE. The radially inward force mv^2/r required for motion of a particle of mass m , traveling at speed v in a curved path of radius of curvature r . When the motion is circular with angular velocity ω , the *centripetal acceleration* v^2/r may be written as $r\omega^2$.

CENTRODE. The path of the instantaneous center of a plane figure having plane motion, that is, motion resulting when all points in the figure move in parallel fixed planes. Any plane body having plane motion which is neither entirely rectilinear nor entirely rota-

tive, but a combination of the two, may be considered at any instant as having rotary motion about a moving point called the *instantaneous center of rotation*.

CENTROIDAL AXIS. An axis through the centroid of a cross section, therefore a possible neutral axes for bending.

CENTROID (OF A GEOMETRICAL FIGURE). The centroid of a given geometrical figure (curve arc, portion of a plane or curved surface, or solid) is the point whose **coordinates** are the **mean values** of the coordinates of the points of the given figure; it is independent of the choice of axes. The centroid of a geometrical figure corresponds to the **center of gravity** (or **center of mass**) of a homogeneous material body of similar form.

For a plane curve arc, the centroid is given by formulae:

$$\bar{x} = \frac{\int_a^b x ds}{L}, \quad \bar{y} = \frac{\int_a^b y ds}{L},$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2} \cdot dx = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} \cdot dy,$$

where L is the length of the arc, and ds is the length of an infinitesimal portion of the curve.

For a plane area, the centroid is given by:

$$\bar{x} = \frac{\iint_A x dS}{A}, \quad \bar{y} = \frac{\iint_A y dS}{A},$$

where A is the area of the given region, and dS is the area of an infinitesimal portion of the surface.

For a solid, the centroid is given by:

$$\bar{x} = \frac{\iiint_V x dV}{V}, \quad \bar{y} = \frac{\iiint_V y dV}{V}, \quad \bar{z} = \frac{\iiint_V z dV}{V},$$

where V is the volume of the given region, and dV is the volume of an infinitesimal portion of the solid.

CENTROID OF MASS. See **center of mass**.

CEPHEID VARIABLES. See **variable stars**.

CERENKOV RADIATION. A charged particle moving with uniform velocity v through a medium in which the velocity of light is less

than v will produce radiation which is sharply peaked along a cone in the forward direction. This cone is defined by the angle $\theta = \cos^{-1}(c/nv)$, where c is the velocity of light *in vacuo* and n is the **index of refraction** of the medium. This phenomenon is roughly analogous to the formation of shock waves in a fluid medium.

CETANE NUMBER. (Correctly cetene number.) The use of the term cetane number is becoming widespread in (misplaced) analogy with octane number. (See **knocking**.)

CHAIN RULE OF DIFFERENTIATION.

Let $w(u_1, u_2, \dots, u_m)$ be a function of m variables u_j , each of which is a function $u_j(x_1, x_2, \dots, x_n)$ of n variables x_i . Then w is a (composite) function of the variables x_i and, under mild conditions of continuity on w and u_j , the partial derivative $\partial w/\partial x_i$ of w with respect to x_i is given by the chain rule:

$$\frac{\partial w}{\partial x_i} = \frac{\partial w}{\partial u_1} \cdot \frac{\partial u_1}{\partial x_i} + \frac{\partial w}{\partial u_2} \cdot \frac{\partial u_2}{\partial x_i} + \dots + \frac{\partial w}{\partial u_m} \cdot \frac{\partial u_m}{\partial x_i}.$$

Many similar chain rules can be formulated for derivatives of higher order.

CHANNEL. (1) A combination of transmission media and equipment capable of receiving signals at one point and delivering related signals at another point. (2) In electronic computers, that portion of a storage medium which is accessible to a given reading station. (See also **track**.) (3) A **bar** having a section in the shape of a rectangle lacking one side. (4) A closed or partly-closed duct through which fluid flows.

CHANNEL CAPACITY. The maximum possible **information rate** through a **channel** subject to the constraints of that channel; channel capacity may be either per second or per symbol.

CHANNEL, ENERGY LENGTH OF. See **energy length for pipe or channel**.

CHANNEL STORAGE. The water volume within a specified portion of a stream channel.

CHANNEL, TRANSITION TO TURBULENT FLOW IN. See **transition to turbulent flow in a pipe or channel**.

CHANNEL, TWO-DIMENSIONAL, LAMINAR FLOW IN. For two-dimensional laminar flow, in a straight channel formed by paral-

lel walls at a distance $2b$ apart, the velocity distribution is given by

$$u = \frac{p'}{2\mu} (b^2 - y^2), \tag{1}$$

where u is the velocity at a distance y from the center, p' is the longitudinal pressure gradient and μ is the viscosity.

If τ_o is the shear stress at the wall, the skin friction coefficient is

$$C_f = \frac{\tau_o}{\frac{1}{2}\rho V^2} = \frac{12}{R}, \tag{2}$$

where V is the mean velocity and R is the Reynolds number based on this velocity and the channel width $2b$. This type of flow is often known as *plane Poiseuille flow*.

CHANNEL UTILIZATION INDEX. The ratio of the information rate (per second) through a channel to the channel capacity (per second).

CHAPMAN-JOUGET CONDITION. In steady-state detonation the lowest possible shock wave velocity is given by

$$v = \frac{1}{\rho_1} \left(\frac{P_2 - P_1}{\frac{1}{\rho_1} - \frac{1}{\rho_2}} \right)^{1/2}$$

where ρ_1 and ρ_2 are the densities in front of and behind the incident shock, respectively, and P_1 and P_2 are the corresponding pressures.

CHARACTER. (1) The trace (sum of the diagonal elements) of a matrix representation of a group. The character of all elements in a given class of the group is the same. (2) One of a set of elementary symbols (marks or events) which may be arranged in ordered aggregates to express information. A group of characters in one context, may be considered as a single character in another, as in the binary-coded-decimal system.

CHARACTERISTIC. (1) A synonym for eigen-, as in **eigenfunction** and **eigenvalue**. (2) For a first-order partial differential equation of the form

$$f(x,y) \frac{\partial z}{\partial x} + g(x,y) \frac{\partial z}{\partial y} = h(x,y),$$

any curve given in parametric form by $x = x(t)$, $y = y(t)$, $z = z(t)$ such that

$$\frac{dx}{dt} = f(x,y), \quad \frac{dy}{dt} = g(x,y), \quad \frac{dz}{dt} = h(x,y)$$

is called a characteristic. The importance of these characteristics is due to the fact that every integral surface $z = z(x,y)$ for the given partial differential equation is generated by a one-parameter family of characteristics; that is, every characteristic having a point in common with the surface lies entirely on it. In general, boundary conditions may be assigned at will along any non-characteristic curve. The concept can be generalized for equations of higher order in a greater number of variables. (See **characteristics, method of**.) (3) See **characteristic equation of a matrix**. (4) See **equation of state**.

CHARACTERISTIC EQUATION (OF GEOMETRICAL OPTICS). The differential equation of geometrical optics

$$V_x^2 + V_y^2 + V_z^2 = n^2$$

is referred to as the characteristic equation or *eikonal equation*. The solutions are **Hamilton's characteristic** (eikonal) of the optical system with index of refraction $n = n(x, y, z)$.

CHARACTERISTIC EQUATION OF A GAS. See **equation of state**.

CHARACTERISTIC EQUATION OF A MATRIX. The characteristic equation of a square matrix $\mathbf{A} = \{a_{ij}\}$, $i, j = 1, 2, \dots, n$, is $\det \{\mathbf{A} - \lambda \mathbf{I}\} = 0$ regarded as an algebraic equation in λ , where \mathbf{I} is the identity matrix and \det denotes determinant, so that in expanded form the equation is

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{vmatrix} = 0.$$

This determinant is called the *secular determinant*, and this equation is often called the *secular equation* because of their application to the study of secular perturbations of the planets. (See also **matrix**.)

CHARACTERISTIC EQUATION OF A SYSTEM. The equation which determines the

normal modes and hence the degree of **stability** of a system.

If the system is envisaged as comprising n physical quantities $q_1(t), \dots, q_n(t)$, whose Laplace transforms $Q_1(s), \dots, Q_n(s)$ are related by the n equations

$$a_{1m}(s)Q_1(s) + a_{2m}(s)Q_2(s) + \dots + a_{nm}(s)Q_n(s) = f_m(s), \quad (m = 1, 2, \dots, n)$$

in which $f_m(s)$ is a function of s depending on initial conditions or external inputs or both, then the equation

$$\Delta(s) \equiv \begin{vmatrix} a_{11} & a_{21} & \dots & a_{n1} \\ a_{12} & a_{22} & \dots & a_{n2} \\ \vdots & \vdots & \ddots & \vdots \\ a_{1n} & a_{2n} & \dots & a_{nn} \end{vmatrix} = 0$$

is the characteristic equation of the system. As a particular case, for a control system in which

$$\begin{aligned} \theta_o(s) - G(s)\theta_e(s) &= 0 \\ \theta_o(s) + \theta_e(s) &= \theta_i(s) \end{aligned}$$

in which $\theta_i, \theta_o, \theta_e$, are the **Laplace transforms** of the input, the output and the error respectively and $G(s)$ is the output to error **transfer function** the characteristic equation is

$$1 + G(s) = 0.$$

CHARACTERISTIC FUNCTION. If $F(x)$ is a **probability distribution function**, its characteristic function is given by the **Fourier transform**

$$\phi(t) = \int_{-\infty}^{\infty} e^{itx} dF(x).$$

The characteristic function is a **moment generating function**,

$$\phi(t) = \sum \mu'_r (it)^r / r!$$

where μ'_r is the r th **moment** of $F(x)$ about the origin. The characteristic function uniquely determines the distribution function by the formula

$$F(x) - F(0) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \phi(t) \frac{1 - e^{-ixt}}{it} dt.$$

If the frequency function (probability density function), $f(x)$, exists this is equivalent to

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{-ixt} \phi(t) dt.$$

Likewise the characteristic function of a function $g(x_1, x_2, \dots, x_n)$ of variates x_1, x_2, \dots, x_n distributed with function $F(x_1, x_2, \dots, x_n)$ is

$$\int \exp\{itg\} dF(x_1, x_2, \dots, x_n).$$

If the variates are independent the characteristic function of their sum is the product of their individual characteristic functions, a fact which gives the characteristic function a fundamental role in sampling theory.

The logarithm of the characteristic function is called the **cumulant** generating function.

CHARACTERISTIC FUNCTION OF A SET. The function whose value is unity for points of the **set** and zero elsewhere.

CHARACTERISTIC FUNCTION OF OPTICS. See **Hamilton's characteristic** and **eikonal**.

CHARACTERISTIC FUNCTIONS IN THERMODYNAMICS. See **thermodynamics, characteristic functions in**.

CHARACTERISTIC IMPEDANCE. See **impedance, characteristic**.

CHARACTERISTIC LENGTH. In propulsion, the ratio of the volume of a combustion chamber to its nozzle throat area. It is a measure of the length of travel available for the combustion of propellants. To obtain the best performance possible, the chemical reaction should be completed before the gaseous combustion products reach the entrance to the exhaust nozzle. The objective is to minimize the characteristic length without introducing any significant reduction in the measured value of **characteristic velocity** due to incomplete reaction of propellants. The relationship is:

$$L^* = \frac{V_c}{A_t}$$

where L^* is the characteristic length, V_c is the chamber volume, and A_t is the nozzle throat area.

CHARACTERISTIC OF A LOGARITHM. See **logarithm**.

CHARACTERISTIC OF A SURFACE. See **surface characteristic**.

CHARACTERISTIC POINTS (FOR A TWO-PARAMETER FAMILY OF SURFACES).

The points defined by

$$\begin{aligned} f(x,y,z,a,b) &= 0, \\ \partial f(x,y,z,a,b)/\partial a &= 0, \\ \partial f(x,y,z,a,b)/\partial b &= 0, \end{aligned}$$

for particular values of the parameters a and b , where $f(x,y,z,a,b) = 0$ is a two-parameter family of surfaces. The *envelope of the two-parameter family of surfaces* is the surface whose equation is obtained by eliminating a and b from these equations. It is the locus of the characteristic points.

CHARACTERISTIC POLYNOMIAL OF A MATRIX. See *matrix*.

CHARACTERISTICS, METHOD OF. (See also *characteristic*.) A method of calculating, for instance, supersonic flow fields for an inviscid gas. In its usual form, as considered here, the method applies only to irrotational flow, but it has also been extended to rotational flow. The method can be used for steady flow, either in two dimensions or with axial symmetry, or for unsteady one-dimensional flow.

In steady supersonic flow the **Mach lines** are characteristics, in the sense used in the theory of hyperbolic equations. In the method of characteristics the network formed by the two families of Mach lines is computed numerically, step-by-step, the **Mach angle** and flow direction being found for each intersection point as the computation proceeds. It is usually necessary to start the computation at a line on which the properties of the flow are known.

In steady two-dimensional flow the conditions that must be satisfied along the Mach lines are

$$\nu + \theta = \text{constant for } a + \text{ Mach line}$$

and

$$\nu - \theta = \text{constant for } a - \text{ Mach line,}$$

where θ is the inclination of the velocity vector to the axis. ν is the Prandtl-Meyer function given by

$$\nu = \mu - \frac{\pi}{2} + \lambda^{-1} \tan^{-1} (\lambda \cot \mu),$$

where μ is the **Mach angle** and

$$\lambda^2 = \frac{\gamma - 1}{\gamma + 1}.$$

The method of characteristics has been much used for computing the shapes of supersonic nozzles to give regions of uniform flow. (See also **Prandtl-Meyer expansion**.) (See H. W. Liepmann and A. Roshko, *Elements of Gas-dynamics*, Wiley, New York, 1957.) It is quite generally employed in the solution of problems governed by **hyperbolic partial differential equations**; for instance, in the theory of plasticity.

CHARACTERISTIC TEMPERATURE. See **Debye temperature**.

CHARACTERISTIC VALUE OF TENSOR. See **eigenvalue equation**.

CHARACTERISTIC VECTOR OF A MATRIX. See **matrix**; **principal vector**; and **eigenvalues and eigenvectors of matrices**.

CHARACTERISTIC VELOCITY. See **velocity, characteristic**.

CHARACTERISTIC WAVELENGTH OF A MEDIUM. The characteristic velocity (see **velocity, characteristic**) divided by the frequency:

$$1/f\sqrt{\mu\epsilon}.$$

CHARACTERISTIC X-RAY SPECTRUM. See **x-ray spectrum, characteristic**.

CHARGE, BOUND. An electric charge which appears at the surface of a **dielectric** by virtue of its **polarization**. The surface density of these charges is equal to the polarization within the dielectric. Such charges are bound in the sense that they result from the creation and reorientation of dipoles within the dielectric. The total amount of bound charge throughout the volume of any dielectric is zero. Bound charges are distinguished from free charges. (See **charge, free**.)

CHARGE CONJUGATION OPERATION. The operation which transforms any state containing n particles of type i , momenta \mathbf{p}_j , spin σ_j into a state of m anti-particles of type i , momenta \mathbf{p}_j , spin σ_j .

This operation is a unitary one and is usually denoted by **C**. The transformation laws of a charged spin 0, spin $\frac{1}{2}$ and spin 1 field under this operation is as follows:

$$\begin{aligned}
 \text{Spin } 0 \quad & C\phi(x)C^{-1} = \eta_0^C \phi^*(x) \\
 & C\phi(x)^*C^{-1} = \bar{\eta}_0^C \phi(x) \\
 \text{Spin } \frac{1}{2} \quad & C\psi(x)C^{-1} = \eta_{1/2}^C C\bar{\psi}^T(x) \\
 & C\bar{\psi}(x)C^{-1} = \bar{\eta}_{1/2}^C \psi^T(x)C^* \\
 \text{Spin } 1 \quad & C\phi_\mu(x)C^{-1} = \eta_1^C \phi_\mu^*(x) \\
 & C\phi_\mu^*(x)C^{-1} = \bar{\eta}_1^C \phi_\mu(x).
 \end{aligned}$$

For the spin $\frac{1}{2}$ case C is the matrix satisfying $C\gamma_\mu^T C = -\gamma_\mu$ ($\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2g_{\mu\nu}$).

If the Hamiltonian for these field theories is invariant under the operation $C:HC^{-1}=H$, then C is a constant of the motion. The total charge of the system, however, anti-commutes with C . One can assume that $C^2 = 1$. It is known that the weak interactions violate C -invariance.

CHARGE CONSERVATION. See **gauge invariance**.

CHARGE-CURRENT DENSITY FOUR VECTOR. The four functions of position and time $j_\mu(r,t)$ where $j_1, j_2, j_3,$ are the components of electric current density, and $j_4 = ic\rho$, where ρ is the charge density. For a Dirac electron field,

$$j_\mu = ic\psi\gamma_\mu\bar{\psi}$$

where $\bar{\psi}$ is the **adjoint** wave-function.

CHARGE DENSITY. Electrostatics deals both with finite, point charges, and distributed charges. (1) A distributed charge can be described in terms of its density (coulombs per cubic meter) at each point of the region of interest. The density is a three-dimensional analog of a simple derivative. (See the **Poisson equation**.) (2) The term is also used to refer to the surface charge density, the charge per unit area.

CHARGE DENSITY OF NUCLEI. Experiments in which atomic nuclei are bombarded with high energy electrons have shown that the electric charge of the nucleus, which arises from the protons, is distributed consistently with the formula

$$\rho(r) = \rho_0[1 + e^{(r-R)/a}]^{-1}$$

where $R = r_0A^{1/3}$ is a measure of the nuclear radius, $r_0 \approx 1.1 \times 10^{-13}$ cm, $A =$ mass number, and $a \approx 0.6 \times 10^{-13}$ cm is a measure of the thickness of the nuclear surface; ρ_0 is a constant.

CHARGE, ELECTRIC. A quantity of **electricity**, measured in **coulombs** or related units. The flow of charge per unit time is the **electric current**. The elementary unit of charge is the charge on the **electron**. (See **electron**.)

CHARGE, FREE. An electric charge on the surface of a conductor or those charges on the surface of a dielectric that are not bound (cf. **charge, bound**). When free charges occur at the interface of dielectrics, there is a discontinuity in the normal component of the **electric displacement vector** at the interface, the magnitude of the change being equal to the surface density of free charges.

CHARGE INDEPENDENCE. The property whereby the nuclear forces between a pair of protons, a pair of neutrons, or a neutron and a proton, are the same in corresponding states of relative motion. Nuclear forces appear to possess this property to a high degree of accuracy. The result is that closely corresponding states can be found in nuclei of the same mass number A but possessing different numbers of neutrons and protons, and this has led to a considerable simplification in the description of the states of light nuclei. In heavier nuclei ($A > 40$) the electrostatic forces between protons (which are clearly not charge-independent) destroy this simplification. The term is also used in an extended sense to describe interactions between mesons and nucleons (neutrons and protons) which also appear to be charge-independent. (See **isotopic spin**.)

CHARGE IN SPACE, FIELD OF MOVING. See **field of moving charge in space**.

CHARGE INVARIANCE. Hypothesis that nucleon-nucleon **interactions** are invariant under rotations in isotopic spin space. One consequence of this is the charge independence of nuclear forces.

CHARGE-MASS RATIO. The ratio of the electric charge of a particle and its mass, important in the physics of electrons, ions, and other electrified bodies of molecular order.

CHARGE NUMBER. The charge number of an ion is the ratio of its charge to that of the proton H^+ . Thus Ba^{++} has the charge number $+2$; PO_4^{---} has the charge number -3 . (See **electroneutrality**.)

CHARGE RENORMALIZATION. In quantum electrodynamics a procedure which allows the removal of certain **divergences** connected with the vacuum fluctuations of the charged matter field. Every charged particle induces a vacuum polarization, i.e., a reordering of the charge distribution of the vacuum. More specifically, a charged particle of charge Q as a result of its interaction with the electron-positron field, surrounds itself with a cloud of (virtual) electrons and positrons. Some of these, with net charge δQ , of the same charge as Q , escape to infinity (due to the infinite range of the Coulomb field), leaving a net charge $-\delta Q$ in the part of the cloud which is closely bound to the test body (i.e., within a distance \hbar/mc). In the present formulation of quantum electrodynamics, δQ is infinite (the aforementioned divergence). The recognition that it is not Q but $Q - \delta Q$ which is the experimentally observed charge of the particle (as long as one only probes distances greater than \hbar/mc), and that it is this quantity which must be set equal to the observed charge, allows one to bypass the divergence difficulty. (See **S matrix, evaluation of.**)

CHARGE-TRANSFER SPECTRUM. See **spectrum, charge-transfer.**

CHARLES LAW. (Also known as the Gay-Lussac law.) Gases heated (or cooled) at constant pressure expand (or contract) by the same fraction of their volume for the same rise of temperature:

$$v = v_0(1 + \alpha t) \quad \text{at } p = \text{const.}$$

The law is found to be an asymptotic one for real gases being valid for $p \rightarrow 0$; $V \rightarrow \infty$ (**perfect gases**). It is contained in the perfect gas law $pv = RT$. Here t is measured on the Celsius or Fahrenheit temperature scales, whereas T is measured on the Kelvin or Rankine scales, respectively. Charles' law is now only of historical significance.

CHATELIER(LE)-BRAUN PRINCIPLE. Let us perturb a system which is initially in stable equilibrium (see **thermodynamic stability conditions**) to a neighboring non-equilibrium state. Since the initial equilibrium is supposed to be stable, the system will return to an equilibrium state.

Theorems governing the behavior of perturbed systems are often known as *theorems*

of constraint or theorems of moderation. The best known thermodynamic theorem of moderation is that of Le Chatelier-Braun (also called the Le Chatelier-Braun Principle) which in the form stated by Le Chatelier is:

"Any system in chemical equilibrium undergoes, as a result of a variation in one of the factors governing the equilibrium, a *compensating* change in a direction such that, had this change occurred alone it would have produced a variation of the factor considered in the *opposite direction*."

However this principle suffers from a number of important exceptions. It is therefore preferable to study the "inoderation" starting from the usual thermodynamic formalism without invoking a special principle. (See also **van't Hoff theorem; Chatelier theorem.**)

CHATELIER(LE) THEOREM. If a reaction is accompanied by an increase in volume, an increase in pressure leads to a reduction of the extent of reaction; correspondingly, if the reaction is accompanied by a decrease in volume, an increase in pressure advances the equilibrium value of the extent of reaction.

Quantitatively this theorem may be expressed by

$$\left(\frac{d\xi}{dp}\right)_T = \frac{V_{T,p}}{a_{T,p}} \quad (1)$$

where ξ is the **extent of reaction**, and

$$V_{T,p} = \left(\frac{\partial^2 V}{\partial \xi^2}\right)_{T,p} \quad a_{T,p} = -\left(\frac{\partial^2 G}{\partial \xi^2}\right)_{T,p} \quad (2)$$

where G is the Gibbs free energy (see **thermodynamics, characteristic functions**). The **chemical stability** condition is

$$a_{T,p} < 0 \quad (3)$$

This theorem is also closely related to the **Chatelier(Le)-Braun principle**.

CHEBYSHEV APPROXIMATION. See **minimax approximation**.

CHEBYSHEV EQUATION. A special case of the Gauss hypergeometric equation

$$(1 - x^2)y'' - xy' + n^2y = 0$$

where n is an integer. The name is often spelled differently, particularly *Tchebycheff*. (See **Chebyshev polynomials**.)

CHEBYSHEV EXPANSION. An expansion in Chebyshev polynomials. Since $T_n = x^n +$ terms of lower degree, $x^n = T_n +$ Chebyshev polynomials of lower degree. Let $f(x)$ have the expansion

$$f(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \dots, \quad -1 \leq x \leq 1.$$

By expressing each x^n in terms of Chebyshev polynomials and collecting terms one has

$$f(x) = \beta_0 + \beta_1 T_1(x) + \beta_2 T_2(x) + \dots.$$

It generally happens that for a given degree of accuracy this Chebyshev expansion can be truncated much sooner than the power series expansion, hence for computing purposes is more effective. (See **minimax approximation**.)

(For more details see Zdenek Kopal, *Numerical Analysis*, John Wiley & Sons, Inc., 1955; Cornelius Lanczos, *Applied Analysis*, Prentice-Hall Company, 1956.)

CHEBYSHEV INEQUALITY. Let $g(x)$ be a non-negative function of a variate x . Then

$$P\{g(x) > k\} \leq \frac{E\{g(x)\}}{k}, \quad k > 0.$$

If $g(x) = (x - m)^2$ where m is the mean of x , and $k = t^2 \sigma^2$, where σ^2 is the variance, the inequality reduces to the **Bienaymé-Chebyshev inequality**.

CHEBYSHEV POLYNOMIALS. The class of special functions obtained by orthogonalizing over the interval $[-1, 1]$ the sequence of powers $1, x, x^2, \dots$ with the weighting function $w(x) = (1 - x^2)^{-1/2}$. They satisfy the differential equation

$$(1 - x^2)y'' - xy' + n^2y = 0, \quad n = 0, 1, 2, \dots.$$

They are given explicitly by:

$$T_0(x) = 1, \quad T_n(x) = 2^{1-n} \cos(n \cos^{-1} x),$$

so that

$$T_1(x) = x, \quad T_2(x) = x^2 - \frac{1}{2},$$

$$T_3(x) = x^3 - \frac{3}{4}x, \dots.$$

Their normalizing factor is $2^{n-1/2} \pi^{-1/2}$ and their generating function is $2^{-n}(1 - t^2)(1 - 2tx + t^2)$. They satisfy the recursion formula

$$T_{n+1}(x) - xT_n(x) + \frac{1}{4}T_{n-1}(x) = 0, \quad n > 1.$$

Their Rodrigues formula is:

$$T_n(x) = (1 - x^2)^{1/2} \frac{(-1)^n 2n!}{(2n)!} \frac{d^n}{dx^n} (1 - x^2)^{n-1/2}.$$

They have the minimum property, among all polynomials of degree n with unity for leading coefficient, of having the least deviation from the real axis; that is: $\max |T_n(x)| = \min$. This property makes the Chebyshev polynomials important, e.g., in arranging a physical process so that its oscillations, which after a certain time may become very great, are a minimum in some fixed interval.

By some authors the polynomials

$$P_n(x) = 2^{n-1} T_n(x) = \cos(n - \cos^{-1} x)$$

are called the Chebyshev polynomials. For these the recursion is

$$P_{n+1} = 2xP_n - P_{n-1}.$$

(See Zdenek Kopal, *Numerical Analysis*, John Wiley & Sons, Inc., 1955; Cornelius Lanczos, *Applied Analysis*, Prentice-Hall Company, 1956.)

CHEBYSHEV QUADRATURE FORMULA. A formula of the form

$$\int_a^b w(x)f(x)dx = \omega \sum_0^n f(x_i) + R_n,$$

where the points x_i are so chosen that the integration is exact when $f(x)$ is a polynomial of degree as high as possible. It turns out the x_i can be determined as roots of an algebraic equation of degree $n + 1$, but unless $n \leq 7$ or $n = 9$ they do not all lie on the interval of integration. The formula is advantageous in particular in cases where the $f(x_i)$ are experimentally measured, since in the formula they are equally weighted.

CHEBYSHEV SYSTEM (OF ORDER n). A set of functions

$$\phi_0(x), \phi_1(x), \dots, \phi_n(x),$$

linearly independent and continuous on an interval (a, b) with the property that if the linear combination

$$F_n(x) = \alpha_0 \phi_0(x) + \dots + \alpha_n \phi_n(x)$$

vanishes more than n times on the interval, then it vanishes identically, and hence $\alpha_0 = \alpha_1 = \dots = \alpha_n = 0$. Such a system is

$$\phi_0 = 1, \quad \phi_1 = x, \quad \dots, \quad \phi_n = x^n;$$

for the interval from 0 to 2π , the system

$$\begin{aligned} \phi_0 &= 1, & \phi_1 &= \sin x, & \phi_2 &= \cos x, & \dots, \\ \phi_{2n-1} &= \sin nx, & \phi_{2n} &= \cos nx \end{aligned}$$

is a Chebyshev system of order $2n$. Such systems are of importance in minimax approximation.

CHECK. A process of partial or complete testing of (1) the correctness of machine operations, (2) the existence of certain prescribed conditions within the computer, or (3) the correctness of the results produced by a routine. A check of any of these conditions may be made automatically by the equipment or may be programmed. (See also **marginal checking**; **verification**.)

CHECK, AUTOMATIC. A check performed by equipment built into the computer specifically for that purpose, and automatically accomplished each time the pertinent operation is performed. Sometimes referred to as a built-in check. Machine check can refer to an automatic check, or to programmed check of machine functions.

CHECK DIGITS. See **check, forbidden-combination**.

CHECK, FORBIDDEN-COMBINATION. A check (usually an automatic check) which tests for the occurrence of a nonpermissible code expression. A *self-checking code* (or *error-detecting code*) uses code expressions such that one or more errors in a code expression produces a forbidden combination. A *parity check* makes use of a self-checking code employing binary digits in which the total number of 1's (or 0's) in each permissible code expression is always even or always odd. A check may be made for either even parity or odd parity. A *redundancy check* employs a self-checking code which makes use of redundant digits called *check digits*.

CHECKING, MARGINAL. A preventive maintenance procedure in which certain operating conditions, e.g., supply voltage or frequency, are varied about their normal values in order to detect and locate incipient defective units. (See also **check**.)

CHECK, MACHINE. See **check, automatic**.

CHECK, MATHEMATICAL. See **check, programmed**.

CHECK, PARITY. See **check, forbidden-combination**.

CHECK POINT. See **point, way**.

CHECK PROBLEM. See **check, programmed**.

CHECK, PROGRAMMED. A check consisting of tests inserted into the program of the problem and accomplished by appropriate use of the machine's instructions. A *mathematical check* (or *control*) is a programmed check of a sequence of operations which makes use of the mathematical properties of that sequence.

CHECK, REDUNDANCY. See **check, forbidden-combination**.

CHECK ROUTINE OR CHECK PROBLEM. A routine or problem which is designed primarily to indicate whether a fault exists in a digital computer, without giving detailed information on the location of the fault. (See also **routine diagnostic**; **routine test**; **check, programmed**.)

CHECK, SELECTION. A check (usually an automatic check) in a digital computer to verify that the correct register, or other device, is selected in the performance of an instruction.

CHECK, TRANSFER. A check (usually an automatic check) on the accuracy of the transfer of a word.

CHEMICAL AFFINITY. The entropy production due to a chemical reaction has the form (see **thermodynamics, second law of**).

$$\frac{d_i S}{dt} = \frac{1}{T} A v \geq 0, \quad (1)$$

where A is the chemical affinity and v , the reaction rate (see **chemical reaction rate**); A is related to the characteristic functions U , H , A , G and to the chemical potentials μ by the relations (see **thermodynamics, characteristic functions of**; **chemical potentials**).

$$\left. \begin{aligned} A &= -\left(\frac{\partial U}{\partial \xi}\right)_{S,V} = -\left(\frac{\partial H}{\partial \xi}\right)_{S,p} \\ &= -\left(\frac{\partial A}{\partial \xi}\right)_{T,V} = -\left(\frac{\partial G}{\partial \xi}\right)_{T,p} \\ &= -\sum_i \nu_i \mu_i \end{aligned} \right\} \quad (2)$$

when ξ is the extent of reaction and ν_i the stoichiometric coefficient.

The basic properties of the affinity A are that it is always of the same sign as the re-

action rate, and that if the affinity is zero the reaction rate is also zero, i.e., the system is in equilibrium.

This definition of affinity is essentially due to De Donder and is called De Donder's fundamental inequality. In the notation used by G. N. Lewis and his school, it is supposed that ξ increases by unity, therefore the relations (2) are written in the form

$$\begin{aligned} A &= -(\Delta U)_{S,V} = -(\Delta H)_{S,p} = -(\Delta A)_{T,V} \\ &= -(\Delta G)_{T,p}. \end{aligned} \quad (3)$$

Note that in this entry, A is the affinity and A , the **Helmholtz function** (work function.)

CHEMICAL CONSTANT. A constant occurring in the **vapor pressure equation** and determined by the introduction of an absolute entropy. (See **thermodynamics, third law of.**)

CHEMICAL EQUILIBRIUM. A system is said to be in chemical equilibrium where the **chemical affinities** of all the possible chemical reactions are equal to 0. As long as one may neglect possible secondary interactions between the reactants one can use the formula for the equilibrium constant K (see **chemical equilibrium, calculation of**) which is related to change in the Gibbs free energy ΔG by the formula

$$\beta \Delta G = -N \ln K,$$

where $\beta = 1/kT$ (k is Boltzmann's constant, T is the absolute temperature). This relation as well as the relation between K and the concentrations of the reactants can be derived by the method of **grand ensembles**. The same method can also be used to evaluate corrections for the case of non-ideal gases.

CHEMICAL EQUILIBRIUM, CALCULATION OF. This can be performed by using the method of **grand ensembles**, from which one obtains a formula for the equilibrium constant K , which is the ratio

$$K = \frac{\Pi C_i}{\Pi C_j},$$

where the C_i are the concentrations of the reactants on the one side of the reaction equation and the C_j those on the other side. Another formula for K is

$$K = \frac{\Pi Z_i}{\Pi Z_j},$$

where the Z_i and Z_j are the **partition functions** of the reactants. (See also **Saha equilibrium formula** and **chemical equilibrium.**)

CHEMICAL INTERACTION. The understanding of the types of bonds has improved to such a point in the last fifty years that it has become extremely difficult to distinguish between what used to be called chemical and physical interactions. An accurate definition of either term is practically impossible now.

If one limits the definition of physical interactions to the van der Waals forces only, one must define the ionic and metallic lattices as "chemical molecules." Even in the crystals of dipolar molecules, electrostatic interactions are important. If, on the other hand, one includes the electrostatic forces in the definition of "physical interactions," a major part of the bond energy of many molecules (NaCl for example) must be attributed to so-called physical interactions.

Steric factors (intramolecular van der Waals forces) affect the molecular structure and bond energy of many molecules; on the other hand, it is the hydrogen bonds, which are quasi-covalent in character (see **coordinate bonds**) which are largely responsible for the cohesion of water, HF, organic acids and alcohols in the liquid and solid state.

The classification of some interactions as chemical or physical is thus mainly a matter of convenience and tradition; it is more accurate to speak of the interaction as being mainly covalent, electrostatic, metallic or van der Waals in type. (See **bond types.**)

CHEMICAL POTENTIAL, INFLUENCE OF CURVATURE ON. Consider a small spherical drop of curvature r . Its **chemical potential** μ_r is related to the chemical potential μ_0 of a bulk liquid phase at the same temperature (for the bulk phase $1/r = 0$) by the relation

$$\mu_r - \mu_0 = \frac{2\gamma}{r} V \quad (1)$$

where V is the molar volume of the liquid (supposed to be independent of pressure) and γ , the surface tension. We see that the chemical potential increases when the drop becomes smaller.

CHEMICAL POTENTIALS. Chemical potentials are defined in terms of the entropy by

the relationship (see **thermodynamics, characteristic functions of**)

$$\mu_i = -T \left(\frac{\partial S}{\partial n_i} \right)_{U,V} \quad (1)$$

Apart from the factor T (the **absolute temperature**) the chemical potential is equal to the change of the entropy dS due to the introduction of the mole number dn_i into the system, at constant total energy U and volume V .

Other sets of independent variables than U and V are often much more convenient. One has also (see **thermodynamics, characteristic functions of**),

$$\begin{aligned} \mu_i &= \left(\frac{\partial U}{\partial n_i} \right)_{S,V} = \left(\frac{\partial H}{\partial n_i} \right)_{S,p} \\ &= \left(\frac{\partial A}{\partial n_i} \right)_{T,V} = \left(\frac{\partial G}{\partial n_i} \right)_{T,p} \end{aligned} \quad (2)$$

The first and last members of Equation (2) show that μ_i is the **partial molar quantity** associated with the Gibbs free energy, G . **Euler's theorem** gives then

$$G = \sum_i n_i \mu_i \quad (3)$$

The relation to **chemical affinity** is also very direct

$$A = - \sum_i \nu_i \mu_i \quad (4)$$

It follows that the condition for chemical equilibrium is

$$\sum_i \nu_i \mu_i = 0 \quad (5)$$

where the ν_i are **stoichiometric coefficients**. This formula expresses the **law of mass action**. Similarly the condition for two phases α and β to be in equilibrium with respect to species i is

$$\mu_i^\alpha = \mu_i^\beta \quad (6)$$

The chemical potential has then the same value in the two phases. (See also **membrane equilibrium; Gibbs-Duhem equation; ideal systems; absolute activity; thermal potential**.)

CHEMICAL REACTION RATE. The quantity ν defined by the equation:

$$\nu = d\xi/dt.$$

where ξ is the **extent of reaction**, and t , the time. (See **conservation of mass; extent of**

reaction; absolute reaction rate theory; equilibrium theory in chemical kinetics; collision theory of chemical kinetics.)

CHEMICAL REACTIONS, ORDER OF. See **order of chemical reactions**.

CHEMICAL STABILITY CONDITION. (See **thermodynamic stability conditions**.) The chemical stability condition expresses the stability of the system with respect to perturbations of the **chemical equilibrium**. At constant pressure and temperature the stability is expressed by

$$\left(\frac{\partial^2 G}{\partial \xi^2} \right)_{p,T} > 0 \quad (1)$$

where G is the Gibbs free energy (see **thermodynamics, characteristic functions of**) and ξ , the extent of reaction. This condition shows that $G(\xi)$ is minimum when the chemical equilibrium is stable. (See also **Chatelier (Le) theorem**.)

CHIEF RAY. See **principal ray**.

CHILD-LANGMUIR-SCHOTTKY EQUATION. See **perveance**.

CHIÓ'S METHOD (FOR EVALUATING A DETERMINANT). A method identical in principle with Gaussian elimination for **matrix inversion** and the solution of linear equations. If the elements are α_{ij} and $\alpha_{11} \neq 0$, then from the i th row subtract the first multiplied by α_{i1}/α_{11} . The result is a determinant of equal value with a zero in position $(i, 1)$. After this has been done for each row other than the first, the determinant reduces to one of lower order.

CHI-SQUARE. If a sample of n values is drawn from a **normal distribution** with variance σ^2 , and if the **variance** is estimated as

$$s^2 = \Sigma(x - \bar{x})^2 / (n - 1),$$

then the ratio $(n - 1)s^2/\sigma^2$ is known as χ^2 , and has the **probability function**

$$P(\chi^2)d\chi^2 = \frac{1}{\Gamma(\frac{1}{2}\nu)} e^{-\frac{1}{2}\chi^2} \left(\frac{1}{2}\chi^2\right)^{\frac{\nu}{2}-2} d\left(\frac{1}{2}\chi^2\right),$$

$$0 \leq \chi^2 \leq \infty,$$

when $\nu = (n - 1)$ is the **degrees of freedom**. This is a special case of the Pearson Type III

function; the probability integral is an **incomplete gamma function**.

If an hypothesis completely specifies the frequencies f_t to be expected in n classes, the goodness of fit of a sample providing frequencies f_o can be tested by calculating

$$\chi^2 = \sum \frac{(f_o - f_t)^2}{f_t}$$

and entering a table of χ^2 with $(n - 1)$ degrees of freedom. This quantity closely approximates the above distribution provided that none of the f_t 's are too small—say, less than 5; to avoid this it is often necessary to combine two or more classes with small expectations. If the hypothesis specifies a probability distribution in which p parameters have to be estimated from the sample in order to compute the expected frequencies, the degrees of freedom are reduced to $(n - p - 1)$.

With **discrete variates**, χ^2 provides a sensitive test of departure from the **Poisson distribution**. If we put

$$\chi^2 = (n - 1)s^2/\bar{x}$$

where \bar{x} is the sample mean and n the number of observations the standard distribution is closely followed for a true mean ≥ 2 .

A similar test can be applied to the **binomial distribution**. In this form, χ^2 is sometimes referred to as the *index of dispersion*.

The χ^2 distribution tends to normality for large values of ν . The usual tables do not extend above $\nu = 30$; for larger values, it is ordinarily sufficient to take $\sqrt{(2\chi^2)}$ as *normally distributed* with mean $\sqrt{(2\nu - 1)}$ and unit standard deviation.

CHOKING. The condition wherein the mass of a compressible fluid flowing through a duct has reached its maximum value. It is always associated with the attainment of sonic velocity (local Mach number = 1) in some cross section of the duct.

CHOLESKI'S METHOD (FOR INVERTING A POSITIVE DEFINITE HERMITIAN MATRIX A). This method is to express the matrix first in the form

$$A = R^*R$$

where R is upper triangular and R^* its conjugate transpose, and thence to compute R^{-1} and multiply by its conjugate transpose. The

method is mathematically equivalent to the method of elimination, but requires the extraction of n square roots.

CHORD. (1) A segment of a straight line between two specified points of intersection of the line with a given curve or surface. (2) In topology, an **element** belonging to the complement of a **tree**. (See also **pitch**.)

CHORD METHOD. A method for the calculation of collision probabilities, introduced by P. A. M. Dirac. The principal result is:

$$1 - P_0 = \frac{1}{\Sigma R_{av}} \int (1 - e^{-\Sigma R}) \phi(R) dR.$$

Here, P_0 is the first flight collision probability in a body characterized by macroscopic cross-section Σ and chord distribution function $\phi(R)$.

CHORD OF AIRFOIL. The section of any airfoil may be formed by superposing a symmetrical *fairing* on a curved *mean line*. In the figure, the broken curved line is the mean line and the chord, or chord line, is the straight line AB joining the ends of the mean line.



The chord of an airfoil section has sometimes been defined as the straight line joining the centers of curvature of the leading and trailing edges. For the usual airfoils of small camber, the difference between the two definitions is small.

CHRISTOFFEL (3-INDEX) SYMBOLS. Let g_{ij} and g^{ij} be the components in the coordinate system x of the covariant and contravariant **metric tensors** respectively. The quantities

$$\frac{1}{2} \left(\frac{\partial g_{ik}}{\partial x^j} + \frac{\partial g_{kj}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^k} \right)$$

are called the *Christoffel (3-index) symbols of the first kind*, or *three-index symbols of the first kind*. They are usually denoted $\Gamma_{k;ij}$, or $[ij,k]$. The quantities

$$\frac{1}{2} g^{km} \left(\frac{\partial g_{im}}{\partial x^j} + \frac{\partial g_{mj}}{\partial x^i} - \frac{\partial g_{ij}}{\partial x^m} \right)$$

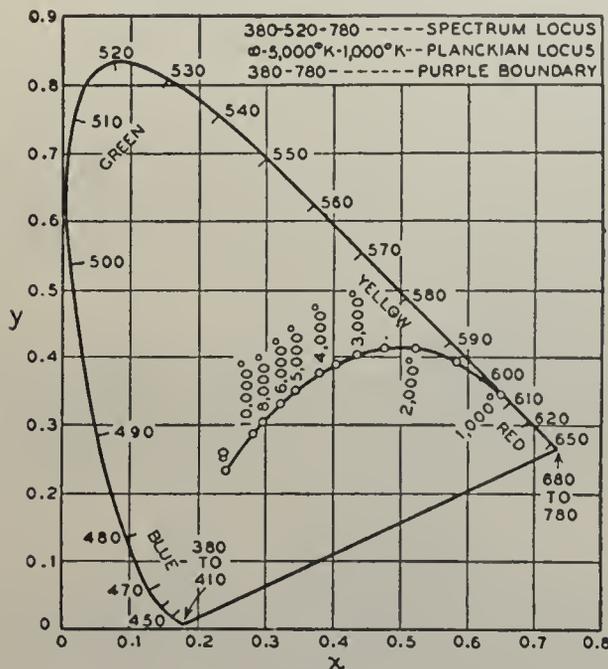
are called the *Christoffel (3-index) symbols of the second kind*, or *three-index symbols of the second kind*. They are usually denoted Γ_{ij}^k or $\{ij\}^k$.

CHROMATIC ABERRATION. The failure of all the light from one point of an object to form an image at a point because of the dispersion of the material of the lenses. The focal length of a lens is, in general, greater for red light (longer wavelength) than for blue light (shorter wavelength). Systems consisting entirely of reflection optics do not show chromatic aberration. Hence, reflection optics are used extensively in infrared instruments. With compound lenses a proper choice of kinds of glass and of lens curvatures can partially compensate for chromatic aberration over a not-too-large wavelength region.

CHROMATICITY. The color quality of a light definable by its **chromaticity coordinates**, or by its **dominant** (or **complementary**) **wavelength** and its **purity** taken together.

CHROMATICITY COORDINATES. The ratio of each of the three **tristimulus values** of a light to their sum.

CHROMATICITY DIAGRAM. A chromaticity diagram or color triangle is a plane diagram showing the results of mixing color



CIE (x,y) chromaticity diagram plotted in rectangular coordinates.

stimuli, every color being represented unambiguously by a single point on the diagram. The most common chromaticity diagram at present is the CIE (x,y) diagram, plotted in rectangular coordinates.

CHROMINANCE. The colorimetric difference between any color and a reference color of equal **luminance**, the reference color having a specified **chromaticity**. In three-dimensional color space, chrominance is a vector which lies in a plane of constant luminance. In that plane it may be resolved into components, called *chrominance components*.

CHRONOLOGICAL OPERATOR. The Dyson chronological operator P is defined as follows: Operating on a product of time-labeled operators, it rearranges them in the same order as the time sequence of their label, the latest one in time occurring first in the product

$$P(A_1(t_1) \cdots A_n(t_n)) = A_i(t_i)A_j(t_j) \cdots A_l(t_l) \quad t_i > t_j > \cdots > t_l. \quad (1)$$

The Wick chronological product T is defined in the same way as Dyson's chronological operator P , except that the T operator includes in its definition the sign of the permutation of fermion operator factors in going from the left to the right-hand side of (1), i.e.,

$$T(A_1(t_1) \cdots A_n(t_n)) = \delta_p A_i(t_i) \cdots A_l(t_l) \quad t_i > \cdots > t_l$$

δ_p is ± 1 according to whether the permutation is even or odd.

CIE-DISTRIBUTION COEFFICIENTS. The **tristimulus values** of the spectral components of an **equi-energy spectrum** in the CIE (XYZ) system. These coefficients $\bar{x}_\lambda, \bar{y}_\lambda, \bar{z}_\lambda$ are so chosen that the values of \bar{y}_λ are identical with the relative luminous efficiencies V_λ .

CIE (1931) STANDARD COLORIMETRIC OBSERVER. The receptor of radiation whose colorimetric characteristics correspond to the distribution coefficients $\bar{x}_\lambda, \bar{y}_\lambda, \bar{z}_\lambda$ adopted by the International Commission on Illumination in 1931.

CIE (1931) STANDARD COLORIMETRIC SYSTEM. A linear and single-valued colorimetric system for evaluating any spectral distribution of energy with the aid of three functions of wavelength, the distribution coefficients: $\bar{x}_\lambda, \bar{y}_\lambda, \bar{z}_\lambda$.

CIRCLE, GEODESIC. See **geodesic circle on a surface**.

CIRCLE OF CONFUSION. See **circle of least confusion**.

CIRCLE OF CONVERGENCE. See **convergence**.

CIRCLE OF CURVATURE. See **curvature**.

CIRCLE OF LEAST CONFUSION. Due to imperfect imagery and to the fact that the images of points at different object distances are commonly observed on a single image-plane (e.g., the film in a camera), the image of a **point object** is, in general, a small region approximately circular, the *circle of confusion*, or blur circle. The smallest blur circle as the image plane is varied is the circle of least confusion. In thin lens theory the center of the circle of least confusion is the midpoint of the **tangential** and **sagittal foci** on a ray. The **locus of centers** is the surface of least confusion. For a thin lens this surface is a sphere centered at the center of the lens.

CIRCLE, OSCULATING. See **curvature**.

CIRCLE POLYNOMIALS. The polynomials

$$R_n^m(x) = \sum_{k=0}^{\frac{1}{2}(n-m)} \frac{(-1)^k (n-k)! x^{n-2k}}{k! \{\frac{1}{2}(n+m) - k\}! \{\frac{1}{2}(n-m) - k\}!}$$

were used by F. Zernike in his classic paper (1934) on the knife-edge test and the phase-contrast method in optics. The R_n^m are orthogonal and are normalized upon multiplying by the factor $(2n+2)^{-1/2}$. Thus it follows that

$$R_n^m(r) \cos m\theta, \quad R_n^m(r) \sin m\theta,$$

for $n \geq m \geq 0$, $n - m$ even, is orthogonal over the unit circle. This set is also complete. The aberration function of an axially symmetrical optical system can be expanded in terms of the functions $R_n^m(r) \cos m\theta$. Using this expansion, small aberrations of the same generic type may be balanced against each other to obtain maximum **Strehl definition**. (See **Nijboer-Zernike aberration functions**.)

CIRCUIT. A **connected graph** or **subgraph** in which each **vertex** is of degree two. It can be shown that **loop** and **circuit** are equivalent concepts. Unfortunately the words **circuit** and **network** have been used interchangeably in the older electrical engineering literature and has been a constant source of confusion. The

above definition substitutes precision for vagueness, and at the same time retains the full intuitive flavor of the word. (See also entries under **network**.)

CIRCUIT, ALTERNATING CURRENT. See **alternating current circuit**.

CIRCUIT ELEMENT. See **element, circuit**.

CIRCUIT MATRIX. See **matrix, circuit**.

CIRCUIT, ORIENTED. The e elements of a **circuit** of a *finite graph* (see **graph, finite**) can be arranged in cyclic order either as $\epsilon_1, \epsilon_2, \dots, \epsilon_e, \epsilon_1$ or $\epsilon_e, \epsilon_{e-1}, \epsilon_{e-2}, \dots, \epsilon_1, \epsilon_e$ and these are, to within cyclic permutations, the only cyclic orders possible. If one of these two possible orientations is assigned to the circuit, the circuit is oriented accordingly.

CIRCUITS, FUNDAMENTAL. Let G be a connected graph containing v vertices and e edges and T a **tree** of G . The end points of each **chord** (with respect to T) are connected by a unique *tree path*. A chord together with its corresponding tree path forms a fundamental circuit. The number of fundamental circuits is equal to the number of chords or $e - v + 1$. If G is finite but not *connected* it consists of P maximal connected subgraphs, each of which possesses a **tree** and its associated fundamental circuits.

CIRCULAR APERTURE. For the **Fraunhofer diffraction** pattern of a circular aperture, see the discussion of the **Airy disc**.

CIRCULAR CURVATURE. See **curvature**.

CIRCULAR DICHROISM. The unequal absorption of the two circular components of plane-polarized light in an optically active medium.

CIRCULAR FUNCTION. A **trigonometric function**.

CIRCULAR MOTION, UNIFORM. See **uniform circular motion**.

CIRCULAR POINT (ON A SURFACE). A point at which the principal curvatures (see **curvature**) of the surface are equal.

CIRCULAR POLARIZATION. Polarization with a cycle that is a circle. (See **polarization cycle**.)

CIRCULATION. The circulation around a closed path of any vector field is the line integral of that vector around the path. The vector ordinarily considered is the fluid velocity, and the circulation is

$$\oint \mathbf{v} \cdot d\mathbf{s}$$

where \mathbf{v} is the fluid velocity, and $d\mathbf{s}$ is a vector element of the path. It is equal to the flux of vorticity through the circuit $\int \boldsymbol{\omega} \cdot d\mathbf{S}$ where $d\mathbf{S}$ is the element of any surface bounded by the circuit.

CIRCULATION THEOREM (BJERKNES).

A theorem stating that the rotation of a mass of horizontally moving air in the atmosphere is inversely proportional to the area of its projection on the equatorial plane. Application of this theorem is extremely limited because it assumes strictly horizontal motion, and when subsidence or upward motion occur much greater changes in rotation are produced than are due to changes in latitude as envisaged in the theorem. The theorem is, however, of some historical interest.

CIRCULATION THEOREM (KELVIN). In an inviscid fluid in which the density is uniquely determined by the pressure, and any body forces are conservative (and proportional to the mass to which they are applied), the **circulation** round a circuit always composed of the same fluid particles remains constant during the motion, i.e.,

$$\frac{D}{Dt} \oint \mathbf{v} \cdot d\mathbf{S} = 0.$$

In an inviscid fluid the resultant of conservative body forces (e.g., gravity) and inertia forces is seen from the equation of motion to be in the direction of the pressure gradient, which, in the above case, is parallel to the density gradient: these forces cannot therefore change the vorticity. The only means of changing the vorticity is by distorting the fluid; but if the circuit moves with the fluid the flux of vorticity remains constant because the vortex lines move with the fluid.

CIRCULATORY INTEGRAL. See **integral, circulatory.**

CIRCUMHOROPTER. See **Vieth-Müller circle.**

CLAIRAUT EQUATION. A first-order differential equation

$$y = xp + f(p)$$

where $p = dy/dx$. Its general solution is $y = cx + f(c)$. It also has a (singular) solution obtained by elimination of p from the equations

$$y = px + F(p); \quad x + F'(p) = 0.$$

CLAPEYRON EQUATION. See **Clausius-Clapeyron equation.**

CLAPEYRON THEOREM. The equation of three moments was given by Clapeyron and Bertot:

$$M_{n-1} \frac{l_n}{E_n I_n} + 2M_n \left[\frac{l_n}{E_n I_n} + \frac{l_{n+1}}{E_{n+1} I_{n+1}} \right] + M_{n+1} \frac{l_{n+1}}{E_{n+1} I_{n+1}} = - \frac{6A_n a_n}{E_n I_n l_n} - \frac{6A_{n+1} b_{n+1}}{E_{n+1} I_{n+1} l_{n+1}}$$

where $n - 1, n, n + 1$ are three adjacent supports of a continuous beam of constant flexural rigidity EI span by span. $A_n a_n$ is the moment of the lateral load moment diagram between $n - 1$ and n about $n - 1$ and $A_{n+1} b_{n+1}$ of the lateral load moment diagram between n and $n + 1$ about $n + 1$.

CLASS. A complete set of mutually conjugate elements of a group.

CLASSICAL. The adjective "classical" is in general applied to any physical theory or treatment that is based on the assumption of Newtonian mechanics. Such treatments include all electrical and magnetic discussions based on the **Coulomb law**, the **Ampere law**, and the **Maxwell equations**. Classical theory is usually to be distinguished from **relativity theory** and **quantum theory**, although relativistic treatments not involving the quantum hypothesis are occasionally referred to as classical, as are treatments involving quantum theory but not **quantum mechanics**.

CLASSICAL APPROXIMATION. See **Wentzel-Kramer-Brillouin approximation.**

CLASSICAL ELASTICITY THEORY. The theory of the relation between the forces and deformation in bodies of elastic or perfectly plastic material, in which it is assumed that the

displacement gradients are infinitesimally small. The strain-energy function is then a quadratic function of the displacement gradients, the stress components are linear functions of the displacement gradients and, throughout the theory, terms of the second and higher degrees in the displacement gradients are neglected in comparison with those of the first degree. Also called *linear elasticity theory*.

CLASSICAL ELECTRON ENERGY. See **electron energy (classical)**.

CLASSICAL ELECTRON ORBIT. See **electron orbit (classical)**.

CLASSICAL ELECTRON RADIUS. See **electron radius (classical)**.

CLASSICAL ELECTRON THEORY. The theory, primarily due to H. A. Lorentz (1906), to account for the varied phenomena (e.g., optics, electricity, chemistry) involving electrons. The fundamental assumptions of the theory were the following: The electron is an elementary particle with a charge e and a mass m ; the motion of the electrons is determined by classical mechanics with the force acting on the electron given by the expression

$$\mathbf{F} = e\mathbf{E} + \frac{e}{c}\mathbf{v} \times \mathbf{H}$$

where e and \mathbf{v} are the charge and velocity of the electron and \mathbf{E} and \mathbf{H} the electric and magnetic field strength. The dynamics of the electromagnetic field in turn is governed by Maxwell's equations

$$\frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{H} = 4\pi \mathbf{j} \quad \nabla \cdot \mathbf{E} = 4\pi \rho$$

$$\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} + \nabla \times \mathbf{E} = 0 \quad \nabla \cdot \mathbf{H} = 0.$$

The source of the field strengths are the charge density ρ and current density \mathbf{j} which are produced by the electrons.

To account for the constitution of matter Lorentz assumed that there are several electrons in each atom which are elastically bound to an equilibrium position and thus are able to perform harmonic vibrations with a given frequency. In electric conductors additional electrons were assumed to move freely about. With these assumptions Lorentz was able to

account qualitatively and often semi-quantitatively for such diverse effects as the absorption, scattering and refraction of light by matter, the Zeeman effect, the optical properties of metals and many more.

In the explanation of the aforementioned phenomena it is possible to consider the electron as a point particle. Lorentz investigated how far such a picture is tenable. If the electric and magnetic energy of the electron's own field is given by E_{em} , then from Einstein mass-energy relation, $E = mc^2$, this electromagnetic self-energy will contribute to the mass, m , of the electron an amount E_{em}/c^2 and one can write

$$m = m_o + E_{em}/c^2$$

where m_o is the "mechanical" mass of the electron, by which is meant all the contributions which are not of electromagnetic origin. For an electron of radius a the electrostatic self-energy is proportional to e^2/a , and thus in the limit as $a \rightarrow 0$ (point-electron) the self-energy is infinite. Within the framework of the Lorentz theory one is forced to abandon the motion of a point electron. In fact, no classical theory is able to account for or answers the questions concerned with the structure of the electron. Attempts at describing the electron as a charged sphere of radius $r_o = e^2/mc^2$, whose mass is entirely of electromagnetic origin runs into the difficulty that it cannot be stable (Earnshaw's theorem). Also this procedure does not allow a consistent explanation of the mass of a moving electron, since the electromagnetic field that the moving particle generates carries energy and momentum which are related to each other differently from the energy and momentum of a particle.

CLASSICAL LIMIT. See **principle of correspondence**.

CLASSICAL STATISTICS. See **Boltzmann statistics**.

CLAUDE REFRIGERATOR. See **reversed Rankine cycle**.

CLAUSIUS. The unit of **entropy** in calculations in which heat is measured in calories and temperature is measured on the Kelvin thermodynamic temperature scale. Hence 1 clausius = 1 cal/°K.

CLAUSIUS-CLAPEYRON EQUATION. An equation relating the latent heat l to the dis-

continuous change in volume accompanying a phase change and to the slope dp/dT of the pressure-temperature relation for the respective phase equilibrium:

$$l = (v'' - v')T \frac{dp}{dT}.$$

Here v'' is the specific volume of one phase, v' is the specific volume of the other phase, and l is reckoned algebraically. For example, for evaporation $l > 0$; $v'' > v'$.

When one phase (say v'') is a vapor, the equation can be used to calculate the vapor pressure curve approximately. For evaporation or sublimation $v'' \gg v'$, and the equation becomes

$$\frac{dp}{p} = \frac{l}{R} \frac{dT}{T^2}$$

where R is the gas constant for the vapor phase assumed to obey the perfect gas law $PV = RT$. This can be integrated with a suitable assumption concerning l . For $l = \text{const.}$, we have

$$\log_e p = -\frac{l}{RT} + \text{const.}, \quad (\text{a})$$

and for $l = a - bT$, we have

$$\log_e p = -\frac{a}{RT} - \frac{b}{R} \log_e T + \text{const.} \quad (\text{b})$$

Equation (a) shows that a plot of a sublimation or a vapor pressure curve in the coordinates $\log_e p$, T^{-1} will yield approximately a straight line whose slope is proportional to the latent heat. Equations (a) or (b) can be used for the interpolation of experimental results in ranges sufficiently far removed from the critical point where the assumptions break down.

A similar relation is valid for chemical reactions in the gaseous phase.

CLAUSIUS CONDITION. If $\Delta\sigma$ is a line element inclined at an angle ϕ to the normal plane of the principal ray of a pencil of angular half-aperture θ in object space and if n , n' are the indices in object, image space, respectively, then in order that the image of $\Delta\sigma$ be a line element in image space $\Delta\sigma'$ with corresponding angles ϕ' , θ' it is sufficient that

$$(n \cos \phi \sin \theta) \Delta\sigma = (n' \cos \phi' \sin \theta') \Delta\sigma'.$$

This is Clausius' condition for the sharp imagery of curves by an axially symmetrical optical system.

CLAUSIUS EQUATION OF STATE. A form of the equation of state, relating the pressure, volume, and temperature of a gas, and the gas constant. The Clausius equation applies a correction to the van der Waals equation to correct the pressure-correction term a for its variation with temperature. The Clausius equation takes the form

$$\left[p + \frac{a}{T(V+c)^2} \right] (V-b) = RT$$

in which p is the pressure of the gas, T is the absolute temperature, V is the volume, R is the gas constant, b is a constant, a is a temperature-dependent constant, and c is a function of a and b .

CLAUSIUS EQUATION OF STATE FOR STEAM. As applied to steam, the Clausius equation of state takes the form

$$\left[p + \frac{\phi(T)}{(V+c)^2} \right] (V-b) = RT'$$

or

$$p(V-b) = RT - \phi(T) \frac{V-b}{(V+c)^2}.$$

In the lower ranges of pressures and temperatures this can be simplified to

$$V-b = \frac{RT}{p} - \psi(T).$$

(See also Berthelot equation.)

CLAUSIUS EQUATIONS. The Clausius equations give the general relations which exist between the thermal coefficients. For the variables T , p , and ξ (extent of reaction) they are

$$\left(\frac{\partial C_{p,\xi}}{\partial p} \right)_{T,\xi} = \left[\frac{\partial(h_{T,\xi} + V)}{\partial T} \right]_{p,\xi} \quad (1)$$

$$\left[\frac{\partial h_{T,p}}{\partial T} \right]_{p,\xi} = \left(\frac{\partial C_{p,\xi}}{\partial \xi} \right)_{T,p} \quad (2)$$

$$\left(\frac{\partial h_{T,p}}{\partial p} \right)_{T,\xi} = \left[\frac{\partial(h_{T,\xi} + V)}{\partial \xi} \right]_{T,p}. \quad (3)$$

The second equation is especially important; it relates the temperature coefficient of the

heat of reaction to the heat capacities of the components which take part in the reaction. It may also be written

$$\frac{\partial}{\partial T} h_{T,p} = \sum_i \nu_i c_{p,i} \quad (4)$$

where $c_{p,i}$ is the partial molar specific heat at constant pressure of component i (see **partial molar quantities**) and ν_i the **stoichiometric coefficient** of i in the reaction.

Equations (2) and (4) are also called the *Kirchhoff equations*.

Similar equations exist for the thermal coefficients in variables V , T , ξ .

For non-reactive systems consisting of pure substances, equation (1) above gives

$$\left(\frac{\partial C_p}{\partial p}\right)_T = -T \left(\frac{\partial^2 V}{\partial T^2}\right)_p.$$

CLAUSIUS FORMULATION OF THE SECOND LAW OF THERMODYNAMICS.

Clausius derived the full contents of the second law of thermodynamics from the following statement of experimental fact: Heat cannot pass spontaneously from a body of lower temperature to a body of higher temperature. For a comparison with other statements, see second law of **thermodynamics**.

CLAUSIUS INEQUALITY. The statement that the **Clausius integral** taken around any cycle is negative, or

$$\oint \frac{dQ}{T} \leq 0$$

where the sign of equality refers to *reversible* cycles only. It is a consequence of the second law of **thermodynamics**. Alternatively, depending on the sequence in the exposition of the second law, it plays an important part in the argument. (See **thermodynamics, second law of**.)

CLAUSIUS INTEGRAL. The expression

$$\int_1^2 \frac{dQ}{T}$$

Along any *reversible* path, the Clausius integral is independent of the path, and hence

$$\oint \frac{dQ}{T} = 0,$$

or

$$dS = \frac{dQ}{T},$$

which can be used as a definition of entropy. (See **Clausius inequality; thermodynamics, second law of**.)

CLAUSIUS-MOSSOTTI EQUATION. The **polarizability** α of simple *non-polar* molecules and atoms can be calculated from the density N_o , in particles per unit volume, and from the static dielectric constant ϵ_s by the Clausius-Mossotti equation

$$\frac{\epsilon_s - 1}{\epsilon_s + 2} = -\frac{4\pi}{3} N_o \alpha. \quad (1)$$

For a constant density, the dielectric constant is, according to (1), practically independent of temperature, and, at constant temperature, the dielectric constant of a gas depends strongly on the pressure. This formula is valid as long as the short range intermolecular forces can be neglected (see **internal fields in dielectrics**).

An alternate form for (1) is:

$$\frac{\epsilon_s - 1}{\epsilon_s + 2} \frac{M}{\rho} = [P] \quad (2)$$

where M is the molecular mass, ρ the density, and $[P] = \frac{4\pi}{3} N\alpha$ is the *molar polarization* (N is Avogadro's constant).

In this form, the Clausius-Mossotti equation is practically identical with the *Lorentz-Lorenz relation*:

$$\frac{n^2 - 1}{n^2 + 2} \frac{M}{\rho} = [R]$$

where n is the index of refraction, $[R] = \frac{4\pi}{3} N\alpha(\omega)$, in the molar refraction, and $\alpha(\omega)$ is the polarizability for an electrical field of angular velocity ω . In fact, the Clausius-Mossotti equation is the limiting value of the Lorentz-Lorenz equation for non-polar substances and for $\omega \rightarrow 0$, since, according to Maxwell's law, the static dielectric constant can then be identified with the square of the index of refraction for an electromagnetic field of 0 frequency.

CLAUSIUS-RANKINE CYCLE. See **Rankine cycle**.

CLAUS STRUCTURE. See **benzene, structures of**.

CLEAR. To restore a storage or memory device in a digital computer to a prescribed state, usually that denoting zero. (See also **reset**.)

CLEARANCE RATIO. See **clearance volume**.

CLEARANCE VOLUME. The volume V_c contained between the piston and cylinder head of a reciprocating engine when the piston is at inner dead center position (maximum travel on inward stroke). The ratio of clearance volume V_c to swept volume V_s is known as the *clearance ratio*

$$\epsilon_o = V_c/V_s.$$

CLEBSCH-GORDAN COEFFICIENTS. Coefficients $C_{jj'}(JMmm')$ in

$$\mathcal{Y}_{J_{jj'}}^M = \sum_{m=-j}^j \sum_{m'=-j'}^{j'} C_{jj'}(JMmm') \mathcal{Y}_{jm}(1) \mathcal{Y}_{j'm'}(2)$$

where

$$\begin{aligned} J_z^{(1)} \mathcal{Y}_{jm}(1) &= m \mathcal{Y}_{jm}(1), \\ J_z^{(2)} \mathcal{Y}_{j'm'}(2) &= m' \mathcal{Y}_{j'm'}(2), \\ \{J^{(1)}\}^2 \mathcal{Y}_{jm}(1) &= j(j+1) \mathcal{Y}_{jm}(1), \\ \{J^{(2)}\}^2 \mathcal{Y}_{j'm'}(2) &= j'(j'+1) \mathcal{Y}_{j'm'}(2) \end{aligned}$$

and

$$\begin{aligned} \{J_z^{(1)} + J_z^{(2)}\} \mathcal{Y}_{J_{jj'}}^M &= M \mathcal{Y}_{J_{jj'}}^M, (M = m + m') \\ \{J^{(1)} + J^{(2)}\}^2 \mathcal{Y}_{J_{jj'}}^M &= J(J+1) \mathcal{Y}_{J_{jj'}}^M, \end{aligned}$$

the operators $J^{(1)}$, $J^{(2)}$ being vector **angular momentum operators**.

CLERK-MAXWELL RELATION. See **Maxwell relation**.

CLOCK PARADOX. If two identical clocks A , B are synchronized and A is accelerated arbitrarily and then brought back to compare with B , which has not been accelerated, then A will record a shorter time interval than B . This conclusion is a consequence of, and is understandable in terms of, relativity theory:

$$t_B = t_A \left(1 + \frac{v^2}{2c^2} \right).$$

t_A is the time interval as it appears to A and t_B is the time interval as it appears to B . v is the relative velocity of A with respect to B during its unaccelerated motion. (See also **relativity, general**.)

CLOSED. A set of points in a **topological space** is closed if it contains all its limit points.

A set of functions $f_1, f_2, \dots, f_n, \dots$ in **Hilbert space** is closed if every function g in the space can be approximated in the form

$$g = a_1 f_1 + a_2 f_2 + \dots + a_n f_n + \dots,$$

with suitable constants a_i .

CLOSED CURVE. See **curve**.

CLOSED EDGE TRAIN. See **edge train, closed**.

CLOSED LOOP. (1) Portion of a **Feynman diagram** describing the virtual creation of a number of electron-positron pairs together with the annihilation of all of them, as for example in the scattering of light by light. (2) See **contour; Ampere law**.

CLOSED QUADRATURE FORMULA. A quadrature formula which explicitly includes the values of the ordinates at the end points, to be contrasted with an **open quadrature formula** in which the ordinate of at least one end point does not appear. Closed formulas are used as **correctors**, open formulas as **predictors**, in the numerical solution of **ordinary differential equations**.

CLOSED SHELLS. If particles obeying **Fermi-Dirac statistics** (i.e., electrons or nucleons) move to a first approximation independently of each other in orbits about a common central field of force, each particle will possess certain discrete energy levels characterized by definite values of the angular momentum l . The **Pauli principle** restricts the number of identical particles having a given energy and the angular momentum to a maximum of $2(2l + 1)$. Such a collection of $2(2l + 1)$ identical particles is called a closed shell, since no further particles having the same energy and angular momentum can be added to it. Owing to the similarity of the radial wave functions of the particles in a closed shell, the structure acquires a special stability. The term is usually used in a slightly enlarged sense, to take account of all the particles allowed by the Pauli principle to occupy a group of levels lying close together in energy. Thus, for example, in a hydrogenic potential ($V = e^2/r$) the $2s$ and $2p$ levels are degenerate. The corresponding closed shell would be considered to be made up of all the particles, 8 in this case, which can simultaneously be put into these two levels. In the

case of light atoms whose potentials do not differ too much from the hydrogenic potential, one obtains closed shells of electrons corresponding to the filling of the 1s level (atomic number $Z = 2$, i.e., He), then the 2s and 2p levels are filled at $Z = 10$ ($= 2 + 8$), i.e., at Ne. In this way one obtains the series of noble gases He, Ne, Ar, Kr, Xe, and Rn. In a similar manner, one obtains especially stable nuclei containing 2, 8, 20, 28, 50, 82, or 126 neutrons or protons. (See **magic numbers**.)

CLOSED SYSTEMS. Thermodynamic systems which can exchange energy but not matter with the outside world. (See also **isolated systems**; **open systems**; **thermodynamic systems**.)

CLOSURE. The closure of a set of points is the set containing the given set and all its limit points. (See **topological space**.)

CLOTHOID. See **Cornu spiral**.

CLUSTER. (1) A group of atoms in a gas where each of the atoms in the group can be reached from any other one by traveling over a chain of atoms, of which any two consecutive atoms are lying within each other's sphere of action. (2) A small group of liquid molecules with much the same spatial arrangement as in a solid crystal. Clusters are found in liquids for temperatures not far removed from the melting point.

CLUSTERING PHENOMENA IN GASES. At any finite temperature, and especially near the condensation point, real gases contain clusters of atoms.

CLUSTER INTEGRALS. Integrals over clusters of atoms occurring in the Mayer theory of virial coefficients.

CLUSTERS, STAR. See **star clusters**.

CLUSTER THEORY OF VIRIAL COEFFICIENTS. In statistical mechanics the virial coefficients are evaluated by considering clusters of atoms.

c-NUMBER. Physical quantities correspond in quantum mechanics to linear operators which do not commute in general. Ordinary numbers which commute with all operators are called sometimes c-numbers to distinguish them from the operator quantities.

COANDA EFFECT. When a two-dimensional jet of fluid is discharged tangentially along a convex solid surface, the jet tends to follow the curve of the boundary. The jet may remain attached to the surface for a considerable distance and be deflected through a large angle. Deflections up to about 180° have been observed.

COCHRAN THEOREM. A theorem in statistics concerning the distribution of quadratic forms. If x_i ($i = 1, 2, \dots, n$) are independent standardized normal variates and q_j ($j = 1, 2, \dots, k$) are quadratic forms in the x 's with ranks n_j ($j = 1, 2, \dots, k$) and if

$$\sum_{j=1}^k q_j = \sum_{i=1}^n x_i^2,$$

then a necessary and sufficient condition for the q_j to be distributed as χ^2 (see **chi-square**) with n_j degrees of freedom respectively is that

$$\sum_{j=1}^k n_j = n.$$

CODAZZI EQUATIONS OF THE SURFACE. See **equations of Gauss and Codazzi**.

CODDINGTON SHAPE AND POSITION FACTORS. Coddington introduced dimensionless factors in third order optical theory that lead to expressions for aberrations in terms of homogeneous polynomials. The shape factor of a lens is

$$\sigma = \frac{r_2 + r_1}{r_2 - r_1}$$

where r_1 and r_2 are the radii of the first and second surface of the lens. The position factor of a lens is

$$\tau = \frac{q - v}{q + p}$$

where p and q are object and image distances.

CODE. For any given **stored-program computer**, each operation of its repertory is represented by a code symbol, which is a particular sequence of characters. Also each cell in the storage unit is designated by a code symbol, the **address** of the cell. An ordered sequence of symbols representing operations and operands (in the form of addresses) when represented in the storage unit, directs the computer to perform the operations upon the operands

in the specified sequence, and this ordered set of code symbols is a machine code, and it defines a **routine** (or **program**).

If more than one address is used, the code is called a *multiple-address code*. In a typical instruction of a four-address code the addresses specify the location of two operands, the destination of the result, and the location of the next instruction in the sequence. In a typical three-address code, the fourth address specifying the location of the next instruction is dispensed with and the instructions are taken from storage in a preassigned order.

In a typical one-address or single-address code, the address may specify either the location of an operand to be taken from storage, the destination of a previously prepared result, or the location of the next instruction. The arithmetic element usually contains at least two storage locations, one of which is an accumulator. For example, operations requiring two operands may obtain one operand from the main storage and the other from a storage location in the arithmetic element which is specified by the operation part.

CODE, ALPHANUMERIC. A binary coded decimal system in which the space combinations are employed to represent the alphabet, quotation marks, and other special symbol requirements.

CODE, BINARY. Any code employing two distinguishable types of **code elements**.

CODE CHARACTER. A particular arrangement of code elements used in a code, to represent a single value.

CODE ELEMENT. One of the discrete events in a code, such as the presence or absence of a pulse, or of a dot or a dash or space as used in a Morse code.

CODE, EXCESS-THREE. A number code in which the decimal digit n is represented by the four-bit binary equivalent of $n + 3$.

CODE, HOLLERITH. The IBM card code in which information is represented on a punched card by the position and number of punches.

CODE OPERATION. (1) The list of operation parts occurring in an instruction code, together with the names of the corresponding operations (e.g., "add," "unconditional trans-

fer," "add and clear," etc.). (2) Synonym for operation part of an instruction.

COEFFICIENT. (1) An algebraic factor. (2) A quantity or parameter characteristic of a substance or a system. With the exception of some entries immediately following, coefficients are listed in this book under the appropriate characteristics.

COEFFICIENT OF ASSOCIATION. See **association**.

COEFFICIENT OF COMPRESSIBILITY. See **compressibility**.

COEFFICIENT OF DETERMINATION. See **determination, coefficient of**.

COEFFICIENT OF HEAT TRANSFER. See **heat transfer**.

COEFFICIENT OF MUTUAL DIFFUSION. See **mutual diffusion, coefficient of**.

COEFFICIENT OF PERFORMANCE. See **refrigeration; Carnot cycle**.

COEFFICIENT OF PROBABILITY. Gibbs' name for the **ensemble density**.

COEFFICIENT OF TENSION. The relative change of pressure with temperature at constant volume of a pure substance, or the expression

$$\beta = \frac{1}{p} \left(\frac{\partial p}{\partial T} \right)_v$$

when p is given at an arbitrary reference state. For a **perfect gas** the coefficient of tension β is equal to the **coefficient of thermal expansion** α , and for both we have

$$\alpha = \beta = \frac{1}{T}$$

COEFFICIENT OF THERMAL CONDUCTIVITY. See **Fourier law**.

COEFFICIENT OF THERMAL EXPANSION. (Also called coefficient of expansion.) The expression

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p$$

or the relative rate of change of volume with temperature at constant pressure.

For perfect gases

$$\alpha = \frac{1}{T}$$

In general there is no particular advantage in using the coefficient of expansion to describe the properties of gases in preference to the **equation of state**, but in the case of liquids or solids the coefficient of thermal expansion is independent of pressure (except for very high pressures) and nearly independent of temperature (over sufficiently wide temperature ranges). When this is the case, the use of the coefficient of thermal expansion offers some practical advantages.

COEFFICIENT OF VARIATION. The coefficient of variation of a distribution, V , is defined as the **standard deviation** divided by the **mean**,

$$V = \frac{\sigma}{m}$$

It is appropriate only where m is not close to zero. Some authors define it as

$$V = 100 \frac{\sigma}{m}$$

in order to avoid decimals.

COEFFICIENT OF VISCOSITY. See **viscosity**, **kinetic theory of**.

COERCIVE FORCE, H_c . The magnetizing force at which the magnetic flux density is zero when the material is in a symmetrical cyclically magnetized condition. Coercive force is not a unique property of a magnetic material, but depends upon the conditions of measurement.

COERCIVITY. The property of a magnetic material measured by the **coercive force** corresponding to the saturation induction for the material. This is a quasi-static property only.

COFACTOR. The cofactor of an element a_{ik} of a square **matrix** is the complementary minor of a_{ik} , namely the determinant of the matrix obtained by crossing out the i^{th} row and the k^{th} column, multiplied by $(-1)^{i+k}$.

COGRADIENT. A word used to describe the relationship of two tensorial quantities which have the same transformation properties. (See **tensor field**; **contragradient**.)

COHERENT RADIATION. In coherent radiation there is a definite statistical correlation between radiation at different positions in a cross section of the radiant energy beam (or beams), whereas in noncoherent radiation these relationships are random. For example, a slit is filled with approximately coherent radiation when it receives light from a small distant source. Interference bands are observed only between coherent beams. (See **partial coherence**.)

COHESION. Forces between the particles of any given mass by virtue of which it resists physical disintegration. The connotation of the term cohesion implies a difference from adhesion, in which the forces are operative chiefly in surfaces.

COHESION ENERGY. See **energy**, **cohesion**.

COHESION, FORCES OF. Attractive forces acting between molecules; these appear when the distance between two molecules is greater than a certain magnitude; below that magnitude the **intermolecular forces** are *repulsive*.

COHESION PRESSURE. The addition term a/V^2 used in the **van der Waals equation** to correct the pressure by adding the attractive force of the molecules. V is the volume of the gas, and a is approximately constant for a given gas.

COHESION, WORK OF. The work required to separate a column of liquid 1 cm^2 in cross section into two parts. It is given by

$$W_c = 2\gamma_L$$

γ_L is the **surface tension** between liquid and vapor in ergs per cm^2 .

COLLAPSE AND COLLAPSE LOAD. The term collapse is applied primarily to excessive deflection or actual collapse in the plastic range. Often perfect plasticity is assumed. (See **limit theorems**.)

COLLAPSE, INCREMENTAL. When live loads of sufficient intensity are applied repeatedly to a structure, it is possible that plastic deformation will continue to occur and that deflections will grow. If the deflections grow without limit the structure is said to suffer incremental collapse. (See **shakedown**.)

COLLECTIVE ELECTRON THEORY OF FERROMAGNETISM. The electrons re-

sponsible for ferromagnetism are supposed to be more or less free. Parallel alignment of their spins is favored by the **exchange interaction**. With the introduction of **Fermi-Dirac statistics**, it is possible to show that a transition from the ferromagnetic to the paramagnetic state occurs at a certain temperature, which is identified with the **Curie point**.

COLLINEATION EQUATIONS OF OPTICS. The mapping of object points into image points in **Gaussian optics** is a non-singular collineation, so the equations of this transformation in an appropriate coordinate system are the collineation equations of optics. One form of these equations is discussed under the **paraxial ray tracing equations**.

COLLISION. Any **interaction** between free particles, aggregates of particles, or rigid bodies in which they come near enough to exert a mutual influence, generally with exchange of energy. The term does not necessarily imply actual contact. The process is always subject to conservation of momentum, and in an elastic collision (see **collision, elastic** and **collision, inelastic**) also to conservation of energy. In the latter case, if the initial velocities are given, the velocities of the bodies after collision can be calculated by applying these two conservation principles. The subject is of special significance in atomic physics, where a collision is defined as a close approach of two or more photons, particles, atoms or nuclei during which an interchange occurs of charge, energy, momentum or other quantities.

COLLISIONAL TRANSFER. The momentum or energy transfer in a fluid can be resolved into two main contributions:

(1) Energy and momentum are carried through a surface element dS by those molecules whose trajectories cross this surface. This is the main mechanism of heat and momentum transfer in dilute gases, i.e., in the density domain for which the **Boltzmann equation** is valid.

(2) At higher densities, one must also consider the collisional transfer which becomes increasingly important: it is the instantaneous transfer of energy and momentum from the *center* of one molecule to the *center* of the other during the collision. If these centers occupy opposite sides of the surface element

dS during the collision, energy and momentum may be carried across dS even though neither molecule crosses the surface.

COLLISION DENSITY, NEUTRON. The number of collisions of neutrons with matter taking place in unit time in unit volume of material. Quantitatively, it is the product of total macroscopic cross section of the material, and the neutron flux.

COLLISION DIAMETER OF MOLECULES. The distance of closest approach between the centers of any two molecules in a collision.

COLLISION, DIRECT. See **direct and inverse collision**.

COLLISION, ELASTIC. A collision during which no change occurs in the internal energy of the participating systems; or in the sum of their kinetic energies of translation. The total mechanical energy is conserved, hence the **coefficient of restitution** is unity.

COLLISION FREQUENCY. The average number of collisions undergone by a particle per unit time. If v is the particle velocity and λ its **mean free path**, the collision frequency ν_c is given by the equation

$$\nu_c = v/\lambda.$$

COLLISION FREQUENCY BY MAXWELL'S EQUATION. See **Stosszahlansatz**.

COLLISION, INELASTIC. A collision during which changes occur both in the internal energy of one or more of the participating systems, and in the sums of their kinetic energies of translation before and after the collision.

COLLISION INTEGRAL. See **Boltzmann transport equation**.

COLLISION INTERVAL. The average period between two consecutive collisions. It is equal to the reciprocal of the **collision frequency**.

COLLISION, INVERSE. See **direct and inverse collision**.

COLLISION OF THE FIRST KIND. The collision of an accelerated particle (e.g., electron) with an atom resulting in a transfer of energy whereby the atom becomes excited and the electron is slowed.

COLLISION OF THE SECOND KIND.

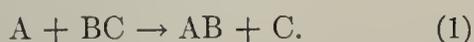
The **collision** of an excited atom with a slow particle (e.g., electron) whereby the atom undergoes transition to a lower energy-state and the other particle is accelerated.

COLLISION PARAMETER. See **impact parameter**.

COLLISION PROBABILITY. (1) When a beam of particles is passing through matter, the probability that any single particle be removed from the beam in a distance dx is $k dx$, where k is in general the inverse of the **mean free path**. (2) The fraction of the particles removed from a collimated beam per centimeter path length and per millimeter of pressure at 0°C. (3) For incident electrons, the number of electrons removed per unit electron current sometimes replaces the fraction.

COLLISION PROBABILITY, FIRST FLIGHT. The probability that a neutron, originating with equal probability at any point in the interior of a body and having random direction of velocity, v , will escape from the body without undergoing a collision. It is denoted by the symbol P_0 .

COLLISION THEORY OF CHEMICAL KINETICS. Consider a chemical reaction involving only one step, of the form



A well studied example is



This is a second order reaction (see **order of a chemical reaction**). The elementary collision theory of chemical kinetics assumes that the probability of reaction in a collision depends only on the *translational degrees of freedom*. For example one may assume that this probability is equal to a constant α when the relative translational energy along the line of centers has a specified value greater than E^x and is otherwise zero.

One obtains then for the rate constant (see **order of a chemical reaction**)

$$k = \alpha^2 D^2 \left(\frac{2\pi kT}{\mu} \right)^{1/2} \cdot \exp \left(- \frac{E^x}{kT} \right) \quad (3)$$

where D is the collision diameter, and μ the reduced mass; E^x is called the *activation energy*, and k is **Boltzmann's constant**.

This expression gives often the correct order of magnitude for simple processes. One may write (3) in the more general form

$$k = \alpha \Gamma e^{-E^x/kT}. \quad (4)$$

$\alpha \Gamma$ is known as the *frequency factor*. In some cases one finds, however, values of α larger than 1 or abnormal temperature dependence. In such cases the simple collision theory cannot be applied. (See **absolute reaction rate theory**.)

COLOGARITHM. The **logarithm** of the reciprocal of a number. Sometimes used in logarithmic computation to avoid the use of negative **mantissas** or of subtraction of logarithms.

COLOR. Color is (1) that characteristic of visual sensation which enables the observer to distinguish differences in the quality of the sensation, of the kind which can be caused by differences in the spectral composition of the light; (2) that characteristic of the light stimulus (light source or object), which gives rise to the visual sensation in (1), e.g., a red light, a white light, a red face, etc.; (3) as defined in (1) and (2), but restricted to the appearance of redness, greenness, etc., as distinct from whiteness, grayness or blackness; i.e., chromatic color in contradistinction to achromatic color. (See **additive mixture of colors**; **additive complementary colors**.)

COLORED BODY. A body for which the **spectral transmission** or **reflection factor** varies with wavelength.

COLOR EQUATION. An algebraic representation of the match of two stimuli, one of which, for instance, can appear as the result of an additive mixture.

COLORIMETRIC PURITY. Colorimetric purity is calculated from the formula

$$p_c = p_e y_d / y$$

where the symbols have the same meanings as in the definition of **excitation purity**.

COLOR INDEX. See **stellar magnitudes** and **spectral classification of the stars**.

COLOR SOLID. That part of the **color space** which is occupied by **surface colors**.

COLOR SPACE. A manifold of three dimensions for the geometrical representation of colors.

COLOR STIMULUS FUNCTION. The relative **spectral distribution** of the radiation from a reflecting or transmitting body. If the light incident on the body has the radiation function S_λ then the color stimulus function of a reflecting body is given by $\phi_\lambda = S_\lambda \cdot \beta_\lambda$, where β_λ is the *spectral luminance factor*, and that of a transmitting body is $\phi_\lambda = S_\lambda \cdot \tau_\lambda$, where τ_λ is the *spectral transmission factor*.

COLOR STIMULUS SPECIFICATION. A set of **tristimulus values** of a light (monochromatic or otherwise).

COLOR TEMPERATURE. The absolute temperature of the full radiator for which the ordinates of the **spectral distribution** curve of emission are proportional (or approximately so), in the visible region, to those of the distribution curve of the radiation considered, so that both radiations have the same chromaticity. (In German: *Verteilungstemperatur*.) In certain countries, by extension, the term color temperature is used in the case of a selective radiator when, for the **colorimetric standard observer**, this radiator has the same color (or at least approximately the same color) as a full radiator at a certain temperature; this temperature is then called the color temperature of the selective radiator. (In German: *Farbtemperatur*.)

COLOR TRIANGLE. The color triangle (**chromaticity diagram**) is a plane diagram showing the results of mixing color stimuli, every color being represented unambiguously by a single point on the diagram.

COLUMN. A column is that part of a structure whose purpose is to transmit, through compression, the weight of the structure and the superimposed loads to the foundation. Other **compression members** are often termed columns because of the similar stress conditions. The ratio of the length of the column to the least **radius of gyration** of its cross section is called the *slenderness ratio*. This ratio affords a means of classifying columns. All the following are approximate values used for convenience. A short steel column is one whose slenderness ratio does not exceed 50; an intermediate length steel column has a slenderness ratio ranging from 50 to about

200, while a long steel column may be assumed as one having a slenderness ratio greater than 200. A short concrete column is one having a ratio of unsupported length to least dimension of the cross section not greater than 10. If the ratio is greater than 10 it is a long column.

If the load on a column is applied through the center of gravity of its cross section it is called an **axial load**. A load at any other point in the cross section is known as an **eccentric load**. A short column under the action of an axial load will fail by direct **compression** but a long column loaded in the same manner will fail by buckling (bending), the buckling effect being so large that the effect of the direct load may be neglected. The intermediate length column will fail by a combination of direct stress and bending.

Euler derived a formula which gives the maximum axial load that a long, slender ideal column can carry without buckling. An ideal column is one which is perfectly straight, homogeneous and free from initial stress. This maximum load, sometimes called the critical load, causes the column to be in a state of unstable equilibrium, that is, any increase in the loads or the introduction of the slightest lateral force will cause the column to fail by buckling. The Euler formula for columns is given below.

$$P = \frac{K\pi^2 EI}{l^2}$$

in which P is the maximum or critical load. E is the modulus of elasticity. I is the moment of inertia of cross-sectional area. l is the unsupported length of column. K is a constant whose value depends upon the conditions of end support of the column. For both ends free to turn $K = 1$; for both ends fixed, $K = 4$; for one end free to turn and the other end fixed $K = 2$ approximately, and for one end fixed and the other end free to move laterally $K = 1/4$.

Since the moment of inertia of a surface is its area multiplied by the square of a length called the **radius of gyration**, the above formula may be rearranged as follows. Using the **Euler** formula for hinged ends, and substituting Ar^2 for I the following formula results:

$$\frac{P}{A} = \frac{\pi^2 E}{\left(\frac{l}{r}\right)^2}$$

$\frac{P}{A}$ is the allowable unit stress of the column, and the quantity $\frac{l}{r}$ is the slenderness ratio.

Since the structural column is generally an intermediate length column and it is impossible to obtain an ideal column, the Euler formula has little practical value for ordinary design. Consequently, various empirical column formulae have been developed to agree with test data, all of which embody the slenderness ratio. For design, appropriate factors of safety are introduced into these formulae.

COLUMN ANALOGY. Hardy Cross gave the analogy between the bending moment in an arch or frame fixed at its ends and the total stress in an eccentrically-loaded stub column. The actual calculations are equivalent to the method of the **elastic center**, and in turn both can be arrived at simply through the general **theorem of virtual work**.

COLUMN-BEAM. See **beam-column**.

COLUMN BUCKLING. See **buckling, column**.

COLUMN, EFFECTIVE LENGTH OF. The Euler buckling load for elastic columns hinged top and bottom, $K = 1$, is arbitrarily chosen as the standard (see **columns**). The effective length of a column is the length of a hinged-ended elastic column of the same cross section which has the same buckling load. A cantilever column of length L has an effective length of $2L$; a fixed-ended column, an effective length of $L/2$.

COLUMN FORMULAS. See **column**.

COLUMN, PIN-ENDED. A pin-ended column or hinged-ended column is free to rotate, but not free to translate laterally at each end.

COMA. (1) Qualitatively, coma is the image error due to the variation of **magnification** in a pencil of rays outside the paraxial region. The image point of intersection of a ray with the focal plane will vary in its distance from the axis according to the ray. In third order optics a comatic pattern is formed by a bundle of rays in the form of **comatic circles** whose envelope is a pair of lines intersecting in an angle of $\pi/3$. (2) Quantitatively, the second of the five **Scidel aberrations** is the coma function of a system.

COMATIC CIRCLES. Rays from an off-axis point through any zone of a lens meet the focal plane in a comatic circle, the radius of which is proportional to the radius of the lens zone.

COMBINATION. An assignment of a group of objects into two or more mutually exclusive sets. The **binomial coefficient** $\binom{n}{k}$ is the number of **combinations** or ways of selecting k objects from a set of n objects. If the k objects are permuted among themselves, no new combinations are formed, but there are $k!$ new arrangements of each combination.

COMBINATION DIFFERENCES AND COMBINATION SUMS. In the analysis of molecular spectra, correctly chosen combinations of the wave numbers of different lines form the basis for the determination of the individual molecular constants and of the distances between different energy levels. For instance the combination differences

$$\begin{aligned} R(J-1) - P(J+1) &= F_v''(J+1) - F_v''(J-1) \\ &= \Delta_2 F''(J) \\ &= (4B_v'' - 6D_v'')(J + \frac{1}{2}) - 8D_v''(J + \frac{1}{2})^3 \\ R(J) - P(J) &= F_v'(J+1) - F_v'(J-1) \\ &= \Delta_2 F'(J) \\ &= (4B_v' - 6D_v')(J + \frac{1}{2}) - 8D_v'(J + \frac{1}{2})^3 \end{aligned}$$

are used to separate the rotational energy levels of the upper and lower vibrational states and to determine the rotational constants.

The combination sums

$$\begin{aligned} R(J-1) + P(J) &= 2\nu_0 + 2(B_v' - B_v'')J^2 \\ &\quad - 2(D_v' - D_v'')J^2(J^2 + 1) \end{aligned}$$

are used to determine the band origins (**zero lines**) and the differences $(B_v' - B_v'')$ and $(D_v' - D_v'')$ of the rotational constants.

Similar combination relations apply to the S and O branches observed in the **Raman effect**.

COMBINATION PRINCIPLE. The principle, first recognized empirically by Ritz, that the frequency of every spectral line can be represented as the difference (combination) of two "terms"

$$\nu = T_1 - T_2$$

selected from a comparatively small number of such terms which are characteristic for the atom in question.

In its modern form

$$\nu = \frac{\nu'}{c} = \frac{1}{\lambda} = \left[\frac{E_1}{hc} - \frac{E_2}{hc} \right] \text{cm}^{-1}$$

the combination principle expresses the fact that every spectral line corresponds to a transition in the atom (or molecule, etc.) between two energy levels and that the energy of the emitted or absorbed photon is equal to the difference in energy between the initial and final level. The quantities

$$\frac{E_1}{hc}; \frac{E_2}{hc}, \text{ etc.}$$

are the spectroscopic term values. ν , ν' , and λ stand, respectively, for wave number, frequency, and wavelength of the spectral line, E for the energy of the level, h for Planck's constant, and c for the velocity of light.

COMBINATION SUMS. See combination differences and combination sums.

COMBINATION VIBRATION. A vibration in a polyatomic molecule in which two or more normal vibrations are simultaneously excited.

COMBINED LOADING. Combinations of bending, torsion, axial load, interior or exterior pressure, etc.

COMBINED STRESS. See stress, combined.

COMBUSTION CALCULATIONS IN ENGINEERING. Combustion is the exothermic reaction between a fuel and oxygen carried out at elevated or high temperature in an industrial appliance, such as a furnace, burner, internal combustion engine, combustion chamber, etc. By far the greatest proportion of the heat utilized in industry is obtained from combustion. The required oxygen is supplied chiefly by feeding atmospheric air (for composition, see air, atmosphere). Sometimes oxygen is supplied directly or in the form of easily decomposed compounds (oxygenizers in rockets). The products of combustion are, as a rule, gaseous, and leave the device at a relatively high temperature.

The chief components of all industrial fuels are carbon, C, and hydrogen, H; small amounts

of sulfur, S, may also be present. Solid and liquid fuels may consist of a mixture of the above elements and their compounds. Liquid fuels are mostly mixtures of hydrocarbons. Gaseous fuels are mostly mixtures of gaseous combustible and inert compounds.

Most combustion processes can be reduced to the following basic chemical reactions:

- (a) $\text{C} + \text{O}_2 \rightarrow \text{CO}_2 + 174,960 \text{ BTU}$
(+97,200 kcal)
- (b) $\text{C} + \frac{1}{2} \text{O}_2 \rightarrow \text{CO} + 53,316 \text{ BTU}$
(+29,620 kcal)
- (c) $\text{CO} + \frac{1}{2} \text{O}_2 \rightarrow \text{CO}_2 + 121,644 \text{ BTU}$
(+67,580 kcal)
- (d) $\text{H}_2 + \frac{1}{2} \text{O}_2 \rightarrow (\text{H}_2\text{O})_{\text{liq}} + 123,300 \text{ BTU}$
(+68,500 kcal)
- (e) $\text{H}_2 + \frac{1}{2} \text{O}_2 \rightarrow (\text{H}_2\text{O})_{\text{vap}} + 103,950 \text{ BTU}$
(+57,750 kcal)
- (f) $\text{S} + \text{O}_2 \rightarrow \text{SO}_2 + 127,548 \text{ BTU}$
(+70,860 kcal)
- (g) $\text{CH}_4 + 2 \text{O}_2 \rightarrow \text{CO}_2 + 2 (\text{H}_2\text{O})_{\text{liq}}$
+ 382,320 BTU (+212,400 kcal)
- (h) $\text{C}_2\text{H}_4 + 3 \text{O}_2 \rightarrow 2 \text{CO}_2 + 2 (\text{H}_2\text{O})_{\text{liq}}$
+ 623,160 BTU (+346,200 kcal)
- (i) $\text{C}_2\text{H}_6 + 3.5 \text{O}_2 \rightarrow 2 \text{CO}_2 + 3 (\text{H}_2\text{O})_{\text{liq}}$
+ 671,220 BTU (+372,900 kcal)

The quantities of heat released are for the equations as written, i.e., per mole of fuel at 0°C (32°F). It may be noted that reactions (a) and (f) involve no change in volume, i.e., no positive or negative contraction (the volume of solid carbon or sulfur can be neglected) if the products of combustion are at the same temperature and pressure as the reactants. In the other cases the volume either increases or decreases on combustion. (In reaction (g), there is no actual change in volume because the H₂O is produced as vapor.)

The heat released by the combustion of a fuel and by cooling the products of combustion to the initial temperature is known as its **calorific value, H**. When the fuel is a mixture of simple components, its calorific value can be calculated by proportional parts. Otherwise it must be measured. (See also **VDI formula**.) The calorific value of a fuel depends on whether combustion is at constant volume (e.g., bomb calorimeter) or at constant pres-

sure (Boys calorimeter, Junkers calorimeter). Denoting reactants by subscript 1 and products by subscript 2, we have

$$H_v = U_1 - U_2 \quad (1)$$

$$H_p = H_1 - H_2 \quad (2)$$

and, since $H = U + PV$,

$$H_p - H_v = P(V_1 - V_2). \quad (3)$$

The difference $H_p - H_v$ depends on whether or not there is contraction.

The calorific value depends on the reference temperature, i.e., on the common temperature of the reactants and products at which it has been determined. (Normally these are listed for $t_0 = 0^\circ\text{C} = 32^\circ\text{F}$.) Given H_p at t_0 , we can calculate H_p at any temperature t from the law of conservation of energy. Assuming constant specific heats for products and reactants, we have:

$$H_p(t) - H_p(t_0) = (t - t_0) \left\{ \sum_{\text{react}} m_1 C_{p1} - \sum_{\text{prod}} m_2 C_{p2} \right\} \quad (4)$$

For H_v , C_p in (4) should be replaced by C_v . The calorific value in a **heat tone**, and Equation (4) applies to any heat tone at constant pressure. The rate of change of H_p or H_v with temperature, is not very large. As long as it is assumed that the air used in combustion as well as the products of combustion are perfect gases, the calorific value is independent of pressure.

All fuels which contain hydrogen or sulfur produce combustion gases with the easily condensable vapors, H_2O and SO_2 . Their calorific values are measured at temperatures at which H_2O or SO_2 appear as liquids and, therefore, include the latent heats of condensation, l . On the other hand, in practical appliances, the combustion gases are discharged at a higher temperature and the latent heats are not released. The temperature at which this condensation occurs is called the *dew point* of the combustion gas. Accordingly, it is necessary to distinguish between the *higher* or *gross calorific value* H_g which includes the latent heat, and the lower or net calorific H_n value which excludes it. If only steam is present, we have

$$\begin{aligned} H_{pg} - H_{pn} &= Ml = 19,381.6 \text{ Btu/lbmol} \\ &= 10,750 \text{ kcal/lbmol}, \end{aligned}$$

where M is the molar weight of steam ($M = 18.016 \text{ lbm/lbmol} = 18.016 \text{ kg/kmol}$; and

$l = 1075.8 \text{ Btu/lbm} = 592.1 \text{ kcal/kg}$ is the latent heat at $0^\circ\text{C} = 32^\circ\text{F}$). For pure hydrogen, H_n is approximately 19% lower than H_g . For pure carbon the two are equal. In practical calculations preference is given to the use of the lower calorific value.

Combustion is a highly complex, irreversible chemical process in which the dynamic conditions (motion of the gases), and the conditions for heat and mass transfer (diffusion) play important parts. The composition of the products of combustion and the actual amount of heat released depend on all those factors, and can seldom be predicted with certainty.

The highest possible amount of heat is released if all chemical elements in the fuel are oxidized to form the highest oxides. When this occurs, combustion is called *complete*. In the case of carbon, this implies that only CO_2 and no CO is present in the products. In the contrary case, combustion is called *incomplete*. In practice, hydrogen always oxidizes to H_2O .

Owing to the operation of the **law of mass action**, combustion can never be complete in practice, because at chemical equilibrium CO will always be present, even if only in minute amounts. On the other hand, owing to the highly irreversible nature of combustion, it must not be assumed that in a practical device the reactants and products are necessarily in chemical equilibrium. Moreover, owing to the inevitable motion of the gases, rapid compression or expansion may take place, leading to states of **frozen equilibrium**.

In practical devices an attempt is always made to approximate complete combustion as closely as possible. This could not be assumed if only enough air, X_{min} , were supplied to provide the exact, stoichiometric quantity of oxygen, O_{min} , necessary for combustion. Accordingly the actual amount of air supplied, X , exceeds X_{min} and their ratio

$$\lambda = X/X_{min} \quad (\lambda > 1)$$

is called the air ratio. Sometimes, the excess of air is described by the related quantity

$$\delta = 1 - \lambda,$$

called *excess air ratio*. An excess of air is required in order to provide the necessary concentration gradients within the combustion zone, and to secure an excess of oxygen in order to shift the chemical equilibrium in the

direction of more complete combustion in accordance with (Le) Chatelier's principle.

Since complete combustion constitutes a desirable, if unattainable, ideal, calculations relating to it are important in the design of combustion equipment. A complication arises in that it is customary to indicate the composition of solid and liquid fuels by specifying mass fractions (denoted c , h , o , etc., for the elements, and w for the moisture content); on the other hand, the composition of gaseous fuels as well as that of all products of combustion is customarily given in mole-fractions or parts by volume, denoted, e.g., $(\text{CH}_4)_f$ for the fuel gas and, e.g., $(\text{CO}_2)_p$ for the products of combustion. It is always assumed that all gases are perfect. Often, in addition, it is stipulated that their specific heats are constant.

The combustion gases are normally analyzed at a temperature below the dew point and, therefore, the analysis does not contain any indication about the H_2O and SO_2 content. The analysis is then said to refer to dry (combustion) gases. The gases actually leaving the combustion zone, being at a temperature above the dew point, are referred to as wet (combustion) gases.

The composition of a fuel can be conveniently characterized by two dimensionless quantities σ and ν . For solid or liquid fuels:

$$\sigma = 1 + \frac{3}{c} \left(h - \frac{o - s}{8} \right)$$

$$\nu = \frac{3n}{7c}$$

Very often ν is small and we may put $\nu \approx 0$. For a gaseous fuel of the following general composition:

$$(\text{CO})_f + (\text{H}_2)_f + (\text{CH}_4)_f + (\text{C}_2\text{H}_4)_f + (\text{O}_2)_f + (\text{N}_2)_f + (\text{CO}_2)_f = 1, \quad (4)$$

$$\sigma = \frac{\left\{ \begin{array}{l} o \cdot s [(\text{CO})_f + (\text{H}_2)_f] + 2(\text{CH}_4)_f \\ + 3(\text{C}_2\text{H}_4)_f - (\text{O}_2)_f \end{array} \right\}}{(\text{CO})_f + (\text{CH}_4)_f + 2(\text{C}_2\text{H}_4)_f + (\text{CO}_2)_f};$$

$$\nu = \frac{(\text{N}_2)_f}{(\text{CO})_f + (\text{CH}_4)_f + 2(\text{C}_2\text{H}_4)_f + (\text{CO}_2)_f}.$$

For solid and liquid fuels, we can write (all volumes refer to normal temperature and pressure):

theoretical (stoichiometric) oxygen:

$$O_{min} = 29.92c\sigma \text{ ft}^3/\text{lbm} = 1.566 \text{ m}^3/\text{kg}$$

theoretical air:

$$X_{min} = 142c\sigma \text{ ft}^3/\text{lbm} = 7.461 \text{ m}^3/\text{kg}$$

volume of combustion gases:

$$V_p = \lambda X_{min} + \frac{3.59}{12}(3h + \frac{3}{8}o + \frac{2}{3}w) \text{ ft}^3/\text{lbm}$$

$$= \lambda X_{min} + \frac{1.87}{120}(3h + \frac{3}{8}o + \frac{2}{3}w) \text{ m}^3/\text{kg}$$

$$\text{H}_2\text{O content: } \left(\frac{h}{2} + \frac{w}{18} \right) \text{ lbmol/lbm}$$

$$\text{SO}_2 \text{ content: } \frac{s}{32} \text{ lbmol/lbm.}$$

For gaseous fuels of the general composition (4) we have:

$$O_{min} = 0.5[(\text{CO})_f + (\text{H}_2)_f] + 2(\text{CH}_4)_f + 3(\text{C}_2\text{H}_4)_f - (\text{O}_2)_f \text{ ft}^3/\text{ft}^3 \text{ (m}^3/\text{m}^3)$$

$$X_{min} = \frac{1}{0.21} O_{min} \text{ ft}^3/\text{ft}^3 \text{ (m}^3/\text{m}^3)$$

$$V_p = 1 + \lambda X_{min} - 0.5[(\text{CO})_f + (\text{H}_2)_f]$$

$$\text{H}_2\text{O content: } (\text{H}_2)_f + 2(\text{CH}_4)_f + 2(\text{C}_2\text{H}_4)_f.$$

In all cases, the molar (volumetric) composition of the dry gases is given by the equations:

$$(\text{CO}_2)_p = 0.21/y$$

$$(\text{O}_2)_p = 0.21(\lambda - 1)\sigma/y$$

$$(\text{N}_2)_p = (0.79\lambda\sigma + 0.21\nu)/y$$

$$y = (\lambda - 0.21)\sigma + 0.21(\nu + 1)$$

$$(\text{CO}_2)_p + (\text{O}_2)_p + (\text{N}_2)_p = 1.$$

The excess air used can be calculated from the composition of the dry gases, with the aid of the formula:

$$\lambda = \frac{0.21}{\sigma} \left[\frac{1}{(\text{CO}_2)_p} + \sigma - 1 - \nu \right].$$

When combustion is incomplete, the above equations do not apply, but owing to the complexity of the process, no general rules for calculations can be indicated. Very often the assumption is made that the reactants and products in the combustion chamber are in chemical equilibrium, and calculations are made with the aid of the law of mass action. This leads to several equations with several

unknowns and to considerable computational difficulties. The temperature prevailing in the combustion chamber appears as an important, unknown parameter. Since it is usually high, **dissociation** must be allowed for.

The highest possible temperature of combustion is obtained when it is assumed that the process occurs in an adiabatic vessel, that combustion is complete, and that dissociation does not occur. The resulting (quite unrealistically high) temperature t is known as the *adiabatic flame temperature* or the theoretical temperature of combustion. With the usual assumptions, it can be determined from the following equations:

Solid or liquid fuels (H_n per unit mass)

$$H_n + [\bar{c}]_{t_r}^{t_f}(t_f - t_r) + \lambda X_{min}[\bar{C}_p]_{t_r}^{t_a}{}_{air}(t_a - t_r) \\ = \left\{ \frac{c}{12} [\bar{C}_p]_{t_r}^{t_f}{}_{CO_2} + \left(\frac{h}{2} + \frac{w}{18} \right) [\bar{C}_p]_{t_r}^{t_f}{}_{H_2O} \right. \\ \left. + 0.21(\lambda - 1)X_{min}[\bar{C}_p]_{t_r}^{t_f}{}_{O_2} \right. \\ \left. + 0.79\lambda X_{min}[\bar{C}_p]_{t_r}^{t_f}{}_{N_2} \right\} (t - t_r).$$

Gaseous fuels (H_n per mole)

$$H_n + (t_f - t_r) \sum_{fuel\ gas} x[\bar{C}_p]_{t_r}^{t_f} \\ + (t_a - t_r)\lambda X_{min}[\bar{C}_p]_{t_r}^{t_a} \\ = (t - t_r) \sum_{burnt\ gases} X[\bar{C}_p]_{t_r}^{t_f}.$$

Notation: t_r is an arbitrary reference temperature; H_n is assumed given at t_r ; t_a is the temperature of air supplied; t_f is the temperature of fuel supplied; X is the mole fraction; $[\bar{c}]_{t_r}^{t_f}$ is the mean specific heat per unit mass within temperature limits as indicated; $[\bar{C}_p]_{t_r}^{t_f}$ is the mean molar specific heat at constant pressure within temperature limits indicated; and t is the unknown adiabatic flame temperature.

Dissociation (important above 1500°C) and heat losses through radiation, convection and incomplete mixing cause the real flame temperature to be non-uniform throughout the combustion zone and always lower than the adiabatic flame temperature t above. The combustion temperature increases with H_n and t_f and t_a (pre-heating); it is higher with pure O_2 than with air, because no nitrogen need be heated. The highest combustion temperatures of about 3100°C (5600°F) are attained in

welding (oxy-acetylene) burners. Higher temperatures can be obtained in electric arcs. The flame temperature is reduced if λ is large, but as a rule there exists an optimum value for it. This, roughly, corresponds to the maximum excess of air which produces the best approximation to complete combustion. Chemical losses reflect on the calorific value and bear more heavily on the final result than do physical losses, consequently it is better to have too much air than too little.

The components of combustible mixtures can exist side by side indefinitely at normal temperatures. In order to start combustion the mixture must first be heated to a definite temperature, the *ignition temperature* (see **ignition limit**). The heat released on ignition must be sufficient for the combustion, initiated by ignition, to propagate itself through the combustible mixture. The velocity of propagation of the combustion zone relative to combustible mixture is called *flame* or *combustion velocity*; it depends on the physical and chemical properties of the mixture, and ranges from about 1 m/sec (3 ft/sec) for CO-air mixtures, to 30 m/sec (100 ft/sec) for H_2 - O_2 mixtures. The velocity of combustion can be reduced considerably, even to zero, by vigorous cooling (as in a Davy lamp); it is also strongly affected by the air-fuel ratio (see **ignition limit**), and can be appreciably increased by inducing turbulence in the combustible mixture (as in internal combustion engines). The slowly progressing mode of combustion, also known as *deflagation*, can be replaced by an extremely fast propagation, known as *detonation*, which is induced in a tube owing to the compressibility of the gases involved.

COMBUSTION VELOCITY. See **combustion calculations in engineering**.

COMMAND (COMPUTER). (1) One of a set of several signals (or groups of signals) which occurs as a result of an **instruction**; the commands initiate the individual steps which form the process of executing the instruction. (2) Synonym for instruction.

COMMENSURABLE. The term used to describe the relationship between several quantities, e.g., b_1 , b_2 , and b_3 , for which a number C , not necessarily an integer, can be found such that $b_1 = n_1C$, $b_2 = n_2C$, and $b_3 = n_3C$ where n_1 , n_2 , and n_3 are integers. If no such

number C exists, then b_1 , b_2 , and b_3 are said to be non-commensurable.

COMMUNAL ENTROPY. The contribution to the entropy of a system arising from the disorder when the molecules are sufficiently free to change positions frequently.

COMMUTATION RELATIONS. Physical quantities are represented in quantum mechanics by linear operators. Two linear operators A and B do not commute in general, $AB \neq BA$. The commutation relations give an expression for the commutator $AB - BA$. Canonically conjugate dynamical variables q , p satisfy the commutation relations $qp - pq = i\hbar$, where $\hbar = h/2\pi$ and h is Planck's constant. Components M_x , M_y , M_z of an angular momentum operator satisfy the commutation relations

$$M_x M_y - M_y M_x = i\hbar M_z,$$

$$M_y M_z - M_z M_y = i\hbar M_x,$$

$$M_z M_x - M_x M_z = i\hbar M_y.$$

(See Jordan-Wigner commutation rules.)

COMMUTATION RULES FOR BOSONS. The commutation rules satisfied by the creation and destruction operators for particles obeying Bose-Einstein statistics. If a_n and a_n^* are the destruction and creation operators for a Bose particle in the (one-particle) state characterized by the observable n , the Jordan-Klein commutation rules for them state that

$$[a_n, a_m] = [a_n^*, a_m^*] = 0 \quad (1)$$

$$[a_n, a_m^*] = \delta_{nm}.$$

Consider the case of neutral (uncharged) spin 0 particles of mass μ in greater detail. In a quantum field theoretic description, to a system of free spin 0 particles corresponds a Hermitian field operator $\phi(x) = \phi^*(x)$ satisfying the equation

$$(\square + \mu^2)\phi(x) = 0.$$

Let f_j be a set of positive energy solutions of the Klein-Gordon equation orthonormal in the scalar product

$$(f, g) = -i \int d\sigma^\mu(x) \left(\frac{\partial \bar{f}}{\partial x^\mu} g(x) - \bar{f} \frac{\partial g}{\partial x^\mu} \right) \quad (2)$$

then one defines the annihilation operator as

$$a_j = -i \int d\sigma^\mu \left[\frac{\partial \bar{f}_j}{\partial x^\mu} \phi(x) - \bar{f}_j(x) \frac{\partial \phi(x)}{\partial x^\mu} \right] \quad (3)$$

and the creation operator a_j^* as the adjoint of the above. The commutation rules of the a 's and a^* 's are given by (1). It can be demonstrated that the inequivalent discrete representations of the commutation rules (1) are \aleph_j in number. For just one of the irreducible representations does a no particle-particle state and an operator $N = \sum a_j^* a_j$ corresponding to the total number of particles, exist.

COMMUTATIVE LAW. An operator, denoted by the small centered circle \circ , obeys the commutative law if $a \circ b = b \circ a$ for all a , b in its domain. Thus, in elementary arithmetic $a + b = b + a$, and $ab = ba$.

COMMUTATOR. The commutator of two linear operators A and B is the expression $[A, B] = AB - BA$.

COMMUTING VARIABLES. In quantum mechanics those dynamical variables whose quantum mechanical operators commute. The set of variables whose operators commute with the Hamiltonian operator are constants of motion.

COMPACT. A subset S of a metric space is compact if every infinite sequence of elements in S contains a sub-sequence which is convergent to an element of S . Thus the unit-sphere in a Euclidean space is compact, while the unit-sphere (the set of all vectors of unit norm) in Hilbert space is not compact.

COMPARISON THEOREM (FOR CONVERGENCE). A series $a_1 + a_2 + \dots$ is absolutely convergent if $|a_i| < |b_i|$ for $i = 1, 2, 3, \dots$, where $b_1 + b_2 + \dots$ is an absolutely convergent series. Similarly $a_1 + a_2 + \dots$ is divergent if $|a_i| > |b_i|$ for some divergent series b_i .

COMPATIBLE DISPLACEMENTS. See displacements, compatible.

COMPENSATION THEOREM. If a network is modified by making a change, ΔZ , in the impedance of one of its branches, the current increment thereby produced at any point in the network is equal to the current that would be produced at that point by a compensating electromotive force acting in series with the modified branch, whose value is $-i\Delta Z$, where i is the original current which flowed in the modified branch.

COMPLEMENT. (1) In decimal representation, the complement of a number x is $10^{\nu} - x$, where ν is some fixed integer, positive, negative, or zero. In binary representation it is $2^{\mu} - x$, where μ is an integer. The subtraction of a number is thus essentially equivalent to the addition of its complement, and in computer construction it is generally easier to mechanize the formation of a complement than that of an arbitrary difference. In binary notation, the one's-complement of x is the number obtained when each digit x_i of x is replaced by $1 - x_i$. (2) The complement of a subset S of a set A is the set of all elements of A not included in S . (3) For the algebraic complement of a minor determinant, see "minor."

COMPLEMENTARITY. A concept introduced by Niels Bohr to express an essential feature of the quantum description of atomic phenomena. Though these phenomena transcend the scope of classical physical explanation, the account of all observations must be expressed in classical terms. Because of the impossibility of any sharp separation between the behavior of atomic objects and the interaction with the measuring instrument, evidence obtained under different experimental conditions cannot be comprehended within a single picture. Different conditions lead to the observation of complementary aspects of the phenomenon, complementary in the sense that only the totality of mutually exclusive observations exhausts the possible information about the atomic object. Wave and particle aspects are complementary aspects in the description of light or elementary particles. The description of a particle motion in terms of position coordinates is complementary to that in terms of momentum components.

COMPLEMENTARY ACCELERATION. Another name for **Coriolis acceleration**.

COMPLEMENTARY COLORS. See **additive complementary colors**.

COMPLEMENTARY DIFFRACTION SCREENS. See **Babinet principle**.

COMPLEMENTARY ENERGY. Consider a body of elastic material, to which classical elasticity theory is applicable, to be in equilibrium under the action of specified body

forces and specified surface tractions applied over part Σ_T of the surface Σ , while displacements are specified over the remainder Σ_u . If the components of the corresponding displacements, in a rectangular Cartesian coordinate system, are u_i ($i = 1, 2, 3$) and the components of the surface tractions acting on the portion Σ_u of the surface are T_i , then the complementary energy is defined by

$$U = \int_{\Sigma_u} T_i u_i d\Sigma,$$

where U is the strain-energy associated with an arbitrary stress field and the **summation convention** is used. The *theorem of minimum complementary energy* states that the complementary energy is an absolute minimum for all variations of the stress field which leave the equations of equilibrium throughout the body and the boundary conditions on Σ_T satisfied. Also called *principle of complementary energy*. In the particular case when surface tractions are specified over the whole surface, the theorem is called the *theorem of minimum strain energy*, or *theorem of least work*, or *Castigliano second theorem*.

COMPLEMENTARY FUNCTION (OF A LINEAR DIFFERENTIAL EQUATION). A solution of the corresponding homogeneous equation. (See **linear**.)

COMPLEMENTARY WAVELENGTH. The complementary wavelength (of a light) is the wavelength of a spectrum light that, when combined in suitable proportions with the light considered, yields a match with the specified **achromatic light**.

COMPLEMENT OF A SUBGRAPH. See **subgraph complement**.

COMPLETE COMBUSTION. See **combustion**.

COMPLETE RADIATOR. A temperature radiator of uniform temperature whose **radiant flux** in all parts of the spectrum is the maximum obtainable from any temperature radiator at the same temperature. Such a radiator is called a **black body** because it will absorb all the radiant energy that falls upon it. No known surface is truly black at all wavelengths.

COMPLETE SET OF ORTHOGONAL FUNCTIONS, EXPANSION IN. (1) Consider the complete set of functions $S_n(x)$ that are orthogonal over an interval L , i.e.,

$$\int_0^L S_n(x)S_m^*(x)dx = 0$$

where $S_m^*(x)$ is the complex conjugate of $S_m(x)$ and that are normalized to unity, i.e.,

$$\int_0^L S_n(x)S_n^*dx = 1.$$

Then in the expansion

$$F(x) = \sum_n a_n S_n(x)$$

of an arbitrary function $F(x)$, satisfying the same boundary conditions as S_n , the coefficients are given by

$$a_n a_n^* = |a_n|^2 = \int_0^L F(x)S_n^*(x)dx.$$

Examples of complete orthogonal sets are the $\sin nx$ and $\cos nx$ functions of the **Fourier series** over the interval 0 to π ; the complex exponential functions e^{inx} , $n = \dots -2, -1, 0, +1, +2 \dots$, over the interval 0 to π ; the Legendre polynomials $P_n(x)$ over the interval 0 to 1; the functions $\sqrt{x} J_p(r_n x)$ over the interval 0 to 1, where J_p is the p^{th} order **Bessel function** of first kind and r_n is its n^{th} root; etc.

(2) The idea of expansion in a complete orthogonal set can be extended to the case where the expansion functions form a continuous, rather than a discrete, orthogonal set. Again the orthogonality of the functions allows the determination of the expansion coefficients which are now continuous functions rather than discrete constants. An example of such an expansion is the **Fourier transform**.

(3) In many problems of quantum mechanics, the quantum mechanical operators have both discrete and continuous **eigenvalues**. The complete set of **eigenfunctions** then includes both those belonging to the discrete spectrum and those belonging to the continuous spectrum of the eigenvalues. The expansion of an arbitrary state function is then a sum over the discrete eigenfunctions plus an integral over the continuous eigenfunctions.

COMPLETE SPACE. A metric space in which every **Cauchy sequence** is convergent to an element in that space.

COMPLEX COMPLIANCE. See **compliance, complex**.

COMPLEX INTEGRATION. See **contour integration**.

COMPLEXION. A microstate of a system.

COMPLEX MODULUS. See **modulus, complex**.

COMPLEX NUMBER. A complex number, usually written $a + bi$ (see below), is a pair of real numbers (a,b) with addition and multiplication defined as follows: $(a,b) + (c,d) = (a + c, b + d)$ and $(a,b) \times (c,d) = (ac - bd, ad + bc)$. The real number a is the *real part*, and the real number b is the *imaginary part*, of the complex number (a,b) . The names “real” and “imaginary” are historical accidents; since a complex number is a pair of real numbers, it is exactly as “real” as they are and, in fact, complex numbers are often useful for the representation of phenomena in the “real” world. A complex number $(a,0)$ with zero imaginary part may be identified with the corresponding real number a . A complex number $(0,b)$ with zero real part is called a pure imaginary. The pure imaginary $(0,1)$ is usually denoted by the symbol i ; since $(0,1) \times (0,1) = (-1,0)$, the number i is a square root of -1 . Since $(a,b) = (a,0) + (b,0)(0,1) = a + bi$, every complex number may be written, as indicated above, in the form $a + bi$. In books on electricity, the letter i is usually replaced by j . Choosing r and θ so that $a = r \cos \theta$ and $b = r \sin \theta$ leads to the polar or trigonometric form of a complex number $z = r(\cos \theta + i \sin \theta)$, and defining $e^{i\theta}$ by the series $e^{i\theta} = 1 + (i\theta) + (i\theta)^2/2! + \dots + (i\theta)^n/n! + \dots$, gives the exponential form $z = re^{i\theta}$, from which all the usual properties of complex numbers may be quickly derived. E.g., the **De Moivre identity** $(re^{i\theta})^n = r^n e^{in\theta}$.

COMPLEX OF A GROUP. Any set of elements of a **group**.

COMPLEX POTENTIAL (IN HYDRODYNAMIC FLOW IN TWO DIMENSIONS). If the motion of an incompressible fluid is plane, two-dimensional, and irrotational there exist a velocity potential ϕ and a stream function ψ such that the velocity is given by

$$u = \frac{\partial \phi}{\partial x} = \frac{\partial \psi}{\partial y}$$

$$v = \frac{\partial \phi}{\partial y} = -\frac{\partial \psi}{\partial x}.$$

These are the **Cauchy-Riemann equations**.

The complex potential then is $w = \phi + i\psi$. Also $\text{div } \mathbf{v} = \nabla^2\phi$ and $\text{curl } \mathbf{v} = \nabla^2\psi$, where $\mathbf{v} = (u, v)$. ϕ only exists in irrotational motion ($\text{curl } \mathbf{v} = 0$); ψ exists even if $\text{curl } \mathbf{v} \neq 0$, provided $\text{div } \mathbf{v} = 0$.

The velocity is given by

$$u - iv = \frac{dw}{dz}$$

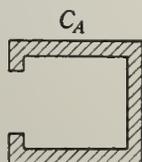
when $z = x + iy$.

Some authors place a minus sign in front of ϕ and ψ as given here.

COMPLEX RADIATION. Radiation composed of several monochromatic radiations.

COMPLEX TRUSS. See **truss, complex**.

COMPLIANCE, ACOUSTIC. The acoustic compliance of an enclosed volume of gas is equal to the magnitude of the ratio of the



Graphical representation of the element, acoustical compliance, C_A .

volume displacement of a piston forming one side of the volume to the pressure causing the displacement. It is measured in cm^5/dyne . It is also called the *acoustic capacitance*.

COMPLIANCE, BULK. Compliance based on **hydrostatic stress** and **dilatational strain**.

COMPLIANCE, COMPLEX. The reciprocal of the *complex modulus*, that is, the ratio of strain to stress in a steady-state oscillatory test, with oscillatory variables expressed in complex notation:

$$k^*(\omega) = k_1(\omega) - ik_2(\omega).$$

COMPLIANCE, CREEP. Subjection of a specimen to a **step function** of stress is known as a creep test. The resulting ratio of strain to stress, $k(t)$, is called the creep compliance, and is a non-decreasing function of the time. The superposition principle enables the strain response, $\epsilon(t)$, to a general stress history $\sigma(\tau)$ to be expressed in terms of $k(t)$:

$$\epsilon(t) = \int_{-\infty}^t k(t - \tau) \frac{\partial \sigma(\tau)}{\partial \tau} d\tau.$$

This represents the constitutive equation of linear viscoelasticity in an integral form.

COMPLIANCE, EQUILIBRIUM. The limiting value of the **compliance** of a viscoelastic material for slow loading or zero frequency.

COMPLIANCE, GLASS. The limiting value of the **compliance** of a viscoelastic material for instantaneous loading or infinite frequency.

COMPLIANCE, LOSS. The imaginary part of the complex compliance $k_2(\omega)$.

COMPLIANCE, MECHANICAL. Compliance in a mechanical vibrating system is that coefficient which, when multiplied by 2π times the frequency, is the reciprocal of the negative imaginary part of the **mechanical impedance**. The unit is the centimeter per dyne.

COMPLIANCE, RECTILINEAL. See **rectilinear compliance**.

COMPLIANCE, ROTATIONAL. See **rotational compliance**.

COMPLIANCE, SHEAR. Compliance based on **shear stress** and **strain** components.

COMPLIANCE, STORAGE. The real part of the complex compliance, $k_1(\omega)$.

COMPONENT. (1) In its general usage, one of the ingredients of a mixture, or one of the distinct molecular or atomic species composing a mixture. In physical chemistry, one among the smallest number of chemical substances which need to be specified in order to reproduce a given chemical system. (2) The projection of a **vector** on a particular coordinate **axis** or along some specified direction. (3) For component of a tensor, see **tensor (tensor field)**.

COMPONENT ANALYSIS. In statistics, a branch of multivariate analysis which seeks to exhibit a multivariate complex as dependent on a number of independent components, each component being a linear function of the original variables. If the components are chosen so that the first accounts for as much of the variation as possible, the second as much of the variation which remains, and so on, they are known as *principal components*.

COMPONENT OF A SCALAR. See **scalar**.

COMPONENT OF A VECTOR. In many problems it is useful to replace a vector, such as a force, by two or more vectors equivalent to it. The two or more vectors are called

components of the original vector. The process of finding the components is called the *resolution of the vector into components*.

COMPONENTS, INDEPENDENT. See independent components.

COMPONENTS OF SYSTEM. See indicial notation.

COMPONENTS OF TENSOR (TENSOR FIELD). See tensor (tensor field).

COMPOSITE BEAM. See beam, composite.

COMPOSITION OF FORCES. See forces, composition of.

COMPOSITION OF TWO TENSORS (TENSOR FIELDS). The operation of forming an inner product of two tensors (tensor fields). Also called *inner multiplication*. The inner product is said to be *compounded* of the two tensors from which it is formed.

COMPOSITION OF VECTORS. See vector addition.

COMPOSITION SERIES (OF A GROUP).

Let G_1 be a normal subgroup of a finite group G which is maximal, that is, such that the only normal subgroup of G which includes G_1 is G itself; similarly, let G_2 be a maximal normal subgroup of G_1 and so on until in n steps we reach $G_n = E$, the group consisting of the identity element alone. Then the sequence of groups so obtained

$$G, G_1, G_2, \dots, G_{n-1}, E$$

is called a composition series for G .

"COMPOUND" DISTRIBUTION. See convolution.

COMPOUND MULTIPLET. See multiplet.

COMPOUND NUCLEUS. The intermediate state produced in a nuclear reaction between capture of the bombarding particle and emission of the reaction product. For example, in the reaction $N^{14} + n \rightarrow O^{14} + p$ the compound nucleus is N^{15} . The appearance of sharp resonances in the cross section (i.e., reaction probability) as a function of energy of the bombarding particle can be interpreted only as the formation of an intermediate state long-lived relative to the time it takes a particle to traverse the nucleus, the mean life being $\tau = \hbar/\Gamma$, where $2\pi\hbar$ is Planck's constant and

Γ the width of the resonance. For example, level widths of about 0.1 eV are frequently observed in the interaction of slow neutrons, of energy about 1 eV, with heavy nuclei. The time required for a 1 eV neutron to cross a distance of about 10^{-12} cm is about 10^{-18} sec, whereas the mean life of the state τ is about 7×10^{-15} sec, which is much longer. During this relatively long time the compound state "forgets" its mode of formation, and can be expected to break up in a manner independent of how it was formed. Thus, the properties of the compound nucleus dominate the behavior of nuclear reactions in the neighborhood of resonances of the cross section (i.e., at low energies). At higher energies, when the de Broglie wavelength of the bombarding particle becomes comparable with the average spacing of particles in the nucleus, only a small portion of the target nucleus interacts with the bombarding particle, so that a compound nucleus is no longer formed. The concept of the compound nucleus was introduced in 1936 by N. Bohr, since when it has had much success in the understanding of nuclear reactions.

COMPOUND TRUSS. See truss, compound.

COMPRESSIBILITY. The coefficient of (isothermal) compressibility is defined by

$$\kappa \text{ (or } \kappa_T) = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_T. \quad (1)$$

The adiabatic (**isentropic**) compressibility coefficient κ_S is similarly defined by

$$\kappa_S = -\frac{1}{V} \left(\frac{\partial V}{\partial p} \right)_S. \quad (2)$$

One has

$$\frac{\kappa_S}{\kappa} = \frac{C_v}{C_p} \quad (3)$$

where C_v/C_p is the ratio of the heat capacities at constant volume and at constant pressure.

COMPRESSIBILITY FACTOR. (1) The semi-theoretical factor Y in the standard-nozzle discharge formula which takes into account the compressibility of the fluid being metered:

$$\dot{m} = \frac{CY}{\sqrt{1 - m^2}} A_o \sqrt{2\rho(p_1 - p_2)}.$$

(See **standard nozzle**.) (2) Value of the ratio

$$z = \frac{pV}{RT}$$

for a **real gas**. Since for a **perfect gas** $z = 1$, the compressibility factor can serve as a measure of the deviation of the properties of a real gas from perfect-gas behavior.

COMPRESSION. (1) In general usage, compression is descriptive of the decrease of volume of a compressible substance (solid, liquid or gaseous) due to the application of pressure. The common example of compression is found in cylinders filled with gas being compressed by the motion of a piston moving in the cylinder. The pressure increase required for any given reduction of volume of a gas depends upon the particular condition surrounding the act of compression; for example, if the cylinder is thoroughly insulated against heat transmission, the compression will be of a type known as **isentropic**, and the temperature of the gas will rise during compression because the mechanical work expended on the piston in obtaining the compression is converted into internal energy in the gas. On the other hand, if the cylinder is a good conductor of heat, and passes heat to the atmosphere as rapidly as it is received by the gas, the temperature at the end of compression may be the same as that at the beginning. In this isothermal compression the pressure at the end of compression is less than that of an equivalent isentropic compression. In general, actual compressions are neither strictly isentropic nor isothermal. (2) In structural engineering compression is used to denote the type of stress which causes the fibers of a member to shorten. (3) In electronics, the ratio of the small-signal power gain g_0 of a device to the power gain g_1 at some higher power level. Expressed in decibels:

$$\text{Compression} = 10 \log_{10} \frac{g_0}{g_1}$$

(See **compressor**.) (4) In television, the reduction in **gain** at one level of a picture signal with respect to the gain at another level of the same signal. (See also **black compression** and **white compression**.) The gain referred to in the definition is for a signal amplitude small in comparison with the total peak-to-peak, **picture signal** involved. A quantitative

evaluation of this effect can be obtained by a measurement of **differential gain**.

COMPRESSION EFFICIENCY. See **enthalpy drop**.

COMPRESSION-IGNITION ENGINE. See **diesel engine**.

COMPRESSION MODULUS. The ratio between the applied hydrostatic pressure and the decrease in volume per unit volume of an isotropic elastic material. Often denoted k and, when the **generalized Hooke's law** is applicable to the material, given in terms of the Lamé constants by

$$k = \frac{1}{3}(3\lambda + 2\mu).$$

Also called *bulk modulus*, or *modulus of compression*.

COMPRESSION RATIO. In an internal combustion engine (gasoline, diesel), the ratio of the volume of the compression chamber (total of the swept volume, V_s , and the volume at end of compression, V_c) to the volume at end of compression

$$r = \frac{V_s + V_c}{V_c}$$

The designation is somewhat illogical, but its use is universal.

COMPRESSION WAVE. A **simple wave** or progressive disturbance in the one-dimensional **isentropic** flow of a compressible fluid, such that the pressure and density of a fluid particle increase on crossing the wave in the direction of its motion.

COMPRESSOR. A machine whose function it is to provide a supply of gas at a pressure higher than that available. Compressors can operate on the static principle, as when compression is effected in a cylinder with the aid of a reciprocating piston driven from a crankshaft, the gas being suitably led into and out of the cylinder by automatic (spring-loaded) or governed valves. Such machines are called reciprocating compressors or simply compressors. Compression can also be achieved by the dynamic principle, as when the gas is first accelerated with the aid of blades on a rotor and then decelerated in a suitably-shaped passage so that its pressure becomes (approximately) equal to the dynamic pressure of the stream. Such machines are called rotary compressors or turbo-compressors.

The thermodynamic process of compression is, in principle, independent of the details of the compressor, except that compression in a rotary compressor is more nearly **isentropic** than in a reciprocating compressor.

The work of compression w per unit mass of gas, assuming the process to be reversible, depends strongly on the type of process undergone by the gas, being highest for isentropic compression, lowest for isothermal compression, and intermediate for polytropic compression. In the case of perfect gases, we have: Ideal work of polytropic compression

$$w = \frac{n}{n-1} p_1 V_1 \left[\left(\frac{p_2}{p_1} \right)^{\frac{n-1}{n}} - 1 \right]$$

(for isentropic compression substitute $n = \gamma$).

Ideal work of isothermal compression

$$w = p_1 V_1 \ln \left(\frac{p_2}{p_1} \right)$$

where 1 is the state at inlet; $p_1 V_1 = mRT_1$; p_2 is the pressure at exit.

Reciprocating compressors necessarily possess a **clearance volume**. The ideal diagram of operation for such a compressor with clearance volume $\epsilon_o V_s$ (ϵ_o being the clearance ratio)

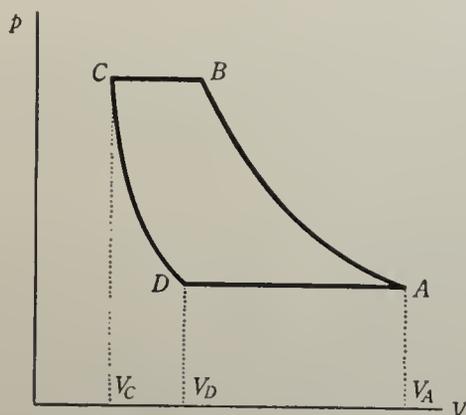


Fig. 1. Ideal p, V diagram of reciprocating compressor. In this diagram, the clearance volume is V_C , the swept volume V_s is equal to $V_A - V_C$, the clearance ratio ϵ_o is V_C/V_s , and the effective swept volume is $V_A - V_D$.

is shown in Figure 1. The work of compression is here

$$w = \frac{n}{n-1} p_A (V_A - V_D) \left[\left(\frac{p_B}{p_A} \right)^{\frac{n-1}{n}} - 1 \right]$$

for polytropic compression. The work of compression per unit mass is the same as without

a clearance volume, but the compressor behaves as if its swept volume had been reduced to $(V_A - V_D) = \mu V_s$ where μ is the **apparent volumetric efficiency**. Thus a larger compressor must be used and the actual work required increases because the losses are higher.

Owing to the existence of a clearance volume, the highest exit pressure attainable becomes

$$p'_B = p_A \left(\frac{1}{\epsilon_o} + 1 \right)^n$$

when the air filling volume $V_s(1 + \epsilon_o)$ attains pressure p'_B on reducing the volume to $\epsilon_o V_s$ so that none is left for delivery.

In order to achieve high exit pressures it is necessary to build multi-stage compressors by joining several of them (two, three, at most, four in practice) in series. When this is done, the gas can be cooled between the stages. In this manner the overall process of compression more nearly approaches an isothermal one, and the required work is reduced. The maximum saving is achieved when the work per unit mass in each stage is the same, which implies equal pressure ratios in each stage. In a two-stage compressor, the intermediate pressure

$$p_x = \sqrt{p_1 p_2}$$

where the gas is compressed to p_1 in the first stage, and to p_2 in the second. Dividing into stages also reduces thermal stresses.

The output of the compressor is judged by its **volumetric efficiency**

$$\lambda = \frac{\text{mass of gas delivered}}{\text{mass filling swept volume}}$$

The volumetric efficiency λ differs from the apparent volumetric efficiency μ because at the end of the inlet stroke (point A in Figure 1) the temperature of the gas T_A is higher than that at inlet T_o . Approximately

$$\frac{\lambda}{\mu} = \frac{T_o}{T_A}$$

The volumetric efficiency λ of a compressor can be improved and even made to exceed unity if a suitable long pipe is placed in front of its inlet. The periodic excitation from the piston causes high-amplitude pressure waves to travel through the inlet pipe. By proper steering it is possible to make the inlet pres-

sure higher than that outside. This process is sometimes called dynamic changing, Figure 2.

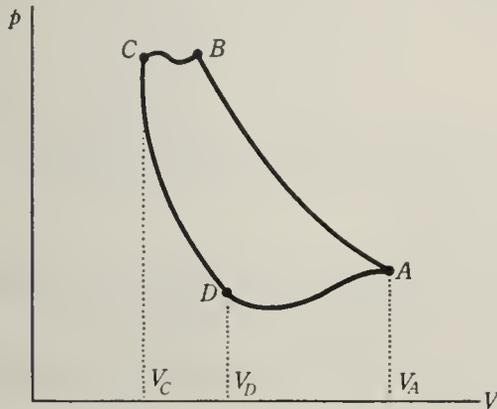


Fig. 2. p, V diagram of reciprocating compressor with dynamic changing.

A **vacuum pump** actually operates on the same principle as a compressor. The chamber to be evacuated is connected to the compressor inlet. The compressor draws gas at a decreasing pressure p_1 , compresses it to the constant atmospheric pressure p_2 and ejects it.

COMPTON EFFECT. Essentially, the scattering of a photon (quantum of electromagnetic radiation) by a free electron in which, owing to the conservation of energy and momentum, the scattered radiation suffers a wavelength change, increasing by

$$\Delta\lambda = \lambda_0(1 - \cos \theta),$$

where θ is the angle through which the beam is scattered and λ_0 , the Compton wavelength of the electron, is given by $\lambda_0 = h/mc$, where h is Planck's constant, m the rest mass of the electron, and c the velocity of light. At the same time, the electron recoils. Thus, the fractional change of wavelength $\Delta\lambda/\lambda$ increases with decreasing wavelength, i.e., with increasing photon energy. The effect was discovered in 1923 by Compton who observed the change of wavelength of a beam of X-rays scattered by carbon. Although electrons are normally bound, at energies $h\nu \gg \epsilon$, the binding energy, they can be considered as essentially free. The Compton effect is one of the famous effects which first demonstrated the corpuscular behavior of light. At long wavelengths, the Compton scattering becomes simply the classical, or Thomson, scattering. Compton scattering is peaked at forward angles compared with Thomson scattering. (See **scattering**.)

COMPUTATIONAL INSTABILITY. An instability in a finite difference equation which does not exist in the differential equation approximated thereby.

COMPUTER. Any physical device used as an aid in computing. It is customary to distinguish two general classes, **analog computers** and **digital computers**. (Also see **calculator**.)

COMPUTER, ANALOG. See **analog computer**.

COMPUTER, DIGITAL. See **digital computer**.

COMPUTER FLOW DIAGRAM. A graphical representation of a **program** or a **routine**.

CONCENTRATED FORCE. See **force, concentrated**.

CONCENTRATION. (1) The process of increasing the quantity of a substance or form of energy or other entity that exists in a volume of space, as in increasing by evaporation the amount of solute contained in unit volume in a solution, or as in gathering heat, light, electricity, or other energy or form of energy. (2) The quantity itself of matter or other entity that exists in a unit volume, as the strength of a solution in mass of solute per unit mass of solution, or in the number of moles, hydrogen ions, etc., contained per unit volume or per unit mass. (3) In statistics, the measure of **dispersion** known as the coefficient of concentration is defined as G where

$$G = \frac{\Delta_1}{2\mu_1}$$

Δ_1 being Gini's mean difference and μ_1 being the mean of the **distribution**.

CONCRETE, REINFORCED. Plain concrete will carry relatively heavy compressive stresses, but any attempt to impose tensile stresses of appreciable magnitude will result in rupture and consequent failure. For this reason plain concrete cannot be used for structural members subjected to a bending action or to a direct tensile action. However, if steel bars are incorporated in such a manner as to carry such tensile action as may develop in the member, then a safe and satisfactory "reinforced concrete" member may be used where otherwise a plain concrete member of the same

size, or even larger size, would be wholly inadequate and unsafe.

There are two physical phenomena which are primarily responsible for the successful use of steel and concrete jointly to form structural units or members: (1) The **coefficients of expansion** and contraction for concrete and steel are very nearly the same, thus preventing undesirable, or even disastrous, internal stresses due to differential expansion or contraction. (2) When the concrete hardens it grips the steel bars very tightly and securely and thus permits stress action between the two materials, the result of this joint action being exhibited as the strength of the structural member to resist the imposed load. Usually the steel bars are roughened to give the so-called "corrugated bar" which further assists in strengthening the adhesion between the concrete and steel by affording a sort of mechanical or interlocking bond.

CONDENSATE. The liquefied working fluid in a steam power installation.

CONDENSATION. (1) Change from gaseous (or vapor) state to liquid state. (See **change of state**.) (2) In general, increase in density. (3) The local increase in density in a sound wave, as contrasted with **rarefaction**: quantitatively,

$$s = \frac{\rho - \rho_0}{\rho_0},$$

where s is the condensation, ρ the local instantaneous density, ρ_0 the constant mean density at a point. (4) Accumulation of electric charge, as of electrons in a **capacitor**. (5) Convergence of light, or formation of a beam of parallel light rays. (6) In chemistry, various combination reactions of molecules to form larger molecules.

CONDENSATION, COEFFICIENT OF. It can be shown from kinetic theory that for a solid or liquid in equilibrium with its vapor at pressure p and temperature T , the mass of vapor molecules striking (and therefore also leaving) unit area per second is

$$\mu = p \sqrt{M/2\pi RT}$$

where M is the molecular weight of the vapor, and R the gas constant. If it can be assumed that every molecule striking the surface is condensed, this expression will also give the rate

of loss of weight of the solid or liquid at temperature T in a vacuum. If the assumption is not justified, the rate of loss of weight can be written

$$\mu = \alpha p \sqrt{M/2\pi RT}$$

the constant α being known as the coefficient of condensation. Experiments appear to show that this may be considerably less than unity in some cases.

CONDENSATION CURVE. See **boiling curve and condensation curve**.

CONDENSATION, EINSTEIN. See **Einstein condensation**.

CONDENSED SYSTEM. A substance or mixture of substances in the liquid or solid state. The term has also been applied to the condensation into a state of zero momentum of the particles in an ideal gas obeying **Bose-Einstein statistics**. (See **Einstein condensation**.)

CONDENSER. (1) For the electric condenser, see **capacitor**. (2) A device for liquefying gases or vapors. In power plant practice, in particular, a device for liquefying steam, especially the steam leaving a steam engine or steam turbine. In this manner the exit pressure is kept low, being roughly equal to the **saturation pressure** of steam which corresponds to the temperature of the available cooling water. This increases the efficiency of the power plant by a considerable amount.

CONDITIONAL CONVERGENCE. A series which is convergent but is not absolutely convergent is said to be conditionally convergent. (See **convergence**.)

CONDITION NUMBER. An indicator of **numerical stability**. In particular, the condition number of a matrix is usually taken to be a number that is equal to unity for a scalar matrix, and that is large when the matrix is in some sense nearly singular. Hence if the problem is to solve the linear system

$$\mathbf{Ax} = h,$$

the problem is unstable in case the condition number of \mathbf{A} is large. Among the several functions of the matrix that have been proposed, the most suitable seems to be

$$\gamma(\mathbf{A}) = \|\mathbf{A}\| \|\mathbf{A}^{-1}\|$$

for some appropriately chosen **norm**. The reason for this is that for small changes in \mathbf{A} , the ratio of the relative change in the solution to the relative change in \mathbf{A} can be shown not to exceed $\gamma(\mathbf{A})$. This is to say that if $\delta\mathbf{A}$ represents a small change in \mathbf{A} , and $\delta\mathbf{x}$ the induced change in \mathbf{x} , then

$$\begin{aligned} (\|\delta\mathbf{x}\|/\|\mathbf{x}\|)/(\|\delta\mathbf{A}\|/\|\mathbf{A}\|) &\leq \|\mathbf{A}\| \|\mathbf{A}^{-1}\| \\ &= \gamma(\mathbf{A}), \end{aligned}$$

provided second-order effects are neglected. More exactly, if $\mathbf{E} = \mathbf{A}^{-1}\delta\mathbf{A}$ and $\|\mathbf{E}\| < 1$, then

$$\begin{aligned} (\|\delta\mathbf{x}\|/\|\mathbf{x}\|)/(\|\delta\mathbf{A}\|/\|\mathbf{A}\|) \\ \leq \gamma(\mathbf{A})/(1 - \|\mathbf{E}\|). \end{aligned}$$

It should be observed that this condition number relates only to the invertibility of a matrix, and has no relation to the finding of its **eigenvalues and eigenvectors**. If \mathbf{A} is diagonalizable,

$$\mathbf{A} = \mathbf{P}\mathbf{\Lambda}\mathbf{P}^{-1}$$

where $\mathbf{\Lambda}$ is diagonal, then $\|\mathbf{P}\| \|\mathbf{P}^{-1}\|$ seems to be the appropriate condition number for this problem.

CONDON PARABOLA. In the Deslandres table of the **band system** of a diatomic molecule, the most intense bands are found to lie roughly on a parabola, the so-called Condon parabola. This effect finds a natural explanation in the **Franck-Condon principle**.

CONDUCTANCE. A term used in electrical circuits and by analogy in acoustical or mechanical systems to denote the real part of admittance. If the circuit or system has no **reactance**, its impedance consisting only of resistance, then its conductance is the reciprocal of its **resistance**.

CONDUCTION. The transmission of energy (heat, sound, electricity) by means of a medium without movement of the medium itself, as distinguished from convection, in which such movement occurs, or from radiation in which the energy quanta pass through the medium and so the transmission does not occur by means of the medium. (See **conduction, electric; thermal conduction**.)

CONDUCTION BAND. In the **band theory of solids**, materials are assumed to have a band of energy levels where electrons can move

freely and carry current. In semiconductors and insulators, this band, known as the conduction band, is normally empty, and is separated from the filled bands below it by an energy gap.

CONDUCTION BAND, DEGENERATE. See **degenerate conduction band**.

CONDUCTION BAND, DENSITY OF QUANTUM STATES IN. See **density of quantum states in conduction band**.

CONDUCTION, ELECTRIC. The transfer of electricity is accomplished by the motion of electric charge. In most metals, the moving charges are electrons; in semiconductors, both electrons and holes are important; in ionic crystals and electrolytes, positive and negative ions are the carriers; in gas discharges both ions and electrons play their parts.

CONDUCTION ELECTRONS, ORBITAL DIAMAGNETISM OF. See **orbital diamagnetism of conduction electrons**.

CONDUCTION ELECTRONS, SPIN PARAMAGNETISM OF. See **spin paramagnetism of conduction electrons**.

CONDUCTION, EXCESS. In a semiconductor, conduction by excess electrons, for example, by electrons introduced by donor impurities and promoted to the conduction band.

CONDUCTION, IONIC, IN SOLIDS. Since the ions of an ionic crystal may diffuse through the lattice, a small electrical conductivity is possible even though the material should otherwise be insulating. The process is very sensitive to temperature, and at low temperatures is also strongly dependent on the structure and purity of the material.

CONDUCTION, METALLIC. The conduction of electricity through solids wholly (or almost so) by the migration of electrons.

CONDUCTION OF HEAT. See **heat transfer; Fourier law**.

CONDUCTION, THERMAL. See **heat transfer; Fourier law**.

CONDUCTIVITY, ACOUSTIC. The ratio of the volume current through an orifice to the difference in **velocity potential** between the two sides of the orifice.

CONDUCTIVITY, ELECTRICAL. Strictly defined only for a material which obeys the Ohm law and equal to the ratio:

$$\sigma = J/E,$$

where **J** is the **current density** and **E** the **electrical field vector**. In any system of units, except the Gaussian, the conductivity is numerically equal to the inverse of the resistance of a cubical block of material having a current flow perpendicular to one pair of faces. The most commonly used unit is the mho per centimeter. In anisotropic materials the conductivity is a **tensor**, instead of a scalar.

CONDUCTIVITY OF HEAT AND ELECTRICITY, STATISTICAL TREATMENT. See **electron theory of metals**.

CONE IN SUPERSONIC FLOW. For a cone at zero incidence in a supersonic stream, provided the open angle is not too large, a conical shock wave is formed with axis and apex coinciding with those of the cone. There is then no characteristic length in the problem, and conditions are constant over any conical surface whose axis and apex coincide with those of the cone. Numerical solutions for a wide range of cone angles and Mach numbers have been tabulated.

This type of flow cannot occur if the cone angle is so large that the equations cannot be satisfied with an *attached* shock wave. The shock wave then becomes *detached* from the apex of the cone and is no longer conical. (See Z. Kopal, *Tables of Supersonic Flow about Cones*, M.I.T. Center of Analysis, Technical Report No. 1, 1947.)

CONFIDENCE LIMITS. Suppose that we are given a series of samples from a population whose distribution contains a parameter θ , and that we can find a rule of procedure which enables us to derive from each sample an assertion of the form $t_1 \leq \theta \leq t_2$. If the rule is such that this assertion has a probability of being correct of $\alpha\%$, then t_1, t_2 are called $\alpha\%$ confidence limits for θ .

CONFIGURATIONAL COMPRESSIBILITY. See **configurational heat capacity**.

CONFIGURATIONAL ELASTICITY OF A LIQUID. The compressibility of a liquid depends in part on resistance to compression

owing to the thermal agitation of the molecules and in part on disturbance of the **short range order**. The contribution from the latter is the configurational elasticity.

CONFIGURATIONAL EXPANSION. See **configurational heat capacity**.

CONFIGURATIONAL HEAT CAPACITY. Consider the heat capacity at constant pressure of a system in which a transformation defined by a parameter ξ may occur: we have (see **thermal coefficients**)

$$C_p = \left(\frac{dQ}{dT} \right)_p = C_{p,\xi} + h_{T,p} \left(\frac{d\xi}{dT} \right)_p. \quad (1)$$

The second term on the right-hand side refers to the effect of the temperature on the **reaction coordinate** ξ . This second term may be called the configurational heat capacity, C_p^{conf} , because in many cases ξ refers to the internal configuration of the system.

Similar definitions may be given for the *configurational compressibility* κ^{conf} and the *configurational thermal expansion* α^{conf} . If there exists a single parameter ξ then one has

$$C_p^{conf} = VT \frac{(\alpha^{conf})^2}{\kappa^{conf}}. \quad (2)$$

CONFIGURATION, ATOMIC. The arrangement in space of the atoms of a molecule.

CONFIGURATION, ELECTRON. See **electron configuration**.

CONFIGURATION FACTOR. See **shape factor**.

CONFIGURATION, INTERACTION. Calculations of bond energy of molecules depend on assuming that the electrons are to be found in certain **orbitals**, for which approximate forms must be assumed. To improve the calculation it is necessary to consider the admixture into the wave function of functions corresponding to other electronic configurations, supposed to represent excited states of the molecule.

CONFIGURATION SPACE. The space of the coordinates of a system of particles. In the configuration space method in quantum mechanics, the state of a system is described by a **wave function** which is a function of a

point in configuration space, that is, a function of all the coordinates of the particles.

CONFLUENCE. (1) If a_1, a_2 are distinct singular points (see **regular singularity**) of a differential equation, another differential equation may frequently be obtained as a limiting form when a_2 approaches a_1 . The second equation is then said to result from the first by confluence. The importance of this procedure arise chiefly from the fact that many of the linear equations of mathematical physics can be obtained by confluence from the generalized **Lamé equation**. (2) In fluid flow, a confluence is a region in which the spacing between streamlines decreases in the direction of flow.

CONFLUENT HYPERGEOMETRIC FUNCTIONS. There are four functions known as confluent hypergeometric functions: Kummer's function, ${}_2F_1[a; b; x]$, its associated solution $U(a; b; x)$ and the two Whittaker functions $M_{k,m}(x)$ and $W_{k,m}(x)$.

Most of the commonly used functions of mathematical physics, such as the **Weber functions** and **Bessel functions**, are special cases of confluent hypergeometric functions.

They can be derived from the *generalized hypergeometric function*

$${}_A F_B [a_1, a_2, \dots, a_A; b_1, b_2, \dots, b_B; x] = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n \dots (a_A)_n x^n}{(b_1)_n (b_2)_n \dots (b_B)_n n!} \quad (1)$$

where $(a)_n \equiv a(a+1)(a+2)\dots(a+n-1)$ and $(a)_0 \equiv 1$, for all a_r, b_r and x , real or complex, provided that no b is a negative integer; it is the solution of the *generalized hypergeometric differential equation*

$$\left\{ x \frac{d}{dx} \left(x \frac{d}{dx} + b_1 - 1 \right) \left(x \frac{d}{dx} + b_2 - 1 \right) \dots \left(x \frac{d}{dx} + b_B - 1 \right) - x \left(x \frac{d}{dx} + a_1 \right) \left(x \frac{d}{dx} + a_2 \right) \dots \left(x \frac{d}{dx} + a_A \right) \right\} y = 0. \quad (2)$$

If $A = B = 1$, Kummer's equation results:

$$x \frac{d^2 y}{dx^2} + (b-x) \frac{dy}{dx} - ay = 0. \quad (3)$$

Its simplest solution is *Kummer's function*

$${}_1F_1(a; b; x) \equiv 1 + \frac{a}{b} x + \frac{a(a+1)x^2}{b(b+1)2!} + \frac{a(a+1)(a+2)x^3}{b(b+1)(b+2)3!} + \dots \quad (4)$$

If $A = B + 1$ in Equation (2), the equation becomes a *Fuchsian equation* of the general type

$$x^B(1-x) \frac{d^{B+1} y}{dx^{B+1}} + \sum_{n=1}^B x^{n-1} (a_n x - b_n) \frac{d^n y}{dx^n} + a_0 y = 0. \quad (5)$$

If $B = 1$, this equation becomes

$$x(1-x) \frac{d^2 y}{dx^2} + \{c - (1+a+b)x\} \frac{dy}{dx} - aby = 0, \quad (6)$$

the **Gauss hypergeometric equation** and one of its solutions is the **Gauss hypergeometric function**

$${}_2F_1[a; b; c; x]. \quad (7)$$

If we replace x by x/b and let $b \rightarrow \infty$ in (6) and (7), then $(b)_n x^n / b^n \rightarrow x^n$ and (6) reduces to Kummer's equation (3), (7) to Kummer's function (4).

An alternative form of solution of Kummer's equation is

$$y = U(a; b; x) = \frac{\Gamma(1-b)}{\Gamma(1+a-b)} {}_1F_1[a; b; x] + \frac{\Gamma(b-1)}{\Gamma(a)} x^{1-b} {}_1F_1[1+a-b; 2-b; x] \quad (8)$$

Putting

$$y = \exp \left\{ \frac{1}{2} \int \left(1 - \frac{b}{x} \right) dx \right\} z = x^{-1/2 b} e^{1/2 x} z \quad (9)$$

in Kummer's equation, so that the coefficient of $\frac{dy}{dx}$ becomes zero, the equation assumes the form of *Whittaker's normalized equation*

$$\frac{d^2 z}{dx^2} + \left(-\frac{1}{4} + \frac{k}{x} + \frac{\frac{1}{4} - m^2}{x^2} \right) z = 0 \quad (10)$$

where $m = 1/2 b - 1/2$ and $k = 1/2 b - a$. The inverse transform from Whittaker's equation to

Kummer's equation is $z = x^{1/2} b e^{-1/2 x} y$ where $a = 1/2 + m - k$ and $b = 1 + 2m$.

One of the solutions of Equation (10) is *Whittaker's function*

$$z = M_{k,m}(x) \\ = x^{1/2+m} e^{-1/2 x} {}_1F_1\left[\frac{1}{2} + m - k; 1 + 2m; x\right].$$

Any general solution of Whittaker's equation is called a Whittaker function.

An alternative solution for Whittaker's equation is the second Whittaker function

$$W_{k,m}(x) = \frac{\pi}{\sin 2m\pi} \\ \left\{ \frac{-M_{k,m}(x)}{\Gamma(\frac{1}{2} - m - k)\Gamma(1 + 2m)} \right. \\ \left. + \frac{M_{k,-m}(x)}{\Gamma(\frac{1}{2} + m - k)\Gamma(1 - 2m)} \right\}$$

for all values real or complex of k , m and x .

Using the standard notation for **Bessel functions**, they can be expressed in terms of confluent hypergeometric functions, in particular Whittaker functions, as follows:

$$M_{o,m}(2x) = \Gamma(1 + m) 2^{2m+1/2} x^{1/2} I_m(x)$$

$$W_{o,m}(x) = \sqrt{\frac{x}{\pi}} K_m\left(\frac{1}{2}x\right)$$

$$M_{o,m}(\pm 2ix)$$

$$= \Gamma(1 + m) \exp\left\{\pm \frac{1}{2}\pi i\left(\frac{1}{2} + m\right)\right\} 2^{2m+1/2} x^{1/2} J_m(x)$$

$$W_{o,m}(2ix)$$

$$= \sqrt{\frac{1}{2}\pi x} \exp\left\{-\frac{1}{4}i\pi(2m + 1)\right\} H_m^{(2)}(x)$$

$$W_{o,m}(-2ix)$$

$$= \sqrt{\frac{1}{2}\pi x} \exp\left\{\frac{1}{4}i\pi(2m + 1)\right\} H_m^{(1)}(x).$$

(For a discussion of all main properties of confluent hypergeometric functions, and tables of ${}_1F_1[a;b;x]$, see *Confluent Hypergeometric Functions* by L. J. Slater, Cambridge University Press, 1960.)

CONFORMAL MAPPING. A mapping of one region upon another that is one-to-one and continuous, and such that angles are preserved. Thus let arcs C_1 and C_2 in one region intersect in a point P ; and let these be mapped into arcs C'_1 and C'_2 intersecting in P' . Let P_1 be any point on C_1 , mapped into P'_1 on C'_1 . Let ϕ be the angle between tangents to C_1 and

C_2 at P , ϕ' that between tangents to C'_1 and C'_2 at P' . Preservation of angles means that $\phi' = \phi$. In case the regions are two-dimensional and simply connected, they can be regarded as regions of two complex planes, and the mapping is then defined by $z' = f(z)$, where $f(z)$ is analytic in z throughout the region.

CONFORMAL REPRESENTATION. See **representation**.

CONFORMAL SOLUTIONS. Suppose that one may describe the interaction between one molecule of species r and one molecule of species s by the expression

$$\epsilon_{rs}(r) = f_{rs}\epsilon(g_{rs}r) \quad (1)$$

where f_{rs} and g_{rs} are constants depending only on the chemical nature of r and s , and where $\epsilon(r)$ is some simple two parameter law of interaction (see **intermolecular forces**). If all constants f_{rs} , g_{rs} are of the same order we may consider a "first order" treatment in which differences $\left(\frac{f_{rs} - f_{r's'}}{f_{rs}}\right)^n$ are neglected for $n > 1$.

In this case it is possible to develop a simple perturbational treatment (Longuet-Higgins) without any supplementary assumptions. The final **excess functions** are expressed in terms of the thermodynamic properties of one component (called the reference component) and in terms of the differences in the f and the g 's. Solutions which satisfy the conditions which we have stated above are called conformal solutions.

CONFOUNDING. In the **design of experiments**, a method which reduces the size of a factorial experiment by sacrificing some of the comparisons which could be made with the full factorial layout. The comparisons usually relate to higher order interactions and are said to be confounded.

CONGRUENCE. See **rectilinear congruence; congruent**.

CONGRUENCE, NORMAL. See **rectilinear congruence**.

CONGRUENCE OF CURVES. A family of curves whose equations involve two independent parameters. Also called *curvilinear congruence*, or *congruence*.

CONGRUENCE OF STRAIGHT LINES. See **rectilinear congruence**.

CONGRUENT. (1) Two geometric figures are congruent if they differ only in their position in space; that is, they are congruent with respect to a given geometry if they can be transformed into each other by transformations belonging to the group of the geometry; e.g., in Euclidean geometry by translations, rotations and reflections. (2) Two elements a, b of a ring are congruent modulo m , written $a \equiv b \pmod{m}$, if there exist elements p, q, r in the ring such that $a = mp + r$, $b = mq + r$; roughly speaking, if they leave the same remainder when divided by m . Thus $3 \equiv 2 \pmod{5}$. (3) Two square matrices A, B are congruent if A can be transformed into B by a congruent transformation; that is, if there exists a non-singular matrix C such that $B = \hat{C}AC$, where \hat{C} is the transpose of C .

CONIC. The locus of a point in the plane such that the ratio of its distance from a fixed point, called the focus, to a fixed line, called the directrix, is a constant. The general equation of the second degree represents a conic. (See **ellipse**, **hyperbola**, **parabola**.)

CONICAL COORDINATES. A degenerate system of **curvilinear coordinates** obtained from an ellipsoidal system. The coordinate surfaces are: spheres with center at the origin of a rectangular system and radii u ($u = \text{const.}$); two sets of **cones** with apexes at the origin, one along the Z -axis and the other along the X -axis ($v, w = \text{const.}$). Conical coordinates are related to rectangular coordinates by the equations

$$x^2 = \frac{u^2 v^2 w^2}{b^2 c^2}$$

$$y^2 = \frac{u^2 (v^2 - b^2) (w^2 - b^2)}{b^2 (b^2 - c^2)}$$

$$z^2 = \frac{u^2 (v^2 - c^2) (w^2 - c^2)}{c^2 (c^2 - b^2)}$$

where $c^2 > v^2 > b^2 > u^2$.

CONICAL REFRACTION. A singular refraction where a single ray of light is refracted into the set of generators of a cone (see **cone**, **visual**). This can occur, for example, in a **biaxial crystal** for a suitable angle of incidence.

CONJUGATE. (1) See **complex number**. (2) See **transform**; **matrix**.

CONJUGATE BEAM. See **beam**, **conjugate**.

CONJUGATE BERTRAND CURVES. See **Bertrand curves**.

CONJUGATE DIRECTIONS (AT A POINT P ON A SURFACE). The directions of the straight line joining P to a neighboring point Q on the surface and the line of intersection of the tangent planes at P and Q , in the limiting case as Q tends to coincidence with P . If these two directions are the same, the direction is said to be a *self-conjugate direction*, or *asymptotic direction*. There are two real asymptotic directions at each point of a surface for which the two principal curvatures are of opposite signs. If the two principal curvatures have the same sign then the two asymptotic directions are imaginary. If one of the principal curvatures is zero, the two asymptotic directions at the point coincide.

CONJUGATE ELEMENTS OF A GROUP.

In a group G an element a is said to be conjugate to an element b if there exists an element p in G such that $b = pap^{-1}$. The relation of conjugacy is easily proved to be an equivalence, so that the whole group G is thereby divided into conjugate classes. A subgroup N is called a self-conjugate, or normal, subgroup if all the conjugates of every element of n (that is, all elements of the form pap^{-1} with a in N and p in G) are included in N .

CONJUGATE FOCI. The interdependent distances between object and lens and lens and image are known as the conjugate distances. As the distance from the object to the lens increases, that from the lens to the image decreases and vice versa. The formula expressing the geometrical relation between these distances is:

$$1/f = 1/v + 1/u$$

where f is the **focal length** of the lens, v , the lens-to-image distance, and u , the object-to-lens distance. The ratio of the conjugate distances determines the size of the image. Thus:

$$R = v/u$$

where R is the relative size of the image to the object. If $u = v$, $1/f = 2/u$ and $u = 2f$. Then the total $u + v = 4f$, which is the minimum

possible distance between real image and real object for a lens with positive focal length.

The definitions of v and u given here are applicable only to a thin lens. The equations apply to any lens system, however, if v and u are measured respectively from and to the appropriate **principal planes** of the lens or the lens system.

CONJUGATE GRADIENTS, METHOD OF.

A method of successive approximations for solving a system of linear equations which is exact after, at most, n steps. The successive approximations are so chosen that each residual is orthogonal to all preceding ones.

CONJUGATE IMPEDANCES. See **impedances, conjugate**.

CONJUGATE NUCLEI. A pair of nuclei of the same mass number for which the numbers of neutrons and protons are interchanged. Thus Be^7 , containing 4 protons and 3 neutrons is the conjugate of Li^7 containing 3 protons and 4 neutrons. Owing to the neutron excess of heavy nuclei, only light nuclei can be conjugate. Also known as *mirror nuclei*. A *self-conjugate nucleus* is one containing equal numbers of neutrons and protons, e.g., B^{10} .

CONJUGATE NUMBERS. Two algebraic numbers are conjugate over a given field if they are roots of the same irreducible equation with coefficients in the field. Thus the complex numbers $a + bi$, $a - bi$ are conjugate over the real field, since they are roots of the equation $x^2 - 2ax + (a^2 + b^2) = 0$.

CONJUGATE POINT. (1) A **singular point** on a curve which is the real intersection of imaginary **branches** of the curve. The condition which must be met is

$$\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 - \frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} < 0.$$

Also called an isolated point.

(2) The conjugate points of a compound pendulum are two support points on a line through the center of gravity for which the period of oscillation is the same.

(3) The correspondence of object points and image points in **Gaussian optics** is one-to-one, and a pair of corresponding points are called conjugate points. The correspondence is linear so we also have the notions of *conjugate lines* and *conjugate planes*.

CONJUGATE SUBGROUP. See **group**.

CONJUGATE SYSTEM OF CURVES ON A SURFACE. Two one-parameter families of curves on a surface such that at a point of intersection of two curves, one from each family, the directions of the two curves are **conjugate directions**.

CONJUGATE VARIABLES. See **canonically conjugate variables**.

CONJUGATION OF DOUBLE BONDS.

Two double bonds are said to be conjugated when they are separated by one single bond. Butadiene, hexatriene, benzene are typical examples of molecules with two or three conjugated double bonds. They are characterized by a high electron mobility, and a lowering of the heat of hydrogenation, i.e., an increased stability. (See **delocalization energy**.)

CONNECTED. A set of W of points in a **topological space** is said to be connected if it is not the union of two disjoint closed sets.

CONNECTED GRAPH. See **graph, connected**.

CONSECUTIVE MEAN. (Also called moving average, overlapping mean, running mean.) A smoothed representation of a **time series** derived by replacing each observed value with a mean value computed over a selected interval. For example, if the observations are of daily maximum temperature and the selected interval is five days, then the value assigned to February 5th is the mean of the daily maxima from February 3rd through 7th, etc.

Consecutive means are used in **smoothing** to eliminate unwanted periodicities or minimize irregular variations.

CONSERVATION LAWS. A conservation law is a statement that some physical quantity remains constant during processes or interactions occurring within an isolated (closed) system. Among the major conservation laws of science are those of energy (including rest mass energy), momentum, and angular momentum. (See following entries.) Some conservation laws such as **conservation of mass** in the non-relativistic limit and the conservation of parity in strong interactions, are of limited applicability. Other conservation

laws are discussed in the entries for the particular conserved quantities.

There is a close relationship between many of the conservation laws and the symmetry properties of nature, a given conserved quantity remaining invariant (conserved) under a given symmetry transformation. Thus the energy of a system remains constant under time inversion (replacing t by $-t$); the momentum of a system is invariant under a linear transformation of the space coordinates; angular momentum is invariant under a rotational transformation. In situations where parity is conserved, it remains invariant under an inversion of the space coordinates. (See also **Lagrangian formalism**.)

CONSERVATION OF ANGULAR MOMENTUM. If the total external **torque** or moment on a rigid body or a system of particles about a point vanishes, the total **angular momentum** of the system about this point is a constant. (See also **moment of momentum**.)

CONSERVATION OF ENERGY. In a closed system the form and availability of energy may change, but the total energy is constant. Conservation of energy in an isolated mechanical system is an idealization. Friction and other dissipative forces and actions are supposed negligible so that the sum of the **potential energy** and the **kinetic energy** is constant. For a mechanical system, which is non-dissipative, work done on the system = increase in mechanical energy of the system. (See **energy conservation in closed systems**; **energy conservation in continuous systems**; **energy conservation in thermodynamics**.)

CONSERVATION OF MASS IN CLOSED SYSTEMS. Consider an homogeneous closed system containing c components ($\gamma = 1 \cdots c$) among which a single chemical reaction is possible. In such a system any variation in the masses will result only from the chemical reaction. Thus the change of the masses m_γ of component γ during the time interval dt can be written as

$$dm_\gamma = \nu_\gamma M_\gamma d\xi \quad (1)$$

where M_γ is the molar mass of component γ and ν_γ its stoichiometric coefficient in the chemical reactions (see **stoichiometric coefficients**). This coefficient is generally counted positive when γ appears in the right hand member of

the reaction equation, negative when it appears in the left hand member; ξ is the degree of advancement or **extent of reaction**.

The total mass of the system is given by $m = \sum_\gamma m_\gamma$. Summing (1) over γ , the conservation of mass for a closed system is expressed by

$$dm = \left(\sum_\gamma \nu_\gamma M_\gamma \right) d\xi = 0. \quad (2)$$

The equation

$$\sum_\gamma \nu_\gamma M_\gamma = 0 \quad (3)$$

is called the equation of the chemical reaction or, more briefly, the stoichiometric equation.

Instead of the mass of the components it is often useful to consider the mole numbers $n_1 \cdots n_c$. We then have, instead of (1)

$$dn_\gamma = \nu_\gamma d\xi. \quad (4)$$

Equations (1) and (4) are extended easily to r simultaneous reactions. We shall always designate the different reactions by indices ($\rho = 1 \cdots r$). We then have instead of (1) and (4)

$$dm_\gamma = M_\gamma \sum_{\rho=1}^r \nu_{\gamma\rho} d\xi_\rho \quad (5)$$

$$dn_\gamma = \sum_{\rho=1}^r \nu_{\gamma\rho} d\xi_\rho \quad (6)$$

where $\nu_{\gamma\rho}$ denotes the stoichiometric coefficient of γ in the reaction ρ .

CONSERVATION OF MASS IN CONTINUOUS SYSTEMS. The conservation of mass in a **continuous system** is expressed by the equation of continuity for the density

$$\frac{\partial \rho}{\partial t} = - \operatorname{div} \rho \omega \quad (1)$$

where ω is the macroscopic velocity. This equation expresses the fact that the local change of the density is equal to the negative divergence of the flow of matter.

Equation (1) holds also for a mixture; ω is then related to the macroscopic velocities of the different components by

$$\omega = \left(\sum_\gamma \rho_\gamma \omega_\gamma \right) / \rho. \quad (2)$$

Thus ω is simply the velocity of the center of gravity in an element of volume.

In general, the local change of a physical quantity is due not only to the divergence of the current which is associated with it, but a "source" term has also to be taken into account. For instance, the equation of continuity for the density ρ_γ of a component γ participating in a chemical reaction is

$$\frac{\partial \rho_\gamma}{\partial t} = -\operatorname{div} \rho_\gamma \omega_\gamma + \nu_\gamma M_\gamma V_v \quad (3)$$

where V_v is the rate of the chemical reaction per unit volume.

CONSERVATION OF MASS IN OPEN SYSTEMS. In an open system we may split the change of mass of component γ into an external part $d_e m_\gamma$ supplied from the exterior and an internal part $d_i m_\gamma$ due to changes inside the system

$$dm_\gamma = d_e m_\gamma + d_i m_\gamma. \quad (1)$$

Taking into account the Equations (5) and (6) introduced in the article on **conservation of mass in closed systems** we have

$$dm_\gamma = d_e m_\gamma + M_\gamma \sum_{\rho=1}^r \nu_{\gamma\rho} d\xi_\rho \quad (2)$$

or

$$dn_\gamma = d_e n_\gamma + \sum_{\rho=1}^r \nu_{\gamma\rho} d\xi_\rho. \quad (3)$$

Summing (2) over γ and taking into account the stoichiometric equations $\sum_\gamma \nu_{\gamma\rho} M_\gamma = 0$, we obtain for the total change of mass

$$dm = d_e m. \quad (4)$$

This relation expresses the conservation of mass in open systems and indicates that the change of the total mass is equal to the mass exchanged with the outside world.

The process of splitting the total change of mass of a component into an external part and an internal part may be used on arbitrary extensive property. (See **energy conservation in thermodynamics; thermodynamics, second law of.**)

CONSERVATION OF MASS, LAW OF. This law has been put in the form that matter can neither be created nor destroyed. More accurately, the total mass of any system remains constant under all transformations. (For the interpretation of this law in the light of relativity theory, see **mass-energy relation.**)

CONSERVATION OF MOMENTUM. For a dynamical system consisting of n material particles of masses $m_1, m_2 \cdots m_n$ respectively and position vectors $\mathbf{r}_1, \mathbf{r}_2 \cdots \mathbf{r}_n$ respectively, if the only forces acting are the mutual interaction forces of the particles the total momentum of the system remains constant; for example,

$$\sum m_i \frac{d\mathbf{r}_i}{dt} = \text{constant.}$$

The law of conservation of momentum is as fundamental to science as the law of **conservation of mass** or the law of **conservation of energy**. Like these laws, it holds in **quantum theory** and **relativity theory** as well as in more classical theories.

CONSERVATION OF VORTICITY. (1) The statement that in the horizontal flow of an inviscid autobarotropic fluid, the vertical component of **absolute vorticity** of each individual fluid particle remains constant. The principle was first applied to the atmosphere by Rossby and is the dynamical principle underlying the **equivalent-barotropic model** of the atmosphere. (2) The hypothesis that the **vorticity** of individual **eddies** is conserved during the turbulent mixing of a fluid. (See **vorticity transport hypothesis.**)

CONSERVATIVE FORCE (FIELD). If the work integral $\int_A^B \mathbf{F} \cdot d\mathbf{r}$ is independent of the path, the force \mathbf{F} is said to be conservative. \mathbf{F} is a function of position \mathbf{r} only and a potential energy V must exist, $\mathbf{F} = -\nabla V$.

CONSERVATIVE SYSTEM. A closed system in which mechanical energy is constant is called a conservative system, as is an **open system** in which all external work is available as potential and kinetic energy of the system.

CONSERVED QUANTITIES. Those quantities which obey conservation laws, e.g., **mass-energy**, **momentum**.

CONSISTENT DEFORMATION, PRINCIPLE OF. See **compatibility**.

CONSISTENT ESTIMATOR. In statistics, an estimator which converges in probability, as the sample size increases, to the parameter which it estimates.

sumer's risk arises from the error of accepting lots which should be rejected.

CONTACT ANGLE. The angle formed by a liquid on the surface of a solid at the gas-solid-liquid interface, measured as the dihedral angle in the liquid. Its value depends on the relative surface energies of the three interfaces, vapor-solid, vapor-liquid and solid-liquid.

CONTACT POTENTIAL. The potential difference observed between the surfaces of two metals in contact. It is due to the differences in **work function**; the electrons in each metal tend to reach the same **Fermi level**, and potential builds up until this is established. A distinction must be made between the contact potentials in air and the so-called "intrinsic" contact potentials in a vacuum with all adsorbed gases removed. According to Millikan, the intrinsic potential difference between two metals A and B is expressed by $V_{AB} = h(\nu_A - \nu_B)/e$, in which h is the **Planck constant**, ν_A and ν_B are the critical frequencies of photoelectric emission for the two metals, and e is the electronic charge. In any case, if the electronic work functions of the metals are p_A and p_B , the contact potential difference is $V_{AB} = (p_A - p_B)/e$. The work functions, and hence V_{AB} , are in general dependent upon the medium surrounding the metals. Accurate measurements of these potentials are, unfortunately, very difficult.

CONTACT RECTIFICATION CURRENT. For a metal-semiconductor contact, the relation between the current density, j , and the voltage, V , applied across the contact, is of the form

$$j = j_o(E)[\exp(eV/kT) - 1]$$

where e is the electronic charge, k is Boltzmann's constant, and T is the absolute temperature. E is the electric field strength at the contact due to surface charge, and $j_o(E)$ is given by the approximate expression

$$j_o(E) \sim \sigma_o |E| \exp\left(\frac{-e|V_b|}{kT}\right)$$

where σ_o is the conductivity of the semiconductor in its deep interior, and V_b is the difference in electric potential between the inner boundary of the space-charge region and the peak of the space-charge barrier.

CONTACT TRANSFORMATION. Let $(q_1, q_2, \dots, q_n, p_1, p_2, \dots, p_n)$ be a set of $2n$ variables

and let $(Q_1, Q_2, \dots, Q_n, P_1, P_2, \dots, P_n)$ be $2n$ other variables which are defined in terms of them by $2n$ equations. If the equations connecting the 2 sets of variables are such that the differential form $P_1dQ_1 + P_2dQ_2 + \dots + P_ndQ_n - p_1dq_1 - p_2dq_2 - \dots - p_ndq_n$ is, when expressed in terms of $(q_1, q_2, \dots, q_n, p_1, p_2, \dots, p_n)$ and their differentials, the perfect differential of a function of $(q_1, q_2, \dots, q_n, p_1, p_2, \dots, p_n)$ then the change from the set of the variables $(q_1, q_2, \dots, q_n, p_1, p_2, \dots, p_n)$ to the other set $(Q_1, Q_2, \dots, Q_n, P_1, P_2, \dots, P_n)$ is called a contact transformation. The name derives from its property in 3-dimensional space: If two surfaces σ and σ' touch at a point, and if Σ and Σ' are the new surfaces resulting from them under a contact transformation, then Σ and Σ' touch again at the image point of the original point. The **canonical transformation** is a special case of a contact transformation. The **canonical equations of motion** preserve their form under such a transformation.

CONTINGENCY. If the members of an aggregate can bear the qualities A_1, A_2, \dots, A_p of one characteristic and B_1, B_2, \dots, B_q of a second, they can be arranged in a $p \times q$ table, the number in the j^{th} row and k^{th} column, say f_{jk} , being the frequency of individuals bearing A_j and B_k . This is called a *contingency table*. If the frequency in the marginal total of the i^{th} row is f_i and that in the k^{th} column is f_j ; the *square contingency* is defined as

$$\phi^2 = \sum_{i,j} \frac{(f_{ij} - f_i \cdot f_j / n)^2}{f_i \cdot f_j / n}$$

The mean-square contingency, ϕ^2/n is equal to χ^2 (**chi-square**) for the table and is used as a test of independence of A and B .

Pearson's coefficient of contingency is defined as

$$C = \left(\frac{\phi^2}{1 + \phi^2}\right)^{1/2}$$

CONTINGENCY TABLE. See **contingency**.

CONTINUANT. A determinant of the form

$$K \begin{pmatrix} a_1 & \dots & a_n \\ b_0 b_1 & \dots & b_n \end{pmatrix} = \begin{vmatrix} b_0 & a_1 & 0 & \dots & 0 \\ -1 & b_1 & a_2 & \dots & 0 \\ 0 & -1 & b_2 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & b_n \end{vmatrix},$$

so called because the n th numerator and n th denominator of the **continued fraction**

$$F = b_0 + a_1/\sqrt{b_1} + \dots$$

are

$$A_n = K \begin{pmatrix} a_0 & \dots & a_n \\ b_0 b_1 & \dots & b_n \end{pmatrix}$$

$$B_n = K \begin{pmatrix} a_1 & \dots & a_n \\ b_1 b_2 & \dots & b_n \end{pmatrix}.$$

CONTINUATION. See **analytic continuation**.

CONTINUED FRACTION. See **fraction, continued**.

CONTINUITY EQUATION. A continuity equation can be applied to the flow of any conserved, indestructible quantity. The continuity equation may be written in general form as

$$\nabla \cdot (\rho \mathbf{v}) = -\dot{\rho} \quad \text{or} \quad \nabla \cdot \mathbf{J} = -\dot{\rho},$$

where \mathbf{v} is a flow velocity, ρ is a density, and $\mathbf{J} = \rho \mathbf{v}$ is a current density.

The continuity equation may be alternatively written in integral form as

$$\int \mathbf{J} \cdot d\mathbf{a} = -\frac{d}{dt} \int \rho dV$$

which may be interpreted as the statement that the net outward current of a quantity from a closed surface is equal to the rate of decrease of that quantity in the volume enclosed by the surface.

(1) In fluid flow, mass is conserved and the equation of continuity holds, where ρ is mass per unit volume and \mathbf{v} is the velocity of mass flow. In the case of an incompressible fluid, ρ is constant and the continuity equation becomes $\nabla \cdot \mathbf{v} = 0$.

(2) In electric currents, charge is conserved and the equation of continuity holds, where ρ is the charge per unit volume, \mathbf{v} is the velocity of charge flow, and $\mathbf{J} = \rho \mathbf{v}$ is the electric current density. If the current is in a steady state, $\nabla \cdot \mathbf{J} = 0$.

(3) In the flow of electromagnetic energy in charge-free space, this energy is conserved and the equation of continuity becomes:

$$\nabla \cdot \mathbf{P} = -\frac{\partial}{\partial t} \left(\frac{\mathbf{E} \cdot \mathbf{D}}{2} + \frac{\mathbf{B} \cdot \mathbf{H}}{2} \right).$$

Here $\mathbf{P} = \mathbf{E} \times \mathbf{H}$ is **Poynting's vector**, the intensity of the radiation or power flow per unit

area, and $\frac{1}{2}(\mathbf{E} \cdot \mathbf{D} + \mathbf{B} \cdot \mathbf{H})$ is the electromagnetic energy density.

If the space is not charge free, energy can be transferred to the charges and be lost from the field. In this case a term $\mathbf{E} \cdot \mathbf{J}$ must be added to the left hand side of the equation, representing the rate of dissipation of energy per unit volume.

(4) In quantum mechanics, probability is conserved, and the continuity equation can be written: $\nabla \cdot \mathbf{S} = -\dot{P}$, where P is the probability density, $\psi\psi^*$, and \mathbf{S} is the probability density current given by $\mathbf{S} = (\hbar/2mi)(\psi^*\nabla\psi - \psi\nabla\psi^*)$.

CONTINUITY OF A BEAM. When applied to a beam, the term continuity means continuity of slope and deflection at junction and support points. Transmission of moment is implied as well.

CONTINUITY OF PATH, PRINCIPLE OF. See **ergodic theorem**.

CONTINUITY OF STATE. Transition between two states, as between the gaseous and liquid states, in either direction, without discontinuity, or abrupt change in physical properties. The existence of a **critical point** makes it possible to pass from the liquid state to the vapor or vice versa, without at any stage observing the appearance of a new phase. This is the principle of the continuity of state first pointed out by James Thomson.

CONTINUOUS BEAM. See **beam, continuous**.

CONTINUOUS CREATION HYPOTHESIS. Hypothesis that the universe is being created continuously by the formation, without apparent mechanism, of one nucleon per 10^9 year-liter, a rate too small to be directly detected, but sufficient to keep the average density constant despite the expansion of the universe. Galaxies which have receded to a distance of more than 2×10^9 years are supposed to disappear from view, and the whole universe is in dynamic equilibrium, recently created matter forming into galaxies to replace those which are lost. Thus the universe is in a steady state, although the matter which composes it is being continually changed. This creation of matter throughout all space, without local energy conservation, is so great a mystery that the theory has not been generally accepted.

CONTINUOUS FUNCTION. The function $f(x)$ is continuous at the point p if, for every $\epsilon > 0$, there exists a δ such that if $|x - p| < \delta$, then $|f(x) - f(p)| < \epsilon$. (Compare **absolute continuity**.)

CONTINUOUS SPECTRUM. See **spectrum, continuous**.

CONTINUOUS SYSTEMS. Systems in which the intensive state variables are not only functions of time but also continuous functions of the space coordinates. A piece of metal heated at one end and cooled at the other is an example of a continuous system. (See also **discontinuous systems**.)

CONTINUOUS SYSTEMS, LAGRANGIAN AND HAMILTONIAN FORMALISM FOR.

In a continuous system one introduces a Lagrangian density $L = L(\psi, \text{grad } \psi, \frac{\partial \psi}{\partial t} \cdot t)$ where the ψ 's are the field variables, e.g., the **metric tensor** in general relativity or the **scalar and vector potentials** in electrodynamics. The corresponding **Lagrangian equations** (of the second type) are,

$$\frac{\partial L}{\partial \psi} - \sum_{xyz} \frac{\partial}{\partial x} \left[\frac{\partial L}{\partial \left(\frac{\partial \psi}{\partial x} \right)} \right] - \frac{\partial}{\partial t} \left[\frac{\partial L}{\partial \left(\frac{\partial \psi}{\partial t} \right)} \right] = 0.$$

The *Hamiltonian density* is defined as

$$H = \frac{\partial L}{\partial \left(\frac{\partial \psi}{\partial t} \right)} - L.$$

The quantities $\frac{\partial \psi}{\partial t}$ are the *generalized velocity-densities* and $\pi \equiv \frac{\partial L}{\partial \left(\frac{\partial \psi}{\partial t} \right)}$ are the *generalized*

momentum-densities. Thus one can also write

$$H = \pi \frac{\partial \psi}{\partial t} - L.$$

The **canonical equations of motion** are,

$$\frac{\partial H}{\partial \psi} - \sum_{xyz} \frac{\partial}{\partial x} \frac{\partial H}{\partial \left(\frac{\partial \psi}{\partial x} \right)} = - \frac{\partial \pi}{\partial t}$$

and

$$\frac{\partial H}{\partial \pi} - \sum_{xyz} \frac{\partial}{\partial x} \frac{\partial H}{\partial \left(\frac{\partial \pi}{\partial x} \right)} = + \frac{\partial \psi}{\partial t}.$$

CONTINUUM. A **compact connected** set containing at least two points.

CONTOUR INTEGRATION. This term usually refers to **integration** over a (closed) simple rectifiable curve in the complex plane. An integral $\int_C f(z) dz$ of a continuous function $f(z)$ around such a curve C is most easily defined in case C is given in parametric form $z = z(t)$ with real t , $a \leq t \leq b$, when $\int_C f(z) dz$ is defined by

$$\int_C f(z) dz = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(\eta_j) \left[z(t_i) - z(t_{i-1}) \right]$$

where the maximum length of the subintervals $(t_i - t_{i-1})$, with $t_0 = a$ and $t_n = b$, tends to zero.

CONTRACTED RIEMANN-CHRISTOFFEL TENSOR. The tensor $G_{\mu\nu} = B_{\mu\nu\epsilon}{}^\epsilon$ where $B_{\mu\nu\sigma}{}^\epsilon$ is the **Riemann-Christoffel tensor**. Summation on the repeated index ϵ is implied.

CONTRACTION (AS A RESULT OF COMBUSTION). See **combustion**.

CONTRACTION (WHEN INDICIAL NOTATION IS USED). The operation of setting two indices on a symbol or product of symbols equal to each other and summing over the values which may be taken by the indices. In tensor analysis one of the symbols must be a superscript and the other a subscript. In Cartesian tensor analysis they may both be subscripts.

CONTRACTION, COEFFICIENT OF. The ratio of the sectional area of the parallel jet of liquid issuing from an orifice to the area of the orifice. Its value depends on the nature of the orifice and varies between 0.5 and 1.

CONTRAFLOW. See **heat exchanger**.

CONTRAGRADIANT. A word used to describe the relationship of two tensors which have different transformation properties. (See **tensor field**.)

CONTRAST. This term is used in physical science and statistics with at least four meanings. (1) In photography, the slope of the curve plotted between **density** and the log of **exposure** (see **H and D curve**). (2) In television, the ratio between the maximum and minimum brightness values in a picture. (3) In psychophysics, the change in the response to a stimulus as a result of the proximity in space or time of other stimuli. In general, the response to a stimulus of given physical intensity is reduced if neighboring stimuli have greater intensities, and vice versa. A measure of contrast is the contrast sensitivity which may be expressed, for example, for brightness by the expression $\Delta B/B$, where B is the brightness of a background, and ΔB is the difference in brightness of a small spot that is just discernible against that background. (4) In statistics, a contrast between a set of independent observations x_i of equal accuracy is a linear function $\Lambda = \sum \lambda_i x_i$ in which the sum of the coefficients $\sum \lambda_i = 0$. If σ^2 is the **variance** of a single observation and Λ, Λ' are two contrasts with coefficients λ_i, λ'_i , then the variance of Λ is $\sum \lambda_i^2 \sigma^2$ and the **covariance** of Λ and Λ' is $\sum \lambda_i \lambda'_i \sigma^2$. If $\sum \lambda_i \lambda'_i = 0$, the contrasts are said to be **orthogonal**.

CONTRAVARIANCE, ORDER OF. See **tensor field**.

CONTRAVARIANT METRIC TENSOR. See **metric tensor**.

CONTRAVARIANT STRESS TENSOR. See **stress tensor**.

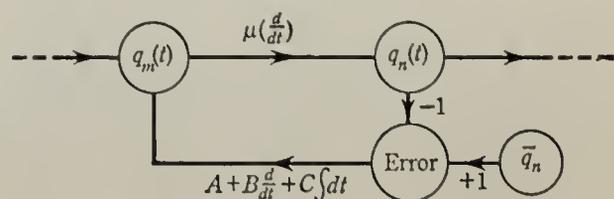
CONTRAVARIANT TENSOR (TENSOR FIELD). See **tensor field**.

CONTRAVARIANT VECTOR (VECTOR FIELD). Contravariant tensor (tensor field) of order one. (See **tensor field**.)

CONTROL. (1) A system or device which exerts a restraining, governing or direction influence. (2) An experiment or test done to confirm or to rule out error in experimental observations. (3) Usually, those parts of a digital computer which effect the carrying out of instructions in proper sequence, the interpretation of each instruction, and the application of the proper signals to the arithmetic unit and other parts in accordance with this interpretation. (4) Frequently, one or more of the components in any mechanism respon-

sible for interpreting and carrying out manually-initiated directions. Sometimes called **manual control**.

CONTROL, PROPORTIONAL, DERIVATIVE AND INTEGRAL. Terms used mainly in connection with process control engineering. Most process control systems are regulating systems, that is, their aim is to keep the controlled quantity constant in the face of internal or external disturbances. In such systems the "input" is therefore constant and corresponds to some pre-set value of the output or controlled quantity; deviation of the output from this set point constitutes the **error**, which must be measured and some function of which must be fed back to a suitable quantity occurring earlier in the process which thus modifies the output in such a way as to reduce the error. The function of error fed back normally consists of not more than three terms, the first (which must be present), being proportional to the error, the second proportional to its time derivative, the third proportional to its time integral. A proportional control system has only the first term present, a proportional plus derivative control system has only the first two terms present and so on.



From a signal flow diagram point of view, such systems may be typified by the general diagram shown, in which the through line of dependence represents the process, $q_n(t)$ is the controlled quantity, \bar{q}_n its constant desired value, $q_m(t)$ is the quantity receiving the feedback from error and A, B, C , determine the magnitudes of the proportional, derivative and integral feed-back terms respectively.

In **Laplace transform** notation the output is therefore contributed to by the error via the relation

\mathcal{L} contribution to output

$$\mathcal{L} \text{ error} = \mu(s)(A + Bs + C/s).$$

The disturbance, which brings the control into operation by setting up an error, may occur prior to q_m or between q_m and q_n .

CONTROL ROD, EFFECTIVE RADIUS OF. See radius, effective, of control rod.

CONTROL SYSTEM EQUATIONS. See system equations.

CONTROL SYSTEM SIGNAL FLOW DIAGRAMS. See signal flow diagrams.

CONTROL SYSTEM, STABILITY OF. See stability (of system).

CONTROL SYSTEM, TYPE NUMBER. The number of integrations contained in the relation connecting output to error, i.e., the (non-negative) power of $\frac{1}{s}$ contained in the output to error transfer function. Also called by some authors the "order" of the system.

The type number is related to the ultimate error (as $t \rightarrow \infty$) of the system when the input has the form of an impulse function or any of its integrals, as follows:

Let the output to error transfer function be $G(s)$ and let this transfer function be expressed in the form

$$G(s) = \frac{KG_o(s)}{s^n}$$

where n is the type number of the system and the gain factor K has such a value that $G_o(0) = 1$. Then, denoting error and input by $\theta_e(t)$, $\theta_i(t)$ respectively

$$\frac{\Theta_e(s)}{\Theta_i(s)} = \frac{1}{1 + G(s)} = \frac{s^n}{s^n + KG_o(s)}$$

If, further, $\Theta_i(s) = \frac{1}{s^m}$ ($m \leq 0$), so that $\theta_i(t)$ is the m th integral of unit impulse function at $t = 0$,

$$\Theta_e(s) = \frac{s^{n-m}}{s^n + KG_o(s)}$$

Hence by the Laplace transform limit theorem, assuming the system is stable,

$$\lim_{t \rightarrow \infty} \theta_e(t) = \lim_{s \rightarrow 0} s\Theta_e(s) = \lim_{s \rightarrow 0} \frac{s^{n-m+1}}{s^n + KG_o(s)} = \left. \begin{aligned} &= \lim_{s \rightarrow 0} \frac{s^{1-m}}{1 + K} \text{ if } n = 0 \\ &= \lim_{s \rightarrow 0} \frac{s^{n+1-m}}{K} \text{ if } n > 0 \end{aligned} \right\}.$$

These results are best stated in the form of a table of the ultimate error:

Input		$n=0$	$n=1$	$n=2$	$n=3$
$m=0$	$\theta_i(t) = \delta(t)$	0	0	0	0
$m=1$	$\theta_i(t) = u(t)$	$\frac{1}{1+K}$	0	0	0
$m=2$	$\theta_i(t) = tu(t)$	∞	$\frac{1}{K}$	0	0
$m=3$	$\theta_i(t) = \frac{t^2}{2} u(t)$	∞	∞	$\frac{1}{K}$	0

In practical systems the number of integrations is never greater than 2.

CONTROL VOLUME. See open system.

CONVECTED COORDINATE SYSTEM. A coordinate system embedded in a body undergoing deformation, so that as the body deforms the coordinates of a particle in the coordinate system remain unchanged.

CONVECTION. Transfer by the motion of a fluid, but often restricted to mean transfer by eddy motions only; sometimes used to refer to the eddy motion itself. Thermal, buoyant, or free convection refers to the transfer of heat (or buoyancy) by the motions which buoyancy forces produce. Forced convection is eddying motion produced mechanically as in a turbulent boundary layer or in a wind over a rough surface. (See also heat transfer.)

CONVECTION CELLS. See Bénard cells.

CONVECTION CURRENT, ELECTRIC. See discussion under current, electric.

CONVECTION, FORCED. See heat transfer.

CONVECTION, NATURAL. See heat transfer.

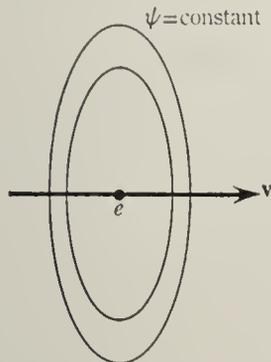
CONVECTIVE POTENTIAL. The convective potential, first introduced by Oliver Heaviside, is the scalar potential rigidly moving with an electric charge of uniform velocity v . For a point-charge e this potential is

$$\psi = \frac{e(1 - \beta^2)}{\sqrt{x^2 + (1 - \beta^2)(y^2 + z^2)}}$$

where

$$\beta = \frac{v}{c}$$

and the center of the coordinate system is at the location of the point charge. The surfaces of constant potential are ellipsoids of revolu-



tion (see figure). The axis of revolution coincides with the direction of the velocity v . (See also **field of moving charge in space**.)

CONVECTIVE (OR ADVECTIVE) RATE OF CHANGE. The rate of change produced by motion in a non-uniform field; equal to

$$\begin{aligned} \mathbf{v} \cdot \text{grad} &= u_i \frac{\partial}{\partial x_i} \\ &= u \frac{\partial}{\partial x} + v \frac{\partial}{\partial y} + w \frac{\partial}{\partial z} \end{aligned}$$

according to the notation used.

CONVENTIONAL YIELD LIMIT. See **limit, yield**.

CONVERGENCE. A sequence of real or complex numbers $a_1, a_2, \dots, a_n, \dots$ is convergent, or converges, to a number l , called the **limit** of the sequence, if, corresponding to each number ϵ , there exists an integer N , such that $|l - a_n| < \epsilon$ for all $n > N$, the vertical strokes denoting **absolute value**.

A **series** $a_1 + a_2 + \dots + a_n + \dots$ is convergent if the sequence of its partial sums s_1, s_2, \dots, s_n converges where $s_n = a_1 + a_2 + \dots + a_n$. A sequence or series which is not convergent is called *divergent*.

A series is *absolutely convergent* if the series of corresponding absolute values $|a_1| + |a_2| + \dots + |a_n| + \dots$ is convergent. A series such as $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$, which is convergent but not absolutely convergent, is said to be *conditionally convergent*. Of the many tests for convergence of a series the following

are most often used: **comparison test**, **ratio test** (D'Alembert's), **Cauchy integral test**. Absolutely convergent series are important because they can be rearranged, multiplied together, and so forth, in very much the same way as finite sums. For every power series $a_0 + a_1z + a_2z^2 + \dots$ in a complex variable z , there exists a non-negative real number ρ (which may be zero or infinity) such that for $|z| < \rho$ the series converges absolutely and for $|z| > \rho$ the series diverges. The circle with center at the origin and radius equal to ρ is called the *circle of convergence* of the series.

An improper integral of the form $\int_c^\infty f(x)dx$, with $f(x)$ continuous for $x \geq c$, is convergent if, for every sequence $c < d_1 < d_2 < \dots$ of real numbers which is not bounded, the sequence $\int_c^{d_n} f(x)dx$ is convergent. Similarly, $\int_c^\infty f(x)dx$ is absolutely convergent if $\int_0^\infty |f(x)|dx$ is convergent, with corresponding definitions for improper integrals of other forms.

A sequence of functions $u_1(x), u_2(x), \dots$ defined over an interval $a \leq x \leq b$ is convergent over the interval if, for every p with $a \leq p \leq b$ the sequence of numbers $u_1(p), u_2(p), \dots$ is convergent. The sequence of functions is *uniformly convergent* to a function $f(x)$ if corresponding to each positive ϵ , there exists an integer N , depending on ϵ but not on x , such that

$$|l(x) - f_n(x)| < \epsilon \quad \text{for all } x,$$

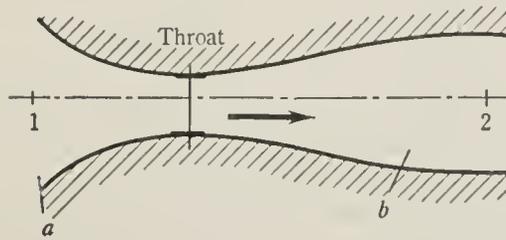
provided only that $n > N$. Uniformly convergent series are important because they allow **termwise integration**, a property which is particularly valuable in the applications of **Fourier series**. The most important test for uniform convergence is the **Weierstrass M-test**.

CONVERGENCE, ANGLE OF. See **angle of convergence**.

CONVERGENCE, CONDITIONAL. See **conditional convergence**.

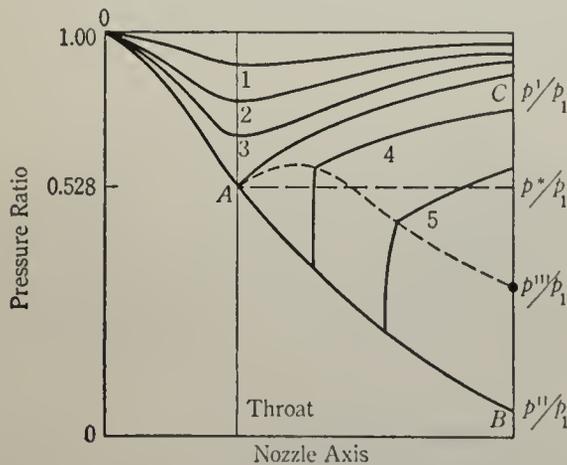
CONVERGENT-DIVERGENT NOZZLE. A channel shaped so that a convergent portion a is followed by a divergent portion b , Figure 1. The narrowest part of the convergent-divergent nozzle is called a *throat*. Convergent-divergent nozzles are used in turbines to accelerate the working fluid (steam or gas) to

high velocities. They are also used in wind tunnels to produce a parallel stream of air at very high velocities.



Convergent-divergent nozzle.

In view of these applications it is important to understand the behavior of a compressible fluid as it flows through a convergent-divergent nozzle. Assuming ideal flow (isentropic process of expansion), it is found that essentially two regimes of flow are possible. (1) Flow when the velocity at the throat is subsonic (throat Mach number < 1). The flow through the entire nozzle is subsonic, and the mass flow depends on the pressure ratio p_2/p_1 , where p_1 is the stagnation pressure in section 1, and p_2 is then the static pressure at 2. Hence, by measuring this pressure ratio one can determine the mean flow. Actually, the pressure at the throat is then measured (by Venturi meter). The variation of pressure along the nozzle axis for different back-pressure ratios p_2/p_1 , is shown in Figure 2. As



the back pressure decreases from unity to p'/p_1 the preceding flow regime prevails, curves 1, 2, 3.

When the back-pressure is reduced to p' , the velocity in the throat becomes sonic (throat Mach number = 1) and the nozzle becomes choked. Any further reduction in back-pressure has no effect on the pressure (or velocity or temperature) distribution in the convergent part of the nozzle. The pressure at the throat

then becomes equal to the so-called critical pressure.

$$p^* = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \times p_1$$

$$(\text{= } 0.528p_1 \text{ for } \gamma = 1.4, \text{ i.e., for air})$$

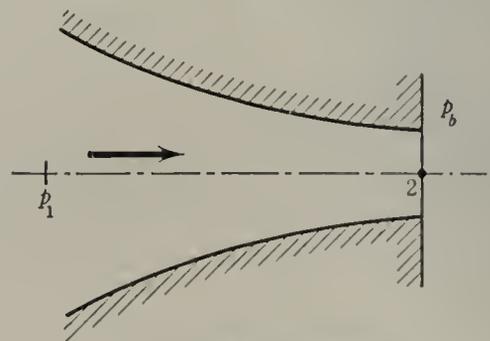
Isentropic flow all along the nozzle is then only possible for two back-pressures, p' and p'' whose values depend on the area ratio A_2/A_t (curves OAC and OAB). For p' the flow at exit is subsonic, for p'' it is supersonic, the Mach number depending also on the area ratio A_2/A_t . Any reduction of back-pressure below p'' has no effect on the flow in the nozzle at all.

The flow patterns for back-pressures between p' and p'' are complex. In a range p' to p''' (where p''' also depends on the area ratio A_2/A_t for a given gas) a normal shock appears in the divergent portion (curves 4,5) moving from the throat towards the exit and gathering strength as p_2 varies from p' down to p''' . For back-pressures between p''' and p'' , ideally, the flow pattern is the same as for $p_2 = p''$, but boundary layer effects and the possibility of separation complicate the process and give rise to more complex patterns.

Supersonic flow in a compressible fluid can be achieved in a steady-state only if a convergent-divergent nozzle is used for the purpose. If the nozzle is rigid (fixed-geometry nozzle) only one particular supersonic Mach number can be reached at exit. In order to regulate the exit Mach number it is possible to use a flexible nozzle, which varies A_t , and so varies the area ratio A_2/A_t for a given A_2 .

A convergent-divergent nozzle used to produce a supersonic stream is also called a deLaval nozzle.

CONVERGENT NOZZLE. A channel in which the cross-sectional area decreases along the flow axis. Convergent nozzles are used in



Convergent nozzle.

turbines to produce high-velocity streams of steam or gases. They are also used in many other engineering devices.

When a compressible fluid flows through a convergent nozzle, the behavior of the nozzle depends essentially on the ratio of back-pressure p_b to the upstream stagnation pressure p_1 . The (static) pressure in the exit section p_2 need not be equal to the back-pressure p_b . There are two distinct regimes of flow: (1) When the exit velocity is subsonic ($M_2 < 1$), we have $p_b = p_2$ and $p_2 > p^*$, where p^* is the so-called *critical pressure*.

$$p^* = \left(\frac{2}{\gamma + 1} \right)^{\frac{\gamma}{\gamma - 1}} \times p_1$$

(= 0.528 p_1 for $\gamma = 1.4$, i.e., for air)

(2) When the back-pressure p_b becomes equal to or smaller than the critical pressure p^* the nozzle becomes choked, and $p_b \neq p_2$. The exit velocity is then sonic ($M_2 = 1$). In fact, the exit pressure $p_2 = p^*$ and changes in the back-pressure (provided only that $p_b < p^*$) have no effect on the operation of the nozzle.

Thus in the range $p^* \leq p_b < p_1$ the mass flow varies, increasing from 0 to a maximum \dot{m}^* at $p_b = p^*$, increasing with decreasing p_b . In the range $p_b < p^*$ the mass flow remains constant at \dot{m}^* and unaffected by p_b .

In a convergent nozzle it is not possible to accelerate the stream to a supersonic velocity ($M > 1$).

CONVERSION. In general, a change, often with the force of a directed or induced change. One specific use is a change in numerical value of a quantity resulting from the use of a different unit in the same or a different system of measurement.

CONVERSION FRACTION. The ratio of the number of internal conversion electrons to the number of quanta emitted plus the number of conversion electrons emitted in a given mode of de-excitation of a nucleus. Partial conversion fractions refer to conversion fractions for various electron shells, e.g., *K*-conversion fractions, *L*-conversion fractions, etc. Sometimes called *conversion coefficient*.

CONVERSION GAIN. In nuclear reactor theory, synonymous with **breeding gain**, but often used when the fissionable nucleus produced is different from the nucleus consumed.

CONVERSION RATIO. (1) The ratio of the number of internal conversion electrons to the number of gamma quanta emitted in the de-excitation of a nucleus of an atom. (See **conversion fraction**; **internal conversion coefficient**.) (2) In nuclear reactor theory, synonymous with **breeding ratio**, but often used when the fissionable nucleus produced is different from the nucleus consumed.

CONVOLUTION. Given two functions $f(t)$, $g(t)$ of some independent variable t , the convolution integral of $f(t)$ and $g(t)$ is (provided the integral exists)

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)g(t - \tau)d\tau$$

or, changing the variable of integration from τ to $(t - \tau)$

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(t - \tau)g(\tau)d\tau.$$

The convolution integral is therefore a function of t and is symmetrical with respect to the functions $f(t)$ and $g(t)$.

The range of integration is sometimes restricted by limitations on $f(t)$ and $g(t)$. For instance if $f(t)$, $g(t) = 0$ when $t < 0$, then clearly the only valid range of integration is $\tau = 0$ to $\tau = t$.

Convolutions are of particular importance in the theory of the Laplace transform, since the transform of the convolution of two functions is given by the product of their transforms.

In probability theory, a term denoting the "sum" of a number of random variables. If ξ_1 and ξ_2 are two independent random variables with distribution functions $F_1(x_1)$ and $F_2(x_2)$, their convolution is the random variable ζ with distribution function $F(z)$ where

$$F(z) = \iint_{x_1 + x_2 < z} dF_1(x_1)dF_2(x_2).$$

The definition is easily extended to n variables which may be dependent.

The expression "compound" distribution in the sense of convolution is to be avoided. It has been used in two senses: (1) where a statistical distribution is mixed in the sense that its members emanate from two or more different populations: (2) when a random variable ξ has a distribution depending on a parameter α , cases arise where α itself may be regarded as having a probability distribution: the distribu-

tion of ξ obtained by integrating over the course of α may then be described as compound.

CONVOLUTION INTEGRAL. See first paragraph under convolution.

CONVOLUTION THEOREM. See Laplace transform theorems.

CONWELL-WEISSKOPF FORMULA. The mobility μ of electrons in a semiconductor in the presence of donor or acceptor impurities is given by

$$\mu = [2^{7/2} \epsilon^2 (kT)^{3/2} \log(1 + x^2)] / N e^{\pi^2/4} m^{3/2}$$

where

$$x = 6\epsilon dkT/c^2$$

and ϵ is the dielectric constant of the medium, N is the concentration of ionized donors (or acceptors), $2d$ is the average distance between ionized donors, e and m the electronic charge and mass, etc.

COOLANT. The fluid used to carry heat away from a system or device, such as an internal combustion engine, etc.

COOLING. The process of reducing the temperature of a body or of extracting heat by circulating a fluid (coolant).

COOLING BY ADIABATIC DEMAGNETIZATION. See adiabatic demagnetization.

COOPERATIVE PHENOMENA. Such processes as order-disorder transformations in binary alloys, ferromagnetism, and melting, where certain subsystems will combine to form units which hold together in spite of the disrupting influence of thermal agitation.

It is to a large extent possible to treat all these phenomena at the same time. The simplest model for such systems, and the one most investigated, is the **Ising model**.

If one considers a simple cubic **AB substitutional alloy**, depending on whether the energy of an **AB** pair is less or larger than the average of an **AA** and a **BB** pair, there will be a tendency for order, that is, for the formation of a superstructure of two sublattices, one a pure **A** and one a pure **B** lattice, where each **A** atom is surrounded by **B** atoms and vice versa, or a tendency for phase separation. At the absolute zero the order will be complete, but when the temperature is increased, there will

be a chance that an **A** atom will move to the place of a **B** atom (β -site) and thus be replaced by a **B** atom on an α -site. The presence of an **A** atom on a β -site will induce a tendency for the neighboring α -sites to be occupied by **B** atoms, and a disordering tendency will set in which will lead to an avalanche effect, and at and above the **Curie temperature** there will no longer be a tendency of **A** atoms to be on α -sites, or **B** atoms to be on β -sites, but there will still be a tendency for **A** atoms to be surrounded by **B** atoms and vice versa. The short range order parameter measures this latter tendency while the long range order parameter measures the tendency of **A** atoms to be on α -sites.

In any statistical treatment of cooperative phenomena one derives an (approximate) expression of the free energy in terms of the two order parameters, and by evaluating its extremum, the equilibrium values of the order parameters are found. The statistical approximations which are used most are the Bragg-Williams, and the **quasi-chemical approximation**.

COORDINATE. One of a set of numbers which determine the position in space of a geometric entity, usually a point but sometimes a line, surface, etc., in such a way that a continuous change in the coordinates corresponds to a continuous change of position, and conversely. The commonest system of coordinates is the Cartesian (or rectangular Cartesian) system in the plane or in 3-dimensional space. In a plane, the homogeneous coordinates of a point, whose Cartesian coordinates are x and y , are any three (not all zero) numbers (x_1, x_2, x_3) for which $x_1/x_3 = x$ and $x_2/x_3 = y$. The points for which $x_3 = 0$ form the **line at infinity**. Homogeneous coordinates are defined analogously for spaces of 3 or more dimensions. If $u = u(x, y)$ and $v = v(x, y)$, then p, q are the **curvilinear coordinates** (the u, v coordinates) of a point P if P is the intersection of the curve $u(x, y) = p$ with the curve $v(x, y) = q$. Commonly used coordinates of this sort are **polar, cylindrical, spherical, elliptic and parabolic coordinates**.

COORDINATE BOND. Some molecules, such as H_2O , NH_3 , HF , $\text{HCl} \cdots$ have in their outer shell one or more free electron doublets which they can eventually share with an atom

or molecule which has an incomplete outer shell. The resulting bond, where the doublet is shared between the two particles, is called a coordinate bond. Its properties are quite similar to those of covalent bonds: such bonds have directed valencies, and limited coordination numbers. The reactions $\text{NH}_3 + \text{H}^+ \rightarrow \text{NH}_4^+$ and $\text{HCl} + \text{O} \rightarrow \text{HClO}$ are typical of the formation of such bonds. Coordinate links are also important in the formation of many ion-solvent complexes in aqueous and ammoniacal solutions. (See **directed valency; bond types.**)

COORDINATE LINES ON A SURFACE. See **surface.**

COORDINATES AND MOMENTA, GENERALIZED. Generalized coordinates are coordinates chosen to describe the motion of a mechanical system, without having their exact nature immediately specified. The generalized coordinates associated with a system having n degrees of freedom are denoted by q_1, q_2, \dots, q_n .

The generalized momenta p_1, p_2, \dots, p_n are defined by the equation:

$$p_i = \frac{\partial L}{\partial \dot{q}_i}$$

where L is the **Lagrangian function** for the system. The generalized momentum p_i is said to be conjugate to the generalized coordinate q_i . The **Lagrange equations** and the **Hamilton canonical equations** are expressible in terms of generalized coordinates and momenta.

COORDINATES IN POLYATOMIC MOLECULES, NORMAL. See **normal vibrations** and **normal coordinates in polyatomic molecules.**

COORDINATES, NORMAL. See **normal modes.**

COORDINATE SYSTEM, ROTATING. See **rotating coordinate system.**

COORDINATION NUMBER. (1) For an atom or group of atoms, the maximum number of coordinate bonds which the atom, or group of atoms can form. (See **coordinate bond.**) (2) For a covalent crystal, the coordination number is the same as (1), but for an ionic crystal, it is the number of atoms at

closest distance (nearest neighbors) of the given atom.

COORDINATIVE VALENCY OF AN ION. See **principal valency of an ion.**

CORE. See also **kern.**

CORIOLIS EFFECTS. The Coriolis force, which is alternatively called the **geostrophic force**, or the deviating force, is the apparent force on particles moving relative to the earth when observed from the earth. For unit mass it is equal to $2\mathbf{v} \times \boldsymbol{\Omega}$ where \mathbf{v} is the velocity relative to the earth and $\boldsymbol{\Omega}$ is the earth's rotation vector. The vertical component is negligible compared with gravity and the horizontal component is equal to $\mathbf{v} \times \mathbf{f}$ where \mathbf{f} is twice the vertical component of the earth's rotation.

A generalized discussion of Coriolis effects, not limited to the earth as a frame of reference, is given in polar coordinates as follows: A particle which is subject to no forces in a polar coordinate system r, θ experiences a radial acceleration $r(d\theta/dt)^2$ and a tangential acceleration $-2(dr/dt)(d\theta/dt)$. These accelerations result from the fact that every point fixed in a coordinate system rotating with an angular velocity $d\theta/dt$ (with the exception of the origin) is accelerated with respect to an inertial frame. The radial acceleration is known as the **centrifugal acceleration**; the tangential acceleration is called the **Coriolis acceleration**. If Newton's second law of motion is applied to the particle without correction for the motion of the coordinate system, a **centrifugal force** $mr(d\theta/dt)^2$ and a **Coriolis force** $-2m(dr/dt)(d\theta/dt)$, where m is the mass of the particle, must be postulated to account for the behavior of the particle.

CORIOLIS PARAMETER. Twice the vertical component of the earth's angular velocity about the local vertical Ω ; usually denoted by

$$f = 2\Omega \sin \phi$$

where ϕ is the latitude.

CORNU SPIRAL. James Bernoulli introduced the plane curve, defined intrinsically by the curvature being proportional to the arc length, and known as the Cornu spiral or *clothoid*. The Cornu spiral is represented parametrically by $x = C(v)$, $y = S(v)$ in terms of the **Fresnel integrals**.

$$C(v) = (2\pi)^{-1/2} \int_0^v t^{-1/2} \cos t dt$$

$$= \int_0^v \cos \frac{\pi}{2} u^2 du,$$

$$S(v) = (2\pi)^{-1/2} \int_0^v t^{-1/2} \sin t dt$$

$$= \int_0^v \sin \frac{\pi}{2} u^2 du.$$

Böhmer has considered the generalization

$$C(v, \alpha) = \int_v^\infty t^{\alpha-1} \cos t dt,$$

$$S(v, \alpha) = \int_v^\infty t^{\alpha-1} \sin t dt.$$

The curve $x = C(v, \alpha)$, $y = S(v, \alpha)$ for fixed α , $0 < \alpha < 1$ is a spiral and becomes the Cornu spiral when $\alpha = 1/2$. The intrinsic equation is

$$\rho = (\alpha s)^{1-1/2}$$

where ρ is the radius of curvature and s is the arc length.

Distances on the Cornu spiral are used in computing intensities in the diffraction pattern resulting from **Fresnel diffraction**. Here the parameter v is **wave velocity** and

$$v = l \sqrt{\frac{2B}{D\lambda(B + D)}}$$

where l is the distance along the diffracting screen, B the distance from the source to the screen, D the distance from the screen to the point of observation, and λ is the wavelength.

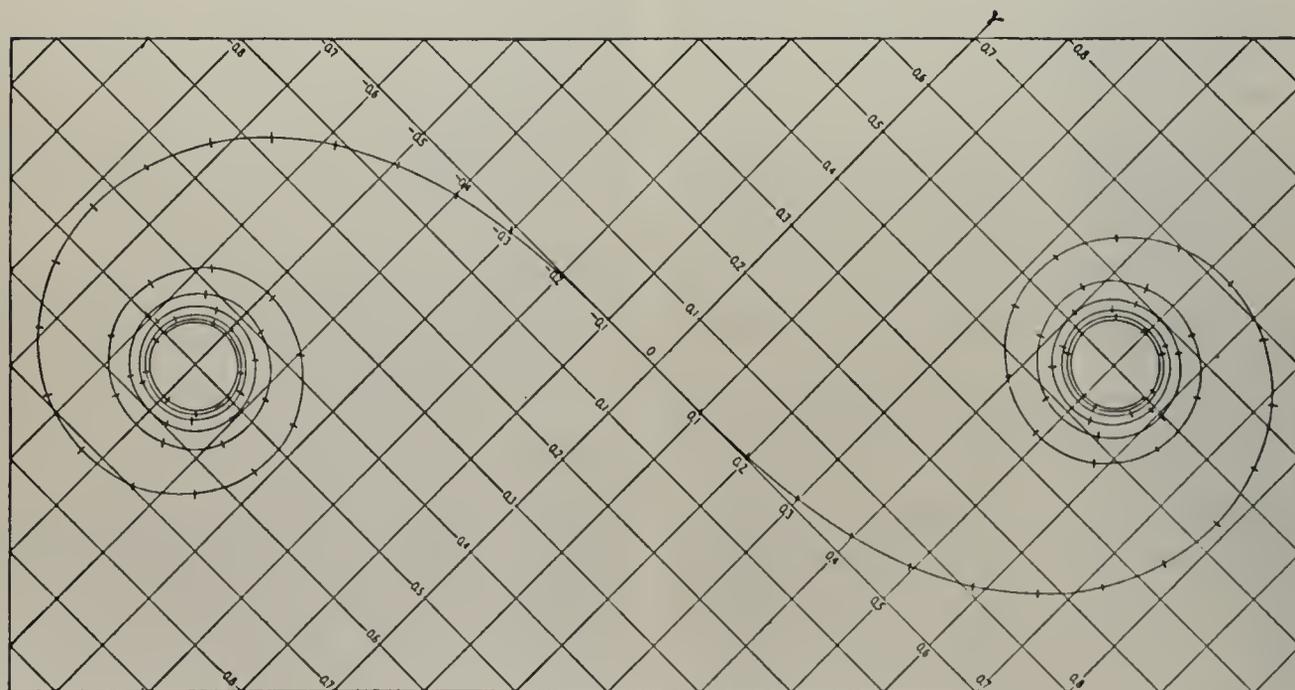
CORONA DISCHARGE. Leakage of charge brought about by the ionization of a gas surrounding a charged conductor. This occurs when the potential gradient exceeds a certain value, but is not sufficient to cause sparking.

CORPUSCULAR THEORY OF LIGHT. See dynamical equations of light rays.

CORRECTION. A quantity added to a previously obtained approximation to yield a better approximation. Sometimes the term is applied to $x - x^*$, where x is the true value and x^* the approximation, and is contrasted with the difference $x^* - x$ which is then designated the **error**. This sense is not the one adopted in this book.

CORRECTION TO VACUUM. The correction of wavelengths, or of the speed of light, as measured in air to the appropriate values *in vacuo*. The index of refraction of air is 1.00029 at 0° Centigrade, 1 atmosphere pressure, and at the wavelength of the mean of two yellow sodium lines, D (5892.9 Å).

CORRECTOR. In solving initial value problems in **ordinary differential equations**, a quadrature formula to be used for improving a given approximate value of the dependent



Cornu spiral.

variable at an advance point, and contrasted with a predictor used to obtain a first approximation. A predictor formula uses an **open quadrature formula**, a corrector formula a **closed quadrature formula**.

CORRELATION. In a general sense in statistics this expression denotes the co-relationship of two variables. A correlation coefficient attempts to measure the interdependence of the variables.

More specifically the correlation or product-moment correlation of two variates x and y is defined as

$$\rho = \frac{\text{cov}(x,y)}{\sqrt{(\text{var } x)(\text{var } y)}}$$

or, for a finite series of values $(x_1, y_1), \dots, (x_n, y_n)$ as

$$r = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\left\{ \sum_{i=1}^n (x_i - \bar{x})^2 \sum_{i=1}^n (y_i - \bar{y})^2 \right\}^{1/2}}$$

where \bar{x} and \bar{y} are the respective means.

Other definitions are possible. When the observations are ranked but not measured, coefficients of *rank correlation* of an analogous kind can be defined. For qualitative variables joint relationship is usually described in terms of *association* or *contingency*.

For variables which are distributed in the bivariate normal (Gaussian) form, the correlation defined above has a clear interpretation, ρ being the parameter or (in the case of a sample) an estimate of the parameter, in the equation of the frequency function,

$$f(x,y) = k \exp \left\{ -\frac{1}{2(1-\rho^2)} \left(\frac{x^2}{\sigma_1^2} - \frac{2\rho xy}{\sigma_1\sigma_2} + \frac{y^2}{\sigma_2^2} \right) \right\}.$$

In a multivariate complex, say for simplicity the three-variate case involving x_1, x_2, x_3 , each pair of variables has a correlation coefficient. It is also possible to consider the correlation of x_1 and x_2 for fixed x_3 . This is known as a *partial correlation*. If the correlation between x_j and x_k is ρ_{jk} , the partial correlation for fixed x_l , ρ_{jkl} is given by

$$\rho_{jkl} = \frac{\rho_{jk} - \rho_{jl}\rho_{kl}}{\left\{ (1 - \rho_{jl}^2)(1 - \rho_{kl}^2) \right\}^{1/2}}$$

In regression analysis, if a variable y is expressed as a linear regression on ρ variables x_1, x_2, \dots, x_ρ , say $\beta_1 x_1 + \beta_2 x_2 + \dots + \beta_\rho x_\rho$, the correlation between y and the regression $\sum \beta_i x_i$ is called a *multiple correlation*. It measures the closeness with which y is "predicted" by the regression equation. The multiple correlation is expressible in terms of the correlations between y and the individual x 's and of the x 's among themselves.

CORRELATION, BISERIAL. See **biserial correlation**.

CORRELATION, CANONICAL. See **canonical correlation**.

CORRELATION COEFFICIENT. (As used in the statistical theory of **turbulence**.) A measure of the **correlation** between two fluctuating quantities in the turbulent motion of a fluid. If u_P and $v_{P'}$ are the values of two fluctuating quantities u and v at points P and P' their correlation coefficient is defined as

$$\overline{u_P v_{P'}} / (\overline{u_P^2} \overline{v_{P'}^2})^{1/2}.$$

When $u = v$ this becomes the **autocorrelation coefficient**. The coefficient may also be defined to include a time interval which elapses between the measurement of u at P and of v at P' . For the turbulence to be represented by statistics such as correlation coefficients the mean values must be properly definable (see **autocorrelation coefficient**).

If u and v are the fluctuations of two components of velocity in isotropic turbulence $\overline{u_P^2} = \overline{v_{P'}^2}$, and if the turbulence is homogeneous $\overline{u_P^2} = \overline{v_{P'}^2}$, so that for homogeneous isotropic turbulence the correlation coefficient becomes

$$\overline{u_P v_{P'}} / \overline{u_P^2}.$$

The coefficient is used to measure transport by turbulence. Thus \overline{uv} is a measure of the transport of x -momentum in the y -direction (see **Reynolds stress**).

The transport of heat in the direction of the axis of a **buoyant jet** is not given simply by the integral

$$\int_{\text{area of jet section}} \bar{w} \bar{T} \rho c dS$$

where w , T , ρ , and c are the velocity, temperature, density, and specific heat of the fluid and

dS the element of area of the section; but there may also be an appreciable contribution from

$$\int \overline{w'T'} \rho dS$$

where w' and T' are the fluctuations of velocity and temperature, because the upward velocity fluctuations are probably correlated with the buoyancy which produces them. (Here the secondary effect of the simultaneous fluctuations of ρ are neglected.)

Likewise in a **turbulent jet** in which there are no buoyancy forces, in the transport of effluent along the direction of the axis, namely

$$\int \{\bar{w}\bar{\sigma} + \overline{w'\sigma'}\} dS$$

where $\sigma (= \bar{\sigma} + \sigma')$ is the concentration of effluent, there may be an appreciable contribution from the second term because the elements of greater concentration will tend to be those of greater velocity, so that w' and σ' will be correlated.

The transport of momentum along a turbulent jet of uniform density is

$$\int \{\bar{w}^2 + \overline{w'^2}\} \rho dS$$

and the second term may often be appreciable.

Correlation coefficients are usually only properly definable when the mean flow is approximately steady because only then can a satisfactory time mean value of a fluctuating quantity (denoted by a bar) be defined.

CORRELATION DIAGRAM. In molecular physics, a diagram showing the correlation between the electronic orbitals in the united atom and in the separated atoms, respectively (see also **building-up principle**). Such diagrams show the (approximate) energetic order of the different orbitals also for internuclear distances r intermediate between the limiting cases of $r = 0$ and $r = \infty$. (For more detail, see G. Herzberg, *Molecular Spectra and Molecular Structure, I. Spectra of Diatomic Molecules*, 2d ed., D. Van Nostrand Co., Inc., Princeton, 1945, p. 326 ff.)

CORRELATION ENERGY. The energy associated with the correlation effect in a metal, i.e., the tendency for electrons to keep apart, and hence for the positions of any two not to be entirely independent. It represents a cor-

rection to a naive calculation of **Coulomb energy**.

CORRELATION RATIO. In a one-way classification of observations y with classes defined by a quantitative variable x , the correlation ratio of y on x , η_{yx} , is defined by

$$\eta_{yx}^2 = \frac{\text{Sum of squares between classes}}{\text{Total sum of squares}},$$

the sum of squares being derived from an **analysis of variance**.

Problems for which the correlation ratio was formerly used can now be treated more simply by analysis of variance techniques.

CORRELATION, SERIAL. See **autocorrelation**.

CORRELOGRAM. The graph of the **autocorrelation coefficient** ρ_k as ordinate against k as abscissa. If the autocorrelations are estimated from a finite series as serial correlations r_k , the graph of r_k against k is also called a **correlogram**.

CORRESPONDENCE. See **mapping**.

CORRESPONDENCE PRINCIPLE. See **principle of correspondence**.

CORRESPONDING STATES, LAW OF. All gases whose constituent atoms interact according to a **Lennard-Jones potential** will have the same **equation of state** if this equation is expressed in terms of **reduced variables**, that is, in the form

$$\frac{p}{p_c} = \phi\left(\frac{T}{T_c}, \frac{V}{V_c}\right) \quad (1)$$

where p_c , T_c , V_c are the critical pressure, temperature and volume, and where ϕ is the same function for all substances of the group. The quantities p/p_c , T/T_c , V/V_c are also called the *reduced* pressure, temperature, and volume.

All equations of state containing two independent parameters like the **van der Waals equation** or the **Dieterici equation** (see **real gases**) may be put into the form (1) above, or using the van der Waals constants,

$$f(p, V, T, a, b) = 0. \quad (2)$$

The critical state (p_c , V_c , T_c) is given by the two conditions

$$\left(\frac{\partial p}{\partial V}\right)_T = 0 \quad (3)$$

$$\left(\frac{\partial^2 p}{\partial V^2}\right)_T = 0. \quad (4)$$

Thus, the two (and no more) parameters can be eliminated, and the **equation of state** can be written in terms of the reduced parameters

$$\pi = p/p_c$$

$$\phi = V/V_c$$

$$\theta = T/T_c$$

in the form

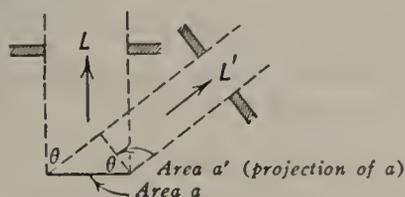
$$F(\pi, \phi, \theta) \quad (5)$$

which does not contain the parameters a and b . Thus Equation (5) does not contain any quantities which depend on the nature of the gas and should be a *universal* form of the equation of state. Such a universal equation of state written in terms of the reduced parameters or corresponding states (expressing the principle of corresponding states) is satisfied with a good degree of precision by simple gases like argon, krypton, xenon, carbon monoxide and xenon, while the corresponding equations of state when not expressed in terms of reduced variables show greater or less divergence.

If quantum effects become important, the law of corresponding states must be suitably altered.

COSET. If H is a subgroup of the group G , a set of elements of the form ah , where a is in G and h runs through H is called a (left) coset of H in G and a set of the form ha is a right coset. The number of such cosets is called the *index* of H in G .

COSINE EMISSION LAW. The energy emitted by a radiating surface in any direction is proportional to the cosine of the angle which that direction makes with the normal. Thus, let the radiating surface, of area a , be emitting a luminous flux L (lumens) in the



normal direction (see figure), and, in any other direction making an angle θ with the

normal, the smaller quantity L' . Then since the apparent brightness is unchanged, $L'/a' = L/a$. This gives $L'/L = a'/a$. But $a'/a = \cos \theta$, hence $L' = L \cos \theta$.

This law applies to thermal radiation as well as to light, and to diffusely reflected as well as directly emitted radiation. The law is true only for a perfectly diffusing surface, strictly, for a **black body**, but it is a good approximation to the behavior of many surfaces.

COSINE LAW. (1) See **cosine emission law**. (2) See **Knudsen cosine law**.

COSINE LAW, KNUDSEN. See **Knudsen cosine law**.

COTTON-MOUTON CONSTANT. See **Cotton-Mouton law**.

COTTON-MOUTON LAW. A dielectric placed in a magnetic field may become double-refracting with retardation δ of the ordinary ray over the extraordinary ray given by

$$\frac{\delta}{\lambda} = \frac{l(n_e - n_o)}{\lambda} = C_m l H^2$$

in which C_m is the Cotton-Mouton magnetic birefringence constant, l is the distance in the dielectric traversed by the light of wavelength λ , and H is the magnetic field strength. Further, n_e and n_o are the refractive indices of light polarized in planes parallel and perpendicular, respectively, to the direction of the magnetic field.

COUETTE FLOW. A two-dimensional steady flow without pressure gradient in the direction of flow and caused by the tangential movement of the bounding surfaces. The only practicable type is the flow between concentric rotating cylinders, although the flow between parallel planes with uniform relative velocity is used in the elementary discussion of **viscosity**.

COULOMB. A unit of electrical charge, abbreviation coul or Cb. (1) The absolute coulomb is defined as the amount of electrical charge which crosses a surface in one second if a steady current of one absolute ampere is flowing across the surface. The absolute coulomb has been the legal standard of quantity of electricity since 1950. (2) The International coulomb, the legal standard before 1950,

is the quantity of electricity which, when passed through a solution of silver nitrate in water, in accordance with certain definite specifications, deposits 0.00111800 gm of silver. 1 Int. coul = 0.999835 abs coul.

COULOMB BARRIER. A potential barrier produced by repulsive electrostatic forces, particularly just beyond the surface of an atomic nucleus.

COULOMB BOND ENERGY. See **Coulomb integral**.

COULOMB DEGENERACY. Identity of the energy levels of a charged particle bound in a Coulomb field for different values of the orbital angular momentum, provided that the principal quantum number and spin state is the same, e.g., the $2S_{\frac{1}{2}}$ and $2P_{\frac{1}{2}}$ states of the hydrogen atom. (See **Lamb shift**.)

COULOMB ENERGY. The contribution to the binding energy of a substance made by electrostatic interactions of ions and electrons. For a metal lattice, the Coulomb energy of a unit cell is approximately that due to a uniform distribution of electrons over an s -sphere of equal volume and is given by

$$U(r) = -\frac{3Ze}{2r_s} + \frac{Zer^2}{2r_s^3}$$

where Z is the number of conducting electrons, and r_s is the radius of the sphere.

COULOMB EXCITATION. If a high energy charged particle passes close to the nucleus of an atom, the time varying electric field felt by the nucleus can cause it to undergo a transition to an excited state. The necessary energy for the transition can be looked upon as having been radiated by the passing particle and absorbed by the nucleus. This excitation by the Coulomb field of the passing particle is called coulomb excitation.

COULOMB FIELD. The field of an electric charge which can be described by a **Coulomb potential**. The electrons of an atom move in the Coulomb field of the atomic nucleus.

COULOMB INTEGRAL. Consider a valence bond formed between two nuclei: A and B, carrying each a valence electron 1 or 2. If the bond energy is calculated by assuming that electron 1 is attached to atom A and electron

2 to atom B, one obtains for the potential interaction energy of the two atoms

$$H_{11} = \int \psi_A(1)\psi_B(2)W\psi_A(1)\psi_B(2)d\tau_1d\tau_2$$

where the ψ -terms represent the wave functions and where W is the perturbation potential. W is the sum of the repulsive Coulomb energy of the two electrons located in volume elements $d\tau_1$ and $d\tau_2$; of that of the two nuclei, and the attractive energy between electron 1 and nucleus B and electron 2 and nucleus A.

H_{11} expresses the electrostatic interactions of the system and is called the Coulomb integral. The corresponding energy is negligible for large internuclear distances; it becomes attractive for intermediate ranges, and leads to a strong repulsion at very small intermolecular distance.

The Coulomb energy behaves thus qualitatively just as one would expect a bond energy to do. However in absolute values, the bond energies calculated are usually much too small. The main contribution to the covalent bond energy results usually from the **exchange integral**. (See **exchange energy**.)

COULOMB LAW (ELECTROSTATICS). The force between two point charges in free space is a pure attraction or repulsion, and is given in rationalized MKSA units by

$$F = \frac{q_1q_2}{\epsilon_0r^2}$$

where q_1, q_2 are the magnitudes of the charges, r is their separation, and ϵ_0 is a constant of nature. The numerical value of ϵ_0 depends upon the system of units used (see **electric constant**). When the charges are immersed in a homogeneous medium, which extends to distances much greater than r in all directions

$$F = \frac{q_1q_2}{\epsilon r^2}$$

where ϵ is the **permittivity**, or absolute dielectric constant of the medium.

COULOMB LAW (MAGNETISM). See **magnetic pole strength**.

COULOMB POTENTIAL. The potential describing the field of an electrostatic charge. The potential energy acting between a charge e and a charge $-Ze$ is given by $V = -Ze^2/r$,

where r is the distance between the two charges.

COULOMB WAVE FUNCTION. The wave function of a charged particle moving in a **Coulomb field**. In non-relativistic quantum mechanics the radial part of a Coulomb wave function is related to **Laguerre polynomials**; in relativistic electron theory to **confluent hypergeometric functions**. (See **Gauss hypergeometric equation**.)

COUNTABLE. A set of objects is said to be countable if it can be put into one-to-one correspondence with the set of positive integers 1, 2, 3, \dots . Synonyms are enumerable, denumerable.

COUNTABLE SET. See **cardinal**.

COUNTER EFFICIENCY. Of a radiation counter tube, the probability that a tube count will take place with a specified particle or quantum incident in a specified manner.

COUNTER-FLOW. See **heat exchanger**.

COUPLE. A couple is a pair of equal and parallel but oppositely directed forces. The moment of a couple is a vector perpendicular to the plane of the two forces. Its magnitude is the same for all **center(s) of moment**.

COUPLED CHEMICAL REACTIONS. The **entropy** production due to two simultaneous chemical reactions is (see **thermodynamics**, **second principle of**)

$$T \frac{d_i S}{dt} = A_1 v_1 + A_2 v_2 \geq 0 \quad (1)$$

where A_1 , A_2 are the affinities of the two reactions (see **chemical affinity**) and v_1 , v_2 the reaction rates (see **chemical reaction rate**). Only the sum in (1) is positive (or zero at equilibrium). Thus it is possible to have

$$A_1 v_1 < 0 \quad A_2 v_2 > 0. \quad (2)$$

The first reaction is then called the coupled and the second the coupling reaction. Such reactions are of great biological interest.

COUPLE, THEOREMS ABOUT. (1) The moment of a couple has the same value when moments of each force are taken about any axis perpendicular to the plane of the couple. (2) A rigid body acted upon by a couple is

in translational equilibrium, but not rotational equilibrium. (3) A system of forces acting on a rigid body can be replaced by a single force acting at an arbitrary point of the body and a couple whose axis is parallel to the line of action of the force. The combination is called a *wrench*.

COUPLING. An interaction between systems, or between properties of a system. When there is little interaction, the coupling is sometimes said to be loose; with considerable interaction, it is called tight.

COUPLING CASES, HUND'S. See **Hund's coupling cases**.

COUPLING CONSTANT. A number, of the dimension of electric charge, measuring the strength of the interaction between interacting fields, defined in analogy with the electronic charge e which is the coupling constant between electrons and electromagnetic radiation. The interaction term between a Fermi field $\psi(x)$ and a boson field $\phi(x)$ is thus usually taken to be of the form $g\bar{\psi}(x)\Gamma\psi(x)\phi(x)$, where $\bar{\psi}$ is the Hermitian conjugate of ψ , Γ is some operator, and g is the coupling constant. In the case that g is small, transition probabilities are then proportional to the pure number $(g^2/\hbar c)$ in lowest order, where $2\pi\hbar$ is Planck's constant and c the velocity of light. In the case of electrodynamics, this quantity is the fine-structure constant $(e^2/\hbar c) \approx 1/137$.

COUPLING, IN ATOMIC AND MOLECULAR STRUCTURE. The energy of a stable state of an atom or molecule is affected by the strength of interaction or coupling of the spins and the orbital angular momenta of the individual electrons with each other, with the angular momentum of molecular rotation and with the electric field along the molecular axis ("coupling to the molecular axis"). In an actual case these interactions may present a very complicated picture. In general, however, a good approximation is obtained if it is assumed that certain interactions predominate. One can work therefore with simplified vector models of atoms or molecules in which the individual angular momentum vectors are added together (see **angular momentum vectors**, **quantum theoretical addition of**) in the order of their strength of coupling. (See **Russell-Saunders coupling**; **(j, j) coupling**; **Hund's coupling cases**.)

COUPLING, (j, j). See (j, j) coupling.

COUPLING, RUSSELL-SAUNDERS. See Russell-Saunders coupling.

COUPLING, SPIN-ORBIT. See Russell-Saunders coupling.

COUPLING, WEAK. Interaction between two particles by means of one or more fields which is such that the **interaction energy** may be expanded in powers of a dimensionless parameter which is small compared with unity, e.g., coupling of charged particles with each other through an electromagnetic field, where the dimensionless parameter is the **fine-structure constant**.

COURSE (NAVIGATION). The direction in which a ship is moving relative to the surface of the earth is known as the course of the ship. The term course is also applied to the direction (or directions) that a ship is supposed to follow from one place to another. During the long history of navigation the term "course" has become very loosely used. For example, we say that a ship is following a great circle course from point *A* to point *B* when, theoretically, its direction of motion relative to the surface of the earth is continually changing. In the discussion of modern navigation several different types of courses are recognized, e.g., Great Circle course, Mercator course, Rhumb Line course, Lambert line, etc. For each of these we have a compass (or gyro) course, a magnetic course, and a true course.

In a number of cases the terminology for courses is different for air- and sea-borne ships. Furthermore the different air services of the United States (Commercial, Army, Navy, or Air Force) use different terminology. For uniformity in this book, the terminology of the U.S. Navy surface navigation is used throughout.

COVALENT BOND. See **homopolar bond**; **atomic bond**; **bond types**.

COVARIANCE. (1) See entries under **covariant**. (2) In statistics, the covariance between two variates *x* and *y* is defined as $E(x - \xi)(y - \eta)$, where ξ , η are the means of *x*, *y* and *E* denotes the expected value. If the population is ρ and the standard deviations σ_x , σ_y , the covariance is equal to $\rho\sigma_x\sigma_y$. (See **analysis of covariance**.)

COVARIANCE MATRIX. See **dispersion**.

COVARIANCE, ORDER OF. See **tensor field**.

COVARIANT COMMUTATION RULES. There exist general methods for obtaining the commutation rules between functions *A*(*x*), *B*(*x*) of the field variables $\phi_s(x)$ due to Feynman, Peierls, Schrödinger and others. We outline here the method due to Peierls based on the **Lagrangian formalism**.

Consider the modified Lagrangian density

$$\mathcal{L}'(x) = \mathcal{L}(x) + \lambda A(x_1)\delta(x - x_1). \quad (1)$$

This Lagrangian \mathcal{L}' will yield modified field equations. The new solution for the field variable *B*(*x*) will be written in the form

$$B'(x) = B(x) + \lambda\delta_1 B(x) \quad (2)$$

where *B'* is completely defined by the equations of motion together with the boundary conditions $\delta_1 B \rightarrow 0$ as $t \rightarrow -\infty$. If *A* contains no time derivatives, the modified Hamiltonian will be

$$\begin{aligned} P'_0 &= H'(t) = H - \lambda A(x_1)\delta(t - t_1) \\ &= H - \delta H. \end{aligned} \quad (3)$$

It can be shown that *B'*(*x*) which is the solution of

$$i\hbar\partial_t B' = [H', B'] \quad (4)$$

and *B* which is a solution of

$$i\hbar\partial_t B = [H, B] \quad (5)$$

are related by the transformation

$$B'(x) = S(t)B(x)S(t)^{-1}, \quad x_0 = t \quad (6)$$

where *S*(*t*) satisfies the equation

$$i\hbar\frac{\partial S(t)}{\partial t} = \delta HS(t). \quad (7)$$

To first order in λ , the solution of (7) for *S* is

$$S(t) = 1 - \frac{i\lambda}{\hbar}\theta(t - t_1)A(x_1) \quad (8)$$

where $\theta(t) = 0$ for $t < 0$ and $\theta(t) = 1$ for $t > 0$. If we substitute this expression in Equation (6) and compare the result with (2) we find

$$i\hbar\delta_1 B(x_2) = [A(x_1), B(x_2)]\theta(t_2 - t_1) \quad (9)$$

and similarly

$$-i\hbar\delta_2 A(x_1) = [A(x_1), B(x_2)]\theta(t_1 - t_2) \quad (10)$$

combining these results in the symmetrical expression

$$[A(x_1), B(x_2)] = i\{\delta_1 B(x_2) - \delta_2 A(x_1)\}. \quad (11)$$

The above is as it stands applicable to boson fields. For example, consider the scalar meson field $\phi(x)$ [whose Lagrangian is $\mathcal{L} = -\frac{1}{2}(\mu^2\phi^2 - \partial_\mu\phi\partial^\mu\phi)$]. We wish to compute the commutator $[\phi(x_1), \phi(x_2)]$. Let us therefore construct the modified Lagrangian

$$\mathcal{L}' = \mathcal{L}(x) + \lambda\delta(x - x_1)\phi(x). \quad (12)$$

The equation determining $\delta_1\phi$ is then

$$(\square + \mu^2)\delta_1\phi(x) = \delta(x - x_1) \quad (13)$$

with $\delta_1\phi = 0$ for $t < t_1$. The solution of this equation is

$$\delta_1\phi(x) = \Delta_R(x - x_1) \quad (14)$$

where

$$\Delta_R(x) = -\frac{1}{(2\pi)^4} \int_{C_R} \frac{e^{-ik_\mu x^\mu}}{k^2 - \mu^2} d^4k \quad (15)$$

with C_R a contour in the complex k_o plane which goes from $-\infty$ to $+\infty$ above the singularities $k_o = \pm\sqrt{\mathbf{k}^2 + \mu^2}$. Similarly if

$$\Delta_A(x) = -\frac{1}{(2\pi)^4} \int_{C_A} \frac{e^{-ik_\mu x^\mu}}{k^2 - \mu^2} d^4k \quad (16)$$

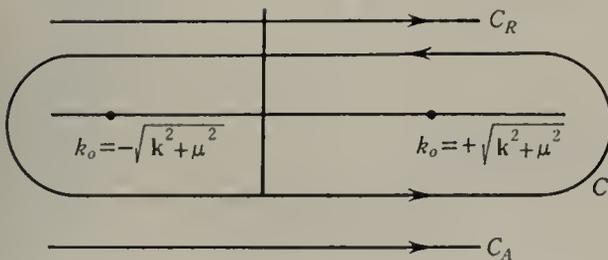
where C_A is a contour in the k_o plane from $-\infty$ to $+\infty$ going below the singularities $k_o = \pm\sqrt{\mathbf{k}^2 + \mu^2}$, the Peierls prescription implies that

$$[\phi(x_1), \phi(x_2)] = i\hbar[\Delta_A(x_1 - x_2) - \Delta_R(x_1 - x_2)] \\ = i\hbar\Delta(x_1 - x_2) \quad (17)$$

where

$$\Delta(x) = \frac{-1}{(2\pi)^4} \int_C \frac{e^{-ik_\mu x^\mu}}{k^2 - \mu^2} d^4k \quad (18)$$

and C is a contour in the k_o plane enclosing both singularities in the counterclockwise sense.



For half integral spin fields, it is necessary to alter the above prescriptions so as to obtain

anticommutation rules between the field operators. We illustrate the necessary alterations for the case of the free Dirac field, whose Lagrangian is

$$\mathcal{L} = -\frac{1}{2}\bar{\psi}(-i\gamma^\mu\partial_\mu + m)\psi \\ - \frac{1}{2}(i\partial_\mu\gamma^\mu + m)\bar{\psi}\cdot\psi. \quad (19)$$

The addition to the Lagrangian of an infinitesimal term of the form $\lambda\psi(x)\delta(x - x_1)$ would lead to (Bose) commutation rules. Instead one must consider the addition of terms of the type $\lambda\theta\psi(x)\delta(x - x_1)$ where θ anticommutes with ψ . The quantity $\delta_2\psi(x)$ is then determined by the equation

$$(-i\gamma^\mu\partial_\mu + m)\delta_2\psi(x) = \theta_2\delta(x - x_2) \quad (20)$$

$$\delta_2\psi = 0 \quad \text{for } t < t_2 \quad (20a)$$

whose solution is

$$\delta_2\psi = S_R(x - x_2)\theta_2 \quad (21)$$

which

$$S_{R/A}(x) = -(i\gamma^\mu\partial_\mu + m)\Delta_{R/A}(x; m). \quad (22)$$

Combining this solution with the one for $\delta_1\bar{\psi}$ yields the following result for the commutator

$$[\theta_1\psi(x_1), \bar{\psi}(x_2)\theta_2] = -i\theta_1\{S_A(x_1 - x_2) \\ - S_R(x_1 - x_2)\}\theta_2. \quad (23)$$

If now θ_1 and θ_2 are chosen to anti-commute with each other and with all field operators, the commutator in (23) reduces to $\theta_1[\psi(x_1), \bar{\psi}(x_2)]_+\theta_2$, where $[A, B]_+ = AB + BA$; whence

$$[\psi(x_1), \bar{\psi}(x_2)]_+ = iS(x_1 - x_2) \quad (24)$$

where

$$S(x) = S_A(x) - S_R(x) \\ = -(i\gamma^\mu\partial_\mu + m)\Delta(x; m). \quad (25)$$

COVARIANT CURVATURE TENSOR. See Riemann-Christoffel tensor.

COVARIANT DERIVATIVE OF TENSOR FIELD. $t_{pq\dots r}^{ij\dots k}$ are the components in a generic coordinate system x of a tensor field of weight W . We define $t_{pq\dots r, m}^{ij\dots k}$ by

$$t_{pq\dots r, m}^{ij\dots k} = \frac{\partial t_{pq\dots r}^{ij\dots k}}{\partial x^m} - W t_{pq\dots r}^{ij\dots k} \Gamma_{ma}^a \\ + t_{pq\dots r}^{aj\dots k} \Gamma_{ma}^i + \dots + t_{pq\dots r}^{ij\dots a} \Gamma_{ma}^k \\ - t_{aq\dots r}^{ij\dots k} \Gamma_{pm}^a - \dots - t_{pq\dots a}^{ij\dots k} \Gamma_{rm}^a,$$

where Γ_{jk}^i denotes the **Christoffel (3-index) symbol** of the second kind. The aggregate of functions $t_{pq\dots r,m}^{ij\dots k}$ so defined in each coordinate system is a **tensor field** of weight W with covariant order one greater than the original tensor field. It is called the covariant derivative of the tensor field. Since the Christoffel symbols are all zero for a rectangular Cartesian coordinate system, the components of the covariant derivative of a tensor field in a rectangular Cartesian coordinate system are the partial derivatives, with respect to the coordinates, of the components of the tensor field in that coordinate system.

COVARIANT DIFFERENTIATION. The operation of forming the **covariant derivative**.

COVARIANT METRIC TENSOR. See **metric tensor**.

COVARIANT PERTURBATION THEORY. The time displacement operator in the **interaction picture** $U(t, t_0)$, defined by

$$U(t, t_0)\Psi(t_0) = \Psi(t)$$

$$U(t_0, t_0) = 1$$

where $\Psi(t)$ is the state vector of the system in the interaction picture, satisfies the equation

$$U(t, t_0) = 1 - \frac{i}{\hbar} \int_{t_0}^t dt' H_I(t') U(t', t_0)$$

where $H_I(t)$ is the interaction Hamiltonian ($H = H_0 + H_I(t)$). It is unitary

$$U^*(t, t_0) = U^{-1}(t, t_0)$$

for finite times t, t_0 , and in addition satisfies the relation

$$U^{-1}(t, t_0) = U(t_0, t)$$

for finite times t, t_0 . It also obeys the group property

$$U(t, t_1)U(t_1, t_0) = U(t, t_0)$$

which allows us to express a finite transformation as a product of infinitesimal ones

$$U(t, t') = U(t, t_1)U(t_1, t_2) \cdots U(t_{n-1}, t_n)U(t_n, t')$$

where $U(t_j, t_{j+1})$ is an infinitesimal transformation from t_j to t_{j+1} . For the infinitesimal case $U(t_j, t_{j+1})$ can be approximated by

$$U(t_j, t_{j+1}) \approx 1 - \frac{i}{\hbar} \int_{t_{j+1}}^{t_j} dt' H_I(t')$$

so that in the limit as $n \rightarrow \infty$

$$U(t, t') = 1 - \frac{i}{\hbar} \int_{t_0}^t dt_1 H_I(t_1) + \left(-\frac{i}{\hbar}\right)^2 \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 H_I(t_1) H_I(t_2) + \cdots + \left(-\frac{i}{\hbar}\right)^n \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n H_I(t_1) H_I(t_2) \cdots H_I(t_n).$$

To write this expansion in a more symmetrical form Dyson has introduced the **chronological ordering operator** P defined so as to order any product of time-dependent operators in chronological sequence, with the one with the earliest time label standing furthest to the right

$$P[A_i(t_i)A_j(t_j) \cdots A_k(t_k)] = A_1(t_1) \cdots A_1(t_n)$$

if $t_1 > t_2 > \cdots t_n$.

Consider the expression

$$I_n(t, t_0) = \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n P[H_I(t_1) \cdots H_I(t_n)]$$

since the integrand of I_n is symmetric with respect to $t_1 t_2 \cdots t_n$ the total contribution is $n!$ times a contribution from a particular time ordering of the t 's. If we take the sequence $t_1 > t_2 > \cdots > t_n$ as representative

$$I_n(t, t_0) = n! \int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n H_I(t_1) \cdots H_I(t_n)$$

whence

$$U(t, t_0) = \sum_{n=0}^{\infty} \left(-\frac{i}{\hbar}\right)^n \int_{t_0}^t dt_1 \cdots \int_{t_0}^{t_{n-1}} dt_n P(H_I(t_1) \cdots H_I(t_n)).$$

In field theory

$$H_I(t) = \int_{-\infty}^{\infty} \mathcal{H}_I(x) d^3x$$

so that the **S matrix** has the following expansion

The normalization of the one-particle state now demands that

$$\begin{aligned} \langle \mathbf{k}' | \mathbf{k} \rangle &= \langle 0 | a_{\mathbf{k}'} a_{\mathbf{k}}^* | 0 \rangle \\ &= \langle 0 | [a_{\mathbf{k}'}, a_{\mathbf{k}}^*] | 0 \rangle = k_0 \delta(\mathbf{k} - \mathbf{k}'). \end{aligned} \quad (7)$$

If we require the commutator of the a and a^* operators to be a c -number then

$$[a_{\mathbf{k}}, a_{\mathbf{k}'}^*] = \delta(\mathbf{k} - \mathbf{k}') k_0. \quad (8)$$

One verifies that the orthonormality conditions (1c) are in fact generally satisfied with these commutation rules.

A representation of these commutation rules is easily obtained from the defining Equations (3)-(4) and one finds

$$\begin{aligned} a^*(k) | \mathbf{k}_1 \cdots \mathbf{k}_n \rangle &= \sqrt{n+1} | \mathbf{k}, \mathbf{k}_1, \mathbf{k}_2 \cdots \mathbf{k}_n \rangle \\ a(k) | \mathbf{k}_1 \cdots \mathbf{k}_n \rangle &= \frac{1}{\sqrt{n}} \sum_i \delta(\mathbf{k} - \mathbf{k}_i) \\ &\quad | \mathbf{k}_1 \cdots \mathbf{k}_{i-1} \mathbf{k}_{i+1} \cdots \mathbf{k}_n \rangle. \end{aligned}$$

This representation is irreducible.

For an assembly of spin $\frac{1}{2}$ particles (which obey the **Pauli exclusion principle**) the state of a single particle is specified by its momentum (energy) and spin quantum number which can take on two values, say ± 1 . We denote such a one-particle state by $|\mathbf{p}, s\rangle$. An n -particle state is denoted by $|\mathbf{p}_1 s_1, \cdots, \mathbf{p}_n s_n\rangle$ and the no-particle state by $|0\rangle$. The Pauli principle requires that no two particles have the same momenta and spin; more generally, that it be antisymmetric under the interchanges of any two particles. The normalization of these states is

$$\begin{aligned} \langle 0 | 0 \rangle &= 1 \\ \langle \mathbf{p} s | \mathbf{p}' s' \rangle &= p_0 \delta(\mathbf{p} - \mathbf{p}') \delta_{ss'} \\ \langle \mathbf{p}_1 s_1 \cdots \mathbf{p}_n s_n | \mathbf{p}'_1 s'_1 \cdots \mathbf{p}'_m s'_m \rangle \\ &= \delta_{nm} \frac{1}{n!} \det | \delta(\mathbf{p}_i - \mathbf{p}_j) \delta_{s_i s_j} |. \end{aligned}$$

We can again define creation and annihilation operators $a^*(\mathbf{p}s)$ and $a(\mathbf{p}s)$ by

$$\begin{aligned} a^*(\mathbf{p}s) | 0 \rangle &= | \mathbf{p}s \rangle \\ \frac{1}{\sqrt{n!}} a^*(\mathbf{p}_1 s_1) \cdots a^*(\mathbf{p}_n s_n) | 0 \rangle &= | \mathbf{p}_1 s_1 \cdots \mathbf{p}_n s_n \rangle \\ a(\mathbf{p}s) | \mathbf{p}s \rangle &= | 0 \rangle \\ a(\mathbf{p}s) | 0 \rangle &= 0. \end{aligned}$$

The antisymmetry of the vector $|\mathbf{p}_1 s_1 \cdots \mathbf{p}_n s_n\rangle$ under the interchange of any two particles implies that $a^*(\mathbf{p}s)$ and $a^*(\mathbf{p}'s')$ must anticommute

$$\begin{aligned} \{a^*(\mathbf{p}s), a^*(\mathbf{p}'s')\} &= 0 \\ \{a(\mathbf{p}s), a(\mathbf{p}'s')\} &= 0 \end{aligned}$$

where $\{A, B\} = AB + BA$. The normalization of the states now implies that

$$\{a(\mathbf{p}s), a^*(\mathbf{p}'s')\} = \delta(\mathbf{p} - \mathbf{p}') p_0 \delta_{ss'}.$$

CREEP (OR CREEP FUNCTION). The strain variation due to loading by a **step function** of stress. For a linear viscoelastic material this is equal to the creep compliance (see **compliance, creep**) multiplied by the stress.

CREEP COMPLIANCE. See **compliance, creep**.

CREEPING MOTION. Flow in which the viscous forces are much larger than the inertia forces ($Re \ll 1$). In terms of the velocity potential ψ , such that

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

the motion satisfies the differential equation

$$\nabla^4 \psi = 0$$

whose particular solutions are sometimes called **biharmonic functions**.

CRIPPLING. See **buckling, local**.

CRITICAL ANGLE. See **total reflection**.

CRITICAL ASSEMBLY. An assembly of fissionable material plus moderator which is capable of maintaining a fission chain reaction at very low power level and which is used mainly to study the behavior of the fissionable material under various conditions of geometry, composition, etc.

CRITICAL COEFFICIENT. An additive property of substances which is also a measure of the space actually occupied by the molecules and is proportional to the **critical volume**. It is expressed as the ratio between the critical temperature T_c and the critical pressure P_c , or

$$k = \frac{T_c}{P_c}.$$

(Some writers define the critical coefficient as the product: $RT \frac{T_c}{P_c}$.)

CRITICAL COMPOSITION. See **critical solution phenomena**.

CRITICAL CONSTANTS. The temperature, pressure and density at the **critical point**.

CRITICAL DAMPING. See **damping, critical**.

CRITICAL DENSITY. The density of a substance which is at its **critical temperature** and **critical pressure**.

CRITICAL EQUATION. In nuclear reactor theory, the equation obtained by equating the effective multiplication (see **multiplication, effective**) to unity. When the effective multiplication is given an explicit expression, the critical equation determines the arrangements of material necessary for the maintenance of a chain reaction.

CRITICAL FIELD. The magnetic field H_c below which the **superconducting transition** takes place at a given temperature. Empirically, a relation is observed of the form

$$H_c = H_0 \{1 - (T/T_c)^2\}$$

where H_0 is a parameter characteristic of the material, and T_c is the critical temperature of the superconductor.

CRITICALITY FACTOR. In the theory of neutron chain reactions, criticality factor is synonymous with effective multiplication. (See **multiplication, effective**.)

CRITICAL LOAD, ABSOLUTE. The smallest **critical load** (eigenvalue) for a structure is sometimes called the absolute critical load.

CRITICAL LOAD OR STRESS. A critical load or stress is the solution to an eigenvalue problem for the existence of neighboring configurations (see **buckling**). There may be an infinite number of successively higher critical stresses in any problem, corresponding to more and more complicated buckled configurations generally of shorter and shorter wavelengths.

CRITICAL LOAD, SOUTHWELL METHOD. The experimental determination of the critical load P_{cr} for a geometrically perfect

structure is always obscured by the imperfections of loading and geometry. For simple structures such as columns the magnification factor for the deflections is determined essentially by the reciprocal of $1 - (P/P_{cr})$. Therefore if the deflection δ is plotted against δ/P where P is the load applied, points will fall on a straight line whose slope gives P_{cr} .

$$\delta = (const.) \frac{P}{1 - \frac{P}{P_{cr}}}, \quad \text{or} \quad \frac{\delta}{P} = \frac{\delta}{P_{cr}} + const.$$

(See **beam column**.)

CRITICAL MACH NUMBER. For a body in a uniform stream, the critical Mach number is the **Mach number** of the undisturbed stream for which sonic velocity is just reached at some point on the body. At stream Mach numbers above the critical value, there is a region of supersonic flow near the body, although the flow in the undisturbed stream may still be subsonic.

CRITICAL MASS. The mass of fissionable material in a critical reactor.

CRITICAL MASS PROBLEM, MINIMUM. See **minimum critical mass problem**.

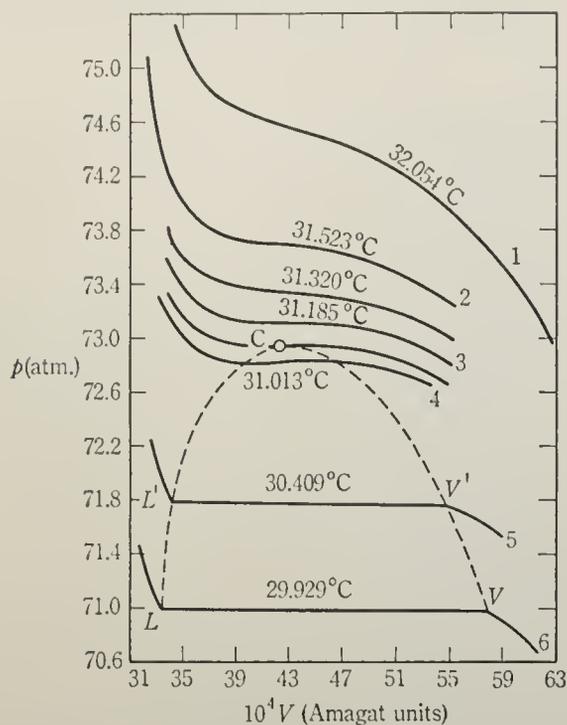
CRITICAL PARAMETERS. The critical temperature, pressure and volume.

CRITICAL PHENOMENA IN THE VAPORIZATION OF MIXTURES. The critical point for a given composition is the point where the liquid and the gaseous phases become identical, as if they were a single component. (See **critical point**.) However, the critical pressure and critical temperature have a different significance for a mixture, than they do in the case of a pure substance. Indeed there may exist a range of pressure and of temperature above the critical point when condensation may still occur.

Closely related to this behavior are the phenomena of *retrograde vaporization* and *retrograde condensation*. In the first case, when we raise the temperature at constant pressure, the vaporization begins, and then is followed by condensation until finally vaporization starts again. The description of retrograde condensation is similar. (See also **binodals**; **spinodal curve**.)

CRITICAL POINT. Consider the family of isotherms of a pure substance in the fluid range (liquid or gas) such for example as shown in the figure for carbon dioxide.

At sufficiently high temperatures each isotherm is a continuous curve, but at low tem-



Isotherms of carbon dioxide in the neighborhood of the critical point.

peratures the isotherm consists of three portions. The first section of the curve at high pressures corresponds to the liquid state, while that at low pressures refers to the gaseous state. These two curves are joined by a horizontal line corresponding to the simultaneous presence of two phases, liquid and gas.

The isotherm between those numbered 3 and 4 in the figure represents the transition between isotherms corresponding to the gas phase only, and those including a horizontal portion corresponding to a liquid-gas equilibrium. In this isotherm the horizontal line has contracted to a single point of inflection C. This is the critical point characterized by the relations

$$\left(\frac{\partial p}{\partial V}\right)_{T_c} = 0; \quad \left(\frac{\partial^2 p}{\partial V^2}\right)_{T_c} = 0; \quad \left(\frac{\partial^3 p}{\partial V^3}\right)_{T_c} < 0. \quad (1)$$

The curve $LL'C$ gives the molar volume of the liquid. Similarly $VV'C$ gives the molar volume of the gas.

At the critical point the molar volumes of the liquid and of the gas become equal. In

general a critical state is characterized by the fact that the two coexistent phases (here the liquid and the vapor) are identical.

The curve $VV'CL'L$ is called the *saturation curve*.

The experimental data do not indicate the existence of a critical point for the liquid-solid transition.

Above the critical point the substance can no longer exist in the liquid state. The critical temperature is thus the highest temperature at which the liquid and vapor can coexist. (See also **critical phenomena in the vaporization of mixtures; critical solution phenomena; continuity of state; Maxwell theorem.**)

CRITICAL POTENTIAL. The minimum electrical potential which has to be supplied to an electron to bring about, on collision with an atom, the emission of a certain spectral line. The critical potential is equal to the energy above the ground state of the upper electronic state taking part in the transition.

CRITICAL PRESSURE. (1) The pressure of a vapor at its **critical point**.

(2) If a compressible fluid flows from a container through an orifice or other constriction, the volume flow depends on the pressure difference across the flow system only if the final pressure exceeds a definite fraction of the initial pressure. If either the initial or the final pressure is held constant, there is a critical pressure separating a regime of flow dependence on pressure difference from one of independence. The fraction depends on the nature of the fluid. (3) See **convergent nozzle; convergent-divergent nozzle**.

CRITICAL PROPERTIES. See **critical parameters**.

CRITICAL REGION. (1) The region in the **diagram of state** of a substance in the neighborhood of the **critical point**. (2) In the statistical theory of testing hypotheses, the space of possible events is divided into two regions, one regarded as concordant with the hypothesis, the other as discordant. If an event, e.g., the outcome of an experiment, falls into the second region the hypothesis is rejected and that region is therefore called critical.

CRITICAL SIZE. In nuclear reactor theory, the value of some geometrical quantity (height, radius, ...) necessary to establish a steady chain reaction. In the case of a bare

reactor, the critical size is closely related to the lowest eigenvalue of the equation

$$(\nabla^2 + B^2)\phi = 0, \quad \phi = 0 \text{ on bounding surface.}$$

CRITICAL SOLUTION PHENOMENA. In addition to critical phenomena in vaporization when the liquid phase becomes identical with the gas phase (see **critical phenomena in the vaporization of mixtures**) there also exist critical solution phenomena in which two liquid phases or two solid phases each of which is a mixture, become identical at the critical point.

For a binary mixture such a critical point is given by the conditions

$$\left(\frac{\partial^2 g}{\partial x_B^2}\right)_c = 0; \quad \left(\frac{\partial^3 g}{\partial x_B^3}\right)_c = 0; \quad \left(\frac{\partial^4 g}{\partial x_B^4}\right)_c > 0 \quad (1)$$

where

$$g = \frac{G}{n} \quad (2)$$

is the molar Gibbs free energy (see **thermodynamics, characteristic functions**) and x_B is the mole fraction.

If the critical temperature is the highest at which the two phases can coexist it is called an *upper critical solution temperature*, if it is the lowest, then the critical point is called a *lower critical solution temperature*.

There are also a few systems which exhibit both an upper and a lower critical solution temperature.

CRITICAL SPEED. See **speed, critical**.

CRITICAL STABILITY (OF SYSTEM). See **stability (of system)**.

CRITICAL TEMPERATURE. (1) The temperature of a substance at its **critical point**. (2) Of a **superconductor**, the temperature T_c at which the **superconducting transition** takes place in zero magnetic field.

CRITICAL VOLUME. The volume occupied by unit mass, commonly one mole, of a substance at its **critical temperature** and **critical pressure**.

CROCCO THEOREM. For an inviscid gas,

$$T \text{ grad } s + \mathbf{v} \times \text{curl } \mathbf{v} = \text{grad } h_o + \frac{\partial \mathbf{v}}{\partial t},$$

where s and h_o are the entropy and stagnation enthalpy per unit mass, T is the temperature and \mathbf{v} is the velocity.

If the flow is steady and adiabatic, the terms on the righthand side vanish and the equation becomes

$$T \text{ grad } s = -\mathbf{v} \times \text{curl } \mathbf{v}.$$

Then, if the flow is irrotational the entropy must be constant throughout. Conversely, if the entropy is not constant the flow cannot be irrotational.

CROSSCORRELATION FUNCTION. A function used in information theory of the form

$$\phi_{12}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f_1(t)f_2(t - \tau)dt$$

where τ is a time-delay parameter. The function $f(t)$ is considered to be continuous, and it retains phase information, provided either or both the functions, $f_1(t)$ or $f_2(t)$ are periodic. It can be shown by use of the **convolution** integral that the linear response of any network or system is given by the crosscorrelation function between output and input for an input of wide-band random noise.

CROSS-DIFFERENTIATION IDENTITY. Alternative term for the **Euler reciprocity relation**.

CROSS EFFECT. See **normal stress effect**.

CROSS-FLOW. See **heat exchanger**.

CROSS-LINKING. Chemical bonding between the molecular chains of a high polymer, which prevent unrestricted viscous flow.

CROSS METHOD. The method of solving elastic rigid frame problems by a successive approximation technique called **moment distribution** as introduced by Hardy Cross.

CROSS-OVER DESIGN. A class of experimental design in which two treatments to be compared are first applied to two sets of individuals and then "crossed over" so that an individual receiving one treatment on one occasion receives the other treatment on the second. Various elaborations of the idea are possible.

CROSS POLARIZATION. The component of the **electric field vector** normal to the desired polarization component.

CROSS PRODUCT OF VECTORS. See vector multiplication.

CROSS SECTION. From its general meaning of a section at right angles to an axis, the term cross section has been extended to mean a measure of the probability of a particular process. It is expressed in units of area, although it is not usually identical with the geometric cross section across which the process occurs. For a collision reaction between nuclear or atomic particles or systems, the cross section is an area such that the number of reactions occurring is equal to the product of number of target particles or systems multiplied by the number of incident particles which would pass through this area at normal incidence. If n_t is the number of target nuclei or other particles per unit area of a substance exposed to an incident beam consisting of n_o particles, then $\sigma = N/n_o n_t$, where N is the number of reactions of a specified type. (While the cross section per atom of a medium is expressed in dimensions of area, the cross section per unit volume of medium is a reciprocal length.) Nuclear cross sections include the cross section for fission, the cross section for slow neutron capture, and the cross section for elastic scattering. Atomic cross sections include the cross section for Compton collision and the cross section for ionization by electron impact. The total cross section is the sum of the separate cross sections by which a particle can be removed from a beam.

CROSS SECTION, ABSORPTION. The total cross section for removal of an incident particle from a beam, minus the elastic scattering cross section. Sometimes therefore called the total inelastic cross section. In the case of neutrons, for example, it includes the cross sections for inelastic scattering, radiative capture, fission, and such reactions as (n, p) , (n, α) , and $(n, 2n)$.

CROSS SECTION, ACTIVATION. The cross section of formation for a particular radionuclide. It is most commonly used for neutron-induced reactions.

CROSS SECTION, BREIT-WIGNER. In the theory of slow neutron resonance reactions, the cross sections for scattering and x-ray emission due to the formation of a compound nucleus with widely separated energy levels are given by

$$\begin{aligned}\sigma_s(E) &= 4\pi(1-g)R^2 \\ &+ 4\pi\lambda_o^2 g \left| \frac{\Gamma_n/2}{E - E_o + i\Gamma/2} + \frac{R}{\lambda_o} \right|^2 \\ \sigma_\gamma(E) &= \pi\lambda_o^2 g \left(\frac{E_o}{E} \right)^{1/2} \frac{\Gamma_n \Gamma_\gamma}{(E - E_o)^2 + (\Gamma/2)^2} \\ g &= \frac{1}{2} \left(\frac{2J + 1}{2I + 1} \right), \\ J &= I + \frac{1}{2} \text{ (for slow neutrons)}\end{aligned}$$

when the neutron energy, E is in the vicinity of E_o , the energy required for the formation of a level. Here R , the nuclear "radius" is defined through the requirement that $\sigma_s(E) = 4\pi R^2$ far from E_o . $2\pi\lambda_o$ is the neutron wavelength at resonance, Γ_n and Γ_γ are the scattering and x-ray widths, and Γ is the total width, all evaluated at resonance. I is the spin of the target nucleus.

CROSS SECTION, DIFFERENTIAL. The cross section for a nuclear process whereby an angle is specified (relative to the direction of incidence) for the emission of particles or photons per unit angle or per unit solid angle.

CROSS SECTION, DOPPLER BROADENED. In the theory of neutron reactions, thermal motion of the target atoms causes the effective cross sections encountered by neutrons to deviate from the Breit-Wigner values. Averaging over these motions gives a Doppler broadened cross section. $\sigma_\gamma(E)$ for example, becomes:

$$\begin{aligned}\sigma_\gamma(x, \theta) &= 4\pi\lambda_o^2 \frac{\Gamma_n \Gamma_\gamma}{\Gamma^2} \\ &\times \int_{-\infty}^{\infty} \frac{dy}{1 + y^2} \exp \left\{ -\frac{(x - y)^2}{4\theta} \right\}\end{aligned}$$

where

$$x = \frac{E - E_o}{\Gamma/2} \quad \theta = \frac{4E_o k T}{A \Gamma^2}$$

A is the mass number of the target nucleus. (For other notation, see **cross section, Breit-Wigner.**)

CROSS SECTION, ELASTIC SCATTERING. The cross section for a scattering process in which, in the center of mass system, no energy is transferred to the scattering center, nor is there any change in the spin motions of particle and scatterer.

CROSS SECTION, INELASTIC SCATTERING. The cross section for the scattering of an incident particle accompanied by excitation of the scattering center. The scattered particle loses an amount of energy equal to the amount needed to raise the scattering center (nucleus, atom, or molecule) to one of its excited states. Inelastic scattering is an important process of energy loss for neutrons passing through a medium of high mass number.

CROSS SECTION, INTERACTION. See **interaction cross section**.

CROSS SECTION, IONIZATION. In general, the probability that a particle or photon traversing a gas, or other form of matter, will undergo an ionizing collision while passing through unit length of a volume just sufficient to contain one atom. The term is used especially in radiation counter technology, where it is a measure of the probability that an electron traversing a radiation counter will make an ionizing collision with one of the counter gas atoms. In the low voltage region the ionization cross section increases linearly with the electron energy.

CROSS SECTION, MACROSCOPIC. Symbolized by Σ , the product of the number of scatterers per unit volume and the appropriate cross section. Given a mixture of scattering materials,

$$\Sigma = \sum_i N_i \sigma_i.$$

When the number of scatterers per unit volume is unknown, the macroscopic cross section is sometimes expressed per unit weight of scattering material.

CROSS SECTION, MICROSCOPIC. Cross section per atom or molecule.

CROSS SECTION OF BAR. The area cut by a plane perpendicular to the axes of the bar.

CROSS SECTION, PARTIAL. A cross section for a particular process among several competing processes. (See **cross section, total**.)

CROSS SECTION, RADIATIVE CAPTURE. The cross section for a process in which the incident particle is captured by the target nucleus to form a compound nucleus which rids itself of excess energy through emission of photons. In the case of neutrons, radiative

capture is the chief absorption process for most elements for neutrons of low energies. The cross section is often inversely proportional to neutron velocity for sufficiently low energies.

CROSS SECTION, REMOVAL. In the study of the penetration of radiation through matter, consider a shielding system composed of a layer of water of thickness w into which a slab of heavier material of thickness, t , is inserted. The removal cross section, Σ_r , characteristic of the heavy material and type of radiation involved, is defined by the assertion that attenuation of radiation by the water shield is augmented by the factor $\exp(-\Sigma_r t)$ when the slab is inserted.

CROSS SECTION, RUTHERFORD. The cross section for elastic scattering of two electrically charged particles through their mutual Coulomb force. The total cross section is infinite owing to the infinite range of the Coulomb potential which gives rise to an infinite cross section for scattering into infinitely small angles. However, the differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \left(\frac{z_1 z_2 e^2}{2\mu v^2} \right)^2 \text{cosec}^4 \frac{1}{2}\theta$$

where θ is the scattering angle in the center of mass system, $z_1 e$ and $z_2 e$ are the charges, and μ the reduced mass, of the two particles, and v is their relative velocity. This celebrated result, true only in the non-relativistic limit, was first given by Rutherford in his work on the scattering of α -particles by matter. The differential scattering cross section for charged particles by nuclei are often expressed in units of the Rutherford cross section to show clearly the departures in scattering due to true nuclear scattering.

CROSS SECTION, SCATTERING. The cross section for scattering of a particle or a photon out of a beam. It is the integral over the total solid angle of the differential scattering cross section. (See **cross section, differential scattering**, and the various entries under scattering.)

CROSS SECTION, STOPPING. The same as atomic stopping power. (See **stopping power**.)

CROSS SECTION, TOTAL. The cross section effective for removing an incident particle from a beam. It is the sum of the separate cross sections for all processes by which the particle can be removed from the beam. For example, the total cross section of an atom for a nuclear particle is essentially the total of the absorption and scattering cross sections; for a photon it is essentially the sum of the Compton scattering, photoelectric and pair production cross sections.

CROUT METHOD. A method of triangularization for matrix inversion.

CRYOSCOPIC CONSTANT. Let us consider the **crystallization curve** of a dilute solution. We define the depression of the freezing point θ of the solvent by

$$\theta = T_1^\circ - T. \quad (1)$$

T_1° is the freezing point of the solvent and T of the solution.

If a solution is both very dilute and ideal one has

$$\theta = \frac{R(T_1^\circ)^2}{\Delta_f h_1^\circ} \frac{M_1}{1000} \sum_s m_s = \theta_c \sum_s m_s. \quad (2)$$

$\Delta_f h_1^\circ$ is the latent heat of fusion of the pure solvent at temperature T_1° , M_1 the molar mass of the solvent, m_s the **molality** of the solute s .

The quantity θ_c is called the cryoscopic constant (or freezing point constant). Its value depends only on the nature of the solvent. For water one has

$$\theta_c = 1.86 \text{ }^\circ\text{C/mole in 1000 g}$$

and for cyclohexanol

$$\theta_c = 41.6 \text{ }^\circ\text{C/mole in 1000 g.}$$

This wide range of values is related to the fact that θ_c contains the entropy of fusion

$$\Delta_f h_1^\circ / T_1^\circ \quad (3)$$

which varies considerably from substance to substance.

CRYSTAL ANGLES. The characteristic constant angles between the faces of any given crystal form.

CRYSTAL ELEMENTS. The angles, plus the **axial ratios** or **intercept ratios**, in terms of which the position of any crystal face may be described.

CRYSTAL INDICES, MILLER. Three indices used to represent any crystal face in terms of the **crystallographic axes** and the **crystallographic axial ratios**. These integers give the ratio of the intercepts of the unit plane to those of the particular face.

CRYSTAL INTERCEPTS. See **crystallographic parameters**.

CRYSTAL LATTICE. See **space lattice**.

CRYSTALLIZATION CURVE. Consider a system of c components and suppose first that these components are completely miscible in the liquid state, but completely immiscible in the solid state. On freezing, the solution yields various crystals, each variety of which consists of a single component, since the formation of mixed crystals is excluded. The coexistence curve of the solution with crystals of component l is given by the equation

$$\ln x_1^l \gamma_1^l = \int_{T_1^\circ}^T \frac{\Delta_f h_1^\circ}{RT^2} dT \quad (1)$$

where x_1^l and γ_1^l are respectively the mole fraction and activity coefficient of component 1 in the solution, $\Delta_f h_1^\circ$ the latent heat of fusion at temperature T_1 and T_1° the melting point of pure component 1.

If the solution is ideal and the latent heat of fusion may be considered as independent of temperature this equation becomes

$$-\ln x_1^l = \frac{\Delta_f h_1^\circ}{R} \left(\frac{1}{T} - \frac{1}{T_1^\circ} \right). \quad (2)$$

This equation is due to Schröder and Van Laar.

If we apply (1) or (2) to a binary mixture we obtain two crystallization curves which intersect at the *eutectic point*. At the eutectic point crystals of both 1 and 2 and the solution coexist in equilibrium.

If the components are miscible in the solid phase, one has **mixed crystals**.

CRYSTALLIZATION RATE EQUATIONS. Attempts have been made from time to time to establish equations for the linear velocity of crystallization (v) of substances. Empirical relationships of the form

$$v = \text{const. } \Delta t / \eta$$

have been used, where Δt is the degree of supercooling and η is the viscosity of the liquid. Theoretical equations have expressed v as exponential functions of the work of formation of a nucleus, w , and activation energy q , e.g.,

$$v = Ae^{-\frac{w}{kT}}e^{-\frac{q}{kT}}$$

where A is a constant, T is the absolute temperature, and k is the Boltzmann constant.

CRYSTALLOGRAM. The x-ray diffraction pattern of a crystal, whence the **crystal structure** may be obtained.

CRYSTALLOGRAPHIC AXES. Usually three, sometimes four, lines meeting at a point which are so chosen as to bear a definite relationship to the characteristic features of the crystallographic **symmetry**. They may, for example, be normal to the plane of symmetry, or parallel to, or coincident with, the edge between principal faces or the axes of symmetry. Although the choice of axes is to some extent arbitrary, some sets produce a simpler representation than others. Whenever possible, the three axes are chosen to be at right angles to each other.

CRYSTALLOGRAPHIC AXIAL RATIOS. The ratios of the crystal intercepts, i.e., the ratios of distances from the origin of the **crystallographic axes** to the points where they are intercepted by the faces of the **unit cell**.

CRYSTALLOGRAPHIC GROUP. Any **subgroup** of the three-dimensional real orthogonal group which leaves invariant (as a whole) the set of lattice-points (i.e., with integral coordinates) in Euclidean 3-space. There are 32 such groups, 11 being crystallographic groups of the first kind (i.e., subgroups of the rotation group), 11 being extended groups of the first kind (i.e., rotations followed by reflection through the origin) and 10 others.

CRYSTALLOGRAPHY. The branch of physical science which deals with the external shapes of crystals and with the geometrical relations between the **atomic planes** within them. If a solid crystal is broken, it is found to have separated along certain cleavage planes into polyhedral fragments, and measurements show that these planes were all originally parallel to one or another of a few standard planes. In most **crystal systems**, each of the more prominent crystal faces be-

longs to one of three plane-families, intersecting along the **crystal axes** (in hexagonal crystals there are four). It is observed (**Haüy law**) that if the ratio of the intercepts of two crystal planes on one of these axes is a simple fraction, the ratios of the intercepts on the other axes are also simple fractions, and in this way it is established that the intercepts on any one axis must be multiples of a common unit. A given family of planes is then labeled by its Miller indices or Bravais-Miller indices, the reciprocals of the ratios of its intercepts with each axis, expressed in terms of the common unit for that axis, and reduced to lowest integral terms. Study of the **axial ratios**, indices, and angles shows that all crystals may be classified into seven **crystal systems**. All the above properties are satisfied if a crystal consists of a regular array of atoms, molecules, ions, etc., in a space lattice, and this is verified by the x-ray analysis of crystal structure.

CRYSTAL MOMENTUM. An expression sometimes used for the quantity obtained by multiplying a **wave vector** by **Planck's constant**, h . It has the dimensions of momentum, but when it refers to an excitation or electron wave in a crystal there is no real momentum associated with it, and it is not necessarily conserved in a collision.

CRYSTAL, NORMAL MODES OF. See **normal modes of a crystal**.

CRYSTAL PARAMETERS. The lengths of the intercepts on the **crystallographic axes** of the standard plane—that is, the lengths of the sides of the **unit cell** of the lattice. In general, only the **crystallographic axial ratios** can be determined directly.

CRYSTAL PHASES (α , β , γ , \dots ϵ , etc.). Phases resulting from the ability of some alloys to crystallize in more than one form, depending on the relative quantities of the constituents. Alloys of different metals may nevertheless give the same crystal lattices, so that a term such as the α -phase shows that there is a structural connection between them. (See **Hume-Rothery rules**.)

CRYSTAL, RESONANCE FREQUENCY OF. The frequency at which a piezoid has either a maximum or minimum impedance, depending upon whether parallel or series resonance is employed.

CRYSTAL SATURATION MAGNETO-STRICTION. When a cubic crystal such as iron is magnetized to saturation in a direction defined by the direction cosines $\alpha_1, \alpha_2, \alpha_3$, the fraction changes in length in a direction $\beta_1, \beta_2, \beta_3$, is given to a first approximation by the expression

$$\frac{\Delta l}{l} = \frac{3}{2}\lambda_1(\alpha_1^2\beta_1^2 + \alpha_2^2\beta_2^2 + \alpha_3^2\beta_3^2 - \frac{1}{3}) + 3\lambda_2(\alpha_1\alpha_2\beta_1\beta_2 + \alpha_2\alpha_3\beta_2\beta_3 + \alpha_3\alpha_1\beta_3\beta_1)$$

where λ_1 and λ_2 are material properties. For the polycrystalline material with random orientation

$$\frac{\Delta l}{l} = \frac{2\lambda_1 + 3\lambda_2}{5}$$

CRYSTALS, EQUATION OF STATE FOR. See **Mie-Grüneisen equation of state.**

CRYSTAL STRUCTURE. It was early suggested that the regular structure of crystals, embodied in the laws of crystallography could be explained if they were thought of as built up by the repetition of equal polyhedral cells, fitting together to fill space, each cell representing a characteristic group of particles, perhaps the atoms and molecules of the compound. A rough calculation showed that the spacing of these units in many ionic crystals might be of the same order of magnitude as the wavelengths of x-rays, as deduced from quantum theory. Von Laue suggested, and verified, that diffraction of the x-rays occurs when they are passed through a crystal, suitably oriented. This diffraction effect is characteristic of the type of crystal, and by observing the relative magnitudes and orientations of the various diffracted beams one has clues to the crystal structure.

Although it is easy to calculate the diffraction pattern for a given crystal structure, it is quite another matter to deduce the crystal structure directly from the diffraction pattern. The first step is to determine the spacing of the atomic planes from the Bragg equation, and hence the dimensions of the unit cell. Any special symmetry of the space group of the structure will be apparent from space group extinction. A trial analysis may then solve the structure, or it may be necessary to measure the structure factors and try to find the phases for a Fourier synthesis. Various techniques can be used, such as the

F² series, the heavy atom, the isomorphous series, anomalous atomic scattering, expansion of the crystal and other methods. (See also **x-ray analysis.**)

CRYSTAL SYMMETRY. It is an obvious characteristic of a crystal that it possesses the property of appearing unchanged after the performance of some simple geometrical operation, such as rotation about an axis, or reflection in a plane. This **symmetry** (see the various classifications under the term **symmetry**) is a consequence of the identical property of the underlying crystal **lattice.**

CRYSTAL SYSTEMS. It can be shown by geometry that there exist 32 different classes of crystal symmetry, or point groups. These are conveniently classified into seven (by some authors, six) systems, characterized by their axial angles and ratios. (See **crystallography**, see table on page 199.)

CUBATURE FORMULAS. See **multivariate interpolation.**

CUBICAL DILATATION. The ratio between the change of volume of an element of a body in passing from an unstrained to a strained state (see **state of strain**) and the volume of the element in the unstrained state.

CUBIC SYSTEM. One of the seven **crystal systems.** In this system, the axes are of equal length and intersect at right angles.

CUMULANTS. The eumulants $\kappa_1, \kappa_2 \dots$ of a probability distribution function are defined by the identity

$$\begin{aligned} \exp \left[it\kappa_1 + \frac{(it)^2}{2!} \kappa_2 + \frac{(it)^3}{3!} \kappa_3 + \dots \right] \\ \equiv \left[1 + it\mu'_1 + \frac{(it)^2}{2!} \mu'_2 + \frac{(it)^3}{3!} \mu'_3 + \dots \right] \\ = \phi(t) \end{aligned}$$

where $\mu'_1, \mu'_2 \dots$ are the moments of the distribution about the origin and $\phi(t)$ is the **characteristic function.** $\Psi(t) = \log \phi(t)$ is called the **cumulant generating function.** κ_1 is equal to the mean μ'_1 and the other cumulants can be expressed in terms of the moments about the mean; the first four can be given by $\kappa_1 = \mu'_1, \kappa_2 = \mu_2, \kappa_3 = \mu_3, \kappa_4 = \mu_4 - 3\mu_2^2$.

The cumulants other than the first are invariant under a change of origin and semi-

invariant under a change of scale. For the **normal distribution** all cumulants after the second are equal to 0; for the **Poisson distribution** all the cumulants are equal to μ . $\gamma_1 = \kappa_3/\kappa_2^{3/2}$ and $\gamma_2 = \kappa_4/\kappa_2^2$ provide measures of **skewness** and **kurtosis**.

CUMULATIVE DISTRIBUTION FUNCTION. See **distribution function**.

CUMULATIVE FREQUENCY FUNCTION. See **distribution**.

CUMULATIVE PROBABILITY FUNCTION. See **distribution**.

CURIE. A unit of radioactivity which was originally defined as the amount of emanation (radon) from or in equilibrium with one gram

of radium. Because of experimental uncertainties this unit has been redefined by the revised recommendations of the International Commission on Radiological Units (July, 1953) as follows: "The curie is a unit of radioactivity defined as the quantity of any radioactive nuclide in which the number of disintegrations per second is 3.700×10^{10} ."

CURIE EQUATION. See **Curie-Weiss law**.

CURIE LAW. See **Curie-Weiss law**.

CURIE POINT. The temperature of the ferromagnetic-paramagnetic change defined in the Curie-Weiss law. By analogy, the term is also used for the **transition temperature** of any **cooperative phenomenon**, such as that exhibited by ferroelectric substances.

CRYSTAL SYSTEMS

Systems	Crystallographic Elements	Essential Symmetry	Number of Point Groups
Cubic, or regular	Three axes at right angles: all equal. $\alpha = \beta = \gamma = 90^\circ$ $a:b:c = 1:1:1$	4 triad axes; 3 diad, or 3 tetrad axes	5
Tetragonal	Three axes at right angles: two equal. $\alpha = \beta = \gamma = 90^\circ$ $a:b:c = 1:1:y$	1 tetrad axis	7
Orthorhombic or Rhombic	Three axes at right angles: unequal. $\alpha = \beta = \gamma = 90^\circ$ $a:b:c = x:1:y$	3 diad axes, or 1 diad axis and 2 perpendicular planes intersecting in a diad axis	3
Monoclinic	Three axes, one pair not at right angles: unequal. $\alpha = \gamma = 90^\circ$ $\beta \neq 90^\circ$ $a:b:c = x:1:y$	1 diad axis, or 1 plane	3
Triclinic or Anorthic	Three axes not at right angles: unequal. $\alpha, \beta, \gamma \neq 90^\circ$ $a:b:c = x:1:y$	No axes or planes	2
Hexagonal	Three axes coplanar at 60° : equal. Fourth axis at right angles to other three. $a_1:a_2:a_3:b = 1:1:1:x$	1 hexad axis	7
Rhombohedral or Trigonal	Three axes equally inclined, not at right angles: all equal. $\alpha = \beta = \gamma \neq 90^\circ$ $a:b:c = 1:1:1$	1 triad axis	5

CURIE SYMMETRY PRINCIPLES. See symmetry requirements in coupling of irreversible processes.

CURIE TEMPERATURE. See Curie point.

CURIE-WEISS LAW. The Curie law states that, above the Curie point θ the magnetic susceptibility of a paramagnetic substance is inversely proportional to the absolute temperature, the constant of proportionality being the Curie constant C' .

The Curie law was modified by Weiss to state that the susceptibility (χ) of a paramagnetic substance above the Curie point is inversely proportional to the temperature excess above that point, i.e.,

$$\chi = C'/(T - \theta).$$

The law does not hold near the melting point of a substance.

CURL. A vector resulting from the action of the operator **del** on a vector, **V**. It can be written in Cartesian coordinates in the following forms:

$$\begin{aligned} \text{curl } \mathbf{V} &= \nabla \times \mathbf{V} \\ &= \mathbf{i} \left\{ \frac{\partial V_z}{\partial y} - \frac{\partial V_y}{\partial z} \right\} + \mathbf{j} \left\{ \frac{\partial V_x}{\partial z} - \frac{\partial V_z}{\partial x} \right\} \\ &\quad + \mathbf{k} \left\{ \frac{\partial V_y}{\partial x} - \frac{\partial V_x}{\partial y} \right\} \\ &= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ V_x & V_y & V_z \end{vmatrix}. \end{aligned}$$

The curl of a position vector is $\mathbf{3}$, $\mathbf{R} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$, $\nabla \times \mathbf{R} = \mathbf{3}$.

If the curl of a vector function vanishes everywhere in a certain region, the function is said to be an irrotational vector (or a lamellar vector), in this region. It follows that if \mathbf{V} is an irrotational vector so that $\nabla \times \mathbf{V} = 0$, then $\mathbf{V} = \nabla\phi$ (\mathbf{V} is the gradient of ϕ), where ϕ is some scalar function.

European writers often use the word rotation instead of curl and the symbol **rot V**.

CURRENT. (1) The term usually refers to an electric current, which is the time rate of flow of electric charge across a surface (see **current, electric**). (2) The term is also used for the time rate of flow of other physical quantities such as mass, quantum mechanical

probability, etc. (See **continuity equations; current density**.)

CURRENT, AVERAGE. The average value of a current (I) over a time interval (t_1, t_2) is

$$\langle I \rangle = \frac{1}{t_2 - t_1} \int_{t_1}^{t_2} I dt.$$

CURRENT, BEAM. The current carried by a beam of charged particles such as positive ions or electrons, e.g., the proton current in a high energy accelerator. The beam current I is given by $I = zNe$ where z is the number of electronic charges (e) carried by the ion and N is the number of ions crossing a plane, perpendicular to the direction of flow, per unit time. The beam current density, the time rate at which charges cross a unit area perpendicular to their velocity is $znev$, where n is the number of particles per unit volume and v is their velocity.

CURRENT, CONDUCTION. Electric current consisting of the flow of free charges, as distinct from **displacement current**.

CURRENT DENSITY. (1) A vector representing the time rate of flow of electric charge per unit area. The direction of the vector is the direction of positive charge flow; the magnitude is the limit of the flow rate per unit area as the area approaches zero. The area considered is perpendicular to the direction of flow. (2) In nuclear physics, a vector such that its component along the normal to a surface equals the net number of particles crossing that surface per unit area and unit time. Commonly referred to simply as current, as in neutron current. (3) By analogy with electric current, a vector representing the time rate of flow of any quantity such as mass or momentum, per unit area in a transport process.

CURRENT DENSITY, DISPLACEMENT.

In a volume distribution of steady current in space, the **current density j** is everywhere solenoidal, i.e., $\nabla \cdot \mathbf{j} = 0$. For time-varying phenomena, the **conduction current density, j**, must satisfy the **continuity equation**, $\nabla \cdot \mathbf{j} + \frac{\partial \rho}{\partial t} = 0$, where ρ is the **electric charge density**. ρ is related to the **electric displacement vector, D**, by the equation

$\nabla \cdot \mathbf{D} = \rho$ (see **Maxwell equations**). The continuity equation then becomes

$$\nabla \cdot \left(\mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \right) = 0$$

where $\frac{\partial \mathbf{D}}{\partial t}$ is called the **displacement current density**. The sum of the conduction current density and the displacement current density is therefore everywhere solenoidal even when currents and fields vary with time.

In a dielectric \mathbf{D} may be written $\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P}$, where \mathbf{P} is the **polarization** (see **electric displacement vector**). The displacement current density can therefore be written in two parts

$$\frac{\partial \mathbf{D}}{\partial t} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \frac{\partial \mathbf{P}}{\partial t}.$$

The first member is called the *vacuum displacement current density* and does not have the significance of a current in the sense of being the motion of charges. The second term is called the *polarization current density* and corresponds to the actual motion of charge within the dielectric.

CURRENT DENSITY, POLARIZATION. See discussion under **current density, displacement**.

CURRENT, EDDY. Any change of magnetic flux in a conducting medium induces an **electromotive force** which causes currents to flow in the medium. (See **Faraday law of electromagnetic induction**.) Currents generated in this way are called eddy currents, and contribute to the power losses in conductors, conductive shields and magnetic cores.

CURRENT, ELECTRIC. Broadly, the flow of electric charge. More specifically, the time rate at which charge crosses a given surface is the current across the surface. There are actually three types of current: conduction current, convection current, and displacement current. Conduction current is due to the motion of charges in a neutral system (as electrons in a conductor, or the motion of electrons and "holes," which contribute largely to the current in semiconductors); convection current is due to the motion of unneutralized charge, as the motion of electrons in a vacuum tube; displacement current is an effect of a changing electric flux. Current is a scalar.

The current through a specified surface is given by the integral over that surface of the normal component of **current density**. In loose usage, we often speak of the direction of a current; actually, being a scalar, current has a sense (plus or minus) but not a direction. The positive sense of a current is conventionally taken as the direction in which positive charges would move if they were the carriers of the current. In metals, where the current is actually carried by negative electrons, the flow of electrons is opposite to the positive direction of the current. E.g., one coulomb of negative charge passing to the left per second is one ampere of (positive) current to the right.

CURRENT (OR STREAM) FUNCTION.

The stream or current function is a scalar function of position used to describe steady two-dimensional flow of an incompressible fluid. The current function, ψ , is defined by

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

where u, v are the components of the velocity parallel to Ox, Oy . It will be noticed that the flow across any line from the point A to the point B is

$$\begin{aligned} \int_A^B (udy - vdx) &= \int_A^B \nabla \psi \cdot d\mathbf{s} \\ &= \psi_B - \psi_A. \end{aligned}$$

CURRENT, NEUTRON. The total neutron current is the vector quantity:

$$\mathbf{J}(r, t) = \int_0^\infty dE \int d\hat{\Omega} \hat{\Omega} N(r, E, \hat{\Omega}, t) v,$$

where the quantities appearing in the integral are defined under **flux, neutron, angular**. The current of neutrons of energy E is:

$$\mathbf{J}(r, E, t) = \int d\hat{\Omega} \hat{\Omega} N(r, E, \hat{\Omega}, t) v.$$

CURRENT, NOISE, EQUIVALENT. A quantitative representation in current units of the spectral density of a noise-current generator at a specified frequency. The relation between the equivalent noise current I_n and the spectral density W_i of the noise-current generator is

$$I_n = (2\pi W_i)/e$$

where e is the magnitude of the electronic charge. The equivalent noise current in terms of the mean square noise-generator current \bar{i}^2 within a frequency increment Δf is

$$I_n = \bar{i}^2 / (2e\Delta f).$$

CURRENT, PROBABILITY. In wave mechanics, the probability of finding a particle at a given point is given by a **probability density**. The change in time of this probability density is given by the divergence of a current. In the case of a single particle described by a Schrödinger wave function $\psi(x,y,z)$, the probability density is $|\psi(x,y,z)|^2$, and the expression of the probability current is given by

$$\frac{\hbar}{2mi} (\psi^* \text{grad } \psi - \psi \text{grad } \psi^*),$$

where ψ^* is the conjugate complex of ψ , m is the mass of the particle, and $\hbar = h/2\pi$, h being Planck's constant.

CURRENT SHEET. An infinitely thin sheet carrying finite current per unit length normal to the lines of flow. This current may be either electric or magnetic. (See **magnetic current**.)

CURRENT SOURCE, IDEAL. An energy source that provides at a given terminal-pair (port), a current function that is independent of the voltage at that terminal-pair (port). An ideal current source is idle when its environment is a short circuit.

CURVATURE. The curvature κ of a plane curve $y = f(x)$ at a point x_0, y_0 on the curve is defined as the rate of change of the inclination of the tangent to the curve with respect to arc length. Thus

$$\kappa = \frac{d}{ds} \text{arc tan } y',$$

or in Cartesian coordinates

$$\kappa = y''(1 + y'^2)^{-3/2}.$$

The reciprocal $1/\kappa$ of the curvature is called the *radius of curvature*. The circle with this radius, with center on the normal, and passing through x_0, y_0 is called the *circle of curvature*, or the *osculating circle*, and its center is the *center of curvature*, while the plane in which it lies is called the *plane of curvature* or the

osculating plane. The curvature of a curve in three dimensional space is the rate of change of direction of the tangent with respect to arc length, and the *torsion* (also called second curvature) is the corresponding rate of change of direction of the **binormal**. The *principal curvatures of a surface* are the maximum and the minimum of the normal curvatures (i.e., curvatures of normal sections of the surface). The product and the sum of the principal curvatures are called the *total curvature* and the *mean curvature*, respectively, of the surface at the given point. The mean value of the principle curvatures is the *mean normal curvature* at the point. The total curvature is also called the *specific curvature* or the *Gauss curvature*. The mean curvature is also called the *first curvature* or the *circular curvature*. The *amplitude of normal curvature* is half the difference between the two principal curvatures. (See also **curvature of surface, center of**.)

CURVATURE, ANTICLASTIC. Longitudinal strain in a beam is accompanied by strain of opposite sign. Therefore curvature of longitudinal fibers produces transverse curvature of opposite sign (saddle surface) known as anticlastic curvature. The ratio of the curvatures is the ratio of the strains when the beam is narrow and is given by **Poisson's ratio**. When the beam is wide, the anticlastic curvature is constrained and the beam is stiffer.

CURVATURE, EULER THEOREM ON NORMAL. See **Euler theorem on normal curvature**.

CURVATURE, GEODESIC. See **geodesic curvature**.

CURVATURE, INFLUENCE OF. See **chemical potential, influence of curvature on**; and **vapor pressure, influence of curvature on**.

CURVATURE, NORMAL. See **normal curvature**.

CURVATURE OF BEAM. The curvature of the line of centroids of the cross section of the beam. When bending is in a single plane x,y and the beam is initially straight, the curvature usually is approximated by

$$\frac{d^2y}{dx^2}$$

as the square of the slope generally is small compared with unity.

CURVATURE OF FIELD. One of the five **Seidel geometrical aberrations** of lenses. If a system is corrected so that there is no spherical aberration, no coma, and no astigmatism, the third order theory images of off-axis points will lie on a curved surface called the **Petzval surface**.

CURVATURE OF LENS, TOTAL. A lens with spherical surfaces of radii r_1, r_2 has total curvature

$$K = \frac{1}{r_1} - \frac{1}{r_2}$$

where an appropriate sign convention is used, such as $r_1 > 0$ if the center of curvature is to the right of the vertex (axial intersection of the surface). Thus for a thin lens, the power is

$$\frac{1}{f} = (n - 1)K$$

where n is the relative index of refraction and f the focal length.

CURVATURE (GAUSSIAN) OF SPACE-TIME. $G = g^{\mu\nu}G_{\mu\nu}$ where $g^{\mu\nu}$ are the contravariant metric components and $G_{\mu\nu}$ is the **contracted Riemann-Christoffel tensor**. The vanishing of G does not imply that space-time is flat.

CURVATURE OF SURFACE, CENTER OF (FOR A POINT). Let P and Q be two points on a line of curvature on the surface. As Q approaches P , the normal to the surface at Q intersects that at P and the point of intersection is called a *center of curvature of the surface* for the point P . For each point on the surface there are, in general, two centers of curvature, corresponding to the two lines of curvature through the point. The distance of a center of curvature from P , measured in the direction of the unit normal at P , is called a (*principal*) *radius of curvature* of the surface at P . There are, in general, two radii of curvature to a surface at each point. The inverse of a radius of curvature of the surface at P is called a *principal curvature of the surface* at P .

CURVATURE, PRINCIPAL DIRECTIONS OF. See **principal directions of curvature**.

CURVATURE, RELATIVE. See **relative curvature**.

CURVATURE, SCALAR. See **scalar curvature**.

CURVATURE, SCREW (OF A CURVE AT A POINT). The magnitude of $d\mathbf{n}/ds$, where \mathbf{n} denotes the unit normal and s the distance measured along the curve from a fixed point.

$$d\mathbf{n}/ds = \sqrt{\kappa^2 + \tau^2},$$

where κ and τ are the **curvature** and **torsion** of the curve at the point.

CURVATURE, SPHERE OF. See **sphere of curvature**.

CURVATURE, SPHERICAL. See **sphere of curvature**.

CURVATURE, SURFACE OF NEGATIVE TOTAL. See **anticlasic surface**.

CURVATURE, TANGENTIAL. See **geodesic curvature**.

CURVATURE TENSOR. In each coordinate system x we define B^i_{ajk} by

$$B^i_{ajk} = \frac{\partial \Gamma^i_{aj}}{\partial x^k} - \frac{\partial \Gamma^i_{ak}}{\partial x^j} + \Gamma^b_{aj}\Gamma^i_{bk} - \Gamma^b_{ak}\Gamma^i_{bj},$$

where Γ^i_{jk} is the **Christoffel three-index symbol** of the second kind. The aggregate of quantities B^i_{ajk} so defined form a **tensor field**, contravariant of order one and covariant of order two, which is called the **curvature tensor**. Also sometimes referred to as the *Riemann-Christoffel tensor*, or *Riemann-Christoffel tensor of the second kind*, or *mixed Riemann-Christoffel tensor*.

CURVATURE, VECTOR. See **vector curvature**.

CURVE. For the purposes of applied mathematics, a curve in n -dimensional Euclidean space may be most conveniently defined as the locus of a point moving with one degree of freedom; that is, its path is defined with the aid of one parameter, $x_1 = x_1(t)$, $x_2 = x_2(t)$, \dots , $x_n = x_n(t)$, with $a \leq t \leq b$. If $x_1(a) = x_1(b)$, $x_2(a) = x_2(b)$, \dots , $x_n(a) = x_n(b)$, the curve is *closed*. If, with this possible exception, distinct values of t produce distinct points, the curve is *simple*. If the total length of an inscribed polygonal line approaches a finite limit as the length of each side approaches zero, the curve is *rectifiable*.

CURVE, CENTER OF. See **centroid** (of a geometrical figure).

CURVED BEAM. See **beam, curved**.

CURVED TUBES. See **tubes, curved**.

CURVE FITTING. Suppose that a physical variable y is known to bear a particular functional relation to another variable x :

$$y = f(x; \alpha, \beta, \dots),$$

where the form of f is known, but where f contains certain parameters α, β, \dots , that are unknown. Thus in radioactive decay the amount of a constituent is known to be an exponential function of time, $y = \alpha \exp(-\beta t)$, where t takes the place of the above x . If it is possible to measure the y associated with any x , one can make a series of measurements, x_i, y_i , and hope to solve the equations

$$y_i = f(x_i; \alpha, \beta, \dots)$$

for the parameters. Since, in general, the measurements will be subject to error one must make more measurements than there are parameters, and since the equations will then be inconsistent, in general, it is necessary to establish some criterion for making the "best" selection of the parameters so that, in some sense, all equations are satisfied as nearly as possible, even though possibly none is satisfied exactly.

A possible criterion is the **minimax approximation**: to select the parameters so as to minimize the greatest of the quantities

$$|y_i - f(x_i; \alpha, \beta, \dots)|.$$

This is sometimes used, but is very difficult computationally in most cases. Sometimes one resorts to trial and error, with visual inspection of the graphs. The method of **least squares** can often be justified on grounds of probability, and usually leads to the simplest computations.

The same mathematical problem arises when a known function $\phi(x)$ is to be approximated in some fashion by a simpler one. For this purpose **interpolation** is the most usual method of procedure.

Sometimes one has only the empirical measurements x_i, y_i , but no theory to prescribe a particular functional form $f(x, \dots)$. In that case one can only apply intuition, and possibly trial and error, in selecting a convenient form.

(See **smoothing**.) (See Leo Nielsen Kaj, *Methods in Numerical Analysis*, Macmillan, 1956.)

CURVE, INTRINSIC EQUATIONS OF. A curve can be completely specified, apart from a rigid motion, by specifying the dependence of the **curvature** and **torsion** on the distance measured along the curve from a fixed point. The equations expressing this dependence are called the *intrinsic equations of the curve*.

CURVE, LOXODROMIC. See **loxodrome**.

CURVE, SKEW. See **twisted curve**.

CURVE, SPACE. See **space curve**.

CURVES, SYSTEM OF CONJUGATE. See **conjugate system of curves on a surface**.

CURVE, TWISTED. See **twisted curve**.

CURVILINEAR CONGRUENCE. See **congruence of curves**.

CURVILINEAR MOTION. Unless the resultant force \mathbf{F} on a particle of mass m is constant in direction and the velocity of the particle is initially in the same direction, curvilinear motion results. Sometimes it is convenient to write Newton's second law (as for the motion of a particle of mass m in a plane) $\mathbf{F} = m\mathbf{a} = m\ddot{\mathbf{r}}$ in Cartesian form $F_x = m\ddot{x}$, $F_y = m\ddot{y}$, $F_z = m\ddot{z}$, where the two dots denote the second derivative with respect to time. Often it is better to use intrinsic coordinates (see **acceleration**) and write $\mathbf{F} = m\mathbf{a}_t + m\mathbf{a}_n = m \frac{dv}{dt} \mathbf{T} + m \frac{v^2}{r} \mathbf{N}$, where \mathbf{T} and \mathbf{N} are unit vectors along the tangent and inward normal, respectively.

CURVILINEAR ORTHOGONAL COORDINATES. A curvilinear system of coordinates α, β, γ is generated by a system of three mutually orthogonal families of surfaces $\alpha = \text{constant}$, $\beta = \text{constant}$, $\gamma = \text{constant}$. It is assumed that the unit tangent vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$ along the coordinate curves form everywhere a right-handed system, i.e., $\mathbf{a} \times \mathbf{b} = \mathbf{c}$.

An infinitesimal line vector $d\mathbf{l}$ is given by the expression

$$d\mathbf{l} = \mathbf{a}M d\alpha + \mathbf{b}N d\beta + \mathbf{c}P d\gamma.$$

A volume element dV is given by

$$dV = MNP d\alpha d\beta d\gamma.$$

A vector surface element whose normal is along the \mathbf{a} direction is given by

$$d\mathbf{S} = \mathbf{a}NPd\beta d\gamma.$$

The unit vectors \mathbf{a} , \mathbf{b} , \mathbf{c} are, of course, not constant. Their derivatives are:

$$\frac{\partial \mathbf{a}}{\partial \alpha} = -\frac{\mathbf{b}}{N} \frac{\partial M}{\partial \beta} - \frac{\mathbf{c}}{P} \frac{\partial M}{\partial \gamma}; \quad \frac{\partial \mathbf{b}}{\partial \alpha} = \frac{\mathbf{a}}{N} \frac{\partial M}{\partial \beta};$$

$$\frac{\partial \mathbf{c}}{\partial \alpha} = \frac{\mathbf{a}}{P} \frac{\partial M}{\partial \gamma};$$

$$\frac{\partial \mathbf{a}}{\partial \beta} = \frac{\mathbf{b}}{M} \frac{\partial N}{\partial \alpha}; \quad \frac{\partial \mathbf{b}}{\partial \beta} = -\frac{\mathbf{a}}{M} \frac{\partial N}{\partial \alpha} - \frac{\mathbf{c}}{P} \frac{\partial N}{\partial \gamma};$$

$$\frac{\partial \mathbf{c}}{\partial \beta} = \frac{\mathbf{b}}{P} \frac{\partial N}{\partial \gamma};$$

$$\frac{\partial \mathbf{a}}{\partial \gamma} = \frac{\mathbf{c}}{M} \frac{\partial P}{\partial \alpha}; \quad \frac{\partial \mathbf{b}}{\partial \gamma} = \frac{\mathbf{c}}{N} \frac{\partial P}{\partial \beta};$$

$$\frac{\partial \mathbf{c}}{\partial \gamma} = -\frac{\mathbf{a}}{M} \frac{\partial P}{\partial \alpha} - \frac{\mathbf{b}}{N} \frac{\partial P}{\partial \beta}.$$

The vector differential operator is defined by

$$\nabla = \frac{\mathbf{a}}{M} \frac{\partial}{\partial \alpha} + \frac{\mathbf{b}}{N} \frac{\partial}{\partial \beta} + \frac{\mathbf{c}}{P} \frac{\partial}{\partial \gamma}$$

$$\nabla \cdot \mathbf{A} = \text{div } \mathbf{A} = \frac{1}{MNP} \left\{ \frac{\partial(A_1NP)}{\partial \alpha} + \frac{\partial(A_2PM)}{\partial \beta} + \frac{\partial(A_3MN)}{\partial \gamma} \right\}$$

$$\nabla \times \mathbf{A} = \text{curl } \mathbf{A} = \frac{\mathbf{a}}{NP} \left\{ \frac{\partial(PA_3)}{\partial \beta} - \frac{\partial(NA_2)}{\partial \gamma} \right\}$$

$$+ \frac{\mathbf{b}}{PM} \left\{ \frac{\partial(MA_1)}{\partial \gamma} - \frac{\partial(PA_3)}{\partial \alpha} \right\}$$

$$+ \frac{\mathbf{c}}{MN} \left\{ \frac{\partial(NA_2)}{\partial \alpha} - \frac{\partial(MA_1)}{\partial \beta} \right\}$$

$$\nabla f = \text{grad } f = \frac{\mathbf{a}}{M} \frac{\partial f}{\partial \alpha} + \frac{\mathbf{b}}{N} \frac{\partial f}{\partial \beta} + \frac{\mathbf{c}}{P} \frac{\partial f}{\partial \gamma}$$

$$\nabla^2 f = \frac{1}{MNP} \left\{ \frac{\partial \left(\frac{NP}{M} \frac{\partial f}{\partial \alpha} \right)}{\partial \alpha} + \frac{\partial \left(\frac{PM}{N} \frac{\partial f}{\partial \beta} \right)}{\partial \beta} \right.$$

$$\left. + \frac{\partial \left(\frac{MN}{P} \frac{\partial f}{\partial \gamma} \right)}{\partial \gamma} \right\}.$$

The operator $\mathbf{A} \cdot \nabla$ is defined by

$$\mathbf{A} \cdot \nabla = \frac{A_1}{M} \frac{\partial}{\partial \alpha} + \frac{A_2}{N} \frac{\partial}{\partial \beta} + \frac{A_3}{P} \frac{\partial}{\partial \gamma}.$$

Examples:

(1) Cartesian coordinates $M = N = P = 1$;
 $\alpha = x$; $\beta = y$; $\gamma = z$.

(2) Cylindrical coordinates $\alpha = r$; $\beta = \phi$;

$\gamma = z$; $M = 1$; $N = r$; $P = 1$; $\frac{\partial \mathbf{a}}{\partial \phi} = \mathbf{b}$;

$$\frac{\partial \mathbf{b}}{\partial \phi} = -\mathbf{a}.$$

(3) Spherical coordinates $\alpha = \phi$; $\beta = \theta$; $\gamma = r$;

$M = r$; $N = r \sin \phi$; $P = 1$; $\frac{\partial \mathbf{a}}{\partial \phi} = -\mathbf{c}$;

$$\frac{\partial \mathbf{c}}{\partial \phi} = \mathbf{a}; \quad \frac{\partial \mathbf{a}}{\partial \theta} = \mathbf{b} \cos \phi; \quad \frac{\partial \mathbf{b}}{\partial \theta} = -\mathbf{c} \sin \phi - \mathbf{a}$$

$$\cos \phi; \quad \frac{\partial \mathbf{c}}{\partial \theta} = \mathbf{b} \sin \phi.$$

If a vector field \mathbf{A} satisfies the condition

$$\mathbf{A} \cdot \text{curl } \mathbf{A} \equiv 0,$$

it is possible to construct a family of surfaces that is everywhere perpendicular to the vector lines \mathbf{A} . It is therefore possible to choose these surfaces as the family $\gamma = \text{constant}$ and the \mathbf{A} lines themselves as the coordinate lines perpendicular to this family, i.e., $\mathbf{A} \equiv \mathbf{c}A_3$ ($|\mathbf{A}| = |A_3|$)

$$A_1 = A_2 \equiv 0.$$

Let an element of arc length along the \mathbf{A} -lines be denoted by dl and R_1 and R_2 be the principal radii of curvature of a surface element perpendicular to \mathbf{c} . Then

$$\text{div } \mathbf{A} = \frac{\partial A_3}{\partial l} + A_3 \left(\frac{1}{R_1} + \frac{1}{R_2} \right).$$

If $\mathbf{A} = \text{grad } f$, then

$$A_3 = \frac{\partial f}{\partial l}$$

and

$$\text{div } \mathbf{A} = \nabla^2 f = \frac{\partial^2 f}{\partial l^2} + \frac{\partial f}{\partial l} \left(\frac{1}{R_1} + \frac{1}{R_2} \right)$$

and

$$\text{curl } \mathbf{A} = |\mathbf{A}|k\mathbf{B} - [\mathbf{T} \times \text{grad } |\mathbf{A}|]$$

where k is the curvature of \mathbf{A} lines

\mathbf{T} is the unit tangent vector along \mathbf{A} lines

\mathbf{B} is the binormal-unit vector of the \mathbf{A} lines.

CUSP. A **singular point** on a curve where there are two coincident tangents. If there is a branch of the curve on each side of the double tangent, the cusp is of the first kind (e.g., the **semicubical parabola**); if the two branches lie on the same side of the double tangent, the cusp is of the second kind. If the curve is represented by $f(x,y) = 0$, the condition for a cusp, and also for a point of osculation, is

$$\left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 - \frac{\partial^2 f}{\partial x^2} \frac{\partial^2 f}{\partial y^2} = 0.$$

CUT-OFF. (1) In a reciprocating steam engine, the point at which the supply of steam to the cylinder is arrested. (2) In a **diesel engine**, the point at which fuel injection ceases. Ideally, combustion also ceases at this point. It is measured by the *cut-off ratio*

$$\rho = \frac{V_c + V_i}{V_c}$$

where V_c is the volume at the end of compression, V_i the volume increase due to combustion, and hence $V_c + V_i$ is the volume at fuel cut-off.

CUT-OFF FREQUENCY. (1) Of a wave filter, the frequency at which the attenuation begins to increase sharply. (2) For a given characteristic field configuration (mode) in a non-dissipative waveguide, the frequency below which the **propagation constant** becomes real.

CUT-OFF RATIO. See *cut-off*; **diesel engine**.

CUT SET. In the theory of **linear graphs** the concept of a cut set is almost as important as that of a **tree**. A cut set of a connected graph G is a set of **edges** such that the deletion of these edges reduces the **rank** of G by one. Moreover, no proper subset of this set possesses this property.

Clearly, the removal of the cut set of edges must yield an unconnected graph (see **graph, connected**) since the number of vertices v is invariant and $\text{rank } G = v - 1$. Thus by "cutting" this set of edges the graph is separated into two pieces. One of the pieces can be an isolated vertex. This latter case occurs for example if all the edges incident at a vertex are removed. As a matter of fact, the totality of edges incident at a given vertex is a cut set if and only if the vertex is not a **cut vertex**.

Another and deeper characterization of a cut set is as a minimal set of **elements** which contains at least one **branch** from every **tree**. Again, a single non-circuit element constitutes a cut set whereas a single **circuit** element does not.

CUT SET MATRIX. The cut set matrix $Q_a = (q_{ij})$ of a *directed graph* G is defined in the following manner:

- (a) Q_a has one row for each cut set of the graph and one column for each **edge**;
- (b) $q_{ij} = 1$ if edge j is in cut set i and the orientations agree (see **cut set, oriented**);
- (c) $q_{ij} = -1$ if edge j is in cut set i and the orientations disagree;
- (d) $q_{ij} = 0$ if edge j is not in cut set i .

The rank of Q_a is $v - 1$, v denoting the number of vertices in G .

The cut set matrix Q_a and **vertex matrix** A_a are rather intimately related. For example:

- (1) If G is **non-separable**, Q_a contains A_a (with some rows possibly multiplied by -1) as a submatrix.
- (2) If Q is a cut set matrix of $v - 1$ rows and rank $v - 1$ of a connected directed graph G of v vertices and A the vertex matrix of G ,

$$Q = DA$$

where D is non-singular.

- (3) Under the same restrictions as in 2, the non-singular submatrices of Q of order $v - 1$ are in one-to-one correspondence with the **trees** of G . That is to say, each such submatrix is the *fundamental cut set matrix* of some tree. Conversely, any fundamental cut set matrix appears as a submatrix of Q .

CUT SET, ORIENTED. Let G be a linear connected graph. (See **graph, connected**.) The removal of a **cut set** of edges decomposes G into two connected pieces A and B . The cut set is oriented by ordering A and B either as (A, B) or (B, A) . Each **element** of the cut set must have one vertex in A and one vertex in B . Suppose the cut set is oriented as (A, B) . Then, an oriented element of the cut set is said to have the same orientation as the cut set if it is directed away from its vertex in A and toward its vertex in B .

CUT SETS, FUNDAMENTAL. Suppose G is a linear connected graph (see **graph, connected**) possessing v vertices and T is one of its trees. Each cut set of G must contain at least one branch of T . Those $v - 1$ cut sets of G which include exactly one branch of T are said to be fundamental with respect to T . The orientation of a fundamental cut set of a directed graph (see **digraph**) is usually chosen to agree with that of the defining branch.

CUT VERTEX. See **vertex, cut**.

CYCLE. (1) The complete sequence of values of a periodic quantity which occur during a period. (2) A series of changes or processes executed in orderly sequence, by means of which a mechanism, a working substance, or a system is caused periodically to return to the same initial state. In particular consider a (closed) thermodynamic system, represented by the cycle

$$\oint dQ = \oint dW.$$

A cycle is represented by a closed (but not necessarily continuous) curve in a thermodynamic diagram. When $\oint dW$ is positive, the cycle produces work at the expense of heat. It is then called a *forward* or *power cycle*. When $\oint dW$ is negative, the cycle consumes work; it can then be used as a refrigerator or a heat pump; it is called a *reversed cycle*.

The *thermal efficiency* of a power cycle is defined as the ratio of the work produced $W =$

$\oint dW$ to the positive heat added to the cycle,

$Q \neq \oint dQ$ but $Q = \int (dQ)_{pos}$. Hence

$$\eta = \frac{W}{Q}.$$

If the total amount of heat $\oint dQ$ is represented as

$$\oint dQ = Q - Q_o, \quad Q_o > 0, \text{ is the sum of negative heats,}$$

we have

$$W = Q - Q_o$$

and

$$\eta = 1 - \frac{Q_o}{Q}.$$

The operation of a reversed cycle is described by the coefficient of performance

$$e = \frac{Q_o}{W}$$

when it is used for refrigeration. The quantity of heat Q_o absorbed by the reversed (refrigerating) cycle is known as the *refrigerating effect*. In the case of a heat pump no generally accepted single coefficient describing its efficiency is available.

CYCLE, MAJOR. In a storage device which provides serial access to storage positions, the time interval between successive appearances of a given storage position.

CYCLE, MINOR. In a storage device which provides serial access to storage positions, the time interval between the appearance of corresponding parts of successive words.

CYCLE OF STRESS. The stress variation on a particular plane through a specific point in a body which is subjected to a repeated load is called a cycle of stress. If the stress varies alternately between **tension** and **compression**, the variation is known as reversal of stress. The reversal is complete when the alternate stresses are equal in magnitude.

The algebraic difference between the maximum and minimum stresses of a cycle is the range of stress. The endurance limit depends on the range of stress.

CYCLE, THERMAL EFFICIENCY OF. See **thermal efficiency of cycle**.

CYCLIC. See **permutation; group**.

CYCLING. A periodic change of the controlled variable in an automatic controller.

CYCLOID. A cycloid is the plane curve traced by the motion of a point fixed on the circumference of a rolling wheel. If the wheel turns at the rate of one radian per minute, then the equations of the cycloid, using the time t as a parameter, are

$$x = a(t - \sin t), \quad y = a(1 - \cos t),$$

where a is the radius of the wheel. (See also **brachistochrone**.)

The teeth of gears are often cut with faces which are arcs of cycloids, so that there is rolling contact where the gears are in mesh.

CYCLOMATIC NUMBER. The same as the nullity of a graph.

CYCLONIC MOTION. Circulatory motion relative to the earth in the direction of the vertical component of the earth's rotation; anticlockwise when viewed from above in the northern hemisphere. Such motion is produced in the atmosphere by horizontal convergence in air at rest, the vortex lines being stretched vertically.

CYCLONIC VORTICITY. Vorticity relative to the earth in a cyclonic direction, i.e., in the direction of the vertical component of the earth's rotation.

CYCLOSTROPHIC FLOW. A form of gradient flow in which the centripetal acceleration exactly balances the horizontal pressure force. (See **cyclostrophic wind**.)

CYCLOSTROPHIC FORCE. The centrifugal force due to the curvature of the path of the air over the earth's surface. (See **gradient wind**.)

CYCLOSTROPHIC WIND. That horizontal wind velocity for which the centripetal acceleration exactly balances the horizontal pressure force:

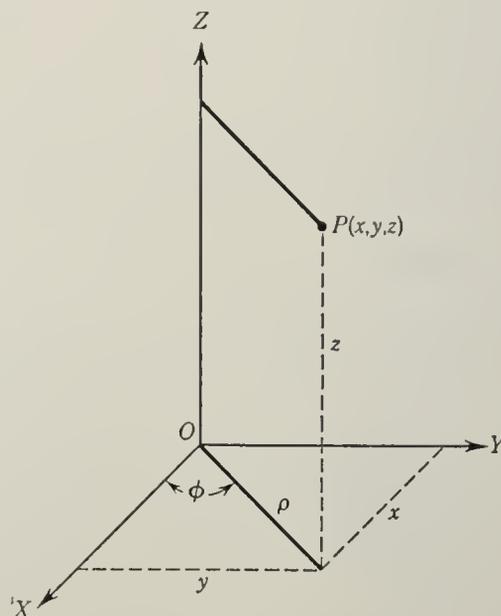
$$\frac{V_c^2}{R} = -\alpha \frac{\partial p}{\partial n}$$

where V_c is the cyclostrophic wind speed, R the radius of curvature of the path, α the specific volume, p the pressure, and n the direction normal to the streamlines toward the center of curvature of the path.

The cyclostrophic wind can be an approximation to the real wind in the atmosphere only near the equator, where the Coriolis acceleration is small; or in cases of very great wind speed and curvature of the path, so that the centripetal acceleration is the dominant one. (See **Eulerian wind**.)

CYLINDRICAL COORDINATES. A curvilinear system ρ, ϕ, z , whose coordinate surfaces

consist of: right-circular cylindrical surfaces based on circles about the origin in the XY -plane ($\rho = \text{const.}$); half-planes from the Z -



Cylindrical coordinates.

axis ($\phi = \text{const.}$); planes parallel to the XY -plane ($z = \text{const.}$). The position of a point in this system is given by (ρ, ϕ, z) where

$$x = \rho \cos \phi; \quad y = \rho \sin \phi; \quad z = z.$$

More precisely the system should be called circular cylindrical, because *elliptic* and *parabolic* cylindrical coordinates are also used. For example, elliptic cylindrical coordinates are defined as follows. Consider the equations

$$\frac{x^2}{a^2 \cosh^2 u} + \frac{y^2}{a^2 \sinh^2 u} = 1$$

$$\frac{x}{a^2 \cos^2 v} - \frac{y^2}{a^2 \sin^2 v} = 1.$$

Elliptic cylindrical coordinates are then defined by the three systems of coordinate surfaces: elliptic cylinders, given by $u = \text{const.}$; hyperbolic cylinders, given by $v = \text{const.}$; planes parallel to the xy -plane.

CYLINDRICAL FUNCTION. Synonym of **Bessel function**.

D

D'ALEMBERTIAN. Analogous to the differential operator del in ordinary **vector** analysis, a four-dimensional operator called **quad**, and denoted by the symbol \square , has components $\partial/\partial x_i$. The contracted second-order **derivative** of a scalar function ϕ can then be written $\square^2\phi$. It is known as the d'Alembertian operator

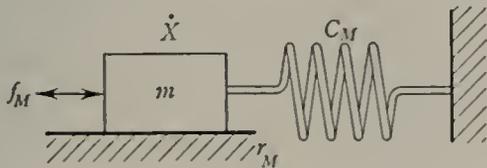
$$\square^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2} \frac{\partial^2}{\partial t^2}$$

and it is thus similar to the **Laplacian** operator in three dimensions. The equation $\square^2\phi = 0$ is a wave equation for waves traveling with the velocity of light, c . The d'Alembertian is Lorentz invariant. (See also Minkowski's world in **relativity, special theory of**.)

D'ALEMBERT PARADOX. A hydrodynamical paradox arising from the neglect of viscosity. This paradox may be traced to the neglect of viscous forces, which are those responsible for fluid resistance by modifying the velocity field close to a solid body.

D'ALEMBERT PRINCIPLE. The equations of motion of the form $\mathbf{F} = m\mathbf{a}$, $M = I\alpha$, etc., may be written $\mathbf{F} - m\mathbf{a} = 0$, etc. If $-m\mathbf{a}$ is called an **inertia** or reversed **effective force**, the sum of the forces is zero. Dynamics is thus reduced to statics by the d'Alembert principle. The equal and opposite internal forces which appear in a system of particles need not be considered. (See **inertia forces**.)

D'ALEMBERT PRINCIPLE APPLIED TO A MECHANICAL RECTILINEAL SYSTEM OF ONE DEGREE OF FREEDOM. The differential equations for mechanical systems



may be set up employing the d'Alembert principle; namely, the algebraic sum of the forces applied to a body is zero. The mechan-

ical forces due to the elements in a mechanical rectilinear system are

Mechanomotive force of mass reaction

$$= -m \frac{d^2x}{dt^2} \quad (1)$$

Mechanomotive force of mechanical

$$\text{rectilinear resistance} = -r_M \frac{dx}{dt} \quad (2)$$

Mechanomotive force of mechanical

$$\text{compliance} = -\frac{x}{C_M} \quad (3)$$

In addition to the above mechanomotive forces are the mechanomotive forces applied externally.

The above principle may be used to derive the differential equation of the mechanical rectilinear system of the figure. From the d'Alembert principle the algebraic sum of the forces applied to a body is zero. The equation may be written

$$m \frac{d^2x}{dt^2} + r_M \frac{dx}{dt} + \frac{x}{C_M} = F_M e^{j\omega t} \quad (4)$$

where $f_M = F_M e^{j\omega t}$ is the external applied mechanical force.

Equation 4 is the same as Equation 8 of **mechanical system of one degree of freedom**.

D'ALEMBERT PRINCIPLE APPLIED TO A MECHANICAL ROTATIONAL SYSTEM OF ONE DEGREE OF FREEDOM. The differential equations for mechanical rotational systems may be set up by employing the **d'Alembert principle**; namely, the algebraic sum of the applied torques applied to a body is zero. The rotational mechanical forces due to the elements in a mechanical rotational system are

Rotatomotive force of moment

$$\text{of inertia reaction} = -I \frac{d^2\phi}{dt^2} \quad (1)$$

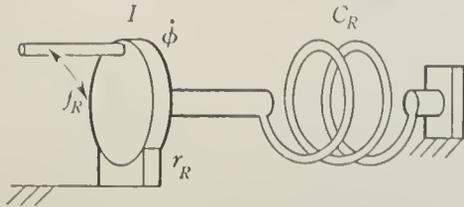
Rotatomotive force of mechanical

$$\text{rotational resistance} = -r_R \frac{d\phi}{dt} \quad (2)$$

Rotatomotive force of rotational compliance

$$= -\frac{\phi}{C_R} \quad (3)$$

In addition to the above rotatomotive forces are the rotatomotive forces applied externally.



Applying the d'Alembert principle the equation for the rotational system of the figure may be written

$$I \frac{d^2\phi}{dt^2} + r_R \frac{d\phi}{dt} + \frac{\phi}{C_R} = f_R \epsilon^{j\omega t} \quad (4)$$

where $f_R = f_R \epsilon^{j\omega t}$ is the external applied torque.

Equation 4 is the same as Equation 8 of mechanical rotational system of one degree of freedom.

D'ALEMBERT PRINCIPLE APPLIED TO AN ACOUSTICAL SYSTEM OF ONE DEGREE OF FREEDOM.

The differential equations for acoustical systems may be set up by employing the d'Alembert principle; namely, the algebraic sum of the applied pressures is zero. The acoustical pressures due to the elements in an acoustical system are

Acoustomotive force of inertive reaction

$$= -M \frac{d^2X}{dt^2} \quad (1)$$

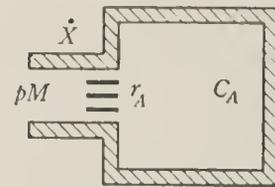
Acoustomotive force of acoustical resistance

$$= -r_A \frac{dX}{dt} \quad (2)$$

Acoustomotive force of acoustical capacitance

$$= -\frac{X}{C_A} \quad (3)$$

In addition to the above acoustomotive forces are the acoustomotive forces applied externally.



Applying the d'Alembert principle, the equation for the acoustical system of the figure may be written

$$M \frac{d^2X}{dt^2} + r_A \frac{dX}{dt} + \frac{X}{C_A} = P \epsilon^{j\omega t} \quad (4)$$

where $p = P \epsilon^{j\omega t}$ is the external applied pressure.

Equation 4 is the same as Equation 7 of acoustical system of one degree of freedom.

D'ALEMBERT TEST. If

$$\lim_{n \rightarrow \infty} |a_{n+1}/a_n| = r$$

and $r < 1$, the series

$$\sum_{n=1}^{\infty} a_n$$

converges; if $r > 1$, it diverges; if $r = 1$, the test fails and the series must be investigated in some other way (see convergence).

This test is often known as Cauchy's convergence test of the second kind but it was published by d'Alembert in 1768, seems to have been known to Waring in 1781, and was not given by Cauchy until 1821. It is essentially the same as the ratio test.

DALTON LAW. (Also known as the Gibbs-Dalton law.)

If a number of chemically inert gases are contained in the same space, each of them expands into the whole volume as if the other component were not present. The total pressure of the mixture is equal to the sum of the partial pressures of the individual gases. The partial pressure of an individual gas is that pressure which would prevail if it alone occupied the total volume.

The Dalton law has only asymptotic validity ($p \rightarrow 0$). Macroscopically, it can be considered an empirical law. Microscopically, it can be derived on the assumption that gases consist of point-like molecules which undergo perfectly elastic collisions but otherwise exert no force on each other.

In spite of its limited validity, the Dalton law forms the basis of all engineering calcula-

tions concerning gaseous mixtures, because more exact formulations are still lacking.

DAMPED VIBRATIONS. See **vibrations, damped.**

DAMPING. This term usually refers to the checking of a motion due to resistance, as by friction or similar cause. It is of especial significance in connection with the diminishing amplitude of an **oscillation**, as that of a pendulum swinging in the air, or that of the electricity vibrating in an oscillating circuit. Unless energy is supplied during each cycle, the amplitude of such a vibrator falls off at each successive oscillation by an amount commonly expressed in terms of the decrement, or *damping factor*, which is the ratio of any one amplitude to that next succeeding it in the same sense or direction.

In so-called *logarithmic damping*, this decrement is constant; in a simple oscillating electric circuit, the amplitude decays as $e^{-\delta t}$, where t is the time and δ , the logarithmic decrement, is a constant depending upon the effective resistance, inductance, and capacitance of the circuit. (See **electric oscillations and waves.**) An important instance of damping is found in the reading of an oscillating index, like a balance pointer, on a scale. If one may assume that the amplitude falls off by equal amounts at each swing (*linear damping*), in order to find the equilibrium position, one has only to average an even number of readings at one extreme and an odd number at the other, and then find the mean of the two averages. This linear damping assumption is an approximation to the relationship $e^{-\delta t} \doteq 1 - \delta t$ for small damping. (See **time-constant.**)

DAMPING, ACOUSTIC. When a sound wave travels through a medium it is attenuated by a number of different mechanisms. The four principal causes of attenuation are heat conduction, scattering, viscous friction, and elastic hysteresis. It is often difficult to separate the effects of viscous friction and elastic hysteresis, and the term internal friction is used to describe them jointly. (See **friction, internal.**)

DAMPING CAPACITY, SPECIFIC. See **specific damping capacity.**

DAMPING COEFFICIENT. See **oscillation, damped harmonic.**

DAMPING, CRITICAL. The value of **damping** of a single degree of freedom system above which free oscillatory motion ceases and the system has an asymptotic approach to equilibrium. In a system in which the variable x obeys the differential equation

$$a \frac{d^2x}{dt^2} + b \frac{dx}{dt} + cx = 0$$

the condition for critical damping is that

$$b = 2\sqrt{ac}.$$

DAMPING FACTOR. Defined in entry on **damping.**

DAMPING, LINEAR. Defined in entry on **damping.**

DAMPING, LOGARITHMIC. Defined in entry on **damping.**

DAMPING, OPTIMAL. That value of **damping**, with the damping ratio slightly less than unity, at which an indicating system such as a galvanometer will overshoot its final deflection by less than the desired uncertainty in reading the instrument. In general, optional damping results in faster readings than does critical damping, with no real loss in accuracy.

DAMPING RATIO. The ratio of the actual **damping** to the critical damping. (See **damping, critical.**)

DAMPING, STRUCTURAL. Damping caused by structural **impedance** in an oscillating mechanical system. It is expressed by the relationship:

$$C = \frac{F}{e_F} \left(\frac{X_F}{X_o} \right)^2 = 2 \frac{c}{c_c}$$

where c is the damping coefficient, c_c is the critical damping coefficient, F is the force, e_F is the velocity of the force, X_F is the displacement of the force, X_o is the zero frequency deflection, and $2 \frac{c}{c_c}$ is the structural damping.

DAMPING, VISCOUS. The damping in a mechanical system in which the force varies in proportion to the velocity. The viscous-damping coefficient is equal to the ratio of the viscous-damping force to the velocity.

DANCOFF CORRECTIONS. In nuclear reactor theory, quantities which enable one to estimate interaction effects among the fuel rods comprising a lattice. The Dancoff correction may be written as $1 - T_1$ where T_1 a *moderator transmission factor*, expresses the probability that a neutron emerging from one fuel rod will be transmitted through the intervening moderator, without colliding, to impinge upon a nearest neighbor rod.

DANDELIN METHOD. A **root-squaring method** for solving **algebraic equations** in which the equation $\phi(x) = 0$ is replaced by

$$\phi(\sqrt{x})\phi(-\sqrt{x}) = 0,$$

differing only mildly from the later **Graeffe method**. Dandelin applied this first in conjunction with the **Newton method** for accelerating convergence, but also proposed repeated application as a method in itself.

DANGER COEFFICIENT. The danger coefficient of a substance for a particular nuclear reactor is the change in reactivity caused by inserting that substance in the reactor. The danger coefficient depends on the amount and distribution of the substance inserted and is usually quoted as reactivity change per unit mass for a standard position in the reactor.

DARBOUX VECTOR. The Darboux vector δ of a curve is defined as the vector

$$\delta = \tau\mathbf{T} + \kappa\mathbf{B}$$

where τ is the torsion, κ is the curvature, \mathbf{T} is the unit tangent vector along the positive sense of the curve, and \mathbf{B} is the unit binormal vector.

If ds denotes an element of arc length along the curves then

$$\frac{d\mathbf{T}}{ds} = \delta \times \mathbf{T}$$

$$\frac{d\mathbf{N}}{ds} = \delta \times \mathbf{N}$$

$$\frac{d\mathbf{B}}{ds} = \delta \times \mathbf{B}$$

where \mathbf{N} is the unit normal vector, i.e., $\mathbf{T} \times \mathbf{N} = \mathbf{B}$.

The Darboux vector can be interpreted kinematically. If a rigid body moves freely in space, its most general motion can be de-

scribed as a screw motion, i.e., the center of mass moves along a space curve and at the same time the "moving trihedron" $\mathbf{T}, \mathbf{N}, \mathbf{B}$ which is rigidly anchored to the center of mass, rotates about the Darboux vector. (The function of the curve must be continuously differentiable for the relations to apply.) In other words, the Darboux vector coincides with the instantaneous axis of rotation. If ω denotes the instantaneous angular velocity vector and v the instantaneous speed of the center of mass, then

$$\omega = v\delta.$$

DARCY. A unit, the permeability of a medium through which a liquid of one centipoise viscosity will flow at a rate of one cubic centimeter per square centimeter of area when the pressure gradient is one atmosphere per centimeter.

DARCY'S LAW. A law which governs the flow of a viscous fluid through a porous medium. The flow is supposed sufficiently slow for the inertia forces to be negligible (i.e., **Stokes flow**, with low **Reynolds number**), and is therefore in the direction towards decreasing pressure and proportional to the pressure gradient. Thus

$$\mathbf{v} = -\frac{k}{\mu} \text{grad } p$$

where k is a constant of the medium. In a medium of constant k and fluid of constant viscosity μ , there is a **velocity potential** given by

$$\phi = -\frac{k}{\mu} p.$$

DARK-LINE SPECTRUM. See **spectrum**, **dark-line**.

DARWIN-FOWLER METHOD. See **method of steepest descents**.

DAVISSON CHART. A chart expressing thermionic-emission current density in terms of cathode power dissipation. The coordinates are skewed in a manner which causes the log of current density as a function of the log of power to be a straight line.

DAY, JULIAN. See **Julian Day**.

DBA. Abbreviation for *dbRN adjusted*. This is a unit used to show the relationship between the interfering effect of a noise fre-

quency or band of noise frequencies, and a fixed amount of noise power commonly called *reference noise*. (See **dbRN**.)

DBRN. Abbreviation for *decibels above reference noise*. This is a unit used to show the relationship between the interfering effect of a noise frequency, or a band of noise frequencies, and a fixed amount of noise power commonly called *reference noise*. A tone of 1000 cycles per second, having a power level of -90 dbm, was selected as the reference noise power because it appeared to have negligible interfering effect, and would permit measurement of interfering effect in positive numbers. By way of explanation, dbRN was originally determined by measuring the interfering effect of noise in telephone circuits which used Type 144 handsets. Later, an improved type of handset (Western Electric Type F1A) came into general use. This required an adjustment of the reference noise level and a different unit. The reference level was changed to -85 dbm and the new unit was called **dba**.

DBX. Abbreviation for *decibels above reference coupling*. *Reference coupling* is defined as the coupling between two circuits that would be required to give a reading of 0 **dba** on a 2-type noise measuring set connected to the disturbed circuit when a test tone of 90 **dba** (using the same weighting as that used on the disturbed circuit) is impressed on the disturbing circuit.

DEAD LOAD. The load due to the weight of the permanent parts of a structure, the immovable part of the **total load**.

DEAD RECKONING (NAVIGATION). The determination of the position of a ship by advancing a known, or assumed, position by any of the methods of the sailings, for the courses and distances traversed by the ship in a given interval of time. In the determination of a dead reckoning position (*DR*), the true headings (i.e., corrected magnetic or corrected gyro) and speed through the water are used.

If the navigator can estimate the effects of the current and winds on the direction and distance of travel of the ship in the given interval, these may be applied to the *DR* position to obtain what is called an *estimated position* (*EP*).

Within recent years several types of dead reckoning equipment have been devised that

receive information from the compass and from the patent log and transform that to a tracing that shows the *DR* position continuously. However, such devices are seldom as accurate as the plot kept by the navigator. (Cf. the sailings.)

DE BROGLIE HYPOTHESIS. The hypothesis that a wave is associated with material particles, with the wavelength, λ , and momentum $p = mv$, of the particle related by $\lambda = h/p$. At high energies the relativistic mass

$$m = m_0/\sqrt{1 - v^2/c^2}$$

must be used.

DE BROGLIE WAVELENGTH. See **de Broglie hypothesis**.

DEBYE EFFECT. A selective absorption of electromagnetic waves in a dielectric medium, due to molecular dipoles. (See **Debye energy**; **induction forces**.)

DEBYE ENERGY. This correction to the dipole-dipole (Keesom) interaction energy results from the interaction of the permanent dipole μ of one molecule with the induced dipole of another molecule with a **polarizability** α .

The electrical field strength ϕ at a distance r from the dipole is proportional to μ/r^3 . It induces in the other molecule an induced dipole moment

$$u_i \approx \alpha\phi \approx \alpha\mu/r^3.$$

The total interaction energy due to this induction effect is

$$\epsilon_D = \mu_i\phi \approx \alpha\mu^2/r^6.$$

The *Debye*, or *induction energy*, is practically independent of temperature. It is, however, usually smaller than the **Keesom energy**. Neither the Keesom nor the Debye energy is capable of interpreting the attractive forces between non-polar molecules. (See **dispersion forces**, **induction forces**.)

DEBYE EQUATION FOR THE DISPERSION OF THE DIELECTRIC CONSTANT. Whereas for static fields the dielectric constant is a real number, for periodic fields it is usually a complex quantity (see **dielectric constant**; **electrical displacement in dielectrics**). Debye has shown that, if in a static

field, equilibrium is reached exponentially in time, then

$$\epsilon(\omega) - \epsilon_\infty = \frac{(\epsilon_s - \epsilon_\infty)}{1 - i\omega\tau}. \quad (1)$$

$\epsilon(\omega)$ is the dielectric constant in a periodic electric field $\mathbf{E} = \mathbf{E}_0 \cos \omega t$. ϵ_∞ and ϵ_s are the limiting values of $\epsilon(\omega)$ for very large and zero frequencies, respectively. τ is the relaxation time of the dielectric.

The real and imaginary parts of the dielectric constant are respectively:

$$\epsilon_1(\omega) - \epsilon_\infty = \frac{\epsilon_s - \epsilon_\infty}{1 + \omega^2\tau^2}$$

and

$$\epsilon_2(\omega) = \frac{(\epsilon_s - \epsilon_\infty)\omega\tau}{1 + \omega^2\tau^2}.$$

The last equation shows that the dielectric losses (proportional to ϵ_2) vanish for very high and very low frequencies, and present a maximum value $\epsilon_2(\omega_m) = (\epsilon_s - \epsilon_\infty)/2$ for an angular frequency $\omega_m = 1/\tau$.

The condition that equilibrium be reached exponentially in time is in general valid only for dilute solutions of dipolar molecules.

DEBYE EQUATION OF STATE (OF SOLIDS). The relation between pressure p , and volume V ,

$$p = \frac{\partial U_0}{\partial V} + \gamma \frac{U_D}{V}$$

where γ is Grüneisen's constant, U_0 is the internal energy at 0°K, and U_D is the contribution to the internal energy due to the lattice vibrations.

DEBYE FREQUENCY. In his derivation of an equation for specific heat, Debye treated a crystalline material as an homogeneous isotropic solid to which classical vibration laws applied. He was then able to calculate the number of vibrations with frequencies between ν and $\nu + d\nu$. The atomic nature of the structure was then accounted for by the postulate of a maximum frequency, ν_0 , of the order of the atomic frequency. ν_0 is the Debye frequency, and is characteristic of a given solid. (See Debye temperature.)

DEBYE HEAT CAPACITY EQUATION. The Debye theory considers the specific heat of a solid as due to thermal vibrations of the crystal lattice, and the molar heat capacity

C_v is given by this theory in terms of the Debye temperature Θ , the relation being

$$C_v = 3Nkf(\Theta/T)$$

where

$$f(x) = 3x^{-3} \int_0^x \frac{e^y y^4}{(e^y - 1)^2} dy$$

(the Debye function).

N is Avogadro's number and k , the Boltzmann constant.

At high temperatures, this relation leads to the Dulong and Petit law $C \rightarrow 3Nk$, and at low temperatures to the Debye T^3 law.

DEBYE-HÜCKEL THEORY. Debye and Hückel have developed a semi-phenomenological theory to take account of the effect of electrostatic forces in such media as solutions of strong electrolytes or plasmas.

It is based on a combination with the Boltzmann formula, of the Poisson equation

$$\nabla^2 \Psi = -\frac{4\pi}{\epsilon} \rho \quad (1)$$

where ρ is the charge density, Ψ , the electrostatic potential, ϵ , the dielectric constant. If one combines these two relations, assumes the electrical potential energy to be small in respect to thermal energy and uses the electro-neutrality condition, one obtains the basic equation of the Debye-Hückel theory

$$\nabla^2 \Psi = \kappa^2 \Psi(r). \quad (2)$$

$\Psi(r)$ is the electric potential at a distance r around some central ion, $1/\kappa$ is a characteristic length, called the Debye length; it is the distance at which the field of the central ion is screened by the other charges to $1/e$ of its value. It is also often called the mean thickness of the ionic atmosphere.

$$\kappa^2 = \frac{4\pi \sum_i N_i z_i^2 e^2}{\epsilon k T}. \quad (3)$$

N_i is the number of ions of species i , z_i is the charge number, and e , the charge of a proton.

The spherically symmetrical solution of (2) which is finite at $r = \infty$ is

$$\Psi(r) = \frac{A}{r} e^{-\kappa r} \quad (4)$$

where A is a constant.

From (4) all thermodynamic properties can easily be deduced, as well as the main trans-

port properties in solutions of strong electrolytes. The main success of the Debye-Hückel theory is the calculating of the **limiting law** for strong electrolytes.

DEBYE LENGTH. See Debye-Hückel theory.

DEBYE TEMPERATURE. A parameter Θ having the dimensions of temperature appearing in the Debye theory of specific heats. It is defined by the equation

$$\Theta = \frac{h\nu_0}{k}$$

where h is Planck's constant, k is the Boltzmann constant, and ν_0 is the **Debye frequency** and is characteristic of a given solid. In the Debye treatment, atomic vibrations are treated as elastic waves, so that ν_0 can be related to the velocities of these waves in the solid. Thus,

$$\nu_0^3 = \frac{9N}{4\pi V} \left/ \left(\frac{2}{c_2^3} + \frac{1}{c_1^3} \right) \right.$$

where c_1 and c_2 are the velocities of longitudinal and transverse elastic waves respectively, V is the volume of the crystal, and N the total number of atoms in the crystal.

DEBYE T^3 LAW. According to the Debye theory of specific heat, the molar specific heat of a solid C_v should tend to be proportional to the cube of the absolute temperature at temperatures below about $\Theta/10$ where Θ is the **Debye temperature**. The expression for C_v is then

$$C_v = \frac{12\pi^4 Nk}{5} (T/\Theta)^3$$

where N is Avogadro's number and k is the Boltzmann constant.

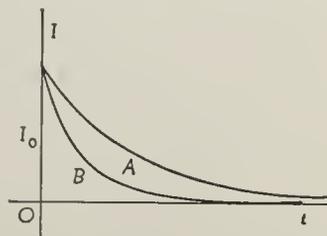
DEBYE THEORY OF CRYSTAL VIBRATIONS. A theory which assumes a spectrum for lattice vibrations similar to that of an elastic continuum, except that it is cut off at a maximum frequency in such a way that the total number of degrees of vibrational modes is equal to the total number of **degrees of freedom** of the lattice.

DEBYE UNIT. A unit of electrical dipole moment equal to 10^{-18} electrostatic units. It is of the order of magnitude of the electronic charge multiplied by the interatomic distance

of chemical compounds. (See **dipole, electrical**.)

DEBYE-WALLER EQUATION. See **thermal motions in a lattice**.

DECAY COEFFICIENT. Certain processes found in nature progress at a rate diminishing in accordance with an exponential function of the time; such, for example, as phosphorescence and radioactive emission. The falling



Typical exponential decay curves. The decay coefficient in *B* is greater than in *A*.

off or "decay" of such a process may be represented by an equation giving the intensity at time t as $I = I_0 e^{-Ct}$, in which I_0 is the intensity at the beginning of the time t and C is the "decay coefficient." Closely related to C is the half-value period, which is the time required for I to fall to $1/2$ its original value I_0 ; it is equal to $0.69315/C$. Thus, if the half-value period of radium B is 1608 sec, its decay coefficient C is

$$\frac{0.69315}{1608 \text{ sec}} = 0.000431/\text{sec.}$$

This means that approximately 0.000431 of the substance existing at any instant disintegrates during the ensuing second. The reciprocal of C , called the "decay modulus," represents the time required for I to diminish to $1/e$ or 0.3697 of its original value I_0 . It is equal to 1.4427 times the half-value period; for the decay of radium B its value is therefore 2320 sec.

This relationship may also be written in terms of the number of atoms present of a particular species, as

$$N = N_0 e^{-\lambda t}.$$

Thus, the decay law is a statistical law, so that if N is the number of radioactive atoms present, the number of which will disintegrate on the average in unit time is λN . The number which will disintegrate in any particular unit of time may not be exactly λN , but if a

large number of measurements of the number of disintegrations per unit time is made, the values will show a **Poisson distribution** with λN as the average value.

DECAY CONSTANT. See **decay coefficient**; **decay constant**, **acoustic**.

DECAY CONSTANT, ACOUSTIC. The exponential power by which the sound decays after the source is stopped.

DECAY MODULUS. In a damped harmonic oscillator, the time for the amplitude of oscillation to diminish to $1/e$ of its initial value is called the decay modulus. For an oscillator with the equation of motion

$$m\ddot{x} + R\dot{x} + fx = 0$$

the decay modulus is $2m/R$. (See **oscillation**, **damped**.)

DECAY TIME OF A MAGNETIC FIELD. The characteristic time of decay of a **magnetic field** due to **Joule** heating in a conductor or a plasma is given by

$$\tau = \frac{4\pi L^2 \sigma / C}{c},$$

where σ is the electrical conductivity in Gaussian units, L is a length such that $L = H/|\nabla H|$, H being the magnitude of the magnetic field, and c is the speed of light. (See **relaxation**.)

DECIBEL. See **gain ratio**.

DECIBELS ABOVE REFERENCE COUPLING. See **DBX**.

DECIBELS ABOVE REFERENCE NOISE. See **DBRN**.

DECIMAL. Characterized by ten; compounded of ten parts. The decimal system of number representation is the ordinary one using the **radix** 10.

DECISION FUNCTION. In statistics, a function of observations determining which way a decision should go according to the values which the function attains.

DECLINATION (ASTRONOMIC). The declination of a heavenly body is the angle measured north or south from the celestial equator along the hour circle through the object to the object. Declination is sometimes referred to as $+$ for north declination and $-$ for south

declination. In using this coordinate, care must be exercised to indicate the proper side of the equator, e.g., N 35° , 35° N, $+35^\circ$.

Star catalogues give the declination of the star for the epoch of the catalogue. Due to **precession** and **nutation**, the declinations are continually changing. Constants to facilitate the reduction of position given in the catalogue to those of another date are published for every star.

DECODER. In computer terminology, a network or system in which a combination of inputs is excited at one time to produce a single output. Sometimes called *matrix*.

DECOMPOSITION OF A VECTOR. Every **vector field** $\mathbf{B}(x,y,z)$ can be decomposed into a sum of a **gradient** and a **curl**, i.e., $\mathbf{B} = \mathbf{N} + \mathbf{E}$, where $\mathbf{N} = \text{grad } f$ and $\mathbf{E} = \text{curl } \mathbf{D}$.

The scalar field f and the vector field \mathbf{D} satisfy the following equations:

$$f = -\frac{1}{4\pi} \iiint_V \frac{\text{div } \mathbf{B}}{r} dV + \frac{1}{4\pi} \oiint_S \frac{\mathbf{B} \cdot \mathbf{n}}{r} dS$$

$$\mathbf{D} = \frac{1}{4\pi} \iiint_V \frac{\text{curl } \mathbf{B}}{r} dV + \frac{1}{4\pi} \oiint_S \frac{\mathbf{B} \times \mathbf{n}}{r} dS$$

r is the distance from the point at which \mathbf{B} is computed to a surface element dS or volume element dV . S is the surface boundary of the volume V ; \mathbf{n} is the outward normal of S .

DEDEKIND CUT. A Dedekind cut in the rational numbers is any division of the set of rational numbers into two subsets A and B such that: (1) neither A nor B is empty; (2) every rational number is either in A or in B ; (3) every number in B is greater than any number in A ; (4) A contains no largest number.

If B contains no smallest number, the cut defines an irrational number.

DE DONDER AFFINITY. See **chemical affinity**.

DE DONDER INEQUALITY. See **chemical affinity**.

DEEP BEAM. See **beam**, **deep**.

DEFECT. A term used to denote various types of point imperfections in solids, such as vacancies, interstitial atoms, etc., as distinct from extended imperfections such as dislocations. Lattice defects are particularly im-

portant in ionic crystals, where they may be created by heating in the vapor of one constituent, thus creating a stoichiometric excess, by bombardment with **x-rays** and energetic particles, etc. Any crystal must contain a certain equilibrium concentration of defects, as a function of the temperature, purely as a result of thermal agitation (see **Frenkel defect**, **Schottky defect**). Defects are responsible for the **diffusion** of ions, ionic conductivity, and the complex phenomena related to color centers.

DEFERRED APPROACH TO THE LIMIT.

Let ϕ represent a quantity to be calculated by a finite difference method (e.g., an integral); let ϕ_h represent the approximation obtained when $\Delta x = h$. Often it is approximately true that

$$\phi_h = \phi_0 + \phi_1 h^2 + \phi_2 h^4 + \dots$$

Let ϕ_{h_1} and ϕ_{h_2} represent similarly values obtained with $h = h_1$ and $h = h_2$. If h_1 and h_2 are small enough so that terms beyond h^2 can be neglected, then

$$\phi_{h_1, h_2} \equiv |h_1^2 \phi_{h_2} - h_2^2 \phi_{h_1}| / |h_1^2 - h_2^2|$$

is a much better approximation to ϕ than is either ϕ_{h_1} or ϕ_{h_2} . Commonly one takes $h_2 = h_1/2$.

DEFINITE. See **form**, **integral**.

DEFINITION CIRCLE. In **impedance matching**, a circle of constant standing-wave ratio surrounding a desired impedance on a complex impedance plane.

DEFLAGRATION. Slowly progressing mode of **combustion** in a flame.

DEFLATION. A transformation applied to a **matrix A** when one **eigenvalue and eigenvector** are known, for the purpose of reducing the computations required for finding the others. The general theorem is, if eigenvalues of **A** are $\lambda_1, \lambda_2, \dots, \lambda_n$, and if $\mathbf{A}\mathbf{v}_1 = \lambda_1\mathbf{v}_1$, then eigenvalues of

$$\mathbf{A}_1 = \mathbf{A} - \mathbf{v}_1 \mathbf{k}^T$$

are $\lambda_1 - \mathbf{k}^T \mathbf{v}_1, \lambda_2, \dots, \lambda_n$, whatever the vector **k**. By choosing \mathbf{k}^T to be a multiple of the first row of **A**, and scaled so that

$$\mathbf{k}^T \mathbf{v}_1 \equiv \lambda_1,$$

the first row of **A**₁ is null.

DEFLECTION. Loads acting on a structure cause displacement of the structure from its original position. The term deflection is used for the displacement of joint and supports, and for the line of centroids of bars of which structures are composed. Deflection rather than strength sometimes governs design.

DEFLECTION, ANGLE OF. See **angle of deflection**.

DEFLECTION, BENDING. The motion or displacement of the line of centroids of a beam or plate in a transverse direction.

DEFLECTION CURVE. The deflection curve of a beam is the position of the line of centroids (axis) of the beam which results from transverse loading.

DEFLECTION, LARGE. The term large deflection is a relative one. Deflections are large if the non-linear effects they produce must be taken into account in the **strain-displacement relations**. In the case of a beam the exact expression for curvature would be required rather than d^2y/dx^2 . The need to take deflection into account in the equations of equilibrium alone, as in buckling of columns, is not considered to make the problem one of large deflection.

DEFLECTION, LATERAL. See **lateral deflection**.

DEFLECTION, SHEAR. The deformations of a beam, frame or arch are in part produced by transverse shear. Energy methods or their equivalent give close approximations to the deflection associated with the shearing stress. Except for specially designed beams with thin or open webs, the shear deflection is small compared with the bending deflection. (See **deep beams**.)

DEFLECTION THEORY. A name given to a successive approximation analysis technique for obtaining consistent deflections of cable and stiffening truss in a suspension bridge.

DEFORMATION. (1) A transformation $f_1(p)$ between two sets *A, B* of elements of a **topological space** is called a deformation of *A* into *B* if $f_1(p)$ is such that, letting $f_0(p)$ denote the identical transformation of *A* into itself, it is possible for all $0 \leq t \leq 1$ and all the points *p* of *A*, to define transformations $f_t(p)$ which are continuous in *t* and in *p*. Two

point sets which can be deformed into each other are said to be *homotopic*. Thus a torus is homotopic to a section of a pipe, but not to a sphere. (2) The change in the shape or size of a body which accompanies a stressed condition is called deformation or *strain*.

DEFORMATION POTENTIAL. The effective electric potential acting on a free electron in a metal or semiconductor as a result of a local deformation of the crystal lattice. The scattering of electrons by lattice vibrations may be analyzed in terms of the deformations produced by the vibrations, and hence the corresponding potentials.

DEFORMATION, PURE HOMOGENEOUS. See pure homogeneous deformation.

DEFORMATION THEORY OF PLASTICITY. A plastic flow law which expresses total plastic strain components in terms of the applied stress, and possibly stress increments. This type of law has been discredited for plastic flow with varying principal stress ratios or rotating principal stress directions in favor of **incremental theories**.

DEGENERACY. (1) The term used to describe the situation in which a quantum mechanical system has more than one **eigenstate** characteristic of the same **eigenvalue(s)** of the operators corresponding to a given set of dynamical variables. The existence of degeneracy is an indication that one has not specified values for as many dynamical variables as quantum mechanics requires for a unique determination of the state of the system. The additional dynamical variables that must be specified can be found by finding all of the additional operators which will commute with the set chosen and with themselves. Specifying eigenvalues for all of the dynamical variables found in this way will eliminate the degeneracy. (2) Sometimes the term degeneracy is used in the more limited sense of describing the ease in which there is more than one energy eigenfunction of the **Hamiltonian** corresponding to the same eigenvalue. The removal of degeneracy in this more limited sense can often be accomplished by the application of a perturbing field leading to a different energy shift for each one of the eigenfunctions involved in the degeneracy. (3) The term degeneracy is also used in certain classical problems, such as those of vibrat-

ing systems in which different modes have the same frequency.

DEGENERACY, EXCHANGE. See exchange degeneracy.

DEGENERACY TEMPERATURE. A temperature usually defined as the temperature for which the de Broglie wavelength of a particle with kinetic energy kT (k is Boltzmann's constant, T is absolute temperature) is equal to the mean separation of the particles in the gas. (See also **degenerate gas**.)

DEGENERATE CONDUCTION BAND. A band in which two or more orthogonal quantum states exist which have the same energy and spin, and zero mean velocity.

DEGENERATE ELECTRON GAS. An electron gas which is far below its **Fermi temperature**, that is, which must be described by the **Fermi distribution**. The essential characteristic of this state is that a very large proportion of the electrons completely fill the lower energy levels, and are unable to take part in any physical processes until excited out of these levels.

DEGENERATE GAS. A gas for which the condition

$$n \frac{h^3}{(2\pi mkT)^{3/2}} \ll 1$$

(n is number of gas particles per unit volume, h is the Planck constant, m is the mass of a gas particle, k is the Boltzmann constant, T is absolute temperature) is not satisfied. This condition is equivalent to

$$e^{\nu} \ll 1,$$

where $\nu = \mu/kT$ (μ is **thermal potential**). (See also **quantum statistics**.)

DEGENERATE LEVELS. Two or more energy levels of a molecule or atom are degenerate when their wave functions are equivalent, and yet independent. The degeneracy is usually split in an external electrical or magnetic field.

DEGENERATE OSCILLATING SYSTEM. A vibrating system with several **degrees of freedom** in which the frequencies associated with two or more degrees of freedom may be equal in magnitude. (See **degeneracy (3)**.)

DEGENERATE STATE. In quantum mechanics, when different states of motion correspond to the same **energy level**, the states are said to be degenerate. The degeneracy can often be removed by the application of a perturbing field with the effective introduction of a new quantum condition, i.e., the breakup of one **eigenvalue** into several. (See **degeneracy** (1) and (2).)

DEGRADATION OF ENERGY. When a quantity of heat Q is allowed to pass from a source at temperature T to the surroundings at temperature $T_o < T$, it is possible to convert some, but not all, of it into mechanical work W . The maximum amount which can be converted into work is that obtainable from a **Carnot cycle** operating between the sources at T and T_o , that is

$$W = \frac{T - T_o}{T} Q.$$

The quantity of heat

$$Q_o = Q - W = \frac{T_o}{T} Q$$

cannot be converted into work, as is expressed by the second law of **thermodynamics**, and becomes *unavailable* or *degraded*. When a real engine operates between the sources at T and T_o it will produce less work than W , say W' , owing to the irreversibility of real processes. The quantity

$$Q'_o = Q - W'$$

becomes then *unavailable* or *degraded*.

DEGREE MATRIX. See **matrix**, **degree**.

DEGREE OF ADVANCEMENT IN CHEMICAL REACTIONS (EXTENT OF REACTIONS). See **conservation of mass in closed systems**.

DEGREE OF A VERTEX. See **vertex**, **degree of**.

DEGREE OF COHERENCE. See **partial coherence**.

DEGREE OF SUPERHEAT. See **superheated vapor**.

DEGREES OF FREEDOM. (1) The number of independent coordinates necessary for the unique determination of the position of every particle in a dynamical system is the number

of degrees of freedom. Each degree of freedom is represented by a coordinate which can vary with time independently of all the rest. A particle constrained to move on a surface has two degrees of freedom. A single particle which may move anywhere in three-dimensional space has three degrees of freedom. A system composed of three particles has 9 degrees of freedom; for it takes nine independent coordinates to specify the positions of the particles in space, and their arrangement may therefore be changed in nine different ways. A single rigid body, on the other hand, has 6 degrees of freedom, since it may have motions of translation in three coordinate directions and it may also rotate about any one of the three coordinate axes through its center of mass. Any actual motion of the body is in general made up of all six, its linear motion being the resultant of three linear components and its rotation the resultant of three angular components. Each molecule of a diatomic gas has 7 degrees of freedom; viz., the six just mentioned for the molecule as a whole (regarded as a rigid body), and, in addition, one corresponding to the possible vibration of the two atoms toward and from each other. If the body is not rigid, the number of degrees of freedom may be virtually infinite. It should, however, be added that, because of restrictions imposed by the **quantum theory**, not all of the possible degrees of freedom can in general be expected to participate in changes of molecular energy. (See **equipartition of energy**.)

(2) The number of independent variables in a thermodynamic system. Thus, in the statement of the **phase rule**, that number of variable factors such as pressure, temperature or concentration, which must be fixed to define completely the state of the system.

(3) The number of independent **meshes**, or the number of independent **cut-sets** that may be selected in a **network**.

(4) A statistic which can (by linear transformation if necessary) be regarded as the sum of squares of ν independent variates is said to have ν degrees of freedom. Thus a sample sum of squares $\sum_{i=1}^n (x_i - \bar{x})^2$, where \bar{x} is the mean, has $n - 1$ degrees of freedom. By extension, a statistic which depends on several such quadratic forms is said to have the degrees of freedom appropriate to the forms; e.g., the **F-ratio**, which is the ratio of two forms with ν_1

and ν_2 degrees of freedom respectively, is spoken of as possessing ν_1 and ν_2 degrees of freedom.

DEGREES OF FREEDOM, FREEZING IN OF. See **freezing in of degrees of freedom.**

DEL. The differential operator used in **vector** analysis and sometimes also called nabla, since its usual symbol ∇ is thought to resemble an Assyrian harp with the latter name. In Cartesian coordinates it is $(i\partial/\partial x + j\partial/\partial y + k\partial/\partial z)$. When applied to a scalar function it gives the **gradient**; to vectors, it can give the **divergence** or the **curl**.

In the following relations ϕ is a scalar; \mathbf{U} , \mathbf{V} are vectors; the asterisk (*) can be either a dot or a cross and, depending on that choice, A , B are either scalars or vectors. $\nabla * (A + B) = \nabla * A + \nabla * B$; $\nabla * (\phi A) = \nabla \phi * A + \phi \nabla * A$; $\nabla(\mathbf{U} \cdot \mathbf{V}) = (\mathbf{V} \cdot \nabla)\mathbf{U} + (\mathbf{U} \cdot \nabla)\mathbf{V} + \mathbf{V} \times (\nabla \times \mathbf{U}) + \mathbf{U} \times (\nabla \times \mathbf{V})$; $\nabla \cdot (\mathbf{U} \times \mathbf{V}) = \mathbf{V} \cdot \nabla \times \mathbf{U} - \mathbf{U} \cdot \nabla \times \mathbf{V}$; $\nabla \times (\mathbf{U} \times \mathbf{V}) = (\mathbf{V} \cdot \nabla)\mathbf{U} - \mathbf{V}(\nabla \cdot \mathbf{U}) - (\mathbf{U} \cdot \nabla)\mathbf{V} + \mathbf{U}(\nabla \cdot \mathbf{V})$. If $\mathbf{R} = ix + jy + kz$, a position vector, $\nabla \cdot \mathbf{R} = 3$; $\nabla \times \mathbf{R} = 0$; $\mathbf{U} \cdot \nabla \mathbf{R} = \mathbf{U}$.

There are six possible combinations where the operator is applied twice, although two of them equal zero identically. They are: (1) $\nabla^2 \phi$, the **Laplacian**; (2) $\nabla^2 \mathbf{V}$; (3) $\nabla(\nabla \cdot \mathbf{V})$; (4) $\nabla \times (\nabla \times \mathbf{V}) = \nabla(\nabla \cdot \mathbf{V}) - \nabla \cdot \nabla \mathbf{V}$; (5) $\nabla \times \nabla \phi = 0$; (6) $\nabla \cdot \nabla \times \mathbf{V} = 0$.

DE LAVAL TURBINE. A single-stage impulse turbine. In order to have a reasonable efficiency a dc Laval turbine must operate at a very high rotational speed (of order of 15,000 rpm).

DELAYED ELASTICITY. See **elasticity, delayed.**

DELAYED NEUTRONS. See **neutrons, delayed.**

DELAY LINE. An electrical **network** used to hold back for a definite time the passage of a signal applied to the input terminals, but which thereafter transmits the delayed signal without distorting it.

DELAY TIME. The response of a **Kelvin material** to a **step function** of stress, of amplitude σ_0 , is a strain which varies with time according to

$$\epsilon(t) = \frac{\sigma_0}{m} (1 - e^{-t/\tau})$$

where m is the modulus and τ is a constant known as the delay time. When a more general material is represented by a sequence of Kelvin elements in series, the associated sequence of delay times, τ_i , gives the discrete delay time spectrum. For such a material, step function stressing gives the response:

$$\epsilon = \sigma_0 \sum_i \frac{1}{m_i} (1 - e^{-t/\tau_i}).$$

General linear viscoelastic behavior corresponds to the continuous generalization of this:

$$\epsilon = \sigma_0 \int_0^\infty L(\tau)(1 - e^{-t/\tau})d\tau.$$

$L(\tau)$ is then the retardation spectrum, sometimes expressed in terms of the variable $\ln \tau$ instead of τ .

DELOCALIZATION ENERGY. The removal of the wall separating two identical boxes containing equal numbers of electrons lowers the **zero point energy** of the system. In the simplest one-dimensional case, the kinetic energy levels, which were given for each box by the sequence $k^2(h^2/8ml^2)$, become $k'^2(h^2/8m(2l)^2)$ for the united box. k and k' are whole numbers, l is the length of the primitive box, h , the Planck constant and m , the mass of the electron. Each level can be occupied by two electrons. The respective zero point energies, before and after the removal of the box, are therefore

$$2 \cdot (2^2 + 4^2 + 6^2 + \dots)(h^2/4m(2l)^2)$$

and

$$(1^2 + 2^2 + 3^2 + 4^2 + \dots)(h^2/4m(2l)^2).$$

The increased mobility of the π -electrons along a skeleton molecule with **conjugated double bonds** leads to an analogous lowering of the heat of formation, i.e., to an increased stability of these molecules. This effect is usually called the *delocalization effect*, and the corresponding decrease in energy is called the *delocalization*, or *enlargement energy*.

DELTA (Δ). In the theory of the slowing down of fast neutrons through collisions with matter, the parameter

$$\Delta = 2 \frac{\sigma_a(kT)}{\mu\sigma_s}$$

gives the relative importance of capture to moderating power. σ_s is the high-energy scattering cross-section of the material, $\sigma_a(kT)$ is the value of the capture cross-section at $E = kT$, k being Boltzmann's constant and T the moderator temperature, and μ is the ratio of neutron mass to atom mass.

DELTA FUNCTION. Commonly written as $\delta(x)$, it lies outside the usual scope of function theory. It can be defined by the relation

$$\int_a^b \phi(x)\delta(x - x_o)dx = \phi(x_o)$$

where $\phi(x)$ is continuous and $a \leq x_o \leq b$. It can thus be thought of as a function which is zero for every value of x except the origin, where it is infinite in such a way that

$$\int_a^b \delta(x)dx = 1, \quad a < 0 < b.$$

The delta function has played an important role in mathematical physics and applied mathematics since it can be used to represent idealizations such as point charge, or point load, which lead to **Green's functions** of the differential equations representing various field theories. Analysis using delta functions has been put on a sound mathematical basis by L. Schwartz in his theory of distributions (see **distribution theory**). $\delta(x)$ can be considered as the derivative of the **Heaviside step function** $H(x)$.

DELTA OPERATOR. See **delta function**.

DELTA-SQUARE PROCESS. Let a_i, a_{i+1}, a_{i+2} represent three consecutive terms in a sequence, possibly three consecutive approximations to a root of an equation obtained by Newton's method, Bernoulli's method, or some other. In many situations, when the sequence has a limit, then $a_i - (\Delta a_i)^2/\Delta^2 a_i$ will be much closer to this limit than will a_{i+3} . Here Δ represents the **difference operator**. If the approach is geometric, that is, if $a_i = a + \alpha\rho^i$, then this gives the limit a exactly.

The formula can be derived by assuming the above form for a_i and solving for a . A somewhat more general formula in terms of $a_i, a_{i+1}, a_{i+2}, a_{i+3}, a_{i+4}$ can be obtained on the assumption that $a_i = a + \alpha_1\rho_1^i + \alpha_2\rho_2^i$. The result has a meaning even when $|\rho| \geq 1$, hence the method is sometimes effective for summing

divergent series. Generalizations have been developed by Shanks and by Wynn.

DELTA WING. A wing whose plan form is an isosceles triangle with its line of symmetry parallel to the direction of flight. If the apex angle is small, the pressure distribution and lift may be estimated from the theory of **wings of small aspect ratio**. For larger apex angles it is necessary to use the more general **lifting surface theory**. In their usual forms, however, both these theories neglect the effects of separation at the leading edges. These separations often have important effects on the real flow.

DEMAGNETIZING FIELD. A body of magnetic material subject to an **applied magnetizing force** \mathbf{H}_a is acted upon by a net magnetizing force,

$$\mathbf{H} = \mathbf{H}_a - \Delta\mathbf{H}$$

where the demagnetizing field $\Delta\mathbf{H}$ is interpreted as being due to the poles induced on the surface of the body. In the case of a permanent magnet with $\mathbf{H}_a = 0$, the demagnetizing field is readily seen to be the field of the magnet itself, which always has such a direction as to oppose the magnetization. The existence of a demagnetizing field is a necessary consequence of the fact that energy is stored in the field external to the magnetized body, and that the sum of this energy and the internal energy must be minimized at equilibrium. The demagnetizing field is approximately proportional to the magnetization, \mathbf{M} .

$$\Delta\mathbf{H} = N\mathbf{M}.$$

The proportionality factor N is called the demagnetizing factor, and depends primarily on the shape of the body.

DEMAGNETIZATION, ADIABATIC. See **adiabatic demagnetization**.

DEMAGNETIZATION CURVE. The portion of the **hysteresis loop** that lies in the second quadrant.

DEMAGNETIZATION FACTOR. See **depolarization factor; demagnetizing field**.

DE MOIVRE IDENTITY. A useful and easily proved rule (see **complex number**) for

raising a complex number to an integral power, namely,

$$\{r(\cos \theta + i \sin \theta)\}^n = r^n(\cos n\theta + i \sin n\theta);$$

in other words, $re^{i\theta} = r^n e^{in\theta}$.

DE MOIVRE-LAPLACE THEOREM. A particular case of the **central limit theorem** applied to the **binomial distribution**. If π is the probability of success the observed proportion p of successes in n independent trials tends, for large n , to be distributed in the normal (Gaussian) form with **mean** and **variance** $\pi(1 - \pi)/n$.

DENSE. A subset T of a **metric space** S is dense in S if every element of S is the limit of a sequence of elements in T .

DENSITY. (1) The ratio of mass m to volume V in a homogeneous system:

$$\rho = m/V.$$

In an inhomogeneous system, the density at a point is given by

$$\rho = \lim dm/dV$$

if the system can be treated as a continuum. (2) In length-force-time systems of units, the term density is sometimes used to denote weight per unit volume, rather than mass per unit volume. (3) By analogy, the ratio of the number of particles or total amount of such a quantity as energy or momentum, carried by or contained in a volume to that volume. Thus one speaks of energy density, luminous density, electron density, charge density, etc. (See entries following.)

DENSITY, DIFFUSE. A quantity defined as the logarithm of the reciprocal of the diffuse transmittance. (See **transmittance**, **diffuse**.)

DENSITY, ENSEMBLE. See **ensemble density**.

DENSITY, LUMINOUS. The luminous energy found in a unit volume of space.

DENSITY MATRIX. If a system can be described by the quantum mechanical wave function

$$\psi = \sum_{\alpha jm} A_m^{\alpha j} U_m^{\alpha j},$$

where the $U_m^{\alpha j}$ constitute a complete, orthogonal set of eigenfunctions of the total angular

momentum with eigenvalue $j(j+1)\hbar^2$, the z -component of the total angular momentum with **eigenvalue** $m\hbar$, and any other operators whose eigenvalues are designated by α ; then the matrix having as its elements

$$\rho_{\alpha jm, \alpha' j' m'} = A_m^{\alpha j} A_{m'}^{\alpha' j'}$$

is known as the density matrix. It is the quantum mechanical counterpart of the classical **ensemble density**.

DENSITY, NEUTRON. The neutron density, $N(r, t)$ is generally defined as:

$$N(r, t) = \int_0^\infty dE \int d\Omega N(r, E, \hat{\Omega}, t)$$

where the angular density $N(r, E, \hat{\Omega}, t)$ is defined under **flux, neutron, angular**.

DENSITY OF MOBILE CHARGES IN SEMICONDUCTOR. If the density of conduction electrons in a non-degenerate conduction band (see **degenerate conduction band**) of a crystal is sufficiently low for mutual interactions to be neglected, the equilibrium value of this density, n_e , can be obtained by integration of the expression for the **density of quantum states** over the **Fermi-Dirac distribution**. This gives

$$n_e = \frac{2(2\pi m_e^* kT)^{3/2}}{h^3} \exp [(\epsilon_F - \epsilon_C)/kT]$$

where ϵ_F is the **Fermi level**, ϵ_C is the energy at the bottom of the conduction band, m_e^* is the effective mass of the electron, T is the absolute temperature, and h and k are the Planck and Boltzmann constants. In obtaining this expression, it is assumed that $\epsilon_F - \epsilon_C \gg kT$.

The same relation may be used for obtaining the density of electron "holes" in a crystal, except that $(\epsilon_v - \epsilon_F)$ must now be substituted for $(\epsilon_F - \epsilon_C)$. (ϵ_v is the energy at the top of the valence band; m_e^* represents the effective mass of the hole.)

DENSITY OF QUANTUM STATES IN CONDUCTION BAND. If, in a non-degenerate conduction band (see **degenerate conduction band**), a crystal of volume Ω_N contains $\nu(\epsilon) d\epsilon$ quantum states with energies between $\epsilon_C + \epsilon$ and $\epsilon_C + \epsilon + d\epsilon$ (ϵ_C is the energy of the bottom of the conduction band), then, for small ϵ , the following asymptotic relation holds

$$\nu(\epsilon) = \frac{2^{1/2}\pi(m_e^*)^{3/2}\Omega_N}{h^3} \epsilon^{1/2}$$

where m_e^* is the effective mass of the electron, and h is Planck's constant.

DENSITY, SCALAR. See **scalar density**.

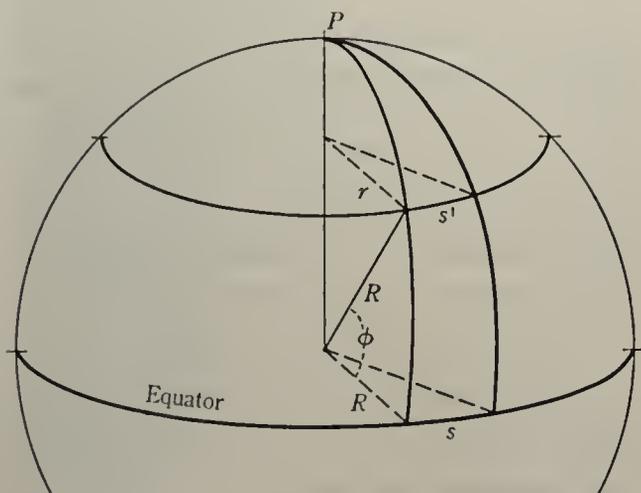
DENSITY, SPECULAR. The logarithm of the reciprocal of the specular transmittance. (See **transmittance, specular**.)

DENUMERABLE. A synonym of **countable**.

DENUMERABLE SET. See **cardinal**.

DEPARTURE. Any course and distance made good by a ship in a given time may be resolved into two components at right angle to each other, one in the north-south direction, known as the difference of latitude, and the other in the east-west direction, known as the departure. Because of errors in the methods available for determining the course and distance made good by the ship, one may assume in most calculations that the earth is a sphere. With this assumption, and with the distance expressed in nautical miles, the north-south component is the difference of latitude in minutes of arc.

Departure cannot be directly expressed as a difference of longitude, for it is measured along a small circle parallel to the equator. In the figure s' is a departure measured in



nautical miles along a parallel of latitude ϕ . The arc s , on the equator, represents the difference of longitude, measured in minutes of arc on the equator, or in nautical miles. Referring to the figure, it is seen that s' is shorter in length than s . If r is the radius of the small circle in latitude ϕ , and R is the equatorial

radius of the earth, then $r/R = \cos \phi$. The length of an arc, in units of linear measure, is proportional to the radius of the arc. Hence $s'/s = r/R = \cos \phi$. s' is referred to as the departure, expressed in nautical miles. Also $s' = s \cos \phi$.

DEPOLARIZATION FACTOR OR DEMAGNETIZATION FACTOR. The factor N relating the **depolarization field** \mathbf{E} to the **polarization** \mathbf{P} of the specimen, by the relation $\mathbf{E} = -N\mathbf{P}$. For example, the depolarization factor for a sphere is $4\pi/3$ about any axis. For a circular cylinder (long) it is 2π about a transverse axis, and 0 about a longitudinal axis. For a thin slab, it is 4π about a normal axis, and 0 about an axis in a plane of the slab, etc. All the factors are given in unratinalized units.

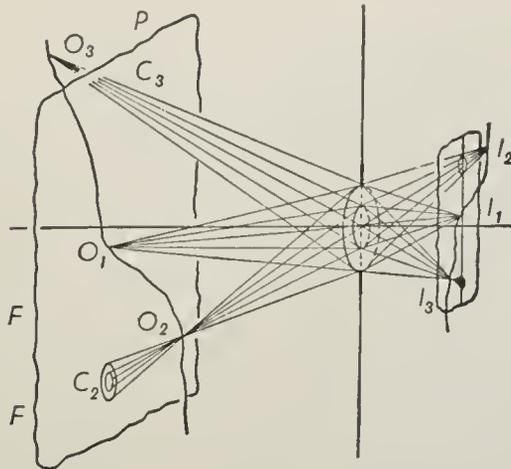
DEPOLARIZATION FIELD. When an electric or magnetic field is applied to a macroscopic specimen, the field acting on a given atom contains a contribution due to the charges or poles induced on the surface of the specimen. This field opposes the external applied field, and hence tends to reduce the **polarization** of the material.

DEPTH, EFFECTIVE. The distance between centroids of the top and bottom flanges of a beam. When the web is designed to carry the shear so that the flanges must carry the entire bending moment, the effective depth multiplied by the area of one flange and by the yield stress gives the moment carrying-capacity for a symmetric beam.

DEPTH OF FIELD. In photography the term depth of field refers to the distance over which satisfactory definition is obtained when the lens is in focus for a certain distance. If, for example, a lens is in focus for an object at a distance of 25' and the definition is satisfactory on objects from 20–40', the depth of field extends from 20–40'. Depth of field is frequently but incorrectly termed depth of focus, which is the range of image distances corresponding to the range of object distances covered by the depth of field.

The depth of field depends upon: 1. The standard adopted for "satisfactory" definition. 2. The distance of the plane on which the lens is in focus. 3. The foetal length of the lens. 4. The relative aperture (f /number). So far as the last three are concerned we may note

(1) that the depth of field increases with the distance of the plane on which the lens is in focus and (2) that it becomes less as the focal length or (3) as the aperture increases.



Let O_3, O_2, O_1 represent an object, parts of which are at different distances from the lens L . Suppose the lens to be focused on O_1 , a point image I_1 will be formed on the focusing screen, or sensitive plate. With the focusing screen, or the sensitive plate, at this point it is clear that the image of O_2 is formed at I_2 behind the screen and that of O_3 at I_3 in front of the screen. In other words, the position of the image point varies with the distance of the object point and the lens cannot produce a point for point image upon a plane surface, such as the sensitive film, unless the subject itself is a plane. When this is not the case, the image of points nearer or further from the lens is a circular disk rather than a point.

Any disk, however, will appear to the eye as a point if the viewing distance is sufficiently great. At a distance of 10", for example, a circle with a diameter of $\frac{1}{100}$ " appears as a point to the average eye. On an angular basis this corresponds to about 2 minutes of an arc.

Since any unsharpness in any part of the negative is increased when an enlargement is made, the disk, or circle of confusion, in an enlargement is equal to the diameter of the disk in the negative multiplied by the degree (times) of enlargement. Thus, if the largest circle allowable for sharp definition is assumed to be $\frac{1}{100}$ " and the print is a $5\times$ enlargement, the maximum diameter of the circle of confusion in the negative is $\frac{1}{100} \times 5$ or $\frac{1}{500}$ " and the depth of field should be calculated accordingly.

If it is assumed that the viewing distance is the distance at which the proper perspective is

obtained, i.e., a distance equal to the focal length of the lens, then the circle of confusion can be expressed as a fraction of the focal length of the lens. The value generally used is 0.00058 or $\frac{1}{1720}$ of the focal length.

If u is the distance focused on, θ is the angular size of circle of confusion ($\frac{1}{1720}$ of the focal length), and τ is the effective diameter of lens or the focal length divided by the f/number , then the nearest point sharply defined in front of the plane in focus is

$$\frac{u^2 \tan \theta}{\tau + u \tan \theta}$$

and the greatest distance beyond the plane in focus is

$$\frac{u^2 \tan \theta}{\tau - u \tan \theta}$$

Tables of depth of field are included in most camera manuals and in reference books.

DERIVATIVE. The derivative $f'(z)$ of a function $f(z)$ of a **complex variable** is the limit, provided it exists and is independent, of the path by which z approaches z_0 , of the difference-quotient, thus,

$$f'(z) = \lim_{z \rightarrow z_0} \frac{f(z) - f(z_0)}{z - z_0},$$

the derivative of a real function being defined analogously.

If $u = f(x, y, z, \dots)$ is a function of several variables, the derivative of u calculated with respect to one of the variables, the others being kept constant, is called a **partial derivative** of u . The partial derivative of u with respect to x

is written in the forms $\frac{\partial u}{\partial x}, u_x$. In case each of the variables x, y, z, \dots on which u depends is itself a function of a single variable t , then

$$\frac{du}{dt} = \frac{\partial u}{\partial x} \frac{dx}{dt} + \frac{\partial u}{\partial y} \frac{dy}{dt} + \frac{\partial u}{\partial z} \frac{dz}{dt} + \dots,$$

which is called a **total derivative** of u . (See also **directional derivative**.)

DERIVATIVE CONTROL. See **control proportional, derivative and integral**.

DERIVATIVE, COVARIANT. See **covariant derivative**.

DERIVATIVE, DIRECTIONAL. If u is a **unit vector**, the quantity $u \cdot \nabla \phi$, its **scalar**

product with the gradient of a scalar function, is the rate of change of ϕ in the direction of \mathbf{u} or the directional derivative of ϕ .

DERIVATIVE, HELMHOLTZ. See Helmholtz derivative.

DERIVATIVE OF TENSOR FIELD, COVARIANT. See covariant derivative of tensor field.

DERIVATIVE, PARTIAL. See partial derivative.

DERIVATIVE, TOTAL. See derivative.

DERIVED SET. The derived set of a given point set is the set of its limit points.

DESCARTES RULE OF SIGNS. The number of positive real roots of the algebraic equation $a_0x^n + a_1x^{n-1} + \cdots + a_{n-1}x + a_n = 0$ is equal to the number of changes of sign in the sequence of coefficients $a_0, a_1, \cdots, a_{n-1}, a_n$ or is less by an even number. The number of negative real roots is equal to the number of changes of sign in $a_0, -a_1, a_2, -a_3, \cdots$, or less by an even number. A special case of the Budan theorem.

DESCRIPTIVE GEOMETRY. The representation on the two-dimensional Euclidean plane of three-dimensional objects. The subject was essentially invented and much developed by Gaspard Monge about 1795.

DESIGN DIAMETER. See blade, twisted.

DESIGN LIFT COEFFICIENT. See thin-airfoil theory.

DESIGN OF EXPERIMENTS. The design of experiments is the science of obtaining unambiguous results of the highest possible accuracy from experiments on variable material. The variability is taken account of in the results by assessing standard errors and fiducial or confidence limits for the estimates derived from an experiment. Three main principles of experimental design may be distinguished:

1. *Replication.* Any treatment must normally be applied to more than one experimental unit. This provides greater accuracy than can be obtained from a single observation, since the experimental errors tend to cancel each other; it also provides a measure of the experimental error derived from the variability between replicates.

2. *Randomization.* The decision as to which experimental unit shall receive which treatment should include a random element. This is designed to ensure that every treatment shall have its fair share of the particularly favorable and the particularly unfavorable experimental units.

3. *Local Control.* Any structure in the properties of the experimental units should be utilized fully. Thus if the units fall into relatively homogeneous groups (neighboring plots in a field, animals from the same litter, etc.) comparisons between treatments should be made as far as possible between units in the same group.

(For particular designs embodying these principles, see randomized blocks, latin squares, lattice designs, balanced incomplete blocks, partially balanced incomplete blocks, Youden squares; see also factorial experiments.)

DESLANDRES TABLE (OF A BAND SYSTEM). See band system.

DESTRUCTION OPERATOR. See creation and annihilation operators.

DETAILED BALANCING, PRINCIPLE OF. A principle which states that in equilibrium the number of processes which destroy a situation A and produce a situation B is equal to the number of processes which destroy B and produce A . This principle is often valid, but not always; in the presence of a magnetic field, for instance, it does not hold for all processes A and B . It was called by Tolman the principle of microscopic reversibility and plays an important role in the derivation of the Onsager relations.

DETECTION OF TABULAR ERRORS (BY MEANS OF DIFFERENCES). This method is based upon the following table, readily verified and extended:

y	δy	$\delta^2 y$	$\delta^3 y$
0		0	
	0		ϵ
0	ϵ	ϵ	-3ϵ
ϵ	$-\epsilon$	-2ϵ	3ϵ
0	0	ϵ	
			$-\epsilon$
0		0	

This shows that the effect of small errors becomes accentuated progressively in the columns of differences.

DETERMINANT. The determinant $|A|$ of an n -rowed square matrix $A = \{a_{ij}\}$ is the sum

$$|A| = \Sigma(\pm)a_{1i_1}a_{2i_2} \cdots a_{ni_n}$$

of the $n!$ products $a_{1i_1}a_{2i_2} \cdots a_{ni_n}$ where $i_1, i_2 \cdots i_n$ is a permutation of the integers $1, 2, \cdots n$, the plus sign being taken if the permutation is even, and the minus sign if it is odd (see **permutation, even or odd**). Thus, if $n = 2$,

$$A = \begin{Bmatrix} a_{11}, & a_{12} \\ a_{21}, & a_{22} \end{Bmatrix}, \text{ then } |A| = a_{11}a_{22} - a_{12}a_{21}.$$

(For various special determinants, **Fredholm, Gram**, etc.: see under the respective names. See also **Laplace expansion**.)

DETERMINANTAL EQUATION. The equation obtained by setting the value of a determinant equal to zero.

DETERMINANTAL WAVE FUNCTION. For a system of N non-interacting particles a **wave function** which is antisymmetric with respect to interchange of any two particles is given in terms of the one particle wave functions by the determinant:

$$\psi = \frac{1}{N!} |\phi_i(r_j)|,$$

where $\phi_i(r_j)$ is the ij entry in the determinant, and represents the j th particle in the i th one particle wave function. For a system of identical particles of half-integral **spin (fermions)** the total wave function of the system (space and spin) must be antisymmetric with respect to interchange of the particles.

DETERMINANT, SLATER. See **Slater determinant**.

DETERMINATE STRUCTURE. A static structure is said to be determinate or statically determinate when the support reactions and the forces and moments at each section of each bar are uniquely determined from the applied loads by statics alone. Forces and moments therefore are independent of the properties of the materials employed. Also, once the load carrying capacity of any member is reached, the structure will fail. (See **indeterminate structure**.)

DETERMINATION, COEFFICIENT OF. The square of the (product-moment) **correlation coefficient** between two variables. More generally, the square of the multiple correlation coefficient between one variate and a set of others.

DETONATION. The process of propagating the reaction front in a combustible mixture which occurs at a very high velocity and during which heating and ignition are done by a shock wave which is created ahead of the combustion front. Detonation is the characteristic mode of combustion which occurs, among others, in explosives.

DEVELOPABLE (SURFACE). A surface that can be rolled out, without stretching or shrinking, onto a plane; it may be defined as a surface whose total **curvature** vanishes everywhere; it is the **envelope** of a one-parameter family of planes. It is also called a *torse*.

DEVELOPABLE, OSCULATING (OF A TWISTED CURVE). The envelope of the **osculating planes** of the curve.

DEVELOPABLE, POLAR (OF A TWISTED CURVE). The envelope of the normal planes of the curve.

DEVELOPABLE, RECTIFYING (OF A TWISTED CURVE). The envelope of the **rectifying planes** of the curve.

DEVIATE, DEVIATION. The deviation of x from a is defined as $x - a$.

If a is the mean, $x - a$ is the deviation from the mean (not the **mean deviation**). The absolute value of the deviation of x from a is $|x - a|$.

If x is a **variate**, the value $x - a$ is often described as a deviate, especially if a is the **mean**.

If P is a probability and $F(x)$ the **distribution function** of a variate x , the variate y defined by

$$P = \int_{-\infty}^y dF(x)$$

is called the *equivalent deviate* of x . y may be regarded as a transformed variate. In particular, if $F(x)$ is the distribution function of a *normal variable*, y is called the *normal equivalent deviate* (N.E.D.).

DEVIATING FORCE. See **geostrophic forc.**

DEVIATION. (1) See **deviate, deviation.**
(2) The angle through which a ray or beam of radiation is bent by diffraction or refraction. (See **angle of deviation.**)

DEVIATION, ANGLE OF. See **angle of deviation.**

DEVIATION LOSS, ANGULAR. Of a **transducer** used for sound emission or reception, an expression, in **decibels**, of the ratio of the reference **response** observed on the principal axis to the transducer response at a specified angle from the principal axis. (See also **directivity pattern.**)

DEVIATION, MINIMUM ANGLE OF. The angle of deviation of a prism (not necessarily triangular) is a minimum if the entrant and exit rays make equal angles with the entrant and exit surfaces of the prism. This angle is of particular interest in prism spectroscopes because it can be easily determined and a simple formula connects the angle A between the planes of the entrant and exit surfaces, the angle of minimum deviation D , and the index of refraction n of the prism,

$$n \sin \frac{1}{2}A = \sin \frac{1}{2}(A + D).$$

If A is small then $D = (n - 1)A$. At the angle of minimum deviation the **resolving power** is independent of A .

DEVIATOR. See **stress deviator; strain deviator.**

DEWAR STRUCTURES. See **benzene, structures of.**

DEW POINT. The temperature at which the actual content of water vapor in an atmosphere is sufficient to saturate the air with water vapor. If the atmosphere contains much water vapor the dew point is higher than in the case of drier air, so that the dew point is an indication of the humidity of the atmosphere. (See also **combustion.**)

DIACAUSTIC. A **caustic** produced by refraction.

DIAGONAL. Any inclined web member of a **truss** other than the end post is known as a diagonal. Under various live load conditions certain web members of a bridge truss may

be subjected to a reversal of stress. Stiff diagonals are web members designed to carry either **tension** or **compression**. Tension diagonals are assumed to be incapable of resisting any appreciable amount of compression.

When the diagonals are designed to take tension only (are not stiff) two are required, sloping in opposite directions, in a panel where either one, acting alone, would have its stress reversed. When one of these diagonals is acting the other is out of action for all practical purposes. The one which carries the **dead load** stress when no other loads are on the bridge is called the main diagonal. The other which comes into play when the main diagonal goes out of action is the counter.

DIAGONAL MATRIX. See **matrix.**

DIAGONAL TENSION. The shear stresses in the web of a beam are equivalent to tension in one 45-degree direction and compression in the other. Concrete must be reinforced to carry this tension. Thin webs of metal may buckle in compression unless they are stiffened by horizontal, vertical or diagonal stiffeners. (See **tension field.**)

DIAGRAM EFFICIENCY. See **blade efficiency.**

DIAGRAM FACTOR. Ratio of area of the real indicator diagram of a reciprocating engine to its ideal value obtained from an idealized diagram. It is equal to the efficiency ratio, η_r .

DIALYTIC. Given two equations for one unknown, namely $f(x) = a_0x^n + a_1x^{n-1} + \dots + a_n = 0$, and $g(x) = b_0x^m + b_1x^{m-1} + \dots + b_m = 0$, let it be required to find the condition on the coefficients a_i, b_j that these equations may be consistent, i.e., may have a common root. The *Sylvester dialytic method* consists of regarding them as linear homogeneous equations in the various powers, including the zeroth, of x . Then multiplying both equations by x gives two new equations with only one new unknown. This process is continued until the number of equations is equal to the number of unknowns. Setting the **determinant** of this system of equations equal to zero gives the desired result.

DIAMAGNETISM. In diamagnetic materials the magnetic susceptibility, χ , is negative and the relative permeability, μ , is less than 1.

Unlike **paramagnetic materials**, whose atoms or molecules have a net magnetic moment, the atoms or molecules of a diamagnetic material have zero magnetic moment in the absence of an external applied field. With the application of an external magnetic field a net **dipole moment**, opposing the field, is induced in the atoms or molecules. In paramagnetic materials there is also a diamagnetic effect but it is much weaker than that of the paramagnetism.

DIAMAGNETISM, LANGEVIN - PAULI FORMULA. The equation for the diamagnetic susceptibility for an assembly of N atoms per unit volume is

$$\chi = - \frac{Ze^2N}{6mc^2} \overline{r^2}$$

where $\overline{r^2}$ is the mean square radial distance of the electrons from their nuclei, and Z is the atomic number. e and m are the electronic charge and mass respectively, and c is the velocity of light. This formula was originally derived classically by Langevin, and corrected quantum-mechanically by Pauli. (See also **Langevin theory of diamagnetism**.)

DIAMAGNETISM OF CONDUCTION ELECTRONS, ORBITAL. See **orbital diamagnetism of conduction electrons**.

DIAPPOINT. The diapoint of a ray in an axially symmetrical optical system is defined as the intersection of the image ray with the plane that passes through an object point and the axis; in the case of a meridional ray, the diapoint is the **sagittal focus**. M. Herzberger has shown that this is a unifying conception in geometric optics, and in particular leads to a simple development of Gullstrand's first-order theory for a **normal congruence**.

DIATHERMAL WALL (ALSO ADIATHERMAL, DIATHERMANEOUS). A perfect heat conductor; assuring equality of temperature on both sides of it. Since in a rigorous development of the principles of thermodynamics it is necessary to introduce the concept of a diathermal wall before the concept of heat, it is necessary to adopt the following alternative definition. If two closed systems are placed in contact (i.e., interact) through a diathermal wall, their states cannot be varied independently of one another. If the state of one system is changed, then in general, the

state of the other system will also change; the systems are coupled. If the state of one system is described by k properties x_i and the state of the other system is described by l properties y_i , then coupling the two systems with the aid of a diathermal wall implies a functional relationship

$$f(x_1, x_2, \dots, x_i, \dots, x_k; y_1, y_2, \dots, y_i, \dots, y_l) = 0.$$

The number of degrees of freedom (independent properties) of the combined system is now $k + l - 1$. (See **adiabatic wall**.)

DIATHERMANEOUS WALL. See **diathermal wall**.

DIATOMIC GAS, DISSOCIATION OF. See **Saha equilibrium formula**.

DIATOMIC MOLECULES, NON-CROSSING RULE. See **non-crossing rule for the potential curves of diatomic molecules**.

DIATOMIC MOLECULES, PARTITION FUNCTION OF. A function which consists to a fair approximation of four factors, relating respectively to the translational, vibrational, rotational, and electronic degrees of freedom. (See **partition function**.)

DIATOMIC ROTATOR, ENERGY LEVELS OF. See **rigid diatomic rotator, energy levels of**.

DICHROIC. (1) A medium is dichroic if it transmits light of different colors depending on either the thickness traversed or on the concentration of the coloring matter in the medium. (2) A medium is dichroic if the reflected light is of a different color than the transmitted light. (3) A crystal is dichroic if it is **pleochroic** with respect to exactly two axes.

DICHROISM, CIRCULAR. See **circular dichroism**.

DIDO'S PROBLEM. The original problem of Dido appears to have been that of finding the curve of given length which encloses a maximum area, the answer being a circle. The problem which is important in the calculus of variations (see **variations, calculus of**), has been generalized in various ways; e.g., by asking that the curve enclose a field of maxi-

imum value, where the fertility varies from point to point.

DIELECTRIC. A material characterized by its relatively poor electrical conductivity, hence an insulator.

DIELECTRIC, ANISOTROPIC. A dielectric medium in which the dielectric constant depends upon the direction along which the electric field is applied. This property is displayed by various crystalline substances and by an ionized gas in a magnetic field.

DIELECTRIC BOUNDARY FORMULAS. Let two different dielectric media, numbered 1 and 2 respectively, meet at a surface S with unit normal \hat{n} drawn from medium 1 into medium 2. If \mathbf{E}_1 and \mathbf{E}_2 are the respective electric vectors and \mathbf{D}_1 and \mathbf{D}_2 the respective electric displacement vectors in these media, then the following relations are satisfied at S :

$$\hat{n} \times (\mathbf{E}_2 - \mathbf{E}_1) = 0, \quad \hat{n} \cdot (\mathbf{D}_2 - \mathbf{D}_1) = \omega$$

where ω is the surface density of free charge distributed over S .

DIELECTRIC CAVITY FIELD. See cavity field in a dielectric.

DIELECTRIC CONSTANT. (1) The absolute dielectric constant is identical with the permittivity ϵ , and is the ratio of the electric field strength \mathbf{E} to the electric displacement \mathbf{D} . In isotropic media ϵ is a scalar, in anisotropic media ϵ is a tensor (see **constitutive equations**). The absolute dielectric constant may also be defined through the Coulomb law for two charges (q_1 and q_2) immersed in a homogenous and isotropic dielectric medium that extends in all directions to distances much greater than r , the distance between the charges: $F = q_1 q_2 / 4\pi\epsilon r^2$ in rationalized units. In non-rationalized units, the corresponding expression is $F = q_1 q_2 / \epsilon r^2$.

(2) The relative dielectric constant is the ratio of the absolute dielectric constant for a medium to that for free space (ϵ/ϵ_0). In the esu system of units the relative and absolute dielectric constants are the same.

Whereas for static fields the dielectric constant is a real number, for periodic fields, it is usually a complex quantity (see **dielectric constant; electrical displacement in dielectrics**). Debye has shown that, if in a static field, equilibrium is reached exponentially in

time, then

$$\epsilon(\omega) - \epsilon_\infty = \frac{(\epsilon_s - \epsilon_\infty)}{1 - i\omega\tau}$$

$\epsilon(\omega)$ is the dielectric constant in a periodic electric field $\mathbf{E} = \mathbf{E}_0 \cos \omega t$. ϵ_∞ and ϵ_s are the limiting values of $\epsilon(\omega)$ for very large and zero frequencies, respectively. τ is the relaxation time of the dielectric. (See also **Kirkwood formula for the static dielectric constant; Debye equation for the dispersion of the dielectric constant**.)

DIELECTRIC CONSTANT, COMPLEX. The ratio of the static capacitance C of a capacitor filled with a dielectric medium to its geometrical or vacuum capacitance is $C/C_0 = \epsilon/\epsilon_0 = k$, where ϵ and ϵ_0 are the permittivities of the medium and of free space, respectively, and k is the relative dielectric constant. If the medium is dissipative (either because it is not a perfect insulator, so that real currents can flow through it, or because there is frictional resistance to induced polarization currents), the a-c behavior of the capacitance is equivalent to that of a resistance in parallel with a capacitance. The effect of this resistance can be incorporated into the dielectric constant, which then becomes a complex quantity denoted by $k^* = \epsilon^*/\epsilon_0$, which can be written as $k^* = k' - jk''$. The imaginary coefficient (k'') is called the *loss factor*, $\tan \delta = k''/k'$ is called the *loss tangent*, and δ is called the *loss angle*.

DIELECTRIC CONSTANT, PLASMA. A plasma in a magnetic field of magnitude \mathbf{B} , can be considered to have a dielectric constant ϵ given by

$$\epsilon = 1 + \frac{4\pi\rho c^2}{\mathbf{B}^2}$$

where ρ is the mass density and c is the velocity of light.

DIELECTRIC CONSTANT, RELATION TO INDEX OF REFRACTION. See Maxwell relation between dielectric constant and refractive index.

DIELECTRIC DISSIPATION FACTOR. The cotangent of the dielectric phase angle of a dielectric material.

DIELECTRIC HYSTERESIS. An effect in a dielectric material analogous to the hysteresis found in magnetic materials. (See **ferroelectricity**.)

DIELECTRIC, INVERSION FIELDS IN. See **inversion fields in dielectrics**.

DIELECTRIC, ISOTROPIC. A dielectric medium in which the **dielectric constant** is independent of the direction of the applied field.

DIELECTRIC LOSS ANGLE. See **dielectric constant, complex**.

DIELECTRIC PHASE ANGLE. The angular difference in phase between the sinusoidal alternating voltage applied to a dielectric and the component of the resulting alternating current having the same period as the voltage.

DIELECTRIC POWER FACTOR. The cosine of the **dielectric phase angle**.

DIELECTRIC, REACTION FIELD IN. See **reaction field in a dielectric**.

DIELECTRIC RELAXATION. That part of the **dielectric constant** of a solid which depends on the orientation of the **dipole moments** of the molecules is subject to the phenomenon of relaxation. A certain time is required for the assembly of dipoles to come into equilibrium with the applied field. (See **relaxation** and related entries.)

DIELECTRIC, SELF ENERGY IN. See **self energy in a dielectric**.

DIELECTRIC STRENGTH. The maximum potential gradient (**electric field strength**) that a material can withstand without rupture.

DIESEL CYCLE. See **diesel engine**.

DIESEL ENGINE. (Also known as a compression-ignition engine.) A type of reciprocating internal combustion engine in which air is induced into, and then compressed nearly adiabatically in a cylinder by a piston moving in it. The compression raises the temperature of the air to a point where the subsequent, timed injection of a spray of liquid fuel causes the latter to ignite and to burn, thus producing useful mechanical work.

A diesel engine can operate either as a two-stroke or as a four-stroke engine. The operation of a diesel engine is illustrated by the

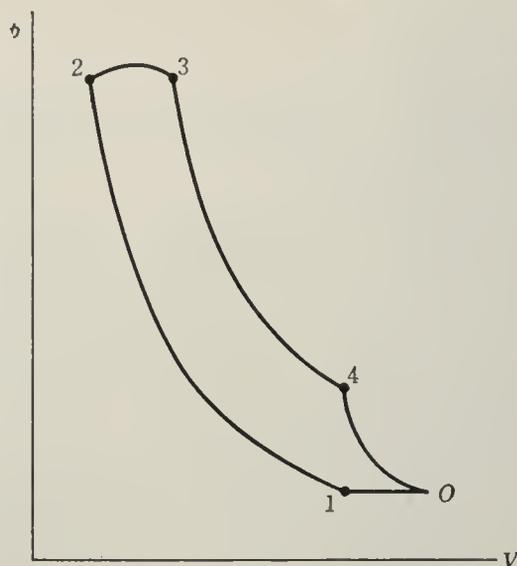


Fig. 1. p,V diagram of two-stroke diesel cycle.

two-stroke cycle shown in Figure 1. In this case, one working stroke occurs for every one revolution, and the sequence of operations is as follows:

- 0-1. Scavenging and introduction of fresh intake of air.
- 1-2. Compression to a temperature higher than ignition temperature of fuel.
- 2-3. Combustion, which occurs in some diesel-engine types nearly at constant pressure.
- 3-4. Expansion; working stroke.
- 4-0. Release.

In a four-stroke engine, the intake of air and release of the burnt gases is governed by valves

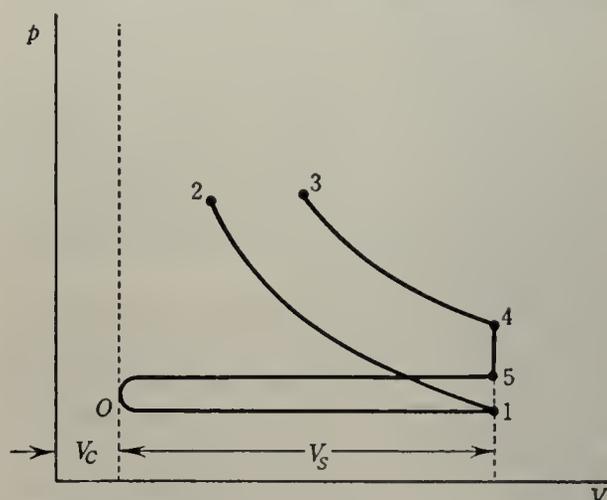


Fig. 2. p,V diagram of four-stroke diesel cycle.

and affects only the lower portion of the diagram, Figure 2. Here the following operations occur:

- 0-1. Introduction of air. Inlet valve open.
- 1-2. Compression of air. Valves closed.
- 2-3. Combustion as before; not shown.
- 3-4. Expansion. Working stroke; valves closed.
- 4-5. Release. Exhaust valve open.
- 5-0. Expulsion of burnt gases. Exhaust valve open.

It is customary to analyze diesel engines by making the following assumptions:

During the entire cycle the gas contained in the cylinder remains unchanged both in quantity and in composition. The addition of heat is effected by transfer from an outside source, instead of by combustion. The rejection of heat is effected by transfer to an outside sink instead of by expulsion of exhaust gases (during this cooling, the volume of the gas in the cylinder remains constant). It is further assumed that the gas in the cylinder obeys the **ideal gas law** (i.e., is a perfect gas) and that its specific heat does not vary, but is constant throughout the cycle.

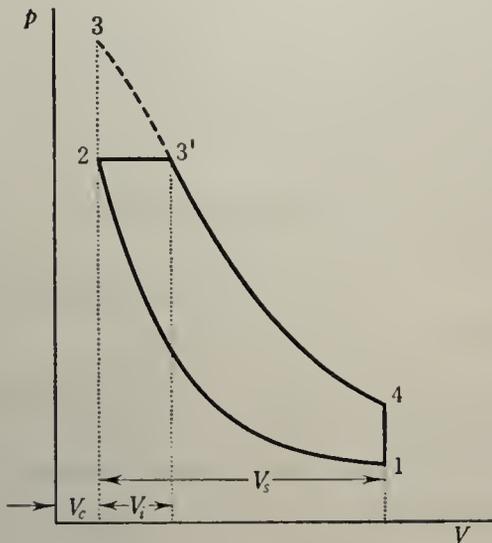


Fig. 3.

In this manner the efficiency of the engine is judged in relation to the diesel cycle shown in Figure 3. In it

$$r = \frac{V_1}{V_2} = \frac{V_c + V_s}{V_c}$$

denotes the compression ratio (which is a ratio of volumes, not pressures), and

$$\rho = \frac{V_c + V_i}{V_c}$$

is the cut-off ratio, which is a measure of the length of injection cycle. Then, the thermal efficiency of the cycle is given by

$$\eta_{th} = 1 - \frac{1}{\gamma r^{\gamma-1}} \frac{\rho^\gamma - 1}{\rho - 1}$$

where γ is the ratio of specific heats c_p/c_v , on the assumption that compression and expansion are isentropic.

The combustion process in a modern high-speed diesel engine does occur not even approximately at constant pressure. In order

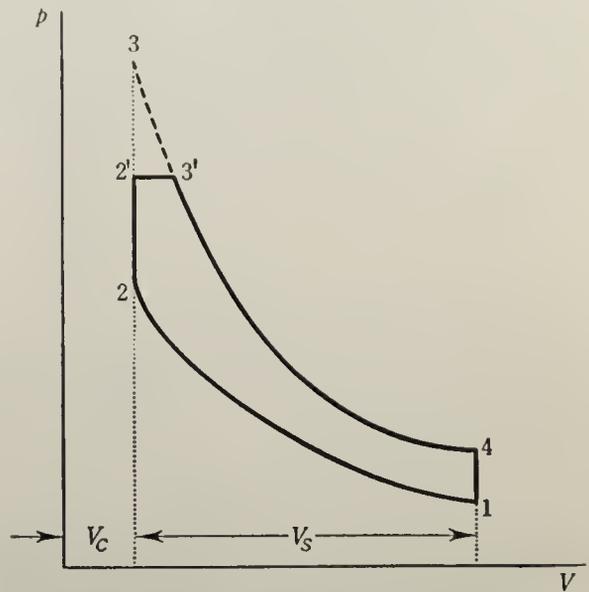


Fig. 4. p, V diagram of dual (combustion) diesel cycle.

to analyze such engines it is assumed that some combustion occurs at constant volume, which is followed by some combustion at constant pressure. The resulting cycle is called a *dual (combustion) cycle*, Figure 4. Its efficiency is

$$\eta_{th} = 1 - \frac{\beta \rho^\gamma - 1}{r^{\gamma-1} [\beta - 1 + \gamma^\beta (\rho - 1)]}$$

Here $\beta = P_{2'}/P_2$ is a measure of the length of time during which the fuel burned at constant volume.

Modern engines operate on compression ratios in the range $r = 10$ to 16 or 18. There exist many varieties of diesel engines which differ from each other in their details of design. At the present time the diesel engine has the highest overall efficiency.

DIETERICI EQUATION. An equation of state relating pressure, volume, and temperature of gas, and the gas constant. The

Dieterici equation applies a correction to the **van der Waals equation** to allow for variation in density throughout a gas, due to the higher potential energies of molecules on or near the boundaries. One form of this equation is

$$P = \frac{RT}{V - b} e^{-a/RTV}$$

in which P is the pressure of the gas, T is the absolute temperature, V is the volume, R is the gas constant, e is the natural log base 2.718 . . . , and a and b are constants.

DIFFERENCE, CENTRAL. If h is the interval between equally spaced values of the **argument** in a **difference table** constructed from $y = f(x)$, it is convenient to define

$$\delta f(x) = f\left(x + \frac{h}{2}\right) - f\left(x - \frac{h}{2}\right);$$

$$\mu f(x) = \frac{1}{2} \left[f\left(x + \frac{h}{2}\right) + f\left(x - \frac{h}{2}\right) \right]$$

A central difference, formed by the first of these operators, is related to a **finite difference** by the equation

$$\delta^m y_{n/2} = \Delta^m y_{(n-m)/2}$$

where m and n are integers, both even or both odd. These quantities are used near the middle of a difference table, as in the **Bessel** and **Stirling formulas for interpolation**.

DIFFERENCE, DIVIDED. If y_0, y_1, y_2, \dots are values of $y = f(x)$, corresponding to x_0, x_1, x_2, \dots , not necessarily evenly spaced, then

$$[x_i x_j] = \frac{y_i - y_j}{x_i - x_j}$$

is a first-order **divided difference** of $f(x)$. Second-order differences, third-order differences, etc., are defined in a similar way, thus the n th order divided difference is

$$[x_0 x_1 \dots x_n] = \frac{[x_0 x_1 \dots x_{n-1}] - [x_1 x_2 \dots x_n]}{x_0 - x_n}$$

These quantities are used for **interpolation** of tabulated functions when the given data are unequally spaced.

DIFFERENCE EQUATION. Any equation involving finite differences. The usual methods for solving **differential equations** reduce to the solution of difference equations, with either

the derivatives or the integrals expressed approximately in terms of finite differences. (See **differentiation formulas, quadrature formulas, ordinary differential equations, and partial differential equations.**)

DIFFERENCE LIMEN (DIFFERENTIAL THRESHOLD) (JUST NOTICEABLE DIFFERENCE). The increment in a stimulus which is just detected in a specified fraction of the trials. The relative difference limen is the ratio of the difference limen to the absolute magnitude of the stimulus to which it is related. According to the Weber law, the relative difference limen is a constant. This law is only approximately true.

DIFFERENCE NUMBER. The same as **neutron excess**.

DIFFERENCE OPERATORS. For interpolation based upon equally spaced points, most standard formulas, as well as others, can be developed and expressed easily in terms of a few difference operators which can be manipulated formally as algebraic quantities. In their definition given below it is to be understood that they have no existence independent of the functions upon which they operate. Understanding that $\Delta x = h$ is fixed, the operators are defined as follows:

$$E^u f(x) = f(x + uh)$$

[displacement operator];

$$\Delta f(x) = f(x + h) - f(x)$$

[forward difference operator];

$$\nabla f(x) = f(x) - f(x - h)$$

[backward difference operator];

$$\delta f(x) = f(x + h/2) - f(x - h/2)$$

[central difference operator];

$$2\mu f(x) = f(x + h/2) + f(x - h/2)$$

[central mean operator];

$$Df(x) = df(x)/dx$$

[differential operator];

$$\theta f(x) = hdf(x)/dx.$$

The last operator is not in common use, but simplifies the writing of some formulas and may be called the *logarithmic displacement operator*. For functions of several variables, subscripts can be used to specify the variable with respect to which the operator acts. Thus

$$E_x f(x, y, \dots) = f(x + h, y, \dots).$$

All operators can be defined formally in terms of one of them. Thus

$$\Delta = E - 1, \quad \nabla = 1 - E^{-1}, \quad E = \exp \theta,$$

the last being the operational expression of the Taylor series.

Examples of the application of these relations are to be found under appropriate headings, but the following illustration may be in order here. Let

$$x = x_0 + uh, \quad u = (x - x_0)/h, \\ f(x) = E^u f(x_0).$$

Since $E = 1 + \Delta$, therefore, by the binomial theorem,

$$E^u = 1 + \binom{u}{1} \Delta + \binom{u}{2} \Delta^2 + \binom{u}{3} \Delta^3 + \dots$$

and therefore, formally, taking $x_i = x_0 + ih$, one has

$$f(x) = f(x_0) + \frac{x - x_0}{h} \Delta f(x_0) \\ + \frac{(x - x_0)(x - x_1)}{2!h^2} \Delta^2 f(x_0) + \dots$$

Unless u is a non-negative integer this is an infinite series, but for purposes of interpolation the series is truncated by dropping all but the first several terms. If all terms are dropped beyond that containing $\Delta^n f(x_0)$, then there is retained on the right that n th degree polynomial that passes through the $n + 1$ points whose coordinates are

$$x_i, f(x_i), \quad i = 0, 1, \dots, n,$$

and the formula is *Newton's interpolation formula* with forward differences. If one writes $E = (1 - \nabla)^{-1}$ and proceeds similarly one obtains *Newton's interpolation formula* with backward differences:

$$f(x) = f(x_0) + \frac{x - x_0}{h} \nabla f(x_0) \\ + \frac{(x - x_0)(x - x_{-1})}{2!h^2} \nabla^2 f(x_0) + \dots$$

It is to be noted that the binomial coefficients

$$\binom{u}{r} = \frac{u(u-1) \cdots (u-r+1)}{r!}$$

are defined even when u is not a positive integer, and they are polynomials in u . Some-

times the symbol $u_{(r)}$ is used, and related to these polynomials are the factorial polynomials

$$u^{(r)} = u(u-1) \cdots (u-r+1) = r! u_{(r)}.$$

Note that

$$\Delta u^{(r)} = r u^{(r-1)}, \quad \Delta u_{(r)} = u_{(r-1)}.$$

DIFFERENTIAL. The differential dx of an independent variable x is itself an independent variable (the increment of x may be taken at will). The differential dy of a dependent variable $y = f(x)$ is defined as the product $dy = f'(x)dx$. The differential dy is a useful approximation to the actual increment of y . Similarly, if $z = f(x, y)$, then the differential dz of z (also called the total or exact differential) is defined by $dz = f_x(x, y)dx + f_y(x, y)dy$, where f_x, f_y are partial derivatives (see **derivative**). The differential may also be defined for more general cases.

DIFFERENTIAL ANALYZER. A computer, generally an **analog computer**, designed for the purpose of solving **differential equations**.

DIFFERENTIAL EQUATION. An equation which relates, for each choice of the independent variable(s), the values of one or more dependent variables and one or more **derivatives** of each. It is an **ordinary differential equation** in case only a single independent variable occurs; a **partial differential equation** when there are two or more. For a dependent variable to be uniquely defined as a function of the independent variable(s), there must be associated a sufficient number of initial or boundary conditions, or a combination of both. (For specific differential equations, Bessel, Laplace, etc., see respective names. For methods of numerical solution see **ordinary differential equations**, **partial differential equations**, **elliptic equations**, **parabolic equations**, and **hyperbolic equations**.)

DIFFERENTIAL EQUATIONS, BOUNDARY VALUE PROBLEM IN. See **boundary value problem**.

DIFFERENTIAL EQUATIONS, HOLO-NOMIC. See **conservative force**.

DIFFERENTIAL EQUATIONS, NON-HOLONOMIC. See **conservative force**.

DIFFERENTIAL EQUATIONS, RHEONOMIC. See **conservative force**.

DIFFERENTIAL EQUATIONS, SCLERONOMIC. See conservative force.

DIFFERENTIAL FORM (OF DEGREE r IN n -DIMENSIONS). A homogeneous polynomial of degree r in n differentials, e.g., dx^i or dx_i ($i = 1, 2, \dots, n$), the coefficients in which are functions of x^i or x_i .

DIFFERENTIAL HEAT OF MIXING. See heat of dilution.

DIFFERENTIAL OPERATOR. See operator; difference operators.

DIFFERENTIAL OPERATORS, SPINOR. See spinor calculus.

DIFFERENTIAL PARAMETERS, BELTAMI. See Beltami differential parameters.

DIFFERENTIAL PROCESS. In statistics, a synonymous term for additive process.

DIFFERENTIAL THEOREM. See Laplace transform theorems.

DIFFERENTIATION. The process of finding a derivative. *Logarithmic differentiation* consists of finding the derivative of a function by first differentiating its logarithm.

DIFFERENTIATION FORMULAS (NUMERICAL). A quite general procedure for carrying out numerical differentiation is to use interpolation, or least squares, or some other method for approximating the function of interest, and then to differentiate the approximating function. When the points of interpolation are unequally spaced one can use divided differences to form an interpolation polynomial.

When the points are uniformly spaced, a number of convenient formulas can be derived by means of difference operators. Since

$$hD = \theta = \log(1 + \Delta) = -\log(1 - \nabla),$$

one obtains

$$\begin{aligned} \theta &= \Delta - \Delta^2/2 + \Delta^3/3 - \dots \\ &= \nabla + \nabla^2/2 + \nabla^3/3 + \dots, \end{aligned}$$

in terms of forward and backward differences, respectively. Derivatives of higher order can be obtained by raising to higher powers the expressions for θ .

Expressions in terms of central differences require more manipulation to obtain, but one finds

$$\begin{aligned} \theta &= \mu[\delta - \delta^3/3! + (2!)^2\delta^5/5! \\ &\quad - (3!)^2\delta^7/7! + \dots], \end{aligned}$$

$$\begin{aligned} \theta^2 &= 2[\delta^2/2! - \delta^4/4! + (2!)^2\delta^6/6! \\ &\quad - (3!)^2\delta^8/8! + \dots]. \end{aligned}$$

To obtain higher powers write $\theta^3 = \theta \cdot \theta^2$, $\theta^4 = (\theta^2)^2$, $\theta^5 = \theta \cdot \theta^4$, \dots . To apply, one truncates the series at some point, and if desired one can replace Δ , ∇ , δ and $\mu\delta$ by their expressions in terms of E .

In case the derivative is required at a point other than a tabulated one, say at $x_0 + uh$, one must apply E^u , expressing E in terms of the appropriate difference operator. However, the error can be expected to be substantially greater. (See Stirling numbers.)

DIFFERENTIATION UNDER THE INTEGRAL SIGN. The derivative, with respect to the parameter, of a definite integral of a function $f(x, m)$ containing a parameter m , when the limits of the integral are constants a and b , is given by

$$\frac{d}{dm} \int_a^b f(x, m) dx = \int_a^b \frac{\partial f}{\partial m} dx$$

and when the limits of the integral are u and v , functions of m , by

$$\begin{aligned} \frac{d}{dm} \int_u^v f(x, m) dx \\ = \int_u^v \frac{\partial f}{\partial m} dx + f(v, m) \frac{dv}{dm} - f(u, m) \frac{du}{dm}. \end{aligned}$$

DIFFLUENCE. The rate at which adjacent flow is diverging along an axis oriented normal to the flow at the point in question; the opposite of confluence. A difffluence formula, especially useful in meteorology, is

$$\frac{\partial v_n}{\partial n} \quad \text{or} \quad V \frac{\partial \psi}{\partial n},$$

where V is the speed of the wind, the n axis is oriented 90 degrees clockwise from the direction of the wind vector, v_n is the wind component in the n direction and ψ is the wind direction measured in degrees clockwise from a reference direction.

DIFFLUENCE REGION. A region in which the spacing between streamlines increases in the direction of flow.

DIFFRACTION. The deviation of the direction of propagation of a wave, determined by its undulatory character, and occurring when the wave passes the edge of an obstacle, or through a restricted aperture. Diffraction phenomena are characteristic of all wave systems. They are found in water waves, matter waves (e.g., wave phenomena exhibited by electrons, neutrons, etc.), sound waves and electromagnetic waves of all frequencies. They produce patterns which are geometrically similar whenever the relation of the wavelength to the geometry of the obstacle or aperture is the same. (See **wave, diffracted; Fresnel diffraction; Fraunhofer diffraction; Airy disc; rectangular aperture; diffraction grating; slit.**)

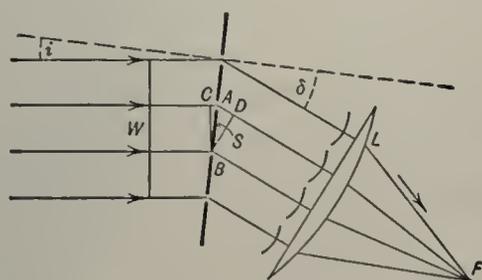
DIFFRACTION, ANGLE OF. See **angle of diffraction.**

DIFFRACTION EVOLUTE. When an object casts a shadow on a plane due to the illumination from a point source, the predominant diffraction figure formed within the shadow is the evolute of the boundary of the shadow and so called a diffraction evolute. When the shadow is circular, the diffraction evolute is an **Arago spot.**

DIFFRACTION, FRAUNHOFER. See **Fraunhofer diffraction.**

DIFFRACTION, FRESNEL. See **Fresnel diffraction.**

DIFFRACTION GRATING, ACTION OF IDEAL PLANE WAVE. A plane, monochromatic light wave W , incident at angle i (see figure), reaches the slits at different times. A



Diffraction by a plane grating.

lens L receives the waves emerging from any two adjacent slits, A and B (among many others), after they have traveled paths differing by $CA + AD$; that is, by $S \sin i + S \sin \delta$, in which $S = AB$. If the lens is so placed that this path difference is a whole number of

wavelengths, $n\lambda$, the successive wave-trains will reach it in the same phase, so that when they are brought to the focus F , they will be in synchronism and will produce a bright image of the distant source. Therefore any angle δ for which this result is possible is subject to the condition

$$S \sin i + S \sin \delta = n\lambda,$$

or

$$\sin \delta = \frac{n\lambda}{S} - \sin i.$$

Bright images will be produced for those angles δ which correspond to $n = 1, 2, 3, 4, \dots$; the numbers denote the "orders" of the images. It is easily shown that for any order the total deviation ($i + \delta$) is least when $\delta = i$ and therefore when

$$\sin \delta = \frac{n\lambda}{2S}.$$

If the incident light is composed of various wavelengths, the corresponding images of any order will appear at different points, since δ varies with λ ; and the result is a **spectrum.**

DIFFRACTION OF NEUTRONS, ANGULAR. When a well collimated beam of slow neutrons (whose de Broglie wavelength λ is of the order of angstroms) falls at a glancing angle upon a crystal, it is preferentially scattered, or reflected, when the angle between the beam and the crystal plane, θ , obeys the Bragg relation

$$n\lambda = 2d \sin \theta$$

where d is the spacing of the crystal planes and n is an integer. Thus slow neutrons behave much as do x-rays in this respect, and have consequently been much used for the investigation of crystal structure. Whereas x-rays are scattered most strongly by atoms of high atomic number Z , however, neutrons are scattered most strongly by protons, i.e., hydrogen atoms. Thus, neutron and x-ray diffraction techniques are largely complementary. In addition neutron diffraction may be used to study the structure of magnetic crystals owing to the scattering which can take place due to the neutron's magnetic moment.

DIFFRACTION PATTERNS. Interference patterns produced by the diffraction of light or x-rays by crystals, slits, liquids, and so on.

DIFFRACTION SPECTRUM. See spectrum, diffraction.

DIFFRACTION THEORY OF ABERRATIONS. See Nijboer-Zernike aberration functions.

DIFFUSER. A duct in which the velocity of a fluid is gradually reduced, with a corresponding increase of pressure. For an effectively incompressible fluid the *efficiency of a diffuser* is defined as the ratio of the actual pressure rise to the pressure rise calculated for the given shape of diffuser with an inviscid fluid. The simplest form of diffuser for subsonic flow consists of a straight tapered duct of circular section. For this case, if the subscripts 1 and 2 refer to the inlet and outlet, respectively, the efficiency for an incompressible fluid is

$$\frac{p_2 - p_1}{\frac{1}{2}\rho V_1^2 \left[1 - \left(\frac{d_1}{d_2} \right)^4 \right]}$$

where V_1 is the velocity at inlet (assumed uniform), d is diameter and p is pressure. Efficiencies up to about 90% can be obtained, with cone angles of about 6 to 8 degrees. For large cone angles separation occurs and the efficiency is much reduced.

For a compressible fluid the efficiency of a diffuser is usually defined as

$$\left\{ \begin{array}{l} \text{Work required per unit mass of fluid for} \\ \text{isentropic compression from inlet pressure} \\ v_1 \text{ to actual outlet pressure } p_2 \end{array} \right\}$$

Kinetic energy of fluid per unit mass, at entry

For a perfect gas this efficiency is equal to

$$\frac{(p_2/p_1)^{\frac{\gamma-1}{\gamma}} - 1}{\left(\frac{\gamma-1}{2} \right) M_1^2}$$

where γ is the ratio of specific heats and M_1 is the inlet Mach number.

When the flow is supersonic at the inlet to the diffuser, as in a diffuser for a supersonic wind tunnel or an air intake for an engine in supersonic flight, part of the compression takes place through one or more shock waves. The efficiency can then be improved by designing the diffuser so that the compression is shared between a number of relatively weak shock waves instead of occurring in one strong one.

DIFFUSE REFLECTANCE. The ratio of the luminous flux diffusely reflected in all directions (other than that of direct reflection) to the total incident flux.

DIFFUSE REFLECTION. A matte surface is continuous but not smooth, i.e., belongs to class C_0 but not class C_1 . When a matte surface is an interface between two media, the resulting reflection of light is diffuse. **Interfaces** belonging to class C_1 produce **regular** or **specular reflections** or refractions.

DIFFUSE SERIES. Series of lines in the spectra of the simpler atoms, the alkalis, alkaline earths, helium, etc., corresponding to transitions from an upper D state to a lower P state.

DIFFUSE TRANSMITTANCE. The ratio of the luminous flux diffusely transmitted in all directions (other than that of direct transmission) to the total incident flux.

DIFFUSION. (1) The process by which molecules intermingle as a result of their random thermal motion. In gases and in liquids in the neighborhood of the **critical point**, the molecular motion resembles a **random walk**, and the diffusion coefficient is of order $\frac{1}{3}\bar{c}\lambda$, where \bar{c} is the mean thermal velocity of a molecule and λ is the **mean free path** of a free molecule. If the liquid is more highly condensed, diffusion depends on thermally induced movements from one stable position in the local lattice to another one, as in diffusion of solids. Diffusion in liquids is extremely slow, but it may be accelerated by inducing a turbulent flow which mingles the separate components intimately and allows molecular diffusion to take place down greatly increased local intensity gradients.

(2) The passage of particles through matter in such circumstances that the probability of scattering is large compared with that of leakage or absorption. It is often limited to phenomena described by a member of the class of differential equations known as diffusion equations.

DIFFUSION AREA, NEUTRON. This area is the square of the **diffusion length** and is equal to one-sixth of the mean square distance traveled by a diffusing neutron, from source to capture.

DIFFUSION BY TURBULENCE. The transport by turbulent motion of a property of the fluid (e.g., momentum, heat, pollution). The process might be described precisely if sufficient detail were permitted, but in the complicated motions conveniently described as turbulence only the mean properties are of importance and the mixing processes are then regarded as equivalent to molecular diffusion, though with a different coefficient.

The distance X traveled by a particle (e.g., of pollution) carried by the motion in time T is given by

$$\overline{X^2} = 2\overline{u'^2} \int_0^T \int_0^t R_\xi d\xi dt$$

where R_ξ is Taylor's autocorrelation coefficient.

DIFFUSION COEFFICIENT. See diffusion, Fick law; transport coefficient.

DIFFUSION COEFFICIENT-MOBILITY RELATION. See Einstein relationship between mobility and diffusion coefficient.

DIFFUSION COEFFICIENT OF ELECTRONS. The variation of concentration n_e of mobile electrons in a semiconductor (see density of mobile charges in semiconductors) obeys the generalized diffusion equation

$$\frac{\partial n_e}{\partial t} = \nabla \cdot (D_e \nabla n_e) + \nabla \cdot (n_e \mu_e \mathbf{E}) - s$$

where D_e is the diffusion coefficient, $\mu_e = eD_e/kT$ where e is the electronic charge, k is Boltzmann's constant, T is the absolute temperature, \mathbf{E} is the electric field strength, and s is the net rate of disappearance of electrons from the conduction band due to trapping, recombination, etc.

An analogous equation holds for the diffusion of holes where the sign of \mathbf{E} is changed.

DIFFUSION COEFFICIENT OF GASES. See diffusion, kinetic theory of.

DIFFUSION COEFFICIENT OF NEUTRONS. When neutron transport through matter is described by diffusion equations governed by Fick's law, the diffusion constant, D , is given as:

$$D = \frac{1}{3\Sigma_s(1 - \overline{\cos \theta})}$$

where Σ_s is the macroscopic scattering cross-section and $\overline{\cos \theta}$ is the average value of neutron scattering angle, θ , in the scattering process.

$\overline{\cos \theta}$ is often put equal to $\frac{3}{2M}$ for scattering by

a nucleus of mass M . For diffusion in a mixture the average diffusion coefficient \bar{D} is given by

$$\frac{1}{\bar{D}} = \sum_i \frac{1}{D_i}$$

DIFFUSION CONSTANT IN A HOMOGENEOUS SEMICONDUCTOR. The quotient of diffusion current density by the charge carrier concentration gradient. It is equal to the product of the drift mobility and the average thermal energy per unit charge of carriers.

DIFFUSION COOLING. In the transport theory of neutrons (see neutron transport theory), this effect refers to the change in the spectrum of thermal neutrons in a moderator caused by the preferential leakage of the faster neutrons.

DIFFUSION EQUATION. See diffusion, Fick law; neutron diffusion theory.

DIFFUSION FACTOR. The diffusion factor (of a secondary source) is the ratio of the arithmetic mean of the values of luminance measured at 20° and 70° to the luminance measured at 5° from the normal, when the secondary source considered is illuminated normally. (1) This quantity is intended to give an indication of the spatial distribution of the diffused flux. The diffusion factor is unity for every uniform diffuser, whatever the value of the diffuse reflection factor. (2) This conception applies both to reflection and to transmission. (3) The C. I. E. at its 10th session (1939) agreed that this form of definition can only be applied to materials for which the indicatrix of diffusion does not differ appreciably from that of ordinary depolished and opal glasses.

DIFFUSION, FICK LAW OF. An empirical relation which states that the diffusion flux, i.e., the quantity of matter or number of particles crossing area A in the direction of the normal n to a surface in a mass transfer process is proportional to the gradient of the concentration c . Hence

$$\mathbf{Q} = -DA \text{ grad } c \quad (1)$$

$$= -DA \frac{dc}{dn}, \quad (2)$$

Formula (2) for flow in one dimension. Making use of the principle of conservation of mass, this equation may be written

$$\nabla^2 c = -\frac{1}{D} \frac{\partial c}{\partial t}. \quad (3)$$

The factor D (in ft²/hr or m²/hr) is known as the *diffusion coefficient*. The Fick law for diffusion is similar to the **Fourier law** for conduction.

DIFFUSION, FORCED. If, in addition to a concentration gradient, there is a steady average force \mathbf{F} acting on the molecules, the simple Fick law must be modified to read

$$\mathbf{J} = -D(\text{grad } n - \frac{n\mathbf{F}}{kT}),$$

where n is the concentration in molecules per unit volume (see **diffusion, Fick law**). The force \mathbf{F} may, for example, be the force of gravity mg as in sedimentation, or the electric force $ze\mathbf{E}$ on an ion of valence z in an electric field of intensity \mathbf{E} .

DIFFUSION INDICATRIX. See **indicatrix of diffusion**.

DIFFUSION KERNEL. See **kernel, diffusion**.

DIFFUSION, KINETIC THEORY OF. In a gas in which there exists a concentration gradient, more particles will move towards the regions of lower concentration than away from them. The diffusion coefficient D is defined by the equation

$$w = -D \frac{dn}{dx},$$

where dn/dx is the concentration gradient and w the net flux of particles in the x -direction. From a consideration of the number of particles crossing a unit area perpendicular to the x -direction either in the positive or in the negative x -direction one finds

$$D = c\lambda v,$$

where c is a numerical constant of order unity, λ the **mean free path**, and v the mean velocity of the particles.

DIFFUSION LENGTH, GENERAL. The mean distance traveled by a diffusing particle from the point of its formation to the point at which it is absorbed. In a homogeneous **semi-conductor**, the particle is a minority carrier.

DIFFUSION LENGTH, NEUTRONS. The quantity $L = \sqrt{D/\Sigma_a}$ where D is the diffusion coefficient and Σ_a the macroscopic absorption cross section characterizing the medium through which neutrons diffuse. The diffusion equation, describing steady-state transport in a homogeneous source-free medium assumes the form,

$$\left(\nabla^2 - \frac{1}{L^2}\right)\Phi = 0.$$

DIFFUSION OF LIGHT. The alteration of the spatial distribution of a beam of light which, after reflection at a surface or passage through a medium, travels on in numerous directions. (See **perfect diffusion; indicatrix of diffusion**.)

DIFFUSION OF SOLIDS. Solids in close contact may diffuse into each other, the rates of diffusion being exponentially related to the absolute temperature and to the time.

A solid may diffuse into itself, the coefficient of self-diffusion depending on temperature according as

$$D = D_0 \exp(-E_A/RT)$$

where E_A is an **activation energy**, and R is the gas constant.

DIFFUSION STABILITY. (See **thermodynamic stability conditions, stability of phases**.) In systems containing more than one component, the initial uniform phase is stable in respect to a perturbation consisting of the appearance of a heterogeneity in composition. Stability conditions with respect to perturbations of this kind are called **diffusion stability conditions**.

In a binary system at constant p and T the diffusion stability condition is

$$\frac{\partial^2 g}{\partial x_A^2} > 0 \quad (1)$$

where

$$g = \frac{G}{n} \quad (2)$$

is the Gibbs free energy per mole (see **thermodynamics, characteristic functions** of) and x_A , the mole fraction of component A .

DIFFUSION THEORY, BOUNDARY. See boundary conditions, diffusion theory.

DIFFUSION THEORY, MULTIGROUP NEUTRON. See neutron diffusion theory, multigroup.

DIFFUSION THEORY, NEUTRON. See neutron diffusion theory.

DIFFUSION TIME, NEUTRON. In nuclear reactor theory, the average time spent by a thermal neutron from its appearance as thermal, to its capture. It is approximately equal to $(\Sigma_a(vp)vp)^{-1}$ where vp is the most probable neutron speed in the distribution of thermal neutrons and Σ_a is the macroscopic absorption cross-section characteristic of the moderator.

DIFFUSION VELOCITY. The flow of component γ (see Equation (3) of **conservation of mass in continuous systems**) can be decomposed into a flow with the average mass velocity ω (see Equation (2) of **conservation of mass in continuous systems**) and a diffusion flow relative to ω

$$\rho_\gamma \omega_\gamma = \rho_\gamma \omega + \rho_\gamma (\omega_\gamma - \omega) = \rho_\gamma \omega + \rho_\gamma \Delta_\gamma \quad (1)$$

where Δ_γ denotes the diffusion velocity with respect to ω . We may note that

$$\sum_\rho \rho_\gamma \Delta_\gamma = 0. \quad (2)$$

(See **flux vectors**.)

DIFFUSIVITY. See **Fourier law**.

DIGITAL COMPUTER. As contrasted with **analog computer**, a computing aid that represents numbers digitally, in decimal, binary, or (conceivably) other base. The simplest artificial example is the abacus, which represents numbers digitally in such a way as to facilitate carrying out additive operations, hence multiplication and division as sequences of these. Desk computers perform these operations automatically, and sometimes also square-rooting or other special sequences. The very high speed, general purpose computers are **stored-program computers** and are digital.

DIGITAL COMPUTER, PARALLEL. A digital computer in which the digits are

handled in parallel. Mixed serial and parallel machines are frequently called serial or parallel according to the way arithmetic processes are performed. (See also **digital, computer, serial**.)

DIGITAL COMPUTER, SERIAL. A digital computer in which the digits are handled serially. Mixed serial and parallel machines are frequently called serial or parallel according to the way arithmetic processes are performed. (See also **digital, computer, parallel**.)

DIGITS (SIGNIFICANT). The significant digits of a number like 2301 or 0.02301, etc., are those digits which determine the ratio of the error (5 in the next decimal place) to the number. Thus, the accuracy, in this sense, of the two numbers given above is the same in each case, namely 5/23010, and each of them is said to have 4 significant digits. The significant digits can be defined as the digits beginning with the first non-zero digit on the left and ending with the last digit on the right. Thus 0.023010 has 5 significant digits. The number of significant digits in a number like 230 is usually taken to be 3, but may actually be 2, depending on whether the meaning is that $229.5 < 230 < 230.5$ or $225 < 230 < 235$. There is need, therefore, of two different symbols for zero, but a second such symbol, \emptyset , is very seldom used.

DIGRAPH. A digraph D is a finite set of vertices $\beta_1, \beta_2, \dots, \beta_v$, together with certain directed lines $\beta_1\beta_2$ (from vertex β_1 to β_2). The possibility that both $\beta_1\beta_2$ and $\beta_2\beta_1$ are lines of the digraph is not excluded. Thus an **oriented graph** is a digraph but not conversely.

The adjacency matrix (see **matrix, adjacency**) $A = (a_{ij})$ of a digraph D is defined as follows: $a_{ij} = +1$ if line $\beta_i\beta_j$ is in D and $a_{ij} = 0$ otherwise. It is important to note that A is not necessarily symmetric ($a_{ij} = a_{ji}$) as is the case for the adjacency matrix of a **non-oriented graph**. A is square and of order v , the number of vertices.

A **tree** T of D with sink β (a vertex) is

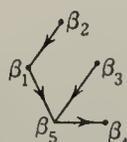
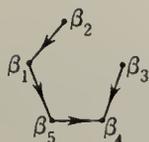
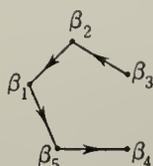
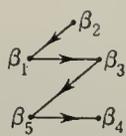
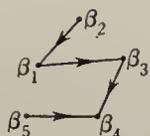
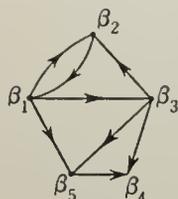
- a subgraph of D which contains all the vertices of D ;
- in T there is a directed path leading to β from every other vertex in D ;
- every vertex other than β is the initial vertex (see **vertex, initial**) of exactly one directed line in T ;

(d) there is no directed line in T for which β is the initial vertex.

An example of a digraph and its five trees is given in the accompanying figure.

$$M = \begin{vmatrix} 3 & -1 & -1 & 0 & -1 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 3 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{vmatrix}.$$

The degree matrix B for D is defined to be that diagonal matrix of order v which is such that the difference $M = B - A$ has every row sum to zero. The matrix tree theorem for digraphs states that the co-factors of the elements in the i^{th} row of M are all equal and each of them gives the number of trees of D with sink β_i . The matrix M associated with the digraph D of the illustrative example above also appears in the figure.

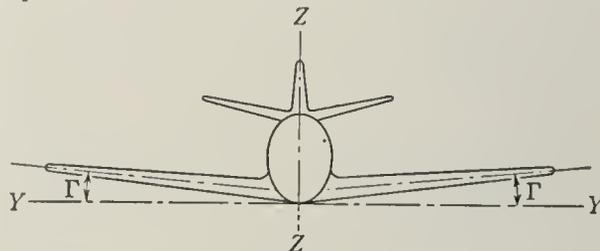


DIHEDRAL ANGLE. The angle between two planes. The dihedral angle is zero if the planes are parallel. If the direction cosines of perpendiculars to two planes are λ, μ, ν and λ', μ', ν' the dihedral angle between the planes is given by

$$\cos \theta = \lambda\lambda' + \mu\mu' + \nu\nu'.$$

DIHEDRAL ANGLE OF WING. The figure shows a front view of an aircraft. ZZ is the plane of symmetry and YY is a perpendicular plane. The dihedral angle of the wing

is the angle Γ in the figure. In general, this may vary along the span of the wing. The dihedral angle has an important effect on the derivative L_v , the rolling moment due to side-slip.



Front view of aircraft, showing dihedral angle of wing.

DIHEDRAL GROUP. The group generated by the relations $C^n = E, S^2 = E, SC = C^{-1}S$, designated by D_n and of order $2n$. Consider a regular polygon in the XY -plane with coordinates of the n corners $x_k = r \cos 2\pi k/n, y_k = r \sin 2\pi k/n, k = 0, 1, \dots, (n - 1)$. The proper rotations of this object through $2\pi/n$, indicated by $C(\phi)$, and the improper rotations, $S(\phi)$, which transform the polygon into itself constitute D_n .

DILATATION. (1) The increase of volume per unit volume of a continuous material. (See also **cubical dilatation**.) (2) Dilatation of time, see **relativity**.

DILATATION RATE. The rate of extension of fluid elements. The cubical dilatation at a point is equal to the divergence of velocity.

DIMENSION. For the purposes of applied mathematics, the dimension of a set of elements is most conveniently defined as the number of parameters necessary to give an **analytic** description of the set. Thus, a space is said to have n dimensions when n coordinates are necessary to fix the position of a point in the space.

DIMENSIONAL ANALYSIS. Since the quantities represented by the two sides of an equation must have the same dimensions, it is often possible to arrive at the form of an equation connecting physical quantities by a consideration only of the dimensions of the quantities involved, without a detailed theory and without consideration of magnitudes. The equations obtained in this way may be in error by a multiplying constant, which can often be determined empirically. In order that two physical quantities may be equal, or that one may be added to or subtracted from the other,

it is obvious that they must have the same makeup and be expressible by the same combination of fundamental units. It follows that in an equation expressing relationship between physical magnitudes, both members and all terms of each member must have the same dimension formula. For example, the total area of a right circular cone of altitude h and having a base of radius r is $a = \pi r^2 + \pi r \sqrt{r^2 + h^2}$, each term of which has the dimension formula L^2 (since π is abstract). Again, the phase angle ϕ of an **alternating current** of frequency n (per second) in a circuit of resistance R (ohms), inductance L (henrys), and capacitance C (farads) is given by

$$\tan \phi = \frac{4\pi^2 n^2 LC - 1}{2\pi n RC}$$

Since $\tan \phi$ is an abstract quantity, the fraction on the right must also be abstract. Since the 1 in the numerator is abstract, the other term $4\pi^2 n^2 LC$ and the whole numerator must be also; hence the denominator is abstract. That is, n , R , and C should have such dimensions that the component fundamental magnitudes cancel when the product nRC is formed. This is true; for the dimension formula of n is $1/T$, and (in electromagnetic measure) that of R is $\mu L/T$, and of C , $T^2/\mu L$; in which μ represents magnetic permeability.

DIMENSIONLESS GROUPS, OR NUMBERS. Expressions of the type

$$G = A^\alpha B^\beta C^\gamma \dots$$

in which A , B , C , etc., denote some physical quantities, and the exponents α , β , γ , etc., are pure numbers, which have the property that their physical dimensions in terms of some chosen system of fundamental dimensions reduces to a pure number. (For examples of dimensionless groups, see **Reynolds number**, **Nusselt number**, **Prandtl number**, **Mach number**, **Froude number** and **Grashof number**.)

DIMENSIONLESS NUMBER. See **dimensionless groups**.

DIOPHANTINE EQUATIONS. A set of **algebraic equations**, usually indeterminate (i.e., with fewer equations than unknowns) and with integral coefficients, for which integral solutions are desired.

DIOPTER. The metric unit of refractive power. The power of an optical system in

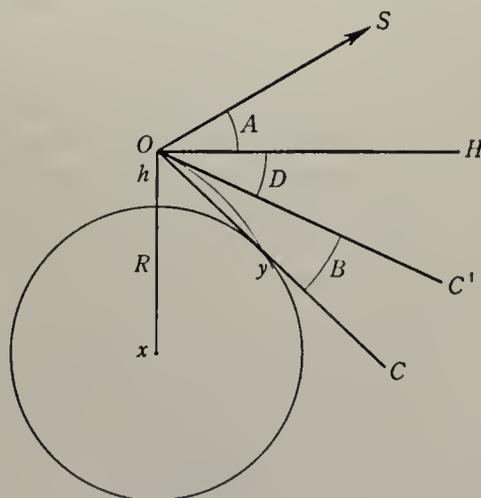
diopeters is numerically equal to the reciprocal of the **focal length** in meters. In ophthalmic optics the deviation of a prism is measured in terms of a similarly named but different unit, the *prism diopter*.

DIOPTICS. That branch of optics dealing with the refraction of light, as opposed to **catoptics** (dealing with reflection).

DIP OF THE HORIZON. The fundamental observation in nautical astronomy is the determination of the altitude of a celestial object. At sea the mariner's sextant is used to measure the angle from the sensible horizon OC' to the direction of the object OS . The observer is a height h above the surface of the assumed spherical earth. OC is the direction of the line from the observer tangent to the earth at y . The light from y to O passes through the atmosphere of the earth in the strata of varying index of refraction. This light follows a curved path and enters the sextant as though coming from C' , the direction of the sensible horizon.

OH is the direction of the astronomic horizon and A is the astronomic altitude of S , uncorrected for parallax or for semidiameter if the altitude of a limb of an object of finite size is observed, e.g., moon, sun, etc.

The direction from the sensible horizon to the astronomic horizon is known as the dip of the horizon, and it is evident that $A = SOC' - D$ in which A is the astronomic altitude uncorrected for parallax or semidiameter.



There is no accurate method known for determining the value of the dip of the horizon from theoretical methods. Long series of observations have resulted in an empirical expression for dip as $D = 0.97h^{1/2}$, with h the height

of the eye above sea level in minutes of arc. There is still uncertainty in the value of the dip as determined from this expression. The Carnegie Institution of Washington found that for 5,000 determinations of the dip at sea no value differed from that computed from the formula by more than 2'5 except one difference of 10'6. Difference from the computed values have been reported as great as 30' and in the polar regions differences as great as several degrees have been reported.

DIPOLE-DIPOLE INTERACTION ENERGY. The mutual energy of interaction of two permanent dipoles of moments P_1 and P_2 , separated by a displacement \mathbf{r} , is (in rationalized units):

$$\frac{1}{4\pi\epsilon_0 r^3} \left[\mathbf{P}_1 \cdot \mathbf{P}_2 - \frac{3(\mathbf{P}_1 \cdot \mathbf{r})(\mathbf{P}_2 \cdot \mathbf{r})}{r^2} \right].$$

DIPOLE, ELECTRIC. Two electrically-charged particles of equal magnitude and opposite sign, separated by a very small distance. The electric potential due to static electric dipole is (in rationalized mksa units):

$$V = p \cos \theta / 4\pi\epsilon_0 r^2$$

where $p = qd$ is the magnitude of the dipole moment.

The electric field due to a static dipole is given by $\mathbf{E} = -\nabla V$ (see **grad**). The torque on a dipole in a field \mathbf{E} is given by $\mathbf{L} = \mathbf{p} \times \mathbf{E}$, and the force on a dipole in a non-uniform inhomogeneous field is given by $\mathbf{F} = \nabla(\mathbf{p} \cdot \mathbf{E})$ where $(-\mathbf{p} \cdot \mathbf{E})$ is the energy of the dipole due to its orientation in the field \mathbf{E} .

Asymmetrical molecules behave as dipoles, since the centers of gravity of the positive and negative charges do not coincide. They tend to orient themselves in external electrical fields or by mutual interactions. The molecular dipole moments are usually expressed in **Debye units** D , where $D = 10^{-18}$ esu.

DIPOLE FIELD (STATIC). At the position r, θ, ϕ , in a spherical polar coordinate system having its polar axis along the direction of the dipole moment, the field due to a dipole has the form, in rationalized units:

$$E_r = \frac{p \cos \theta}{4\pi\epsilon_0 r^3}; \quad E_\theta = \frac{p \sin \theta}{4\pi\epsilon_0 r^3}; \quad E_\phi = 0$$

where p is the magnitude of the dipole moment ($p = qd$ for an electric dipole). For a mag-

netic dipole, $p = iA$ and H replaces E . These expressions are valid only when r , the distance from the dipole to the point at which the field is measured, is very much larger than any dimension of the dipole. If this condition is not satisfied, the effects of **multipole moments** must be included in the field.

DIPOLE, MAGNETIC. A circulating current loop, of dimensions small compared to the distance at which it is being observed, acts like a dipole of moment of magnitude $m = iA$, where i is the current and A is the area of the loop. The *magnetic dipole moment* can be considered as a vector whose direction is normal to the plane of the loop and whose sense is taken as the direction of progression of a right-handed screw rotating with the current.

The *magnetic scalar potential* $\phi [= \mathbf{m} \cdot \nabla \left(\frac{1}{r} \right)]$,

where \mathbf{m} is the magnetic moment] and the *magnetic flux density* \mathbf{B} produced by a static magnetic dipole are given by expressions that are identical, except for obvious changes of symbols, to those for the potential V and electric field \mathbf{E} produced by an electric dipole. (See **dipole, electric**.)

DIPOLE MOMENT, ELECTRIC. For an electric dipole consisting of two charges, $+q$ and $-q$, the dipole moment is the vector $\mathbf{p} = q\mathbf{d}$, where \mathbf{d} is the displacement of the positive charge with respect to the negative charge.

DIPOLE MOMENT, INDUCED. A dipole moment induced in a system (e.g., in an atom or molecule) because it is brought into an electric or magnetic field. (See **polarizability**.)

DIPOLE MOMENT, MAGNETIC. See **dipole, magnetic; dipole field (static)**.

DIPOLE MOMENT, MOLECULAR. It is found from measurements of **dielectric constant** (i.e., by its temperature dependence, as in the Debye equation for total polarization) that certain molecules have permanent dipole moments. These moments are associated with the distribution of charge within the molecule, and provide valuable information as to the molecular structure.

DIPOLE ORIENTATION. The effect by which the free rotation of the molecules in a solid may be hindered, so that their **dipole moments** may only take on certain discrete

orientations. This may have a marked effect on the dielectric constant. (See steric hindrance.)

DIPOLE, OSCILLATING. A dipole whose dipole moment oscillates harmonically with time, $\mathbf{p} = p_0 e^{j\omega t}$. Oscillating electric and magnetic dipoles radiate electromagnetic waves. (See dipole radiation entries.)

DIPOLE, PERMANENT. A dipole whose dipole moment does not vanish in the absence of an externally applied field.

DIPOLE RADIATION, ELECTRIC. The electromagnetic radiation sent out by an oscillating electric dipole, $\mathbf{p} = \mathbf{p}_0 e^{-j\omega t}$. The electric and magnetic fields at a distance r from the dipole are (in spherical polar coordinates and in rationalized mksa units):

$$E_\theta = \frac{-k^2 p_0 \sin \theta e^{j(k-\omega t)}}{4\pi \epsilon_0 r},$$

$$H_\phi = \frac{\omega k p_0 \sin \theta e^{j(k-\omega t)}}{4\pi r},$$

$$H_r = 0,$$

where $k = \omega/c$. The term is often applied to radiation from quantized atomic or molecular systems whose transitions produce radiation having the same dependence on θ as does the radiation from a classical electric dipole.

DIPOLE RADIATION, MAGNETIC. The electromagnetic radiation sent out by an oscillating magnetic dipole, $\mathbf{m} = \mathbf{m}_0 e^{-j\omega t}$ where $m_0 = i_0 A$. The structure of the field is identical with that of the electric dipole (see dipole radiation, electric) except that H replaces E . Thus, the field is given by

$$E_\phi = \frac{k^2}{4\pi} \sqrt{\frac{\mu}{\epsilon}} \frac{m_0 \sin \theta e^{j(k-\omega t)}}{r},$$

$$H_\theta = \frac{k^2 m_0 \sin \theta e^{j(k-\omega t)}}{4\pi r},$$

$$H_r = 0.$$

DIPOLE TRANSITION, ELECTRIC AND MAGNETIC. Transitions from one energy state to another, accompanied by the absorption or emission of dipole radiation. Depending on the nature of the initial and final states, the radiation may be electric dipole radiation or magnetic dipole radiation.

DIRAC DELTA FUNCTION. See distribution theory.

DIRAC EQUATION. The relativistic wave equation discovered by Dirac in 1928 while trying to overcome the difficulties of negative probabilities associated with the one-particle interpretation of the Klein-Gordon equation if one does not adopt the manifold of positive energy solutions as the set of realizable states. Dirac was led to the equation bearing his name by requiring that it be of first order in the time like the Schrödinger equation. Relativistic covariance requires that there be complete symmetry in the treatment of the space and time components so that the equation must likewise be of first order in the space derivatives. The superposition principle of quantum mechanics requires that it be linear. Finally the requirement that in the correspondence limit classical relativity be valid implies that the wave function ψ also satisfy the equation

$$(\square + m^2 c^2 / \hbar^2) \psi(x) = 0 \quad (a)$$

since this is the statement that ψ describes a free particle of mass m for which the energy momentum equation $p^2 = m^2 c^2$ is to hold. The most general equation which satisfies these conditions is then of the form

$$\begin{aligned} \frac{1}{c} i \frac{\partial \psi}{\partial t} &= i \boldsymbol{\alpha} \cdot \nabla \psi + \frac{mc}{\hbar} \beta \psi \\ &= H \psi \end{aligned} \quad (b)$$

where $\boldsymbol{\alpha} = (\alpha_1, \alpha_2, \alpha_3)$, β are square matrices and ψ an N -component quantity. The right-hand side of (b) can be taken as the Hamiltonian, H , which must be hermitian. The matrices α and β must therefore be hermitian. The requirement that (a) be satisfied requires that these matrices satisfy the following commutation rules:

$$\begin{aligned} \alpha^k \alpha^l + \alpha^l \alpha^k &= 2\delta^{kl} & k, l = 1, 2, 3 \\ \alpha^k \beta + \beta \alpha^k &= 0 \\ \beta^2 &= 1. \end{aligned} \quad (c)$$

These in turn imply that there exist but one irreducible representation of these matrices. This representation is 4-dimensional. The wave function ψ thus has 4 components. The equation can be shown to be form invariant under Lorentz transformations

$$x \rightarrow x' = \Lambda x + a, \quad (d)$$

if the transformed wave function ψ' , is related to ψ by

$$\psi'(x') = S(\Lambda)\psi(\Lambda^{-1}(x' - a)) \quad (e)$$

where $S(\Lambda)$ is a 4×4 matrix operating on the components of ψ and which satisfies the equation

$$S^{-1}\gamma^\lambda S = \sum_{\mu=0}^{\infty} \Lambda_\mu^\lambda \gamma^\mu. \quad (f)$$

The 4×4 matrices γ^μ , $\mu = 0, 1, 2, 3$, are related to the α and β matrices by $\gamma^i = \beta\alpha^i$, $i = 1, 2, 3$ and $\gamma^0 = \beta$. They satisfy the following commutation rules

$$\gamma^\mu\gamma^\nu + \gamma^\nu\gamma^\mu = 2g^{\mu\nu} \quad (g)$$

where $g^{\mu\nu}$ is the metric tensor.

A particle described by the Dirac equation has in addition to its orbital angular momentum $\mathbf{r} \times \mathbf{p}$, an intrinsic angular momentum, or spin, of magnitude $\hbar/2$. The spin is described by an operator $\frac{\hbar}{2}\Sigma$ (a 4×4 matrix) whose component in any direction has eigenvalues $\pm\hbar/2$. A Dirac particle is therefore a spin $\frac{1}{2}$ particle. The total angular momentum of a Dirac particle is the vector sum of orbital and spin angular momentum.

The Dirac equation admits of both positive and negative energy solutions, i.e., solutions of the form $\psi \sim \exp(-ip_\mu x^\mu)u(\mathbf{p})$ with $p_0 = +c\sqrt{\mathbf{p}^2 + mc^2}$ positive energy solutions and $p_0 = -c\sqrt{\mathbf{p}^2 + mc^2}$ for negative energy solutions. For each three dimensional momenta \mathbf{p} there are two linearly independent solutions of positive energy and two linearly independent solutions of negative energy. These linearly independent solutions can be chosen to be eigenfunctions of the helicity operator $S(p) = \Sigma \cdot \mathbf{p}/|\mathbf{p}|$, whose eigenvalues are $+1$ and -1 depending on whether the spin of the particle is parallel or anti-parallel to the direction of motion. In the presence of an external electromagnetic field the Dirac equation for an electrically charged particle of charge e is

$$\gamma^\mu \left(i\hbar\partial_\mu - \frac{e}{c}A_\mu(x) \right) \psi(x) = mc\psi(x). \quad (h)$$

This equation predicts that a charged spin $\frac{1}{2}$ particle has a magnetic moment, μ , of one Bohr magnetron: $\mu = \mu_0 = \frac{e\hbar}{2mc}$. The non-relativistic **Pauli theory** had to ascribe arbitrarily a magnetic moment μ to the spin, whose value it

took from the experimental results. Dirac's equation on the other hand contains implicitly an interaction of the spin with a magnetic field. The resulting magnetic moment $\mu = \mu_0 = e\hbar/2mc$ is in almost exact agreement with experiment in the case of the electron. (See **anomalous magnetic moment of a spin $\frac{1}{2}$ particle.**) Equation (h) may be applied to the hydrogen atom by choosing $A_i = 0$, $i = 1, 2, 3$, $A_0 = Ze^2/r$, i.e., by considering the nucleus as the source of the Coulomb field in which the electron moves. For this situation the equation can be solved exactly and the energy levels, $E_{n,j}$, for the positive energy states, are described by the principal quantum number n and the total angular momentum quantum number j with

$$E_{n,j} = mc^2 \left\{ 1 + \frac{\alpha^2 Z^2}{(n' + \sqrt{(j + \frac{1}{2})^2 - \alpha^2 Z^2})^2} \right\}^{\frac{1}{2}} \quad (i)$$

$$n' = 0, 1, 2, \dots, \quad j = \frac{1}{2}, \frac{3}{2}, \dots$$

or approximately

$$E_{n,j} \approx mc^2 - \frac{\alpha^2 mc^2 Z^2}{2n^2} \left[1 + \frac{\alpha^2 Z^2}{n} \left(\frac{1}{j + \frac{1}{2}} - \frac{3}{4n} \right) + \dots \right] \quad (j)$$

$$n = n' + j + \frac{1}{2} = 1, 2, \dots$$

The observed fine structure of the levels of hydrogen and hydrogen-like atoms, particularly He^+ , is in good agreement with the Dirac equation. This agreement includes the degeneracy of levels with the same j except for one case, namely the $S_{\frac{1}{2}}$ and $P_{\frac{1}{2}}$ level. In this case Lamb found the $2S_{\frac{1}{2}}$ state of hydrogen to be about 1057.77 megacycles higher than the $2P_{\frac{1}{2}}$ state, a separation which has been explained by radiative corrections to the simple Dirac equation. (See **level displacements of hydrogen-like atoms.**)

Although by adopting the manifold of positive energy solutions as the set of realizable states for a spin $\frac{1}{2}$ particle, good agreement is obtained with many experimental facts relating to the behavior of such particles, nonetheless there remains within the framework of a one-particle interpretation of the Dirac equation, several unacceptable features connected with the negative energy solutions.

These states are not realized in nature. The Dirac equation, however, predicts the possibility of radiative transitions with the emission of light from positive to negative energy states. No regular (positive energy) state could thus be stable since there are an infinite number of negative energy states to which such a positive energy particle could make a transition with the emission of a quantum of light. To overcome these difficulties, Dirac put forth the hypothesis that the vacuum corresponds to the state in which all negative energy states are occupied (by just one particle, in view of Pauli's exclusion principle) but that this infinite "sea" of particles does not produce any electromagnetic effects. Only a deviation from this normal condition, e.g., the presence of a positive energy particle or the absence of a negative energy particle (a "hole" in the sea of particles) leads to observable effects. Such a hole, corresponding to a negative energy $-E$, behaves like a positively charged particle of positive energy $+E$. The Dirac hypothesis is known as the **hole theory**. While Dirac first meant to identify the holes as protons, it was shown by Oppenheimer and Weyl that relativistic invariance requires that hole and particle have the same mass. This prediction of the theory was confirmed by Anderson's discovery of positrons, particles of mass m_e , the electronic mass, but having a charge equal and opposite to that of electrons.

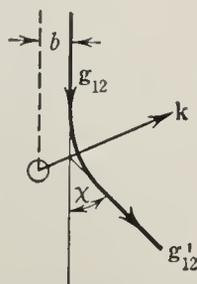
Thus although Dirac's equation was designed to describe one particle, the above makes it clear that this interpretation cannot be maintained. Since particles may be created and annihilated by transitions from and to negative energy states, the number of particles is not a constant and a field theoretical description is called for. Accordingly, the arguments advanced for the particular form of Dirac's equation are no longer valid although it is undoubtedly the correct equation for "simple" spin $\frac{1}{2}$ systems such as the electron and μ meson (where by simple spin $\frac{1}{2}$ systems we understand particles which undergo only weak or electromagnetic interactions).

DIRAC FORMULA FOR HYDROGEN FINE STRUCTURE. See **hydrogen fine structure**.

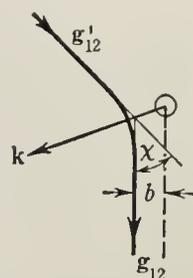
DIRAC \hbar . See **h-bar**.

DIRAC OPERATORS. See **Dirac equation**.

DIRECT AND INVERSE COLLISION. For any molecular collision characterized by an initial relative velocity g_{12} and a final relative velocity g'_{12} , there exists an analogous *inverse collision* or *encounter* with initial relative velocity g'_{12} and final relative velocity g_{12} . By definition, the last of these processes is called



Direct Collision



Inverse Collision

Direct and inverse collision. The unit vectors along the apse line and the angle of deflection are of opposite sign and equal magnitude for the direct and inverse processes.

the *direct collision* or *encounter*. (See figure.) Equilibrium is established when the frequencies of direct and inverse collision are equal.

DIRECT CURRENT. Unidirectional current as produced from batteries, from dynamo machinery equipped with **commutators**, or by means of **rectifiers**.

DIRECT-CURRENT CIRCUITS. See **electric circuits**; **Ohm's law**; **Kirchhoff laws**.

DIRECTED GRAPH. See **digraph**.

DIRECTED PATH. See **path, directed**.

DIRECTED VALENCY. One of the characteristics of the covalent bonds is that they are usually directional in character and rapidly saturated. (See **bond types**.)

In the case of H_2O , for example, the O atom reacts with two H atoms only, and forms two bonds at an angle of 105° to one another.

Directional valency results from the non-spherical symmetry of the p , d and f electron orbitals and of their hybrid **wave functions** with s orbitals. The bonds tend to form with a maximum interaction, i.e., a maximum overlap of the wave functions. (See **hybridization of electron orbitals**.)

DIRECT INTERACTION. The process whereby a nuclear reaction can sometimes take place without the formation of a compound

nucleus. The most notable type of reaction is the (d,p) reaction, for example



in which the relatively loosely-bound deuteron (d) loses its neutron to the O^{16} target nucleus to form O^{17} , the proton continuing on its way deflected only by a small angle. The delicate balance of angular momentum and linear momentum in such a reaction gives rise to a highly characteristic angular distribution of the protons relative to the bombarding deuteron beam, and this may be used to measure spin and parity of the states of the residual nucleus so formed.

DIRECTION. The position of one point in space relative to another without reference to the distance between them. Direction may be either three-dimensional or two-dimensional. Direction is not an angle but is often indicated in terms of its angular difference from a reference direction. The *direction angles* of a line are the 3 angles it makes with the positive directions of the coordinate axes. Its *direction cosines* are the cosines of these angles. Any set of 3 numbers proportional to its direction cosines are called *direction numbers* for the line. A direction at a point on a surface in which the curvature of a normal section is a maximum or a minimum is called a *principal direction*.

DIRECTIONAL. The directional derivative of function f of any number of variables is its rate of change with respect to arc-length along a curve in the given direction. Thus, if the curve is given parametrically by $x = x(t)$, $y = y(t)$, $z = z(t)$, the directional derivative of $f(x,y,z)$ is given by,

$$\frac{df}{dt} = \frac{\partial f}{\partial x} \frac{dx}{dt} + \frac{\partial f}{\partial y} \frac{dy}{dt} + \frac{\partial f}{\partial z} \frac{dz}{dt}.$$

DIRECTIONAL CHARACTERISTIC OF A CIRCULAR-RING SOUND SOURCE. The directional characteristic of a circular-ring sound source of uniform strength and the same phase at all points on the ring is

$$R_\alpha = J_0 \left[\left(\frac{2\pi R}{\lambda} \right) \sin \alpha \right] \quad (1)$$

where R_α is the ratio of the sound pressure for an angle α to the sound pressure for an angle $\alpha = 0$, J_0 is the Bessel function of zero order,

R is the radius of the circle, in centimeters, and α is the angle between the axis of the circle and the line joining the point of observation and the center of the circle.

DIRECTIONAL CHARACTERISTIC OF A CURVED-LINE SOUND SOURCE (ARC OF A CIRCLE). A curved-line sound source may be made up of a large number of point sound sources vibrating in phase on the arc of a circle separated by very small distances. The directional characteristics of such a line in the plane of the arc are,

$$R_\alpha = \frac{1}{2m+1} \left| \sum_{k=-m}^{k=m} \cos \left[\frac{2\pi R}{\lambda} \cos(\alpha + k\theta) \right] + j \sum_{k=-m}^{k=m} \sin \left[\frac{2\pi R}{\lambda} \cos(\alpha + k\theta) \right] \right| \quad (1)$$

where R_α is the ratio of the pressure for an angle α to the pressure for an angle $\alpha = 0$, α is the angle between the radius drawn through the central point and the line joining the source and the distant observation point, λ is the wavelength, in centimeters, R is the radius of the arc, in centimeters, $2m+1$ is the number of points, θ is the angle subtended by any two points at the center of the arc, and k is variable.

Another method is to break up the arc into a large number of equal chords. The strength is assumed to be uniform over each chord. Also the phase of all the chords is the same. In this case the result takes the form,

$$R_\alpha = \frac{1}{2m+1} \left| \sum_{k=-m}^{k=m} \cos \left\{ \frac{2\pi R}{\lambda} \cos(\alpha + k\theta) \right\} \times \frac{\sin \left[\frac{\pi d}{\lambda} \sin(\alpha + k\theta) \right]}{\frac{\pi d}{\lambda} \sin(\alpha + k\theta)} + j \sum_{k=-m}^{k=m} \sin \left\{ \frac{2\pi R}{\lambda} \cos(\alpha + k\theta) \right\} \times \frac{\sin \left[\frac{\pi d}{\lambda} \sin(\alpha + k\theta) \right]}{\frac{\pi d}{\lambda} \sin(\alpha + k\theta)} \right| \quad (2)$$

where R_α is the ratio of the pressure for an angle α to the pressure for an angle $\alpha = 0$, λ is

the wavelength, in centimeters, R is the radius of the arc, in centimeters, $2m + 1$ is the number of chords, θ is the angle subtended by any of the chords at the center of circumscribing circle, and d is the length of one of the chords, in centimeters.

DIRECTIONAL CHARACTERISTIC OF A NONUNIFORM STRAIGHT LINE SOUND SOURCE. The directional characteristic of a line sound source, all parts vibrating in phase, in which the strength varies as a function of the distance x along a line is given by

$$R_\alpha = \frac{\int_{-\frac{d}{2}}^{+\frac{d}{2}} f(x) \epsilon^{-j(2\pi x/\lambda) \sin \alpha} dx}{\int_{-\frac{d}{2}}^{+\frac{d}{2}} f(x) dx} \quad (1)$$

where R_α is the ratio of the sound pressure for the angle α to the sound pressure for the angle $\alpha = 0$. The direction $\alpha = 0$ is normal to the line, x is the distance from the center of the line, in centimeters, d is the total length of the line, in centimeters and $f(x)$ is the strength distribution function.

DIRECTIONAL CHARACTERISTICS OF A PLANE CIRCULAR-PISTON SOUND SOURCE. The directional characteristics of a circular-piston sound source mounted in an infinite baffle with all parts of the surface of the piston vibrating with the same strength and phase are

$$R_\alpha = \frac{2J_1\left(\frac{2\pi R}{\lambda} \sin \alpha\right)}{\frac{2\pi R}{\lambda} \sin \alpha} \quad (1)$$

where R_α is the ratio of the sound pressure for an angle α to the sound pressure for an angle $\alpha = 0$, J_1 is the Bessel function of the first order, R is the radius of the circular piston, in centimeters, α is the angle between the axis of the circle and the line joining the point of observation and the center of the circle, and λ is the wavelength, in centimeters.

DIRECTIONAL CHARACTERISTIC OF A PLANE RECTANGULAR-SURFACE SOUND SOURCE. The directional characteristic of a rectangular-surface source with all

parts of the surface vibrating with the same strength and phase are

$$R_{\alpha\beta} = \frac{\sin\left(\frac{\pi l_a}{\lambda} \sin \alpha\right)}{\frac{\pi l_a}{\lambda} \sin \alpha} \cdot \frac{\sin\left(\frac{\pi l_b}{\lambda} \sin \beta\right)}{\frac{\pi l_b}{\lambda} \sin \beta} \quad (1)$$

where $R_{\alpha\beta}$ = ratio of sound pressure for the angles α, β to the sound pressure for the angle $\alpha, \beta = 0$, l_a is the length of the rectangle, l_b is the width of the rectangle, α is the angle between the normal to the surface source and the projection of the line joining the middle of the surface and the observation point on the plane normal to the surface and parallel to l_a , and β is the angle between the normal to the surface source and the projection of the line joining the middle of the surface and the observation point on the plane normal to the surface and parallel to l_b .

The directional characteristic of a plane rectangular-surface source with uniform strength and phase is the same as the product of the characteristic of two line sources at right angles to each other and on each of which the strength and phase are uniform.

DIRECTIONAL CHARACTERISTIC OF A SERIES OF POINT SOUND SOURCES. The directional characteristic of a sound source made up of any number of equal point sound sources, vibrating in phase, located on a straight line and separated by equal distances is given by

$$R_\alpha = \frac{\sin\left(\frac{n\pi d}{\lambda} \sin \alpha\right)}{n \sin\left(\frac{\pi d}{\lambda} \sin \alpha\right)} \quad (1)$$

where R_α is the ratio of the sound pressure for an angle α to the sound pressure for an angle $\alpha = 0$. The direction $\alpha = 0$ is normal to the line, n is the number of sources, d are distances between the sources, in centimeters, and λ is wavelength, in centimeters.

DIRECTIONAL CHARACTERISTIC OF A STRAIGHT-LINE SOUND SOURCE. A straight-line sound source may be made up of a large number of points of equal strength and phase on a line separated by equal and very small distances. If the number of sources n approach infinity and d , the distance between the sources, approaches zero in such a way that

$$nd = l \quad (1)$$

the limiting case is the line source. If this is carried out, Equation 1 of **directional characteristic of a series of point sound sources** becomes

$$R_\alpha = \frac{\sin\left(\frac{\pi l}{\lambda} \sin \alpha\right)}{\frac{\pi l}{\lambda} \sin \alpha} \quad (2)$$

where R_α is the ratio of the sound pressure for the angle α to the sound pressure for the angle $\alpha = 0$. The direction $\alpha = 0$ is normal to the line, l is the length of the line, in centimeters and λ is wavelength, in centimeters.

DIRECTIONAL CHARACTERISTIC OF A TAPERED STRAIGHT-LINE SOUND SOURCE. The directional characteristic of a line sound source, all parts vibrating in phase, in which the strength varies linearly from its value at the center to zero at either end, is given by

$$R_\alpha = \frac{\sin^2\left(\frac{\pi l}{2\lambda} \sin \alpha\right)}{\left(\frac{\pi l}{2\lambda} \sin \alpha\right)^2} \quad (1)$$

where R_α is the ratio of the sound pressure for an angle α to the sound pressure for an angle $\alpha = 0$. The direction $\alpha = 0$ is normal to the line, l is the total length of the line in centimeters, and λ is the wavelength, in centimeters.

DIRECTIONAL CHARACTERISTICS OF END FIRED LINE SOUND SOURCE. An end fired line sound source is one in which there is progressive phase delay between the elements of the line. In the case in which the time delay of excitation between the elements corresponds to the time of wave propagation in space for this distance the maximum directivity occurs in direction corresponding to the line joining the elements. The directional characteristics of an end fired line of this type and of uniform strength is given by

$$R_\alpha = \frac{\sin \frac{\pi}{\lambda} (l - l \cos \alpha)}{\frac{\pi}{\lambda} (l - l \cos \alpha)} \quad (1)$$

where R_α is the ratio of the pressure for an angle α to the pressure for the angle $\alpha = 0$. The direction $\alpha = 0$ is along the line, l is the length of the line, and λ is the wavelength.

DIRECTION COSINE. See **direction**.

DIRECTION COSINES, OPTICAL. See **optical direction cosines**.

DIRECTION OF PROPAGATION. At any point in a homogeneous, isotropic medium, the direction of time-average energy-flow. In a uniform **waveguide**, the direction of propagation is often taken along the axis. In the case of a uniform lossless waveguide, the direction of propagation at every point is parallel to the axis, and in the direction of time-average energy-flow.

DIRECTIONS, CONJUGATE. See **conjugate directions**.

DIRECTION, SELF-CONJUGATE. See **conjugate directions**.

DIRECTIONS, PRINCIPAL. See **principal directions**.

DIRECTIONS, PRINCIPAL, OF SYMMETRIC SECOND-ORDER TENSOR. See **principal directions of symmetric second-order tensor**.

DIRECT METHODS. Methods usually contrasted with **iterative methods** or methods of successive approximation for **matrix inversion** and the solution of equations. The exact inverse or solution is expressed as the result of a finite sequence of operations on scalars.

DIRECT METHODS IN GEOMETRIC OPTICS. The application of the **Hamiltonian theory**. For theoretical purposes it is pre-eminent but for practical design its use is limited.

DIRECTOR SURFACE. A reference surface which is cut by all the curves of a **congruence of curves**.

DIRECT PRODUCT (OF SUBGROUPS). A **group** G is said to be the direct product of two of its normal subgroups A and B having only the identity in common if every element in G can be represented as the product of an element of A with an element of B , and simi-

larly for the case of more than two subgroups. The G is said to be completely reducible if it is the direct product of simple groups.

DIRECT REFLECTANCE. The ratio of the luminous flux reflected in accordance with the laws of regular **reflection** to the total incident flux.

DIRECTRIX (OF A RULED SURFACE). A reference curve on the **ruled surface** cutting every generator. (See also **conic**.)

DIRECT STRESS. See **stress, direct**.

DIRECT SUM. Let \mathfrak{H}_i ($i = 1, 2, \dots, n$) be a sequence of **Hilbert spaces**. The set of sequences $\{f_1, f_2, \dots\}$ with $f_i \in \mathfrak{H}_i$ can be made into a vector space with the definition

$$\alpha\{f_1, f_2, \dots, f_n\} + \beta\{g_1, g_2, \dots, g_n\} \\ = \{\alpha f_1 + \beta g_1, \alpha f_2 + \beta g_2, \dots\}.$$

The set of all such sequences with $\sum_{i=1}^n (f_i, f_i) < \infty$ where (f_i, f_i) denote the scalar product in \mathfrak{H}_i is then a Hilbert space with the scalar product

$$(\{f_1, f_2, \dots, f_n\}, \{g_1, g_2, \dots, g_n\}) = \sum_{i=1}^n (f_i, g_i).$$

We denote this Hilbert space by $\mathfrak{H}_1 \oplus \mathfrak{H}_2 \oplus \dots \oplus \mathfrak{H}_n$. If $T_g^{(i)}$ is a **representation** of the group G in \mathfrak{H}_i then the direct sum representation

$$T_g = T_g^{(1)} \oplus T_g^{(2)} \oplus \dots \oplus T_g^{(n)}$$

in $\mathfrak{H}^{(1)} \oplus \mathfrak{H}^{(2)} \oplus \dots$ is defined by the equation

$$T_g\{f_1, f_2, \dots\} = \{T_g^{(1)}f_1, T_g^{(2)}f_2, \dots\}.$$

DIRECT TRANSMITTANCE. The ratio of the luminous flux transmitted in accordance with the laws of direct **transmission** to the total incident flux.

DIRICHLET INTEGRAL. The integral

$$\frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \frac{\sin(n + \frac{1}{2})(t-x)}{2 \sin \frac{1}{2}(t-x)} dt$$

expressing the n th partial sum of a **Fourier series**.

DIRICHLET PRINCIPLE. The principle that the solution of the equation $u_{xx} + u_{yy} = 0$, satisfying given boundary conditions, will be given by the function, among all functions

satisfying the boundary conditions, which minimizes the integral

$$\iint (u_x^2 + u_y^2) dx dy.$$

The principle, which requires certain reservations, was named after Dirichlet by Riemann but ought to have been named after Gauss and Lord Kelvin. It is important as a link between **partial differential equations** and the calculus of **variations**. The name is sometimes also given to the corresponding principle for more general elliptic differential equations.

DIRICHLET PROBLEM. See **boundary value problem**.

DIRICHLET SERIES. A series of the form,

$$a_1 + \frac{a_2}{2^x} + \frac{a_3}{3^x} + \dots + \frac{a_n}{n^x} + \dots$$

DISADVANTAGE FACTOR. In **neutron transport theory**, in a situation in which one body is embedded in another (usually a fuel plate or foil in moderator) the disadvantage factor is the ratio of the average neutron **flux** in the first body to that in the second.

DISCONTINUITY. A point at which a function is not continuous. If the point $x = a$ is such that $f(x)$ approaches distinct finite limits as x approaches a from the left or from the right, then a is said to be a **jump discontinuity** of $f(x)$, or $f(x)$ is said to have a jump discontinuity at a . If $f(x)$ can be made continuous at a by giving a suitable definition to $f(a)$, then $f(x)$ is said to have a **removable discontinuity** at a .

DISCONTINUOUS SPECTRUM. See **spectrum, discontinuous**.

DISCONTINUOUS SYSTEM. A term used in thermodynamics for systems which consist of a finite number of homogeneous regions. The intensive **state variables** have the same value throughout each homogeneous region, but they have different values in different regions. As a result they are discontinuous at the boundary. (See also **continuous systems**.)

DISCRETENESS. (1) A term referring to the distribution of allowed values of a physical quantity over a given interval. The distribution is discrete if only a denumerable set of

values is permitted. This is the case, for example, with the allowed frequencies of vibration of a finite stretched string, and quantized quantities such as the energy states of a bound system or the quantized values of angular momentum. (2) The atomicity of mass and charge.

DISCRETE ORDINATES METHOD. In neutron transport theory, a method synonymous with the Wick-Chandrasekhar method.

DISCRETE SPECTRUM. See **spectrum, discrete**.

DISCRETE VARIATE. A variate which occurs only in terms of integral values of a certain unit. Examples are the number of votes cast in an election, or the number of beta particles emitted by a radioactive substance.

DISCRIMINANT. The discriminant of an algebraic equation $x^n + a_1x^{n-1} + \dots + a_n = 0$ is the product of the squares of all the differences of the roots taken in pairs. It is rational in the coefficients.

DISCRIMINANT FUNCTION. Suppose we have two normal multivariate populations with equal variances and covariances but different means, and an observation which is known to come from one or other of the populations. Overlap in the distributions of the individual variates may make it difficult to allot the observation to its correct population with certainty, but we can construct a function of the variates, the discriminant function, which will allocate a member to one class or the other with a known probability of correctness. The function is usually chosen so as to maximize this probability, and is then linear in the variates for normal variation with equal dispersions. In other cases, more complicated discriminants are required. The procedure extends the allocation to more than two populations.

DISINTEGRATION. (1) Loss of form or powdering. (2) The passage of a metal into colloidal solution when it is made an electrode under certain conditions. (3) Transformations of radioactive elements are termed disintegrations when they result from radioactivity.

DISINTEGRATION CONSTANT. The probability per unit time, λ , that a given unstable

particle of system, such as a radioactive atom, will undergo spontaneous transformation. It is defined by the equation $dN/dt = -\lambda N$, where N is the number of untransformed particles or systems existing at time t . It is the reciprocal of the **mean life** of the given system before undergoing transformation.

DISINTEGRATION CONSTANT, PARTIAL. One of **disintegration constants** of a particle or system that undergoes more than one mode of disintegration. (See **branching**.)

DISINTEGRATION ENERGY, GROUND STATE. The disintegration energy of a nuclear disintegration when all the reactant and product nuclei end in their ground states. (See **disintegration energy, nuclear**.)

DISINTEGRATION ENERGY, NUCLEAR. The energy evolved, or the negative of the energy absorbed, in a nuclear disintegration; symbol Q . It is equal to the energy equivalence of the sum of the masses of the reactants minus the sum of the masses of the products. (For each reactant or product which is a nucleus, the appropriate mass is that of the corresponding neutral atom.) If the disintegration energy is positive, the disintegration is exothermic; if it is negative, the disintegration is endothermic. Radioactive disintegrations have positive Q -values; nuclear reactions may have positive values of either sign. Sometimes the term nuclear disintegration energy is applied to the ground-state disintegration energy. (See **disintegration energy, ground-state**.)

DISJOINT SUBGRAPHS. See **subgraphs, disjoint**.

DISLOCATION. A discontinuity in the displacement field of the type produced in a solid body by making a cut in it which does not bisect the body, displacing the cut surfaces relative to each other and joining them. Alternatively, two portions of the surface of the body may be brought together and joined.

An important type of dislocation is that occurring in a lattice. It is a region of departure from the normal lattice arrangement localized in the neighborhood of a line or curve, called the *dislocation line*. The atomic arrangement remote from the dislocation line is identical with that for a perfect crystal lat-

tice, except for a small elastic distortion. One can associate a vector \mathbf{t} with the line joining an atom to one of its nearest neighbors, $\mathbf{t}^{(0)}$ being the corresponding vector in the undistorted lattice. If we now consider a closed path enclosing a dislocation line, we have

$$\begin{aligned}\Sigma \mathbf{t} &= 0 \text{ by definition} \\ \Sigma \mathbf{t}^{(0)} &= \mathbf{t}' \neq 0\end{aligned}$$

where \mathbf{t}' is the same for all paths encircling the dislocation line in the same sense, and is called the **slip vector**, or **Burger's vector**, of the dislocation.

DISLOCATION LINE. See **dislocation**.

DISPERSION. (1) In a medium in which the velocity of progressive waves of small amplitude varies with the wavelength, a disturbance of arbitrary form, which (according to **Fourier theorem**) may be regarded as composed of superposed trains of waves of all wavelengths, changes shape as it progresses because the different component wave trains travel at different speeds. The disturbance is then said to be dispersed. The time required for a pulse (or concentrated disturbance) which is subsequently dispersed to pass a point is the difference between the times of arrival of the fastest and slowest moving component waves traveling with their **group velocities**. Since the group velocity differs from the phase velocity in a dispersive medium, a dispersed pulse has waves moving through it with their appropriate wave velocity. Each wave changes its length as it moves through the disturbance, at each point having a length determined by the time of arrival of these waves traveling with the group velocity from the original pulse.

All gravity waves in fluids are dispersive.

As defined above, dispersion is the separation of a complex wave into components. The term is also applied to the property of an optical device or medium giving rise to the phenomenon, or the numerical value of this property.

(2) In statistics, dispersion is a general term denoting the spread of a series of values, usually about some central point such as the mean or median. The chief measure of dispersion in use is the **standard deviation** but the **mean deviation** and the **mean difference** are also employed.

In a multivariate situation, dispersion is a useful word to cover measures of **variance** and **covariance**. Thus, the dispersion matrix of a set of variates is the matrix (α_{ij}) where α_{ij} is the covariance of the i^{th} and j^{th} variates. This is also known as the *covariance matrix* or the *variance-covariance matrix*.

DISPERSION, ANOMALOUS. See **anomalous dispersion**.

DISPERSION FORCES. The dispersion forces make an essential contribution to the attractive part of the **intermolecular forces**. At any instant the electrons in molecule A are in some configuration which results in an instantaneous **dipole moment**. This instantaneous dipole moment induces a dipole moment in molecule B and *vice versa*. The net result is an *attraction* between the molecules A and B . Quantum mechanics gives the following first approximation for this attraction

$$\phi(r) = -\frac{3}{4} \left(\frac{h\nu_A h\nu_B}{h\nu_A + h\nu_B} \right) \frac{\alpha_A \alpha_B}{r^6} \quad (1)$$

in which $h\nu_A$, $h\nu_B$ are characteristic energies of the order of the **ionization potentials** of molecules A and B and α_A , α_B are the **polarizabilities**.

Formula (1) was first derived by F. London, and so these forces are also called the *London forces*. In the derivation, concepts somewhat similar to those of the optical dispersion theory appear. (See entries following.) Therefore these forces are commonly called dispersion forces.

DISPERSION FORMULA. An empirical expression for the refractive index of a substance as a function of wavelength is a dispersion formula. (See **Hartmann dispersion formula**; **Helmholtz-Ketteler formula**; **Herzberger Dispersion formula**; **Eykman formula**; **Cauchy formula for refractive index**.)

DISPERSION OF LIGHT, CLASSICAL THEORY OF (FOR GASES). In this theory atoms are considered as including electrons, not rigidly fixed, which can oscillate about equilibrium positions when acted up by oscillatory forces (see **atom, Thomson**). The classical laws of motion and of electromagnetism are used. An electron set into oscillatory motion by an incident electromagnetic wave will in turn emit electromagnetic radia-

tions (see **dipole, oscillating**). It will thus act like a forced damped harmonic oscillator and the radiation it emits will be out of phase with the incident radiation (see **oscillator, forced**). If there are N_k electrons per unit volume with natural frequency ω_k and damping constant g_k , the refractive index of the gas will be given (in mksa units) by

$$n = 1 + \frac{1}{2} \sum_k \frac{(N_k e^2 / m \epsilon_0)(\omega_k^2 - \omega^2)}{(\omega_k^2 - \omega^2)^2 + \omega^2 g_k^2},$$

where the summation is carried over the different types of dispersion electrons. This equation accounts for the phenomena of anomalous dispersion at frequencies close to $\omega_k/2\pi$.

DISPERSION OF ROTATION. The **angle of rotation** of the plane of vibration in optically active substances is $\rho = K/\lambda^2 + a$ where K is constant and a depends on natural free periods of vibration in the crystal. This relation is called the dispersion of the rotation.

DISPERSION, PARTIAL. In the correction of the secondary spectrum of chromatic aberration in an optical system the partial dispersion of a medium $P_\lambda = (n_\lambda - n_F)/(n_F - n_C)$ occurs as a discrete substitute for $d^2n/d\lambda^2$. The indices of refraction n_F, n_C are for light of the wavelength of the blue F and red C hydrogen lines, respectively. (See **Abbe number**.)

DISPERSION, QUANTUM THEORY OF. See **quantum theory of dispersion**.

DISPERSION, RECIPROCAL LINEAR. The derivative $d\lambda/dx$, where λ is **wavelength** and x is the distance along the **spectrum**. The reciprocal linear dispersion is usually expressed in Ångströms per millimeter.

DISPERSION, RECIPROCAL MEAN. See **Abbe number**.

DISPERSION RELATIONS. Dispersion relations are integral formulas connecting the real and imaginary parts of **scattering amplitudes** (S matrix elements). Kronig and Kramers were the first to point out that the coherent forward scattering amplitude of light by matter, $f(\omega)$, has the property that its real and imaginary part are related by

$$\text{Re}[f(\omega) - f(0)] = \frac{2\omega^2}{\pi} P \int_0^\infty \frac{\text{Im} f(\omega')}{\omega'(\omega'^2 - \omega^2)} d\omega'$$

where P denotes the principal value. It was also pointed out by Kramers that these "dis-

persions relations" were a consequence of the principle of **causality**, i.e., that all interactions (signals) propagate with velocities not greater than the velocity of light. The quantity $\text{Im} f(\omega)$ is furthermore related to the total cross section through the **optical theorem**

$$\sigma(\omega) = \frac{4\pi}{k} \text{Im} f(\omega),$$

where k is the wave number corresponding to ω ($\omega = kc$). Thus the real part of the forward scattering amplitude is determined by an integral over the total cross section.

In establishing dispersion relations in field theory, the following assumptions are usually made:

- (a) that local fields can be used to describe the particles, and that these fields have well defined transformation properties under Lorentz transformations;
- (b) that these operators satisfy the principle of microscopic **causality**, i.e., if $A(x), B(y)$ are two observable Heisenberg fields then $[A(x), B(y)] = 0$ if $(x - y)^2 < 0$;
- (c) that the totality of all possible physical states of the field system forms a set of basis vectors which is complete in the quantum mechanical sense, and that these states have an energy-momentum spectrum which is positive time-like. Furthermore, that there exists a unique invariant state of energy-momentum zero, the vacuum state;
- (d) that the field operators satisfy **asymptotic conditions**.

Axioms (b) and (c) allow us to infer that the scattering amplitudes are boundary values of analytic functions in momentum space. The dispersion relations are a reflection of these analytic properties.

In the case of meson-nucleon scattering (see **S-matrix** and **quantized field theory**) the matrix element of the S -matrix can be written in the following form

$$T(k_0, \Delta^2) = i \int d^4x \exp [ik_0 x_0 - i\mathbf{k} \cdot \mathbf{x}] \theta(x_0) \left\langle \frac{\Delta}{2} \left| \left[j\left(\frac{x}{2}\right), j\left(-\frac{x}{2}\right) \right] \right| - \frac{\Delta}{2} \right\rangle$$

where $j = (\square + \mu^2)\phi$ and ϕ is the field operator for the mesons (isotopic spin considerations

have been neglected). The matrix element of the commutator in the above is taken between one-nucleon states, in the particular coordinate frame where the initial and final nucleon have momenta $+\Delta/2$ and $-\Delta/2$ the initial and final value of the meson momenta \mathbf{k} are $\pi \pm \frac{\Delta}{2\mathbf{k}'}$, respectively. The quantity k_0 is the meson energy given by

$$k_0 = \left(\mu^2 + \pi^2 - \frac{\Delta^2}{4} \right)^{1/2}$$

where

$$|\pi| = \left(k_0^2 - \mu^2 - \frac{\Delta^2}{4} \right)^{1/2}.$$

It will be noted that in the expression defining T , the energy variable occurs only in the exponent. The vanishing of the commutator outside the light cone allows one to extend the energy dependence into the complex k_0 plane. This extension is justified if the mass μ is made imaginary and sufficiently large in absolute value. The use of **Dyson representation** for this causal commutator and the spectral properties allow one to derive the following analytic properties of T considered as function of the invariants $W^2 = (p+k)^2$ and $\Delta^2 = \frac{1}{4}(p-p')^2$ where p, p' and k, k' are the initial and final nucleon and meson momenta. M is an analytic function in a certain region of the Δ^2 plane, the boundary of which can be expressed as of function of W^2 and μ^2 , and for $\mu^2 < -\frac{\Delta^2}{4}$ is an analytic function in the upper half of the complex W^2 plane. Hence for this case

$$T(W^2, \Delta^2, \mu^2) = \frac{1}{\pi} \int dW'^2 \frac{\text{Im } T(W'^2, \Delta^2, \mu^2)}{W'^2 - W^2}.$$

Symmetry properties and spectral properties of $\text{Im } T$ allow us to rewrite the right-hand side only over the range W'^2 from $(\mu + M)^2$. It is further necessary to show that this relation holds for μ^2 extrapolated to its physical value; this can be done.

The problem of proving the analytic properties relevant for the dispersion relations is a special case of the general task of finding what analytic properties of **Green's functions** follow from (a) Lorentz invariance, (b) causality, and (c) spectral conditions. For the two-particle scattering amplitude, one is dealing with the vacuum expectation value of the retarded or time ordered product of four operators and thus with a function of the 6 scalar

products which can be formed from the 3 independent vectors $p + p', p + k', p' + k$. Local commutativity and spectral conditions assert that these functions are analytic in a certain domain D in the space of 6 complex variables. The problem is then to characterize this domain and to compute its envelope of holomorphy $E(D)$ and to derive a representation of the most general function in the class of interest which is analytic in $E(D)$ and has singularities everywhere on the boundary.

DISPERSION, ROTATORY. See **rotatory dispersion**.

DISPERSIVE POWER. (1) The ratio of the difference in **deviation** of light of two different wavelengths relative to the deviation for light whose wavelength is an average of the two. Thus, if D_A, D_B are the **deviations** for wavelength $\lambda = A$ and $\lambda = B$ and if D_C is that for light of some intermediate wavelength $\lambda = C = \frac{1}{2}(A + B)$, then the dispersive power is given by

$$d = \frac{D_A - D_B}{D_C}.$$

(2) A quantity which is approximately equal to the reciprocal of the dispersive power, also referred to as the Abbe number or as ν -value, is defined by lens designers and manufacturers of optical glass as

$$\nu = \frac{n_D - 1}{n_F - n_C}$$

where n_D, n_F and n_C are the refractive indices of the glass for the **D, F, and C Fraunhofer lines**.

DISPERSIVITY, MOLAR. The difference in molar **refraction** at two wavelengths.

DISPERSIVITY, SPECIFIC. The difference in specific **refraction** at two wavelengths.

DISPLACEMENT. (1) The vector quantity denoting change in position of a point is called displacement. The term *linear displacement* is employed occasionally to distinguish displacement from *angular displacement*. Angular displacement, the change in angular position, is not a vector quantity.

(2) In a piston and cylinder mechanism, displacement means the volume swept out by the piston face. Given the bore and stroke as D

and L , the number of cylinders n , the displacement is:

$$\frac{\pi D^2 L n}{4}$$

(3) The term displacement is also applied to ships. The portion of a ship which is immersed in water is buoyed up by a force equal to the weight of water displaced by the body. Hence the displacement of a ship in tons of water is equal to the weight of the ship and of its contents. Various legal and conventional definitions of the displacement of a ship are related, but not equal, to the buoyancy.

(4) For electric displacement, see **electric induction** and **current, displacement**.

DISPLACEMENT, COMPLEX. In solving for the transient and steady state behavior of a vibrating system undergoing **forced oscillations**, it is often mathematically useful to construct a complex displacement $x_c = x + ix'$, where x is the real, and x' the imaginary part.

DISPLACEMENT COMPONENTS. Change of the coordinates, in some reference system, of a particle of a body arising from a **deformation** of the body.

DISPLACEMENT CURRENT. See **current, displacement**.

DISPLACEMENT, ELECTRIC. See **electric induction** and **current, displacement**.

DISPLACEMENT FIELD (IN A BODY). The vector field defined by the **displacement** vector throughout the body.

DISPLACEMENT, GENERALIZED. Angular displacement, rotation, curvature, change in volume, change in length, etc., may be considered as generalized displacements. Each has a corresponding generalized force (moment, pressure, force, etc.) which multiplied by the generalized displacement, gives a virtual or real work quantity.

DISPLACEMENT LAW, SOMMERFELD-KOSSEL. See **Sommerfeld-Kossel displacement law**.

DISPLACEMENT OPERATOR. See **difference operators**.

DISPLACEMENTS, COMPATIBLE. Displacements compatible with the **constraints** is a phraseology employed in some statements of

the theorem of **virtual work**. Its meaning is that the displacements chosen for the system do not violate the actual conditions of support or constraint of the system.

DISPLACEMENT THICKNESS OF BOUNDARY LAYER. The displacement thickness δ^* for a boundary layer is defined by

$$\delta^* = \int_0^\delta \left(1 - \frac{u}{U}\right) dy,$$

where u is the velocity at a distance y from the wall and U is the velocity outside the boundary layer, i.e., for $y > \delta$.

The streamlines in the potential flow outside the boundary layer are displaced away from the wall by an amount δ^* , due to the reduced velocity in the boundary layer.

DISPLACEMENT, VIRTUAL. Any displacement of a system may be termed a virtual displacement. Often the term is restricted to infinitesimal displacements compatible with the constraints.

DISSIPATION. (1) The interaction between matter and energy incident upon it, such that the portion of the energy used up in the interaction is no longer available for conversion into useful work. (2) A persistent loss of mechanical energy because of the presence of frictional or frictionlike forces or resistances. (3) In free oscillatory motion, a persistent loss of mechanical energy due to presence of friction-like resistance to motion which eventually exhausts the total energy of the system and causes it to come to rest. Such motion is said to be damped. (See **oscillations, damped**.)

DISSIPATION FACTOR. The reciprocal of **Q**, the **storage factor**.

DISSIPATION FUNCTION. A mathematical expression related to the loss of the mechanical energy in a system. For example, if a system consists of a number of particles, and the equations of motion for a typical particle are

$$m\ddot{x} = -R_1\dot{x} + X$$

$$m\ddot{y} = -R_2\dot{y} + Y$$

$$m\ddot{z} = -R_3\dot{z} + Z$$

(X , Y , and Z are the components of the total force, external and molecular, on the particle,

except the force of resistance; R_1, R_2, R_3 are the friction constants for the typical particle), then the function $F = \frac{1}{2}\Sigma R_1\dot{x}^2 + R_2\dot{y}^2 + R_3\dot{z}^2$ is the dissipation function of the system. If q_1, q_2, \dots, q_n are the generalized coordinates of the system and if Q_r denotes the expression

$$Q_r = \sum_1^N \left[X \frac{\partial x}{\partial q_r} + Y \frac{\partial y}{\partial q_r} + Z \frac{\partial z}{\partial q_r} \right],$$

and if T is the kinetic energy of the system, the equation of motion can be written in the form

$$\frac{d}{dt} \left(\frac{\partial T}{\partial \dot{q}_r} \right) - \frac{\partial T}{\partial q_r} + \frac{\partial F}{\partial \dot{q}_r} = Q_r \quad (r = 1, 2, \dots, n).$$

Dots mean derivatives with respect to time. (See also **time variation of entropy production**; **Rayleigh dissipation function**.)

DISSIPATIVE FORCE. Non-conservative forces are dissipative. The work they do is path-dependent. Part or all of this work is converted to non-available energy and does not appear as kinetic or potential energy.

DISSIPATIVE FUNCTION. See **dissipation function**.

DISSOCIATION ENERGY OF A MOLECULE. The energy necessary for the complete separation from each other of the two atoms forming a diatomic molecule, or for the removal of an atom or a group of atoms from a polyatomic molecule. More exactly, the dissociation energy is the difference between the energy of the molecule and its dissociation products in their respective ground states. If therefore the actual dissociation process leads to atoms (or radicals) in excited states the dissociation energy is the energy needed for their separation minus the excitation energy. (See also **potential functions**; **atomic heats of formation**; **bond energies**.)

DISSOCIATION LAWS. See **Saha equilibrium formula**.

DISSOCIATION OF DIATOMIC GAS. See **Saha equilibrium formula**.

DISSYMMETRY FACTOR OR ANISOTROPY FACTOR. A quantity used to express conveniently the magnitude of **circular dichromism**. It is defined by the following formula:

$$g = \frac{(\kappa_l - \kappa_r)}{\kappa}$$

where g is the dissymmetry factor, κ_l and κ_r are the absorption indices for the left- and right-circularly polarized light, and κ is the absorption index for ordinary light of the same wavelength.

DISTANCE, GEODESIC. Geodesic distance. (See **geodesic parallels**.)

DISTORTION. In general, a change of form, shape, or wave form. The last usage applies to acoustics and electromagnetics. In optics, the following specific usages apply: (1) Qualitatively, the **image error** due to the departure of optical imagery from a similarity transformation. (2) Quantitatively, one of the five **Seidel aberrations**.

DISTORTIONAL STRAIN ENERGY. See **Mises yield condition**.

DISTORTIONAL WAVE. See **wave(s), shear**.

DISTORTION, AMPLITUDE. A type of distortion that occurs in an amplifier or other device when the amplitude of the output is not exactly a linear function of the input amplitude. (See also **distortion, amplitude-frequency**; and **distortion, harmonic**.)

DISTORTION, AMPLITUDE-FREQUENCY. Distortion due to an undesired amplitude-frequency characteristic. The usual desired characteristic is flat over the frequency range of interest. Amplitude-frequency distortion is sometimes called *amplitude distortion* or *frequency distortion*.

DISTORTION ENERGY. The energy stored elastically may be considered to be the sum of a volumetric and a shear strain or distortional energy. The volumetric energy is that associated with the mean stress, the distortion energy with the stress deviation. For an isotropic material the distortion energy per unit volume is

$$\frac{1}{12G} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

where $\sigma_1, \sigma_2, \sigma_3$ are principal stresses and G is the shear modulus or modulus of rigidity.

DISTORTION, ENVELOPE DELAY. That form of distortion which occurs when the rate of change of phase shift with frequency of a circuit or system is not constant over the fre-

quency range required for transmission. Envelope delay distortion is usually expressed as one-half the difference in microseconds between the maximum and minimum envelope delays existing between the two extremes of frequency defining the channel used.

DISTORTION, GEOMETRICAL. See **geometrical distortion**.

DISTORTION, HARMONIC. Nonlinear distortion characterized by the appearance in the output of harmonics other than the fundamental component when the input wave is sinusoidal. Harmonic distortion is sometimes called amplitude distortion.

DISTRIBUTED FORCE. See **force, distributed**.

DISTRIBUTION. If a variable can take a set of values $\{x\}$ with relative frequency or probability $f(x)$, it is said to be distributed over these values and to have a distribution.

$f(x)$ is called the *frequency function* and its sum or integral $\int_{-\infty}^x f(x)dx = F(x)$ is called the *distribution function*. Similar definitions apply if x is continuous and mathematically it may be more convenient to define $F(x)$ and to regard $f(x)$ as its derivative when existent.

There are many variants of these names. The frequency function is often called the *probability density function* and the distribution function is called the *cumulative frequency function* or *cumulative probability function* or even the *probability function*.

The terminology extends immediately to more than one variable. (See also **binomial distribution**; **Edgeworth's series**; **Gram-Charlier series**; **normal distribution**; **Pearson distribution**; **Poisson distribution**; **Maxwell distribution**; **Maxwell-Boltzmann distribution**; **distribution-in-momentum**; **distribution theory**.)

DISTRIBUTION, ARC-SINE. See **arc-sine distribution**.

DISTRIBUTION, CAUCHY. See **Cauchy distribution**.

DISTRIBUTION COEFFICIENTS (CHEMICAL). See **Nernst distribution law**.

DISTRIBUTION COEFFICIENTS (OPTICAL). The tristimulus values, on any given colorimetric system, of the monochromatic

components of an equal energy spectrum. The set of distribution coefficients defines the *distribution functions* or *distribution curves*. (See **CIE-distribution coefficients**.)

DISTRIBUTION FACTORS. The **moment distribution factors** which multiply the unbalanced moment at a joint to give the moments to be distributed to the members framing into the joint.

DISTRIBUTION-FREE INFERENCE. A method of inference in statistics which does not depend on the precise form of the parent distribution. For example, if treatments A and B are applied to n pairs of plots, the n differences in yield typified by $x_A - x_B$ can be tested for significance by considering their sign only, a large preponderance of positive signs indicating that A is better than B ; and this does not depend on the **distribution** of the x values.

DISTRIBUTION FUNCTION. See **distribution**.

DISTRIBUTION FUNCTION, FERMI-DIRAC. See **Fermi-Dirac distribution function**.

DISTRIBUTION IN ENERGY. A probability distribution function for various energy values.

DISTRIBUTION IN MOMENTUM. A probability distribution function for various values of momentum.

DISTRIBUTION IN PHASE. A probability distribution function in phase space.

DISTRIBUTION, MOMENT OF. See **moment of a distribution**.

DISTRIBUTION OF MOLECULAR VELOCITIES, LAW OF. At any given temperature different molecules of a gas have different velocities which range from very small to very large values. A large majority of molecules have velocities falling in a narrow range, but a small fraction of molecules have very low or high velocity. As the temperature is raised the maximum in the velocity distribution curve shifts to higher velocities and flattens out. (See **Maxwell distribution law**.)

DISTRIBUTION THEORY. The usual definition of the Dirac δ function given by

$$\delta(x) = 0, x \neq 0 \quad \text{and} \quad \int_{-\infty}^{+\infty} \delta(x)dx = 1,$$

is not a mathematically valid definition since if a function is zero everywhere except at a point, its integral, with the usual Lebesgue definition of the integral, vanishes. The theory of distributions of L. Schwartz generalizes the notion of the ordinary (measurable) function and defines the notion of a "generalized function." Such generalized functions encompass δ functions and their derivatives and justify their use. In the following a complex valued function $\phi(x) = \phi(x_1 \cdots x_n)$ of n real variables $x_1, x_2 \cdots x_n$, where $x_1 \cdots x_n$ range over n dimensional Euclidean space R^n , is said to be a "test function" if it is infinitely differentiable, and vanishes outside some compact set K (i.e., a closed and bounded interval in the case $n = 1$). The complement of the largest open set where $\phi(x)$ is zero is called the *support* of $\phi(x)$. We denote by D the set of these test functions: D is the vector space of all C^∞ functions that have compact support. A sequence $\{\phi_n(x)\}$ of test functions is said to converge to zero if the functions $\phi_n(x)$ and all their derivatives converge uniformly to zero and if all ϕ_n have the same compact support. (Uniform convergence is required for each fixed order of the derivatives, not for all orders collectively.) This notion of convergence defines a (pseudo-) topology in D . A **functional** T on D is an operation that associates with every $\phi \in D$ a complex number. This associated number is denoted by $T(\phi)$. A functional T on the space of test function D is said to be linear if

- (a) $T(\phi_1 + \phi_2) = T(\phi_1) + T(\phi_2)$ for every ϕ_1, ϕ_2 in D .
- (b) $T(\lambda\phi) = \lambda T(\phi)$ for every ϕ in D and every complex number λ .

A functional is said to be continuous if $T(\phi_j)$ converges to zero for any sequence $\{\phi_j\}$ of functions ϕ_j in D that converge to zero as defined above.

A *distribution* as defined by Schwartz is a continuous linear functional on the space of test functions D . The space of all distributions is denoted by D' and is the dual space of D . A distribution T is said to be zero in an open set Ω of R^n if $T(\phi) = 0$ for all test functions ϕ in D whose support is contained in Ω . The support

of a distribution is the complement of the largest open set in which T is zero.

Let E be the vector space of all C^∞ (infinitely differentiable) functions with arbitrary support. Convergence in E is defined as follows: A sequence $\{\phi_j\}$, ϕ_j in E , converges to zero if the ϕ_j converge to zero in every compact subset of R^n and the same is true for the derivatives of any order. (Uniform convergence is again required only for fixed order of the derivatives, not for all order collectively.) If ϕ_0 is a fixed function in E , we can define as neighborhoods of the sets $\{\phi \mid |\phi(x) - \phi_0(x)| < \epsilon\}$ for x in K , where $\epsilon > 0$ and K is a compact set. This definition of neighborhoods defines a topology in that a sequence is convergent in this topology if it is convergent as defined above. The dual space of E , that is, the space of continuous linear functional on E is denoted by E' . Schwartz has shown that E' consists exactly of those distributions in D' that have compact support.

It is sometimes convenient to write a distribution as an integral over a "generalized function" as follows:

$$T_f(\phi) = \int_{-\infty}^{+\infty} f(x)\phi(x)dx$$

where the integral and the "generalized function" $f(x)$ is defined by this equation. The Dirac δ -function as well as the singular functions $\Delta, \Delta_{(1)}$ etc., are such generalized functions.

On the other hand if $f(x)$ is a summable function, then the Lebesgue integral $\int_{-\infty}^{+\infty} f(x)\phi(x)dx$ defines a distribution which is again denoted by $T_f(\phi)$.

The derivative of a generalized function is defined in such a way that it remains valid when the generalized function is also a differentiable function. One defines the derivative of a generalized function $f'(x)$ by

$$\int_{-\infty}^{+\infty} f'(x)\phi(x)dx = - \int_{-\infty}^{+\infty} f(x)\phi'(x)dx.$$

For every test function ϕ the right hand side always exists and hence the left hand side gives a definition of $f'(x)$. The distribution derivative DT_f is defined by

$$DT_f(\phi) = -T_f(d\phi/dx).$$

A function $f(x)$ is said to be "rapidly decreasing" if it tends to zero faster than any polynomial as $|x| \rightarrow \infty$. Let S be the set of these functions. A sequence $\{\phi_n(x)\}$ in S tends to

zero if $\{D^l P(x)\phi_n(x)\}$ tends to zero uniformly for all x as $n \rightarrow \infty$, for each polynomial P and each l .

A "tempered distribution" T is a linear functional on S such that $T(\phi_n) \rightarrow 0$, if $\{\phi_n\} \rightarrow 0$. The set of tempered distributions, S' , is the largest set in which Fourier transforms map the space into itself. The Fourier transform \tilde{T} of any T in S' is defined to be that distribution for which

$$T(\hat{\phi}) = \tilde{T}\bar{\phi}$$

where $\hat{\phi}(x) = \phi(-x)$ and $\bar{\phi}$ is the Fourier transform of ϕ which, if ϕ is S , is also in S . When T corresponds to an absolutely integrable function, this above definition reduces to the usual Fourier transformation.

DISTRIBUTIVE LAW. An operator A is distributive over addition (obeys the distributive law) if $A(a + b) = Aa + Ab$. Thus, in elementary arithmetic, multiplication is distributive over addition.

DIVERGENCE. The scalar product of the vector differential operator, **del** (∇) and a **vector**. In Cartesian coordinates

$$\nabla \cdot \mathbf{v} = \text{div } \mathbf{v} = \frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z}.$$

The divergence of a position vector is constant; $\nabla \cdot \mathbf{R} = 3$; $\mathbf{R} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$.

If \mathbf{v} represents at each point in space the direction and magnitude of a vector, then $\text{div } \mathbf{v}$ is the outward flux of the vector per unit volume.

If the divergence of a vector function of position vanishes everywhere in a certain region, the function is said to be a solenoidal vector in that region. It follows that if \mathbf{v} is a solenoidal vector so that $\nabla \cdot \mathbf{v} = 0$, then $\mathbf{v} = \nabla \times \mathbf{w}$, or \mathbf{v} is the **curl** of some vector \mathbf{w} .

DIVERGENCE LOSS (SOUND). That part of the **transmission loss** which is due to the divergence or spreading of the sound rays in accordance with the geometry of the system (e.g., spherical waves emitted by a point source).

DIVERGENCES IN QUANTIZED FIELD THEORY, REMOVAL OF. In any quantized

field theory corresponding to the interaction of fermions with bosons based on a Lagrangian with an interaction term of the form $G\bar{\psi}\Gamma\psi\phi$ where ψ is the fermion field operator and ϕ that of the boson field, the **S-matrix** expansion in powers of G will contain divergences in the higher order terms. In particular, for the case of quantum electrodynamics the rules for evaluating Feynman diagrams (see **S matrix, evaluation of**) imply that three types of divergences can occur in evaluating an n^{th} order diagram consisting of F_e external and F_i internal electron lines, P_e external and P_i internal photon lines, and n_s vertices at which the **mass renormalization** term act. Of these, those arising because the integrand does not fall off rapidly enough for large values of the momentum of one of the internal lines are called ultraviolet divergences and are the ones with which the **renormalization** technique is designed to eliminate. The other divergences are connected with the fact that photons have zero mass (infrared divergences) or the onset of inelastic processes and can be handled in a known fashion. Any ultraviolet divergence will be removed if a sufficient number of the internal momenta are held fixed. A *primitive divergent* is defined as a divergent graph made convergent by the cutting of any internal line. Dyson has shown that any graph for which $\frac{3}{2}F_e + P_e + n_s > 4$ has the matrix element which is convergent. The number of primitive Feynman divergent graphs in quantum electrodynamics are finite and are indicated in Figure 1. In that figure,

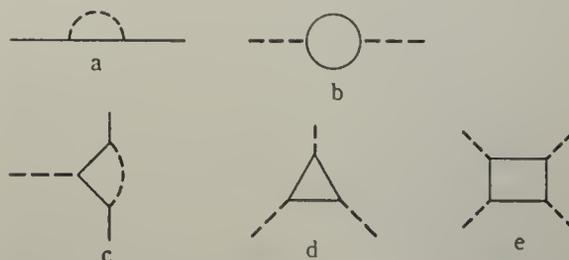


Fig. 1.

a and b are an electron and a photon self-energy diagram, respectively. Diagram a gives rise to a possible linearly divergent contribution, b , to a possibly quadratically divergent contribution, though in fact the quadratically divergent contribution vanishes by **gauge invariance**. Diagram c is a vertex part, and

is at most logarithmically divergent. The diagram d actually gives a zero contribution to the S -matrix (Furry's theorem) and therefore need not be considered. Diagram e correspond to the scattering of light by light and is potentially logarithmically divergent. However, due to gauge invariance it is in fact convergent. There are therefore just three primitive divergents in quantum electrodynamics. (In meson theory with a coupling of the form $(G\bar{\psi}\lambda\gamma\phi)$ diagram e is divergent and must be compensated by a counter term $\delta\phi^4$ in \mathcal{L} , diagram d vanishes if ϕ is a pseudoscalar field, otherwise it is divergent and must also be compensated.)

The possible divergences of the theory having been isolated, the next stage is to devise a prescription for the removal of these divergences. Consider an arbitrary graph G . Associated with this graph is a skeleton graph G_o formed by omitting all self-energy and vertex parts from G . A graph which is its own skeleton is irreducible. Starting from an irreducible graph, the insertion of self-energy and vertex parts in all possible ways give rise to a set Γ , of diagrams. It is possible to give a prescription to find the contribution of $M(\Gamma)$ of Γ to S from a specification of G_o . This has the consequence that only irreducible diagrams need be considered.

To establish this prescription, consider the effect of the insertion of an electron self-energy part into the internal electron line of momentum p of any graph. By the rules previously established the effect of this insertion is to replace the factor $S_F(p)$ by

$$S_F(p) \frac{ie^2}{(2\pi)^4} \left\{ \int d^4k D_F(k) \gamma^\mu S_F(p+k) \gamma_\mu \right\} S_F(p),$$

i.e., $S_F(p)$ by $S_F(p) \Sigma^{(2)}(p) S_F(p)$. The factor

$$\begin{aligned} \Sigma^{(2)}(p) &= \frac{e^2}{(2\pi)^4} \int d^4k D_F(k) \gamma^\mu S_F(p+k) \gamma_\mu \\ &= \int d^4k R(p,k) \end{aligned}$$

is at most linearly divergent. Therefore, the integrand $R(p,k)$ when expanded into a **Taylor series**

$$R(p,k) = R(0,k) + p \frac{\partial}{\partial p} R(0,k) + R_c(p,k)$$

is such that $\int R_c(p,k) d^4k$ converges. Relativistic invariance further asserts that

$$\begin{aligned} \Sigma^{(2)}(p) &= A^{(2)} + B^{(2)}(\gamma p - m) \\ &\quad + (\gamma p - m) S_c^{(2)}(p) \end{aligned}$$

where $A^{(2)}$ and $B^{(2)}$ are infinite constants and $S_c^{(2)}(p)$ vanishes for $\gamma p = m$, i.e., if p corresponds to a free electron line. For a free electron $\Sigma^{(2)}$ reduces to $A^{(2)}$, which corresponds to the change in mass of the electron due to its interaction with the radiation field. The only physical change which can occur for a single particle with the radiation field is a change of mass so that one would expect the modified propagator to be derived in this case to be S_F the function corresponding to the mass $m_o + \delta m$. Since

$$\begin{aligned} &\frac{1}{\gamma p - (m_o + \delta m)} \\ &= \frac{1}{\gamma p - m_o} + \delta m \frac{1}{\gamma p - m_o} \cdot \frac{1}{\gamma p - m_o} + \dots \\ &= S_F(p) + \delta m S_F(p) \cdot S_F(p) + \dots \end{aligned}$$

$A^{(2)}$ is equal to δm to order e^2 .

Now in the Lagrangian formulation of the theory, the mass m_o which one writes down for the mass of the electron-positron field is the bare mass. The experimentally determined mass, m is equal to $m_o + \delta m$ where δm is the correction to the mass due to the interaction with the radiation field. One can formulate the theory in terms of m , by writing for the mass term in the Hamiltonian, $m_o \bar{\psi}\psi = m \bar{\psi}\psi - \delta m \bar{\psi}\psi$ and include the $\delta m \bar{\psi}\psi$ term with the interaction term $\bar{\psi}\gamma_\mu \psi A^\mu$. H_I is then equal to $j^\mu A_\mu - \delta m \bar{\psi}\psi$. If this formulation is adopted there will also arise an insertion into the internal electron line due to the action of the $\delta m \bar{\psi}\psi$ term. The contribution of this graph is obtained by replacing S_F (which is now expressed in terms of the mass m) by $S_F \delta m S_F$. The contribution of both these insertions thus requires S_F to be replaced by

$$\begin{aligned} S_F(p) &\rightarrow S_F(p) + S_F(p)[(A^{(2)} - \delta m) \\ &\quad + B^{(2)}(\gamma p - m) + (\gamma p - m) S_c] S_F(p). \end{aligned}$$

Therefore if δm to order e^2 is set equal to $A^{(2)}$, this divergence is eliminated. This is called **mass renormalization**. Since

$$S_F(p)(\gamma p - m) = 1,$$

after mass renormalization to order e^2

$$S_F(p) \rightarrow (1 + B^{(2)})S_F(p) + S_F(p)(\gamma p - m)S_c(p)S_F(p).$$

The $B^{(2)}$ divergence has the effect of multiplying S_F by a factor $1 + B$, which renormalization is usually called *wave function renormalization*.

A similar treatment holds for the replacement of a simple vertex by the diagram c . The vertex factor γ_μ is found to be modified by the replacement of γ_μ by $\Lambda_\mu(p, p')$ with

$$\Lambda_\mu^{(2)}(p, p') = \frac{e^2}{(2\pi)^4} \int D_F(k) \{ \gamma^\lambda S_F(p + k) \gamma_\mu S_F(p' + k) \gamma_\lambda \}$$

which is at most logarithmically divergent. Relativistic invariance asserts that $\Lambda_\mu^{(2)}$ can be written in the form $\Lambda_\mu^{(2)}(p, p') = L^{(2)}\gamma_\mu + \Lambda_{\mu c}^{(2)}(p, p')$ with $\Lambda_{\mu c}^{(2)}$ convergent and equal to zero for $\gamma p = \gamma p' = m$. It can be shown that $-L^{(2)} = B^{(2)}$. This is a special case of the theorem proved by Ward that $-L = B$ to all orders.

Finally the insertion of a photon self-energy part (figure 1b) into an internal photon line of momentum k implies the replacement of $D_F(k)$ by $D_F(k)\Gamma(k)D_F(k)$ where to second order

$$\begin{aligned} \Pi^{(2)}k &= \frac{e^2}{(2\pi)^4} \int d^4p \text{Tr} \{ \gamma_\mu S_F(p) \gamma^\mu S_F(p + k) \} \\ &= \frac{e^2}{(2\pi)^4} \int d^4p P(p, k) \end{aligned}$$

is at most quadratically divergent. The Taylor expansion of P , then yields for Π an expression of the form $\Pi(k) = C^{(2)} + D^{(2)}k^2 + k^2\Pi_c^{(2)}(k^2)$ since from relativistic invariance $\Pi = \Pi(k^2)$ where Π_c is convergent and vanishes for $k^2 = 0$. The term $C^{(2)}$ vanishes by gauge invariance. The divergent term $D^{(2)}$ gives rise by renormalization of the electromagnetic field strength or equivalently to a renormalization of the charge. The remaining convergent term leads to observable effects and is the physical manifestation of the **polarization of the vacuum**.

The above rules outlined for the e^2 correction can be generalized for arbitrary insertions and lead to the following rules to obtain from G^o the complete contribution of the graphs of Γ to $S^{(n)}$. For every internal electron line of the irreducible diagram G_o from which Γ is ob-

tained, make the replacement $S_F(p)$ by $S'_F(p)$ where

$$S'_F(p) = S_F(p) + S_F(p)\Sigma(p)S_F(p)$$

where $\Sigma(p)$ denotes the sum of contributions of all possible self-energy diagrams. For the photon lines, replaces D_F by D'_F , where

$$D'_F(R) = D_F(k) + D_F(k)\Pi(k)D_F(k)$$

and each vertex factor γ_μ by $\Gamma_\mu(p, p')$ where

$$\Gamma_\mu(p, p') = \gamma_\mu + \Lambda_\mu(p, p').$$

The removal of the divergences from self-energy diagrams which are more complicated than the irreducible ones drawn above is somewhat more involved, but consistent rules can nonetheless be given. For example, the diagram in Figure 2

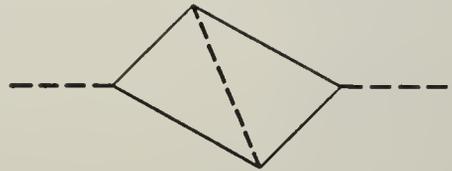


Fig. 2.

obtained by adding a vertex part to an irreducible photon self-energy diagram gives rise to "overlapping" divergences. As stated above, consistent rules for subtracting these divergences can be given.

The culmination of these rules due to Dyson and Salam is the proof that the finite contributions D'_{F1} , S'_{F1} , $\Gamma_{\mu 1}$ left after the subtraction of infinities are related to the complete quantities Γ_μ , S'_F , D'_F by multiplicative constants, i.e.,

$$\Gamma_\mu = Z_1^{-1}\Gamma_{\mu 1}(e_1)$$

$$S'_F = Z_2 S'_{F1}(e_1)$$

$$D'_F = Z_3 D'_{F1}(e_1)$$

where

$$e_1 = Z_1^{-1}Z_2Z_3^{1/2}e.$$

Charge conservation, or equivalently gauge invariance implies that $Z_1 = Z_2$ (Ward's identity). The quantity e_1 is called the renormalized charge.

The above has the result that for any irreducible diagram G_I^n of order n which gives rise to a finite contribution to S_n , one can obtain the complete radiative connections to this diagram by the replacement $S_F \rightarrow S'_F$, $D_F \rightarrow D'_F$, $\gamma_\mu \rightarrow \Gamma_\mu$. These contributions are divergent. However, if the integrand of G_I^n is $e^n \mathbf{I}(D'_F, S'_F, \gamma_\mu)$ then it can be shown that

$$e^n \mathbf{I}(D'_F, S'_F, \Gamma_\mu) = (eZ_1^{-1}Z_2Z_3^{1/2})^n \mathbf{I}(D'_{F1}S'_{F1}\Gamma_{\mu1}) \\ = e_1^n \mathbf{I}(D'_{F1}S'_{F1}\Gamma_{\mu1})$$

where $\mathbf{I}(D'_{F1}, S'_{F1}, \Gamma_{\mu1})$ leads to a convergent integral. Thus a finite result is obtained for the graph considered with all its added radiative corrections. The quantity e_1 can be identified with the experimentally measured charge of the electron, since the above rules lead to the conclusion that two slowly moving electrons separated by a large distance r interact according to a potential e_1^2/r . Thus all the infinities in quantum electrodynamics are isolated in two infinite constants, the mass and charge parameters of the electron.

DIVERGENT. Not convergent.

DIVIDED DIFFERENCES. For *interpolation* with unequally spaced abscissae, let $f(x)$ be the function to be interpolated, and let its values be given at

$$x_0, x_1, \dots, x_n,$$

where it is not required that they be in order or even distinct. Form the table as follows:

x_0	$f(x_0)$				
x_1	$f(x_1)$	$f(x_0, x_1)$			
x_2	$f(x_2)$	$f(x_1, x_2)$	$f(x_0, x_1, x_2)$		
x_3	$f(x_3)$	$f(x_2, x_3)$	$f(x_1, x_2, x_3)$	$f(x_0, x_1, x_2, x_3)$	\dots
x_4	$f(x_4)$	$f(x_3, x_4)$	$f(x_2, x_3, x_4)$	$f(x_1, x_2, x_3, x_4)$	

where

$$f(x_i, x_j) = [f(x_i) - f(x_j)]/(x_i - x_j);$$

$$f(x_i, x_j, x_k) = [f(x_i, x_j) - f(x_j, x_k)]/(x_i - x_k);$$

.....

When there are coincident abscissae they should be placed in adjacency, and then

$$f(x, x) = f'(x),$$

$$f(x, x, x) = f''(x)/2!,$$

.....

It can be shown that any divided difference is a symmetric function of all its arguments, which means that

$$f(\dots, x_i, \dots, x_j, \dots) = f(\dots, x_j, \dots, x_i, \dots),$$

it being understood that dots represent arguments that are the same on the two sides.

In terms of divided differences one can write the following interpolation formula due to Newton:

$$f(x) = f(x_0) + (x - x_0)f(x_0, x_1) \\ + (x - x_0)(x - x_1)f(x_0, x_1, x_2) + \dots \\ + (x - x_0) \dots (x - x_n)f(x_0, x_1, \dots, x_n, x),$$

where the last term written is the **remainder** and previous ones constitute the interpolation formula.

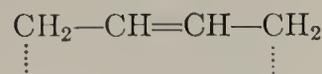
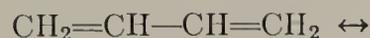
DOMAIN. The domain of a function or operator is the **set** of its arguments, i.e., the values that the independent variable (or variables) may assume. An *integral domain* is a **ring** with unit element in which division, when it exists, is unique. An **open non-empty** (connected) **point-set** is called a domain.

DOMINANT MODE. In a **waveguide**, that characteristic field configuration (mode) having the lowest **cutoff frequency**. For excitation frequencies between this cutoff and the next higher, this is the only mode by which energy will be propagated along the waveguide.

DOMINANT WAVELENGTH. Dominant wavelength (of a colored light, not purple) is the wavelength of the **spectrum** light that, when combined in suitable proportions with the specified **achromatic** light, yields a match with the light considered. When the dominant wavelength cannot be given (this applies to purples) its place is taken by the **complementary wavelength**.

DONNAN MEMBRANE EQUILIBRIUM. Suppose two solutions α, β to be separated by a membrane permeable to some of the ions present and to the solvent, but not to all of the ions. The equilibrium which is eventually reached is called the Donnan membrane equilibrium.

DONOR BOND. A double or triple bond which has a tendency to share some of its multiple bond character with a neighboring bond. The latter is an *acceptor bond*. For example, in the fundamental state of butadiene: $\text{CH}_2=\text{CH}-\text{CH}=\text{CH}_2$, the 1-2 and 3-4 bonds are donor bonds, while the 2-3 bond is an acceptor bond. Compare the two resonant formulas:



.....

The dotted line in the second configuration represents a bond between two unpaired electrons of opposite spin on the terminal C atoms. (See also **bond character**.)

DOOLITTLE METHOD. A particular scheme of recording intermediate results in applying triangularization to solve a system of normal equations. (See **matrix inversion**.)

DOPPLER EFFECT. The effects upon the apparent frequency of a wave train produced (1) by motion of the source toward or away from the stationary observer, and (2) by motion of the observer toward or from the stationary source; the motion in each case being with reference to the (supposedly stationary) medium.

For sound waves, the observed frequency f_0 , in cycles/sec, is given by

$$f_0 = \frac{v + w - v_0}{v + w - v_s} f_s$$

where v is the velocity of sound in the medium, v_0 is the velocity of the observer, v_s is the velocity of source, w is the velocity of wind in the direction of sound propagation, and f_s is the frequency of source.

For optical waves

$$f_0 = f_s \sqrt{\frac{c + v}{c - v}}$$

where v is the velocity of the source relative to the observer and c is the speed of light.

DOPPLER WIDTH OF A SPECTRAL LINE.

The contribution to the total line width of the Doppler shifts due to the thermal motions of the absorbing or emitting atoms.

The whole Doppler width ($2 \times$ Doppler half width) of a spectral line is given by

$$2(\Delta\lambda)_D = \frac{2\xi_0\lambda}{c}$$

where λ is the wavelength at the center of the line, c the velocity of light, and

$$\xi_0 = \sqrt{\frac{2RT}{\mu}}$$

the most probable thermal velocity of the atoms of atomic weight μ at temperature T . (R stands for the gas constant.)

DOPPLER WIDTH OF BREIT-WIGNER CROSS-SECTION. In the theory of the resonant capture of neutrons by heavy nuclei, the temperature motion of the nuclei causes a broadening of the Breit-Wigner cross-section profile (see **cross-section, Doppler broadened**). The broadening is characterized by the quantity

$$\theta = \frac{4E_0kT}{A\Gamma^2},$$

while $2\sqrt{\theta}$ is the Doppler width, w . Strongly perturbed widths have Gaussian form and fall to $1/e$ of their maximum when $|E - E_0| = w\Gamma$.

DOT PRODUCT OF VECTORS. See **vector multiplication**.

DOUBLE EXPONENTIAL DISTRIBUTION. See **exponential distribution**.

DOUBLE INTEGRATION PROCEDURE. The name given to the straightforward but often cumbersome method of finding **beam deflections** by integrating

$$d^2y/dx^2 = M/EI$$

with support and junction conditions taken into account.

DOUBLE MODULUS. See **modulus, reduced or double**.

DOUBLE REFRACTION. When a beam of light traverses an anisotropic medium, double refraction occurs so that the beam is separated into two components, polarized at right angles to each other, having different velocities within the medium and usually being propagated in different directions. The two components are distinguished as the *ordinary* and the *extraordinary rays*.

DOUBLE SOUND SOURCE. See **sound source, double**.

DOUBLE STARS. A celestial object that appears to be a single star to the unaided eye, but in a telescope is found to be two, or more, stars apparently very close together. If long-continued observation of the system indicates no evidence of relative motion it is safe to assume that the system is merely an optical double, i.e., it appears because the line of sight from one star is close to the line of sight from one very much farther away. If relative

motion of one star to the other is observed, the star is assumed to be a physically connected pair known as a binary system. (See **binary stars**.)

DOUBLE SYSTEM. See **indicial notation**.

DOUBLET. (1) Two electrons which are shared by two atoms so as to form a non-polar valence bond. (2) A pair of spectral lines resulting from transitions between a common state and two states which differ only in total angular momentum (**J**), i.e., have identical values of orbital (**L**) and spin (**S**) angular momenta. (3) Two stationary states having common values of (**L**) and (**S**), but different values of (**J**). (4) A **lens**, particularly an **achromat**, having two components. (5) In fluid mechanics, a source of strength m and a sink of strength $-m$ at distance δx apart constitute a doublet. The potential of the velocity due to a doublet is obtained by differentiating the potential of a source in the direction in which the doublet is oriented. For a doublet of moment $M_1 = m\delta x$, the potential, at distance r at angle θ from the axis of the doublet, is

$$\phi = \frac{M \cos \theta}{r^2}.$$

The potential of a fixed doublet in a uniform stream U , for which $\phi = Ur \cos \theta$, is

$$\phi = \left(Ur + \frac{M}{r^2} \right) \cos \theta$$

which, beyond a distance a , where $a^3 = \frac{2M}{U}$, is the same as the potential for inviscid flow round a sphere of radius a fixed in a uniform stream U .

In two-dimensional flow a line source of strength m per unit length and a parallel line sink of equal strength at distance δx constitute a line doublet of moment $\mu = m\delta x$. The complex potential of a line doublet is

$$\phi + i\psi = \frac{\mu}{2} = \frac{\mu}{r} (\cos \theta + i \sin \theta).$$

DOUBLE TENSOR. See **tensor, double**.

DOUBLING, INVERSION. See **inversion doubling**.

DOUBLING, LAMBDA-TYPE. See **Λ -type, doubling**.

DOUBLING, I-TYPE. See **i-type doubling**.

DRAG (FLUID). The resultant force exerted by the fluid on a body in a uniform stream may be resolved into two components, the *drag* in the direction of the stream and the *lift* perpendicular to the stream.

The force exerted by the fluid on an *element* of the body surface may be resolved into two components, normal and tangential to the surface. The drag due to the integrated effect of the normal components over the whole body surface is the *normal-pressure drag*, while that due to the tangential components is the *surface-friction drag*. In an inviscid fluid, of course, the latter is always zero.

In the absence of shock waves and trailing vortices the normal-pressure drag is also zero for an inviscid fluid, so that the total drag is zero. In a real fluid there is a drag due to effects of viscosity, which appears partly as a normal-pressure drag (sometimes called form drag) and partly as surface-friction drag. For a body of good "streamline shape," with no separation of the boundary layer, the normal-pressure drag is usually very small and most of the drag is due to surface friction, especially at high Reynolds numbers. On a bluff body, separation of the boundary layer leads to a large normal-pressure drag, and the surface-friction drag is then a small proportion of the total.

When lift is produced by a wing or other body, trailing vortices are formed. Kinetic energy must be continually added to the trailing-vortex system, and this energy can only be derived from work done against an associated drag, the *trailing-vortex drag*. Trailing vortex drag has often been called "induced drag," but the latter term has also been used commonly with another meaning, and it is therefore not recommended.

At transonic and supersonic speeds, shock waves are formed and there is then an additional component of drag, the *wave drag*, associated with the dissipation of energy in the shock waves. (See also **drag coefficient; trailing-vortex drag; profile drag of a wing**.)

DRAG (IONIC). The effect of interionic attraction in reducing the freedom of an ion to move in an electrical field, because of the interference of the ions of opposite charge by which a given ion is surrounded. The drag effect is an essential part of the explanation

of the Debye-Hückel theory of the anomalous properties of concentrated solutions of strong electrolytes.

DRAG (OPTICAL). See Fresnel coefficient of drag.

DRAG COEFFICIENT. If D is the drag of a body in a uniform stream of density ρ and velocity V , the drag coefficient is

$$C_D = \frac{D}{\frac{1}{2}\rho V^2 S},$$

where S is a representative area of the body. For bodies of revolution and for bluff bodies, S is usually taken as the "frontal area," i.e., the area of the body's projection on a plane normal to the stream. For a wing, S is usually the plane area, while for a complete airplane it is usually the gross wing area.

For a two-dimensional airfoil, if D' is the drag per unit span, the drag coefficient is

$$C_D = \frac{D'}{\frac{1}{2}\rho v^2 c},$$

where c is the chord. (See also drag.)

DRAG OF A WING, PROFILE. See profile drag of a wing.

DRAG, PROFILE. See profile drag of a wing.

DRAG, TRAILING-VORTEX. See trailing-vortex drag.

DRAG, WING. See slender-body theory.

DRIVING-POINT IMPEDANCE. See impedance, driving point.

DRIVING SYSTEM, ELECTRODYNAMIC.

A driving system in which the mechanical forces are developed by the interaction of currents in a conductor and the magnetic field in which it is located. The system is depicted in the figure. The force f_M , in dynes, due to the interaction of the current in the voice coil and the polarizing magnetic field is

$$f_M = Bli \quad (1)$$

where B is the flux density, in gauss, l is the length of the conductor, in centimeters, and i is the current, in abamperes.

The electromotive force e , in abvolts, developed by the motion of the conductor is

$$e = Bl\dot{x} \quad (2)$$

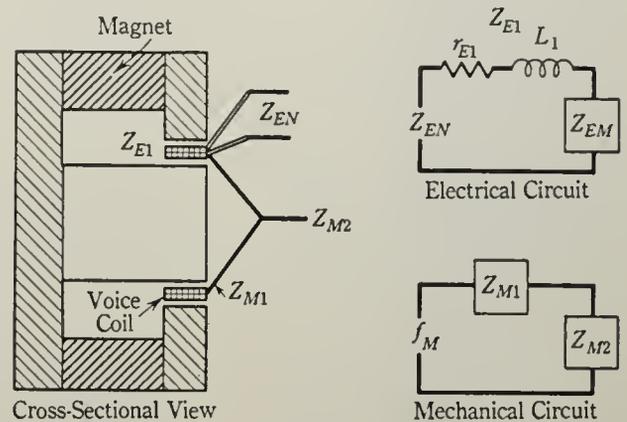
where \dot{x} is the velocity, in centimeters per second.

From Equations 1 and 2

$$\frac{e}{i} = (Bl)^2 \frac{\dot{x}}{f_M} \quad (3)$$

$$\frac{e}{i} = z_{EM} \quad (4)$$

where z_{EM} is the electrical impedance, in abohms, due to motion, termed motional electrical impedance.



Electrodynamic driving system. In the electrical circuit: z_{EN} , the normal electrical impedance of the voice coil. z_{EM} , the motional electrical impedance of the voice coil. z_{E1} , the damped electrical impedance of the voice coil. $z_{E1} = r_{E1} + j\omega L_1$. L_1 and r_{E1} , the damped inductance and electrical resistance of the voice coil. In the mechanical circuit: f_M , the driving force. z_{M1} , the mechanical rectilinear impedance of the voice coil. z_{M2} , the mechanical rectilinear impedance of the load.

From the mechanical circuit of the figure, the mechanical rectilinear impedance of the vibrating system at the voice coil is

$$z_M = z_{M1} + z_{M2} \quad (5)$$

where z_M is the total mechanical rectilinear impedance at the conductor, in mechanical ohms, z_{M1} is the mechanical rectilinear impedance of the voice coil and suspension system, in mechanical ohms, and z_{M2} is the mechanical rectilinear impedance of the load, in mechanical ohms.

The mechanical rectilinear impedance z_M , in mechanical ohms, at the voice coil is

$$z_M = \frac{f_M}{\dot{x}} \quad (6)$$

The electrical impedance due to motion from Equations 3, 4 and 6 is

$$z_{EM} = \frac{(Bl)^2}{z_M} \tag{7}$$

The motional electrical impedance of a transducer is the vector difference between its normal and blocked electrical impedance.

The normal electrical impedance of a transducer is the electrical impedance measured at the input to the transducer when the output is connected to its normal load.

The blocked electrical impedance of a transducer is the electrical impedance measured at the input when the mechanical rectilinear system is blocked, that is, in the absence of motion.

The normal electrical impedance z_{EN} , in abohms, of the voice coil is

$$z_{EN} = z_{E1} + z_{EM} \tag{8}$$

where z_{E1} is the damped electrical impedance of the voice coil, in abohms, and z_{EM} is the motional impedance of the voice coil, in abohms.

The motional electrical impedance as given by Equation 8 may be represented as in series with the blocked or damped electrical impedance of the conductor, as depicted by the electrical circuit in the figure.

The dynamic driving system is almost universally used for all types of direct radiator and horn loud speakers.

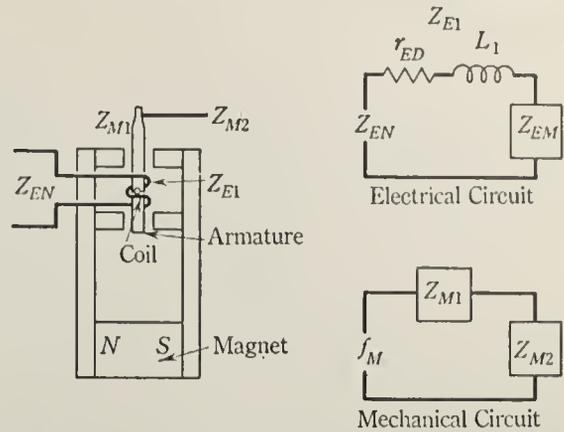
DRIVING SYSTEM, ELECTROMAGNETIC.

A driving system in which the mechanical forces result from magnetic reaction. A typical balanced armature magnetic driving system is shown in the figure. The steady field is usually supplied by a permanent magnet. The armature is located so that it is in the equilibrium with the steady forces. The alternating current winding is wound around the armature.

The result of the forces f_M , in dynes at the four poles acting upon the armature due to a current in the coil is

$$f_M = \frac{4\phi_1 Ni}{R_2 A} \tag{1}$$

where ϕ_1 is the total flux, in maxwells, at each pole due to the permanent magnet, A is the effective area, in square centimeters, of the pole piece, N is the number of turns in the coil, i is the current in the coil, in abamperes,



Polarized balanced armature electromagnetic driving system. In the electrical circuit: z_{EN} is the normal electrical impedance of the coil; z_{EM} , the motional electrical impedance of the coil; z_{E1} , the damped electrical impedance of the coil; $z_{E1} = r_{E1} + j\omega L_1$; L_1 and r_{E1} , the damped inductance and electrical resistance of the coil. In the mechanical circuit: f_M is the driving force; z_{M1} , the mechanical rectilinear impedance of the armature; z_{M2} , the mechanical rectilinear impedance of the load; z_{M1} and z_{M2} are referred to a point on the armature directly over a pole piece.

and R_2 is the reluctance of the magnetic circuit, in oersteds, which the coil energizes.

The electromotive force, e , in abvolts, generated in the coil is

$$e = \frac{NMA}{a^2} \dot{x} \tag{2}$$

where \dot{x} is the velocity of the armature, in centimeters per second, M is the magnetomotive force, in gilberts, of the steady field, a is the spacing between the armature and pole, in centimeters, and A is the effective area of a pole piece, in square centimeters.

From the mechanical circuit of the figure, the mechanical rectilinear impedance of the vibrating system is

$$z_M = z_{M1} + z_{M2} \tag{3}$$

where z_M is the total mechanical rectilinear impedance, in mechanical ohms, z_{M1} is the mechanical rectilinear impedance of the armature, in mechanical ohms, and z_{M2} is the mechanical rectilinear impedance of the load, in mechanical ohms.

The mechanical rectilinear impedance z_M , in mechanical ohms, at the armature directly over a pole piece is

$$z_M = \frac{f_M}{\dot{x}} \tag{4}$$

Combining Equations 1 and 2

$$\frac{e}{i} = \frac{4N^2\phi_1 M}{a^2 R_2} \frac{\dot{x}}{f_M} \quad (5)$$

From Equations 3, 4 and 5

$$z_{EM} = \frac{4N^2\phi_1 M}{a^2 R_2 z_M} \quad (6)$$

where z_{EM} is the motional electrical impedance, in abohms, and z_M is the total mechanical rectilinear impedance including the armature with reference to a point on the armature directly over one of the pole pieces.

If the entire reluctance is assumed to reside in the air gap, Equation 6 may be written

$$z_{EM} = \frac{4N^2\phi_1^2}{a^2 z_M} \quad (7)$$

The normal electrical impedance z_{EN} , in abohms, of the coil is

$$z_{EN} = z_{E1} + z_{EM} \quad (8)$$

where z_{E1} is the damped electrical impedance of the coil, in abohms, and z_{EM} is the motional electrical impedance of the coil, in abohms.

The motional electrical impedance as given by Equation 7 may be represented as in series with the blocked or damped electrical impedance of the coil as depicted by the electrical circuit in the figure.

DRIVING SYSTEM, ELECTROSTATIC. A driving system in which the mechanical forces result from electrostatic reactions. Consider the system of the figure consisting of a vibrating surface moving normal to its plane and separated from a fixed conductor. The force f_M , in dynes, between the plates is

$$f_M = \frac{e_0 e}{4\pi a^2} A \quad (1)$$

where e is the alternating electromotive force between plates, in statvolts, e_0 is the polarizing electromotive force between the plates, in statvolts, a is the normal distance between the plates, in centimeters, and A is the area of the plate, in square centimeters.

The generated current i , in statamperes due to the motion of the movable plate or diaphragm is

$$i = \frac{e_0 A}{4\pi a^2} \dot{x} \quad (2)$$

where \dot{x} is the velocity of the diaphragm, in centimeters per second.

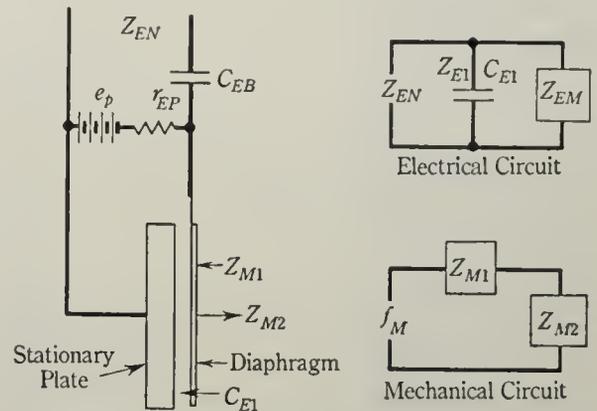
The mechanical rectilinear z_M , in mechanical ohms impedance at the movable plate is

$$z_M = \frac{f_M}{\dot{x}} \quad (3)$$

From the mechanical circuit of the figure, the mechanical rectilinear impedance of the vibrating system is

$$z_M = z_{M1} + z_{M2} \quad (4)$$

where z_M is the total mechanical rectilinear impedance of the vibrating system, in mechanical ohms, z_{M1} is the mechanical rectilinear



Electrostatic driving system. In the electrical network: z_{EN} is the normal electrical impedance of the condenser; z_{EM} , the motional electrical impedance of the condenser; z_{E1} , the damped electrical impedance of the condenser; $z_{E1} = 1/j\omega C_{E1}$; C_{E1} , the damped electrical capacitance of the condenser. In the mechanical circuit: f_M is the driving force; z_{M1} , the mechanical rectilinear impedance of the movable plate; z_{M2} , the mechanical rectilinear impedance of the load.

impedance of the vibrating plate, in mechanical ohms, and z_{M2} is the mechanical rectilinear impedance of the load, in mechanical ohms.

From Equations 1 and 2

$$\frac{e}{i} = \frac{16\pi^2 a^4 f_M}{e_0^2 A^2 \dot{x}} \quad (5)$$

From Equations 3, 4 and 5

$$z_{EM} = \frac{16\pi^2 a^4}{e_0^2 A^2} z_M \quad (6)$$

where z_{EM} is the motional electrical impedance, in statohms, and z_M is the total mechanical rectilinear impedance presented to the vibrating surface including the vibrating surface.

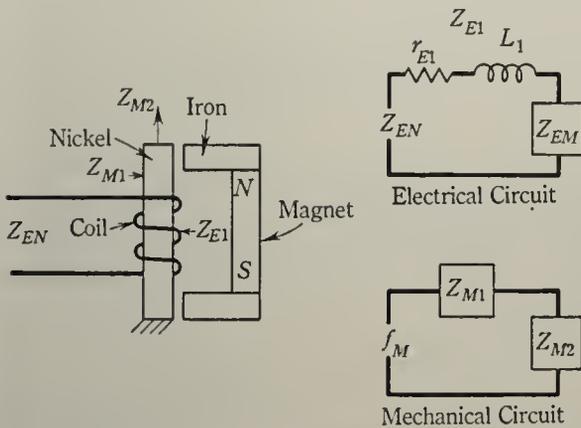
The normal electrical impedance z_{EN} , in statohms, of the condenser is

$$z_{EN} = \frac{z_{E1}z_{EM}}{z_{E1} + z_{EM}} \quad (7)$$

where z_{E1} is the damped electrical impedance of the condenser, in statohms, and z_{EM} is the motional electrical impedance of the condenser, in statohms.

The motional electrical impedance as given by Equation 6 may be represented as in parallel with the blocked or damped electrical impedance of the condenser as depicted by the electrical network in the figure.

DRIVING SYSTEM, MAGNETOSTRICTION. A driving system in which the mechanical forces result from the deformation of a ferromagnetic material having magnetostriction properties. The term "Joule effect" is applied to the phenomena in which a change in linear dimensions occurs when a magnetic field is applied along a specified direction. The term "Villari effect" is applied to the phenomena in which a change in magnetic induction occurs when a mechanical stress is applied along a specified direction.



Magnetostriction driving system. In the electrical circuit: z_{EN} is the normal electrical impedance of the coil; z_{EM} , the motional electrical impedance of the coil; z_{E1} , the damped electrical impedance of the coil; $z_{E1} = r_{E1} + j\omega L_1$; L_1 and r_{E1} , the damped inductance and electrical resistance of the coil. In the mechanical circuit: f_M is the driving force. z_{M1} , the mechanical rectilinear impedance of the rod. z_{M2} , the mechanical rectilinear impedance of the load.

Consider the system shown in the figure. Assume that the rod is clamped so that no motion is possible and that a current is applied to the winding; then the mechanical stress f_M , in dynes, due to the Joule effect, is

$$f_M = \frac{4\pi NiK}{R} \quad (1)$$

where K is the constant representing the dynamical Joule magnetostriction effect, R is the reluctance of the magnetic circuit, N is the number of turns in the coil, and i is the current in the coil, in abamperes.

The induced voltage, e in abvolts, due to a change in dimensions of the rod is

$$e = \frac{4\pi NK}{R} \dot{x} \quad (2)$$

where \dot{x} is the velocity of the change in the longitudinal dimension of the rod, in centimeters per second.

Combining Equations 1 and 2

$$\frac{e}{i} = \frac{16\pi^2 N^2 K^2}{R^2} \frac{\dot{x}}{f_M} \quad (3)$$

In the above consideration it has been assumed that the stress and driving force are uniform over the length of the rod. Under these conditions the rod is a compliance given by

$$C_{M1} = \frac{l}{EA} \quad (4)$$

where C_{M1} is the compliance of the rod, in centimeters per dyne, A is the cross-sectional area of the rod, in square centimeters, l is the length of the rod, in centimeters, and E is Young's modulus.

The mechanical rectilinear impedance z_{M1} , in mechanical ohms, of the rod is

$$z_{M1} = \frac{1}{j\omega C_{M1}} \quad (5)$$

For the conditions under consideration the mechanical rectilinear impedance of the vibrating system, from the mechanical circuit of the figure, is

$$z_M = z_{M1} + z_{M2} \quad (6)$$

where z_M is the total mechanical rectilinear impedance, in mechanical ohms, z_{M1} is the mechanical rectilinear impedance of the rod, in mechanical ohms, and z_{M2} is the mechanical rectilinear impedance of the load, in mechanical ohms.

The mechanical rectilinear impedances z_M , z_{M1} and z_{M2} are referred to one end of the rod with the other end rigidly fixed. The dimensions of the rod are assumed to be small compared to the wavelength.

The mechanical rectilinear impedance at the end of the rod is

$$z_M = \frac{f_M}{\dot{x}} \tag{7}$$

From Equations 3, 6 and 7

$$z_{EM} = \frac{16\pi^2 N^2 K^2}{R^2 z_M} \tag{8}$$

where z_{EM} is the motional electrical impedance, in abohms, and z_M is the total mechanical rectilinear impedance load upon the rod, including the effective mechanical rectilinear impedance of the rod, in mechanical ohms.

The normal impedance of the coil is

$$z_{EN} = z_{E1} + z_{EM} \tag{9}$$

where z_{E1} is the damped impedance of the voice coil, in abohms, and z_{EM} is the motional impedance, in abohms—Equation 8.

The damped impedance of the coil of most magnetostriction systems comprises a resistance in series with an inductance (see figure). The damped impedance and the motional impedance are effectively in series, as shown by Equation 9 and depicted by the electrical circuit in the figure.

In the above considerations the length of the rod is assumed to be a small fraction of the wavelength. In general, magnetostriction driving systems are operated at resonance. The three most common systems are as follows: a rod fixed on one end and loaded on the other, a rod free on one end and loaded on the other and a free rod.

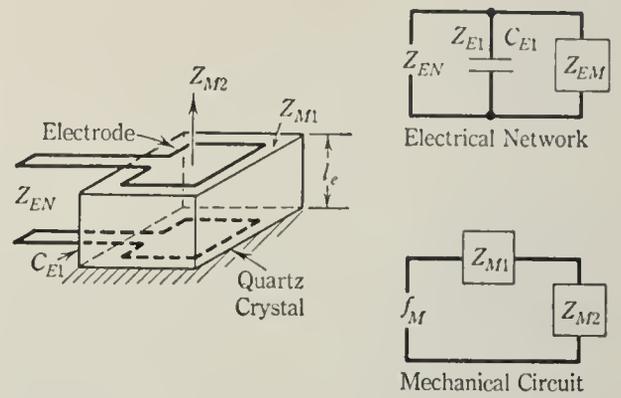
DRIVING SYSTEM, PIEZOELECTRIC. A driving system in which the mechanical forces result from the deformation of a crystal having converse piezoelectric properties. Among the crystals that exhibit piezoelectric phenomena are quartz, tourmaline, and Rochelle salt.

The consideration will be that of an X cut quartz crystal as shown in the figure.

The force f_M , in dynes, due to the application of an electromotive force is

$$f_M = \frac{KEAe}{l_e} \tag{1}$$

where K is the constant of the crystal, 6.4×10^{-8} for quartz, e is the applied voltage, in statvolts. l_e is the length of the crystal, in centimeters, E is Young's modulus, and A is the cross-sectional area, in square centimeters.



Piezoelectric driving system. In the electrical network: z_{EN} is the normal electrical impedance of the crystal; z_{EM} , the motional electrical impedance of the crystal; z_{E1} , the damped electrical impedance of the crystal; $z_{E1} = 1/j\omega C_{E1}$. C_{E1} , the damped electrical capacitance of the crystal. In the mechanical circuit: f_M is the driving force; z_{M1} , the mechanical rectilinear impedance of the crystal; z_{M2} , the mechanical rectilinear impedance of the load.

The generated current i , in statamperes due to a change in dimension of the crystal is

$$i = \frac{KEA\dot{x}}{l_e} \tag{2}$$

where \dot{x} is the velocity of the change in the longitudinal dimension of the crystal.

From Equations 1 and 2

$$\frac{e}{i} = \frac{l_e^2 f_M}{K^2 E^2 A^2 \dot{X}} \tag{3}$$

In the above consideration it has been assumed that the stress and driving force are uniform over the length l_e of the crystal. Under these conditions the crystal has a compliance given by

$$C_{M1} = \frac{l_e}{EA} \tag{4}$$

where A is the cross-sectional area of the crystal, in square centimeters, l_e is the length of the crystal, in centimeters, and E is Young's modulus.

The mechanical rectilinear impedance of the crystal is

$$z_{M1} = \frac{1}{j\omega C_{M1}} \tag{5}$$

For the conditions under consideration the mechanical rectilinear impedance of the vibrating system, from the mechanical circuit of the figure is

$$z_M = z_{M1} + z_{M2} \tag{6}$$

where z_M is the total mechanical rectilinear impedance, in mechanical ohms, z_{M1} is the mechanical rectilinear impedance of the crystal, in mechanical ohms, and z_{M2} is the mechanical rectilinear impedance of the load, in mechanical ohms.

The mechanical rectilinear impedances z_M , z_{M1} and z_{M2} are referred to one end of the crystal with the other end rigidly fixed. The dimensions of the crystal are assumed to be small compared to the wavelength.

The mechanical rectilinear impedance at the end of the crystal is

$$z_M = \frac{f_M}{\dot{x}} \quad (7)$$

From Equations 3, 6 and 7

$$z_{EM} = \frac{l_e^2}{K^2 E^2 A^2} z_M \quad (8)$$

The normal electrical impedance of the crystal system is

$$z_{EN} = \frac{z_{EM}}{1 + j\omega C_{E1} z_{EM}} \quad (9)$$

where z_{EM} is the motional impedance and C_{E1} is the capacitance of the crystal in the absence of motion.

The damped impedance and the motional impedance are effectively in parallel as shown by Equation 9 and depicted by the electrical circuit in the figure.

In the above considerations the length of the crystal is assumed to be a small fraction of the wavelength. In general, piezoelectric driving systems are operated at resonance. The three most common systems are as follows: a crystal fixed on one end and loaded on the other, a crystal free on one end and loaded on the other and a free crystal.

DROP-WISE CONDENSATION. When a vapor is cooled by a solid surface at sufficiently low temperature to cause condensation, the process of condensation may occur in one of two modes. Discrete drops may be formed on the surface, a process called drop-wise condensation. Alternatively, all drops may coalesce into a continuous film covering the solid surface. This is called *film-wise condensation*. In the case of drop-wise condensation, the coefficient of heat transfer is much higher than in the case of film-wise condensation, sometimes by factors as high as 1000.

DRUDE EQUATION. A relationship between the specific rotation of an optically-active substance and the wavelength of the light, of the form:

$$[\alpha] = \frac{k}{\lambda^2 - \lambda_0^2}$$

where $[\alpha]$ is the specific rotation, λ the wavelength, and k and λ_0^2 are constants, known as the rotation constant and the dispersion constant of the substance.

DRUDE THEORY OF ELECTRONS IN METALS. The original form of the free electron theory of metals in which the electrons were treated as a gas of classical particles. This theory is capable of explaining the high electrical conductivity of metals, but attributes to them far too large a specific heat, and has to be supplemented by the postulate that the electrons obey **Fermi-Dirac**, rather than **Maxwell-Boltzmann** statistics.

DRY-ADIABATIC LAPSE RATE. A special process lapse rate of temperature, defined as the rate of decrease of temperature with height of a parcel of **dry air** lifted adiabatically through an atmosphere in **hydrostatic equilibrium**. This lapse rate is g/c_{pd} , where g is the acceleration of gravity and c_{pd} is the specific heat of dry air at constant pressure; numerically equal to 9.767°C per km or about 5.4°F per thousand feet.

Potential temperature is constant with height in an atmosphere with this lapse rate.

DRY-BULB TEMPERATURE. See **adiabatic saturation**.

DRY COMBUSTION GASES. See **combustion**.

DRY GASES. See **combustion**.

DRY (SATURATED) STEAM. See **dry (saturated) vapor**.

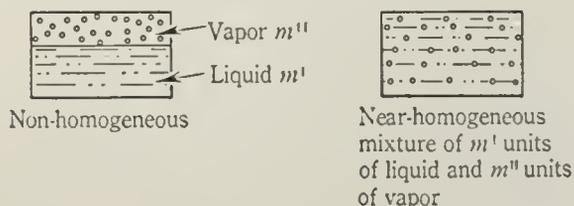
DRY (SATURATED) VAPOR. The vapor in a two-phase system which is in equilibrium with its own liquid, in contrast with superheated vapor. The pressure p and temperature T of dry, saturated vapor are not independent but lie on the vapor-pressure curve for the substance.

DRYNESS FRACTION. In a two-phase, vapor-liquid mixture (for example, in wet

steam), the ratio x of the mass of the vapor m'' to that of the whole system m ; thus

$$x = m''/m, \quad m = m' + m''$$

(where m' is the mass of liquid). The concept is applied to two-phase systems irrespective of whether or not they are homogeneous.



The extensive thermodynamic properties are determined by the dryness fraction. If ϕ is any extensive property, then

$$\begin{aligned} \phi &= \phi' + x(\phi'' - \phi') \\ &= \phi'' - (1 - x)(\phi'' - \phi'). \end{aligned}$$

Here ϕ' refers to the liquid and ϕ'' to the vapor. The intensive properties have the same values for both components in equilibrium.

The dryness fraction of the saturated liquid $x' = 0$, and of saturated vapor $x'' = 1$.

D²-STATISTIC. In multivariate analysis, a statistic which measures the "distance" between two populations with identical dispersions. If (α_{ij}) is the dispersion matrix of a p -variate complex, (α^{ij}) is its inverse and δ_i is the difference of means of the i^{th} variate, the distance Δ is defined by

$$\Delta^2 = \sum_{i,j=1}^p \alpha^{ij} \delta_i \delta_j$$

and for a sample the definition of the squared distance is similar with estimated means and dispersions on the right.

DUAL-COMBUSTION CYCLE. See diesel engine.

DUAL CYCLE. See diesel engine.

DUAL GRAPHS. See graphs, dual.

DUAL NETWORK. See network, dual.

DUAL TENSORS, RELATION TO SPINORS. See spinor calculus.

DUANE AND HUNT LAW. X-rays generated by electrons striking a target cannot have a frequency greater than eV/h where e is the electronic charge, V the exciting voltage and

h is Planck's constant. This is a simple consequence of quantum theory.

DUCT, FLOW OF GAS IN. See nozzle or duct, flow of gas in.

DUCTILITY. The ability of material to undergo large plastic deformation without cracking. It varies greatly with the conditions of test or fabrication. Quite brittle materials become ductile at high hydrostatic pressures and elevated temperatures. Ductile materials become brittle at low temperatures and with triaxial tension.

DUFFIN-KEMMER MATRICES. Matrices β_μ ($\mu = 1, 2, 3, 4$) satisfying

$$\beta_\mu \beta_\nu \beta_\sigma + \beta_\sigma \beta_\nu \beta_\mu = \delta_{\mu\sigma} \beta_\sigma + \delta_{\nu\sigma} \beta_\mu.$$

The equation $\left[\sum \beta_\mu \frac{\partial}{\partial x_\mu} + \frac{mc}{\hbar} \right] \psi = 0$ then describes particles of spin 0 or \hbar .

DUHAMEL INTEGRAL. See transient response of a dynamical system to an arbitrary force.

DUHEM-MARGULES EQUATION. The partial pressures p_1 and p_2 of the components of a binary solution are related by the equation

$$(1 - x_2) \left(\frac{\partial \ln p_1}{\partial x_2} \right)_T + x_2 \left(\frac{\partial \ln p_2}{\partial x_2} \right)_T = 0 \quad (1)$$

where x_2 is the mole fraction of component 2 in the solution. This equation, called the Duhem-Margules equation, is valid for ideal and non-ideal solutions as long as the gas phase may be considered as a perfect gas mixture, and as the total pressure remains sufficiently small not to affect the properties of the condensed phase.

This relation which is a form of the Gibbs-Duhem equation yields important information about one of the partial pressures if the other is known.

By combining it with the equation

$$p = p_1 + p_2 \quad (2)$$

one may calculate the partial pressures p_1, p_2 when the total pressure is known as a function of composition.

DUHEM THEOREM. The state of a thermodynamic system formed by ϕ phases is com-

pletely determined if one knows: (1) The physico-chemical state of each phase, defined for example by the intensive variables

$$T, p, x_1^1 \cdots x_c^1, x_1^2 \cdots x_c^{\phi} \quad (1)$$

where x_i^α is the mole fraction of component i in phase α ; (2) The extensive variables for each system. One may take as the independent extensive variables, the masses

$$m^1 \cdots m^{\phi} \quad (2)$$

of the different phases.

Duhem's theorem states that whatever the number of phases, of components, or of chemical reactions, the equilibrium state of a **closed system** for which one knows the initial masses is completely determined by *two independent variables*.

The basic difference between the Duhem theorem and the Gibbs **phase rule** is that the phase rule is concerned only with *intensive* variables while Duhem's theorem is a statement about the *total* number of variables (intensive + extensive) determining the state of a *closed* system. (See also **Saurel's theorem**.)

DULONG-PETIT RULE. A rule, discovered in 1819, which states that the product of the specific heat per unit mass of a crystalline solid element and its atomic weight (=atomic specific heat) is approximately the same for all such elements and has a value of about 6.2 kcal/C = 6.2 Btu/F. According to statistical mechanics this rule is an asymptotic one and is valid for temperatures T which are large compared with a characteristic temperature Θ ($T \gg \Theta$), different for each element. See **Debye temperature**, **Debye T^3 law**.

DUMMY. In experimental design it is sometimes convenient to regard certain plots as subject to treatment when no treatment is, in fact, applied. Such a "treatment" is then called dummy. Likewise, in certain forms of analysis it is convenient to regard constants as dummy variables.

DUMMY INDEX. See **summation convention**.

DUMMY UNIT LOAD METHOD. A method of finding deflection of a structure at a given point in a given direction by placing a unit load at the point in the direction desired. In essence it is a special application of the general method of virtual work in which the ac-

tual system is chosen as the compatible one, and the unit load system as the equilibrium one. Therefore, any system of forces and stresses in equilibrium with the unit load is permissible.

DUPIN INDICATRIX. See **indicatrix**.

DUPIN THEOREM. See **Euler theorem on normal curvature**.

DYADIC OPERATOR. A symbolic operator containing functions of a **dyadic**.

DYADIC. In ordinary **vector** analysis no meaning is attached to a symbol such as **AB**, where neither dot nor cross stands between two vectors. Such a quantity is called a **dyad** and the symbolic sum of two or more dyads is a dyadic **polynomial** or, more briefly, a **dyadic**. It is useful in the study of linear vector functions and has applications to theories of rotation and strain, electromagnetism, and optical phenomena in non-isotropic solids.

Consider the vector function $\mathbf{r}' = \mathbf{i}(c_1\mathbf{i}\cdot\mathbf{r}) + \mathbf{j}(c_2\mathbf{j}\cdot\mathbf{r}) + \mathbf{k}(c_3\mathbf{k}\cdot\mathbf{r})$, which is readily interpreted since the quantities in parentheses are scalars. The same equation, however, can be more simply written as $\mathbf{r}' = (\mathbf{i}c_1\mathbf{i} + \mathbf{j}c_2\mathbf{j} + \mathbf{k}c_3\mathbf{k})\cdot\mathbf{r}$ or, generally, if $\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3, \dots, \mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3, \dots$ are two sets of vectors, as $\mathbf{r}' = (\mathbf{a}_1\mathbf{b}_1 + \mathbf{a}_2\mathbf{b}_2 + \mathbf{a}_3\mathbf{b}_3 + \dots)\cdot\mathbf{r} = \Phi\cdot\mathbf{r}$. In the last equation, the direct or scalar product of the dyadic Φ and the vector \mathbf{r} yields a new vector \mathbf{r}' . The scalar product is not always commutative, for it does not follow that $\Phi\cdot\mathbf{r} = \mathbf{r}\cdot\Phi$. A special dyadic is the idempfactor, $\mathbf{I} = \mathbf{ii} + \mathbf{jj} + \mathbf{kk}$, which has the property that $\mathbf{I}\cdot\mathbf{r} = \mathbf{r}\cdot\mathbf{I} = \mathbf{r}$.

Nine fundamental dyads can be formed from unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$. The nonian form of any dyadic can be written in terms of them and scalar quantities a_{mn} as

$$\begin{aligned} \Phi = & a_{11}\mathbf{ii} + a_{12}\mathbf{ij} + a_{13}\mathbf{ik} + a_{21}\mathbf{ji} + a_{22}\mathbf{jj} \\ & + a_{23}\mathbf{jk} + a_{31}\mathbf{ki} + a_{32}\mathbf{kj} + a_{33}\mathbf{kk}. \end{aligned}$$

One can continue the algebra of dyadics, defining several kinds of products, reciprocals, conjugates, and other properties. They are discussed at length in Gibbs-Wilson, *Vector Analysis*, Yale University Press, New Haven, Conn., 1925, or, as related to a special physical problem, by W. H. Zachariasen, *Theory of X-Ray Diffraction in Crystals*, John Wiley and Sons, Inc., New York, 1945. A more sophisticated approach to the dyadic regards

it as a special kind of **tensor** with nine components. Each of them is a function of three coordinates and they transform from one coordinate system to another according to rules which can be derived. A dyadic can also be written in **matrix** form and can be generalized to the concept of triad, tetrad, polyad, and polyadic.

DYNAMICAL ANALOGIES, CLASSICAL.

The fundamental electrical elements of **electrical resistance, inductance, and electrical capacitance**; the mechanical rectilinear elements of **mechanical rectilinear resistance, mass and compliance**; the mechanical rotational elements of **mechanical rotational resistance, moment of inertia, and rotational compliance**; and the acoustical elements of **acoustical resistance, inertance, and acoustical capacitance** are defined by means of the fundamental equations. These equations establish the analogies between the three fundamental elements in the four dynamical systems. The consideration of electrical, mechanical rectilinear, mechanical rotational, and acoustical systems of one degree of freedom establishes further analogies between the elements and the quantities in the four systems. Electrical, mechanical rectilinear, mechanical rotational, and acoustical wave filters provide additional data on the analogies in the four systems. The subject of analogies is extended further by treatment of the **reciprocity theorems** in the four systems.

The analogies between the elements and quantities in the four systems are shown in the table.

Mathematically the elements in an electrical network are the coefficients in the differential equations describing the network. When the electric circuit theory is based upon Maxwell's dynamics the network forms a dynamical system in which the currents play the role of velocities. In the same way the coefficients in the differential equations of a mechanical or acoustical system may be looked upon as mechanical or acoustical elements. Kirchhoff's electromotive force law plays the same role in setting up the electrical equations as d'Alembert's principle does in setting up the mechanical and acoustical equations. That is to say, every electrical, mechanical or acoustical system may be considered as a combination of electrical, mechanical or acoustical elements.

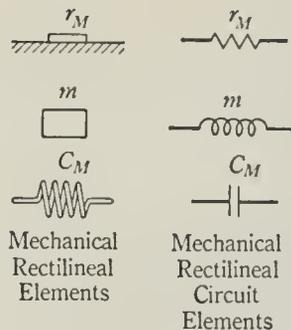


Fig. 1.

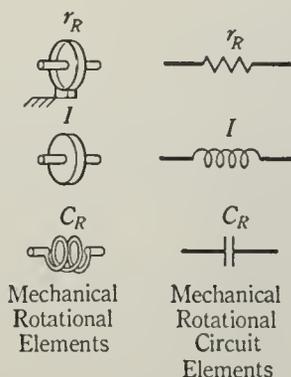


Fig. 2.

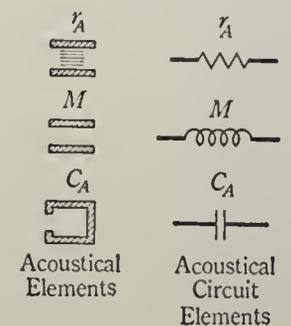


Fig. 3.

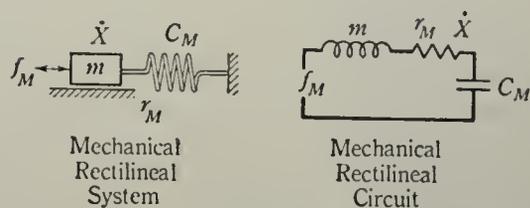


Fig. 4.

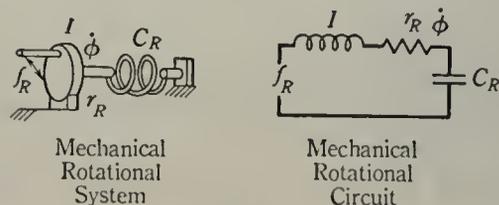


Fig. 5.

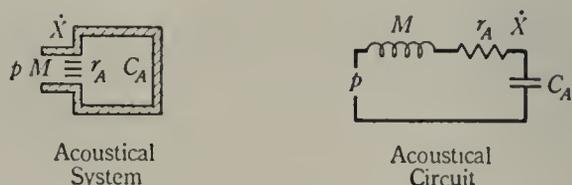


Fig. 6.

Therefore, any mechanical or acoustical system may be reduced to an electrical network and the problem may be solved by electrical circuit theory.

In view of the tremendous amount of study which has been directed towards the solution of circuits, particularly electrical circuits, and the engineer's familiarity with electrical circuits, it is logical to apply this knowledge to the solution of vibration problems in other fields by the same theory as that used in the solution of electrical circuits.

To carry out the solution of a problem in a **mechanical rectilinear, mechanical rotational or acoustical system**, the elements in these systems are converted to mechanical rectilinear circuit elements, mechanical rotational cir-

cuit elements or acoustical circuit elements, as shown in Figures 1, 2, and 3. The procedure is illustrated in Figures 4, 5, and 6 for the solution of a mechanical rectilinear, mechanical rotational, and acoustical systems. In Figure 4, the mechanical rectilinear system of one degree of freedom is converted to an acoustical circuit. The constants of the elements and quantities used in the mechanical rectilinear, mechanical rotational and acoustical circuits are shown in the table. The performance of the circuits is obtained by the application of electrical circuit theory. The equations for the performance of the systems of Figures 4, 5, and 6 will be found in **mechanical rectilinear, mechanical rotational, and acoustic systems of one degree of freedom**.

QUANTITIES AND ELEMENTS IN ELECTRICAL, MECHANICAL RECTILINEAL, MECHANICAL ROTATIONAL, AND ACOUSTICAL SYSTEMS

Electrical			Mechanical Rectilinear			Mechanical Rotational			Acoustical		
Quantity	Unit	Sym- bol	Quantity	Unit	Sym- bol	Quantity	Unit	Sym- bol	Quantity	Unit	Sym- bol
Electromotive Force	Volts $\times 10^8$	e	Force	Dynes	f_M	Torque	Dyne Centimeter	f_R	Pressure	Dynes per Square Centimeter	p
Charge or Quantity	Coulombs $\times 10^{-1}$	q	Linear Displacement	Centimeters	x	Angular Displacement	Radians	ϕ	Volume Displacement	Cubic Centimeters	X
Current	Amperes $\times 10^{-1}$	i	Linear Velocity	Centimeters per Second	\dot{x} or v	Angular Velocity	Radians per Second	$\dot{\phi}$ or θ	Volume Current	Cubic Centimeters per Second	\dot{X} or U
Electrical Impedance	Ohms $\times 10^9$	z_E	Mechanical Impedance	Mechanical Ohms	z_M	Rotational Impedance	Rotational Ohms	z_R	Acoustical Impedance	Acoustical Ohms	z_A
Electrical Resistance	Ohms $\times 10^9$	r_E	Mechanical Resistance	Mechanical Ohms	r_M	Rotational Resistance	Rotational Ohms	r_R	Acoustical Resistance	Acoustical Ohms	r_A
Electrical Reactance	Ohms $\times 10^9$	x_E	Mechanical Reactance	Mechanical Ohms	x_M	Rotational Reactance	Rotational Ohms	x_R	Acoustical Reactance	Acoustical Ohms	x_A
Inductance	Henries $\times 10^9$	L	Mass	Grams	m	Moment of Inertia	(Gram) (Centimeter) ²	I	Inertance	Grams per (Centimeter) ⁴	M
Electrical Capacitance	Farads $\times 10^{-9}$	C_E	Compliance	Centimeters per Dyne	C_M	Rotational Compliance	Radians per Dyne per Centimeter	C_R	Acoustical Capacitance	(Centimeter) ⁵ per Dyne	C_A
Power	Ergs per Second	P_E	Power	Ergs per Second	P_M	Power	Ergs per Second	P_R	Power	Ergs per Second	P_A

DYNAMICAL ANALOGIES, MOBILITY.

The analogies which have been presented in the section on classical dynamical analogies

mechanical mobility is defined as the complex ratio of velocity to force. Although the mobility dynamical analogy can be applied to all types of vibrating systems, the most direct and useful application of the mobility dynamical analogy resides in the field of mechanical vibrating systems.

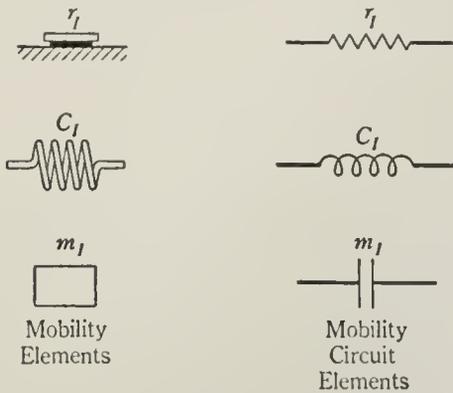


Fig. 1.

(see dynamical analogies, classical) and related sections have been formal ones due to the similarity of the differential equations of the electrical, mechanical, and acoustical systems. For this reason these analogies have been termed the classical impedance dynamical analogies. These are, however, not the only ones possible of development for useful applications. In this connection, the **mobility** analogy has been developed and used on a wide scale to solve problems in mechanical vibrating systems. In the mobility analogy,

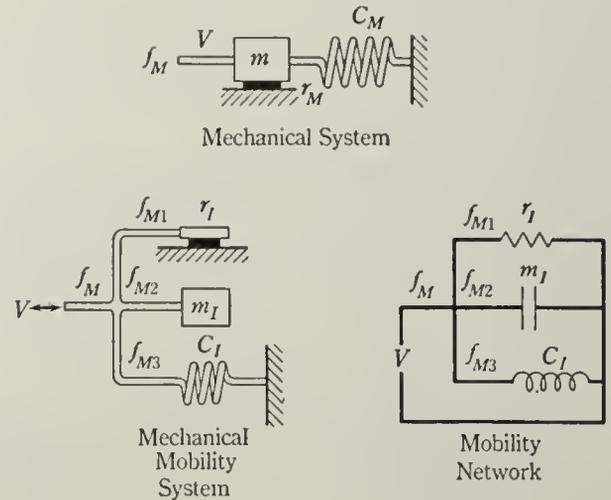


Fig. 2.

As in the case of the solution of a problem in a mechanical vibrating system by applying the principles of classical dynamical analogies, the mechanical elements in the mechanical sys-

Electrical			Mechanical Rectilinear Mobility		
Quantity	Unit	Symbol	Quantity	Unit	Symbol
Electromotive Force	Volts $\times 10^{-8}$	e	Velocity	Centimeters per Second	\dot{x} or v
Charge or Quantity	Coulombs $\times 10^{-1}$	q	Impulse or Momentum	Gram Centimeter per Second	Q
Current	Amperes $\times 10^{-1}$	i	Force	Dynes	f_M
Electrical Impedance	Ohms $\times 10^9$	z_E	Mechanical Mobility	Mechanical Mhos	z_I
Electrical Resistance	Ohms $\times 10^9$	r_E	Responsivity	Mechanical Mhos	r_I
Electrical Reactance	Ohms $\times 10^9$	x_E	Excitability	Mechanical Mhos	x_I
Inductance	Henries $\times 10^9$	L	Compliance or Mobility Inertia	Centimeters per Dyne	C_I
Electrical Capacitance	Farads $\times 10^9$	C_E	Mass or Mobility Capacitance	Grams	m_I
Power	Ergs per Second	P_E	Power	Ergs per Second	P_I

tem are converted to mobility circuit elements as depicted in Figure 1.

The analogies between the elements and the quantities in the electrical and mechanical rectilinear mobility systems are shown in the table.

The procedure for the application of the mobility analogy is illustrated in Figure 2. The mechanical system is converted to a mechanical mobility system. Then it is a simple matter to convert the mechanical mobility system to a mobility network. The performance of the network is obtained by the application of electrical circuit theory.

DYNAMICAL EQUATIONS OF LIGHT RAYS. The optical paths, or rays, in a medium of refractive index $n = n(x, y, z)$ are the solutions of the system of dynamical equations

$$\ddot{x} = \frac{\partial}{\partial x} \left(\frac{1}{2}n^2 \right),$$

$$\ddot{y} = \frac{\partial}{\partial y} \left(\frac{1}{2}n^2 \right),$$

$$\ddot{z} = \frac{\partial}{\partial z} \left(\frac{1}{2}n^2 \right),$$

which satisfy

$$\dot{x}^2 + \dot{y}^2 + \dot{z}^2 = n^2,$$

where one dot denotes the first derivative with respect to time, and two dots, the second. Thus, the rays are paths of "corpuseles" moving in a potential field $\phi = -\frac{1}{2}n^2$ with velocity n . This last conclusion is in conflict with Foucault's measurements of the relative velocities of light in air and water.

Descartes proved **Snell's law** on the basis of the corpuscular theory using the continuity of the tangential component of velocity.

DYNAMICAL SIMILARITY. Two systems are dynamically similar if the ratio of corresponding forces is the same. This requires that all possible **Froude numbers** must be the same in the two systems. The inertia forces, gravity forces, viscous forces, etc., must be in the same ratio.

In making model experiments the **Reynolds number** is usually the most important when gravity does not directly affect the flow, but the **Richardson** and ordinary Froude numbers are important in modeling flows of stratified

fluids, and the **Rayleigh number** in Stokes flows which transfer buoyancy (or heat).

DYNAMICAL SURFACE TENSION. In a surface phase whose age is low with respect to the characteristic times involved in diffusion, the adsorption (see **Gibbs division surface**) may be taken equal to zero. The surface tension of such a "fresh" surface is called the dynamical surface tension.

DYNAMICAL VARIABLES. Those variables in terms of which classical mechanics is built up, and which can be given an operational definition. Examples are the coordinates and components of velocity, momentum, and angular momentum of particles, and functions of these quantities.

DYNAMIC BOUNDARY CONDITION. The condition that the pressure must be continuous across an internal boundary or **free surface** in a fluid.

DYNAMIC EQUILIBRIUM. See **equilibrium, dynamic**.

DYNAMIC LOADING. The loading applied as a consequence of the motion of a live load and often expressed as a multiple of the static value of that load. A suddenly applied load such as would be exerted by the release of a weight in contact with a horizontal elastic beam gives a dynamic load that is twice the static load.

DYNAMIC PARALLAX (OF BINARY STARS). If the masses of the two components of a visual **binary star**, m_1 and m_2 , are found from the **mass-luminosity relation**, it is possible to determine the parallax of the system. From the Kepler laws, one can write

$$m_1 + m_2 = \frac{a^3}{P^2 \pi^3}$$

in which π is the parallax of the system in seconds of arc, and P is given in sidereal years. In a visual binary, a is the separation of the components and this can be measured in seconds of arc. π is the only remaining unknown and therefore

$$\pi = \frac{a}{\sqrt[3]{P^2(m_1 + m_2)}}.$$

DYNAMIC PRESSURE. The dynamic pressure at a given point in a fluid is the difference

between the total pressure p_o and the static pressure p at that point. For an incompressible fluid, **Bernoulli's equation** shows that the dynamic pressure is equal to $\frac{1}{2}\rho v^2$, where ρ is the density of the fluid and v is the velocity. For a compressible fluid the dynamic pressure ($p_o - p$) is given by

$$\frac{p_o - p}{p} = \left[1 + \left(\frac{\gamma - 1}{2} \right) M^2 \right]^{\frac{\gamma}{\gamma - 1}} - 1.$$

(See also **Pitot tube**.)

DYNAMICS. The science that deals with the motion of systems of particles under the influence of forces. Dynamics deals with the causes of motion, as opposed to **kinematics**, which deals with its geometrie description, and to **statics**, which deals with the conditions for lack of motion.

DYNAMICS OF A FREE MASS POINT (RELATIVITY). The relativistic force **F** acting on an accelerated mass point can be defined, just as in classical mechanics, as the time rate of change of linear momentum.

$$\mathbf{F} = \frac{d}{dt} \left(\frac{m_o v}{\sqrt{1 - v^2/c^2}} \right)$$

m_o is the rest mass. If the force is parallel to the velocity, its magnitude is

$$\left(1 - \frac{v^2}{c^2} \right)^{-3/2} m_o \frac{d^2x}{dt^2}.$$

If the force is perpendicular to the velocity, its magnitude is

$$\left(1 - \frac{v^2}{c^2} \right)^{-1/2} m_o \frac{d^2y}{dt^2} \quad \text{or} \quad \left(1 - \frac{v^2}{c^2} \right)^{-1/2} m_o \frac{d^2z}{dt^2}.$$

(It is assumed that the velocity is parallel to the x -direction.) The coefficients

$$\left(1 - \frac{v^2}{c^2} \right)^{-3/2} m_o \quad \text{and} \quad \left(1 - \frac{v^2}{c^2} \right)^{-1/2} m_o$$

are called the "longitudinal" and the "transversal" mass.

DYNAMICS, RAREFIED GAS. See **rarefied gas dynamics**.

DYNAMIC UNBALANCE. See **balancing**.

DYNAMIC VISCOSITY. See **viscosity, dynamic**.

DYNE. The cgs unit of force. See **mechanical units**.

DYSON REPRESENTATION. Consider two fields $A(x)$ and $B(x)$ obeying the **causality condition**

$$[A(x), B(y)] = 0 \quad \text{for } (x - y)^2 < 0 \quad (1)$$

where x, y are four vectors and the scalar product is defined by

$$x \cdot y = x_\mu y^\mu = x_0 y_0 - \mathbf{x} \cdot \mathbf{y}. \quad (2)$$

An important problem is to determine the analytic form of the (**generalized**) function

$$\tilde{f}(x) = \langle P, \alpha | \left[A \left(\frac{x}{2} \right), B \left(-\frac{x}{2} \right) \right] | Q, \beta \rangle \quad (3)$$

or its Fourier transform

$$f(q) = \frac{1}{(2\pi)^4} \int d^4x e^{-iqx} \tilde{f}(x) \quad (4)$$

where $|P, \alpha\rangle$ and $|Q, \beta\rangle$ are any two physical states specified by the energy momentum vectors P and Q and by other quantum numbers α and β . By (1) the function \tilde{f} in x space has the property

$$\tilde{f}(x) = 0 \quad \text{for } x^2 < 0. \quad (5)$$

From Equation (3) and the assumption that the physical states of the system are complete in \mathcal{H} and that they can be characterized in part by energy momentum four-vectors $p^{(n)}$ for which $p^{(n)} p^{(n)} \geq 0, p_o^{(n)} \geq 0$, the function f in q space also has the property of vanishing in a large region. In the coordinate system in which

$$\frac{1}{2}(P + Q) = (a, 0, 0, 0) \quad a > 0. \quad (6)$$

$f(q) = 0$ for all $q = (q_0, q)$ satisfying

$$a - [\mathbf{q}^2 + b^2]^{1/2} < q_0 < [\mathbf{q}^2 + c^2]^{1/2} - a \quad (7)$$

where b and c are the lowest masses of the possible intermediate states $|n\rangle, |m\rangle$ in the decomposition

$$\begin{aligned} \langle P\alpha | A \left(\frac{x}{2} \right) B \left(-\frac{x}{2} \right) | Q\beta \rangle &= \sum_n \langle P\alpha | A \left(\frac{x}{2} \right) | n \rangle \langle n | B \left(-\frac{x}{2} \right) | Q\beta \rangle \\ \langle P\alpha | B \left(-\frac{x}{2} \right) A \left(\frac{x}{2} \right) | Q\beta \rangle &= \sum_m \langle P\alpha | B \left(-\frac{x}{2} \right) | m \rangle \langle m | A \left(\frac{x}{2} \right) | Q\beta \rangle. \end{aligned} \quad (8)$$

Jost and Lehmann have shown that the class of generalized function $f(q)$ which vanish in the region

$$|q_0| > [\mathbf{q}^2 + c^2]^{1/2} - a \quad (9)$$

and have Fourier transforms vanishing for $x^2 < 0$ is identical with the class of functions of the form

$$f(q) = \int d^3u \int d\kappa^2 \epsilon(q_0) \delta[q_0^2 - (\mathbf{q} - \mathbf{u})^2 - \kappa^2] \\ \times [\Phi_1(\mathbf{u}, \kappa^2) + q_0 \Phi_2(\mathbf{u}, \kappa^2)] \quad (10)$$

with the functions $\Phi_1(\mathbf{u}, \kappa^2)$ and $\Phi_2(\mathbf{u}, \kappa^2)$ each vanishing except in the region

$$|\mathbf{u}| < a \quad \kappa \geq c - (a^2 - \mathbf{u}^2)^{1/2}. \quad (11)$$

This theorem provides a representation for functions $f(q)$ vanishing in the region (7) for the case $b = c$. An important Lemma also due to Jost and Lehmann is that the class of func-

tion $f(q)$ satisfying $\tilde{f}(x) = 0$ for $x^2 < 0$ and vanishing in the region

$$|q_0| + |\mathbf{q} - \mathbf{v}| < \lambda$$

is identical with the class represented by (10) with the functions Φ_1 , and Φ_2 vanishing in $\kappa^2 + (\mathbf{u} - \mathbf{v})^2 < \lambda^2$.

Dyson has given the solution to general case in the following theorem: for a function $f(q)$ to vanish in the region $h(\mathbf{q}) < q_0 < g(\mathbf{q})$ and to have a Fourier transform vanishing for $x^2 < 0$ it is necessary and sufficient that

$$f(q) = \int d^4u \int ds \epsilon(q_0 - u_0) \delta[(q - u)^2 - s] \Psi(u, s)$$

where $\Psi(u, s)$ is a function vanishing outside the region S defined by

$$m(\mathbf{u}, s) \leq u_0 \leq M(\mathbf{u}, s)$$

with

$$m(\mathbf{u}, s) = \max_{\mathbf{q}} \{g(\mathbf{q}) - [(\mathbf{q} - \mathbf{u})^2 + s]^{1/2}\}$$

$$M(\mathbf{u}, s) = \min_{\mathbf{q}} \{h(\mathbf{q}) + [(\mathbf{q} - \mathbf{u})^2 + s]^{1/2}\}.$$

E

e (THE NUMBER). A transcendental number, used as the base of the system of natural or Napierian logarithms. It is defined by

$$e = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{n}\right)^n$$

or by

$$e = \lim_{x \rightarrow 0} (1 + x)^{1/x}.$$

It is represented by the infinite series

$$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \frac{1}{4!} + \cdots + \frac{1}{n!} + \cdots$$

and has the approximate value of 2.71828.

EARNSHAW THEOREM. No charge can be in stable equilibrium in an electric field under the influence of electric forces alone.

EARTHQUAKE RESISTANCE. Resistance to failure due to horizontal motion of the supports of a structure. Building codes may call for design against a static horizontal force of between 0.025 and 1.0 times the **dead load**, depending upon location and part of structure. The actual problem involves the dynamic response of a structure to a random sequence of forced motions at its base and is still under study.

EARTHQUAKE SCALES. The intensity and magnitude of an earthquake is rated by a number of scales. The Rossi-Forel intensity scale from the barely felt I to the disaster X and the Mercalli scale from 1 to total damage 12 are in common use.

EBULLIOSCOPIC CONSTANT. See **boiling constant**.

ECCENTRIC LOADING. A force applied parallel to the axis of a bar (line of centroids of the cross sections) but at some distance from it, produces both direct stress and bending. (See **kern**.)

ECHELON. A highly specialized form of **diffraction grating**. It consists of a row of glass plates of exactly equal thickness, packed

together to form a miniature stairway of equal risers. The light enters normally to the largest plate at one end (see figure) and emerges at various deviations through the low "risers." It is easily shown that if the thickness of the plates is a , the height of the "risers" b , and the



refractive index of the glass n , the equivalent path difference between successive emergent streams for any angle of deviation Δ is $na - a \cos \Delta + b \sin \Delta$; or since Δ is in practice always small, $\cos \Delta = 1$ and $\sin \Delta = \Delta$ (in radians), giving $(n - 1)a + b\Delta$. This must be equal to an integral multiple, N , of the wavelength λ for any spectrum line, the deviation of which is therefore

$$\Delta = N \frac{\lambda}{b} - (n - 1) \frac{a}{b}.$$

The smallest value N can have (for $\Delta = 0$) is $(n - 1) \frac{a}{\lambda}$, which, since a is usually several mm. and $(n - 1)$ is 0.5 or more, is of the order of several thousand. The **dispersion**, viz.,

$$D = \frac{d\Delta}{d\lambda} = \frac{N}{b} - \frac{a}{b} \frac{dn}{d\lambda},$$

is correspondingly large. The echelon is thus especially adapted to the study of the **hyper-fine structure** of spectrum lines.

ECKERT NUMBER. See **dimensionless groups**.

ECLIPSING BINARY STARS. An eclipsing binary star is both a **binary star** and also a **variable star**. The light curve is characterized by a relatively sharp and symmetric drop to minimum. The curve is not always flat at maximum, but may show a slow rise and then a decline to a point at which the drop to minimum occurs. In some cases there are two minima, one deeper than the other. It is believed that, in this class of variable star, there

are two stars that differ in size and brightness, and that are revolving about the center of gravity of the system in elliptic orbits, with the plane of the orbit passing through or relatively close to the sun, i.e., with inclination close to 90° . If the inclination differs appreciably from 90° the eclipse will be only partial. All available evidence points to this type of variability.

When the masses and relative dimensions of the components are known from the orbit, light curve, and parallax, the mass of the system is

$$m_b + m_f = \frac{(a_b + a_f)^3}{25P^2}$$

with a in millions of kilometers and P in days.

The radius of the brighter component is given by $(a_1 + a_2)r_b$ where r_b is given by the photometric data. The volume of this star in terms of the volume of the sun, is $2.98 (a_1 + a_2)^3 r_b$. Therefore, the density of the brighter star, in terms of the density of the sun as unity, is given by

$$\rho_b = \frac{m_b}{V_b} = \frac{1}{74.4P^2 r_b^3} \cdot \frac{m_b}{m_b + m_f}$$

In the second right-hand member of this equation, $\frac{m_b}{m_b + m_f}$ may be estimated with considerable accuracy from the mass-luminosity relation:

$$\frac{m_b}{m_f} = \left(\frac{L_b}{L_f} \right)^{2/5}$$

Much more information about the atmospheres of the two stars may be determined by spectrophotometric observations of the changes as one star passes over the photosphere of the other and leaves just the atmosphere exposed. This and other information is needed for the development of various theories relative to stellar structure.

ECLIPTIC. The great circle on the celestial sphere cut out by the plane containing the orbit of the earth. The ecliptic intersects the **celestial equator** at the vernal and autumnal equinoxes. The angle between the plane of the ecliptic and the plane of the equator is known as the *obliquity of the ecliptic* and is about 23.5° , but is slowly changing. The accurate value of the obliquity is published in almanacs. The celestial latitudes and longitudes of astronomic objects are referred to the

plane of the ecliptic and the equinoxes. (C.f. **precession**.)

ECONOMISER. See **air-preheating**.

EDDY. (1) By analogy with a molecule, a "glob" of fluid within the fluid mass that has a certain integrity and life history of its own; the activities of the bulk fluid being the net result of the motion of the eddies.

The concept is applied with varying results to phenomena ranging from the momentary spasms of the wind to storms and anticyclones.

(2) Any circulation drawing its energy from a flow of much larger scale, and brought about by pressure irregularities as in the lee of a solid obstacle.

EDDY CONDUCTIVITY. See **turbulent transfer coefficient**.

EDDY CURRENT. See **current, eddy**.

EDDY FLUX. (Or turbulent flux.) The rate of transport (or flux) of fluid properties such as momentum, mass, heat, or suspended matter by means of **eddies** in a turbulent motion; the rate of turbulent exchange. (See **mixing length, exchange coefficients, interaction**.)

EDDY HEAT CONDUCTION. (Or eddy heat flux; also called eddy conduction.) The transfer of heat by means of **eddies** in turbulent flow, treated analogously to molecular conduction.

If temperature is represented by the sum of the mean temperature of the fluid plus its instantaneous departure from the mean, substituted into the Fick equation and the mean value taken, then the form of the resulting equation for the mean temperature is unchanged except that the molecular heat transfer terms

$$K_x \frac{\partial T}{\partial x}, \quad K_y \frac{\partial T}{\partial y}, \quad \text{and} \quad K_z \frac{\partial T}{\partial z}$$

are replaced by

$$K_x \frac{\partial \bar{T}}{\partial x} - c_p \overline{\rho u' T'}, \quad K_y \frac{\partial \bar{T}}{\partial y} - c_p \overline{\rho v' T'}$$

and

$$K_z \frac{\partial \bar{T}}{\partial z} - c_p \overline{\rho w' T'},$$

where ρ is the density, c_p is the specific heat at constant pressure, and u' , v' , w' , and T' the departures of the velocity components and the

temperature. These are analogous to the **Reynolds stresses** of the correlation term as expressed as an **exchange coefficient**. Then (considering only one-dimensional variability)

$$-\overline{w'T''} = K_z \frac{\partial \bar{T}}{\partial z},$$

where K is the eddy conductivity coefficient (see **turbulent transfer coefficient**) and, unlike the molecular coefficient, is usually a function of the coordinates.

On the basis of the **mixing length** theory, the mean rate of transfer of heat q in an incompressible fluid normal to the direction of z is

$$q = -c_p \rho l^2 \sqrt{w'^2} \frac{\partial T}{\partial z},$$

where l is the mixing length. Because the atmosphere is compressible, temperature is not conservative and in the previous equations the **potential temperature** must be used when the transfer is considered in the vertical direction. Another expression for the transfer of heat, now across an isobaric surface, is

$$q = -K c_p \rho \left(\Gamma + \frac{\partial T}{\partial z} \right),$$

where Γ is the dry- or saturation-adiabatic lapse rate, as the circumstance requires.

EDDY KINETIC ENERGY. (Also called turbulent energy.) The **kinetic energy** of that component of fluid flow which represents a departure from the average kinetic energy of the fluid, the mode of averaging depending on the particular problem. This eddy kinetic energy is represented by $\overline{\rho u'^2}$ where ρ is the density, u' is the **eddy velocity**, and the superior bar denotes an average.

In general circulation studies, for example, the zonal average along a fixed latitude circle is usually considered. As another example, in small-scale turbulence studies it is frequently desirable to consider the average with respect to time at a fixed point in the fluid.

EDDY SPECTRUM. The distribution of the frequency of **eddies** of various sizes or scales in a turbulent flow, or the distribution of kinetic energy among eddies of various frequencies or sizes.

EDDY STRESS. See **Reynolds stress**.

EDDY TRANSPORT (EDDY TRANSFER). See **turbulent transfer coefficient**.

EDDY VELOCITY. (Also called fluctuation velocity.) The difference between the **mean velocity** of fluid flow and the instantaneous velocity at a point. For example,

$$u' = u - \bar{u},$$

where u' is the eddy velocity, u is instantaneous velocity, and \bar{u} is mean velocity.

Over the same interval which defines the mean velocity, the average value of the eddy velocity is necessarily zero.

EDDY VISCOSITY. See **turbulent transfer coefficient**.

EDGE. Two distinct points (endpoints) and a line segment joining them. Considering the edge as point-closed (including endpoints) is the conventional procedure although open line segments are also used by some authors. Similarly the insistence upon the distinctness of the endpoints of an edge is motivated by the desire to exclude closed "loops" in which the endpoints coincide.

EDGE CONDITION. A condition introduced in problems involving diffraction of electromagnetic waves by obstacles with sharp edges. The edge must be a **singularity** of the field with the following characteristics: (1) No energy is radiated from the edge itself. (2) The field components parallel to the edge shall be bounded.

EDGE FORCE. Surface traction applied to the cylindrical bounding surface of a plate or shell. Usually measured per unit length along the periphery of the plate or shell, the detailed distribution on the bounding surface in the thickness direction being unspecified. Also called *edge traction*.

EDGE MULTIPLICITY. The number of times an **edge** appears in an **edge sequence**.

EDGE OF REGRESSION. The locus of the points of intersection of successive characteristics of a one-parameter family of surfaces as the parameter is varied. If $f(x, y, z, a) = 0$ is the one-parameter family of surfaces then the equations for the edge of regression can be obtained by eliminating a from the equation

$$f(x, y, z, a) = 0, \quad \partial f(x, y, z, a) / \partial a = 0,$$

$$\partial^2 f(x, y, z, a) / \partial a^2 = 0.$$

EDGE SEQUENCE. A **subgraph** whose edges admit an ordering possessing the following property: Each edge has one **vertex** in common with the preceding **edge** and the other vertex in common with the succeeding edge. The words "preceding" and "succeeding" are defined with respect to the ordering imposed on the edges.

EDGE TRAIN. An **edge sequence** in which each **edge** has **multiplicity** one.

EDGE TRAIN, CLOSED. An **edge train** whose **terminal** vertices coincide.

EDGE TRAIN, OPEN. An **edge train** whose **terminal** vertices do not coincide.

EDGEWORTH SERIES. A form of expansion of a frequency function in terms of derivatives of the **normal distribution**. For a distribution in standard form (i.e., with zero mean and unit variance) the expansion is

$$f(x) = \left[\exp \left\{ \sum_{j=3}^{\infty} \kappa_j \frac{(-D)^j}{j!} \right\} \right] \alpha(x),$$

where κ_j is the j^{th} **cumulant**, D is the operator $\frac{d}{dx}$ and $\alpha(x)$ is the normal function $e^{-\frac{1}{2}x^2}/\sqrt{(2\pi)}$.

(See also **Gram-Charlier series**.)

EFFECTIVE ANGULAR VELOCITY, EFFECTIVE FORCE, ETC. See entries under **angular velocity**, **effective**, etc.

EFFECTIVE AREA (ANTENNA). The square of the wavelength times the **power gain** (or **directive gain**) in a specified direction, and divided by 4π . When power gain is used, the effective area is that for power reception; when directive gain is used, the effective area is that for **directivity**.

EFFECTIVE BANDWIDTH. See **bandwidth**, **effective**.

EFFECTIVE EXHAUST VELOCITY. See **exhaust velocity**, **effective**.

EFFECTIVE HEIGHT (ANTENNA). In its present usage, this term means the height of the antenna center of radiation above the effective ground level. For an antenna with a symmetrical current distribution, the center of radiation is the center of the distribution. For an antenna with an asymmetrical current distribution, the center of radiation is the

center of current moments when viewed from directions near the direction of maximum radiation.

EFFECTIVE MASS. A parameter of the dimensions of a mass which is often used in the band theory of solids. An exact definition covering all its uses is impossible, but one may say that electrons at the bottom of a band, and holes near the top of a band, behave in many respects as if they were free particles with masses rather different from the mass of a free electron.

EFFECTIVE MULTIPLICATION. See **multiplication**, **effective**.

EFFECTIVE PHASE RETARDATION. See **partial coherence**.

EFFECTIVE POWER OF A LENS. The reciprocal of the **back foal length**.

EFFECTIVE PRINCIPAL QUANTUM NUMBER. The quantity

$$n^* = n + \mu$$

in the **Rydberg equation** for the line series in atomic spectra.

EFFECTIVE RADIUS OF CONTROL ROD. See **radius**, **effective**, of **control rod**.

EFFECTIVE VALUE. A term used in electricity, mechanics, etc., to denote the root-mean square value of a periodically-variable (usually sinusoidal) quantity.

EFFECTIVE WAVELENGTH. The wavelength of a monochromatic beam having the same penetration in a given medium as a beam of complex radiation.

EFFECTIVE WIDTH. See **width**, **effective**.

EFFICIENCY. (1) A ratio used to assess the performance of a machine or a prime mover in relation to its idealized version. (2) The ratio of some desired output quantity, such as work, to the (usually larger) equivalent which must be supplied to a device or prime mover, for example, heat. (3) The ratio of energy or work input to output in a transformation process. Thus, in a power cycle, the ratio of work done, W , to the heat required Q . Or

$$\eta = \frac{W}{Q}$$

Since some heat, say Q_o , must always be rejected, the efficiency is also

$$\eta = 1 - \frac{Q_o}{Q}$$

because by the first law of thermodynamics $W = Q - Q_o$ for a cycle. Efficiencies are so defined as to give a number smaller than unity for real engines. (4) In the theory of statistical estimation, the efficiency of an estimator is measured by its **dispersion** about the parameter under estimate or by its **variance**; the smaller the dispersion the more efficient the estimator. If there exists an optimum estimator, i.e., one with minimum dispersion, the efficiency of any other estimator is measured by the inverse ratio of their variances.

Measures of efficiency of an experimental design can similarly be set up in terms of the smallness of the experimental error.

EFFICIENCY, AVAILABLE POWER. Of an electroacoustic transducer used for sound reception, the ratio of the electric power available at the electric terminals of the **transducer** to the **acoustic power** available to the transducer. For an electroacoustic transducer which obeys the **reciprocity principle**, the available power efficiency in sound reception is equal to the transmitting efficiency. In a given narrow-frequency band, the available power efficiency is numerically equal to the fraction of the open-circuit mean-square thermal noise voltage present at the electric terminals which is contributed by thermal noise in the acoustic medium.

EFFICIENCY, CONVERSION. In vacuum tube **amplifier** and **oscillator** circuits, the a-c (usually radio frequency) power output divided by the d-c power input to the plate circuit. It is also called the plate efficiency. It varies quite widely with different classes of amplifiers, running as high as about 80% in some class C amplifiers, from 30 to 70% (depending upon conditions) in class B and being of the order of 20% in the usual class A amplifier.

EFFICIENCY, FROUDE. See **Froude momentum theory**.

EFFICIENCY, INDICATED. See **indicated efficiency**.

EFFICIENCY, LUMINOUS. The ratio of the luminous flux to the radiant flux.

EFFICIENCY OF A DIFFUSER. See **diffuser**.

EFFICIENCY OF ROCKET ENGINE. The efficiency of a rocket motor may be expressed as the product of a number of factors. One relationship between these factors is the following:

$$\eta = \eta_d \eta_F \eta_v$$

where η is the (overall) efficiency of the motor, η_d is the discharge correction factor, η_F is the thrust correction factor and η_v is the velocity correction factor. These factors are expressed, in turn, by the following relationships:

$$\eta_d = (dm/dt)/(dm/dt)_{th}$$

$$\eta_F = F/F_{th} = \eta_v/\eta_d$$

$$\eta_v = c/c_{th}$$

where (dm/dt) is the mass flow rate, F is the thrust, c is the effective exhaust velocity, and subscript *th* denotes the theoretical values. The ideal efficiency of a rocket motor is found from the relationship

$$\eta_i = 1 - (p_e/p_c)^{\frac{\gamma-1}{\gamma}} = 1 - T_e/T_c$$

where η_i is the ideal efficiency, p_e is the exhaust pressure, p_c is the chamber pressure, γ is the ratio of specific heats, T_e is the exhaust temperature and T_c is the chamber temperature. The efficiency of any jet engine can be regarded as the product of its thermal efficiency, η_T , its mechanical efficiency, η_M , and its propulsive efficiency, η_P , provided that these terms are defined as follows: thermal efficiency (η_T) = energy input to nozzle / energy in fuel; mechanical efficiency (η_M) = energy available for propulsion / energy input to nozzle; propulsive efficiency (η_P) = propulsive work / energy available for propulsion.

EFFICIENCY, QUANTUM. See **quantum efficiency**.

EFFICIENCY RATIO. Ratios of indicated work W_i to ideal work W . The ideal work is calculated on the basis of an idealized diagram, for example, an air-standard cycle in the case of internal combustion engines.

$$\eta_r = W_i/W.$$

It is equal to the so-called **diagram factor**. It is also equal to the ratio of indicated thermal efficiency, η_i , to the thermal efficiency of the ideal cycle, η_{th} :

$$\eta_r = \eta_i / \eta_{th}$$

It is sometimes also called the relative efficiency.

EHRENFEST ADIABATIC LAW. See **adiabatic law for quantized states (Ehrenfest)**.

EHRENFEST RELATIONS. The Ehrenfest relations describe the influence of pressure on a lambda-point considered as a second order transition (see **transitions of higher order**).

Using the superscripts $-$ and $+$ to denote the value of quantities immediately below and immediately above the temperature T of the λ -point, the Ehrenfest relations are

$$\frac{dT_\lambda}{dp} = \frac{(\alpha^- - \alpha^+)V_\lambda T_\lambda}{C_p^- - C_p^+} \quad (1)$$

$$\frac{dT_\lambda}{dp} = \frac{\kappa^- - \kappa^+}{\alpha^- - \alpha^+} \quad (2)$$

where C_p is the molar heat capacity, α , the thermal expansion coefficient and κ , the compressibility coefficient.

EHRENFEST THEOREM. See **adiabatic law for quantized states (Ehrenfest)**.

EIGENFUNCTION. If a differential or integral equation possesses solutions satisfying the given boundary conditions for only certain values of a parameter λ , such a value of λ is an **eigenvalue** and the corresponding solution is the eigenfunction belonging to the eigenvalue. If the set of all eigenfunctions of a linear operator are non-degenerate they will form an **orthogonal** set which is **normalizable**. No specific case has been found where the set is not also complete.

(2) As used in quantum mechanics, the eigenfunctions of a quantum mechanical operator (see **operators, quantum mechanical**).

(3) Often used as a synonym for **energy eigenfunction**.

EIGENFUNCTIONS, ELECTRONIC OF MOLECULES. See **molecular eigenfunctions**.

EIGENFUNCTIONS, MOLECULAR. See **molecular eigenfunctions**.

EIGENFUNCTIONS, MOLECULAR, SYMMETRY PROPERTIES OF. See **symmetry properties of molecular eigenfunctions**.

EIGENFUNCTIONS, ROTATIONAL. See **molecular eigenfunctions**.

EIGENFUNCTIONS, VIBRATIONAL. See **molecular eigenfunctions**.

EIGENSTATE. In quantum mechanics, measurable quantities correspond to linear operators. An eigenstate of a linear operator A is an **eigenvector** of A . The corresponding **eigenvalue** gives the result of a measurement, if the quantity A is measured in the eigenstate.

EIGENVALUE (PROPER VALUE). The concept described by this hybrid word has become extremely important in pure and applied mathematics, and in engineering, physics and chemistry. The German word "*eigen*" means "characteristic," a term already overburdened in mathematical English. The "characteristic values" of a physical system are numbers which describe, for example, the critical frequencies of a suspension bridge or of a rotating shaft, the critical load of a supporting column, or the energy levels of a system in quantum mechanics. In corresponding mathematical language, we shall define the eigenvalues of a square **matrix**, of the kernel of an **integral equation**, of a **differential equation** with **boundary conditions**, and of an **operator** or **transformation**, the last case including all the others.

If $\mathbf{A} = \{a_{ik}\}$ is a square matrix, then the number λ is an eigenvalue (also called a characteristic number, a proper value, a latent root, etc.) of the matrix \mathbf{A} if the **determinant**

$$\begin{vmatrix} a_{11} - \lambda & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} - \lambda & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \cdots & a_{nn} - \lambda \end{vmatrix}$$

is equal to zero. In other words, an eigenvalue of a matrix is a root of the **characteristic equation** of the matrix.

As a physical example, consider a suspension bridge which is vibrating with n degrees of freedom about a position of stable equilibrium. Its potential energy at any time will be a function, call it $A(q_1, q_2, \dots, q_n)$, of its n position coordinates q_1, q_2, \dots, q_n , which is approxi-

mated by the quadratic form $\Sigma a_{ik}q_iq_k$, the constants a_{ik} being given by $a_{ik} = \frac{1}{2}(\partial^2 A / \partial q_i \partial q_k)$ evaluated at the position of equilibrium. Then, the eigenvalues of the matrix $\{a_{ik}\}$ are the squares of the critical frequencies of the bridge, namely the frequencies at which an impressed force will cause dangerous resonance.

From the theory of linear equations it follows that this definition of the eigenvalues of a matrix can be restated as follows. The constant λ is an eigenvalue of the matrix $\{a_{ik}\}$ if the n homogeneous linear equations in the n unknowns q_1, q_2, \dots, q_n

$$\begin{aligned} a_{11}q_1 + a_{12}q_2 + \dots + a_{1n}q_n &= \lambda q_1 \\ a_{21}q_1 + a_{22}q_2 + \dots + a_{2n}q_n &= \lambda q_2 \\ \vdots & \\ a_{n1}q_1 + a_{n2}q_2 + \dots + a_{nn}q_n &= \lambda q_n \end{aligned}$$

have a non-trivial solution (i.e., one for which not all the q_i vanish). In other words, regarding the matrix as a linear operator in n -dimensional Euclidean space, the constant λ is an eigenvalue of the operator A which transforms a vector $\mathbf{q} = (q_1, q_2, \dots, q_n)$ into a vector denoted by $A\mathbf{q}$, if there exists a non-zero vector $\mathbf{q} = (q_1, q_2, \dots, q_n)$ such that $A\mathbf{q} = \lambda\mathbf{q}$. Such a non-zero vector is called an *eigenvector* (of the operator A) belonging to the eigenvalue λ .

Similarly, let $A(x,y)$ be a continuous function of two variables (the infinite-dimensional analogue of a matrix with rows and columns) defined for $a \leq x, y \leq b$, and let A be the integral operator which transforms a function $\phi(x)$ into a function $f(x)$ according to the formula $f(x) =$

$\iint A(x,y)\phi(y)dy$, where the integration is taken over the square $a \leq x, y \leq b$ and the functions $\phi(x)$ and $f(x) = A\phi(x)$ are infinite-dimensional analogues (in **Hilbert space**) of the above vectors \mathbf{q} and $A\mathbf{q}$. Then the constant λ is defined to be an eigenvalue of the integral operator A if the integral equation

$$\iint A(x,y)\phi(y)dy = \lambda\phi(x),$$

which we may also write in the form $A\phi = \lambda\phi$, has a non-trivial solution $\phi(x)$. The solution $\phi(x)$ is called an eigensolution, or **eigenfunction** belonging to λ .

The type of eigenvalue problem occurring most frequently in practical applications involves a **differential operator**. For example, the critical frequencies of a vibrating plate (im-

portant in the theory of microphones and elsewhere) are the squares of the eigenvalues of the biharmonic operator discussed below.

In general, let A be any differential operator acting on functions ϕ of any number of variables; e.g., for the clamped plate, $\phi = \phi(x,y)$ and A is the biharmonic operator defined by the formula

$$A\phi = \partial^4\phi/\partial x^4 + 2\partial^4\phi/\partial x^2\partial y^2 + \partial^4\phi/\partial y^4.$$

Also, let us consider as admissible only those functions ϕ which satisfy certain boundary conditions; e.g., for the clamped plate, ϕ and its normal derivative must vanish on the boundary. Then the constant λ is an eigenvalue of the operator A if there exists an admissible function ϕ , called an eigenfunction of A , such that $A\phi = \lambda\phi$. Differential eigenvalue problems of this sort are of fundamental importance in the theory of vibrations and buckling and in quantum mechanics.

EIGENVALUE EQUATION. An equation of the form $P\psi = \lambda\psi$ where P is a linear operator and λ is a constant. ψ is called the eigenfunction of the operator P corresponding to the **eigenvalue**, λ .

EIGENVALUE EQUATION OF TENSOR. For a covariant second-order **tensor** t_{ij} , the equation $|t_{ij} - \lambda g_{ij}| = 0$. For a contravariant second-order tensor t^{ij} , the equation $|t^{ij} - \lambda g^{ij}| = 0$. For a mixed second-order tensor t_j^i , the equation $|t_j^i - \lambda \delta_j^i| = 0$. g_{ij} and g^{ij} are the covariant and contravariant **metric tensors** respectively and δ_j^i is the **Kronecker delta**. For a second-order Cartesian tensor t_{ij} , the equation $|t_{ij} - \lambda \delta_{ij}| = 0$, where δ_{ij} is the Kronecker delta. The solutions for λ of the eigenvalue equation are called the **eigenvalues**, *characteristic values*, or *principal values of the tensor*.

EIGENVALUES AND EIGENVECTORS OF MATRICES, METHODS OF COMPUTING.

For a given square matrix \mathbf{A} (singular or not), non-null vectors \mathbf{x} exist satisfying $\mathbf{A}\mathbf{x} = \lambda\mathbf{x}$ only when the scalar λ satisfies $\det(\lambda\mathbf{I} - \mathbf{A}) = \phi(\lambda) = 0$, called the characteristic equation (cf., the entry **eigenvalue**) where $\phi(\lambda)$ is a polynomial of degree n , the order of \mathbf{A} , called the characteristic polynomial (see **matrix**). Any root λ of this equation is called an eigenvalue; associated with any root λ there is at least one non-null vector \mathbf{x} called eigenvector belonging to λ . The number of independent

eigenvectors belonging to an eigenvalue may equal, but cannot exceed, the multiplicity of λ as a root of the characteristic equation. For normal matrices, including **Hermitian**, the number equals the multiplicity. For a non-normal matrix, if λ is a root of multiplicity $\nu > 1$, and if fewer than ν independent vectors belong to λ , then there are principal vectors $\mathbf{y} \neq 0$ satisfying $(\lambda \mathbf{I} - \mathbf{A})^\mu \mathbf{y} = 0$ for $1 < \mu \leq \nu$. It is always true that $\phi(\mathbf{A}) = 0$. If there is a polynomial $\psi(\lambda)$ of degree $m < n$ for which $\psi(\mathbf{A}) = 0$, then that polynomial of lowest degree for which this is true is a minimal polynomial, and \mathbf{A} is said to be derogatory; otherwise nonderogatory. The condition for being derogatory is rather stringent and not often satisfied in practice, and the same is true for the presence of principal vectors, but when the conditions are nearly satisfied the corresponding eigenvectors are poorly defined and computational difficulties arise, possibly even insurmountable.

Implicitly or explicitly, to evaluate an eigenvalue requires the solution of an algebraic equation of degree n . The problem is much simpler for a **Hermitian matrix**, and for these is discussed under that heading. Some of these methods can also be adapted to normal matrices. Here the general case is considered. A method may purport to yield only the characteristic, or possibly the minimal, polynomial, leaving this to be solved by any of the standard methods for solving algebraic equations. Such a method is called direct. Once an eigenvalue is known, an eigenvector belonging to it can be found by solving a system of homogeneous equations (see **matrix inversion**). A direct expansion of the determinant is unthinkable when n is at all large.

Since any $n + 1$ vectors in n -space are linearly dependent, if $\mathbf{b} = \mathbf{b}_1 \neq 0$, then in the sequence

$$\mathbf{b}_{i+1} = \mathbf{A}\mathbf{b}_i$$

there is a smallest index $m \leq n$ such that \mathbf{b}_{m+1} is linearly dependent upon those preceding:

$$\mathbf{b}_{m+1} + \beta_1 \mathbf{b}_m + \beta_2 \mathbf{b}_{m-1} + \dots + \beta_m \mathbf{b}_1 = 0. \tag{1}$$

This represents n equations in $m \leq n$ unknowns, and they are consistent by hypothesis, even if $m < n$. But this is

$$(\mathbf{A}^m + \beta_1 \mathbf{A}^{m-1} + \dots + \beta_m \mathbf{I})\mathbf{b} = 0. \tag{2}$$

Hence, if

$$\psi(\lambda) \equiv \lambda^m + \beta_1 \lambda^{m-1} + \dots + \beta_m, \tag{3}$$

then $\psi(\mathbf{A})\mathbf{b} = 0$ and $\psi(\lambda)$ is either a minimal polynomial or a factor of it, and except for very special choices of \mathbf{b} , $\psi(\lambda)$ is a minimal polynomial.

If one forms the matrix

$$\mathbf{B} = (\mathbf{b}_1, \mathbf{b}_2, \dots, \mathbf{b}_m) \tag{4}$$

whose columns are the \mathbf{b}_i , then

$$\mathbf{A}\mathbf{B} = \mathbf{B}\mathbf{F} \tag{5}$$

where \mathbf{F} is the companion matrix whose form is

$$\mathbf{F} = \begin{pmatrix} 0 & 0 & \dots & 0 & -\beta_m \\ 1 & 0 & \dots & 0 & -\beta_{m-1} \\ & & \dots & & \\ & & & & \\ 0 & 0 & \dots & 1 & -\beta_1 \end{pmatrix}. \tag{6}$$

Its characteristic polynomial is $\psi(\lambda)$. With $\mathbf{b}_1 = \mathbf{c}_1$, the first column of \mathbf{I} , this is the method of *Krylov*.

This method is effective for matrices of fairly low order. For matrices of high order the equations to be solved for the β_i tend to become ill-conditioned. A method due to *Hessenberg* yields a matrix \mathbf{B} such that

$$\mathbf{A}\mathbf{B} = \mathbf{B}\mathbf{G}, \tag{7}$$

where \mathbf{G} is not the companion matrix, but whose form is such that the characteristic polynomial can be expanded directly, and from which it is rather easy to solve for the eigenvectors. To apply this method one selects auxiliary vectors $\mathbf{c}_1, \mathbf{c}_2, \mathbf{c}_3, \dots$ subject to two mild restrictions but otherwise arbitrary. Ordinarily one can take (with *Hessenberg*) each $\mathbf{c}_i = \mathbf{e}_i$, a column of \mathbf{I} . One starts again with a vector \mathbf{b} , possibly \mathbf{c}_1 . Now \mathbf{b}_2 is a linear combination of \mathbf{b}_1 and $\mathbf{A}\mathbf{b}_1$ orthogonal to \mathbf{c}_1 ; \mathbf{b}_3 a linear combination of $\mathbf{b}_1, \mathbf{b}_2$, and $\mathbf{A}\mathbf{b}_2$ orthogonal to both \mathbf{c}_1 and \mathbf{c}_2 ; \dots . The restrictions on the \mathbf{c}_i are, first, linear independence, and, second,

$$\mathbf{c}_i^* \mathbf{b}_i \neq 0.$$

With these conditions fulfilled one forms

$$\mathbf{A}\mathbf{b}_1 = \mathbf{b}_1 \gamma_{11} + \mathbf{b}_2$$

where

$$\mathbf{c}_1^* \mathbf{A}\mathbf{b}_1 = \mathbf{c}_1^* \mathbf{b}_1 \gamma_{11}.$$

Hence γ_{11} is obtained, and therefore \mathbf{b}_2 . Next

$$\mathbf{A}\mathbf{b}_2 = \mathbf{b}_1 \gamma_{12} + \mathbf{b}_2 \gamma_{22} + \mathbf{b}_3,$$

where

$$\begin{aligned} \mathbf{c}_1^* \mathbf{A} \mathbf{b}_2 &= \mathbf{c}_1^* \mathbf{b}_1 \gamma_{12}, \\ \mathbf{c}_2^* \mathbf{A} \mathbf{b}_2 &= \mathbf{c}_2^* \mathbf{b}_1 \gamma_{12} + \mathbf{c}_2^* \mathbf{b}_2 \gamma_{22}. \end{aligned}$$

From the first of these one obtains γ_{12} , from the second γ_{22} , and finally \mathbf{b}_3 . The process continues, and \mathbf{G} is seen to have the form

$$\mathbf{G} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \cdots \\ 1 & \gamma_{22} & \gamma_{23} & \cdots \\ 0 & 1 & \gamma_{33} & \cdots \\ & & \dots & \dots \end{pmatrix}. \tag{8}$$

It may be observed that when the choice $\mathbf{c}_i = \mathbf{c}_i$ is made, then the first element of \mathbf{b}_2 is null as are the first two elements of \mathbf{b}_3 , the first three of \mathbf{b}_4 , \dots .

A further refinement of the method is the biorthogonalization method of *Lanczos*. In this method only \mathbf{b}_1 and \mathbf{c}_1 are arbitrary. Thereafter, just as \mathbf{b}_2 is a linear combination of \mathbf{b}_1 and $\mathbf{A}\mathbf{b}_1$ orthogonal to \mathbf{c}_1 , so is \mathbf{c}_2 a linear combination of \mathbf{c}_1 and $\mathbf{A}^*\mathbf{c}_1$ orthogonal to \mathbf{b}_1 , and so for the others. The matrix \mathbf{G} is then tridiagonal, being null everywhere except along, just below, and just above the diagonal. The recursion then never contains more than three terms in any equation.

Another modification is due to *Arnoldi*, who chooses $\mathbf{b}_i = \mathbf{c}_i$ and requires \mathbf{B} to be an orthogonal matrix. Then (7) is satisfied, but with a matrix \mathbf{G} of the form

$$\mathbf{G} = \begin{pmatrix} \gamma_{11} & \gamma_{12} & \gamma_{13} & \cdots \\ \gamma_{21} & \gamma_{22} & \gamma_{23} & \cdots \\ 0 & \gamma_{32} & \gamma_{33} & \cdots \\ & & \dots & \dots \end{pmatrix} \tag{9}$$

similar to (8) but with elements other than ones on the subdiagonal. The development of the characteristic polynomial from \mathbf{G} is almost equally straightforward.

When only one, or few, of the eigenvalues of largest modulus are required a simple iteration scheme is advisable, and is preferred by some even for a complete reduction of rather large matrices. If there is a single proper value exceeding all others in modulus, λ_1 , vectors \mathbf{b}_ν in the continued sequence

$$\mathbf{b}_{\nu+1} = \mathbf{A} \mathbf{b}_\nu$$

approach the eigenvector belonging to λ_1 , so that for large ν ,

$$\mathbf{b}_{\nu+1} \doteq \lambda_1 \mathbf{b}_\nu.$$

Hence, with any vector \mathbf{u} , an approximate value of λ_1 is given by

$$\mathbf{u}^* \mathbf{A} \mathbf{b}_\nu = \lambda_1 \mathbf{u}^* \mathbf{b}_\nu.$$

Convergence, when it occurs, can be accelerated by application of the **delta-square process** termwise to the vectors in the sequence.

In the case of a pair of complex roots, convergence does not occur, but in the limit \mathbf{b}_ν becomes parallel to the plane of the two eigenvectors. Then if

$$\mu_\nu = \mathbf{u}^* \mathbf{b}_\nu$$

the roots of the equation

$$\begin{vmatrix} 1 & \mu_\nu & \mu_{\nu+1} \\ \lambda & \mu_{\nu+1} & \mu_{\nu+2} \\ \lambda^2 & \mu_{\nu+2} & \mu_{\nu+3} \end{vmatrix} = 0$$

approach these roots λ_1 and λ_2 as ν increases. In fact, whether or not λ_1 and λ_2 are equal in modulus, but provided only both exceed all others in modulus, the statement is true. Moreover, the roots of

$$\begin{vmatrix} 1 & \mu_\nu & \mu_{\nu+1} & \mu_{\nu+2} \\ \lambda & \mu_{\nu+1} & \mu_{\nu+2} & \mu_{\nu+3} \\ \lambda^2 & \mu_{\nu+2} & \mu_{\nu+3} & \mu_{\nu+4} \\ \lambda^3 & \mu_{\nu+3} & \mu_{\nu+4} & \mu_{\nu+5} \end{vmatrix} = 0$$

approach the three proper values of largest modulus, if such exist (cf. **Bernoulli method**).

Returning to the two roots, for large ν ,

$$\mathbf{b}_\nu \doteq \mathbf{v}_1 + \mathbf{v}_2,$$

$$\mathbf{b}_{\nu+1} \doteq \lambda_1 \mathbf{v}_1 + \lambda_2 \mathbf{v}_2,$$

approximately, where \mathbf{v}_1 and \mathbf{v}_2 are eigenvectors belonging to λ_1 and λ_2 . Knowing λ_1 and λ_2 , as well as \mathbf{b}_ν and $\mathbf{b}_{\nu+1}$, these equations can be solved for \mathbf{v}_1 and \mathbf{v}_2 .

When any eigenvalue and eigenvector are known it is possible to apply *deflation* as follows: The matrix

$$\mathbf{A} - \lambda_1 \mathbf{v}_1 \mathbf{u}^*,$$

where \mathbf{u} is any vector satisfying $\mathbf{u}^* \mathbf{v}_1 = 1$ has the same eigenvalues as \mathbf{A} except that instead of λ_1 the new matrix has a zero. Moreover, if \mathbf{w} is an eigenvector of $\mathbf{A} - \lambda_1 \mathbf{v}_1 \mathbf{u}^*$, belonging to the eigenvalue λ , then $(\mathbf{A} - \lambda_1 \mathbf{I}) \mathbf{w}$ is a proper vector of \mathbf{A} belonging to the eigenvalue λ . And finally, it is possible to choose \mathbf{u}^* so that every element but the first in the first row

of $\mathbf{A} - \lambda_1 \mathbf{v}_1 \mathbf{u}^*$ is zero. To do this, normalize \mathbf{v}_1 to have 1 as the first element (should this be zero, permute rows and corresponding columns of \mathbf{A} , and the same elements of \mathbf{v}_1 to make the first element non-null). Choose the last $n - 1$ elements of \mathbf{u}^* to be $1/\lambda_1$ times the corresponding elements of the first row of \mathbf{A} , and choose the first element of \mathbf{u}^* to satisfy $\mathbf{u}^* \mathbf{v}_1 = 1$. It remains now to consider only a matrix of order $n - 1$, and further reductions are made as further eigenvalues and eigenvectors are found.

In case μ is an approximation to a particular eigenvalue λ_i , closer than to any other λ_j , then $\mu - \lambda_i$ is the numerically largest eigenvalue of $(\mathbf{A} - \mu \mathbf{I})^{-1}$. Hence the solution of the system

$$(\mathbf{A} - \mu \mathbf{I}) \mathbf{x}_1 = \mathbf{x}_0$$

for an almost arbitrary \mathbf{x}_0 will give a good approximation to the eigenvector belonging to λ_i , and $\mathbf{x}_1^* \mathbf{A} \mathbf{x}_1 / \mathbf{x}_1^* \mathbf{x}_1$ will be a better approximation to λ_i . This is *Wielandt's broken iteration*.

Unless it is known a priori that a matrix has **principal vectors** that are not eigenvectors, and has auxiliary conditions available for determining them, rounding errors will almost inevitably conceal them, but the problem may turn out to be highly unstable. If $\mathbf{A} = \mathbf{P} \mathbf{\Lambda} \mathbf{P}^{-1}$ where $\mathbf{\Lambda}$ is diagonal, the **numerical stability** decreases as $\|\mathbf{P}\| \|\mathbf{P}^{-1}\|$ increases in some **norm**, hence this seems to be an appropriate **condition number** with respect to this problem. But as \mathbf{A} approaches the nondiagonalizable form, this number becomes infinite, hence at least some of the eigenvalues and eigenvectors become ill defined.

EIGENVECTOR. See **eigenvalue**.

EIKONAL. H. Brun's term for the **eigenfunction** of an optical system. Bruns in 1895 developed a **Hamiltonsion theory** for geometric optics. At the time continental Europe was well aware, through Jacobi, of Hamilton's contributions to classical mechanics but was apparently unaware that Hamilton first developed these methods in optical theory. The alternative spelling *ikonal* is now sometimes used.

EIKONAL EQUATION. The differential equation of geometrical optics

$$V_x^2 + V_y^2 + V_z^2 = n^2$$

is referred to either as the eikonal equation, or as the characteristic equation. The solutions are the eikonal or Hamilton's characteristic of the optical system with refractive index $n = n(x, y, z)$.

EINSTEIN COEFFICIENTS. Discussed under **transition probabilities**, **Einstein**.

EINSTEIN CONDENSATION. A perfect gas obeying **Bose-Einstein statistics** will show at low temperatures a third order phase transition which in many respects resembles the condensation of an imperfect gas. The condensed phase consists of particles in their ground state. This phenomenon has been studied extensively in connection with a possible relation to the **lambda-transition** of liquid helium.

EINSTEIN DIFFUSION EQUATION. An equation for the mean square displacement of spherical colloidal particles in a gas or liquid, due to Brownian movement. The mean square displacement from its original position after a time τ is

$$\overline{x^2} = \frac{RT}{3\pi\eta r N} \tau,$$

where R is the gas constant, T is the absolute temperature, r is the radius of the particle, η is the viscosity, N is Avogadro's number. This relationship is only valid for particles of such size that they obey Stokes' resistance law.

EINSTEIN EFFECTS. See **relativity**, **general**.

EINSTEIN EQUATION FOR HEAT CAPACITY. See **Einstein specific heat function**.

EINSTEIN MASS-ENERGY RELATIONSHIP. See **mass-energy relationship**.

EINSTEIN PHOTOELECTRIC EQUATION. An equation giving the kinetic energy of an electron ejected from a system in the **photoelectric effect**, where the electron is ejected by an incident photon, absorbing all the energy of the latter. This equation is:

$$E_k = h\nu - \omega$$

where E_k is the kinetic energy of the ejected electron, h is the Planck constant, ν is the frequency associated with the absorbed photon, and ω is the energy necessary to remove the electron from the system.

EINSTEIN RELATIONSHIP BETWEEN MOBILITY AND DIFFUSION COEFFICIENT. The mobility μ of charges in a semiconductor, or ionic solution is related to the diffusion coefficient D_e by

$$\mu = eD_e/kT$$

where e is the magnitude of the charge, k is the Boltzmann constant, and T is the absolute temperature.

EINSTEIN SPECIFIC HEAT FUNCTION. The heat capacity of an assembly of N simple harmonic oscillators, all having the same frequency is given by

$$c_v = Nk(h\nu/kT)^2 e^{h\nu/kT} / (e^{h\nu/kT} - 1)^2,$$

where h and k are the Planck and Boltzmann constants. This type of function represents the contribution of the optical modes to the specific heat of a solid, which is otherwise more accurately given by a Debye heat capacity equation. By replacing Nk in the equation by $3R$, where R is the gas constant, its value is C_v , the heat capacity at constant volume of one gram-atomic weight of an element.

EINSTEIN TRANSITION PROBABILITY. See transition probability, Einstein.

EISNER-LACHS THEOREM. The theorem, proved by Eisner and Lachs in 1947, stating that in a nuclear reaction produced with an unpolarized beam of given orbital angular momentum incident on an unpolarized target, the angular distribution of the outgoing intensity cannot be more complicated than that of the incoming intensity.

EKMAN LAYER. (Sometimes called spiral layer.) The layer of transition between the surface boundary layer, where the shearing stress is constant, and the free atmosphere, where the atmosphere is treated as an ideal gas in approximate geostrophic equilibrium. In Ekman's analysis, the coefficient of eddy viscosity is assumed constant within this layer; subsequent calculations have relaxed this assumption.

EKMAN SPIRAL. The equiangular spiral of 45° traced out by a point whose position vector is the steady horizontal ocean current vector (with varying depth) produced by a steady surface stress due to the wind. The water at great depth is at rest and is represented by the

center of the spiral. The surface water moves in a direction at 45° to the wind (to the right in the northern hemisphere). The total momentum of any column of ocean is at right angles to the wind, and the total momentum below any depth is at right angles to the direction of shear at that depth.

ELASTANCE. The reciprocal of capacitance, measured in darafs.

ELASTICA. A plane curve into which the central line of a thin, initially cylindrical, elastic rod can be bent by means of forces and couples applied to its ends only.

ELASTIC AFTER EFFECT. See after effect, elastic.

ELASTICA, INFLEXIONAL. An elastica on which there are one or more points of inflexion.

ELASTICA, NON-INFLEXIONAL. An elastica on which there are no points of inflexion.

ELASTIC AXIS. The line joining the shear centers of the cross sections of an elastic beam.

ELASTIC CENTER. The point on the rigid extension from one end of, or a cut through, an elastic arch or frame which will not translate when a moment is applied there. Correspondingly a force applied at the elastic center will not rotate the rigid extension. It is, therefore, also the centroid in the plane of the M_s/EI diagram erected perpendicular to the plane of the structure, where M_s is the moment due to the applied loads with zero force and moment at the elastic center.

ELASTIC COEFFICIENTS, LATTICE THEORY OF. The elastic constants and elastic moduli of crystals may be calculated on the assumption that the only forces are those between near neighbors in the lattice. Such a calculation gives reasonable results for ionic crystals, but is quite unsatisfactory for metals, where the Cauchy relations are not obeyed. The free electron gas in a metal is not easily compressed but scarcely opposes shear.

ELASTIC CONSTANTS. Physical constants occurring in the strain-energy function or stress-deformation relation for a homogeneous, elastic material, which characterize the elastic behavior for the material, e.g., the Lamé con-

stants in the case when the material is isotropic and obeys the **generalized Hooke's law**.

Two independent elastic constants characterize the small strain behavior of a linearly elastic isotropic material having the same properties in tension and compression. Four related elastic constants are in common use. The modulus of elasticity in simple tension or compression given by the ratio of axial stress to strain is called **Young's modulus**. The ratio of transverse strain magnitude to longitudinal strain magnitude is called **Poisson's ratio**. The ratio of shear stress to strain in simple shear is the *shear modulus* or *modulus of rigidity* and the ratio of pressure to unit volume change is the *bulk modulus*.

ELASTIC CURVE. The curve of the neutral surface of a structural member subjected to loads which cause bending is called the elastic curve. The ordinates between this curve and the original position of the neutral surface represent the deflections due to bending.

ELASTIC FOUNDATION. The term elastic foundation generally means linearly elastic, and usually refers to the simplest type in which the local foundation reaction is proportional to the local deflection.

ELASTIC HYSTERESIS CONSTANT. The ratio of the area (expressed in energy units) of the stress-strain loop, for a unit volume of the material, to the square of the maximal strain.

ELASTICITY, DELAYED. Mechanical behavior which is elastic for slow changes, but **anelastic** for changes at finite stress and strain rates. It is represented by the **Kelvin** or **Voigt viscoelastic model**.

ELASTICITY, HIGH. Viscoelastic deformation of polymers.

ELASTICITY, RECIPROCITY THEOREM FOR. See **Maxwell and Betti reciprocity theorem**.

ELASTICITY, RETARDED. See **elasticity, delayed**.

ELASTICITY THEORY. See **classical elasticity theory; finite elasticity theory**.

ELASTIC LIMIT. In simple tension or compression the maximum stress which can be carried without measurable plastic deformation

is called the elastic limit. More generally it is a limiting state of stress σ_{ij} such that $K\sigma_{ij}$, where $K > 1$, will cause plastic deformation. Actually, the more precise the measuring instruments, the lower the stress at which plastic deformation is observed. An offset **yield strength** or a **tangent modulus** limitation gives a repeatable and more useful definition.

ELASTIC LINE. See **elastic curve; elastic axis**.

ELASTIC LOAD METHOD. A conjugate beam or truss procedure in which the loads are M/EI or similar quantities and the forces and moments in the conjugate structure are slopes and deflections.

ELASTIC MATERIAL. A material for which the work done by the applied forces in a quasi-static **deformation** is a single-valued function of the initial and final states of deformation.

ELASTIC WEIGHTS. Although sometimes employed as an alternate term for elastic loads, elastic weights denote ds/EI where s is the distance along the structure.

ELASTOSTATIC. Adjective describing condition in elastic or perfectly elastic material in which neither the **displacement field** nor the **stress field** varies with time. Hence, *elastostatic problem*.

ELECTRIC(AL) CAPACITANCE. See **capacitance**.

ELECTRIC CIRCUITS. Electric charges, at rest and/or in motion, are the fundamental sources of electric and magnetic **fields**. Broadly speaking, there are three more or less distinct situations involving the spatial distribution of the fields produced by distributions of charge and current. Many important instances arise where these fields are distributed throughout a region of space of vast extent and differ accordingly at separated points in the region. On the other hand, important applications are made of devices in which the electric and magnetic fields are confined to a much more limited region of space although the fields still undergo a significant variation in magnitude from point to point throughout the region, as in a transmission line. Here the fields are confined to the immediate vicinity of the line conductors but the fields vary significantly as one moves along the line.

Finally, however, there are applications utilizing pieces of apparatus in which the electric and magnetic fields are confined to regions of space which are so restricted in spatial extent that one may speak of an electric or magnetic field as having an essentially constant value in the immediate region of the device which is very much greater than at all other points of space.

It is found, however, that when one deals with changing currents (charges in motion with varying velocity), the spatial dimension alone is not an adequate measure to establish which of the three situations suggested above is involved in a particular application. For sinusoidally varying currents one can define a quantity known as the wavelength which in free space is numerically equal to the velocity of light divided by the frequency of the sinusoidal variation. If spatial extent is measured in wavelengths, then an assemblage of apparatus which is confined to a region which is less than about one tenth of a wavelength in greatest dimension results in a good approximation to the third situation considered above. This possibility, one in which the electric fields and the magnetic fields may be considered to be concentrated in individual pieces of apparatus, is the domain of electric circuits. An electric circuit may be defined as a characterization of an electrical system in terms of the integrated effects of the electric and magnetic fields present in the system. The characterization is an approximation to the actual field problem in which one replaces the actual system by elements having **resistance**, **capacitance**, and **inductance** and by sources of **electric potential** and **electric current**. Systems in which the approximation cited is permissible are sometimes called "lumped constant" circuits. Antennas and transmission lines, on the other hand, are often referred to as "distributed constant" circuits. The distinction between these two designations comes from the spatial variation of the electric and magnetic fields as outlined above.

As may be inferred from their definitions, the parameters of resistance, capacitance, and inductance are not necessarily independent of the currents and voltages impressed upon the elements of an electrical system. Whether the resistance of an element is a function of the current through it or not, the relation $v = Ri$ (R is resistance in ohms) is still valid for the

connection between the voltage across an element and the current through it. On the other hand, corresponding relations for the coil and the capacitor become more involved if their parameters are dependent on current or voltage. For constant parameters of inductance (L in henrys) and capacitance (C in farads), the voltages and currents in pure inductive and capacitive components assume the form

$$v = L \frac{di}{dt} \quad \text{or} \quad i = \frac{1}{L} \int v dt$$

and

$$i = C \frac{dv}{dt} \quad \text{or} \quad v = \frac{1}{C} \int i dt$$

respectively. If the parameters L and C vary with impressed voltage and current, the above relations are not valid and recourse must be made to the basic law of **electromagnetic induction** when coils are involved and to the expression for electric current as the rate of change of charge (derivative of charge with respect to time) for circuits containing capacitors.

Whether elements with constant or variable parameters are involved, one may employ **Kirchhoff laws of networks** to formulate equations representing conditions of equilibrium between the applied voltages and/or currents and the quantities that result. The equilibrium equations may be formulated on the basis of voltages as independent variables and currents as dependent quantities, the system of equations being known as the mesh equations for the circuit. Alternatively, they may be written on the nodal basis, where the sources are current generators and the dependent quantities are nodal voltage differences with respect to a reference node. As an example of the process involved, consider the simple series circuit shown in Figure 1 and the simple parallel circuit shown in Figure 2 (page 291). The single mesh equation characterizing the series circuit and the single nodal equation describing equilibrium in the parallel circuit are, respectively

$$V(t) = Ri(t) + L \frac{di(t)}{dt} + \frac{1}{C} \int i(t) dt$$

and

$$I(t) = \frac{1}{R} v(t) + \frac{1}{L} \int v(t) dt + C \frac{dv(t)}{dt}$$

The mesh and nodal equations for more complicated circuits have a form similar to these equations with added dependent variable terms and, in general, additional independent

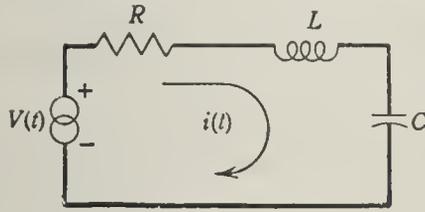


Fig. 1.

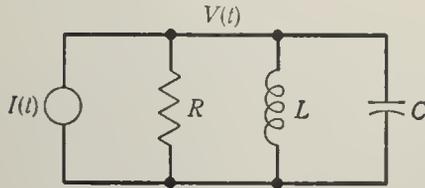


Fig. 2.

variable terms. The equations may be solved by various mathematical means to yield the desired unknown quantities, currents for mesh equations and voltages for nodal equations.

If the applied voltage sources are batteries or alternative sources of electric potential which are constant with time, the solution to the network equations described above will consist of constant terms (including zero as a special case) plus terms which decay exponentially with time. The latter terms are called the transient terms or the transient solution. They correspond to the solution of the homogeneous **differential equations**. The constant terms represent the **steady state** solution of the problem. In the case of **direct currents**, these particular solutions may be obtained in terms of the resistances involved, that is, in the computation of the steady state response for circuits with constant potential applied, inductors may be replaced by elements of zero resistance and capacitors by resistors of infinite resistance. If, on the other hand, the applied sources vary sinusoidally with time, the solution to the network problems which have been considered will have transient terms which decay with time, as was stated above, but the steady state terms will be sinusoidally varying quantities. The steady state solutions will involve all elements, those having inductance and capacitance as well as the resistive elements.

ELECTRIC CONSTANT. (Symbol ϵ_0 or γ_e . The electric constant pertinent to any system

of units in the scalar dimensional factor ϵ_0 appearing in the **Coulomb law of force** between two charges in vacuo:

$$F = q_1 q_2 / n \epsilon_0 r^2$$

where $n = 1$ for unrationalized units, and $n = 4\pi$ for rationalized units.

In the esu system, ϵ_0 is assigned the value unity, with no dimensions. In all systems, $\epsilon_0 \mu_0 = 1/c^2$, where c is the velocity of light in the appropriate system. Note that c is dimensional. (See **magnetic constant**.) In the mksa system, ϵ_0 has the dimensions: farad/meter.

ELECTRIC CURRENT. See **current, electric; alternating currents; direct currents**.

ELECTRIC DIPOLE. See **dipole, electric**.

ELECTRIC(AL) DIPOLE MOMENT. See **dipole, electric**.

ELECTRIC DISPLACEMENT VECTOR. In a polarizable medium, (see **dielectric constant**), the measured **electric field vector** is due both to **free charges** and to the **bound charges** resulting from the polarization of the medium. Since the electric field vector is the fundamental measurable quantity, it is convenient to express it in terms of two auxiliary field quantities associated respectively with the free and bound charges. The field associated with the free charges is called the electric displacement vector, **D**, and satisfies the equation $\nabla \cdot \mathbf{D} = \rho$, where ρ is the free charge density (see **Maxwell equations**). The field associated with the bound charges is the **polarization vector P**. These three vectors are related as follows:

$$\mathbf{D} = \epsilon_0 \mathbf{E} + \mathbf{P},$$

where ϵ_0 is the **electric constant**. The vector **D** is sometimes called the **electric induction**, or the **electric flux density**, and defined by $\mathbf{D} = \epsilon \mathbf{E}$, ϵ being the dielectric constant. (See **constitutive relations**.)

ELECTRIC FIELD. The field of force having its source either in **electric charges**, or in time-varying **magnetic fields**.

ELECTRIC FIELD STRENGTH (INTENSITY). The magnitude of the **electric field vector**. In the mksa system of units it is measured in newtons/coulomb, or equivalently

in volts/meter. For its units in other systems, see **electromagnetic units**. The magnitude of the electric field strength at a distance r from a point of charge Q is, from Coulomb's law, $E = Q/4\pi\epsilon_0 r^2$ and its direction along \mathbf{r} , outward from the charge Q , if Q is positive. The electrostatic field strength due to a number of stationary charges is

$$\mathbf{E} = \sum_i q_i \mathbf{r}_i / 4\pi\epsilon_0 r_i^3$$

where the summation is a vector sum. The field due to stationary charges is a conservative field and the field strength may therefore be represented as the negative gradient of a potential V , $\mathbf{E} = -\text{grad } V = -\nabla V$. An electric field is also produced by a changing magnetic field, and is then given by

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},$$

or alternatively

$$\phi \mathbf{E} \cdot d\mathbf{s} = -\frac{d}{dt} \int \mathbf{B} \cdot d\mathbf{A}.$$

(See Maxwell equations; Faraday law of electromagnetic induction.)

ELECTRIC FIELD VECTOR. The fundamental electric field quantity, defined as the force per unit charge on a positive charge in the limit as the charge approaches zero. This quantity has the dimensions newtons per coulomb, or volts per meter, and is denoted by \mathbf{E} .

ELECTRIC FLUX DENSITY. See **electric displacement vector**.

ELECTRIC(AL) IMPEDANCE. See **impedance; impedance, electrical**.

ELECTRIC(AL) INDUCTION. See **electric displacement vector**.

ELECTRIC INTENSITY. See **electric field strength**.

ELECTRICITY. (1) Under certain conditions physical objects separated in space can exert non-gravitational forces on each other. The source of these forces was originally thought to reside in an excess or deficiency of some substance possessed by all bodies. This substance, called "electricity," was later found to consist of discrete particles (charges) individually capable of "electrical" behavior. (2) A term loosely applied to physical phenomena associated with **electric charges**.

ELECTRIC(AL)-MECHANICAL-ACOUSTICAL RECIPROCITY THEOREM. See **reciprocity theorem, electric(al)-mechanical-acoustical**.

ELECTRIC(AL)-MECHANICAL RECIPROCITY THEOREM. See **reciprocity theorem, electric(al)-mechanical**.

ELECTRIC MULTIPOLES. The potential due to a static charge distribution ρ can be written

$$\phi(\mathbf{R}) = \frac{1}{4\pi\epsilon_0} \int_V \frac{\rho(\mathbf{R}') dV'}{r}$$

where $r = |\mathbf{R} - \mathbf{R}'|$ and the integral extends over the volume of the charge. (See **electrostatics, laws of**.) At distances $R = |\mathbf{R}|$ far from the charge distribution, $1/r$ can be expanded in a **Taylor series** as follows:

$$\begin{aligned} \frac{1}{r} &= \frac{1}{R} + \sum_{j=1}^3 x'_j \left[\frac{\partial}{\partial x'_j} \left(\frac{1}{r} \right) \right]_{r=R} \\ &+ \frac{1}{2!} \sum_{j,k=1}^3 x'_j x'_k \left[\frac{\partial^2}{\partial x'_j \partial x'_k} \left(\frac{1}{r} \right) \right]_{r=R} + \dots \end{aligned}$$

Inserting into the integral yields the multipole expansion,

$$\begin{aligned} \phi(\mathbf{R}) &= \frac{1}{4\pi\epsilon_0} \left\{ \frac{1}{R} \int_V \rho dV' \right. \\ &+ \sum_{j=1}^3 \left[\frac{\partial}{\partial x'_j} \left(\frac{1}{r} \right) \right]_{r=R} \int_V x'_j \rho dV' \\ &+ \frac{1}{2!} \sum_{j,k=1}^3 \left[\frac{\partial^2}{\partial x'_j \partial x'_k} \left(\frac{1}{r} \right) \right]_{r=R} \int_V x'_j x'_k \rho dV' + \dots \end{aligned}$$

The integral coefficients in this expansion represent the moments of the charge distribution: $\int_V \rho dV'$ is the total charge, $\int_V x'_j \rho dV'$ is the j^{th} component of the **dipole moment**, $\int_V x'_j x'_k \rho dV'$ is the jk^{th} component of the **quadrupole moment**, etc.

ELECTRIC NETWORK, LAWS OF. See **Kirchhoff laws; reciprocity theorem, electric network; Thévenin theorem**.

ELECTRIC(AL) NETWORK RECIPROCITY THEOREM. See **reciprocity theorem, electric(al) network**.

ELECTRIC(AL) POTENTIAL. See **potential, electric.**

ELECTRIC(AL) QUANTUM NUMBERS. See **Stark effect.**

ELECTRIC(AL) RESISTANCE. See **resistance.**

ELECTRIC(AL) SYSTEM OF ONE DEGREE OF FREEDOM. An electrical system of one degree of freedom is shown in Figure 1. In one degree of freedom the activity in

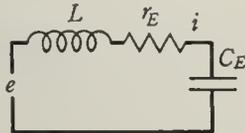


Fig. 1. Electric(al) system of one degree of freedom.

every element of the system may be expressed in terms of one variable. In the electrical system an electromotive force e acts upon an inductance L , an electrical resistance r_E and an electrical capacitance C_E connected in series.

The principle of the conservation of energy states that the total store of energy of all forms remains constant if the system is isolated so that it neither receives nor gives out energy; in the case of transfer of energy the total gain or loss from the system is equal to the loss or gain outside the system. The sum of the kinetic, potential and heat energy during an interval of time is, by the principle of conservation of energy, equal to the energy delivered to the system during that interval. In the electrical system of Fig. 1, there are three forms of energy, namely, kinetic, potential, and heat energy.

In the following derivations, the symbol, ϵ , is used for the base of natural logarithms, to avoid confusion with electromotive force, e .

Kinetic Energy. The kinetic energy T_{KE} stored in the magnetic field of the electrical circuit is

$$T_{KE} = \frac{1}{2} Li^2 \quad (1)$$

where L is the inductance, in abhenries, and i is the current through the inductance L , in abamperes.

Potential Energy. The potential energy V_{PE} stored in the electrical capacitance of the electrical circuit is

$$V_{PE} = \frac{1}{2} \frac{q^2}{C_E} \quad (2)$$

where C_E is the capacitance, in abfarads, and q is the charge on the capacitance, in abcoulombs.

Kinetic and Potential Energy. The total energy stored in the system is

$$W_E = T_{KE} + V_{PE} = \frac{1}{2} Li^2 + \frac{1}{2} \frac{q^2}{C_E} \quad (3)$$

The rate of change of energy in the system is

$$\frac{dW_E}{dt} = Li \frac{di}{dt} + \frac{q\dot{q}}{C_E} = Lq\dot{q} + \frac{q\dot{q}}{C_E} \quad (4)$$

Dissipation of Energy. The rate at which electromagnetic energy D_E is converted into heat is

$$D_E = r_E i^2 \quad (5)$$

where r_A is the electrical resistance, in abohms, and i is the current, in abamperes.

Equation of Motion. The power delivered to the system must be equal to the rate of kinetic energy storage plus the rate of potential energy storage plus the power loss due to dissipation. The rate at which work is done or power delivered to the electrical system by the applied electromotive force is $\dot{q}E\epsilon^{j\omega t} = e\dot{q}$.

The rate of increase of energy ($T_{KE} + V_{PE}$) of the system plus the rate at which work is done on the system or power delivered to the system by the external forces must be equal to the rate of dissipation of energy D_E . Writing this sentence mathematically yields the equation of motion for the electrical system of Figure 1.

$$Lq\dot{q} + r_E \dot{q}^2 + \frac{q\dot{q}}{C_E} = E\epsilon^{j\omega t} \dot{q} \quad (6)$$

$$L\dot{q} + r_E \dot{q} + \frac{q}{C_E} = E\epsilon^{j\omega t} \quad (7)$$

Solution of Equation of Motion. The steady state solution of the differential Equation 7 is

$$\dot{q} = i = \frac{E\epsilon^{j\omega t}}{r_E + j\omega L - \frac{j}{\omega C_E}} = \frac{e}{z_E} \quad (8)$$

Electrical Impedance. From Equation 8 the vector electrical impedance z_E in abohms, is

$$z_E = r_E + j\omega L - \frac{j}{\omega C_E} \quad (9)$$

Electrical Reactance. Electrical reactance x_E is the imaginary part of the electrical imped-

ance z_E , of Equation 9. The electrical reactance x_E , in abohms, is

$$x_E = \omega L - \frac{1}{\omega C_E}. \quad (10)$$

Response. The response of the electrical system of Figure 1 can be obtained from Equation 8. The response is the current i , in abamperes, for a constant applied voltage e , in abvolts. A typical response frequency characteristic is shown in Figure 2.

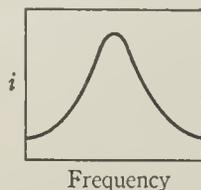


Fig. 2. Response frequency characteristic of the electric(al) system of Fig. 1.

Resonant Frequency. For a certain value of L and C_E , there will be a certain frequency at which the imaginary component of the electrical impedance is zero. This frequency is called the resonant frequency. At this frequency the ratio of the current to the applied voltage is a maximum. At the resonant frequency the current and voltage are in phase. The resonant frequency f_r , in cycles per second, is given by

$$f_r = \frac{1}{2\pi\sqrt{LC_E}}. \quad (11)$$

Power. The power P_E , in ergs per second, dissipated in the electrical system of Figure 1 is given by

$$P_E = r_E i^2. \quad (12)$$

ELECTROACOUSTICAL RECIPROcity THEOREM. See reciprocity theorem, electroacoustical.

ELECTROCHEMICAL CELL. The essential characteristic of an electrochemical cell is that a chemical process involving ions can take place in it in such a manner that the process is accompanied by a transfer of charge from one terminal to the other. (See **electromotive force**.)

ELECTROCHEMICAL POTENTIAL. The expression

$$\mu_i = \mu_i + z_i \mathfrak{F} \phi \quad (1)$$

is called the electrochemical potential. It consists of the ordinary **chemical potential**

μ_i , and the electrical part $z_i \mathfrak{F} \phi$, z_i is the charge number (see **electroneutrality**), ϕ , the electrostatic potential and \mathfrak{F} , the Faraday ($= 0.9649 \times 10^5$ coulombs, the electrical charge associated with one gram of a species having a charge number 1).

The electrochemical potential determines the thermodynamic equilibrium between phases α and β at different electric potentials and having different compositions. At equilibrium we have for all components i which may be transferred from α to β ,

$$\mu_i^\alpha = \mu_i^\beta \quad (2)$$

ELECTRODYNAMICS, MINKOWSKI. See Minkowski electrodynamics.

ELECTRODYNAMICS, QUANTUM. See quantum electrodynamics.

ELECTROKINETIC (ZETA) POTENTIAL. The difference in potential between the immovable liquid layer attached to the surface of a solid phase and the movable part of the diffuse layer in the body of the liquid.

ELECTROLYTES, STRONG. See limiting law for strong electrolytes.

ELECTROMAGNETIC CONSTANT. The speed of propagation of electromagnetic waves in a vacuum, commonly denoted by c . A long history of direct measurements culminated in the method which was perfected and refined by Michelson and others. Michelson's final value (1930) was 299,772 kilometers per sec. The most probable value, as of 1941, is given by Birge as 299,776 kilometers per sec. Later experiments using microwaves and other techniques indicate that 299,793 kilometers per sec is a better value.

ELECTROMAGNETIC DRIVING SYSTEM. See driving system, electromagnetic.

ELECTROMAGNETIC FIELD. According to the behavior of electric and magnetic fields described by the **Maxwell equations**, a time-varying **electric field** is always accompanied by a **magnetic field**, and conversely, a time-varying magnetic field is accompanied by an electric field. The **field of force** associated with these coupled electric and magnetic effects is called an electromagnetic field, and is considered to have an objective existence in space apart from the electric or magnetic

sources which may be considered to have generated it.

ELECTROMAGNETIC FIELD QUANTIZATION. See **quantization of electromagnetic field**.

ELECTROMAGNETIC FIELD TENSOR.

In terms of the covariant four-vector potential $\phi_i = (-c\mathbf{A}, \phi)$, the covariant electromagnetic field tensor is defined by

$$F_{ij} = \frac{\partial\phi_j}{\partial x^i} - \frac{\partial\phi_i}{\partial x^j}.$$

In terms of electric and magnetic field quantities, this antisymmetric tensor takes the form

$$F_{ij} = \begin{pmatrix} 0 & -cB_z & cB_y & -E_x \\ cB_z & 0 & -cB_x & -E_y \\ -cB_y & cB_x & 0 & -E_z \\ E_x & E_y & E_z & 0 \end{pmatrix}.$$

The corresponding contravariant tensor F^{ij} is obtained by changing the signs of the elements in the fourth row and fourth column.

ELECTROMAGNETIC FIELD, TRANSFORMATION EQUATIONS FOR. The Lorentz transformation equations for the electric and magnetic fields, \mathbf{E} and \mathbf{B} , are

$$\mathbf{B}'_{\parallel} = \mathbf{B}_{\parallel}$$

$$\mathbf{B}'_{\perp} = \gamma \left(\mathbf{B}_{\perp} - \frac{\mathbf{v} \times \mathbf{E}}{c^2} \right)$$

$$\mathbf{E}'_{\parallel} = \mathbf{E}_{\parallel}$$

$$\mathbf{E}'_{\perp} = \gamma (\mathbf{E}_{\perp} + \mathbf{v} \times \mathbf{B}),$$

where, for example, \mathbf{B}_{\parallel} and \mathbf{B}_{\perp} represents the components of \mathbf{B} parallel and perpendicular respectively to the direction of the velocity vector \mathbf{v} , and $\gamma = (1 - v^2/c^2)^{1/2}$.

When these transformed fields are used in Maxwell's equations, these equations are relativistically invariant.

ELECTROMAGNETIC GENERATING SYSTEM. See **generating system, electromagnetic**.

ELECTROMAGNETIC INDUCTION. See **Faraday law of electromagnetic induction**.

ELECTROMAGNETIC MASS. A charged particle in motion is accompanied by an elec-

tromagnetic field having a certain **electromagnetic momentum**. A force acting on the particle must change not only the momentum of the "bare" mechanical mass, but also the momentum of the electromagnetic field associated with the charge. The contribution of this latter effect to the total inertia of the particle is expressible as an **electromagnetic mass**.

ELECTROMAGNETIC MOMENTUM. See **energy-momentum tensor**.

ELECTROMAGNETIC RADIATION. The **electromagnetic field** associated with variable current and charge systems can generally be resolved into several components, each falling off with distance from the source according to a different power of distance R . (See the discussion under **field, radiation**.) The radiation field, which falls off as $1/R$, is the only component that contributes to the integral of the **Poynting vector** at large distances. It is this property of the electromagnetic field that is usually referred to as **electromagnetic radiation**.

ELECTROMAGNETIC RECIPROCITY THEOREM. See **reciprocity theorem, electromagnetic**.

ELECTROMAGNETIC SPECTRUM. The ordered array of all known electromagnetic radiations, extending from the shortest gamma rays, through x-rays, ultraviolet radiation, visible radiation, infrared radiation, and including microwave and all other wavelengths of radio energy.

The division of this continuum of wavelengths (or frequencies) into a number of named subportions is rather arbitrary and, with one or two exceptions, the boundaries of the several subportions are only vaguely defined.

ELECTROMAGNETIC STRESS. The **energy-momentum tensor** T_{ik} may be viewed as a **stress tensor** for the electromagnetic field of forces. If the forces between, say, two isolated charges are considered to be transmitted through the intervening space, then this space must be in a state of **stress** in the sense that the electromagnetic forces are transmitted across any element of surface located in it. From this point of view, the component dF_i of the force $d\mathbf{F}$ transmitted across a surface element

$d\mathbf{S}$ whose component in the k^{th} direction is dS_k is given by

$$dF_i = \sum_{k=1}^3 T_{ik} dS_k.$$

ELECTROMAGNETIC UNITS. At least eight or ten different systems of electrical and magnetic units are in common use. Each of these is based on a particular choice of a constant of proportionality in an experimentally verified physical law. Some systems start with Coulomb's law, which states that the force, f , between two electrical charges, q_1 and q_2 , separated by a distance r in empty space is directly proportional to the product of the charges and inversely proportional to the square of the distance between them, i.e.:

$$f = K_e q_1 q_2 / r^2,$$

where K_e is a constant that may be chosen for convenience. Such systems are known as electrostatic systems. The choice of K_e as unity and dimensionless, together with the use of the dyne and the centimeter as units of force and length, leads to the *cgs electrostatic system* in which the unit of charge, the *statcoulomb*, is the charge which repels an exactly similar charge, separated from it by one centimeter *in vacuo*, with a force of one dyne. This system is frequently called the *esu system*. Another choice, which is sometimes convenient, takes K_e as a dimensionless constant equal to $1/4\pi$. This leads to the *rationalized cgs electrostatic system*. In either of the two systems, charge has the dimensions of $\text{dyne}^{1/2} \text{cm}$, equivalent to $\text{cm}^{3/2} \text{gm}^{1/2} \text{sec}^{-1}$. All other electrical quantities also have identical dimensions in the two systems, although the sizes of their units differ.

In distinction to the electrostatic systems are the *electromagnetic systems* (emu systems), which start with the law of attraction between currents. If two currents of magnitudes I_1 and I_2 flow in long parallel wires, separated by a distance d *in vacuo*, they attract each other with a force per unit length, f_l , given by

$$f_l = K_m I_1 I_2 / d.$$

Here the constant of proportionality, K_m , may be chosen quite arbitrarily. The *cgs emu system* is based on the dyne and the centimeter as units of force and length and on the choice of K_m as a dimensionless constant of magni-

tude two. The emu of current, the *abampere*, then has the dimensions of $\text{dyne}^{1/2}$, or $\text{cm}^{1/2} \text{gm}^{1/2} \text{sec}^{-1}$.

The emu of charge, the *abcoulomb*, is defined as the charge which passes a given surface in one second if a steady current of one abampere flows across the surface. Its dimensions are therefore $\text{cm}^{1/2} \text{gm}^{1/2}$, which differ from the dimensions of the *statcoulomb* by a factor which has the dimensions of a speed. This relationship is connected with the fact that the ratio $2 K_e / K_m$ must have the value of the square of the speed of light in any consistent system of units. It follows further that

$$1 \text{ abcoulomb} = 2.998(10)^{10} \text{ statcoulomb},$$

the speed of light in *vacuo* being $2.998(10)^{10}$ cm/sec.

A *rationalized emu system*, in which K_m is taken as $1/2\pi$, has also been developed, but it is not widely used.

The electrostatic system is convenient for problems in which the principal equations may be deduced from Coulomb's law. Similarly, the electromagnetic system is convenient for problems involving the interactions between currents. In many physical problems, both electrical and magnetic interactions take place. Both systems suffer from certain inconveniences under the circumstances, and the *Gaussian system* of units has, as a result, gained wide popularity. In this system, magnetic quantities, such as magnetic field strength and magnetic flux density, are expressed in emu, while electric field strength, charge, and current are expressed in esu. To maintain self-consistency, it is essential that a factor c , the speed of light, be introduced into many of the equations which describe electromagnetic phenomena.

In all of the systems discussed thus far, *cgs mechanical units* have been employed. New systems of electrical units evolve if other sets of mechanical units are substituted. Only two such systems, both based on the *mks mechanical units*, have found wide acceptance. Of these two, only the *rationalized mksa system* is given here. The arbitrary choice which leads to this system is that of the unit of current. The *absolute ampere* is defined as exactly one-tenth of an abampere. With this choice, and with the newton and the meter as the units of force and length, the two constants which were chosen arbitrarily in the esu and

RELATIONS AMONG THE SYSTEMS OF ELECTRICAL AND MAGNETIC UNITS

(The dimensions of the various quantities are shown in square brackets)

Quantity	mksa (Absolute) System	Equivalents in Other Systems			
		Old International System	egs esu System	egs emu System	Gaussian System
Permittivity of empty space (ϵ_0)	$8.855(10)^{-12}$ Farad/M [m ⁻³ k ⁻¹ s ⁴ a ²]	$8.859(10)^{-12}$ Int. Farad/M	1 [Dimensionless]	$1.1126(10)^{-21}$ [cm ⁻² S ²]	1 [Dimensionless]
Permeability of empty space μ_0	$1.2566(10)^{-6}$ Henry/M [mks ⁻² a ⁻²]	$1.2560(10)^{-6}$ Int. Henry/M	$1.1126(10)^{-21}$ [cm ⁻² S ²]	1 [Dimensionless]	1 [Dimensionless]
Charge (Q)	1 Coulomb [sa]	1.000165 Int. Coulomb	$2.998(10)^9$ Statcoulomb [cm ^{3/2} gm ^{1/2} s ⁻¹]	0.1 Abcoulomb [cm ^{1/2} gm ^{1/2}]	$2.998(10)^9$ Statcoulomb [cm ^{3/2} gm ^{1/2} s ⁻¹]
Potential difference (V)	1 Volt [m ² ks ⁻³ a ⁻¹]	0.999670 Int. Volt	$3.336(10)^{-2}$ Statvolt [cm ^{1/2} gm ^{1/2} s ⁻¹]	(10) ³ Abvolt [cm ^{3/2} gm ^{1/2} s ⁻²]	$3.336(10)^{-2}$ Statvolt [cm ^{1/2} gm ^{1/2} s ⁻¹]
Current (I)	1 Ampere [a]	1.000165 Int. Ampere	$2.998(10)^9$ Statampere [cm ^{3/2} gm ^{1/2} s ⁻²]	0.1 Abampere [cm ^{1/2} gm ^{1/2} s ⁻¹]	$2.998(10)^9$ Statampere [cm ^{3/2} gm ^{1/2} s ⁻²]
Resistance (R)	1 Ohm [m ² ks ⁻³ a ⁻²]	0.999505 Int. Ohm	$1.1126(10)^{-12}$ Statohm [cm ⁻¹ s]	(10) ⁹ Abohm [cms ⁻¹]	$1.1126(10)^{-12}$ Statohm [cm ⁻¹ s]
Electric displacement (D)	1 Coulomb/M ² [m ⁻² sa]	1.000165 Int. Coulomb/M ²	$2.998(10)^5$ Statcoulomb/cm ² [cm ^{-1/2} gm ^{1/2} s ⁻¹]	(10) ⁻⁵ Abcoulomb/cm ² [cm ^{-3/2} gm ^{1/2}]	$2.998(10)^5$ Statcoulomb/cm ² [cm ^{-1/2} gm ^{1/2} s ⁻¹]
Capacitance (C)	1 Farad [m ⁻² k ⁻¹ s ⁴ a ²]	1.000495 Int. Farad	$8.988(10)^{11}$ cm [cm]	(10) ⁻⁹ Abfarad [cm ⁻¹ s ²]	$8.988(10)^{11}$ cm [cm]
Magnetic dipole moment	1 Ampere M ² [m ² a]	1.000165 Int. Ampere M ²	$3.336(10)^{-6}$ Statmaxwell/cm [cm ^{-1/2} gm ^{1/2}]	(10) ⁵ Maxwell/cm [cm ^{1/2} gm ^{1/2} s ⁻¹]	(10) ⁵ Maxwell/cm [cm ^{1/2} gm ^{1/2} s ⁻¹]
Magnetic field strength (H)	1 Ampere turn/M [m ⁻¹ a]	1.000165 Int. Ampere turn/M	$3.767(10)^8$ Statoersted [cm ^{1/2} gm ^{1/2} s ⁻²]	$1.257(10)^{-2}$ Oersted [cm ^{-1/2} gm ^{1/2} s ⁻¹]	$1.257(10)^{-2}$ Oersted [cm ^{-1/2} gm ^{1/2} s ⁻¹]
Magnetic flux density (B)	1 Weber/M ² [ks ⁻² a ⁻¹]	0.999670 Int. volt S/M ²	$3.336(10)^{-7}$ Statmaxwell/cm ² [cm ^{-3/2} gm ^{1/2}]	(10) ⁴ Maxwell/cm ² [cm ^{-1/2} gm ^{1/2} s ⁻¹]	(10) ⁴ Gauss [cm ^{-1/2} gm ^{1/2} s ⁻¹]
Inductance (L)	1 Henry [m ² ks ⁻² a ⁻²]	0.999505 Int. Henry	$1.1126(10)^{-12}$ Stathenry [cm ⁻¹ s ²]	(10) ⁹ cm [cm]	(10) ⁹ cm [cm]
Power	1 Watt [m ² ks ⁻³]	0.999835 Int. Watt	(10) ⁷ erg/s [cm ² gm s ⁻³]	(10) ⁷ erg/s [cm ² gm s ⁻³]	(10) ⁷ erg/s [cm ² gm s ⁻³]

emu systems are determined and have dimensions. They become:

$$K_e = 8.986(10)^9 \text{ km}^3\text{s}^{-4}\text{a}^2,$$

and

$$K_m = 2.000(10)^{-7} \text{ kms}^{-2}\text{a}^2.$$

One virtue of the mksa system is that nearly all of the electrical quantities expressed in it coincide closely with the *practical system* of units which grew up during the nineteenth century. Thus the volt, the ampere, the henry, the farad, and the ohm are all units in the mksa system. In fact, the legal electrical units have been fixed by international agreement since 1950 as the absolute mksa units. Prior to that time, the *International system* of electrical units had been used. This system had been intended to coincide with the absolute system, but had been defined in terms of fixed standards, which are slightly in error. There are therefore small differences between the two sets of electrical quantities, of the order of a few parts in ten thousand. Because many quantities stated in the literature are expressed in international units, these obsolescent definitions are included here.

One more remark needs to be made in regard to the dimensions of certain electromagnetic units. Two electrical quantities, the field strength \mathbf{E} and the displacement \mathbf{D} , are closely related, as are two magnetic quantities, the field strength \mathbf{H} and the flux density \mathbf{B} . In the electrostatic system, \mathbf{E} and \mathbf{D} have the same dimensions and are identical in magnitude in empty space; in the electromagnetic system, \mathbf{H} and \mathbf{B} have a corresponding relation. Thus in air, the electrical properties of which are practically those of empty space, the flux density is identical with the magnetic field strength if both are expressed in emu. The old unit of field strength, the *gauss*, has therefore been used to denote both \mathbf{H} and \mathbf{B} . In an attempt to avoid confusion, the name of the emu unit of \mathbf{H} was changed to the *oersted* about 30 years ago. The gauss had become so well established, however, that it is still used, and its meaning (either oersted or maxwell per square cm) must be judged from context.

The relations among the five systems of electrical units discussed are displayed in the table on page 297.

ELECTROMAGNETIC WAVES. Waves characterized by varying electric and mag-

netic fields. By differentiating Maxwell's equations and substituting from one equation to the other, the electric and the magnetic field vectors can be separated and each shown to satisfy the **wave equation**. Its solution is an electromagnetic wave. (See **electromagnetic spectrum**).

ELECTROMAGNETISM. A term generally used in reference to the generation of magnetic fields by electric currents. (See **Ampère law**, **Biot-Savart law** and **Maxwell equations**.)

ELECTROMOTIVE FORCE. Because of energy losses, a conservative electrostatic field cannot support a steady current. In order to account for steady currents, a non-conservative field having its origin in sources known as electromotive forces must be introduced. The electromotive force \mathcal{E} acting in a closed circuit may be defined as

$$\mathcal{E} = \oint \mathbf{E}' \cdot d\mathbf{l}$$

where \mathbf{E}' represents the non-conservative fields. Such fields can be produced by diffusion of ions in an electrolyte, for example, or by changing the magnetic flux through a circuit. (Cf. **Faraday law of electromagnetic induction**.) If the circuit is opened, the potential difference between the terminals is equal to the total electromotive force in the circuit.

The electromotive force of an **electrochemical cell** is equal to the affinity (see **chemical affinity**) of the chemical change associated with a flow of electricity of one **faraday**.

ELECTROMOTIVE FORCE, AVERAGE. For an alternating voltage (with no d-c component) the average value is zero. The term is often misused to mean the average absolute emf:

$$\frac{1}{T} \int_0^T |\mathcal{E}| dt.$$

For a sine wave, this is $2/\pi$ times the peak value.

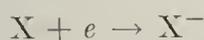
ELECTROMOTIVE FORCE, MOTIONAL. The **electromotive force** induced in a circuit by virtue of motion of the conductor across a magnetic field:

$$\mathcal{E} = \int \mathbf{v} \times \mathbf{B} \cdot d\mathbf{l},$$

where \mathbf{V} is the velocity of the conductor, \mathbf{B} is the magnetic flux density, and the integral extends over the length of the conductor, $d\mathbf{l}$ being an element of length.

ELECTRON. An elementary particle of rest mass $m_e = 9.107 \times 10^{-28}$ g and charge of 4.802×10^{-10} statcoulomb, and a **spin** of one-half unit, i.e., of $\hbar/2 = \hbar/4\pi$. Its charge may be positive or negative, although the term electron is commonly used for the negative particle, which is also called the negatron. The positive electron is called the **positron**.

ELECTRON AFFINITY. The electron affinity of an atom or of a molecule X is the energy evolved in the reaction:



or, conversely, it is the energy of ionization of the negative ion X^- . In the above definition, the atom or molecule, and the atomic or molecular ion, are assumed in their fundamental state and without kinetic energy. X and e are initially at rest at an infinite distance from one another.

ELECTRON(S), ANTIBONDING. See molecular orbital theory of valence.

ELECTRON, BONDING. See molecular orbital theory of valence.

ELECTRON CONCENTRATION. The ratio of the number of valence electrons to the number of atoms in a molecule. This quantity is useful in studying the intermetallic compounds, where it is correlated with the crystal structure.

ELECTRON CONFIGURATION. The totality of the orbitals of the single electrons in an atom or molecule. One writes the electron configuration symbolically by giving the quantum numbers corresponding to the different orbitals, and indicating the number of electrons in each orbital by a right hand superscript. The lowest electron configuration of the Na atom, for instance, is $1s^2 2s^2 2p^6 3s$, that of the B_2 molecule $(\sigma_g 1s)^2 (\sigma_u 1s)^2 (\sigma_g 2s)^2 (\sigma_u 2s)^2 (\pi_u 2p)^2$. For atoms, as a rule, only the orbitals of electrons outside closed shells are given, i.e., $3s$ in the above case. For molecules, closed shells of the separate atoms which, to a first approximation, are not affected by the molecule formation (see **building-up principle**) are indicated only by the symbols $K, L,$

etc. (see **electron shells in an atom**), i.e., $KK(\sigma_g 2s)^2 (\sigma_u 2s)^2 (\pi_u 2p)^2$. (For the lowest electron configurations of atoms see *American Institute of Physics Handbook*, McGraw-Hill, Inc., 1957, 7-14, 15. For the lowest electron configurations of molecules see G. Herzberg, *Molecular Spectra and Molecular Structure. I. Spectra of Diatomic Molecules*. D. Van Nostrand Company, Inc., Princeton, 1950, pp. 341 and 343.)

ELECTRON ENERGY (CLASSICAL). An electron of charge $-e$ (emu) moving from point x_1 to point x_2 in an electric field E (emu) loses potential energy or gains kinetic energy according to

$$U = -e \int_{x_1}^{x_2} E dx = -e(V_1 - V_2) \text{ ergs,}$$

where V_1 and V_2 are the potentials in emu at x_1 and x_2 respectively.

The kinetic energy of an electron moving with velocity v cm/sec is

$$U = m_0 c^2 \left[\frac{1}{1 - (v/c)^2} - 1 \right] \text{ ergs}$$

$$= \frac{1}{2} m_0 v^2 \left[1 + \frac{3}{4} \left(\frac{v}{c} \right)^2 + \frac{5}{8} \left(\frac{v}{c} \right)^4 + \dots \right] \text{ ergs,}$$

where m_0 stands for the **rest mass** of the electron in gram and c for the velocity of light in cm/sec.

The transverse mass m_t and the longitudinal mass m_l of an electron moving with velocity v are given by

$$m_t = \frac{m_0}{[1 - (v/c)^2]^{1/2}}; \quad m_l = \frac{m_0}{[1 - (v/c)^2]^{3/2}}.$$

ELECTRONEUTRALITY. If one describes the properties of electrolytic solutions in terms of ionic species, one has to take account of the fact that the concentrations of all species are not independent because the solution as a whole is neutral.

One generally uses the symbol z_i to denote the charge on an ion measured in units of the charge of a proton (for example, for Na^+ , $z = 1$; for La^{3+} , $z = 3$; for PO_4^{3-} , $z = -3$); z is also called the *charge number* of the ion.

If n_i is the number of moles of the ionic species i , the condition of electrical neutrality is

$$\sum_i z_i n_i = 0. \quad (1)$$

Alternatively if one uses the subscript $+$ to denote positively-charged ions or *cations* and $-$ to denote negatively charged ions or *anions*, then one may write (1) in the form

$$\sum_{+} z_{+}n_{+} = \sum_{-} z_{-}n_{-}. \quad (2)$$

ELECTRON GAS, DEGENERATE. See **degenerate electron gas**.

ELECTRONIC CHARGE, SPECIFIC. The ratio e/m of the electronic charge to the rest mass of the electron, i.e.,

$$e/m = 5.2731 \times 10^{17} \text{ esu g}^{-1}.$$

ELECTRONIC CONFIGURATION. See **electron configuration**.

ELECTRONIC EIGENFUNCTIONS OF MOLECULES. See **molecular eigenfunctions**.

ELECTRONIC ENERGY LEVELS OF A MOLECULE. The different electronic states of a molecule are characterized by certain quantum numbers and symmetry properties of their eigenfunctions. For diatomic and linear polyatomic molecules the orbital angular momentum Λ about the internuclear axis is defined and has the magnitude $\Lambda h/2\pi$, where Λ is the corresponding quantum number which can assume only integral values. Depending on whether $\Lambda = 0, 1, 2, \dots$, we distinguish $\Sigma, \Pi, \Delta, \dots$ states. For nonlinear molecules different types (species) of electronic states arise depending on the symmetry properties of the nuclear frame. For example, if the molecule has a single plane of symmetry there are two species of electronic energy levels, those whose eigenfunctions are symmetric with respect to that plane and those whose eigenfunctions are antisymmetric with respect to it. They are designated A' and A'' , respectively. For a molecule with two mutually perpendicular planes of symmetry there are four species which may be characterized by $++, --, +-, -+$ where the two signs indicate the behavior with respect to the two planes. These four species are designated $A_1, A_2, B_1,$ and B_2 , respectively.

Each electronic state has a multiplicity $(2S + 1)$ depending on the value of the quantum number S of the resultant electron spin of the molecule.

No general formulas for the energies of the

electronic states of a molecule can be given except for those states in which one electron is excited to orbitals of increasing principal quantum number n . In this case one has to a good approximation

$$T_e = A - \frac{R}{(n - a)^2}$$

where A is the ionization potential, R the Rydberg constant, and a the Rydberg correction. (See also **fine structure**, **Λ -type doubling**.)

ELECTRONIC GYROMAGNETIC RATIO. See **gyromagnetic ratio, electronic**.

ELECTRONIC SPECIFIC HEAT. According to classical statistics, the *free* electrons in a metal should contribute an amount $3Nk$ to the specific heat, where N is the number of electrons per unit volume and k is Boltzmann's constant. The observed contribution is very much smaller than this, and the reason for this is that quantum statistics must be applied. Sommerfeld has pointed out that the electrons should be treated as a **Fermi-Dirac gas** for which the specific heat per unit volume is given by

$$c_v = \frac{1}{2}\pi^2 NKT/T_F$$

where T_F is the **Fermi temperature**. The contribution is thus only $(\pi^2/6) (T/T_F)$ of the value predicted classically, and at ordinary temperatures this fraction is very small. At very low temperatures, the specific heat resulting from lattice vibrations is proportional to T^3 so that, as the absolute zero is approached, the electronic contribution to the specific heat becomes relatively more important.

ELECTRONIC SPECTRA OF MOLECULES. The electronic spectrum of a molecule corresponds to the (allowed) transitions between the vibrational and rotational sub-levels of different electronic states.

A transition between the electronic states i and k is allowed as dipole radiation if there is at least one component of the dipole moment $M_x, M_y,$ or M_z which has the same symmetry properties as the product of the electronic eigenfunctions $\psi_e^i \psi_e^{k*}$. The electronic selection rules therefore are of the same form as the vibrational selection rules. The symmetry of the products $\psi_e^i \psi_e^{k*}$ can be determined from tables given by H. Sponer and E. Teller, *Rev.*

Mod. Phys. 13, 75 (1941) or by G. Herzberg, in *Molecular Spectra and Molecular Structure, II. Infrared and Roman Spectra of Polyatomic Molecules*, D. Van Nostrand Co., Inc., Princeton, 1945, though the latter were originally prepared for vibrational transitions.

For diatomic and linear polyatomic molecules the selection rule

$$\Delta\Lambda = 0, \pm 1$$

results from the above general rule.

(a) *Vibrational structure.* The totality of vibrational transitions for a given electronic transition is a band system. The wave numbers of the bands of a band system are represented by the formula

$$\nu = \nu_e + G'(v'_1, v'_2, \dots) - G''(v''_1, v''_2, \dots)$$

or in the case of diatomic molecules

$$\nu = \nu_e + \omega'_e(v' + \frac{1}{2}) - \omega'_e x'_e(v' + \frac{1}{2})^2 + \dots - [\omega''_e(v'' + \frac{1}{2}) - \omega''_e x''_e(v'' + \frac{1}{2})^2 + \dots]$$

where $\nu_e = T'_e - T''_e$ is the origin of the band system.

Which vibrational transitions are possible and what intensities they have is determined by the integral

$$\int \psi'_v \psi''_v d\tau_v.$$

As long as the interaction of vibration and electronic motion is negligible the relative intensities of the various vibrational transitions are proportional to the square of this integral. Only those vibrational transitions are possible for which the product $\psi'_v \psi''_v$ is symmetric with respect to all symmetry operations permitted by the symmetry of the molecule.

(b) *Rotational structure.* The rotational structure of the individual vibrational transitions (bands) of an electronic transition is essentially the same as that of rotation-vibration bands (see **rotation-vibration spectra of molecules**) as long as there is no spin splitting (see **fine structure**). If spin splitting is present, additional selection rules apply; for example, for diatomic and linear polyatomic molecules, if N the angular momentum apart from spin is defined, one has

$$\Delta N = 0, \pm 1.$$

For Σ — Σ transitions $\Delta N = 0$ does not occur. For details about the formulas for the branches in such cases, see G. Herzberg, *Molecular*

Spectra and Molecular Structure, I. Spectra of Diatomic Molecules, 2d ed., D. Van Nostrand Company, Inc., Princeton, 1950.

ELECTRONIC TRANSITIONS, VIBRATIONAL SUM RULE FOR. See vibrational sum rule for electronic transitions.

ELECTRON-OPTICAL INDEX OF REFRACTION. The path of an electron is determined by **Fermat's principle** using the electron-optical index of refraction n where

$$m_0 c n = m s - e A \cos \chi$$

where m is the mass, m_0 the rest mass, e , the charge of the electron, s the image distance (from the image principal plane to the image plane) and χ is the angle between the path of the electron and the magnetic vector potential A .

ELECTRON-OPTICAL LENS EQUATION. The general lens equation in electron optics is

$$\frac{f_i}{\rho_i} - \frac{f_o}{\rho_o} = 1, \quad \frac{f_i}{f_o} = \sqrt{\frac{\phi_i}{\phi_o}}, \quad \frac{\rho_i}{\rho_o} = M \sqrt{\frac{\phi_i}{\phi_o}}$$

where ρ_o is the object distance (from object principal plane to object plane), f_o is the object-side focal length, ϕ_o is the object-side electric potential along the axis of symmetry, and ρ_i , f_i , ϕ_i are the corresponding image-side quantities; finally, M is the **magnification**.

ELECTRON ORBIT (CLASSICAL). An electron having a velocity of v cm/sec in a magnetic field of flux density B moves in a stable circular orbit of radius r cm, when acted upon by a force

$$F = -m_t v^2 / r,$$

where $m_t = \frac{m_o}{1 - (v/c)^2}$ is the transverse mass of the electron (m_o represents the **rest mass** of the electron, c the velocity of light).

The cyclotron condition therefore is given by

$$\frac{m_t v}{r} = e B,$$

where e is the electronic charge in emu.

$$r = 10^4 \frac{2m_t V}{e B} \text{ cm,}$$

where V is the electron velocity v expressed in equivalent volts. The periodic time of one

revolution of an electron in the cyclotron orbit is

$$T = \frac{2\pi m_t}{eB}.$$

ELECTRON ORBITS IN AN ATOM (BOHR ORBITS). In the Bohr-Sommerfeld model of the atom the electrons described circular and elliptical orbits about the atomic nucleus. These orbits are selected from the continuum of classically possible orbits by means of certain quantum conditions. For hydrogen and the hydrogen-like ions the major and minor axes of these orbits are

$$a = \frac{h^2}{4\pi^2\mu e^2} \frac{n^2}{Z} = \frac{a_0}{Z} n^2$$

$$b = \frac{h^2}{4\pi^2\mu e^2} \frac{nk}{Z} = \frac{a_0}{Z} nk.$$

Here n is the principal quantum number, taking on the values

$$n = 1, 2, 3, \dots,$$

and k the azimuthal quantum number which, for a given n , can take the values

$$k = 1, 2, \dots, n.$$

h stands for Planck's constant, e , for the electronic charge, and Z , for the atomic number.

$$\mu = \frac{mM}{m + M}$$

is the reduced mass of the electron (m and M , electronic and nuclear mass respectively).

The energy of the electron orbits, neglecting relativistic effects, is given, for hydrogen and hydrogen-like ions by

$$E_n = \frac{2\pi^2\mu e^4}{h^2} \frac{Z^2}{n^2} = \frac{RZ^2}{n^2} hc$$

where R is the **Rydberg constant**. For hydrogen and the hydrogen-like ions therefore the energy is the same for all Bohr orbits with the same principal quantum number n , independently of the value of the azimuthal quantum number k . (For the effect of relativity corrections, see **hydrogen fine structure**.)

In the more general case of an electron moving in the field of an atomic core (nucleus plus p electrons) the orbits are ellipses whose axes rotate uniformly about the center of

gravity (rosette motion). In this case the energy of the electron orbits varies with both the quantum numbers n and k , and is given by the expression

$$E_{n,k} = \frac{R(Z - p)^2}{(n + a)^2} hc,$$

where a is the so-called **Rydberg correction** which depends on the azimuthal quantum number k and which rapidly approaches zero with increasing value of k .

ELECTRON, PI (π -ELECTRON). See **wave function, pi (π)**.

ELECTRON RADIUS, CLASSICAL. A quantity given by the equation,

$$r_0 = \frac{e^2}{mc^2} = 2.81785 \times 10^{-13} \text{ cm},$$

where e and m are respectively the charge and mass of the electron and c is the velocity of light.

ELECTRONS, EQUIVALENT. See **equivalent electron**.

ELECTRON SHELLS IN AN ATOM. The group of energy levels (**orbitals, orbits**) of the single electrons in an atom characterized by the same principal quantum number n . The number of electrons in a shell with a certain n is limited by the **Pauli principle**. A shell containing the maximum possible number of electrons is called a closed shell. Traditionally shells with $n = 1, 2, 3, \dots$ are designated K, L, M, \dots shells. (Sometimes the subgroups with the same values for both the quantum numbers n and l are referred to as shells (or subshells), e.g., L_1 (sub)shell for $n = 2, l = 0$; L_2 (sub)shell for $n = 2, l = 1$.) In general, energy levels belonging to the same shell lie close together, while successive shells differ considerably in their energy.

ELECTRON, SIGMA (σ -ELECTRON). See **wave function, sigma (σ)**.

ELECTRON SPIN. The intrinsic angular momentum s of the electron, that is, the rotation about its own axis. s has the magnitude

$$\sqrt{s(s + 1)} (h/2\pi) = \frac{1}{2} \sqrt{3} (h/2\pi)$$

where $s = \frac{1}{2}$ is the corresponding quantum number.

The magnetic moment associated with the electron spin is

$$\mu_e = 0.92837 \times 10^{-20} \text{ erg gauss}^{-1}$$

or

$$1.001145358 \mu_o$$

where μ_o is the **Bohr magneton**.

The ratio of μ_e to μ_o , the anomalous electron moment correction, is given by

$$\mu_e/\mu_o = 1 + \frac{\alpha}{2\pi} - 2.973 \frac{\alpha^2}{\pi^2}$$

(α is Sommerfeld's fine structure constant).

The **gyromagnetic ratio** of the spinning electron is

$$g_s \approx 2$$

("double magnetism" of electron spin).

ELECTRON THEORY, DIRAC. See **Dirac electron theory**.

ELECTRON THEORY OF METALS. A theory considering the system of conduction electrons to be a nearly **degenerate Fermi-Dirac gas**. When there are no external forces or temperature gradients the Fermi-distribution of the electrons will give rise to the correct order of magnitude of the electronic specific heat.

If there are electric or magnetic fields acting on the metal, or if there is a temperature gradient, the distribution function $f(x, y, z; u, v, w)$ (x, y, z : position coordinates; u, v, w : velocity components) is found as a solution of the **Boltzmann equation**. The electrical conductivity σ and the thermal conductivity k are thus obtained by evaluating the electrical current j and heat current w from the equations

$$j_x = \int f(x, y, z; u, v, w) e u \, du \, dv \, dw$$

$$w_x = \int f(x, y, z; u, v, w) \frac{1}{2} m (u^2 + v^2 + w^2) u \, du \, dv \, dw,$$

where m is the mass of the electrons and e their charge, and using the definitions of σ and k .

ELECTRON-VOLT (e.v.). An energy unit widely used in atomic and molecular physics. 1 electron volt is the energy equal to the kinetic energy of an electron which has been accelerated through a potential difference of 1 volt, or

$$1 \text{ e.v.} = 1.60206 \times 10^{-12} \text{ ergs/molecule.}$$

A photon of energy 1 e.v. has the wavelength

$$\lambda' = 12397.7 \text{ \AA}$$

and the wavenumber

$$\nu = \frac{1}{\lambda} = 8066.0 \text{ cm}^{-1}.$$

ELECTROPHILIC AGENT. See **inductive effect**.

ELECTROSTATIC DRIVING SYSTEM. See **driving system, electrostatic**.

ELECTROSTATIC FIELD. An electric field produced by stationary charges. (See **electrostatics; electric field strength**.)

ELECTROSTATIC GENERATING SYSTEM. See **generating system, electrostatic**.

ELECTROSTATIC INTERACTIONS AND VALENCE THEORY. See **bond types; atomic bonds**.

ELECTROSTATICS. That branch of electromagnetism that deals with the effects of stationary (as opposed to moving) electric charges. The basic law of electrostatics is the **Coulomb law** (see entry following). Materials are conveniently classed into conductors and non-conductors. In the former, charges are free to move within the conductor and any charge placed on a conductor will so distribute itself over the surface that the electric field within the conductor is zero and so that the conductor is an equipotential. When an uncharged conductor is placed in an electric field, produced, for example, by a neighboring charged body, a separation of charge on the surface will occur. A charge placed on a non-conductor will remain where it was placed. No material is a perfect non-conductor, but may approximate one very closely. When a non-conductor is placed in an electric field, **electric dipoles** are induced within it.

Static electric charges may be built up on a body by friction, by electrostatic induction, and by other means.

ELECTROSTATICS, LAWS OF. The laws of electrostatics are expressed in terms of electric charges, q , charge densities $\rho = dq/dV$, electric field strengths $\mathbf{E} = \mathbf{F}/q$, and electrostatic potentials $\phi = -\int \mathbf{E} \cdot d\mathbf{s}$. The basic law

of electrostatics is the Coulomb law of force between charges:

$$\mathbf{F} = \frac{q_1 q_2 \mathbf{r}}{4\pi\epsilon_0 r^3},$$

with the resulting field due to a single point charge q :

$$\mathbf{E} = \frac{q\mathbf{e}_r}{4\pi\epsilon_0 r^2} \quad (\text{rationalized mksa units}),$$

where \mathbf{e}_r is a unit vector pointing in the direction of increasing \mathbf{r} . The field due to a number of discrete charges is the vector sum

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i(\mathbf{e}_r)_i}{r_i^2},$$

while that due to a continuous distribution of charges is

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \frac{\rho\mathbf{e}_r dV}{r^2}.$$

The potential at some point in space due to a single point charge, referred to an origin of potential at infinity, is $\phi = q/4\pi\epsilon_0 r$, while the potentials respectively due to a number of discrete charges or to a continuous distribution of charges are

$$\phi = \frac{1}{4\pi\epsilon_0} \int \frac{\rho dV}{r} \quad \text{and} \quad \phi = \frac{1}{4\pi\epsilon_0} \sum_i \frac{q_i}{r_i^2},$$

again referred to an origin of potential at infinity. If the potential function ϕ has been determined, then the electric field may be determined from $\mathbf{E} = -\nabla\phi$ where $\nabla\phi$ is the gradient of ϕ . Alternatively, if the field \mathbf{E} has been determined, the potential ϕ may be obtained from $\phi = \int_{\infty}^R \mathbf{E} \cdot d\mathbf{s}$.

The basic law of electrostatics, the Coulomb law, may alternatively be expressed either as the **Gauss law** or as the **Poisson equation**. In the absence of local charges, the Poisson equation reduces to the special case of the Laplace equation.

The electrostatic field is a conservative field, which leads to the fact that it is possible to set up a potential function ϕ . This fact may be alternatively stated in terms of the closed line integral $\oint \mathbf{E} \cdot d\mathbf{s} = 0$, or in terms of the curl of \mathbf{E} : $\nabla \times \mathbf{E} = 0$.

ELECTROVALENT BOND. See **heteropolar bond**.

ELEMENT. (1) A quantity identified by two symbols designating a row and column in an array, such as a **matrix** element or an element of a **determinant**. (See also **integral**, **line**, etc.) (2) A collection of atoms of one type which cannot be decomposed into any simpler units by any chemical transformation. To date 102 different elements are known; these may be grouped into an ascending series according to the nuclear charge; some elements (those in the so-called radioactive series) spontaneously decompose into simpler elements; radioactive decomposition can be induced artificially where it does not occur in nature and for each element there is known a number of isotopes, i.e., atoms with the same nuclear charge but different nuclear masses which may vary within certain limits. There are also instances in which atoms have the same nuclear charge and nuclear mass, but differ only in nuclear energetics, and hence stability and behavior. (3) In network topology, an **edge**. (4) Any electrical device with terminals at which it may be directly connected to other electrical devices. (5) An integral part of a device (e.g., of an electron tube or a semiconductor) which contributes to its operation. (6) A parameter in an acoustical system which defines a distinct activity in its part of the system.

ELEMENTARY. The elementary operations on a **matrix** consist of the interchange of two rows (or columns), the multiplication of a row (or column) by a constant and the addition to any one row (or column) of any multiple of a different row (or column).

The elementary **functions** are the algebraic functions (that is, functions $y = f(x)$ defined by an algebraic equation in x and y) and the exponential functions (of the form $y = e^{f(x)}$) and their inverses (logarithmic functions). Since, for example, $2 \cos x = e^{ix} + e^{-ix}$, the trigonometric functions are included, as well as combinations of the above functions. The derivative of an elementary function is always an elementary function, but not every elementary function can be so obtained. Examples of non-elementary functions are the **gamma function**, **Bessel's functions**, etc.

ELEMENTARY SYSTEM. A system whose manifold of states constitutes a representation space for an irreducible representation of the **inhomogeneous Lorentz group**. Equivalently,

all the states of the system can be obtained from the Lorentz transforms of any *one* state by superposition. In other words, there exists no relativistically invariant distinction between the various states of the system. Elementary systems are of particular interest because they have the transformation properties of what is usually called a particle, e.g., a proton or α -particle.

ELEMENT, CIRCUIT. An **element** of a **graph** G which is contained in some **circuit** of G .

ELEMENT, NON-CIRCUIT. An **element** of a **graph** G which is not contained in a **circuit** is a non-circuit element of G . The removal of a non-circuit element from a **connected** graph G leaves G unconnected. The removal of a circuit element leaves the connectivity and the number of **vertices** invariant.

ELEMENT, ORIENTED. An oriented element of a **graph** is an **element** with an orientation assigned by ordering the **vertices** of the element. If the vertices β_1 and β_2 of element ϵ_1 are ordered as (β_1, β_2) , ϵ_1 is said to be oriented away from β_1 and toward β_2 .

ELEMENTS, CONJUGATE. See **conjugate elements of a group**.

ELEMENTS OF SYSTEM. See **indicial notation**.

ELIMINATION. A method of solving a system of equations in which one of the equations is solved for one of the unknowns in terms of the others, and the solution used to replace this unknown in all the other equations. Thus if the original system was one of n equations in n unknowns, one obtains a new system of which $n - 1$ of the equations contain only $n - 1$ of the unknowns.

ELLIPSE. A **conic section** obtained by a cutting plane parallel to no element of a right-circular conical surface. It is the **locus** of a point which moves so that the sum of its distances from two foci is a constant. Its eccentricity (see **conic section**) is less than unity. The standard equation may be taken as $x^2/a^2 + y^2/b^2 = 1$. The curve is a central conic for it is symmetric about both the X - and Y -axes. When placed in its standard position, the center of the ellipse is the coordinate origin, the major axis of length $2a$ is along

the X -axis, and the minor axis of length $2b$ is along the Y -axis. The distance from the center to either focus is $\sqrt{a^2 - b^2}$; the eccentricity e is given by $ae = \sqrt{a^2 - b^2}$; the length of the latus rectum is $2b^2/a$; the equations for the directrices are $x = \pm a/e$. The distance from any point on the ellipse to a focus is a focal radius and the sum of the two focal radii equals $2a$.

If the semi-major axis equals the semi-minor axis ($a = b$), the ellipse degenerates into a circle.

The polar equation of an ellipse is

$$r = \frac{a(1 - e^2)}{1 - e \cos \theta}$$

and its parametric equations are $x = a \cos \phi$, $y = b \sin \phi$.

ELLIPSE, GEODETIC. See **geodesic ellipse on a surface**.

ELLIPSOID. A **central quadric**, given in its standard form, with center at the coordinate origin, as

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

where a, b, c are the semi-axes. Any plane section is an ellipse.

If two of the axes become equal, the surface is a spheroid, which can be generated as a surface of revolution. Consider an ellipse in the XZ -plane

$$\frac{x^2}{a^2} + \frac{z^2}{c^2} = 1$$

with $a > c$, so that its major axis is along the X -axis of the coordinate system and its minor axis along the Z -axis. There are then two possibilities: (1) rotate the ellipse about its major axis and the result is a prolate spheroid with $a > b = c$; (2) rotate about the minor axis and obtain an oblate spheroid, $a = b > c$.

In the final degenerate case, $a = b = c$, the surface is a **sphere**.

ELLIPSOIDAL COORDINATES. A system of coordinates λ, μ, ν based on confocal quadrics. If constants a, b, c are taken with $a > b > c$, the surfaces are: (1) **ellipsoids**, $\lambda = \text{const.}$, $c^2 > \lambda > -\infty$; (2) **hyperboloids** of one sheet, $\mu = \text{const.}$, $b^2 > \mu > c^2$; (3) **hyperboloids** of two sheets, $\nu = \text{const.}$, $a^2 > \nu > b^2$.

The relation between the rectangular Cartesian coordinates and the ellipsoidal coordinates of a point are

$$x^2 = \frac{(a^2 - \lambda)(a^2 - \mu)(a^2 - \nu)}{(b^2 - a^2)(c^2 - a^2)}$$

$$y^2 = \frac{(b^2 - \lambda)(b^2 - \mu)(b^2 - \nu)}{(a^2 - b^2)(c^2 - b^2)}$$

$$z^2 = \frac{(c^2 - \lambda)(c^2 - \mu)(c^2 - \nu)}{(a^2 - c^2)(b^2 - c^2)}$$

Since x, y, z occur as squares in these relations they give eight points symmetrically located in the Cartesian system. Some convention must then be adopted for the signs of the ellipsoidal coordinates in order to locate a point uniquely.

Most of the curvilinear systems used in mathematical physics (including rectangular and spherical polar coordinates) are degenerate cases of the ellipsoidal system.

ELLIPSOID, MOMENTAL (ELLIPSOID OF POINSON). For a rotating body with no resultant external torque acting on it

$$\boldsymbol{\omega} \cdot \mathbf{H} = 2T$$

where \mathbf{H} is the vector angular momentum, $\boldsymbol{\omega}$ is the resultant vector angular velocity and T is the total kinetic energy. When expressed in component form, this becomes the equation of an ellipsoid in angular velocity space.

$$I_{xx}\omega_x^2 + I_{yy}\omega_y^2 + I_{zz}\omega_z^2 = 2T.$$

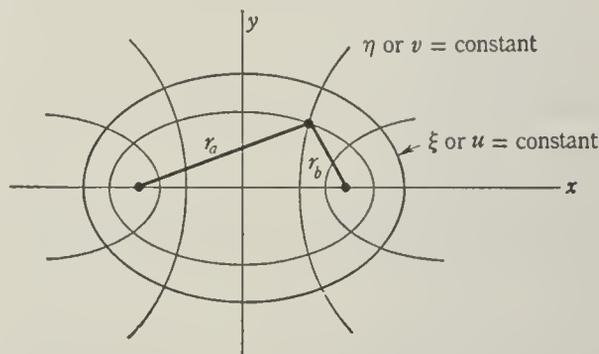
I_{xx}, I_{yy} and I_{zz} are the moments of inertia with respect to the principal axes fixed in the body. $\omega_x, \omega_y, \omega_z$ are the components of angular velocity with respect to the principal axes fixed in the body. The length of the radius vector from the origin to the surface of the ellipsoid represents a value of the angular speed consistent with the total kinetic energy and angular momentum. (See inertia, moments and products of; rigid body, kinetic energy of.)

ELLIPSOID, POLARIZABILITY. See polarizability.

ELLIPTICAL COORDINATES. A plane, two dimensional, coordinate system in which the coordinate curves are the mutually orthogonal sets of confocal ellipses and hyperbolas. The transformation equations are, with the foci at $x = \pm a, y = 0$:

$$x = a \cosh u \cos v,$$

$$y = a \sinh u \sin v.$$



Alternatively, the two coordinates $\xi = \cosh u$ and $\eta = \cosh v$ may be used, in which case the following hold:

$$\xi = \frac{r_A + r_B}{2a}, \quad \eta = \frac{r_A - r_B}{2a}.$$

These coordinates are useful in some two-center problems of physics.

ELLIPTICALLY LOADED WING. A wing for which the distribution of circulation across the span is given by

$$\Gamma = \Gamma_0 [1 - (y/s)^2]^{1/2}, \quad (1)$$

where s is the semi-span and y is measured from the mid-span position.

With this type of loading the induced angle of downwash, due to the trailing vortices, is constant across the span. The coefficient of trailing-vortex drag has the minimum value $\frac{C_L^2}{\pi A}$, for given values of the lift coefficient C_L and aspect ratio A .

(See also **lifting-line theory**.)

ELLIPTICALLY POLARIZED WAVE. See polarization cycle.

ELLIPTIC CYLINDRICAL COORDINATES. A degenerate case of ellipsoidal coordinates where the coordinate surfaces are: (1) elliptic cylindrical with semi-axes $a = c \cosh u, b = c \sinh u, u = \text{const.}$; (2) hyperbolic cylindrical with $a = c \cos v, b = c \sin v, v = \text{const.}$; (3) planes parallel to the XY -plane, $z = \text{const.}$ A point in this system has rectangular Cartesian coordinates

$$x = c \cosh u \cos v$$

$$y = c \sinh u \sin v$$

$$z = z$$

and $0 \leq u \leq \infty$; $0 \leq v \leq 2\pi$; $-\infty < z < \infty$.

ELLIPTIC FUNCTION. A single-valued, doubly-periodic, meromorphic function of a complex variable. The functions which are inverses of **elliptic integrals** are elliptic functions. Elliptic functions satisfy many of the non-linear differential equations arising in mathematical physics.

ELLIPTIC INTEGRAL. Any integral of the type

$$\int f(x, \sqrt{R}) dx,$$

where f is a rational function of its two arguments and R is a third or fourth degree polynomial in x , with no repeated roots. It may be reduced, by suitable change of variable, to a sum of elementary integrals and one or more of the following types:

$$u_1 = \int_0^x \frac{dt}{\sqrt{(1-t^2)(1-k^2t^2)}}$$

$$= \int_0^\phi \frac{dw}{\sqrt{1-k^2 \sin^2 w}}$$

$$u^2 = \int_0^x \sqrt{\frac{1-k^2t^2}{1-t^2}} dt = \int_0^\phi \sqrt{1-k^2 \sin^2 w} dw$$

$$u_3 = \int_0^x \frac{dt}{(t^2-a)\sqrt{(1-t^2)(1-k^2t^2)}}$$

$$= \int_0^\phi \frac{dw}{(\sin^2 w - a)\sqrt{1-k^2 \sin^2 w}}.$$

These are incomplete elliptic integrals of the first, second, third kind, respectively. When expressed in terms of $t = \sin w$, they are Legendre's normal forms. The constant k ($0 < k^2 < 1$) is the modulus and a is an arbitrary constant. If $\phi = \pi/2$, the integrals are called complete.

Series evaluation of the elliptic integrals may be made and numerical tables for them are available. They are called elliptic because they were first studied in order to determine the circumference of the ellipse. Their properties are best studied in terms of their inverse functions.

ELLIPTIC PARABOLOID. See paraboloid.

ELLIPTIC PARTIAL DIFFERENTIAL EQUATION. Consider the general linear partial differential equation of second order with two independent variables,

$$A \frac{\partial^2 u}{\partial x^2} + 2B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D \frac{\partial u}{\partial x} + E \frac{\partial u}{\partial y} + F = 0$$

where the coefficients A, B, C, D, E, F are continuous functions of x, y . This equation, which includes the majority of the differential equations of physics, is said to be of *elliptic*, *hyperbolic* or *parabolic* type at a point x, y according as the discriminant $B^2 - AC$ is negative, positive, or zero. By a real transformation of variables it can be brought, respectively, into one of the following three forms,

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \dots \quad \text{elliptic}$$

$$\frac{\partial^2 u}{\partial x \partial y} + \dots \quad \text{hyperbolic}$$

$$\frac{\partial^2 u}{\partial x^2} + \dots \quad \text{parabolic}$$

where in each case the dots denote terms of lower order than the second in the derivatives of u . These concepts can easily be generalized for equations of higher order.

For an elliptic partial differential equation, it is customary, if the solution is to be well defined, to impose conditions along the boundary, such as the value of the dependent variable (Dirichlet condition), or its normal derivative (Neumann condition). In the simplest case, namely, the Laplace equation,

$$\nabla^2 \phi = 0,$$

the solution may be approximated by finite difference methods by representing the Laplace operator (also written $D_x^2 + D_y^2$) by $h^{-2}(\delta_x^2 + \delta_y^2)$ which gives rise to the coefficient scheme

$$\begin{array}{ccc} & & 1 \\ & \bullet & \\ & & \\ 1 & & -4 & & 1 \\ & & & & \\ & & & & 1 \end{array}$$

which is always implicit (see **matrix inversion**). (See also Lothar Collatz, *Numerische Behandlung von Differentialgleichungen*, 2nd edition, Springer, 1955; Kaiser S. Kunz, *Numerical Analysis*, McGraw-Hill Book Company, 1957.)

ELLIPTIC POINT (ON A SURFACE). A point at which the product of the principal curvatures of the surface (see **curvature of surface**, **radius of**) is positive.

ELONGATION OR EXTENSION. The increase in length of a line joining two points, especially the increase of the gauge length of a tension specimen.

ELONGATION QUADRIC. See **strain ellipsoid**.

EMAGRAM. A thermodynamic diagram, designed by A. Refsdal, with temperature as abscissa and the logarithm of pressure, increasing downward, as ordinate. Since work or energy in a cyclic process is expressible as

$$R \oint T \frac{dp}{p}$$

where T is temperature, p pressure, and R the gas constant, it is proportional to the area enclosed on the emagram by the curve representing the process. (See **skew T — log p diagram**.)

EMDEN EQUATION (EMDEN FUNCTION). Various models of the stars have been developed for various conditions. A number of polytropic models have been proposed, subject to the condition that pressure and density would be determined in accord with the adiabatic gas law

$$P = k\rho^\gamma.$$

One of the fundamental equations which a gaseous sphere in adiabatic equilibrium must satisfy is known as the Emden equation

$$\frac{1}{x^2} \frac{d}{dx} \left(x^2 \frac{dy}{dx} \right) + y^n = 0$$

in which n is defined by

$$\gamma = 1 + \frac{1}{n}.$$

Tables of Emden functions have been published by the British Association of Science for various values of the polytropic index n .

EMERGENT STEM CORRECTION. See **exposed stem correction**.

EMISSION COEFFICIENT. The emission coefficient ϵ_ν measures the energy emitted per second in the frequency range $d\nu$ by a volume element dV into the solid angle $d\omega$. The total energy per second emitted by a volume element is therefore

$$E = dV \int_0^\infty \int_\omega \epsilon_\nu d\nu d\omega.$$

ϵ_ν depends on the physical properties of the emitting substance, on the frequency ν , and, sometimes, on the direction of the flow of radiation.

The total energy per second emitted by a volume element may also be expressed in terms of wavelengths, as is the usage in certain other forms of emission functions, by the relationship

$$E = dV \int_0^\infty \int_\omega \epsilon_\lambda d\lambda d\omega$$

where ϵ_λ measures the energy emitted per second in the wavelength range $d\lambda$ by the volume element dV into the solid angle $d\omega$.

EMISSION SPECTRUM. See **spectrum**, **emission**.

EMISSIVE POWER. In radiation, energy emitted per unit time and area from a surface over the waveband spectrum. When the energy is computed at a given wavelength, it is called the **monochromatic emissive power**. I_λ , where

$$I_\lambda = \frac{dq}{d\lambda}.$$

(See **black body radiation**.)

EMISSIVITY. (More precisely, hemispheric emissivity.) The ratio of the radiant energy q emitted by a surface to that, q_b , emitted by a **black body** at the same temperature T ,

$$\epsilon = \frac{q}{q_b}.$$

The emissivity at one particular wavelength is called **monochromatic (hemispheric) emissivity**. The monochromatic emissivity of a surface is a function of the wavelength λ , the angle of emission ϕ , and the temperature T . (See also **spectral emissivity**; **total emissivity**.)

EMITTANCE. The radiant emittance of a source is the power radiated per unit area of the surface. This may be either the radiant emittance per unit range in wavelength, the spectral radiant emittance, or its integral over all wavelengths, the total radiant emittance. If the radiant emittance is evaluated by the **standard luminosity function**, it is called luminous emittance. For a perfectly diffusing surface, the luminous emittance is equal to π times the **luminance intensity**.

EMU SYSTEM OF UNITS. See **electromagnetic units**.

ENCASTRÉ BEAM. See **beam, fixed-ended**.

END MOMENT. See **moment, end**.

ENDOMORPHISM. See **homomorphism**.

ENDOTHERMIC REACTIONS. See **thermal coefficients**.

END-POINT METHOD. A method of approximation for treating the integral equations with displacement kernel, which appear in problems of **neutron transport theory** characterized by plane or spherical geometry. In this method, the solution near each boundary is treated as though no other boundary existed, in which case the integral equation is amenable to the Wiener-Hopf technique of solution. A complete solution is then obtained by combining the one-boundary solutions in such a way that their asymptotic values coincide.

ENDURANCE LIMIT. See **fatigue**.

ENERGY. Energy is often defined as the ability to do work. Thus, a capacitance of magnitude C , carrying a charge Q , possesses electrical energy in the amount $Q^2/2C$, and can do this much work in the process of being discharged. In this instance, the energy is considered to reside in the electric field between the plates of the capacitor. In other cases, e.g., that of kinetic energy (see **energy, kinetic**), the energy is considered to reside in the body itself. The mechanical or electrical energy of a system is always measured as the maximum amount of work that the system can do in coming to static equilibrium.

In thermodynamics, energy is a general term embracing heat, work (both being "energy in transition") and internal energy. Some authors tend to restrict the term energy to be synonymous with internal energy. In that

terminology, heat and work are distinct from energy and are not regarded as "forms of energy." The authors who use the term energy for internal energy tend to use the term "internal energy" in a more restricted sense too. In that terminology, internal energy means that part which describes the internal energy (in the wider sense) of a pure, homogeneous stable substance at rest in a constant (or very weak) gravitational, electric and magnetic field and free from surface tension.

In special relativity, energy and mass are equivalent, being connected by the Einstein equation $E = mc^2$ where c is the speed of light. Hence, when relativistic mechanics must be applied, e.g., when speeds comparable to c are involved, the energy of a system includes the rest energy m_0c^2 of all the bodies in the system. Here m_0 is the mass of a body at rest with respect to the other bodies of the system and to the observer.

Energy is particularly important because it is a conserved quantity, which can be neither created nor destroyed. It may, however, be exchanged among various bodies or may be converted from one form to another, or interconverted with mass as in the Einstein equation above. (See also **work; conservation of energy**.)

ENERGY, ACTIVATION. See **activation energy**.

ENERGY, BINDING. See **binding energy**.

ENERGY, CLASSICAL ELECTRON. See **electron energy (classical)**.

ENERGY, COHESION. The energy which would be required to break up a solid or liquid into its constituent atoms or molecules. The cohesion energy is often expressed as the energy per mole of a body of such extent that the surface energy is negligibly small.

ENERGY CONSERVATION. See **conservation of energy**.

ENERGY CONSERVATION IN CLOSED SYSTEMS. In closed systems and in the absence of an external field, the energy U supplied from the outside during the time interval dt is equal to the sum of the heat dQ and the mechanical work dW performed at the boundaries of the system. If the pressure is normal to the surface, the mechanical work is simply

$-pdV$ and the expression of the energy conservation becomes (see also **energy conservation in thermodynamics**)

$$dU = dQ - pdV. \quad (1)$$

From a purely phenomenological point of view (1) may be considered as the definition of the heat received by the system. The extension of the mechanical principle of conservation of energy to include the flow of heat is due mainly to Carnot, Joule, Helmholtz and Clausius.

ENERGY CONSERVATION IN CONTINUOUS SYSTEMS. In order to express the energy conservation in continuous systems, it is useful to introduce the energy density per unit volume

$$u_v = \frac{\delta U}{\delta V}. \quad (1)$$

In agreement with the general formulation of the principle of energy conservation (see **energy conservation in thermodynamics**) we shall have

$$\frac{\partial u_v}{\partial t} = -\operatorname{div} \Phi[U] \quad (2)$$

where $\Phi[U]$ is the energy flow. This flow contains in general different contributions, among which are

(1) the convection flow corresponding to a center of mass motion ω

$$u_v \omega \quad (3)$$

(2) a heat flow

$$\mathbf{Q} \quad (4)$$

(3) a flow of energy corresponding to the pressure tensor p_{ij} in the fluid, its i component being

$$\sum_j p_{ij} \omega_j \quad (5)$$

(4) a flow of potential energy (for example the **Poynting** flow of electromagnetic energy)

(5) a flow of energy related to diffusion.

The total energy U may be split into an internal energy, a potential energy and a macroscopic kinetic energy. Each contribution taken separately does no more satisfy an equation of the simple form (2), because of the possible transformation of one form of energy to another.

ENERGY CONSERVATION IN THERMODYNAMICS. The principle of conservation of energy plays in thermodynamics a fundamental role. It is therefore also called the first law of thermodynamics. (See **thermodynamics, first law of**.) In its most general form it postulates the existence of a function of state, called the internal energy of the system U , such that its change per unit time is equal to some flow, called the energy flow from the surroundings.

This statement can be expressed symbolically by the formula (see **conservation of mass in open systems**)

$$dU = d_e U \quad \text{or} \quad d_i U = 0 \quad (1)$$

$d_e U$ is the energy received during the time dt from the outside and $d_i U$ is the energy "creation" inside the system.

The explicit form of the energy flow depends on the nature of the system considered. (See **energy conservation in closed systems; energy conservation in continuous systems**.)

ENERGY, COULOMB. See **Coulomb energy**.

ENERGY DENSITY (ELECTROMAGNETIC). The energy required to set up electric and magnetic fields can be thought of as residing in the fields. The energy density is then the field energy per unit volume.

The energy required to set up an electric field, E , in space, or in an isotropic medium, is given, if rationalized units are used, by $\int \frac{1}{2} \epsilon E^2 d\tau$, where $d\tau$ is a volume element. The expression $\frac{1}{2} \epsilon E^2$ is the electric energy density.

Similarly, the energy required to set up a magnetic field, H , is $\int \frac{1}{2} \mu H^2 d\tau$, and the expression $\frac{1}{2} \mu H^2$ is the magnetic energy density. In an electromagnetic wave the instantaneous value of the energy density is $\frac{1}{2} \epsilon E^2 + \frac{1}{2} \mu H^2$ where E and H are the instantaneous magnitudes of the electric and magnetic fields.

For anisotropic media the terms ϵE^2 and μH^2 used above should be replaced by $\mathbf{E} \cdot \mathbf{D}$ and $\mathbf{H} \cdot \mathbf{B}$ respectively. If non-rationalized units are employed, the factor $\frac{1}{2}$ is replaced by $\frac{1}{8\pi}$.

ENERGY DENSITY SPECTRUM. (Sometimes called energy spectrum.) The square of the amplitude of the (complex) **Fourier transform** of an aperiodic function. Thus, if

$f(t)$ is the given function, its Fourier transform is

$$F(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f(t)e^{-i\omega t} dt,$$

and the energy density spectrum is $|F(\omega)|^2$. It is assumed that the total energy

$$\int_{-\infty}^{\infty} |f|^2 dt$$

is finite. (See **power spectrum**.)

ENERGY, DISTORTION. See **distortion energy**.

ENERGY, DISTRIBUTION IN. See **distribution in energy**.

ENERGY EIGENFUNCTION. An **eigenfunction** of the **Hamiltonian operator**. The associated eigenvalue is often called the *energy eigenvalue*.

ENERGY EIGENSTATE. See **energy eigenfunction**.

ENERGY EQUATION FOR BOUNDARY LAYER. The energy equation for steady flow of an incompressible fluid in a two-dimensional boundary layer is

$$\frac{1}{2U^3} \frac{d}{dx} (U^3 \delta^{**}) = \frac{1}{\rho U^2} \int_0^{\delta} \tau \frac{\partial}{\partial y} \left(\frac{u}{U} \right) dy, \quad (1)$$

where δ^{**} is the energy thickness, τ is the shear stress, u is the local velocity at a distance y from the wall and U is the velocity outside the boundary layer, i.e., for $y > \delta$.

Equation (1) applies both to laminar and to turbulent boundary layers. The right-hand side represents, non-dimensionally, the rate at which energy is taken from the mean flow by the shear stresses, while the left-hand side represents the rate of increase of kinetic energy defect of the mean flow.

In the case of a laminar boundary layer,

$$\tau = \mu \frac{du}{dy},$$

so that Equation (1) may be written

$$\frac{1}{2} \frac{d}{dx} (U^3 \delta^{**}) = \frac{\mu}{\rho} \int_0^{\delta} \left(\frac{\partial u}{\partial y} \right)^2 dy. \quad (2)$$

ENERGY EQUATION FOR STEADY ADIABATIC FLOW. For any adiabatic gas flow,

i.e., with no heat transfer across the boundaries of a stream tube,

$$h + \frac{1}{2}v^2 = \text{constant}, \quad (1)$$

where v is velocity and h is specific enthalpy. For a perfect gas, Equation (1) becomes

$$C_p T + \frac{1}{2}v^2 = C_p T_o, \quad (2)$$

where T is the temperature, C_p is the specific heat at constant pressure, and suffix o refers to "stagnation" conditions (with $v = 0$).

Equation (2) may also be written in the alternative forms

$$\left(\frac{\gamma}{\gamma - 1} \right) \frac{p}{\rho} + \frac{1}{2}v^2 = \left(\frac{\gamma}{\gamma - 1} \right) \frac{p_o}{\rho_o} \quad (3)$$

or

$$\frac{a^2}{\gamma - 1} + \frac{1}{2}v^2 = \frac{a_o^2}{\gamma - 1}, \quad (4)$$

where a is the velocity of sound.

It should be noted that Equations (1) to (4) are valid even when viscous effects are important, e.g., in flow through a shock wave.

ENERGY, EQUIPARTITION OF. See **equipartition of energy**.

ENERGY, EXCITATION. See **excitation energy**.

ENERGY, FREE. See **free energy**.

ENERGY GAP. The energy gap ϵ_g is the minimum energy required to excite an electron from the normally filled **valence band** to the normally empty **conduction band**. For thermal equilibrium at temperature T the following relation enables ϵ_g to be determined:

$$np = 2.33 \times 10^{31} \left(\frac{m_n m_p}{m^2} \right)^{3/2} T^3 \exp(-\epsilon_g/kT)$$

where n and p are the number of electrons and the number of holes per cm^3 respectively, m_n and m_p are the effective masses of electrons and holes, m is the free electron mass, and k is Boltzmann's constant. The value of ϵ_g for exciting an electron from a valence band to a conduction band optically is generally higher than that for thermal excitation.

ENERGY INTEGRAL. The first integration of the **Newton equation of motion** $m\dot{\mathbf{r}} = \mathbf{F}$, in which \mathbf{r} is the position vector of the particle of mass m and \mathbf{F} is the resultant (vector) force acting in it, yields an integral of the form

$$\frac{1}{2}mv^2 - \int \mathbf{F} \cdot d\mathbf{r} = C$$

where v is the velocity at any instant and C is a constant of integration. This is usually called the energy integral, since $\frac{1}{2}mv^2$ is the kinetic energy. If \mathbf{F} is a conservative force

$-\int \mathbf{F} \cdot d\mathbf{r}$ depends only on the position of the particle and is therefore called the potential energy. The constant C becomes the total energy and the existence of the energy integral implies the conservation of mechanical energy.

ENERGY, INTERNAL. See internal energy.

ENERGY, IONIZATION. See ionization energy.

ENERGY, KINETIC. The scalar quantity or measure of the energy of motion is called kinetic energy. It is given by $\frac{1}{2}mv^2$ for a particle of mass m traveling with speed v , and by $\frac{1}{2}I_o\omega^2$ for a rigid body of moment of inertia I_o rotating about a fixed axis O with angular velocity ω . The kinetic energy for general motion of a rigid body is

$$\frac{1}{2}mv_c^2 + \frac{1}{2}I_{cx}\omega_x^2 + \frac{1}{2}I_{cy}\omega_y^2 + \frac{1}{2}I_{cz}\omega_z^2$$

where m is the total mass and x, y, z , are principal axes through the center of mass denoted by subscript c .

ENERGY, KINETIC, OF A MASS POINT. In classical mechanics it is $\frac{1}{2}m_o v^2$, but in relativistic mechanics it is

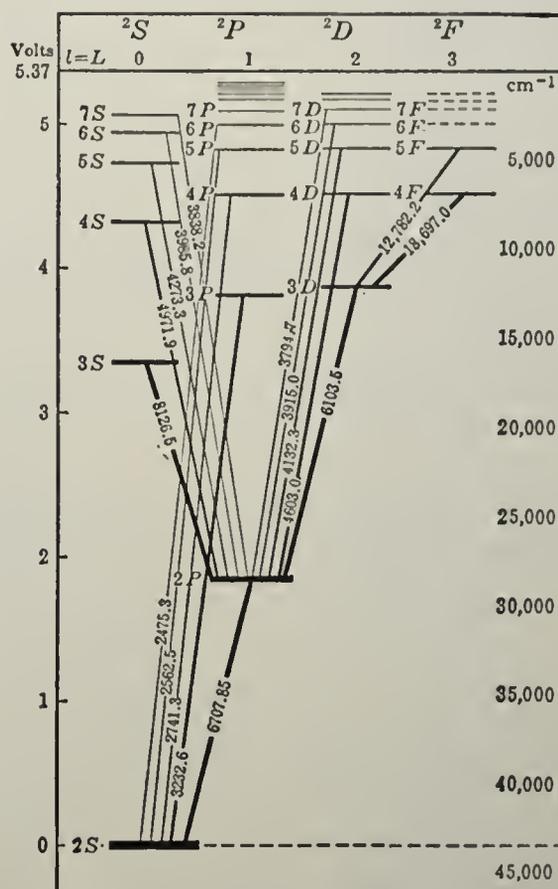
$$\frac{m_o c^2}{\sqrt{1 - v^2/c^2}} = m_o c^2 + \frac{1}{2}m_o v^2 + \dots \text{ for small } v/c;$$

where m_o is the rest mass, v is the speed of the particle, and c is the speed of light.

ENERGY LEVEL. A stationary state of energy of any physical system. The existence of many stable, or quasi-stable states, in which the energy of the system stays constant for some reasonable length of time, is an essential characteristic of quantum-mechanical systems, and is the basis of large areas of modern physics.

ENERGY LEVEL DIAGRAM. The graphical presentation of the energy levels of an atom, molecule, nucleus, etc., with respect to an arbitrary zero level, as a rule the ground state, or, for atoms, the ionized state. Atomic

and molecular energy level diagrams are in general designed to give a picture of the energetic order of the different types of energy state terms (S, P, D , etc.) and of the terms of different multiplicity. Usually important transitions are indicated by lines connecting the energy levels involved. See for example the energy level diagram of the lithium atom (figure). (The numbers on the lines indicating the transitions are the corresponding wavelengths.)



Energy level diagram of the lithium atom (after Grotian). (From G. Herzberg, *Molecular Spectra and Molecular Structure I Spectra of Diatomic Molecules*, 2nd ed., D. Van Nostrand Company, Inc., Princeton, 1950).

ENERGY LEVELS, ATOMIC, STATISTICAL WEIGHT OF. See statistical weight of atomic energy levels.

ENERGY LEVELS, MOLECULAR. See molecular energy levels; electronic energy levels of a molecule; rotational energy levels of a molecule; vibrational energy levels of a molecule.

ENERGY LEVELS, MOLECULAR, STATISTICAL WEIGHT OF. See statistical weight of molecular energy levels.

ENERGY LEVELS OF HARMONIC OSCILLATOR. See harmonic oscillator, energy levels of.

ENERGY LEVELS OF RIGID DIATOMIC ROTATOR. See rigid diatomic rotator, energy levels of.

ENERGY, MECHANICAL. For a conservative dynamical system the mechanical energy is the invariant of the motion which is equal to the sum of the kinetic energy (see **energy, kinetic**) and the potential energy (see **energy, potential**). This invariant or constant results from a first integration of the equations of motion and the first integral is usually known as the **energy integral**. Strictly speaking the mechanical energy does not exist for a non-conservative system. However, the concept is so useful that it is customary to treat a dissipative system as a system whose energy (defined for the equivalent non-dissipative system) decreases with the time. This makes possible an important connection with non-mechanical forms of macroscopic energy, for example, heat.

ENERGY MOMENTUM TENSOR. See Lagrangian.

ENERGY, MUTUAL. See interaction energy.

ENERGY, NEGATIVE KINETIC. (1) In quantum mechanics there may be a finite probability of finding a particle in the classically forbidden region where the total energy of the particle is less than its potential energy. In such a region, the kinetic energy, given as the difference between the total energy and the potential energy, would be negative. If an attempt is made to observe a particle in this classically forbidden region, one gives to the particle an indeterminate amount of energy (see **uncertainty principle**), with the result that negative kinetic energy is not observable. Particles pass through such classically forbidden regions of negative kinetic energy in the *tunnel effect*.

(2) The term is also used to describe the energy states of particles in those energy levels that were first predicted by the **Dirac electron theory**.

ENERGY OF A CHARGED SYSTEM. If the charges q_i are varied on a set of conductors at potentials V_i , the resulting change of energy of the system is given by

$$dU = \sum_i V_i dq_i.$$

Expressed in terms of coefficients of potential (see **potential, coefficients of**)

$$dU = \sum_i \sum_j p_{ij} q_j dq_i$$

which yields upon integration

$$\begin{aligned} U &= \frac{1}{2} \sum_i \sum_j p_{ij} q_i q_j = \frac{1}{2} \sum_i V_i q_i \\ &= \frac{1}{2} \sum_i \sum_j c_{ij} V_i V_j \end{aligned}$$

where the c_{ij} are the coefficients of induction. (See **induction, coefficients of**.)

ENERGY OF ACTIVATION. See activation energy.

ENERGY OF A SYSTEM OF CURRENT CIRCUITS. The magnetic fluxes through a set of circuits carrying steady currents I_j can be written

$$\dot{\phi}_i = \sum_j M_{ij} I_j$$

where the $M_{ij} = M_{ji}$ are the **mutual inductances**, and $M_{ii} \equiv L_i$ is the **self-inductance** of the i th circuit. If the circuits are of constant length, i.e., they may be non-rigid, but have no sliding contacts, the induced emf's are

$$\begin{aligned} \varepsilon_i &= -d\phi_i/dt \\ &= -\sum_j M_{ij} dI_j/dt - \sum_j I_j dM_{ij}/dt. \end{aligned}$$

Establishing the currents requires doing work at the rate

$$\frac{dW}{dt} = -\sum I_i \varepsilon_i$$

hence

$$\begin{aligned} dW &= -\sum I_i \varepsilon_i dt \\ &= \sum \sum M_{ij} I_i dI_j + \sum \sum I_i I_j dM_{ij}. \end{aligned}$$

If the circuits are held stationary so that no mechanical work is done, the work becomes stored energy of the system, and

$$\begin{aligned} dU &= \sum \sum M_{ij} I_i dI_j \\ U &= \frac{1}{2} \sum \sum M_{ij} I_i I_j. \end{aligned}$$

ENERGY OF n-PARTICLE SYSTEM (THE VIRIAL). If in a system of n particles confined to a finite region of space whose position vectors with respect to a given origin are \mathbf{r}_i the resultant force on the i th particle is \mathbf{F}_i , the virial of the system is defined to be

$$-\frac{1}{2} \overline{\sum_{i=1}^n \mathbf{r}_i \cdot \mathbf{F}_i}$$

The bar over the sum refers to a time average over a time interval long compared with the time taken by a particle to traverse the region in which the particles are confined. It is understood that not only are the magnitudes of the position vectors \mathbf{r}_i bounded, but that the same is true of the velocities of the particles. According to the virial theorem the virial of the system is equal to the average kinetic energy of the system.

ENERGY, POTENTIAL. The potential energy of a system is energy of configuration or position measured with respect to a reference configuration. For a static conservative system, the potential energy is the work needed to bring the system from the reference configuration to the configuration it is in. The potential energy is a minimum for stable equilibrium. (See **conservative force field**.)

ENERGY PRODUCT. The product of the magnitudes of \mathbf{B} and \mathbf{H} at any point on the demagnetization curve. The maximum value of this product is the best single criterion for a material to be used in permanent magnets. (See **hysteresis**.)

ENERGY RELATIONS IN UNIFORM FORCE FIELD. A uniform force field is a special case of central force field (see **force, central**). In such a field the force is constant in direction and magnitude. Examples: the gravitational field near the surface of the earth, and the electrostatic field between the two parallel plates of a condenser.

ENERGY, RELATIVE. The energy of particles in the **center of mass** (coordinate) system.

ENERGY STATE. See **state, energy and energy eigenfunction**.

ENERGY STATE, NEGATIVE. (1) Any bound state, in which the sum of the kinetic energy and the potential energy, the latter reckoned relative to zero at infinity, is less than zero. The existence of such states is essential for the stability of any system that is not surrounded by a region of positive potential energy, such as the Coulomb barrier.

(2) A consequence of the **Dirac electron**

theory is that there exist electron states of negative total energy (including both rest mass energy and kinetic energy). Electrons in such states of negative energy are unobservable, only electrons of positive total energy being observable. The allowed positive and negative states are restricted (only $E > m_0c^2$ and $E < m_0c^2$ are allowed in a field-free region). If a γ -ray photon of energy greater than $2m_0c^2$ (where m_0 is the rest mass energy of the electron) is absorbed by an electron of negative energy, it will be lifted into a positive energy state and will become observable. The positron is identified with the hole that is left behind.

The negative energy states are ordinarily completely filled with electrons, with no room for further electrons, a consequence of the **Pauli exclusion principle**. However, when a hole is created, an electron of positive energy near by may "fall" into it (pair annihilation) with the emission of γ -rays.

A similar picture may be developed for any pair composed of a particle and its anti-particle, the latter being the result of the lifting of the particle from a state of negative energy to one of positive energy.

ENERGY STATE TERMS. Terms designating the discrete energy states of a particle in a system. Thus the energy states of an atom are called S, P, D, F, \dots terms, respectively, corresponding to the values 0, 1, 2, 3, \dots of L , the resultant angular momentum **quantum number** of the atom. The energy states of a molecule are called $\Sigma, \Pi, \Delta, \Phi \dots$ terms, respectively, corresponding to the values 0, 1, 2, 3, \dots of Λ , the electronic orbital angular momentum (about internuclear axis) quantum number.

The letters indicating the value of L are usually preceded by a superscript denoting the **multiplicity** and followed by a subscript denoting the total angular momentum quantum number J . In addition, the principal quantum number is often written as a coefficient. Energy state terms and their transitions are shown in **energy level diagrams**.

ENERGY THICKNESS OF BOUNDARY LAYER. The energy thickness δ^{**} for a boundary layer is defined by

$$\delta^{**} = \int_0^{\delta} \frac{u}{U} \left[1 - \left(\frac{u}{U} \right)^2 \right] dy,$$

where u is the velocity at a distance y from the wall and U is the velocity outside the boundary layer, i.e., for $y > \delta$.

The defect of flux of kinetic energy, per unit width of boundary layer, is $\frac{1}{2}\rho U^3\delta^{**}$. (See also **energy equation for boundary layer**.)

ENGINE. (1) A mechanical device which produces some desired effect. (2) In a more restricted sense, a mechanical device which produces power, as distinct from a machine which produces some desired effect (e.g., the compression of a gas by a compressor; the maintenance of a body at a temperature lower than that of the surroundings by a refrigerator, etc.) other than the production of power. In that sense the term engine is synonymous with *prime mover*.

ENGINE CAPACITY. See **swept volume**.

ENGINEERING SYSTEM OF UNITS. (Also known as gravitational system of units.) A system of units based on the following three fundamental units: force, length, time. There exist several gravitational systems of units. The metric gravitational system is based on the kilogram (kg) as unit of force. In order to avoid confusion this unit has now been renamed kilopond (kp). The unit of length is the meter (m), and the unit of time is the second. The English gravitational system (also called the f lbf s system) is based on the pound (lb or lbf) for force, the foot (ft) for length and the second (sec) for time. The table below lists the principal derived units in the above systems.

GRAVITATIONAL
SYSTEM

Derived unit	Metric	English
Mass	1 $\frac{\text{kp sec}^2}{\text{m}}$ (no name)	1 slug = 1 $\frac{\text{lbf sec}^2}{\text{ft}}$
Work, energy	1 kp m (no name)	1 lbf ft = 1 foot pound
Pressure	1 $\frac{\text{kp}}{\text{m}^2}$ or 1 at = $10^4 \frac{\text{kp}}{\text{m}^2}$	1 $\frac{\text{lbf}}{\text{ft}^2}$ or 1 $\frac{\text{lbf}}{\text{in}^2} = 144 \frac{\text{lbf}}{\text{ft}^2}$
Power	1 $\frac{\text{kp m}}{\text{sec}}$ or 1 PS = $75 \frac{\text{kp m}}{\text{sec}}$	1 $\frac{\text{lbf ft}}{\text{sec}}$ or 1 HP = $550 \frac{\text{lbf ft}}{\text{sec}}$

(See also **acoustical units; electrical units; mechanical units; thermal units**.)

ENLARGEMENT ENERGY. See **delocalization energy**.

ENLARGEMENT, METHODS OF. Methods of **matrix inversion** by which the inverse of a matrix of order m is expressible in terms of that of a submatrix of order $m - 1$.

ENSEMBLE. A collection of similar systems considered in statistical mechanics. Ensembles were introduced by Gibbs, and their importance lies in the fact that the average behavior of a system in an ensemble can often be used to predict the behavior of an actual physical system. Usually all systems in an ensemble are supposed to have the same number of constituent particles. Such ensembles are called *petit ensembles*. Examples of petit ensembles which are used extensively are the **micro-** and **macrocanonical** ensembles. In order to study systems in which chemical reactions can take place, Gibbs introduced the so-called *grand ensembles* where the different systems do not have all the same number of particles. The most often used grand ensemble is the **grand canonical ensemble**.

In the Fowler-Darwin formulation of statistical mechanics, an ensemble is a single macrocanonical (macroscopic) collection of identical microcanonical systems. Average values are taken over the microcanonical possible states of the single macrocanonical ensemble of microcanonical systems, rather than over a set of replicas of a macrocanonical system.

ENSEMBLE, CANONICAL. See **canonical ensemble**.

ENSEMBLE, COOPERATIVE. An ensemble in which the interactions between the systems composing the ensemble are not negligible. The state of a given system is largely determined by the states of the neighboring systems.

ENSEMBLE DENSITY. A quantity ρ defined in such a way that $\rho d\Omega$ is the fraction of systems in an **ensemble** which have values of the momenta and position coordinates of all the particles in the system corresponding to a point in **gamma-space** within the **extension in phase** $d\Omega$. (For **grand ensembles** one must suitably alter this definition.)

ENSEMBLE, IDEAL. An ensemble in which the interactions between the systems composing the assembly can be neglected, such as a perfect gas or an ideal solution.

ENSEMBLE, MICROCANONICAL. See microcanonical ensemble.

ENSKOG-MAXWELL EQUATIONS OF CHANGE. The evaluation of any molecular property ϕ depending on velocity, position or time, can be obtained from the Boltzmann equation on multiplication by ϕ , and integration over the whole velocity space.

One obtains thus the Enskog-Maxwell equations of change

$$\frac{\partial(n\bar{\phi})}{\partial t} = -\frac{\partial}{\partial \mathbf{r}} \cdot (n\bar{\phi}\mathbf{v}) + n \left(\frac{\partial \bar{\phi}}{\partial t} + \mathbf{v} \frac{\partial \bar{\phi}}{\partial \mathbf{v}} + \frac{\mathbf{X}}{m} \frac{\partial \bar{\phi}}{\partial \mathbf{v}} \right) + 2\pi \sum_j \iiint \phi(f'_i f'_j - f_i f_j) \mathbf{g}_{ij} b db dv_i dv_j \quad (1)$$

$\bar{\phi}$ is the average value of ϕ ; n is the concentration in molecules per cc, \mathbf{v} , the molecular velocity, \mathbf{X} , the external force, f_i, f_j, f'_i and f'_j are the molecular distribution functions of the molecules involved in the direct and inverse collisions respectively. \mathbf{g}_{ij} is the relative velocity of the collision partners.

Equation (1) expresses the fact that the local variation of the property ϕ can result: (a) from the streaming of the molecules into and out of the volume element under consideration; (b) from the dependence of ϕ for each molecule on time, on its position and on its velocity, and (c) from the variation of ϕ through collisions between molecules.

(1) takes a much simpler form when the molecular property considered is a **summational invariant**, such as mass, momentum or energy. The last term of (1) is then equal to 0, and one obtains the **equation of continuity**, the **equation of motion**, and the **equation of energy conservation**, respectively. (See **Navier-Stokes equations, general treatment for mixtures.**)

ENSKOG SERIES FOR THE SOLUTIONS OF THE BOLTZMANN EQUATION. Enskog's solution of the Boltzmann equation is based on the introduction of a perturbation parameter ϵ which increases or decreases the total number, but does not change the rela-

tive number of collisions of various types. Enskog proposes an equation of the types:

$$\frac{\partial f_i}{\partial t} + \mathbf{v}_i \frac{\partial f_i}{\partial \mathbf{r}} + \mathbf{A}_i \frac{\partial f_i}{\partial \mathbf{v}_i} = \frac{1}{\epsilon} \sum_j J(f_i, f_j) \quad (1)$$

f_i is the distribution function of the molecule of velocity \mathbf{v}_i . \mathbf{A}_i is the acceleration caused by the external forces. $J(f_i, f_j)$ are the collision integrals (see **Boltzmann transport equation**).

The solution for small ϵ corresponds then to very frequent collisions, which tend to establish the local equilibrium.

The distribution function f_i is written as a power series in ϵ :

$$f_i = f_i^{(0)} + \epsilon f_i^{(1)} + \epsilon^2 f_i^{(2)} + \dots \quad (2)$$

When this expansion is introduced in Equation (1), one obtains, by equating the coefficients of equal powers of ϵ , a set of integro-differential equations:

$$0 = \sum_j J(f_i^{(0)}, f_j^{(0)}) \quad (3)$$

$$\frac{\partial f_i^{(0)}}{\partial t} + \mathbf{v}_i \frac{\partial f_i^{(0)}}{\partial \mathbf{r}} + \mathbf{A}_i \frac{\partial f_i^{(0)}}{\partial \mathbf{v}_i} = \sum_j [J(f_i^{(0)}, f_j^{(1)}) + J(f_i^{(1)}, f_j^{(0)})] \quad (4)$$

These equations can be solved, and the distribution function is obtained as a series of successive approximations:

$f^{(0)}$ is the local equilibrium distribution, leading to the Euler equation of change.

$f^{(1)}$ leads to the **Navier-Stokes equations**.

$f^{(2)}$ yields the **Burnett equations**, etc.

The convergence of this series has often been questioned, but the excellent agreement of the transport coefficients calculated in the first approximation with the experimental values and with the coefficients obtained by other methods for the solution of the Boltzmann equation (see **thirteen moments approximation**) indicate that the first Enskog approximation is fairly accurate for dilute gases under small relative gradients of the macroscopic quantities (temperature, concentration, pressure).

ENSKOG THEORY FOR THE TRANSPORT PHENOMENA IN DENSE GASES. Enskog has extended the classical theory for the trans-

port properties of diluted gases to the case of dense gas consisting of *hard spheres*. In this hypothesis, binary collision only must be retained in the collision integral of the **Boltzmann equation**; but these collision integrals are modified because of the **collisional transfer** which is important at high densities, and because of the finite volume of the molecules.

An equation of state for hard sphere gases was used to estimate the effect of the latter factor. When the experimental conditions are such that the intermolecular repulsive forces are predominant, the agreement between the experimental values and the values predicted by Enskog's theory, coupled with the above-mentioned equation of state, is surprisingly good, taking into account the rough approximation made.

ENTHALPY. Thermodynamic potential defined as

$$H = U + pV$$

where U is the internal energy of the system. In older text-books, the term total heat, to be discouraged, was used for enthalpy.

The principle of conservation of energy for closed systems expressed in terms of the enthalpy is

$$dH = dQ + Vdp.$$

(See also **thermodynamics, characteristic functions of**.)

ENTHALPY DROP. The difference between the final and initial specific enthalpy of the working fluid in a turbine or turbocompressor. Since the process of expansion in a turbine, or the process of compression in a turbocompressor is very nearly adiabatic (but not necessarily reversible), the application of the first law of thermodynamics to the respective open system gives

$$W = (h_1 - h_2)\dot{m}.$$

The work in a turbine or turbocompressor stage per unit mass flow \dot{m} is equal to the enthalpy drop. (Hence the importance of the concept in engineering thermodynamics.)

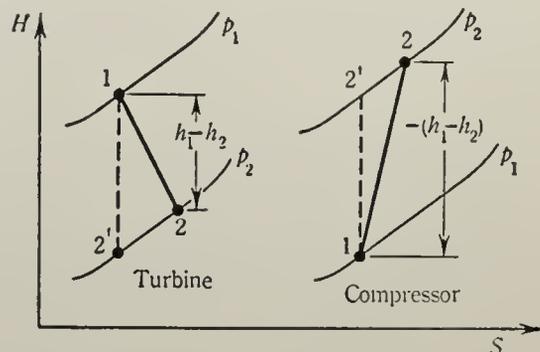
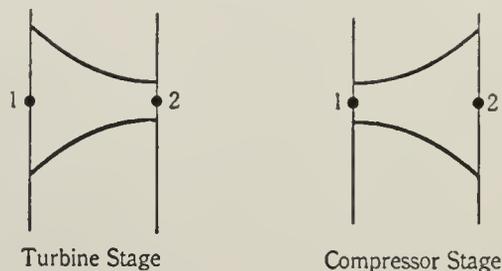
In the ideal case, the process of expansion or compression is also **isentropic**: in a real process, as shown in the figure, the entropy must increase; it cannot decrease according to the second law of **thermodynamics**. The ideal enthalpy drop is known as the *isentropic*

enthalpy drop. (In older text-books it is called adiabatic enthalpy drop.) Hence:

$$\text{enthalpy drop} = h_1 - h_2.$$

$$\text{isentropic enthalpy drop} = h_1 - h'_2.$$

The isentropic enthalpy drop serves as a standard of comparison for the operation of turbines, turbine stages, rotary compressors



and compressor stages. This is achieved by defining an expansion efficiency

$$\eta_e = \frac{h_1 - h_2}{h_1 - h'_2} \quad (\eta_e < 1)$$

and a compression efficiency

$$\eta_c = \frac{h'_2 - h_1}{h_2 - h_1} \quad (\eta_c < 1).$$

Values of η_e or η_c approaching unity signify low losses and good design.

The enthalpy drop is sometimes still called heat drop because in older texts the term "total heat" was used for "enthalpy"; however, such terminology is to be discouraged.

ENTHALPY OF ACTIVATION. See **absolute reaction rate theory**.

ENTHALPY THEOREM. The formulation of the first law of thermodynamics for open systems. (See **thermodynamics, first law of**.)

ENTIRE FUNCTION. An entire function of a complex variable for example, $\sin z$, e^z , is a function which has no singularities in the finite plane; that is, it is represented everywhere by its Taylor series about any point.

ENTRAINMENT. A process whereby fluid adjacent to a moving stream is caused to move with the stream, either on account of the viscous drag in the shearing layer which bounds the stream in laminar flow, or on account of the turbulent mixing at the edge of the stream. Turbulent entrainment is particularly important at the boundaries of a jet.

Entrainment also occurs on the boundary of a mass of buoyant fluid as it rises through otherwise stationary surroundings provided the fluids are miscible. The volume of the buoyant mass is thereby increased but the whole mass is decelerated.

ENTRANCE PUPIL. The Gaussian image of the aperture of an optical system in object space.

ENTROPIES, STATISTICAL METHOD OF. See statistical method of entropies.

ENTROPY. In thermodynamics, a property of a system which remains constant during a reversible adiabatic process, and which is defined by the equation

$$dS = dQ/T$$

where dS is an infinitesimal change in the entropy of a system, dQ is the infinitesimal quantity of heat that enters the system, and T is the Kelvin temperature of the system.

In statistical mechanics, entropy is a quantity proportional to the logarithm of the probability of the state considered, which is defined by the equation

$$S = k \log_e P + \text{constant}$$

where S is the entropy, k is the Boltzmann constant, and P is the statistical probability. It can be shown that the two definitions are equivalent.

For use of the term entropy in information theory, see information content, average.

(See also thermodynamics, second law of; thermodynamics, characteristic functions of; Carathéodory treatment of the second law; second law in one-component closed systems.)

ENTROPY CHART. A thermodynamic diagram in which entropy is used as one of the

coordinates, mostly restricted to systems with two independent variables. Entropy charts are often drawn exactly to scale and fulfill the function of an equation of state. They are widely used for practical calculations. The most widespread entropy chart is the **Mollier chart** drawn in the coordinates h,s (enthalpy-entropy). Mollier charts are very useful in calculations involving steam engines. Another useful entropy chart is that drawn in the coordinates T,s (absolute temperature-entropy).

ENTROPY, COMMUNAL. The contribution to the entropy of a gas due to the fact that the molecules occupy a common volume. Its value is nk , where n is the number of molecules present and k is the Boltzmann constant.

ENTROPY CONSTANT. An additive constant entering into the equation for the entropy which is determined by the requirement that entropy vanishes at the absolute zero of temperature. (See also thermodynamics, third law of.)

ENTROPY INCREASE, PRINCIPLE OF. It is a corollary of the second law of thermodynamics that in an isolated system a decrease in the total entropy is impossible—the total entropy of the system increasing when the process within it is irreversible, and remaining constant when the process is reversible.

An alternative and entirely equivalent formulation asserts that the change in entropy dS in any infinitesimal process in a closed system can be split into two parts

$$dS = d_e S + d_i S.$$

The part $d_e S$ is due to an exchange of heat with the surroundings and is given by

$$d_e S = \frac{dQ}{T}.$$

The part $d_i S$ is generated inside the system by the process, and can never be negative; it is positive for irreversible processes and zero for reversible processes. In a homogeneous system, the quantity

$$\dot{\theta} = \frac{d_i S}{dV dt}$$

is called the (local rate of) entropy production.

ENTROPY OF ACTIVATION. See absolute reaction rate theory; activated complex.

ENTROPY OF MONATOMIC GAS. Its value is given by one of the following three equations:

(Boltzmann statistics)

$$S = kN\left(\frac{5}{2} - \nu\right),$$

(Bose-Einstein statistics)

$$S = -\nu kN + \frac{5}{2}k \left(\frac{2\pi m}{\beta h^2}\right)^{\frac{3}{2}} V \sum_{n=1}^{\infty} \frac{e^{n\nu}}{n^{\frac{5}{2}}},$$

(Fermi-Dirac statistics)

$$S = -\nu kN - \frac{5}{2}k \left(\frac{2\pi m}{\beta h^2}\right)^{\frac{3}{2}} V \sum_{n=1}^{\infty} \frac{(-e^\nu)^n}{n^{\frac{5}{2}}},$$

where S is the entropy, k is Boltzmann's constant, m is the mass of the gas atoms, N is the total number of atoms in the system, V is the volume occupied by the gas, $\beta = 1/kT$ (T is absolute temperature), h is Planck's constant, and $\nu = \beta g$ (g is the partial **thermal potential**) which is related to N by the following equations:

(Boltzmann statistics)

$$N = \left(\frac{2\pi m}{\beta h^2}\right)^{\frac{3}{2}} V e^\nu,$$

(Bose-Einstein statistics)

$$N = \left(\frac{2\pi m}{\beta h^2}\right)^{\frac{3}{2}} V \sum_{n=1}^{\infty} \frac{e^{n\nu}}{n^{\frac{3}{2}}},$$

(Fermi-Dirac statistics)

$$N = - \left(\frac{2\pi m}{\beta h^2}\right)^{\frac{3}{2}} V \sum_{n=1}^{\infty} \frac{(-e^\nu)^n}{n^{\frac{3}{2}}}.$$

Expressed in terms of the pressure p and the temperature T one has in the case of Boltzmann statistics the **Sackur-Tetrode equation**,

$$S = k \left(\frac{5}{2} \ln T - \ln p + \frac{3}{2} \ln \frac{2\pi m}{h^2} + \frac{5}{2} \ln k + \frac{5}{2} \right),$$

and similar expressions for Bose-Einstein and Fermi-Dirac gases.

ENTROPY, STANDARD. The total **entropy** of a substance in a state defined as standard. Thus, the standard states of a solid or a liquid are regarded as those of the pure solid or the pure liquid, respectively, and at a

stated temperature. The standard state of a gas is at 1 atmosphere pressure and specified temperature, and its standard entropy is the change of entropy accompanying its expansion to zero pressure, or its compression from zero pressure to 1 atmosphere. The standard entropy of an ion is defined in a solution of unit activity, by assuming that the standard entropy of the hydrogen ion is zero.

ENTROPY, STATISTICAL DEFINITION. See **entropy** and **entropy of monatomic gas**.

ENTROPY VECTOR. The **four-vector** obtained by multiplying the proper entropy density by the local **four-velocity** in the relativistic theory of classical thermodynamics.

ENTROPY, VIRTUAL. See **virtual entropy**.

ENTRY LENGTH FOR PIPE OR CHANNEL. The usual expressions for velocity distribution and skin friction in pipe or channel flow refer to conditions at a large distance from the entry. Near the entry the **boundary layer** forms on the wall of the pipe or channel, and there is a central core of fluid which is unaffected by viscosity. As the boundary layer grows the central core is accelerated and the pressure falls. The central core ceases to exist when the boundary-layer thickness becomes equal to the radius of the pipe or the half-width of the channel. Still further downstream the velocity distribution becomes approximately parabolic (for laminar flow), the exact parabolic distribution being attained only asymptotically.

In a pipe of circular section the velocity distribution becomes approximately parabolic at a distance x from the entry given by $\frac{x}{dR} \approx 0.05$, where d is the diameter and R is the Reynolds number based on diameter and mean velocity. In a two-dimensional channel the corresponding condition is $\frac{x}{bR} \approx 0.05$, where b is the channel width and R is the Reynolds number based on this width and the mean velocity.

When the Reynolds number is so large that the flow eventually becomes turbulent, the flow near the entry depends on the initial disturbances. When the entry is almost undisturbed, transition to turbulent flow occurs when the Reynolds number based on distance from the entry is of order 10^5 to 10^6 . Still further downstream the velocity distribution

assumes its final form. For more disturbed entry conditions, the positions both for transition to turbulent flow and for attainment of the fully developed velocity distribution are further upstream.

ENUMERABLE. Synonym of **countable**, denumerable.

ENUMERABLE SET. See **cardinal**.

ENVELOPE. A curve that is tangent to each of a given family of curves. The envelope of the family of curves $f(x,y,t) = 0$, where t is the variable parameter of the family, is given by the pair of equations

$$f(x,y,t) = 0; \quad \frac{\partial f}{\partial t} = f_t(x,y,t) = 0.$$

These parametric equations of the envelope may be used to eliminate the parameter. This definition may be at once generalized to higher dimensions; e.g., the envelope of a family of surfaces. (See **surface**, **characteristic of**.)

ENVELOPE OF FAMILY OF SURFACES. See **surface**, **characteristic of**.

ENVELOPE OF TWO-PARAMETER FAMILY OF SURFACES. See **characteristic points for a two-parameter family of surfaces**.

ENVIRONMENTAL LAPSE RATE. The rate of decrease of temperature with elevation, $-\partial T/\partial z$, or occasionally $\partial T/\partial p$, where p is pressure. The concept may be applied to other atmospheric variables (e.g., lapse rate of density) if these are specified. The environmental lapse rate is determined by the distribution of temperature in the vertical at a given time and place and should be carefully distinguished from the **process lapse rate**, which applies to an individual air parcel. (See **supcradiabatic lapse rate**.)

EOLOTROPIC. An alternative spelling of **acolutropic** and synonymous with **anisotropic**, meaning not **isotropic**.

EPSILON-SYSTEM (IN n -DIMENSIONS). The system $\epsilon_{i_1 i_2 \dots i_n}$ or $\epsilon^{i_1 i_2 \dots i_n}$ defined by

$$\epsilon_{i_1 i_2 \dots i_n} = \sqrt{g} e_{i_1 i_2 \dots i_n}$$

$$\epsilon^{i_1 i_2 \dots i_n} = \frac{1}{\sqrt{g}} e^{i_1 i_2 \dots i_n}$$

where g is the determinant of the components of the covariant **metric tensor** for an n -dimensional coordinate system and $e_{i_1 i_2 \dots i_n}$ and $e^{i_1 i_2 \dots i_n}$ are **e-systems**.

EQUATION. For particular equations, **differential**, **integral**, **Laplace**, **Abcl**, etc., see under the respective names.

EQUATION OF CHANGE. See **Enskog-Maxwell equation of change**.

EQUATION OF CONTINUITY. The flux of fluid across any closed surface always composed of the same fluid particles is zero, or the rate of accumulation of mass within a closed surface fixed in space is equal to the flux across the surface. The corresponding forms of the equation are

$$\frac{D\rho}{Dt} + \rho \operatorname{div} \mathbf{v} = 0$$

$$\frac{\partial \rho}{\partial t} + \operatorname{div} \rho \mathbf{v} = 0.$$

For an incompressible fluid $D\rho/Dt = 0$ and the equation is

$$\operatorname{Div} \mathbf{v} = \frac{\partial u_i}{\partial x_i} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0.$$

If the flow is in parallel planes or axisymmetric then the equation contains only two terms and there exists a **stream function**. (See also **equations of fluid motion**.)

EQUATION OF ENERGY BALANCE. See **Navier-Stokes equations**; **Enskog-Maxwell equation of change**.

EQUATION OF MOTION OF GAS. See **Navier-Stokes equations**; **Enskog-Maxwell equation of change**.

EQUATION OF PIEZOTROPY. (Also called **physical equation**.) An equation relating the thermodynamic variables in processes of a **piezotropic fluid**. In its general form it expresses the density ρ as a function of the pressure p .

$$\rho = f(p).$$

The derivative $d\rho/dp$ is called the **coefficient of piezotropy**. The most familiar such equation is that for polytropic changes of state in an ideal gas:

$$p\rho^{-\lambda} = \text{constant},$$

where λ is the modulus of the polytropic process.

In many meteorological problems the equation of piezotropy may be obtained by combination of the **equation of state** and the first law of **thermodynamics** for an adiabatic ($\lambda = 1.4$), isobaric ($\lambda = 0$), or isothermal ($\lambda = 1$) process. The equation of piezotropy, when taken together with the **equations of fluid motion** and the **equation of continuity**, forms a complete system of five equations in the unknowns p , ρ , and the three velocity components.

The equation of piezotropy refers to processes undergone by individual parcels and not to the spatial distribution of the state variable; it is to be distinguished from the equation of **barotropy**.

EQUATION OF STATE. (Also called characteristic equation.) A relation, empirical or derived, between thermodynamic properties of a substance or system.

The equation of state must be single-valued in terms of its variables. This is a direct consequence of the concept of a **state**.

There exist systems, namely systems which undergo processes involving hysteresis, (plastic deformation, ferromagnetism, etc.) for which no equation of state can be indicated. Although it is believed that the laws of thermodynamics apply to such systems, the rigorous results of classical thermodynamics cannot be applied to such systems because the science of thermodynamics is developed on the assumption of the existence of the single-valued function.

In the realm of classical thermodynamics, equations of state are assumed given. They can be derived from first principles only by the methods of statistical mechanics and quantum mechanics. These rely on the adoption of suitable molecular models for substances, and so far no universal, generally applicable model has been discovered even for narrow classes of substances such as gases.

It is an experimental fact that every thermodynamic system possesses a definite number n of independent properties which determine its state. Consequently, an equation of state is a relation between n properties (mutually independent) chosen (otherwise arbitrarily) as the independent properties ($x_1, x_2 \dots x_n$) of the system and one more property, the dependent

property y . Hence the equation of state is a function of the form

$$F(y, x_1, x_2, \dots x_n) = 0. \quad (a)$$

The simplest thermodynamic systems possess two independent properties, consequently the simplest equation of state is written in terms of three variables. When it is written in terms of pressure p , volume V , and absolute temperature T , it is called the p - V - T relation for the system or the thermal equation of state. When one of the caloric-thermodynamic properties (better called caloric properties, because p, V, T are thermodynamic properties also), such as **enthalpy**, **entropy**, **Gibbs function**, or work function (**Helmholtz function**) are given, the equation is called a thermodynamic equation, or better, a **caloric equation of state**, although the latter is not a commonly accepted designation.

Even in the case of a simple system, one equation of state, e.g., the equation $f(p, V, T) = 0$, does not necessarily determine the form of all the other equations of state. This is connected with the fact that the derivation of the other equations of state may involve the integration of partial derivatives which leads to the appearance of whole functions in the integration "constant." An equation from which other equations of state can be derived by differentiation only, is called a **fundamental equation** (of state). In the case of a pure substance in a specified phase, the p, V, T relation does not constitute a fundamental equation with respect to the properties U, H, S, G, A , or their derived properties C_p, C_v, γ , etc. Consequently, it is possible to have two or more substances whose p, V, T relations are identical but whose specific heats, for example, are different.

In the case of continuous systems, for which the state changes from point to point, for example, a flow field of a viscous fluid, it is assumed that at every point, the equation of state is the same as for a homogeneous system and does not involve the gradients of the thermodynamic properties. Hence, such systems can only be studied with the aid of thermodynamics if local departures from equilibrium are small (near-equilibrium processes), i.e., if the gradients of the thermodynamic properties are not too great.

An equation of state must necessarily involve a finite (even if very large) number of

independent variables. The particular variables which are chosen as independent is immaterial, on condition that they are mutually independent, and that their number is appropriate to the physical nature of the system.

For the equations of state of various types of systems, see the corresponding entries in this book. Thus, the **Curie equation** is the equation of state of a paramagnetic solid. The equation of state, in p, V, T , for a **perfect gas** is given under that heading. The p, V, T equations of state for imperfect (real) gases are given under the headings **imperfect gases**; **virial equation of state**; **Beattie and Bridgman equation**; **Berthelot equation**; **Clausius equation**; **Dieterici equation**; **Keyes equation**, **van der Waals equation**, etc.

It should also be noted that equations of state for systems which consist of several components, rather than a single substance, can be written by introducing the variables n_1, n_2, \dots, n_c , which are the mole numbers of the components present.

Finally, it should be recognized that the data represented by equations of state of substances can be obtained in the form of diagrams or tables, e.g., **steam charts** and **steam tables**.

EQUATION OF STATE, ADIABATIC. See **adiabatic equation of state**.

EQUATION OF STATE FOR CRYSTALS. See **Mie-Grüneisen equation of state**.

EQUATIONS, HOLONOMIC. See **conservative force**.

EQUATIONS, NON-HOLONOMIC. See **conservative force**.

EQUATIONS OF COMPATIBILITY OF STRAIN (FOR INFINITESIMAL STRAIN). Differential equations which are satisfied by the infinitesimal strain components and which express the necessary and sufficient condition that there exists a displacement field consistent with them. In a rectangular Cartesian coordinate system x they may be written, using the **indicial notation** and **summation convention**,

$$\frac{\partial^2 e_{ij}}{\partial x_k \partial x_l} + \frac{\partial^2 e_{kl}}{\partial x_i \partial x_j} - \frac{\partial^2 e_{ik}}{\partial x_j \partial x_l} - \frac{\partial^2 e_{jl}}{\partial x_i \partial x_k} = 0,$$

where e_{ij} denotes the infinitesimal strain components in the system x . Only six of these

equations are independent. Also called *Saint-Venant compatibility equations*.

EQUATIONS OF FLUID MOTION. For a viscous fluid these are the **Navier-Stokes equations**. For an incompressible fluid $\text{div } \mathbf{v} = 0$; for a homogeneous, or well-stirred, fluid $\frac{1}{\rho} \text{grad } p = \text{grad } \int \frac{dp}{\rho} = \text{grad } \frac{p}{\rho}$ if the fluid is also incompressible; if the body forces are conservative, $F = -\text{grad } \Omega$; (for gravity $\Omega = -gz$, where z is height) and writing $\text{curl } \mathbf{v} = \boldsymbol{\omega}$ the equations reduce to

$$\frac{\partial \mathbf{v}}{\partial t} - \mathbf{v} \times \boldsymbol{\omega} = -\text{grad} \left(\Omega + \frac{p}{\rho} + \frac{1}{2} \mathbf{v}^2 \right) - \nu \text{curl } \boldsymbol{\omega}. \quad (1)$$

If the motion is irrotational $\boldsymbol{\omega} = 0$ and $\mathbf{v} = \text{grad } \phi$, ϕ being the velocity potential, and then

$$\text{grad} \left(\frac{\partial \phi}{\partial t} + \Omega + \frac{p}{\rho} + \frac{1}{2} \mathbf{v}^2 \right) = 0.$$

This may be integrated to give

$$\frac{\partial \phi}{\partial t} + \Omega + \frac{p}{\rho} + \frac{1}{2} \mathbf{v}^2 = \mathcal{F}(t).$$

The left hand side may be put equal to a constant if $\mathcal{F}(t)$ may be regarded as absorbed into $\frac{\partial \phi}{\partial t}$ and this is then the general form of **Bernoulli's equation** for irrotational flow.

Equation (1) may also be integrated for the steady motion $\left(\frac{\partial \mathbf{v}}{\partial t} = 0 \right)$ of inviscid fluids for then

$$\begin{aligned} \mathbf{v} \cdot \text{grad} \left(\Omega + \frac{p}{\rho} + \frac{1}{2} \mathbf{v}^2 \right) \\ = \boldsymbol{\omega} \cdot \text{grad} \left(\Omega + \frac{p}{\rho} + \frac{1}{2} \mathbf{v}^2 \right) \\ = 0 \end{aligned}$$

showing that

$$\Omega + \frac{p}{\rho} + \frac{1}{2} \mathbf{v}^2$$

= constant along a streamline or vortex line. The expression is therefore constant in the surface composed of all the vortex lines through a streamline. Such a surface is called a *Bernoulli surface*.

Conservative body forces may be eliminated when the fluid is incompressible and of uniform

density by subtracting out the hydrostatic pressure p_0 given by

$$\frac{p_0}{\rho} + \Omega = 0.$$

Writing $p' = p - p_0$ the equation is

$$\frac{D\mathbf{v}}{Dt} = -\text{grad} \frac{p'}{\rho} + \nu(\text{div grad})\mathbf{v}$$

or

$$\frac{Du_i}{Dt} = -\frac{1}{\rho} \frac{\partial p'}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j}$$

according to the notation used.

EQUATIONS OF GAUSS AND CODAZZI.

Equations which must be satisfied by the fundamental magnitudes of the first and second orders, $a_{\alpha\beta}$ and $b_{\alpha\beta}$, for a surface. The *Codazzi equations of the surface*, or *Mainardi-Codazzi relations* are

$$b_{\alpha\beta,\gamma} - b_{\alpha\gamma,\beta} = 0.$$

This tensor equation represents two independent relations

$$b_{11,2} - b_{12,1} = 0 \text{ and } b_{22,1} - b_{21,2} = 0.$$

The *Gauss equation of the surface*, or the *Gauss characteristic equation*, is

$$R_{\sigma\alpha\beta\gamma} = b_{\alpha\gamma}b_{\sigma\beta} - b_{\sigma\gamma}b_{\alpha\beta},$$

where $R_{\sigma\alpha\beta\gamma}$ is the **Riemann-Cristoffel tensor** for the surface and is defined in terms of the **fundamental magnitudes of the first order**. The tensor equation represents a single equation

$$R_{1212} = b_{11}b_{22} - b_{12}^2.$$

EQUATIONS OF MOTION. A set of equations, generally in differential form, which when solved yield information concerning the motion of a particle or system of particles. Initial or boundary conditions on position and/or velocity and a knowledge of the forces acting on the system at any instant is necessary. There are several equivalent forms in which the equations of motion may be expressed. (See **Newton laws of motion**; **Lagrange equations of motion**; **canonical equations of motion**; and **Euler equations of motion** (for rotational motion); **kinematics**; **equations of motion, parametric**.)

EQUATIONS OF MOTION, PARAMETRIC.

In certain cases of motion in two and three

dimensions, the solutions of the differential equations of motion yield the displacements of the components along the coordinate axes as a function of time with the general form of these parametric equations:

$$x = f_1(t)$$

$$y = f_2(t)$$

$$z = f_3(t).$$

These are called parametric equations of the path with the time t as the parameter. To determine the path of the motion in the particular two or three dimensional space, it is necessary to eliminate the time and obtain a function of the form $\Phi(x,y,z) = 0$.

The motion of a projectile and the composition of simple harmonic motions in a plane are examples. (See **kinematics**.)

EQUATIONS, RHEONOMIC. See **conservative force**.

EQUATIONS, SCLERONOMIC. See **conservative force**.

EQUATOR. (1) An imaginary line cut on the surface of the earth by a plane perpendicular to the axis of rotation, and passing through the center of the earth. (2) An imaginary line cut by this plane on the **celestial sphere**. This line is often called the "celestial equator" or "equinoctial" to differentiate it from the terrestrial equator.

EQUICONTINUOUS. A set of functions $f_n(x)$ is said to be equicontinuous over their common domain of definition if, for every $\epsilon > 0$ there exists a $\delta > 0$ such that, for all x in the domain of definition,

$$|f(x_1) - f(x)| < \epsilon \text{ if } |x_1 - x| < \delta.$$

(Compare **Aseoli's theorem**.)

EQUIDISTANT ALLEYS. In the Luneberg geometry of binocular vision, equidistant alleys are pairs of curves that are the loci of points apparently equidistant from the median. Blumenfeld's experiment showing that equidistant alleys are not parallel alleys shows that binocular visual space is non-Euclidean.

EQUI-ENERGY SPECTRUM. A spectrum in which the spectral concentration of energy evaluated on a wavelength basis is constant throughout the visible region.

EQUILIBRANT. A force or force system which is equivalent to the negative of an existing force system. The existing force system plus its equilibrant is equivalent to zero force and zero couple.

EQUILIBRIA, CHEMICAL. See **chemical equilibria, calculation of.**

EQUILIBRIUM. (1) A state of a system characterized by the fact that no spontaneous processes occur in it. (a) When chemical processes are involved, the system may be in a state of *retarded equilibrium* (for example, a stoichiometric mixture of hydrogen and oxygen at atmospheric pressure and temperature). In such a system, a spontaneous process can occur by the removal of constraints (in the example, by the application of a catalyst (or catalyzer) or by a local increase in temperature. (b) When mechanical entities are involved, all forces are counterbalanced by equal and opposite forces. (See various mechanical systems in following entries.) In a state of equilibrium, at least some of the quantities describing the system are independent of time. (See also **equilibrium, dynamic.**)

(2) In statistical mechanics an **ensemble** is in equilibrium when it is in the state of most probable distribution. Fluctuations about this most probable state upset the statistical equilibrium but not the macroscopic equilibrium.

Some special or qualified types of equilibria are *apparent equilibrium*, a condition in which some factor prevents the system from reaching true equilibrium, and *metastable equilibrium*, an equilibrium which is not the *most stable* under the given conditions. Two other terms sometimes used are *stable equilibrium*, in which the potential energy of the system is a minimum, and *unstable equilibrium*, in which the potential is not a minimum, or is a maximum. (See **equilibrium, dynamic; equilibrium, metastable; equilibrium, neutral.**)

EQUILIBRIUM COMPLIANCE. See **compliance, equilibrium.**

EQUILIBRIUM CONSTANT. See **chemical equilibria, calculation of.**

EQUILIBRIUM, DYNAMIC. The application of D'Alembert's principle, or the introduction of inertia forces, reduces dynamics to statics in a formal sense. $\mathbf{F} = m\mathbf{a}$ becomes $\mathbf{F} - m\mathbf{a} = 0$. Equations of motion then may be called equations of dynamic equilibrium.

EQUILIBRIUM, FROZEN. See **frozen equilibrium.**

EQUILIBRIUM, METASTABLE. A definite equilibrium state which is not the most stable equilibrium under the given conditions. A metastable system will often undergo a spontaneous change upon addition of the stable phase, or frequently under the action of vibratory forces.

EQUILIBRIUM MODULUS. See **modulus, equilibrium.**

EQUILIBRIUM, NEUTRAL. A system is said to be in neutral equilibrium when it is not **unstable** and when some displacement from the equilibrium configuration produces no tendency to return to the original equilibrium position or to depart farther from it. Therefore, such a system, while in a **steady state**, cannot be said to exhibit either stability or instability.

This term is often used in connection with the **parcel method** of stability analysis; if the perturbation is a wave, "neutral equilibrium" and "stability" are often used interchangeably. (See **neutral stability.**)

EQUILIBRIUM OF A PARTICLE. A particle is said to be in equilibrium if the vector sum of all the forces acting on the particle is equal to zero. This condition can be expressed in component form by the equations $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$. Since a single particle does not possess the property of extension by definition, every force acting on the particle must act through the same point. Hence, the condition for equilibrium of a particle is identical with the first condition for equilibrium of rigid body as given under **equilibrium of forces.**

EQUILIBRIUM OF A RIGID BODY WITH RESPECT TO ROTATION. A rigid body, initially at rest, is in rotational equilibrium if the algebraic sums of the **torques** or moments about each of three mutually-perpendicular axes through the center of mass are zero. This is identical with the second condition for **equilibrium of forces on a rigid body.**

EQUILIBRIUM OF A RIGID BODY WITH RESPECT TO TRANSLATION. A rigid body is said to be in equilibrium with respect to translation if the vector sum of all the forces acting on the body is equal to zero. This

condition is identical with the first condition for **equilibrium of forces on a rigid body**, and also with the single condition for equilibrium of a particle.

EQUILIBRIUM OF A SYSTEM OF CONNECTED PARTICLES. If a system of connected particles (e.g., joined by strings or rods, etc.) is acted on by external forces, the equilibrium of the whole can be studied by examining the equilibrium of each particle separately under the influence of *all* the forces which act on it. This is called the principle of separate equilibrium. It is useful in the study of flexible jointed cables. Each particle must be in equilibrium separately for the system to be in equilibrium. $\Sigma \mathbf{F} = 0$ and $\Sigma \mathbf{M} = 0$ for the system is necessary but not sufficient.

EQUILIBRIUM OF FORCES ON A RIGID BODY. A state of balance between or among forces. The much used term equilibrium is here confined to its dynamical sense; such subjects as thermal equilibrium, radioactive equilibrium, etc., are treated in appropriate places elsewhere. Unless otherwise specified, the term refers to that set of conditions to which a system of forces must be adjusted in order that a free body acted upon by them will experience no acceleration. This is termed "static equilibrium," to distinguish it from the "kinetic or dynamic equilibrium" with which **d'Alembert's principle** is concerned.

Two conditions are necessary for the equilibrium of a set of forces: (1) The vector sum of the forces must be zero; then if they are resolved into rectangular components, the algebraic sums of the *X*, the *Y*, and the *Z* components must separately reduce to zero, or symbolically, $\Sigma F_x = 0$, $\Sigma F_y = 0$, and $\Sigma F_z = 0$. (2) The algebraic sum of the torques or moments of the forces about each of any three mutually perpendicular axes must be zero; the body then has no tendency to accelerate in any direction or about any axis.

EQUILIBRIUM, PRINCIPLE OF SEPARATE. The principle whereby the equilibrium of several particles can be studied by examining the equilibrium of a single particle in the group with respect to all the forces acting on it. (See **statics**.)

EQUILIBRIUM, STABLE. An equilibrium state of a system of one or more particles such

that the **potential energy** of the system is a minimum. (Contrast **equilibrium, unstable**.)

EQUILIBRIUM THEORY IN CHEMICAL KINETICS. The standard theories of chemical kinetics are equilibrium theories in which a **Maxwell-Boltzmann distribution** of reactants is postulated. This is true both for the **collision theory** and for the **absolute rate theory**. Because of the existence of an **activation energy** for most chemical reactions, "energetic molecules" play a dominant role in the reaction rate. The condition for the applicability of the equilibrium theory is then that the process

Average molecules \rightleftharpoons Energetic molecules
is much faster than the process:

Energetic molecules \rightarrow Products of chemical reactions.

EQUILIBRIUM, THERMAL. A system is said to be in thermal equilibrium if the temperature is uniform throughout the system and is the same as that of the surroundings.

EQUILIBRIUM, THERMODYNAMIC. A system is said to be in thermodynamic equilibrium if it is simultaneously in mechanical, thermal and chemical equilibrium. For a chemical system this means that the thermodynamic variables temperature, pressure and thermodynamic potentials are constant throughout the system and are the same as those of the surroundings.

EQUILIBRIUM, UNSTABLE. If the **resultant** of the forces acting on a rigid body is zero, the body is in equilibrium. The equilibrium is unstable if the system does not tend to return to the equilibrium position after a static displacement, but tends instead to depart farther from it. A pencil balanced on its point is an illustration of unstable **equilibrium**.

EQUIPARTITION OF ENERGY. In a system which can be treated by classical physics and which consists of a large number of particles, the average kinetic energy per particle is found to be proportional to the absolute temperature and to the number of **degrees of freedom** of one particle, the coefficient of proportionality being $\frac{1}{2}k$, where *k* is Boltzmann's constant. The average kinetic energy is thus independent of the mass of the particles.

The principle of equipartition of energy can

be generalized under certain circumstances to include degrees of freedom corresponding to terms in the potential energy. It breaks down when degrees of freedom begin to be frozen in. (See **freezing in of degrees of freedom**.)

EQUIPHASE SURFACE. In a three-dimensional wave, $A(x,y,z)e^{j[\omega t - \Phi(x,y,z)]}$, the surfaces $\Phi(x,y,z) = \text{constant}$ are called *equiphase surfaces*.

EQUIPOTENTIAL REGION. A field-free region. If the **potential** is uniform in a region,

$$\nabla\phi = 0,$$

hence there is no electric field. Conversely if

$$\mathbf{E} = -\nabla\phi = 0,$$

ϕ is independent of position.

EQUIPOTENTIAL SURFACE. A surface on which the **potential** is independent of position. At each point of the surface, the **gradient** $\nabla\phi$ crosses perpendicularly, i.e., equipotential surfaces are **normal** to the lines of force.

EQUIVALENCE. A (binary) relation R for the elements a, b, c of a set S is an equivalence, if it is *reflexive*, *symmetric*, and *transitive*. (For the meaning of these terms, and for the fundamental importance in mathematics of the concept of equivalence, see **relation**.)

EQUIVALENCE THEOREM. The field in a source-free region bounded by a surface could be produced by a distribution of electric and magnetic currents (**current-sheets**) on that surface that would be equivalent, for points inside the surface, to the actual external sources. (See **induction theorem**.)

EQUIVALENT-BAROTROPIC MODEL. A model atmosphere characterized by (1) frictionless and adiabatic flow, (2) hydrostatic and **quasi-geostrophic** equilibrium, and in which (3) the vertical shear of the horizontal wind is assumed to be proportional to the horizontal wind itself. It is, accordingly, an atmosphere in which the wind does not change direction with height and consequently one in which the contours and isotherms (on isobaric surfaces, for example) are everywhere parallel. In such an atmosphere, the vertically averaged motions are presumably equivalent to those at some intermediate level, the *equivalent-barotropic level*. In terms of the motion at this level, assumed to be an isobaric sur-

face, the behavior of the equivalent-barotropic model may be described by a single equation (the *vorticity equation*) in a single unknown (the height of the isobaric surface). (See **barotropic vorticity equation**.)

EQUIVALENT DEVIATE. See **deviate**, **deviation**.

EQUIVALENT ELECTRONS. Equivalent electrons in an atom or molecule are electrons in states with the same quantum numbers, apart from spin, and eigenfunctions having the same symmetry with respect to a reflection at the center (*even* or *odd* for atoms, g or u for molecules with a center of symmetry). The **Pauli principle** restricts the number of equivalent electrons to a maximum of $2(2l + 1)$ (l is the azimuthal quantum number).

EQUIVALENT FORCE SYSTEMS. Two force systems are said to be equivalent when they have the same resultant. Summation of forces in any direction and summation of moment about any axis are equal. Both systems can be reduced to the same wrench. (For definition of wrench, see **couple**, **theorems about**.)

EQUIVALENT POTENTIAL TEMPERATURE. The **potential temperature** corresponding to the *adiabatic equivalent temperature*.

$$\theta_E = T_{ac} \left(\frac{1000}{p} \right)^{0.286},$$

where θ_E is the equivalent potential temperature, T_{ac} the adiabatic equivalent temperature, and p the pressure in millibars. This temperature is conservative with respect to **dry-** and **pseudo-adiabatic processes**.

EQUIVALENT REPRESENTATIONS OF GROUPS. Two representations $g \rightarrow T_g$ and $g \rightarrow T'_g$ of a group G are said to be equivalent if there exists a non-singular operator M (i.e., one for which M^{-1} exists) such that $T'_g = MT_gM^{-1}$ for all g . Two equivalent representations can be considered as the realizations of the same representation in terms of two different bases in the vector space.

EQUIVALENT SYSTEMS IN THERMODYNAMICS OF IRREVERSIBLE PROCESSES. See **transformation theory of thermodynamics of irreversible processes**.

EQUIVALENT TEMPERATURE. (1) *Isobaric equivalent temperature:* The temperature that an air parcel would have if all water vapor were condensed out at constant pressure, the **latent heat** released being used to heat the air.

$$T_{ie} = T \left(1 + \frac{Lw}{c_p T} \right),$$

where T_{ie} is the isobaric equivalent temperature, T the temperature, w the mixing ratio, L the latent heat, and c_p the specific heat of air at constant pressure. (2) *Adiabatic equivalent temperature (or pseudo-equivalent temperature):* The temperature that an air parcel would have after undergoing the following (physically unrealizable) process: **dry-adiabatic expansion** until saturated; **pseudo-adiabatic expansion** until all moisture is precipitated out; dry-adiabatic compression to the initial pressure. This is the equivalent temperature as read from a **thermodynamic chart** and is always greater than the *isobaric equivalent temperature*.

$$T_{ae} = T \exp \frac{Lw}{c_p T},$$

where T_{ae} is the adiabatic equivalent temperature. (3) A term used in British engineering for that temperature of a uniform enclosure in which, in still air, a sizeable **black body** at 75°F would lose heat at the same rate as in the environment.

EQUIVALENT THICKNESS. The thickness, expressed in terms of the mass per unit area, of a foil which will just prevent the passage of α -particles of known range in air. The equivalent thickness in mg/cm² is equal to the product of the range (in cm) and the density (in mg/cm³). (See **stopping power**.)

EQUIVALENT WIDTH OF A SPECTRAL LINE. A practical measure for the intensity of an observed absorption line which is independent of the instrumental resolving power. The equivalent width of a spectral line is defined as

$$W_\nu = \int \frac{I_o - I_\nu}{I_o} d\nu$$

or

$$W_\lambda = W_\nu \lambda^2,$$

where I_o is the intensity of the continuous background at the wavelength λ of the absorp-

tion line and I_ν the measured intensity at wavenumber ν . The equivalent width of a spectral line is given by the width (in wavenumber or wavelength units) of a rectangle of height I_o and an area equal to the total area under the line profile. (Cf. **line width**.)

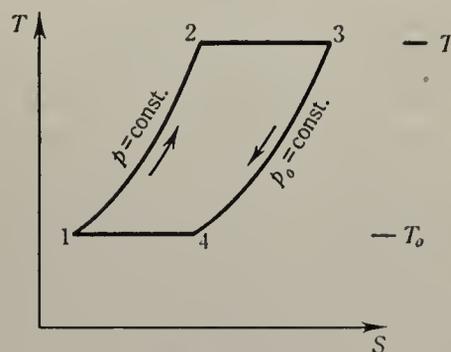
ERGODIC AND QUASI-ERGODIC HYPOTHESES.

Hypotheses which were introduced to justify the use of statistical mechanics in describing mechanical systems. The ergodic hypothesis introduced by Boltzmann (and by Maxwell who called it the principle of continuity of path) assumed that any system will after a sufficiently long time return to its original state. In this form it is certainly incorrect, but the quasi-ergodic hypothesis, introduced by P. and T. Ehrenfest, which states that any system will after a sufficiently long time return arbitrarily closely to its original state, is probably correct for all systems of physical interest.

ERGODICITY. A property of dynamic stochastic systems which, in general terms, may be expressed by saying that a system is ergodic if it tends in probability to a limiting form which is independent of the initial conditions.

ERGODIC THEOREM. See **ergodic and quasi-ergodic hypotheses**.

ERICSSON CYCLE. A cycle consisting of: 1-2 heating at constant pressure p ; 2-3 isothermal expansion (T); 3-4 cooling at constant pressure p_o ; 4-1 isothermal compression (T_o).



T,S diagram of Ericsson cycle.

The Ericsson cycle is approached by a closed cycle gas turbine when compression and expansion occur in a large number of stages with interstage cooling or heating, respectively.

The efficiency of this cycle is equal to that of a Carnot cycle

$$\eta = 1 - \frac{T_o}{T}$$

(See **gas turbines**.)

ERROR. In general, any deviation of a computed quantity from the theoretically correct value. In a restricted sense, that deviation due to unavoidable random disturbances, or to the use of finite approximations to what is defined by an infinite series, and hence to be contrasted with **blunder**. Errors of measurement arise from the use of instruments of less than infinite precision, and from random, uncontrollable disturbances. Truncation errors are due to the use of a term as an approximation for the limit of an infinite sequence, or, in general, to the use of any approximation in place of the theoretically correct quantity (cf. **remainder**, **roundoff**). If $f(x)$ is to be computed but only the approximation x^* to x is known, then $f(x) - f(x^*)$ is the *propagated error*. (See **error**, **propagation of**.) Bounds for the propagated error can be had by applying mean value theorems when f has suitable continuity properties. Generated errors are those which arise in the computation itself because of rounding. They depend upon the details of the routine.

If f is the true and f^* the computed value, sometimes $f - f^*$ is called the **correction**, $f^* - f$ the **error**. The term correction, however, is used in a different way in this book, to designate a quantity to be added to f^* for reducing the error, hence an approximation to the error. The word error occurs in several specialized senses in different sciences. We may distinguish two main fields: errors of measurement (or, more generally, errors of observation) and sampling errors, apart from such things as copying errors, interpretation errors and so forth. Errors of observation are themselves capable of classification, e.g., into *instrumental errors*, due to imperfections in instruments, and *personal errors*, attributable to the observer. *Sampling errors* are variations due to incompleteness of the sample coverage; they are not "mistakes" or imperfections in measurement. In control systems, the error is the difference, expressed as a function of time, between the input or command and the output or controlled variable. In most of the literature the error is defined as "input

minus output" but some authors use the definition "output minus input" derived from the instrument field, in which a meter with positive error is one that reads high, that is, one for which the reading (or output) is greater than the desired value (or input).

ERROR FUNCTION. The definite **integral**, also called the Gauss error function,

$$\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_0^t e^{-y^2} dy.$$

When the results of a series of measurements are described about an average by a Gaussian curve, $\operatorname{erf}(ha)$ is the probability that the error of a single measurement lies between $\pm a$, where h is the precision index.

The function is mainly employed in the theory of observational errors. In statistics the related **normal distribution** is almost universal.

ERROR, PROPAGATION OF. When a quantity is calculated as a function of one or more measured quantities, instead of being measured directly, it is often necessary to estimate the uncertainty in the calculated quantity which results from the estimated errors of the measured quantities. In other words, if z is a function of x_1, x_2, \dots, x_p , say $f(x_1, x_2, \dots, x_p)$ and it is desired to find the **distribution** of z given that of the x 's, this is usually done by finding large sample approximations to the **variance** of z in terms of the **variances** and **covariances** of the x 's.

ERRORS, DETECTION OF. See **detection of tabulation errors**.

ESCALATOR METHOD. A method of **enlargement** for **matrix inversion**, and for computing **eigenvalues** and **eigenvectors** of **matrices**.

ESCAPE PROBABILITY, FIRST FLIGHT. The complement of the first flight collision probability (see **collision probability**, **first flight**), thus, $1 - P_o$.

ESCAPE PROBABILITY, RESONANCE. See **resonance escape probability**.

ESCAPE SPEED (VELOCITY). The minimum speed at which an inert body must be shot from the earth to avoid recapture by the earth's gravitational field is called the escape speed or escape velocity.

A projectile accelerated from the earth's surface at a speed below that of escape travels away from the earth in an elliptical path gradually losing speed until at the point farthest from the earth (apogee), it turns earthward and begins to travel back to the earth, under the influence of its gravitational field; but still following the elliptical path. If the velocity of the projectile were sufficient (orbital velocity) it would establish a permanent orbit beyond the sensible atmosphere of the earth, becoming a satellite. When the projectile velocity is sufficient for escape from the earth's gravity field, its path becomes a parabola (or hyperbola) hence the term parabolic velocity. The escape velocity from the earth is given by the following relationship:

$$V_e = R_e \sqrt{\frac{2g_e}{R_e + h}}$$

where R_e is radius of the earth, g_e is the gravitational acceleration of earth at the surface, and h is the altitude at take-off.

ESSENTIAL SINGULARITY. See **singularity**.

ESTIMATED POSITION. See **dead reckoning**.

ESTIMATES, CONSISTENT. See **consistent estimates**.

ESTIMATION, THEORY OF. Given a **sample** from a **population** whose specification involves one or more **parameters**, it is necessary to form estimates of the parameters. Usually, many different estimates of a given parameter can be derived, and the theory of estimation is concerned with the properties of these different estimates. Together with the estimate itself, it is useful to provide some idea of its precision and this is commonly done by specifying an interval which is intended to contain the true value of the parameter (see **confidence limits, fiducial inference**).

A basic result in the theory of estimation states that, under general conditions, a **consistent** estimator has a sampling variance in large samples which is not less than a certain lower bound. Statistics whose variance attains this lower bound are said to be *efficient*.

ESU. See **electromagnetic units**.

e-SYSTEM (IN n -DIMENSIONS). The system $e^{i_1 i_2 \cdots i_n}$ or $e_{i_1 i_2 \cdots i_n}$, the components of which take the values 1 or -1 accordingly as $i_1 i_2 \cdots i_n$ is an even or odd permutation of $1, 2, \cdots, n$ and 0 otherwise.

ETA (H OR η). In reactor theory, and nuclear physics, the quantity η is the average number of fission neutrons emitted as a result of the capture of one thermal neutron by a fissionable nucleus.

ETTINGSHAUSEN COEFFICIENT. See **galvanometric and thermometric effects**.

ETTINGSHAUSEN EFFECT. See **galvanometric and thermometric effects**.

EUCKEN CORRECTION. According to the kinetic theory of dilute gases, the thermal conductivity coefficient λ , the viscosity coefficient η and the heat capacity per gram at constant volume \hat{c}_v for a monoatomic gas should be related by the equation:

$$\lambda = 2.5\eta\hat{c}_v. \quad (1)$$

This equation is in excellent agreement with the experimentally observed values in the rare gases.

For diatomic and polyatomic molecules, Equation (1) does not hold: whereas the transfer of internal energy hardly affects the viscosity coefficient, it increases both the heat capacity and the heat flux (and therefore the thermal conductivity coefficient). The two increases are not proportional.

Eucken has calculated the thermal conductivity of a polyatomic gas, assuming that the collisions maintain the internal degrees of freedom at their equilibrium energy value at the local temperature. Then, for a perfect gas;

$$\lambda = \eta\hat{c}_v \cdot a = \frac{1}{4} \left(g \frac{c_p}{c_v} - 5 \right) \eta\hat{c}_v. \quad (2)$$

a in this equation is called the Eucken correction or Eucken factor.

Other models have been proposed to calculate the energy transfer when the collisional exchange of energy between the translational and internal degrees of freedom is slow. Molecules with a higher energy content are formed in the warm parts of the apparatus, and diffuse towards the colder regions where they are deactivated with a liberation of energy. The molecules with a lower energy content diffuse

in the opposite direction. The Eucken factor calculated for such models differs by about 20% from that deduced from (2). This explains why, in spite of its rough theoretical basis, the Eucken equation has been fairly successful for the prediction of λ of the polyatomic molecules.

EUCLIDEAN ALGORITHM. The algorithm for finding the **highest common factor** of two numbers. It consists of forming a sequence of numbers of which the first two are the given numbers (the larger one first), the third is the remainder when the first is divided by the second, the fourth is the remainder when the second is divided by the third and so forth until zero is reached. The last non-zero number in the sequence is the desired highest common factor.

EUCLIDEAN SPACE. An n -dimensional Euclidean space is described abstractly by the following three properties, which determine that it is a linear, metric, n -dimensional space.

1(a). For every pair of points p, q which may also be called vectors, i.e., with base point at the origin, there exists a unique point $p + q$ such that $p + q = q + p$ and $p + (q + r) = (p + q) + r$ for every point r .

1(b). For every point p and every real number m there exists a unique point mp such that for all points q, r and numbers n

$$m(p + q) = mp + mq; \quad (m + n)p = mp + np;$$

$$(mn)p = m(np); \quad 1.p = p.$$

1(c). There exists a point 0 such that $p + 0 = 0, 0p = 0$ for every point p .

2. For every pair of points p, q there exists a unique **scalar product** (p, q) which is a real number such that, for all points r and numbers m , we have: $(mp, q) = m(p, q); (p + q, r) = (p, r) + (q, r); (q, p) = (p, q); (p, p) > 0$ if $p \neq 0$.

3. There exist in the space n linearly independent points p_1, p_2, \dots, p_n but no set of $(n + 1)$ linearly independent points.

As a concrete realization of a Euclidean n -space we may consider as points p the sets of n real numbers (p_1, p_2, \dots, p_n) with scalar product of p and q defined as

$$p_1q_1 + p_2q_2 + \dots + p_nq_n.$$

EULER ANGLE. One of three parameters describing the orientation of a rigid body relative to a **Cartesian coordinate system** (x, y, z) fixed in space. Suppose another coordinate

system (x', y', z') is fixed in the body. Then the two systems may be made coincident by three successive rotations, applied in the appropriate order, and the three angles of rotation are the **Euler angles**. The order of performing the rotations and the symbols for the angles have been given in different ways by various authors so that some confusion exists in the literature of mechanics, where these parameters are most frequently used, as to the definition of the angles.

EULER CONSTANT. See **Euler-Mascheroni constant**.

EULER EQUATION. The condition that the integral

$$\int_{x_1}^{x_2} I(x, y, y') dx$$

have a stationary value is

$$\frac{\partial I}{\partial y} - \frac{d}{dx} \frac{\partial I}{\partial y'} = 0$$

the latter being known as the Euler equation or the Euler-Lagrange equation in the calculus of variations. (See **variations, calculus of**.)

EULER EQUATIONS OF MOTION. For a rigid body with one point fixed and coordinates fixed in the body and coinciding with the principal axes, the equations of motion of the body can be written as

$$I_{xx}\dot{\omega}_x + (I_{zz} - I_{yy})\omega_y\omega_z = M_x$$

$$I_{yy}\dot{\omega}_y + (I_{xx} - I_{zz})\omega_z\omega_x = M_y$$

$$I_{zz}\dot{\omega}_z + (I_{yy} - I_{xx})\omega_x\omega_y = M_z$$

where I_{xx}, I_{yy} and I_{zz} are the moments of inertia about the principal axes, L_x, L_y and L_z are the components of moment about the principal axes, ω_x, ω_y and ω_z are the components of the angular velocity about the principal axes and the dot indicates the time derivative. (See **moments of inertia; rotational motion**.)

EULER FORMULA (COLUMNS). A formula which gives the maximum axial load that a long, slender ideal column can carry without buckling. An ideal column is one which is perfectly straight, homogeneous and free from initial stress. This maximum load, sometimes called the critical load, causes the column to be in a state of unstable equilib-

rium, that is, any increase in the loads or the introduction of the slightest lateral force will cause the column to fail by buckling. The Euler formula for columns is given below.

$$P = \frac{K\pi^2 EI}{l^2}$$

in which P is maximum or critical load, E is modulus of elasticity, I is moment of inertia of cross-sectional area, l is unsupported length of column, K is a constant whose value depends upon the conditions of end support of the column. For both ends free to turn $K = 1$; for both ends fixed, $K = 4$; for one end free to turn and the other end fixed $K = 2$ approximately, and for one end fixed and the other end free to move laterally $K = 1/4$. (See column.)

EULER FORMULA (TURBINE). See velocity diagram.

EULERIAN ANGLES. See Euler angles.

EULERIAN METHOD OF ANALYSIS. A method in which the fluid within an arbitrary surface fixed in space is considered. By considering the local rates of change of density and momentum the equations of continuity and fluid motion are deduced. All properties of the fluid are expressed as functions of three coordinates of position, in a fixed frame of reference, and time.

EULERIAN WIND. In the classification of Jeffreys, a wind motion only in response to the pressure force. It is defined mathematically by

$$\frac{dV}{dt} = -\alpha \nabla_H p,$$

where V is the horizontal velocity, α the specific volume, ∇_H the horizontal vector differential operator and p the pressure.

The **cyclostrophic wind** is a special case of the Eulerian wind, which is limited in its meteorological applicability to those situations in which the **Coriolis effect** is negligible.

EULER-LAGRANGE EQUATIONS. See Lagrange equations.

EULER-MACLAURIN FORMULA. The formal expansion

$$\int_{x_0}^{x_m} f(x) dx = h(f_0/2 + f_1 + f_2 + \dots + f_{m-1} + f_m/2) - \sum_2^{\infty} B_\nu h^\nu (f_m^{(\nu-1)} - f_0^{(\nu-1)})/\nu!,$$

where

$$f_i = f(x_i) = f(x_0 + ih)$$

and the B_ν are the **Bernoulli numbers**.

EULER-MASCHERONI CONSTANT. A number, also often called simply Euler's constant, which occurs, for example, in one definition of the **gamma function**. It can be defined by several equivalent infinite integrals, one example being

$$C = \int_0^{\infty} e^{-t} \ln t dt.$$

but most often its definition is given by the equation,

$$C = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{n} - \ln n \right).$$

Its numerical value is 0.577215665... but the quantity $\gamma = 1.781072 \dots$, defined by $\ln \gamma = C$ is sometimes defined as the Euler Mascheroni constant.

EULER METHOD (FOR SOLVING AN ORDINARY DIFFERENTIAL EQUATION).

$$y' = f(x, y)$$

takes

$$y_{\nu+1} = y_\nu + hf_\nu,$$

$$f_\nu = f(x_\nu, y_\nu).$$

The "improved Euler method" obtains sequentially

$$y_{\nu+1/2} = y_\nu + hf_\nu/2,$$

$$y_{\nu+1} = y_\nu + hf_{\nu+1/2}$$

and can be considered an elementary form of the **Runge-Kutta method**. The "improved Euler-Cauchy method" forms

$$y_{\nu+1}^* = y_\nu + hf_\nu, \quad f_{\nu+1}^* = f(x_{\nu+1}, y_{\nu+1}^*),$$

$$y_{\nu+1} = y_\nu + h(f_\nu + f_{\nu+1}^*)/2.$$

(See Lothar Collatz, *Numerische Behandlung von Differentialgleichungen*, 2nd edition, Springer, 1955.)

EULER RECIPROCITY RELATION. If Z is a single-valued function of the variables x and y , the total or exact differential dZ may be written

$$dZ = Xdx + Ydy,$$

where X and Y are also functions of x and y . Then it can be shown that

$$\left(\frac{\partial X}{\partial y}\right)_x = \left(\frac{\partial Y}{\partial x}\right)_y.$$

This is known as the Euler criterion, or reciprocity relation. It is useful for deriving thermodynamic relationships, e.g., the **Maxwell relations**.

EULER-RODRIGUES PARAMETER. One of four parameters used to describe the orientation of a rigid body. They are functions of three **direction cosines** and they form the components of a **quaternion**.

EULER SUMMATION FORMULA. The **Euler-Maclaurin formula** when used for summing a series.

EULER THEOREM FOR HOMOGENEOUS FUNCTIONS. The function $f(x,y,z)$ is called homogeneous of the m^{th} degree in variables x,y,z,\dots if one has identically

$$f(kx,ky,kz,\dots) = k^m f(x,y,z). \quad (1)$$

By differentiating in respect to k and putting $k = 1$, one obtains the second identity

$$\frac{\partial f}{\partial x} x + \frac{\partial f}{\partial y} y + \dots \equiv m f(x,y,z). \quad (2)$$

This is Euler's theorem. Conversely it can be shown that any function which satisfies (2) is homogeneous of the m^{th} degree in x,y,z,\dots .

In thermodynamics one is concerned mainly with the two simplest cases $m = 1$ and $m = 0$. For example, the volume $V(p,T,n_1 \dots n_c)$ is an **extensive variable** of the mole numbers $n_1 \dots n_c$. Therefore applying (2) with $m = 1$, there results

$$\sum_{i=1}^c \left(\frac{\partial V}{\partial n_i}\right)_{T,p} n_i = V(T,p,n_1 \dots n_c). \quad (3)$$

On the contrary a partial molar volume v_j (see **partial molar quantities**) is an **intensive variable**. Therefore ($m = 0$)

$$\sum_{i=1}^c n_i \left(\frac{\partial v_j}{\partial n_i}\right)_{T,p} = 0.$$

(See also **Gibbs-Duhem relations**.)

EULER THEOREM ON NORMAL CURVATURE. The normal curvature κ_n at a point on a surface in a direction which makes angles θ and $\frac{1}{2}\pi - \theta$ with the principal directions at the point, corresponding to which the principal curvatures of the surface are κ_a and κ_b respectively, is given by

$$\kappa_n = \kappa_a \cos^2 \theta + \kappa_b \sin^2 \theta.$$

Dupin's theorem follows immediately; the sum of the normal curvatures at a point in two perpendicular directions is constant and equal to the sum of the principal curvatures at the point.

EULER TRANSFORMATION. (Of an infinite series whose partial sums are

$$S_n = a_0 + a_1 x + \dots + a_n x^n.)$$

This transformation utilizes the identity

$$(1-x)S_n = a_0 - a_n x^{n+1} + x \sum_0^{n-1} \Delta a_i x^i.$$

It is a method for the **summation of series**.

EULER TURBINE FORMULA. See **velocity diagram**.

EUTECTIC POINT. See **crystallization curve**.

EV. Symbol for one electron volt, a unit of energy equal to 1.602×10^{-12} erg, being the energy gained by a single electronic charge passing *in vacuo* through a potential difference equal to one volt.

EVAPORATION. See **boiling**.

EVEN AND ODD VARIABLES IN THERMODYNAMICS OF IRREVERSIBLE PROCESSES. See **reciprocity relations**.

EVEN FUNCTION. A function such that $f(x) = f(-x)$, typical examples being x^2 and $\cos x$. (See **odd function**.)

EVENT. A physical occurrence involving the interaction between two or more physical entities. For example, the collision between two bodies; or the emission of a photon by an excited atom as a result of its interaction with the radiation field. Events are described as

occurring at specified positions and times in a given coordinate system, or frame of reference. The special theory of relativity treats such problems as the descriptions of the same event, or the simultaneity of two events, as viewed by observers in two equivalent inertial frames of reference.

EVEN TERM OF AN ATOM. A term for which Σl_i , summed over all the electrons of an atom, is even. The eigenfunctions of even terms remain unchanged for a reflection of all particles at the center.

EVERETT INTERPOLATION FORMULA.

In the notation of **difference operators**

$$E^u = (1 - u) + \left(\begin{matrix} 2 - u \\ 3 \end{matrix} \right) \delta^2 + \left(\begin{matrix} 3 - u \\ 5 \end{matrix} \right) \delta^4 \\ + \dots + \left[u + \left(\begin{matrix} 1 + u \\ 3 \end{matrix} \right) \delta^2 \right. \\ \left. + \left(\begin{matrix} 2 + u \\ 5 \end{matrix} \right) \delta^4 + \dots \right] E.$$

It thus requires differences of even orders only, but they must be taken both at x_0 and at $x_1 = x_0 + h$.

EVJEN METHOD. A method for the evaluation of **lattice sums** in which charges of opposite signs are taken together in neutral groups, so that the contribution of each group is small and the sum converges rapidly.

EVOLUTE. The evolute of a given curve is the locus of its centers of **curvature**. (Discussed in entry on **involute**.)

EVOLUTE OF A SURFACE. See **involute of a surface**. Also called *surface of centers of the surface*.

EXACT. An exact differential equation of the form $f(x, dx, y, dy, \dots) = 0$ is one whose left side is an exact **differential**. For an equation of the form $M(x, y)dx + N(x, y)dy = 0$ to be exact, it is necessary and sufficient that

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}.$$

EXALTATION. The positive difference between the observed and the calculated values of the **molar refractivity** of a substance. (See **optical exaltation**.)

EXCESS FUNCTIONS. The difference between the **thermodynamic functions of mixing** for an actual system and the value corresponding to a perfect solution at the same temperature, pressure and composition is called the thermodynamic excess function (denoted by superscript E). Thus the excess Gibbs free energy (see **thermodynamics, characteristic functions of**) is (per mole)

$$g^E = g^M - RT(x_1 \ln x_1 + x_2 \ln x_2). \quad (1)$$

In terms of activity coefficients (1) becomes (see **reference systems**)

$$g^E = RT x_1 \ln \gamma_1 + RT x_2 \ln \gamma_2. \quad (2)$$

Similar definitions hold for the other thermodynamic quantities.

Very often the excess functions have a simple parabolic form. (They must vanish at $\alpha_1 = 0$ and $\alpha_2 = 0$.)

EXCHANGE. (1) A quantum mechanical concept based on the idea of identical particles. For example, suppose that two electrons are in **states** that allow them to come close together. Then, because they are indistinguishable particles, one could not tell the difference if they exchanged states. Thus the wave function of the system must be such that an exchange of the electrons leaves the magnitude of the wave function unchanged, except possibly for sign, i.e., the **wave function** must be either symmetric or antisymmetric to an exchange of the two particles. Particles whose total wave function (including both space and spin coordinates) is symmetric under an exchange operator obey the **Bose-Einstein statistics**. Particles whose total wave function is antisymmetric obey the **Fermi-Dirac statistics**.

(2) Exchange is also used more specifically as the exchange of one particle between two others, as in the exchange of the single electron between the two identical protons in the hydrogen molecular ion, or the exchange of a meson between two nucleons.

EXCHANGE COEFFICIENTS. (Also called *austausch coefficients*, *eddy coefficients*, *interchange coefficients*.) Coefficients of **eddy flux** (e.g., of momentum, heat, water vapor, etc.) in turbulent flow, defined in analogy to those of the kinetic theory of gases (see **eddy**). The *exchange-coefficient hypothesis* states that the mean eddy flux per unit area of a conservative

quantity (suitably expressed) is proportional to the **gradient** of the mean value of this quantity, that is,

$$\text{mean flux per unit area} = -A \frac{d\bar{E}}{dn},$$

where A is the exchange coefficient, \bar{E} the mean value of the quantity, and n the direction normal to the surface. In strict analogy to molecular properties A would be constant, for turbulent flow A turns out to depend on time and location.

EXCHANGE DEGENERACY. An exchange process which does not entail a change in value or configuration. For example, by the Heitler-London theory, the essential reason for the strong attraction (or repulsion), of the two H-atoms in the H_2 molecule is the exchange degeneracy, i.e., the fact that for very large internuclear distance, by exchange of the two electrons of the two atoms a configuration results that is indistinguishable from the original configuration. Therefore, as they approach, an interaction between them arises which may be treated mathematically as electron exchange.

EXCHANGE ENERGY. A specifically quantum-mechanical effect (see **quantum mechanics**) which has no classical analogue. It is due to the interaction between two systems that arises, or could arise, from the continuous exchange of a particle between them. Exchange energy is the origin of **covalent bonding**, of **ferromagnetism** and **antiferromagnetism**, probably of **nuclear forces** (where exchange energy could arise by exchange of π -mesons between nucleons, giving rise to an effective potential which involves an operator which exchanges the spins, isotopic spins and/or positions of the particles) and of numerous other physical phenomena.

Specifically in the case of a covalent bond, the major part of the energy is due to the "electron exchange" between the atoms. The wave function representing two *isolated* atoms may be represented by

$$\psi = \psi_A(1)\psi_B(2)$$

where electron 1 is supposed in interaction with nucleus A and electron 2 interacts with nucleus B. The alternate configuration $\psi_A(2)\psi_B(1)$ is another good description of the system at large internuclear distances. Both

wave functions lead to the same energy (**exchange degeneracy**).

At shorter distances, neither of these approximations alone is an adequate description of the system, since both electrons interact with both nuclei. The total wave function must be constructed as either a symmetrical, or an antisymmetrical linear combination of $\psi_A(1)\psi_B(2)$ and $\psi_A(2)\psi_B(1)$. (In what follows the spin contribution to the equation of the wave function is neglected.)

In a first approximation, the total energy is split up into three contributions: the first is the energy of *interaction of each electron with its proton*, that is the energy (see **Coulomb integral**) of two isolated H atoms; the second is a **Coulomb energy**

$$\int \psi_A(1)\psi_B(2)W\psi_A(1)\psi_B(2)d\tau_1d\tau_2,$$

representing the average electrostatic interaction of each electron cloud with the other, with the nucleus of the other electron, and the electrostatic repulsion of the two nuclei, where W is the perturbation potential; the third contribution is the *exchange integral*:

$$\int \psi_A(1)\psi_B(2)W\psi_A(2)\psi_B(1).$$

At intermediate distance, the contribution of this integral to the total energy (exchange energy) is much larger than that of the Coulomb integral. It is attractive for the symmetrical wave function, and repulsive for the antisymmetrical wave function.

The electron exchange leads thus to a splitting of the energy levels. The exchange energy depends essentially on the *overlaps* of the wave functions $\psi_A(1)$, $\psi_B(1)$ and $\psi_A(2)$, $\psi_B(2)$; it decreases rapidly (exponentially) as the internuclear distance increases. (See **overlap integral**.)

The name "exchange energy" is somewhat misleading since there is no real frequency of exchange of the electrons from one nucleus to the other. In fact the exchange energy results from a greater mobility of the electrons in the fields of the two nuclei, whence there results a lowering of the zero point energy. (See **delocalization energy**.)

EXCHANGE FORCES. If, in the interaction between two nucleons, their charges or spins, or both, are interchanged, the forces which

give rise to the interaction are called exchange forces. They may be pictured as arising through the exchange or virtual mesons between the nucleons. Phenomenologically, however, they are represented in terms of an ordinary potential function $V(r)$, where r is the separation between the nucleons, multiplied by a permutation operator P which exchanges charges, spin, or both. If the wave function of the two particles is totally antisymmetric (in space, spin, and isotopic spin), then the combined exchange of spin and charge is equivalent to the exchange of position, except for a change of sign. A space exchange potential may therefore be written $V(r)P_r$, where $P_r = -P_\sigma P_\tau$, and P_σ, P_τ exchange spin and isotopic spin respectively. The operator P_r has eigenvalues $+1$ or -1 according as the relative orbital angular momentum L of the pair of nucleons is even or odd. This potential is therefore equivalent to $(-1)^L V(r)$, giving rise to attractive or repulsive forces according to the parity of L . The space exchange force is known as a *Majorana force*; forces which exchange spin or charge separately are called *Bartlett* and *Heisenberg* forces, respectively. Nuclear forces are known to have exchange properties of this general kind.

EXCHANGE INTEGRAL. See **exchange energy**.

EXCITATION. (1) Addition of energy to a system, whereby it is transferred from its ground state to a state of higher energy, called an excited state. (2) The field excitation of dynamo machines, meaning the current or voltage of the field circuit. (3) In electron-tube circuits, the input signal of any stage is commonly called the excitation. Thus in a radio receiver, the signal picked up by the antenna supplies the excitation for the first stage, the output of the first supplies the excitation for the next, and so on.

EXCITATION CURVE. In nuclear physics, a graphical relationship between the energy of the incident particles or photons, and the relative yield of a specified nuclear reaction.

EXCITATION ENERGY (EXCITATION POTENTIAL). The energy necessary to bring an atom, molecule, atomic nucleus, etc., from the ground state into an excited state.

EXCITATION FUNCTION, ATOMIC. The **cross section** for the excitation of an atom to a particular excited state expressed as a function of the energy of the incident electrons.

EXCITATION FUNCTION, NUCLEAR. The **cross section** for a particular nuclear reaction expressed as a function of the energy of the incident particle or photon. The term excitation function is sometimes used also as a synonym for **excitation curve**.

EXCITATION LEVELS. See **excited state**.

EXCITATION POTENTIAL. See **excitation energy**.

EXCITATION PURITY. A quantity denoted by p_e and defined by the following relations

$$p_e = \frac{y - y_w}{y_d - y_w} \quad \text{or} \quad p_e = \frac{x - x_w}{x_d - x_w}$$

x and y are the **chromaticity coordinates** of the light considered, x_d and y_d those of the spectral light which has the same hue as the light considered; x_w and y_w are the chromaticity coordinates of the adopted achromatic light. Whether the formula in x or that in y is to be taken depends on which gives the numerator the greater numerical value. For colors for which no spectral light of the same hue can be found, the chromaticity coordinates to be taken for x_d and y_d are those of the corresponding point on the line joining the ends of the spectrum locus (the line of pure purples).

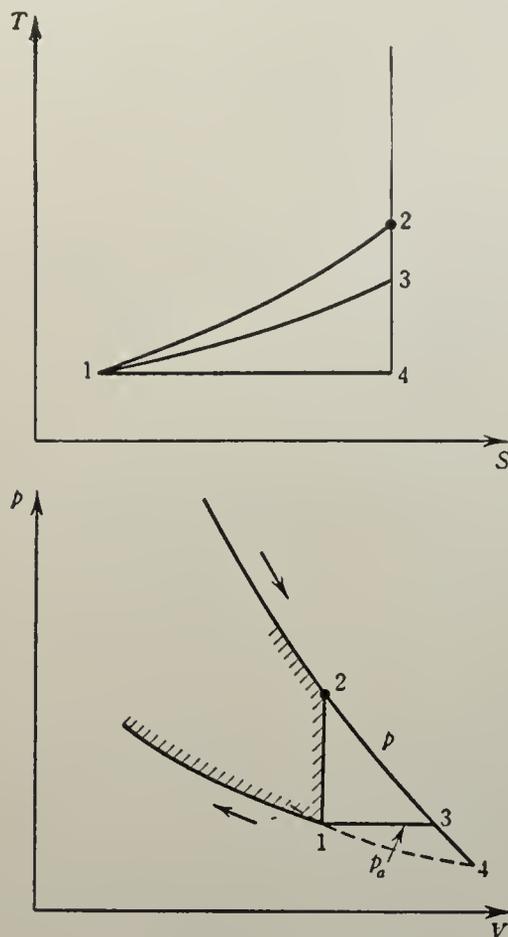
EXCITED STATE. A stationary state having an energy higher than that of the ground state.

EXCITON. A combination of an electron and a hole in a semiconductor or insulator in an excited state. The hole behaves as a positive charge, and it is supposed that the electron is attracted to it to form a state akin to that of a hydrogen atom. The probability of the electron falling into the hole is limited by the difficulty of losing the excess energy, so that the exciton may have a relatively long life. The existence of these states may be inferred from the absorption of light associated with their excitation. (Cf. **positronium**.) Alternatively, an exciton may be thought of as an excited state of an atom or ion, the excitation wandering from one cell of the lattice to another.

EXCLUSION PRINCIPLE. See Pauli exclusion principle.

EXHAUST GAS TURBINE. A gas turbine driven with the exhaust gases from a reciprocating internal combustion engine. Historically, exhaust gas turbines were the first successful gas turbines: they were easier to build than separate efficient gas turbines owing to the comparatively low temperature of the combustion gases.

The exhaust gas turbine utilizes the energy available in the working fluid of an internal combustion engine at release, state 2 in the figure. Ideally, the expansion through the tur-



T,S diagram and p,V diagram for exhaust gas turbine.

bine is isentropic along 23, and the ideal work W_t of the turbine is

$$W_t = \dot{m}(h_2 - h_3),$$

where \dot{m} is the mass rate of flow, and $h_2 - h_3$ is the isentropic enthalpy drop. The reciprocating engine is incapable of utilizing this enthalpy drop because losses due to piston friction along 23 would exceed the work obtainable, as the engine stroke would become excessively large, and the piston force, propor-

tional to the pressure difference $p - p_a$ would be small over most of the additional stroke.

In principle, more work could be obtained if expansion from state 2 could be continued below the atmospheric pressure p_a , to state 4, such that $T_4 = T_1$. However, such a scheme is not practicable owing to the additional complications in the design of the plant which it would entail.

An exhaust steam turbine (Bauer-Wach turbine) operates on the same principle.

EXHAUST LOSS. Loss of work associated with the fact that the working fluid from a reciprocating steam or internal combustion engine is capable of performing further work after exhaust. (See **exhaust-gas turbine**.)

EXHAUST-STEAM TURBINE. See **exhaust-gas turbine**.

EXHAUST (JET) VELOCITY. See **exhaust velocity**.

EXHAUST VELOCITY. The velocity at which a propelling fluid is discharged from a jet-type engine. The exit or exhaust velocity of a thermal engine depends on several parameters which can be subdivided into thermochemical parameters and motor parameters. Thermochemical parameters are: energy content of the fuel, mixing ratio of the propellants, expansion pressure ratio, and degree of combustion. Motor parameters are: thrust, mass of weight flow of propellant, nozzle divergence, difference between nozzle exit pressure, and ambient pressure. Although some of the thermochemical parameters, such as expansion ratio and degree of combustion, are strongly influenced by the motor design, they must be known, or at least be assumed for thermochemical performance calculations. The second group of parameters contains quantities which serve to measure the exhaust velocity (thrust, weight flow), or which depend on the motor design (divergence angle), or on operational conditions, such as the exit-ambient pressure difference. In a given motor, the nozzle dimensions are fixed, hence, for a given chamber pressure, the exit pressure is determined. Therefore, a variation of the pressure difference is due to change of ambient pressure as it occurs during powered ascent.

The exhaust velocity defined by the first group of parameters describes the propellant performance, independent of motor design and

operational conditions, and indicates the impulse, that is, the thrust for a given mass flow and length of time, which can be expected from the propellant in question. Therefore, this velocity is termed "impulse exhaust velocity." If divided by the gravitational constant, it yields the specific impulse, a parameter which depends only on thermochemical data and the expansion ratio.

The second group of parameters defines an exhaust velocity which describes the motor performance. It is determined by the thrust, obtained at a given propellant consumption, and varies with changing ambient pressure. For instance, the over-all thrust increases as the difference of exit minus ambient pressure increases (pressure thrust). Instead of accounting separately for the pressure thrust (as distinguished from the momentum thrust, produced by the discharge of matter), one can add this effect to the momentum thrust (defined as the product of mass flow and exhaust velocity). If, in an ascending rocket the over-all thrust increases and the propellant mass flow remains constant, the exhaust velocity necessarily increases. This illustrates that exhaust velocity is not the physical or thermochemical exhaust velocity but an equivalent velocity which converts instantaneous operational conditions into equivalents of a standard condition (namely exit pressure equal to ambient pressure, i.e., no pressure thrust). For this reason exhaust velocity is better termed *equivalent exhaust velocity*. If this velocity is divided by the gravitational constant, one obtains the specific thrust.

EXHAUST VELOCITY, EFFECTIVE. In jet propulsion terminology, the thrust divided by the mass of propellant flowing per second of time. It appears in the equations:

$$c = \frac{F}{w/g}$$

or

$$c = \frac{F}{\dot{m}} = gI_{sp} = \frac{Fg}{\dot{w}} = c^*C_f = \eta_v c_{th}$$

$$= V_e + \frac{A_e}{\dot{m}}(p_e - p_0)$$

where c is the effective exhaust velocity, F is the thrust, w is the weight of propellant, g is the acceleration of gravity, \dot{m} is the mass flow through motor, I_{sp} is the specific impulse, \dot{w} is

the weight flow of propellant through the motor, c^* is the characteristic velocity, C_f is the thrust coefficient, η_v is the motor velocity correction factor, and $c_{th} = \lambda V_e$, where λ is the nozzle correction factor, and V_e is the exhaust velocity. A_e is the exit area, p_e is the exhaust pressure, and p_0 is the ambient pressure.

The theoretical effective exhaust velocity is given by the equation:

$$c_{th} = \lambda V_e + (p_{eth} - p_0) \frac{A_e}{\dot{m}_{th}}$$

Theoretical and actual values differ because of the losses due to friction and heat transfer. Logical theoretical values of c might range between 3300-9000 feet/second. A typical actual value is on the order of 6300 feet/second. The effective exhaust velocity is an experimentally determined value, but it is not the exhaust velocity actually attained by the motor. In the flow of gas through a rocket nozzle, variations occur in the thermodynamic parameters because of friction, of the continued combustion of the propellant, and of the secondary effects of compressible fluid flow, such as shock waves. For these reasons the effective exhaust velocity is used primarily in solving actual problems.

EXHAUST VELOCITY, THEORETICAL.

Theoretically, the full amount of the lower heat of **combustion** could be utilized in an engine to provide an expansion of the gas down to the datum temperature; this yields a theoretical exhaust velocity. For actual operation, however, this would not be practical, even if the resulting low exit pressure could be attained without overexpansion, since it requires an extremely large area ratio of the nozzle. This ratio could be obtained either by using a short nozzle with a large angle of divergence, or a very long nozzle with a moderate angle of divergence. In the first case, considerable losses result from a decrease in the magnitude of the useful velocity component which lies parallel to the motor axis, in favor of the component normal to the axis. The latter component is of no use for the propulsion. In the second case a very long nozzle would increase the motor weight excessively and thus reduce the overall rocket performance more than the additional heat utilization would increase it. There exists an optimum

compromise between nozzle geometry and heat energy utilization.

EXIT APERTURE. The exit aperture, or image numerical aperture, of an axially symmetrical optical system is $n' \sin \theta'$ where n' is the refractive index in image space and θ' is the angle subtended, at the Gaussian image point, by the radius of the **exit pupil**. (See **numerical aperture**.)

EXIT PUPIL (OR IMAGE-SIDE PUPIL). The Gaussian image of the aperture of an optical system in image space.

EXOTHERMIC REACTIONS. See **thermal coefficients**.

EXOTHERMIC REACTION (EXOERGIC). (1) For exothermic reactions of molecules or between atoms, see **thermal coefficients**.

(2) A nuclear reaction in which, roughly speaking, the maximum kinetic energy of the products exceeds that of the reactants; i.e., one in which there is a net production of energy. More precisely, one for which the Q -value is positive. (See **reaction energy**.)

EXPANSION. (1) For mathematical expansions, see **Taylor series**, **Laplace expansion**, **binomial theorem**, etc. (2) The process of increasing the volume of a system either reversibly, as in a cylinder provided with a piston, or irreversibly, as in the **Joule-Thomson** throttling experiment through a porous plug.

Sometimes the term expansion is applied to describe any process involving a reversible or irreversible decrease in pressure. In most instances both meanings are identical. They are different when either pressure or volume is constant, but it is customary to use the phrases constant-pressure expansion (meaning an increase in volume) and constant-volume expansion (meaning a decrease in pressure).

EXPANSION, COEFFICIENT OF. See **coefficient of thermal expansion**.

EXPANSION AREA RATIO. The ratio of the cross-sectional area at the exit of a rocket motor, or the exit section of a nozzle, to the cross-sectional area of the throat of the motor or nozzle. This quantity is sometimes denoted by the shorter expression, **expansion ratio**, but in that case it should be distinguished from the other meaning of the latter term.

EXPANSION EFFICIENCY. See **enthalpy drop**.

EXPANSION, PARTIAL FRACTION. See **partial fraction expansion**.

EXPANSION RATIO. (1) The ratio of the pressure at a point just outside the rear lip of an exhaust nozzle to the pressure in the combustion chamber. This ratio is important because of its relationship to the **exhaust velocity** of the rocket. (2) The ratio of the area of the exit section of a nozzle to its throat. However, the more common term for this ratio is **expansion area ratio**.

EXPANSION WAVE. (Also called *rarefaction wave*.) A simple wave or progressive disturbance in the isentropic flow of a compressible fluid, such that the pressure and density of a fluid particle decrease on crossing the wave in the direction of its motion.

EXPECTATION. If $F(x)$ is the **distribution function** of a variate x , the expected value of a quantity t depending on x is

$$\int_{-\infty}^{\infty} t dF(x).$$

It is in fact the average or mean value of t over the distribution of x . Analogous expressions apply to multivariate situations.

EXPECTATION VALUE. In quantum mechanics, the expectation value or mean value of a quantity A in a normalized state ψ , for which $(\psi, \psi) = 1$, is given by a scalar product $(\psi, A\psi)$. If ψ is an **eigenstate** of A , then the expectation value is equal to the corresponding **eigenvalue**.

EXPLOSION ENGINE. See **spark ignition engine**.

EXPLOSION WAVES. Gravity waves produced in the atmosphere by large explosions, of which the classical examples are the eruption of Krakatoa in 1883 and the fall of the great Siberian Meteorite in 1908. Each produced barometric oscillations detected at distances of thousands of miles. They are tidal waves and are dispersed, although the velocity of each component wavelength is equal to the velocity of sound at some altitude in the atmosphere. They differ from solar and lunar diurnal and semi-diurnal oscillations in the atmosphere, whose nodes are lines of longi-

tude or latitude in that their amplitude is greatest in the troposphere, the others having their maximum amplitude at several times the height of the tropopause.

EXPONENT. Also called index, it is the power to which an expression is raised. The concept of exponent plays an important role in obtaining the solution of a second-order linear **differential equation** in the neighborhood of a regular **singular point**. For if we assume a formal solution

$$w = (z - c)^\alpha [1 + a_1(z - c) + a_2(z - c)^2 + \dots],$$

and seek to determine the coefficients a_i by substituting into the differential equation and comparing coefficients, we get an equation (called the **indicial equation**), determining two values of α , which are called the exponents of the equation at the point c . The concept can be generalized directly for equations of higher order.

"EXP-6" POTENTIAL. See **interatomic potential**.

EXPONENTIAL DISTRIBUTION. A distribution of the form

$$dF = \frac{1}{\sigma} \exp \left\{ -\frac{|x - m|}{\sigma} \right\} dx, \quad m \leq x \leq \infty.$$

The distribution

$$dF = \frac{1}{2\sigma} \exp \left\{ -\frac{|x - m|}{\sigma} \right\} dx, \quad -\infty \leq x \leq \infty$$

is sometimes called the double exponential distribution.

EXPOSED THREAD CORRECTION.

When temperatures are measured with the aid of a liquid-in-glass (e.g., mercury) thermometer, it is necessary for the whole of the thermometric fluid to be at the measured temperature. Normally this is impossible, because the thread must emerge and be visible to the eye. Consequently, in accurate measurements it is usual to apply a correction, known as the exposed or emergent stem correction. The correction is proportional to the average temperature of the stem outside the bath; the latter can be measured with the aid of a special thermometer (known as a Mahlke (thread) thermometer) provided with a very large bulb.

The reading of the main thermometer should be corrected by an amount

$$a(t - t_0)c.$$

Here a is the length of the exposed thread; t is the temperature reading; t_0 is the mean temperature of the exposed thread (measured by the auxiliary thermometer); and c is a coefficient which depends on the properties of the glass and the thermometer fluid.

With temperatures of the order of 300°C, the exposed thread correction may amount to as much as 10°C. The correction can be reduced or nearly eliminated by completely submerging the thermometer, but that is possible only very rarely.

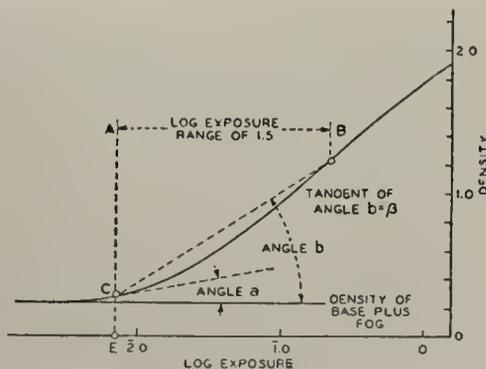
EXPOSURE. A measure of photographic stimulus defined as

$$\int I(t) dt,$$

where $I(t)$ is the **illumination** at time t .

EXPOSURE-DENSITY RELATIONSHIP.

The response of a sensitive material, as shown by the relationship between density and exposure, is usually represented by a curve in which density is plotted against the *logarithm* of the exposure. This curve is known as (1) the *D log E curve*; (2) the *H & D curve*, after Hurter & Driffield, two English investigators who were the first to plot such curves; and (3) as the *characteristic curve* since it indicates the principal characteristics of a sensitive material insofar as the relationship between exposure, development and density is concerned.



D log E curve. (From Neblette, *Photography, Its Materials and Processes*, 5th ed., D. Van Nostrand Company, Inc., Princeton, 1952).

EXPOSURE, PHOTOGRAPHIC. The product of **illuminance** I and time t of exposure. This definition assumes that the reciprocity

law holds. Sometimes the exposure is taken as It^p , where p is a constant of the particular photographic emulsion and has a magnitude approximately equal to unity.

EXTENDED SINE RELATIONSHIP. If n, n' are indices of refraction, y, y' are distances from the optical axis, θ, θ' are the angles light rays make with the optical axis in object, image space respectively and ϕ is the power of the optical system, then

$$ny \sin \theta + n'y' \sin \theta' = yy'\phi$$

is the extended sine relationship. When the approximation

$$yy'\phi \cong 0$$

is valid this is equivalent to Abbe's sine condition.

EXTENSION IN PHASE. Term introduced by Gibbs for volume elements in gamma-space.

EXTENSION, SIMPLE. See simple shear; simple extension.

EXTENSIVE VARIABLES. Properties such as mass m and volume V are defined for the system as a whole, and called in thermodynamics extensive variables or extensive properties. They always may be expressed as integrals over the whole system; i.e.,

$$m = \iiint_V \rho dx dy dz$$

where ρ is the density. (See also **thermodynamic system**.)

EXTENT OF REACTION (OR DEGREE OF ADVANCEMENT OF CHEMICAL REACTION.) (See conservation of mass in closed systems.) Consider a closed system, i.e., a system which does not exchange mass with the external world. If a reaction, either chemical or physico-chemical (e.g., a change of phase), takes place inside this system, the mass balance equation for the individual species taking part in the reaction can be written:

$$\begin{aligned} m_1 - m_1^0 &= \xi \nu_1 M_1 \\ m_2 - m_2^0 &= \xi \nu_2 M_2 \\ &\dots \dots \dots \\ m_n - m_n^0 &= \xi \nu_n M_n \end{aligned}$$

where m_1^0, m_2^0, m_n^0 are the initial masses of components 1, 2 \dots n and $\nu_1, \nu_2, \dots, \nu_n$ are the **stoichiometric coefficients**. ξ is called the extent of the reaction (Th. De Donder). The extension of this concept to several simultaneous reactions is immediate: to each distinct reaction ρ corresponds a distinct value ξ_ρ . The rate V_ρ of reaction ρ is the time derivative of ξ_ρ

$$V_\rho = \frac{d\xi_\rho}{dt}$$

EXTERNAL FORCE. See force, external or active.

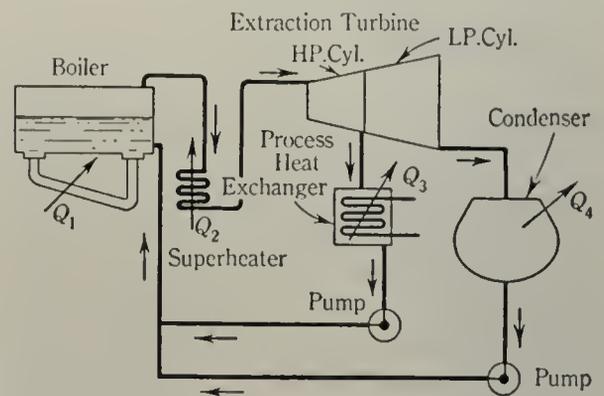
EXTERNAL HEAT OF EVAPORATION. See latent heat.

EXTERNAL OPTICAL DENSITY. External optical density or reflection density is the logarithm to base ten of the reciprocal of the (total) reflection factor.

EXTINCTION. The decrease in intensity of radiation traversing a medium due to both absorption and scattering.

EXTINCTION COEFFICIENT (ABSORPTION COEFFICIENT.) The internal absorption factor of an infinitesimally thin layer of a medium divided by the thickness of the layer.

EXTRACTION TURBINE. In order to increase the overall economy of a plant which requires process heat, it is advantageous to use a back-pressure turbine. The use of a back-pressure turbine becomes inconvenient when



Extraction turbine and related equipment.

the heat and power requirements are not well matched. Furthermore, if the temperature difference between the maximum temperature available and that required for process heating is small, the power produced will be small

and may not warrant the capital cost. In such cases it is possible to use an extraction turbine (or *pan-out turbine*) in which some steam is extracted from a turbine between stages at an appropriate temperature. The remainder of the steam is expanded to the lowest temperature available. This scheme enables variations in power and heat requirements to be easily accommodated.

EXTRANEIOUS. An extraneous root of a method of solving an **algebraic equation** E_1 is a root of an equation E_2 introduced by the method and having some of its roots in common with E_1 , which is not also a root of E_1 where the equation E_2 , introduced by the method, has some of its roots in common with E_1 . For example, let E_1 be the equation $\sqrt{x^2} = 3$ and E_2 the equation $x^2 = 9$ obtained by squaring both sides of E_1 .

EXTRAPOLATED END-POINT. In the theory of **neutron transport**, the flux distribution far from sources, boundaries, interfaces, appears as identical with the solution to an appropriate diffusion equation, adjusted to vanish at a point (z_0) outside the boundary-surface of the medium. This point is known as the extrapolated end-point.

EXTRAPOLATION. This process may be contrasted with **interpolation** when the function is required at a point outside the range of the x_i . The same methods are applied.

EXTREMAL. See **variations, calculus of.**

EXTREME PATH. See **Fermat principle.**

EYKMAN FORMULA. An empirical relationship for the molar refraction of a liquid, of the form:

$$R = \frac{M(n^2 - 1)}{\rho(n + 0.4)} = \frac{V(n^2 - 1)}{n + 0.4}$$

in which R is the **molar refraction** for a given optical frequency, n is the **index of refraction** for that frequency, M is the molecular weight, V is the **molecular volume**, and ρ is the density.

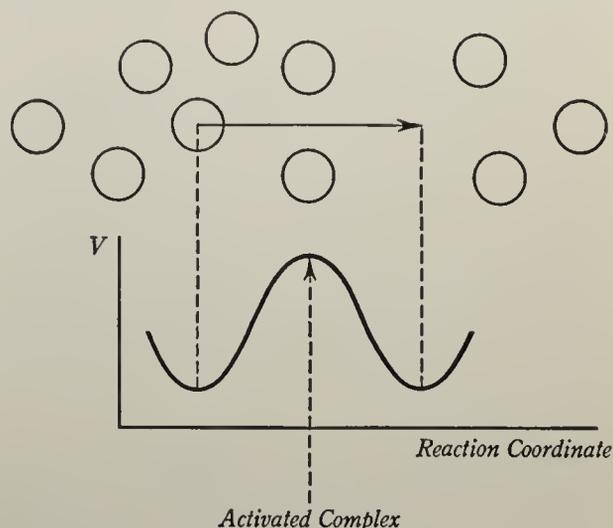
EYRING ACOUSTICAL ENERGY EQUATION. See **Franklin equation.**

EYRING THEORY OF REACTION RATES. See **absolute reaction rate theory.**

EYRING THEORY OF TRANSPORT PROCESSES. The **absolute reaction rate theory** has been applied by Eyring and others to transport processes in dense gases and liquids. The fundamental equation is

$$k = \left(\frac{kT}{h}\right) \exp\left(-\frac{\Delta G^\ddagger}{kT}\right). \quad (1)$$

It is assumed that in all these transport processes (viscosity, diffusion, thermal conductivity) the basic kinetic process is the motion of a molecule to a vacant site near it. In order to squeeze through the "bottleneck" formed by its nearest neighbors, the wandering molecule must pass through a region of high energy (see figure). One may apply the concepts of



activated complex, activation free energy, etc., exactly as in the theory of chemical rate processes (see **absolute reaction rate theory**) to obtain explicit expressions for the transport coefficients. The main weakness of the theory is the use of the equilibrium assumption for rates with relatively small activation energies (see **equilibrium theory.**) It is also not certain that one may consider the motion of simple molecules without taking account the perturbation of their neighbors.

F

FACTOR. See factor group, integrating factor, etc.

FACTOR ANALYSIS. Suppose we have a multivariate complex of variables x_1, x_2, \dots, x_k . We may be prepared to assume an underlying set of variables z_1, z_2, \dots, z_p , with $p < k$ such that each x is a linear function of the z 's together with a part specific to itself,

$$x_i = a_{i1}z_1 + a_{i2}z_2 + \dots + a_{ip}z_p + s_i.$$

Factor analysis is the name given to the techniques for estimating the various parameters in a model of this kind. Many such techniques exist, the most satisfactory from a theoretical point of view being usually laborious in computation. Electronic computers have proved very useful here.

The z 's are known as common factors, the s 's as specific factors. If one of the z 's contributes to all the x 's, it is called a *general factor*, while one that contributes to some but not to others is called a *group factor*. The a 's are called the *factor loadings* or *saturations*; a common factor with both positive and negative loadings is called *bipolar*. The proportion of the variance of a particular x accounted for by the common factors is called the *communality* of that x .

FACTOR-GROUP. If the subgroup N of a group G is normal (see **conjugate elements**) we can define a multiplication for its cosets as follows: If q, r are elements of G , and Q and R are cosets of N with q in Q and r in R , then the product QR is defined (uniquely since N is normal) as that coset which contains the element qr . Such a group is denoted by G/N and is called a factor-group; its number of elements is equal to the index (see **coset**) of N in G .

FACTORIAL. If n is a positive integer, factorial n means the product $1 \cdot 2 \cdot 3 \cdots n$. It may be denoted by the symbols \underline{n} or $n!$, but the latter form is generally preferred. For the general complex number z , we define $z! = \int_0^\infty u^z e^{-u} du$. Thus $z! = \Gamma(z + 1)$, where Γ denotes the **gamma function**.

FACTORIAL COEFFICIENTS. Stirling numbers, so called from their definition.

FACTORIAL EXPERIMENT. An experiment whose treatments are made up of combinations of the variants of several factors. The factor variants may be qualitative—e.g., crop varieties—or quantitative—e.g., amounts of fertilizer applied—but in either case are referred to as levels. Referring to the observations for convenience as yields, any **contrast** between the mean yields for different levels of one factor, averaged over all levels of the other factors in the experiment, is said to belong to the *main effect* of the factor. The mean yields for all combinations of the levels of two different factors, averaged over all levels of the other factors, can be set out in a two-way table; a contrast between the entries in this table which is orthogonal to all the main effect contrasts is said to belong to the first order interaction between the two factors. Interactions of higher order are similarly defined.

The underlying notion behind the analysis into main effects and interactions is that of additivity of the effects of the different factors. If the effects are perfectly additive, all interactions are zero; the presence of first or higher order interactions discloses the presence of less or more complicated departures from additivity. In many fields of application, interactions involving 3 or more factors are often small enough to be neglected.

One of the most useful types of factorial experiment is that in which each factor has the same number of levels. An experiment with s factors each at p levels is called an s^p experiment, and each replicate will contain s^p treatments. In practice, p is often 2 or 3, but if several factors are to be investigated s^p may be large and heterogeneity of the experimental material may lead to loss of accuracy. To avoid this, each replicate may be laid down in a number of blocks in such a way that the contrasts between blocks coincide with treatment contrasts that are of little interest, such as the higher order interactions. This device is known as *confounding*.

If the true values of the high order interactions are zero, their estimates will provide estimates of experimental error. An experiment with many factors can then be carried out in a single replication, a suitable estimate of error still being available. Extending this notion, it is possible to carry out an experiment using only a suitably chosen fraction $1/g$ of all the possible treatment combinations. In this case it is found that the contrasts that can be estimated are sums of groups of g main effect and interaction contrasts; each member of such a group is called an *alias* of all the others. If it can be arranged that all the aliases of the contrasts that are of interest are high order interactions, the interpretation of the results remains reasonably unambiguous. Experiments with single or fractional replication enable a large number of factors to be investigated in a single experiment without the amount of experimental material reaching unmanageable proportions.

FACTOR OF SAFETY. See **safety factor**.

FACTOR THEOREM OF ALGEBRA. If $(x - r)$ is a factor of a polynomial $P(x)$, then $x = r$ is a *zero* of the polynomial function $P(x)$, and a *root* of the polynomial equation $P(x) = 0$. Conversely, if $x = r$ is a zero of the polynomial or a root of the equation, then $(x - r)$ is a factor of $P(x)$. (See also **algebraic equations**.)

FAHRENHEIT TEMPERATURE SCALE. See **temperature**.

FAILURE. The inability of a **structure** or a structural member to perform its proper function causes a condition known as failure. This condition may be the result of sudden fracture as in the case of brittle materials or the excessive **deformation** of ductile materials. Another cause of failure is a lack of equilibrium between the external **loads** and resisting forces such as exists in structures which fail by sliding or overturning.

FAILURE, THEORIES OF. A term most often applied to yielding rather than more complete failure. Historically, in part because of a confusion between yielding and fracture, the criteria of failure include maximum normal stress, maximum extensional strain, maximum elastic energy, maximum shear strain or distortional energy, maximum shear stress, all possible combinations of the

invariants of stress for isotropic materials, and many others. Yielding of metals is only slightly effected by hydrostatic pressure and thus depends upon shear stress. For isotropic metals, the data seems to lie between the maximum shear stress hypothesis of Tresca and the J_2 criterion of Maxwell-Huber-Henky-Mises-Nadai.

FAIR GAME. In **games theory**, a game in which the participants have the same expectation of gain. This is not necessarily the same thing as having the same probability of success, since their stakes may be different.

FALSE POSITION, METHOD OF. See *Regula falsi*.

FALTUNG. Synonym of **convolution**.

FAMILY. A synonym of **aggregate**, **set**, etc., but usually used of a set (called an n -parameter family), which is determined by the values of n real (or complex) parameters. Thus the set of all circles in the plane is a 3-parameter family.

FAMILY OF SURFACES. See **surfaces**, **family of**.

FARAD. A unit of capacitance, abbreviation f or fd. The farad is the capacitance of a **capacitor** (condenser) which acquires a charge of one coulomb when a steady potential difference of one volt is maintained across its terminals. The microfarad (10^{-6} f) and the micromicrofarad (picofarad) (10^{-12} f) are used as units of capacitance much more often than is the farad.

FARADAY. The quantity of electric charge associated with one gram equivalent of an electrochemical reaction. It is equal to the product of the **Avogadro constant** and the charge on the **electron**. It has the value 9.651 abamperes per gram equivalent or 96.51 coulombs per gram equivalent.

FARADAY EFFECT. When a transparent isotropic medium is in a strong magnetic field there is a rotation of the plane of vibration of polarized light which is transmitted through the medium in the direction of the field. The direction of rotation depends on whether the velocity of the light is parallel or antiparallel to the magnetic field. The angle of rotation α is given by

$$\alpha = \omega H$$

where l is the length of the path traversed in a magnetic field of strength H and ω (sometimes C is known as the *Verdet constant*. The Faraday effect is one of the special cases mentioned under **rotation of the vibration plane**. (See Robertson, *Introduction to Optics*, 4th ed., D. Van Nostrand Co., Inc., Princeton, 1954.)

FARADAY LAW OF ELECTROMAGNETIC INDUCTION. When the magnetic flux, ϕ , linking a circuit, changes, an electromotive force given by

$$\varepsilon = - \frac{d\phi}{dt}$$

is induced in the circuit.

FAST EFFECT, INTERACTION. In reactor theory, that part of the **fast fission factor** stemming from fast fissions in one fuel rod caused by neutrons born in fission in a different rod.

FAST FISSION FACTOR. In reactor theory, this factor, denoted by ϵ , expresses the average number of fast neutrons slowing down past the fast fission threshold of U-238 or U-235, per neutron produced by fission in the system. A different definition, encountered occasionally, is the ratio of all neutrons produced per unit time by fission to the number of neutrons produced per unit time by thermal neutron induced fission.

FATIGUE. (1) The tendency for a metal to fracture under repeated stressing considerably below the ultimate tensile strength. In a common method of test, a polished specimen is rotated rapidly and subjected to alternate flexure. The *fatigue limit* or *endurance limit* of a metal or alloy is that stress below which failure will presumably not occur in an infinite number of cycles. When such a limit does not exist, the term is often applied to the stress causing fracture in a specified large number of cycles.

When alternating cycles cause appreciable alternating plastic deformations, fatigue occurs in a small number of cycles. This behavior is known as low-cycle fatigue. (See **shakedown**.)

FECHNER FRACTION. If the eye can just distinguish an object whose **brightness** differs by an amount dB from a large field of brightness B , the contrast sensitivity may be meas-

ured by dB/B , sometimes called the Fechner fraction.

FEEDBACK. General Concept. Feedback is said to be present in a system if at least one pair of physical quantities in the system, say $q_1(t)$, $q_2(t)$ are interrelated in such a way that q_1 affects q_2 and q_2 affects q_1 , the two relationships being independent. Thus if the relationships between the system quantities may be written

$$q_1(t) = f_1\left(\frac{d}{dt}\right) \cdot q_2(t) + \text{possible contributions from other quantities}$$

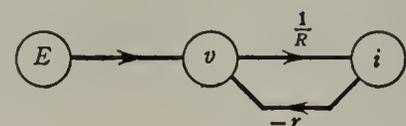
$$q_2(t) = f_2\left(\frac{d}{dt}\right) \cdot q_1(t) + \text{possible contributions from other quantities}$$

where $f_1(x)f_2(x) \neq 1$, then the feedback exists. Which direction of dependence is forward and which backward is largely a matter of arbitrary convention, though it is usual to associate the forward direction with a main causatory chain from the input to the output of the system: then any dependence by which a quantity earlier in the causatory chain is affected by a later one is a feedback. The essential point however, as stated above, is the existence of a two-way dependence of the above form. Surprisingly perhaps, the majority of systems are feedback systems. To illustrate this, consider a battery of e.m.f E and internal resistance r connected to a load resistance R . The current i is related to E by the single relation $i = E/(r + R)$ and therefore no feedback exists between i and E . If we introduce the voltage drop v across the load, however, we have the equations

$$v = E - ir$$

$$i = v/R$$

and we note that there exist two independent relations between v and i . Specifically, v controls i through the medium of the load resistance R , whereas i controls v through the medium of the battery internal resistance r . These equations may be pictorially depicted in a dependence or **signal flow diagram**:



Signal flow diagram.

The existence of the feedback existing between v and i is displayed here in the form of a closed loop threading the two quantities, called a *feedback loop*.

Although even such a simple system as this can be therefore considered as a feedback system, the more important class of feedback systems are those in which a feedback loop exists which includes an active element, i.e., a system element associated with a source of power, for instance, an electronic tube (with its source of anode voltage supply), a rotary machine amplifier (with its prime mover), a magnetic amplifier (with its a.c. power supply) or their equivalents in other fields of engineering. It is to such systems that feedback theory is mainly applied. It is also only such systems which are capable of sustained oscillations or instability, a property which distinguishes them from purely passive systems or systems including active elements which are not included in feedback loops.

This last statement is, however, only applicable to what might be called engineering systems. Feedback theory has been applied with some measure of success to other fields, notably economics and ecology. It is known that economic systems are liable to sustained oscillations (the so-called booms and slumps) as also are, for instance, the population densities of certain interacting animal species (e.g., lynxes and foxes). What, if anything, in these systems corresponds to an active element is difficult to assess.

The main reasons for introducing feedback into a system are as follows: (1) to modify the gain-frequency characteristic of a system; this includes, for instance, modifying the degree of stability of the system, modifying its cut-off frequency, making the gain more uniform over the working band of frequencies, etc.; (2) to minimize the effects of non-linearities of a system; (3) to minimize the effects of the variation of system parameters; (4) to minimize the output noise level of a system; (5) to modify, particularly in feedback amplifiers, the input and output impedances of the amplifier; (6) to limit the amplitude of one or more system quantities, e.g., torque in an electro-mechanical control system.

Single Loop Analysis. In a system containing a single feedback loop relating two quantities $q_1(t)$ and $q_2(t)$ we may write

$$q_1(t) = \theta_1(t) + \beta \left(\frac{d}{dt} \right) \cdot q_2(t)$$

$$q_2(t) = \mu \left(\frac{d}{dt} \right) \cdot q_1(t)$$

where $\theta_1(t)$ is an external input to the quantity $q_1(t)$. These equations, eliminating $q_1(t)$, may be written, dropping the brackets,

$$\frac{q_2}{\theta_1} = \frac{\mu}{1 - \mu\beta}$$

or, if we limit our attention to steady-state conditions at an angular frequency ω , writing $q_2(t) = Q_2 e^{i\omega t}$, $\theta_1(t) = \Theta_1 e^{i\omega t}$

$$\frac{Q_2}{\Theta_1} = \frac{\mu(i\omega)}{1 - \mu(i\omega)\beta(i\omega)} \equiv G(i\omega)$$

where $G(i\omega)$ is the output-to-input or gain frequency-function of the system, its modulus giving the amplitude ratio of q_2 and θ_1 , its phase, the phase advance of q_2 relative to θ_1 .

The chief effects of the feedback dependence β may be stated as follows:

(1) $G(i\omega)$ is multiplied at every frequency by the factor $\{1 - \mu(i\omega)\beta(i\omega)\}^{-1}$ i.e., the output-to-input amplitude ratio is multiplied by $|1 - \mu\beta|^{-1}$ and the output-to-input phase advance is reduced by $\text{ph}(1 - \mu\beta)$.

(2) Considering β to be a constant function but μ to vary (due, for instance, in an electronic amplifier to tube aging), logarithmic differentiation of the above expression for G gives

$$\begin{aligned} d \log G &= d \log \mu + \frac{\mu\beta}{1 - \mu\beta} d \log \mu \\ &= \frac{1}{1 - \mu\beta} d \log \mu \end{aligned}$$

or

$$\begin{aligned} d\{\log|G| + i \text{ph } G\} \\ &= \frac{1}{1 - \mu\beta} d\{\log|\mu| + i \text{ph } \mu\}. \end{aligned}$$

Thus, if and only if $\mu\beta$ is real and $\neq 1$,

$$\begin{aligned} d \log|G| &= \frac{1}{1 - \mu\beta} d \log|\mu| \\ d \text{ph } G &= \frac{1}{1 - \mu\beta} d \text{ph } \mu. \end{aligned}$$

That is, the variation in the gain (expressed in nepers or decibels), for a given variation in μ

expressed in the same units, is multiplied by $\frac{1}{1 - \mu\beta}$ and so is the variation in the phase of the gain function for a given phase change in μ .

It should be noted here that normally, over the pass-band of a feedback amplifier $|\mu\beta| \gg 1$ and $\text{ph } \mu\beta \doteq \pi$ i.e., $\mu\beta$ is sensibly large and negative real. Note further that as $|\mu\beta| \rightarrow \infty$, $G \rightarrow \frac{1}{\beta}$ and is thus unaffected by μ variation.

(3) The output noise level due to any source of noise injected into or developed in any part of the μ -sequence is multiplied by $|1 - \mu\beta|^{-1}$, assessed at noise frequency. The same is true of the effect of non-linearities in the μ -sequence, since these, by creating signal components at new frequencies (modulation products) may be considered as "noise" sources.

The simple concepts and properties above, however, are inadequate for the analysis of more complex, particularly multi-loop systems, if only for the reasons that in practice the addition of the β -sequence often modifies the μ -sequence, whereas μ has been assumed the same whether β is present or not, and that in practice it is very often difficult to disentangle the μ -sequence from the β -sequence. Some aspects of the more advanced theory are given under various separate headings, notably **stability**, **sensitivity**, **return difference**, **signal flow diagrams**, etc.

FEED-WATER HEATING. The process whereby the feed-water to the boiler is pre-heated by the use of steam bled from the power plant (turbine) between stages. This causes an increase in the efficiency of the plant. (See **carnotization**.)

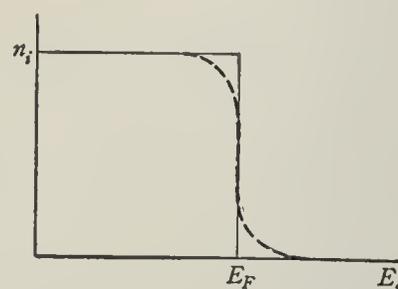
FERMAT'S PRINCIPLE. See **optical path**.

FERMI CONSTANT. A universal constant appearing in **β -disintegration theory**, expressing the strength of the interaction between the transforming nucleon and the electron-neutrino field. Its value is between 10^{-48} and 10^{-49} g cm⁵ sec⁻², and it is usually denoted by the symbol g or G . (See also **coupling constant**.)

FERMI-DIRAC DISTRIBUTION FUNCTION. In a **Fermi-Dirac gas** the number of particles, n_i , in a state of energy E_i , at the absolute temperature T is

$$n_i = \frac{1}{e^{(E_i - E_F)/kT} + 1}$$

where k is the **Boltzmann constant** and E_F is the **Fermi energy**. If $kT \ll E_F$ this function is nearly unity for $E_i < E_F$ and nearly zero



Fermi-Dirac distribution function for a gas at very low temperatures.

for $E_i > E_F$. The gas is then said to be degenerate, and only the particles in the range of energy levels of width kT about E_F are available for conduction, etc.

FERMI-DIRAC GAS. An assembly of independent particles obeying **Fermi-Dirac statistics (fermions)**. The essential property of such particles is that they obey the **Pauli exclusion principle** of quantum mechanics, which requires that no two particles may be in exactly the same state. If the gas is very dense, this requirement may mean that the particles must occupy a wide range of **energy levels**, much larger perhaps than can be supplied by thermal energy. It can be shown by statistical mechanics that in such a degenerate gas (of which the free electrons in a metal are the standard example) only a small proportion of the particles contribute to such properties as the susceptibility, specific heat, conductivity. (See also **quantum statistics**.)

FERMI-DIRAC LIQUID. A liquid whose constituent particles are **fermions**. The best known example is liquid He₃.

FERMI-DIRAC STATISTICS. In the case where the particles in a system of independent particles satisfy the Pauli exclusion principle, there will be either just one possible **macrostate** or none for a given set of numbers n_i of particles in the i -th single particle level, depending on whether or not all n_i are equal to zero or unity. In the first case the possible macrostate is the one corresponding to an anti-symmetric superposition of all possible **microstates**. This means that in Fermi-Dirac statistics all macrostates corresponding to a set of n_i , none of which is larger than unity, have the same weight, while any macrostate corre-

sponding to a set of n_i of which at least one is larger than unity has zero weight. (See also **quantum statistics**.)

FERMI ENERGY. According to the free electron theory of metals the electrons form a dense **Fermi-Dirac gas**, whose energy is very high even at very low temperatures. The Fermi energy is a measure of this, but the term seems to be used indiscriminately to refer either to (a) the energy E_F occurring as a parameter in the **Fermi-Dirac distribution function** and measuring the highest occupied level at very low temperatures, or to (b) the average energy of the electrons, which is $\frac{3}{5}E_F$. In most metals the Fermi energy is of the order of a few electron volts. The usage (b) is recommended, (a) being referred to as the **Fermi level** (it being, in fact, the **chemical potential** of the electrons).

FERMI HOLE. The depletion region surrounding an electron in the energy band theory of solids. As a result of electron correlations and **exchange** effects, there is a region surrounding each electron in which there is less than normal probability of finding other electrons.

FERMI LEVEL. The point on an **energy level diagram** corresponding to the top of the Fermi distribution; or the energy level (in a semi-conductor) for which the **Fermi-Dirac distribution function** has a value of $\frac{1}{2}$. It is thus the **thermal potential** of a Fermi-Dirac system of independent particles. Usually applied to a **degenerate gas**, where the Fermi level coincides with the top of the particle distribution.

FERMI LEVELS, QUASI. In a **semiconductor** a potential energy defined so as to give either the number of **holes**, or of electrons in the **conduction band**, when the material is not in thermal equilibrium, as if it were the **Fermi level**.

FERMION. A particle obeying **Fermi-Dirac statistics**.

FERMI OSCILLATOR. A system capable of only two **energy levels**.

FERMI PERTURBATION. See **Fermi resonance**.

FERMI RESONANCE. In a polyatomic molecule the accidental degeneracy of two

vibrational energy levels belonging to the same species, leading to **perturbation** (mutual "repulsion") of the "original" energy levels E_n^0 and E_i^0 . If ψ_n^0 and ψ_i^0 are the "original" eigenfunctions, the matrix element W_{ni} of the perturbation function is given by

$$W_{ni} = \int \psi_n^0 W \psi_i^0 d\tau$$

where W is essentially given by the anharmonic (cubic, quartic, ...) terms in the potential energy (see **vibrational energy levels of a molecule**).

If the separation $\delta = E_n^0 - E_i^0$ is small (close resonance), the energy E of the perturbed energy levels is given by

$$E = \bar{E}_{ni} \pm \frac{1}{2} \sqrt{4|W_{ni}|^2 + \delta^2}$$

where $\bar{E}_{ni} = \frac{1}{2}(E_i^0 + E_n^0)$. If δ is large compared to $2|W_{ni}|$ the energy of the perturbed levels is given by

$$E = \bar{E}_{ni} \pm \left[\frac{\delta}{2} + \frac{|W_{ni}|^2}{\delta} \right].$$

The eigenfunctions corresponding to the perturbed energy levels are given by

$$\psi_n = a\psi_n^0 - b\psi_i^0$$

$$\psi_i = b\psi_n^0 + a\psi_i^0$$

where

$$a = \left[\frac{\sqrt{4|W_{ni}|^2 + \delta^2} + \delta}{2\sqrt{4|W_{ni}|^2 + \delta^2}} \right]^{\frac{1}{2}}$$

$$b = \left[\frac{\sqrt{4|W_{ni}|^2 + \delta^2} - \delta}{2\sqrt{4|W_{ni}|^2 + \delta^2}} \right]^{\frac{1}{2}}.$$

Due to the interaction of rotation and vibration the Fermi resonance will as a rule also cause a change in the effective B values (see **rotational energy levels of a molecule**) of the interacting vibrational levels. If B_n^0 and B_i^0 are the B values for the unperturbed levels, the actual B values are given by

$$B_n = a^2 B_n^0 + b^2 B_i^0,$$

$$B_i = b^2 B_n^0 + a^2 B_i^0,$$

where a and b are the constants defined above. Since $a^2 + b^2 = 1$, it follows that

$$B_n + B_i = B_n^0 + B_i^0$$

that is the sum of the B values is unchanged by the perturbation.

FERMI TEMPERATURE. The degeneracy temperature of a **Fermi-Dirac** gas, defined by E_F/k , where E_F is the energy of the **Fermi level**, occurring as a parameter in the **Fermi-Dirac distribution function**. For the free electrons in a metal, this temperature is of the order of tens of thousands of degrees, so that the gas is highly degenerate.

FERMI THEORY OF BETA DECAY. The Fermi theory of β -decay was proposed in 1934 in analogy with quantum electrodynamics. It gives the probability P of the ejection, in unit time, of a β -particle whose energy lies between E and $E + dE$ as

$$PdE = f(Z, E)pE(E_{max} - E)^2dE$$

where p is the momentum of the electron. Here E is expressed in units of mc^2 and p in units of mc . $f(Z, E)$ is a slowly varying function of nuclear charge Z and energy.

FERMI THEORY OF COSMIC RAY ACCELERATION. A theory in which "collisions" are assumed to take place between charged cosmic ray particles and the magnetic fields associated with turbulent interstellar gas clouds. A charged particle will be reflected with increased energy if the gas cloud is moving toward it, and with decreased energy in an overtaking collision. Since more collisions per unit time will occur with gas clouds moving toward a particle than away from it, an average net gain of energy per collision results. Many such collisions can account for the high energies found in cosmic ray particles.

FERROELECTRIC EFFECT. The phenomenon whereby certain crystals may exhibit a spontaneous **dipole moment** (which is called ferroelectric by analogy with **ferromagnetic**—exhibiting a permanent magnetic moment). The effect in the most typical case, barium titanate, seems to be due to a **polarization catastrophe**, in which the local electric fields due to the polarization itself increase faster than the elastic restoring forces on the ions in the crystal, thereby leading to an asymmetrical shift in ionic positions, and hence to a permanent dipole moment. Ferroelectric crystals often show several **Curie points**, **domain structure hysteresis**, etc., much as do ferromagnetic crystals.

FERROMAGNETIC RESONANCE. The apparent **permeability** of a magnetic material

at **microwave frequencies** is affected (in the presence of a transverse, steady field) by the precession of electron orbits in the atoms. If the microwave frequency equals the precession frequency, **resonance** occurs and the apparent permeability reaches a sharp maximum. The resonance frequency depends upon the strength of the transverse field. Thus, a thin film of a ferromagnetic substance placed in a static magnetic field H is found to be capable of absorbing from an oscillating field whose magnetic vector is perpendicular to H at a frequency given by

$$\omega = \left(\frac{ge}{2mc} \right) (BH)^{1/2}$$

where B is the magnetic induction associated with H , e and m are the charge and mass of the electron, c is the velocity of light, and g is very near to 2, the **Landé factor** for free electrons.

FERROMAGNETISM, HEISENBERG THEORY. See Heisenberg theory of ferromagnetism.

FERROMAGNETISM, STATISTICAL TREATMENT. See cooperative phenomena.

FEYNMAN METHOD. In **neutron transport theory**, a method for the analysis of the integral equation form of the theory. We assume that for the system in question we can solve the one velocity integral equation (with isotropic scattering) for each velocity v . Thus, we generate a set of eigenfunctions $u_i(\mathbf{r};v)$ for any fixed v . The solution to the full integral equation is expanded in terms of these functions, thus reducing the full equation to a system of integral equations in v only. In certain cases the set u_i may be replaced by the set of solutions to the diffusion equation without severe loss of accuracy. (See **integral transport equation**.)

FEYNMAN DIAGRAM. See Feynman positron theory.

FEYNMAN POSITRON THEORY. In 1949 Feynman developed a theory the purpose of which was to simplify all calculations involving positrons and electrons interacting with external fields. The theory was then extended to include interactions between the particles, and the resulting theory was proved equivalent to the standard procedure of **quantized**

field theory. Feynman's theory gives a completely new physical viewpoint for the interacting process, permitting visualization and ease of understanding. We first illustrate it for the non-relativistic theory.

Consider the Schrödinger equation for a single particle, $i\hbar\partial_t|t\rangle = H|t\rangle$. The state vector at time $|t_2\rangle$ is related to the one at time $|t_1\rangle$ by the time-translation operator $U(t_2, t_1) = \exp(-iH(t_2 - t_1)/\hbar)$

$$|t_2\rangle = U(t_2, t_1)|t_1\rangle = \exp[-iH(t_2 - t_1)/\hbar]|t_1\rangle. \quad (1)$$

In the Schrödinger position representation $\psi(x, t) = \langle x|t\rangle$ is the Schrödinger wave function at time t , in terms of which (1) becomes

$$\begin{aligned} \langle \mathbf{x}_2|t_2\rangle &= \psi(\mathbf{x}_2, t_2) \\ &= \int \langle \mathbf{x}_2|U(t_2, t_1)|\mathbf{x}_1\rangle \langle \mathbf{x}_1|t_1\rangle d^3x_1 \\ &= \int d^3x_1 K(\mathbf{x}_2 t_2; \mathbf{x}_1 t_1) \psi(\mathbf{x}_1 t_1) \end{aligned} \quad (2)$$

where

$$\begin{aligned} K(\mathbf{x}_2 t_2; \mathbf{x}_1 t_1) &= \langle \mathbf{x}_2|U(t_2, t_1)|\mathbf{x}_1\rangle \\ &= \sum_E \langle \mathbf{x}_2|E\rangle \langle E|\exp[-iH(t_2 - t_1)/\hbar]|\mathbf{x}_1\rangle \\ &= \sum_E u_E(\mathbf{x}_2) \overline{u_E(\mathbf{x}_1)} \exp[-iE(t_2 - t_1)/\hbar] \end{aligned} \quad (3)$$

in the Green's function for Equation (1). In (3) the states $|E\rangle$ are the eigenfunctions of H , which is assumed to be time-independent. We have adopted the notation $u_E(\mathbf{x}) = \langle \mathbf{x}|E\rangle$. From the completeness of the set $\{|E\rangle\}$, i.e., from the fact that $\sum_E |E\rangle\langle E| = 1$, we note that (3) implies

$$K(\mathbf{x}_2 t; \mathbf{x}_1 t) = \langle \mathbf{x}_2|\mathbf{x}_1\rangle = \delta(\mathbf{x}_2 - \mathbf{x}_1) \quad (4)$$

as should be. From the fact that $U(t_2, t_1)$ vanishes for $t_2 < t_1$, $K(\mathbf{x}_2 t; \mathbf{x}_1 t)$, likewise vanishes for $t_2 < t_1$. The above Green's functions K is therefore that solution of

$$\begin{aligned} [i\hbar\partial_{t_2} - H(\mathbf{x}_2)]K(\mathbf{x}_2 t_2; \mathbf{x}_1 t_1) \\ = i\hbar\delta(\mathbf{x}_2 - \mathbf{x}_1)\delta(t_2 - t_1) \end{aligned} \quad (5)$$

which vanishes for $t_2 < t_1$. A perturbation theory can easily be developed in this formulation. Assume that $H = H_0 + V$ and that the problem is soluble for $H = H_0$, i.e., when $V = 0$. Let us denote by K_0 the Green's function

for $V = 0$, i.e., for the free particle. Then the solution of (5) which satisfies the boundary condition $K = 0$ for $t_2 < t_1$ is the solution of the integral equation

$$\begin{aligned} K(2, 1) &= K_0(2, 1) \\ &- \frac{i}{\hbar} \int K_0(2, 3)V(3)K(3, 1)d^4x_3 \end{aligned} \quad (6)$$

where $K(2, 1)$ stands for $K(\mathbf{x}_2 t_2, \mathbf{x}_1 t_1)$ and $d^4x = d^3x dt$. That (6) is a solution of (5) is readily verified by operating with $[i\hbar\partial_{t_2} - H_0(2)]$ on both sides of the equation; also since $K_0(2, 1)$ is zero if $t_2 < t_1$, $K(2, 1)$ likewise vanishes for $t_2 < t_1$. The **Neuman-Liouville** expansion of (6) yields

$$\begin{aligned} K(2, 1) &= K_0(2, 1) - \frac{i}{\hbar} \int d^4x_3 K_0(2, 3)V(3) \\ &\cdot K_0(3, 1) + \left(\frac{-i}{\hbar}\right)^2 \int d^4x_3 \int d^4x_4 K_0(2, 3) \\ &\cdot V(3)K_0(3, 4)V(4)K_0(4, 1) \end{aligned} \quad (7)$$

which can be interpreted as follows: the first term represents the particle traveling freely from 1 to 2. The second term represents the particle moving freely from 1 to 3 being scattered at 3 (due to $V(3)$) and again traveling as a free particle from 3 to 2. (The integration over 3 is evident.) The third term represents double scattering at 3, and at 4, etc.

A corresponding treatment of the **Dirac equation** can be given. For a free particle the latter reads

$$(i\gamma_\mu\partial^\mu - m)\psi(x) = 0 \quad (8)$$

(where we have set $\hbar = c = 1$). The Green's function K_0 connecting the wave function ψ at 2 to that at 1 is now a 4×4 matrix, which satisfies

$$\begin{aligned} (i\gamma_\mu\partial_2^\mu - m)K_0(2, 1) &= i\delta(2, 1) \\ [\delta(2, 1) &= \delta(\mathbf{x}_2 - \mathbf{x}_1)\delta(t_2 - t_1)]. \end{aligned} \quad (9)$$

A particular solution of this differential equation is given by

$$\begin{aligned} K_0 &= \sum_{E_n > 0} u_n(\mathbf{x}_2)\bar{u}_n(\mathbf{x}_1) \exp[-iE_n(t_2 - t_1)] \\ &+ \sum_{E_n < 0} u_n(\mathbf{x}_1)\bar{u}_n(\mathbf{x}_2) \exp[-iE_n(t_2 - t_1)], \end{aligned}$$

for $t_2 > t_1$

$$K_0 = 0, \quad \text{for } t_2 < t_1 \quad (10)$$

where u_n are energy eigenfunctions of the Dirac equation. The negative energy solutions must be included in order that for time $t_2 = t_1$, $K_0(2,1) = \delta(x_2 - x_1)$ [since only with the inclusion of the negative energy solutions is the set $u_n(x)$ complete].

This choice of K_0 corresponds to the one-electron theory rather than to the correct **hole theory**. This can be seen as follows: Consider an electron being scattered by a potential V which vanishes for $t < -T$ and for $t > T$. The result of a perturbation calculation [Equation (7)] is then that for $t > T$ the wave function of the electron will contain components corresponding to positive as well as negative energy solutions of the Dirac equation, implying a non-vanishing probability for finding the particle in a negative energy state which is clearly unphysical. The correct Green's function can be obtained by noting that according to the hole theory, the negative energy states are occupied, hence they are not available to the electron after scattering. This requires that for $t_2 > t_1$, K is a sum over positive energy states only; K must, however, still satisfy (9) in order that it be a Green's function. This can be done by subtracting from the solution K_0 given by (10) the following solution of the homogeneous equation: $\sum_{E_n < 0} u_n(x_2)\bar{u}_n(x_1) \exp[-iE_n(t_2 - t_1)]$. This yields the new Green's function K_+ with the properties that

$$K_+ = \sum_{E_n > 0} u_n(x_2)\bar{u}_n(x_1) \exp[-iE_n(t_2 - t_1)], \quad \text{for } t_2 > t_1$$

$$K_+ = - \sum_{E_n < 0} u_n(x_1)\bar{u}_n(x_2) \exp[-iE_n(t_2 - t_1)], \quad \text{for } t_2 < t_1$$

$$= - \sum_{E_n < 0} u_n(x_1)u_n(x_2) \exp[-i|E_n||t_2 - t_1|], \quad \text{for } t_2 < t_1. \quad (11)$$

If a potential is present, an integral equation which is the analogue of (7) can be written. (K_+ replaces K_0 .) The various terms in the series expansion of this integral equation can

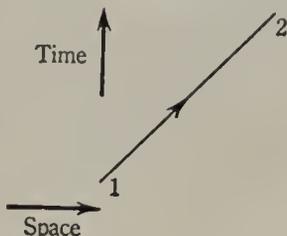


Fig. 1.

be visualized by means of space-time *Feynman diagrams*. The first term $K_+(2,1)$ represents a free particle traveling from 1 to 2 and will be represented by a directed straight line (Figure 1). The second term

$$(-i) \int K_+(2,3)V(3)K_+(3,1)d^4x_3 \quad (12)$$

represents a single scattering. If the region where V is different from zero is represented by a bounded region S in a space-time plot, then this term (12) can be represented by the diagram indicated in Figure 2.

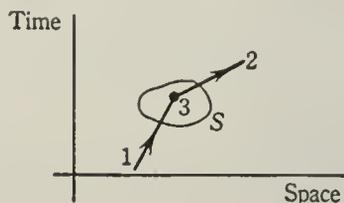


Fig. 2.

The diagrams drawn in Figure 3 represent the two possibilities represented by the third term in the perturbation expansion:

$$(-i)^2 \int d^4x_3 K_+(2,3)V(3)K_+(3,4)V(4)K_+(4,1). \quad (13)$$

Case (a). As one follows the world line from 1 to 2 time always increases, so that in K_+ only positive energies occur.

Case (b). As one follows the world line, in going from 4 to 3 time goes "backward" and

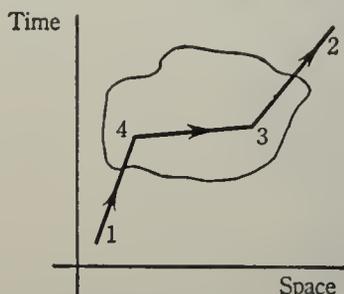


Fig. 3a. Case (a).

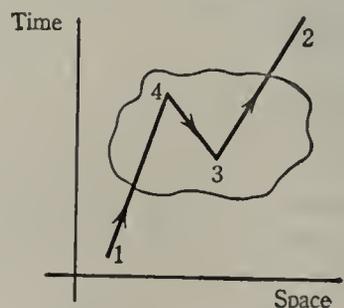


Fig. 3b. Case (b).

in K_+ (4,3) negative energy sums occur. In Feynman's interpretation this represents a positron propagating from 3 to 4. Alternatively, proceeding in the direction of increasing time one can interpret the sequence of events as follows: An electron-positron pair is created at 3; the electron propagates toward 2, the positron propagates toward 4, at which point it annihilates the incoming electron.

Both situations are included in the single term (13) since one is to integrate over all times 3 and 4. The term (13) is therefore only represented by a single diagram, either Figure 3a or 3b, since whichever is chosen the other can be obtained from it by letting the time component of 3 move relative to that of 4.

The above prescription for the propagation kernel K_+ can be shown to be equivalent to the hole theoretic formulation, the minus sign in 10(b) being a reflection of the **Pauli principle**. The formalism can be extended to the case of many particles interacting through the quantized electromagnetic field.

The connection of the Feynman formalism to the quantized electron-positron field can be established as follows:

In the presence of an external electromagnetic field, the electron-positron field is described by a Heisenberg field operator $\psi(x)$ which obeys the following field equation and equal time anticommutation rules

$$(-i\gamma^\mu\partial_\mu + m)\psi_e(x) = \frac{e}{\hbar i}\gamma^\mu A_\mu^e(x)\psi_e(x)$$

$$\{\psi(x), \psi^*(x')\}_{x_0 = x'_0} = \delta(\mathbf{x} - \mathbf{x}')$$

and which has the following expansion in terms of creation and annihilation operators

$$\psi_F(x) = \sum_{n, E^+} b_n \phi_n(x) + \sum_{n, E^-} \phi_n(x) d_n^*$$

where the first term runs over the positive energy solutions and the second over negative energy solutions of the Dirac equation in this external field; b_n is a destruction operator for an electron in the positive energy state $\phi_n(x)$ and d_n^* a creation operator for a positron. We shall assume A^e time independent and sufficiently weak so that stationary solution $\phi_n(x) = u_n(x) \exp(-iE_n t)$ can be defined for which a gap exists between positive and negative energy eigenvalues. Under these circumstances a vacuum state Ψ_0 exists which has the property that $b_n \Psi_0 = d_n \Psi_0 = 0$, for all n . The

probability amplitude of finding an electron at time t_2 at \mathbf{x}_2 if there was one at time t_1 at \mathbf{x}_1 with $t_2 > t_1$ can then be obtained from the quantity

$$(\bar{\psi}(2)\Psi_0, \bar{\psi}(1)\Psi_0) = (\Psi_0, \psi(2)\gamma_0\psi(1)\Psi_0)$$

or more generally from the quantity

$$S_{Fe}(2,1) = (\Psi_0, T(\psi(2)\bar{\psi}(1)\Psi_0)$$

where T is the Wick **chronological product**. One can verify that $-1/2iS_{Fe}(2,1) = K_+(2,1)$ where K_+ is the Feynman propagator in the presence of the potential $\gamma^\mu A_\mu^e$.

FEYNMAN THEORY OF LIQUID HELIUM. An extension of the Landau theory of liquid helium II, in which the liquid is treated as more like a solid than a gas. The presence of phonons and rotons is predicted as well as the "freezing out" of the rotons below 0.6°K .

FIBER STRESS. Lines parallel to the axis of a **bar** are called fibers. The normal stress on the cross section of a bar is called the fiber stress. In the elastic range, the extreme fiber stress in a straight bar bent about a principal axis is given by $P/A + Mc/I$ where P is the axial force, A , the cross-sectional area, M , the bending moment, I , the moment of inertia about the principal axis of bending and c , the distance of the extreme fiber from the neutral axis.

FICKIAN DIFFUSION. Diffusion in which the transfer of a quantity Q is in a direction down the gradient of Q at a rate proportional to the gradient. The flux of Q across unit area perpendicular to the unit vector \mathbf{l} is $\kappa \mathbf{l} \cdot \text{grad } Q$, κ being the diffusion coefficient which is, however, usually the same in all directions, in which case

$$\frac{\partial Q}{\partial t} = \kappa \text{div grad } Q = \kappa \nabla^2 Q.$$

But if the medium is not isotropic κ may vary with direction, and then the lines of flow of Q are not parallel to the gradient of Q .

FICK LAW OF DIFFUSION. See **Fickian diffusion**; **diffusion**, **Fick law of**.

FIDUCIAL INFERENCE. A type of estimation which, in a defined probabilistic sense, locates a parameter in a certain range; or a

type of inference which purports to decide questions of significance in the light of a "fiducial probability" distribution.

Intervals of estimation based on fiducial probability are different in general from **confidence** intervals, but the two are sometimes confused. The fiducial argument has been subject to much controversy.

FIELD. A set of elements for which addition and multiplication are defined (and lead to elements in the set) such that: both operations are **commutative** and **associative**, and multiplication is **distributive** over addition; also there are two elements, conveniently called 0 and 1, such that $0 + a = 1a = a$ for all a in the set; finally, subtraction and division are always possible and unique (except that division by zero is not possible); that is, for every a there exists a unique x such that $a + x = 0$ and a unique y (unless $x = 0$) such that $ay = 1$. Well-known examples of fields are: the set of real numbers, of complex numbers, of rational numbers, of residue classes modulo a prime, and so forth. (See also entries following.)

FIELD, ELECTRIC. (1) The attraction (or repulsion) exerted by one electric charge on another can be described quantitatively in terms of the electric field produced by the first charge. By definition, the field at any point is the force (a vector, having magnitude and direction) that would be exerted on a unit positive test charge placed at that point. Hence an electric field is a specific type of **field of force**. The path that would be followed by an inertia-less test particle is called a "line of force." It follows from the **Coulomb law** that the field of a point charge in free space has the magnitude

$$E = \frac{q}{4\pi\epsilon_0 r^2}$$

where q is the charge, ϵ_0 is the **permittivity**, r is the distance from the charge, and rationalized units are employed. (See **flux, electric**.)

(2) An electric field is produced also by a varying magnetic field. (See **Maxwell's equations**; **Faraday law of electromagnetic induction**; **field of moving charge in space**.)

FIELD EQUATIONS, LAGRANGE. See **Lagrange field equations**.

FIELD EQUATIONS, RELATIVISTIC. See **relativity, general**.

FIELD, FREE. A field (wave or potential) in a homogeneous, isotropic medium free from boundaries. In practice it is a field in which the effects of the boundaries are negligible over the region of interest. In acoustics, the actual pressure impinging on an object (e.g., electroacoustic transducer) placed in an otherwise free **sound field** will differ from the pressure which would exist at that point with the object removed, unless the acoustic impedance (see **impedance, acoustic**) of the object matches the acoustic impedance of the medium.

FIELD, IMPRESSED. The **electromagnetic field** which would exist if the body of interest were not present, or the field that would exist in free space with the given source distribution.

FIELD, INDUCTION. See the discussion under **field, radiation**.

FIELD, LEPTON. In the **Fermi theory of beta-decay**, the nuclear transition involving the emission of an electron and an antineutrino is treated in a way which is analogous to the treatment of electromagnetic transitions. As an analog to the electromagnetic field, Fermi introduced the concept of the electron-neutrino field or, as it is now called, the **lepton field**. This field is described mathematically as being the sum of products of electron and antineutrino **wave functions** plus a sum of products of positron and neutrino wave functions.

FIELD OF FORCE. Electric charges exert force upon other electric charges, magnets exert forces upon other magnets, matter exerts gravitational force upon other matter. All these action-at-a-distance phenomena are conveniently described in terms of the "force field" set up by a source. The force exerted on a unit test-particle (unit charge, unit pole, unit mass) is a vector function of position of the test-particle. That is, the force on a unit test-particle has a definite magnitude and direction for each possible location of the test-particle. The totality of these force vectors, or the vector function of position, is called a "field of force." Any path that would be followed by an inertia-less test-particle in a force field is called a "line of force." It is con-

venient to decide arbitrarily how close together possible lines of force are independent, so that there will be a finite number of lines, and the concept can be made quantitative. Conventionally a unit source radiates 4π lines, or one per unit solid angle in unrationalized systems, and a single line ($1/4\pi$ per unit solid angle) in rationalized systems.

FIELD OF MOVING CHARGE IN SPACE.

It is assumed that the charge density $\rho(\mathbf{r}, t)$ and the current density $\rho\mathbf{v} = \mathbf{g}(\mathbf{r}, t)$ are known functions of space and time. It is furthermore assumed that the charges are imbedded in empty space. Maxwell's equations are in this case:

$$\text{curl } \mathbf{H} = \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} + \frac{4\pi}{c} \rho \mathbf{V}; \quad \text{div } \mathbf{E} = 4\pi\rho$$

$$\text{curl } \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{H}}{\partial t}; \quad \text{div } \mathbf{H} = 0.$$

To solve these equations, one introduces the scalar potential ϕ and the vector potential \mathbf{A} where

$$\mathbf{H} = \text{curl } \mathbf{A}$$

$$\mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t} - \text{grad } \phi$$

and

$$\text{div } \mathbf{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0 \quad (\text{Lorentz gauge condition}).$$

The potentials satisfy the equations

$$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = -\frac{4\pi}{c} \mathbf{g}$$

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = -4\pi\rho.$$

The solutions (retarded potentials) are

$$\mathbf{A}(x, y, z, t) = \frac{1}{c} \iiint \frac{\mathbf{g}(\xi, \eta, \zeta, t - r/c)}{r} d\xi d\eta d\zeta$$

$$\phi(x, y, z, t) = \frac{1}{c} \iiint \frac{\rho(\xi, \eta, \zeta, t - r/c)}{r} d\xi d\eta d\zeta$$

$$r = \sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}.$$

If the charge is a point charge of magnitude e the solution reduces to the so-called Liénard-Wiechert potentials:

$$\mathbf{A} = \frac{e}{c} \left[\frac{\mathbf{v}}{r - \frac{\mathbf{r} \cdot \mathbf{v}}{c}} \right]_{\tau = t - \frac{r}{c}}$$

$$\phi = \left[\frac{e}{r - \frac{\mathbf{r} \cdot \mathbf{v}}{c}} \right]_{\tau = t - \frac{r}{c}}$$

\mathbf{r} and \mathbf{v} are the "effective" position and velocity of the charge at the retarded time $t - \frac{r}{c}$.

FIELD OF VIEW. The region constituted by all points on which the gaze can be fixed when the eye is allowed to move while the head is kept immobile. The field of view may be monocular or binocular. (See **Lunenburg geometry**.)

FIELD OPERATOR. In quantized field theory, an operator which represents the creation or annihilation of a particle.

FIELD QUANTIZATION. The transition from a classical field theory to a quantized field theory. To any quantized field theory there corresponds a semi-classical field theory with the same field equations. The quantized field theory is obtained from this classical field theory by replacing the classical field quantities by operators satisfying simple **commutation** or **anti-commutation** relations. (See **field theory, quantized** and **quantum electrodynamics**.)

FIELD, QUASI-STATIC. See the discussion under **field, radiation**.

FIELD, RADIATION. The electromagnetic field associated with oscillating **electric** and **magnetic dipoles** can be written as the sum of three components. The first of these has an inverse-cube dependence on distance from the dipole, resembling the static field of the dipole but multiplied by a time-varying factor. It is in phase with the source current and is often called the "*quasi-static*" field. The second component falls off inversely as the square of the distance and is in quadrature with the source current. For this reason there is no net loss of energy due to this component. It is called the "*induction*" field, a term which emphasizes its reactive character. The last component is again in phase with the source current but varies inversely as the distance, thus

predominating over the other two at large distances. The in-phase relation represents real power loss, and its dependence on distance means that its contribution to the integral of the **Poynting vector** does not vanish for extremely large distances. This component is called the "*radiation*" field. A somewhat similar decomposition of the field occurs also in the more general case of an accelerated charge. (See also **dipole radiation, electric**; and **dipole radiation, magnetic**.)

FIELD RAY. Two linearly independent **optical paths**, one from an axial object point and one from an off-axis object point, determine all paraxial optical paths in the **Gaussian optics** of a system. These are referred to as the *axial* and *field rays*, respectively. The field ray is usually taken parallel to the axis in object space.

FIELD, SELF-CONSISTENT. See **self-consistent field**.

FIELD STRENGTH. The **vector** characterizing a **field**. Thus an electric field has a strength (volts per meter) at a given point, as well as a direction; its strength is, therefore, the magnitude of its **electric field vector**, **E**. (See **field of force**.)

FIELD STRENGTH TRANSFORMATION LAWS. See **transformation laws for field strengths**.

FIELD, TENSOR. See **tensor field**.

FIELD THEORY. Theory of the dynamical motion of a set of electromagnetic or matter fields, in which the electromagnetic potentials A_μ and the matter wave functions ψ_a are represented by functions of position and time. Usually but not always a field theory is developed in accordance with the postulates of special relativity theory so that if it is designed to describe nuclear forces the principle of relativity will be extended to include the impossibility of detecting absolute uniform motion of the laboratory by means of nuclear experiments conducted therein. In this type of field theory the equations of motion are derived from a **Lagrangian density**

$$L\left(\psi_a, A_\mu, \frac{\partial\psi_a}{\partial x_\nu}, \frac{\partial A_\mu}{\partial x_\nu}\right)$$

being, for the ψ_a ,

$$\sum_{\mu=1}^4 \frac{\partial}{\partial x_\mu} \left(\frac{\partial L}{\left(\frac{\partial\psi_a}{\partial x_\mu}\right)} \right) = \frac{\partial L}{\partial\psi_a}$$

For other properties of L see **gauge invariance**, **energy-momentum tensor**. Since it is possible to derive the **Maxwell equations** and the **Dirac electron theory** in this way, it has been hoped to formulate in analogous fashion a satisfactory meson theory of nuclear forces.

Field theories may also be developed as **relativistic** and as non-relativistic theories, as well as for quantized and non-quantized descriptions.

FIELD THEORY, QUANTIZED. Classical field theories, like that of the electromagnetic field, have to be modified in quantum theory in order to include the description of quantum phenomena. In the same way as in particle mechanics the classical quantities are replaced by operators in the transition to quantum mechanics, the classical field quantities become field operators in a quantized field theory. The particles appear in the theory as the quanta of the field. Photons are the quanta of a radiation field, electrons the quanta of a quantized electron field, the different mesons quanta of meson fields. If the particles obey **Bose-Einstein statistics**, the quantized field operators satisfy simple **commutation** relations; if the particles are fermions obeying **Fermi statistics**, the field operators satisfy analogous anti-commutation relations. The best known example of quantized field theory is **quantum electrodynamics** which describes the interaction of electrons and the radiation field. Nuclear forces are related to the interaction of nucleons with meson fields. Many problems of elementary particle physics belong to quantum field theory. The quantum field equations satisfied by the field operators are in general non-linear equations, which means that it is a difficult mathematical problem to arrive at quantitative predictions. In quantum electrodynamics, the coupling between the fields is weak, and one can apply perturbation methods. In many other important cases, however, the coupling between the fields is strong, and the perturbation techniques which have been developed in recent years in quantum field theory are of little help.

FIELD THEORY, WEYL UNIFIED. Theory of gravitation and electromagnetism based on a non-Riemannian geometry in which the length of a vector may change under parallel displacement.

$$\frac{dl}{l} = A_\mu dx^\mu.$$

The A_μ are interpreted as the electromagnetic potentials. With this interpretation, multiplying a length as (r,t) by $\exp[X(r,t)]$ leads to the gauge transformation

$$A'_\mu = A_\mu + \partial_\mu X.$$

(See **gauge invariance**, second kind.)

FIGURED SURFACE. In lens design and fabrication, a surface that departs from a specified surface (often a sphere) by small amounts (of the order of a few wavelengths).

FIGURE OF MERIT. (1) General term for various graphical relationships which summarize certain desirable features of amplifying devices. (2) The current required to produce one division deflection (usually 1 mm on a scale at a distance of 1 m) of a galvanometer. (3) Any quantity which expresses quantitatively the performance of a measuring device.

FILM. In **heat transfer**, the name given to the **boundary layer**.

FILM TEMPERATURE. In **heat transfer**, the mean between the temperature of a duct or an immersed body, and the **bulk temperature**.

FILMWISE CONDENSATION. See **dropwise condensation**.

FILTER. An electrical **network** designed to eliminate certain frequencies in a signal by attenuation, leaving the remaining frequencies little changed. Specific adjectives, e.g., low-pass, high-pass, band-pass and band-elimination indicate the particular behavior for which the filter was designed.

FILTERING APPROXIMATIONS. A set of mathematical approximations introduced into a system of hydrodynamical partial differential equations to filter out or exclude solutions corresponding to those physical disturbances which are believed to contribute only negligibly to the problem at hand.

FINAL VERTEX. See **vertex**, final.

FINE STRUCTURE. A. Rotational fine structure in band spectra (see **molecular spectra**).

B. Multiplet splitting

(I) Atomic Spectra

An energy level with a given electronic angular orbital momentum \mathbf{L} and electron spin $\mathbf{S} \neq 0$ will split into a number of fine structure components with different total electronic angular momenta \mathbf{J} . The corresponding quantum numbers are

$$J = (J + S), (J + S - 1), \dots, (J - S).$$

The number of fine structure components is $2S + 1$ for $J > S$, and $2J + 1$ for $J < S$.

Transitions between fine structure levels follow the selection rule

$$\Delta J = 0, \pm 1, \quad \text{and} \quad J = 0 \leftarrow \rightarrow J = 0.$$

The magnitude of the fine structure splitting depends on the **coupling** between \mathbf{S} and \mathbf{L} . For strict **Russell-Saunders coupling** the energy of the fine structure levels is given by

$$W = W_0 + A \frac{J(J + 1) - L(L + 1) - S(S + 1)}{2}$$

where A is a constant characteristic of the multiplet considered (see Γ -value). In this case the interval between successive fine structure components (J and $J + 1$) is proportional to $J + 1$ (**Landé's interval rule**). (See also **hydrogen fine structure**, **multiplet**.)

(II) Molecular Spectra

The total angular momentum \mathbf{J} is the vector sum of the angular momentum of the nuclear frame, the electronic orbital angular momentum \mathbf{L} in the direction of the internuclear axis, and the electron spin \mathbf{S} . The total angular momentum apart from spin is designated \mathbf{N} . For the corresponding quantum numbers we have

$$J = (N + S), (N + S - 1), \dots, (N - S).$$

The interaction of rotation and electron spin (which increases with increasing rotation) causes a variation of the multiplet splitting with N or J . The following formulas give the rotational term values for some important cases

referring to diatomic and linear polyatomic molecules:

$^2\Sigma$ states ($\Lambda = 0, S = \frac{1}{2}$)

$$F_1(N) = B_v N(N + 1) + \frac{1}{2}\gamma N$$

$$F_2(N) = B_v N(N + 1) - \frac{1}{2}\gamma(N + 1).$$

Here $F_1(N)$ and $F_2(N)$ refer to the levels with $J = N + \frac{1}{2}$ and $J = N - \frac{1}{2}$, respectively, and γ is a small coupling constant ($\gamma \ll B_v$).

$^3\Sigma$ states ($\Lambda = 0, S = 1$) (Schlapp's formula).

$$F_1(N) = B_v N(N + 1) + (2N + 3)B_v - \lambda$$

$$- \sqrt{(2N + 3)^2 B_v^2 + \lambda^2 - 2\lambda B_v}$$

$$- \gamma(N + 1)$$

$$F_2(N) = B_v N(N + 1)$$

$$F_3(N) = B_v N(N + 1) - (2N - 1)B_v - \lambda$$

$$+ \sqrt{(2N - 1)^2 B_v^2 + \lambda^2 - 2\lambda B_v}$$

$$- \gamma N.$$

Here $F_1(N)$, $F_2(N)$, $F_3(N)$ refer to the levels with $J = N + 1$, $J = N$, and $J = N - 1$, respectively, and λ and γ are small coupling constants.

$^2\Pi$, $^2\Delta$, \dots states ($\Lambda = 1, 2, \dots, S = \frac{1}{2}$) (Hill and Van Vleck's formula).

$$F_1(J) = B_v[(J + \frac{1}{2})^2 - \Lambda^2$$

$$- \frac{1}{2}\sqrt{4(J + \frac{1}{2})^2 + Y(Y - 4)\Lambda^2}]$$

$$- D_v J^4$$

$$F_2(J) = B_v[(J + \frac{1}{2})^2 - \Lambda^2$$

$$+ \frac{1}{2}\sqrt{4(J + \frac{1}{2})^2 + Y(Y - 4)\Lambda^2}]$$

$$- D_v(J + 1)^4.$$

Here $Y = A/B_v$, where the coupling constant A is a measure of the strength of the **coupling** between the spin \mathbf{S} and the orbital angular momentum $\mathbf{\Lambda}$; $F_1(J)$ is the term series that forms for large rotation the levels with $J = N + \frac{1}{2}$, while $F_2(J)$ forms for large rotation the levels with $J = N - \frac{1}{2}$.

FINE STRUCTURE CONSTANT. See **Sommerfeld fine structure constant**.

FINE STRUCTURE, HYDROGEN. See **hydrogen fine structure**.

FINITE AMPLITUDE EQUATIONS (ACOUSTIC). For plane waves of infinitesimal amplitude travelling in an ideal gas, the particle displacement ξ obeys the wave equation

$$\frac{\partial^2 \xi}{\partial t^2} = \frac{\gamma p_o}{\rho_o} \frac{\partial^2 \xi}{\partial x^2} \quad (1)$$

where γ is the ratio of the specific heats, p_o is the mean pressure, and ρ_o is the mean density. Such waves are propagated with constant velocity $c_o = \sqrt{(\gamma p_o / \rho_o)}$. For waves of finite amplitude, (1) becomes

$$\frac{\partial^2 \xi}{\partial t^2} = c_o^2 \left(1 + \frac{\partial \xi}{\partial x} \right)^{-(\gamma+1)} \frac{\partial^2 \xi}{\partial x^2}. \quad (2)$$

The velocity of propagation c is then given by

$$c = c_o \left(1 + \frac{\partial \xi}{\partial x} \right)^{-(\gamma+1)/2}$$

and c thus depends on the amplitude. Thus, waves of large amplitude travel faster than waves of smaller amplitude, and as a finite disturbance is propagated through a gas, a shock front develops. (See **wave, shock**.)

FINITE ELASTICITY THEORY. The theory of the relation between the forces and deformation in bodies of elastic or perfectly-elastic material, in which it is not assumed that the displacement gradients are infinitesimally small (cf. **classical elasticity theory**).

FINITE GRAPH. See **graph, finite**.

FIRST BETTI NUMBER. See **graph, nullity**.

FIRST CURVATURE OF A SURFACE. See **curvature**.

FIRST FLIGHT COLLISION PROBABILITY. See **collision probability, first flight**.

FIRST FLIGHT ESCAPE PROBABILITY. See **escape probability, first flight**.

FIRST FOCAL POINT. The first focal point F in **Gaussian optics** is the object point so situated that all rays through F have image rays parallel to the axis of the system.

FIRST FUNDAMENTAL THEOREM. In nuclear reactor theory, the statement that in a critical bare reactor, (a) the neutron flux—averaged over angle—is separable in space

and energy, i.e., $\Phi(r, E) = \chi(E)\psi(\mathbf{r})$; and (b), the spatial factor satisfies the equation:

$$(\nabla^2 + B^2)\psi(\mathbf{r}) = 0$$

with the condition that $\psi(\mathbf{r})$ be everywhere positive and vanish on the boundary. B^2 , the lowest eigenvalue of the equation, is called the **buckling**.

FIRST LAW OF THERMODYNAMICS. See **thermodynamics, first law of**.

FIRST LIMIT THEOREM. If a sequence of **distribution functions** $\{F_n\}$ converges to a limit F then the corresponding sequence of **characteristic functions** $\{\phi_n\}$ converges to a limit ϕ and ϕ is the characteristic function of F . This is the first limit theorem of probability and statistics.

FIRST ORDER OPTICS. See **Gaussian optics**.

FIRST ORDER SOLUTION THEORY. See **conformal solutions**.

FISHER'S DISTRIBUTION. If s_1^2, s_2^2 are two independent estimates of variance, with degrees of freedom ν_1 and ν_2 , in data from a normal population, the distribution of one half the logarithm of their ratio,

$$z = \frac{1}{2} \log (s_1^2/s_2^2)$$

is called Fisher's z -distribution. It has the form

$$dF = \frac{2\nu_1^{1/2}\nu_2^{1/2} e^{\nu_1 z} dz}{B(\frac{1}{2}\nu_1, \frac{1}{2}\nu_2) (\nu_1 e^{2z} + \nu_2)^{1/2(\nu_1 + \nu_2)}},$$

$$-\infty \leq z \leq \infty.$$

Fisher chose the logarithm, rather than the ratio $F = s_1^2/s_2^2$ itself, to facilitate interpolation in tables. The ratio F has a **beta distribution** of the second kind. Tests of significance based on F or z are widely used in the **analysis of variance**.

FISHER-YATES TEST. A test of independence of data arranged in a 2×2 **contingency table**. It is an exact test, used for small frequencies where the **chi-square** test is feared to be too approximate.

FISSION SPECTRUM. The distribution in energy of prompt neutrons emitted at fission, measured in the coordinate system of the fissioning nucleus. Within present experimental

accuracy the probability that a neutron is emitted in the range $E, E + dE$ is:

$$f(E) = \frac{a^{3/2}}{\Gamma(\frac{3}{2})} \sqrt{E} e^{-aE}, \quad a = 0.775$$

where E is to be measured in Mev. (See also **Watt spectrum**.)

FISSION YIELD CURVE. A curve obtained by plotting the fraction of fissions, usually expressed as a percentage, which give rise to a particular nuclide, against the mass number of that nuclide. Since there are two product nuclei per fission, the total fission yield is usually taken to be 200%. The typical fission yield curve contains two broad peaks around mass numbers 90 and 140 owing to the fact that fission is predominantly asymmetric. The trough between them, representing symmetric fission, may correspond to only 0.1% of the most probable yield, but this depends rather sensitively upon the actual fission process considered (for example, upon the energy of the neutrons giving rise to fission).

FIX. The point of intersection of two **lines of position** is known as a fix. This fix may be the actual position of the ship or plane if the lines of position are obtained by observation of celestial objects (celestial navigation), or by radio or other electronic means (electronic navigation), or by observations of terrestrial objects in known geographic positions (pilotage).

FIXED ARCH. See **arch**.

FIXED-ENDED BEAM. See **beam, fixed-ended**.

FIXED-END MOMENTS. A term used in the **moment distribution** procedure to describe the moments which would be produced by the transverse loads if the ends of the elastic beam were fully restrained against rotation. Clockwise end moments are taken as positive.

FIXED POINT. (1) A system of computing in which all quantities must be scaled so as to be representable by a fixed number of digits and with the **radix** point in a fixed position. It may be contrasted with floating-point computations in which a scale factor is explicitly computed and associated with each number, and represented as a power of the radix. In some **digital computers** floating-point operations are automatic; in others they are not.

(2) In the analysis of beams, fixed points are points of zero moment (points of inflection or contraflexure) in an unloaded portion of a continuous elastic beam with supports $A, B, C, D \dots$ when a moment is applied at A . They are fixed points in the sense that the point of zero moment in open CD will not change as the loads on ABC are altered.

FIXED POINT THEOREM. See **Brouwer fixed point theorem**.

FIXED THERMOMETRIC POINTS. See **temperature**.

FLAME VELOCITY. See **combustion**.

FLAT SPACE-TIME. Space-time for which the Riemann-Christoffel tensor vanishes. The metric can then always be chosen thus

$$g_{\mu\nu} = \delta_{\mu\nu}, \text{ if } x_4 = ict.$$

FLEXURE. A term used to denote the bending of beams, plates, and shells.

FLEXURE PROBLEM. In the theory of the bending of a beam of isotropic elastic material, satisfying the generalized Hooke's law, by a force applied to its end and perpendicular to its length, the tangential components of strain and stress in the cross-sectional planes are determined by a two-dimensional stress function. This may be expressed in terms of the torsion function for the beam and another harmonic function of position on the cross section of the beam, often denoted χ and called the *flexure function*, the normal derivative $\partial\chi/\partial n$ of which is given by

$$\frac{\partial\chi}{\partial n} = \left[-\frac{1}{2}\sigma x^2 + \left(1 - \frac{1}{2}\sigma\right)y^2\right]l - (2 + \sigma)zym$$

on the cylindrical surface of the beam, where the xy -planes are the cross-sectional planes of the beam, l and m are the direction cosines of the outward-drawn normal to the surface of the beam, σ is Poisson's ratio for the material of the beam and it is assumed that the applied force acts parallel to the x -axis. The problem of determining χ is called the *flexure problem*.

FLEXURE, SIMPLE (OF A BEAM). Also called *pure flexure*, *simple bending* or *pure bending*. A state of deformation of the beam in which straight lines parallel to the length of the beam become arcs of circles all of which lie parallel to a plane, called the *plane of*

bending or *plane of flexure*, and in which planes normal to the length of the beam remain plane and intersect in a single line. Usually, the state of *deformation* is one in which the cylindrical surface of the beam is force-free. Then, for isotropic elastic materials and for elastic materials with certain other types of symmetry, such a deformation may be supported by equal and opposite *couples*, applied to the two ends of the beam, provided that the plane of bending is normal to one of the principal axes of inertia of the cross-section of the beam. These couples are called the *bending couples* or *flexural couples*. For an isotropic elastic material obeying the generalized Hooke's law, the bending couple is EI/R , where E is Young's modulus for the material of the beam, I is the square of the radius of gyration of the cross section of the beam about an axis perpendicular to the plane of bending, and R is the radius of curvature of the beam (i.e., the radius of curvature of the arc into which a line parallel to the length of the beam is bent). This relation between the bending couple and the curvature $1/R$ is called the *Bernoulli-Euler law*. The constant of proportionality EI is called the *modulus of flexural rigidity* for the beam.

FLOATING POINT. See **fixed point**.

FLOQUET THEOREM. Given a second-order linear differential equation with periodic coefficients that are one-valued functions of the independent variable, such as **Mathieu's equation**, then there exists a solution of the form $y = e^{\mu x}P(x)$, where $P(x)$ is a periodic function and μ is a constant. The general solution of the differential equation is $y = Ae^{\mu x}P(x) + Be^{-\mu x}P(-x)$, where the constants of integration are A and B .

FLORY-HUGGINS EFFECT. See **polymer mixtures**.

FLOW DIAGRAM. A graphical representation of a sequence of operations. In machine computation, a diagram with labeled boxes, arrows, etc., showing the logical pattern of a problem, but not ordinarily including machine commands.

FLOW, ENERGY EQUATION FOR STEADY ADIABATIC. See **energy equation for steady adiabatic flow**.

FLOW, FREE-MOLECULE. See free-molecule flow.

FLOW, HYPERSONIC. See hypersonic flow.

FLOW IN A PIPE OR CHANNEL, TRANSITION TO TURBULENT. See transition to turbulent flow in a pipe or channel.

FLOW IN PIPE, LAMINAR. See pipe, laminar flow in.

FLOW IN PIPE, TURBULENT. See pipe, turbulent flow in.

FLOW OF GAS IN NOZZLE OR DUCT. See nozzle or duct, flow of gas in.

FLOW OF GAS IN STREAM TUBE. See nozzle or duct, flow of gas in.

FLOW PAST ROTATING CYLINDER. See rotating cylinder, flow past.

FLOW, PLUG. See plug flow.

FLOW RULE. A rule which establishes the relation between strain-rate and stress for plastic flow. For a prescribed yield limit $f(\sigma_{11}, \sigma_{12}, \dots)$, the flow rule is given by the plastic potential principle:

$$\dot{\epsilon}_{ij} \propto \frac{\partial f}{\partial \sigma_{ij}}$$

FLOW, SLIP. See slip flow.

FLows OF IRREVERSIBLE PROCESSES. See rates of irreversible processes.

FLOW THEORY OF PLASTICITY. Synonymous with incremental theory.

FLOW THROUGH GAUZE. See gauze, flow through.

FLOW, TURBULENT, IN A BOUNDARY LAYER, TRANSITION OF. See transition of turbulent flow in a boundary layer.

FLOW WORK. The product pV . The origin of the term is as follows: when writing the first law of thermodynamics for an open system operating at a steady state and admitting the working fluid at a (constant) state 1 and discharging it at a (constant, but different) state 2, it is necessary to select a hypothetical closed system. This is usually done by adding the two cylinders and piston shown in broken line

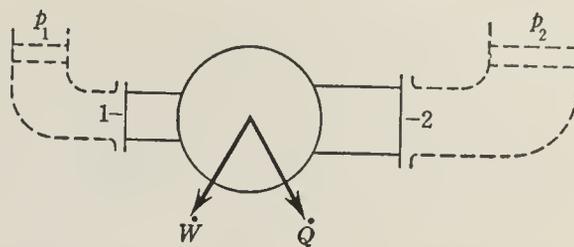


Fig. 1.

in Figure 1. The rate at which work is performed at the boundaries of the closed system is then

$$\dot{W}_c = \dot{W} - p_1 \dot{V}_1 + p_2 \dot{V}_2.$$

Here \dot{W} is the rate at which work is done by the open system, $-p_1 \dot{V}_1$ is the rate at which the surroundings ("the atmosphere") perform work on the closed system (left-hand cylinder and piston) pressure p_1 , and $+p_2 \dot{V}_2$ is the rate at which the closed system performs work on the surroundings ("against the atmosphere"). Consequently in many flow problems which are conveniently analyzed in terms of an open system the product

$$pV \text{ or } pv \text{ (per unit mass)}$$

appears. Since it has dimensions of work, and since it is associated with flow problems, it is called flow work. It must be realized that in the preceding argument it has been assumed that the state of the working fluid is uniform across sections 1 and 2 ("one-dimensional flow") and that no shearing stresses are involved in the flow through the additional hypothetical cylinders.

A similar term occurs in the analysis of a reciprocating-engine cylinder. When the valves are closed the work of expansion or compression is equal to

$$W_{12} = \int_1^2 p dV.$$

In addition to expanding (or compressing) the working fluid, it is necessary to induce it

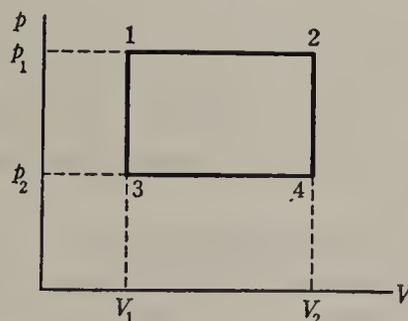


Fig. 2 Ideal pV cycle.

through the open inlet valve when “the atmosphere” performs work $-p_1V_1$ on the piston, and to expel it through the open exhaust valve when the piston performs work $+p_2V_2$ “against the atmosphere.” Hence the total work of a complete cycle of operations is equal to

$$W = \int_1^2 pdV - p_1V_1 + p_2V_2 = - \int_1^2 Vdp.$$

The total work, sometimes called technical work (particularly in German textbooks) consists of the work of expansion (or compression) and the two flow-work terms pV .

FLUCTUATION(S). In thermodynamics one deals with matter in bulk and hence usually considers uniform systems. However, all matter is built up of atoms and its atomistic nature will produce fluctuations which can be studied by statistical means.

For most physical quantities a system consisting of a large number of particles will show a Gaussian or **normal distribution**, and the relative fluctuations of a quantity G will usually be given by an equation of the type

$$\frac{\langle G^2 \rangle - \langle G \rangle^2}{\langle G \rangle^2} = \frac{1}{\langle G \rangle} \quad (1)$$

where the $\langle \rangle$ indicate average values. As most $\langle G \rangle$ will be proportional to the number of particles in the system, the relative fluctuations will usually be negligibly small.

There are, however, cases where fluctuations become experimentally observable. The first case is that of Brownian motion where small particles in suspension will undergo fluctuations in the uniform pressure and hence show a **random walk** phenomenon. The second case is where Equation (1) breaks down. This will happen near a critical temperature when, for instance, critical opalescence occurs.

If we are dealing with systems of **bosons** or **fermions**, Equation (1) must be slightly changed, but the conclusion that the left hand side is usually very small for systems consisting of many particles remains valid.

FLUCTUATION (FROM AN EQUILIBRIUM STATE). See **natural and unnatural processes**.

FLUCTUATION-DISSIPATION THEOREM. A general relation between the amplitude of fluctuations, the spectral distribution of the resistance for such fluctuations, and

the absolute temperature. An example is the **Nyquist theorem**.

FLUCTUATIONS, ZERO POINT. See **zero point fluctuations**.

FLUORESCENCE. Luminescence that persists for less than about 10^{-8} second after excitation.

FLUORESCENCE YIELD. The probability that an atom in an excited state will emit an x-ray photon, rather than an Auger electron (see **Auger effect**). The value of the fluorescence yield lies between 0 and 1, characteristic of the particular state of excitation of the atom. The K -shell fluorescence yield increases with increasing atomic number, and is the sum of the $K \rightarrow L_{II}$, $K \rightarrow L_{III}$... transitions.

FLUTTER (IN STRUCTURES). Resonant vibration of any part of an aircraft, or of any structural element, in a fluid stream, maintained by oscillatory aerodynamic or hydrodynamic forces induced by deflection of the structure.

FLUX. (1) A quantity proportional to the surface integral of the normal (perpendicular) force field intensity over a given area.

$$\text{Flux} = K \int F_N dS$$

where F_N is the normal component of a **field** (e.g., gravitational, electric, magnetic), and K is the constant of proportionality between the field and the flux density (**permittivity, permeability, etc.**). (See also **electric flux density; magnetic flux density**.) (2) A term which denotes the volume or mass of fluid or particles transferred across a given area perpendicular to the direction of flow in a given time.

There are many specific applications in science of the term flux. For electromagnetic radiation, it signifies the energy per unit time, or the power passing through a surface. For photons or particles, flux is the number per unit time passing through a surface. In nuclear science, flux commonly means the product nv , where n is the number of particles per unit volume and v is their mean velocity.

FLUX, ADJOINT. In the general formulation of time-dependent neutron transport theory, the substitution $\Phi(r, E, \hat{\Omega}, t) = \phi(r, E, \hat{\Omega})e^{\lambda t}$ leads to an eigenvalue problem of the form:

$$\mathcal{L}\phi_n = \lambda_n v^{-1} \phi_n + \text{boundary conditions.}$$

Here, the operator will depend upon the mathematical model under consideration (multi-group diffusion theory, one-velocity transport theory, ...) and v is the neutron velocity variable. Using the conventional scalar product notation, the adjoint operator, \mathfrak{M} is defined through $(\phi, \mathfrak{M}\psi) = (\psi, \mathcal{L}\phi)$ and the adjoint flux satisfies $\mathfrak{M}\phi_n = \lambda_n v^{-1} \psi_n + \text{boundary conditions}$. It is generally assumed in neutron transport theory that the eigenvalues of \mathfrak{M} are identical with those of \mathcal{L} . From the foregoing, it is easy to prove the orthogonality relationship

$$(\psi_m, v^{-1} \phi_n) = \delta_{nm}.$$

In special cases, e.g., one-group diffusion theory, the operator \mathcal{L} is self-adjoint, $\mathcal{L} \equiv \mathfrak{M}$ and $\psi_n \equiv \phi_n$.

FLUX DENSITY. A vector representing the flux passing per unit area normal to the vector. (See **electric flux density**; **magnetic flux density**; **Poynting vector**.)

FLUX DENSITY, INTRINSIC (B_i) (INTENSITY OF MAGNETIZATION). The intrinsic flux density (induction) or magnetic polarization of a medium is the vector difference between the **magnetic flux density** at the point of interest, and the flux density which would exist at that point, for the same magnetizing force, if the medium were a vacuum:

$$\mathbf{B} = \mu_0 \mathbf{H} + \mathbf{B}_i,$$

where μ_0 is the magnetic permeability of free space. In unrationalized systems of units, the intrinsic flux density is defined by

$$\mathbf{B}_i = 4\pi\mu_0 \mathbf{M},$$

whereas in rationalized systems,

$$\mathbf{B}_i = \mu_0 \mathbf{M}$$

where \mathbf{M} is the **magnetization** of the medium.

FLUX DENSITY, PEAK (B_m). The maximum flux density in a magnetic material in a specified cyclically magnetized condition.

FLUX DENSITY, RESIDUAL (B_r). The magnetic flux density at which the magnetizing force is zero when the material is in a symmetrically cyclically magnetized condition.

FLUX, ELECTRIC. See **electric displacement vector**.

FLUXES OF IRREVERSIBLE PROCESSES. See **rates of irreversible processes**.

FLUX LINKAGE. Since **magnetic flux density** is solenoidal

$$(\nabla \cdot \mathbf{B} = 0),$$

any closed contour links a definite amount of flux. This flux linkage

$$\phi = \int \mathbf{B} \cdot d\mathbf{S}$$

plays an important role in the basic relations of electromagnetism.

FLUX, LUMINOUS. See **luminous flux**.

FLUX, MAGNETIC. See **magnetic flux**.

FLUX, NEUTRON, ANGULAR. Let $N(r, E, \hat{\Omega}, t) dV dE d\hat{\Omega} dt$ be the number of neutrons in volume element dV of matter, located at position r , which, during the time interval between t and $t + dt$ have energies lying between E and $E + dE$ and travel in directions lying within the element of solid angle $d\hat{\Omega}$ about unit vector $\hat{\Omega}$. The quantity Nv , where v is the neutron velocity corresponding to E , is the neutron angular flux.

FLUX, NEUTRON, TOTAL. The total neutron flux is the integral of the angular flux over all energies and angles. Thus,

$$\Phi(r, t) = \int_0^\infty dE \int d\hat{\Omega} N(r, E, \hat{\Omega}, t) v.$$

FLUX VECTORS. In a gas which is not at equilibrium, there exist either separately or simultaneously gradients of density, of molecular momentum or of temperature. These gradients lead to molecular transports of mass (m), molecular momentum (mv) or kinetic energy ($mv^2/2$) through the gas. Let $\phi(\mathbf{V})$ design any *scalar* quantity such as m , mv_x , mv_y , mv_z , $mv^2/2$, etc. The amount of ϕ which crosses in unit time a surface element dS moving with the mass average velocity of the gas is:

$$(\mathbf{n} \cdot \int \phi f \mathbf{V} d\mathbf{V}) dS = (\mathbf{n} \cdot \Psi) dS. \quad (1)$$

f is the distribution function, \mathbf{V} the difference between the molecular velocity and the mass average velocity, and \mathbf{n} a unit vector normal to dS .

Ψ is called the flux vector associated with the property ϕ . The transport of vectorial quantities (mv for example) leads to the introduction of *three* flux vectors, one for each component of the vectorial property. The components of these three flux vectors form a tensor.

(1) *Mass flux vector.* For the j^{th} component of a mixture,

$$\Psi_{mj} = m_j \int f_j \mathbf{V}_j d\mathbf{V}_j = n_j m_j \bar{\mathbf{V}}_j. \quad (2)$$

$\bar{\mathbf{V}}_j$ is the difference between the average velocity of the molecules j and the mass average velocity of the gas. It is called the *diffusion velocity* of component j . Because of the definition of \mathbf{V} , the mass flux vector in a pure gas is always 0.

(2) *Momentum flux vectors.* If $\phi = mV_x$ (V_x is the difference between the x component of the molecular velocity and that of the mass average velocity), the flux vector is:

$$\Psi(mV_x) = m_j \int V_x \mathbf{V} f d\mathbf{V} = nm \overline{V_x \mathbf{V}}. \quad (3)$$

The nine scalar quantities appearing in the right hand side of (3) and of the similar equations for the transport of mV_y and mV_z form the *pressure tensor*. They have the form

$$r_{ij} = nm \overline{V_i V_j}.$$

When $i = j$, these components are called the *normal stresses* p_{xx} is the pressure in the x -direction across a surface which is itself normal to the x -axis.

When $i \neq j$, the components of the pressure tensor are called the *shearing stresses*, because they cause neighboring layers of the gas to slip with respect to each other.

In a gas at equilibrium, the shearing stresses are always 0, and the normal stresses are equal. The shearing stresses are important for viscous flow.

(3) *The flux vector associated with the transport of kinetic energy* ($\phi = mv^2/2$).

$$\begin{aligned} \Psi_{\frac{1}{2}mv^2} &= \mathbf{q} = \left(\frac{1}{2}\right)m \int V^2 \mathbf{V} f dV \\ &= \left(\frac{1}{2}\right)nm \overline{V^2 \mathbf{V}} \end{aligned} \quad (4)$$

It is usually called the *heat flux vector*, but it is only for monoatomic gases in their fundamental electronic state that (4) represents the total heat carried through the gas. For polyatomic molecules, one must introduce *internal energy flux vectors* which also contribute to

the energy flow in the gas. (See **Eucken correction**.)

f/NUMBER. The angle of projection of a camera lens is measured by the **f/number** which is the ratio of the **effective focal length** to the diameter of the **exit pupil**. Thus, the **f/number** is the reciprocal of twice the tangent of the **angle of projection**. The reciprocal of the square of the **f-number** of a lens is a measure of the speed of the lens. Following the recommendation of a committee of the Royal Photographic Society of Great Britain, at ratios greater than 1:4 ($f/4$) lenses are marked so that the exposures increase as the power of 2. Thus:

f/number	4	5.6	8	11.3	16	22
Relative exposure	1	2	4	8	16	32

FOCAL LENGTH. The distance from the object or image **principal point** to its corresponding **focal point** is the *object* or *image focal length*, respectively, of an optical system. If the term focal length is used without qualification then the image focal length is meant.

FOCAL LENGTH, BACK. See **back focal length**.

FOCAL LENGTH, REDUCED. See **reduced focal length**.

FOCAL LINES. The rays of a **normal congruence** in the neighborhood of a ray R intersect to the first order in two focal lines, one through each focus of R . The focal lines are called the *tangential* and the *sagittal focal lines* of R . Normals to a wave-front belonging to the congruence do not, in general, intersect to the first order. However, the normals to a line of curvature on the wave-front do intersect to the first order in a focal line. Thus R and the focal lines are mutually orthogonal.

FOCAL PLANES. In the **Gaussian optics** of an axially symmetrical optical system, planes perpendicular to the axis passing through the **focal points**.

FOCAL POINT, FIRST. See **first focal point**.

FOCAL POINTS. In **Gaussian optics**, the **object** and the **image points** conjugate to infinity in the system. They are referred to as the **first** and **second**, respectively, **focal points**.

FOCAL POINT, SECOND. See **second focal point**.

FOKKER-PLANCK EQUATION. An equation representing one of the limiting cases of an **additive stochastic process**. It first occurred in the theory of diffusion when drift is present and may be written

$$\frac{\partial}{\partial t} v(x,t) = -2c \frac{\partial}{\partial x} v(x,t) + D \frac{\partial^2}{\partial x^2} v(x,t)$$

where v is the probability density for displacement at time t , D is the diffusion coefficient and c represents drift.

The Fokker-Planck equation describes the Brownian motion of a free particle in velocity space.

FOLDY-WOUTHUYSEN REPRESENTATION. In the Dirac theory for a spin $\frac{1}{2}$ particle, a positive energy particle can realize two independent states for each value of its physical momentum. These correspond to the two possible directions of the spin. According to quantum mechanics each such pair of physical states is to be represented by exactly two vectors in Hilbert space. There therefore exists a redundancy in the representation of these vectors in the usual formulation of the Dirac theory, where the corresponding wave functions have four components. It must therefore be possible to find a transformation such that the wave functions of a free Dirac electron of definite momentum have just two components, as in the **non-relativistic Pauli theory**. This problem was solved by Foldy and Wouthuysen by performing the canonical transformation

$$\begin{aligned} \psi &\rightarrow e^{iS} \psi = \psi' \\ H &\rightarrow e^{iS} H e^{-iS} = H' \end{aligned}$$

$$S = - \left(\frac{i}{2mc} \right) \beta \boldsymbol{\alpha} \cdot \mathbf{p} \frac{mc}{|\mathbf{p}|} \tan^{-1} \frac{|\mathbf{p}|}{mc}$$

on the Dirac equation

$$(\boldsymbol{\alpha} \cdot \mathbf{p} + \beta mc) \psi = E \psi$$

with the result that $H' = \beta \sqrt{m^2 + \mathbf{p}^2}$. In the representation in which β is diagonal the equation $H' \psi' = E \psi'$ has solutions where the upper components represent positive energies and the lower components negative energies. Furthermore, they have only two non-vanishing components. In this F - W representation the physical content of the Dirac equation is

greatly clarified. For example, in the F - W representation the position operator \mathbf{X}_{FW}

$$\mathbf{X}_{FW} = e^{-iS} \mathbf{x} e^{iS}$$

has the property that its time rate of change is given by \mathbf{p}/E , the velocity of the particle (in contrast to the old representation where the time rate of \mathbf{x} was $c\boldsymbol{\alpha}$ which would imply a velocity c for the particle, and different components of the velocity not being simultaneously measurable since $[\alpha_x, \alpha_y] \neq 0$). Furthermore $\mathbf{X}_{FW} \times \mathbf{p}$ the orbital angular momentum is a constant of the motion (in contrast to $\mathbf{x} \times \mathbf{p}$ which is not, in the Dirac representation). In the F - W representation there exists a spin angular momentum which is also separately a constant of the motion (in contrast to Σ which was not a constant of the motion in the Dirac representation).

FOOT-CANDLE. A unit of **illuminance** or **luminous flux density** when the foot is taken as the unit length. It is the illuminance on a surface one square foot in area on which there is a uniformly-distributed flux of one **lumen**, or the illuminance at a surface all points of which are at a distance of one foot from a uniform source of one **candle**. The term is synonymous with lumen per square foot.

FOOTING. A **foundation** structure used to distribute wall or column loads to the bearing soil or to the piling.

FOOT-LAMBERT. A unit of **luminance** equal to $1/\pi$ **candle** per square foot, or to the uniform luminance of a perfectly diffusing surface emitting or reflecting light at the rate of one **lumen** per square foot. A foot-candle is a unit of incident light and a foot-lambert is a unit of emitted or reflected light. For a perfectly reflecting and perfectly diffusing surface, the number of foot-candles is equal to the number of foot-lamberts.

FOOT-POUND. See **mechanical units**.

FORBIDDEN TRANSITION. In quantum mechanical terms, a transition between two states of a system for which the change in quantum numbers is one that, under the appropriate **selection rules**, makes the transition less probable than competing allowed transition (see **transition, allowed**)—other things, such as the energy change, being equal. As a case in point, transitions involving changes of 2 or more in angular momentum, in units of

$h/2\pi$, have often a smaller probability of occurrence than a possible competing "allowed" transition involving a change of only 1. Cases in which "forbidden" transitions occur are:

(1) Intercombination lines such as the 2537 Å line ($2^3P_1 - 1^1S_0$) in the spectrum of mercury violate the selection rule of $\Delta S = 0$. However, this rule is true only for pure Russell-Saunders coupling (see **coupling, Russell-Saunders**), and with increase in atomic number, spin-orbit interaction becomes more effective.

(2) The selection principles are deduced by analogy of the behavior of an electrical dipole, and therefore systems having a **quadrupole** moment exhibit the effects of transitions forbidden for dipoles, but not for **quadrupole radiation**.

(3) Forbidden transitions may occur as the result of variable magnetic dipole moment.

(4) Forbidden transitions may occur by the action of strong electric fields, either applied externally or produced by neighboring atoms or ions (enforced dipole moment).

FORBIDDEN TRANSITION, NUCLEAR.

The term forbidden transition has been taken over from optical spectra and applied to β -ray spectra. Those transformations of a nucleus which result in a change of more than one unit of angular momentum are characterized by this term even when they are highly probable as a result of the orbital angular momentum of the electron and the neutrino.

FORCE. (1) In dynamics, the physical agent which causes a change of momentum, measured by the time rate of change of momentum. If the speeds involved are low compared with that of light, a force may be defined as proportional to the mass m of a body and to the acceleration a of the body which is produced by the force. Thus $\mathbf{f} = kma$, where k is a constant for a given system of units, and has the dimensionless value unity in length-mass-time systems or $1/g$ in length-force-time systems, g being the acceleration due to gravity. Force is a vector quantity, requiring both a magnitude and a direction for its complete specification.

(2) In statics, the physical agent which produces a strain in a body.

(3) From its initial conception, which was purely mechanical as expressed in (1) and (2), above, the term force extends to denote loosely

any operating agency. (See special entries under this term which follow.)

FORCE, ACTIVE. See **force, external or active**.

FORCE, APPLIED. Forces exerted on a body by bodies in contact or bodies at a distance (as, for example, the earth). When the body is considered together with its support, the supporting **reactions** exerted on the body are not considered applied forces.

FORCE, CENTRAL. (1) A force which is always directed toward a fixed point. If a particle is acted upon by a central force only, its **angular momentum** with respect to the fixed point is a constant vector. The motion, therefore, must be **plane motion**. (2) The term is used in regard to **nuclear forces** to denote a force which acts along the line joining a pair of particles.

FORCE, CENTRIFUGAL. See **centrifugal force**.

FORCE, CENTRIPETAL. See **centripetal force**.

FORCE, CONCENTRATED. See **concentrated force**.

FORCE, CONSERVATIVE. See **conservative force**.

FORCE CONSTANT. In an elastic system obeying the **Hooke law**, the magnitude of the applied force F , and the elongation x , are related by $F = kx$, where the constant k is called the force constant.

FORCE CONSTANTS OF LINKAGES. Expressions of the forces acting between nuclei to restrain relative displacement. They provide the means of measurement of the resistance to stretching of the valence bond and the resistance to deformation of the valence angle, and they express these factors mathematically.

FORCE, CORIOLIS. See **Coriolis force**.

FORCED CONVECTION. See **heat transfer**.

FORCE, DISTRIBUTED. A force which acts over a line, surface, or volume of a body is called a distributed force. A *concentrated force* is an idealization in which the total force is considered to act at a point.

FORCED VIBRATIONS. See vibrations, forced.

FORCED WAVE. Any wave which is required to fit irregularities at the boundary of a system or satisfy some impressed force within the system. The forced wave will not in general be a characteristic mode of oscillation of the system. It cannot be exhibited independently unless the system admits of no free waves. (A homogeneous incompressible fluid bounded by two rigid surfaces is an example of such a system.)

The gravity lee wave in the atmosphere is an example of mixed forced and free waves.

FORCE, EFFECTIVE (EFFECTIVE MECHANOMOTIVE FORCE). The root mean square of the instantaneous force (see force, instantaneous) over a complete cycle. The unit is the dyne.

FORCE, EXTERNAL OR ACTIVE. A force applied to the given system by bodies in contact or at a distance.

FORCE FACTOR. (a) The force factor of an electromechanical transducer is: (1) the complex quotient of the force required to block the mechanical system divided by the corresponding current in the electric system; (2) the complex quotient of the resulting open-circuit voltage in the electric system divided by the velocity in the mechanical system. Force factors (a1) and (a2) have the same magnitude when consistent units are used and the transducer satisfies the principle of reciprocity. (See reciprocity theorem, acoustical.) It is sometimes convenient in an electrostatic or piezoelectric transducer to use the ratios between force and charge or electric displacement, or between voltage and mechanical displacement.

(b) The force factor of an electroacoustic transducer is: (1) the complex quotient of the pressure required to block the acoustic system divided by the corresponding current in the electric system; (2) the complex quotient of the resulting open-circuit voltage in the electric system divided by the volume velocity in the acoustic system. Force factors (b1) and (b2) have the same magnitude when consistent units are used and the transducer satisfies the principle of reciprocity.

FORCE FIELD. See field of force.

FORCE FIELD, CONSERVATIVE. A field whose force can be derived by taking the negative gradient of a scalar potential function, V , i.e., $\mathbf{F} = -\nabla V$. The work done by a conservative force on a particle in its motion between two given points is independent of the path. Alternative definitions are (a) that $\oint \mathbf{F} \cdot d\mathbf{s} = 0$ (see integral, line), and (b) that $\nabla \times \mathbf{F} = 0$ (see Stokes theorem). An example of a conservative force field is the inverse square law field of electric charges or of gravitational masses.

FORCE, GENERALIZED. Moment, pressure, force pairs, etc., may be considered as generalized forces (see displacement, generalized). The product of generalized force and corresponding generalized displacement gives a virtual or real work term.

FORCE, HEISENBERG. See Heisenberg force.

FORCE, IMPRESSED. Any external force acting on a particle in a dynamical system. The resultant force on each such particle can always be resolved into a resultant external impressed force and a resultant internal constraint force.

FORCE, INTERNAL. The equal and opposite forces between particles or bodies of a system. Their vector sum is zero so that they do not contribute to the total impulse and do not change the momentum of the system. They do enter into the energy balance because mechanical energy may be dissipated or stored.

FORCE, LORENTZ. See Lorentz force.

FORCE, "LOST." In the application of the d'Alembert principle by the method of Mach, the "lost" force on the j th particle of a dynamical system is equal to the difference between the external impressed force and the effective force, the latter being equal to the mass times the acceleration of the j th particle.

FORCE ON A CONDUCTOR. A charge dq moving with velocity \mathbf{v} through a steady magnetic field of flux density \mathbf{B} experiences a force

$$\mathbf{F} = dq(\mathbf{v} \times \mathbf{B}).$$

If the charge is moving along a conductor, $dq\mathbf{v} = Id\mathbf{l}$, so the force on an element $d\mathbf{l}$ of a conductor is

$$\mathbf{F} = Id\mathbf{l} \times \mathbf{B}.$$

FORCE ON A RIGID CIRCUIT. (See **energy of a system of current circuits**.) If the circuits are rigid, motion will change the **mutual inductances**, M_{ij} , but not the **self-inductances**, $L_i \equiv M_{ij}$. The change of energy of the current system, with motion of the circuits, is:

$$\begin{aligned} dU &= \sum \sum M_{ij} I_i dI_j + \sum \sum I_i I_j dM_{ij} - \sum F_l d\xi_l \\ &= \sum \sum M_{ij} I_i dI_j + \sum \sum \sum I_i I_j \frac{dM_{ij}}{d\xi_l} d\xi_l \\ &\quad - \sum F_l d\xi_l \end{aligned}$$

so for constant currents:

$$\left(\frac{\partial U}{\partial \xi_l} \right)_I = \sum \sum I_i I_j \frac{dM_{ij}}{d\xi_l} - F_l$$

and

$$\left(\frac{\partial U}{\partial \xi_l} \right)_I = \frac{1}{2} \sum \sum I_i I_j \frac{dM_{ij}}{d\xi_l}$$

from the expression for the energy of a system of circuits. Therefore

$$F_l = \frac{1}{2} \sum \sum I_i I_j \frac{dM_{ij}}{d\xi_l}$$

and the forces are such as to increase the mutual inductances. As in the case of a charged system, the energy of the system increases as it does work; the set of electromotive forces maintaining the currents does work at twice the rate that mechanical work is done. Substitution of torque for force, and rotation for displacement, in the foregoing yield the torques on the rigid elements of a system of circuits.

FORCE ON A SUBMERGED SURFACE.

Consider a *plane* surface of area A , submerged in a stationary liquid. Let the surface intersect the free surface of the liquid in a line XX . Then the force on the plane surface due to fluid pressure is pA , where p is the pressure at the centroid. The center of pressure is at a distance from XX equal to the ratio of the second and first moments of the surface area about XX .

The component in any required direction of the force on a *curved* surface due to fluid pressure is the same, in magnitude and line of action, as that computed for the projection of

the given surface on a plane normal to the required direction.

FORCE ON MOVING CHARGES AND CURRENTS. A charge dq moving with velocity \mathbf{v} through a steady magnetic field of flux density \mathbf{B} experiences a force

$$\mathbf{F} = dq(\mathbf{v} \times \mathbf{B}).$$

If the charge is moving along a conductor, $dq\mathbf{v} = Id\mathbf{l}$, so the force on an element $d\mathbf{l}$ of a conductor is

$$\mathbf{F} = Id\mathbf{l} \times \mathbf{B}.$$

FORCE POLYGON (POLYGON OF FORCES). The figure obtained by considering force vectors as free vectors and adding them successively in any order by placing the tail of each additional one at the head of the previous one. The resultant force is given in magnitude and direction by the line from the tail of the first to the head of the last. Closure of the force polygon is a necessary but not sufficient condition for equilibrium. The resultant may be a couple. (See **funicular polygon**.)

FORCE, RESTORING. The force which acts on a particle or portion of a mechanical system when displaced from equilibrium and whose direction is such as to return the system to equilibrium. In simple cases, the restoring force is linear, i.e., proportional to the first power of the distance. (See **Hooke law; force constant**.) For some physical systems, however, the restoring force may be proportional to the second or higher power of the distance.

FORCES, COMPOSITION OF. The combination of two or more forces to a simpler system with the same **resultant** force and the same **moment** about any point or axis.

FORCES, CONCURRENT. Forces which meet at a point. **Equilibrium** is assured if a sufficient number of independent equations are satisfied, for example,

$$\sum \mathbf{F}_i = 0,$$

or in three-dimensional space

$$\sum F_x = 0$$

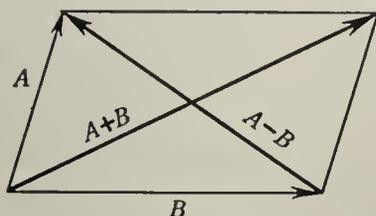
$$\sum F_y = 0$$

$$\sum F_z = 0.$$

FORCES, COPLANAR. A system of forces such that the lines of action of all the forces lie in a single plane.

FORCES IN THERMODYNAMICS OF IRREVERSIBLE PROCESSES. A number of causes can give rise to the occurrence of irreversible phenomena. These causes are, for example, temperature gradients, concentration gradients or **chemical affinities**. In the thermodynamics of irreversible processes, these quantities are usually called "forces" or "generalized affinities" and are denoted by X_i .

FORCES, PARALLELOGRAM OF. A geometrical representation by which the sum or difference of two concurrent forces **A** and **B** can be considered as the diagonals of a parallelogram. The two adjacent sides are proportional in length to the magnitude of the forces and their directions coincide with the lines of action of the forces. The diagonal represent-



Parallelogram of forces.

ing the sum passes through the origin of the two vectors. The diagonal representing the difference passes through the extremities of the two vectors.

FORCES, RESOLUTION OF. Replacement of a force by two or more **components** whose **resultant** is the given force. (See **composition of forces**.)

FORCES, RESULTANT OF. See **resultant**.

FORCES, TRIANGLE OF. Three concurrent forces which act such that the common point is in translational equilibrium, can be represented by a triangle. The sides of the triangle are proportional in length to the magnitude of the forces and the directions of the sides coincide with the lines of action of the forces. (See **equilibrium of a point; statics**.)

FORCE, SURFACE. See **surface force**.

FORCE SYSTEMS, EQUIVALENT. See **equivalent force systems**.

FORCE, TANGENTIAL. A force parallel to and in a surface under consideration, e.g., frictional force between a belt and a pulley wheel, or shear force on a cross-sectional plane.

FORCE, TRANSMISSIBILITY OF. See **transmissibility of force**.

FORCE, UNITS OF. See **mechanical units**.

FORCE WITHIN A CHARGED SYSTEM. If the conductors of a charged set are allowed to move, hence to let the system do work, the energy change of the system is given by:

$$dU = \sum V_i dq_i - \sum F_i d\xi_i$$

where F_i is the force, in the direction of the displacement $d\xi_i$, on the i th conductor. If the charges are held constant, $dq_i = 0$, and we have

$$\left(\frac{\partial U}{\partial \xi_i}\right)_q = -F_i.$$

If the potentials are held constant, write

$$d(U - \sum V_i q_i) = -\sum q_i dV_i - \sum F_i d\xi_i$$

but

$$U - \sum V_i q_i = \frac{1}{2} \sum V_i q_i - \sum V_i q_i = -U$$

(see **energy of a charged system**) so that

$$dU = \sum q_i dV_i + \sum F_i d\xi_i$$

and

$$\left(\frac{\partial U}{\partial \xi_i}\right)_V = F_i.$$

Note that for constant potential, the energy of the system increases at the same rate mechanical work is done; the sources of potential supply twice as much energy as needed for the mechanical work. The derivatives

$$\left(\frac{\partial U}{\partial \xi_i}\right)_q \quad \text{and} \quad \left(\frac{\partial U}{\partial \xi_i}\right)_V$$

can readily be expressed in terms of

$$\frac{\partial p_{ij}}{\partial \xi_i} \quad \text{and} \quad \frac{\partial c_{ij}}{\partial \xi_i},$$

using the expressions for the energy of a charged system in terms of coefficients of potential and induction. Replacing force (F_i) by torque (T_i), and linear displacement ($d\xi_i$) by rotation ($d\theta_i$), the foregoing relations yield the torques of the charged system.

FOREST. The set of **trees** formed by choosing a tree for each maximal connected **subgraph** of a graph G . If G has v **vertices**, e **edges** and P maximal connected subgraphs, the number of **elements** in a forest is $v - P$. The number of elements not in a forest is equal to $\mu = e - v + P$.

FORM. A homogeneous polynomial in two or more variables. A form in two variables, e.g., $a_{11}x_1^2 + 2a_{12}x_1x_2 + a_{22}x_2^2$ is called a binary form; the corresponding adjectives for a greater number of variables are ternary, quaternary, quinary, senary, etc. A form is definite if all its values are of one sign, except that its value for $x_1 = x_2 = \dots = x_n = 0$ is, of course, zero.

A form of degree two is quadratic, the adjectives for higher degrees being cubic, quartic, quintic, sextic and so forth.

FORM, BILINEAR. A homogeneous polynomial in $2n$ variables $x_1, x_2, \dots, x_n; y_1, y_2, \dots, y_n$, which is linear in the x 's and in the y 's considered separately; thus $a_{11}x_1y_1 + a_{12}x_1y_2 + a_{21}x_2y_1 + a_{22}x_2y_2$ is a bilinear form. If the x_i are identified with the y_i , the result is a quadratic form.

FORM, DIFFERENTIAL. See differential form.

FORM DRAG. See drag.

FORM FACTOR. (1) Factor introduced into a theory, usually by physical and non-rigorous arguments, to allow consequences of the theory to be computed without contributions from values of a parameter for which the theory is not applicable (see **cut-off**). (2) A means for describing one attribute of the shape of an **alternating-current** wave. The strength of a-c constantly varies in magnitude and direction. The effective value of a-c is the root-mean-square (rms) value. The form factor is the ratio of the rms value of the wave to its full-wave rectified average value; the factor is unity for a square wave, and 1.11 for a pure sine wave. It is of importance in the measurement of waves containing harmonics, for it is the ratio of the reading on an rms meter to that on an ideal rectifier-type meter.

FORM PARAMETER FOR TURBULENT BOUNDARY LAYER. The form parameter for a **turbulent boundary layer** is usually defined as

$$H = \frac{\delta^*}{\theta},$$

where δ^* is the **displacement thickness** and θ is the **momentum thickness**.

In calculations on turbulent boundary layers the usual assumption is that the velocity profile, expressed non-dimensionally, depends only on H . The value of H varies from about 1.4 in zero pressure gradient to about 2 or more at separation. (See also **turbulent boundary layer**.)

FORWARD DIFFERENCE OPERATOR. See **difference operators**.

FORWARD INTERPOLATION. See **Newton interpolation formulas**.

FORWARD SCATTER. The scattering of **radiant energy** into the hemisphere of space bounded by a plane normal to the direction of the incident radiation and lying on the side toward which the incident radiation was advancing; the opposite of **backward scatter**.

In **Rayleigh scattering**, forward scatter accounts for half of the total. As the particle size increases above the Rayleigh limit, an increasing fraction of the total scattering is forward scattering.

FOSTER REACTANCE THEOREM. For a two-terminal, dissipationless network, consisting of n meshes, the driving point impedance has a reactance which is zero at frequencies $f_1, f_3 \dots f_{2n-1}$, and which is infinite at frequencies $f_2, f_4 \dots f_{2n-2}$; the reactance is proportional to the expression

$$\frac{(f^2 - f_1^2)(f - f_3^2) \dots (f^2 - f_{2n-1}^2)}{(f^2 - f_2^2)(f^2 - f_4^2) \dots (f^2 - f_{2n-2}^2)};$$

where $0 < f_1 < f_2 \dots < f_{2n-1} < \infty$.

Since the derivative of the reactance with respect to frequency is greater than zero for frequency values between zero and infinity, its **poles** and **zeros** alternate.

FOUNDATION, ELASTIC. See **elastic foundation**.

FOUNDATION MODULUS. The ratio of the local pressure to the local deformation for the simplest type of **elastic foundation**. (See **beam on elastic foundation**.)

FOUR-DIMENSIONAL VECTOR ANALYSIS. See **Minkowski world**.

FOUR-FACTOR FORMULA. In chain reacting systems, the infinite multiplication factor, k_x can often be represented by the product $\eta\epsilon pf$. η is the average number of fast fission neutrons emitted as a result of the capture of one thermal neutron in the fuel material, ϵ is the **fast-fission factor**, p is the **resonance escape probability** for an infinite system, and f is the **thermal utilization** for an infinite system.

FOUR-FORCE. The **four-vector** describing the rate of change of four-momentum along the space-time path of a particle:

$$m_0 \frac{dv_\mu}{ds}$$

(m_0 is the rest mass.)

FOURIER ANALYSIS. The process of finding the representation of a function as a **Fourier series**. (See **harmonic analysis**.)

FOURIER-BESSEL TRANSFORM. The functions defined by

$$f(y) = \int_0^\infty F(x) J_m(xy) x dx;$$

$$F(x) = \int_0^\infty f(y) J_m(xy) y dy$$

are a pair of Fourier-Bessel transforms, the function J_m being a **Bessel function** of order m .

FOURIER EQUATION. See **Fourier law**.

FOURIER LAW. An empirical law relating to the conduction of heat. It states that the heat flux (rate of heat transfer per unit time and area) is proportional to the temperature gradient, or

$$\frac{\dot{Q}}{A} = \dot{q} = -k \text{ grad } T$$

or, in one dimension

$$\dot{q} = -k \frac{dT}{dn}$$

where n is the normal.

The coefficient of proportionality k is called the (*coefficient of*) **thermal conductivity**; it is a property of the medium conducting the heat, and depends on the local state.

The Fourier law leads to the formulation of the Fourier equation for heat conduction:

$$\frac{\partial T}{\partial t} = a \nabla^2 T$$

where t is time, and where

$$a = \frac{k}{\rho c_p}$$

is called the (*coefficient of*) **thermal diffusivity** of the material conducting the heat.

The Fourier law, and hence the Fourier equation, gives a description of the process of heat conduction which agrees well with experiment. However, it has been pointed out that the equation is parabolic, implying an infinite rate of propagation of local changes in temperature, which is physically improbable. When the changes in temperature are extremely rapid, the Fourier equation must be expected to break down. It has been proposed to replace the Fourier law by an alternative empirical assumption.

When dealing with problems of heat conduction in liquids and gases, as opposed to solids, it must be borne in mind that the above equations describe the physical phenomena adequately only in very thin layers of fluid. When the extent of the fluid is not extremely small, the temperature field causes density gradients to appear. In turn, these will cause the fluid to move in the gravitational field of the earth. The resulting motion attributable to conduction causes much larger rates of heat transfer than those termed *natural convection*.

FOURIER SERIES. A single-valued function, continuous except possibly for a finite number of finite discontinuities in the interval $-\pi$ to π , and with only a finite number of **maxima** or **minima** in that interval, may, according to the Fourier theorem, be represented by a **Fourier series**

$$f(x) = \sum_{n=1}^{\infty} a_n \sin nx + \frac{b_0}{2} + \sum_{n=1}^{\infty} b_n \cos nx$$

where the coefficients are given by

$$\pi a_n = \int_{-\pi}^{\pi} f(t) \sin ntdt;$$

$$\pi b_n = \int_{-\pi}^{\pi} f(t) \cos ntdt.$$

A change of variable may be made so that the interval extends from 0 to n , 0 to $2n$, L to $-L$, etc. The series may be generalized to permit the expansion of a function of several variables. Fourier analysis is the process of representing a function in a Fourier series.

If the range of the variable is $-\infty < x < \infty$, the Fourier **integral** is

$$2\pi f(x) = \int_{-\infty}^{\infty} f(z)e^{iy(x-z)}dydz$$

or, in a more common form, if $f(x)$ is real the exponential function may be replaced by $\cos y(x - z)$.

FOURIER SERIES, COMPLEX. If in the Fourier series

$$f(x) = \sum_{n=1}^{\infty} a_n \sin nx + \frac{b_0}{2} + \sum_{n=1}^{\infty} b_n \cos nx,$$

we set

$$\begin{aligned} 2\beta_n &= b_n - ia_n & n > 0 \\ 2\beta_n &= b_{-n} + ia_{-n} & n < 0 \\ 2\beta_0 &= b_0 \end{aligned}$$

then the series takes the convenient complex form

$$f(x) = \sum_{-\infty}^{\infty} \beta_n e^{inx}.$$

FOURIER SERIES, HALF-RANGE. In a full-range Fourier series the length of the fundamental interval being $2L$, the $f(x)$ is expanded in terms of the functions $\sin \frac{n\pi x}{L}$ and $\cos \frac{n\pi x}{L}$, the coefficients being given by integrals taken over the fundamental interval. In a half-range series, $f(x)$ is expanded either in terms of $\sin \frac{n\pi x}{L}$ or of $\cos \frac{n\pi x}{L}$, the coefficients being given by integrals taken over an interval of length L .

FOURIER SYNTHESIS. The process of computing the form of a function from the values of its Fourier coefficients.

FOURIER TRANSFORM. Subject to certain restrictions, $f(y)$ is the Fourier transform of $F(x)$, where

$$f(y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ixy} F(x) dx;$$

$$F(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ixy} f(y) dy.$$

In some applications, the factors $1/\sqrt{2\pi}$ are modified. (See also **integral transform**.)

In feedback and control-system theory the Fourier transform is used almost exclusively for converting a function of time t , $f(t)$, into a function of angular frequency ω , $F(i\omega)$, which is the Fourier transform of $f(t)$ and defined by

$$F(i\omega) = \int_{-\infty}^{+\infty} e^{-i\omega t} f(t) dt$$

the integral existing provided $\int_{-\infty}^{+\infty} |f(t)| dt$ converges. Then

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{+i\omega t} F(i\omega) d\omega.$$

The convergence limitation on $f(t)$ excludes many frequent functions, notably the **step function**, a fact which is partly responsible for the much wider use of the **Laplace transform**.

FOURIER TRANSFORM THEOREMS.

(a) *Even and odd relationships.* If $f(t)$ is a real, even function of t , $F(i\omega)$ is a real, even function of ω .

If $f(t)$ is a real, odd function of t , $F(i\omega)$ is an imaginary, odd function of ω .

(b) *Conjugacy relationships.* If $f(t)$ is real, $F(i\omega)$ and $F(-i\omega)$ are complex conjugates.

(c) *Time-shift theorem.* If $F(i\omega)$ is the Fourier transform of $f(t)$ then the Fourier transform of $f(t + t_0)$ is $e^{+i\omega t_0} F(i\omega)$, where t_0 is real and finite.

(d) *Frequency-shift theorem.* If $F(i\omega)$ is the Fourier transform of $f(t)$ then $F(i\omega + \omega_0)$ is the Fourier transform of $e^{-it\omega_0} f(t)$.

(e) *Parseval's theorem.* If $F_1(i\omega)$, $F_2(i\omega)$ are the Fourier transforms of $f_1(t)$, $f_2(t)$ then

$$\begin{aligned} \int_{-\infty}^{\infty} F_1(i\omega) F_2(i\omega) d\omega &= 2\pi \int_{-\infty}^{\infty} f_1(t) f_2(-t) dt \\ &= 2\pi \int_{-\infty}^{\infty} f_1(-t) f_2(t) dt \end{aligned}$$

and conversely

$$\begin{aligned} \int_{-\infty}^{\infty} f_1(t) f_2(t) dt &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(i\omega) F_2(-i\omega) d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} F_1(-i\omega) F_2(i\omega) d\omega. \end{aligned}$$

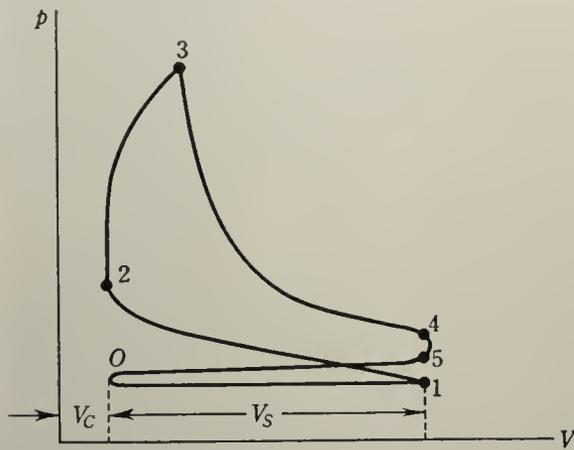
In particular, if $f_1(t) \equiv f_2(t) \equiv f(t)$ and $f(t)$ is real

$$\int_{-\infty}^{\infty} \{f(t)\}^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(i\omega)F(-i\omega)d\omega$$

$$= \frac{1}{2\pi} \int_{-\infty}^{\infty} |F(i\omega)|^2 d\omega.$$

FOUR-MOMENT EQUATION. A relation between the four end moments of two adjacent beams which takes into account the possibility of more than two members meeting at a joint. It reduces to the **three-moment equation** when the moment is continuous through the joint. Stability and vibration problems of frames lead to and are solved conveniently by four-moment equations.

FOUR-STROKE ENGINE. A reciprocating engine in which one working stroke is executed per two revolutions (four strokes). A four-stroke engine (see schematic figure) is



Four-stroke engine cycle.

equipped with an inlet valve i and an exhaust valve e and the sequence of operations is as follows:

- 0-1. Introduction of working fluid (air in diesel engine, combustible mixture in gasoline engine). Valve i open, valve e closed. First stroke.
- 1-2. Compression of working fluid. Valves i and e closed. Second stroke.
- 2-3-4. Combustion and expansion. Combustion nearly at $V = \text{constant}$ in gasoline engine and nearly at $p = \text{constant}$ in diesel engine. Valves i and e closed. Third stroke.

4-5-0. Release and subsequent expulsion of burnt gases. Valve e open. Fourth stroke.

In practice the i and e valves do not open at dead centers, as assumed in the idealized description above. They open earlier and close later to allow for the finite time of opening and closing and for the acceleration of gas columns, etc.

FOUR-VECTORS AND TENSORS. The four-dimensional line element is assumed to have the form

$$(ds)^2 = g_{ik}dx^i dx^k$$

$$= -[(dx)^2 + (dy)^2 + (dz)^2] + c^2 dt^2$$

$$g_{11} = g_{22} = g_{33} = -1 \quad g_{44} = c^2$$

$$g_{ik} = 0 \quad \text{for } i \neq k \quad \det g_{ik} = |g| = -c^2.$$

The relationship between the covariant and contravariant components of a four-vector are ($i, k = 1, 2, 3$)

$$a_i = -a^i$$

$$a_4 = c^2 a^4$$

the relationships between the components of a tensor t of second rank are

$$t_{ik} = t^{ik} \quad t_k^i = -t^{ik}$$

$$t_{i4} = -c^2 t^{i4} \quad t_4^i = c^2 t^{i4}$$

$$t_{44} = c^4 t^{44} \quad t_i^4 = -t^{4i}$$

$$t_4^4 = c^2 t^{44}.$$

If it is assumed that Σ' moves with the velocity v relative to Σ in the direction of the positive x -axis, the transformation laws for the contravariant components a^i are

	a^1	a^2	a^3	a^4
$a^{1'}$	$\frac{1}{\sqrt{1 - \beta^2}}$	0	0	$-\frac{v}{\sqrt{1 - \beta^2}}$
$a^{2'}$	0	1	0	0
$a^{3'}$	0	0	1	0
$a^{4'}$	$-\frac{v}{c^2 \sqrt{1 - \beta^2}}$	0	0	$\frac{1}{\sqrt{1 - \beta^2}}$

Under this motion, the transformation matrix for the contravariant components t^{ik} of a tensor of rank 2 is:

	11	12	13	14	21	22	23	24	31	32	33	34	41	42	43	44
11	$\frac{1}{1-\beta^2}$	0	0	$\frac{v}{1-\beta^2}$									$\frac{-v}{1-\beta^2}$	0	0	$\frac{v^2}{1-\beta^2}$
12	0	$\frac{1}{\sqrt{1-\beta^2}}$	0	0									0	$\frac{-v}{\sqrt{1-\beta^2}}$	0	0
13	0	0	$\frac{1}{\sqrt{1-\beta^2}}$	0									0	0	$\frac{-v}{\sqrt{1-\beta^2}}$	0
14	$\frac{-v}{c^2(1-\beta^2)}$	0	0	$\frac{1}{1-\beta^2}$									$\frac{\beta^2}{1-\beta^2}$	0	0	$\frac{-v}{1-\beta^2}$
21					$\frac{1}{\sqrt{1-\beta^2}}$	0	0	$\frac{-v}{\sqrt{1-\beta^2}}$								
22					0	1	0	0								
23		0			0	0	1	0				0			0	
24					$\frac{-v}{c^2\sqrt{1-\beta^2}}$	0	0	$\frac{1}{\sqrt{1-\beta^2}}$								
31									$\frac{1}{\sqrt{1-\beta^2}}$	0	0	$\frac{-v}{\sqrt{1-\beta^2}}$				
32									0	1	0	0				
33		0				0			0	0	1	0			0	
34									$\frac{-v}{c^2\sqrt{1-\beta^2}}$	0	0	$\frac{1}{\sqrt{1-\beta^2}}$				
41	$\frac{-v}{c^2(1-\beta^2)}$	0	0	$\frac{\beta^2}{1-\beta^2}$									$\frac{1}{1-\beta^2}$	0	0	$\frac{-v}{1-\beta^2}$
42	0	$\frac{-v}{c^2\sqrt{1-\beta^2}}$	0	0									0	$\frac{1}{\sqrt{1-\beta^2}}$	0	0
43	0	0	$\frac{-v}{c^2\sqrt{1-\beta^2}}$	0									0	0	$\frac{1}{\sqrt{1-\beta^2}}$	0
44	$\frac{\beta^2}{c^2(1-\beta^2)}$	0	0	$\frac{-v}{c^2(1-\beta^2)}$									$\frac{-v}{c^2(1-\beta^2)}$	0	0	$\frac{1}{1-\beta^2}$

An antisymmetric tensor of rank 2 is also called a six-vector. The transformation matrix

for a contravariant six-vector is, in the case of the motion stated above:

	14	24	34	23	31	12
14	1	0	0	0	0	0
24	0	$\frac{1}{\sqrt{1-\beta^2}}$	0	0	0	$\frac{+v}{c^2\sqrt{1-\beta^2}}$
34	0	0	$\frac{1}{\sqrt{1-\beta^2}}$	0	$\frac{-v}{c^2\sqrt{1-\beta^2}}$	0
23	0	0	0	1	0	0
31	0	0	$\frac{-v}{\sqrt{1-\beta^2}}$	0	$\frac{1}{\sqrt{1-\beta^2}}$	0
12	0	$\frac{+v}{\sqrt{1-\beta^2}}$	0	0	0	$\frac{1}{\sqrt{1-\beta^2}}$

Transformation law for a symmetric contravariant tensor of rank 2:

$$\begin{aligned}
 t^{11'} &= \frac{t^{11} - 2vt^{14} + v^2t^{44}}{1 - \beta^2} \\
 t^{22'} &= t^{22}; \quad t^{33'} = t^{33} \\
 t^{44'} &= \frac{\beta^2t^{11} - 2vt^{14} + c^2t^{44}}{c^2(1 - \beta^2)} \\
 t^{12'} &= \frac{t^{12} - vt^{24}}{\sqrt{1 - \beta^2}}; \quad t^{13'} = \frac{t^{13} - vt^{34}}{\sqrt{1 - \beta^2}} \\
 t^{23'} &= t^{23} \\
 t^{14'} &= \frac{-vt^{11} + (c^2 + v^2)t^{14} - c^2vt^{44}}{c^2\sqrt{(1 - \beta^2)}} \\
 t^{24'} &= \frac{-vt^{12} + c^2t^{24}}{c^2(1 - \beta^2)}.
 \end{aligned}$$

Relativistic invariants are

$$t^{11} + t^{22} + t^{33} - c^2t^{44} = -g_{i\lambda}t^{i\lambda}$$

and

$$\begin{aligned}
 (t^{11})^2 + (t^{22})^2 + (t^{33})^2 + c^4(t^{44})^2 \\
 + 2 \left[(t^{12})^2 + (t^{23})^2 + (t^{31})^2 \right. \\
 \left. - c^2(t^{14})^2 - c^2(t^{24})^2 - c^2(t^{34})^2 \right] \\
 = g_{i\mu}g_{\lambda\rho}t^{i\lambda}t^{\mu\rho}.
 \end{aligned}$$

For further details, see Erwin Madelung, *Die mathematischen Hilfsmittel des Physikers*, Third Edition, 1936, Springer (Berlin).

FOWLER FUNCTION. The photoelectric yield for a metallic surface can be expressed as $y = \alpha AT^2\phi(x)$, where $\phi(x)$ is the Fowler function. A is the constant appearing in the **Richardson-Dushman equation**. The parameter x is given by $x = (h\nu - h\nu_0)/kT$, where ν_0 is the photoelectric threshold frequency. The fraction of those electrons incident at the surface per unit area per unit time that absorb a quantum and escape (for unit intensity of incident radiation) is denoted by α .

FRACTIONAL BOND ORDER. See **bond character**; **mobile bond order**.

FRACTIONAL PARENTAGE COEFFICIENT. A coefficient introduced by Raeh in the theory of spectroscopy to describe the way in which several particles obeying **Fermi-Dirac statistics** (e.g., electrons) can be coupled

together to form antisymmetric states. Suppose Φ_β denotes a set of antisymmetric eigenfunctions for $n - 1$ particles moving independently in a common field of force, and let ϕ_γ denote the eigenfunctions for a single particle in the same field of force. Then any of the products $\Phi_\beta\phi_\gamma$ are eigenfunctions for the whole collection of n particles. However, these products are not in general antisymmetric; only certain linear combinations

$$\Psi_\alpha = \sum_{\beta\gamma} c_{\beta\gamma} \Phi_\beta\phi_\gamma$$

will be antisymmetric. The coefficients c are the fractional parentage coefficients, each eigenfunction α of the n -particle problem possessing a set of "parent" eigenfunctions β of the $(n - 1)$ -particle problem. In practice, the summation is restricted to those eigenfunctions which are degenerate in energy in the common field of force. Straightforward methods then exist for computing their values.

FRACTION, CONTINUED. A continued fraction is an infinite **sequence** $\{s_n\}$ with members formed from the sequences a_1, a_2, \dots and b_0, b_1, b_2, \dots according to the following directions. The term s_n is found by replacing the denominator of s_{n-1} by $(b_{n-1} + a_n/b_n)$. When written in the conventional way such a fraction is rather awkward, for it would read

$$\begin{aligned}
 b_0 + \frac{a_1}{b_1 + \frac{a_2}{b_1 + \frac{a_3}{b_3 + \dots}}}
 \end{aligned}$$

It can be simplified by writing it on one line with the plus sign properly placed, as

$$b_0 + \frac{a_1}{b_1} + \frac{a_2}{b_2} + \frac{a_3}{b_3} + \dots$$

or, in a two-line form

$$\left(\begin{array}{c} a_1, a_2, \dots \\ b_0, b_1, b_2, \dots \end{array} \right).$$

Some modifications of these notations also appear. (See use of $\overline{\quad}$ on next page.)

When there are a finite number of terms, the fraction is called **terminating**; if there are an infinite number of terms, **non-terminating**. **Convergence** behavior of such fractions can be studied by generalization of the methods used for **series**.

Sometimes continued fractions are suitable as solutions of a linear second-order **differential equation**, which will be assumed given in the form

$$y = A_0(x)y' + B_1(x)y''.$$

If this equation is differentiated n times the result is

$$y^{(n)} = A_n y^{(n+1)} + B_{n+1} y^{(n+2)},$$

where

$$A_n = (A_{n-1} + B'_n)/(1 - A'_{n-1})$$

and

$$B_{n+1} = B_n/(1 - A'_{n-1}).$$

The ratio y/y' is a continued fraction and it is the reciprocal of the logarithmic derivative of the solution to the differential equation. The result is found to be

$$y/y' = A_0 + \frac{B_1}{A_1} + \frac{B_2}{A_2} + \dots$$

The method does not seem to be readily extended to differential equations of higher order. Its most familiar application in mathematical physics is to the **Mathieu equation**.

In the systematic evaluation of continued fractions, consider the (possibly infinite) system of differences

$$\begin{aligned} x_0 &= b_0x_1 + a_1x_2, \\ x_1 &= b_1x_2 + a_2x_3, \\ x_2 &= b_2x_3 + a_3x_4, \\ &\dots \end{aligned}$$

So long as no x_i vanishes or becomes infinite one can write

$$\begin{aligned} x_0/x_1 &= b_0 + a_1/(x_1/x_2), \\ x_1/x_2 &= b_1 + a_2/(x_2/x_3), \\ x_2/x_3 &= b_2 + a_3/(x_3/x_4), \end{aligned}$$

and with repeated substitution

$$x_0/x_1 = b_0 + a_1/\overline{b_1} + a_2/\overline{b_2} + \dots$$

where each horizontal bar signifies the inclusion of all that follows. The right-hand number is a continued fraction. If one arbitrarily sets $x_{i+1} = 0$ and $x_i = 1$, considering only the first i equations, the finite continued fraction can be evaluated by solving the i th equation for x_{i-1} ; substituting in the preceding and solving

for x_{i-2}/x_{i-1} ; and continuing until x_0/x_1 is obtained. In many applications the a_i and b_i are functions of a variable x , and one truncates and evaluates in the manner just described.

A finite continued fraction

$$F_n = b_0 + a_1/\overline{b_1} + \dots + a_n/b_n \equiv A_n/B_n,$$

where A_n and B_n are polynomials in the a_i and b_i can be otherwise evaluated by means of the recursions (see **continuant**)

$$\begin{aligned} A_n &= b_n A_{n-1} + a_n A_{n-2}, \\ B_n &= b_n B_{n-1} + a_n B_{n-2}. \end{aligned}$$

These relations, however, though of great theoretical importance, are not to be recommended for computation in general since they tend to be highly unstable numerically.

The fraction F_n is called the n th approximant to F ; A_n and B_n the n th numerator and denominator. If all A_n and B_n are given, and if

$$A_n/B_n \neq A_{n+1}/B_{n+1}$$

for every n , then the a_n and b_n are determined uniquely. In particular the approximants F_{2n} are called the even parts, F_{2n+1} the odd parts, of F , and it is possible to form a continued fraction whose approximants are the even parts of F , and one whose approximants are the odd parts.

For applications of continued fractions, see **reciprocal differences**; **Padé table**.

FRACTION, PARTIAL. When a given rational fraction is resolved into a sum of simpler fractions the individual terms in the sum are called partial fractions. The process of resolving a fraction in this way is the inverse process to that of reducing to a common denominator. If the fraction is of the form $f(x)/g(x)$, where the degree of the numerator is less than that of the denominator, $g(x)$ may be written as

$$g(x) = a_0(x - x_1)^{r_1}(x - x_2)^{r_2} \dots$$

$$(x^2 + 2b_1x + c_1)^{s_1}(x^2 + 2b_2x + c_2)^{s_2} \dots$$

Then the fraction $f(x)/g(x)$ may be written as the sum of $r_1 + r_2 + \dots + s_1 + s_2 + \dots$ in the form

$$\frac{A_1}{x - a} + \frac{A_2}{(x - a)^2} + \dots + \frac{A_r}{(x - a)^r}$$

with constant partial numerators, and $s_1 + s_2 + \dots$ in form

$$\frac{B_1 + C_1x}{Q} + \frac{B_2 + C_2x}{Q^2} + \dots + \frac{B_s + C_sx}{Q^s}$$

with linear partial numerators where the partial denominators have the form $Q = (x^2 + 2bx + c)$. The undetermined coefficients A_i, B_i, C_i in these fractions may be obtained by clearing of fractions and equating coefficients of like powers of x on both sides of the equality, which is an identity in x ; or by other special devices, such as substituting special values of x . An **integral** may often be evaluated by converting the integrand into a sum of partial fractions.

FRACTURE, BRITTLE. See **brittle fracture**.

FRACTURE LINE THEORY. See **rupture line theory**.

FRAME. A structure composed of **beams** and **beam-columns** joined in such a manner that load can be carried only if the joints are able to transmit or take appreciable moment (see **framework**). In a *rigid frame* the rotation of all members framing into a joint is the same. The word rigid refers to the absence of relative rotation at a joint, whereby the frame itself would deform. (See **inequalities, method of**.)

FRAME OF REFERENCE. A coordinate system in which position measurements can be made.

FRAME OF REFERENCE, INERTIAL. A frame of reference in which a free body (i.e., one subject to no forces) is not accelerated. Newton's laws of motion are obeyed in all inertial frames. The special theory of relativity is based on the postulates: (a) that all inertial frames of reference are equivalent for the description of physical phenomena, and (b) that the speed of light has the same value in all inertial frames of reference.

FRAME, PORTAL. See **portal frame**.

FRAMES, MECHANISM METHOD OF ANALYSIS. See **mechanism method of analysis of frames**.

FRAMEWORK. A system of rigid or deformable bodies of arbitrary sizes and shapes fastened together to form a whole, which can be in equilibrium under the action of suitable forces. For example, a bridge truss. Moment carrying capacity at the joints is not essential, as it is for a frame.

FRANCK-CONDON PRINCIPLE. The Franck-Condon principle forms the basis for an understanding of the intensity distribution in a molecular **band-system** (see also **Condon parabola**). In its elementary form the F.C.P. is based on the fact that an electron "jump" in a molecule takes place so rapidly in comparison with the vibrational motion that immediately afterwards the nuclei have very nearly the same relative position and velocity as before. A transition between two different electronic levels of a molecule therefore takes place between points on the respective potential curves (see **potential functions of a molecule**) which lie very nearly vertically above each other, that is, for vibrational levels for which the nuclear distance at the turning point of the vibrational motion is nearly the same.

In the wavemechanical form of the F.C.P. the transition probability depends on the magnitude of the overlap integral of the vibrational eigenfunctions $\psi_{v'}$ and $\psi_{v''}$

$$R^{v'v''} = \bar{R}_e \int \psi_{v'} \psi_{v''} dr$$

where \bar{R}_e is the mean electronic transition moment which varies only very slowly with the internuclear distance r . The intensity of an electronic (vibronic) band is given in emission by

$$I_{em.}^{v'v''} = \frac{64}{3} \pi^4 c N_{v'} v^4 \bar{R}_e^2 \left[\int \psi_{v'} \psi_{v''} dr \right]^2$$

and in absorption

$$I_{abs.} = \frac{8\pi^3}{3hc} I_o \Delta x N_{v''} v \bar{R}_e^2 \left[\int \psi_{v'} \psi_{v''} dr \right]^2$$

where $N_{v'}$ and $N_{v''}$ are the numbers of molecules in the vibrational levels v' and v'' respectively, I_o , the intensity of the incident beam of light, Δx , the thickness of the absorbing layer, h , Planck's constant, and c , the velocity of light.

FRANKLIN EQUATION. The energy level E of sound in a room decays exponentially with time t . Thus

$$E = E_o e^{-\mu t}$$

where μ is the decay constant, and E_o is the initial sound level. The Franklin equation states that

$$\mu = c\bar{\alpha}S/4V$$

where c is the velocity of sound, $\bar{\alpha}$ is the mean sound absorption coefficient, S is the exposed surface, and V is the volume of the room. For dead rooms (i.e., rooms which are exceptionally non-reverberant and $\bar{\alpha}$ is comparable with unity), Franklin's equation must be replaced by the **Eyring equation**, where $\bar{\alpha}$ is replaced by $\log(1 - \bar{\alpha})$, thus

$$\mu = -c \log(1 - \bar{\alpha})S/4V.$$

F-RATIO. See **degrees of freedom**.

FRAUNHOFER DIFFRACTION. Diffraction phenomena observed when both the point of observation and (usually) the source are effectively at infinite distance from the aperture or other diffracting agency. (For the Fraunhofer diffraction pattern of a circular aperture, see the discussion of the **Airy disc**; see also **rectangular aperture** and **slit**.)

FREDHOLM DETERMINANT. Consider a Fredholm integral equation of the second kind:

$$\phi(x) = f(x) + \lambda \int_a^b K(x, \xi) \phi(\xi) d\xi.$$

The Fredholm determinant for the kernel $K(x, \xi)$ is

$$D(\lambda) = 1 - \lambda \int_a^b K(\xi_1, \xi_1) d\xi_1 + \frac{\lambda^2}{2!} \int_a^b \int_a^b \begin{vmatrix} K(\xi_1, \xi_1) & K(\xi_1, \xi_2) \\ K(\xi_2, \xi_1) & K(\xi_2, \xi_2) \end{vmatrix} d\xi_1 d\xi_2 + \dots$$

If we write

$$D(x, \xi; \lambda) = \lambda K(x, \xi) - \lambda^2 \int_a^b \begin{vmatrix} K(x, \xi) & K(x, \xi_1) \\ K(\xi_1, \xi) & K(\xi_1, \xi_1) \end{vmatrix} d\xi_1 + \dots,$$

then the solution of the above integral equation is given by

$$\phi(x) = f(x) + \int_a^b f(\xi) \frac{D(x, \xi; \lambda)}{D(\lambda)} d\xi,$$

in analogy with **Cramer's rule** for n equations in n unknowns.

FREDHOLM THEOREM (FOR INTEGRAL EQUATIONS). The theorem of the alternative for integral equations (*cf.* **alternative, theorem of**), namely: for fixed λ , a Fredholm equation of the second kind

$$\phi(x) = f(x) + \lambda \int_a^b K(x, z) \phi(z) dz$$

either has a unique continuous solution $\phi(x)$ for every continuous $f(x)$ or else the corresponding equation of the first kind (i.e., with $f(x) \equiv 0$) has a positive finite number r of linearly **independent** solutions $\psi_1(x), \dots, \psi_r(x)$, the number λ then being said to be an eigenvalue of multiplicity r of the kernel $K(x, z)$. In this case the transposed equation

$$\chi(x) = \lambda \int_a^b K(z, x) \chi(z) dz$$

also has r linearly independent solutions $\chi_1(x), \chi_2(x), \dots, \chi_r(x)$ and the original (non-homogeneous) equation is solvable only if $f(x)$ is orthogonal to every $\chi_i(x)$, that is, only if

$$\int_a^b f(x) \chi_i(x) dx = 0, \quad (i = 1, 2, \dots, r),$$

the solution $\phi(x)$ being then uniquely determined except that any linear combination of the $\chi_i(x)$ may be added to it.

FREE ATMOSPHERE. (Sometimes called free air.) That portion of the earth's atmosphere, above the **planetary boundary layer**, in which the effect of the earth's surface friction on the air motion is negligible, and in which the air is usually treated (dynamically) as an **ideal gas**. The base of the free atmosphere is usually taken as the **geostrophic wind level**.

FREE-BODY DIAGRAM. A diagram of an isolated body or system of bodies showing all the external forces acting.

FREE CONVECTION. See **heat transfer**.

FREEDOM, DEGREES OF. See **degrees of freedom**.

FREE ENERGY. There are two quantities to which this term has been applied. (1) The Gibbs free energy, G , which is also called the **Gibbs function** or free enthalpy to distinguish it from (2) the Helmholtz free energy, A , which is also called the work function.*

* Note that most American writers use F for the Gibbs function and A (as above) for the Helmholtz function, while the usage elsewhere is generally G (as above) for the Gibbs function and F for the Helmholtz function. To avoid confusion, the symbol F was avoided in this book by use of the mixed notation.

(1) The Gibbs free energy is defined by the equation

$$G = U - TS + pV = H - TS$$

where U is the internal energy, T , the absolute temperature, S , the entropy, p , the pressure, V , the volume and G , the Gibbs free energy.

(2) The Helmholtz free energy which is also called the **Helmholtz function** is defined by the equation

$$A = U - TS$$

where U is the internal energy, T , the absolute temperature, S , the entropy and A , the Helmholtz free energy. For a constant-temperature, reversible change in which the work is due to expansion only, $dG = Vdp$ and $dA = -pdV$. In terms of the partition function, $A = -RT \ln Z$. Like the Gibbs free energy, the Helmholtz free energy is a thermodynamic potential. (See also **thermodynamics, characteristic functions of.**)

FREE ENERGY OF GERM FORMATION.

The difference in the **Helmholtz functions** in the presence and in the absence of a germ or nucleus. (See **germs, kinetics of phase formation.**)

Gibbs has given different explicit expressions for this quantity, which depends essentially on the surface tension γ and the size of the germ. One of them is

$$(\Delta A)_{T,V} = \frac{1}{3}\gamma\Omega > 0 \quad (1)$$

where Ω is the area of the germ. This free energy is positive. Therefore the formation of a germ can only be due to *statistical fluctuations*. Moreover its value is proportional to

$$N_g^{2/3} \quad (2)$$

where N_g is the number of molecules in the germ.

The free energy of germ formation plays, in the kinetics of phase formation, a role somewhat similar to that of the free energy of activation in the kinetics of ordinary reactions. (See **absolute reaction rate theory.**)

FREE ENTHALPY. See **Gibbs function.**

FREE MASS POINT. See **dynamics of a free mass point.**

FREE-MOLECULE FLOW. Flow of a gas in which collisions between molecules have no appreciable effects, so that the only collisions of importance are those between the gas mole-

cules and the solid boundaries. Free-molecule flow occurs for values of the **Knudsen number** greater than about 10. (See also **rarefied gas dynamics; slip flow.**)

FREE STREAMLINES. The boundaries of a region of stagnant fluid in the wake of a body, the fluid outside being in steady motion. The free streamlines are presumed to originate at sharp (or salient) edges on the body. Many problems in two-dimensional flow are soluble assuming free streamlines to exist, the pressure along them being constant. The free streamlines are on **vortex sheets** which are in fact unstable configurations of motion; but nevertheless the solutions obtained are of value in the neighborhood of the body.

FREE VALENCE INDEX. Daudel and other workers have calculated the bond character of benzene and other aromatic molecules taking into account, not only the Kekulé formula, but also more excited configurations. (See **benzene, structure of.**) In the case of benzene, for instance, they obtain a double bond character of 0.462 for each bond, whereas assuming only Kekulé formulae, one would calculate a bond character of 0.500. This means that at each corner of the benzene hexagon, there remains approximately .076 of an electron which is not used in the double or single bonds. The "Dewar" bonds across the hexagon are not true bonds, but rather free electrons with opposite spins; Daudel defines then a free valence index of .076 for each corner of the benzene molecule. Similar free valence indices have been calculated for each corner of more complicated molecules; the knowledge of these indices is important for the theoretical prediction of the reactivity of aromatic molecules. (See **free valency; molecular diagrams; double bond character.**)

FREE VALENCY. Refined molecular models indicate that the total **bond character** (see also **valence bond method**) or the total **mobile bond order** (see also **molecular orbital method**) are smaller than the value predicted under the assumption that each **bonding electron** participates in a bond.

In the molecular orbital theory, the free valency of an atom has been defined by Coulson as the difference between the maximum bond number which that atom can manifest and the sum of the mobile bond orders of the bonds in which it participates. Although nu-

merical values of the free valency thus defined and of the **free valence index** calculated by the valence bond method differ, they are usually proportional to each other.

FREE VECTORS. See **affine tensors and free vectors.**

FREE VIBRATIONS. See **vibrations (free) of undamped system.**

FREE VOLUME THEORY OF LIQUIDS. See **cell model of the liquid state.**

FREE-VORTEX BLADE. See **blade, twisted.**

FREE WAVE. Any wave not acted upon by any external force except for the initial force that created it; a wave solution satisfying a homogeneous equation of motion and homogeneous boundary conditions. In a system with no impressed forces, a free wave has zero amplitude at the boundaries of the system. The phase speed, wavelength, etc. of the free wave or waves are characteristics of the system. A simple example of such a wave in meteorology is a **billow cloud** layer over level ground.

In a **steady-state** solution, free waves have arbitrary amplitude. These may be specified by **initial conditions** to determine the solution completely.

FREEZING IN OF DEGREES OF FREEDOM. Some **degrees of freedom** will be related to energies which are large compared to kT (k is Boltzmann's constant; T is absolute temperature). In that case the system will behave as if it possessed fewer degrees of freedom. A diatomic molecule will, for instance, behave like a particle of 5 degrees of freedom (3 translational and 2 rotational) instead of 6, the vibrational degree of freedom corresponding to a change in the distance between the two atoms being frozen in.

FRENET FORMULAE. See **Serret-Frenet formulae.**

FRENKEL DEFECT. A lattice **vacancy** created by removing an ion from its site and placing it at an **interstitial position** within the lattice. The equilibrium concentration of Frenkel defects is

$$n = C_F(NN')^{1/2}e^{-W/2kT}$$

where N , N' are the densities of lattice points and interstitial positions, W is the work

necessary to make the defect, and C_F is a numerical factor of the order of 100.

FREQUENCY, ATOMIC. See **atomic frequency.**

FREQUENCY, COMPLEX. The constant of proportionality relating the time rate of change of an exponential wave to its value at every instant. The real part of a complex frequency expresses the growth of the wave in **nepers** per unit time (and is sometimes called the *neper frequency*) and the imaginary part, the angular velocity in radians per unit time (sometimes called the *radian frequency*).

As used in this book, the complex frequency is denoted by the symbols:

$$s = \sigma + j\omega.$$

No standard notation has ever been adopted for the complex frequency. The earlier literature in network theory made almost exclusive use of the symbol p , and whereas some authors still prefer this notation, modern usage favors the symbol, s . The symbol, λ (lambda), at one time had limited acceptance.

FREQUENCY EQUATION. (Also called dispersion equation.) An equation relating **phase speed** to **wavelength** and to the physical parameters of the system (fluid depth, current speed, temperature, etc.) in a linear oscillation.

Mathematically, the frequency equation is the result of substituting a simple harmonic solution in the homogeneous differential **equations of motion** and the homogeneous **boundary conditions**. The frequency equation thus describes the **free waves** of the system. If the wavelength does not appear in the expression for the phase speed, the system is non-dispersive.

FREQUENCY FACTOR. The rate of an elementary chemical reaction can usually be written as a product of a function of the concentrations of the participants, and of a **rate constant** k_r which depends on temperature. k_r itself can be written in the form: $k_r = A \cdot \exp(-E_a/kT)$, where k is the **Boltzmann constant**, T , the **absolute temperature**, E_a , the **activation energy** and A is called the **frequency factor**.

The frequency factor depends on the **collision number**, and on the entropy of activation. This latter dependence is often called the

“steric factor,” since in an oversimplified image, it expresses the fact that the mutual orientation of the collision partners during the collision may be unfavorable for the reaction to take place.

FREQUENCY FACTOR FOR CHEMICAL REACTION. See collision theory.

FREQUENCY FUNCTION. See distribution.

FREQUENCY, INSTANTANEOUS. The time rate of change of the phase of a traveling wave.

FREQUENCY MODULATION. A type of modulation in which the frequency of a continuous radio carrier wave is varied in accordance with the properties of a second (modulating) wave.

FREQUENCY, NATURAL. Of a body or system, a frequency of free oscillation. This term is commonly applied to coils and antennas in communication circuits. In the former it designates the frequency at which the inductance of the coil and its distributed capacity produce resonance. Referred to the antenna, it means the lowest frequency at which there will be a standing wave on the antenna.

FREQUENCY OF A PERIODIC QUANTITY. The frequency of a periodic quantity, in which time is the independent variable, is the number of periods occurring in unit time. If a periodic quantity, y , is a function of the time, t , such that

$$y = f(t) = A_0 + A_1 \sin(\omega t + a_1) + A_2 \sin(2\omega t + a_2) + \dots,$$

then the frequency is $\omega/2\pi$. Unless otherwise specified, the unit is the cycle per second.

FREQUENCY OF ENERGY RESONANCE. For a forced harmonic oscillator the frequency for which the velocity and the energy dissipation of the system are maxima is given by

$$f = \frac{1}{2\pi} \sqrt{\frac{k}{m}}$$

where k is the stiffness and m is the effective mass.

FREQUENCY OF OSCILLATIONS. See oscillation, frequency of.

FREQUENCY, PLASMA (LANGMUIR FREQUENCY). A characteristic frequency of a plasma defined by

$$\omega_p = \left(\frac{4\pi n e^2}{m} \right)^{1/2}$$

where n is the electron density, m , the electron mass, and e , the electronic charge.

FREQUENCY RESPONSE. In general, the relationship between output and input in a system, or, by extension, between any two system quantities, on the assumption that both quantities are sinusoids (or cisoids) of the same frequency. If, due to the system interrelations, the relation between two quantities $q_1(t)$ and $q_2(t)$ is expressible by

$$a \left(\frac{d}{dt} \right) \cdot q_1(t) = b \left(\frac{d}{dt} \right) \cdot q_2(t)$$

(where, in a real system, a and b are real functions of $\frac{d}{dt}$) and if it is supposed that $q_1(t) = \bar{q}_1 e^{i\omega t}$, $q_2(t) = \bar{q}_2 e^{i\omega t}$ where \bar{q}_1 , \bar{q}_2 may be complex, then

$$a(i\omega)\bar{q}_1 = b(i\omega)\bar{q}_2$$

or

$$\frac{\bar{q}_2}{\bar{q}_1} = \frac{a(i\omega)}{b(i\omega)} \equiv G(i\omega), \text{ say.}$$

The frequency response is therefore obtained from the differential equation connecting output and input by expressing the ratio $q_2(t)/q_1(t)$ in operational form, i.e., as a function of the operator $\frac{d}{dt}$, and substituting $i\omega$ for $\frac{d}{dt}$ as the argument of this function.

Alternatively, for the above differential relationship, taking Laplace transforms subject to the condition $q_1(t) = 0 = q_2(t)$ for $t < 0$

$$a(s)Q_1(s) = b(s)Q_2(s)$$

and

$$\frac{Q_2(s)}{Q_1(s)} = \frac{a(s)}{b(s)} = G(s)$$

the transfer function of Q_2 to Q_1 . Hence the frequency response is obtained by substituting $i\omega$ for s in the transfer function relating the two quantities.

Indeed, using the transfer function relationship of the last equation, and, as before, writing $q_1(t) = \bar{q}_1 e^{i\omega t}$ so that $Q_1(s) = \frac{\bar{q}_1}{s - i\omega}$,

$$\begin{aligned} \frac{Q_2(s)}{\bar{q}_1} &= \frac{G(s)}{s - i\omega} \\ &= \frac{G(i\omega)}{s - i\omega} + \text{other partial fractions with} \\ &\quad \text{denominators corresponding} \\ &\quad \text{to the poles of } G(s) \end{aligned}$$

on the supposition that $G(s)$ is bounded as $s \rightarrow \infty$ and does not have poles at $s = \pm i\omega$. Thus, taking inverse Laplace transforms, $\frac{q_2(t)}{\bar{q}_1} = G(i\omega)e^{i\omega t} + \text{terms involving normal modes of the system.}$ Thus the only component of $q_2(t)$ at frequency ω is

$$G(i\omega)\bar{q}_1 e^{i\omega t} = G(i\omega)q_1(t)$$

showing again that the frequency response, as defined, is the value of the transfer function $G(s)$ with s replaced by $i\omega$. This is still true if $G(s)$ is unstable, i.e., if some of the normal modes tend to infinity with t , though the value of this information for unstable systems is questionable. If $G(s)$ is stable, however, and the normal modes therefore $\rightarrow 0$ as $t \rightarrow \infty$, $G(i\omega)$ gives the "steady-state" relationship between the cisoids representing q_2 and q_1 .

Finally, it should be noted that if $\bar{q}_2 = |\bar{q}_2|e^{i\phi_2}$ and $\bar{q}_1 = |\bar{q}_1|e^{i\phi_1}$ then

$$G(i\omega) = \left| \frac{\bar{q}_2}{\bar{q}_1} \right| e^{i(\phi_2 - \phi_1)}$$

so that the modulus and phase of the frequency response give respectively the amplitude ratio and the relative phase of the output and input cisoids (or sinusoids).

If $q_1(t)$ is expressed by its **Fourier transform**, if this exists,

$$q_1(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Q_1(i\omega) e^{i\omega t} d\omega$$

each elemental component of the right-hand side is a cisoid of frequency ω and amplitude $Q_1(i\omega)d\omega/2\pi$. Such an element is therefore multiplied by $G(i\omega)$ in forming its contribution to $q_2(t)$. Thus

$$q_2(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} Q_1(i\omega)G(i\omega)e^{i\omega t}d\omega$$

or, more concisely,

$$Q_2(i\omega) = G(i\omega)Q_1(i\omega).$$

FREQUENCY RESPONSE REPRESENTATION. (a) *Nyquist Locus.* If, as ω varies, the frequency response $G(i\omega)$ is plotted as a complex quantity in its own complex plane, the locus traced out by the point representing G is called the *Nyquist locus* of G (or, similarly, of any other function of frequency). The range of ω for a complete Nyquist locus is from $-\infty$ to $+\infty$. In this connection, if G is a real function (as it always is for a real system) $G(i\omega)$ and $G(-i\omega)$ are complex conjugates; hence the part of the Nyquist locus corresponding to the negative range of ω is, point for point, the mirror image in the real axis of the part-locus for the positive range of ω .

The complete Nyquist locus should be provided also with a scale of ω along the locus.

Similar loci drawn for the function $G(s)$ with $s = \alpha + i\omega$ (α constant) are sometimes used to determine the degree of damping associated with the normal modes of the system and may be referred to as modified Nyquist loci. Whereas the Nyquist locus is the conformal map of the imaginary axis of the s -plane ($s = \alpha + i\omega$), such modified loci are the corresponding maps of vertical lines in the s -plane. Since constant ω loci (with variable α) are horizontal lines in the s -plane, it follows that the constant $-\omega$ loci will cut the modified Nyquist loci orthogonally except at points corresponding to **singularities** of $G(s)$ in the s -plane.

In control system analysis, the frequency function plotted is usually that relating the output $\theta_o(t) = \bar{\theta}_o e^{i\omega t}$ to the error $\theta_e(t) = \bar{\theta}_e e^{i\omega t}$. Denoting therefore

$$\frac{\theta_o(t)}{\theta_e(t)} = \frac{\bar{\theta}_o}{\bar{\theta}_e} \equiv G(i\omega)$$

and remembering that $\theta_i(t) = \theta_e(t) + \theta_o(t)$ and therefore $\bar{\theta}_i = \bar{\theta}_e + \bar{\theta}_o$, it is clear that, in the diagram for the plane of G shown in Figure 1,

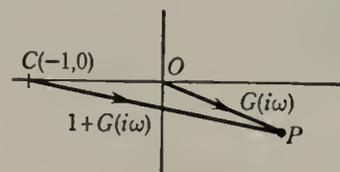


Fig. 1.

if P is any point on the Nyquist locus for a particular frequency so that OP is the phasor $G(i\omega)$, then CP drawn from the point C is the phasor $1 + G(i\omega)$. In other words since OP represents $\bar{\theta}_o/\bar{\theta}_e$, CP represents $1 + \bar{\theta}_o/\bar{\theta}_e = \bar{\theta}_i/\bar{\theta}_e$, CO being the phasor $+1$. Multiplying these phasors CO , OP , CP by $\bar{\theta}_e$, which does not alter the shape of the triangle, we obtain the important property that the directed sides of the triangle CO , OP , CP , represent the relative magnitudes and relative phases of the cisoids (or sinusoids) representing error, output and input respectively.

(b) *Log |G| - Log ω and Phase G - Log ω Graphs (Bode Diagrams)*. Here instead of plotting $G(i\omega)$ in a complex Cartesian plane, using ω as a parameter, $\log |G(i\omega)|$ and $\text{ph } G(i\omega)$ are plotted separately on a base of $\log \omega$, the base of the logarithms being usually 10, $\log |G|$ being normally expressed in decibels. It is to be noted that $\log_e |G|$ and $\text{ph } G$ are respectively the real and imaginary parts of $\log_e G$.

If the function $f(\omega) = \log(i\omega \pm A)$, with A real, is considered, then $f(\omega) = \frac{1}{2} \log(\omega^2 + A^2)$. The following properties of the graph of $f(\omega)$ to a base of $\log \omega$ are immediately deducible:

(i) It is asymptotic to the constant value $\log |A|$ as $\log \omega \rightarrow -\infty$.

(ii) It is asymptotic to the straight line through the origin, $f(\omega) = \log \omega$ as $\log \omega \rightarrow \infty$; the slope of this line is unity but is more often expressed as 6 (more correctly $20 \log_{10} 2$) decibels per octave (i.e., two-fold increase in frequency) or 20 decibels per decade (ten-fold increase in frequency).

(iii) The asymptotes meet at $\omega = |A|$ called the *break- or knee-frequency*, and the height of the graph above the two-line profile formed by the asymptotes is, at this value of ω , $\frac{1}{2} \log 2$ (3.010 db) which is its maximum value; the slope of the graph at this point is 3 db per octave.

(iv) The shape of the graph is not affected by A , which only modifies the position of the break point.

These properties are shown in Figure 2.

Thus if the transfer function $G(s)$ were expressible as $(s + A_1)(s + A_2) \cdots / (s + B_1)(s + B_2) \cdots$ in which all A 's and B 's were real, so that $\log |G(i\omega)| = \log |K| + \sum_r \log |s + A_r| - \sum_r \log |s + B_r|$, the resultant graph of $\log |G(i\omega)|$ could be obtained by adding or sub-

tracting the identically shaped but differently positioned graphs corresponding to individual terms in each of the summations, finally dropping the origin by $\log |K|$. As a preliminary

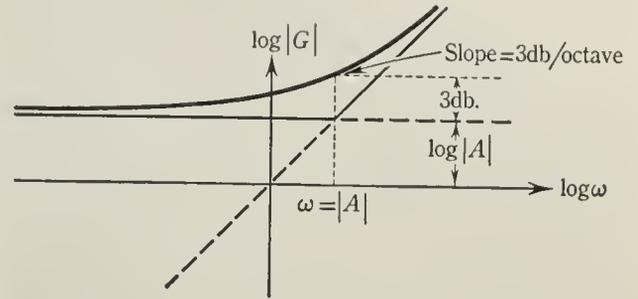


Fig. 2.

approximation at any rate, the two-asymptote approximation to the individual graphs could be used. However, $G(s)$ more generally has complex zeros and poles as well as real ones, occurring in conjugate complex pairs.

If the function $g(\omega) = \log |(i\omega)^2 + 2kA i\omega + A^2|$ is therefore considered, with A real as before and with $|k| < 1$ (if $|k| \geq 1$ the quadratic form can be factorized into linear factors of the form already considered), it is found that the graph of $g(\omega)$ on a base of $\log \omega$ has the following essential properties:

(i) It is asymptotic to the constant value $2 \log |A|$ as $\log \omega \rightarrow -\infty$.

(ii) It is asymptotic to the straight line through the origin $g(\omega) = 2 \log \omega$ (slope 12 db per octave) as $\log \omega \rightarrow \infty$.

(iii) The asymptotes cut at $\omega = |A|$ and at this frequency $g(\omega) = \log K(2A^2|k|) = 2 \log |A| + \log 2|k|$, the height of the graph at this frequency being therefore $\log 2|k|$ (which may be positive or negative) above the asymptote profile, but this is not in general the maximum deviation between the two; the slope of the graph at this point is always 6 db/octave.

Thus the graph of $g(\omega)$ although it has the asymptotes which might be expected from the analysis of $f(\omega)$ may diverge considerably from the asymptote profile (say for values of $|k| < 0.3$). Provided due attention is paid to this point, if $G(s)$ can be expressed in the form considered previously but with some A 's and B 's now occurring in conjugate complex pairs, the graph of $\log |G|$ may still be obtained by superposing the log modulus graphs of the component factors (linear or quadratic) as previously. The use of the asymptote outlines is, however, dangerous as an approximation except where $\log \omega \rightarrow \pm\infty$.

With the same precautions, the $\text{ph } G$ graphs

may also be obtained by a superposition process, for if $G(s)$ as before is in the form $K(s + A_1)(s + A_2) \cdots / (s + B_1)(s + B_2) \cdots$, then

$$\text{ph } G(i\omega) = \sum_r \text{ph } (i\omega + A_r) - \sum_r \text{ph } (i\omega + B_r).$$

Moreover $\text{ph } (i\omega + A)$, with A real, increases from 0 to $\frac{\pi}{2}$ when $A > 0$, having the value $\frac{\pi}{4}$ at the break-frequency $\omega = |A|$ and similarly decreases from π to $\frac{\pi}{2}$ with a break-frequency value of $\frac{3\pi}{4}$ when $A < 0$; again the *shape* of the graph is not affected by A , though in this case changing the sign of A turns the graph upside down. The phase of a quadratic factor, $\text{ph } \{(i\omega)^2 + 2i\omega kA + A^2\} = \text{ph } (A^2 - \omega^2 + 2i\omega kA)$ is always zero at $\omega = 0$ and is $\pm\pi$ as $\omega \rightarrow \infty$ according as $kA \gtrless 0$; it has the value $\pm \frac{\pi}{2}$ at break-frequency $\omega = |A|$ according as $kA \gtrless 0$ but the shape of its graph depends on $|k|$.

However, the phase graph is not very widely used since the value of $\text{ph } G(i\omega)$ in the case of **minimum-phase G functions**, can be derived from $\log |G(i\omega)|$ from a relationship established by Bode for such functions:

$$\{\text{ph } G(i\omega)\}_{\omega=\Omega} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{d \log |G|}{du} \cdot \log_e \coth \frac{|u|}{2} \cdot du$$

where $u = \log_e \left(\frac{\omega}{\Omega}\right)$. In the integrand, the first factor is the slope of the $\log |G| - \log \omega$ graph at any frequency ω , the second factor constituting a weighting factor associated with these slopes which has a peak value at $\omega = \Omega$ and decays rapidly and symmetrically on either side (when plotted on a $\log \omega$ base). Thus the phase at $\omega = \Omega$ is largely determined by the slope of the $\log |G| - \log \omega$ graph in the neighborhood of $\omega = \Omega$, though the contributions from the slopes at more remote values, particularly if these are high compared with that at $\omega = \Omega$, can seldom be neglected. With some practice, however, it is not difficult to sketch the phase graph reasonably accurately from the graph of $\log |G|$, the phase being roughly $\frac{\pi}{2}$ per unit slope of the $\log |G|$ graph at any point.

FREQUENCY RESPONSE TO INDICIAL RESPONSE CONVERSION. If the frequency response relating output to input for a system is $G(i\omega)$, so that $\Theta_o(i\omega) = G(i\omega)$ $\Theta_i(i\omega)$, the corresponding normal response in the time domain to an input $\delta(t)$, unit impulse function at $t = 0$, may be found from the fact that the **Fourier transform** of $\delta(t)$ is unity. Hence with this input

Fourier transform of $\theta_o(t) = \Theta_o(i\omega) = G(i\omega)$ and hence

$$\theta_o(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} e^{i\omega t} G(i\omega) d\omega.$$

If $\theta_o(t)$ is required and if the evaluation of the integral is a complicated process or if $G(i\omega)$ is obtained experimentally so that G is not known in functional form, the evaluation of $\theta_o(t)$ may be achieved, generally to a high degree of accuracy, by Floyd's method.

The expression for $\theta_o(t)$ is first simplified by writing

$$G(i\omega) \equiv R(\omega) + iI(\omega)$$

in which, since G is a real function, $R(\omega)$ and $I(\omega)$, its real and imaginary parts, are respectively even and odd functions of ω . Hence

$$\begin{aligned} \theta_o(t) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} (\cos \omega t + i \sin \omega t)(R + iI) d\omega \\ &= \frac{1}{2\pi} \int_{-\infty}^{\infty} (R \cos \omega t - I \sin \omega t) d\omega \\ &= \frac{1}{\pi} \int_0^{\infty} (R \cos \omega t - I \sin \omega t) d\omega. \end{aligned}$$

Further, it is known that if $G(i\omega)$ is the frequency response function of a physical system, $\theta_o(t)$ must vanish for $t < 0$. It follows that

$$\int_0^{\infty} R \cos \omega t d\omega = \int_0^{\infty} I \sin \omega t d\omega, \quad t < 0$$

and hence, since the left-hand side is even in t but the right-hand side is odd,

$$\int_0^{\infty} R \cos \omega t d\omega = - \int_0^{\infty} I \sin \omega t d\omega, \quad t > 0.$$

Hence

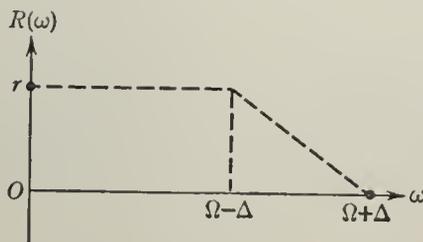
$$\theta_o(t) = \frac{2}{\pi} \int_0^{\infty} R \cos \omega t d\omega, \quad t > 0.$$

The graph of $R(\omega)$, which is at once obtainable from a knowledge, graphical or functional,

of $G(i\omega)$, is next approximated to by a polygonal outline. In this connection, since for most systems $G(i\omega) \rightarrow 0$ as $\omega \rightarrow \infty$, the upper bound of frequency for the polygonal approximation is fixed at such a value, say ω_m that the range of integration ω_m to ∞ is judged to contribute a negligible amount to the above expression for $\theta_0(t)$, i.e., ω_m is such that

$$\theta_0(t) \doteq \frac{2}{\pi} \int_0^{\omega_m} R \cos \omega t d\omega, \quad t > 0.$$

Floyd next analyzes the contribution made to the integral on the assumption that the graph of $R(\omega)$ is of the trapezoidal form shown in the diagram.



For this value of R

$$\int_0^{\infty} R \cos \omega t d\omega = r \int_0^{\Omega-\Delta} \cos \omega t d\omega + r \int_{\Omega-\Delta}^{\Omega+\Delta} \frac{\Omega + \Delta - \omega}{2\Delta} \cos \omega t d\omega$$

which on evaluation simplifies to

$$r\Omega \cdot \frac{\sin \Omega t}{\Omega t} \cdot \frac{\sin \Delta t}{\Delta t}$$

and can therefore easily be evaluated for any value of t and given Ω and Δ by using tables of $(\sin x)/x$.

Finally therefore, if the polygonal approximation to the graph of $R(\omega)$ is considered as made up of a number of elements of the above form,

$$\theta_0(t) = \frac{2}{\pi} \sum_k r_k \Omega_k \frac{\sin \Omega_k t}{\Omega_k t} \cdot \frac{\sin \Delta_k t}{\Delta_k t}$$

the summation extending over all the elemental trapezoids.

If the value of the output is required when the input is unit step-function at $t = 0$ (the **indicial response**), then the above expression for the output must be integrated with respect to t over the range 0 to t , either graphically or analytically.

FREQUENCY-SHIFT THEOREM. See Laplace transform theorems.

FRESNEL. A unit of frequency equal to 10^{12} cycles per second.

FRESNEL DIFFRACTION. The diffraction pattern in a plane near the **aperture**.

FRESNEL EQUATIONS. Equations giving the intensity of each of the two polarization components of light in a less dense medium reflected at the surface of a denser transparent medium.

If the light is polarized in the plane of incidence, i.e., the magnetic vector is in the plane of incidence:

$$\frac{I}{I_o} = \frac{\sin^2 (i - r)}{\sin^2 (i + r)}$$

If the light is polarized in a plane perpendicular to the plane of incidence:

$$\frac{I}{I_o} = \frac{\tan^2 (i - r)}{\tan^2 (i + r)}$$

These may be combined for normal incidence into the more familiar equation

$$\frac{I}{I_o} = \left(\frac{n - 1}{n + 1} \right)^2$$

for air as one medium.

FRESNEL EQUATIONS FOR METALLIC REFLECTION. For strongly absorbing materials like metals, the index of refraction n of the simple Fresnel equation should be replaced by its complex form

$$\bar{n} = n(1 - ik),$$

where k is the **absorption index**. Introducing this gives for the reflection coefficient of light normally incident *in vacuo* onto a metallic medium

$$R = \frac{(n - 1)^2 + n^2 k^2}{(n + 1)^2 + n^2 k^2}$$

Note that when k is very large, the reflection coefficient is nearly unity.

FRESNEL INTEGRALS. See Cornu spiral; noise level reduction of barriers.

FRESNEL INTEGRALS, TABLE OF. See noise level reduction of barriers.

FRESNEL ZONE. See half-period element.

FRICTION. The resistance offered to the motion of one body upon or through another.

FRICTION, ANGLE OF. The limiting angle between the direction of the force resisting sliding and the normal to the surface of sliding is called the angle of friction ϕ ; $\tan \phi = \mu$, the coefficient of friction. (See **friction, coefficient of**.)

FRICTION, COEFFICIENT OF. The maximum ratio of the frictional or tangential force F to the normal force N between two bodies at rest with respect to each other is called the coefficient of static friction, μ . $F \leq \mu N$. When there is sliding motion between the bodies, μ is called the coefficient of kinetic friction and $F = \mu N$. The coefficient of kinetic friction is generally less than the coefficient of static friction, and tends to decrease with the speed of sliding. The static coefficient, as stated by Coulomb, is essentially independent of the area of contact for ordinary surfaces. For steel on steel, a value of 0.15 is reasonable, reducing to 0.03 at appreciable relative motion.

FRICTION, CONE OF. A conical surface which always contains the resultant of the force of friction between two surfaces and the normal force pressing the two surfaces together. The half angle at the apex of the cone is equal to the angle of friction.

FRICTION, INTERNAL. A generic term for the processes in solids which result in the conversion of elastic energy into heat or other forms of energy which are neither kinetic nor elastic. The most general method of measuring internal friction is in terms of the specific damping capacity or specific loss. This is defined as the ratio $\Delta W/W$ where ΔW is the energy loss when a specimen is taken round a stress cycle, and W is the maximum elastic energy stored in it during the cycle. Internal friction can be measured in other ways, such as the logarithmic decrement Δ' of free vibrations, where the specimen is the elastic element in the vibrating system (see **logarithmic decrement**); the breadth at half amplitude (ΔN) of the resonance peak (see **resonance, sharpness of**) of such a system; or the loss angle δ for forced vibrations (see **loss factor, mechanical**). It can also be measured by the amplitude attenuation coefficient (see **acoustic attenuation coefficient**), of elastic waves in

the solid. For low dissipation coefficients ($\Delta W/W < 0.1$), the following approximate relations can be used between the different quantities listed above. For harmonic motion at one frequency,

$$\frac{\Delta W}{W} = 2\Delta' = \frac{2\pi}{\sqrt{3}} \frac{\Delta N}{N} = 2\pi \tan \delta = \frac{4\pi\alpha c}{\omega}$$

α is here the amplitude attenuation coefficient for waves of angular frequency ω whose phase velocity of propagation is c .

FRICTION VELOCITY. A velocity v_* derived from the shearing stress τ_0 at a boundary and the fluid density ρ .

$$v_* = (\tau_0/\rho)^{1/2}$$

In the turbulent flow of an incompressible fluid of uniform density parallel to a plane boundary, in which the shearing stress is the same at all distances from the boundary and is equal to the Reynolds stress,

$$\tau_0 = \overline{u'w'}$$

and v_* is thus related to the velocity fluctuations u' and w' along and parallel to the boundary; and if the viscous shearing stresses are negligible compared with the **Reynolds stresses** in the turbulent region, v_* gives a measure of the "velocity of slip" in the laminar boundary layer. (See also **logarithmic profile** and **skin friction**.)

FRINGE. A local extreme of an interference or diffraction pattern.

FRINGES, VISIBILITY OF. See **visibility of fringes**.

FROBENIUS METHOD. See **indicial equation**.

FRÖHLICH-BARDEEN THEORY (1955). An attempt to explain superconductivity in terms of the interaction between the conduction electrons and the lattice vibrations of the metal, as suggested by the isotope effect. (See **Bardeen-Cooper-Schrieffer theory of superconductivity**.)

FRONTAL HOROPTER. See **horopter**.

FRONTOGENETICAL FUNCTION. A kinematic measure of the tendency of the flow in an air mass to increase the horizontal gradient

of a conservative property α , defined by the equation

$$F = \frac{d}{dt} |\nabla \alpha|.$$

FROUDE EFFICIENCY. See **Froude momentum theory**.

FROUDE MOMENTUM THEORY. In this theory the steady-flow momentum equation (see **momentum equation for boundary layer**) is used to obtain an expression for the thrust of a screw propeller. The theory is also applicable to any other propulsive device, such as a turbo-jet or ram-jet engine, which takes in fluid from the stream and discharges it again with increased total pressure.

In the simplest form of the theory it is assumed that the velocity is uniform across the slipstream or jet. At a station far downstream, where the static pressure is the same as in the undisturbed stream, let the velocity in the slipstream or jet relative to the propulsive device be $V + v$, where V is the free-stream velocity. Then, if m is the mass of fluid passing through the propulsive device per unit time, the propulsive thrust is

$$T = mv. \quad (1)$$

The power required ideally, to produce the slipstream or jet, is equal to the rate of increase of kinetic energy of the stream. This is

$$E = \frac{1}{2}m[(V + v)^2 - V^2] = TV + \frac{1}{2}mv^2.$$

The useful propulsive power is TV , and the *Froude efficiency* is defined as

$$\frac{TV}{E} = \frac{1}{1 + \frac{1}{2} \frac{v}{V}}.$$

FROUDE NUMBER. A non-dimensional number, F , representing the ratio of the inertia forces to mechanical forces. In particular the name is used in connection with gravity forces so that

$$F^2 = u^2/gh$$

where u is a representative velocity and h a representative height. For horizontal flow in a stream of uniform density, of depth h , flow of speed u is **super-undal** if $F > 1$.

In a stably stratified fluid of static stability β

$$F^2 = u^2/g\beta h^2$$

where h is a suitably chosen representative length, which might be the wavelength of standing waves.

The Froude number also represents the ratio of the kinetic energy to the potential energy of the mechanical forces or to the energy dissipated by them. The **Reynolds number** is a special case of the Froude number, the mechanical forces being due to viscosity in that case. The **Richardson number** is a case in which the kinetic energy available is represented by the shear.

FROZEN EQUILIBRIUM. Owing to the slowness of attainment of some chemical equilibria, it can happen that the reaction rates are negligible during a time sufficient for other kinds of equilibria (i.e., thermal equilibria) to be reached. The system is then in a metastable equilibrium sometimes called frozen equilibrium.

F-TEST. A statistical test of significance based on the ratio of two independent quadratic estimators of variance. The name derives from the initial letter of the surname of Sir Ronald Fisher, who introduced it. (See **Fisher's distribution**.)

FUCHS THEOREM. If a second-order linear differential equation has regular singular points at ∞ and at x_k , $k = 1, 2, \dots, n$, and no other singularities, its general form is

$$y'' + \frac{p_{(n-1)}(x)y'}{F(x)} + \frac{p_{2(n-1)}(x)y}{F^2(x)} = 0$$

where $F(x) = (x - x_1)(x - x_2) \cdots (x - x_n)$ and $p_i(x)$ is a polynomial in x of degree $\leq i$. Such an equation is said to be of Fuchsian type. The theorem may be extended to equations of any order.

FUEL. See **combustion**.

FUGACITY. In a mixture of perfect gases (see **perfect gas mixtures**; **partial pressure**) the **chemical potential** may be written in the form

$$\mu_i = \mu_i^+(T) + RT \ln p_i \quad (1)$$

where p_i is the partial pressure of component i . By analogy one may write for a mixture of real gases

$$\mu_i = \mu_i^+(T) + RT \ln p_i^z \quad (2)$$

where $\mu_i^+(T)$ is the same function as for the perfect gas, while all the effects of molecular

interactions are included in the p_i^x . The function $p_i^x(T, p, n_1 \cdots n_c)$ is called the fugacity of component i . This definition, due to G. N. Lewis, permits the preservation for **real gases** of the general form of the equations for perfect gases, with the fugacities replacing partial pressures.

In the low pressure limit, p_i^x reduces to p_i

$$\lim_{p \rightarrow 0} \frac{p_i^x}{p_i} = 1. \quad (3)$$

FULL LINEAR GROUP. The collection of all non-singular **matrices** of order n , with matrix multiplication as the law of combination. Its order is infinite since its elements are the infinite number of linear **transformations** of one n -dimensional **vector** into another.

It contains several subgroups, also of infinite order: (1) The *unitary group*, with elements all n -dimensional unitary matrices. (2) The *orthogonal group*, with elements all real n -dimensional square orthogonal matrices. The determinants of these matrices can only be ± 1 ; if $+1$, they are proper orthogonal matrices, if -1 , improper orthogonal matrices. The subgroup of the orthogonal group containing only proper orthogonal matrices is the *rotation group* or order n . If $n = 3$, the proper orthogonal matrices correspond to rotations in three-dimensional **Euclidean space**, while the improper matrices correspond to such rotations followed by reflections in a plane perpendicular to the axis of rotation. These conceptions may be generalized for n -dimensions and specialized for two dimensions. (See **Lie group**.)

FULL RADIATOR (BLACK BODY). The thermal radiator which absorbs completely all incident radiation, whatever the wavelength or direction of incidence. This radiator has the maximum spectral concentration of radiant **emittance** at a given temperature.

FULLY-ROUGH FLOW. Turbulent flow for which

$$\frac{v_* \epsilon}{\nu} \geq 100,$$

where v_* is the **friction velocity**, ν is the **kinematic viscosity** and ϵ is the average height of the surface irregularities. When a flow is fully rough, it becomes almost independent of the viscosity.

FUNCTION. A mathematical expression describing the relation between **variables**; the function taking on a definite value, or values, when special values are assigned to certain other quantities, called the arguments, or independent variables of the function. If there is one independent variable, the dependent variable y may be determined explicitly by the equation $y = f(x)$ or implicitly by $f(x, y) = 0$. If there are several independent variables the forms are $y = f(x_1, x_2, \cdots, x_n)$ or $f(x_1, x_2, \cdots, x_n, y) = 0$. The precise definition of a function, namely as a set of ordered pairs, the first element of each pair being an argument of the function and the second its corresponding value, was first introduced by Dirichlet. A set-theoretical definition is simply a many-one relation, that is, a relation which to any **element** in its **domain** relates exactly one element in its **range**. (For various functions, **elliptic**, **algebraic**, etc., see the relevant terms.)

FUNCTIONAL. A functional is a real-valued function $F(f)$ defined for some set X of functions (usually X is a **Banach space**). A functional is linear if $F(f_1 + f_2) = F(f_1) + F(f_2)$ and bounded if $|F(f)| \leq M \|f\|$ for some constant M . If $f(x)$ is continuous and $g(x)$ is of bounded variation, the **Stieltjes integral** $\int_0^1 f(x) dg(x)$ is a continuous linear functional of f on C . A theorem of Riesz says that every continuous linear functional can be so represented by suitable choice of $g(x)$.

FUNCTIONAL ITERATION. See **iterative methods**.

FUNCTION, EVEN. See **even function**.

FUNCTION, ODD. See **odd function**.

FUNDAMENTAL AFFINE CONNECTION. The aggregate of sets of **Christoffel three-index symbols** of the second kind defined in every coordinate system. The Christoffel symbols Γ_{ij}^k of the second kind in a specified coordinate system x are *the components of the fundamental affine connection* in the system x .

FUNDAMENTAL BANDS. The bands in the **infrared spectrum of a molecule** corresponding to transitions from the vibrationless ground state to a state in which one **normal vibration** is singly excited.

FUNDAMENTAL CIRCUITS. See **circuits, fundamental.**

FUNDAMENTAL COMPONENT. The fundamental frequency component in the **harmonic analysis** of a wave.

FUNDAMENTAL CONTRAVARIANT TENSOR. Contravariant metric tensor. (See **metric tensor.**)

FUNDAMENTAL COVARIANT TENSOR. Covariant metric tensor. (See **metric tensor.**)

FUNDAMENTAL CUT SETS. See **cut sets, fundamental.**

FUNDAMENTAL EQUATIONS OF HYDRODYNAMICS. The **equations of motion**, the **equation of continuity**, the **energy equation**, the **equation of state**, and the **equation of continuity** for water, considered together as a **closed system** of equations.

A simplified physical model can dispense with certain of these equations without sacrificing completeness, e.g., in two-dimensional homogeneous incompressible flow, kinetic energy is the only form of energy, and the equations of motion and continuity form a closed system.

FUNDAMENTAL EQUATIONS OF THERMODYNAMICS. When a certain number n of properties of a thermodynamic system (in particular a homogeneous system) are fixed, the state of the system is fixed, and all remaining properties are fixed. It does not follow, however, that it is possible to calculate all equilibrium thermodynamic properties of a system, when one equation of state is given. In general, this is not the case, but it is possible to indicate several general relations, called fundamental equations (or fundamental equations of state) which have this property. An **equation of state** is called fundamental if all equilibrium properties of the system can be derived from it mathematically by processes which do not involve integration. All thermodynamic consistency relations involve partial derivatives. In the case of simple systems, these are of the form

$$\left(\frac{\partial z}{\partial y}\right)_x = \phi(x,y)$$

where z is the dependent, and x,y are the independent properties. In this case

$$z = \int \phi(x,y)dy + \psi(x).$$

Here, the integral can be evaluated, but the arbitrary (from the mathematical point of view) function $\psi(x)$ cannot be determined from the equation of state.

In the case of simple systems, the only four forms which allow us to calculate all equilibrium properties by processes involving differentiation and algebraic calculations exclusively, are:

$$F_1(U,S,V) = 0$$

$$F_2(H,S,p) = 0$$

$$F_3(A,T,V) = 0$$

$$F_4(G,T,p) = 0.$$

Here U is the **internal energy**, $H = U + pV$, is the **enthalpy**, $A = U - TS$ is the **Helmholtz function**, $G = H - TS$ is the **Gibbs function**.

It is noteworthy that the p - V - T relation, $F(p,V,T) = 0$, is not a fundamental equation. (See **equation of state.**)

In the case of complex systems, involving several independent components and phases, the above relations must also contain the masses (or numbers of moles) of the constituents

$$F_1(U,S,V,n_i) = 0, \text{ etc.}$$

In addition, the equation

$$F_5(p,T,\mu_i) = 0$$

where μ_i are the chemical potentials, is also a fundamental equation. (See also **thermodynamics, characteristic functions in.**)

FUNDAMENTAL FORM FOR A SURFACE. See **ground form of a surface.**

FUNDAMENTAL FREQUENCY. See **frequency, fundamental; vibration, fundamental mode of.**

FUNDAMENTAL HORN EQUATION. See **horn equation, fundamental.**

FUNDAMENTAL MAGNITUDES OF THE FIRST ORDER (FOR A SURFACE). See **ground forms of a surface.**

FUNDAMENTAL MAGNITUDES OF THE SECOND ORDER (FOR A SURFACE). See **ground forms of a surface.**

FUNDAMENTAL METRIC TENSOR. See **metric tensor**.

FUNDAMENTAL MODE. See **mode, fundamental**.

FUNDAMENTAL PROBABILITY SET. The basic events and their probabilities forming the units of a probabilistic situation, any other set of events under consideration being expressible in terms of them. It follows that all the probabilities of derived sets are expressible in terms of those of the fundamental set.

FUNDAMENTAL SERIES. See **Bergmann series**.

FUNDAMENTAL STATE. The fundamental state of a system (atom, molecule, or ensemble of particles) is that state of the system which has the lowest possible energy (i.e., the **zero point energy**). The term may be used in a more restricted meaning: a molecule in its fundamental *electronic* state may still pos-

sess vibrational, rotational, and, eventually, translational energy.

FUNDAMENTAL TENSOR. See **metric tensor**.

FUNDAMENTAL THEOREM OF ARITHMETIC. See **unique factorization theorem**.

FUNDAMENTAL THEOREM, REACTOR. See **first fundamental theorem** and **second fundamental theorem**.

FUNICULAR POLYGON. A funicular or string polygon is a geometric figure employed to obtain a point on the line of action of the resultant of a coplanar force system. The **force polygon** gives the magnitude and direction of the resultant force. Funicular polygons are graphical representations of shapes a flexible string would assume when loaded by the actual forces. For equilibrium, both the force and funicular polygons must close.

FUSION. See **solidification**.

f-VALUE. See **oscillator strength**.

G

GAIN-BANDWIDTH PRODUCT. The gain-bandwidth product is equal to the product of the amplification of an amplifier stage at mid-band (its maximum value) and the bandwidth of the amplifier in megacycles. The bandwidth is defined as the difference Δf between the two frequencies at which the power output is a specified fraction, usually one-half, of the midband value.

GAIN, BREEDING. See **breeding gain**.

GAIN, CONVERSION. See **conversion gain**.

GAIN MARGIN. A partial indication of the degree of stability of a feedback loop based upon the extent by which the loop **gain ratio** falls short of unity at the frequency (or frequencies) at which the loop phase shift is zero. Two definitions are in use:

(1) Gain Margin = $1 -$ (loop gain ratio at zero phase shift).

(2) Gain Margin (in decibels) = $-$ (loop gain in decibels at zero phase shift).

Both definitions given zero gain margin for a critically stable system.

Note. To avoid confusion it should be noted that if $Y(s)$ is the output/error transfer function in an error-monitored control system, the loop transfer function is $-Y(s)$. Thus *zero phase* of the loop frequency-function $-Y(i\omega)$ implies 180° *phase* (lag) of $Y(i\omega)$.

GAIN RATIO (GAIN). Measures of amplification used in frequency response work and preferably restricted to the comparison of the amplitudes of one and the same physical quantity (voltage, current, etc.) at two different points of a system.

Gain ratio from point A to point B

$$= \frac{\text{amplitude of specified quantity at } B}{\text{amplitude of specified quantity at } A}$$

Gain from point A to point B

$$\text{in decibels} = 20 \log_{10} (\text{gain ratio})$$

$$\text{in nepers} = \log_e (\text{gain ratio})$$

$$\text{Thus } 1 \text{ neper} = 20 \log_{10} e \text{ or } 8.686 \text{ decibels (dbs)}$$

The bel (and decibel) are strictly units of power amplification, the power amplification in bels being \log_{10} (power gain ratio). When comparing, say, voltages or currents, these are assumed to be applied across, or flowing through, equal hypothetical resistances at A and at B so that the power gain ratio is the square of the voltage or current gain ratio, giving:

Gain (i.e., power gain) in dbs

$$= 10 \log_{10} (\text{power gain ratio})$$

$$= 20 \log_{10} (\text{voltage or current gain ratio})$$

as above.

The use of the above terms and units when comparing, say, a voltage at B with a current at A or *vice-versa* (so that the "gain ratio" is not dimensionless) is to be deprecated, as is also the measurement of gain in dbs when applied to non-electrical quantities other than power.

GALACTIC CLUSTERS. See **star clusters**.

GALERKIN METHOD. See **Ritz method**.

GALERKIN STRESS FUNCTIONS. Any solution for the displacement components u_i ($i = 1, 2, 3$) in a rectangular Cartesian coordinate system x , of the **Navier equations** of equilibrium for a body of isotropic, elastic material satisfying the **generalized Hooke's law**, on which no body forces are acting, may, provided it has sufficient regularity, be written in the form

$$\mu u_i = (1 - \sigma) \nabla^2 F_i - \frac{\partial F_j}{\partial x_j \cdot \partial x_i},$$

where F_i ($i = 1, 2, 3$) are three biharmonic functions of position in space, the Galerkin stress functions. μ and σ are the **rigidity modulus** and **Poisson's ratio** for the material. Such a solution of the Navier equations is called a *Galerkin solution*.

GALILEAN TRANSFORMATION. The transformation to a system moving with constant relative velocity according to non-relativistic kinematics:

$$dx' = dx - v_x dt$$

$$dy' = dy - v_y dt$$

$$dz' = dz - v_z dt$$

$$dt' = dt.$$

GALVANOMAGNETIC AND THERMO-MAGNETIC EFFECTS. There are three other effects similar to the Hall effect (see **Hall coefficient**) which relate flow of electric current, flow of heat, and magnetic field. These are the *Nernst effect*, the *Ettingshausen effect*, and the *Righi-Leduc effect*. All these effects may be summarized mathematically as follows:

Let \mathbf{j} and \mathbf{q} be the vector densities of electric current and heat current, respectively, let \mathbf{H} be the magnetic field strength. Let \mathbf{E}_t and $\nabla_t T$ be the projections of the electric field vector and the temperature gradient, respectively, on the plane normal to \mathbf{j} or \mathbf{q} , whichever is non-vanishing. Then the **Hall coefficient** R_H may be defined by the relation

$$\mathbf{E}_t = R_H \mathbf{j} \times \mathbf{H},$$

the *Nernst coefficient* Q by the relation

$$\mathbf{E}_t = -Q \nabla T \times \mathbf{H}, \quad \text{when } \mathbf{j} = 0,$$

the *Ettingshausen coefficient* P by

$$\nabla_t T = P \mathbf{j} \times \mathbf{H}, \quad \text{under conditions } \mathbf{j} \cdot \nabla T = 0 \text{ and } \mathbf{q} = 0,$$

and the *Righi-Leduc coefficient* S by

$$\nabla_t T = S \mathbf{H} \times \nabla T, \quad \text{when } \mathbf{j} = 0.$$

It may be noted that the Righi-Leduc effect bears the same relation to the Ettingshausen effect as the Nernst effect does to the Hall effect.

GALVANOMETER EQUATION. The equation of motion of a **galvanometer**:

$$P \frac{d^2\theta}{dt^2} + \left(K + \frac{G^2}{R} \right) \frac{d\theta}{dt} + U\theta = \frac{GE}{R},$$

where P is the moment of inertia, K , the mechanical damping coefficient, U , the restoring torque constant, G , the motor constant, E , the applied voltage and R , the total circuit resistance.

GAMES THEORY. The branch of mathematics which deals with the theory of contests

between players under certain specified rules. If the game is such that the winnings of one player are derived solely from the losses of others, so that the algebraical sum of winnings is zero, the game is a *zero-sum game*.

If two players are engaged, and one has a choice of p courses of action, the other a choice of q courses, a $p \times q$ table can be drawn up showing how much one player wins if he chooses the i^{th} course and his adversary chooses the j^{th} . This is called a *pay-off matrix*. The object of the theory is to find the best strategy for a given player to pursue. If he acts so as to minimize his maximum risk of loss the strategy is called *minimax*, while if he acts so as to maximize his minimum chance of gain, the strategy is called *maximin*.

GAMMA. (1) The one-millionth part of a gram. (2) A unit of **magnetic field intensity**. (3) The tangent of the angle of the straight-line portion of the $D \log E$ curve and the $\log E$ axis is a measure of the degree of development and was termed by Hurter and Driffeld the *development factor* and designated by the Greek letter γ (gamma). Gamma may also be defined as

$$\gamma = \frac{D_2 - D_1}{\log E_2 - \log E_1}$$

where D_2 and D_1 are **densities** on the straight-line portion of the $D \log E$ curve produced by $\log E_2$ and $\log E_1$, respectively, and where E_2 and E_1 are exposures (see **exposure-density relationship**). (4) In a color or monochrome television channel, or part thereof, the coefficient expressing the selected evaluation of the slope of the used part of the log vs. log plot relating input (abscissa) and output (ordinate) **signal magnitudes** as measured from the point corresponding to some reference **black level**. As the log vs. log plot is usually not entirely straight in the used region, it is necessary to formalize that evaluation of the slope, for example, by the use of the value at a particular point, maximum, mean, or other value. The method of evaluation should be stated. At some points, the signal may be in terms of **light intensity** or **light transmission**.

GAMMA CORRECTION. The modification of a **transfer characteristic** for the purpose of changing the value of gamma. (See **gamma (4)**.)

GAMMA DISTRIBUTION. A frequency (probability) distribution of the form

$$dF = \frac{e^{-x} x^{\lambda-1}}{\Gamma(\lambda)} dx, \quad 0 \leq x \leq \infty.$$

It is also known as *Pearson's Type III*.

GAMMA FACTOR. See gamma value.

GAMMA FUNCTION. The improper integral, sometimes called Euler's second integral

$$\Gamma(z) = \int_0^{\infty} e^{-t} t^{z-1} dt.$$

It converges for all positive, real values of z . Its properties include:

$$\Gamma(z+1) = z\Gamma(z);$$

$$\Gamma(z)\Gamma(1-z) = \pi \csc \pi z;$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi};$$

when $z = n$, a positive integer, $\Gamma(n) = (n-1)!$, hence this is often called the **factorial function**.

The Weierstrass definition of the function is

$$1/\Gamma(z) = ze^{Cz} \prod_{n=1}^{\infty} (1+z/n)e^{-z/n}$$

where C is the **Euler-Mascheroni constant**;

$$C = \lim_{n \rightarrow \infty} \left(1 + \frac{1}{2} + \cdots + \frac{1}{n} - \ln n\right) \\ = 0.577215 \dots$$

Another definition is that of Euler

$$\Gamma(z) = \lim_{n \rightarrow \infty} \frac{(n-1)!}{z(z+1)(z+2)\cdots(z+n-1)} n^z.$$

(See also **beta function**, which is Euler's first integral, and **Stirling formula**.)

GAMMA MATRICES. See Dirac equation.

GAMMA PERMANENCE RULE. See gamma value.

GAMMA SPACE. Phase space of $2fN$ dimensions, the coordinates being f generalized coordinates and f generalized momenta for each of the N particles of the system, each particle having f degrees of freedom. It is the phase space of the whole gas and was called Γ -space by Ehrenfest to distinguish it from the phase space of one molecule (μ -space).

GAMMA SUM RULE. See gamma value.

GAMMA VALUE. The energy of electronic spin-orbit interaction in an atom, designated Γ (sometimes referred to as Γ -factor). Without specific assumptions as to the **coupling conditions**

$$\Gamma = \sum_i a_i l_i s_i \overline{\cos(\mathbf{l}_i \mathbf{s}_i)},$$

where

$$a_i = \frac{R\alpha^2 Z_{eff,i}^4}{n_i l_i (l_i + \frac{1}{2})(l_i + 1)}.$$

For **Russell-Saunders coupling**

$$\Gamma = ALS \cos(\mathbf{LS}) \\ = A \frac{J(J+1) - L(L+1) - S(S+1)}{2},$$

where

$$A = \sum_i a_i \frac{s_i}{S} \overline{\cos(\mathbf{s}_i \mathbf{S})} \frac{l_i}{L} \overline{\cos(\mathbf{l}_i \mathbf{L})}.$$

Here \mathbf{l}_i and \mathbf{s}_i are orbital angular momenta and spins of the individual electrons, \mathbf{L} and \mathbf{S} the resultant electronic orbital angular momentum and the resultant electron spin. l_i , s_i , L , and S are the corresponding quantum numbers. α is **Sommerfeld's fine structure constant**, R the **Rydberg constant**, $Z_{eff,i}$ the **effective atomic number**.

In a weak magnetic field of field strength H the vector \mathbf{J} has a component \mathbf{M} in the field direction, in a stronger field the vectors \mathbf{L} and \mathbf{S} have separate components \mathbf{M}_L and \mathbf{M}_S in the field direction (quantum numbers M , M_L , M_S . $M = M_L + M_S$).

Under these conditions

$$\overline{\cos(\mathbf{SL})} = \overline{\cos(\mathbf{SH})} \overline{\cos(\mathbf{LH})} = \frac{M_S M_L}{SL}$$

and

$$\overline{\cos(\mathbf{SL})} = ALS \cos(\mathbf{LS}) = AM_L M_S.$$

For this case Landé's **Γ -permanence rule** states: for the levels of a given multiplet corresponding to the same value of the quantum number M , the sum of the Γ -values is independent of the field strength.

In a very strong magnetic field the vectors \mathbf{l}_i and \mathbf{s}_i have separate components \mathbf{m}_{l_i} and \mathbf{m}_{s_i} in the field direction (quantum numbers m_{l_i} , m_{s_i} ; $M = \sum_i (m_{l_i} + m_{s_i})$). Under these conditions

$$\Gamma = \sum_i \gamma_i = \sum_i a_i m_{s_i} m_{l_i}$$

where γ_i represents the Γ -value for a single electron.

For this case Goudsmit's Γ -sum rule states: for a given electron configuration the sum of all Γ -values corresponding to a given value of M is independent of the field strength. (For more details see L. Pauling and S. Goudsmit, *The Structure of Line Spectra*, McGraw-Hill, Inc., New York, 1930, p. 157 ff.)

GAS, BOSE-EINSTEIN TREATMENT. See Bose-Einstein gas.

GAS, CLASSICAL MONATOMIC, SECOND VIRIAL COEFFICIENT. See second virial coefficient.

GAS CONSTANT. The constant of proportionality R in the equation of state of a perfect gas $pV = RT$. When referring to one gram-molecule of gas, R has the value, 8.3149×10^7 ergs/mole $^{\circ}\text{C}$ or 2782.77 ft lbf/lbmole $^{\circ}\text{C}$.

Values of the gas constant for real gases, as given in engineering tables, are usually expressed in units of mass, rather than moles. Their equivalent values may be calculated from the (universal) gas constant above by dividing it by the molecular weight of the gas, provided it is recognized that the values so obtained are in error by the departure of the particular gas from ideality over the range given. The gas constant per unit mass for a mixture of non-reactive gases may be obtained by averaging their tabulated gas constants, by multiplying them by their respective percentages in the mixture, and adding the products.

GAS DEGENERACY. See degenerate gas.

GAS DYNAMICS. Aerodynamics of compressible fluid flow. (See also **subsonic and supersonic flow**, **rarefied gas dynamics**.)

GAS DYNAMICS, RAREFIED. See rarefied gas dynamics.

GASES, SEPARATION OF. See reversible separation of gases.

GASES, SPECIFIC HEATS OF. See specific heats of gases.

GAS, FERMI-DIRAC TREATMENT. See Fermi-Dirac gas.

GAS FLOW IN NOZZLE OR DUCT. See nozzle or duct, flow of gas in.

GAS, IDEAL. See perfect gas.

GAS, IMPERFECT. See real gas.

GAS, PERFECT. See perfect gas.

GAS, REAL. See real gas.

GAS-VAPOR MIXTURES. See air-steam mixtures.

GAUGE-INVARIANCE. Classical electromagnetic field theory is invariant under the gauge transformation

$$A_{\mu}(x) \rightarrow A'_{\mu}(x) = A_{\mu}(x) + \frac{\partial \chi(x)}{\partial x^{\mu}} \quad (1)$$

as a consequence of the fact the field strengths

$$F_{\mu\nu} = \frac{\partial A_{\nu}}{\partial x^{\mu}} - \frac{\partial A_{\mu}}{\partial x^{\nu}} \quad (2)$$

(which are the observables of the system) remain invariant under this transformation. The arbitrary function $\chi(x)$ must however satisfy

$$\square \chi(x) = 0 \quad (3)$$

to ensure the **Lorentz condition**

$$\frac{\partial A_{\mu}}{\partial x_{\mu}} = 0. \quad (4)$$

In quantum field theory, gauge invariance is the requirement of the invariance of the theory under the simultaneous transformation

$$A_{\mu}(x) \rightarrow A'_{\mu}(x) = A_{\mu}(x) - \partial_{\mu} \chi(x) \quad (5)$$

$$\phi_C(x) \rightarrow \phi'_C(x) = e^{ie\chi(x)} \phi_C(x) \quad (6a)$$

$$\phi_C^*(x) \rightarrow \phi_C'^*(x) = e^{-ie\chi(x)} \phi_C^*(x) \quad (6b)$$

where ϕ_C is any charged field operator and $\chi(x)$ is a (c -number) function. This requirement determines almost uniquely the interaction of charged fields with the electromagnetic field.

For if one notes that

$$(\partial_{\mu} - ieA'_{\mu})\phi'_C = e^{ie\chi}(\partial_{\mu} - ieA_{\mu})\phi_C \quad (7)$$

it follows that if the replacement

$$\partial_{\mu} \phi_C \rightarrow (\partial_{\mu} - ieA_{\mu})\phi_C \quad (8a)$$

$$\partial_{\mu} \phi_C^* \rightarrow (\partial_{\mu} - ieA_{\mu})\phi_C^* \quad (8b)$$

is made for all charged fields in the Lagrangian without the presence of the electromagnetic field, then the total Lagrangian (includ-

ing the electromagnetic interactions) will remain unchanged by the simultaneous transformations (5) and (6).

GAUGE INVARIANCE AND CONSERVATION LAWS. Assume that a given Lagrangian (and hence the resulting field equations) is invariant under the following gauge transformations: for any field ϕ_i ,

$$\phi_i \rightarrow \phi'_i \equiv \phi_i e^{i\epsilon q_i} \quad (1)$$

where ϵ is a real number and q_i a real number characteristic of the field ϕ_i . For the case ϵ is infinitesimal

$$\phi'_i = \phi_i(1 + i\epsilon q_i)$$

or

$$\delta\phi_i = i\epsilon q_i \phi_i.$$

The observed invariance of \mathcal{L} under (1) means that

$$\begin{aligned} 0 = \delta\mathcal{L} &= \partial_\mu \sum_i \frac{\partial\mathcal{L}}{\partial\phi_{i,\mu}} \delta\phi_i \\ &= i\epsilon \partial_\mu \sum_i q_i \frac{\partial\mathcal{L}}{\partial\phi_{i,\mu}} \phi_i \end{aligned}$$

so that the current

$$J_\mu = i\epsilon \sum_i q_i \frac{\partial\mathcal{L}}{\partial\phi_{i,\mu}} \phi_i$$

is conserved, i.e.,

$$\partial_\mu J^\mu = 0.$$

GAUSS. See **electromagnetic units**.

GAUSS AND CODAZZI, EQUATIONS OF. See **equations of Gauss and Codazzi**.

GAUSS CHARACTERISTIC EQUATION. See **equations of Gauss and Codazzi**.

GAUSS CURVATURE. See **Gauss theorem (on curvature); curvature**.

GAUSS EQUATION OF THE SURFACE. See **equations of Gauss and Codazzi**.

GAUSS FORMULAS. Formulas which provide expressions for the tensor derivatives of the **base vectors** x_α^i ($i = 1, 2, 3$) for a surface coordinate system in terms of the fundamental magnitudes of the second order, $b_{\alpha\beta}$, for the surface and the unit normal n^i :

$$x_{\alpha,\beta}^i = b_{\alpha\beta} n^i.$$

They show that the tensor derivatives of the base vectors are parallel to the **unit normal**.

GAUSS HYPERGEOMETRIC EQUATION.

A canonical form of the **Riemann-Papperitz equation**, where the regular singular points have been shifted to $x = 0, 1, \infty$. Its form is $x(1-x)y'' + [c - (a+b+1)x]y' - aby = 0$, in which a, b, c are parameters related to the exponents of the equation. Its general solution around the point $x = 0$ and convergent for $|x| < 1$, provided c is not a positive or negative integer or zero, is

$$\begin{aligned} y &= AF(a, b, c; x) \\ &+ Bx^{1-c}F(1+a-c, 1+b-c, 2-c; x). \end{aligned}$$

In this solution $F(x)$ is the Gauss hypergeometric function, and A, B are constants of integration.

The Gauss hypergeometric function can be written as an infinite series in the form

$$\begin{aligned} F(a, b, c; x) &= \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)\Gamma(b+n)}{\Gamma(c+n)n!} x^n \end{aligned}$$

where Γ is the **gamma function**. The coefficient of the general term x^k is

$$\frac{a(a+1)(a+2)\cdots(a+k-1)b(b+1)\cdots(b+k-1)}{c(c+1)\cdots(c+k-1)k!}.$$

Many well-known functions, including the **Legendre** and other **polynomials** are compactly represented by the hypergeometric series. Thus $F(1, b, b; x)$ is the ordinary geometric series and $F(a, b, b; x)$ is the **binomial expansion** of $(1-x)^{-a}$.

If one lets $b \rightarrow \infty$ in the hypergeometric equation the singular points at 1 and ∞ merge by **confluence** to give an irregular singular point at ∞ . The result, called the confluent hypergeometric equation (also the Pochhammer-Barnes equation) is

$$xy'' + (c-x)y' - ay = 0.$$

Its general solution is

$$y = AF(a, c; x) + Bx^{1-c}F(1+a-c, 2-c; x)$$

where

$$F(a, c; x) = \frac{\Gamma(c)}{\Gamma(a)} \sum_{n=0}^{\infty} \frac{\Gamma(a+n)x^n}{\Gamma(c+n)n!}$$

is the **confluent hypergeometric series**. The notation ${}_1F_1$ is often used for this series and ${}_2F_1$ for the Gauss hypergeometric series. The first subscript indicates the number of fac-

torial terms in the numerator and the second subscript the number of such terms in the denominator. The more general series ${}_nF_m$ has also been studied.

Any differential equation with three regular singular points or less can be expressed as a special case of the Gauss equation or its confluent form.

GAUSSIAN BRACKETS. In Gauss' *Disquisitiones Arithmeticae* a bracket function is introduced in an algorithm for solving a linear diophantine equation. M. Herzberger has shown that these provide a convenient notational and calculational device for ray tracing an optical system in Gaussian optics.

The Gaussian brackets are defined recursively by

$$[a_1 \cdots a_n] = [a_1 \cdots a_{n-2}] + [a_1 \cdots a_{n-1}]a_n$$

with the initial conditions

$$[] = 1, [a] = a.$$

The principal properties are:

- (1) Symmetry: $[a_n \cdots a_1] = [a_1 \cdots a_n]$.
- (2) The expansions:

$$\begin{aligned} [a_1 \cdots a_m \cdots a_n] &= a_m[a_1 \cdots a_{m-1}][a_{m+1} \cdots a_n] \\ &\quad + [a_1 \cdots a_{m-1} + a_{m+1} \cdots a_n], \end{aligned}$$

$$\begin{aligned} [a_1 \cdots a_m \cdots a_n][a_1 \cdots a_m][a_{m+1} \cdots a_n] & \\ &\quad + [a_1 \cdots a_{m-1}][a_{m+2} \cdots a_n]. \end{aligned}$$

- (3) The determinant rule:

$$\begin{vmatrix} [a_1 a_{n-1}] & [a_2 a_{n-1}] \\ [a_1 a_n] & [a_2 a_n] \end{vmatrix} = (-1)^{n-1}.$$

GAUSSIAN DISTRIBUTION. See normal distribution.

GAUSSIAN ELIMINATION. Matrix inversion by triangularization.

GAUSSIAN INTERPOLATION FORMULAS. (Also called the Newton-Gauss formulas.) In the notation of difference operators, the forward formula is

$$\begin{aligned} E^u &= 1 + u\delta E^{1/2} + \binom{u}{2} \delta^2 \\ &\quad + \binom{u+1}{3} \delta^3 E^{1/2} + \binom{u+1}{4} \delta^4 + \dots \end{aligned}$$

and the backward formula is

$$\begin{aligned} E^{-u} &= 1 - u\delta E^{-1/2} + \binom{u}{2} \delta^2 \\ &\quad - \binom{u+1}{3} \delta^3 E^{-1/2} + \binom{u+1}{4} \delta^4 - \dots \end{aligned}$$

GAUSSIAN KERNEL. See kernel, Gaussian.

GAUSSIAN LENS EQUATION. If u is the object distance, u' the image distance, f the focal length, n, n' the refractive indices of object and image space, then a basic theorem of Gaussian optics states

$$\frac{n}{u} + \frac{n'}{u'} = \frac{n' - n}{f}.$$

GAUSSIAN OPTICS. The manifold of rays through an optical system is four-dimensional. For axially symmetrical systems, the geometrical optics in the tangent hyperplane to the ray coinciding with the axis is Gaussian optics or first order optics.

Expressed otherwise, if the coordinates of a ray are the rectangular coordinates x, y of its intersection with some plane orthogonal to the axis and the optical direction cosines p, q , then the mixed characteristic of an axially symmetric system is a function of

$$u = x^2 + y^2, v = px + qy, w = p^2 + q^2.$$

The constant term of the power series expansion in u, v, w is the characteristic function of Gaussian optics.

Thus, Gaussian optics is the theory of the first order approximation $\sin \theta = \theta$. The next approximation $\sin \theta = \theta - \frac{1}{6}\theta^3$ corresponds to third order optics, the theory of the approximate characteristic W including terms of the second degree in u, v, w .

GAUSSIAN QUADRATURE FORMULA. Any formula of the form

$$\int_a^b w(x)f(x)dx = \sum_0^n \mu_i f(x_i) + R$$

where the density function $w(x) \geq 0$; the coefficients μ_i are constant and depend only upon the function $w(x)$ and the range of integration; and the abscissae x_i are selected so that the remainder R vanishes whenever $f(x)$ is a polynomial of degree $2n + 1$ or less. It can be shown that these x_i exist and satisfy an alge-

braic equation of degree $n + 1$; that they are real; and that when ordered they satisfy $a < x_0 < x_1 < \dots < x_n < b$. They are, in fact, zeros of one of a set of orthogonal polynomials.

If a and b are both finite, one can make a substitution of variable, if necessary, and suppose $b = -a = 1$. Then if $w(x) = 1$, the x_i are zeros of the **Legendre polynomial** of degree $n + 1$, and one has for

$$n = 1: \mu_0 = \mu_1 = 1;$$

$$n = 2: \mu_0 = \mu_2 = \frac{5}{9}, \quad \mu_1 = \frac{8}{9};$$

$$n = 3: \mu_0 = \mu_3 = (18 - \sqrt{30})/36, \quad \mu_1 \\ = \mu_2 = (18 + \sqrt{30})/36.$$

By choosing $w(x)$ appropriately either a or b or both can be infinite. (For tabulation of x_i and μ_i , and for a discussion of the theory, see Zdenek Kopal, *Numerical Analysis*, John Wiley & Sons, Inc., 1955.)

GAUSSIAN REPRESENTATION. See **spherical representation of a surface**.

GAUSSIAN UNITS. A nonrationalized mixed system of units in which electrical quantities, such as electric charge and electric potential, are carried in the esu system, while currents and magnetic quantities are carried in the emu system. The constant c is then carried in the equations where it is needed, as a conversion factor. (See **electromagnetic units**.)

GAUSSIAN WAVE GROUP. A wave group for which

$$q(\kappa) = ae^{-\alpha(\kappa - \kappa_0)^2}.$$

Here κ is the "wavelength constant" but actually the independent variable, while α and κ_0 are arbitrary constants.

GAUSS LAW. The total **electric flux** passing out from a closed surface (c.s.) is (in rationalized units) equal to the total charge, q , enclosed within the surface. In unrationalized units the flux is equal to 4π times the enclosed charge q .

In integral form, the Gauss law may be written (in rationalized MKS units)

$$\int_{c.s.} \mathbf{D} \cdot d\mathbf{A} = \int_{c.s.} \epsilon_0 \mathbf{E} \cdot d\mathbf{A} = q = \int \rho dV,$$

where \mathbf{D} is the electric flux density, \mathbf{E} is the electric field strength, ρ is the electric charge density, and ϵ_0 is the permittivity of free space.

Applying Gauss theorem, the law may be put in the differential form,

$$\nabla \cdot \mathbf{D} = \rho, \quad \text{or} \quad \nabla \cdot \mathbf{E} = \rho/\epsilon_0.$$

Using $\mathbf{E} = -\nabla V = -\rho/\epsilon_0$, the law may be put into still another form, the **Poisson equation**: $\nabla^2 V = -\rho/\epsilon_0$.

GAUSS LAW FOR A MAGNETIC MEDIUM. Applying the Gauss law to the magnetic case:

$$\int \mathbf{B} \cdot d\boldsymbol{\sigma} = 4\pi \int \rho dT$$

$$\nabla \cdot \mathbf{B} = 4\pi\rho$$

where \mathbf{B} is the **magnetic induction**, and ρ the density of magnetic charge. Since free magnetic charge does not exist in nature, $\rho = 0$, and the Gauss law states:

$$\int \mathbf{B} \cdot d\boldsymbol{\sigma} = 0$$

$$\nabla \cdot \mathbf{B} = 0.$$

GAUSS LAW OF NORMAL GRAVITATIONAL FORCE. The surface integral of the normal component of the gravitational force on a particle of unit mass, taken over any closed surface is equal to $-4\pi G$ times the total mass enclosed by the surface.

GAUSS-MARKOV THEOREM. A theorem to the effect that an unbiased linear estimator of a parameter has minimal **variance** when obtained by the method of **least squares**.

GAUSS PLANE. See **Argand plane**.

GAUSS PRINCIPLE OF LEAST CONSTRAINT. See **least constraint**, **Gauss principle of**.

GAUSS-SEIDEL METHOD. A term sometimes applied to the total step iteration for solving linear equations (see **matrix inversion**), although both Gauss and Seidel used a **relaxation method**.

GAUSS-STOKES THEOREMS, FOUR-DIMENSIONAL. See **Minkowski world**.

GAUSS THEOREM. See **integral theorems of vector analysis**.

GAUSS THEOREM ON CURVATURE. The product of the **principal curvatures** of a surface at a point of the surface is a surface invariant (the Gauss curvature).

GAUZE, FLOW THROUGH. Gauzes made of woven wire are commonly used for reducing both the spatial non-uniformity and the **turbulence** of a stream of fluid.

Considering first the effect of a gauze on spatial disturbances in a stream, it is assumed that the scale of these disturbances is large compared with the mesh size. The essential properties of the gauze may be expressed in terms of two dimensionless coefficients, defined as

$$k = \frac{p_1 - p_2}{\frac{1}{2}\rho U^2}$$

and

$$F_\theta = \frac{\left\{ \begin{array}{l} \text{Force component in plane of gauze,} \\ \text{per unit area} \end{array} \right\}}{\frac{1}{2}\rho U^2}$$

U is the velocity of a uniform stream approaching the gauze and $(p_1 - p_2)$ is the pressure drop at the gauze when it is placed normal to the stream. In the definition of F_θ , the normal to the gauze makes an angle θ with the approaching stream. F_θ is, of course, a function of θ , and is zero when $\theta = 0$. It is found from experiments that F_θ may be expressed in the convenient form

$$\frac{F_\theta}{\theta} = 2(1 - \alpha),$$

where α is a constant for small values of θ .

It can be shown theoretically that when a gauze is placed normal to the mean flow the effect on a *small* spatial disturbance is to reduce all longitudinal velocity increments in the ratio

$$\frac{1 + \alpha - \alpha k}{1 + \alpha + k},$$

and to reduce all lateral velocities in the ratio α . It follows that for $k = 1 + 1/\alpha$ small longitudinal disturbances are entirely removed. For the types of gauze normally used this corresponds to a value of k of about 2.8.

The effect of a gauze on the turbulence of a stream is twofold. First, the wakes from the individual wires of the gauze may introduce turbulence into the stream. Secondly, the resistance introduced by the screen causes a

reduction of turbulence. The first effect is small if the gauze is woven from fine wire with a small mesh size, and may even be zero if the Reynolds number based on the wire diameter is sufficiently small. Theoretical studies of the second effect have shown that the reduction in turbulent energy depends on the three-dimensional energy spectrum of the turbulence in the approaching stream. If the turbulence is isotropic upstream of the gauze it becomes axi-symmetric after passing through the gauze.

GAVIOLA CAUSTIC TEST. A test of the deviation of a mirror from a **paraboloid** of revolution, based on the caustic of a parallel axial bundle being the locus of centers of curvature of the parabolic sections. (See **caustic**.)

GAY-LUSSAC LAW. See **Charles law**.

GEE-POUND. The unit of mass which a force of one pound will accelerate by one foot per second per second is called the *slug* or *gee-pound*.

GEGENBAUER FUNCTION. A solution of the **differential equation**

$$(z^2 - 1)u'' + (2n + 1)zu' - a(a + 2n)u = 0.$$

For integral values of a , the solution becomes the Gegenbauer polynomial. This equation is a special case of the **Gauss hypergeometric equation**.

GEIGER-NUTALL RELATION. An empirical relationship between the range of an α -particle in air and **disintegration constant** of natural α -emitting radionuclides of the form:

$$\log R = A + B \log \lambda,$$

where R is the α -particle range, λ is the **disintegration constant**, A is a parameter having different values for the three natural radioactive series, and B is constant. This relationship was more closely applicable to the members of the natural series than it is to the more recently discovered α -emitters, especially those produced artificially. Current theory does not substantiate the relationship except for the general trend.

GENERALIZED AFFINITIES. See **forces in thermodynamics of irreversible processes**.

GENERALIZED COORDINATES AND MOMENTA. See coordinates and momenta, generalized.

GENERALIZED FUNCTION. See distribution.

GENERALIZED HOOKE'S LAW. See Hooke's law, generalized.

GENERALIZED HYDROSTATIC EQUATION. The vertical component of the vector equation of motion in natural coordinates when the acceleration of gravity is replaced by the virtual gravity. For most purposes it is identical to the hydrostatic equation.

GENERALIZED KRONECKER DELTA. See Kronecker delta, generalized.

GENERALIZED PLANE STRESS. A state of stress in a plate, the major surfaces of which are formed by two parallel planes. If a rectangular Cartesian reference system x, y, z is chosen with its z -axis perpendicular to the major surface of the plate and the normal component σ_{zz} of the stress vanishes throughout the plate, while the tangential components σ_{zx} and σ_{yz} vanish on its major surfaces, the plate is said to be in a state of generalized plane stress.

GENERALIZED VELOCITIES. See velocities, generalized.

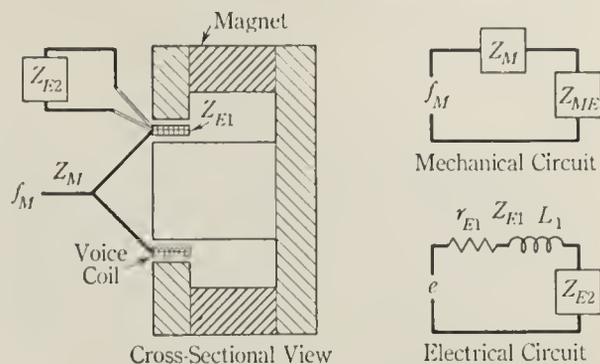
GENERATED ERROR. See error.

GENERATING FUNCTION. If the Taylor expansion of a function $f(x, u)$ expanded in powers of u is

$$f(x, u) = p_0(x) + p_1(x)u + p_2(x)u^2 + \dots$$

then $f(x, u)$ is called a generating function for the sequence of functions $p_0(x)$, $p_1(x)$, $p_2(x)$, \dots . Thus $(1 - 2xu + u^2)^{-1/2}$ is a generating function for the **Legendre polynomials**; $\exp[x^2 - (u - x)^2]$ for the **Bessel functions**; $(1 - u)^{-1} \exp\left(\frac{-xu}{1 - u}\right)$ for the **Laguerre polynomials**; $(1 - xu)(1 - 2xu + u^2)^{-1}$ for the **Chebyshev polynomials**, etc.

GENERATING SYSTEM, ELECTRODYNAMIC. A moving conductor or a moving coil generating system is a generating system in which the electromotive force is developed by motion of a conductor through a magnetic field.



Electrodynamic generating system. In the mechanical circuit f_M is the external driving force; z_M is the total mechanical rectilinear impedance of the mechanical portion of the vibrating system actuated by f_M ; z_{ME} is the mechanical rectilinear impedance due to the electrical circuit. In the electrical circuit e is the internal electromotive force generated in the voice coil; z_{E1} is the damped electrical impedance of the voice coil. $z_{E1} = r_{E1} + j\omega L_1$; L_1 and r_{E1} are the damped inductance and electrical resistance of the voice coil. z_{E2} is the electrical impedance of the external load.

The voltage e , in abvolts, due to the motion of the conductor in the magnetic field (see figure), is

$$e = Bl\dot{x} \quad (1)$$

where B is the flux density, in gaussses, l is the length of the conductor, in centimeters, and \dot{x} is the velocity of the conductor, in centimeters per second.

The velocity of the conductor is governed by the mechanical driving force, the mechanical rectilinear impedance of the mechanical system, and the mechanical rectilinear impedance due to the electrical system. The vibrating system is shown in the figure. In the mechanical circuit z_M represents the mechanical rectilinear impedance, in mechanical ohms, of the mechanical portion of the vibrating system actuated by f_M including the mechanical rectilinear impedance of the coil at the voice coil. f_M represents the mechanomotive force, in dynes, at the voice coil. The mechanical rectilinear impedance z_{ME} , in mechanical ohms, due to the electrical system is, from Equation 7 of driving system, electrodynamic

$$z_{ME} = \frac{(Bl)^2}{z_E} \quad (2)$$

where B is the flux density, in gaussses, l is the length of the conductor, in centimeters, $z_E = z_{E1} + z_{E2}$, z_{E1} is the electrical impedance of the voice coil, in abohms, and z_{E2} is the electrical impedance of the external load, in abohms.

The velocity of the voice coil is

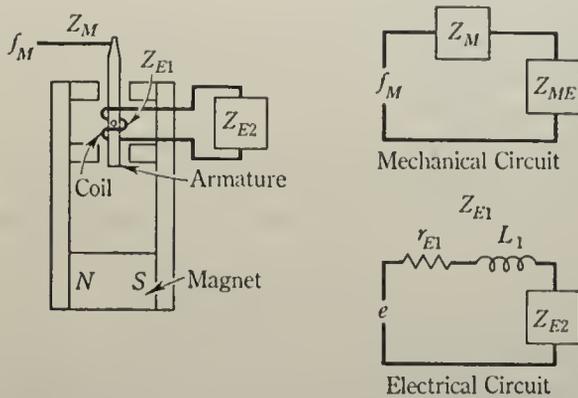
$$\dot{x} = \frac{f_M}{z_M + z_{ME}} \tag{3}$$

From Equations 1 and 3 the generated electromotive force e , in abvolts, is

$$e = Bl\dot{x} = \frac{Blf_M}{z_M + z_{ME}} \tag{4}$$

The generated electromotive force is effectively in series with the electrical impedance z_{E1} of the voice coil and the electrical impedance z_{E2} of the external load, as depicted by the electrical circuit in the figure.

GENERATING SYSTEM, ELECTROMAGNETIC. In the simple generating system of the preceding entry both the steady magnetic flux and the change in flux, due to the deflection of the armature, flows through the armature. Consider a balanced armature type of generating system in which only the alternating flux flows longitudinally through the armature as shown in the figure.



Balanced armature electromagnetic generating system. In the mechanical circuit z_M is the total mechanical rectilinear impedance of the mechanical portion of the vibrating system actuated by f_M ; z_{ME} is the mechanical rectilinear impedance due to the electrical circuit. In the electrical circuit e is the internal electromotive force generated in the coil; z_{E1} is the damped electrical impedance of the coil; $z_{E1} = r_{E1} + j\omega L_1$. L_1 and r_{E1} are the damped inductance and electrical resistance of the coil. z_{E2} is the electrical impedance of the external load.

The electromotive force e , in abvolts, generated in the coil due to motion of the armature from Equation 2 of **driving system, electromagnetic** is

$$e = \frac{NMA}{a^2} \dot{x} \tag{1}$$

where N is the number of turns in the coil, M is the magnetomotive force, in gilberts, of the steady field, A is the area of a pole piece, in square centimeters, a is the spacing between the armature and pole, in centimeters, and \dot{x} is the velocity of the armature, in centimeters per second.

The velocity of the armature is governed by the mechanical driving force, the mechanical rectilinear impedance of the mechanical system, and the mechanical rectilinear impedance due to the electrical system. The vibrating system is shown in the figure. In the mechanical circuit z_M represents the mechanical rectilinear impedance in mechanical ohms, of the mechanical portion of the vibrating system including the mechanical rectilinear impedance of the armature. f_M represents the mechanomotive force, in dynes, on the armature. The mechanical rectilinear impedance z_{ME} , in mechanical ohms, due to the electrical system is, from Equation 6 of **driving system, electromagnetic**

$$z_{ME} = \frac{4N^2\phi M}{a^2 R_2 z_E} \tag{2}$$

where N is the number of turns in the coil, ϕ is the total flux in the air gap at one of the poles, in maxwells, M is the magnetomotive force, in gilberts, of the magnet, a is the spacing between armature and pole, in centimeters, R_2 is the reluctance, in oersteds, of the alternating magnetic circuit, $z_E = z_{E1} + z_{E2}$, z_{E1} is the electrical impedance of the coil, in abohms, and z_{E2} is the electrical impedance of the external load, in abohms.

The velocity of the armature, in centimeters per second, is

$$\dot{x} = \frac{f_M}{z_M + z_{ME}} \tag{3}$$

From Equations 1 and 3

$$e = \frac{NMA}{a^2} \dot{x} = \frac{NMAf_M}{a^2(z_M + z_{ME})} \tag{4}$$

The generated electromotive force is effectively in series with the electrical impedance z_{E1} of the coil and the electrical impedance z_{E2} of the external load, as depicted by the electrical circuit in the figure.

GENERATING SYSTEM, ELECTROSTATIC. A condenser or electrostatic generating system is a generating system in which

the electromotive force is developed by the relative motion between two plates carrying different electrostatic charges.

The current i_1 , in statamperes, generated by the motion of the movable plate of the condenser is, from Equation 2 of **driving system, electrostatic**

$$i_1 = \frac{e_0 A \dot{x}}{4\pi a^2} \quad (1)$$

where e_0 is the polarizing voltage, in statvolts, A is the area of the plate, in square centimeters, a is the spacing between the plates, in centimeters, and \dot{x} is the velocity of the movable plate, in centimeters per second.

The current i_2 , in statamperes, due to the electromotive force e across the electrical impedances z_{E1} and z_{E2} of the figure is

$$i_2 = -\frac{e}{z_E} \quad (2)$$

where e is the electromotive force, in statvolts,

$$z_E = \frac{z_{E1} z_{E2}}{z_{E1} + z_{E2}}$$

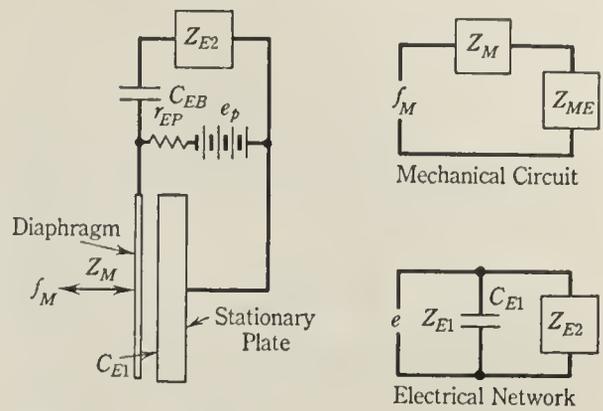
$$z_{E1} = \frac{1}{j\omega C_{E1}}$$

C_{E1} is the electrical capacitance of the condenser, in statfarads, and z_{E2} is the electrical impedance of the external load, in statohms.

Since there is no external current applied on the electrical side of the system the sum of the currents i_1 and i_2 is zero. From Equations 1 and 2 the generated electromotive force, e , in statvolts, across the electrical impedances z_{E1} and z_{E2} is

$$e = \frac{e_0 A \dot{x}}{4\pi a^2} z_E. \quad (3)$$

The velocity of the movable plate is governed by the mechanical driving force, the mechanical rectilinear impedance of the mechanical system and the mechanical rectilinear impedance due to the electrical system. The vibrating system is shown in the figure. In the mechanical circuit z_M represents the mechanical rectilinear impedance of the mechanical portion of the vibrating system actuated by f_M including the mechanical impedance of the movable plate. f_M represents the mechanomotive force at the movable plate. The me-



Electrostatic generating system. In the mechanical circuit z_M is the total mechanical rectilinear impedance of the mechanical portion of the vibrating system actuated by f_M ; z_{ME} is the mechanical rectilinear impedance due to the electrical circuit. In the electrical network e is the electromotive force generated across the condenser. z_{E1} , the damped electrical impedance of the condenser. $z_{E1} = 1/j\omega C_{E1}$. C_{E1} is the damped electrical capacitance of the condenser. Z_{E2} is the electrical impedance of the external load.

chanical rectilinear impedance z_{ME} , in mechanical ohms, due to the electrical system is, from Equation 4 of **driving system, electrostatic**

$$z_{ME} = \frac{e_0^2 A^2}{16\pi^2 a^4} z_E \quad (4)$$

$$\text{where } z_E = \frac{z_{E1} z_{E2}}{z_{E1} + z_{E2}}$$

$$z_{E1} = \frac{1}{j\omega C_{E1}}$$

C_{E1} is the capacity of the generator, in statfarads, z_{E2} is the electrical impedance of the external load, in statohms.

The velocity of the movable plate \dot{x} , in centimeters per second, is

$$\dot{x} = \frac{f_M}{z_M + z_{ME}}. \quad (5)$$

From Equations 3 and 5 the electromotive force e in abvolts, across z_{E1} and z_{E2} in parallel, depicted by the electrical network of the figure, is

$$e = \frac{e_0 A z_E f_M}{4\pi a^2 (z_M + z_{ME})}. \quad (6)$$

The electromotive force e_1 , in abvolts, in series with z_{E1} and z_{E2} which will produce the electromotive force e across z_{E2} is of interest in the design of generating systems.

Equation 3 may be written

$$e = \frac{e_0 A \dot{x}}{4\pi a^2} \left(\frac{z_{E1} z_{E2}}{z_{E1} + z_{E2}} \right). \quad (7)$$

The electrical capacitance of the condenser C_{E1} from Equation 4 is

$$C_{E1} = \frac{A}{4\pi a}. \quad (8)$$

The electrical impedance z_{E1} is

$$z_{E1} = \frac{1}{j\omega C_{E1}} = \frac{4\pi a}{j\omega A}. \quad (9)$$

Substituting Equation 9 in 8,

$$e = \frac{e_0 \dot{x}}{a j\omega} \left(\frac{z_{E2}}{z_{E1} + z_{E2}} \right). \quad (10)$$

The amplitude x in centimeters, in terms of the velocity is

$$x = \frac{\dot{x}}{j\omega}. \quad (11)$$

Substituting Equation 11 in 10,

$$e = \frac{e_0 x}{a} \left(\frac{z_{E2}}{z_{E1} + z_{E2}} \right). \quad (12)$$

The electromotive force e in terms of e_1 and the impedances z_{E1} and z_{E2} is

$$e = \frac{e_1 z_{E2}}{z_{E1} + z_{E2}}. \quad (13)$$

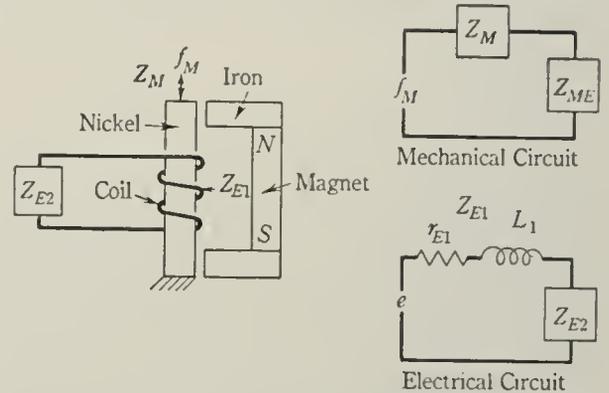
Comparing Equations 12 and 13,

$$e_1 = \frac{e_0 x}{a}. \quad (14)$$

The electrostatic generating system may be considered to consist of a generator having an internal or open circuit electromotive force e_1 as given by Equation 14 and an internal impedance z_{E1} . Equation 14 shows that this electromotive force is independent of the frequency if the amplitude is independent of the frequency. However, the voltage e across the load may vary with frequency depending upon the nature of load z_{E2} .

GENERATING SYSTEM, MAGNETOSTRICTION. A magnetostriction generating system is a generating system in which the electromotive force is developed in a stationary coil by a change in magnetic flux due to the deformation of a ferromagnetic material

having magnetostriction properties. The magnetostriction generator, shown in the figure, consists of a coil surrounding a magnetic circuit which includes a ferromagnetic material



Magnetostriction generating system. In the mechanical circuit z_M is the total mechanical rectilinear impedance of the mechanical portion of the vibrating system actuated by f_M ; z_{ME} is the mechanical rectilinear impedance due to the electrical circuit. In the electrical circuit e is the internal electromotive force generated in the coil; z_{E1} is the damped electrical impedance of the coil. $z_{E1} = r_{E1} + j\omega L_1$. L_1 and r_{E1} are the damped inductance and electrical resistance of the coil; z_{E2} is the electrical impedance of the external load.

having magnetostriction properties. The voltage e , in abvolts, developed in the coil due to deformation of the rod, from equation

$$e = \frac{4\pi N K}{R} \dot{x} \quad (1)$$

where N is the number of turns in the coil, R is the reluctance of the magnetic circuit, K is the constant representing the dynamical Villari magnetostriction effect, and \dot{x} is the velocity at the point of application of the driving force to the rod, in centimeters per second.

The velocity of the rod is governed by the mechanical driving force, the mechanical impedance of the mechanical system and the mechanical impedance due to the electrical system. The vibrating system is shown in the figure. In the mechanical circuit z_M represents the mechanical rectilinear impedance, in mechanical ohms, of the mechanical portion of the vibrating system actuated by f_M including the mechanical rectilinear impedance of the magnetostriction rod. f_M represents the mechanomotive force, in dynes, on the rod. It is assumed that the force f_M is the same at all points along the length of the rod and that the phase of the amplitude is constant along the rod. The mechanical recti-

lineal impedance due to the electrical system is, from Equation 8 of **driving system, magnetostriction**

$$z_{ME} = \frac{16\pi^2 N^2 K^2}{z_E R^2} \quad (2)$$

where $z_E = z_{E1} + z_{E2}$, z_{E1} is the electrical impedance of the coil, in abohms, and z_{E2} is the electrical impedance of the external circuit, in abohms.

The dimensions of the rod are assumed to be a small fraction of a wavelength. Under these conditions the rod is a compliance C_{M1} , in centimeters per dyne, given by

$$C_{M1} = \frac{l}{EA} \quad (3)$$

where A is the cross-sectional area of the rod, in square centimeters, l is the length of the rod, in centimeters, and E is Young's modulus.

The mechanical rectilinear impedance of the rod is

$$z_{M1} = \frac{1}{j\omega C_{M1}} \quad (4)$$

For the conditions under consideration the mechanical rectilinear impedance of the vibrating system is

$$z_M = z_{M1} + z_{M2} \quad (5)$$

where z_M is the total mechanical rectilinear impedance, in mechanical ohms, z_{M1} is the mechanical rectilinear impedance of the rod, in mechanical ohms, and z_{M2} is the mechanical rectilinear impedance of the load, in mechanical ohms.

The velocity of the rod, in centimeters per second, at the driving point is

$$\dot{x} = \frac{f_M}{z_M + z_{ME}} \quad (6)$$

From Equations 1 and 6 the generated electromotive force, in abvolts, is

$$e = \frac{4\pi N K f_M}{(z_M + z_{ME})R} \quad (7)$$

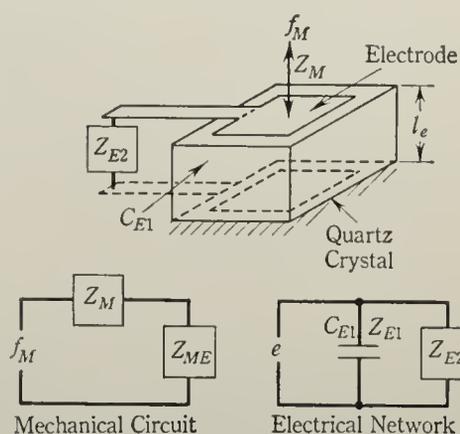
The generated electromotive force is effectively in series with the electrical impedance z_{E1} , in abohms, of the coil and the electrical impedance z_{E2} , in abohms, of the external load as depicted by the electrical circuit in the above figure.

In the above considerations the length of

the rod is assumed to be a small fraction of the wavelength. In general, magnetostriction generating systems are operated at resonance. The two most common systems are as follows: a rod fixed on one end and driven on the other and a rod free on one end and driven on the other.

GENERATING SYSTEM, PIEZOELECTRIC.

A piezoelectric generating system is a generating system in which the electromotive force is developed by the deformation of a crystal having converse piezoelectric properties. The crystal generating system, shown in the figure, consists of a suitably ground crystal



Piezoelectric generating system. In the electrical circuit z_M is the total mechanical rectilinear impedance of the mechanical portion of the vibrating system actuated by f_M ; z_{ME} is the mechanical rectilinear impedance due to the electrical network. In the electrical circuit e is the electromotive force generated across the crystal; z_{E1} is the damped electrical impedance of the crystal. $z_{E1} = 1/j\omega C_{E1}$. C_{E1} is the damped electrical capacitance of the crystal; z_{E2} is the electrical impedance of the external load.

having converse piezoelectric properties fitted with appropriate electrodes.

The current, in statamperes, generated by the motion of the crystal from Equation 4 of **driving system, piezoelectric**, is

$$i_1 = \frac{KEA}{l_e} \dot{x} \quad (1)$$

where K is a constant of the crystal, 6.4×10^{-8} for quartz, E is Young's modulus, l_e is the length of the crystal, in centimeters, A is the cross-sectional area of the crystal, in square centimeters, length of the crystal, in centimeters, and \dot{x} is the velocity of the crystal, in centimeters per second.

The current, in statamperes, due to the elec-

tromotive force e across the electrical impedances z_{E1} and z_{E2} is

$$i_2 = -\frac{e}{z_E} \quad (2)$$

where e is the electromotive force, in statvolts,

$$z_E = \frac{z_{E1}z_{E2}}{z_{E1} + z_{E2}}$$

$$z_{E1} = \frac{1}{j\omega C_{E1}}$$

C_{E1} is the electrical capacitance of the crystal, in statfarads, and z_{E2} is the electrical impedance of the external load, in statohms.

Since there is no external current applied to the electrodes of the crystal, the sum of the currents i_1 and i_2 is zero. From Equations 1 and 2 the generated electromotive force e , in statvolts, across the electrical impedance z_{E1} and z_{E2} is

$$e = \frac{KEA\dot{x}}{l_e} z_E \quad (3)$$

The velocity at the end of the crystal is governed by the mechanical driving force, the mechanical rectilinear impedance of the mechanical system and the mechanical rectilinear impedance due to the electrical system. The vibrating system is shown in the figure. In the mechanical circuit z_M represents the mechanical rectilinear impedance of the mechanical portion of the vibrating system actuated by f_M including the mechanical rectilinear impedance of the crystal. f_M represents the mechanomotive force at the end of the crystal. It is assumed that the force f_M is the same at all points along the length of the crystal and that the phase of the amplitude is constant along the crystal. The mechanical impedance z_{ME} , in mechanical ohms, due to the electrical system is

$$z_{ME} = \frac{K^2 E^2 A^2}{l_e^2} z_E \quad (4)$$

where $z_E = \frac{z_{E1}z_{E2}}{z_{E1} + z_{E2}}$

$$z_{E1} = \frac{1}{j\omega C_{E1}}$$

C_{E1} is the capacitance of the generator, in statfarads, z_{E2} is the electrical impedance of the external load, in statohms.

The dimensions of the crystal are assumed to be a small fraction of a wavelength. Under these conditions the crystal is a compliance given by

$$C_{M1} = \frac{l_e}{EA} \quad (5)$$

where A is the cross-sectional area of the crystal, in square centimeters, l_e is the length of the crystal, in centimeters, and E is Young's modulus.

The mechanical rectilinear impedance of the crystal is

$$z_{M1} = \frac{1}{j\omega C_{M1}} \quad (6)$$

For the conditions under consideration the mechanical rectilinear impedance of the vibrating system is

$$z_M = z_{M1} + z_{M2} \quad (7)$$

where z_M is the total mechanical rectilinear impedance, in mechanical ohms, z_{M1} is the mechanical rectilinear impedance of the crystal, in mechanical ohms, and z_{M2} is the mechanical rectilinear impedance of the load, in mechanical ohms.

The velocity at the end of crystal, in centimeters per second, is

$$\dot{x} = \frac{f_M}{z_M + z_{ME}} \quad (8)$$

From Equations 3 and 8 the electromotive force across z_{E1} and z_{E2} in parallel, depicted by the electrical network of the figure, is

$$e = \frac{KEA z_E f_M}{l_e (z_M + z_{ME})} \quad (9)$$

The electromotive force e_1 in series with z_{E1} and z_{E2} which will produce the electromotive force e across z_{E2} is of interest in the design of generating systems.

Equation 3 may be written

$$e = \frac{KEA\dot{x}}{l_e} \left(\frac{z_{E1}z_{E2}}{z_{E1} + z_{E2}} \right) \quad (10)$$

The electrical capacitance of the crystal is

$$C_{E1} = \frac{AD}{4\pi l_e} \quad (11)$$

where D is the dielectric constant of the crystal.

The electrical impedance z_{E1} is

$$z_{E1} = \frac{1}{j\omega C_{E1}} = \frac{4\pi l_e}{j\omega AD}. \quad (12)$$

Substituting 12 in 10,

$$e = \frac{4\pi KE\dot{x}}{j\omega D} \left(\frac{z_{E2}}{z_{E1} + z_{E2}} \right). \quad (13)$$

The amplitude x , in centimeters, in terms of the velocity is

$$x = \frac{\dot{x}}{j\omega}. \quad (14)$$

Substituting 14 in 13,

$$e = \frac{4\pi KE\dot{x}}{D} \left(\frac{z_{E2}}{z_{E1} + z_{E2}} \right). \quad (15)$$

The electromotive force e in terms of e_1 is

$$e = \frac{e_1 z_{E2}}{z_{E1} + z_{E2}}. \quad (16)$$

Comparing Equations 15 and 16,

$$e_1 = \frac{4\pi KE\dot{x}}{D}. \quad (17)$$

The piezoelectric generating system may be considered to consist of a generator having an internal or open circuit electromotive force e_1 as given by Equation 17 and an internal impedance z_{E1} . Equation 17 shows that this electromotive force is independent of the frequency if the amplitude is independent of the frequency. However, the voltage e across the load may vary with frequency depending upon the nature of the load z_{E2} .

In the above considerations the length of the crystal is assumed to be a small fraction of the wavelength. In general piezoelectric generating systems are operated at resonance. The two most common systems are as follows: a crystal fixed on one end and driven on the other and a crystal free on one end and driven on the other.

GENERATION RATE (IN A SEMICONDUCTOR). The time rate of creation of electron-hole pairs.

GENERATION TIME. In reactor theory, the time between the appearance of successive neutron generations, synonymous with neutron lifetime. (See **lifetime**, **neutron**.)

GENERATORS OF A SURFACE. See ruled surface.

GENERATORS OF ENVELOPE OF ONE-PARAMETER FAMILY OF PLANES. The characteristics of a one-parameter family of surfaces.

GENTILE STATISTICS. Gentile introduced the so-called intermediate statistics in which in a system of independent particles each single particle level can be occupied by up to l particles, where l is finite and larger than unity (compare $l = 1$ for **Fermi-Dirac statistics** and $l = \infty$ for **Bose-Einstein statistics**). These statistics have not found any application.

GEODESIC. See **geodesic line**.

GEODESIC CIRCLE ON A SURFACE. The locus of a point P on the surface such that geodesic distance (see **geodesic parallels**) r of P from a fixed point C on the surface is constant. This is an orthogonal trajectory of the geodesics drawn through C . C is the *center* of the geodesic circle and r its *radius*.

GEODESIC COORDINATES (PARAMETERS) FOR A SURFACE. Parameters u, v such that the curves $v = \text{constant}$ are a singly-infinite family of geodesics and the curves $u = \text{constant}$ are the **geodesic parallels** orthogonal to them.

GEODESIC CURVATURE (AT A POINT OF A CURVE ON A SURFACE). The curvature of the curve at the point relative to the geodesic which touches it at that point. Also called *tangential curvature* of the curve at the point.

GEODESIC DISTANCE (BETWEEN TWO GEODESIC PARALLELS). See **geodesic parallels**.

GEODESIC ELLIPSE ON A SURFACE. The curve $\frac{1}{2}(u + v) = \text{constant}$, where u and v are the geodesic distances of a point on the surface from two independent *geodesic parallels*, $u = 0$ and $v = 0$.

GEODESIC FORM FOR FIRST FUNDAMENTAL FORM. See **geodesic parallels**.

GEODESIC HYPERBOLA ON A SURFACE. The curve $\frac{1}{2}(u - v) = \text{constant}$, where u and v are the geodesic distances of a point on the

surface from two independent geodesic parallels, $u = 0$ and $v = 0$.

GEODESIC LINE ON A SURFACE (OR GEODESIC). The curve on a surface joining two points whose length between these points is less than that of any other curve on the surface joining them. A curve on the surface whose principal normal (see **normal, principal**) at each point coincides with the normal to the surface at that point.

GEODESIC, NULL. See **null-geodesic**.

GEODESIC PARALLELS ON A SURFACE. Consider a singly-infinite family of geodesics (see **geodesic line**) on a surface. The singly-infinite family of curves on the surface which cut these orthogonally at each point are called geodesic parallels. The distance between two geodesic parallels measured on the surface along any geodesic of the family is the same and is called the *geodesic distance* between the two geodesic parallels. If the geodesics are taken as the parametric curves $v = \text{constant}$ and their orthogonal trajectories are taken as the curves $u = \text{constant}$, then the first fundamental form for the surface takes the form

$$a(du)^2 + b(dv)^2$$

with a independent of v . In terms of the parameters \bar{u}, \bar{v} defined by

$$\bar{u} = \int \sqrt{a} du \quad \text{and} \quad \bar{v} = v,$$

the first fundamental form for the surface takes the form

$$(d\bar{u})^2 + b(d\bar{v})^2.$$

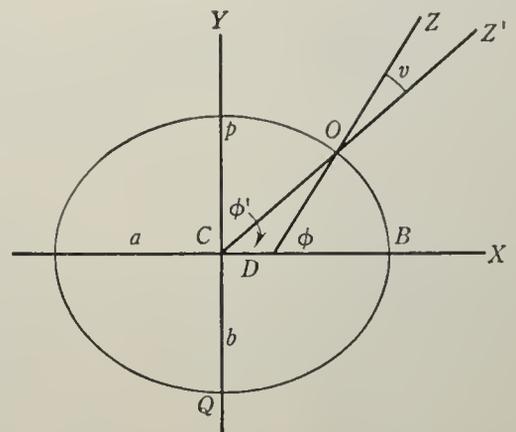
This is called the *geodesic form* for the first fundamental form.

GEODESIC POLAR COORDINATES FOR A SURFACE. Geodesic parameters u, v for the surface such that the curves $u = \text{constant}$ are geodesic circles of radius u on the surface having a common center, the pole of the coordinate system, and the curves $v = \text{constant}$ are geodesics joining the point u, v on the surface with the pole.

GEODESIC TANGENT (TO A CURVE ON A SURFACE AT A POINT). The geodesic (see **geodesic line**) on the surface which touches the curve on the surface at the point.

GEODESIC TRIANGLE ON A SURFACE. A curvilinear triangle on a surface bounded by geodesics (see **geodesic line**) on the surface.

GEOID. The shape of the surface of the rotating earth, as defined by the level of the ocean. For astronomical purposes it is sufficient to consider the geoid as a spheroid of revolution with the minor axis the line joining the poles of rotation. A plane containing the axis of rotation cuts the surface of the earth in a terrestrial meridian, which is elliptical. The major axis of the ellipse is in the plane of the equator and a , the length of the semi-major axis of the ellipse, is the equatorial



radius of the geoid (earth). Denoting the polar radius as b , and the eccentricity of the meridional ellipse as e , then $b^2 = a^2(1 - e^2)$. As is evident from the figure (axes CX and CY),

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1.$$

For an observer on the earth at O there are two types of latitude: The geocentric latitude represented by ϕ' and the astronomic latitude represented by ϕ .

The angle $\angle ZOZ'$ is known as the angle of the vertical and is represented by v . Now define a quantity m as $\frac{e^2}{(2 - e^2)}$ and thus

$$\phi - \phi' \equiv v = m \left(\frac{\sin 2\phi}{\sin 1''} \right)$$

$$- \frac{1}{2} \left(\frac{m^2 \sin 4\phi}{\sin 1''} \right) + \frac{1}{3} \left(\frac{m^3 \sin 6\phi}{\sin 1''} \right) \dots$$

For the sake of uniformity the so-called "International Geoid" is used by astronomers and its dimensions are:

a	equatorial radius.....	3963.339	statute miles
b	polar radius.....	3949.994	“ “
e	eccentricity.....	0.08199	
o	oblateness.....	$\frac{1}{297}$	
m		0.003373	

With this notation,

$$v \equiv \phi - \phi' = 695''.65 \sin 2\phi - 1''.17 \sin 4\phi.$$

In the various fields of science slightly different values for the dimensions of the geoid are used. However, the variations so caused in the various computed quantities are not significantly important.

GEOMAGNETIC COORDINATES. A system of **spherical coordinates** based on the best fit of a centered dipole to the actual magnetic field of the earth.

GEOMAGNETIC LATITUDE. (Or magnetic latitude.) A coordinate used in **geomagnetism** bearing the same relation to the geomagnetic equator as geographic latitude does to the geographic equator.

GEOMAGNETIC POLE. A pole of the system of **geomagnetic coordinates**; that is, an axis pole of the mathematical magnetic field of closest fit to the actual magnetic field of the earth. (Cf. **magnetic pole**.)

GEOMETRIC ASSOCIATION, PRINCIPLE OF. A principle used in forecasting the thickness of a layer between two given constant-pressure surfaces. In practical use of the principle, specific thicknesses are considered to be associated with **corresponding points** on the lower of the two constant-pressure surfaces (the upper one can also be used). A prognostic chart is prepared for the lower surface and the assumption is made that the thickness of the layer at the corresponding points is unchanged at the same corresponding points on the prognostic map. The prognostic **thickness chart** is then constructed from thickness values at the corresponding points.

GEOMETRIC(AL) BUCKLING. See **buckling**.

GEOMETRIC(AL) DISTORTION. Any monochromatic aberration which causes the reproduced image to be geometrically dissimilar to the perspective plane-projection of the object. If the image of a square in the object plane is a concave star-shaped region, the distortion is referred to as *pin cushion distortion*.

If, on the other hand, the image is convex, it is said to have *barrel distortion*.

GEOMETRIC(AL) ENERGY FLUX. If $L(u,v)$ is the optical differential invariant of a two-dimensional manifold of rays with parameters u,v and if S is a simply connected region in the (u,v) -plane bounded by a simple closed curve, then the integral invariant

$$\iint_S L(u,v) dS$$

is the geometrical energy flux through S .

GEOMETRIC MEAN. The geometric mean, G , of N positive **variates** is the N th root of the product of the N variates.

$$G = (x_1 x_2 x_3 \cdots x_N)^{\frac{1}{N}}.$$

It is found by use of logarithms.

$$\log G = \frac{1}{N} \sum_{j=1}^N \log x_j.$$

GEOMETRIC(AL) OPTICS. Visual phenomena are man's richest and most extensive contact with the physical world, so it is not surprising that geometrical optics as a systematic science antedates Euclid's *Catoptrics* of about 300 B.C. and includes such early monumental works as the 10-volume *Handbook on Optics* of Witela, about A.D. 1270.

Maxwell's discovery that the propagation of light is an electromagnetic phenomenon made optics a branch of electromagnetism. However, geometrical optics continues to be a useful description of many optical properties, always with the understanding that there is a limit to the physical accuracy of the results. Geometrical optics is always an extremely valuable intuitive tool. It is customary to use the name "physical optics" for the more complex and physically accurate theory based on electromagnetic theory.

A systematic development of geometrical optics can be carried out in a number of ways: (1) as a postulational system, (2) as derived from **Fermat's principle**, (3) as derived from **Huygens' principle**, (4) as a limiting form of physical optics (see **Kirchhoff's geometrical optics theorem**), or (5) by introducing the light rays as bicharacteristics of **Maxwell's equations**. (See **dynamical equations of light**

rays; paraxial ray tracing equations; direct method in geometrical optics.)

GEOMETRIC(AL) OPTICS, DIRECT METHOD. See **direct method in geometric optics.**

GEOMETRY. The subject-matter of geometry has become so general that it is difficult to give a definition suitable for the present day. In general, a geometry is the theory of properties of a set of points, lines, etc., which are invariant under a pre-assigned group of transformations. Classically, the methods of Euclid, who probably lived from 330–275 B.C., were used. These were based on a number of definitions, five postulates, and nine general axioms. The definitions included statements on point, line, solid, proposition, hypothesis, theorem, etc. The axioms were accepted without proof, the following being typical: if equals are added to or subtracted from equals the sums or remainders are equal; the whole is equal to the sum of all its parts and greater than any of its parts. Among the postulates, typical examples are: a straight line can be drawn from one point to another and can be produced indefinitely; a circumference can be described from any point as a center and with any given radius.

The development of mathematics and physical sciences gradually showed that there were logical deficiencies in the Euclidean definitions and postulates and, in 1899, the German mathematician David Hilbert resolved these difficulties with an improved set of geometric axioms. This work has been of profound significance in many branches of mathematics and science.

More specifically, non-Euclidean geometries discard one or more of the Euclidean axioms. The best-known of these are: hyperbolic geometry in which it is assumed that through any point there are two or more parallel lines which do not intersect a given line in the plane (it leads to the conclusion that the sum of three angles in a triangle is less than two right angles); elliptic geometry, in which the sum of the angles is greater than two right angles and all the lines through a given point intersect a given line in the plane so that no line has a parallel; parabolic geometry, namely that of Euclid. The names of mathematicians most prominently connected with these geometries are: hyperbolic, C. F. Gauss (1777–1855),

German; Wolfgang and John Bolyai, Hungarians, father and son; N. I. Lobatchewski, Russian (1793–1856); elliptic, G. Riemann (1826–1866), German.

There are several specialized branches of classical geometry or mathematical techniques related to it. Analytical geometry, developed by the French mathematician and philosopher, René Descartes (1596–1650) and Pierre Fermat (1601–1665), another Frenchman, is an application of algebraic results to geometry. It is also called coordinate geometry.

Descriptive and projective geometries developed from the interest of both painters and mathematicians in the problem of describing three-dimensional figures on a plane. Prior to the time of the Renaissance artists in general had been satisfied with symbolic representations of persons and objects but subsequently they became increasingly desirous of greater realism in their work. Albrecht Dürer, the German painter and engraver (1471–1528), is thought by some mathematical historians to be the inventor of descriptive geometry. Somewhat later, the French mathematician Gaspard Monge (1746–1818) placed the subject on a firm mathematical basis.

A more generalized development of geometric figures is projective geometry. Its founders include Gaspard Desargues, a French engineer (1593–1662); Blaise Pascal, French geometer and philosopher (1623–1662); Jean Victor Poncelet, French mathematician and general in the armies of Napoleon (1788–1867). One of its important objects is the study of properties invariant under projection and section.

Differential geometry is essentially the application of differential and integral calculus to the study of **curves** and surfaces. Methods of **tensor** calculus are frequently used and it is the chief mathematical apparatus of relativity theory.

Finally, mention should be made of **topology**, which is a study of one-to-one bicontinuous transformations. The usual description of topology as rubber-sheet geometry is sufficiently suggestive for two-dimensional space only, but the topological spaces of most importance in applied (or pure) mathematics, have an infinite number of dimensions; see, e.g., **Hilbert space**.

GEOMETRY FACTOR (RADIATION). The average solid angle at the source sub-

tended by the aperture or sensitive volume of the detector, divided by the complete solid angle (4π). Frequently used loosely to denote counting yield or **counter efficiency**.

GEOPOTENTIAL. The potential energy of a unit mass relative to sea level, numerically equal to the work that would be done in lifting the unit mass from sea level to the height at which the mass is located; commonly expressed in terms of **dynamic height** or **geopotential height**. The geopotential Φ at height z is given mathematically by the expression,

$$\Phi = \int_0^z g dz,$$

where g is the acceleration of **gravity**.

GEOPOTENTIAL HEIGHT. The height of a given point in the atmosphere in units proportional to the potential energy of unit mass (**geopotential**) at this height, relative to sea level. The relation, in the c.g.s. system, between the geopotential height Z and the geometric height z is

$$Z = \frac{1}{980} \int_0^z g dz,$$

where g is the acceleration of gravity, so that the two heights are numerically interchangeable for most meteorological purposes.

GEOPOTENTIAL SURFACE. (Also called **equigeopotential surface**, **level surface**.) A surface of constant **geopotential**, i.e., a surface along which a parcel of air could move without undergoing any changes in its **potential energy**. Geopotential surfaces almost coincide with surfaces of constant geometric height. Because of the poleward increase of the acceleration of **gravity** along a constant geometric-height surface, a given geopotential surface has a smaller geometric height over the poles than over the equator.

GEOSTROPHIC ACCELERATION. The acceleration experienced by a particle moving so that its velocity is always equal to the **geostrophic wind**. A term usually applied only in steady pressure fields.

If the wind is approximately geostrophic the geostrophic acceleration, which is roughly the convective rate of change of the geostrophic wind, is a measure of the ageostrophic wind, and

$$v_A = \frac{1}{f} v_G \cdot \text{grad } v_G$$

the direction being towards low pressure for an acceleration.

Alternatively, in a steady wind field

$$v_A \times \mathbf{f} = (v_G \cdot \text{grad}_H) v_G.$$

GEOSTROPHIC FORCE. Alternatively called the deviating force, or Coriolis force; the apparent force on particles moving relative to the earth when observed from the earth. For unit mass it is equal to $2\mathbf{v} \times \boldsymbol{\Omega}$ where \mathbf{v} is the velocity relative to the earth and $\boldsymbol{\Omega}$ is the earth's rotation vector. The vertical component is negligible compared with gravity and the horizontal component is equal to $\mathbf{v} \times \mathbf{f}$ where \mathbf{f} is twice the vertical component of the earth's rotation.

GEOSTROPHIC VORTICITY. The vorticity of the geostrophic wind, which is equal to $\frac{g}{f} \nabla_H^2 h$ if the variations of f with latitude are ignored, where ∇_H^2 is the two dimensional horizontal Laplacian operator, and h is the height of an isobaric surface.

The rate of change of geostrophic vorticity is a practical measure of the large scale horizontal convergence or divergence of the air which cannot be computed with comparable accuracy from wind measurements.

GEOSTROPHIC WIND. A measure of the horizontal pressure field in the atmosphere. If the isobars are at right angles to the x -axis the geostrophic wind v_G is given by

$$v_G = \frac{1}{\rho f} \frac{\partial p}{\partial x}$$

the direction being along the isobars in a cyclonic direction around low pressure centers, where ρ is the air density and $f = 2\Omega \sin \phi$, is twice the vertical component of the earth's rotation Ω , ϕ being the latitude.

Alternatively the geostrophic wind is given by

$$v_G = \frac{g}{f} \frac{\partial h}{\partial x}$$

where h is the height of an isobaric surface.

In vector form $v_G \times \mathbf{f} = \frac{1}{\rho} \text{grad}_H p$ where \mathbf{f} is a vertical vector and grad_H denotes the horizontal gradient.

The wind is approximately equal to the geostrophic wind when the acceleration is negligible and there is no exchange of momentum with the air above or below.

GERMS; KINETICS OF PHASE FORMATION. Consider a supersaturated vapor. The vapor is in metastable equilibrium with respect to the liquid phase (see **stability of phase**). This means that the **chemical potential** of the vapor is larger than that of the liquid considered as the bulk phase (see **chemical affinity**),

$$\mu^\alpha - \mu_\infty^\beta > 0 \quad (1)$$

where α denotes the vapor phase and β , the liquid phase, and the subscript ∞ indicates that the liquid phase is considered as a bulk phase. The chemical potential of a spherical drop increases when its radius r decreases (see **influence of curvature on chemical potential**). Therefore for sufficiently small drops

$$\mu^\alpha - \mu_r^\beta < 0 \quad (2)$$

where r is the radius of the drop. The vapor is therefore stable with respect to the formation of such drops. There exists a value of r such that

$$\mu^\alpha = \mu_r^\beta. \quad (3)$$

This relation defines a critical size for the drops. Drops of this size are called *condensation germs* (or *condensation nuclei*).

The critical size of the germ decreases rapidly when at constant temperature the vapor pressure is increased above the vapor tension, or when at a given pressure the temperature is decreased below the equilibrium coexistence value. Therefore at some point germs may be formed spontaneously as a consequence of statistical fluctuations. They then grow and the new phase appears. This mechanism explains why the kinetics of phase formation has a highly cooperative character. It also explains why it may be deeply influenced by the presence of impurities or walls. (See also **free energy of germ formation**.)

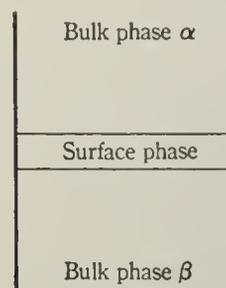
GEV. See **BEV**.

g-FACTOR. See **Landé g-factor**.

GIBBS-DALTON LAW. See **Dalton law**.

GIBBS DIVISION SURFACE. Consider a system consisting of two homogeneous bulk phases α and β separated by a surface phase.

The concentrations vary continuously through the surface phase from those of the interior of one phase to those of the interior of the other. In order to give a well-defined meaning to the thermodynamic functions of the surface phase, independently of the exact position of the boundaries of the surface layer, it is useful, following Gibbs, to replace the real surface phase by a geometrical surface. The bulk phases are considered to be homogeneous up to this geometrical surface, which is called the Gibbs division surface. (See figure.)



Gibbs division surface.

Call C_i^α , C_i^β the concentrations of component i in the bulk phases α , β and V' , V'' the volumes of the bulk phases in the description of the system in terms of the Gibbs division surface. The numbers of moles of component i in the phases prime and second prime will be defined by

$$n'_i = C_i^\alpha V'; \quad n''_i = C_i^\beta V''. \quad (1)$$

The number of moles of component i adsorbed at the surface is then defined by

$$n_i^a = n_i - n'_i - n''_i. \quad (2)$$

This number may be positive, zero or *negative*.

The adsorption of component i is defined as the ratio of n_i^a to the area of the Gibbs surface

$$\Gamma_i = \frac{n_i^a}{\Omega}. \quad (2)$$

An inconvenience of this definition is that the numerical value of the adsorption depends strongly on the exact position of the Gibbs surface. Therefore it is important to express the thermodynamic quantities in terms of *combinations* of Γ_i which are independent of this position. For example it can be shown that the *relative adsorption* of component i in respect to component 1

$$\Gamma_{i1} = \Gamma_i - \Gamma_1 \frac{C'_i - C''_i}{C'_1 - C''_1} \quad (3)$$

is independent of the position of the Gibbs surface. The adsorption of component i may alternatively be defined directly in terms of the molar content of the surface phase itself, by a formula similar to (2). Its numerical value then depends on the assumed position of the boundaries of the surface phase. Again, however, combinations of the adsorptions may be introduced which are independent of the exact specification of these boundaries.

GIBBS-DUHEM EQUATION. This equation expresses a connection between the increments of the **intensive variables** which characterize a thermodynamic system. It is

$$SdT - Vdp + \sum_i n_i d\mu_i = 0. \quad (1)$$

The variations occur in the intensive variables, absolute temperature T , pressure p , and **chemical potential** μ_i , while their coefficients are the extensive variables, entropy S , volume V , and number of moles n_i .

For changes taking place at constant temperature and pressure, Equation (1) reduces to

$$\sum_i n_i d\mu_i = 0. \quad (2)$$

This is simply a form of the **Euler theorem** for the chemical potentials, which are intensive variables.

GIBBS FORMULA FOR THE SURFACE TENSION. The total differential of the surface tension γ in the variables temperature T , and **chemical potentials** μ_i is

$$d\gamma = -s^a dT - \sum_i \Gamma_i d\mu_i \quad (1)$$

where s^a is the entropy per unit area of the surface phase (see **thermodynamic functions of surface phases**) and Γ_i the adsorption of component i (see **Gibbs division surface**).

This formula is the analog for a surface phase, of the **Gibbs-Duhem equation** for a bulk phase.

Another basic formula also due to Gibbs which relates the surface tension to the thermodynamic functions of the surface phase is

$$\gamma = A^a - \sum_i \Gamma_i \mu_i \quad (2)$$

where A^a is the Helmholtz free energy (**Helmholtz function**) per unit area of the surface phase. This expression is the analog of the

relation between the Gibbs free energy and the chemical potentials

$$G = \sum_i n_i \mu_i \quad (3)$$

valid for a bulk phase. Both relations (2) and (3) are related to the **Euler theorem**.

GIBBS FREE ENERGY. See **Gibbs function**.

GIBBS FUNCTION. The potential given by

$$G = H - TS = U + pV - TS$$

where $H = U + pV$ is the **enthalpy**; U is the internal energy; p is the pressure; V is the volume; T is the absolute temperature; and S is the entropy. Some authors use the graphic term free enthalpy, and some others call it free energy. Since the name **free energy** is also given to the **Helmholtz function**, confusion results and it is preferable to use the neutral term Gibbs function. (See also **thermodynamics, characteristic functions of**.)

GIBBS-HELMHOLTZ EQUATIONS. Equations relating the **Helmholtz function** A (or the **Gibbs function** G) to the total energy U (or the enthalpy H) (see **thermodynamics, characteristic functions of**).

$$\left(\frac{\partial \frac{A}{T}}{\partial T} \right)_{V, \xi} = - \frac{U}{T^2} \quad (1)$$

$$\left(\frac{\partial \frac{G}{T}}{\partial T} \right)_{p, \xi} = - \frac{H}{T^2} \quad (2)$$

where p is the pressure, T , the absolute temperature, V , the volume, ξ , the **extent of reaction**.

GIBBSIAN ENSEMBLE. See **ensemble (1)**.

GIBBS-KONOVALOV THEOREMS. Consider a binary system containing two phases (i.e., liquid and vapor). Both components can pass from one phase to another. The Gibbs-Konovalov theorems refer to the properties of the phase diagrams of such systems (see **azotropic systems**). The first theorem is: *At constant pressure, the temperature of coexistence passes through an extreme value (maximum, minimum or inflexion with a horizontal value) if the composition of the two phases is the same, and conversely at a point at which the*

temperature passes through an extreme value, the phases have the same composition. The second theorem is similar. It refers to the co-existence pressure at constant temperature. (See also **indifferent states**.)

GIBBS PARADOX. When two samples of the same gas at a given temperature and pressure are allowed to mingle by the removal of a separating partition, the entropy of the resulting system is equal to the sum of the entropies of the two original parts, and there is no extra term which arises when the two original systems are composed of different gases. This paradoxical absence is called the Gibbs paradox; it can be explained by using the theory of **grand canonical ensembles**.

GIBBS PHENOMENON. Behavior shown by a **function** with a discontinuity, when it is approximated by a finite number of terms in a **Fourier series**. As a simple example, consider $f(x) = 1$ for $0 > x > \pi$; $f(x) = -1$ for $\pi < x < 2\pi$. The sum of the first n terms in the Fourier series converges to ± 1 as $n \rightarrow \infty$, except at the discontinuity, $x = \pi/(2n + 1)$. There the sum approaches 1.179, overshooting the correct value by about 18%.

GILBERT. See **electromagnetic units**.

GINI MEAN DIFFERENCE. See **concentration**.

GIVENS METHOD. A method for computing **eigenvalues** of a **Hermitian matrix**.

GLADSTONE-DALE LAW. If the **refractive index** of a medium is a linear function of its **density**, then the medium satisfies the Gladstone-Dale law.

GLANCING ANGLE. (1) The angle between a ray and the tangent plane to a surface. The complement of the **angle of incidence**. (2) The term is often used as a modifier, to indicate the incidence of a beam at a very small angle with the surface.

GLASS COMPLIANCE. See **compliance, glass**.

GLASS MODULUS. See **modulus, glass**.

GLASS TRANSITION TEMPERATURE. See **glassy state**.

GLASSY STATE. Below the glass transition temperature of a polymer, local order is in-

creased and **viscoelastic** effects are less prominent.

GLIDE PLANE. (1) A **symmetry element** of a **space lattice**, such that the lattice remains unchanged after a **reflection** in the plane, followed by a **translation** parallel to the same plane. (2) Same as **slip plane** in theory of **dislocations**.

GLOBULAR CLUSTERS. See **star clusters**.

GOLDSCHMIDT LAW. The structure of a crystal is determined by the ratio of the numbers, the ratio of the sizes, and the properties of polarization of the atoms or ions from which it is formed.

Thus, there is a change from the face-centered to the body-centered lattice in the alkali chlorides as the cation radius increases. Similarly, there is a change from the rock-salt to the zincblende lattice in the series CdO, CdS, CdSe, CdTe, corresponding to the increasing size and polarizability of the anion.

GÖRTZEL-GREULING APPROXIMATION. In the theory of the slowing down of neutrons in matter, an approximate relationship between the **slowing-down density** q and neutron **flux** $\phi(E)$, first presented by G. Görtzel and E. Greuling, is:

$$q(E) = (\xi\Sigma_s + \gamma\Sigma_a)E\phi(E).$$

Here, Σ_s and Σ_a are macroscopic scattering and absorption cross sections, respectively, ξ is the average logarithmic energy loss for a neutron colliding with a nucleus of mass M , and

$$\gamma = 1 - \frac{\frac{1}{2}\alpha\epsilon^2}{1 - \alpha(1 + \epsilon)}; \quad \epsilon = \ln \frac{1}{\alpha}, \quad \alpha = \left(\frac{M - 1}{M + 1} \right)^2.$$

GÖRTZEL-SELENGUT METHOD. A method of analysis of the energy-dependent transport equation in **neutron transport theory**. A spherical harmonics expansion is made for the angular distribution of flux, and is cut off at the P_1 approximation. Then, in treating the collision integrals, those referring to scattering by hydrogen are treated exactly, while in those describing scattering by "heavy elements" the age approximation is used. (See **spherical harmonics method; age approximation**.)

GÖTHERT'S RULE. This rule is obtained from the linear equation of subsonic flow and applies to any body in a uniform subsonic stream, provided the perturbation velocities are small and the stream Mach number is not close to 1. The **Prandtl-Glauert rule**, which applies only to two-dimensional bodies, is a special case of the Göthert rule.

The Göthert rule relates the pressure coefficient C_p at a point on a given body A in a stream of Mach number M to the pressure coefficient C_{p_0} at the corresponding point on a related body B at zero Mach number. The body B is obtained by an **affine transformation** of the body A , in which all dimensions in directions perpendicular to the undisturbed stream are multiplied by $(1 - M^2)^{1/2}$. The relation between the pressure coefficients is then

$$\frac{C_{p_0}}{C_p} = 1 - M^2.$$

As is the case for the Prandtl-Glauert rule, the Göthert rule does not apply at Mach numbers above the critical Mach number.

GOUDSMIT AND UHLENBECK ASSUMPTION. To explain the **multiplet** structure of spectral lines and the anomalous **Zeeman effect** Goudsmit and Uhlenbeck made, at first independently of quantum mechanics, the assumption that the electron has an intrinsic angular momentum (rotation about its own axis) or **spin** s of magnitude $\frac{1}{2}h/2\pi$ with an associated magnetic moment of 1 **Bohr magneton**.

GOUDSMIT GAMMA-SUM RULE. See **gamma-value**.

g-PERMANENCE RULE. According to Pauli's **g-permanence rule** the sum of the **Landé g-factors** for the components of a given **multiplet** corresponding to a given value of M is independent of the field strength. For a weak field M is the quantum number corresponding to the component in the field direction of the resultant total electronic angular momentum J . For a moderately strong field $M = M_S + M_L$, where M_S and M_L are the quantum numbers of the components in the field direction of the resultant electronic orbital angular momentum L and electron spin S .

GRADE. For a continuous frequency distribution with **distribution** function $F(x)$ the

grade of a value is simply $F(x)$. It is a quantity analogous to that of **rank**, which cannot, of course, be defined for a continuous variable.

GRADIENT. (1) A vector denoting the maximum rate of change of a **scalar** ϕ and directed along the direction of the maximal change. Formally the vector obtained by the application of the vector differential operator **del** ∇ to a scalar point function. In rectangular coordinates it is

$$\text{grad } \phi = \nabla\phi = \mathbf{i} \frac{\partial\phi}{\partial x} + \mathbf{j} \frac{\partial\phi}{\partial y} + \mathbf{k} \frac{\partial\phi}{\partial z}$$

where \mathbf{i} , \mathbf{j} , \mathbf{k} are unit vectors along the X -, Y -, and Z -axes. At any point, P , it is normal to the surface $\phi(x,y,z) = \text{constant}$, which passes through P . (2) The **vector** gradient of a vector field \mathbf{V} is written $\text{grad} \cdot \mathbf{V}$. It is a **tensor** given by the **dyadic** multiplication of the del operator with the vector field \mathbf{V} .

GRADIENT COUPLING. Type of postulated coupling between nucleons and other particles (**mesons**, **β -particles** and **neutrinos**) in which the interaction energy depends explicitly on the first order derivatives of the wave functions with respect to position and time.

GRADIENT WIND. The wind v defined by

$$vf = \frac{1}{\rho} \frac{\partial p}{\partial x} \mp \frac{v^2}{R}$$

where the motion is steady, frictionless, and in circles of radius R , where the x -axis is taken locally towards high pressure. The motion is cyclonic around low pressure and anticyclonic around high pressure, the upper or lower sign being taken, respectively.

The deviating force and the cyclostrophic force balance the pressure gradient force if the air moves with the gradient wind velocity.

As a measure of the actual wind it is an improvement on the **geostrophic wind**, since it takes some account of the curvature of the path of the air, if R is taken as the local radius of curvature of the isobars.

GRADUATION. Another term for **smoothing**.

GRAEFFE METHOD. A **root-squaring method** for solving algebraic equations. Given the algebraic equation, separate odd and even powers of x on opposite sides of the equation:

$$x^n + a_2x^{n-2} + a_3x^{n-4} + \dots = a_1x^{n-1} + a_3x^{n-3} + \dots$$

If both sides are squared, all powers will be even, and x^2 can be replaced by y . Separate in the same way and repeat. After p such operations, let the equation be

$$z^n + c_2z^{n-2} + \dots = c_1z^{n-1} + \dots$$

Then the roots z_i of this equation, and the roots x_i of the first, are related by

$$z_i = x_i^p, \quad \nu = 2^p.$$

Then if

$$|x_1| > |x_2| \geq \dots,$$

it follows that, approximately,

$$c_1 \doteq x_1^p.$$

If

$$|x_1| \geq |x_2| > |x_3| \geq \dots,$$

then, approximately,

$$c_2 \doteq x_1^p x_2^p.$$

And, if

$$|x_1| = |x_2| > |x_3| \geq \dots,$$

then $z_1 = x_1^p$ and $z_2 = x_2^p$ satisfy, approximately,

$$z^2 - c_1z + c_2 \doteq 0.$$

When all roots are distinct,

$$x_2^p \doteq c_2/c_1, \quad x_3^p \doteq c_3/c_2, \quad x_4^p \doteq c_4/c_3, \dots$$

approximately. An enumeration of all possible contingencies resulting from the presence of conjugate pairs equal in modulus is out of the question, but generally speaking, a term that continues to oscillate rather than to increase after continued squaring marks the presence of a conjugate pair, and the pair z_i and z_{i+1} will satisfy approximately

$$c_{i-1}z^2 - c_i z + c_{i+1} = 0$$

(see **symmetric functions**). The method can be generalized to apply to a function expressible in the form

$$f(x) = 1 + a_1/x + a_2/x^2 + \dots$$

If the series converges for $|x| > R$, then all zeros of $f(x)$ exceeding R in modulus can be found. (See Alston S. Householder, *Principles of Numerical Analysis*, McGraw-Hill Book Company, 1953.)

GRAIN BOUNDARY. The surface separating two regions of a solid in which the crystal axes

are differently oriented. It has been shown that such a boundary may be thought of as built up of an array, or network of **dislocations**, whose spacing depends on the tilt θ of the axes across the surface. The energy (per unit area) of a grain boundary is given by

$$E/E_m = (\theta/\theta_m) \{1 - \ln(\theta/\theta_m)\}$$

where E_m and θ_m are parameters depending on the material.

GRAM-CHARLIER SERIES. The series, useful in statistics,

$$F(x) = \sum_{k=0}^{\infty} c_k e^{-x^2/2} H_k(x),$$

where the constants c_k depend on the frequency function (see **distribution**) represented over the interval $[-\infty, \infty]$ and the $H_k(x)$ are the **Hermite polynomials**. The Gram-Charlier series is similar to the **Edgeworth series**, and indeed the two are identical for infinite series; their difference arises in regard to the stoppage point when a finite number of terms only is taken as an approximation, in which case Edgeworth's form is probably preferable.

GRAM DETERMINANT. The Gram determinant of n functions (or vectors) f_1, f_2, \dots, f_n is the determinant of the $n \times n$ matrix whose general term is (f_i, f_j) , namely the scalar product of f_i and f_j . The vanishing of the Gram determinant is necessary and sufficient for linear dependence of the functions.

GRAM-SCHMIDT PROCESS. A method for converting a given set of vectors $\mathbf{u}_1, \mathbf{u}_2, \dots, \mathbf{u}_n$ into an orthonormal set $\mathbf{v}_1, \mathbf{v}_2, \dots, \mathbf{v}_n$. We begin with normalizing the vector \mathbf{u}_1 , by dividing it by its norm $\|\mathbf{u}_1\|$ and setting $\mathbf{v}_1 = \mathbf{u}_1/\|\mathbf{u}_1\|$. We then define a vector \mathbf{w}_2 , orthogonal to \mathbf{v}_1 by setting $\mathbf{w}_2 = \mathbf{u}_2 - (\mathbf{u}_2, \mathbf{v}_1)\mathbf{v}_1$. Next we normalize \mathbf{w}_2 by setting $\mathbf{v}_2 = \mathbf{w}_2/\|\mathbf{w}_2\|$. Again we define \mathbf{w}_3 orthogonal to \mathbf{v}_1 and \mathbf{v}_2 by setting $\mathbf{w}_3 = \mathbf{u}_3 - (\mathbf{u}_3, \mathbf{v}_1)\mathbf{v}_1 - (\mathbf{u}_3, \mathbf{v}_2)\mathbf{v}_2$ and then set $\mathbf{v}_3 = \mathbf{w}_3/\|\mathbf{w}_3\|$ and so forth. (See also **orthogonalization**.)

The process can be equally well applied to functions.

GRAND CANONICAL ENSEMBLE. A grand ensemble with a density ρ given by the equation

$$\rho = e^{-q + \nu n - \beta \epsilon},$$

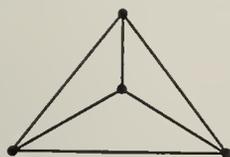
where q is a (normalizing) constant, $\beta = 1/kT$ (k , Boltzmann's constant; T , absolute temperature), ϵ , the energy of the system, $\nu = \beta g$ (g , the partial thermal potential), and n the number of particles of the system.

GRAND ENSEMBLE. See ensemble.

GRAND PARTITION FUNCTION. Same as density of a grand canonical ensemble.

GRAPH, AUTOMORPHISM. An isomorphism (see graph, isomorphic) of a graph with itself.

GRAPH, COMPLETE. A complete graph G is a linear graph in which every two distinct vertices are endpoints of an edge in G . The figure shown below is a complete graph with four vertices.



The total number N of distinct labeled trees in a complete graph containing v vertices is

$$N = v^{v-2},$$

a result due to Cayley. Thus, the above example has 16 trees. They are enumerated under tree.

GRAPH COMPONENT. A component of a graph G is a non-separable maximal connected subgraph. The decomposition of a graph into components is unique.

GRAPH, CONNECTED. A graph is connected if there exists a path between any two vertices. Stated differently, any two distinct vertices β_1 and β_2 are the terminal vertices of some path.

GRAPH CONNECTIVITY. The nullity of a graph G is also known as its connectivity.

GRAPH, DIRECTED. See digraph.

GRAPH, FINITE. Graphs containing only a finite number of line segments and vertices.

GRAPHICAL STATICS. The equilibrium of forces is often treated graphically in such practical problems as the stresses in the members of a framed structure. If three concurrent forces are in equilibrium, the three vectors drawn to a common scale to represent

them may be made to form a closed triangle (Figure 1); or if more than three, a closed



Fig. 1. Three forces in equilibrium.

polygon (Figure 2). The principle may be extended and is much used in the calculation of the forces in a truss by means of the so-called

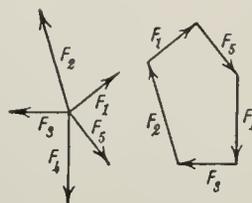


Fig. 2. Five forces in equilibrium.

stress diagram. A simple example is shown in Figure 3, which represents a small roof-truss with equal loads resting on it at the

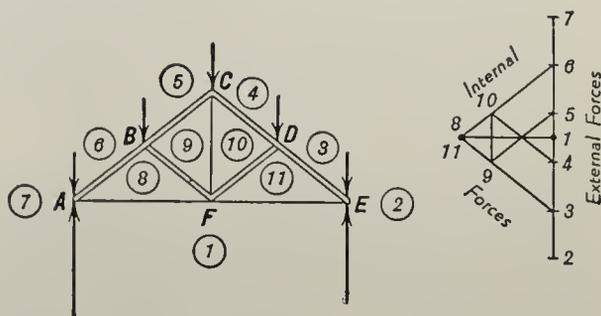


Fig. 3. Elevation of truss with corresponding phase diagram.

joints A, B, C, D, E , and supported by the upward reactions of the walls at A and E . The several compartments of the figure are numbered, and both the external forces and the forces acting along the members between these compartments are represented, both in magnitude and direction, by the lines joining the corresponding numbers in the stress diagram. For example, the compressive force in the strut BF is represented by the line 8-9, while the tension in the vertical rod CF is given by 9-10. The closed figure 5-4-10-9-5 in the stress diagram indicates the equilibrium of the forces acting at the joint C . This method of analysis is attributed to Maxwell.

In the graphical solution of some types of trusses it is found on reaching a particular joint that the arrangement of members is such that there are more than two unknowns. It

is then necessary to replace the unknowns by a substitute system which reduces the number of unknowns at the joint to two. The substitute system consists of a single member inserted in such a way that the truss remains stable and determinate. This member is called a substitute, fictitious or phantom member. When the solution reaches a joint where the stress in the members is unaffected by the substitution, the substitute arrangement is replaced by the original arrangement. The graphic procedure is reversed in direction to find the stress in the original members.

GRAPH, INFINITE. Graphs containing an infinite number of line segments and **vertices**. Such graphs have many interesting mathematical properties but have not as yet found practical application.

GRAPH, LINEAR. A collection of **edges** no two of which have a point in common that is not a **vertex**. The words linear-complex and 1-complex are frequently used alternatives.

As defined here a graph is an abstract graph devoid of any geometric significance whatsoever. It is true, however, that a graph can be interpreted as a **configuration** in 3-dimensional euclidean space.

GRAPH, NON-ORIENTED. A linear graph in which the **elements** have not been assigned an orientation is said to be non-oriented. (See **element, oriented**.) Such a graph is also called *ordinary*.

GRAPH, NON-SEPARABLE. A graph of which every **subgraph** has at least two **vertices** in common with its **complement**.

GRAPH, NULLITY. The nullity μ of a graph G possessing v **vertices**, e **edges** and P **maximal connected subgraphs** is

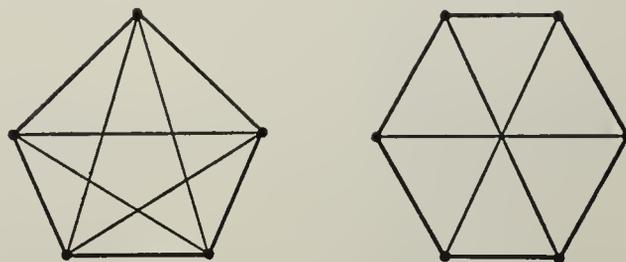
$$\mu = e - v + P \geq 0.$$

GRAPH, ORDINARY. See **graph, non-separable**.

GRAPH, ORIENTED. A linear graph is oriented when an orientation has been assigned to each of its elements (see **element, oriented**). By long-standing convention the phrase "oriented graph" is applied only to graphs which possess at most one directed segment between any two vertices. (For the more general case in which parallel edges are permitted see **digraph** or **graph, directed**.)

GRAPH, PLANAR. A linear graph G can be viewed from either a geometric or a topological standpoint. In the first it is considered to be a collection of **edges** no two of which have a point in common that is not a **vertex**. In the latter it is thought of as defining a set of points in three dimensions whose members are the points which make up the edges of the graph. This point set is the topological graph G^* corresponding to the linear graph G . G is said to be planar if G^* can be mapped on a plane by a one-to-one continuous transformation in such a way that no two image edges have a point in common that is not the image of a vertex in G .

It has been shown by Kuratowski that a linear graph is planar if and only if it does not contain either of the following two graphs as subgraphs:



GRAPH RANK. The rank of a graph G is $v - P$ where v is the number of **vertices** and P the number of **maximal connected subgraphs** of G .

GRAPHS, DUAL. The linear graph G_2 is the dual of the linear graph G_1 if the conditions enumerated below are satisfied:

(a) The **edges** of G_1 and G_2 are in one-to-one correspondence.

(b) If H_1 is any subgraph of G_1 and H_2 is the complement of the corresponding subgraph in G_2 ,

$$r_2 = R_2 - n_1,$$

where r_2 is the **rank** of H_2 , R_2 is the rank of G_2 and n_1 is the **nullity** of H_1 .

It follows easily from this definition that $\text{rank } G_1 = \text{nullity } G_2$ and $\text{rank } G_2 = \text{nullity } G_1$. Furthermore if G_2 is the dual of G_1 , G_1 is the dual of G_2 .

Two extremely useful and significant results are that the dual of a non-separable graph is non-separable and that a linear graph is planar if and only if it possesses a dual.

The usual geometric procedure for finding

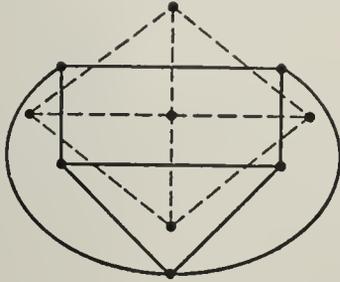
the dual of a planar graph G involves three steps:

(a) Choose a set of fundamental circuits. (See **circuit, fundamental**.)

(b) Put a **node** in each such circuit and a node outside the graph.

(c) Connect any two nodes which are on opposite sides of a branch by a line segment.

The resulting graph is the dual of G . These rules are illustrated in the accompanying diagram in which the dual appears dotted.



GRAPH, SEPARABLE. A connected graph is separable if it contains at least one **sub-graph** which has only one **vertex** in common with its complement. Otherwise the graph is non-separable.

GRAPHS, HOMEOMORPHIC. Two graphs G and G' are homeomorphic if there exists a one-to-one bicontinuous mapping between the two point-sets defined by G and G' . (See also **graph, planar**.)

GRAPHS, ISOMORPHIC. Two graphs G and G' are said to be isomorphic if there exists a one-to-one transformation which maps the **vertices** of G onto the vertices of G' and the **edges** of G onto the edges of G' in such a way as to preserve **incidence** relationships. In other words if vertex β and edge ϵ are incident in G , the respective images β' and ϵ' are incident in G' . The one-to-one transformation referred to above is an isomorphism of G with G' .

GRAPHS, THEORY OF. See various entries under **graph(s)**.

GRAPH, TOPOLOGICAL. See **graph, planar**.

GRASHOF NUMBER. A non-dimensional number G , relating buoyancy forces to viscous forces:

$$G = g\alpha \frac{\Delta T}{T} h \sqrt{\frac{h}{\nu^2}} = gBh^3/\nu^2$$

where B is the buoyancy force per unit mass. It is equal to (**Reynolds number** \div **Froude number**)² and to **Rayleigh number** \div **Prandtl number**.

GRAVITATIONAL CONSTANT. The law of universal gravitation, as stated by Newton, may be expressed as

$$F = G \frac{m_1 m_2}{d^2}$$

When m_1 and m_2 are unit masses and the distance between the centers of mass, d , is unit distance, $F = G$, which is defined as the constant of universal gravitation.

In the c.g.s. system of units $G = 6.670 \times 10^{-8}$. It follows that the attraction between two gram masses one centimeter apart is of the order of a 15-millionth of a dyne.

GRAVITATIONAL FIELD STRENGTH. The force per unit mass resulting from the gravitational attraction of other masses which produce a gravitational field.

GRAVITATIONAL FLATTENING. The constant

$$\frac{g_p - g_e}{g_e}$$

where g_p is the gravitational acceleration at the poles, and g_e is the gravitational acceleration at the equator. This constant occurs in the formula for finding the value of the **gravitational acceleration** at a given latitude.

GRAVITATIONAL POTENTIAL. For a point in a gravitational field (see **field, gravitational**), the amount of work which must be done on a particle of unit mass to move it from the point to infinity. It becomes equal to the potential energy per unit mass at the point in question if the potential energy is taken to be equal to zero at infinity, which is usually the case. If x, y, z are the coordinates of the attracted body of mass m and ξ, η, ζ are the coordinates of the attracting body and $\rho(\xi, \eta, \zeta)$ its density at that point, then the potential energy of m is

$$V^* = -Gm \iiint_B \frac{\rho d\xi d\eta d\zeta}{\sqrt{(\xi - x)^2 + (\eta - y)^2 + (\zeta - z)^2}}$$

the integration to be extended over the entire body. G is Newton's **gravitational constant**. The potential function is

$$\frac{V^*}{m} \equiv V = -G \iiint_B \frac{\rho d\xi d\eta d\zeta}{\sqrt{(\xi-x)^2 + (\eta-y)^2 + (\zeta-z)^2}}$$

V satisfies the Poisson equation:

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 4\pi G\rho(x,y,z).$$

In empty space, this becomes Laplace's equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \frac{\partial^2 V}{\partial z^2} = 0.$$

The force on a point mass is located at the point (x,y,z) is

$$\mathbf{F} = -m \text{grad } V.$$

The solution of the two-dimensional Poisson equation

$$\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} = 2\pi G\rho(x,y)$$

is

$$V = -G \iint \rho(\xi,\eta) \left| \ln \sqrt{(x-\xi)^2 + (y-\eta)^2} \right| d\xi d\eta.$$

The potential of a homogeneous spherical shell with inner radius b , outer radius a and density ρ is:

$$V = -G \cdot 2\pi\rho(a^2 - b^2) \quad \text{for } 0 \leq r \leq b$$

$$V = -G \cdot 4\pi\rho \left(\frac{a^2}{2} - \frac{b^3}{3r} - \frac{r^2}{6} \right) \quad \text{for } b \leq r \leq a$$

$$V = -G \cdot \frac{4\pi}{3} \rho \left(\frac{a^3 - b^3}{r} \right) \quad \text{for } a \leq r$$

the potential of an infinitely long, thin, homogeneous rod if density ρ (= mass per unit length)

$$V = -2G\rho |\ln r|$$

where r is the perpendicular distance from the rod.

GRAVITATIONAL RADIUS. The gravitational radius of a mass m is the length Gm/c^2 (G is the gravitational constant, c , the velocity of light in vacuo). For the sun this is 1.47 km, for the earth 5 mm.

GRAVITATIONAL SYSTEM OF UNITS. See mechanical units.

GRAVITATION, NEWTONIAN. See Newton law of universal gravitation.

GRAVITY, CENTER OF. See center of gravity.

GRAVITY WAVE. (Also called gravitational wave.) A wave disturbance in which buoyancy (or *reduced gravity*) acts as the restoring force on parcels displaced from **hydrostatic equilibrium**. There is a direct oscillatory conversion between potential and kinetic energy in the wave motion. Pure gravity waves are stable for fluid systems which have **static stability**. This static stability may be (a) concentrated in an interface or (b) continuously distributed along the axis of gravity.

GRAY BODY (NON-SELECTIVE RADIATOR). A gray body or a non-selective radiator (over the visible region of the spectrum) is a radiator whose spectral emissivity is independent of wavelength within this region.

GRAY SCALE. A series of achromatic tones ranging from black to white. A gray scale may be divided into three or more steps but 10 is a common number of divisions. A gray scale is sometimes included with the subject when making a color photograph so that measurements of its densities on the separation negatives or tripack will give the density range of that stage in the reproduction. A gray scale is helpful in controlling the processing stages in the analysis and synthesis of a color photograph.

GREAT CIRCLE CHART. (GREAT CIRCLE SAILING, COMPOSITE SAILING.) The great circle chart, on the gnomonic projection, is of considerable value to navigators in planning long voyages by sea or by air. The earth is presumed to be a sphere. The projection is perspective in character and corresponds to the picture of the spherical earth that would be received on a plane tangent to the surface of the earth by rays of light from the center.

Any plane through the center of the earth intersects the surface in a great circle. The trace of this plane on the tangent plane is a straight line. On the chart meridians of longitude are straight lines but not parallel, while parallels of latitude are curved. The graticule of latitude and longitude is printed on the chart, together with outlines of the land masses. Some directions relative to the use of the chart are also printed.

A straight line between any two points on the chart is the great circle track between the two points. This is neither the shortest distance nor the geodesic distance, but the difference between the two is a very small percentage of the total.

Any two points 180° apart on the great circle have the same latitude numerically, but with different signs, unless the great circle is the equator. The two points of greatest latitude along the great circle are called vertices. On either side of a vertex the latitude decreases until the equator is reached 90° from the longitude of the vertex.

Great circle sailing is employed when a ship wishes to take advantage of the shortest distance, rather than to follow the rhumb line of constant course. Because of distortion, the rhumb line appears to be shorter than the great circle, on the Mercator chart.

Composite track. When the great circle track between two points passes into regions unsuited for navigation (e.g., the great circle from north-western ports of the United States to Japan passes north of the Aleutian Islands) a composite track is used as the shortest available track. First, a limiting parallel is selected as being the highest latitude that should be used. Then great circles are computed from the point of departure and the point of destination that are parallel to the limiting parallel. The ship leaves and follows the great circle to the point of tangency. Then she steers east or west along the limiting parallel until the point where the great circle from destination is tangent to the limiting parallel. The ship now leaves the limiting parallel and follows the great circle to destination.

The problems relative to great circle and composite sailing may be solved on the great-circle chart. However, various methods have been devised for arriving at the same result by computation, and the results are far more accurate. Various methods for solving great circle and composite tracks are found in treatises on navigation such as *U.S. Hydrographic Office Publication #9*.

GREEN'S FORMULA. See Green's theorem.

GREEN'S FORMULA, FOUR DIMENSIONAL. See Minkowski world.

GREEN'S FUNCTION. A function $K(x, \xi)$ associated with a given boundary value prob-

lem $L[y] = \phi(x)$ that has the property that the solution is expressible in the form

$$y(x) = \int K(x, \xi) \phi(\xi) d\xi,$$

the integration extending over the interval on which the solution is to be defined. As an example, if $L[y] \equiv y''$, and the boundary conditions are $y(0) = y(1) = 0$, then

$$K(x, \xi) = \begin{cases} (1 - \xi)x & \text{for } x \leq \xi, \\ (1 - x)\xi & \text{for } x > \xi. \end{cases}$$

The limits of integration are then from 0 to 1. The notion can be generalized to spaces of higher dimensionality. For theoretical investigations the function is of fundamental importance, but for numerical computation it is seldom used.

GREEN'S IDENTITY. A class of integral identities useful in the study of **linear partial differential equations**. In particular, for a linear partial differential equation of the second order

$$L\phi = a\phi_{xx} + 2b\phi_{xy} + c\phi_{yy} + 2d\phi_x + 2e\phi_y + f\phi = g \quad (1)$$

where a, b, c, d, e, f, g are known functions of x and y , and (1) is a **hyperbolic equation** (i.e., such that $ac - b^2 < 0$), Green's identity is given by

$$\iint_R (\psi L\phi - \phi \bar{L}\psi) dx dy = \int_C \left[\sigma \left(\psi \frac{\partial \phi}{\partial x} - \phi \frac{\partial \psi}{\partial x} \right) + B\phi\psi \right] ds \quad (2)$$

where the adjoint operator \bar{L} is defined by

$$\bar{L}\psi = (a\psi)_{xx} + (2b\psi)_{xy} + (c\psi)_{yy} - (2d\psi)_x - (2e\psi)_y + f\psi, \quad (3)$$

R is a region with boundary curve C for which the exterior normal has direction cosines

$$\xi, \eta; \sigma = \sqrt{(a\xi + b\eta)^2 + (b\xi + c\eta)^2};$$

$$B = (2d - a_x - b_y)\xi + (2e - b_x - c_y)\eta;$$

and $\partial/\partial\nu$ is the directional derivative in the binormal direction, which is the direction with direction cosines

$$\xi' = \frac{a\xi + b\eta}{\sigma}, \quad \eta' = \frac{b\xi + c\eta}{\sigma},$$

i.e.,

$$\frac{\partial}{\partial v} = \xi' \frac{\partial}{\partial x} + \eta' \frac{\partial}{\partial y}.$$

This identity is useful in the **Riemann method** of solving the **Cauchy problem**.

GREEN'S THEOREM. Any one of several theorems relating an integral over a domain in n dimensions to an integral over a domain in $(n - 1)$ dimensions. In its most usual form it reads

$$\int_C \{P(x,y)dx + Q(x,y)dy\} = \int_A \left\{ \frac{\partial Q(x,y)}{\partial x} - \frac{\partial P(x,y)}{\partial y} \right\} dA,$$

where $P(x,y)$ and $Q(x,y)$ have continuous first partial derivatives in the domain A .

Green theorems for more than two dimensions are closely related to various theorems called by other names, e.g., Gauss theorem. For example, if u, v are scalar functions, S indicates a double and τ a triple integral, the Gauss theorem in vector form is

$$\int_{\tau} \nabla u \cdot \nabla v d\tau + \int_{\tau} u \nabla^2 v d\tau = \int_S u \nabla v \cdot dS,$$

and on exchanging u and v and subtracting the result from this equation, the Green theorem results

$$\int_{\tau} (u \nabla^2 v - v \nabla^2 u) d\tau = \int_S (u \nabla v - v \nabla u) \cdot dS.$$

In one dimension, the Green theorem runs

$$\int_a^b [vL(u) - u\bar{L}(v)] dx = [P(u,v)]_a^b$$

where \bar{L} is the operator adjoint to L (see **adjoint of an operator**) which in unintegrated form is also called *Lagrange's identity*. In general, Green theorems are equivalent to integration by parts.

GREGORY FORMULA (GREGORY-NEWTON FORMULA). A formula for numerical evaluation of an integral. It is obtained from the **Newton formula for interpolation** and may be written

$$\begin{aligned} \int_{x_0}^{x_r} f(x) dx &= h \left(\frac{1}{2} f_0 + f_1 + f_2 + \cdots + \frac{1}{2} f_r \right) \\ &- (\Delta f_{r-1} - \Delta f_0) / 12 - (\Delta^2 f_{r-2} + \Delta^2 f_0) / 24 \\ &- 19(\Delta^3 f_{r-3} - \Delta^3 f_0) / 720 \\ &- 3(\Delta^4 f_{r-4} + \Delta^4 f_0) / 160 + \cdots \end{aligned}$$

where h is the interval between equally-spaced values of the independent variable x and the quantities $\Delta^m f_k$ are **finite differences**.

GRID, AS MEANS OF GENERATING TURBULENCE. In many of the experiments that have been made on turbulence, a square-mesh grid of bars has been used in a wind tunnel to generate the turbulence. Until recently it was thought that the turbulence produced in this way, at a distance from the grid greater than about 20 mesh lengths, was very nearly homogeneous and isotropic. It has been shown recently, however, that there is a lack of isotropy in the large scale components of the turbulence and that even at large distances from the grid the variation of intensity across the stream is of order $\pm 10\%$.

GROSS CALORIFIC VALUE. See **combustion**.

GROSS SECTION. When a prismatic member contains holes or transverse slots or notches in a limited region, the cross-sectional area of the prism is called the gross area or gross section. A cross-sectional area from which material has been removed is called a *net area* or *net section*.

GROUND EFFECT (ON AIRPLANE). When an airplane is flying near the ground the air flow is modified by the presence of the ground. The effect may be calculated by considering the effect on the flow of an inverted image of the airplane below the ground, so that the condition of zero vertical velocity component at the ground is satisfied. Since the most important effects are caused by the trailing vortices of the image airplane, it is usually sufficiently accurate to represent the image airplane by a simple horse-shoe vortex, i.e., a uniformly loaded lifting line with trailing vortices springing from the tips.

The trailing vortices of the image airplane induce an upwash at the real airplane. This leads to a reduction of drag and incidence for a given value of the lift coefficient. An effect that is more important in practice is the reduction of downwash angle at the tail, which means that the upward elevator deflection required to maintain equilibrium at a given lift coefficient is increased.

GROUND FORMS OF A SURFACE. u^α ($\alpha = 1, 2$) are curvilinear coordinates on a

surface. The indicial notation and summation convention are used, repeated Greek indices denoting summation over the values 1, 2. \mathbf{r} denotes the vector from an origin to the point u^α on the surface; \mathbf{n} denotes the **unit normal** to the surface at that point.

First ground form: The quadratic differential form $a_{\alpha\beta}du^\alpha du^\beta$, where $a_{\alpha\beta}$ is the covariant **metric tensor** for the surface, or more loosely the metric tensor for the surface, defined by

$$a_{\alpha\beta} = \frac{\partial \mathbf{r}}{\partial u^\alpha} \cdot \frac{\partial \mathbf{r}}{\partial u^\beta}.$$

$a^{\alpha\beta}du^\alpha du^\beta$ is the square of the length of a linear element on the surface joining the points u^α and $u^\alpha + du^\alpha$. It is also called the *first fundamental form for the surface*. The quantities $a_{\alpha\beta}$ are also called the *fundamental magnitudes of the first order for the surface*. The contravariant **metric tensor** for the surface $a^{\alpha\beta}$ is defined by $a^{\alpha\beta}a_{\beta\gamma} = \delta_\gamma^\alpha$, where δ_γ^α is the **Kronecker delta**.

Second ground form: The quadratic differential form $b_{\alpha\beta}du^\alpha du^\beta$, where $b_{\alpha\beta}$ is defined by

$$b_{\alpha\beta} = -\frac{1}{2} \left(\frac{\partial \mathbf{n}}{\partial u^\alpha} \cdot \frac{\partial \mathbf{r}}{\partial u^\beta} + \frac{\partial \mathbf{n}}{\partial u^\beta} \cdot \frac{\partial \mathbf{r}}{\partial u^\alpha} \right),$$

and \mathbf{n} is the **unit normal** to the surface. Also called *second fundamental form for the surface*. The quantities $b_{\alpha\beta}$ are the *fundamental magnitudes of the second order for the surface*.

Third ground form: The quadratic differential form $c_{\alpha\beta}du^\alpha du^\beta$, where $c_{\alpha\beta}$ is defined by

$$c_{\alpha\beta} = a^{\sigma\tau} b_{\sigma\alpha} b_{\tau\beta}.$$

Also called *third fundamental form for the surface*.

GROUND STATE. The lowest energy state of a stationary quantum mechanical system, particularly of a nucleus.

GROUP. A set of elements, finite or infinite in number, satisfying the following conditions: (1) There is a defined operation by which to each ordered pair of elements A and B in the group \mathbf{G} there is associated an element C of \mathbf{G} , denoted by $C = AB$, and called the **product** of A and B . (2) For this operation the **associative** law holds: $(AB)C = A(BC) = ABC$ for any three elements A, B, C of \mathbf{G} . There exists: (3) a unit element E in \mathbf{G} such that $EA = A$ for every element A of \mathbf{G} , and (4) to

each element A of \mathbf{G} a **reciprocal** (or inverse) element A^{-1} of \mathbf{G} such that $A^{-1}A = E$.

It must be understood that product, as defined in (1), is a convenient word to use for the result of combining two or more elements in a group but the law of combination is not confined to multiplication. For example, let the group elements be the integers $0, \pm 1, \pm 2, \dots$ and let the combination law be addition, then the product of any two elements is their algebraic sum. These integers, regarded as elements of a group, will be seen to satisfy the requirements (1)–(4).

A finite group containing n elements is of order n . If $m \leq n$ elements satisfy the requirements of (1)–(4), they form a subgroup. Every group contains at least two subgroups: the unit element and the group itself.

If X is an element of a group \mathbf{G} and \mathbf{H} is a subgroup of \mathbf{G} , then the set of elements \mathbf{HX} consisting of all products HX with H in \mathbf{H} is called a right coset and $X\mathbf{H}$ is a left coset. In general, cosets are not groups because they do not contain E , the unit element. If A, B, X are three elements of a group, then $B = X^{-1}AX$ is the **transform** of A by X and A, B are conjugate to each other. The complete set of group elements conjugate among themselves is called a *conjugate class*, or merely a *class*, of the group.

If \mathbf{H} is a subgroup of the group \mathbf{G} and X is an element of \mathbf{G} , not necessarily contained in \mathbf{H} , then $X^{-1}\mathbf{H}X$ is also a subgroup of \mathbf{G} and is said to be conjugate to \mathbf{H} . A subgroup \mathbf{H} which is identical with every one of its conjugate subgroups is said to be *invariant*, or to be a *normal subgroup* or *normal divisor* of \mathbf{G} .

Suppose \mathbf{H} is an invariant subgroup of a group \mathbf{G} and that $\mathbf{HX}, \mathbf{HY}, \dots$ are its cosets. If the product of \mathbf{HX} and \mathbf{HY} is defined as the coset containing the product XY , then the set of cosets of \mathbf{H} is itself a group, called a *quotient* or *factor group* and denoted by \mathbf{G}/\mathbf{H} . For particular groups, permutation, symmetric, etc., see under the respective names. (For groups of linear, full linear, unitary transformations, etc., see under **Lie group**.)

GROUP, ALTERNATING. See **alternating group**.

GROUP DIFFUSION METHOD. See **neutron diffusion theory, multi-group**.

GROUP, LIE. See **Lie group**.

GROUP, NEUTRON. In neutron transport theory, the collection of all neutrons in a system whose energies lie within a specified band.

GROUP OF RESTRICTED HOMOGENEOUS LORENTZ TRANSFORMATIONS.

That subgroup of the **homogeneous Lorentz group** for which $\det \Lambda = +1$ and $\Lambda_0^0 \geq 1$. The following are some important facts about the restricted homogeneous Lorentz group: The group has both finite and infinite dimensional irreducible representations. However, the only *finite* dimensional irreducible representation is the one-dimensional trivial representation $\Lambda \rightarrow 1$. There exist a denumerable infinity of non-equivalent finite dimensional irreducible representations of the restricted homogeneous Lorentz group. (In general, these are non-unitary representations.) These finite dimensional irreducible representations can be labeled and characterized by two non-negative indices n, m , where $n, m = 0, \frac{1}{2}, 1, \frac{3}{2}, \dots$. The representation $\mathbf{D}(n, m)$ has the dimension $(2n + 1)(2m + 1)$; it is single valued if $n + m$ is integral and double valued otherwise. The basis vectors in the vector spaces, $V^{(n, m)}$ and $V^{(m, n)}$, on which the representations $\mathbf{D}(n, m)$ and $\mathbf{D}(m, n)$, respectively, act can be so chosen that the representation matrices for $\mathbf{D}(n, m)$ are the complex conjugates of those for $\mathbf{D}(m, n)$. A quantity which transforms under $\mathbf{D}(0, 0)$ is a scalar, one which transforms under $\mathbf{D}(\frac{1}{2}, \frac{1}{2})$ a 4-component vector, one which transforms under $\mathbf{D}(\frac{1}{2}, 0)$ a spinor, one which transforms under $\mathbf{D}(0, \frac{1}{2})$ a conjugate spinor, etc.

GROUP RELAXATION. A variant of the total-step iteration for solving a linear system (see **matrix inversion**) in which at each step two or more equations are satisfied simultaneously. Same as **block relaxation**.

GROUPS, TRANSFORMATION. See **transformation groups**.

GROUP VELOCITY. The velocity C of propagation of a group of waves formed by the superposition of wave trains of adjacent wavelength. If c is the phase velocity and C the group velocity,

$$C = c - \lambda \frac{\partial c}{\partial \lambda}$$

where λ is the wavelength. $C = c$ in a non-dispersive medium.

C is the velocity with which the wave energy is propagated. If a disturbance of finite duration is propagated and is dispersed into a series of waves, that part of the disturbance in which the oscillations are of wavelength λ travels with the corresponding group velocity, while the individual crests travel with their phase velocity, changing their length as they travel, and advancing or moving backwards relative to the disturbance (the group) according as $\frac{\partial c}{\partial \lambda} \geq 0$. In the former case the wave amplitude dies away as the waves approach the front of the group and new crests appear at the rear.

GROWTH CURVE. (1) A curve showing the general pattern of growth of a variable. (2) An **activity curve** in which the activity increases with time, or that portion of an activity curve showing such an increase. (3) A theoretical or experimental curve showing, as a function of time, the number of atoms, or the mass, or the activity of a nuclide being produced in a **radioactive transformation** or in an **induced nuclear reaction**.

GRÜNEISEN CONSTANT (OR GRÜNEISEN GAMMA). The constant γ occurring in the relation connecting the linear expansion coefficient β with the compressibility K and the specific heat C_V , i.e.,

$$\beta = K\gamma C_V / 3V$$

where V is the volume. This relation is reasonably well satisfied for cubic crystals. According to the theory by which it is deduced,

$$\gamma = - \frac{V d\Theta}{\Theta dV},$$

where Θ is the **Debye temperature**. (See **Mie-Grüneisen equation of state**.)

GRÜNEISEN FORMULA. An empirical formula for the variation of the **electrical resistivity** of a very pure metal with temperature, of the form

$$\rho \propto TG(\Theta/T)$$

where

$$G(x) = x^{-4} \int_0^x \frac{s^2 ds}{(e^s - 1)(1 - e^{-s})}.$$

There is good theoretical justification for a formula of this type, but the parameter Θ

should not be taken literally as a measure of the Debye temperature.

g-SUM RULE. (Pauli's g-sum rule.) In a weak magnetic field, for the group of atomic states arising from a given **electron configuration**, the sum of the Landé g-factors for all levels with the same value of J , the quantum number of resultant total electronic angular momentum, is a constant independent of the **coupling** conditions. In a stronger magnetic field where J is no longer defined, the rule applies to the levels with a given M , the quantum number of the component in the field direction of the resultant total electronic angular momentum.

In a field of intermediate strength M is to be taken as $M_S + M_L$, the sum of the quantum numbers of the components in the field direction of the resultant spin and resultant orbital angular momentum of the electrons. In a very strong field M stands for $\sum_i (m_{l_i} + m_{s_i})$, the sum of the quantum numbers of the components in the field direction of the spins and angular momenta of the individual electrons.

GUEST YIELD CONDITION. A linear version of the Mohr criterion:

$$\sigma_1 - \sigma_3 = c_1 + c_2(\sigma_1 + \sigma_3), \quad \sigma_1 \geq \sigma_2 \leq \sigma_3$$

where σ_1 , σ_2 and σ_3 are principal stresses.

GUIDED WAVE. A wave whose energy is concentrated near a boundary, or between substantially parallel boundaries, separating materials of different properties, and whose **direction of propagation** is effectively parallel to these boundaries.

GUIDING CENTER. The motion of a charged particle in a slowly varying (both spatially and temporally) electromagnetic field can be approximated by a rapid gyration around a moving point. This point is called the guiding center of the particle. This approximation is termed the guiding center theory.

GUIDING CENTER DRIFTS. In the **guiding center** approximation, the motion of the guiding center is described in terms of a drift velocity. An electric field \mathbf{E} produces a zeroth order drift velocity \mathbf{v} given by

$$\mathbf{v} = c\mathbf{E} \times \mathbf{B}/B^2,$$

where \mathbf{B} is the magnetic field and c is the velocity of light.

GULDBERG AND WAAGE LAW. See law of mass action.

GYROMAGNETIC RATIO. The ratio of the magnetic moment to the mechanical angular momentum of a system. In spectroscopy, as a rule, denoted by g and given in units $e/2mc$ (Bohr magneton divided by $h/2\pi$) or denoted by g_I and given in units $e/2m_p c$ (nuclear magneton divided by $h/2\pi$).

I. *Gyromagnetic ratio, electronic*

(a) Orbital motion of an electric point charge. An electron traveling around a circular orbit of radius r f times per second has a mechanical angular momentum of magnitude

$$p = 2\pi f m r^2$$

and generates a magnetic moment of magnitude

$$\mu = e f \pi r^2 / c = \frac{e}{2mc} p$$

(e and m are electronic charge and mass, c is velocity of light). The classical gyromagnetic ratio of an electron moving in an orbit is therefore

$$\mu/p = e/2mc \quad \text{or} \quad g = 1.$$

In an atom, for an orbital angular momentum \mathbf{L} of magnitude

$$p_L = \sqrt{L(L+1)} \frac{h}{2\pi}$$

the magnetic moment has the magnitude

$$\mu_L = \frac{e}{2mc} \frac{h}{2\pi} \sqrt{L(L+1)} = \mu_o \sqrt{L(L+1)}$$

(μ_o , Bohr magneton). The gyromagnetic ratio is, therefore as above,

$$\mu_o/p = \frac{e}{2mc} \quad \text{or} \quad g_o = 1.$$

(b) Electron spin. The magnetic moment associated with the intrinsic electronic angular momentum (spin) \mathbf{s} , of magnitude

$$p_s = \sqrt{s(s+1)} h/2\pi \quad (s = \frac{1}{2})$$

is given by

$$\mu_s = \sqrt{s(s+1)} \frac{e}{mc} \frac{h}{2\pi}.$$

The gyromagnetic ratio of the spinning electron is therefore

$$\mu_s/p_s = 2 \times \frac{e}{2mc} \quad \text{or} \quad g_s = 2.$$

(Actually, on account of the quantum electro-dynamical electron moment correction

$$g_s = 2 \times \left[1 + \frac{\alpha}{2\pi} - 2.973 \frac{\alpha^2}{\pi^2} \right] \\ = 2 \times 1.00114536.)$$

(c) Orbital motion of spinning electron. See **Landé g-factor**.

II. Magnetic ratio, nuclear

The magnetic moment associated with the intrinsic nuclear angular momentum (spin) \mathbf{I} , of magnitude

$$p_I = \sqrt{I(I+1)} h/2\pi$$

is given by

$$\mu_I = \frac{e}{2m_p c} \frac{h}{2\pi} \sqrt{I(I+1)}.$$

Here m_p is the mass of the proton and g_I the nuclear g-factor.

The nuclear gyromagnetic ratio depends in a complicated way on the structure of the nucleus concerned, and cannot be expressed by a simple formula.

For the proton the g-factor is

$$g_I = 5.5853 \quad (I = \frac{1}{2}).$$

(For further data on nuclear g-factors see H. Kopfermann, *Nuclear Moments*, Academic Press, Inc., New York, 1958.)

GYROSCOPE. A gyroscope is a rotating object with a very high angular momentum about a principal axis through its center of gravity. The spin does not diminish rapidly

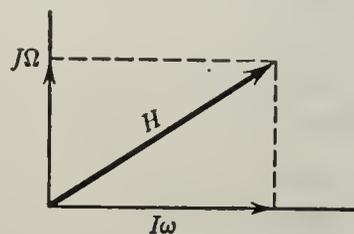
with time. When suspended freely, the moment about the center of gravity is zero and the spin axis remains fixed in space. The change in total angular momentum is then very closely the change in the spin momentum. It is denoted by:

$$\mathbf{M} = I\boldsymbol{\omega}$$

where \mathbf{M} is the angular momentum due to rotation, I is the moment of inertia and $\boldsymbol{\omega}$ is the spin. ($I = \frac{mk^2}{g}$, where m is the mass of the rotating element, g is the acceleration of gravity, and k is the radius of gyration.) Similarly, a gyroscope also has a precession momentum given by:

$$\mathbf{N} = J\boldsymbol{\Omega}$$

where \mathbf{N} is the precession momentum, J is moment of inertia of the gyroscope about the precession axis, and $\boldsymbol{\Omega}$ is the precession. The



vector sum of \mathbf{M} and \mathbf{N} is the total angular momentum, \mathbf{H} . That is,

$$\mathbf{H} = \mathbf{M} + \mathbf{N} = I\boldsymbol{\omega} + J\boldsymbol{\Omega}.$$

The time rate of change of the vector \mathbf{H} , ($d\mathbf{H}/dt$), is defined as the torque. Thus,

$$d\mathbf{H}/dt = \boldsymbol{\Omega} \times \mathbf{H}$$

$$\boldsymbol{\Omega} \times \mathbf{H} = \boldsymbol{\Omega} \times I\boldsymbol{\omega} + \boldsymbol{\Omega} \times J\boldsymbol{\Omega}$$

$$\boldsymbol{\Omega} \times \mathbf{H} = d\mathbf{H}/dt = \boldsymbol{\Omega} \times I\boldsymbol{\omega} \quad (\text{since } \boldsymbol{\Omega} \times J \rightarrow 0)$$

$$\mathbf{T} = \boldsymbol{\Omega} \times I\boldsymbol{\omega}.$$

H

HAAG'S THEOREM. The statement that any **relativistic quantum field theory** which has the following four properties: (a) relativistic transformation properties, (b) a unique, normalizable invariant vacuum state Ψ_0 and no negative energy states or states of space-like momenta, (c) canonical commutation rules at equal times, and (d) relation to the free field theory at a given time by a unitary transformation, is completely equivalent to a free non-interacting field theory.

HALF-LIFE. The half-life of a substance is the time required for one-half of it to undergo a disintegration or, in some cases, a reaction process. Thus, the half-life of a radioactive substance is the time required for disintegration of one-half the atoms of a sample of it. The relation of the half-life to the **disintegration constant** and the **mean life** is as follows:

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda} = 0.693\tau$$

where $t_{1/2}$ is the half-life, λ is the disintegration constant and τ is the mean life.

Again, the half-life of a free radical is the time required for one-half of those of a single kind present in the substance or system to undergo transformation. The biological half-life of a substance is the time in which a living tissue, organ or individual eliminates, through biological processes, one-half of a given amount of a substance which has been introduced into it. The effective half-life is a term usually applied to a radioactive substance in a biological organism. It is defined in terms of the half-life of the radioactive substance itself, and its biological half-life in the organism, by the following expression:

Effective half-life

$$= \frac{\text{Radioactive half-life} \times \text{Biological half-life}}{\text{Radioactive half-life} + \text{Biological half-life}}$$

HALF-PERIOD ELEMENT. The portion of a wave front contained between any two con-

centric spheres so that the difference of their radii equals one half wavelength. It is also called a *Fresnel zone* or a *Huygens' zone* of the **wave front**.

HALF-POWER WIDTH OF A RADIATION-LOBE. In a plane containing the direction of the maximum of the lobe, the full angle between the two directions in that plane about the maximum in which the radiation intensity is one-half the maximum value of the lobe.

HALF-SPACE PROBLEM. See **problem of Boussinesq and Cerruti**.

HALF-THICKNESS. The thickness of a particular absorber that will reduce the intensity of a beam of radiation to one-half its initial value. If the absorption is exponential, the half-thickness is related to the linear or mass **absorption coefficient** and the mean free path as follows:

$$d_{1/2} = \frac{\ln 2}{\mu} = \frac{0.693}{\mu} = 0.693l$$

where $d_{1/2}$ is the half-thickness, μ is the absorption coefficient and l is the mean free path.

HALF-WIDTH. If $y = f(x)$ is a function such that it has a maximum y_m at x_m and falls off rapidly on each side of this maximum, the half-width of the function is the difference $x_2 - x_1$ between the two values of x for which $y = y_m/2$; e.g., the half-width of the **error function** integrand e^{-y^2} is 1.67.

HALF-WIDTH OF A SPECTRAL LINE. A measure for the range in wavelength, or in units related to the wavelength, over which a spectral line has a measurable intensity. The definition of the term is essentially a matter of convenience for the purpose in hand. The term therefore occurs in the literature with several different denotations. For emission lines it may refer to (a) the distance from the center of the line to the point where the intensity has fallen to $1/2$ times its maximum value (also called "half-intensity-width"), or (b) the distance from the center of the line to the

point where the intensity has fallen to $1/e$ times its maximum value. For absorption lines it refers to the distance from the center of the line to the point where the **absorption coefficient** has fallen to $1/2$ times its maximum value. Twice the half-width is called "whole half-width," or simply "width" or "breadth" of the spectral line. For the limiting case of pure thermal broadening the half-width $\Delta\lambda$ for emission lines according to definition (a) and for absorption lines is given by

$$\Delta\lambda = \sqrt{\ln 2} (\Delta\lambda)_D$$

and for emission lines according to definition (b) by

$$\Delta\lambda = (\Delta\lambda)_D$$

where $(\Delta\lambda)_D$ is the **Doppler half-width** of the spectral line. For the limiting case of negligible thermal broadening the corresponding values are

$$\Delta\lambda = \frac{\gamma}{4\pi c} \lambda^2$$

and

$$\Delta\lambda = \frac{\gamma}{2\pi c} \lambda^2$$

where γ is a parameter depending on the gas pressure, and, in the presence of an external electric or magnetic field, the field strength.

The expression $\frac{\gamma}{2\pi c} \lambda^2$ sometimes is referred to as one half the "natural line width." However the latter term is more commonly used in the restricted sense given under the heading **natural line width**.

HALF-WIDTH OF RESONANT PEAK. See resonance, sharpness of.

HALL ANGLE. The ratio of the electric field E_y , developed across the current, to the field E_x , generating the current, in the magnetic field H_z , as a result of the Hall effect. (See **galvanometric and thermometric effects**.)

HALL COEFFICIENT (HALL EFFECT). See galvanometric and thermometric effects.

HALL MOBILITY. The mobility of electrons or holes in a semiconductor as measured by the Hall effect. (See **galvanometric and thermometric effects**.) It is given by

$$\mu_H = c\theta/H_z$$

where θ is the **Hall angle** in the magnetic field H_z , and c is the velocity of light.

HAMILTON-CAYLEY THEOREM. Every matrix satisfies its own characteristic equation.

HAMILTON CHARACTERISTIC. If $P(x,y,z)$ is a point in object space, and $P'(x',y',z')$ is a point in image space, then the optical path length $V(x,y,z,x',y',z')$ of the ray joining P and P' is the **characteristic function** of Hamilton, or the **cikonal** of Bruns, of an optical system. If n and n' are the refractive indices of object and image spaces then

$$\frac{\partial V}{\partial x} = p, \quad \frac{\partial V}{\partial y} = q, \quad \frac{\partial V}{\partial z} = r,$$

$$\frac{\partial V}{\partial x'} = -p', \quad \frac{\partial V}{\partial y'} = -q', \quad \frac{\partial V}{\partial z'} = -r',$$

where (p,q,r) and (p',q',r') are the optical direction cosines of the ray at the object and image points, respectively; thus V satisfies

$$\left(\frac{\partial V}{\partial x}\right)^2 + \left(\frac{\partial V}{\partial y}\right)^2 + \left(\frac{\partial V}{\partial z}\right)^2 = n^2,$$

$$\left(\frac{\partial V}{\partial x'}\right)^2 + \left(\frac{\partial V}{\partial y'}\right)^2 + \left(\frac{\partial V}{\partial z'}\right)^2 = (n')^2.$$

Any solution of this system of equations is a characteristic function. To distinguish V it is referred to as the *point characteristic*. Other commonly occurring characteristics are called the mixed characteristic, the angle characteristic, and the wave front characteristic.

A characteristic function forms the basis of the direct method in geometrical optics; all properties of the system can be derived from the characteristic. (See **Hamilton theorem**.)

HAMILTONIAN (HAMILTONIAN FUNCTION OF A SYSTEM). Generally denoted by the symbol H . It is defined by the equation

$$H(q_k, p_k, t) = -L(q_k, p_k, t) + \sum_{l=1}^{3n} p_l \dot{q}_l(q_k, p_k, t).$$

L is the **Lagrangian function** of the system, expressed as a function of the coordinates, momenta and time. \dot{q}_l stands for the generalized velocities, also expressed as functions of the coordinates, momenta and time, where q are the coordinates of position, p , those of momentum, and the dot means the derivative

with respect to time. n is the number of particles of the system. If the time does not occur explicitly, the system is called conservative and H is identical with the total energy of the system.

HAMILTONIAN DENSITY. See continuous systems, Lagrangian and Hamiltonian formalism for.

HAMILTONIAN FORMALISM FOR CONTINUOUS SYSTEMS. See continuous systems, Lagrangian and Hamiltonian formalism for.

HAMILTONIAN FUNCTION OF OPTICS. If the unit tangent vector of an optical path in a medium with refractive index $n = n(x, y, z)$ is $(\cos \alpha, \cos \beta, \cos \gamma)$ then the Hamiltonian function is

$$H(x, y, p, q) = -\sqrt{n^2 - p^2 - q^2} = -n \cos \gamma$$

where $p = n \cos \alpha$, $q = n \cos \beta$ are optical direction cosines. This leads to the canonical form of **Fermat's principle**: the optical paths are the curves $x = x(z)$, $y = y(z)$ for which

$$V = \int_{z_0}^{z_1} \left(p \frac{dx}{dz} + q \frac{dy}{dz} - H \right) dz$$

has an extreme value. (See **Hamilton theorem**.)

HAMILTONIAN OPERATOR. In quantum mechanics, an operator associated with the **Hamiltonian (Hamiltonian function of a system)** of classical physics. For a single particle the classical Hamiltonian function in rectangular Cartesian coordinates is given by

$$H = \frac{p_x^2 + p_y^2 + p_z^2}{2m} + V(x, y, z)$$

where p_x is the x -component of the momentum, etc., m is the mass and $V(x, y, z)$ is the potential energy of the particle. By the rules of Schrödinger wave mechanics for associating operators with dynamical variables (see **operators, quantum mechanical; momentum operator; and position operator**) the Hamiltonian operator in the Schrödinger representation becomes

$$H = -\frac{\hbar^2}{2m} \nabla^2 + V(x, y, z),$$

where ∇^2 is the Laplacian. Since the operator associated with the total energy of a system is $i\hbar \frac{\partial}{\partial t}$ one obtains, as the analogue to the equation $H = E$ in classical physics, the time-dependent Schrödinger equation (for one particle)

$$H\psi \equiv -\frac{\hbar^2}{2m} \nabla^2 \psi + V(x, y, z)\psi = i\hbar \frac{\partial \psi}{\partial t}.$$

HAMILTON-JACOBI PARTIAL DIFFERENTIAL EQUATION. A first order partial differential equation for the **action function**

$$S \left(q_k, \frac{dq_k}{dt}, t \right),$$

where q_k and $\frac{dq_k}{dt}$ are the generalized coordinates

and velocities of the system. The differential equation is

$$\frac{\partial S}{\partial t} + H \left(q_k, \frac{\partial S}{\partial q_k}, t \right) = 0.$$

H is the **Hamiltonian (Hamiltonian function of a system)** of the system, in which the momenta p_k are replaced by the expressions $\frac{\partial S}{\partial q_k}$.

HAMILTON PRINCIPLE. See **action principle**.

HAMILTON THEOREM. If V is the point characteristic of an optical system, i.e., V is the **optical path length** from an object to its Gaussian image point, if \mathbf{a} , \mathbf{a}' are position vectors of object and image points, and if \mathbf{s} , \mathbf{s}' are tangent vectors of length, n , n' of the object and image rays, then

$$\mathbf{s}' \frac{d\mathbf{a}'}{du} - \mathbf{s} \frac{d\mathbf{a}}{du} = \frac{dV}{du}$$

for any differentiable one-dimensional manifold of rays, with parameter u .

H AND D CURVE. See **exposure-density relationship**.

HANKEL FUNCTION. See **Bessel functions of the third kind**.

HANKEL TRANSFORM. Provided certain conditions are satisfied, $f(y)$ is the Hankel transform of $F(x)$ of order n if

$$f(y) = \int_0^{\infty} J_n(xy)F(x)xdx,$$

$$F(x) = \int_0^{\infty} J_n(xy)f(y)ydy,$$

where J_n is a Bessel function. (See also **integral transform**.)

HARDNESS (INDENTATION AND REBOUND). A measure of yield strength and work hardening which serves to identify, to control and to select materials and their treatment.

Brinell. The indenter is a 10-mm. diameter hardened steel ball. A sintered tungsten-carbide ball is also coming into use, especially for testing hard metals. The load applied is generally 500 kg. for soft metals and 3000 kg. for steels and hard metals. The Brinell hardness number is defined by the relation

$$\begin{aligned} BHN &= \frac{W}{\text{Area of indentation}} \\ &= \frac{2W}{\pi D[D - (D^2 - d^2)^{1/2}]} \end{aligned}$$

where W is the load, D is the diameter of the indenter, and d is the mean chordal diameter of the indentation. Tables are available for direct conversion to hardness from the diameter of the indentation as measured with a calibrated magnifier after removal of the piece from the testing machine.

Rockwell. Indenter is $\frac{1}{16}$ "-, $\frac{1}{8}$ "-, or $\frac{1}{4}$ "-diameter steel ball or a conical diamond having an apex angle of 120° and a slightly rounded point. The various scales used are designated by letters. Rockwell "B," for example, indicates a 100-kg load on a $\frac{1}{16}$ "-diameter ball. Rockwell "C" indicates a 150-kg load on the diamond indenter. Rockwell "30T" designates a load of 30 kg on a $\frac{1}{16}$ "-diameter ball. (An instrument of higher sensitivity known as the Rockwell Superficial Tester is used for loads of 15, 30, and 45 kg.) The size of the indentation is measured by a dial gauge as the final depth minus a small preliminary penetration produced by a minor preload of 10 kg. The Rockwell hardness values are arbitrary numbers having an inverse relationship to the depth of the indentation.

Vickers. Also known as Diamond Pyramid

Hardness. Indenter is a square-based diamond pyramid with included angle between faces of 136° . Loads may vary from 1 to 120 kg with 10, 30, and 50 kg in common use. Hardness is equal to load (kg) divided by surface area (sq mm) of the permanent indentation. It is determined directly from optical measurements of the diagonals of the indentation which appears square at the surface of the metal.

Tukon. A specially developed instrument for determining hardness under very light loads down to 25 grams. The small indentations are measured at high magnifications up to 1000 times. The indenter is a diamond pyramid that makes an elongated impression, one diagonal being 7 times the other in length.

Eberbach. Also used for very light loads. It consists of a spring-loaded, Vickers-type diamond pyramid indenter arranged for use on a metallurgical microscope.

Scleroscope. Values depend on the height of rebound of a diamond-tipped body falling under the force of gravity from a fixed height. The instrument is relatively small and is portable. One type reads directly on a graduated dial.

HARD SPHERE GAS. See **rigid sphere model**.

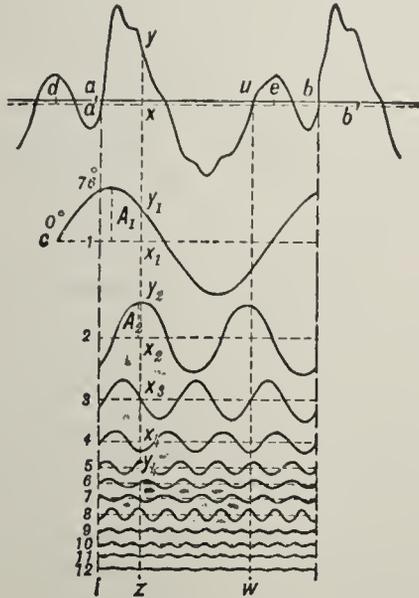
HARD SPHERE MODEL. See **rigid sphere model**.

HARMONIC ANALYSIS. Not only is it possible to combine two or more simple **harmonic motions** of different period, amplitude, and phase to form a complex motion, but there are also means of analyzing the resultant motion, when the latter is given, to find its component harmonics. Fourier showed that the analysis is possible for any periodic motion, however complicated. The equation, called the **Fourier series**, may be written

$$y = a \sin 2\pi nt + b \cos 2\pi nt + c \sin 4\pi nt + d \cos 4\pi nt + e \sin 6\pi nt + f \cos 6\pi nt + \dots,$$

in which y is the displacement of the vibrating particle and t is the time. The fundamental frequency n and the constants $a, b, c, d, \text{etc.}$, must be calculated from the given wave form or the data from which it is plotted. There is a type of instrument, called a "harmonic analyzer," which automatically computes the coefficients; or it may be done

mathematically, though the process is laborious and much aided by digital computer techniques. The accompanying figure shows the wave form and the twelve components of a complex tone.



Records of a complex sound and 12 of its components (after D. C. Miller).

HARMONIC FUNCTION. A function is harmonic within a given domain if it and its first derivative are continuous functions there and if it satisfies Laplace's equation. The most familiar ones are solid spherical harmonics which are homogeneous in the variables x, y, z . They must therefore also satisfy the equation of the Euler theorem for homogeneous functions. A spherical surface harmonic is the special set taken on the surface of a unit sphere and usually given in spherical polar coordinates. The angular dependent parts can be taken as

$$Y(\theta, \phi) = \begin{pmatrix} \cos m\phi \\ \sin m\phi \end{pmatrix} P_n^m(\cos \theta),$$

where P_n^m is the associated Legendre polynomial.

HARMONIC MEAN. Given the N positive numbers $x_1, x_2, x_3, \dots, x_N$, the harmonic mean, H , is defined as the reciprocal of the arithmetic mean of the reciprocals,

$$\frac{1}{H} = \frac{\frac{1}{x_1} + \frac{1}{x_2} + \dots + \frac{1}{x_N}}{N}.$$

The arithmetic mean of N unequal positive numbers is greater than the geometric mean which in turn is greater than the harmonic mean. The harmonic mean is used in time and rate problems.

HARMONIC MOTION. Simple harmonic motion is the to-and-fro motion of a body in a straight line, of such character that the displacement of the body from its equilibrium position (or mid-point of the motion) varies sinusoidally with time, i.e.,

$$x = a \cos(\omega t + \phi)$$

where x is the displacement and a and ϕ are constants. The frequency of the motion is $\omega/2\pi$. Such a motion occurs whenever a body moves on a straight line under the action of a restoring force proportional to the displacement, i.e., $f = -ks$, with no other forces acting. A mass suspended by a spring that obeys the Hooke law is the simplest example. The term is also applied to motions that approximate simple harmonic; for example, the motion of a pendulum bob approximates simple harmonic motion very closely for small amplitudes. Similarly, the addition of a small dissipative frictional force results in a motion which may be described as a simple harmonic motion with an amplitude that decreases with time.

The term harmonic motion (without the qualifier "simple") is used to describe rotary and other motions in which some variable, such as an angular displacement, varies sinusoidally with time.

Two simple harmonic motions of the same frequency, at right angles to each other, e.g.,

$$x = a \cos \omega t \quad \text{and} \quad y = b \cos(\omega t + \phi),$$

lead to an elliptical motion. Special cases occur for $\phi = 0$, when the ellipse degenerates to a straight line, and for $a = b$ and $\phi = 90^\circ$, when it degenerates to a circle. If the frequencies are unequal but commensurate, the motion is a Lissajou figure.

HARMONIC MOTION, DAMPED. The differential equation of a system which will oscillate with a harmonic motion is

$$M \frac{d^2x}{dt^2} + kx = F(t),$$

where M is the mass of the moving particle, k is the restoring force constant, x is the in-

stantaneous displacement and $F(t)$ is the external force applied to the mass. If frictional forces or other dissipative agencies act, the behavior of the system may often be approximated closely by the equation

$$M \frac{d^2x}{dt^2} + R \frac{dx}{dt} + kx = F(t)$$

where R is a constant, often called the resistance.

HARMONIC MOTION, PERIOD OF. The time for one complete oscillation, that is, the reciprocal of the frequency.

HARMONIC MOTION, SIMPLE ELLIPTIC. A compounded oscillatory motion consisting of (simple) harmonic motion in two fixed perpendicular directions with equal frequencies. The resultant trace of the motion is in general an ellipse, which depending on the relative phase and amplitudes of the two components may take the special form of a circle or straight line.

HARMONIC OSCILLATOR. See oscillator, harmonic.

HARMONIC OSCILLATOR, PARTITION FUNCTION OF. The function given by the equation

$$Z = \sum_n e^{-\beta h\nu(n+\frac{1}{2})} = e^{+\frac{1}{2}\beta h\nu} (e^{\beta h\nu} - 1)^{-1},$$

where $h\nu$ is the energy distance between successive oscillator levels, $\beta = 1/kT$ (k is Boltzmann's constant and T is absolute temperature), and Z is the partition function. Note that Planck's constant h is used in above formula, while the Dirac $\hbar = \frac{h}{2\pi}$, is used in the entry oscillator, harmonic.

HARMONIC PROGRESSION. The sequence a, b, c, \dots if their reciprocals $1/a, 1/b, 1/c, \dots$ form an arithmetic progression. The harmonic mean between two numbers is the middle term of a harmonic progression whose first and last terms are the given numbers. The harmonic mean between a and b is given by $H = 2ab/(a + b)$. If A, G , and H are respectively the arithmetic, geometric, and harmonic mean of two numbers then $G^2 = AH$.

HARMONIC SERIES OF SOUNDS. A series of sounds in which each basic frequency

in the series is an integral multiple of a fundamental frequency. (See harmonic analysis.)

HARTLEY. A unit of information content equal to the information content of a message, the *a priori* probability of which is one-tenth. If, in the definition of information content, the logarithm is taken to the base ten, the result will be expressed in hartleys.

HARTLEY FORMULA FOR TIME-FREQUENCY DUALITY. As implied by the Fourier transform, a time function cannot be confined within a small region on the time scale when the steady-state transmission characteristic is confined to a narrow range on the time scale. For example, it is well known that, if a telegraph dot is made narrower and narrower, its corresponding significant-frequency spectrum becomes broader and broader until, in the limit when the dot becomes an impulse, its significant-frequency spectrum is of infinite extent. Two mathematical equations which express this relationship are

$$g(t) = \int_{-\infty}^{\infty} c(f) \text{cas } 2\pi ftdf$$

$$c(f) = \int_{-\infty}^{\infty} g(t) \text{cas } 2\pi ftdt$$

where f is the cyclic frequency, t is time, and cas denotes cosine added to sine.

HARTLEY PRINCIPLES (WITH REGARD TO INFORMATION CAPACITY OF A TRANSMISSION CHANNEL). The amount of information that can be transmitted is proportional to the width of the frequency range, and the time it is available. Information content is equated to the total number of code elements, multiplied by the logarithm of the number of possible values a code element may assume. Information content is independent of how the code elements are grouped. By quantizing, the continuous magnitude-time function used in ordinary telephony may be transmitted by a succession of code symbols such as are employed in telegraphy. And, to obtain the maximum rate of transmission of information, the signal elements need to be spaced uniformly.

HARTMANN DISPERSION FORMULA. A very useful dispersion formula due to Hartmann which suggests the anomalous disper-

sion near an absorption band, but does not provide for more than one such band and hence can be used over only a limited range of wavelengths. It is of the form:

$$n = n_0 + \frac{c}{(\lambda - \lambda_0)^a},$$

where n_0 , c and λ_0 are constants to be determined from measurements. For best fit to empirical results $a = 1.2$, but for convenience $a = 1$ is often used.

HARTREE APPROXIMATION. A method used to determine energy levels of atomic systems. The wave function of the system is assumed to be the product of one-particle wave functions $\psi_k(x)$, each particle occupying a definite one-particle state. The anti-symmetry of the wave function is not taken into account. The expectation value of the energy in a state described by such a product wave function is then calculated and minimized with respect to variations of the functions $\psi_k(x)$. This gives equations to determine the one-particle wave functions $\psi_k(x)$. The potential appearing in these equations is determined by the states of the other particles. The particles move in a self-consistent field which determines their motion and which is determined by this motion.

HARTREE-FOCK METHOD. This is an improvement on the Hartree approximation for calculating energy levels of atomic systems. The wave function with respect to which the expectation value of the energy of the system is minimized is a Slater determinant, the anti-symmetrized product of one-particle wave functions $\psi_k(x)$. The equations determining these functions $\psi_k(x)$ include exchange effects due to the anti-symmetry of the wave function of the system.

HAUNCHED BEAM. See beam, haunched.

HAUSDORFF SPACE. See topological space.

HÄUÛY LAW. The fundamental law of crystallography stating that for a given crystal there exists a set of axial ratios such that the intercepts of every crystal plane on the crystal axes are expressible as rational fractions of these ratios. (See Miller indices.)

h-BAR OR \hbar . Symbol for Dirac- \hbar , the universal constant $h/2\pi$, where h is the Planck constant.

HEAT. The term has been used in two related, but distinctly different, senses. (1) According to best modern usage, heat is energy in the process of transfer between a system and its surroundings as a result of temperature differences. (2) According to the older (and still current) usage, heat is the energy contained in a sample of matter, including both potential energy resulting from interatomic forces and kinetic energy associated with the chaotic motion of the molecules of a substance. In the range of temperatures in which quantum effects are not of importance, the average kinetic energy per molecule is one-half of the Boltzmann constant times the Kelvin temperature for each degree of freedom, in accordance with the principle of the equipartition of energy. The average potential energy per molecule in monatomic solids, under the same conditions, is equal to the average kinetic energy; the Dulong and Petit law is explained by this fact.

In accordance with the second definition, terms such as heat content, specific heat, etc., are used as measures of the internal energy of matter.

Although we now recognize that heat is energy, it is still customary to express quantity of heat in the old water-temperature measure, by means of British thermal units or of calories; and whenever heat quantities so expressed are used in thermodynamic calculations, it is necessary to use the mechanical equivalent of heat as a conversion factor between these and the ordinary dynamic units of energy (foot-pounds, joules, or ergs).

HEAT BALANCE. A method of accounting for all heat units in a process or change during which heat is transferred. Examples of cases where heat balances might be undertaken are: (1) Determining the nature and the magnitude of the various losses which occur when coal is burned in a steam boiler furnace. (2) Accounting for all heat units during the operation of a prime mover such as a Diesel engine or a steam turbine. (3) Determining the distribution of heat in a static heating device such as a water heater supplied with steam.

Heat balance work is based upon the first

law of thermodynamics. (See **conservation of energy**.)

HEAT CAPACITIES, RELATION BETWEEN. (See **thermal coefficients**.) The molar heat capacity at constant pressure and the molar heat capacity at constant volume are related by

$$C_p = C_v + \frac{\alpha^2 TV}{\kappa}$$

where α is the **thermal expansion coefficient**, and κ , the **isothermal compressibility coefficient**.

HEAT CAPACITY. The amount of heat necessary to raise the temperature of a system, entity, or substance by one degree of temperature. It is most frequently expressed in calories per degree centigrade. If the mass of a substance is specified, then certain derived values of the heat capacity can be obtained, such as the atomic heat, molar heat, or specific heat. (See **thermal coefficients**.)

HEAT CAPACITY EQUATION. See **Einstein specific heat function**.

HEAT CAPACITY, QUANTUM THEORY OF. See **quantum theory of heat capacity**.

HEAT CARRIED AWAY. The **enthalpy** of combustion gases (in a chimney, stack, etc.) with reference to the enthalpy of the fuel and air entering the device; it is a measure of the energy lost through inability to cool the products of combustion to the temperature of the surrounding atmosphere. It is usually assumed that the atmosphere is at a standard temperature t_s (0°C or 60°F) and that the combustion gases are perfect. Hence

$$Q_c = \left\{ \frac{c}{12} [\bar{C}_p]_{t_s}^c \text{CO}_2 + \left(\frac{h}{2} + \frac{w}{18} \right) [\bar{C}_p]_{t_s}^c \text{H}_2\text{O} + 0.21(\lambda - 1) X_{min} [\bar{C}_p]_{t_s}^c \text{O}_2 + 0.79\lambda X_{min} [\bar{C}_p]_{t_s}^c \text{N}_2 \right\} t_c - [c]_{t_s}^f t_f - \lambda X_{min} [\bar{C}_p]_{t_s}^a t_a$$

for solid or liquid fuel, or

$$Q_c = t_c \sum_{\text{burnt gases}} x [\bar{C}_p]_{t_s}^c - t_f \sum_{\text{fuel gas}} x [\bar{C}_p]_{t_s}^f - \lambda X_{min} [\bar{C}_p]_{t_s}^a t_a$$

for a gaseous fuel.

Symbols: t_c is the temperature of burnt gases leaving the device; t_a is the temperature of air entering the device; t_f is the temperature of fuel entering the device; X_{min} is the stoichiometric quantity of air in volume units; λ is the **air ratio**; $[\bar{C}_p]_{t_1}^2$ is the **mean molar specific heat** between the temperatures indicated; x is the mole fraction and $c, h, \text{etc.}$, is the fuel composition in mass units of respective element per unit mass of fuel; w refers to moisture.

In a boiler or furnace the heat carried away is also known as the **stack loss**.

HEAT CONSUMPTION. See **specific heat consumption**.

HEAT CONTENT. See **enthalpy**.

HEAT DROP. See **enthalpy drop**.

HEAT EXCHANGER. A device for exchanging heat between two fluids. In a *heater* the temperature of the main fluid is raised; it is lowered in a *cooler*. The second fluid in a cooler is known as the *coolant*. The most widely used coolant is water (cooling water).

There are three principal types of heat exchanger, depending on the relative direction of the two fluids on the two sides of the heating surface: parallel flow, Figure 1, counter-flow or contra-flow, Figure 2, and cross-flow, Figure 3.

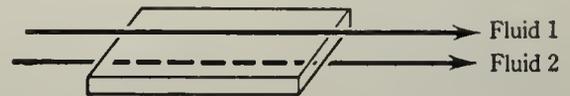


Fig. 1.



Fig. 2.

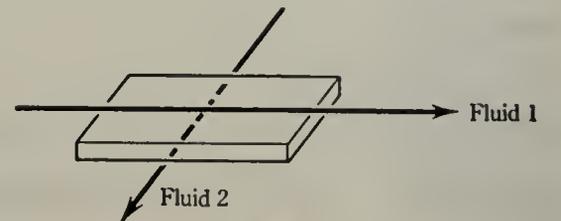


Fig. 3.

Practical heat exchangers are seldom strictly of one particular type and necessarily contains elements of each type. They are described by the designation which is related to the conditions prevailing over the greatest portion of the heating surface.

HEAT FLUX VECTOR. See *flux vector*.

HEAT FUNCTION. See *enthalpy*.

HEAT INDEX. As used by C. W. Thornthwaite in his 1948 *climatic classification*, a function of temperature designed to have low magnitude under cold conditions, increasing exponentially with increasing temperature. For a given station, it is numerically equal to the sum of the twelve monthly values of the expression

$$\left(\frac{t}{5}\right)^{1.514},$$

where t is the normal monthly temperature in °C.

HEATING SURFACE. The metal surface separating two fluids exchanging heat in a **heat exchanger**. In particular, the sum of the surfaces across which heat is added to the working fluid in a boiler.

HEATING VALUE. The heating, or caloric, value of a **fuel** is the quantity of heat produced by the combustion, under specified conditions, of unit weight or volume of the fuel. The heating value of a fuel may be calculated by a formula which may be derived for any fuel by multiplying the percentage of each chemical element present by its heating value per unit weight, and adding the products for all combustible elements in the fuel. Thus for coal, whose combustible elements consist of carbon, hydrogen, and sulfur, the heating value is:

$$14,540C + 62,000H + 4000S$$

(B.T.U. per lb of coal).

The numbers in the above formula are the heating values per lb respectively, of carbon, hydrogen, and sulfur. In the use of this formula, it is essential that only that portion of the element that is actually free to burn be employed. For example, all coal contains some moisture. Now the hydrogen present in this water is not free to burn (i.e., it is already combined with oxygen). Therefore the figure used for H in the foregoing formula should not include the hydrogen present as water.

Heating value by formula will not necessarily be the same as that obtained experimentally with the fuel calorimeter. The difference lies not in the accuracy of the experiment, nor of the calculation, but in the possible endo-

thermic or exothermic reactions which take place when a compound fuel, such as a hydrocarbon, is burned. The volatile matter of coal must be broken down into the elements of carbon and hydrogen by heat-absorbing action before they may reunite with the oxygen during combustion. For this reason, experimentally determined heating values are less than those which are computed by formulas, which contain no terms for such endothermic reactions. Approximate heating values of some of the common fuels are: coal, 13,000 B.T.U. per lb; natural gas, 1000 B.T.U. per cu ft; artificial gas, 300 B.T.U. per cu ft; gasoline, 19,000 B.T.U. per lb; wood, 5000 B.T.U. per lb.

Many thermodynamic analyses require the use of a "lower heating value" which may be obtained from the above values by subtracting an allowance for the latent heat of evaporation of the steam found in the products of combustion. (See also **combustion calculations in engineering**.)

HEAT, LATENT. The heat per unit mass or mole required to produce a change of phase at constant pressure and temperature; it is equal to the difference in the specific **enthalpies** of the substance at the two phases. Usually, latent heats are reckoned positive for the transition from a phase of lower to a phase of higher energy. (For example liquid → vapor.) Often an adjective is added to specify the type of transition involved, and the adjective "latent" may be omitted, e.g., (latent) heat of evaporation, (latent) heat of sublimation, (latent) heat of fusion, etc. Sometimes, particularly in the case of evaporation, the latent heat per unit mass

$$l = h_2 - h_1$$

is split up into two terms in view of the definition $H = U + pV$:

$$l = (u_2 - u_1) + p(v_2 - v_1) \quad (\text{since } p = \text{const}).$$

The first term, the change in internal energy

$$p = u_2 - u_1$$

is termed *internal (latent) heat* and the second term

$$\psi = p(v_2 - v_1)$$

which represents the work done at the boundaries of the system against the external pressure is called *external (latent) heat*.

HEAT, MECHANICAL EQUIVALENT OF.

The conversion factor between any unit commonly employed to express mechanical energy and a unit commonly employed to express thermal energy, e.g., 4.1840 joules/calorie.

HEAT, MOLECULAR. (Heat, Molar.) The product of the **gram-molecular weight** of a compound and its specific heat. (See **heat, specific.**) The result is the **heat capacity** per gram-molecular weight.

HEAT OF DILUTION. The *total* or *integral heat of dilution* is the difference in enthalpy between a solution and its components. The *differential heat of dilution* or of *mixing*, which is more commonly meant by the term heat of dilution, is the heat absorbed (change in partial molar enthalpy) when one mole of a specified component is dissolved in a large bulk of solution at constant temperature and pressure.

HEAT OF MIXING. See **excess functions.**

HEAT OF REACTION. See **thermal coefficients.**

HEAT OF TRANSFER. See **thermomolecular pressure.**

HEAT PROPAGATION IN CONDUCTING MEDIUM. A process governed by the general equation:

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{c\rho}{K} \frac{\partial T}{\partial t}$$

where T is the temperature, c , the specific heat, ρ , the density, K , the thermal conductivity, x, y, z , rectangular coordinates, and t , the time. For the case of radial heat flow (spherical coordinates), the equation is

$$\frac{\partial^2 T}{\partial r^2} + \frac{2}{r} \frac{\partial T}{\partial r} = \frac{\rho c}{K} \frac{\partial T}{\partial t}$$

where r is the radius.

For the case of radial heat flow in one plane (cylindrical coordinates), the equation is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} = \frac{\rho c}{K} \frac{\partial T}{\partial t}$$

HEAT REGENERATION. The process of exchanging heat internally between the working fluid at two different states in a particular cycle. Heat is best exchanged between two ranges of states in the same temperature range

and in a counter-flow **heat exchanger**. The process of heat regeneration results in an increase in efficiency. (For specific examples see **carnotization** and **Joule cycle**.)

HEAT, SPECIFIC. Also called the specific heat capacity. The quantity of heat required to raise the temperature of unit mass of a substance by one degree of temperature. The units commonly used for its expression are the unit mass of one gram, the unit quantity of heat in terms of the **caloric**. The unit mass of one **mole** is also used, although the preferred term for the quantity of heat is then the *molar heat* or *molecular heat*.

HEAT, SPECIFIC, AT CONSTANT PRESSURE. The amount of heat required to raise unit mass of a substance through one degree of temperature without change of pressure. Usually denoted by C_p , when the **mole** is the unit of mass, and c_p when the **gram** is the unit of mass.

HEAT, SPECIFIC, AT CONSTANT VOLUME. The amount of heat required to raise unit mass of a substance through one degree of temperature without change of volume. Usually denoted by C_v , when the **mole** is the unit of mass, and c_v when the **gram** is the unit of mass.

HEAT, SPECIFIC, OF SOLIDS. See **Dulong and Petit law**; **Debye theory of specific heat**; **Born-Einstein equation for heat capacity**; **Born-von Kármán theory of specific heat**.

HEAT THEOREM (NERNST HEAT THEOREM). An alternative name given to the third law of **thermodynamics**.

HEAT TONE. Latent heat associated with a change of phase, an allotropic transformation or a chemical reaction. The common feature of these processes is that they occur at $p = \text{constant}$ and $T = \text{constant}$ and that the overall system consists of a mixture of two systems 1 and 2 whose thermodynamic properties differ by finite amounts and whose relative masses change during the process. The heat tone refers to a complete transformation of a unit mass from 1 to 2.

HEAT TRANSFER. (1) The physical process whereby heat passes from one system to another. (2) The branch of science which describes the process of heat transfer, that is

also known as heat transmission. The central problem in heat transfer is to calculate the rate at which heat passes from one system to another under a variety of conditions. An auxiliary problem intimately connected with the preceding one is the calculation of the temperature field which exists in the system of interest while heat is being transferred.

Heat can be transferred by three essentially different physical mechanisms, known as *modes of heat transfer*. In *conduction* heat is transferred through a material medium at rest. It occurs in solid bodies and very thin layers of fluid. When the layers of fluid (liquid or gas) become extensive, the temperature field in the system which accompanies the transfer of heat causes the density to vary from point to point. A mass of fluid of varying density placed in the gravitational field of the earth will not be in equilibrium, unless the density stratification satisfies a definite relationship. Hence, the buoyancy forces acting on the fluid particles will give rise to a velocity field. In *convection* heat is transferred principally by virtue of the physical motion of the fluid. Convection is called *natural* when the velocity field in the fluid is caused predominantly by the interaction of the temperature field with the gravitational field of the earth. Convection is called *forced* when the velocity field is imposed on the fluid, e.g., by pressure gradients, etc. Convection does not occur in a solid, because in it, a temperature field causes varying degrees of *thermal expansion* which, in turn, may give rise to *thermal stresses*, but not to motion. Heat can also be transferred from one system to another without the intervention of a material body in between. It is then called *radiation*. Radiation is the transfer of energy by electromagnetic waves, and at the speed of light. We speak of *thermal radiation* when the bulk of the energy is carried in a particular range of wavelengths: 0.8 to 400 microns (visible light: 0.4 to 0.8 μ ; ultraviolet radiation: 0.02 to 0.4 μ ; X-rays: 10^{-6} to 0.02 μ).

Conduction. When a homogeneous system of arbitrary shape transfers heat by conduction, it is found that a (scalar) temperature field $T(x,y,z,t)$ is present throughout the system. The directed (vectorial) quantity $\dot{\mathbf{q}}$ which represents the rate of heat (energy) transfer per unit time and area at a point in the system is called *heat flux*. The principle of conservation

of energy, in the absence of heat sources, leads to the general equation of conduction

$$\rho c \frac{\partial T}{\partial t} + \text{div } \dot{\mathbf{q}} = 0 \quad (1)$$

(ρ is density; c is specific heat). The relation between the heat flux vector $\dot{\mathbf{q}}$ and the gradient of the temperature field is given by **Fourier's empirical law**:

$$\dot{\mathbf{q}} = -k \text{ grad } T. \quad (2)$$

In accordance with the second law of **thermodynamics**, the coefficient of proportionality $k(T)$, known as the *thermal conductivity* of the material, must always be positive

$$k > 0. \quad (2a)$$

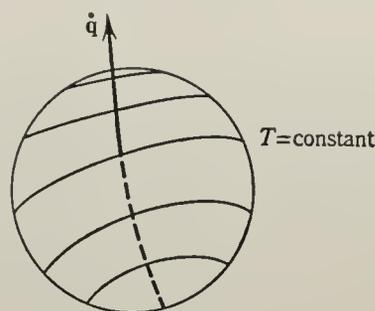


Fig. 1.

The thermal conductivity of known materials ranges from 0.003 for some gases to 300 Btu/hr ft F for some metals, i.e., over a ratio of 1:100,000.

Substitution of (2) into (1) leads to *Fourier's equation of heat conduction*

$$\rho c \frac{\partial T}{\partial t} = \text{div } \{k(T) \text{ grad } T\}. \quad (3)$$

In many applications, the thermal conductivity can be assumed constant, when Equation (3) simplifies to

$$a \frac{\partial T}{\partial t} = \nabla^2 T. \quad (4)$$

$$a = \frac{\rho c}{k} \quad (5)$$

is known as the *thermal diffusivity* of the material.

In the presence of heat sources, the source strength $Q'(x,y,z,t)$ must be added to the right-hand side of Equation (1); hence the term $Q'/\rho c$ will appear on the right-hand side of Equation (4). Fourier's equation, together with the ini-

tial and boundary conditions, determines the temperature field $T(x,y,z,t)$. Fourier's law then determines the heat flux $\dot{\mathbf{q}}$ at every point.

The instantaneous rate \dot{Q} at which heat is transferred across a solid body is found by integrating the heat flux $\dot{\mathbf{q}}$ over its surface, but *not* over its entire surface. It is necessary to integrate *separately* the portions with positive and negative heat fluxes. As a rule one part of the surface, Figure 2, will be in contact with

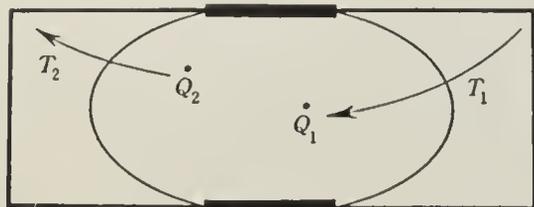


Fig. 2.

system 1 at T_1 and another part will be in contact with system 2 at T_2 , the remainder of the surface being adiabatic. The quantity of heat \dot{Q} transferred by conduction is measured separately for systems 1 and 2.

When the flow of heat is steady, the temperature field satisfies **Laplace's equation**

$$\nabla^2 T = 0 \quad (6)$$

in the absence of sources, and the function $T(x,y,z)$ is independent of the properties of the material, being a function of the geometrical arrangement only. It is easy to see that then in Figure 2 we have $\dot{Q}_1 = \dot{Q}_2$. The quantity is the rate at which heat is conducted through the solid body, because in the steady state the solid body neither stores nor discharges energy.

When the body is anisotropic (crystals), the vectors $\dot{\mathbf{q}}$ and $\text{grad } T$ are no longer collinear, but they remain proportional. Fourier's law (in Cartesian coordinates x_1, x_2, x_3) assumes the form

$$\begin{aligned} \dot{q}_1 &= -k_{11} \frac{\partial T}{\partial x_1} - k_{12} \frac{\partial T}{\partial x_2} - k_{13} \frac{\partial T}{\partial x_3} \\ \dot{q}_2 &= -k_{21} \frac{\partial T}{\partial x_1} - k_{22} \frac{\partial T}{\partial x_2} - k_{23} \frac{\partial T}{\partial x_3} \\ \dot{q}_3 &= -k_{31} \frac{\partial T}{\partial x_1} - k_{32} \frac{\partial T}{\partial x_2} - k_{33} \frac{\partial T}{\partial x_3}. \end{aligned} \quad (7)$$

The three fluxes $\dot{q}_1, \dot{q}_2, \dot{q}_3$ are seen to be coupled. It is a consequence of the second Law of thermodynamics and **Onsager's principle**

of **reciprocal relations** that the tensor k_{ij} is non-negative, definite and symmetrical

$$k_{ij} = k_{ji}. \quad (8)$$

The problem of calculating the transfer of heat by conduction has, therefore, been reduced to the purely mathematical problem of solving some form of Fourier's equation subject to appropriate boundary and initial conditions. Many particular problems have been solved and the reader will find an excellent compendium in *Conduction of Heat in Solids* by H. S. Carslaw and V. C. Jaeger, 2nd edition, Clarendon Press, 1959. The boundary conditions at the surface differ in accordance with the constraints imposed on it. Very seldom the surface temperature is prescribed by a function $T = f(x,y,z,t)$ on the surface. More frequently, the boundary condition is given in the form of a requirement on the heat flux $\dot{\mathbf{q}}$. When there is no heat flux (adiabatic surface) the condition is

$$\frac{\partial T}{\partial n} = 0, \text{ on surface} \quad (9)$$

(n is the normal to the surface). If forced or natural convection take place at the surface, the condition is written in the form of Newton's law of cooling (see below, *Convection*)

$$\dot{q}_{surf} = -k \frac{\partial T}{\partial n} = h(T_w - T_\infty). \quad (10)$$

Problems involving radiation at the surface occur seldom and are sometimes approximated by a condition identical with Equation (10).

Fourier's law and equation describe actual processes in an excellent manner, but the objection has been raised that Fourier's equation, being parabolic, admits of an infinite rate of propagation of temperature changes, contrary to physical reality. Proposed modifications of Fourier's law do not, however, materially alter the solution in practical cases, but necessarily lead to grave mathematical complications.

Convection. Since convection always involves a fluid in motion, it is possible to solve problems in convection from first principles only in the case of laminar motion. In most important applications, the accompanying motion is turbulent and it is necessary to rely to a large extent on empirical information. The fundamental empirical assumption, known as *Newton's law of cooling*, asserts that the rate of heat transfer per unit area \dot{q}'' is proportional to the difference between the temperature of the solid

surface, T_w , and that of the fluid at a large distance from the wall, T_∞ . Hence

$$\dot{q}'' = h(T_w - T_\infty) \quad (11)$$

where the coefficient h (Btu/ft² hr F), known as the *coefficient of heat transfer* or *film coefficient*, depends on the properties of the fluid and its state of motion. It is, therefore, a complicated function in any particular problem. Newton's law of cooling holds in the large majority of cases, but breaks down where the surface is not isothermal. In the case of compressible flows (at high Mach numbers), it is replaced by the relation

$$\dot{q}'' = h(T_w - T_a) \quad (12)$$

where T_a denotes the adiabatic wall temperature, i.e., the temperature which would be assumed by the fluid at the wall if the latter were non-conducting. This difference is due to the large amount of heat dissipated by the flow near a wall and is often referred to as the *thermal barrier* because, generally, it impedes the transfer of heat as $T_a = T_\infty + rU_\infty^2/2c_p$ (r is the recovery factor which depends on the flow; c_p is the specific heat at constant pressure).

Owing to the difficulties associated with the calculation of the coefficient of heat transfer, it is often necessary to confine oneself to a *dimensional analysis* of the problem. It is then found that for geometrically similar bodies in similar flow fields with similar boundary conditions, the relation between the coefficient of heat transfer h and the remaining parameters can be expressed in terms of various dimensionless groups (numbers). To show their relationships, certain of these numbers are given below in the dimensions used in the present discussion. They can, of course, be defined in numerically and dimensionally equivalent terms. For the definitions most commonly used, see the corresponding entries. They are:

the Nusselt number

$$Nu = \frac{hL}{k}$$

the Reynolds number

$$Re = \frac{\rho U_\infty L}{\mu} = \frac{U_\infty L}{\nu}$$

the Grashof number

$$Gr = \frac{g\beta(T_w - T_\infty)L^3}{\nu^2} \quad (13)$$

the Prandtl number

$$Pr = \frac{\nu}{a} = \frac{\mu c_p}{k}$$

the Eckert number

$$Ec = \frac{U_\infty^2}{c_p(T_w - T_\infty)}$$

(L is the characteristic length; μ is the dynamic viscosity; ν is the kinematic viscosity; U_∞ is the fluid velocity at large distance from surface; ρ is the density of fluid; β is the coefficient of thermal expansion of fluid; c_p is the specific heat of fluid at constant pressure.) The general relation is

$$Nu = f(Re, Gr, Pr, Ec). \quad (14)$$

The exact form of the function f must be determined, theoretically or empirically, for each class of dynamically similar problems. Equation (14) formulates the problem in terms of the minimum number of independent variables. Several specialized forms of Equation (14) are of importance. In general $(T_w - T_\infty) \ll U_\infty^2/c_p$ and the Eckert number can be omitted. In natural convection the Reynolds number can be omitted, when

$$Nu = f_1(Gr, Pr). \quad (\text{natural convection}) \quad (15)$$

When the natural motion is very slow,

$$Nu = f_2(Gr \times Pr) = f_2(Ra) \quad (16)$$

where Ra is known as the *Rayleigh number*. In forced convection, the Grashof number can be omitted, and we have

$$Nu = f_3(Re, Pr) \quad (\text{forced convection}) \quad (17)$$

when a more accurate analysis is performed, it is found that the turbulence intensity Tu of the free stream must be added to the independent variables in Equation (14). In particular, equation (17) must be rewritten to read

$$Nu = f_4(Re, Pr, Tu). \quad (18)$$

The Stanton number is $St = \frac{Nu}{Re \times Pr}$ (19)

In high-speed flow, the Eckert number cannot be omitted, and it or the

$$\text{Mach number } M = \frac{U_\infty}{c} = \frac{Ec}{\gamma - 1} \quad (20)$$

(c is the velocity of sound; γ is the ratio of specific heats) must be retained as an independent variable.

The velocity profile in a fluid near a wall, as well as the temperature distribution, are of the boundary layer type. Consequently, in many cases the relations for the Nusselt number (and hence for h) can be derived from boundary layer theory. In cases of laminar flow in the boundary layer, the problem is a mathematical one, because the boundary layer equations can then be written and analyzed. In cases when turbulent flow is involved, additional information of an empirical nature must be supplied.

Boundary layer theory leads to a general relation between the shearing stress τ (a characteristic of the velocity field) and the local heat flux \dot{q} (a characteristic of the temperature field) known as Reynolds' analogy. The analogy is usually expressed in the form of a relation between the dimensionless skin friction coefficient

$$c_f = \frac{\tau}{\frac{1}{2}\rho U_\infty^2} \quad (21)$$

and the Stanton number. In the simplest case of two-dimensional flow past a flat plate at zero incidence, and for $Pr = 1$, the analogy has the form

$$St = \frac{1}{2}c_f. \quad (22)$$

The relation is an expression of the fact that under those conditions the velocity and temperature profiles are identical functions of the coordinate perpendicular to the surface. For $Pr \neq 1$, the two are similar.

Boundary layer theory leads to the formulation of another important analogy, namely that between heat and mass transfer. The mechanism of the diffusion of a vapor into a flowing stream is basically identical with that of heat, and this reflects in the respective equations for laminar or turbulent flow. Hence a complete analogy can be established between the transfer of heat in forced convection and the transfer of mass into it.

Radiation. The transfer of heat by radiation is studied in terms of and in relation to the ideal case of black body radiation. Black body radiation constitutes an upper limit for the heat flux from any surface at the same temperature. The amount of heat emitted by a unit surface at temperature T is given by the **Stefan-Boltzmann law** $E = \sigma T^4$. Real surfaces are characterized by a coefficient $\epsilon < 1$, called its *emissivity* (equal to its absorptivity, see **Kirchhoff's law**). It is assumed that real surfaces are gray, i.e., that they have the same

spectral energy distribution as a black body. Hence for a gray surface

$$E = \epsilon\sigma T^4. \quad (23)$$

The quantity of heat exchanged between two surfaces at T_1 and T_2 , respectively, is obtained by considering that each one of them emits energy in accordance with Equation (23) and that, being gray, it reflects a fraction $1 - \epsilon$ of the energy emitted by the other surface. In this manner it can be calculated that the heat flux \dot{q}_{12} due to radiation between two infinite, parallel plane surfaces 1 and 2 is

$$\dot{q}_{12} = \frac{\sigma}{(1/\epsilon_1) + (1/\epsilon_2) - 1} (T_1^4 - T_2^4). \quad (24)$$

For two gray surfaces 1 and 2, Figure 3, we obtain

$$\dot{Q}_{12} = \frac{\epsilon_1 \epsilon_2}{\pi} \sigma (T_1^4 - T_2^4) \times \iint \frac{\cos \phi_1 \cos \phi_2}{r^2} dA_1 dA_2 \quad (25)$$

where the double integral constitutes a purely geometrical shape factor. The formula in Equation (25) is approximate only because it disregards the reflected quantities of energy. It is exact for two black surfaces $\epsilon_1 = \epsilon_2 = 1$.

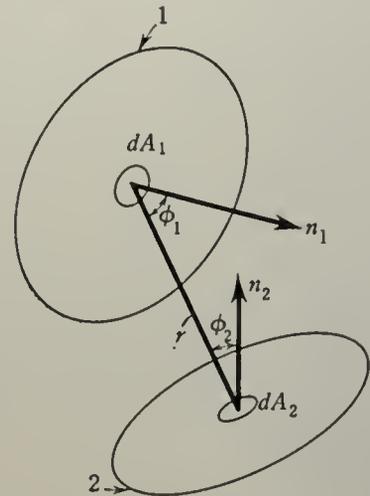


Fig. 3.

For concentric cylinders or spheres, the exact equation is

$$\dot{Q}_{12} = A_1 \frac{\sigma}{(1/\epsilon_1) + (A_1/A_2)\{(1/\epsilon_2) - 1\}} \times (T_1^4 - T_2^4). \quad (26)$$

(A_2 is the outer surface; A_1 is the inner surface.)

When the outer surface is very large, $A_2 \rightarrow \infty$, the rate of heat transferred becomes independent of its emissivity and depends only on the emissivity of the small surface.

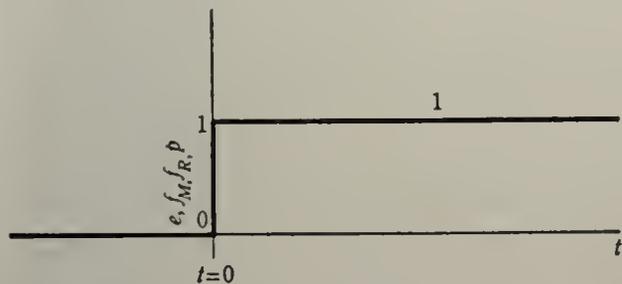
Radiation from flames poses special problems because gases are selective emitters (and absorbers) and emit (or absorb) energy in preferred wave bands. Consequently the relations become complex; they have not yet been fully explored.

HEAT, UNITS OF. See **thermal units**.

HEAVISIDE OPERATIONAL CALCULUS.

The behavior of a vibrating system may be analyzed by solving the differential equations of the dynamical system. In other words find the currents or velocities of the elements which when substituted in the differential equations will satisfy the initial and final conditions. The solution of the differential equation may be divided into the steady-state term and the transient term. The operational calculus is of great value in obtaining the transient response of an electrical, mechanical or acoustical system to a suddenly impressed voltage, force or pressure. The general analysis used by Heaviside is applicable to any type of vibrating system whether electrical, mechanical or acoustical. The response of a system to a unit force can be obtained with the Heaviside calculus.

Heaviside's unextended problem is as follows: given a linear dynamical system of n degrees of freedom in a state of equilibrium, find its response when a unit force is applied at any point. The unit function, **1**, depicted in the figure, is defined to be a force which is zero for $t < 0$ and unity for $t \geq 0$.



The unit function. The electromotive force, force, torque or pressure is zero before and unity after $t = 0$.

The response of a dynamical system to a unit force is called the indicial admittance of the system. It is denoted by $A(t)$. $A(t)$ represents the current, linear velocity, angular velocity, or volume current when a unit electromotive force, force, torque or pressure is

suddenly applied in a system which was initially at rest.

In the Heaviside calculus the differential equations are reduced to an algebraic form by replacing the operator d/dt by the operator p and the operation $\int dt$ by $1/p$. Tables of operational formulas have been compiled which serve for operational calculus the same purpose that tables of integrals serve the integral calculus. Operational formulas may be modified, divided or combined by various transformation schemes. This is similar to integration by parts or change of variable in the integral calculus.

The procedure in the Direct Heaviside Operational Method to be followed in obtaining an operational solution of an ordinary differential equation is as follows: Indicate differentiation with respect to the independent variable by means of the operator p . Indicate integration by means of $1/p$. Manipulate p algebraically and solve for the dependent variable in terms of p . Interpret and evaluate the solution in terms of known operators. (See **transient response of a dynamical system to an arbitrary force—Duhamel's integral**.)

HEAVISIDE UNIT FUNCTION. The function $f(x)$ defined by $f(x) = 0$ if x is negative, and $f(x) = 1$ if x is zero or positive.

HEINE-BOREL THEOREM. If an infinite set S of open intervals covers the finite closed interval $[a, b]$ in the sense that every point of $[a, b]$ is contained in at least one interval of S , then there exists a finite subset of S having the same property. The theorem is easily generalized in various ways.

HEINE FORMULA. An integral representation for **Legendre polynomials**:

$$P_n^m(x) = (n+1)(n+2)\cdots(n+m)(-1)^{m/2} \times \frac{1}{\pi} \int_0^\pi [x + \sqrt{x^2 - 1} \cos \phi]^n \cos m\phi d\phi.$$

HEISENBERG FORCE. A central force between two nucleons which is derivable from a potential containing an operator which exchanges both the spins and the positions of the two particles. Nuclear forces are known to have exchange properties of this general type.

HEISENBERG PICTURE. The description of the evolution in time of a quantum mechani-

cal system by the use of a time-independent state vector $|\Phi_H\rangle$

$$\partial_t |\Phi_H\rangle = 0 \quad (\text{a})$$

but by time-dependent operators, $F_H(t)$, corresponding to the observables of the system. The time dependence of $F_H(t)$ is given by

$$\partial_t F_H(t) = \frac{i}{\hbar} [H_H, F_H(t)] \quad (\text{b})$$

where H_H is the Hamiltonian of the system. The state vector, $|\Phi_H\rangle$, in the Heisenberg picture is the same for all time and describes the entire history of the system, i.e., the results of all possible experiments on the system throughout its history. However, if an actual experiment is performed on the system, the state vector will be changed. Although the Heisenberg state vector Φ_H does not depend on time, it may be specified by the results it predicts for some experiments at a given time. It can, for example, be specified as that state vector which corresponds to the **Schrödinger** picture state vector at time $t = 0$, i.e., $|\Phi_H\rangle = |\Psi_S(0)\rangle$. For a closed system, i.e., one for which the Hamiltonian H_S in the **Schrödinger picture** is time-independent, there exists a unitary transformation

$$V(t) = e^{iH_S t/\hbar} \quad (\text{c})$$

which effects the transition from the Schrödinger to the Heisenberg picture, with

$$|\Phi_H\rangle = V(t) |\Psi_S(t)\rangle \quad (\text{d})$$

$$F_H(t) = V(t) F_S V^{-1}(t) \quad (\text{e})$$

where it is assumed that the Schrödinger and Heisenberg pictures coincide at time $t = 0$. (It is to be noted that for a closed system $H_S = H_H = H$.) The Heisenberg picture is the one in which the investigation and expression of the **relativistic invariance** of the quantum mechanical equation of motion is usually made. The reason for this is principally that the Schrödinger picture deals only with experiments which are instantaneous in the **inertial frame** which it uses. Such experiments are difficult to express in terms of similar experiments in different inertial systems. Furthermore, in order to define an instantaneous state it must be possible to define a **complete set of commuting observables** which correspond to compatible experiments at a single instant of time. Since it is not at all clear that such a set exists for a relativistic (field) system one

cannot expect that instantaneous states can be defined for such systems. For this reason, as well as for the reason that relativistic covariance can be simply discussed in terms of Heisenberg states, the Heisenberg picture plays an important role in the formulation of relativistic quantum mechanics.

HEISENBERG THEORY OF FERROMAGNETISM. The exchange interaction (see discussion of **exchange energy**) between electrons in neighboring atoms can be shown to depend on the relative orientations of the electronic **spins**. If it should turn out that parallel spins are favored, there is a strong tendency for all the spins in the lattice to become aligned, the transition to the ordered state corresponding to the **Curie point**. The concept of localized spins (e.g., *d*-electrons in the transition metals) is confirmed by neutron diffraction, but the theory is incomplete at the stage of calculating the actual magnitude and sign of the interaction.

HEITLER - LONDON - SLATER - PAULING METHOD. See **valence bond method**.

HEITLER-LONDON THEORY OF VALENCE. Application of the Heitler-London method (see **hydrogen molecule, Heitler-London theory of; valence bond method**) to molecules formed from more complicated atoms for which, however, the calculations can be carried out less rigorously. In the case of molecular states of different multiplicities resulting from two atoms in *S* states, Heitler and London were able to show that the states with the lowest total spin **S** always lie lowest and that the others lie in the order of their multiplicities. In other words, the greater the number of electron pairs with antiparallel spins formed from the unpaired electrons of the separate atoms, the more stable the resulting molecular state will be. Within the limits to which this result can be generalized the valency of a **homopolar bond** is equal to the number of newly formed electron pairs and the valency of an atom equal to the number of unpaired electrons. The representation of the saturation of a homopolar bond as a saturation of electron spins by forming antiparallel pairs corresponds to the empirical concept of chemical bonding by electron pair formation.

HELE-SHAW CELL. A cell enclosing a thin layer of viscous fluid between parallel bound-

aries. If the layer is of uniform thickness and the viscosity large enough for the inertia forces to be negligible (i.e., **Stokes flow**, with low **Reynolds number**) the drag, which is in the opposite direction to the velocity and proportional to it, exactly balances the pressure gradient force and so

$$\mathbf{v} = -\kappa \text{grad } p.$$

The velocity profile across the layer is parabolic and so \mathbf{v} is a representative velocity. κ depends on how \mathbf{v} is defined, on the thickness of the layer, and on the fluid viscosity. The isobars are equipotentials if κ is a constant, and an analogy can be drawn with any two-dimensional problem requiring the solution of **Laplace's equation**, e.g., in magnetism, electrostatics, etc. The lines of flow which may be revealed by coloring the fluid and having glass walls to the cell are analogous with lines of force, etc., in problems with similar boundaries. Variable permeability, etc., may be represented by variable thickness of the cell.

HELICITY. The scalar product $\hat{\sigma} \cdot \hat{\mathbf{p}}$ where $\hat{\sigma}$ is a unit vector in the direction of the particle's spin and $\hat{\mathbf{p}}$ is a unit vector in the direction of the particle's momentum. (See **polarization**.)

HELIUM GAS. Because of its low atomic mass, helium has an appreciable vapor pressure even at low temperatures, its boiling point being 4.2°K. Because of this the **equation of state** of helium can only be obtained properly if one takes quantum effects into account.

HELIUM, LIGHT ISOTOPE OF. The most abundant helium atoms, He_4 , are **bosons**, but the He_3 atoms are **fermions**. This has as a consequence that liquid He_3 does not show superfluidity—a property very probably connected with the **Bose-Einstein statistics** obeyed by the He_4 atoms.

HELIUM, LIQUID. Because of its very large **zero-point energy** helium does not solidify under its own vapor pressure. At 2.2°K it undergoes a phase transition, the **lambda transition**. Below the lambda point it exhibits superfluidity and its viscosity behaves like that of a gas.

Liquid helium near the absolute zero is a quantum-liquid and can be approximately described as being a system of phonons and rotons.

HELIX. A curve traced on a cylindrical or conical surface in such a way that all elements of the surface are cut at a constant angle. A circular helix lies on a right circular cylindrical surface. In **parametric form**, its equation is

$$x = a \cos \theta, \quad y = a \sin \theta, \quad z = b\theta$$

where a, b are constants and θ is a parameter. The thread of a screw is often a circular helix.

HELMHOLTZ DERIVATIVE (Of a scalar $U(x,y,z,t)$ or a vector $\mathbf{A}(x,y,z,t)$.) (Also called Lorentz derivative.) The Helmholtz derivative is defined by the expressions

$$\frac{dU}{dt} = \frac{dU}{dt} + U \text{div } \mathbf{v}$$

and

$$\frac{d\mathbf{A}}{dt} = \frac{d\mathbf{A}}{dt} + \mathbf{A} \text{div } \mathbf{v} + (\mathbf{A} \cdot \nabla) \mathbf{v}.$$

$\frac{dU}{dt}$ and $\frac{d\mathbf{A}}{dt}$ are the total or substantial derivatives with respect to time, \mathbf{v} is the instantaneous velocity of a material particle, the carrier of the scalar property U or the vector property \mathbf{A} . If S is a moving material surface, then

$$\frac{d}{dt} \iint_S U dS = \iint_S \frac{dU}{dt} dS$$

and

$$\frac{d}{dt} \iint_S \mathbf{A} \cdot d\mathbf{S} = \iint_S \frac{d\mathbf{A}}{dt} \cdot d\mathbf{S}.$$

$$\frac{d}{dt} \int_C \mathbf{A} \cdot d\mathbf{r} = \int \left[\frac{\partial \mathbf{A}}{\partial t} + \left\{ (\text{curl } \mathbf{A}) \times \mathbf{v} \right\} \right] \cdot d\mathbf{r}$$

$$\frac{d}{dt} \iiint_V U dV = \iiint_V \left[\frac{\partial U}{\partial t} + \text{div} (U\mathbf{v}) \right] dV$$

HELMHOLTZ EQUATION (FOR OPTICAL MAGNIFICATION). An equation of the form

$$n_1 y_1 \tan \theta_1 = n_2 y_2 \tan \theta_2,$$

expressing the relation between the linear and the angular magnification at a spherical refracting interface. y_1, y_2 are linear dimensions of object and image, θ_1, θ_2 the angles made by focal rays and axis at object and image points and n_1, n_2 are refractive indices of object and image space. Also called

Lagrange-Helmholtz equation. (See, however, the **Abbe sine condition**.) A spherical surface cannot satisfy both these equations for finite angles. Hence a spherical surface can never make a perfect image. (See **Lagrange theorem**.)

HELMHOLTZ EQUATION FOR VORTICITY. An equation which describes the rate of change of vorticity ω in a homogeneous incompressible viscous fluid.

$$\begin{aligned} \frac{D\omega}{Dt} &= \frac{\partial\omega}{\partial t} + (\mathbf{v} \cdot \nabla)\omega \\ &= (\omega \cdot \nabla)\mathbf{v} + \nu \nabla^2 \omega. \end{aligned}$$

In the right-hand side the first term represents the change in vorticity due to stretching or shearing motion in the fluid, and the second term the conduction of vorticity by viscosity.

HELMHOLTZ FLOW. Flow with free streamlines or vortex sheets.

HELMHOLTZ FORMULATION OF HUYGENS PRINCIPLE. Solutions of the wave equation

$$\nabla^2 u = \frac{1}{c^2} \frac{\partial^2 u}{\partial t^2}$$

of the form $u = ve^{-ikct}$ where v is independent of t and solutions of $(\nabla^2 + k^2)v = 0$ are called monochromatic waves. Helmholtz is responsible for the theorem:

Suppose v is a solution of $(\nabla^2 + k^2)v = 0$ with first and second partial derivatives continuous outside and on a closed surface S , and such that $|rv|$ is bounded and

$$r \left(\frac{\partial v}{\partial r} - ikv \right) \rightarrow 0$$

uniformly with respect to θ and ϕ as $r \rightarrow \infty$. Let

$$I(P) = \iint_S \left\{ \frac{e^{ikr}}{r} \frac{\partial v}{\partial \nu} - v \frac{\partial}{\partial \nu} \left(\frac{e^{ikr}}{r} \right) \right\} dS$$

where r is the distance from a fixed point P and $\partial/\partial \nu$ denotes differentiation along the outward normal to S . The value of $I(P)$ is $-4\pi v(P)$ or zero according as P is outside or inside S .

HELMHOLTZ FREE ENERGY. See **Helmholtz function**.

HELMHOLTZ FUNCTION. The potential given by

$$A = U - TS$$

where U is the internal energy, T is the temperature, and S is the entropy. Some authors call this function **free energy** (or the work function). However, it is preferable to use the neutral term **Helmholtz function** because other authors use the term **free energy** to denote the **Gibbs function**, with resulting confusion. (See also **thermodynamics, characteristic functions of**.)

HELMHOLTZ INSTABILITY. (Also called shearing instability.) The hydrodynamic **instability** arising from a shear, or discontinuity, in current speed at the interface between two fluids in two-dimensional motion. The perturbation gains kinetic energy at the expense of that of the basic currents.

According to the theory of **small perturbations**, waves of all wavelengths on such an interface are unstable, their rate of growth being $\exp \mu t$ with μ given by

$$\mu = \frac{\pi}{\lambda} |U - U'|,$$

where λ is the wavelength and U and U' the current speeds of the two fluids. Such waves are called **Helmholtz waves** or *shear waves*, and move with a phase speed c equal to the mean of the current speeds

$$c = \frac{1}{2}(U + U').$$

HELMHOLTZ-KETTELER FORMULA. The dispersion formula:

$$n^2 = 1 + \sum \frac{D'\lambda^2}{\lambda^2 - \lambda_s^2 + G\lambda^2}$$

where D' is a constant, $G\lambda^2$ is the term representing the frictional force, λ is the wavelength and the sum is taken over s , the λ_s being the resonant wavelengths of the medium.

HELMHOLTZ-LAGRANGE FORMULA. See the **Lagrange theorem** and **Helmholtz' equation (for optical magnification)**.

HELMHOLTZ THEOREM. The statement that if \mathbf{F} is a vector field satisfying certain quite general mathematical conditions, then \mathbf{F} is the sum of two vectors, one of which is **irrotational** (has no vorticity), the other **solenoidal** (has no divergence).

HELMHOLTZ WAVES. Unstable waves at a discontinuity of velocity in a fluid, i.e., at a **vortex sheet**. In the simplest case the discontinuity is horizontal and the density above (ρ_1) is less than below (ρ_2) so that the surface is statically stable; then waves of length $2\pi/k$ given by

$$\frac{g}{k} < \frac{\rho_1 \rho_2}{\rho_2 - \rho_1} (U_1 - U_2)^2$$

are unstable when the amplitude is small, where U_1 and U_2 are the velocities of the upper and lower streams, and the streams are deep compared with the wavelength.

Helmholtz waves have been invoked by some authors to explain the flapping of flags, the ruffling of a water surface by wind, and the formation of some forms of billow clouds.

HENCKY-PRANDTL NET. Net of orthogonal systems of slip lines in plane plastic flow of a perfectly plastic material. (See **slip line** for their properties.)

HENNEBERG METHOD. A method of analyzing statically determinate complex trusses by substituting for one of the bars to arrive at a simple truss. This substitute truss first is analyzed under the given loads and next under the action of equal and opposite forces along the bar which was removed. The magnitude and sign of the pair of forces is adjusted to make the total force in the substitute bar equal to zero.

HENRY. See **electromagnetic units**.

HENRY LAW. See **vapor pressure of ideal solutions**.

HEREDITARY MATERIAL. See **viscoelasticity**.

HERMANN-MANGUIN SYMBOLS. A notation sometimes used to describe the **symmetry classes** of crystals. Two-, three-, four- and six-fold **rotation axes** are represented by the numbers 2, 3, 4 and 6. Three-, four- and six-fold **inversion axes** have symbols $\bar{3}$, $\bar{4}$, $\bar{6}$. Asymmetry has the symbol 1. A center of **symmetry** has the symbol $\bar{1}$. A plane of **symmetry** is represented by m (mirror). The first number denotes the **principal axis**. If a plane of symmetry is perpendicular to an axis, this is represented by n/m (e.g., $2/m$, $4/m$, $6/m$). Then follow the symbols for the sec-

ondary axes, if any, and then any other symmetry planes.

HERMITE EQUATION. A second-order differential equation

$$y'' - 2xy' + 2ny = 0$$

where n is a constant. (See **Hermite polynomials**.)

HERMITE INTERPOLATION FORMULA. An **interpolation** formula which makes use of the polynomial of degree $2n + 1$ with prescribed value and prescribed slope at each of the $n + 1$ abscissae

$$x_0, x_1, \dots, x_n,$$

assumed distinct. The polynomial* is

$$H(x) = \sum_0^n [f_i v_i(x) + f'_i w_i(x)] L_i^2(x),$$

where f_i and f'_i are the prescribed values and slopes; where

$$v_i(x) = x - x_i,$$

$$w_i(x) = 1 - w_i(x)\omega''(x_i)/\omega'(x);$$

and where the $L_i(x)$ and $\omega(x)$ occur in the **Lagrange interpolation formula**. More generally, higher derivatives may also be prescribed.

HERMITE POLYNOMIALS. The class of **special functions** obtained by orthogonalizing over the interval $[-\infty, \infty]$ the sequence of powers $1, x, x^2, \dots$, with the weighting function $w(x) = e^{-x^2}$. (Sometimes the weighting function is taken to be $e^{-x^2/2}$, and notations also differ by a factor of $(-1)^n$ or $n!$.) They satisfy the (Hermite) differential equation,

$$y''(x) - 2xy' + 2ny = 0, \quad n = 0, 1, 2, \dots$$

Explicitly they are given by (the normalizing factor is $2^n n! \sqrt{\pi}$)

$$H_0(x) = 1, \quad H_1(x) = 2x,$$

$$H_2(x) = 4x^2 - 2, \quad H_3(x) = 8x^3 - 12x, \dots,$$

$$H_n(x) = (2x)^n - \frac{n(n-1)}{1!} (2x)^{n-2} + \frac{n(n-1)(n-2)(n-3)}{2!} (2x)^{n-4} + \dots$$

Their generating function is $e^{x^2} e^{-(t-x)^2}$. They satisfy the recursion formula

$$H_{n+1}(x) - 2xH_n(x) + 2nH_{n-1}(x) = 0, \quad n \geq 1.$$

Their Rodrigues formula is:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n e^{-x^2}}{dx^n}.$$

They are useful, e.g., in the problem of the linear oscillator in quantum mechanics. (For a closely associated series much used in statistics, see **Gram-Charlier series**.)

HERMITIAN CONJUGATE. (1) See **Hermitian matrix**; **Hermitian operator**; **representation theory**, **quantum mechanical**.

HERMITIAN MATRIX. A matrix **A** such that $A^* = A$. Its **eigenvalues** (proper values) are all real and it can always be diagonalized. In fact, there exists a unitary matrix **V** such that

$$A = V\Lambda V^*$$

where Λ is diagonal. The computation of the **eigenvalues and eigenvectors** is much simpler, therefore, than in the more general case.

One of the most successful methods is that of *Givens*, which will be described for the symmetric (real) case: Let R_{23} be a matrix that differs from the identity only in the elements (2, 2), (3, 3), (2, 3), and (3, 2), and let

$$\rho_{22} = \cos \phi = \rho_{33}; \quad \rho_{23} = -\rho_{32} = \sin \phi.$$

Then R_{23} is orthogonal, and ϕ can be chosen so that in

$$R_{23}^T A R_{23}$$

the elements in positions (1, 3) and (3, 1) vanish. If a matrix R_{24} is formed analogously, the elements in (1, 4) and (4, 1) can be annihilated without disturbing the zeros previously created. By continuing this process, the final matrix will be tridiagonal in form:

$$T = \begin{pmatrix} \alpha_1 & \beta_1 & 0 & 0 & \cdots \\ \beta_1 & \alpha_2 & \beta_2 & 0 & \cdots \\ 0 & \beta_2 & \alpha_3 & \beta_3 & \cdots \\ \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

The characteristic polynomial can be obtained by means of a very simple recursion:

$$\begin{aligned} \phi_0(\lambda) &= 1, \\ \phi_1(\lambda) &= \lambda - \alpha_1, \\ \phi_2(\lambda) &= (\lambda - \alpha_2)\phi_1(\lambda) - \beta_1^2\phi_0(\lambda), \\ \phi_3(\lambda) &= (\lambda - \alpha_3)\phi_2(\lambda) - \beta_2^2\phi_1(\lambda), \\ &\dots \end{aligned}$$

$\phi_n(\lambda)$ being the required polynomial. However, if every $\beta_i \neq 0$ the polynomials $\phi_i(\lambda)$ form a Sturm sequence, possessing all the properties required for satisfying the **Sturm theorem**.

Consequently, for any μ , by counting the variations in sign of the sequence of values $\phi_i(\mu)$ one can ascertain the precise number of eigenvalues exceeding and exceeded by μ . It is therefore advantageous not to expand the polynomial $\phi(\lambda)$ explicitly, but to proceed as follows for solving the equation: Having found an interval on which one or more roots are known to lie, take consecutive midpoints, evaluating the terms of the sequence for each by means of the recursion.

If the vectors are required, it will be necessary to form the product **R** of all the rotations applied. Then

$$AR = RT,$$

and if **t** is an eigenvector of **T** belonging to λ :

$$Tt = \lambda t,$$

then

$$ARt = RTt = \lambda Rt$$

so that **Rt** is an eigenvector of **A** belonging to λ . (See also **Lanzos method of biorthogonalization**.)

Another method, often called the *Jacobi method*, for computing eigenvalues is iterative in character, and makes use of the fact that the matrix **A** can be transformed to the diagonal form Λ as the limit of an infinite sequence of plane rotations. The rotation matrices R_{ij} are of the same form as before, but the angle ϕ is chosen so that in the matrix

$$R_{ij}^T A R_{ij}$$

the elements vanish in the positions (i, j) and (j, i) . Zeros are not preserved, in general, but the sum of the squares of the diagonal elements is increased by the squares of the annihilated elements at each step. Optimally one should, at each step, annihilate the pair of largest off-diagonal elements, but to reduce the searching one can go through the elements above the diagonal in some sequential order, annihilating all elements greater than some specified magnitude. With successive cycles the level should be decreased. In the limit the product of all the rotation matrices R_{ij} is the matrix **V** of eigenvectors. This method has

been described in detail by von Neumann, Goldstine, and Murray.

HERMITIAN OPERATOR. An operator P , operating in a complex **Hilbert space**, is Hermitian if, for all elements u, v in the space, the following relation holds for scalar products $(Pv, u) = (v, Pu)$. Thus, if (u, v) is defined by

$$(u, v) = \int_a^b u^* P v dx,$$

where u^* is the complex function conjugate to u , then P is Hermitian if

$$\int_a^b u^* \cdot (Pv) dx = \int_a^b v \cdot (Pu)^* dx.$$

All operators of interest in quantum mechanics have this property.

HERMITIAN SCALAR PRODUCT. See **vector space**.

HERMITIAN SPACE. See **vector space**.

HERPOLHODE. The curve along which the cone traced out by the **angular velocity vector** intersects the invariable plane tangent to the momental ellipsoid (see **ellipsoid, momental**) and perpendicular to the angular momentum vector, in the case of a rotating rigid body not subject to any external torque. (See **polhode**.)

HERSCHEL CONDITION. To obtain **Abbe's sine condition** the change in aberration due to a transverse displacement of the object point is investigated and this change is zero if the sine condition is satisfied. Similarly, Herschel's condition is sufficient for the vanishing of the change in aberration due to a longitudinal displacement of the object point. The condition for an axially symmetric optical system is

$$\frac{\sin \frac{1}{2}\theta}{u} = \frac{\sin \frac{1}{2}\theta'}{u'}$$

where u, u' are the angles from the optical axis made by a paraxial ray between the paraxial foci O, O' , and θ, θ' are the angles with the optical axis made by any ray from O to O' . If the object and image are at infinity, i.e., the system is telescopic, the Herschel condition becomes

$$\frac{\rho}{y} = \frac{\rho'}{y'}$$

where ρ, ρ' are the incident heights of a paraxial ray at finite aperture in the plane of the entrance and exit pupils, respectively, and y, y' are the incident heights of the ray in object and image spaces, respectively.

Herschel's condition and Abbe's sine condition cannot simultaneously hold, except for telescopic systems or systems with transverse magnification ± 1 . In these cases the two conditions are equivalent.

HERTZ. A unit of frequency equal to one cycle per second.

HERTZ VECTOR. The integration of Maxwell's equations can be reduced to the determination of a vector potential \mathbf{A} and a scalar potential Φ which in a homogeneous medium satisfy the wave equations

$$\nabla^2 \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = 0$$

$$\nabla^2 \phi - \frac{1}{c^2} \frac{\partial^2 \phi}{\partial t^2} = 0$$

and the Lorentz gauge condition

$$\nabla \cdot \mathbf{A} + \frac{1}{c} \frac{\partial \Phi}{\partial t} = 0.$$

(Whenever the operator ∇^2 precedes a vector quantity it is to be understood that the vector is expressed in Cartesian coordinates.)

Heinrich Hertz has shown that it is possible under ordinary conditions to define the electromagnetic field in terms of a single vector function \mathbf{F} , viz.

$$\mathbf{A} = \frac{1}{c^2} \frac{\partial \mathbf{F}}{\partial t}$$

$$\Phi = \text{div } \mathbf{F}$$

and

$$\mathbf{H} = \frac{1}{c^2} \text{curl } \frac{\partial \mathbf{F}}{\partial t}; \quad \mathbf{E} = -\text{grad } \text{div } \mathbf{F} - \frac{1}{c^2} \frac{\partial^2 \mathbf{F}}{\partial t^2}$$

$$\nabla^2 \mathbf{F} - \frac{1}{c^2} \frac{\partial^2 \mathbf{F}}{\partial t^2} = 0.$$

\mathbf{F} is the Hertz vector.

If the field is generated by a charge distribution $\rho = \rho(\xi, \eta, \zeta, t)$ and a velocity distribution $\mathbf{v} = \mathbf{v}(\xi, \eta, \zeta, t)$ and if

$$\mathbf{p} = \int_0^t \rho \mathbf{v} dt,$$

then

$$F(x,y,z,t) = \iiint \frac{dx dy dz}{r} \mathbf{p}_{t-r/c}$$

where $r = \sqrt{(x - \xi)^2 + (y - \eta)^2 + (z - \zeta)^2}$ and the subscript $t - \frac{r}{c}$ signifies that \mathbf{p} is to be evaluated at the retarded time $\frac{t - r}{c}$. If the charge is concentrated in one point and is oscillating about a fixed point in a fixed direction, the quantity \mathbf{p} represents the dipole moment with respect to this fixed point. If the fixed point is chosen as the origin of the coordinate system ($\xi = \eta = \zeta = 0$) and the fixed direction as the z -axis, then

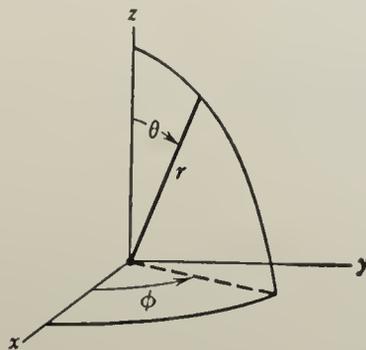
$$p_x = p_y = 0$$

$$p_z = p(t)$$

and

$$\mathbf{F} = \mathbf{k} \frac{p(t - r/c)}{r}$$

In this case it is best to use polar coordinates:



The potentials are given by

$$A_r = \left(\frac{\dot{p}}{cr} \cos \theta \right)_{t-r/c}$$

$$A_\theta = - \left(\frac{\dot{p}}{cr} \sin \theta \right)_{t-r/c}$$

$$A_\phi = 0$$

$$\Phi = \left\{ \frac{\dot{p}}{cr} + \frac{p}{r^2} \right\}_{t-r/c} \cos \theta$$

$$H_r = H_\theta = 0; \quad H_\phi = \frac{\sin \theta}{r} \left\{ \frac{\ddot{p}}{c^2} + \frac{\dot{p}}{cr} \right\}_{t-r/c}$$

$$E_\phi = 0; \quad E_r = 2 \cos \theta \left\{ \frac{\dot{p}}{cr^2} + \frac{p}{r^3} \right\}_{t-r/c}$$

$$E_\theta = \left[\frac{\dot{p}}{c^2 r} \sin \theta + \left(\frac{\dot{p}}{cr^2} + \frac{p}{r^3} \right) \sin \theta \right]_{t-r/c}$$

A dot means partial derivative with respect to time. The field in the "near-zone" is given by

$$H_\phi = \frac{\dot{p}}{cr^2} \sin \theta$$

$$E_r = \frac{2p}{r^3} \cos \theta; \quad E_\theta = \frac{p}{r^3} \sin \theta.$$

It is understood that unretarded values are used in this case. This is the field of a current \dot{p} and a static dipole p . The field in the "far-zone" or "wave-zone" is given by

$$H_r = H_\theta = 0; \quad E_r = 0; \quad E_\phi = 0;$$

$$H_\phi = \left\{ \frac{\ddot{p}}{rc^2} \sin \theta \right\}_{t-r/c}; \quad E_\theta = \left\{ \frac{\ddot{p}}{rc^2} \sin \theta \right\}_{t-r/c}.$$

The total radiation per unit time is given by

$$S = \frac{2}{3c^3} (\ddot{p})^2_{t-r/c}.$$

An accelerated charge emits not only radiation but also angular momentum. The amount of angular momentum radiated per unit time is

$$\mathbf{M} = \frac{2}{3c^3} \left[\mathbf{p} \times \dot{\mathbf{p}} \right]_{t-r/c}$$

HERZBERGER DISPERSION FORMULA.

The dispersion formula for the index of refraction

$$n = \mu_0 + \mu_1 \lambda^2 + \mu_2 (\lambda^2 - 0.035)^{-1} + \mu_3 (\lambda^2 - 0.035)^{-2}$$

is due to Herzberger. The μ_i are empirical constants.

HESSEBERG METHOD. See **eigenvalues and eigenvectors.**

HESSE NORMAL FORM.

Most of the standard forms, $y = mx + b$; $Ax + By + C = 0$, etc., of equation for the straight line in analytic geometry suffer from the defect that they fail to represent certain lines ($y = mx + b$ does not represent a vertical line) or else that many different equations (e.g., $Ax + By + C = 0$ and $2Ax + 2By + 2C = 0$) represent the same straight line. Hesse's normal form is constructed with the purpose that exactly one such equation shall correspond to each straight line; it is obtained from $Ax + By + C = 0$ in the following way. By division, if necessary, by -1 it is arranged that: if

$C \neq 0$, then $C > 0$; if $C = 0$, $B \neq 0$, then $B > 0$; if $C = B = 0$, then $A > 0$; and the equation is then divided by the normalizer factor $\sqrt{A^2 + B^2}$. Hesse's normal form is of considerable practical importance in the problem of finding the distance from a point to a line.

HESSIAN. The Hessian of a function $F(x_1, x_2, \dots, x_n)$ of n variables is the functional determinant,

$$\begin{vmatrix} F_{1,1} & F_{1,2} & \cdots & F_{1,n} \\ F_{2,1} & F_{2,2} & \cdots & F_{2,n} \\ \vdots & \vdots & & \vdots \\ F_{n,1} & F_{n,2} & \cdots & F_{n,n} \end{vmatrix}$$

where $F_{ij} = \partial^2 F / \partial x_i \partial x_j$. It is to be noted that the Hessian of F is the **Jacobian** of the n functions $\partial F / \partial x_1, \partial F / \partial x_2, \dots, \partial F / \partial x_n$.

HESS LAW. The heats of reaction may be added in the same way as the equations for chemical reactions. This fact is of use when the heat of a particular reaction is difficult to measure experimentally, but the reaction can be split into other reactions whose heats are more easily determined.

HETEROPOLAR BOND. (Also called ionic bond or electrovalent bond.) The chemical bond between two ions of opposite charge, produced by the classical electrostatic attraction. For two pointlike ions of charge ϵ ($= 4.8024 \times 10^{-10}$ esu) at distance r from each other, the potential energy, according to Coulomb's law, is

$$V = -\frac{\epsilon^2}{r} = -\frac{11.615}{r} \times 10^4,$$

where the numerical factor gives V in cm^{-1} if r is substituted in Angstrom units. As a result of the finite extent of the actual ions, a deviation from the simple Coulomb law takes place at small internuclear distances. The deviation is always in the sense of a repulsion and can be represented by an exponential term, so that

$$V = -\frac{\epsilon^2}{r} + Be^{-r/\rho},$$

where B and ρ are constants. ρ is a measure of the sum of the radii of the ions under consideration. (For more detail see G. Herzberg,

Molecular Spectra and Molecular Structure, I, Spectra of Diatomic Molecules, 2d ed., D. Van Nostrand Co., Inc., Princeton, 1950.)

HEXAGONAL CLOSE-PACKED STRUCTURE. A crystal structure obtained by packing together equal spheres as follows: A layer is made by placing each sphere in contact with six others. A second layer is added by placing each sphere in contact with three spheres of the bottom layer. In the third layer, the spheres go directly above those in the first layer—and so on. (See also **close-packed structure**.)

HEXAGONAL SYSTEM. One of the seven crystal systems. In this system, three equal axes are coplanar at 60° , fourth axis is at right angles to the other three.

HICKS FORMULA. A formula developed from the original Rydberg formula for a more accurate representation of the observed wave numbers ν_n in atomic line series. Hicks' formula is:

$$\nu_n = \nu_\infty - \frac{R}{\left[n + \mu + \frac{a}{n} + \frac{b}{n^2} + \frac{c}{n^3} + \cdots \right]^2}$$

where a, b, c, \dots are constants. (For the meaning of the other symbols, see **Rydberg equation**.)

HIDDEN COORDINATES. (See derivation of **Hamilton-Jacobi partial differential equation**.) It may happen that some of the coordinates $q_1 q_2 \dots$ are not explicitly contained in the **Lagrangian**, although the velocities $\frac{dq_1}{dt}, \frac{dq_2}{dt}, \dots$ are so contained. Such coordinates are called *hidden, ignorable* or *cyclic*. Their corresponding conjugate momenta are constants of motion: Example: the angular position of a particle rotating along a circle with uniform angular velocity. The angle does not occur in the Lagrangian.

HIGH ELASTICITY. See **elasticity, high**.

HIGHER IGNITION LIMIT. See **ignition limit**.

HIGHEST COMMON FACTOR. The highest common factor of two polynomials, P_0 and P_1 , is the polynomial of greatest degree that divides both. In case this is a constant, the polynomials are said to be relatively prime.

It can be found by applying the Euclidean algorithm: Supposing the degree of P_1 not exceeding that of P_0 , divide P_1 into P_0 and let $-P_2$ represent the remainder; divide P_2 into P_1 with $-P_3$ the remainder; \dots ; then if $P_i \neq 0$, $P_{i+1} = 0$, P_i is the highest common factor. This is represented

$$\begin{aligned} P_0 &= Q_1 P_1 - P_2, \\ P_1 &= Q_2 P_2 - P_3, \\ &\dots \dots \dots \\ P_{i-1} &= Q_i P_i. \end{aligned}$$

HILBERT SPACE. A space which is linear, metric and complete. (See under Euclidean space, and under complete.) Usually a Hilbert space is also taken to be infinite-dimensional; that is, for any integer n there exist n linearly independent points (or vectors) in the space; and also separable, that is, there exists a sequence of points which is everywhere dense in the space. The most important realizations of a Hilbert space are the set of all sequences $(x_1, x_2, \dots, x_n, \dots)$ such that $(x_1^2 + x_2^2 + \dots + x_n^2 + \dots)$ is convergent, and the set of measurable functions f integrable in square over a fundamental interval; that is, such that $\int_a^b f^2(x) dx$ is integrable in the sense of Lebesgue.

HILBERT TRANSFORM. The integral transform defined by

$$\begin{aligned} f(x) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[1 + \cot \frac{(x-y)}{2} \right] \phi(y) dy \\ \phi(y) &= \frac{1}{2\pi} \int_{-\pi}^{\pi} \left[1 + \cot \frac{(x-y)}{2} \right] f(x) dx. \end{aligned}$$

(See also integral transform.)

HINDERED ROTATION AROUND A SINGLE BOND. According to elementary theories, the σ C—C single bond in ethane, propane, acetaldehyde, etc., should be axially symmetrical. One observes, however, that the relative rotation of the two groups around this bond is not free, but hindered by a potential barrier of the order of 1–3 kcal, which must be crossed over whenever the mutual orientation of the groups is changed by about 20° .

HINGE. A support or junction point in a structure at which there is no restraint against rotation. The bending moment or end moment is zero at a hinge, but shear and axial

force generally can be transmitted across a hinge.

HINGED ARCH. See arch.

HINGE, PLASTIC. When the limit moment is reached in a beam of perfectly plastic material, rotation can take place under constant moment. At such a plastic hinge, no additional moment is required to produce rotation, while at an actual hinge no moment at all is needed. (See mechanism; method of analysis of frames.)

HITCHCOCK METHOD. A method for finding complex roots of an algebraic equation. (See Bairstow method.)

HITCHCOCK TRANSPORTATION PROBLEM. See transportation problem.

H-L-S-P (HEITLER-LONDON-SLATER-PAULING) METHOD. See valence bond method.

H-M-H (HUND-MULLIKEN-HÜCKEL) METHOD. See molecular orbitals method.

HODOGRAPH. The curve formed by the ends of the velocity vectors of a moving particle, when all the vectors are drawn from a common point.

HODOGRAPH EQUATIONS, TRANSONIC. See transonic hodograph equations.

HODOGRAPH TRANSFORMATION. A transformation of the x, y -plane into the u, v -plane where x, y are the independent, and u, v the dependent variables in the differential equations of gas dynamics. In certain gases, it transforms the equations into linear ones.

HOFF CONVERGENCE METHOD. The stability of a rigid-jointed truss under a given set of loads is checked by applying an arbitrary moment at any joint and determining the end moments everywhere by moment distribution. Stiffness and carry-over factors are computed, taking axial force into account. If the moment distribution process converges and the answer is unique, the truss is stable. Uniqueness can be checked by changing the order of balancing of joints.

HOHLRAUM. See black body.

HÖLDER CONDITION. A function $f(x)$ satisfies a Hölder condition at a point a if

there exist constants c and k such that $|f(x) - f(a)| < c|x - a|^k$ for all x in some neighborhood of a . (Compare **Lipschitz condition**.)

HÖLDER INEQUALITY. A generalization of the **Schwarz inequality**; thus for real numbers,

$$|a_1b_1 + \cdots + a_nb_n| \leq \{ |a_1|^p + \cdots + |a_n|^p \}^{1/p} \{ |b_1|^q + \cdots + |b_n|^q \}^{1/q},$$

and for real functions,

$$\left| \int_a^b f(x)g(x)dx \right| \leq \left\{ \int |f(x)|^p dx \right\}^{1/p} \cdot \left\{ \int |g(x)|^q dx \right\}^{1/q}$$

with

$$\frac{1}{p} + \frac{1}{q} = 1.$$

These formulas give the **Schwarz inequality** for the case $p = q = 2$.

“HOLE.” In general, a state or **energy level** not occupied by a particle, particularly when adjacent levels are filled. The term is applied particularly to the case of electrons in a metal, or **semiconductor**, where it is more convenient to describe an **energy band** as containing just a few holes, rather than as nearly full of electrons. This usage is reinforced by the circumstance that the electrons in such states, near the top of a band, have anomalous properties, such as negative **effective mass**, whereas the holes, representing the absence of such anomalous electrons, are mobile, and behave otherwise as if they were normal particles (although, of course, of opposite electric charge).

For the application of the term in particle theory, see entry following.

HOLE THEORY. The theory proposed by Dirac in 1930 to resolve the difficulty associated with the negative energy solutions of the **Dirac equation**. The latter admits both positive and negative energy solutions. The meaning of a negative energy particle is, however, not clear. A negative energy particle if it existed, would have a negative rest mass, and it would be accelerated in a direction opposite to that of the external force. No such particles are known to exist in nature. In the presence of an external field the Dirac equa-

tion predicts a finite probability for a transition from a positive energy state to a negative energy one. Furthermore, in order to guarantee energy conservation, such transitions (from a positive to a negative energy state) would be to have to release an amount of energy of the order of $2mc^2$, where m is the mass of the particle. Such transitions are never observed. To resolve this difficulty Dirac advanced the **hole theory** in which he assumed that all the negative energy states are occupied with one particle in each state in accordance with the **Pauli exclusion principle**. If this is so, the exclusion principle also makes it impossible for positive energy particles to make transitions to negative energy states (since they are all occupied), unless they are first emptied by some means. Such an unoccupied negative energy state will now appear as something with positive energy, since to make it disappear, i.e., to fill the unoccupied state up, we would have to add to it a negative energy particle. Similarly, the charge of the “hole” (i.e., of the unoccupied negative energy state) would be opposite of that of the positive energy particle.

In his original formulation, which identified the electron as the positive energy particle, Dirac envisaged an electron distribution of negative density everywhere in the world, with a perfect vacuum having all states of negative energy occupied, and all states of positive energy unoccupied. One difficulty encountered with this assumption involves the enormous charge density contributed by the negative energy states. In the modern version of the theory, as developed by Heisenberg, Kramers and others, this difficulty is overcome and there exists a complete symmetry between particles and **anti-particles** (“hole”).

Dirac’s hole theory predicts the possibility of “pair creation” (i.e., the simultaneous creation of a particle anti-particle pair) if enough energy is supplied by a γ -ray (or otherwise) to lift an electron from a negative energy state to a positive energy state, i.e., if the energy of the γ -ray is greater than $2mc^2$. The hole theory, in fact, predicts electromagnetic effects even if the energy is less than $2mc^2$ because the electromagnetic field will cause redistribution of charge and thus give rise to a “**polarization of the vacuum**.”

The correctness of the hole theoretic interpretation of the spin $\frac{1}{2}$ Dirac equation as

applied to electrons, neutrinos, μ -mesons, protons, neutrons, Λ and Σ particles has been established experimentally by the verification of the existence of positrons (the anti-particle in the case of electrons), anti-neutrons, μ -particles, anti-protons, anti-neutrons, anti- Λ and anti- Σ particles. The existence of electronic vacuum polarization effects has also been established experimentally, particularly through a contribution to the **Lamb shift**, by the fine structure of the positronium ground state and by the level shifts of mesic atoms.

HOLLOW BOX. See **black body**.

HOLOMORPHIC FUNCTION. A function is said to be holomorphic (or regular) in a region S of the complex plane if it has a derivative at every point of S .

HOLONOMIC MOTION. Motion is holonomic when the differential equations of constraint are integrable, *non-holonomic* if they are not.

HOLTE METHOD. In the theory of the slowing down of neutrons at large distances from a source, this method is the most powerful and general of the analytical methods. It is based upon analytical continuation in the plane of the energy-transform variable, and unlike the methods of **Bethe** and of **Wick** may be extended to the case of a medium having energy-dependent cross sections.

HOMEOMORPHIC GRAPHS. See **graphs**, **homeomorphic**.

HOMOCENTRIC. A pencil of rays is homocentric or *stigmatic* if the rays are concurrent in a point other than the **object point**.

HOMOGENEOUS. A function $f(x_1, x_2, \dots, x_n)$ is homogeneous in all of its variables if, for any parameter t , $f(tx_1, tx_2, \dots, tx_n) = t^n f(x_1, x_2, \dots, x_n)$. The exponent n is the order of the function. The Euler theorem on homogeneous functions states that for such a function

$$x_1 \frac{\partial f}{\partial x_1} + \dots + x_n \frac{\partial f}{\partial x_n} = nf.$$

The term is used with two meanings for a differential equation: (1) A first-order equation, $y' = M(x,y)/N(x,y)$ is a homogeneous equation if M, N are homogeneous functions of the same degree. (2) The general equation,

$f(x,y,y',y'',\dots) = 0$ is homogeneous and linear if f is a homogeneous linear function of y and all its derivatives. If the right-hand side equals a function of x , the independent variable, it is still linear but now inhomogeneous.

An **integral equation**, a **boundary condition**, or a system of simultaneous linear algebraic equations can also be homogeneous or inhomogeneous in a similar way.

HOMOGENEOUS AFFINE TRANSFORMATION. See **affine transformation**.

HOMOGENEOUS DEFORMATION. A deformation in which a particle initially at X_i in a rectangular Cartesian reference system x moves to x_i in the same system, where

$$x_i = b_i + a_{ij}X_j$$

where b_i and a_{ij} are constants and the **indicial notation** and **summation convention** are used.

HOMOGENEOUS FUNCTIONS IN THERMODYNAMICS. See **Euler theorem**.

HOMOGENEOUS LORENTZ TRANSFORMATION. The linear transformation which relates the space-time coordinates $x^\mu = \{x^0 = ct, x^1, x^2, x^3\}$ of an event as seen by an observer S to the space-time coordinates x'^μ of the same event as seen by an observer S' . The most general homogeneous Lorentz transformation is the linear transformation

$$x'^\mu = \sum_{\nu=0}^3 \Lambda^\mu{}_\nu x^\nu \quad (a)$$

(where $x^\mu = \{x^0 = ct, x^1, x^2, x^3\}$) which leaves the quadratic form

$$\begin{aligned} x^2 &= \sum_{\mu=0}^3 x^\mu x_\mu = (x^0)^2 - \mathbf{x}^2 \\ &= (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 \end{aligned} \quad (b)$$

invariant, i.e., for which $x^2 = x'^2$. The requirement that $x'^2 = x^2$ is the mathematical statement of the fact that light propagates with the same speed, c , with respect to both 0 and $0'$. In (a) the transformation coefficients Λ_μ^ν are all real. The condition that the quadratic form x^2 be invariant requires that

$$\sum_{\mu,\rho=0}^3 \Lambda_\nu{}^\mu g_{\mu\rho} \Lambda_\sigma{}^\rho = g_{\nu\sigma} \quad (c)$$

or in matrix form

$$\Lambda^T g \Lambda = g \quad (d)$$

where $g_{\mu\nu}$ is the **metric tensor** with components

$$g_{00} = -g_{11} = -g_{22} = -g_{33} = +1$$

$$g_{\mu\nu} = 0 \quad \text{for } \mu \neq \nu. \quad (e)$$

It follows from Equation (d) that $\det \Lambda = \pm 1$ and therefore for every homogeneous Lorentz transformation there exists an inverse transformation Λ^{-1} such that $\Lambda^{-1}\Lambda = 1$, the identity transformation corresponding to $S = S'$. The product of two homogeneous Lorentz transformations is again a homogeneous Lorentz transformation so that the set of all homogeneous Lorentz transformations form a **group**: the **homogeneous Lorentz group**.

By setting $\sigma = \nu = 0$ in Equation (c) one obtains

$$(\Lambda_0^0)^2 = 1 + \sum_{i=1}^3 (\Lambda_0^i)^2$$

$$\geq 1$$

so that $\Lambda_0^0 \geq 1$ or $\Lambda_0^0 \leq -1$. A Lorentz transformation for which $\Lambda_0^0 \geq 1$ is called an **orthochronous homogeneous Lorentz transformation**.

The set of all Λ can be divided into four subsets according to whether $\det \Lambda$ equals plus or minus one and Λ_0^0 is greater than +1 or less than -1. The subset with $\det \Lambda = +1$ and $\Lambda_0^0 \geq 1$ is called the group of restricted homogeneous Lorentz transformations and is a 6-parameter continuous group. The other subsets can be obtained by adjoining to the restricted homogeneous Lorentz group the following three (discrete) transformations:

(a) Space Inversion: $x_0 \rightarrow x'_0 = x_0$; $\mathbf{x} \rightarrow \mathbf{x}' = -\mathbf{x}$

$$\Lambda(i_s) = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\det \Lambda(i_s) = -1 \quad \Lambda_0^0 \geq 1$$

(b) Time Inversion: $x_0 \rightarrow x'_0 = -x_0$; $\mathbf{x} \rightarrow \mathbf{x}' = \mathbf{x}$

$$\Lambda(i_t) = \begin{pmatrix} -1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}$$

$$\det \Lambda(i_t) = -1 \quad \Lambda_0^0 \leq -1$$

(c) Space-Time Inversion: $x' = -x$

$$\Lambda(i_{st}) = \Lambda(i_s)\Lambda(i_t)$$

$$= \begin{pmatrix} -1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

$$\det \Lambda(i_{st}) = +1 \quad \Lambda_0^0 = -1$$

These subsets are disjoint and are not continuously connected.

HOMOGENEOUS PROCESS. A **stochastic process** $\{x(t)\}$ is homogeneous in t if the probability of transition from state $x(t_1)$ to $x(t_2)$ depends only on the difference $t_2 - t_1$.

HOMOGRAPHIC TRANSFORMATION. A homographic (also called linear or linear fractional) transformation of a complex variable z is a transformation of the form

$$w = \frac{az + b}{cz + d}$$

with constants a, b, c, d such that $ad - bc \neq 0$. Any three points α, β, γ may be transformed by a (uniquely determined) homographic transformation into any three others α', β', γ' . (See also **homogeneous Lorentz transformation**.)

HOMOMETRIC PAIRS. Two **crystal structures** having the same X-ray diffraction pattern. This is possible because, basically, a diffraction pattern depends only on the relative vector distances between the atoms in the lattice, not on their absolute positions in space.

HOMOMORPHISM. A mapping of a group G onto a group G' such that products are mapped onto products, i.e., such that, if $a \rightarrow a'$ and $b \rightarrow b'$, then $ab \rightarrow a'b'$. If the mapping is one-to-one it is called an *isomorphism*. A homomorphism (isomorphism) of a group into itself is called an *endomorphism* (*automorphism*).

HOMOPOLAR BOND. See **atomic bond**.

HOMOTOPIC. See **deformation**.

HOOKE'S LAW. The concept of the proportionality of stress to strain as a suitable description of the behavior of most materials in

the elastic range. (See Hooke's law, generalized; elastic constants.)

HOOKE'S LAW, GENERALIZED. Each of the six components of stress at a point of a body is a linear function of the six components of (infinitesimal) strain at that point. The law is valid for a large number of hard materials over a range of strain of practical interest. For isotropic materials the law takes the following form in a rectangular Cartesian coordinate system:

$$\sigma_{ij} = 2\mu e_{ij} + \lambda e_{kk} \delta_{ij},$$

where σ_{ij} and e_{ij} are the components of stress and strain respectively in the system, δ_{ij} is the **Kronecker delta** and the **summation convention** is used. λ and μ are then physical constants for the particular material to which the equation applies and are called the Lamé constants. μ is also known as the *rigidity modulus*, *rigidity*, or *sheer modulus* for the material.

HOOP STRESS. See stress, circumferential.

HORIZONTAL SHEAR. See shear, horizontal.

HORN EQUATION, FUNDAMENTAL. A horn is a passive acoustical transducer consisting of a tube of varying sectional area. The fundamental wave equation for axial motion in tube of varying section is

$$\ddot{\phi} - c^2 \frac{\partial \phi}{\partial x} \frac{\partial}{\partial x} (\log S) - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0$$

where ϕ is the velocity potential, c is the velocity of sound, in centimeters per second, S is the cross-sectional area at a point x along the axis, in square centimeters, and x is the distance along the axis of the horn, in centimeters.

HORNER METHOD (FOR OBTAINING IRRATIONAL ROOTS OF AN ALGEBRAIC EQUATION). Given that a particular root is located between consecutive integers n_0 and $n_0 + 1$, reduce the roots by n_0 . Then locate the root between consecutive tenths, say $n_1/10$ and $(n_1 + 1)/10$, $0 \leq n_1 \leq 9$, and reduce by $n_1/10$. By continuing, the root can be determined as

$$n_0 + n_1 10^{-1} + n_2 10^{-2} + \dots$$

to as many decimals as desired (see **synthetic division**). For hand computation the method

is not to be recommended, but with a binary machine the method takes the form of successive bisection of an interval on which a root is known to lie, in order to obtain successive binary digits, and the method is often very effective. (See **algebraic equations**.)

HORN, FINITE CONICAL. The acoustical impedance at the throat of a finite conical horn may be obtained in a manner similar to the procedure for the finite cylindrical horn in the following entry by employing the equations for the pressure and velocity in an infinite conical horn and applying the proper boundary conditions. The expression for the acoustical impedance, z_{A1} , at the throat in terms of the dimensions of the horn and the acoustical impedance, z_{A2} , at the mouth is

$$z_{A1} = \frac{\rho c}{S_1} \left[\frac{jz_{A2} \frac{\sin k(l - \theta_2)}{\sin k\theta_2} + \frac{\rho c}{S_2} \sin kl}{\left\{ \begin{array}{l} z_{A2} \frac{\sin k(l + \theta_1 - \theta_2)}{\sin k\theta_1 \sin k\theta_2} \\ - \frac{j\rho c \sin k(l + \theta_1)}{S_2 \sin k\theta_1} \end{array} \right\}} \right] \quad (1)$$

where z_{A1} is acoustical impedance at the throat, in acoustical ohms, S_1 is the area of the throat, in square centimeters, S_2 is the area of the mouth, in square centimeters, l is the length of the horn, in centimeters, $k\theta_1$ is $\tan^{-1} kx_1$, $k\theta_2$ is $\tan^{-1} kx_2$, x_1 is the distance from the apex to the throat, in centimeters, x_2 is the distance from the apex to the mouth, in centimeters, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, c is the velocity of sound, in centimeters per second, ρ is the density of air in grams per cubic centimeter, z_{A2} is the acoustical impedance at the mouth, in acoustical ohms.

The acoustical impedance at the mouth of the horn is usually assumed to be the same as that of a piston in an infinite baffle. In this case the mouth acoustical impedance, z_{A2} , is given by Equation 5 of **mechanical and acoustical impedance load upon a vibrating piston**.

HORN, FINITE CYLINDRICAL. The acoustical impedance at the throat of the finite cylindrical horn is

$$z_{A1} = \frac{p_1}{U_1} \quad (1)$$

where z_{A1} is the acoustical impedance, in acoustical ohms, p_1 is the pressure at the throat, in dynes per square centimeter, and U_1 is the volume current, in cubic centimeters per second.

The acoustical impedance, at the mouth of a cylindrical horn is

$$z_{A2} = \frac{p_2}{U_2} \quad (2)$$

where z_{A2} is the acoustical impedance, in acoustical ohms, p_2 is the pressure at the mouth, in dynes per square centimeter, and U_2 is the volume current, in cubic centimeters per second.

From Equations 4 and 5 of **horn, infinite cylindrical** the expressions for the pressures and volume currents at the throat and mouth are given by

$$\text{At } x = 0, \quad p_1 = kc\rho A e^{jkt} \quad (3)$$

$$U_1 = S_1 k A e^{jkt} \quad (4)$$

$$\text{At } x = l, \quad p_2 = kc\rho A e^{jk(ct-l)} \quad (5)$$

$$U_2 = S_1 k A e^{jk(ct-l)}. \quad (6)$$

From Equations 1, 2, 3, 4, 5 and 6 the expression for the acoustical impedance, z_{A1} , at the throat in terms of the length and cross-sectional area of the horn and the acoustical impedance, z_{A2} , at the mouth is

$$z_{A1} = \frac{\rho c}{S_1} \left(\frac{S_1 z_{A2} \cos(kl) + j\rho c \sin(kl)}{jS_1 z_{A2} \sin(kl) + \rho c \cos(kl)} \right) \quad (7)$$

where ρ is the density of the medium, in grams per cubic centimeter, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, c is the velocity of sound, in centimeters per second, S_1 is the cross-sectional area of the pipe, in square centimeters, l is the length of the pipe, in centimeters, and z_{A2} is the acoustical impedance at the mouth, in acoustical ohms.

The acoustical impedance at the mouth of the horn is usually assumed to be the same as that of a piston in an infinite baffle. In this case the mouth acoustical impedance, z_{A2} , is given by Equation 5 of **mechanical and acoustical impedance load upon a vibrating piston**.

HORN, FINITE EXPONENTIAL. The acoustical impedance at the throat of a finite exponential horn may be obtained in a manner similar to the procedure for the finite cylindrical horn in the preceding entry by employing the equations for the pressure and

velocity in an infinite exponential horn and applying the proper boundary conditions. The expression for the acoustical impedance, z_{A1} , at the throat in terms of the length and flare constant of the horn and the acoustical impedance, z_{A2} , at the mouth is

$$z_{A1} = \frac{\rho c}{S_1} \left[\frac{S_2 z_{A2} [\cos(bl + \theta)] + j\rho c [\sin(bl)]}{jS_2 z_{A2} [\sin(bl)] + \rho c [\cos(bl - \theta)]} \right] \quad (1)$$

where z_{A1} is the acoustical impedance at throat, in acoustical ohms, S_1 is the area of the throat, in square centimeters, S_2 is the area of the mouth, in square centimeters, l is the length of the horn, in centimeters, z_{A2} is the acoustical impedance of the mouth, in acoustical ohms, θ is $\tan^{-1} a/b$, a is $m/2$, and b is $\frac{1}{2}\sqrt{4k^2 - m^2}$.

For $b = 0$, Equation 1 is indeterminate. To evaluate, take the derivative of the numerator and denominator with respect to b and set $b = 0$. Then the expression for the throat acoustical impedance becomes

$$z_{A1} = \frac{\rho c}{S_1} \left[\frac{z_{A2} \left(1 - \frac{ml}{2}\right) + j \frac{\rho c}{S_2} \frac{lm}{2}}{jz_{A2} \frac{lm}{2} + \frac{\rho c}{S_2} \left(1 + \frac{ml}{2}\right)} \right] \quad (2)$$

Below the frequency range corresponding to $b_1 = 0$, b_1 is imaginary. For evaluating this portion of the frequency range the following relations are useful:

$$\begin{aligned} \tan^{-1} jA &= j \tanh^{-1} A \\ &= \frac{1}{2}j[\log \epsilon(1 + A) - \log \epsilon(1 - A)] \end{aligned} \quad (3)$$

$$\begin{aligned} \log \epsilon(-1) &= \pm j\pi(2K + 1), \\ K &= \text{any integer} \end{aligned} \quad (4)$$

$$\cos(A \pm jB) = \cos A \cosh B \mp j \sin A \sinh B \quad (5)$$

$$\sin jA = j \sinh A \quad (6)$$

The acoustical impedance at the mouth of the horn is usually assumed to be the same as that of a piston in an infinite baffle. In this case the mouth acoustical impedance, z_{A2} , is given by Equation 5 of **mechanical and acoustical impedance load upon a vibrating piston**.

HORN, INFINITE CONICAL. The equation expressing the cross-sectional area as a function of the distance along the axis is

$$S = S_1 x^2 \tag{1}$$

where S is the cross-sectional area at x , in square centimeters, x is the distance along the horn axis, in centimeters, and S_1 is the cross-sectional area at x_1 , in square centimeters.

The general horn equation for the conical horn from Equation 1 and the fundamental horn equation (see **horn equation, fundamental**) is

$$\ddot{\phi} - \frac{2c^2}{x} \frac{\partial \phi}{\partial x} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0. \tag{2}$$

The velocity potential, pressure, and volume current are

$$\phi = \frac{A}{x} \epsilon^{j(\omega t - kx)} \tag{3}$$

$$p = -\frac{j\omega\rho A}{x} \epsilon^{j(\omega t - kx)} \tag{4}$$

$$U = -\frac{AS(1 + jkx)\epsilon^{j(\omega t - kx)}}{x^2} \tag{5}$$

where ϕ is the velocity potential, A is the amplitude of ϕ , p is the sound pressure, in dynes per square centimeter, U is the volume current, in cubic centimeters per second, ρ is the density of the medium, in grams per cubic centimeter, ω is $2\pi f$, f is the frequency, in cycles per second, t is the time, in seconds, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, c is the velocity of sound, in centimeters per second.

The real and imaginary components of the acoustical impedance, in acoustical ohms, at the throat are

$$r_A = \frac{\rho c}{S_1} \frac{k^2 x_1^2}{1 + k^2 x_1^2} \tag{6}$$

$$x_A = \frac{\rho c}{S_1} \frac{kx_1}{1 + k^2 x_1^2} \tag{7}$$

where r_A is the acoustical resistance, in acoustical ohms, x_A is the acoustical reactance, in acoustical ohms, S_1 is the area at x_1 , in square centimeters, x_1 is the distance of throat from $x = 0$, in centimeters.

HORN, INFINITE CYLINDRICAL (INFINITE PIPE). The equation expressing the cross-sectional area as a function of the distance along the axis is

$$S = S_1 \tag{1}$$

where S_1 is the cross section of the pipe, in square centimeters.

The general horn equation for the infinite pipe from Equation 1 and the fundamental horn equation is

$$\ddot{\phi} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0. \tag{2}$$

The velocity potential, pressure, and volume current are

$$\phi = A \epsilon^{jk(ct-x)} \tag{3}$$

$$p = kc\rho A \epsilon^{jk(ct-x)} \tag{4}$$

$$U = S_1 k A \epsilon^{jk(ct-x)} \tag{5}$$

where ϕ is the velocity potential, A is the amplitude of ϕ , x is the distance along the axis, in centimeters, p is the sound pressure, in dynes per square centimeter, U is the volume current, in cubic centimeters per second, t is time, in seconds, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, ρ is the density of the medium, in grams per cubic centimeter and c is the velocity of sound, in centimeters per second.

The real and imaginary components of the acoustical impedance at the throat or input end of the pipe are

$$r_A = \frac{\rho c}{S_1} \tag{6}$$

$$x_A = 0 \tag{7}$$

where r_A is the acoustical resistance, in acoustical ohms, and x_A is acoustical reactance, in acoustical ohms.

HORN, INFINITE EXPONENTIAL. The equation expressing the cross-sectional area as a function of the distance along the axis

$$S = S_1 \epsilon^{mx} \tag{1}$$

where S is the cross-sectional area at x , in square centimeters, x is the distance along the axis, in centimeters, S_1 is the cross-sectional area at the throat, that is, $x = 0$, in square centimeters, and m is the flaring constant.

The general horn equation for the exponential horn from Equation 1 and the fundamental horn equation is

$$\ddot{\phi} - c^2 m \frac{\partial \phi}{\partial x} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0. \tag{2}$$

The velocity potential, pressure, and volume current are

$$\phi = \epsilon^{-(m/2)x} \left[A \epsilon^{-j \frac{\sqrt{4k^2 - m^2}}{2} x} \right] \epsilon^{j\omega t} \tag{3}$$

$$p = -j\omega\rho\epsilon^{-(m/2)x} \left[A\epsilon^{-j\frac{\sqrt{4k^2-m^2}}{2}x} \right] \epsilon^{j\omega t} \quad (4)$$

$$U = -AS \left[\frac{m}{2} + j\frac{\sqrt{4k^2-m^2}}{2} \right] \left[\epsilon^{-\frac{m}{2}x-j\frac{\sqrt{4k^2-m^2}}{2}x+j\omega t} \right] \quad (5)$$

where ϕ is the velocity potential, A is the amplitude of ϕ , p is the sound pressure, in dynes per square centimeter, U is the volume current, in cubic centimeters per second, ω is $2\pi f$, f is the frequency, in cycles per second, t is the time, in seconds, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, c is the velocity of sound in centimeters per second and ρ is the density of the medium, in grams per cubic centimeter.

The real and imaginary components of the acoustical impedance, in acoustical ohms, at the throat are

$$r_A = \frac{\rho c}{S_1} \sqrt{1 - \frac{m^2}{4k^2}} \quad (6)$$

$$x_A = \frac{\rho c}{S_1} \frac{m}{2k} \quad (7)$$

where r_A is the acoustical resistance, in acoustical ohms and x_A is the acoustical reactance, in acoustical ohms.

When $m = 2k$ or $4\pi f = mc$ the acoustical resistance is zero. This is termed the cutoff frequency of the exponential horn.

Below the cutoff frequency the acoustical impedance is entirely reactive and

$$x_A = \frac{\rho c}{S_1} \left(\frac{m}{2k} - \sqrt{1 - \frac{m^2}{4k^2}} \right). \quad (8)$$

HORN, INFINITE PARABOLIC. The equation expressing the cross-sectional area as a function of the distance along the axis is

$$S = S_1 x \quad (1)$$

where S is the cross-sectional area at x , in square centimeters, x is the distance along the horn axis, in centimeters and S_1 is the cross-sectional area at x_1 , in square centimeters.

The general horn equation for the parabolic horn from Equation 1 and the fundamental horn equation (see **horn equation, fundamental**) is

$$\ddot{\phi} - \frac{c^2}{x} \frac{\partial \phi}{\partial x} - c^2 \frac{\partial^2 \phi}{\partial x^2} = 0. \quad (2)$$

The velocity potential, pressure, and volume current are

$$\phi = A[J_0(kx) - jY_0(kx)]\epsilon^{j\omega t} \quad (3)$$

$$p = -j\omega\rho A[J_0(kx) - jY_0(kx)]\epsilon^{j\omega t} \quad (4)$$

$$U = ASk[-J'_0(kx) + jY'_0(kx)]\epsilon^{j\omega t} \quad (5)$$

where ϕ is the velocity potential, A is the amplitude of ϕ , p is the sound pressure, in dynes per square centimeter, U is the volume current, in cubic centimeters per second, ω is $2\pi f$, f is the frequency, in cycles per second, t is the time, in seconds, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, J_0 , J_1 are **Bessel functions** of the first kind of order zero and one, and Y_0 , Y_1 are Bessel functions of the second kind of order zero and one.

The real and imaginary components of the acoustical impedance, in acoustical ohms, at the throat are

$$r_A = \frac{\rho c}{S_1} \frac{2}{\pi k x_1 [J_1^2(kx_1) + Y_1^2(kx_1)]} \quad (6)$$

$$x_A = \frac{\rho c}{S_1} \frac{J_0(kx_1)J_1(kx_1) + Y_0(kx_1)Y_1(kx_1)}{J_1^2(kx_1) + Y_1^2(kx_1)} \quad (7)$$

where r_A is the acoustical resistance, in acoustical ohms, and x_A is the acoustical reactance, in acoustical ohms, ρ is the density of the medium, in grams per cubic centimeter, c is the velocity of sound, in centimeters, S_1 is the area at x_1 , in square centimeters, x_1 is the distance of the throat from $x = 0$, in centimeters, k is $2\pi/\lambda$, and λ is the wavelength, in centimeters.

HOROPTER. (1) A horopter, or *frontal horopter*, is a **geodesic** in the **Luneburg geometry** of visual space that is apparently perpendicular to the median.

(2) The *horopter of single vision* is defined for a given point of fixation and attitude of the eyes as the set of points in the binocular field of view whose retinal images lie on corresponding points of curves on the two retinas.

HOSPITAL RULE. See under **L'Hospital**.

HOTELLING'S T. A generalization to the multivariate case by H. Hotelling (1931) of the statistic t known as "**Student's distribution**." It tests simultaneously a set of means of variates which may be interdependent.

HOUR ANGLE (LHA) (SHA). The hour angle of a celestial object (*LHA*) is the angle measured along the celestial equator from the

observer's celestial meridian to the point of intersection of the hour circle through the object with the celestial equator. Hour angle is usually measured to the west (in the direction of apparent rotation of the celestial sphere) from the local meridian to the hour circle through the object. If the hour angle is measured toward the east it *must* be carefully labeled.

Sidereal hour angle (SHA) is the hour angle of a celestial object measured from the vernal equinox toward the west.

HOUR CIRCLE. Any great circle on the celestial sphere that passes through the poles of rotation is perpendicular to the celestial equator and is known as an hour circle.

H THEOREM. The *H function* has been introduced by Boltzmann:

$$H = \int f \log f d\mathbf{v}$$

where f is the molecular distribution function, and \mathbf{v} the velocity. For uniform systems (f independent of the position) and in the absence of external forces, it can be shown from the Boltzmann equation that:

$$\frac{dH}{dt} = -\frac{\pi}{2} \sum_{ij} \iiint \ln \frac{f'_i f'_j}{f_i f_j} (f'_i f'_j - f_i f_j) \mathbf{g}_{ij} b db d\mathbf{v}_i d\mathbf{v}_j \quad (1)$$

where \mathbf{g}_{ij} is the relative velocity and b the impact parameter.

Whether $f'_i f'_j$ be larger or smaller than $f_i f_j$, each term of the sum on the right hand side of (1) is always positive or 0. Therefore, one has the fundamental relationship:

$$\frac{dH}{dt} \leq 0.$$

The irreversible behavior of H can be paralleled with that of the entropy. The demonstration of the H theorem was the first "proof" of irreversible behavior from microscopic arguments, but the demonstration rested on the validity of the Boltzmann equation, which, at the time, was based on intuitive probabilistic arguments.

HUBBLE CONSTANT. The reciprocal of T in the equation $d = vT$, where v is the velocity

of recession of a galaxy which is at a distance d . $T \doteq 8 \times 10^9$ years is sometimes called the age of the universe, although there is no evidence that the term should be taken literally.

HUME-ROTHERY RULES. (See **crystal phases**.) The ratio of the number of valence electrons to the total number of atoms in an alloy is characteristic of a given crystal phase. Thus CuZn, AgCd, Cu₃Al, are all in the β -phase with ratio 3:2, while Cu₅Zn₈, Cu₉Al₄, are in the γ -phase with ratio 21:13.

HUMIDITY. (1) Absolute. The mass of water vapor present in unit volume of gas, usually expressed in grams of water vapor per cubic meter. (2) Mixing ratio. The ratio of the mass of water vapor to the mass of other gas in a mixture, expressed in meteorology in grams of water vapor per kilogram of dry air. (3) Relative. The percentage of the saturation mixing ratio of water vapor actually present in a gas mixture.

HUND COUPLING CASES. A set of relatively simple coupling cases proposed by F. Hund, representing limiting cases of the various possible coupling conditions in diatomic and linear polyatomic molecules (see also **coupling in atomic and molecular structure**). Of most practical importance are Hund's cases (a) and (b). In case (a) the resultant electronic angular momentum \mathbf{L} and the resultant electron spin \mathbf{S} are strongly coupled to the internuclear axis with components Λ and Σ respectively, so that the total electronic angular momentum in the direction of the internuclear axis $\Omega = \Lambda + \Sigma$ is well defined. In case (b) the vector \mathbf{L} has a well-defined component Λ in the direction of the internuclear axis. However \mathbf{S} is no longer coupled to the internuclear axis, but to the resultant \mathbf{N} of Λ and the molecular rotation. (For more detail see G. Herzberg, *Molecular Spectra and Molecular Structure, I. Spectra of Diatomic Molecules*, 2d ed., D. Van Nostrand Co., Inc., Princeton, 1950, p. 219 ff.)

HUND-MULLIKEN-HÜCKEL METHOD. See **molecular orbitals method**.

HUND RULES FOR ATOMIC MULTIPLETS. (1) Of the atomic terms resulting from **equivalent electrons**, those with greatest **multiplicity** $2S + 1$ (S , quantum number of resultant electron spin) lie lowest, and of these the lowest is the one with the highest value of

L (L , quantum number of resultant orbital electronic angular momentum).

(2) Multiplets formed from equivalent electrons are regular when less than half the shell is occupied, but inverted when more than half the shell is occupied. (See **electron shells in an atom**.)

HURWITZ CRITERION. See **stability (of system)**.

HURWITZ POLYNOMIAL. A polynomial of order n in some variable s having real coefficients,

$$H(s) = \sum_{r=0}^n a_r s^r$$

is known as a Hurwitz polynomial if the n roots of the equation

$$H(s) = 0$$

all have negative real parts. The constraints on the coefficients required for this to be true are discussed under **stability**, algebraic criteria.

An interesting property of Hurwitz polynomials is that the component polynomials consisting of the odd and even terms in $H(s)$, namely,

$$H_1(s) = a_0 + a_2 s^2 + a_4 s^4 + \dots$$

and

$$H_2(s) = s(a_1 + a_3 s^2 + a_5 s^4 + \dots)$$

have zeros which all lie on the imaginary axis of s , the zeros of H_1 and H_2 interlacing.

HUYGENS' PENDULUM. See **tantochrone**.

HUYGENS' PRINCIPLE. Hadamard has analyzed Huygens' principle in the syllogism:

(Major premise.) To determine the effect at time t_1 of a luminous phenomenon caused by a given disturbance at time $t_0 < t_1$ we may find the state at time t^1 , $t_0 < t^1 < t_1$, and from that deduce the state at t_1 .

(Minor premise.) If during $t_0 - \epsilon \leq t \leq t_0$ a luminous disturbance is localized in the immediate neighborhood of a point O , the effect at the subsequent time t^1 is localized in a very thin spherical shell with center O and radius $c(t^1 - t_0)$, where c is the velocity of light.

(Conclusion.) The effect at time t_1 due to a luminous disturbance localized at O at time t_0 is determined when the initial disturbance is replaced by a suitable system of luminous disturbances taking place at time t^1 , $t_0 < t^1 < t_1$,

and distributed over the surface of the sphere with center O and radius $c(t^1 - t_0)$.

Huygens' principle gives a satisfactory account of the laws of reflection and of refraction but is not sufficient for the explanation of diffraction phenomena though it provides a good approximation in some cases. Fresnel extended Huygens' principle, by adjoining assumptions concerning the amplitude of the secondary disturbances, and thereby explained some diffraction phenomena. Helmholtz, for monochromatic wave functions, and Kirchhoff in the general case, showed that these special assumptions were satisfied by all sufficiently well-behaved solutions of the wave equation. (See **Helmholtz formulation of Huygens' principle** and **Kirchhoff's formulation of Huygens' principle**.)

HUYGENS' WAVELETS. The secondary disturbances in **Huygens' principle** are referred to as Huygens' wavelets. If $V(x, y, z, u, v)$ is **Hamilton's point characteristic**, where u, v are curvilinear coordinates on the surface of a primary disturbance, then Huygens' wavelets are given by

$$V(x, y, z, a, b) = ct$$

where c is the velocity of light and t is time.

If the wave fronts of the primary disturbance are

$$\phi(x, y, z) - ct = 0$$

then

$$\phi_x^2 + \phi_y^2 + \phi_z^2 = n^2$$

but $V(x, y, z, u, v)$ is also an integral of this differential equation, depending on two parameters, and it follows that the envelope of the wavelets also is a solution of this equation. That is, both the wavelets and their envelope are **wave fronts**.

HUYGENS' ZONE. See **half-period element**.

HYBRIDIZATION OF EIGENFUNCTIONS.

Strictly speaking, this term means any linear combination of the **eigenfunctions** of one problem used to represent an eigenfunction of another problem. It is applied to problems of electronic **bonding**, where the true **bonding orbitals** may be hybrids of, for example, both s and p type **atomic orbitals**. (See next entry.)

HYBRIDIZATION OF ELECTRON ORBITALS. Some atoms, such as boron and carbon, have 3 or 4 singlet electrons in their

maximum valency state. From elementary considerations, one of these should be in the $2s$, and the others in the $2p$ states. One would then observe a marked difference between the properties of the bonds formed by these different electrons. The fact that the three BH bonds in the BH_3 molecule, or the four CH bonds of methane are found to be equivalent can be understood when one takes into account the possible hybridization of the wave functions of the three or four electrons of these atoms. Each of the $2s$ and $2p$ wave functions is itself a solution of the wave equation for the maximum valency state (5S for carbon); a linear combination of these is therefore also a solution. Pauling has shown that such hybrid orbitals can be constructed, and that they lead to the formation of equivalent bonds and to the observed valency angles.

HYBRID ORBITALS. See hybridization of electron orbitals.

HYDRAULIC JUMP, OR BORE. A transition at which the mass transport of a stream moving under gravity is unchanged but the horizontal momentum flux is decreased by a rise in pressure due to an increase in depth of the stream. There is a loss of total head at the jump so that the transition is irreversible: energy is dissipated at the jump in a breaking wave.

For a jump to be possible the flow must be super-undal, i.e., the **Froude number**

$$F = U/(gh)^{1/2}$$

must exceed unity. If u_1 , u_2 and h_1 , h_2 are the speeds and depths of the stream moving on a horizontal bed, the bore being stationary, then

$$u_2^2 = \frac{g}{2} \frac{h_1}{h_2} (h_1 + h_2).$$

If $H_1 = h_1 + u_1^2/2g$ is the total head of the oncoming stream, the loss of head at the jump is

$$H_1 - H_2 = (h_2 - h_1)^3/4h_1h_2.$$

By impressing on the system a velocity $-u_1$ it is seen that a bore invades still water with speed u_1 where

$$u_1^2 = \frac{g}{2} \frac{h_2}{h_1} (h_1 + h_2).$$

HYDRODYNAMIC ANALOGY (FOR THE TORSION PROBLEM IN CLASSICAL ELASTICITY THEORY). A hydrodynamic problem the solution to which is determined by differential equations and boundary conditions equivalent to those of the **torsion** problem. For example, the differential equations and boundary conditions determining the torsion function ϕ and conjugate torsion function ψ are those determining the **velocity potential** and **stream function**, respectively, for the irrotational motion of an incompressible inviscid fluid, contained in a vessel of the same shape as the cylinder which undergoes torsion and rotated with uniform angular velocity (vorticity) about the line corresponding to the axis of torsion. Again, the differential equation and boundary condition determining $\psi - \frac{1}{2}(x^2 + y^2)$ are the same as those determining the velocity distribution in a **Newtonian fluid** which flows under a constant pressure head through a straight pipe, the cross section of which is the same as that of the cylinder which undergoes torsion.

HYDRODYNAMICS. The science of the dynamics of fluid motion, especially the steady motions of an incompressible, inviscid fluid.

HYDRODYNAMICS, RELATIVISTIC. See relativistic hydrodynamics.

HYDROGEN BOND. The hydrogen atoms in such groups as HF, HO, HN and sometimes HC tend to be shared between the electronegative atom or group to which they are attached and similar groups on other molecules. These hydrogen bonds increase the intermolecular forces and the boiling points of HF, H_2O , NH_3 (to a certain extent), as well as those of organic alcohols and acids, etc.

The formation of these hydrogen bridges is usually explained by the fact that the H atom, attached to an electronegative atom or group, becomes more or less positively charged. It tends to acquire a coordination number 2 instead of 1, and to share an electron doublet with an O, N, or F atom of another molecule. It is only when the C atom is linked to strongly electronegative atoms such as Cl or F that the CH bond is sufficiently polarized to form hydrogen bonds with other molecules.

HYDROGEN FINE STRUCTURE. If in the Bohr-Sommerfeld model of the hydrogen atom and the hydrogen-like ions relativity effects

are taken into account, the elliptical electron orbits (see **electron orbits in an atom**) are found to describe a rosette motion (Sommerfeld). The energy corresponding to the different orbits depends in this case on both the principal quantum number n and the azimuthal quantum number k , and is given by

$$E_{n,k} = -\mu c^2 \left[1 + \frac{\alpha^2 Z^2}{(n-k + \sqrt{k^2 - \alpha^2 Z^2})^2} \right]^{-1/2} - \mu c^2$$

or, in series expansion,

$$E_{n,k} = -\frac{2\pi^2 \mu e^4 Z^2}{h^2 n^2} \left[1 + \frac{\alpha^2 Z^2}{n} \left(\frac{1}{k} - \frac{3}{4n} \right) \right] = -\frac{RZ^2}{n^2} hc - \frac{R\alpha^2 Z^4}{n^4} \left[\frac{n}{k} - \frac{3}{4} \right] - \dots$$

Here h stands for Planck's constant, c for the velocity of light, Z for the atomic number, μ for the reduced mass

$$\mu = \frac{mM}{m+M}$$

(m and M , for masses of electron and nucleus respectively), α , for Sommerfeld's fine structure constant

$$\alpha = \frac{2\pi e^2}{hc} = 7.29729 \times 10^{-3}$$

(e , electronic charge, and R for the **Rydberg constant**). Since the fine structure constant α is small, higher terms in Z in the above expansion can, as a rule, be neglected. They are, however, important in the x-ray spectra of hydrogen-like ions of high atomic number Z . (For the higher terms see H. E. White, *Introduction to Atomic Spectra*, McGraw-Hill, Inc., New York, 1934, p. 134.) The first term of the expansion formula gives the main part of the energy which is the same as that of the circular Bohr orbit.

Dirac's formula for the hydrogen fine structure is the same as Sommerfeld's if k is replaced by $j + \frac{1}{2}$ ($j + \frac{1}{2} = k = 1, 2, \dots, n$). To each value of j , except for the j value corresponding to $k = n$, there are two possible values of the orbital quantum number $l = j + \frac{1}{2}$ and $l = j - \frac{1}{2}$. Therefore to each level with the quantum number n and the quantum number $k \neq n$ in Sommerfeld's the-

ory, there corresponds, in Dirac's theory, a pair of levels with the quantum numbers n and $j \pm \frac{1}{2}$. However the energy of these two levels, e.g., $2^2S_{1/2}$ and $2^2P_{1/2}$ (for notation see **energy levels of atoms**) is strictly the same. (For removal of this degeneracy see **Lamb shift**.)

HYDROGEN MOLECULE, HEITLER-LONDON THEORY OF. Solution of the wave equation for the H_2 molecule using the state of the separated atoms as zero approximation and then introducing the interaction of the two atoms as a perturbation.

The potential energy of two electrons 1 and 2 in the field of two protons A and B is given by

$$V = \frac{e^2}{R} - \frac{e^2}{r_{1A}} - \frac{e^2}{r_{2A}} - \frac{e^2}{r_{1B}} - \frac{e^2}{r_{2B}} + \frac{e^2}{r_{12}}$$

where R, r_{1A}, \dots are defined as in Figure 1. If a first order perturbation calculation is carried

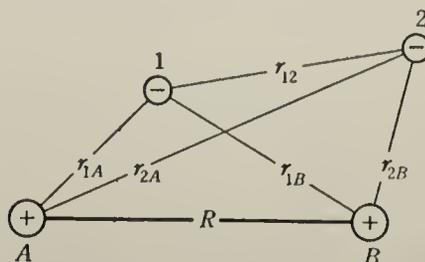


Fig. 1. Designations for the H_2 molecule. (From G. Herzberg, *Molecular Spectra and Molecular Structure. I. Spectra of Diatomic Molecules*, 1950, D. Van Nostrand Company, Inc., Princeton.)

out using as perturbation function W , the deviation from the potential energy for large R

$$W = +\frac{e^2}{R} + \frac{e^2}{r_{12}} - \frac{e^2}{r_{1B}} - \frac{e^2}{r_{2A}}$$

one obtains two energy levels E_s and E_a with eigenfunctions ψ_s and ψ_a given by

$$\psi_s = N_s[\phi_A(1)\phi_B(2) + \phi_A(2)\phi_B(1)]$$

$$\psi_a = N_a[\phi_A(1)\phi_B(2) - \phi_A(2)\phi_B(1)].$$

Here $\phi_A(1), \phi_B(1), \phi_A(2), \phi_B(2)$ are the hydrogen eigenfunctions for each of the electrons moving in the field of nuclei A and B , respectively, and N_s and N_a are normalization factors. The eigenfunction ψ_s is symmetric, that is, remains unchanged for an exchange of the electrons. ψ_a is antisymmetric, that is, it changes sign for such an exchange.

For the energies one finds

$$E_s = 2E_H + \frac{K + J}{1 + S}$$

$$E_a = 2E_H + \frac{K - J}{1 - S}$$

Here E_H is the energy of the ground state of the H atom, and

$$S = \int \phi_A(1)\phi_B(1)\phi_A(2)\phi_B(2)d\tau_1d\tau_2$$

$$K = \int \phi_A(1)\phi_B(2)W\phi_A(1)\phi_B(2)d\tau_1d\tau_2$$

$$J = \int \phi_A(1)\phi_B(2)W\phi_A(2)\phi_B(1)d\tau_1d\tau_2.$$

All three integrals, K , J , and S , occurring in the energy expressions, depend on the value of the internuclear distance R . Actual evaluation of these integrals gives the lower broken-line curve in Figure 2 for E_s and the upper for

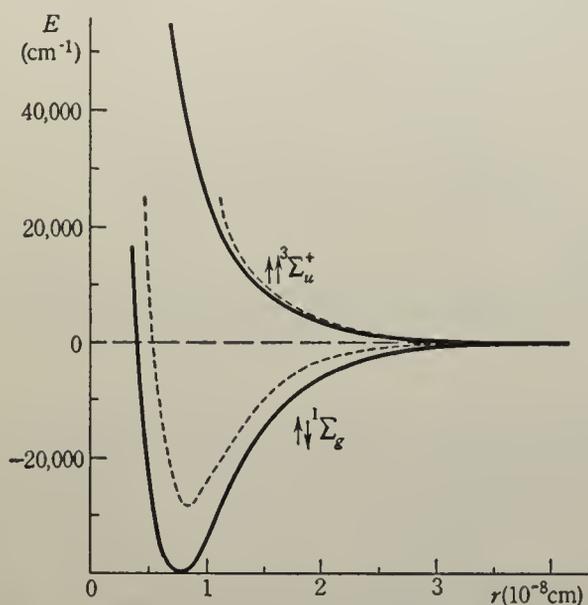


Fig. 2. Potential curves of the two lowest states of the H_2 molecule. (From G. Herzberg, *Molecular Spectra and Molecular Structure. I. Spectra of Diatomic Molecules*, 1950. D. Van Nostrand Company, Inc., Princeton.)

E_a (omitting the constant term $2E_H$), that is for the state characterized by the symmetric eigenfunction ψ_s a potential minimum arises leading to molecule formation while for the state characterized by the antisymmetric eigenfunction ψ_a there is repulsion for all R values. (The full line curves in Figure 2 result from calculations by James and Coolidge

which carried the original calculations of Heitler and London to higher approximations.)

The integral K (Coulomb integral) gives the interaction of the two charge clouds on a classical basis. The integral J (exchange integral) arises from the fact that the two electrons are indistinguishable and can be exchanged. The latter integral gives the major contribution to the potential energy of the molecule.

Since, according to the **Pauli principle**, only those states can occur whose total eigenfunction is antisymmetric, the stable state E_s can occur only with antisymmetric spin function, that is, with antiparallel electron spins. According to this theory, the ground state of H_2 should therefore be a singlet state. This is in agreement with spectroscopic observation.

(For more details see W. Heitler, *Elementary Wave Mechanics*, Oxford University Press, 1945.)

HYDROGEN, SPECIFIC HEAT OF. See specific heat of hydrogen.

HYDROSTATIC EQUATION. The form assumed by the vertical component of the **Navier-Stokes equation** when all Coriolis, earth curvature, frictional, and vertical acceleration terms are considered negligible compared with those involving the vertical **pressure force** and the force of **gravity**. Thus

$$\frac{\partial p}{\partial z} = -\rho g,$$

where p is the pressure, ρ the density, g the acceleration of gravity, and z the geometric height.

HYDROSTATIC EQUILIBRIUM. The state of a fluid whose surfaces of constant pressure and constant mass (or density) coincide and are horizontal throughout. Complete balance exists between the force of **gravity** and the **pressure force**. The relation between the pressure and the geometric height is given by the **hydrostatic equation**.

The analysis of atmospheric stability has been developed most completely for an atmosphere in hydrostatic equilibrium.

HYDROSTATICS. The study of the distribution of fluid pressure, and the forces on submerged bodies, in a fluid at rest.

Since there can be no shear stresses in a fluid at rest, the pressure gradient at any point

is vertical and equal to the weight of fluid per unit volume. That is,

$$\frac{dp}{dz} = -\rho g,$$

where z is measured vertically upward and ρ is the density of the fluid. (See also **Archimedes' principle**; **force on a submerged surface**; **stability of a floating body**.)

HYDROSTATIC STRESS. See **stress, hydrostatic**.

HYLLERAAS COORDINATES. A two-particle set of coordinates useful in the quantum mechanical problems of the helium atom, the singly ionized lithium atom, etc. The coordinates are $u = r_{12}$, where r_{12} is the separation between the two particles; $s = r_1 + r_2$, where r_1 and r_2 are the respective distances of the two particles from the origin; and $t = r_1 - r_2$. The volume element is then given by $dv_1 dv_2 = \pi 2u(s^2 - t^2) du ds dt$. The ranges of the variables are: $0 \leq u \leq s$; $-u \leq t \leq u$; $0 \leq s < \infty$.

HYPERBOLA. A conic section obtained by a plane cutting both nappes of a right-circular conical surface. It is the locus of a point which moves so that the difference of its distances from two foci is a constant. Its eccentricity is greater than unity.

The standard equation may be taken as $x^2/a^2 - y^2/b^2 = 1$. The curve is a central conic for it is symmetric about both the X - and Y -axes when placed in standard position with the coordinate origin at its center. The transverse axis, coincident with the X -axis, is of length $2a$; the conjugate axis, along the Y -axis, has length $2b$ ($b < a$). The distance from the center of the hyperbola to either focus is $\sqrt{a^2 + b^2}$; the eccentricity, $e = \sqrt{a^2 + b^2}/a$; the length of the latus rectum is $2b^2/a$; the equations for the directrices are $x = \pm a/e$, the same as for the **ellipse**. The distance from any point on the hyperbola to a focus is a focal radius and the differences between any two focal radii equals $2a$. The lines $y = \pm bx/a$ are asymptotes to the hyperbola. If the length of the transverse axis becomes equal to that of the conjugate axis ($a = b$), the curve is an equilateral or rectangular hyperbola. In this case, the asymptotes are perpendicular to each other. If the coordinate axes are rotated so that they coin-

cide with the asymptotes, the equation for the rectangular hyperbola becomes $xy = a^2/2$.

The polar equation of the hyperbola is $r = a(e^2 - 1)/(e \cos \theta - 1)$ and parametric equations are $x = a \operatorname{eosh} u$, $y = b \sinh u$ or $x = a \sec \phi$, $y = b \tan \phi$.

HYPERBOLA, GEODESIC. See **geodesic hyperbola on a surface**.

HYPERBOLIC EQUATION. See **hyperbolic partial differential equation**.

HYPERBOLIC FUNCTION. Combinations of $e^{\pm z}$ with properties similar to those of the trigonometric functions. They are defined by:

$$\sinh z = (e^z - e^{-z})/2 = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \dots$$

$$\cosh z = (e^z + e^{-z})/2 = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \dots$$

$$\tanh z = \sinh z / \cosh z; \quad \coth z = 1 / \tanh z$$

$$\operatorname{sech} z = 1 / \cosh z; \quad \operatorname{csch} z = 1 / \sinh z.$$

If n is a positive integer, $i = \sqrt{-1}$, $u = n\pi i$; $\sinh u = \tanh u = 0$; $\cosh u = (-1)^n$; $\sinh(z + u) = (-1)^n \sinh z$; $\cosh(z + u) = (-1)^n \cosh u$.

For real z , the hyperbolic functions are related to the **hyperbola** as the trigonometric functions are related to a **circle**.

Again with real z , hyperbolic and circular (trigonometric) functions are related as follows: $\sinh iz = i \sin z$; $\operatorname{eosh} iz = i \cos z$; $\tanh iz = i \tan z$. Additional formulas, similar to those familiar from trigonometry, are: $\operatorname{eosh}^2 z - \sinh^2 z = 1$; $1 - \tanh^2 z = \operatorname{sech}^2 z$; $\cosh^2 z + \sinh^2 z = \operatorname{eosh} 2z$; $2 \sinh z \operatorname{eosh} z = \sinh 2z$.

The inverse hyperbolic functions are also denoted in a manner similar to that for the inverse trigonometric functions. Thus, if $y = \sinh z$, the inverse function is the angle whose hyperbolic sine is y , or $z = \sinh^{-1} y = \operatorname{arc} \sinh y$.

HYPERBOLIC LOGARITHM. A logarithm to the base e ; a **natural logarithm**.

HYPERBOLIC PARABOLOID. See **paraboloid**.

HYPERBOLIC PARTIAL DIFFERENTIAL EQUATION. A second order partial differential equation. For the solution to be well defined ordinarily initial and bound-

any conditions are needed. In the simple case the equation can be reduced to the form $u_{xx} - u_{yy} + \dots = 0$, where omitted terms involve no derivatives of second or higher order. The usual difference scheme is representable

$$\begin{array}{c} 1 \\ \tau^2 \quad 2(1 - \tau^2) \quad \tau^2 \\ 1 \end{array}$$

where $\tau = \Delta y / \Delta x$. If initial conditions are given along a horizontal line, it is necessary that $\tau \leq 1$ for adequate **numerical stability**. (See **elliptic partial differential equation**; **partial differential equation**; **Cauchy problem for partial differential equations**.)

HYPERBOLIC POINT (ON A SURFACE).

A point at which the product of the principal curvatures (see **curvature of surface**, **radius of**) of the surface is negative. Hence, a singular point in a **streamline** field, constituting the intersection of a **convergence line** and a **divergence line**.

HYPERBOLOID. A **central quadric** with one or two negative terms in its equation. If there is only one, so that

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

the surface is a hyperboloid of one sheet, given this name because any point on the surface may be reached from any other point on the surface. A plane parallel to the XY -plane gives an **ellipse** but if the sections are parallel to the XZ - or YZ -planes the results are **hyperbolas**. When $a = b$, the sections by planes $z = \text{constant}$ are **circles** and the surfaces can be generated by revolving the hyperbola, $x^2/a^2 - z^2/c^2 = 1$ about its conjugate axis, the Z -axis.

If there are two negative terms in the equation

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$$

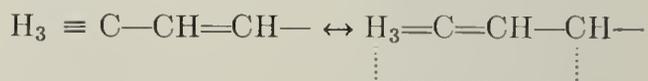
the surface is a hyperboloid of two sheets, separated into two parts symmetrically located above and below the planes $x = \text{constant}$. Traces parallel to the XY - and XZ -planes are hyperbolas and traces parallel to the YZ -planes are ellipses, provided $x > a$. When $b = c$, the sections by planes $x = \text{const.}$

are circles and a surface of revolution results when the hyperbola $x^2/a^2 - y^2/c^2 = 1$ is rotated about its X - or transverse axis.

HYPERCIRCLE METHOD. See **method of the hypereircle**.

HYPERCOMPLEX NUMBER. See **quaternion**.

HYPERCONJUGATION. The hypothesis of hyperconjugation has been advanced to interpret some properties of substances containing but 1 double bond by analogy with those of substances containing conjugated double bonds. Consider a substance with a terminal structure $\text{H}_3\text{C}-\text{CH}=\text{CH}-\dots$. One of the possible resonating structures of this group is



the dotted line indicating two unpaired electrons with opposite spins. Some authors have questioned the fact that hyperconjugation alone is sufficiently large to explain the observed anomalous properties of such substances (bond lengths and heats of formation).

HYPERFINE SPECTRUM. See **hyperfine structure**.

HYPERFINE STRUCTURE. The (in general, narrow) splitting of atomic and molecular spectral lines or energy levels due to the influence of the **nuclear spin**; also the narrow structure of atomic lines due to the **isotope effect**.

(a) Hyperfine structure due to the presence of nuclei with the same **atomic number** Z , but different masses M (isotope effect).

For the isotopes of the hydrogen atom the effect of the nuclear mass on the atomic energy levels can be accounted for by the dependence of the **Rydberg constant** on the nuclear mass M according to

$$R = \frac{2\pi^2 e^4}{ch^3} \frac{mM}{m + M},$$

where e and m are respectively charge and mass of the electron, c is the velocity of light, and h is Planck's constant. For more complicated atoms this approach is inadequate. According to the above formula the isotope effect for elements of moderately high atomic number Z would rapidly approach zero. However relatively large isotope shifts (of an order

similar to that of the effect of nuclear spin, see below) have been observed for heavy elements (e.g., Zn, P). Wave mechanical calculation of the hyperfine structure on the basis of different masses alone produce a result of the right order of magnitude for the lighter elements (e.g., Li, Ne). However for heavier elements the change of nuclear radius with nuclear mass has to be taken into account. (For more details see H. Kopfermann, *Nuclear Moments*, Academic Press, Inc., New York, 1958.)

(b) Hyperfine structure due to nuclear spin.

If the nucleus of an atom or one nucleus in a molecule has a spin $\mathbf{I} \neq 0$, the total angular momentum \mathbf{F} is the vector sum of \mathbf{I} and \mathbf{J} , the total angular momentum apart from spin. If more than one nucleus in a molecule has $\mathbf{I} \neq 0$, \mathbf{F} is the vector sum of \mathbf{J} and \mathbf{T} , the resultant nuclear spin. The corresponding quantum numbers are

$$F = J + I, J + I - 1, \dots, |J - I|$$

or

$$F = J + T, J + T - 1, \dots, |J - T|.$$

The number of hyperfine structure components, for large J , is $2I + 1$ and $2T + 1$ respectively. (Because of the very small interaction between the nuclear spins the additional splitting corresponding to the different possible values of T is very small. See below.)

The selection rules for F are

$$\Delta F = 0, \pm 1 \quad \text{and} \quad F = 0 \leftarrow | \rightarrow F = 0.$$

In general, the hyperfine structure splitting of an energy level is due to a combination of magnetic dipole and electric quadrupole interaction. If the magnetic interaction due to the nuclear magnetic moment

$$\mu I = \mu_{on} g_I$$

where μ_{on} is the **nuclear magneton**, g_I , the nuclear g -factor (see **gyromagnetic ratio**) predominates, the intervals between successive hyperfine structure components (F and $F + 1$) are proportional to $F + 1$ (**Landé's interval rule**).

(1) *Atoms*. For purely magnetic interaction of the nucleus with the rest of the atom the energy of the hyperfine structure levels is given by

$$W = W_o - A \frac{G}{2}$$

where W_o is the energy without hyperfine structure splitting,

$$G = F(F + 1) - I(I + 1) - J(J + 1)$$

and

$$A = \frac{\overline{\mu_I H(0)}}{IJ}.$$

Here $\overline{H(0)}$ is the mean value of the magnetic field produced at the position of the nucleus by the motion of the electrons.

For purely electrostatic interaction of a nucleus possessing an electric quadrupole moment \mathbf{Q} with the rest of the atom the energy of the hyperfine structure components is given by

$$W = W_o - B \frac{\frac{3}{8}G(G + 1) - \frac{1}{2}I(I + 1)J(J + 1)}{I(2I - 1)J(2J - 1)}$$

where

$$B = eQ\overline{\phi_{JJ}(0)}.$$

Here e stands for the charge of the electron and $\overline{\phi_{JJ}(0)}$ for the vector gradient of the electric field of the electrons at the position of the nucleus. $\overline{\phi_{JJ}(0)}$ is rotationally symmetric about the J -axis.

(2) *Molecules*. For purely magnetic interaction between one nucleus with $\mathbf{I} \neq 0$ and the rest of the molecule, the energies of the hyperfine structure components are given by

$$W = W_o + \left[\frac{aK^2}{J(J + 1)} + b \right] G$$

Here a and b are constants depending on the nuclear magnetic moment and on the magnetic moment due to molecular rotation; J and K are, respectively, the quantum numbers corresponding to the total angular momentum of the molecule apart from nuclear spin and its component in the direction of the figure axis.

For purely electrostatic interaction of a nucleus having $\mathbf{I} \neq 0$ and $\mathbf{Q} \neq 0$, with the rest of the molecule, the energy of the hyperfine structure components in diatomic, linear, and symmetric top molecules is given, to a good approximation, by

$$W = W_o - B \left[1 - \frac{3K^2}{J(J + 1)} \right] \left[\frac{\frac{3}{8}G(G + 1) - \frac{1}{2}J(J + 1)I(I + 1)}{I(2I - 1)(2J - 1)(2J + 3)} \right].$$

In general, for $Q \neq 0$, the electrostatic interaction of the nucleus with the rest of the molecule is predominant, and the magnetic interaction can be taken into account as a small perturbation. (For more detail, see H. Kopfermann, *l.c.*)

Influence of a magnetic field. In a magnetic field the hyperfine structure levels of an atom or molecule show a **Zeeman effect** and **Paschen-Baek effect** similar to that for ordinary fine structure. However because of the relatively weak interaction between \mathbf{J} and \mathbf{I} , the transition from the former to the latter effect takes place at lower field strength.

At moderate field strength each level corresponding to a total angular momentum \mathbf{F} will split into $2F + 1$ component levels with quantum numbers

$$M_F = F, F - 1, F - 2, \dots, -F.$$

The selection rules for M_F are

$$\Delta M_F = 0, \pm 1,$$

and

$$M_F = 0 \leftarrow | \rightarrow M_F = 0 \quad \text{for } F = 0.$$

At higher field strength, each level corresponding to a total angular momentum \mathbf{J} will split into $2J + 1$ equidistant component levels with quantum numbers

$$M_J = J, J - 1, J - 2, \dots, -J.$$

This is the ordinary Zeeman effect with line separations which, at sufficiently high field strengths, are considerably greater than those of the field-free hyperfine structure components. Each term with a given J is however once more split into $2I + 1$ components with quantum numbers

$$M_I = I, I - 1, I - 2, \dots, -I.$$

The selection rules for the quantum numbers M_J and M_I are

$$\Delta M_J = 0, \pm 1 \quad \text{and} \quad \Delta M_I = 0.$$

The energy of the component levels with different values of M_F is given by

$$W = W_0 - g_F M_F \mu_0 n H$$

where W_0 is the energy in zero field, H the magnetic field strength and

$$g_F = \frac{\left\{ \begin{array}{l} [F(F+1) + J(J+1) - I(I+1)]g_r \\ + [F(F+1) + I(I+1) - J(J+1)]g_I \end{array} \right\}}{2F(F+1)}.$$

Here g_r is the rotational g -factor, a number of the order 1, characteristic of the particular molecular state, and g_I the nuclear g -factor. (See **gyromagnetic ratio**.)

HYPERGEOMETRIC DISTRIBUTION. If a finite population of N units contains a proportion p of units of one type ("successes") and a proportion $q = 1 - p$ of units of another type ("failures"), the probability of obtaining r "successes" in a sample of n is

$$\frac{1}{N^{[n]}} \frac{n!}{r!(n-r)!} (Np)^{[r]} (Nq)^{[n-r]}$$

where $a^{[n]}$ denotes $a(a-1)(a-2)\dots(a-n+1)$.

The corresponding frequency distribution is called the hypergeometric distribution, as its terms are those of a hypergeometric series. The mean is np , the variance $\left(\frac{N-n}{N-1}\right) npq$.

The variance is smaller than that of the corresponding **binomial distribution** to which it tends as N tends to infinity.

HYPERGEOMETRIC EQUATION. See **Gauss hypergeometric equation**.

HYPERGEOMETRIC FUNCTION. The function, usually denoted by $F(a, b; c; z)$ defined by the hypergeometric series

$$1 + \frac{a \cdot b}{1 \cdot c} z + \frac{a(a+1)b(b+1)}{1 \cdot \dots \cdot c(c+1)} z^2 + \dots,$$

or also, equivalently, in other ways, e.g., by use of the **gamma function**. (See **Gauss hypergeometric equation**; and **confluent hypergeometric function**.)

HYPERMULTIPLY. In an atom with non-zero nuclear spin \mathbf{I} , each energy level with a given value of J , the quantum number of the resultant total electronic angular momentum, splits into a number of hyperfine structure levels with quantum numbers

$$F = J + I, J + I - 1, \dots, |J - I|.$$

The resulting hypermultiplet has $2I + 1$ components for $J > I$, and $2J + 1$ components for $J < I$. The intervals between successive hyperfine structure levels (F and $F + 1$) are proportional to $F + 1$ (Landé's interval rule).

(For more details, including hypermultiplet structure in molecules, see **hyperfine structure**, and the literature quoted under this heading.)

HYPersonic FLOW. If τ is the thickness ratio of a body in a stream of Mach number M , the product $M\tau$ is known as the *hypersonic similarity parameter*. Hypersonic flow is often defined as flow for which this parameter is of order 1, or greater. The linearized equation of supersonic flow is then no longer valid, because although the perturbation velocities may be small compared with the speed of flow they are not small compared with the speed of sound. This means that there are some additional non-linear terms in the equation which cannot be neglected.

Alternatively, hypersonic flow is sometimes defined as flow in which the Mach number M exceeds some arbitrarily specified value, often taken as 5.

In hypersonic flow the bow shock wave lies close to the body, and since flow disturbances are confined to the region between the body and the bow shock wave, the region of disturbed flow is a narrow one. The strength of the bow shock wave varies considerably with distance from the nose of the body, so that the flow in the disturbed region is rotational, even outside the viscous boundary layer. The variations of *speed* in the flow are quite small, the changes of Mach number being associated mainly with changes of temperature, and hence of the speed of sound.

HYPersonic SIMILARITY LAW. For hypersonic flow past a series of slender bodies or thin airfoils, related to each other by an **affine transformation**, the pressure coefficients at corresponding points are given by

$$C_p = \tau^2 P(M\tau), \quad (1)$$

where M is the stream Mach number, τ is the thickness ratio of the body, and P is a function depending on the shape of the body, when τ has been specified. The product $M\tau$ is known as the *hypersonic similarity parameter*.

Since, at hypersonic speeds, $M^2 \gg 1$, Equation (1) may also be written as

$$C_p = \tau^2 P[\tau(M^2 - 1)^{1/2}] \quad (2)$$

and it can be shown that in this form the law is consistent with the linearized equation of

supersonic flow. (See **hypersonic flow; subsonic and supersonic flow, linear equations**.) Thus Equation (2) applies over the whole range of supersonic and hypersonic speeds, for a slender body or thin airfoil.

HYPersonic SIMILARITY PARAMETER. See **hypersonic flow; hypersonic similarity law**.

HYPOTHESIS, STATISTICAL. A hypothesis concerning the form of a statistical (probability) **distribution** or some of its parameters, or of a process generating such a distribution. Many scientific hypotheses are of such a type and the testing of hypotheses forms a large and important part of statistical theory.

If the hypothesis concerns a parameter which, if known, would determine the parent population completely, it is said to be simple; in the contrary case it is composite.

HYSTERESIS. In general, the phenomenon exhibited by a system whose state depends on its previous history. This term usually refers to magnetic hysteresis, as exhibited by a ferromagnetic material. A quantitative study of the process indicates that, as the field strength, \mathbf{H} , is increased and then decreased, the magnetic induction, \mathbf{B} , follows the magnetization curve, which is of a type shown in the figure of the entry for **magnetization curve**. Note that the initial portion (solid line) of the curve is not retraced.

Electric hysteresis is a somewhat analogous phenomenon exhibited by dielectrics in the electric field.

Some solids exhibit either elastic hysteresis, in which the variables corresponding to \mathbf{H} and \mathbf{B} in the magnetic case are the stress and the strain or deformation; or plastic hysteresis, for which see **unloading**.

Hysteresis of an oscillator is a behavior that may appear in an oscillator wherein multiple values of the output power and/or frequency will correspond to give values of an operating parameter.

Hysteresis of a radiation counter tube is the temporary change in the counting rate versus voltage characteristic caused by previous operation.

I

ICE POINT. Temperature of equilibrium between air-saturated water and ice at a pressure of one standard atmosphere.

ICE-POINT DEPRESSION. The glass used in liquid-in-glass thermometers shows a considerable thermal lag: the volume change of the glass lags behind the temperature change, reaching its final value only after several hours. Thus, if a thermometer is immersed in an ice-water bath shortly after having been used at a higher temperature, the liquid (e.g., mercury) column falls somewhat below the 0°C (32°F) mark. This effect is known as ice-point depression. Good quality thermometers have an ice-point depression of less than 0.05°C on cooling from 100°C.

IDEAL ANGLE OF INCIDENCE. See **thin-airfoil theory**.

IDEAL CURRENT SOURCE. See **current source, ideal**.

IDEAL GAS. See **perfect gas**.

IDEALLY PLASTIC. Synonymous with **perfectly plastic**.

IDEAL SYSTEM. A thermodynamic system is called an ideal system when the **chemical potentials** of all the components are of the form

$$\mu_i = \mu_i(T, p) + RT \ln x_i \quad (1)$$

where $\mu_i(T, p)$ is a function only of the variables **absolute temperature**, T , and pressure, p . The x_i are the **mole fractions** of the components.

Systems for which μ_i has this form possess remarkably simple properties (see **law of mass action**; **vapor pressure of perfect solutions**). Moreover mixtures of **perfect gases** and very dilute **solutions** have these properties.

According to Equation 1, a system is called ideal if the chemical potential of component i varies linearly with the logarithm of the mole fraction of i , with a slope RT . This linear relation need not necessarily extend over the whole concentration range, so that the quantity

$\mu_i(T, p)$ is in general the value of μ_i extrapolated to $x_i = 1$ at constant T, p . If the system is ideal in a concentration range which extends to $x_i = 1$, then

$$\mu_i(T, p) = \mu_i^\circ(T, p) \quad (2)$$

where μ_i° is the chemical potential of the pure component i . They are two important cases: (a) The mixture is ideal for all values of x_i and for all i . It is then called a *perfect mixture* and Equation 2 is verified for all i . (b) The mixture is ideal when all components but one (index 1) are present in very small amount. Such systems are called *ideal dilute solutions*. Then Equation 2 is only valid for component 1.

Different kinds of ideal systems are distinguished by the form of $\mu_i(T, p)$. In a mixture of perfect gases $\mu_i(T, p)$ varies logarithmically with pressure, while for a liquid or solid solution one can, to a first approximation, regard μ_i as independent of pressure.

IDEAL VOLTAGE SOURCE. See **voltage source, ideal**.

IDEMPOTENT. A **matrix** which remains unchanged when multiplied by itself; e.g., the 3-rowed matrix,

$$\begin{pmatrix} 2 & -2 & -4 \\ -1 & 3 & 4 \\ 1 & -2 & -3 \end{pmatrix}.$$

IDENTIFYING INDEX. See **index, free**.

IDENTITY. An *equality* in which both members are equal for all values of the symbols for which the members are defined. Either member can be transformed into the other by use of the fundamental rules of operation. An identity involves a difference of form but not of value. It is frequently indicated by putting the symbol \equiv between the two members. Thus,

$$(a + b)(a - b) \equiv (a^2 - b^2).$$

I EFFECT. See **inductive effect and reactivity**.

IGNITION LIMIT. The velocity of combustion in a combustible mixture depends on its composition. It is reduced to zero at both ends of the scale, when the ignition limit is reached. The lower ignition limit corresponds to fuel deficiency, and the higher or upper ignition limit corresponds to oxygen deficiency.

IGNITION TEMPERATURE. See **combustion**.

IGNORABLE COORDINATE. A coordinate which does not appear in the **Lagrangian** function, although its time derivative may appear.

IKONAL. See **eikonal**.

ILLUMINANCE. See **illumination**.

ILLUMINANCE, RETINAL. A psychophysiological quantity, partially correlated with the **brightness** attribute of visual sensation, and measured in **trolands**.

ILLUMINATION (ILLUMINANCE). Illumination (at a point of a surface) is the quotient of the **luminous flux** incident on an infinitesimal element of surface containing the point under consideration by the area of that element. (See **quantity of illumination**.)

ILLUMINATION INTENSITY. The **flux density** of light incident upon a surface.

ILLUMINATION, QUANTITY OF. The product of an **illumination** and its duration.

IMAGE FOCAL LENGTH. See **focal length**.

IMAGE IMPEDANCE. That impedance which causes maximum power transfer, as dictated by the **Thevenin theorem**, from the **transducer** to which it is connected.

IMAGE NUMERICAL APERTURE. See **exit aperture**.

IMAGE PHASE CONSTANT. The imaginary part of the **transfer constant**. The prefix "image" may be omitted when there is no danger of confusion.

IMAGE RAY. See **ray**.

IMAGE-SIDE PUPIL. See **exit pupil**.

IMAGES, METHODS OF. (1) When electric charges are placed in the neighborhood of conductors, charges are induced on the surfaces of the conductors. The surface distribution of charges (together with the externally placed

charges) is such as to make the conducting surface an equipotential surface. The electric field and electrostatic potential external to the conductors is then the resultant field due to both the externally placed charges and the induced surface charges. In the method of images the field and potential external to the conductors are determined by imagining that the conductors are removed and that additional image charges are placed in such a way that equipotential surfaces are formed at what had been the conducting surfaces. All images must be placed in regions of space that are separated from the actual charges by the surface or surfaces at which the boundary conditions are to be satisfied. The method of images may also be used to solve problems involving dielectric rather than conducting boundaries. (See **boundary conditions on the electromagnetic field vectors**.)

(2) Similarly, the method of images may be used for the solution of magnetostatic, hydrodynamic, and other problems in which boundary conditions at the interface between two media determine the solution. In such cases, fictitious image magnetic dipoles, sources or sinks of fluid, etc., are introduced to satisfy the boundary conditions over the surfaces of the interfaces. (See following entry.)

(3) In the solution of problems in the theory of the diffusion and **slowing down** of **neutrons** in matter, this method is useful. In using it, the material under consideration is extended from finite to infinite extent, and the boundary condition satisfied by placing appropriate sources of positive or negative strength throughout the medium.

IMAGE SOURCES, SINKS, LINE VORTICES, ETC. The irrotational flow of an incompressible inviscid fluid in the presence of simply-shaped boundaries (e.g., planes, spheres, circular cylinders) can often be computed by finding a system of **sources**, **sinks**, **vortices**, etc., with the whole of space occupied by fluid, which together produce a flow in which the boundaries of the given system are in the same position as streamlines in the new system. The sources, sinks, etc., required in addition to those of the given system must be outside the region of the given fluid, and are the images of the given system in the given boundaries. There is a complete analogy with simple electrostatic systems in which the boundaries are equipotentials.

IMAGINARY. See **complex number**.

IMMITTANCE. A group of electrical quantities consisting of **impedances** and **admittances**.

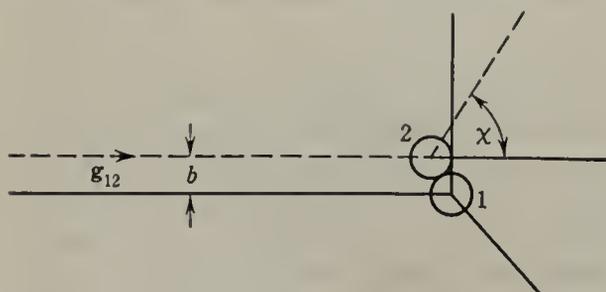
IMPACT. The action of two bodies in collision, whereby the velocity of one or both bodies is changed. In the case of direct impact, the velocity of the moving bodies is in the direction of the normal (perpendicular) to the bodies at the point of contact. Otherwise the impact is oblique. The impact is central when the centers of mass of the two bodies lie on the line of impact (normal to the bodies at the point of contact). Impact is a term which implies a short time of contact during which the impulse of the external forces, with the possible exception of some reactions, is negligible. The impulse of the (large) equal and opposite forces of impact is strictly zero. Therefore the momentum of a system of unsupported bodies is not changed during the impact.

IMPACT BUCKLING. See **buckling, impact**.

IMPACT, ELASTIC. In the impact of elastic bodies some energy appears as vibratory motion. Elementary theory ignores this loss and assumes that rigid body kinetic energy is conserved. The coefficient of restitution (see **restitution, coefficient of**) is taken as unity. (See **impact**.)

IMPACT, INELASTIC. During inelastic impact, the rigid body kinetic energy is spent partly in vibratory or wave motion of the impacting bodies and partly in inelastic deformation in the region of contact. In the extreme case the coefficient of restitution (see **restitution, coefficient of**) is zero.

IMPACT PARAMETER. Consider (see figure) a collision between two molecules. In the



Impact parameter. Collision between two hard spheres. The trajectory of sphere 2 in the coordinate system in which 1 is stationary is indicated by the broken line. χ is the angle of deflection.

system of coordinates chosen, molecule 1 is at rest, while molecule 2 moves with the relative velocity g_{12} . The axis is parallel to g_{12} . The *impact parameter* b is the minimum distance at which molecule 2 would pass by molecule 1 if the two molecules did not interact.

For hard spheres, the amount of momentum and energy exchanged during the collision depends on b alone; for other more realistic models, where the interaction potential energy depends on distance, the momentum and energy exchange depends both on the impact parameter and on the initial relative velocity. (See also **angle of deflection**.)

IMPACT, PLASTIC. A collision between two bodies whose coefficient of **restitution** has a value less than unity. In such a collision kinetic energy is lost. The amount of energy lost is given by the formula:

$$\Delta K = \frac{1}{2}M_1(e^2 - 1)u_{1e}^2 + \frac{1}{2}M_2(e^2 - 1)u_{2e}^2$$

where M_1 is the mass of the first body, M_2 is the mass of the second body, e is the coefficient of restitution between the two bodies, u_{1e} is the final velocity of the first body with respect to center of mass, u_{2e} is the final velocity of the second body with respect to center of mass.

The total momentum remains unchanged in a plastic impact.

A collision in which the coefficient of restitution is zero is often called a perfectly inelastic impact.

IMPEDANCE. The term expresses the relation between a sinusoidally varying quantity (such as force, pressure, voltage, electric field strength, temperature) and a second quantity (such as velocity, current, magnetic field strength, heat flow) which is a measure of the response of a physical system to the first. For most systems, the impedance depends on the frequency of the disturbance. When the impedance is stated for a given frequency, it connects the disturbing quantity and the response only when a steady state is reached—during the transient period the response to a sinusoidal disturbance is not, in general, itself sinusoidal.

The most common use of the term is in connection with alternating current circuits, in which the impedance Z is defined as the (complex) ratio of the (complex) potential differ-

ence V applied across a circuit element to the (complex) current I flowing in the element:

$$Z = V/I = (V_0/I_0)e^{i\phi},$$

where V_0 and I_0 are the amplitudes of the sinusoidally varying potential differences and current, respectively, and ϕ is the phase angle.

If the disturbing quantity is a sinusoidally varying force acting on a mechanical system, the force may be written in the complex form $F = F_0e^{i\omega t}$ and the velocity of motion as $v = v_0e^{i(\omega t - \phi)}$. The mechanical impedance is then

$$Z_m = F/v = (F_0/v_0)e^{i\phi}.$$

Thus, for a mass M , subject to a restoring force equal to $-kx$ (where x is the displacement of the mass from its equilibrium position and k is a constant) and to a dissipative force equal to $-R_m dx/dt$, where R is a constant, the mechanical impedance is

$$Z_m = R_m + i(\omega M - k/\omega).$$

Here R_m is exactly analogous to electrical resistance (R), M to inductance (L), and k to inverse capacitance (C) in an electric circuit having R , C , and L in series. The elements of the mechanical system (mass, springs, resistive effects) are thus analogs of circuit elements. The quantity $(\omega M - k/\omega)$ is the mechanical reactance of the system.

The concept of impedance as outlined above is satisfactory as long as the physical dimensions of the electrical circuit elements, or their mechanical analogs, are sufficiently small so that the currents or velocities may be considered to be the same everywhere throughout them. When this condition is not satisfied, it must be recognized that any disturbance will be propagated as a wave. Under such circumstances, the impedance offered to an electromagnetic wave is taken as \mathbf{E}/\mathbf{H} , where \mathbf{E} is the (complex) electric field strength at any point and \mathbf{H} is the (complex) magnetic field strength at the same point. The impedance is measured in ohms, as is the impedance for circuits with lumped parameters. For a longitudinal acoustic wave, the impedance is defined as the ratio of the (complex) pressure excess to the (complex) volume velocity, i.e., the velocity times the area perpendicular to the direction of motion. This ratio is known as the acoustic impedance. In the cgs system of units it is measured in acoustical ohms ($\text{gm cm}^{-4} \text{sec}^{-1}$).

(See also **impedance, transform** and other entries immediately following.)

IMPEDANCE, ACOUSTIC. The acoustic impedance of a sound medium on a given surface lying in a wave front is the complex quotient of the **sound pressure** (force per unit area) on that surface by the flux (**volume velocity**, or linear velocity multiplied by the area), through the surface. When concentrated, rather than distributed, impedance of a portion of the medium is defined by the complex quotient of the pressure difference effective in driving that portion, by the flux (volume velocity), the acoustic impedance may be expressed in terms of mechanical impedance, acoustic impedance being to the mechanical impedance divided by the square of the area of the surface considered. The commonly used unit is the **acoustical ohm**. Velocities in the direction along which the impedance is to be specified are considered positive. This definition pertains to single-frequency quantities in the steady state and to systems whose properties are independent of the magnitudes of these quantities. Various other definitions have been proposed, but their application has been less widespread: (a) the quotient of the force by the **particle velocity** (Crandall); (b) the quotient of the sound pressure by the **volume displacement** (Webster); (c) the quotient of the sound pressure by the particle velocity (Brillié).

IMPEDANCE, BLOCKED. Of a transducer, the impedance at the input when the impedance of the output system is made infinite. For example, in the case of an electromechanical transducer, the blocked electric impedance is the impedance measured at the electric terminals when the mechanical system is blocked or clamped; the blocked mechanical impedance is measured at the mechanical side when the electric circuit is open-circuited.

IMPEDANCE, CHARACTERISTIC, ACOUSTIC. The characteristic impedance of a medium is given by the ratio of the effective **sound pressure** and the effective **particle velocity** for a plane progressive sound wave propagated through that medium. It is equal to the product of the density of the medium and the velocity of sound in the medium. The unit is the **rayl** (dyne-sec/cm^2) (or the MKS rayl newton-sec/m^2).

IMPEDANCE, CHARACTERISTIC OF A LINE OR GUIDE. Electromagnetic waves are often transmitted over lines or waveguides. The impedance of such a guide, at any particular frequency, depends on the length of the guide and on the conditions at the far end of the line. For example, a coaxial cable that is one-eighth wavelength long and has a short circuit at the far end, acts like an **inductance**, while one three-eighths of a wavelength long, similarly short-circuited, acts like a capacitance. These variations in impedance result from reflections of the wave from the far end. Were the guide infinitely long, there would be no effect of reflections. The impedance of a guide of infinite wavelength is known as the characteristic impedance of the guide. It follows that a finite guide having as a termination an element with impedance equal to the characteristic impedance will have no reflections from its far end—such a termination is equivalent to an infinite length of line extending beyond the end. The characteristic impedance of a lossless line is equal to the square root of the product of the inductance per unit length and the capacitance per unit length.

The characteristic impedance of either an electromagnetic or an acoustic waveguide is a pure resistance, whose magnitude is dependent on the dimensions of the cross section of the guide and on the medium in which the wave is transmitted, but is independent of frequency.

IMPEDANCE CHARACTERISTIC, OF A MEDIUM. As indicated under **impedance, characteristic, of a line or guide**, this quantity depends on the dimensions of the cross section of the guide. If a plane sinusoidal wave is being propagated through space without a guide, the ratio of the amplitude of the electric field strength to that of the magnetic field strength (or of the pressure amplitude to the velocity amplitude) at any point is the same as at any other. This ratio is the characteristic impedance of the medium in which the wave travels. For an electromagnetic wave in a non-dissipative medium it is equal to the square root of the ratio of the absolute permeability to the permittivity, i.e., $Z_c = \sqrt{\mu/\epsilon}$ (377 ohms for empty space).

IMPEDANCE CIRCLE. Consider a nondissipative transmission line terminated by a fixed **impedance**. As the line length is varied,

the (complex) input impedance describes a circle in the R, X plane.

IMPEDANCE, DRIVING-POINT. At a driving point of a transducer, the complex ratio of the applied sinusoidal potential difference, force, or pressure to the resultant current, velocity, or volume velocity, respectively, at this point, all inputs and outputs being terminated in any specified manner.

IMPEDANCE, FREE. Of a transducer, the impedance at the input of the transducer when the impedance of its load is made zero. The approximation is often made that the free electric impedance of an electroacoustic transducer designed for use in water is that measured with the transducer in air.

IMPEDANCE, FREE MOTIONAL. Of a transducer, the complex remainder after the blocked impedance has been subtracted from the free impedance.

IMPEDANCE, IMAGE. See **image impedance**.

IMPEDANCE, ITERATIVE. Of a transducer, that impedance which, when connected to one pair of terminals, produces a like impedance at the other pair of terminals.

IMPEDANCE LEVEL. The value of circuit **impedance** which is used in matching several **networks** to each other or to their terminal impedances is called the impedance level in that part of the circuit.

IMPEDANCE, MECHANICAL. The complex quotient of the alternating force applied to a system by the resulting linear velocity in the direction of the force at its point of application. The unit is the **mechanical ohm**.

IMPEDANCE, MECHANICAL ROTATIONAL (ROTATIONAL IMPEDANCE). The complex quotient of the alternating torque applied to the system by the resulting angular velocity in the direction of the torque at its point of application. The unit is the **rotational ohm**.

IMPEDANCE, MOTIONAL (LOADED MOTIONAL IMPEDANCE). Of a transducer, the complex remainder after the blocked impedance has been subtracted from the loaded impedance.

IMPEDANCE, MUTUAL. The impedance that is common to two meshes of a network is their mutual impedance. Equivalently, if a set of voltages and currents are related by

$$E_1 = Z_{11}I_1 + Z_{12}I_2 + \cdots + Z_{1n}I_n$$

$$E_n = Z_{n1}I_1 + Z_{n2}I_2 + \cdots + Z_{nn}I_n$$

the coefficient $Z_{ij} = Z_{ji}$ is called the mutual impedance between the i th and j th meshes.

IMPEDANCE, NORMALIZED (WITH RESPECT TO A WAVEGUIDE). An impedance divided by the characteristic impedance of the waveguide.

IMPEDANCES, CONJUGATE. Impedances having resistance components which are equal and reactance components which are equal in magnitude but opposite in sign. Conjugate impedances are expressible by conjugate complex quantities.

IMPEDANCES, IMAGE, OF A TRANSDUCER. The impedances which will simultaneously terminate all of the inputs and outputs of the transducer in such a way that at each of its inputs and outputs the impedances in both directions are equal.

IMPEDANCE, SPECIFIC ACOUSTIC (UNIT AREA ACOUSTIC IMPEDANCE). At a point in a medium, the complex ratio of sound pressure to particle velocity.

IMPEDANCE, SURFACE TRANSFER. When an electromagnetic wave is guided along the surface of a conductor having finite conductivity, the ratio of tangential electric field at the surface to the current induced at the surface of the conductor in the conductor.

IMPEDANCE, THROAT ACOUSTIC. The acoustic impedance (see **impedance, acoustic**) at the input end of a horn.

IMPEDANCE, TRANSFER. Between two points of a transducer, the complex ratio of the applied sinusoidal potential difference, force, or pressure at one point to the resultant current, velocity, or volume velocity at the other point, all inputs and outputs being terminated in any specified manner.

IMPEDANCE, TRANSFORM. The analogy for transient situations of the steady state impedances, viz., the ratio of the one-sided

Laplace transforms of two quantities such as voltage and current.

IMPERFECT GAS. See **real gas**.

IMPLICIT FUNCTION. A function y of x defined by an equation such as $F(x,y) = 0$. The *implicit function theorem* can be stated in varying degrees of generality; usually in applications something like the following. If the function $F(x,y) = 0$ is continuous and has continuous first partial derivatives in a neighborhood of a point $x = a, y = b$, where $F(a,b) = 0$, and if $\partial F/\partial y \neq 0$ at (a,b) , then there exists a unique function $y = f(x)$, continuous in a neighborhood of $x = a$, such that $F(x,f(x))$ is identically zero and $f(a) = b$. Generalizations to a larger number of variables can be made directly.

IMPORTANCE FUNCTION. In the theory of neutron chain reactors, the increase of the overall level of neutron population due to the introduction of a single neutron of speed v at point \mathbf{r} in the system.

IMPROPER FRACTION. A fraction whose numerator is not less in absolute value than its denominator.

IMPROPER INTEGRAL. A definite integral such that the integrand becomes infinite at one or more points in the interval of integration, or at least one of the limits of integration is infinite.

IMPULSE. A vector quantity defined by the time integral of the force \mathbf{F} acting on a particle over a finite interval, for example,

$$\int_{t_1}^{t_2} \mathbf{F} dt$$

for the interval from t_1 to t_2 . The impulse-momentum theorem states that the impulse equals the change in momentum experienced by a particle during the corresponding time interval.

IMPULSE, ANGULAR. See **angular impulse**.

IMPULSE FUNCTIONS. (1) *Definitions.* A class of functions defined not by their values, but by their integral properties.

Unit impulse function of t , of order one (see below for higher orders), at $t = t_0$, denoted by $\delta(t - t_0)$ is such that

$$\int_{\alpha}^{\beta} \delta(t - t_0) dt = 1 \quad \text{for all } \alpha, \beta \text{ such that } \alpha < t_0 < \beta$$

$$= 0 \quad \text{for all } \alpha, \beta \text{ such that } \alpha < \beta < t_0$$

$$\text{or } t_0 < \alpha < \beta.$$

An alternative definition, which includes the above as a particular case, is

$$\int_{\alpha}^{\beta} \delta(t - t_0) f(t) dt = f(t_0) \quad \text{for all } \alpha, \beta \text{ such that } \alpha < t_0 < \beta$$

$$= 0 \quad \text{for all } \alpha, \beta \text{ such that } \alpha < \beta < t_0$$

$$\text{or } t_0 < \alpha < \beta$$

provided $f(t)$ is continuous at $t = t_0$ and bounded elsewhere.

(2) *Right-Handed, Left-Handed and Symmetrical Impulse Functions.* It will be noticed that the above definitions do not cover the possibilities $t_0 = \alpha$ or $t_0 = \beta$. The value of the integrals in either of these cases demands a more precise definition. If $\delta(t)$ is a "right-handed" function, then, in addition to the above,

$$\int_{\alpha}^{\beta} \delta(t - t_0) dt = 1 \quad \text{when } t_0 = \alpha$$

$$= 0 \quad \text{when } t_0 = \beta.$$

If $\delta(t)$ is a "left-handed" function, then, in addition to the definitions in (1) above,

$$\int_{\alpha}^{\beta} \delta(t - t_0) dt = 1 \quad \text{when } t_0 = \beta$$

$$= 0 \quad \text{when } t_0 = \alpha.$$

If $\delta(t)$ is a "symmetrical" function, then, in addition to the definitions in (1) above,

$$\int_{\alpha}^{\beta} \delta(t - t_0) dt = \frac{1}{2} \quad \text{when } t_0 = \alpha \text{ or } t_0 = \beta.$$

(3) *The Impulse Function as a Limit Function.* The implications of the above definitions are perhaps clarified by considering the impulse function as the limiting form of a normal function when a suitable function parameter tends to a limiting value.

If for instance any function $\phi(t)$ is integrable over any range of t and is such that $\int_{-\infty}^{\infty} \phi(t) dt = 1$

then the same is clearly true of the function $k\phi(kt)$, where k is any positive constant. The effect of increasing k , however, is to compress the abscissae while expanding the ordinates and as $k \rightarrow \infty$ the function $k\phi(kt)$ will tend to $\delta(t)$.

Instances of such limit approximations to $\delta(t)$ are

$$\lim_{k \rightarrow \infty} \frac{\sin kt}{\pi t}, \quad \lim_{k \rightarrow \infty} \frac{k \operatorname{sech} kt}{\pi}, \quad \lim_{k \rightarrow \infty} \frac{k}{2} e^{-k|t|},$$

which are all symmetrical, and

$$\lim_{k \rightarrow \infty} f(k, t)$$

where $f(k, t) = k$ in the interval $0 < t < \frac{1}{k}$ and $f(k, t) = 0$ outside this interval, which is a widely used form of right-handed impulse function.

(4) *The Impulse Function of Order One as the Derivative of the Step-Function.* It is clear

from the above definition of $\delta(t)$ that $\int_{-\infty}^t \delta(t) dt = 0$ or 1 according as $t \leq 0$, and is therefore unit step function at the origin, $u(t)$. Conversely it would appear that $\delta(t)$ is therefore the derivative of $u(t)$. Although $u(t)$ is not differentiable at $t = 0$, it may be made so by considering it also as a limit function. For instance if $\delta(t)$ is identified with $\lim_{k \rightarrow \infty} \frac{\sin kt}{\pi t}$ then $u(t)$ may be expressed as the limit form of

$$\int_{-\infty}^t \frac{\sin kt}{\pi t} dt = \frac{1}{2} + \int_0^t \frac{\sin kt}{\pi t} dt$$

$$= \frac{1}{2} + \frac{1}{\pi} \operatorname{Si}(kt)$$

where $\operatorname{Si}(x) = \int_0^x \frac{\sin x}{x} dx$. Any objections raised on the ground that the integral of a limit is here identified with the limit of an integral may be discounted by arguing that k need not be infinite, that the limit is therefore never reached, that we can produce two differentially related functions which approximate as closely as we please to $u(t)$ and $\delta(t)$ and finally that in any case true discontinuous functions such as $u(t)$ and $\delta(t)$ never occur in physical problems, but only approximations to them.

(5) *Impulse Functions of Higher Order.* Unit impulse function of order $n (> 1)$ at $t = t_0$ may be defined by

$$\int_{\alpha}^{\beta} \delta_n(t - t_0) dt = \delta_{n-1}(t) \quad \text{if } \alpha < t_0 < \beta$$

$$= 0 \quad \text{if } \alpha < \beta < t_0 \text{ or } t_0 < \alpha < \beta$$

with similar extended definitions to cover the cases $t_o = \alpha$ or $t_o = \beta$ according as right-handed, left-handed or symmetrical functions are envisaged. Thus, using the above limit approach $\delta_n(t - t_o)$ may be considered as the derivative of $\delta_{n-1}(t - t_o)$. From this point of view, identifying $\delta_1(t) \equiv \delta(t)$ with such a limit function as $\frac{\sin kt}{\pi t}$ has the obvious advantage that all the derivatives of the limit function exist; if a right- (or left-) handed limit function is used, this property is lost.

(6) *Laplace Transform of Impulse Functions.* The Laplace transform of $\delta(t)$, by virtue of its definition integral, is unity provided either (i) the lower limit of the Laplace integral is 0 — or (ii) the lower limit is 0 and $\delta(t)$ is defined in a right-handed manner.

Under the same conditions the Laplace transform of $\delta_n(t)$ is s^{n-1} .

IMPULSE-MOMENTUM PRINCIPLE. The time integral of Newton's second law for a mass particle,

$$\mathbf{F} = m\mathbf{a} = \frac{d}{dt}(m\mathbf{v}),$$

is the impulse-momentum principle,

$$\int_{t_1}^{t_2} \mathbf{F} dt = (m\mathbf{v})_2 - (m\mathbf{v})_1.$$

This statement that impulse equals change in momentum applies also to the generalization

$$\mathbf{F} = \frac{d}{dt}(m\mathbf{v}_c)$$

where \mathbf{F} represents the sum of all forces acting on a system of total mass m and \mathbf{v}_c is the velocity of the center of mass.

IMPULSE-WEIGHT RATIO. A measure of the efficiency of propulsive systems that is used to compare **propellants**. It is defined as the ratio of total **impulse** to take-off weight, and may be found from the expression:

$$I/W = \frac{Ft}{W}$$

where I/W is the impulse-weight ratio, F is the thrust, t is the time of burning of the motor, and W is the total weight of the rocket at take-off.

IMPULSIVE MOTION. Motion induced in a fluid by the impulsive motion of the bound-

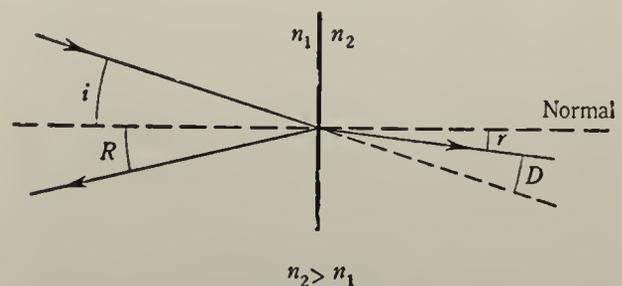
aries. If the fluid is incompressible and of uniform density and the impulsive pressure is $\omega (= P\Delta t)$, the impulsive motion is irrotational with velocity potential $\phi = -\omega/\rho$.

IMPULSIVE SOUND EQUATION.

$$\frac{E}{V} = e^{\frac{c\bar{\alpha}S}{4V}},$$

where E is the total sound energy produced by a single impulse, V is the volume of room, $\bar{\alpha}$ is the mean **sound absorption coefficient**, S is the exposed surface area, t is the duration of impulsive sound, and c is the velocity of sound.

INCIDENCE, ANGLE OF. (1) The angle between the chord line of an airfoil and the direction of the relative air stream. (Also known as **angle of attack**.) In the case of a twisted wing the angle of incidence varies along the span. It is usual then to specify the angle of incidence at a particular spanwise position, e.g., the wing root or the center line. (2) The angle at which one body or material or radiation strikes a surface, measured from the line of direction of the moving entity to a line perpendicular to the surface at the point of impact. The term is used commonly in regard



to the impact of radiant energy upon a material surface. In the figure, i is the angle of incidence, R , the angle of reflection, D , the angle of deviation, and r the angle of refraction.

INCIDENCE, ANGLE OF, IDEAL. See **thin-airfoil theory**.

INCIDENCE FOR ZERO LIFT. See **thin-airfoil theory**.

INCIDENCE, PRINCIPAL, ANGLE OF. See **angle of principal incidence**.

INCIDENT. A **vertex** and an **edge** are incident with each other if the vertex is an end-point of the edge.

INCOMMENSURABLE. Two numbers a and b are said to be incommensurable if there exists no number contained in each of them an integral number of times. Thus 1 and $\sqrt{2}$ are incommensurable. A number, such as $\sqrt{2}$, which is incommensurable with the integers is irrational.

INCOMPLETE BETA FUNCTION. The incomplete beta function is defined by

$$B_x(p, q) = \int_0^x x^{p-1}(1-x)^{q-1}dx, \quad 0 \leq x \leq 1, \\ p > 0, \quad q > 0.$$

INCOMPLETE BLOCK. A layout used in the design of experiments. If the experiment is divided into blocks and there are more treatments than can be inserted in any one block, the blocks are called incomplete. If the design is such that there are the same number of treatments in each block and every pair of treatments occurs together in the same number of blocks, the design is said to be *balanced*.

INCOMPLETE COMBUSTION. See **combustion**.

INCOMPLETE GAMMA FUNCTION. The function defined by

$$\Gamma_x(n) = \int_0^x e^{-x}x^{n-1}dx, \quad n > 0, \quad 0 \leq x \leq \infty.$$

INCREMENTAL THEORY OF PLASTICITY. A plastic flow law which expresses strain increments of plastic flow in terms of the applied stress and possibly stress increments. This is the more natural type of plastic flow law since plastic strain is permanent and does not have to be maintained by the applied stress. (See also **deformation theory of plasticity**.)

INDEPENDENCE. See **independent**.

INDEPENDENT. (1) A set of n quantities p_1, p_2, \dots, p_n (e.g., vectors, functions, etc.) for which addition is defined and also multiplication by a scalar, is *linearly independent* if for any set of n scalars a_1, a_2, \dots, a_n not all zero, we have

$$a_1p_1 + a_2p_2 + \dots + a_np_n \neq 0.$$

A set of n functions f_1, f_2, \dots, f_n of the n independent variables x_1, x_2, \dots, x_n is (functionally) independent if there does not exist any identically vanishing function $F(f_1, f_2, \dots,$

$f_n)$ with not all $\partial F/\partial f_i$ equal to zero. If f_1, f_2, \dots, f_n have continuous first partial derivatives, they will be independent in the neighborhood where not all the $\partial F/\partial f_i$ vanish, provided their **Jacobian** does not vanish identically.

(2) In the theory of probability, A and B are independent if the probability of A for given B is equal to the probability of A with B unspecified, viz.,

$$P(A/B) = P(A).$$

It follows that

$$P(A \text{ and } B) = P(A)P(B).$$

Likewise two **random variables** are independent if their frequency functions are related by

$$f(x_1, x_2) = f_1(x_1)f_2(x_2).$$

Analogous definitions apply to more than two variables.

INDEPENDENT COMPONENTS. The number of independent components of a thermodynamical system is the total number of components c less the number of **independent reactions** r' other than transfers of molecules from one phase to another

$$c' = c - r'.$$

INDEPENDENT REACTIONS. In a system containing c components which can undergo r' reactions other than transfers of molecules from one phase to another, there is for each reaction a **stoichiometric equation**

$$\sum_i \nu_{i\rho} M_i = 0, \quad (i = 1, \dots, c; \quad \rho = 1, \dots, r'). \quad (1)$$

The reactions $1, \dots, r'$ are said to be independent (from a thermodynamic point of view) if none of these equations can be derived by linear combination of the others.

INDEPENDENT VARIABLE. (1) For the mathematical meaning of this term, see **function**. (2) That independent quantity or condition which, through the action of the control system of an automatic controller, directs the change in the controlled variable according to a predetermined relationship.

INDETERMINACY PRINCIPLE. A consequence of quantum mechanics that asserts that in the simultaneous determination of the values of two canonically conjugated variables, the product of the smallest possible un-

certainties in their values is of the order of magnitude of the Planck constant h . If Δq is the uncertainty in the value for the coordinate q of a particle, and Δp is the uncertainty in the simultaneous determination of the corresponding component of its momentum p , then $\Delta p \cdot \Delta q = h/4\pi$. Similarly if ΔE and Δt are the uncertainties in the simultaneous determination of the energy and the time, $\Delta E \cdot \Delta t = h/4\pi$.

INDETERMINATE FORM. Limiting processes applied to special combinations of **functions** sometimes result in meaningless expressions such as $0/0$, ∞/∞ , $0 \cdot \infty$, $\infty - \infty$, 1^∞ , 0^∞ , ∞^0 , etc. These are called indeterminate forms. To evaluate them, **L'Hospital's rule**, or modifications of it may be used, as will now be shown for several special cases.

(1) The case $0/0$. The limiting value of $f(x)/\phi(x)$ for $x = a$ is $f^{(n)}(a)/\phi^{(n)}(a)$, where the lowest-order **derivatives** which do not vanish are to be evaluated.

(2) The case ∞/∞ . Let $f(x) = 1/g(x)$, $\phi(x) = 1/h(x)$ and evaluate $h^{(n)}(a)/g^{(n)}(a)$ as in case (1). The same procedure works when $a = -\infty$.

(3) The case $0 \cdot \infty$. If $f(x)\phi(x)$ becomes indeterminate, reduce the expression to case (1) or case (2) by writing it as $f(x)h(x)$ or $g(x)\phi(x)$.

(4) The case $\infty - \infty$. The indeterminate form is $f(x) - \phi(x) = 1/g(x) - 1/h(x) = (h - g)/gh$, which is now case (1).

(5) The cases 0^0 , ∞^0 , 1^∞ . The function has the form $f(x)^{\phi(x)}$. Its **logarithm**, however, is $\phi(x) \ln f(x)$ which is case (3).

INDETERMINATE STRUCTURE. A statically indeterminate structure is one that cannot be analyzed by means of the equations of static equilibrium alone. The properties of the material of which the structure is composed enter the problem through the stress-strain relations. Also the requirements of geometry or continuity must be considered.

INDEX. (1) An exponent. (2) A subscript or superscript used in **indicial notation**. (For index of a subgroup, see **eoset**.)

INDEX, DUMMY. See **summation convention**.

INDEX, FREE. An index on a symbol or product of symbols which is not repeated (cf. dummy index in entry on **summation convention**). Also called *identifying index*.

INDEX, IDENTIFYING. See **index, free**.

INDEX OF PROBABILITY. Gibbs' name for the natural logarithm of the **coefficient of probability**.

INDEX OF REFRACTION, ABSOLUTE. For a non-absorbing medium the absolute index of refraction may be defined by either: (1) The velocity of radiation *in vacuo* divided by the velocity of the same radiation in a specified medium. Because of **Snell's law**, the index of refraction may also be defined as the ratio of the sine of the angle of **incidence** (*in vacuo*) to the **angle of refraction**. Because the difference is small, the index of refraction is frequently measured with respect to air rather than with respect to free space (vacuum). (See **index of refraction, relative**.) Excepting a few very special cases (x-rays; light in metal films) the index of refraction is a number greater than unity. A few representative values are:

$$n_{\text{water}} = 1.34, n_{\text{glass}} = 1.5-1.9,$$

$$n_{\text{germanium}} = 4.25 \text{ (infrared radiation).}$$

(2) The square root of the relative dielectric constant of a medium $\sqrt{\epsilon/\epsilon_0}$. The index of refraction is invariably a function of the frequency or wavelength of the radiation. When definition (2) is used, the appropriate value of ϵ is often not the permittivity observed with static fields.

(For absorbing media, see **index of refraction, complex**.)

INDEX OF REFRACTION, COMPLEX. The dynamic dielectric constant k of a medium is in general a complex number and the complex index of refraction is $\sqrt{k} = n + im = n(1 - ik)$ where the real part n is the refractive index and m , or alternatively, k is the absorption constant or **absorption index** of the medium. The second form is especially useful in the study of metallic reflection. (See **Fresnel equations for metallic reflection**.)

INDEX OF REFRACTION, ELECTRON-OPTICAL. See **electron-optical index of refraction**.

INDEX OF REFRACTION FOR METALS. See **index of refraction, complex**.

INDEX OF REFRACTION, RELATIVE. If v_i is the velocity of light in a non-absorbing medium M_i then the relative index of refraction

tion M_2 in M_1 is $n_1 = v_1/v_2$. In many applications M_1 is air and since the absolute index of refraction of air under common conditions of temperature and pressure is approximately 1.00029, the absolute index of M_2 is often used as an approximation to the relative index of M_2 in air and referred to as the index of M_2 .

INDEX OF REPULSION. See **point centers of repulsion**.

INDEX, UMBRAL. See **summation convention**.

INDICATED EFFICIENCY. In a reciprocating gas compressor, the ratio of the indicated power IP_i , measured on an indicator diagram, to the ideal power consumption. The latter can be calculated on two alternative bases: (a) assuming an isothermal process, IP_T , (b) assuming an isentropic process IP_S . Hence

$$\eta_i = \frac{IP_T}{IP_i} \quad \text{or} \quad \eta'_i = \frac{IP_S}{IP_i}.$$

The indicated efficiency is a measure of how well the actual engine performs the stipulated ideal process ($T = \text{const.}$ or $S = \text{const.}$). Since compressors are usually cooled, and since IP_T represents the minimum work, it is found that $\eta_i < 1$ always, but η'_i may become larger than 1.

INDICATED MEAN EFFECTIVE PRESSURE. See **mean effective pressure**.

INDICATED THERMAL EFFICIENCY. In a reciprocating prime mover (internal combustion engine, steam engine), the ratio of indicated work W_i to heat Q supplied to working fluid:

$$\eta_i = \frac{W_i}{Q}.$$

The indicated work is measured on an *indicator diagram* and, consequently, excludes mechanical losses. The indicated thermal efficiency is a measure of the perfection of the thermodynamic design of the engine, i.e., a measure of how well the heat supplied Q has been transformed into mechanical work. In an internal combustion engine

$$Q = BH_n$$

per unit time; here B is the fuel consumption and H_n its lower **calorific value**. In a steam engine

$$Q = h - h_w$$

per unit mass of working fluid, where h is the enthalpy of the steam at admission, and h_w is the enthalpy of the feed-water to the boiler.

INDICATRIX (AT A POINT P OF A SURFACE). In a two-dimensional rectangular Cartesian coordinate system x, y with axes in the principal directions at P , for which the principal curvatures are κ_x and κ_y , the ellipse $|\kappa_x|x^2 + |\kappa_y|y^2 = 1$, if P is an elliptic point; the two hyperbolas $\kappa_x x^2 + \kappa_y y^2 = \pm 1$, if P is a hyperbolic point; a pair of parallel straight lines in the tangent plane at P , parallel to the principal direction for which the principle curvature is zero, if P is a parabolic point. $|\kappa_x|$ and $|\kappa_y|$ denote the absolute values of κ_x and κ_y .

These curves are similar and similarly situated to the real curves of intersection with the surface of planes parallel to the tangent plane at P on both sides of it and infinitely close to it. Also called *Dupin's indicatrix*.

INDICATRIX OF DIFFUSION. The surface formed by the extremities of the radii vectors drawn in all directions from an element of a surface of a secondary source, or from an element of volume of a translucent solid, when each radius vector represents the (relative) **luminous intensity** or the (relative) **luminance** in the corresponding direction. (1) In many cases only a meridian section of this indicatrix is required. (2) The term indicatrix is often used to denote, instead of the surface, the curve obtained in a similar manner in a plane normal to the element concerned.

INDICATRIX OF REFRACTION. An indicatrix of refraction at a point O of an anisotropic medium is a set of points P so that the distance OP is proportional to the index of refraction of the medium in the direction OP .

INDICIAL EQUATION. As an attempt to find a series solution valid near a point $x = c$ of a linear homogeneous ordinary differential equation

$$y^{(n)} + a_1 y^{(n-1)} + \cdots + a_{n-1} y' + a_n y = 0,$$

we may substitute

$$y = \sum_{k=0}^{\infty} a_k (x - c)^{k+s}$$

in the equation. When k is put equal to zero, and the coefficient of the lowest power of x is

set equal to zero, the result is called the indicial equation, since it serves to determine the values of s . These values are called the **exponents** of the differential equation.

The indicial equation may be: (1) independent of s , in which case no series solution of the type assumed exists; (2) a **polynomial** in s , of degree equal to the order of the differential equation; depending on the nature of the roots of the indicial equation, there may then be n distinct series solutions of the differential equation or fewer; (3) a polynomial in s of degree less than the order of the differential equation in which there will be fewer than n distinct series solutions.

In case (2) the point c is a regular singularity (see **singular point of a differential equation**) of the differential equation, and setting successive coefficients equal to zero in the above power series in x , obtained by substituting the above y into the given differential equation, will, in general, determine the successive a_k . But if the roots of the indicial equations differ by an integer, the general solution will involve logarithms. This method is often named after Frobenius, who made an important contribution to its theoretical justification.

INDICIAL NOTATION. The notation in which a set of quantities, called a *system*, is denoted by a symbol carrying one or more indices which may be superscripts or subscripts. These are usually lower case Latin italics and it is understood that each of the indices may take independently the integral values 1, 2, \dots , n (say). For example, with $n = 3$, the symbol a_j^i denotes the nine quantities $a_1^1, a_2^2, a_3^3, a_3^2, a_2^3, a_1^3, a_3^1, a_2^1, a_1^2$. With $n = 4$, the symbol $a_{pq}b^r$ denotes the sixty-four quantities obtained by assigning to p, q and r the values 1, 2, 3, 4 independently.

The individual quantities obtained by assigning values from 1 to n to each of the indices are called the *elements* or *components* of the system. Systems of quantities depending on one index are called *systems of the first order* or *simple systems*. Those depending on two indices are called *systems of the second order* or *double systems*, and so on.

INDICIAL RESPONSE. The **normal response** of a system to an input in the form of **unit step function** at the origin.

If the output quantity is $q_o(t)$ the input $\theta(t)$ and their **Laplace transforms** for an initially quiescent system are related by

$$Q_o(s) = F(s) \cdot \Theta(s)$$

where $F(s)$ is the transfer function connecting them, then if $\theta(t)$ is unit step function, $\Theta(s) = \frac{1}{s}$ and therefore

$$Q_o(s) = F(s)/s.$$

Hence

$$\begin{aligned} q_o(t) &= \int_0^t \{\mathcal{L}^{-1}F(s)\} dt \\ &= \int_0^t W(t) dt \end{aligned}$$

where $W(t)$ is the **weighting function** (the response to unit **impulse function** at $t = 0$). Thus indicial response is the time integral of the weighting function. Similarly, the response to unit ramp function input is the integral of the indicial response.

INDIFFERENT LINE. See **Saurel theorem**.

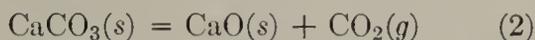
INDIFFERENT STATES. Let us consider a closed system whose state (see **Duhem theorem**) is determined completely by T, p , the weight fractions w_i^α of each component ($i = 1 \dots c$) in the various phases α ($\alpha = 1 \dots \phi$) and the mass of each phase, i.e., by the variables

$$T, p, w_1^1, \dots, w_c^\phi, m^1 \dots m^\phi. \quad (1)$$

Suppose the system is initially in the state (1).

Let us then consider the set of states accessible to this closed system, i.e., compatible with the conservation of mass (see **conservation of mass in closed systems**). If there exists in this set states which differ from (1) in the mass of at least one of the phases, but in which all the weight fractions are the same, the state (1) is called an **indifferent state**. The system is then called an *indifferent system*. This terminology is due to Duhem.

A simple example is given by the decomposition of calcium carbonate



where (s) means a solid phase and (g) a gas. CaCO_3 and CaO are two distinct solid phases.

If a molecule of CaCO_3 decomposes, then it increases the amount of CaO and CO_2 without altering the composition of any of the phases, each phase being formed by a simple com-

ponent. All states of the system are indifferent.

The **azeotropic systems** are also special cases of indifferent systems.

The properties of indifferent systems are very similar to those of azeotropic systems. For example, one has the generalized **Gibbs-Konovalov theorems**: *If in any isothermal (isobaric) equilibrium change the system passes through an indifferent state, then the pressure (temperature) passes through an extreme value, and conversely.* (See also **Saurel theorem**.)

INDUCED DRAG. See **drag**.

INDUCED POLARIZATION. The **polarization** brought about by the action of an electric field in a dielectric that does not contain permanent **dipoles**.

INDUCTANCE. See **inductance, self**; and **inductance, mutual**.

INDUCTANCE, MUTUAL. The mutual inductance, M , between two circuits or two circuit elements is defined as the ratio $M = \phi/I$ where ϕ is the flux linked by one and being produced by a current I in the other. From the Faraday law of electromagnetic induction, the voltage, V , induced in the first circuit or circuit element is given by $V = -MdI/dt$, where dI/dt is the time rate of change of the current in the second circuit or element. Mutual inductances are measured in henries. (See also **inductance, self**; **circuits**.)

INDUCTANCE, SELF. The self inductance, L , is defined as the ratio $L = \phi/I$, where ϕ is the magnetic flux linked and produced by a circuit or circuit element and I is the current in the same circuit or element. From the Faraday law of electromagnetic induction the back emf is given by $V = -LdI/dt$. Self inductances are measured in henries. (See also **inductance, mutual**.)

INDUCTION DRAG. A velocity-dependent resistive force which acts on a conducting fluid to dampen its motion perpendicular to the magnetic lines of force.

INDUCTION EFFECT. See **van der Waals forces**.

INDUCTION, ELECTRIC AND MAGNETIC. The word induction is often used to indicate the generation or alteration of charge

or current distributions, or magnetic dipole distributions in a substance, brought about by the presence or approach of an electrified body, a magnet or any other source of electric or magnetic field.

INDUCTION ENERGY. See **Debye energy**.

INDUCTION FIELD. See **field, induction**.

INDUCTION FORCES. When a charged particle a (for example an ion) interacts with a neutral molecule, the charged particle a *induces* on the neutral molecule b a dipole moment. If the **polarizability** of molecule b is α_b the energy of interaction between the charge e_a and this induced moment is

$$\phi(r) = -\frac{e_a\alpha_b}{2r^4}. \quad (1)$$

Similarly, there exists a potential energy of interaction between a point dipole μ_a and an induced dipole produced in a neutral molecule of polarizability α_b . When averaged over the angles, the result is

$$\phi(r) = -\frac{\mu_a^2\alpha_b}{r^6}. \quad (2)$$

It is important to note that (1) and (2) correspond always to an *attraction*. This effect was discussed for the first time by Debye.

INDUCTION, INTRINSIC (B_i) (INTENSITY OF MAGNETIZATION). The intrinsic induction or magnetic polarization of a medium is the vector difference between the **magnetic induction** at the point of interest, and the induction which would exist at that point, for the same **magnetizing force** if the medium were a vacuum there.

$$\mathbf{B} = \gamma_m \mathbf{H} + \mathbf{B}_i,$$

where γ_m is the magnetic constant (μ_0). In unrationalized systems of units the magnetic polarization is defined by

$$\mathbf{P}_m = \mathbf{B}_i/4\pi,$$

whereas in rationalized systems,

$$\mathbf{P}_m = \mathbf{B}_i.$$

In any system, the polarization is related to the **magnetic moment density** by $\mathbf{P}_m = \gamma_m \mathbf{M}$.

INDUCTION THEOREM. Consider a medium comprising two homogeneous regions (which may have different or identical prop-

erties) separated by a closed surface S . An impressed electromagnetic field $\mathbf{E}_i, \mathbf{H}_i$ gives rise to an induced field; the induced field is composed of a field reflected from the surface and a field transmitted by the surface. The induced field is the same as that produced by electric and magnetic current-sheets on the surface, of densities

$$\mathbf{J} = \mathbf{n}_i \times \mathbf{H}_i$$

$$\mathbf{M} = \mathbf{E}_i \times \mathbf{n}.$$

This is a consequence of **Green's theorem** applied to **Maxwell's equations**.

INDUCTIVE CAPACITY. See **permittivity** or **dielectric constant**.

INDUCTIVE EFFECT AND REACTIVITY.

The term is used to describe the tendency of electronegative or electropositive groups or atoms in organic molecules to polarize the remainder of the molecule, i.e., to decrease or increase the electron density of the neighboring atoms. For molecules containing double bonds (and particularly **conjugated double bonds**), the inductive effect can be quite important, because of the mobility of the π -electrons. It affects the tendency of the molecule to react with *electrophilic* or *nucleophilic* atoms or groups of atoms. (See also **resonance effect**.)

INDUCTOR. A device whose primary purpose is to introduce **inductance** into an electric circuit or network.

INEQUALITIES, METHOD OF. A procedure for computing the plastic limit load of a frame based upon the solution of systems of linear inequalities. The inequalities express the fact that at each critical section the bending moment cannot exceed the positive moment or be algebraically less than the negative limit moment.

INEQUALITY. For various inequalities, e.g., **Abel**, **Hölder**, **Minkowski**, see the respective names.

INEQUALITY THEOREMS, METHOD OF. A technique for the **X-ray analysis of crystal structure** in which use is made of certain fundamental inequalities, which must be satisfied by the **structure factors**, to estimate their phases.

INERTANCE, ACOUSTIC (ACOUSTIC MASS). The quantity which, when multiplied by 2π times the frequency, gives the acoustic reactance (see **reactance, acoustic**) associated with the kinetic energy of the medium. It is measured in acoustic ohms.

INERTANCE, SPECIFIC ACOUSTIC. The coefficient which, when multiplied by 2π times the frequency, gives the positive imaginary part of the specific acoustic impedance. (See **impedance, specific acoustic**.)

INERTIA. A property manifested by all matter, representing the resistance to any alteration in its state of motion. **Mass** is the quantitative measure of inertia.

INERTIA, AREA MOMENT OF. If the area lies in the xy -plane, the moment of inertia about the y -axis is $I_y = \int_A x^2 dA$, about the x -axis is $I_x = \int_A y^2 dA$ and about a perpendicular or polar axis $I_p = \int_A r^2 dA$, where r is measured from the polar axis. If the polar axis is erected at the intersection of the x and y axes, $I_p = I_x + I_y$.

INERTIA, AREA PRODUCT OF. If the area is taken to be in the xy -plane the product of inertia $I_{xy} = \int_A xy dA$. The product of inertia is zero if either axis is a line of symmetry or, more generally, a principal axis.

INERTIA ELLIPSOID. The quadric surface associated with the inertia tensor. (See **angular momentum**.)

INERTIA FORCES. Forces equal and opposite to the reaction of matter on its surroundings when it is accelerated; represented in an equation of motion by the rate of change of momentum; equal to the resultant of all the other forces.

INERTIAL COORDINATE SYSTEM. See **inertial frame**.

INERTIAL FLOW. Flow in the absence of external forces.

INERTIAL FRAME. A **frame of reference** in which Newton's laws are valid, is termed an inertial frame. Axes fixed on the earth, a rotating body in **curvilinear motion**, are most

often taken to be a sufficiently good approximation to an inertial frame. **Coriolis** and **centrifugal** effects, however, do play a significant role in problems of missiles and satellites, wind and weather.

INERTIAL INSTABILITY. (1) (Also called dynamic instability.) Generally, **instability** in which the only form of energy transferred between the steady state and the disturbance is **kinetic energy**.

(See **Helmholtz instability**, **barotropic instability**.)

(2) The hydrodynamic **instability** arising in a rotating fluid mass when the velocity distribution is such that the kinetic energy of a disturbance grows at the expense of kinetic energy of the rotation.

INERTIA, MOHR CIRCLE FOR. See **Mohr circle for inertia**.

INERTIA, MOMENTS AND PRODUCTS OF. In the general case of the motion of a particle or aggregate of particles with respect to a single fixed point, the angular momentum can be written as having three components with respect to a coordinate system based at the fixed point.

$$H_x = \omega_x \sum m_i (y_i^2 + z_i^2) - \omega_y \sum m_i x_i y_i - \omega_z \sum m_i x_i z_i$$

$$H_y = -\omega_x \sum m_i x_i y_i + \omega_y \sum m_i (x_i^2 + z_i^2) - \omega_z \sum m_i y_i z_i$$

$$H_z = -\omega_x \sum m_i x_i z_i - \omega_y \sum m_i y_i z_i + \omega_z \sum m_i (x_i^2 + y_i^2)$$

where $\omega_x, \omega_y, \omega_z$ are components of angular velocity, m_i is the mass of i th particle, x_i, y_i, z_i are coordinates of i th particle.

The terms $\sum m_i (y_i^2 + z_i^2), \sum m_i (x_i^2 + z_i^2), \sum m_i (x_i^2 + y_i^2)$ are called moments of inertia with respect to the $x, y,$ and z axes, respectively, and are symbolized by $I_{xx}, I_{yy},$ and I_{zz} .

The terms $\sum m_i x_i y_i, \sum m_i x_i z_i,$ etc., are called the products of inertia and are symbolized by $I_{xy}, I_{xz},$ etc.

For a continuous rigid body the summations are replaced by integrals over the volume of the body. In a rigid body, it is sometimes easier to choose coordinate axes, called moving axes, which are fixed in the body. There always exists one set of such axes, called principal axes, such that the products of inertia

vanish and the angular momentum can be expressed in terms of the moments of inertia alone. Following are formulae for the moments of inertia of certain homogeneous solids with respect to the axes specified (M is the total mass of the body in each case):

Particle distant r from axis	Mr^2
Sphere of radius R , with respect to any diameter	$\frac{2}{5}MR^2$
Cube of edge L , with respect to axis through center parallel to edge	$\frac{1}{6}ML^2$
Rectangular plate, dimensions $A \times B$, with respect to axis perpendicular to it at center	$\frac{M}{12}(A^2 + B^2)$
Cylinder of length L and radius R , with respect to axis perpendicular to its length at center	$M \left(\frac{L^2}{12} + \frac{R^2}{4} \right)$
Cylinder of radius R , with respect to its own longitudinal axis	$\frac{1}{2}MR^2$
Any body with respect to any axis distant r from the center of mass, the value for a parallel axis through that point being I_0	$I_0 + Mr^2$

Experimental methods of obtaining moments of inertia by the use of a torsion pendulum are explained in any laboratory manual of elementary dynamics.

For certain purposes it may be desirable to know at what one distance from the axis all the particles of the body of mass M would have to be placed to give it the same moment of inertia I that it actually has. This distance is the radius of gyration, and is expressed by the formula

$$k = \sqrt{\frac{I}{M}}$$

The "principal axes" of a body through a given point are axes of maximum or minimum moment of inertia.

The quantity expressed by

$$I = \int r^2 da,$$

in reference to any plane figure, is called the areal moment of inertia of the figure with

respect to a given straight line in its plane. The figure is divided into elements of area da , each element multiplied by the square of its distance r from the axis, and the products summed as indicated above to get the areal moment of inertia. This quantity is purely geometric and has of course no actual connection with inertia or mass. One of its important applications is in the theory of **flexure** of elastic rods or beams. If E is the (Young's) elastic modulus of the material and I the areal moment of inertia of the cross-section with respect to the **neutral axis**, the bending moment or flexural torque required to bend the rod to a **curvature** κ is given by

$$M = EI\kappa.$$

INERTIA, ROUTH RULE OF. See **Routh rule of inertia**.

INFINITE ABSORBER MODEL. In the theory of resonance capture of neutrons in a chain reacting system, the calculational model in which it is assumed that the heavy atom which does the capturing may be ascribed infinite mass, i.e., the capturing atom does not moderate. (See **narrow resonance model**.)

INFINITE GRAPH. See **graph, infinite**.

INFINITE MULTIPLICATION. See **multiplication, infinite**.

INFINITESIMAL. The word infinitesimal is properly defined only as part of a phrase. Thus a function $y = y(x)$ is sometimes said to be an infinitesimal if there is a value of x , call it a (usually $a = 0$ or $a = \infty$), such that $y \rightarrow 0$ as $x \rightarrow a$, but more properly one should say that y becomes infinitesimal at a . The idea of the *order of an infinitesimal* is often convenient. Thus if y and z become infinitesimal at a , but their quotient remains between finite (non-zero) limits, then y and z are said to be infinitesimals of the same order. If y^n and z are of the same order, then z is said to be of the n th order relative to y , and so forth.

INFINITESIMAL DEFORMATION. A **deformation** in which the displacement gradients in a rectangular Cartesian coordinate system are sufficiently small so that terms of second degree in them may be systematically neglected in comparison with terms of first-degree; terms of third degree may be systematically neglected in comparison with terms of second degree and so on.

INFINITESIMAL RING OF A GROUP OF LINEAR TRANSFORMATIONS. Let G be a group of linear transformations (see, e.g., **Lie group**), i.e., a group of $n \times n$ (real or complex) matrices M whose elements (at least for small values of the p_i) are differentiable functions of r parameters p_1, p_2, \dots, p_r so chosen that $M(0, 0, \dots, 0)$ is the unit **matrix**. E.g., for rotations through an angle ϕ in the plane $n = 2, r = 1, p_1 = \phi$ and the matrices are given by

$$\begin{pmatrix} \cos \phi, & -\sin \phi \\ \sin \phi, & \cos \phi \end{pmatrix}.$$

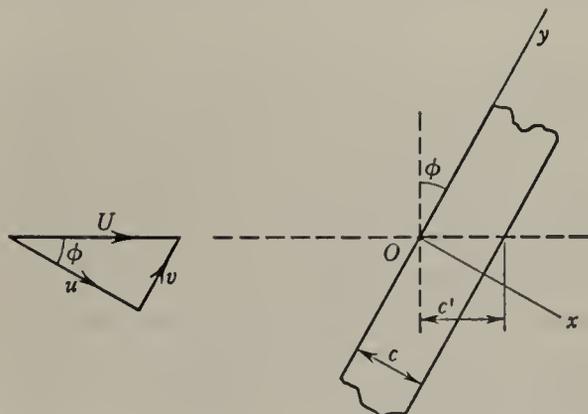
Consider all possible ways of selecting a one-parameter family $M(\theta)$ from G with $M(0) = 0$, and form the corresponding matrix $M'(0)$. The set of all such matrices $M'(0)$ is called the infinitesimal ring G^0 of the group G . E.g., in the above case the infinitesimal ring for rotations in the plane consists of multiples of the matrix

$$\begin{pmatrix} 0, & -1 \\ 1, & 0 \end{pmatrix}.$$

It is easy to prove that if G^0 contains the matrices U and V , then it also contains $W \equiv UV - VU$, and is actually a (non-commutative, non-associative) ring if W is defined as the product of U and V . The ring G^0 is in fact the most important special case of an abstract **Lie ring**.

INFINITE YAWED WING. The theory of an infinite yawed wing of constant chord is of interest, because it is relatively simple and gives some insight into the more complicated case of a swept back wing of finite span.

The system to be considered is shown in the figure. The wing is yawed at an angle ϕ and is placed in an infinite uniform stream



Infinite yawed wing.

of velocity U . Resolving the velocity U into components u and v , parallel to the axes Ox and Oy , it is clear that if viscous effects are neglected the pressure distribution on the wing depends only on the component u , and is independent of v . Thus the pressure, lift and moment coefficients on the yawed wing (denoted by symbols with primes) can be related simply to the coefficients on the same wing at zero yaw (symbols without primes) by the equation

$$\frac{C'_p}{C_p} = \frac{C'_L}{C_L} = \frac{C'_m}{C_m} = \cos^2 \phi. \quad (1)$$

Except in cases where the compressibility of the fluid can be neglected, this equation must be applied to the yawed wing at a Mach number M' and to the unyawed wing at a Mach number $M = M' \cos \phi$. This illustrates the important effect of sweep back in delaying to higher Mach numbers the changes that occur on an unswept wing at Mach numbers approaching 1.

In deriving Equation (1) it has been assumed that the incidence of the unyawed wing is the same as that of the yawed wing measured in a plane normal to the axis OY . Also, the chord used in defining C'_m is c' (and not c) and the pitching moment is measured about an axis perpendicular to the velocity U .

It can be shown that for a **laminar boundary layer** on an infinite yawed wing the equations for flow in the plane xz are independent of the flow in the plane yz . Hence the flow in the plane xz can be calculated without considering the flow in the plane yz , and the results then used to calculate the latter.

When the boundary layer is **turbulent** the evidence regarding this "independence principle" is conflicting, but at present it seems likely that the principle does *not* apply.

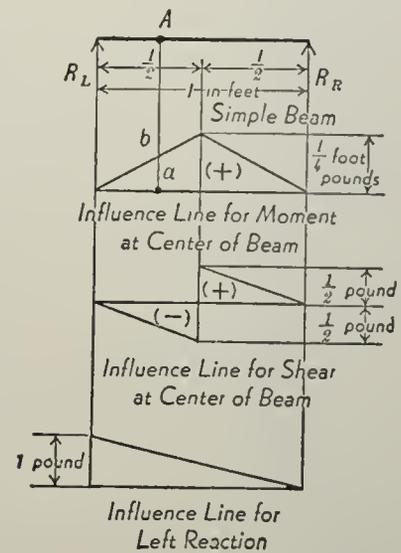
INFINITY. The word infinity is usually defined only as part of a phrase. Thus a function $f(x)$ is said to approach infinity at a point $x = a$ if after choice of any number N , a number $\delta > 0$ can be found such that $f(x) > N$ for all x if $|x - a| < \delta$.

INFLECTION, POINT OF. A point on a curve, $y = f(x)$, where the **derivative** dy/dx changes sign but where neither a maximum nor a minimum occurs. It may be distinguished from these two cases, for, at a point of inflection, $d^2y/dx^2 = 0$.

The bending moment in a beam is zero at a point of inflection.

INFLUENCE COEFFICIENTS. The magnitude of the effect produced at one point by a unit disturbance at another, as for example, a component of stress, force, moment, or deflection at one point of a structure produced by a unit force or deflection at another.

INFLUENCE LINE. An influence line is a graphical way of representing the effect of a certain variable circumstance upon a given condition. In particular, the influence line as applied in structural engineering represents the



variable effect of a single moving unit concentrated load upon the shear, bending moment, reaction, or any other function of a structure such as a beam, **truss**, or **bridge**. The influence line is plotted in reference to a base or zero line. Positive or tensile effects are represented above the line and negative or compressive effects below. The ordinate of the influence line is the ratio of the effect to the concentrated load producing it. If the load is in lbs. or tons the effect is in lbs. or tons. It is very useful for locating the position of the load which will produce maximum effect. For instance, the influence line for **bending moment** at the center of the beam shown above indicates that the maximum moment for this point will occur when the moving load which may be taken as unity is directly over the point. Any other ordinate such as ab represents the bending moment at the center due to a load of unity at point A . The maximum moment at the center of this beam due to a **uniform** load of w lbs. per linear ft., covering the

entire length, may be computed by multiplying the area of the influence line by w .

$$\text{Area} = \frac{l}{4} \times l \times \frac{1}{2} = \frac{l^2}{8}$$

$$\text{Maximum moment at center} = \frac{wl^2}{8}$$

Such quantitative results can also be obtained by the usual analytical methods. However, since influence lines are almost invariably drawn to indicate how a structure subjected to moving loads should be loaded, it is usually convenient, and faster, to use the ordinates and areas of the influence line to obtain moments, stresses, etc.

Influence lines for statically **determinate structures** are composed of straight lines. Those for **indeterminate structures** are curved or have straight segments the intersections of which lie on curves. (See **Mueller-Breslau principle**.)

INFORMATION. (1) See **information content** and **information theory**. (2) Apart from its ordinary meaning, this term occurs in statistics in a specialized sense. If a sample of n values is drawn from a frequency function $f(x, \theta)$ the amount of information about θ in the sample is defined as

$$n \int_{-\infty}^{\infty} \left(\frac{\partial}{\partial \theta} \log f(x, \theta) \right)^2 f(x, \theta) dx.$$

In a multivariate situation the matrix whose $(i, j)^{th}$ element is the expectation of $n \frac{\partial \log f}{\partial \theta_i} \times \frac{\partial \log f}{\partial \theta_j}$ is called the *information matrix*.

INFORMATION CONTENT (OF A MESSAGE OR A SYMBOL FROM A SOURCE). The negative of the logarithm of the probability that this particular message or symbol will be emitted from the source. The choice of logarithmic base determines the unit of information content. (See **bit** and **hartley**.) The probability of a given message or symbol being emitted may depend on one or more preceding messages or symbols. The quantity has been called *self-information*.

INFORMATION CONTENT, AVERAGE. (PER SYMBOL) (INFORMATION RATE FROM A SOURCE, PER SYMBOL.) The average of the information content per symbol

emitted from a source. The terms entropy and negentropy are sometimes used to designate average information content.

INFORMATION CONTENT, CONDITIONAL. (OF A FIRST SYMBOL GIVEN A SECOND SYMBOL.) The negative of the logarithm of the conditional probability of the first symbol, given the second symbol. The choice of logarithmic base determines the unit of information content. (See **bit** and **hartley**.) The conditional information content of an input symbol given an output symbol, averaged over all input-output pairs, is the *equivocation*. The conditional information content of output symbols relative to input symbols, averaged over all input-output pairs, has been called *spread*, *prevarication*, *irrelevance*, etc.

INFORMATION MATRIX. See **matrix, information**.

INFORMATION THEORY. A mathematical theory relating to the problems of the processing and transmission of information. Information is measured quantitatively in terms of equally likely yes or no decisions called **bits**. A communication channel is characterized by a capacity measured in bits per second. The capacity determines the maximum rate at which the channel can convey information. The central problem in information theory is that of finding methods of coding information which make for the most efficient use of a channel.

INFRARED RADIATION. Any radiation whose monochromatic components lie for practical purposes within the wavelength range 780–10⁵ millimicrons.

INFRARED SPECTRA OF MOLECULES. See **rotation spectra of molecules**, **rotation-vibration spectra of molecules**.

INFRARED SPECTRUM. See **spectrum, infrared**.

INHOMOGENEOUS LORENTZ TRANSFORMATION. An inhomogeneous Lorentz transformation, $L = \{a, \Lambda\}$ is defined by

$$x'_\mu = \sum_{\nu=0}^3 \Lambda_\mu^\nu x_\nu + a_\mu$$

where Λ is a **homogeneous Lorentz transformation** and the term a_μ corresponds to a space-

time translation by the amount $a_\mu = \{a_0, a_1, a_2, a_3\}$. The multiplication law for Lorentz transformations is

$$\{a_1, \Lambda_1\} \cdot \{a_2, \Lambda_2\} = \{a_1 + \Lambda_1 a_2, \Lambda_1 \Lambda_2\}$$

The set of all inhomogeneous Lorentz transformations form a 10 parameter continuous group. If one denotes the **generators** for infinitesimal space time-translations by p_μ , and the generators for rotations in the $x^\mu x^\nu$ plane by $M_{\mu\nu} = -M_{\nu\mu}$ the commutation rules obeyed by these operators are

$$\begin{aligned} [p_\mu, p_\nu] &= 0 \\ [M_{\mu\nu}, M_{\rho\sigma}] &= -i(g_{\mu\rho}M_{\nu\sigma} - g_{\nu\rho}M_{\mu\sigma} \\ &\quad + g_{\mu\sigma}M_{\rho\nu} - g_{\nu\sigma}M_{\rho\mu}) \\ [M_{\mu\nu}, p_\sigma] &= i(g_{\nu\sigma}p_\mu - g_{\mu\sigma}p_\nu). \end{aligned}$$

IN HOUR (INVERSE HOUR). In the theory of neutron chain reactors, a unit of **reactivity**. It is the amount of positive reactivity required to give a particular reactor a steady positive period of one hour or 3600 seconds, and so, as a unit, varies from system to system. The inhour formula, expressed in terms of inhours, becomes:

$$\rho^{(ih)} = \frac{\frac{l}{k_{eff}T} + \sum_i \frac{\beta_i}{1 + \lambda_i T}}{\frac{l}{3600k_{eff}} + \sum_i \frac{\beta_i}{1 + 3600\lambda_i}}$$

if λ_i and l are expressed in terms of seconds.

IN HOUR (INVERSE HOUR) FORMULA. In the theory of neutrons chain reactions, a formula relating reactor period (see **period, reactor**) to effective **multiplication**. A form very often used is:

$$\rho = \frac{k_{eff} - 1}{k_{eff}} = \frac{l}{k_{eff}T} = \sum_i \frac{\beta_i}{1 + \lambda_i T}$$

where ρ denotes the reactivity, k_{eff} the effective multiplication, l the neutron lifetime, T the reactor period, β_i the yield fraction and λ_i the decay constant for the i^{th} group of delayed neutrons. If N groups of delayed neutrons are considered, N real periods T_j emerge as solutions, but no more than one may be positive.

INITIAL MODULUS. The value of the **tangent modulus** at zero stress.

INITIAL-VALUE PROBLEMS. For **ordinary differential equations**, those for which the

initial values of the dependent variables are specified; for **partial differential equations**, those of parabolic or hyperbolic type, requiring the specification of initial conditions as well as possibly conditions along the boundary.

In mechanics and fluid mechanics, initial-value problems include particularly those dynamical or transient problems in which the state of the system changes, as contrasted with **steady-state** problems, and is determinable at any time from the initial conditions.

INITIAL VERTEX. See **vertex, initial**.

INNER MULTIPLICATION OF TENSORS. See **tensors, inner product of**.

IN, OUT FIELD. See **S matrix and QT**.

INSPECTIONAL ANALYSIS. The reduction of the mathematical equations of a problem to non-dimensional units of space, time, and mass; or testing such equations for invariance under any group of transformations. The procedure stands in close relation to **dimensional analysis** and usually gives rise to a set of **non-dimensional numbers** appearing as coefficients in the governing equations. These can always be arranged to be the same as the non-dimensional numbers or parameters of a corresponding dimensional analysis.

INSTABILITY. A property of the **steady state** of a system such that certain disturbances or perturbations introduced into the steady state will increase in magnitude, the maximum perturbation amplitude always remaining larger than the initial amplitude.

The method of **small perturbations**, assuming **permanent waves**, is the usual method of testing for instability; unstable perturbations then usually increase exponentially with time. An unstable non-linear system may or may not approach another steady state; and the method of small perturbations is incapable of making this prediction. (See also entries under **stability**.)

INSTANTANEOUS AXIS OF ROTATION. In rigid body motion, the line in the body or an extension of the body which is instantaneously at rest. For a cylinder rolling down an inclined plane without slipping, the instantaneous axis is the line of contact between the cylinder and the plane.

INSTANTANEOUS CENTER. A term applicable to plane motion. It is the point of

intersection of the **instantaneous axis of rotation** with the **plane of motion**. The instantaneous center is a point about which the body may be considered to be in pure **rotation** (i.e., without **translation**) at any instant. It is not necessarily on the body. In the case of **rectilinear motion** it is infinitely distant.

INSTANTANEOUS FREQUENCY. See **frequency, instantaneous**.

INTEGRABLE SQUARE. A function $f(x)$ having the property that

$$\int f^*(x)f(x)dx < \infty$$

is integrable square (f^* is the complex conjugate of f). The definition is very often used of real functions $f(x)$, where it becomes

$$\int f^2(x)dx < \infty;$$

the type of integration referred to is usually that of Lebesgue.

INTEGRAL. The function $\phi(x)$ is an integral of $f(x)$ if $d\phi/dx = f(x)$. The process of finding an integral is integration or the inverse of **differentiation**. If C is any real number, then $\phi(x) + C$ is also an integral of $f(x)$. (See **constant of integration**.) Thus, if one integral exists, an infinite number of others may be obtained by adding an arbitrary constant. These are called indefinite integrals and indicated symbolically as

$$\int f(x)dx = \phi(x) + C.$$

(For various types of definite integral, e.g., **Riemann**, **Stieljes**, **Lebesgue**, and others, see the respective names.)

INTEGRAL, CAUCHY. See **Cauchy integral**.

INTEGRAL, CIRCULATORY. An integral of a vector function \mathbf{F} :

$$\oint \mathbf{F} \cdot d\mathbf{s}$$

over a closed **contour**. It is a measure of the tendency of lines of force to close up. If \mathbf{F} refers to a fluid, then the circulatory integral is a measure of the flow around the path chosen. When the vector field has a potential, this integral is zero and the field is said to be **irrotational**.

INTEGRAL CONTROL. See **control, proportional, derivative and integral**.

INTEGRAL, CONVOLUTION. See **convolution integral**.

INTEGRAL, DEFINITE. See **integral**.

INTEGRAL DOMAIN. See **domain**.

INTEGRAL, ELLIPTIC. See **elliptic integral**.

INTEGRAL EQUATION. An equation in which the unknown function occurs under an integral sign. A *linear integral equation of the third kind* is of the form

$$g(x)\phi(x) = f(x) + \lambda \int_a^b K(x,z)\phi(z)dz,$$

where the functions $g(x)$, $f(x)$ and $K(x,z)$ are known and it is desired to find the unknown ϕ as a function of x . The function $K(x,z)$ is called the kernel (or nucleus) of the equation. The limits of integration a and b are either constants or known functions of x , and λ is a parameter. When one or both of the limits become infinite or when the kernel becomes infinite at one or more points in the range of integration, the equation is called singular.

The most important special cases are:

(1) Fredholm equation of the first kind:
 $g(x) = 0$; a and b constant.

(2) Fredholm equation of the second kind:
 $g(x) = 1$; a and b constant.

(3) Volterra equation of the first kind:
 $g(x) = 0$; $a = 0$, $b = x$.

(4) Volterra equation of the second kind:
 $g(x) = 1$; $a = 0$, $b = x$.

(See also **Fredholm determinant**; **Liouville-Neumann series**.)

INTEGRAL FUNCTION. Synonym of **entire function**.

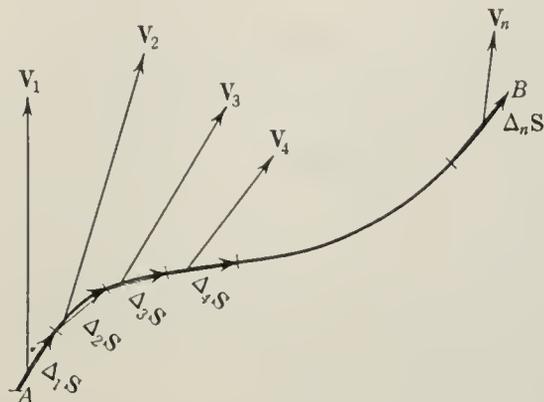
INTEGRAL, IMPROPER. See **improper integral**.

INTEGRAL, LEBESGUE. See **Lebesgue integral**.

INTEGRAL, LEBESGUE-STIELJES. See **Lebesgue-Stieljes integral**.

INTEGRAL, LINE. Given a **vector function** of position $\mathbf{V}(x,y,z)$ which is defined for all

points on a curve such as $A-B$ in the figure, one may replace the curve approximately by a series of equal, directed chords $\Delta_1S, \Delta_2S, \dots, \Delta_nS$. The magnitude and direction of the vector \mathbf{V} may then be determined at some point in each segment of the curve. The sum of the scalar products:



$$\sum_{j=1}^n \mathbf{V}_j \cdot \Delta_j S,$$

can then be obtained. The line integral is defined as

$$\int_A^B \mathbf{V} \cdot d\mathbf{S} = \lim_{n \rightarrow \infty} \sum_{j=1}^n \mathbf{V}_j \cdot \Delta_j S.$$

The usefulness of the line integral will be immediately recognized if a special case is considered. Suppose that \mathbf{V} is the force acting on a particle in a field of force. Then the line integral is just the work done on the particle as it moves from A to B under the action of the force.

When the line integral is taken over a closed path, or contour, starting and ending at the same point, it is usually denoted as

$$\int_C \mathbf{V} \cdot d\mathbf{S} \quad \text{or} \quad \oint \mathbf{V} \cdot d\mathbf{S}$$

and is sometimes called a *contour integral* or a *circulatory integral*.

The choice of the direction in which $d\mathbf{S}$ shall be counted as positive is a matter of convention, but cases arise (e.g., in connection with **Stokes theorem**) when consistency of convention must be assured. The magnitude of the integral is related by Stokes theorem to the integral of $\nabla \times \mathbf{V}$ (curl \mathbf{V}) over any surface bounded by the path.

In the particular case in which \mathbf{V} is the gradient of a potential ϕ ,

$$\int_A^B \mathbf{V} \cdot d\mathbf{S} = \phi_B - \phi_A,$$

i.e., to the difference of potential between B and A . Over a closed contour, then

$$\oint \mathbf{V} \cdot d\mathbf{S} = 0$$

and the field of \mathbf{V} is described as irrotational.

INTEGRAL, LOGARITHMIC. The definite integral

$$Li(z) = \int_0^z \frac{dt}{\ln t}.$$

If $z = e^x$, the function is also called the exponential integral

$$Ei(x) = Ci(x) + i Si(x) + \frac{i\pi}{2}$$

where $Ci(x)$, $Si(x)$ are the improper integrals called the cosine integral and sine integral, respectively:

$$\begin{aligned} Ci(x) &= -\int_x^\infty \frac{\cos t}{t} dt \\ &= C + \ln x - \int_0^x \frac{(1 - \cos t)dt}{t} \\ Si(x) &= \int_0^x \frac{\sin t}{t} dt = \frac{\pi}{2} - \int_x^\infty \frac{\sin t}{t} dt \end{aligned}$$

and C is the **Euler-Mascheroni constant**.

INTEGRAL, MULTIPLE. The definition of an **indefinite integral** can be extended to cover the case of a function of several variables and then more than one **integration** will be required to evaluate the integral. The subject can be considered in various ways but one simple approach is that of partial integration as the inverse to partial **differentiation**. Thus, given the double integral

$$u = \iint f(x,y) dx dy$$

one wishes to determine u so that it will satisfy the partial **differential equation** $u_{xy} = f(x,y)$. The first integration is performed with respect to x , for example, holding y constant, and the second with respect to y , although the order of integrating does not matter. Constants of integration added to the result complete the work, although these are not really constants but arbitrary functions of the variables.

Further generalization to triple, quadruple, etc., integrals offers no further difficulty in principle. Often, for ease in printing, a single integral sign is used for multiple integrals.

The definite multiple integral is commonly of more importance and it may be interpreted geometrically. For example, a function of three variables $f(x,y,z) = 0$ can be considered as a surface. Double integrals, with the appropriate limits, can then be formulated to give the volume of a solid bounded by two or more surfaces, the area of the surface itself, and the moment of inertia of a plane area. Similarly, a triple integral may be used to obtain the volume of a solid or of a closed surface. When given these geometrical interpretations double and triple integrals are often called surface and volume integrals.

INTEGRAL OPERATOR. An operator L involving integrals, usually of some such form as

$$Lf(x) = \int_a^b f(x)K(x,s)ds,$$

or similarly in several variables, where the function $K(x,s)$ is called the kernel of the operator. (Compare **integral transform**.)

INTEGRAL, RESONANCE. See **resonance integral**.

INTEGRAL, RIEMANN. See **Riemann integral**.

INTEGRAL, STIELJES. See **Stieljes integral**.

INTEGRAL, SURFACE. See **integral, multiple**.

INTEGRAL THEOREMS OF VECTOR ANALYSIS.

$$(1) \oint_C \mathbf{F} \cdot d\mathbf{R} = \iint_S (\mathbf{n} \cdot \nabla) \times \mathbf{F} da$$

(Stokes Theorem)

$$(2) \oint \oint_S \mathbf{F} \cdot \mathbf{n} da = \iiint_V \nabla \cdot \mathbf{F} dv$$

(Gauss Theorem)

$$(3) \oint_C f d\mathbf{R} = \iint_S \mathbf{n} \times \nabla f da$$

$$(4) \oint_C d\mathbf{R} \times \mathbf{F} = \iint_S (\mathbf{n} \times \nabla) \times \mathbf{F} da$$

$$(5) \oint \oint_S f \mathbf{n} da = \iiint_V \nabla f dv$$

$$(6) \oint \oint_S \mathbf{n} \times \mathbf{F} da = \iiint_V \nabla \times \mathbf{F} dv$$

$$(7) \iiint_V \mathbf{A}(\nabla \cdot \mathbf{B}) dv$$

$$= \oint \oint_S \mathbf{A}(\mathbf{n} \cdot \mathbf{B}) da - \iiint_V (\mathbf{B} \cdot \nabla) \mathbf{A} dv$$

Time Derivatives of Integrals

$$(8) \frac{d}{dt} \iint_S \mathbf{A} \cdot \mathbf{n} da$$

$$= \iint_S \left\{ \frac{\partial \mathbf{A}}{\partial t} + \mathbf{v} \operatorname{div} \mathbf{A} + \operatorname{curl} [\mathbf{A} \times \mathbf{v}] \right\} \cdot \mathbf{n} da$$

$$= \iint_S \left\{ \frac{d\mathbf{A}}{dt} + \mathbf{A} \operatorname{div} \mathbf{v} + (\mathbf{A} \cdot \nabla) \mathbf{v} \right\} \cdot \mathbf{n} da$$

$$(9) \frac{d}{dt} \iiint_V f dv = \iiint_V \left\{ \frac{\partial f}{\partial t} + \operatorname{div} (f\mathbf{v}) \right\} dv$$

$$= \iiint_V \left\{ \frac{df}{dt} + f \operatorname{div} \mathbf{v} \right\} dv$$

Explanation of symbols:

$d\mathbf{R}$ is a vector element of arc-length,

da is a surface element, \mathbf{n} its normal,

dv is an element of volume,

\mathbf{v} is the velocity of a surface element or volume element,

C is the boundary curve of surface S ,

S is the surface boundary of volume V .

Green's Formulae

$$(10) \iiint_V u \nabla^2 w dv + \iiint_V (\nabla u) \cdot (\nabla w) dv$$

$$= \oint \oint_S u \frac{\partial w}{\partial n} da$$

$$(11) \iiint_V \{u \nabla^2 w - w \nabla^2 u\} dv$$

$$= \iint_S \left\{ u \frac{\partial w}{\partial n} - w \frac{\partial u}{\partial n} \right\} da$$

If f is a scalar depending on x,y,z,t , it can be represented as the following integral:

$f(x,y,z,t)$

$$= \frac{1}{4\pi} \iiint_V \frac{1}{r} \left\{ b^2 \frac{\partial^2 f}{\partial t^2} - \text{div grad } f \right\}_{t+br} dv$$

$$- \frac{1}{4\pi} \oint \oint_S \left\{ \frac{b}{r} \mathbf{r}_o \cdot \mathbf{n} \frac{\partial f}{\partial t} + f \mathbf{n} \cdot \nabla \frac{1}{r} - \frac{\mathbf{n} \cdot \nabla f}{r} \right\}_{t+br} da.$$

If \mathbf{B} is a vector depending on x,y,z,t , it can be represented as the following integrals:

$\mathbf{B}(x,y,z,t)$

$$= \frac{1}{4\pi} \iiint_V \frac{1}{r} \left\{ b^2 \frac{\partial^2 \mathbf{B}}{\partial t^2} + \text{curl curl } \mathbf{B} \right\}_{t+br} dv$$

$$- \frac{\nabla}{4\pi} \iiint_V \left\{ \frac{\text{div } \mathbf{B}}{r} \right\}_{t+br} dv$$

$$+ \frac{\nabla}{4\pi} \oint \oint_S \left\{ \frac{\mathbf{B} \cdot \mathbf{n}}{r} \right\}_{t+br} da$$

$$+ \frac{\text{curl}}{4\pi} \oint \oint_S \left\{ \frac{\mathbf{B} \times \mathbf{n}}{r} \right\}_{t+br} da$$

$$- \frac{1}{4\pi} \oint \oint_S \left\{ \frac{\mathbf{n} \times \text{curl } \mathbf{B}}{r} \right\}_{t+br} da$$

$$= \frac{1}{4\pi} \iiint_V \frac{1}{r} \left\{ b^2 \frac{\partial^2 \mathbf{B}}{\partial t^2} + \text{curl curl } \mathbf{B} \right\}_{t+br} dv$$

$$- \frac{\nabla}{4\pi} \iiint_V \left\{ \frac{\text{div } \mathbf{B}}{r} \right\}_{t+br} dv$$

$$+ \frac{b}{4\pi} \frac{\partial}{\partial t} \oint \oint_S \left\{ \frac{\mathbf{r}_o \times [\mathbf{n} \times \mathbf{B}]}{r} \right\}_{t+br} da$$

$$- \frac{1}{4\pi} \oint \oint_S \left\{ \mathbf{B} \mathbf{n} \cdot \nabla \frac{1}{r} \right\}_{t+br} da$$

$$+ \frac{1}{4\pi} \oint \oint_S \left\{ \left(\nabla \frac{1}{r} \right) \times (\mathbf{n} \times \mathbf{B}) \right\}_{t+br} da$$

$$- \frac{b}{4\pi} \frac{\partial}{\partial t} \oint \oint_S \left\{ \frac{\mathbf{r}_o}{r} (\mathbf{n} \cdot \mathbf{B}) \right\}_{t+br} da$$

$$- \frac{1}{4\pi} \iiint_S \left\{ \frac{\mathbf{n} \times \text{curl } \mathbf{B}}{r} \right\}_{t+br} da.$$

The subscript $t + br$ means that after the expression has been evaluated in terms of x,y,z,t , the time t is to be replaced by $t + br$. The constant b is the reciprocal of the wave-velocity with which the physical disturbance is propagated; r is the distance from the point x,y,t to a point of V or S ; \mathbf{r}_o is a unit vector along r ; \mathbf{n} is the outer normal of the closed surface, S .

INTEGRAL TRANSFORM. Consider a homogeneous integral equation

$$f(y) = \int K(x,y)F(x)dx$$

with kernel $K(x,y)$. The function $f(y)$ is an integral transform of $F(x)$. Given $F(x)$, presumably $f(y)$ may be found explicitly. Regarding the equality as an integral equation, however, one wishes to solve for $F(x)$, or invert the transform. Thus, if the transform can be inverted the result will be the solution of the integral equation for the given kernel.

Many special cases have been studied and given special names. The reciprocal relations, written in the form

$$f = C \int K F dx \quad \text{and} \quad F = C' \int K' f dy$$

are shown in the following table for the more familiar integral transforms.

Name	Constants	Integration Limits
1. Fourier	$C = C' = 1/\sqrt{2\pi}$	$\pm \infty$
2. Sine (cosine)	$C = C' = \sqrt{2/\pi}$	$\infty, 0$
3. Laplace	$C = 1, C' = 1/2\pi i$	$\infty, 0; c \pm i\infty$
4. Mellin	$C = 1, C' = 1/2\pi i$	$\infty, 0; c \pm i\infty$
5. Hankel	$C = C' = 1$	$\infty, 0$
6. Hilbert	$C = C' = 1/2\pi$	$\pm \pi$

Name	K	K'
1. Fourier	e^{ixy}	e^{-ixy}
2. Sine (cosine)	$\sin (\cos) xy$	$\sin (\cos) xy$
3. Laplace	e^{-xy}	e^{xy}
4. Mellin	x^{y-1}	x^{-y}
5. Hankel	$J_m(xy)$	$J_m(xy)$
6. Hilbert	$1 + \cot (x - y)/2$	K

Notes for table: (1) The constant factor is sometimes omitted or modified. (3) The constant c is any real number such that $\int_0^\infty e^{-cx}|f(x)|dx$ is bounded. The symbol p is almost invariably used in place of y . (5) This is often called the Fourier-Bessel transform. The kernel, J_m is a **Bessel function** of order m . (6) The principal value of the integral is to be taken, that is:

$$\int_{-\pi}^{\pi} = \lim_{\delta \rightarrow 0} \left[\int_{-\pi}^{y-\delta} + \int_{y+\delta}^{\pi} \right].$$

All of these relations must be used with proper care as to the conditions under which they are valid. These properties, as well as tables of the pairs $f(y)$ and $F(x)$ in explicit form, may be found in *Formulas and Theorems for the Special Functions of Mathematical Physics*, W. Magnus and F. Oberhettinger, translated by John Vermer, Chelsea Publishing Co.

INTEGRAL TRANSPORT EQUATION.

The Boltzmann transport equation, an integro-differential equation for the angular flux $vN(\mathbf{r}, E, \hat{\Omega})$ may, in certain cases, be integrated to yield an integral transport equation, (1) for the total flux, or (2) the angular flux integrated over angle. The conditions for (2) are: the neutron sources be isotropic, and the scattering in the laboratory system must be independent of angle. The further condition that scattering produces no change in neutron energy makes (1) possible. Equation (2) has the form:

$$vN(\mathbf{r}, E) = \frac{1}{4\pi} \int \frac{d^3\mathbf{r}'}{|\mathbf{r} - \mathbf{r}'|^2} \exp(-\Sigma(E)|\mathbf{r} - \mathbf{r}'|) \\ \times \left[4\pi S(\mathbf{r}', E) \right. \\ \left. + \int dE' \Sigma(E') v' N(\mathbf{r}', E') C(E') f(E', E) \right]$$

in a geometrically simple homogeneous body. See **neutron transport theory** for definition of symbols.

INTEGRAL, VOLUME. See **integral, multiple**.

INTEGRAND. A function which is to be integrated.

INTEGRATING FACTOR. An integrating factor for a differential equation $M(x, y)dx + N(x, y)dy = 0$ is a function $\mu(x, y)$ such that $\mu M dx + \mu N dy = 0$ is exact, that is, such that

$$\frac{\partial(\mu M)}{\partial y} = \frac{\partial(\mu N)}{\partial x}.$$

INTEGRATION. See **integral**.

INTEGRATION, CONTOUR. See **contour integration**.

INTEGRATION, NUMERICAL. See **quadrature formulas**.

INTENSITY DISTRIBUTION, CURVE OF.

A curve, generally polar, which represents the **luminous intensity** in a plane passing through the source, as a function of the angle measured from some given direction. (1) When the source has an axis of symmetry, the plane is generally a meridian plane. (2) The pole of the polar curve is at the point representing the position of the light source. (3) When the reference direction is vertical, angles are measured from the downward vertical.

INTENSITY DISTRIBUTION, SURFACE OF.

The surface formed by the extremities of all the radii vectors drawn from a common origin, the length of each radius vector being proportional to the luminous intensity of the source in the corresponding direction.

INTENSITY, MEAN HORIZONTAL. See **mean horizontal intensity**.

INTENSITY, MEAN SPHERICAL. See **mean spherical intensity**.

INTENSITY METHOD OF MEASURING TOTAL ABSORPTIVITY.

A method, due to Knudsen, of measuring the total absorptivity (see **sound absorption coefficient**) of a room, in which the maximum steady state energy density is measured before and after the addition of a known amount of absorption. The total absorptivity α can then be determined by the relation

$$\alpha = \frac{\alpha'}{\frac{E_{max}}{E'_{max}} - 1}$$

where E'_{max} is the maximum steady state energy density after α' units of absorption have been added.

INTENSITY OF A SOURCE OF PARTICLES. The total number of particles emitted per unit area per unit time.

INTENSITY OF A SPECTRAL LINE. The intensity of a spectral line in emission (energy emitted by the source per second) is given by

$$I_{em.}^{nm} = N_n h c \nu_{nm} A_{nm}.$$

Here N_n is the number of atoms in the initial (upper) state, h is Planck's constant, c is the velocity of light, ν_{nm} is the wave number, and A_{nm} is Einstein's transition probability for spontaneous emission

$$A_{nm} = \frac{64\pi^4 \nu_{nm}^3}{3h} |R^{nm}|^2$$

where R^{nm} is the matrix element of the transition (see **quantum mechanical transition**).

The intensity of a spectral line in absorption (energy absorbed from an incident beam of 1 cm cross section) is, for a sufficiently thin absorbing layer Δx , given by

$$I_{abs.}^{nm} = \rho_{nm} N_m B_{mn} \Delta x h c \nu_{nm}.$$

Here N_m is the number of atoms per cm^3 in the initial (lower) state, ρ_{nm} is the density of radiation of the incident beam, and B_{mn} is Einstein's transition probability for absorption

$$B_{mn} = \frac{8\pi^3}{3h^2 c} |R^{nm}|^2.$$

INTENSITY OF A SPHERICAL SOUND WAVE, FORMULA FOR. The intensity of a spherical sound wave in a dissipative medium is given by

$$I = \frac{P e^{-\alpha r}}{4\pi r^2},$$

where I is intensity in $\text{ergs/cm}^2\text{-sec}$, P is power output of source in ergs/sec , r is distance from source in cm , α is amplitude absorption coefficient in nepers/cm .

INTENSITY OF RADIATION. The energy or the number of photons or of particles, flowing through unit area per unit time. For parallel radiation, the area usually refers to a surface normal to the direction of propagation.

INTENSITY OF RADIOACTIVITY. The number of atoms disintegrating per unit time, or derivatively, the number of scintillations or other effects (roentgens per hour at one meter) observed per unit time.

INTENSITY, RADIANT (OF A SOURCE). The rate of transfer of radiant energy per unit solid angle.

INTENSITY RULES FOR MULTIPLETS. In the case of **Russell-Saunders coupling** (small multiplet splitting) the line intensities in a **multiplet** are determined by the following rule (Burger-Dorgelo-Ornstein's sum rule): The sum of the intensities of all the lines of a multiplet which belong to the same initial or final state is proportional to the statistical weight $2J + 1$ of the initial or final state, re-

spectively. From the sum rule the following rules can be derived: (a) The components of a multiplet for which the quantum numbers J and L alter in the same manner are more intense than those for which they alter unequally. (b) The components belonging to a large J value are more intense than those with small J .

The detailed multiplet intensity rules, as derived from quantum mechanics, are:

For transitions $(L - 1) \rightarrow L$

$(J - 1) \rightarrow J$

$$I = \frac{\left\{ \begin{array}{l} B(L + J + S + 1)(L + J + S) \\ (L + J - S)(L + J - S - 1) \end{array} \right\}}{J}$$

$J \rightarrow J$

$$I = \frac{\left\{ \begin{array}{l} -B(L + J + S + 1)(L + J - S) \\ (L - J + S)(L - J - S - 1)(2J + 1) \end{array} \right\}}{J(J + 1)}$$

$(J + 1) \rightarrow J$

$$I = \frac{\left\{ \begin{array}{l} B(L - J + S)(L - J + S - 1) \\ (L - J - S - 1)(L - J - S - 2) \end{array} \right\}}{J + 1}$$

For transitions $L \rightarrow L$

$(J - 1) \rightarrow J$

$$I = \frac{\left\{ \begin{array}{l} -A(L + J + S + 1)(L + J - S) \\ (L - J + S + 1)(L - J - S) \end{array} \right\}}{J}$$

$J \rightarrow J$

$$I = \frac{\left\{ \begin{array}{l} A[L(L + 1) + J(J + 1)] \\ -S(S + 1)^2(2J + 1) \end{array} \right\}}{J(J + 1)}$$

$(J + 1) \rightarrow J$

$$I = \frac{\left\{ \begin{array}{l} -A(L + J - S + 2)(L + J - S + 1) \\ (L - J + S)(L - J - S - 1) \end{array} \right\}}{(J + 1)}$$

The constants A and B depend on temperature and wave-number and can be omitted in the calculation of the relative intensities, as long as the multiplet splitting is small.

INTENSIVE PROPERTIES. The properties which are independent of the mass of a system, such as the pressure p and the temperature T , and which take well-defined values at each point of a system.

INTENSIVE VARIABLE. See **intensive property**.

INTERACTION. Whenever two or more bodies exert mutual forces on one another, they are said to interact. The term interaction is often used loosely for the force of interaction, i.e., the mutual force exerted by one body on another. (See the various types of interaction and also the various entries under **coupling**.)

INTERACTION CONFIGURATION. See **configuration, interaction**.

INTERACTION CROSS SECTION. The interaction cross section is given by $\sigma = 1/n\lambda$, where n is the number of nuclei per unit volume and λ is the **interaction mean free path**.

INTERACTION CURVE. A limiting relation between two independent quantities, especially one required by design specifications. The curve giving the combinations of axial force P and bending moment M which will cause failure in a particular column provides an example of actual interaction. The straight line design formula

$$\frac{P}{P_{max}} + \frac{M}{M_{max}} \leq 1$$

is an example of a specification requirement where P_{max} is the maximum axial force allowed when $M = 0$, and M_{max} the maximum moment allowed when $P = 0$.

INTERACTION ENERGY. The energy of **interaction** is the work that must be done against the force(s) of interaction to establish a physical system referred to infinity as an origin.

INTERACTION, EXCHANGE. See **exchange force**.

INTERACTION FAST EFFECT. See **fast effect, interaction**.

INTERACTION, FERMI. See **universal Fermi interaction**.

INTERACTION, MAJORANA. See **majorana force**.

INTERACTION, MANY-BODY. See **many-body force**.

INTERACTION MEAN FREE PATH. The average distance which a given particle travels before experiencing an **interaction** with another particle.

INTERACTION PICTURE. Suppose the **Hamiltonian** H of a system is composed of two parts: an unperturbed Hamiltonian H_o , and an interaction (perturbation) Hamiltonian H_I

$$H = H_o + H_I \quad (1)$$

so that in the **Schrödinger picture** the time behavior of the state vector describing the system is governed by the equation

$$i\hbar\partial_t\Phi_S(t) = (H_o + H_I)\Phi_S(t). \quad (2)$$

Introduce the vector

$$\Psi_{IR}(t) = e^{iH_o t/\hbar}\Phi_S(t), \quad (3)$$

which by virtue of (2) satisfies the equation

$$\begin{aligned} i\hbar\partial_t\Phi_{IR}(t) &= e^{iH_o t/\hbar}H_I e^{-iH_o t/\hbar}\Phi_{IR}(t) \\ &= H_I^{(IR)}(t)\Psi_{IR}(t). \end{aligned} \quad (4)$$

The scheme which describes the physical system by the vector $\Psi_{IR}(t)$ and operators $Q_{IR}(t)$

$$Q_{IR}(t) = e^{+iH_o t/\hbar}Q_S e^{-iH_o t/\hbar} \quad (5)$$

is called the interaction picture. In the interaction picture the time dependence of the operators is determined by the unperturbed Hamiltonian, with

$$-i\hbar\partial_t Q_{IR}(t) = [H_o, Q_{IR}(t)]. \quad (6)$$

(Note $H_o^{IR} = H_o^S$.) The time dependence of the state vector is determined by the interaction Hamiltonian as indicated by Equation (4).

In relativistic field theoretic application the advantage of the interaction picture stems from the fact that the field operators satisfy field-free equations, so that **covariant commutation rules** can be written down for all times. It also permits a generalization of Equation (4) which makes it covariant. Equation (4) is not covariant, since it singles out a special Lorentz frame in defining the time derivative. (See **Tomanaga-Schwinger equation**.)

INTERATOMIC POTENTIAL. The potential energy of two atoms. The three most important potentials used are the Lennard-Jones "12-6" potential,

$$U(r) = Ar^{-12} - Br^{-6},$$

the "exp-6" potential,

$$U(r) = Ce^{-\alpha r} - Dr^{-6},$$

and the Morse potential,

$$U(r) = E\{1 - \exp[-\beta(r - r_0)]\}^2,$$

where α , β , A , B , C , D , E , and r_0 are constants, and $U(r)$ is the potential energy for a distance apart r .

INTERCOMBINATION LINES. Spectral lines corresponding to transitions between electronic energy levels of different **multiplicity**. Such transitions involve a violation of the (approximate) **selection rule** $\Delta S = 0$ which applies as long as **Russell-Saunders coupling** is a good approximation. As a rule, therefore, intercombination lines are very weak, especially for elements of low **atomic number**. For heavier elements where Russell-Saunders coupling does not hold, the intensity of intercombination lines may approach that of ordinary lines from the same energy level.

INTERFERENCE. The variation of wave amplitude with distance or time, caused by the superposition of two or more waves. As most commonly used, the term refers to the interference of waves of the same or nearly the same frequency. Wave interference is characterized by the phenomenon of the occurrence of local maxima and minima of **wave amplitude**, which cannot be described by the ray approximation to solutions of the wave equation. In terms of the **Huygens approximation**, interference can occur whenever wave disturbance can be propagated from a source to a region of space by two or more paths of different length. There is (destructive) interference if the phases and amplitudes of the disturbances arriving by the various routes are such as to reduce the square of the resultant amplitude below the sum of the squares of the amplitudes of the components. If the square of the resultant amplitude exceeds the sum of the squares of the component amplitudes, constructive interference occurs. Interference effects are observed with electromagnetic waves, sound waves, water waves, as well as with

matter waves (i.e., electron and neutron waves).

INTERFERENCE COEFFICIENTS OF THERMODYNAMICS OF IRREVERSIBLE PROCESSES. See **phenomenological relations and reciprocity relations**.

INTERMITTENCY EFFECT. The departure from the **reciprocity law** when the **exposure** of a photographic emulsion is made in a series of discrete installments rather than in a continuous exposure to the same total energy.

INTERMOLECULAR FORCES. Our knowledge of the intermolecular forces comes mainly from the following sources:

(1) Thermodynamic properties of gases at low densities (see **real gases**; **statistical mechanics**).

(2) Transport processes in dilute gases, for example, viscosity, diffusion, etc. (see **kinetic theory of gases**).

(3) Thermodynamic properties of crystals near $^{\circ}\text{K}$.

It is convenient, though somewhat arbitrary, to classify intermolecular forces in two types (excluding **chemical interactions**):

(1) *Repulsive short range forces.*

(2) *Attractive long range forces.*

The short range forces become significant when the molecules come close enough together for their electron clouds to overlap. The exact analytical form of these forces is complicated (see **Lennard-Jones potential**, **Buckingham potentials**). The corresponding energy is called the *overlap energy*.

The various contributions to the long range forces are:

(1) The electrostatic contribution.

(2) The dispersion contribution (see **dispersion forces**).

(3) The induction contribution (see **induction forces**).

INTERNAL ABSORPTANCE. The ratio of the **luminous flux** absorbed between the entry and exit surface of the filter or the plate, to the flux which leaves the entry surface.

INTERNAL COMBUSTION ENGINE. A reciprocating, usually high-speed, engine working on the two-stroke or four-stroke principle, which produces power by virtue of the internal combustion of a spray of liquid fuel, or a

vaporized liquid fuel, in the engine cylinder. (See **diesel engine**; **spark-ignition engine**.)

INTERNAL CONVERSION. In an electromagnetic transition between two states of an atomic nucleus differing by energy E , a γ -ray of energy E is usually emitted. An alternative mode of decay, however, is that an atomic electron of energy $E - \epsilon$ may be emitted from one of the inner shells of the atom through direct interaction with the electromagnetic field of the nucleus, where ϵ is the binding energy of the electron. Thus, one speaks of K -conversion, L -conversion, etc., corresponding to electrons ejected from the K , L , \dots , shells. If the transition in question follows a β -decay transition, the spectrum of electrons observed will therefore consist of a number of discrete energy groups, corresponding to the conversion electrons, superimposed upon the continuous spectrum coming from the β -decay itself. The process is more probable the lower the energy E of the transition.

INTERNAL CONVERSION COEFFICIENT. The ratio of the number of internal conversion electrons to the number of quanta emitted plus the number of conversion electrons emitted in a given mode of de-excitation of a nucleus. Partial conversion coefficients refer to conversion fractions for various electron shells, e.g., K -conversion fractions, L -conversion coefficients, etc. Sometimes called conversion fraction.

INTERNAL ENERGY. The energy ascribed to a given state of a system, which is determined only by the state itself (and is thus a scalar quantity) and is not accounted for by the kinetic energy of bulk motion or potential energy in external force fields. By thermodynamics, the change in the internal energy when the system goes adiabatically from one state to another is equal to the external work performed in bringing about the change. On a molecular scale, the internal energy is the sum of the kinetic energy of the thermal motion of the molecules and the sum of their potential energies in each other's fields of force. By the first law of thermodynamics, the change of internal energy in any process is equal to the difference of the heat gained and the external work done. (See **thermodynamics**, **first law of**.)

INTERNAL ENERGY FLUX VECTOR. See **flux vector**.

INTERNAL FIELDS IN DIELECTRICS. The interactions between the molecules in a dielectric can be broadly subdivided in two classes: (1) the short range intermolecular forces (**dispersion**, **repulsion forces**, and eventually, **chemical interactions**) which result practically in nearest neighbor interactions only; (2) the dipolar interactions having longer ranges: in order to predict their effect, one must consider a very large number of interacting pairs.

Lorentz and later authors have evaluated the electric field acting on the individual charges inside the dielectric (inner or internal field) as follows: a small sphere is chosen, large enough to manifest the same properties as a macroscopic system, yet small enough for the dipolar interactions within the sphere to be calculable exactly. The long range dipolar interactions of the molecules within the sphere with the surrounding material are treated by means of macroscopic equations.

The internal field, appears thus as a sum of two terms, one of which is evaluated by microscopic equations, the other by assuming the whole system to be continuous. One assumes often that the sum of the interaction within the sphere leads to a negligible field; this hypothesis is justified exactly for some models. (See also under **reaction** and **cavity field**.)

INTERNAL FRICTION. See **friction**, **internal**.

INTERNAL HEAT OF EVAPORATION. See **latent heat**.

INTERNAL OPTICAL DENSITY. The logarithm to base ten of the reciprocal of the internal transmission factor.

INTERNAL TRANSMITTANCE. The ratio of the **luminous flux** reaching the exit surface of a filter or a plate to the flux which leaves the entry surface.

INTERNAL VERTEX. See **vertex**, **internal**.

INTERNATIONAL ANGSTROM. See **angstrom**.

INTERPOLATION. The approximation from tabulated values of a function, and possibly of its derivatives, of values not included in a

table. Let $f(x)$, and possibly derivatives, be tabulated at x_0, \dots, x_m . Usually one selects a set of functions ϕ_0, \dots, ϕ_n , which may be polynomials, trigonometric functions, exponentials, or of any other convenient class, and applies some criteria for selecting coefficients α_i , such that $\alpha_0\phi_0 + \dots + \alpha_n\phi_n$ is accepted as an approximation $P[f|x]$ to $f(x)$. The most common criteria are agreement of P with f (and possibly derivatives) at the x_i . Most commonly the term "interpolation" is applied only when criteria of this type are applied. (Cf. **difference operators**; **Aitken method**; **Lagrange interpolation formula**; **Hermite interpolation formula**; **Newton interpolation formula**; and, for other criteria, **minimax approximation**; **least squares**.)

INTERQUARTILE RANGE. The interquartile range is defined as $Q_3 - Q_1$ where Q_3 and Q_1 are the third and first **quartiles** in a distribution. It is sometimes used as a measure of **dispersion**.

INTERVAL BETWEEN EVENTS. If two **events** occur at the positions and times represented by

$$x^\mu \text{ and } x^\mu + dx^\mu \ (\mu = 1, 2, 3, 4),$$

the interval between is given by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu,$$

where $g_{\mu\nu}$ is the metric of space-time. In Minkowski space, the interval is given by:

$$ds^2 = c^2dt^2 - dr^2.$$

INTERVAL RULE, LANDE'S. See **Landé's interval rule**.

INTERVAL, SOUND. The interval between two sounds is their spacing in pitch or frequency, whichever is indicated by the context. The frequency interval is expressed by the ratio of the frequencies or by a logarithm of this ratio.

INTRINSIC DERIVATIVE OF TENSOR FIELD. $t_{pq\dots r}^{ij\dots k}$ are the components in a generic coordinate system x of a tensor field of weight W , which is defined along a curve by its dependence on a parameter τ . The intrinsic derivative of the tensor field with respect to τ is the tensor with components $\delta t_{pq\dots r}^{ij\dots k}/\delta\tau$ in the coordinate system x , defined by

$$\frac{\delta t_{pq\dots r}^{ij\dots k}}{\delta\tau} = t_{pq\dots r,m}^{ij\dots k} \frac{dx^m}{d\tau},$$

where $t_{pq\dots r,m}^{ij\dots k}$ are the components in the system x of the covariant derivative of the tensor field. Also called *absolute derivative*.

INTRINSIC DERIVATIVE, SURFACE. See **surface, intrinsic derivative**.

INTRINSIC EQUATIONS OF A CURVE. See **curve, intrinsic equations of**.

INTRINSIC GEOMETRY OF A SURFACE. See **intrinsic properties of a surface**.

INTRINSIC PROPERTIES OF A SURFACE. Those properties of the surface which can be described without reference to the surrounding space. The description of these properties is the *intrinsic geometry of the surface*.

INVARIANCE, GAUGE. See **gauge invariance**.

INVARIANCE PRINCIPLE. A principle whereby a relationship must be invariant under certain transformations. For special cases, see discussions of **Lorentz invariants** and various articles on **relativity**.

INVARIANT, OR ABSOLUTE INVARIANT (UNDER A TRANSFORMATION OF COORDINATES). An entity which is unaltered by the transformation. For example, a point in space, as distinct from the coordinates of the point; an absolute **tensor** or **vector** as distinct from the components of the tensor or vector in a particular coordinate system. Most frequently used to signify **scalar field** and in this sense often referred to as *scalar invariant*. Entities such as relative tensor fields or relative scalar fields are also called *relative invariants*. The term invariant may be used to signify either an absolute or relative invariant.

INVARIANT POINT. State of a given system where the number of degrees of freedom is zero, e.g., the triple point: the condition that solid, liquid, and vapor phases shall exist in equilibrium completely determines the state of the system. (See **freedom, degree of (2)**.)

INVERSE ELEMENT. The inverse of a group element A is the element A^{-1} such that $A^{-1}A = AA^{-1} = I$, where I is the unit element.

INVERSE FUNCTION. If $y = f(x)$, the inverse function is $x = g(y)$.

INVERSE HYPERBOLIC FUNCTION. See *hyperbolic function*.

INVERSE INTERPOLATION. The approximation, from an interpolating polynomial, of the value of an abscissa corresponding to a given ordinate. This amounts to solving an algebraic equation of degree equal to the polynomial. (See Zdenek Kopal, *Numerical Analysis*, John Wiley & Sons, Inc., 1955; E. T. Whittaker and G. Robinson, *The Calculus of Observations*, Blackie & Sons, Ltd., 1940.)

INVERSE MATRIX. The inverse of an $n \times n$ matrix A whose **determinant** is not zero is the unique $n \times n$ matrix A^{-1} such that $AA^{-1} = A^{-1}A = I$, where I is the unit matrix of order n .

INVERSE OPERATOR. An operator symbol which cancels the process directed by another operator. Thus A, B are inverse operators if they mean addition and subtraction of a constant, respectively, or if they are defined as differentiation and integration.

INVERSE SQUARE FIELD, MOTION IN. See *motion in inverse square field*.

INVERSE SQUARE LAWS. (1) Laws of force in which the force of interaction between two particles varies as $1/r^2$ where r is the distance of separation between the particles. The gravitational force and the **Coulomb law of force** between electrical charges follow inverse square laws. (2) The intensity of radiation from a point source emitting isotropic radiation decreases as $1/r^2$ from the source in a non-absorbing medium. This law is a consequence of the law of conservation of energy.

INVERSE SURFACES. Let \mathbf{r} be the vector drawn from a fixed point C to a generic point P on a surface S . Let Q be a point on the line CP such that the vector drawn from C to Q is $(c^2/r^2)\mathbf{r}$, where c is a real constant and r is the length of the vector \mathbf{r} . Then the locus of the point Q is a surface S' . The surfaces S and S' are inverse surfaces. C is the *center of inversion*, c the *radius of inversion*. The process by which the surface S' is generated from S is called *inversion*.

INVERSE TRIGONOMETRIC FUNCTIONS. See *trigonometric functions, inverse*.

INVERSION. (1) See *inverse surfaces*. (2) Literally, a condition in which the temperature

of the air increases upwards, this being an inversion of the more normal condition of a decrease with height. More loosely it is used to refer to a shallow layer of great static stability which inhibits the mixing of the air above and below and therefore acts as a ceiling to convection from warm ground or to the dispersal of pollution from the ground. Alternatively the term is sometimes used to indicate a state in which the **potential temperature** increases upwards, i.e., one of static stability. (3) The alteration of the normal **Boltzmann distribution** in such a manner that the population of the higher energy levels is greater than that of the lower. (See *temperature, negative*.) (4) See *spectrum, inversion*. (5) See *inversion of coordinates*.

INVERSION, CENTER OF. See *inverse surfaces*.

INVERSION DOUBLING. In a nonplanar symmetric top molecule a reflection of all particles at the origin (inversion) leads to a configuration which cannot also be obtained by rotation of the molecule. Corresponding to these two configurations of the molecule, each rotational level J, K is doubly degenerate as long as the potential hill separating these two configurations is infinitely high. For a finite potential hill, a splitting occurs into two sublevels which have opposite symmetry with respect to an inversion. The eigenfunctions of these sublevels contain equal contributions from both "original" levels.

INVERSION INTEGRAL. See *Laplace transform*.

INVERSION OF COORDINATES. Strictly, the transformation of a coordinate system that carries each coordinate into the negative of itself; for example, in the rectangular Cartesian coordinate system, the transformation $x \rightarrow -x, y \rightarrow -y, z \rightarrow -z$.

The term is sometimes used loosely to indicate a transformation that reverses the sign of one or more, but not all, of the coordinates. For example in a mirroring transformation only one of the three spatial coordinates is inverted and the other two are left invariant. Another example is in *Minkowski* four-dimensional space, where *spatial inversion* inverts the three spatial coordinates and leaves the time coordinate invariant; or alternately, *time*

inversion, which inverts the time coordinate but not the spatial coordinates.

INVERSION, RADIUS OF. See **inverse surfaces**.

INVERSION SPECTRA. See **microwave spectra of molecules**.

INVERSION TEMPERATURE. At sufficiently low temperatures, the Joule-Thomson coefficient is positive (see **throttling experiment**), and cooling by throttling is the result. This coefficient becomes negative at higher temperatures, and the temperature at which the effect changes sign is called the *inversion temperature*.

INVERTED MULTIPLY. See **multiplet**.

INVISCID FLUID. Alternatively called a "perfect" or "ideal" fluid, a fluid in which the only stress across any element of area is normal to it and is the fluid pressure. There are no shearing stresses, and vorticity is not diffused.

INVOLUTE (OF A CURVE). If the tangents to the curve C are normals to the curve C' , then C' is an involute of C and C is an *evolute* of C' .

INVOLUTE (OF A SURFACE). Take a singly-infinite system of geodesics (see **geodesic line**) on a surface S . At each point P on S draw the tangent to the geodesic of the family which passes through P . On this tangent take a point Q such that the distance PQ is constant. Then the locus of Q is a surface S' which is called an involute of the surface S . The surface S is called the *evolute* of the surface S' .

ION. An atom or molecularly-bound group of atoms which has gained or lost one or more electrons, and which has thus a negative or positive electric charge; sometimes a free electron or other charged subatomic particle. Ions may be produced in gases by the action of radiation of sufficient energy; ionic solids are built up of ions bound together by their electrostatic forces, and when dissolved in a polar liquid, such as water, the salt dissociates into its ions, which have an independent existence.

IONIC BOND. See **heteropolar bond; atomic bond; bond type**.

IONIC BOND CHARACTER. Consider a diatomic molecule AB , of which A is the more electronegative atom. Let ψ_{cov} be the symmetrical wave function $\psi_A(1)\psi_B(2) + \psi_A(2)\psi_B(1)$ representing the pure covalent bond and ψ_{ion} the "ionic" wave function $\psi_A(1)\psi_A(2)$. Neglecting the ionic configuration with both electrons attached to atom B , one can write for the total wave function:

$$\psi = \psi_{cov} + \lambda\psi_{ion}$$

where λ is a constant. The percentage ionic character of the bond (Coulson) is then $100\lambda^2/(1 + \lambda^2)$.

A rough experimental determination of the ionic character can be obtained by comparing the dipole moment of the bond to that which one would observe if atoms A and B carried, respectively, a whole negative and positive charge each.

IONIC CONFIGURATION. See **atomic bond energy**.

IONIC STRENGTH. The ionic strength I of an electrolyte solution is defined by

$$I = \frac{1}{2}\sum z_i^2 m_i \quad (1)$$

where z_i is the charge number of ion i (see **electroneutrality**) and m_i , its **molality**.

Instead of molalities, other concentration scales, for example, molar concentrations, may also be used in (1).

IONIZATION, DEGREE OF. The ratio, usually stated as a percentage, of the number or the concentration of the particles in a system which become ionized, to those which remain unionized. If the system is a solution, the reference particles are the molecules of a specified component.

IONIZATION ENERGY. See **ionization potential**.

IONIZATION, MINIMUM. The smallest possible value of the specific ionization (see **ionization, specific**) that a charged particle can produce in passing through a particular substance.

IONIZATION POTENTIAL. The ionization potential of an electron is the energy absorbed when the electron is detached from an atom, molecule or ion and brought to rest at infinity, resulting in an ion in its ground electronic state. When it is not otherwise specified, the

ionization potential of an atom, molecule or ion is the lowest of the ionization potentials of its electrons.

For polyatomic molecules, one distinguishes often between the *vertical* and the *adiabatic ionization potential*. The *vertical ionization potential* is the energy absorbed during the instantaneous removal of the electron: the bond lengths of the molecule remaining unaltered, i.e., the resulting ion is not necessarily in its **fundamental state**. It may still contain a large amount of vibrational energy.

The *adiabatic ionization potential* is the energy absorbed when the electron is detached adiabatically, i.e., infinitely slowly and reversibly. The vibrational degrees of freedom of the molecule have time to adjust continually during the ionization. At the end of this process, the ion is in its fundamental state. Whence

$$I_{\text{adiabatic}} \leq I_{\text{vertical}}.$$

IONIZATION, SPECIFIC. The number of **ion pairs** formed per unit distance along the track of an ion passing through matter. This is sometimes called the total specific ionization to distinguish it from the primary specific ionization, which is the number of ion clusters produced per unit track length. The relative specific ionization is the specific ionization for a particle of a given medium relative either to that for (1) the same particle and energy in a standard medium, such as air at 15°C and 1 atm, or (2) the same particle and medium at a specified energy, such as the energy for which the specific ionization is a maximum.

IONIZATION, TOTAL. (1) The total electric charge on the ions of one sign when the energetic particle that has produced these ions has lost all of its kinetic energy. For a given gas the total ionization is closely proportional to the initial energy and is nearly independent of the nature of the ionizing particle. It is frequently used as a measure of particle energy. (2) The total number of ion pairs produced by the ionizing particle along its entire path.

ION PAIR. (1) A positive ion and a negative ion or electron, having charges of the same magnitude, and formed from a neutral atom or molecule by the action of radiation or by any other agency that supplies energy. (2) As postulated in the **Debye-Hückel theory**,

in concentrated solutions of strong electrolytes (two or more), ions may occasionally approach each other so closely that they may form pairs (or groups) without entering into permanent chemical combination.

ION(S), STANDARD ENTROPY OF. The quantity called the standard **entropy** of an ion is really the relative partial entropy of the ion in a solution of unit activity, that of the hydrogen ion being assumed to be zero.

ION(S), STANDARD FREE ENERGY OF. The **free energy** of formation of the particular ions, taking that for the formation of hydrogen ions at unit activity from the gas at 1 atmosphere pressure to be zero.

IRRADIANCE. Irradiance (at a point of a surface) is the quotient of the radiant power incident on an infinitesimal element of surface containing the point under consideration, by the area of that element.

IRRADIATION. (1) Descriptively, irradiation is the exposure of an object to radiation. (2) Quantitatively, irradiation is the product of an **irradiance** and its duration.

IRREVERSIBLE PROCESS. A process occurring in a system such that, in order to reverse the direction of the process, a finite change in the parameters of the system must be made, e.g., the compression or expansion of a gas in a cylinder by means of a piston, when friction is present between piston and cylinder.

IRREVERSIBLE PROCESSES, LINEAR RELATIONS IN THERMODYNAMICS OF. See **phenomenological relations in thermodynamics of irreversible processes**.

IRROTATIONAL FLOW. Flow in which the **vorticity**, or rotation, of the fluid is zero (curl $\mathbf{v} = 0$) and the velocity is the gradient of a potential, i.e., $\mathbf{v} = \text{grad } \phi$. It is therefore called potential flow. If the fluid is also incompressible the equation of continuity is **Laplace's equation**, $\nabla^2 \phi = 0$, and the flow is everywhere uniquely determined if ϕ , or the normal component of the velocity, is specified at all points of the boundary.

In general the motion can only remain irrotational if the density is uniform and there is no tangential force at the boundaries (which in practice requires that the fluid be inviscid).

IRROTATIONAL VECTOR. See **vector, irrotational**.

IRROTATIONAL WAVES. See **wave(s), dilatational**.

ISALLOBAR. A line along which the local rate of change of pressure is constant, usually drawn as lines of constant pressure tendency.

ISALLOBARIC WIND. The ageostrophic component of the wind due to a changing pressure field. It is equal to

$$v_I = -\frac{1}{\rho f^2} \text{grad}_H \frac{\partial p}{\partial t}$$

where ρ is the density, f is twice the vertical component of the earth's rotation, and grad_H denotes the horizontal gradient. It is therefore directed towards regions of greatest pressure fall.

ISENTROPIC. Occurring at constant **entropy**. Any reversible process in an isolated system.

ISENTROPIC BULK MODULUS. See **bulk modulus**.

ISENTROPIC ENTHALPY DROP. See **enthalpy drop**.

ISENTROPIC HEAT DROP. See **enthalpy drop**.

ISING MODEL. A model of a binary alloy, in which it is assumed that the two types, A and B , of atoms are arranged in a regular array. In the state of perfect order, all A atoms are on α -sites and all B atoms on β -sites, so chosen that each atom has the same neighbors as do others of the same type. Thus, in one dimension, the array



represents perfect order. When any two atoms are interchanged, e.g., if the above array becomes



the change in the internal energy of the crystal may be computed by considering the number of pairs of the types $A-B$, $A-A$, and $B-B$ that have been created or destroyed. Thus in the case considered above, the increase in internal energy is

$$\epsilon_{BB} + \epsilon_{AA} - 2\epsilon_{AB}$$

where ϵ_{BB} , ϵ_{AA} , and ϵ_{AB} are the energies of the pairs of the types indicated by the subscripts.

The model has been of great value in the consideration of order-disorder transitions, but is of limited validity, even when next nearest neighbors are considered. (See also **cooperative phenomena**.)

ISOBAR. (1) A line along which the pressure is constant. In meteorology isobars are drawn for horizontal surfaces. (2) One of two or more atomic species, or elements, which have the same **mass number**, but differ in other respects, such as **atomic number** or **energy state**.

ISOBARIC SURFACE. A surface of constant pressure; represented on a meteorological chart by contours of height above sea level. The **geostrophic wind** is then given by the spacing and orientation of the contours.

ISOCANDELA CURVE. (1) A curve traced on an imaginary sphere with the source at its center and joining all the points corresponding to those directions in which the luminous intensity is the same. (2) A plane projection of this curve.

ISOCHORIC. Of equal or constant volume, usually applied to a thermodynamic process during which the volume of the system remains unchanged.

ISOCHROMATIC STIMULI. Color stimuli which, when acting simultaneously, give rise to identical color sensations. The identity of color may result from the identity of spectral distribution or it can occur when the spectral distributions are different if the **tristimulus values** are the same (metameric lights). In photometry it is usual to describe lights as isochromatic if they have the same **chromaticity**.

ISOCHRONE. A line connecting points having the same time values, as points of the same gelation time for colloidal solutions.

ISOELECTRONIC PRINCIPLE. Similar molecules which have the same number of valence electrons should have similar molecular orbitals. Their properties should therefore be quite analogous. Such similarities are evident for carbon monoxide and nitrogen, for example.

ISOGONIC. Having equal magnetic declination.

ISOLATED SYSTEM. A system which is isolated from, and therefore does not exchange matter or energy with, its surroundings. The conservation laws of energy, momentum, and angular momentum only hold in systems which are isolated with respect to the agents that cause changes in the conserved quantities. Thus an isolated system for the conservation of energy is one on whose boundaries no work is done and across whose boundaries no heat flows. Similarly, momentum is conserved only in a system isolated in the sense that no net force acts across its boundaries. (See also **closed system**; **open system**; **thermodynamic system**.)

ISOLUX. A curve or surface of equal light intensity. (Also called **isophot**.)

ISOMAGNETICS. Lines connecting points at which some property of the earth's magnetic field (such as the magnitude, the vertical component, or the horizontal component) remains constant. Isomagnetic lines may indicate local magnetic anomalies such as caused by magnetic ore bodies, magnetic minerals in sediments, or the vertical rather than the horizontal deviation of the compass or magnetic needle.

ISOMETRIC CONFORMAL REPRESENTATION. See **representation**.

ISOMETRIC LINES ON A SURFACE. If in terms of the parameters u and v , the first ground forms for a surface takes the form

$$\lambda[U(du)^2 + V(dv)^2],$$

where λ is a function of u and v , U is a function of u only and V is a function of v only, then the curves $u = \text{constant}$ and $v = \text{constant}$ are called **isometric lines**; u and v are called **isometric parameters**. The term **isothermal** or **isothermic** is also used instead of **isometric**.

ISOMETRIC PARAMETERS. See **isometric lines on the surface**.

ISOMORPHIC GRAPHS. See **graphs**, **isomorphic**.

ISOMORPHISM. See **homomorphism**.

ISOPERIMETRIC PROBLEM. In the calculus of variations, the problem of making an integral **stationary**, while one or more integrals involving the same variables are to be kept constant (accessory conditions). It is so-

called from a particular example, that of finding the figure of maximum area with a fixed **perimeter**. The method of **Lagrange multipliers** may be used.

ISOPHOT. A curve or surface of equal light intensity. (Also called **isolux**.)

ISOPLANASIE CONDITION. See **Staeble-Lihotzky condition**.

ISOTHERMATURE LOCI. Lines on a **chromaticity diagram** connecting points having equal correlated color temperatures. (See **temperature**, **correlated color**.)

ISOTHERMAL. (1) Occurring at, or pertaining to, constant temperature. (2) See also **isometric lines on the surface**.

ISOTHERMAL ATMOSPHERE. (Also called **exponential atmosphere**.) An atmosphere in **hydrostatic equilibrium** in which the temperature is constant with height and in which, therefore, the pressure decreases exponentially upward. In such an atmosphere the thickness between any two levels is given by

$$z_B - z_A = \frac{R_d T_v}{g} \ln \frac{P_A}{P_B},$$

where R_d is the gas constant for dry air, T_v the virtual temperature ($^{\circ}\text{K}$), g the acceleration of gravity, and P_A and P_B the pressures at the heights z_A and z_B , respectively. In the isothermal atmosphere there is no finite level at which the pressure vanishes.

ISOTHERMAL BULK MODULUS. See **bulk modulus**.

ISOTHERMAL COMPRESSION. Compression during which the temperature remains constant. In general, this will entail flow of heat into or out of the system.

ISOTHERMAL EXPANSION. Expansion during which the temperature remains constant. In general, this will entail flow of heat into or out of the system.

ISOTHERMIC. See **isometric lines on the surface**.

ISOTHERMS. Lines (or surfaces) consisting of points at the same temperature, e.g., corresponding to a given temperature in the equa-

tion of state. (See **critical point**; **Maxwell theorem**.)

ISOTOPE. One or two or more nuclides having the same atomic number, hence constituting the same element, but differing in mass number.

ISOTOPE CHARTS (Z-N, Z-A, A-Z, A-2Z, TRILINEAR). Any of a set of charts in which the properties of atomic nuclei, including their modes of radioactive decay, are summarized. In the trilinear chart, the neutron number $A-Z$ is plotted against the atomic number Z , the axes being inclined at an angle of 60 degrees; the result is that nuclides with the same mass number are found in the same vertical row, while each horizontal row contains species with the same neutron excess $A-2Z$.

ISOTOPE EFFECT IN ATOMIC SPECTRA. See **hyperfine structure(a)**.

ISOTOPE EFFECT IN MOLECULAR SPECTRA. The internuclear potential field of a molecule is, to a very good approximation, not affected by isotopic substitution of the nuclei. As a consequence, an isotope effect on the molecular electronic energy levels can, as a rule, be neglected, and the vibrational and rotational effects be treated on the assumption that the equilibrium internuclear distances are the same in all isotopic molecules.

In the case of a *diatomic molecule* the **vibrational constants** of an isotopic molecule [designated by the superscript (i)] are related to those of the "normal" molecule by the formulas

$$\omega_e^{(i)} = \rho\omega_e, \quad \omega_e^{(i)}x_e^{(i)} = \rho^2\omega_e x_e,$$

$$\omega_e^{(i)}y_e^{(i)} = \rho^3\omega_e y_e, \dots$$

where

$$\rho = \sqrt{\frac{\mu}{\mu^{(i)}}}$$

and μ and $\mu^{(i)}$ are the **reduced masses** of the "normal" and the isotopic molecules, respectively. The vibrational absorption bands of an isotopic molecule are given by the formula

$$\nu_{\text{abs}}^{(i)} = \rho\omega_e[(v + \frac{1}{2}) - \frac{1}{2}]$$

$$- \rho^2\omega_e x_e[(v + \frac{1}{2})^2 - \frac{1}{4}]$$

$$+ \rho^3\omega_e y_e[(v + \frac{1}{2})^3 - \frac{3}{8}] + \dots$$

The vibrational isotope shift is therefore

$$\Delta\nu = \nu_{\text{abs}} - \nu_{\text{abs}}^{(i)}$$

$$= \omega_e(1 - \rho)v - \omega_e x_e(1 - \rho^2)(v^2 + v)$$

$$+ \omega_e y_e(1 - \rho^3)(v^3 + \frac{3}{2}v^2 + \frac{3}{4}v) + \dots$$

If ρ is close to one, the vibrational isotope shift is approximately given by the expression

$$\Delta\nu = (1 - \rho)v\Delta G_{v+\frac{1}{2}}$$

where $\Delta G_{v+\frac{1}{2}}$ stands for the separation of successive **vibrational energy levels**.

The **rotational constants** of an isotopic molecule are related to those of the "normal" molecule by the formulas

$$B_e^i = \rho^2 B_e, \quad \alpha_e^i = \rho^3 \alpha_e, \quad D_e^i = \rho^4 D_e,$$

$$\beta^i = \rho^5 \beta, \dots$$

The rotational isotope shift, neglecting the rotational constant β is therefore

$$\Delta\nu_r = \nu_r - \nu_r^i = (1 - \rho^2)[B'_e J'(J' + 1)$$

$$- B''_e J''(J'' + 1)]$$

$$- (1 - \rho^3)[\alpha'_e(v' + \frac{1}{2})J'(J' + 1)$$

$$- \alpha''_e(v'' + \frac{1}{2})J''(J'' + 1)]$$

$$- (1 - \rho^4)[D'J'^2(J' + 1)^2$$

$$- D''J''^2(J'' + 1)^2].$$

In the case of a rotation-vibration spectrum the above formula is greatly simplified since $B'_e = B''_e$, $\alpha'_e = \alpha''_e$, $D' \approx D''$. The second and third terms are, in general, very small compared to the first. Therefore, to a first approximation,

$$\Delta\nu_r = (1 - \rho^2)\nu_r.$$

The rotational isotope shift is thus, to a good approximation, proportional to the distance from the **zero line**.

For two isotopic *polyatomic molecules* the product of the $\omega^{(i)}/\omega$ values for all vibrations of a given symmetry type is independent of the potential constants and depends only on the masses of the atoms and the geometrical structure of the molecule according to the following formula (Teller-Redlich product rule):

$$\frac{\omega_1^{(i)} \omega_2^{(i)} \cdots \omega_f^{(i)}}{\omega_1 \omega_2 \cdots \omega_f} = \sqrt{\left[\left(\frac{m_1}{m_1^{(i)}} \right)^\alpha \left(\frac{m_2}{m_2^{(i)}} \right)^\beta \cdots \left(\frac{M}{M^{(i)}} \right)^t \right] \times \left(\frac{I_x^{(i)}}{I_x} \right)^{\delta x} \left(\frac{I_y^{(i)}}{I_y} \right)^{\delta y} \left(\frac{I_z^{(i)}}{I_z} \right)^{\delta z}}$$

Here quantities with the superscript (*i*) refer to one of the isotopic molecules, quantities without superscript to the other; $\omega_1, \omega_2, \dots, \omega_f$ are the zero order frequencies of the *f* (genuine) vibrations of the symmetry type considered; m_1, m_2, \dots are the masses of the representative atoms of the various sets (each set consisting of those identical atoms that are transformed into one another by the symmetry operations permitted by the molecule); α, β, \dots are the numbers of vibrations (inclusive of nongenuine vibrations) that each set contributes to the symmetry type considered; M is the total mass of the molecule; t is the number of translations of the symmetry type considered; I_x, I_y, I_z are the moments of inertia about the *x, y, and z* axes; $\delta x, \delta y, \delta z$ are 1 or 0 depending on whether or not the rotation about the *x, y, or z* axis is a nongenuine vibration of the symmetry type considered. Both on the left and right hand side (in $\alpha, \beta, \dots, t, \delta x, \delta y, \delta z$) a degenerate vibration is counted only once.

ISOTOPE EFFECT IN SUPERCONDUCTIVITY. The critical temperature T_c of superconductors varies with the isotopic mass M . The relation $T_c \propto M^{-1/2}$ suggests that the superconducting transition depends on the velocity of sound, a result predicted by the Fröhlich-Bardeen theory.

ISOTOPE MIXTURE, VALUE OF. A measure of the difficulty of preparing a quantity of an isotope mixture. It is proportional to the amount of the mixture, and is also a function of the composition of the mixture. The change in value created by a cascade is directly proportional to the number of separating elements it contains. For a binary mixture, the value of Q moles of mixture, taking the equimolar mixture as the reference state, is given by

$$Q(2N - 1) \ln R$$

when N is the mole fraction of either component, R is $N/1 - N$, the molecular abundance ratio.

When referred to an arbitrary mole fraction N_0 as the reference state the value of Q moles of mixture is

$$V = \left\{ (2N - 1) \ln \frac{N(1 - N_0)}{N_0(1 - N)} + \frac{(N - N_0)(1 - 2N_0)}{N_0(1 - N_0)} \right\} Q.$$

The total value created by a separating plant is the difference between the total value of all outgoing materials and the total value of all ingoing materials. This resultant is independent of N_0 .

ISOTOPE SEPARATION FACTOR. The ratio of the abundance ratio of two isotopes after processing to their abundance ratio before processing. It is given by the following equation:

$$r = \frac{n'_1/n'_2}{n_1/n_2}$$

where n_1 and n_2 are the mole fractions of isotopes of mass numbers m_1 and m_2 respectively, and n'_1 and n'_2 are the corresponding quantities after processing. The term "enrichment factor" is sometimes used for $r - 1$.

ISOTOPIC ABUNDANCE. The relative amount (expressed as number of atoms) of a particular isotope in a sample of an element.

ISOTOPIC ABUNDANCE, FRACTIONAL. The ratio of the number of atoms of a particular isotope to the total number of atoms of the element, both in a given sample. It is usually expressed as a percentage.

ISOTOPIC ABUNDANCE, RELATIVE. The number of atoms of a particular isotope relative to a specified number of atoms of a specified isotope, both in a given sample. Usually the meaning is more specific, as the number of atoms of a particular isotope relative to 100 atoms of the most abundant isotope, or to 1 atom of the least abundant isotope.

ISOTOPIC (ISOBARIC) NUMBER. Synonymous with **neutron excess**.

ISOTOPIC SPIN (ISOBARIC SPIN). A concept introduced by Heisenberg as a convenient device for the description of states of particles between which there are charge-independent forces. The isotopic spin is defined in analogy with the ordinary spin angular mo-

momentum of elementary particles, and is usually denoted by T . The isotopic spin of the nucleon (neutron and proton) is taken to be $\frac{1}{2}$: its two "magnetic" substates having $M_T = \pm \frac{1}{2}$ then refer to neutron and proton respectively. States of two nucleons may then be characterized by isotopic spin zero or unity, the former of which can only refer to a neutron and a proton since $M_T = 0$ necessarily in this case, while the latter can refer to a pair of neutrons ($M_T = +1$) or it can refer to a pair of protons ($M_T = -1$). If forces are charge-independent, they may depend upon T but not upon M_T . Similarly, one ascribes isotopic spin unity to the π -mesons whose substates with $M_T = \pm 1, 0$ then refer to the two charged, and the single uncharged, pions. One is then able to describe the charge independent interactions between pions and nucleons in a correspondingly concise manner. The concept of isotopic spin has been extremely useful in classifying the quantum states of light nuclei, and it is now proving of even greater significance in the theory of fundamental particles.

ISOTOPIC SPIN QUANTUM NUMBER.

See **quantum number, isobaric spin**.

ISOTROPIC. A medium M is isotropic with respect to a function f defined on M if f is a scalar field over M . If M is not isotropic, then M is anisotropic or anelotropic.

ISOTROPIC MATERIAL. A material in which different directions are indistinguishable by physical experiment. In elasticity theory, a material in which the elastic properties are independent of direction.

ISOTROPIC STATE OF STRESS (AT A POINT). A state of stress in an element of material at a point, such that the normal components of the Cartesian stress tensor at the point are all equal and the tangential components are all zero. Also called *spherical state of stress*. If the equal normal components are negative, the state of stress is called a *hydrostatic pressure*; if they are positive, *negative hydrostatic pressure* or *hydrostatic tension*. For more general states of stress at a point, the negative of the mean of the three normal components of the stress tensor at the point is called the *hydrostatic pressure at the point*. Alternatively, a system of surface forces acting on a body, everywhere directed

normally inwards to the surface and such that the surface force per unit area measured in the state of deformation existing is constant over the surface is called a *hydrostatic pressure*.

ISOTROPIC TENSOR. See **tensor, isotropic**.

ITERATED FISSION EXPECTATION. In a **critical assembly**, the value after large time of the number of fissions per generation time arising from the daughter neutrons of a given neutron. This is a specific normalization of the **importance function**.

ITERATED LOGARITHM, LAW OF. A theorem in probability to the effect that in a series of n binomial trials with probability p , the difference between the number of successes S and the expected number np is of the order $(2npq \log \log n)^{1/2}$ and tends to that quantity with probability unity as n tends to infinity.

ITERATION, METHOD OF. In **neutron transport theory**, a specific application of the general idea of constructing progressively more accurate solutions to an integral equation by iteration. In the case of the one-velocity integral equation it can be shown that an increase in accuracy does indeed occur. An initial trial function might be obtained from the diffusion approximation.

ITERATIVE IMPEDANCE. See **impedance, iterative**.

ITERATIVE METHODS (FOR SOLVING EQUATIONS, WHETHER ALGEBRAIC OR TRANSCENDENTAL). These methods are, in fact, methods of successive approximation in which, having given one or more approximations to a solution, it is used in computing an improved one. Only the case of a single equation in a single variable will be considered here (see **matrix inversion** and the solution of linear equations, and **simultaneous equations**; also Alston S. Householder, *Principles of Numerical Analysis*, McGraw-Hill Book Co., 1953, for further development of the theory).

If the equation to be solved is

$$f(x) = 0,$$

let

$$\phi(x) = x - g(x)f(x),$$

where throughout some region containing α , the root to be determined, $g(x)$ nowhere vanishes or becomes infinite. Then

$$\alpha = \phi(\alpha),$$

and if, for some x_0 in this region, every

$$x_{i+1} = \phi(x_i)$$

is again in the region and the sequence of x_i converges, it necessarily converges to a root. A sufficient condition for this is that

$$|\phi'(x)| \leq k < 1$$

at every point of the region. Moreover, if ϕ is analytic in some circle about x_0 , and if it can be shown that for some positive $k < 1$,

$$|\phi(x') - \phi(x'')| < k|x' - x''|,$$

whenever both x' and x'' are in the circle, then it can be concluded that every x_i will, in fact, fall within the circle and that the equation has a root α to which the sequence converges.

Newton's method is obtained with

$$g(x) = 1/f'(x),$$

and one is assured of convergence if, for real α ,

$$f(x_0)f''(x_0) > 0,$$

and neither f' nor f'' changes sign between α and x_0 .

J

JACOBIAN. Let F_1 and F_2 be two functions of u and v . Then

$$J = \begin{vmatrix} \frac{\partial F_1}{\partial u} & \frac{\partial F_1}{\partial v} \\ \frac{\partial F_2}{\partial u} & \frac{\partial F_2}{\partial v} \end{vmatrix} = \frac{\partial F_1}{\partial u} \frac{\partial F_2}{\partial v} - \frac{\partial F_1}{\partial v} \frac{\partial F_2}{\partial u}$$

is the functional **determinant** or Jacobian of F_1 and F_2 with respect to u and v . It is frequently denoted by $\partial(F_1, F_2)/\partial(u, v)$.

In general if F_1, F_2, \dots, F_n are functions of u_1, u_2, \dots, u_n , then

$$J = \frac{\partial(F_1, F_2, \dots, F_n)}{\partial(u_1, u_2, \dots, u_n)} = \begin{vmatrix} \frac{\partial F_1}{\partial u_1} & \frac{\partial F_1}{\partial u_2} & \dots & \frac{\partial F_1}{\partial u_n} \\ \frac{\partial F_2}{\partial u_1} & \frac{\partial F_2}{\partial u_2} & \dots & \frac{\partial F_2}{\partial u_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial F_n}{\partial u_1} & \frac{\partial F_n}{\partial u_2} & \dots & \frac{\partial F_n}{\partial u_n} \end{vmatrix}.$$

(See also **Hessian**.)

JACOBI METHOD. A method for computing **eigenvalues** of a **Hermitian matrix**.

JACOBI POLYNOMIALS. The class of **special functions** $J_n(\alpha, \beta, x)$ obtained by orthogonalizing over the interval $[-1, 1]$ the sequence of powers, $1, x, x^2, \dots$ with the weighting function $w(x) = (1-x)^\alpha (1+x)^\beta$, with $\alpha, \beta > -1$. (Thus, for $\alpha = \beta = 0$, they reduce to the **Legendre polynomials**.) They satisfy the differential equation

$$(1-x^2)y'' + [\beta - \alpha - (\alpha + \beta + 2)x]y' + n(\alpha + \beta + n + 1)y = 0.$$

Their **Rodrigues formula** is

$$J_n(\alpha, \beta, x) = \frac{(-1)^n}{2^n n!} (1-x)^{-\alpha} (1+x)^{-\beta} \frac{d^n}{dx^n} \times [(1-x)^{\alpha+a} (1+x)^{\beta+b}].$$

JACOBI THEOREM IN OPTICS. There is a general theorem of Jacobi that applies to the differential equation of geometrical optics: Let $V(x, y, z, a, b)$ be a complete integral of the equation

$$V_x^2 + V_y^2 + V_z^2 = n^2$$

where the subscript notation is used for partial derivatives. A complete integral is defined as a set of solutions depending on two parameters a, b and such that the matrix

$$\begin{bmatrix} V_{xa} & V_{ya} & V_{za} \\ V_{xb} & V_{yb} & V_{zb} \end{bmatrix}$$

is of rank two. Then the light rays of the medium of refractive index $n = n(x, y, z)$ are given by

$$\frac{\partial}{\partial a} V(x, y, z, a, b) = c_1, \quad \frac{\partial}{\partial b} V(x, y, z, a, b) = c_2$$

where c_1, c_2 are arbitrary constants.

JEANS LAW. See **black body radiation**.

JET, BENT OVER. A turbulent jet emerging into a smooth cross-wind gradually assumes the speed of the wind as it mixes with it and therefore becomes bent over. It becomes bifurcated so that it appears to consist of two contra-rotating parallel rolls with exterior fluid passing up between.

After becoming bent over the momentum in the direction of emission (z -axis) is still retained but the velocity and concentration decrease as z^{-2} , as it entrains exterior air.

If it possesses a significant amount of buoyancy and emerges vertically into a horizontal cross-wind, ultimately the upward momentum acquired from the buoyancy becomes dominant and the upward velocity decreases as $z^{-1/2}$ and the concentration as z^{-2} .

Bent over jets can also be formed when heavy or light fluid emerges horizontally into stationary fluid.

JET, BUOYANT. Sometimes called a *plume*, similar in appearance to a turbulent jet (see

jet, turbulent) but necessarily having a vertical axis. If the motion is produced by buoyancy and not significantly by a large efflux velocity the entrainment velocity and mean upward velocity are proportional to $z^{-1/2}$, while the concentration decreases as $z^{-5/2}$ in the axis-symmetric case, z being measured from the apex of the cone which envelopes the jet. In a two-dimensional plume rising from a horizontal line source of buoyancy up a wedge, the mean velocity is independent of height and the concentration decreases like z^{-1} . The plumes from two equal parallel horizontal line sources flow together if there is a rigid boundary below them and then rise as a single plume as if from a line midway between them. (See **correlation coefficient**.)

JET, TURBULENT. The configuration of motion produced by the emergence of fast moving fluid into otherwise stationary surroundings. In the case of a round orifice the motion becomes turbulent at the boundary of the issuing fluid and the region of turbulence grows outwards and inwards.

The region of smooth flow in the center, called the *potential core*, decreases conically until the turbulence reaches the axis. Exterior fluid is entrained by the mixing on the outside and is carried in the direction of the axis. There is therefore a velocity towards the axis of the jet in the exterior fluid.

From the place where the turbulence occupies the whole section the motion is similar at all stages. The turbulent region occupies a cone of semi-vertical angle about 9° . The inflow velocity, the mean velocity within the jet in the direction of its axis, and the mean turbulent velocity, is proportional to z^{-1} , z being measured from the apex of the cone enveloping the jet. This is proved from the assumption that the flux of momentum along the jet is constant.

The emerging fluid is diluted by mixing with the environment, and the concentration is proportional to z^{-1} .

The profiles of time-mean velocity and time-mean concentration across the jet are not identical but rather similar, the profile being almost Gaussian except at the edges.

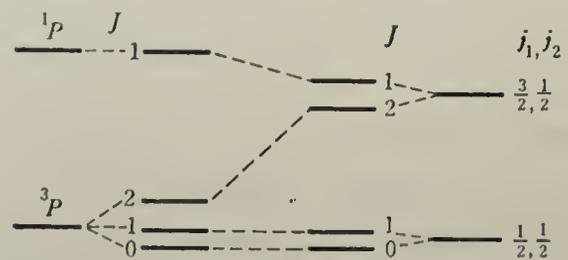
Plane jets have similar properties but the velocity and concentration are proportional to $z^{-1/2}$, z being measured from the edge of the enveloping wedge. (See **correlation coefficient**.)

JEZEK METHOD. An approximate solution of the instability problem for eccentrically-loaded columns, based on an elastic-perfectly plastic material and the assumption of a sine wave deflection. Jezek also gave an exact solution based upon the idealized stress-strain relation.

(j, j) COUPLING. Theoretical limiting case for the **coupling** of the spins s_i and the orbital angular momenta l_i of the individual electrons in an atom: strong coupling between the s_i and l_i of each individual electron, adding up (in two possible ways, see **angular momentum vectors, quantum theoretical addition of**) to the total angular momentum j_i of the individual electrons; weak coupling between the individual j_i , resulting in the total angular momentum J of magnitude $J(h/2\pi)$ of the atom. Written symbolically:

$$(l_1s_1)(l_2s_2)(l_3s_3) \cdots = (j_1j_2j_3 \cdots) = J.$$

Pure (j, j) coupling holds for some of the excited states of atoms of high **atomic number** Z , but occurs relatively seldom. More frequent are transitions cases between (j, j) coupling and **Russell-Saunders coupling**. The figure illustrates (for a *ps* electron configuration) the



Relative positions of the terms of a *ps* electron configuration. To the left, Russell-Saunders coupling; to the right, (j, j) coupling.

relative positions of the energy levels in the two limiting cases, and their correlation.

JOHANSEN THEORY. See **rupture line theory**.

JOHNSON AND LARK-HOROWITZ FORMULA. A formula for the resistivity of a metal or degenerate **semiconductor** due to the presence of impurities which scatter the electrons. It may be written

$$\rho = 6270n^{-1/2} \text{ ohm-cm}$$

where n is the number density of impurity atoms per cm^3 .

JOINT. The intersection or junction of members of a **truss** or a **frame** is called a joint. A *pinned joint* permits freedom of rotation and no moment at the ends of the members at the joint. A *rigid joint* permits no relative rotation between the members framing into the joint.

JOINT EFFICIENCY. This term refers to a strength ratio in connection with either riveted or welded joints. The joint efficiency is defined as the ratio of the strength of a section of the joint to the strength of an analogous section of solid plate. The joint efficiency, in pressure vessel design, usually requires an increase in the theoretical thickness of the plate which would otherwise be required for a seamless vessel. To illustrate, if the theoretical thickness of a seamless vessel, for a given set of conditions, is 1", the same vessel with a welded joint having an efficiency of 80% would require a wall thickness of 1¼". Welded joint efficiencies vary from 55 to 95%; riveted joint efficiencies from 45 to 95%, but most of them have values between 60 and 70%.

JOINTS, METHOD OF. The method of solving for the forces in the members of a **truss** by summing forces at each joint. If the number of joints is j there will be $2j$ independent joint equations for a plane truss and $3j$ for a space truss.

JORDAN CURVE. A simple, closed **curve**.

JORDAN-HÖLDER THEOREM. Any two **composition-series** of a **group** consist of the same number of subgroups, and lead to two sets of factor groups which, except for the order in which they occur, are identical with each other.

JORDAN MATRIX. A **matrix** whose elements are equal and non-zero along the principal diagonal, unity along the diagonal immediately above, and zero elsewhere.

JORDAN METHOD. A method of **matrix inversion** in which by a succession of row operations one reduces the given matrix to diagonal form.

JORDAN NORMAL FORM. See **Jordan matrix**.

JOST-LEHMANN REPRESENTATION. See **Dyson representation**.

JOUKOWSKI AIRFOILS. Two-dimensional **airfoil theory** is based mainly on the use of conformal mapping to transform the known solution for flow past a circular cylinder into flow past an airfoil. If z and ζ are the complex variables in the planes of the circle and the airfoil, the shape of the airfoil is defined by the mapping function $\zeta = f(z)$ and by the radius and position of the center of the circle in the z -plane.

Joukowski airfoils are those which are given by the simple mapping function

$$\zeta = z + \frac{c^2}{z} \quad (1)$$

The conditions of the transformation are chosen so that the airfoil has a sharp trailing edge, this being a singular point. The circulation round the airfoil is then determined by the **Joukowski condition**.

The family of Joukowski airfoils was the first to be studied theoretically. Because the airfoils have a cusp at the trailing edge, they are not really suitable for practical use, and are now mainly of historical interest.

Karman and Trefftz introduced a generalized Joukowski transformation, giving airfoils with a finite angle at the trailing edge. This is

$$\frac{\zeta + nc}{\zeta - nc} = \left(\frac{z + c}{z - c} \right)^n \quad (2)$$

The trailing-edge angle is then equal to $\pi(2 - n)$. The simple Joukowski transformation of Equation (1) is a special case of Equation (2), obtained by putting $n = 2$. (See H. Glauert, *The Elements of Aerofoil and Air-screw Theory* (2nd Edition), Cambridge, 1947.)

JOUKOWSKI CONDITION. In calculating the flow past a two-dimensional airfoil, results may be obtained for any arbitrarily chosen value of the circulation. For an airfoil with the usual sharp trailing edge, however, there is only one value of the circulation for which the theoretical velocity at the trailing edge is not infinite. Joukowski suggested that the circulation should be chosen to have this value; the circulation is then said to satisfy the Joukowski condition.

Experiments show that the circulation round an airfoil is slightly less than that given by the Joukowski condition, because of the effects of the boundary layers. The boundary layer near

the trailing edge of a lifting airfoil is usually thicker on the upper surface than on the lower, so that the circulation is that given by the Joukowski condition for an incidence slightly lower than the actual incidence. At high Reynolds numbers, when the boundary layers are very thin relative to the airfoil chord, the discrepancy is small and the Joukowski condition gives a good approximation to the circulation found in experiments.

JOULE. See **mechanical units**.

JOULE CYCLE. A quantity of ideal gas is taken through the following reversible processes: (a) from a pressure P_2 and volume V , compressed adiabatically to pressure P_1 ; (b) heated at constant pressure P_1 ; (c) expanded adiabatically to pressure P_2 ; (d) cooled at constant pressure P_2 to initial volume V .

The efficiency of a Joule engine is

$$1 - \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}},$$

where γ is the ratio of the specific heats.

JOULE LAW. See **perfect gas**.

JOULE-THOMSON COEFFICIENT. The ratio of the change in temperature to the change in pressure when a gas expands at constant **enthalpy** to a lower pressure through a small aperture or porous plug. Its mathematical form is therefore

$$\left(\frac{\partial T}{\partial p}\right)_H.$$

JOULE-THOMSON INVERSION TEMPERATURE. The temperature, or one of the two possible temperatures, at which the **Joule-Thomson coefficient** changes its sign for a given gas.

J-SHAPED DISTRIBUTION. A frequency function $f(x)$ which, when graphed with $f(x)$

as ordinate against x as abscissa, has the shape roughly of part of the letter J or its mirror image.

JULIAN DAY. A system of chronological reckoning by days has many advantages in the simplification of calculations which involve long periods of time with the resulting confusion in calendar dates. The "Julian Era" is arbitrarily set to begin at Greenwich Mean Noon January 1, 4713 B.C. Julian days are reckoned consecutively from that era. Fractions of a day are recorded on the decimal system from Greenwich Mean Noon. January 1, 1960, is JD 2,436,934.00000 at GMT noon. Julian Day numbers are tabulated in the *American Ephemeris and Nautical Almanac*, together with tables to facilitate the determination of the Julian Day Number for any date.

JUMP. To (conditionally or unconditionally) cause the next **instruction** to be selected from a specified storage location.

JUMP, CONDITIONAL. An **instruction** which will cause the proper one of two (or more) **addresses** to be used in obtaining the next instruction, depending upon some property of one or more numerical expressions or other conditions.

JUMP DISCONTINUITY. See **discontinuity**.

JUMP, UNCONDITIONAL. An **instruction** which interrupts the normal process of obtaining instructions in an ordered sequence, and specifies the **address** from which the next instruction must be taken.

JURY PROBLEM. A differential equation solved numerically by a **method of successive approximations** which fits the solution to given **boundary conditions**.

Elliptic equations, such as the Poisson equation, lead to jury problems. A partial differential equation, such as the barotropic vorticity equation, may combine a jury problem and a **marching problem**.

K

KÁRMÁN CONSTANT. An absolute numerical constant, k , relating the mean velocity U and distance z from a plane wall to the shear stress τ_o and density ρ in rectilinear steady turbulent flow parallel to it, according to

$$(\tau_o/\rho)^{1/2} = kz \frac{\partial U}{\partial z}.$$

It is required that the shear stress be independent of distance from the wall. The **velocity profile** is logarithmic.

To demonstrate the existence of this constant it is necessary to assume that the motion is similar at all distances from the wall (or, in **mixing length** terminology that the mixing length, or eddy size, is proportional to distance from the wall), and that the level of turbulence is in equilibrium with the pressure field producing the motion and that it is at large Reynolds number (viscous stresses negligible in the mean motion). $k = 0.4$ approximately, according to experiments.

KÁRMÁN-FRIEDRICH EQUATIONS. In the analysis of two-spar wings, a spar may be considered as a beam on elastic supports provided by the ribs. The ribs are assumed to have infinite bending stiffness but finite torsional stiffness. The resulting equations are of the **three-moment** type.

KÁRMÁN-TSIEN RELATION. A theoretical relationship of the Mach 1 pressure coefficient to the corresponding pressure coefficient at low speed. The relationship takes into account the compressibility effects of high speeds. It gives a better correlation with experimental findings of actual pressure coefficient than the Glauert theory. The pressure coefficient according to the Kármán-Tsien relation is:

$$C_p = \frac{C_{p0}}{\sqrt{1 - M_1^2} + \frac{1}{2}C_{p0}[1 - \sqrt{1 - M_1^2}]}$$

where C_p is the pressure coefficient at Mach 1, and C_{p0} is the low speed pressure coefficient. The above relation refers to two-dimensional flow.

KÁRMÁN VORTEX STREET. See **vortex street**.

KATABATIC WIND. A down-slope wind, usually a shallow layer (2–3 meters deep) of air, cooled by contact with the ground, draining down a hillside.

KATOPTICS. Variant spelling of **catoptics**.

KEESOM ENERGY. The average interaction energy of two dipoles at a distance r . For two dipoles in line, the interaction energy is given by $\pm 2\mu^2/r^3$ where μ is the dipole moment. The \pm sign depends on whether the dipoles are oriented in opposite or parallel directions on the line. For two dipoles in a three-dimensional space, the interaction energy is $(\mu^2/r^3)f$, where f is a factor depending on the mutual orientation. For a **Boltzmann distribution**, if $\mu^2/r^3 \ll kT$, the average interaction energy of the two dipoles is

$$\epsilon_K = -\frac{2}{3} \frac{1}{r^6} \frac{\mu^4}{kT}.$$

ϵ_K is the Keesom energy.

At very high temperatures, the orientation becomes completely disordered, and ϵ_K becomes 0.

KEESOM POTENTIAL. This intermolecular potential applies to **rigid spheres** containing a **point dipole** (or a symmetrical quadrupole). Its analytical expression is

$$\phi(r, \theta_a, \theta_b, \phi_b - \phi_a) = \infty \quad r < \sigma \quad (1)$$

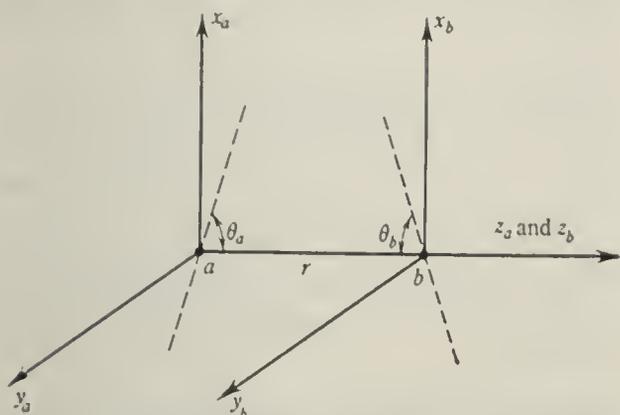
$$= -\frac{\mu_a \mu_b}{r^3} g(\theta_a, \theta_b, \phi_b - \phi_a) \quad r > \sigma \quad (2)$$

where

$$\begin{aligned} g(\theta_a, \theta_b, \phi_b - \phi_a) \\ = 2 \cos \theta_a \cos \theta_b - \sin \theta_a \sin \theta_b \cos(\phi_b - \phi_a). \end{aligned} \quad (3)$$

The figure represents two rigid spheres (e.g., molecules) having the centers at a and b . The axes of their components of moment, μ_a and μ_b ,

are indicated by the dashed lines shown at angles to the z -axis of θ_a and θ_b , respectively. The other two coordinates in the derivation,



Keesom potential.

ϕ_a and ϕ_b , define the corresponding axis orientations about the z -axis. The distance r between a and b is the distance between the centers of the molecules, or effectively the internuclear distance.

KEKULÉ STRUCTURE. See benzene, structures of.

KELLOGG EQUATION. An equation of state, relating the pressure, absolute temperature, and density of a gas. It is of the form:

$$p = RT\rho + \left(B_0RT - A_0 - \frac{C_0}{T^2} \right) \rho^2 + \left(bRT - a - \frac{c}{T^2} \right) \rho^3$$

in which p is the pressure, T the absolute temperature, ρ the density, R the gas constant, and A_0 , B_0 , C_0 , a , b , and c are constants.

KELVIN CIRCULATION THEOREM. See circulation theorem.

KELVIN EQUATION FOR SURFACE TENSION.

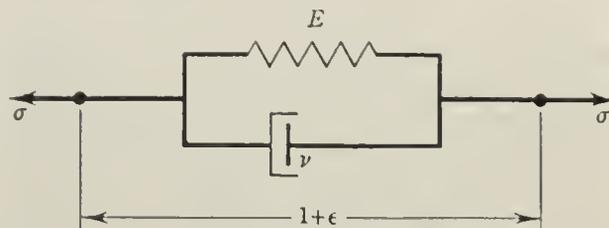
$$U_s = \gamma - T \frac{d\gamma}{dT}$$

where U_s is the surface energy per unit area, γ is the surface tension, T is absolute temperature.

KELVIN FORMULA FOR THE VAPOR PRESSURE OF SMALL DROPS. See influence of curvature on vapor pressure.

KELVIN FORMULATION OF THE SECOND LAW OF THERMODYNAMICS. See Planck-Kelvin formulation of the second law of thermodynamics.

KELVIN MATERIAL. Material whose mechanical behavior exhibits delay elasticity only. It is represented by the Kelvin model:



In the linear case the constitutive equation takes the form:

$$\sigma = E\epsilon + \nu\dot{\epsilon}.$$

(See viscoelasticity models.)

KELVIN MODEL. See Kelvin material for the basic Kelvin or Voigt model. More general viscoelastic materials can be represented by a sequence of Kelvin models in series with a spring and a dashpot, and this is sometimes referred to as the general Kelvin model. (See delay time for properties of this model.)

KELVIN TEMPERATURE SCALE. See temperature; thermodynamics, second law in closed systems.

KEPLER EQUATION. See anomaly, true and eccentric.

KEPLER LAWS. After much study of available data, Kepler drew three conclusions about the motion of the planets: (1) The orbit of each planet is an ellipse with the sun at one focus. (2) The radius vector drawn from the sun to the planet sweeps out equal areas in equal times. (3) The square of the period of a planet is proportional to the cube of the major axis of the elliptical orbit. Newton deduced the law of gravitation from Kepler's laws. (For further details see planetary motions.)

KERN (OR KERNEL OR CORE). A straight bar loaded by an eccentric force P parallel to its axis will bend as well as stretch or compress. The distribution of strain over the cross section is linear. In the elastic range the stress, too, is linearly distributed

$$\sigma = \frac{P}{A} + \frac{Pe_x y}{I_x} + \frac{Pe_y x}{I_y}$$

where x and y are principal axes and e_x and e_y are the eccentricities of P about the x and y -axes, respectively. The limits of the combination of e_x, e_y for the stress to have the same sign at each point of the cross section defines the kern. A compressive force within the kern produces no tension.

KERNEL. A known function, also called a **nucleus**, $K(x,z)$ which occurs in an **integral equation**. It is **symmetric** if $K(x,z) = K(z,x)$; **Hermitian** if $K(x,z) = K^*(z,x)$; **skew Hermitian** if $K(x,z) = -K^*(z,x)$, where the asterisk indicates the **complex conjugate**. A **polar kernel** has the form $K(x,z) = u(z)G(x,z)$, where $G(x,z)$ is symmetric. It may be transformed into a symmetric kernel by a change of the dependent variable. A **definite kernel**, positive or negative, satisfies the requirement

$$\int f(x)dx \int K(x,z)f(z)dz \geq 0,$$

where $f(x)$ is any finite function defined over the range for which the kernel is defined. A **singular kernel** has one or more **singular** points within its limits of integration. A kernel of the form

$$K(x,z) = \sum_{i=1}^k f_i(x)g_i(z)$$

is called degenerate. A solving kernel or resolvent is an infinite series of iterated kernels which appears in the **Liouville-Neumann series**. (See also **integral transform**.) (For use of the term kernel in structural engineering, see **kern**.)

KERNEL, DIFFUSION. In the diffusion of neutrons, these kernels may refer to point, line, plane, etc., sources. As an example, the point kernel

$$K_p(\mathbf{r},\mathbf{r}') = \frac{\exp\{-\kappa|\mathbf{r}-\mathbf{r}'|\}}{4\pi D|\mathbf{r}-\mathbf{r}'|}$$

with D being the diffusion constant and κ , the reciprocal of diffusion length, gives the monoenergetic neutron flux at \mathbf{r} due to unit source at \mathbf{r}' .

KERNEL, GAUSSIAN. In reactor theory, the Gaussian kernel,

$$K(\mathbf{r},\mathbf{r}') = (4\pi\tau)^{-3/2} \exp\left\{-\frac{|\mathbf{r}-\mathbf{r}'|^2}{4\tau}\right\}$$

with τ the age-to-thermal characteristic of the slowing-down medium being often used for a slowing-down kernel. It may be derived directly from Fermi age theory. (See **age equation, Fermi**.)

KERNEL, MULTIGROUP. In the multigroup formulation of the slowing-down of neutrons, in which neutrons, distributed continuously with respect to energy are assigned to a finite number of energy groups, the group kernel $K_j(\mathbf{r},\mathbf{r}')$ denotes the slowing-down kernel from point \mathbf{r}' in energy group $j-1$ to point \mathbf{r} in group j .

KERNEL, POINT. Of particular interest in the study of the penetration of radiation through matter (shields) this kernel, denoted $G(R)$ gives the total flux of radiation at a point separated by a distance R from a point source which emits one particle per second.

KERNEL, SCATTERING. In scattering theory, the microscopic cross-section describing the scattering of neutrons from energy E to E' through an angle θ . Often denoted by $\sigma_s(E \rightarrow E', \theta)$, the quantity, or its average over angle, acts as kernel in the transport equation describing the slowing down of neutrons.

KERNEL, SLOWING DOWN. In reactor theory, $K(\mathbf{r},E; \mathbf{r}',E')$, the flux of neutrons of energy E at point \mathbf{r} due to a unit point source of neutrons of energy E' at \mathbf{r}' is a quite general slowing-down kernel. Its average, $\bar{K}(\mathbf{r},\mathbf{r}')$, representing the probability that a fission neutron born at \mathbf{r}' , becomes thermal at \mathbf{r} , is often called the slowing-down kernel.

KERNEL, SYNTHETIC. An *ad-hoc* slowing-down kernel for neutrons, constructed to agree with experimental results. It is often expressed as the convolution of a number of multigroup kernels. (See **kernel, multigroup**.)

KERNEL, TRANSPORT. In the transport theory of monoenergetic neutrons, the transport kernel,

$$K(\mathbf{r},\mathbf{r}') = \sum_s \frac{\exp\{-\Sigma|\mathbf{r}-\mathbf{r}'|\}}{4\pi|\mathbf{r}-\mathbf{r}'|^2}$$

gives the flux of neutrons at \mathbf{r} which have suffered n collisions due to unit flux at \mathbf{r}' of neutrons which have suffered $n-1$ collisions.

KERR ELECTRO-OPTICAL EFFECT AND LAW. This effect occurs when an isotropic substance (e.g., nitrobenzene) becomes anisotropic in the presence of an electric field. Kerr's electro-optical law then applies: if n_p and n_s are the refractive radius for light polarized in planes parallel and perpendicular, respectively, to the electric vector of intensity E , then

$$n_p - n_s = B\lambda E^2$$

where B is the Kerr electro-optical constant for the given substance, wavelength and temperature.

KERR MAGNETO-OPTICAL EFFECT. See magneto-optical effect.

KET VECTOR. Vector in Hilbert space describing the state of a dynamical system in quantum mechanics. Denoted by the symbol $|A\rangle$. (See also bra vector.)

KEV. Symbol for one thousand electron volts, a unit of energy equal to 1.602×10^{-9} erg.

KEYES EQUATION. An equation of state for a gas, deduced from the concept of the nuclear atom. This equation is designed to correct the van der Waals equation for the effect upon the term b of the surrounding molecules. The equation is written as

$$P = \frac{RT}{V - Be^{-\alpha_l V}} - \frac{A}{(V + l)^2}$$

in which P is pressure, T is absolute temperature, V is volume, R is the gas constant, e is the base of natural logarithms, 2.718..., and A , α , B , and l are constants for each gas.

KEYES-SMITH-GERRY EQUATION. An empirical (thermal) equation of state for steam. It is given below in a dimensionally correct form, after E. Schmidt:

$$v = \frac{RT}{p} + B \left[1 + \left(\frac{Bp}{RT} \right) f_1(T) + \left(\frac{Bp}{RT} \right)^3 f_2(T) - \left(\frac{Bp}{RT} \right)^{12} f_3(T) \right]$$

where

$$R = 47.063 \frac{\text{kp m}}{\text{kg C}} = 85.78 \frac{\text{ft lbf}}{\text{lbm F}}$$

$$B = \left[1.890 - \frac{2641.6 \text{ C}}{T} \times 10^{(284.38 \text{ C}/T)} \right] \times 10^{-3} \frac{\text{m}^3}{\text{kg}}$$

$$= \left[3.0274 - \frac{7616.4 \text{ F}}{T} \times 10^{(511.88 \text{ F}/T)} \right] \times 10^{-2} \frac{\text{ft}^3}{\text{lbm}}$$

$$f_1(T) = \frac{367.00 \text{ C}}{T} - 0.7400 \left(\frac{1000 \text{ C}}{T} \right)^2$$

$$= \frac{676.80 \text{ F}}{T} - 0.7400 \left(\frac{1800 \text{ F}}{T} \right)^2$$

$$f_2(T) = 20.630 - 12.000 \left(\frac{1000 \text{ C}}{T} \right)^2$$

$$= 20.630 - 12.000 \left(\frac{1800 \text{ F}}{T} \right)^2$$

$$f_3(T) = 28994 - 5.398 \left(\frac{1000 \text{ C}}{T} \right)^{24}$$

$$= 28994 - 5.398 \left(\frac{1800 \text{ F}}{T} \right)^{24}$$

(See also steam tables.)

KILOMOLE. That quantity of substance whose mass expressed in kilograms is numerically equal to the molecular weight (mass) of the substance.

KILOPOUND. See kip.

KIND OF TENSOR. See tensors, type of.

KINEMATICALLY ADMISSIBLE. A term applied to pattern of deformation or rate of deformation which is geometrically permissible. In plastic limit analysis the deformations are taken as plastic only and may be discontinuous. In elastic analysis, kinematically admissible fields of displacement and corresponding strains are required for the theorem of minimum potential energy.

KINEMATIC BOUNDARY CONDITION. The condition that the fluid velocity directed perpendicular to a solid boundary must vanish on the boundary itself. This may be stated mathematically by the expression

$$\mathbf{n} \cdot \mathbf{V} = 0,$$

where \mathbf{n} is a unit vector normal to a solid surface, and \mathbf{V} is the fluid velocity vector. In meteorology, this **boundary condition** is often employed in considering flow near the earth's surface.

When the boundary is a fluid surface or **interface**, this condition applies to the vector difference of velocities across the interface and requires that the interface, although in motion, will at all times consist of the same fluid parcels. In meteorology, such a condition must be applied at fronts and other surfaces of discontinuity.

(See also **dynamic boundary condition**.)

KINEMATICS. The geometry of motion, without regard to forces or bodies of matter. Two of the aspects given special consideration are: the motion of points and the motion of rigid figures. Whatever system of space coordinates is found simplest may be used, and may be transformed from one system to another as desired.

The motion of a point may be completely specified by giving each of its three rectangular coordinates (for example) as a function of the time; that is, by writing its "equations of motion":

$$\left. \begin{aligned} x &= f_1(t), \\ y &= f_2(t), \\ z &= f_3(t). \end{aligned} \right\} \quad (1)$$

KINEMATIC VISCOSITY. See **absolute viscosity**.

KINETIC ENERGY. See **energy, kinetic**.

KINETIC ENERGY FLUX VECTOR. See **flux vector**.

KINETIC ENERGY, NEGATIVE. See **energy, negative kinetic**.

KINETIC ENERGY, OF IRROTATIONAL MOTION. If ϕ is the velocity potential then the kinetic energy, T , of the fluid contained within a bounding surface S is

$$T = \frac{1}{2}\rho \int \phi \operatorname{grad} \phi \cdot dS = \frac{1}{2}\rho \int \phi \frac{\partial \phi}{\partial n} dS.$$

In two dimensional motion with complex potential $\phi + i\psi$,

$$T = \frac{1}{2}\rho \int \phi d\psi$$

per unit length, the integral being taken round the boundary.

The *minimal theorem* of Kelvin states that for an incompressible fluid of uniform density the kinetic energy of the actual (irrotational) motion (satisfying $\operatorname{div} \mathbf{v} = 0$ and $\operatorname{curl} \mathbf{v} = 0$) is less than that of any other motion (not irrotational) satisfying the same boundary conditions.

KINETIC ENERGY OF RIGID BODY. The kinetic energy T of rigid body consists of two parts T' and T'' , i.e., $T = T' + T''$ where T' is the translational energy and T'' , the rotational energy.

$$T' = \frac{1}{2}m\mathbf{v}_o^2$$

where \mathbf{v}_o is the velocity of the center of mass and

$$T'' = \frac{1}{2}(A\omega_1^2 + B\omega_2^2 + C\omega_3^2)$$

where A, B, C are the principal moments of inertia and $\omega_1, \omega_2, \omega_3$ the components of the angular velocity $\boldsymbol{\omega}_o$ along the directions of A, B, C .

KINETIC MOMENTUM. Kinetic momentum of a charged particle in an electromagnetic field \mathbf{A}, ϕ , the vector $\mathbf{p} - (e/c)\mathbf{A}$, where \mathbf{p} is the momentum. Name arises from analogy with the kinetic energy $W - e\phi$, where W is the total energy.

KINETIC POTENTIAL. See **Lagrangian function**.

KINETICS. The science dealing with the effects of forces and moments on the motion of material bodies.

KINETICS, REACTOR. In reactor theory, the study of the time behavior of a chain-reacting system. If the diffusion approximation to neutron transport theory be used, the relevant time-dependent differential equations are:

$$\begin{aligned} D\nabla^2\Phi + [(1 - \beta)k_\infty P_\infty(B^2) - 1] \Sigma_a \Phi \\ + pP_\infty(B^2) \sum_{i=1}^m \lambda_i C_i = \frac{1}{v} \frac{\partial \Phi}{\partial t} \\ \frac{\partial C_i}{\partial t} = -\lambda_i C_i + \frac{k_\infty}{p} \beta_i \Sigma_a \Phi, \quad i = 1, \dots, m. \end{aligned}$$

The first equation expresses the diffusion of thermal neutrons (one-group theory, speed = v) in a medium characterized by diffusion

constant D , absorption Σ_a , infinite multiplication k_∞ , non-leakage probability (during slowing down) $P_\infty(B^2)$ and resonance escape probability p . The second equation gives the time variation of concentration of the i^{th} type of excited nucleus which acts as a source (“precursor”) of delayed neutrons. $\beta = \sum_i^m \beta_i$

is the total fraction of fission neutrons which are delayed and λ_i is the decay constant for i^{th} type of precursor.

KINETIC THEORY. A theory which explains the phenomena in gases described macroscopically by thermodynamics as due to the kinetic motion and collisions of atoms and molecules.

First introduced by Krönig and Clausius to explain the **perfect gas law**, it can be extended to furnish expressions for the **viscosity, thermal conductivity, diffusion coefficient**, and other transport coefficients of gases. Its natural extension is **statistical mechanics** where the attempt to deduce properties of matter in bulk from atomic properties is brought to its logical conclusion and is made applicable to liquids and solids as well as gases.

To derive the perfect gas law in kinetic theory one defines the pressure in a gas as the force per unit area, and thus, by Newton’s second law, as the (net) transport of momentum through a unit area. In this way one obtains

$$p = \frac{1}{3}nmv^2,$$

where p is the pressure, n the number of particles per unit volume, m their mass, and v their **root mean square velocity**.

Similarly one can derive **van der Waals’ equation** for an **imperfect gas**. (See also **viscosity, kinetic theory of; diffusion, kinetic theory of; and thermal conductivity, kinetic theory of**.)

KINETIC THEORY OF LIQUIDS. For many purposes liquids can be treated as if they were dense gases, and **kinetic theory** can be applied for such problems as the (kinetic) viscosity of liquids, the thermal conductivity of liquids, and diffusion through liquids.

KINETIC THEORY OF VISCOSITY. See viscosity, kinetic theory of.

KINETIC THEORY TEMPERATURE. Temperatures defined by the ratio of the aver-

age (random) kinetic energy of a (point) particle to $\frac{3}{2}k$ (k is Boltzmann’s constant).

KIP (OR KILOPOUND). One thousand pounds.

KIRCHHOFF EQUATION FOR THE HEAT OF REACTION. See Clausius equations.

KIRCHHOFF FORMULATION OF HUYGENS PRINCIPLE. Two theorems, due to Kirchhoff, contain a formulation of **Huygens’ principle** that generalize **Helmholtz’s formulation** for monochromatic wave-functions to heterochromatic wave-functions.

(1) If $u(x, y, z, t)$ is a wave-function with continuous first and second partial derivatives inside and on a closed surface S and (x_1, y_1, z_1) is a point inside S , then

$$4\pi u(x_1, y_1, z_1, t) = \iint_S \left\{ [u] \frac{\partial}{\partial n} \left(\frac{1}{r} \right) - \frac{1}{cr} \frac{\partial r}{\partial n} \left[\frac{\partial u}{\partial t} \right] - \frac{1}{r} \left[\frac{\partial u}{\partial n} \right] \right\} dS$$

where r is the distance from (x_1, y_1, z_1) to a generic point on S , $\partial/\partial n$ denotes differentiation along the inward normal to S , and

$$[f(x, y, z, t)] = f \left(x, y, z, t - \frac{r}{c} \right).$$

If, however, (x_1, y_1, z_1) is outside S , then the integral is zero.

(2) If $u(x, y, z, t)$ is a wave function having no singularities outside a closed surface S for all t' , $-\infty < t' < t$, and if u behaves like $F(ct - R)/R$ at large distances R from the origin, where $F(ct)$ and $F'(ct)$ are bounded near $t = -\infty$, then the above integral formula holds for points (x_1, y_1, z_1) outside S provided that $\partial/\partial n$ means differentiation along the outward normal to S . If (x_1, y_1, z_1) is outside S , in this case, the integral is zero.

KIRCHHOFF GEOMETRIC OPTICS THEOREM. A theorem due to Kirchhoff stating that geometrical optics is a limiting form of physical optics. More precisely, the diffuse boundary of the shadow in diffraction phenomena becomes the sharp shadow of geometrical optics as the wavelength of light tends to zero.

KIRCHHOFF LAW OF RADIATION. The monochromatic **emissivity** of a surface at temperature T is equal to its monochromatic **ab-**

sorptivity. It is a consequence of the second law of thermodynamics.

KIRCHHOFF LAWS OF NETWORKS. Two laws relating to electric networks carrying steady currents. The general case is that of n points or junctions, each one of which is connected with each of the $n - 1$ remaining points by a conductor containing a source of electromotive force. Kirchhoff's two statements are as follows:

(1) If conductors forming part of a network carrying a steady current meet at one point, the sum of the currents flowing toward the point is equal to the sum of those flowing away from it; or the algebraic sum of all the currents in these conductors is zero.

(2) Starting at any one of the junctions of such a network and following any succession of the conductors which form a closed path, around either way to the starting point, the algebraic sum of the products formed by multiplying the resistance of each conductor by the current through it is equal to the algebraic sum of the electromotive forces encountered on the journey. (In this reckoning, we call all currents moving with us positive, and all electromotive forces tending to cause such currents positive.)

Maxwell has set forth a general method of calculating the currents and the relative potentials of the junctions when the resistances and electromotive forces in the several branches of a network are given. For a network of n points, this method involves the solution of $n - 1$ simultaneous, first-degree equations. The work is often simplified, however, by the circumstance that some of the conductors or some of the electromotive forces are absent. (3) The Kirchhoff laws also hold when alternating rather than direct currents are flowing in circuits if the instantaneous currents and voltages are used. They do not necessarily hold for the average currents and voltages, since for circuit elements other than resistances (i.e., capacitance and inductance) the current and voltage are not in phase. Formally, if complex voltages, complex currents and complex impedances are used, with $V = IZ$, the Kirchhoff laws are applicable to alternating current circuits.

KIRKWOOD APPROXIMATION. An approximation used in the kinetic theory of liquids. The force on one molecule of a set

of molecules is assumed to be the sum of the forces exerted, neglecting in turn all but one other molecule of the set. In this way an equation may be obtained for the radial distribution function.

KIRKWOOD EQUATION FOR THE DIELECTRIC CONSTANT. Kirkwood has shown that for quasi-spherical molecules of dipole moment μ , whose polarizability is isotropic, the static dielectric constant ϵ_s is given by:

$$\epsilon_s - n^2 = \frac{3\epsilon_s}{2\epsilon_s + n^2} \frac{4\pi N_o \mu \cdot \mu^*}{3kT}$$

where n is the index of refraction of the molecule, N_o , the concentration in molecules per cubic centimeter, T , the absolute temperature, k , the Boltzmann constant, μ the dipole moment of a given dipole, and μ^* the average dipole moment of the neighboring dipoles, when this dipole is maintained in a fixed position.

If nearest neighbor interactions only are considered, the scalar product

$$\mu \cdot \mu^* = \mu^2 (1 + z \overline{\cos \gamma})$$

where μ is the magnitude of the dipole moment; z , the average number of nearest neighbors, and $\overline{\cos \gamma}$ is the average value of the cosine of the angle between the molecule and its neighbors.

In this case, the Kirkwood formula can be written in a form which is very similar to the **Onsager equation** (which neglects the mutual orientation of the dipoles):

$$\epsilon_s - n^2 = \frac{3\epsilon_s}{2\epsilon_s + n^2} \left(\frac{n^2 + 2}{3kT} \right) \frac{4\pi N_o \mu_v^2}{3kT} (1 + z \overline{\cos \gamma}).$$

μ_v is the magnitude of the dipole moment when the molecule is in a vacuum.

The $\overline{\cos \gamma}$ of Equation (2) depends on the detailed interactions between neighboring dipoles. If these tend to orient the molecular dipoles in parallel directions, the dielectric constant will be larger than the value predicted by the Onsager equation. In the non-parallel case, it will be smaller.

KLEIN-GORDON EQUATION. The relativistic equation which describes the motion of a (scalar or pseudoscalar) spinless particle, of mass m . It is derived by inserting the operator substitutions $E \rightarrow i\hbar\partial_t$, $\mathbf{p} = -i\hbar\nabla$ into the

relativistic relation between the energy and momentum for a free particle

$$E^2 = c^2 \mathbf{p}^2 + m^2 c^4 \quad (\text{a})$$

where m is the mass of the particle. This procedure then yields the equation

$$-\hbar^2 \frac{\partial^2 \phi(x)}{\partial t^2} = (-\hbar^2 c^2 \nabla^2 + \mu^2 c^4) \phi(x) \quad (\text{b})$$

or in relativistic notation

$$\left(\square + \frac{m^2 c^2}{\hbar^2} \right) \phi(x) = 0 \quad (\text{c})$$

(where $\square = \partial_\mu \partial^\mu$, $\partial^\mu = (c \partial_t, \partial_x, \partial_y, \partial_z)$ denotes the d'Alembertian). Equation (b) and (c) is known as the Klein-Gordon equation. The amplitude $\phi(x)$ is a one-component scalar quantity which under an **inhomogeneous Lorentz transformation**

$$x' = \Lambda x + a$$

transforms according to

$$\phi(x) \rightarrow \phi'(x') = \phi(x).$$

The amplitude ϕ describes a scalar particle if under a spatial inversion $\mathbf{x} \rightarrow -\mathbf{x}$, $x_0 \rightarrow x_0$, $\phi \rightarrow \phi$. If on the other hand, under a spatial inversion, $\phi \rightarrow -\phi$, ϕ is said to describe a pseudoscalar particle.

A consistent interpretation of the Klein-Gordon equation as the equation describing a spinless particle of mass m results if one adopts the manifold of positive energy solutions as the set of states which are physically realizable. This vector space is made into a Hilbert space by defining the scalar product of his vectors as

$$(f, g) = i \int_\sigma d\sigma_\mu(x) \left\{ \frac{\partial \bar{f}(x)}{\partial x_\mu} g(x) - \bar{f}(x) \frac{\partial g(x)}{\partial x_\mu} \right\}.$$

This scalar product does not depend on the space-like surface σ for f and g which obey the Klein-Gordon equation.

KLEIN-NISHINA FORMULA. (1) Expression derived from the Dirac electron theory without radiative corrections for the differential cross section for scattering by an electron at rest of a quantum with momentum \mathbf{k}_0 to give a quantum with momentum \mathbf{k} in the element of solid angle $d\Omega$:

$$d\phi = \frac{r_0^2}{4} d\Omega \frac{k^2}{k_0^2} \left(\frac{k_0}{k} + \frac{k}{k_0} - 2 + 4 \cos^2 \Theta \right)$$

(Θ = angle between directions of polarization of \mathbf{k} and \mathbf{k}_0).

(2) The integral Klein-Nishina formula gives the total cross section for scattering, not being restricted to a solid angle.

KLEIN-RYDBERG METHOD. Method introduced by O. Klein (*Z. Phys.* **76**, 226, (1932)) and R. Rydberg (*Z. Phys.* **73**, 376 (1932); **80**, 514 (1933)) for constructing the potential curve (see **potential functions of molecules**) of a diatomic molecule point by point from the observed vibrational and rotational levels without assuming an analytical expression for the potential function. The exact curves obtained in this way are generally fairly closely approximated by the simple **Morse curve**.

KNEE-FREQUENCY. See **frequency response representation**.

KNUDSEN COSINE LAW. It can be shown from kinetic theory that for a gas at rest at a uniform temperature, the number of molecules striking or leaving an area dS of the wall in a solid angle $d\omega$ making an angle θ with the normal is given by

$$\frac{dS}{4\pi} n \bar{c} \cos \theta d\omega,$$

where n is the number of molecules per cm^3 , and \bar{c} the mean velocity. This is known as the cosine law. Knudsen assumed that it holds for each molecule individually, that the direction of a molecule on leaving the wall is independent of its direction before striking, and that the law gives the probability of a molecule leaving the wall in a given direction. The law is probably true for irregular surfaces, but does not hold in such cases as that of the diffraction of molecules from crystal lattices, which have a regular arrangement.

KNUDSEN GAS. At very low densities, the mean free path for molecular collisions is quite large. The molecules enclosed in a container move on linear trajectories, and suffer many more collisions with the walls than between themselves. The transport properties (diffusion, viscosity, thermal conductivity) of such gases depend essentially on the collisions with the walls, and not on the gaseous collisions. The properties of these gases have been studied extensively by M. Knudsen, and a gas which is placed under experimental conditions

such that the *mean free path of its molecules is much larger than the dimensions of the measuring apparatus* is called a *Knudsen gas*.

KNUDSEN NUMBER. The ratio of the mean free path of the molecules of a gas to a length characteristic of the flow field. For values of the Knudsen number less than about 0.01, the gas may be treated as a continuum. (See also *rarefied gas dynamics*.)

KOCH EQUATION. An empirical (thermal) equation of state for steam:

$$v = \frac{RT}{p} - \frac{A}{(T/D)^{2.82}} - p^2 \frac{B}{(T/D)^{14}} + \frac{C}{(T/D)^{31.6}}$$

The equation is dimensionally correct, and the constants have the following values:

$$R = 47.06 \frac{\text{kp m}}{\text{kg } ^\circ\text{C}} = 85.77 \frac{\text{ft lbf}}{\text{lbm } ^\circ\text{F}}$$

$$A = 0.09172 \frac{\text{m}^3}{\text{kg}} = 14.692 \frac{\text{ft}^3}{\text{lbm}}$$

$$B = 1.3088 \times 10^{-4} \frac{\text{m}^3}{\text{kg}} \left(\frac{\text{m}^2}{\text{kp}} \right)^2 \\ = 1036.6 \frac{\text{ft}^3}{\text{lbm}} \left(\frac{\text{in}^2}{\text{lbf}} \right)^2$$

$$C = 4.379 \times 10^7 \frac{\text{m}^3}{\text{kg}} \left(\frac{\text{m}^2}{\text{kg}} \right)^2 \\ = 3.467 \times 10^{14} \frac{\text{ft}^3}{\text{lbm}} \left(\frac{\text{in}^2}{\text{lbf}} \right)^2$$

$$D = 100^\circ\text{C} = 180^\circ\text{F}.$$

The equation breaks down in the neighborhood of the **critical point**. (See **steam tables**.)

KOLMOGOROFF SIMILARITY HYPOTHESES. (Also called local similarity hypotheses, universal equilibrium hypotheses.) Statements of the factors determining the transfer and dissipation of **kinetic energy** at the high wave-number end of the spectrum of turbulence.

(1) At large **Reynolds numbers** the local average properties of the small-scale components of any turbulent motion are determined entirely by **kinematic viscosity** and average rate of **dissipation** per unit mass.

(2) There is an upper subrange (the *iner-*

tial subrange) in this bandwidth of small eddies in which the local average properties are determined only by the rate of dissipation per unit mass.

KRONECKER DELTA. In Cartesian tensor analysis, the function δ_{ij} , usually defined for positive integral i and j , such that $\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$.

KRONECKER DELTA, GENERALIZED (IN n DIMENSIONS). The system $\delta_{j_1 j_2 \dots j_k}^{i_1 i_2 \dots i_k}$, defined as having the values 1 or -1 when the k superscripts are distinct numbers chosen from 1 to n and the k subscripts form an even or odd permutation respectively of the superscripts, and as having the value 0 in all other cases.

Often denoted δ_{ij} in Cartesian tensor analysis or in contexts other than those of tensor analysis.

KRONIG-PENNEY MODEL. A one-dimensional periodic potential for which the wave equation may be solved exactly, and shows **band structure**, thus providing a mathematical model in which certain properties of metals may be verified by rigorous calculations.

KRYLOV METHOD. See **eigenvalues and eigenvectors**.

k-SPACE. An abbreviation for momentum space, or **wave-vector** space, i.e., the space mapped out by the wave-vectors \mathbf{k} , e.g., of the various electronic **wave functions** in a metallic crystal. It has the same symmetry properties as the reciprocal lattice of the crystal.

k-STATISTICS. The p th k -statistic of a **sample** is defined to be the unique polynomial, symmetric in the observations, whose expectation is κ_p , the p th **cumulant**. The k -statistics are readily computed from the moments or from the sums of powers of the observations—thus, in a sample of n

$$k_1 = s_1/n = m_1$$

$$k_2 = (ns_2 - s_1^2)/n(n - 1) = nm_2/(n - 1)$$

$$k_3 = (n^2s_3 - 3ns_2s_1 + 2s_1^3)/n(n - 1)(n - 2) \\ = n^2m_3/(n - 1)(n - 2)$$

$$\begin{aligned}
 k_4 &= ((n^3 + n^2)s_4 - 4(n^2 + n)s_3s_1 \\
 &\quad - 3(n^2 - n)s_2^2 + 12ns_2s_1^2 \\
 &\quad - 6s_1^4)/n(n-1)(n-2)(n-3) \\
 &= n^2((n+1)m_4 - 3(n-1)m_2^2)/(n-1) \\
 &\quad \times (n-2)(n-3)
 \end{aligned}$$

where $s_p = \Sigma x^p$, m_1 is the mean and m_2, m_3, m_4 the moments about the mean. The sampling properties of the cumulants are much simpler than those of the moments.

KUNDT CONSTANT. The **Verdet constant** divided by the **magnetic susceptibility**. In the case of ferromagnetic substances having variable susceptibility, the Verdet factor is not constant, but is proportional to the **susceptibility**. For such substances the Kundt factor has a constant value.

KURIE PLOT. A graph of the **β -particle spectrum** in which the Fermi probability equation is rearranged in the form:

$$K(N/f)^{1/2} = C - (E + 1)$$

where N is the number of β -particles of momentum (or energy) lying within a certain narrow range, f is a complicated function of

the corresponding β -particle energy E (in units of mc^2) as worked out by Fermi, and K and C are constants. The plot of $(N/f)^{1/2}$ against $(E + 1)$, often called a Kurie plot, is generally a straight line for allowed transitions and some **forbidden transitions**, in other words, the equation holds; and is used in determining the character of the β -transition and the maximum energy.

KURTOSIS. Kurtosis is a property of a **distribution** purporting to express its relative peakedness. The only measure of kurtosis in use is the ratio μ_4/μ_2^2 , where μ_4 and μ_2 are the central **moments** of order four and two. In fact, this is now realized to be an indifferent measure of "peakedness."

KUTTA-JOUKOWSKI LAW. For a two-dimensional airfoil in a steady uniform subsonic stream of velocity V , with circulation Γ round the airfoil, the lift force per unit span is $\rho V\Gamma$, where ρ is the fluid density in the undisturbed stream.

The result can be obtained in various ways; perhaps the simplest is by considering the rate of change of momentum of the fluid passing across a cylindrical surface of large radius with its center at the airfoil.

L

LABORATORY COORDINATE SYSTEM. See L-System and C-System.

LAG, ANGLE OF. When two related quantities, such as an alternating voltage and an alternating current, vary sinusoidally with time and have the same frequency, they may be expressed as

$$Q_1 = A \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} (\omega t + \phi)$$

$$Q_2 = B \begin{Bmatrix} \sin \\ \cos \end{Bmatrix} \omega t$$

where A , B , and ω are constants. It is then said that Q_2 lags (behind) Q_1 and ϕ is known as the angle of lag if it is positive. If ϕ is negative its magnitude is the angle of lead and Q_2 is said to lead Q_1 .

LAG CORRELATION. The correlation between two time-series when one is lagged with respect to the other; if u_t and v_t are the values at time t , the correlation of u_t and v_{t+k} would be a lag correlation with lag k .

LAGRANGE BRACKET. If u and v are two functions of the generalized coordinates q and momenta p , the expression

$$\{u, v\} = \sum_{i=1}^N \frac{\partial q_i}{\partial u} \frac{\partial p_i}{\partial v} - \frac{\partial p_i}{\partial u} \frac{\partial q_i}{\partial v}$$

is called the Lagrange bracket of u and v . It remains invariant under a canonical transformation. (See also **Poisson bracket**.)

LAGRANGE BRACKET OF OPTICS. See the **optical differential invariant**.

LAGRANGE DENSITY. Integrand L of an integral which is such that the conditions that it should be an extreme are the equations of motion of a dynamical system of fields. L is a function of the fields, their time and space derivatives, and the coordinates and time. (See **continuous systems, Lagrangian and Hamiltonian formalism for**.)

LAGRANGE EQUATIONS OF MOTION (SOMETIMES CALLED EULER-LAGRANGE EQUATIONS). A set of equations of motion for a dynamical system. In the case where the system is conservative the equation of motion for the i th particle is written:

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} = 0$$

where L is the Lagrangian function or kinetic potential; q_i is the generalized coordinate of i th particle ($i = 1, 2, 3, \dots, n$, where n is the number of **degrees of freedom** of the system); \dot{q}_i is the generalized velocity of i th particle.

These equations have the advantage over Newton's equations that any kind of coordinates may be used. (See **coordinates and momenta, generalized**.)

For non-conservative systems a set of equations can be written:

$$\frac{d}{dt} \frac{\partial K}{\partial \dot{q}_i} - \frac{\partial K}{\partial q_i} = Q_i$$

where Q_i is the generalized force such that $Q_i \delta q_i$ represents work done by Q_i when the coordinate q_i changes by δq_i ; K is the kinetic energy of the system. (See also **continuous systems, Lagrangian and Hamiltonian formalism for; variations, calculus of**.)

LAGRANGE FIELD EQUATIONS. See **Lagrangian formalism for field systems**.

LAGRANGE IDENTITY. See **adjoint equation**, and **Sturm-Liouville problem**.

LAGRANGE INTERPOLATION FORMULA. A formula for interpolation that may be used when the abscissae x_0, x_1, \dots, x_n at which $f(x)$ is known are distinct but not necessarily equally spaced. Let

$$\omega(x) = (x - x_0)(x - x_1) \cdots (x - x_n),$$

$$L_i(x) = \omega(x) / [(x - x_i)\omega'(x_i)]$$

$$= \prod_{\substack{j=0, n \\ j \neq i}} [(x - x_j) / (x_i - x_j)].$$

Then the Lagrange formula is

$$P[f|x] = \sum f_i L_i(x).$$

The polynomials $L_i(x)$ have been tabulated for particular spacings of the abscissae, since they are independent of the function f to be interpolated. In case some x_i are coincident, confluent forms exist (see Alston S. Householder, *Principles of Numerical Analysis*, McGraw-Hill Book Company, 1953, for a general theory), but in practice it is simpler than to use **divided differences**.

LAGRANGE MULTIPLIERS. Terms used in a method of solving **isoperimetric problems** in the calculus of **variations**. Suppose $u = f(x_1, x_2, \dots, x_n)$ and that $m < n$ accessory conditions $\phi_1(x_1, x_2, \dots, x_n) = 0$; $\phi_2 = 0$; \dots ; $\phi_m = 0$ are also given. Using m constants L_i , called the Lagrange multipliers, form the equation

$$F = f + L_1\phi_1 + L_2\phi_2 + \dots + L_m\phi_m.$$

If the n derivatives are required to vanish

$$\partial F/\partial x_1 = 0; \quad \partial F/\partial x_2 = 0; \quad \dots; \quad \partial F/\partial x_n = 0$$

these relations, together with the m conditions $\phi_i = 0$, make it possible to determine the $(m + n)$ variables so that an **extremal** or stationary value of u results.

LAGRANGE THEOREM. If y, y' are distances from the axis, θ, θ' angles of inclination with the axis, n, n' refractive indices in object, image space, respectively, then Lagrange's theorem in Gaussian optics states

$$n\theta y = n'\theta' y'.$$

This is the first order form of both **Abbe's sine condition** and the **Helmholtz equation**. It is also referred to as the *Helmholtz-Lagrange formula*, the *Smith-Helmholtz equation*, or *Helmholtz equation*.

LAGRANGIAN COORDINATES. (1) (Also called material coordinates.) A system of coordinates by which fluid parcels are identified for all time by assigning them coordinates which do not vary with time. Examples of such coordinates are (a) the values of any properties of the fluid conserved in the motion; or (b) more generally, the positions in space of the parcels at some arbitrarily selected moment. Subsequent positions in space of the parcels are then the dependent variables,

functions of time and of the Lagrangian coordinates.

Few observations in meteorology are Lagrangian: this would require successive observations in time of the same air parcel. Exceptions are the constant-pressure balloon observation, which attempts to follow a parcel under the assumption that its pressure is conserved, and certain small-scale observations of diffusing particles. (Cf. **Eulerian coordinates**; see also **Lagrange equations of motion**.)

(2) Same as **generalized coordinates**.

LAGRANGIAN CORRELATION. The correlation between the properties of a flow following a single parcel of fluid through its space and time variations.

(See **correlation coefficient**.)

LAGRANGIAN DIFFERENTIATION. See **material differentiation**.

LAGRANGIAN EQUILIBRIUM POINT. A point in space where the resultant gravitational field is zero. This condition may be produced by the equilibrium between the gravitational fields of two or more bodies, or it may be experienced by a body in a free-fall orbit, where its angular acceleration is equal and opposite to the acceleration of the resultant gravitational field. Theoretically, of course, every body has an infinite gravitational field; the Lagrangian Equilibrium Point is therefore limited in its validity to the gravitational fields which are assumed to be significant.

LAGRANGIAN FORMALISM FOR CONTINUOUS SYSTEMS. See **continuous systems**, **Lagrangian and Hamiltonian formalism** for.

LAGRANGIAN FORMALISM FOR FIELD SYSTEMS. Recall the Lagrangian formalism for classical particle mechanics. If we form the action

$$I = \int_{t_2}^{t_1} L(q_i, \dot{q}_i, t) dt \quad (1)$$

where L is the Lagrangian, then the **action principle** states that I is stationary for those variations of the q_i for which $\delta q_i(t_0) = \delta q_i(t_1) = 0$. The variation of I induced by variations δq_i is

$$\delta I(t_1, t_2) = \int_{t_2}^{t_1} \sum_i \left(\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} \right) \delta q^i dt + \sum_i \left[\frac{\partial L}{\partial \dot{q}_i} \delta q^i \right]_{t_2}^{t_1}. \quad (2)$$

The last term in (2) is zero when $\delta q^i = 0$ at times, t_1 and t_2 . The action principle is thus equivalent to the **Lagrange equations of motion**

$$\frac{\partial L}{\partial q_i} - \frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} = 0. \quad (3)$$

A more general variation consists in varying not only the q^i 's but also the times t_1 and t_2 , i.e., $t_1 \rightarrow t_1 + \delta t_1$ and $t_2 \rightarrow t_2 + \delta t_2$. If we denote by ${}_n q^i$ the "new" q^i resulting from the variation of the "old" ${}_o q^i$, then the variation of the q^i at a fixed time can be written as

$$\delta q^i(t) = {}_n q^i(t) - {}_o q^i(t). \quad (4)$$

The variation of the $q^i(t_1)$ produced by varying both q^i and t_1 will be denoted by

$$\begin{aligned} \Delta q^i(t_1) &= {}_n q^i(t_1 + \delta t_1) - {}_o q^i(t_1) \\ &= \delta q^i(t_1) + \delta t_{1n} \dot{q}^i(t_1). \end{aligned} \quad (5)$$

Under this double variation,

$$\begin{aligned} \delta I(t_1, t_2) &= \int_{t_1}^{t_1 + \delta t_1} L(t) dt - \int_{t_2}^{t_2 + \delta t_2} L(t) dt \\ &\quad + \int_{t_2}^{t_1} \delta L(t) dt \\ &= L(t_1) \delta t_1 - L(t_2) \delta t_2 + \sum_i \left[\frac{\partial L}{\partial \dot{q}_i} \delta q^i \right]_{t_2}^{t_1} \end{aligned} \quad (6)$$

where we have dropped those terms which vanish when the equations of motion (3) are satisfied. With the help of (4) and (5) Equation (6) can be rewritten as

$$\begin{aligned} \delta I &= [L(t_1) - \sum p_i(t_1) \dot{q}^i(t_1)] \delta t_1 \\ &\quad - [L(t_2) - \sum p_i(t_2) \dot{q}^i(t_2)] \delta t_2 \\ &\quad + \sum_i p_i(t_1) \Delta q^i(t_1) - \sum p_i(t_2) \Delta q^i(t_2) \end{aligned} \quad (7)$$

where $p_i(t)$, the momentum conjugate to $q^i(t)$, is defined as the coefficient of $\Delta q^i(t)$, i.e.,

$$p_i(t) = \frac{\partial L}{\partial \dot{q}^i(t)}. \quad (8)$$

Similarly the Hamiltonian is defined as the coefficient of δt

$$H(t) = \sum p_i(t) \dot{q}^i(t) - L(t). \quad (9)$$

In effecting the transition to quantum mechanics, the commutation relations and the Heisenberg equations of motion are defined in terms of these canonical variables by making them operators and requiring them to satisfy the commutation rules

$$[q^l(t), p_j(t)] = i\hbar \delta_{lj} \quad (10a)$$

$$[q^l(t), q^m(t)] = [p^m(t), p^n(t)] = 0 \quad (10b)$$

and that their time dependence be determined by the equation

$$i\hbar \frac{dO(t)}{dt} = [O(t), H] \quad (11)$$

where $O(t)$ is any dynamical variable not involving the time explicitly.

The above considerations are easily generalized to the case of a field system. A field requires for its specification a set of field functions $\phi_s(x)$, [$x = (x^0, x^1, x^2, x^3)$; the index s refers to the different components]. The finite number of degrees of freedom of particle mechanics is now replaced by a continuous infinity. One can consider $\phi_s(\mathbf{x}, t)$ as the "coordinates" of the field, and the behavior of $\phi_s(\mathbf{x}, t)$ as time evolves is the description of the evolution of the field system. In a field theory one can again define a Lagrangian density $\mathcal{L} = \mathcal{L}(\phi_s, \phi_{s\mu})$, where $\phi_{s\mu} = \partial_\mu \phi_s$, such that the variation of action integral I

$$\begin{aligned} I &= \int_{t_2}^{t_1} dt \int_v d^3x \mathcal{L}(\phi_s, \phi_{s\mu}) \\ &= \int_\Omega d^4x \mathcal{L}(\phi_s, \phi_{s\mu}) \end{aligned} \quad (12)$$

with respect to ϕ_s yield the field equations. More precisely one requires that I be an extremum for arbitrary variations of ϕ_s , provided only that $\delta \phi_s(\mathbf{x}, t_1) = \delta \phi_s(\mathbf{x}, t_2) = 0$ where t_1 and t_2 are the time limit of integration in (12). This action principle is the natural extension of Hamilton's principle in particle mechanics, since the aggregate of values which ϕ_s and $\dot{\phi}_s$ take on can be thought of as the coordinates q^i and velocities \dot{q}^i occurring in particle mechanics. The variation of I due to $\phi_s \rightarrow \phi_s + \delta \phi_s$ is given by

$$\delta I = \int_{\Omega} d^4x \sum_s \delta\phi_s \left[\frac{\partial \mathcal{L}}{\partial \phi_s} - \sum_{\mu=0}^3 \frac{\partial}{\partial x^\mu} \left(\frac{\partial \mathcal{L}}{\partial \phi_{s,\mu}} \right) \right] + \int_{\Sigma} d^3\sigma \sum_s \delta\phi_s \left(\frac{\partial \mathcal{L}}{\partial \phi_{s,\mu}} \right) n^\mu(x) \quad (13)$$

where Σ denotes the bounding surface of Ω (the four-volume of integration over which the field extends) and $n^\mu(x)$ is the outward normal to Σ at x . For the case that Σ consists of two planes $t = t_1$ and $t = t_2$ and that the fields tend to zero for large spatial distance, the action principle, $\delta I = 0$ for variations which vanish on Σ , requires that

$$\frac{\partial \mathcal{L}}{\partial \phi_s} - \sum_0^3 \frac{\partial}{\partial x^\mu} \frac{\partial \mathcal{L}}{\partial \phi_{s,\mu}} = 0 \quad (14)$$

which are the field equations.

Under a variation $\delta\phi_s$ of the field functions together with a rigid body displacement δx_μ of Ω , the variation of I is

$$\delta I = \int_{\Omega} \sum_s \left(\frac{\partial \mathcal{L}}{\partial \phi_s} - \sum_{\mu=0}^3 \frac{\partial}{\partial x_\mu} \frac{\partial \mathcal{L}}{\partial \phi_{s,\mu}} \right) \delta\phi_s d^4x + \delta x^\mu \int_{\Omega} \sum_s \frac{\partial}{\partial x_\nu} \left\{ \mathcal{L} g_{\mu\nu} - \phi_{s,\mu} \frac{\partial \mathcal{L}}{\partial \phi_{s,\nu}} \right\} d^4x + \int_{\Omega} \frac{\partial}{\partial x^\nu} \left\{ \frac{\partial \mathcal{L}}{\partial \phi_{s,\nu}} \Delta\phi_s \right\} d^4x \quad (15)$$

where $\delta\phi_s = {}_n\phi_s(x) - {}_o\phi_s(x)$ and $\Delta\phi_s = \delta\phi_s + \phi_{s,\mu}\delta x^\mu$. If the equations of motion are satisfied, the first term vanishes and δI can be re-written using Gauss theorem, $\int_{\Omega} \partial F^\mu / \partial x^\mu d^4x = \int_{\Sigma} d\sigma^\mu F_\mu$, as follows:

$$\delta I = \int_{\Sigma} d\sigma_\mu \left[\delta x_\nu \left(\mathcal{L} g^{\mu\nu} - \sum_s \phi_s \frac{\partial \mathcal{L}}{\partial \phi_{s,\nu}} \right) + \sum_s \Delta\phi_s \frac{\partial \mathcal{L}}{\partial \phi_{s,\mu}} \right]. \quad (16)$$

The momentum canonically conjugate to ϕ_s is now defined as the coefficient of $\Delta\phi_s$

$$\pi_s(x) = n_\mu \frac{\partial \mathcal{L}}{\partial \phi_{s,\mu}} \quad (17)$$

where n_μ is the unit vector normal to Σ . Similarly, the canonical energy-momentum tensor is defined by

$$T^{\mu\nu} = \sum_s \phi_s^\mu \frac{\partial \mathcal{L}}{\partial \phi_{s,\nu}} - \mathcal{L} g^{\mu\nu}. \quad (18)$$

It has the property that

$$\frac{dT^{\mu\nu}}{dx^\nu} = 0 \quad (19)$$

by virtue of the equations of motion, if \mathcal{L} does not explicitly depend on the space-time coordinates. Upon integrating (19) over 3-space one obtains

$$\frac{d}{dx^0} \int d^3x T^{\mu 0} + \sum_{k=1}^3 \int \frac{d}{dx^k} T^{\mu k} d^3x = 0. \quad (20)$$

If the fields vanish at large spatial distances, then so will $T^{\mu k}$, and the second term does not contribute since it can be recast into a surface integral. One therefore obtains the four conservation laws

$$\frac{d}{dx^0} \int T^{\mu 0} d^3x = 0, \quad \mu = 0, 1, 2, 3. \quad (21)$$

The first corresponds to conservation of the energy P_0 ,

$$P^0 = \int d^3x T^{00} \quad (22)$$

and the other three to the conservation of the three quantities

$$P^k = \int T^{k0} d^3x = \int d^3x \sum_s \frac{\partial \mathcal{L}}{\partial \phi_{s,0}} \phi_s^k = \sum_s \int d^3x \pi_s \phi_s^k \quad (23)$$

which are the three components of the total momentum of the field. If \mathcal{L} is a scalar, $T^{\mu\nu}$ is a second rank tensor and the four quantities

$P_\mu = \int d^3x T_{\mu 0}$ transform like a four-vector. In the transition to the quantized field theory, the field quantities ϕ_s and π_s become operators satisfying the canonical commutation rules and

$$[\phi_s(x), \pi_{s'}(x')] |_{x_0=x'_0} = i\hbar \delta(\mathbf{x} - \mathbf{x}') \delta_{ss'} \\ [\phi_s(x), \phi_{s'}(x')] |_{x_0=x'_0} = 0 \\ = [\pi_s(x), \pi_{s'}(x')] |_{x_0=x'_0} = 0. \quad (24)$$

The Heisenberg equations of motion are now given by

$$[P_\mu, O_x] = i\hbar \frac{\partial O_x}{\partial x^\mu} \quad (25)$$

where O_x is any operator function.

As an illustration of the above formalism we briefly treat the case of a neutral meson field ϕ which satisfies the equation of motion

$$(\square + \mu^2)\phi(x) = (\partial^\mu\partial_\mu + \mu^2)\phi(x) = 0. \quad (26)$$

The Lagrangian density for this case can be taken to be

$$\mathcal{L} = -\frac{1}{2}(\mu^2\phi^2 - \partial_\mu\phi\partial^\mu\phi) \quad (27)$$

since the Euler equations (14) for this \mathcal{L} are precisely Equation (26). The momentum

canonically conjugate to ϕ is $\pi(x) = \frac{\partial\mathcal{L}}{\partial\dot{\phi}} = \dot{\phi}(x)$

$= \phi_0(x)$. The Hamiltonian, or energy operator is given by

$$\begin{aligned} H = P_0 &= \int d^3x T^{00} \\ &= \frac{1}{2} \int d^3x \{ \pi^2 + \mu^2\phi^2 + (\nabla\phi)^2 \} \end{aligned}$$

since

$$T_{\mu\nu} = \phi_\mu\phi_\nu + \frac{1}{2}g_{\mu\nu}(\mu^2\phi^2 - \phi_\lambda\phi^\lambda).$$

In the quantized theory the commutation rules for the field operators $\pi(x)$ and $\phi(x)$ are

$$[\pi(x), \phi(x')]_{x_0=x'_0} = -i\hbar\delta(\mathbf{x} - \mathbf{x}').$$

(See also **covariant commutation rules.**)

LAGRANGIAN FUNCTION OR KINETIC POTENTIAL. The difference between the kinetic energy and potential energy of a dynamic system. It is generally symbolized by L .

LAGRANGIAN METHOD OF ANALYSIS.

A method whereby the motion of a body of fluid always consisting of the same fluid particles is considered. By considering the rate of change of its volume and momentum the equations of continuity and fluid motion are deduced. Properties of the fluid are expressed in terms of their values at one moment (or, in steady flow, at one value of one of the space coordinates, i.e., their values on a given surface or plane) and time from that moment (or displacement from the given surface). The Lagrangian coordinates of a particle are the values of the coordinates of the particle at the specified moment or when the particle crosses the specified surface.

LAG, SHEAR. See **shear lag.**

LAGUERRE EQUATION. The linear equation $xy'' + (1-x)y' + ny = 0$, having a

simple **pole** at the origin. Its solutions are the **Laguerre polynomials**. Differentiation of the equation k times and replacement of the k th derivative by y gives

$$xy'' + (k+1-x)y' + (n-k)y = 0$$

which is the associated Laguerre equation whose solutions are associated Laguerre polynomials. These functions occur in the quantum mechanical problem of the hydrogen atom.

The associated polynomials may be defined by the equivalent expressions

$$\begin{aligned} L_n^{(k)}(x) &= \frac{e^x x^{-k}}{n!} \frac{d^n}{dx^n} (e^{-x} x^{n+k}) \\ &= \sum_{i=0}^n \binom{n+k}{n-i} \frac{(-x)^i}{i!}. \end{aligned}$$

The special case of $k=0$ gives the Laguerre polynomials

$$L_n(x) = 1 - \binom{n}{1}x + \binom{n}{2}\frac{x^2}{2!} - \binom{n}{3}\frac{x^3}{3!} + \dots$$

LAGUERRE POLYNOMIALS. The class of special functions $L_n(x)$ obtained by orthogonalizing over the interval $[0, \infty]$ the sequence of powers $1, x, x^2, \dots$ with the weighting function $w(x) = e^{-x}$. They are the eigenfunctions of the eigenvalue problem

$$xy'' + (1-x)y' + \lambda y = 0$$

under the boundary conditions $y(0) = \text{finite}$ and $y(\infty) = O(x^k)$ for some positive k , where O is the **ordersymbol**. The eigenvalues are $\lambda = n = 0, 1, 2, \dots$. The first few Laguerre polynomials are:

$$L_0(x) = 1, \quad L_1(x) = -x + 1,$$

$$L_2(x) = x^2 - 4x + 2, \dots,$$

whose normalizing factor is $(n!)^{-2}$. Their generating function is $(1-t)^{-1}e^{-xt(1-t)^{-1}}$. They satisfy the recursion formula

$$\begin{aligned} L_{n+1}(x) - (2n+1-x)L_n(x) \\ + n^2L_{n-1}(x) = 0, \quad n > 0. \end{aligned}$$

Their Rodrigues formula is

$$L_n(x) = e^x \frac{d^n}{dx^n} (x^n e^{-x}).$$

The more general polynomials with weighting function $w(x) = x^\alpha e^{-x}$, $\alpha > -1$, are also often called Laguerre polynomials.

(For associated Laguerre polynomials see Laguerre equation.)

LAMB CONSTANT L . The quantity $\frac{8}{3\pi} \alpha^3 R_\infty$, where α is the fine structure constant, $\alpha = e^2/\hbar c$, and $R_\infty = \frac{1}{2}\alpha^2 mc^2 = e^2/2a_0$ is the Rydberg constant for an infinitely heavy nucleus.

LAMBDA POINT (λ -POINT). See **transition of higher order**. (For dependence of to lambda point on pressure, see **Ehrenfest relations**.)

LAMBDA TRANSITION. See **lambda point**.

LAMBDA-TYPE DOUBLING. The Π , Δ , \dots electronic states of diatomic and linear polyatomic molecules are doubly degenerate if the molecule is not rotating. In the rotating molecule the interaction of rotation and electronic motion causes a splitting of this degeneracy which in general increases with increasing rotation (Λ -type doubling). The **rotational levels** of the two term series, distinguished by superscripts c and d , are in the case of a ${}^1\Pi$ state

$$F_c(J) = B_v^c J(J+1) + \dots,$$

$$F_d(J) = B_v^d J(J+1) + \dots$$

that is, the splitting is given by

$$\Delta\nu_{cd} = (B_v^c - B_v^d)J(J+1) = qJ(J+1)$$

where the splitting constant q depends on the position of nearby Σ states. For Δ states the splitting is usually negligibly small.

LAMBERT. A unit of **luminance** equal to $1/\pi$ **candle** per square centimeter, and, therefore, equal to the uniform luminance of a perfectly diffusing surface emitting or reflecting light at the rate of one **lumen** per square centimeter.

LAMBERT COSINE LAW. See **cosine emission law**.

LAMBERT PROJECTION (LAMBERT LINE). The Lambert conformal conic projection with two standard parallels was devised by Johan Heinrich Lambert during the last half of the 18th century. It has become

of increasing importance with the development of air navigation.

It is a conic projection but differs from others in that the cone, with apex on the axis of rotation of the earth, enters the surface of the earth at one parallel of latitude and emerges at another. These two parallels are referred to as the *limiting* or the *standard parallels*. The projection is conformal. Thus areas on the earth retain their relative sizes and shapes on the chart. The scale of distance is nearly constant over the chart. On a Lambert Chart of the United States with limiting parallels 45°N and 29°N the scale of distance in latitude 49°N is 1.01, at 45° is exact (or 1.00), on the 37th parallel is 0.99, on the 29th parallel is again 1.00, and in latitude 25°N is 1.01.

Meridians of longitude are converging straight lines and parallels are curved. Rhumb lines are curved and this means that navigation by the various **sailings** becomes very difficult on the Lambert Chart. However, great circles are nearly flat and for considerable distances may be considered as straight. For example: on a Lambert projection with limiting parallels 36° and 54° north latitude the Lambert line, i.e., a straight line on the Lambert Chart, from Cape Hatteras to the English Channel has a maximum distance of only 15.6 miles from the great circle.

LAMB SHIFT. According to the Dirac theory for the **fine structure** of hydrogen and hydrogen-like ions, levels with the same quantum numbers n and j , but different quantum

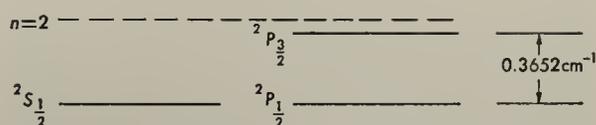


Fig. 1. Energy levels for $n = 2$ level of hydrogen according to Dirac theory. (Dashed line represents position of level according to Schrödinger theory.)

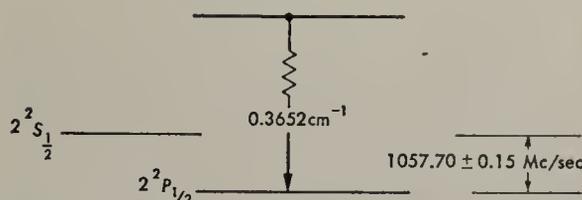


Fig. 2. Energy levels for $n = 2$ level of hydrogen when radiative corrections are included. (The experimental value for $2^2S_{1/2} - 2^2P_{1/2}$ level shift is 1057.70 Mc/sec). The Lamb shift for the $n = 2$ level in cm^{-1} is 0.0353 cm^{-1} .

numbers l , have the same energy. The fact that this does not hold strictly was first demonstrated by Lamb and Retherford (1947), who measured a shift of about 1050 Mc between the $2^2S_{1/2}$ level and the $2^2P_{1/2}$ level of hydrogen. (See Figs. 1 and 2.) According to Bethe, Schwinger, Weisskopf and others, all levels of H, He^+ , etc., are shifted compared to the Dirac levels on account of the interaction of the electron with the radiation field. This quantum electro-dynamical shift is largest for S states for which it is given by

$$\Delta E(n, 0, \frac{1}{2}) = \frac{8Z^4 \alpha^3}{n^3 3\pi} R_\infty \left(1 - \frac{3m}{M}\right) \left\{ \log \frac{mc^2}{2k_0(n, 0)} + \frac{1}{3} \frac{9}{0} + 3\pi Z \alpha \left[\frac{4}{3} \frac{2}{8} \frac{7}{4} - \frac{1}{2} \log 2 \right] \right\}.$$

For P, D, \dots states the shift is given by

$$\Delta E(n, l, j) = \frac{8Z^4 \alpha^3}{n^3 3\pi} R_\infty \left(1 - \frac{3m}{M}\right) \left[\log \frac{R}{k_0(n, l)} \pm \frac{3}{8} \frac{1}{(j + \frac{1}{2})(2l + 1)} \right],$$

where the \pm sign corresponds with $j = l \pm 1$. Here α is **Sommerfeld's fine structure constant**, and k_0 is the so-called average excitation energy ($k_0 = 16.646 RhcZ^2$ for $2^2S_{1/2}$), m and M are the masses of electron and nucleus respectively, R is the **Rydberg constant**, and Z is the **atomic number**. As seen from the above formula the Lamb shifts are proportional to Z^4 and inversely proportional to n^3 .

The experimental methods of Lamb depend on the long life of the metastable $2^2S_{1/2}$ states (~ 3 sec.). Roughly speaking, the experiments consist in making a beam of hydrogen atoms and exciting the atoms by electron bombardment. Most of the excited atoms in the $2^2S_{1/2}$ state, but not those in the $2^2P_{1/2}$ or $2^2P_{3/2}$ levels, would live long enough to reach a detector. If now a radio frequency field of the correct frequency to induce transitions from the $2^2S_{1/2}$ level to one of the 2^2P levels is placed in the path of the atoms, then the decay of the atoms in the 2^2P states would result in a reduction of the atoms detected and thus indicate a radio frequency resonance. In the actual experiment, the beam along most of its path transverses a homogeneous magnetic field provided by an external magnet. In this way the Zeeman components of $2^2S_{1/2}$ and 2^2P are separated and the likelihood of decay from the $2^2S_{1/2}$ through mixing with the $2^2P_{1/2}$ is de-

creased. There exists also an important experimental advantage in performing the experiment in a magnetic field. This is that the radio frequency source can then be used at a fixed frequency, as the resonances of the Zeeman components can be explored by varying the strength of the magnetic field keeping the frequency fixed.

(For further treatment of the Lamb shift see **level displacement in hydrogen-like atoms**.)

LAMÉ CONSTANTS. See **Hooke's law**, generalized.

LAMÉ EQUATION (GENERALIZED). The most general second-order (linear) differential equation with five regular singularities, one of them being the point at infinity, with preassigned exponents differing from each other by $\frac{1}{2}$ at each singularity, all other points of the complex plane being ordinary points. This equation is remarkable because of the large number of important equations (**Legendre**, **Bessel**, etc.) obtainable from it by **confluence**. Letting a_1, a_2, a_3, a_4 and ∞ be the singular points, with exponents $\alpha_1, \alpha_1 + \frac{1}{2}, \dots, \alpha_4, \alpha_4 + \frac{1}{2}, \mu_1, \mu_1 + \frac{1}{2}$, the equation has the form

$$\frac{d^2w}{dz^2} + P \frac{dw}{dz} + Qw = 0$$

where

$$P = \sum_{i=1}^4 \frac{\frac{1}{2} - 2\alpha_i}{z - a_i},$$

$$Q = \sum \frac{\alpha_i(\alpha_i + \frac{1}{2})}{(z - a_i)^2} + \frac{Az^2 + 2Bz + C}{(z - a_1) \cdots (z - a_4)}.$$

A is expressible in terms of the α_i , and B, C are arbitrary constants. For example if the confluence takes the form $a_1 = a_2 = 0, a_3 = a_4 = \infty$, then choosing all $\alpha_i = 0$ and setting $z = \zeta^2$, with proper choice of B and C , we get

$$\zeta^2 \frac{d^2w}{d\zeta^2} + \zeta \frac{dw}{d\zeta} + (\zeta^2 - n^2)w = 0,$$

which is **Bessel's equation**.

LAMELLAR VECTOR FIELD. A vector field \mathbf{B} is called lamellar if it satisfies the condition

$$\mathbf{B} \cdot \text{curl } \mathbf{B} \equiv 0.$$

Such a vector field \mathbf{B} can always be written in the form

$$\mathbf{B} = f \text{ grad } g.$$

A lamellar vector field can be represented as a family of surfaces (lamina) which are orthogonal to the vector field \mathbf{B} . (See Figure 1.)

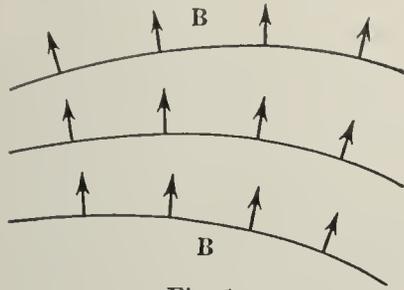


Fig. 1.

If in addition $\text{div } \mathbf{B} = 0$, the field can be split up into tubes along which the "flux" is constant; by "flux" is meant $\iint_S \mathbf{B} \cdot \mathbf{n} da$, where S is the top or bottom boundary of the tube. (See Figure 2.)

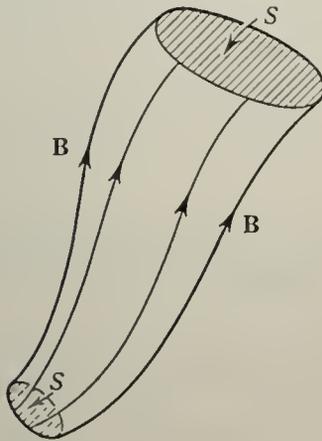


Fig. 2.

LAMÉ RELATIONS. Six independent relations which must be satisfied by the covariant metric tensor for a three-dimensional space, as necessary and sufficient conditions that it be Euclidean. They are usually written, in tensor notation,

$$S^{ij} = 0,$$

where S^{ij} is defined by

$$S^{ij} = \frac{1}{4} \epsilon^{ikl} \epsilon^{jmn} R_{klmn}.$$

R_{klmn} is the Riemann-Christoffel tensor and ϵ^{ikl} is the ϵ -system (epsilon system).

LAMÉ THEORY OF THICK WALLED CYLINDERS. An isotropic elastic circular cylinder of constant wall thickness subjected to uniform internal and external pressures p_i , p_e has the following state of stress

$$\sigma_t = \frac{a^2 p_i}{b^2 - a^2} \left(1 + \frac{b^2}{r^2} \right) - \frac{b^2 p_e}{b^2 - a^2} \left(1 + \frac{a^2}{r^2} \right)$$

$$\sigma_r = \frac{a^2 p_i}{b^2 - a^2} \left(1 - \frac{b^2}{r^2} \right) - \frac{b^2 p_e}{b^2 - a^2} \left(1 - \frac{a^2}{r^2} \right)$$

$$\sigma_z = \frac{F}{\pi(b^2 - a^2)}$$

where a is the internal radius, b is the external radius, σ_t is the hoop or circumferential stress, σ_r the radial, and σ_z the axial stresses. F is the total axial force.

LAMINAR BOUNDARY LAYER. See boundary layer.

LAMINAR BOUNDARY-LAYER EQUATIONS. The Navier-Stokes equations for steady two-dimensional flow of an incompressible fluid, in the absence of body forces, are

$$\rho \left(u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} \right) = - \frac{\partial p}{\partial x} + \mu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right), \quad (1)$$

$$\rho \left(u \cdot \frac{\partial v}{\partial x} + v \cdot \frac{\partial v}{\partial y} \right) = - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right), \quad (2)$$

with the equation of continuity

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \quad (3)$$

In these equations, u and v are velocity components parallel to the x and y -axes, ρ is density, p is pressure and μ is viscosity. To derive the equations for flow in a boundary layer, the x -axis is taken along the solid boundary, the y -axis being normal to the boundary. If l is a characteristic length along the boundary, it can be shown that the terms involving μ in Equations (1) and (2) are negligible except in a layer adjacent to the wall whose thickness δ is of order $lR^{-1/2}$, where R is the Reynolds number based on the length l . This layer is, of course, the boundary layer, and at high Reynolds numbers its thickness δ is small compared with l .

It can also be shown that $\frac{v}{u}$ is $O(R^{-1/2})$ so that

at high Reynolds numbers all the terms in Equation (2) are small compared with those in Equation (1). Thus the pressure is almost constant across the boundary layer and the pressure distribution along the boundary layer is determined by the external flow. Equation (2) is not required, since it only expresses a relationship between terms which are all small and unimportant. Moreover, in Equation (1) $\frac{\partial^2 u}{\partial x^2} \ll \frac{\partial^2 u}{\partial y^2}$, so that the boundary layer equations are simply Equation (3) and

$$u \cdot \frac{\partial u}{\partial x} + v \cdot \frac{\partial u}{\partial y} = -\frac{1}{\rho} \cdot \frac{\partial p}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad (4)$$

where $\nu = \mu/\rho$, the kinematic viscosity. The boundary conditions are $u = v = 0$ at $y = 0$, and $u = U(x)$ at $y = \infty$, where $U(x)$ is the velocity of the potential flow outside the boundary layer.

By introducing a system of curvilinear coordinates it can be shown that these boundary-layer equations also apply when the solid boundary is curved, provided the radius of curvature of the boundary is always large compared with the thickness of the boundary layer.

It should be emphasized that the assumptions made in deriving the boundary-layer equations are only valid at high Reynolds numbers. In a **turbulent boundary layer** the Reynolds stresses are of importance and the equations derived here for steady laminar flow do not apply.

LAMINAR BOUNDARY LAYER, INSTABILITY OF. See **transition of turbulent flow in a boundary layer.**

LAMINAR BOUNDARY LAYER, SOLUTION OF EQUATIONS. Exact analytical solutions of the laminar boundary-layer equations can only be obtained for a few special cases. The general problem of flow in a boundary layer with any prescribed pressure distribution can be solved by numerical step-by-step methods, although the work is laborious and there are considerable difficulties. For practical purposes, when a large number of cases is to be computed, an approximate method must be used.

One of the simplest approximate methods is that introduced by Pohlhausen. It is assumed

in this method that the velocity distribution in the boundary layer can be expressed as

$$\frac{u}{U} = a\eta + b\eta^2 + c\eta^3 + d\eta^4,$$

where $\eta = y/\delta$, u is the velocity at a distance y from the wall, U is the velocity outside the boundary layer, and δ is the boundary-layer thickness. In general, the thickness δ varies with distance x along the wall. By introducing the boundary conditions at the wall and at the edge of the boundary layer, the quantities a , b , c and d can all be expressed in terms of a single dimensionless parameter

$$\Lambda = \frac{\delta^2}{\nu} \cdot \frac{du}{dx},$$

where ν is the kinematic viscosity.

The displacement and momentum thicknesses can each be expressed in terms of Λ , and the shear stress at the wall is

$$\tau_0 = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0} = \frac{\mu U}{\delta} \left(2 + \frac{\Lambda}{6} \right).$$

The solution is completed by substituting these quantities into the momentum equation (see **momentum equation for boundary layer**) and integrating.

The Pohlhausen method gives fairly good agreement with more exact calculations in regions where the pressure gradient $\frac{dp}{dx}$ is negative,

but is less satisfactory in regions of positive pressure gradient. Methods depending on two free parameters, instead of the single one Λ in the Pohlhausen method, have been introduced to give better results in regions of positive pressure gradient. (See H. Schlichting, *Boundary Layer Theory* (English Translation by J. Kestin), Pergamon, 1955.)

LAMINAR FLOW. Flow in which there is no turbulence. (See **channel, two-dimensional, flow in; pipe, laminar flow in.**)

LAMINAR SEPARATION BUBBLE. See **transition of turbulent flow in a boundary layer.**

LAMINAR SUB-LAYER. See **turbulent boundary layer.**

LANCZOS METHOD OF BIORTHOAGONALIZATION. A method of forming sequentially the columns of matrices **B** and **C** such

that $\mathbf{B}^T\mathbf{C} = \mathbf{D}$ is diagonal, and, for a given matrix \mathbf{A} ,

$$\mathbf{AB} = \mathbf{BT}, \quad \mathbf{A}^T\mathbf{C} = \mathbf{CT},$$

$$\mathbf{T} = \begin{pmatrix} z_{11} & z_{12} & 0 & 0 & \cdots \\ 1 & z_{22} & z_{23} & 0 & \cdots \\ 0 & 1 & z_{33} & z_{34} & \cdots \\ \cdot & \cdot & \cdot & \cdot & \cdots \end{pmatrix}$$

a tridiagonal matrix. If the initial columns of \mathbf{B} and \mathbf{C} are chosen so that no diagonal element of \mathbf{D} vanishes, the reduction can be carried through to completion barring special properties of \mathbf{A} . (See **eigenvalues and eigenvectors of matrices, computation of**.) (For **Hermitian matrices** the method is equivalent to that of Givens.)

LANDAU FORMULA. A formula which gives the contribution to the magnetic susceptibility of metals produced by changes in the orbital wave functions of the free electrons which occur when a magnetic field is applied. (See **orbital diamagnetism of conduction electrons**.)

LANDÉ g -FACTOR. Landé's g -factor stands for the **gyromagnetic ratio** of the electrons in an atom due to orbital motion and electron spin (see also **Zeeman effect**). It can be stated in explicit form only for simple coupling conditions. For **(L, S) coupling** it is given by

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}.$$

Here J , L , and S are the quantum numbers corresponding to the resultant total angular momentum, resultant orbital angular momentum and resultant spin of the electrons, respectively. (For more detail, see H. E. White, *Introduction to Atomic Spectra*, McGraw-Hill, New York, 1934, p. 216 ff.) (See also **g-sum rule, g-permanence rule**.)

LANDÉ INTERVAL RULE. (1) The intervals between successive **fine structure** components of an atomic energy level (J and $J+1$) are proportional to $J+1$. (J is the quantum number of resultant total electronic angular momentum.)

(2) The intervals between successive **hyper-fine structure** components of an atomic energy level (F and $F+1$) are proportional to $F+1$. (F is the quantum number of the resultant

total angular momentum, including nuclear spin.)

LANGEVIN FUNCTION. The function $L(x)$ giving the average dipole moment $\bar{\mu}$ in a gas of dipoles μ in a magnetic field H at a temperature T ,

$$\bar{\mu} = \mu L(x), \quad x = \beta\mu H,$$

$\beta = 1/kT$, k being Boltzmann's constant. The Langevin function is given by the equation

$$L(x) = \coth x - \frac{1}{x}.$$

The Langevin function is not restricted to gases, but applies generally to systems of polarized entities. (See **Weiss theory of ferromagnetism**.)

LANGEVIN-PAULI FORMULA. See Langevin theory of diamagnetism.

LANGEVIN THEORY OF DIAMAGNETISM. Langevin considered the orbital motion of an electron in an atom to be an equivalent current loop of radius r . The application of a magnetic field \mathbf{H} causes a precession which increases the magnetic moment by the amount

$$\Delta\mathbf{M} = \frac{e^2\mathbf{H}r^2}{4mc^2}$$

where e and m are the electronic charge and mass, and c is the velocity of light. This implies a magnetic susceptibility of $-e^2r^2/4mc^2$. For N atoms per unit volume, each with Z orbital electrons (i.e., of atomic number Z), the susceptibility is given by

$$\chi = -\frac{Ze^2N}{6mc^2}\bar{r}^2$$

where \bar{r}^2 is the mean square radial distance of the electrons from their nuclei.

This equation is known as the *Langevin-Pauli formula*.

LANGLEY. A unit of energy per unit area commonly employed in radiation theory; equal to one gram-calorie per square centimeter.

LANGMUIR ADSORPTION ISOTHERM. A relation between the density (ρ) and the pressure (p) of the adsorbed substance given by the equation

$$\rho = \frac{Ap}{1 + Ap}$$

where A is a constant. (For further discussion, see **adsorption isotherm**.)

LANGUAGE (IN ELECTRONIC COMPUTERS). (1) A system consisting of (a) a well defined, usually finite, set of **characters**; (b) rules for combining characters with one another to form words or other expressions; and (c) a specific assignment of meaning to some of the words or expressions, usually for communicating information or data among a group of people, mechanisms, etc. (2) A system similar to the above but with any specific assignment of meanings. Such systems may be distinguished from (1) above, when necessary, by referring to them as formal or uninterpreted languages. Although it is sometimes convenient to study a language independently of any meanings, in all practical cases at least one set of meanings is eventually assigned. (See also **code**; **language, machine**.)

LANGUAGE, MACHINE. (1) A language, occurring within a machine, ordinarily not perceptible or intelligible to persons without special equipment or training. (2) A translation or transliteration of (1) above into more conventional characters but frequently still not intelligible to persons without training.

LAPLACE EQUATION. A second-order partial differential equation of elliptic type which, in vector form, is $\nabla^2\phi = 0$. In Cartesian coordinates it reads

$$\frac{\partial^2\phi}{\partial x_1^2} + \frac{\partial^2\phi}{\partial x_2^2} + \cdots + \frac{\partial^2\phi}{\partial x_n^2} = 0.$$

It is the homogeneous case of **Poisson's equation**. Its solutions, the scalar quantity ϕ , occur in problems involving steady-state temperatures, gravitational and electric potentials, hydrodynamics of ideal fluids, and many other physical phenomena. The equation is usually solved by the method of **separation of variables** in a suitable curvilinear coordinate system and with **boundary conditions** imposed by physical requirements. Such solutions are called *harmonic functions*. In two dimensions, the real and the imaginary part of an analytic function of a complex variable must satisfy Laplace's equation. Thus if $w = u + iv$ is an analytic function of $z = x + iy$, then $u_{xx} + u_{yy} = 0$ and $v_{xx} + v_{yy} = 0$. Under usual conditions, gravitational and electrostatic poten-

tials, as well as many other physical functions, are harmonic functions of three variables.

LAPLACE EXPANSION. From the n by n matrix \mathbf{A} select any k rows (the theorem could also be stated with interchange of the words "row" and "column"). Form all the k -rowed determinants which can be formed by selecting k columns from k rows; multiply each of these determinants by its algebraic complement (see **minor**); then the sum of all these products is equal to the determinant of \mathbf{A} .

LAPLACE FORMULA. See **surface tension of curved interfaces**.

LAPLACE LAW. See **Ampere law**.

LAPLACE TRANSFORM. The Laplace transform of a function $f(x)$ is the function $F(y)$ defined by

$$F(y) = \int_0^{\infty} e^{-yx}f(x)dx,$$

where x and y are not necessarily real variables.

Specializing to the real domain, the unilateral Laplace transform of $f(t)$, a function of a real variable t , is defined as

$$F(s) = \mathcal{L}f(t) = \int_0^{\infty} e^{-st}f(t)dt,$$

and in this case $f(t)$ is given in terms of $F(s)$ by the inversion integral,

$$f(t) = \mathcal{L}^{-1}F(s) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} e^{-ts}F(s)ds$$

in which c is any real number such that $\int_0^{\infty} e^{-ct}|f(t)|dt$ is bounded. (The bilateral transform, in which the integration range of t is from $-\infty$ to $+\infty$ has few applications in the control field, though it is used in some aspects of network and system synthesis.)

The zero lower limit of the t -integral for $F(s)$ is often for clarity, modified to $0-$ or $0+$, meaning respectively

$$\lim_{\epsilon \rightarrow 0} = \mp |\epsilon|,$$

it being understood that any other limiting process which may occur in the integrand is carried out first. The distinction is important when $f(t)$ contains an **impulse function** of any order at $t = 0$, as it often does. (See also **integral transform**.)

LAPLACE TRANSFORM THEOREMS. In the theorems that follow $F(s)$ denotes the unilateral Laplace transform of $f(t)$. Where desirable, according as the lower limit of integration is taken as $0+$ or $0-$, the Laplace transform of $f(t)$ will be denoted by $F_{\pm}(s)$, and the operation of Laplace transformation by \mathcal{L}_{\pm} .

(a) *The Superposition Theorem.*

$$\mathcal{L} \sum_n A_n f_n(t) = \sum_n A_n \mathcal{L} f_n(t)$$

where $f_1(t), f_2(t) \dots$ are functions of t and A_1, A_2, \dots are constants.

(b) *The Frequency-shift Theorem.*

$$\mathcal{L}_{\pm} e^{at} f(t) = F_{\pm}(s - a)$$

where a is any finite constant, real or complex.

(c) *The Time-shift Theorem.*

(i) Time-delay.

$$\mathcal{L}_{\pm} f(t - t_0) = e^{-t_0 s} F_{\pm}(s)$$

where t_0 is real and ≥ 0 , provided $f(t) = 0$ for $t < 0$ and, in the case of the $0+$ transform, $f(t)$ in addition contains no impulse functions at $t = 0$.

(ii) Time-advance.

$$\mathcal{L}_{\pm} f(t + t_0) = e^{t_0 s} F_{\pm}(s)$$

where t_0 is real and ≥ 0 , provided $f(t) = 0$ for $t < t_0$ and, in the case of the $0+$ transform, $f(t)$ in addition contains no impulse functions at $t = t_0$.

(d) *Limit Theorems.*

(i) $\lim_{t \rightarrow 0+} f(t)$

$$= \lim_{s \rightarrow \infty} s F_{+}(s)$$

$$= \lim_{s \rightarrow \infty} s \{ F_{-}(s) - K_1 - K_2 s - K_3 s^2 \dots \}$$

where K_1, K_2, \dots are the amplitudes of any impulse functions of order 1, 2, \dots at $t = 0$ which $f(t)$ may contain.

(ii) $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} s F_{\pm}(s)$

provided $F(s)$ has no singularities to the right of or on the imaginary axis of the s plane.

(e) *The Convolution Theorem.*

If $f(t), g(t)$ have Laplace transforms $F(s), G(s)$, then

$$\begin{aligned} F_{\pm}(s) \cdot G_{\pm}(s) &= \mathcal{L}_{\pm} \int_{0\pm}^{t\mp} f(t - \tau)g(\tau)d\tau \\ &= \mathcal{L}_{\pm} \int_{0\pm}^{t\mp} f(\tau)g(t - \tau)d\tau \end{aligned}$$

(f) *The Integral Theorem.*

$$\mathcal{L}_{\pm} \int_{0\pm}^t f(t)dt = \frac{1}{s} F_{\pm}(s)$$

(g) *The Differential Theorem.*

(i) Basic Form. If $f(t)$ is differentiable over the integration range of the Laplace transform integral, then

$$\mathcal{L}_{\pm} \frac{d}{dt} f(t) = s F_{\pm}(s) - f(0\pm).$$

(ii) Modified Form. If the t -derivative of a discontinuity is interpreted as a first order **impulse function** of amplitude equal to the discontinuity, and if, extending the concept, the t -derivative of an impulse function of any order is interpreted as an impulse function of order one higher and having the same amplitude, then provided $f(t)$ is differentiable in this extended sense over the integration range, the above statement of the theorem is still true.

(iii) Extended Form. Iteration of the theorem gives

$$\mathcal{L}_{\pm} \frac{d^n}{dt^n} f(t) = s^n F_{\pm}(s)$$

$$- \left\{ s^{n-1} f + s^{n-2} \frac{df}{dt} + \dots + \frac{d^{n-1} f}{dt^{n-1}} \right\}_{t=0\pm}.$$

If $f(t) = 0$ for $t < 0$

$$\mathcal{L}_{-} \frac{d^n}{dt^n} f(t) = s^n F_{-}(s).$$

(h) *Basic Laplace Transforms.*

(i) $f(t) = 1$ $F_{\pm}(s) = \frac{1}{s}$

(ii) $f(t) = e^{at}$ $F_{\pm}(s) = \frac{1}{s - a}$ for all values of $|a|$

(iii) $f(t) = \delta(t)$ $F_{+}(s) = 0$
 = unit impulse $F_{-}(s) = 1$
 of first order
 at $t = 0$

LAPLACIAN. (1) The **Laplacian operator**. (2) In nuclear terminology, the negative of buckling.

LAPLACIAN OPERATOR. An operator sometimes represented by Δ , sometimes by ∇^2 , and of the form

$$\partial^2/\partial x^2 + \partial^2/\partial y^2 + \dots$$

In vector notation, the operator is written

$$\text{div grad} = \nabla \cdot \nabla = \nabla^2.$$

In **rectangular coordinates** its components are $\partial^2/\partial x^2$, $\partial^2/\partial y^2$, $\partial^2/\partial z^2$; in spherical **polar coordinates** $\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right)$, $\frac{1}{r^2 \sin^2 \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial}{\partial \theta} \right)$, $\frac{1}{r^2 \sin^2 \theta} \frac{\partial^2}{\partial \phi^2}$ and in **cylindrical coordinates** $\frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\rho \frac{\partial}{\partial \rho} \right)$, $\frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2}$, $\frac{\partial^2}{\partial z^2}$. (See also **curvilinear orthogonal coordinates**.)

LAPORTE RULE. A strict selection rule for dipole radiation which is stated in the form: **Even terms** can combine only with odd, and odd only with even. A special case of the Laporte rule is the prohibition of the combinations of two terms with the same **electron configuration**.

LAPSE RATE. The rate of decrease with height, usually referring to temperature in the atmosphere and therefore equal to $-\partial T/\partial z$.

Throughout this book, a number of lapse rates for specified conditions are defined under the corresponding headings (e.g., **dry-adiabatic lapse rate**.)

LARGE ION. (Also called *slow ion*, *heavy ion*.) An ion of relatively large mass and low mobility which is produced by the attachment of a **small ion** to an **Aitken nucleus**. They were discovered by P. Langevin, and are sometimes referred to as "Langevin ions."

LARGE NUMBERS. See **law of large numbers**.

LARMOR PRECESSION. The motion which a charged particle or system of charged particles subject to a central force directed towards a common point experiences when under the influence of a small uniform magnetic field. If a coordinate system is chosen which rotates

about the direction of the magnetic field with an angular velocity

$$\omega = -\frac{q}{2mc} H,$$

where ω is the angular velocity, q is electric charge in esu, m is mass, c is the velocity of light, H is magnetic field strength in emu, then the motion in this system of coordinates is the same as the motion referred to a coordinate system fixed in space without a magnetic field. This is called the Larmor theorem.

The application of this principle arises almost exclusively in the description of the motion of atoms and electrons in magnetic fields. (See **precession**.)

LATENT HEAT. See **heat, latent**.

LATENT HEAT OF PHASE CHANGE. See **Clausius-Clapeyron equation**.

LATENT HEAT OF VOLUME CHANGE. See **thermal coefficients**.

LATENT ROOTS. See **eigenvalues and eigenvectors**; and **matrix**.

LATERAL BUCKLING. A beam bent about its strong axis may fail by combined bending and twisting. The compression flanges or fibers then undergo appreciable lateral displacements. Lateral buckling can be avoided by providing enough lateral supports or by using cross sections with sufficiently large torsional rigidity and strength.

LATERAL DEFLECTION. The component of **deflection** perpendicular to the applied load. A beam whose section has markedly different moments of inertia about its principal axes shows large lateral deflections when the plane of the loads is almost perpendicular to the strong axis. (See **unsymmetrical bending**.) For example, when the cross section is rectangular, if the loads are directed along one diagonal, the other diagonal becomes the neutral axis and deflection is in the direction perpendicular to the neutral axis. Therefore, if the cross section is a narrow rectangle, a load directed almost perpendicular to the strong axis almost will cause lateral deflection alone.

LATERAL LOAD. Lateral load or transverse load is a force perpendicular to the axis of a beam, or perpendicular to the middle surface of a membrane, plate, or shell.

LATERAL MAGNIFICATION. The linear magnification normal to the optical axis of an axially symmetric system is the radial or lateral magnification dr'/dr where dr is in the radial direction in object space and dr' is the conjugate element in image space. If dr'_i/dr_i is the lateral magnification of the i th surface, $i = 1, 2, \dots, N$ of an optical system, then the lateral magnification of the system is

$$\frac{dr'}{dr} = \prod_{i=1}^N \frac{dr'_i}{dr_i}$$

In **Gaussian optics**, at each surface of an axially symmetrical system, the lateral magnification is

$$m_i = r'_i/r_i$$

where r_i is the distance of an object point from the axes.

LATERAL SPHERICAL ABERRATION.

The difference between the reciprocals of the image distances for paraxial and rim rays. (See also **longitudinal spherical aberration**.) If S' and S'_h are the image distances for paraxial rays and rays which strike a lens a distance h from its center, respectively, then $S' - S'_h$ is the longitudinal spherical aberration, and $(1/S'_h) - (1/S')$ is the lateral spherical aberration.

LATIN SQUARE. An experimental design based on a $p \times p$ array of p letters such that each letter occurs once and only once in each row and column; e.g., for $p = 4$ and the letters A, B, C, D

A	B	C	D
B	D	A	C
C	A	D	B
D	C	B	A

Such a layout might, for example, correspond to 16 plots and four treatments represented by the letters.

The design may be generalized to allow for a further treatment represented by Greek letters and is then known as a Graeco-Latin square, e.g.

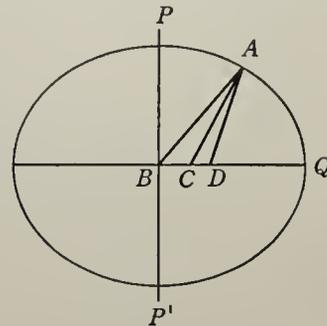
$A\alpha$	$B\beta$	$C\gamma$	$D\delta$
$B\gamma$	$A\delta$	$D\alpha$	$C\beta$
$C\delta$	$D\gamma$	$A\beta$	$B\alpha$
$D\beta$	$C\alpha$	$B\delta$	$A\gamma$

Here no combination of Roman and Greek letters occurs more than once. More general designs are sometimes known as Hyper-Graeco-Latin squares.

The purpose of all these designs is to provide independent comparisons of row, column and treatment effects.

LATITUDE. 1. *Terrestrial Latitude.* As a first approximation consider the earth as a sphere rotating about an axis passing through the north and south poles of rotation. The great circles on the sphere passing through the poles of the earth are known as meridians. The plane passing through the center of the earth perpendicular to the axis of rotation is the plane of the equator and cuts the surface of the earth in the great circle known as the equator. The geographic latitude of a point on the surface of the earth is the angular distance north or south of the equator through 90° .

When the earth is considered as a **geoid**, there are *astronomic*, *geocentric* and *geodetic latitudes* for a point on the surface of the geoid. The figure shows a cross section of the



geoid in the plane of a meridian. PB represents the axis of rotation, $PAQP'$ is an elliptical meridian of the geoid through the point A , on the surface of the geoid. AB is a line from A to the center of the geoid and ABQ is the geocentric latitude of the point A . AC is a normal to the surface of the geoid, and ACQ is the geodetic latitude of A . AD is the direction of gravity at A , and ADQ is the astronomic latitude of A .

Astronomic and geodetic latitude differ slightly at various places on the earth due to uneven distribution of mass within the earth. The difference is referred to as the *deflection of the vertical* and can be determined by precise determination of astronomic latitudes at stations arranged about a circle centered upon the point for which geodetic latitude is desired.

Conversion from astronomic to geocentric latitude and *vice versa* can be accomplished on the basis of the dimensions of the standard geoid. If we call astronomic latitude ϕ and geocentric latitude ϕ' we have

$$\phi - \phi' \equiv v = m \frac{\sin 2\phi}{\sin 1''} - \frac{1}{2} \left(m^2 \frac{\sin 4\phi}{\sin 1''} \right) + \frac{1}{3} \left(m^3 \frac{\sin 6\phi}{\sin 1''} \right) - \dots$$

$$m = \frac{e^2}{2 - e^2} = 0.003373$$

in which e is the eccentricity of the geoid. For all practical purposes one need only the first two terms of the expansion and (using the data for Hayford's geoid) we have

$$\phi - \phi' \equiv V = 695''.65 \sin 2\phi - 1''.17 \sin 4\phi.$$

The astronomic latitude ϕ may be determined observationally to any desired accuracy.

2. *Celestial Latitude.* The position of a heavenly body may be referred to the **ecliptic** as the fundamental great circle. Celestial latitude is the angular distance north or south of the ecliptic measured along a great circle passing through the poles of the ecliptic.

3. *Galactic Latitude.* The position of a heavenly body may be referred to the plane of the galaxy (milky way) as the fundamental great circle. Galactic latitude is the angular distance north or south of the galaxy measured along a great circle passing through the poles of the galaxy.

LATTICE. (1) A set S of elements a, b, \dots , is *partially ordered* if a binary relation often denoted by the symbol \leq , which is reflexive, antisymmetric and transitive (see **relation**), is defined for certain of its elements. For example, let a, b, \dots denote the subsets of S and let a stand in the given relation to b if the subset a is included in the subset b . A partially ordered set is a **lattice** if for any two elements a, b there exists an element c which is a least upper bound for a, b ; that is, such that $a \leq c, b \leq c$ and if $a \leq e, b \leq e$, then $c \leq e$, and also an element d which is a greatest lower bound for a, b ; that is, such that $d \leq a, d \leq b$ and if $f \leq a, f \leq b$ then $f \leq d$. These elements c and d are called the **join** (or **sum**) and the **meet** (or **intersection**) respectively of a and b and are denoted by $c = a \cup b$ and $d = a \cap b$. The terms **cup** and **cap** are also used and it is also

common to write $a + b$ for $a \cup b$ and ab or $a \times b$ for $a \cap b$. (See **Boolean algebra**.)

(2) Atomic lattices are important in the determination of **crystal structure**; while lattices of fissionable and non-fissionable materials are used in heterogeneous nuclear reactors.

LATTICE CONSTANT. A length representing the size of the **unit cell** in a **crystal lattice**. In a **cubic crystal**, this is just the length of the side of the unit cell, but such a simple definition is not in general possible, and the lattice constant must be chosen according to the geometry of the structure in each case.

LATTICE DIMENSIONS. According to the **Bragg formula** the spacing of the **atomic planes** can be deduced from the X-ray diffraction pattern and a knowledge of the X-ray wavelength, which can itself be measured by diffraction from a ruled grating.

LATTICE ENERGY OF CRYSTALS. The energy required to dissociate a crystal completely into gaseous ions at infinite distance from each other. (For method of calculation see **Born-Mayer equation**.)

LATTICE MODEL. Apart from its importance for the solid state, a **lattice** model is also often used for calculations in the liquid state. This is partly justified by the fact that the available volume per particle in the liquid phase near the melting point is only slightly ($\sim 10\%$) larger than in the solid. The main difference is that the number of nearest neighbors has a well-defined value in crystals, but only an average temperature-dependent value in liquids.

LATTICE, RECIPROCAL. See **reciprocal lattice**.

LATTICE SUM. In general, any sum whose terms are functions of the positions of points in a space **lattice**. The term is applied particularly to those sums involved in evaluating the **Coulomb energy** of an ionic crystal, where the magnitude of each term depends on the inverse distance of the lattice point from the origin, with a sign depending on the charge of the ion at that point. Such sums are difficult to evaluate directly, as they converge only very slowly, but special methods have been devised to calculate them. (See **Evjen method**.)

LATTICE THEORY OF ELASTIC COEFFICIENTS. See elastic coefficients, lattice theory of.

LATTICE, THERMAL MOTIONS IN. See thermal motions in a lattice.

LATTICE VIBRATION CONTRIBUTION TO SPECIFIC HEAT. The contribution to the molar specific heat at constant volume, due to lattice vibrations, is given by Debye's equation

$$C_v = 3Nkf(x)(\Theta/T)$$

where

$$f(x) = 3x^{-3} \int_0^x \frac{e^y y^4}{(e^y - 1)^2} dy$$

where N is Avogadro's number, k is Boltzmann's constant, and Θ is the **Debye temperature**. (See **Debye heat capacity equation**.)

LAUE EQUATIONS. The Laue equations are necessary conditions for an extremal of the intensity of radiation (x-rays) diffracted by a crystal. They have the form

$$(\mathbf{u} - \mathbf{u}_0) \cdot \mathbf{a}_1 = n_1 \lambda$$

$$(\mathbf{u} - \mathbf{u}_0) \cdot \mathbf{a}_2 = n_2 \lambda$$

$$(\mathbf{u} - \mathbf{u}_0) \cdot \mathbf{a}_3 = n_3 \lambda$$

where \mathbf{u}_0 is a unit vector in the direction of the incident beam, \mathbf{u} is a unit vector in the direction of the scattered beam, λ is the wavelength, and a_1, a_2, a_3 are the crystallographic axial vectors in the Zachariasen notation.

LAURENT SERIES. A generalization of the **Taylor series**, making it possible to develop a function of the **complex variable** about an isolated **singular point** $z = z_0$. The result is a two-way power series (positive and negative powers)

$$f(z) = \sum_{n=-\infty}^{\infty} a_n (z - z_0)^n$$

with coefficients given by

$$a_n = \frac{1}{2\pi i} \int_C \frac{f(t) dt}{(t - z_0)^{n+1}}$$

The **contour** consists of two concentric circles around the singular point, in the positive and negative directions, respectively. The series converges in the annulus determined by the two circles. If there are an infinite number of negative powers in the series, $f(z)$ has an essential **singularity** at z_0 ; if a_{-k} is not zero

and all other coefficients $a_n = 0$ for $n < -k$, the singular point is a **pole** of order k . The coefficient

$$a_{-1} = \frac{1}{2\pi i} \int_C f(t) dt$$

is called the **residue** of the function $f(z)$ about the point z_0 .

LAW OF AREAS. See **Keplerian motion**.

LAW OF COMPOSITION OF FORCES. **Equilibrium; forces, parallelogram of; forces, polygon of.**

LAW OF CONSERVATION OF LEPTONS.

The statements that to each **lepton** one can assign a leptonic number, $+1$ or -1 , and to any other particle a leptonic number zero, and that in any physical process the total leptonic number is conserved. One can assign the lepton number $+1$ to e^- , then because $e^+ + e^- \rightarrow 2\gamma$, the positron, e^+ , has leptonic number -1 . The following assignments are then consistent with experiments:

<i>Leptonic number +1</i>	<i>Leptonic number -1</i>
e^- (electron)	e^+ (positron)
ν (neutrino)	$\bar{\nu}$ (anti-neutrino)
μ^- (negatively charged μ -meson)	μ^+ (positively charged μ -meson)

LAW OF CONSTANT HEAT SUMMATION. See **Hess law**.

LAW OF COSINES. In a plane triangle: $a^2 = b^2 + c^2 - 2bc \cos A$. In a spherical triangle: $\cos a = \cos b \cos c + \sin b \sin c \cos A$, or $\cos A = -\cos B \cos C + \sin B \sin C \cos a$.

LAW OF ERROR. See **normal distribution**.

LAW OF HESS. See **Hess law**.

LAW OF LARGE NUMBERS. A theorem in probability which takes various forms; the general effect is that if an event has probability p , then in a large number n of trials, the proportion of events is ever closer to np in a probabilistic sense.

More precisely the "weak" law of large numbers states that if $\{n_k\}$ is a sequence of independent random variables with a common distribution and mean m , then for every $\epsilon > 0$

$$P_{n \rightarrow \infty} \left\{ \left| \frac{\sum_{i=1}^n x_i}{n} - \mu \right| > \epsilon \right\} \rightarrow 0.$$

The "strong" law of large numbers states that **Bernoulli's theorem** and the **Gauss-Laplace theorem** are special cases of the law. More general forms are also studied.

LAW OF MASS ACTION. The condition for chemical equilibrium (see **chemical affinity**, **chemical potentials**) gives for an ideal system

$$K(T,p) = x_1^{\nu_1} \cdots x_c^{\nu_c}. \quad (1)$$

The quantity $K(T,p)$ is the *equilibrium constant* of the reaction, $x_1 \cdots x_c$ are the **mole fractions** and $\nu_1 \cdots \nu_c$ the **stoichiometric coefficients**. For a non-ideal system (see **reference systems**) one has the generalized law of mass action

$$K(T,p) = (x_1\gamma_1)^{\nu_1} \cdots (x_c\gamma_c)^{\nu_c} \quad (2)$$

where $\gamma_1 \cdots \gamma_c$ are the activity coefficients.

LAW OF SINES. In a plane triangle:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}.$$

In a spherical triangle

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

LAW OF TANGENTS. In a plane triangle:

$$\frac{a-b}{a+b} = \frac{\tan \frac{1}{2}(A-B)}{\tan \frac{1}{2}(A+B)}.$$

LCAO APPROXIMATION. See **molecular orbitals method**.

LEAST ACTION, PRINCIPLE OF. This principle, first enunciated rather loosely by Maupertuis in the 18th century, states that the actual motion of a conservative dynamical system from P_1 to P_2 takes place in such a way that the **action** has a stationary value with respect to all other possible paths between P_1 and P_2 corresponding to the *same energy*. (Cf. **Hamilton principle**.)

LEAST CIRCLE OF ABERRATION. The smallest circular cross section of a bundle of rays forming a radially symmetrical caustic surface is the **least circle of aberration** of the bundle.

"LEAST CONSTRAINT," GAUSS PRINCIPLE OF. (1) The motion of connected points

is such that, for the elementary motion actually taken, the sum of the products of the mass of each particle into the square of the distance of its deviation from the position it would have reached if free, is a minimum.

(2) The motion of a system of material points interconnected in any way and submitted to any influences, agrees at each instant as closely as possible with the motion the points would have if they were free. The actual motion takes place so that the constraints on the system are the least possible. For the measurement of the constraint, during any infinitesimal element of time, take the sum of the products of the mass of each point by the square of its deviation from the position the point would have occupied at the end of the element of time, if it had been free.

LEAST-ENERGY PRINCIPLE. A principle relating to stable equilibrium, and having very wide application. If a conservative system is in stable equilibrium, any slight change in its condition or configuration requiring the performance of work will put it out of equilibrium, so that, if the system is now left to itself, it will return to its former state and in so doing will give up the energy imparted when it was disturbed.

(Cf. **potential energy**.)

LEAST SQUARES. A method of **smoothing** or **curve fitting** which selects the fitted curve (or surface) so as to minimize the sum of the squares of the deviations of the given points from the curve (or surface). In the linear case one has a system which can be expressed in matrix form as

$$\mathbf{Xc} = \mathbf{y} + \mathbf{d},$$

where the matrix \mathbf{X} is $N \times n$, with $N > n$; \mathbf{y} is the vector of measured quantities; and the length of the residual vector \mathbf{d} is to be minimized. If the measurements are equally weighted, this means that c is chosen so that $\mathbf{d}^T\mathbf{d}$ is as small as possible; if not, then $\mathbf{d}^T\mathbf{W}\mathbf{d}$ is to be made as small as possible, where \mathbf{W} is the diagonal matrix whose elements are the weights. The solution is that of the system

$$\mathbf{X}^T\mathbf{W}\mathbf{X}\mathbf{c} = \mathbf{X}^T\mathbf{W}\mathbf{y},$$

where $\mathbf{W} = \mathbf{I}$ if the weights are equal (see **matrix inversion**). For fitting a polynomial the matrix \mathbf{X} has the form

$$\mathbf{X} = \begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ 1 & x_3 & x_3^2 & \cdots & x_3^{n-1} \\ 1 & x_4 & x_4^2 & \cdots & x_4^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \end{pmatrix}$$

where x_i is the value of the independent variable for the i th measurement. However, the use of **orthogonal polynomials** is advisable for this problem.

In polynomial fitting the unknown parameters are the coefficients of the polynomial, and they occur linearly in the equations they satisfy. In case the theoretical relationship is of the form

$$\eta = f(\xi; \alpha, \beta, \dots)$$

where the parameters α, β, \dots , do not occur linearly, it is generally necessary to linearize the equations in order to solve them. Two methods can be suggested, and either requires initial, sufficiently close, approximations α_0, β_0, \dots , which are to be improved progressively. By one method one forms the quantity to be minimized,

$$\delta(\alpha, \beta, \dots) = \Sigma [\eta_i - f(\xi_i; \alpha, \beta, \dots)]^2,$$

the summation extending over all observations, and applies a method of **steepest descent**. In the other method one expands in **Taylor series**

$$f(\xi_i; \alpha_0, \beta_0, \dots) + (\alpha - \alpha_0)f_\alpha(\xi_i; \alpha_0, \beta_0, \dots) \\ + (\beta - \beta_0)f_\beta(\xi_i; \alpha_0, \beta_0, \dots) + \dots = \eta_i + \delta_i.$$

Here one wishes to minimize $\Sigma \delta_i^2$, or possibly $\Sigma \omega_i \delta_i^2$ where the ω_i are weights. If α_0, β_0, \dots are sufficiently close to the true α, β, \dots , one can omit quadratic and higher terms in $\alpha - \alpha_0, \beta - \beta_0, \dots$, and solve as a linear least squares problem for these differences. The result could be added as partial corrections to α_0, β_0, \dots , and the process repeated. (See W. Edwards Deming, *Statistical Adjustment of Data*, John Wiley & Sons, Inc., 1938.)

LEAST WORK, THEOREM OF. A special form of the theorem of minimum **complementary energy** which applies to a linear elastic system whose boundary displacements are zero wherever they are specified. Under these conditions the complement of the strain energy is a minimum $V = V'$, and the strain energy U is equal to V , which in turn equals the work done by the applied forces. This work is then

minimum and if R_i is one of the independent unknown components of reaction or internal force

$$\partial V' / \partial R_i = 0.$$

(See Castigliano's theorem.)

LEBESGUE INTEGRAL. The Lebesgue integral

$$\int_{(E)} f(x) dx$$

of the bounded **measurable function** $f(x)$, taken over the **measurable set** E which is of finite measure, is defined as the limit to which either

$$\sum_{r=1}^{r=n} a_{r-1} m(e_r), \text{ or } \sum_{r=1}^{r=n} a_r m(e_r),$$

tends, as the greatest of the numbers a_r, a_{r-1} converges to zero; where (L, U) is divided into n parts (a_{r-1}, a_r) , for $r = 1, 2, 3, \dots, n$; and $a_0 = L, a_n = U$; and where e_r is that set of points of E , for all of which

$$a_{r-1} \leq f(x) < a_r; \quad r = 1, 2, 3, \dots, n-1;$$

and e_n is that set for which $a_{n-1} \leq f(x) \leq a_n$. U and L denote the upper and lower bounds of $f(x)$ in E .

LEBESGUE MEASURE. See **measure**.

LEBESGUE-STIELTJES INTEGRAL. Whenever the **Lebesgue integral**

$$\int_{\alpha}^{\beta} F(\xi) d\xi$$

exists, where $\alpha = \phi(a), \beta = \phi(b)$, its value may be said to define that of the **Lebesgue-Stieltjes integral**

$$\int_a^b f(x) d\phi(x),$$

of the measurable function $f(x)$, with respect to the monotone function $\phi(x)$.

LE CHATELIER PRINCIPLE. See **Chatelier (Le)-Braun principle**.

LE CHATELIER THEOREM. See **Chatelier (Le) theorem**.

LEDUC LAW. See **Amagat law**.

LEE WAVES. Standing waves formed in a fluid stream under gravity in the lee of an obstacle, usually on the bottom boundary of

the fluid. Theoretically there exists a system of lee waves in an infinitely deep stream of uniform velocity and static stability in which the wave amplitude increases upwards and decreases downstream: but in a nonuniform stream in which the coefficient

$$l^2 = \frac{g\beta}{U^2} - \frac{1}{U} \frac{\partial^2 U}{\partial z^2}$$

varies with height (z) in a suitable way (making it, in this respect, equivalent to a stream made up of superposed layers), a train of waves whose amplitude is at a maximum at some middle level in the fluid (i.e., at a height comparable with the size of the obstacle) may extend far downstream.

The simplest case is that of surface waves on water behind a ship or immersed obstacle, this exemplifying the more general result that if the static stability is concentrated into a single discontinuity the amplitude is a maximum at that level.

Lee waves are waves which travel through the fluid with a velocity equal and opposite to that of the stream so that they remain stationary relative to the obstacle producing them, and cannot occur in super-undal flow.

LEGENDRE EQUATION. The differential equation

$$(1 - x^2)y'' - 2xy' + n(n + 1)y = 0.$$

It is a special case of the associated Legendre equation

$$(1 - x^2)y'' - 2xy' + \left[n(n + 1) - \frac{m^2}{1 - x^2} \right] y = 0.$$

(See Legendre functions.)

LEGENDRE FUNCTIONS. The only solutions, with continuous first derivative for all x in the interval $-1, 1$, of the Legendre equation

$$(1 - x^2)y'' - 2xy' + n(n + 1)y = 0, \\ n = 0, 1, 2, \dots,$$

are the Legendre polynomials, $P_0(x), P_1(x), P_2(x), \dots$. But if we allow $y(x)$ to become infinite at the end-points, then the equation is also satisfied by functions $Q_0(x), Q_1(x), \dots$ called Legendre functions of the second kind. On the other hand, if we drop the restriction that n be a non-negative integer, we get

Legendre functions $P_\nu(x)$ of the first kind (the restriction is usually added that $P_\nu(1) = 1$), which are finite throughout. Similarly, an associated Legendre function of the first kind is a (bounded) solution of the associated Legendre equation (see above), while an associated function of the second kind is a solution (allowed to become infinite at the end-points) of the associated equation for certain particular values of n and m .

LEGENDRE POLYNOMIALS. The class of special functions obtained by orthogonalizing over the interval $[-1, 1]$ the sequence of powers $1, x, x^2, \dots$ with the weighting function $w(x) \equiv 1$. They are the eigenfunctions of the eigenvalue problem:

$$(1 - x^2)y'' - 2xy' + \lambda y = 0$$

under the boundary condition that y is to remain finite at the end-points $x = \pm 1$. The eigenvalues here are $\lambda = n(n + 1)$, with $n = 0, 1, 2, \dots$ and the n th Legendre polynomial belongs to the eigenvalue $n(n + 1)$. The first few polynomials (in the form usually given; here the normalizing factor is $(2n + 1)^{1/2}$):

$$P_0(x) = 1, \quad P_1(x) = x, \quad P_2(x) = \frac{1}{2}(3x^2 - 1), \\ P_3(x) = \frac{1}{2}(5x^3 - 3x) \dots$$

If we set $x = \cos \theta$, these can also be expressed conveniently in terms of multiple angles, thus

$$P_0(\cos \theta) = 1, \quad P_1(\cos \theta) = \cos \theta; \\ P_2(\cos \theta) = \frac{1}{4}(3 \cos 2\theta + 1); \\ P_3(\cos \theta) = \frac{1}{8}(5 \cos 3\theta + 3 \cos \theta), \dots$$

It is in this form that they naturally arise in problems of temperature, potential, etc. for a sphere.

Their generating function is $(1 - 2xt + t^2)^{-1/2}$. They satisfy the recursion formula

$$(n + 1)P_{n+1}(x) - x(2n + 1)P_n(x) \\ + nP_{n-1}(x) = 0.$$

Their Rodrigues formula is:

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n.$$

They have the minimum property, among all polynomials of degree n with the same coefficient of x^n , of having the least mean distance from the x -axis; that is,

$$\int_{-1}^{+1} P_n(x) dx = \min.$$

(For an integral representation of the Legendre polynomials, see **Schl afli formula**.)

LEHMAN-FILHES METHOD. See spectroscopic binary stars.

LEHMANN REPRESENTATION. A representation of the vacuum expectation value of two Heisenberg fields (**Wightman function** of order 2), first obtained by Lehmann. The integral representation for the vacuum expectation of the Wightman function $W^{(2)}(x,y) = \langle \Psi_0, \phi(x)\phi(y)\Psi_0 \rangle$, where $\phi(x)$ is a real scalar field (i.e., which under an inhomogeneous Lorentz transformation $\{a,\Lambda\}$ transforms according to

$$U(a,\Lambda)\phi(x)U(a,\Lambda)^{-1} = \phi(\Lambda x + a) \quad (1)$$

where $U(a,\Lambda)$ is a unitary (antiunitary if Λ includes a time inversion) representation of the Lorentz group can be found as follows: By virtue of Lorentz invariance

$$W^{(2)}(x,y) = W^{(2)}(\Lambda x + a, \Lambda y + a) \quad (2)$$

From pure translation invariance ($\Lambda = I$) one infers that $W^{(2)}(x,y) = W^{(2)}(x - y)$. On the assumption that there exist a unique, invariant vacuum state Ψ_0 , such that $U(a,\Lambda)\Psi_0 = \Psi_0$ and that the set of states $|p_\mu^{(n)}, \alpha\rangle$ with $p_0^{(n)} \geq 0$, $p_\mu^{(n)}p^{(n)\mu} \geq 0$, which are eigenstates of the total energy momentum operator P_μ and some other observables (denoted by α) are complete, one deduces that

$$\begin{aligned} W^{(2)}(x,y) &= \sum_{p^{(n)}, \alpha} \langle \Psi_0 | \phi(x) | p_\mu^{(n)}, \alpha \rangle \\ &\quad \langle p_\mu^{(n)}, \alpha | \phi(y) | \Psi_0 \rangle \\ &= \sum_{p^{(n)}, \alpha} \langle \Psi_0 | \phi(0) | p_\mu^{(n)}, \alpha \rangle \\ &\quad \langle p_\mu^{(n)}, \alpha | \phi(0) | \Psi \rangle \exp[ip^{(n)}(x - y)] \end{aligned} \quad (3)$$

where use has been made of the fact that $U(a,1) = \exp(-iP_\mu a^\mu)$. Defining

$$\begin{aligned} \rho(p^2) &= (2\pi)^3 \sum_\alpha \langle \Psi_0 | \phi(0) | p, \alpha \rangle \langle p, \alpha | \phi(0) | \Psi_0 \rangle \\ &= (2\pi)^3 \sum_\alpha |\langle \Psi_0 | \phi(0) | p, \alpha \rangle|^2 \geq 0 \end{aligned} \quad (4)$$

where the sum runs over all states with fixed energy momentum p , one obtains

$$\begin{aligned} W^{(2)}(x - y) &= -\frac{1}{(2\pi)^3} \int \theta(k_0) \rho(k^2) \exp[-ik(x - x')] d^4k \end{aligned} \quad (5)$$

where the factor $\theta(k_0)$ (which is equal to +1 if $k_0 \geq 0$, and equal to 0 if $k_0 < 0$) enters, since due to the spectrum condition only positive energy vectors k are involved in the sum (3). If we write

$$\rho(k^2) = \int_0^\infty \rho(\kappa^2) \delta(k^2 - \kappa^2) d\kappa^2 \quad (6)$$

the limits are from 0 to ∞ since by (4) k is the energy momentum of a physical state hence k^2 (which corresponds to the square of the mass of the state) must be greater than zero. Finally therefore

$$\begin{aligned} W^{(2)}(x - y) &= \int_0^\infty d\kappa^2 \rho(\kappa^2) \left(\frac{1}{(2\pi)^3} \right) \\ &\quad \int_0^\infty d^4k \exp[-ik(x - y)] \delta(k^2 - \kappa^2) \epsilon(k_0) \end{aligned} \quad (7)$$

Alternatively, since the second integral is usually denoted by

$$\begin{aligned} \Delta^{(+)}(x - y; \kappa^2) &= -\frac{1}{(2\pi)^3} \int d^4k \epsilon(k_0) \\ &\quad \exp[-ik(x - y)] \delta(k^2 - \kappa^2) \end{aligned} \quad (8)$$

one deduces that

$$W^{(2)}(x - y) = \int_0^\infty d\kappa^2 \rho(\kappa^2) \Delta^{(+)}(x - y; \kappa^2) \quad (9)$$

Equation (9) is the Lehmann representation for the two-fold vacuum expectation value of a scalar field. It is a special case of the **Dyson representation** of a causal commutator. For the vacuum expectation value of the time-ordered product (one-particle Green's function),

$$G(x,y) = \langle \Psi_0, P(\phi(x)\phi(y))\Psi_0 \rangle,$$

where

$$\begin{aligned} P(A(x)B(y)) &= A(x)B(y), \quad \text{if } x_0 > y_0 \\ &= B(y)A(x), \quad \text{if } y_0 > x_0 \end{aligned}$$

the following result holds

$$G(x,x') = \int_0^\infty d\kappa^2 \rho(\kappa^2) \int_{-\infty}^{+\infty} d^4k \frac{\exp[-ik(x-x')]}{k^2 - \kappa^2 - i\epsilon}$$

$$= \int_0^\infty d\kappa^2 \rho(\kappa^2) \Delta_F(x-x'; \kappa^2)$$

where $\rho(\kappa^2)$ is the same weight function as defined above. For a spinor field $\psi(x)$, the following representation is obtained for the one-particle Green's function:

$$G(x,x') = i(\Psi_0, T[\psi(x)\bar{\psi}(x')]\Psi_0)$$

$$= i(\Psi_0, P[\psi(x)\bar{\psi}(x')]\Psi_0)\epsilon(x,x')$$

where

$$\epsilon(x,x') = 1 \quad \text{for } x_0 > x_0'$$

$$= -1 \quad \text{for } x_0 < x_0'$$

$$G(x,x') = \frac{1}{(2\pi)^4} \int d^4p \exp[-ip(x-x')]G(p)$$

where

$$G(p) = \frac{A_0}{\gamma p + m}$$

$$+ \int_{m+\mu}^\infty d\kappa \left(\frac{A_+(\kappa)}{\gamma p + \kappa + i\epsilon} + \frac{A_-(\kappa)}{\gamma p - \kappa + i\epsilon} \right)$$

and the A 's are real and non-negative.

LEIBNITZ RULE. A formula for the n th derivative of the product of two functions in terms of the successive derivatives of the factors:

$$\frac{d^n}{dx^n}(uv) = \frac{d^n u}{dx^n} \cdot v + \binom{n}{1} \frac{d^{n-1}u}{dx^{n-1}} \cdot \frac{dv}{dx}$$

$$+ \binom{n}{2} \frac{d^{n-2}u}{dx^{n-2}} \frac{d^2v}{dx^2} + \dots + u \frac{d^n v}{dx^n}$$

$$= \sum_{k=0}^n \binom{n}{k} \frac{d^{n-k}u}{dx^{n-k}} \frac{d^k v}{dx^k}$$

The coefficients are **binomial coefficients**.

LENGTH, LINEAR EXTRAPOLATION. See **linear extrapolation length**.

LENGTH OF A CURVE. The length of a straight line is interpreted experimentally to mean the number of times another straight line of unit length can be superimposed on the given line. Since this operation cannot be conveniently applied to a curve the concept is generalized to the **limit** of the sum of chords to the curve. As the number of chords in-

creases without limit each chord separately approaches zero as a limit.

In rectangular coordinates, if the curve is described by $f(x,y) = 0$, its length from the point (a,c) to the point (b,d) is given by either one of the **definite integrals**

$$s = \int_a^b \sqrt{1+y'^2} dx = \int_c^d \sqrt{1+x'^2} dy$$

where $y' = dy/dx$ and $x' = dx/dy$. If t is a parameter, $x = f_1(t)$, $y = f_2(t)$, the arc length between $t = a$ and $t = b$ is

$$s = \int_a^b \sqrt{x_t^2 + y_t^2} dt$$

where $x_t = dx/dt$ and $y_t = dy/dt$. In **polar coordinates**, if $r = f(\theta)$, $r' = dr/d\theta$ and $\theta' = d\theta/dr$,

$$s = \int_{\theta_1}^{\theta_2} \sqrt{r^2 + r'^2} d\theta = \int_{r_1}^{r_2} \sqrt{r^2 \theta'^2 + 1} dr$$

LENGTH, RELAXATION. See **relaxation length**.

LENNARD-JONES POTENTIAL. See **interatomic potential**.

LENS, ELECTRON. A device using either electric fields or an arrangement of coils or magnets to produce a focusing force on a beam of electrons.

LENS EQUATION, ELECTRON-OPTICAL. See **electron-optical lens equation**.

LENS EQUATION, GAUSSIAN. See **Gaussian lens equation**.

LENS POWER. The power of a lens is the reciprocal of its **foveal length** and is usually measured in diopters (i.e., reciprocal meters). Thus for an axially symmetric lens having spherical surfaces of radii r_1, r_2 and axial separation t the power is

$$\phi = \frac{1}{f} = \phi_1 + \phi_2 - \frac{t}{n} \phi_1 \phi_2,$$

where f is the focal length, n , the relative index of refraction of the lens medium to the surrounding medium and ϕ_1, ϕ_2 , the powers of the surfaces. An appropriate sign convention would be to have $r_i > 0$ when the center of curvature is to the right of the vertex (i.e., axial intersection of the surface), so that

$$\phi_1 = \frac{n-1}{r_1}, \quad \phi_2 = -\frac{n-1}{r_2}$$

The thin lens approximation, $t \approx 0$, is often used so that

$$\phi = \phi_1 + \phi_2 = (n-1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right).$$

LENZ LAW. A general law of electromagnetic induction stating that when the **electromotive force** induced by a variation in **magnetic flux** produces an electric current, the magnetic effect of that current tends to oppose the change in magnetic field which produced it.

LEPTON(S). The generic name of the “elementary” particles having spin $\frac{1}{2}$ and mass less than that of the nucleons. Three such types of leptons are known:

Particle	Mass (in electron masses)	Mean Life	Charge
μ^\pm (μ -meson)	206.9 ± 0.1	2.22 ± 0.02 $\times 10^{-6}$ sec	$\pm e$
e^\pm (electron and positron)	1	stable	$\pm e$
$\nu, \bar{\nu}$ (neutrino and antineutrino)	0	stable	0

LEPTON CONSERVATION. See **law of conservation of leptons**.

LETHARGY. In neutron transport theory the lethargy variable, denoted u , is often used in place of the neutron energy. They are related according to

$$E = E_0 e^{-u}, \quad u = \ln \left(\frac{E_0}{E} \right)$$

where E_0 is an arbitrarily chosen “source energy.”

LEVEL. The difference of a quantity from an arbitrarily specified reference quantity or position. In this general sense, the word level is used widely throughout science and engineering. In these specific applications, its meaning is readily inferred from the context. In quantum mechanics it is essentially synonymous with energy **eigenstate**.

LEVEL ABOVE THRESHOLD (SENSATION LEVEL). Of a sound, the pressure level of the sound in decibels above its threshold of audibility (see **audibility, threshold of**) for the individual observer. Similar definitions are used for the other psychophysical stimuli, such as light.

LEVEL DISPLACEMENTS IN HYDROGEN-LIKE ATOMS. Owing to its interaction with the quantized electromagnetic field, an electron placed in the Coulomb field of a nucleus will experience effects not predicted by the simple **Dirac equation**. Another effect not taken into account by the simple Dirac equation is the motion of the nucleus, which in the Dirac equation is represented by a Coulomb field and assumed infinitely heavy. Finally corrections to the level scheme predicted by the Dirac equation will arise due to the structure of the nucleus, i.e., to its finite size, magnetic moment, etc.

The interaction of the electron with the quantized electromagnetic field (see **quantization of electromagnetic field**) is characterized by the coupling constant

$$\alpha = e^2/\hbar c = 1/137$$

which is small, so that perturbation methods can be applied with α as the development parameter. The convergence of the expansion is quite good (although probably it is only an asymptotic expansion) and calculations up to the fourth order in α are enough to compare the calculated energy displacements with experiments for the case of hydrogen and deuterium ($Z = 1$). The corrections to the dynamics of the electron due to these interactions with the quantized electromagnetic field are called “**radiative corrections**.”

The second order radiative corrections, those for which the electron has interacted twice with the quantized electromagnetic field (i.e., has emitted and then reabsorbed a virtual photon), are listed in Table 1 for the $2^2S_{1/2} - 2^2P_{1/2}$ shift. The corrections to order $\alpha(Z\alpha)^4$ are those in which the electron has interacted once with the external field in between the emission and subsequent absorption of the virtual photon, whereas the contributions to order $\alpha(Z\alpha)^5$ are those in which the electron has interacted twice with the external field in between the emission and absorption. The reduced mass corrections have been included. The symbols $K_0(2.0)$, $K_0(2.1)$ represent the average excitation energies for the 2S and 2P levels, respectively, with

$$K_0(2.0) = 16.160 Ry_\infty$$

$$K_0(2.1) = 0.9704 Ry_\infty$$

Ry_∞ being the Rydberg constant for an infinitely heavy nucleus. The quantity

$\left(\frac{8}{3\pi} \alpha^3 Ry_\infty\right)$ is known as **Lamb's constant** and denoted by L .

The fourth order radiative corrections are listed in Table 2, corresponding to the correction due to the virtual emission and absorption of two photons, but during which time only one interaction with the nuclear field has taken place.

The finite mass effects besides the one already included in Tables 1 and 2 $\left[\text{factors } \left(1 + \frac{m}{M}\right)^{-3} \approx \left(1 - 3 \frac{m}{M}\right) \right]$ have been computed to contribute 0.33 mc/s to the hydrogenic $2S_{1/2}-2P_{1/2}$ level shift, 0.17 ± 0.01 mc/sec in D and 2.8 mc/sec in He^+ .

The finite sizes of the proton and deuteron give rise to corrections for S levels similar to the small energy differences which give rise to the **isotope shift** in heavy nuclei. These "finite" size contribute to the $2^2S_{1/2}-2^2P_{1/2}$

level shift an amount of 0.12 ± 0.02 Mc/s for H, 0.73 ± 0.02 Mc/s for D and 7.1 Mc/s for He.

The comparison between the theoretically calculated level shift and the experimentally observed value are listed in Table 3. The agreement for the case of H is nothing short of amazing, as theoretical and experimental values check within an overall inaccuracy of 0.25 Mc. This compared with the total energy of the levels considered represents a precision of almost 1 part in 10^{10} . The theoretical value for the Lamb shift exceeds the experimental value for the case of H and D by about 0.2 mc. It is expected that the $(\alpha Z)^2$ corrections to the 2nd order radiative corrections will account for this discrepancy. The same contribution to the He^+ would turn out to be 64 ($Z^6 = 64$) times larger or about 13 Mc/s, which is just the excess of the theoretical over experimental value.

For the case of the $n = 3$ level the value of the $3^2S_{1/2}-3^2P_{1/2}$ shift has been experimen-

TABLE 1
2ND ORDER RADIATIVE CORRECTIONS TO $2S_{1/2}-2P_{1/2}$ INTERVAL

	Contribution (Theoretical)	Magnitude of Shift of $2S_{1/2}-2P_{1/2}$ Levels on Mc/s		
		H	D	He^+
Radiative shift (order $\alpha(\alpha Z)^4$)	$Z^4L \left(-2 \log \alpha + \frac{m}{M} + \frac{11}{24} - 2 \log Z - \log \frac{K_0(2.0)}{K_0(2.1)} \right) \left(1 - 3 \frac{m}{M} \right)$	1009.84	1010.63	13168.3
Radiative shift (order $\alpha(\alpha Z)^5$) (2nd order in the external potential)	$Z^5L(3\pi\alpha) \left(1 + \frac{1}{128} - \frac{1}{2} \log 2 + \frac{5}{192} \right) \left(1 - 3 \frac{m}{M} \right)$	7.14	7.13	228.4
Vacuum polarization (order $\alpha(\alpha Z)^4$)	$Z^4L \left(-\frac{1}{5} \right) \left(1 - 3 \frac{m}{M} \right)$	-27.08	-27.11	-433.9
Magnetic moment (order $\alpha(\alpha Z)^4$)	$Z^4L \left(\frac{1}{2} \right) \left(1 - 2.75 \frac{m}{M} \right)$	67.71	67.77	1084.7

TABLE 2
4TH ORDER RADIATIVE CORRECTIONS TO $2^2S_{1/2}-2^2P_{1/2}$ INTERVAL

	Contribution	Magnitude of Shift in Mg/sec		
		H	D	He^+
Radiative shift (order $\alpha^2(\alpha Z)^4$)	$Z^4L \left(\frac{3\alpha}{2\pi} \right) (0.52)$	0.24 ± 0.13	0.24 ± 0.13	3.9
Magnetic moment (order $\alpha^2(\alpha Z)^4$)	$Z^4L \left(-0.328 \frac{\alpha}{\pi} \right)$	-0.11	-0.11	-1.7
Vacuum polarization (order $\alpha^2(\alpha Z)^4$)	$Z^4L \left(-\frac{41}{54} \frac{\alpha}{\pi} \right)$	-0.24	-0.24	-3.8

TABLE 3
TOTAL SHIFT OF $2^2S_{1/2}-2^2P_{1/2}$ INTERVAL

	H	D	He ⁺
Theoretical	1057.95 ± 0.15	1059.21 ± 0.16	14055.8 ± 3.0
Experimental	1057.77 ± 0.10	1059.00 ± 0.11	14043.0 ± 3.0

tally measured by Lamb with the result of (315.0 ± 0.3) Mc/s which compares extremely well with the theoretically computed value of 314.94 ± 0.05 Mc/s. The theoretical value for the $4^2S_{1/2}-4^2P_{1/2}$ Lamb shift is 133.10 ± 0.02 Mc/sec. No precise experimental value is available.

Finally, Table 4 lists the level shifts for fixed Coulomb field ($Z = 1$) for several S , P and D states, in first approximation in the external field and without 4th order radiative corrections.

TABLE 4
LEVEL SHIFT FOR FIXED COULOMB FIELD

State	Level Shift in Mc/sec
1 S	8126.87
2 S	1039.24
3 S	309.69
4 S	130.95
2 $P_{1/2}$	-12.88
2 $P_{3/2}$	12.55
3 $P_{1/2}$	-3.49
3 $P_{3/2}$	4.05
4 $P_{1/2}$	-1.41
4 $P_{3/2}$	1.77
3 $D_{1/2}$	-1.30
3 $D_{3/2}$	1.21

LEVEL OF ENERGY. The value of an energy, as for example as electrostatic potential, thought of as measured vertically above some fixed origin. Not the same as energy level.

LEVEL POPULATION. The number of systems is an ensemble which are in each of the energy levels permissible for a quantum mechanical system.

LEVEL, QUANTIZATION. See quantization level.

LEVEL SCHEME. See energy level diagram.

LEVEL, SENSATION. See level above threshold.

LEVEL WIDTH. The energy spectrum of the bound states of a stationary quantum me-

chanical system consists of a number of discrete values of energy called **energy levels**. However, if the system is not quite stationary, but decays in time with a mean life τ , then by the uncertainty principle each energy level becomes slightly indeterminate in energy, covering an energy range of the order of $\Gamma = \hbar/\tau$, where $2\pi\hbar$ is Planck's constant, and Γ is the level width. If a level may decay in any of a number of different ways a, b, \dots , the width may be considered to be the sum of a number of partial widths, $\Gamma = \Gamma_a + \Gamma_b + \dots$, where Γ_a is proportional to the probability per second that the level will decay according to mode a . If the lifetime of the level is directly measurable, then the width may be inferred from the above relation. More usually, however, the level appears as a resonance, say in a nuclear reaction, the spread in energy of the resonance being proportional to Γ . (See **Breit-Wigner formula**, for example, where Γ has its usual definition of the full width of the resonance at half height.)

LÉVY-MISES MATERIAL. Synonymous with **Saint Venant-Mises material**.

LEWIS ACID, LEWIS BASE, OR LEWIS SALT. See acids and bases, definitions of **Brönsted and Lewis**.

L'HOSPITAL RULE. If two functions $f(x)$ and $g(x)$ together with their derivatives up to order $(n - 1)$ vanish at $x = a$, and if their derivatives of n th order do not both vanish there or both become infinite, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{f^{(n)}(a)}{g^{(n)}(a)}$$

LIBRATION. In general terms, a quivering or swaying motion, applied more specifically to an oscillating rotary motion, as of the moon about its axis or of a molecule in a solid where it has insufficient energy to make complete free rotations.

LIEBMANN METHOD. The **Gauss-Seidel method** applied to the solution of the differ-

ence equations approximating the Laplace or the Poisson equation. (See **matrix inversion**; **partial differential equations**.)

LIE GROUP. A group which is also an analytic manifold in which the group operations, multiplication and formation of inverse, are analytic. Historically, the first Lie groups were continuous transformations of the points of a manifold. Thus, consider the set of transformations of the points of a Euclidean plane,

$$x_1 = \phi(x, y, a), \quad y_1 = \psi(x, y, a),$$

for some range of values of the parameter a , given by functions ϕ and ψ such that, if two transformations of the set are carried out in succession, the result is again a transformation of the set. An easily visualized example is the group of rotations

$$x_1 = x \cos a - y \sin a, \quad y_1 = x \sin a + y \cos a.$$

Lie groups of transformations owe their practical importance partly to their usefulness in systematizing the solutions of differential equations, both ordinary and partial, and partly to the fact that many of the standard groups of linear transformations are Lie groups. For example:

The *full linear group* is the group of all non-singular matrices with complex numbers as elements.

The *real linear group* is the group of all non-singular $n \times n$ matrices with real numbers as elements.

The *unimodular group* is the group of all complex matrices with determinant equal to unity.

The *real unimodular group* is the group of all real matrices with determinant equal to unity.

The *unitary group* is the group of all unitary matrices.

The *unitary modular group* is the group of all unitary matrices with determinant equal to unity.

The *real orthogonal group* is the group of all real orthogonal matrices.

The *rotation group* is the group of all real orthogonal matrices with determinant equal to plus one.

LIENARD-WIECHERT POTENTIALS. See **field of a moving charge in space**.

LIE RING. A real vector space in which a product ab , linear in both factors, is defined for every pair of elements a, b in such a way that $ba = -ab$ and $(ab)c + (bc)a + (ca)b = 0$ for all c . (See **infinitesimal ring of a group of linear transformations**).

LIFE, MEAN. The average time during which an atom or other system exists in a particular form, e.g., the mean time between the appearance and disappearance (birth and death) of a particle. Five examples are: (1) The mean life of mesons before undergoing transformation. (2) The mean life of excited nuclei or atoms before losing their energy of excitation. (3) For a radionuclide, the mean life is the reciprocal of the **disintegration constant**. For branching decay, it is given by

$$\tau = \frac{1}{\lambda} = \frac{1}{\lambda_1 + \lambda_2 + \lambda_3 \dots}$$

where $\lambda_1, \lambda_2, \lambda_3, \dots$ are the partial disintegration constants for the various modes involved.

(4) For a thermal neutron, the mean time interval between the instant when the neutron becomes thermal, and when it disappears from the reactor by absorption or leakage. For a homogeneous medium, the infinite and finite lifetimes refer to the lifetime in infinite- and given finite-sized regions of the medium. (5) When excess carriers are injected into a semiconductor, they will eventually recombine with others of the opposite sign. Experiment shows that in the bulk material this mean life may be rather long, because the recombination process between holes and electrons requires the emission of a photon or phonon. The process is much enhanced at the surface, as shown by the Suhl effect.

LIFETIME. (1) The mean life (see **life, mean**). (2) The **half-life**.

LIFETIME, COMPARATIVE. The product of the **half-life** or half-period of a β -disintegrator and a function (commonly represented by the symbol f) which expresses the probability per unit time that a β -transition will occur in a given nucleus. The function f depends chiefly upon the **β -disintegration energy**, and to some extent, also upon the atomic number of the product nucleus. The use of the comparative lifetime values puts all the β -disintegrators on the same basis so far as the disintegration energies and atomic numbers are

concerned, and these values are measures of the inherent forbiddenness of the β -transitions.

LIFETIME, MEAN, OF ELECTRON IN TRAP. See mean lifetime of electron in trap.

LIFETIME, NEUTRON. In nuclear reactor theory, the average time spent by a neutron from its appearance via fission to its ultimate capture. When neutron leakage is small, the lifetime is approximately the sum of the **slowing down** and **diffusion times**.

LIFT AND MOMENT ON AN AIRFOIL, GENERAL FORMULAE. For a two-dimensional airfoil in a uniform stream, let the flow be given by the complex potential function $w(z)$, where $z = x + iy$. Then if X and Y are the components of the resultant force acting on unit span of the airfoil, it may be shown by considering the change of momentum of the fluid passing across a closed curve round the airfoil that

$$X - iY = \frac{1}{2}i\rho \int \left(\frac{dw}{dz}\right)^2 dz, \quad (1)$$

where the integral is taken round a closed curve surrounding the airfoil.

From Equation (1), the **Kutta-Joukowski law** can be derived. The couple per unit length of cylinder (**pitching moment** per unit span), about an axis through the origin, is

$$M = -\frac{1}{2}\rho\Re \int z \left(\frac{dw}{dz}\right)^2 dz, \quad (2)$$

where \Re denotes the real part.

Equations (1) and (2) were derived by Blasius, and are sometimes known as **Blasius' formula**, or the *theorem of Blasius*.

LIFT COEFFICIENT. The lift L on a body in a stream of fluid is the component of the resultant force on the body, due to the fluid, in a direction perpendicular to the stream. The lift coefficient is

$$C_L = \frac{L}{\frac{1}{2}\rho V^2 S},$$

where ρ is the density of the fluid, V is the velocity of the undisturbed stream, and S is a representative area of the body. For a wing, S is usually taken as the plan area, and for a complete aircraft the gross wing area is used.

For a two-dimensional airfoil, if L' is the lift per unit span, the lift coefficient is

$$C_L = \frac{L'}{\frac{1}{2}\rho V^2 c},$$

where c is the chord.

LIFT COEFFICIENT, DESIGN. See **thin-airfoil theory**.

LIFTING-LINE THEORY. The theory is applicable in subsonic flow to wings of small sweep-back and fairly large aspect ratio (not less than about 4). It is assumed that the wing can be replaced by a set of bound vortices lying along the Y -axis (Figure 1), the

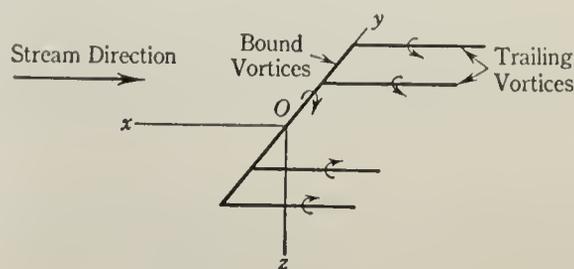


Fig. 1.

combined strength of these bound vortices at any point y , being equal to the circulation around the wing at that position. Since the bound vortices cannot end except on a boundary, they must turn downstream to form a system of trailing vortices, as indicated in Figure 1.

If $\Gamma(y)$ is the circulation, the strength of the trailing vortex from an element of length dy is $-\frac{d\Gamma}{dy} \cdot dy$, since this is the change of strength of the bound vortex in the length dy .

This model of the flow, assumed in the theory, is generally in accordance with observation. In fact, the sheet of trailing vortices behind a wing is unstable, and "rolls up" to form a pair of vortices at a distance apart rather less than the span of the wing. This rolling up of the vortex sheet is neglected in calculating the behavior of the wing but should be considered in some other problems, e.g., in estimating the angle of downwash at the position of the tailplane on an aircraft.

In lifting-line theory, the component w of the velocity induced by the trailing vortices, in the direction of the Z -axis, is calculated for points along the Y -axis. This velocity is added vectorially to the stream velocity V , and it is

assumed that the flow past each section of the wing is the same as for *two-dimensional* flow with this new velocity.

The velocity component w at a point y_1 is

$$w(y_1) = \frac{1}{4\pi} \int_{-s}^{+s} \frac{\frac{d\Gamma}{dy} \cdot dy}{y_1 - y}, \quad (1)$$

where s is the semi-span of the wing.

Thus a section of the wing at y_1 experiences the same lift as a two-dimensional airfoil of the same section at an angle of incidence

$$\alpha_o = \alpha - \frac{w}{V}, \quad (2)$$

where α is the true angle of incidence of the wing section and both α and α_o are measured relative to the zero-lift line. This lift force is

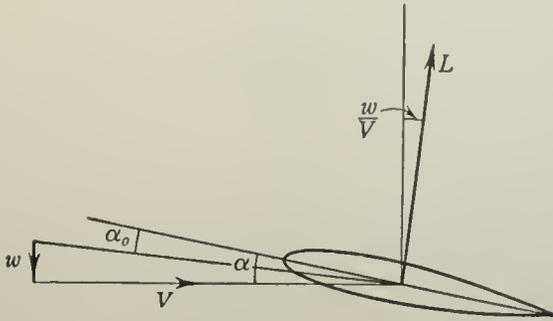


Fig. 2.

inclined downstream as shown in Figure 2, making an angle $\frac{w}{V}$ with the normal to the stream direction. Thus the drag coefficient of this wing section is increased by an amount $\frac{w}{V} \cdot C_L$, where C_L is the local lift coefficient. Also, if a_o is the lift gradient $\frac{dC_L}{d\alpha_o}$ in two-dimensional flow, the lift coefficient at the section of the three-dimensional wing under consideration is

$$C_L = a_o \alpha_o, \quad (3)$$

so that the circulation at this section is

$$\Gamma = \frac{1}{2} C_L c V = \frac{1}{2} a_o c (\alpha V - w), \quad (4)$$

where c is the local wing chord.

The problem of determining the lift distribution on a given wing at a given angle of incidence is essentially that of finding a distribution of Γ across the span that satisfies both the equations (1) and (4). In the usual

method of solution, the coordinate y is replaced by a new variable θ , defined by

$$y = -s \cos \theta.$$

The circulation $\Gamma(y)$ is then expressed as a **Fourier series**

$$\Gamma = 4sV \sum_{n=1}^{\infty} A_n \sin n\theta, \quad (5)$$

the coefficients A_n being chosen to satisfy Equations (1) and (4). It is found that the lift coefficient C_L for the whole wing depends only on the coefficient A_1 , and this is given by

$$A_1 = \frac{C_L}{\pi A}, \quad (6)$$

where A is the **aspect ratio** of the wing.

It has already been mentioned that the drag coefficient of each section of the wing is increased by an amount $\frac{w}{V} \cdot C_L$, where C_L is the local lift coefficient of the wing section. The corresponding increase in the drag coefficient of the whole wing is known as the coefficient of *trailing-vortex drag*. In the past, this quantity has usually been called the *induced drag* coefficient, but as the term "induced drag" is now used with another meaning "trailing-vortex drag" is to be preferred.

The coefficient of trailing-vortex drag, for a wing developing a lift coefficient C_L , is

$$C_{Di} = \frac{C_L^2}{\pi A} (1 + \delta), \quad (7)$$

where δ is a small quantity given by

$$1 + \delta = \frac{\sum n A_n^2}{A_1^2}. \quad (8)$$

It is clear from Equation (2) that the angle of incidence required to obtain a given lift coefficient is greater for a wing of finite span than for a two-dimensional airfoil. When the wing of finite span is *untwisted*, so that the true angle of incidence α is constant across the span, it is easily shown from Equation (6) that

$$\alpha = \alpha_o + \frac{C_L}{\pi A} (1 + \tau), \quad (9)$$

where α and α_o are the angles of incidence for the wing of finite span and for a two-dimensional airfoil, each measured relative to the zero-lift line and giving the lift coefficient C_L . τ is a small quantity given by

$$1 + \tau = \frac{\alpha}{A_1} - \frac{\pi A}{2a_0}, \quad (10)$$

where a_0 is the lift gradient $\frac{dC_L}{d\alpha_0}$ for the two-dimensional airfoil.

An important case is that of *elliptic loading* for which Equation (5) takes the simple form

$$\Gamma = 4sVA_1 \sin \theta = 4sVA_1 \sqrt{1 - \frac{y^2}{s^2}}. \quad (11)$$

All the other coefficients A_n are then zero, and δ and τ in Equations (7) and (9) are also zero. Both the trailing-vortex drag coefficient and the "induced incidence" ($\alpha - \alpha_0$) then have minimum values, for given values of C_L and A . The induced velocity component w is constant across the span and has a value given by

$$\frac{w}{V} = \frac{C_L}{\pi A} = \alpha - \alpha_0. \quad (12)$$

The trailing-vortex drag coefficient is simply $\frac{C_L^2}{\pi A}$.

For all the wing planforms used in practice it is found that both δ and τ are fairly small compared with 1. Thus the case of elliptic loading, which implies $\delta = \tau = 0$, may be regarded as a reasonable first approximation for any of the usual wing planforms.

LIFTING-SURFACE THEORY OF WINGS.

The simplest form of three-dimensional wing theory is **lifting-line theory**, in which the wing is represented by a bound vortex lying along a line normal to the stream. Lifting-line theory is not satisfactory, however, for wings which are swept back, or which have a small aspect ratio, and for these cases a more accurate representation of the wing must be considered.

In lifting-surface theory the induced velocities are usually assumed to be small so that the equations can be linearized. The flow field for a wing with thickness at incidence is then calculated by superposition of the flow fields for (1) a symmetrical wing with thickness at zero incidence and (2) a thin (possibly cambered and twisted) wing at incidence.

For the calculation of (1), the wing is represented by a distribution of sources and sinks over the plane of symmetry of the wing. For (2), a distribution of vortices over the surface

of the wing *and* its wake is used. In various approximate forms of the vortex theory (2), which are useful for purposes of calculation, the distributed vorticity is replaced by a finite number of discrete vortices.

In supersonic flow the linearized equation for the velocity potential can be used, in conjunction with distributions of sources, to calculate flow fields for case (2) as well as for case (1). (See A. Robinson and J. A. Laurmann, *Wing Theory*. Cambridge, 1956.)

LIGHT. Light is (1) an attribute of all the perceptions or sensations which are peculiar to the organ of vision and which are produced through the agency of that organ; and (2) radiation capable of stimulating the organ of vision. (See **quantity of light**.)

LIGHT ABSORPTION IN SPACE. See **stellar luminosities**.

LIGHTHILL METHOD. See **airfoil theory (two-dimensional)**.

LIGHT, QUANTITY OF. The product of luminous flux and the time during which it is maintained.

LIGHT SCATTERING FUNCTIONS. The light scattering functions for spherical particles are discussed under **spherical wave functions**.

LIGHT SOURCES, STANDARD. See **specified achromatic light**.

LIGHT YEAR. See **astronomical distance units**.

LIKELIHOOD. If $f(x, \theta)$ denotes a **frequency (probability) function** depending on one or more parameters collectively denoted by θ , the likelihood of a sample of independent values x_1, x_2, \dots, x_N is defined as $L = f(x_1, \theta) \cdot f(x_2, \theta) \cdot \dots \cdot f(x_N, \theta)$.

The method of maximum likelihood consists in estimating the parameters θ by choosing those values which maximize L (or $\log L$). R. A. Fisher has shown, under general conditions, that maximum likelihood estimates are **consistent** and **efficient** and tend to be **normally distributed** in large samples; further, that they are **sufficient** if sufficient statistics exist.

LIMIT. See **convergence**.

LIMIT ANALYSIS AND DESIGN (PLASTIC). Plastic limit analysis and design is concerned with the load-carrying capacity of a structure or other body composed of elastic-perfectly plastic material. Although still idealized, it is a far more realistic approach than the use of an arbitrary limitation of the maximum stress computed from an elastic analysis. (See **limit theorems**; **limit load**.)

LIMIT, ELASTIC. See **elastic limit**.

LIMITING CURVES. Any line on a **phase diagram**, or other graphical representation of the conditions of a system, at which two phases are coexistent. Usually these limiting curves are the plotted curves that separate phases.

LIMITING DENSITY. The value which the density of a gas approaches as its pressure-volume relationship approaches the constant value (at constant temperature) of an **ideal gas**.

LIMITING LAW FOR STRONG ELECTROLYTES. The activity coefficients in a strong electrolyte solution exhibit characteristics quite different from those of non-electrolytes. Indeed in highly dilute solutions we have for the **osmotic coefficient** of the solvent

$$(1 - \phi) \sim C_s \quad C_s \rightarrow 0 \text{ (non-electrolytes)} \quad (1)$$

$$(1 - \phi) \sim C_s^{1/2} \quad C_s \rightarrow 0 \text{ (strong electrolytes)} \quad (2)$$

where C_s is the concentration of the solute.

As a consequence, the derivative of $1 - \phi$ at the origin is finite in the first case, but infinite in the second. This behavior of strong electrolytes is related to the long range electrostatic forces between the ions in the solution.

The statistical theory due to Debye and Hückel (see **Debye-Hückel theory**), leads indeed to the following *limiting law* for the activity coefficient of an ion with the charge z_i , in a very dilute solution in which the **ionic strength** is I

$$\log_{10} \gamma_i = - z_i^2 \frac{\sqrt{2\hbar}}{2.3026} \frac{e^2 N^2}{(DRT)^{3/2}} I^{1/2}.$$

where N is the Avogadro number, e the electronic charge, D the dielectric constant of the solvent and R the gas constant. The ionic strength is in units of moles/cm³.

LIMIT LOAD. The plastic limit load is the maximum or collapse load of a structure composed of elastic-perfectly plastic material computed on the basis of the original geometry. (See **limit theorem**.) Other limiting conditions may be employed to define different kinds of limit loads.

LIMIT MOMENT. See **moment, limit or plastic**.

LIMIT OF RESOLUTION, RAYLEIGH CRITERION. See **Rayleigh criterion of resolving power**.

LIMIT, PROPORTIONAL. The stress in a simple tension test at which the deviation of the stress-strain curve becomes noticeable. It may differ from the **elastic limit** in that elasticity may be non-linear, but the distinction is not very meaningful, in fact, for most metals and other structural materials.

LIMIT THEOREMS. The limit theorems of plasticity established by Drucker, Prager, and Greenberg hold for any stable elastic-perfectly plastic material when geometry changes of the body are negligible prior to collapse and during the early stages of collapse. They hold therefore for the initial motion of a rigid-perfectly plastic material.

The preliminary theorem is: Collapse occurs under constant load and at constant stress; plastic strains only take place. The limit theorems then are:

I. (Lower bound.) If an equilibrium distribution of stress can be found which balances the applied load and is everywhere below yield or at yield, the structure will not collapse or will just be at the point of collapse.

II. (Upper bound.) The structure will collapse if there is any compatible pattern of plastic deformation for which the rate at which the external forces do work equals or exceeds the rate of internal dissipation.

Theorem I expresses the ability of the structure to adjust itself to carry the applied load if at all possible. It gives lower bounds on, or safe values of, the plastic limit loading. The maximum lower bound is the plastic limit load itself. Theorem II states that if a path of failure exists the structure will not stand up. It gives upper bounds on, or unsafe values of, the plastic limit loading. The minimum upper bound is the plastic limit load itself. The two theorems enable reasonably close bracketing

of the answer to problems of practical importance.

As the lower bound theorem permits any distribution of stress satisfying equilibrium and the loading conditions, it must be true, within the limitation of negligible effect of change in geometry, that residual, thermal or initial stresses or deflections do not influence the plastic limit load. (See also **first limit theorem**; **second limit theorem**; **sinusoidal limit theorem**.)

LIMIT, YIELD. The stress at which plastic flow is initiated. When this stress is sharply defined as for mild steel it is termed a *natural yield limit*. When the onset of plastic flow occurs gradually, as for most metals, a *conventional yield limit* is defined below which plastic strains are neglected.

LINEAR. An adjective used in many related senses. Given the quantities $x_1, x_2, x_3, \dots, x_n$, a linear combination of them is $a_1x_1 + a_2x_2 + \dots + a_nx_n$. (See **linearly independent**.)

A linear function is a polynomial of the first degree in its variables. Thus, with the linear function, $y = mx + b$, a plot of it would be a straight line of slope m and intercept b on the y -axis. The general case of a linear algebraic equation would be $a_1x_1 + a_2x_2 + \dots + a_nx_n = a_0$, where the x_i are variables and the a_i are constants. (See also **linear equations (algebraic)**.)

A set of *simultaneous linear equations* is

$$\sum_{j=1}^n a_{ij}x_j = b_i;$$

$i = 1, 2, \dots, n$ and a_{ij}, b_i are constants.

A linear **differential equation** is $A_0(x)y + A_1(x)y' + A_2(x)y'' + \dots + A_n(x)y^{(n)} = f(x)$ where the $A_i(x)$ are functions of the independent variable only and y', y'', \dots are the first, second, etc., derivatives.

LINEAR ABSORPTION COEFFICIENT. See **absorption coefficient**.

LINEAR COMBINATIONS OF WAVE FUNCTIONS. See under **valence bond method**; **molecular orbitals method**; **exchange energy**.

LINEAR DIFFERENTIAL EQUATION. See **linear**.

LINEAR DIFFERENTIAL FORM. Differential form of degree one.

LINEAR EQUATIONS (ALGEBRAIC).

Equations in which the unknowns occur only linearly, hence of the form

$$a_{11}x_1 + a_{12}x_2 + \dots = h_1,$$

$$a_{21}x_1 + a_{22}x_2 + \dots = h_2.$$

They are homogeneous if $0 = h_1 = h_2 = \dots$, otherwise nonhomogeneous. In matrix form the equations can be written

$$\mathbf{A}x = h,$$

and the condition for solvability is that the rank of \mathbf{A} and the rank of the augmented matrix (\mathbf{A}, h) be the same. (For methods of solving see **matrix inversion**.)

LINEAR EXTRAPOLATION LENGTH. In the theory of neutron transport, the flux distribution far from sources, boundaries, interfaces, appears as identical with the solution to an appropriate diffusion equation, which, if extended to a boundary surface, S , would satisfy

$$\left. \frac{1}{\Phi} \frac{\partial \Phi}{\partial n} \right|_S = \frac{1}{l}$$

l is the linear extrapolation length. Only in very simple cases is l equal to z_0 , the extrapolated end point. For an infinite half-space composed of non-capturing, spherically scattering atoms,

$$l = 0.7104\lambda_s$$

where λ_s is the scattering mean free path.

LINEAR GRAPH. See **graph, linear**.

LINEAR GROUP. See **Lie group**.

LINEAR HYPOTHESIS. In statistics this expression occurs in a special sense concerning normally distributed random variables. Given p such variables with means μ_i ($i = 1, 2, \dots, p$) connected with parameters θ_i ($i = 1, 2, \dots, p$) by linear equations

$$\mu_k = \sum_{j=1}^p c_{jk}\theta_j$$

a hypothesis specifying r of the θ 's ($r < p$) is a linear hypothesis. Many common types of statistical test can be reduced to this form.

LINEAR INEQUALITIES. A system of relations among variables x_i , possibly including

linear equations among them, but also including at least one inequality of the form

$$\sum a_i x_i \geq b$$

(in practice a strict inequality is seldom required). Such a system may be incompatible (e.g., $x_1 \geq 0, x_2 \geq 0, -x_1 - x_2 - 1 \geq 0$), may define a unique point (e.g., $x_1 \geq 0, x_2 \geq 0, -x_1 - x_2 \geq 0$), or else will define a region in space, not necessarily bounded (e.g., $x_1 \geq 0, x_2 \geq 0, -x_1 - x_2 + 1 \geq 0$ define a bounded region; $x_1 \geq 0, x_2 \geq 0, x_1 + x_2 - 1 \geq 0$ an unbounded region).

The inequality written above can be replaced by the equivalent pair

$$x_0 + \sum a_i x_i = b_i, \quad x_0 \geq 0,$$

and in general it is possible to replace a system of inequalities by a system in the special form

$$\mathbf{Ax} = \mathbf{b}, \quad \mathbf{x} \geq 0,$$

where \mathbf{A} is a rectangular matrix, \mathbf{x} and \mathbf{b} are vectors.

The principal applications are in **linear programming**.

LINEARLY INDEPENDENT VECTORS.

The vectors $\mathbf{A}_1, \mathbf{A}_2, \dots, \mathbf{A}_m$ are linearly independent if the equation $c_1 \mathbf{A}_1 + c_2 \mathbf{A}_2 + \dots + c_m \mathbf{A}_m = 0$ implies that $c_1 = c_2 = \dots = c_m = 0$. Any n -dimensional space contains n linearly independent vectors. They are a **base** of the vector space. Any other vector can be written as a linear combination of these base vectors. It is convenient to choose the base vectors mutually perpendicular or orthogonal and of unit length. In ordinary three-dimensional space these base vectors are denoted by $\mathbf{i}, \mathbf{j}, \mathbf{k}$.

LINEAR MAGNIFICATION. (1) See **representation**. (2) If ds is the length in the object space of an optical system and ds' is the conjugate element in image space then ds'/ds is the linear magnification of the system. If ds'_i/ds_i is the linear magnification of the i^{th} surface, $i = 1, 2, \dots, N$, of an optical system then the linear magnification of the system is

$$\frac{ds'}{ds} = \prod_{i=1}^N \frac{ds'_i}{ds_i}$$

LINEAR PASSIVE NETWORK. See **network, linear passive**.

LINEAR POLARIZATION. See **polarization cycle**.

LINEAR PROGRAMMING. The problem of minimizing or maximizing a linear function $C = c_0 + \sum c_i x_i$, subject to constraints in the form of **linear inequalities** to be satisfied by the x_i . The function C may represent costs or profits or losses in a business activity or a competition (possibly against nature), and the constraints represent limitations upon resources or their availability.

If the inequalities are compatible and satisfied by more than a single point x_i , the solution lies on the boundary, and, if unique, is the intersection of some subset of the planes whose equations are formed by using equality instead of inequality signs. Hence, in principle, the problem could be solved by solving the equations in all possible combinations and comparing the values of C . However, the difficulty of the problem comes from the fact that if the numbers of variables and relations are at all large, as they often are in practice, the number of sets to be solved may be astronomical. For such cases the **simplex method** may be used.

LINEAR SYSTEMS. Systems such that the inter-related quantities comprising the system are related by linear differential or differential-integral equations.

Such equations and therefore such systems obey the principle of superposition, namely, the combined effect of a number of causes acting together is the sum of the effects of the several causes acting separately.

LINEAR SYSTEM (NETWORK). See **network, linear system**.

LINEAR TRANSFORMATION (OF A COMPLEX VARIABLE). See **homographic transformation**.

LINEAR VISCOELASTICITY. See **viscoelasticity, linear**.

LINE INTENSITY. See **intensity of a spectral line**.

LINE INTENSITY FORMULAS. See **intensity rules for multiplets; rotational sum rules (b)**.

LINE, LOXODROMIC. See **loxodrome**.

LINE OF APSIDES. A line which contains the major axis of an **ellipse** is known as the line of apsides of the ellipse. In astronomy the

term is used to indicate the line joining perihelion and aphelion points in an orbit and extending to infinity to cut the celestial sphere.

LINE OF CURVATURE (ON A SURFACE).

A curve on a surface such that the tangent to the curve at any point is parallel to a principal direction of curvature of the surface at that point.

LINE OF FLOW. A line whose tangent is, at a given time, everywhere parallel to the velocity (u, v, w). It is given by

$$\frac{dx}{u} = \frac{dy}{v} = \frac{dz}{w}.$$

The paths of the particles are the time-envelopes of the lines of flow.

LINE OF NODES. The astronomical term applied to a line of intersection of any two fundamental planes. The line of nodes for the moon is the line of intersection of the plane containing the moon's orbit with the plane of the ecliptic. The line of nodes for any member of the solar system, other than satellites, is the line of intersection of the plane of the orbit of the object with the plane of the ecliptic. The line of nodes for the earth is the line of intersection of the plane of the earth's equator with the plane of the ecliptic.

LINE OF POSITION. In navigation a line of position is any line on the surface of the earth on which a ship may be presumed to be located. Lines of position may be determined in a variety of ways.

In pilotage (navigation by use of fixed objects on shore) a line of position is established by the direction of a fixed object. This direction may be established by visual observation or by radio. If the distance can be determined as well as the direction (e.g., by synchronized audible and radio signals, or by measuring the angle subtended by an object of known size (e.g., height of a light house) a second line of position is established as the circle centered on the object with radius equal to the determined distance. These two observations of the same object provides a complete position or a **fix**.

In celestial navigation lines of position are determined by observing the altitude of a celestial object and computing the direction and distance of the substellar point from the

assumed position of the ship. (See **navigational astronomy**.)

Loran lines of position are determined by the use of special equipment.

LINE OF STRICTION OF A SURFACE. See **ruled surface**.

LINE, POLAR. See **polar line**.

LINE, RECTIFYING. See **rectifying line**.

LINES OF LATITUDE. See **surface of revolution**.

LINES OF LONGITUDE. See **surface of revolution**.

LINE SPECTRUM. See **spectrum, line**.

LINE STRENGTH. The strength of a spectral line is defined (Condon and Shortley) as the sum of the squared matrix elements of the transitions contributing to the intensity of the line, that is

$$S^{nm} = \sum |R^{n_i m_k}|^2.$$

Different from the **transition probability**, the line strength is the same for absorption and emission lines.

LINE VORTEX. A line vortex, or vortex filament, is a vortex tube of finite strength but of infinitesimal section (i.e., of zero substance). The circulation round a line vortex surrounded by fluid in irrotational motion is the same for all circuits encircling it.

In the special case of a rectilinear vortex the complex potential is

$$w = \phi + i\psi = -\frac{i\kappa}{2\pi} (\log r + i\theta) = -\frac{i\kappa}{2\pi} \log z$$

where κ is the strength of the vortex, or circulation round it. It produces a velocity $\kappa/2\pi r$, at right angles to the radius, at distance r . The fields of motion of parallel rectilinear vortices are additive, and each moves with the velocity at its position due to all the others.

More generally the velocity at a point due to a vortex filament is perpendicular to the surface on which the solid angle, A , subtended by the filament is constant and is equal to $\kappa \text{ grad } A$. The fluid velocity is equal to the sum of the velocities due to all filaments in the fluid.

LINE WIDTH. See **Doppler width of a spectral line; level width**.

LINE WIDTH, MAGNETIC RESONANCE.

See magnetic resonance line width.

LINE WIDTH, NATURAL. See natural line width.

LINK. Alternate term for chord.

LIN METHOD. See polynomial factorization.

LIOUVILLE EQUATION. In the statistical mechanics of an ensemble of systems, each containing N particles of mass m , it is useful to introduce a density or probability function $P_N(q_N, p_N)$, representing the probability that a system of the ensemble will have its point in phase space fall within the volume bounded by $q_1, q_2, \dots, p_1, p_2, \dots, p_N$, and $q_1 + \delta q_1, q_2 + \delta q_2, \dots, p_1 + \delta p_1, p_2 + \delta p_2, \dots, p_N + \delta p_N$. If no new systems are created or destroyed, P_N will satisfy the Liouville equation:

$$\frac{\partial P_N}{\partial t} + \sum_{j=1}^N \left(\nabla_j \frac{P_N p_j}{N} + \delta_j P_N \dot{p}_j \right) = 0,$$

where

$$\nabla_j = \partial/\partial q_j \quad \text{and} \quad \delta_j = \partial/\partial p_j.$$

LIOUVILLE-NEUMANN SERIES. Given a set of n linear algebraic equations in n unknowns, ϕ_i , in the form in which they naturally arise in the study of forced vibrations with n degrees of freedom, namely

$$\phi_i = f_i + \lambda \sum_{j=1}^n k_{ij} \phi_j \quad (i = 1, 2, \dots, n),$$

where the f_i and k_{ij} are given constraints and λ is a parameter.

In matrix form, this system of equations becomes

$$f = (I - \lambda K)\phi,$$

where f and ϕ are the vectors (f_1, f_2, \dots, f_n) and $(\phi_1, \phi_2, \dots, \phi_n)$, I is the unit matrix and K is the matrix $\{k_{ij}^{-1}\}$. Then the solution, under certain convergence conditions, is given by

$$\begin{aligned} \phi &= (I - \lambda K)^{-1} f \\ &= f + \lambda K f + \lambda^2 K^2 f + \lambda^3 K^3 f \dots, \end{aligned}$$

which is called the Liouville-Neumann series for the system of equations.

Now consider a Fredholm integral equation of the second kind (the same concepts may be

used to advantage for the integral equations as well),

$$\phi(x) = f(x) + \lambda \int_a^b K(x,z)\phi(z)dz.$$

By analogy with the above, let us attempt to write the solution as a power series in λ ,

$$\begin{aligned} \phi(x) &= \phi_0(x) + \lambda_1 \phi_1(x) \\ &\quad + \lambda_2 \phi_2(x) + \lambda_3 \phi_3(x) + \dots \end{aligned}$$

Substituting into the integral equation and equating coefficients of equal powers of λ , we get

$$\begin{aligned} \phi(x) &= f(x) + \lambda \int K(x,z)f(z)dz \\ &\quad + \lambda^2 \int K_2(x,z)f(z)dz + \dots, \end{aligned}$$

where

$$K_2(x,z) = \int K(x,y)K(y,z)dy, \text{ etc.},$$

which is the Liouville-Neumann series for the given integral equation.

If we set

$$K(x,z;\lambda) = \sum_{n=0}^{\infty} \lambda^n K_{n+1}(x,z),$$

then the above solution (again under certain convergence conditions) may be written

$$\phi(x) = f(x) + \lambda \int K(x,z;\lambda)f(z)dz.$$

Thus $K(x,z;\lambda)$ is called the *resolvent kernel* of the given equation.

LIOUVILLE SURFACE. A surface for which the first fundamental form (see **geodesic parallels on a surface**) may be written in the form

$$[f(u) + g(v)] [F(u)(du)^2 + G(v)(dv)^2],$$

by a suitable choice of the surface parameters u, v , where f and F are functions of u only and g and G are functions of v only.

LIOUVILLE THEOREMS. (1) If $f(z)$ is analytic and bounded for all finite values of the complex variable z , then $f(z)$ is a constant. (2) The **extension in phase** occupied by a collection of representative points in **phase space** is constant in time. (See also **Liouville equation; ensembles.**)

LIPSCHITZ CONDITION. A condition somewhat stronger than simple continuity. In the simplest form, a function $f(x)$ is said to satisfy a Lipschitz condition on the interval from a to b in case there exists a constant k such that

$$|f(x_1) - f(x_2)| \leq k|x_1 - x_2|$$

whenever x_1 and x_2 both lie on the interval. If the derivative exists and is bounded on the closed interval then the Lipschitz condition is satisfied, but not conversely. The condition is a useful one in the study of the solution of a differential equation. Various generalizations are possible, in particular to functions of several variables.

LIQUEFACTION. The process of change of phase from gaseous to liquid. The opposite change is called boiling or evaporation.

LIQUID, BOSE-EINSTEIN TREATMENT. See Bose-Einstein liquid.

LIQUID HELIUM. See helium, liquid.

LIQUIDS, KINETIC THEORY OF. See kinetic theory of liquids.

LIQUID SPHERE, RELATIVISTIC FIELD OF. See relativity, general.

LIQUID STATE. See cell model of the liquid state.

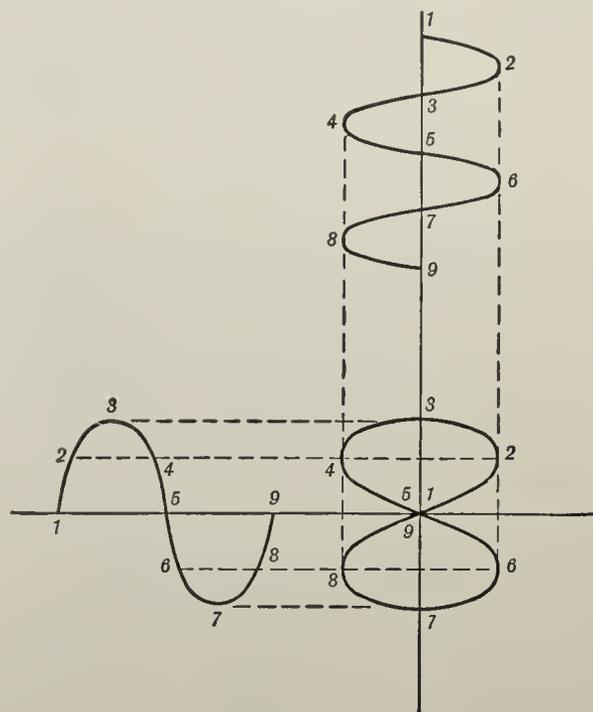
LIQUIDS, STATISTICAL TREATMENT. See liquids, theory of.

LIQUIDS, THEORY OF. Although statistical mechanics can deal with crystals and gases, liquids still pose many insoluble problems. One can either treat a liquid as a disordered crystal or as a dense gas, and either of these methods of approach can claim some successes, but an all-embracing theory of liquids does not yet exist. (See also **kinetic theory of liquids**; **Bose-Einstein liquids**; **helium, liquid**.)

LIQUIDUS CURVE. See mixed crystals.

LISSAJOUS FIGURES. When two sine waves, varying about axes at right angles, are combined, the resultant figure is no longer a sine wave but varies with the relative time phase of the waves and with their relative frequency. For example, if the waves have the same frequency the resultant is a straight

line when they are in time phase (or 180° out of phase) and is an ellipse for all other values of phase position. For equal amplitudes of the original waves and 90° phase the ellipse is the special case of the circle. If the frequencies of the two waves are not the same the resultant becomes more complicated but gives a definite pattern whenever the frequencies are in the ratio of whole numbers to one another. The figure shows a graphical construction for a frequency ratio of 1:2.



Construction of Lissajous figure.

Such figures are obtained when the resultant motion of two **simple harmonic motions** at right angles to each other is examined. The figures can be produced in a cathode-ray tube by supplying each deflection-circuit with harmonically-related voltages. The figures are used thus for phase and frequency measurements.

LIVE LOAD. Variable or moving loads are classified as live loads. Snow, wind and stored merchandise constitute variable loads. Moving loads are made up of the weight of people, trains, trucks, etc. The dead weight of traveling cranes must be considered as a live load on the structure (see **dead load**). Live loads are treated conveniently by the use of **influence lines**.

LJUNGSTRÖM TURBINE. See axial-flow turbine.

LOAD. (1) Any force which is supported by a body is called a load. The forces which in turn support the given body are called reactions. A concentrated load is a theoretical force having a contact area negligibly small compared with the area of the surface of the body upon which the force acts. A distributed load is one whose area of contact covers, wholly or partially, the area of the supporting surface of the body. Distributed loads are uniform if the intensity is the same for each unit of area covered by the load. When this intensity varies, the distributed load is non-uniform. (2) In communications, the load is the **signal power** delivered by a **transducer**.

LOAD, AXIAL. See **axial load**.

LOAD FACTOR. The factor of safety against plastic collapse or other ultimate failure of a structure. It is given by the **collapse load** divided by the design load, and thus generally is quite different from the nominal factor of safety, which is equal to the yield stress divided by the working stress.

LOADING, DYNAMIC. See **dynamic loading**.

LOAD, LATERAL. See **lateral load**.

LOAD, LIMIT. See **limit load**.

LOAD, LIVE. See **live load**.

LOAD, MOVING. See **live load**.

LOADS, METHOD OF FICTITIOUS. Another name for the method of elastic loads. See also **conjugate beam**.

LOBAČEVSKIĀ METHOD. In current Russian literature, any **root-squaring method** as a method of solving algebraic equations numerically. Actually the idea of root-squaring for this purpose had already been proposed by Dandelin, and the specific algorithm proposed by LobačevskiĀ was slightly more cumbersome than **Graeffe's method** which is the one in common use at present.

LOCAL BUCKLING. See **buckling, local**.

LOCAL DERIVATIVE. See **local rate of change; partial derivative**.

LOCALIZED MONOLAYERS. In the discussion of adsorbed films, an important role is

played by films that are at most one molecule thick, and which are attached to the surface of a solid or of a liquid in which the molecules are practically insoluble. Such a layer is often called a *monolayer*.

In the model of a localized monolayer, there are definite points of attachment on the solid surface capable of accommodating just one adsorbed molecule. Such points on the surface are called "*sites*." (See **mobile monolayers; adsorption isotherms**.)

LOCALIZED STATES. The states which represent a system localized at a definite space point at a given time. In a quantum mechanical description the states which represent a system localized at time $t = 0$ at $x = y = z = 0$ can be determined from the following (physically plausible) postulates:

(a) They form a linear set, S , i.e., the superposition of two such localized states again be localized in the same manner.

(b) The set S be invariant under rotations and under space inversions and time inversions.

(c) If a state is localized at the origin at time $t = 0$, then a spatial displacement of this state shall make it orthogonal to all states of S .

In non-relativistic quantum mechanics these states are essentially delta functions times spin functions in configuration space. In the relativistic situation since a particle must be described by a superposition, positive energy wave functions of a localized state of an elementary system (i.e., one whose states transform under some irreducible representation of the Lorentz group) have non-vanishing values in a region of linear dimension \hbar/mc , where m is the mass of the particle. (δ -functions cannot be constructed out of only positive energy solutions since the latter do not form a complete set.)

LOCALLY EUCLIDEAN. A topological space T is said to be locally Euclidean at a point p if there exist a Cartesian coordinate system at p ; that is, a **homeomorphism** between a neighborhood of p and the unit cube in a **Euclidean** n -space. Then each point p corresponds to a set of numbers x_1, x_2, \dots, x_n which are called the coordinates of p . Thus the surface of a sphere, say of the Earth, is locally Euclidean of two dimensions at every point; at the north pole, for example, since if the south frigid zone is removed it is clear that the remainder can be distorted topologically

into a square on the Euclidean plane. A locally Euclidean space is a space which is locally Euclidean at each of its points.

LOCAL RATE OF CHANGE. The rate of change at a fixed point in a frame of reference; denoted by $\partial/\partial t$.

LODE'S VARIABLES. To express the flow rule for plastic flow Lode devised stress and strain variables which express the influence of the intermediate stress or strain principal values in comparison with the extreme values.

$$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}, \quad \sigma_1 \geq \sigma_2 \geq \sigma_3.$$

$$\nu = \frac{2\epsilon_2 - \epsilon_1 - \epsilon_3}{\epsilon_1 - \epsilon_3}, \quad \epsilon_1 \geq \epsilon_2 \geq \epsilon_3.$$

For plastic flow with varying principal stresses, the components in the strain variable should be replaced by strain increment components. For the flow rule associated with **Mises flow condition**, then $\mu = \nu$.

LOGARITHM. If B is an arbitrarily chosen number greater than unity, then the logarithm L of any other number N is defined by $N = B^L$; $L = \log_B N$. The chosen number B is the base of the system of logarithms. For any base, $\log_B 1 = 0$; $\log_B B = 1$. Two systems of logarithms are generally used: common and natural.

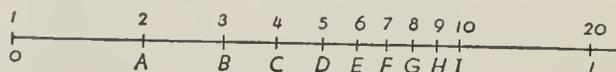
The former, also called *Briggs logarithms*, uses the base 10 and is particularly useful for numerical calculations. The common logarithm of a number N could be indicated by $\log_{10} N$ but the notation $\log N$ is more usual. Since any number may be written in the form $N = 10^n \times M$, where n is an integer, positive or negative, its common logarithm is $\log N = n + \log M$. The first part of this sum is the *characteristic* of the logarithm and it may be obtained by inspection of the given number. The second part, called the *mantissa*, is an irrational number, less than unity and usually given in decimal form.

If a logarithm is given, the number which corresponds to it is the *antilogarithm*. The *cologarithm* is the logarithm of the reciprocal of a number. Thus $\text{colog } N = \log (1/N) = -\log N$.

The *natural logarithms* have the base $e = 2.71828 \dots$ and are also called the Napierian or hyperbolic system. Such logarithms occur

as the result of **integration**, etc., and they often appear in equations representing physical phenomena. Instead of the more exact symbol $\log_e N$, the abbreviated notation $\ln N$ is customary. The **modulus** of the common system relative to the natural system of logarithms is $\log e = 0.434294 \dots$ and, inversely, $\ln 10 = 2.302585 \dots$ is the modulus of the natural system relative to the common system.

A logarithmic scale is one in which the distance from the origin to any scale mark is proportional to the **logarithm** of the number attached to that mark.



Logarithmic scale.

Thus, the accompanying figure shows a (common) logarithmic scale with the numbers 1, 2, 3, \dots , 10, \dots attached to the division marks; if we take OI as the unit of length, then the distances OA , OB , OC , etc., are represented by $\log 2$, $\log 3$, $\log 4$, etc., so that $OI = \log 10 = 1$. In going from left to right, the scale marks will become closer and closer together.

The logarithmic scale is applied in the **slide-rule** and in logarithmic paper. In the latter case, if both abscissa and ordinate are marked logarithmically, the paper is called log-log paper; if the abscissa is equally spaced and only the ordinate is logarithmic, it is semi-log paper. The logarithm of a complex number $z = r(\cos \theta + i \sin \theta) = re^{i\theta}$ is given by $\ln z = \ln r + i(\theta + 2\pi n)$, where $n = 0, \pm 1, \pm 2, \dots$. It may also be defined by the integral $\ln z = \int_1^z \frac{dz}{z}$ taken over any path which avoids the origin.

LOGARITHMIC DECREMENT. In a mechanical system which is performing damped harmonic oscillations, the natural logarithm of the amplitude of successive excursions on the same side of the position of equilibrium is called the logarithmic decrement. Thus, if the displacement u is given by the expression

$$u = u_0 e^{-\alpha t} \cos \omega t$$

the logarithmic decrement is $2\pi\alpha/\omega$. The logarithmic decrement has, however, sometimes been defined as the natural logarithm of the ratio of the amplitude of successive excursions on opposite sides of the position of equilibrium; it then has half the value given above,

When a specimen of a solid acts as the elastic element in a vibrating mechanical system, the value of the logarithmic decrement observed is characteristic of that solid, although in most materials this value is found to depend to some extent on the amplitude of the vibrations, and on their frequency. When the damping is low, the logarithmic decrement may be related to other measures of internal friction. (See **friction, internal**.)

LOGARITHMIC DIFFERENTIATION.

Sometimes a **derivative** is found most easily by taking **logarithms** on both sides of the defining functional equation and then differentiating. The result of logarithmic differentiation is $d \ln f = df/f$.

LOGARITHMIC DISPLACEMENT OPERATOR. See **difference operators**.

LOGARITHMIC DISTRIBUTION. A frequency distribution for which the frequency at the (integral) value j ($= 1, 2, \dots \infty$) is $\frac{\lambda^j}{j}$ / $\{-\log(1 - \lambda)\}$ that is to say, is the coefficient of t^j in the expansion of

$$\frac{\log(1 - \lambda t)}{\log(1 - \lambda)}$$

It is the limiting form of a negative **binomial distribution**.

LOGARITHMIC INTEGRAL. See **integral, logarithmic**.

LOGARITHMIC PROFILE OF VELOCITY.

The profile of the mean velocity \bar{u} assumed near a plane boundary by fluid in turbulent motion in which the turbulence is similar at all distances. This condition is represented in the **mixing length** theory by assuming either that the mixing length is proportional to distance from the boundary ($l = kz$) or that it is determined locally and is equal to the simplest length derivable from the local features of the velocity profile, viz.

$$l = k \frac{\partial \bar{u}}{\partial z} / \frac{\partial^2 \bar{u}}{\partial z^2}$$

Also the shearing stress τ_0 is assumed to be independent of distance from the boundary. Then

$$\bar{u} = \frac{\tau_0^{1/2}}{k\rho^{1/2}} \log(z/z_0)$$

where k is Kármán's constant, and z_0 is called a "roughness length," not necessarily related to the "roughness height."

The profile is not applicable very close to the boundary where there is a layer in laminar flow because the turbulence is not similar, the smallest eddies being destroyed by viscosity. (See **turbulent boundary layer; roughness effect on skin friction**.)

LOGARITHMIC SCALE. See **logarithm**.

LOGICAL DESIGN. (1) The planning of a computer or data-processing system prior to its detailed engineering design. (2) The synthesizing of a network of **logical elements** to perform a specified function. (3) The result of (1) and (2) above, frequently called the logic of the system, machine, or network.

LOGICAL DIAGRAM. In **logical design**, a diagram representing the **logical elements** and their interconnections without necessarily expressing construction or engineering details.

LOGICAL ELEMENT. In a computer or data-processing system, the smallest building blocks which can be represented by operators in an appropriate system of symbolic logic. Typical logical elements are the **and-gate** and the **flip-flop**, which can be represented as operators in a suitable symbolic logic.

LOGICAL OPERATION. (1) Any nonarithmetical operation. Examples are: **extract**, logical (bit-wise) multiplication, **jump**, data transfer, etc. (2) Sometimes, only those nonarithmetical operations which are expressible bit-wise in terms of the propositional calculus or a two-valued Boolean algebra.

LOGICAL SYMBOL. A symbol used to represent a **logical element** graphically.

LOGISTIC CURVE. The logistic curve is a growth curve used to describe functions which continually increase, gradually at first, more rapidly in the middle growth period, and slowly again, reaching a maximum at the end of the growth. Its equation is

$$y = \frac{k'}{1 + e^{a+bx}}, \quad b < 0,$$

the symmetrical logistic. A more general form is

$$y = k_1 + \frac{k_2}{1 + e^{a+bx+cx^2}}, \quad c < 0,$$

the asymmetrical or skew logistic.

LOGIT. In biological response, the probability of response P to dose x is sometimes represented by the logistic curve

$$P = \frac{1}{1 + e^{-(\alpha + \beta x)}}.$$

The logit of P is defined as

$$Y = \log \frac{P}{1 - P}$$

and in this case is the linear function $-(\alpha + \beta x)$. Thus the theory of linear relations and **regressions** can be applied to Y and x .

LONDON FORCES. Weak forces of attraction (also called **dispersion forces**) exerted on each other by inert atoms. London, on the basis of quantum mechanics, has shown that they are due to the perturbation of the repulsive ground state by the higher electronic states of the system consisting of the two atoms. This perturbation at large internuclear distances r , gives a potential energy decreasing as $-(1/r^6)$ toward smaller r values. At smaller distances r the strong repulsion of the zero-valent atoms sets in, so that only a very shallow minimum at a relative large internuclear distance results. Analogous forces also add to the mutual attraction of atoms with free valences and of molecules with or without permanent dipole moments. (See also **Van der Waals forces**.)

LONDON SUPERCONDUCTIVITY EQUATIONS. In the treatment due to F. and H. London, the charge density ρ and the current density \mathbf{j} in superconducting matter are each considered as the sum of two parts, so that

$$\frac{\partial \rho^{(n)}}{\partial t} + \nabla \cdot \mathbf{j}^{(n)} = 0$$

and

$$\frac{\partial \rho^{(s)}}{\partial t} + \nabla \cdot \mathbf{j}^{(s)} = 0.$$

The superscript n corresponds to the "normal part," and the superscript s to the superconducting part. If \mathbf{E} and \mathbf{B} are, respectively, the electric field in electrostatic units, and

magnetic induction in electromagnetic units, we have

$$\mathbf{j}^{(n)} = \sigma \mathbf{E}, \quad \frac{\partial}{\partial t} (\Lambda \mathbf{j}^{(s)}) = \mathbf{E}$$

and

$$\nabla \times (\Lambda \mathbf{j}^{(s)}) = -\mathbf{B}/c$$

where c is the velocity of light, Λ is a constant of the material which depends on temperature, and σ is a conductivity which is finite at all temperatures.

LONGITUDE. 1. *Terrestrial Longitude.* The longitude of a point on the surface of the earth is measured from the terrestrial meridian through Greenwich, England east or west along the equator, or along a parallel of latitude, to the terrestrial meridian through the point. This angle may be expressed in units of angle (usually degrees) or units of time. In units of time longitude is expressed toward the west from Greenwich through 24 hours.

2. *Celestial Longitude.* The position of a heavenly body may be referred to the plane of the **ecliptic** as the fundamental plane. Great circles passing through the poles of the ecliptic are known as *circles of latitude*. The celestial longitude of an object is the angular distance measured along the ecliptic from the vernal equinox, in the direction of the sun's motion on the ecliptic, to the point of intersection of the circle of latitude through the object, with the ecliptic.

3. *Galactic Longitude.* The position of a heavenly body may be referred to the plane of the galaxy as the fundamental plane. Galactic longitude is measured eastward from an intersection of the celestial equator and the galactic equator at about Right Ascension 18h 40m.

LONGITUDINAL MAGNIFICATION. In an axially symmetric optical system longitudinal magnification is the **linear magnification** measured in the axial direction. Thus dx'/dx is longitudinal magnification if dx is measured parallel to the optical axis in object space and dx' is the conjugate distance in image space.

LONGITUDINAL SHEAR. See **shear, longitudinal**.

LONGITUDINAL SPHERICAL ABERRATION. The difference between the **focal length** of rim rays and paraxial rays. (See also **lateral spherical aberration**.)

LONGITUDINAL WAVES. See wave(s), dilational.

LONGITUDINAL WAVES IN A ROD. It is assumed that the rod is of homogenous material and constant cross section. The velocity of sound propagation in a rod is

$$c = \sqrt{\frac{Q}{\rho}} \quad (1)$$

where c is the velocity of sound propagation, in centimeters per second, Q is Young's modulus, in dynes per square centimeter, and ρ is the density, in grams per cubic centimeter.

LONG PERIOD VARIABLES. See variable stars.

LONG RANGE ORDER. A system may be said to possess long range order if it is possible to assign letters A, B, C , etc., to the sites of the lattice in such a way that there is a greater probability of finding an atom of type A on an A -site, of type B on a B -site, and so on, than any other arrangement. Such order is characteristic of **order-disorder transitions** in binary alloys. It is measured by the parameter

$$S = \frac{r - w}{r + w}$$

where r and w are the numbers of atoms on right and wrong sites respectively. (See also **cooperative phenomena**.)

LONG WAVES. Waves of infinitely long wavelength, or waves whose velocity is, for practical purposes, the same as those of infinite wavelength. They are gravity waves whose length is large compared with the fluid depth so that the horizontal displacement of the fluid particles is the same at all depths and the vertical acceleration is negligible (the pressure is hydrostatic at all times). The velocity of long waves in a single layer of fluid of depth h is $(gh)^{1/2}$.

In the atmosphere standing waves of great length (circa 3000 km) which occur in the upper air westerly winds and in which the air-stream executes a wavy pattern around a line of latitude with a few (usually between 3 and 7) waves encircling the pole, are called long waves.

LOOP. A closed path. An immediate consequence of this definition is that a finite graph

contains only a finite number of loops, a conclusion which is crucial for the practical application of Kirchhoff's voltage law.

LORENTZ CONDITION. The condition

$$\nabla \cdot \mathbf{A} + \frac{1}{c} \frac{\partial \phi}{\partial t} = 0$$

which may be imposed on the **vector** and **scalar potentials** describing an electromagnetic field. In **Minkowski space** the condition may be written

$$\frac{\partial A_\mu}{\partial x_\mu} = 0.$$

LORENTZ DERIVATIVE. See Helmholtz derivative.

LORENTZ FIELD. The fictitious field introduced in the theory of dielectric and magnetic polarization in order to find the actual local field acting on the molecules. It is defined as the field produced inside a spherical cavity in a uniformly polarized medium, and has the value $(4\pi/3)\mathbf{P}$, where \mathbf{P} is the **polarization** of the medium.

LORENTZ FORCE. The force exerted by an electric field \mathbf{E} and a magnetic field \mathbf{B} on a charge q moving with velocity \mathbf{v} . It is given by the formula

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}).$$

LORENTZ GAS. A simplified model of a fully ionized gas, which is useful for the calculation of plasma transport properties. The electron-electron interactions are neglected and the ions are assumed to be infinitely massive.

LORENTZ LAW. See Lorentz number.

LORENTZ-LORENZ LAW. If n is the refractive index and ρ the density of a dielectric, then the Lorentz-Lorenz law is

$$n^2 - 1 = C(n^2 + 2)\rho$$

where C is a constant for the dielectric known as the *specific refractive power*. (See **Clausius-Mossotti equation**.)

LORENTZ NUMBER. The ratio

$$L = K/\sigma T$$

where K is the **thermal conductivity** and σ the **electrical conductivity** of a metal at absolute

temperature T . This ratio was observed to be nearly constant for many metals by Wiedemann and Franz, in agreement with the **free electron theory of metals** which predicts the value

$$L = \frac{\pi^2}{3} \left(\frac{k}{e} \right)^2 = 2.7 \times 10^{-13} \text{ statvolt}^2/\text{deg}^2$$

where k is **Boltzmann's constant** and e the electronic charge. This constancy is not maintained down to low temperatures, at least until the regime of **residual resistance** is attained.

LORENTZ TRANSFORMATION. The coordinate system $\Sigma'(x', y', z', t')$ moves with the constant velocity \mathbf{v} along the positive x -direction of the coordinate system $\Sigma(x, y, z, t)$. At the time $t = t' = 0$, the two coordinate systems coincide. The transformation laws are

$$\mathbf{r}' = \mathbf{r} - \mathbf{v} \left\{ \frac{\mathbf{r} \cdot \mathbf{v}}{v^2} \left(1 - \frac{1}{\sqrt{1 - \beta^2}} \right) + \frac{t}{\sqrt{1 - \beta^2}} \right\}$$

$$t' = \frac{t - \frac{\mathbf{r} \cdot \mathbf{v}}{c^2}}{\sqrt{1 - \beta^2}}, \quad \beta = \frac{v}{c}$$

The transformation matrix is:

	x	y	z	t
x'	$\frac{1}{\sqrt{1 - \beta^2}}$	0	0	$\frac{-v}{\sqrt{1 - \beta^2}}$
y'	0	1	0	0
z'	0	0	1	0
t'	$\frac{-v}{c^2 \sqrt{1 - \beta^2}}$	0	0	$\frac{1}{\sqrt{1 - \beta^2}}$

If the components of \mathbf{v} are not $(v, 0, 0)$ but (v_x, v_y, v_z) the transformation matrix is

	x	y	z	t
x'	$1 - \frac{v_x^2}{v^2} M$	$-\frac{v_x v_y}{v^2} M$	$-\frac{v_x v_z}{v^2} M$	$-v_x / \sqrt{1 - \beta^2}$
y'	$-\frac{v_x v_y}{v^2} M$	$1 - \frac{v_y^2}{v^2} M$	$-\frac{v_y v_z}{v^2} M$	$-v_y / \sqrt{1 - \beta^2}$
z'	$-v_x v_z M$	$-v_y v_z M$	$1 - \frac{v_z^2}{v^2} M$	$-v_z / \sqrt{1 - \beta^2}$
t'	$-\frac{v_x}{c^2 \sqrt{1 - \beta^2}}$	$-\frac{v_y}{c^2 \sqrt{1 - \beta^2}}$	$-\frac{v_z}{c^2 \sqrt{1 - \beta^2}}$	$1 / \sqrt{1 - \beta^2}$

where

$$M = 1 - \frac{1}{\sqrt{1 - \beta^2}}$$

and

$$\beta^2 = \frac{v^2}{c^2} = \frac{v_x^2 + v_y^2 + v_z^2}{c^2}$$

Kinematic consequences of the Lorentz-transformation:

(A) If a material point has the velocity $\mathbf{w} = \frac{d\mathbf{r}}{dt}$ in Σ and the velocity $\mathbf{w}' = \frac{d\mathbf{r}'}{dt'}$ in Σ' ,

then

$$\mathbf{w}' = \frac{\mathbf{w} \sqrt{1 - \beta^2} - \mathbf{v} \left\{ \frac{\mathbf{w} \cdot \mathbf{v}}{v^2} (\sqrt{1 - \beta^2} - 1) + 1 \right\}}{1 - \frac{\mathbf{w} \cdot \mathbf{v}}{c^2}}$$

Special cases:

(1) If \mathbf{w} is parallel to \mathbf{v} , then

$$\mathbf{w}' = \mathbf{v} \frac{\left(\frac{w}{v} - 1 \right)}{\left(1 - \frac{wv}{c^2} \right)} = \frac{\mathbf{w} - \mathbf{v}}{1 - \frac{wv}{c^2}}$$

(Addition law of velocities)

(2) If \mathbf{w} is perpendicular to \mathbf{v} , then

$$\mathbf{w}' = \mathbf{w} \sqrt{1 - \beta^2} - \mathbf{v};$$

and

$$w'^2 = w^2 + v^2 - \frac{v^2 w^2}{c^2}$$

(B) *Time Dilatation (Slowing of Clocks)*

If a clock ticks n times per second in a rest frame, then from the point of an observer moving with velocity v relative to the clock, it will appear to tick

$$n \sqrt{1 - \frac{v^2}{c^2}}$$

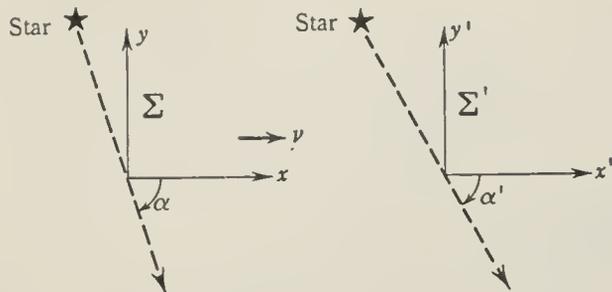
times per second.

(C) *Lorentz-Fitzgerald Contraction*

If a body moves through space with speed v , its dimension parallel to the direction of motion is reduced in the ratio

$$\sqrt{1 - \frac{v^2}{c^2}} : 1.$$

(D) Aberration of light



$$\tan \alpha' = \frac{\sin \alpha \sqrt{1 - \beta^2}}{(\cos \alpha) - \beta}. \text{ For small values of } \beta, \text{ the angle } \Delta \alpha = \alpha' - \alpha \text{ (} \alpha = \text{angle of aberration) is } \Delta \alpha = \beta \sin \alpha.$$

(E) Doppler Effect

Assume an observer at rest and a light source moving with the velocity v and the component of this velocity in the direction to the observer to be $v \cos \alpha$, then

$$\frac{\lambda}{\lambda'} = \frac{1 - \frac{v}{c} \cos \alpha}{\sqrt{1 - \beta^2}}$$

or

$$\frac{\Delta \lambda}{\lambda} = \frac{\lambda' - \lambda}{\lambda} = \frac{\sqrt{1 - \beta^2} - 1 + \frac{v \cos \alpha}{c}}{1 - \frac{v \cos \alpha}{c}}$$

where λ = wavelength of light if the source is at rest, and λ' = wavelength of light as it appears to the observer if the light source is moving.

Special cases:

(1) Longitudinal Doppler effect ($\alpha = 0$)

$$\frac{\Delta \lambda}{\lambda} = \frac{\sqrt{1 - \beta^2} - 1 + \beta}{1 - \beta} \sim \beta.$$

(2) Transverse Doppler effect ($\alpha = 90^\circ$)

$$\frac{\Delta \lambda}{\lambda} = (\sqrt{1 - \beta^2} - 1) \sim -\frac{\beta^2}{2}.$$

(F) Fresnel Coefficient of Drag

The correction to be applied to the velocity of light, in a moving transparent medium, to correct for the motion of the medium. It is given by

$$\pm v \left(\frac{n^2 - 1}{n^2} - \frac{\lambda}{n} \cdot \frac{dn}{d\lambda} \right)$$

where v is the velocity of the medium, n its index of refraction and λ is the wavelength of the light in vacuum.

LORENTZ TRANSFORMATION, PROPER. See proper Lorentz transformation.

LOSCHMIDT NUMBER. See Avogadro law.

LOSS ANGLE IN A DIELECTRIC MATERIAL. When a dielectric substance is placed for a sufficient time in a periodic field $E = E_0 \cos \omega t$, the electric displacement D becomes itself periodic in time. In general, the displacement and the field are not in phase, and one may write:

$$D = D_0 \cos (\omega t - \phi).$$

The phase angle ϕ is related to the energy losses in the dielectric (which are proportional to $\tan \phi$), and is therefore usually called the loss angle. (See dielectric constant, complex.)

LOSS, COMPLIANCE. See compliance loss.

LOSS, DIVERGENCE. See divergence loss.

LOSS FACTOR, DIELECTRIC. The rate at which heat is generated in a dielectric is proportional to its loss factor, which is equal to the product of its dielectric constant by its power factor. Both the dielectric constant and power factor are usually functions of frequency, therefore the loss factor changes with changing frequency.

LOSS FACTOR, MECHANICAL. When a stress $\sigma_0 \cos \omega t$ is applied to an elastic solid in which damping is present, the strain lags behind the stress, so that $\epsilon = \epsilon_0 \cos (\omega t - \delta)$, $\tan \delta$ is called the mechanical loss factor. This characterizes the damping at the angular frequency ω and can be related to other measures of internal friction. (See friction, internal.)

LOSS FUNCTION. In decision theory, a function of the decision and the true underlying distributions which expresses the loss incurred in taking that decision. If there are a number of possible situations, the array of losses according to situation and decision is called the *loss matrix*. It is analogous to the pay-off matrix of games theory.

LOSS, INSERTION. See insertion loss.

LOSS MATRIX. See loss function.

LOSS MODULUS. See modulus, loss.

LOSS TANGENT. See dielectric constant, complex.

LOSS, TRANSMISSION. A general term used to denote a decrease in signal power in transmission from one point to another. Loss is usually expressed in **decibels**.

LOUDNESS CONTOURS. Curves which show the related values of **sound pressure level** and frequency required to produce a given loudness sensation for the typical listener. (See **loudness of a sound**.)

LOUDNESS OF A SOUND. The magnitude of the auditory sensation produced by the sound. The relation between the loudness and the **intensity** of a sound should agree with common experience in estimates of the sensation magnitude. A true loudness scale should be constructed so that when the units are double the sensation will be doubled and when the scale is trebled the sensation will be trebled, etc. The unit on the loudness scale is termed the *son*. The loudness level of the reference tone, expressed in *phons*, is the intensity level of the reference tone of 1000 cycles per second expressed in decibels. By definition, a loudness of one *son* has been arbitrarily selected to correspond to a loudness level of 40 phons.

The loudness of a tone of 1000 cycles is given by the empirical equation,

$$\log_{10} N = 0.03L_N - 1.2$$

where N is the loudness, in *sones*, and L_N is the loudness level, in *phons*.

The loudness level is given by,

$$L_N = 10 \log \frac{I}{I_0}$$

where L_N is the loudness level, in *phons*, I is the intensity of the sound, in watts per square centimeter, I_0 is the intensity of the reference sound, that is, the threshold of hearing ($= 10^{16}$ watts per square centimeter).

LOWER BOUND THEOREM FOR PLASTICITY. See **limit theorems**.

LOWER CALORIFIC VALUE. See **combustion**.

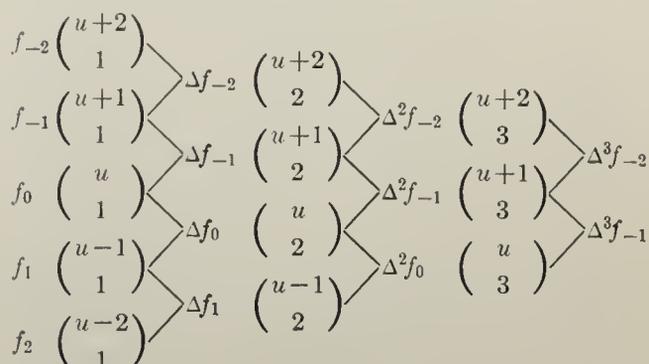
LOWER IGNITION LIMIT. See **ignition limit**.

LOWERING INDICES ON A TENSOR. See **raising and lowering indices on a tensor**.

LOXODROME (CURVE). A curve on a surface of revolution which cuts the meridians at a constant angle, other than a right angle. Also called *loxodromic spiral*, or *loxodromic line*, or *loxodromic curve*. (See also **rhumb line**.)

LOZENGE DIAGRAM. A mnemonic scheme from which can be read off a great variety of forms of the interpolating polynomial for equal-interval interpolation. (For generalization, see Zdenek Kopal, *Numerical Analysis*, John Wiley & Sons, Inc., 1955.)

For forward difference formulas write



where the table can be extended upwards, downwards, and to the right in an obvious way. A valid polynomial for interpolation can be constructed by starting with any f_i in the first column, multiplying the binomial coefficient on the right by either of the two Δf_i to which it is joined, multiplying the coefficient on the right of that by either $\Delta^2 f_i$, continuing thus and adding results. Moreover, all polynomials that end with the same $\Delta^n f_i$ on the right are identically equal.

The same diagram can be used for central differences if, for example, one replaces Δf_{-2} by $\delta f_{-3/2}$, $\Delta^2 f_{-1}$ by $\delta^2 f_{-1}$, \dots .

LR TRANSFORMATION. A method for obtaining the **eigenvalues** and **eigenvectors** of a **matrix** \mathbf{A} , consists of taking $\mathbf{A}_0 = \mathbf{A}$ and forming

$$\mathbf{A}_i = \mathbf{L}_i \mathbf{R}_i, \quad \mathbf{A}_{i+1} = \mathbf{R}_i \mathbf{L}_i,$$

where \mathbf{R}_i is upper triangular and \mathbf{L}_i unit lower triangular. Under rather general circumstances the sequence of matrices \mathbf{A}_i approaches an upper triangular matrix in the limit, and

since all A_i have the same proper values as has A itself, the diagonal elements of the limit matrix are these proper values. The method is due to Rutishauser, who has also devised methods of accelerating convergence and for treating exceptional cases. For a discussion of the factorization see **matrix inversion**. (See also National Bureau of Standards, *Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues*, NBS Appl. Math. Series 49, 1958.)

(L, S) COUPLING. See **Russell-Saunders coupling**.

L-SYSTEM AND C-SYSTEM. In analyzing collision processes, it is convenient to use two frames of reference. In one, the *L*-system, the coordinate system is assumed to be fixed in the laboratory, or with respect to the observer. In the other, the *C*-system, the coordinate system is assumed to be fixed with respect to the center of mass of the colliding particles.

I-TYPE DOUBLING. In a linear polyatomic molecule, energy levels with vibrational angular momentum $l = 1, 2, \dots$ are doubly degenerate if the molecule is not rotating. The splitting of these degenerate states, brought about by interaction with the molecular rotation, is called *l*-type doubling. *l*-type doubling produces two series of rotational terms with slightly different constants, B_v^c and B_v^d . The difference $\Delta\nu$ between the two term series is given by

$$\Delta\nu = q_l J(J+1) = (B_v^c - B_v^d)J(J+1).$$

The two rotational levels belonging to a given J have opposite **parity** (+, -).

LÜDER'S LINES. Loci of concentrated shear strain which appear after plastic flow of metals with an upper yield point. They occur along **slip lines**, or trajectories of maximum shear stress.

LUMEN. One lumen is the luminous flux emitted within unit solid angle (one **steradian**) by a point source having a uniform intensity of one **candela**. Definition adopted by the 9th General Conference on Weights and Measures (1948).

LUMEN-HOUR (LUMEN-SECOND). Units of quantity of light; the quantity of light

equal to one lumen radiated or received for a period of one hour (second).

LUMEN PER SQUARE METER (LUX). A unit of **illumination**, recommended by the C.I.E.

LUMEN-SECOND. See **lumen-hour**.

LUMERG. A unit of luminous energy (see **energy, luminous**). One erg of radiant energy having a luminous efficiency of x **lumens** per watt is x lumergs of luminous energy.

LUMINANCE. Luminance (at a point of a surface and in a given direction) is the quotient of the **luminous intensity** in the given direction of an infinitesimal element of the surface containing the point under consideration, by the orthogonally projected area of the element on a plane perpendicular to the given direction.

LUMINANCE FACTOR. Luminance factor (of a non-luminous body—secondary light source—under specified conditions of illumination and observation) is the ratio of the **luminance** of the body considered, illuminated and observed under these conditions, to the luminance of a **perfect diffuser** receiving the same illumination. (See **spectral luminance factor**.)

LUMINANCE PRIMARY. That one of a set of three **transmission primaries** whose amount determines the **luminance** of a color.

LUMINANCE TEMPERATURE FOR A GIVEN WAVELENGTH. The absolute temperature of a **full radiator** for which the **luminance** at the specified wavelength has the same spectral concentration as for the radiator considered. In practice, particularly in pyrometry, the reference wavelength is generally 655 millimicrons.

LUMINANCE TRANSMISSION CONSTANT. That type of transmission in which the transmission primaries are a **luminance primary** and two **chrominance primaries**.

LUMINOSITY COEFFICIENTS. The constant multipliers for the respective **tristimulus values** of any color, such that the sum of the three products is the **luminance** of the color.

LUMINOSITY FUNCTION (STANDARD). Because of the variable sensitivity of the eye to radiation of different wavelengths, a stand-

ard function has been established by the Commission Internationale de l'Éclairage (CIE), formerly called in English translations International Commission on Illumination (ICI), for converting radiant energy into luminous (i.e., visible) energy.

For the standard conditions chosen in establishing this standard luminosity function (**photopic vision**) the luminously effective radiant intensity in lumens of radiation of spectral intensity J_λ watts/unit wavelength is given by

$$680 \int_{\lambda=0}^{\lambda=\infty} y_\lambda J_\lambda d\lambda$$

where y_λ is the standard luminosity function normalized to a value of unity at 555 millimicrons. The numerical values for y_λ are commonly given as a **luminosity curve**.

For very low levels of intensity (**scotopic vision**) the peak of the luminosity function curve shifts toward the violet for young eyes (507 $m\mu$) with an absolute value of 1746 lumens/watt.

LUMINOSITY, RELATIVE. The ratio of the value of the **luminosity** at a particular wavelength to the value at the wavelength of maximum luminosity. (See also **stellar luminosities**.)

LUMINOSITY STELLAR. See **stellar luminosities**.

LUMINOUS COEFFICIENT. A coefficient which measures the integrated fraction of the **radiant power** that contributes to its luminous properties as evaluated by means of the standard **luminosity function**.

Luminous coefficient

$$= \frac{\int_{\lambda=0}^{\lambda=\infty} y_\lambda J_\lambda d\lambda}{\int_{\lambda=0}^{\lambda=\infty} J_\lambda d\lambda}$$

Where y_λ is the standard luminosity function and J_λ is the spectral power distribution of the radiant intensity. The luminous coefficient is unity for a narrow band of wavelengths at 555 millimicrons.

LUMINOUS EFFICIENCY. (1) The luminous efficiency of radiation is the ratio of **luminous flux** to the corresponding **radiant flux**. (2) The luminous efficiency of a source is the ratio of the total flux emitted to the total power consumed.

LUMINOUS EFFICIENCY, RELATIVE. See **relative luminous efficiency**.

LUMINOUS EMITTANCE (FROM A POINT OF A SURFACE). The quotient of the **luminous flux** emitted from an infinitesimal element of surface containing the point under consideration, by the area of that element.

LUMINOUS FLUX. The quantity characteristic of radiant flux which expresses its capacity to produce a luminous sensation, evaluated according to the values of relative **luminous efficiency**. Unless otherwise indicated, the luminous flux in question relates to photopic vision, and is connected with the radiant flux in accordance with the formula adopted in 1948 by the C.I.E., i.e., by the relation

$$\phi = K_m \int P_\lambda \cdot V_\lambda \cdot d\lambda$$

in which $P_\lambda \cdot d\lambda$ is the radiant flux corresponding to the radiation comprised between λ and $\lambda + d\lambda$, and V_λ is the relative luminous efficiency, the values of which as a function of λ are contained in colorimetric tables.

LUMINOUS INTENSITY (IN A GIVEN DIRECTION). The quotient of the **luminous flux** emitted by a source, or by an element of a source, in an infinitesimal cone containing the given direction, by the solid angle of that cone.

LUMINOUS REFLECTANCE, DIRECTIONAL. The ratio of the **luminance** of an imperfectly diffusing surface to the **illuminance** of a perfectly diffusing (and perfectly reflecting) surface.

LUMINOUS REFLECTANCE, SPECULAR. The ratio of (1) the **illuminance** of a surface observed by reflection in a mirror to (2) the **illuminance** of the surface seen directly in the specular luminous reflectance of the mirror.

LUNDQUIST SERIES CRITERION. A criterion for the stability of rigid-jointed frameworks based upon a **moment distribution** procedure. (See also **Hoff convergence method**.)

LUNEBURG GEOMETRY. R. K. Lunenburg developed the geometry of the binocular field of view, called *visual space*. The apparently straight lines (as opposed to the physically

straight lines) are geodesics in a Riemannian space of constant curvature. Early psychological experiments seemed to indicate that the space was hyperbolic, but this conclusion seems to be in doubt at present. (See **equidistant alleys**; **horopter**; **parallel alleys**; **Victh-Müller torus**.)

LUNEBURG LENS. Luneburg has shown that a medium with index of refraction

$$n = \frac{r^{m-1}}{1 + r^{2m}}, \quad m \geq 1,$$

is a perfect optical system, where r is the distance from a fixed point. When $m = 1$ the system is **Maxwell's fisheye**.

LUX. See **lumen per square meter**.

LUX-SECOND. A unit of quantity of illumination; a quantity of illumination of one **lux** continued for one second.

LYMAN SERIES. A series of lines, in the ultraviolet region, of the spectra of hydrogen and the hydrogen-like ions He^+ , Li^{++} , \dots . The wave numbers

$$\nu_n = \frac{1}{\lambda_n}$$

are represented by the formula

$$\nu_n = RZ^2 \left[\frac{1}{1^2} - \frac{1}{n^2} \right], \quad (n = 2, 3, \dots),$$

where R stands for the **Rydberg constant** and Z for the **atomic number**.

M

MACAULEY METHOD. A method of simplifying the double integration procedure for calculating the deflections of beams of uniform cross section when the applied loading is discontinuous. Two constants of integration only are introduced. Junction conditions at points of discontinuity of loading $x = a$ are satisfied by integrating $x - a$ as a unit, and by extending all distributed loads to the right-hand end of the beam, introducing negative loads as needed.

MACH ANGLE. At a point in a supersonic gas flow where the Mach number is M the Mach angle is

$$\mu = \sin^{-1} \frac{1}{M}.$$

(See also **Mach cone**, **Mach line**.)

MACH CONE. In a *uniform* supersonic gas flow, lines drawn through any point P , making an angle with the streamlines equal to the **Mach angle** μ , defines the surfaces of two cones, with their apices at P . These are the *upstream* and *downstream* Mach cones for P . Any small disturbance at P cannot affect any part of the flow outside the downstream Mach cone, correspondingly, P cannot be affected by any small disturbance outside the upstream Mach cone.

If the flow is not uniform, lines may still be drawn making an angle μ with the stream direction at each point, but these lines will not be straight, and the surface defined by the lines will be a conoid having a tangent cone at its vertex of apex angle 2μ .

In uniform two-dimensional flow the Mach cone becomes a wedge, whose faces make equal angles μ with the stream. (See also **Mach angle**; **Mach line**.)

MACH CRITERION. Only those propositions should be employed in physical theory from which statements about observable phenomena can be deduced.

MACHE UNIT. A unit of quantity of radioactive emanation, defined as the quantity of

emanation which sets up a saturation current equal to one one-thousandth of the electrostatic unit of current. It is equal to 3.6×10^{-10} curie.

MACH LINE. A line drawn in a supersonic gas flow making an angle with the local flow direction equal to the **Mach angle** μ . The velocity component normal to a Mach line is sonic and small disturbances in a supersonic flow are propagated along Mach lines.

The Mach lines through a given point in a uniform flow define the **Mach cone** at that point. In two-dimensional flow there are two families of Mach lines, running downstream to the right and left of the streamlines. The former are usually known as $+$ Mach lines, the latter as $-$ Mach lines. (See also **characteristics**, **method of**; **Mach angle**; **Mach cone**.)

MACH NUMBER. The ratio of the speed of flow at a point in a fluid to the speed of sound in the fluid at the same point. For the important case of a body in a uniform stream, the Mach number of the undisturbed stream is usually given.

Since the speed of sound is $a = (\gamma p / \rho)^{1/2}$, the Mach number M (or Ma) corresponding to a speed of flow v is given by

$$M^2 = \left(\frac{v}{a}\right)^2 = \frac{\rho v^2}{\gamma p}.$$

Since the pressure changes due to the flow are $O(\rho v^2)$, the resulting density changes will be negligible if $M^2 \ll 1$. The fluid may then be considered to be incompressible, if the only density changes to be considered are those due to the flow.

MACH NUMBER, CRITICAL. See **critical Mach number**.

MACLAURIN SERIES. A special case of the **Taylor series**. If $f(x)$ and all of its **derivatives** remain finite at $x = 0$, then $f(x)$ may be expanded to

$$f(x) = f(0) + f'(0)x + f''(0) \frac{x^2}{2!} + \dots + f^{(n-1)}(0) \frac{x^{n-1}}{(n-1)!} + R_n$$

where R_n is the remainder after n terms. When the remainder converges towards zero as the number of terms increases, the result is the **Maclaurin series** for $f(x)$ at $x = 0$.

Expressions for R_n are:

$$\frac{x^n}{n!} f^{(n)}(\theta x), \quad 0 < \theta < 1;$$

$$\frac{1}{(n-1)!} \int_0^x f^{(n)}(x-t)t^{n-1} dt.$$

MACROCANONICAL ENSEMBLE. A term denoting a **canonical ensemble**, but especially used to distinguish it from a **microcanonical ensemble**.

MACROSCOPIC. A term used in **statistical mechanics** to describe any of the constants, properties, or variables of a system which are directly observable by the methods of measurement used in an actual or hypothetically realizable experiment on the system.

MACROSTATE. (1) A term which describes a state of a system of independent particles where the total number of particles in each single particle level is given. A single macrostate may correspond to many different **microstates**. (2) Any state of a system which can be distinguished from all other states by observations made on the system in an actual or in a realizable hypothetical experiment. That is, the **state** of a **thermodynamic system** described in terms of macroscopic properties, i.e., in terms of properties, such as pressure, volume, temperature, etc., which refer to the whole system. The macrostate usually involves a small number of independent properties and the description is independent of the system's molecular or atomic structure, in contradistinction to its **microstate**.

MADLUNG CONSTANT. A constant appearing in the term for the **Coulomb energy** ϕ of an ionic crystal (see **Born-Mayer equation**). It may be defined by A in the equation

$$\phi = - \frac{Ae^2}{R}$$

where R is the separation of nearest neighbors in the crystal, and is evaluated by adding the mutual electrostatic potential energies of all the ions in the particular lattice. The Madelung constant also appears in the expression for the wavelength of the residual radiation (Reststrahlen) selectively reflected by a given heteropolar crystal.

MADLUNG EXPRESSION FOR ELECTRICAL POTENTIAL DUE TO INFINITE LINEAR ARRAY. Consider an infinite line array of charges consisting of p kinds of charge e_j , the j^{th} kind being located at $x_j \pm t, x_j \pm 2t$, etc., along a line, and let the array be neutral, so that

$$\sum_{j=1}^p e_j = 0.$$

Then Madelung's method gives for the potential V , at a point whose coordinate along the line is x and whose distance from it is r ,

$$V(x,r) = \frac{4}{t} \sum_{n=1}^{\infty} \sum_{j=1}^p e_j K_0 \left(\frac{2\pi nr}{t} \right) \cos \left[\frac{2\pi n(x - x_j)}{t} \right]$$

where

$$K_0(z) = (\pi i/z) H_0^{(1)}(iz)$$

is the modified **Bessel function** of the second kind.

MAGIC NUMBERS. Certain numbers of neutrons or protons which give rise to a nucleus of especially high stability (i.e., high binding energy). Thus, nuclei containing 2, 8, 20, 28, 50, 82, or 126 neutrons or protons are especially stable. For example, tin ($Z = 50$) has as many as ten stable isotopes, while lead ($Z = 82$) is the end-product of several radioactive series. By analogy with the structure of atoms, these numbers represent **closed shells** of neutrons or of protons in the nucleus.

MAGNETIC ANISOTROPY. The dependence of magnetic properties on direction; e.g., a crystal exhibits "hard" and "easy" directions for **magnetization**. As one result, a magnetizing force \mathbf{H} not parallel to a crystal axis, will produce induction \mathbf{B} that is not parallel to \mathbf{H} . The "ratio" of \mathbf{B} to \mathbf{H} is no longer a number (μ), but becomes a **tensor**.

MAGNETIC CHARGE DENSITY (ρ°). The divergence of the magnetic induction \mathbf{B} ,

by analogy with electric charge density which is the divergence of the **dielectric displacement D**. A fictitious concept sometimes used in writing the **Maxwell equations** in a symmetric form.

MAGNETIC CIRCUIT. See **Bosanquet law**.

MAGNETIC CURRENT DENSITY, (\mathbf{J}^*) . A vector quantity related to the **magnetic charge density** by the equation of continuity. Hence

$$\mathbf{J}^* = -\nabla \times \mathbf{E} - \frac{\partial \mathbf{B}}{\partial t}.$$

Since \mathbf{J}^* , being a fictitious quantity, is identically zero, the quantity

$$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

is sometimes called the **magnetic current density**.

MAGNETIC DIPOLE. See **dipole, magnetic**.

MAGNETIC DOUBLE REFRACTION. See **Cotton-Mouton effect**.

MAGNETIC ENERGY. See **energy; Poynting theorem**.

MAGNETIC FIELD. A vector field associated with distributions of currents and magnetized materials. (See **magnetic induction; magnetic field intensity**.)

MAGNETIC FIELD STRENGTH, \mathbf{H} . A derived vector, associated with a magnetic field, which is independent of the magnetic properties of the material. Thus for any given distribution of currents the measurable magnetic effects vary with the material within the system, but the field strength \mathbf{H} does not. The unit is the **ampere-turn/meter**.

MAGNETIC FLUX. The magnetic flux through any bounded surface S is the surface integral of the normal component of the **magnetic induction** taken over the surface.

$$\phi = \int_S \mathbf{B}_n \cdot d\mathbf{S}.$$

MAGNETIC FLUX DENSITY, \mathbf{B} . See **magnetic induction**.

MAGNETIC INDUCTION (\mathbf{B}) (MAGNETIC FLUX DENSITY). The observable vector associated with a magnetic field. By introducing a circuit element of length dl which carries a current I , the mechanical force is given by the quantity $I dl \times \mathbf{B}$. The unit is the **weber/square meter**. (For other units, see **electromagnetic units**.)

MAGNETIC MOMENT DENSITY. See **magnetization**.

MAGNETIC MOMENT, \mathbf{m} . The magnetic moment of a body is the volume integral of its **magnetization, \mathbf{M}** . The torque exerted on a magnet or current loop of moment \mathbf{m} in a uniform field of magnetic induction \mathbf{B} is $\mathbf{m} \times \mathbf{B}$.

MAGNETIC MOMENT, NUCLEAR OR ATOMIC. The magnetic moment of a nuclear or atomic particle or system of particles usually denotes the magnetic dipole moment. For a particle or system in a magnetic field, the interaction energy is the negative of the product of the magnetic field strength (see **magnetizing force**) by the component of the magnetic dipole moment of the particle in the direction of the field ($-\mu_H H$). A magnetic moment is associated with the intrinsic spin of a particle and with the orbital motion of a particle in a system, e.g., nuclei with finite spins have finite magnetic moments between about -2 and $+6$ nuclear Bohr magnetons.

MAGNETIC POLE. (1) (Also called dip pole.) In geomagnetism, either of the two points on the earth's surface at which the magnetic meridians converge, i.e., where the magnetic field is vertical. (Cf. **geomagnetic pole**.)

(2) In magnetic theory, a fictitious entity analogous to a unit electric charge of electrostatic theory. In nature only dipoles, not isolated magnetic poles, exist.

MAGNETIC POTENTIAL. Since the curl of a magnetostatic field in a current-free region vanishes, i.e.,

$$\nabla \times \mathbf{B} = 0, \quad \nabla \times \mathbf{H} = 0,$$

the vectors \mathbf{H} and \mathbf{B} are derivable from scalar potentials,

$$\mathbf{H} = -\nabla U.$$

(See **magnetic field**.)

MAGNETIC RESONANCE. See *resonance*, *magnetic*.

MAGNETIC RESONANCE, LINE WIDTH.

In magnetic resonance experiments, the width of the absorption lines depends on the interactions of the spins with each other and with the crystal lattice. It may be measured by the random fluctuating magnetic field exerted on a spin by its neighbors, i.e., is of the order of

$$\Delta H = \mu/a^3$$

where μ is the magnetic moment of each spin and a is the interatomic spacing. In liquids, however, the motion of the molecules is so rapid that this effect averages out to nearly zero. The line width in **ferromagnetic resonance** is anomalously large.

MAGNETIC RIGIDITY ($H\rho$). A measure of the momentum of a particle equal to the product of the **magnetic field intensity** perpendicular to the path of the particle, and the resultant radius of curvature of the path of the particle.

MAGNETIC SCALAR POTENTIAL. For a steady magnetic field in the absence of currents, the magnetic field intensity \mathbf{H} is irrotational (see **Maxwell's equations**). Hence a scalar potential exists.

MAGNETIC SHELL (MAGNETIC DIPOLE SHEET, DOUBLE LAYER). A surface distribution of **magnetic dipoles** associated with discontinuities in the normal component of the **magnetic field intensity** across the surface. The magnetic field of any circuit of arbitrary size and geometry may be identified with that of an equivalent magnetic shell bounded by the circuit.

MAGNETIC SPECTRUM. See *spectrum*, *magnetic*.

MAGNETIC STRAIN ENERGY (E_σ). A component of **potential energy** in a magnetic domain, given by:

$$E_\sigma = \frac{3}{2}\lambda_s\sigma \sin^2\theta$$

where λ_s is the **magnetostriction** expansion occurring between the demagnetized state and saturation, σ the tensile stress to which the domain is subject, and θ the angle between the magnetization and the tension.

MAGNETIC SURFACES. The magnetic lines of force of an arbitrary magnetic field which does not possess a high degree of symmetry will not in general close on themselves in any finite volume but will wander ergodically throughout all space. If, however, a field possesses lines of force which approximately close on themselves, a single line of force may trace out an approximately closed surface over which the line wanders ergodically. Such a surface is termed a magnetic surface.

MAGNETIC SUSCEPTIBILITY (χ). The ratio

$$\chi = |\mathbf{M}|/|\mathbf{H}| \text{ in rationalized units or}$$

$$\chi = |\mathbf{M}|/4\pi|\mathbf{H}| \text{ in non-rationalized units,}$$

where \mathbf{M} is the magnetization and \mathbf{H} is the magnetizing force. The susceptibility χ is related to the relative permeability μ_r by:

$$\mu_r = (1 + 4\pi\chi)\mu_0,$$

and again the factor 4π is dropped in rationalized units. Note that the magnitude of the susceptibility differs by a factor of 4π in rationalized and unrationalized units, but that μ_r is unaffected by rationalization. (See also **magnetization**; **paramagnetism**; **diamagnetism**; **ferromagnetism**.)

MAGNETIC VECTOR POTENTIAL, \mathbf{A} . The vector \mathbf{B} , the magnetic induction, is always solenoidal. Hence \mathbf{B} may always be written in the form $\mathbf{B} = \nabla \times \mathbf{A}$.

MAGNETISM, LAWS OF. The laws of magnetism are those of electromagnetism under steady-state conditions. The pertinent **Maxwell equations** reduce to

$$\nabla \times \mathbf{H} = \mathbf{J}$$

$$\nabla \cdot \mathbf{B} = 0.$$

In addition, the equation of continuity becomes

$$\nabla \cdot \mathbf{J} = 0$$

since there is no displacement current. By the **Stokes theorem**, the first equation can be expressed in integral form as

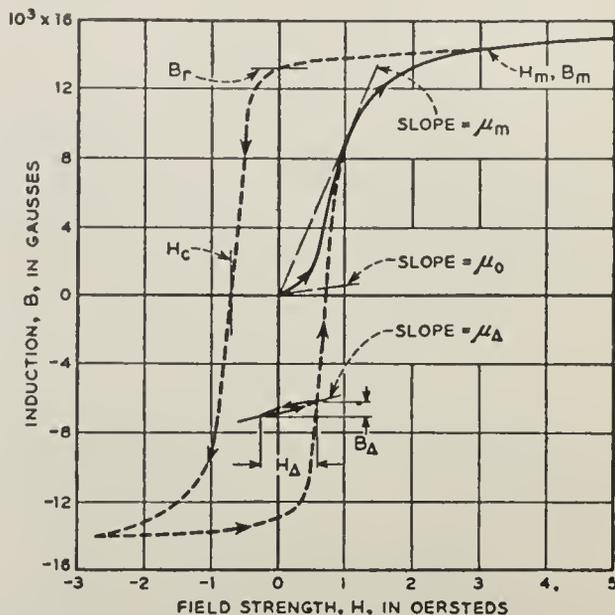
$$\oint \mathbf{H} \cdot d\mathbf{l} = I$$

(the *Ampère law*).

MAGNETISM, QUANTUM THEORY OF. As magnetic properties of matter are related to atomic phenomena, many of the magnetic properties can be understood only on the basis of quantum theory. The magnetic moment of an electron is partly due to its orbital motion. The corresponding contribution of the electrons to the magnetic moment of an atom is proportional to the orbital angular momentum M of the atom, $\mu = eM/2mc$, where e is the charge, m the mass of an electron, c is the speed of light. Since the components of M can take only values which are integral multiples of $k/2\pi$, the magnetic moment μ has components which are integral multiples of $ek/4\pi mc$. This is called the *Bohr magneton*. In addition to their orbital magnetic moment, electrons have a magnetic moment connected with their spin. The magnitude of this moment is equal to one Bohr magneton. Quantum theory gives a satisfactory explanation of the diamagnetism of atoms which is due to an electronic current distribution opposing the effect of the external magnetic field, of the temperature independent paramagnetism of many metals due to a re-orientation of spins in previously unoccupied states of a zone, of the temperature dependent paramagnetism of certain substances related to an orientation of existing **dipole moments**, of many features of ferromagnetism which is related to the living up of a large number of spins on neighboring atoms, and of many other magnetic phenomena. (See **ferromagnetic resonance**; **paramagnetic susceptibility**, **Langevin theory**; **Brillouin zone**; **band theory of solids**.)

MAGNETIZATION CURVE. An originally unmagnetized specimen of magnetic material, when subjected to an increasing magnetizing force (\mathbf{H}) develops the induction (\mathbf{B}) shown in the solid curve. This solid curve is called the magnetization curve, and is not retraced as \mathbf{H} is reduced. The initial slope of the magnetization curve is the initial permeability (μ_0). If \mathbf{H} is carried to some maximum value \mathbf{H}_m and then reduced (to $-\mathbf{H}_m$), \mathbf{B} follows the dotted hysteresis curve. The residual induction \mathbf{B}_r is the induction remaining when \mathbf{H} has been reduced to zero; the reverse \mathbf{H} needed to reduce \mathbf{B} to zero is called the coercive force (\mathbf{H}_c). If \mathbf{H} (and therefore also \mathbf{B}) goes through a small cyclic change starting from an arbitrary point on the hysteresis loop (see figure), a minor loop is described, with an

average slope μ_Δ , called incremental permeability. If the cyclic change of \mathbf{H} approaches zero, μ_Δ approaches a limit μ_r , the reversible permeability. If the positive and negative



Magnetization curve (solid) and hysteresis loop (dotted). (From Bozorth, *Ferromagnetism*, 1951. D. Van Nostrand Company, Inc., Princeton.)

peaks of \mathbf{H} are equal in a cyclic magnetization, the hysteresis loop is symmetrical. As the peak value \mathbf{H}_m is increased (making larger loops), the loop tips trace a path called the normal magnetization curve. (See also **hysteresis**.)

MAGNETIZATION, \mathbf{M} (MAGNETIC MOMENT DENSITY). The magnetic polarization vector \mathbf{M} describes the change in the magnetic field intensity due to the presence of matter. For a material with linear magnetic properties, it is proportional to the applied field intensity \mathbf{H} , and the ratio $|\mathbf{M}|/|\mathbf{H}|$ defines the magnetic susceptibility χ_m of the material. For materials with non-linear magnetic properties

$$\chi_m = \partial|\mathbf{M}|/\partial|\mathbf{H}|.$$

Magnetization is measured in **ampere-turns/meter**

MAGNETIZATION, SATURATION. The application of an increasing magnetizing force to a ferromagnetic substance yields a resulting intrinsic induction (\mathbf{B}_m) that asymptotically approaches a constant value (\mathbf{B}_s), the saturation magnetization.

MAGNETIZING FORCE (\mathbf{H}). An auxiliary vector field introduced for mathematical convenience in the discussion of magnetic phenomena. Its introduction is prompted by the fact that the Maxwell equation:

$$\nabla \times \mathbf{B} = \mu \mathbf{J} + \mu \frac{\partial \mathbf{D}}{\partial t},$$

which depends, through the permeability μ , upon the medium in which a solution is desired, may be made independent of the medium by introducing the magnetizing force field $\mathbf{H} = \mathbf{B}/\mu$. This leads to the equation:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}$$

which, with the application of Stoke's Theorem, gives directly the integral form of Ampere's law

$$\oint \mathbf{H} \cdot d\mathbf{s} = \int \mathbf{J} \cdot d\mathbf{A} = I,$$

for stationary currents. \mathbf{H} is measured in ampere-turns per meter (mksa) or oersteds (emu). The terms *magnetic field*, *magnetic field strength* and *magnetic field intensity* are often used for \mathbf{H} . (See *electromagnetic units*.)

MAGNETIZING FORCE, DYNAMIC. The applied magnetomotive force per unit length required to provide the flux and the rate-of-change of flux specified by the conditions for which a dynamic hysteresis loop is determined.

MAGNETOCALORIC EFFECT. Consider a specimen of matter whose magnetic moment is \mathbf{M} in an applied magnetic field \mathbf{H} . Then, if the field \mathbf{H} is changed, so that the entropy S remains constant, the absolute temperature T will change according to the relation

$$\left(\frac{\partial T}{\partial \mathbf{H}}\right)_s = -\frac{T}{C^{(H)}} \left(\frac{\partial \mathbf{M}}{\partial T}\right)_H$$

where $C^{(H)}$ is the heat capacity of the specimen at constant \mathbf{H} , and is equal to $T(\partial S/\partial T)_H$. The equation is valid if the derivatives are all taken at constant pressure or all at constant volume.

MAGNETOHYDRODYNAMICS. (Also called hydromagnetics.) The science of the motion of an electrically conducting fluid (plasma) in a magnetic field.

MAGNETOMECHANICAL DAMPING. A component of energy loss associated with the elastic vibration of a magnetic material, produced by interaction of magnetic effects with stress and strain.

MAGNETOMECHANICAL FACTOR. The number g' occurring in the definition of the magnetomechanical ratio. The theory of the gyromagnetic effect predicts that g' should be nearly 2 for electron spins, and this is actually observed in ferromagnetic substances, showing that the orbital moments (for which g' is unity) are effectively quenched.

MAGNETOMECHANICAL RATIO. The ratio of the magnetic moment to the angular momentum, as observed in the gyromagnetic effect. This ratio is usually expressed in the form $g'e/2mc$ where g' is the magnetomechanical factor.

MAGNETOMOTIVE FORCE. The line integral

$$\int_A^B \mathbf{H} \cdot d\mathbf{S}$$

when \mathbf{H} is the magnetic field intensity, defines the magnetomotive force between two points

A and B . For a closed contour $\oint \mathbf{H} \cdot d\mathbf{S} = I$ (*Ampere law*).

MAGNETON, NUCLEAR. See *nuclear magneton*.

MAGNETO-OPTICAL DISPERSION. An expression of the Faraday effect as the wavelength derivative of the product of the Verdet constant and the refractive index.

MAGNETO-OPTICAL EFFECT (KERR MAGNETO-OPTICAL EFFECT). Kerr discovered that if plane-polarized light (see *light*, *plane-polarized*) is reflected normally from the polished pole of a strong electromagnet, the light becomes slightly elliptically-polarized. The ellipticity is so small that the effect may be considered merely as a rotation of the plane of vibration through an angle θ given by

$$\theta = KH$$

where H is the magnetizing force and K is the magnetic Kerr constant.

MAGNETOSTATIC FIELD. A field in which the magnetizing force and magnetic induction are independent of time.

MAGNETOSTATICS. The study of magnetic fields which are independent of time.

MAGNETOSTRICTION. The elastic deformation of a material under forces exerted by a magnetostatic field.

MAGNETOSTRICTION, CRYSTAL SATURATION. See crystal saturation magnetostriction.

MAGNETOSTRICTION DRIVING SYSTEM. See driving system, magnetostriction.

MAGNETOSTRICTION GENERATING SYSTEM. See generating system, magnetostriction.

MAGNETRON, BOHR. See Bohr magnetron.

MAGNIFICATION. The ratio of the size of an image formed by an optical system to the size of the object is the most frequent meaning of magnification. However, for a large and distant object, the ratio at the eye of the angle subtended by the image to the angle which would be subtended at the eye by the object itself is also called magnification, although it is better stated as **magnifying power**.

MAGNIFICATION, ANGULAR. See angular magnification.

MAGNIFICATION, AXIAL. See axial magnification.

MAGNIFICATION, EMPTY. See empty magnification.

MAGNIFICATION, LATERAL. See lateral magnification.

MAGNIFICATION, LINEAR. See linear magnification; representation.

MAGNIFICATION, LONGITUDINAL. See longitudinal magnification.

MAGNIFICATION, NORMAL. See normal magnification.

MAGNIFICATION, OPTIMUM. See optimum magnification.

MAGNIFICATION, RADIAL. See radial magnification.

MAGNIFICATION RATIO OF A PROJECTOR. The ratio of the maximum luminous intensity of a projector to the mean spherical luminous intensity of the projector lamp.

MAGNIFICATION, ZONAL. See zonal magnification.

MAGNIFYING POWER. The ratio of the apparent size of an image to the apparent size of the object as seen without the magnifying device or instrument. However, to define "apparent size of an object" in terms that will permit of rigorous discussion requires certain assumptions. One may define the apparent size of an object as the angle which the object subtends at the eye, or as the size of the image which the lens of the eye forms of the object on the retina of the eye (retinal image). With any definition, however, the magnifying power of a complex device or instrument depends somewhat upon the accessory equipment.

MAGNITUDE. (1) See order. (2) The length of a vector.

MAGNUS EFFECT. See rotating cylinder, flow past.

MAHALANOBIS' DISTANCE. See D^2 statistic.

MAINARDI-CODAZZI RELATIONS. See equations of Gauss and Codazzi.

MAJORANA FORCE. A central force between two nucleons which is derivable from a potential containing an operator which exchanges the positions of two particles but not their spins. Nuclear forces are known to have exchange properties of this general type. (See majorana operator.)

MAJORANA OPERATOR. The quantum mechanical operator P_{ij} which exchanges the positions of the i^{th} and j^{th} particles when it operates upon the wave function of a system which contains these particles. For example, if $\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \dots, \mathbf{r}_n)$ represents the wave function for a system containing n particles located by the position vectors $\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \dots, \mathbf{r}_n$, then

$$P_{13}\psi(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3, \dots, \mathbf{r}_n) = \psi(\mathbf{r}_3, \mathbf{r}_2, \mathbf{r}_1, \dots, \mathbf{r}_n).$$

The operator can be written explicitly as

$$P_{ij} = -\frac{1}{4}(1 + \sigma_i \cdot \sigma_j)(1 + T_i \cdot T_j),$$

where $\sigma_i \cdot \sigma_j$ is the scalar product of the vector operators representing the spin of the i^{th} and j^{th} particles and $T_i T_j$ is the similar quantity for the isotopic spin. The majorana potential

$$V(\mathbf{r}_j - \mathbf{r}_i) = \frac{1}{2} \sum_{i,j} J(\mathbf{r}_j - \mathbf{r}_i) P_{ij},$$

in which $J(r_j - r_i)$ is some attractive **potential** function of the vector separation of the particles, has shown the greatest success of any of the simple **interactions** used to explain some of the general properties of nuclear structure, such as the apparent saturation of nuclear forces.

MALUS COSINE-SQUARED LAW. A law applying to the intensity of polarized light as affected by the polarizing apparatus. If a beam of plane-polarized light is passed through a Nicol prism, for example, the intensity (flux density) of the emergent beam falls off, as the prism is rotated, from a maximum value when the transmission plane of the prism coincides with the plane of vibration of the light, to zero when it is at right angles to that direction. The intensity varies as the square of the cosine of the angle through which the prism has been thus rotated. The same law applies to the effect of a glass reflector, reflecting always at the polarizing angle, as the plane of reflection is rotated around the stationary, polarized incident beam.

MALUS-DAUPIN THEOREM. See **normal congruence**.

MANIFOLD (ANALYTIC MANIFOLD). A manifold is a connected locally **Euclidean** topological **space**. The simplest example of a manifold which is not Euclidean as a whole is the surface of a sphere, say of the Earth. Now suppose that a manifold has been covered by a set of overlapping neighborhoods, each of which is associated with a (Cartesian) coordinate system. Then, in any region common to two systems, say in the torrid zone, the coordinates x, y of a point p in the one system will be functions, $x = \phi(x, y)$; $y = \psi(x, y)$ of the coordinates x, y of the same point in the other system. Two coordinate systems are said to be analytically related if either they do not overlap at all or else in overlapping regions these functions $\phi(x, y)$ and $\psi(x, y)$ are analytic, i.e., if they are expansible in a Taylor series.

Now suppose we have defined a family F of coordinate systems S :

- (1) F contains at least one coordinate system at every point p .
- (2) Any two coordinate systems in F are analytically related to each other.
- (3) F is complete; i.e., any coordinate system which is analytically related to

every coordinate system in F is itself in F . A manifold with such a family of coordinate systems is called an analytic manifold.

Then a function defined in a neighborhood of a point p is said to be analytic at p if it is an analytic function of its coordinates in a coordinate system at p . A simple example is given by the entire Euclidean plane, provided with a complete set of equivalent coordinate systems, one coordinate system being called equivalent to another if the systems are analytically related. (Compare **Lie group**.)

MANTISSA. The decimal part of a common logarithm. (See **logarithm, common**.)

MANY-BODY FORCE. An interaction between two particles that become modified when a third particle is present, e.g., the forces between polarizable molecules.

MAPPING. If S and T are two sets of elements, then a set of ordered pairs (s, t) where s runs through S and t runs through all or part of T is a mapping of S into T ; t is the image of s and s is said to be mapped onto t . If t also runs through the whole of T , the mapping is also said to be onto T . Synonyms such as *function*, *transformation*, *correspondence*, etc., are used in certain cases for historical reasons; thus, a function $f(x, y, z) = x^2 + y^2 + z^2$ of three real variables is a mapping of the set of ordered triples (x, y, z) of real numbers into the set of real numbers. If S is mapped onto T in such a way that distinct elements s have distinct images t , then the mapping is said to be *one-to-one*, or to be a *one-to-one correspondence* between S and T . The *product* ST of two mappings S and T is the mapping obtained by carrying out first T and then S (some writers use the reverse order, which seems less natural in view of the following example). Example: let S be the mapping of the set of real numbers onto itself given by $S(x) = x^3$ and let T be given by $T(x) = x + 1$; then $ST(x) = S\{T(x)\} = (x + 1)^3$ and $TS(x) = x^3 + 1$. If a set of mappings forms a group it is usually called a *group of transformations*; e.g., consider all one-to-one mappings of a set of n objects onto itself (usually called *permutations*); clearly it forms a group S_n with $n!$ elements, which is called the *symmetric group* of degree n .

MAPPING, CONFORMAL. See conformal mapping; representation.

MAPPING, ONE-TO-ONE. See representation.

MARCHING PROBLEM. A differential equation with initial conditions solved numerically by computing the values of the dependent variable recursively for systematically increasing values of the independent variable. For example, the wave equation is solved at each time-step before advancing to the next time-step. Hyperbolic equations may be formulated as marching problems. (See **jury problem**.)

MARGUERRE LARGE DEFLECTION THEORY. An approximate but quite accurate method of solution of problems of large deflection of simply supported rectangular elastic plates in membrane compression. The energy method is used to determine the free parameters in the assumed deflection shape.

MARGULES EQUATION. An equation for the equilibrium inclination of an **interface** separating two homogeneous air masses in a steady geostrophic motion parallel to the interface,

$$\tan \alpha = \frac{f(T_2 v_1 - T_1 v_2)}{g(T_2 - T_1)},$$

where α is the angle of inclination of the surface to the horizontal, f the Coriolis parameter, g the gravitational acceleration, and T_1 and T_2 the absolute temperatures of the colder and warmer air masses, respectively, with speeds v_1 and v_2 . This equilibrium condition has been used to calculate the slope of atmospheric frontal surfaces.

MARKOV CHAIN. See **Markov process**; **transition probability**.

MARKOV PROCESS. A **stochastic process** of a simple kind. The distribution of future states depends only on the present state and not on previous history; that is to say the conditional probabilities of events for $t > t_0$ given values at t_0 are independent of values assumed at times $t < t_0$.

The term Markov chain is often reserved for discrete time Markov processes.

MARKOV SCHEME. See **autoregression**.

MARK'S BOUNDARY CONDITIONS. See boundary conditions, Mark's.

MARSHAK'S BOUNDARY CONDITIONS. See boundary conditions, Marshak's.

MARTINGALE. A **stochastic process** $\{x(t)\}$ is called a martingale if the expectation $E|x(t)|$ is finite for all t and the conditional probability

$$E\{n(t_{n+1})/x(t_1), \dots, x(t_n)\} \\ = n(t_n), \quad n > 1, \quad t_1 < t_2 < \dots < t_{n+1}.$$

with probability unity. In short, the expected value of what is to happen next (t_{n+1}) is what has just happened (t_n).

MASOTTI FIELD. The electric field which would exist at the position of a molecule in a polarized dielectric if all of the molecules within a sphere of radius R around the molecule in question were removed without disturbing the polarization of the remaining dielectric. The radius R must be chosen large enough to be able to view, from the position of the molecule in question, all of the dielectric outside of the sphere as a continuum.

MASS. The physical measure of the principal inertial property of a body, i.e., its resistance to change of motion. At speeds small compared with the speed of light, the mass of a body is independent of its speed. Under these circumstances, the masses m_1 and m_2 of two bodies may be compared by allowing the two bodies to interact. Then

$$m_1/m_2 = |a_2|/|a_1|,$$

where $|a_1|$ and $|a_2|$ are the magnitudes of the respective accelerations of the two bodies as a result of the interaction. This permits the measurement of the mass of any particle with respect to a standard particle (for example, the standard kilogram). At higher speeds, the mass of a body depends on its speed relative to the observer according to the relation:

$$m = m_0/\sqrt{1 - v^2/c^2}$$

where m_0 is the mass of the body as found by an observer at rest with respect to the body, v is the speed of the body relative to the observer who finds its mass to be m , and c is the speed of light in empty space (see entries on **relativity**).

As a consequence of the Newton law of uni-

versal gravitation or of the Einstein demonstration of the equivalence of inertial and gravitational masses, equal masses at the same location in a gravitational field have equal weights. Because of this, masses may be compared with a platform balance or a spring balance.

Mass is particularly important because it is a conserved quantity, which can neither be created nor destroyed. Thus, the mass of any isolated system is a constant. When relativistic mechanics is appropriate, e.g., when speeds comparable to the speed of light are involved, mass may be converted into energy and *vice versa*, hence the energy of the system must be converted into mass through the Einstein equation.

$$E = mc^2$$

where c is the speed of light in empty space, before the conservation law may be applied.

MASS ABSORPTION COEFFICIENT. See **absorption coefficient**.

MASS, ACOUSTIC (ACOUSTIC INERTANCE). The quantity which, when multiplied by 2π times the frequency, gives the acoustic reactance (see **reactance, acoustic**) associated with the kinetic energy of the medium.

MASS ACTION, LAW OF. See **chemical equilibria, calculation of**.

MASS, CENTER OF. See **center of mass**.

MASS DEFECT. The difference between the atomic mass and the mass number of a nuclide. (See **packing fraction**.)

MASS FLUX VECTOR. See **flux vector**.

MASS FORMULA. An equation for the atomic mass of a nuclide as a function of its atomic number and mass number. (See **mass formula, empirical and mass formula, semi-empirical**.)

MASS FORMULA, EMPIRICAL. A formula for the mass of an atom of the form:

$$M(A, Z) = A(1 + f_A) + \frac{1}{2}B_A(Z - Z_A)^2 + \Delta,$$

where M is the mass of an atom of atomic number Z and mass number A ; f_A is an idealized packing fraction; Z_A is the (nonintegral) atomic number of maximum stability for mass

number A ; and Δ is a term reflecting the "pairing energy" of like nucleons, and therefore having four possible values which are

$$\Delta = \begin{cases} +\frac{1}{2}\delta_A & \text{for even } A, \text{ odd } Z \\ +\frac{1}{2}\epsilon_A & \text{for odd } A, \text{ odd } Z \\ -\frac{1}{2}\epsilon_A & \text{for even } A, \text{ odd } Z \\ -\frac{1}{2}\delta_A & \text{for even } A, \text{ odd } Z \end{cases}.$$

The terms δ_A , ϵ_A , as well as f_A , B_A and Z_A are usually taken to be parameters varying smoothly with A , which are adjusted to fit the experimental data.

MASS FORMULA, SEMIEMPIRICAL. A mass formula (due principally to von Weizsäcker, Bethe, Becker, Bohr and Wheeler) based on the liquid-drop model of the nucleus. It may be written:

$$M(A, Z) = ZM_H + (A - Z)M_n - aA + b\frac{(A - 2Z)^2}{A} + 4\pi cr_0^2 A^{2/3} + \frac{3Z(Z - 1)e^2}{5r_0 A^{1/2}} + \Delta.$$

Here $M(A, Z)$ is the mass of an atom of mass number A and atomic number Z , M_H and M_n are the masses of the hydrogen atom and the neutron, e is the electronic charge, r_0 is the radius parameter, and a , b and c are adjustable parameters. The terms represent, in order: the mass of the constituent protons and electrons; the mass of the constituent neutrons; the "bulk energy of condensation" due to short-range attractive forces between nucleons and proportional to the volume of the nucleus; the "asymmetry energy" corresponding to the tendency of protons and neutrons to be equal in number; the "surface energy"; the "electrostatic energy" due to repulsive forces between protons; and the "pairing energy" as given under **mass formula, empirical**.

MASSIEU FUNCTION. See **thermodynamics, characteristic functions of**.

MASS-LUMINOSITY RELATION, STELLAR. See **stellar mass-luminosity relation**.

MASS NUMBER. The whole number nearest in value to the atomic mass when that quantity is expressed in atomic mass units.

In light of present-day theory, the mass number represents the total number of nucleons in the nucleus, and is therefore equal to the sum of the atomic number and the neutron number. The mass number is commonly written as a superscript after or before the symbol of the atom, such as O^{16} or ^{40}K .

MASS POINT, FREE. See **dynamics of a free mass point**.

MASS RENORMALIZATION. The self energy of an electron (i.e., its change in mass due, for example, to its interaction with the quantized electromagnetic field) is logarithmically divergent when calculated in perturbation theory. The way to overcome, or rather bypass, this difficulty is based on an observation made by Kramers (1947) in connection with classical electrodynamics. (See **classical electron theory**.) He pointed out that one can never experimentally observe m_0 , the bare mass of the electron, but only $(m_0 + \delta m)$ the observed mass, $m = m_0 + \delta m$, where δm is the change in mass due to the interaction with the electromagnetic field. Kramers therefore suggested that only the observable quantity m should play a role in the theory and not the separate quantities m_0 or δm . This principle is known as the principle of mass renormalization. One accepts the fact that the change in mass may be infinite (hoping, however, that a future theory might make it finite) but one does not consider this of consequence since the physically observable quantities such as energy levels and cross sections are finite when expressed in terms of the observable mass m . (See **divergences**.)

MASS SPECTRUM. See **spectrum, mass**.

MATCHING STIMULI (INSTRUMENTAL STIMULI, PRIMARIES). The defined **stimuli** of an additive colorimeter.

MATERIAL BUCKLING. See **buckling**.

MATERIAL DERIVATIVE. Also called total or substantial derivative. (See **total derivative**.)

MATHEMATICAL PHYSICS, EQUATIONS OF. The name is sometimes given to a set of partial differential equations of second order, of which the following are the most commonly met with: (1) the **Laplace equation**,

$$\nabla^2\phi = 0$$

and its inhomogeneous analogue, the **Poisson equation**; (2) the equation of wave motion,

$$c^2\nabla^2\phi = \partial^2\phi/\partial t^2;$$

(3) the **diffusion equation**, which also applies to thermal conduction,

$$a^2\nabla^2\phi = \partial\phi/\partial t;$$

(4) the equation of **telegraphy**,

$$a\partial^2\phi/\partial t^2 + b\partial\phi/\partial t = \partial^2\phi/\partial x^2.$$

The **parameters** are observable quantities and t is the time. In modern theoretical physics, the differential equations of quantum mechanics, particularly the **Schrödinger wave equation**, must be included.

Many special functions (which see) owe their importance to the fact that they are useful in constructing solutions of these equations. In fact, the name "equations of mathematical physics" is sometimes also given to the linear ordinary differential equations (see **Legendre's equation**, **Bessel's equation**, etc.) which arise when the above partial differential equations are solved by the method of separation of variables. These ordinary equations are all specializations arising from **confluence** from the generalized **Lamé equation**.

MATHIEU EQUATION. A **differential equation** resulting from the separation, in **elliptical cylindrical coordinates**, of **partial differential equations** like **Laplace's** or the wave equation. It also occurs in the quantum mechanical problem of a molecule with restricted internal rotation. The usual form of the equation is

$$y'' + (a + b \cos 2x)y = 0$$

but related forms are sometimes given. Since the coefficient of y is periodic, the **Floquet theorem** applies and possible solutions are

$$y = e^{\mu x}P(x),$$

where $P(x)$ is periodic and can be written as an infinite series of sine or cosine terms. Substitution of this assumed solution into the differential equation gives a three-term **recursion formula** or the coefficients in the series. The exponent μ is obtained as a complicated function of the parameters a and b , which may be written as a **continued fraction**. If the final solution is required to be periodic, a situation occurring frequently for physical reasons,

$\mu = ik$, with k an integer. The allowed values of a are the **eigenvalues** and the solutions are called Mathieu functions.

MATRIX. A rectangular array of scalars (real or complex) called elements of the matrix, together with certain rules of combination. If

$$\mathbf{A} = (\alpha_{ij}) = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \alpha_{13} & \cdots \\ \alpha_{21} & \alpha_{22} & \alpha_{23} & \cdots \\ \dots & \dots & \dots & \dots \end{pmatrix}, \quad \mathbf{B} = (\beta_{ij})$$

and if \mathbf{A} and \mathbf{B} have the same dimensions, then the sum is

$$\mathbf{A} + \mathbf{B} = (\alpha_{ij} + \beta_{ij}),$$

that is, corresponding elements are added together.

If \mathbf{A} has n columns and \mathbf{B} has n rows, the product is

$$\mathbf{AB} = \left(\sum_k \alpha_{ik} \beta_{kj} \right).$$

If ρ is a scalar,

$$\rho \mathbf{A} = (\rho \alpha_{ij}).$$

The transpose of \mathbf{A} is

$$\mathbf{A}^T = (\alpha_{ji}) = \begin{pmatrix} \alpha_{11} & \alpha_{21} & \alpha_{31} & \cdots \\ \alpha_{12} & \alpha_{22} & \alpha_{32} & \cdots \\ \dots & \dots & \dots & \dots \end{pmatrix};$$

the conjugate transpose \mathbf{A}^* is the result of replacing each element in \mathbf{A}^T by its complex conjugate. If $\mathbf{A}^T = \mathbf{A}$, \mathbf{A} is symmetric; if $\mathbf{A}^* = \mathbf{A}$, \mathbf{A} is hermitian. The square matrix $\mathbf{I} = (\delta_{ij})$, where δ_{ij} is the **Kronecker delta**, is called the identity. If it is $n \times n$, then for any matrix \mathbf{A} with n rows, $\mathbf{IA} = \mathbf{A}$, and for any matrix \mathbf{B} with n columns, $\mathbf{BI} = \mathbf{B}$. A square matrix, with n rows and n columns, is said to be of order n . The square matrix \mathbf{C} such that

$$\mathbf{CA} = \mathbf{AC} = \mathbf{I},$$

if such exists, is called the inverse of \mathbf{A} and denoted $\mathbf{C} = \mathbf{A}^{-1}$. When \mathbf{A}^{-1} exists then \mathbf{A} is said to be nonsingular. If \mathbf{A} is hermitian, of order n , and for every vector \mathbf{x} of n dimensions, which is to say, matrix of n rows and one column,

$$\mathbf{x}^* \mathbf{A} \mathbf{x} \geq 0,$$

then \mathbf{A} is nonnegative semidefinite; if for every vector $\mathbf{x} \neq 0$,

$$\mathbf{x}^* \mathbf{A} \mathbf{x} > 0,$$

then \mathbf{A} is positive definite. For nonhermitian matrices these notions are not defined.

The elements α_{ii} of a matrix $\mathbf{A} = (\alpha_{ij})$ constitute the diagonal, or main diagonal. If all other elements are null, \mathbf{A} is a diagonal matrix. If, in addition,

$$\alpha_{ii} = \alpha,$$

that is, all diagonal elements are equal, then \mathbf{A} is a scalar matrix and $\mathbf{A} = \alpha \mathbf{I}$.

If $\mathbf{V}^* \mathbf{V} = \mathbf{V} \mathbf{V}^* = \mathbf{I}$, then \mathbf{V} is a unitary matrix, and if \mathbf{V} is unitary and real it is orthogonal.

If \mathbf{A} is square matrix of order n , then $\det \mathbf{A}$ will represent the determinant of \mathbf{A} . The polynomial

$$\phi(\lambda) = \det (\lambda \mathbf{I} - \mathbf{A})$$

is a polynomial in λ of degree n whose leading coefficient is unity, and whose constant term is $\det \mathbf{A}$. This is called the characteristic polynomial, and the Cayley-Hamilton theorem states that

$$\phi(\mathbf{A}) = 0.$$

There is a polynomial $\psi(\lambda)$ of minimal degree and leading coefficient unity for which

$$\psi(\mathbf{A}) = 0.$$

This is called the minimal polynomial, and $\psi(\lambda)$ may or may not be the same as $\phi(\lambda)$, but in all cases $\psi(\lambda)$ divides $\phi(\lambda)$, and every zero of ϕ is also a zero of ψ . (These zeros are variously called proper values, characteristic values, **eigenvalues**, latent roots.) If λ is any eigenvalue, there exists a nonnull vector \mathbf{x} satisfying

$$(\mathbf{A} - \lambda \mathbf{I}) \mathbf{x} = 0,$$

and any such vector is called an eigenvector, or characteristic vector, or proper vector, or modal column. Naturally there exists also a nonnull vector \mathbf{y} such that

$$\mathbf{y}^* (\mathbf{A} - \lambda \mathbf{I}) = 0.$$

Either \mathbf{x} or \mathbf{y} is said to belong to λ . If \mathbf{A} is hermitian, then all eigenvalues are real. In this event there exists a unitary matrix \mathbf{V} such that

$$\mathbf{V}^* \mathbf{A} \mathbf{V} = \mathbf{\Lambda},$$

where $\mathbf{\Lambda}$ is diagonal, and its elements are the eigenvalues of \mathbf{A} . Moreover, the columns of \mathbf{V} are eigenvectors, and, in fact, if \mathbf{v}_i is the i th column of \mathbf{V} , and λ_i the i th element in the diagonal of $\mathbf{\Lambda}$, then \mathbf{v}_i belongs to λ_i . The matrix \mathbf{V} is said to diagonalize \mathbf{A} .

If $\mathbf{A}^*\mathbf{A} = \mathbf{A}\mathbf{A}^*$, then \mathbf{A} is said to be a normal matrix. Such a matrix can also be diagonalized by a unitary matrix \mathbf{V} , but the diagonal form $\mathbf{\Lambda}$ is not real unless \mathbf{A} is hermitian.

For a more general matrix \mathbf{A} , there exists a nonsingular matrix \mathbf{V} such that

$$\mathbf{V}^{-1}\mathbf{A}\mathbf{V} = \mathbf{\Lambda},$$

where $\mathbf{\Lambda}$ has the form

$$\mathbf{\Lambda} = \begin{pmatrix} \Lambda_1 & 0 & 0 & \cdots \\ 0 & \Lambda_2 & 0 & \cdots \\ 0 & 0 & \Lambda_3 & \cdots \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix},$$

and each Λ_i is either a scalar matrix or of the form

$$\Lambda_i = \begin{pmatrix} \lambda_i & 0 & 0 & \cdots \\ 1 & \lambda_i & 0 & \cdots \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix}$$

with all diagonal elements equal, with ones just below the diagonal, and zeros elsewhere. This $\mathbf{\Lambda}$ is the Jordan normal form. Each λ_i is an eigenvalue, each column \mathbf{v} of \mathbf{V} is a **principal vector**, which is to say that for some λ and some ν ,

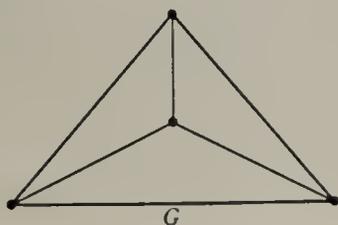
$$(\mathbf{A} - \lambda\mathbf{I})^\nu \mathbf{v} = 0.$$

Also each row of \mathbf{V}^{-1} is a principal row vector.

(For computational methods see **eigenvalues and eigenvectors of matrices, computation of and matrix inversion**.)

MATRIX, ADJACENCY. Let G be a **non-oriented graph** possessing e elements $\epsilon_1, \epsilon_2, \dots, \epsilon_e$ and v vertices $\beta_1, \beta_2, \dots, \beta_v$. The adjacency matrix \mathbf{A} of G is a square matrix of order v in which $a_{ij} = 1$ if β_i and β_j are adjacent (see **vertices, adjacent**) and zero otherwise; a_{ij} denotes the entry in \mathbf{A} located in the i^{th} row and j^{th} column of \mathbf{A} . Note that $a_{ii} = 0$ ($i = 1, 2, \dots, v$), i.e., the diagonal of \mathbf{A} is composed solely of zeros.

Let \mathbf{B} be the degree matrix of G . The *Matrix Tree Theorem* states that all the primary co-factors of the matrix $\mathbf{M} = \mathbf{B} - \mathbf{A}$ of a connected **non-oriented graph** G are equal to



the number of trees in G . A graph having the associated matrices \mathbf{A} , \mathbf{B} , \mathbf{M} below is shown in the accompanying figure.

$$\mathbf{A} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix},$$

$$\mathbf{B} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 3 \end{bmatrix},$$

$$\mathbf{M} = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 3 & -1 & -1 \\ -1 & -1 & 3 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix}.$$

Observe that the (1, 1) co-factor of \mathbf{M} is precisely equal to 16, the number of trees in G (see **tree**). (See also **digraph**.)

MATRIX, ADJOINT OF. See **adjoint of a matrix**.

MATRIX, CIRCUIT. Let G be a linear oriented graph with elements ϵ_j ($j = 1, 2, \dots, e$) and oriented circuits C_i ($i = 1, 2, \dots, k$). The circuit matrix $\mathbf{B}_a = (b_{ij})$ has k rows and e columns and is defined as follows:

- $b_{ij} = +1$ if element ϵ_j is in circuit C_i and agrees in orientation with that of C_i ;
- $b_{ij} = -1$ if element ϵ_j is in circuit C_i and is oppositely oriented to C_i ;
- $b_{ij} = 0$ if element ϵ_j is not in circuit C_i .

The circuit matrix corresponding to a set of **fundamental circuits** of a connected graph is denoted by \mathbf{B}_c . It contains $e - v + 1$ rows and e columns where v is the number of **vertices**. The orientation of a fundamental circuit is usually chosen to agree with that of its unique **chord**. The rank of a fundamental circuit matrix is $e - v + 1$.

MATRIX, CUT SET. See **cut set matrix**.

MATRIX, DEGREE. Let G be a graph with v vertices $\beta_1, \beta_2, \dots, \beta_v$. The degree matrix \mathbf{B} is a square, diagonal matrix of order v in which b_{ii} is the degree of the vertex β_i ($i = 1, 2, \dots, v$). (See **vertex, degree**; b_{ii} is the entry in \mathbf{B} located in the i^{th} row and i^{th} column. An

example of a degree matrix is given under **matrix, adjacency.**)

MATRIX ELEMENT. (1) A quantity a_{ij} at the intersection of the i^{th} row and the j^{th} column of a **matrix**. It may be of a general functional nature, real or complex.

(2) The quantity resulting when a quantum mechanical operator O operates on an element ψ_n of a complete, orthogonal set of wave functions from the left and is then multiplied from the left by the complex conjugate of another element of the set, ψ_m^* , and integrated over the position variables. The set of quantities

$$O_{m,n} = \int \psi_m^*(x,y,z) O \psi_n(x,y,z) dx dy dz$$

can be considered to be the m,n elements of a matrix O which represents the operator O . In quantum mechanics it is usually not the practice to introduce a special notation to distinguish between the operator O and the matrix O representing the operator, as is done in this book by the use of **bold-face** letters for matrices.

MATRIX, IDENTITY. A diagonal matrix with all non-zero elements equal to unity.

MATRIX, INCIDENCE. See **matrix, vertex.**

MATRIX INVERSION. For a given square **matrix** A , the determination of a matrix denoted A^{-1} such that $AA^{-1} = A^{-1}A = I$, the identity, assuming A^{-1} to exist. A closed method of inversion would yield the exact inverse, except for errors due to rounding, after a finite number of operations in prescribed sequence performed upon the elements. These operations are usually arithmetic, but may include square-rooting. Other methods, sometimes called **iterative** are such as to produce a sequence of matrices C_v which approach A^{-1} in the limit, again apart from rounding errors. Since $x = A^{-1}h$ satisfies the system of linear equations $Ax = h$, and since, conversely, each column of A^{-1} is the solution of such a system with a particular h , the problems of inversion and solution are mathematically equivalent and will be spoken of interchangeably.

The most common direct methods are known as (Gaussian) **elimination**, or triangularization, with variants due to Crout, Banachiewicz, Doolittle, and many others, differing only in detailed arrangements and systems of recording intermediate results.

Let A be partitioned in any way in the form

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix},$$

where A_{11} is required only to be square and nonsingular. In particular A_{11} may be a scalar, A_{21} therefore a column vector, A_{12} a row vector; or A_{22} may be a scalar, A_{12} a column vector, and A_{21} a row vector. The methods seek a factorization in one of the two forms:

$$(1) \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} L_{11} & 0 \\ L_{21} & L_{22} \end{pmatrix} \begin{pmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{pmatrix},$$

$$(2) \begin{pmatrix} P_{11} & 0 \\ P_{21} & P_{22} \end{pmatrix} \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{pmatrix},$$

where the partitioning is conformable throughout. The first factorization requires that

$$L_{11}R_{11} = A_{11}, \quad L_{11}R_{12} = A_{12}, \\ L_{21}R_{11} = A_{21}, \quad L_{21}R_{12} + L_{22}R_{22} = A_{22},$$

and the second that

$$P_{11}A_{11} = R_{11}, \quad P_{11}A_{12} = R_{12}, \\ P_{21}A_{11} + P_{22}A_{21} = 0, \quad P_{21}A_{12} + P_{22}A_{22} = R_{22}.$$

The second form is more usual but in fact they are entirely equivalent computationally. If A_{11}^{-1} is known, which is certainly the case if A_{11} is a scalar, then all matrices can be determined and one can even choose arbitrarily either P_{11} or R_{11} , and either P_{22} or R_{22} . One common choice (Gaussian elimination) is to take P_{11} the identity (= 1 if a scalar), and P_{22} the identity. Then

$$R_{11} = A_{11}, \quad R_{12} = A_{12}, \\ P_{21} = -A_{21}A_{11}^{-1}, \quad R_{22} = A_{22} - A_{21}A_{11}^{-1}A_{12}.$$

Thereafter one proceeds with R_{22} as with the original A , that is, one multiplies on the left by a matrix

$$\begin{pmatrix} 1 & 0 \\ 0 & P' \end{pmatrix},$$

where P' and R_{22} are to be partitioned. The result is the formation of $n - 1$ matrices P_i each unit lower **triangular**, such that

$$P_{n-1}P_{n-2} \cdots P_1 = P$$

is also unit lower triangular, and

$$PA = R$$

is upper triangular and hence readily inverted. Note that \mathbf{P} has a unit determinant, whence the determinant of \mathbf{A} is given by the product of the diagonal elements of \mathbf{R} . If one is interested only in solving equations, the \mathbf{P}_i need not be retained, but each must be multiplied by the vector on the right.

While ordinarily one partitions off a row and a column, taking \mathbf{A}_{11} a scalar, it sometimes happens that the matrix \mathbf{A} has a submatrix \mathbf{A}_{11} of order greater than 1, but whose inverse is already known or readily obtained. The above formulas apply equally to this case. Analogous formulas can be written for taking \mathbf{R}_{11} and \mathbf{R}_{22} to be identities.

In the methods of **enlargement** one applies the above formulas first to a second order submatrix; next, this is taken to be \mathbf{A}_{11} and bordered to form a third order submatrix to be inverted by the same formulas. Eventually the entire matrix is inverted. This method is to be recommended when the matrix to be inverted is a finite segment of an infinite matrix and the size of the segment to be inverted is not determined in advance.

When \mathbf{A} is positive definite the factorization (1) can be modified:

$$\begin{pmatrix} \mathbf{A}_{11} & \mathbf{A}_{12}^T \\ \mathbf{A}_{21} & \mathbf{A}_{22} \end{pmatrix} = \begin{pmatrix} \mathbf{L}_{11} & 0 \\ \mathbf{L}_{21} & \mathbf{I} \end{pmatrix} \begin{pmatrix} \mathbf{L}_{11}^T & \mathbf{L}_{21}^T \\ 0 & \mathbf{L}_{22} \end{pmatrix},$$

with \mathbf{L}_{22} symmetric. Evidently

$$\begin{aligned} \mathbf{L}_{11}\mathbf{L}_{11}^T &= \mathbf{A}_{11}, & \mathbf{L}_{21}\mathbf{L}_{11}^T &= \mathbf{A}_{21}, \\ \mathbf{L}_{22} &= \mathbf{A}_{22} - \mathbf{L}_{21}\mathbf{L}_{21}^T. \end{aligned}$$

Ordinarily \mathbf{A}_{11} is a scalar. For obvious reasons this is called the square-root method. It is due to Cholesky, and to Banachiewicz.

Other factorizations are to apply a Schmidt **orthogonalization** to the columns, $\mathbf{A} = \mathbf{Q}\mathbf{T}$, where \mathbf{T} is a unit upper triangle and \mathbf{Q} is orthogonal by columns. Hence $\mathbf{Q}^T\mathbf{Q} = \mathbf{D}$, a diagonal, and

$$\mathbf{A}^{-1} = \mathbf{T}^{-1}\mathbf{D}^{-1}\mathbf{Q}^T.$$

It might seem more natural to form $\mathbf{A}\mathbf{U} = \mathbf{Q}$ with $\mathbf{U} = \mathbf{T}^{-1}$ formed directly. This can be done but it turns out the same computations are required to form \mathbf{U} directly as to form first \mathbf{T} and then \mathbf{T}^{-1} .

Alternatively the rows of \mathbf{A} can be orthogonalized by applying the same process to \mathbf{A}^T .

Still another possibility is to obtain an orthogonal matrix $\mathbf{\Omega}$ such that $\mathbf{\Omega}\mathbf{A} = \mathbf{S}$, where \mathbf{S} is upper triangular. This is done by applying first a rotation in the (1, 2)-plane to eliminate the element α_{21} of \mathbf{A} ; then in the (1, 3)-plane to eliminate α_{31} , and continuing until all sub-diagonal elements are eliminated.

In the triangular factorization significance can be optimized by interchanging rows and columns at each stage so that the divisor in every division has as many significant figures as possible, or, when there is a choice, is as large as possible. In forming plane rotations, this interchange is unnecessary, but the method requires more arithmetic operations than triangular factorization, and requires a square root in the selection of each rotation.

Quite different in appearance is the method of modification based upon the following identity:

$$\begin{aligned} (\mathbf{B} - \sigma\mathbf{u}\mathbf{v}^T)^{-1} &= \mathbf{B}^{-1} - \tau\mathbf{B}^{-1}\mathbf{u}\mathbf{v}^T\mathbf{B}^{-1}, \\ \sigma^{-1} + \tau^{-1} &= \mathbf{v}^T\mathbf{B}^{-1}\mathbf{u}, \end{aligned}$$

readily verified, where \mathbf{u} and \mathbf{v} are column vectors. If \mathbf{u} and \mathbf{v} are columns \mathbf{e}_i and \mathbf{e}_j of \mathbf{I} , then $\sigma\mathbf{u}\mathbf{v}^T$ is a matrix whose only nonnull element is σ in the i^{th} row and j^{th} column. Hence the formula gives the effect on the inverse of modifying a single element of the matrix. Or if $\sigma = 1$, $\mu = e_i$, it gives the result of modifying the entire i^{th} row by v . Evidently one could start with any matrix, perhaps \mathbf{I} or the diagonal of \mathbf{A} , whose inverse is known, and modify row by row, or column by column, to build up to \mathbf{A}^{-1} . In case \mathbf{A} is symmetric, take $u = v = e_i \pm e_j$, $\sigma = \alpha_{ij} = \alpha_{ji}$. This affects also the diagonal elements so that one starts by inverting a suitably demodified diagonal of \mathbf{A} . This method seems to be in principle the method of "tearing" advocated by Kron.

If \mathbf{C} is an approximation to \mathbf{A}^{-1} which one wishes to improve, form either of

$$\mathbf{H} = \mathbf{I} - \mathbf{A}\mathbf{C}, \quad \mathbf{K} = \mathbf{I} - \mathbf{C}\mathbf{A}.$$

Then either of

$$\mathbf{C}_1 = \mathbf{C}(\mathbf{I} + \mathbf{H}), \quad \mathbf{C}'_1 = (\mathbf{I} + \mathbf{K})\mathbf{C}$$

may be a better approximation, and will be if in any **norm**, one of the following is true:

$$\|\mathbf{H}\| < 1, \quad \|\mathbf{K}\| < 1.$$

Convergence is then quadratic. A rigorous

bound for the error in the inverse is given by either of

$$\|A^{-1} - C\| \leq \|CH\| / (1 - \|H\|),$$

$$\|A^{-1} - C\| \leq \|KC\| / (1 - \|K\|),$$

under the same condition.

For solving the system

$$Ax = h,$$

if A is of high order but with most elements null, one usually prefers to generate a sequence of vectors x_ν approaching the solution x in the limit. Let

$$S_\nu = x - x_\nu, \quad r_\nu = h - Ax_\nu = AS_\nu,$$

Starting with any x_0 , one forms recursively

$$x_{\nu+1} = x_\nu + C_\nu r_\nu,$$

where C_ν is some matrix which may be fixed in advance, or may change from step to step. One can always arrange, if convenient, so that every diagonal element of A is unity. If this is done, and $C_\nu = I$, one has the simple, or total step, iteration. If one writes, then,

$$A = I - B,$$

so that B has a null diagonal, it is necessary and sufficient that the **spectral radius** $\rho(B) < 1$. For the single-step (or Seidel, or Gauss-Seidel, or Liebmann) iteration let

$$A = L + R,$$

where L is null except below the diagonal, and R is null below the diagonal. Then

$$C_\nu = R^{-1},$$

or, as the computation is usually made, one solves

$$Rx_{\nu+1} = h - Lx_\nu.$$

Convergence is assured when A is positive definite, and, more generally, when $\rho(R^{-1}L) < 1$, the latter condition being necessary and sufficient. Note that in solving for the i^{th} element of $x_{\nu+1}$, one solves the i^{th} equation for that element in terms of the most recent values of all other elements, and the equations are taken in fixed cyclic order. In the method of **relaxation** there is no fixed order, but one selects at each stage the particular equation which is least well satisfied, and if this is the j^{th} , one solves for the j^{th} element in terms of most recent values of the others. This inspection can be done readily in computing by hand, but not readily by machine, whence relaxation is not recommended for machine computation (al-

though some authors apply the term more generally to any method, or at least some other methods, of generating converging sequences). In the method of steepest descent

$$C_\nu = (r_\nu^T A r_\nu)^{-1} r_\nu r_\nu^T.$$

The method is more usually written

$$x_{\nu+1} = x_\nu + \mu_\nu r_\nu,$$

$$\mu_\nu = r_\nu^T r_\nu / r_\nu^T A r_\nu.$$

It applies in this form only when A is positive definite, and then it always converges. Note, however, that any system with a nonsingular matrix A can be converted to an equivalent system with a positive definite matrix:

$$A^T A x = A^T h.$$

(See **condition number**, and **norm**.)

MATRIX, MODAL. The modal **matrix** of a given matrix A is that matrix H satisfying the condition that

$$(H^{-1}AH)_{ij} = \lambda_i \delta_{ij}$$

where $(H^{-1}AH)_{ij}$ is the i, j **matrix element** of the matrix $H^{-1}AH$ and λ_i are the **eigenvalues** of the matrix A .

MATRIX, PRINCIPAL DIAGONAL OF. The set of **matrix elements**, A_{ij} , for which $i = j$.

MATRIX, REAL. A matrix which has only real numbers or functions as elements.

MATRIX, REDUCIBLE. See **representation of a group**.

MATRIX, REDUCING. See **representation of a group**.

MATRIX, S. See **S-matrix**.

MATRIX, SCATTERING. See **S-matrix**.

MATRIX, SECONDARY DIAGONAL OF. The set of elements A_{ij} for which $i + j = n + 1$, where n is the order of the matrix.

MATRIX, STOCHASTIC. A real square matrix with non-negative elements and unit row sums. These matrices have important applications as **transition probability matrices** in the theory of **Markov chains**.

MATRIX, TRANSMISSION. See **transmission, matrix**.

MATRIX TREE THEOREM. See **matrix, adjacency**.

MATRIX, VERTEX. The vertex matrix $A_a = (a_{ij})$ of a linear oriented graph G possessing v vertices β_i , ($i = 1, 2, \dots, v$) and e elements ϵ_j , ($j = 1, 2, \dots, e$), is a matrix with v rows and e columns such that

- (a) $a_{ij} = 1$ if element ϵ_j is incident at vertex β_i and oriented away from β_i ;
- (b) $a_{ij} = -1$ if element ϵ_j is incident at β_i and oriented toward β_i ;
- (c) $a_{ij} = 0$ if element ϵ_j is not incident at vertex β_i .

The vertex matrix has exactly two non-zero elements, one $+1$ and one -1 in each column and at least one non-zero element in each row. The rank of the vertex matrix of a connected graph is $v - 1$.

For a non-oriented graph the vertex matrix is defined similarly:

- (a) $a_{ij} = 1$ if element ϵ_j is incident at vertex β_i and
- (b) $a_{ij} = 0$ otherwise.

A graph is completely defined by its vertex matrix.

MATRIZANT. The matrizant of a matrix A whose elements are functions of some variable x is the series:

$$\mathbf{I} + \int_{t_0}^t \mathbf{A}(x_1) dx_1 + \int_{t_0}^t \mathbf{A}(x_1) \int_{t_0}^t \mathbf{A}(x_2) dx_2 dx_1 + \int_{t_0}^t \mathbf{A}(x_1) \int_{t_0}^t \mathbf{A}(x_2) \int_{t_0}^t \mathbf{A}(x_3) dx_3 dx_2 dx_1 + \dots$$

where \mathbf{I} is the identity matrix.

MATTE SURFACE. A matte surface is continuous but not smooth, i.e., it belongs to class C_0 but not class C_1 . When a matte surface is an interface between two media, the resulting reflection of light is diffuse.

MATTHIESSEN RULE. An approximate rule stating that the total electrical or thermal resistivity of a metal is the sum of the separate resistivities due to scattering of the electrons by thermal vibrations of the lattice, by impurities, by imperfections, etc.

MAUPERTUIS THEOREM. If a static system is in equilibrium and every change in configuration consistent with the constraints increases the potential energy, the equilibrium is stable. (See minimum potential energy.)

MAXIMAL CONNECTED SUBGRAPH. See subgraph, maximal connected.

MAXMIN. See games theory.

MAXWELL. The unit of flux in the electromagnetic system of units. (See electromagnetic units.)

MAXWELL AND BETTI RECIPROCITY THEOREM. See reciprocity theorem in classical elasticity theory.

MAXWELL-BOLTZMANN DISTRIBUTION LAW. A more general relationship than the Maxwell distribution law. It may be expressed as follows

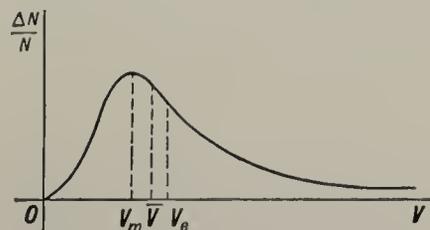
$$f(p, q) = C e^{-\beta \epsilon(p, q)},$$

where $f(p, q)$ is the number of particles in the system per unit volume of phase space with given values of the generalized momenta, p , and coordinates, q , C is a normalizing constant, $\beta = 1/kT$ (k : Boltzmann's constant, T : absolute temperature), and $\epsilon(p, q)$ is the particle energy.

MAXWELL-BOLTZMANN STATISTICS. See Boltzmann statistics.

MAXWELL-BOLTZMANN TRANSPORT EQUATION. See Boltzmann transport equation.

MAXWELL DISTRIBUTION LAW. A law expressing the relative numbers of molecules in a gas which have various given speeds, or various given kinetic energies, of thermal agitation at any instant. In its usual forms, it is limited by certain simplifying conditions, viz., uniformity of temperature, absence of turbulence or convection currents, negligible effect of gravity, and purity of the gas (molecules



Graph of the Maxwell distribution law.

all of equal mass). The law may be expressed in various terms. For example, if N is the total number of molecules, the proportion of them having speeds confined to the interval Δv between $v - \frac{1}{2}\Delta v$ and $v + \frac{1}{2}\Delta v$ (and hence having v cm per sec as their representative speed), is

$$\frac{\Delta N}{N} = \frac{4h^3(\Delta v)}{\sqrt{\pi}} v^2 e^{-h^2 v^2}. \quad (1)$$

h is a constant which may be shown to be equal to

$$h = 6.034 \times 10^7 \sqrt{\frac{m}{T}}, \quad (2)$$

in which T is the absolute temperature and m is the mass of one molecule in grams. From (1) it is easily deduced that the modal (most frequent) speed of the molecules is

$$v_m = \frac{1}{h}; \quad (3)$$

the mean speed is

$$\bar{v} = \frac{2}{\sqrt{\pi}h}; \quad (4)$$

and the effective speed (corresponding to average kinetic energy) is

$$v_e = \sqrt{\frac{3}{2}} \cdot \frac{1}{h}. \quad (5)$$

This last quantity, and hence h , can be calculated from the density and the pressure of the gas. (See **kinetic theory, equipartition of energy, Maxwell-Boltzmann distribution law, perfect gas law.**)

MAXWELL EFFECT. The optical birefringence shown by viscous liquids having anisotropic molecules due to the existence of a shearing velocity gradient. If the liquid is flowing in the x -direction with a velocity v_x , which is a function of y only, then the difference between the indices of refraction for the two principal planes of polarization for a light wave traveling in the z direction will be

$$\Delta n = c(\partial v_x / \partial y).$$

The constant c is known as the Maxwell constant. The two principal planes are the ones which are perpendicular to each other, with the z -axis as their line of intersection, and each makes an angle of 45° with the x -axis.

MAXWELL EQUATIONS. A set of four classic formulae of the **electromagnetic theory**. They deal with certain **vector quantities** pertaining to any point of a region under varying electric and magnetic influence. If the point is in empty space, the equations are somewhat

simplified; in general, provision must be made for the presence of dielectrics, conductors, or magnetizable bodies. In these equations, **H** is **magnetizing force**, **B** is **magnetic induction**, **E** is **electric intensity**, **D** is **electric induction**, ρ is **electric charge density**, **J** is **conduction current density**, t is time. The "curl" and the "divergence" of a function are well-known operators of vector analysis. The equations, in rationalized mks units, are

$$\text{Curl } \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t} + \mathbf{J}$$

$$\text{Curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

The additional relations

$$\text{Div } \mathbf{B} = 0$$

$$\text{Div } \mathbf{D} = \rho$$

are frequently included as part of Maxwell's system, although they are not independent relations if one assumes the conservation of charge. The last two are also known as the **Gauss law**. For linear homogeneous isotropic media, $\mathbf{B} = \mu \mathbf{H}$; $\mathbf{D} = \epsilon \mathbf{E}$. The values of μ and ϵ for a vacuum satisfy

$$\mu_v \epsilon_v = 1/c^2$$

where c is the speed of light, μ is **permeability** and ϵ is **permittivity**.

The equations and the expressions of conservation are often stated in the integral form:

$$\oint \mathbf{H} \cdot d\mathbf{S} = \int_S \left(\frac{\partial \mathbf{D}}{t} + \mathbf{J} \right) \cdot d\mathbf{S};$$

$$\oint \mathbf{E} \cdot d\mathbf{S} = -\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{S};$$

$$\oint \mathbf{B} \cdot d\mathbf{S} = 0; \quad \int_S \mathbf{D} \cdot d\mathbf{S} = \int_V \rho dV.$$

(See **integral, line; integral, surface; Gauss theorem; Stokes theorem.**)

MAXWELL FISHEYE. An optical medium with the index of refraction

$$n = \frac{a}{b^2 + r^2}$$

where r is the distance from a fixed point and a, b are constants. In this system, any sphere

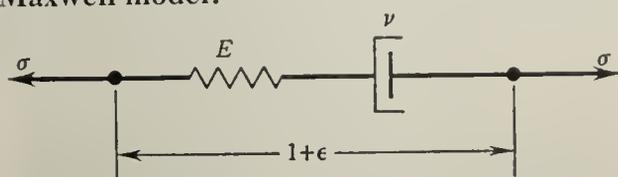
of radius R has a conjugate sphere, of radius $1/R$, which is a perfect and undistorted image of the sphere of radius R . This image is inverted; its magnification is $-1/R^2$. Maxwell's fisheye is a perfect optical system.

MAXWELLIAN MOLECULES. See point centers of repulsion.

MAXWELLIAN VELOCITY DISTRIBUTION. See Maxwell distribution law.

MAXWELLIAN VIEW. A method of observing an integrating photometric sphere (i.e., a sphere with a white, diffusing inner wall, in which a light source is placed). In this method the source is focussed on the pupil of the eye; a method for weak sources.

MAXWELL MATERIAL. A viscoelastic material exhibiting instantaneous elasticity and viscous flow only. It is represented by the Maxwell model:



In the linear case the constitutive equation takes the form:

$$\dot{\epsilon} = \frac{\dot{\sigma}}{E} + \frac{\sigma}{\nu}$$

At constant strain, the stress in a Maxwell material relaxes to zero.

MAXWELL MODEL. See Maxwell material for the basic Maxwell model. More general viscoelastic materials can be represented by a sequence of Maxwell models in parallel with a spring, and this is sometimes referred to as the general Maxwell model. (See relaxation time for properties of this model.)

MAXWELL-MOHR METHOD. A method of calculating the deflection δ of a pin-connected elastic truss at a given point in a given direction which introduces a unit load at the point. It is equivalent to the method of virtual work in which the actual truss is taken as the geometric or compatible system and the forces s'_i set up by the unit load are taken as the equilibrium system

$$\delta = \sum \frac{S_i l_i}{A_i E_i} s'_i$$

MAXWELL RELATION(S). From the basic relations for the characteristic functions (see thermodynamics, characteristic functions of) it may be easily deduced that

$$\left(\frac{\partial S}{\partial V}\right)_T = \frac{\alpha}{\kappa} \quad (1)$$

$$\left(\frac{\partial S}{\partial p}\right)_T = -\alpha V \quad (2)$$

where S is the entropy, α , the coefficient of thermal expansion and κ , the (isothermal) compressibility. These two relations are due to Maxwell.

MAXWELL RELATION BETWEEN DIELECTRIC CONSTANT AND INDEX OF REFRACTION. According to Maxwell's identification of light with electromagnetic radiation, the dielectric constant ϵ and the refractive index, n , of a substance should be related by the formula $\epsilon = n^2$. This relation only holds under rather restrictive conditions, such as the absence of permanent dipoles in the substance, measurement with light of very long wavelength, etc.

MAXWELL STRESS FUNCTIONS. Using the indicial notation, the stress components in a rectangular Cartesian coordinate system x , at a point x_i of a body, may be denoted σ_{ij} . Provided the body is in equilibrium and no body forces are acting, the stress field must satisfy equations of equilibrium, which may be written, using the summation convention,

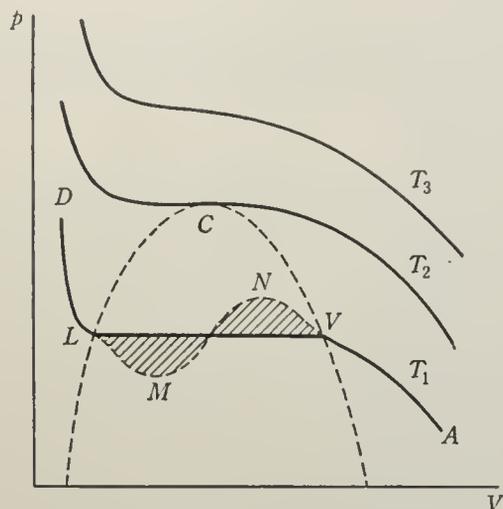
$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0.$$

Any stress field, of sufficient regularity, satisfying these equations may be expressed in terms of three functions of position in the body, ϕ_1, ϕ_2, ϕ_3 (say), called the Maxwell stress functions, by

$$\begin{aligned} \sigma_{11} &= \frac{\partial^2 \phi_2}{\partial x_3^2} + \frac{\partial^2 \phi_3}{\partial x_2^2}, & \sigma_{22} &= \frac{\partial^2 \phi_3}{\partial x_1^2} + \frac{\partial^2 \phi_1}{\partial x_3^2}, \\ \sigma_{33} &= \frac{\partial^2 \phi_1}{\partial x_2^2} + \frac{\partial^2 \phi_2}{\partial x_1^2}, & \sigma_{23} &= -\frac{\partial^2 \phi_1}{\partial x_2 \partial x_3}, \\ \sigma_{31} &= -\frac{\partial^2 \phi_2}{\partial x_3 \partial x_1}, & \sigma_{12} &= -\frac{\partial^2 \phi_3}{\partial x_1 \partial x_2}. \end{aligned}$$

MAXWELL THEOREM FOR ISOTHERMS. Consider an isotherm in the region where a liquid can coexist with its vapor (see critical

point). The figure represents such an isotherm $DLVA$; C is the critical point, LCV is the saturation curve. In accordance with the **continuity of state** one may treat the segments AV and LD of the isotherms as two parts of a single continuous curve such as $AVNMLD$. The part NM of this "extrapolated" isotherm corresponds to unstable phases (see **mechanical stability**).



p, V isotherms in region of the critical point.

Maxwell has shown that the condition of equilibrium (see **chemical potential**) between the liquid L and the vapor V requires

$$\int_{VNML} V dp = 0. \quad (1)$$

The two shaded areas in the figure are therefore equal.

MAYER THEORY OF VIRIAL COEFFICIENTS. See **cluster theory of virial coefficients**.

MEAN ACTIVITY COEFFICIENTS. See **mean chemical potentials**.

MEAN, ARITHMETIC. See **arithmetic mean**.

MEAN CHEMICAL POTENTIALS. Consider an electrolyte which dissociates into ν_+ cations each with a charge z_+ and ν_- anions of charge z_- (see **electroneutrality**). We then have

$$z_+ \nu_+ + z_- \nu_- = 0. \quad (1)$$

Because of the electroneutrality conditions the chemical potentials of the anions and of the cations μ_+, μ_- cannot be measured separately.

For this reason it is useful to define a mean chemical potential by the relation

$$\mu_{\pm} = \frac{\nu_+ \mu_+ + \nu_- \mu_-}{\nu_+ + \nu_-}. \quad (2)$$

To this mean chemical potential corresponds in the usual way a mean **activity coefficient**.

MEAN CURVATURE. See **curvature**.

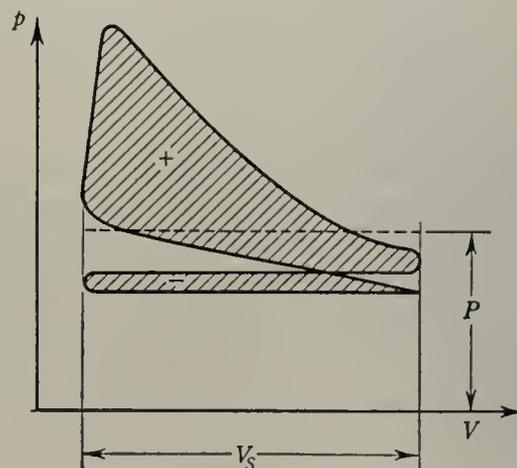
MEAN DEVIATION. The mean deviation of N sample values x about a point a is defined as $\Sigma|x - a|/N$. Used as a **measure of dispersion**, a is usually chosen to be the sample mean, or the sample median about which the mean deviation is a minimum. An alternative but less desirable expression is "average **deviation**."

MEAN DIFFERENCE. A measure of **dispersion**, not to be confined with mean deviation. For a set of values x_1, x_2, \dots, x_N the mean difference is

$$\frac{1}{N(N-1)} \sum_{i,j=1}^N |x_i - x_j|.$$

Sometimes a divisor N^2 is used. The quantity can also be defined for continuous variation.

MEAN EFFECTIVE PRESSURE. (OR INDICATED MEAN EFFECTIVE PRESSURE.) In an indicator diagram, that pressure P which acting uniformly along one stroke would produce a quantity of work equal to that produced (or consumed) during one cycle of operation of the engine. Hence, on the indicator diagram, the product PV_s is equal to the area enclosed by the diagram, with due regard to signs.



The mean effective pressure is directly related to the power of an engine. If the number

of revolutions per working stroke is denoted by k ($k = 1$ for a two-stroke engine, and $k = 2$ for a four-stroke engine), the engine speed is denoted by n (revolutions per minute) we have

$$HP_i = \frac{PnV_s}{k}$$

In a multi-cylinder engine, V_s denotes the sum of the swept volumes of all cylinders.

Hence

$$P = K \frac{HP_i}{nV_s}$$

MEAN FREE PATH. (1) The average distance that a particle travels between successive collisions, or that a wave (train) travels between successive reflections. Under collisions one must understand any scattering process which can include processes such as absorption, inelastic scattering, and so on. If the **cross section** for such processes is σ and n the number of scatterers per unit volume, the mean free path l is given by the equation

$$l = \frac{1}{n\sigma}$$

(1a) The average distance covered by a gaseous molecule between two successive collisions. There is no difficulty in defining a mean free path Λ in a gas consisting of **hard spheres**; elementary kinetic considerations show that for a pure hard sphere gas, Λ depends on the concentration n in molecules per cm^{-3} , and on the collision diameter d , but not on temperature (for a given n):

$$\Lambda = \frac{1}{\sqrt{2} \pi n d^2}$$

For more realistic potentials, when the intermolecular potential energy varies smoothly with intermolecular distance, one meets the same difficulty as in the definition of the **molecular cross section**; classically, the interaction decreases rapidly with distance, but does not vanish, except at infinity. Even in a dilute gas, molecules are then in slight interaction with each other, and there is no clearcut definition of a collision or a mean free path. A comparison of the equations for the transport coefficient obtained from the potential curve considered, and from the hard sphere model, allows one to define a "mean free path" for a molecule and for a given set of experi-

mental conditions. In contrast with result obtained for hard spheres, the mean free path depends now slightly on the temperature, and also on the transport coefficient from which it has been derived.

(1b) In neutron transport theory, the average distance traveled by a neutron before experiencing a collision of a particular type (scattering, capture, etc.). It is equal to the reciprocal of the macroscopic cross-section $(N\sigma_i)^{-1}$ for the process under consideration. The reciprocal of the mean free path in a mixture is equal to the sum of the reciprocals of the component mean free paths.

(1c) The mean free path of sound waves in an enclosure is defined as the average distance between successive reflections of a sound wave traveling in the enclosure.

MEAN FREE PATH, TRANSPORT. The transport mean free path is closely related to mean free path for scattering, namely

$$l_{tr} = \frac{l_s}{1 - \langle \cos \theta \rangle_{av}}$$

where $\langle \cos \theta \rangle_{av}$ is the average cosine of the scattering angle.

In nuclear reactors, the term is used with two meanings: (1) Where the **Fick law** is applicable, three times the diffusion coefficient of **neutron flux**. (2) A modified mean free path used to correct for the persistence of velocities and anisotropy of **scattering**, the latter being expressed in the above formula.

MEAN FREE TIME. The average time between collisions. In solid state physics, an important example of the use of the term is to denote the average time between collisions of an electron with impurities in a **semiconductor**.

MEAN HORIZONTAL INTENSITY. The average value of the **luminous intensity** of a source in all directions in a horizontal plane passing through the center of the source.

MEAN LIFETIME. See **life, mean**.

MEAN LIFETIME OF AN ATOMIC STATE. The time τ after which the number N_n of atoms left in a given atomic state n is $1/e$ times the original number N_n^0 . If A_{nm} is the Einstein transition probability of spontaneous emission

(see **transition probability**) for the transition from the state n to a lower state m , we have

$$N_n = N_n^0 \exp - \left(\sum_m A_{nm} \right) t$$

and

$$\tau = \frac{1}{\sum_m A_{nm}}$$

For allowed (electric dipole) transitions τ is of the order 10^{-8} sec. If no allowed transitions can occur from the state n to any lower state, the mean life is much larger; it is of the order 10^{-3} sec if magnetic dipole transitions are possible, and of the order 1 sec if only quadrupole transitions are possible. States with such long lifetimes are called **metastable states**.

MEAN LIFETIME OF ELECTRON IN TRAP. This can be defined as τ where the rate of thermal release of electrons is $(1/\tau) \times$ number in traps. We then have the relation

$$\tau = \frac{h^3(g_1/g_0)}{16\pi m_{\epsilon}^* (kT)^2 \sigma} \exp \left(\frac{\epsilon_c - \epsilon_t}{kT} \right)$$

where h and k are Planck's and Boltzmann's constants, respectively, m_{ϵ}^* is the effective electron mass, T is the absolute temperature, $(\epsilon_c - \epsilon_t)$ the energy change involved in transferring an electron from the bottom of the conduction band into an electron trap, σ is the average cross-section for capture of a free electron by the trap, and g_1, g_0 are the statistical heights of full and empty traps respectively.

MEAN MOLECULAR VELOCITY. See **molecular velocity, mean**.

MEAN NORMAL CURVATURE. See **curvature, mean normal**.

MEAN OF DISTRIBUTION. See **concentration**.

MEAN SPHERICAL INTENSITY. The average value of the **luminous intensity** of a source in all directions. It is also the quotient of the total **luminous flux** by the total solid angle, 4π steradians.

MEAN-SQUARE MOLECULAR VELOCITY. See **molecular velocity, mean-square**.

MEAN SUN. A fictitious object that is assumed to move eastward in the plane of the

celestial equator with constant angular velocity. It completes the circuit of the equator in the same period of time that the true sun completes its circuit of the ecliptic. (See **time-keeping**.)

MEAN VALUE OF A FUNCTION. Defined for a function $f(x)$ over an interval (a,b) by

$$\frac{\int_a^b f(x) dx}{b - a}$$

Over an area S it is defined by

$$\frac{\iint_S f(x,y) dS}{S}$$

Over a region V of space it is defined by

$$\frac{\iiint_V f(x,y,z) dV}{V}$$

MEAN VALUE THEOREMS. The first law of the mean for **integrals** is

$$\int_a^b f(x) dx = (b - a)f(z)$$

where $a \leq z \leq b$ and $f(x)$ is a **continuous function**.

The second law of the mean is

$$\int_a^b f(x)\phi(x) dx = \phi(a) \int_a^b f(x) dx$$

with z and $f(x)$ restricted as before; $\phi(x)$ is also continuous and a positive monotonic decreasing function in the interval (a,b) . Another form of the second law is

$$\begin{aligned} \int_a^b f(x)\phi(x) dx \\ = \phi(a) \int_a^b f(x) dx + \phi(b) \int_z^b f(x) dx \end{aligned}$$

where $\phi(x)$ is not necessarily always positive.

There are similar formulas for the case where $\phi(x)$ is an increasing function. The two forms of the second theorem are known as the forms of *Bonnet* and of *DuBois-Reymond*, respectively.

A mean value theorem also exists for a **derivative**. Let $f(x)$ be a function which has

a finite derivative at all points of the interval (a, b) . Then there exists a value of z between a and b such that $f(b) - f(a) = f'(z)(b - a)$. The theorem may be interpreted geometrically, for it states that the tangent to a smooth curve is parallel to an intermediate point on a chord of the curve. The procedure can be generalized to give the extended mean value theorem

$$f(b) = f(a) + (b - a)f'(a) + \frac{(b - a)^2}{2!}f''(a) + \cdots + \frac{(b - a)^{n-1}}{(n-1)!}f^{(n-1)}(a) + \frac{(b - a)^n}{n!}f^{(n)}(z).$$

(See also **Taylor Series**.)

MEAN VELOCITY. The time average of the velocity at a fixed point, over a somewhat arbitrary time interval T counted from some fixed time t_0 . For example, the mean velocity of the u component is

$$\bar{u} = \frac{1}{T} \int_{t_0}^{t_0+T} u dt.$$

MEAN, WEIGHTED. See **arithmetic mean**.

MEASURABLE FUNCTION. A function $f(x)$, defined in any interval (a, b) , is said to be measurable, provided that, for every value of A , the set of points x , of (a, b) , at which $f(x) \geq A$, is a measurable set of points.

MEASURABLE SET OF POINTS. When the exterior and interior measures $m_e(G)$ and $m_i(G)$ of a set G , of points in p dimensions, are equal to one another, the set G is said to be measurable, and the number $m_e(G) \equiv m_i(G)$ is defined to be the **measure** of G . When G is measurable its measure is denoted by $m(G)$.

MEASURE OF LOCATION. A value which determines the location of a frequency **distribution**. It is, in some defined sense, typical or central. The customary measures are the **arithmetic mean** and the **median**.

MECHANICAL-ACOUSTICAL RECIPROCITY THEOREM. See **reciprocity theorem, mechanical-acoustical**.

MECHANICAL AND ACOUSTICAL IMPEDANCE LOAD UPON AN OSCILLATING SPHERE. An oscillating sphere is a sphere whose radius remains constant while

the sphere executes a movement of translation as a function of the time. The mechanical impedance, in mechanical ohms, of the air load upon an oscillating sphere is

$$z_M = \frac{4\pi R^2 \rho c}{3} \left[\frac{k^4 R^4 + j(2kR + k^3 R^3)}{4 + k^4 R^4} \right] \quad (1)$$

where R is the radius of the sphere, in centimeters, ρ is the density, in grams per cubic centimeter, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, and c is the velocity of sound, in centimeters per second.

The acoustical impedance, in acoustical ohms, of the air load upon an oscillating sphere is

$$z_A = \frac{\rho c}{12\pi R^2} \left[\frac{k^4 R^4 + j(2kR + k^3 R^3)}{4 + k^4 R^4} \right]. \quad (2)$$

The acoustical impedance per unit area of an oscillating sphere is

$$z_1 = \frac{\rho c}{3} \left[\frac{k^4 R^4 + j(2kR + k^3 R^3)}{4 + k^4 R^4} \right]. \quad (3)$$

The oscillating sphere is an acoustical doublet. Therefore, the acoustical resistance component is proportional to the fourth power of the frequency when the dimensions are small compared to the wavelength. The oscillating sphere represents the direct radiator loud-speaker without a baffle.

MECHANICAL AND ACOUSTICAL IMPEDANCE LOAD UPON A PULSATING CYLINDER. The pulsating cylinder is a cylinder whose radius increases and decreases with time. The motion of the air around the cylinder will, like the motion of the cylinder itself, take place in radial directions in planes normal to the axis of the cylinder and will have the same radial velocity in all directions but will depend upon the distance from the center of the cylinder.

The mechanical impedance, in mechanical ohms, of the air load, per unit length, upon a pulsating cylinder is

$$z_M = 2\pi R \rho c \left[\frac{(2kR)^2 + j2kR}{1 + (2kR)^2} \right] \quad (1)$$

where R is the radius of the cylinder, in centimeters, ρ is the density of air, in grams per cubic centimeter, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, and c is the velocity of sound in centimeters per second.

The acoustical impedance, in acoustical

ohms, of the air load per unit length upon a pulsating cylinder is given by

$$z_A = \frac{\rho c}{2\pi R} \left[\frac{(2kR)^2 + j2kR}{1 + (2kR)^2} \right]. \quad (2)$$

The acoustical impedance per unit area is

$$z_1 = \rho c \left[\frac{(2kR)^2 + j2kR}{1 + (2kR)^2} \right]. \quad (3)$$

MECHANICAL AND ACOUSTICAL IMPEDANCE LOAD UPON A PULSATING SPHERE.

The pulsating sphere is a sphere whose radius increases and decreases with time. The motion of the air around the sphere will, like the motion of the sphere itself, take place only in radial directions and will have the same velocity in all directions, but will depend upon the distance from the center of the sphere.

The mechanical impedance, in mechanical ohms, of a pulsating sphere is

$$z_M = 4\pi R^2 \rho c \left[\frac{(kR)^2 + jkR}{1 + (kR)^2} \right] \quad (1)$$

where R is the radius of the sphere, in centimeters, ρ is the density, in grams per cubic centimeter, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, and c is the velocity of sound, in centimeters per second.

The acoustical impedance, in acoustical ohms, of the air load upon pulsating sphere is

$$z_A = \frac{\rho c}{2\pi R^2} \left[\frac{(kR)^2 + j(kR)}{1 + (kR)^2} \right]. \quad (2)$$

The acoustical impedance per unit area is

$$z_1 = \rho c \left[\frac{(kR)^2 + jkR}{1 + (kR)^2} \right]. \quad (3)$$

MECHANICAL AND ACOUSTICAL IMPEDANCE LOAD UPON A VIBRATING PISTON.

The mechanical impedance, in mechanical ohms, of the air load upon one side of a vibrating piston set in an infinite baffle is

$$z_M = \pi R^2 \rho c \left[1 - \frac{J_1(2kR)}{kR} \right] + j \frac{\pi \omega \rho}{2k^3} K_1(2kR) \quad (1)$$

where R is the radius of piston, in centimeters, ρ is the density, in grams per cubic centimeter, c is the velocity of sound, in centimeters per second, k is $2\pi/\lambda$, λ is the wavelength, in centi-

meters, ω is $2\pi f$, and f is the frequency, in cycles per second.

J_1 and K_1 may be found in treatises on **Bessel functions**. They are also defined by the series,

$$1 - \frac{J_1(2kR)}{kR} = \frac{k^2 R^2}{2} - \frac{k^4 R^4}{2^2 \cdot 3} + \frac{k^6 R^6}{2^2 \cdot 3^2 \cdot 4} \dots \quad (2)$$

$$K_1(2kR) = \frac{2}{\pi} \left[\frac{(2kR)^3}{3} - \frac{(2kR)^5}{3^2 \cdot 5} + \frac{(2kR)^7}{3^2 \cdot 5^2 \cdot 7} \dots \right]. \quad (3)$$

The acoustical impedance, in acoustical ohms, of the air load upon one side of a vibrating piston in an infinite baffle is

$$z_A = \frac{\rho c}{\pi R^2} \left[1 - \frac{J_1(2kR)}{kR} \right] + \frac{j\omega \rho}{2\pi R^4 k^3} K_1(2kR). \quad (4)$$

The acoustical impedance per unit area of the piston is

$$z_1 = \rho c \left[1 - \frac{J_1(2kR)}{kR} \right] + \frac{j\omega \rho}{2R^2 k^3} K_1(2kR). \quad (5)$$

The above equation may be used to determine the mechanical or acoustical radiation resistance and the mechanical or acoustical reactive component of the air load upon the cone in a direct radiator loudspeaker. It is customary to use the above equations for the resistive and reactive components of the mechanical or acoustical impedance at the mouth of a horn in computing the mechanical or acoustical impedance at the throat of the horn.

MECHANICAL AND ACOUSTICAL IMPEDANCE LOAD UPON A VIBRATING PISTON IN FREE SPACE.

The mechanical impedance, in mechanical ohms, of the air load upon one side of a vibrating piston in free space is given by the approximate equation

$$z_M = \pi R^2 \rho c \left[\frac{\left(\frac{5R}{\lambda}\right)^4 + j \left[\frac{10R}{\lambda} + \left(\frac{5R}{\lambda}\right)^3 \right]}{4 + \left(\frac{5R}{\lambda}\right)^4} \right] \quad (1)$$

where R is the radius of the piston, in centimeters, ρ is the density, in grams per cubic centi-

meter, c is the velocity of sound in centimeters per second, and λ is the wavelength, in centimeters.

The acoustical impedance, in acoustical ohms, of the air load upon one side of a vibrating piston in free space is

$$z_A = \frac{\rho_c}{\pi R^2} \frac{\left(\frac{5R}{\lambda}\right)^4 + j \frac{10R}{\lambda} + \left(\frac{5R}{\lambda}\right)^3}{4 + \left(\frac{5R}{\lambda}\right)^4}. \quad (2)$$

The acoustical impedance per unit area of the piston is

$$z_1 = \rho_c \left[\frac{\left(\frac{5R}{\lambda}\right)^4 + j \left[\frac{10R}{\lambda} + \left(\frac{5R}{\lambda}\right)^3 \right]}{4 + \left(\frac{5R}{\lambda}\right)^4} \right]. \quad (3)$$

MECHANICAL AND ACOUSTICAL IMPEDANCE LOAD UPON A VIBRATING PISTON IN THE END OF AN INFINITE TUBE. The mechanical impedance, in mechanical ohms, of the air load upon a vibrating piston in the end of an infinite tube is given by the approximate equation

$$z_M = \pi R^2 \rho_c \left[1 - \frac{J_1\left(\frac{8.8R}{\lambda}\right)}{\frac{4.4R}{\lambda}} \right] + j \frac{\pi \omega \rho}{2k^3} K_1\left(\frac{8.8R}{\lambda}\right) \quad (1)$$

where R is the radius of the piston, in centimeters, ρ is the density, in grams per square centimeter, c is the velocity of sound, in centimeters per second, and λ is the wavelength, in centimeters.

J_1 and K_1 may be found in treatises on **Bessel functions**. They are also defined by the series

$$1 + J_1\left(\frac{8.8R}{\lambda}\right) = \frac{\left(\frac{4.4R}{\lambda}\right)^2}{2} + \frac{\left(\frac{4.4R}{\lambda}\right)^4}{2^2 \cdot 3} + \frac{\left(\frac{4.4R}{\lambda}\right)^6}{2^2 \cdot 3^2 \cdot 4} \dots \quad (2)$$

$$K_1\left(\frac{8.8R}{\lambda}\right) = \frac{2}{\pi} \frac{\left(\frac{8.8R}{\lambda}\right)^3}{3} + \frac{\left(\frac{8.8R}{\lambda}\right)^5}{3^2 \cdot 5} + \frac{\left(\frac{8.8R}{\lambda}\right)^7}{3^2 \cdot 5^2 \cdot 7} \dots \quad (3)$$

The acoustical impedance, in acoustical ohms, upon a vibrating piston in the end of an infinite tube is

$$z_A = \frac{\rho_c}{\pi R^2} \left[1 - \frac{J_1\left(\frac{8.8R}{\lambda}\right)}{\frac{4.4R}{\lambda}} \right] + \frac{j \omega \rho}{2\pi R^4 K^3} K_1\left(\frac{8.8R}{\lambda}\right). \quad (4)$$

The acoustical impedance per unit area of the piston is

$$z_1 = \rho_c \left[1 - \frac{J_1\left(\frac{8.8R}{\lambda}\right)}{\frac{4.4R}{\lambda}} \right] + \frac{j \omega \rho}{2R^2 K^3} K_1\left(\frac{8.8R}{\lambda}\right). \quad (5)$$

MECHANICAL AND ACOUSTICAL IMPEDANCE LOAD UPON A VIBRATING STRIP. The mechanical impedance, in mechanical ohms, of the air load, per unit length, upon one side of an infinitely long vibrating strip set in an infinite baffle is

$$z_M = 2\rho c D \left[\frac{(2kD)^{3/2} + (2kD)^{5/2}}{1 + (2kD)^{3/2}} \right] \quad (1)$$

where $2D$ is the width of the vibrating strip, in centimeters, ρ is the density, air grams per cubic centimeter, c is the velocity of sound, in centimeters per second, k is $2\pi/\lambda$, and λ is the wavelength, in centimeters.

The acoustical impedance, in acoustical ohms, of the air load per unit length, upon a vibrating strip set in an infinite baffle is

$$z_A = \frac{\rho c}{2D} \left[\frac{(2kD)^{3/2} + (2kD)^{5/2}}{1 + (2kD)^{3/2}} \right]. \quad (2)$$

The acoustical impedance per unit area of the strip is

$$z_1 = \rho c \left[\frac{(2kD)^{\frac{3}{2}} + (2kD)^{\frac{5}{2}}}{1 + (2kD)^{\frac{3}{2}}} \right]. \quad (3)$$

MECHANICAL EFFICIENCY. In a reciprocating prime mover, the ratio of the work available on the driving shaft, W_e , to the indicated work W_i measured on an indicator diagram

$$\eta_m = \frac{W_e}{W_i}. \quad (1)$$

In a turbine, the ratio of W_e to the enthalpy drop,

$$W_i = h_1 - h_3 \quad (2)$$

for the real process. In a reciprocating compressor, the ratio of the indicated work W_i to the work W_e supplied to the driving shaft,

$$\eta_m = \frac{W_i}{W_e}. \quad (3)$$

In a turbocompressor, the indicated work is replaced by the (negative) enthalpy of the real process,

$$W_i = h_1 - h_3. \quad (4)$$

In all cases the mechanical efficiency is a measure of the perfection of the mechanical design and condition of the engine; its value is always less than unity and decreases as the mechanical losses (friction, power required to drive ancillary mechanisms such as fuel pump, scavenge pump, lubrication pump, etc.) increase.

MECHANICAL EQUIVALENT OF HEAT.

If the units of work and heat (see first law of thermodynamics) are defined separately, it becomes necessary to determine the relation between them, the mechanical equivalent of heat

$$J = \frac{\text{unit of work}}{\text{unit of heat}}$$

or the heat equivalent of mechanical energy

$$A = \frac{\text{unit of heat}}{\text{unit of work}}$$

The modern tendency is to define heat in terms of work and the first law of thermodynamics, and either to measure heat in units of work, or to define the mechanical equivalent of heat by convention. (See **thermal units**.)

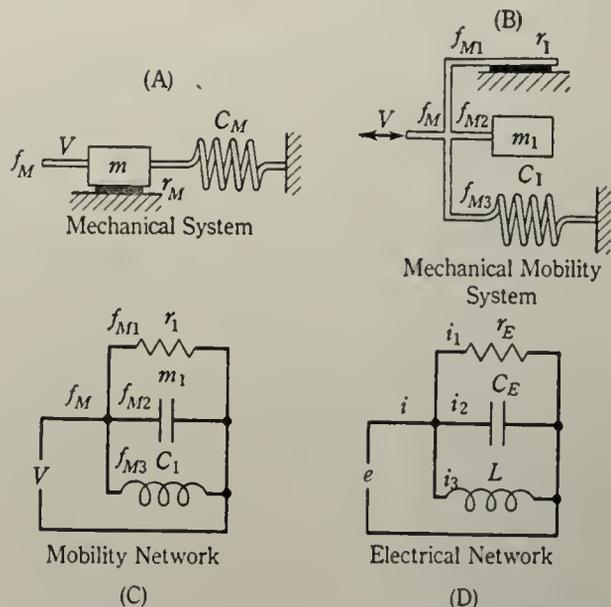
MECHANICAL EQUIVALENT OF LIGHT.

The experimental problem in this determination is to separate the visible from the infrared and ultraviolet radiation. One procedure is to enclose a lamp, of known power output, in a jacket which absorbs the invisible radiation and transmits the visible, the former being measured by the temperature-rise of the jacket and the latter photometrically. Measurements by Ives in 1926 with white light gave 1.6×10^{-3} watts per lumen. More recent measurements made at $555 \text{ m}\mu$ gave 1.46×10^{-3} watts per lumen, or 680 lumens per watt. (See also **luminosity function, standard**.)

MECHANICAL IMPEDANCE. See **impedance; impedance, mechanical**.

MECHANICAL LOSS FACTOR. See **loss factor, mechanical**.

MECHANICAL MOBILITY SYSTEM. A vibrating system of one degree of freedom consisting of a mass, compliance and mechanical resistance is considered in this book from the standpoint of the classical dynamical analogy (see **dynamical analogy, classical**). It is the purpose of this section to consider the same



A mechanical vibrating system consisting of a mass, compliance and mechanical resistance. A. Mechanical system. B. Mechanical mobility system equivalent to the mechanical system of A. C. Mobility network of the mechanical system. D. Electrical network analog of the mobility system.

system from the standpoint of the mechanical mobility analogy.

The mechanical system consisting of a mass, compliance and mechanical resistance is shown

in Figure 1A. The mechanical vibrating system may be rearranged to the equivalent mechanical system which is termed the mechanical mobility system of Figure 1B. The development of the mobility network of Figure 1C follows in a simple and direct manner from the mechanical mobility system of Figure 1B.

The sum of the forces through the three branches of the mechanical mobility system and the mobility network of Figure 1B and C is

$$f_m = f_{m1} + f_{m2} + f_{m3} \quad (1)$$

where
$$f_{M1} = \frac{v}{r_I} \quad (2)$$

$$f_{M2} = m_I \frac{dv}{dt} \quad (3)$$

$$f_{M3} = \frac{1}{C_I} \int v dt \quad (4)$$

The mobility network and the electrical analogy are developed below.

From the sum of Equations 2, 3 and 4 the differential equation of the mobility network of Figure 1C is

$$f_M = m_I \frac{dv}{dt} + \frac{v}{r_I} + \frac{1}{C_I} \int v dt. \quad (5)$$

The sum of the electrical currents of the electrical network of Figure 1D is

$$i = i_1 + i_2 + i_3 \quad (6)$$

where
$$i_1 = \frac{e}{r_E} \quad (7)$$

$$i_2 = C_E \frac{de}{dt} \quad (8)$$

$$i_3 = \frac{1}{L} \int e dt \quad (9)$$

From the sum of Equations 7, 8 and 9 the differential equation of the electrical network of Figure 1D is

$$i = C_E \frac{de}{dt} + \frac{e}{r_E} + \frac{1}{L} \int e dt. \quad (10)$$

Comparing the variables and coefficients of the mobility and electrical networks in the Differential Equations 5 and 10 establishes the analogous variables and quantities in the two systems.

MECHANICAL OHM. See ohm, mechanical.

MECHANICAL RECTILINEAL MOBILITY. Mechanical rectilinear mobility is the inverse of mechanical rectilinear impedance. Mechanical rectilinear mobility z_I , in mechanical mhos, is defined as the complex ratio of linear velocity to linear force as follows:

$$z_I = \frac{v}{f_M} \quad (1)$$

where v is the velocity, in centimeters per second, and f_M is the force, in dynes.

It is evident that a mechanical element in the mechanical mobility sense is analogous to the electrical element if velocity difference across the mechanical element is analogous to the voltage difference across the electrical element and if the force through the mechanical element is analogous to the electrical current through the electrical element.

Mechanical rectilinear mobility z_I , in mechanical mhos, is a complex quantity and may be written as follows:

$$z_I = r_I + jx_I \quad (2)$$

where r_I is the responsivity, in mechanical mhos, and x_I is the excitability, in mechanical mhos.

MECHANICAL RECTILINEAL RECIPROCALITY THEOREM. See reciprocity theorem, mechanical rectilinear.

MECHANICAL RECTILINEAL RESISTANCE. Mechanical rectilinear energy is changed into heat by a rectilinear motion which is opposed by linear resistance (friction). In a mechanical system dissipation is due to friction. Energy is lost by the system when a mechanical rectilinear resistance is displaced a distance x by a force f_M .

Mechanical rectilinear resistance (termed mechanical resistance) r_M , in mechanical ohms, is defined as

$$r_M = \frac{f_M}{u} \quad (1)$$

where f_M is the applied mechanical force, in dynes, and u is the velocity at the point of application of the force, in centimeters per second.

Equation 1 states that the driving force applied to a mechanical rectilinear resistance is

proportional to the mechanical rectilinear resistance and the linear velocity.

Mechanical rectilinear resistance is represented by sliding friction which causes dissipation of energy in the figure.



Graphical representation of the element, mechanical rectilinear resistance, r_M .

MECHANICAL RECTILINEAL SYSTEM OF ONE DEGREE OF FREEDOM.

A mechanical rectilinear system of one degree of freedom is shown in Figure 1. In one degree

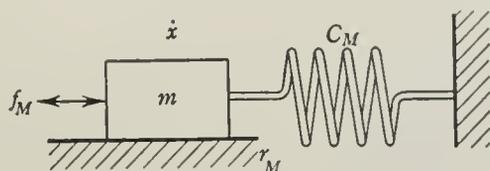


Fig. 1. Mechanical rectilinear system of one degree of freedom.

of freedom the activity in every element of the system may be expressed in terms of one variable.

In the mechanical rectilinear system a driving force f_M acts upon a particle of mass m fastened to a spring or compliance C_M and sliding upon a plate with a frictional force which is proportional to the velocity and designated as the mechanical rectilinear resistance r_M .

The principle of the conservation of energy states that the total store of energy of all forms remains constant if the system is isolated so that it neither receives nor gives out energy; in the case of transfer of energy the total gain or loss from the system is equal to the loss or gain outside the system. The sum of the kinetic, potential, and heat energy during an interval of time is, by the principle of conservation of energy, equal to the energy delivered to the system during that interval. In the mechanical rectilinear system of Figure 1, there are three forms of energy, namely, kinetic, potential, and heat energy.

Kinetic Energy. The kinetic energy T_{KM} stored in the mass of the mechanical rectilinear system is

$$T_{KM} = \frac{1}{2}m\dot{x}^2 \tag{1}$$

where m is the mass, in grams, and \dot{x} is the velocity of the mass m , in centimeters per second.

Potential Energy. The potential energy V_{PM} stored in the compliance or spring of the mechanical rectilinear system is

$$V_{PM} = \frac{1}{2} \frac{x^2}{C_M} \tag{2}$$

where $C_M = 1/s$ is the compliance of the spring, in centimeters per dyne, s is the stiffness of the spring, in dynes per centimeter, and x is the displacement, in centimeters.

Kinetic and Potential Energy. The total energy stored in the system is

$$W_M = T_{KM} + V_{PM} = \frac{1}{2}m\dot{x}^2 + \frac{1}{2} \frac{x^2}{C_M} \tag{3}$$

The rate of change of energy in the system is

$$\frac{dW_M}{dt} = m\dot{x}\ddot{x} + \frac{x\dot{x}}{C_M} \tag{4}$$

Dissipation of Energy. Assume that the frictional force f_M upon the mass m as it slides back and forth is proportional to the velocity as follows:

$$f_M = r_M\dot{x} \tag{5}$$

where r_M is the mechanical resistance, in mechanical ohms, and \dot{x} is the velocity, in centimeters per second.

The rate at which mechanical rectilinear energy D_M is converted into heat is

$$D_M = f_M\dot{x} = r_M\dot{x}^2 \tag{6}$$

Equation of Motion. The power delivered to the system must be equal to the rate of kinetic energy storage plus the rate of potential energy storage plus the power loss due to dissipation.

The rate at which work is done or power delivered to the mechanical rectilinear system by the applied mechanical force is $\dot{x}F_M e^{j\omega t} = f_M\dot{x}$.

The rate of increase of energy ($T_{KM} + V_{PM}$) of the system plus the rate at which work is done on the system or power delivered to the system by the external forces must be equal to the rate of dissipation of energy D_M . Writing this sentence mathematically yields the equation of motion for the mechanical rectilinear system of Figure 1.

$$m\dot{x}\ddot{x} + r_M\dot{x}^2 + \frac{x\dot{x}}{C_M} = F_M e^{j\omega t} \dot{x} \tag{7}$$

$$m\ddot{x} + r_M\dot{x} + \frac{x}{C_M} = F_M e^{j\omega t} \tag{8}$$

Solution of Equation of Motion. The steady state solution of the differential Equation 8 is

$$\dot{x} = \frac{F_M \epsilon^{j\omega t}}{r_M + j\omega m - \frac{j}{\omega C_M}} = \frac{f_M}{z_M}. \quad (9)$$

Mechanical Rectilinear Impedance. From Equation 9 the vector mechanical rectilinear impedance z_M , in mechanical ohms, is

$$z_M = r_M + j\omega m - \frac{j}{\omega C_M}. \quad (10)$$

Mechanical Rectilinear Reactance. Mechanical rectilinear reactance x_M is the imaginary part of the mechanical rectilinear impedance z_M , of Equation 10. The mechanical rectilinear reactance x_M , in mechanical ohms, is

$$x_M = \omega m - \frac{1}{\omega C_M}.$$

Response. The response of the mechanical rectilinear system of Figure 1 can be obtained from Equation 9. The response is the velocity \dot{x} , in centimeters per second, for a constant applied force f_M , in dynes. A typical response frequency characteristic is shown in Figure 2.

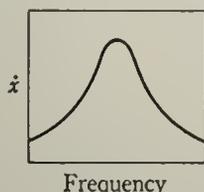


Fig. 2. Response frequency characteristic of the mechanical rectilinear system of Figure 1.

Resonant Frequency. For a certain value of m and C_M , there will be a certain frequency at which the imaginary component of the mechanical rectilinear impedance is zero. This frequency is called the resonant frequency. At this frequency the ratio of the current to the applied voltage is a maximum. At the resonant frequency the current and voltage are in phase. The resonant frequency f_r , in cycles per second, is given by

$$f_r = \frac{1}{2\pi\sqrt{mC_M}}. \quad (11)$$

Power. The power P_M , in ergs per second, dissipated in the mechanical rectilinear system of Figure 1 is given by

$$P_M = r_M \dot{x}^2. \quad (12)$$

MECHANICAL RECTILINEAR SYSTEM OF ONE DEGREE OF FREEDOM, D'ALEMBERT'S PRINCIPLE APPLIED TO. See D'Alembert's principle applied to mechanical rectilinear system of one degree of freedom.

MECHANICAL ROTATIONAL RECIPROCALITY THEOREM. See reciprocity theorem, mechanical rotational.

MECHANICAL ROTATIONAL RESISTANCE. Mechanical rotational energy is changed into heat by a rotational motion which is opposed by a rotational resistance (rotational friction). Energy is lost by the system when a mechanical rotational resistance is displaced by an angle ϕ by a torque f_R .

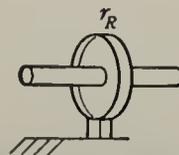
Mechanical rotational resistance (termed rotational resistance) r_R , in rotational ohms, is defined as

$$r_R = \frac{f_R}{\theta} \quad (1)$$

where f_R is the applied torque, in dyne centimeters, and θ is the angular velocity at the point of application about the axis, in radians per second.

Equation 1 states that the driving torque applied to a mechanical rotational resistance is proportional to the mechanical rotational resistance and the angular velocity.

Mechanical rotational resistance is represented by a wheel with a sliding friction brake which causes dissipation of energy. (See figure.)



Graphical representation of the element, mechanical rotational resistance, r_R .

MECHANICAL ROTATIONAL SYSTEM OF ONE DEGREE OF FREEDOM. A mechanical rotational system of one degree of freedom is shown in Figure 1. In one degree of freedom the activity in every element of

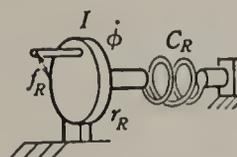


Fig. 1. Mechanical rotational system of one degree of freedom.

the system may be expressed in terms of one variable.

In the mechanical rotational system a driving torque f_R acts upon a flywheel of moment of inertia I connected to a spring or rotational compliance C_R and the periphery of the wheel sliding against a brake with a frictional force which is proportional to the velocity and designated as the mechanical rotational resistance r_R .

The principle of the conservation of energy states that the total store of energy of all forms remains constant if the system is isolated so that it neither receives nor gives out energy; in the case of transfer of energy the total gain or loss from the system is equal to the loss or gain outside the system. The sum of the kinetic, potential, and heat energy during an interval of time is, by the principle of conservation of energy, equal to the energy delivered to the system during that interval. In the mechanical rotational system of Figure 1, there are three forms of energy, namely, kinetic, potential and heat energy.

Kinetic Energy. The kinetic energy T_{KR} stored in the moment of inertia of the mechanical rotational system is

$$T_{KR} = \frac{1}{2} I \dot{\phi}^2 \quad (1)$$

where I is the moment of inertia, in gram (centimeter)² and $\dot{\phi}$ is the angular velocity of I , in radians per second.

Potential Energy. The potential energy V_{PR} stored in the rotational compliance or spring of the mechanical rotational system is

$$V_{PR} = \frac{1}{2} \frac{\phi^2}{C_R} \quad (2)$$

where C_R is the rotational compliance of the spring, in radians per dyne per centimeter, and ϕ is the angular displacement, in radians.

Kinetic and Potential Energy. The total energy stored in the system is

$$W_R = T_{KR} + V_{PR} = \frac{1}{2} I \dot{\phi}^2 + \frac{1}{2} \frac{\phi^2}{C_R} \quad (3)$$

The rate of change of energy in the system is

$$\frac{dW_R}{dt} = I \dot{\phi} \ddot{\phi} + \frac{\phi \dot{\phi}}{C_R} \quad (4)$$

Dissipation of Energy. Assume that the frictional torque f_R upon the flywheel I as the

periphery of the wheel slides against the brake is proportional to the velocity as follows:

$$f_R = r_R \dot{\phi} \quad (5)$$

where r_R is the mechanical rotational resistance, in rotational ohms, and $\dot{\phi}$ is the angular velocity, in radians per second.

The rate at which mechanical rotational energy D_R is converted into heat is

$$D_R = f_R \dot{\phi} = r_R \dot{\phi}^2 \quad (6)$$

Equation of Motion. The power delivered to the system must be equal to the rate of kinetic energy storage plus the rate of potential energy storage plus the power loss due to dissipation.

The rate at which work is done or power delivered to the mechanical rotational system by the applied mechanical torque is $\phi F_R e^{j\omega t} = f_R \phi$.

The rate of increase of energy ($T_{KR} + V_{PR}$) of the system plus the rate at which work is done on the system or power delivered to the system by the external forces must be equal to the rate of dissipation of energy D_R . Writing this sentence mathematically yields the equation of motion for the mechanical rotational system of Figure 1.

$$I \dot{\phi} \ddot{\phi} + r_R \dot{\phi}^2 + \frac{\phi \dot{\phi}}{C_R} = F_R e^{j\omega t} \dot{\phi} \quad (7)$$

$$I \ddot{\phi} + r_R \dot{\phi} + \frac{\phi}{C_R} = F_R e^{j\omega t} \quad (8)$$

Solution of Equation of Motion. The steady state solution of the differential Equation 8 is

$$\dot{\phi} = \frac{F_R e^{j\omega t}}{r_R + j\omega I - \frac{j}{\omega C_R}} = \frac{f_R}{z_R} \quad (9)$$

Mechanical Rotational Impedance. From Equation 9 the vector mechanical rotational impedance z_R , in rotational ohms, is

$$z_R = r_R + j\omega I - \frac{j}{\omega C_R} \quad (10)$$

Mechanical Rotational Reactance. Mechanical rotational reactance x_R is the imaginary part of the mechanical rotational impedance z_R of Equation 10. The mechanical rotational reactance x_R , in rotational ohms, is

$$x_R = \omega I - \frac{1}{\omega C_R} \quad (11)$$

Response. The response of the mechanical rotational system of Figure 1 can be obtained from Equation 9. The response is the velocity $\dot{\phi}$, in radians per second, for a constant applied torque f_R , in dynes centimeters. A typical response frequency characteristic is shown in Figure 2.

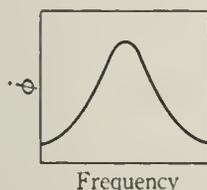


Fig. 2. Response frequency characteristic of the mechanical rotational system of Figure 1.

Resonant Frequency. For a certain value of I and C_R , there will be a certain frequency at which the imaginary component of the mechanical rotational impedance is zero. This frequency is called the resonant frequency. At this frequency the ratio of the current to the applied voltage is a maximum. At the resonant frequency the current and voltage are in phase. The resonant frequency f_r , in cycles per second, is given by

$$f_r = \frac{1}{2\pi\sqrt{IC_R}} \quad (12)$$

Power. The power P_R , in ergs per second, dissipated in the mechanical rotational system of Figure 1 is given by

$$P_R = r_R \dot{\phi}^2 \quad (13)$$

MECHANICAL ROTATIONAL SYSTEM OF ONE DEGREE OF FREEDOM, D'ALEMBERT'S PRINCIPLE APPLIED TO. See D'Alembert's principle applied to mechanical rotational system of one degree of freedom.

MECHANICAL STABILITY. (See *thermodynamic stability conditions; stability of phases.*) A phase must satisfy certain conditions if it is to be stable (or metastable). For systems consisting of a single component the necessary and sufficient conditions are the conditions of **thermal stability** and of mechanical stability. The condition of mechanical stability is

$$-\left(\frac{\partial p}{\partial V}\right)_T > 0.$$

The isothermal compressibility coefficient is thus always negative for all stable (or metastable) phases.

MECHANICAL UNITS. All mechanical measurements involve the motion of material bodies, described in terms of space and time coordinates. Hence length and time are almost universally chosen as fundamental quantities. The present standard of time is the *mean solar day*, defined as the average period between two successive transits of the sun across the meridian at any given spot on the earth's surface. The most commonly used unit of time is the *second*, defined as 1/86,400 part of a mean solar day. This is not a completely satisfactory definition (see the entry on **time** for further discussion of the subject). It seems probable that the second will be redefined in the near future, as a specified multiple of a period of vibration of some particular molecule, probably ammonia.

Two independent length standards, the meter and the yard have been used widely. By recent international agreement, however, the inch has been redefined as exactly 0.0254 meters, and the yard as exactly 36 inches. The change became effective in the United States in July 1959.

In physics, chemistry, and electrical engineering, as well as in much of mechanical engineering, the commonly employed systems of mechanical units use length, mass, and time as fundamental quantities. These are known as *length-mass-time* systems. Three such systems, the *meter-kilogram-second* (MKS), the *centimeter-gram-second* (cgs), and the *foot-pound-second* (f lbn s) systems, are in use. In all three systems the constant of proportionality in Newton's law is chosen as a dimensionless quantity of unit magnitude.

In structural engineering and in some mechanical engineering applications, forces play a more important part than do masses. Systems which use length, force, and time as fundamental units are therefore convenient. An example is the *foot-pound-second* system. The unit of force, the *pound (force)*, is defined as the weight of a pound mass at a point on the earth's surface at a point where the acceleration due to gravity is 32.174 ft/sec². In this system the unit of mass, the *slug*, is a derived unit, equal to 1/32.174 lbn. In order that confusion may not be caused by the use of the pound both as a unit of mass and as a unit of force, the pound (mass) is abbreviated as lbn, the pound (force) as lbf.

A third type of system defines both the mass

RELATIONS AMONG THE SYSTEMS OF MECHANICAL UNITS

Quantity	MKS System	Equivalents in Other Systems				
		egs System	f lbm s System	f lbf s System	f lbm lbf s System	cm gm gf s System
Length	1 Meter	10 ² cm	3.281 ft	3.281 ft	3.281 ft	10 ² cm
Mass	1 Kilogram	10 ³ gm	2.205 lbm	70.94 slug	2.205 lbm	10 ³ gm
Density	1 K/M ³	10 ⁻³ gm/cm ³	62.43(10) ⁻³ lbm/ft ³	2.009 slug/ft ³	62.43(10) ⁻³ lbm/ft ³	10 ⁻³ gm/cm ³
Force	1 Newton	10 ⁵ dyne	7.233 poundal	0.2248 lbf	0.2248 lbf	102.0 gf
Work (Energy)	1 Joule	10 ⁷ erg	22.93 ft poundal	0.7376 ft lbf	0.7376 ft lbf	1.020(10) ⁴ gf cm
Power	1 Watt	10 ⁷ erg/sec	22.93 ft poundal/sec	1.341(10) ⁻³ horse power	0.7376 ft lbf/sec	1.020(10) ⁴ gf cm/sec

and force as well as units of length and time. In such *mass-force-length-time systems*, the constant of proportionality in Newton's second law takes on dimensions and a non-unitary value. The abbreviation gf is often used to indicate a *gram (force)*, the unit of force, which is defined as the weight of a one-gram (gm) mass under the action of a gravitational acceleration of 980.665 cm/sec².

The more important units in each system and the relations among these units are shown in the table. As an example of the use of this and similar tables, suppose that a moment of inertia is specified as 1050 gm cm², and that it is desired to express this quantity in the f lbm s system. The use of conversion factors from the table shows that

$$\begin{aligned}
 1050 \text{ gm cm}^2 &\times \frac{2.205 \text{ lbm}}{1000 \text{ gm}} \times \frac{(3.281 \text{ ft})^2}{\text{M}^2} \\
 &\times \frac{\text{M}^2}{(100 \text{ cm})^2} = 0.002492 \text{ lbm ft}^2.
 \end{aligned}$$

MECHANICS, QUANTUM. See **quantum mechanics**.

MECHANISM METHOD OF ANALYSIS OF FRAMES. As shown by Symonds and Neal, the collapse or plastic **limit load** of frames can be determined simply from the **upper bound theorem** point of view alone. A frame collapses as a **mechanism** with **plastic hinges**. For a given frame and loading, every possible collapse mechanism may be regarded as some combination of a limited number of independent mechanisms. Only a few combinations need be examined in most practical problems.

MECHANISM OF SOUND ABSORPTION BY ACOUSTICAL MATERIALS. The mechanism of sound absorption by acoustical materials may be illustrated by the acoustical impedance concept. Expressions have been developed for the normal acoustical impedance per unit area of the acoustical material. These considerations are confined to the frequency range in which the thickness of the material is small compared to the wavelength. It is assumed that the back of the acoustical material is placed in contact with the wall. The acoustical impedance of the acoustical material is given by

$$z_{A1} = \frac{r_{A1D}d}{3} + \frac{j\omega d m \rho}{3} - \frac{j\rho c^2}{\omega P d} \quad (1)$$

where z_{A1} is the aeoustical impedance, in acoustical ohms per unit area, r_{A1D} is the d-c aeoustical resistance of the material per unit cube, in aeoustical ohms, d is the thickness of the material, in centimeters, ρ is the density of air, in grams per cubic centimeter, m is the ratio of the effective density of the air in the pores to its density in the open, P is the porosity, the ratio of the volume of air in the pores to the total volume, c is the velocity of sound, in centimeters per second, ω is $2\pi f$, and f is the frequency, in cycles per second.

The aeoustical resistance, in acoustical ohms, per unit area is

$$r_{A1} = \frac{r_{A1D}d}{3} \quad (2)$$

The inertance per unit area is

$$M_1 = \frac{d m \rho}{3} \quad (3)$$

The aeoustical capacitance per unit area is

$$C_{A1} = \frac{P d}{\rho c^2} \quad (4)$$

MEDIAN. (1) The median of a set of values is that value which divides the frequency into two halves. If the set is even in number the median is usually taken as midway between the two **central values**. (2) In the **Luneburg geometry** of binocular vision, the median is the line in the horizontal plane that is the Euclidean perpendicular bisector of the line segment joining the **nodal points** of the eyes of the observer.

MELLIN TRANSFORM. This transform, $f(y)$, and its inverse, $F(x)$, are defined, subject to certain conditions, by the relations

$$f(y) = \int_0^{\infty} x^{y-1} F(x) dx;$$

$$F(x) = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} x^{-y} f(y) dy.$$

(See also **integral transform**.)

MELTING. The process of change of phase from solid to liquid. The opposite process is called solidification.

MEMBRANE. A flat or curved sheet of material of negligible bending rigidity and strength. It carries load by stresses in its middle surface.

MEMBRANE ANALOGY (FOR TORSION OF A ROD). A membrane is stretched under uniform biaxial tension T and is clamped on a plane curve in the xy -plane of a rectangular Cartesian coordinate system, x, y, z , identical with the periphery of the cross section of the rod. A uniform surface force, p per unit area, directed normally to one of the surfaces of the membrane is applied. The displacement w of a generic particle of the membrane parallel to the z -axis is governed by the differential equation

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) \frac{2Tz}{p} + 2 = 0,$$

subject to the condition $2Tz/p = 0$, on the clamped boundary. Thus, the dependence of $2Tz/p$ on x and y is precisely that of the **Prandtl stress function** for the torsion problem.

MEMBRANE ANALOGY-BENDING. The differential equation and boundary conditions for the stress function for cantilever bending of an elastic prismatic bar are the same as for deflection of a uniformly stretched membrane having the same level boundary as the cross section of the beam and loaded transversely by a pressure distribution determined from the beam problem.

MEMBRANE ANALOGY-TORSION. The differential equation and boundary conditions for the stress function for torsion of an elastic prismatic bar are the same as for deflection of a uniformly stretched membrane with the same boundary as the cross section and loaded transversely with uniform pressure. In multiply connected cross sections, all boundaries are level but not at the same height. Membrane slope gives shear stress; membrane contours give the direction of shear stress; and the volume between the membrane and the level of the boundary is proportional to the twisting moment. (See **Batho** and **Bredt formulas**.)

MEMBRANE EQUILIBRIUM. If two phases α and β are separated by a fixed wall permeable to some components i but not to other components j (i.e., a **semipermeable membrane**), the condition for the two phases to be

in equilibrium as regards i is (see **chemical potentials; chemical equilibrium; chemical affinity**)

$$\mu_i^\alpha = \mu_i^\beta \quad (1)$$

where μ_i^α , μ_i^β are the chemical potentials of the components i in the phases α and β . But in this case in general the chemical potentials of the components j in the two phases will be different

$$\mu_j^\alpha \neq \mu_j^\beta \quad (2)$$

and the pressure in phase α will be different from the pressure in phase β .

$$p^\alpha \neq p^\beta. \quad (3)$$

The difference $p^\alpha - p^\beta$ is also called the *osmotic pressure*.

Such a partial equilibrium is called a *membrane equilibrium*.

MEMBRANE THEORY (OF THIN SHELLS). A theory of the deformation of thin shells by applied forces in which the couples, in directions tangential to the shell, acting on elements of the shell, are neglected.

MEMORY UNIT (OF A COMPUTER). See *storage unit*.

MERCALLI SCALE. See *earthquake scales*.

MERCATOR PROJECTION. The Mercator projection of the present day is a development of the conformal chart of the earth that was invented by Gerhard Kramer (Latin surname Mercator) about the middle of the 16th century. For nautical purposes this type of projection will probably be used by navigators as long as they use the rhumb line, i.e., follow the **loxodrome**, from one place to another.

Contrary to many loose statements, the Mercator projection should not be considered as a perspective projection of the earth from the center to a cylinder tangent to the earth at the equator. Instead it is a **conformal projection**. The equator is represented by a straight line with parallels of latitude straight and parallel to the equator. Meridians of longitude are straight lines perpendicular to the equator and the meridians. Since the chart is to be conformal at every point, the distance scale must be the same in all directions at every point. In other words the parallels must be so spaced that the scale along a meridian is the same as that along a parallel.

Since the earth is considered to be a **geoid**,

an element of distance along a parallel, dp , is given as

$$dp = \frac{a \cos \phi dLo}{(1 - e^2 \sin^2 \phi)^{1/2}}$$

in which a is the equatorial radius of the earth, ϕ is the latitude, Lo the longitude, and e the eccentricity.

We may assume that the meridians are at their actual distances apart at the equator, in which case $dp = a dLo$ and we have

$$\frac{d\phi}{a dLo} = \frac{\cos \phi}{(1 - e^2 \sin^2 \phi)^{1/2}}.$$

An element of length along the elliptical meridian is

$$dm = \frac{a(1 - e^2)d\phi}{(1 - e^2 \sin^2 \phi)^{3/2}}.$$

If we consider ds an element of length on the completed projection, that is to represent this element of length on the meridian, we must also have the ratio of dm to ds equal the scale along the parallel. Thus we have

$$\frac{dm}{ds} = \frac{a(1 - e^2)d\phi}{ds(1 - e^2 \sin^2 \phi)^{3/2}} = \frac{\cos \phi}{(1 - e^2 \sin^2 \phi)^{1/2}}$$

or

$$ds = \frac{a(1 - e^2)d\phi}{(1 - e^2 \sin^2 \phi) \cos \phi}.$$

The distance of any parallel ϕ from the equator on the completed chart is

$$S = \int_0^\phi \frac{a(1 - e^2)d\phi}{(1 - e^2 \sin^2 \phi) \cos \phi}.$$

The values of this integral have been computed many times and a number of tables of values are available. For example Table 5 of *U.S. Hydrographic Office #9* contains the value of the integral for every minute of latitude from the equator to the pole. These are referred to by navigators as **meridional parts** (see **sailings, mercator**).

The greatest virtue of the Mercator chart for navigators is the fact that the **rhumb line** is straight. Within reasonable limits the chart is conformal over large areas, particularly in the lower latitudes. This is shown by the comparative areas at various latitudes of a unit square projected on the chart

ϕ	0°	15°	30°	45°	60°	75°
Area	1	1.02	1.15	1.4	2	3.9

For a small square the differential stretching of the north-south side is hardly noticeable. Beyond 75° other types of projection should be used, e.g., modified Lambert **conformal projection**.

MERCURY, ADVANCE OF PERIHELION OF. See **relativity, general**.

MERIDIAN(S). See **surface of revolution**.

MERIDIAN SECTIONS. See **surface of revolution**.

MERIDION(AL) PLANE. In an axially symmetric optical system a plane containing the axis.

MERIDION(AL) RAY. In an axially symmetric optical system, a ray that is coplanar with the axis. If the ray also intersects the axis in the object plane, it is an *axial ray*, otherwise it is an oblique meridional ray. Sometimes the term meridional ray is used in the restricted sense of an axial ray.

MEROMORPHIC FUNCTION. A function of a complex variable for which every point in the finite plane is either a regular point or a pole. For example,

$$f(z) = \frac{P(z)}{Q(z)},$$

where P and Q are polynomials, without common factor, is a meromorphic function.

MESH. (1) For mathematical meaning, see **net**. (2) As used in electricity, a mesh is a set of branches forming a closed path in a **network**, provided that if any one branch is omitted from the set, the remaining branches of the set do not form a closed path. The term loop is sometimes used in the sense of mesh.

MESON. A fundamental particle intermediate in mass between the electron and the proton. Those identified so far are the μ -meson (or muon) of rest mass $207m$ (where m is the electron mass), which is found with positive and negative charge; the π -meson (or pion), of which the positive and negative forms have mass $273m$, while the neutral form has mass $264m$; and the K -meson of mass $966m$. Of these mesons, the pion is known to interact

very strongly with nuclei and is thought to be largely responsible for nuclear forces in the same sense that the photon is responsible for electromagnetic forces. All these particles are unstable, decaying radioactively ultimately to photons and neutrinos. They are seen in the cosmic radiation, and can be produced artificially by accelerating machines such as the cyclotron.

MESOPIC VISION. Vision in conditions intermediate between those of **photopic** and **scotopic vision**.

METACENTER. See **stability of a floating body**.

METACENTRIC HEIGHT. See **stability of a floating body**.

METALLIC BOND. See **bond types**.

METALLIC REFLECTION. See **Fresnel equations for metallic reflection**.

METALS, ELECTRON THEORY OF. See **electron theory of metals**.

METASTABLE PHASES. See **stability of phases**.

METASTABLE STATE. An excited state of an atom, molecule, etc., from which all transitions to a lower state are "forbidden" (see **forbidden transition**), for instance the lowest triplet state in helium and the alkaline-earth atoms. Metastable states have a **mean lifetime** of 10^5 sec or more.

METER-CANDLE. A unit of **illuminance**, the same as the **lux**.

METER-CANDLE-SECOND. A unit of photographic exposure corresponding to an **illumination** of 1 lux acting for 1 second.

METHOD OF IMAGES. See **images, method of**.

METHOD OF ITERATION. See **iteration, method of**.

METHOD OF MOMENTS. See **moments method**.

METHOD OF POLYNOMIAL APPROXIMATIONS. A method of analysis in the theory of neutron chain-reacting systems. It is assumed that the **cross-sections** appearing in the Boltzmann transport equation (see **neutron transport theory**) may be approximated,

in their energy dependence, by polynomial expressions of low order. When this is the case, it is convenient to expand the energy variation of the neutron density in terms of a similar set of polynomials. The energy expansion may be carried out through the use of the **Mellin transformation**.

METHOD OF SMALL PERTURBATIONS.

(Also called method of perturbations, perturbation method.) The linearization of the appropriate equations governing a system by the assumption of a steady state, with departures therefrom limited to small perturbations.

METHOD OF STEEPEST DESCENTS. See steepest descents, method of.

METHOD OF SUCCESSIVE APPROXIMATIONS. See iterative methods.

METHOD OF THE HYPERCIRCLE. A method for finding the upper and lower bounds for quadratic functionals which arise in the solution of linear boundary value problems occurring in connection with certain **partial differential equations** of mathematical physics. The method was originally developed in connection with classical elasticity theory.

METHOD OF VERDE AND WICK. See Verde and Wick, method of.

METHOD, S_N . See S_N method.

METHOD, SOURCE-SINK. See source-sink method.

METHOD, SPENCER-FANO. See Spencer-Fano method.

METHOD, SPHERICAL HARMONICS. See spherical harmonics method.

METHOD, WICK-CHANDRASEKHAR. See Wick-Chandrasekhar method.

METHOD, YVONNE. See Yvonne method.

METRIC SPACE. See topological space.

METRIC TENSOR. In n -dimensional space, let \bar{x}^i ($= \bar{x}_i$) be the coordinates in a rectangular Cartesian coordinate system \bar{x} of a point which has contravariant coordinates x^i in the generic curvilinear coordinate system x . Define the quantities g_{ij} by

$$g_{ij} = \frac{\partial \bar{x}^k}{\partial x^i} \frac{\partial \bar{x}_k}{\partial x^j},$$

in each curvilinear coordinate system y . The aggregate of quantities so obtained forms an absolute covariant tensor of second order, which is called the *covariant metric tensor* or *metric tensor* or *fundamental metric tensor* or *fundamental tensor*. g_{ij} , as defined above, are the components of this tensor in the coordinate system x . The set of quantities g_{ij} is often referred to as the *covariant metric tensor* (or *metric tensor*) for the coordinate system x .

Define the quantities g^{ij} by

$$g^{ij} = \frac{\partial x^i}{\partial \bar{x}^k} \frac{\partial x^j}{\partial \bar{x}_k}$$

in each curvilinear coordinate system x . The aggregate of quantities so obtained forms an absolute contravariant tensor of second order, which is called the *contravariant metric tensor*. g^{ij} , as defined above, are the components of this tensor in the coordinate system x . The set of quantities g^{ij} is often referred to as the *contravariant metric tensor* for the coordinate system x . g_{ij} and g^{ij} are related by

$$g^{ik} g_{kj} = \delta_j^i,$$

where δ_j^i denotes the n -dimensional **Kronecker delta**.

MEUNIER THEOREM (MEUSNIER THEOREM). κ_n is the normal curvature at a point of the surface in the direction of some tangent line and κ is the curvature, at the same point, of the curve of intersection of the surface with a plane containing the same tangent line and making an angle θ with the plane defined by the tangent line and the normal to the surface. Then

$$\kappa_n = \kappa \cos \theta.$$

MEV. Symbol for one million electron volts, a unit of energy equal to 1.602×10^{-6} erg.

MHO. A unit of electrical conductance, the reciprocal ohm. (See **electromagnetic units**.)

MICROBAR. A unit of pressure used in acoustics. It is equal to one dyne per square centimeter.

MICROCANONICAL ENSEMBLE. An **ensemble**, the density of which is zero except in a very narrow range of values of the energy of the system.

MICROGRAM. One millionth of a gram, sometimes denoted by γ .

MICROMICRO-. Prefix denoting 10^{-12} . Thus 1 micromicrofarad ($\mu\mu f$) = 10^{-12} farad (f).

MICROMICRON. One millionth of a **micron**, or one trillionth of a meter, sometimes denoted by $\mu\mu$, one micromicron = 10^{-6} micron = 10^{-12} meter.

MICRON. Unit of length, abbreviation μ . Exactly 10^{-6} meter and 10^{-4} centimeter.

MICRORECIPROCAL DEGREE. A convenient unit for the expression of the reciprocal of the color temperature (see **temperature, color**). The value is expressed in microreciprocal degrees by the expression M.D. = 1,000,000/color temperature ($^{\circ}\text{K}$).

MICROSCOPIC. (1) Of such a size as to be observable with the aid of an optical microscope but not with the unaided eye or a simple magnifier. (2) In statistical mechanics, the term is used to describe any of the constants, properties, or variables of a system which are below the level of individual detection by the methods of measurement used in an actual or hypothetically realizable experiment.

MICROSCOPIC REVERSIBILITY, PRINCIPLE OF. A postulate that each microscopic process occurring must be accompanied by an inverse process. As stated in a specific case by Mitchell and Zemansky: "At equilibrium, the total number of molecules leaving a given quantum state in unit time shall equal the number arriving at that state in unit time, and also in unit time the number leaving by any one particular path shall be equal to the number arriving by the reverse path."

MICROSTATE. The **state** of a **thermodynamic system** described in terms of its microscopic structure. It consists, in principle, of a set of coordinates of all its atoms or molecules, which describes their instantaneous positions, and of a set of momenta describing the instantaneous velocities of all the molecules. Hence a microstate involves a very large number of coordinates and is contingent on the adoption of a molecular model for the thermodynamic system. In **statistical and quantum mechanics** it is found more convenient to invert the description of the microstate. A system of coordinates (phase space) is selected which consists of three space coordinates, x_1 ,

x_2 , x_3 , and of three momentum coordinates, p_x , p_y , p_z , and the description is achieved by indicating the *occupation numbers* n of all the molecules, i.e., by specifying the number of molecules n_i which are instantaneously present in the cells of length Δx_1 , Δx_2 , Δx_3 , Δp_1 , Δp_2 , Δp_3 into which the phase space can be subdivided. This description is in contrast with the **macrostate** which relies on the indication of a small number of measurable properties (such as pressure, volume, temperature, etc.). Interpreted in microscopic terms, the macrostate makes use of suitably-formed averages of the microscopic coordinates. Hence one macrostate corresponds to a large number of different microstates, but not *vice versa*. To each microstate is assigned the same a priori weight in **statistical mechanics**.

MICROWAVE SPECTRA OF MOLECULES. Molecular spectra in the microwave region arise from transitions between narrowly spaced energy levels. Depending on the types of energy levels involved one distinguishes:

(a) Pure rotation spectra, corresponding to transitions between the rotational energy levels belonging to the same electronic and vibrational state (see **rotation spectra of molecules**).

(b) Spin-reorientation spectra, corresponding to transitions between the multiplet components of an electronic energy level (see **fine structure II**).

(c) Inversion spectra, corresponding to transitions between the two different modifications of a non-planar molecule (see **rotation-vibration spectra of molecules**).

(d) Spectra corresponding to transitions between the Λ -doublet components of Π , Δ , \dots electronic states of diatomic and linear polyatomic molecules, or to transitions between the l -doublet components of Π , Δ , \dots vibrational levels of linear polyatomic molecules. (See **Λ -type doubling, l -type doubling**.)

MIDPOINT FORMULA (FOR NUMERICAL QUADRATURE)

$$\int_a^b f(x)dx = (b-a)f\left(\frac{a+b}{2}\right) + \frac{1}{24}(b-a)^3 f''(\xi), \quad a \leq \xi \leq b.$$

MIE FUNCTIONS. See **spherical vector wave functions**.

MIE-GRÜNEISEN EQUATION OF STATE.

This is an approximate equation of state for crystals, and relates the pressure p , the volume v , and the total energy U . It may be written

$$p = -\frac{dU_o}{dv} + \gamma \frac{(U - U_o)}{v} \quad (1)$$

where U_o is the value of the energy in the absence of thermal vibrations, but including zero point of energy

$$\gamma = \frac{d \ln \omega}{d \ln v}$$

where ω is the frequency of a normal mode. In deriving Equation (1), γ is assumed to have the same value for all normal modes. (See **Grüneisen constant**.)

Equation (1) leads to the following expression for the linear thermal expansion coefficient β

$$\beta = \frac{1}{3v} \left(\frac{\partial v}{\partial T} \right)_p = \frac{\kappa \gamma}{3V_m} C_v$$

where κ is the volume compressibility, V_m is the molar volume, and C_v the molar specific heat at constant volume.

MIE SCATTERING. Any scattering produced by spherical particles without special regard to comparative size of radiation wavelength and particle diameter; to be contrasted, therefore, with **Rayleigh scattering**. (See **Mie theory**.)

MIE THEORY. A complete mathematical-physical theory of the scattering of electromagnetic radiation by spherical particles, developed by G. Mie in 1908. In contrast to the theory of **Rayleigh scattering**, the Mie theory is not restricted to scattering by particles small compared to the wavelength of the scattered radiation, but rather embraces all possible ratios of diameter to wavelength.

MIGRATION AREA FOR NEUTRONS.

Generally referring to the squared-distance traveled by a neutron during its lifetime in matter, migration area (M^2) is given by:

$$M^2 = \frac{1}{6}(\langle r_s^2 \rangle + \langle r_t^2 \rangle)$$

where $\langle r_s^2 \rangle$ is the average squared-distance between the point at which the neutron appears in the medium and the point at which it becomes "thermal" and $\langle r_t^2 \rangle$ is the average

squared-distance between the point at which the neutron becomes thermal and the point at which it is captured. M^2 is often defined as

$$M^2 = (\tau + L^2)$$

where τ is the characteristic age-to-thermal for the medium, and L^2 is the diffusion area. The two definitions are not equivalent.

MIGRATION LENGTH FOR NEUTRONS.

The migration length is the square root of the migration area.

MILLER INDICES.

According to the **Häuy law**, the intercepts of any crystal plane on the crystal axes may be expressed as rational fractional multiples of the crystal parameters. The Miller indices are the reciprocals of these fractions, reduced to integral proportions. Thus, the symbol (211) means the plane having intercepts $\frac{1}{2}a, b, c$ on the three axes respectively. The notation $\bar{2}$ is used to mean -2 . In the hexagonal crystal system, four indices, the **Bravais-Miller indices**, are used.

MILLIBAR. The unit of pressure used in meteorology. 1 bar = 10^6 dynes cm^{-2} = 1000 mb = 1 atmosphere approx.

MILLIMASS UNIT. One-thousandth of an atomic mass unit; symbol mmu.

MILNE METHOD (FOR SOLVING AN ORDINARY DIFFERENTIAL EQUATION).

$$y' = f(x, y)$$

uses the **predictor**

$$y_{\nu+1} = y_{\nu-3} + 4h(2f_{\nu} - f_{\nu-1} + 2f_{\nu-2})$$

and the **corrector**

$$y_{\nu+1} = y_{\nu-1} + h(f_{\nu+1} + 4f_{\nu} + f_{\nu-1})/3,$$

the latter formula being the **Simpson rule** applied to the evaluation of the integral. The formulas are meaningful when y and f are vectors. The method is one of the simpler ones and often quite accurate, but for some problems exhibits serious lack of stability. (See F. B. Hildebrand, *Introduction to Numerical Analysis*, McGraw-Hill Book Company, 1956; Lothar Collatz, *Numerische Behandlung von Differentialgleichungen*, 2nd edition, Springer, 1955.)

MILNE'S PROBLEM. The problem of finding the distribution, in space and angle, of neutrons (or photons) in a homogeneous

source-free medium filling the half-space $x < 0$. The particles are assumed to originate at $x = -\infty$ and do not undergo a change in energy upon scattering.

MINIMAL SURFACE. A surface at each point of which the principal curvatures (see **curvature of surface, center of**) are equal in magnitude and opposite in sign.

MINIMAX. See **games theory**.

MINIMAX APPROXIMATION. The representation of a function $f(x)$ by an approximation $P[f|x]$ selected from a class of functions in such a way that if $R = f - P$, then the maximum of $|R|$ over all points x of some assigned set is minimized over all functions of the class. Often called *Chebyshev approximation*. There is no simple algorithm for forming the Chebyshev approximation to an arbitrary continuous function, but there are techniques of successive approximation to it.

In case the class of functions is the class of linear combinations of functions $\phi_0(x), \phi_1(x), \dots, \phi_n(x)$ on an interval from a to b , then the basic theorem is the following: If the functions ϕ_i form a **Chebyshev system** of order n for the interval from a to b , and if f is continuous on this interval, then the minimax approximation $P_n(x)$ is unique; moreover $|f(x) - P_n(x)|$ achieves its maximum at least $n + 2$ times at points x_0, x_1, \dots, x_{n+1} on the interval and at these points $f - P_n$ alternates in sign.

The **Chebyshev polynomial** $T_n(x)$ is that polynomial of degree n with leading coefficient unity that gives the minimax approximation to zero on the interval from -1 to $+1$; hence $x^n - T_n$ is that polynomial of degree $n - 1$ that gives the minimax approximation to x^n . This explains the utility of **Chebyshev expansions**. (See Zdenek Kopal, *Numerical Analysis*, John Wiley & Sons, Inc., 1955; Cornelius Lanczos, *Applied Analysis*, Prentice-Hall Company, 1956; Cecil Hastings, Jr., Jeanne T. Hayward, and James P. Wong, Jr., *Approximations for Digital Computers*, Princeton University Press, 1955.)

MINIMAX METHOD OF ESTIMATION. Suppose we wish to estimate a **population parameter** θ from a sample, and that we can specify the amount of loss incurred if we adopt any value θ_i when the true value is θ_j . Denoting the sample values collectively by X , suppose also that we have some rule R that tells

us which value of θ to adopt for any particular sample X . We can then calculate the expected loss or risk as a function of θ . For any given rule R we can find the maximum value of the risk, and the minimax estimate is provided by the particular rule R_0 that minimizes this maximum risk. (See also **loss function; games theory**.)

MINIMUM. A point x_0 for a function $y = f(x)$ where the value of y is less than at any other point in the neighborhood of $x = x_0$ (see **maximum**). The test for a minimum is: $dy/dx = 0$ or ∞ at $x = x_0$; $dy/dx < 0$ for $x < x_0$; $dy/dx > 0$ for $x > x_0$; $d^2y/dx^2 > 0$.

MINIMUM CRITICAL MASS PROBLEM. In the theory of neutron chain reactors, the problem of finding that distribution of fissionable material which makes a reactor of fixed size just critical with the smallest amount of fuel.

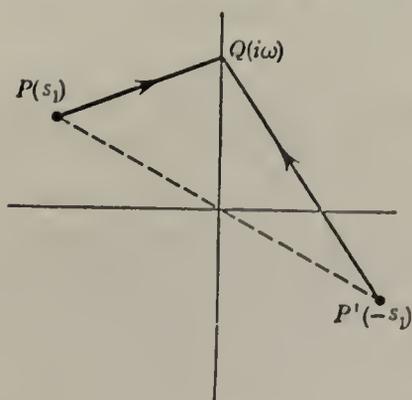
MINIMUM ENERGY PRINCIPLE. See **variation principle and bond energies**.

MINIMUM ENTROPY PRODUCTION. See **time variation of the entropy production**.

MINIMUM PHASE FUNCTIONS. A transfer function $F(s)$ is a minimum phase function if it is a stable function and if there exists no other stable function $G(s)$ such that $|G(i\omega)| \equiv |F(i\omega)|$ at all ω , but such that the total change in $\text{ph } G(i\omega)$ as ω goes from $-\infty$ to $+\infty$ is smaller than the total change in $\text{ph } F(i\omega)$ as ω goes from $-\infty$ to $+\infty$.

The minimum phase property may be interpreted in terms of the pole-zero distribution of the function $F(s)$.

Consider the linear factor $(s - s_1)$ where s_1 has a negative real part (see figure). If $s = i\omega$



the phasor $(s - s_1) = (i\omega - s_1)$ is representable in the figure by **PQ**. As ω varies from

$-\infty$ to $+\infty$ this phasor increases in phase by π . If however we consider the factor $(s + s_1)$, represented by $\mathbf{P}'\mathbf{Q}$ this factor will decrease in phase by π . Thus if $F(s)$ may be written in the form

$$F(s) = K \frac{(s - a_1)(s - a_2)\cdots}{(s - b_1)(s - b_2)\cdots}$$

the total increase in $\text{ph } F(i\omega)$ as ω goes from $-\infty$ to $+\infty$ will be

$$\pi(A_- - A_+ - B_- + B_+)$$

where A_- , A_+ are respectively the number of zeros of $F(s)$ with negative and positive real parts, B_- , B_+ the corresponding number of poles. However, if $F(s)$ is a stable transfer function B_+ must be zero and the increase of phase reduces to

$$\pi(A_- - A_+ - B) = -\pi(B - A + 2A_+)$$

where A , B are the total numbers of zeros and poles. Since for any practical transfer function $B \geq A$, the phase swing is clearly reduced by making A_+ vanish, i.e., $F(s)$ is a minimum phase function if all its poles and zeros are in the left half plane. For if $F(s)$ had a zero at $s = a$ in the right half plane (and a conjugate one at $s = a^*$ if a is complex) a function $G(s)$ could be constructed having zeros at $s = -a$ (and $-a^*$), the other factors being left unchanged; thus $|G(i\omega)|$ would be identical with $|F(i\omega)|$ (since if a is real $|i\omega - a| = |i\omega + a|$ whereas if a is complex $|i\omega - a||i\omega - a^*| = ||a|^2 - \omega^2 - 2i\omega Ra| = ||a|^2 - \omega^2 + 2i\omega Ra| = |i\omega + a||i\omega + a^*|$) but the phase swing for $G(i\omega)$ would be less than for $F(i\omega)$.

For a minimum phase function, the absence of zeros and poles of $F(s)$ in the right half plane, and thus the absence of singularities of $\log F(s)$ in this region, permits the phase of $F(i\omega)$ to be obtained as a single valued function of $\log |F(i\omega)|$.

MINIMUM POTENTIAL ENERGY THEOREM. See potential energy, theorem of minimum.

MINIMUM WEIGHT DESIGN. The economy of a design depends upon material consumption and cost of fabrication. Although a design of minimum weight is not necessarily the best it is always of interest. Following in the spirit of Michell, general theorems for minimum weight plastic design were formulated by Drucker and Shield. The theorems

follow from the plastic limit theorems and give upper and lower bounds on minimum weight. Earlier studies of elastic structures were made by Wasiubynski, and of plastic beams, frames and trusses by Heyman and by Prager.

MINKOWSKI ELECTRODYNAMICS FOR MOVING BODIES. A body is assumed at rest in a coordinate system Σ' (with the coordinates x' , y' , z' , t'). In it Maxwell's equations are

$$\frac{\partial \mathbf{B}'}{\partial t'} = -\text{curl}' \mathbf{E}'$$

$$\frac{\partial \mathbf{D}'}{\partial t'} + \mathbf{j}' = \text{curl}' \mathbf{H}'$$

$$\text{div}' \mathbf{D}' = \rho' \quad \text{div}' \mathbf{B}' = 0$$

with the auxiliary material equations (for isotropic bodies)

$$\mathbf{D}' = \epsilon \mathbf{E}'$$

$$\mathbf{B}' = \mu \mathbf{H}'$$

$$\mathbf{j}' = \sigma \mathbf{E}'$$

the system Σ' is assumed to move with the velocity \mathbf{v} with respect to the laboratory system Σ . The following relation exists between the field variables in both systems (\parallel means a component parallel to \mathbf{v} and \perp means a component perpendicular to \mathbf{v}):

$$\mathbf{D}'_{\parallel} = \left(\mathbf{D} + \frac{1}{c^2} (\mathbf{v} \times \mathbf{H}) \right)_{\parallel}$$

$$\mathbf{D}'_{\perp} = \left(\frac{\mathbf{D} + \frac{1}{c^2} (\mathbf{v} \times \mathbf{H})}{\sqrt{1 - \beta^2}} \right)_{\perp}$$

$$\mathbf{H}'_{\parallel} = (\mathbf{H} - [\mathbf{v} \times \mathbf{D}])_{\parallel};$$

$$\mathbf{H}'_{\perp} = \left(\frac{\mathbf{H} - [\mathbf{v} \times \mathbf{D}]}{\sqrt{1 - \beta^2}} \right)_{\perp}$$

$$\mathbf{E}'_{\parallel} = (\mathbf{E} + [\mathbf{v} \times \mathbf{B}])_{\parallel}$$

$$\mathbf{E}'_{\perp} = \left(\frac{\mathbf{E} + [\mathbf{v} \times \mathbf{B}]}{\sqrt{1 - \beta^2}} \right)_{\perp}$$

$$\mathbf{B}'_{\parallel} = \left(\mathbf{B} - \frac{1}{c^2} [\mathbf{v} \times \mathbf{E}] \right)_{\parallel}$$

$$\mathbf{B}'_{\perp} = \left(\frac{\mathbf{B} - \frac{1}{c^2} [\mathbf{v} \times \mathbf{E}]}{\sqrt{1 - \beta^2}} \right)_{\perp}$$

Furthermore

$$\begin{aligned} \mathbf{D}_{\parallel} &= \epsilon \mathbf{E}_{\parallel}; & \mathbf{B}_{\parallel} &= \mu \mathbf{H}_{\parallel} \\ \left\{ \begin{aligned} \mathbf{D}_{\perp} &= \epsilon(1 - \beta^2) \mathbf{E}_{\perp} \\ &+ \left(\epsilon\mu - \frac{1}{c^2} \right) (\mathbf{v} \times \mathbf{H}) \\ \mathbf{B}_{\perp} &= \mu(1 - \beta^2) \mathbf{H}_{\perp} \\ &+ \left(\frac{1}{c^2} - \epsilon\mu \right) (\mathbf{v} \times \mathbf{E}) \end{aligned} \right. \end{aligned}$$

$$\mathbf{j}'_{\parallel} = \left(\frac{\mathbf{j} - \rho \mathbf{v}}{1 - \beta^2} \right)_{\parallel}; \quad \mathbf{j}'_{\perp} = \mathbf{j}_{\perp};$$

$$\rho' = \frac{\rho - \frac{1}{c^2} (\mathbf{v} \times \mathbf{j})}{\sqrt{1 - \beta^2}}$$

Ohm's law:

$$(\mathbf{j} - \rho \mathbf{v})_{\parallel} = \sigma \sqrt{1 - \beta^2} (\mathbf{E} + (\mathbf{v} \times \mathbf{B}))_{\parallel}$$

$$(\mathbf{j} - \rho \mathbf{v})_{\perp} = \frac{\sigma}{1 - \rho^2} (\mathbf{E} + (\mathbf{v} \times \mathbf{B}))_{\perp}$$

$\rho \mathbf{v}$ is called the convection current and $\mathbf{j} - \rho \mathbf{v}$ is called the conduction current.

MINKOWSKI INEQUALITY. A generalization of the **Schwarz inequality**, easily deduced from **Hölder's inequality**, thus, for real numbers,

$$\begin{aligned} \{ |a_1 + b_1|^p + \dots + |a_n + b_n|^p \}^{1/p} \\ \leq \{ |a_1|^p + \dots + |a_n|^p \}^{1/p} \\ + \{ |b_1|^p + \dots + |b_n|^p \}^{1/p}, \end{aligned}$$

and for real functions

$$\begin{aligned} \left\{ \int_a^b |f + g|^p dx \right\}^{1/p} \\ \leq \left\{ \int_a^b |f|^p dx \right\}^{1/p} + \left\{ \int_a^b |g|^p dx \right\}^{1/p}. \end{aligned}$$

MINKOWSKI WORLD. The metric of this world is given by the four-dimensional distance formula $s^2 = x^2 + y^2 + z^2 + l^2$ where $l = ict$. The radius vector has the components $(x; y; z; l)$. Two vectors $(x'; y'; z'; l')$ and $(x''; y''; z''; l'')$ are perpendicular to each other if

$$x'x'' + y'y'' + z'z'' + l'l'' = 0.$$

In this quasi-Euclidean space, it is unnecessary to distinguish between covariant and contra-

variant tensors, or between tensors or tensor densities. The most important geometric objects of this space are vectors of the first kind P_i (of which the radius vector is a typical example), six-vectors (antisymmetric tensors of rank 2, also called surface tensors) $f_{ik} = -f_{ki}, f_{ii} = 0$, symmetric tensors of the second rank $T_{ik} = T_{ki}$ and three-vectors (also called volume tensors) A_{ikm} ; these are antisymmetric tensors of rank 3. In all these cases $i, k, m = x, y, z, l$.

Dual Tensors. The dual of a given tensor will be denoted by an asterisk.*

$$f_{yz}^* = f_{xl}; f_{zx}^* = f_{yl}; f_{xy}^* = f_{zl}; f_{xl}^* = f_{yz}.$$

In general

$$f_{ik}^* = f_{(ik)}$$

where (ik) denote the missing indices of ik and $ik(ik)$ form an even permutation of the array $(xyzl)$.

The two invariants of a six-vector are

$$|f|^2 \equiv f_{yz}^2 + f_{zx}^2 + f_{xy}^2 + f_{xl}^2 + f_{yl}^2 + f_{zl}^2$$

and

$$\begin{aligned} ff^* \equiv f_{yz}f_{yz}^* + f_{zx}f_{zx}^* + f_{xy}f_{xy}^* \\ + f_{xl}f_{xl}^* + f_{yl}f_{yl}^* + f_{zl}f_{zl}^*. \end{aligned}$$

The components (or projections) of a six-vector f_{ik} on a three dimensional subspace are defined in the following way:

on the sub-space (x, y, z) as (f_{yz}, f_{zx}, f_{xy}) .

or

on the sub-space (y, z, l) as (f_{zl}, f_{ly}, f_{yz}) , etc.

The components of the four-vector f_j , which is perpendicular to the j -axis, are defined by

$$f_j = (f_{jx}; f_{jy}; f_{jz}; f_{jl}).$$

Similarly for

$$f_{ik}^*:$$

$$f_j^* = (f_{jx}^*; f_{jy}^*; f_{jz}^*; f_{jl}^*).$$

The only non-vanishing components of a three-vector are

$$A_{yzl}, A_{zlx}, A_{lxy}, A_{xyz}.$$

Their duals are the component of an ordinary four-vector **B**, i.e.,

$$A_{yzl}^* = -B_x$$

$$A_{zlx}^* = B_y$$

$$A_{lxy}^* = -B_z$$

$$A_{xyz}^* = B_l$$

(the four indices $klmn$ in $A_{klm}^* = B_n$ must form an even permutation of the array $xyzl$).

The product of a 4-vector P and a six-vector f is defined as new 4-vector A , such that

$$A_j = P_x f_{jx} + P_y f_{jy} + P_z f_{jz} + P_l f_{jl}.$$

The product of a six-vector f with itself is defined as a symmetric tensor T of second rank:

$$\begin{aligned} T_{jh} &\equiv \mathbf{f}_j \cdot \mathbf{f}_h \\ &= f_{jx} f_{hx} + f_{jy} f_{hy} + f_{jz} f_{hz} + f_{jl} f_{hl}. \end{aligned}$$

Similarly

$$\begin{aligned} T_{jh}^* &= \mathbf{f}_j^* \cdot \mathbf{f}_h^* \\ &= f_{jx}^* f_{hx}^* + f_{jy}^* f_{hy}^* + f_{jz}^* f_{hz}^* + f_{jl}^* f_{hl}^*. \end{aligned}$$

4-Dimensional Vector Analysis. The ordinary nabla operator $\nabla = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right)$ is generalized to the Lorentz operator \diamond , where

$$\diamond = \left(\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}, \frac{\partial}{\partial t} \right).$$

The 3-dimensional Laplace operator ∇^2 is generalized to the D'Alembert operator \square , where

$$\square = \diamond^2 = \left(\frac{\partial^2}{\partial x^2}, \frac{\partial^2}{\partial y^2}, \frac{\partial^2}{\partial z^2}, \frac{\partial^2}{\partial t^2} \right).$$

Four possible operations exist: (a) the scalar divergence of a 4-vector, written Div . (b) the vector divergence of a 6-vector, written $\mathcal{D}iv$. (c) the rotation of a 4 vector, written Curl . (d) the gradient of a scalar, written Grad . In what follows, (one-dimensional) line elements will be denoted by lower-case English letters, e.g., ds, dl ; two-dimensional surface-elements by the use of lower case Greek letters, e.g., $d\sigma$; elements of three-dimensional subspace by capital English letters, e.g., dS ; and elements of the four-dimensional world by the use of capital Greek letters, e.g., $d\Sigma$.

(1) The scalar divergence of a 4-vector P is defined as

$$\text{Div } P = \frac{\partial P_x}{\partial x} + \frac{\partial P_y}{\partial y} + \frac{\partial P_z}{\partial z} + \frac{\partial P_l}{\partial l}.$$

(2) The vector divergence of a six vector f is a 4-vector, whose j -component is

$$\begin{aligned} (\mathcal{D}iv f)_j &= \frac{\partial f_{jx}}{\partial x} + \frac{\partial f_{jy}}{\partial y} + \frac{\partial f_{jz}}{\partial z} \\ &\quad + \frac{\partial f_{jl}}{\partial l} \quad (j = x, y, z, l). \end{aligned}$$

(3) We will assume that the letters s', s'', n, s shall denote 4 directions mutually perpendicular and that they also form an even permutation of $xyzl$. Furthermore, we shall assume that the 4-vector B is the dual of the three-vector A . We then define the eurl in the following way:

$$\begin{aligned} (\text{Curl } B)_{ns} &= (\text{Curl } B)_{s's''}^* = (\text{Curl } A)_{s's''} \\ &= (\text{Curl } A)_{ns}^* = \frac{\partial B_s}{\partial n} - \frac{\partial B_n}{\partial s}. \end{aligned}$$

(4) The gradient of a scalar V is the four-vector

$$\text{Grad } V = \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}, \frac{\partial V}{\partial l} \right).$$

The dual of the vector divergence $(\mathcal{D}iv f)^*$ is defined to be a 4-vector whose s component is:

$$(\mathcal{D}iv f)_s^* = \frac{\partial f_{sx}^*}{\partial x} + \frac{\partial f_{sy}^*}{\partial y} + \frac{\partial f_{sz}^*}{\partial z} + \frac{\partial f_{sl}^*}{\partial l}$$

e.g.,

$$\begin{aligned} (\mathcal{D}iv f)_x^* &= \frac{\partial f_{xy}^*}{\partial y} + \frac{\partial f_{xz}^*}{\partial z} + \frac{\partial f_{xl}^*}{\partial l} \\ &= \frac{\partial f_{zl}}{\partial y} + \frac{\partial f_{ly}}{\partial z} + \frac{\partial f_{yz}}{\partial l}. \end{aligned}$$

Differential Identities. (a) If B is a 4-vector, then

$$\mathcal{D}iv [(\text{Curl } B)^*] \equiv 0.$$

(b) If f is a 6-vector, then

$$\text{Div } [\mathcal{D}iv f] \equiv 0.$$

Integral Theorems. (I) *Gauss-Theorem.* Let P be a 4-vector, Σ a four-dimensional domain, S its three-dimensional boundary with the outer normal n , then

$$\oint\!\!\!\oint_S P_n dS = \iiint\!\!\!\int_\Sigma \text{Div } P d\Sigma.$$

(II) *The Two Gauss-Stokes Theorems.* Let f be a 6-vector, S a 3-dimensional volume imbedded in the 4-dimensional world. Let s be normal to S and σ the closed 2-dimensional boundary of S . Let s', s'' be two directions, perpendicular to each other and tangential to σ and s (perpendicular to S) and dn (within S , but perpendicular to σ) be two directions perpendicular to each other, then

$$(a) \quad \oint\!\!\!\oint_\sigma f_{sn} d\sigma = \iiint\!\!\!\int_S (\mathcal{D}iv f)_s dS$$

$$(b) \quad \oint\!\!\!\oint_{\sigma} f_{s's''} d\sigma = \iiint_S (\text{Div } f^*)_{s'} dS.$$

(III) *Stokes Theorem.* Let B be a 4-vector, s a closed "world" curve, σ a 2-dimensional surface bounded by s , and s' , s'' two directions, perpendicular to each other, but tangential to $d\sigma$, then

$$\oint_s B_s ds = \iint_{\sigma} (\text{Curl } B)_{s's''} d\sigma.$$

If the surface σ is closed, then s vanishes and

$$\oint\!\!\!\oint_{\sigma} (\text{Curl } B)_{s's''} d\sigma = 0.$$

(IV) *Green's Formula.* If U and V are two sufficiently differentiable scalar functions, then

$$\begin{aligned} \oint\!\!\!\oint_S \left(U \frac{\partial V}{\partial n} - V \frac{\partial U}{\partial n} \right) dS \\ = \iiint_{\Sigma} (U \square V - V \square U) d\Sigma. \end{aligned}$$

n is the outer normal of S . If

$$V = \frac{1}{R^2} = \frac{1}{(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2 + (l - l_o)^2}$$

where (x_o, y_o, z_o, l_o) is an interior point of Σ , then

$$(a) \quad 4\pi^2 U_o = - \iiint_{\Sigma} \frac{\square U}{R^2} d\Sigma - \oint\!\!\!\oint_S \left[U \frac{\partial}{\partial n} \left(\frac{1}{R^2} \right) - \left(\frac{1}{R^2} \right) \frac{\partial U}{\partial n} \right] dS$$

where $U_o = U(x_o, y_o, z_o, l_o)$.

If $V \equiv 1$, then Green's formula yields the identity

$$\iiint_{\Sigma} \square U d\Sigma = \oint\!\!\!\oint_S \frac{\partial U}{\partial n} dS.$$

If Σ is an infinitely large sphere, then from (a) we obtain

$$4\pi^2 U_o = - \iiint_{\Sigma} \frac{\square U}{R^2} d\Sigma + U_m$$

where U_m is the average value of U over the infinite 3-dimensional closed boundary S of Σ .

(For further details, see A. Sommerfeld, *Annalen der Physik*, 32, 749, 1910; 33, 649, 1910.)

MINOR (OF A MATRIX). If a matrix B is formed from a matrix A by striking out certain rows and columns, then B is called a minor of A . The matrix C formed by the deleted rows and columns is called the complementary minor of B in A . If A , B , C are all square, then $(-1)^k C$ is called the algebraic complement of B , where k is the sum of the indices of the rows and columns of C .

MINUTE. (1) Unit of time, abbreviation m or min. One 1440th part of a mean solar day; 60 seconds. (2) Unit of angle, abbreviation '. One 60th part of a degree. (Cf. **time and time-keeping.**)

MIRROR NUCLEI. See **conjugate nuclei.**

MISES MATERIAL. Synonymous with **Saint Venant-Mises material.**

MISES YIELD CONDITION. This principle states that plastic flow occurs when the second invariant of the stress deviator reaches the square of the yield stress in shear:

$$J_2 = \frac{1}{2} s_{ij} s_{ji} = k^2.$$

This is equivalent to a prescribed limit on the elastic strain energy associated with shear deformation, the so-called *distortional strain energy*, or a prescribed limit on the *octahedral shear stress*, the shear stress on a plane equally inclined to all three principal directions.

MIXED CHARACTERISTIC FUNCTION.

If $P(x, y, z)$ is a point in object space, on a ray, and if F' is the foot of a perpendicular through the origin onto the ray in image space, then the mixed characteristic function W is the optical path length along the ray from P to F' . If (p', q', r') are the optical direction cosines of the ray in image space, then

$$W = W(x, y, z, p', q', r')$$

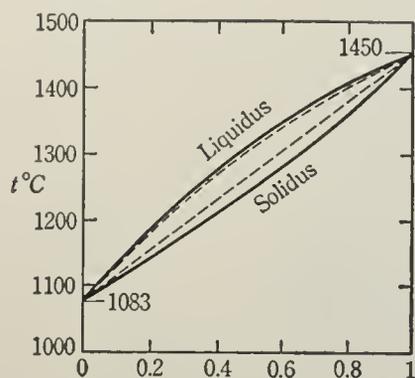
and W is simply related to the point characteristic function (see **Hamilton's characteristic**).

$$W = V(x, y, z, x', y', z') - (p'x' + q'y' + r'z').$$

Another characteristic function, also called a mixed characteristic function, is obtained by interchanging the object and image spaces in the above definition, and is denoted by

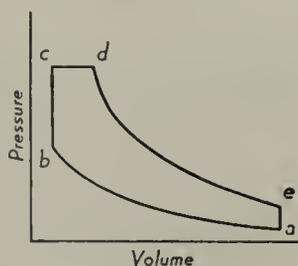
$$W' = W'(p, q, r, x', y', z').$$

MIXED CRYSTALS. The most common type of freezing point diagram is that exhibiting a eutectic (see **crystallization curve**). Another type is that corresponding to a binary system which forms a continuous series of mixed crystals. The diagram is completely similar to that of the condensation of a binary mixture, the boiling curves and the condensation curves being replaced by the liquidus and solidus curves (see figure).



Freezing point diagram for a binary system forming a continuous system of mixed crystals.

MIXED CYCLE. High-speed internal combustion engines perform their thermodynamic cycles of operation so rapidly that, whether originally intended to operate on the Diesel or the Otto principle, the actual cycle has a combustion that exhibits both constant volume and constant pressure phases. Spark ignition engines must receive the timed spark in advance of piston dead center position. Compression ignition engines must have the injector timed to begin the fuel spray also ahead of dead center position. In either case, on account of high rotative engine speed the combustion is only partially completed before the piston begins the expansion stroke. The remaining fuel is burned approximately at constant pressure. A conventionalized cycle is shown in the fig-



p, V diagram of mixed cycle.

ure. Combustion extends from b to d . The air standard efficiency expression is:

$$E = 1 - \frac{1}{r^{\gamma-1}} \left[\frac{ZR^{\gamma} - 1}{(Z - 1) + Z\gamma(R - 1)} \right]$$

where r is the ratio of compression along ab ; γ is the specific heat ratio—constant pressure to constant volume; R is the cutoff ratio V_d/V_c ; Z is the combustion pressure ratio P_c/P_b .

All factors which influence either Otto or Diesel cycle efficiency appear here, and in addition a factor introducing characteristics of the constant volume phase of combustion. (See **Diesel engine**.)

MIXED DISTRIBUTION. (Also called compound distribution.) A frequency distribution which is composed of a mixture of several unlike populations of data, or populations with different parameters.

MIXED REFLECTION. The simultaneous occurrence of regular reflection and of diffuse reflection.

MIXED STRESS TENSOR. See **stress tensor**.

MIXED TENSOR (TENSOR FIELD). See **tensor field**.

MIXED TRANSMISSION. The simultaneous occurrence of direct transmission and diffuse transmission.

MIXING LENGTH. (Ger. *Mischungsweg*.) The distance which a parcel of a fluid in turbulent motion is supposed, in the mixing length theory, to travel through the surroundings before becoming mixed into them, (analogous to the mean free path of molecules). The parcel possesses on the average the mean properties of its region of origin so that there is a correlation between its properties (e.g., temperature, concentration, momentum, etc.) and its direction of motion in a region where there is a gradient of the property in question.

If applied to the transfer of x -momentum in the z -direction by turbulence when the mean motion is \bar{u} in the x -direction and varies only in the z -direction, it is found that the shearing stress, or **Reynolds stress** $-\rho\overline{u'w'}$, is

$$\tau_{xz} = -\rho l^2 \left| \frac{\partial \bar{u}}{\partial z} \right| \left| \frac{\partial \bar{u}}{\partial z} \right|$$

or more generally that the transfer across unit area of a quantity Q in the z -direction is equal to

$$-l^2 \left| \frac{\partial \bar{u}}{\partial z} \right| \left| \frac{\partial \bar{Q}}{\partial z} \right|,$$

for the fluctuations u' , w' of the velocity are both assumed to be of magnitude $l \left| \frac{\partial \bar{u}}{\partial z} \right|$ in steady state (locally isotropic) turbulence, and the fluctuation Q' of Q is $l \frac{\partial \bar{Q}}{\partial z}$.

Alternatively if the level of turbulent velocity fluctuations is represented by $(\overline{w'^2})^{1/2}$ and the scale, or mixing length, by l' , the eddy transfer coefficient, which is presumed to depend only on these two quantities, must necessarily be of the form

$$\begin{aligned} \kappa M &= l' (\overline{w'^2})^{1/2}, \\ &= l'^2 \left| \frac{\partial \bar{u}}{\partial z} \right| \end{aligned}$$

according to the previous definition. The mixing length is thus seen to be a characteristic length of the turbulent motion which occurs in any dimensional analysis of turbulence, and its precise value depends upon its definition and it does not express an obvious physical feature of the turbulence.

If the mixing length is assumed either to be proportional to distance from the boundary (Prandtl), or to be determined by the local properties of the turbulence (Karman) and equal to $k \frac{\partial \bar{u}}{\partial z} / \frac{\partial^2 \bar{u}}{\partial z^2}$, where k is **Karman's constant**, the **velocity profile** is logarithmic, either assumption being equivalent to supposing that the motion is similar at all distances from the boundary.

MIXTURES. See **average potential model for mixtures**.

MKS UNITS. See **electromagnetic units**.

MOBILE BOND ORDER. Whereas in the valence bond approximation, the definition of a double bond character is immediate (it is the total weight in percent of the structures in which the particular bond appears as double), the definition of the *bond order* in the MO method is slightly more difficult: each electron has an orbital which extends over many (n) nuclei. The contribution of the electron l to a particular bond 1-2 is the coefficient of ψ_{1l} , ψ_{2l} in the development of ψ_l^2 ,

where the wave function ψ_l of electron l is given by:

$$\psi_l = c_{1l}\psi_{1l} + c_{2l}\psi_{2l} + \cdots + c_{nl}\psi_{nl}.$$

If there are π electrons in the molecule, Coulson defines *the mobile or double bond order* of the 1-2 bond as the sum of $c_{1l}c_{2l}$ over all the possible orbitals of the m electrons.

The contribution of one of the orbitals of an electron can be negative, if c_1 and c_2 have opposite signs (unfavorable overlap). In the case of benzene, one obtains equal mobile bond orders, as should be expected, but in this definition, the sum of the *mobile bond orders* is larger than three. (Mobile bond order is 0.667 for each bond.)

MOBILE MONOLAYERS. (See also **localized monolayers**.) In the model of a mobile monolayer some point in the adsorbed molecule (for example its center of mass) is bound tightly to the surface in the normal direction, with complete freedom of the movement in the two directions in the surface. The treatment of mobile monolayers is very similar to that of usual three-dimensional bulk phases. For example, a mobile monolayer of a simple substance may be gaseous, liquid or solid, according to the degree of interaction between the adsorbed molecules. (See also **spreading pressure**.)

MOBILE OPERATOR. See **material differentiation**.

MOBILITY. (1) Random motion of various particles, such as sub-atomic particles, atoms, ions, molecules and colloidal particles. (2) Directed motion of charged particles subject to the action of forces and fields of force. Hence, the term mobility applies to all processes of electrical conduction, whether by ions, by electrons, by "holes," etc. The mobility, μ , is given by the expression

$$\mu = \frac{\sigma}{ne} = \frac{\langle v \rangle_{av}}{E},$$

where σ is the conductivity, e is the charge of the carriers, n is their number-density and $\langle v \rangle_{av}$ is the average drift velocity, while E is the **electric field** producing the drift. The mobility is expressed in cm/see per volt/cm, or in similar units, and it gives the drift velocity of the carriers under the influence of a unit electric field. In **semiconductors** the mobility

may be directly determined from the Hall effect (see **galvanometric and thermometric effects**), and is related to the **mean free time** τ , between collisions by the formula

$$\mu = e\tau/m$$

and to the Hall coefficient, R_H , by the formula

$$\mu = R_H\sigma$$

where σ is the **conductivity**.

MOBILITY CAPACITANCE (MASS). In the mechanical rectilinear mobility system the mass (mobility capacitance) m_I , in grams, is analogous to electrical capacitance C_E .

The mechanical rectilinear excitability x_I of a mass (mobility capacitance), in mechanical mhos, is defined as

$$x_I = \frac{1}{\omega m_I} \quad (1)$$

where ω is $2\pi f$, and f is the frequency, in cycles per second.

Equation 1 shows that the mass (mobility capacitance) m_I in the mechanical rectilinear mobility system is analogous to electrical capacitance C_E in the **electrical system**.

Mass (mobility capacitance) m_I in the mechanical rectilinear mobility system may also be defined as follows:

$$f_M = m_I \frac{dv}{dt} \quad (2)$$

$$v = \frac{1}{m_I} \int f_M dt. \quad (3)$$

In the electrical system electrical capacitance C_E may be defined as follows:

$$i = C_E \frac{de}{dt} \quad (4)$$

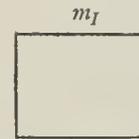
where i is the electrical current, in abamperes, C_E is electrical capacitance, in abfarads, e is electromotive force, in abvolts, and t is time, in seconds.

$$e = \frac{1}{C_E} \int idt \quad (5)$$

where i is the current, in abamperes.

It will be seen that Equations 2 and 3 in the mechanical rectilinear mobility system are analogous to Equations 4 and 5 in the electrical system.

Mobility capacitance (mass) in the mechanical rectilinear mobility system is represented by a mass in the figure.



Graphical representation of the element, mobility capacitance, m_I .

MOBILITY-DIFFUSION COEFFICIENT RELATION. See Einstein relationship between mobility and diffusion coefficient.

MOBILITY, HALL (OF AN ELECTRICAL CONDUCTOR). The quantity μ in the relation

$$\mu = R_H\sigma$$

where μ is the Hall mobility, R_H is the Hall coefficient, and σ is the conductivity. (See **mobility; Hall effect**.)

MOBILITY INERTIA (COMPLIANCE). In the mechanical rectilinear mobility system the compliance (mobility inertia) C_I , in centimeters per dyne, is analogous to electrical inductance L .

The mechanical rectilinear excitability x_I of a compliance (mobility inertia), in mechanical mhos, is defined as

$$x_I = \omega C_I \quad (1)$$

where ω is the $2\pi f$, and f is the frequency, in cycles per second.

Equation 1 shows that compliance (mobility inertia) C_I , in centimeters per dyne, is analogous to inductance.

Compliance (mobility inertia) C_I in the mechanical rectilinear mobility system may also be defined as

$$v = C_I \frac{df_M}{dt} \quad (2)$$

In the electrical system inductance may be defined as

$$e = L \frac{di}{dt} \quad (3)$$

where L is the inductance, in abhenries.

It will be seen that Equation 2 in the mechanical rectilinear mobility system is analogous to Equation 3 in the electrical system.

Mobility inertia (rectilinear compliance) in the mechanical rectilinear mobility system is represented by a spring in the figure.



Graphical representation of the element, mobility inertia, C_I .

MOBILITY OF IONS IN CHAMBER. The mobility of ions in a gas is defined by the relationship:

$$v = \frac{kE}{p}$$

where v is the drift velocity, k is the mobility at unit pressure, E the electric field, and p the pressure.

MOBILITY OF IONS IN SOLIDS. Conduction of electricity in ionic crystals is due to the motion of lattice defects, either of the Schottky or Frenkel type. The mobility is given by

$$\mu = (eD_0/kT)e^{-E/kT}$$

where D_0 is a numerical constant, and E is an activation energy, which depends on the energy required to make a defect and on the height of the energy barrier that must be surmounted in order that the defect may move.

MOBILITY RESISTANCE. See **responsivity**.

MODAL COLUMN OF A MATRIX. A term sometimes applied to an eigenvector. (See **eigenvalues and eigenvectors**.)

MODE. (1) The modes of a population are the values of the variate for which the relative frequency attains a local maximum. In most populations there is only one such maximum; such populations are said to be unimodal. (2) A mode of propagation (transmission) is a form of propagation of guided waves that is characterized by a particular field pattern in a plane transverse to the direction of propagation, which field pattern is independent of position along the axis of the waveguide. In the case of uniconductor waveguides, the field pattern of a particular mode of propagation is also independent of frequency. (3) A mode of resonance is a form of natural electromagnetic oscillation in a resonator, characterized by a

particular field pattern that is invariant with time.

MODE, DOMINANT. See **dominant mode**.

MODE, FUNDAMENTAL. In neutron transport theory, the solution to the equation

$$(\nabla^2 + B^2)\Phi = 0$$

that is everywhere positive and vanishes at the extrapolated boundary of the system. (See **first fundamental theorem**.)

MODERATION THEOREM. See **Chatelier, le-Braun principle**.

MODERATOR TRANSMISSION FACTOR. See **Dancoff corrections**.

MODIFIED REVERSED RANKINE CYCLE. See **reversed Rankine cycle**.

MODULATION. The process or result of the process whereby some characteristic of one wave is varied in accordance with another wave.

MODULUS. See **complex number**; **logarithm**; **elliptic function**.

MODULUS, BULK. A modulus based on hydrostatic stress and dilatational strain.

MODULUS, COMPLEX. Steady-state oscillatory loading of a linear viscoelastic material produces steady-state strain response which in general will be out of phase with the stress. Using complex notation with the stress given by: $\sigma_0 e^{i\omega t}$, the ratio of stress to strain is a complex function of frequency called the complex modulus:

$$m^*(\omega) = m_1(\omega) + im_2(\omega).$$

MODULUS, EQUILIBRIUM. The limiting value of the modulus of a viscoelastic material for slow loading or zero frequency.

MODULUS, GLASS. The limiting value of the modulus of a viscoelastic material for instantaneous loading or infinite frequency.

MODULUS, LOSS. The imaginary part of the complex modulus, $m_2(\omega)$.

MODULUS OF COMPRESSION. See **compression modulus**.

MODULUS OF CONTINUITY. If the function $f(x)$ is continuous in the closed interval $[a, b]$, then, for every $\delta > 0$, the maximum of

the difference $|f(x) - f(y)|$ for $|(x - y)| < \delta$ is a monotone-decreasing function of δ , call it $w(\delta)$. This function $w(\delta)$ is called the modulus of continuity of the function $f(x)$, and is important, e.g., in theorems of goodness of approximation to functions by simpler functions such as polynomials.

MODULUS OF ELASTICITY. The ratio of the unit stress to the unit deformation of a structural elastic material is a constant, as long as the unit stress is below the **proportional limit**, and is called the modulus of elasticity. The shearing modulus of elasticity is frequently called the *modulus of rigidity*.

MODULUS OF RIGIDITY. See **elastic constants; modulus of elasticity**.

MODULUS OF RUPTURE. The modulus of rupture in bending of a material is found by testing a transversely loaded beam of constant cross section to failure and substituting the maximum **bending moment**, **moment of inertia** of the cross section and the distance from the neutral axis to the extreme fiber in the flexure formula:

$$\text{Modulus of rupture} = \frac{Mc}{I}$$

The torsional modulus of rupture is obtained by testing a shaft of constant, circular cross-section to failure and then substituting the maximum **torque**, polar moment of inertia of the cross section and the radius in the torsion formula:

$$\text{Torsional modulus of rupture} = \frac{Tc}{J}$$

(See **torsional stress**.)

The bending or torsional modulus of rupture may be used to predict the maximum bending or torsional moment which a member can resist.

It should be kept in mind, however, that the actual maximum stress existing in the beam or shaft is less than the modulus of rupture because the **proportional limit** is exceeded prior to fracture.

MODULUS, REDUCED OR DOUBLE. The curvature in bending of an elastic beam is given by M/EI . When the **elastic limit** is exceeded, the curvature can be taken as $M/E_r I$ where E_r is a reduced or double modulus. E_r

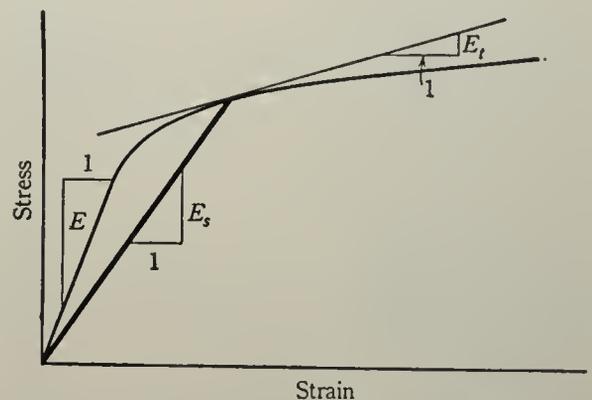
will depend upon both the elastic modulus and the tangent modulus. When column buckling in the plastic range is treated (incorrectly) as an eigenvalue problem, the reduced modulus controls the computed buckling load.

MODULUS, RELAXATION. Subjection of a specimen to a **step function** of strain is known as a relaxation test. The resulting ratio of stress to strain, $m(t)$ is called the relaxation modulus, and is a non-increasing function of time after application of the strain. The superposition principle enables the stress response, $\sigma(t)$ to a general strain history $\epsilon(t)$ to be expressed in terms of $m(t)$:

$$\sigma(t) = \int_{-\infty}^t (t - \tau) \frac{\partial \epsilon(\tau)}{\partial \tau} d\tau.$$

This represents the constitutive equation of linear viscoelasticity in an integral form.

MODULUS, SECANT. The ratio of stress to strain in simple tension, compression or shear is called the secant modulus, E_s . It lies between the modulus of elasticity, E and the tangent modulus, E_t .



MODULUS, SHEAR. A modulus based on shear stress and strain components.

MODULUS, STORAGE. The real part of the **complex modulus**, $m_1(\omega)$.

MODULUS, TANGENT. The slope of the stress-strain curve for increasing stress is called the tangent modulus, E_t (see also **modulus, secant**). As a true local physical property, the tangent modulus is significant in plastic buckling. Replacing **Young's modulus** in the **Euler buckling load** by the tangent modulus gives useful results for columns, as shown by Shanley.

MOHR CIRCLE FOR INERTIA. The moment and product of inertia of an area transform as a second order tensor (see **Mohr circle for stress**). A Mohr circle can be drawn therefore with $I_x = \int_A y^2 dA$ replacing σ_x , $I_y = \int_A x^2 dA$ replacing σ_y and $I_{xy} = \int_A xy dA$ replacing τ_{xy} with substitution of moment of inertia for the normal stress coordinate σ and product of inertia for τ .

MOHR CIRCLE FOR STRAIN. A Mohr circle can be drawn for the components of strain in a plane if the tensor components, that is, the internal strain components ϵ_x, ϵ_y , and the shear strain components $\epsilon_{xy} = \gamma_{xy}/2$, are employed. In the circle for stress, and of course in the corresponding equations, ϵ replaces σ and $\gamma/2$ replaces τ . (See **Mohr circle for stress**.)

MOHR CIRCLE FOR STRESS. The transformations of a second order symmetric tensor in two dimensions can be represented by a circle in a plot of the diagonal terms against the off diagonal term. The situation for plane stress components is shown in the accompany-

$$\sigma_n = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \cos 2\phi - \tau_{xy} \sin 2\phi$$

$$\tau = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2} \sin 2\phi + \tau_{xy} \cos 2\phi$$

the magnitude of the principal stresses

$$\sigma_1, \sigma_2 = \frac{\sigma_x + \sigma_y}{2} \pm \left[\left(\frac{\sigma_x - \sigma_y}{2} \right)^2 + \tau_{xy}^2 \right]^{1/2}$$

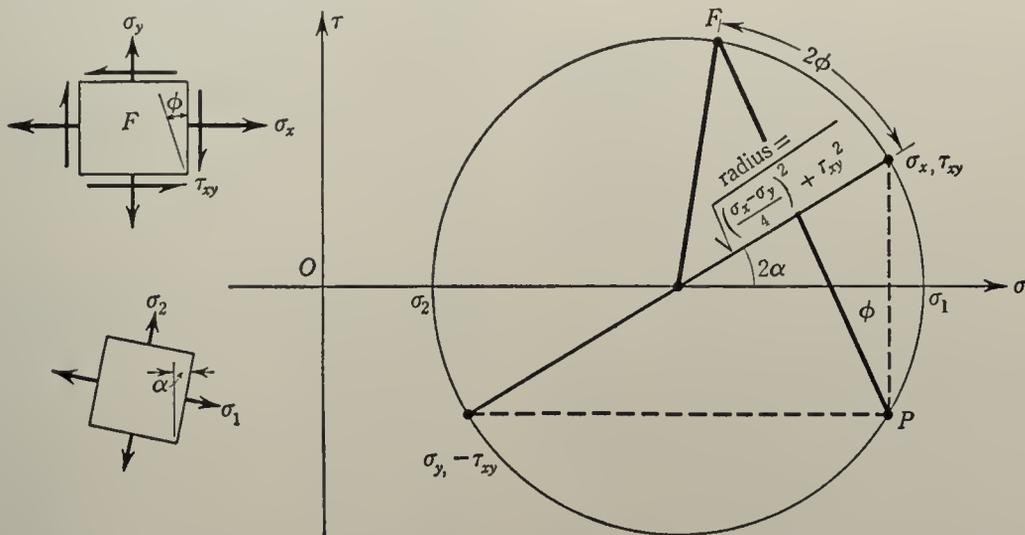
and their orientation

$$\tan 2\alpha = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

Point P is called the pole of the circle. A line drawn from P to any point such as F is parallel to the trace of plane F in the plane of the paper.

Although the circle constructions are most often associated with the name of Mohr they are in fact due to Culmann. Mohr generalized Culmann's results to three dimensions.

MOHR YIELD CONDITION. This law was devised both for plastic flow and fracture, and stipulates a limiting shear stress as a function of the normal stress on the plane of failure.



ing figure where normal stress σ on any plane perpendicular to the paper is plotted against shearing stress τ on the same plane. Tensile normal stress is positive, as is shear, which is directed in a clockwise sense around the block on which it acts. Angular rotations on the circle are twice those in the actual plane. The special convention for positive shear makes the direction of rotation the same. The transformation equations (coordinates of F)

MOISTURE-CONTINUITY EQUATION.

The storage equation as applied to the atmosphere. The general form of the equation is written

$$\frac{dS}{dt} = I + E - O - P,$$

where I is the atmospheric moisture inflow, E the evapotranspiration from the ground, O the atmospheric moisture outflow, P precipitation,

and $\frac{dS}{dt}$ the time rate of change of moisture storage in the portion of the atmosphere under consideration. In practice the equation is more commonly applied to a finite interval of time and the various terms become mean values in this interval.

MOLALITY. The molality of a solute in a mixture is defined as the number of moles of the solute dissolved in 1,000 g of solvent. This unit is frequently used for electrolytic solutions.

MOLARITY. The molarity of a solute in a mixture is defined as the number of moles of solute contained in 1 liter of solution.

MOLAR PROPERTIES (HEAT CAPACITY, VOLUME, POLARIZATION, . . . ETC.).

The heat capacity, volume, polarization, . . . etc., of one mole (gram-molecular weight) of a substance. Since a mole contains always the same number of molecules, whatever the substance, molar quantities usually occur in laws connecting atomic and macroscopic phenomena in rather simpler ways than do quantities expressed "per unit mass," or "per unit volume."

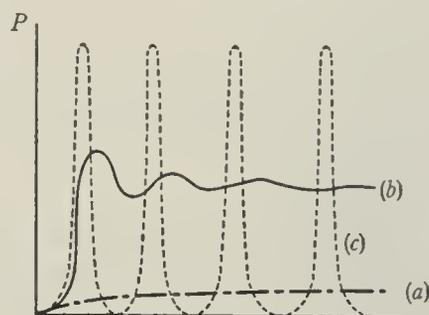
MOLE (OR MOL). That quantity of substance whose mass (in any units) is numerically equal to the molecular weight of the substance. When the mass is in grams, the unit is called the gram-mole, when the mass is in kilograms, the unit is the kilomole (kmol), and when the mass is pounds, the unit is called the pound-mole (lb-mole).

MOLECULAR CHAOS. For dilute gases, the hypothesis is usually made that there is *no correlation* between the velocities and positions of the various molecules; i.e., it is assumed that at any time t , the probability $f(\mathbf{r}_1 \cdots \mathbf{r}_N, \mathbf{v}_1 \cdots \mathbf{v}_N, t)$ that the N molecules of a system occupy the positions \mathbf{r}_1 to \mathbf{r}_N with velocities $\mathbf{v}_1 \cdots \mathbf{v}_N$ can be factorized into a product of independent probabilities $f(\mathbf{r}_1, \mathbf{v}_1, t) \cdots f(\mathbf{r}_N, \mathbf{v}_N, t)$.

$f_i(\mathbf{r}_i, \mathbf{v}_i, t)$ is the probability of finding a molecule i in a volume element $d\mathbf{r}_i d\mathbf{v}_i$ around the point $\mathbf{r}_i, \mathbf{v}_i$ of phase space, when this molecule is alone in the system.

This hypothesis of molecular chaos is not valid in dense gases, in liquids, and *a fortiori* in solids, where correlations are quite impor-

tant: the probability P of finding a second molecule at a distance \mathbf{r} from a given molecule varies markedly with distance, and a complete factorization of the distribution function is not possible.



Probability P of finding the center of a molecule B in unit volume as a function of the distance r from the center of a molecule A, in the case of a dilute gas, curve (a), a liquid, curve (b), or a solid, curve (c).

MOLECULAR DIAGRAMS. The molecular diagram technique has been introduced by Daudel to represent in a compact and schematic way the results of wave-mechanical calculations on molecules. The carbon skeleton of the molecule is drawn. The **bond characters** are indicated along the bonds. The **free valence index**, and eventually the ionic character of the atoms at the summits are shown. The reactivity of the molecule can then be visualized rapidly.

MOLECULAR EIGENFUNCTIONS. The total eigenfunction ψ of a molecular energy level can be expressed, to a first approximation, as

$$\psi = \psi_e \psi_v \psi_r$$

where ψ_e is the electronic eigenfunction, ψ_v the vibrational eigenfunction, and ψ_r the rotational eigenfunction.

I. Electronic Eigenfunctions

(a) *Diatomic and linear polyatomic molecules.* If the nuclei are considered as fixed on the z -axis, and $z_i, \rho_i,$ and ϕ_i are the (cylindrical) coordinates for each electron (ρ_i , distance from the axis; ϕ_i , azimuth), then neglecting electron spin the electronic eigenfunction ψ_e has the form

$$\psi_e = \chi e^{+i\Lambda\phi_1} \quad \text{or} \quad \psi_e = \bar{\chi} e^{-i\Lambda\phi_1}.$$

Here χ and $\bar{\chi}$ are functions of the electronic coordinates $z_i, \rho_i,$ and ϕ_i' , where the $\phi_i' = \phi_i - \phi_1$ are the relative azimuths of all but the first electron. The function χ differs from the function $\bar{\chi}$ only in that the ϕ_i' are replaced by

their negatives. The constant Λ is a positive integer. Λ is the quantum number of the electronic angular momentum $\mathbf{\Lambda}$ about the z -axis. $\mathbf{\Lambda}$ has the magnitude $\Lambda(h/2\pi)$.

As long as the coupling of spin and orbital motion of the electrons is small the electronic eigenfunction including electron spin ψ_{es} may be written as a product of a coordinate function ψ_e , as above, and a spin function β , that is

$$\psi_{es} = \psi_e \beta.$$

(For the form of the spin function, see E. Wigner and E. E. Witmer, *Z. Phys.* **51**, 859 (1928).)

(b) *Polyatomic molecules.* For the general case of polyatomic molecules, the above simple expressions for the electronic eigenfunctions cannot be used. However, it is possible to make exact statements about their **symmetry properties**.

II. Vibrational Eigenfunctions

(a) *Diatomic molecules.* As long as the anharmonicity of the vibration is small, the vibrational eigenfunctions are approximated by the harmonic oscillator eigenfunctions. These are the **Hermite** orthogonal functions

$$\psi_v(x) = N_v e^{-\frac{1}{2}\alpha x^2} H_v(\sqrt{\alpha}x)$$

where N_v is a normalization factor, $H_v(\sqrt{\alpha}x)$ the Hermite polynomial of the v th degree and

$$\alpha = \frac{4\pi^2 \mu \nu_{osc}}{h} = \frac{2\pi \sqrt{\mu k}}{h}.$$

Here μ stands for the reduced mass, ν_{osc} for the (classical) frequency of oscillation, k for the force constant, and h for Planck's constant. (For the anharmonic oscillator functions see A. S. Coolidge, H. M. James, and R. D. Present, *J. Chem. Phys.* **4**, 193 (1936), and J. L. Dunham, *Phys. Rev.* **34**, 438 (1929).)

(b) *Polyatomic molecules.* The total vibrational eigenfunction of a polyatomic molecule is, to a first approximation, the product of $3N - 6$, or, in the case of a linear molecule, $3N - 5$, harmonic oscillator functions.

$$\psi_v = \Pi \psi_i(\xi_i) = \Pi N_{v_i} e^{-\frac{1}{2}\alpha_i \xi_i^2} H_{v_i}(\sqrt{\alpha_i} \xi_i)$$

where $\alpha_i = 2\pi \nu_i/h$. Here ν_i is the (classical) oscillating frequency of the i th normal vibration.

III. Rotational Eigenfunctions

(a) *Diatomic and linear polyatomic molecules.* The rotational eigenfunctions of a di-

atomic or linear polyatomic molecule are the so-called surface harmonics

$$\psi_r = N_r P_J^{|M|}(\cos \vartheta) e^{iM\phi}$$

where ϕ is the azimuth of the line connecting the mass point to the origin, taken about the z axis; ϑ is the angle between this line and the z axis; J is the rotational quantum number corresponding to the angular momentum \mathbf{J} of magnitude $(h/2\pi)\sqrt{J(J+1)} \approx (h/2\pi)J$ (h is Planck's constant). M is a second quantum number (the so-called magnetic quantum number) which takes the values $M = J, J-1, J-2, \dots, -J$, and which represents in units $h/2\pi$ the component of the angular momentum \mathbf{J} in the direction of the z -axis; $P_J^{|M|}(\cos \vartheta)$ is a function of the angle ϑ , the so-called associated **Legendre function**, N_r is a normalization constant.

The probability of finding the system oriented in the direction (ϑ, ϕ) is

$$\psi_r \psi_r^* = N_r^2 [P_J^{|M|}(\cos \vartheta)]^2$$

that is, the probability is independent of ϕ .

(b) *Symmetric top (and spherical top molecules.* The rotational eigenfunctions of the symmetric top are given by

$$\psi_r = \theta_{JKM}(\vartheta) \cdot e^{iK\chi} \cdot e^{iM\phi}.$$

Here ϑ , ϕ , and χ are the so-called **Eulerian angles**, ϑ is the angle of the figure axis of the top with the fixed z axis, ϕ is the azimuthal angle about the z axis, and χ is the azimuthal angle measuring the rotation about the figure axis; J and M are quantum numbers as defined above. K is a quantum number corresponding to the component of \mathbf{J} in the direction of the figure axis; therefore $J = K, K+1, K+2, \dots$. The function $\theta_{JKM}(\vartheta)$ depends in a somewhat complicated way on the angle ϑ ; it contains the so-called **Jacobi** (hypergeometric) **polynomials**.

(c) *Asymmetric top molecules.* In this case the expressions for the rotational eigenfunctions are very complicated. However, exact statements can be made about their **symmetry properties**.

MOLECULAR EIGENFUNCTIONS, SYMMETRY PROPERTIES OF. See **symmetry properties of molecular eigenfunctions**.

MOLECULAR ENERGY LEVELS. The energy levels of a molecule correspond to the **eigenvalues** of the Schrödinger equation of the

system. To a first approximation, the total energy can be treated as the sum of the energies of electronic motion, vibration, and rotation of the molecule, that is

$$E = E_e + E_v + E_r.$$

The corresponding term values are

$$\frac{E}{hc} = T = T_e + G + F.$$

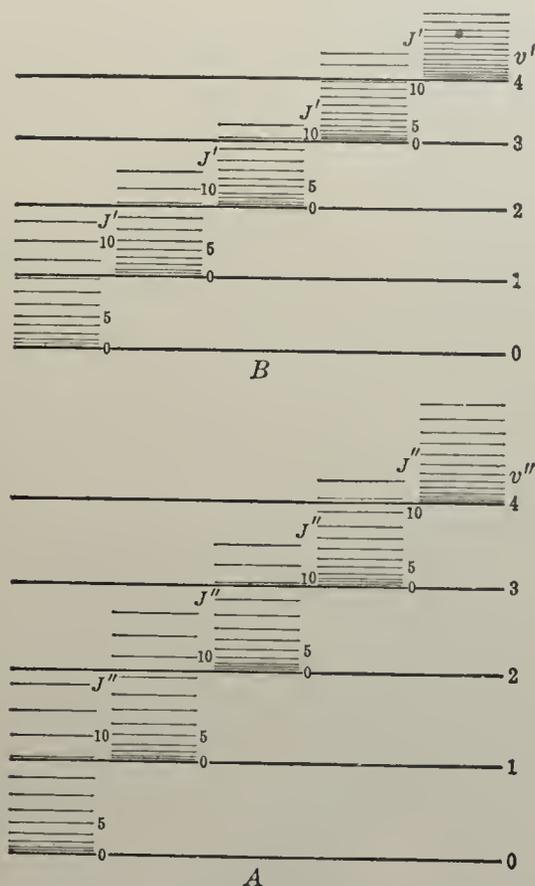
In the simplest case, that of a diatomic molecule, we have

$$G = \omega_e(v + \frac{1}{2}) - \omega_e x_e(v + \frac{1}{2})^2 + \omega_e y_e(v + \frac{1}{2})^3 \dots$$

and

$$F = B_v J(J + 1) - D_v J^2(J + 1)^2 + \dots$$

Here ω_e , $\omega_e x_e$, $\omega_e y_e$, \dots are vibrational constants, B_v , D_v , \dots rotational constants, and v and J , respectively, the vibrational and rotational quantum numbers.



Vibrational and rotational levels of two electronic states A and B of a molecule (schematic). Only the first few rotational and vibrational levels are drawn in each case. (From G. Herzberg, *Molecular Spectra and Molecular Structure. I. Spectra of Diatomic Molecules*, 1950. D. Van Nostrand Company, Inc., Princeton.)

In general the differences between neighboring electronic energy levels are much larger than those between neighboring vibrational energy levels. The latter, in turn, are spaced much more widely than the rotational energy levels. A schematic diagram giving the vibrational and rotational energy levels of two electronic states of a diatomic molecule is shown in the figure.

(For more detail see **electronic energy levels of a molecule**, **vibrational energy levels of a molecule**, **rotational energy levels of a molecule**.)

MOLECULAR ENERGY LEVELS, STATISTICAL WEIGHT OF. See **statistical weight of molecular energy levels**.

MOLECULAR FIELD APPROXIMATION.

A simple and often reasonably accurate method for treating problems of **ferromagnetism**, **antiferromagnetism**, etc. Each magnetic ion is treated independently as if in a field consisting of any external field, H together with the field H' created by the net polarization of its neighbors. H' , in its turn, gives the ion an average polarization, which must be put equal to that assumed for any equivalent neighbors. This self-consistency condition is sufficient to give the polarization as a function of temperature. The assumed field H' may not be just a magnetic field, but the equivalent effect of other processes, such as the **exchange interaction** between the spins.

MOLECULAR FREE PATH. The average free path or distance traveled by a molecule between collisions in a gas or in a solution. (See **mean free path**.)

MOLECULAR NUMBER. (1) The sum of the **atomic numbers** of the atoms in a molecule. (2) An integral number denoting the position occupied by a given molecule in a series obtained by arranging molecules in order of increasing molecular frequency.

MOLECULAR ORBITAL. See **orbital**.

MOLECULAR ORBITALS METHOD. The solution of the Schrödinger equation for a diatomic or polyatomic molecule is very difficult; it has been tackled mainly by two techniques: the **valence bond method** and the molecular orbital method, also called the *Hund-Mulliken-Hückel Method*.

In this latter method, each electron is de-

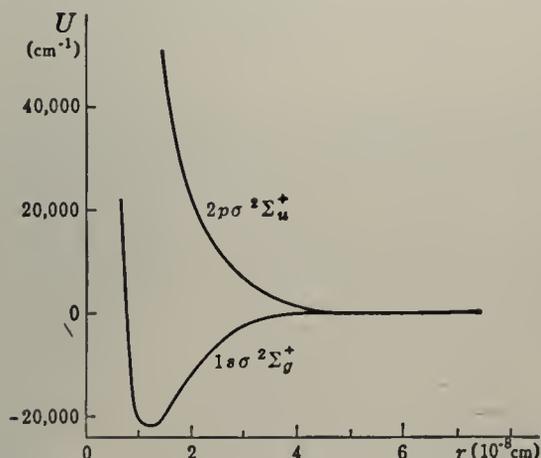
scribed by a wave function ψ which characterizes its orbit in the molecule and is therefore called the molecular orbital. These wave functions are defined by quantum numbers which characterize their energies and shapes.

The molecule is built up by introducing the electrons one by one in the various lowest-lying orbitals, taking into account the **Pauli exclusion principle**.

In order to compute the molecular orbitals, the assumption is usually made, after Mulliken, that these orbitals can be approximated by a *linear combination of atomic orbitals* (i.e., of the orbitals which would be accessible to the electrons in the isolated atoms). This is called the *LCAO approach*. Thus if the molecule consists of atoms A, B, \dots N, the molecular orbitals for an electron will be: $\psi = c_A\psi_A + c_B\psi_B + \dots + c_N\psi_N$. The coefficients $c_A \dots c_N$ are found by minimizing the integral

$$E = \int \psi^* H \psi d\tau / \int |\psi|^2 d\tau$$

(Cf. **variation principle**.) The choice of the valence bond or of the molecular orbital method as a starting point for the study of a molecule is usually governed by physical arguments: if the electrons in the molecule tend to pair up and form fairly localized bonds, (H_2 , CH_4 \dots), the valence bond method is usually a better starting point; for the H_2^+ molecular bond, where a one-electron bond is formed, or in molecules where some electrons (like the π -electrons in conjugated double bonds) are spread out over many neighboring



Potential curves of the two lowest states of H_2^+ , after Teller (theoretical). (From G. Herzberg, *Molecular Spectra and Molecular Structure. I. Spectra of Diatomic Molecules*, 1950. D. Van Nostrand Company, Inc., Princeton.)

atoms, the molecular orbital method is usually a better first approximation.

The only molecule which has been treated rigorously by this method is H_2^+ . Starting from large internuclear distance in zero-approximation, a resonance degeneracy exists for the two possible states of the electron. At smaller internuclear distances this degeneracy is removed in such a way, that the energy of one of the two resulting components becomes smaller, that of the other larger with decreasing r , that is, a stable and an unstable state result. See the figure.

It is obvious that, ultimately, both methods should, after a sufficient amount of refinement, lead to identical wave functions and energies for the molecules; and it is highly encouraging that they agree qualitatively in most cases, even when only the first and still rather rough approximation can be made.

MOLECULAR ORBITALS, NONLOCALIZED. See nonlocalized molecular orbitals.

MOLECULAR POTENTIAL FUNCTIONS. See potential functions of molecules.

MOLECULAR ROTATION. (1) (See **rotational energy levels of a molecule**.) (2) The power of a non-crystallized substance (gas, pure liquid, solution) to rotate the plane of polarization of light transmitted through it, by interaction between the radiation and the single molecules. This type of optical activity occurs in all substances whose molecules (in general complex organic molecules, e.g., various sugars) have neither a center of symmetry nor a plane of symmetry. In practical work one defines the *specific rotations* $[\alpha]$ as the rotation produced by a 1 cm column of liquid containing 1 gm of active substance per em^{-3} of solution. Therefore

$$[\alpha] = \frac{\theta}{ld}$$

where θ is the angle of rotation in degrees, l the length of the light path in cm, and d the concentration of the active substance in $gm\ em^{-3}$. By *molecular rotation* one understands the product of the specific rotation $[\alpha]$ with the molecular weight M .

MOLECULAR SPECIES. See species.

MOLECULAR SPECTRA. Molecular spectra consist of radiation absorbed or emitted by

(free) molecules, corresponding to transitions between different **molecular energy levels** (see also **combination principle**). In general, a molecular spectrum can be easily distinguished from an atomic spectrum by the presence of a great many narrowly spaced lines, usually grouped into **bands** ("band spectrum"). While a transition of an atom from one electronic state to another gives rise to a single spectral line (or line multiplet), a corresponding transition of a molecule produces a whole **band system**. However the number of band systems observable for a particular molecule is in general much smaller than the number of lines in an atomic spectrum, since molecules have only comparatively few stable electronic states (see also **predissociation**). Molecular spectra involving two different electronic states (**electronic spectra of molecules**) have been observed at wavelengths from the near infrared to the vacuum ultraviolet. Spectra corresponding to transitions between the rotational energy levels of two different vibrational levels of the same electronic state (**rotation-vibration spectra of molecules**) are found in the infrared, while spectra corresponding to transitions between the rotational levels of the same electronic and vibrational state (pure rotation spectra, see **rotation spectrum of molecules**) occur in the far infrared and in the centimeter and millimeter wavelength region. At these long wavelengths are also observed spectra arising from transitions between the components of a multiplet electronic energy level. (See **fine structure B II**.)

MOLECULAR SPECTRA, COMBINATION DIFFERENCES AND COMBINATION SUMS IN. See **combination differences and combination sums**.

MOLECULAR SPECTRA, FINE STRUCTURE OF. See **fine structure**.

MOLECULAR SPECTRA, ISOTOPE EFFECT IN. See **isotope effect in molecular spectra**.

MOLECULAR SPECTRA, MICROWAVE. See **microwave spectra of molecules**.

MOLECULAR SPECTRA, PERTURBATIONS IN. See **perturbations in molecular spectra**.

MOLECULAR STRUCTURE, COUPLING IN. See **coupling, in atomic and molecular structure**.

MOLECULAR TERM SYMBOLS. In molecular term symbols, as in **atomic term symbols**, the **multiplicity** $2S + 1$ (S is the quantum number of resultant electron spin) is indicated as a left hand superscript. The main part of the term symbol, a Greek or roman capital letter, is identical with the symbol for the symmetry species of the energy level in question. (See also **symmetry properties of molecular normal vibrations** and the literature quoted with that entry.) For diatomic and linear polyatomic molecules the electronic term symbols are $\Sigma, \Pi, \Delta, \dots$, for $\Lambda = 0, 1, 2, \dots$ (Λ is the quantum number of resultant electronic orbital angular momentum in the direction of the figure axis), in close analogy to the symbols S, P, D, \dots for $L = 0, 1, 2, \dots$ of atoms (L is the quantum number of resultant electronic orbital angular momentum).

MOLECULAR VELOCITIES, DISTRIBUTION LAW. See **distribution of molecular velocities, law of**.

MOLECULAR VELOCITY. In general, the velocity of a molecule due to random thermal motion, i.e., the quantity v appearing in the **Maxwell distribution law**.

MOLECULAR VELOCITY, MEAN. If a system of particles contains a number n_1 moving with velocity c_1 , n_2 moving with velocity $c_2 \dots$, the mean velocity \bar{c} is defined as

$$\bar{c} = \frac{n_1 c_1 + n_2 c_2 + \dots}{n_1 + n_2 + \dots}$$

Strictly speaking, the mean molecular **velocity** in a gas in equilibrium is zero. The term is often used, however, to indicate the mean speed, in which case the values of c are averaged without regard to sign. It may then be shown from the **Maxwell distribution law** that the mean molecular velocity is

$$\bar{c} = \sqrt{\frac{8kT}{\pi m}}$$

MOLECULAR VELOCITY, MEAN-SQUARE. If a system of particles contains a number n_1 moving with velocity c_1 , n_2 with velocity c_2, \dots , etc., the mean square velocity $\overline{c^2}$ is defined as

$$\overline{c^2} = \frac{n_1 c_1^2 + n_2 c_2^2 + \dots}{n_1 + n_2 + \dots}$$

This quantity appears, for example, in the expression for the pressure of an ideal gas according to the kinetic theory. It can be shown from the **Maxwell distribution law** that the mean square velocity is connected with the mean velocity (see **molecular velocity, mean**) by the relation

$$\overline{c^2} = \frac{3\pi}{8} (\bar{c})^2 = \frac{3kT}{m}$$

MOLECULAR VELOCITY, MOST PROBABLE. If the function given by the **Maxwell distribution law** is plotted against the velocity c , a smooth curve is obtained which has a maximum at a value of c which may be denoted by α , where α is called the most probable velocity. It increases with temperature, and is connected with the mean velocity (see **molecular velocity, mean**) by the relation

$$\bar{c} = \frac{2\alpha}{\sqrt{\pi}}$$

MOLECULE. The smallest indivisible particle of a substance which still exhibits most of the properties of the substance.

MOLECULE, DISSOCIATION ENERGY OF. See **dissociation energy of a molecule**.

MOLECULE, ROTATIONAL CONSTANTS OF. See **rotational constants of a molecule**.

MOLECULE, ROTATIONAL ENERGY LEVELS IN. See **rotational energy levels in a molecule**.

MOLECULES, DIATOMIC, NON-CROSSING RULE. See **non-crossing rule for the potential curves of diatomic molecules**.

MOLECULES, EIGENFUNCTIONS OF. See **molecular eigenfunctions**.

MOLECULES, ELECTRONIC SPECTRA OF. See **electronic spectra of molecules**.

MOLECULES, MOMENTS OF INERTIA OF. See **moments of inertia of molecules**.

MOLECULES, NON-COMBINING MODIFICATIONS OF. See **non-combining modifications of**.

MOLECULES, POLYATOMIC. See **polyatomic molecules, types of**.

MOLECULES, POTENTIAL FUNCTIONS OF. See **potential functions of molecules**.

MOLECULES, PRINCIPAL AXES OF. See **moments of inertia of molecules**.

MOLECULES, ROTATION SPECTRA OF. See **rotation spectra of molecules**.

MOLECULES, SYMMETRY PROPERTIES OF NORMAL VIBRATIONS IN. See **symmetry properties of normal vibrations in molecules**.

MOLECULES, VALENCE FORCES IN POLYATOMIC. See **valence forces in polyatomic molecules**.

MOLECULES, VIBRATIONAL ENERGY LEVELS OF A. See **vibrational energy levels of a molecule**.

MOLECULES, VIBRATIONAL SPECTRA OF. See **vibrational spectra of molecules**.

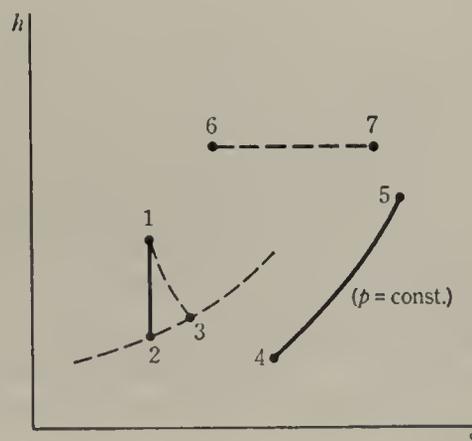
MOLE FRACTIONS. The mole fraction of the component i is defined by

$$x_i = \frac{n_i}{\sum_i n_i} = \frac{n_i}{n} \quad (1)$$

where n_i is the number of moles of component i . Clearly

$$\sum_{i=1}^c x_i = 1. \quad (2)$$

MOLLIER CHART. A thermodynamic diagram for a homogeneous system possessing two independent properties, in which **enthalpy** is the ordinate and **entropy** is the abscissa. Mollier charts are used widely in engine cal-



culations, particularly those in which the working fluid is steam. The chart then contains the vapor-pressure curve, curves of constant pressure, curves of constant **dryness function**, curves of constant temperature, and sometimes curves of constant specific volume.

The usefulness of the chart is predicated on the following properties:

1. An **isentropic** process is a straight vertical line.
2. The work of an isentropic process is the length of this straight line. $W_{\text{isentropic}} = h_1 - h_2$.
3. The work of an adiabatic (irreversible) process between the equilibrium states 1 and 3 ($s_3 > s_1$) is $W_{ad} = h_1 - h_3$, i.e., the vertical distance between 1 and 3.
4. In a process at constant pressure the heat exchanged is $Q_{45} = h_5 - h_4$, that is, again, the vertical distance between 4 and 5.
5. In an adiabatic throttling (irreversible) process between the equilibrium states 6 and 7, the two end-states must lie on one horizontal line. Knowing one value at 7 for a given state 6, one can determine the others with reference to the lines on the diagram.

MOLLIER EQUATION. An empirical (thermal) equation of state for steam:

$$v = 47.1 \frac{T}{p} - \frac{2}{(T/100^\circ\text{C})^{1.95}} p - \frac{1.9}{(T/100^\circ\text{C})^{14} \times 10^4} p^2.$$

It is sufficiently accurate at lower pressures and temperatures, even though it is not dimensionally correct. (See also **steam tables**.)

MOMENT. The product of a quantity and a distance to some significant point related to that quantity. The principal moments are **moments of forces**, moments of lines, moments of areas, and moments of masses. Two types of moment are static moment and moment of inertia, the latter being discussed in the entry **inertia, moments and products of**. For other moments see the entries following.

MOMENT, STATISTICAL. In statistics, the n th moment of a distribution function $F(x)$ about a point a is defined as the **expected value** of $(x - a)^n$:

$$\mu'_n = \int (x - a)^n dF(x).$$

The first moment μ'_1 is the **mean** of the distribution. Of particular importance are the moments about the mean, or central moments, denoted by μ_n ; μ_2 is called the **variance**. Central moments can be obtained from moments about an arbitrary point by formulae of the type

$$\begin{aligned} \mu_2 &= \mu'_2 - \mu'_1{}^2 \\ \mu_3 &= \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'_1{}^3 \\ \mu_4 &= \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'_1{}^2 - 3\mu'_1{}^4. \end{aligned}$$

The ratios $\beta_1 = \mu_3^2/\mu_2^3$ and $\beta_2 = \mu_4/\mu_2^2$ are used as measures of **skewness** and **kurtosis** respectively; for a **normal curve**, $\beta_1 = 0$, $\beta_2 = 3$.

The absolute moment of order n about the point a is defined as

$$\nu'_n = \int |x - a|^n dF.$$

The factorial moment of order n of a discrete distribution about the point a is defined as

$$\mu'_{[n]} = \Sigma (x - a)^{[n]} f(x)$$

where $f(x)$ is the probability at the point x , and $(x - a)^{[n]} = (x - a - h) \cdots (x - a - (n - 1)h)$, h being the interval between successive values of x . The factorial moments are readily calculated by successive summation; they are related to the ordinary moments by formulae of the type

$$\begin{aligned} \mu'_1 &= \mu'_{[1]} \\ \mu'_2 &= \mu'_{[2]} + h\mu'_{[1]} \\ \mu'_3 &= \mu'_{[3]} + 3h\mu'_{[2]} + h^2\mu'_{[1]} \\ \mu'_4 &= \mu'_{[4]} + 6h\mu'_{[3]} + 7h^2\mu'_{[2]} + h^3\mu'_{[1]}. \end{aligned}$$

MOMENTA, GENERALIZED. See **coordinates and momenta, generalized**.

MOMENTAL ELLIPSOID. See **ellipsoid, momental**.

MOMENT AREA METHOD. The elastic deflections of beams are calculated conveniently by the moment-area method. Two theorems are employed. The first is the direct integration of curvature $= M/EI$ between definite limits $x = a$ and $x = b$. In words, the change in slope from a to b is the area of the M/EI diagram between a and b . The second

theorem states that the deflection at b measured vertically upward (beam horizontal) from the tangent to the beam at a is the moment of the M/EI diagram between a and b about b . (See **conjugate beam**.)

MOMENT, BALANCING. The method of **moment distribution** is a moment balancing procedure.

MOMENT, BENDING. See **bending moment**.

MOMENT DIAGRAM. The moment diagram is a pictorial representation of the variation in bending moment along the axis of a beam or frame. It may be constructed directly by taking summation of moments to one side of each section, or constructed from the shear diagram, taking into account whatever concentrated moments may be introduced. Shear V is related to moment M by $V = dM/dx$. Therefore the ordinate to the shear diagram gives the slope of the moment diagram and the change in moment between two points is the area of the shear diagram, plus any concentrated moments between the two points. (See **bending moment**.)

MOMENT DISTRIBUTION. A method introduced by Hardy Cross for solving elastic frame problems by successive approximations or relaxations. The procedure may be visualized as follows: All joints of the structure are locked against rotation and translation. The loads are applied to the structure and the fixed end moments at the joints computed for each beam and column. Next one or more joints which can rotate are permitted to rotate freely so that the moments at these joints balance. The distribution of moments at a joint is governed by *distribution factors* equal to the stiffness of the member framing into the joint divided by the sum of the stiffness of all members framing into the joint. Unlocking of a joint not only alters the end moment in a member at that joint but also changes the moment at the opposite end of the member. This carryover moment is given by the *carryover factor* multiplied by the distributed moment. For a uniform beam or column the carryover factor is $1/2$. The method of moment distribution consists of this repeated locking and unlocking of joints until the unbalanced moment is everywhere small enough to be ignored. (See **slope-deflection equation**.)

If a joint also is free to translate, as for example, in the sidesway of a structure, the negative of the restraining forces implied in the procedure as described above must be applied to the structure and the result added to the previous solution. If desired, both rotation and translation relaxation can proceed together.

MOMENT, END. The moment at the end A of a beam or column AB is written as M_{AB} in the **slope-deflection equations**. The relation between end moments and end rotations in elastic structures is the basis of such methods as **moment distribution**, **angle balancing** and **slope deflection**.

MOMENT, LIMIT OR PLASTIC. When a beam of elastic-perfectly plastic material is bent more and more, the state of stress approaches uniform compression equal to the yield stress on one side of the neutral axis and uniform tensile yield on the other. During the bending, the neutral axis remains fixed in position for a symmetric beam but moves for an unsymmetric one. The final position for equal yield stress in tension and compression divides the cross-sectional area in half so that the axial force is zero. The asymptotic value of the moment is called the limit or plastic moment and is equal to the sum of the yield stress σ_y times the **static moment** of each half of the area or $\sigma_y bh^2/4$ for a rectangle $b \times h$.

MOMENT OF A DISTRIBUTION. The n -th moment M_n of a distribution $f(x)$ is defined as

$$M_n = \int x^n f(x) dx.$$

MOMENT OF FORCE. The vector quantity $\mathbf{r} \times \mathbf{F}$ where \mathbf{r} is the position vector from the center of moments O to any point on the line of action of the force \mathbf{F} is called the **moment** of \mathbf{F} about O . Its magnitude is the product of the magnitude of \mathbf{F} and the perpendicular distance from O to the line of action of \mathbf{F} . Its direction is perpendicular to the plane of O and the line of action of \mathbf{F} .

MOMENT OF INERTIA. See **inertia**, **moments** and **products of**.

MOMENT OF INERTIA, ROUTH RULE. See **Routh rule of inertia**.

MOMENT OF INERTIA, TRANSFER THEOREM FOR. The moment of inertia I about

any axis equals the moment of inertia I_c about a parallel axis through the center of mass plus the total mass M , of the body times the square of the distance, d , between the axes. $I = I_c + Md^2$.

MOMENT OF MOMENTUM (ANGULAR MOMENTUM). For a collection of r particles and a given origin, the quantity

$$\sum_{j=1}^{j=r} \mathbf{r}_j \times \left(m_j \frac{d\mathbf{r}_j}{dt} \right)$$

is called the moment of momentum about the origin, where \mathbf{r}_j is the position vector from origin to j th particle; m_j is the mass of j th particle; $\frac{d\mathbf{r}_j}{dt}$ is the time rate of change of position vector; \times indicates the vector cross product operation. (See **momentum**.)

For a rigid body, the total moment of momentum is designated as the angular momentum and written

$$\begin{aligned} \mathbf{H} = & \mathbf{i}(\omega_x I_{xx} - \omega_y I_{xy} - \omega_z I_{xz}) \\ & + \mathbf{j}(-\omega_x I_{yx} + \omega_y I_{yy} - \omega_z I_{yz}) \\ & + \mathbf{k}(-\omega_x I_{zx} - \omega_y I_{zy} + \omega_z I_{zz}) \end{aligned}$$

where $\omega_x, \omega_y, \omega_z$ are components of angular velocity; I_{xx}, I_{yy}, I_{zz} are moments of inertia; and I_{xy}, I_{xz} , etc., are products of inertia. (See **inertia, moments and products of**; and **angular momentum**.)

MOMENT ON AN AIRFOIL. See **lift and moment on an airfoil, general formulae**.

MOMENTS, CENTER OF. See **center of moments**.

MOMENTS METHOD. In the analysis of the space and energy distribution of neutrons slowing down in a large homogeneous system, one of the effective methods for treating the space dependence. The angular flux is first expanded in terms of **spherical harmonics**, but it is not these spherical harmonic moments which are studied, but rather their spatial moments. One obtains a set of coupled integral equations for the space angle moments ϕ_{ne} , and since the equation for ϕ_{ne} contains only moments of order n and $n - 1$ no special assumption about higher moments need be made.

MOMENTS OF INERTIA OF MOLECULES. The moment of inertia of a molecule (or generally of a rigid body) about an axis is defined by

$$I = \sum m_i \rho_i^2$$

where ρ_i is the perpendicular distance of a nucleus of mass m_i from the axis. There are three mutually perpendicular axes for which the moment of inertia is a maximum or a minimum. These are called *principal axes* and the corresponding moments of inertia the *principal moments of inertia*.

The magnitudes of the principal moments of inertia determine the spacing of the **rotational energy levels of a molecule**. For the discussion of their rotational properties one distinguishes therefore different types of molecules according to the relative magnitude of their principal moments of inertia (see **polyatomic molecules, types of**).

For a linear molecule the moment of inertia about an axis perpendicular to the figure axis is given by

$$I_B = \sum m_i r_i^2$$

where r_i stands for the distance of the i th nucleus from the center of mass. The moment of inertia about the figure axis I_A is very small.

For the special case of a diatomic molecule one has

$$I_B = \mu r^2$$

where

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

is the **reduced mass** and r is the internuclear distance.

MOMENT, STATIC. The static moment of an area in the xy -plane about x -axis is

$$\int_A y dA.$$

MOMENTUM. For a single particle of mass m whose position vector is \mathbf{r} the momentum is the vector quantity

$$m \frac{d\mathbf{r}}{dt} \text{ or } m\mathbf{v}, \text{ where } \mathbf{v} = \frac{d\mathbf{r}}{dt}$$

is the velocity. For a system of n particles of masses $m_1 \cdots m_n$ respectively and position vectors $\mathbf{r}_1 \cdots \mathbf{r}_n$ respectively the total momentum is

$$\sum_{i=1}^n m_i \cdot \frac{d\mathbf{r}_i}{dt}$$

From the fundamental principles of mechanics, the time rate of change of the total momentum of a system of particles is equal to the vector sum of all the external applied forces. For a system subject only to interaction forces between the particles, the total momentum remains constant. (See **conservation of momentum**.)

MOMENTUM, ANGULAR. See **angular momentum**.

MOMENTUM, CONSERVATION OF. See **conservation of momentum**.

MOMENTUM DENSITY. See **continuous systems, Lagrangian and Hamiltonian formalism for**.

MOMENTUM, DISTRIBUTION IN. See **distribution in momentum**.

MOMENTUM EIGENFUNCTION. An **eigenfunction** of the quantum mechanical operator representing the linear momentum. In the Schrödinger representation it is

$$\psi \sim e^{i\mathbf{p} \cdot \mathbf{r} / \hbar}$$

where \mathbf{p} is the momentum vector, \mathbf{r} is the position vector, and \hbar the **Dirac** \hbar .

MOMENTUM EQUATION FOR BOUNDARY LAYER. For steady flow of an incompressible fluid in a two-dimensional *laminar* boundary layer the momentum equation is

$$\frac{\tau_o}{\rho U^2} = \frac{d\theta}{dx} + (H + 2) \frac{\theta}{U} \cdot \frac{dU}{dx}, \quad (1)$$

where τ_o is the shear stress at the wall, x is distance along the wall, U is the velocity at the edge of the boundary layer, θ is **momentum thickness** and $H\theta$ is **displacement thickness**.

For a **turbulent boundary layer** the momentum equation contains two additional terms, as follows:

$$\frac{\tau_o}{\rho U^2} = \frac{d\theta}{dx} + (H + 2) \frac{\theta}{U} \cdot \frac{dU}{dx} - \frac{1}{\rho U^2} \int_0^\delta (P - p) dy - \frac{d}{dx} \int_0^\delta \frac{\overline{u'^2}}{U^2} dy \quad (2)$$

In this equation, P is the pressure at the edge of the boundary layer, y is the distance from

the wall to a point in the boundary layer where the pressure is p and the component of turbulent velocity along the x -axis is u' , δ is the boundary-layer thickness and $\overline{u'^2}$ denotes the mean value of u'^2 .

In many cases the difference between Equations (1) and (2) is small enough to be neglected, and Equation (1) is commonly applied to a turbulent boundary layer. There are some cases, however, particularly near separation, where the additional terms in Equation (2) are of the same order as $\frac{\tau_o}{\rho U^2}$ and cannot be neglected.

Equations (1) and (2) are obtained by integration of the usual boundary-layer equations with respect to y , from 0 to δ .

MOMENTUM FLUX VECTOR. See **flux vector**.

MOMENTUM, KINETIC. See **kinetic momentum**.

MOMENTUM, MOMENT OF. See **moment of momentum**.

MOMENTUM OPERATOR. The quantum mechanical operator representing the momentum variable. In the Schrödinger representation, operation by this operator consists of partial differentiation with respect to the canonically conjugate position variable and multiplication by $i\hbar$.

MOMENTUM THEORY, FROUDE. See **Froude momentum theory**.

MOMENTUM THICKNESS OF BOUNDARY LAYER. The momentum thickness θ for a boundary layer is defined by

$$\theta = \int_0^\delta \frac{u}{U} \left(1 - \frac{u}{U}\right) dy,$$

where u is the velocity at a distance y from the wall and U is the velocity outside the boundary layer, i.e., for $y > \delta$.

The deficiency of momentum flux in the boundary layer, compared with a layer of constant velocity U , is $\rho U^2 \theta$ per unit width of the layer. (See also **momentum equation for boundary layer**.)

MOMENTUM-TRANSPORT HYPOTHESIS. The hypothesis that **momentum** is conserved in turbulent eddy transfer. This hypothesis, together with that of the **mixing length**, leads

to an expression for the variation of the **shearing stress** τ with height

$$\frac{\partial \tau}{\partial z} = \frac{\partial}{\partial z} \left(\mu \frac{d\bar{u}}{dz} \right),$$

where μ is the eddy viscosity and \bar{u} the mean horizontal wind.

This hypothesis is to be compared with the **vorticity-transport hypothesis**, the respective results being identical only if the **eddy viscosity** is constant.

MONGE'S FORM FOR EQUATION OF SURFACE. An equation of a surface of the form

$$z = f(x, y)$$

where x, y, z is a rectangular Cartesian coordinate system.

MONOCHROMATIC RADIATION. Radiation having one frequency or one wavelength. Actually, no finite amount of radiation will ever be strictly monochromatic. It will, at best, contain a narrow band of wavelengths.

MONOCHROMATIC WAVE. See **Helmholtz's formulation of Huygens' principle**.

MONOCLINIC SYSTEM. One of the seven crystal systems. In this system, the three axes are of unequal length, and two of them are not at right angles to each other.

MONOGENIC (FUNCTION OF A COMPLEX VARIABLE). Synonym of **analytic**.

MONOLAYER. See **localized monolayer**; **mobile monolayer**.

MONOMOLECULAR REACTIONS. See **unimolecular reactions**.

MONOTONIC. A sequence a_1, a_2, \dots , is monotonic increasing if $a_m \leq a_n$ when $m < n$. A function $f(x)$ is monotonic increasing if $f(a) \leq f(b)$ when $a < b$. Monotonic decreasing is similarly defined.

MONOVARIANT SYSTEM. See under **phase rule**.

MONTE CARLO METHOD. Any method for obtaining a statistical estimate of a desired quantity by random sampling. In the most successful applications the desired quantity is in fact a statistical parameter, and the sampling is made from an artificial population

that is in some sense a model of the physical system itself: for example, the fraction of neutrons, of given initial distribution in direction and energy, that can be expected to penetrate to a given depth in a medium of known composition and geometric form. The term is also used, rather loosely, to denote the solution of distributional problems by sampling experiments. (For more details see Alston S. Householder, *Principles of Numerical Analysis*, McGraw-Hill Book Company, 1953; Alston S. Householder, G. E. Forsythe, and H. H. Germond (eds.), *Monte Carlo Method*, NBS Appl. Math. Series 12, 1951; H. A. Meyer (ed.), *Symposium on Monte Carlo Methods*, John Wiley & Sons, Inc., 1956.)

MONTGOMERY NOISE TRANSMISSION EFFECT. When three successive points are taken along a **semiconduction filament**, the noise voltages V_{12}, V_{23}, V_{13} do not combine so that the noise power V_{13}^2 equals the sum of V_{12}^2 and V_{23}^2 . This is interpreted as being due to the **lifetime** of a **hole** being so long that most of the holes entering the first segment also enter the second, and hence the voltages themselves should be added.

MONTH (SIDEREAL, SYNODIC). Astronomically, the month is the period of revolution of the moon about the earth. However, several kinds of month are in common use depending upon the reference points used. The following are the two most commonly used.

The *sidereal month* is the period of revolution of moon from one right ascension back to the same right ascension again. This averages 27d 7hr 43m 11.5s (ca 27 $\frac{1}{3}$ days) but varies by as much as seven hours due to perturbations in the moon's orbit.

The *synodic month* is the period of revolution of the moon from new moon to new moon again (the interval between successive conjunctions with the sun) and averages 29d 12hr 44m 2.8s (ca 29 $\frac{1}{2}$ days) but varies by more than half a day.

MORERA THEOREM. The converse of the **Cauchy integral theorem**. If $f(z)$ is a continuous function of the complex variable z , defined in a finite simply-connected domain D , and if

$$\int_C f(z) dz = 0$$

for any closed contour in D , then $f(z)$ is an analytic function of z in D .

MORERA STRESS FUNCTIONS. Using the indicial notation, the stress components in a rectangular Cartesian coordinate system x , at a point x_i of a body, may be denoted σ_{ij} . Provided the body is in equilibrium and no body forces are acting, the stress field must satisfy equations of equilibrium which may be written, using the summation convention,

$$\frac{\partial \sigma_{ij}}{\partial x_j} = 0.$$

Any stress field, of sufficient regularity, satisfying these equations may be expressed in terms of three functions of position in the body ψ_1, ψ_2, ψ_3 (say), called the Morera stress functions, by

$$\sigma_{11} = -2 \frac{\partial^2 \psi_1}{\partial x_2 \partial x_3}, \quad \sigma_{22} = -2 \frac{\partial^2 \psi_2}{\partial x_3 \partial x_1},$$

$$\sigma_{33} = -2 \frac{\partial^2 \psi_3}{\partial x_1 \partial x_2},$$

$$\sigma_{23} = \frac{\partial}{\partial x_1} \left(\frac{\partial \psi_2}{\partial x_2} + \frac{\partial \psi_3}{\partial x_3} \right),$$

$$\sigma_{31} = \frac{\partial}{\partial x_2} \left(\frac{\partial \psi_3}{\partial x_3} + \frac{\partial \psi_1}{\partial x_1} \right),$$

$$\sigma_{12} = \frac{\partial}{\partial x_3} \left(\frac{\partial \psi_1}{\partial x_1} + \frac{\partial \psi_2}{\partial x_2} \right).$$

MORSE CURVE. An empirical representation of the variation of the interaction energy of a chemical bond as a function of interatomic distances.

The Morse potential energy $E(R)$ for a diatomic molecule is given by:

$$E(R) = D_e \{ \exp [-2a(R - R_e)] - 2 \exp [-a(R - R_e)] \}$$

where D_e is the dissociation energy of the bond, and R_e , the interatomic separation at the minimum of the curve (equilibrium interatomic distance). (See also potential functions of molecules; interatomic potential.)

MORSE FUNCTION. See Morse curve.

MORSE POTENTIAL. See Morse curve.

MOSELEY DIAGRAM FOR X-RAY LEVELS. The graphical representation of

the dependence of the x-ray energy levels or x-ray absorption limits on the atomic number Z . It is usual to plot

$$\sqrt{\frac{\nu}{R}}$$

where $\nu (= 1/\lambda)$ is the wavenumber of the absorption limit and R is the Rydberg constant, against Z . One obtains for the K and L series nearly straight lines, while for the M and higher series more complicated curves are obtained. (For more detail see H. E. White, *Introduction to Atomic Spectra*, McGraw-Hill Book Company, Inc., New York, 1934.)

MOSELEY LAW. Moseley's law states that for different elements the wavenumbers

$$\nu = \frac{1}{\lambda} (\text{cm}^{-1})$$

of corresponding spectral lines in the x-ray region are approximately proportional to the squares of the atomic numbers Z . For the lines of the K and L series this law can, to a good approximation, be represented by the formula

$$\nu = KR(Z - \sigma)^2,$$

where R is the Rydberg constant, and K and σ are constants characteristic for the line in question (e.g., for K_α , $K = 3/4$ and $\sigma = 1$; for L_β , $K = 5/36$ and $\sigma = 7.4$).

MOST PROBABLE MOLECULAR VELOCITY. See molecular velocity, most probable.

MOTION. A change in position with respect to a reference system of a material particle or aggregate of such particles.

MOTION, CANONICAL EQUATION OF. See canonical equation of motion.

MOTION, EQUATIONS OF. See equations of motion; canonical equations of motion.

MOTION, EULER EQUATIONS OF. See Euler equations of motion.

MOTION IN REPULSIVE INVERSE SQUARE FIELD. The motion is along a hyperbola. One of the focal points is the center of the repulsive force. A classical example is the path described by an α -particle as it passes through the vicinity of a positively charged nucleus.

MOTION, NEWTON LAWS OF. See Newton laws of motion.

MOTION, OSCILLATORY. See oscillatory motion.

MOTION, PARAMETRIC EQUATIONS OF. See parametric equations of motion.

MOTION, RECTILINEAR. See rectilinear motion.

MOTION, RELATIVE. See relative motion.

MOTION, SIMPLE HARMONIC. See simple harmonic motion.

MOTION, UNIFORM. See uniform motion.

MOUTIER LAW. The work done in any isothermal reversible cyclic process is zero; as is also the algebraic sum of the quantities of heat absorbed and emitted in such a process. The Moutier law is a direct consequence of the first law of thermodynamics.

MOVING AVERAGE. The moving average of a series u_t ($t = 1, 2, \dots$) of extent $k + 1$ and weights w_j ($j = 0, 1, \dots, k$), $\sum w_j = 1$, is given by

$$v_t = \sum_{j=0}^k w_j u_{t+j}, \quad t = 1, 2, \dots$$

If all the weights are equal to $1/(k + 1)$ the average is said to be simple.

MOVING AXES, EQUATION OF MOTION REFERRED TO. Let x_o, y_o, z_o , represent a set of axes fixed in an inertial system, and let x, y, z be a second set of rigid rectangular axes moving relative to x_o, y_o, z_o . ρ is the radius vector OO' and \mathbf{r} the position vector of a moving point relative to the moving coordinate system. Let \mathbf{f}_o be the absolute acceleration (i.e., measured in the x_o, y_o, z_o system) and \mathbf{f} the relative acceleration (i.e., measured in the moving system x, y, z). Let $\boldsymbol{\omega}$ be the angular velocity of the moving system and $\dot{\boldsymbol{\omega}}$ the absolute time rate of change of $\boldsymbol{\omega}$ and \mathbf{v} the relative velocity. Then

$$\mathbf{f} = \mathbf{f}_o - \ddot{\boldsymbol{\rho}} + \boldsymbol{\omega} \times (\mathbf{r} \times \boldsymbol{\omega}) + \mathbf{r} \times \dot{\boldsymbol{\omega}} + 2\mathbf{v} \times \boldsymbol{\omega}$$

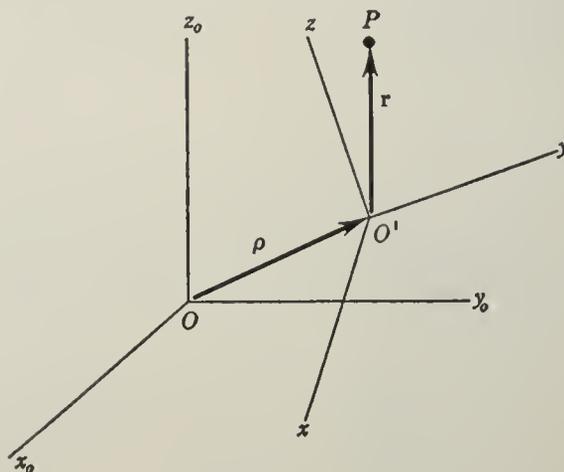
the term $\boldsymbol{\omega} \times (\mathbf{r} \times \boldsymbol{\omega})$ is the centrifugal acceleration and the term $2\mathbf{v} \times \boldsymbol{\omega}$ is the Coriolis acceleration. If $\mathbf{F} = m\mathbf{f}_o$ is the absolute resultant force acting on the mass m , then the relative force $\mathbf{F}_{rel} \equiv m\mathbf{f}$ is

$$\mathbf{F}_{rel} = \mathbf{F} - m\ddot{\boldsymbol{\rho}} + m\boldsymbol{\omega} \times (\mathbf{r} \times \boldsymbol{\omega}) + m\mathbf{r} \times \dot{\boldsymbol{\omega}} + 2m\mathbf{v} \times \boldsymbol{\omega}.$$

$m\boldsymbol{\omega} \times (\mathbf{r} \times \boldsymbol{\omega})$ is the centrifugal force and $2m\mathbf{v} \times \boldsymbol{\omega}$ is the Coriolis force. If $\boldsymbol{\omega}$ is constant and if O' coincides permanently with O , then,

$$\mathbf{F}_{rel} = \mathbf{F} + m\boldsymbol{\omega} \times (\mathbf{r} \times \boldsymbol{\omega}) + 2m\mathbf{v} \times \boldsymbol{\omega}.$$

(See also Coriolis force.)



MOVING CLUSTERS. See star clusters.

M-THEORY. A theory describing the dynamic behavior of a compressible fluid, with infinite electrical conductivity, upon the application of a constant electric field at its boundaries, in the absence of an external magnetic field.

MUELLER-BRESLAU PRINCIPLE. A very useful principle which can be proved directly by the theorem of virtual work: The ordinates of the influence line for a force or moment at any section of a structure are given by the deflection curve for the structure due to a unit deformation introduced at the section in place of the force or moment. The unit deformation



must be chosen so that no other force or moment at the section does work. As shown in the figure the displacement which gives the influence line for shear shows no change in slope across the break.

MULTICOLLINEARITY. In multivariate analysis, a situation in which linear relations exist among the variables. Analysis by regression methods is then impaired and coefficients become quite unreliable.

MULTIGROUP KERNEL. See **kernel, multigroup.**

MULTIGROUP NEUTRON DIFFUSION THEORY. See **neutron diffusion theory, multigroup.**

MULTINOMIAL SERIES. A polynomial in several variables is often called a multinomial. When raised to a power, it may be expanded by the multinomial theorem of which the **binomial series** is a special case. The result is

$$(x_1 + x_2 + x_3 + \cdots + x_r)^n = \sum \frac{n!}{a_1! a_2! \cdots a_r!} x_1^{a_1} x_2^{a_2} \cdots x_r^{a_r}$$

where a_k is an integer and $\sum_{k=1}^r a_k = n$.

MULTIPLE CORRELATION. See **correlation.**

MULTIPLE INTEGRAL. See **integral, multiple.**

MULTIPLE SCATTERING. In contrast to **primary scattering**, any **scattering** in which radiation is scattered more than once before reaching the eye, antenna, or other sensing element. *Secondary scattering* is the simplest and most commonly cited form of multiple scattering.

MULTIPLY. In an atom, energy levels with a given resultant electronic angular momentum L and non-zero resultant electron spin S split into a number **fine structure** components with quantum numbers

$$J = (L + S), (L + S - 1), \cdots, |L - S|.$$

The resulting multiplet has $2S + 1$ components for $L > S$, and $2L + 1$ components for $L < S$. (See also **multiplicity; alternation of multiplicities.**)

Multiplets whose components lie in the order of their J values (smallest J value lowest) are called regular (normal), those whose components lie in the opposite order (largest J value lowest) inverted multiplets. (See **Hund's rules for atomic multiplets.**)

For atoms with low atomic number Z (**Russell-Saunders coupling**) the multiplets have narrow spacing, while for heavier elements the spacing may be very wide and the concept of multiplets meaningless. According to **Landé's**

interval rule (holding strictly only for strict Russell-Saunders coupling) the intervals between successive multiplet components (J and $J + 1$) are proportional to $J + 1$.

The term "multiplet" is applied also to the narrowly spaced groups of lines corresponding to transitions between the multiplet components of the same or of two different atomic energy levels. A line multiplet may have more components than either energy state involved (**Compound multiplet**).

For more detail and for the analogous case of multiplet structure in molecular spectra, see **fine structure. B.** (See also **intensity rules for multiplets.**)

MULTIPLETS, HUND RULES FOR ATOMIC. See **Hund rules for atomic multiplets.**

MULTIPLICATION, EFFECTIVE. Pertaining to a chain reacting system, this quantity is also known as the *criticality factor*. It is an extensive quantity, and is the ratio of the average number of neutrons present in the $k + 1$ generation to the average number present in the k^{th} generation. It may also be defined as the ratio of the number of neutrons produced in the system per unit time to the number captured and escaping per unit time, in a state free of transients. It is denoted by the symbol k_{eff} . (See **multiplication, infinite.**)

MULTIPLICATION, INFINITE. The effective multiplication (see **multiplication, effective**) for a chain-reacting system may be expressed as the product of two factors, $k_{\text{eff}} = k_{\infty} P$. The infinite multiplication, k_{∞} , is the ratio of neutrons produced in the system per unit time to neutrons captured in the system per unit time, when the system is infinitely large. It is an intensive quantity. P is closely related to the probability that a typical neutron does not escape from the system during its lifetime.

MULTIPLICATION OF TENSORS. The operation of forming the **product of (two) tensors.**

MULTIPLICITIES, ALTERNATION OF. The **multiplicity** $2S + 1$ of an energy level of an atom or ion is integral or half integral if the number of electrons is even or odd, respectively. Therefore the multiplicity of an electronic term is odd (singlet, triplet \cdots) for an

even number of electrons, and even (doublet, quartet, ...) for an odd number of electrons. As a consequence even and odd multiplicities alternate for successive elements in the periodic system, and, similarly, for the series of ions with increasing charge of a given element.

MULTIPLICITY. The multiplicity of an atomic or molecular state has the value $2S + 1$, where S is the quantum number of the resultant electron spin. In the vector model of the atom, $2S + 1$ is the number of ways in which the resultant **electron spin** S can align itself (**space quantization**) with respect to L , the resultant electronic angular momentum. (An electronic energy level has only $2L + 1$ components if $L < S$.) (See also **alternation of multiplicities; multiplet**.)

MULTIPLICITY, EDGE. See **edge multiplicity**.

MULTIVARIATE INTERPOLATION. Interpolation of functions of two or more variables. Since

$$E_x = 1 + \Delta_x, \quad E_y = 1 + \Delta_y$$

one can expand formally

$$E_x^u E_y^v = (1 + \Delta_x)^u (1 + \Delta_y)^v$$

and derive equal interval formulas. Likewise analogues of the Lagrange formula can be derived. To write them explicitly is hardly worth while since they amount to operating on the two (or more) variables separately. (See Kaiser S. Kunz, *Numerical Analysis*, McGraw-Hill Book Company, 1957; E. T. Whittaker and G. Robinson, *The Calculus of Observations*, D. Van Nostrand Company, Inc., Princeton, 1940.)

MURAOUR RELATIONSHIP. See **burning rate, linear**.

MUSKINGUM METHOD. A method of **streamflow routing** in which the **channel storage** S is assumed to conform to the equation

$$S = K[xI + (1 - x)O],$$

where I and O are instantaneous values of inflow and outflow respectively, x is a dimensionless constant reflecting the relative importance of I and O in determining storage, and K is a storage constant with the dimension of time. K and x are characteristics of a particular **reach** and are determined by analysis of past floods in the reach. The method assumes that storage is a linear function of the weighted flow in the reach and is adaptable to a simple mathematical solution.

MU SPACE. **Phase space** of $2f$ dimensions, the coordinates being the f generalized coordinates and f generalized momenta of one particle. It is the phase space of one mmolecule and was called μ -space by Ehrenfest to distinguish it from Γ -space.

MUTUAL CAPACITANCE. See **capacitance**.

MUTUAL DIFFUSION, COEFFICIENT OF. A quantity in the kinetic theory of gases which measures the tendency of gases to diffuse into one another in non-turbulent flow. This diffusion coefficient is a property of the gases in question and of the assumed nature of the molecular impacts in the **diffusion** process. For rigid, perfectly elastic, spherical molecules the coefficient of mutual diffusion d_{12} is

$$d_{12} = \frac{3}{4n(\sigma_1 + \sigma_2)^2} \left[\frac{2kT(m_1 + m_2)}{\pi m_1 m_2} \right]^{1/2} \text{ cm}^2/\text{sec},$$

where n is **Loschmidt's number** (the number of molecules per cm^3), σ_1 , σ_2 , and m_1 , m_2 are the effective molecular diameters and masses of the two gases, respectively, T is the temperature in degrees Kelvin, and k is **Boltzmann's constant**. (For further details, see Lettau, *Compendium of Meteorology*.)

MUTUAL IMPEDANCE. See **impedance**.

MUTUAL INDUCTANCE. See **inductance**.

MUTUAL INTENSITY. See **partial coherence**.

N

NABLA. A name used for ∇ , the **vector** differential operator generally called **del**. The shape of the symbol is thought to be similar to that of an Assyrian harp with that name.

NAPIERIAN LOGARITHMS. See **logarithms**.

NARROW RESONANCE MODEL. In the theory of resonance capture of neutrons in a chain reacting system, the calculational model in which it is assumed that a single collision with the heavy, absorbing atom is sufficient to degrade the neutron to energies below the resonance in question. (See **infinite absorber model**.)

NATURAL AND UNNATURAL PROCESSES. Processes which bring a thermodynamic system nearer to its equilibrium are often called natural processes. An unnatural process is then one in a direction away from equilibrium. While there is no limitation in regard to the scale or the duration of a natural process, an unnatural process can only occur over a limited range of space and of time. An unnatural process may be considered as a *fluctuation*. It has to be considered in the statistical treatment, while it may be neglected under many circumstances in a purely phenomenological thermodynamical treatment.

There are, however, cases in which fluctuations may acquire a macroscopical character (e.g., fluctuations near the critical point, critical opalescence).

Reversible processes are in a sense a limiting case between natural and unnatural processes (see **reversible and irreversible processes**) which consist in the passage of a system through a continuous series of equilibrium states.

NATURAL LINE WIDTH. The irreducible finite width of a spectral line which remains after all causes of line broadening other than radiation damping have been removed.

According to classical theory the natural line width, or (natural) "whole half width," is given by

$$(\Delta\lambda)_{\text{natural}} = \frac{1}{2\pi c} \lambda^2 \gamma_{cl}$$

where

$$\gamma_{cl} = \frac{8\pi^2 e^2}{3mc} \frac{1}{\lambda^2}$$

is the classical radiation damping constant. Therefore

$$(\Delta\lambda)_{\text{natural}} = \frac{4\pi e^2}{3mc^2} = 1.18 \times 10^{-4} \text{A}$$

independent of wavelength λ .

(For a different usage of the term "natural line width" see **half width of a spectral line**. This meaning of the term is less common than the one used here, but corresponds more closely to the quantum mechanical concept (see below).)

In the quantum mechanical treatment of the problem γ_{cl} has to be replaced by

$$\gamma_m + \gamma_n = \frac{1}{T_m} + \frac{1}{T_n},$$

where T_m and T_n are the **mean life times** of the upper and lower energy levels involved in the transition. The natural line width, defined this way, is not constant and has a different value for each spectral line. For "**allowed**" transitions in the optical region, however, $(\gamma_m + \gamma_n)$ is of the same order of magnitude as γ_{cl} , so that the value for the natural line width obtained from classical theory remains a good approximation.

NATURAL LOGARITHMS. See **logarithms**.

NATURAL WIDTH OF LEVEL. The width of an atomic or nuclear energy level due to its spontaneous transition lifetime.

$$\Gamma = \hbar/\tau$$

where τ is the lifetime of the state.

NATURAL YIELD LIMIT. See **limit, yield**.

NAUTICAL ASTRONOMY. (See also **line of position**.) The fundamental problem in the determination of the position of a ship at sea

is to find the direction of a line of position on which the ship is located and the point on the line where the ship is found. The data available to the navigator are (1) the dead reckoning position ($DR \phi$ and $DR Lo$), (2) the Greenwich mean time, (3) the altitude of a celestial object for which the equatorial coordinates are known.

The Greenwich Mean Time (GMT) is carried by the ship's chronometer. The chronometer is checked so frequently by radio time signals that GMT is known within five seconds. The altitude of the celestial object is obtained by measuring the apparent altitude with a sextant and correcting the observation for instrumental errors, dip of the horizon, refraction, etc. The errors inherent in the observation itself and the uncertainty of some of the corrections are such that the so-called observed altitude (h_o) has a probable error of about 1'.

In the various methods for computing the altitude and azimuth of the celestial object for a given dead reckoning position and time, slightly different terminology is used from that found in astronomical definitions. The term bearing, B , is used instead of the astronomical horizontal coordinate, azimuth. Bearing is measured in the plane of the horizon from the north toward the east to the point of intersection of the vertical circle through the object with the horizon. In many cases bearing is measured from the north to the east or west through 180° in which case the symbol B' , should be used. Astronomers measure local hour angle, LHA , from the south to the west along the equator. Navigators frequently use $360^\circ - LHA$. There is no confusion in the mind of the navigator for he knows from observation whether the object that he is using is east or west of the meridian.

In the development of the various forms for computing the altitude and bearing of the observed object from the DR position, navigators use a trigonometric function known as the haversine, hav , which is defined as

$$hav \theta = \frac{1 - \cos \theta}{2}$$

Tables of haversines, with arguments in units of both time and arc, are published in various tables for navigators such as *The American Practical Navigator*, H.O. (U. S. Hydrographic

Office) Pub. No. 9, a volume that is frequently known as "the navigator's bible."

The following notation will be used in writing the various equations used by navigators for transforming coordinates.

- LHA is the local hour angle of the object.
- dec is the declination of the object. ($90^\circ - dec$) is polar distance P .
- ϕ is the latitude.
- h is the corrected altitude. ($90^\circ - h$) is zenith distance, Z .
- B is the bearing.

1. *General equations*

$$\begin{aligned} \cos z &= \sin dec \sin \phi \\ &\quad + \cos dec \cos \phi \cos LHA \\ \cos B \sin z &= \sin dec \cos \phi \\ &\quad - \cos dec \sin \phi \cos LHA \end{aligned}$$

$$\sin B \sin z = \cos dec \sin LHA \text{ (check equation).}$$

2. *Astronomer's formulae for logarithmic computation*

M is a parametric angle

$$\begin{aligned} \tan M &= \cotan dec \cos LHA \\ \tan B &= \sin M \tan LHA \sec(\phi + M) \\ \tan z &= \sec B \cotan(\phi + M). \end{aligned}$$

3. *The so-called navigator's method*

This has been used for nearly two hundred years and is still preferred by some navigators.

$$\text{let } s = \frac{\phi + P + h}{2} \text{ (} P \text{ is polar distance)}$$

$$\begin{aligned} \text{hav } LHA &= \cos s \sec \phi \operatorname{cosec} P \sin(s - h) \\ \text{hav } B &= \sec h \sec \phi \sin(s - h) \sin(s - \phi). \end{aligned}$$

It is more convenient to define two other quantities:

t is the meridian angle measured either east or west from the meridian through 180°
 B' is measured from the elevated pole either east or west through 180° .

Then

$$\begin{aligned} \text{hav } t &= \cos s \sec \phi \operatorname{cosec} P \sin(s - h) \\ \text{hav } B' &= \sec h \sec \phi \sin(s - h) \sin(s - \phi). \end{aligned}$$

4. *Cosine-haversine method*

This is most commonly used at the present time for the determination of a line of position (*LOP*).

In this method we define a parametric angle θ :

$$\text{hav } \theta = \cos \text{dec} \cos \phi \text{ hav } t$$

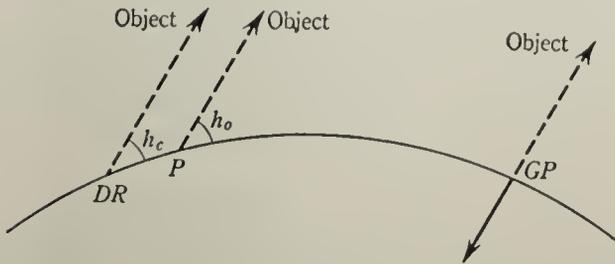
$$\text{hav } z = \text{hav } \theta + \text{hav } (\phi - \text{dec})$$

$$\sin B' = \cos \text{dec} \sin t \text{ cosec } z.$$

This may lead to some ambiguity in B' and B but it is easily cleared up in practice.

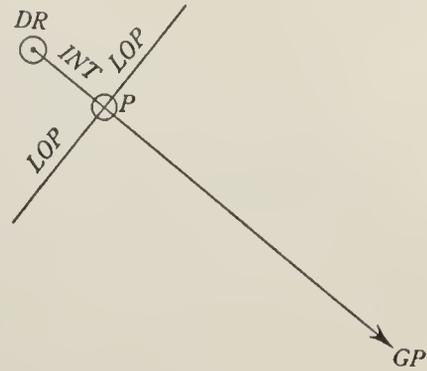
In addition to the computational methods for finding the altitude and bearing of a celestial object using DR ϕ and Lo , Hydrographic Office Publication 214 (H.O. 214) solves the problem by tables. The publication is in eight large volumes and is a bit cumbersome in a small chart room or plane, but they solve the problem both quickly and accurately.

Once we have the computed altitude and bearing of the observed object the determination of the line of position (*LOP*) becomes an exercise in geometry. The figure below shows



a great circle representing the surface of the earth in the plane containing the observer and the observed object. GP represents the point where a line from the object to the center of the earth would pass through the surface of the earth. For all astronomical objects, other than the moon, the distances are so great in comparison with the radius of the earth that we may assume that they are parallel. Therefore one may assume that $DR-O$ and $P-O$ are parallel. However the angles h_o and h_c are not the same. The difference $h_o - h_c$, which is known as the intercept, is equal to the distance in minutes of arc, or nautical miles, between DR and P . This distance is measured in the direction of the object or along a line of bearing. The figure in next column is a representation of the surface of the earth on a plane representing the region in which the navigator is working. The computed direction of the observed object is represented by the line from DR toward GP , which is the computed bearing

of the object. The intercept ($h_o - h_c$) is marked off along the bearing line either toward or away from GP (toward when $h_o - h_c$



is positive). Usually the problem is plotted on a chart or a small area plotting sheet and the coordinates of the most probable position (MPP) are taken from the chart or sheet. However the coordinates of MPP may be determined by computation or by traverse tables treating the intercept as course and distance from the DR position.

NAUTICAL MILE. The unit of distance used in navigation. The length of a nautical mile is 6076.103 U.S. feet and is equivalent to the length of a minute of latitude.

NAVIER-STOKES EQUATIONS. Equations of motion for a fluid in which the viscous stress is proportional to the rate of strain, viz.

$$\begin{aligned} \frac{Du_i}{Dt} &= \frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} \\ &= -\frac{1}{\rho} \frac{\partial p}{\partial x_i} + F_i + \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} + \frac{1}{3} \nu \frac{\partial \Delta}{\partial x_i}. \end{aligned}$$

The upper line represents the inertia force of unit mass and the terms in the lower line respectively the pressure gradient force, the body force per unit mass, the viscous stresses resulting from straining or distorting the fluid, and the contribution to the gradient of normal pressure resulting from a spatial variation of the dilation Δ (or rate of cubical expansion;

$$\Delta = \frac{\partial u_i}{\partial x_i} = \text{div } \mathbf{v}.$$

The last term is zero in incompressible fluids and is usually neglected except in compressible flows with large accelerations. ν is the dynamic viscosity.

In vector notation, the equations are written

$$\begin{aligned} \frac{D\mathbf{v}}{Dt} &= \frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v} \cdot \text{grad})\mathbf{v} \\ &= -\frac{1}{\rho} \text{grad } p + \mathbf{F} + \nu(\text{div grad})\mathbf{v} \\ &\quad + \frac{1}{3}\nu \text{grad div } \mathbf{v}. \end{aligned}$$

which may be rearranged in the form

$$\begin{aligned} \frac{\partial\mathbf{v}}{\partial t} + \frac{1}{2} \text{grad } v^2 - \mathbf{v} \times \text{curl } \mathbf{v} \\ = -\frac{1}{\rho} \text{grad } p + \mathbf{F} + \frac{4}{3}\nu \text{grad div } \mathbf{v} \\ - \nu \text{curl curl } \mathbf{v}. \end{aligned}$$

NAVIER-STOKES EQUATIONS, GENERAL TREATMENT FOR MIXTURES. The Navier-Stokes equations are the most important of the equations of change (see **Enskog-Maxwell equations of change**). They are derived from the first approximation to the solution of the **Boltzmann equation** by either the Chapman-Enskog method (See **Enskog series**) or the **13-moments approximation**. They are:

(1) The *equations of continuity* of the individual components of a mixture:

$$\frac{dn_i}{dt} = n_i \left(\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{v} \right) - \left(\frac{\partial}{\partial \mathbf{r}} \cdot n_i \mathbf{v}_i \right) + \mathbf{r}. \quad (1)$$

(2) The *equation of continuity* for the system as a whole:

$$\frac{d\rho}{dt} = -\rho \left(\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{v} \right)$$

or

$$\frac{\partial\rho}{\partial t} = -\left(\frac{\partial}{\partial \mathbf{r}} \cdot \rho \mathbf{v} \right). \quad (2)$$

(3) The *equation of motion*:

$$\frac{\partial\boldsymbol{\omega}}{\partial t} = -\frac{1}{\rho} \left(\frac{\partial}{\partial \mathbf{r}} p \right) + \frac{1}{\rho} \sum_i n_i \mathbf{X}_i. \quad (3)$$

(4) The *equation of energy balance*:

$$\begin{aligned} \frac{d\hat{U}}{dt} &= -\frac{1}{\rho} \left(\frac{\partial}{\partial \mathbf{r}} \cdot \mathbf{q} \right) - \frac{1}{\rho} \left(p \cdot \frac{\partial}{\partial \mathbf{r}} \boldsymbol{\omega} \right) \\ &\quad + \frac{1}{\rho} \sum_i n_i (\mathbf{v}_i \cdot \mathbf{X}_i). \quad (4) \end{aligned}$$

$d/dt = \partial/\partial t + \mathbf{v} \partial/\partial \mathbf{r} + (\mathbf{X}/n)(\partial/\partial \mathbf{v}_i)$ is the **total derivative** as measured by an observer

which follows an element of volume of the fluid; n_i is the concentration of the molecular species i in particles per cubic centimeter; ρ is the density; \mathbf{v}_i is the diffusion velocity of component i ; \mathbf{r}_i is the rate of production of i by chemical reactions; $\boldsymbol{\omega}$ is the mass average velocity; p is the pressure tensor; \mathbf{X}_i is the external force acting on a molecule of i , \hat{U} is the energy per unit mass of the system; \mathbf{q} is the heat flux vector (including internal, and eventually radiation energy).

(1) The equation of continuity for the individual components of a mixture expresses the fact that the concentration of component i , as measured by an observer moving at the mass average velocity of the fluid, can change either because of an expansion or contraction of the fluid during its movement, or because of chemical reactions producing this species.

(2) The overall equation of continuity shows that the total density measured by the same observer can only change if the fluid expands or contracts during its movement.

(3) The equation of motion shows that the velocity changes either because of the existence of a pressure gradient (both normal and shearing stresses appear in the pressure tensor, p) or because of external forces acting on the molecules.

(4) The equation of energy balance expresses the balance of energy; the variation of the energy content of a volume element moving at the velocity of the fluid can result from the divergence of the heat flux, from the pV work and from the work of viscous forces, or finally because of the work accomplished by the external forces.

The Navier-Stokes equations are the starting point for the hydrodynamic theory when the energy, velocity or density gradients are small. (These gradients appear to the first order in the right-hand side of Equations (1, 2, 3, 4), and no second or higher order derivatives appear.)

NEAREST NEIGHBOR. Any atom whose distance from a given atom in a **crystal lattice** is such that no other atom is closer to the given atom. The number of nearest neighbors is the **coordination number** of the lattice.

NÉEL TEMPERATURE. See **antiferromagnetism**.

NEGATIVE ENERGY STATE. See *energy state, negative*.

NEGATIVE TOTAL CURVATURE, SURFACE OF. See *antielastic surface*.

NEIGHBORHOOD. See *topological space*.

NEPER. A dimensionless unit used for expressing the ratio of two voltages or two currents, the number of nepers being the natural logarithm of such a ratio. When conditions are such that the power ratio is proportional to the square of the voltage or current ratio, the number of nepers is one half of the natural logarithm of the power ratio. One neper is thus equal to 8.686 *decibels*. Its use has been extended to acoustics and mechanics, where quantities similar to current and voltage are involved. (See *gain ratio*.)

NERNST COEFFICIENT. See *galvanometric and thermometric effects*.

NERNST DISTRIBUTION LAW. Consider two phases α and β . The equilibrium condition for the distribution of component i between the two phases is (see *chemical potentials* and *activity coefficients*).

$$\frac{x_i^\beta \gamma_i^\beta}{x_i^\alpha \gamma_i^\alpha} = K_i(T, p) \quad (1)$$

where x_i^α , x_i^β are the mole fractions of i in the phases α , β and γ_i^α , γ_i^β , the activity coefficients of i . The equilibrium constant $K_i(T, p)$ is called the distribution or partition coefficient of the substance i between the phases α and β .

This equation expresses the Nernst distribution law.

NERNST EFFECT. See *galvanometric and thermometric effects*.

NERNST HEAT THEOREM. See *thermodynamics, third law of*.

NET. A set of intervals such that every point of a closed linear interval $[a, b]$ is contained in at least one interval of the set, each interval being called a *mesh* of the net.

NET CALORIFIC VALUE. See *combustion*.

NET SECTION. See *gross section*.

NETWORK, ACTIVE. A network having generators or other energy sources among its components.

NETWORK, ANNULING. An arrangement of *impedance* elements connected in parallel with filters to annul or cancel capacitive or inductive reactance at the extremes of a filter pass-band.

NETWORK, BALANCING. An arrangement of impedances connected to one branch of a hybrid *network* to match the impedance of a line connected to the opposite branch.

NETWORK, CONNECTED. A *network* in which there exists at least one path, composed of *branches* of the network, between every pair of *nodes* of the network.

NETWORK, CUT-SET. A set of *branches* of a *network* such that the cutting of all the branches of the set increases the number of separate parts of the network, but the cutting of all the branches except one does not.

NETWORK, ELECTRIC. See *electric network*.

NETWORK ELEMENT. A two-port system (electrical, mechanical, or a combination of both) whose input-output characteristics may be expressed

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \end{bmatrix}.$$

The element is a *reciprocal element* if $a_{12} = a_{21}$, and is *antireciprocal* if $a_{12} = -a_{21}$. When neither of these conditions exists, the network is *non-reciprocal*.

NETWORK FUNCTION, STABLE LINEAR. A linear *network* function all of whose *poles* of transmittance lie in the interior of the left-half of the complex-frequency plane.

NETWORK, HYBRID. A bridge-type circuit or connecting device that combines the functions of providing impedance matching between certain circuits and isolation between other circuits. A hybrid is often used to connect a four-wire line to a two-wire line so that both directions of transmission on the four-wire line are isolated from each other, but are connected to the two-wire line.

NETWORK, LINEAR. A system (network) for which the superposition of *excitation functions* results in a response which is the superposition of the individual *response functions*, and additionally, which exhibits the property

of homogeneity, i.e., the magnitude scale factor of the excitation is preserved in the response.

NETWORK, LINEAR PASSIVE. A network such that (1) if currents of any waveform are fed to the terminals of the network, the total energy delivered to the network is non-negative; (2) no voltages appear between any pair of terminals before a current is fed to the network.

NETWORK MESH. A set of branches forming a closed path in a network, provided that if any one branch is omitted from the set, the remaining branches of the set do not form a closed path. The term loop is sometimes used in the sense of mesh.

NETWORK NODE (JUNCTION POINT), (BRANCH POINT), (VERTEX). A terminal of any branch of a network or a terminal common to two or more branches of a network.

NETWORK, NONLINEAR. A network (circuit) not specifiable by linear differential equations with time as the independent variable.

NETWORK, PASSIVE. A network whose output waves are independent of any sources of power which is controlled by the actuating waves.

NETWORK, RECIPROCAL. The term reciprocal network is widely used with two different meanings. One is the meaning that refers to the applicability of the **reciprocity principle**, whereas the other refers to a network whose function is the reciprocal of the function of another network. Inasmuch as neither term is standardized, it is largely a matter of personal preference among authors. It can be argued that the networks that obey the reciprocity principle are a larger class and therefore the term reciprocal is more effectively utilized in that sense. (Cf. discussion of the term **bilateral**.)

NETWORKS, DUAL. Networks having the same type of characterizing equations, except that one is expressed in terms of **node voltages** and the other in terms of **loop currents**.

NETWORKS, KIRCHHOFF LAWS OF. See Kirchhoff laws of networks.

NEUBER-PAPKOVICH STRESS FUNCTIONS. Any solution for the displacement components u_i ($i = 1, 2, 3$), in a rectangular Cartesian coordinate system x , of the Navier equations of equilibrium for a body of isotropic, elastic material satisfying the **generalized Hooke's law**, on which no body forces are acting, may, provided it has sufficient regularity, be written in the form

$$2\mu u_i = (3 - 4\sigma)\phi_i - x_j \frac{\partial \phi_j}{\partial x_i} - \frac{\partial \phi_o}{\partial x_i},$$

where ϕ_o and ϕ_i ($i = 1, 2, 3$) are four harmonic functions of position in space, the *Neuber-Papkovich stress functions*, or *Papkovich stress functions*. μ and σ are the **rigidity modulus** and **Poisson's ratio** for the material. Such a solution of the Navier equations is called a *Neuber-Papkovich solution* or *Papkovich solution*.

NEUMANN FUNCTION. Also called a Bessel function of the second kind, it is defined by

$$N_n(x) = \csc n\pi [\cos n\pi J_n(x) - J_{-n}(x)]$$

where $J_n(x)$ is a **Bessel function**. The general solution of the Bessel differential equation can be taken as $y = AJ_n(x) + BN_n(x)$, where A, B are integration constants. The index n may be either non-integral or integral but in the latter case $N_n(x)$ contains a logarithmic term so that this solution is usually unsuitable for a physical problem because of its behavior at $x = 0$.

NEUMANN'S FUNCTION. A function of the form

$$N(p, q) = \frac{1}{4\pi r} + V(p),$$

where R is a region with a boundary surface S , q is an interior point of R , r is the distance pq , $V(p)$ is harmonic, V is the potential (hence the name potential function), $\partial N/\partial n$ is constant on S and $\iint_S N dS_p = 0$. The solution of the Neumann problem is

$$\phi(q) = \iint_S f(p)N(p, q) dS_p.$$

NEUMANN'S PROBLEM. See boundary value problem.

NEUMANN'S PRINCIPLE. The physical properties of a crystal cannot be of lower symmetry than the symmetry of the external form of the crystal. Thus, for example, all the properties of a cubic crystal must have cubic symmetry—which means that any tensor property such as susceptibility, resistivity, thermal expansion, etc., must be **isotropic**.

NEUMANN'S SERIES. See **Liouville-Neumann series**.

NEUTRAL AXIS. When a **beam** undergoes a simple flexure, a line initially parallel to the length of the beam which does not change its length in the deformation is called a neutral axis. If all the lines parallel to the length of the beam, which lie in a plane, remain unchanged in length, the plane is called a neutral surface or plane. Therefore, the neutral axis is the line of intersection of the neutral surface of a beam with a cross-sectional plane. Fibers to one side of the neutral surface are extended, to the other side they are compressed. To avoid confusion, the neutral axis and neutral surface usually are defined for pure bending alone.

In a straight linearly-elastic beam the neutral axis passes through the **centroid** of the cross section. In a curved elastic beam the neutral axis is displaced toward the center of curvature (see **Winkler-Bach formula**). In a curved, or straight, perfectly plastic beam the neutral axis divides the cross-sectional area into two equal parts.

NEUTRAL EQUILIBRIUM. See **equilibrium, neutral**.

NEUTRAL SURFACE. See **neutral axis**.

NEUTRAL WAVE. Any wave whose amplitude does not change with time. In most contexts these waves are referred to as *stable waves*, the term "neutral wave" being used when it is important to emphasize that the wave is neither damped nor amplified. (See **permanent wave**.)

NEUTRINO. A particle of zero rest mass and spin $\frac{1}{2}$ (in units of $h/2\pi$), postulated in order to conserve energy and momentum in the process of β -decay. Since its interaction with matter takes place only through the extremely weak β -decay interaction it can pass through many kilometers of matter without suffering a collision.

NEUTRINO, TWO-COMPONENT EQUATION. See **two-component equation of the neutrino**.

NEUTRON. An electrically neutral elementary particle of mass very nearly equal to that of the proton, and a constituent of all nuclei other than that of hydrogen. In the free state it is unstable with respect to **beta-decay**, decaying to a proton with a **half-life** of about 12 min. Being uncharged it interacts with matter mainly by direct collisions with nuclei, although to a lesser extent it can also interact magnetically through its magnetic moment. Compared with an ionizing particle like a proton it can therefore pass through relatively large amounts of matter before being brought to rest or captured. Its rest mass is 1.00894 amu; spin quantum number $\frac{1}{2}$; magnetic moment -1.9125 nuclear magnetons; Fermi-Dirac statistics.

NEUTRON COLLISION DENSITY. See **collision density, neutron**.

NEUTRON CURRENT. See **current, neutron**.

NEUTRON DENSITY. See **density, neutron**.

NEUTRON DIFFERENTIAL AREA. See **differential area, neutron**.

NEUTRON DIFFUSION COEFFICIENT. See **diffusion coefficient, neutron**.

NEUTRON DIFFUSION LENGTH. See **diffusion length, neutron**.

NEUTRON DIFFUSION THEORY. A simplified version of **neutron transport theory**, based upon the **Fick law** and the conservation of neutrons. For neutrons of speed v and flux $\Phi(r,t)$ the diffusion equation is:

$$\frac{1}{v} \frac{\partial \Phi}{\partial t} = \text{div} (D \text{ grad } \Phi) - \sum_a \Phi + S(r,t).$$

The formal deduction of diffusion theory from transport theory is based upon retention of only the lowest order terms obtained in a **spherical harmonics** analysis of the latter.

NEUTRON DIFFUSION THEORY, BOUNDARY CONDITIONS. See **boundary conditions, diffusion theory**.

NEUTRON DIFFUSION THEORY, MULTI-GROUP. An approximation to polycenergetic

neutron transport theory in which the spatial migration of neutrons is described by the **Fick law** and the energy distribution is replaced by the assignment of neutrons to a number of energy "groups." The result is a set of coupled differential equations for the group fluxes $\Phi_j(r)$.

NEUTRON DIFFUSION TIME. See *diffusion time, neutron*.

NEUTRON EXCESS. The difference between the number of neutrons and the number of protons in the nucleus; found by subtracting the atomic number of that nuclide from the neutron number; or by subtracting twice the atomic number from the mass number.

NEUTRON FLUX, ANGULAR. Defined under *flux, neutron, total*.

NEUTRON FLUX, TOTAL. See *flux, neutron, total*.

NEUTRON LIFETIME. See *lifetime, neutron*.

NEUTRON MEAN FREE PATH. See *mean free path*.

NEUTRON MIGRATION AREA. See *migration area for neutrons*.

NEUTRON MIGRATION LENGTH. See *migration length for neutrons*.

NEUTRONS, DELAYED. Certain nuclei undergo β -decay to such a highly excited state of the product nucleus that a neutron may be emitted from the latter. These neutrons therefore appear exponentially in time with the half-life of the parent β -emitter. An example is N^{17} which decays to states in O^{17} which are above the neutron emission threshold. A number of fission products also have this property, so that following fission a certain small fraction of the neutrons produced will appear after the fission has taken place. Since the delayed neutron is emitted by the parent nucleus with imperceptible delay in time, the time interval between fission and delayed neutron emission depends only upon the radioactive lifetimes of the intervening members of the fission chain. The identified half-lives of delayed neutrons are 55.6, 22.0, 4.51, 1.52, and 0.43 secs. The β -decaying parents of the two longest activities are known to be Br^{87} and I^{137} . Delayed

neutrons amount to about 0.75% of the total number of neutrons produced in the thermal neutron fission of U^{235} . Although a small fraction, they are instrumental in making it possible to control easily the power level of a nuclear reactor.

NEUTRON SLOWING-DOWN AREA. See *slowing-down density*.

NEUTRON SLOWING-DOWN LENGTHS. See *slowing-down lengths*.

NEUTRON SLOWING-DOWN TIME. See *slowing-down time*.

NEUTRON SPECTRUM. See *spectrum, neutron*.

NEUTRONS, PROMPT. Neutrons emitted simultaneously with the event of nuclear fission.

NEUTRONS, RECIPROCITY THEOREM FOR. See *reciprocity theorem for neutrons*.

NEUTRONS, THERMAL. In the steady-state distribution-in-energy of neutrons diffusing through matter, it is often possible to isolate a component strongly peaked about the energy kT , where k is Boltzmann's constant and T the Kelvin temperature of the moderator. Neutrons in this group, which extends to about 0.3 ev, are called thermal neutrons.

NEUTRONS, THERMALIZATION OF. See *thermalization of neutrons*.

NEUTRON STREAMING. See *streaming, neutron*.

NEUTRON TEMPERATURE. See *temperature, neutron*.

NEUTRON TRACK LENGTH. See *track length, neutron*.

NEUTRON TRANSPORT THEORY. The theory describing the migration of neutrons through matter. Let $N(\mathbf{r}, E, \hat{\Omega}, t) dV dE d\hat{\Omega} dt$ be the number of neutrons in volume element dV at position \mathbf{r} during the time interval between t and $t + dt$ which have energies lying between the E and $E + dE$ and travel in directions lying within the element of solid angle $d\hat{\Omega}$ about unit vector $\hat{\Omega}$. N satisfies the transport equation

$$\frac{\partial N}{\partial t} + v\hat{\Omega} \cdot \nabla N + \Sigma N v$$

$$= \int_0^\infty dt' \int_0^\infty dE' \int d\hat{\Omega}' \Sigma(E') v' N(\mathbf{r}, E, \hat{\Omega}', t - t')$$

$$\times C(E', t') f(E', \hat{\Omega}'; E, \hat{\Omega}; t') + S(\mathbf{r}, E, \hat{\Omega}, t).$$

Here, v is the neutron velocity, $\Sigma(E)$ is the total macroscopic cross-section for collision with atoms composing the material and Cf is the average number of secondary neutrons appearing within the energy range $E, E + dE$ and directions $\hat{\Omega}, \hat{\Omega} + d\hat{\Omega}$ with a delay time of t' after the collision of a neutron with an atom. This equation, which is a linear version of the Boltzmann transport equation, is sometimes referred to as the "Boltzmann Equation" (for neutrons). In most applications effects of delay time are neglected.

NEUTRON TRANSPORT THEORY, METHOD OF ITERATION. See method of iteration.

NEUTRON TRANSPORT THEORY, ONE-VELOCITY FORM. An approximate mathematical theory describing the migration of neutrons through matter, in which it is assumed that all neutrons have the same speed, v .

$$\frac{1}{v} \frac{\partial N}{\partial t} + \hat{\Omega} \cdot \nabla N + \Sigma N$$

$$= C \Sigma \int d\hat{\Omega}' f(\hat{\Omega}'; \hat{\Omega}) N(\mathbf{r}, \hat{\Omega}') + S.$$

(For notation see neutron transport theory, where a more general formulation is presented.)

NEVILLE METHOD OF INTERPOLATION. See Aitken method of interpolation.

NEWTON. A unit of force in MKS system which will impart an acceleration of one meter per second per second to a mass of one kilogram (equals 10^5 dynes).

NEWTON-COTES QUADRATURE FORMULAS. Formulas of the form

$$\int_{x_0}^{x_n} f(x) dx = \eta_0 f_0 + \eta_1 f_1 + \cdots + \eta_n f_n + R_n,$$

where the coefficients η_i are obtained by integrating the coefficients of the **Lagrange interpolation formula**. The best known and most

useful of these are the **trapezoidal rule**, **Simpson's rule**, and the **three-eighths rule**.

NEWTON EQUATION. If the intersection of the optical axis with a plane through the principal focal point of object space is taken as the origin of coordinates for points in the object space and a similar origin is taken for coordinates in image space, the collineation equations of **Gaussian optics** reduce to

$$xx' = ff'$$

where x, x' are distances from their respective origins parallel to the optical axis and f, f' are focal distances measured from their respective origins. This form of the **collineation equations** is called Newton's equation.

NEWTON-GAUSS INTERPOLATION FORMULAS. See Gaussian interpolation formulas.

NEWTONIAN FLUID. A fluid in which the stresses resulting from viscosity are linearly related to the instantaneous rate of strain and are independent of its time variations. The **Navier-Stokes equations** apply to an isotropic Newtonian fluid.

NEWTONIAN MECHANICS. A self consistent mechanics based on the **Newton laws** of motion and assuming the validity of the **Galilean transformation**.

NEWTONIAN VISCOSITY. Viscous flow for which the rate of shear strain is proportional to the shear stress, that is, the coefficient of viscosity is constant.

NEWTON INTERPOLATION FORMULAS. See difference operators.

NEWTON LAW FOR HEAT LOSS (COOLING). The heat loss by convection from one body to another is proportional to the temperature difference between the two bodies. This law holds only for small temperature differences and then only approximately. Nevertheless, in heat transfer by forced or neutral convection it is postulated that the rate Q of heat transferred per unit time is

$$Q = Ah(t - t_o) \quad (1)$$

where A is the area, h , the coefficient of heat transfer, t the temperature of the fluid and t_o the temperature of the solid wall. The *coefficient of heat transfer* h is a weak function of

temperature. When expressed in dimensionless form (see **dimensionless numbers**) as a Nusselt number

$$Nu = \frac{hl}{k} \quad (2)$$

where l is the characteristic length and k , the thermal conductivity. It is a function of the **Reynolds**, **Prandtl** and **Grashof** numbers in forced and natural convection

$$Nu = f(Re, Pr, Gr). \quad (3)$$

Hence the Nusselt number depends on temperature only, insofar as the properties of the gas, expressed in Reynolds, Prandtl and Grashof numbers, depend on temperature.

Newton's law of cooling, Equation (1), breaks down in the case of compressible flow and in cases when the wall temperature t_o is not uniform. In compressible flow it can still be used if t_o is interpreted as the adiabatic temperature given by

$$T_a = T + \frac{1\mathcal{V}^2}{2c_p},$$

where T is the absolute temperature of the stream.

NEWTON LAW OF FLUID RESISTANCE (FRICTION). The force on a solid body in a steady stream is proportional to the square of the velocity. It can be proved by dimensional arguments for large **Reynolds numbers** provided that no large gravity or sound waves are produced (i.e., provided that the body leaves only a turbulent wake) and that the Mach number is small compared with unity.

NEWTON LAW OF UNIVERSAL GRAVITATION. Every particle in the universe attracts every other particle with a force that is directly proportional to the product of their masses, and inversely proportional to the square of the distance between their centers of mass. The constant of proportionality in the cgs system of units is

$$G = 6.670 \times 10^{-8} \frac{\text{dyne cm}^2}{g^2}.$$

(See **Newton theory of gravitation**.)

NEWTON LAWS OF MOTION. (1) A particle remains at rest or in a state of uniform motion in a straight line unless acted upon by an external force. (2) The acceleration

produced by a force is directly proportional to the force and inversely proportional to the mass of the particle which is being accelerated. (3) To every action there is an equal and opposite reaction.

Newton's laws of motion may be considered as the basic postulates of the theory of mechanics. They may be extended to systems of particles and to continuous bodies by the assumption that these bodies may be treated as collections of particles. They are also limited to instances in which classical mechanics apply, i.e., to cases in which the speeds of motion are small compared to the speed of light.

NEWTON METHOD (FOR SOLVING EQUATIONS, WHETHER ALGEBRAIC OR TRANSCENDENTAL). A particular **iterative method**, and the one most commonly used. The method can also be extended to **simultaneous equations** and to **matrix inversion**. (See also **polynomial factorization**.)

NEWTON-STIRLING INTERPOLATION FORMULAS. See **Stirling interpolation formulas**.

NEWTON THEORY OF GRAVITATION. Newton's conception of gravitation was expressed by his statement to the effect that every particle of matter attracts every other particle with a force proportional to the product of the masses and to the inverse square of the distance. We are thus left to picture an infinitely complex network of attractions joining every two particles in the universe and tending to pull them together. Newton did not specify what the "particles" were assumed to be, whether atoms or otherwise.

The Newtonian law may be expressed by the equation $f = Gm_1m_2/r^2$, in which m_1 and m_2 are the masses of two particles, r the distance between them, and G the gravitation constant. The value of G in the cgs system of **units** is

$$6.670 \times 10^{-8} \frac{\text{dyne cm}^2}{g^2}.$$

For practical purposes the "particles" may be homogeneous spheres, r being the distance between their centers. Other bodies of finite size, such as cubes or cylinders, would not do, as they are not "centrobaric"; that is, there is no one point toward which their attraction is di-

rected. The planets and stars, being sensibly spherical, may be treated approximately as particles. It was from the study of the two-body problem as applied to such objects that Newton deduced the conclusion expressed in his law.

NEYMAN-PEARSON THEORY. A general theory of testing statistical hypotheses due to J. Neyman and E. S. Pearson. It is based upon the fact that errors of two different kinds can be made: (1) the rejection of a hypothesis when it is true, (2) the acceptance of a hypothesis when it is false. Methods are developed for controlling errors of the first kind at fixed probabilities and of minimizing the probability of the second kind of error. Tests are compared by the power function, which is the probability of rejecting the hypothesis under examination when some other hypothesis is true—the complement of the probability of errors of the second kind.

NIJBOER-ZERNIKE ABERRATION FUNCTIONS. Let V be the optical path length from an object point through an axially symmetrical optical system to its Gaussian image point, expressed as a function of the polar coordinates r, ϕ of the point of intersection with the exit pupil. The individual terms of the expression

$$V = \sum a_{mr} r^m (\cos \phi)^n$$

are the classical **aberration** functions, including the third order **Seidel** and fifth order **Schwarzschild** aberrations.

Zernike developed the expansion in orthogonal functions

$$V = \sum b_{mn} R_n^m(\rho) \cos m\phi$$

where $R_n^m(\rho)$ are the circle polynomials, $\rho = r/r_{max}$ for r_{max} the radius of the **exit pupil**. Nijboer has classified the aberration functions corresponding to the individual terms of this expansion. The resulting theory is called the *diffraction theory of aberrations*.

NILPOTENT. An **operator**, generally represented in **matrix** form, which satisfies the relation

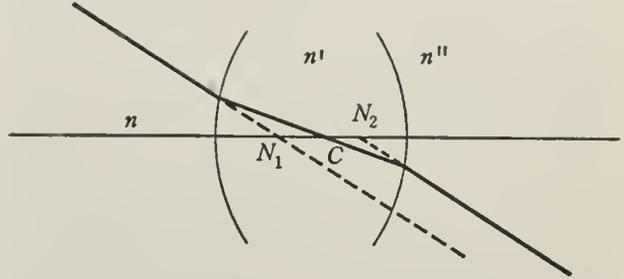
$$A^n = 0$$

for some value of n . (See also **idempotent**.)

NIT. A unit of **luminance**, equal to 1 **candle**/sq m.

NODAL PLANE. See discussion of **nodal points**.

NODAL POINT(S). (1) Of all the rays that pass through a lens from an off-axis object point to its corresponding image point, there will always be one for which the direction of the ray in the image space is the same as that in the object space. The two points at which



these segments, if projected, intersect the axis are called the **nodal points**, and the transverse planes through them are called the **nodal planes**. Only if n and n'' , the indices of refraction in the object and image spaces, are identical are the nodal planes also the principal planes. C is the optical center of the lens. (2) For uses of this term in other applications to mathematics, see **node(s)**.

NODE. (1) A singular point on a curve having the property that two branches of the curve, with distinct tangents, pass through the point. Also called a *crunode*. (2) See **vertex**. (3) The points, lines, or surfaces in a standing wave system (see **wave, standing**) where some characteristic of the wave field has essentially zero amplitude. The appropriate modifier should be used with the word "node" to signify the type that is intended (pressure node, velocity node, etc.).

NODE(S), PARTIAL. The points, lines, or surfaces in a standing wave system (see **wave, standing**) where some characteristic of the wave field has a minimum amplitude differing from zero. The appropriate modifier should be used with the words "partial node" to signify the type that is intended (pressure partial node, velocity partial node, etc.).

NOISE. Any undesired sound. By extension, noise is any unwanted disturbance within a useful frequency band, such as undesired electric waves in any transmission channel or device. Such disturbances, when produced by other services, are called **interference**. Noise is also accidental or random fluctuation in

electric circuits due to motion of the current carriers.

NOISE, BACKGROUND. (1) Noise due to audible disturbances of periodic and/or random occurrence. (2) In receivers, the noise in absence of signal modulation on the carrier. (3) In recording and reproducing, background noise is the total system noise independent of whether or not a signal is present. The signal should not be included as part of the noise.

NOISE FACTOR (NOISE FIGURE). Of a linear system at a selected input frequency, the ratio of (1) the total noise power per unit bandwidth (at a corresponding output frequency) available at the output terminals, to (2) the portion thereof engendered at the input frequency by the input termination, whose noise temperature is standard (290°K) at all frequencies. For heterodyne systems there will be, in principle, more than one output frequency corresponding to a single input frequency, and vice versa; for each pair of corresponding frequencies a noise factor is defined. The phrase, "available at the output terminals," may be replaced by "delivered by the system into an output termination," without changing the sense of the definition.

NOISE FACTOR (NOISE FIGURE), AVERAGE. Of a linear system, the ratio of (1) the total noise power delivered by the system into its output termination when the noise temperature of its input termination is standard (290°K) at all frequencies to (2) the portion thereof engendered by the input termination. For heterodyne systems, portion (2) includes only that noise from the input termination which appears in the output via the principal frequency transformation of the system, and does not include spurious contributions such as those from image-frequency transformations. A quantitative relation between average noise factor, F , and spot noise factor, $F(f)$, is

$$\bar{F} = \frac{\int_0^\infty F(f)G(f)df}{\int_0^\infty G(f)df},$$

where f is the input frequency and $G(f)$ is the ratio of (a) the signal power delivered by the system into its output termination to

(b) the corresponding signal power available from the input termination at the input frequency. For heterodyne systems, (a) comprises only power appearing in the output via the principal frequency transformation of the system; in other words, power via image-frequency transformations is excluded.

NOISE INSULATION FACTOR. A factor which represents the difference between the noise level outside and inside an acoustic enclosure, and thus gives a rating of the overall noise reduction provided by the enclosure. When measured in decibels, this factor is given by $10 \log (a/T)$, where a is the total absorption of the enclosure measured in sabins, and T the total transmittance, where

$$T = \sum \tau_n \Delta s_n$$

and τ_n is the transmission coefficient of an element of area Δs_n .

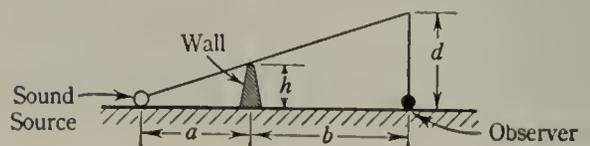
NOISE LEVEL. The value of noise integrated over a specified frequency range with a specified frequency weighting and integration time. It is expressed in decibels relative to a specified reference.

NOISE LEVEL IN A ROOM, TOTAL. The total noise level in a room for a specified frequency range may be obtained from the spectrum level (see noise, spectrum level of room) as follows,

$$N.L. = 10 \log_{10} \frac{\int_{f_1}^{f_2} I df}{I_0}$$

where I is the sound intensity, in watts per square centimeter, in the frequency band df , df is the frequency band, in cycles per second, I_0 is the reference sound intensity of 10^{-16} watts per square centimeter, f_1 and f_2 are frequency limits of the bandwidth under consideration.

NOISE LEVEL REDUCTION OF BARRIERS. The sound level reduction of barriers such as solid walls and fences interposed between the source of sound or noise and the point of observation can be calculated from the analogy with the optical case. (See figure.)



The fundamental equation for the sound reduction of a barrier is given by

$$SRL = -3 + 10 \log \left[\left(\frac{1}{2} - x \right)^2 + \left(\frac{1}{2} - y \right)^2 \right] \quad (1)$$

where *SLR* is sound reduction, in decibels.

TABLE OF FRESNEL INTEGRALS

<i>v</i>	<i>x</i>	<i>y</i>	<i>v</i>	<i>x</i>	<i>y</i>
0.00	0.0000	0.0000	4.50	0.5261	0.4342
0.10	0.1000	0.0005	4.60	0.5673	0.5162
0.20	0.1999	0.0042	4.70	0.4914	0.5672
0.30	0.2994	0.0141	4.80	0.4338	0.4968
0.40	0.3975	0.0334	4.90	0.5002	0.4350
0.50	0.4923	0.0647	5.00	0.5637	0.4992
0.60	0.5811	0.1105	5.05	0.5450	0.5442
0.70	0.6597	0.1721	5.10	0.4998	0.5624
0.80	0.7230	0.2493	5.15	0.4553	0.5427
0.90	0.7648	0.3398	5.20	0.4389	0.4969
1.00	0.7799	0.4383	5.25	0.4610	0.4536
1.10	0.7638	0.5365	5.30	0.5078	0.4405
1.20	0.7154	0.6234	5.35	0.5490	0.4662
1.30	0.6386	0.6863	5.40	0.5573	0.5140
1.40	0.5431	0.7135	5.45	0.5269	0.5519
1.50	0.4453	0.6975	5.50	0.4784	0.5537
1.60	0.3655	0.6389	5.55	0.4456	0.5181
1.70	0.3238	0.5492	5.60	0.4517	0.4700
1.80	0.3336	0.4508	5.65	0.4926	0.4441
1.90	0.3944	0.3734	5.70	0.5385	0.4595
2.00	0.4882	0.3434	5.75	0.5551	0.5049
2.10	0.5815	0.3743	5.80	0.5298	0.5461
2.20	0.6363	0.4557	5.85	0.4819	0.5513
2.30	0.6266	0.5531	5.90	0.4486	0.5163
2.40	0.5550	0.6197	5.95	0.4566	0.4688
2.50	0.4574	0.6192	6.00	0.4995	0.4470
2.60	0.3890	0.5500	6.05	0.5424	0.4689
2.70	0.3925	0.4529	6.10	0.5495	0.5165
2.80	0.4675	0.3915	6.15	0.5146	0.5496
2.90	0.5626	0.4101	6.20	0.4676	0.5398
3.00	0.6058	0.4963	6.25	0.4493	0.4954
3.10	0.5616	0.5818	6.30	0.4760	0.4555
3.20	0.4664	0.5933	6.35	0.5240	0.4560
3.30	0.4058	0.5192	6.40	0.5496	0.4965
3.40	0.4385	0.4296	6.45	0.5292	0.5398
3.50	0.5326	0.4152	6.50	0.4816	0.5454
3.60	0.5880	0.4923	6.55	0.4520	0.5078
3.70	0.5420	0.5750	6.60	0.4690	0.4631
3.80	0.4481	0.5656	6.65	0.5161	0.4549
3.90	0.4223	0.4752	6.70	0.5467	0.4915
4.00	0.4984	0.4204	6.75	0.5302	0.5362
4.10	0.5738	0.4758	6.80	0.4831	0.5436
4.20	0.5418	0.5633	6.85	0.4539	0.5060
4.30	0.4494	0.5540	6.90	0.4732	0.4624
4.40	0.4383	0.4622	6.95	0.5207	0.4591

To obtain *x* and *y* as given in the Table of Fresnel Integrals, the value of *v* may be determined from the following equation.

$$v = d \sqrt{\frac{2a}{\lambda b(a+b)}} \quad (2)$$

where λ is wavelength, in feet, and *a*, *b*, and *d*, in feet, are the dimensions of the figure.

When *v* is known, the values of *x* and *y* can be determined from the Table of Fresnel Integrals. The values of *x* and *y* can be substituted in Equation 1 to obtain the noise reduction of the wall.

NOISE POWER, AVAILABLE. The maximum noise power that may be drawn from a network by a load whose impedance is the complex conjugate of the impedance of the network itself.

NOISE RATIO (NR). The ratio of the available noise power (see **noise power, available**) at the output of a transducer divided by the noise power at the input.

NOISE, SPECTRUM LEVEL OF ROOM. The spectrum level of a complex sound is given by

$$B = 10 \log_{10} \frac{I}{WI_0}$$

where *B* is the spectrum level, in decibels, *I* is the sound intensity, in watts per square centimeter, in the frequency bandwidth *W*, *W* is the bandwidth, in cycles per second, and *I*₀ is the reference sound intensity of 10⁻¹⁶ watts per square centimeter.

The spectrum level of an average room for the frequency range 100 to 10,000 cycles is given by the empirical equation,

$$B = \frac{140}{B^{1/4}} + 15 - b$$

where *B* is the spectrum level, in decibels, *f* is the frequency, in cycles per second, and *b* is a constant. (*b* = 0 for factories; *b* = -20 for offices; *b* = -35 for residences.)

NOISE TEMPERATURE. At a pair of terminals and at a specific frequency, the temperature of a passive system having an available noise power per unit bandwidth equal to that of the actual terminals.

NOISE, THERMAL (JOHNSON NOISE).

The **noise** produced by thermal agitation of charges in a conductor. The **available thermal noise power** produced in a resistance is independent of the resistance value, and is proportional to the absolute temperature and the frequency bandwidth over which the noise is measured, as indicated by the formula:

$$N_t = 1.37 \times 10^{-23} T \Delta f$$

in which N_t is the available thermal noise power, T is the temperature of the resistance in degrees Kelvin, and Δf is the bandwidth in cycles per second.

NOISE TRANSMISSION EFFECT. See **Montgomery noise transmission effect**.

NOMOGRAPH OR NOMOGRAM. (Also called an alignment chart or isopleth.) Two or more scales, drawn and arranged so that results of calculation may be found from the relation of points on them. For example, suppose the relation $f(x, y, z) = 0$ is given. Three graduated scales or lines are then constructed so that, if selected values of two variables are located with a straight-edge, the third scale will be cut at a value which satisfies the given equation. The method is readily extended to more than three variables.

NON-BONDING ORBITALS. See **bonding orbitals**.

NON-CENTRAL DISTRIBUTIONS. The **chi-square** distribution and the related **Student's distribution** and **Fisher's distribution** are based on sums of squares of normal variables with unit variance and zero mean. If these sums relate to variables with non-zero mean the distributions are called non-central.

NON-CENTRAL FORCES. Forces which depend upon the spin directions as well as upon the distance separating a pair of particles. The most important examples are the vector force, or spin-orbit force, the potential of which contains a factor $\mathbf{S} \cdot \mathbf{L}$, where \mathbf{S} is the resultant **spin** vector of the particles, and \mathbf{L} is their relative orbital angular momentum; and the tensor force, or dipole force, the potential of which contains a factor $3(\mathbf{s}_1 \cdot \mathbf{r})(\mathbf{s}_2 \cdot \mathbf{r}) - (\mathbf{s}_1 \cdot \mathbf{s}_2)$, where \mathbf{s}_1 and \mathbf{s}_2 are the spin vectors of the two particles, and \mathbf{r} is a unit vector in the direction of the vector joining the two particles. The first may be realized

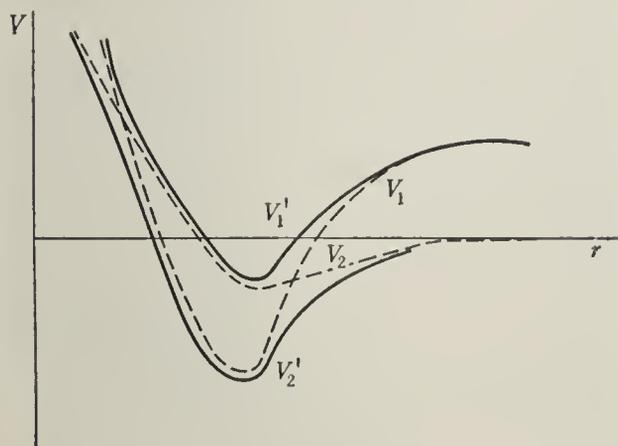
through the interaction between the spin magnetic moment with the orbital magnetic moment, the second through the interaction of two magnetic dipoles. The spin-orbit force is thought to be largely responsible for the **polarization** produced in scattering of neutrons or protons by nuclei while the tensor force is responsible for the **quadrupole moment** of the deuteron.

NON-CIRCUIT ELEMENT. See **element, non-circuit**.

NON-COMBINING MODIFICATIONS OF MOLECULES. In a symmetrical molecule, groups of rotational energy levels are distinguished from each other by the symmetry properties of their total eigenfunctions, apart from nuclear spin, for instance (*s*) and (*a*) in diatomic and linear polyatomic molecules, *A*, *E*, etc. in symmetric top molecules. (See **symmetry properties of molecular eigenfunctions**.) If the identical nuclei have zero spin only one group of rotational levels can occur, e.g., (*s*) for diatomic and linear polyatomic molecules, *A* for symmetric top molecules. For non-zero spin of the nuclei, all the rotational levels occur. However, on account of the smallness of the nuclear magnetic moments, the different groups of levels do not combine with each other to any significant extent, even by collision. Such molecules, therefore, exist in several different modifications. For pairs of identical nuclei with non-zero spin there are only two different modifications (e.g., H_2 , D_2 , N_2 , H_2O , H_2CO). As a rule, the modification with the greater statistical weight is referred to as *ortho-modification*, the one with the smaller statistical weight as *para-modification*. (For more detail see G. Herzberg, *Molecular Spectra and Molecular Structure, I. Spectra of Diatomic Molecules*, 2d ed., and *Infrared and Raman Spectra of Polyatomic Molecules*, D. Van Nostrand Co., Inc., Princeton, 1950 and 1945, respectively.)

NON-CROSSING RULE FOR THE POTENTIAL CURVES OF DIATOMIC MOLECULES. For an infinitely slow change of the internuclear distance in a diatomic molecule, two electronic states of the same **species** cannot cross each other ("avoid" each other). In other words, the potential curves of two electronic states of the same species cannot cross each other.

Consider two solutions, ψ_1 and ψ_2 of the wave equation of a molecule. V_1 and V_2 are the potential energy curves (see figure), for these two wave functions separately.



If a linear combination of ψ_1 and ψ_2 is possible, the wave functions

$$\psi'_1 = a_1\psi_1 + a_2\psi_2$$

and

$$\psi'_2 = a_3\psi_1 - a_4\psi_2$$

are more general solutions of the wave equation. The corresponding potential energy curves V'_1 and V'_2 are also plotted in the figure.

The non-crossing rule states that the curves $V'_1(r)$ and $V'_2(r)$ will always lie respectively above and below the two curves $V_1(r)$ and $V_2(r)$; i.e., if a transition between these two states is possible, V'_1 and V'_2 can never intersect.

NON-DETERMINATION, COEFFICIENT OF. The square of the coefficient of alienation.

NON-DIMENSIONAL EQUATION. An equation in which each member has been rendered free of physical dimensions by the systematic introduction of a set of reference constants. The dynamical equations for two dimensional flow, for example, may be made non-dimensional by introducing the constants

$$l = \frac{x}{\xi}, \quad \delta = \frac{z}{\zeta}, \quad U = \frac{\tau l}{t},$$

where l is a characteristic length in the x -direction, δ a characteristic depth, and U a reference velocity; ξ , ζ , and τ are new non-dimensional variables or parameters replacing x , z , and t , respectively. (See **dimensional analysis**.)

NON-DISPERSIVE WAVES. Waves whose speed of propagation is independent of wavelength. A medium in which small amplitude waves do not redistribute the mass in the direction of propagation is non-dispersive, e.g., a stretched membrane in transverse oscillation, a liquid film maintained by surface tension in which the two surfaces remain at the same distance apart. Waves of small amplitude in which the motion is entirely in the direction of propagation (longitudinal waves) are also non-dispersive, e.g., compression (sound) waves in a gas.

A disturbance or pulse is propagated without change of shape in a non-dispersive medium.

NON-EQUILIBRIUM PHENOMENA. See **fluctuations**.

NON-EQUIVALENT ELECTRONS. Electrons not belonging to the same orbital, i.e., differing in the values of either the quantum number n or l or both.

NON-EUCLIDEAN GEOMETRY. See **geometry**.

NON-GENUINE NORMAL VIBRATIONS OF POLYATOMIC MOLECULES. The equation for the determination of the frequencies of the normal vibrations (see **normal vibrations and normal coordinates in polyatomic molecules**) has, in the general case six, for linear molecules five solutions which are equal to zero. These solutions correspond to the so-called non-genuine vibrations of the molecule, that is to translations and rotations of the molecule.

NON-HOLONOMIC. A motion is termed non-holonomic when the differential equations of constraint are not integrable. A sphere slipping and rolling on a plane is an example of non-holonomic motion. (See also **conservative force**.)

NON-LOCALIZED MOLECULAR ORBITALS. Molecular orbitals for which the probability density distribution does not have a maximum between two nuclei but is spread over a large part of the molecule, as for instance around a benzene ring.

NON-ORIENTED GRAPH. See **graph, non-oriented**.

NON-PARAMETRIC INFERENCE. An older and less preferable term for **distribution-free inference**.

NON-RELATIVISTIC QUANTUM MECHANICS. See **quantum mechanics**.

NON-SELECTIVE RADIATOR. See **grey body**.

NON-SEPARABLE GRAPH. See **graph, non-separable**.

NON-UNIFORM GAS, DIFFUSION FORMULA. See **diffusion, kinetic theory of**.

NON-UNIFORM GAS, KINETIC THEORY TREATMENT. See **kinetic theory; diffusion, kinetic theory of; thermal conduction, kinetic theory of; viscosity, kinetic theory of**.

NON-UNIFORM GAS, THERMAL CONDUCTION IN. See **thermal conduction, kinetic theory of**.

NORM. In some literature the norm $\|x\|$ of the vector x , and the norm $\|A\|$ of the matrix A refers specifically to what will be called the Euclidean norm, defined by the non-negative square roots of

$$\|x\|^2 = x^*x; \quad \|A\|^2 = \text{trace}(A^*A).$$

(See **trace**.) More generally, however, a norm $\|x\|$ of a vector x is taken to be any real valued function of the elements satisfying the conditions

- (1) $x \neq 0 \Rightarrow \|x\| > 0$;
- (2) $\|\alpha x\| = |\alpha| \|x\|$;
- (3) $\|x + y\| \leq \|x\| + \|y\|$;

and the associated norm $\|A\|$ of a square matrix A is

$$\|A\| = \sup_{\|x\|=1} \|Ax\|.$$

Such a matrix norm possesses properties (1), (2), and (3), with matrix arguments, and also, for any two matrices A and B ,

$$(4) \quad \|AB\| \leq \|A\| \|B\|.$$

The Euclidean matrix norm possesses these properties, but does not satisfy the definition. The Euclidean matrix and vector norms are further related, as are all associated norms, by the property that for any matrix A and vector x ,

$$\|Ax\| \leq \|A\| \|x\|.$$

But for norms as here defined, for any A there exists an $x \neq 0$ such that an equality holds, and this is not true for the Euclidean norms. In fact, it is the **spectral norm** that is associated with the Euclidean vector norm.

Norms are important in studying convergence properties of sequences of vectors and matrices, and in obtaining error bounds (see **matrix inversion**). In fact, a sequence of vectors x_n has a limit x if and only if the sequence of norms $\|x_n - x\|$ vanishes in the limit, whatever norm may be used. A sufficient condition for the sequence of matrix powers B^n to have the limit 0 is that $\|B\| < 1$, in any norm. Also for any matrix A and any norm, the *spectral radius* satisfies

$$\rho(A) \leq \|A\|.$$

If x satisfies $Ax = h$, if $y = x - s$ is any approximation, and if $r = h - Ay = As$, then

$$\|s\| \leq \|A^{-1}\| \|r\|,$$

which gives a measure of the departure of the approximate solution y from the true solution x . In some cases $\|A^{-1}\|$ can be evaluated *a priori*, as when the equations arise from a partial differential equation. Otherwise if $H = I - AC$, and $\|H\| < 1$, then

$$\|A^{-1}\| \leq \|C\| / (1 - \|H\|).$$

As other examples of norms, if $x^T = (\xi_1, \dots, \xi_n)$ and $A = (\alpha_{ij})$, then

$$\|x\|_e = \max_i |\xi_i|, \quad \|A\|_e = \max_i \sum_j |\alpha_{ij}|;$$

$$\|x\|_{e'} = \sum_i |\xi_i|, \quad \|A\|_{e'} = \max_i \sum_j |\alpha_{ji}|.$$

Again, given any norm, and any nonsingular matrix G (for example, a diagonal matrix),

$$\|x\|_G = \|G^{-1}x\|; \quad \|A\|_G = \|G^{-1}AG\|$$

are again norms.

NORMAL. (1) A perpendicular to a line or to a curve at a given point. In a plane, its slope equals the negative reciprocal of the slope of the given line; in space, if the given line has **direction cosines** L, M, N and its normal has direction cosines λ, μ, ν , then $\lambda L + \mu M + \nu N = 0$. (2) A **partial differential equation** is in normal form when written in terms of coordinates defined by its characteristic curves. (3) If A^\dagger is the **matrix** obtained from A by exchanging rows and columns and taking the complex conjugate of

each element, then A is normal if $AA^\dagger = A^\dagger A$. Every normal matrix can be transformed by a unitary matrix to diagonal form and conversely if a matrix can be so transformed it is normal. (4) If a quadratic form is reduced to a sum of squared terms by proper choice of coordinate system, the new coordinates are called normal coordinates (see **diagonalization of a matrix**). (5) A directional derivative is called a normal derivative if it is taken in a direction normal to a given curve. (6) For a normalized function, see **orthogonal function**.

NORMAL ACCELERATION. See **centripetal acceleration**.

NORMAL CONGRUENCE. (1) The **rectilinear congruence** formed by the normals to a surface. (2) In optics, the normal congruence of light rays to a smooth wave surface. The *Malus-Dupin* theorem asserts that the surface remains smooth under reflection or refraction.

NORMAL COORDINATES. See **normal modes**.

NORMAL CURVATURE (AT A POINT OF A SURFACE IN THE DIRECTION OF A TANGENT LINE). The **curvature**, at the point, of the curve of intersection of the surface with the plane defined by the normal to the surface and the tangent line.

NORMAL CURVATURE, EULER THEOREM ON. See **Euler theorem on normal curvature**.

NORMAL CURVATURE, RADIUS OF (AT A POINT OF A SURFACE IN THE DIRECTION OF A TANGENT LINE). The reciprocal of the **normal curvature**.

NORMAL DISTRIBUTION. The normal (or Gaussian) distribution is that with frequency function

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp [-(x - \mu)^2/2\sigma^2].$$

The distribution is symmetrical with mean μ and standard deviation σ ; its graph is called the normal curve. Its central importance in statistics stems from three facts: (1) Many actual populations approximate closely to normal form. (2) It forms the limiting distribution of many widely used statistics. (3) Under general conditions, the mean of any distribution tends to be normally distributed in

large samples (**Central Limit Theorem**). To physicists and astronomers the distribution is more familiar as the *Gaussian distribution* or *law of error*. The distribution of errors of observation is usually assumed to be of this form, which may also be written

$$f(x) = \frac{h}{\sqrt{\pi}} \exp (-h^2 x^2)$$

where h is now regarded as a measure of precision in the measurements and it is assumed that the mean is zero.

NORMAL EQUATIONS. The equations which arise in estimation by **least-squares**. They are a set of simultaneous equations in the unknowns and are often linear, especially in applications to the combination of observations involving errors of measurement.

NORMAL EQUIVALENT DEVIATE. See **deviate, deviation**.

NORMAL FORM. See **Hesse normal form**.

NORMAL FREQUENCIES. See **normal modes**.

NORMALIZATION. If $f(x)$ is real and defined for $a \leq x \leq b$, the **norm** of f is

$$N(f) = \int_a^b f^2 dx.$$

Then if a new function

$$\phi(x) = f(x)/\sqrt{N(f)}$$

is defined, it follows that

$$N(\phi) = \int_a^b \phi^2 dx = 1$$

and $f(x)$ is said to be **normalized**. The procedure is readily generalized to include complex functions. (See also **orthonormal**.)

Similarly, the normalization of a vector consists of dividing its components (a_1, a_2, \dots, a_n) by its norm $\sqrt{(a_1^2 + a_2^2 + \dots + a_n^2)}$, so as to produce a vector of unit norm.

NORMALIZED INTENSITY OF A DIFFRACTION PATTERN. See **Strehl definition**.

NORMALIZING FACTOR. The reciprocal of the norm of a vector or a function. (See **normalization**.)

NORMAL MAGNIFICATION. In telescopes and microscopes the eye is usually placed at the **exit pupil** of the system, and if the full brightness of the object is to be represented in the image, the exit pupil must be of such size as just to fill the pupil of the eye. The particular **magnification** which just meets this condition is called normal.

NORMAL MATRIX. See **matrix**.

NORMAL MODES. A mechanical system, composed of coupled oscillators, such that each oscillator is subject to an elastic restoring force, when displaced from its equilibrium configuration, can be described in terms of normal coordinates Q_1, Q_2, \dots, Q_n and normal velocities $\dot{Q}_1, \dot{Q}_2, \dots, \dot{Q}_n$ (n is the number of degrees of freedom of the system).

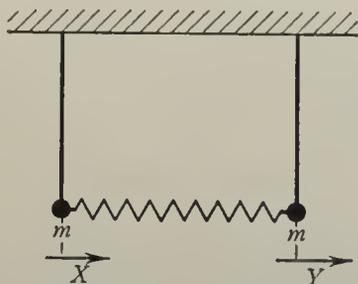
The total energy of the system is given by,

$$E = \sum_{i=1}^n \alpha_i \dot{Q}_i^2 + \beta_i \dot{Q}_i^2$$

the α_i and β_i are arbitrary constants, dependent on the initial conditions only.

The normal coordinates are independent of each other, i.e., each can be excited while the others remain at rest. A vibration in which only one normal coordinate is excited, is called a **normal mode of vibration**. The frequency associated with a normal mode is called a **characteristic** or eigenfrequency of the system.

Consider the following example:



Two simple pendulums of mass m and length L are connected by a weightless spring whose force constant is k . Let x and y be the displacements of the left hand and right hand masses from their equilibrium positions. The normal modes are

$$\begin{aligned} \bar{\bar{X}} &= x + y \\ \bar{\bar{Y}} &= x - y. \end{aligned}$$

The normal mode $\bar{\bar{X}}$ has the angular frequency $\sqrt{\frac{g}{L}}$ and the normal mode $\bar{\bar{Y}}$ has the angular frequency $\sqrt{\frac{g}{L} + \frac{2k}{m}}$.

The total energy is,

$$E = \frac{m}{4} (\dot{\bar{\bar{X}}}^2 + \dot{\bar{\bar{Y}}}^2) + \frac{mg}{4L} \bar{\bar{X}}^2 + \left(\frac{mg}{4L} + \frac{g}{2} \right) \bar{\bar{Y}}^2.$$

In the $\bar{\bar{X}}$ mode the masses vibrate in phase with the same amplitude; in the $\bar{\bar{Y}}$ mode the masses vibrate in opposite phase with the same amplitude.

NORMAL MODES OF A CRYSTAL. Let \mathbf{r}^{jl} be the position vector of the j^{th} atom in the l^{th} unit cell of a crystal in equilibrium. Let \mathbf{u}^{jl} be the displacement of this atom from its equilibrium position. Then the displacements associated with the normal modes of vibration are given by

$$u_{\mu}^{jl}(\mathbf{k}, n) = \text{Re} \{ A_{\mu}^{i}(\mathbf{q}, n) \exp [i(\mathbf{q} \cdot \mathbf{r}^{jl} - \omega_{qn}t)] \}$$

where Re means "real part of," the index μ goes from 1 to 3, the vector \mathbf{q} and the index n label each normal mode, ω_{qn} being the angular frequency for that mode. n takes on $3s$ values corresponding to $3s$ bands or branches of the vibrational spectrum, where s is the number of atoms in the unit cell.

NORMAL MODES OF A SYSTEM. The possible forms of time-function constituting the time-behavior of the quantities in a linear system which is originally disturbed but then deprived of external input.

If the **characteristic equation** of the system $\Delta(s) = 0$ can be written

$$(s - s_1)^{m_1} \dots (s - s_r)^{m_r} \dots = 0$$

then the normal modes are

$$e^{s_1 t}, t e^{s_1 t}, \dots, t^{m_1-1} e^{s_1 t}; e^{s_2 t}, t e^{s_2 t}, \dots, t^{m_2-1} e^{s_2 t}; \text{ etc.}$$

The number of distinct modes is equal to the order of the characteristic equation when expressible as a polynomial equation in s . Any quantity in the system can, under these conditions, be represented by a linear combination of the normal modes, the coefficients associated with the various modes depending upon the initial conditions.

NORMAL MULTIPLET. See **multiplet**.

NORMAL PLANE. See **principal planes**.

NORMAL PLANE TO A CURVE. See **normal to a curve at point P**.

NORMAL PRESSURE. In a fluid the normal pressure is the normal component of the stress across unit area within the fluid. In an inviscid fluid there is no other component. In a static fluid the pressure is independent of the orientation of the area across which the normal stress is measured, and this is usually also assumed to be the case in isotropic fluids in motion. (See **static pressure**.)

NORMAL-PRESSURE DRAG. See **drag**.

NORMAL, PRINCIPAL (TO A CURVE AT A POINT P). The normal to the curve at P which lies in the osculating plane (see **curvature, plane of**) at P . A unit vector in the direction of the principal normal is called the *unit (principal) normal*. It is usually taken as directed from P to the concave side of the curve.

NORMAL PRODUCT. The normal product of a number of creation and annihilation operators is defined by

$$N(UV \cdots W) = (-1)^p U'V' \cdots W' \quad (1)$$

where $U', V', \cdots W'$ denote the same set of operators as the set $U, V, \cdots W$, but ordered so that all the annihilation operators stand to the right of all creation operators, and p is equal to the number of interchanges of Fermion operators required to go from the original ordering to the one on the right side of (1). Sometimes the normal product is denoted by $:UV \cdots W:$. By definition the distributive law is assumed to hold for a normal product

$$:(U + V)W \cdots: = :UW \cdots: + :VW \cdots:$$

NORMAL REACTION (I.E., NORMAL TO PATH). The reactive thrust with which a constraining surface acts on a contacting object which in turn is subjected to a force with a component perpendicular to the surface. If there is no friction between the surface and the object, the reactive thrust is perpendicular or normal to the surface. An example is an object moving on a frictionless inclined plane. The normal reaction is perpendicular to the plane and has the magnitude $W \cos \theta$ where W is the weight of the object and θ is the angle of the plane. The presence of **friction** intro-

duces a component of force parallel to the surface.

NORMAL RESPONSE. The normal response of a system to a specified input is the output produced by that specified input when the system is quiescent for $t < 0$.

NORMAL SPACE. See **topological space**.

NORMAL SPECTRUM. See **spectrum, normal**.

NORMAL STATE. In nuclear physics, a term sometimes used for **ground state**.

NORMAL STRESS. (1) A component of stress perpendicular to the plane on which it acts. Tension is ordinarily taken as positive but in soil mechanics, which deals primarily with compressive stress, compression is usually called positive. (2) See **flux vector**.

NORMAL STRESS EFFECT. In non-linear theory, shear strains alone demand the application of normal stresses as well as shear stresses. These are also called *cross effects*.

NORMAL SUBGROUP. See **group**.

NORMAL TEMPERATURE AND PRESSURE. Abbreviated to NTP. An agreed standardized reference state, specified by a pressure of 760 millimeters of mercury (one atmosphere) and the ice-point temperature ($0^\circ\text{C} = 32^\circ\text{F}$). Although the use of NTP as a reference state is to be encouraged for the sake of uniformity, sometimes slightly different reference states may be encountered, in particular the standard temperature and pressure, abbreviated to STP and specified by a pressure of 30 inches of mercury and a temperature of 60°F .

NORMAL TO A CURVE AT A POINT P. Any line through P perpendicular to the tangent at P . In the case when the curve is a plane curve, the line through P perpendicular to P and lying in the plane of the curve. The *normal plane to a curve at P* is the plane through P perpendicular to the tangent at P .

NORMAL TO SURFACE. See **tangent plane to surface**.

NORMAL, UNIT. See **unit normal**.

NORMAL VIBRATIONS AND NORMAL COORDINATES IN POLYATOMIC MOLECULES The potential and kinetic energies

of a system of N particles of masses m_i for small displacements from the equilibrium position are given by

$$V = \frac{1}{2} \sum_{ij} k_{ij} q_i q_j$$

and

$$T = \frac{1}{2} \sum_{ij} b_{ij} \dot{q}_i \dot{q}_j$$

where the q_i may either be $3N$ Cartesian displacement coordinates or, for nonlinear molecules, $3N - 6$, for linear molecules, $3N - 5$ internal displacement coordinates such as changes of internuclear distances. The k_{ij} ($=k_{ji}$) are force constants, the b_{ij} ($=b_{ji}$) are constants depending on the masses and geometrical parameters of the molecule. By the linear transformation

$$q_i = c_{i1} \xi_1 + c_{i2} \xi_2 + c_{i3} \xi_3 + \dots$$

new coordinates ξ_i , so-called normal coordinates, can be formed such that both V and T are sums of squares.

$$V = \frac{1}{2} (\lambda_1 \xi_1^2 + \lambda_2 \xi_2^2 + \lambda_3 \xi_3^2 + \dots)$$

$$T = \frac{1}{2} (\xi_1^2 + \xi_2^2 + \xi_3^2 + \dots)$$

that is, the motion in the molecule in this approximation may be considered as a superposition of $3N$ or $3N - 6$ or $3N - 5$ independent harmonic oscillators described by the normal coordinates ξ_i such that

$$\xi_i = \xi_i^0 \cos(2\pi\nu_i t + \phi_i).$$

In each such normal vibration all nuclei in the molecule carry out simple harmonic motions about their respective equilibrium positions with one and the same frequency ν_i which is related to λ_i by

$$\lambda_i = 4\pi^2 \nu_i^2.$$

The λ_i , that is, the frequencies of the different normal vibrations, are determined by the secular equation:

$$\begin{vmatrix} k_{11} - b_{11}\lambda & k_{12} - b_{12}\lambda & k_{13} - b_{13}\lambda & \dots \\ k_{21} - b_{21}\lambda & k_{22} - b_{22}\lambda & k_{23} - b_{23}\lambda & \dots \\ k_{31} - b_{31}\lambda & k_{32} - b_{32}\lambda & k_{33} - b_{33}\lambda & \dots \\ \dots & \dots & \dots & \dots \end{vmatrix} = 0.$$

If Cartesian coordinates are used, six or five of the λ_i are found to be zero depending on whether the molecule is nonlinear or linear, respectively. These zero roots correspond to the nongenuine normal vibrations (null vibra-

tions): the translations and rotations. When two or three λ_i are equal, we have doubly or triply degenerate normal vibrations.

The form of a given normal vibration ξ_j can be obtained from the transformation equations by putting all other ξ_i equal to zero. The coefficients c_{ij} are the minors of the above determinant.

The general relation between the force constants and the frequencies of the normal vibrations is given by the determinantal equation above. In the most general case there are $\frac{1}{2}n(n+1)$ force constants, ($n = 3N - 6$ or $3N - 5$) while there are only n normal frequencies. (See also **symmetry properties of normal vibrations in molecules.**)

NORMAL VIBRATIONS OF POLYATOMIC MOLECULES, NON-GENUINE. See **nongenuine normal vibrations of polyatomic molecules.**

NORTON THEOREM. The current in any terminating impedance Z_T connected to any network is the same as if Z_T were connected to a parallel combination of a generator whose current is the short-circuit current of the network, and an impedance Z_R which is the impedance looking back into the network from the terminals of Z_T , with all generators replaced by impedances, equal to the internal impedances of these generators. (See also **Thévenin theorem.**)

NOVAE OR EXPLOSIVE STARS. The so-called new stars, or novae, are designated by the constellation and year in which they appear, or by the constellation name preceded by letters. For example: Nova Herculis 1934 is also known as DQ Herculis. It is highly probable that a nova is not a "new star." In a number of cases, after a nova has appeared, examination of previously taken photographs of the region showed a relatively faint star at the position of the "nova." The light curve of a nova is characterized by a spectacular rise to maximum, of the order of 12 magnitudes in two days, and then a slow decline to approximately its former brightness in five or six hundred days. Immediately after maximum the decline is relatively rapid, of the order of about half the rise in about two weeks. From this point on the decline is more gradual, accompanied by oscillations of a magnitude or so with periods of a few days. These oscilla-

tions cease to be conspicuous by the end of a hundred days. Then comes a slow and somewhat irregular decline to the vicinity of the original brightness.

The changes in brightness of a nova are accompanied by equally spectacular changes in the spectra. McLaughlin has made a very thorough study of these changes and, while there are differences among the various novae, a general sequence may be outlined.

In cases where the nova is discovered before reaching maximum, the spectrum is comparable to that of a B5 star, but with superimposed emission bands of He I, C II, and N III. The absorption lines are broad and diffuse with a strong displacement toward the violet. However, the emission bands of H and the other elements are not displaced. There is strong evidence that we have a star from which the reversing layer has been torn exposing the hotter layer below.

After maximum is passed the spectrum changes rapidly. Another set of absorption lines appear. The lines are displaced to the violet. Together with the appearance of the post-maximum absorption spectrum, emission bands are evident but they are so broad and diffuse that identification is difficult. As the nova fades in light the emission bands become more prominent. In the next stage a new set of absorption lines of H, Ca II, and others make their appearance as diffuse lines. Changes of this sort continue as the star fades in brightness. With this fading of the star the emission bands begin to gain in relative intensity. In the final stage we find the emission spectra of the bright-line nebulae, such wavelengths as 5007A, 4959A, etc., dominating the final spectrum of the nova. The existence of nebulae in the vicinity of a faded nova has been confirmed in a number of cases.

NOZZLE OR DUCT, FLOW OF GAS IN.

It is assumed here that the flow in the nozzle or duct is steady and can be treated as one-dimensional, i.e., that the conditions are uniform over any section of the duct. The continuity equation is then

$$\rho v A = \text{constant}, \quad (1)$$

where A is the area of cross section of the duct.

If there is no heat transfer to or from the walls of the duct, the steady-flow energy equation can be applied. This may be written

$$1 + \left(\frac{\gamma - 1}{2}\right) M^2 = \frac{T_o}{T} = \left(\frac{a_o}{a}\right)^2, \quad (2)$$

where M is Mach number, T is absolute temperature, a is velocity of sound and suffix o refers to "stagnation" conditions (with $v = 0$).

If there are no significant effects of viscosity, the flow will be isentropic and the additional relation

$$\frac{p}{\rho^\gamma} = \text{constant} \quad (3)$$

may be used. (It should be noted that this implies not only that viscous effects at the walls are negligible, but also that there are no shock waves).

Combining Equations (2) and (3) gives

$$\frac{p_o}{p} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{\gamma}{\gamma - 1}} \quad (4)$$

and

$$\frac{\rho_o}{\rho} = \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right]^{\frac{1}{\gamma - 1}}. \quad (5)$$

Introduction of Equation (1) leads to the area equation

$$\frac{A}{A_s} = \frac{1}{M} \left\{ \frac{2}{\gamma + 1} \left[1 + \left(\frac{\gamma - 1}{2}\right) M^2\right] \right\}^{\frac{\gamma + 1}{2(\gamma - 1)}}, \quad (6)$$

where A_s is the area of the duct for $M = 1$. It may be shown also that the area A_s is the minimum value of A . Thus sonic velocity can only be reached at the section of minimum area, i.e., at the "throat" of a convergent-divergent nozzle. With the assumptions already stated, Equations (1) to (6) can also be applied to the flow in any **stream tube**.

NTP. Abbreviation for **normal temperature and pressure**.

NU(ν). The average number of neutrons released in a nuclear fission event.

NUCLEAR CHARGE. The positive charge on the atomic nucleus due to the protons it contains. The sum of the charges of the protons in a nucleus, equal to $+Ze$.

NUCLEAR ENERGY LEVELS. The bound states of atomic nuclei exhibit a discrete energy spectrum. Each of these bound states

is commonly referred to as an energy level. The energy of the state is given relative to the energy of the ground state as zero. Further energy levels appear in the continuum of the energy spectrum which, although not strictly speaking discrete, nevertheless show up as more or less sharp resonances in reaction processes.

NUCLEAR FORCES. In spite of the electrostatic repulsion between protons, nuclei are stable. This indicates that there must be strong attractive forces between nucleons. Considerations of the binding energies of the lightest nuclei, followed by analysis of scattering experiments in which protons and neutrons are made to collide at high energies, shows the forces between pairs of nucleons to be of short range, falling rapidly to zero beyond a separation distance of about 2×10^{-13} cm. Furthermore, these forces are known to be *charge-independent*; that is, the forces between a pair of neutrons, or a pair of protons, or a neutron and a proton, are the same (excluding the electrostatic forces which act between protons alone) in equivalent states of motion. As proposed by Yukawa, such forces may qualitatively be understood in terms of the exchange of virtual mesons between the nucleons in a collision process. According to Yukawa's original proposal this exchange would give rise to a potential of the form $r^{-1}e^{-\mu r}$, where r is the separation, and $\mu = mc/\hbar$, $2\pi\hbar$ being Planck's constant, c the velocity of light, and m the mass of the meson. Although the details of this mechanism are still obscure it is now fairly certain that the mesons involved are the π -mesons having a mass of about 286 times the electron mass. However, sufficient experimental information is available to show that the forces are more complicated than this in reality. They certainly depend upon the relative spin directions of the two particles and upon the direction of spin relative to the direction of relative motion of the pair. Furthermore, although attractive at larger distances, as required to give rise to nuclear binding, they appear to become strongly repulsive at distances less than about 0.6×10^{-13} cm. Theory would also indicate that the force between a pair of particles may depend upon the presence of further particles; in other words, many-body forces may be pres-

ent in nuclei. Nothing is known of the properties of these forces at the present time.

NUCLEAR GYROMAGNETIC RATIO. See *gyromagnetic ratio, nuclear*.

NUCLEAR MAGNETIC MOMENT. An electrically charged quantum mechanical system possessing angular momentum will contain electric currents which give rise to a magnetic dipole moment of order $eh/4\pi mc$, where e is the charge in esu, h is Planck's constant, m is the mass, and c is the velocity of light. Most atomic nuclei, for example, possess magnetic dipole moments of this order of magnitude (provided that they have non-zero spin). Even an electrically neutral particle, however, can possess a magnetic moment, owing to its interaction with virtual charged fields of particles. Thus, the neutron possesses a magnetic moment of the same order of magnitude as a proton owing to its interaction with the π -meson field. The magnetic moments of the ground states of nuclei were extremely helpful as a clue to the structure of nuclei simply because they are structure sensitive and can be measured with high accuracy.

NUCLEAR MAGNETIC RESONANCE. The resonance phenomenon met with in energy transfer between a radiofrequency alternating magnetic field and a nucleus placed in a constant magnetic field H that is sufficiently strong to decouple the nuclear spin from the influence of the atomic electrons. Resonance is encountered when $\omega = gh$, where ω is the angular frequency of the alternating field, g the nuclear *gyromagnetic ratio*, and h is the Planck constant. Transitions can then be induced between the various possible substates corresponding to different quantized orientations of the nuclear spin relative to the direction of H . The phenomenon has found application in connection with measurements of g both by molecular beams and by the use of macroscopic samples of solids or liquids. In the latter case, ω is commonly held constant, and H is varied back and forth through the resonance value for the sake of convenience in producing a display upon an oscilloscope.

NUCLEAR MAGNETON. The nuclear magneton μ_{on} is the unit used to measure the magnetic moments of atomic nuclei. It is defined as

$$\begin{aligned}\mu_{on} &= \frac{he}{4\pi m_p c} = \frac{m}{m_p} \mu_o = 1/1836 \quad \mu_o \\ &= 0.505038 \times 10^{-23} \text{ erg gauss}^{-1}.\end{aligned}$$

Here h stands for Planck's constant, e , for the charge of the electron, m and m_p , respectively, for the masses of electron and proton, μ_o for the **Bohr magneton**, and c for the velocity of light.

NUCLEAR MATTER. An idealized picture of the interior of a nucleus, represented by an infinite homogeneous medium of neutrons and protons in equal densities in which the electrostatic forces between protons are ignored, and surface effects can be neglected. A concept much used in the study of nuclear structure, particularly in the theory of saturation of nuclear forces.

NUCLEAR NUMBER. The same as **mass number**.

NUCLEAR POTENTIAL. The potential energy V of a nuclear particle as a function of its position in the field of a nucleus or of another nuclear particle. A central potential is one that is spherically symmetric, so that V is a function only of the distance r of the particle from the center of force. A non-central potential, on the other hand, is one that is not spherically symmetrical, or that depends upon the relative directions of the angular momenta associated with the particle and the center of force as well as upon the distance r . A negative potential corresponds to an attractive force, while a positive potential corresponds to a repulsive force.

Although the expression can certainly be applied to the problem of nuclear forces, the usual meaning of a nuclear potential refers to the interaction of a nucleon (neutron or proton) with a complex nucleus. Although the potential energy of a single nucleon inside a nucleus is clearly a rapidly varying function of position and time (since it represents the interaction with a large number of closely packed, fast moving particles) one may nevertheless speak of the average potential energy, and regard this as a smoothly varying function. For a neutron, the nuclear potential is essentially negative inside the nucleus, rising rapidly to zero outside the nuclear radius R . For a proton the long-range electrostatic repulsion must, of course, be added. Owing to the

Pauli exclusion principle, and to the exchange nature of nuclear forces, however, such a potential cannot in general be regarded simply as a function of position, $V = V(\mathbf{r})$; it depends in addition upon the momentum of the particle, which in quantum mechanics does not commute with the position. Hence, the potential must be regarded as a non-diagonal matrix operator,

$$V = \langle \mathbf{r} | V | \mathbf{r}' \rangle$$

in configuration space, or a similar operator in momentum space.

Although the concept of a nuclear potential in this latter sense cannot be defined in a precise way, it has nevertheless been extremely useful, both qualitatively and quantitatively, in the investigations of nuclear structure and nuclear reactions. It has been of particular usefulness in the optical model of nuclear reactions.

NUCLEAR RADIUS. The density distribution of matter in the atomic nucleus approximates to a uniform sphere whose radius is given by $R = r_o A^{1/3}$, where A is the mass number of the nucleus, and r_o is a constant length of about 1.2×10^{-13} cm. Actually the surface of the nucleus is somewhat diffuse, and departures from spherical symmetry can be important, so the concept of a nuclear radius is none too precise. Further, measurements of the nuclear radius by means of scattering experiments, for example, give results depending upon the method of measurement. Nevertheless, the concept is a useful one for most practical purposes.

NUCLEAR REACTOR REACTIVITY. See **reactivity**.

NUCLEAR SPIN. The intrinsic angular momentum of the atomic nucleus due to rotation about its own axis. It is usually designated I and has the magnitude

$$\sqrt{I(I+1)}h/2\pi \approx I(h/2\pi),$$

where I is the nuclear spin quantum number which has different (integral or half-integral) values (including zero) for different nuclei. (For data on the spins of stable nuclei, see *American Institute of Physics Handbook*, McGraw-Hill, Inc., 1957, p. 8-6 ff.)

In spectroscopy the nuclear spin is of importance for the explanation of the **hyperfine**

structure, and of the intensity alternation in band spectra.

NUCLEAR STRUCTURE. The nucleus of an atom of atomic number Z and mass number A contains Z protons and $A-Z$ neutrons, bound together under the influence of short-range nuclear forces much as molecules are bound together in a drop of liquid. The strength of binding may be determined by subtracting the actual mass of the atom from the mass of its constituent particles considered as free particles. The binding energy E_B is then related to this mass defect ΔM by Einstein's relation $E_B = \Delta Mc^2$, where c is the velocity of light. The precise value of E_B depends upon the nucleus concerned, and upon how many neutrons and protons it contains, but it is of the order of 8 mev per nucleon in most nuclei.

The binding energy determines whether the nucleus is stable or unstable. Amongst the lighter nuclei the ones which are stable are those in which the number of protons is approximately equal to the number of neutrons, so that $A \approx 2Z$. In heavier stable nuclei there is an excess of neutrons over protons owing to the repulsive electrostatic forces between the protons. Thus, the most stable oxygen nucleus is O^{16} containing 8 protons and 8 neutrons, while the most nearly stable uranium nucleus is U^{238} containing 92 protons and 146 neutrons. Nuclei containing a disproportionate number of neutrons tend to be unstable and decay radioactively by emission of electrons whereby neutrons are converted into protons; those containing an excess of protons similarly tend to decay by emission of positrons or by capture of orbital electrons.

It has been shown that the nucleus is approximately spherical in shape and of volume proportional approximately to its mass. It is, however, capable of executing oscillations about the spherical form, and in certain circumstances may even acquire a permanent deformation. The heaviest nuclei are unstable under deformation, as a result of which they undergo spontaneous fission. These properties may be described qualitatively by regarding the nucleus as an electrically charged drop of liquid possessing volume energy and surface tension.

Although the nucleus is normally found in its lowest energy state it may be produced as the result of a nuclear reaction, or through

radioactivity in a number of excited states whose detailed properties may differ quite markedly from the lowest state. If formed in an excited state it will decay, normally by the emission of electromagnetic radiation (γ -rays) to the lowest state, or by the emission of particles to another nucleus.

NUCLEOPHILIC REAGENT. See *inductive effect*.

NUCLEUS OF AN INTEGRAL EQUATION. See *kernel*.

NUISANCE PARAMETERS. Parameters of a system which bedevil the estimation of certain other parameters in virtue of being unknown. Either they must be themselves estimated, or methods must be found to get rid of them from the estimation procedure.

NULL-GEODESIC. Curve drawn in space-time such that the infinitesimal interval between two neighboring points on the curve vanishes. This represents a possible space-time path of a light ray.

The equation of a null-geodesic are

$$g_{ik}dx^i dx^k = 0$$

and

$$\frac{Dx^i}{Du} = 0$$

u is an arbitrary parameter along the curve; the left-hand member of the second equation is the *covariant derivative* with respect to u .

NULLITY GRAPH. See *graph, nullity*.

NULL MATRIX. A matrix having only zero elements.

NULL SEQUENCE. A sequence having zero as a limit.

NULL VECTOR. A vector A_μ of zero length ($A_\mu A_\mu = 0$). In special relativity theory the displacement between two *events* on the path of a photon is a null vector.

NUMBER. The concept of a cardinal number is difficult to define, and we should perhaps content ourselves with defining what cardinal numbers do (relations between them being merely a convenient method of expressing relations between sets), leaving the question of what they are to philosophers. Thus, we say that two sets have the same cardinal number if they can be put into one-to-one correspond-

ence with each other. The set of natural numbers 0, 1, 2, \dots , being given in this way, it is convenient to define rational numbers (pairs of natural numbers; e.g., a,b ; c,d , etc., with addition defined by $a,(b) + (c,d) = (ad + bc, bd)$, and so on), so that division (except by zero) is always possible, and negative numbers, so that subtraction is always possible. The next step is to define irrational numbers (see **Dedekind cut**), so as to be able to take roots, and **complex numbers**, so as to have roots of negative numbers. For these and other types of numbers, see the respective headings.

NUMBER OPERATOR. See **creation and annihilation operators**.

NUMBERS, CONJUGATE. See **conjugate numbers**.

NUMERICAL ANALYSIS. The theory underlying the development of numerical processes. In Lanczos's terms, numerical analysis is distinguished from **parexic analysis** which has as its aim the development of approximate formulas. In more common usage it would include parexic analysis.

NUMERICAL APERTURE. Abbe named the quantity $n \sin u$ the numerical aperture (N.A.) of a lens system, where u is the angular radius of the objective lens as seen from a point on the optical axis at the object, and n is the index of refraction of the medium between the object and the objective lens. The importance of the N.A. is that the optical path difference of two rays, one to each opposite edge of the objective, is n -fold greater in an immersion medium of index n than it would be in air, making the diffraction pattern of the objective aperture n -fold narrower. (See **Rayleigh criterion of resolving power** and **Abbe sine condition**.)

NUMERICAL CUBATURE. See **multivariate interpolation**.

NUMERICAL DIFFERENTIATION. See **differentiation formulas**.

NUMERICAL FORECASTING. (Also called mathematical forecasting, dynamic forecasting, physical forecasting, numerical weather prediction.) The forecasting of the behavior of atmospheric disturbances by the numerical solution of the governing **fundamental equations of hydrodynamics**, subject to observed initial conditions. When applied to the cy-

clonic-scale atmospheric disturbances in the form of a dynamical model, these solutions form a method of forecasting the behavior of the migratory pressure systems of middle latitude. Numerical forecasting is usually performed with the aid of high-speed computing devices. (See **barotropic model**, **thermotropic model**.)

NUMERICAL QUADRATURE. See **quadrature**.

NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS. See **differential equations**.

NUMERICAL SOLUTION OF EQUATIONS. See **algebraic equations**, **matrix inversion**, **Newton method**, **iterative methods**, **Bernoulli method**, **Graeffe method** and **Horner method**.

NUMERICAL STABILITY. A problem is numerically stable in case small errors in the data produce moderate errors at most in the required quantities. A method is numerically stable if it does not permit the accumulation of large **generated errors**, or if it does not involve a transformation of the original problem into another one that may be substantially less stable. The term is applied in particular to certain methods for solving **ordinary and partial differential equations** for which the approximating system of difference equations is stable. (See **condition number**.)

NUMERICAL TENSOR. See **tensor, isotropic**.

NUSSELT NUMBER. A non-dimensional number, N , relating the heat flux H per unit area to conductivity k and temperature gradient $\Delta T/h$

$$N \text{ (or } Nu) = hH/k\Delta T.$$

The Nusselt number can be applied to heat transfer by forced turbulent convection as well as to ordinary conductivity.

NUTATION. In the case of a spinning top or **gyroscope**, the inclination of the top's axis to the vertical will vary periodically between certain limiting angles. This motion is called nutation. In general a spinning top or gyroscope experiences both nutation and **precession**.

NUTATION OF EARTH AND MOON. If the sun and the moon apparently moved about the earth in fixed circular orbits and in the plane of the ecliptic, the changes produced by **precession** would be maintained. However, this is far from being the case. The motion of the vernal equinox along the ecliptic is not quite uniform. In an interval of time, t , the displacement of the vernal equinox can be expressed by $a_1 t + S \sin 2L$, where a_1 and S are constants, and L is the longitude of the sun. The term $a_1 t$ indicates a progressive and uniform motion along the ecliptic. Such a term is frequently called a *secular term*, but in precessional theory it is known as a *precessional term*, and is part of the precessional motion. The term $S \sin 2L$ is obviously periodic in character, oscillating between $+S$ and $-S$ during the year. Terms of this character are called *periodic terms*.

The same treatment may be given to terms caused by the moon, yielding both secular and periodic terms. Moreover, similar treatment is accorded other objects for which these effects are significant. A general expression for nutation has the form

$$at + b \sin N + c \sin 2N \\ + P \sin 2L + m \sin 2\lambda + \dots$$

in which N is the longitude of the ascending node of the moon's orbit and λ is the longitude

of the moon. In the computations for almanacs, many terms are used and the computational work is so great that it is often shared by several nations.

NUTTING EQUATION. A power law equation giving **creep strain** as a product of a power of the stress and a power of the time.

NYQUIST CRITERION FOR A CONTROL SYSTEM. See *stability (of system)*.

NYQUIST FREQUENCY. (Also called turn-over frequency.) The highest frequency which can be determined in a **Fourier analysis** of a discrete sampling of data. If a time series is sampled at interval Δt , this frequency is $1/2\Delta t$ cps.

NYQUIST LOCUS. See *frequency response representation*.

NYQUIST THEOREM. The relation between the mean square voltage $\langle V^2 \rangle$ of an equipotential conductor carrying zero current, the electrical resistance $\rho(\omega)$ as a function of frequency ω of the currents, and the absolute temperature T_ξ ,

$$\langle V^2 \rangle = (4/\beta) \int_0^\infty \rho(\omega) d\omega,$$

where $\beta = 1/kT$, k being Boltzmann's constant. This is a special case of the **fluctuation-dissipation theorem**.

O

OBJECT FOCAL LENGTH. See focal length.

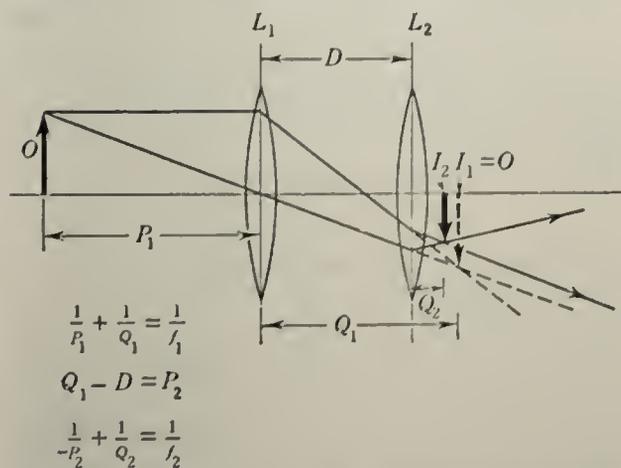
OBJECT POINT. The real or virtual point of intersection of a pencil of rays incident upon an optical system.

OBJECT RAY. See ray.

OBJECT, REAL. In geometrical optics, an object is called "real" if from each point of it, light diverges towards the optical system. (See **object, virtual**.)

OBJECT-SIDE PUPIL. See entrance pupil.

OBJECT, VIRTUAL. An example of a virtual object is given below. The image I_1 which would have been formed by lens L_1



had not lens L_2 been interposed acts as a virtual object for lens L_2 .

OBLATE. Flattened or depressed at the poles. The ellipsoid

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

with $a \geq b \geq c$ is a sphere if $a = b = c$, and a spheroid if $a = b > c$ or $a > b = c$. In the case $a = b > c$ (the two equal axes are greater than the unequal one) the spheroid is oblate; in the case $a > b = c$, it is prolate.

OBLATE SPHEROIDAL COORDINATES. The coordinate surfaces are: families of oblate

ellipsoids of revolution around the Z -axis with semi-axes $a = c\sqrt{1 + \xi^2}$, $b = c\xi$ ($\xi = \text{const.}$); hyperboloids of revolution of one sheet with $a = c\sqrt{1 - \eta^2}$, $b = c\eta$ ($\eta = \text{const.}$); and planes from the Z -axis ($\phi = \text{const.}$). The following additional relations hold:

$$0 < \xi < \infty; \quad -1 < \eta < 1; \quad 0 < \phi < 2\pi$$

$$x = c\sqrt{(1 + \xi^2)(1 - \eta^2)} \cos \phi$$

$$y = c\sqrt{(1 + \xi^2)(1 - \eta^2)} \sin \phi$$

$$z = c\xi\eta.$$

Alternative variables often used are

$$\xi = \sinh u; \quad \eta = \cos v;$$

$$0 < u < \infty; \quad 0 < v < \pi.$$

OBSERVABLE. In quantum mechanics, any quantity that can be observed. Observables are related to linear operators, and the possible values that the observable can take are given by the spectrum of the **eigenvalues** of the operator.

OCCUPATION NUMBERS. A set of states of a system of identical particles can be given by enumerating first a complete set of states of a single particle, and giving then the occupation numbers which describe how many particles occupy each of these single particle states. In a system of fermions there cannot be more than one particle in a single particle state, and the possible values of the occupation number are 0 and 1. For bosons, the possible values of the occupation numbers are 0, 1, 2, 3, \dots . (See also **microstate**.)

OCTAHEDRAL SHEAR STRESS. See **Mises yield condition**.

OCTAHEDRAL STRESS. See **stress, octahedral**.

OCTANE NUMBER. See **knocking**.

ODD-EVEN RULE OF NUCLEAR STABILITY. The rule, based on the number of

stable nuclides, that nuclides with even numbers of both protons and neutrons are most stable; those with an even number of protons and an odd number of neutrons, or *vice versa*, are somewhat less stable; while those with odd numbers of both protons and neutrons are least stable.

ODD FUNCTION. A function which changes sign when the sign of the dependent variable is changed: $f(-x) = -f(x)$. Typical examples are x^3 and $\sin x$. (See **even function**.)

ODD TERM OF AN ATOM. A term for which $\sum l_i$, summed over all the electrons in an atom is odd (l , azimuthal quantum number). The eigenfunctions of odd terms are changed in sign by reflection of all particles at the center. The odd terms are distinguished from the even ones by a superscript o (e.g., ${}^2P^o$).

OFFENSE AGAINST THE SINE CONDITION. Abbreviated to OSC. A quantity introduced into lens design by A. E. Conrady in order to have a numerical measure of **coma**, and defined as the ratio of the sagittal coma to the distance of the image point from the optical axis. (See **Abbe sine condition**.)

OHM, MECHANICAL. By analogy with the concept of electrical impedance, the complex ratio of the force acting in a mechanical system to the velocity is often termed the mechanical impedance and is measured in mechanical ohms. In the cgs system, the mechanical ohm is equal to 1 dyne per cm/sec, and has the dimensions of gm/sec. (See also **acoustic units**.)

OHM'S LAW. The statement that in a conductive medium the current I is proportional to the impressed electromotive force E , when this is constant. The constant of proportionality (E/I) is termed the **resistance**.

OHM'S LAW IN HYDROMAGNETICS. In a conducting fluid, Ohm's Law takes the form

$$\mathbf{E} + \mathbf{V} \times \mathbf{B}/c = \mathbf{j}\eta,$$

where \mathbf{E} is the electric field, \mathbf{V} is the fluid velocity, \mathbf{B} is the magnetic field, \mathbf{j} the electric current density and η the electrical resistivity. This reduces to the usual form of Ohm's Law in the coordinate system moving with the fluid.

ONE-TO-ONE MAPPING. See **representation**.

ONE-VELOCITY NEUTRON TRANSPORT THEORY. See **neutron transport theory, one velocity form**.

ONSAGER EQUATION FOR THE DIELECTRIC CONSTANT. If the short range interactions in a dielectric can be neglected (see **internal fields in dielectrics**), and if the **polarizability** α of the dipolar molecules is practically isotropic, Onsager has shown that the static dielectric constant ϵ_s of a pure liquid is given by

$$\epsilon_s - n^2 = \frac{3\epsilon_s}{2\epsilon_s + n^2} \frac{4\pi\mu_v^2 N_o}{3kT} \left(\frac{n^2 + 2}{3} \right)^2$$

where n is the refractive index, μ_v is the magnitude of the dipole moment of the molecule in vacuum; N_o is the concentration in particles per cubic centimeter, k is the Boltzmann constant, and T the absolute temperature. Onsager has obtained similar equations for the case of mixtures. (See also **Kirkwood equation for the dielectric constant**.)

ONSAGER RELATIONS. If X_i ($i = 1, 2, \dots$) denote forces acting on a system (such as electric field strength, temperature gradient, \dots), and j_i the fluxes produced by the forces (such as electrical current, heat current, \dots), one finds phenomenologically relations of the type

$$j_i = \sum_j L_{ij} X_j.$$

The Onsager relations state that

$$L_{ij} = L_{ji}.$$

An example of the Onsager relations are the Kelvin relations for thermoelectric phenomena, in which case the L_{ij} are related to the Peltier heat and the thermoelectric power. (See also **reciprocity relations**.)

OPEN EDGE TRAIN. See **edge train, open**.

OPEN QUADRATURE FORMULA. A quadrature formula in terms of ordinates drawn at points within the interval of integration, and excluding at least one endpoint, e.g., for integrating from x_0 to x_n , but based upon f_0, f_1, \dots, f_{n-1} only and not involving f_n . Such a formula is useful as a **predictor** formula in the numerical solution of **ordinary differential equations**.

OPEN SET. The concept of an open set of points in a **topological space** is often taken as a fundamental concept in terms of which other

concepts are defined. If the notion of neighborhood is already defined, as, e.g., in a metric space, an open set S may be defined as a set S such that, for every point p in S , the set S includes an entire neighborhood of p . Thus the interior of a circle in the **Euclidean plane** is an open set, the interior plus the circumference is a **closed** set, while the interior plus half the circumference is neither open nor closed.

OPEN SYSTEM. (Alternatively known as a *control volume*.) A **thermodynamic system** whose walls may be crossed by matter and energy (heat or work). (Cf. **isolated system** and **closed system**.)

OPERATING CHARACTERISTIC. If an inspection procedure provides a rule for accepting or rejecting batches on the basis of some observed quantity (such as the proportions of defective items in a sample from the batch), the operating characteristic is a graph of the probability of accepting a batch as ordinate against the value of the observed quantity as abscissa.

OPERATION. See **operator**. For **Hermitian operator**, etc., see the respective headings.

OPERATIONAL DEFINITION. A definition which can be reduced to the experimental steps that must be taken to measure the quantity being defined.

OPERATIONALISM. The principle that any quantity which is to be used in scientific considerations must be given an **operational definition**.

OPERATIONAL METHODS. The treatment of differential and other operators as though they were algebraic symbols in order to derive formal identities. Most often the term applies to the use of Fourier, Laplace, or other transforms in the solution of functional equations, but see **difference operators** for the formal derivation of formulas for **interpolation**, **quadrature**, and **differentiation**.

OPERATOR. In its widest sense the word operator is synonymous with **function**, **transformation**, etc., denoting a mapping of one set of elements (the domain of the operator) onto another (its range). But when the word operator is used, it is generally understood that both the domain and the range consist of functions (i.e., functions from number to number).

Thus, a **differential operator** transforms a function into its derivative, an **integral operator** has the form

$$g(s) = \int K(x,s)f(x)dx,$$

and so forth. A **linear operator** is a common case. It obeys the **distributive** law, thus for the operator A , $A[f(x) + g(x)] = Af(x) + Ag(x)$ and $A \cdot cf(x) = cAf(x)$, where c is any constant. If the order of applying operators to a function is immaterial, the operators are commutative. Suppose $A = a+$ and $B = b+$, with a, b constant, are applied to a function of x , then $ABf(x) = a + b + f(x) = BAF(x)$ and A, B commute. However, if $P = \partial/\partial x$ and $Q = x \cdot$, then P and Q are not commutative for $PQf(x) = f(x) + QPf(x)$.

If A and B are non-commutative, their commutator is $(A, B) = AB - BA$. According to quantum theory, if the commutator vanishes for two operators that represent dynamical variables, then the measurement of one of these variables does not interfere with that of the other.

As stated above, a differential operator involves one or more differentiations. Examples are $D = d/dx$, $D^2 = d^2/dx^2$, $D^{(n)} = d^n/dx^n$; $F(u) = f(x)u'' + g(x)u' + h(x)u$. In the more general case of an n -th order operator

$$L(u) = \sum_{i=0}^n [f_i(x)u^{(i)}]^{(s_i)}$$

the adjoint operator is

$$\bar{L}(u) = \sum_{i=0}^n (-1)^{r_i+s_i} [f_i(x)u^{(s_i)}]^{(r_i)}$$

If $L(u) = \bar{L}(u)$, the operators are self-adjoint. Any second-order differential operator can be made self-adjoint with an integrating factor

$$\exp \int \frac{(g - f')}{f} dx.$$

(See also **difference operators**; **creation and destruction operators**.)

If the elements of the domain of the transformation in question are other than functions, then, as stated above, it is customary to refer to the transformation by some other word than operator; e.g., addition of a number a , or multiplication by a group element a , etc., are usually called "operations."

OPERATOR, ADJOINT OF. See **operator**; **adjoint of an operator**.

OPERATOR, ANNIHILATION. See **creation and destruction operators**.

OPERATOR, CENTRAL MEAN. See **difference operators**.

OPERATOR, CREATION. See **creation and destruction operators**.

OPERATOR, DESTRUCTION. See **creation and destruction operators**.

OPERATOR, DIRAC. See **Dirac operator**.

OPERATOR, DISPLACEMENT. See **difference operators**.

OPERATOR, DYADIC. See **dyadic operator**.

OPERATOR, EXCHANGE. A quantum-mechanical operator, which exchanges the positions of two particles in a **wave function**.

OPERATOR, HERMITIAN. Any **operator** whose **matrix** representation is a Hermitian matrix.

OPERATOR, PAULI SPIN. See **Pauli spin operator**.

OPERATOR, POTENTIAL. See **potential operator**.

OPERATORS, DIFFERENCE. See **difference operators**.

OPERATOR, VECTOR. See **vector operator**.

OPTICAL AXIS. Of an anisotropic medium (e.g., a crystal), a direction for which **double refraction** does not occur.

OPTICAL CENTER. In an optical system a point so located that every **optical path** through that point will emerge from the system parallel to its direction of incidence.

OPTICAL DENSITY. The logarithm to base ten of the reciprocal of the **transmission factor**. (See **internal optical density**.)

OPTICAL DENSITY, EXTERNAL. See **external optical density**.

OPTICAL DENSITY, INTERNAL. See **internal optical density**.

OPTICAL DEPTH. The optical depth, sometimes referred to as "optical thickness," of an

absorbing layer of geometrical thickness is defined as

$$\tau_\nu = \int \kappa_\nu ds,$$

where κ_ν is the absorption coefficient for frequency ν . τ_ν is a non-dimensional quantity, introduced in the solution of problems of radiative transfer in cases where the absorption coefficient κ_ν varies along the line of sight. A beam of light of intensity $I_{\nu,0}$, after passing through an absorbing layer of optical depth τ_ν , has the intensity

$$I_\nu = I_{\nu,0} e^{-\tau_\nu}.$$

A layer of optical depth $\tau = 1$ therefore reduces the intensity of transmitted radiation to $1/e$ times its initial value.

OPTICAL DIFFERENTIAL INVARIANT. Let a two-dimensional manifold of rays, with parameters u, v , be specified by the position vectors \mathbf{a} of the object points (origin arbitrary) and the normalized direction vector \mathbf{s} of a ray, where the length of \mathbf{s} is the refractive index in object space. Let \mathbf{a}' , \mathbf{s}' be the corresponding image position vector and normalized direction vector of the image ray. The optical differential invariant is

$$L(u, v) = \mathbf{a}_u \mathbf{s}_v - \mathbf{a}_v \mathbf{s}_u = \mathbf{a}'_u \mathbf{s}'_v - \mathbf{a}'_v \mathbf{s}'_u$$

where the subscripts indicate partial derivatives. This is the **Lagrange bracket** of the calculus of variations.

OPTICAL DIRECTION COSINES. If $(\cos \alpha, \cos \beta, \cos \gamma)$ is the unit tangent vector of an **optical path** in a medium with refractive index n , then

$$p = n \cos \alpha, \quad q = n \cos \beta, \quad r = n \cos \gamma$$

are referred to as optical direction cosines. Since

$$p^2 + q^2 + r^2 = n^2,$$

there are two independent **direction cosines** and so sometimes the pair p, q are called the direction cosines of a ray.

OPTICAL DISTANCE. In the theory of neutron transport, the distance between two points, multiplied by the averaged total **cross section**, or averaged inverse **mean free path**, where the averaging is extended along the line-segment joining the points.

OPTICAL MODE. A type of **thermal vibration** of a **crystal lattice** whose frequency is nearly independent of wave number. The optical modes may be thought of as internal vibrations of the molecules or **unit cells** of the lattice, loosely coupled from cell to cell. In ionic crystals, this leads to strong absorption in the infrared because of the fluctuating dipole moment as the ions of opposite sign move relative to one another. The optical modes contribute to the specific heat with **Einstein specific heat functions**.

OPTICAL MODEL OF THE NUCLEUS. A model which represents the effect of the target nucleus on the bombarding particle in a nuclear reaction by means of a potential which is a complex function of position or shape approximating to the nuclear density distribution, the real part representing the elastic scattering, the imaginary part the absorption. This model has been quite successful in explaining the total cross sections of nuclei for neutrons and protons with energies in the 100 Mev region. Its most striking success, however, has been in predicting the total neutron cross sections for neutrons in the region of 1 Mev energy, where resonances are present in principle but are averaged out, either deliberately, or through poor energy resolution of the experiment. Whereas the compound nucleus theory gives the actual, complicated cross section of a process involving an actual, complicated nucleus, the optical model gives, surprisingly accurately, the average cross section in terms of the average properties of nuclear matter.

OPTICAL PATH. According to **Fermat's principle** an optical path C , or ray, in an isotropic medium is an extremal of the optical path length

$$\int_C n(x,y,z) ds$$

where $n(x,y,z)$ is the index of refraction and s is arc length on C . Thus C is an integral curve of the differential equations,

$$\begin{aligned} \frac{d}{ds} \left(n \frac{dx}{ds} \right) - \frac{\partial n}{\partial x} &= \frac{d}{ds} \left(n \frac{dy}{ds} \right) - \frac{\partial n}{\partial y} \\ &= \frac{d}{ds} \left(n \frac{dz}{ds} \right) - \frac{\partial n}{\partial z} = 0 \end{aligned}$$

which are the **Euler equations** of the variational problem. But, these are also the dif-

ferential equations of the **geodesics** of a **Riemannian space** with line element $[n(x,y,z)]^2 = [(dx)^2 + (dy)^2 + (dz)^2]$, which is a further characterization of an optical path.

Finally, in an isotropic medium an optical path can be characterized as an orthogonal trajectory of a one-parameter family of wave fronts. In an anisotropic medium the optical paths are the lines of energy flow or the curve whose tangent vector field is the **Poynting vector** $\mathbf{E} \times \mathbf{H}$.

OPTICAL THEOREM. The **S-matrix** connects the initial and final states, $|i\rangle$ and $|f\rangle$ of a quantum-mechanical system by

$$|f\rangle = S|i\rangle \quad (1)$$

Conservation of probability requires that

$$\langle f|f\rangle = \langle i|i\rangle \quad (2)$$

so that S is unitary:

$$S^*S = SS^* = 1. \quad (3)$$

(A proof of this unitarity depends on the properties of $H = H_0 + H_I$, the Hamiltonian of the system.) If one defines the operator **T** by

$$S = 1 + \mathbf{T} \quad (4)$$

the unitary condition implies

$$\mathbf{T}^* + \mathbf{T} = -\mathbf{T}^*\mathbf{T}. \quad (5)$$

The S -matrix has only matrix elements between states of equal energy so that if we write

$$\langle a|\mathbf{T}|b\rangle = 2\pi i \delta(E_a - E_b) \langle a|T|b\rangle \quad (6)$$

then Equation (5) asserts that

$$\text{Im} \langle i|T|i\rangle = \pi \sum_f \delta(E_i - E_f) |\langle f|T|i\rangle|^2. \quad (7)$$

The right hand side is connected with the total cross section, whereas the left hand side is connected to the forward elastic scattering amplitude in the case of two particle scattering. More generally if we write

$$\langle a|\mathbf{T}|b\rangle = 2\pi i \delta^{(4)}(p_a - p_b) \langle a|M|b\rangle \quad (8)$$

where $p_a = (E_a, \mathbf{p}_a)$ is the energy-momentum of the initial state, then unitarity asserts that

$$2\text{Im} \langle i|M|i\rangle = 2\pi \sum_f \delta^{(4)}(p_f - p_i) |\langle f|M|i\rangle|^2. \quad (9)$$

Now the transition probability per unit volume per unit time for the process $i \rightarrow f$ is

$$w = \frac{|\langle f|T|i\rangle|^2}{\lim_{r,r\rightarrow\infty} \tau v} = \frac{1}{(2\pi)^2} \delta^{(4)}(\rho_i - \rho_f) |\langle f|M|i\rangle|^2. \quad (10)$$

The cross section for the process $i \rightarrow f$ is then given by $\sigma(i \rightarrow f) = w/\text{flux}$. For a two particle collision the flux is given by $\text{flux} = v/(2\pi)^6$ where v is the relative velocity between particles, whence

$$\sigma(i \rightarrow f) = \frac{(2\pi)^4}{v} \delta^{(4)}(\rho_i - \rho_f) |\langle f|M|i\rangle|^2. \quad (11)$$

Summing over all final states gives the total cross section

$$\sigma_{total} = \frac{(2\pi)^4}{v} \sum_f \delta^{(4)}(\rho_i - \rho_f) |\langle f|M|i\rangle|^2 \quad (12)$$

so that comparing (12) and (9) verifies our previous assertion. Equation (9) is sometimes called the optical theorem. For the two-particle scattering case it can be reduced to the following expression

$$\text{Im } f(0) = \frac{k}{4\pi} \sigma_{total} \quad (13)$$

where $f(0)$ is the non-spin flip (coherent) forward scattering amplitude and k is the magnitude of the projective particle's momentum (in the laboratory system).

OPTICS, CANONICAL EQUATIONS OF. See canonical equations of optics.

OPTICS, COLLINEATION EQUATIONS OF. See collineation equations of optics.

OPTIMUM-INTERVAL INTERPOLATION.

The use of interpolation formulas based upon intervals of varying size adjusted so as to optimize the result in some way. Usually the interval size will be taken as great as possible so that the error of an interpolating polynomial of fixed degree will not surpass a specified limit. (See Alston S. Householder, *Principles of Numerical Analysis*, McGraw-Hill Book Company, 1953.)

OPTIMUM MAGNIFICATION. The maximum value of the numerical aperture of a dry

lens is 1.0. Oil-immersion objectives with numerical apertures up to 1.65 have been constructed. The minimum distance between points which are just resolved is thus 2.7×10^{-5} cm for a dry lens, and 1.6×10^{-5} cm for an oil-immersion lens, using oblique illumination and a wavelength of 5500 Å. The maximum useful magnification is thus about 800 for dry objectives, and 1200 for oil-immersion objectives.

ORBIT. Strictly, the closed trajectory followed by a particle under the influence of a central force. The noun is often applied to trajectories that are not closed, such as the parabolic path of a comet or the path of an earth satellite that is falling and speeding up as a result of dissipative forces.

ORBITAL. Term introduced by Mulliken for the states of single electrons in an atom or molecule which are characterized by certain values of their quantum numbers apart from spin, and certain orbital wave functions.

In atoms, orbitals with $l = 1, 2, \dots$ are designated s, p, d, \dots orbitals, and the electrons in them s, p, d, \dots electrons. Placing the value of n in front of the symbol for l , one speaks, for example, of a $2p$ orbital, or $2p$ electrons.

Similarly, in molecules, one distinguishes orbitals with certain values of n, l , and m_l , and speaks for example, of $3d\sigma, 3d\pi, 3d\delta, \dots$ orbitals or electrons, respectively, for $n = 3, l = 2$, and $m_l = 0, 1, 2, \dots$. In many cases, the eigenfunctions of a single electron in a molecule can be approximated by a linear combination of the orbital eigenfunctions of the separated atoms, that is, molecular orbitals are linear combinations of atomic orbitals. (n, l , and m_l are the principal, azimuthal and magnetic quantum numbers, respectively.) (See also quantum numbers of the individual electrons in an atom.)

ORBITAL, ANTIBONDING. A molecular orbital whose energy increases as the two atoms are brought together, corresponding to a net repulsion. In an antibonding orbital the electron density is least in the region between the two nuclei.

ORBITAL, BONDING. A molecular orbital coupling two atoms in such a way that the energy has a minimum value when the interatomic distance is small, thus favoring a bond

between them. In a bonding orbital the electron density is greatest in the region between the two nuclei.

ORBITAL DIAMAGNETISM OF CONDUCTION ELECTRONS. There is a contribution to the magnetic susceptibility of a metal which results from changes in the orbital wave functions when a magnetic field is applied. **Landau's formula** for the susceptibility χ produced by the free conduction electrons as a result of this is

$$\chi = -\frac{4m\mu_B^2}{h^2} \left(\frac{N\pi^2}{9}\right)^{1/2} \quad (1)$$

where N is the number of free electrons per unit volume, and μ_B is the **Bohr magneton**. Classical theory would give instead of (1), $\chi = 0$. The contribution to the susceptibility given by Equation (1) must be added to that which results from **spin paramagnetism of conduction electrons**. The total value of the susceptibility is then twice as great, and of the opposite sign to that given by Equation (1).

ORBITAL ELEMENTS. See **orbit, astronomical, elements of**.

ORBITAL MOTION OF ELECTRONS IN AN ATOM. See **electron orbits in an atom (Bohr orbits)**.

ORBITAL, P. An atomic orbital using a p -state eigenfunction (1 unit of orbital angular momentum).

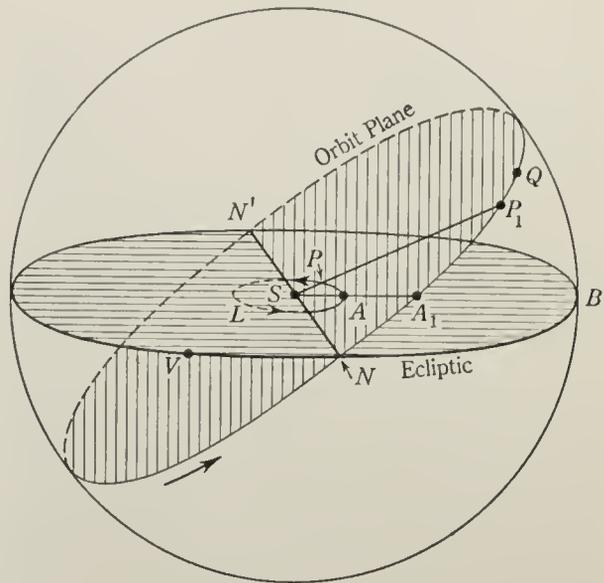
ORBITAL, S. An atomic orbital using an s -state eigenfunction (zero orbital angular momentum).

ORBIT, ASTRONOMICAL, ELEMENTS OF.

In describing the motions of the planets and other members of the solar system about the sun, and to locate the position of such an object at any past or future time, seven so-called elements of the orbit are required.

The plane of the ecliptic is the fundamental plane of reference for the solar system, and the direction of the vernal equinox is the fundamental direction. In the figure is drawn the celestial sphere, showing the plane of the ecliptic and the position of the vernal equinox, V , for an arbitrary date. The sun is at the center of the sphere and the orbit plane of a planet or other celestial object is represented by the plane

NA_1P_1PQN' . The orbit itself is shown as the ellipse APL with perihelion at A . The line of intersection of the orbit plane and the ecliptic is known as the line of nodes, NN' . The angle



VN in the plane of the ecliptic is one of the elements of the orbit and is known as the *longitude of the ascending node*. This angle is usually symbolized by θ . A_1NB is another element of the orbit known as the *inclination*, designated by i . The third orbital element is the sum of θ , in the plane of the ecliptic, and NA_1 (which is designated by ω) in the orbit plane. This sum, $\bar{\omega} = \theta + \omega$, is the *longitude of perihelion*. These three elements, θ , i , and $\bar{\omega}$ are three elements of the orbit. The longitude of the nodes θ determines the points N and N' on the celestial sphere at which the plane of the orbit intersects the ecliptic. The angle i is the angle at which the plane of the orbit is inclined to the plane of the ecliptic. The longitude of perihelion, $\bar{\omega}$, determines the direction of the perihelion point.

Two other elements define the ellipse itself: e is the *eccentricity* of the orbit, and a is the *semi-major axis of the ellipse*, usually expressed in astronomical units. Two more elements are required to determine the position of the object in the orbit at a given time. They are: P , the complete *period of revolution* about the ellipse, and μ , the *mean daily motion*, which is simply $360^\circ/P$ in days.

In addition to the seven elements discussed above it is desirable to specify some starting time for the motion in the ellipse. The quantity used for this purpose is the mean longitude of the object as seen from the sun on some

particular date called the epoch (E), e.g., January 1, 1960.

T is the precise time at which the planet was at perihelion.

ORBIT, BOHR. See Bohr orbit.

ORBIT THEORY, PLANETARY. See planetary orbit theory.

ORDER (OF SYSTEM). For notational system, see indicial notation; or for control system, see control system, type number.

ORDER-DISORDER TRANSFORMATION. Many alloys with a characteristic lattice structure show a specific heat anomaly at a critical temperature which corresponds to a change from an ordered to a disordered arrangement of the constituent atoms of the alloy. This occurs without any significant change in lattice type. For a binary alloy AB , which contains an atomic fraction a of constituent A , the degree of order s is defined by

$$s = (p - a)(1 - a)$$

where p is the fraction of "A" atoms in their correct ordered positions.

If ϵ is the energy of exchange of a pair of A and B atoms, from wrong to right positions, ϵ will have a maximum value ϵ_0 when there is complete order, and will be zero for complete disorder. If it is assumed that $\epsilon = s\epsilon_0$, then the total energy arising from disorder ΔE is given by

$$\Delta E = \Delta E_0(1 - s^2)$$

and

$$\Delta E_0 = \frac{1}{2}Na(1 - a)\epsilon_0$$

where N is the total number of atoms. (See also cooperative phenomena.)

ORDER OF CHEMICAL REACTIONS. A chemical reaction is said to be of the n^{th} order if its rate is directly proportional to the product of n concentrations. Therefore the decomposition of A , if described by the equation

$$\frac{dC_A}{dt} = -kC_A \quad (1)$$

is a *first order* reaction. Similarly if it is described by the equation

$$\frac{dC_A}{dt} = -kC_AC_B \quad \text{or} \quad \frac{dC_A}{dt} = -kC_A^2 \quad (2)$$

it is a *second order* reaction.

The coefficient k which appears in (1) or (2) is called the *rate constant*. Generally the temperature variation of a rate constant may be expressed by

$$k = Pe^{-\frac{Q}{kT}} \quad (3)$$

where k is the rate constant and k is the Boltzmann constant. This equation is called the *Arrhenius equation*. (See also collision theory; absolute reaction rate theory.)

ORDER OF CONTRAVARIANCE. See tensor field.

ORDER OF CONTROL SYSTEM. See control system, type number.

ORDER OF COVARIANCE. See tensor field.

ORDER OF DIFFERENTIAL EQUATION. See differential equation entries.

ORDER OF GROUP. The number of elements in the group.

ORDER OF POLYNOMIAL. A polynomial in x is said to be of order n when n is the highest power of x appearing in it.

ORDER (TOTAL) OF TENSOR. See tensor field.

ORDER PARAMETERS. See cooperative phenomena.

ORDER STATISTICS. Variate values in a sample determined according to the order in the array of sample-values when arrayed according to magnitude; e.g., the smallest value, the **median** (middle value). More generally, any function of such statistics, e.g., the **range** (largest less smallest value).

ORDINARY DIFFERENTIAL EQUATION.

An equation in which there occur, along with each dependent variable, one or more of its derivatives with respect to a single independent variable. In general there must be as many equations as there are dependent variables. Also, in general, if any derivatives higher than the first occur, it is possible to enlarge the system to one containing only first derivatives. Thus if the equation is of the form

$$f(x, y, y', y'') = 0,$$

where x is the independent variable, one can set

$$y_1 = y, \quad y_2 = y',$$

and the given equation is equivalent to the simultaneous equations

$$\begin{aligned} f(x, y_1, y_2, y'_2) &= 0, \\ y'_1 - y_2 &= 0. \end{aligned}$$

Finally, it is generally possible to solve for first derivatives and write the system in the form

$$dy_i/dx = \phi_i(x, y_1, y_2, \dots, y_n), \quad i = 1, 2, \dots, n.$$

The equations are then said to be in normal form. In order that the $y_i(x)$ be uniquely defined as functions of x it is necessary to adjoin n further conditions relating the values assumed by the y_i at certain discrete points. For an initial value problem, all values are specified at a single point, for a **boundary-value problem**, relations between values at the two boundaries are specified. Here it will be assumed that initial values are given.

A system in normal form can be compactly represented by

$$y' = f(x, y),$$

where, if $n > 1$, y , y' , and f represent vectors in n -space. Methods for numerical solution in the general case are rather obvious generalizations of the case $n = 1$, and attention will be confined to this. It will be assumed that the initial value

$$y(x_0) = y_0$$

is given. In case $f(x, y)$ is analytic in x and in y at (x_0, y_0) , then it is possible to obtain the coefficients of the series expansion

$$y = y_0 + (x - x_0)y'_0 + \frac{1}{2}(x - x_0)^2 y''_0 + \dots$$

In fact, y_0 is given, from the equation itself one obtains

$$y'_0 = f(x_0, y_0),$$

and by differentiation one obtains

$$y''_0 = f_x(x_0, y_0) + y'_0 f_y(x_0, y_0)$$

and other derivatives of higher order are similarly found.

Most numerical methods, however, combine interpolation with the **Picard method** in the sense that the quadratures required by this method are carried out numerically. This idea is capable of almost infinitely many variations, but to begin with it is generally assumed that by series expansion or otherwise one has first obtained suitable values $y_1 = y(x_1)$, $y_2 =$

$y(x_2)$, \dots where, in general, $x_\nu = x_0 + \nu h$, and h represents a sufficiently fine subdivision. Thereafter one can write

$$y_{\nu+i} = y_\nu + \int_{x_\nu}^{x_{\nu+i}} f(x, y) dx,$$

where $y_\nu, y_{\nu+1}, \dots, y_{\nu+i-1}$ have been found. At this point one uses first a predictor formula which expresses the integral in terms of values of f at points already known (see **quadrature, numerical**). This gives an approximate value for $y_{\nu+i}$, hence permits the calculation of an approximate value of $f_{\nu+i}$, and one can then apply a corrector formula which utilizes this value. Application of the corrector formula gives a new approximation for $y_{\nu+i}$, and the corrector formula should be reapplied until the changes are negligible. It is advisable to have h small enough so that at most a second application of the corrector formula will be sufficient. Methods of this type are the **Euler method**, the **Adams-Bashford method**, and the **Milne method**. (See National Physical Laboratory, *Modern Computing Methods*, Notes on Applied Science No. 16, Her Majesty's Stationery Office, 1957; George M. Murphy, *Ordinary Differential Equations and Their Solution*, D. Van Nostrand Company, 1960.) (For questions of *stability* see F. B. Hildebrand, *Introduction to Numerical Analysis*, McGraw-Hill Book Company, 1956; Lothar Collatz, *Numerische Behandlung von Differentialgleichungen*, 2nd edition, Springer, 1955.)

The method of *Runge-Kutta* is somewhat different in character.

For the explicit treatment of an equation of higher order

$$y'' = f(x, y, y')$$

without reducing it to normal form in two independent variables, see **Störmer method**.

ORDINARY POINT. An ordinary point of a function $f(z)$ of the **complex variable** z is a point $z = z_0$ at which $f(z)$ is analytic. Any point which is not an ordinary point is a **singular point**.

ORDO-SYMBOL. If a function $f(x)$ is such that, as x approaches 0 or ∞ , the function $|x^k f(x)|$ is bounded by a positive constant k which is independent of x , then $f(x)$ is said to be of **order** x^{-k} . In symbols, $f(x) = O(x^{-k})$, where the O is called the **ordo-symbol**. The limiting process with 0 or ∞ as the limit is not

always stated explicitly but inferred from the context. In the special case where $\lim |x^k f(x)| = 0$, one writes $f(x) = o(x^{-k})$, read as: of smaller order than (x^{-k}) .

ORIENTATION EFFECT. See **van der Waals forces**.

ORIENTATION ENERGY. See **Keesom energy**.

ORIENTED CIRCUIT. See **circuit, oriented**.

ORIENTED CUT SET. See **cut set, oriented**.

ORIENTED ELEMENT. See **element, oriented**.

ORIENTED GRAPH. See **graph, oriented**.

ORIGIN. A basic reference point being used in some consideration. Examples are the reference point for the measurement of potential energy; the point of intersection of the coordinate axes in a coordinate system; the point from which distances are measured.

ORR-SOMMERFELD EQUATION. See **stability of laminar flow**.

ORTHOCHRONOUS HOMOGENEOUS LORENTZ TRANSFORMATION. A homogeneous Lorentz transformation

$$x' = \Lambda x,$$

for which $\Lambda_0^0 \geq 1$. A homogeneous Lorentz transformation is orthochronous if and only if it transforms every positive time-like vector into a positive time-like vector. The set of all orthochronous homogeneous Lorentz transformations form a group.

ORTHOCHRONOUS INHOMOGENEOUS LORENTZ TRANSFORMATION. An inhomogeneous Lorentz transformation

$$x' = \Lambda x + a$$

for which $\Lambda_0^0 \geq 1$.

ORTHOGONAL COORDINATES, CURVILINEAR. See **curvilinear orthogonal coordinates**.

ORTHOGONAL FUNCTION. The word orthogonal means, in general, perpendicular; thus the three axes of a rectangular Cartesian coordinate system are mutually orthogonal.

In n -dimensional space, two vectors **A**, **B** with components A_i, B_i ($i = 1, 2, \dots, n$) are orthogonal if

$$\sum_{i=0}^n A_i B_i = 0.$$

Analogously, if the vector space involved has an infinite number of dimensions two functions **A**(x), **B**(x) are orthogonal if

$$\int_a^b \mathbf{A}(x) \cdot \mathbf{B}(x) dx = 0.$$

The limits of integration are needed to specify the range of x for which the functions are defined. They may be finite or infinite.

Arbitrary functions may be made orthogonal by the **Gram-Schmidt process**.

ORTHOGONAL GROUP. A subgroup of the **unitary group**; its elements are all real n -dimensional square **unitary matrices**. The determinant of these matrices is ± 1 ; if $+1$, they are proper orthogonal matrices, if -1 , improper orthogonal matrices. The subgroup of the orthogonal group containing only proper orthogonal matrices is the **rotation group** of order n . If $n = 3$, the proper orthogonal matrices correspond to rotations about an axis passing through the origin, while the improper matrices correspond to such rotations, followed by reflections in a plane perpendicular to the axis of rotation. These conceptions may be generalized for n -dimensions and specialized for two dimensions.

ORTHOGONALIZATION. Given an **inner product** operation (ϕ, ψ) , defined for any pair of functions ϕ and ψ of a specified class, if $(\phi, \psi) = 0$ then ϕ and ψ are said to be orthogonal. Given a sequence of functions of the class,

$$\phi_0, \phi_1, \phi_2, \dots$$

one can ask for a sequence

$$\psi_0, \psi_1, \psi_2, \dots$$

such that $(\psi_i, \psi_j) = 0$ when $i \neq j$, and

$$\phi_0 = \psi_0,$$

$$\phi_1 = \alpha_{10}\psi_0 + \psi_1,$$

$$\phi_2 = \alpha_{20}\psi_0 + \alpha_{21}\psi_1 + \psi_2,$$

$$\dots \dots \dots$$

where the α_{ij} are constants. One finds easily that

$$(\psi_0, \phi_1) = \alpha_{10}(\psi_0, \psi_0),$$

$$(\psi_0, \phi_2) = \alpha_{20}(\psi_0, \psi_0),$$

$$(\psi_1, \phi_2) = \alpha_{21}(\psi_1, \psi_1),$$

and all α_{ij} and ψ_i can be obtained sequentially unless for some i , $(\psi_i, \psi_i) = 0$. (See also **Gram-Schmidt process**.)

ORTHOGONAL MATRIX. See **matrix**.

ORTHOGONAL POLYNOMIALS. Any set of polynomials that are mutually orthogonal with respect to an **inner product** operation. Such an operation can be defined quite formally in terms of an abstract moment operator Ω defined by

$$\begin{aligned} \Omega(\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \dots) \\ = \alpha_0 \mu_0 + \alpha_1 \mu_1 + \alpha_2 \mu_2 + \dots \end{aligned}$$

where the μ_i are arbitrarily assigned constants. Most often one defines the result of Ω operating on a function ϕ to be

$$\Omega[\phi(x)] = \int_a^b \phi(x) \omega(x) dx$$

where a and b are fixed, finite or infinite, and $\omega(x)$ is a fixed density function. The inner product of two polynomials ϕ and ψ is

$$(\phi, \psi) = \Omega[\phi(x)\psi(x)].$$

The orthogonal polynomials can be obtained by applying the process of **orthogonalization** to the powers of x :

$$1, x, x^2, x^3, \dots$$

Orthogonal polynomials arise in **Gaussian quadrature** in that their zeros are the points x_i at which the integrand function must be evaluated. They occur as the denominators of the entries in a **Padé table** along a line parallel to the diagonal.

Given a finite set of discrete points x_0, x_1, \dots , if the operator Ω is defined by

$$\Omega[\phi(x)] = \sum \phi(x_i) \omega_i$$

where ω_i are non-negative weight functions, then the corresponding orthogonal polynomials are finite in number. Let these be $\psi_0, \psi_1, \psi_2, \dots$. For **least square** polynomial fitting to

data taken at these points x_i , it is advantageous to express the required polynomial in the form

$$f(x) = \gamma_0 \psi_0 + \gamma_1 \psi_1 + \gamma_2 \psi_2 + \dots$$

and to solve for the γ 's by least squares. (For representative examples, see **Lagrange**, **Laguerre**, **Hermitic polynomials**.)

ORTHOGONAL VECTORS. See **vector(s)**, **orthogonal**.

ORTHOHELIUM. Early investigators believed that the triplet and singlet lines in the spectrum of the helium atom belonged to two kinds of helium, orthohelium and parhelium, respectively. The triplet and singlet terms of helium, that is, the energy levels with parallel and anti-parallel electron spins, are therefore traditionally referred to, respectively, as orthohelium terms and parhelium terms. (In analogy the triplet and singlet term systems of the alkaline earth atoms are referred to as ortho and para systems.)

ORTHO MODIFICATION. See **non-combining modifications of molecules**.

ORTHONORMAL. Suppose a set of functions of the complex variable z is defined over the range $a \leq z \leq b$, so that the members of the set, $f_1(z), f_2(z), \dots$ are orthogonal

$$\int_a^b f_i^*(z) f_j(z) dz = 0, \quad i \neq j.$$

Then in general,

$$\int_a^b f_i^*(z) f_i(z) dz = c_i^2 \neq 0$$

is not equal to unity (i.e., the functions $f_i(z)$ are not normalized). It is frequently convenient to redefine the functions so that they are normalized by taking $c_i \phi_i(z) = f_i(z)$ such that

$$\int_a^b \phi_i^*(z) \phi_j(z) dz = \delta_{ij}$$

functions $\phi_i(z)$ are then said to form an orthonormal set. For an arbitrary set of functions, $f_i(z)$, conversion to orthonormal functions may be effected by the **Schmidt process**. If z is real, obvious simplifications in the equations are possible.

ORTHOPOSITRONIUM. The state of **positronium** in which the spins of electron and

positron are parallel. The 1s state annihilates into three gamma-rays with a mean life of 1.4×10^{-7} sec.

ORTHORHOMBIC SYSTEM. One of the seven fundamental systems of crystallography; in this system the axes are of unequal length, and intersect at right angles. (See crystal systems.)

ORTHOSEOPIC SYSTEM (RECTILINEAR SYSTEM). An optical system corrected for both distortion and spherical aberration.

ORTHOTOMIC SYSTEM. An optical system that admits only a normal congruence of rays.

OSCILLATING SYSTEM, DEGENERATE. See degenerate oscillating system.

OSCILLATION. One complete period of vibratory or periodic motion, for example, the whole succession of states that takes place before the motion begins to repeat itself. For example, one oscillation of a pendulum bob is a complete excursion from where it started back to its original position with the same velocity (magnitude and direction). The time of one oscillation is called one period and the number of oscillations per second (the reciprocal of the period) is called the frequency. The definition cannot be applied strictly to nonperiodic motions (e.g., see oscillation, damped harmonic). In such motions the period is usually taken as the time between successive zeros of the displacement.

OSCILLATION, CENTER OF. See center of oscillation.

OSCILLATION, DAMPED ELECTRICAL. (See also oscillations in electric circuits.) If a simple series circuit, consisting of an inductance L , a capacitance C , and a resistance R , is temporarily disturbed, by the induction of currents or the temporary introduction of potentials, the differential equation which applies is

$$L \frac{d^2q}{dt^2} + R \frac{dq}{dt} + \frac{q}{C} = 0,$$

where q is the charge on the condenser at any time t . Comparison of this equation with that given under oscillation, damped harmonic shows that the two are identical if L replaces m and $1/C$ replaces f . Hence the charge on

the condenser (and also the current, equal to dq/dt) will oscillate with a natural frequency

$$= \frac{1}{2\pi} \sqrt{1/LC - R^2/4L^2}$$

if $1/LC > R^2/4L^2$. Under these circumstances, the charge or current amplitude will decrease exponentially with time, in proportion to $e^{-Rt/2L}$. When $1/LC < R^2/4L^2$, no oscillation will take place. However, if energy is supplied to the circuit at the natural frequency and in the proper phase, steady oscillations may be maintained.

OSCILLATION, DAMPED HARMONIC. An oscillation in which there is resistance to the motion. This resistance is opposite to the direction of motion and to a good approximation is proportional to the first power of the velocity. The presence of the resistance leads to a continuous dissipation of the total mechanical energy of motion.

The differential equation of motion for a dissipative oscillatory system with one degree of freedom and no external driving force is

$$m \frac{d^2x}{dt^2} + R \frac{dx}{dt} + fx = 0$$

where m is the mass of particle, R is the damping coefficient, f is the stiffness coefficient. The solution is of the form

$$x = e^{-\frac{R}{2m}t} \left(A e^{\sqrt{R^2/4m^2 - \frac{f}{m}}t} + B e^{-\sqrt{R^2/4m^2 - \frac{f}{m}}t} \right)$$

where A and B are arbitrary constants.

If $\frac{R^2}{4m^2} < \frac{f}{m}$, oscillations of steadily decreasing amplitude take place with frequency $\nu = \frac{1}{2\pi} \sqrt{f/m - R^2/4m^2}$.

If $\frac{R^2}{4m^2} = \frac{f}{m}$, no oscillations take place and motion is said to be "critically damped."

If $\frac{R^2}{4m^2} > \frac{f}{m}$, no oscillations take place and motion is said to be "overdamped."

(See damping, mechanical; logarithmic decrement.)

OSCILLATION, FORCED. The oscillation which results when an external periodic driv-

ing force is applied to a system capable of free oscillations. In the one-dimensional case for a dissipative system the differential equation of motion is

$$m \frac{d^2x}{dt^2} + R \frac{dx}{dt} + fx = F_0 \cos 2\pi\nu t.$$

F_0 is the amplitude of driving force; ν is frequency of driving force.

The solution to this equation has two parts, a transient which eventually damps to zero, and a steady state which is the dominant part when the transient diminishes. The steady state solution has the form

$$x = \frac{F_0 \cos(\omega t - \alpha)}{\sqrt{\omega^2 R^2 + (f - m\omega^2)^2}}$$

where $\omega = 2\pi\nu$, $\alpha = \tan^{-1} \frac{R\omega}{f - m\omega^2} =$ phase difference between force and displacement.

Of special interest in forced oscillations is the phenomenon of resonance. Velocity resonance is a state of maximum velocity and occurs when the frequency of the driving force

has the value $\nu_0 = \frac{1}{2\pi} \sqrt{\frac{f}{m}}$. ν_0 is called the

resonance frequency. *Amplitude resonance* is a state of maximum displacement and occurs when the frequency of the driving force has the

value $\nu_1 = \frac{1}{2\pi} \sqrt{f/m - R^2/4m^2}$.

When the damping coefficient is small, the resonance frequency amplitude ν_1 and the free oscillation frequency become approximately the same. At the resonance frequency ν_0 , the force and velocity are exactly in phase while the force and displacement are 90° out of phase. The word resonance in general usage means amplitude resonance. (See **harmonic motion**.)

OSCILLATION(S), FREE OR NATURAL.

One of three types of oscillations: 1. **Oscillations** that continue in a circuit or system after the applied force has been removed, the frequency of the oscillations being determined by the parameters in the system or circuit, commonly referred to as shock-excited oscillations. 2. The oscillation of some physical quantity of a body or system when the externally applied forces consist either of those which do no work, or of those which are de-

rivable from a potential that is invariant during the time under consideration, or both. 3. That type of oscillatory motion into which a suitable system not subject to external driving forces is capable of being excited by a displacement from an equilibrium position. (See **oscillations, damped harmonic**.)

OSCILLATION, FREQUENCY OF. The number of complete **oscillations** of a given system per unit time, commonly symbolized by ν or f . The frequency is the reciprocal of the period, the time for one complete oscillation. Sometimes the angular frequency, symbolized by ω , is used for greater convenience in manipulating trigonometric functions. The angular frequency has the unit radian per unit time, equal to $2\pi \times$ frequency, and is sometimes called the **pulsatance**.

OSCILLATION, HARMONIC. See **oscillator, harmonic**.

OSCILLATION, MODES OF. In the case of standing waves the boundary conditions restrict the possible frequencies of oscillation to a discrete set of values. The whole set constitutes the modes of oscillation of the system.

OSCILLATIONS IN ELECTRIC CIRCUITS.

An ideal electric circuit composed of a capacitance C and a dissipationless self-inductance L , is an electrical analog to the mechanical harmonic oscillator. (See **oscillator, harmonic; oscillation, damped harmonic; oscillation, damped electrical**.) Equating the magnitudes of the potential differences across L and C gives, $Q/C = -LdI/dt$. Another form of the equation is $Q/C = -Ld^2Q/dt^2$, where use has been made of the fact that the current I and charge Q are related by $I = dQ/dt$. This second form of the circuit equation is the harmonic oscillator equation. Its solution may be written $Q = Q_0 \cos(\omega t + \phi)$ where Q_0 and ϕ are determined by the initial conditions and Q , and $I = dQ/dt$. The radian frequency of oscillation is $\omega = 1/\sqrt{LC}$, or the frequency is $f = \omega/2\pi = 1/2\pi\sqrt{LC}$. The period of the oscillations is $T = 1/f = 2\pi\sqrt{LC}$.

No self-inductance is ideal, since all self-inductances contain resistance. As a result, the energy stored in the circuit, $(Q^2/2C + LI^2/2)$ is dissipated. In order to maintain

the oscillations, power has to be fed into the circuit.

OSCILLATIONS OF SYSTEM WITH SEVERAL DEGREES OF FREEDOM. If a system consists of N mass points all of which vibrate with very small amplitude and if no damping or internal forces are present, then the mass points vibrate sinusoidally (or “harmonically”) about their respective equilibrium positions. In this case there exist in general $3N$ different “characteristic” frequencies with which the particle can oscillate (if the number of frequencies is smaller, one speaks of “degenerate” systems). The state of motion in which all particles vibrate with the same frequency, is called a *normal mode*. In every mode the ratio of the various amplitudes bears a definite relation to each other. The general state of motion can be described as a linear superposition of the different normal modes. Each normal mode can be excited separately. The analysis of the possible normal modes depends on the symmetry properties of the system and is greatly facilitated by the use of **group theory**.

OSCILLATION, STEADY-STATE (VIBRATION). The oscillation of a body or system in which the motion at each point is a periodic quantity. This is frequently a special case of forced oscillation.

OSCILLATOR, ANHARMONIC. A mechanical system with one degree of freedom which, when displaced from equilibrium, is subject to a restoring force corresponding to a superposition of first and higher powers of the displacement. A simple example of an anharmonic oscillator has the equation of motion

$$m \frac{d^2x}{dt^2} + k_1x + k_2x^2 = 0.$$

The vibrations of the human ear drum may be described by an equation of this form, which means physically that the restoring force is not symmetrical about the equilibrium position of the oscillator. The equation is mathematically non-linear (because of the presence of the square term k_2x^2) and leads to a solution involving harmonics of the fundamental oscillation frequency (if the anharmonic term k_2x^2 is always small compared with the harmonic term k_1x).

OSCILLATOR, HARMONIC. (See also **oscillation, damped; oscillation, forced; oscillator, anharmonic.**) A particle or mechanical system with one degree of freedom which when displaced from equilibrium, is subject to a restoring force varying directly as the displacement. If the effective mass is m , the stiffness is k , and the displacement is x , the differential equation of motion of the harmonic oscillator is

$$m \frac{d^2x}{dt^2} + kx = 0$$

leading to solution

$$x = Ae^{i(2\pi\nu t + B)}$$

where $\nu = \frac{1}{2}\pi\sqrt{k/m}$ is the **frequency**, A is the **amplitude** and B is the initial phase. This system is undamped and performs simple harmonic motion. It should be noted that x need not be a linear displacement but may be an angle. Harmonic oscillations in different directions in a plane may be superposed and the result is often referred to as a two-dimensional harmonic oscillator. The motion of the resultant system is, however, not necessarily periodic and the use of the name therefore somewhat questionable.

The wave mechanical solution of the problem analogous to that of the classical harmonic oscillator plays an important role in atomic and molecular physics. If a particle of mass m is constrained to linear motion with its potential energy V being proportional to the square of the displacement x from some equilibrium position, the **Schrödinger equation** is

$$\left\{ \frac{d^2}{dx^2} + \frac{m}{\hbar^2} \left(E - \frac{1}{2}kx^2 \right) \right\} \psi = 0$$

where \hbar is the Dirac h , k is a constant, and E is the energy of the particle. This equation has solutions only for discrete **eigenvalues** ($\mathbf{1}$) of E , these being

$$E_n = \hbar\sqrt{k/m} \left(n + \frac{1}{2} \right).$$

For a two- or three-dimensional isotropic harmonic oscillator we have, respectively, $E_n = \hbar\sqrt{k/m} (n + 1)$, $g_n = n + 1$, $E_n = \hbar\sqrt{k/m} (n + \frac{3}{2})$, $g_n = \frac{1}{2}(n + 1)/(n + 2)$, where g_n is the degree of degeneracy of the level E_n , where n is an integer ($0, 1, 2 \dots$). The **eigenfunctions** are

$$\psi_n = e^{-\frac{1}{2}\alpha x^2} H_n(x)$$

where $H_n(x)$ is a **Hermite polynomial** and $\alpha = \sqrt{km/\hbar}$. As the expression for E_n shows, the wave mechanical harmonic oscillator differs from the classical in that only discrete, equally spaced energy levels are possible and that the lowest state has an energy $(\hbar/2)\sqrt{k/m}$. This leads to the concept of **zero point energy**. (See also **harmonic oscillator**, **partition function** of.)

OSCILLATOR STRENGTH. The oscillator strength f^{nm} (also called number of dispersion electrons or f -value) represents the ratio of the quantum theoretical and classical contributions of the transition $n \rightarrow m$ to the refractivity $N - 1$ ($N =$ index of refraction), where the classical value is calculated on the assumption of a single oscillating electron of frequency ν_{nm} . The oscillator strength is given by

$$f^{nm} = \frac{\mu\hbar c^2 \nu_{nm}}{\pi e^2} B_{mn} = \frac{8\pi^2 \mu c \nu_{nm}}{3\hbar e^2} |R^{nm}|^2.$$

Here h stands for Planck's constant, c for the velocity of light, μ and e , respectively, for mass and charge of electron, ν_{nm} for the wave-number, B_{mn} for the Einstein **transition probability** of absorption and R^{nm} for the matrix element of the transition.

For a single emission electron there is a sum rule

$$\sum_n f^{nm} = 1.$$

For the strongest electronic transitions f may therefore be of the order 1.

OSCILLATORY, MOTION. Persistent motion which is confined to a finite region of space. Such motion is not necessarily periodic (see, for example, **Lissajous figures**), but when it is, there is precise repetition after the elapse of a finite time called the period. The motion of a pendulum, of a weight on a stretched spring and of a point in a plucked string are examples of oscillatory motion. (See **oscillator**.)

OSCULATING CIRCLE. See **curvature**.

OSCULATING DEVELOPABLE. See **developable**, **osculating**.

OSCULATING PLANE. The plane, if it exists, through a point P of a space curve which has contact of higher order with the curve than any other plane through the point; the limiting position of the plane through

P , P' , P'' as the points P' , P'' approach P along the curve.

OSCULATING SPHERE. See **sphere of curvature**.

OSEEN METHOD. A method of solving the equations of motion of a viscous fluid past a solid body at low **Reynolds number**. The method ignores the squares of the velocities of the disturbance produced by the body, i.e., treats the motion as a small perturbation of a uniform stream. It is inaccurate near stagnation points in particular, but is an improvement on the assumption that the inertia forces are negligible (**Stoke's flow**).

OSMOTIC COEFFICIENT. In a solution, instead of characterizing deviations from ideality of the solvent by its **activity coefficient**, one may also use the osmotic coefficient ϕ , due to Bjerrum and Guggenheim. It is defined by

$$\mu_1 = \mu_1^0(T, p) + \phi RT \ln x_1 \quad (1)$$

where μ_1 is the **chemical potential** of the solvent, μ_1^0 its value for the pure solvent at given T and p , and x_1 its mole fraction.

OSMOTIC EQUILIBRIUM. See **osmotic pressure**.

OSMOTIC PRESSURE. Consider a solution (denoted by α) separated from the pure solvent (denoted by β) by a membrane which is permeable only to the solvent. A membrane of this kind is called a *semi-permeable membrane*. The chemical equilibrium eventually established between the phases is called osmotic equilibrium (see also **membrane equilibrium**).

At osmotic equilibrium, the pressures applied to the phases are different, and the difference

$$\Pi = p^\alpha - p^\beta \quad (1)$$

is called the osmotic pressure. The osmotic pressure is given by

$$\Pi = - \frac{\phi RT \ln x_1}{\bar{V}_1^0}. \quad (2)$$

ϕ is the osmotic coefficient (see **osmotic coefficient**) of the solvent, x_1 its mole fraction in the solution, \bar{V}_1^0 is the molar volume of the pure solvent at the average pressure $(p^\alpha + p^\beta)/2$. For solutions which are both dilute and ideal, Equation (2) reduces to the *Van't Hoff equation*

$$\Pi = RT \sum_{s=2}^c C_s \quad (3)$$

where C_s is the concentration of the solute s . In this limit the osmotic pressure becomes independent of the nature of the solvent and formally identical with the equation of state of a perfect gas.

OSTWALD DILUTION LAW. Consider the ionization of a weak electrolyte. As an example, consider a 1-1 electrolyte like HCl, which dissociates according to the formula



The degree of dissociation, which one may denote by ϵ , is the ratio of the number of molecules dissociated to the total number of molecules whether dissociated or not.

The law of mass action gives

$$K = \frac{\gamma_{\pm}}{\gamma_{AB}} \frac{\epsilon^2 C_o}{1 - \epsilon} \quad (1)$$

where K is the equilibrium constant, γ_{AB} the activity coefficient of AB and γ_{\pm} the mean activity coefficient of the ions (see **mean chemical potential**); C_o is the total concentration of AB.

If the solution is sufficiently dilute to be treated as ideal,

$$\gamma_{\pm} = \gamma_{AB} = 1,$$

and there results the Ostwald dilution law

$$K = \frac{\epsilon^2 C_o}{1 - \epsilon}. \quad (2)$$

OSTWALD STATEMENT OF THE SECOND LAW OF THERMODYNAMICS. See perpetual motion engine of the second kind.

OTTO CYCLE. See spark-ignition engine.

OVERALL EFFICIENCY. See brake thermal efficiency.

OVERHANGING BEAM. See beam, overhanging.

OVERLAP ENERGY. See intermolecular forces; exchange energy; overlap integral.

OVERLAP INTEGRAL. The integral $\int \psi' \psi'' d\tau$, where $d\tau$ is a volume element, measures the extent to which the two different wave functions ψ' and ψ'' overlap. It is called the "overlap integral." Such integrals play an

important role in the quantum mechanical calculations of bond energies because the exchange integrals, which are responsible for the main part of the covalent bond energy, can only be important when the overlapping of the wave functions is large.

OVERRELAXATION. For solving the system

$$\mathbf{x} = \mathbf{k} + \mathbf{B}\mathbf{x}$$

(see **matrix inversion**) by the simple iteration scheme one writes

$$\mathbf{x}_{\nu+1} = \mathbf{x}_{\nu} + \mathbf{r}_{\nu}, \quad \mathbf{r}_{\nu} = \mathbf{k} - (\mathbf{I} - \mathbf{B})\mathbf{x}_{\nu}.$$

In case of slow convergence it is suggested to write $\alpha_{\nu} \mathbf{r}_{\nu}$ in place of \mathbf{r}_{ν} , where α_{ν} is a scalar. When $\alpha_{\nu} > 1$ this is known as overrelaxation, when $\alpha_{\nu} < 1$, underrelaxation. When $\rho(\mathbf{B}) < 1$ (which is necessary for convergence) and all **eigenvalues** are real, an optimal choice for α_{ν} can be found by the use of **Chebyshev polynomials**.

OVERTONE. (1a) A physical component of a complex sound having a frequency higher than that of the basic frequency. (1b) A component of a complex tone having a pitch higher than that of the fundamental pitch. (1c) The term "overtone" has frequently been used in place of "harmonic," the n th harmonic being called the $(n - 1)$ st overtone. There is, however, ambiguity sometimes in the numbering of components of a complex sound when the word overtone is employed. Moreover, the word "tone" has many different meanings so that it is preferable to employ terms which do not involve "tone" wherever possible.

(2) In a mechanical vibrating system with a set of normal modes of oscillation (for example, a vibrating string or organ pipe) an overtone is a mode of frequency higher than the fundamental. The first overtone is the mode of frequency next higher than the fundamental, etc.

OVERTONE BANDS. The first, second, third, \dots overtone bands (or second, third, fourth, \dots harmonics) are bands in the **infrared spectrum of a molecule** corresponding to transitions in which the quantum number ν_i of the i th **normal vibration** changes by two, three, four, \dots units. (See also **fundamental bands**.)

OXYGENIZER. See combustion.

P

PACKING FRACTION. The ratio of the mass defect to the mass number of a nuclide. It is positive for nuclides having mass numbers less than 16, and greater than 180, and negative for most other nuclides.

PADÉ TABLE. A double-entry table associated with a formal power series

$$f(z) = s_0 + s_1z + s_2z^2 + \dots$$

The entry in row p and column q , where $p, q \geq 0$, is the fraction $A_{p,q}(z)/B_{p,q}(z)$, where $A_{p,q}$ and $B_{p,q}$ are polynomials of degree q and p , respectively, such that the formal expansion of the fraction as a power series in z agrees with that of f to and including the term in z^{p+q} . The consecutive entries taken in a diagonal direction are approximants of a **continued fraction** expansion of $f(z)$. Often the continued fraction will converge more rapidly than the power series, or may even converge where the power series diverges, and so provides an effective computational device. To form a particular entry in the Padé table, let

$$B_{p,q} = b_0 + b_1z + \dots + b_pz^p,$$

and observe that in the product $f(z)B_{p,q}(z)$, terms in $z^{q+1}, z^{q+2}, \dots, z^{q+p}$ all drop out. This provides p homogeneous equations in the $p + 1$ unknowns, b_0, \dots, b_p . If a solution exists, then the polynomial $A_{p,q}$ can be obtained by direct multiplication. (See **quotient-difference algorithm, orthogonal polynomials.**) The Shanks generalizations of the **delta-square process** applied to the sequence of partial sums of a power series lead to entries in the Padé table.

PAIR PRODUCTION. The conversion of a photon into an electron and a positron, thus conserving electric charge and spin angular momentum. Of the energy of the photon $h\nu$, a part equal to $2mc^2$, where m is the rest mass of the electron (or positron) and c the velocity of light, is accounted for by the rest mass of the pair. This energy, $2mc^2$, is therefore the threshold for the process. The remaining energy, $h\nu - 2mc^2$, then appears as kinetic en-

ergy of the pair. It is not possible to conserve energy and momentum unless this process takes place in the presence of a charged particle capable of taking away momentum (and energy). It therefore occurs in the presence of an atomic nucleus, and the cross section for the process rises steeply with atomic number. Pair production is one of three distinct processes whereby a photon can eject an electron from matter, the other two being the photoelectric effect and the Compton effect. Of these, pair production predominates for high energies (above a few Mev).

PANEL POINT. See **truss**.

PAN-OUT TURBINE. See **extraction turbine**.

PARABOLA. A **conic section** obtained by a cutting plane parallel to an element of a right circular conical surface. It is the locus of a point which moves so that its distance from the *directrix* equals its distance from the *focus*, thus the *eccentricity* is unity.

The standard equation in rectangular **Cartesian coordinates** is $y^2 = 2px$. The coordinates of its focus are $x = p/2, y = 0$ and its directrix is parallel to the Y -axis at $x = -p/2$. The straight line through the focus and perpendicular to the directrix is the *axis* of the parabola. The point where the parabola crosses its axis is the *vertex*. When the curve is placed in its standard position, the axis is the X -axis and the vertex is the coordinate origin.

The *latus rectum* of the parabola, namely the chord through the focus perpendicular to the axis, has length $2p$. Its center is at infinity, hence it is a non-central conic.

The parabola has no **asymptote**. Its equation in **polar coordinates** is $r = 2a/(1 - \cos \theta) = a \sec^2 \theta/2$. If the tangents at the extremities of the latus rectum are taken as coordinate axes, the parabola is tangent to the new coordinate system and its equation is $x^{1/2} + y^{1/2} = a^{1/2}$.

Occasionally the name parabola is given to a curve with the equation $y^2 = ax^n$; thus, if $n = 3$, the curve is a *semi-cubical parabola*,

just as the equation $y = ax^3$ is that of a *cubical parabola*. The *evolute* of a parabola is a semi-cubical parabola.

PARABOLIC COORDINATES. The coordinate surfaces are: paraboloids of revolution extending in the direction of the positive Z -axis; paraboloids of revolution extending in the direction of the negative Z -axis; planes through the Z -axis. That is, they are the surfaces $\xi = \text{const.}$, $\eta = \text{const.}$, and $\phi = \text{const.}$, where ξ, η, ϕ are related to **Cartesian coordinates** by the equations

$$\begin{aligned} x &= \xi\eta \cos \phi \\ y &= \xi\eta \sin \phi \\ z &= \frac{1}{2}(\eta^2 - \xi^2). \end{aligned}$$

PARABOLIC CYLINDRICAL COORDINATES. A curvilinear system similar to **parabolic coordinates** with coordinate surfaces consisting of two families which are parabolic cylindrical ($\xi, \eta = \text{const.}$) and a family of planes ($z = \text{const.}$). A point in this system is given by

$$\begin{aligned} x &= \xi\eta \\ y &= \frac{1}{2}(\eta^2 - \xi^2) \\ z &= z. \end{aligned}$$

PARABOLIC PARTIAL DIFFERENTIAL EQUATION. A partial differential equation that can be reduced to the form

$$\partial^2 u / \partial x^2 = \partial u / \partial t$$

when the variables are suitably chosen. (See also under **partial differential equation; elliptic partial differential equation.**)

PARABOLIC POINT (ON A SURFACE). A point at which one of the principal curvatures of the surface (see **curvature of a surface, center of**) is zero.

PARABOLOID. A quadric whose equation may be written in the form

$$\frac{x^2}{a} + \frac{y^2}{b} = z,$$

where a and b are real constants at least one of which is positive, by a suitable choice of the rectangular Cartesian reference system x, y, z . If a and b are both positive, the quadric is an *elliptic paraboloid*. If one of the con-

stants a is positive and the other is negative, the quadric is a *hyperbolic paraboloid*.

PARABOLOIDAL COORDINATES. A curvilinear coordinate system of confocal paraboloids. If λ, μ, ν are the real roots of a cubic equation in a parameter describing such surfaces, they are *elliptic paraboloids* ($\lambda, \mu = \text{const.}$); $-\infty < \lambda < b^2$; $a^2 < \nu < \infty$ and *hyperbolic paraboloids* ($\mu = \text{const.}$), $b^2 < \mu < a^2$ where $a > b > c$ are constants.

As in the case of ellipsoidal coordinates a convention is needed for the sign of the coordinates. They are related to rectangular coordinates by the equations

$$\begin{aligned} x^2 &= \frac{(a^2 - \lambda)(a^2 - \mu)(a^2 - \nu)}{(b^2 - a^2)} \\ y^2 &= \frac{(b^2 - \lambda)(b^2 - \mu)(b^2 - \nu)}{(a^2 - b^2)} \\ z &= \frac{1}{2}(a^2 + b^2 - \lambda - \mu - \nu). \end{aligned}$$

PARALLAX. Apparent change of position of distant objects, due to the actual change of position of the observer. The amount of apparent shift is known as the *parallactic shift*, which is inversely proportional to the distance of the object.

PARALLAX, DYNAMIC. See **dynamic parallax (of binary stars).**

PARALLAX, GEOCENTRIC; HORIZONTAL; LUNAR; SOLAR.

1. *Geocentric Parallax.* The angle between the direction of an astronomic object from a point on the surface of the earth and the direction of the same object from the center of the earth is known as the geocentric parallax of the object. Both directions are in the same plane.

In Figure 1, O is the position of the observer and Z' represents his geocentric zenith. M represents the moon, or any other member

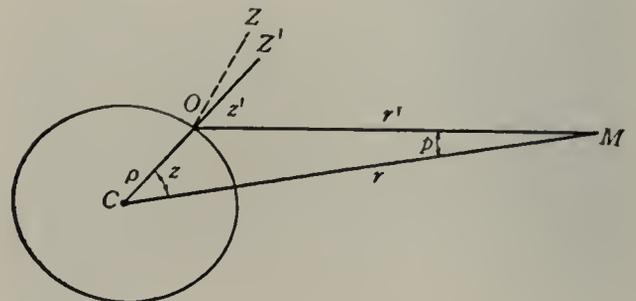


Fig. 1. Geocentric parallax.

of the solar system, C is the center of the earth.

Calling the geocentric zenith distances of the object at the surface of the earth and at the center z' and z , and denoting by r' and r , the distances of M from the observer and the center of the earth, we have $\sin p = \rho/r \sin z'$. The angle p is called the parallax of M .

2. *Horizontal Parallax.* If the object is on the geocentric horizon of the observer $z' = 90^\circ$ and the parallax is known as the horizontal parallax of the object. Calling this p' we have

$$\sin p' = \rho/r.$$

If the observer is on the equator we have the equatorial horizontal parallax $P_o = a/r$ in which a is the equatorial radius of the earth or 3963.339 miles (Hayford's Geoid). Owing to the elliptical motions of the members of the solar system, the geocentric distances will vary. The average of these distances, r_o , yields a quantity known as the Mean Equatorial Horizontal Parallax, P , where $\sin P = a/r_o$.

Finally we have

$$\sin p = (\rho/a)(r_o/r) \sin P \sin z'.$$

3. *Lunar Parallax.* The Moon is the closest to the earth of all astronomic objects and its parallax may be determined by direct observation. Let us assume that we have two observatories on the same meridian of longitude, one in the northern hemisphere in astronomic latitude ϕ and the other in the southern hemisphere in astronomic latitude ϕ_1 . Assume further that the Moon is on the meridian of both observatories.

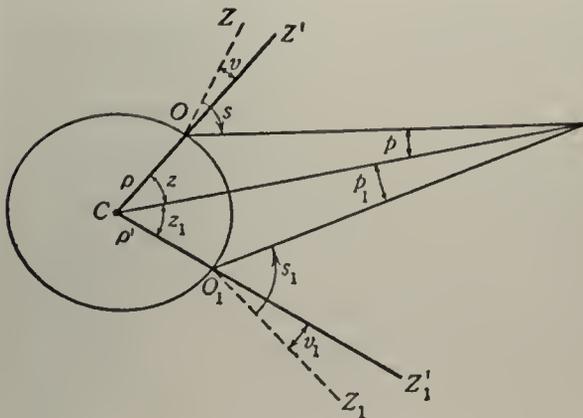


Fig. 2. Lunar parallax.

The observed zenith distances, after correction for refraction, are s and s_1 . Reference to Figure 2 shows that

$$p + p_1 = s + s_1 - \phi - \phi_1.$$

s and s_1 are measured from the astronomic zenith and must be referred to the geocentric zenith by subtracting the so-called angles of the vertical v and v_1 from the astronomic values. Now

$$\sin p = \rho/r \sin (s - v)$$

$$\sin p_1 = \rho_1/r_1 \sin (s_1 - v_1)$$

in which ρ and ρ_1 are the radii of the geoid at points of observation.

Finally calling $p = p_1 = \theta$ we obtain by trigonometrical analysis

$$\tan \phi = \frac{\rho \sin \theta \sin (s - v)}{\rho \cos \theta \sin (s - v) + \rho_1 \sin (s_1 - v_1)}.$$

The mean equatorial horizontal parallax of the moon may now be written

$$\sin p = (\rho/a)(r_o/r)(\sin P \sin (s - v)).$$

In the above it is assumed that the observations made at O and O_1 were referred to the moon's center. In actual practice the two observers would agree to observe the zenith distances of some small object on the surface of the moon. Furthermore it is very rarely, if ever, possible to have two observing stations on the same meridian and in different hemispheres.

The moon's parallax may also be determined by theoretical considerations combined with observations of the moon's motion. At the present time the distance is close to 238,900 miles, corresponding to a mean equatorial horizontal parallax of 3422".7.

4. *Solar Parallax.* Solar parallax is far too small to be determined by direct observational methods. The method which has yielded the best results is a combination of observation and of the application of the laws of planetary motion. Two of the members of the solar system, Mars and Eros (a minor planet or asteroid), have been used. The latter is the better of the two because it has a starlike image.

If we apply Kepler's third law of planetary motion we have

$$\frac{4\pi^2 a^3}{T^2} = G(M + m)$$

in which a is the semi-major axis of the orbit, T is the orbital period, m is the mass of the planet and M is the mass of the sun. In the

case of the earth and Mars or, better still in the case of Eros and the earth, we can neglect the differences in the planetary masses and have

$$a^3 : a_1^3 :: T^2 : T_1^2.$$

The value of T and T_1 are known with accuracy, and consequently the ratio of distance from the sun is known with accuracy. The most favorable time for observing is when the planet is at perigee and the earth at apogee. These combinations occur at infrequent intervals, the last was in 1931 and the next will be in 1975. At the time of a close approach to the earth the celestial coordinates of right ascension and declination are accurately measured at stations all over the globe. Parallax displacements from the predicted position yield the horizontal parallax, and from that distance from the earth may be determined.

PARALLEL ALLEYS. In the **Luneburg geometry** of binocular vision, parallel alleys are pairs of **geodesics** apparently parallel to the median and passing through two points on opposite sides and equidistant from the median. (See **equidistant alleys**.)

PARALLEL AXIS THEOREM. See **moment of inertia, transfer theorem for**.

PARALLEL CIRCLES. See **surface of revolution**.

PARALLEL DISPLACEMENT OF A VECTOR. The infinitesimal parallel displacement of a contravariant vector a^i is defined by

$$\delta a^i = -\Gamma_{kl}^i a^k \delta \xi^l$$

and of a covariant vector a_k by

$$\delta a_k = \Gamma_{kl}^i a_i \delta \xi^l.$$

The Γ_{kl}^i are the Christoffel symbols of the second kind. If the space possesses a metric g_{ik} , then

$$\Gamma_{ik}^l = \frac{1}{2} g^{ls} \left[\frac{\partial g_{is}}{\partial \xi^k} + \frac{\partial g_{ks}}{\partial \xi^i} - \frac{\partial g_{ik}}{\partial \xi^s} \right].$$

If $\oint \delta a^i \equiv 0$ along any closed curve, the space is said to possess distant parallelism. A necessary and sufficient condition for distant parallelism in a metric space is the identical vanishing of the Riemannian curvature tensor.

PARALLELEPIPED. A **polyhedron** with parallelograms as faces, thus a six-sided **prism**

with parallelograms as bases. Sometimes the figure is also called a **parallelepipedon** and, incorrectly, **parallepiped** or **parallelopipedon**.

The volume of a **parallelepiped** is conveniently written in **vector analysis** as the scalar triple product $[ABC]$ where **A, B, C** are three vectors describing the edges of the **parallelepiped**.

PARALLELISM WITH RESPECT TO A SURFACE S ALONG A CURVE C ON S.

If surface vectors on the surface S are defined at each point of a curve C lying in the surface, they are said to constitute a system of parallel vectors with respect to S along C if and only if there the surface **intrinsic derivative** of the vector with respect to the parameter defining position on C (which may be taken as the length measured along C) vanishes.

PARALLEL LINE ASSAY. A method of testing a certain preparation against a standard. If the response to a treatment is a linear function of the dose, the **regressions** of response on dose will often be parallel lines at a distance which measures the difference in potency of the two treatments.

PARALLELOGRAM. A **quadrilateral** with opposite sides parallel and equal in length. Its consecutive angles are complementary. If a, b are the lengths of its sides and C is the acute angle between them, the area of a parallelogram is $A = ab \sin C = bh$, where h is the distance between the bases, b . Its area is also given by $2A = d_1 d_2 \sin D$, where d_1, d_2 are the lengths of the two diagonals and D is the acute included angle.

PARALLELOGRAM OF FORCES. See **forces, parallelogram of**.

PARALLELS. See **surface of revolution**.

PARALLELS (TERRESTRIAL AND CELESTIAL). 1. *Terrestrial.* *Parallels of latitude* are small circles on the surface of the earth produced by planes passing through the earth parallel to the plane of the equator.

2. *Celestial.* *Parallels of declination* are small circles on the celestial sphere produced by planes passing through the celestial sphere parallel to the equator.

Parallels of altitude are small circles on the celestial sphere produced by planes passing through the celestial sphere parallel to the

horizon. *Almucantor* is an obsolete name for a parallel of altitude.

Parallels of celestial latitude are small circles produced on the celestial sphere by planes passing through the earth parallel to the ecliptic.

PARALLEL SURFACES. The surface S' is said to be parallel to the surface S if the distance measured from S to S' along the normal to S at a point P is independent of the position of P on S . If P' is the point at which the normal to S at P intersects S' , then the normals to S and S' at P and P' respectively, have the same directions. If S' is parallel to S , then S is parallel to S' .

PARAMAGNETIC RESONANCE. See resonance, paramagnetic.

PARAMAGNETIC SUSCEPTIBILITY, LANGEVIN THEORY. The average magnetic moment of an assembly of magnetic dipoles in the magnetic field is calculated from the **Langevin function**, that is, as if each were a magnet capable of any orientation in the field but subject to averaging over a Boltzmann distribution. The result is

$$\chi = N\mu^2/3kT$$

where μ is the moment of each atom, N is the number of atoms per unit volume, and k is the Boltzmann constant. The relation does not hold except at very low temperatures.

PARAMAGNETISM. The property of materials having a positive magnetic susceptibility.

PARAMAGNETISM OF CONDUCTION ELECTRONS, SPIN. See spin paramagnetism of conduction electrons.

PARAMAGNETISM, QUANTUM THEORY OF. Certain atoms, ions and molecules contain electrons with unpaired spins. Such electrons may also be in states whose orbital angular momenta are not compensated. In a magnetic field H the energy levels corresponding to the two directions of spin will be split by the amount

$$\Delta E = g\mu_B H$$

where μ_B is the Bohr magneton and g is the **Landé factor** for the particular state concerned. If there are, as there may well be, J

levels, all equally spaced, the magnetization is then given by

$$M = NgJ\mu_B B_J(x) \rightarrow NJ(J+1)g^2\mu_B^2 H/3kT$$

where $x = gJ\mu_B H/kT$, and B_J is the **Brillouin function**, whence the susceptibility can be calculated. The theory for the rare earth ions, and other complicated atoms is very complex, and requires a detailed consideration of the **spin-orbit coupling**, **crystal field**, and all the apparatus of the theory of **spectral terms**.

PARAMETER. An arbitrary constant, as distinguished from a fixed or absolute constant. Any desired numerical value, subject in some cases to certain restrictions. Thus, in the set of equations $(x-a)^2 + (y-b)^2 = r^2$ for circles in the plane, there are 3 parameters, two describing the position of the center and one giving the radius, and for each choice of a , b , and r , a corresponding circle is obtained. Similarly, the equations of a curve $x = \phi(t)$, $y = \psi(t)$, are said to be in parametric form, and each choice for the parameter t produces a corresponding point on the curve. (See **constant**.)

PARAMETERIZATION. The representation, in a dynamic model, of physical effects in terms of admittedly oversimplified **parameters**, rather than realistically requiring such effects to be consequences of the dynamics of the system.

PARAMETER OF STATE. See thermodynamic property.

PARAMETRIC CURVES ON A SURFACE. See surface.

PARAMETRIC EQUATIONS OF SURFACE. See surface.

PARAMETRIC LINES ON A SURFACE. See surface.

PARA MODIFICATION. See non-combinig modifications of molecules.

PARAPOSITRONIUM. The state of **positronium** in which the spins of electron and positron are antiparallel. The $1s$ state annihilates into two gamma-rays with a mean life of 1.25×10^{-10} sec. If one of these gamma rays is polarized in a particular plane, the other is polarized in a perpendicular plane.

PARAXIAL. A ray, or optical path, in **Gaussian optics**.

PARAXIAL RAY TRACING EQUATIONS.

In an axially symmetrical optical system, let k_ν be the curvature at the vertex of the ν^{th} surface S_ν , n_ν the index of refraction of the medium between S_ν and $S_{\nu+1}$, and t_ν is the distance between the vertices of S_ν and $S_{\nu+1}$. If T_ν is the tangent plane of S_ν at the vertex then the projected position vectors $\mathbf{A}_\nu, \mathbf{A}'_\nu$ of a ray are vectors from the vertex to, respectively, the intersection with T_ν of the ray before and after refraction (or reflection) at S_ν , and the projected direction vectors $\mathbf{S}_\nu, \mathbf{S}'_\nu$ are the projections on T_ν of direction vectors in the ray before and after, respectively, its intersection with S_ν .

At each surface, define $\beta_\nu = t_\nu/n_\nu$ and $\gamma_\nu = (n_{\nu-1} - n_\nu)k_\nu$ or $\gamma_\nu = 2n_{\nu-1}k_\nu$ accordingly as refraction or reflection occurs at S_ν . Then for an optical system of N surfaces with object surface S_o and image surface S_{N+1} the paraxial ray tracing equations of Gaussian optics are

$$\begin{aligned} \mathbf{A}'_{N+1} &= a\mathbf{A}_o + b\mathbf{S}_o \\ \mathbf{S}'_{N+1} &= c\mathbf{A}_o + d\mathbf{S}_o \end{aligned}$$

where the coefficients are given by the Gaussian brackets

$$\begin{aligned} a &= [\beta_N \gamma_N \beta_{N-1} \cdots \beta_o \gamma_o] \\ b &= [\beta_N \gamma_N \cdots \beta_o] \\ c &= [\gamma_N \beta_{N-1} \cdots \beta_o \gamma_o] \\ d &= [\gamma_N \beta_{N-1} \cdots \beta_o]. \end{aligned}$$

PARAXIAL SINGLE-SURFACE EQUATION. For a spherical optical surface separating two media

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n' - n}{r}$$

Here n and n' are the indices of refraction, s and s' the object and image distance from the vertex of the surface, and r is the **radius of curvature** of the surface. In terms of the **focal length** of the surface, the above equation becomes

$$\frac{n}{s} + \frac{n'}{s'} = \frac{n}{f} = \frac{n'}{f'}$$

If distances of object and image are measured from their focal points, the equation takes the Newtonian form $xx' = ff'$.

PARCEL METHOD. A test for atmospheric instability in which a displacement is made from a steady state under the assumption that only the parcel or parcels displaced are affected, the environment remaining unchanged. Although this method has been applied to various problems (e.g., inertial instability), its most familiar context is with vertical displacements from hydrostatic equilibrium, in which the parcel displaced is assumed to undergo adiabatic temperature changes, and the buoyant force resulting from its contrast with the unchanged environment leads to the criterion for stability, $\gamma < \Gamma$, where γ is the lapse rate of virtual temperature with height, and Γ is the dry- or saturation-adiabatic lapse rate, according to the condition of the parcel. (Cf. **slice method**.)

PARENT OF A STATE. If a quantum mechanical system consists of N tightly bound particles plus one loosely bound particle, the wave function for the N tightly bound particles, the “core of the configuration,” is called the parent of the state of the entire system.

PAREXIC ANALYSIS. A term coined by Cornelius Lanczos to designate the branch of analysis concerned with the derivation of “nearby” (i.e., approximating) but numerically manageable relations. It is distinguished, on the one hand, from “pure” analysis, which is concerned with limiting processes and exact relations. On the other hand it is distinguished from **numerical analysis** which is concerned with the translation into numerical processes of the approximations provided by parexic analysis. The aim of parexic analysis is to reduce **truncation errors**; that of numerical analysis to reduce **generated error** and the number of operations required.

PARFOCAL. A set of optical systems arrayed so that their focal points are all in the same plane is parfocal. The term describes a circumstance frequently met by the interchangeable objective lenses of a camera, or sets of eyepieces so mounted that they may be interchanged without varying the focus of the instrument with which they are used.

PARHELIUM. See **orthohelium**.

PARITY. A non-classical symmetry property of a quantum mechanical system related to its wave function, and expressing the equiva-

lence between left and right. The parity is even if the wave function is unchanged by an inversion (i.e., reflexion in the origin) of the coordinate system, and odd if it changes only in sign. Until recently all non-degenerate stationary states were believed to have a definite parity, as a result of which parity conservation was regarded as a fundamental principle of physics. Experiments have indeed shown that the parity of a nuclear state is a very well-defined concept, the state containing less than 10^{-8} of the "wrong" parity. However, recent experiments have shown that parity conservation is violated completely in the very weak interactions of which β -decay is the typical example. The reason for this breakdown in weak interactions is not yet understood.

PARITY OPERATOR. If the quantum mechanical laws of a system are assumed invariant under the transformation $\mathbf{x}_i \rightarrow -\mathbf{x}_i, x_o \rightarrow x'_o$, i.e., under the transformation from a right-handed to a left-handed coordinate system, there then exists a connection between the operators representing the physical observables in the two coordinate systems which is such that the commutation rules and equations of motion have the same form in the two systems. This connection is a unitary one (see **invariance and quantum mechanics**) and is denoted by P . If $|\psi\rangle$ denotes the state of the system in the right-handed system $P|\psi\rangle$ is the mirror image state and is again a possible state of the system due to the assumed symmetry between right and left. Clearly, one therefore has $P^2 = 1$, since two inversions must lead back to the original state.

In a field theoretic treatment the connection between the operators describing various kinds of particles in the two coordinate systems are listed below.

Scalar, Spin 0

$$P^{-1}\phi(x)P = \phi(-\mathbf{x}, x_o)$$

Pseudoscalar, Spin 0

$$P^{-1}\phi(x)P = -\phi(-\mathbf{x}, x_o)$$

Spin $\frac{1}{2}$

$$P^{-1}\phi(x)P = \epsilon\gamma_0\psi(-\mathbf{x}, x_o) \quad |\epsilon| = 1$$

Spin 1 (electromagnetic field)

$$P^{-1}\epsilon_j(x)P = -\epsilon_j(-\mathbf{x}, x_o)$$

$$P^{-1}\mathcal{H}_j(x)P = \mathcal{H}_j(-\mathbf{x}, x_o)$$

P is unitary and satisfies $P^2 = 1$, hence $P^* = P$ so that P is also Hermitian, hence observable. For systems whose laws of motion are invariant under reflections, $PHP^{-1} = H$, where H is the Hamiltonian, whence $[P, H] = 0$ and P is a constant of the motion (law of parity conservation).

It has been experimentally established that the **weak interactions** are not P invariant. A theory is invariant under space reflection if no experiment enables one to distinguish between left and right.

PARSEC. See **astronomical distance units**.

PARSEVAL EQUATION. See **Bessel inequality**.

PARTIAL COHERENCE. If light from an extensive primary source L reaches two points P_1, P_2 , either directly or through an optical system, then let an element ΔL_m of the source give rise to the complex displacements u_{m1}, u_{m2} at P_{1m}, P_{2m} , respectively. Suppose that ΔL_m is small enough so that u_{m1}, u_{m2} are coherent, but not necessarily in phase. Let $J_{\alpha\beta} = \sum_m u_{m\alpha}\bar{u}_{m\beta}$ where $\alpha, \beta = 1, 2$ and $\bar{u}_{m\alpha}$ is the complex conjugate of $u_{m\alpha}$. Then J_{12} is called the *mutual intensity* of P_1 and P_2 due to L , and J_{11}, J_{22} are the ordinary intensities at P_1, P_2 . The *phase coherence factor* between P_1 and P_2 is

$$\gamma = \frac{J_{12}}{\sqrt{J_{11}J_{22}}}$$

where the modulus $|\gamma_{12}|$ is the *degree of coherence* and $\arg \gamma_{12}$ is the *effective phase retardation* of P_2 with respect to P_1 .

If L is a self-luminous source with flux density $l(Q)$ at point Q and if the optical paths QP_1, QP_2 are nearly normal to L then

$$J_{\alpha\beta} = \iint_L l(Q)w_1(Q)\widetilde{w_2(Q)}dL$$

where $w_\alpha(Q)$ is the displacement caused at $P_\alpha, \alpha = 1, 2$ by a point source of unit flux at $Q \in L$.

If through further imagery a point P is reached by light from both P_1 and P_2 and if f_α is the displacement produced at P by displacements of unit amplitude and zero phase at P_α then the intensity at P is given by the bilinear law

$$I(P) = J_{11}|f_1|^2 + 2R(J_{12}f_1\bar{f}_2) + J_{22}|f_2|^2.$$

PARTIAL CORRELATION. See **correlation**.

PARTIAL DERIVATIVE. See derivative.

PARTIAL DIFFERENTIAL EQUATION.

A differential equation containing two or more independent variables, hence containing partial derivatives. The order of the equation is that of a derivative of highest order. It is linear if linear in the dependent variable(s) and all derivatives; quasilinear if linear in the partial derivatives of highest order. Second-order linear differential equations

$$au_{xx} + 2bu_{xy} + cu_{yy} + \dots = 0$$

in two independent variables are classified as

- elliptic*, $ac - b^2 > 0$,
- parabolic*, $ac - b^2 = 0$,
- hyperbolic*, $ac - b^2 < 0$.

The coefficients a , b , and c are functions of x and y , and the inequalities must hold throughout the region under consideration. Methods of solution differ slightly for the three types. Most often, however, for any type and any order, a method of differences is used by which the differential operators are replaced by approximating expressions in terms of **difference operators**. This requires dividing the space of the independent variables, in the case of two independent variables, by equally spaced lines parallel to each axis; for three independent variables by equally spaced planes parallel to each coordinate plane; and similarly in higher dimensions.

The method is best described by an example. Consider the simple parabolic equation

$$u_{xx} = u_y,$$

or, in operator form,

$$(D_x^2 - D_y)u = 0.$$

The natural, and simplest, difference representation of D_x^2 is by central differences

$$D_x^2 \doteq h^{-2}\delta_x^2,$$

where h represents the constant difference along the x -axis. There are two equally natural representations of D_y :

$$D_y \doteq k^{-1}\Delta_y, \quad D_y \doteq k^{-1}\nabla_y.$$

If applied to $u_{ij} = u(x_i, y_j)$, these lead to somewhat different difference equations which can be written:

$$\begin{aligned} \gamma u_{i+1,j} + (1 - 2\gamma)u_{i,j} + \gamma u_{i-1,j} - u_{i,j+1} &= 0, \\ \gamma &= k/h^2, \end{aligned}$$

in the first case, and, in the second case

$$\gamma u_{i+1,j} - (1 + 2\gamma)u_{i,j} + \gamma u_{i-1,j} + u_{i,j-1} = 0.$$

The coefficients in the two approximations can be represented schematically by the following **stars**:

$$\begin{array}{cccccc} & & -1 & & & \\ \gamma & 1 - 2\gamma & \gamma & \gamma & -(1 + 2\gamma) & \gamma \\ & & & & & 1 \end{array}$$

These schemes show the relative locations of the points to which the coefficients are applied. The first scheme is simplest to apply, since one has only to solve for u at the upper point in terms of its values at the points below. It turns out, however, that to insure adequate **numerical stability** it is necessary that $\gamma \leq 1$. This is an example of an explicit (or marching) scheme. To apply the other method no restriction is needed on γ , but to obtain the value of u along any horizontal line it is necessary to solve a system of linear equations, although the matrix is tridiagonal, and hence readily inverted. This is an example of an implicit (or jury) scheme.

To start the solution it is necessary that suitable initial and boundary conditions be given. If these involve derivatives, then the derivatives in the boundary conditions must also be represented by approximating differences. (See special partial differential equations under individual names.)

PARTIAL DIFFERENTIAL EQUATIONS, CAUCHY PROBLEM FOR. See **Cauchy problem for partial differential equations**.

PARTIAL FRACTION EXPANSION. A widely used method of inverse Laplace transformation. If $F(s)$ is a **Laplace transform** whose inverse $f(t)$ is required, then provided $F(s)$ can be written in the form

$$F(s) = \sum_0^m a_r s^r / \sum_0^n b_r s^r$$

where $m < n$, and provided the linear factors of the denominator polynomial are known, $F(s)$ can be directly expanded in partial fractions by the following rules:

In the most general case of repeated factors in the denominator, if

$$\sum_0^n b_r s^r \equiv B(s - \beta_1)^{n_1} (s - \beta_2)^{n_2} \dots$$

where therefore $n_1 + n_2 + \dots = n$, then

$$F(s) = \sum_1^{n_1} \frac{B_{1,r}}{(s - \beta_1)^r} + \sum_1^{n_2} \frac{B_{2,r}}{(s - \beta_2)^r} + \dots$$

an expansion in the form of n partial fractions, in which

$$B_{1,n_1} = \lim_{s \rightarrow \beta_1} \{F(s) \cdot (s - \beta_1)^{n_1}\}$$

and generally

$$B_{1,r} = \frac{1}{(n_1 - r)!} \lim_{s \rightarrow \beta_1} \left(\frac{d}{ds} \right)^{n_1 - r} \{F(s) (s - \beta_1)^{n_1}\}$$

with similar expressions for $B_{2,r}$, $B_{3,r}$, etc. Since moreover, when r is integral and positive

$$\mathcal{L}^{-1} \frac{1}{(s - \beta)^r} = \frac{t^{r-1}}{(r-1)!} e^{\beta t}$$

the inverse Laplace transform of $F(s)$ may at once be written down as the sum of n functions of t of this form.

If in the first place $m \geq n$, the denominator must be divided into the numerator until the numerator is of lower order than the denominator. The result of this diversion will be to give additional terms forming a polynomial in s of order $(m - n)$, the inverse Laplace transform of each term of which is found from the fact that, when $r \geq 0$

$$\mathcal{L}^{-1} s^r = \delta_{r+1}(t),$$

unit impulse function of order $(r + 1)$ at $t = 0$.

PARTIAL MOLAR QUANTITIES. To an extensive variable Y correspond intensive variables y_i defined by

$$y_i \equiv \left(\frac{\partial Y}{\partial n_i} \right)_{T,p} \quad (1)$$

The y_i are the partial molar quantities associated with Y .

The Euler theorem for homogeneous functions gives

$$Y = \sum_i n_i y_i, \quad (2)$$

$$\sum_i n_i \left(\frac{\partial y_j}{\partial n_i} \right)_{T,p} = 0 \quad \text{and} \quad \sum_i n_i \left(\frac{\partial y_i}{\partial n_j} \right)_{T,p} = 0. \quad (3)$$

An important example is the partial molar volume

$$v_i \equiv \left(\frac{\partial V}{\partial n_i} \right)_{T,p} \quad (4)$$

It is the limit for $\partial n_i \rightarrow 0$ of the ratio of the increase in volume ∂V to the number of moles ∂n_i which added to the system at constant T and p , produce this increase in volume.

It sometimes happens that dn_i is negative. Thus, for example, when a little magnesium sulfate is added to an aqueous solution of this salt the volume decreases; whence $dn_{\text{MgSO}_4} < 0$.

PARTIAL MOLAR VOLUME. See partial molar quantities.

PARTIAL PRESSURE. In a mixture of perfect or real gases, the partial pressure p_i of component i is defined as the product of the total pressure p and the mole fraction x_i

$$p_i = x_i p. \quad (1)$$

Therefore

$$p = \sum_i p_i. \quad (2)$$

In a mixture of perfect gases

$$p_i = \frac{n_i R T}{V}. \quad (3)$$

The partial pressure of i is then the same as if component i occupies the same volume at the same temperature in the absence of the other gases. This is the Dalton law, which is treated more fully under that heading. (See also Duhem-Margules equation.)

PARTICLE IN CENTRAL FORCE FIELD VARYING INVERSELY AS R^2 , MOTION OF.

The motion of a particle is in a plane and must be a conic section, with the center of force at one of the focal points. The particle radius vector describes equal angles in equal times (law of constant areal velocities).

PARTICLE INTERPRETATION. A particle interpretation of a quantum mechanical system is a complete description of the system in terms of particle observables. This requires the existence of operators $E^{(m,n,\dots)}(S)$ and $F^{(m,n,\dots)}(T)$ giving the probability of finding at any time m particles of the first kind, n of the second kind, etc, with properties succinctly indicated by S or T . S stands for any region of $(3m + 3n + \dots)$ -dimensional position configuration space, together with a certain dis-

tribution of particle spin. T stands for a region of $(3m + 3n + \dots)$ -dimensional momentum configuration space, together with an arrangement of components of particle spins.

One calls a particle interpretation *simple* if (a) the no-particle state has definite mass, zero; (b) the one-particle states have definite mass and the appropriate spin; (c) in a state with more than one particle, the energy is the sum of all free particle energies when the particles are far apart.

PARTICLE, MATERIAL. A concept which is introduced when developing the fundamental principles of mechanics. A material particle is characterized by the properties of mass and an observable position in space and time. It does not possess the property of geometrical extension, forces acting on a particle are necessarily concurrent. From the standpoint of mechanics, **matter** is considered to be made up of an aggregate of particles.

PARTICULAR. A particular solution (or particular integral) of a **differential equation** is a solution which can be obtained from the general solution by assigning particular values to the constants of integration.

PARTITION. Consider a permutation.

$$\begin{pmatrix} a_1, a_2, \dots, a_n \\ b_1, b_2, \dots, b_n \end{pmatrix}$$

of n objects, where the notation indicates that a_i is replaced by b_i . This permutation can also be expressed by cycles $(a b c \dots d)(e f) \dots$, etc., indicating that a is to be replaced by b , b by c , \dots , d by a , e by f , f by e , and so forth. Suppose now that there are α cycles of degree (i.e., length) 1, β cycles of degree 2, etc. Then we can conveniently denote this property of the original permutation by the symbol $(1^\alpha 2^\beta 3^\gamma \dots)$, which is called a partition of the permutation. This concept is useful in **group theory**, since every finite group can be represented as a group of permutations.

PARTITION COEFFICIENT. See **Nernst distribution law**.

PARTITION FUNCTION. Defined by the equation

$$Z = \sum_i g_i e^{-\beta \epsilon_i},$$

where $\beta = 1/kT$ (k is Boltzmann's constant, T is absolute temperature) and where ϵ_i and g_i

are the energy levels of the system and their degree of degeneracy. If the energy levels can be split into several independent parts,

$$\epsilon_i = \epsilon_a + \epsilon_b + \epsilon_c,$$

while

$$g_i = g_a g_b g_c,$$

the partition function can be written as a product,

$$Z = Z_a Z_b Z_c,$$

where Z_a , Z_b , and Z_c refer to the independent degrees of freedom. These independent parts are usually the translational, vibrational, rotational, electronic, and spin parts.

Z is called the partition function since each term in the sum is proportional to the number of particles in that particular state. It can be shown that the Helmholtz **free energy** (work function) A is related to the partition function,

$$A = -kT \ln Z.$$

PARTITION FUNCTION FOR ATOMS.

(See **partition function**.) As the distance of the first excited level ϵ_1 of an atom from the ground state level ϵ_0 ($=0$) is usually large compared to kT (k is Boltzmann's constant, T is absolute temperature), the partition function Z can often be approximated by

$$Z = g_0 + g_1 e^{-\beta \epsilon_1},$$

where g_0 and g_1 are the degrees of degeneracy of the ground state and first excited state, and $\beta = 1/kT$.

PARTITION FUNCTION FOR DIATOMIC MOLECULES. See **diatomic molecules, partition function for**.

PARTITION FUNCTION OF HARMONIC OSCILLATOR. See **harmonic oscillator, partition function of**.

PASCAL DISTRIBUTION. Another name for the negative **binomial distribution**, though why Pascal should be credited with it is a mystery.

PASCAL TRIANGLE. If the coefficients of $(x + y)^k$ in the **binomial series** are arranged as shown, successive coefficients can be obtained as a sum of two numbers in the preceding line. The second figure in each line is the value of k .

1						
1	1					
1	2	1				
1	3	3	1			
1	4	6	4	1		
1	5	10	10	5	1	
1	6	15	20	15	6	1

Other forms of the triangle are often shown, especially that in the shape of an isosceles triangle, with unity at the apex and unities along the sides.

The triangle can easily be extended by simple additions, hence the coefficient in a binomial expansion can be determined to any order with a minimum of effort.

The triangle can be used to find the number of **combinations** when selecting k objects from n objects, since this also is equal to the binomial coefficient, $\binom{n}{k}$.

PASCHEN-BACK EFFECT. In a strong magnetic field where the magnetic splitting becomes greater than the multiplet splitting a transition of the anomalous Zeeman effect to the pattern of one closely resembling that of the normal Zeeman effect takes place. The latter, called Paschen-Back effect, can be treated, to a first approximation, by assuming independent space quantization of the resultant electronic orbital angular momentum \mathbf{L} and the resultant electron spin \mathbf{S} ("uncoupling of \mathbf{L} and \mathbf{S} in a strong magnetic field"). Each level with a given L therefore is split into $2L + 1$ components characterized by the magnetic quantum numbers

$$M_L = L, L - 1, L - 2, \dots, -L,$$

and each level with a given M_L into $2S + 1$ components, characterized by the magnetic quantum numbers

$$M_S = S, S - 1, S - 2, \dots, -S.$$

The energy of the component levels is given, to a first approximation, by

$$W = W_0 + h\omega M_L + 2h\omega M_S.$$

Here W_0 is the energy without field, h is Planck's constant, ω is the Larmor frequency

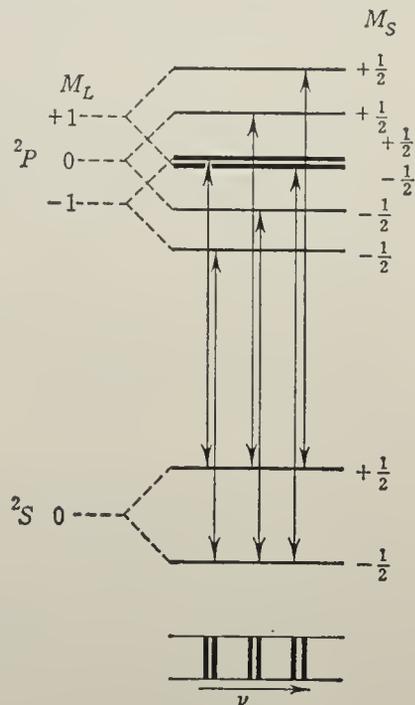
$$\omega = \frac{1}{2\pi} \frac{eH}{2mc},$$

where e and m are, respectively, the electronic charge and mass, c is the velocity of light, and H is the magnetic field strength.

The selection rules are

$$\Delta M_L = 0, \pm 1; \quad \Delta M_S = 0.$$

The resulting splitting pattern is very similar to that of the normal Zeeman effect (see the figure). Lines corresponding to transitions



Paschen-Back effect for a $2P \rightarrow 2S$ transition.

with $\Delta M_L = 0$ are plane polarized with the electric vector parallel to the field direction (π -components), those with $\Delta M_L = \pm 1$ are plane polarized with the electric vector perpendicular to the field direction (σ -components). (For more detail see H. E. White, *Introduction to Atomic Spectra*, McGraw-Hill, Inc., New York, 1934, p. 223 ff.)

PASCHEN SERIES. Series of lines in the spectra of the hydrogen atom and the hydrogen-like ions He^+ , Li^{++} , \dots , also called Ritz-Paschen series. The wavenumbers $\nu_n = \frac{1}{\lambda_n}$ are given by the formula

$$\nu_n = RZ^2 \left[\frac{1}{3^2} - \frac{1}{n^2} \right], \quad (n = 4, 5, \dots),$$

where R stands for the **Rydberg constant** and Z for the **atomic number**.

PASSIVE NETWORK, LINEAR. See network, linear passive.

system of probability and frequency functions from 1895 to 1916. We follow C. C. Craig's simplified exposition. The Pearson system is useful in fitting an actual **frequency distribution** by means of the **sample moments**. The sample moments in conjunction with the criterion, δ , tell which Pearson type will explain the sample results. This Pearson type is then considered as a hypothesis which is tested against the actual frequencies by the **chi-square** test of goodness of fit.

The second use of the system is to approximate a theoretical **probability function** by means of the first 4 moments of the theoretical probability function. This use is regarded as empirical and a convenience until more exact results are obtained.

In fitting an actual frequency distribution either the method of **moments** or the method of maximum **likelihood** is available. While the method of maximum likelihood is preferable on theoretical grounds, it is somewhat arduous in practice. We shall explain the method of moments. .

The Pearson system is based on the differential equation,

$$\frac{dy}{ydt} = \frac{a - t}{b_0 + b_1t + b_2t^2},$$

where $t = \frac{x - \bar{X}}{\sigma_x}$, **standard units**.

This differential equation may be derived from the **hypergeometric distribution** and consequently the Pearson system may be considered as having a basis in probability. The system consists of 3 main types, 9 transitional types, and the **normal curve**. The types cover a very wide range of bell-shaped, J-shaped, and U-shaped functions. These will be briefly explained in terms of the criterion,

$$\delta = \frac{2\alpha_4 - 3\alpha_3 - 6}{\alpha_4 + 3}.$$

If δ is negative, we have main *Type I*; if δ is positive and the roots of $b_0 + b_1t + b_2t^2$ are complex, we have main *Type IV*; if δ is positive and the roots of $b_0 + b_1t + b_2t^2$ are real, we have main *Type VI*. The types occurring most frequently are *I, III, IV, and VII*. The values of $a, b_0, b_1,$ and b_2 are

$$a = \frac{-\alpha_3}{2(1 + 2\delta)},$$

$$b_0 = \frac{2 + \delta}{2(1 + 2\delta)},$$

$$b_1 = \frac{\alpha_3}{2(1 + 2\delta)},$$

$$b_2 = \frac{\delta}{2(1 + 2\delta)}.$$

We assume hereafter that $\alpha_3 \geq 0$. If $\alpha_3 < 0$, we can make it positive by changing the signs of all **variates**.

TYPE I, $\delta < 0, y = C(t - r_1)^{m_1}(r_2 - t)^{m_2}, r_1 \leq t \leq r_2$

$$r_1 = \frac{-\alpha_3 + \sqrt{D}}{2\delta}, \quad r_2 = \frac{-\alpha_3 - \sqrt{D}}{2\delta},$$

$$D = \alpha_3^2 - 4\delta(\delta + 2),$$

$$m_1 = \frac{1 + \delta}{\delta} \frac{\alpha_3}{\sqrt{D}} - \frac{1 + 2\delta}{\delta},$$

$$m_2 = -\frac{1 + \delta}{\delta} \frac{\alpha_3}{\sqrt{D}} - \frac{1 + 2\delta}{\delta},$$

$$C = \frac{\Gamma(m_1 + m_2 + 2)}{\Gamma(m_1 + 1)\Gamma(m_2 + 1)(r_2 - r_1)^{m_1 + m_2 + 1}}.$$

The *Type I* function may be J-shaped, U-shaped, or bell-shaped.

TYPE IV, $\alpha_3 \neq 0, \delta > 0, \alpha_3^2 < 4\delta(\delta + 2)$.

$$y = Ce^{\frac{v\pi}{2}} [(t + r)^2 + s^2]^{-m} e^{-v \tan^{-1} \frac{t+r}{s}},$$

$$-\infty < t < \infty, \quad \frac{v}{2} = -\frac{1 + \delta}{\delta} \frac{\alpha_3}{\sqrt{-D}},$$

$$m = \frac{1 + 2\delta}{\delta}, \quad r = \frac{\alpha_3}{2\delta},$$

$$s = \frac{\sqrt{-D}}{2\delta}, \quad D = \alpha_3^2 - 4\delta(\delta + 2).$$

$$C = \frac{s^{2m-1}}{G(2m - 2, v)},$$

$$G(2m - 2, v) = \int_0^\pi \sin^{2m-2} \varphi e^{v \cos \varphi} d\varphi.$$

The *Type IV* function is always bell-shaped.

TYPE VI, $\alpha_3 > 0, \delta > 0$.

$$y = Cz^{m_2}(z - \alpha)^{m_1}, \quad \alpha \leq z < \infty, \quad t - r_2 = z,$$

$$t - r_2 = z, \quad r_1 - r_2 = \alpha,$$

$$r_1 = \frac{-\alpha_3 + \sqrt{D}}{2\delta},$$

$$r_2 = \frac{-\alpha_3 - \sqrt{D}}{2\delta}, \quad D = \alpha_3^2 - 4\delta(\delta + 2),$$

$$m_1 = \frac{1 + \delta}{\delta} \frac{\alpha_3}{\sqrt{D}} - \frac{1 + 2\delta}{\delta},$$

$$m_2 = -\frac{1 + \delta}{\delta} \frac{\alpha_3}{\sqrt{D}} - \frac{1 + 2\delta}{\delta},$$

$$C = \frac{\Gamma(-m_2)}{\alpha^{m_1+m_2+1}\Gamma(m_1+1)\Gamma(-m_1-m_2-1)}.$$

The *Type VI* function may be J-shaped or bell-shaped.

TYPE III

$$y = Ce^{-\frac{2}{\alpha_3}t} \left(\frac{\alpha_3}{2}t + 1 \right)^{\frac{4}{\alpha_3^2}-1}.$$

$$-\frac{2}{\alpha_3} \leq t < \infty. \quad C = \left(\frac{4}{\alpha_3^2} \right)^{\frac{4}{\alpha_3^2}-\frac{1}{2}} \cdot \frac{1}{\Gamma\left(\frac{4}{\alpha_3^2}\right)}.$$

The *Type III* function may be J-shaped or bell-shaped. It is an excellent approximation to the **binomial distribution**.

TYPE II, $\alpha_3 = 0, -1 < \delta < 0$.

$$y = C(S^2 - t^2)^M, \quad -S \leq t \leq S,$$

$$S = \sqrt{-(1 + 2/\delta)},$$

$$M = -\left(2 + \frac{1}{\delta}\right),$$

$$C = \frac{\Gamma(2M + 2)}{(2S)^{2M+1}[\Gamma(M + 1)]^2}.$$

Type II may be bell-shaped or U-shaped.

TYPE VII, $\alpha_3 = 0, \delta > 0$.

$$y = C(t^2 + s^2)^{-m}, \quad -\infty > t > \infty.$$

$$s = \sqrt{1 + \frac{2}{\delta}},$$

$$m = 2 + \frac{1}{\delta}, \quad C = \frac{s^{2m-1}\Gamma(m)}{\sqrt{\pi}\Gamma(m - 1/2)}.$$

Type VII is always bell-shaped.

TYPE V, $\alpha_3 \neq 0, \delta > 0, \alpha_3^2 = 4\delta(\delta + 2)$.

$$y = C(t + r)^{-2m} e^{-\frac{2r(m-1)}{t+r}}, \quad -r \leq t < \infty,$$

$$m = 2 + \frac{1}{\delta}, \quad r = \frac{\alpha_3}{2\delta},$$

$$C = \frac{[2r(m-1)]^{2m-1}}{\Gamma(2m-1)}.$$

Type V is always bell-shaped.

The remaining special types *VIII, IX, X, XI, and XII*, occur quite infrequently. In fact, the terminology can be said to have survived in the **Type I** or **beta distribution**, the **Type III** or **gamma distribution**, the **Type IV** distribution and, to a smaller extent, the **Type VI** distribution. The distributions are determined by their first four **moments**, a fact used in fitting them to observed data. The **normal distribution** is also a member of the family.

PELL EQUATION. The diophantine equation $x^2 - Dy^2 = 1$, where D is a positive integer which is not a perfect square.

PENDULUM. An oscillatory device whose configuration is specified by one or more **angular positions** is called a pendulum. (See **pendulum, ballistic**.)

PENDULUM, BALLISTIC. An instrument for measuring the high speed of a projectile moving horizontally. It consists of a **pendulum** mass which is heavy compared with the projectile and is supported by relatively long strings or a rod. **Momentum** is conserved as the projectile embeds itself in the mass and **energy** is conserved as the pendulum swings. The speed of the projectile is given by the height of rise of the pendulum mass, which is

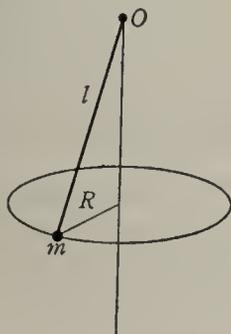
measured accurately by the horizontal excursion.

PENDULUM, COMPOUND. A rigid body free to oscillate about a fixed axis under the action of a restoring moment (usually produced by gravity) is called a compound or *physical pendulum*. The plane through the **center of gravity**, C , and perpendicular to the axis of rotation may be termed the *plane of motion* and the intersection of the axis of rotation with this plane is called the *point of support*, C . The period of small oscillations is $2\pi\sqrt{I_o/mgh}$ where I_o is the moment of inertia about O , m is the total mass, and h is the distance OC . The length of a simple pendulum of the same period of oscillation is $L = I_o/mh = r_o^2/h$ where r_o is the **radius of gyration** for O . L is called the *equivalent length* of the compound pendulum. Replacing r_o^2 by $r_c^2 + h^2$ where r_c is the radius of gyration for C , $L = h + r_c^2/h$. The point on the extension of OC at a distance L from O is called the *center of oscillation*. As L is given by r_c and h , the period of oscillation will be the same for all support points on a circle of radius h with center at C . Except for the minimum period condition $h = r_c$, $L = 2r_c$, there are two values of h , h_1 and h_2 , which give the same L . Two points on a line through C , one at h_1 and the other at h_2 from C are called *conjugate points*. Their distance apart $h_1 + h_2 = L$.

PENDULUM, CONICAL. A mass m suspended from a fixed point O by a string of fixed length l , rotating along a circle of radius R with uniform angular velocity ω . The value of the angular speed is given by the equation

$$\omega^2 = \frac{g}{l^2 - R^2}.$$

while the period is $2\pi\sqrt{h/g}$, where h is the perpendicular distance from O to the circle of motion.



Conical pendulum.

The conical pendulum is a special case of the spherical pendulum. (See **pendulum, spherical**.)

PENDULUM DAY. The time required for the plane of a freely suspended (Foucault) pendulum to complete an apparent rotation about the local vertical. This period τ is given by the formula

$$\tau = \frac{2\pi}{\Omega \sin \phi} = \frac{\text{sidereal day}}{\sin \phi},$$

where Ω is the angular speed of the earth and ϕ the latitude. (See **inertial flow**.)

PENDULUM, EQUIVALENT LENGTH OF. The length of a simple pendulum having the same period of oscillation as a compound pendulum. (See **pendulum, compound**.)

PENDULUM, FOUCAULT. Any pendulum suspended in such a way that its (vertical) plane of oscillation is free to rotate. Because of the rotation of the earth, there is an apparent rotation of this plane, as observed by a terrestrial observer. The rotation is clockwise in the northern hemisphere and counterclockwise in the southern hemisphere. Its angular speed is $\omega \sin \phi$ where Ω = angular speed of earth due to its daily rotation, ϕ = latitude (0° at the equator).

PENDULUM, HUYGENS. See **tantochrone**.

PENDULUM, KATER. Kater devised a number of rigid **pendulums** for comparing the accelerations of gravity at different places on the earth. Any pendulum of fixed length, carried from place to place, will swing in periods inversely proportional to the square root of the value of this acceleration, g , at the respective stations. Therefore if the pendulum has been timed at a station at which the value of g is accurately known, its period at any field station gives at once the value of g at that station. Kater's best known pendulum is reversible. (See **pendulum, reversible**.)

PENDULUM, PHYSICAL. See **pendulum, compound**.

PENDULUM, REVERSIBLE. A pendulum which is used for accurate determinations of the acceleration of gravity. By measuring the period corresponding to two different suspension lengths, there is no need for a measure-

ment of the moment of inertia. The acceleration of gravity is given by

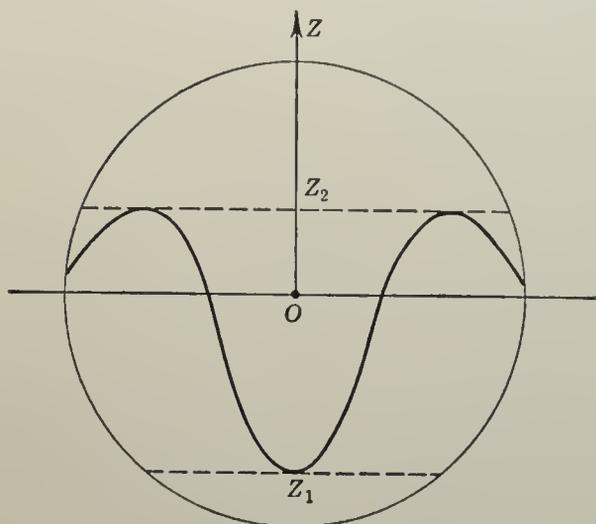
$$g = \frac{4\pi^2[l_A^2 - l_B^2]}{l_A P_A^2 - l_B P_B^2}$$

where l_A, l_B are two different lengths of suspension to center of mass; P_A, P_B are periods corresponding to the above lengths.

PENDULUM, SIMPLE. A simple pendulum consists of a point-mass, suspended from a horizontal axis by a light rod or inextensible string of length L . For small oscillations the period T is

$$T = 2\pi \sqrt{\frac{l}{g}}$$

PENDULUM, SPHERICAL. A particle constrained to slide frictionless on the surface of a sphere under the sole influence of gravity represents a spherical pendulum. The motion oscillates between the two levels z_1 and z_2 . The arithmetic mean of z_1 and z_2 lies below the center O of the sphere. The motion is given



Spherical pendulum.

by $z = z_1 + (z_2 - z_1)\text{sn}^2(\omega t)$. (It is assumed that at the time O the particle is at the lower level.) z has the period T , where $a =$ radius of sphere; $T = \frac{2K}{\omega}$; $\omega = \sqrt{\frac{g(z_2 - z_1)}{2a^2}}$. $\text{sn}(x)$ is the Jacobian elliptic function.

$$K = \int_0^1 \frac{dx}{\sqrt{(1-x^2)(1-k^2y^2)}}$$

$$k = \sqrt{\frac{z_2 - z_1}{z_3 - z_1}}$$

PENDULUM, TORSION. The torsion pendulum consists of a body suspended by a fine wire or elastic fiber in such a manner that it will execute rotational oscillations as the suspension twists and untwists. If I is the **moment of inertia** of the body with respect to the axis of oscillation, and if K is the torsion coefficient of the fiber (torque required to twist is through an angle of one radian), the period of oscillation is given by the simple formula

$$T = 2\pi \sqrt{\frac{I}{K}} \tag{1}$$

Both I and K may have to be determined by experiment in actual laboratory practice. This is easily done by measuring the period T and then adding to the suspended body another of known moment of inertia I' , giving a new period of oscillation T' , which is also measured. From (1),

$$T' = 2\pi \sqrt{\frac{I + I'}{K}}; \tag{2}$$

and the solution of the two equations now gives $K = 4\pi^2 I' / (T'^2 - T^2)$, $I = T^2 I' / (T'^2 - T^2)$.

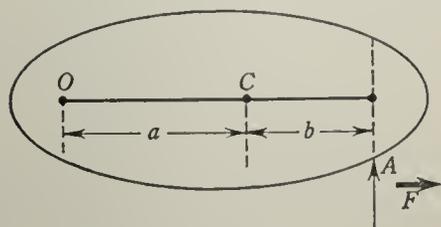
PENETRATION DEPTH. (1) The distance from the surface of a **superconductor** in a magnetic field at which the intensity of the field has fallen to $1/e$ of its value outside. (2) The distance to which an external magnetic field penetrates into a superconductor from whose bulk it is excluded by the Meissner effect. The **London equations** predict a depth $(\Lambda c^2 / 4\pi)^{1/2} \sim 10^{-5}$ cm in a static field, a result confirmed by observations on small particles. At high frequencies the effect goes over into the eddy-current skin-depth appropriate to the density and mobility of the "normal" electrons. (3) In induction heating usage, the thickness of a layer extending inward from the surface of a conductor, which has the same resistance to direct current as the conductor as a whole has to alternating current of a given frequency.

PENETRATION FACTOR. See **skin depth**.

PENETRATION PROBABILITY (PENETRABILITY). The probability of passage of a particle through a potential barrier, defined as a region of finite extent in which the potential energy of the particle exceeds its total energy. An example of barrier penetration is

the passage of α -particles through the Coulomb barrier at the nuclear surface. Penetration of this type is a quantum mechanical phenomenon which cannot be explained in terms of classical physics. In general, the penetration probability increases as the energy of the particle approaches the height of the barrier and as the thickness of the barrier decreases. Interference effects, however, play an important role when the thickness of the barrier is a multiple of one-quarter of the wavelength of the de Broglie wavelength associated with the particle. The half-width γ of any nuclear process involving a barrier is the product of the intrinsic probability that the particles will appear at the surface of the barrier times the penetrability of the barrier.

PERCUSSION, CENTER OF. A rigid lamina subject to no constraints is given a blow at point A by the force F . The lamina will start to rotate about point O . This point is called the center of percussion. Let C be the center of mass and k_c the radius of gyration about C .



Then, as shown in the figure,

$$a = \frac{k_c^2}{b}.$$

No shock will be felt at O when a blow is struck at A .

PERFECT GAS. A perfect gas may be defined by the following two laws: The **Joule law**: the energy per mole, U , depends only on the temperature. The **Boyle law**: at constant temperature, the volume V occupied by a given number of moles of gas varies in inverse proportion to the pressure.

By combination of these two laws we obtain the **equation of state** for perfect gas

$$pV = nRT \quad (1)$$

where R is the **gas constant**, T , the absolute temperature. (It is also called the *perfect gas law*.)

The perfect gas is an abstraction to which any real gas approximates according to the na-

ture of the gas and the conditions. For a given temperature and composition the perfect gas condition is approached when the density tends to zero. From a molecular point of view the perfect gas laws correspond to the behavior of a system of molecules whose interactions may be neglected in expressing the thermodynamic equilibrium properties. However, even at a low density the transport properties depend essentially on the interactions. (See **Boltzmann transport equation, kinetic theory of gases**.)

The thermodynamic properties of a perfect gas are, of course, especially simple. For example, the difference between the molar heat capacities at constant pressure and constant volume is equal to the gas constant R

$$C_p - C_v = R. \quad (2)$$

The value of R is 0.08205 liter-atm. degree⁻¹ mole⁻¹, which in cgs units is equal to 8.314×10^7 g cm² sec⁻² degree⁻¹ mole⁻¹. This relationship, Formula (2), applies only approximately to real gases.

However, the way in which either C_p or C_v depend on the temperature can only be calculated from statistical mechanics.

PERFECT GAS MIXTURES. A mixture of gases in volume V at temperature T is called a **perfect gas mixture** if the Helmholtz free energy A (see **thermodynamics, characteristic functions of**) is equal to the sum of the free energies which the separate constituents would have if each were confined alone in the same volume at the same temperature.

The equation of state of a perfect gas mixture is

$$pV = nRT \quad (1)$$

with

$$n = \sum_i n_i. \quad (2)$$

The **chemical potential** is of the form (see also **ideal systems**)

$$\mu_i = \mu_i^0(T, p) + RT \ln x_i \quad (3)$$

where μ_i^0 is the chemical potential for the pure gas i at the same T and p ; μ_i^0 is of the form

$$\mu_i^0 = \mu_i^\dagger(T) + RT \ln p \quad (4)$$

it depends in a logarithmic way on the pressure.

The temperature dependence of $\mu_i^\dagger(T)$ can only be fixed by considerations of statistical mechanics.

PERFECTLY ELASTIC MATERIAL. A material for which the excess of the work done by the applied forces in a deformation over the kinetic energy generated in the deformation is a single-valued function of the initial and final states of deformation.

PERFECTLY PLASTIC MATERIAL. A material which flows indefinitely at constant yield stress.

PERFECT MIXTURES. See **ideal systems**.

PERFECT OPTICAL SYSTEMS. An optical system is perfect if the mapping of object points into image points by the system is a non-singular collineation. A system is plane-perfect if the mapping from one object plane to its image plane is non-singular and linear. (See the **Abbe-Maxwell theorem**, **Maxwell's fisheye**, the **Luneberg lens**.)

PERFECT SOLUTIONS. A liquid or solid mixture which remains ideal at all concentrations (see **ideal systems**). This situation is realized when the intermolecular forces between the different molecules are identical. Moreover if quantum effects have to be taken account, the masses of the molecules and their statistics (see **quantum statistics**) have to be the same.

Perfect solutions have specially simple properties. (See **vapor pressure in perfect solutions**; **osmotic pressure**; **excess functions**.)

PERIHELION OF MERCURY, ADVANCE OF. See **relativity, general**.

PERIOD. (1) If, for a non-constant function $f(x)$, there exists a constant a such that $f(x + a) = f(x)$ for all x , then a is said to be a period of the function $f(x)$; if a is a period, then any multiple of a is clearly also a period, a period which is not a multiply of any other period being called a *primitive* or *fundamental period*. A function of a real variable can have at most one primitive period, but a function of a complex variable may have two.

(2) An interval, especially one established by repeated or regular recurrence, or the entities contained within such an interval.

PERIODIC, MULTIPLY. A function of several variables $\theta_1, \theta_2, \dots, \theta_f$, each of which depends linearly upon the time, $\theta_i = V_i t$, is said to be multiply periodic with period b in the variables $\theta_1, \theta_2, \dots, \theta_f$ if

$$F(\theta_1 + n_1 b, \theta_2 + n_2 b, \dots, \theta_f + n_f b) \\ = F(\theta_1, \theta_2, \dots, \theta_f),$$

where n_1, n_2, \dots, n_f are integers. If the various V_i are not all equal and are not commensurable, it is clear that F will not be periodic in time. A system for which this is true is frequently called *conditionally periodic*.

PERIODIC STAR. See **variable stars**.

PERIOD, NATURAL. The period of a free oscillation of a body or system.

PERIOD OF DEFORMATION. Time from initial impact until bodies cease to approach. (See **period of restitution**; **impact**.)

PERIOD OF RESTITUTION. Time from maximum compression until bodies are ultimately separated and moving with final velocities. (See **impact**.)

PERIOD OF VARIATION. See **variable stars**.

PERIODOGRAM. A form of diagram used in the analysis of oscillatory time-series. The values of the series are correlated with a trigonometric term $\sin(\lambda t + \epsilon)$ for varying values of λ . The correlation (or some linear function of it) is plotted as ordinate against λ as abscissa and if the resultant has a sharp peak at $\lambda_1, \lambda_2, \dots$, etc., the series is analyzed into the sum of terms of period $2\pi/\lambda_1, 2\pi/\lambda_2, \dots$, plus a residual. Modern practice is in favor of graphing the correlation (or some similar measure) against the frequency $2\pi/\lambda$, in which case we have the **spectral function**.

Other forms of periodogram are sometimes used.

PERIOD, REACTOR. In the theory of neutron chain reactors, the reactor period T is related by $\omega T = 2\pi$ to any of the "natural" frequencies, ω , characteristic of a particular system. (See **inhour equation**.) The period corresponding to the algebraically greatest frequency is called the steady or stable period. The dependent variable in these considerations is, of course, the neutron flux.

PERMANENT WAVE. A wave (in a fluid) moving with no change in streamline pattern, and which, therefore, is a **stationary wave** relative to a coordinate system moving with the wave.

PERMEABILITY, μ . (1) Absolute permeability is the ratio of the magnitude of **magnetic induction** to the **magnetic field intensity**. Thus $\mathbf{B} = \mu\mathbf{H}$; for an isotropic material μ is a scalar, but for an anisotropic material μ is a tensor. It is related to the magnetic susceptibility χ_m by the equation $\mu = 1 + \chi_m$ (in rationalized units). In unrationalized units, the relation is $\mu = 1 + 4\pi\chi_m$.

(2) Specific or relative permeability is the ratio (μ/μ_0) of the absolute permeability to the permeability (μ_0) of free space. In MKS units $\mu_0 = 4\pi \times 10^{-7}$ henry/meter. Also known as *magnetic constant*.

PERMITTIVITY. The constant ϵ appearing in **Coulomb's law**. The ratio ϵ/ϵ_0 is the relative permittivity or **dielectric constant**, where ϵ_0 is the permittivity of free space.

PERMUTATION. Given n distinguishable objects or elements, each different arrangement of the elements is a permutation. The number of permutations is $n(n-1)(n-2)\cdots 3\cdot 2\cdot 1 = n!$. Several different symbols, such as ${}_nP_n$, $P_{n,n}$ or $P(n,n)$ are used to indicate this result. If the n things are taken r at a time ($r < n$), ${}_nP_r = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$. When n_1 of the elements are all alike

of the first kind, n_2 of the second kind, etc., so that $n_1 + n_2 + \cdots + n_m = n$, the number of permutations is $\frac{n!}{n_1!n_2!\cdots n_m!}$. This result also

applies if the elements are separated into m parts with n_i elements in the i th part. If the number of elements in each of the m parts is not specified but each part must contain at least one element, the number of permutations is

$\frac{n!(n-1)!}{(n-m)!(m-1)!}$. This number is increased to $\frac{(m+n-1)!}{(m-1)!}$ if empty parts are permitted.

A *combination* is an arrangement of objects or elements, where the order of arrangement is not distinguished. Thus given the three letters a, b, c the possible permutations are $(abc), (acb)$, etc., six in number but there is only one combination. With symbols as before,

$${}_nC_r = \frac{n(n-1)(n-2)\cdots(n-r+1)}{r!} \\ = \frac{n!}{r!(n-r)!}$$

This number is identical with the $\binom{n}{r}$, the coefficient in the **binomial series**. Moreover, ${}_nC_r = {}_nP_{n-r}$; ${}_nP_r = r!{}_nC_r$.

The number of combinations of n different elements into m specified parts, with empty parts allowed is m^n ; of n identical elements into m different parts with empty parts is $\frac{(n+m-1)!}{n!(m-1)!}$

but when at least one element is in each part, the number is $\frac{(n-1)!}{(m-1)!(n-m)!}$. Finally, the

total number of combinations of n things taken 1, 2, 3, \cdots , n at a time is $\sum_{i=1}^n {}_nC_i = 2^n - 1$.

(See also **permutation group**.)

PERMUTATION, CYCLIC. See **permutation group**.

PERMUTATION, EVEN OR ODD. A permutation may always be written as a product of **transpositions**, either even or odd in number. The permutation is then said to be even or odd.

PERMUTATION GROUP. Its elements, $n!$ in number, are the various **permutations** or rearrangements of a standard arrangement of n symbols or objects. A typical element is

$$S = \begin{pmatrix} s_1 & s_2 & \cdots & s_n \\ 1 & 2 & \cdots & n \end{pmatrix}$$

meaning that the operation S replaces 1 by s_1 , 2 by s_2 , etc. If another element is indicated by T , then ST , the rearrangement designated by T followed by S , the resulting permutation is also in the **group**.

A permutation sending s_1 into s_2 , s_2 into s_3 , etc., and finally s_n into s_1 is called a *cycle* on n letters. It is usually written as (s_1, s_2, \cdots, s_n) . The degree of a cycle equals the number of symbols permuted. A cyclic permutation of degree two is a *transposition*. Any permutation may always be written as a product of transpositions, either even or odd in number. The permutation is then said to be even or odd.

The group of all permutations of n letters or objects, of order $n!$, is called the *symmetric group*. The even permutations of n objects form a *subgroup* of the symmetric group. Its order is $n!/2$ and it is called the *alternating group*.

Every group is *isomorphic* to some permutation group. It is easy to find a **representation** of a permutation group by using a permutation **matrix**. Each row and column of such a matrix has but one non-zero element and that is unity. The row and column thus designate the initial and final locations of the object permuted.

PERMUTATION SYMBOL. A symbol carrying indices, taking the values (e.g.) 1, 2, \dots , n , the value of which is 1, -1 or 0 depending on the values assigned to the indices, e.g., generalized **Kronecker delta**, alternating tensor. (See **tensor**, **alternating**.)

PERPENDICULAR AXIS THEOREM. The moment of inertia of a plane figure about a perpendicular or polar axis is the sum of the moments of inertia about two axes at right angles in the plane passing through the polar axis.

PERPETUAL MOTION ENGINE. A hypothetical engine which could produce work continuously without expense. It is necessary to discern two types of perpetual motion engines, as defined in the two entries following.

PERPETUAL MOTION ENGINE OF THE FIRST KIND. A hypothetical engine which would operate continuously (i.e., perform **cycles**) and which would produce mechanical work without consuming energy in any form, particularly heat. The first law of **thermodynamics** asserts that a perpetual motion engine of the first kind is impossible.

PERPETUAL MOTION ENGINE OF THE SECOND KIND. A hypothetical engine which operates periodically (i.e., performs **cycles**) and continuously, and produces work from heat drawn from a single source at fixed temperature. The second law of **thermodynamics** asserts that a perpetual motion engine of the second kind is impossible. In fact W. Ostwald used the preceding statement as an independent formulation of the second law. (For a comparison with other statements, see second law of **thermodynamics**.)

PERTURBATION. (1) A small contribution to a physical quantity, such that the problem into which the quantity enters can be solved exactly or in a far simpler manner than otherwise if the perturbation is neglected. (See **perturbation theory**.) The form in which a

perturbation is most frequently used in both classical and quantum mechanics is a small additional energy, called the *perturbation energy*. (2) Any departure introduced into an assumed **steady state** of a system. The magnitude of the departure is often assumed to be small so that product terms in the dependent variables may be neglected; the term "perturbation" is therefore sometimes used as synonymous with small perturbation. The perturbation may be concentrated at a point or in a finite volume of space; or it may be a wave (sine or cosine function); or, in the case of a rotating system, it may be symmetric about the axis of rotation.

PERTURBATION EQUATION. Any equation governing the behavior of a **perturbation**. This may or may not be a linearized differential equation.

PERTURBATION FORMULA, RAYLEIGH-SCHRÖDINGER. See **Rayleigh-Schrödinger perturbation formula**.

PERTURBATION HAMILTONIAN. That part of the **Hamiltonian** of a system which represents the perturbation energy of the system.

PERTURBATION MOTION. The motion of a disturbance (usually but not necessarily assumed infinitesimal), as opposed to the motion of the steady state of the system on which the perturbation is superimposed.

PERTURBATION QUANTITY. Any parameter of a system, e.g., velocity components or temperature, which may or may not have been assumed to be small perturbations from a mean or steady-state value.

PERTURBATIONS IN MOLECULAR SPECTRA. The displacement of a band from its regular position in the **band system** (vibrational perturbation) or the displacement (and/or weakening) of corresponding lines in the different branches of a band (rotational perturbation). A perturbation observed in the spectrum is indicative of the presence of a perturbation (shift) of one of the energy levels involved due to interaction with another level of the same, or nearly the same, energy.

One distinguishes homogeneous and heterogeneous perturbations, corresponding to interaction between energy levels of the same or

different **species**, respectively. Homogeneous perturbations arise from interaction between states with the same quantum number Λ ($\Delta\Lambda = 0$) in diatomic and linear polyatomic molecules, or from Fermi-interaction (see **Fermi-resonance**) in polyatomic molecules. Heterogeneous perturbations correspond to interaction between states differing by $\Delta\Lambda = \pm 1$ in diatomic and linear polyatomic molecules, or to Coriolis interaction in polyatomic molecules.

The selection rules for perturbation are in all cases

$$\Delta S = 0; \quad \Delta J = 0; \quad + \leftarrow | \rightarrow -; \quad s \leftarrow | \rightarrow a.$$

(J and S are quantum numbers of the total angular momentum and of the electron spin.)

PERTURBATION THEORY. Any problem in which the effects can be divided into large contributions and (small) corrections can be dealt with by means of perturbation methods. The problem is first solved for the unperturbed system for which the smaller corrections are neglected, and these corrections are taken into account only in the next approximations. Such perturbation methods have been used, for instance, in astronomy to calculate small deviations in the orbits of planets due to the effect of other planets. A much more frequent use is made of perturbation methods in quantum mechanics.

PERTURBATION THEORY, QUANTUM MECHANICAL. The mathematical complexity of many quantum mechanical problems is such that one cannot hope to obtain exact solutions. However, good predictions can sometimes be obtained by means of perturbation theory, if one can assume that the actual system differs only slightly from a simpler system for which the problem can be solved, and the neglected difference can be dealt with as a perturbation of this simpler unperturbed system. The effect of a weak electromagnetic field on an atom, for instance, can be dealt with as a perturbation, and the transition probabilities between the energy states of the unperturbed atom can be calculated by means of perturbation theory. A weak interaction between two particles can be dealt with as a perturbation in the collision process of the two particles. The perturbation methods can be time-independent or time-dependent, according to whether the unperturbed states are described

by time-independent wave functions or time-dependent ones. If the strength of a weak interaction between two systems is proportional to a constant parameter, the wave functions and energy values of the wave equation can be expanded in powers of this constant. The zero-order approximation is given by the unperturbed wave functions and energy values which are independent of this parameter. These determine the first-order approximations together with that part of the wave equation which is linear in the coupling parameter. By successive approximations one obtains expressions for second, third and higher order perturbations in the wave function and the energy. If an unperturbed energy state is degenerate, that is, if two or more states have the same unperturbed energy, the effect of the perturbation has to be taken into account first between these degenerate states.

PERTURBATION VELOCITY POTENTIAL. See **slender-body theory**.

PERVEANCE. The quotient of the space-charge-limited cathode current by the $\frac{3}{2}$ -power of the anode voltage in a **diode**. Perveance is the constant G appearing in the Child-Langmuir-Schottky equation.

$$i_k = Ge_b^{3/2}.$$

When the term perveance is applied to a triode or multi-grid tube, the anode voltage e_b is replaced by the composite controlling voltage e' of the equivalent diode.

PETIT ENSEMBLE. See **ensemble**.

PETZVAL'S CONDITION. If a lens system is free of astigmatism, then $P = 0$, where P is the **Petzval sum** of the system, is a sufficient condition for the system to image plane objects as planes. This is Petzval's condition. To ensure $P = 0$ is to eliminate the aberration of curvature of field.

PETZVAL SUM. If $1/r_i$ is the curvature of the i^{th} spherical surface of an axially symmetric lens system and n_i is the index of refraction of the medium following the i^{th} surface, then the Petzval sum of the system is

$$P = \sum_i \frac{1}{r_i} \left(\frac{1}{n_{i-1}} - \frac{1}{n_i} \right).$$

For a system of thin lenses, if ϕ_i is the power of the i^{th} lens then

$$P = \sum_i \phi_i/n_i$$

(See **Petzval's condition** and **Petzval surface**.)

PETZVAL SURFACE. The mean image in the third order theory of a rotationally symmetrical lens system lies on a sphere of radius P , where P is the **Petzval sum** of the system. The sphere, with center on the axes and tangent to the paraxial focal plane, is the Petzval surface of the system.

PFAFF PROBLEM. A differential expression

$$dW = \sum_{i=1}^n X_i dx_i$$

where the X_i are functions of the independent variables is called a *Pfaff expression* or a *Pfaffian*. In general the value of W depends upon the path of **integration**, whereupon the expression is an inexact differential. If the value of W is independent of the path of integration, the expression is an *exact* or perfect differential. In this case, the equation is said to be integrable. The non-integrable case was once thought to be meaningless since it did not seem to be obtainable from a single primitive. However, if it contains $2n$ or $(2n - 1)$ variables it is equivalent to not more than n algebraic equations the finding of which is called Pfaff's problem. Thus, if there are three variables, the general solution is an arbitrarily chosen relation $f(x, y, z) = 0$ and a second relation containing an arbitrary constant $g(x, y, c) = 0$. Pfaff's problem may be interpreted geometrically and is of some interest in advanced theories of mechanics and thermodynamics.

PFUND SERIES. Line series in the spectra of hydrogen and the hydrogen-like ions He^+ , Li^{++} , \dots . The wavenumbers $\nu_n = \frac{1}{\lambda_n}$ are given by the formula

$$\nu_n = RZ^2 \frac{1}{5^2} - \frac{1}{n^2}, \quad (n = 6, 7, \dots),$$

where R stands for the **Rydberg constant** and Z for the **atomic number**. For hydrogen the Pfund series lies in the infrared region of the spectrum.

PHASE. A region of space in a thermodynamic system which is separated from the remainder by a clearly definable surface and

within which the thermodynamic properties differ from the remainder when the whole system is in thermal equilibrium, i.e., when, in particular, the pressure and temperature are uniform throughout the system. Examples: (a) the liquid and vapor regions of a system consisting of both in thermal equilibrium, constitute two distinct phases; (b) two allotropic forms of a solid which exist side by side at a given pressure and temperature constitute two distinct phases, (c) in a solution, the pure components, and the regions differing in concentration, all constitute separate phases of the system.

Since all gases can be mixed in any proportion, a given system can possess only one gaseous phase; it can, however, possess several liquid and solid phases. (See also **phase of a periodic quantity**.)

PHASE ANGLE. A conventional representation of simple harmonic motion is

$$y = A \sin(\omega t + \alpha) = A \sin\left(\frac{2\pi t}{T} + \alpha\right)$$

in which A is the amplitude, $\omega t + \alpha$ or $\frac{2\pi t}{T} + \alpha$ is the phase angle, and α is the epoch angle, t is the time variable, T is the period and ω is the angular velocity.

Two coherent harmonic motions, described by

$$y_1 = A \sin(\omega t + \alpha)$$

and

$$y_2 = A \sin(\omega t + \beta)$$

are said to have a phase difference of $(\beta - \alpha)$.

PHASE ANGLE, ACOUSTIC. The angle whose tangent is the ratio of the acoustic reactance to the acoustic resistance. (See **reactance, acoustic** and **resistance, acoustic**.)

PHASE COHERENCE FACTOR. See **partial coherence**.

PHASE CONSTANT. See **propagation constant**.

PHASE DIFFERENCE VS. PATH DIFFERENCE. The difference in phase between two coherent wave disturbances that have traveled over different **optical paths**, from some point at which they had the same phase, is $2\pi/\lambda$ times the path difference, where λ is the wavelength.

PHASE, DISTRIBUTION IN. See **distribution in phase**.

PHASE FUNCTIONS, MINIMUM. See **minimum phase functions**.

PHASE INTEGRAL. If a coordinate q is a periodic function of time with the period T and if p is its conjugate momentum, then the integral

$$\oint pdq,$$

taken over one period, is called a phase integral. A fundamental postulate of the older quantum theory was that such an integral must be an integral multiple of Planck's constant. (See **partition function**.)

PHASE MARGIN. A partial indication of the degree of stability of a feed-back loop defined by:

Phase margin = phase of the loop frequency-function at that frequency for which the loop gain ratio is unity (i.e., the loop **gain** in logarithmic units is zero).

To avoid confusion it should be noted that if $Y(s)$ is the output/error transfer function in an error-monitored control system, the loop transfer function is $-Y(s)$. Thus the phase of the loop frequency-function differs by 180° from the phase of $Y(i\omega)$.

PHASE MIXING OR FINE SCALE MIXING.

A general phenomenon first pointed out by Gibbs in demonstrating that coarse graining of the n -particle distribution function in phase space leads to a decrease in the H -function for the system. Landau damping of electron plasma oscillations can be looked upon as a phenomenon of this kind. It may also be interpreted as a mixing and cancellation of the phases of the **normal modes** of the system.

PHASE OF A PERIODIC QUANTITY. The phase of a **periodic quantity**, for a particular value of the independent variable, is the fractional part of a **period** through which the independent variable has advanced, measured from an arbitrary origin. In the case of a simple sinusoidal quantity, the origin is usually taken as the last previous passage through zero from the negative to positive direction.

The origin is generally so chosen that the fraction is less than unity. The phase is expressed in **radians**, rather than in periods, in which use its numerical value is increased by a factor of 2π .

PHASE RETARDATION. See **partial coherence**.

PHASE RULE. The phase rule, due to Gibbs, gives the number ω of intensive variables which can be fixed arbitrary in a system in equilibrium. This number is also called the variance or the number of degrees of freedom of the system. It is given by

$$\omega = 2 + (c - r') - \phi \quad (1)$$

where c is the number of components, r' the number of **independent chemical reactions**, other than transfers of molecules from one phase to another, and ϕ , the number of phases.

In terms of **independent components**, c' , Equation (1) may also be written

$$\omega = 2 + c' - \phi. \quad (2)^*$$

If $\omega = 0$ the system is invariant. We cannot fix either temperature or pressure arbitrarily. Equilibrium can only be established at isolated points. An example is the *triple point* at which pure solid, liquid and vapor are in equilibrium ($\omega = 2 + 1 - 3 = 0$).

If $\omega = 1$ the system is *monovariant*. We can, for example, fix the temperature, but the equilibrium pressure is then fixed. This is the situation for a system containing one component and two phases.

If $\omega = 2$ the system is *bivariant*. Within certain limits both pressure and temperature can be given arbitrarily. This is the situation for $c = 1$ and $\phi = 1$, or $c = 2$ and $\phi = 2$.

* By many authorities, Equation (2) is written in the notation, $f = 2 + c - p$, using f for the number of independent variables, and p for the number of phases.

PHASE SEPARATION OF SUBSTITUTIONAL ALLOYS, STATISTICAL TREATMENT. See **cooperative phenomena**.

PHASE SLOWNESS. Consider a harmonic wave

$$V = A(x, y, z) e^{j[\omega t - \Phi(x, y, z)]}.$$

The phase slowness is the vector

$$\mathbf{S} = \frac{1}{\omega} \nabla \Phi.$$

The reciprocals of the components of \mathbf{S} are the speeds of propagation of constant phase along the corresponding axes. Since these speeds are *not* the components of a velocity vector, **phase velocity** is not conveniently generalized from one dimension to three.

PHASE SPACE. A multidimensional space, each point of which determines completely the phase of the system, that is, the values of all the generalized coordinates and generalized momenta. (See **mu-space** and **gamma-space**.)

If a system contains N degrees of freedom, i.e., needs N generalized coordinates q to describe it, then its phase space is a Cartesian space with $2N$ dimensions, viz., the coordinates q_1, \dots, q_N and the corresponding momenta p_1, \dots, p_N . The state of the system at any instant is represented by a representative point in phase space with the coordinates $q_1, \dots, q_N, p_1, \dots, p_N$. As time goes on, this representative point describes a curve in the phase space. (See **Liouville's theorem**.)

PHASE VELOCITY (WAVE VELOCITY).

The velocity of propagation of an equiphase surface along the direction normal to that surface (see also **waves, group velocity of**). Thus the phase velocity is the velocity of propagation of wave crests, nodes, or troughs. In a dispersive medium this differs from the velocity of propagation of a disturbance of finite extent as a whole.

PHASE, WAVE. See **wave phase**.

PHENOMENOLOGICAL COEFFICIENTS IN THERMODYNAMICS OF IRREVERSIBLE PROCESSES. See the following entry.

PHENOMENOLOGICAL RELATIONS IN THERMODYNAMICS OF IRREVERSIBLE PROCESSES. The entropy production can be written as the sum of the products of generalized forces or affinities and the corresponding rates (or generalized fluxes) of the irreversible processes (see **entropy production; thermodynamics, second law of**)

$$\frac{d_i S}{dt} = \sum_k J_k X_k \geq 0. \quad (1)$$

At thermodynamic equilibrium one has simultaneously for all irreversible processes

$$J_k = 0 \quad \text{and} \quad X_k = 0. \quad (2)$$

One can assume that near equilibrium one has linear relations between the forces and the rates. Such a scheme automatically includes empirical laws as the **Fourier law** for heat flow or the **Fick law** for diffusion. Linear laws of this kind are called the phenomenological relations. Indeed the existence of such relations is an extrathermodynamic hypothesis.

In the case of two simultaneous irreversible processes, the phenomenological relations are

$$\begin{aligned} J_1 &= L_{11}X_1 + L_{12}X_2 \\ J_2 &= L_{21}X_1 + L_{22}X_2. \end{aligned} \quad (3)$$

The coefficients in L are the phenomenological coefficients. The coefficients L_{ii} may stand for the heat conductivity, the electrical conductivity, the chemical drag coefficient, while the coefficients L_{ik} (with $i \neq k$) describe the interference of the two irreversible processes i and k . For example, if the two irreversible processes represent thermal conductivity and diffusion, the coefficient L_{ik} is connected with thermodiffusion. Such coefficients may be called the "interference coefficients," or "coupling coefficients," since they couple a mass transfer with an energy gradient or a heat flux with a density gradient, for example.

Replacing the fluxes by their value (2) in (1) we obtain the quadratic form

$$\begin{aligned} \frac{d_i S}{dt} &= L_{11}X_1^2 \\ &+ (L_{12} + L_{21})X_1X_2 + L_{22}X_2^2 \geq 0. \end{aligned} \quad (4)$$

This quadratic form has to be positive for all positive or negative values of X_1X_2 , except when $X_1 = X_2 = 0$, which corresponds to equilibrium. Therefore

$$L_{11} > 0, \quad L_{22} > 0, \quad (L_{12} + L_{21})^2 < 4L_{11}L_{22}. \quad (5)$$

Hence the "proper" phenomenological coefficients (L_{11}, L_{22}) are positive. On the other hand, the interference coefficients (L_{12}, L_{21}) may be positive or negative, their magnitude being limited only by (5). This is in agreement with the observation that coefficients, like thermal conductivity or electrical conductivity, are always positive, while for example, the thermodiffusion coefficient has no definite sign.

The domain of validity of the linear phenomenological relations can of course only be

discussed when a statistical or kinetic theory is available. This is the case of dilute gases (see **Boltzmann's equation; kinetic theory of gases**). It can be shown that for transport processes the linear laws are valid when the variation of the properties of the gas are small over a **mean free path**.

On the other hand for chemical reactions the linear laws are valid when the **chemical affinity** is small in respect to the thermal energy RT .

PHON. The unit of **loudness level** of a sound, defined as numerically equal to the **sound pressure level in decibels**, relative to 0.0002 microbar, of a simple tone of frequency 1000 cycles per second which is judged by the listeners to be equivalent in loudness.

PHONON. (1) A sound quantum. (2) A quantum of the lattice vibrations of a crystal. The theory of lattice vibrations is in many respects similar to that of the electromagnetic field, and it is convenient to introduce "particles," defined by **wave-packets**, moving through the medium with the **group velocity**, and capable of being annihilated, created, scattered, etc., by interaction with electrons and lattice imperfections. Like a **photon**, a phonon is quantized to have the energy $h\nu$, where ν is its vibrational frequency, and h is Planck's constant. The concept of phonons allows a great formal simplification in theories of thermal and electrical conduction in solids. (See also **phonon, longitudinal, or transverse**, and subsequent entries.)

PHONON GAS, STATISTICAL TREATMENT. The **phonons** as quantized harmonic oscillator vibrations obey Bose-Einstein statistics, and can be treated by standard methods. The treatment is similar to that of a **photon gas**, the only difference being that while all photons move with the same velocity and are always transverse, phonons can be either longitudinal or transverse and their velocity depends both on their polarization and on their wavelength.

PHONON, LONGITUDINAL, OR TRANSVERSE. The **acoustic modes** of crystals fall into three distinct branches, which may be identified with the three types of **polarization** of the vibration, i.e., parallel to the **wave vector** (longitudinal), or in one of the two perpendicular directions normal to the wave

vector (transverse). The actual polarizations are never exactly thus, except along certain symmetry directions in the crystal, but usually this assumption is a reasonable approximation. The distinction between the different types is not negligible in **phonon-phonon** and **phonon-electron** interactions.

PHONON MEAN FREE PATH. A quantity Λ defined from the **thermal conductivity** K of a dielectric solid, from its volume heat capacity C , and from the average velocity of sound v in the solid, by the formula

$$K = \frac{1}{3}Cv\Lambda.$$

Λ does indeed represent roughly the mean free path of a phonon as it carries thermal energy through the crystal, but it is not easy to assign to it an exact microscopic significance in terms of the various processes by which a phonon is scattered. However, in glasses, Λ has about the dimensions of the unit of structure (e.g., the SiO_2 tetrahedron), while in good crystals it varies as $e^{-\theta/2T}$ at low temperatures, according to the theory of Umklapp processes. In the region of boundary scattering, Λ is about the diameter of the specimen.

PHONON-PHONON INTERACTION. The **thermal vibrations** of a **crystal lattice** are only independent of one another at first approximation. When the **anharmonic terms** are taken into account, it is found that there are processes by which the modes interact, for example, by the mutual interference of two phonons to make a third. These "collisions," especially the Umklapp processes, are responsible for the thermal resistance of a dielectric solid at high temperatures.

PHONON SPECTRUM. See **spectrum, phonon**.

PHOTOCURRENT COEFFICIENT. The change in photocurrent generated by a phototube per unit change in radiant flux producing it.

PHOTOELASTICITY. The study of the distribution of stress in an elastic body by means of the birefringence induced in a model of optically sensitive material. Two and three dimensional problems can be treated in a complete manner not possible by any other technique.

PHOTOELECTRIC CONSTANT. A quantity equal to h/e where h is the **Planck constant**, and e , the **electronic charge**, and which multiplied by the frequency of any radiation exciting photoemission gives the potential difference corresponding to the quantum energy absorbed by the escaping photoelectron.

$$\begin{aligned} h/e &= 4.1349 \times 10^{-7} \text{ erg} \cdot \text{sec} \cdot \text{emu}^{-1} \\ &= 1.3793 \times 10^{-17} \text{ erg} \cdot \text{sec} \cdot \text{esu}^{-1}. \end{aligned}$$

PHOTOELECTRIC EFFECT. In its earlier use, this term covered broadly all changes in the electrical characteristics of substances due to radiation, generally in the form of light. Nowadays, however, the term refers almost exclusively to the emission of bound electrons following absorption of photons. Their kinetic energy is given by the **Einstein photoelectric equation**, $E = h\nu - \epsilon$. The cross section for this process rises extremely rapidly, as a function of $h\nu$, from the threshold $h\nu_0 = \epsilon$ to a peak; after which it falls gradually until the threshold corresponding to another bound state appears. It is the mechanism mainly responsible for the absorption of X-rays and low energy γ -rays.

PHOTOELECTRIC EQUATION, EINSTEIN. See **Einstein photoelectric equation**.

PHOTOELECTRIC SENSITIVITY. The ratio of photoelectric emission current to the radiant flux density which caused the emission.

PHOTOELECTRIC THRESHOLD. The quantum of energy $E_0 = h\nu_0$, which is just enough to release an electron from a given system in the photoelectric effect. If the incident photon has greater energy, the excess will appear as kinetic energy of the ejected electron. The frequency ν_0 is known as the threshold frequency, and the corresponding wavelength λ_0 , as the threshold wavelength. Thus, $E_0 = h\nu_0 = hc/\lambda_0$.

PHOTOELECTRIC WORK FUNCTION. The energy required to secure the release of electrons from a given surface by photoelectric emission.

PHOTOELECTRIC YIELD. See **photoelectric sensitivity**.

PHOTON. The quantum of electromagnetic radiation, of energy $h\nu$, where ν is the fre-

quency of the radiation and h is **Planck's constant**, and momentum $hk/2\pi$, where \mathbf{k} is the propagation vector (or wave vector). For many purposes, the photon may be regarded as a particle of spin 1 (in units of $h/2\pi$), although it can exhibit only two states of polarization. It has zero rest mass.

PHOTON GAS, STATISTICAL TREATMENT. See **Planck's radiation law** and **Bose-Einstein statistics applied to photons**.

PHYSICAL COMPONENTS OF STRAIN (IN AN ORTHOGONAL, CURVILINEAR COORDINATE SYSTEM). The components of **strain** in a local rectangular Cartesian coordinate system, the axes of which are tangential to the coordinate lines of the curvilinear coordinate system at the point considered. The *normal* and *tangential* components of strain in the curvilinear coordinate system are the normal and tangential components respectively in the local rectangular Cartesian coordinate system. Also referred to, more loosely, as *components of strain*, or *strain components*, or *strain* in the orthogonal, curvilinear coordinate system.

PHYSICAL COMPONENTS OF STRESS (IN AN ORTHOGONAL, CURVILINEAR COORDINATE SYSTEM). The components of **stress** in a local rectangular Cartesian coordinate system the axes of which are tangential to the coordinate lines of the curvilinear coordinate system at the point considered. The *normal* and *tangential* components of stress in the curvilinear coordinate system are the normal and tangential components respectively in the local rectangular Cartesian coordinate system. Also referred to, more loosely, as *components of stress*, or *stress components* or *stress*, in the orthogonal, curvilinear coordinate system.

PHYSICAL INTERACTIONS. See **chemical interactions**.

PI. Consider the differential equation

$$\frac{d^2x}{dt^2} = -x,$$

with initial conditions

$$x(0) = 1, \left. \frac{dx}{dt} \right|_0 = 0,$$

which describes a simple oscillating system in one of its extreme positions. The time taken for it to reach its opposite extreme position (that is, the smallest positive value of t for which $\frac{dx}{dt} = 0$) is denoted by the Greek letter π , called pi and approximately equal to 3.14159. It is easy to show that π is also equal to the ratio of the circumference of a circle to its diameter.

PICARD METHOD. A numerical method of successive approximation or iteration for solving differential equations. If the given equation is $y' = f(x,y)$ subject to the condition that $y = y_0$ when $x = x_0$, the solution may be written in the form of an integral equation

$$y = y_0 + \int_{x_0}^x f(x,y)dx.$$

An approximate solution is

$$y_1 = y_0 + \int_{x_0}^x f(x,y_0)dx,$$

and a sequence of approximations can be formed:

$$\begin{aligned} y_2 &= y_0 + \int_{x_0}^x f(x,y_1)dx; \\ &\dots \dots \dots \\ y_n &= y_0 + \int_{x_0}^x f(x,y_{n-1})dx. \end{aligned}$$

Under quite general circumstances this sequence can be shown to converge to the solution. Unless f is a polynomial it is not generally feasible to effect the quadratures analytically, but most methods of numerical solution of **differential equations** take their departure from this and integrate numerically. Moreover, y and f can be interpreted as vectors and the methods apply equally to systems of differential equations.

PICO-. Prefix meaning 10^{-12} .

PIEZOELECTRIC CONSTANTS. Let p_i, u_i ($i = 1$ to 6) be, respectively, the stress and strain components. Let E_α, P_α ($\alpha = 1$ to 3) be the components of electric field and electric polarization respectively. The piezoelectric constants or stress coefficients $e_{\alpha i}$ and the piezoelectric moduli or strain coefficients $d_{\alpha i}$ are then defined by

$$\begin{aligned} e_{\alpha i} &= \left(\frac{\partial P_\alpha}{\partial u_i} \right)_E = - \left(\frac{\partial p_i}{\partial E_\alpha} \right)_u \\ d_{\alpha i} &= \left(\frac{\partial P_\alpha}{\partial p_i} \right)_E = \left(\frac{\partial u_i}{\partial E_\alpha} \right)_p \end{aligned}$$

If c_{ij} and s_{ij} are the elastic constants and moduli respectively for $E = 0$, then

$$d_{\alpha i} = \sum_{j=1}^6 s_{ji}e_{\alpha j}$$

and

$$e_{\alpha j} = \sum_{i=1}^6 c_{ij}d_{\alpha i}.$$

PIEZOELECTRIC DRIVING SYSTEM. See **driving system, piezoelectric.**

PIEZOELECTRIC EFFECT. The interaction between electrical and mechanical stress-strain variables in a system. (See **piezoelectric constants.**)

PIEZOELECTRIC GENERATING SYSTEM. See **generating system, piezoelectric.**

PIEZOTROPY. The property of a fluid in which processes are characterized by a functional dependence of the thermodynamic functions of state:

$$\frac{d\rho}{dt} = b \frac{dp}{dt},$$

where ρ is the density, p the pressure, and b a function of the thermodynamic variables, called the *coefficient of piezotropy*. With the **equation of state** and the **first law of thermodynamics** this relation determines a path for each parcel on a **thermodynamic diagram**, given the initial state.

The simplest forms of piezotropy are those in which all changes in the fluid are incompressible ($b = 0$); adiabatic ($b = 1/C^2 = c_v/c_p RT$), where C is the local speed of sound, c_v and c_p the specific heats at constant volume and pressure, respectively, R the gas constant, and T the Kelvin temperature; or isothermal ($b = 1/RT$). These processes are also **polytropic**.

In dynamic work, except in some studies devoted to diabatic phenomena, it has usually been assumed that the atmosphere is piezotropic, with adiabatic processes.

(Cf. **barotropy, autobarotropy.**)

PINCUSHION DISTORTION. See geometric distortion.

PIPE, ENERGY LENGTH OF. See energy length for pipe or channel.

PIPE, LAMINAR FLOW IN. For laminar flow in a straight pipe of uniform circular section, at a large distance from the entry, the velocity distribution across the pipe is given by

$$u = \frac{p'}{4\mu} (a^2 - r^2) \quad (1)$$

where u is the velocity at a radius r , p' is the longitudinal pressure gradient, μ is the viscosity and a is the internal radius of the pipe. The volume flowing per unit time is

$$Q = \frac{\pi a^4 p'}{8\mu}, \quad (2)$$

and the mean velocity $\left(\frac{Q}{\pi a^2}\right)$ is half the velocity at the center.

If τ_o is the shear stress at the wall, the skin friction coefficient is

$$C_f = \frac{\tau_o}{\frac{1}{2}\rho V^2} = \frac{16}{R}, \quad (3)$$

where V is the mean velocity and R is the Reynolds number based on this velocity and the internal diameter of the pipe. This type of flow is often known as *Poiseuille flow*.

PIPE, ROUGHNESS HEIGHT OF. See roughness, effect on skin friction.

PIPE, TRANSITION TO TURBULENT FLOW IN. See transition to turbulent flow in a pipe or channel.

PIPE, TURBULENT FLOW IN. Most of the essential features of turbulent flow in a pipe are the same as for a **turbulent boundary layer**. An important difference is that the flow is turbulent right across the pipe (except near the entry) so that there is no wavy "edge" to the turbulent region and there can be no fixed points at which turbulence only occurs intermittently. A consequence of this is that the logarithmic velocity profile applies almost entirely across a pipe, although it does not apply in the outer part of a boundary layer.

For fully developed flow in a pipe of constant section there is a simple relation between

the shear stress at the wall and the longitudinal pressure gradient. For a pipe of circular section this relation is

$$\frac{dp}{dx} = 4 \frac{\tau_o}{d},$$

where d is the diameter of the pipe and τ_o is the shear stress at the wall.

PITCH. (1) That attribute of auditory sensation in terms of which sounds may be ordered on a scale extending from low to high, such as a musical scale. Pitch depends primarily upon the frequency of the sound stimulus, but it also depends upon the sound pressure and wave form of the stimulus. The pitch of a sound may be described by the frequency of that simple tone, having a specified **sound pressure** or **loudness level**, which seems to the average normal ear to produce the same pitch. (2) See **pitch of screw**. (3) In a turbine or turbocompressor stage, or more generally in a cascade of blades, the distances between identical points on two adjoining blades, measured on an agreed diameter (pitch diameter), usually the mean diameter. The distance c in the figure is known as the *chord*, and the pitch chord ratio s/c constitutes an important similarity parameter of the **cascade**. (See figure.)

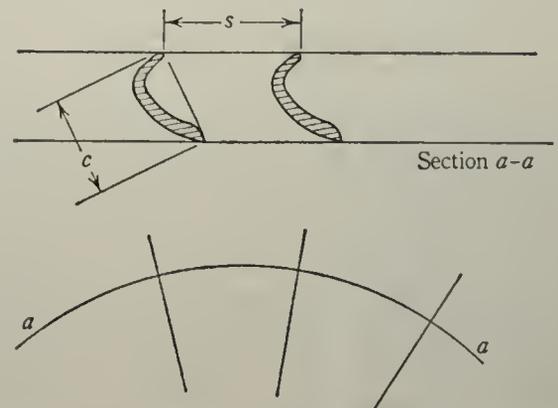


Diagram showing pitch and chord of a cascade of blades.

PITCH CHORD RATIO. See **pitch**.

PITCH DIFFERENCE, MINIMUM PERCEPTIBLE. The smallest difference in **pitch** that can be discerned by the average single ear.

PITCHING MOMENT COEFFICIENT. For a complete aircraft, the pitching moment M

is the moment, due to all the aerodynamic forces, about an axis passing through the center of gravity and normal to the plane of symmetry. (The plane of symmetry is vertical in normal straight flight.) The pitching moment is positive when it tends to increase the angle of incidence. For steady flight the elevator must be adjusted to make the pitching moment zero.

The pitching moment coefficient is

$$C_M = \frac{M}{\frac{1}{2}\rho V^2 S \bar{c}},$$

where S is the gross wing area and \bar{c} is a mean wing chord, usually the aerodynamic mean chord.

For an isolated wing, the pitching moment coefficient is defined in the same way, except that the position of the axis must be specified. The axis is still normal to the plane of symmetry, i.e., the plane containing the velocity and lift vectors, and is often chosen to pass through the mean **quarter-chord point** of the wing.

For a two-dimensional airfoil, if M' is the pitching moment per unit span (usually about an axis through the quarter-chord point), the pitching moment coefficient is

$$C_M = \frac{M'}{\frac{1}{2}\rho V^2 c^2},$$

where c is the chord.

PITCH LEVEL. The pitch level P of a sound is given by

$$P = A \log_2 f,$$

where P is in octaves, f in kilocycles and A is a constant.

PITCH OF SCREW. Axial distance between adjacent turns of a single thread on a screw.

PI-THEOREM. A principal theorem in **dimensional analysis** that may be stated as follows: Suppose we have a dimensionally homogeneous relation $G(\alpha, \beta, \gamma, \dots) = 0$ in n dimensional variables, $\alpha, \beta, \gamma, \dots$, valid for a certain system of m **fundamental units**. The equation may then be put in the form $F(\pi_1, \pi_2, \dots) = 0$, where the π 's are the $n - m$ independent products of the variables $\alpha, \beta, \gamma, \dots$, which are dimensionless in the fundamental units.

PLACZEK FUNCTION. In the theory of the slowing-down of neutrons in large, homogeneous systems, let $\Phi(E, E_s)$ be the neutron flux at E due to a monoenergetic source of unit energy at E_s . Then, the Placzek function, P_A , which depends upon the atomic mass of the scattering material is defined through

$$\Phi(E_s, E) = \frac{1}{E_s \Sigma(E)} P_A(E/E_s).$$

PLANAR GRAPH. See graph, planar.

PLANAR POINT (ON A SURFACE). A point at which the principal curvatures (see **curvature of surface, center of**) of the surface are both zero.

PLANCK CONSTANT. A universal constant, h , that has the value, by least squares adjusted output values, of $6.62377 \pm 0.00018 \times 10^{-27}$ erg second. It is the factor of proportionality relating the energy of a photon to its frequency, i.e., $E = h\nu$. (See **Planck law**.)

PLANCK FORMULATION OF THE SECOND LAW OF THERMODYNAMICS. See **Planck-Kelvin formulation of the second law of thermodynamics**.

PLANCK FUNCTION. See **thermodynamics, characteristic functions of**.

PLANCKIAN COLOR. The color or wavelength-intensity distribution of the light emitted by a **black body** at a given temperature.

PLANCKIAN LOCUS. The locus of **chromaticities** of Planckian (**black body**) radiators having various temperatures. (See figure in definition of **chromaticity diagram**.)

PLANCK-KELVIN FORMULATION OF THE SECOND LAW OF THERMODYNAMICS. M. Planck and W. Thomson derived the full contents of the second law of thermodynamics from two slightly different formulations which can be unified into the following statement of experimental fact. It is impossible to construct an engine that would extract heat from a given source and transform it into mechanical energy, without bringing about some additional changes in the bodies involved. (For a comparison with other statements, see **thermodynamics, second law of**.)

PLANCK LAW. The fundamental law of the quantum theory, expressing the essential concept that energy transfers associated with radiations such as light or x-rays are made up of definite quanta or increments of energy proportional to the frequency of the corresponding radiation. This proportionality is usually expressed by the quantum formula $E = h\nu$, in which E is the value of the quantum in units of energy and ν is the frequency of the radiation.

h , the constant of proportionality, is known as the elementary quantum of action or more commonly, the **Planck constant**. Since E is energy and ν is frequency, h has the dimensions of energy \times time, or **action**.

PLANCK RADIATION FORMULA. A relationship given by the equation

$$\rho(\nu)d\nu = \frac{8\pi h\nu^3 d\nu}{c^3(e^{\beta h\nu} - 1)},$$

where $\rho(\nu)d\nu$ is the energy of the radiation field per unit volume in the frequency range $\nu, \nu + d\nu$, h is Planck's constant, c the velocity of light, $\beta = 1/kT$ with k , Boltzmann's constant and T , the absolute temperature. This formula describes the spectral distribution of the radiation from a complete radiator or "black body."

PLANE MOTION. If every element of a rigid body moves parallel to a fixed plane the body is said to have plane motion. For a general system of particles, the motion of all particles must be parallel to the plane of motion and all particles on lines perpendicular to the plane of motion must have the same displacement.

PLANE OF CURVATURE. See **curvature, plane of**.

PLANE, OSCULATING. See **curvature, plane of; principle planes**.

PLANE-PERFECT OPTICAL SYSTEMS. See **perfect optical systems**.

PLANE POLARIZATION. If the vector field representing the vibrations in a beam of radiation is linearly dependent then the radiation is plane polarized. The **polarization eye** is then a line segment.

PLANE SOUND WAVES. See **sound waves, plane**.

PLANES, PRINCIPAL. See **principal planes**.

PLANE STRAIN. Consider a right cylinder, the cross section of which may or may not be circular, with its generators parallel to the z -axis of a rectangular Cartesian coordinate system x, y, z . The cylinder undergoes a deformation in which the displacement components u, v, w , parallel to the axes x, y, z , respectively, of a generic point, initially at x, y, z , are of the form

$$u = u(x, y), \quad v = v(x, y), \quad w = \text{constant}.$$

Such a deformation is called a plane strain.

Within the framework of **classical elasticity theory**, it can be shown that the two normal components of stress σ_{xx} and σ_{yy} in the x and y directions, respectively, and the shearing component σ_{xy} are expressible in terms of a bi-harmonic function of x and y , $\phi(x, y)$, which is called the *Airy stress function*, by the formulae

$$\sigma_{xx} = \frac{\partial^2 \phi}{\partial y^2}, \quad \sigma_{yy} = \frac{\partial^2 \phi}{\partial x^2}, \quad \sigma_{xy} = -\frac{\partial^2 \phi}{\partial x \partial y}.$$

PLANE STRESS. A body is in a state of plane stress parallel to the xy -plane of a rectangular Cartesian coordinate system x, y, z when the stress vector on planes normal to the z -axis vanishes.

PLANE STRESS, GENERALIZED. See **generalized plane stress**.

PLANETARY BOUNDARY LAYER. (Also called friction layer, atmospheric boundary layer.) That layer of the atmosphere from the earth's surface to the **geostrophic wind level** including, therefore, the **surface boundary layer** and the **Ekman layer**. Above this layer lies the **free atmosphere**.

PLANETARY MOTION. Motion in a plane under the action of a central force. (See **force, central**.)

PLANETARY MOTIONS. (For terms, see **orbit, astronomical elements of**.) In 1609 Kepler published the first two of his now famous laws of planetary motion, and the third followed in 1618. They may be expressed as follows:

(1) The orbit of every member of the solar system is a conic section with the sun at one focus.

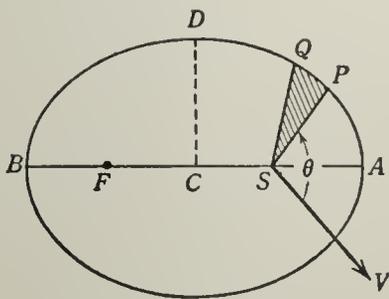
(2) The radius vector, i.e., the line from the

sun to the object, sweeps out equal areas in equal times.

(3) The squares of the periods of any two objects revolving about the sun are in the same ratio as the cubes of their mean distances.

In 1687 Newton published the *Principia* in which he enunciated his famous laws of motion and, in particular for our purposes, the fundamental law of gravitation: Every particle in the universe attracts every other with a force that is proportional to the product of their masses, and inversely proportional to the square of the distance between them.

First Law of Kepler. The figure shows an ellipse with foci at S (the sun) and F . The



direction of the vernal equinox is SV . P and Q represent two positions of an object moving on the ellipse. When the object is at A it is said to be at perihelion and at B it is at aphelion. CA is the semi-major axis of the ellipse, a , and CD is the semi-minor axis, b . Calling the eccentricity of the ellipse, e , then

$$b^2 = a^2(1 - e^2). \quad (1)$$

The perihelion distance $SA = a(1 - e)$, and the aphelion distance $SB = a(1 + e)$. P is any position of the planet in its orbit and SP is the radius vector, r , sometimes referred to as the heliocentric distance. Assuming that SV is in the orbit plane, the position of the object in the orbit is defined by r and the angle θ , measured from the direction of the vernal equinox to the radius vector in the direction of motion of the object in the orbit. Now let ω be the value of θ when the planet is at perihelion, i.e., at A . Then the angle $ASP = \theta - \omega$. The equation of the ellipse is known to be

$$r = \frac{p}{1 - e \cos(\theta - \omega)} \quad (2)$$

where

$$p = b^2/a = a(1 - e^2). \quad (3)$$

The time required for the planet to complete a circuit of its orbit is known as the period, usually designated by T . For example, for the earth $T = 365.24 \dots$ days.

Second Law of Kepler. Differentiating the polar equation of the ellipse there results

$$r^2 \frac{d\theta}{dt} = \frac{h}{2}$$

in which r and θ have already been defined and h is a constant. Now the whole area of the ellipse is πab and the total time is T , the complete period from perihelion back to perihelion. Using (1) above,

$$\frac{2\pi ab}{T} = h = \frac{2\pi a^2 \sqrt{1 - e^2}}{T}.$$

In the time T , the radius vector sweeps out 360° or 2π , hence the average rate would be $n = \frac{2\pi}{T}$, where n is the mean angular motion of the planet. Finally there results

$$na^2 \sqrt{1 - e^2} = h. \quad (4)$$

Third Law of Kepler. This law may be expressed as

$$\frac{a^3}{T^2} = \frac{a_1^3}{T_1^2} \quad \text{or} \quad n^2 a^3 = n_1^2 a_1^2.$$

Using this expression for the third law,

$$\frac{a_1}{a} = \left(\frac{T_1}{T}\right)^{2/3},$$

and if a and T refer to the earth's orbit about the sun, then $a_1 = (T)^{2/3}$ astronomical units.

Newton Law of Gravitation. (C.f. **gravitational constant**.) Newton's law is

$$F = G \frac{mm_1}{r^2}.$$

Its application to the motions of the planets about the sun leads to the three laws of Kepler. Let M and m represent the masses of the sun and a planet and let μ be defined by $\mu = G(M + m)$ where G is the gravitational constant. It can be shown that the constant, h , defined above, is given by $h^2 = \mu p = \mu a(1 - e^2)$. We also have from above the relation $h^2 = n^2 a^4(1 - e^2)$. From these two, there results

$$n^2 a^3 = \mu \equiv G(M + m).$$

For another planet of mass m_1 , we have

$$\begin{aligned} \mu_1 &= G(M + m_1) \\ \frac{n^2 a^3}{n_1^2 a_1^3} &= \frac{M + m}{M + m_1} \end{aligned}$$

Since

$$n = \frac{2\pi}{T}$$

Then

$$\frac{a^3}{a_1^3} = \frac{M + m}{M + m_1} \frac{T^2}{T_1^2}$$

In the case of the planets, m and m_1 are so small in comparison with M , the mass of the sun, that $\frac{M + m}{M + m_1}$ is very close to unity and Kepler's third law results.

However, the mass of a planet may be determined in terms of the mass of the sun provided the planet has one or more satellites. If m , a , and T refer to the earth, we have $G(M + m) = 4\pi^2 \frac{a_1^3}{T_1^2}$. The motion of a satellite about a planet will follow the same general principles of orbital motion as those of a planet about the sun, and we have

$$\frac{m_1 + m'}{M + m_1} = \left(\frac{a_1}{a}\right)^3 \left(\frac{T}{T_1}\right)^2$$

in which m_1 is the mass of the planet and m' is the mass of the satellite. The mass m' of the satellite is small in comparison with m_1 , the mass of the planet, and m_1 , in turn, is small in comparison with the mass of the sun, M . Hence we have

$$\frac{m_1}{M} = \left(\frac{a_1}{a}\right)^3 \left(\frac{T}{T_1}\right)^2$$

In the above we have assumed that, in each case, there were only the few objects involved in the universe. As a matter of fact there are gravitational attractions from all members of the universe. Some of these forces, while much smaller than those we have considered, do produce appreciable effects known as **perturbations**.

Collecting the principal formulae for elliptic motion,

$$\begin{aligned} r &= \frac{a(1 - e^2)}{1 + e \cos v} \\ n^2 a^3 &= \mu \equiv G(M + m) \\ h^2 &= \mu a(1 - e^2) \end{aligned}$$

$$E - e \sin E = M \equiv n(t - \tau)$$

$$r = a(1 - e \cos E)$$

$$\tan \frac{v}{2} = \sqrt{\frac{1 + e}{1 - e}} \tan \frac{E}{2}$$

In the above a , e and τ are the so-called elements of the elliptic orbit, τ being the time of passing perihelion.

PLANIMETER. An instrument for measuring the area enclosed by a curved boundary, hence an **analog computer** for the purpose of performing **quadratures**.

PLASMA. (1) An assembly of ions, electrons, neutral atoms and molecules in which the motion of the particles is dominated by electromagnetic interactions. This condition occurs when the macroscopic electrostatic shielding distance (**Debye length**) is small compared to the dimensions of the plasma. Because of the large electrostatic potentials which would result from an inhomogeneous distribution of unlike charges, a plasma is effectively neutral. Thus there are equal numbers of positive and negative charges in every macroscopic volume of a plasma. Also, because a plasma is a conductor, it interacts with electromagnetic fields. The study of these interactions is called *hydro-magnetics* or *magnetohydrodynamics*. (2) A collection of electrons and ions, usually at a high enough temperature so that the ionization level is above 5% and at densities such that the Debye shielding distance is much smaller than the macroscopic dimensions of the system.

PLASTIC ANALYSIS. An analysis based upon plastic stress-strain relations. The term is often used to denote the computation of the load-carrying capacity of a given structure under the idealization of perfectly plastic behavior. (See **meehanism method of analysis of frames; limit theorems**.)

PLASTIC BENDING. See **moment, limit or plastic**.

PLASTIC DESIGN. A design based upon plastic stress-strain relations. Most often the term implies the assumption of perfect plasticity. (See **limit theorems; minimum weight design**.)

PLASTIC FLOW. A term of many different meanings but which, in all cases, denotes an irreversible deformation process. In the mathematical theory of plasticity, plastic flow or plastic deformation is time-independent. The term also is used to describe the creep or time-dependent deformation of viscous materials and of concrete or clay masses under constant load.

PLASTIC HYSTERESIS. See *hysteresis, plastic*.

PLASTICITY. Irreversible deformation of a material subjected to a **yield stress** which is essentially independent of the rate of strain. (See *ductility; limit analysis*.)

PLASTICITY, ALTERNATING. The successive plastic deformation of opposite sign produced by a repeated load cycle of sufficient range (see *shakedown*). Low cycle fatigue is associated with alternating plasticity.

PLASTICITY, DEFORMATION THEORY OF. See *deformation theory of plasticity*.

PLASTICITY, FLOW THEORY OF. See *flow theory of plasticity*.

PLASTICITY, INCREMENTAL THEORY OF. See *incremental theory of plasticity*.

PLASTICITY, LIMIT THEOREMS OF. See *limit theorems*.

PLASTICITY, UPPER BOUND THEOREM OF. See *limit theorems*.

PLASTIC LIMIT ANALYSIS AND DESIGN. See *limit analysis and design, plastic*.

PLASTIC LIMIT LOAD. See *limit load*.

PLASTIC MODULUS. A ratio of stress to strain or increase of stress to increase of strain in the plastic range (see *modulus, secant* and *modulus, tangent*). Also a pseudo modulus for bending based upon the formulas for elastic bending (see *modulus, reduced*).

PLASTIC MOMENT. See *moment, limit or plastic*.

PLASTIC POTENTIAL. See *flow rule*.

PLASTIC-RIGID MATERIAL. A material which is considered to be rigid for stresses below the **yield limit**, and flows plastically when subjected to the yield limit stress.

PLASTIC WAVES. See *waves, plastic*.

PLATE. (1) A body bounded by a right cylindrical (or prismatic) surface and by two parallel planes which are normal to the generators of the cylindrical surface, the distance between these parallel planes being generally small compared with the characteristic dimensions of the body parallel to these planes.

(2) A transverse load-carrying structure or element whose middle surface is plane and whose thickness h is small compared with its overall dimensions in the plane of the middle surface (see *sheet*). *Thin-plate theory*, just as ordinary beam theory, considers curvatures due to moments and ignores transverse shear deformation. Strains are linear with distance from the middle surface. The differential equation governing small elastic deflections w is

$$\frac{\partial^4 w}{\partial x^4} + 2 \frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = \frac{q}{D}$$

where q is the transverse pressure and $D = \frac{Eh^3}{12(1-\nu^2)}$, E is Young's modulus and ν is Poisson's ratio.

Moderately thick-plate theory developed by Reissner includes a first order correction for shear deformation and permits independent satisfaction of the boundary conditions on shear force and twisting moment. Vibrations of thin and thick plates have been studied extensively by Mindlin and co-workers.

Deflections of the order of the thickness of the plate may induce very large **membrane** stresses in the elastic or plastic range.

PLATES, CIRCULAR. Circular plates, symmetrically loaded, provide simple examples of plate bending of practical importance. Small elastic deflections w are governed by the ordinary differential equation

$$\frac{1}{r} \frac{d}{dr} \left\{ r \frac{d}{dr} \left[\frac{1}{r} \frac{d}{dr} \left(r \frac{dw}{dr} \right) \right] \right\} = \frac{q}{D}$$

which may be integrated directly given the variation of the load intensity q and the flexural rigidity D with the distance r from the center of the plate.

For uniform load and constant D

$$w = \frac{q}{64D} (a^2 - r^2)^2 \text{ for a clamped plate}$$

and

$$w = \frac{q}{64D} (a^2 - r^2) \left(\frac{5 + \nu}{1 + \nu} a^2 - r^2 \right)$$

for a simply supported plate

where a is the radius of the plate and ν is **Poisson's ratio**.

The moment at the center of the plate is $\frac{(1 + \nu)}{16} qa^2$ for the clamped plate and $\frac{3 + \nu}{16} qa^2$ for the simply supported plate. The clamping moment for the clamped plate is $\frac{qa^2}{8}$.

Based on the Tresca yield criterion, the **limit load** q for a uniformly loaded clamped circular plate is $11.3M_o/a^2$, and for a simply supported plate is $6M_o/a^2$, where $M_o = \sigma_o h^2/4$, σ_o is the yield stress in simple tension or compression.

PLATES, RECTANGULAR. The great practical importance of rectangular plates in ships and buildings as well as other structures has led to solutions of a wide variety of problems. Large and small deflections in both the elastic and plastic range have been investigated analytically and experimentally. (See **rupture line theory**.)

PLUG FLOW. When a **Bingham material** flows down a tube, shear may occur only in an annulus with a central core remaining rigid, because the stress in the core is below the yield limit. This situation is known as plug flow.

PLUME. The region occupied by effluent, e.g., the region made visible by smoke from a chimney. Sometimes used to refer to *buoyant* effluents as distinct from a *jet* of effluent having the same density as the environment.

P_N APPROXIMATION. In the **spherical harmonics** method of **neutron transport theory**, the system of coupled equations describing neutron migration is often made finite by neglecting moments of order greater than N . The approximate solution so obtained is referred to as the " P_N approximation" to the correct solution.

POHLHAUSEN METHOD. See **laminar boundary layer equations, solution of**.

POINCARÉ INVARIANT. In geometric optics, the integral

$$I = \int_C n(pdx + qdy + rdz)$$

is an invariant for all closed curves C around a given tube of rays where n is the index of refraction and p, q, r the direction cosines of the tube ray through (x, y, z) . In particular if the tube is contained in a **normal congruence** of rays, then $I = 0$.

POINSON ELLIPSOID. See **ellipsoid, momental**.

POINT. See **vertex**.

POINT BISERIAL COEFFICIENT. See **biserial correlation**.

POINT BRILLANCE. A quantity involved in the visual observation of a source of light when viewed directly from such a distance that the apparent diameter is very small. The point brilliance is measured by the illumination produced by the source on a plane at the observer's eye normal to the incident light.

POINT CENTERS OF REPULSION. The potential function

$$\phi(r) = dr^{-\delta} \quad (1)$$

corresponds to a repulsion between molecules, where δ is called the *index of repulsion* and has for most molecules a value between 9 and 15.

When $\delta = 4$ the molecules are known as *Maxwellian* molecules. This potential gives a good representation of the repulsive forces, but does not take account of the attractive part of the intermolecular forces. (See also **intermolecular forces; Born repulsion**.)

POINT CHARACTERISTIC FUNCTION. See **Hamilton's characteristic**.

POINT GROUP. One of the 32 different **symmetry classes** to which a crystal may belong, in virtue of the occurrence of different elements of symmetry passing through a single point, together with the necessity of obeying the **Häuy law** of rational indices.

POINT KERNEL. See **kernel, point**.

POINT SOURCE. (1) A single point emitting **radiation**. No finite source of radiation is a true point, but any source viewed from a distance sufficiently great compared to the linear size of the source may be considered as a point source. In the distance range in which meas-

urements of the radiation from a source show that it obeys the **inverse square law** (no absorption), then to the accuracy of the measurements, the source may be considered as a point source. (See **uniform source**.) (2) A single point emitting particles. The analogous concepts of *line source* and *plane source* are sometimes useful. (3) A single point in a mass flow emitting mass, i.e., a point at which the **equation of continuity** fails.

POISE. Unit of viscosity in the **egs system of units**, named after J. Poiseuille. 1 poise = 1 g/cm sec.

POISEUILLE FLOW. See **pipe, laminar flow in**.

POISEUILLE FLOW, PLANE. See **channel, two-dimensional, laminar flow in**.

POISEUILLE-HAGEN LAW. The relation between velocity and pressure gradient in **Poiseuille flow**.

POISSON BRACKET. If u and v are two functions of the generalized coordinates q and momenta p , the expression

$$[u, v]_{q,p} = \sum_{i=1}^N \frac{\partial u}{\partial q_i} \frac{\partial v}{\partial p_i} - \frac{\partial v}{\partial q_i} \frac{\partial u}{\partial p_i}$$

is called the Poisson bracket of the two functions. N is an invariant under a canonical transformation.

If $\{u, v\}_{q,p}$ is the **Lagrange bracket** of the functions u and v , then

$$\{u, v\}_{q,p} \cdot [u, v]_{q,p} = 1.$$

POISSON CONSTANT. The ratio κ of the **gas constant** R to the **specific heat** at constant pressure c_p .

For dry air this is

$$\kappa = 0.286.$$

For moist air this must be multiplied by $(1 - 0.29q)$, where q is the specific humidity. The Poisson constant enters into meteorology in the so-called Poisson equation for a dry-adiabatic process:

$$\frac{T}{T_0} = \left(\frac{p}{p_0}\right)^\kappa,$$

where T is temperature, p , pressure, and (T_0, p_0) the initial state.

POISSON DISTRIBUTION. The Poisson distribution is a discrete distribution with one parameter whose frequency function is given by $f(r) = e^{-\mu} \mu^r / r! (r = 0, 1, 2, \dots \infty)$. The mean and variance are both equal to μ , and are best estimated from the sample mean. For large μ , the distribution approaches normality. The **binomial distribution** $(q + p)^n$ approaches the Poisson distribution as a limiting form when $n \rightarrow \infty$ and $p \rightarrow 0$ in such a way that $np = \mu$ remains constant. If events occur in such a way that the probability of an occurrence in a small interval of space or time dt is $\lambda dt + O(dt^2)$, independently of other intervals, the numbers of events in equal finite intervals follow the Poisson distribution. For this reason such a series of events is known as a Poisson process.

POISSON EQUATION. The **partial differential equation**

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = f(x, y, z).$$

It occurs in (1) electrostatics, where ϕ is potential due to a charge distribution of volume density ρ and $f = -4\pi\rho$; (2) thermal conductivity, where ϕ is the temperature in a homogeneous medium of thermal conductivity k and in which $A(x, y, z)$ calories of heat are generated per unit of volume and time, so that $f(x, y, z) = -A/k$. When no heat is generated in the medium, $A = 0$ or when there is no charge, $\rho = 0$, hence Laplace's equation results in each special case. (See **Laplace equation**.)

POISSON INTEGRAL. The solution of the **Laplace equation** taking assigned continuous values $f(\phi)$ on the circumference of the unit-circle in the x, y plane is given by the Poisson integral

$$u(r, \phi) = \frac{1}{2\pi} \int_{-\pi}^{\pi} f(\theta) \frac{1 - r^2}{1 - 2r \cos(\theta - \phi) + r^2} d\theta.$$

POISSON RATIO. The ratio of lateral strain to longitudinal strain in simple tension or compression. Often denoted ν (or σ) and, for an isotropic elastic material which obeys the generalized **Hooke's law**, may be defined in terms of the Lamé constants by

$$\nu = \frac{\lambda}{2(\lambda + \mu)}.$$

Poisson's ratio is sometimes denoted also by μ , or m or $1/m$. In the plastic range the volume change tends toward zero and Poisson's ratio tends toward 0.5 for an isotropic material. (See also **Cauchy relations**.)

POLAR COORDINATES. If r is the distance from the origin of a **rectangular Cartesian coordinate system** to a point (x,y,z) and if the direction angles of a line drawn from the origin to the point are α,β,γ then the **polar coordinates** of the point are given by

$$x = r \cos \alpha; \quad y = r \cos \beta; \quad z = r \cos \gamma;$$

$$r^2 = x^2 + y^2 + z^2.$$

This system is generally called **spherical polar coordinates**.

If the point lies in a plane determined by a pair of the coordinates, the XY -plane for instance, then $z = 0$, and with the usual symbols, $\alpha = \beta = (\pi - \theta)$

$$x = r \cos \theta; \quad y = r \sin \theta, \quad \theta = \tan^{-1} y/x.$$

The coordinate origin is called the *pole*; the X -axis is the *polar axis*; the angle θ is the *polar* or *vectorial angle* (sometimes the *azimuth* of the point); r is the **radius vector**. **Complex numbers** are often plotted in this way, the vectorial angle then being called the *amplitude*, *argument* or *phase* and the radius vector is the *modulus*.

POLAR COORDINATES, GEODESIC. See **geodesic polar coordinates for a surface**.

POLAR DEVELOPABLE. See **developable, polar**.

POLAR DISTANCE (ASTRONOMIC) (PD).

The polar distance of a celestial object is the angle measured from the nearest pole of rotation of the celestial sphere along the hour circle through the object to the object. The pole from which the polar distance is expressed should be carefully noted, e.g., S 48°. The sum of polar distance and **declination** is 90°.

POLARITY. (1) A line segment is said to exhibit polarity when its two ends are distinguishable. (2) By analogy with (1), a physical system has polarity when two points in the system have different characteristics. For example, an electric **cell** has polarity, usually indicated by the plus and minus markings of its terminals. A coil has no polarity but a **transformer** does, since one of the two

secondary terminals is positive at those times when a particular one of the two primary terminals is positive, and *vice versa*.

POLARIZABILITY. The polarizability α is defined as the electric dipole moment $|\mathbf{P}|$ induced in an atom, molecule, crystal, or amorphous solid by the unit electric field strength \mathbf{F} , therefore

$$|\mathbf{P}| = \alpha |\mathbf{F}|.$$

In a molecule or crystal the magnitude of the induced dipole moment depends on the orientation of the system to the field, and in general the direction of \mathbf{P} does not coincide with the direction of \mathbf{F} , that is

$$\mathbf{P}_F = \alpha_F |\mathbf{F}|$$

(polarizability tensor). However for reasons of symmetry these two directions coincide if \mathbf{F} has the direction of one of the axes of symmetry of the system. Choosing these axes as coordinate axes we have

$$\mathbf{P}_x = \alpha_{xx} \mathbf{F}_x; \quad \mathbf{P}_y = \alpha_{yy} \mathbf{F}_y; \quad \mathbf{P}_z = \alpha_{zz} \mathbf{F}_z.$$

If $1/\sqrt{\alpha_F}$ is plotted in the various directions from the origin, a surface is obtained which has the form of an ellipsoid (**polarizability ellipsoid**). (For more detail see J. P. Mathieu, *Spectres de Vibration et Symétrie des Molecules et des Cristaux*, Hermann, Paris, 1945.)

The polarizability changes with internuclear distance. Therefore a change in polarizability, that is, a change in the amplitude of the induced dipole moment, is associated with both the vibration and the rotation of a molecule (or crystal, as far as applicable). For vibration, to a good first approximation one can put

$$\alpha = \alpha_{ov} + \alpha_{1v} \sin 2\pi\nu_{osc}t$$

where α_{ov} is the polarizability in the equilibrium position, α_{1v} is the amplitude of the change in polarizability during the vibration ($\alpha_{1v} \ll \alpha_{ov}$), and t is the time. Correspondingly, for rotation,

$$\alpha = \alpha_{0r} + \alpha_{1r} \sin 2\pi 2\nu_{rot}t,$$

where α_{0r} is the average polarizability and α_{1r} is the amplitude of the change in polarizability for rotation about the rotational axis considered. The frequency with which the polarizability changes during the rotation is twice the rotational frequency, since the polar-

izability is the same for opposite directions of the applied field. The polarizability can be determined from the index of refraction by means of the **Lorentz-Lorenz law**, and, in the case of non-polar substances, from the **Clausius-Mosotti equation**. (See under these headings.)

POLARIZABILITY CATASTROPHE. According to the standard theory of the **dielectric constant** of an assembly of dipoles, using the **Lorentz field** concept, at a certain temperature the dielectric constant would become infinite. It has been proposed that at this temperature saturation effects enter, and the substance should become spontaneously polarized, or ferroelectric. This "4 π /3 catastrophe," as it is sometimes called, is avoided by Onsager's theory, but the problem has not been completely settled. There is reason to believe that an assembly of dipoles might become antiferroelectric under their mutual interactions, although this has never been observed in practice.

POLARIZABILITY, ELECTRONIC. That part of the **polarizability** of an atom which arises from the displacement of the electrons relative to the nucleus. It is given by

$$\alpha = \frac{e^2}{m} \sum_j \frac{f_{ij}}{\omega_{ij}^2 - \omega^2}$$

where f_{ij} and ω_{ij} are the **oscillator strength** and frequency for the transition from the ground state i to the excited state j and ω is the frequency of the external measuring field.

POLARIZABILITY ELLIPSOID. A representation of the **electromagnetic theory** in anisotropic dielectric media may be given by the ellipsoid described by the equation:

$$\frac{x^2}{k_x} + \frac{y^2}{k_y} + \frac{z^2}{k_z} = 1$$

in which x, y, z are coordinate axes and $k_x, k_y,$ and k_z are the principal dielectric constants. If v_a represents the velocity of waves traveling perpendicular to the x -axis with their electric displacements parallel to the x -axis, then $v_a = c/\sqrt{k_x}$ (c = velocity of light in a vacuum) on the assumption that the magnetic permeability of the medium is that of empty space.

POLARIZABILITY, IONIC. That part of the **polarizability** in ionic crystals due to the

relative displacement of ions of opposite sign when an electric field is applied, and giving rise to **infrared absorption**.

POLARIZATION. (1) The state induced by the separation of positive and negative electric charges by an external electric field.

(2) A vector quantity **P** representing the electric dipole moment per unit volume induced within a dielectric material. The ratio $\left| \frac{\mathbf{P}}{\mathbf{E}} \right|$ defines the electric susceptibility χ_E . The unit of polarization is coulomb/sq. meter.

(3) The state in which the disturbances of the magnetic or electric field vector is confined to one plane. (See **polarization cycle**.)

When a dielectric material is placed between the plates of a loaded condenser, the electrical charges within the dielectric are displaced from their equilibrium position. Since these charges cannot move freely, as in a conductor, they cannot move into the condenser plates, but there appears a net charge of surface density $+\mathbf{P}$ and $-\mathbf{P}$ on the dielectric surfaces facing the condenser plates. These charges tend to cancel the effect of the condenser charges. The polarization can be calculated from the charge density on the condenser plates, σ , and from the dielectric constant ϵ by the equation

$$\mathbf{P} = \sigma \frac{\epsilon - 1}{\epsilon}$$

(4) The formation of localized regions near the electrodes of an electric cell during electrolysis of products which modify the current through the cell.

POLARIZATION CURVE. The current-voltage relationship of an electrolytic system as plotted graphically, especially as it shows the progressive effect of the growth of a counterelectromotive force due to **polarization** phenomena.

POLARIZATION CYCLE. If the **vector field** representing the vibrations in a beam of radiation is contracted into a field of bound vectors having a common origin then the locus of terminal points of the bound vector field is the polarization cycle of the radiation. The radiation is *plane polarized*, if the cycle is a line segment, *elliptically polarized* if the cycle is an ellipse, etc. (See **Stokes polarization theorem**.)

POLARIZATION, INDUCED. See **induced polarization**.

POLARIZATION, NUCLEAR. (1) Alignment of the spin of a nucleus along a specified direction (or more generally, a non-isotropic distribution of nuclear spins) as a result of a nuclear reaction. (2) Alignment of the magnetic moments of many nuclei in the same direction, giving a net macroscopic magnetic moment.

POLARIZATION OF PARTICLES. Orientation of particles so that one component of spin angular momentum is positive, that is

$$P = \frac{\langle J_z \rangle_{av}}{J}$$

POLARIZATION, PHI (Φ). The state of an electromagnetic wave in which the E-vector is tangential to the lines of latitude of reference. The usual frame of reference has the polar axis vertical, and the origin at or near the antenna. Under these conditions, a vertical dipole will radiate only theta (θ) polarization (see **polarization, theta**) and a horizontal loop will radiate only phi (ϕ) polarization.

POLARIZATION PLANE. For a **linearly polarized wave**, the direction of the **electric vector**. This usage is common in radio and microwave discussions. The classical usage in optics is that the plane of polarization (see **plane of vibration**) is in the direction of the magnetic vector, and so is perpendicular to the electric vector. Because of these contradictory usages, confusion is best avoided if the direction of polarization is specifically stated in terms of the direction of the electric or of the magnetic vector.

POLARIZATION, PROPORTION OF. See **proportion of polarization**.

POLARIZATION, THETA (Θ). The state of an electromagnetic wave in which the E-vector is tangential to the meridian lines of some given spherical frame of reference. The usual frame of reference has the polar axis vertical, and the origin at or near the antenna. Under these conditions, a vertical dipole will radiate only theta (θ) polarization, and the horizontal loop will radiate only phi (ϕ) polarization. (See **polarization, phi**.)

POLARIZATION UNIT VECTOR (FOR A FIELD VECTOR). At a point, a complex field vector divided by its magnitude. For a field vector of one frequency at a point, the polarization unit vector completely describes the state of polarization, that is, the **axial ratio** and orientation of the polarization ellipse and the sense of rotation on the ellipse. A complex vector is one each of whose components is a complex number. The magnitude is the positive square root of the scalar product of the vector and its complex conjugate.

POLARIZING ANGLE. See **Brewster's angle**.

POLAR LINE (FOR A POINT OF A CURVE). The generator of the polar developable (see **developable, polar**) formed as the intersection of successive normal planes (see **normal to a curve at point P**) at the point. The straight line normal to the osculating plane and passing through the center of curvature (see **curvature, center of**) of the curve at the point.

POLAR MOMENT OF INERTIA. See **moment of inertia**.

POLAR VECTOR. See **vector, polar**.

POLE. (1) An arbitrarily chosen point in the plane of the **polygon of forces**. Lines joining the pole to the head and to the tail of each force vector are called **rays**. The strings of the **funicular polygon** are drawn parallel to the rays. The moment of a force about a point is equal to the pole distance times the intercept on the funicular diagram. Pole distance is the perpendicular distance from the pole to the line of action of the force. (2) See **magnetic pole**. (3) The interaction of an axis of rotation or of symmetry with a surface, often spherical. (4) One electrode of an electric cell.

POLE OF AN ANALYTIC FUNCTION. The point z_0 of the complex plane is a pole of order k of the function $f(z)$ if z_0 is an isolated singular point of $f(z)$ such that the **Laurent expansion** of $f(z)$ in an annulus about z_0 has only a finite number k of non-vanishing terms $a_n(z - z_0)^n$ with negative n . (Roughly expressed, $f(z)$ increases like $(z - z_0)^{-k}$ as z approaches z_0 .) But if for infinitely many negative n the coefficient a_n does not vanish, then z_0 is an essential **singularity** of $f(z)$.

E.g., $\sin 1/z$ has an essential singularity at the origin.

POLE OF MOHR CIRCLE. See **Mohr circle for stress.**

POLHODE. For a rotating rigid body subject to no external resultant **torque**, the line of intersection of the cone traced out by the angular velocity vector with the momental ellipsoid. (See **ellipsoid, momental.**) (See also **herpolhode.**)

POLYATOMIC MOLECULES, NORMAL VIBRATIONS AND COORDINATES IN. See **normal vibrations and normal coordinates in polyatomic molecules.**

POLYATOMIC MOLECULES, PARTITION FUNCTION OF. Values of these functions can often be approximated, since many of the sums can be replaced by integrals, and since the vibrational part is often practically equal to unity. Each case has to be considered separately.

POLYATOMIC MOLECULES, TYPES OF. For the discussion of their rotational properties one distinguishes different types of polyatomic molecules, according to the relative values of the principal **moments of inertia** I_A , I_B , and I_C , or according to the symmetry of the momental ellipsoid, with axes A , B , and C .

The *asymmetric top molecule* is defined as one in which all three principal moments of inertia are different from each other.

$$I_A \neq I_B \neq I_C, \quad (I_A < I_B < I_C).$$

A *symmetric top molecule* is characterized by the fact that two of its principal moments of inertia are the same (I_B), and that the third (I_A) is of the same order of magnitude. The axis of the third moment of inertia is called the *figure axis* of the molecule. If $I_A < I_B$, we speak of a *prolate symmetric top*; if $I_A > I_B$, of an *oblate symmetric top*. In this case the momental ellipsoid is a rotational ellipsoid.

A *spherical top molecule* is defined as a rotating body in which all three principal moments of inertia are equal, that is,

$$I_A = I_B = I_C = I.$$

The momental ellipsoid of the spherical top is a sphere.

In addition there is the special case of the symmetrical top molecule of which one of the

principal moments of inertia (I_A) is equal to zero, or extremely small, while the two others (I_B) are equal, that is, the momental ellipsoid is a circular cylinder. This is the case for linear polyatomic (and for diatomic) molecules. (See also **moments of inertia of molecules.**)

POLYATOMIC MOLECULES, VALANCE FORCES IN. See **valance forces in polyatomic molecules.**

POLYGON. A plane figure with n vertices and n sides, also called an n -gon. Depending on the value of n , the following names are used: 3, triangle; 4, quadrilateral; 5, pentagon; 6, hexagon; 7, heptagon; 8, octagon; 9, nonagon; 10, decagon, etc.

POLYGON OF FORCES. See **force polygon.**

POLYHEDRON. A solid with faces formed from plane polygons. The intersections of faces are edges and the points where three or more edges meet are vertices. If the faces are congruent regular polygons and the polyhedral angles are congruent, the polyhedron is regular. There are only five regular polyhedra, which are called the Platonic solids. Their names and the nature of their faces are: tetrahedron, 4 equilateral triangles; hexahedron or cube, 6 squares; octahedron, 8 equilateral triangles; dodecahedron, 12 pentagons; icosahedron, 20 equilateral triangles.

An important equation for polyhedra, discovered by Descartes and Euler, is $V - E + F = 2$, where V is the number of vertices, E the number of edges, and F the number of faces. With this equation it is easy to see that there are only five regular polyhedra.

POLYMER. A material with large chain-like molecules which commonly lead to **viscoelastic** behavior.

POLYMER MIXTURES. The most characteristic effect observed in the thermodynamical properties of polymer mixtures is the existence of large deviations from the laws of perfect solutions, due mainly to the difference in size between the particles of solute and the solvent. (See **perfect solutions; athermal solutions.**) If one uses a **lattice model**, the main problem is to calculate the combinatorial factor corresponding to the number of ways of arranging the molecules on the lattice. For open chain

polymers satisfactory approximations exist (Flory, Huggins, Guggenheim).

For large coordination numbers, the entropy of mixing (see **thermodynamic functions of mixing**) is given by

$$\frac{S^M}{k(N_A + N_B)} = -x_A \ln \phi_A - x_B \ln \phi_B \quad (1)$$

where N_A, N_B are the number of monomers and of polymers, x_A, x_B the mole fractions, ϕ_A, ϕ_B the volume fractions, k , the Boltzman constant.

To (1), there corresponds always a *negative* excess entropy (see **excess functions**). This explains, for example, why polymer mixtures have a smaller vapor tension than that predicted by the laws of **perfect solutions**.

POLYNOMIAL. A rational integral function, sometimes also called a multinomial, in n variables of the form

$$c_1 x_1^{a_1} x_2^{b_1} \cdots x_n^{r_1} + c_2 x_1^{a_2} x_2^{b_2} \cdots x_n^{r_2} + \cdots + c_k x_1^{a_k} x_2^{b_k} \cdots x_n^{r_k}.$$

For any term in this expression c_i is the coefficient, a_i is the degree with respect to x_1 , b_i with respect to x_2 , etc., and the total degree of that term is $a_i + b_i + \cdots + r_i$. The highest degree in x_i of any term whose coefficient is not zero is the degree of the polynomial in x_i , and the highest total degree of any term with non-vanishing coefficient is the degree of the polynomial. The coefficients, c_i , which are constants, may be real or complex.

If the terms of a polynomial all have the same degree, the polynomial is **homogeneous**. The expressions **quantic** and **form** are also used for a homogeneous polynomial.

The commonest case is the n th degree polynomial in one variable, which may be written as

$$a_0 x^n + a_1 x^{n-1} + \cdots + a_{n-1} x + a_n.$$

For such a polynomial, the fundamental theorem of **algebra** implies that the polynomial may be factored into linear factors

$$a_0(x - x_1)(x - x_2)(x - x_3) \cdots (x - x_n) = 0,$$

where the x_1, x_2, \cdots, x_n , called the *roots* of the polynomial, may be real or complex and need not all be distinct. If $k \leq n$ roots are equal to each other, the root is said to be k -fold or k -tuple. The following relations hold for the roots

$$x_1 + x_2 + \cdots + x_n = -a_1/a_0;$$

$$x_1 x_2 + x_2 x_3 + \cdots + x_{n-1} x_n = a_2/a_0;$$

$$x_1 x_2 x_3 + \cdots + x_{n-2} x_{n-1} x_n = -a_3/a_0; \cdots;$$

$$x_1 x_2 x_3 \cdots x_n = (-1)^n a_n/a_0.$$

(For many important special polynomials, **Hermite**, **Laguerre**, etc., see the respective names.)

POLYNOMIAL APPROXIMATIONS, METHOD OF. See method of polynomial approximations.

POLYNOMIAL FACTORIZATION. The characteristic equation of all lumped-parameter systems being of polynomial form, it is often required to find the roots of such an equation (i.e., the zeros of the polynomial) even when the equation is of degree > 4 . A number of methods, usually iterative in nature and necessarily laborious in execution, exist for the solution of this problem, of which two are here outlined.

In all cases the equation will be taken as

$$A(s) \equiv a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0 = 0.$$

(a) *Newton's Method.* This is based on the argument that if $s = s_0$ is an approximate root of the equation $A(s) = 0$, then $s = s_0 + h$, where

$$h = - \left[A(s) / \frac{dA(s)}{ds} \right]_{s=s_0}$$

is a better approximation; if $(s_0 + h)$ were a root, by **Taylor's theorem**,

$$A(s_0 + h) = A(s_0) + h \left[\frac{dA(s)}{ds} \right]_{s=s_0} + \frac{h^2}{2!} \left[\frac{d^2 A(s)}{ds^2} \right]_{s=s_0} + \cdots = 0.$$

Provided h is small enough therefore, i.e. *provided the approximate root s_0 is near enough to the true root*, a good approximation to h is obtained by writing

$$A(s_0) + h \left[\frac{dA(s)}{ds} \right]_{s=s_0} = 0$$

which gives the value of h given above. Iteration of the process gives (provided the series of approximations converges to a limiting value) the required root to any accuracy de-

sired. If the series of approximations does not converge, a closer initial approximation s_0 must be made by trial and error.

The method may be extended to complex roots, the Taylor expansion being still valid for complex s and h since $A(s)$ is analytic.

A variant on the method is to use the first *three* terms of the Taylor expansion, using the numerically smaller of the two resulting solutions for h . In either case, provided s_0 is initially close enough, the convergence is normally fairly rapid. (See also **Newton's method**.)

(b) *Lin's Method*. If n is even, take as the first approximation to a quadratic factor of $A(s)$ the function

$$Q(s) = s^2 + \frac{a_1 s}{a_2} + \frac{a_0}{a_2},$$

and divide $A(s)$ by $Q(s)$ until a quadratic remainder is left,

$$Q_1(s) = b_2 s^2 + b_1 s + b_0 = b_2 Q(s) + k_1 s + k_0.$$

If $k_1 s + k_0$ is not considered a close enough approximation to zero, repeat the process substituting

$$\frac{Q_1(s)}{b_2} = s^2 + \frac{b_1 s}{b_2} + \frac{b_0}{b_2}$$

for $Q(s)$. The process may be continued to give any desired degree of accuracy; quadratic factors are thus removed from $A(s)$ one at a time until only one is left.

If n is odd, at least one root is real. Take as an approximation to the corresponding factor of $A(s)$ the linear factor

$$L(s) = \left(s + \frac{a_0}{a_1} \right);$$

divide $A(s)$ by $L(s)$ until a linear remainder $L_1(s) = b_1 s + b_0 = b_1 L(s) + k_0$ is left. If k_0 is not near enough to zero, repeat the process, using $L_1(s)/b_1$ instead of $L(s)$. The process ultimately gives one real root to any accuracy required, leaving a polynomial of even degree which may then be factorized into quadratic factors as above.

POLYTROPIC ATMOSPHERE. A model atmosphere in **hydrostatic equilibrium** with a constant nonzero **lapse rate**. The vertical distribution of pressure and temperature is given by

$$\frac{p}{p_0} = \left(\frac{T}{T_0} \right)^{g/R\gamma},$$

where p is the pressure, T the Kelvin temperature, g the acceleration of gravity, R the gas constant for air, and γ the environmental lapse rate, the subscript zeros denoting values at the earth's surface.

POLYTROPIC PROCESS. A thermodynamic process in which changes of pressure p and density ρ are related according to the formula

$$p\rho^{-\lambda} = p_0\rho_0^{-\lambda},$$

where λ is a constant and subscript zeros denote initial values of the variables. Therefore pressure and temperature are similarly related:

$$\frac{p}{p_0} = \left(\frac{T}{T_0} \right)^k,$$

where k is the *coefficient of polytropy*. For isobaric processes, $k = 0$; for isosteric processes, $k = 1$; for adiabatic processes $k = c_p/R$, where c_p is the specific heat at constant pressure and R is the gas constant.

In meteorology this formula is applied to individual air parcels, and should be distinguished from that for a **polytropic atmosphere**, which describes a distribution of pressure and temperature in space.

PONDEROMOTIVE EQUATION. (Lorentz force on a particle as a Euler-Lagrange equation.) The classical law (the Lorentz force) acting on a particle of mass m_0 and charge e in an electromagnetic field is given by

$$m_0 \frac{d\mathbf{v}}{dt} = e \left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{H}}{c} \right]. \quad (\text{A})$$

The relativistic expression takes the mass-variability into account:

$$\frac{d}{dt} \left(\frac{m_0}{\sqrt{1 - v^2/c^2}} \mathbf{v} \right) = e \left[\mathbf{E} + \frac{\mathbf{v} \times \mathbf{H}}{c} \right] \quad (\text{A}')$$

where m_0 is the rest mass of the particle.

Equation (A') can be derived from a Lagrangian principle

$$\delta \left(\int_{P_1}^{P_2} L dt \right) = 0, \quad (\text{B})$$

where the Lagrange function L is expressed in terms of the electromagnetic potentials \mathbf{A} , ϕ , viz.,

$$L = -mc^2 \sqrt{1 - \frac{v^2}{c^2}} - e\phi + \frac{e}{c} \mathbf{v} \cdot \mathbf{A} \quad (\text{B}')$$

Equation (A') represents the Euler-Lagrange equations of (B). Equation (A') can also be written as the Hamiltonian equations of the Hamiltonian function \mathcal{H} , where

$$\mathcal{H} = m_0 c^2 \left[1 + \frac{\left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right) \cdot \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right)}{m_0^2 c^2} \right]^{\frac{1}{2}} + e\phi.$$

The generalized momentum vector \mathbf{p} is defined by

$$\mathbf{p} = \frac{m_0 \mathbf{v}}{\sqrt{1 - v^2/c^2}} + \frac{e}{c} \mathbf{A}.$$

The Hamilton equations are

$$\begin{aligned} \frac{dx_i^\nu}{dt} &= v_i = \frac{\partial \mathcal{H}}{\partial p_i} \\ &= \frac{1}{m_0} \left[1 + \frac{\left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right) \cdot \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right)}{m_0^2 c^2} \right]^{-\frac{1}{2}} \\ &\quad \cdot \left(p_i - \frac{eA_i}{c} \right) \end{aligned}$$

$$\begin{aligned} \frac{dp_i^\nu}{dt} &= - \frac{\partial \mathcal{H}}{\partial x_i} \\ &= \frac{e}{m_0} \left[1 + \frac{\left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right) \cdot \left(\mathbf{p} - \frac{e}{c} \mathbf{A} \right)}{m_0^2 c^2} \right]^{-\frac{1}{2}} \\ &\quad \cdot \left(p_k - \frac{eA_k}{c} \right) \frac{\partial A_k}{\partial x_i} - e \frac{\partial \phi}{\partial x_i} \\ &= \frac{d}{dt} \left(\frac{m_0 v_s}{\sqrt{1 - v^2/c^2}} \right) \\ &\quad + \frac{e}{c} \left(\frac{\partial A_i}{\partial x_k} v_k + \frac{\partial A_i}{\partial t} \right) \quad (i = 1, 2, 3) \end{aligned}$$

(the summation convention for repeated indices is assumed; $x_1 = x$; $x_2 = y$; $x_3 = z$).

(For explanation of electromagnetic potentials see **field of moving charge in space**.) (See also **relativity, general**.)

POPULATION OF LEVELS. A term used to describe the specification of the number of

systems in an **ensemble** which are in each of the quantum-mechanically allowed **energy levels** for such a system.

PORT. (1) A place of access to a **network** (system) where energy may be supplied or withdrawn or where network variables may be observed or measured. A designated pair of terminals is an example of a port. (2) An opening for the inlet or outlet of fluid, especially in a valve face or engine cylinder surface.

PORTAL FRAME. A continuous line frame consisting of two columns and a pitched roof or horizontal beam. (See **rigid frame**.)

PORTER-THOMAS DISTRIBUTION. In the theory of neutron resonance reactions a proposed distribution function for the reduced neutron width, γ_n , to be found with a given compound-nucleus resonance. It is

$$P(\gamma_n) = (2\pi \langle \gamma_n^2 \rangle_{av})^{-\frac{1}{2}} \exp \left(- \frac{\gamma_n^2}{2 \langle \gamma_n^2 \rangle_{av}} \right)$$

γ_n is related to the neutron width, Γ_n by

$$\Gamma_n = 2\gamma_n^2 \sqrt{\frac{2mE}{\hbar^2}}.$$

PORT-PAIR. Any two **ports** of a network (system). A two-terminal-pair is an example of a port-pair.

POSITIONAL NOTATION. One of the schemes for representing real numbers, characterized by the arrangement in sequence of digits (symbols for integers) with the understanding that the successive digits are to be interpreted as the coefficients of successive integral powers of a number called the radix or base of the notation. The representation of a real number by the notation

$$A_n A_{n-1} \cdots A_2 A_1 A_0 . A_{-1} A_{-2} \cdots A_{-m},$$

which is an abbreviation for the sum

$$\sum_{i=-m}^n A_i r^i$$

where the $.$ is called the radix point, the A_i are integers ($0 \leq |A_i| \leq r$) called digits, and r is an integer greater than one called the radix (or base). The signs of all of the A_i are the same as the sign of the number represented. In the decimal number system, the radix is ten and

the radix point is called the decimal point. In the binary number system, the radix is two and the radix point is called the binary point.

POSITION ANGLE. The term position angle is used in astronomy to denote the angle between the great circle joining any two celestial objects, and the **hour circle** through one of the objects. In measuring **double stars** the position angle is the angle between the great circle joining the two stars and the **hour circle** through the brighter of the pair, the angle being measured from the north to the east through 360°. (Cf. **binary stars**.)

POSITION, DEAD RECKONING. See **dead reckoning**.

POSITION, ESTIMATED. See **dead reckoning**.

POSITION FACTOR. See **Coddington shape and position factors**.

POSITION OPERATOR. The quantum mechanical operator representing the position variable. In the **Schrödinger representation**, operation by this operator amounts to simple multiplication by the position variable.

POSITION VECTOR. See **vector, position**.

POSITIVE (SEMI-) DEFINITE MATRIX. A matrix that is Hermitian with all eigenvalues positive (non-negative).

POSITRON EMISSION, CONDITIONS FOR. If $M(A)$ is the isotopic (atomic) weight of the parent element A and $M(B)$ that of the product B, the condition that positron emission be energetically possible is that

$$M(A) - M(B) \geq 2m_e \quad (2m_e = 1.02 \text{ Mev})$$

where m_e is the rest mass of an electron.

POSITRONIUM. The system of one electron and one positron in a bound state. In the 1s state it may exist as **parapositronium** with a mean life of 1.25×10^{-10} sec or as **orthopositronium** with a mean life of 1.4×10^{-7} sec.

POSTERIOR PROBABILITY. Let the unconditional probability of q on data h be $P(q/h)$, and let p be some other event. Then $P(q/h)$ is called the prior probability of q , and $P(q,p/h) = P(q/h) P(p/q,h)$ is the pos-

terior probability, i.e., the probability of q given that p is known. $P(p/q,h)$ is the **likelihood** of p . Put loosely, the distinction is that prior probability is the probability of q before it is known whether p holds or not, the posterior probability is that of q after it is known that p holds. The knowledge concerning p is usually given by experiment and the transition from prior to posterior may be interpreted as the extent to which an attitude towards a hypothesis q is altered by an experimental result.

POST-TENSIONING. See **prestressed concrete**.

POSTULATE. It is difficult to make any clear-cut distinction between postulate and axiom.

POTENTIAL BARRIER. A region of space in which the potential energy of a particle exceeds the total energy, so that classically the particle would be unable to enter the region. For example, in α -decay, an α -particle is prevented from leaving the nucleus by the potential barrier representing the electrostatic forces. Quantum-mechanically the barrier can be penetrated, but the wave function is exponentially damped through the region.

POTENTIAL, CONNECTIVE. See **connective potential**.

POTENTIAL CORE. The region near the orifice in the center of a turbulent jet which is not turbulent. It is approximately conical, the apex being $2\frac{1}{2}$ –3 orifice diameters from the orifice, for a circular orifice.

POTENTIAL, CRITICAL. See **critical potential**.

POTENTIAL CURVE DIAGRAM. See **potential functions of molecules**.

POTENTIAL DENSITY. The density of parcel of air would attain if compressed adiabatically by descent to the standard pressure of 1000 mb. The potential density ρ' is most easily defined in relation to the **potential temperature** θ as

$$\rho' = p/R\theta,$$

where p is a pressure of 1000 mb and R the gas constant, in appropriate units.

POTENTIAL DIFFERENCE. See **potential, electric.**

POTENTIAL, ELECTRIC. The work which has to be done against the forces in an **electric field** in bringing a positive unit charge from infinity to some fixed point, is called the potential at that point. The *potential difference* between two points is the difference between the potentials at the two points.

POTENTIAL ENERGY. See **energy, potential.**

POTENTIAL ENERGY OF DEFORMATION, OR POTENTIAL ENERGY (OF AN ELASTIC BODY). Consider a body of elastic material in equilibrium under the action of surface tractions with components T_i ($i = 1, 2, 3$) in a rectangular Cartesian coordinate system x , per unit area of surface measured in the undeformed state, and body forces with components F_i in the system x , per unit volume measured in the undeformed state. Let u_i be the components of a displacement field. Using the summation convention, the potential energy of deformation is defined as

$$U = \int_{\Sigma} T_i u_i d\Sigma - \int_V F_i u_i dV,$$

where U is the strain-energy associated with the deformation defined by the displacement field u_i and the first and second integrals are evaluated over the surface and volume of the body, respectively, in the undeformed state. If the body forces are specified throughout the body and the surface tractions over part (or the whole) of the surface, the displacement components being specified over the remainder of the surface, then the theorem of minimum potential energy applies.

POTENTIAL ENERGY, THEOREM OF MINIMUM. The theorem of minimum potential energy, or *principle of potential energy*, states that of all the displacement fields which satisfy the prescribed boundary conditions, those which satisfy the equations of equilibrium provide *stationary values for the potential energy of deformation*. For elastic materials to which classical elasticity theory is applicable, there is only one such displacement field and it is that which gives the absolute minimum value for the potential energy. In a linear or non-linear elastic body, of all the **compatible** or **kinematically admissible** states

of strain and displacement u_i^* satisfying boundary conditions on displacement u_i , the actual state minimizes the potential energy or

$$U - \int_{A_T} T_i u_i dA - \int_V F_i u_i dV \leq U^* \int_{A_T} T_i u_i^* dA - \int_V F_i u_i^* dV$$

where A_T is the region of the boundary where surface traction T_i is specified, U is the strain energy, and F_i is the body force per unit of volume. (See **complementary energy**.)

POTENTIAL FLOW. Alternatively called irrotational, flow in which the velocity \mathbf{v} is given by $\mathbf{v} = \text{grad } \phi$. A necessary and sufficient condition for this to be so is that $\text{curl } \mathbf{v} = 0$, i.e., the **vorticity** is everywhere zero.

POTENTIAL FUNCTIONS OF MOLECULES. In a molecule, the potential energy V under whose influence the nuclei carry out their vibrations is the sum of the electronic energy and the Coulomb potential of the nuclei. For a molecular state to be stable, it is necessary that the potential energy in its dependence on the internuclear distance (or distances) have a minimum.

The potential energy V of a molecule can be represented by a power series

$$2V = \sum_i \sum_j k_{ij} q_i q_j + \sum_i \sum_j \sum_k f_{ijk} q_i q_j q_k + \sum_i \sum_j \sum_k \sum_l g_{ijkl} q_i q_j q_k q_l + \dots$$

corresponding to a hypersurface in a $(N + 1)$ dimensional space where N is the number of vibrational degrees of freedom of the molecule considered.

In the case of a diatomic molecule the power series reduces to

$$V = \frac{1}{2} k x^2 (1 + a_1 x + a_2 x^2 + \dots)$$

and the hypersurface to a curve in a two-dimensional coordinate system.

In the above equation k is the force constant (see **vibrational energy levels of a molecule**) and $x = r - r_e$ is the displacement from the equilibrium position of the nuclei (r_e). If large internuclear distances are considered the potential energy may be represented by a *Morse function*

$$V(r - r_e) = D_e[1 - e^{-\beta(r-r_e)}]^2.$$

Here D_e is the dissociation energy referred to the minimum of the potential energy of the electronic state considered, and

$$\beta = \sqrt{\frac{2\pi^2 c \mu}{D_e h}} \omega_e = 1.2177 \times 10^7 \omega_e \sqrt{\frac{\mu_A}{D_e}}.$$

In this expression $\omega_e = \frac{\nu_{osc}}{c}$ is, apart from the factor c , the vibrational frequency for infinitesimal amplitude. μ_A is the reduced mass in atomic weight units ($O^{16} = 16$) and D_e is in cm^{-1} .

POTENTIAL, GRAVITATIONAL. See gravitational potential.

POTENTIAL IN INVERSE SQUARE FIELD. The potential in an inverse square

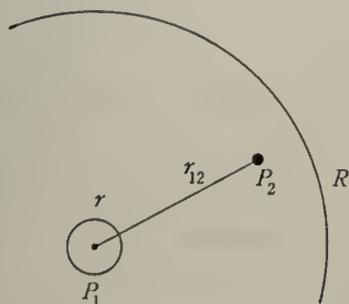
field is of the form $\frac{k}{r}$ where r is the distance from the center of force. If the force is attractive, k is negative and if the force is repulsive, k is positive. The force is given by $\mathbf{F} = -\nabla \left(\frac{k}{r} \right)$.

POTENTIAL, INTERATOMIC. See interatomic potential.

POTENTIAL OPERATOR. Let $u(x_2 y_2 z_2)$ be a finite, continuous and single-valued function of the coordinates x_2, y_2, z_2 of a point P_2 (see figure) and let r_{12} be the distance

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

of P_2 from another point P_1 with the coordinates x_1, y_1, z_1 . Describe two spheres about P_1 as center, one a very small sphere of radius r



Potential operator.

and the other a very large sphere of radius R . The potential of u , designated by $\text{Pot } u$, is defined as the limit of the integral

$$\int_r^R \frac{u(x_2, y_2, z_2)}{r_{12}} d\tau_2$$

where $d\tau_2 \equiv dx_2 dy_2 dz_2$ taken over the volume between the two spheres, as $r \rightarrow 0$ and $R \rightarrow \infty$. In terms of the potential operator the vector \mathbf{B} can be written

$$\mathbf{B} = \frac{1}{4\pi} \text{Pot } \nabla \times \nabla \times \mathbf{B} - \frac{1}{4\pi} \text{Pot } \nabla \nabla \cdot \mathbf{B}.$$

The first term is solenoidal and the second term is irrotational. (See also **vector resolution**.)

POTENTIAL PLASTIC. See flow rule.

POTENTIAL SCATTERING. In the compound nucleus theory of nuclear reactions, the non-resonant part of the elastic scattering. It can interfere with the resonant scattering, making the resonance in the cross section look somewhat distorted. The cross section for potential scattering is approximately equal to the cross section between resonances, provided they are not too close together.

POTENTIAL, STANDARD. The potential of an electrode composed of a substance in its standard state, in equilibrium with its ions, which are all in their standard states. (See **hydrogen scale**.)

POTENTIAL, STOPPING. That potential difference between two electrodes which is just sufficient to prevent any electrons emitted from the positive electrode from reaching the negative one, thus a measure of the initial velocity or the energy of the electron. The magnitude of this quantity when the electrons are emitted from a surface by the **photoelectric effect** of light, is equal to $(h\nu - h\nu_0)/e$ where h is the Planck constant, ν is the frequency, and e is the charge of an electron (in consistent units).

POTENTIAL TEMPERATURE. The temperature, denoted by θ , acquired by air brought adiabatically to a standard pressure, usually taken as 1000 millibars. If p is the actual pressure, p_0 the standard, γ the ratio of the specific heats, and T the temperature

$$\theta = T \left(\frac{p_0}{p} \right)^{\frac{\gamma-1}{\gamma}} = T \left(\frac{p_0}{p} \right)^{\frac{R}{c_p}}$$

R being the gas constant and c_p the specific heat at constant pressure.

The air is statically stable or unstable according as the potential temperature increases or decreases upwards. (See **adiabatic lapse rate**.)

POTENTIAL, VECTOR. See **vector potential**.

POUND. See **mechanical units**.

POUND-MOLE. The quantity of substance whose mass expressed in pounds (lbm) is numerically equal to the molecular weight of the substance. (See **mole**.)

POWER. (1) See **cardinal**. (2) The time rate of doing work. The defining equation for power is $P = dW/dt$, where W is the work done and t is the time.

It may be expressed in units of work per unit time (e.g., foot-pounds per minute or ergs per second) or more arbitrarily, as in horsepower or watts. One horsepower is 33,000 foot-lbs per minute. One watt is 10^7 ergs (or 1 joule) per second. (See **mechanical units**; **electromagnetic units**.)

POWER DENSITY SPECTRUM. (Sometimes called power spectrum.) A measure of the contribution to the total variance from a given frequency band in the generalized Fourier representation of a random function. If $f(t)$ is a random function, the total energy $\int_{-\infty}^{\infty} f^2 dt$ is infinite, so the Fourier integral representation is inadequate. If a transform is defined over a finite interval

$$F_T(\omega) = \frac{1}{2\pi} \int_{-T}^T f(t)e^{-i\omega t} dt,$$

under suitably restrictive conditions the power density spectrum may be defined as

$$\lim_{T \rightarrow \infty} \frac{\pi}{T} |F_T(\omega)|^2.$$

The theorem, proved by N. Wiener, establishing the analogy between the analysis of random functions and ordinary Fourier analysis, is that the power density spectrum is the **Fourier transform** of the **autocorrelation function**, which is defined for random functions as

$$\lim_{T \rightarrow \infty} \frac{1}{2T} \int_{-T}^T f(t)f(t + \tau) dt.$$

POWER FACTOR. In alternating current networks, the (mean) power is $EI \cos \theta$, where θ is the phase difference between E and I . The multiplying factor, $\cos \theta$, is the power factor. (See also **alternating current circuits**.)

POWER FLOW, COMPLEX. The complex power flow through a surface is the integral of the normal component of the complex Poynting vector (see **Poynting vector, complex**) over that surface.

POWER, LENS. See **lens power**.

POWER OF A SURFACE. The optical power of a surface separating two media with relative index of refraction n is $(n - 1)$ times the mean **curvature** of the surfaces or $\frac{1}{2}(n - 1)(k_1 + k_2)$ where k_1, k_2 are the principal curvatures of the surface. For a surface given explicitly, $z = z(x, y)$, the power is approximately $\frac{1}{2}(n - 1)\nabla^2 z$ if the surface is approximately flat in the sense that the squares of the first partial derivatives of z are negligible.

POWER SERIES. An infinite series of the form

$$\sum_{n=0}^{\infty} a_n x^n = a_0 + a_1 x + a_2 x^2 + \dots + a_n x^n + \dots,$$

where $a_0, a_1, \dots, a_n, \dots$ are **constants** and x is a **variable**, is called a power series.

For every power series $\sum a_n x^n$ there exists a constant l (which may be zero or infinity) such that the series is absolutely convergent for all values of x with $|x| < l$, and is **divergent** for all values of x with $|x| > l$. This number l is called the limit of convergence of the power series, and the interval $(-l, l)$ is called the interval of convergence. The series may converge or diverge at the ends of this interval, for $x = l$ and $x = -l$.

If $\lim_{n \rightarrow \infty} \left| \frac{a_n}{a_{n+1}} \right|$ exists, it is the limit of convergence.

If $\lim_{n \rightarrow \infty} \sqrt[n]{|a_n|}$ exists, it is the reciprocal of the limit of convergence.

A power series is uniformly convergent within the interval $(-l', l')$, where $l' < l$ and l is the limit of convergence of the given power series.

The function defined as the sum of a power series $\sum a_n x^n$ is a continuous function of x at all points within the interval of convergence.

For the operations with power series, we have the following theorems:

Two power series may be added together for all values of x for which both series are convergent, i.e., for the smaller of the two intervals of convergence.

Two power series may be multiplied together for all values of x for which both series are absolutely convergent, i.e., for the smaller of the two intervals of convergence.

A power series may be differentiated term by term for all values of x within its interval of convergence.

A power series may be integrated term by term between any limits lying within the interval of convergence.

Power series are very useful in the analytical representation of functions by the expansion of functions in series, and thereby for calculation of their values.

POWER SPECTRUM. See *spectral function*.

POYNTING THEOREM. Manipulation of the **Maxwell equations** yields a relation known as the Poynting theorem:

$$\int_S (\mathbf{E} \times \mathbf{H}) \cdot \mathbf{n} da + \int_V \mathbf{E} \cdot \mathbf{J} dV \\ = - \int_V \left\{ \mathbf{E} \cdot \frac{\partial \mathbf{D}}{\partial t} + \mathbf{H} \cdot \frac{\partial \mathbf{B}}{\partial t} \right\} dV,$$

where \mathbf{n} is the unit vector normal to the surface S that bounds the volume V . The customary interpretation of this result is that the right-hand side represents the rate of loss of electric and magnetic energy stored in the volume V , that the second term on the left represents the rate of dissipation of electrical energy as Joule heat in V , and that the first term represents the balancing flow of electromagnetic energy through the surface S bounding the volume. It is often convenient to assume that the Poynting vector

$$\mathbf{P} = \mathbf{E} \times \mathbf{H}$$

represents the power flow per unit area in the direction \mathbf{P} . This resolution of the total outward power flow is not unique, and is not acceptable when applied to an electrically-charged permanent magnet, but is notably successful in treating electromagnetic radiation problems. (See *continuity equation*; *energy-momentum tensor*.)

POYNTING VECTOR. See *Poynting theorem*.

POYNTING VECTOR, COMPLEX. When for steady harmonic fields the electromagnetic field vectors are represented by complex quantities \mathbf{E} and \mathbf{H} , the **Poynting vector** is given the form

$$\mathbf{P} = \frac{1}{2} \mathbf{E} \times \mathbf{H}^*,$$

and the real part of \mathbf{P} is the average power flow per unit area.

PRACTICAL WIDTH. In the theory of resonance capture of neutrons in a chain reacting system, the practical width of a cross-section resonance is the energy range for which the "resonance" part of the total, Doppler-broadened, Breit-Wigner cross section is larger than the sum of the background "hard-sphere" scattering cross sections of all elements in the system, including the fuel.

PRANDTL-GLAUERT RULE. This rule is obtained from the linear equation of subsonic flow and applies to a two-dimensional body in a uniform stream, with small perturbation velocities. The rule does not apply for Mach numbers near 1, because the usual subsonic linear equation is then not valid.

If C_p is the pressure coefficient at a given point on a two-dimensional body in a stream of Mach number M (less than 1), while C_{p_0} is the pressure coefficient at the same point in incompressible flow ($M = 0$), then

$$\frac{C_p}{C_{p_0}} = (1 - M^2)^{-1/2}.$$

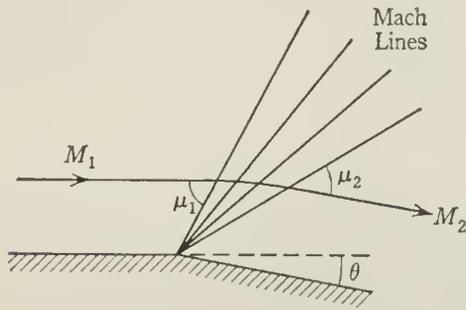
It follows from this that the coefficients of **lift** and **pitching moment** C_L and C_m , on a two-dimensional airfoil at a Mach number M , are related to the coefficients C_{L_0} and C_{m_0} in incompressible flow by the equation

$$\frac{C_L}{C_{L_0}} = \frac{C_m}{C_{m_0}} = (1 - M^2)^{-1/2}.$$

The Prandtl-Glauert rule is not applicable for stream Mach numbers above the critical Mach number, because the flow then contains regions which are supersonic. (See also *Göthert rule*; *subsonic and supersonic flow*, *linear equations of*.)

PRANDTL-MEYER EXPANSION. A two-dimensional supersonic flow in which an ini-

tially uniform flow is expanded around a sharp convex corner (see figure). The **Mach lines** are straight, as shown in the diagram, and



Prandtl-Meyer expansion.

conditions are uniform along any one Mach line. The angle of deflection is

$$\theta = \nu(M_2) - \nu(M_1) \text{ radians,}$$

where $\nu(M)$ is the *Prandtl-Meyer function* given by

$$\nu(M) = \mu - \frac{\pi}{2} + \lambda^{-1} \tan^{-1} (\lambda \cot \mu),$$

$$\lambda^2 = \frac{\gamma - 1}{\gamma + 1}$$

and μ is the **Mach angle** ($= \sin^{-1} \frac{1}{M}$).

The maximum possible value of $\nu(M)$, corresponding to $M = \infty$ and zero pressure, is

$$\nu_{max.} = \frac{\pi}{2} (\lambda^{-1} - 1)$$

and for $\gamma = 1.4$, this is about 130° . For this value of γ , this is the maximum possible angle of deflection, for a stream initially at a **Mach number** of 1.

PRANDTL-MEYER FUNCTION. See Prandtl-Meyer expansion.

PRANDTL NUMBER. The ratio σ of dynamic viscosity to thermometric conductivity: $\sigma = \nu/\kappa$, is non-dimensional. For a gas, σ is always less than unity.

PRANDTL-REUSS MATERIAL. An elastic perfectly plastic material with elastic strain components given by standard linear isotropic theory, yield point stress according to the **Mises condition**, and plastic strain increment components according to the associated **flow rule**. Thus, for plastic flow the relation between stress and strain deviators is

$$2G\dot{\epsilon}_{ij} = \lambda s_{ij}$$

$$J_2 = \frac{1}{2} s_{ij} s_{ji} = k^2, \quad \dot{J}_2 = 0, \quad \lambda \geq 0.$$

Plastic flow does not modify the dilatation.

PRANDTL STRESS FUNCTION. The function Ψ (say) defined in terms of the conjugate torsion function $\psi(x,y)$ by

$$\Psi = \psi(x,y) - \frac{1}{2}(x^2 + y^2).$$

PRECESSION. An effect manifested by a rotating rigid body when a **torque** is applied to it in such a direction that the orientation of the axis of rotation would change in the absence of **angular momentum**. If the speed of rotation and the magnitude of the applied torque are constant, the axis, in general, slowly describes a cone, its motion at any instant being at right angles to the direction of the torque.

A familiar example of precession is an ordinary symmetrical spinning top. If the axis of spin is not exactly vertical, the force of gravity exerts a torque tending to overturn the top; but instead of tipping over, it “wobbles” with a precessional motion about the vertical through the pivotpoint. The **gyroscope** exhibits similiar behavior. The period of precession is given by

$$T_p = \frac{4\pi^2 I_s}{QT_s}$$

in which I_s is the moment of inertia about the axis of spin, Q is the torque due to gravity, and T_s the period of the spin. (See also **Larmor precession**.)

PRECESSION (ASTRONOMIC). The plane of rotation of the earth, the equatorial plane, is inclined to the plane of the ecliptic by about $23^\circ 5'$. Since the sun is in the plane of the ecliptic and the moon always close to it, their gravitational pull on the rotating geoid produces a precessional motion. The pole of the equator describes a small circle on the celestial sphere with the pole of the ecliptic as center. The radius of this small circle is about $23^\circ 5'$, and the period of revolution is about 26,000 years. The average combined effect of the attractions of the sun and moon result in a retrograde motion of the vernal equinox along the ecliptic of about $50''$ per year.

This so-called luni-solar precession produces changes in the coordinates of the stars. The

latitude of a star is not changed but the longitude increases at the rate of about 50'' per year. The equatorial angle of right ascension, α , and declination, δ , are changed at the following rates per year

$$\Delta\alpha = 50'' (\cos \epsilon + \sin \epsilon \sin \alpha \tan \delta)$$

$$\Delta\delta = 50'' \sin \epsilon \cos \alpha$$

in which ϵ is the obliquity of the ecliptic. (See also **nutations**.)

PRECESSION, LARMOR. See **Larmor precession**.

PRECESSION, RELATIVITY. See **relativity precession**.

PRECESSION, THOMAS. See **relativity precession**.

PRECISION. See **accuracy**.

PREDICTOR. A quadrature formula used in the numerical solution of **ordinary differential equations** in which the integral is expressed in terms of equally spaced ordinates to the left of but not including the endpoint. This provides an initial approximation to the new ordinate. Thereafter the approximation is used in a **corrector** to improve or check the approximation. A predictor formula is **open**, a corrector formula **closed**. (See **Adams-Bashford method** for an example.)

PREDISSOCIATION. The radiationless transition of a molecule from a stable (discrete) state to an unstable (continuous) state of the same energy, with consequent broadening of the absorption lines, or breaking-off of the emission lines. Corresponding to the three forms of energy of the molecule (electronic, vibrational, or rotational), three kinds of overlapping of discrete by continuous states, that is three cases of predissociation, are possible:

I. Overlapping of a certain electronic state (that is, of its vibrational or rotational levels) by the dissociation continuum belonging to another electronic state—radiationless transition into this other dissociated electronic state.

II. Overlapping of the higher vibrational levels of an electronic state of a polyatomic molecule by a dissociation continuum joining onto a lower dissociation limit of the same electronic state—radiationless splitting off of the particular atom or group of atoms (predissociation by vibration).

III. Overlapping of the higher rotational levels of a given vibrational level of a diatomic molecule by the dissociation continuum belonging to the same electronic state—radiationless decomposition of the molecule with no change of electronic state (predissociation by rotation).

The place in the energy level diagram of a molecule at which the predissociation begins (**predissociation limit**) gives at least an upper limiting value for the corresponding dissociation limit. (For more detail see G. Herzberg, *Molecular Spectra and Molecular Structure, I. Spectra of Diatomic Molecules*, 2d ed., D. Van Nostrand Company, Inc., Princeton, 1950.)

PRE-IONIZATION. See **Auger effect**.

PRESSURE. A type of stress, characterized by its uniformity in all directions (as distinguished from compressive stress in one direction). Pressure may be defined analytically as that part of the **stress tensor** that does not depend directly on the rate of strain and which arises from the molecular movements appropriate to the local density and temperature, i.e., the pressure satisfying the equation of state. (See **static pressure**; **flux vector**.)

PRESSURE, ABSOLUTE. The term applied to the true pressure of a substance or system, commonly to distinguish it from partial pressure, gauge pressure, etc.

PRESSURE, ATMOSPHERIC. The normal atmospheric pressure, or pressure of one atmosphere, is 1.013×10^6 dynes per square centimeter or about 14.7 pounds per square inch. This will support a column of mercury (in a barometer) 76 centimeters high at 0°C.

PRESSURE, CENTER OF. See **center of pressure**.

PRESSURE COEFFICIENT. For a body in a uniform stream of fluid, the pressure coefficient at a point where the pressure is p is defined as

$$C_p = \frac{p - p_1}{\frac{1}{2}\rho V^2},$$

where ρ , V and p_1 are the density, velocity and **static pressure** in the undisturbed stream. For incompressible flow, $C_p = 1$ at a stagnation point.

PRESSURE COEFFICIENT OF CHEMICAL EQUILIBRIUM. See *Chatelier, le, theorem*.

PRESSURE FORCE. (Or pressure-gradient force.) The force due to differences of **pressure** within a fluid mass. The (vector) force per unit volume is equal to the pressure gradient $-\nabla p$, and the force per unit mass (specific force) is equal to the product of the volume force and the specific volume $-\alpha\nabla p$.

In the atmosphere, the vertical component of this force is of the order of 10^4 times the horizontal components; in meteorological literature the term "pressure force" usually refers only to the latter horizontal pressure force.

PRESSURE, GAUGE. Pressure measured relative to atmospheric pressure taken as zero.

PRESSURE, IMPACT. Pressure exerted by a moving fluid on a plane perpendicular to its direction of flow. In other words, it is measured *in* the direction of flow.

PRESSURE LINE. A line of thrust or funicular polygon in an **arch**. When the arch is statically determinate (3-hinged arch) the pressure line is unique. In a two-hinged arch it passes through the hinges but its rise decreases with the horizontal thrust. In a fixed arch the position of the supports also is not determined by statics alone. A *Voussoir* or unreinforced arch is considered safe if a pressure line can be drawn lying within the middle third or *kern* of the arch.

PRESSURE, MEAN EFFECTIVE. See *mean effective pressure*.

PRESSURE, PARTIAL. The pressure that is exerted by a single gaseous **component** of a mixture of gases.

PRESSURE, STATIC. See *static pressure*.

PRESSURE TENSOR. See *flux vector*.

PRESSURE, TOTAL. See *total pressure*.

PRESSURE, VELOCITY. The component of the pressure of the moving fluid that is due to its velocity and is commonly equal to the difference between the impact pressure and the static pressure. (See *pressure impact*, and *pressure, static* (1).)

PRESTRESSED CONCRETE. Prestressed concrete resulted from the desire to overcome

the disadvantage of the low tensile strength of concrete. By means of high strength steel wires, cables, or rods—particularly the first two—a concrete member is pre-compressed. Then, when the structure receives its load, the compression is relieved on that portion which would normally be in tension. Thus, a beam is prestressed so that under load the concrete on the side normally in tension has no tensile forces acting on it. The ultimate strength, however, of a prestressed beam is but little if any higher than that of an ordinary reinforced concrete beam with the same steel reinforcement and concrete strength. Prestressing improves behavior at and below working loads.

PRE-TENSIONING. See *prestressed concrete*.

PRIGOGINE THEOREM OF MINIMUM ENTROPY PRODUCTION. See *time variation of the entropy production*.

PRIMARY SPECTRUM. See *spectrum, primary*.

PRIME MOVER. An engine capable of producing power continuously, usually at the expense of heat supplied to it from a source (by **combustion**, or from a **nuclear reactor**). Examples: internal combustion engine, steam or gas turbine, steam engine. No agreed single term exists to describe engines whose function it is to produce a desired change of state at the expense of a continuous supply of work (such as a compressor).

PRIMITIVE. Given a function of one or more arbitrary constants or functions, the constants may be eliminated by direct differentiation one or more times, or the constants and functions may be eliminated by differentiation and algebraic operations. The result in either case is a **differential equation** and the given function is its primitive or general solution. A primitive n^{th} root of unity is an n^{th} root of unity which is not also a root of lower order. Thus $i = \sqrt{-1}$ is a primitive fourth root of unity and -1 is an imprimitive fourth root. (For primitive period, see *period*.)

PRIMITIVE EQUATIONS OF FLUIDS. The Eulerian **equations of fluid motion** in which the primary dependent variables are the fluid's velocity components. These equations govern a wide variety of fluid motions

and form the basis of most hydrodynamical analysis.

PRIMITIVE TRANSLATION. A space lattice has the property of exactly repeating itself if it is carried through any one of a number of different translations in space. These translations may all be constructed by the repeated application of three primitive translations, which thus constitute the sides of the **unit cell** of the lattice.

PRINCIPAL AXES OF A QUADRIC. See **central quadric**.

PRINCIPAL AXIS. See **axis, principal**.

PRINCIPAL AZIMUTH, ANGLE OF. See **angle of principal azimuth**.

PRINCIPAL COMPONENTS (STATISTICAL). See **component analysis**.

PRINCIPAL CURVATURE. See **curvature**.

PRINCIPAL CURVATURE OF SURFACE. See **curvature of surface, center of**.

PRINCIPAL DIRECTIONS OF CURVATURE AT A POINT OF A SURFACE. Two directions on the surface, at the point, at right angles to each other, such that the normal to the surface at a consecutive point in either of these directions intersects the normal at P . They are the directions of greatest and least normal curvature at the point.

PRINCIPAL DIRECTIONS OF SYMMETRIC SECOND-ORDER TENSOR. The directions of the axes of a rectangular Cartesian coordinate system chosen in such a way that the components of the tensor (see **tensor field**) with unequal indices are zero.

PRINCIPAL E PLANE. In electromagnetics, a plane containing the direction of maximum radiation, and in which the electric vector everywhere lies in the plane.

PRINCIPAL EXTENSION RATIOS. The displacement field in an infinitesimal element of volume at a point of a body which undergoes a deformation may be considered as the resultant of three displacement fields in the element arising from a translation, rigid rotation and pure, homogeneous deformation. The principal extension ratios at the point of the body considered are the principal extension ratios for this pure, homogeneous deformation.

PRINCIPAL EXTENSIONS (FOR INFINITESIMAL DEFORMATIONS). At any point of a body which is subjected to an infinitesimal deformation, a rectangular Cartesian coordinate system can be chosen in which the tangential components of strain at that point vanish. The normal components of strain in this system are the *principal extensions*, or *principal components of strain*, at the point. They are the eigenvalues of the infinitesimal strain tensor at the point. The directions of the axes of this rectangular Cartesian coordinate system are the *principal directions of strain*, or *principal axes of strain at the point*. (See also **principal extension ratios**.)

PRINCIPAL H PLANE. In electromagnetics, a plane containing the direction of maximum radiation, and in which the electric vector is everywhere normal to the plane, while the magnetic vector lies in the plane.

PRINCIPAL INCIDENCE, ANGLE OF. See **angle of principal incidence**.

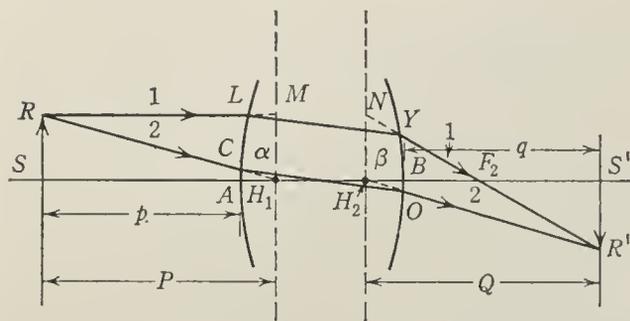
PRINCIPAL NORMAL. That normal at a point P of a space curve which lies in the **osculating plane** of the curve. The **tangent**, the principal normal and the **binormal** are mutually perpendicular and form the moving trihedral of the curve.

PRINCIPAL PLANES. (1) At a point of a twisted curve, the three planes defined by the **tangent**, **principal normal** and **binormal** at the point taken two together. The plane defined by the tangent and principal normal is the *osculating plane*, or *plane of curvature*; that defined by the principal normal and binormal is the *normal plane*, and that defined by the tangent and binormal is the *rectifying plane*.

(2) In mechanics, a principal plane of stress at a point is a plane in a body at that point on which the shearing stress is zero and the normal stress has a stationary value. There are three mutually perpendicular principal planes at each point. (See **stress, principal**.)

(3) In **Gaussian optics** the principal planes, or *unit planes* are the pair of conjugate planes where the **lateral magnification** is unity. There is a one-to-one correspondence between the points of the first principal plane in object space and the second principal plane in image space so that the object and image are of the same size and on the same side of the optical axis. If object distances are measured from

the first principal plane and image distance from the second principal plane, then the **thin lens formulae** will hold. (See figure.)



Principal plane of lens. H_1 and H_2 are the principal points; MH_1 and NH_2 the principal planes of a thick lens. In locating images by a graphical method, principal rays may be used as for thin lenses if the incident light falls on an imaginary thin lens coincident with the first principal plane, the lens being shifted to coincide with the second principal plane for the emergent light. (*Introduction to Optics, Geometrical and Physical* by Robertson, 4th ed., D. Van Nostrand Company, Inc., 1954.)

(4) For a ray of light traversing a double-refracting crystal, the principal plane is determined by the direction of the ray and the axial direction of the crystal.

PRINCIPAL POINTS (GAUSS POINTS). In **Gaussian optics**, axial conjugate points for which the **lateral magnification** is unity.

PRINCIPAL QUANTUM NUMBER. See quantum numbers of the individual electrons in an atom.

PRINCIPAL QUANTUM NUMBER, EFFECTIVE. See effective principal quantum number.

PRINCIPAL RADIUS OF CURVATURE OF SURFACE. See curvature of surface, center of.

PRINCIPAL RAY. The ray through a specified **object point** P and the center of the **entrance pupil** of an optical system, is the principal ray, or the *chief ray* through P .

PRINCIPAL ROOT. The principal root of a complex number is that n^{th} root which has smallest non-negative amplitude.

PRINCIPAL SECTION OF A CRYSTAL. A plane through a crystal which contains the **optic axis** of the crystal and the ray of light under consideration.

PRINCIPAL SERIES. Series of lines in the spectra of the simpler atoms, the alkalis, alkaline earths, helium, etc., corresponding to transitions from a series of upper P states to a lower S state (frequently the ground state).

PRINCIPAL STRESS OR STRAIN. See stress, principal; strain, principal.

PRINCIPAL VALENCE FORCES. See subsidiary valence forces.

PRINCIPAL VALUE OF TENSOR. See eigenvalue equation.

PRINCIPAL VECTOR—OF A MATRIX A. Any non-null vector x such that for some scalar λ and integer ν ,

$$(\lambda I - A)^\nu x = 0.$$

In case $\nu = 1$, x is an eigenvector (proper vector), and in any event λ is an **eigenvalue** (proper) value.

PRINCIPAL OF CORRESPONDENCE. A statement by Bohr of the assumption that in the limit, for very low frequencies of radiation, the results obtained by quantum theory must be in quantitative agreement with those obtained on the basis of classical theory. According to this principle, the predictions of quantum theory must coincide with those of classical theory for sufficiently high quantum numbers.

PRINCIPLE OF ENTROPY INCREASE. See entropy increase, principle of.

PRINCIPLE OF MICROSCOPIC REVERSIBILITY. See microscopic reversibility, principle of.

PRINCIPLE OF SUPERPOSITION. See superposition, principle of.

PRINCIPLE OF THE UNATTAINABILITY OF THE ABSOLUTE ZERO. See thermodynamics, third law of.

PRIOR PROBABILITY. Defined under posterior probability.

PRISM. (1) A polyhedron with two congruent and parallel faces, the *bases*. Its other faces, called *lateral*, are produced by drawing lines from the corresponding vertices of the bases and they are therefore parallelograms. The prism is named triangular, quadrangular, etc., descriptive of its base. (2) Glass and

other transparent materials are cut into many different forms of prism for various optical purposes.

PRISM, RESOLVING POWER OF. The resolving power of a prism (to resolve near-by spectral lines) is given by

$$R = (\text{Base of Prism})(dn/d\lambda)_\lambda$$

where $(dn/d\lambda)_\lambda$ is the rate of change of refractive index with wavelength, in the wavelength-region under consideration.

PRISM, TOTAL DEVIATION OF. If the light is incident at angle i on the first prism face, and if the prism angle is α and the refractive index is n , the total deviation after passage through the prism in a plane at right angles to the prism edge is given by

$$\Delta = i - \alpha + \sin^{-1} \left[n \sin \left(\alpha - \sin^{-1} \frac{\sin i}{n} \right) \right].$$

PRIVILEGED DIRECTIONS. When a plane-polarized beam falls on a thin plate of an anisotropic material which is not optically active, there are in general two and only two mutually-perpendicular directions for the plane of polarization (privileged directions) such that the emergent beam is also plane-polarized. With other orientations of the polarization of the entrant beam, the emergent beam is circularly or elliptically polarized.

PROBABILITY. Suppose that a trial may have one of two mutually exclusive outcomes—say “success” and “failure”—and that we associate with it a number p , $0 \leq p \leq 1$, called the probability of a success. The calculus of probability is concerned with the manipulation of such numbers, and is largely based on the following results:

(A) If two trials with probabilities p_1, p_2 are independent, in that the outcome of either has no effect on that of the other, the probability that both will result in successes is $p_1 p_2$.

(B) If the probabilities of a set of mutually exclusive events are $p_1, p_2, p_3 \dots$, the probability that at least one will occur is $p_1 + p_2 + p_3 + \dots$.

The application of statistical techniques usually results in a statement couched in terms of probabilities, but the application of probability theory to the real world encounters formidable logical difficulties. Two main schools of thought may be distinguished.

(1) In the frequency theory, the trial is supposed capable of indefinite repetition, either real or hypothetical, and the probability is identified with the limiting frequency of successes in this infinite sequence. However, the existence of a limiting value is difficult to establish theoretically, and impossible to establish empirically because of the infinite length of the sequence.

(2) The other school identifies probability with the strength of belief that a success will occur. The main difficulty here is the subjective nature of the definition. (See also **statistical probability**.)

PROBABILITY CURRENT. See **current, probability**.

PROBABILITY DENSITY. The quantity

$$\Phi\Phi^*dr = |\Phi|^2dr = |\Phi^*|^2dr$$

where Φ is the Schroedinger wave function and Φ^* is its complex conjugate. As was postulated by Born, the quantity set out above gives the probability that the particle under consideration will be found in the volume element dr at the position given by the coordinates.

PROBABILITY DENSITY FUNCTION. An alternative name for the frequency function (see **distribution**). If $F(x)$ is the distribution function of a continuous variate x , the probability density function is defined as $p(x) = dF(x)/dx$, where this exists. Roughly speaking, $p(x)$ gives the probability of obtaining a value “near” x .

PROBABILITY DISTRIBUTION. If a quantity may take any one of a set of values with different probabilities, it is said to possess a probability distribution. (See **distribution**.)

PROBABILITY FUNCTION. See **distribution**.

PROBABILITY, INDEX OF. Gibb's name for the natural logarithm of the coefficient of probability.

PROBABILITY, LAW OF COMPOUND. If the events $x_1, x_2 \dots x_n$ are independent with probabilities $p_1, p_2 \dots p_n$ respectively, then the probability that all of these events should happen at the same time when all are in question is the product of the probabilities $\prod_{i=1}^n p_i$.

PROBABILITY MATRIX. See *matrix, stochastic*.

PROBABILITY SAMPLING. A form of sampling such that the probability of any individual being selected is determined beforehand, as distinct from methods of selection depending on judgment or choice of a more or less arbitrary kind.

PROBABILITY SET, FUNDAMENTAL. See *fundamental probability set*.

PROBABLE ERROR. An obsolescent measure of variability equal to 0.6745 times the **standard error**, which has replaced it. The reason for the numerical factor is that in the **normal distribution** one half of the frequency lies in a range 0.6745σ on either side of the mean, σ being the **standard deviation**.

PROBIT. The normal equivalent deviate increased by 5 so as to avoid negative values.

PROBLEM OF BOUSSINESQ AND CERUTI. (Also called *problem of Boussinesq*, or *problem of the half-space*.) The general problem of the solution of the boundary value problem for an isotropic elastic material obeying the generalized **Hooke's law** occupying the half-space bounded by an infinite plane.

PROCA EQUATIONS. One of the four inequivalent sets of equations to describe particles of spin unity, developed by analogy with the **Maxwell equations** for free space.

$$\kappa F_{\mu\nu} = \frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu}; \quad \kappa A_\nu = \frac{\partial F_{\mu\nu}}{\partial x_\mu}$$

κ being proportional to the mass of the particle.

PROCESS LAPSE RATE. The rate of decrease of the temperature of an air parcel as it is lifted, $-dT/dz$, or occasionally dT/dp , where p is pressure. The concept may be applied to other atmospheric variables, e.g., the process lapse rate of density. The process lapse rate is determined by the character of the fluid processes and should be carefully distinguished from the **environmental lapse rate**, which is determined by the distribution of temperature in space. In the atmosphere the process lapse rate is usually assumed to be either the **dry-adiabatic lapse rate** or the **saturation-adiabatic lapse rate**.

PRODUCT. The result obtained when two or more quantities, such as **numbers**, **functions**, or **equations** are combined by multiplication. In **vector** and **tensor** analysis, the concept of a product must be generalized for there are several kinds of products and multiplication does not always obey the **commutative law**.

In **matrix** algebra, if **A** is a rectangular matrix of order $(m \times h)$ and **B** of order $(h \times n)$, the product **C** = **AB** is of order $(m \times n)$ and its elements are $C_{ij} = \sum A_{ik}B_{kj}$. Another matrix combination is the direct product. If **A** and **B** are square, of order m and n , respectively, the direct product is of order $(m \times n)$ and defined by the relation

$$\mathbf{A} \times \mathbf{B} = [A_{ij}B_{rs}].$$

The index pairs (i, r) and (j, s) refer to row and column, respectively. They are customarily arranged in dictionary order so that (j, s) precedes (j', s') if $j < j'$, $s < s'$ or if $j = j'$, $s < s'$, etc.

In **group** theory, multiplication means any defined combination law and the result is a product. For example multiplication might be defined as addition and then the product is that result commonly known in algebra as a sum.

The product of two infinite **series** is called a **Cauchy product**.

An expansion of the form

$$u_1 u_2 u_3 \cdots u_n \cdots = \prod_{k=1}^{\infty} u_k$$

is an infinite product. The partial products form a **sequence** $\{p_n\}$ such that $p_1 = u_1$; p_2

$= u_1 u_2$; $p_3 = u_1 u_2 u_3$; \cdots , $p_n = \prod_{k=1}^n u_k$. The

convergence of infinite products is studied by methods similar to those used for infinite series. An infinite product of the form $(1 + u_1)(1 + u_2) \cdots$ may be converted into an infinite series $1 + x_1 + x_2 + x_3 + \cdots$; where $u_n = x_n / (1 + x_1 + x_2 + \cdots + x_{n-1})$. The product converges only if the positive infinite series, $u_1 + u_2 + u_3 + \cdots$ converges.

PRODUCT-MOMENT. If x_1, x_2, \cdots, x_n are **random variables**, the product-moment of order (r_1, r_2, \cdots, r_n) is the expected value of

$$x_1^{r_1} x_2^{r_2} \cdots x_n^{r_n}.$$

For one variable this reduces to the definition of **moment**. For two variables measured

about their means, the first product-moment, $E(x_1x_2)$, is the **covariance**.

PRODUCT-MOMENT CORRELATION. See **correlation**.

PRODUCT OF INERTIA. See **inertia, moments and products of**.

PRODUCT OF TENSORS. If

$$a_{j_1j_2\cdots j_N}^{i_1i_2\cdots i_M} \quad \text{and} \quad b_{l_1l_2\cdots l_S}^{k_1k_2\cdots k_R}$$

are the components of two tensors (tensor fields) of orders $M + N$ and $R + S$ respectively in a generic n -dimensional coordinate system, then the quantities $a_{j_1j_2\cdots j_N}^{i_1i_2\cdots i_M} b_{l_1l_2\cdots l_S}^{k_1k_2\cdots k_R}$ are the components in the coordinate system of a tensor (tensor field) of order $M + N + R + S$, which is called the *product* (or *outer product*) of the two tensor (tensor fields). (See also **tensors, inner product of**.)

PROFILE DRAG OF A WING. The difference between the total **drag** of a wing and the **trailing-vortex drag**. The term is normally used only for speeds of flow so low that there are no regions of supersonic flow and hence no shock waves.

PROGRAM. A plan or design of a computational **routine** to be performed by a **stored-program computer**. Sometimes this term is loosely used interchangeably with **code**.

PROGRESSION. A simple type of **sequence** or **series**, the most common ones being known as arithmetic, geometric, and harmonic.

(1) *Arithmetic progression.* The form is $a, a + d, a + 2d, a + 3d, \dots$ where a is the first term and d is the constant difference between two successive terms. If there are n terms in the series, its sum $S_n = n(a + l)/2 = n[a + (n - 1)d]/2$ where l is the last term.

(2) *Geometric progression.* Its form is a, ar, ar^2, ar^3, \dots and its last term is $l = ar^{n-1}$. If written as $a(1 + r + r^2 + r^3 + \dots)$ its sum to n terms is $a(1 - r^n)/(1 - r)$. When $r < 1$, the infinite series **converges** and its sum is $a/(1 - r)$. The series diverges for other values of r . The n th root of the product of n positive quantities x_i is the geometric mean, $G = (x_1x_2x_3 \cdots x_n)^{1/n}$.

(3) *Harmonic progression.* The sequence a, b, c, \dots is a harmonic progression if the reciprocals $1/a, 1/b, 1/c, \dots$ form an arithmetic progression. The harmonic mean between two numbers is the middle term of a harmonic pro-

gression whose first and last terms are the given numbers. The harmonic mean between a and b is given by $H = 2ab/(a + b)$. If A, G , and H are respectively the arithmetic, geometric, and harmonic mean of two numbers then $G^2 = AH$. If the series is

$$\sum_{n=1}^{\infty} 1/n^r, \quad \text{with } r \text{ real,}$$

it converges for $r > 1$, but diverges for $r \leq 1$. This is often called the hyperharmonic series.

PROGRESSION OF BANDS. See **band system**.

PROJECTION ANGLE. The angle subtended at an axial image point by a radius of the exit pupil of an axially symmetric optical system. (Cf. **aperture angle**.)

PROJECTION, STEREOGRAPHIC OF A SPHERE. See **stereographic projection of a sphere**.

PROMPT NEUTRONS. See **neutrons, prompt**.

PROOF TEST. A test which subjects a structure or part to a load above the design load but below failure or distress so that the structure remains usable.

PROPAGATION CONSTANT. A complex quantity associated with harmonic plane waves, whose real part is the *attenuation constant*, and whose imaginary part is the *phase constant*. The attenuation constant is the logarithmic derivative of the amplitude of a field component in the direction of propagation. The phase constant is the derivative of the phase angle of a field component in the direction of propagation.

PROPELLER. See **airscrew**.

PROPER. In special **relativity** theory, the adjective proper is used to denote that a property or quantity is measured by an observer in a **Lorentz frame** in which the body being measured and/or the measuring devices are at rest. Examples are proper length, proper time, proper mass, proper density.

PROPER FRACTION. A fraction whose numerator is less in absolute value than its denominator.

PROPER LORENTZ TRANSFORMATION.

Any Lorentz transformation

$$x' = \Lambda x + a$$

for which $\det \Lambda = +1$. The set of all Lorentz transformations whose determinant is $+1$ form a group, the proper Lorentz group.

PROPER MOTION. See **stellar motions**.

PROPER SUBGRAPH. See **subgraph, proper**.

PROPERTY. See **thermodynamic property**.

PROPER VALUE. See **eigenvalues and eigenvectors**.

PROPER VECTOR OF A MATRIX. See **principal vector of a matrix and eigenvalues and eigenvectors**.

PROPORTIONAL CONTROL. See **control proportional, derivative and integral**.

PROPORTIONAL LIMIT. See **limit, proportional**.

PROPORTIONAL LOADING. All loads on a body are kept in constant ratio with each other; $P_i = KP_i^0$ where P_i^0 is a fixed system of loads and K varies with time. (See also **radial loading**.)

PROPORTION OF POLARIZATION. Light is only partially polarized when it is passed through a pile of plates. The proportion of polarization is given by:

$$PP = \frac{I_p - I_s}{I_p + I_s}$$

where I_p is the intensity of the light vibrating parallel to the plane of incidence and I_s is the intensity of the light vibrating perpendicular to the plane of incidence. For a single surface the proportion of polarization is only about 8.1%. For m plates ($2m$ surfaces) of refractive index n ,

$$PP = \frac{m}{m + \left(\frac{2n}{1 - n^2}\right)^2}$$

PROTON. The nucleus of the hydrogen atom, and a constituent of all other nuclei. It is a positively charged elementary particle whose charge is equal in magnitude to the electronic charge. The number of protons in the nucleus of each atom of an element is equal to the

atomic number Z of the element. It has a rest mass of 1.67×10^{-24} gm, or 1.0075 amu; magnetic moment, 2.79245 nuclear magnetons, or 1.52×10^{-3} dyne cm gauss $^{-1}$; spin quantum number $\frac{1}{2}$; and Fermi-Dirac statistics.

PSEUDO-ADIABATIC EXPANSION. A **saturation-adiabatic process** in which the condensed water substance is removed from the system, and therefore best treated by the thermodynamics of **open systems**. Meteorologically, this process corresponds to rising air from which the moisture is precipitating. Descent of air so lifted becomes by definition a **dry-adiabatic process**.

PSEUDOSCALAR. A scalar quantity which changes its sign when the coordinate system, to which the quantity is referred, is changed from a right-handed to a left-handed one, or vice versa. An example is the scalar product of a polar vector and a **pseudovector**. A typical pseudoscalar in 3-dimensional space is the triple scalar product of three polar vectors **A**, **B**, **C**, i.e., $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$.

PSEUDOSCALAR COUPLING. Type of interaction energy postulated between a π -meson and a nucleon which consists of the product of the pseudoscalar field of the π -meson and a bilinear pseudoscalar function of the nucleon wave-functions.

PSEUDOVECTOR. The **vector** product of two vectors does not completely satisfy the formal requirements of a vector for it does not change sign if the **coordinate system** is changed from a right-handed to a left-handed one, or vice versa. It is a typical example of a pseudovector (also called an axial vector). Thus the vector for an element of area, represented by the vector product $d\mathbf{S} = d\mathbf{x} \times d\mathbf{y}$ is not determined with respect to direction unless an arbitrary convention is established for the positive side of the surface element. The three components of a pseudovector are actually the components of a three-dimensional antisymmetric **tensor** of second rank. Physical quantities which are pseudovectors include angular momentum (vector product of momentum and radius vector); moment of a force (vector product of force and distance).

The multiplication relations of pseudovectors and (true) vectors are as follows (Various products of (true) vectors which are discussed at length in the article on **vector multiplia-**

tion are repeated here, to contrast them with the pseudovector products):

The product of a true scalar and a true vector is a true vector.

The product of a true scalar and a pseudovector is a pseudovector.

The product of a pseudoscalar and a true vector is a pseudovector.

The scalar product of two true vectors is a true scalar.

The scalar product of a true vector and a pseudovector is a pseudoscalar.

The vector product of two true vectors is a pseudovector.

The vector product of two pseudovectors is a pseudovector.

The vector product of a true vector and a pseudovector is a true vector.

The gradient of a true scalar is a true vector.

The gradient of a pseudoscalar is a pseudovector.

The curl of a (true) vector is a pseudovector.

The curl of a pseudovector is a (true) vector.

PSEUDOVECTOR COUPLING. Type of interaction energy postulated between a nucleon and other particles in which a bilinear pseudovector function of the nucleon wavefunctions appears.

PSYCHROMETRIC FORMULA. The semi-empirical relation giving the **vapor pressure** in terms of the barometer and psychrometer readings. For Fahrenheit temperatures, this formula is

$$e = e' - \left[3.67 \times 10^{-4} \left(1 + \frac{t' - 32}{1571} \right) \right] p(t - t'),$$

where t is the dry-bulb temperature, t' the wet-bulb temperature, e' the saturation vapor pressure at t' , p the barometric pressure, and e the vapor pressure; all pressure units being the same.

(For details and tables see: List, R. J., Ed., *Smithsonian Meteorological Tables*, 6th rev. ed., 1951.)

PSYCHROMETRIC TABLES. Tables prepared from the **psychrometric formula** and used to obtain **vapor pressure**, **relative humidity**, and **dew point** from values of wet-bulb and dry-bulb temperature.

PULSATANCE. Product of 2π and the frequency in cycles per second. It is expressed in radians per second. (See **oscillation, frequency of**.)

PULSATING STARS. See **variable stars**.

PULSE. (1) A variation of a quantity whose value is normally constant; this variation is characterized by a rise and decay, and has a finite duration. (2) A waveform whose duration is short compared to the time-scale of interest, and whose initial and final values are the same. The word "pulse" normally refers to a variation in time; when the variation is in some other dimension, it should be so specified, such as "space pulse." This definition is broad so that it covers almost any transient phenomenon. The only features common to all pulses are rise, finite duration, and decay. It is necessary that the rise, duration, and decay be of a quantity that is constant (not necessarily zero) for some time before the pulse and has the same constant value for some time afterwards. The quantity has a normally constant value and is perturbed during the pulse. No relative time scale can be assigned.

PULSE AMPLITUDE. A general term indicating the magnitude of a **pulse**. For specific designation, adjectives such as average, instantaneous, peak, rms (effective), etc., should be used to indicate the particular meaning intended. Pulse amplitude is measured with respect to the normally constant value, unless otherwise stated.

PULSE AMPLITUDE, AVERAGE. The average of the instantaneous amplitude taken over the **pulse duration**.

PULSE AMPLITUDE, AVERAGE ABSOLUTE. The average of the absolute value of the instantaneous amplitude taken over the **pulse duration**.

PULSE AMPLITUDE, PEAK. The maximum absolute peak value of the **pulse**, excluding those portions considered to be unwanted, such as spikes. Where such exclusions are made, it is desirable that the amplitude chosen be illustrated pictorially.

PULSE AMPLITUDE, RMS (EFFECTIVE). The square root of the average of the square

of the instantaneous amplitude, taken over the **pulse duration**.

PULSE BANDWIDTH. The smallest continuous frequency interval outside of which the amplitude of the spectrum does not exceed a prescribed fraction of the amplitude at a specified frequency. Caution—This definition permits the spectrum amplitude to be less than the prescribed amplitude within the interval. Unless otherwise stated, the specified frequency is that at which the spectrum has its maximum amplitude. This term should be "Pulse Spectrum Bandwidth" because it is the spectrum and not the pulse itself that has a bandwidth. However, usage has caused the contraction.

PULSE CARRIER, CREST FACTOR OF. The ratio of the peak pulse amplitude (see **pulse amplitude, peak**) to the root-mean-square amplitude. (See **pulse amplitude, rms**.)

PULSE DECAY TIME. The interval between the instants at which the instantaneous amplitude last reaches specified upper and lower limits, namely, 90 per cent and 10 per cent of the peak-pulse amplitude (see **pulse amplitude, peak**) unless otherwise stated.

PULSE DURATION. (1) The time interval between the first and last instants at which the instantaneous amplitude reaches a stated fraction of the peak pulse amplitude. (See **pulse amplitude, peak**.) (2) The duration of a rectangular pulse whose energy and peak power equal those of the pulse in question. When determining the peak power, any **transients** of relatively short duration are frequently ignored.

PULSE DUTY FACTOR. The ratio of the average **pulse duration** to the average **pulse spacing**. This is equivalent to the product of the average pulse duration and the **pulse repetition rate**.

PULSE INTERVAL. See **pulse spacing**.

PULSE MODE. (1) A finite sequence of **pulses** in a prearranged pattern, used for selecting and isolating a communication channel. (2) The prearranged pattern.

PULSE REPETITION FREQUENCY. The **pulse repetition rate** of a periodic pulse train.

PULSE REPETITION PERIOD. The reciprocal of the **pulse repetition frequency**

PULSE REPETITION RATE. The average number of **pulses** per unit of time.

PULSE RISE TIME. The interval between the instants at which the instantaneous amplitude first reaches specified lower and upper limits, 10 per cent and 90 per cent of the peak-pulse amplitude (see **pulse amplitude, peak**) unless otherwise stated.

PULSE SEPARATION. The interval between the trailing-edge pulse-time of one **pulse** and the leading-edge pulse-time of the succeeding pulse.

PULSE SPACING. The interval between the corresponding pulse times of two consecutive pulses, sometimes called the *pulse interval*.

PULSE TIME, LEADING-EDGE. The time at which the instantaneous amplitude first reaches a stated fraction of the peak pulse amplitude. (See **pulse amplitude, peak**.)

PULSE TIME, MEAN. The arithmetic mean of the leading edge pulse time and the trailing edge pulse time. For some purposes the importance of a **pulse** is that it exists (or is large enough) at a particular instant of time. For such applications the important quantity is the mean pulse time. The leading edge pulse time and trailing edge pulse time are significant primarily in that they may allow a certain tolerance in timing.

PULSE TIME, TRAILING-EDGE. The time at which the instantaneous amplitude last reaches a stated fraction of the peak pulse amplitude. (See **pulse-amplitude, peak**.)

PURE HOMOGENEOUS DEFORMATION. A homogeneous deformation in which a particle initially at X_i in a rectangular Cartesian reference system x moves to x_i in the same system, where

$$x_i = a_{ij}X_j + b_i, \quad a_{ij} = a_{ji},$$

b_i and a_{ij} are constants and the **indicial notation** and **summation convention** are used. By a suitable choice \bar{x} , say, of the rectangular Cartesian reference system, the deformation may be described by

$$\bar{x}_1 = \lambda_1 \bar{X}_1, \quad \bar{x}_2 = \lambda_2 \bar{X}_2 \quad \text{and} \quad \bar{x}_3 = \lambda_3 \bar{X}_3,$$

where \bar{X}_i and \bar{x}_i are the coordinates in the system \bar{x} corresponding to X_i and x_i respectively in the system x . Then, λ_1 , λ_2 , and λ_3 are the solutions for λ of the determinantal equation.

$$|a_{ij} - \lambda\delta_{ij}| = 0$$

in which δ_{ij} denotes the Kronecker delta, λ_1 , λ_2 and λ_3 are called the *principal extension ratios* for the pure homogeneous deformation. Sometimes the absolute values of λ_1 , λ_2 and λ_3 , rather than their actual values are called the principal extension ratios. The directions of the axes of the system x are called the *principal directions* of the pure homogeneous deformation.

PURE PURPLES. Color stimuli represented on the **chromaticity diagram** by the straight line joining the ends of the spectrum locus.

PURE ROTATION SPECTRUM. See rotation spectrum of molecules.

PURITY, COLORIMETRIC. See colorimetric purity.

PURITY, EXCITATION. See excitation purity.

p , v , T RELATION. See equation of state.

P WAVE. Seismological name for a dilatational elastic wave. (See **wave(s)**, dilatational.)

PYTHAGOREAN SCALE. See scale, Pythagorean.

PYTHAGOREAN THEOREM. See Bessel inequality.

Q

Q. (1) A **figure of merit** of an energy-storing system equal to

$$2\pi \left(\frac{\text{average energy stored}}{\text{energy dissipated per half cycle}} \right),$$

which is equal to $\omega L/R$ for an inductor, where R is the equivalent series resistance. For a capacitor, Q is $1/\omega CR$, again the ratio of reactance to effective resistance. For a medium, Q is the ratio of displacement **current density** to conduction current density. (2) See **q factor**.

Q FACTOR. A measure of the sharpness of resonance of a resonant system (see **resonance**, **sharpness of**), often referred to simply as the "Q" of the system.

Q_M. In the theory of the slowing down of neutrons through collisions with matter, the maximum lethargy loss possible for a neutron colliding with a stationary, free atom of mass, M .

$$\exp(-q_M) = \left(\frac{M-1}{M+1} \right)^2.$$

QUADRATIC DIFFERENTIAL FORM. Differential form of degree two.

QUADRATURE, NUMERICAL. A method for the (approximate) evaluation of a definite integral of a function of a single variable, the term cubature being used for two variables. The most obvious procedure is to obtain some sufficiently close **approximation** to the integrand $f(x)$, whether by **interpolation**, by **least squares**, or otherwise, but in terms that lend themselves readily to analytic integration, and to integrate the result. Any formulas derivable by equal-interval interpolation can be derived also by the application of **difference operators**. To do this, let $F(x)$ be the indefinite integral, whence

$$\int_a^b f(x)dx = F(b) - F(a)$$

and apply the operator $\exp \theta$ to $F(x)$. A

somewhat more sophisticated and more general approach is to seek an approximation of the form

$$\int_a^b f(x)w(x)dx = \sum_0^n \mu_i f(x_i),$$

where the μ_i are constant coefficients, independent of the function f , and where the expression on the right is to equal the integral on the left exactly whenever $f(x)$ is a polynomial of some maximal degree. When the x_i are specified in advance, this degree can be no greater than n ; if they are left free it can be of degree $2n+1$, and the result is a **Gaussian quadrature formula**. More general formulas, not based upon polynomial representations, are also possible. (See Alston S. Householder, *Principles of Numerical Analysis*, McGraw-Hill, 1953. For special formulas see quadrature formulas under the names **Chebyshev**; **Euler-Maclaurin**; **Gaussian**; **three-eighths rule**; **Simpson rule**; **trapezoidal rule**; **Weddle rule**; see also **remainder formulas**.)

For solving **ordinary differential equations** the integrand is of the form $f(x, y)$, where $y = y(x)$ is the dependent variable to be determined, and considered as a function of x ,

$$f[x, y(x)] = f(x),$$

it is to be integrated over a range beyond that for which its values are known. Most often it is necessary to express

$$\int_{x_\nu}^{x_{\nu+n}} f(x)dx$$

in terms of values of f at points x_j , for $j < \nu + n$, in order to obtain an approximate value of $y_{\nu+n}$. Such a formula is called a predictor, or an open quadrature formula. To simplify the formulas let $\nu = 0$. Given an approximation to $y_{\nu+n}$, one applies a corrector, which is a quadrature formula expressed also in terms of $f_{\nu+n}$.

There is no restriction in taking $\nu = 0$. To obtain a predictor formula, one applies to $F(x_{n-1})$ the operator

$$E - E^{-(n-1)} = e^\theta - e^{-(n-1)\theta}$$

$$= n\theta - [(n-1)^2 - 1]\theta^2/2! + \dots,$$

and hence to $f(x_{n-1})$ the operator

$$nh[1 - (n-2)\theta/2! + (n^2 - 3n + 3)\theta^2/3! - \dots],$$

thereafter substituting

$$\theta = -\ln(1 - \nabla)$$

and retaining as many powers of ∇ in the expansion as may be required. This is the basis for the **Adams-Bashford method**. To obtain the corrector one applies to $F(x_n)$ the operator $1 - E^{-n}$.

QUADRIC, CENTRAL. See **central quadric**.

QUADRIC SURFACE. Any of the **surfaces** represented in **Cartesian coordinates** by an algebraic **equation** of the second degree in three variables

$$Ax^2 + By^2 + Cz^2 + 2Fyz + 2Gxz + 2Hxy$$

$$+ 2Px + 2Qy + 2Rz + D = 0.$$

Every **plane** section of a quadric surface is a **conic section**.

When squared terms only appear in the equation, so that its form is $Ax^2 + By^2 + Cz^2 = 1$, one speaks of a central quadric. Thus the surface is symmetric about the coordinate origin and can be an ellipsoid or a hyperboloid. The other quadric surfaces: a paraboloid and the degenerate quadrics are non-central quadrics.

QUADRUPLE POINT. The temperature at which four **phases** are in **equilibrium**. An example is the case in which an anhydrous salt, a hydrate of the salt, its saturated solution, and its water vapor are in equilibrium in a salt-water system.

QUADRUPLE PRODUCT OF VECTORS. If **A, B, C, D** are any four vectors, two types of **products** are possible:

$$(1) \quad (\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D})$$

$$= (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C});$$

$$(2) \quad (\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D})$$

$$= (\mathbf{A} \cdot \mathbf{C} \times \mathbf{D})\mathbf{B} - (\mathbf{B} \cdot \mathbf{C} \times \mathbf{D})\mathbf{A}.$$

The latter equation is also conveniently written in the equivalent form

$$[\mathbf{ACD}]\mathbf{B} - [\mathbf{BCD}]\mathbf{A} = [\mathbf{ABD}]\mathbf{C} - [\mathbf{ABC}]\mathbf{D},$$

where the bracket signifies the scalar **triple product**. (See also **vector system, reciprocal**.)

QUADRUPOLE. Let a collection of electric or magnetic charges be distributed around a point; for example, the center of mass of a system of atoms, molecules, or nuclei. The potential at a distance r from this point may be represented by an infinite series of terms in inverse powers of r . The term in the inverse first power is the Coulomb potential, the inverse second power term is the dipole potential, the inverse third power term is the quadrupole potential, etc. A typical example is an array of four charges of equal magnitude so spaced that they coincide with the vertices of a parallelogram. Charges located on opposite vertices are of the same sign; the distance of separation between charges is taken to be of the order of molecular or infinitesimal dimensions.

QUADRUPOLE MOMENT. When the radiation field due to a set of moving electric or magnetic charges is expanded in a series of powers of the product of the charges times space coordinates, the sum of the quadratic terms is the quadrupole moment.

QUADRUPOLE MOMENT (NUCLEAR). A measure of the deviation of the electric charge distribution of a nucleus from a spherical distribution. If M is the z -component of the total angular momentum \mathbf{J} of the nucleus, the quadrupole moment is defined as the expectation value of $\sum_p(3z_p^2 - r_p^2)$ in the substate having $M = J$, where the summation is over all the protons. It is commonly expressed in barns. Being a structure sensitive quantity, the nuclear quadrupole moment has been a valuable clue in the development of the theory of nuclear structure. The large quadrupole moments of many rare earth nuclei, in particular, have shown that nuclei are capable of acquiring large permanent deformations, leading to rotational spectra.

QUADRUPOLE RADIATION. Radiation emitted by a **quadrupole**. Since the selection rules were deduced by analogy between the behavior of a classical electric dipole and the emission of radiation by quantum transitions, then there are arrangements of two dipoles (e.g., a linear arrangement) in which the posi-

tive charges coincide and are located at the center of two negative charges separated by a distance $2x$ ($x =$ distance from center), so $\sum e_i x_i = 0$) when the quadrupole moment ($\sum e_i \cdot x_i^2$) not zero but varies as $1/r^3$, and therefore acts as a source of radiation.

QUANTIC. A homogeneous algebraic function of two or more variables, in general containing only positive integral powers of the variables, and usually a polynomial in several variables. Quantics are classified into quadratic, cubic, quartic, quintic, etc., according to degree, and into binary, ternary, quaternary, etc., according to the number of variables involved. (See also **multinomial series**.)

QUANTILE. If a set of N observations forming a frequency distribution are arranged in order of magnitude and every (N/p) th observation is marked off, these observations are called quantiles. Special cases are (1) $p = 2$, the central observation, called the **median**; (2) $p = 4, 5, 6, 10$ giving the quartiles, quintiles, sextiles, deciles; (3) $p = 100$, giving the centiles or percentiles.

QUANTIZATION. Transition from a classical theory or a classical quantity to a quantum theory or the corresponding quantity in quantum mechanics.

QUANTIZATION OF ELECTROMAGNETIC FIELD. In order to describe in a consistent manner the interaction between matter and radiation, it is necessary to quantize not only the motion of the material particles but also the electromagnetic field. (By "quantizing" is meant the consistent application of certain rules which lead from classical to quantum mechanics.) The reason for the necessity of quantizing the electromagnetic field lies in the fact that if the mechanical parameter (co-ordinates, momenta) of particles are to be quantized, then the fields coupled to them must also be quantized. This is because the **Heisenberg uncertainty principle** cannot possibly be valid for part of nature. For if the uncertainty principle did not apply to the field, then it has been shown by Bohr and Rosenfeld that it would be possible to set up a (gedanken) experiment which, in principle, would permit an accurate simultaneous determination of the position and momentum of a particle by observing and measuring the fields generated by the particle. Hence the

quantization of the electromagnetic field is an inescapable consequence of the quantization of particle dynamics.

The electromagnetic field is described by the electric and magnetic field intensity, $\mathbf{E}(\mathbf{x}, t)$ and $\mathbf{H}(\mathbf{x}, t)$ respectively. In the absence of charges their dynamic behavior is governed by Maxwell's equation

$$\nabla \times \mathbf{E} + \frac{1}{c} \frac{\partial \mathbf{H}}{\partial t} = 0 \quad \nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{H} - \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} = 0 \quad \nabla \cdot \mathbf{H} = 0.$$

By a Fourier analysis of the fields, the latter can be thought of as a superposition of monochromatic waves. The dynamic properties of such a monochromatic wave can be translated into that of a harmonic oscillator. Thus the dynamic behavior of the electromagnetic field can be transcribed into that of a doubly (due to the polarization) infinite set of harmonic oscillators. In the absence of charges and currents all these modes are uncoupled and the total field energy

$$E = \frac{1}{2} \int d^3 \times (\mathbf{E}^2 + \mathbf{H}^2)$$

is the sum of the energy of all these oscillators. The application of the quantum rules (replacement of the Poisson brackets of the canonical variables describing the system by commutation rules) to the electromagnetic field, i.e., the quantization of the electromagnetic field, is equivalent to the **quantization** of a set of **harmonic oscillators**. Hence, the energy in one monochromatic wave can only change by multiple of $\hbar\omega$, where ω is the frequency of the wave. Electromagnetic energy of a frequency ω must always appear in "quanta" of the size $\hbar\omega$. This is the **light quantum hypothesis**. Another important consequence of the quantization is the **zero point fluctuation(s)**. As a result of the uncertainty principle a harmonic oscillator even in its lowest energy state still has a finite amplitude of vibration. This fact when transcribed to the electromagnetic field implies that even in its lowest energy state, i.e., in the state in which there are no photons (light quanta) present, the electromagnetic field fluctuates about the mean value zero, but the mean square values of the field strengths are different from zero and do not vanish. The strength of the

field fluctuation in the vacuum, averaged over a volume V of linear dimensions a , is approximately given by

$$E_{fluct}^2 = H_{fluct}^2 \approx \hbar c/a^4.$$

These fluctuating fields will affect the motion of charged particles. A major part of the **Lamb shift** in a hydrogen atom can be understood as the contribution to the energy from the interaction of the electron with these zero-point oscillations.

QUANTIZATION OF SIGNALS. A process in which the range of values of a wave is divided into a finite number of smaller sub-ranges, each of which is represented by an assigned or "quantized" value within the sub-range. "Quantized" may be used as an adjective modifying various forms of modulation, for example, quantized **pulse-amplitude modulation**.

QUANTIZATION, SECOND. This is a method to describe the behavior of a quantum mechanical system of identical particles. Physical quantities correspond in the description to linear operators of a space the elements of which describe states of systems with different numbers of particles. Advantages of the method come from the fact that the states considered are in this way not restricted to a definite number of particles, but states of similar systems of different size are dealt with together. The elementary operations of the method are the **creation** and **annihilation operators** which change the number of particles by one unit. The creation and annihilation operators satisfy simple **commutation relations** in the case of a system of bosons, and simple **anticommutation relations** in the case of fermions. There is a simple correspondence between the description in terms of the second quantization method and the **configuration space** method in which the state of the system is described by a wave function depending on the coordinates of a definite number of particles.

QUANTIZATION, SPACE. See **space quantization**.

QUANTIZE. To restrict the possible values of a variable to a discrete number of possible values.

QUANTIZED FIELD THEORY. See **field theory, quantized; quantization of electromagnetic field**.

QUANTUM. An observable quantity is said to be quantized when its magnitude is, in some or all of its range, restricted to a discrete set of values. If the magnitude of the quantity is always a multiple of a definite unit, then that unit is called the quantum (of the quantity). For example, the quantum or unit of orbital angular momentum is \hbar (**Dirac** \hbar), and the quantum of energy of electromagnetic radiation of frequency ν is $h\nu$, where h is the **Planck constant**. In field theories, a field (or the field equations) is quantized by application of a proper quantum-mechanical procedure and this results in the existence of a fundamental field particle, which may be called the field quantum. Thus the **photon** is a quantum of the electromagnetic field and in nuclear field theories, the **meson** is considered to be the quantum of the nuclear field.

QUANTUM EFFICIENCY. A measure of the efficiency of conversion or utilization of light or other energy, being in general the ratio of the number of distinct events produced in a radiation sensitized process to the number of quanta absorbed (the intensity-distribution of the radiation in frequency or wavelength should be specified). In the **photoelectric** and **photoconductive effects**, the quantum efficiency is the number of electronic charges released for each **photon** absorbed. For a **phototube**, the quantum efficiency is defined as the average number of electrons photochemically emitted from the **photocathode** per incident photon of a given wavelength. In a photochemical reaction, the quantum efficiency is the ratio of the number of molecules transformed to the number of quanta of radiation absorbed.

QUANTUM ELECTRODYNAMICS. In classical physics the laws of an electromagnetic field are expressed by **Maxwell's equations** which determine the change of the field in space and time in the presence of a given charge and current distribution. The motion of a charged particle in a given electromagnetic field is described by the **Lorentz equation**. In quantum physics this description is to be modified to take into account the existence of quanta. Quantum electrodynamics is the subject which describes the interaction of

charged particles with the electromagnetic field according to the rules of quantum mechanics.

In the absence of charges the classical Maxwell equations in vacuum remain valid in the quantized theory. The field quantities, the electromagnetic potential $A_\mu(x, y, z, t)$ and the field strength $F_{\mu\nu}(x, y, z, t)$, $\mu, \nu = 0, 1, 2, 3$, $F_{\mu\nu} = -F_{\nu\mu}$ of the classical equations become, however, operators in the quantum theory. The Maxwell equations are valid between these operators which satisfy simple **commutation relations**. One obtains in this way a quantum theory of radiation, in which the energy of the radiation field has eigenvalues

$$\sum_k m_k h\nu_k.$$

The numbers m_k are integers and represent the number of quanta $h\nu_k$ of frequency ν_k present, h is Planck's constant.

The motion of an electron moving in a given electromagnetic field can be described in a relativistic way by the **Dirac equation**. By replacing the wave function by an operator $\psi(x, y, z, t)$ in this equation, one can describe a system of electrons and positrons moving independently in the given external field. The electron field operators $\psi(x, y, z, t)$ and their **adjoints** satisfy simple anticommutation relations which express the exclusion principle for the electrons. The current due to the electrons and positrons is given by an operator expressed by $\psi(x, y, z, t)$ and its adjoint and is the source of an electromagnetic field. If one inserts this current operator into the Maxwell equations, and consider A_μ and ψ both as operators, the Dirac equation and Maxwell equations form a coupled set of operator equations, and are the basic equations of quantum electrodynamics.

This (system of equations) is non-linear in the operator fields A_μ and ψ and can be approached only by approximation methods. As the coupling between the radiation field and the electrons is weak, **perturbation methods** lead to good theoretical predictions. One can describe the emission and absorption of electromagnetic radiation, different scattering processes, the creation and annihilation of electron-positron pairs and many other phenomena.

In the course of calculations with the field operators, mathematical complications are met which are related to the appearance of divergent integrals. For a long time this stood in the way of calculating any higher order cor-

rections to electromagnetic processes. In recent years, quantum electrodynamics has undergone a considerable development. Many new techniques and concepts have been introduced, through the work of **Feynman, Schwinger, Dyson** and others. One of the results of this development is that one can give unambiguous prescriptions for obtaining finite predictions concerning electromagnetic corrections. The two most important examples are perhaps the correction to the energy levels of the hydrogen atom due to the interaction of the electron with the radiation field, which is called the **Lamb shift**, and the change in the magnetic moment of the **electron** due to radiation corrections. In both cases there is an excellent agreement between theory and experiment to a very high accuracy.

QUANTUM FIELD THEORY AND S-MATRIX THEORY. See **S-matrix theory** and **quantum field theory**.

QUANTUM FIELD THEORY, RELATIVISTIC. See **relativistic quantum field theory**.

QUANTUM-MECHANICAL PERTURBATION THEORY. See **perturbation theory, quantum-mechanical**.

QUANTUM-MECHANICAL RESONANCE. See **resonance, quantum-mechanical**.

QUANTUM-MECHANICAL SYSTEM. Any physical system or model for which the application of classical mechanics yields non-valid results and which can be treated by the application of **quantum mechanics**.

QUANTUM MECHANICS. It is one of the great discoveries of the twentieth century that the various atomic phenomena are governed by the laws of a mechanics which is different from Newton's classical mechanics, and incorporates into a consistent mathematical scheme both the laws of the old classical mechanics and the existence of quanta. In engineering problems, and whenever the effect of the existence of quanta is negligible, Newton's mechanics remains valid. However, in most atomic phenomena the role of Planck's constant h becomes important, and the description needs the new mechanics, quantum mechanics. The development of the new mechanics is linked with the names of Schrödinger, Heisenberg, Dirac, and many others. The new disci-

pline was discovered in the years 1925–28, and has been developing since, finding an ever-increasing field of application.

Historically, the development came from two rather different directions. **Schrödinger** discovered his wave equation which is based on Louis de Broglie's idea that the motion of atomic particles is related to matter waves. The state of an atomic system is described by a wave function which satisfies the wave equation. These wave functions play a dominant role in the theory, and the new mechanics acquired the name wave mechanics. Classical mechanics is a limiting case of wave mechanics in an analogous way to that in which geometrical optics is a limiting case of classical wave optics. Heisenberg arrived at a correct description of quantum phenomena by introducing **matrices**, or **linear operators**, to describe dynamical quantities. The **eigenvalues** of these matrices represent the possible values of the physical quantities.

One obtains in this way a simple representation of quantities which can have only definite quantized values, like the energy of an atom. This form of the new mechanics acquired the name matrix mechanics or quantum mechanics. The two approaches, Schrödinger's wave mechanics and Heisenberg's quantum mechanics, turned out, however, to represent different aspects of the same theory. The wave functions of wave mechanics can be considered as elements of a **function space** the linear operators of which have the properties postulated in Heisenberg's matrix mechanics. Quantum mechanics and wave mechanics are used today as synonymous expressions.

The **Hamiltonian function** of classical mechanics corresponds in quantum mechanics to an operator H , the eigenvalues of which give the possible energy values E of the system, $H\psi = E\psi$. In the wave mechanical form, this Hamiltonian operator H is obtained from the classical Hamiltonian function $H = H(x,p)$ by replacing the momenta p by differential operators

$$p = \frac{h}{2\pi i} \frac{\partial}{\partial x}$$

with respect to the canonically conjugate coordinates x . In the simplest case of a single particle of mass m moving in a given external potential $V(x,y,z)$, the classical Hamiltonian function is

$$H = \frac{1}{2m} (p_x^2 + p_y^2 + p_z^2) + V(x,y,z).$$

The corresponding wave mechanical eigenvalue equation, $H\psi = E\psi$, is the wave equation

$$\left\{ -\frac{h^2}{8\pi^2 m} \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) + V(x,y,z) \right\} \psi(x,y,z) = E\psi(x,y,z).$$

This is, for instance, the wave equation of an electron with coordinates x,y,z , moving in the **Coulomb field** of a proton, as in the hydrogen atom. The state of the electron is described by the wave function $\psi(x,y,z)$ and the eigenvalues E of the equation give the energy levels of the hydrogen atom. The wave functions corresponding to a definite energy value, E , represent standing waves. The quantization of the energy, the existence of discrete energy levels in the atom, results from the wave equation in an analogous way to the definite value of sound frequencies on a violin string.

The wave function $\psi(x,y,z)$ replaces the concept of the classical orbit of the electron in the description; in the case of the hydrogen atom the elliptical orbits of the Bohr model. It does not make any sense to inquire about the exact position of an electron at a given time in a state of definite energy. One can only give the probability of finding the electron in a volume element $d\Omega$ near the point x,y,z . In the state given by the wave function $\psi(x,y,z)$, this probability is $|\psi(x,y,z)|^2 d\Omega$. The integral of this expression over the whole space, the probability of finding the electron somewhere in space, has to be equal to one. This gives a normalization condition on the wave function $\psi(x,y,z)$.

The momentum operator $p = \frac{h}{2\pi i} \frac{\partial}{\partial x}$ and the coordinate x satisfy the commutation relation $xp - px = ih/2\pi$. Expressing any dynamical quantity in classical mechanics as a function of coordinates and momenta, and replacing x and p in a suitably symmetrized form of the expression, by operators satisfying the commutation relation $xp - px = ih/2\pi$, one obtains the corresponding quantum mechanical operators. In general, eigenvalues of two operators A and B correspond to the same state only if A and B commute, $AB - BA = 0$. These eigenvalues correspond to the possible values

of the related physical quantities, and two physical quantities are simultaneously measurable only if their operators commute. In the previous example of the hydrogen atom, the Hamiltonian operators representing the energy and the coordinates of an electron are non-commuting quantities.

With the help of quantum mechanics one can obtain a large number of quantitative predictions about phenomena in atoms, molecules, solids; in nuclear physics; and many other branches of physics. In many cases it helps to form qualitative ideas about a physical problem, in other cases the complexity of the mathematical problem involved needs new techniques in order to make further advance.

QUANTUM MECHANICS (NON-RELATIVISTIC). That form of quantum mechanics which does not take into account relativistic effects. It is valid whenever all velocities are small in comparison with the velocity of light. It is a subject the principles of which are much more reliably established than these of relativistic quantum mechanics. In the entry **quantum mechanics** the non-relativistic form of quantum mechanics is assumed.

QUANTUM MECHANICS (RELATIVISTIC). Quantum mechanics in which the requirements of the special theory of **relativity** are taken into account. Its equations are invariant with respect to **Lorentz transformations**. Particles of spin $\frac{1}{2}$ can be described in a relativistic way by the **Dirac equation**, particles of 0 spin by the **Klein-Gordon equation**. The application of the Dirac equation to the problem of the hydrogen atom gives relativistic corrections to the energy levels and describes their **fine structure**. The relativistic description of a particle in quantum mechanics is closely related to the possibility of creation and annihilation of pairs of particles and of other changes in the number of particles, and an adequate interpretation requires the description in terms of systems of particles. **Quantum electrodynamics** and **quantum field theory** deal with this problem, among others.

QUANTUM MECHANICS AND RELATIVISTIC INVARIANCE. See **relativistic invariance** and **quantum mechanics**.

QUANTUM MECHANICS AND REPRESENTATION OF GROUPS. See **representation of groups** and **quantum mechanics**.

QUANTUM MECHANICS AND SYMMETRY. See **symmetry** and **quantum mechanics**.

QUANTUM NUMBER. A number assigned to each of the various values of a quantized quantity in its discrete range. The **quantum numbers** arise from the mathematics of the eigenvalue problem and may be related to the number of **nodes** in the **eigenfunction**. A state may be described by giving a sufficient set of compatible quantum numbers. In the customary formulations, each quantum number is either an integer (which may be positive, negative or zero) or an odd half-integer. (See entries following.)

QUANTUM NUMBER, EFFECTIVE PRINCIPAL. See **effective principal quantum number**.

QUANTUM NUMBER, ISOTOPIC SPIN. A nuclear **quantum number** based upon the concept that the proton and the neutron are different states of the same elementary particle, the nucleon. The nucleon is assigned an isobaric spin quantum number of $\frac{1}{2}$, and its two possible orientations, $\tau_z = +\frac{1}{2}$ and $-\frac{1}{2}$, are assigned to the neutron and proton, respectively. The isobaric spin vectors of all nucleons in a given nucleus combine as do angular momentum vectors, to give a total isobaric spin vector T , which is equal to $\sum \tau$; and it has an orientation T_z , which is equal to $\sum \tau_z$. The symbolic space in which these orientations occur is called isobaric space; its Z -axis corresponds to the direction of observable (negative) charge. Nuclei with a common value of T_z have the same neutron excess, equal to $2T_z$, by the relationship:

$$T_z = \frac{1}{2}(N - Z)$$

where N is the number of neutrons in the nucleus and Z is its number of protons. Both T and T_z are integral for even values of A , and half-integral for odd values.

QUANTUM NUMBERS OF THE INDIVIDUAL ELECTRONS IN AN ATOM. To a first approximation each electron in an atom can be considered as moving independently of the other electrons, in a centrally symmetric but, in general, non-Coulomb field, resulting from the overlapping of the Coulomb field of the nucleus and the mean field of the other electrons. To this approximation each elec-

tron is characterized by four quantum numbers, as follows:

Principal quantum number n . Approximate measure of the region in which the electron preferably remains. Different values of n (1, 2, 3, ...) correspond to widely different energy values.

Azimuthal quantum number l . Corresponding to the angular momentum of the electron in its orbit. Can take on all integral values $l = 0, 1, \dots, n - 1$ for a given value of n . In general, the energy difference between electrons with different values of l but the same value of n is smaller than the difference between electrons with different values of n .

Magnetic quantum number m_l . Corresponding to the component of the electronic orbital angular momentum in the direction of an external magnetic field. Can take on the values $m_l = l, l - 1, l - 2, \dots, -l$ for a given value of l .

Magnetic quantum number m_s . Corresponding to the component of the electron spin in the direction of an external magnetic field. Can take on the values $+\frac{1}{2}$ and $-\frac{1}{2}$.

In some cases it is more suitable to use instead of the magnetic quantum numbers m_l and m_s the following two quantum numbers:

j , corresponding to the total angular momentum of the electron (spin plus orbital motion). Can take on the values $l + \frac{1}{2}$ and $l - \frac{1}{2}$.

m_j , corresponding to the component of the total angular momentum of the electron in the direction of an external magnetic field. Can take on the values $m_j = j, j - 1, \dots, -j$.

QUANTUM STATISTICS. The diffractive quantum effects which lead to energy levels are taken into account by using, for instance, in **partition functions**, sums over energy levels rather than integrals over phase space. The symmetry effects are taken into account by using **Bose-Einstein** or **Fermi-Dirac statistics** rather than **Boltzmann statistics**.

In the case of independent particles the **Boltzmann distribution** is replaced by either the **Bose-Einstein** distribution,

$$f(\epsilon) = \frac{1}{e^{-\nu+\beta\epsilon} - 1},$$

or by the **Fermi-Dirac** distribution

$$f(\epsilon) = \frac{1}{e^{-\nu+\beta\epsilon} + 1}, \quad (1)$$

where $f(\epsilon)\rho(\epsilon)d\epsilon$ is the number of particles with energies between ϵ and $\epsilon + d\epsilon$, $\rho(\epsilon)$ the density of energy levels, $\beta = 1/kT$ (k : Boltzmann's constant, T : absolute temperature), and $\nu = \beta g$ (g : partial **thermal potential**).

Of some interest is the case of a **degenerate gas** which occurs in the Bose-Einstein case when $\nu \rightarrow 0$. In that case the so-called **Einstein condensation** occurs. In the **Fermi-Dirac** case degeneracy occurs when $e^{-\nu} \rightarrow \infty$. This happens at sufficiently low temperatures. In fact, the "gas" of the conduction electrons in a metal is practically completely degenerate well above room temperature. In a degenerate gas, the distribution (1) is such that $f(\epsilon) \doteq 1$ for all ϵ up to the **Fermi level** ϵ_0 , and $f(\epsilon) \doteq 0$ for all energies above the Fermi-level.

In the case of interacting particles one uses **ensemble** theory where now the **density matrix** replaces the classical **ensemble density**.

(See also **Bose-Einstein statistics** and **Fermi-Dirac statistics**.)

QUANTUM THEORETICAL ADDITION OF ANGULAR MOMENTUM VECTORS. See **angular momentum vectors**, **quantum theoretical addition of**.

QUANTUM THEORY OF DISPERSION. The dispersion of light in a dielectric medium can be described in quantum theory by means of perturbation methods. The electromagnetic field of the light wave polarizes the atoms of the medium, and the refractive index of light waves of a given frequency can be calculated from the polarizability of the atoms. Historically, this problem played an important part, as the derivation of the dispersion formula of Kramers and Heisenberg was one of the first steps in the development of **quantum mechanics**. The electric field of a light wave of frequency ω induces a dipole moment per atom proportional to the field. The constant of proportionality gives the polarizability of the atom. This is given by the Kramers-Heisenberg dispersion formula as

$$\alpha = \sum_k \frac{f_k e^2 / m}{\omega_k^2 - \omega^2}$$

with

$$f_k = \frac{4\pi m}{e^2 \hbar} \omega_k |M_{k0}|^2,$$

where $\hbar\omega_k$ are the energy levels of the atom which is assumed to be in the state $k = 0$ in the

absence of the field, M_{ko} is the matrix element of the electric moment between states k and o , m is the mass, e , the charge of an electron. A classical oscillator of a single frequency ω_o gives a polarizability

$$\alpha = \frac{e^2/m}{\omega_o^2 - \omega^2 + i\omega g}$$

and the quantum formula is a linear combination of such terms. The imaginary term $i\omega g$ determines the absorption of light waves. A similar term appears in the more exact quantum mechanical formula in which g is related to the **width** of the energy levels.

QUANTUM THEORY OF HEAT CAPACITY. A theory which explains on the basis of energy quantization the decrease of specific heats at low temperatures to values below their classical values. For the case of regular solids, the **Debye** theory of specific heat forms a good approximation.

QUANTUM THEORY OF MAGNETISM. See magnetism, quantum theory of.

QUANTUM THEORY OF PARAMAGNETISM. See paramagnetism, quantum theory of.

QUANTUM THEORY OF RADIATION. See quantum electrodynamics.

QUANTUM THEORY OF SPECTRA. The present theory of spectra, which is based on an idea that there exist in each atom or molecule certain permitted energy levels. An atom or molecule absorbs or radiates energy as it moves from one energy level to another. The frequency (ν) of the radiation associated with such change of energy level is given by

$$E_1 - E_2 = h\nu$$

E_1, E_2 are the energy levels and h is the **Planck constant**.

QUANTUM, VIRTUAL. See virtual quantum.

QUANTUM YIELD. The number of photon-induced reactions of a specified type per photon absorbed. In the **photoelectric effect**, the quantum yield is more commonly called the **photoelectric efficiency**. In photochemistry it is the ratio of the number of reactions induced both directly and indirectly by light to the number of photons absorbed; a quantum yield greater than unity indicates a chain reaction.

In plant physiology it is the ratio of the number of reactions of the primary photochemical step of photosynthesis (as yet undetermined) to the number of photons absorbed.

QUARTER-CHORD POINT. At a section of an airfoil of chord c , the point on the chord line at a distance $c/4$ from the leading edge is known as the quarter-chord point. According to two-dimensional **thin airfoil theory** this is the position of the aerodynamic center (in subsonic flow). (See also **aerodynamic mean chord**.)

QUARTILE. One of a set of numbers (a **quantile**) on the random-variable axis which divide a **probability distribution** into four equal areas. The three quartile points which lie between the extremes of the distribution are designated as Q_1, Q_2, Q_3 and are defined in terms of the **distribution function** $F(x)$ as follows:

$$F(Q_1) = .25; \quad F(Q_2) = .50; \quad F(Q_3) = .75.$$

Thus Q_2 coincides with the **median**. In empirical **relative frequency** tables, the quartiles are estimated by interpolation.

The **interquartile range** $2Q$ is the distance from Q_1 to Q_3 ; half of this distance Q is called the **semi-interquartile range** (or **quartile deviation**) and is sometimes used as a crude measure of variability or spread.

QUASI-CHEMICAL APPROXIMATION. Statistical method of treating **cooperative phenomena** in **substitutional alloys** where the pairs of nearest neighbors in the lattice are considered as independent units.

QUASI-ERGODIC HYPOTHESIS. See ergodic and quasi-ergodic hypotheses.

QUASI FERMI LEVEL. See Fermi level, quasi.

QUASI-GEOSTROPHIC APPROXIMATION. (Also called *geostrophic approximation*, *pseudogeostrophic approximation*.) The use of the assumption of **geostrophic equilibrium** in certain contexts of the equations of motion, but not in others. This compromise arose from the fact that the horizontal **divergence** of the geostrophic wind grossly fails to estimate the real divergence, which is better represented by the field of vertical motion. When the divergence has been thus eliminated from the

vorticity equation, the assumption that all (remaining) horizontal wind velocities are geostrophic has greatly increased validity. The dynamical effect of this approximation is to eliminate or filter out the higher frequencies, such as those of all sound and gravity waves, from the system of equations, while retaining the frequencies associated with cyclonic-scale motions.

QUASI-HYDROSTATIC APPROXIMATION.

(Or *quasi-hydrostatic assumption*; also called *hydrostatic approximation*.) The use of the **hydrostatic equation** as the vertical **equation of motion**, thus implying that the vertical accelerations are small without constraining them to be zero. This compromise takes advantage of the smallness of organized vertical accelerations in cyclonic-scale motions while allowing theoretically for a realistic distribution of vertical velocities, which may be computed from the other equations of a closed system. Dynamically, the effect of the quasi-hydrostatic approximation is to eliminate or filter out the higher frequencies, corresponding to sound waves and certain (but not all) **gravity waves**, from the fundamental equations, while retaining the frequencies corresponding to cyclonic-scale motions.

QUASI-LAGRANGIAN COORDINATES.

A system of mixed Eulerian and Lagrangian coordinates. At least one coordinate of each fluid parcel must therefore be unvarying with time. Such a system takes advantage of the fact that in many atmospheric models there is one property (but not three properties) conserved in the motion. The most frequently used system of this type is the (x, y, θ) -system under adiabatic motions, where x and y are **Cartesian coordinates**, and θ the **potential temperature**. If water vapor in all phases is admitted to the system, **wet-bulb potential temperature** or a similar conservative temperature may be used.

QUATERNION. A generalization of the **complex number** is called a *hypercomplex* number. It has the form $Q_0 + Q_i e_i$, where Q_0 and Q_i are real and e_i is related to $\sqrt{-1}$. If $i = 1, 2, 3$ so that the hypercomplex number has four components, it is a special kind of four-dimensional **dyadic** and called a quaternion. In that case,

$$e_i^2 = -1; \quad e_i e_j = \epsilon_{ijk} e_k, \quad i \neq j,$$

where ϵ_{ijk} is a set of 27 quantities, which can take only the values 0, ± 1 . A given ϵ_{ijk} vanishes if two of its indices are identical; it equals $+1$, if the subscripts are obtained by an even permutation of (123) and -1 , if by an odd permutation.

Quaternions add like ordinary complex numbers but their multiplication is not, in general, commutative. Thus,

$$\begin{aligned} PQ &= (P_0 + P_i e_i)(Q_0 + Q_j e_j) \\ &= (P_0 Q_0 - P_i Q_i + P_0 Q_i e_i + Q_0 P_i e_i \\ &\quad + \epsilon_{ijk} P_i Q_j e_k), \end{aligned}$$

which does not equal QP unless $P_i = kQ_i$, where k is a real number.

It is often useful to write a quaternion in the form $Q = Q_0 + \mathbf{Q}$, where Q_0 is a scalar and \mathbf{Q} is a **vector** with components Q_1, Q_2, Q_3 . More generally let \mathbf{u} be a unit vector, then $Q = Q_0 + q\mathbf{u}$, showing the analogy between a quaternion and a complex number, for Q_0 is the real part of Q and q the imaginary part.

Consider in classical mechanics a rigid body rotated through an angle θ about a line through the origin of a rectangular coordinate system with direction angles a, b, c . The coordinates of a point before and after the rotation are conveniently given in terms of the Euler-Rodrigues parameters. They are $Q_0 = \cos \theta/2$, $Q_1 = \sin \theta/2 \cos a$, $Q_2 = \sin \theta/2 \cos b$, $Q_3 = \sin \theta/2 \cos c$. They form the components of a quaternion and $Q_0^2 + Q_1^2 + Q_2^2 + Q_3^2 = 1$. These parameters are also related to the **Euler angles**, the **Pauli spin matrices** of quantum mechanics, and the rotation **group** in group theory.

QUEUEING THEORY. See **queues, theory of**.

QUEUES, THEORY OF. That part of the theory of **stochastic processes** which deals with individuals lining up for service. Many industrial problems can be expressed in terms of queues, e.g., bottlenecks in production lines, congestion problems in traffic, servicing of customers at counters or stations, storage and stocking in warehouses, etc.

QUOTA SAMPLING. A method of drawing a sample which purports to make it more representative in the qualities under inquiry by forcing it to be representative in certain other respects. For example, a quota sample of human beings might be sought such that (a)

one half were male, (b) the proportions in income-groups were the same as those in the population, and so on.

QUOTIENT-DIFFERENCE ALGORITHM.

An algorithm due to Rutishauser and having numerous applications. Let

$$f(z) = s_0/z + s_1/z^2 + s_2/z^3 + \dots$$

represent a formal power series, and form the table

$s_1^{(0)}$									
$s_1^{(1)}$	$q_1^{(0)}$	$d_1^{(0)}$	$e_1^{(0)}$	$s_2^{(0)}$					
$s_1^{(2)}$	$q_1^{(1)}$	$d_1^{(1)}$	$e_1^{(1)}$	$s_2^{(1)}$	$q_2^{(0)}$				
$s_1^{(3)}$	$q_1^{(2)}$	$d_1^{(2)}$	$e_1^{(2)}$	$s_2^{(2)}$	$q_2^{(1)}$	$d_2^{(0)}$	$e_2^{(0)}$	$s_3^{(0)}$	
	$q_1^{(3)}$					$d_2^{(1)}$	$e_2^{(1)}$	$s_3^{(1)}$	
.....									

by the following rules:

$$s_1^{(\nu)} = s_\nu,$$

$$q_\sigma^{(\nu)} = s_\sigma^{(\nu+1)} / s_\sigma^{(\nu)}$$

$$d_\sigma^{(\nu)} = q_\sigma^{(\nu+1)} - q_\sigma^{(\nu)}$$

$$e_\sigma^{(\nu)} = d_0^{(\nu)} + e_{\sigma-1}^{(\nu+1)}, \quad e_0^{(\nu)} = 0$$

$$s_{\sigma+1}^{(\nu)} = s_\sigma^{(\nu+1)} \cdot e_\sigma^{(\nu)}.$$

If the entries of a column of q 's approaches a limit, this limit is a pole of $f(z)$ (see **Bernoulli method**). Among other properties of the table, only the following will be mentioned here, that

$$f(z) = s_0\sqrt{z} - q_1^{(0)}\sqrt{1} - e_1^{(0)}\sqrt{z} - q_2^{(0)}\sqrt{1} - \dots$$

is a particular **continued fraction** expansion of $f(z)$, and others are readily obtainable. (See National Bureau of Standards, *Further contributions to the solution of simultaneous linear equations and the determination of eigenvalues*, NBS Appl. Math. Series 49, 1958.)

QUOTIENT GROUP. See **factor group**.

QUOTIENT LAW FOR TENSORS. Systems $a_{j_1 j_2 \dots j_N}^{i_1 i_2 \dots i_M}$ are given in each coordinate system. If, using the **summation convention**, $v^{j_1} b_{j_1 j_2 \dots j_N}^{i_1 i_2 \dots i_M}$ are the components of a tensor, contravariant of order M and covariant of order $N - 1$, where v^{j_1} are the components of an arbitrary contravariant vector, then $b_{j_1 j_2 \dots j_N}^{i_1 i_2 \dots i_M}$ are the components of a tensor, contravariant of order M and covariant of order N . If $v_{i_1} b_{j_1 j_2 \dots j_N}^{i_1 i_2 \dots i_M}$ are the components of a tensor, contravariant of order $M - 1$ and covariant of order N , where v_{i_1} are the components of an arbitrary covariant vector, then $b_{j_1 j_2 \dots j_N}^{i_1 i_2 \dots i_M}$ are the components of a tensor, contravariant of order M and covariant of order N . Also used for particular cases of this law and for more general laws of a similar type.

Q-VALUE. A synonym for **reaction energy, nuclear**.

R

RACAH COEFFICIENT. A coefficient which occurs in the quantum mechanical theory of angular momentum, concerned with the various equivalent modes of coupling of three commuting angular momenta, and expressible as a sum over four **Wigner coefficients**. The coefficient is of particular importance in the theory of spectroscopy and of angular correlations in nuclear reactions.

RAD. A unit of absorbed dose, recommended by the International Commission on Radiological Units (July, 1953). It is 100 ergs per gram.

RADIAL DISTRIBUTION FUNCTION. The radial distribution function for a liquid is defined as the function $\rho(r)$, where $4\pi r^2 \rho(r) dr$ is the average number of molecule centers at distances between r and $r + dr$ from some selected molecule. If the liquid is isotropic, it is the average number density at distance r from the selected molecule. The radial distribution function may be computed from measurements of X-ray diffraction patterns, and it is of central importance in the **kinetic theory** of liquids.

RADIAL LOADING. When all loads on a body are increased from zero in constant ratio with each other, the loading path in a load space will be a radial line from the origin, $P_i = KP_i^0$. The term *proportional loading* also is used.

RADIAL MAGNIFICATION. The **linear magnification** normal to the optical axis of an axially symmetric system is the radial or lateral magnification dr'/dr , where dr is in the radial direction in object space and dr' is the conjugate element in image space.

RADIAL SYMMETRY. The symmetry of a configuration whose properties are functions only of radial distance from an origin, and thus independent of the azimuthal coordinate in two dimensions and of azimuthal and latitudinal coordinates in three dimensions.

Radial symmetry in two dimensions is often

called *circular symmetry*; in three dimensions, *spherical symmetry*.

RADIAL VELOCITY. See **stellar motions**.

RADIANCE. Radiance or the radiant intensity per unit area (at a point on a surface and in a given direction) is the quotient of the radiant intensity in the given direction of an infinitesimal element of the surface containing the point under consideration, by the area of the orthogonal projection of this element on a plane perpendicular to the given direction.

RADIANT EFFICIENCY. The ratio of the radiant flux emitted to the power consumed.

RADIANT EMITTANCE (AT A POINT OF A SURFACE). The quotient of the radiant power emitted by an infinitesimal element of surface containing the point under consideration, by the area of that element.

RADIANT INTENSITY. Radiant intensity (of a source in a given direction) is the quotient of the radiant power emitted by a source, or by an element of source, in an infinitesimal cone containing the given direction, by the solid angle of that cone. For a source which is not a point source: the quotient of the radiant flux received at an elementary surface by the solid angle which this surface subtends at any point of the source, when this quotient is taken to the limit as the distance between the surface and the source is increased.

RADIATION. (1) The emission and propagation of energy through space or through a material medium in the form of waves; for instance, the emission and propagation of electromagnetic waves, or of sound and elastic waves. (2) The energy propagated through space or through a material medium as waves; for example, energy in the form of electromagnetic waves or of elastic waves. The term radiation, or radiant energy, when unqualified, usually refers to electromagnetic radiation; such radiation commonly is classified, accord-

ing to frequency, as Hertzian, infra-red, visible (light), ultraviolet, x-rays, and γ -rays. (3) Corpuscular emissions, such as α - and β -radiation, or rays of mixed or unknown type, as cosmic radiation.

RADIATION, ACOUSTIC. The sound energy set up in a medium by a vibrating source depends not only on the physical characteristics of the medium and of the oscillating volume displacement set up by the vibrating source, but also on the size and shape of the generator. The acoustic power P generated by a vibrating source can be expressed as

$$P = u^2 R_A \times 10^{-7} \text{ watts}$$

where u is the rate of volume displacement of the fluid in cm^3 per sec., and R_A is the acoustic radiation resistance of the source in acoustic ohms.

RADIATION, COMPLEX. See **complex radiation**.

RADIATION CONDITION. The assumption in a great many boundary-value problems that at great distances from a source of energy the field has the form of a diverging wave. It is necessary to introduce such a condition before any uniqueness theorem can be proved.

RADIATION DAMPING. (1) The damping due to the loss of energy through radiation. Such damping creates an apparent frictional force and is often expressed in terms of a radiation resistance. (2) In **quantum electrodynamics** the virtual interaction of particles with the **zero point field** leads to damping terms which effect such quantities as natural level widths and scattering cross sections.

RADIATION DENSITY CONSTANT. The constant used in the **Stefan-Boltzmann law** to express the total energy density of **black-body radiation** as a function of the temperature.

$$\begin{aligned} a &= 8\pi^5 k^4 / 15c^3 h^3 \\ &= 7.569 \times 10^{-15} \text{ erg cm}^{-3} \text{ deg}^{-4}. \end{aligned}$$

RADIATION, ELECTROMAGNETIC. See **electromagnetic radiation**.

RADIATION FIELD. See **field, radiation**.

RADIATION LAWS. See **Kirchhoff law**, **Planck law**, **Stefan-Boltzmann law**, **Wien law** and **quantum electrodynamics**.

RADIATION LENGTH. The mean path length required for the reduction, by the factor $1/e$, of the energy of relativistic charged particles as they pass through matter. Such particles lose their energy mainly by radiating (see **bremstrahlung**). The radiation length for relativistic electrons in air is 330 m; in lead it is 0.5 cm.

RADIATIONLESS TRANSITION. A transition taking place between two **energy states** of a system in which the necessary energy is taken from it or given up to it by direct interaction with another system or particle, rather than by absorption or emission of electromagnetic radiation. Several examples of radiationless transitions might be mentioned. In internal conversion, a nucleus makes a transition to a lower energy state by giving up the excess energy E_0 directly to a planetary electron which leaves the atom with a kinetic energy which is E_0 minus the binding energy of the atomic electron. In the Auger effect, the energy released when an atomic electron makes a transition to a lower energy state is given directly to one or more electrons which are weakly enough bound to be separated from the atom by this energy. Another example is the **excitation** or de-excitation of an atom or molecule by a collision of the first kind or of the second kind with another atom or molecule.

RADIATION PATTERN. The graphical representation of the radiation field at large distances from an **antenna** as a function of direction.

RADIATION POTENTIAL. The potential difference in volts corresponding to the energy in electron-volts required to excite an atom or a molecule to emit one of its characteristic radiation-frequencies.

RADIATION PRESSURE. (1) For pressure of electromagnetic radiation, see **Stefan-Boltzmann law**; **adiabatic changes of radiation pressure**. (2) In acoustics, radiation pressure is the unidirectional pressure force exerted at an interface between two media due to the passage of a **sound wave**.

RADIATION, QUANTUM THEORY OF. See **quantum electrodynamics**.

RADIATION RESISTANCE. (1) The radiation resistance of an antenna is the value of an ohmic resistance which dissipates energy

at the same rate as the antenna radiates it. (2) The (acoustic) radiation resistance of a medium is the acoustic impedance (see **impedance, acoustic**) of a plane wave, equal to $\rho_0 c/S$ where ρ_0 is the mean density of the medium in gm/cm³, c is the velocity of sound in cm/sec, and S is the area of the **wave front** under consideration in cm².

RADIATIVE CORRECTION. Difference between the theoretical values of some property of a dynamical system as computed from the quantized field theory (see **field theory, quantized**) of the system and from the corresponding unquantized field theory. Applied particularly to the theory of electrons, positrons and the electromagnetic field. (See **Lamb shift, anomalous magnetic moment (electron), vacuum polarization, self-energy, renormalization.**)

RADIOACTIVE DISPLACEMENT LAW. A law originally stated by Soddy and Fajans. In its more modern and inclusive form it may be stated as follows: When a nucleus emits an α -particle the new nucleus formed has an **atomic number** two less than the parent and a **mass number** of four less. When a nucleus emits a negative β -particle the atomic number of the new nucleus formed is one greater than the parent and the mass number remains the same. The emission of a positron or the capture of an orbital electron decreases the atomic number by one without changing the mass number. Isomeric transition and γ -emission lead to no change in atomic number or mass number.

RADIOACTIVE EQUILIBRIUM. A condition which may obtain in the course of the decay of a radioactive parent having shorter-lived descendants, in which the ratio of the activity of the parent to that of a descendant is independent of time. This condition can exist only when no activity longer-lived than that of the parent is interposed in the decay chain. If the **half-life** of the parent is long compared to the time of the experiment then the state is called secular equilibrium. When secular equilibrium exists among members A , B , C , \dots etc., of a decay series, then $N_A \lambda_A = N_B \lambda_B = N_C \lambda_C = \dots$ where N_A , N_B , N_C , \dots are the numbers of atoms of A , B , C , \dots , respectively, and λ_A , λ_B , λ_C , \dots are the corresponding **disintegration constants**. If the half-

life of the parent is short and a decline in parent activity is observable, the state is called transient equilibrium. When transient equilibrium exists between a parent, A , and daughter, B , then $N_A \lambda_A = N_B (\lambda_B - \lambda_A)$ where N_A and N_B are the number of atoms and λ_A and λ_B the respective **decay constants**.

RADIOACTIVITY. Radioactivity denotes the spontaneous disintegration of the atomic nucleus with the emission of corpuscular or electromagnetic radiation. There are essentially two different kinds of radioactivity, depending upon the type of particle emitted. In the simplest, the "parent" nucleus of mass number A and atomic number Z emits an **alpha particle**, thereby transforming into a "daughter" nucleus of mass number $A-4$ and atomic number $Z-2$. This process is called *alpha-decay*. The energy of the alpha particle depends upon the parent nucleus which may often emit a number of distinct energy groups corresponding to transitions to various excited states of the daughter nucleus. The second kind of radioactivity is characterized by the emission of **electrons** or **positrons** from the nucleus, and is called *beta-decay*. In the simplest case the beta particle is an ordinary negative electron and the parent nucleus of atomic number Z transforms into one of atomic number $Z+1$, to balance electric charge, and of the same mass number A . In this case the spectrum of electrons is continuous in energy, in spite of the fact that the transformation takes place between two nuclear states of well defined energy (i.e., rest mass), and in order to conserve energy, momentum, and also angular momentum, it is necessary to postulate that a further particle, the **neutrino**, is emitted simultaneously. In each type of radioactive decay the daughter nucleus may be left in an excited state from which it will usually emit electromagnetic radiation (γ -rays). One further variant of beta-decay is found in which, instead of the emission of a positron, the absorption of an electron takes place. The electron is found in the tightly bound inner shells surrounding the nucleus, and the process is therefore known as orbital electron capture.

RADIUS, EFFECTIVE, OF CONTROL ROD. In the theory of control rods and their effects on neutron chain reactors, the boundary condition to be satisfied by the thermal flux at the surface of such a rod is most accurately

given by a relation between logarithmic derivative of flux and extrapolation distance, d . If, however, the extrapolation distance is small compared with the rod radius, a , the boundary condition may be replaced by the simpler requirement that the thermal flux vanish at the effective radius, $a_{eff} = a - d$.

RADIUS OF CURVATURE OF A CURVE. The reciprocal of the curvature. (See **curvature**; **normal curvature of a surface**; **total curvature**.)

RADIUS OF CURVATURE OF SURFACE. See **normal curvature of a surface**.

RADIUS OF GYRATION. The radius of gyration of a mass with respect to a particular axis is the square root of the quotient of the moment of inertia divided by the mass. It is the distance at which the entire mass must be assumed to be concentrated in order that the product of the mass and the square of this distance will equal the moment of inertia of the actual mass about the given axis. The numerical value of the radius of gyration, k , is given by the following formula in which I is the moment of inertia and M , the mass.

$$k = \sqrt{\frac{I}{M}}$$

The radius of gyration of a geometrical figure is similar except that the moment of inertia of the geometrical figure is involved. (See **inertia**, **products and moments of**.)

RADIUS OF INVERSION. See **inverse surfaces**.

RADIUS OF NORMAL CURVATURE. See **normal curvature**, **radius of**.

RADIUS OF SPHERICAL CURVATURE. See **spherical curvature**, **radius of**.

RADIUS OF TORSION. See **torsion**, **radius of**.

RADIUS, VECTOR. See **vector radius**.

RADIX. An integer used in a system of numeration whereby all numbers are expressed as sums of powers of it. Thus in the radix r , a number would be written in the form of a sequence of integers

$$\alpha_r \alpha_{r-1} \cdots \alpha_1 \alpha_0 \cdot \alpha_{-1} \alpha_{-2} \alpha_{-3} \cdots$$

with the understanding that $0 \leq \alpha_i \leq r - 1$ for every i , and the interpretation

$$\alpha_r r^r + \cdots + \alpha_1 r + \alpha_0 + \alpha_{-1} r^{-1} + \alpha_{-2} r^{-2} + \cdots$$

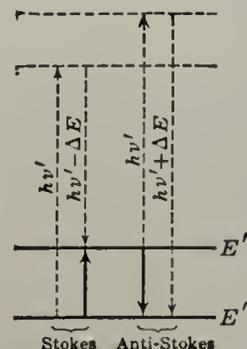
The radix point stands between α_0 and α_{-1} . In the ordinary decimal system the radix is 10; in the binary system it is 2. A digital computer using the radix 3 is said to be under construction in the U.S.S.R. (See **bit**; **binary**; **decimal**.)

RADIX POINT. See **radix**.

RAISING AND LOWERING INDICES ON A TENSOR (TENSOR FIELD). The operation of **inner multiplication** of the tensor (tensor field), contravariant of order M and covariant of order N , by the contravariant **metric tensor** to obtain an associated tensor (tensor field), contravariant of order $M + 1$ and covariant of order $N - 1$, is called *raising the index* on which the inner multiplication is applied. The operation of inner multiplication by the covariant **metric tensor** to obtain an associated tensor (tensor field), contravariant of order $M - 1$ and covariant of order $N + 1$, is called *lowering the index* on which the inner multiplication is applied.

RAMAN EFFECT. An effect occurring in the scattering of light by molecules and crystals, resulting in wavelength displacements between part of the scattered radiation and the incident light.

If the energy of the light quantum incident on the system is equal to $h\nu'$ (h is Planck's constant, ν' is frequency of incident radiation), the energy of the scattered light quantum will be



Quantum theory of the Raman effect. The heavy arrows give the transition actually taking place in the system considered. (From G. Herzberg, *Molecular Spectra and Molecular Structure. I. Spectra of Diatomic Molecules*. D. Van Nostrand Company, Inc., Princeton, 1950.)

$\nu' \pm \Delta E$, where $\Delta E = E' - E''$ is the difference in energy between two stable states of the system. If frequencies and energy differences are measured in wave number units, the Raman shifts give directly the energy differences of the system in cm^{-1} . Raman lines displaced toward longer wavelengths are sometimes referred to as Stokes lines, those displaced to shorter wavelengths as anti-Stokes lines. (See figure.)

For the mathematical formulation of the wave mechanical theory of the Raman effect it is necessary to consider the matrix element of the scattering moment

$$[P]^{nm} = \int \Psi_n^* \mathbf{P} \Psi_m d\tau$$

where Ψ_n and Ψ_m are the wave functions of two states of the system considered, and where \mathbf{P} , the induced electrical dipole moment is given by

$$|\mathbf{P}| = \alpha |\mathbf{F}|$$

where α stands for the **polarizability** of the system and

$$\mathbf{F} = \mathbf{F}_0 \sin 2\pi\nu't$$

for the electric field strength (t is time). Since Ψ_n^* , Ψ_m and \mathbf{P} have the time factors $e^{2\pi i(E_n/h)t}$, $e^{-2\pi i(E_m/h)t}$, and $e^{2\pi i\nu't}$, respectively, $[P]^{nm}$ varies with the frequency $\nu' + (E_n - E_m)/h$. For the amplitude one obtains

$$[P^0]^{nm} = |\mathbf{F}| \int \psi_n^* \alpha \psi_m d\tau.$$

If for two states E_n and E_m this integral is different from zero a transition from n to m can take place under the influence of the incident light and at the same time the scattered light quantum will have the frequency $\nu' + (E_n - E_m)/h$, that is Raman lines displaced by the amount $(E_n - E_m)/h$ appear. The square of the integral is proportional to the transition probability which determines the intensity of the particular Raman lines. (For $n = m$ the undisplaced frequency ν' is obtained. In this case the above integral determines the intensity of the Rayleigh scattering.)

If the polarizability of the system (oscillator or rotator) is constant (independent of rotation or vibration) the constant α can be put in front of the integral sign in the above expression. Owing to the orthogonality of the eigenfunctions, the integrals are then all equal to zero, except for $m = n$; that is if α is constant only

Rayleigh scattering and no Raman effect appears. A transition from one state to another and therefore a Raman shift can occur only if the polarizability changes during the process under consideration.

(For more detail see the entries **rotation spectra of molecules**, **rotation-vibration spectra of molecules**, **vibrational spectra of molecules** and G. Herzberg, *Molecular Spectra and Molecular Structure, I. Spectra of Diatomic Molecules*, 2d ed., and *II. Infrared and Raman Spectra of Polyatomic Molecules*, D. Van Nostrand Company, Inc., Princeton, 1950 and 1945, respectively.)

RAMAN SPECTRUM. See Raman effect.

RAMBERG-OSGOOD PARAMETERS. The stress-strain curve in simple tension for many structural materials can be represented reasonably well by the equation

$$\epsilon = \sigma/E + K(\sigma/E)^n$$

where the parameters K and n are adjusted to fit the data. E is Young's modulus, σ is stress, and ϵ is strain.

RAM-JET. A form of propulsive device for high-speed flight, having no moving parts (except for fuel pumps). Air entering at the intake has its velocity reduced in a **diffuser**, with consequent rise of pressure. The compressed air is then heated by combustion of fuel and expanded through a nozzle to produce a high velocity jet. The principle is similar to that of a **turbo-jet**, except that all the compression of the air before combustion takes place in the intake and diffuser, so that there is no need for a mechanical compressor or for a turbine. At zero forward speed, a ram-jet cannot operate. A supersonic ram-jet develops thrust according to the equation

$$F_t = \rho_0 V_0 S_0 \left[\left(1 + \frac{F}{A} \right) V_e - V_0 \right] + S_e (P_e - P_0).$$

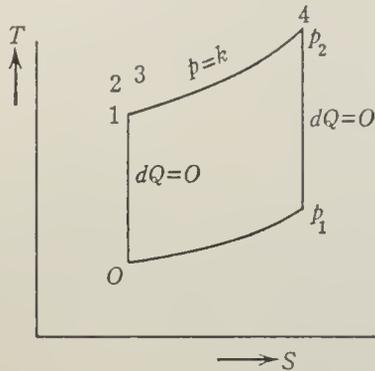
Where F_t is the gross thrust, ρ_0 is the entering air density, V_0 is the inlet velocity, S_0 is the inlet area, $F = FV + F_p$ is the thrust due to mass flow and pressure differential, A is the internal area upon which the thrust works, V_e is the exit velocity, S_e is the exit cross section, P_e is the exhaust pressure, and P_0 is the free stream pressure.

Drag of a supersonic ram-jet is given by:

$$D = \int_s [(P - P_0)_x + \sigma_x] ds$$

where σ_x is the skin friction stress parallel to the free stream direction. Net thrust is $F_n = F_T - D$.

An S-T diagram showing the ideal thermodynamic cycle of a ram-jet appears in the figure.



Ideal ram-jet or turbo-jet cycle.

RAMP FUNCTION. If $f(t)$ is a ramp function of t ,

$$f(t) = kt, t > 0$$

$$= 0, t \leq 0.$$

The ramp function is therefore the integral of a step function of amplitude k . The **Laplace transform** of unit ramp function ($k = 1$) is $\frac{1}{s^2}$.

RANDOM ERROR. The inherent imprecision of a given process of measurement; the unpredictable component of repeated independent measurements on the same object under sensibly uniform conditions. It is found experimentally that given sufficient refinement of reading, a series of independent measurements x_1, x_2, \dots, x_n will vary one from another even when conditions are most stringently controlled. Hence, any such measurement x_i may be regarded as composed of two terms:

$$x_i = \mu + v_i,$$

where μ (ordinarily the true value) is a numerical constant common to all members of the series and v_i , the random error, is an unpredictable deviation from μ .

RANDOMIZED BLOCKS. The commonest type of experimental design is that known as randomized blocks. Suppose that there are t

treatments and that the experimental units can be divided into blocks of t units each in such a way that units within a block are relatively homogeneous. Then we may allocate the treatments to the units at random, subject to the restriction that each treatment occurs just once in each block. All treatment comparisons can now be made within blocks and any differences between blocks can be eliminated from the estimate of experimental error.

RANDOMNESS. The concept of randomness is sometimes taken as an undefinable, but is more usually defined in terms of **probability**. A method of selection is random if it gives to each of the members at choice a predeterminate probability of being chosen. (See also **random variable**.)

RANDOM VARIABLE. A quantity which may take one of a specified set of values with a probability varying, in general, from value to value. A particular value which the random variable may take is called a variate-value, and often the expression "variate" is used to denote the random variable itself, e.g., in terms such as *multivariate analysis*.

RANDOM WALK PROCESS. See **additive process**.

RANGE. (1) In mathematics, the **set** of values which may be taken by a variable, a function or a transformation. Thus, the range of a transformation is the set which includes every point which results from that transformation. (2) In statistics, the most general measure of dispersion; the interval or the difference between the greatest and the least of a set of **values**. It is a type of **order-statistic**. (3) In physics, the distance that a charged particle will penetrate a given substance before its kinetic energy is reduced to a value below which it can no longer produce ionization. For a heavy ion, such as a proton or an α -particle, the range usually refers to the component of displacement in the initial direction; it is only slightly shorter than the path length, or distance measured along the track of the particle. For a meson, whose track shows moderate deflections near the end, the range usually refers to the path length. For an electron, whose track may be quite tortuous because of frequent deflections, the range usually refers to the greatest distance of penetration in a specified direction; this distance may be con-

siderably shorter than the path length. The residual range is the distance over which the particle can still produce ionization after having already lost some of its energy in passing through matter. Because of the phenomenon of straggling, particles of a given kind and of the same initial energy do not all have the same range. For heavy ions, the ranges are distributed, in a manner similar to the **Gaussian distribution**, about the mean range, or range that is exceeded by half of the particles.

RANGE-ENERGY RELATION. A relation between the distance a particle will travel through matter (its **range**) before giving up all of its energy, and its initial energy. The term is used with regard to charged particles. Since the energy loss per unit path length is a rather complicated function of the energy, the range-energy relation is not linear. For α -particles and similar heavy ions, the range involved is usually the mean range, most commonly in dry air at 15°C and 1 atm. For electrons, it is usually the maximum range, most commonly in aluminum. For β -particles the energy involved is the maximum kinetic energy of the β -particle spectrum.

RANK. (1) See **rank of a matrix**. (2) A system of homogeneous **linear equations** is of rank n if the matrix of its coefficients has that rank. (See also **tensor**; **graph rank**.)

RANK CORRELATION. Suppose we have a sample of n pairs of ranked observations, (x_i, y_i) , $i = 1, 2, \dots, n$. Two measures of correlation between the samples are in current use.

(1) Spearman's ρ is the ordinary **correlation** between the ranks regarded as variate values. It may readily be calculated as

$$1 - \frac{6\sum(x_i - y_i)^2}{n(n^2 - 1)}$$

(2) Kendall's τ may be defined as $1 - 4s/n(n - 1)$, where s is the smallest number of interchanges needed to transform one reaching into the other. Both ρ and τ take values in the range -1 to $+1$.

ρ is somewhat easier to calculate than τ , but τ has some practical and many theoretical advantages.

RANKINE CYCLE. (Also known as the Clausius-Rankine cycle.) An ideal cycle used

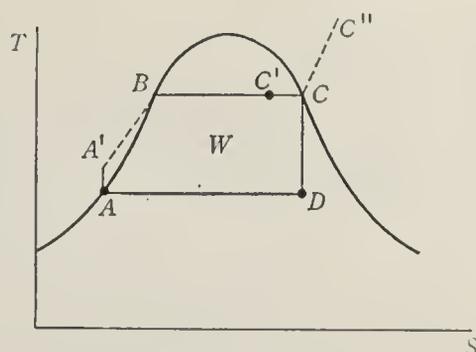


Fig. 1. T - S Diagram of ideal Rankine cycle.

as a standard of comparison in steam plants. The sequence of operations (see Figure 1), all assumed to be reversible, is as follows:

C - D . Isentropic expansion of vapor.

D - A . Liquefaction of vapor at constant pressure in a condenser.

A - A' . Isentropic compression of liquid; the distance A - A' is usually very small and cannot be distinguished on a diagram drawn to scale, because the points A and A' nearly coincide.

A' - B . Heating of liquid in boiler.

B - C . Evaporation in boiler; the end point can be dry saturated, point C ; wet, point C' ; or (in the majority of cases) superheated, point C'' .

The thermal efficiency of the cycle is

$$\eta = \frac{h_C - h_D}{h_C - h_W}$$

where

$$h_W = h_A \approx h_{A'}$$

Figure 2 shows the cycle in H , S coordinates (**Mollier diagram**), which is used more often in engineering practice than the temperature-entropy diagram.

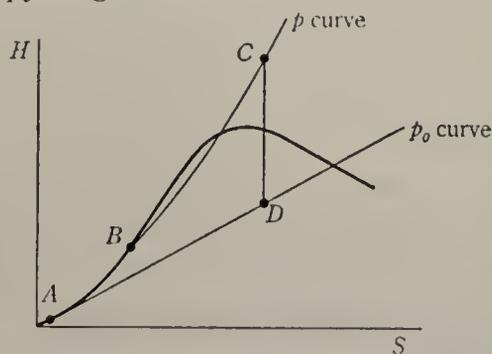


Fig. 2. H - S Diagram of ideal Rankine cycle.

The Rankine cycle is of great practical importance because it represents the maximum amount of work W which can be obtained at

the expense of the heat $Q = h_c - h_w$ under given circumstances.

The variation of the Rankine efficiency η for steam with admission pressure and back-pressure is shown in Figure 3. (See also reversed Rankine cycle.)

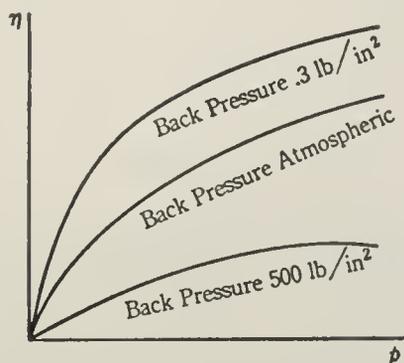


Fig. 3. Graphs showing relation of Rankine cycle efficiency to admission pressure at various back pressures (initial superheat constant).

RANKINE EFFICIENCY. See Rankine cycle.

RANKINE-HUGONIOT RELATION. In supersonic aerothermodynamics, a mathematical relationship between pressure and density through (i.e., before and after) a normal shock wave. Four forms of this relation appear below:

$$\frac{p_1}{p_0} = \frac{(\gamma + 1)p_1 - (\gamma - 1)p_0}{(\gamma + 1)p_0 - (\gamma - 1)p_1}$$

$$= \frac{(\gamma + 1)V_0 - (\gamma - 1)V_1}{(\gamma + 1)V_1 - (\gamma - 1)V_0}$$

$$\frac{V_0}{V_1} = \frac{\rho_1}{\rho_0} = \frac{(\gamma + 1)p_1 + (\gamma - 1)p_0}{(\gamma + 1)p_0 + (\gamma - 1)p_1}$$

$$\frac{p_1 - p_0}{p_0} = \frac{p_1}{p_0} - 1 = \frac{2\gamma(\rho_1 - \rho_0)}{(\gamma + 1)\rho_0 - (\gamma - 1)\rho_1}$$

$$\frac{\rho_1 - \rho_0}{\rho_0} = \frac{\rho_1}{\rho_0} - 1 = \frac{2(p_1 - p_0)}{(\gamma + 1)p_0 + (\gamma - 1)p_1}$$

where p is pressure, γ is the ratio of specific heats, ρ is density and V is volume.

RANKINE TEMPERATURE SCALE. See temperature.

RANKINE VORTEX. (1) A particular kind of vortex having a vertical axis and circular motion. The vorticity is uniformly distributed within a cylinder of radius a outside which the

circulation is constant. The velocity is at right angles to the radius and equal to ωr for $r < a$ (solid rotation) and to $\omega a^2/r$ for $r > a$.

The depression z of the free surface is given by

$$z = a^2\omega^2/2gr^2 \quad r > a$$

$$= a^2\omega^2(1 - r^2/2a^2)/g \quad r < a,$$

the depression at the center being $a^2\omega^2/g$. (2) See V-R vortex.

RANK OF A GRAPH. See graph rank.

RANK OF A MATRIX. The number of column (or row) vectors in the smallest set that can be selected from among the columns (or rows) of the matrix that are such that all other columns (or rows) are expressible as linear combinations of them. The number is the same, whether one counts rows or columns. Also it is the order of the largest nonnull determinant that can be formed from the rows and columns of the matrix.

RAOULT LAW. See vapor pressure in perfect solutions.

RAREFIED GAS DYNAMICS. When the density of a gas is so low that the mean free path λ is not negligible in comparison with a length l characteristic of the flow field, the usual continuum theory of gas dynamics is no longer applicable. The ratio $\lambda/l = K$ is called the Knudsen number, and the gas may only be treated as a continuum for values of K less than about 0.01.

From the usual relations of the kinetic theory of gases it may be shown that $K \approx \frac{M}{R}$, where

M is the **Mach number** of the flow and R is the **Reynolds number** based on the length l . When the Reynolds number is not too small, the thickness δ of the boundary layer is a more appropriate length to use in defining Knudsen number than a characteristic dimension of the body. In this case, if L is a characteristic dimension of the body and R_L is the Reynolds number based on this length, $\delta/L \approx R_L^{-1/2}$, so that $K = \lambda/\delta \approx M/R_L^{1/2}$. For very low Reynolds numbers the boundary-layer concept is not applicable and the Knudsen number is $K = \lambda/L \approx M/R_L$.

The range $0.01 < K < 1$ is often known as the *slip-flow regime*, because in this range of K the most important noncontinuum effect is the existence of a velocity of slip at a solid bound-

ary. The *velocity of slip* is of order λ times the velocity gradient at the boundary, so that if λ/δ is very small the velocity of slip is negligible in comparison with the velocity outside the boundary layer.

For values of K greater than about 1 other non-continuum effects become important and the regime for $K > 10$ is known as *free-molecule flow*. In this type of flow collisions between molecules have a negligible effect in comparison with direct impact of molecules on the body.

RATE CONSTANT. See **order of chemical reactions**; **frequency factor**.

RATE OF DECAY OF SOUND. The time rate at which the **sound pressure level** (or **velocity level**, or **sound-energy density level**) is decreasing at a given point and at a given time. The practical unit is the **decibel** per second.

RATES OF IRREVERSIBLE PROCESSES. (See also **chemical reaction rates**.) The thermodynamic forces of irreversible processes give rise to irreversible changes, as for example, heat flow, electrical current and chemical change. These are called the "rates" (or "fluxes," "flows" or "currents") of the irreversible processes. They are generally denoted by J_i .

RATIO. The **quotient** of two numbers, frequently written as $a:b$, a/b , or $a \div b$.

RATIONAL NUMBER. Consider all ordered pairs a,b of integers with $b \neq 0$, the first and second members of the pair being called its numerator and denominator respectively. If we define addition of two pairs (a,b) and (c,d) by $(a,b) + (c,d) = (ad + bc, bd)$ and multiplication by $(a,b) \times (c,d) = (ac, bd)$, then each pair is called a fraction and is usually written a/b , or similarly. If we now define a binary relation R for fractions by saying that a/b is in the relation R to c/d if $ad = bc$, then this relation is easily proved to be an equivalence (i.e., to be reflexive, symmetric and transitive: see **relation**), so that the set of all fractions falls into mutually exclusive equivalence classes, each of which is called a rational number.

RATIO TEST. See **D'Alcembert test**.

RAY. (1) A normal to a **wavefront**. In **geometrical optics** a ray or light ray is defined as: A directed **optical path** from an object point O to an image point I . (2) In case the index of refraction is sectionally constant over the medium so that the optical path is polygonal, then a directed half-line (or sometimes a directed line) that coincides in part with a segment of the path is also called a ray. According to the second meaning, there are image rays or object rays if the half-line (line) contains the **image** or **object point**, respectively. (3) In acoustics, a term defined under the following entry, **ray acoustics**. (4) A direction in **Hilbert space**. A ray of unit length is an equivalence class of vectors of unit length, two vectors being equivalent if they differ only by a multiplicative constant of modulus one. In the mathematical formulation of quantum mechanics, the state function (from which all the physically relevant information about a physical system can be derived) is represented by a ray in a complex Hilbert space: If $|\psi\rangle$ is a vector in Hilbert space which corresponds to a physically realizable state, then $|\psi\rangle$ and a constant multiple of $|\psi\rangle$ both represent this state. It is therefore, customary to choose an (arbitrary) representative vector of the ray which is normalized to one to describe the state. If the states are normalized, only a constant factor of modulus one is left undetermined and two vectors which differ by such a phase factor represent the same state. Rays representing states which can be prepared are called physically realizable.

RAY ACOUSTICS. The analysis of acoustical problems under the assumption that sound travels along straight lines or rays in passing through homogeneous material. **Diffraction** effects are neglected. The methods of ray acoustics are applicable only if the state of the medium and the boundaries of the medium change only slightly over a distance equal to the sound wavelength.

RAY, FIELD. See **field ray**.

RAYL. The magnitude of a specific acoustic resistance, reactance or impedance for which a sound pressure of one microbar produces a linear particle velocity of one cm. per second. Its dimensions are $ML^{-2}T^{-1}$, and it is equal to 1 dyne-second per (centimeter)³.

RAYLEIGH CRITERION FOR RESOLUTION. Rayleigh assumed that two primary

point sources are at the limit of resolution of an optical system if the diffraction patterns of their images are located so that the central maximum of one lies over the first minimum of the other. Thus, this limit is specified by the radius of the **Airy disk**, $r_1 = 0.61 \lambda/\rho_o$ where λ is the wavelength, $\rho_o = n \sin \theta$ is the exit aperture.

RAYLEIGH DISK. Rayleigh observed that when a disk was suspended by a light fiber it would tend to turn at right angles to the impinging sound wave. Koenig developed the formula for the turning moment of the disk as

$$M = \frac{4}{3} \rho a^3 u^2 \sin 2\theta \quad (1)$$

where M is the turning moment acting upon the disk, in dyne centimeters, ρ is the density of air, in grams per cubic centimeter, a is the radius of the disk, in centimeters, θ is the angle between the normal to the disk and the direction of propagation of the sound wave, in degrees, and u is the particle velocity of the sound wave, root-mean-square, in centimeters, per second.

When a sound wave falls upon the disk the angular deflection will be

$$\phi = \frac{M}{S} \quad (2)$$

where S is the moment of torsion of the suspension, in dyne centimeters.

The moment of torsion of the suspension is given by

$$S = \frac{I}{T^2} [4\pi^2 + (\log_e \gamma)^2] \quad (3)$$

where T is the periodic time of the suspended disk, in seconds, I is the moment of inertia of the disk, $I = ma^2/4$, m is the mass of the disk, in grams, a is the radius of the disk, in centimeters, and γ is the damping factor, the ratio of two successive swings.

From equations 1, 2, and 3 it is possible to determine the particle velocity u in the sound wave.

(References: Rayleigh, *Phil. Mag.*, Vol. 14, p. 186, 1882. Koenig, *Ann. d. Physik*, Vol. 43, p. 43, 1891.)

RAYLEIGH FORMULA. An expression of the **drag coefficient** as a power of the Reynolds number Re

$$C_D = k(Re)^n,$$

where k is a constant.

RAYLEIGH-JEANS EQUATION. From the standpoint of statistical mechanics, using the theorem of the equipartition of energy, Rayleigh and Jeans obtained

$$dE_\lambda = 2\pi ckT\lambda^{-4}d\lambda$$

for the spectral distribution of the radiation from a black body. This formula agrees with experiment only at long wavelengths, and fails completely at short wavelengths. (See **Planck's radiation formula; black body radiation** for the true spectral distribution.)

RAYLEIGH LAW. See Rayleigh scattering.

RAYLEIGH LIMIT FOR SPHERICAL ABERRATION. An optical system is sufficiently corrected for spherical aberration according to the Rayleigh criterion if the mixed characteristic function $W(\rho)$ satisfies

$$|W(\rho) - W(0)| < \lambda/4, \quad \text{for } 0 < \rho \leq \rho_o,$$

where λ is the wavelength and $\rho_o = n \sin \theta$ is the exit aperture. (See the discussion under **Airy disk**.)

RAYLEIGH NUMBER. The non-dimensional number determining the nature of the flow produced by buoyancy forces in a viscous, heat conducting fluid subjected to uneven heating. The buoyancy forces are reduced by the heat conduction and by the motion and increased by increasing the temperature gradients or gravity. The viscous forces are proportional to the viscosity. Velocity does not enter into the number because it is taken to be so small that the inertia forces are negligible. Accordingly the Rayleigh number is

$$g(\Delta\rho/\rho)h^3/\kappa\nu$$

in which h is a representative linear dimension (usually the depth over which the density differences $\Delta\rho$ are maintained), ρ is a mean density, κ is the thermometric conductivity (the thermal conductivity divided by the specific heat) and ν the dynamic viscosity. In a gas $\Delta\rho/\rho$ may be replaced by $\Delta T/T$ because the density differences are produced by temperature differences, and the coefficient of expansion is $1/T$. (See also **Bénard cells**.)

RAYLEIGH-RITZ METHOD. See **Ritz method**.

RAYLEIGH SCATTERING. Any scattering process produced by spherical particles whose

radii are smaller than about one-tenth the **wavelength** of the scattered **radiation**.

In Rayleigh scattering, the **scattering coefficient** varies inversely with the fourth power of the wavelength, a relation known as *Rayleigh's law*. The angular intensity **polarization** relationships for Rayleigh scattering are conveniently simple. For particles not larger than the *Rayleigh limit*, there is complete symmetry of scattering about a plane normal to the direction of the incident radiation, so that the **forward scatter** equals the **backward scatter**. The Rayleigh scattering coefficient k_s is

$$k_s = \frac{2}{3}\pi^5 n \left(\frac{m^2 - 1}{m^2 + 2} \right)^2 \frac{d^6}{\lambda^4},$$

where n is the number of scatterers of diameter d , m the index of refraction, and λ the wavelength of the radiation. (Cf. **Rayleigh-Jeans equation**.)

Unfortunately, in the real atmosphere frequent occurrence of scattering particles with radii in excess of the Rayleigh limit restricts the applicability of this theory. The theory does, however, prove quite useful in describing scattering of radar energy by raindrops, although here, too, significant practical cases arise in which the Rayleigh condition on wavelength-to-radius ratio is not satisfied. In all such cases, recourse must be made to the complete and highly complex **Mie theory** of scattering.

RAYLEIGH-SCHRÖDINGER PERTURBATION FORMULA. In a quantum mechanical system the energy operator of which can be decomposed as $H = H_0 + H_1$ and in which H_1 can be treated as a small perturbation, the Rayleigh-Schrödinger perturbation theory expresses the corrections to the energy and wave functions in terms of the eigenvalues $W_n^{(0)}$ and eigenfunctions $\psi_n^{(0)}$ of the unperturbed energy operator H_0 . The expression of the energy corrections to $W_n^{(0)}$ up to the second-order is

$$\Delta W_n = (n|H_1|n) + \sum_m' \frac{(n|H_1|m)(m|H_1|n)}{W_n^{(0)} - W_m^{(0)}}$$

where $(n|H_1|m)$ is the matrix element of H_1 between the states $\psi_n^{(0)}$ and $\psi_m^{(0)}$ and the summation is extended over all states $\psi_m^{(0)}$ with the exception of $m = n$.

RAYLEIGH-TAYLOR INSTABILITY. The instability of the surface of separation be-

tween two fluids of different densities when the lighter fluid is accelerated toward the heavier fluid.

RAY TRACING. The determination (either analytically, numerically, or geometrically) of a ray, or **optical path**, from a given object ray and data (radii, thickness, distances, and refractive indices) of an optical system. There are paraxial ray traces in **Gaussian optics**, third order ray traces where the approximation $\sin \theta = \theta - \frac{1}{6}\theta^3$ is used in Snell's law, and exact ray traces.

REACTANCE. The imaginary part of **impedance**.

REACTANCE, ACOUSTIC. The imaginary component of the acoustic impedance (see **impedance, acoustic**). The commonly used unit is the **acoustic ohm**.

REACTANCE, SPECIFIC ACOUSTIC. The imaginary component of the specific acoustic impedance. (See **impedance, specific acoustic**.)

REACTION. (1) In general, a response such as the equal and opposite force which, according to the third of the Newton laws of motion, results when a force is applied to a material system. Specifically, the force exerted by the supports or bearings on a loaded mechanical system. (2) A term sometimes used for circuit regeneration. (3) A chemical change. Specifically, a change by which one or more substances are transformed into one or more entirely new substances, the process being accompanied by a noticeable change in energy, but not, in most cases, by a change in the total mass of the system. Radioactive reactions, however, both natural and artificial, do involve a change of the mass of the system, which is accompanied by a release of energy, opposite in sign to the change in mass, and equal to its magnitude multiplied by the square of the velocity of light.

REACTION CHANNEL. In a nuclear reaction, that portion of configuration space in which the compound nucleus can be considered to have divided into two or more parts, each defined by a definite spin direction, and with a definite value of the relative orbital angular momentum. To each such reaction channel there corresponds a phase angle to which the partial cross section for the process relates.

REACTION COORDINATE. See **absolute reaction rate theory**.

REACTION ENERGY, NUCLEAR. (1) The **disintegration energy** of a nuclear reaction; symbol Q . It is equal to the sum of the kinetic or radiant energies of the reactants minus the sum of the kinetic or radiant energies of the products. (If any product of a specified reaction is in an excited nuclear state, the energy of subsequently emitted γ -radiation is not included in the sum.) (2) Often, implicitly, the ground-state nuclear reaction energy, which is the reaction energy when all reactant and product nuclei are in their ground states; symbol Q_0 .

REACTION FIELD IN A DIELECTRIC.

Let a small sphere of volume V be polarized uniformly, leading to a dipole moment PV (where P is the **polarization** vector). The sphere is included in a dielectric of constant ϵ ; one assumes that there is no macroscopic field. The sphere interacts with the surroundings which, in turn, affect the electric field inside the sphere. The modification of this electric field inside the sphere is called the *reaction field* R . Its value is

$$R = \frac{2}{3} \frac{\epsilon - 1}{2\epsilon + 1} 4\pi P.$$

P is the polarization inside the sphere.

In the presence of a macroscopic field in the dielectric, the total internal field can be decomposed into the sum of this reaction field and of the **cavity field**. (See also **internal field**.)

REACTION, NORMAL. See **normal reaction**.

REACTION RATE, CHEMICAL. See **chemical reaction rate**.

REACTION RATE THEORY, ABSOLUTE. See **absolute reaction rate theory**.

REACTIONS, INDEPENDENT. See **independent reactions**.

REACTION STRESSES. Those stresses produced during fabrication and erection which extend over an appreciable region of an externally unloaded structure are distinguished from local **residual stresses** by the name reaction stresses.

REACTIVITY. In the theory of chain reacting systems the reactivity, ρ , is a measure of the effective multiplication (k_{eff}) in excess of unity and is defined by

$$\rho = \frac{k_{eff} - 1}{k_{eff}}.$$

REACTIVITY OF ORGANIC MOLECULES. See under **inductive effect**; **resonance effect**; **molecular diagrams**.

REACTIVITY, TEMPERATURE COEFFICIENT OF. In the theory of neutron chain reactors, change in temperature of the constituent materials cause a change in the steady-state distribution of neutrons in energy and hence a change in reaction rates. We are thus led to an overall temperature coefficient of reactivity, defined as

$$\frac{dk_{eff}}{dT} \text{ or } \frac{d\rho}{dT}.$$

REACTOR. (1) A nuclear reactor is an apparatus in which nuclear fission is sustained in a self-supported chain reaction. (2) An element or device used to introduce **reactance** into a circuit or system.

REACTOR FUNDAMENTAL THEOREM. See **first fundamental theorem** and **second fundamental theorem**.

REACTOR KINETICS. See **kinetics, reactor**.

REACTOR PERIOD. See **period, reactor**.

REACTOR THEORY, ASYMPTOTIC. See **asymptotic reactor theory**.

REACTOR TRANSFER FUNCTION. In the theory of neutron chain reactors, a measure of the response of the neutron density to a sinusoidal time variation in reactivity. More quantitatively, it expresses the ratio of the **Laplace transform** of the change in neutron density to that of the change in reactivity.

REAL GASES. The pressure of a gas can be represented by a series in powers of $1/V_m$ where V_m is the molar volume

$$p = \frac{RT}{V_m} \left(1 + \frac{B}{V_m} + \frac{C}{V_m^2} + \dots \right) \quad (1)$$

in which $B, C \dots$ are functions of temperature only.

For $V_m \rightarrow \infty$ we come back to the equation of state of a perfect gas. This series converges rapidly (except in the critical region, or if long range forces are present; see **plasma**). The coefficients $B, C \dots$ are called the second, third \dots virial coefficients.

The second virial coefficient has been studied very extensively. It is negative at low temperatures but increases and becomes positive at higher temperatures. The transition temperature at which $B = 0$ is called the *Boyle temperature*.

Alternatively we may also write

$$V_m = \frac{RT}{p} (1 + B'p + C'p^2 + \dots). \quad (2)$$

Aside from their practical interest, the virial expansions (1) or (2) are important because each coefficient has a well defined meaning in terms of intermolecular forces (see **Mayer cluster theory**).

Many empirical equations of state have been suggested in the past. One of the best known is the *van der Waals equation of state*

$$p = \frac{RT}{V-b} - \frac{a}{V^2} \quad (3)$$

where a and b are constants called respectively the cohesion and the covolume of the gas.

Another well known equation of state is that of *Dieterici*

$$p(V-b) = RT \exp\left(-\frac{a}{RTV}\right). \quad (4)$$

The distinction between liquid and gas and the existence of a critical point can be discussed *qualitatively* in terms of such simple equations of state. (See **critical point; corresponding states theorem**.) These equations can also be extended for mixtures considering a and b as functions of the composition.

When the equation of state is known (as well as the thermodynamic properties in the low density limit) the general formalism of thermodynamics permits one to calculate completely the thermodynamic functions of a real gas. (See **equation of state** for this discussion, and for listing of other equations of state of real gases given in this book.)

REALIZATION. A **stochastic process** may be regarded as defined by a distribution of infinitely many random variables. A sample from it is called a realization of the process.

It consists of a sequence of observations which may be infinite or finite in extent, e.g., an observed time-series considered as one of the ways in which a given stochastic system could have developed.

REAL LINEAR GROUP. See **Lie group**.

REAL ORTHOGONAL GROUP. See **Lie group**.

REAL UNIMODULAR GROUP. See **Lie group**.

REATTACHMENT OF BOUNDARY LAYER AFTER SEPARATION. When a **laminar boundary layer** separates it is sometimes found that the separated layer becomes turbulent soon after separation and then becomes reattached to the surface. This does not occur if the Reynolds number is low, because then the laminar shear layer is relatively stable, and does not become turbulent for a considerable distance after separation. However, if the Reynolds number of the boundary layer at separation, based on the **displacement thickness** and the velocity at the edge of the layer, is greater than about 500, transition to turbulent flow usually occurs very quickly. Reattachment may then occur, if the shape of the boundary is favorable to the process.

The region enclosed by the boundary and the separated shear layer, between the point of separation and reattachment, is known as a *separation bubble*. The turbulent shear layer extracts fluid from the bubble by entrainment, and an equivalent quantity of fluid must be diverted from the shear layer near the reattachment point and point and flow back near the boundary into the bubble. This back-flow can only be maintained if the pressure within the bubble is less than that near the reattachment point, i.e., there must be a rise of pressure as the separated shear layer approaches reattachment.

RECIPROCAL BASE VECTORS. See **base vectors, reciprocal**.

RECIPROCAL DIFFERENCES. Differences of a function $f(x)$ whose values are given at points x_0, x_1, x_2, \dots , not necessarily uniformly spaced, are utilized for **interpolation** by means of rational fractions instead of polynomials. This is appropriate in regions near a singularity. The reciprocal differences

$$\rho(f|x_0, \dots, x_n|) = \rho(x_0, \dots, x_n)$$

are formed by means of a table:

$$\begin{array}{l} x_0 \quad \rho(x_0) \\ x_1 \quad \rho(x_1) \quad \rho(x_0, x_1) \\ x_2 \quad \rho(x_2) \quad \rho(x_1, x_2) \quad \rho(x_0, x_1, x_2) \\ x_3 \quad \rho(x_3) \quad \rho(x_2, x_3) \quad \rho(x_1, x_2, x_3) \quad \rho(x_0, x_1, x_2, x_3) \\ x_4 \quad \rho(x_4) \quad \rho(x_3, x_4) \quad \rho(x_2, x_3, x_4) \quad \rho(x_1, x_2, x_3, x_4) \end{array}$$

where

$$\rho(x_i, x_j) = (x_i - x_j)/[\rho(x_i) - \rho(x_j)],$$

$$\rho(x_i, x_j, x_k) = \rho(x_j) + (x_i - x_k)/[\rho(x_i, x_j) - \rho(x_j, x_k)],$$

the latter formula being typical. Then the interpolation is made by taking $f(x)$ to be an approximant of the **continued fraction**.

$$f(x) = \rho(x_0) + (x - x_0)/\sqrt{\rho(x_0, x_1)} + (x - x_1)/\sqrt{\rho(x_0, x_1, x_2) - \rho(x_0)} + \dots$$

If the n^{th} approximant is

$$F_n = A_n/B_n,$$

then $A_{2p}, A_{2p+1}, B_{2p+1}, B_{2p+2}$ are of degree p in x . It is not required that the x_i be distinct, and in the extreme case all may coincide. In that event the reciprocal differences are expressed in terms of derivatives, the treatment being analogous to the confluent case of **divided differences**, and the resulting continued fraction is then analogous to a **Taylor series**.

RECIPROCAL LATTICE. Let $\mathbf{a}, \mathbf{b}, \mathbf{c}$, be the **primitive translations** of a given crystal lattice. Let

$$\mathbf{a}^* = \frac{\mathbf{b} \times \mathbf{c}}{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})}, \quad \mathbf{b}^* = \frac{\mathbf{c} \times \mathbf{a}}{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})},$$

$$\mathbf{c}^* = \frac{\mathbf{a} \times \mathbf{b}}{\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c})}.$$

Then $\mathbf{a}^*, \mathbf{b}^*, \mathbf{c}^*$, define the **unit cell** of the reciprocal lattice. This has the properties (i) the vector $\mathbf{r}^*(hkl) = h\mathbf{a}^* + k\mathbf{b}^* + l\mathbf{c}^*$ of the reciprocal lattice is normal to the (hkl) plane of the crystal lattice, (ii) the length of the vector $\mathbf{r}^*(hkl)$ is equal to the reciprocal of the spacing of the (hkl) planes.

RECIPROCAL NETWORK. See **network, reciprocal**.

RECIPROCAL STRAIN ELLIPSOID. See **strain ellipsoid**.

RECIPROCAL THEOREM IN CLASSICAL ELASTICITY THEOREM. See **reciprocity theorem of Maxwell and Betti**.

RECIPROCAL THEOREMS. See **reciprocity theorems**.

RECIPROCAL VECTOR SYSTEM. From the properties of the **quadruple product** of vectors, the following relation is found to hold for any four vectors $\mathbf{r}, \mathbf{a}, \mathbf{b}, \mathbf{c}$:

$$\mathbf{r}[\mathbf{abc}] = [\mathbf{rbc}]\mathbf{a} + [\mathbf{rca}]\mathbf{b} + [\mathbf{rab}]\mathbf{c}$$

which may also be written in the equivalent form

$$\mathbf{r} = \mathbf{r} \cdot \mathbf{a}'\mathbf{a} + \mathbf{r} \cdot \mathbf{b}'\mathbf{b} + \mathbf{r} \cdot \mathbf{c}'\mathbf{c}.$$

The system of three vectors

$$\mathbf{a}' = \frac{\mathbf{b} \times \mathbf{c}}{[\mathbf{abc}]}; \quad \mathbf{b}' = \frac{\mathbf{c} \times \mathbf{a}}{[\mathbf{abc}]}; \quad \mathbf{c}' = \frac{\mathbf{a} \times \mathbf{b}}{[\mathbf{abc}]}$$

is reciprocal to the three non-coplanar vectors $\mathbf{a}, \mathbf{b}, \mathbf{c}$. The unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ form a system which is its own reciprocal. Conversely, a system which is its own reciprocal is a set of mutually perpendicular unit vectors, forming either a right-handed or left-handed Cartesian coordinate system. The two systems satisfy the identity $[\mathbf{abc}][\mathbf{a}'\mathbf{b}'\mathbf{c}'] = 1$.

RECIPROCAL VELOCITY REGION. The energy region (generally from 0 to several eV) in which the capture cross section (see **cross section, capture**) for neutrons by a given element is inversely proportional to the neutron velocity.

RECIPROCITY LAW (PHOTOGRAPHIC). The **optical density** of an exposed emulsion, with standard development, is a function of only the **irradiance** and time of exposure. This law may be applied only in certain situations.

RECIPROCITY PRINCIPLE, ELECTRIC NETWORK. See **reciprocity theorem, electric network**.

RECIPROCITY RELATIONS. The Onsager reciprocity relations state that, provided a proper choice is made of the rates J_i (see **rates of irreversible processes**) and forces X_i (see **forces in thermodynamics of irreversible processes**) the matrix of phenomenological coeffi-

icients L_{ik} is symmetric (see **phenomenological relations**), i.e.,

$$L_{ik} = L_{ki}. \quad (1)$$

The proper choice means that the rates J_i and the forces X_i are such that the entropy production per unit time due to the irreversible processes is

$$\frac{d_i S}{dt} = \sum_i J_i X_i \geq 0. \quad (2)$$

(See **thermodynamics, second law of; time variation of the entropy production**.)

The reciprocity relations are based on the classical **fluctuation theory** (see also **Einstein relation**) supplemented by the property of "**microscopic reversibility**," i.e., the symmetry of all equations of motion of individual particles with respect to time. In other words "**microscopic reversibility**" expresses the invariance of classical or quantum equations of motion in respect to the transformation $t \rightarrow -t$.

The reciprocity relations (1) express the fact that, when the flux corresponding to the irreversible process i is influenced by the force X_k of the irreversible process k , then the flux k is also influenced by the force X_i through the same phenomenological coefficient L_{ik} (see, for example, **thermomolecular pressure difference**).

When an external magnetic field \mathbf{B} is applied to the system the Onsager relations (1) have to be modified to read

$$L_{ik}(\mathbf{B}) = L_{ki}(-\mathbf{B}) \quad (3)$$

This means that the coefficient L_{ik} is the same function of \mathbf{B} as L_{ki} of $-\mathbf{B}$. The origin of (3) lies in the **Lorentz force**, This force being the vector product of the particle velocity and the magnetic field, one has to inverse not only the velocities but also the magnetic field to insure that all particles retrace their former paths.

The rates J_i may generally be expressed as time derivatives of state parameters α_i (which may be local temperatures, pressures, chemical degrees of advancement, etc.). In most applications these parameters are even functions of the particle velocities. The reciprocity relations have to be modified if some of the α_i are "odd variables" which change their sign with a reversal of the velocities. Let us suppose that the rates $J_1 \cdots J_m$ correspond to time derivatives of even variables, while $J_{m+1} \cdots J_n$

correspond to odd variables. We then have instead of (3)

$$L_{ik}(\mathbf{B}) = L_{ki}(-\mathbf{B}) \quad (4)$$

$$L_{i\nu}(\mathbf{B}) = -L_{\nu i}(-\mathbf{B}) \quad (5)$$

$$L_{\lambda\nu}(\mathbf{B}) = L_{\nu\lambda}(-\mathbf{B}) \quad (6)$$

with $i, k = 1 \cdots m$; $\nu, \lambda = m + 1, \cdots n$.

RECIPROCITY THEOREM, ACOUSTICAL.

In an acoustical system comprising a medium of uniform density and propagating irrotational vibrations of small amplitude, if a pressure p' produces a particle velocity v' and a pressure p'' produces a particle velocity v'' , then

$$\iint (v'' p' - v' p'')_n ds = 0 \quad (1)$$

where the surface integral is taken over the boundaries of the volume.

In the simple case in which there are only two pressures, as illustrated in the free field acoustical system of Figure 1, Equation 1 becomes

$$p' v'' = p'' v' \quad (2)$$

where p' , p'' and v' , v'' are the pressures and particle velocities depicted in the free field acoustical system of Figure 1.



Fig. 1.

The above theorem is applicable to all acoustical problems. However, the above theorem can be restricted to lumped constants as follows: In an acoustical system composed of inductance, acoustical capacitance and acoustical resistance let a set of pressures $p'_1, p'_2, p'_3 \cdots p'_n$ all harmonic of the same frequency acting in n points in the system, produce a volume current distribution $\dot{X}_1, \dot{X}_2, \dot{X}_3 \cdots \dot{X}_n$, and let a second set of pressures $p''_1, p''_2, p''_3 \cdots p''_n$ of the same frequency as the first, produce a second volume current distribution $\dot{X}''_1, \dot{X}''_2, \dot{X}''_3 \cdots \dot{X}''_n$. Then

$$\sum_{j=1}^n p'_j \dot{X}''_j = \sum_{j=1}^n p''_j \dot{X}'_j. \quad (3)$$

This theorem is valid provided the acoustical system is invariable, contains no internal source of energy or unilateral device, linearity in the relations between pressures and volume currents and complete reversibility in the elements,

and provided the applied pressures $p_1, p_2, p_3 \dots p_n$ are all of the same frequency.

In the simple case in which there are only two pressures, as illustrated in the acoustical

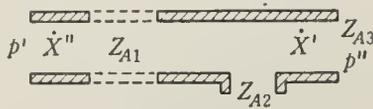


Fig. 2.

system of lumped constants in Figure 2, Equation 3 becomes

$$p' \dot{X}'' = p'' \dot{X}' \tag{4}$$

where p', p'' and \dot{X}', \dot{X}'' are the pressures and volume currents depicted in the acoustical system of lumped constants in Figure 2.

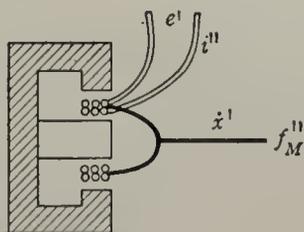
RECIPROCIITY THEOREM, ELECTRIC-(AL)-MECHANICAL. In an interconnected electrical-mechanical system let a set of electromotive forces $e'_1 \dots e'_n$ act in the electrical system, and a set of forces $f'_{M1} \dots f'_{Mn}$ act in the mechanical system with the resultant currents $i'_1 \dots i'_n$ in the electrical system and with the resultant velocities $\dot{x}'_1 \dots \dot{x}'_n$ in the mechanical system; let also, e'', i'', f''_M and \dot{x}'' represent a second set of electromotive forces, currents, forces and velocities. Then

$$\sum_{j=1}^n (e'_j i''_j + f'_{Mj} \dot{x}''_j) = \sum_{j=1}^n (e''_j i'_j + f''_{Mj} \dot{x}'_j). \tag{1}$$

In the simple case in which there is only one electromotive force in the electrical system and one force in the mechanical system

$$e' i'' = f'' M \dot{x}' \tag{2}$$

Equation 2 states that if a unit electromotive force e' in the electrical system produces a certain velocity \dot{x}' in the mechanical system, then a unit force f''_M in the mechanical system will



produce a current i'' in the electrical system which is numerically the same as the velocity previously produced in the mechanical system.

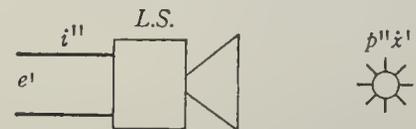
The electrical-mechanical reciprocity theorem is illustrated in the figure.

RECIPROCIITY THEOREM, ELECTRIC-(AL)-MECHANICAL-ACOUSTICAL. Since reciprocity relations hold in electrical-mechanical and mechanical-acoustical systems, they will also hold for three systems interconnected in the order electrical, mechanical, acoustical. This type of system embraces practically all reversible electroacoustic transducers.

For the simple case of a pressure p' in a sound field producing a current i' in the electrical system and a voltage e'' in the electrical system producing a volume current \dot{X}'' in the sound field the reciprocity relation may be written

$$\iint (p' \dot{X}'') ds = e'' i'.$$

This equation states that if, in the electrical system of a loud speaker, a generator of electromotive force e'' produces, at a point in a sound field, a volume current \dot{X}'' , then a nu-



merically equal pressure in the sound field at that point will produce a current i' in the electrical system equal to the previously produced volume current \dot{X}'' in the sound field.

The electrical-mechanical-acoustical reciprocity theorem is illustrated in the figure.

RECIPROCIITY THEOREM, ELECTRIC-(AL) NETWORK. In an electrical system composed of the electrical elements of inductance, electrical capacitance and electrical resistance, let a set of electromotive forces $e'_1, e'_2, e'_3 \dots e'_n$ all harmonic of the same frequency acting in n points in the invariable network, produce a current distribution $i'_1, i'_2, i'_3 \dots i'_n$, and let a second set of electromotive forces $e''_1, e''_2, e''_3 \dots e''_n$ of the same frequency as the first produce a second current distribution $i''_1, i''_2, i''_3 \dots i''_n$. Then

$$\sum_{j=1}^n e'_j i''_j = \sum_{j=1}^n e''_j i'_j. \tag{1}$$

This theorem is valid provided the electrical system is invariable, contains no internal source of energy or unilateral device, linearity in the relations between electromotive forces and currents and complete reversibility in the elements, and provided the electromotive forces $e_1, e_2, e_3 \dots e_n$ are all of the same frequency.

In the simple case in which there are only two electromotive forces, as illustrated in the



electrical system of the figure, Equation 1 becomes

$$e'i'' = e''i' \quad (2)$$

where e' , e'' and i' , i'' are the electromotive forces and currents depicted in the electrical system of the figure.

RECIPROCIITY THEOREM, ELECTRO-ACOUSTICAL. For an electroacoustic transducer satisfying the reciprocity principle, the quotient of the magnitude of the ratio of the open-circuit voltage at the output terminals (or the short-circuit output current) of the transducer, when used as a sound receiver, to the free-field sound pressure referred to an arbitrarily selected reference point on or near the transducer, divided by the magnitude of the ratio of the sound pressure apparent at a distance, d , from the reference point to the current flowing at the transducer input terminals (or the voltage applied at the input terminals), when used as a sound emitter, is a constant called the "reciprocity constant" independent of the type or constructional details of the transducer. The reciprocity constant is given by

$$\left| \frac{M_0}{s_0} \right| = \left| \frac{M_s}{s_s} \right| = \frac{2d}{\rho f} \cdot 10^{-7}$$

where M_0 is the free-field voltage response as a sound receiver, in open-circuit volts per microbar, referred to the arbitrary reference point on or near the transducer; M_s is the free-field current response in short-circuit amperes per microbar, referred to the arbitrary reference point on or near the transducer; s_0 is the sound pressure produced at a distance d centimeters from the arbitrary reference point in microbars per ampere of input current; s_s is the sound pressure produced at a distance d centimeters from the arbitrary reference point in microbars per volt applied at the input terminals; f is the frequency in cycles per second; ρ is the density of the medium in grams per centimeter³; d is the distance in centimeters from the arbitrary reference point on or near

the transducer to the point at which the sound pressure established by the transducer when emitting is evaluated.

RECIPROCIITY THEOREM (ELECTRO-MAGNETIC). Given a harmonic source at a point P_1 which is associated with field vectors \mathbf{E}_1 and \mathbf{H}_1 , and another source at P_2 which is associated with field vectors \mathbf{E}_2 and \mathbf{H}_2 , then whenever these vectors are continuous and finite, they satisfy the equation

$$\nabla(\mathbf{E}_1 \times \mathbf{H}_2 - \mathbf{E}_2 \times \mathbf{H}_1) = 0.$$

There are a number of other reciprocity theorems which may be obtained from this one.

RECIPROCIITY THEOREM FOR NEUTRONS. Consider the transport of monoenergetic neutrons in a scattering and absorbing medium. The theorem states that if $\psi(\mathbf{r}, \hat{\Omega}; \mathbf{r}_o, \hat{\Omega}_o)$ is the angular flux of neutrons in direction $\hat{\Omega}$ at \mathbf{r} due to a source of unit strength in direction $\hat{\Omega}_o$ at \mathbf{r}_o then

$$\psi(\mathbf{r}, \hat{\Omega}; \mathbf{r}_o, \hat{\Omega}_o) = \psi(\mathbf{r}, -\hat{\Omega}; \mathbf{r}_o, -\hat{\Omega}_o).$$

RECIPROCIITY THEOREM, MECHANICAL-ACOUSTICAL. In an interconnected mechanical-acoustical system let a set of forces $f'_{M1} \cdots f'_{Mn}$ act in the mechanical system, and a set of pressures $p'_{11} \cdots p'_{1n}$ act in the acoustical system with the resultant velocities $\dot{x}'_1 \cdots \dot{x}'_n$ in the mechanical system and with the resultant volume currents $\dot{X}'_1 \cdots \dot{X}'_n$ in the acoustical system; let also, f'' , \dot{x}'' , p'' and \dot{X}'' represent a second set of such forces, velocities, pressures and volume currents. Then

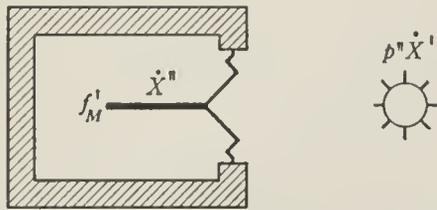
$$\sum_{j=1}^n (f'_{Mj} \dot{x}''_j + p'_j \dot{X}''_j) = \sum_{j=1}^n (f''_{Mj} \dot{x}'_j + p''_j \dot{X}'_j). \quad (1)$$

In the simple case in which there is only one force in the mechanical system and one pressure in the acoustical system

$$f'_M \dot{x}'' = p'' \dot{X}'. \quad (2)$$

Equation 2 states that if a unit force f'_M in the mechanical system produces a certain volume current \dot{X}' in the acoustical system, then a unit pressure p'' acting in the acoustical system will produce a velocity \dot{x}'' in the mechanical system which is numerically the same as the volume current previously produced in the acoustical system.

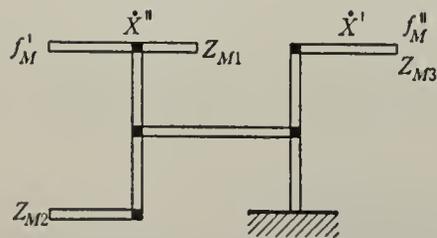
The mechanical-acoustical reciprocity theorem is illustrated in the figure.



RECIPROCITY THEOREM, MECHANICAL RECTILINEAL. In a mechanical rectilinear system composed of mechanical elements of mass, compliance and mechanical resistance, let a set of forces $f'_{M1}, f'_{M2}, f'_{M3} \dots f'_{Mn}$ all harmonic of the same frequency acting in n points in the system produce a velocity distribution $x'_1, x'_2, x'_3 \dots x'_n$, and let a second set of forces $f''_{M1}, f''_{M2}, f''_{M3} \dots f''_{Mn}$ of the same frequency as the first produce a second velocity distribution $x''_1, x''_2, x''_3 \dots x''_n$. Then

$$\sum_{j=1}^n f'_{Mj} x''_j = \sum_{j=1}^n f''_{Mj} x'_j. \tag{1}$$

This theorem is valid provided the mechanical system is invariable, contains no internal source of energy or unilateral device, linearity in the relations between forces and velocities and complete reversibility in the elements, and provided the applied forces $f_{M1}, f_{M2}, f_{M3} \dots f_{Mn}$ are all of the same frequency.



In the simple case in which there are only two forces, as illustrated in the mechanical rectilinear system of the figure, Equation 1 becomes

$$f'_M x'' = f''_M x'. \tag{2}$$

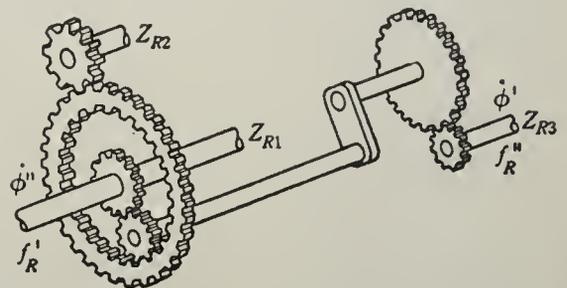
where f'_M, f''_M and x', x'' are the forces and velocities depicted in the mechanical rectilinear system of the figure.

RECIPROCITY THEOREM, MECHANICAL ROTATIONAL. In a mechanical rotational system composed of mechanical rotational elements of moment of inertia, rotational compliance and mechanical rotational resistance, let a set of torques $f'_{R1}, f'_{R2}, f'_{R3} \dots f'_{Rn}$

all harmonic of the same frequency acting in n points in the system, produce a rotational velocity distribution $\phi'_1, \phi'_2, \phi'_3 \dots \phi'_n$, and let a second set of torques $f''_{R1}, f''_{R2}, f''_{R3} \dots f''_{Rn}$ of the same frequency as the first produce a second rotational velocity distribution $\phi''_1, \phi''_2, \phi''_3 \dots \phi''_n$. Then

$$\sum_{j=1}^n f'_{Rj} \phi''_j = \sum_{j=1}^n f''_{Rj} \phi'_j \tag{1}$$

This theorem is valid provided the mechanical rotational system is invariable, contains no internal source of energy or unilateral device, linearity in the relations between torques and rotational velocities, and provided the applied torques $f_{R1}, f_{R2}, f_{R3} \dots f_{Rn}$ are all of the same frequency.



In the simple case in which there are only two torques, as illustrated in the mechanical rotational system of the figure, Equation 1 becomes

$$f'_R \phi'' = f''_R \phi'. \tag{2}$$

where f'_R, f''_R and ϕ', ϕ'' are the torques and angular velocities depicted in the mechanical rotational system of the figure.

RECIPROCITY THEOREM OF MAXWELL AND BETTI. The reciprocal theorem in classical elasticity theory applies to completely linear systems only. The stress-strain relation for the material must be linearly elastic and all geometrical changes which take place must have negligible effect on the conditions of equilibrium. The theorem then states that for a given body subjected independently to two systems of loads (which may include effects of inertial forces) the work done by the first load system on the displacements of the second equals the work of the second on the displacements of the first. An alternate form for a constrained body is that the displacement in the α -direction at A due to a force P in the β -direction at B equals the displacement in the β -direction at B when P is applied at A in the

α -direction. Also called *theorem of reciprocity*.

RECIRCULATION. Reversed flow in a turbulent jet confined in a tube. When the fluid of a confined jet mixes with ambient fluid having a different velocity, the mass flow is conserved but the momentum flux is decreased so that there is a pressure rise in the direction of flow which may cause the slower moving fluid near the wall to be reversed and re-entrained, i.e., recirculated.

Recirculation occurs when the flux of ambient fluid in the direction of the jet is less than what would be entrained into a similar free jet. Recirculation does not usually occur in confined flames because the expansion caused by the generation of heat produces an acceleration with a corresponding increase of momentum flux.

Recirculation often occurs in confined sprays near the nozzle where the droplets are decelerated relative to the surrounding gas.

RECOMBINATION, COEFFICIENT OF. A coefficient A that appears in the law expressing the rate of recombination of ions in a gas. If n^+ and n^- are the respective numbers per unit volume of the ions of the two signs, then

$$\frac{dn^+}{dt} = \frac{dn^-}{dt} = An^+n^-.$$

The constant depends both on the nature of the gas and on the pressure.

RECOMBINATION RATE, SURFACE. The time rate at which free electrons and holes recombine at the surface of a semiconductor.

RECOMBINATION RATE, VOLUME. The time rate at which free electrons and holes recombine within the volume of a semiconductor.

RECOMBINATION VELOCITY (ON A SEMICONDUCTOR SURFACE). The quotient of the normal component of the electron (hole) current density at the surface by the excess electron (hole) charge density at the surface.

RECOVERY. See *after effect, elastic*.

RECTANGULAR APERTURE. If light of wavelength λ illuminates a rectangular area of width a and height b and if ϕ is the angle of

incidence, then the intensity of the Fraunhofer diffraction pattern in a plane parallel to the plane of the aperture is

$$I = I_0 \left(\frac{\sin \beta}{\beta} \frac{\sin \gamma}{\gamma} \right)^2$$

where $\lambda\beta = \pi a \sin \theta$ with θ measured in a plane perpendicular to the side of length b and $\lambda\gamma = \pi b \sin \omega$ with ω measured in a plane perpendicular to the side of length a .

For the case $b \gg a$, see *slit*.

RECTANGULAR COORDINATES. Cartesian coordinates with three mutually perpendicular axes, used for locating the position of a point in space. A convention must be established for the relative arrangement of the three axes. The usual case, which is called a right-handed system, may be described as follows. Calling the axes OX , OY , OZ , choose the XY -plane to lie in the plane of the paper with the positive OX -axis pointing to the reader's right and the positive OY -axis pointing toward the top of the page. The positive OZ -axis is then pointing upward from the page toward the reader. If a pair of axes is exchanged, the system becomes left-handed.

From the analytic point of view, what is important is not whether a given system of coordinates has right-handed orientation, but only whether two given systems have the same or opposite orientation. This is decided as follows. Let (a_{11}, a_{21}, a_{31}) , (a_{12}, a_{22}, a_{32}) , (a_{13}, a_{23}, a_{33}) be the coordinates in the second system of the points whose coordinates in the first system are $(1,0,0)$, $(0,1,0)$, $(0,0,1)$. Then the two systems have the same or opposite orientation according as the (necessarily non-vanishing) determinant,

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

is positive or negative.

RECTANGULAR DISTRIBUTION. A frequency distribution of the form

$$f(x) = \frac{1}{b-a}, \quad a \leq x \leq b.$$

RECTIFIABLE CURVE. See *curve*.

RECTIFYING DEVELOPABLE. See *developable, rectifying*.

RECTIFYING LINE (AT A POINT OF A TWISTED CURVE). The generator of the rectifying developable (see **developable, rectifying**) of the curve which passes through the point.

RECTIFYING PLANE. See **principal planes**.

RECTILINEAL COMPLIANCE. Mechanical rectilinear potential energy is associated with the compression of a spring or compliant element. Mechanical energy increases as the spring is compressed. It decreases as the spring is allowed to expand. It is a constant, and is stored, when the spring remains immovably compressed. Rectilinear compliance is the mechanical element which opposes a change in the applied force. Rectilinear compliance C_M (termed compliance) in centimeters per dyne, is defined as

$$f_M = \frac{x}{C_M} \quad (1)$$

where x is the displacement, in centimeters, and f_M is the applied force, in dynes.

Equation 1 states that the linear displacement of a compliance is proportional to the compliance and the applied force.

Stiffness is the reciprocal of compliance.

Compliance in the mechanical rectilinear system is represented by a spring in the figure.



Graphical representation of the element rectilinear compliance, C_M .

RECTILINEAL RESISTANCE, MECHANICAL. See **mechanical rectilinear resistance**.

RECTILINEAR CONGRUENCE. A family of straight lines whose equation involves two independent parameters. Also called *congruence*, or *congruence of straight lines*. Each of the straight lines is a *ray*. The rectilinear congruence formed by the normals to a surface is called a *normal congruence*.

RECTILINEAR CONGRUENCE, SPHERICAL REPRESENTATION OF. See **spherical representation of a rectilinear congruence**.

RECTILINEAR MOTION. Motion in a straight line.

RECTILINEAR OPTICAL SYSTEM. See **orthoscopic system**.

RECURRENCE PARADOX. Any mechanical system will, after a sufficiently long period, return to any non-equilibrium state it may once have occupied. This is in apparent contradiction to the second law of **thermodynamics** but this recurrence paradox can be resolved by using the statistical approach to the second law.

RECURSION FORMULA. Relations between successive coefficients in a **series**, frequently the series solution of a differential equation, making it possible to calculate the $(n + 1)^{th}$ member of the series if the n^{th} , $(n - 1)^{th}$, etc., members are known. They usually involve only two coefficients but in some more complicated cases involve three. Examples are given under the heading of various polynomials, e.g., **Lagrange**, etc.

REDUCED EQUATION OF STATE. See **principle of corresponding states**.

REDUCED FOCAL LENGTH. The first **focal length** of a spherical-refracting surface, or of a lens, divided by the **index of refraction** of the medium in which the light is incident; or the second focal length divided by the refractive index of the medium into which the rays emerge.

REDUCED MASS. In a two-body system consisting of the masses m_1 and m_2 , the so-called reduced mass is given by

$$\frac{m_1 m_2}{m_1 + m_2}$$

REDUCED MODULUS. See **modulus, reduced or double**.

REDUCED PRESSURE. See **corresponding states, law of**.

REDUCED TEMPERATURE. See **corresponding states, law of**.

REDUCED VOLUME. See **corresponding states, law of**.

REDUCIBLE. See **representation**.

REDUCTION IN AREA. The ductility of metals is often measured in a tension test by

the reduction of cross-sectional area of fracture. When appreciable necking occurs, the so-called *true strain* is the natural logarithm of the ratio of the original cross-sectional area to the minimum area in the necked region.

REDUNDANCY. A statically **indeterminate structure** is a redundant structure. The degree of redundancy is the excess of the number of force and moment unknowns over the total available independent equations of static equilibrium.

REDUNDANT CONSTRAINT. When a body or structure forms a self-contained unit, stable attachment to a support requires six independent constraints in space, three in a plane. The excess of the number of constraints over those needed for stability gives the number of redundant constraints. The choice of the constraints to be labelled as redundant is arbitrary to a considerable extent, and depends upon the procedure adopted for obtaining a solution to the problem.

REFERENCE COUPLING. See **DBX**.

REFERENCE STIMULI. Color stimuli, not collinear but otherwise unrestricted, whose additive mixture can be used to evaluate all other color stimuli. These reference stimuli are necessary and sufficient for this purpose.

REFERENCE SYSTEMS—ACTIVITY AND ACTIVITY COEFFICIENTS. Consider an arbitrary thermodynamic system for which the **chemical potentials** of the various components are given by

$$\mu_i = \mu_i(T, p, x_1 \cdots x_c) \quad (1)$$

where x_i is the **mole fraction** of component i . Let us then consider a system of the same components but under certain (hypothetical) conditions such that it behaves like an **ideal system**. The chemical potentials are then of the form

$$\mu_i^R = \mu_i(T, p) + RT \ln x_i. \quad (2)$$

This latter system is called the *corresponding ideal system* or the *ideal reference system*. (Note that the R -exponent on the left side of the equation is not the gas constant, which appears on the right.)

Let us write

$$RT \ln a_i = \mu_i - \mu_i(T, p) \quad (3)$$

or

$$\mu_i = \mu_i(T, p) + RT \ln a_i. \quad (4)$$

We define a_i as the *activity* of the component i in the non-ideal system, with respect to the particular reference system considered.

The *activity coefficient* γ_i is defined by

$$\gamma_i = a_i/x_i. \quad (5)$$

Finally one calls often

$$\mu_i^{id} = \mu_i(T, p) + RT \ln x_i \quad (6)$$

the "ideal chemical potential." Then

$$\mu_i = \mu_i^{id} + RT \ln \gamma_i. \quad (7)$$

By introducing the activity and activity coefficients, it is possible to extend in a formal manner the laws valid for ideal systems to arbitrary real systems. These concepts were introduced by G. N. Lewis.

REFERENCE VALUE. The reference value of any parameter in a circuit is defined by Bode as that value of the parameter which produces zero steady-state output when all other parameters have their normal values and the input is at a specified frequency.

It is in general a function of frequency.

REFLECTANCE. See **spectral reflectance; direct reflectance; diffuse reflectance**.

REFLECTING POWER. See **absorptive power**.

REFLECTION. A deviation of the direction of radiant flux taking place entirely within or at the surface of a single optical medium. In some cases the term is restricted by the requirement that the wavelength of the monochromatic components of the radiation be invariant under reflection. (See **mixed reflection; reflex reflection; spectral reflection; uniform diffuse reflection**.)

REFLECTION, ANGLE OF. See **angle of reflection**.

REFLECTION COEFFICIENT. See **scattering coefficient**.

REFLECTION DENSITY. See **external optical density**.

REFLECTION, DIFFUSE. See **diffuse reflection**.

REFLECTION, LAWS OF. The laws of reflection in the case of **specular reflection** at a smooth surface may be stated as:

The incident and reflected rays are: (1) in

a normal plane to the surface, (2) on the same side of the surface, and (3) at equal angles with the normal.

If the surface is not smooth the reflection is **diffuse**.

REFLECTION PLANE. A **symmetry element** possessed by certain crystals, whereby one-half of the crystal is the reflection of the other half in a plane drawn through the center of the crystal.

REFLECTION, REFLEX. See **reflex reflection**.

REFLECTIVITY. The total reflection factor of a layer of material of such a thickness that there is no change of reflection factor with increase in thickness. (See also **absorptive power**.)

REFLECTOR SAVINGS. The reflector savings is the decrease in value of the parameter describing the size of a just-critical chain reacting assembly, when the assembly is changed by surrounding it with a scattering material (reflector).

REFLEX REFLECTION (RETRO-REFLECTION). Reflection in which the path of the returning light lies close to the direction in which the light is incident, whatever the **angle of incidence** at the reflecting surface.

REFRACTION. The change in the direction of propagation of radiation determined by change in the velocity of propagation in passing through a non-homogeneous medium, or in passing from one medium to another. (See **specific refraction**; **molar refraction**; **atomic refraction**; **standard refraction**.)

REFRACTION, CONICAL. See **conical refraction**.

REFRACTION, LAWS OF. Both Snell and Descartes must be given credit for discovering the laws of refraction (for specular refraction) which may be stated as:

The incident and refracted rays (1) are in the same plane with the normal to the surface, (2) they lie on opposite sides of the surface, and (3) the sines of their inclinations to the normal bear a constant ratio to one another, the ratio depending only on the two media involved, not on the angles. Condition (3) is usually referred to as **Snell's Law**. (See **specular reflection or refraction**.)

REFRACTION, MOLAR. See **molar refraction**.

REFRACTION OF SOUND. The change in direction of propagation of sound, produced by a change in the nature of the medium which affects the velocity, is termed refraction. Sound is refracted when the density varies over the wave front. For example the velocity of sound is given by

$$c^2 = \frac{\gamma p_0}{\rho} \quad (1)$$

where c is the velocity of sound, in centimeters per second, γ is the ratio of specific heat at constant pressure to that at constant volume, 1.4 for air, ρ is the density of the medium in grams per cubic centimeter, and p_0 is the static pressure, in dynes per square centimeter.

Thus it will be seen that a sound wave may be bent either downward or upward depending upon the relative temperatures (densities) of the air, Figure 1. The distance over which

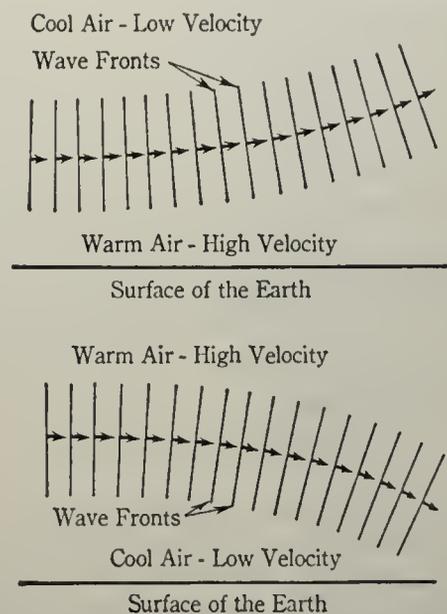


Fig. 1. The refraction of a sound wave in air.

sound may be heard is greater when the wave is bent downward than when it is bent upward. The first condition usually obtains during the early morning hours while the latter condition prevails during the day.

Structures may be built which refract sound waves. Acoustic lenses and prisms employing these structures may be used for various acoustical applications, as for example, loudspeakers and microphones.

Practical systems have been developed

based upon obstacle arrays and path length devices.

Obstacle arrays increase the effective density of the medium and thus produce a reduced propagation velocity of sound waves passing through the array. Three different obstacle arrays are shown in Figure 2.

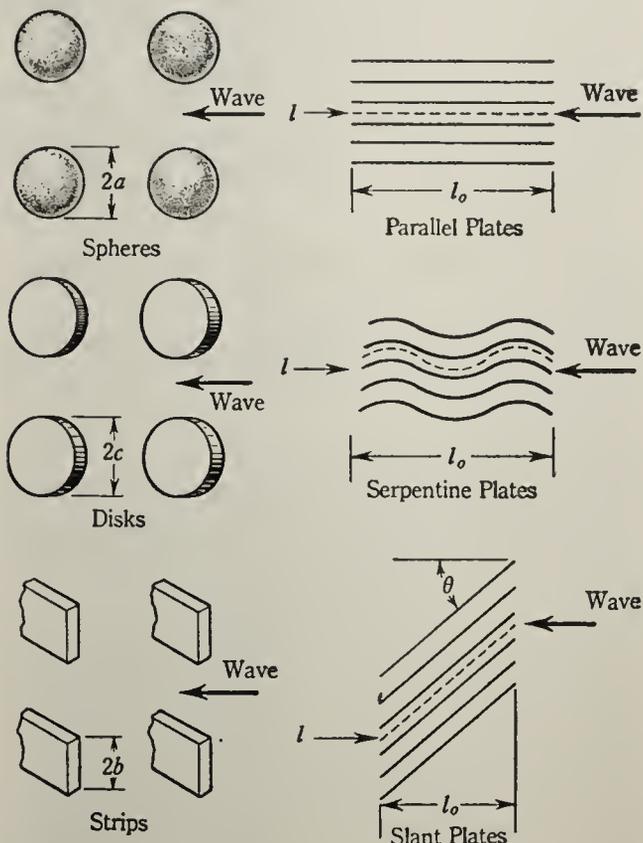


Fig. 2. Obstacle and path length structure for refracting sound waves.

The index of refraction n , of a spherical obstacle array as shown in Figure 2, is given by

$$n^2 = 1 + \frac{2}{3}\pi a^3 N \quad (2)$$

where a is the radius of the sphere, and N is the number of spheres per unit volume.

The index of refraction n , of a disk obstacle array as shown in Figure 2 is given by

$$n^2 = 1 + \frac{8}{3}c^3 N \quad (3)$$

where c is the radius of the disk, and N is the number of disks per unit volume.

The index of refraction n , of a series of strips as shown in Figure 2, is given by

$$n^2 = 1 + \pi b^2 N \quad (4)$$

where b is the half breadth of the strip normal to the direction of propagation of the wave and N is the number of strips per unit area viewed endwise.

Path length devices increase the time of travel of the waves through the path over that in free space. Three different path length devices are shown in Figure 2.

The index of refraction n , of parallel plates as shown in Figure 2, is unity.

The index of refraction n , of the serpentine plates as shown in Figure 2, is given by

$$n = \frac{l}{l_0} \quad (5)$$

where l is the path length through the plates, and l_0 is the path in the absence of the plates.

The index of refraction n , of slant plates as shown in Figure 2, is given by

$$n = \frac{l}{l_0} = \frac{1}{\cos \theta} \quad (6)$$

where θ is the angle between the direction of propagation of the wave and the plane of the plates.

An acoustic lens which converges the impinging sound wave is shown in Figure 3A. The sheet metal is arranged so that the path length through the lens is the greatest at the center of the lens. The action of the lens is

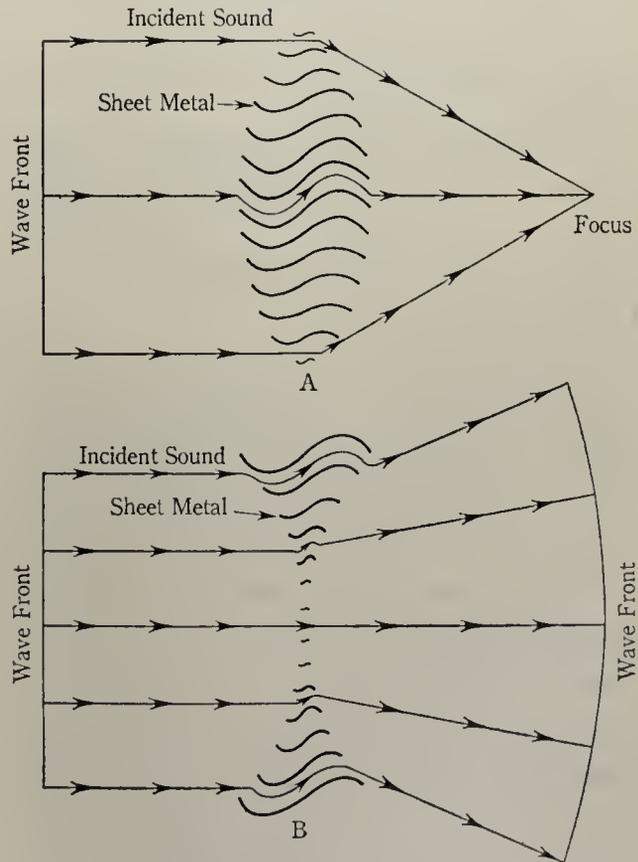


Fig. 3. Acoustic lenses. A, Converging lens. B, Diverging lens.

depicted by the ray and wavefront diagram of Figure 3A. The path lengths of all the pencils of the incident sound wave are all the same at focus. An acoustic lens which diverges the incident sound wave is shown in Figure 3B. The action of the lens is depicted by the ray diagram and wavefront diagram of Figure 3B.

An acoustic prism is shown in Figure 4. The acoustic prism changes the direction of the impinging sound wave. The action of the prism is depicted by the ray and wavefront diagrams of Figure 4.

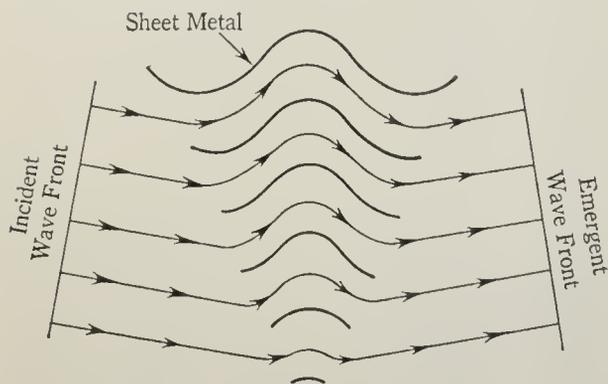


Fig. 4. Acoustic prism.

REFRACTIVE DISPERSIVITY. The derivative of the **refractive index** with respect to wavelength or frequency.

REFRACTIVE INDEX. See entries under **index of refraction**.

REFRACTIVE INDEX, CAUCHY FORMULA. See **Cauchy formula for refractive index**.

REFRACTIVE POWER, SPECIFIC. See **Lorenz-Lorentz law**.

REFRACTIVITY. (1) In general, the property of **refraction**, or a quantitative relationship by which it is expressed, which is commonly some function of the index of refraction. (2) The quantity $(n - 1)$ which enters many optical formulas is sometimes called **refractivity**. Here n is the **index of refraction**.

REFRIGERATING EFFECT. See **cycle** and **reversed Rankine cycle**.

REFRIGERATION CYCLE. A cycle that takes heat at a lower temperature and rejects it at a higher. Such a cycle must receive a power input from an external source, and the amount of heat rejected exceeds that taken in by the amount of work required to effect the

cycle. Theoretically, any power cycle which is reversible could be reversed to create a refrigeration cycle. Actually, practical considerations have caused modification of the reversed power cycle for refrigeration use. Nevertheless, the ordinary vapor compression refrigerating cycle resembles the Rankine power cycle to a close degree. (See **Carust cycle, reversed Rankine cycle**.)

REFRIGERATOR, ABSORPTION. See **absorption refrigerator**.

REFRIGERATOR, CASCADE. See **cascade refrigerator**.

REGENERATIVE CYCLE. See **carnotization**.

REGENERATIVE STEAM CYCLE. See **carnotization**.

REGRESSION. Suppose a variate y is distributed in some form at each of several values of a variable x . If the means, e.g., \bar{y}_x , of the y -distributions are related to the corresponding x -values by a functional relation $\bar{y}_x = f(x)$, this is said to be the regression equation of y on x . In the simplest case, the y distributions are normal with constant variance σ^2 , and $f(x)$ is linear, being most conveniently taken to be $\bar{y}_x = \alpha + \beta(x - \bar{x})$ where \bar{x} is the mean of the x 's. In this case, estimates of α and β (the regression coefficients) are obtained from a sample of n pairs of observations (x, y) by the method of **least squares** as

$$a = \Sigma y/n, \quad b = \Sigma(x - \bar{x})(y - \bar{y})/\Sigma(x - \bar{x})^2$$

while σ^2 is estimated by

$$s^2 = [\Sigma(y - \bar{y})^2 - b\Sigma(x - \bar{x})(y - \bar{y})]/(n - 2)$$

with $(n - 2)$ **degrees of freedom**. The variances of a and b are σ^2/n and $\sigma^2/\Sigma(x - \bar{x})^2$, and to apply tests of significance or to calculate fiducial or confidence limits, we may replace σ^2 by s^2 and use the **t-distribution** with $(n - 2)$ d.f.

In multiple regression, there are several different x 's and the regression equation takes the form $\bar{y} = \beta_1 x_1 + \beta_2 x_2 + \dots + \beta_p x_p$. With similar assumptions, the method of **least squares** provides efficient estimates of the β 's. No assumptions need be made about the interrelationships of the x 's; in particular, they may be successive powers of a variable t , then providing a method of fitting a polynomial relationship $y = \beta_0 + \beta_1 t + \beta_2 t^2 \dots$ (See

orthogonal polynomials, analysis of variance.)

REGRESSION, EDGE OF. See **edge of regression**.

REGULA FALSI. A method of solving an equation $f(x) = 0$, for a real root, in which if $f(x_0)f(x_1) < 0$, and the interval from x_0 to x_1 is known to contain only one real root, one passes the chord from the point $[x_0, f(x_0)]$ to $[x_1, f(x_1)]$, taking x_2 , the intersection of the chord with the real axis, as the next approximation. This is

$$x_2 = (x_0f_1 - x_1f_0)/(f_1 - f_0)$$

where $f_i = f(x_i)$. Repeat, using either x_0 or x_1 with x_2 . This is an **iterative method** which converges more slowly than Newton's, but from the opposite direction, hence if the two are used simultaneously the error is rigorously bounded.

REGULAR FUNCTION (OF A COMPLEX VARIABLE). A function $f(z)$ is regular at a point a if it has a derivative at every point in some neighborhood of a .

REGULARIZATION. Formal device introduced into quantized field theory (see **field theory, quantized**) in order to remove ambiguities arising in the evaluation of certain integrals. It corresponds to adding extra fields with different masses which then are allowed to tend to infinity.

REGULAR MULTIPLET. See **multiplet**.

REGULAR REFLECTION OR REFRACTION. See **specular reflection or refraction**.

REGULAR SINGULARITY (OF A DIFFERENTIAL EQUATION). Consider the homogeneous linear ordinary differential equation,

$$y^{(n)} + a_1y^{(n-1)} + \dots + a_{n-1}y' + a_ny = 0,$$

where the $a_i = a_i(x)$ are functions of x . The point $x = c$ is called a regular singularity of the equation if, although some of the $a_i(x)$ may have poles at c , still the functions $(z - c)^i a_i(x)$ are all analytic there. Any pole of $a_i(x)$ which is of higher than the i^{th} order is called an irregular singularity of the equation. For the importance of this classification see **indicial equation**.

REGULAR SOLUTIONS. A class of non-ideal solutions considered by Hildebrand for which the excess entropy (see **excess func-**

tions) is negligible. Therefore the excess Gibbs free energy is equal to the excess enthalpy (or heat of mixing)

$$G^E \cong H^E.$$

REGULAR SPACE. See **topological space**.

REHEAT FACTOR. In a gas or steam turbine cycle, the ratio R of the sum of the isen-

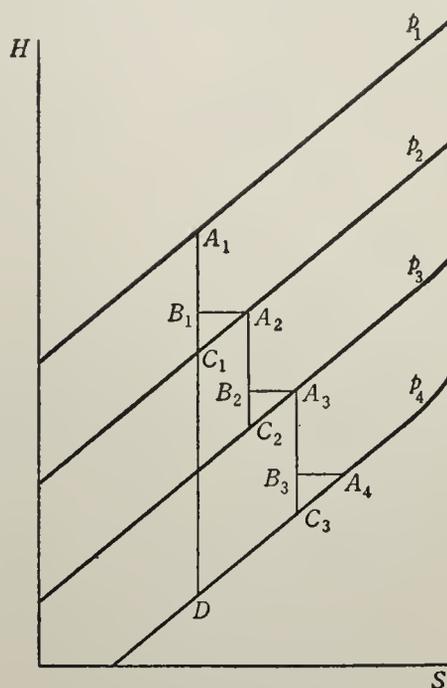


Fig. 1. Enthalpy-entropy relations for stages of turbine.

tropic enthalpy drops for all stages, to the isentropic enthalpy drop for the whole turbine. Hence in Figure 1,

$$R = \frac{\Sigma \overline{AC}}{A_1 D}.$$

The reheat factor $R > 1$ in real turbines.

There exists no single term for the analogous quantity in a turbocompressor, but it is sometimes called the reverse heat factor, C . Hence in Figure 2. (See page 762.)

$$C = \frac{A_1 D}{\Sigma \overline{AC}}.$$

It should be noted that here the ratio is inverted, and again $C > 1$.

RELATION. Let S be a given set of elements. A binary relation for S is a set R of ordered pairs of elements of S (or more generally, of pairs constructed in some way from the subsets of S). To express the fact that the pair a, b is in R , we write aRb , to be read as "a is

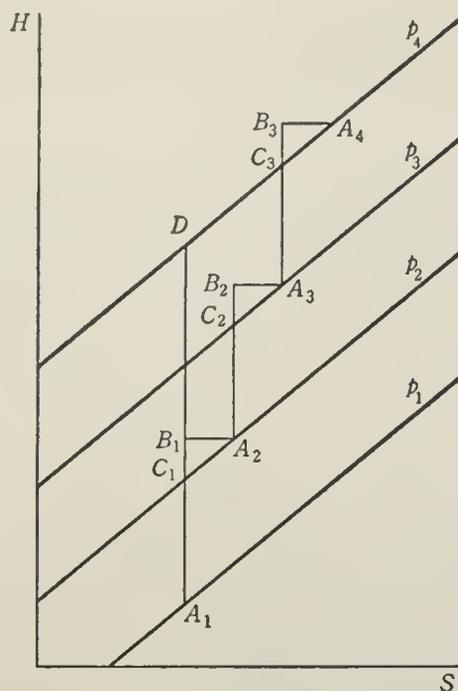


Fig. 2. Enthalpy-entropy relations for stages of turbocompressor. (See reheat factor.)

in the relation R to b ," for example, if the set S is the human race, then the relation of fatherhood is the set of all pairs of human beings in which the first is the father of the second. A binary relation is *reflexive* if aRa for all a is in S , *symmetric* if aRb implies bRa , and *transitive* if aRb and bRc imply aRc . Thus, fatherhood is none of the three, ancestorhood is transitive, etc. A relation may be, e.g., partly non-symmetric, e.g., brother, or completely non-symmetric, e.g., father, in which case it is often called antisymmetric. Or, if S is the set of fractions, then "greater than" is transitive, while "reducing to same fraction" is all three. A binary relation which is reflexive, symmetric and transitive is called an *equivalence*. If R is an equivalence, then the set S can be divided into mutually exclusive subsets of equivalent elements, each of which can then be regarded as a new mathematical entity. For example, a set of equivalent fractions is called a *rational number*. In fact, it is in this way that the majority of mathematical definitions are made; e.g., of quotient **group**.

RELATIVE ADSORPTION. See **Gibbs division surface**.

RELATIVE COORDINATE SYSTEM. Any coordinate system which is moving with respect to an **inertial coordinate system**.

RELATIVE CURVATURE OF CURVE C WITH RESPECT TO CURVE C' AT A POINT AT WHICH THEY TOUCH. The

difference between the **vector curvatures** of C and C' at the point of contact.

RELATIVE EFFICIENCY. See **efficiency ratio**.

RELATIVE ERROR. The ratio of the **error** in an approximation to the true value of the quantity approximated.

RELATIVE FREQUENCY. If the frequency in a class is f_i , and the total frequency in the distribution is Σf_i , then the relative frequency in the i th class is defined as $\frac{f_i}{\Sigma f_i}$. Similarly the relative frequency of an event is defined as $\frac{x}{s}$ where x is the number of successes and s is the number of trials.

RELATIVE INVARIANT. See **invariant**.

RELATIVE LUMINOUS EFFICIENCY.

(1) Relative luminous efficiency (of a monochromatic radiation of wavelength λ) is the ratio of the radiant flux at wavelength λ_m to that at wavelength λ which produces equally intense luminous sensations under specified photometric conditions, λ_m being chosen so that the maximum value of this ratio is unity. Unless otherwise indicated, the values used for the relative luminous efficiency relate to photopic vision by the normal eye having the characteristics laid down by the C.I.E. The values of V_λ were provisionally agreed internationally by the C.I.E. in 1924 and adopted in 1933 by the International Committee of Weights and Measures. (2) Relative luminous efficiency (of a monochromatic radiation, for scotopic vision, for the photometric standard observer) is the ratio of the radiant flux at wavelength λ_m to that at wavelength λ which produces equally intense luminous sensations under specified photometric conditions, λ_m being chosen so that the maximum value of this ratio is unity; the values used for the relative luminous efficiency relate to scotopic vision by the normal eye having the characteristics laid down by the C.I.E.

RELATIVE MOTION. Motion with respect to a **frame of reference** which in turn may be moving with respect to an **inertial frame** is called relative motion.

RELATIVE SCATTER INTENSITY. For scattering of radiation under any given set of physical conditions: the ratio of the **radiant**

intensity scattered in any given direction to the radiant intensity scattered in the direction of the incident beam.

The value of this ratio is a function of the angle between the direction in question and the direction of the incident beam. Thus it may be symbolized as $f(\phi)$, the *relative scattering function*. (See **scatter angle**.)

RELATIVE SENSITIVITY. See **sensitivity**.

RELATIVE SPECTRAL ENERGY DISTRIBUTION. The relative spectral energy distribution is the description of the spectral character of a radiation (description of a light) by the way in which the relative spectral concentration of some radiant quantity (e.g., **radiance**) varies throughout the spectrum.

RELATIVISTIC DYNAMICS OF A FREE MASS POINT. See **dynamics of a free mass point**.

RELATIVISTIC HYDRODYNAMICS. The fundamental laws of motion for an ideal fluid of pressure p and rest density ρ_0 , moving with the velocity \mathbf{w} (components denoted by u, v, w) are

$$\begin{aligned} \operatorname{div} \left\{ \gamma^2 \mathbf{w} \left(\rho_0 + \frac{p}{c^2} \right) \right\} \\ + \frac{\partial}{\partial t} \left\{ \gamma^2 \left(\rho_0 + \frac{p}{c^2} \right) - \frac{p}{c^2} \right\} = 0 \\ \gamma^2 \left(\rho_0 + \frac{p}{c^2} \right) \frac{Du}{Dt} + \frac{\partial p}{\partial x} + \frac{u}{c^2} \frac{\partial p}{\partial t} = F_x \\ \gamma^2 \left(\rho_0 + \frac{p}{c^2} \right) \frac{Dv}{Dt} + \frac{\partial p}{\partial y} + \frac{v}{c^2} \frac{\partial p}{\partial t} = F_y \\ \gamma^2 \left(\rho_0 + \frac{p}{c^2} \right) \frac{Dw}{Dt} + \frac{\partial p}{\partial z} + \frac{w}{c^2} \frac{\partial p}{\partial t} = F_z \\ \gamma = \frac{1}{\sqrt{1 - \frac{|\mathbf{w}|^2}{c^2}}} \end{aligned}$$

F_x, F_y, F_z are the components of the external volume forces (i.e., forces per unit volume). From the transformation laws for forces it can be shown that $p = p_0$, i.e., the pressure is the same in the moving system as in the system at rest. The first equation is the generalized **continuity equation** and the remaining equations are the **laws of motion**.

(See W. H. McCrea, *Relativity Physics*, Methuen Co., Ltd., London, fourth reviewed edition, reprinted 1957.)

RELATIVISTIC INVARIANCE AND QUANTUM MECHANICS. A quantum theory is said to be relativistically invariant if there exists for every ray Φ which describes a possible state of a physical system and every inhomogeneous Lorentz transformation $\{a, \Lambda\}$, a translated ray $\Phi'(a, \Lambda)$. This ray correspondence must satisfy the requirement that transition probabilities between rays and their translations are the same:

$$|(\Phi, \Psi)|^2 = |(\Phi'(a, \Lambda), \Psi'(a, \Lambda))|^2. \quad (a)$$

(See **symmetry principle and quantum mechanics**.) If the Hilbert space of physical states is a direct sum of subspaces \mathcal{H}_i such that every ray in \mathcal{H}_i corresponds to a physically observable state and every physically observable state is represented by a ray lying in one of the \mathcal{H}_i (see **superselection rules**), one can prove that by choosing a suitable representative vector Φ and $\Phi\{a, \Lambda\}$ from the rays $\Phi, \Phi'(a, \Lambda)$, one can define an operator $U(a, \Lambda)$ such that

$$\Phi(a, \Lambda) = U(a, \Lambda)\Phi \quad (b)$$

where U is either **unitary** or **anti-unitary** and is uniquely determined up to a multiplicative factor in each subspace \mathcal{H}_i . By the multiplicative law for Lorentz transformation the vector $\Phi(a_1, \Lambda_1)\Phi(a_2, \Lambda_2)$ and $\Phi(a_1 + \Lambda_1 a_2, \Lambda_1 \Lambda_2)$ are identical, from which follows that

$$\begin{aligned} U(a_1, \Lambda_1)U(a_2, \Lambda_2) \\ = \omega(a_1 \Lambda_1, a_2 \Lambda_2)U(a_1 + \Lambda_1 a_2, \Lambda_1 \Lambda_2) \quad (c) \end{aligned}$$

where ω is a factor of modulus one (which could be different on different subspaces). Since the square of an anti-unitary operator is unitary, $U(a, \Lambda)$ is unitary when (a, Λ) belongs to the restricted group since every such group element can be written as a product of elements which are squares. Since every element of the other pieces of the group can be written as the product of an element of the restricted group and a space inversion i_s , or a time inversion i_t or space time inversion $i_{ts} = i_s i_t$ it suffices to examine $U(0, i_s)$ and $U(0, i_t)$. It turns out on physical grounds that $U(0, i_s)$ must be unitary and $U(0, i_t)$, anti-unitary.

Thus, for each relativistically invariant quantum theory there exists by virtue of (c) a (continuous) representation up to a factor of the

inhomogeneous Lorentz group. To distinguish the different possible behaviors of a relativistic quantum theory under proper inhomogeneous Lorentz transformation one needs only classify the continuous representations (up to a factor) of the inhomogeneous Lorentz group.

RELATIVISTIC INVARIANCE OF A PHYSICAL THEORY. The requirement that the predictions of the theory with regard to the result of any possible observation shall be in accordance with requirements of the principle of special relativity. (See also **symmetry principles and quantum mechanics.**) In practice by relativistic invariance is meant the requirement that the mathematical formalism of the theory shall be covariant for all transformations of the restricted inhomogeneous Lorentz group.

RELATIVISTIC INVARIANTS IN THE SPECIAL THEORY OF RELATIVITY.

- (1) Speed of light in empty space, c .
- (2) Length of a world line element,

$$ds = \pm \sqrt{dx^2 + dy^2 + dz^2 - c^2 dt^2}.$$
- (3) Phase of an electromagnetic wave.
- (4) Rest mass of a particle.
- (5) Electric charge.
- (6) Action integral $\int_{t_1}^{t_2} L dt$.
- (7) Planck's constant, h .
- (8) Entropy $S = S_o$ (S = entropy of moving system; S_o = entropy of system at rest).
- (9) Boltzmann's constant, k .
- (10) Rest temperature = $T_o = \frac{T}{\sqrt{1 - \beta^2}}$.
- (11) Heat, $Q_o = \frac{Q}{\sqrt{1 - \beta^2}}$.

RELATIVISTICALLY INVARIANT WAVE EQUATIONS. From the mathematical theory of the irreducible representations of the inhomogeneous Lorentz group it is known that these representations can be labelled by the mass, spin and sign of energy when one excludes cases of space-like or null momentum. The physical interpretation that their representation space constitutes the manifold of states of an **elementary system** of the given mass and spin implies that invariance arguments suffice to obtain an enumeration of the relativistic equations for elementary systems.

All that is needed is an explicit realization of the irreducible representations in terms of a convenient set of vectors (wave functions of the elementary system) and the verification that within the manifold so defined a Lorentz invariant inner product can be constructed.

One finds that basis vectors of a representation space may be expressed as functions $\psi(p, \xi)$ of the momentum vector p (the eigenvalue of P_μ , the generator for translations), the momenta varying over the set $p^2 = m^2 \geq 0$ and of auxiliary spin variables ξ which may assume a finite or infinite set of values. The representation of the inhomogeneous Lorentz transformation $\{a, \Lambda\}$ within the manifold spanned by the $\psi(p, \xi)$ is given by

$$U(a, \Lambda)\psi(p, \xi) = e^{-ipa}Q(p, \Lambda)\psi(\Lambda^{-1}p, \xi)$$

where Q is an operator which operates only on the variables ξ .

When $p_\mu p^\mu = m^2$, $m > 0$, and j , the spin label of the irreducible representation, is equal to 0, the wave functions are defined on the positive shell of the hyperboloid $p_\mu p^\mu = m^2$, i.e., $p_o = \sqrt{p^2 + m^2}$. The invariant scalar product is

$$(\psi, \phi) = \int \frac{d^3 p}{p_o} \bar{\psi}(p)\phi(p) \quad (a)$$

and the wave equation reduces to $(p_\mu p^\mu - m^2)\phi = 0$.

For particles of higher spin $j = \frac{1}{2}N$ ($N = 1, 2, \dots$) the wave functions are again defined on the positive hyperboloid $p_o^2 = p_1^2 + p_2^2 + p_3^2 + m^2$. In addition they depend on the $2j$ spin variables $\xi_1, \xi_2, \dots, \xi_{2j}$ where each of ξ is four-valued, i.e., $\xi_i = (1, 2, 3, 4)$. The wave functions which describe the possible states of the system are required to be symmetric functions of the ξ 's and satisfy the $2j$ equations

$$\sum_{\mu=0}^3 \gamma_{(\alpha)}^\mu p_\mu \psi = m\psi \quad \alpha = 1, 2, \dots, 2j \quad (b)$$

$$\psi = \psi(p, \xi_1, \xi_2, \dots, \xi_{2j})$$

where the $\gamma_{(\alpha)}$'s operate on the ξ 's and satisfy the relation

$$\gamma_{(\alpha)}^\mu \gamma_{(\alpha)}^\nu + \gamma_{(\alpha)}^\nu \gamma_{(\alpha)}^\mu = 2g^{\mu\nu} \quad (c)$$

where $g^{\mu\nu}$ is the metric tensor, which ensures that ψ also satisfies $(p_\mu p^\mu - m^2)\psi = 0$ and thus describes a particle of definite mass. The scalar product is defined as

$$\begin{aligned}
 (\psi, \phi) &= \int p_0^{-2s-1} \sum_{\xi} \bar{\psi} \phi d^3 p \\
 &= \int p_0^{-2s-1} \sum_{\xi_1, \xi_2, \dots, \xi_{2j}=1}^4 \bar{\psi}(p, \xi_1 \dots \xi_{2j}) \\
 &\quad \phi(p, \xi_1 \dots \xi_{2j}) d^3 p. \tag{d}
 \end{aligned}$$

It can be shown that there exists a set of $2j + 1$ wave functions for a given \mathbf{p} which satisfy (b), (c) and which are linearly independent and that this set is invariant under rotations. The wave function ψ thus describes a particle of mass m , and spin j which can take on $2j + 1$ different total angular momentum substates. The case $j = \frac{1}{2}$ corresponds to the **Dirac equation**.

The representations for the case $m^2 = 0$ can be treated as above except that integrations in the scalar product are only over the light cone. For the higher spin case the essentially new feature is that now the wave equation (b) is invariant under any one of the operators $\gamma_{5(\alpha)} = i\gamma_{(\alpha)}^1 \gamma_{(\alpha)}^2 \gamma_{(\alpha)}^3 \gamma_{(\alpha)}^0$, $\gamma_{5(\alpha)}^2 = 1$ so that the previous manifold is restricted by the condition

$$\gamma_{5(\alpha)} \psi = \pm \psi.$$

The manifold $\gamma_{5(\alpha)} \psi = +\psi$ correspond to right polarization, that for $\gamma_{5(\alpha)} \psi = -\psi$, to left polarization; each of these manifolds being one dimensional for a given momentum \mathbf{p} . A mass zero particle has only two states of polarization. The case $j = \frac{1}{2}$ corresponds to the neutrino equation, the case $j = 1$ to the photon equation.

RELATIVISTIC QUANTUM FIELD THEORY. Almost simultaneously with the formulation of non-relativistic quantum mechanics the period from 1925 to 1935 saw the formulation and development of relativistic quantum mechanics, i.e., the incorporation of the physical requirements of the theory of special relativity into the structure of quantum mechanics. Thus, in 1926, a relativistic wave equation for a spin 0 particle was proposed by Schrödinger, Klein, Gordon and others (see **Klein-Gordon equation**) and in 1928, that for a spin $\frac{1}{2}$ particle by Dirac (see **Dirac equation**). It was soon recognized, however, that a consistent relativistic theory of interacting particles is by necessity a many-particle theory and that the incorporation of the phenomenon of particle creation and annihilation is an integral and irreducible aspect of relativistic quantum the-

ory. This, incidentally, corresponds to what is observed experimentally: light (**photons**) can be emitted and absorbed by charged systems, electron-positron pairs can be created and annihilated, mesons can be created in nucleon-nucleon collisions, etc. A way to describe an assembly of relativistic non-interacting particles is illustrated in the following for the case of spin 0 particles. It is known that a single isolated spin 0 particle is described by a positive energy solution, f_{α} , of the **Klein-Gordon equation**:

$$(\square + \mu^2)f_{\alpha}(x) = 0.$$

The index α labels a complete set of normalizable "wave-packet" positive energy solutions, so that

$$\begin{aligned}
 (f_{\alpha}, f_{\beta}) &= i \int d\sigma^{\mu}(x) \bar{f}_{\alpha}(x) \overleftrightarrow{\partial}_{\mu} f_{\beta}(x) = \delta_{\alpha\beta} \\
 \sum_{\alpha} f_{\alpha}(x) \bar{f}_{\alpha}(x') &= \frac{1}{i(2\pi)^3} \int_{k_0 > 0} \frac{d^3 k}{k_0} e^{-ik(x-x')} \\
 &= i\Delta^{(+)}(x-y).
 \end{aligned}$$

An n -particle state can be characterized by a set of occupation number n_1, n_2, \dots with $\sum_i n_i = n$, which specify the number of particles having quantum numbers α', α'' . Thus $|n_1, n_2, \dots, n_{\beta}, \dots\rangle$, $\sum_i n_i = n$, is an n -particle state with the n_1 particle in the state α' , n_2 in the state α'' , etc. The state $|0, \dots, 0, \dots\rangle$ with all $n_i = 0$, also abbreviated as $|0\rangle$, is called the no-particle state. Creation and annihilation operators a_{β} and a_{β}^* which increase or decrease the number of particles in the state β by one can be defined by the equations

$$\begin{aligned}
 a_{\beta} |\dots n_{\beta} \dots\rangle &= \sqrt{n_{\beta}} |\dots, n_{\beta} - 1 \dots\rangle \\
 a_{\beta} |0\rangle &= 0 \quad (\text{for all } \beta)
 \end{aligned}$$

$$\begin{aligned}
 a_{\beta}^* |\dots n_{\beta} \dots\rangle \\
 &= \sqrt{n_{\beta} + 1} |\dots, n_{\beta} + 1 \dots\rangle
 \end{aligned}$$

and thus satisfy the commutation rules

$$\begin{aligned}
 [a_{\alpha}, a_{\beta}^*] &= \delta_{\alpha\beta} \\
 [a_{\alpha}, a_{\beta}] &= [a_{\alpha}^*, a_{\beta}^*] = 0.
 \end{aligned}$$

The operator $a_{\beta}^* a_{\beta}$ is the number operator for the state β , i.e., operating on the vector $|n_1, n_2, \dots, n_{\beta}, \dots\rangle$ it yields $n_{\beta} |n_1, n_2, \dots, n_{\beta}, \dots\rangle$. The operator $\sum_{\beta} a_{\beta}^* a_{\beta}$ is thus the total number of particle operator.

One can next introduce the operators

$$\begin{aligned} \phi^{(+)}(x) &= \sum_{\alpha} f_{\alpha}(x) a_{\alpha} \\ \phi^{(-)}(x) &= \sum_{\alpha} \bar{f}_{\alpha}(x) a_{\alpha}^* = (\phi^{(+)}(x))^* \end{aligned}$$

which no longer depend on the one-particle basis $\{f_{\alpha}\}$ originally chosen to define occupation number space and which satisfies the following commutation rules:

$$\begin{aligned} [\phi^{(\pm)}(x), \phi^{(\pm)}(x')] &= 0 \\ [\phi^{(+)}(x), \phi^{(-)}(y)] &= i\Delta^{(+)}(x - y). \end{aligned}$$

The operator

$$\begin{aligned} \phi(x) &= \phi^{(+)}(x) + \phi^{(-)}(x) \\ &= \phi^*(x) \end{aligned}$$

satisfies the Klein-Gordon equation

$$(\square + \mu^2)\phi(x) = 0$$

and satisfies the commutation rules

$$\begin{aligned} [\phi(x), \phi(y)] &= i\Delta(x - y) \\ &= i(\Delta^{(+)}(x - y) - \Delta^{(+)}(y - x)). \end{aligned}$$

If one chooses for the set of commuting observables α , the energy and linear momentum and denotes the creation (annihilation) operator for a spin zero particle of momentum \mathbf{k} , energy $\sqrt{\mathbf{k}^2 + m^2}$ by $a_{\mathbf{k}}^*(a_{\mathbf{k}})$, then $\phi(x)$ expressed in terms of these operators becomes

$$\phi(x) = \frac{1}{\sqrt{2(2\pi)^3}} \int_{k_0} \frac{d^3k}{k_0} (a_{\mathbf{k}} e^{-ikx} + a_{\mathbf{k}}^* e^{ikx}).$$

It will be noted that the total number operator can now be written as

$$\begin{aligned} N &= \int \frac{d^3k}{k_0} a_{\mathbf{k}}^* a_{\mathbf{k}} \\ &= i \int d\sigma^{\mu} \left\{ \frac{\partial \phi^{(-)}(x)}{\partial x^{\mu}} \phi^{(+)}(x) - \phi^{(-)}(x) \frac{\partial \phi^{(+)}(x)}{\partial x^{\mu}} \right\} \end{aligned}$$

and the energy momentum operator as

$$\begin{aligned} P_{\mu} &= \int \frac{d^3k}{k_0} k_{\mu} a_{\mathbf{k}}^* a_{\mathbf{k}} \\ &= \int d\sigma^{\nu} T_{\mu\nu}(x) \end{aligned}$$

$$k_{\mu} = (k_0 = \sqrt{\mathbf{k}^2 + m^2}, \mathbf{k})$$

where

$$T_{\mu\nu}(x) = : \frac{\partial \phi}{\partial x^{\mu}} \frac{\partial \phi}{\partial x^{\nu}} + \frac{1}{2} g_{\mu\nu} (\mu^2 \phi^2 - \phi_{\lambda} \phi^{\lambda}) :$$

and $::$ denotes that the **normal product** of the operator is to be taken. The similarity of these operator expressions with those obtained from the **Lagrangian formalism** of a classical field system which is then quantized is to be noted. For the case at hand the identification of $\pi(x)$ with $\partial_0 \phi(x)$ results in a complete equivalence of the two formalisms.

One can thus say that the quantized field described by the operator $\phi(x)$ which satisfies the Klein-Gordon equation corresponds to a system of free non-interacting particles. The particle aspect corresponds to a particular choice of the complete set of observables to describe the system, namely that in which the operators N and P_{μ} ($\mu = 0, 1, 2, 3$) are diagonal.

The above is easily generalized to the case of complex fields which then describe charged particles. The introduction of interaction between particles is achieved by coupling the equations of motion satisfied by the field operators corresponding to the different particles. Thus, in the usual description of the interaction between electrons, positrons and the electromagnetic field, the field operators satisfy the following equations of motion:

$$\begin{aligned} (i\gamma^{\mu} \partial_{\mu} - m)\psi(x) &= \delta m \psi(x) + e\gamma^{\mu} A_{\mu}(x)\psi(x) \\ \square A_{\mu}(x) &= j_{\mu}(x) = \frac{e}{2} [\bar{\psi}(x)\gamma_{\mu}\psi(x)]. \end{aligned}$$

These equations of motion are derivable from the Lagrangian

$$\begin{aligned} \mathcal{L}(x) &= -\frac{1}{2} \frac{\partial A_{\mu}(x)}{\partial x_{\nu}} \frac{\partial A^{\mu}(x)}{\partial x^{\nu}} \\ &\quad - \frac{1}{4} [\bar{\psi}(x), (-i\gamma^{\mu} \partial_{\mu} + m)\psi(x)] \\ &\quad - \frac{1}{2} [i\partial_{\mu} \bar{\psi}(x)\gamma^{\mu} + m\bar{\psi}(x), \psi(x)] \\ &\quad - \frac{e}{2} [\bar{\psi}(x)\gamma^{\mu}\psi(x)]A_{\mu}(x) + \delta m \bar{\psi}(x)\psi(x). \end{aligned}$$

The above operator equations of motion are supplemented by the quantum conditions expressed by the commutation rules:

$$\begin{aligned}
[\psi(x), \bar{\psi}(x')]_+ &= -iS(x - x') \\
[A_\mu(x), A_\nu(x')] &= i\hbar c g_{\mu\nu} D(x - x') \\
[\psi(x), \psi(x')]_+ &= [\bar{\psi}(x), \bar{\psi}(x')]_+ = 0 \\
[\psi(x), A_\mu(x)] &= [\bar{\psi}(x), A_\mu(x)] = 0
\end{aligned}$$

for $(x - x')^2 < 0$ i.e., x', x' spacelike.

The operators carry the entire time dependence of the system and we are thus in the **Heisenberg picture**. The state vector $|\Psi\rangle$ of the system is time-independent with $\partial_t|\Psi\rangle = 0$. The Lagrangian enables one to construct an energy-momentum tensor $T_{\mu\nu}$ from which the total energy four-momentum of the field, P_μ can be defined; the time component, P_0 is the Hamiltonian of the system. The energy-momentum operators satisfy the following commutation rules

$$[P_\mu, P_\nu] = 0$$

and they correspond to the space-time displacement operators in the sense that for an arbitrary Heisenberg operator $F(x)$

$$\frac{i}{\hbar} [P_\mu, F(x)] = \frac{\partial F(x)}{\partial x^\mu}$$

Since all the operators P_μ commute with one another, one can choose a representation in which all the components of the four-vector operator P_μ are diagonal. It is usually assumed that

(a) the eigenvalues of P_μ are time-like or null vectors, i.e., that the total mass of the system be positive or zero.

(b) there exists a state $|0\rangle$ with eigenvalue 0 of P_μ i.e., a state with zero energy and momentum and that this state is the state of lowest energy. It is called the *vacuum state*.

Similar sets of coupled equations describe the interaction of other kinds of particles. For example, the interaction of nucleons (mass M , spin $1/2$, isotopic spin $1/2$, field operator $\psi(x)$) with mesons (mass μ , spin 0, isotopic spin 1, field operator $\phi_i(x)$) is described by the operator equations

$$\begin{aligned}
(i\gamma^\mu \partial_\mu - M)\psi(x) &= \delta M\psi(x) - G\gamma_5 \tau^i \phi^i(x)\psi(x) \\
(\square + \mu^2)\phi_i(x) &= \delta\mu^2 \phi^i(x) - \frac{1}{2}G[\bar{\psi}(x)\gamma_5 \tau^i \psi(x)]
\end{aligned}$$

and the non-vanishing commutators (or anti-commutators) are

$$\begin{aligned}
[\bar{\psi}(x), \psi(x')]_+ &= \gamma_0 \delta(x - x'), \quad (x_0 = x'_0) \\
i \left[\frac{\partial \phi^i(x)}{\partial t}, \phi^j(x') \right] &= \delta^{ij} \delta(x - x'), \quad (x'_0 = x_0)
\end{aligned}$$

It should not be surprising that the basic equations of these theories are inconsistent, since they are patterned after classical theories whose equations are inconsistent, involving as they do the self-interaction of point particles (see **classical electron theory**). Inconsistencies are indeed present on the above quantized theories and reveal themselves in the occurrences of the so-called divergences: the equations of the theory predict ill-defined and infinite results for physical quantities which should be finite. For example, if one computes the **S-matrix** elements for a particular process, it turns out that although the lowest order term in the coupling constant in which the process is possible is finite, all higher order terms (**radiative corrections**) are actually infinite. Until 1947 the procedure was to ignore these divergences and to consider only the lowest order non-vanishing term (which is usually finite). In quantum electrodynamics where the expansion parameter is $e^2/\hbar c = 1/137 \ll 1$, this procedure gives reasonable agreement with experiment.

One of the most important advances in quantum field theory occurred in the post-war decade and consisted in the discovery by Feynman, Kramers and Schwinger that there existed a natural procedure for extracting finite parts of the infinite coefficients in the higher powers of the coupling constant. The method is referred to as *renormalization* because all the infinities appearing in the S-matrix elements are traced to a certain finite number of basic ones in the parameter of the theory (Dyson), e.g., in electrodynamics the infinite corrections to the mass and charge of the particles. One has only to replace these infinite corrected parameters everywhere in the theory by their physical values as observed in nature, and all the coefficients of the power series expansion of the S-matrix elements become finite. The renormalization calculations for processes like the **Lamb shift** and the **anomalous magnetic moment** of the electron yield very good agreement with experiment.

In its original form the renormalization procedure was a considerable advance, but it lies entirely outside the confines of conventional mathematics since it deals with meaningless (inconsistent) equations and manipulates them according to a certain prescription to obtain finite results. Moreover, in its original form it is applicable only to the power series expansion of the theory. This restriction is

particularly constricting in meson theory, where the coupling constant $G^2/\hbar c \approx 10$, so that power series expansions cannot be expected to be a useful representation of the predictions of the theory.

Although generalizations of the renormalization method which free it from the restriction to power series, and which eliminate many of the manipulations with infinities, have been given by Källén and by Valatin, these generalizations lead to equations which are of highly implicit character and for which it is not possible to assert that they are mathematically well defined. The central question of quantum field theory: Do the renormalized equations of quantized field theory (e.g., quantum electrodynamics or meson theory) have any solutions?, remains unanswered at present.

When one passes from the simplest theories, such as quantum electrodynamics, to more ambitious theories which attempt to explain the existence and relations of all the elementary particles in terms of fields, the above features of the equations of motions become almost insurmountable handicaps. It has led to the natural question: is there something fundamentally wrong with local relativistic field theory? Although there have been many attempts to answer this question there is no precise evidence that there is something wrong with field theory. In fact, the qualitative predictions of field theory have been amply verified whenever put to the experimental test.

The difficulties mentioned above have recently motivated investigations of the general structure of local field theory where one avoids any dynamical assumptions and attempts to determine the physical consequences of very general axioms like that of relativistic invariance and the commutativity of fields at space-like separations. They discuss field theories in general, rather than a specific theory such as quantum electrodynamics. They do not use formal tools such as the Hamiltonian and Lagrangian formalism, nor do they use perturbation theoretic methods. They also attempt a higher standard of mathematical rigor than is usual in the bulk of field theoretic investigations. An axiomatic treatment of local relativistic quantum field theory has been given by Garding and Wightman whose version we follow for the case of a scalar field. They note first of all that what can be expected to be a well-defined linear operator is not the field itself $\phi(x)$, but the field operator integrated with a test function $f(x)$

$$\phi(f) = \int d^4x f(x)\phi(x)$$

The **test functions** are complex-valued functions vanishing outside some compact set of space-time points and are infinitely differentiable. The notion of a scalar field is then defined as follows:

Let \mathfrak{H} be the Hilbert space and $\{a, \Lambda\} \rightarrow U(a, \Lambda)$ continuous representation of the inhomogeneous Lorentz group in \mathfrak{H} . It is supposed that no negative energy states exist in \mathfrak{H} and that a unique vector (the vacuum state) exists which is described by a vector satisfying $U(a, \Lambda)\Psi_0 = \Psi_0$. A scalar field is then a linear operator valued functional defined for all test functions f vanishing outside some compact set and satisfying the following axioms:

(I) *The operators $T(f)$ and $T(f)^*$ possess a common dense domain, \mathfrak{D} , which is a manifold in \mathfrak{H} satisfying $\Psi_0 \in \mathfrak{D}$,*

$$U(a, \Lambda)\mathfrak{D} \subset \mathfrak{D}, \quad T(f)\mathfrak{D} \subset \mathfrak{D}, \quad T(f)^*\mathfrak{D} \subset \mathfrak{D}.$$

(Ib) *$T(f)$ is weakly continuous in f : if Φ and Ψ are \mathfrak{D} then $(\Phi, T(f)\Psi)$ is a **distribution** in the sense of Schwartz. (It is usually further assumed that $(\Phi, T(f)\Psi)$ is a tempered distribution.)*

(II) *Under Lorentz transformations*

$$U(a, \Lambda)T(f)U^{-1}(a, \Lambda) = T(f_{[a, \Lambda]})$$

where $f_{[a, \Lambda]}(x) = f(\Lambda^{-1}(x - a))$. (This is the rigorous statement of the operator transformation law $U(a, \Lambda)\phi(x)U^{-1}(a, \Lambda) = \phi(\Lambda x + a)$ which defines ϕ as a scalar field.)

(III) *Local commutativity.* Let f and g be two test functions with the property $f(x)g(y) = 0$ for all x and y such that $(x - y)^2 \geq 0$, then

$$[T(f), T(g)] = 0$$

where $[\quad , \quad]$ denotes the commutator. If one were to consider an odd-half integer spin field the commutator would have to be replaced by the anticommutator (see **connection of spin with statistics**).

Axioms I, II and III define the notion of a local scalar field in relativistic quantum theory. The axioms can easily be generalized to other types of fields. A set of fields T_j constitutes a complete description of a quantum mechanical system if it is locally commuting and if the set of states $\{P(T_j)\Psi\}$ where P runs over all polynomials in T_j and T_j^* , is dense in the set of all physical states. A set of fields is locally commuting if $[T_i(g), T_j(f)] = 0$ for all i and j and

for all test functions with the property $f(x)g(y) = 0$ for all x and y such that $(x - y)^2 \geq 0$.

When a relativistic quantum theory of a physical system possesses a complete description in terms of fields satisfying axioms I, II and III one may call it a field theory, but if it is to describe natural phenomena at all it must contain in addition to fields also some particle observables. As a minimum it must contain those prediction of collision experiments which in the usual formulation are collected in the **S-matrix**. The standard way to do this is to impose the **asymptotic condition** on a set of fields of the theory.

(IV) A scalar field $\phi(x)$ satisfies the asymptotic condition if (a) for every pair of vectors Φ and Ψ in \mathfrak{H} the limit

$$\lim_{t_0 \rightarrow +\infty} (\Phi, \phi(f, t_0)\Psi) = (\Phi, \phi^{out}(f)\Psi)$$

$$\lim_{t_0 \rightarrow -\infty} (\Phi, \phi(f, t_0)\Psi) = (\Phi, \phi^{in}(f)\Psi)$$

where

$$\phi(f, t_0) = i \int_{t_0} d\sigma^\mu f(x) \overleftrightarrow{\partial}_\mu \phi(x)$$

when f a normalizable solution of the Klein-Gordon equation with mass m , exists and (b) the operators $\phi^{in}(f)$ and $\phi^{out}(f)$ defined by this equation are free neutral scalar fields of mass m , i.e., the fields $\phi^{in}(x)$ and $\phi^{out}(x)$ satisfy the differential equations and commutation relations of the free field theory. If the set ϕ^{in} and ϕ^{out} are irreducible, then there exist a unitary operator S such that

$$\phi^{out} = S^{-1} \phi^{in} S.$$

This matrix of S is the **S-matrix**. Using the set of operators ϕ^{in} and ϕ^{out} one can obtain explicit formulae for the stationary states describing the collisions of beams of particle associated with the field. For example, the stationary state in which two beams of spin zero mesons with four-momenta α and β collide and give rise to their collision products is given by

$$\phi_{in}(f_\alpha) \phi_{in}(f_\beta) \Psi_0.$$

In spite of their very general character these axioms are extremely restrictive and already imply the **TPC theorem**, the connection between **spin and statistics** and **Haag's theorem**.

RELATIVITY, COMPOSITION LAW OF ACCELERATION IN. See **acceleration, composition law of**.

RELATIVITY, NEWTONIAN. The assumption that clocks and rigid measuring rods are unaffected by their motion relative to an observer, which leads to the Galilean transformation equations. (Cf. **relativity theory, special**; **relativity theory, general**; **Lorentz transformations**.)

RELATIVITY PRECESSION. (Also called Thomas Precession.) A set of space axes that is both moving and accelerated has, as observed from an inertial reference system, an angular velocity of precession,

$$\omega = \frac{[\mathbf{a} \times \mathbf{v}]}{v^2} \left\{ \left[1 - \frac{v^2}{c^2} \right]^{-1/2} - 1 \right\}.$$

\mathbf{v} is the instantaneous velocity and \mathbf{a} the instantaneous acceleration of the moving coordinate system.

For small velocities,

$$\omega = \frac{\dot{\mathbf{v}} \times \mathbf{v}}{2c^2}.$$

RELATIVITY THEORY, GENERAL. (1) *Ponderomotive law:*

$$m_0 \left(\frac{d^2 x^i}{ds^2} + \Gamma_{rk}^i \frac{dx^r}{ds} \frac{dx^k}{ds} \right) = F^i,$$

where F^i is the impressed, non-gravitational force. (See also **ponderomotive equation**.)

(2) Field equations

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R - \lambda g_{\mu\nu} = -\chi P_{\mu\nu}. \quad (\text{A})$$

λ is the cosmological constant,

$$\chi = \frac{8\pi G}{c^4}.$$

where G is Newton's **gravitational constant**, $R_{\mu\nu}$ is the contracted **Riemann-Christoffel curvature tensor**, R is the curvature scalar, $P_{\mu\nu}$ is the matter tensor (also called stress-energy tensor). In the mass-point representation of matter, $\lambda = 0$ and the field equations are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 0 \quad (\text{A}')$$

outside the mass points. However, the field equations are not satisfied at the locations of the mass points. The field equations (A) can be represented as the Euler-Lagrange equations of a Hamiltonian principle:

$$I = \iiint\!\!\!\int_D R \sqrt{-g} dx^1 dx^2 dx^3 dx^4$$

$$\delta I = 0$$

(g is the determinant of the $g_{\mu\nu}$).

The variations of the $g_{\mu\nu}$ and their first derivatives are assumed to vanish on the boundary of the four-dimensional domain D .

In the presence of an electromagnetic field, the variational principle is

$$I = \iiint\!\!\!\int_D \left(R - \frac{\chi}{c^2} \phi_{\mu\sigma} \phi^{\mu\sigma} \right) \sqrt{-g} dx^1 dx^2 dx^3 dx^4$$

$$\delta I = 0$$

where the covariant vector $\phi_\mu = (\mathbf{A}; -c\phi)$ (\mathbf{A} = vector potential; ϕ = scalar potential) and

$$\phi_{\mu\sigma} = \frac{\partial \phi_\mu}{\partial x^\sigma} - \frac{\partial \phi_\sigma}{\partial x^\mu}$$

In this variational principle, the ϕ_μ and their first derivatives are also assumed to vanish on the surface of D . The resulting field equations are

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{\chi}{c^2} (2\phi_{\mu\rho} \phi_{\nu}{}^\rho - \frac{1}{2} g_{\mu\nu} \phi_{\rho\sigma} \phi^{\rho\sigma})$$

$$\phi_{;\rho}{}^{\mu\rho} = 0$$

(semicolon signifies covariant derivative).

The ponderomotive equations for a charged particle are

$$\frac{dx^\mu}{ds} + \Gamma_{\rho\sigma}{}^\mu \frac{dx^\rho}{ds} \frac{dx^\sigma}{ds} - \frac{e}{m_0 c^3} \phi^{\mu\rho} g_{\rho\lambda} \frac{dx^\lambda}{ds} = 0$$

(e = charge of particle).

Conservation laws. There exist 4 conservation laws which correspond to the conservation of momentum and energy in classical mechanics:

$$\frac{\partial T_\mu{}^\nu}{\partial x^\nu} = 0$$

with

$$T_\mu{}^\nu = \sqrt{-g} \left(P_\mu{}^\nu + \frac{t_\mu{}^\nu}{16\pi G} \right)$$

where

$$t_\mu{}^\nu = \frac{1}{\sqrt{-g}} \left(\frac{\partial H}{\partial g^{\rho\lambda}} g^{\rho\lambda}{}_{,\nu} - \delta_\mu{}^\nu H \right)$$

and

$$H = \sqrt{-g} g^{\mu\nu} (\Gamma_{\rho\sigma}{}^\mu \Gamma_{\mu\nu}{}^\rho - \Gamma_{\mu\rho}{}^\sigma \Gamma_{\nu\sigma}{}^\rho)$$

$t_\mu{}^\nu$ is the pseudo-stress energy tensor, caused by the gravitational field only.

Rigorous solutions of the field equations. (a) Schwarzschild solution of a charged mass point at rest (m_0 = mass; e = electric charge).

$$ds^2 = \left[\delta_{ij} - \left\{ 1 - \frac{1}{1 - \frac{2Gm_0}{r} + \frac{Ge^2}{r^2}} \right\} \xi_i \xi_j \right] dx^i dx^j + \left[1 - \frac{2Gm_0}{r} + \frac{Ge^2}{r^2} \right] dt^2 \quad i, j = 1, 2, 3$$

$$r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2; \quad \xi_i = \frac{x^i}{r}$$

(b) *The field of an incompressible liquid sphere.*

(b-1) *Interior of sphere* ($0 \leq \gamma \leq \gamma_a$)

$$ds^2 = \frac{3}{\chi c^2 \rho} (d\gamma^2 + \sin^2 \gamma d\theta^2 + \sin^2 \gamma \sin^2 \theta d\phi^2) - c^2 \left(\frac{3 \cos \gamma_a - \cos \gamma}{2} \right) dt^2$$

$\gamma = \gamma_a$ = surface of sphere

ρ = density

$$\chi = \frac{8\pi G}{c^2}$$

(b-2) *Exterior of sphere* ($\gamma_a \leq \gamma \leq \frac{\pi}{2}$)

$$ds^2 = \frac{R}{R - \alpha} dR^2 + R^2 (d\theta^2 + \sin^2 \theta d\phi^2) - c^2 \frac{R - \alpha}{R} dt^2$$

$$\alpha = \frac{2GM}{c^2}$$

= "gravitational radius of the total M "

where

$$R^3 = \left(\frac{\chi c^2 \rho}{3} \right)^{-3/2} \left\{ \frac{3}{4} \cos \gamma_a \left(\gamma - \frac{\sin 2\gamma}{2} - \gamma_a + \frac{\sin 2\gamma_a}{2} \right) + \frac{3}{2} \sin^3 \gamma_a - \frac{\sin^3 \gamma}{2} \right\}$$

The volume of the sphere is

$$V = 2\pi \left(\frac{3}{\chi c^2 \rho} \right) \left(\gamma_a - \frac{\sin 2\gamma_a}{2} \right)$$

The pressure in the interior of the sphere is

$$p = \rho c^2 \frac{\cos \gamma - \cos \gamma_a}{3 \cos \gamma_a - \cos \gamma}$$

The Three Einstein Effects. (a) *The advance of the perihelion of Mercury.* One considers the motion of a small body (= "planet") in the field of Schwarzschild mass (= "sun") which is produced by a much larger mass. The particle moves along a geodesic. The motion takes place in a plane. If polar coordinates r and ϕ are used, the resulting differential equations are

$$r^2 \frac{d\phi}{ds} = \frac{h}{c} \tag{1}$$

$$\left(1 - \frac{2mG}{c^2 r}\right) \frac{dt}{ds} = k \tag{2}$$

$$\frac{d^2 u}{d\phi^2} + u = \frac{mG}{h^2} + \frac{3mGu^2}{c^2} \tag{3}$$

where m = Schwarzschild mass; c = velocity of light in flat space; h, k = constants of integration; G = Newton's constant of gravitation; $u = \frac{1}{r}$; ds = four-dimensional, space-like line-element.

Equation (1) is the generalized law of the constancy of the areal velocity and equation (2) represents the conservation law of energy. Equation (3) differs from the Newtonian expression by the small perturbation term $\frac{3mGu^2}{c^2}$. The approximate solution of Equation (3) is found to be

$$u = \frac{Gm}{h^2} [1 + \epsilon \cos(1 - \delta)\phi] \tag{4}$$

where ϵ = eccentricity; $\delta = \frac{3G^2 m^2}{h^2 c^2}$.

From Equation (4) it is seen that ϕ must increase by $2\pi(1 + \delta)$ in order that u shall resume its original value, i.e., the perihelion advances by

$$2\pi\delta = \frac{24\pi^3 a^2}{c^2 T^2 (1 - \epsilon^2)}$$

where a = semi-major axis of orbit; and T = period of revolution.

For mercury it is about 43'' per 100 years.

(b) *Deflection of light in a Schwarzschild field.* The differential equation for the path of a light ray is

$$\frac{d^2 u}{d\phi^2} + u = \frac{3Gmu^2}{c^2}$$

If the right hand term is considered a perturbation, the zeroth order solution is

$$u = \frac{\cos \phi}{R}$$

where R is a constant of integration; i.e., a straight line whose shortest distance from the Schwarzschild center is R . The first order approximation to the solution is

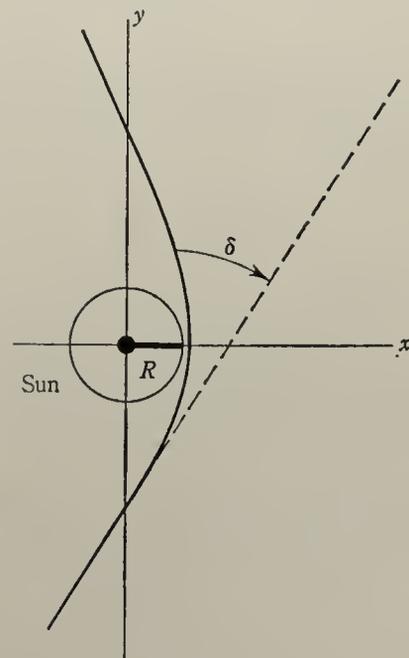
$$u = \frac{\cos \phi}{R} + \frac{Gm}{c^2 R^2} (1 + \sin^2 \phi)$$

or

$$R = r \cos \phi + \frac{Gmr}{c^2 R} (1 + \sin^2 \phi)$$

or in Cartesian coordinates

$$x = R - \left(\frac{Gm}{c^2 R}\right) = \frac{(x^2 + 2y^2)}{\sqrt{x^2 + y^2}}$$



the curve resembles a hyperbola. The deflection δ is

$$\delta = \frac{4Gm}{Rc^2}$$

If R is the radius of the sun, and m its mass, $\delta = 1.75''$.

(c) *Red shift.* An atomic clock defines an invariant four-dimensional time-like interval.

$$l_o^2 \equiv g_{ik} dx^i dx^k.$$

If the clock is at rest, i.e., $dx^1 = dx^2 = dx^3 = 0$, then

$$l_o^2 = g_{44}(dx^4)^2.$$

If two identical clocks, not subject to any non-mass-proportional forces, are located at two different space-time points, then

$$dx_{(1)}^4 : dx_{(2)}^4 = \frac{1}{\sqrt{-g_{44}^{(1)}}} : \frac{1}{\sqrt{-g_{44}^{(2)}}}.$$

Since the periods of vibration are in the inverse ratio of the frequencies, one can also write

$$\nu_1^2 : \nu_2^2 = (-g_{44}^{(1)}) : (-g_{44}^{(2)}).$$

For weak gravitational fields

$$-g_{44} = 1 + \frac{\phi}{c^2}$$

where ϕ is the gravitational potential at the location of the atomic clock. Therefore

$$\frac{\nu_2^2 - \nu_1^2}{\nu_1^2} = \frac{\phi_2 - \phi_1}{c^2 \left(1 + \frac{\phi_1}{c^2}\right)}.$$

If the frequencies ν_2 and ν_1 differ very little from each other and if $\nu_1 \equiv \nu_o$ and $\phi_1 = 0$, then approximately

$$\frac{\nu_2 - \nu_o}{\nu_o} = \frac{\phi_2}{c^2}.$$

Since ϕ_2 is always negative, the frequency shift from $\nu_2 - \nu_o$ is negative, hence a spectral line of an atom, located on the sun, is shifted to the red compared to the same line, if emitted by the same atom, if it is located on the earth.

On the surface of the sun $\phi_2 = -\frac{Gm}{R}$ and

$$\nu_2 - \nu_o = -\frac{Gm}{Rc^2} \nu_o$$

or

$$\lambda_2 - \lambda_o = +\frac{Gm}{Rc^2} \lambda_o$$

where m = mass of sun, and R = radius of sun.

RELATIVITY THEORY, SPECIAL. Theory developed by Einstein based on the hypothesis that the velocity of light is the same as meas-

ured by any one of a set of observers moving with constant relative velocity. According to Newtonian theory and the Galilean transformation, the mechanical motion of an object with respect to an inertial system could be predicted from a knowledge of the forces acting on it and the initial conditions, independently of any knowledge of the motion of the inertial system itself. Einstein extended this to optical phenomena, postulating that these also could be described without knowing the velocity of the laboratory with respect to the rest of the universe. The null result of the Michelson-Morley experiment and of other attempts to measure the velocity of the laboratory relative to the ether was then interpreted as an immediate consequence of a fundamental principle of relativity, that the equations of electrodynamics have the same form in all systems in which the equations of mechanics are valid. From this principle, and the constancy of the velocity of light, it is possible to deduce the **Lorentz transformation** (appropriately modified in its interpretation so as to ignore the ether and to relate the observations of two observers moving with constant relative velocity). Since the **Maxwell equations** are covariant under this transformation, but the equations of Newtonian mechanics are not, one is led to a modified description of the mechanics of particles which is in accord with the relativity principle. This description is indistinguishable from the Newtonian theory for systems in which all relative velocities are small compared with that of light, except that in this limit it also predicts that when a mass m_o is annihilated an amount of energy $E = m_o c^2$ is released. In the more general case the mass of a particle is given by $m = \gamma m_o$ where m_o is its rest-mass and $\gamma = (1 - \beta^2)^{-1/2}$, with $\beta = v/c$, v being the velocity of the particle relative to the observer, and the expansion of $E = mc^2$ in powers of β then yields $E = m_o c^2 + \frac{1}{2} m_o v^2 + \dots$ the second term being the kinetic energy of the particle in Newtonian mechanics. The momentum of a particle then appears as mv , and the rate of change of this expression with respect to the time is equal to the force acting on the particle. The relation between the energy E and momentum p of a free particle then becomes $E^2 = p^2 c^2 + m_o^2 c^4$.

In addition to leading to such well-verified conclusions, special relativity theory was able

to yield also the few valid consequences of the ether hypothesis. It provided the first verification of the Mach principle by its insistence on the role of the observer in the description of phenomena which the observer measures (cf. **quantum mechanics**) and pointed out that even the simultaneity of two events at different positions is not an intrinsic property of those events, but depends also on the motion of the observer who is recording them. This result emerges from Einstein's re-interpretation of the **Lorentz transformation**, referred to above, and is related to the consequent re-interpretation of the Fitzgerald factor and to the relativistic slowing of clocks.

The formal identification of a Lorentz transformation with a rotation in Minkowski space provides a basis for representing the equations of relativistic mechanics as relations between **four-vectors** analogous to representation of the non-relativistic equations as relations between vectors. In the same way that the latter representation ensures that the form of the equations shall be independent of the particular directions in which the chosen set of orthogonal axes happens to point, so the Minkowski representation ensures that the equations of relativistic mechanics shall be independent of those in which Lorentz observer is involved. All observers moving with constant relative velocity thus use equations of the same form to describe the optical and mechanical phenomena which they measure.

The theory has been generalized to include relatively accelerated observers (see **relativity theory, general**) and to include quantum phenomena (see **quantum mechanics, relativistic**).

RELAXATION. (1) A method of successive approximations for solving a system of equations

$$\phi_i(x_1, x_2, \dots, x_n) = 0, \quad i = 1, 2, \dots, n.$$

Given any approximate solution $x_i^{(\nu)}$, one evaluates the residuals $\phi_i(x_j^{(\nu)}) = R_i^{(\nu)}$ and selects the largest. If this is $R_{i'}$, one solves $\phi_{i'} = 0$ for $x_{i'}^{(\nu+1)}$ in terms of the other $x^{(\nu)}$, recomputes the residuals, and repeats. For machine computation, however, it is generally uneconomical to make the search and the steps are carried out in a preassigned cyclic order. Hence the term is sometimes extended to include this method. The term was introduced by Southwell, but the method was used by Gauss for solving the **normal equations**.

Some writers use the term for any **iterative method**. (See also **matrix inversion**.)

(2) A term applied to the process by which a physical system reaches **equilibrium** or a **steady state** after a sudden change in conditions. For example, suppose that a plate of metal is in the presence of a magnetic field **H**, directed perpendicular to the plate. If the source of the field is suddenly removed, the flux inside the metal does not go to zero instantaneously. As it starts to decrease, eddy currents are induced in the metal, in such a direction as to oppose the change in flux. As a result, the flux decays exponentially with time. (See **relaxation time**.)

The term relaxation is generally limited to processes or phenomena in which dissipative agents are present to such an extent that the system does not overshoot and then oscillate about the equilibrium state.

Systems having a relaxation time τ can, in general, respond readily to periodic changes that have a frequency much less than $\frac{1}{2}\pi\tau$; they show very little response if the frequency is much greater than $\frac{1}{2}\pi\tau$. The behavior in the intermediate range of frequencies, $2\pi f \approx 1/\tau$, depends on the parameters of the system.

(3) The term relaxation or relaxation function denotes the stress variation due to loading by a **step function** of strain. For a linear viscoelastic material this is equal to the relaxation modulus multiplied by the strain.

RELAXATION FREQUENCY. In general terms, the inverse of the **relaxation time**. Sometimes, the inverse of $2\pi\tau$, where τ is the relaxation time.

RELAXATION FUNCTION. See **relaxation (3)**.

RELAXATION LENGTH. In the transport of neutrons and/or radiation, the overall **mean free path** for absorption is sometimes called the relaxation length.

RELAXATION MODULUS. See **modulus, relaxation**.

RELAXATION PHENOMENA. Any phenomenon in which a system requires an observable length of time in order to reach equilibrium after sudden changes in conditions, forces, or effects which are applied to the system.

RELAXATION SPECTRUM. See relaxation time.

RELAXATION TIME. In many material phenomena, the response to an abrupt change is often a time-measurable approach to equilibrium, frequently exponential. Examples are: (1) An abrupt change of **magnetizing force** usually does not produce an instantaneous, corresponding change in magnetic induction, but the new value is approached over a period of time. The time constant involved in such a phenomenon is often called relaxation time; if equilibrium is approached exponentially it is the time for the variable quantity to change by the fraction $(1 - 1/e)$ of the original departure from the equilibrium value. (2) A case closely related to the foregoing is that of a crystal in which all the spins are aligned by a magnetic field, which is then removed. The magnetic moment will then decay to zero; if this decay is exponential with time, of the form $e^{-t/\tau}$, then τ is the relaxation time. (3) The time for which an electron may travel in a metal before it is scattered, and loses its momentum. (4) The response of a **Maxwell material** to a step function of strain, of amplitude ϵ_0 , is a stress σ which varies with time according to:

$$\sigma = \epsilon_0 m e^{-t/\tau}.$$

m is a modulus, and τ is a constant known as the relaxation time. When a more general material is represented by a sequence of Maxwell elements in parallel, the associated sequence of relaxation times, τ_i , gives the discrete relaxation spectrum. For such a material, **step function** straining gives the response:

$$\sigma = \epsilon_0 \sum_i m_i e^{-t/\tau_i}.$$

General linear viscoelastic behavior corresponds to the continuous generalization of this:

$$\sigma = \epsilon_0 \int_0^\infty H(\tau) e^{-t/\tau} d\tau.$$

$H(\tau)$ is then the relaxation spectrum, sometimes expressed in terms of the variable $\ln \tau$ instead of τ .

RELIABILITY. If a measured quantity y can be written in the form

$$y = a + b + c$$

where a is constant, b varies from one individual to another with variance σ_b^2 , and c (representing errors of measurement, etc.) varies from one measurement to another on the same individual with variance σ_c^2 , the coefficient of reliability is defined by

$$r^2 = \frac{\sigma_b^2}{\sigma_b^2 + \sigma_c^2}$$

that is to say, it is the proportion of the total **variance** accounted for by the component b . The quantity "reliability" is sometimes measured by σ_b^2 .

RELUCTANCE. In a magnetic circuit, the ratio of the magnetomotive force to the flux. (See **Bosanquet law**.)

REMAINDER. The difference obtained when an **approximation** is subtracted from the quantity being approximated; hence a **correction**. (See **remainder formulas**.)

REMAINDER FORMULAS. This term will be applied here to any formula expressing the **remainder** in approximating a function, or an integral or a derivative of it. A quite general one is the following: Let

$$f(x) = P(x) + R(x)$$

where $P(x)$ is the polynomial of degree n that interpolates $f(x)$ at the points x_0, x_1, \dots, x_n , and let

$$\omega(x) = (x - x_0)(x - x_1) \dots (x - x_n).$$

Then

$$R(x) = \omega(x) \int_0^1 \int_0^{t_n} \dots$$

$$\int_0^{t_1} f^{(n+1)}[x + t_0(x_0 - x) + t_1(x_1 - x_0) + \dots + t_n(x_n - x_{n-1})] dt_0 \dots dt_n.$$

It is not required that the x_i be distinct, and, in fact, if $x_0 = x_1 = \dots = x_n$ one has a form of the remainder for Taylor series.

A somewhat simpler form is

$$R(x) = \omega(x) f^{(n+1)}(\xi) / (n + 1)!,$$

where, however, of ξ one knows only that it lies on the interval containing all $n + 2$ abscissae: x, x_0, x_1, \dots, x_n .

These same formulas provide remainders for numerical differentiation formulas provided

the derivative is to be evaluated at one of the points x_i , since

$$R'(x_i) = \omega'(x_i)f^{(n+1)}(\xi)/(n+1)!$$

For remainders in the quadrature formulas, refer to these by name.

REMAINDER THEOREM. See algebraic equations.

REMANANCE. The residual magnetic induction when the magnetizing field is reduced to zero from a value sufficient to saturate the material. (See magnetization curve.)

REMOVABLE DISCONTINUITY. See discontinuity.

RENEWAL THEORY. A piece of physical equipment may comprise several components, each with a "life" of service. "Death," that is to say, failure of a component, involves renewal, and the theory of such complexes is called renewal theory. Where failure follows a probability law the theory may be regarded as part of the theory of stochastic processes.

RENORMALIZATION. See divergences, removal of; self-charge; mass renormalization; mass and charge renormalization.

REPEATABILITY. A measure of deviation of test results from their mean value, all determinations being carried out by one operator without change of apparatus in those cases where the manner of handling apparatus can alter results.

REPOSE, ANGLE OF. The maximum angle with the horizontal at which an object on an inclined plane will retain its position without tending to slide. The tangent of the angle of repose equals the coefficient of static friction. (Cf. friction, coefficient of.)

REPRESENTATION (SINGLE-VALUED OF A SURFACE S ON A SURFACE S'). If to each point of a surface S there corresponds one and only one point of a surface S' , the surface S' is a *single-valued representation* or *single-valued mapping* of S . If, in addition, to each point of S' there corresponds one and only one point of S , the surface S' is said to be a *one-to-one representation* or *one-to-one mapping* of S . If S' is a one-to-one representation of S and the angle between any two linear elements on S at a generic point P

of S is equal to the angle between the corresponding linear elements of S' , then S' is a *conformal representation* or *conformal mapping* of S . The ratio between the length of a linear element at a point of S' and the length of the element of S to which it corresponds is called the *linear magnification* of the conformal representation at that point. If the linear magnification is unity for all points, the representation is an *isometric conformal representation*. The surfaces S and S' are then *aplicable surfaces*.

REPRESENTATION, IRREDUCIBLE. An irreducible matrix which is a representation of an operator. (See representation theory, quantum mechanical.)

REPRESENTATION OF GROUPS. A representation of a group G is a mapping (correspondence) which associates to every element g of G a linear operator T_g in a certain vector space V , such that group multiplication is preserved and the identity e of G is mapped into the identity I in V . That is, if $e, g_1, g_2 \dots$ are the elements of G and if to these elements are associated the linear operations $T_e, T_{g_1}, T_{g_2}, \dots$ etc., in V , the correspondence $g \rightarrow T_g$ is said to form a representation of the group if

$$T_e = I \quad (\text{a})$$

$$T_{g_1}T_{g_2} = T_{g_1g_2} \quad (\text{b})$$

If T_g is represented by a matrix, one speaks of a matrix representation. A subspace V_1 of V is said to be invariant under the representation $g \rightarrow T_g$, if all the vectors v in V_1 are transformed by T_g into vectors v' again in V_1 and this for all T_g . If the only subspaces of V which are invariant under the representation $g \rightarrow T_g$ consist of the entire space and the subspace consisting of the null vector alone, one calls the representation *irreducible*.

For compact groups, it is always possible to define a scalar product in V such that their representations in V are **unitary**, i.e., such that the operators T_g are all unitary: $T_g^* = T_g^{-1} = T_{g^{-1}}$. Furthermore, the study of the unitary representations of compact groups can be reduced to the study of irreducible representations. For if there exists a subspace V_1 of V invariant under T_g , then the orthogonal complement of V_1 , V_1^\perp , i.e., the set of all vectors orthogonal to V_1 is also invariant under T_g . Thus V has been split onto two invariant subspaces. This process can be continued

until one deals with only irreducible representation. For compact groups it is known (Peter-Weyl theorem) that this inductive process of decomposing invariant subspaces into invariant subspaces terminates: the irreducible representations are all finite dimensional and every representation is a direct sum of irreducible finite dimensional representations. Finally, it should be noted that in general one is interested only in **inequivalent irreducible representations**. It is also usual to only consider continuous representations, i.e., representations $g \rightarrow T_g$ such that $(v, T_g w)$ is a continuous function of g for every pair of vectors v and w in V .

REPRESENTATION OF GROUPS AND QUANTUM MECHANICS. For quantum mechanical applications one is actually only interested in ray representations of G on V , i.e., in the correspondence $g \rightarrow \underline{T}_g$ between elements g of G and operator rays \underline{T}_g ; \underline{T}_g maps onto itself the linear set \mathfrak{R} of rays corresponding to physically realizable states and preserves transition probabilities (see **symmetry principles and quantum mechanics**). It can be shown that for each ray correspondence \underline{T}_g one can determine an essentially unique vector correspondence T'_g which is arbitrary only up to a complex multiplicative factor of modulus 1. The T'_g then satisfies

$$T'_g T'_{g_2} = \omega(g_1 g_2) T'_{g_1 g_2} \quad (a)$$

where $\omega(g_1 g_2)$ is a complex number of modulus one depending on g_1 and g_2 . A correspondence $g \rightarrow T'_g$ satisfying (a) is known as a representation up to a factor of G . It has been shown that by a suitable restriction of the arbitrariness of T'_g (T'_g and $e^{i\alpha g} T'_g$ represent the same correspondence) that for the **three-dimensional rotation group** and the restricted **Lorentz group** ω can be taken as equal to ± 1 and that for the **Galilean group** ω can be expressed by a fairly simple expression.

The importance for quantum mechanical applications of determining all unitary representations of a relativity group G , comes from the fact that the knowledge of such a unitary representation can in effect, replace the wave equations for the system under consideration. For example, if one used a description of the system in the **Heisenberg picture** in which the Heisenberg state $|\psi_H\rangle$ coincides with $|\psi_S(t_0)\rangle$, the **Schrödinger picture** state at time $t = 0$ in a given Lorentz frame, then the Schrödinger state vector at time t_0 can be ob-

tained by transforming to a frame $0'$ for which $t' = t - t_0$, while all other coordinates remain unchanged. If L is this transformation then

$$|\psi_S(t_0)\rangle_S = U(L) |\psi_S(0)\rangle.$$

Thus a determination of all the unitary representations of the **inhomogeneous Lorentz group**, for example, is equivalent to a determination of all possible relativistic wave equation equations.

REPRESENTATIONS OF THE INHOMOGENEOUS LORENTZ GROUP. The representation on a Hilbert space \mathfrak{H} of the group of **inhomogeneous Lorentz transformations** by linear operators $U(a, \Lambda)$ which satisfies

$$U(a_1, \Lambda_1) U(a_2, \Lambda_2) = \omega(a_1, \Lambda_1; a_2, \Lambda_2) U(a_1 + \Lambda_1 a_2; \Lambda_1 \Lambda_2)$$

where $|\omega| = 1$. It can be shown that for the restricted group ω can be chosen to be ± 1 . For the pure translations

$$U(a, 1) = e^{ia_\mu p^\mu}$$

where p^μ , $\mu = 0, 1, 2, 3$ are self adjoint operators which have the physical interpretation of being the observables of energy ($\mu = 0$) and momentum ($\mu = 1, 2, 3$). From the multiplication law of the group it follows that the spectrum of p is invariant under restricted Lorentz transformations. The physical requirement that there be a lower bound on the energy together with the group multiplication law then requires that $U(i_s)$, the representation for the space inversion $i_s \mathbf{x} = (x^0, -\mathbf{x})$, be unitary and that $U(0, i_t)$, the representation for the operation of time inversion, be anti-unitary. It can be shown that every (continuous) representation is a direct integral of irreducible representations.

The irreducible representation of the inhomogeneous Lorentz group can be classified according to whether P_μ is a space-like, time-like or null vector, or P_μ is equal to the zero vector. For this last case $p_\mu = 0$, the complete system of unitary transformations coincides with the complete system of infinite dimensional unitary representations of the homogeneous group. The representations of principal interest for physical applications are those for which $P_\mu P^\mu = m^2 =$ positive constant or $p^\mu p_\mu = 0$.

Each of these irreducible representations of the restricted group has a definite value of $P_\mu P^\mu = m^2$, where the non-negative real num-

ber m is the mass of the system. For $m \neq 0$ the distinct irreducible representations are labeled by the spin j which takes on the value $0, \frac{1}{2}, 1, \frac{3}{2}, \dots$. For $m = 0$ there are two classes of representations; one of these classes can be labeled by a spin $\pm j$ with $j = 0, \frac{1}{2}, 1, \dots$ and which satisfies $u^\mu = \pm j p^\mu$, $W = w^\mu w_\mu = 0$ where $w^\kappa = \epsilon^{\lambda\mu\nu} P_\lambda M_{\mu\nu}$; $M_{\mu\nu}$ being the infinitesimal generator for Lorentz transformations in the $\mu\nu$ plane. The second class are those which $W = w^\mu w_\mu \neq 0$.

For the group including inversions, mass and spin still continue to label distinct irreducible representations, but for $m \neq 0$, there are four irreducible representations for each mass and spin corresponding to the four possibilities $U(i_t)^2 = \pm 1$, $U(0, i_s i_t)^2 = \pm 1$.

The representations for the case $p^2 < 0$ have been classified but they do not appear to be of physical interest, as they would correspond to systems with space-like moments which could have arbitrarily high negative energies.

REPRESENTATION, SPHERICAL. See spherical representation of a surface.

REPRESENTATIVE ENSEMBLE. An ensemble chosen so as to represent as well as possible a physical system about which our knowledge is incomplete.

REPULSIVE POTENTIAL CURVES IN A MOLECULE. A potential curve (see potential functions of a molecule) without a minimum. Repulsive potential curves are characteristic of unstable electronic states of the molecule.

RESIDUAL RESISTANCE. That part of the electrical resistance of a metal which is independent of the temperature, and hence remains when the resistance due to scattering of the electrons by lattice vibrations is made small at low temperatures (see conductivity, electrical). Residual resistance is caused by impurities and imperfections in the crystal, and hence may be reduced by purification, annealing, etc.

RESIDUAL STRESSES. Stresses remaining in a body free of surface traction or other load and at uniform temperature. Residual stress may be produced by fabrication processes such as rolling, welding, riveting, or heat treatment, by local plastic deformation, by autofrettage, etc. (See reaction stresses.)

RESIDUE. If $f(z)$, a function of the complex variable z , has a pole at $z = z_0$ so that it may be expanded in a Laurent series $\sum a_n (z - z_0)^n$, then the coefficient a_{-1} is the residue of the function at the point z_0 . Determination of the residues of a function, makes it possible to evaluate an integral in the complex plane by the Cauchy theorem.

The concept of the residue is relevant to the theory of functions of a complex variable and in particular to the evaluation of contour integrals and, in the feedback field, of the Laplace inversion integral (see Laplace transform).

If, in the neighborhood of a point $s = s_0$ in the complex plane of s , a function $F(s)$ is capable of expansion in the form

$$F(s) = \sum_0^{\infty} a_r (s - s_0)^r + \sum_1^n b_r (s - s_0)^{-r}$$

where $b_n \neq 0$, then $F(s)$ is said to have a pole of order n at $s = s_0$ and b_1 is the residue of $F(s)$ at that pole.

Alternative expressions for the residue b_1 are:

$$(i) \quad b_1 = \frac{1}{2i\pi} \int F(s) ds \text{ around an infinitesimal circle with center } s = s_0$$

$$(ii) \quad b_1 = \frac{1}{(n-1)!} \lim_{s \rightarrow s_0} \left(\frac{d}{ds} \right)^{n-1} \{ F(s) (s - s_0)^n \}.$$

For a first order pole this reduces to

$$b_1 = \lim_{s \rightarrow s_0} \{ (s - s_0) F(s) \}$$

If the order of the pole is unknown the above formula may be used with successively increasing values of n . The first value of n yielding a finite result (whether zero or not) is the order of the pole and the finite result is the value of the residue.

(iii) b_1 = coefficient of $(s - s_0)^{-1}$ in the partial fraction expansion of $F(s)$, assuming that this can be carried out.

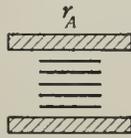
RESIDUE THEOREM. If $F(s)$ is analytic on a contour and is analytic within the contour except at a finite number of poles or essential singularities then the integral of $F(s)$ around the contour (anticlockwise) is equal to $2\pi i$ times the sum of the residues of $F(s)$ at those of its poles within the contour.

RESILIENCE. The resilience of a body measures the extent to which energy may be stored in it by elastic deformation. The implication of the word "stored" in the above definition is that this energy may be released in the form of mechanical work when the force causing the elastic deformation is removed, and that resilience is a property of a material within its **elastic limit**. The "modulus of resilience" is the maximum energy storage in a unit volume of the material. In practical units it is the inch pounds of energy stored in a cu. in. of the material stressed to the elastic limit. The modulus of resilience is directly proportional to the square of the stress, and inversely proportional to the modulus of elasticity. It is equal to the area under the stress-strain diagram up to the elastic limit, or

$$\frac{1}{2} \frac{\sigma^2}{E}$$

in which E is the modulus of elasticity and σ is the elastic limit.

RESISTANCE, ACOUSTIC. Real component of acoustic impedance (see **impedance, acoustic**); the unit generally used is the **acoustic ohm**.



Graphic representation of the element, acoustical resistance, r_A .

tic); the unit generally used is the **acoustic ohm**.

RESISTANCE, ACOUSTIC FLOW (D-C ACOUSTIC RESISTANCE). The quotient of the pressure difference between the two surfaces of a sound absorbing material, in dynes/cm², by the volume current through the material in cm³/sec. The unit is the **acoustic ohm**.

RESISTANCE, ELECTRIC. Electrical resistance is the factor by which the square of the instantaneous conduction current must be multiplied to give the power lost by dissipation as heat or other permanent radiation of energy away from the electric circuit. The unit of electric resistance is the ohm. (See also **Ohm's law; resistance (specific)**.)

RESISTANCE, FLUID. See **viscosity; fluid friction**.

RESISTANCE, MECHANICAL RECTILINEAL. See **mechanical rectilinear resistance**.

RESISTANCE, MECHANICAL ROTATIONAL. See **mechanical rotational resistance**.

RESISTANCES IN A-C CIRCUITS. The resistance of a conductor is not, in general, independent of the frequency of the current flowing in it, because of the effects of distributed **inductance** and **capacitance** and of **skin effects**. To a reasonable approximation, a resistor may be considered equivalent to a pure resistance R in series with an **inductance** L and with a **capacitance** C in parallel with the combination. The impedance of such a combination is

$$Z = (R + i\omega L)/(1 - \omega^2 LC + i\omega CR)$$

where i is $\sqrt{-1}$ and ω is the 2π times the frequency. At very low frequencies the real part of Z , or the effective resistance, is found to be approximately equal to R , but the resistor also has a reactance $\omega(L - CR^2)$. The quantity in parentheses is called the *effective inductance*; its ratio to R is the *time constant* of the resistor. Only when the time constant is small compared with the inverse of the frequency will a resistor behave like a pure resistance. At very high frequencies, the effective resistance is $R(1 - \omega^2 C^2 R^2)$; the resistor acts as a pure reactor when $\omega = 1/CR$. However, at such frequencies the skin effect may increase the value of R so greatly that the approximation involved in assuming this quantity to be constant is a very bad one. The problem must then be considered in greater detail.

RESISTANCE, SPECIFIC (RESISTIVITY). A proportionality factor characteristic of different substances equal to the **resistance** that a centimeter cube of the substance offers to the passage of electricity, the current being perpendicular to two parallel faces. It is defined by the expression:

$$R = \rho \frac{l}{A}$$

where R is the resistance of a uniform conductor, l is its length, A is its cross-sectional area, and ρ is its resistivity. Resistivity is usually expressed in ohm-centimeters.

RESISTANCE, SPECIFIC ACOUSTIC. The real component of the specific acoustic impedance. (See **impedance, specific acoustic**.)

RESOLUTION. A term used in a number of specific cases in science to denote the process of separating closely-related forms or entities or the degree to which they can be discriminated. The term is most frequently used in optics to denote the smallest extension which a magnifying instrument is able to separate or the smallest change in wavelength which a spectrometer can differentiate. In this last sense, it is defined as the ratio of the average wavelength (wave number or frequency) of two spectral lines, which can just be detected as a doublet, to the difference in their wavelengths (wave numbers or frequencies). The term resolution is also applied to such varied processes as the separation of a racemic mixture into its optically-active components or as the breaking up of a vectorial quantity into components.

RESOLUTION OF FORCES. See **forces, resolution of.**

RESOLUTION SENSITIVITY. The minimum change in the measured variable which produces an effective response of the instrument or automatic controller.

RESOLUTION, VECTOR. See **vector resolution into solenoidal and irrotational vectors.**

RESOLVENT OF AN INTEGRAL EQUATION. See **Liouville-Neumann series.**

RESOLVING POWER. (1) The ability of an optical system to resolve, i.e., separate, two entities, such as the ability of a telescope to separate the images of the two stars of a double star, the ability of a microscope to separate the images of two points which are close together, and the ability of a spectroscope to separate two spectral lines. Most studies of resolving power are based on the **Rayleigh criterion of resolving power.** (1a) The resolving power of a microscope for two self-luminous points is given by the relation $d = 1.220\lambda (2 N.A.)^{-1}$, where d is the linear separation of the two points, λ is the wavelength used, and $N.A.$ is the **numerical aperture** of the object lens. (1b) Most telescopes have large objective lenses in order to have large light-gathering power, and to have high resolution. This high resolution may produce resolved images too close together to be resolved by the human eye. Hence an eye-lens or ocular is included in the system for the purpose of

magnifying the initial image so that the eye can see it as resolved. Note that no amount of magnification of the initial image can increase the resolving power of the telescope over the resolving power of the objective lens. The angular resolving power of a telescope is $1.220 \lambda/2r$ radians where r is the radius of the objective lens. (1c) The chromatic resolving power of a prism, or prisms, is $\lambda/\Delta\lambda = tdn/d\lambda$ where t is the total length of the base. (2) By extension, the term applies to instruments separating particles. The resolving power of a mass spectroscope is the highest value of the ratio $m/\Delta m$ for the complete separation of the mass spectrum lines differing by Δm in mass. (3) In a unidirectional antenna, the resolving power is the reciprocal of its beam width, measured in degrees. The resolution of a directional radio system can be different from the resolving power of its antenna, since the resolution is affected by other factors.

RESONANCE. (1) Every physical system, in general, has one or more natural **vibration** frequencies characteristic of the system itself and determined by constants pertaining to the system. Thus a flexible string of length l and mass δ per unit length, and subjected to a tension f , will, if struck or plucked and left to itself, vibrate with frequencies equal to

$$\frac{1}{2l} \sqrt{\frac{f}{\delta}}$$

and to various integral multiples thereof (overtones). If such a system is given impulses with some arbitrary frequency, it will necessarily vibrate with that frequency even though it is not one of those natural to it. These "forced vibrations" may be very feeble; but if the impressed frequency is varied, the response becomes rapidly more vigorous whenever any one of the natural frequencies is approached, its amplitude often increasing many fold as exact synchronism is reached. This effect is known as resonance. The many uses of this conception in present-day science stem from this initial use in mechanics or acoustics to denote a prolongation or reinforcement of sound by induced vibration. Such acoustical (and mechanical) resonance can often be represented by a differential equation of the form

$$M \frac{d^2x}{dt^2} + R \frac{dx}{dt} + Sx = A \cos \omega t$$

TABLE OF PROPERTIES OF RESONANT SYSTEMS

$$M \frac{d^2x}{dt^2} + R \frac{dx}{dt} + Sx = A \cos \omega t$$

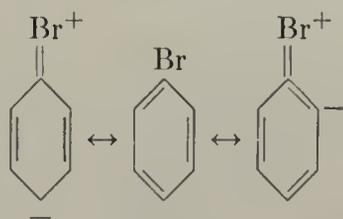
	At Velocity Resonance	At Displacement Resonance	At the Natural Frequency
Frequency	$\frac{1}{2\pi} \sqrt{\frac{S}{M}}$	$\frac{1}{2\pi} \sqrt{\frac{S}{M} - \frac{R^2}{2M^2}}$	$\frac{1}{2\pi} \sqrt{\frac{S}{M} - \frac{R^2}{4M^2}}$
Amplitude of displacement	$\frac{A}{R \sqrt{\frac{S}{M}}}$	$\frac{A}{R \sqrt{\frac{S}{M} - \frac{R^2}{4M^2}}}$	$\frac{A}{R \sqrt{\frac{S}{M} - \frac{3R^2}{16M^2}}}$
Amplitude of velocity	$\frac{A}{R}$	$\frac{A}{R \sqrt{1 + \frac{R^2}{4MS} - 2R^2}}$	$\frac{A}{R \sqrt{1 + \frac{R^2}{16MS} - 4R^2}}$
Phase of displacement with reference to applied force	$\frac{\pi}{2}$	$\tan^{-1} \sqrt{\frac{4MS}{R^2} - 2}$	$\tan^{-1} \sqrt{\frac{16MS}{R^2} - 4}$

For values of R , small compared to \sqrt{SM} , there is little difference between the three cases discussed above.

which permits a mathematical statement of velocity resonance and displacement resonance. In the former, a small change in the frequency of the applied force causes a decrease in velocity at the driving point; while in the former it causes a decrease in the amplitude of displacement.

Resonance phenomena are exhibited by all systems in motion, including atomic, molecular, and electronic systems, which are treated by quantum-mechanical methods. (See **resonance, quantum-mechanical**; **magnetic resonance**; **resonance, electrical**; **resonance effect**.)

RESONANCE EFFECT (R. EFFECT). For many substitution reactions on aromatic nuclei in solutions, the reagent is a positive ion (Br^+ , NO_2^+). Those electrophilic groups react preferentially with the more electronegative sites on the aromatic nucleus. Consider the example of bromobenzene; one can write the resonance formulas



The resonance (R) effect enhances the electronegativity of the para and ortho sites, and leads to a preferential ortho-para electrophilic substitution. (See **induction effect**; **molecular diagrams**.)

RESONANCE, ELECTRICAL. There are two types of electrical resonance, series resonance and parallel resonance, as explained in the following discussion. In an a-c circuit containing inductance and capacitance in series (see Figure 1a) the impedance is given by

$$Z = R + j \left[\omega L - \frac{1}{\omega C} \right]$$

where R is the resistance, ω is 2π times the frequency, L is the inductance, and C , the capacitance. It can readily be seen that at some frequency the terms in the bracket will cancel each other, and the impedance will equal the resistance alone. This condition, which gives a minimum impedance (and thus a maximum current for a fixed impressed voltage) and unity power factor is known as series resonance. Where the resistance is small the current may become quite large. As the voltage drop across the condenser or coil is the product of the current and the impedance of that particular unit, it may also become very large. The condition of resonance may even give rise to a voltage across one of these units which is many times the voltage across the whole circuit, being, in fact, Q times the applied voltage for the condenser and nearly the same for the coil. This is possible since the drops across the coil and condenser are nearly 180° out of phase and thus almost cancel one another, leaving a relatively small voltage across the circuit as shown in Figure 1b. For a

circuit composed of an inductance unit in parallel with a capacitance the opposite effects of these two types of reactance will counteract one another at some frequency and produce

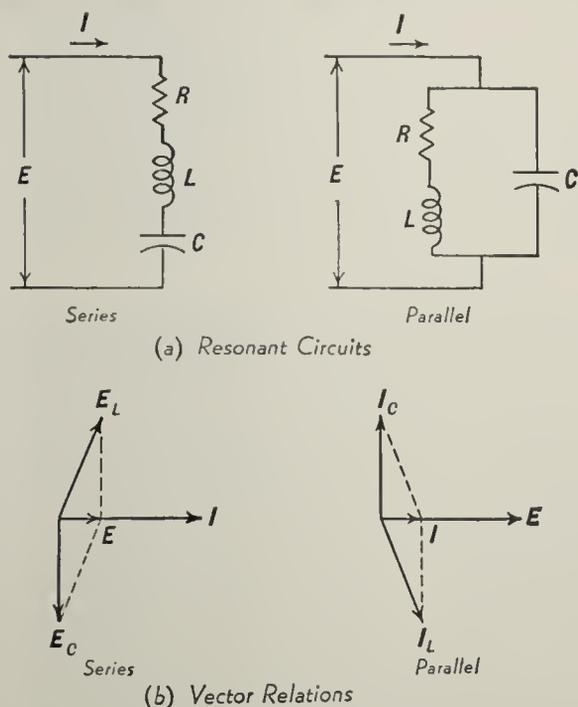


Fig. 1. Diagrams (a) and phase relations (b) of resonant circuits.

unity power factor for the circuit. This is parallel resonance or *antiresonance* as it is sometimes called. In such a circuit the currents in the individual branches may be many times that in the line since they are out of phase and combine vectorially to give the line current. The impedance of a parallel resonant circuit is very high, its behavior being almost identical with that of the current in a series circuit if the Q of the parallel circuit is above 10.

RESONANCE ENERGY. (1) The kinetic energy, measured with reference to the **laboratory system**, of a particle that will be captured or scattered preferentially because of the presence of an appropriate **resonance level** in the compound nucleus (incident particle plus target nucleus). The relation of the resonance energy E_{res} to the energy of excitation E_{exc} of the resonance level above the ground state of the compound nucleus is given by the expression

$$E_{res} = (E_{exc} - E_B) \frac{\text{mass of compound nucleus}}{\text{mass of target nucleus}}$$

where E_B is the binding energy of the incident particle in the compound nucleus. (2) The

added **binding energy** of a molecule due to quantum mechanical resonance. (See **resonance, quantum mechanical**; **valence bond method**.)

RESONANCE ENERGY IN THEORY OF MOLECULAR STRUCTURE. See **resonance, quantum mechanical**.

RESONANCE ESCAPE PROBABILITY. In a system of infinite extent, the probability, denoted p , that a neutron will slow down from an initial energy below the relevant fast-fission thresholds to final thermal energies without being captured. In finite systems, p is dependent upon the effects of neutron leakage, and is an extensive rather than an intensive quantity.

RESONANCE, FERMI. See **Fermi resonance**.

RESONANCE, FERROMAGNETIC. See **ferromagnetic resonance**.

RESONANCE INTEGRAL. (1) In the molecular orbital method, the wave function of an electron in a double bond, or in a set of conjugated double bonds, is usually assumed, in a first approximation, to be given by a linear combination of "atomic orbitals" (LCAO method):

$$\psi = \sum_k c_k \phi_k$$

c_k^2 is a measure of the probability of finding the electron in the vicinity of nucleus k .

When one calculates the energy of the molecule, there appear various integrals of which the resonance integral is of the form

$$\beta_{ik} = \int \phi_i H \phi_k d\tau$$

where H is the Hamiltonian function.

The contribution of the resonance integral to the bond energy is positive or negative depending on the sign of the product $c_i c_k$. The resonance integral may become very large when the energies of the two orbitals are almost equal.

(2) In the study of neutron-absorption nuclear reactions, the quantity $I_r = \int \frac{dE}{E} \sigma_a(E)$, evaluated for a particular nuclide, is known as the resonance integral. In the formula, $\sigma_a(E)$ is the absorption cross section expressed as a

function of neutron energy, E , and the range of integration extends from "cadmium cut-off" energy (~ 0.4 ev) to a characteristic energy of fission. In the case of neutron moderation by an extremely dilute mixture of fuel and moderator, the resonance escape probability is simply related to I_r . Indeed, $p = \exp \left[- \frac{N_o}{N_1} \frac{1}{\xi \sigma_{s1}} I_r \right]$, where subscript (1) refers to the moderator.

RESONANCE INTEGRAL, EFFECTIVE.

In the theory of the slowing down of fast neutrons through interaction with matter, the effective resonance integral is defined in terms of the resonance escape probability, p , through the expression:

$$p = \exp \left[- \frac{N_o V_o}{N_1 V_1} \frac{1}{\bar{\xi} \sigma_s} (I_r)_{eff} \right].$$

Here, V_o , V_1 are the volumes occupied by fuel and moderator, respectively ($V_o = V_1$ in a homogeneous mixture), N_o and N_1 are the number densities of fuel and moderator atoms,

$\bar{\xi} \sigma_s = \sum_{j=1}^n \xi_j \sigma_{s_j}$ for an n -component moderator,

and $(I_r)_{eff}$ is the effective resonance integral. For a dilute, homogeneous system, the effective

integral reduces to $I_r = \int \frac{dE}{E} \sigma_a(E)$. (See

resonance integral.)

RESONANCE, MAGNETIC. Particles with intrinsic magnetic moments have quantized energy levels when placed in a magnetic field of flux density B (see **energy states, magnetic**). The possible energy states are given (in MKSA units) by $E = mg\mu_{B,N}B$, where m is the magnetic quantum number, g the g -factor and $\mu_{B,N}$ is either μ_B , the Bohr magneton for electrons, or μ_N the nuclear magneton for protons or nuclei. A small oscillatory magnetic field $B_1 = B' \sin \omega t$, at right angles to the much stronger constant field B , can induce transitions between these levels if the Bohr frequency condition $\omega_o = E/\hbar = g\mu_{B,N}B/\hbar$ is satisfied.

RESONANCE MODEL, NEUTRON. See **neutron resonance model**.

RESONANCE, PARAMAGNETIC. Magnetic resonance involving electron magnetic moments and their associated spins (see **resonance, magnetic**). A free electron placed in a magnetic field of flux density B can orient its

spin and magnetic moment either parallel or antiparallel to the field. The energies associated with these two states are $E = \pm \frac{1}{2}g\mu_B B$ where the g -factor is 2 for a free electron, and μ_B is the Bohr magneton. A small alternating magnetic field at right angles to B can induce transitions between these two states if its radian frequency is $\omega = g\mu_B B/\hbar$, where \hbar is the Planck constant divided by 2π . For magnetic fields of a few kilogauss (≈ 0.1 Wb/M²), the resonance frequency lies in the centimeter region of microwaves, and measurements are made by microwave absorption methods.

RESONANCE, QUANTUM MECHANICAL.

In the theory of **valence bonds** it is often possible to envisage two or more alternative arrangements of the bonds between the atoms in a molecule, both arrangements being of the same energy because of the symmetry of the geometrical configuration. In the method introduced by L. Pauling, the **wave function** obtained for the actual molecule is a linear combination of the wave functions corresponding to the separate structures. Thus, for example, in the benzene ring, there are two obvious ways of putting in the three double bonds. But such **degenerate states**, in quantum theory, may always be combined, in any proportions. In fact, there will usually be further interaction terms, such as the **Coulomb energy** between the bonding electrons, which will tend to stabilize a mixture of the two or more states, at a lower energy than either separately. One then says that the energy of the system is lowered by "resonance" between the two alternative arrangements of the bonds; or, equivalently, that the molecular **binding energy** is increased thereby, with consequent stabilization of the molecule. The amount of the change is called **resonance energy**, and the structures, **resonating structures**. (See also **exchange energy**.)

RESONANCE, SHARPNESS OF. If a stress which varies sinusoidally with time is applied to a mechanical system, and the frequency of the stress is close to one of the frequencies of free vibration of the mechanical system, the amplitudes of the displacement and of the particle velocity produced in the system are found to be large. This is the phenomenon of resonance. The frequency at which the maximum amplitude occurs is called the **resonant frequency**. The rapidity with which the value

of the displacement amplitude or the particle velocity amplitude falls off, as the frequency of the applied stress departs from the resonant frequency, is a measure of the damping in the system. The sharpness of resonance is generally defined as $N/(N_1 - N_2)$, where N is the resonant frequency and N_1 and N_2 are frequencies on either side of the resonant frequency at which the amplitude has $1/\sqrt{2}$ times its maximum value (so that the power at N_1 and N_2 have half the value at N). The quantity $N/(N_1 - N_2)$ is often denoted by Q , the "Q" of a system being considered as a figure of merit with regard to absence of damping. Conversely, the reciprocal of the sharpness of resonance is used as a measure of damping and is sometimes called the **half width** of the resonance peak. When used in this form, N_1 and N_2 are often taken as the values at half amplitude rather than at half power. ΔN is then used to denote $(N_1 - N_2)$, and $\Delta N/N$ is a measure of the damping, which can be related to other methods of defining internal friction (see **friction, internal**).

In treating mechanical resonance, it should be noted that in the presence of damping the resonant frequency for maximum particle velocity is not the same as the resonant frequency for maximum displacement, and where confusion can arise, displacement resonance or particle velocity resonance should be specified. (See **oscillation, forced**.)

RESONANCE SPECTRUM. See **spectrum, resonance**.

RESONANCE THEORY OF ATMOSPHERIC OSCILLATIONS. A theory, originally put forward by Kelvin, to explain the large amplitude of the semi-diurnal wave of barometric pressure. This wave is of much larger amplitude than either the solar or lunar (diurnal or semi-diurnal) equilibrium tides, and this can only be explained by assuming either that 12 hours is a natural period of free oscillation of the atmosphere so that resonance amplifies waves of that period, or that the force producing it is different, e.g., that the wave is produced thermally and not primarily by gravitational forces.

Both explanations are probably correct, and Holmberg has suggested that they are linked: that the earth's rotation is kept at its present speed by the wave through friction at the ground, and the wave is produced by diurnal

temperature oscillations and amplified by resonance.

RESONANT FREQUENCY. (1) A frequency at which resonance exists. The commonly used unit is the cycle per second. In cases where there is a possibility of confusion, it is necessary to specify the type of resonant frequency, e.g., displacement resonant frequency or velocity resonant frequency. (2) The frequency of a mode of elastic vibration of a crystal such as quartz, which is coupled through the piezoelectric effect to an electrical system. (See **resonance, sharpness of**.)

RESPONSE. Of a device or system, a quantitative expression of the output as a function of the input under conditions which must be explicitly stated. The response characteristic, often presented graphically, gives the response as a function of some independent variable, such as frequency or direction. Modifying phrases must be prefixed to the term "response" to indicate explicitly what measure of the output or of the input is being utilized.

RESPONSE, FREQUENCY. See **frequency response**.

RESPONSE, NORMAL. See **normal response**.

RESPONSE TIME. The time interval required for the output of a control system to approximate to a pre-determined extent to its ultimate (or so called "steady-state") value. The phrase "to a pre-determined extent" may be defined in a number of ways.

For instance, the response time may be defined as the time interval required for the amplitude of every normal mode of the system to fall to a given fraction (or less) of its original value. In this case the response-time is a direct measure of the degree of attenuation of the least damped mode of the system and is independent of the input used.

Alternatively, and more frequently, the response time is defined as the time (after the application of the input) after which the output remains within a specified "deviation limit" of its ultimate value. This form of definition must be associated with a specified input, usually, but not necessarily, a **step function**.

RESPONSE REPRESENTATION, FREQUENCY. See frequency response representation.

RESPONSIVITY (MOBILITY RESISTANCE). In the mechanical rectilinear mobility system, mechanical rectilinear responsivity (mobility resistance) r_I , in mechanical mhos, is defined as

$$r_I = \frac{v}{f_M} = \frac{1}{r_M}$$

where v is the velocity, in centimeters per second, f_M is the force, in dynes, and r_M is the mechanical impedance, in mechanical ohms.

Responsivity mobility resistance in the mechanical rectilinear mobility system is represented in the figure by sliding friction which causes dissipation of energy.



Graphic representation of the element responsivity r_I .

RESPONSIVENESS, ACOUSTIC. The reciprocal of the acoustic resistance. (See resistance, acoustic.)

RESTITUTION, COEFFICIENT OF (COLLISION COEFFICIENT). In a two-body collision involving particles 1 and 2, moving in the same straight line, the coefficient of restitution is defined by

$$e = \frac{v_2 - v_1}{u_1 - u_2}$$

where $u_1 > u_2$ are the velocities with respect to a primary inertial system before collision and $v_2 > v_1$ are the corresponding velocities after collision. For a completely elastic collision $e = 1$. For an inelastic collision $e < 1$. (See impact.)

REST MASS. The mass, in grams, of an elementary particle or nuclide at rest. It is equal to the atomic mass divided by Avogadro's constant ($N = 6.0248 \times 10^{23}$ (g mol)⁻¹). Examples (E. R. Cohen and W. M. DuMond (1955), *Encycl. Phys.*, Springer-Verlag, Berlin):

Electron rest mass:

$$m = (9.1083 \pm 0.0003) \times 10^{-28} \text{ g}$$

Proton rest mass:

$$m_p = M_p/N = (1.67239 \pm 0.00004) \times 10^{-24} \text{ g}$$

Neutron rest mass:

$$m_n = M_n/N = (1.67479 \pm 0.00004) \times 10^{-24} \text{ g}$$

RESTRAINT COEFFICIENT. Columns and beams whose ends are restrained against rotation by elastic members framing into the joint are not fully fixed nor free to rotate. A restraint coefficient of unity indicates a hinged end, a coefficient of 2, a fixed end.

RESULTANT. The resultant of a system of forces is a simpler system of forces which has the same component of force in any direction and the same moment about any axis or point. The dynamic effect of the resultant acting on a rigid body will be the same as the effect of the given system of forces. If all forces are concurrent, the simplest form of the resultant is a single force. If all forces are parallel or coplanar they may be reduced to a single force or to a single couple. In the general three-dimensional case both a force and couple may be needed. (See wrench.)

RETARDATION SPECTRUM. See delay time.

RETARDED ELASTICITY. See elasticity, retarded.

RETARDED POTENTIALS. See field of a moving charge in space.

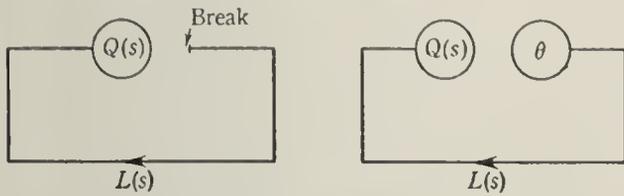
RETROGRADE CONDENSATION AND VAPORIZATION. See critical phenomena in the vaporization of mixtures.

RETRO-REFLECTION. See reflex reflection.

RETURN DIFFERENCE AND RETURN RATIO. A term used in feedback theory to assess the quantitative effects of the various feedback loops at a particular point of or in relation to a particular physical element of the system. (The return difference is also called the feedback, notably by Bode, but the latter term is better reserved for the purely qualitative concept of two quantities each affecting the other through different channels.)

The basic idea of return difference is most easily illustrated in the case of a single feedback loop. If such a loop in signal flow diagram form is imagined broken as it leaves some

quantity $Q(s)$ on it and if some external input $\theta(s)$ be injected into the loop at the break point (see figure) the resulting value of $Q(s)$ is



clearly $\theta(s) \cdot L(s)$ where $L(s)$ is the loop **transfer function**. The difference between the input $\theta(s)$ and the "output" $\theta(s)L(s)$ expressed as a fraction of the input, is the return difference. Hence in this simple case,

$$\begin{aligned} \text{Return difference} &= \frac{\theta(s) - L(s)\theta(s)}{\theta(s)} \\ &= 1 - L(s). \end{aligned}$$

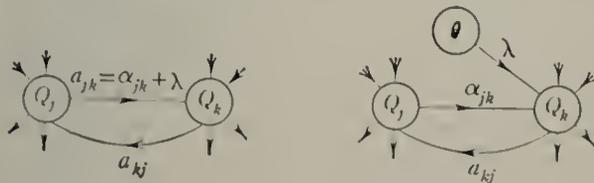
Similarly, the actual loop "output" expressed as a fraction of the input, is known as the return ratio. In this case

$$\text{Return Ratio} = L(s)$$

and always

$$\text{Return Ratio} + \text{Return Difference} = 1.$$

RETURN DIFFERENCE WITH RESPECT TO CIRCUIT PARAMETER. Let the quantities comprising the system be so selected that the parameter λ under consideration is a component of only a single dependence relating two and only two of these quantities. (This is always possible except in the case of mutual inductance, which, by its very nature, will always be part of two dependence relations and is excluded from the concept.) Denote the two quantities in a signal flow diagram by $Q_j(s)$, $Q_k(s)$ and let the link dependence a_{jk} (from Q_j to Q_k) be equal to $(\alpha_{jk} + \lambda)$. The component dependence λ (see figure) is imagined



broken as it leaves Q_j , an external input $\theta(s)$ injected at the break and the resulting value of $Q_j(s)$ obtained. Then return ratio with reference to $\lambda = \frac{\text{resulting value of } Q_j(s)}{\theta(s)}$.

This may be simply interpreted in terms of the system determinant as follows. Let the original system equations in the absence of inputs and initial conditions be

$$\begin{aligned} a_{1m}Q_1 + a_{2m}Q_2 + \dots + a_{nm}Q_n &= 0, \\ (m &= 1, 2, \dots, n) \end{aligned}$$

and let them be rewritten, in order to erect the signal flow diagram,

$$\begin{aligned} Q_m &= a_{1m}Q_1 + a_{2m}Q_2 + \dots \\ &+ (a_{mm} + 1)Q_m + \dots + a_{nm}Q_n = 0. \end{aligned}$$

By supposition λ only occurs in a_{jk} and the above break in the dependence will therefore only affect the k^{th} equation. Q_k will receive a contribution $\lambda\theta$ due to the external input instead of the previous contribution λQ_j (part of $a_{jk}Q_j$). Thus the equation for Q_k becomes

$$\begin{aligned} Q_k &= a_{1k}Q_1 + a_{2k}Q_2 + \dots + \alpha_{jk}Q_j + \dots \\ &+ (1 + a_{kk})Q_k + \dots + a_{nk}Q_n + \lambda\theta \end{aligned}$$

or

$$\begin{aligned} a_{1k}Q_1 + a_{2k}Q_2 + \dots + \alpha_{jk}Q_j + \dots \\ + a_{kk}Q_k + \dots + a_{nk}Q_n &= -\lambda\theta \end{aligned}$$

the other equations, in their original form, being still valid. Solving these equations for Q_j determinantly gives

$$Q_j = -\lambda\theta \frac{\Delta_{jk}}{\Delta_o}$$

where Δ_{jk} is the co-factor of a_{jk} in the system determinant Δ and Δ_o is the value of Δ with α_{jk} replacing a_{jk} , that is, with λ equated to zero. Hence

$$\text{Return Ratio} = -\frac{\lambda\Delta_{jk}}{\Delta_o}$$

and

$$\text{Return Difference} = 1 - \text{Return Ratio}$$

$$\begin{aligned} &= \frac{\Delta_o + \lambda\Delta_{jk}}{\Delta_o} \\ &= \frac{\Delta}{\Delta_o}. \end{aligned}$$

Thus the return difference of a circuit parameter is the ratio of the normal value of the system determinant to its value when the parameter vanishes, provided always the system determinant is linear in the parameter.

Although this result has been obtained on

the basis of setting up the system equations in a restricted manner (so as to ensure that λ only occurs in the single element a_{jk}) it is still valid however the system equations are written down with the only proviso that the system determinant is linear in the parameter considered.

RETURN DIFFERENCE FOR A GENERAL REFERENCE. The concept may be further extended by estimating the return ratio around the loop or loops of which not λ but $(\lambda - \lambda_1)$, the departure of λ from some arbitrary reference value λ_1 , forms a link. The parameter value λ is here considered as $\lambda_1 + (\lambda - \lambda_1)$. In the flow diagram, the component link λ_1 is retained as part of α_{jk} and only $(\lambda - \lambda_1)$ is broken and fed with the external input θ . A similar analysis to the above then gives the result

$$\text{Return difference of } \lambda \text{ for a reference } \lambda_1 = \frac{\Delta}{\Delta_{\lambda_1}}$$

where Δ_{λ_1} is the value of Δ when λ assumes its reference value λ_1 .

RETURN PERIOD. The interval of time expected before a time-series returns to some specified value, such as zero, or some extreme value. The term may, rather more generally, also mean the expected time before the series attains or exceeds a certain limit, e.g., in regard to the flooding of a river.

RETURN RATIO. See return difference and return ratio.

REVERSAL SPECTRUM. See spectrum, reversal.

REVERSED CARNOT CYCLE. See Carnot cycle.

REVERSED CYCLE. See cycle.

REVERSED RANKINE CYCLE. A modified form of the reversed Rankine cycle forms the basis of the operation of refrigerators. The direct reversed Rankine cycle, as shown in the diagram of Figure 1, could not be used for the purpose. In the reversed cycle, the working fluid ("refrigerant") must be cooled at constant pressure from state C' to state A . This implies that a cooling source is available whose temperature is equal to, or lower than, that corresponding to A . If this were the case, no refrigerator to produce temperature $T_o = T_A$ would be necessary. It is, therefore, necessary

to allow for the fact that the liquid refrigerant can be cooled along BA only to a temperature corresponding to an intermediate state, say A'

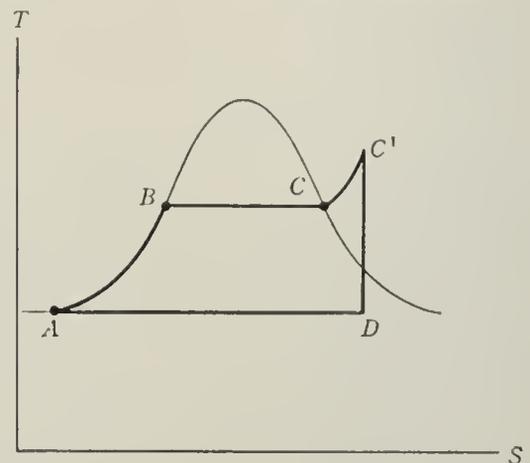


Fig. 1.

in Figure 2. The cycle is closed by throttling the refrigerant from state A' to state β , Figure 2. It would be possible to expand the refrigerant reversibly and adiabatically in an expansion engine along $A'\alpha$ and to heat it along

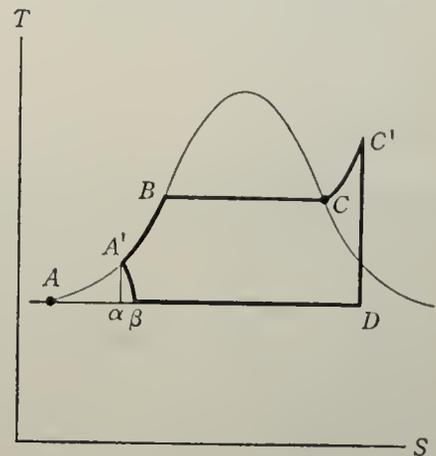


Fig. 2.

αD instead of βD , as would be the case with a throttling valve, and so to increase the efficiency of the operation.

Both cycles, the one with an expansion engine $DC'CBA'\alpha D$, and the one with a throttling valve $DC'CBA'\beta D$, are known as *modified reversed Rankine cycles*. The more efficient cycle is used only if it is necessary to ensure a high refrigerating effect, that is, if the temperature difference $T_{A'} - T_o$ is large ($T_a = T_\beta = T_o$). Otherwise, the cycle with the cheaper expansion valve (which effects the throttling process $A'\beta$) is used.

The coefficient of performance of a modified Rankine cycle with expansion value is given by

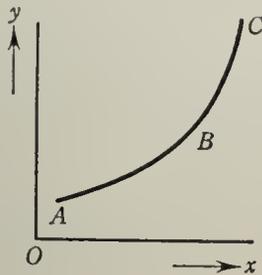
$$\epsilon = \frac{Q_o}{W_o}$$

where the refrigerating effect $Q_o = h_D - h_\beta$ is represented by the area under the line and the work $W_o = h_{C'} - h_D$ is represented by the area $DC'CBAD$. The work of the expansion engine $W' = h_{A'} - h_\alpha = h_\beta - h_\alpha$ ($h_{A'} = h_\beta - \text{throttling}$) is lost, and the refrigerating effect is smaller by $Q'_o = W' = h_\beta - h_\alpha$ compared with the cycle with an expansion engine. (Since $h_{A'} = h_\beta$, we also have: area $A'A\alpha$ = area under the line $\alpha\beta$. The increased performance of the cycle with expansion engine is given by

$$\epsilon' = \frac{Q_o + Q'_o}{W_o - Q'_o}$$

A refrigerator based on the reversed Rankine cycle using an expansion engine was first designed by Claude (Claude refrigerator).

REVERSIBLE AND IRREVERSIBLE PROCESSES. Consider a system which undergoes the transformation ABC (see figure).



Reversible and irreversible processes.

The change is said to be *reversible* if there exists a change CBA such that:

(a) The variables characterizing the state of the system return through the same values, but in the inverse order;

(b) Exchanges of heat, matter and work with the surroundings are of the reverse sign and takes place in the reverse order. Thus, for example, if in the trajectory ABC the system receives a quantity of heat Q , it must give up the same quantity in the inverse trajectory CBA .

All changes which do not satisfy those two conditions are termed *irreversibles*. No changes which take place in nature are reversible. All are irreversible. But in many cases real processes can be derived approaching as nearly as we wish to reversible processes.

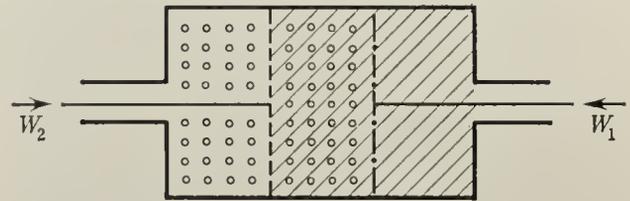
Thus let us consider a wide U-tube containing water in which initially the water level is

higher in one arm than in the other. The level in this arm falls, passes through the equilibrium position, and rises again. If we could reduce friction and viscosity to zero, we should obtain on the limit a reversible change.

A reversible change may always be considered as a succession of **equilibrium states**. However, there are processes in nature which cannot be considered as reversible without suppressing them completely. For example, the equalization of temperatures, chemical reactions, and diffusion are processes of this kind. (See also **natural and unnatural processes**.)

REVERSIBLE SEPARATION OF GASES.

Since the mixing of gasses (diffusion) is an irreversible process, the reverse process of separation involves the expenditure of work.



- ○ ○ ○ ○ Gas 1
- ////// Gas 2
- Membrane 1
Impervious to Gas 2
- · - · - Membrane 2
Impervious to Gas 1

Reversible separation of gases.

The amount of work required will be a minimum when the process occurs reversibly. In actual practice reversible separation cannot be achieved, even approximately, but the process can be imagined if semi-permeable membranes are used. In the arrangement, work W_1 and W_2 is done by pistons assumed to act from each side of the chamber, and the mixture can be separated reversibly.

The minimum work of separation is

$$\begin{aligned} W &= mRT \sum x_i \ln \frac{1}{x_i} \\ &= pV \sum x_i \ln \frac{1}{x_i} \\ &= T \sum m_i R_i \ln p/p_i \end{aligned}$$

(where x is the molar fraction; the subscript i refers to individual components; the terms without subscript refer to the mixture; and the summation extends over the n components i).

REVERSIBILITY PARADOX. All mechanical laws are invariant under a time reversal. This is an apparent contradiction to the second law of **thermodynamics**, but this reversibility paradox can be resolved by using the statistical approach to the second law.

REVERSION OF SERIES. A given power series

$$y = a_0 + a_1x + a_2x^2 + \dots$$

may be reverted to give an explicit representation of x as a function of y . The result is

$$x = z + c_1z^2 + c_2z^3 + \dots$$

where $z = (y - a_0)/a_1$; $c_1 = -a_2/a_1$; $c_2 = -a_3/a_1 + 2(a_2/a_1)^2$; $c_3 = -a_4/a_1 + 5a_2a_3/a_1^2 - 5(a_2/a_1)^3$; \dots

The method of undetermined coefficients can be used to obtain further coefficients in the series but the labor becomes great.

REVOLUTION, SURFACE OF. See **surface of revolution**.

REYNOLDS ANALOGY. If the coefficients of turbulent transfer of momentum and of material or heat are the same in turbulent flow with a steady mean state, Reynolds analogy between the processes of transfer of momentum and matter or heat holds, and in such a case the way in which a given turbulent motion transfers matter or heat can be deduced from measurements of the **velocity profile**. It is impossible to deduce the velocity profile in turbulent motion when the turbulence is produced by the motion in an unknown manner; but in cases in which the presence of temperature gradients, or gradients of fluid-borne matter, do not affect the motion, Reynolds analogy gives a useful indication of the transfer of heat or material. In practice, however, the analogy is known not to hold strictly and the velocity and temperature or concentration profiles of jets and plumes are not identical. When buoyancy forces are important there are theoretical reasons why the analogy should not hold.

REYNOLDS EQUATION. A form of the **Navier-Stokes equation** for the mean motion of an incompressible turbulent fluid in which the shearing stresses due to the turbulence appear as terms analogous to those representing molecular viscosity, viz. for the mean velocity \bar{u} in the x -direction

$$\begin{aligned} \rho \frac{\partial \bar{u}}{\partial t} &= \frac{\partial}{\partial x} (\overline{p_{xx}} - \rho \overline{u^2} - \rho \overline{u'^2}) \\ &+ \frac{\partial}{\partial y} (\overline{p_{xy}} - \rho \overline{uv} - \rho \overline{u'v'}) \\ &+ \frac{\partial}{\partial z} (\overline{p_{xz}} - \rho \overline{vw} - \rho \overline{u'w'}) \end{aligned}$$

(u' , v' , w') being the fluctuation around the mean velocity (\bar{u} , \bar{v} , \bar{w}).

In non-turbulent flow p_{xx} , p_{xy} , p_{xz} represent the normal and shearing stresses, and the equation is written

$$\begin{aligned} \rho \frac{\partial u}{\partial t} &= \frac{\partial}{\partial x} (p_{xx} - \rho uu) \\ &+ \frac{\partial}{\partial y} (p_{xy} - \rho uv) + \frac{\partial}{\partial z} (p_{xz} - \rho uw). \end{aligned}$$

The Reynolds stresses $\overline{\rho u'u'}$, $\overline{\rho u'v'}$, and $\overline{\rho u'w'}$ thus appear as terms which must be added to the equation for the mean motion.

REYNOLDS NUMBER. The terms in the equations for the motion of a viscous fluid all have the same dimensions; when there are no body forces or when the density is uniform the equation can therefore be written in the form

$$\frac{Du_i^*}{Dt^*} = - \frac{\partial p^*}{\partial x_i^*} + \frac{1}{R} \frac{\partial^2 u_i^*}{\partial x_j^* \partial x_j^*}$$

in which

$$\begin{aligned} u_i^* &= u_i/U; & t^* &= tU/d; \\ p^* &= p'/\rho U^2; & x_i^* &= x_i/d. \end{aligned}$$

p' is the departure of the pressure from the hydrostatic value. The starred quantities are dimensionless. U , ρ , and d are a sample velocity, density, and length, chosen appropriately from the problem under study.

The ratio of the first and last terms is R , which is the Reynolds number, and equal to Ud/ν . It is the ratio of the inertia forces, represented by ρU^2 (twice the kinetic energy of unit volume) to the viscous forces, represented by $\mu U/d$ (coefficient of viscosity multiplied by a velocity gradient). It is a special case of the **Froude number** which is the ratio of the inertia forces to the mechanical forces.

In any flow pattern the Reynolds number is not unique because U and d can be chosen in many ways. For flow past a body U is nor-

mally taken as the speed at a great distance from the body, and d as the diameter of the body.

REYNOLDS NUMBER, CRITICAL. See bluff body, flow past.

REYNOLDS NUMBER, ROUGHNESS. See roughness, effect on transition of laminar boundary layer.

REYNOLDS STRESSES. The momentum transferred across unit area by the velocity fluctuations in turbulent motion. In **Reynolds equation** the stresses appear as terms additional to the stresses due to molecular viscosity. If u' and v' are the velocity fluctuations in the x - and y -directions the Reynolds stress in the x -direction across unit area perpendicular to the y -direction is

$$\tau_{xy} = -\overline{\rho u'v'}$$

this being the rate at which x -momentum, $\rho u'$, is being transported in the y -direction by the velocity fluctuations v' .

The work done in the mean motion against the Reynolds stresses is presumed to increase the energy of the turbulence.

There is also a normal Reynolds stress $\overline{\rho u'u'}$ which modifies the normal pressure.

The Reynolds stresses may be evaluated by direct measurement of the velocity fluctuations. They are sometimes represented by an eddy viscosity.

RHEOLOGY. Study of the deformation and flow of materials in response to stress.

RHEONOMIC. See conservative force.

RHOMBOHEDRAL SYSTEM. One of the seven crystal systems. In this system, three equal axes are equally inclined to each other, but not at right angles.

RHUMB LINE. Any line on the surface of the earth that crosses successive meridians at constant angle is called a rhumb line, a loxodromic spiral, or a **loxodrome**.

Because of the convergence of the meridians on the poles of the earth, a ship following a rhumb line (i.e., maintaining constant course) travels around the earth on a spiral that approaches one of the poles as a limit, unless the angle with successive meridians is 0° or 90° . If the angle is 0° the rhumb line is a

meridian. If the angle is 90° the ship follows the equator or a small circle parallel to the equator.

RICCATI EQUATION. An ordinary differential equation of first order

$$y' = A_0(x) + A_1(x)y + A_2(x)y^2.$$

It is the most general form of the first-order differential equation $y' = f(x,y)$, where $f(x,y)$ is rational in y and with a general solution containing no singularities except poles. Every second-order linear differential equation may be transformed into a Riccati equation. The general Riccati equation cannot be solved by quadrature and it defines a set of **transcendental** functions more complicated than the elementary transcendentals, such as $\ln x$ and its inverse, e^x .

RICCI CALCULUS. See absolute differential calculus.

RICCI IDENTITY. $t_{pq\cdots r}^{ij\cdots k}$ and B_{jkl}^i are the components of an arbitrary tensor of weight W and the **curvature tensor**, respectively, in a generic coordinate system. S_{ij} is defined by

$$S_{ij} = R_{ij} - R_{ji},$$

where R_{ij} are the components of the Ricci tensor. The identity

$$\begin{aligned} t_{pq\cdots r,s,t}^{ij\cdots k} - t_{pq\cdots r,t,s}^{ij\cdots k} &= t_{pq\cdots r}^{aj\cdots k} B_{ast}^i \\ &+ t_{pq\cdots r}^{ia\cdots k} B_{ast}^j + \cdots + t_{pq\cdots r}^{ij\cdots a} B_{ast}^k \\ &- t_{aq\cdots r}^{ij\cdots k} B_{pst}^a - t_{pa\cdots r}^{ij\cdots k} B_{qst}^a - \cdots \\ &- t_{pq\cdots ra}^{ij\cdots k} B_{rst}^a - W t_{pq\cdots r}^{ij\cdots k} S_{st} \end{aligned}$$

holds and is known as the Ricci identity.

RICCI LEMMA. See Ricci theorem.

RICCI TENSOR. The second-order **covariant tensor** B_{imj}^m formed by the indicated contraction of the **curvature tensor** B_{ikj}^m ; often denoted R_{ij} .

RICCI THEOREM. The covariant derivative of either of the fundamental tensors is zero. Also called *Ricci lemma*.

RICHARDSON - DUSHMAN EQUATION. The basic equation of thermionic emission. Richardson first developed the equation from the thermodynamic theory of gases, while a later derivation by Dushman based on quan-

tum mechanics gives identical results. This equation gives the thermionic current density, j , resulting from the emission of electrons from a metallic surface at the temperature T .

$$j = AT^2 e^{-\phi/kT}$$

where A is a constant depending on the state of the surface, and ϕ is the **thermionic work function** for the emitter.

RICHARDSON EQUATION. See **Richardson-Dushman equation**.

RICHARDSON NUMBER. A non-dimensional number, R_i , representing the ratio of the energy available from shearing motion to the potential energy of a stratified fluid. Thus

$$R_i = g\beta/(\partial U/\partial z)^2$$

where β is the static stability, U the horizontal velocity, and z is measured vertically.

In turbulent motion the energy extracted from the mean flow by the Reynolds stresses is $K_M(\partial U/\partial z)^2$ and the gain in potential energy is $K_H g\beta$, where K_M is the coefficient of eddy transfer of momentum, the eddy stress then being $K_M \partial U/\partial z$, and K_H is the coefficient of eddy transfer of heat, or buoyancy. If $K_M(\partial U/\partial z)^2 - K_H g\beta > 0$, the excess of energy made available is assumed to take the form of turbulent motion, and so the turbulence will grow. Richardson's criterion of "just no turbulence" is therefore that

$$R_i \leq K_M/K_H.$$

The criterion has been widely misapplied to the case of turbulent motion as a means of differentiating growing from dying turbulence. The ratio K_H/K_M is not known, and probably depends on the Richardson number. A more useful number for some considerations is the Flux Richardson Number

$$R_f = \left(\frac{g}{\rho} \overline{\rho' w'} \right) / \left(\overline{u' w'} \frac{\partial U}{\partial z} \right)$$

where ρ' , u' , w' are the fluctuations of density, horizontal, and vertical velocity.

Alternatively R_f may be defined as K_H/K_M where these coefficients are defined as if the momentum and buoyancy transferred by the eddies were being conducted (with coefficients K_H and K_M) in a medium in steady flow.

In **baroclinic flow** in the atmosphere the Richardson number

$$R_i = \frac{f^2 \theta \frac{\partial \theta}{\partial z}}{g \left(\frac{\partial \theta}{\partial x} \right)^2},$$

in which the shear is represented by the thermal wind, provides a criterion for the stability of geostrophic airstreams.

RICHARDSON PLOT. By plotting the logarithm of the thermionic current per squared Kelvin degree, i.e., $\log I/T^2$, against the reciprocal of the absolute temperature, a straight line is obtained whose slope is a measure of the **activation energy** involved (see **Richardson-Dushman equation**). A similar plot is useful in any process involving thermal activation with a **Boltzmann factor**.

RIDGE (OF HIGH PRESSURE). A form seen on a chart of isobars or chart of contours of an isobaric surface. Ridges have rounded contours. A ridge in the thickness pattern is a ridge of warm air, and usually denotes warm air in the upper air contour pattern.

RIEKE DIAGRAM. A polar-coordinate load diagram for microwave oscillators, particularly klystrons and magnetrons. Constant-power and constant-frequency contours are plotted against the polar plot of load admittance, commonly called the *Smith chart*.

RIEMANN-CRISTOFFEL TENSOR. The tensor with components R_{ijkl} in a generic coordinate system x , defined by

$$R_{ijkl} = g_{im} B_{jkl}^m,$$

where g_{im} is the covariant **metric tensor** and B_{jkl}^m is the **curvature tensor**. Expressed in terms of the **Cristoffel symbols** by

$$R_{ijkl} = \frac{\partial \Gamma_{ijk}}{\partial x^l} - \frac{\partial \Gamma_{ijl}}{\partial x^k} + \Gamma_{ik}^a \Gamma_{a;jl} - \Gamma_{il}^a \Gamma_{a;jk}.$$

Also called *covariant curvature tensor*, or *Riemann-Cristoffel tensor of the first kind*. (See also **curvature tensor**.)

RIEMANN FUNCTION. See **Riemann method**.

RIEMANN INTEGRAL. Let $f(x)$ be a bounded function, defined for the closed linear interval $[a, b]$, where $b > a$; so that there exist an upper bound U , and a lower bound L , of

the values of $f(x)$ in the closed interval. A set of intervals such that every point of $[a,b]$ is contained in at least one interval of the set is called a net, each interval being called a mesh of the net. Let a system of nets, with closed meshes, be applied to the interval $[a,b]$, and let $\delta_1^{(n)}, \delta_2^{(n)}, \dots, \delta_{m_n}^{(n)}$ denote the breadths of the m_n meshes of the net D_n , of the system. Let $M(\delta_r^{(n)})$ denote any number so chosen as to be not greater than the upper bound of the function $f(x)$ in the closed mesh $\delta_r^{(n)}$ and not less than the lower bound of $f(x)$ in the same mesh, and consider the sum

$$S_n = \delta_1^{(n)}M(\delta_1^{(n)}) + \delta_2^{(n)}M(\delta_2^{(n)}) + \dots + \delta_{m_n}^{(n)}M(\delta_{m_n}^{(n)}).$$

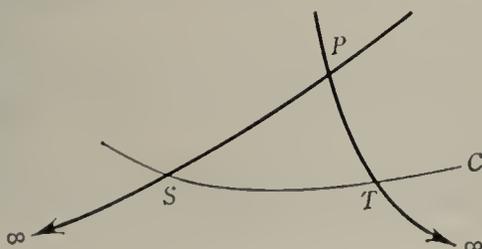
If the sequence $S_1, S_2, \dots, S_n, \dots$ be convergent, and have the same number S for its limit, whatever system of nets, applied to $[a,b]$, be employed, and however the numbers $M(\delta_r^{(n)})$ be chosen, subject only to their limitation in relation to the upper and lower bound of $f(x)$ in the meshes $\delta_r^{(n)}$, then the function $f(x)$ is said to have a Riemann integral in the interval $[a,b]$, and the number S defines the value of its integral. The integral, when the limit S exists, is denoted by $\int_a^b f(x)dx$.

RIEMANN MAPPING THEOREM. Every simply connected domain in the complex plane, whose boundary contains more than one point, can be mapped conformally onto the interior of the unit circle.

RIEMANN METHOD. (For notation, see Green's identity.) A method of solution of the Cauchy problem for hyperbolic partial differential equations. It is based on the application of Green's identity (see Equation (2) under that heading) for a suitable region R and auxiliary function ψ . For the equation

$$L\phi = a\phi_{xx} + 2b\phi_{xy} + c\phi_{yy} + 2d\phi_x + 2e\phi_y + f\phi = g \quad (1)$$

where a, b, c, d, e, f, g are functions of x and y such that $ac - b^2 < 0$ (i.e., the equation is



hyperbolic), Riemann's method leads to the solution (see the figure)

$$2\phi(X,Y) = (\psi\sigma)_T + (\psi\sigma)_S + \int_S^T \left[\sigma \left(\psi \frac{\partial\phi}{\partial Y} - \frac{\partial\phi}{\partial X} \right) + B\phi\psi \right] ds + \iint_R \psi y dx dy. \quad (2)$$

This equation gives the value of ϕ at $P(X,Y)$ when values of ϕ are prescribed along the arc ST of the curve C ; SP and TP are characteristics of (1). This solution shows that the angular region $(\infty SPT \infty)$ is the domain of influence of data at P , it is also the domain of dependence of the solution at P . In (2), $\psi = \psi(x,y;X,Y)$ is the Riemann function, defined as the solution of the adjoint equation

$$L\psi = 0$$

with the boundary conditions

$$\frac{d\psi}{ds} + \frac{d\sigma\psi}{ds} = B, \text{ on } PT$$

$$\frac{d\psi}{ds} + \frac{d\sigma\psi}{ds} = -B, \text{ on } PS$$

$$\sigma\psi = 1, \text{ at } P.$$

For an example, see wave equation.

RIEMANN - PAPPERITZ EQUATION. A second-order linear differential equation, as studied in the Fuchs theorem, with three regular singular points in the finite plane at $x = a, b, c$. It may be symbolized by the Riemann P -function

$$y = P \left\{ \begin{matrix} a & b & c \\ a' & b' & c' \\ a'' & b'' & c'' \end{matrix} ; x \right\}$$

where $a', a'', b', b'', c', c''$ are the exponents at the singularities, y is the dependent variable and x , the independent variable.

The Riemann problem in the theory of linear differential equations consisted of the search for a function to satisfy a given P -function. It is shown that the transformation of variable

$$u = \left(\frac{x-a}{x-b} \right)^k \left(\frac{x-c}{x-b} \right)^l y$$

will shift the exponents of the differential equation to $a' + k, a'' + k, b' - k - l, b'' - k - l, c' + l, c'' + l$, without affecting the singular points. On the other hand, the general linear transformation

$$x = (Az + B)/(Cz + D); \quad (AD - BC) \neq 0$$

will shift the singular points to three new positions, say a_1, b_1, c_1 , as determined by the transformation but the exponents will not be altered. Combination of these two transformations will thus result in a standard form of the linear differential equation of Fuchsian type. The standard form is usually taken as the **Gauss hypergeometric equation**. The solution of any given linear differential equation of second order with three or fewer singular points will therefore be a special case of solutions to the Gauss equation, for the given differential equation can always be converted into the latter by suitable transformation of variables as described.

RIEMANN SPHERE. A sphere with the **complex plane** mapped onto it by stereographic projection. The sphere is so placed that its south pole rests on the plane; then to each point on the sphere (except the north pole) is assigned the complex number where the ray from the north pole through the point cuts the plane. To the north pole itself is assigned the point-at-infinity, an ideal point whose usefulness is determined by the fact that its neighborhoods may be thought of as the exteriors of circles about the origin.

RIEMANN-STIELTJES INTEGRAL. If $(a, x_1, x_2, \dots, x_{m-1}, b)$ be a net, fitted on to (a, b) , and in which the breadth of the greatest mesh is d , let us consider the sum

$$S_d \equiv f(\xi_1)\{\phi(x_1) - \phi(a)\} + f(\xi_2)\{\phi(x_2) - \phi(x_1)\} + \dots + f(\xi_m)\{\phi(b) - \phi(x_{m-1})\},$$

where $f(x), \phi(x)$ are the bounded functions, defined in the interval (a, b) , and $\xi_1, \xi_2, \dots, \xi_m$ are points, assigned in any manner, which are in the closed meshes

$$(a, x_1), (x_1, x_2) \dots (x_{m-1}, b) \text{ respectively.}$$

We may denote x_0 by a , and x_m by b .

If the functions $f(x), \phi(x)$ be such that S_d converges to a definite number, as the number m of the meshes is increased indefinitely, subject to the condition that d converges to zero, and if this limit is independent of the mode of

successive sub-division of the interval by the nets, and of the mode in which the sets of points $\xi_1, \xi_2, \dots, \xi_m$ are assigned in the nets, $f(x)$ is said to have a Stieltjes integral with respect to $\phi(x)$. Such integral is defined to be the limit of S_d , as $d \rightarrow 0$, is denoted by

$$\int_a^b f(x)d\phi(x),$$

and is called a Riemann-Stieltjes integral.

RIEMANN SURFACE. A surface used in representing multivalued functions of the **complex variable**. One sheet is assigned to each **branch** of the function, each sheet is cut at the branch line, and all are joined together so that a closed contour may be traced by passing continuously along the sheets of the surface.

RIEMANN ZETA FUNCTION. The *generalized zeta function* is defined for $\text{Re } s > 0$ by the equation

$$\zeta(s, v) = \sum_{n=1}^{\infty} (v + n)^{-s}, \quad v \neq 0, 1, 2, \dots$$

Riemann's zeta function is obtained by putting $v = 0$ in the definition of the generalized zeta function

$$\zeta(s) = \sum_{n=1}^{\infty} (1/n)^s, \quad \text{Re } s > 1.$$

(See also **Weierstrass' function**.)

RIGHT ASCENSION (R.A.). The right ascension of a celestial object is measured from the vernal equinox along the celestial equator toward the east (counterclockwise) to the point of intersection of the hour circle through the object with the equator. Right ascension may be expressed in units of time or in angular units.

Star catalogues give the right ascension of the object for some particular **epoch**. Due to **precession** and **nutations**, these coordinates are slowly changing. Constants to reduce the positions given in the catalogue to those of any other date are published for each star in the catalogue.

$$\begin{aligned} \text{Right ascension} + \text{sidereal hour angle} \\ = 24 \text{ hr (or } 360^\circ). \end{aligned}$$

RIGID BODY. An aggregate of material **particles** in which the distance between any two particles remains constant with time.

RIGID BODY, GENERAL EQUATION OF MOTION. The motion of a rigid body can be described by the translational motion of the center of mass and the rotational motion about the center of mass. The equation for translational motion has the general form

$$\mathbf{F} = M \frac{d^2\mathbf{r}}{dt^2}.$$

\mathbf{F} is the vector sum of all the forces acting on the rigid body, M is the total mass, and \mathbf{r} is the position vector of the center of mass. The equation for rotational motion has the general form

$$\mathbf{L} = \frac{d\mathbf{H}}{dt}.$$

\mathbf{L} is the resultant **torque** and \mathbf{H} the vector **angular momentum** about the center of mass as an origin.

RIGID BODY, GENERAL EQUATION OF ROTATIONAL MOTION ABOUT A POINT. The fundamental equation of motion for rotational motion of a rigid body about a fixed point is

$$\frac{d\mathbf{H}}{dt} = \mathbf{L}$$

where $\frac{d\mathbf{H}}{dt}$ is the time derivative of total vector angular momentum about the point, \mathbf{L} is the resultant torque about the point.

If the moments of inertia are referred to the principal axes, with origin at fixed point and axes fixed in the body, this equation becomes

$$\begin{aligned} \mathbf{L} = & \mathbf{i}[I_{xx}\dot{\omega}_x + (I_{zz} - I_{yy})\omega_y\omega_z] \\ & + \mathbf{j}[I_{yy}\dot{\omega}_y + (I_{xx} - I_{zz})\omega_z\omega_x] \\ & + \mathbf{k}[I_{zz}\dot{\omega}_z + (I_{yy} - I_{xx})\omega_x\omega_y]. \end{aligned}$$

(This is also known as the **Euler equation** for a rigid body.) (See also **rigid body, general equation of motion.**)

RIGID DIATOMIC ROTATOR, ENERGY LEVELS OF. Values given by the equations

$$\epsilon_j = j(j + 1) \frac{h^2}{8\pi^2 I}, \quad g_j = 2j + 1,$$

where j is the rotational quantum number, h is Planck's constant, I the moment of inertia of the rotator, and g_j the degree of degeneracy of ϵ_j .

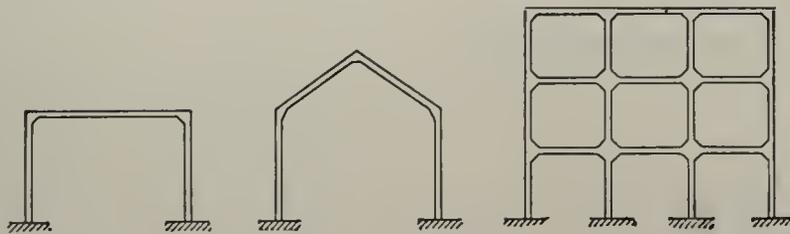
RIGID DIATOMIC ROTATOR, ENTROPY OF. The value is given by the equation

$$S = k[\ln(8\pi^2 I k T / h^2 s) + 1],$$

where k is Boltzmann's constant, T is the absolute temperature, S is the entropy, h is Planck's constant, I is the moment of inertia of the rotator, and s is the symmetry factor which is 2 for a diatomic molecule consisting of two identical atoms, and 1 otherwise.

RIGID FRAME. A rigid frame, also known as a continuous frame, is an **indeterminate structure** in which continuity of action between the intersecting or adjacent members is obtained by means of moment-resisting **joints** (joints capable of resisting **bending moment**).

Rigid frames are usually constructed of



Bents



Vierendeel Girder

Rigid frames.

structural steel or reinforced concrete, although some frames have been built of wood and, particularly in the last few years, of **prestressed concrete**. In the steel structures the joints are either riveted or welded whereas in the concrete structures continuity is obtained by running the main reinforcing rods through the joint.

Rigid frames are used as bridges, and as **bents** in mill and multiple-story buildings. The Vierendeel girder bridge is a rigid frame which is similar in outline to the usual bridge **truss**. However, the diagonals are omitted since the chords are designed for **flexure**. This truss is very useful for special cases of building framing. Rigid frame action is also utilized in the design of reinforced concrete culverts and sewers.

RIGIDITY, FLEXURAL. The ratio of moment to curvature of an elastic beam is the flexural rigidity EI , where E is Young's **modulus of elasticity** and I is the **moment of inertia** about the axis of bending.

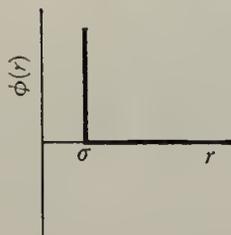
RIGIDITY MODULUS. See **Hooke's law**, **generalized**.

RIGID SPHERE GAS (OR MODEL). A system consisting of particles which interact without distortion (i.e., like rigid spheres). If the particles are **bosons**, this model is a reasonable representation of helium gas. It is described by the potential function

$$\phi(r) = \infty, \quad \text{where } r < \sigma \quad (1)$$

$$\phi(r) = 0, \quad \text{where } r > \sigma \quad (2)$$

which corresponds to the interaction between rigid impenetrable spheres of diameter σ (see figure).



Rigid spheres.

This model which is often used in statistical-mechanical calculations because of its simplicity gives a crude representation of the strong, short-range repulsive forces between molecules.

RING. A mathematical system for which two binary operations are defined, call them addition and multiplication, such that both operations are **commutative** and **associative** (these conditions are sometimes relaxed for multiplication) and multiplication is distributive over addition; also subtraction is always possible (cf. **field**). For example, the even integers $\dots -4, -2, 0, 2, 4, \dots$ form a ring.

RIPPLES. See **capillary waves**.

RITTER METHOD. The determination of the force in a member of a truss by cutting a section and taking moments about the point of concurrence of the remaining unknown forces.

RITZ FORMULA. Expansion of the Rydberg formula (see **Rydberg equation**) when given in the form

$$\nu_n = R \left[\frac{1}{p^2} - \frac{1}{q^2} \right]$$

by setting

$$p = n_1 + a_1 + \frac{b_1}{n_1^2} + \frac{c_1}{n_1^4} + \frac{d_1}{n_1^6} + \dots$$

$$q = n_2 + a_2 + \frac{b_2}{n_2^2} + \frac{c_2}{n_2^4} + \frac{d_2}{n_2^6} + \dots$$

Using only the first two terms of the expansion for p and q , one obtains the general **Rydberg equation**.

RITZ METHOD. A method for solving a boundary value problem which applies when the problem can be replaced by an equivalent minimizing problem. The method is to approximate the required function by a member of a family depending upon certain parameters. The problem is then reduced to that of minimizing a certain function of these parameters. The method is closely related to that of Galerkin. When applied to **buckling** problems, an upper bound is found for the critical load when a finite number of terms is chosen. (See Lothar Collatz, *Numerische Behandlung von Differentialgleichungen*, 2nd edition, Springer, 1955.)

ROBIN FUNCTION. A function of the form

$$R_{g,\psi}(p,q) = \frac{1}{4\pi r} + V(p), \quad r \text{ being the distance}$$

pq , $V(p)$ being harmonic, and the derivative continuous over the boundary. The solution

$\phi(q)$ of the Robin problem (see **boundary value problem**) can be expressed as

$$\phi(q) = \iint_S f(p) R_{g,\psi}(p,q) dS_p.$$

ROCKET. In a rocket motor, gas is produced at high temperature and pressure by chemical reaction of solid or liquid propellants. The gas is expanded through a nozzle to produce a high-velocity jet. If it can be assumed that the velocity is uniform across the jet the thrust is $T = mv$, where m is the rate of mass flow through the propelling nozzle and v is the jet velocity relative to the rocket.

The *specific impulse* of a rocket motor is defined as the ratio of the thrust to the weight of propellant consumed per unit time. Thus the specific impulse is measured in units of time and is equal to

$$\frac{T}{mg} = \frac{v}{g}.$$

ROCKET MOTOR EFFICIENCY. See **efficiency of rocket motor**.

ROCKET MOTOR, EFFICIENCY OF. See **efficiency of rocket motor**.

RODRIGUES FORMULA. (1) A formula which expresses the n^{th} function of a class of special functions as the n^{th} derivative of some polynomial. E.g., the Rodrigues formula for the **Legendre polynomials** is

$$P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n.$$

See **special functions** and the various particular cases listed there.

(2) Let P be a point on a surface whose vector position is \mathbf{r} . Let Q be a neighboring point to P on a line of curvature at P for which the principal curvature at P is κ . If $\mathbf{r} + d\mathbf{r}$ denotes the vector position of Q and \mathbf{n} and $\mathbf{n} + d\mathbf{n}$ denote the unit normals to the surface at P and Q respectively, then

$$d\mathbf{n} + \kappa d\mathbf{r} = 0.$$

ROLLE THEOREM. Let $f(x)$ be a function which vanishes at $x = a$ and at $x = b$, and which has a finite derivative $f'(x)$ at all points in the interval (a,b) . Then $f'(x)$ vanishes at some point x_0 between a and b .

ROLLING RESISTANCE (OR FRICTION).

In the rolling of a wheel on a plane surface there is some distortion of the two surfaces in contact due to the normal force between the surfaces. Such distortion smears out the ideal line contact and effectively introduces a force with a component in opposition to the motion. This component of force is called the rolling resistance or friction F_r and is proportional to the normal force N . A coefficient of rolling resistance or friction u_r can be defined by $u_r = F_r/N$, where F_r can be determined experimentally by observing the deceleration on a horizontal surface.

ROOM CONSTANT. A quantity defined by the expression

$$\frac{S\bar{\alpha}}{1 - \bar{\alpha}}$$

where $\bar{\alpha}$ is the average value of the **sound absorption coefficient**, and S is the total area of the boundaries of the room.

ROOT LOCUS ANALYSIS. A method of analysis normally, but not necessarily, confined to control systems, of which the main purpose is to display graphically the effect of varying a specified parameter on the location of the roots of the characteristic equation and thus on the damping of the modes of the system and on its stability. The loci followed by the various roots as the parameter varies are in fact the root loci from which the analysis derives its title.

The basic principles of the analysis are as follows. If the output to error transfer function of the system is $G(s)$ then the characteristic equation is $1 + G(s) = 0$. Provided the specified parameter λ occurs linearly in this equation, the equation can be rewritten in the form

$$\lambda = F(s), \text{ a real function of } s,$$

where λ is assumed positive and, being a parameter of a real system, real. Assuming further that $F(s)$ is a rational function of s , the factors of its numerator and denominator being known, we may write

$$\lambda = K \frac{(s - a_1)(s - a_2) \cdots (s - a_m)}{(s - b_1)(s - b_2) \cdots (s - b_n)}$$

which has m (or n) roots in s according as $m \geq n$ and will therefore give rise to m (or n) separate root loci. Hence

$$\text{ph } \lambda = \text{ph } K + \sum_1^m \text{ph}(s - a_r) - \sum_1^n \text{ph}(s - b_r)$$

and

$$\lambda = |K| \prod_1^m |s - a_r| \prod_1^n |s - b_r|^{-1}.$$

Since λ is real and positive and K real, the phase equation, which is the basis of the loci themselves, (the modulus equation being merely used to scale the loci in terms of λ) gives

$$\begin{aligned} \sum_1^m \text{ph}(s - a_r) - \sum_1^n \text{ph}(s - b_r) \\ = 2N\pi \text{ if } K > 0 \\ = (2N + 1)\pi \text{ if } K < 0. \end{aligned}$$

If the position of the zeros, a_r , and poles, b_r , is known the problem is therefore to find the permissible paths followed by a moving point s in the plane such that the phase angles of the various phasors $(s - a_r)$, $(s - b_r)$ joining the zeros and poles to the moving point satisfy the above relation.

The distances of the moving point s from the various zeros and poles (namely $|s - a_r|$ and $|s - b_r|$) then give the corresponding value of λ from the modulus equation.

It may be noted that:

(i) As $\lambda \rightarrow 0$, $s \rightarrow a_1, a_2, \dots$, i.e., the loci start from the zeros of $F(s)$.

(ii) As $\lambda \rightarrow \infty$, $s \rightarrow b_1, b_2, \dots$, i.e., the loci terminate at the poles of $F(s)$.

(iii) If $n > m$ then as $|s| \rightarrow \infty$, $\lambda \rightarrow 0$. The direction in which $|s| \rightarrow \infty$ is specified in this case by $\lambda \rightarrow \frac{K}{s^{n-m}}$ i.e., $\text{ph } s = \frac{\text{ph } K}{n - m} = \frac{2N\pi}{n - m}$

or $\frac{(2N + 1)\pi}{n - m}$ according as $K \gtrless 0$. These

relations therefore specify asymptotic directions for $(n - m)$ of the loci as $\lambda \rightarrow 0$ (the other m terminating at the m finite zeros of $F(s)$). Similarly if $m > n$ there are $(m - n)$

asymptotic directions given by $\text{ph } s = \frac{\text{ph } K}{m - n}$,

as $\lambda \rightarrow \infty$. The asymptotes themselves in both cases pass through the "centroid" of the pole-zero assembly in the finite plane, counting a zero say as mass $+1$ and a pole as mass -1 .

(iv) If any of the poles or zeros of $F(s)$ lie on the real axis of s , certain regions of this axis are barred to the loci. For if the moving point s crosses one of these zeros or poles, the phase of the corresponding factor changes by

π , the others remaining unchanged in phase, so that the phase equation cannot be obeyed on both sides of such a pole or zero. Since, when s is on the real axis, the phase contributions of the factors corresponding to conjugate complex zeros or poles is zero, as is the contribution of the factors corresponding to any real zeros or poles to the left of s , whereas that due to any factor corresponding to a real zero or pole to the right of s may be taken as $\pm\pi$, it is at once deducible that s can only lie in those parts of the real axis where the difference between the numbers of real poles and real zeros to the right of s is even or odd according as $K \gtrless 0$.

(v) The loci may leave the real axis at those points, called break points, where an incremental displacement at right angles to the axis causes no change in the sum of the phase contributions.

ROOT-MEAN-SQUARE ERROR. If a random variable has frequency function $f(x)$ the mean-square error around a point a is

$$\int (x - a)^2 f(x) dx.$$

This is the second **moment** μ'_2 . Its square root is the root-mean-square error. If a is the mean of the distribution, the mean-square error is the **variance** and its square root is the **standard deviation**, or, in relation to a sampling distribution, the **standard error**. The mean-square error is a minimum when a is the mean.

ROOT-MEAN-SQUARE VELOCITY. See **molecular velocity**, **mean square**.

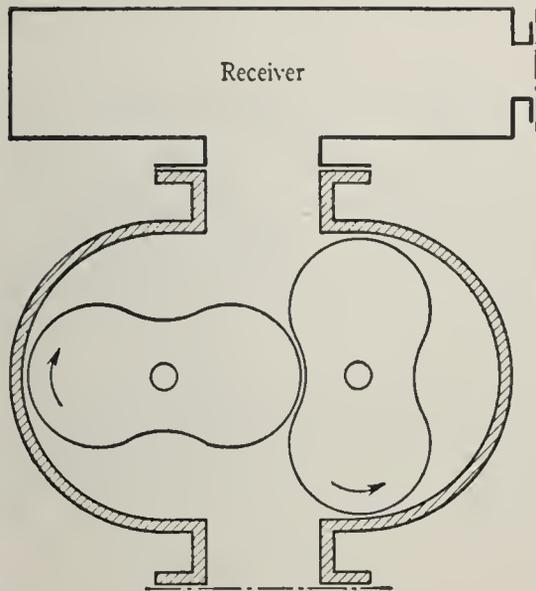
ROOT OF A BLADE. See **blade**.

ROOT OF A CONGRUENCE. Assume a congruence of numbers (or quantities) of the form $f(x) = 0$ (modulus r). Then the root is a number whose substitution into $f(x)$, makes it divisible by r .

ROOT OF AN EQUATION. A number whose substitution for the unknown in the equation yields an identity.

ROOTS BLOWER. A rotary **compressor** in which compression is achieved by positive displacement. Its essential parts are two rotors, each having two or more lobes with profiles formed by epi- and hypocycloids. At all angular positions the high-pressure space (re-

ceiver) is sealed from the low-pressure intake. A volume of air is trapped between one lobe and the casing. When the lobe travels to a



Roots blower.

point where a connection is made with the high pressure receiver, the air is compressed and displaced into the receiver.

ROOT-SQUARING METHODS. Methods of obtaining an equation whose roots are the squares of those of a given one. **Graeffe's method** is the one in common use, although previously Lobačevskiĭ, and still earlier Dandelin, had described other methods and had applied them in the numerical solution of algebraic equations.

ROSETTE. A gauge which measures extensional strain at a point or small region of a surface in three or more directions is called a rosette. It provides sufficient information to calculate the principal strains and their orientation. The two common types are 45° and 60° gauges.

ROSSBY DIAGRAM. A thermodynamic diagram, named after its designer, with **mixing ratio** as abscissa and **potential temperature** as ordinate. Lines of constant **equivalent potential temperature** are added.

This chart has been much used in air-mass analysis.

ROSSBY FORMULA. See Rossby wave.

ROSSBY NUMBER. A non-dimensional number representing the ratio of the inertia forces due to velocity relative to the earth,

represented by U^2/a where a is the earth's radius or the dimension of a wind system, to the **deviating force**, represented by Uf . Thus $R_o = U/af$. For flow with Rossby number in the neighborhood of unity the earth's rotation is an important influence in determining its nature. This is the case with the large scale wind systems in the earth's atmosphere and with ocean currents.

ROSSBY WAVES. Standing waves which can occur in a barotropic airstream subjected to a deviating force which varies across the direction of the stream. If $\beta = \partial f/\partial y$, where f is twice the vertical component of the earth's rotation, and y is measured northwards; an airstream of speed U along lines of latitude would execute standing waves of length $2\pi(U/\beta)^{1/2}$.

ROSSI-FOREL SCALE. See earthquake scales.

ROTATING COORDINATE SYSTEM. If \mathbf{i} , \mathbf{j} , \mathbf{k} are the basis of a right-handed coordinate system Σ and if Σ has the instantaneous angular velocity $\boldsymbol{\omega}$ with respect to a fixed coordinate system Σ^o and if Σ and Σ^o have a common origin, then

$$\frac{d\mathbf{i}}{dt} = \boldsymbol{\omega} \times \mathbf{i}$$

$$\frac{d\mathbf{j}}{dt} = \boldsymbol{\omega} \times \mathbf{j}$$

$$\frac{d\mathbf{k}}{dt} = \boldsymbol{\omega} \times \mathbf{k}.$$

If \mathbf{A} is any time dependent vector, the absolute rate of change of \mathbf{A} , i.e., $\frac{d\mathbf{A}}{dt}$ and the relative change of rate of \mathbf{A} (denoted by $\frac{D\mathbf{A}}{Dt}$), i.e., the change as judged by a rotating observer, are related by the equation

$$\frac{d\mathbf{A}}{dt} = \frac{D\mathbf{A}}{Dt} + \boldsymbol{\omega} \times \mathbf{A}.$$

The absolute velocity \mathbf{v}_o and the relative velocity \mathbf{v} of a moving point are related by the equation

$$\mathbf{v}_o = \mathbf{v} + \boldsymbol{\omega} \times \mathbf{r}$$

(\mathbf{r} is the position vector of the moving point).

Newton's law of motion with respect to the moving observer reads (provided ω is constant):

$$m \frac{D^2 \mathbf{r}}{Dt^2} = \mathbf{F}_o + m[\omega \times (\mathbf{r} \times \omega)] - 2m[\omega \times \mathbf{v}].$$

\mathbf{F}_o is the absolute force as seen by a stationary observer in Σ^o . The second term is the centrifugal force. The last term is the **Coriolis force**.

ROTATING CYLINDER, FLOW PAST.

When a cylinder is held with its axis normal to a stream of fluid, and rotated about its axis, a circulation is developed around the cylinder and hence a transverse force is generated, normal to the stream and to the axis of the cylinder. This is known as the **Magnus effect**.

Considering a cylinder with its axis horizontal, rotating clockwise in a stream flowing horizontally from left to right, separation occurs more readily below the cylinder than above it. Thus when the stream is started from rest the vorticity shed into the stream from below the cylinder exceeds that of opposite sign from above. The result is that a "starting vortex" passes downstream with the fluid, and a circulation is developed around the cylinder of the same sign as the rotation. Thus the transverse force or "lift" acts vertically upwards.

In experiments it is found that the flow is not even approximately two-dimensional unless disks are fitted to the ends of the cylinder. The maximum lift coefficient obtainable, with disks fitted, is about 9, and this occurs when the ratio of the peripheral velocity to the stream velocity is about 4. There is then only one stagnation point on the surface of the cylinder and the flow near the surface is everywhere in the same direction as the motion of the surface. Thus there is no mechanism by which a further increase of rotational speed can lead to any further increase of circulation and lift.

ROTATING REYNOLDS NUMBER. (Or

rotation Reynolds number.) A non-dimensional number arising in problems of a rotating viscous fluid. It may appear either as $\Omega h^2/\nu$, in which case it equals one-half the square root of the **Taylor number**, or as $\Omega r^2/\nu$. r is a suitable radius, h a representative depth, Ω the

absolute angular speed, and ν the kinematic viscosity.

ROTATION. Motion of a rigid body in which one or two points of the rigid body are kept fixed.

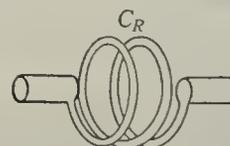
ROTATIONAL COMPLIANCE. Mechanical rotational potential energy is associated with the twisting of a spring or compliant element. Mechanical energy increases as the spring is twisted. It decreases as the spring is allowed to unwind. It is constant, and is stored when the spring remains immovably twisted. Rotational compliance is the mechanical element which opposes a change in the applied torque. Rotational compliance C_R , in radians per centimeter per dyne, is defined as

$$f_R = \frac{\phi}{C_R}$$

where ϕ is the angular displacement in radians, and f_R is the applied torque, in dyne centimeters.

This equation states that the rotational displacement of the rotational compliance is proportional to the rotational compliance and the applied force.

Rotational compliance in the mechanical rotational system is represented as a spring in the figure.



Graphical representation of the element, rotational compliance, C_R .

ROTATIONAL CONSTANTS OF A MOLECULE. The constants occurring in the expressions for the rotational energy of a molecule. For the rigid non-vibrating molecule the rotational constants are

$$A = \frac{h}{8\pi^2 c I_A}; \quad B = \frac{h}{8\pi^2 c I_B}; \quad C = \frac{h}{8\pi^2 c I_C}$$

where h stands for Planck's constant, c for the velocity of light, and I_A , I_B , and I_C are the principal moments of inertia of the molecule.

In the vibrating and rotating molecule the rotational constants vary with the vibrational quantum numbers v_i according to

$$A_{[v]} = A_e - \sum \alpha_i^A \left[v_i + \frac{d_i}{2} \right]$$

$$B_{[v]} = B_e - \sum \alpha_i^B \left[v_i + \frac{d_i}{2} \right]$$

$$C_{[v]} = C_e - \sum \alpha_i^C \left[v_i + \frac{d_i}{2} \right].$$

Here A_e , B_e , and C_e are the rotational constants for the (hypothetical) vibrationless state of the molecule, v_i the quantum numbers of the different normal vibrations, d_i the degree of degeneracy, and α_i^A , α_i^B , and α_i^C small constants.

In the non-rigid molecule the influence of centrifugal stretching is taken into account by the rotational constant $D \ll B$. For a diatomic molecule, to a first approximation,

$$D = \frac{4B^3}{\omega^2}$$

where ω is the vibrational frequency. In a polyatomic molecule, D depends in general on all the vibrational frequencies of the molecule.

ROTATIONAL EIGENFUNCTIONS. See **molecular eigenfunctions.**

ROTATIONAL ENERGY LEVELS OF A MOLECULE. (a) *Pure rotation (non-vibrating molecule).* The rotational energy levels of the *rigid diatomic* or *linear polyatomic molecule* are given by the expression

$$\frac{E_r}{hc} = F(J) = BJ(J+1)$$

where E_r is the rotational energy (in ergs) and $F(J)$ is the rotational term value (in cm^{-1}). The rotational constant B is given by

$$B = \frac{h}{8\pi^2 c I_B} = \frac{27.9830 \times 10^{-40}}{I_B}$$

where I_B is the moment of inertia of the molecule. J is the rotational quantum number corresponding to the angular momentum \mathbf{J} whose magnitude is

$$\frac{h}{2\pi} \sqrt{J(J+1)} \approx \frac{h}{2\pi} J.$$

For the *nonrigid diatomic* or *linear polyatomic molecule* we have

$$\frac{E_r}{hc} = F(J) = BJ(J+1) - DJ^2(J+1)^2 + \dots$$

where D is a rotational constant representing the influence of the centrifugal forces. In the case of a diatomic molecule of vibrational frequency ω (in cm^{-1}), the constant D , in a first approximation, is given by

$$D = \frac{4B^3}{\omega^2}.$$

In a polyatomic molecule, D depends in general on all the vibrational frequencies of the molecule.

The rotational energy levels of a *rigid symmetric top molecule* are given by the expression

$$\frac{E_r}{hc} = F(J,K) = BJ(J+1) + (A-B)K^2.$$

Here E_r is the rotational energy (in ergs), $F(J,K)$ the rotational term value (in cm^{-1}); A and B are rotational constants given by

$$B = \frac{h}{8\pi^2 c I_B}, \quad A = \frac{h}{8\pi^2 c I_A}$$

where I_A and I_B are the principal moments of inertia of the molecule and J and K are rotational quantum numbers. Here J corresponds to the total angular momentum \mathbf{J} , and K corresponds to the component of \mathbf{J} in the direction of the figure axis; therefore $J = K, K+1, K+2, \dots$. All rotational levels with $K > 0$ are doubly degenerate.

For the *nonrigid symmetric top molecule*, the energy formula is

$$\begin{aligned} \frac{E_r}{hc} &= F(J,K) \\ &= BJ(J+1) + (A-B)K^2 \\ &\quad - D_J J^2(J+1)^2 - D_{JK} J(J+1)K^2 \\ &\quad - D_K K^4 + \dots \end{aligned}$$

where D_J , D_{JK} , and D_K are rotational constants corresponding to D in the linear molecule.

The energy levels of a *spherical top molecule* are given by

$$\frac{E_r}{hc} = F(J) = BJ(J+1), \quad B = \frac{h}{8\pi^2 c I}$$

where I is the moment of inertia of the molecule.

The energy levels of the *rigid asymmetric top* are represented by the formula

$$F(J_\tau) = \frac{1}{2}(B + C)J(J + 1) + [A - \frac{1}{2}(B + C)]W_\tau.$$

Here

$$A = h/8\pi^2cI_A, \quad B = h/8\pi^2cI_B, \quad C = h/8\pi^2cI_C$$

where I_A , I_B , and I_C are the three principal moments of inertia of the molecule. The symbol τ numbers the $2J + 1$ levels of a given J in the order of their energy, i.e.,

$$\tau = -J, -J + 1, \dots, +J$$

and W_τ are the roots of algebraic equations containing A , B , and C . For the lowest values of J one has

$$J = 0:$$

$$W_0 = 0$$

$$J = 1:$$

$$W_\tau = 0$$

$$W_\tau^2 - 2W_\tau + (1 - b^2) = 0$$

$$J = 2:$$

$$W_\tau - 1 + 3b = 0$$

$$W_\tau - 1 - 3b = 0$$

$$W_\tau - 4 = 0$$

$$W_\tau^2 - 4W_\tau - 12b^2 = 0$$

$$J = 3:$$

$$W_\tau - 4 = 0$$

$$W_\tau^2 - 4W_\tau - 60b^2 = 0$$

$$W_\tau^2 - (10 - 6b)W_\tau + (9 - 54b - 15b^2) = 0$$

$$W_\tau^2 - (10 + 6b)W_\tau + (9 + 54b - 15b^2) = 0$$

$$J = 4:$$

$$W_\tau^2 - 10(1 - b)W_\tau + (9 - 90b - 63b^2) = 0$$

$$W_\tau^2 - 10(1 + b)W_\tau + (9 + 90b - 63b^2) = 0$$

$$W_\tau^2 - 20W_\tau + (64 - 28b^2) = 0$$

$$W_\tau^3 - 20W_\tau^2 + (64 - 208b^2)W_\tau + 2880b^2$$

$$= 0.$$

Here b stands for

$$b = \frac{C - B}{2[A - \frac{1}{2}(B + C)]}.$$

The average of the levels with a certain J follows accurately (neglecting centrifugal stretching) the formula for the simple rotator with an average rotational constant, that is,

$$\frac{\sum J F(J_\tau)}{2J + 1} = \frac{1}{3}(A + B + C)J(J + 1).$$

When two of the three principal moments of inertia are nearly equal, the formulas for the

symmetric top can be applied if the average of the two corresponding rotational constants (B and C or A and B) is used in place of B .

(For the effect of external fields on the rotational energy levels, see **Zeeman effect**, **Stark effect**; for the influence of the nuclear spin, see **hyperfine structure**.)

(b) *Interaction of rotation and vibration (vibrating and rotating molecule)*. The interaction of rotation and vibration causes the rotational energy of a vibrating molecule to be somewhat different from that of a nonvibrating molecule. One has for the term values of the rotating vibrator

$$T = G(v) + F_v(J)$$

where $G(v)$ is the vibrational term value (see **vibrational energy levels of a molecule**).

For a vibrating diatomic molecule

$$F_v(J) = B_v J(J + 1) - D_v J^2(J + 1)^2 + \dots$$

Here

$$B_v = B_e - \alpha_e(v + \frac{1}{2}) + \dots$$

and

$$D_v = D_e + \beta_e(v + \frac{1}{2}) + \dots$$

The constants B_e and D_e refer to the equilibrium position and are defined by formulas entirely similar to those previously given for B and D . The constants α_e and β_e are small compared to B_e and D_e , respectively, and are determined by the form of the potential function.

The rotational term values of a *vibrating linear polyatomic molecule* are given by the same formula as those of diatomic molecules except that B_v depends now on the vibrational quantum numbers of all the vibrations. We have

$$B_{[v]} = B_{v_1 v_2 v_3 \dots} = B_e - \sum \alpha_i \left(v_i + \frac{d_i}{2} \right)$$

where the α_i are small constants similar to α_e for diatomic molecules and where d_i is the degeneracy of the vibration i . The rotational constant $B_{000\dots}$, obtainable from the pure rotation spectrum, for the lowest vibrational level is given by

$$B_{[0]} = B_{000\dots} = B_e - \sum \alpha_i \frac{d_i}{2}.$$

Vibrational levels with $l = 1, 2, \dots$ (Π, Δ, \dots vibrational levels) are doubly degenerate. With increasing rotation a splitting of this degeneracy arises (l -type doubling). As a

result there are two rotational term series with slightly different rotational constants, B_v^c and B_v^d . The splitting is given by

$$\Delta\nu = q_i J(J+1) = (B_v^c - B_v^d)J(J+1).$$

The splitting constant q_i for a given perpendicular vibration ν_i is of the same order as α_i . The two levels of a given J have opposite parity (+, -).

As for linear molecules, the term values of a *vibrating symmetric top molecule* can be represented as the sum of vibrational and rotational term values

$$T = G(v_1, v_2, v_3, \dots) + F_{[v]}(J, K).$$

In the case of a nondegenerate vibrational level and neglecting the effect of centrifugal forces, the rotational term values are given by

$$F_{[v]}(J, K) = B_{[v]}J(J+1) + (A_{[v]} - B_{[v]})K^2$$

where

$$B_{[v]} = B_e - \sum \alpha_i^B \left(v_i + \frac{d_i}{2} \right) + \dots$$

$$A_{[v]} = A_e - \sum \alpha_i^A \left(v_i + \frac{d_i}{2} \right) + \dots$$

The α_i^B and α_i^A are constants similar to α_e of diatomic molecules and A_e and B_e are the rotational constants corresponding to the equilibrium position.

In a degenerate vibrational level the Coriolis interaction of the degenerate components causes an additional term

$$-2A_{[v]} \sum_i (\pm \zeta_i l_i) K$$

which has to be added to the previous expression for $F_{[v]}(J, K)$. Here ζ_i is a constant, $0 \leq \zeta_i \leq 1$, measuring the magnitude of the vibrational angular momentum of the degenerate vibration ν_i in units $h/2\pi$ and $l_i = v_i, v_i - 2, \dots, 1$ or 0 is the azimuthal quantum number of the degenerate vibration. For a state in which only one degenerate vibration is singly excited ($l_i = 1$) the additional term is

$$\mp 2A_{[v]} \zeta_i K$$

leading to an increasing splitting of the degeneracy with increasing K .

The individual ζ_i are complicated functions of the potential constants and other parameters of the molecule. But the sums of the ζ_i of all vibrations of a given species are indepen-

dent of the potential constants. For example, for axial XY_3 molecules (pyramidal or planar)

$$\zeta_3 + \zeta_4 = \frac{I_A}{2I_B} - 1 = \frac{B}{2A} - 1$$

for axial XYZ_3 molecules

$$\zeta_4 + \zeta_5 + \zeta_6 = \frac{I_A}{2I_B} = \frac{B}{2A}$$

for axial $WXYZ_3$ molecules

$$\zeta_5 + \zeta_6 + \zeta_7 + \zeta_8 = \frac{I_A}{2I_B} + 1 = \frac{B}{2A} + 1.$$

For X_2Y_6 molecules of point group D_{3h} or D_{3d}

$$\zeta_7 + \zeta_8 + \zeta_9 = 0, \quad \text{and}$$

$$\zeta_{10} + \zeta_{11} + \zeta_{12} = \frac{I_A}{2I_B} = \frac{B}{2A}.$$

The energy of a *vibrating spherical top* is the sum of the vibrational energy $G(v_1, v_2, v_3, \dots)$ and the rotational energy

$$F_{[v]}(J) = B_{[v]}J(J+1) + \dots$$

where

$$B_{[v]} = B_e - \sum \alpha_i^B \left(v_i + \frac{d_i}{2} \right) + \dots$$

For a molecule that is a spherical top on account of its symmetry (e.g., CH_4) doubly and triply degenerate vibrational levels occur. In the case of the latter (but not of the former) the Coriolis interaction produces a splitting into three sets of levels given by

$$F_{[v]}^+(J) = B_{[v]}J(J+1) + 2B_{[v]}\zeta_i(J+1)$$

$$F_{[v]}^0(J) = B_{[v]}J(J+1)$$

$$F_{[v]}^-(J) = B_{[v]}J(J+1) - 2B_{[v]}\zeta_i J$$

where ζ_i is a constant giving the vibrational angular momentum in units $h/2\pi$.

To a good approximation the rotational energy levels of a *vibrating asymmetric top molecule* are obtained from those of the non-vibrating asymmetric top molecule by substituting effective values of the various rotational constants corresponding to the vibrational level considered, that is

$$F_{[v]}(J_\tau) = \frac{1}{2}(B_{[v]} + C_{[v]})J(J+1) + [A_{[v]} - \frac{1}{2}(B_{[v]} + C_{[v]})]W_\tau^{[v]}$$

where

$$\begin{aligned} A_{[v]} &= A_e - \sum \alpha_i^A (v_i + \frac{1}{2}) \\ B_{[v]} &= B_e - \sum \alpha_i^B (v_i + \frac{1}{2}) \\ C_{[v]} &= C_e - \sum \alpha_i^C (v_i + \frac{1}{2}) \end{aligned}$$

and where the quantities $W_\tau^{[v]}$ are given by equations similar to those given above, except that the constants A, B, C are to be replaced by $A_{[v]}, B_{[v]},$ and $C_{[v]}$.

(c) *Internal rotation. Free rotation.* When one part of a symmetric top molecule can rotate freely relative to the other about the figure axis, the following term has to be added to the ordinary rotational energy $F(J, K)$

$$F_t(k_1, k) = \frac{A_1 A_2}{A} \left(k_1 - k \frac{A}{A_1} \right)^2$$

Here A_1 and A_2 are the rotational constants corresponding to the partial moments of inertia $I_A^{(1)}$ and $I_A^{(2)}$; $k (= \pm K)$ is the quantum number of the component of the total angular momentum \mathbf{J} about the top axis; k_1 is the quantum number of the angular momentum of part 1 [moment of inertia $I_A^{(1)}$] of the molecule and assumes the values

$$k_1 = 0, \pm 1, \pm 2, \dots$$

For molecules with $I_{A_1} = I_{A_2}$ or $\frac{A}{A_1} = \frac{1}{2}$ the term F_t simplifies to

$$\begin{aligned} F_t(k_1, k) &= A(2k_1 - k)^2 \\ &= A(k_1 - k_2)^2 = AK_i^2 \end{aligned}$$

where $K_i = |k_1 - k_2|$ is the quantum number of internal rotation.

Internal rotation. Hindered rotation. The limiting case of hindered rotation is that of torsional oscillation in a periodic potential field with n potential minima

$$V(\chi) = V \left(\chi \pm \frac{2\pi}{n} \right).$$

If a cosine form is assumed for the hindering potential

$$V = \frac{1}{2} V_0 (1 - \cos n\chi)$$

the energy levels in the neighborhood of the minima for large V_0 are those of a harmonic oscillator:

$$G(v_t) = \omega_t (v_t + \frac{1}{2})$$

where the torsional frequency ω_t is given by

$$\omega_t = n \sqrt{\frac{V_0 A_1 A_2}{A}}$$

or, for a molecule with two equal parts

$$\omega_t = 2n \sqrt{V_0 A}.$$

For small values of V_0 the vibrational motion of the molecule becomes a hindered rotation. The energy levels corresponding to this intermediate case can be found qualitatively by interpolation between those of the two limiting cases, free rotation and torsional oscillation.

(d) *Interaction of rotation and electronic motion.* (See **fine structure BI** (Multiplet splitting in molecular spectra) and **Λ -type doubling**.)

ROTATIONAL LEVEL. See **rotational energy levels of a molecule**.

ROTATIONAL PARTITION FUNCTION. The contribution to the total **partition function** of molecules which is associated with their rotational energy.

ROTATIONAL RESISTANCE, MECHANICAL. See **mechanical rotational resistance**.

ROTATIONAL SPECTRA IN NUCLEI. The motion of individual nucleons in the atomic nucleus are such as to exert a centrifugal pressure upon the surface of the nucleus as a whole. Under certain circumstances, particularly in regions of nuclei far removed from closed shells, this can give rise to a permanent distortion of the nucleus into an approximate ellipsoidal form. It is then found that the nucleus exhibits a series of energy levels resembling the rotational bands well known in molecular spectroscopy. That is, the energy levels can be approximated by the formula

$$E_{IK} = E_K + (\hbar^2/2\mathcal{I}_K)I(I + 1).$$

Here K is the component of angular momentum I along the symmetry axis of the nucleus, $2\pi\hbar$ is Planck's constant, and \mathcal{I}_K is an effective moment of inertia which, in general, depends upon the quantum number K which labels the whole band. For given K and parity, $I = K, K + 1, K + 2, \dots$, except when $K = 0$. In the latter case, $I = 0, 2, 4, \dots$ if the parity is even, or $I = 1, 3, 5, \dots$ if the parity is odd. The value of \mathcal{I}_K is approximately one-half the rigid moment of inertia of the nucleus as a

whole, showing that only a portion of the nucleus takes part in the rotational motion. The energies E_K , usually large compared to the spacings within a given band, may be regarded as the energy levels corresponding to excitation of the distorted nucleus in the absence of the rotational motion.

ROTATIONAL SUM RULES. (a) The average of all rotational levels with a certain rotational quantum number J of an asymmetric top molecule with rotational constants A , B , and C , follows the formula for the simple rotator with rotational constant $1/3(A + B + C)$, i.e.,

$$\frac{\sum_{\tau} F(J_{\tau})}{2J + 1} = 1/3(A + B + C)J(J + 1).$$

Other similar sum rules hold for certain sub-levels of a given J . (For more details see G. Herzberg, *Molecular Spectra and Molecular Structure, II. Infrared and Raman Spectra of Polyatomic Molecules*, D. Van Nostrand Co., Inc., Princeton, 1945, p. 49 ff.)

(b) For a given electronic transition in a molecule the sums of the line strengths of all the transitions from or to a given rotational level with rotational quantum number J are proportional to the statistical weight $(2J + 1)$ of that level.

ROTATIONAL WAVE. See *wave(s)*, *shear*.

ROTATION AXIS. A **symmetry element** possessed by certain crystals, whereby the crystal can be brought into a physically equivalent position by rotation about an axis which can be one-fold, two-fold, three-fold, four-fold or six-fold, according to whether the crystal can be brought into self-coincidence by the operations of rotation through 360° , 180° , 120° , 90° , or 60° about the rotation axis.

ROTATION CAPACITY. The ability of a continuous beam or frame to reach its plastic **limit load** may be restricted if the members do not have sufficient capacity to rotate and maintain the **limit moment** when it is reached. Attention to design details such as stiffness and lateral supports obviates this difficulty.

ROTATION GROUP. The **group** of all real unitary matrices with **determinant** equal to $+1$. (See **Lie group**.)

ROTATION, HINDERED. See **hindered rotation**.

ROTATION-INVERSION AXIS. A **symmetry element** possessed by certain crystals by which the crystal is brought into self-coincidence by a combined rotation about the axis and **inversion**.

ROTATION, MOLECULAR. See **molecular rotation**.

ROTATION-REFLECTION AXIS. A **symmetry element** possessed by certain crystals, whereby the crystal is brought into self-coincidence by combined rotation and reflection in a plane perpendicular to the axis of rotation. Rotation-reflection axes may be one-fold, two-fold, three-fold, four-fold, or six-fold, according to whether the rotation which, with the reflection, brings the crystal into self-coincidence, is through an angle of 360° , 180° , 120° , 90° or 60° .

ROTATION, SPECIFIC. The angular rotation of the plane of polarization as it passes through an optically-active material, divided by the length of the path and the density of the material. (See **molecular rotation**.)

ROTATION SPECTRA OF MOLECULES.

The pure rotation spectrum of a molecule corresponds to transitions between rotational levels belonging to the same electronic and vibrational state of the molecule. Such transitions can give rise to a spectrum in the infrared or microwave region only in molecules with a permanent dipole moment, that is, only in molecules without a center of symmetry. They can give rise to a Raman spectrum only if the polarizability of the molecule changes during the transition, independent of whether or not there is a center of symmetry.

(a) *Diatomic and linear polyatomic molecules.* For the infrared spectrum the selection rules are

$$+ \longleftrightarrow -, \quad + \leftarrow | \rightarrow +, \quad - \leftarrow | \rightarrow -$$

that is, positive levels combine only with negative levels, and

$$\Delta J = J' - J'' = 1$$

where J' and J'' are the rotational quantum numbers of the upper and lower states, respectively. Accordingly, the wave-numbers of the pure rotation spectrum are given by the formula

$$\nu = 2B(J + 1) - 4D(J + 1)^3 + \dots$$

where $D \ll B$ and J stands for J'' .

For the Raman spectrum the selection rules referring to the symmetry of the rotational states are

$$+ \longleftrightarrow +, \quad - \longleftrightarrow -, \quad + \longleftrightarrow -$$

and

$$s \longleftrightarrow s, \quad a \longleftrightarrow a, \quad s \longleftrightarrow a$$

that is, positive levels combine only with positive, negative only with negative, symmetric only with symmetric, and antisymmetric only with antisymmetric levels. The selection rule for the rotational quantum number is

$$\Delta J = 0, \pm 2$$

that is, besides the undisplaced line ($\Delta J = 0$) one observes two lines both with $\Delta J = J' - J'' = \pm 2$, one with the lower state as the initial state (Stokes line) and one with the upper state as the initial state (anti-Stokes line).

The wave number shifts are given by the formula

$$|\Delta\nu| = (4B - 6D)(J + \frac{3}{2}) - 8D(J + \frac{3}{2})^3$$

or, since always $D \ll B$, to a very good approximation

$$|\Delta\nu| = 4B(J + \frac{3}{2})$$

where J , as always, stands for J'' , the rotational quantum number of the lower state involved.

For molecules with a center of symmetry, corresponding to the alternation of statistical weights for the symmetric and antisymmetric rotational levels, an alternation of intensities will occur. If the spins of all nuclei with the possible exception of the one at the center are zero, alternate lines will be missing.

(b) *Symmetric top molecules.* A pure rotation spectrum in the far infrared and microwave region can occur only if the molecule has a permanent dipole moment. For the accidental symmetric top the selection rules are

$$\Delta K = 0, \pm 1; \quad \Delta J = 0, \pm 1;$$

$$+ \longleftrightarrow -, \quad + \longleftrightarrow +, \quad - \longleftrightarrow -.$$

For a molecule which is a symmetric top because of its symmetry the same selection rules hold, but $\Delta K = \pm 1$ is excluded. In addition, only states having the same species of the rotational eigenfunction combine with one another (e.g., $A \longleftrightarrow A$, $E \longleftrightarrow E$, $A \longleftrightarrow E$ for molecules with a threefold axis of symmetry).

The wave-numbers of the pure rotation lines

when the molecule has an axis of symmetry are given in a first approximation by the formula

$$\nu = 2B(J + 1)$$

or, if centrifugal stretching is taken into account,

$$\nu = 2B(J + 1) - 2D_{KJ}K^2(J + 1) - 4D_J(J + 1)^3.$$

(For definition of the constants B , D_{KJ} , and D_J , see **rotational energy levels of a molecule.**)

For the rotational Raman spectrum, in the case of the accidental symmetric top the selection rules are

$$\Delta J = 0, \pm 1, \pm 2; \quad \Delta K = 0, \pm 1, \pm 2$$

and

$$+ \longleftrightarrow +, \quad - \longleftrightarrow -, \quad + \longleftrightarrow -.$$

If the molecule is a symmetric top because of its symmetry, the same selection rules apply except that transitions with $\Delta K = \pm 1, \pm 2$ are no longer possible and, those with $\Delta J = \pm 1$ occur only for $K \neq 0$. In this case the Raman lines form two branches on either side of the undisplaced line with the displacements

$$|\Delta\nu| = F(J + 2, K) - F(J, K) = 6B + 4BJ,$$

$$J = 0, 1, \dots \text{ (S-branches)}$$

and

$$|\Delta\nu| = F(J + 1, K) - F(J, K) = 2B + 2BJ,$$

$$J = 1, 2, \dots \text{ (R-branches)}$$

neglecting centrifugal stretching terms.

(c) *Spherical top molecules.* Molecules which are spherical tops on account of their symmetry have no pure rotation spectrum in the infrared because they have no permanent dipole moment. Accidental spherical top molecules may have a permanent dipole moment and, consequently, a pure rotation spectrum. The selection rule is

$$\Delta J = 0, \pm 1$$

leading to the same wave-number formula as for linear molecules.

Molecules which are spherical tops on account of their symmetry have no pure rotational Raman spectrum, since the polarizability does not change during the rotation. In accidental spherical top molecules a rotational Raman spectrum may occur. The selection rule is

$$\Delta J = 0, \pm 1, \pm 2.$$

The Raman displacements are the same as those of a symmetric top molecule.

(d) *Asymmetric top molecules.* Asymmetric top molecules in general have a permanent dipole moment, and therefore have a pure rotation spectrum in the far infrared or microwave region. The selection rule for J is

$$\Delta J = 0, \pm 1.$$

If the molecule has no symmetry the only further restriction is that levels of the same species do not combine with each other

$$\begin{aligned} ++ \leftarrow | \rightarrow ++, \quad +- \leftarrow | \rightarrow +-, \\ -+ \leftarrow | \rightarrow -+, \quad -- \leftarrow | \rightarrow --. \end{aligned}$$

If the molecule has an axis of symmetry, only those rotational levels can combine with one another whose eigenfunctions have the same behavior with respect to a rotation by 180° about this axis, and opposite behavior with respect to similar rotations about the other two axes. Thus if the dipole moment lies in the axis of least moment of inertia (a axis) only the transitions

$$++ \longleftrightarrow -+ \quad \text{and} \quad +- \longleftrightarrow --$$

can take place. If the dipole moment lies in the axis of intermediate moment of inertia (b axis) only the transitions

$$++ \longleftrightarrow -- \quad \text{and} \quad +- \longleftrightarrow -+$$

can take place. If the dipole moment lies in the axis of largest moment of inertia (c axis), only the transitions

$$++ \longleftrightarrow +- \quad \text{and} \quad -+ \longleftrightarrow --$$

can take place.

The polarizability of an asymmetric top molecule in general changes during the rotation, and therefore as a rule a rotational Raman spectrum will occur. The selection rule for J is

$$\Delta J = 0, \pm 1, \pm 2.$$

If the molecule has no symmetry, transitions between levels of any of the symmetry types ($++$, $+-$, $-+$, $--$) can occur. If the molecule has at least one twofold axis of symmetry, only levels of the same species can combine with each other, that is

$$\begin{aligned} ++ \longleftrightarrow ++, \quad +- \longleftrightarrow +-, \\ -+ \longleftrightarrow -+, \quad -- \longleftrightarrow --. \end{aligned}$$

(e) *Molecules with internal rotation.* For symmetrical molecules there is no pure rota-

tion spectrum corresponding to free internal rotation. In slightly asymmetric molecules the internal rotation is infrared active. For the pure internal rotation spectrum the selection rules are

$$\begin{aligned} \Delta J = 0, \pm 1, \quad \Delta K = \pm 1, \\ \Delta K_1 = \pm 1, \quad \Delta K_2 = 0 \end{aligned}$$

where $K_1 = |k_1|$, and $K_2 = |k_2|$. Therefore the Q "lines" of the free internal rotation spectrum form the double series

$$\nu = A_1 - B \mp 2BK \pm 2A_1K_1$$

where the upper signs hold for positive ΔK and ΔK_1 , the lower signs for negative ΔK and ΔK_1 .

ROTATION-VIBRATION SPECTRA OF MOLECULES. Rotation-vibration spectra correspond to transitions between the rotational levels of two different vibrational states belonging to the same electronic energy level of a molecule.

(a) *Diatomic molecules.* The same selection rules apply as for the pure rotation and the pure vibration spectrum separately. Therefore the vibrational quantum number can change by

$$\Delta v = \pm 1, \pm 2, \dots$$

with $\Delta v = \pm 1$ giving by far the strongest transitions. The selection rules for the rotational quantum number, assuming that there is no electronic angular momentum about the internuclear axis, are, in the case of infrared transitions

$$\Delta J = J' - J'' = \pm 1$$

and in the case of Raman transitions

$$\Delta J = J' - J'' = 0, \pm 2.$$

Hence in the infrared a rotation-vibration band consists of two branches, an R branch ($\Delta J = +1$) and a P branch ($\Delta J = -1$) which are given by (neglecting small terms in D' and D'')

$$\begin{aligned} \nu_R &= \nu_0 + 2B'_v + (3B'_v - B''_v)J \\ &\quad + (B'_v - B''_v)J^2 \quad (J = 0, 1, \dots) \\ \nu_P &= \nu_0 - (B'_v + B''_v)J \\ &\quad + (B'_v - B''_v)J^2 \quad (J = 1, 2, \dots). \end{aligned}$$

Here ν_0 is the vibrational energy difference between the two states (band origin); J is the rotational quantum number J'' of the lower state.

In the Raman effect a rotation-vibration band consists of three branches, an S branch ($\Delta J = +2$), and O branch ($\Delta J = -2$), and a Q branch ($\Delta J = 0$). (Formulas for these branches may be found in G. Herzberg, *Molecular Spectra and Molecular Structure, I. Spectra of Diatomic Molecules*, 2d Edition, D. Van Nostrand Co., Inc., Princeton, 1950.)

(b) *Linear polyatomic molecules.* The selection rules for rotation-vibration spectra of linear polyatomic molecules are the same as for diatomic molecules if the quantum number l of the vibrational angular momentum is zero in both the upper and lower states, i.e., if $l' = l'' = 0$. In this case the same two branches occur.

If l' or l'' or both are different from zero, in addition to the transitions occurring for diatomic molecules (see above) in the infrared, transitions with $\Delta J = 0$ occur; in the Raman effect, transitions with $\Delta J = \pm 1$ occur. That is, the selection rules are

$$\Delta J = 0, \pm 1 \quad (\text{infrared})$$

$$\Delta J = 0, \pm 1, \pm 2 \quad (\text{Raman effect}).$$

At the same time the symmetry selection rules

$$+ \longleftrightarrow -, \quad s \longleftrightarrow a \quad (\text{infrared})$$

$$+ \longleftrightarrow +, \quad - \longleftrightarrow -, \quad s \longleftrightarrow a$$

(Raman effect).

must be obeyed.

The additional possibility $\Delta J = 0$ in the infrared gives rise to a Q branch whose formula is

$$\nu_Q = \nu_0 + (B' - B'')J + (B' - B'')J^2.$$

In the Raman spectrum in such cases P and R branches in addition to the S , O , and Q branches can occur.

According to the preceding selection rules, when l_i is different from zero a transition between the two components of an l -type doublet can occur. Such transitions occur in the microwave region and are represented by the formula

$$\nu = q_i J(J + 1).$$

(c) *Symmetric top molecules.* The selection rules for the vibrational quantum numbers for the rotation-vibration spectrum are the same as for the pure vibration spectrum. If the molecule is a symmetric top on account of its symmetry, the (vibrational) transition moment can only be either parallel or perpendicular to the figure axis. For an accidental symmetric

top any orientation with regard to the figure axis is possible.

If the transition is parallel to the figure axis (\parallel band), the selection rules for the rotational quantum numbers are

$$\Delta K = 0,$$

$$\Delta J = 0, \pm 1 \quad (\Delta J = 0 \text{ forbidden for } K = 0)$$

and if the transition moment is perpendicular to the figure axis (\perp band)

$$\Delta K = \pm 1, \quad \Delta J = 0, \pm 1.$$

If the transition moment has a general direction with respect to the figure axis, changes of the rotational quantum numbers allowed by either set of selection rules may occur, i.e., the resulting band has both a \parallel and a \perp component (hybrid band).

Both \parallel and \perp bands consist of a number of subbands corresponding to the different values of K . Each subband consists of a P , a Q and an R branch corresponding to $\Delta J = -1, 0$, and $+1$, respectively, similar to the bands of linear molecules.

The zero lines of the subbands of a \parallel band or of the \parallel component of a hybrid band are given by

$$\nu_0^{\text{sub}} = \nu_0 + [(A'_{[v]} - A''_{[v]}) - (B'_{[v]} - B''_{[v]})]K^2$$

those of a \perp band or of the \perp component of a hybrid band are given by

$$\nu_0^{\text{sub}} = \nu_0 + (A'_{[v]} - B'_{[v]}) \pm 2(A'_{[v]} - B'_{[v]})K + [(A'_{[v]} - B'_{[v]}) - (A''_{[v]} - B''_{[v]})]K^2.$$

Here it is assumed that both states involved are nondegenerate or, if degenerate, of such a nature that the effect of Coriolis forces can be neglected. If this is not the case the term $-2A_{[v]}\Sigma(\pm\zeta_i l_i)K$ has to be added to the energy formula, and the subband formulas are correspondingly changed. For example, if the upper state is degenerate with $\zeta_i \neq 0$, $l_i = 1$, and the lower state nondegenerate, the subbands of the resulting \perp band are given by

$$\nu_0^{\text{sub}} = \nu_0 + [A'_{[v]}(1 - 2\zeta_i) - B'_{[v]}] \pm 2[A'_{[v]}(1 - \zeta_i) - B'_{[v]}]K + [(A'_{[v]} - B'_{[v]}) - (A''_v - B''_v)]K^2$$

where the upper sign holds for $\Delta K = +1$ and the lower for $\Delta K = -1$. Neglecting the

dependence of A and B on the v_i , the spacing of the subbands is $2[A(1 - \zeta_i) - B]$ instead of $2(A - B)$ for a nondegenerate upper state.

The intensities of the lines in absorption are given by the expression

$$I(J, K) = CA_{KJ} \nu g_{KJ} e^{-F(K, J)hc/kT}$$

where the g_{KJ} are statistical weight factors and the A_{KJ} intensity factors given by

$$\begin{aligned} \Delta J = +1: A_{KJ} &= \frac{(J+1)^2 - K^2}{(J+1)(2J+1)} \\ \Delta J = 0: A_{KJ} &= \frac{K^2}{J(J+1)} \\ \Delta J = -1: A_{KJ} &= \frac{J^2 - K^2}{J(2J+1)} \quad (\Delta K = 0) \\ \Delta J = +1: A_{KJ} &= \frac{(J+2 \pm K)(J+1 \pm K)}{(J+1)(2J+1)} \\ \Delta J = 0: A_{KJ} &= \frac{(J+1 \pm K)(J \mp K)}{J(J+1)} \\ \Delta J = -1: A_{KJ} &= \frac{(J-1 \mp K)(J \mp K)}{J(2J+1)} \quad (\Delta K = \pm 1). \end{aligned}$$

Here K and J refer to the rotational quantum numbers of the lower state. For $K = 0$, $\Delta K = +1$ the values given by the formulas have to be multiplied by 2.

The vibrational selection rules for the Raman effect are also the same as for the pure vibration spectrum. In the most general case of an accidental symmetric top with arbitrary orientation of the polarizability ellipsoid with respect to the momental ellipsoid the selection rules for the rotational quantum numbers are

$$\begin{aligned} \Delta K &= 0, \pm 1, \pm 2; \\ \Delta J &= 0, \pm 1, \pm 2 \quad (J' + J'' \geq 2). \end{aligned}$$

If the molecule has symmetry and if therefore the figure axis coincides with one of the symmetry axes, only certain components of the matrix elements of the polarizability α are different from zero and only certain of the above transitions can occur.

The inversion doubling which occurs for all nonplanar molecules is usually negligibly small. But for molecules like NH_3 for which the two configurations obtained by inversion are separated by only a comparatively small po-

tential barrier, an appreciable doubling arises. The rotational constants in the two component levels are slightly different, that is, one has

$$\begin{aligned} F_{[v]}^s(J, K) &= B_{[v]}^s J(J+1) \\ &\quad + (A_{[v]}^s - B_{[v]}^s) K^2 + \dots \\ F_{[v]}^a(J, K) &= B_{[v]}^a J(J+1) \\ &\quad + (A_{[v]}^a - B_{[v]}^a) K^2 + \dots \end{aligned}$$

where the superscripts s and a refer to the levels whose vibrational eigenfunctions are symmetric and antisymmetric with respect to the inversion. Transitions from one set of levels to the other occur in the microwave region, the selection rules being

$$\Delta J = 0, \quad \Delta K = 0, \quad K \neq 0.$$

The resulting lines are therefore given by the formula

$$\begin{aligned} \nu &= \nu_0 + (B_{[v]}^s - B_{[v]}^a) J(J+1) \\ &\quad + [(A_{[v]}^s - A_{[v]}^a) - (B_{[v]}^s - B_{[v]}^a)] K^2 + \dots \end{aligned}$$

where ν_0 is the inversion splitting for zero rotation. Slight deviations of the observed microwave spectra from this formula can be accounted for by adding higher (quartic) terms to it.

(d) *Spherical top molecules.* For the accidental spherical top the selection rules for J are the same as for the symmetric top, both in the infrared and the Raman effect. For a molecule that is a spherical top on account of its symmetry, additional rules apply.

In the infrared the most common vibrational transitions are $F_2 - A_1$. Of the three components of the upper state the F^+ levels combine with the lower state only with $\Delta J = -1$, the F_0 levels only with $\Delta J = 0$ and the F^- levels only with $\Delta J = +1$. Therefore $F_2 - A_1$ bands have only three branches represented by the formulas

$$\begin{aligned} (RJ) &= \nu_0 + 2B'_{[v]} - 2B'_{[v]}\zeta_i + (3B'_{[v]} \\ &\quad - B''_{[v]} - 2B'_{[v]}\zeta_i)J \\ &\quad + (B'_{[v]} - B''_{[v]})J^2 \\ Q(J) &= \nu_0 + (B'_{[v]} - B''_{[v]})J \\ &\quad + (B'_{[v]} - B''_{[v]})J^2 \\ P(J) &= \nu_0 - (B'_{[v]} + B''_{[v]} - 2B'_{[v]}\zeta_i)J \\ &\quad + (B'_{[v]} - B''_{[v]})J^2. \end{aligned}$$

In the Raman effect, for $A_1 - A_1$ vibrational transitions only $\Delta J = 0$ occurs, that is, only a

Q branch. But in $F_2 - A_1$ vibrational transitions all five ΔJ values are possible for each of the three sublevels of the F_2 state; the resulting bands therefore consist of fifteen branches.

(e) *Asymmetric top molecules.* The selection rules for the rotation-vibration spectra of asymmetric top molecules are the same as those for the vibration and the rotation spectra separately, except that it is now the direction of the *change* of dipole moment and *change* of polarizability that determines the infrared and Raman transitions respectively. The fine structure of the bands is always very complicated and cannot be represented by simple formulas, except if the molecule approaches the limiting case of a symmetric top ($A \simeq B$ or $C \simeq B$). (For more details see G. Herzberg, *l.c.*)

(f) *Molecules with internal rotation.* For symmetrical molecules the selection rules for the quantum number K_i of the internal rotation are $\Delta K_i = 0$ for $\Delta K = 0$ and $\Delta K_i = \pm 1$ for $\Delta K = \pm 1$. As a consequence the \parallel bands of a symmetric top molecule are not affected by the presence of internal rotation, while in the \perp bands each of the line-like Q branches is split into a number of nearly equidistant "lines" of spacing $2B$. For the rotation-vibration spectrum of asymmetrical molecules we have in the case of \parallel bands ($\Delta K = 0$) the selection rule

$$\Delta K_1 = 0, \quad \Delta K_2 = 0$$

and in the case of \perp bands ($\Delta K = \pm 1$)

$$\Delta K_1 = \pm 1, \quad \Delta K_2 = 0 \quad \text{or}$$

$$\Delta K_1 = 0, \quad \Delta K_2 = \pm 1$$

depending on whether the dipole moment of the vibrational transition is in part 1 or part 2 of the molecule. The structure of the \parallel bands is therefore not affected by the presence of internal rotation, while in the \perp bands each subband corresponding to a given K and ΔK is resolved into a number of sub-subbands corresponding to the different K_1 values and $\Delta K_1 = \pm 1$ or to the different K_2 values and $\Delta K_2 = \pm 1$, depending on whether the oscillating dipole moment is in part 1 or part 2. For $\Delta K_1 = \pm 1$ the spacing of the sub-subbands is $2A_1$, for $\Delta K_2 = \pm 1$ it is $2A_2$.

ROTATOR, PARTITION FUNCTION OF. A function given by the equation.

$$Z = \sum_j (2j + 1) e^{-\beta \Theta j(j+1)}$$

where Z is the partition function, $\beta = 1/kT$ (k is Boltzmann's constant and T the absolute temperature), and Θ is the so-called "rotational temperature" (in energy units), given by the relation

$$\Theta = \frac{kh^2}{8\pi^2 I}$$

with h being Planck's constant, and I being the moment of inertia of the rotator.

ROTATORY DISPERSION. An optical **dispersion**, due to the polarization plane of polarized light of different wavelengths being rotated at different rates by optically active substances.

ROTATORY POWER. The angle of rotation produced in a beam of polarized light by an optically active medium per unit distance traversed.

ROTOR. A region in a fluid stream (usually in two-dimensional steady motion) in part of which the flow is in the opposite direction to the main stream. Rotors are characteristic of large amplitude disturbances in horizontal streams of stably stratified fluid such as in an airstream flowing over steep mountains. If clouds are present they may appear to rotate. Rotors most commonly occur in **lee waves**.

If ζ is the displacement at a point of the streamline from its level in the undisturbed horizontal flow, the condition that a rotor shall occur is that $\partial\zeta/\partial z > 1$, where z is height. This condition is most easily satisfied in airstreams with large static stability.

ROUGHNESS, EFFECT ON SKIN FRICTION. With a **laminar boundary layer**, roughness of the surface has no effect on the skin friction provided it is not large enough to cause transition to turbulent flow.

With a **turbulent boundary layer** roughness has no effect on the skin friction provided the roughness height ϵ satisfies approximately the condition

$$\frac{\epsilon v_*}{\nu} < 5, \quad (1)$$

where ν is the kinematic viscosity and ρv_*^2 is the shear stress at the wall. If Equation (1) is satisfied the protrusions are all inside the laminar sub-layer and the surface is then said to be hydraulically smooth.

When Equation (1) is not satisfied the skin

friction is increased by the roughness. The type of roughness that has been most fully investigated is that formed by closely spaced grains of sand of uniform size. With this type of roughness a single length ϵ , defining the size of the sand grains, is sufficient to specify the roughness completely.

With sand roughness two regimes may be distinguished, apart from that given by Equation (1). For $5 < \frac{\epsilon v_*}{\nu} < 70$, the skin friction

coefficient depends not only on the Reynolds number (as for a smooth surface) but also on the relative roughness ϵ/δ , where δ is the boundary-layer thickness. This state is known as the *transition régime*, because it is intermediate between the hydraulically smooth surface already mentioned and the completely rough régime considered below.

The *completely rough régime* occurs for $\frac{\epsilon v_*}{\nu} > 70$, and in this condition the skin-friction coefficient is independent of Reynolds number and depends only on ϵ/δ .

With increasing distance downstream along the surface of a body, the boundary-layer thickness δ increases, so that if the roughness of the surface is uniform the relative roughness ϵ/δ decreases. At the same time v_* decreases with increasing distance downstream, so that the surface may be in the completely rough condition near the upstream end and then change to the transition régime further downstream. Eventually, if the body is sufficiently long, the surface may even become hydraulically smooth.

Attempts have been made to define, for other forms of rough surface, an equivalent sand roughness which would have the same skin friction as the given surface at any Reynolds number. This procedure is useful in some cases, but it is not always satisfactory because some forms of rough surface do not behave even qualitatively in the same way as sand roughness. In particular, with some forms of rough surface it is apparently not possible to obtain "completely rough" conditions in which the skin friction coefficient depends only on ϵ/δ .

For flow in a pipe the effects of roughness are the same as for a boundary layer. The dimensionless *roughness height* is ϵ/r , where r is the radius of the pipe, and of course with uniform roughness this does not change with distance downstream.

ROUGHNESS, EFFECT ON TRANSITION OF LAMINAR BOUNDARY LAYER. Transition from laminar to turbulent flow in a boundary layer may occur further upstream if the solid boundary is roughened. Although the detailed mechanism of the effect of roughness on transition is not yet fully understood, it is clear that roughness on the surface introduces disturbances into the boundary layer, and under certain conditions these may be amplified and cause the flow to become turbulent. For a roughness element of height ϵ , a *roughness Reynolds number* may be defined as

$$R_\epsilon = \epsilon U_\epsilon / \nu,$$

where U_ϵ is the velocity in the undisturbed boundary layer at $y = \epsilon$. For values of R_ϵ less than some critical value $R_{\epsilon crit.}$, the roughness has no effect at all on the position of transition. The value of $R_{\epsilon crit.}$ depends on the shape and arrangement of the roughness elements, but for most forms of roughness it lies in the range 100 to 300.

For values of R_ϵ greater than the critical value the roughness causes transition to occur further upstream than on a smooth surface. For sufficiently large values of R_ϵ , usually about 2 or 3 times $R_{\epsilon crit.}$, transition to turbulent flow occurs almost immediately after the roughness.

ROUGHNESS HEIGHT OF PIPE. See roughness, effect on skin friction.

ROUGH RÉGIME. See roughness, effect on skin friction.

ROUNDING. The process of dropping the less significant digits in the representation of a number, generally with the rule that a unit should be added in the last place whenever the first discarded digit is 5 or greater in decimal representation, 1 in binary. When this rule is not adopted the process is called **truncation**.

ROUNDING ERROR (ROUND OFF). The error which results from **rounding**.

ROUSSEAU DIAGRAM. A graphical construction by means of which the **luminous flux** and the mean spherical luminous intensity of a (point-) source with an axis of symmetry may be calculated from a knowledge of the luminous intensity in a certain number of directions in a meridian plane of the source. The coordinates in one direction are proportional to luminous intensities, in the other direction to solid angles.

ROUTH CRITERION. See **Routh rule of inertia**.

ROUTH RULE OF INERTIA. The moment of inertia about an axis of symmetry is given by

$$I = M \frac{a^2 + b^2}{n}$$

where M is the total mass, a and b are the two semi-axes perpendicular to each other and to the symmetry axis and n is 3 for a rectangular parallelepiped, 4 for an elliptic cylinder, and 5 for an ellipsoid.

This rule is applied as a criterion of the **stability** (of a system).

ROUTINE. A sequence of operations carried out by a **stored-program computer** as specified by a **code** for the purpose of performing a mathematical or logical computation. (Cf. **program**.)

RUBBERY STATE. A state of **polymers** in the range of temperature and time at which **viscoelastic deformation** is dominant.

RULED SURFACE. A surface which can be generated by the motion of a straight line. The straight lines lying in the surface are the *generators of the surface*. The point into which the common perpendicular to two neighboring generators degenerates as these are brought into coincidence is the *central point* of the generator. The locus of the central points of all the generators of the surface is the *line of striction* of the surface. It is, of course, perpendicular to every generator of the surface.

RULED SURFACE, DIRECTRIX OF. See **directrix**.

RULE, FLOW. See **flow rule**.

RUNGE-KUTTA METHOD. A method for the numerical solution of an ordinary differential equation

$$y' = f(x, y),$$

requiring the sequential computation of the following quantities:

$$f_\nu = f(x_\nu, y_\nu),$$

$$k_1 = hf_\nu, \quad k_2 = hf(x_\nu + h/2, y_\nu + k_1/2),$$

$$k_3 = hf(x_\nu + h/2, y_\nu + k_2/2),$$

$$k_4 = hf(x_{\nu+1}, y_\nu + k_3),$$

$$y_{\nu+1} = y_\nu + (k_1 + 2k_2 + 2k_3 + k_4)/6.$$

The formulas have an obvious interpretation when y and f are vectors, hence for a system of equations. The method has the advantage of not requiring special measures at the outset as do the methods of Adams, Milne, and others based upon straight interpolation. It has the disadvantage of being slightly more complicated. Analogous simpler schemes, some of lower and some of higher accuracy, are available. (See Lothar Collatz, *Numerische Behandlung von Differentialgleichungen*, 2nd edition, Springer, 1955.) (In this class, see the **Euler method**.)

RUNS. In a sequence of observations of an attribute, the successive occurrence of members of the same type is called a run. If the observations are values of a continuous variable the expression "run" is often applied to a set which are monotonic.

RUPTURE LINE THEORY (JOHANSEN THEORY). An upper bound technique for determining the **plastic limit load** for plates and shells. The **kinematically admissible** displacement pattern is chosen as a pattern of yield line hinges separating rigid body sections of plate or shell, and is analogous to the yield or **plastic hinge mechanism method** for frames. Although the minimum upper bound which is obtained is normally a very good answer, it is not always so. A continuous distribution of plastic deformation may have to be considered.

RUPTURE, MODULUS OF. See **modulus of rupture**.

RUSSELL ANGLES. A series of angles which define the center lines of zones of equal area on a sphere, so that the determination of total luminous flux from a curve of intensity distribution is reduced to the calculation of the arithmetic mean of values at these angles.

RUSSELL-SAUNDERS COUPLING. (Also called **(L,S) coupling** or **spin-orbit coupling**.) Theoretical limiting case for the coupling of the spins s_i and the orbital angular momenta l_i of the individual electrons in an atom (or molecule): strong coupling between the individual s_i , adding up (in various possible ways, see **angular momentum vectors, quantum theoretical addition of**) to a resultant spin S of magnitude $S(h/2\pi)$; strong coupling between the individual l_i , adding up to a resultant electronic orbital angular momentum L of magnitude $L(h/2\pi)$; weak coupling between the re-

sultant angular momenta \mathbf{S} and \mathbf{L} , adding up to a total electronic angular momentum \mathbf{J} of magnitude $J(h/2\pi)$. Written symbolically:

$$(\mathbf{s}_1, \mathbf{s}_2, \dots)(\mathbf{l}_1, \mathbf{l}_2, \dots) = (\mathbf{S}, \mathbf{L}) = \mathbf{J}.$$

Characteristic for Russell-Saunders coupling is the occurrence of narrowly spaced groups of energy levels with the same values of S and L , but different values of J (**multiplets**). Multiplets with the same value of L but different values of S (different **multiplicity**) have considerably different energy. (Actually the observed energy difference between corresponding terms of different multiplicity is not due to strong magnetic interaction of the respective electron spins s_i , but to Coulomb interaction of the electrons and to the Heisenberg resonance phenomenon. For atoms of not too high **atomic number**, Russell-Saunders coupling constitutes a good approximation to the actual coupling conditions.

RUTHERFORD. (1) A unit of radioactivity, symbol rd, equal to 10^6 disintegrations per sec. (2) A quantity of a nuclide having an activity of 1 rutherford.

RYDBERG CONSTANT. The Rydberg constant for an atom of nuclear mass M is given by

$$R = \frac{2\pi^2\mu e^4}{ch^3}.$$

Here h stands for Planck's constant, e , for the electronic charge, c , for the velocity of light and

$$\mu = \frac{Mm}{M+m}$$

for the reduced mass of the electron, with M and m the nuclear and electronic masses respectively.

The Rydberg constant for an atom of infinite nuclear mass is given by

$$R_\infty = \frac{2\pi^2me^4}{h^3c}.$$

Therefore

$$R = R_\infty \frac{M}{m+M}.$$

RYDBERG CORRECTION. See Rydberg equation.

RYDBERG EQUATION. The individual series in the spectra of most atoms can be represented by the Rydberg equation

$$\nu_n = \nu_\infty - \frac{R}{(n+\mu)^2}$$

where ν_n is the wave number of a line in the series, ν_∞ the wave number of the series limit, R is the Rydberg constant, n is an integer, and $\mu < 1$ a constant, the so-called *Rydberg correction*.

In its more general form the Rydberg equation is given by

$$\nu_n = \frac{R}{(n_2 + \mu_2)^2} - \frac{R}{(n_1 + \mu_1)^2}$$

(see also **combination principle**).

For hydrogen the Rydberg corrections are equal to zero, so that the Rydberg equation in its general form is reduced to

$$\nu_n = \frac{R}{n_2^2} - \frac{R}{n_1^2} = R \left[\frac{1}{n_2^2} - \frac{1}{n_1^2} \right]$$

or, for an individual series, e.g., the Balmer series

$$\nu_n = R \left[\frac{1}{2^2} - \frac{1}{n_1^2} \right].$$

In the form

$$\nu_n = RZ^2 \left[\frac{1}{n_2^2} - \frac{1}{n_1^2} \right],$$

with Z standing for the atomic number, the above formula applies to hydrogen as well as the hydrogen-like ions He^+ , Li^{++} , \dots .

RYDBERG FORMULA. See Rydberg equation.

RYDBERG-RITZ COMBINATION PRINCIPLE. See **combination principle**.

RYDBERG SERIES. A series of energy levels in an atom or molecule in which one electron is excited to **orbitals** of increasing principal quantum number n . The term values of a Rydberg series are given by

$$T_e = A - \frac{R}{(n-p)^2}$$

where A is the **ionization potential**, R , the **Rydberg constant**, and p , the **Rydberg correction**.

S

SABIN. A measure of the sound absorption of a surface. It is the equivalent of 1 square foot of a perfectly absorptive surface.

SABINE LAW. An empirical formula for the reverberation time in a room. It has the same form as the **Franklin equation**.

SACKUR-TETRODE EQUATION. An equation giving the translational **entropy** of an ideal gas. With certain simplifying approximations, it becomes:

$$S_{tr} = R \left[\ln \frac{(2\pi mkT)^{3/2}}{h^3 N} V + \frac{5}{2} \right]$$

in which S_{tr} is the translational **entropy** of one mole of gas, R is the **gas constant**, m is the **molecular mass**, k is the **Boltzmann constant**, T is the absolute temperature, h is the Planck constant, N is the **Avogadro constant**, and V is the molar volume.

SADDLE POINT METHOD. A method for obtaining an asymptotic approximation to a function expressible in the form

$$\int_C e^{zf(\tau)} d\tau$$

where $f(\tau) \rightarrow -\infty$ at either end of the curve C . If $f(z) = u + iv$, the method involves choosing the path C of integration to pass through a saddle point of u , a point that will be among the roots of $f'(\tau) = 0$, and to pass through it in the direction $v = \text{const}$. Points of C in the vicinity of the saddle point then contribute most to the value of the integral, and for large positive z only these points are significant. The method is sometimes called the *method of steepest descent* since the direction $\sigma = \text{const}$ is that which descends most steeply from the saddle point.

SAFETY FACTOR. Although nominally the ratio of the value of a quantity measuring failure to the value at working conditions, the safety factor is most often simply the ratio of yield or ultimate strength to **working stress**. The more realistic ratio of plastic limit load to

working load is called the **load factor**. The factor of safety against yielding of structural steel is in the neighborhood of 1.65.

A statistical point of view is helpful in assessing the significance of safety factors as both the applied loads and the strength of the material will show great scatter. The problem is basically one of extreme values and not well explored as yet.

SAGITTA. In the **Gaussian optics** of spherical surfaces an approximate value for the sagitta to an arc of a circle is frequently useful. When $x \ll y$ the exact equation, $r^2 = (r - x)^2 + y^2$ reduces to $x \approx y^2/2r$, where r is the radius, $2y$ is the chord and x is the sagitta.

SAGITTAL FOCAL LINES. See **focal lines**.

SAGITTAL FOCUS (SECONDARY FOCUS). The first order **focus** of a bundle in the **sagittal plane** of a meridional ray.

SAGITTAL PLANE. A plane containing a meridional ray of an optical system but perpendicular to the tangential or meridional plane of that ray.

SAHA EQUILIBRIUM FORMULA. Equation for the relative concentrations C_i , C_{el} , C_{at} of singly-ionized ions, electrons, and neutral atoms,

$$\frac{C_i C_{el}}{C_{at}} = \left(\frac{2\pi m}{\beta h^2} \right)^{3/2} \frac{2g_i}{g_{at}} e^{-\beta\chi},$$

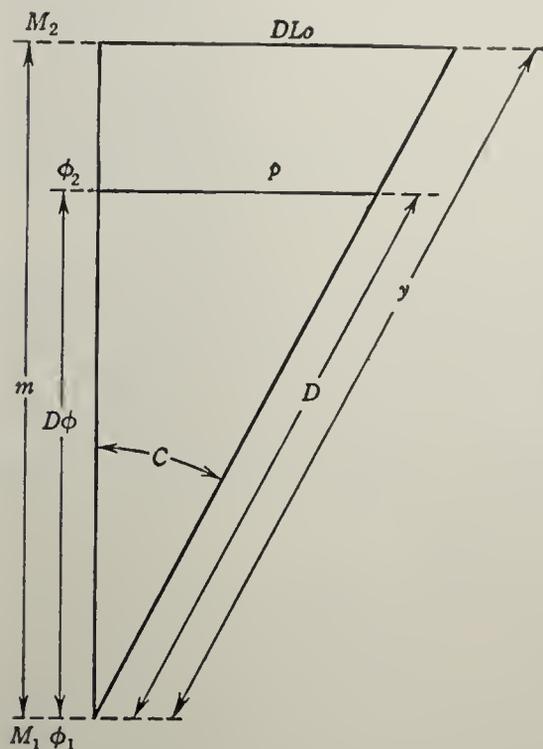
where m is the electron mass, $\beta = 1/kT$ (k : Boltzmann constant, T : absolute temperature), h , Planck's constant, χ , the ionization potential, and g_i and g_{at} the **statistical weights** of the ground states of the ion and the atom, or for the relative concentrations C_A , C_B , and C_{AB} of atoms A and B and the diatomic molecule AB ,

$$\frac{C_A C_B}{C_{AB}} = \frac{g_A g_B}{g_{AB}} \left(\frac{2\pi\mu}{\beta h^2} \right)^{3/2} \frac{\beta h^2}{8\pi^2 I} (1 - e^{-\beta h\nu}) e^{-\beta D},$$

where g_A , g_B , g_{AB} , are the ground state statistical weights, μ is the reduced mass $m_A m_B / m_{AB}$, I is the moment of inertia of the molecule,

$h\nu$ its vibrational energy, and D its dissociation energy.

(THE) SAILINGS (NAVIGATION). The various methods for advancing the position of a ship to obtain the dead reckoning position are referred to as the sailings. While this advancement of position may be accomplished by graphical methods, nevertheless many careful navigators prefer to solve this problem by computational methods.



The following standard notation is used throughout this discussion, and is illustrated in the figure.

Latitude: The latitude of the point of departure is ϕ_1 .

The latitude of the point of arrival is ϕ_2 .

The mid-latitude $\frac{\phi_1 + \phi_2}{2} = \phi_m$.

Difference of latitude is $D\phi$

Meridional part of departure is M_1 and that of arrival is M_2

Meridional distance ($M_1 - M_2$) is m

Longitude: The longitude of departure is Lo_1

The longitude of arrival is Lo_2

Longitude difference is DLo

Departure is p

Course or course angle is C or C_n

Distance is expressed in nautical miles or minutes of arc on the sphere.

1. *Plane Sailing.* This is the simplest of all of the sailings but should not be used for distances of more than a hundred miles. The earth is considered as a plane surface. The triangle to be solved is that in the figure bounded by the sides $d\phi$, p , and D . From the figure it is evident that

$$p = D \sin C, \quad D\phi = D \cos C, \quad D = D\phi \sec C.$$

These relations are so frequently used by navigators that they are published in tabular form and known as the *Traverse Tables*. These tables are published by the U.S. Hydrographic Office and various other publishers. The format of one table may differ from that of others, but they are all designed to serve the same purposes. In the most common form of *Traverse Tables* we have $D\phi$ and p tabulated for every degree of course angle and for distances up to 1000 miles.

2. *Traverse Sailing.* If a ship is frequently altering course or speed or both, as might be the case on tactical maneuvers, or operating in a group of islands, the ship is said to be running a traverse. For each leg of the traverse the departures and changes of latitude are computed or taken from a traverse table. The individual changes of latitude are added together algebraically to find the cumulative p . The cumulative $D\phi$ and p are treated as a problem in plane sailing or, more commonly, by methods discussed below.

3. *Parallel Sailing.* Parallel sailing is another name for the conversion of departure, p , in miles to DLo expressed in angular measure. The assumption is made that a ship sails along a parallel of latitude ϕ for a certain number of miles, p . Then

$$DLo = p \sec \phi \quad \text{or} \quad p = DLo \cos \phi.$$

4. *Middle Latitude Sailing.* Popularly referred to as mid-latitude sailing, this method combines plane and parallel sailing, or traverse and parallel sailing, when considerable distance, particularly in the north-south component, is involved. This method may be used in safety between the 60° parallels, either north or south, for distances of three hundred miles or more. The traverse is completed, providing a total departure, p , and a total latitude distance $D\phi$. However the difference in longitude, computed using ϕ_1 will differ appreciably from that obtained using ϕ_2 . A good approximation is obtained by using the average of the

two latitudes ϕ_m . If the traverse crosses the equator, the sections in the northern and southern hemispheres should be computed separately and the results combined to obtain DLo .

5. *Mercator Sailing* (see **Mercator Chart**). When a traverse is to be solved on a Mercator chart, either by graphical or computational methods, two right triangles, with identical angles are employed. The first is the triangle used in plane sailing ($D\phi, p, D$). Using this triangle we determine $D\phi$ from the equation $D\phi = D \cos C$ and $\phi_1 + D\phi = \phi_2$. We now refer to a table of Mercator meridional parts and find M_1 and M_2 corresponding to ϕ_1 and ϕ_2 . Now the larger triangle (m, DLo, y) is used. The side m is the difference of the meridional parts corresponding to the latitudes $M_2 - M_1 = m$ and the angle C is the course. Now we have:

To find Mercator Course and distance between two points

$$\tan C = \frac{DLo}{m}, \quad D = D\phi \sec C.$$

To find $D\phi$ and DLo when Mercator course and distance are given

$$D\phi = D \cos C, \quad DLo = m \tan C.$$

It should be noted that in Mercator sailing we use longitude difference directly and not departure.

6. *Great Circle Sailing* and 7. *Composite Sailing* are discussed under **great circle**; and under **Lambert chart**, respectively.

SAINT VENANT-MISES MATERIAL. A rigid perfectly plastic material equivalent to the **Prandtl-Reuss material** with negligible elastic strain components.

SAINT VENANT PRINCIPLE. A principle which states that the **strains** produced in a body by application to a small part of its surface or throughout a small part of its volume, of a system of forces which is statically equivalent to zero force and zero couple are of negligible magnitude at distances from the region over which the forces are applied large compared with the linear dimensions of this region.

SAMPLE. A sample is a collection of individuals drawn from a **population**. Ordinarily inferences are to be made from the sample to

the population, and the one must be in some way representative of the other.

When sampling from an actual population, the simplest method is to draw a random sample. The members of the population are numbered off, and the sample selected with the aid of a table of random numbers or some similar device. To ensure more even coverage, stratified sampling may be adopted. The population is divided into a number of homogeneous groups (the strata) and a random sample is selected from each. If only a random selection of the strata are sampled, we have a two-stage sample; three or more stage samples are similarly constructed. In two-phase sampling one type of observation is taken only on a small calibrating sample, while another (which may be easier or cheaper to obtain) is taken on a larger sample.

The introduction of a random element can ensure the absence of **bias** from an estimate based on a sample, and also makes possible a valid estimate of the error to which such an estimate is subject. (See also **quota sample**.)

SAMPLE POINT. The point on a **chromaticity diagram** that represents the **chromaticity** of the sample.

SAMPLING DISTRIBUTION. If a sampling procedure be applied repeatedly to a population there is generated a collection of samples, and the distribution of statistics calculated from these samples is called a sampling distribution.

SAND HEAP ANALOGY. An analogy for torsion of perfectly plastic bars equivalent to the **membrane analogy** for **torsion** of elastic bars. The slope of a sand heap is constant as the yield stress in shear is constant; the contours give the direction of shear stress; and the volume of the sand heap is proportional to the twisting moment. A combination of membrane and sand heap "roof" serves for the elastic-plastic problem.

SANDWICH CONSTRUCTION, AIRCRAFT. A sandwich consisting of a light thick core faced with thin sheet metal has considerably more strength and stability than if the thin metal sheets were used alone. Used extensively for aircraft where light smooth structure with great bending rigidity is required. Core material may be metal or resin-impregnated paper or fabric honeycomb mate-

rial, light porous foam "rubber" or similar materials. The core carries the shear, the facings the bending moment.

SARGENT CURVES. Graphs obtained by plotting the logarithms of the radioactive disintegration constants of β -emitting radioelements, against the corresponding logarithms of their maximum β -particle energies. Most of the points corresponding to the natural radioelements were found to fall on two straight lines.

An interpretation of this result was provided by the Fermi theory of β -decay based upon the neutrino concept, which yielded the relationship between the disintegration constant and the maximum β -particle energy, which in its logarithmic form is

$$\log \lambda = \log k + 5 \log E_{max}.$$

SAROS. The fact that eclipses occur in periodic intervals was known to the ancient Chaldeans, and probably even in prehistoric times. This period of 18 years, $11\frac{1}{3}$ days ($10\frac{1}{3}$ days if there happen to be 5 leap years in the interval) is known as the Saros. If an eclipse should occur on January 1st, 1967, at noon, another similar eclipse would occur on January 12th, 1985, at eight o'clock in the evening. The eclipse would not occur at the same point on the earth but would be about 8 hours farther west in **longitude**.

During the course of a Saros there are about 29 lunar and 41 solar eclipses, each repeated during the next Saros, but not at the same portion of the earth.

SATURATED CHARACTER OF THE COVALENT BOND. See **directed valency**.

SATURATION ADIABAT. (Or moist adiabat, wet adiabat.) On a **thermodynamic diagram**, a line of constant **wet-bulb potential temperature**. In practice, approximate computations are usually employed, and the resulting lines represent, ambiguously, saturation adiabats and **pseudo-adiabats**. (Compare **dry adiabat**.)

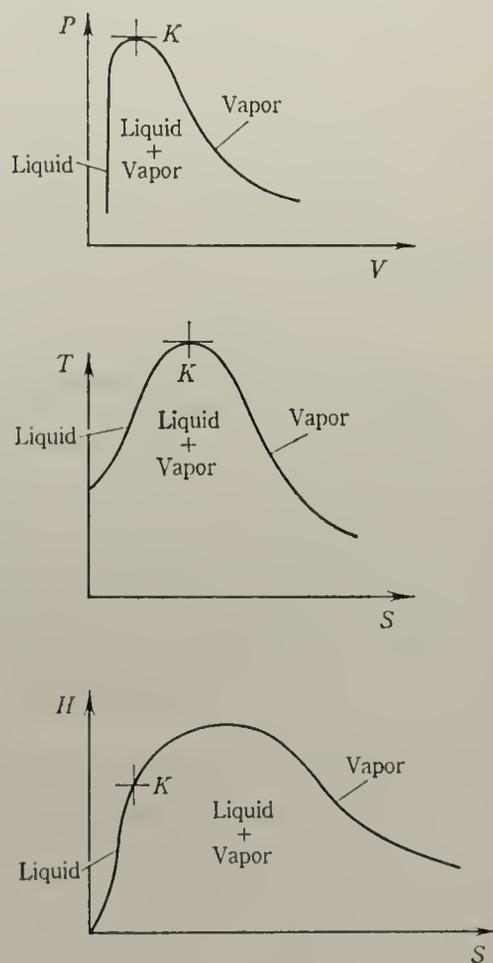
SATURATION-ADIABATIC LAPSE RATE. (Or moist-adiabatic lapse rate.) A special case of **process lapse rate**, defined as the rate of decrease of temperature with height of an air parcel lifted in a **saturation-adiabatic process** through an atmosphere in **hydrostatic equilibrium**. Owing to the release of latent heat,

this lapse rate is less than the **dry-adiabatic lapse rate**, and the differential equation representing the process must be integrated numerically. **Wet-bulb potential temperature** is constant with height in an atmosphere with this lapse rate.

SATURATION-ADIABATIC PROCESS. An **adiabatic process** in which the air is maintained at **saturation** by the evaporation or condensation of water substance, the latent heat being supplied by or to the air, respectively. The ascent of cloudy air, for example, is often assumed to be such a process.

SATURATION CURVE. The boundary curve in a thermodynamic diagram involving at least one extensive property (such as $p, V; T, S; \text{ or } H, S$) which is the locus of all states of equilibrium between two phases of a pure substance. The saturation curves for the liquid-vapor equilibrium of H_2O are shown in the figures.

The **critical point K** which separates the



The saturation curves for the liquid vapor equilibrium of H_2O .

locus for the saturated liquid from the locus for saturated vapor also lies on the saturation curve. The saturation curve encloses the region of two-phase equilibrium.

(See also **binodals** and **spinodal curve**.)

SATURATION MAGNETIZATION. See **magnetization, saturation**.

SATURATION OF NUCLEAR FORCES. The property of nuclear forces whereby the binding energy of a nucleus is approximately proportional to the mass number, i.e., the binding energy per particle is constant.

SAUREL THEOREM. Call ω the variance of a system (see **phase rule**). It can be shown that for $\omega \geq 2$ there are $\omega - 1$ conditions which have to be satisfied for a state to be an **indifferent state**. These $(\omega - 1)$ relations between the ω variables leave only one independent variable. Therefore the indifferent states of the system fall in a line called the *indifferent line*. In this general form this theorem is due to Saurel.

SAVINGS, REFLECTOR. See **reflector saving**.

SCALAR. In each n -dimensional curvilinear coordinate system we define a function of position. Let f and \bar{f} denote the functions defined in the coordinate systems x and \bar{x} respectively and let

$$\bar{f} = |\partial x / \partial \bar{x}|^W f,$$

for all choices of the coordinate systems, where $|\partial x / \partial \bar{x}|$ denotes the Jacobian determinant $\partial(x^1, x^2, \dots, x^n) / \partial(\bar{x}^1, \bar{x}^2, \dots, \bar{x}^n)$, and W is an integer (positive or negative). The aggregate of functions so defined is called an n -dimensional scalar. W is the *weight of the scalar*. By comparing this definition with that of a **tensor field**, it is seen that a scalar may be regarded as a tensor field of order zero. f is called the *component of the scalar* in the system x ; \bar{f} , the component in the system \bar{x} .

SCALAR, ABSOLUTE. Scalar of zero weight. Often called scalar when no confusion is possible.

SCALAR, COMPONENT OF. See **scalar**.

SCALAR CURVATURE. The scalar defined by $g^{ij}R_{ij}$, where g^{ij} denotes the contravariant metric tensor and R_{ij} the Ricci tensor.

SCALAR DENSITY. Scalar of weight unity.

SCALAR FIELD. Consider the space reflection $x' \rightarrow x$; $x' = -x$, $x'^0 = x^0$. Define the transformation of a spinless field operator under space reflection such that

$$\begin{aligned} \phi'(x') &= P\phi(x)P^{-1} \\ &= \eta_p\phi(x') \end{aligned}$$

where η_p is a complex constant and P the unitary operator which induces the transformation. If the operator of space reflection is performed twice, we must revert to the original field so that $\eta_p^2 = 1$ or $\eta_p = \pm 1$. A field which transforms with $\eta_p = +1$ is called a scalar field, while one that transforms with $\eta_p = -1$ is called a **pseudoscalar** field. The particles described by scalar (pseudoscalar) fields have an intrinsic parity $+1$ (-1).

SCALAR INVARIANT. See **invariant**.

SCALAR POTENTIAL. A single-valued function of position and time:

$$\phi(r) = - \int_{r_0}^r \mathbf{F}(r,t) \cdot ds,$$

where r is the position vector, r_0 is a vector locating an arbitrary reference point, t is the time, ds is an infinitesimal vector displacement, and $\mathbf{F}(r,t)$ is any vector field. The requirement that ϕ be single valued insures that **curl** $\mathbf{F}(r,t) = 0$ everywhere, and *vice versa*; this condition on the curl is equivalent to the requirement of single valuedness. (See **integral, line**; **Stokes theorem**.)

If the field is independent of time, $\mathbf{F}(r) = -\mathbf{grad} \phi(r)$. Two special cases of time-independent fields are: (1) the electrostatic field, in which $\phi(r)$ is the electric potential at r , relative to r_0 ; and (2) the force field, in which $\phi(r)$ is the potential energy at r , relative to r_0 .

An example of the time-dependent case occurs in the general electromagnetic field, in which $\phi(r,t)$ is the electromagnetic scalar potential that is used in combination with the magnetic vector potential, to determine the field.

The magnetic field cannot be described completely in terms of a scalar potential, because the curl of the magnetic field vector \mathbf{H} is not zero over all space. However, in limited regions of space in which one can draw no closed contour containing or enclosing a current, it

is possible to introduce the scalar magnetic potential, since $\text{curl } \mathbf{H}$ is zero in such a region.

SCALAR PRODUCT. A product of two geometric vectors, \mathbf{x} and \mathbf{y} , is $|\mathbf{x}| |\mathbf{y}| \cos \theta$, where the bars signify the geometric length, and the angle θ is that between the vectors. If these vectors are referred to an orthonormal coordinate system, and if x and y are the column vectors whose elements are the coordinates of \mathbf{x} and \mathbf{y} in this system, then

$$x^T y = y^T x = |\mathbf{x}| |\mathbf{y}| \cos \theta.$$

In any event, the scalar product is often written $\mathbf{x} \cdot \mathbf{y}$ and hence referred to as the dot product. (See **Euclidean space**; **Hilbert space**.)

SCALAR PRODUCT, HERMITIAN. See **vector space**.

SCALAR PRODUCT OF VECTORS. See **scalar product**; **vector multiplication**.

SCALAR, RELATIVE. Scalar of non-zero weight.

SCALAR TRIPLE PRODUCT. See **triple product of vectors**.

SCALE, EQUALLY TEMPERED. A series of notes selected from a division of the octave (usually) into 12 equal intervals.

EQUALLY TEMPERED INTERVALS

Name of Interval	Frequency Ratio	Cents
Unison	1:1	0
Minor second or semitone	1.059463:1	100
Major second or whole tone	1.122462:1	200
Minor third	1.189207:1	300
Major third	1.259921:1	400
Perfect fourth	1.334840:1	500
Augmented fourth } Diminished fifth }	1.414214:1	600
Perfect fifth	1.498307:1	700
Minor sixth	1.587401:1	800
Major sixth	1.681793:1	900
Minor seventh	1.781797:1	1000
Major seventh	1.887749:1	1100
Octave	2:1	1200

SCALE, JUST. A musical scale which is formed by the combination of three consecutive triads each having the ratio 4:5:6, or 10:12:15. By consecutive triads is meant

triads such that the highest note of one is the lowest note of the other.

SOME JUST INTERVALS

Name of Interval	Frequency Ratio	Cents
Unison	1:1	
Semitone	16:15	111.731
Minor tone or lesser whole tone	10:9	182.401
Major tone or greater whole tone	9:8	203.910
Minor third	6:5	315.641
Major third	5:4	386.314
Perfect fourth	4:3	498.045
Augmented fourth	45:32	590.224
Diminished fifth	64:45	609.777
Perfect fifth	3:2	701.955
Minor sixth	8:5	813.687
Major sixth	5:3	884.359
Harmonic minor seventh	7:4	968.826
Grave minor seventh	16:9	996.091
Minor seventh	9:5	1017.597
Major seventh	15:8	1088.269
Octave	2:1	1200.000

SCALENE. A triangle is scalene if no two of its sides are equal.

SCALE OF TURBULENCE. (1) A measure of the average size of **eddies** in turbulent flow, defined by

$$L = \int_0^{\infty} R(y) dy,$$

where $R(y)$ is the normalized **correlation** between the same simultaneous component of the **eddy velocity** at points separated by a distance y . (See **microscale of turbulence**.) (2) The length of the **period** used to obtain the **mean velocities** of the flow. If the period is approximately one hour, the scale is called large; a few minutes, intermediate; and a few seconds, small.

SCARE, PYTHAGOREAN. A musical scale such that the frequency intervals are represented by the ratios of integral powers of the numbers 2 and 3.

SCATTER DIAGRAM. A method of representing the joint relationship of two variables in statistics. If the variables are x, y the graph of y against x for each observed pair generates a constellation of points. The variation being statistical, these points do not, in general, lie on a smooth curve, but their clustering properties represent the degree of relationship between x and y .

SCATTERING. In its general sense, this term means causing the random distribution of a group of entities, or bringing about a less orderly arrangement, either in position or direction. More specifically, the term denotes the change in direction of particles or photons owing to collision with other particles or systems; it may also be regarded as the diffusion of a beam of sound or light (or other electromagnetic radiation) due to the anisotropy of the transmitting medium. For the various kinds of scattering, and the various entities scattered, see the entries which follow.

SCATTERING AMPLITUDE. A quantity closely related to the intensity of scattering of a wave by a center of force such as an atomic nucleus or an atom. In the simplest case, that of elastic scattering, if e^{ikz} describes an incident plane wave of wave number k , then the scattered wave at large distances from the scattering center can be written $a(\theta, \phi)e^{ikr}/r$, where $a(\theta, \phi)$ is the scattering amplitude as a function of scattering angle. The differential scattering cross section is then given by $d\sigma/d\Omega = |a|^2$, where $d\Omega$ is an element of solid angle about the direction θ, ϕ . In the event that the scattering is isotropic (which it will always be for sufficiently small k) a is independent of angle and the scattering cross section becomes simply $\sigma = 4\pi|a|^2$.

SCATTERING ANGLE. The angle between the initial and final lines of motion of a scattered particle or ray. It may be specified as applying either to the **laboratory** or **center-of-mass system**.

SCATTERING AREA COEFFICIENT. (Or scattering area ratio.) The dimensionless ratio of the **scattering cross section** to the geometric cross section of a scattering particle.

SCATTERING COEFFICIENTS. For plane waves which are incident at a material or geometrical discontinuity, the ratio of the amplitude of the reflected wave to that of the incident wave determines a *reflection coefficient*. The ratio of the amplitude of the transmitted wave to that of the incident wave is the *transmission coefficient*.

SCATTERING, COMPTON. Elastic scattering of photons by electrons. Because the total energy and total momentum are con-

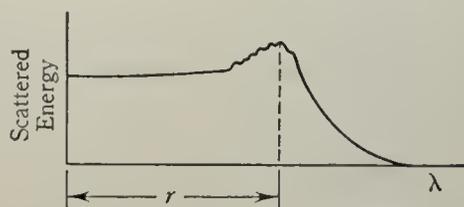
served in the collisions, the wavelength of the scattered radiation undergoes a change that depends in amount on the scattering angle. If the scattering electron is assumed to be at rest initially, the Compton shift is given by the following equation:

$$\lambda' - \lambda = \lambda_0(1 - \cos \theta) = (h/m_e c)(1 - \cos \theta),$$

where λ' is the wavelength associated with the scattered photons, λ is the wavelength of the incident photons, λ_0 is the (Compton) wavelength of the electron and θ is the angle between the paths of incident and scattered photons.

SCATTERING CROSS SECTION. (Also called extinction cross section, effective area.) The hypothetical area normal to the incident **radiation** that would geometrically intercept the total amount of radiation actually scattered by a **scattering particle**. It is also defined, equivalently, as the cross-sectional area of an isotropic scatterer (a sphere) which would scatter the same amount of radiation as the actual amount.

SCATTERING CURVE, EXPERIMENTAL. For wavelengths much shorter than the radius of the particles causing the **scattering**, the scattered energy is nearly independent of the



wavelength. For wavelengths much longer than the radius of the particles, the scattered energy falls off as the inverse fourth power of the wavelength (**Rayleigh scatter**).

For the case where the particles and the wavelengths are nearly the same, the scattered energy is a maximum. Practical calculations on transmission in this last case are very complicated.

SCATTERING FACTOR, ATOMIC. See **atomic scattering factor**.

SCATTERING FUNCTION. The intensity of scattered radiation in a given direction per lumen of flux incident upon the **scattering material**. Suppose a collection of scattering particles have a combined cross-sectional area A to receive radiation of illuminance E . Then,

if the intensity of radiation observed to be scattered in a direction at the angle ϕ with the direction of the incident beam is $I(\phi)$, the scattering function $S(\phi)$ of this collection of particles is such that

$$S(\phi) = \frac{I(\phi)}{EA}.$$

When the collection of scattering particles is taken to be those in one unit volume, the associated scattering function is known as the *volume scattering function*. The *relative scatter intensity* is obtained by simply normalizing either of the preceding functions to the scattering intensity observed in the direction of the incident beam.

SCATTERING KERNEL. See **kernel, scattering**.

SCATTERING LOSS. In acoustics, that part of the **transmission loss** which is due to scattering within the medium or due to roughness of the reflecting surface. In electromagnetic waves, the loss in power at the receiving point, due to **scattering**.

SCATTERING MATRIX. See **matrix, scattering**.

SCATTERING PHASE SHIFT. If a particle is scattered by a center of force of short range, the wave function of the scattered particle at large distances from the scatterer can be expressed in terms of the incident wave at large distances by means of certain phase angles which represent the effect of the scatterer. Consider the scattering of a spinless particle, represented by an incident plane wave along the z axis, $\psi_o = e^{ikz} = e^{ikr \cos \theta}$ of wave number k . We may expand ψ_o into partial waves of orbital angular momentum l about the scattering center,

$$\psi_o = \sum_{l=0}^{\infty} (2l+1) i^l j_l(kr) P_l(\cos \theta)$$

where j_l is a spherical **Bessel function**, and l an integer, and P_l is a **Legendre polynomial**. The scattered wave can then be shown to behave asymptotically as

$$\psi_{sc} \sim \frac{e^{ikr}}{2ikr} \sum_{l=0}^{\infty} (2l+1) e^{2i\delta_l} - 1) P_l(\cos \theta)$$

where δ_l is a constant phase angle, depending upon k but not upon θ , for the l^{th} partial wave,

called the l^{th} phase shift. The differential scattering cross section is then given by $d\sigma/d\Omega = |f(\theta)|^2$, where the scattering amplitude is given by

$$f(\theta) = \frac{1}{k} \sum_{l=0}^{\infty} (2l+1) e^{i\delta_l} \sin \delta_l P_l(\cos \theta),$$

while the total scattering cross section is given by

$$\sigma_{sc} = \frac{4\pi}{k^2} \sum_{l=0}^{\infty} (2l+1) \sin^2 \delta_l(k) = \frac{4\pi}{k} \text{Im } f(0).$$

Although there are an infinite number of phase shifts δ_l appearing in these formulae, only a few will usually be of importance for low energies. If a is the effective range of the scattering force then δ_l will differ appreciably from zero only for $l < ka$. A resonance is produced should any one of these phases pass through $\pi/2$, when $\sin^2 \delta_l$ becomes unity. Corresponding to the partial wave expansion in ψ_o one speaks of the l^{th} partial wave (s-wave, p-wave, etc., for $l = 0, 1, \dots$).

SCATTERING POWER, ATOMIC. See **atomic scattering power**.

SCATTERING, SOUND. When a plane wave of sound encounters an obstacle, part of the wave continues undeflected, and part is scattered. Where the scattering object is a rigid cylinder of radius a , and a plane wave of frequency $(\omega/2\pi)$ is incident normal to the axis of the cylinder, then, for wavelengths long compared with a , the ratio of the intensity (S_ϕ) of the wave scattered at angle ϕ , to the intensity of the incident wave S_o is given by the approximate relation

$$S_\phi/S_o \simeq (1 - 2 \cos \phi)^2 (\pi \omega^3 a^4 / 8c^3 r)$$

where r is the distance from the axis of the cylinder and c is the velocity of sound in the medium.

The total power P scattered per unit length of cylinder is given by

$$P \simeq 3\pi^2 \omega^3 a^4 S_o / 4c^3.$$

The corresponding approximate expressions for scattering by a spherical obstacle of radius a for wavelengths very much greater than a are

$$S_\phi/S_o \simeq (\omega^4 a^6 / 9c^4 r^2) (1 - 3 \cos \phi)^2$$

and

$$P \simeq (16\pi \omega^4 a^6 / 9c^4) S_o.$$

SCHLÄFLI FORMULA. An integral representation of the **Legendre polynomial**

$$P_n(z) = \frac{1}{2\pi i} \int \frac{(t^2 - 1)^n}{2^n(t - z)^{n+1}} dt$$

where the **contour** encircles the point z in the counterclockwise direction in the complex plane.

SCHLAPP FORMULA. See **fine structure**.

SCHMIDT-HILBERT METHOD. A method of solving **integral equations** with symmetric **kernels**. It consists essentially of expanding the right side of the equation in a (generalized Fourier) series of **eigenfunctions** of the kernel.

SCHMIDT MODEL OF NUCLEI. A model of atomic nuclei of odd mass number A in which the whole dynamics of the nucleus is supposed to be invested in the odd particle, neutron for even atomic number Z , or proton for odd Z . This particle is supposed to move in the central field of force representing its average interaction with the rest of the nucleus. Used in conjunction with the **Pauli exclusion principle**, this model gives a remarkably accurate qualitative picture of the spins and magnetic moments of the ground states of nuclei. It breaks down if more detailed properties are required of it. However, it formed the empirical basis of the highly successful shell model which followed it.

SCHMIDT PROCESS. See **Gram-Schmidt process**.

SCHÖNFLIES CRYSTAL SYMBOLS. A notation for the description of the **symmetry classes** of crystals.

SCHOTTKY DEFECT. A lattice vacancy created by removing an ion from its site and placing it on the surface of the crystal. For electric neutrality, the number of cation Schottky defects must equal the number of anion Schottky defects. The number, n , of Schottky defects is given by

$$\frac{n}{N - n} = C_s \exp(-\epsilon/kT)$$

where there are N lattice points, and ϵ is the energy required to remove an ion from a lattice point, and then add it to the surface. C_s is a numerical factor of the order of 10^3 – 10^4 . (Cf. with entry following.)

SCHOTTKY EFFECT. (1) The effect caused by the ability of atoms or molecules in a crystal to occupy two different energy states. If the individual atoms are mutually independent, then the ratio of the number of atoms in the higher energy state to that in the lower is

$$\frac{N_2}{N_1} = g \exp(-\epsilon/kT)$$

where ϵ is the difference in energy of the two states, g is the ratio of the statistical weights, k is the Boltzmann constant, and T is the absolute temperature.

The Schottky effect results in deviations from the **Debye T^3 law** for some substances. It produces an additional term for specific heat per mol.:

$$C'_v = \frac{Ng(\epsilon^2/kT^2)}{(1 + ge^{-\epsilon/kT})^2} e^{-\epsilon/kT}$$

where N is Avogadro's number.

(2) In thermionic emission, the term Schottky effect describes the reduction of the work function of a thermionic emitter by the application of an accelerating field at the emitter's surface. This results in larger thermionic currents than are predicted by the **Richardson-Dushman equation**, and the expression for saturation current becomes

$$I_o \exp \left[\frac{e\sqrt{Ee}}{kT} \right]$$

where I_o is zero field thermionic current, E is the field intensity at the cathode, and e is the electronic charge.

SCHRÖDER-VAN LAAR EQUATION. See **crystallization curve**.

SCHRÖDINGER EQUATION. The basic equation of wave mechanics. It is developed by using the de Broglie wavelength in the description of a particle and then associating with the energy E or the x -component of momentum p_x of the particle a differential operator

$$E = i\hbar \frac{\partial}{\partial t} \quad \text{or} \quad p_x = -i\hbar \frac{\partial}{\partial x}$$

where \hbar is the **Dirac h** . Writing these differential operators in the Hamiltonian function one obtains:

$$-\frac{\hbar^2}{2m}\nabla^2\psi + V(\mathbf{r})\psi = i\hbar\frac{\partial\psi}{\partial t} = E\psi,$$

where ∇^2 is the **Laplacian**, m the mass of the particle, E , its total energy, and $V(\mathbf{r})$ its potential energy (usually a function of position). This is the time dependent Schrödinger equation for ψ .

In many instances, we are interested in the allowed values of E in **stationary states** of the system. Using the **Planck law** we may set $E = h\nu = 2\pi\hbar\nu$ and write

$$\psi = \phi(\mathbf{r})e^{2\pi i\nu t} = \phi(\mathbf{r})e^{iEt/\hbar}$$

where $\phi(\mathbf{r})$ is a function of position only. We then obtain the time independent equation:

$$\left[\nabla^2 + \frac{2m}{\hbar^2}\{E - V(\mathbf{r})\}\right]\phi = 0.$$

It is often found that solutions of the equation exist only for specific **eigenvalues** of E . To each eigenvalue E_n there corresponds an eigenfunction of the coordinates ϕ_n . The probability of finding the particle in a region of volume dV is

$$\int|\phi|^2dV = \int\phi\phi^*dV,$$

assuming that ϕ has been normalized so that the integral over all space is unity. (See also **quantum mechanics, non-relativistic; quantum mechanics, relativistic; Schrödinger picture; simple harmonic oscillator**.)

SCHRÖDINGER PICTURE. The state of a physical system at a given time t can be defined by the results of all possible (compatible) experiments on the system at that time. In quantum mechanics, this information is contained in the state vector $|\Psi(t)\rangle$. In the Schrödinger picture the evolution of the system in time is described by the time dependence of the state vector. This time dependence is governed by the Schrödinger equation,

$$H_s|\Psi_s(t)\rangle = i\hbar\partial_t|\Psi_s(t)\rangle \quad (\text{a})$$

where H_s is the Hamiltonian operator of the system. Furthermore, the operators corresponding to physical observables, F_s , are time independent

$$\partial_t F_s = 0 \quad (\text{b})$$

and are the same for all time. Although the operators are time independent, their expecta-

tion value in any given state will in general be time dependent. Call $|\Psi_s(t)\rangle = \Psi_s(t)$ and

$$\langle F_s \rangle = (\Psi_s(t), F_s\Psi_s(t)) \quad (\text{c})$$

then using (a) one finds that

$$i\hbar\frac{d}{dt}\langle F_s \rangle = (\Psi_s(t), [F_s, H_s]\Psi_s(t)). \quad (\text{d})$$

In the Schrödinger picture, by definition one calls \dot{F}_s that operator for which

$$\langle \dot{F}_s \rangle = d_t\langle F_s \rangle. \quad (\text{e})$$

(See **Heisenberg picture, interaction picture**.)

SCHUR LEMMA. If $D(R)$ and $D'(R)$ are two unitary, irreducible, rotation transformation matrices of dimensions n and n' , and there exists a matrix M with n columns and n' rows such that $MD(R) = D'(R)M$ for every rotation R , then either $M = 0$ or $n = n'$ and the determinant of $M \neq 0$. If $n' = n$, D and D' are equivalent. This was proved by Schur in 1905.

SCHWARZ-CHRISTOFFEL TRANSFORMATION. A conformal transformation $w = f(z)$ which transforms the interior of a polygon in the w -plane into the upper half of the z -plane. (The name is also given to a transformation sending a polygon in z -plane into the upper half of the w -plane.) If the interior angles of the polygon in the w -plane are $\alpha_1, \alpha_2, \dots, \alpha_n$, then the derivative dw/dz has the form

$$\frac{dw}{dz} = k(z - x_1)^{\beta_1}(z - x_2)^{\beta_2}\dots(z - x_n)^{\beta_n},$$

where $\beta_i = (\alpha_i/\pi) - 1$, so that the desired w is obtained by integration. The vertices of the polygon are transformed into points x_1, x_2, \dots, x_n on the real axis, a certain number of which may be chosen at will. One or more of the vertices of the polygon may be at infinity, as for example, in determining the electric potential around a parallel-plate condenser.

SCHWARZ INEQUALITY. For a linear space with a scalar (inner) product (see, e.g., **Euclidean space**), the inequality is $|(a,b)|^2 \leq |a| \cdot |b|$. For complex numbers this takes the form $|a_1\bar{b}_1 + a_2\bar{b}_2 + \dots + a_n\bar{b}_n|^2 \leq a_1\bar{a}_1 + a_2\bar{a}_2 + \dots + a_n\bar{a}_n \cdot |b_1\bar{b}_1 + b_2\bar{b}_2 +$

$\dots + b_n \bar{b}_n|$. For two real functions $f(x)$ and $g(x)$, it becomes $\left| \int_a^b f(x)g(x)dx \right|^2 \leq \left| \int_a^b f^2(x)dx \right| \cdot \left| \int_a^b g^2(x)dx \right|$, while, if \mathbf{x} and \mathbf{y} are two vectors (real or complex), then

$$|\mathbf{x}^*\mathbf{y}|^2 = |\mathbf{y}^*\mathbf{x}|^2 \leq (\mathbf{x}^*\mathbf{x})(\mathbf{y}^*\mathbf{y})$$

where the asterisk designates the conjugate transpose. The inequality is also variously called the Cauchy, Cauchy-Schwarz, Bunia-kovski inequality. (For generalizations, see **Minkowski inequality** and **Hölder inequality**.)

SCHWARZSCHILD - KOHLSCHÜTTER FORMULAS. Formulas often used to compute the **Seidel aberrations** of an axially symmetrical optical system of spherical surfaces. Let $1/R_i$ be the curvature, s_i the object distance, s'_i the image distance, and t_i the **object-side distance** of the stop from the i^{th} surface, $i = 1, \dots, N$. Let n_i be the index of refraction of the medium following the i^{th} surface and d_i the distance from vertex i to vertex $i + 1$. Compute then for $i = 1, \dots, N$

$$K_i = n_{i-1} \left(\frac{1}{R_i} + \frac{1}{s_i} \right),$$

$$s_{i+1} = -s'_i + d_i$$

$$h_{i+1} = -\frac{s_{i+1}}{s'_i} h_i, \quad h_1 = \frac{s_1}{s_1 - t_1},$$

$$k_{i+1} = k_1 + \sum_{j=1}^i \frac{d_j}{n_j h_j h_{j+1}}, \quad k_1 = \frac{t_1}{h_1},$$

$$(kc)_i = k_i + \frac{1}{h_i^2 K_i},$$

$$f_i = \frac{1}{s_i} + \frac{1}{R_i} - \frac{1}{s'_i}.$$

The Seidel aberration functions are:

spherical aberration $B = \sum_{i=1}^N B_i,$

coma $F = \sum_{i=1}^N F_i,$

astigmatism $C = \sum_{i=1}^N C_i,$

Petzval sum $P = \sum_{i=1}^N P_i,$

and distortion $E = \sum_{i=1}^N E_i,$

where the contributions of each surface are

$$B_i = \frac{1}{2} h_i^4 K_i f_i \left(\frac{1}{s_i} + \frac{1}{s'_i} \right),$$

$$F_i = (kc)_i B_i,$$

$$C_i = (kc)_i F_i,$$

$$P_i = \frac{1}{R_i} \left(\frac{1}{n_{i-1}} - \frac{1}{n_i} \right),$$

$$E_i = (kc)_i (C_i + \frac{1}{2} P_i).$$

SCLERONOMIC. See **conservative force**.

SCREENING CONSTANT. (1) A quantity occurring in the relationship between the frequency of a line in a particular x-ray series, and the **atomic number** of the element emitting the rays, of the form:

$$\sqrt{\nu} = a(Z - \sigma)$$

in which ν is the frequency of the line, a is a constant, Z is the atomic number of the element, and σ is the screening constant, that is the same for all the lines in a given series.

SCREW AXIS. A type of **symmetry element** possessed by certain **space groups**, in which the lattice is unaltered after a rotation about the axis, and a simultaneous translation along it.

SCREW CURVATURE. See **curvature, screw**.

SCROLL. See **skew surface**.

S-CURVE. The **hydrograph**, for a given river basin, which would theoretically result from continuous runoff at a constant rate of 1.00 inch per specified time period. The curve has a characteristic S-shaped, or ogive, rising limb and finally represents a constant flow rate sufficient to discharge the specified rate of runoff. An S-curve usually is constructed from a single unit hydrograph by a simple process of successive time-displacement followed by summing the ordinates (discharge values).

In theory, the unit hydrographs for storms of any duration may be derived from the resulting curve; and this is the primary use for it.

SECANT FORMULA FOR COLUMNS. The maximum stress in an elastic initially-straight column of length L and cross-section area A loaded by a force P parallel to the axis but eccentric a distance e about a principal axis for which the moment of inertia is I is

$$\sigma_{max} = \frac{P}{A} + \frac{Pec}{I} \sec \frac{L}{2} \sqrt{\frac{P}{EI}}$$

Test data on strength of columns can be matched by setting σ_{max} equal to the yield strength and choosing e in the secant formula so that the formula is used in some design specifications.

SECANT MODULUS. See **modulus, secant.**

SECOND. (1) Unit of time, abbreviation S, s, or sec. One 86,400th part of a mean solar day. (2) The international Astronomical Union defines the second as the fraction $1/31,556,925.975$ of the length of the tropical year for 1900. The tropical year is the period of revolution of the earth relative to the Vernal Equinox or First Point of Aries. (3) Unit of angle, abbreviation ". One sixtieth part of a minute, or one 3600th part of a degree. (Cf. **time and time-keeping.**)

SECONDARY ELECTROMAGNETIC CONSTANTS. In transmission line theory, the **propagation constant** and the characteristic impedance are important secondary constants. In three-dimensional theory, the important constants are the intrinsic propagation constant σ , and the intrinsic impedance η , defined by

$$\sigma = \sqrt{i\omega\mu(g + i\omega\epsilon)} \quad \eta = \sqrt{\frac{i\omega\mu}{g + i\omega\epsilon}}$$

where μ and ϵ are the **permeability** and the **permittivity**, respectively, g is the specific conductivity, and ω is the radian frequency of the wave. These characteristics of the medium do *not* depend on the geometry of the wave.

SECONDARY FLOW. Flow due to the component of vorticity in the direction of the velocity; representing a rotation round the streamlines. Quantitatively it is equal to ξ/q , where $\xi = \omega \cdot \mathbf{v}/q$ is the component of **vorticity** in the direction of flow and q is the speed of flow, ω and \mathbf{v} being the vorticity and velocity.

In a stream in which the vorticity is normal to the streamlines, e.g., one of uniform shear, secondary flow is induced in curved flow such as the passage round an obstacle or round the bend of a channel or pipe. The vortex lines are wrapped round the obstacle; this is important in the case of wing roots or struts in the boundary layer of an aircraft surface. If a bend continues the secondary flow tends to oscillate about the position in which the **Bernoulli surfaces** of greatest total head lie on the outside of the bend, i.e., on the streamlines of least curvature.

A method due to Hawthorne of computing approximately the secondary flow is to assume that the flow in each Bernoulli surface is the same as if the whole fluid possessed the same total head as in that surface, and to compute the secondary flow from the distortion of the vortex lines in such a primary flow.

SECONDARY STRESSES. Stresses, forces, or moments in a structure which result from changes in geometry from the original configuration or any which are not the primary or essential means of carrying the applied load. Examples are **membrane stresses** induced in plates by deflection, or **bending moments** induced in riveted trusses which are designed basically as pin-connected.

SECOND CURVATURE OF A SURFACE. See **curvature of a surface, second.**

SECOND FOCAL POINT. In **Gaussian optics**, the axial image of an **infinite object point**.

SECOND FUNDAMENTAL THEOREM. In reactor theory, the assertion that in a uniform bare reactor the probability that a neutron does not escape from the system during the slowing-down process is the **Fourier transform** of the slowing-down kernel. (See **kernel, slowing down.**)

SECOND LIMIT THEOREM. A theorem of probability and statistics to the general effect that if a set of moments $\mu_j(n)$, ($j = 0, 1, \dots, \infty$) of a distribution $F(n)$ depending on a parameter n tend to the moments of a distribution F as n tends to a limit then the sequence $F(n)$ tends to F under certain conditions.

SECOND ORDER TRANSITION TEMPERATURE. Synonymous with **glass transition temperature.**

SECOND QUANTIZATION. See *quantization*, second.

SECOND VIRIAL COEFFICIENT. Coefficient of the $1/V$ (V is volume) term in the **equation of state**. For a classical monatomic gas the second virial coefficient B is given by the equation

$$B = 2\pi N \int_0^\infty (1 - e^{-\beta\phi(r)})r^2 dr,$$

where $\phi(r)$ is the **interatomic potential**, N the number of particles in the system, and $\beta = 1/kT$ (k : Boltzmann's constant, T : absolute temperature).

SECTION MODULUS. The ratio of moment of inertia I about a principal axis, or sometimes any neutral axis, to the extreme fiber distance c for that axis is the section modulus, S or Z , for the axis. Section moduli are geometric quantities which are tabulated for rolled and extruded sections. For pure bending about a neutral axis x , with moment about the axis M_x , the maximum stress is

$$M_x c / I_x = M_x / Z_x.$$

SECTIONS, METHOD OF. See *Ritter method*.

SECULAR DETERMINANT. Synonymous with the determinant of the **characteristic equation of a matrix**.

SECULAR EQUATION. See **characteristic equation of a matrix**.

SEIDEL ABERRATIONS. The deviations of third order optical theory from Gaussian optics were analyzed by von Seidel in terms of five constituent aberration functions of an optical system, S_1, \dots, S_5 . They have the property that if $S_1 = \dots = S_{i-1} = 0$, then S_i can be simply interpreted as a monochromatic image defect of the system. To compute the Seidel aberrations from system data, see the **Schwarzschild-Kohlschütter formulas**. (See also **spherical aberration**; **coma**; **astigmatism**; **curvature of field**; and **distortion**.)

SEIDEL METHOD. The term sometimes applied to the single-step iteration for solving linear equations. (See **matrix inversion**.) Actually the method Seidel (and Gauss) described was a method of relaxation.

SELECTION RULES. A radiative transition between two states of an atom, molecule, etc., can take place only if the transition moment R^{nm} is different from zero. Here

$$R^{nm} = \int \psi_n^* M \psi_m d\tau,$$

where ψ_n and ψ_m are the **eigenfunctions** of the states involved in the transition, and M stands for the electric dipole moment, magnetic dipole moment, electric quadrupole moment, etc. of the system. The condition $R^{nm} \neq 0$ is fulfilled only if the quantum numbers of the states n and m obey certain rules, the so-called selection rules. The selection rules depend on the type of eigenfunction of the states involved, and are different for transitions taking place by electric dipole radiation, magnetic dipole radiation, electric quadrupole radiation, etc. Transitions by electric dipole radiation, for example, are governed by the strict selection rules: $J = 0, \pm 1$ with the restriction $J = 0 \leftarrow \rightarrow J = 0$, and the **Laporte rule** (even terms combine only with odd, and odd only with even). In addition, there are approximate selection rules which hold only for weak **coupling** between **L** and **S** (small **multiplet** splitting): $\Delta L = 0, \pm 1$, and $\Delta S = 0$ (Prohibition of intercombinations). For the specific selection rules applying in different cases, see the relevant entries.

SELECTION RULES, NUCLEAR. A set of statements that serve to classify transitions of a given type (emission or absorption of radiation, β -decay, and so forth) in terms of the **spin** and **parity** (I and π) **quantum numbers** of the initial and final states of the systems involved in the transitions, in such a way that transitions of a given order of inherent probability (after making allowance for the influence of varying energy, charge and size of system, and so forth) are grouped together. The group having highest probability of taking place per unit time is said to consist of allowed transitions; all others are called **forbidden transitions**. Table 1 lists the selection rules for radiative transitions: each entry gives the character (E = electric, M = magnetic) and the multipole order (1 for dipole, 2 for quadrupole, 3 for octopole, . . .) of the predominant radiation mechanism for the indicated spin change ΔI and parity change $\Delta\pi$; the entry "no" means that radiative transitions are strictly forbidden.

TABLE 1

$\Delta\pi$	ΔI						
	0 $I = 0$	0 $I \neq 0$	1	2	3	4	5
No	None	M1	M1	E2	M3	E4	M5
Yes	None	E1	E1	M2	E3	M4	E5

Table 2 lists the selection rules for β -decay; the entry *A* means that for the indicated spin and parity change the transition is allowed; I, means that it is first forbidden; II, second forbidden . . .

TABLE 2

$\Delta\pi$	ΔI						
	0	1	2	3	4	5	6
No	A	A	II	II	IV	IV	VI
Yes	I	I	I	III	III	V	V

Fermi selection rules and Gamow-Teller (GT) selection rules are alternative sets of rules for allowed β -transitions; so that a transition allowed according to either set is actually allowed.

SELECTIVE EMITTER. A surface whose monochromatic emissivity varies with wavelength.

SELECTIVE RADIATOR. A radiator whose spectral emissivity depends on the wavelength (in the visible part of the spectrum).

SELF-CONJUGATE DIRECTION. See conjugate directions.

SELF-CONJUGATE NUCLEUS. See conjugate nuclei.

SELF-CONSISTENT FIELD. A simple way of describing the motion of a system of particles is to consider the motion of each particle in the field of the others. This field is self-consistent if the motion of the particles determined in the field leads to a charge distribution which gives rise to the same field. Such

a self-consistent field is the basis of atomic calculations according to the Hartree approximation or the Hartree-Fock method.

SELF-ENERGY IN A DIELECTRIC. The part of the free energy which is required to polarize a dielectric when it is placed an external electric field. It is generally assumed that the self-energy F_s is proportional to the volume of the dielectric and to the square of the polarization:

$$F_s = \gamma VP^2.$$

The coefficient of proportionality γ depends on the shape of the dielectric.

SELF-INDUCTANCE. See inductance.

SELLMEIER EQUATION. See Cauchy formula for refractive index.

SEMICONDUCTOR, DENSITY OF MOBILE CHARGE IN. See density of mobile charge in semiconductor.

SEMICUBICAL PARABOLA. See parabola.

SEMI-EMPIRICAL MASS FORMULA. See mass formula, semi-empirical.

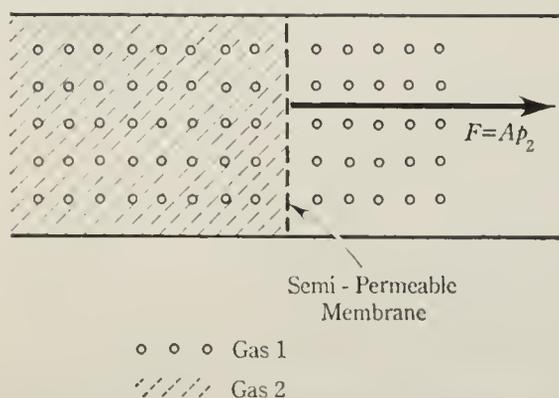
SEMI-INVERSE METHOD (OF SOLUTION OF PROBLEMS IN ELASTICITY THEORY). The method consists in making certain assumptions, usually from intuitive considerations and from arguments concerned with symmetry, regarding the stress, strain or displacement fields while leaving enough freedom in them to enable us to satisfy the equations of equilibrium (or motion), the equations of compatibility and the boundary conditions of the problem. Typical examples occur in classical elasticity theory in the problems of torsion and of flexure.

SEMI-PERMEABLE MEMBRANE (SEMI-PERMEABLE DIAPHRAGM). A membrane or septum through which one (or more) of the substances composing a mixture or solution may pass, but not all.

In osmotic pressure determinations, semipermeable membranes permit the passage of a solvent but not of certain colloidal or dissolved substances. Many natural membranes are semipermeable, e.g., cell walls; other membranes may be made artificially, e.g., by precipitating copper ferrocyanide in the interstices of a porous cup, the cup serving as a

frame to give the membrane stability. (See **membrane equilibrium**.)

Semipermeable membranes are also used in the separation of gases. When a semipermeable membrane is placed in a gas mixture,



Separation of gases by semipermeable membrane.

being impermeable to gas 2 and allowing gas 1 to pass, the force exerted on it will equal the area times the partial pressure of gas 2 only. While there are no ideal semipermeable membranes for gases, there exist in practice reasonable approximations to them, such as incandescent platinum or palladium sheets, which can be penetrated by hydrogen but not by other gases. A film of water also acts as a semipermeable membrane for gases, since it is pervious to NH_3 or SO_2 because of their solubility in water, but gases which are not easily soluble are held back.

SEMI-POLAR DOUBLE BOND. The term "semi-polar double bond" has been used by Lowry and Sidgwick to designate the N—O bond in substances of the type $(\text{R})_3\text{NO}$. This bond is best understood as a coordinate link where a pair of electrons of the N atom is shared with the O atom.

SEMITONE (HALF-STEP). The interval between two sounds whose basic frequency ratio is approximately equal to the twelfth root of two. The interval, in equally tempered semitones, between any two frequencies, is 12 times the logarithm to the base 2 (or 39.86 times the logarithm to the base 10) of the frequency ratio. (See **scale, equally-tempered**; **scale, just**.)

SENSE OF A VECTOR. The direction into which the vector is pointing.

SENSITIVITY. In general, a measure of response to a stimulus, the sensitivity being the

greater as the response to a given stimulus is the greater (or the stimulus to produce a given response is the smaller) e.g. the sensitivity of a galvanometer may be expressed as so many divisions per microampere; if expressed as the number of microamperes for full-scale deflection, it is understood that the greater this number, the smaller (inversely) is the sensitivity.

In feedback theory the term is more specifically applied to the response of the **gain** of a feedback system to the change in some circuit parameter. It is usually defined as the ratio of the fractional change in gain ratio caused by a certain fractional change in the parameter; thus

Sensitivity

$$= \frac{d(\text{Gain Ratio})}{\text{Gain Ratio}} \bigg/ \frac{d(\text{Parameter})}{\text{Parameter}}$$

$$= \frac{\text{Increment in gain (in logarithmic units)}}{\left\{ \begin{array}{l} \text{Increment in log of parameter (in} \\ \text{same units).} \end{array} \right\}}$$

(It should be noted however, that some authors, notably Bode, define sensitivity as the reciprocal of the above expression. Such a definition appears to be in opposition to the general meaning of the term as given above.)

Sensitivity in Terms of System Determinant. Let the external input be θ_1 , the input quantity (of which θ_1 is a component) be q_1 , and the output quantity be q_n . Let the parameter in question be λ and let the system equations be written in such a form that λ occurs as a constituent of a single equation coefficient, say a_{jk} , the equations being written (in Laplace transform form)

$$Q_1 = (1 + a_{11}Q_1) + a_{21}Q_1 + \cdots + a_{n1}Q_n + \theta_1$$

$$0 = a_{1m}Q_1 + a_{2m}Q_2$$

$$+ \cdots + a_{nm}Q_n \quad (1 < m \leq n)$$

in the absence of initial conditions. Then

$$G \equiv \text{gain ratio} = \frac{Q_n}{\theta_1} = - \frac{\Delta_{n1}}{\Delta}$$

where Δ is the system determinant

$$\begin{vmatrix} a_{11} & \cdot & \cdot & \cdot & a_{1n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ a_{n1} & \cdot & \cdot & \cdot & a_{nn} \end{vmatrix}$$

and Δ_{n1} is the co-factor of the element a_{n1} in Δ . Thus

$$\begin{aligned} \frac{dG}{d\lambda} &= -\frac{1}{\Delta^2} \left\{ \Delta \frac{d\Delta_{n1}}{d\lambda} - \Delta_{n1} \frac{d\Delta}{d\lambda} \right\} \\ &= -\frac{1}{\Delta^2} \{ \Delta_{n1,jk} - \Delta_{n1}\Delta_{jk} \} \\ &= +\frac{1}{\Delta^2} \Delta_{nk}\Delta_{j1} \end{aligned}$$

by a well-known identity in determinantal theory. Note that this result is still valid if $k = 1$, i.e., if λ occurs in the first row of Δ and is therefore absent in Δ_{n1} , for in this case

$$\frac{dG}{d\lambda} = \frac{\Delta_{n1}}{\Delta^2} \frac{d\Delta}{d\lambda} = \frac{\Delta_{n1}\Delta_{j1}}{\Delta^2}.$$

Hence,

$$\begin{aligned} \text{sensitivity} &= \frac{d \log G}{d \log \lambda} = -\frac{\Delta_{nk}\Delta_{j1}}{\Delta^2} \cdot \frac{\Delta}{\Delta_{n1}} \cdot \lambda \\ &= -\lambda \frac{\Delta_{nk}\Delta_{j1}}{\Delta\Delta_{n1}}. \end{aligned}$$

Relation to Return Difference. When $\lambda = 0$ let $G = G_o$ and consider the sensitivity assessed with respect to the differential gain ratio, $G - G_o$. Clearly

$$G - G_o = -\frac{\Delta_{n1}}{\Delta} + \frac{\Delta_{n1,o}}{\Delta_o} = \frac{\Delta\Delta_{n1,o} - \Delta_o\Delta_{n1}}{\Delta\Delta_o},$$

the zero suffix indicating throughout the value of the suffixed quantity when $\lambda = 0$. Since moreover G_o by definition does not contain λ ,

$$\frac{d}{d\lambda} (G - G_o) = \frac{dG}{d\lambda} = -\frac{1}{\Delta^2} \{ \Delta_{n1,jk} - \Delta_{n1}\Delta_{jk} \}$$

as before. Moreover

$$\begin{aligned} \Delta_{n1} &= \Delta_{n1,o} + \lambda\Delta_{n1,jk} \\ \Delta &= \Delta_o + \lambda\Delta_{jk}. \end{aligned}$$

Substituting from these expressions for $\Delta_{n1,jk}$ and Δ_{jk}

$$\begin{aligned} \frac{d}{d\lambda} (G - G_o) &= -\frac{1}{\lambda\Delta^2} \{ \Delta(\Delta_{n1} - \Delta_{n1,o}) - \Delta_{n1}(\Delta - \Delta_o) \} \\ &= +\frac{1}{\lambda\Delta^2} \cdot \Delta\Delta_o(G - G_o). \end{aligned}$$

Thus

$$\frac{d \log (G - G_o)}{d \log \lambda} = \frac{\Delta_o}{\Delta} = \frac{1}{\text{return difference of } \lambda}$$

or, the sensitivity, assessed with respect to the difference in gain ratio when the parameter has normal and zero values, is the reciprocal of the **return difference** for the element. As a particular case, if the parameter is such that its vanishing makes the gain also vanish ($G_o = 0$), then the sensitivity (as normally defined) is the reciprocal of the return difference.

Relative Sensitivity. Following a similar line of thought to that in the last section, suppose that λ has its *reference value* $\bar{\lambda}$; then, by definition of the reference value, G vanishes. Consider λ as equal to the sum of the reference value $\bar{\lambda}$ and the deviation therefrom, $(\lambda - \bar{\lambda})$, and assess the sensitivity on the basis of an incremental change not in $\log \lambda$ but in $\log (\lambda - \bar{\lambda})$. This is called the *relative sensitivity*. Thus

$$\text{relative sensitivity} = \frac{d \log G}{d \log (\lambda - \bar{\lambda})}.$$

Since Δ_{n1} is linear in λ and vanishes when $\lambda = \bar{\lambda}$, we have

$$\Delta_{n1} = (\lambda - \bar{\lambda})\Delta_{n1,jk}$$

$$\therefore G = -(\lambda - \bar{\lambda}) \frac{\Delta_{n1,jk}}{\Delta}$$

$$\begin{aligned} \therefore \frac{dG}{d(\lambda - \bar{\lambda})} &= -\frac{1}{\Delta^2} \{ \Delta - (\lambda - \bar{\lambda})\Delta_{jk} \} \Delta_{n1,jk} \\ &= -\frac{\Delta_o + \bar{\lambda}\Delta_{jk}}{\Delta^2} \Delta_{n1,jk} \\ &= -\frac{\Delta_{\bar{\lambda}}\Delta_{n1,jk}}{\Delta^2} \end{aligned}$$

where $\Delta_{\bar{\lambda}}$ is the value of Δ when $\lambda = \bar{\lambda}$.

$$\begin{aligned} \therefore \text{relative sensitivity} &= \frac{d \log G}{d \log (\lambda - \bar{\lambda})} \\ &= \frac{dG}{d(\lambda - \bar{\lambda})} \cdot \frac{\lambda - \bar{\lambda}}{G} \\ &= \frac{\Delta_{\bar{\lambda}}}{\Delta} \end{aligned}$$

or the *relative sensitivity* is the reciprocal of the return difference of the parameter considered

for a reference equal to the **reference value** of the parameter.

SEPARABLE GRAPH. See **graph, separable.**

SEPARABLE SPACE. A metric space is separable if there exists in it a countable subset of points which is everywhere dense. (See **Hilbert space.**)

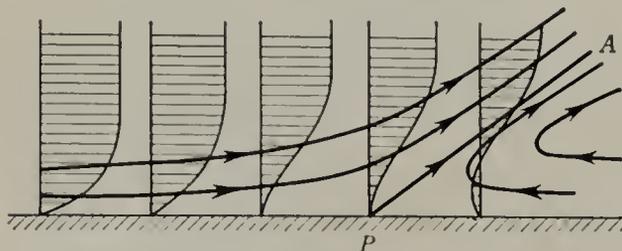
SEPARATION BUBBLE. See **reattachment of boundary layer after separation.**

SEPARATION OF BOUNDARY LAYER.

Separation of a boundary layer can only occur if the pressure gradient in the direction of the stream is positive. A fluid element near the boundary has small kinetic energy, and can only continue to move against a pressure gradient if the net force in the stream direction due to the shear stresses is sufficiently large. Otherwise, the pressure gradient causes the fluid element to come to rest and then to start moving in the reverse direction; eddies are formed further downstream and the streamlines are deflected away from the boundary by the region of eddies and reversed flow. This is known as separation of the boundary layer.

Since the possibility of avoiding separation depends on the magnitude of the shear stresses, there is an important difference between laminar and turbulent boundary layers. In a **laminar boundary layer** the shear stresses are caused only by viscosity and are comparatively small, so that separation usually occurs soon after the pressure minimum on a body. In a **turbulent boundary layer** the shear stresses are much larger, because of the existence of Reynolds stresses, so that separation is delayed for a greater distance after the pressure minimum, and on a streamline body may not occur at all.

The diagram shows stream lines and velocity profiles in a separating boundary layer; separation occurs at the point *P*, and the limiting



Stream lines and velocity profiles in separating boundary layer.

stream line at the boundary diverges from the boundary along the line *PA*. Downstream of *P* the flow near the wall is reversed in direction and hence the velocity gradient at the wall $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is negative. At the separation point $\left(\frac{\partial u}{\partial y}\right)_{y=0}$ is zero and hence the shear stress at the wall is zero.

Unsteady conditions in the eddying region downstream of separation often cause fluctuations in the external flow and in the pressure field, sometimes causing the separation point to fluctuate in position.

The positive pressure gradient required to cause separation of a laminar boundary layer is so small that on most shapes of body, even those of good "streamline" shape, there is a point at which the boundary layer separates if it is still laminar at that point. It can be shown that the position of this laminar separation point is independent of Reynolds number, for a specified pressure distribution along the body. At high Reynolds numbers, separation does not usually occur on bodies of good streamline shape, because the boundary layer becomes turbulent as soon as the pressure gradient becomes positive, and this happens before the laminar separation point is reached. At moderately high Reynolds numbers it is sometimes found that the boundary layer separates, as a laminar layer, at the laminar separation point, but that the separated layer then becomes turbulent almost immediately and reattaches to the surfaces. There is then a small "bubble" of separated flow which has little effect on the flow outside the boundary layer.

A "bluff" body is one for which the positive pressure gradient in potential flow is so large that even a turbulent boundary layer separates. Circular cylinders and spheres are examples of such bodies. On a bluff body the pressure downstream of separation is much lower than it would be in potential flow, so that there is a large "form drag" due to the distribution of normal pressure, the drag due to surface friction being relatively small. Separation occurs further upstream when the boundary layer is laminar at separation than when it is turbulent, so that in the former case the pressure at the rear of the body is lower and the drag is higher. This explains why, within a critical range of Reynolds numbers,

it may be possible to *reduce* the drag of a bluff body by slightly roughening its surface; the roughness makes the boundary layer turbulent and delays separation.

The problem of predicting the position of separation on a body, with a given pressure distribution, is equivalent to finding the position at which the shear stress at the wall falls to zero. For laminar boundary layers there are satisfactory approximate methods of solving the equations, and in most cases the position of separation can be predicted with fair accuracy. For turbulent boundary layers the calculations are less satisfactory, because there is not yet a complete theory of flow in a turbulent boundary layer, but in many cases calculations based on semi-empirical theories give a useful indication of the position of separation.

Separation of a boundary layer may be prevented or delayed by various methods of boundary-layer control. These methods include suction of fluid into a porous wall and blowing fluid tangentially along the wall in the direction of the stream.

SEPARATION OF GASES, REVERSIBLE. See **reversible separation of gases**.

SEPARATION OF VARIABLES. A method of finding a particular solution to a linear **partial differential equation**. Assume a solution which is the product of functions each depending on a single one of the independent variables. Substitution of this product into the original differential equation often results in an equation in which some terms are functions of only one variable, while other terms do not involve this variable. Each term involving a single variable may then be set equal to a constant. The original equation has thus been separated into several ordinary differential equations which may be solved by the usual methods. The constants of integration furnish enough parameters to satisfy the boundary conditions.

The procedure is particularly useful for the equations which occur in physical problems. For example, for **Laplace's equation** in Cartesian coordinates

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = 0,$$

assume $\phi(x,y,z) = X(x)Y(y)Z(z)$ and the particular solution becomes

$$\phi(x,y,z) = \exp(kx + ly + mz),$$

where $k^2 + l^2 + m^2 = 0$. The assumed solution with separated variables is often called the Bernoulli trial solution.

The name "separation of variables" is also used to refer to the much simpler problem of solving an ordinary differential equation $y' = F(x,y)$ by getting all terms involving y onto the left side and all those involving x onto the right.

SEQUENCE. A set of elements $s_1, s_2, \dots, s_n, \dots$, which can be arranged in an order so that when n is given, the n th member of the sequence s_n is completely specified. The elements are usually arranged by matching them up, one by one, with the positive integers 1, 2, 3, \dots, n, \dots . A common symbol for a sequence is $\{s_n\}$. If there exists a number N such that $N \geq |s_n|$ for all n , then the sequence is bounded. If there exists an l such that, after choice of any $\epsilon > 0$, there exists an N such that $|l - s_n| < \epsilon$ for all $n > N$, then the sequence is convergent to the limit l .

SEQUENCE (OF BANDS). See **band system**.

SEQUENCE, EDGE. See **edge sequence**.

SEQUENTIAL ANALYSIS. Sequential analysis refers to data obtained by a sequential procedure, that is to say, a rule under which the sample number is not fixed in advance, but depends to some extent on the outcome of the sampling as it proceeds. For example, it may be decided to toss a penny until 50 heads have appeared and the appearance of the 50th head terminates the sampling. Sequential procedures may be used for estimation or for testing hypothesis. In the latter case, they usually provide a specific rule for making one of the following three decisions: (1) to accept the hypothesis being tested (the null hypothesis), (2) to reject the null hypothesis, (3) to continue the experiment by making an additional observation.

SERBER-WILSON METHOD. In **neutron transport theory**, a method for improving upon the diffusion approximation by improving upon the conditions used to select appropriate solutions. As an example, the critical radius of a bare sphere is determined not by demanding that the diffusion-theory solution vanish at the extrapolated boundary, but that it satisfy

the precise integral equation at the center of the sphere.

SERIAL CORRELATION. See autocorrelation.

SERIES. A series is a pair of sequences $a_1, a_2, \dots, a_n, \dots$ and $S_1, S_2, \dots, S_n, \dots$ such that $S_n = a_1 + a_2 + \dots + a_n$, the first sequence being called the sequence of terms and the second the sequence of partial sums. It is customary to denote a series by the single expression $a_1 + a_2 + \dots + a_n + \dots$. A series is said to be convergent if its sequence of partial sums is convergent. For various types of series, **alternating, asymptotic, binomial**, etc., see under the respective terms.

SERIES INTEGRATION. Sometimes, when an **integral** cannot be evaluated in terms of known functions, the integrand may be expanded in a finite or infinite **series**. Integration of these terms, one by one, is series integration. It is necessary, of course, to investigate the **convergence** properties of the series for it may, or may not, give the correct value of the integral.

The reverse operation is also possible. Thus, given an infinite series, it is desired to represent it by a definite integral. The **Euler-Maclaurin formula** may be used in either way: to sum a series or to evaluate an integral.

SERRET-FRENET FORMULAE. The relations

$$\begin{aligned} dt/ds &= \kappa \mathbf{n}, & dn/ds &= \tau \mathbf{b} - \kappa \mathbf{t}, \\ db/ds &= -\tau \mathbf{n}, \end{aligned}$$

where \mathbf{t} , \mathbf{n} , \mathbf{b} are the unit tangent, unit normal and unit binormal respectively at a generic point of the curve, s is the distance measured along the curve to that point from a fixed point and κ and τ are the **curvature** and **torsion** of the curve at the point. Also called *Frenet formulae*.

SET. A collection of numbers or symbols considered as a whole. For example, the set of all prime numbers or the set of all matrices with determinant equal to unity. Geometrically, the symbols in a set determine a **domain**.

SET, CUT. See cut set.

SET, PERMANENT. When a solid has been strained beyond the **elastic limit** and the de-

forming stress is completely removed, in general the strain does not decrease ultimately to zero but to some non-vanishing value, known as a permanent set. In tension tests on metals the permanent set at stress σ and strain ϵ is taken as the total strain less the elastic recovery, $\epsilon - \sigma/E$.

SETTLEMENT OF SUPPORTS. In an **indeterminate structure** with **redundant constraints** the relative motion or settlement of supports may cause drastic redistribution of stress. The load carrying capacity of structures which are brittle or structures which fail by buckling will be affected greatly. However, the **plastic limit load** is unaffected.

SHAFT. A bar subjected primarily to twisting moment, a moment about the axis of the bar. (See **torsion**.)

SHAKEDOWN. When an elastic-plastic body is subjected to an infinite repetition of varying loads which produce plastic deformation, it is said to shake down if the plastic work has a finite limit. Loads below the shakedown limit eventually will produce elastic response only. Prager has stated and proved the theorem for elastic-perfectly plastic material: Shakedown will occur if a state of self-stress can be found such that the superposition of this state and the purely elastic response to the given variation of surface traction, body force, and temperature will at no point or instant lead to stress at or above the yield limit.

SHALLOW-WATER WAVE. (Also called long wave, Lagrangian wave.) An ocean wave whose length is sufficiently large compared to the water depth (i.e., 25 or more times the depth) that the following approximation is valid:

$$c = \sqrt{gh},$$

where c is the wave velocity, g the acceleration of gravity, and h the water depth. Thus, the velocity of shallow-water waves is independent of **wavelength** L . In water depths between $(1/2)L$ and $(1/25)L$ it is necessary to use the more precise expression

$$c = \sqrt{(gL/2\pi)[\tanh(2\pi h/L)]}.$$

SHANNON FORMULA. A theorem in information theory which states that a method

of coding exists whereby C binary digits per second may be transmitted with arbitrarily small frequency of error where C is given by

$$C = B \log_2 \left(1 + \frac{S}{N} \right)$$

and no higher rate can be transmitted; B is the bandwidth, and S/N is the **signal-to-noise ratio**.

SHAPE FACTOR. The ratio of the fully plastic moment for a perfectly plastic material to the moment which first causes the elastic stress to reach yield. Therefore, it is also the ratio of the sum of the static moments of the areas above and below the neutral axis for plastic bending about that axis, to the section modulus for elastic bending. The shape factor is 1.5 for a rectangular section and about 1.15 for an I-section bent about its strong axis.

SHARP SERIES. A series of lines in the spectra of the simpler atoms, the alkalis, alkaline earths, helium, etc., corresponding to transitions from an upper S state to a lower P state.

SHEAR. The component of resultant force lying in a plane is called the shearing force on the plane. A shear diagram is a graphical representation of the variation in shear along a beam. An illustration of a shear diagram is given below for an overhanging beam with a uniformly **distributed load** covering the entire length of the beam. The points where the shear changes sign are points of maximum **bending moment**. The area of the shear diagram between any two points is equal to the

change of bending moment between these points.

The slope of the shear diagram is the ordinate of the load diagram.

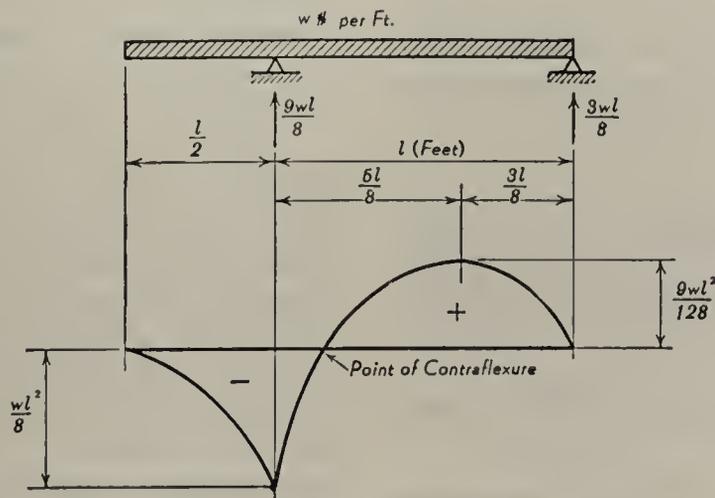
Rivets and welds are also subjected to shear. If the riveted connection is made so that the shear occurs between two lap plates only it is called single shear. If the connection is such that shearing force is carried on two planes as in the case of a butt joint, with one plate coming from one side between two from the other side, the condition is one of double shear.

SHEAR CENTER (CENTER OF TWIST). When the stresses acting on the cross section of a beam are those associated with bending and transverse shear alone, the resultant force is a transverse shear acting through the shear center. Bending by transverse forces without twist thus requires the external forces to be applied through the shear center.

SHEAR COMPLIANCE. See **compliance, shear**.

SHEAR DIAGRAM. See **shear**.

SHEAR FLOW. A thin-walled tube, profile or similar section beam resists transverse **shear** in its cross section primarily by shear stresses directed parallel to the center line of the thin sectional elements. The product of these shear stresses multiplied by the wall thickness is called a shear flow. Also, a closed tubular section resists **torsion** with a constant shear flow, a multi-cell tube, with a constant shear flow in each element. (See **Batho** and **Bredt** formulas.)



Shear diagram.

SHEAR-GRAVITY WAVE. A combination of gravity waves and a Helmholtz wave on a surface of discontinuity of density and velocity. If the densities of the lower and upper layers respectively are ρ and ρ' and the velocities U and U' , the phase speed c of the shear-gravity wave is

$$c = \frac{\rho U + \rho' U'}{\rho + \rho'} \pm \left[\frac{gL}{2\pi} \frac{\rho - \rho'}{\rho + \rho'} - \frac{\rho\rho'(U - U')^2}{(\rho + \rho')^2} \right]^{1/2},$$

where g is the acceleration of gravity and L the wavelength. The motion is unstable if and only if the bracketed quantity is negative; the density difference thus contributes to stability and the velocity difference to instability.

SHEAR, HORIZONTAL. The study of the stresses induced in beams bent by transverse loads usually is begun by considering vertical loads on a horizontal beam with the vertical plane a plane of symmetry of the beam. Shear force V on the cross section then is vertical, and vertical shearing stress is computed from the equal complementary horizontal shearing stress S_s , assumed uniformly distributed on a horizontal plane perpendicular to the cross section: $S_s = V\theta/Ib$, where I is the moment of inertia of the cross section about the neutral axis, b is the width of the cross section at the line of cut by the horizontal plane, and θ is the static moment of the area between this cut and the extreme fiber about the neutral axis.

SHEAR LAG. When shear strains accompanying bending or other normal strains are appreciable, the normal stresses are not distributed in accordance with elementary theory. Stiffeners of shell structures, for example, are not fully effective in carrying tension or compression. The term shear lag conveys this concept.

SHEAR, LONGITUDINAL. A more descriptive term for horizontal shear (see **shear, horizontal**) especially useful for thin-walled sections.

SHEAR MODULUS. See **modulus, shear; elastic constants.**

SHEAR, SIMPLE. A state of homogeneous deformation in a body, which in some rec-

tangular Cartesian reference system x may be described by

$$x_1 = X_1 + kX_2, \quad x_2 = X_2, \quad x_3 = X_3,$$

where k is a constant. k is called the *amount of shear* and $\tan^{-1} k$, the *angle of shear*. Any plane perpendicular to the x_3 axis is called a *plane of shear* or *shear plane*.

SHEAR STRESS. See **stress tensor; also flux vector.**

SHEAR STRESS, CRITICAL. The resolved shear stress which is required to initiate slip in a given crystallographic direction along a given crystallographic plane of a single crystal of a metal.

SHEAR, TRANSVERSE. See **shear.**

SHEAR WAVE. (1) A Helmholtz wave. (2) An elastic wave (see **waves, elastic, in solids**), which causes elements of the medium to change in shape without any change in volume. In a plane shear wave, the particle motion is perpendicular to the direction of propagation, and it is therefore a transverse wave. A shear wave may be defined mathematically as an elastic wave whose velocity field has zero divergence. Such waves travel with the velocity $\sqrt{(G/\rho)}$ through an isotropic medium, where G is the shear modulus and ρ is the density of the medium. Shear waves have also been called *equivoluminal waves*, *distortional waves*, and *rotational waves*.

SHEET. An in-plane load-carrying element whose middle surface is plane and whose thickness h is small compared with its overall dimensions in the plane of the middle surface (see **plate**). Generalized equations of plane stress are assumed to apply, e.g.,

$$\frac{\partial(h\sigma_x)}{\partial x} + \frac{\partial(h\tau_{xy})}{\partial y} + \rho h F_x = \rho h \frac{\partial^2 u}{\partial t^2}.$$

SHELL. A transverse load-carrying member, bounded by two essentially parallel surfaces, whose middle surface is curved and whose thickness h is small compared with its overall dimensions. It resists load through bending action as well as through **membrane stress**. Occasionally membranes, as for example a pressurized sphere, are called shells.

SHEPPARD'S CORRECTIONS. If a continuous frequency distribution is grouped, the

moments derived by replacing each observation by the central value of the group into which it falls differ from those of the original distribution. Provided the distribution tapers smoothly to zero in both directions, average corrections to the grouped moments, known as Sheppard's corrections, may be applied as follows:

$$\mu_1 (\text{corrected}) = \mu_1$$

$$\mu_2 (\text{corrected}) = \mu_2 - h^2/12$$

$$\mu_3 (\text{corrected}) = \mu_3$$

$$\mu_4 (\text{corrected}) = \mu_4 - \frac{1}{2}h^2\mu_2 + 7h^4/240.$$

SHIP MOTION. The complex motion imparted to a ship upon encountering waves. All the motions can be regarded as combinations of three oscillations about horizontal or vertical axes (roll, pitch, and yaw), and three linear displacements of the center of gravity (heave, surge, and sway).

SHOCK-EXPANSION THEORY. A method of calculating the pressure distribution, and hence the lift, pitching moment and wave drag, on a two-dimensional airfoil at supersonic speeds. The method is only applicable to airfoils with sharp leading and trailing edges. The strength of the upper and lower leading-edge shock waves are calculated from the deflection angles at the leading edge. The expansion over the convex surface of the airfoil is then calculated from simple-wave theory (essentially the theory of the **Prandtl-Meyer expansion**). Boundary-layer effects are neglected and it is assumed that the leading-edge shock waves are of uniform strength in the portion of the field which affects the pressure on the airfoil. The latter assumption is not correct, because the leading-edge shock waves are weakened by the expansion waves produced by the convex surface of the airfoil. Nevertheless, it has been shown that the theory gives the pressure distribution on the airfoil with good accuracy, even when the quantities neglected are no longer small.

SHOCK WAVE. (1) If the velocity of propagation of mechanical waves in a medium increases with increasing amplitude of the disturbance, the front of a pulse propagated through the medium becomes increasingly steep, and the limiting steepness is governed by the dissipating properties of the medium.

Such a wave is called a shock wave. Shock waves are produced when the compressibility of the medium decreases with increasing pressure, or when the particle velocity produced by the disturbance is comparable with the velocity of propagation of the wave. (See **finite amplitude equations (acoustic)**).

(2) A region of very small thickness in a flowing gas, across which the velocity of flow and the properties of the gas all change. The normal component of the flow velocity, relative to the shock wave, must always be supersonic on the upstream side and subsonic on the downstream side. The thickness of a shock wave decreases with increasing strength, and for a strong shock wave the thickness is only a little greater than the mean free path of the molecules.

The flow within a shock wave is strongly influenced by viscosity and thermal conductivity, but for calculating the overall changes across a shock wave these effects may be ignored and the shock wave treated as a discontinuity.

A shock wave may be either *normal* or *oblique* to the local flow direction. Considering first a normal shock wave, the equations of momentum, energy and continuity are

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2, \quad (1)$$

$$h_1 + \frac{1}{2}u_1^2 = h_2 + \frac{1}{2}u_2^2 \quad (2)$$

and

$$\rho_1 u_1 = \rho_2 u_2, \quad (3)$$

where h is specific enthalpy, u is velocity and subscripts 1 and 2 refer to conditions before and after the shock wave.

From Equations (1) to (3), the change of velocity across a normal shock wave can be related to the changes of pressure, density and temperature. It can be shown that the velocities u_1 and u_2 are related simply by

$$u_1 u_2 = a^{*2}, \quad (4)$$

where a^* is the speed of sound in the gas at the condition corresponding to a local Mach number of 1.

The gain of specific entropy in a normal shock wave is given by

$$\Delta s = \frac{2\gamma R(M_1^2 - 1)^3}{3(\gamma + 1)^2} + \text{higher-order terms in } (M_1^2 - 1), \quad (5)$$

where M_1 is the Mach number on the upstream side of the shock wave and R is the gas constant.

The equations for an oblique shock wave may be obtained by superposing on the solution for a normal shock wave a velocity component tangential to the shock wave. Consideration of the **momentum equation** shows that this tangential velocity component is the same on both sides of the shock wave. Thus, since the normal velocity component is reduced in passing through the shock wave, the stream is deflected away from the normal in passing through the shock wave. For given upstream conditions and a given angle of deflection of the stream, there are usually two alternative shock waves satisfying the equations. One of these is strong and nearly normal to the incident stream, while the other is weaker and inclined at a smaller angle to the stream. In most cases, the weaker of these two shock waves is found in experiments.

SHOCK-WAVE BOUNDARY-LAYER INTERACTION. Where a **shock wave** extends into a **boundary layer** there is a complicated region of interaction. This region is very difficult to analyze theoretically because the usual assumptions of boundary-layer theory are no longer valid. In many cases the shock wave causes separation of the boundary layer; this modifies the external pressure distribution and often causes another shock wave, starting at the position of separation.

SHORT-RANGE FORCE. A force of interaction between two particles which is essentially ineffective when the interparticle separation exceeds a certain distance; usually applied to nuclear forces which have a range of several times 10^{-13} cm.

SHORT-RANGE ORDER. The type of **order** in which the probability of a given type of atom having neighbors of a given type (for example, that an A atom is surrounded by B atoms) is greater than would be expected on a purely random basis. There is thus a tendency to form small ordered domains, but these do not link together at long distances. Short-range order occurs in binary alloys above the **order-disorder transition** temperature, and is measured by the parameter,

$$\sigma = \frac{q - q(\text{rand})}{q(\text{max}) - q(\text{rand})}$$

where q is the fraction of the total number of nearest-neighbor bonds in the solid between unlike atoms, $q(\text{max})$ is the maximum possible value of q for the given ratio of constituents, and $q(\text{rand})$ is the value of q for an entirely random arrangement.

Short range order is characteristic of liquids and disordered solids, in contrast to the **long range order** found in crystalline solids. (See also **cooperative phenomena**.)

SHOWER UNIT (COSMIC RAYS). The mean path length required for the reduction, by the factor $\frac{1}{2}$, of the energy of relativistic charged particles as they pass through matter. Such particles lose their energy mostly by radiation. The shower unit s is related to the **radiation length** l by the equation:

$$s = l \ln 2 = 0.693l.$$

SHROUD. See **blade**.

SIDEREAL YEAR. See **year**.

SIDESWAY. When a continuous frame structure, such as a portal or a building frame, is subjected to load, its joints will rotate and generally also move sideways, even with vertical load only. This sidesway makes analysis quite elaborate and often is considered separately in the **moment distribution** procedure.

SIEGBAHN UNIT (X-UNIT). A length of 10^{-11} cm, widely used in expressing the wavelength of x-rays.

SIGNAL FLOW DIAGRAMS. A pictorial way of representing the interrelations between the various quantities present in a system as well as the input or inputs applied to the system from outside.

The conventional way of setting up such a diagram is to consider the value of any quantity in the system as the sum of the contributions made to it by the other quantities and the inputs directly, that is, not through the medium of intermediate quantities. The quantities and inputs are usually inserted in the diagram within small circles, the interrelations being shown by line links connecting these circles, such links being labeled with the relevant factor or operator implied in the dependence and provided with an arrow to indicate the direction in which the contribution

is being made. The diagrams are interpreted by two essential rules:

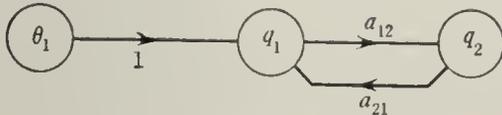
(1) The value of any quantity is the sum of its incoming feeds. (2) The value of a feed is the quantity from which the feed link originates multiplied by the link factor or operator.

For instance the equations

$$q_1(t) = \theta_1(t) + a_{21} \left(\frac{d}{dt} \right) q_2(t),$$

$$q_2(t) = a_{12} \left(\frac{d}{dt} \right) q_1(t)$$

would be represented by

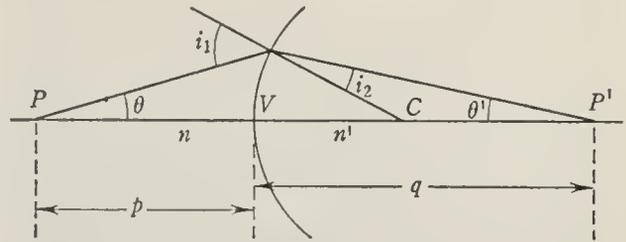


The diagrams are usually drawn in terms of the **Laplace transform** of the quantities rather than in the time domain.

SIGNAL LEVEL. At any point in a transmission system, the difference of the measure of the signal at that point from the measure of an arbitrarily-specified signal chosen as a reference. In audio techniques, the measures of the signal are often expressed in **decibels**, thus their difference is conveniently expressed as a ratio.

SIGN CONVENTION (LENS AND MIRROR). All authors do not follow the same sign convention. The following probably has the largest following: (1) Draw all figures with the light incident on the reflecting or refracting surface from the left. (2) Consider the object distance $p = PV$ positive when P is at the left of the vertex. (3) Consider the image distance $q = VP'$ positive when P' is at the right of the vertex. (4) Consider the radius of curvature $R = CV$ positive when the center of curvature lies to the right of the vertex. (5) Consider the slope angles positive when the axis must be rotated counterclockwise through less than $\pi/2$ to bring it into coincidence with the ray. (6) Consider angles of incidence and refraction positive when the **radius of curvature** must be rotated counterclockwise through less than $\pi/2$ to bring it into coincidence with the ray. (7) Consider distances normal to the axis positive when measured upward.

In the following diagram only θ' is negative.



When the convention is followed, the simple lens formula takes the form

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = (n - 1) \left(\frac{1}{R_1} - \frac{1}{R_2} \right).$$

SIGNIFICANCE TESTS. Suppose that a sample provides an estimate t of a parameter θ , and that a certain hypothesis specifies a certain value for θ , $\theta = \theta_0$ say. t will differ from θ_0 by a discrepancy $d = \theta_0 - t$, and it may be possible to deduce from the sampling distribution of t the probability P that a discrepancy as large as d would have arisen if the hypothesis $\theta = \theta_0$ were true. If this probability is small, the sample may be taken to provide evidence against the truth of the hypothesis. This procedure is called a test of significance; if P is less than some value α (commonly chosen to be 0.05 or 0.01), we say that t is significantly different from θ_0 at the level α , or simply that d is significant at this level. The hypothesis $\theta = \theta_0$ (i.e., $d = 0$) is called the null hypothesis.

Alternative tests of the same hypothesis are often available, based on different statistics. To choose between them, we introduce the notion of an alternative hypothesis, $\theta = \theta_1$ say. We may now calculate, for a given α , the probability β of obtaining a significant result at this level when the alternative hypothesis is true. This probability (a function of α and θ_1) is called the power of the test. A few tests can be shown to be at least as powerful as any alternative test for all values of θ_1 . Such tests are said to be uniformly most powerful and are clearly to be preferred in cases where they exist.

If we imagine some action being taken based on the result of a significance test, this action may be referred to as accepting or rejecting the null hypothesis. Sampling fluctuations may then lead to two types of incorrect action:

(1) We may reject the null hypothesis when it is true.

(2) We may accept the null hypothesis when it is false.

These are referred to as errors of the first and second kind; their probabilities are respectively α and $(1 - \beta)$. A test for which $(1 - \beta) \geq \alpha$ for all values of θ_1 is said to be unbiased. (See also **Neyman-Pearson theory**.)

SIGNIFICANT FIGURES (OR DIGITS).

Those digits that can be considered correct in the approximate representation of a quantity, whether measured or computed, other than zeros to the right of the decimal and to the left of the first nonnull digit. Thus in 0.00123, the significant digits are 1, 2, and 3. To write a number in the form 0.001230 is to imply that the final digit is known to be a zero, and that the four digits, 1, 2, 3, and 0 are all significant.

SIGNUM. The signum function of x , written $\text{sgn } x$ or $\text{sg } x$ is -1 when $x < 0$, 0 when $x = 0$, and 1 when $x > 0$.

SILSBEE RULE. A long circular wire of radius a cannot carry a supercurrent greater than $\frac{1}{2}aH_c$, where H_c is the **critical field** of the material. Evidently the magnetic field of the current itself then destroys the **superconductivity**.

SIMILARITY HYPOTHESES. See **Kolmogoroff similarity hypotheses**.

SIMILARITY LAW, TRANSONIC. See **transonic similarity law**.

SIMILARITY THEOREM. For any given dynamical system consisting of connected particles and rigid bodies it is possible to construct another system exactly similar to it but on a different scale. If the masses and forces in the two systems bear certain ratios to each other, the performance of the two systems will be similar but with different velocities that bear a constant relationship to each other.

Specifically, to determine the relation between the various ratios involved, let the linear dimensions of the systems 1 and 2 be in the ratio $x:1$; let the masses of the corresponding particles be in the ratio $y:1$; let the rates of operation be in the ratio $z:1$, so that time between corresponding phases will be in the ratio $1:z$; and let the forces be in the ratio $w:1$. The equation of motion of a particle is given by

$$f_M = m\ddot{x} \quad (1)$$

where f_M is force, in dynes, m is mass, in grams, and \ddot{x} is acceleration, in centimeters per second per second.

Referring to equation 1 and employing the ratios specified above, it is seen that m will be changed in the ratio $y:1$, \ddot{x} will be changed in the ratio $xz^2:1$, and f_M will be changed in the ratio $w:1$. Thus, it follows that the relation between the four numbers x , y , z and w as specified above is given by

$$w = xyz^2. \quad (2)$$

The principle of similarity in acoustics states: For any acoustical system involving diffraction phenomena it is possible to construct a new system on a different scale, which will exhibit similar performance, providing the wavelength of the sound is altered in the same ratio as the linear dimensions in the new system.

The principle of similarity is useful in predicting the performance of similar acoustical systems from a single model. For example, a small model can be built and tested at very high frequencies to predict the performance of similar large systems at lower frequencies. For example, in the diffraction of sound by objects, if the ratio of the linear dimensions of the two objects is $x:1$, the corresponding frequency characteristics will be displaced $1:x$ in frequency.

SIMPLE BEAM. See **beam, simple**.

SIMPLE CURVE. A curve without double points; that is, a set of points (x,y) with $x = x(t)$, $y = y(t)$ for t in an interval $a \leq t \leq b$, such that x and y do not assume the same pair of values for two different values of t in $a < t < b$. If $x(a) = x(b)$ and $y(a) = y(b)$, the simple curve is said to be closed.

SIMPLE EXTENSION. A state of pure homogeneous deformation in a body, associated with a homogeneous stress system for which, in some rectangular Cartesian reference system, all the stress components are zero, except one of the normal components. The direction of this non-vanishing stress component is called the *direction of the simple extension*.

SIMPLE FLEXURE OF A BEAM. See **flexure, simple (of a beam)**.

SIMPLE HARMONIC MOTION. Motion in which the particle is attracted towards an

origin by a force directly proportional to the instantaneous distance of the particle from the origin. The resulting periodic motion is characterized by a space-time graph of simple sine form.

The differential equation of motion is

$$m \frac{d^2x}{dt^2} = -kx$$

where m is the mass of the particle, k is the constant of proportionality (**stiffness coefficient**), x is the displacement from origin.

The solution of the differential equation yields the displacement as a function of the time.

$$x = A \frac{\sin}{\cos} \{2\pi ft + \epsilon\}$$

where A is the amplitude of motion, f is the frequency = $(1/2\pi)\sqrt{k/m}$, ϵ is the **epoch** determined by the value of the function at $t = 0$.

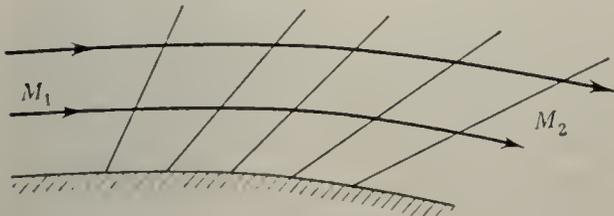
The projection of a particle moving in a circle of radius A and with frequency f onto any straight line in the plane of the circle travels according to the above sinusoidal formula. This is called the circle of reference. (See **harmonic oscillator**.)

SIMPLE SHEAR. See **shear, simple**.

SIMPLE SYSTEM. See **indicial notation**.

SIMPLE TRUSS. See **truss, simple**.

SIMPLE WAVE. In two-dimensional supersonic flow a simple wave is a field of flow in which one family of **Mach lines** consists of straight lines. An example shown in the figure is the expansion of an initially uniform



Simple Wave.

flow along a convex boundary; a special case of this is the **Prandtl-Meyer expansion** or centered wave. The formulae obtained for a Prandtl-Meyer expansion may be applied to the calculation of any simple wave flow.

SIMPLEX METHOD. A method due to George Dantzig for solving the **linear pro-**

gramming problem. Let it be required to find that vector \mathbf{x} satisfying

$$\mathbf{Ax} = \mathbf{h}, \quad \mathbf{x} \geq 0$$

such that $\phi = \alpha + \mathbf{a}^T \mathbf{x}$ is minimized, where the **matrix** \mathbf{A} has n rows and $N > n$ columns, and the vectors \mathbf{x} and \mathbf{h} are of dimensions N and n , respectively. It is assumed that \mathbf{A} is of rank n . It is required that initially one find a particular solution \mathbf{x} of the inequalities, with $N - n$ of the elements null, and such that the matrix of coefficients of the others is nonsingular. After a possible renumbering, it can be supposed that the null elements are the last $N - n$. Partition the system with $\mathbf{x}^T = (\mathbf{y}^T, \mathbf{z}^T)$, where \mathbf{y} and \mathbf{z} are of dimension n and $N - n$, respectively, and write

$$\mathbf{By} + \mathbf{Cz} = \mathbf{h}, \quad \mathbf{y} = \mathbf{B}^{-1}\mathbf{h} - \mathbf{B}^{-1}\mathbf{Cz}.$$

At this stage the elements of \mathbf{y} are called the basic variables, those of \mathbf{z} the nonbasic variables, and the initial solution of the inequalities has $\mathbf{y} = \mathbf{B}^{-1}\mathbf{h} \geq 0$ and $\mathbf{z} = 0$.

Now ϕ can be written

$$\begin{aligned} \phi &= \alpha + \mathbf{b}^T \mathbf{y} + \mathbf{c}^T \mathbf{z} \\ &= (\alpha + \mathbf{b}^T \mathbf{B}^{-1} \mathbf{h}) + (\mathbf{c}^T - \mathbf{b}^T \mathbf{B}^{-1} \mathbf{C}) \mathbf{z}. \end{aligned}$$

If $\mathbf{c}^T - \mathbf{b}^T \mathbf{B}^{-1} \mathbf{C} \geq 0$ the problem is already solved. If not, consider one of its negative elements. By allowing the corresponding element of \mathbf{z} to become positive but holding all others at zero, ϕ can be decreased, but, in general, some element of $\mathbf{y} = \mathbf{B}^{-1}\mathbf{h} - \mathbf{B}^{-1}\mathbf{Cz}$ will eventually vanish. At this point, where the first element of \mathbf{y} has vanished, the \mathbf{z} which has been allowed to increase is adjoined to the set of basic variables, and the \mathbf{y} which has vanished is adjoined to the nonbasic ones, and the process is repeated.

SIMPLY-CONNECTED REGION. A region with the property that any closed curve lying in the region can be continuously deformed into a single point without leaving the region.

SIMPSON RULE (FOR NUMERICAL QUADRATURE).

$$\int_{x_0}^{x_2} f(x) dx = \frac{h}{3} (f_0 + 4f_1 + f_2) + R,$$

where

$$f_i = f(x_0 + ih) = f(x_i)$$

$$R = -h^5 f^{IV}(\xi)/90.$$

Hence

$$\int_{x_0}^{x_{2n}} f(x)dx \doteq \frac{h}{3}[f_0 + 4f_1 + 2f_2 + 4f_3 + \cdots + 4f_{2n-1} + f_{2n}].$$

SIMULTANEOUS EQUATIONS. The solution of simultaneous linear equations is discussed under **matrix inversion**. In case there are more equations than unknowns, it may be that some are redundant, and can be dropped without affecting the theoretical solution. If not, the equations are inconsistent, and only a **least squares** or other approximation can be used. If there are fewer equations than unknowns the determination cannot be unique, and at most it will be possible to leave some of the unknowns arbitrary, solving for the others in terms of these.

Even with nonlinear equations it is sometimes possible to eliminate some of the unknowns by solving and substituting, although this may not be advisable since the analytic expressions tend to be more complicated. In any case, suppose the system has the form

$$\phi_i(\xi_1, \dots, \xi_n) = 0, \quad i = 1, 2, \dots, n,$$

assume no further reduction is possible or desirable, and suppose that a solution $\alpha_1, \alpha_2, \dots, \alpha_n$ actually exists. If

$$\Phi = \sum_i^n \phi_i^2,$$

then $\Phi(\xi_1, \dots, \xi_n) \geq 0$ and $\Phi(\alpha_1, \dots, \alpha_n) = 0$. Hence Φ is minimized for $\xi_i = \alpha_i$. Therefore one may use the method of **steepest descent** as one method of solving the system, provided the necessary derivatives exist.

Another method is a generalization of Newton's method. Suppose $\xi_i^{(0)}$ is a set of numbers sufficiently close to α_i . Then, by expanding in **Taylor's series**,

$$\begin{aligned} \phi_i(\alpha_1, \dots, \alpha_n) &= 0 \\ &= \phi_{i,0} + \sum_j (\alpha_j - \xi_j^{(0)}) \partial \phi_{i,0} / \partial \xi_j + \cdots, \end{aligned}$$

where the subscript 0 signifies evaluation for $\xi_i = \xi_i^{(0)}$. If the differences $\alpha_j - \xi_j^{(0)}$ are small enough and the terms not written (quadratic and higher) are negligible, and if, further, the matrix of first partial derivatives

$$(\partial \phi_i / \partial \xi_j)$$

is nonsingular throughout a sufficiently large neighborhood of the solution α_j , then the linearized equations can be solved and the result will be approximately equal to the differences $\alpha_j - \xi_j^{(0)}$. By adding these to the $\xi_j^{(0)}$ one obtains an improved set, and the process can be continued as many times as seems necessary.

SIMULTANEOUSLY MEASURABLE. In quantum mechanics, physical quantities correspond to linear operators which in general do not commute. The measurement of a quantity A is related to the **eigenvalues** and **eigenstates** of the operator A . Two quantities A and B are simultaneously measurable if the operators A and B commute, $AB = BA$. It is only in this case that any eigenstate of A is an eigenstate of B .

SINE CONDITION. See **Abbe sine condition**.

SINE RELATIONSHIP, EXTENDED. See **extended sine relationship**.

SINGLE-ACTING MACHINE. See **double-acting machine**.

SINGLE-STEP ITERATION. See **matrix inversion**.

SINGLE-VALUED MAPPING. See **representation**.

SINGLE-VALUED REPRESENTATION. See **representation**.

SINGULAR INTEGRAL EQUATION. An integral equation with infinite limits of integration or an infinite **kernel**.

SINGULARITIES. See **singular point of a function**.

SINGULARITY FUNCTIONS. An infinite family of **excitation** time-functions characterized by discontinuities, and obtained by successive differentiation and integration of the **impulse function**.

SINGULAR MATRIX. A matrix whose **determinant** vanishes and where a transformation involving that matrix is also singular.

SINGULAR POINT OF A DIFFERENTIAL EQUATION. Consider the linear differential equation with variable coefficient

$$w^{(n)} + P_1(z)w^{(n-1)} + \cdots + P_{n-1}(z)w' + P_n(z)w = 0.$$

A point Z_0 is called a singular point for the equation if Z_0 is a singular point for at least one of the functions $P_i(z)$. If these coefficients $P_i(z)$ can be written in the form $P_i(z)^{-i} = (Z - Z_0)p_i(z)$ with $p_i(z)$ analytic at Z_0 , then Z_0 is called a regular singular point of the equation. The importance of such points lies in the fact that, for the case $n = 2$ for example, there exist in the neighborhood of regular singular point two linearly independent solutions of the equations which are regular in the sense of Fuchs; that is, which are of the form $w = Z^\alpha h(z) \log z + z^\beta k(z)$, where $h(z)$ and $k(z)$ are analytic at Z_0 .

SINGULAR POINT OF A FUNCTION. A value of the complex variable z , for which $f(z)$ is an analytic function, is called an ordinary point. Any point which is not an ordinary point is a singular point.

Singular points or singularities are classified as: (1) poles or unessential singularities; (2) essential singularities or poles of infinite order; (3) branch points caused by the fact that the function is not single-valued.

Let $w = u + iv$ be a single-valued function of $z = x + iy$, where u, v are real single-valued functions of x and y . Then $z = z_0$ is a pole of order k , provided that $(z - z_0)^k w(z)$ is analytic and not zero at $z = z_0$. The positive integer k is called the order of the pole. Singular points of this kind are non-essential because they are effectively removed if $w(z)$ is multiplied by $(z - z_0)^k$. Typical examples are $w = 1/z(z - a)(z - b)$, which has three simple poles at $z = 0, a, b$, respectively; $w = \csc z$, an infinite number of poles on the real axis.

A singularity which is not a pole or a branch point is called essential. A simple example is $w(z) = \sin 1/z = 1/z - 1/3!z^3 + 1/5!z^5 - 1/7!z^7 \cdots$ and it is seen that no finite value of k in $z^k w(z)$ will remove the singular point of this function at $z = 0$.

The essential or non-essential character of a singularity can be investigated by expansion of the function in a **Laurent series**. In most cases, the result can be obtained more simply by inspection of the function. For **branch points**, see under that heading.

Singular points are of considerable importance in the study of **linear differential equations**, especially when they are to be solved by expansions in series. (See previous entry.)

An irregular singular point may arise from

the **confluence** of two or more regular singular points.

SINGULAR SOLUTION OF A DIFFERENTIAL EQUATION. A solution to a differential equation which is not a special case of the general solution hence not obtainable from the general solution by assigning a value to its parameters. (See e.g., **Clairaut equation**.)

SINGULAR VALUES OF A MATRIX. The singular values of a matrix A are the nonnegative square roots of the proper values of A^*A .

SINK. A point into which fluid disappears. The strength $-m$ of a sink is the total flux of fluid into it. The velocity potential at distance r due to a sink is

$$\phi = \frac{m}{4\pi r}$$

A *line sink*, in two-dimensional flow, is a line into which fluid disappears. The strength $-m$ of a line sink is the total flux of fluid into unit length of it. The complex potential of a line sink is

$$\begin{aligned} w = \phi + i\psi &= -\frac{m}{2\pi}(\log r + i\theta) \\ &= -\frac{m}{2\pi} \log z. \end{aligned}$$

SINUSOIDAL LIMIT THEOREM. A theorem in time-series analysis due to E. Slutsky. If a moving average of two is taken of a random series and the process iterated n times; if m^{th} differences are taken of the result; then the resulting series tends to the form of a sine wave as m and n tend to infinity such that the ratio m/n remains constant.

SIX-VECTORS. See **four vectors and tensors**.

SIZE EFFECT. Properties such as **strength**, **yield stress**, **prone to brittle fracture**, **endurance limit**, etc., will to some extent be sensitive to the volume of material under test or consideration. (See **Barba law**.)

SKEW CURVE. See **twisted curve**.

SKEWNESS. The departure of a frequency distribution from symmetry. It is usually measured by the third mean-moment μ_3 standardized by division by $\mu_2^{3/2}$, where μ_2 is the **variance**.

SKEW RAY. In general, any light ray that does not lie in a plane. For axially symmetric optical systems, all rays not coplanar with the axis are skew rays.

SKEW SURFACE. A ruled surface which is not a developable surface. Also called *scroll*.

SKEW-SYMMETRIC TENSOR. See tensor, skew-symmetric.

SKEW T-LOG p DIAGRAM. An emagram (temperature and logarithm of pressure as coordinates) with the isotherms rotated 45 degrees clockwise to produce greater separation of isotherms and adiabats. This is the **thermodynamic diagram** presently used by the U. S. Air Force.

SKIN DEPTH. For a conductor carrying currents at a given frequency as a result of the **electromagnetic waves** acting upon its surface, the depth below the surface at which the current density has decreased one **neper** below the current density at the surface. Usually the skin depth is sufficiently small so that for ordinary configurations of good conductors, the value obtained for a plane wave falling on a plane surface is a good approximation. The skin depth is given by the relationship:

$$\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$$

where f is given in cycles per second, μ is given in henries per meter, and σ is given in mhos per meter.

SKIN FRICTION. When a fluid flows past a solid boundary, there is a shear stress at the boundary

$$\tau_o = \mu \left(\frac{\partial u}{\partial y} \right)_{y=0}, \tag{1}$$

where μ is viscosity, y is distance measured normally from the boundary and u is the velocity component parallel to the boundary.

Equation (1) is correct even for a turbulent boundary layer, or for turbulent flow in a pipe or channel, because as $y \rightarrow 0$ the Reynolds stresses become negligible in comparison with the viscous shear stress.

The shear stress τ_o is often known as the *skin friction stress*, or simply the *skin friction*. The *skin friction coefficient* is defined as

$$C_f = \frac{\tau_o}{\frac{1}{2}\rho V^2}$$

where V is the velocity outside the boundary layer or, in the case of flow in a pipe or channel, the mean velocity of flow.

In calculations on turbulent boundary layers the *friction velocity* is often introduced. This is defined by

$$\tau_o = \rho v_*^2. \tag{2}$$

SKIN FRICTION, EFFECT OF ROUGHNESS ON. See roughness, effect on skin friction.

SKIN RESISTANCE. The skin resistance of a unit square is equal to $1/\sigma\delta$ where σ is the resistivity of the conductor per unit volume and δ is the **skin depth** at the frequency of measurement. It is thus seen that the skin resistance is the same as the d-c resistance of a unit square of thickness δ .

SLAB. Although efforts have been made to reserve the word slab for an in-plane load-carrying element or **sheet**, the term is still used in bridge and building construction to denote **plate**.

SLATER DETERMINANT. This determinant describes a simple type of antisymmetric wave function for a system of fermions. If $\psi_{r_1}(x), \psi_{r_2}(x), \dots, \psi_{r_n}(x)$ is an **orthonormal** system of N one-particle wave functions and x_i stands for all the variables of the i^{th} particle, their antisymmetrized product gives a Slater-determinant

$$\frac{1}{\sqrt{N!}} \begin{vmatrix} \psi_{r_1}(x_1) & \psi_{r_2}(x_1) & \dots & \psi_{r_n}(x_1) \\ \psi_{r_1}(x_2) & \psi_{r_2}(x_2) & \dots & \psi_{r_n}(x_2) \\ \dots & \dots & \dots & \dots \\ \psi_{r_1}(x_n) & \psi_{r_2}(x_n) & \dots & \psi_{r_n}(x_n) \end{vmatrix}$$

A general state of the n -particle system can be described by a wave function which is a linear combination of such Slater determinants.

SLATER SUM. This quantity is given by the equation

$$\sum_n \phi_n^* e^{-\beta H} \phi_n,$$

where the ϕ_n are a complete set of orthonormal functions, $\beta = 1/kT$ (k is Boltzmann's constant, T is absolute temperature). It plays an important role in **quantum statistics**. (See also **cluster theory of virial coefficients**.)

SLENDER-BODY THEORY. The linearized equation for the *perturbation velocity potential* in subsonic or supersonic flow is

$$(1 - M^2)\phi_{xx} + \phi_{yy} + \phi_{zz} = 0. \quad (1)$$

In the case of a very long slender body, whose cross section varies only slowly with x , the conditions near the body can be approximated by neglecting the first term in Equation (1). The perturbation velocity potential $\phi(x, y, z)$ then has the form

$$\phi(x, y, z) = \phi(y, z; x) + g(x), \quad (2)$$

where $\phi(y, z; x)$ is a velocity potential for two-dimensional flow in a yz -plane, in which x appears only as a parameter determined by the shape of the cross section at x , and $g(x)$ is an arbitrary function of x .

It is clear that the degree of "slenderness" required for this approximation to be valid becomes less as the Mach number approaches 1. At supersonic speeds "slenderness" implies that the body must lie well within the **Mach cone** from the pointed nose of the body.

In the case of a wing of zero thickness, either plane or cambered, the term $g(x)$ affects the upper and lower surfaces equally and so does not affect the distribution of lift. This leads to a particularly simple theory for **wings of small aspect ratio**.

In the general case, where the body has thickness, expressions for $g(x)$ have been derived both for subsonic and for supersonic flow. The expressions involve the Mach number and the first and second derivatives of the area of cross section with respect to x , and for most cases they are independent of the shape of the cross section, for given area. These expressions have been used to calculate the *wave drag* of a large number of different shapes in supersonic flow. (See also **subsonic and supersonic flow, linear equations; wings of small aspect ratio; transonic area rule**.) (See G. N. Ward, *Linearized Theory of Steady High-Speed Flow*, Cambridge, 1955.)

SLENDERNESS. See slender-body theory.

SLENDERNESS RATIO. Columns are classified as long or short by their slenderness ratio l/r where l is the length and r the radius of gyration about the **weak axis**, unless otherwise specified. The **Euler buckling** stress for a hinged-ended elastic column may be written $\pi^2 E / (l/r)^2$. (See **column**.)

SLIP EFFECT. Consider a gas which is placed between two parallel plates. The lower plate is at rest, and the upper one moves at a constant velocity v_u in the x -direction. The molecules which strike the lower wall have undergone their last collision with a gaseous molecule at some distance (on the average one **mean free path**) from this plate. The average x -component of their velocity is therefore different from 0. The average velocity of the reflected molecules cannot be smaller than 0; whence the gas velocity in the immediate vicinity of the wall (average of the velocities of the incident and reflected molecules) is not 0. Viewed from a macroscopic point of view, the first layer of gas in contact with the wall is not at rest, but *slips* on the wall at a velocity v_s .

If dv_x/dy is the velocity gradient in the gas, one may write a linear relationship:

$$v_s = \xi \frac{dv_x}{dy}$$

ξ is the "*slip distance*," or "*slip coefficient*," and is of the order of one mean free path. It depends therefore on pressure, and also on the nature of the surface: for highly polished surfaces, the average velocity of the *reflected* molecules has itself a component in the direction of v_u , and this increases the slip coefficient.

The slip effect becomes predominant when the slip distance is large compared to the separation of the two plates, i.e., in the Knudsen domain. (See **Knudsen gas**.)

SLIP FLOW. Flow of a gas at **Knudsen numbers** between about 0.01 and 1. The velocity of slip at a solid boundary is of order λ times the velocity gradient at the boundary, where λ is the mean free path of the molecules. The slip velocity imposes on the flow a boundary condition that is different from that of no slip in continuum flow. For Knudsen numbers greater than about 1, other non-continuum effects appear. (See also **rarefied gas dynamics; free-molecule flow**.)

SLIP-FLOW REGIME. See rarefied gas dynamics.

SLIP LINE. A trajectory of the directions of maximum shear stress in plane plastic flow. **Hencky** showed that stress distributions associated with perfect plasticity in plane strain could be expressed by simple geometrical prop-

erties of the slip line fields of the orthogonal families of slip lines. The angular change in direction of arcs of one family which span the gap between two members of the other family is constant. This is equivalent to the condition that the radii of curvature of members of one family of slip lines where they are cut by a member of the other family change by the arc length along that member.

SLIP VECTOR. See **dislocations**.

SLIP, VELOCITY OF. See **rarefied gas dynamics**.

SLIT. If light of wavelength λ illuminates a slit of width a and if ϕ is the angle of incidence, then the intensity of the Fraunhofer diffraction pattern in a plane parallel to the plane of the slit is

$$I = I_0 \left(\frac{\sin \beta}{\beta} \right)^2, \quad \beta = \frac{\pi a}{\lambda} (\sin \phi + \sin \theta)$$

where θ is measured in a plane normal to the slit.

SLOPE. The slope of a **beam** or **column**, $dy/dx = \tan \theta$ and the angular position θ are not distinguished in elementary beam theory. In the **slope-deflection** equations, clockwise rotation or change in slope is taken as positive.

SLOPE-DEFLECTION EQUATIONS. In a continuous **frame** with beams and columns of constant flexural rigidity EI between joints, the relation between **end moments** and **end slopes** (both taken as positive when clockwise) is written for each member separately. Calling the generic member of length L , AB , the rotation at A is

$$\theta_{AB} = \frac{M_{ABL}}{3EI} - \frac{M_{BAL}}{6EI} + \phi_{AB} + \frac{\Delta_B - \Delta_A}{L}$$

$$M_{AB} = 2k_{AB}(2\theta_{AB} + \theta_{BA}) + M_{AB}^F - 6k_{AB} \frac{\Delta_B - \Delta_A}{L}$$

where M_{AB} is the end moment at A , M_{BA} , the end moment at B , ϕ_{AB} the rotation at A due to the transverse load acting on a simple beam, Δ_B and Δ_A the **support settlements** at B and A , k_{AB} the **stiffness factor** EI/L , and M_{AB}^F the moment due to the transverse load acting on a fixed beam.

When **sidesway** is prevented, summing the

moments at each joint and equating to the externally applied moment, setting $\theta_{AB} = \theta_{AC}$, etc., for a rigid-jointed structure gives n equations in the n unknown joint rotations. The equations may be solved by the **moment distribution** method of Hardy Cross.

SLOPE STABILITY. One of the important problems of soil mechanics is the determination of the height and slope of an embankment which will not slide under its own weight and the impressed load or surcharge, and which is thus said to have slope stability.

SLOWING-DOWN AREA. (1) In an infinite, homogeneous medium, the slowing-down area is one-sixth the mean square distance between the neutron source and the point where the neutron, generally after many collisions, reaches a given energy. (2) In Fermi age theory, the slowing down area is identical with the age. (See **age equation, Fermi**.)

SLOWING-DOWN DENSITY. In the theory of the slowing down of neutrons in matter, the slowing-down density, denoted most generally by $q(r, t, E)$, is the number of neutrons per unit volume at r , per unit time at t which are degraded by collisions from energies greater than E to energies less than E .

SLOWING-DOWN KERNEL. See **kernel, slowing-down**.

SLOWING-DOWN LENGTH. The square root of the **slowing-down area**.

SLOWING-DOWN TIME. The slowing-down time may be defined as the time required by a neutron to slow down, on the average, from its original energy to the energy in question. It may be shown equal to the Fermi age (see **age equation, Fermi**) divided by the time-average diffusion coefficient encountered during the slowing-down process.

SLOWING OF CLOCKS. See **Lorentz transformation**.

SLUG. A unit of mass in the f lbf s system of units, being the mass which is accelerated at 1 ft/s² by a force of one pound. It is equal to 32.174 lbm.

SLUTSKY-YULE EFFECT. In time-series analysis, the effect by which a moving average of a random series generates, under certain conditions, an oscillatory series.

SMALL AREA PLOTTING SHEET. (See also **sailings**.) Many practicing navigators prefer to solve their problems by graphical rather than analytic methods. The work may be done on charts but everything must be erased later and the charts suffer. The small area plotting sheet provides a convenient method for the solution of problems in plane sailing.

The plotting sheet, which can be purchased in quantity, has printed on it a central meridian that is marked off in minutes of arc (i.e., nautical miles). A circle of 60-miles radius is centered on the sheet and marked off in degrees of angle from the north toward the east. Parallels of latitude are printed passing through the center of the degree circle and tangent to it. Since meridians converge on the poles they must be spaced in accord with the relation of parallel sailing that $p = DLo \cos \phi$ in which p is east-west distance in miles corresponding to DLo in angular measure, and ϕ is the mid-latitude of the region for which the sheet is being used. This is solved graphically

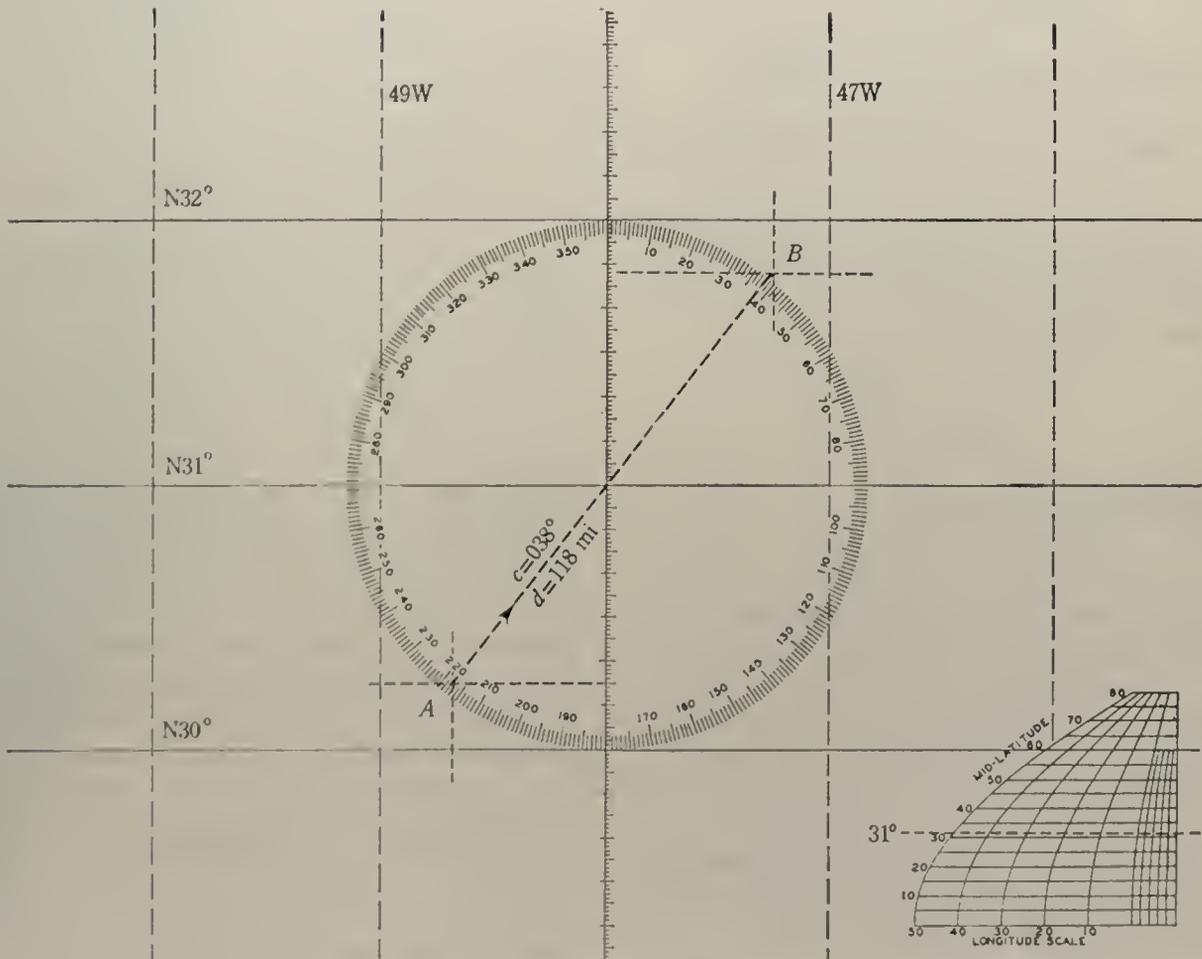
by a diagram in which the longitude scale in miles is plotted against the mid-latitude.

On the diagram a typical problem is solved, dotted lines being used for the lines drawn on the sheet. A ship leaves A ($\phi 30^\circ 15'N$ & $Lo 48^\circ 39'W$) to proceed to a point B ($\phi 31^\circ 48'N$ & $Lo 47^\circ 15'W$). The mid-latitude is taken as 31° and a line is drawn on the departure- DLo diagram at $\phi = 31^\circ$. This establishes the longitude scale for the 60' spacing of the meridians and also the scale for departure in miles.

The points A and B are plotted using the center scale for latitude and the departure scale for longitudes. A line is drawn from A to B and the course and distance determined using the distance scale defined in the north-south direction. The course is found to be 038° and the distance 118 miles.

SMALL ION. (Also called light ion, fast ion.)

An atmospheric ion of the type that has the greatest mobility; and hence, collectively, it is the principal agent of atmospheric **conduction**. The exact physical nature of the small ion has



Solution of navigation problem using small area plotting chart.

never been fully clarified, but much evidence indicates that each is a singly-charged atmospheric molecule (or, rarely, an atom) about which a few other neutral molecules are held by the electrical attraction of the central ionized molecule. Estimates of the number of satellite molecules range as high as twelve.

SMALL PERTURBATION. A disturbance imposed on a system in **steady state**, with amplitude assumed small of the first order; i.e., the square of the amplitude is negligible in comparison with the amplitude, and the derivatives of the perturbation are assumed to be of the same order of magnitude as the perturbation. When the fundamental equations are applied to the perturbed system the non-linear convective and advective terms in the perturbations are therefore omitted, and the equations are said to have been linearized.

SMALL SOURCE THEORY. See **source-sink method**.

S-MATRIX. The equation of motion of the state vector $\Psi(t)$ in the **interaction picture**

$$i\hbar\partial_t\Psi(t) = H_I(t)\Psi(t) \tag{1}$$

can be recast into the integral equation

$$\Psi(t) = \Psi(t_0) - \frac{i}{\hbar} \int_{t_0}^t dt' H_I(t')\Psi(t'). \tag{2}$$

The solution of (2) satisfies (1) together with the boundary condition that at time t_0 , Ψ reduces to the prescribed state vector $\Psi(t_0)$. The operator $U(t, t_0)$ defined by

$$\begin{aligned} \Psi(t) &= U(t, t_0)\Psi(t_0) \\ U(t_0, t_0) &= 1 \end{aligned}$$

can be shown to be unitary

$$U^*(t_0, t_1) = U^{-1}(t_0, t_1)$$

and furthermore to satisfy the relation

$$U(t_1, t_0) = U^{-1}(t_0, t_1).$$

The S matrix is defined by

$$S = \lim_{t \rightarrow \infty} \lim_{t' \rightarrow -\infty} U(t, t') = U(\infty, -\infty)$$

and can likewise be shown to be unitary. It determines the overall change in the state of a system as a result of the interaction:

$$\Psi(+\infty) = S\Psi(-\infty).$$

For a scattering process $\Psi(-\infty)$ denotes the initial state $\Psi(-\infty) = |i\rangle$ and S transforms $|i\rangle$ into all possible final states. The probability amplitude for finding the system in a particular final state $|f\rangle$ is given by $\langle f|S|i\rangle$. The matrix element $\langle f|S|i\rangle$ is thus the probability amplitude for the transition $i \rightarrow f$. The infinite time interval between initial and final states assures that energy is exactly conserved (in accordance with the uncertainty relations $\Delta t \Delta E \geq \hbar$).

If the state f is different from i , we can write

$$\langle f|S|i\rangle = -2\pi i \langle f|T|i\rangle \delta(E_f - E_i)$$

where E_i and E_f are the energies of the initial and final state, respectively. The transition probability for an infinite time interval is therefore

$$\begin{aligned} W &= |\langle f|S|i\rangle|^2 \\ &= (2\pi)^2 [\delta(E_f - E_i)]^2 |\langle f|T|i\rangle|^2. \end{aligned}$$

If we write

$$\delta(E_f - E_i)$$

$$= \lim_{\tau \rightarrow \infty} \frac{1}{2\pi\hbar} \int_{-\tau/2}^{\tau/2} \exp\left[\frac{i}{\hbar}(E_f - E_i)t\right] dt.$$

W can be written as

$$W = \frac{2\pi}{\hbar} \delta(E_f - E_i) |\langle f|T|i\rangle|^2 \tau$$

where τ is the large time during which the transition takes place. The transition probability per unit time is thus

$$w = \lim_{\tau \rightarrow \infty} \frac{W}{\tau} = \frac{2\pi}{\hbar} \delta(E_f - E_i) |\langle f|T|i\rangle|^2.$$

S-MATRIX IN QUANTIZED FIELD THEORY: EVALUATION OF. An elegant formalism for the evaluation of the S -matrix elements in a field theory has been given by Feynman, Schwinger, and Dyson. Consider the n^{th} order term of the S matrix as defined in the interaction picture (see **covariant perturbation theory**)

$$S_n = \frac{1}{n!} \int d^4x_1 \cdots \int d^4x_n P(H_I(x_1) \cdots H_I(x_n)) \tag{1}$$

and let us for simplicity confine our attention to quantum electrodynamics where

$$H_I(x) = -\frac{1}{c} j_\mu(x) A^\mu(x) \quad (2)$$

where $j_\mu(x) = \frac{1}{2} e [\bar{\psi}(x) \gamma_\mu \psi(x)]$ is the current operator of the quantized electron-positron field and $A^\mu(x)$ is the quantized electromagnetic 4-potential. In the above P denotes the Dyson **chronological operator**. The P product may be replaced by the Wick T chronological product, since the Dirac operators occur in pairs in $j_\mu(x)$. Wick has shown how to express a product such as $T(H_I(x_1) \cdots H_I(x_n))$ in normal constituents, i.e., terms in which all creation operators stand to the left of all destruction operators. Thus, given any initial and final state specified by definite numbers of free particles with specified spin and momenta, there exists one and only one normal constituent with a non-zero matrix element between these states. A decomposition of S into normal constituents is thus equivalent to the listing of all the matrix elements of S in a representation in which the free particles are diagonal. A **Feynman diagram** is then simply a concise graphical way of representing a normal product.

Wick's theorem states that a T product of time-labeled creation and annihilation operators can be decomposed into a unique sum of normal constituents as follows:

$$\begin{aligned} T(UV \cdots XYZ) &= N(UVW \cdots XYZ) \\ &+ \langle T(UV) \rangle_o N(W \cdots XYZ) \\ &\pm \langle T(UW) \rangle_o N(V \cdots XYZ) \\ &+ \cdots \langle T(UV) \rangle_o \langle T(\cdot) \rangle_o \cdots \langle T(XZ) \rangle_o \end{aligned}$$

where the sum on the right includes all possible sets of contractions $\langle T(AB) \rangle_o$. These contractions are certain easily computed c -number functions, which are non-vanishing only if the two operators occurring in the contraction $\langle T(UV) \rangle_o$ are both electron or photon operators. Thus the T product in (1) can be written as $T = \sum_N T_N$ where the sum is over all normal constituents. Each normal constituent can be associated with a Feynman diagram as follows: (a) draw n points $x_1 \cdots x_n$. For each associated (contracted) pair of Dirac operators $\psi(y) \bar{\psi}(x)$ draw a directed line from x to y . For each pair of contracted photon operators $A_\mu(x) A_\nu(y)$ draw a dotted undirected line from x to y . For each unpaired electron operator $\psi(x), \bar{\psi}(x)$ draw a line from x to y . For each unpaired electron operator $\psi(x), \bar{\psi}(x)$ draw a line from x to the edge of the diagram outwardly

directed for $\bar{\psi}(x)$ and directed towards x for $\psi(x)$. For an unpaired $A_\mu(x)$ draw a dotted line from x to the edge of the diagram. The unpaired operators annihilate the incoming and create the outgoing particles. There is a one-to-one correspondence between the Feynman diagrams with n vertices and certain group of terms in $S^{(n)}$.

To each diagram with vertices identified (i.e., with specified points $x_1 \cdots x_n$) there corresponds a single normal constituent. It is this correspondence which is the basis of the usefulness of the concept of diagrams. In practice one draws all possible topologically different diagrams consistent with the interaction term.

If the latter is of the form $G \bar{\psi}_\alpha \Gamma_{\alpha\beta} \psi_\beta \phi$ then two fermion lines and one boson line must meet at every vertex. The matrix element corresponding to any n^{th} order diagram can then be obtained by writing down the following factors:

- (1) A factor $(-i/\hbar c)^n$ for the diagram as a whole.
- (2) A factor $G \Gamma_{\alpha\beta}$ for each vertex.
- (3) A factor $\frac{1}{2} \hbar c \Delta_F(x_j - x_l)$ for an internal boson line connecting the point x_j and x_l .
- (4) A factor $-\frac{1}{2} S_{F\alpha\beta}(x_l - x_j)$ for an internal fermion line directed from x_j to x_l . The factor $(G \Gamma)_{\sigma\alpha}$ is assumed to occur at the vertex labeled by x_l and the factor $G \Gamma_{\beta\delta}$ at the vertex labeled by x_j .
- (5) The correct creation and annihilation operators $\psi^\pm(x) \bar{\psi}^\pm(x) \phi^\pm(x)$ for each external fermion or boson line leaving or arriving at x .
- (6) A factor (-1) for each internal closed fermion loop.
- (7) Integrate over $x_1 \cdots x_n$.

For the case of the electromagnetic field, (2) above becomes replaced by: a factor $(e\gamma^\mu)$ for each vertex and (3) by: a factor $\frac{1}{2} \hbar c D_{\mp}(x_j - x_l) g_{\mu\nu}$ for an internal photon line connecting the point x_j and x_l . It is assumed the factors $e\gamma^\mu$ and $e\gamma^\nu$ act at these vertices. Also one is to sum over the indices μ and ν . Similarly in (4), $G \Gamma$ is replaced by $e\gamma^\mu$ and in ϕ by A_μ . It is to be noted that the $1/n!$ in $S^{(n)}$ is not included since we are dealing only with topologically different graphs. Clearly there are $n!$ permutations of the points $x_1 \cdots x_n$ among themselves, which leave the graph topologically unchanged, i.e., which are the same except for a labeling of the points.

In the above $-\frac{1}{2} S_F(x - y)$ is the contrac-

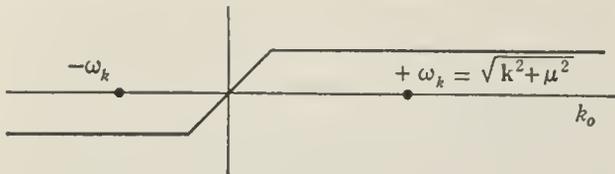


Fig. 1.

tion $\langle T(\psi(x)\bar{\psi}(y)) \rangle$. Similarly $+\frac{1}{2}\hbar c\Delta_F(x-y)$ corresponds to the contraction $\langle T(\phi(x)\bar{\phi}(y)) \rangle$ with

$$\Delta_F(x) = \frac{2i}{(2\pi)^4} \int_{C_F} \frac{e^{-ikx}}{k^2 - \mu^2} d^4k$$

where the contour C_F is indicated in Figure 1. It can also be written in the form

$$\Delta_F(x) = \frac{2i}{(2\pi)^4} \int_{-\infty}^{+\infty} d^4k \frac{e^{-ikx}}{k^2 - \mu^2 + i\epsilon}$$

and

$$\begin{aligned} S_F(x) &= -\frac{2i}{(2\pi)^4} \int_{C_F} \frac{1}{-m + i\epsilon} e^{-ipx} d^4p \\ &= -\frac{2i}{(2\pi)^4} \int_{-\infty}^{+\infty} \frac{\gamma \cdot p + m}{p^2 - m^2 + i\epsilon} e^{-ipx} d^4p. \end{aligned}$$

For example, in quantum electrodynamics, the lowest order Compton scattering diagrams are indicated in Figure 2.

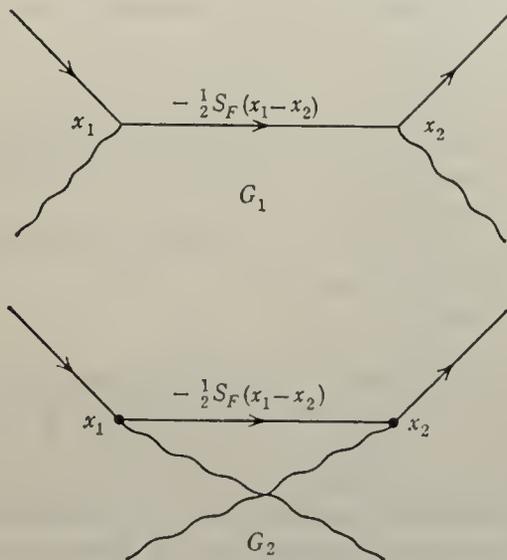


Fig. 2.

The contribution to $S^{(2)}$ corresponding to G_1 is

$$\begin{aligned} &\left(-\frac{i}{\hbar c}\right)^2 e^2 \int_{-\infty}^{+\infty} d^4x_1 \int_{-\infty}^{+\infty} \\ &\times \overline{d^4x_2 \psi^{(+)}(x_1) \gamma_\mu (-\frac{1}{2} S_F(x_1 - x_2))} \\ &\times \gamma_\nu \psi^{(+)}(x_2) A^{\mu(-)}(x_1) A^{\nu(+)}(x_2). \end{aligned}$$

Similarly the contribution to $S^{(2)}$ due to G_2 is

$$\begin{aligned} &\left(-\frac{i}{\hbar c}\right)^2 \int_{-\infty}^{+\infty} d^4x_1 \int_{-\infty}^{+\infty} \\ &\times \overline{d^4x_2 \psi^{(+)}(x_1) \gamma_\mu (-\frac{1}{2} S_F(x_1 - x_2))} \\ &\times \gamma_\nu \psi^{(+)}(x_2) A^{\mu(-)}(x_2) A^{\nu(+)}(x_1). \end{aligned}$$

For the evaluation of transition probabilities the matrix element of S is to be evaluated between a certain initial and final state. For Compton scattering we require $\langle p', s'; k' e' | S | ps, ke \rangle$ where $|ps, ke \rangle$ denotes the (initial) state containing 1 photon of momentum \mathbf{k} , polarization \mathbf{e} and an electron of momentum \mathbf{p} , spin s .

S-MATRIX THEORY AND QUANTUM FIELD THEORY. We base our development on the following assumptions (see quantum theory of fields):

(1) There exist states which correspond to the description of systems having a definite number of particles which are infinitely far apart and which do not interact with one another. For these ‘‘asymptotic states’’ the energy and momenta of the particles are additive. To every such state there corresponds a normalizable vector $|\rangle$ in Hilbert space, \mathfrak{H} . In the subsequent discussion, we shall not distinguish between elementary and compound particles. A particle is characterized as a localized entity having a definite mass and spin which are determined by experiment. The masses, charges, etc., of the particles enter the theory as given parameters. The manifold of one-particle states is a relativistically invariant subspace of the Hilbert space of all possible states. Furthermore, the manifold of states of one particle of a given type belongs to an irreducible representation of the Lorentz group.

(2) The asymptotic states can be built up by taking direct products (properly symmetrized to take into account the **Pauli principle**) of one-particle states. The asymptotic states thus can be labeled by number operators for a complete set of one-particle states. We shall consider two sets of asymptotic states corresponding to observations in the infinite past and future. These will be denoted by $|\rangle_+$ and $|\rangle_-$ respectively.

(3) There exists a group G of transformations L , which includes Lorentz transformations and which is represented by operators $U(L)$ over the states $|\rangle$. The group G corresponds to certain symmetry operations under which the

theory is invariant. (See **invariance and quantum mechanics.**) For Lorentz transformations $\{a, \Lambda\}$, $x' = \Lambda x + a$

$$U(a_1, \Lambda_1)U(a_2, \Lambda_2) = \omega U(a_1 + \Lambda_1 a_2, \Lambda_1 \Lambda_2)$$

$$\omega = \omega(a_2 \Lambda_2, a_1 \Lambda_1) \quad |\omega| = 1$$

where $U(a, \Lambda)$ is unitary if Λ is orthochronous and antiunitary if Λ includes a time inversion. For an infinitesimal Lorentz transformation

$$x'_\mu - x_\mu = \delta x_\mu = \omega_\mu{}^\nu x_\nu$$

$$\omega_{\mu\nu} + \omega_{\nu\mu} = 0$$

the states transform according to

$$| \rangle \rightarrow | \rangle' = U(a, \Lambda) | \rangle$$

where

$$U(a, \Lambda) = 1 + \delta U_L$$

with

$$i\delta U_L = P_\mu a^\mu + \frac{1}{2}\omega^{\mu\nu} M_{\mu\nu}.$$

The hermitian operator P_μ , the generator for translations, corresponds to the energy momentum of the system and $M_{\mu\nu}$, the generator for space-time rotations, to the total angular momentum tensor of the system.

If we choose a set of states which are eigenfunctions of P_μ , $P_\mu |p_\mu^{(n)}, \alpha\rangle = p_\mu |p_\mu^{(n)}, \alpha\rangle$ then since

$$U(a, 1) = \exp(-iP_\mu a^\mu)$$

$$U(a, 1) |P_\mu, \alpha\rangle = \exp(-ip_\mu a^\mu) |p_\mu, \alpha\rangle.$$

(4) There exists a unique vacuum, the state of no particles, which looks identical to all observers

$$U(a, \Lambda)\Psi_o = \Psi_o = |0\rangle.$$

(5) There exists a complete set of states $|p^{(n)}, \alpha\rangle$ which are eigenstates of the energy-momentum operator with $p_o^{(n)} \geq 0$, and $p_\mu^{(n)} p^{(n)\mu} \geq 0$; α designates the other observables necessary to specify the states. Hence for arbitrary operator AB

$$\langle P, \alpha | AB | Q, \beta \rangle = \langle P, \alpha | A | 0 \rangle \langle 0 | B | Q, \beta \rangle$$

$$+ \sum_{n\gamma} \frac{1}{(2\pi)^3} \int \langle P, \alpha | A | n\gamma \rangle \langle n\gamma | B | Q, \beta \rangle d^4k.$$

(6) There exists a unitary matrix S , the S-matrix, characterizing transitions between asymptotic states. The stable states such as the

- (a) vacuum
- (b) one-particle states
- (c) bound states

are unchanged by S . If we denote by $|\sigma\rangle$ such stable states then

$$S|\sigma\rangle = |\sigma\rangle.$$

The above can be recast into an operator formalism which paraphrases the usual free field formalism. For simplicity we consider only one type of particle of mass m and spin 0. Since the asymptotic states correspond to states with a definite number of particles of definite momenta \mathbf{p}_i and energy $p_{io} = \sqrt{\mathbf{p}_i^2 + m^2}$, which do not interact, they may be obtained in the usual way from the vacuum by the application of creation operators to it. In particular, for the ingoing asymptotic states we can write

$$|p_1 p_2 \cdots p_n\rangle_+ = \frac{1}{\sqrt{n!}} a_{in}^*(p_1) \cdots a_{in}^*(p_n) |0\rangle$$

$$a_{in}(p) |0\rangle = 0$$

and since these states correspond to separated non-interacting particles

$$[a_{in}(p), a_{in}^*(p')] = \delta^3(\mathbf{p} - \mathbf{p}') p_o$$

$$[a_{in}(p), a_{in}(p')] = 0.$$

We now identify

$$\left. \begin{aligned} a_{in}(k) &= A_{in}(k) \\ a_{in}^*(k) &= A_{in}(-k) \end{aligned} \right\} \text{for } k_o > 0$$

and define

$$A_{in}(x) = \frac{1}{(2\pi)^{3/2}} \int d^4k \exp(-ikx) \delta(k^2 - m^2) A_{in}(k).$$

The transformation properties of the states now imply that

$$\left. \begin{aligned} U(a, \Lambda) A_{in}(x) U^{-1}(a, \Lambda) &= A_{in}(\Lambda x + a) \\ U(a, \Lambda) A_{in}(k) U^{-1}(a, \Lambda) &= \exp(-ika) A_{in}(\Lambda k) \end{aligned} \right\} \text{for } k^2 = m^2, \Lambda \text{ in } L_+^*$$

(Schrödinger transformation law)

Clearly $A_{in}(x)$ satisfies the free Klein-Gordon equation

$$(\square + m)A_{in}(x) = 0$$

and by virtue of the commutation rules of a_{in} and a_{in}^* , A_{in} satisfies the free field commutation rules

$$[A_{in}(x), A_{in}(y)] = i\hbar c \Delta(x - y).$$

Similar definitions hold for the out fields operators and the outgoing asymptotic states, $|\rangle_-$. Any operator can be expanded in terms of the in field operators. In particular the expansion of the S matrix $S = e^{i\eta}$ will be written in the following fashion

$$\eta = \sum_{n=4}^{\infty} \frac{1}{n!} \int d^4x_1 \int d^4x_2 \cdots \times \int d^4x_n :A_{in}(x_1) \cdots A_{in}(x_n) : h^{(n)}(x_1 \cdots x_n)$$

where $h^{(n)}(x_1 \cdots x_n)$ are c -number functions of the space-time coordinates. Note that the series starts at $n = 4$, corresponding to the fact that the first non-vanishing h corresponds to meson-meson scattering. The $::$ indicate that the normal product of the operators is to be taken.

The invariance of S under Lorentz transformations and the properties of the normal product implies that

- (a) $h(x_1, \cdots x_n) = h(x_1 + a, x_2 + a, \cdots x_n + a)$
- (b) $h(\Lambda x_1, \cdots \Lambda x_n) = h(x_1 \cdots x_n)$
- (c) $P_i h(x_1, x_2, \cdots x_n) = h(x_1 \cdots x_n)$

where P_i is an arbitrary permutation on the labels of the x 's

$$P = \begin{pmatrix} 1 & 2 & 3 & 4 & \cdots & n \\ i_1 & i_2 & i_3 & i_4 & \cdots & i_n \end{pmatrix}.$$

Finally, the unitary of the S -matrix (hermiticity of the η -matrix) and the neutrality of the A field asserts that

$$h(x_1 \cdots x_n) = \bar{h}(x_1 \cdots x_n).$$

In momentum space with

$$h_n(x_1 \cdots x_n) = \frac{1}{(2\pi)^{\frac{5}{2}n}} \int d^4k_1 \cdots \int d^4k_n \times \exp [i(k_1x_1 + k_2x_2 \cdots + k_nx_n)] \times h_n^T(k_1, k_2, \cdots k_n)$$

invariance under translations implies that

$$h^T(k_1 \cdots k_n) = \delta(k_1 + k_2 + \cdots k_n) \bar{h}_n(k_1 k_2 \cdots k_n)$$

so that the η matrix has the following expansion

$$\eta = \sum_{n=4}^{\infty} \frac{1}{n!} \int d^4k_1 \cdots \int d^4k_n \delta(k_1 + \cdots + k_n) \cdot \bar{h}_n(k_1 \cdots k_n) \delta(k_1^2 - m^2) \cdots \delta(k_n^2 - m^2) :A_{in}(k_1) \cdots A_{in}(k_n)::$$

Note only the values of \bar{h} on the "mass shell" ($k_i^2 = m^2$) appear in the expansion for η and hence for S . Off the mass shell the functions may be chosen arbitrarily. The mass shell values can be considered to be "experimentally" determined since they are related to cross sections for the production and scattering processes among the particles. The properties of $\bar{h}_n(k_1, k_2 \cdots k_n)$ on the mass shell are

- (1) $\bar{h}_n(k_1 \cdots k_n)$ is a symmetric function of $k_1 \cdots k_n$
- (2) $\bar{h}_n(k_1 \cdots k_n) = \bar{h}_n(-k_1, \cdots -k_n)$ (hermiticity of η)
- (3) $\bar{h}(\Lambda k) = \bar{h}(k)$. In order that all cross sections and their generalizations for many-particle systems be finite
- (4) $\bar{h}(k_1 \cdots k_n)$ must not contain any four-dimensional δ function and
- (5) $\bar{h}(k_1 \cdots k_n)$ must be a continuous function of its invariant variables.
- (6) Finally the invariance of the theory under space-time inversion implies

$$\bar{h}(k_1, \cdots k_n) = h(-k_1 \cdots -k_n).$$

The above are all requirements for h on the mass shell. The extrapolation off the mass shell is arbitrary thus far. One can demand that conditions 1-6 be satisfied everywhere.

Lehmann, Symanzik and Zimmerman (LSZ), whose development we are following, have also inquired as to the connection between such an S -matrix formalism and Lorentz covariant field operators. Recall that in this framework the S -matrix is defined as

$$A_{out}(x) = S^{-1} A_{in}(x) S.$$

LSZ have shown that given an S -matrix, there exist many invariant fields $A(x)$ which tend asymptotically for $t \rightarrow \pm\infty$ to the given fields $A_{out}(x)$ and $A_{in}(x)$, respectively, more precisely if f is a normalizable K-G solution, there exist many fields $A^f(t)$ such that

$$\lim_{t \rightarrow \pm\infty} (\Phi, A^f(t)\Psi) = (\Phi, A_{out/in}^f\Psi)$$

where

$$A^f(t) = i \int d\sigma^\mu(x) \left[A(x) \frac{\partial \bar{f}(x)}{\partial x^\mu} - \frac{\partial A(x)}{\partial x^\mu} \bar{f}(x) \right].$$

If we express the S -matrix as

$$S = e^{i\eta}$$

since

$$\begin{aligned} & e^{-i\eta}A_{in}(x)e^{i\eta} - A_{in}(x) \\ &= \int_0^1 \frac{d}{d\lambda} [\exp(-i\eta\lambda)A_{in}(x)\exp(+i\eta\lambda)]d\lambda \\ &= -i \int_0^1 \exp(-i\eta\lambda)[\eta, A_{in}(x)]\exp(+i\eta\lambda)d\lambda \end{aligned}$$

then

$$\begin{aligned} A_{out}(x) &= \exp[-i\eta]A_{in}(x)\exp[i\eta] \\ &= A_{in}(x) + i \int_0^1 \exp[-i\eta\lambda][\eta, A_{in}(x)] \\ &\quad \exp[i\eta\lambda]d\lambda \\ &= A_{in}(x) - \int d^4y\Delta(x-y) \int_0^1 d\lambda \\ &\quad \times \exp[-i\eta\lambda]\mathfrak{D}(x)\exp[i\eta\lambda] \end{aligned}$$

whence,

$$\begin{aligned} \mathfrak{D}(y) &= \sum_{n=2}^{\infty} \frac{1}{n!} \int d^4x_1 \cdots \int d^4x_n h^{(n+1)} \\ &\quad \times (y, x_1 \cdots x_n) : A_{in}(x_1) \cdots A_{in}(x_n) : \end{aligned}$$

Note that only $\int \Delta(x-y)\mathfrak{D}(y)d^4y$ is uniquely determined if η is given since only the latter involves values of \bar{h} for k on the mass shell. We now introduce the interacting field operators $A(x)$ by

$$\begin{aligned} A(x) &= A_{in}(x) + \int d^4y\Delta_R(x-y) \int_0^1 d\lambda \\ &\quad \times \exp[-i\lambda\eta]\mathfrak{D}(y)\exp[i\lambda\eta]. \end{aligned}$$

Since $\int \Delta_R(x-y)\mathfrak{D}(y)d^4y$ requires knowledge of \bar{h}_n off the mass shell it is not determined uniquely if η is given. As stated above the extrapolation can be carried out in many ways even if one requires that the symmetry properties (1-6) be satisfied everywhere for all values of k . It then follows that A is hermitian and that

$$U(a, \Lambda)A(x)U^{-1}(a, \Lambda) = A(\Lambda x + a)$$

i.e., that $A(x)$ is a scalar field operator. Moreover, it satisfies the asymptotic conditions

$$\lim_{t \rightarrow \pm\infty} (\Phi, A^f(t)\Psi) = (\Phi, A_{out}^f\Psi)$$

by virtue of the Riemann-Lebesgue lemma.

Define next the current operator

$$j(x) = \int_0^1 d\lambda \exp[-i\lambda\eta]\mathfrak{D}(x)\exp[i\eta\lambda]$$

then

$$A_{out}(x) = A_{in}(x) + \int \Delta(x-y)j(y)dy$$

and

$$A(x) = A_{in}(x) - \int \Delta_R(x-y)j(y)dy$$

or equivalently

$$(\square + m^2)A(x) = -j(x).$$

We have thus defined a Heisenberg field operator, $A(x)$. The operator $A(x)$ defined in this way need not, however, be causal, i.e., need not satisfy

$$[A(x), A(y)] = 0, \quad \text{for } (x-y)^2 > 0.$$

LSZ therefore require the extrapolation off the energy shell to be such that causality be satisfied. In addition they require it to be satisfied for

$$\bar{h}_n(k_1 \cdots k_n) \quad \text{for } k_1^2 = k_2^2 = k_3^2 = k_n^2 = m^2$$

and that the extrapolation be continuous.

This represents an important additional requirement. LSZ then call an S -matrix "causal" if there exists an extrapolation such that the commutator condition

$$[A(x), A(y)] = 0,$$

for $x-y$ space-like is satisfied for the extrapolated field.

It is not clear that this concept of causality is the one realized in nature. It is therefore necessary to confront the properties of causal S -matrices (in the above sense) with experiment. To this end they introduce the concept of retarded product of operators.

The retarded product of two field operators is defined as the retarded commutator, i.e.,

$$\begin{aligned} R(A(x)A(y)) &= R(x,y) \\ &= -i\theta(x-y)[A(x), A(y)] \end{aligned}$$

more generally the retarded product of $n+1$ operators is defined as

$$\begin{aligned}
 n = 0 \quad & R(x) = A(x) \\
 n \geq 1 \quad & R(x; x_1, x_2 \cdots x_n) \\
 & = R(A(x)A(x_1) \cdots A(x_n)) \\
 & = (-i)^n \sum_p \theta(x - x_1) \cdots \theta(x_{n-1} - x_n) \\
 & \quad \times [\cdots [A(x), A(x_1)], \cdots A(x_n)]
 \end{aligned}$$

the summation \sum_p extends over all the permutations of the n coordinates x_i .

The vacuum expectation values of retarded products are denoted by

$$r(x; x_1 \cdots x_n) = (\Omega, R(x; x_1 \cdots x_n)\Omega).$$

It is clear from their definitions that the R -products have the following simple properties:

- (a) retardation: $R(x; x_1 \cdots x_n) = 0$, if $x_{i_0} > x_o$, for $i = 1, \cdots n$. Clearly $R = 0$ if more than one of the $x_{i_0} > x_o$.
- (b) symmetry: $R(x; x_1 \cdots x_n)$ is a symmetric function of $x_1 \cdots x_n$.
- (c) hermiticity: An R -product of hermitian operators is hermitian.
- (d) $R(x; yx_1 \cdots x_n) = -i\theta(x - y) \cdot \sum_{comb} [R(x; x_1 \cdots x_k), R(y; x_{k+1}, \cdots x_n)]$ (A)

where the summation is over all possible divisions of the variables $x_1 x_2 \cdots x_n$ into two groups.

From (A), interchanging x and y and adding one obtains

$$\begin{aligned}
 R(x; yx_1 \cdots x_n) - R(y; x_1 \cdots x_n) \\
 = -i \sum_{comb} [R(x; x_1 \cdots x_k), R(y; x_{k+1}, \cdots x_n)].
 \end{aligned}$$

The invariance properties of the retarded products are as follows:

- (a) Under translations:
since $A(x + a) = e^{-iPa}A(x)e^{iPa}$
 $R(x + a; x_1 + a, x_2 + a, \cdots x_n + a)$
 $= e^{-iPa}R(x; x_1 \cdots x_n)e^{iPa}$
so that
 $r(x, x_1 \cdots x_n) = r(x - x_1, \cdots, x - x_n)$.
- (b) Under homogeneous Lorentz transformations:
since $A(x)$ is Lorentz covariant, i.e.,
 $U(\Lambda)A(x)U(\Lambda)^{-1} = A(\Lambda x)$

and causal, i.e.,

$$[A(x), A(y)] = 0, \quad \text{for } (x - y)^2 < 0$$

therefore

$$\begin{aligned}
 R(\Lambda x, \Lambda x_1, \cdots \Lambda x_n) \\
 = R(\Lambda x, \Lambda x_1 \cdots, \Lambda x_n) \\
 = U(\Lambda)R(x; x_1, x_2 \cdots x_n)U^{-1}(\Lambda)
 \end{aligned}$$

and

$$r(\Lambda x, \Lambda x_1 \cdots \Lambda x_n) = r(x, x_1, \cdots x_n).$$

As a result of the asymptotic condition

$$\lim_{t \rightarrow \pm\infty} (\Phi, A^f(t)\Phi) = (\Phi, A_{in}^f\Psi)_{out}$$

valid for every pair of normalizable states Φ, Ψ in \mathcal{H} , one derives that

$$\begin{aligned}
 (\Phi, [R(x; x_1 \cdots x_n), A_{in}^f]\Psi) \\
 = \lim_{z_o \rightarrow -\infty} \left(\Phi, i \int d\sigma(z) [R(x; x_1 \cdots x_n), A(z)] \right. \\
 \left. \frac{\overleftrightarrow{\partial}}{\partial z_o} \bar{f}(z)\Psi \right) \\
 = - \lim_{z_o \rightarrow -\infty} \int d\sigma(z) \left(\Phi, R(x; x_1 \cdots x_n z) \right. \\
 \left. \frac{\overleftrightarrow{\partial}}{\partial z_o} \bar{f}(z)\Psi \right) \\
 = \int_{-\infty}^{+\infty} d^4z \bar{f}(z) (\square_z + \mu^2) \\
 \quad \times (\Phi, R(x; x_1 \cdots x_n, z)\Psi)
 \end{aligned}$$

where use has been made of Gauss theorem and the fact that

$$\lim_{z_o \rightarrow +\infty} (\Phi, R(x; x_1 \cdots x_n, z)\Psi) = 0$$

due to the retarded character of R . The equation

$$\begin{aligned}
 (\Phi, [R(x; x_1 \cdots x_n), A_{in}^f]\Psi) \\
 = \int d^4z \bar{f}(z) (\square_z + \mu^2) (\Phi, R(x; x_1 \cdots x_n z)\Psi)
 \end{aligned}$$

is known as a recursion relation. In particular for the case $n = 1$, we have

$$\begin{aligned}
 (\Phi, [R(x), A_{in}^f]\Psi) \\
 = \int_{-\infty}^{+\infty} d^4z \bar{f}(z) (\square_z + \mu^2) (\Phi, R(x, z)\Phi)
 \end{aligned}$$

letting $f(z) = \Delta(z' - z)$ we derive

$$[A(x), A_{in}(z)] = -i \int d^4 z' \Delta(z - z') K_z R(x, z')$$

where the notation $K_x = \square_x + \mu^2$ has been introduced. Since

$$A_{out}(x) = A_{in}(x) - \int \Delta(x - y) \times (\square_y + \mu^2) A(y) d^4 y$$

the commutator of A_{out} with A_{in} is

$$\begin{aligned} [A_{out}(x), A_{in}(y')] &= i\hbar c \Delta(x - y') - \int \Delta(x - y') \\ &\times (\square_y + \mu^2) [A(y'), A_{in}(x)] \\ &= i\Delta(x - y) - i \int d^4 x' \int d^4 y' \\ &\times \Delta(x - x') \Delta(y - y') \\ &\times K_x K_{y'} R(x', y'). \end{aligned}$$

Using the above recurrence relation one easily derives that the S -matrix element for the scattering of two particles of initial energy momentum p and k and final momenta $p'k'$ is given by

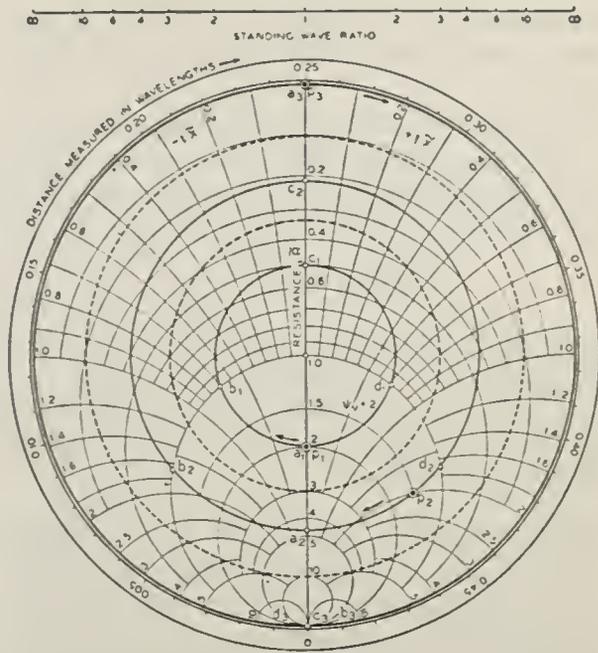
$$\begin{aligned} -\langle p', k', | p, k \rangle_+ &= +\langle p', k' | p, k \rangle_+ \\ &+ 2\pi i \delta(p + k - p' - k') T \end{aligned}$$

where

$$\begin{aligned} T &= T(k + k'; p, p') \\ &= - \int d^4 x \exp \left[i \frac{k + k'}{2} x \right] \\ &\times \langle p' | \theta(x) \left[j \left(\frac{x}{2} \right) \cdot j \left(-\frac{x}{2} \right) \right] | p \rangle \end{aligned}$$

where $j(x) = K_x A(x)$. This is a closed expression for the scattering amplitude from which certain analyticity properties of T can be inferred, which in turn allow the writing of **dispersion relations**.

SMITH DIAGRAM (CHART). A diagram with polar coordinates, developed to aid in the solution of transmission line and waveguide problems. It is composed of the following sets of lines: (1) Constant resistance circles. (2) Constant reactance circles. (3) Circles of constant standing wave ratio. (4) Radius lines representing constant line-angle



Smith diagram.

loci. The chart employs normalized quantities for maximum flexibility.

SMITH-HELMHOLTZ EQUATION. See the Lagrange theorem and Helmholtz equation.

SMOOTHING. The replacement of a curve, or of a sequence of points by another that is in some sense more regular, and yet whose ordinates, for any abscissa, are changed as little as possible. The irregularities in a sequence of points may be due to errors in measurement. If theory requires the theoretically correct points to lie on a given curve, one may apply some method of **curve fitting**, possibly least squares. If not, one may select arbitrarily a simple function, possibly a polynomial, and fit it by least squares. If the purpose is merely to obtain a smooth graph, this may be drawn visually. A somewhat more sophisticated method is to take, say, 5 consecutive points, fit a parabola, and replace the middle point by the one on the parabola. The next parabola requires four of these points and one new one.

S-n DIAGRAM. The number of cycles of stress to produce fatigue failure depends upon the stress level. A plot of stress or log stress vertically against log number of cycles horizontally is called an S - n diagram. (See fatigue.)

SNELL LAWS. (1) When light passes from one medium to another, the incident ray,

the normal to the surface at the point of incidence, and the refracted ray are all in the same plane.

(2) The sine of the angle of incidence bears to the sine of the angle of refraction, a ratio which is constant for the same two media, and depends only on the nature of those media.

$$n_1 \sin i_1 = n_2 \sin i_2,$$

where n_1 and n_2 are the indices of refraction of the first and second media, respectively.

If $n_1 > n_2$, it is impossible to satisfy the Snell law, for $\sin i_1 > n_2/n_1$. Total internal reflection then occurs.

SNELL LAWS APPLIED TO SOUND.

For a sound wave traveling across the boundary between two fluids, the relationship is identical with the Snell laws in optics. For non-dissipative media it can be stated as $\sin \theta_2 / \sin \theta_1 = c_2 / c_1$, where θ_1 is the angle of incidence, θ_2 is the angle of refraction, c_1 is the velocity in the incident medium, and c_2 is the velocity of sound in the refracting medium. Where the boundary is between a solid and a fluid, or between two solid media, the situation is more complex, in that both longitudinal and transverse waves can travel through solids. (See **velocity of sound**, and **waves, elastic in solids**.) The law can still be applied, however, in an extended form; thus, when a longitudinal wave is reflected at the boundary between two elastic solids, two waves (a longitudinal and a transverse) are refracted into the second medium, and two waves (a longitudinal and a transverse) are reflected back. The law now states that the value of $(\sin \theta) / c$ is the same for the incident wave and for each of the four waves generated at the surface. θ is the angle that the direction of propagation of the wave makes with the normal to the boundary, and c is the velocity of propagation of that wave.

S_N METHOD. In neutron transport theory, a method of analysis of the transport equation whereby the integral term describing scattering is approximated by dividing the angular range into N intervals in each of which the neutron flux is assumed to vary linearly. The method was devised by B. Carlson.

SOAP FILM ANALOGY. A soap film is a membrane under uniform tension. (See **membrane analogy**.)

SOLAR ANTAPEX. See **solar motion**.

SOLAR APEX. See **solar motion**.

SOLAR CONSTANT OF RADIATION. The intensity of solar radiation in free space at the earth's mean solar distance. The constant is usually expressed in $\text{cal cm}^{-2} \text{min}^{-1}$, and is equivalent to the amount of radiation incident in unit time on unit surface exposed perpendicularly to unimpeded radiation from the sun. Measurements on earth must be corrected for the effect of the atmosphere.

SOLAR MOTION. If the sun is in motion relative to some fixed point in space, the stars in the direction in which the sun is moving should be opening out while in the opposite direction they should be closing in. This effect was noted by Herschel as early as 1783. The point toward which the sun seems to be moving is called the *solar apex* and the opposite point is called the *solar antapex*.

A statistical study of the apparent motions of the stars indicates that the sun is moving toward a point in the constellation of Hercules in right ascension 18 hr and declination $+30^\circ$, with a velocity of 20 km/sec or 12 mi/sec.

As the apparent motions of fainter and fainter stars were determined, it became evident that the position of the solar apex was being moved progressively toward the northeast.

SOLENOID. (1) A coil which may consist of one or more layers of windings. It is the basis of all forms of the electromagnet, and is thus part of the working-mechanism of many electrically-operated devices.

The magnetic field strength within a solenoid whose length L is much greater than its diameter is nearly uniform and is approximately

$$B = \mu H = \mu NI / L,$$

where N is the number of turns and I is the current (rationalized units) (2) A tube formed in space by the intersection of unit-interval constant-value surfaces of two scalar quantities. The number of solenoids enclosed by a space curve is therefore equal to the flux of the vector product of the two gradients through a surface bounded by the curve, or

$$\iint_S (\nabla \phi_1 \times \nabla \phi_2) \cdot d\mathbf{S} = \oint \phi_1 d\phi_2,$$

where dS is the vector element of area of a surface discussed by the given curve.

Solenoids formed by the intersection of surfaces of equal pressure and density are frequently referred to in meteorology. A **barotropic** atmosphere implies the absence of solenoids of this type, since surfaces of equal pressure and density coincide. (See also **baroclinity**, **vorticity equation**, **circulation theorem**.)

SOLENOIDAL VECTOR. See **vector**, **solenoidal**.

SOLID INJECTION. See **injection**.

SOLIDIFICATION. The process of change of **phase** from liquid to solid. The opposite process is called boiling or evaporation.

SOLIDS, BAND THEORY OF. See **band theory of solids**.

SOLID, THERMAL EXPANSION OF. See **thermal expansion of solid**.

SOLIDUS CURVE. See **mixed crystals**.

SOLITARY WAVE. A finite-amplitude **gravity wave** consisting of a single crest; under certain conditions, a **permanent wave**.

SOLUBILITY. See **crystallization curve**.

SOLUBILITY PRODUCT. The law of **mass action** for the equilibrium between a solid strong electrolyte and the ions which are formed from it in solution, may be written for dilute and ideal solutions

$$C_+^{\nu+} C_-^{\nu-} = K(T, p) \quad (1)$$

The equilibrium constant K is called the solubility product.

SOLUTIONS, ATHERMAL. See **athermal solutions**.

SOMMERFELD FINE STRUCTURE CONSTANT. The dimensionless constant

$$\alpha = 2\pi e^2/hc = (7.29729 \pm 0.00003) \times 10^{-3}$$

introduced by Sommerfeld in the theoretical treatment of the fine structure of the hydrogen atom and the hydrogen-like ions. (See **hydrogen fine structure**.) In the above formula e stands for the charge of the electron, c , for the velocity of light, and h , for the Planck constant.

SOMMERFELD-KOSSEL DISPLACEMENT LAW. The first spark spectrum of an element is similar in all details to the arc spectrum of the element preceding it in the periodic system; similarly, the second spark spectrum is similar to the first spark spectrum of the element preceding it, or to the arc spectrum of the element with **atomic number** two units smaller, and so forth.

SONE. A unit of loudness. By definition, a simple tone of frequency 1000 cycles per second, 40 **decibels** above a listener's threshold, produces a loudness of 1 sone. The loudness of any sound that is judged by the listener to be n times that of the 1-sone tone is n sones. The loudness scale is a relation between loudness and **level above threshold** for a particular listener. In presenting data relating loudness in sones to **sound pressure level**, or in averaging the loudness scales of several listeners, the thresholds (measured or assumed) should be specified. The term "loudness unit" has been used for the basic subdivision of a loudness scale based on group judgment on which a loudness level of 40 **phons** has a loudness of approximately 1000 loudness units. For example, see Figure 1 of *American Standard for Noise Measurement, Z24.2-1942*.

SOUND ABSORPTION COEFFICIENT OF SURFACES. The ratio of the sound energy absorbed by a surface to the sound energy incident upon it. It may be defined for a particular angle of incidence or for random wave incidence. The average sound absorption coefficient for a room $\bar{\alpha}$ is given by the weighted average of the random-incidence absorption coefficients. Thus

$$\bar{\alpha} = \Sigma S_n \alpha_n / \Sigma S_n$$

where α_n is the random-incidence absorption coefficient of the particular section of the surface of the room of area S_n .

SOUND ATTENUATION FACTOR. See **attenuation factor (2)**.

SOUND BAND PRESSURE LEVEL. The **band pressure level** of a sound for a specified frequency band is the effective sound pressure level for the sound energy contained within the band. The width of the band and the reference pressure should be specified. When measuring thermal noise, the **standard deviation**

tion of the sound pressure readings will not exceed about 10 per cent if the product of the band width in cycles per second and the integration time in seconds exceeds 20.

SOUND-ENERGY DENSITY. The sound-energy density at a point in a sound field is the sound energy contained in a given infinitesimal part of the medium divided by the volume of that part of the medium. The commonly used unit is the erg per cubic centimeter. The term instantaneous energy density has a meaning analogous to the related terms used for **sound pressure**. In speaking of average energy density in general, it is necessary to distinguish between the space average (at a given instant) and the time average (at a given point).

SOUND ENERGY FLUX. The average rate of flow of sound energy for one period through any specified area. The commonly used unit is the erg per second. Expressed mathematically, the sound-energy flux J is

$$J = \frac{1}{T} \int_0^T p S v_a dt,$$

where T is an integral number of periods or a time long compared to a period, p is the instantaneous **sound pressure** over the area S , v_a is the component of the instantaneous particle velocity (see **velocity, particle**) in the direction a , normal to the area S . In a medium of density ρ , for a plane or spherical free wave having a speed of propagation c , the sound-energy flux through the area S , corresponding to an effective sound pressure p , is

$$J = \frac{p^2 S}{\rho c} \cos \theta,$$

θ is the angle between the direction of propagation of the sound and the normal to the area S .

SOUND EQUATION, IMPULSIVE. See **impulsive sound equation**.

SOUND INTENSITY (SPECIFIC SOUND-ENERGY FLUX) (SOUND-ENERGY FLUX DENSITY). The sound intensity in a specified direction at a point is the average rate of sound energy transmitted in the specified direction through a unit area normal to this direction at the point considered. The commonly used unit is the erg per second per

square centimeter, but sound intensity may also be expressed in watts per square centimeter. The sound intensity in any specified direction, a , of a sound field is the sound-energy flux through a unit area normal to that direction. This is given by the expression

$$I_a = \frac{1}{T} \int_0^T p v_a dt,$$

where T is an integral number of periods or a time long compared to a period, p is the instantaneous sound pressure, v_a is the component of the instantaneous particle velocity in the direction a . In the case of a free plane or spherical wave having the effective sound pressure, p , the speed of propagation, c , in a medium of density, ρ , the intensity in the direction of propagation is given by:

$$I = \frac{p^2}{\rho c}.$$

SOUND INTENSITY LEVEL (SPECIFIC SOUND-ENERGY FLUX LEVEL) (SOUND-ENERGY FLUX DENSITY LEVEL). The intensity level, in **decibels**, of a sound is 10 times the logarithm to the base 10 of the ratio of the intensity of this sound to the reference intensity. The reference intensity should be stated explicitly. In discussing sound measurements made with pressure or velocity microphones, especially in enclosures involving normal modes of vibration or in sound fields containing standing waves, caution must be observed in using the terms "intensity" and "intensity level." Under such conditions it is more desirable to use the terms sound pressure level or sound velocity level, since the relationship between the intensity and the pressure or velocity is generally unknown.

SOUND INTERVAL. See **interval, sound formula for**.

SOUND LEVEL. The sound level, at a point in a sound field has been defined by the American Standards Association as the weighted sound pressure level determined in the manner specified in the *American Standard Sound Level Meters for Measurement of Noise and Other Sounds*, **Z24.3-1944**, or its latest revision. The meter reading (in decibels) corresponds to a value of the sound pressure integrated over the audible frequency range with a spe-

cified frequency weighting and integration time.

SOUND, LOUDNESS OF. See *loudness of a sound*.

SOUND OCTAVE-BAND PRESSURE LEVEL (OCTAVE PRESSURE LEVEL).

The *band pressure level* for a frequency band corresponding to a specified octave. The location of the octave-band pressure level on a frequency scale is usually specified as the geometric mean of the upper and lower frequencies of the octave.

SOUND POWER OF A SOURCE. The total sound energy radiated by the source per unit of time. The commonly used unit is the erg per second but the power may also be expressed in watts.

SOUND PRESSURE, EFFECTIVE (ROOT-MEAN-SQUARE SOUND PRESSURE). The root-mean-square value of the instantaneous sound pressures, over a time interval, at the point under consideration. In the case of periodic sound pressures, the interval must be an integral number of periods or an interval that is very long compared to a period. In the case of nonperiodic sound pressures, the interval should be long enough to make the value obtained essentially independent of small changes in the length of the interval. The term *effective sound pressure* is frequently shortened to *sound pressure*.

SOUND PRESSURE, INSTANTANEOUS. The total instantaneous pressure at a point under consideration minus the static pressure at that point. The commonly used unit is the *microbar*. Sometimes called *excess sound pressure*.

SOUND PRESSURE LEVEL. The sound pressure level, in decibels, of a sound is 20 times the logarithm to the base 10 of the ratio of the pressure of this sound to the reference pressure. The reference pressure should be explicitly stated. The following reference pressures are in common use: (1) 2×10^{-4} microbar; (2) 1 microbar. Reference pressure (1) has been in general use for measurements dealing with hearing and sound-level measurements in air and liquids, while (2) has gained widespread use for calibrations, and many types of sound-level measurements in liquids. It is to be noted that in

many sound fields the sound pressure ratios are not proportional to the square root of corresponding power ratios and hence cannot be expressed in decibels in the strict sense; however, it is common practice to extend the use of the decibel to these cases.

SOUND PRESSURE, MAXIMUM. The maximum sound pressure for any given cycle of a periodic wave is the maximum absolute value of the instantaneous sound pressure occurring during that cycle. The commonly used unit is the microbar. In the case of a sinusoidal sound wave this maximum sound pressure is also called the *pressure amplitude*.

SOUND PRESSURE, PEAK. For any specified time interval, the maximum absolute value of the instantaneous sound pressure in that interval. The commonly used unit is the microbar. In the case of a periodic wave, if the time interval considered is a complete period, the peak sound pressure becomes identical with the maximum sound pressure. (See *sound pressure, maximum*.)

SOUND PRESSURE SPECTRUM LEVEL. At a specified frequency, the effective sound pressure level for the sound energy contained within a band 1 cycle per second wide, centered at the specified frequency. Ordinarily this has significance only for sound having a continuous distribution of energy within the frequency range under consideration. The reference pressure should be explicitly stated. Since in practice it is necessary to employ filters having an effective band width greater than 1 cycle per second, the pressure spectrum level is in general a computed quantity. For a sound having a uniform distribution of energy, the computation can be made as follows: Let L_{ps} be the desired pressure spectrum level, p be the effective pressure measured through the filter system, p_0 be reference sound pressure, Δf be the effective band width of the filter system, and $\Delta_0 f$ be the reference band width (1 cycle per second), then

$$L_{ps} = 10 \log_{10} \left[\frac{p^2 / \Delta f}{p_0^2 / \Delta_0 f} \right].$$

For computational purposes, if L_p is the band pressure level observed through the filter, the above relation reduces to

$$L_{ps} = L_p - 10 \log_{10} \frac{\Delta f}{\Delta_0 f}.$$

SOUND REFLECTION COEFFICIENT (ACOUSTICAL REFLECTIVITY). The sound reflection coefficient of a surface not a generator is the ratio of the rate of flow of sound energy reflected from the surface, on the side of incidence, to the incident rate of flow. Unless otherwise specified, all possible directions of incident flow are assumed to be equally probable. Also, unless otherwise stated, the values given apply to a portion of an infinite surface, thus eliminating edge effects.

SOUND REFRACTION. See **refraction of sound.**

SOUND SCATTERING. See **scattering, sound.**

SOUND SOURCE, DOUBLET. Two point sources equal in strength $\pm 4\pi A'$, but opposite in phase separated by a vanishingly small distance δr . The strength of the doublet is $4\pi A'\delta r$. Let $A'\delta r = A$. In these considerations A' corresponds to A of Equation 1 of the entry for **sound source, point**, that is $4\pi A' = S\xi_0$.

At a distance r in a direction inclined at an angle α to the axis of the doublet the velocity potential is

$$\phi = \frac{\left(\frac{1}{r} + jk\right) A}{r} e^{jk(ct-r)} \cos \alpha \quad (1)$$

where ϕ is the velocity potential, r is the distance from the source, in centimeters, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, c is the velocity of sound propagation, in centimeters per second, A is defined by Equation 1 of the entry for **sound source, point**, t is the time, in seconds, and α is the angle between the line joining the source and the point of observation and the axis of the doublet.

The sound pressure of a doublet source is

$$p = -\rho \frac{\partial \phi}{\partial t} = -j \frac{\rho ck A}{r} \left(\frac{1}{r} + jk\right) e^{jk(ct-r)} \cos \alpha \quad (2)$$

where p is the sound pressure, in dynes per square centimeter. Retaining the real parts of Equation 2,

$$p = \frac{\rho ck A}{r} \left[\frac{1}{r} \sin k(ct - r) + k \cos k(ct - r) \right] \cos \alpha. \quad (3)$$

At a very large distance

$$p \propto \frac{k^2 A}{r} \cos \alpha. \quad (4)$$

At a very small distance

$$p \propto \frac{k A}{r^2} \cos \alpha. \quad (5)$$

The particle velocity has two components, the radial $\frac{\partial \phi}{\partial r}$ and the transverse $\frac{1}{r} \frac{\partial \phi}{\partial \alpha}$. The radial component of the particle velocity is

$$u = \frac{\partial \phi}{\partial r} = - \left[\left(\frac{2}{r^3} + \frac{jk}{r^2} \right) + jk \left(\frac{1}{r^2} + \frac{jk}{r} \right) \right] A e^{jk(ct-r)} \cos \alpha. \quad (6)$$

Retaining the real parts of Equation 6

$$u = -A \left[\left(\frac{2}{r^3} - \frac{k^2}{r} \right) \cos k(ct - r) - \frac{2k}{r^2} \sin k(ct - r) \right] \cos \alpha. \quad (7)$$

At a very large distance

$$u \propto \frac{A k^2}{r} \cos \alpha. \quad (8)$$

At a very small distance

$$u \propto \frac{A}{r^3} \cos \alpha. \quad (9)$$

The transverse component of the particle velocity is

$$u = \frac{1}{r} \frac{\partial \phi}{\partial \alpha} = - \left(\frac{1}{r} + jk \right) \frac{A}{r^2} e^{jk(ct-r)} \sin \alpha. \quad (10)$$

Retaining the real parts of Equation 10

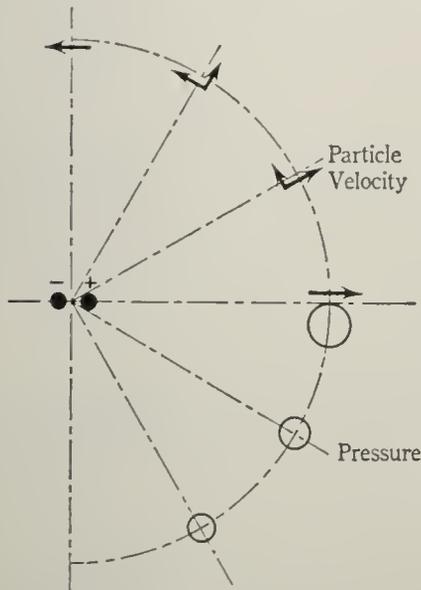
$$u = -A \left[\frac{1}{r^3} \cos k(ct - r) - \frac{k}{r^2} \sin k(ct - r) \right] \sin \alpha. \quad (11)$$

At a very large distance

$$u \propto \frac{Ak}{r^2} \sin \alpha. \tag{12}$$

At a very small distance

$$u \propto \frac{A}{r^3} \sin \alpha. \tag{13}$$



The sound pressure and particle velocity at a constant distance from a doublet source. The magnitude of the pressure is indicated by the circle. The particle velocity has two components, a radial and a transverse component. The direction and magnitude of these two components are indicated by vectors.

The figure shows the velocity components and the pressure for various points around a doublet source.

The total power, in ergs, emitted by a doublet source is

$$P = \iint \frac{p^2}{\rho c} dS \tag{14}$$

where p is the pressure, in dynes per square centimeter, ρ is the density, in grams per cubic centimeter, c is the velocity of sound, in centimeters per second, and dS is the area, in square centimeters, over which the pressure is p .

Taking the value of p from Equation 4 (for r very large), the total average power in ergs per second emitted by a doublet source is

$$P_T = 2\pi r^2 \int_0^\pi \frac{\rho c k^4 A^2}{2r^2} \cos^2 \alpha \sin \alpha d\alpha \tag{15}$$

$$P_T = \frac{2}{3} \pi \rho c k^4 A^2 \tag{16}$$

where ρ is the density, in grams per cubic centimeter, k is $2\pi/\lambda$, λ is the wavelength, in centi-

eters, c is the velocity of sound, in centimeters per second, and A is defined by equation **sound source, point**.

The power output from a simple source (Equation 6 of **sound source, point**) is proportional to the square of the frequency, while the power output from a doublet source (Equation 16) is proportional to the fourth power of the frequency.

SOUND SOURCE, POINT. A small source which alternately injects fluid into a medium and withdraws it.

Assume a point source with a maximum rate of fluid emission of $4\pi A$ cubic centimeters per second. The momentary rate at a time t is $4\pi A \cos \omega t$. The maximum rate of fluid emission may be written

$$-4\pi A = S \dot{\xi}_0 \tag{1}$$

where S is the area of the surface of the source, in square centimeters, and $\dot{\xi}_0$ is the maximum velocity, in centimeters per second over the surface S .

The velocity potential of a point source from Equation 1 of **sound wave, spherical**, is

$$\phi_r = \frac{A}{r} e^{jk(ct-r)} \tag{2}$$

where ϕ_r is the velocity potential, r is the distance from source, in centimeters, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, c is the velocity of sound propagation, in centimeters per second, t is the time in seconds, and A is defined in Equation 1.

The particle velocity at a distance r , from Equation 6 of **sound wave, spherical**, is

$$u = -\frac{Ak}{r} \left[\frac{1}{kr} \cos k(ct - r) - \sin k(ct - r) \right] \tag{3}$$

where u is the particle velocity in centimeters per second.

The pressure at a distance r from Equation 4 of **sound wave, spherical**, is

$$p = \frac{\rho kcA}{r} \sin k(ct - r) \tag{4}$$

where p is the sound pressure in dynes per square centimeter.

The intensity or average power P in ergs per second, transmitted through a unit area at

a distance r , in centimeters, is the product of p and u and is given by

$$P = \frac{\rho c k^2 A^2}{2r^2}. \quad (5)$$

The total average power P_T , in ergs per second emitted by the source is

$$P_T = 2\pi\rho c k^2 A^2 \quad (6)$$

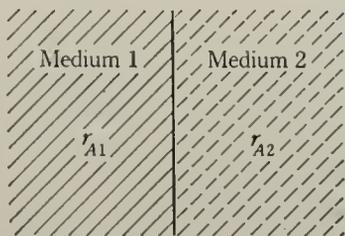
where ρ is the density of the medium, in grams per cubic centimeter, c is the velocity of sound, in centimeters per second, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, and A is defined by Equation 1.

SOUND SOURCES, DIRECTIONAL CHARACTERISTICS. See entries under **directional characteristic**.

SOUND SOURCE, STRENGTH OF (STRENGTH OF A SIMPLE SOURCE).

The maximum instantaneous rate of volume displacement produced by the source when emitting a wave with sinusoidal time variation.

SOUND TRANSMISSION FROM ONE MEDIUM TO ANOTHER MEDIUM. The problem of transmission from one medium to another medium as shown in the figure is the



same as the problem of transmission from one pipe to another.

The ratio of the power transmitted in the medium 2 to incident flow of power in the medium 1 of the figure is

$$P_{12} = \frac{4r_{A1}r_{A2}}{(r_{A1} + r_{A2})^2}$$

where r_{A1} is the acoustical resistance of medium 1, in acoustical ohms and r_{A2} is the acoustical resistance of medium 2, in acoustical ohms.

SOUND TRANSMISSION IN A ROD. See **longitudinal waves in a rod**; **torsional waves in a rod**.

SOUND TRANSMISSION IN TUBES. The transmission loss in tubes of circular section is of interest in problems in acoustics involving the use of tubes. The equation expressing the sound transmission in a tube is

$$A = A_0 e^{-\alpha x}$$

where A is the amplitude (pressure or volume current) at a distance x centimeters from the

amplitude A_0 , α is $\frac{\gamma'}{Rc} \sqrt{\frac{\omega\mu}{2\rho}}$, R is the radius

of the tube, in centimeters, c is the velocity of sound, in centimeters per second, ω is $2\pi f$, f is the frequency, in cycles per second, μ is the viscosity coefficient, 1.86×10^{-4} for air, ρ is the density, in grams per cubic centimeter, γ' is $1 + 1.58(\gamma^{1/2} - \gamma^{-1/2})$, and γ is the ratio of specific heats, 1.4 for air.

SOUND TRANSMISSION IN TUBES LINED WITH ABSORBING MATERIAL.

In ventilator and exhaust systems it is desirable to provide a high degree of attenuation for audio-frequency waves while offering low resistance to continuous flow of air. For that purpose, the most satisfactory systems are ducts lined with absorbing material. Longitudinal isolation of the walls of the duct should be provided to prevent longitudinal transmission of sound by the walls of the duct. This can be accomplished by the use of rubber connectors at regular intervals. The walls of the duct should be rigid so that air-borne sounds are not transmitted through the walls. Very high attenuation can be obtained in ducts of this type.

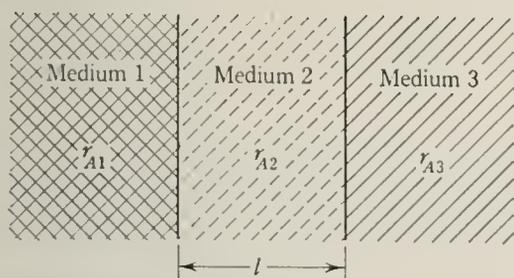
The attenuation $[a]$, in decibels per foot, in a square or rectangular conduit lined with absorbing material may be obtained from the following empirical equation,

$$[a] = 12.6\alpha^{1.4} \frac{P}{A}$$

where P is the perimeter, in inches, A is the cross-sectional area in square inches, and α is the absorption coefficient of the material used for lining the duct.

The above equation holds for square ducts and rectangular ducts in which the ratio between the two sides is not greater than two.

SOUND TRANSMISSION THROUGH THREE MEDIA. The problem of transmis-



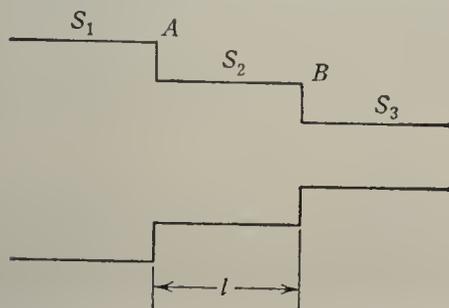
Sound transmission through three media, the middle medium being of finite length l .

sion through the three media of the figure is the same as that through the three pipes below. The ratio of the power transmitted in the medium 3 to the incident flow of power in the medium 1 is

$$P_{13} = \frac{4 \frac{r_{A1}}{r_{A3}}}{\left(\frac{r_{A1}}{r_{A3}} + 1\right)^2 \cos^2 kl + \left(\frac{r_{A1}}{r_{A2}} + \frac{r_{A2}}{r_{A3}}\right)^2 \sin^2 kl} \quad (1)$$

where r_{A1} is the acoustical resistance of the medium 1, in acoustical ohms, r_{A2} is the acoustical resistance of the medium 2, between 1 and 3, in acoustical ohms, r_{A3} is the acoustical resistance of the medium 3, in acoustical ohms, l is the length of the medium 2, in centimeters, k is $2\pi/\lambda$, λ is the wavelength in the medium 2, in centimeters.

SOUND TRANSMISSION THROUGH THREE PIPES. Consider three pipes of cross sectional areas S_1 , S_2 , and S_3 as shown below.



Sound transmission through three pipes, the middle pipe being of finite length l .

Assume that sound travels from pipe 1 to pipe 3. Let the boundary between S_1 and S_2 be denoted by A and between S_2 and S_3 by B .

The ratio of the power transmitted in S_3 to the incident flow of power in S_1 is

$$P_{13} = \frac{4 \frac{S_3}{S_1}}{\left(\frac{S_3}{S_1} + 1\right)^2 \cos^2 kl + \left(\frac{S_2}{S_1} + \frac{S_3}{S_2}\right)^2 \sin^2 kl} \quad (1)$$

If kl is small, the transmission is independent of the cross section of the channel S_2 . If $\sin kl = \pm 1$, the power transmission is

$$P_{13} = \frac{4 \frac{S_3}{S_1}}{\left(\frac{S_2}{S_1} + \frac{S_3}{S_2}\right)^2} = \frac{4S_3S_2^2S_1}{(S_2^2 + S_1S_3)^2} \quad (2)$$

Equation 2 shows that $P_{13} = 1$, if $S_2^2 = S_1S_3$. That is, if $\sin kl = \pm 1$ and providing the area of S_2 is a geometric mean of S_1 and S_3 , the transmission is unity.

SOUND VELOCITY. See velocity of sound.

SOUND WAVE, INTENSITY OF A SPHERICAL. See intensity of a spherical sound wave.

SOUND WAVE, PLANE. A sound wave in which **equiphase surfaces** are a family of parallel planes.

The velocity potential in a plane sound wave is

$$\phi = A \cos k(ct - x) \quad (1)$$

where ϕ is the velocity potential, A is the amplitude of ϕ (see Equation 2 below), k is $2\pi/\lambda$, λ is the wavelength, in centimeters, $c = f\lambda$ is the velocity of sound, in centimeters per second, and f is the frequency, in cycles per second.

The amplitude of the velocity potential is

$$A = \frac{\xi_0}{k} \quad (2)$$

where A is the amplitude of the velocity potential, and ξ_0 is the maximum particle velocity, in centimeters per second.

The particle velocity in a plane sound wave is

$$u = \frac{\partial \phi}{\partial x} = kA \sin k(ct - x) \quad (3)$$

where u is the particle velocity, in centimeters per second.

The particle velocity in a sound wave is the instantaneous velocity of a given infinitesimal

part of the medium, with reference to the medium as a whole, due to the passage of the sound wave.

The sound pressure in a plane sound wave is

$$p = kc\rho A \sin k(ct - r) \quad (4)$$

where ρ is the density, in grams per cubic centimeter, and p is the sound pressure, in dynes per square centimeter.

SOUND WAVE, SPHERICAL. A sound wave in which **equiphase surfaces** are a family of concentric spheres.

The velocity potential in a spherical sound wave is

$$\phi = \frac{A}{r} \epsilon^{jk(ct-r)} \quad (1)$$

where ϕ is the velocity potential, r is the distance from the origin, in centimeters, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, c is the velocity of sound, in centimeters per second, t is the time, in seconds, and A is the amplitude of the velocity potential.

The maximum rate of fluid emission of a small source is given by

$$U = 4\pi A \quad (2)$$

where U is the fluid emission, in cubic centimeters per second.

The sound pressure in a spherical sound wave is

$$p = -\frac{jkcA\rho}{r} \epsilon^{jk(ct-r)} \quad (3)$$

where p is the sound pressure, in dynes per square centimeter, and ρ is the density, in grams per cubic centimeter.

Retaining the real part of Equation 3, the pressure is

$$p = \rho \frac{kcA}{r} \sin k(ct - r). \quad (4)$$

The particle velocity in a spherical sound wave is

$$u = -\left(\frac{1}{r} + jk\right) \frac{A}{r} \epsilon^{jk(ct-r)} \quad (5)$$

where u is the particle velocity, in centimeters per second.

Retaining the real part of Equation 5 the particle velocity is

$$u = -\frac{Ak}{r} \left[\frac{1}{kr} \cos k(ct - r) - \sin k(ct - r) \right]. \quad (6)$$

The phase angle between the pressure and velocity in a spherical wave is given by

$$\theta = \tan^{-1} \frac{1}{kr} \quad (7)$$

where θ is the angle between the pressure and particle velocity.

For very large values of kr , that is, plane waves, the pressure and particle velocity are in phase. The ratio of the absolute value of the particle velocity to the absolute value of the pressure is given by

$$\text{Ratio} = \frac{\sqrt{1 + k^2 r^2}}{\rho c k r}. \quad (8)$$

SOUND WAVE(S), STATIONARY. The wave system resulting from the interference of waves of the same frequencies, which is characterized by the existence of nodes or partial nodes.

Consider two plane waves of equal amplitude traveling in opposite directions; the velocity potential may be expressed as

$$\phi = A[\cos k(ct - x) + \cos k(ct + x)] \quad (1)$$

where ϕ is the velocity potential, k is $2\pi/\lambda$, λ is the wavelength, in centimeters, x is the distance normal to the planes of equal phase, in centimeters, t is the time, in seconds, c is the velocity of sound, in centimeters per second, and A is the amplitude of the velocity potential; see Equation 2 of plane sound waves (in entry **sound waves, plane**).

The sound pressure in the stationary wave system is

$$p = -\rho \frac{\partial \phi}{\partial t} \\ = kc\rho A[\sin k(ct - x) + \sin k(ct + x)] \quad (2)$$

where p is the sound pressure in dynes per square centimeter, and ρ is the density, in grams per cubic centimeter.

Equation 2 may be written,

$$p = 2kc\rho A[\sin ket \cos kx]. \quad (3)$$

The particle velocity in the stationary wave system is

$$u = \frac{\partial \phi}{\partial x} = kA[\sin k(ct - x) - \sin k(ct + x)] \quad (4)$$

where u is the particle velocity, in centimeters per second.

Equation 4 may be written as follows

$$u = -2kA[\cos kct \sin kx] \quad (5)$$

$$u = 2kA \left[\sin \left(kct - \frac{\pi}{2} \right) \cos \left(kx - \frac{\pi}{2} \right) \right] \quad (6)$$

Equations 3 and 6 show that the maxima of the particle velocity and pressure are separated by a quarter wavelength. The maxima of p and u differ by 90° in time phase.

SOURCE. (1) A point from which fluid originates. The strength m of a point source is the total flux of fluid from it. The velocity potential at distance r due to the source is

$$\phi = -\frac{m}{4\pi r}$$

A *line source*, in two-dimensional flow, is a line from which fluid originates. The strength m of a line source is the total flux of fluid from unit length of it. The complex potential of a line source is

$$w = \phi + i\psi = \frac{m}{2\pi} (\log r + i\theta) = \frac{m}{2\pi} \log z.$$

(2) For source of sound, see **sound source**.

SOURCE LEVEL. The **sound intensity** at a point on the axis one yard from a source, measured in **decibels** above a reference level. In symbols, the source level S , in decibels, is given by

$$S = 10 \log \frac{I_0}{I_r}$$

where I_0 is the intensity at a point on the axis one yard from the source, I_r is the reference intensity, usually corresponding to an rms pressure of 1 dyne/cm².

SOURCE-SINK METHOD. In the theory of neutron chain reactors, a method for the analysis of systems composed of regular, heterogeneous arrays of fuel and moderator. Fuel rods are represented by line sources of strength proportional to the thermal neutron flux at the rod, while the contribution of a rod to the

thermal flux at a point in the moderator is the convolution of slowing-down and diffusing **kernels** for a line source. The method is useful in the analysis of lattices containing small numbers of rods and in showing the connection between diffusion in homogeneous and heterogeneous systems. It is sometimes called "small source theory."

SOUTHWELL METHOD FOR CRITICAL LOAD. See **critical load**, **Southwell method**.

SPACE CHARGE LIMITATION OF CURRENTS. It has been shown by Child that the current between a plane cathode and a parallel plane anode at a distance d from it, when the anode potential is V , cannot exceed a certain maximum value, determined by the modification of the electric field near the cathode as a result of the space charge of electrons in that region. If the electrons leave the cathode with zero speed, the maximum current per unit area of the cathode is

$$i = \frac{4\epsilon_0}{9} \sqrt{\frac{2e}{m}} \frac{V^{3/2}}{d^2}$$

where e and m are the electronic charge and mass, respectively, and ϵ_0 is the electric constant. Langmuir has extended the equation to include the case of a cylindrical cathode of radius a , surrounded by a coaxial cylindrical anode of radius b . The maximum current per unit length is then

$$i = \frac{8\pi\epsilon_0}{9} \sqrt{\frac{2e}{m}} \frac{V^{3/2}}{b(\ln b/a)^2}$$

The dependence of the current on the $3/2$ power of the potential difference is general, and is the basis of the definition of **perveance**.

SPACE CURVE. A curve in space, not necessarily a **twisted curve**.

SPACE GROUP. When identical objects are placed at the lattice points of a **space lattice** a regular spacial array is obtained, which has, perhaps, **symmetry elements** over and above those of the **point group** of the original lattice. Thus, because of some symmetry property of the identical objects (that is, of the **unit cells**, in a crystal), there may exist **glide planes** and **screw axes** which define operations turning the structure into itself. It has been shown that there are only 230 different types of sym-

metry possible for such a system; these are the 230 **space groups** which are classified according to the symmetry elements they possess.

SPACE, HERMITIAN. See **vector space**.

SPACE LATTICE. A regular three-dimensional arrangement of points constructed by the repeated application of the **primitive translations** which carry a unit cell into its neighbor. Each point in the lattice has exactly the same environment and the lengths and orientations of the primitive translations fix the type of the lattice. There are only fourteen kinds of simple space lattice, which are classified according to their symmetry type, as follows:

<i>Symmetry Type</i>	<i>Lattice Type</i>
Cubic	Simple cube; face-centered cube; body-centered cube.
Tetragonal	Tetragonal prism; body-centered tetragonal prism.
Orthorhombic	Rectangular prism; body-centered rectangular prism; rhombic prism; body-centered rhombic prism.
Monoclinic	Monoclinic parallelepiped; monoclinic, one-face centered prism.
Triclinic	Triclinic parallelepiped.
Hexagonal	Hexagonal prism.
Rhomboidal	Rhombohedron.

SPACE-LIKE SURFACE. A surface, σ , in space-time such that no two points on it can be connected by a light signal: for any two points x, y , on σ $(x - y)^2 = (x^0 - y^0)^2 - (x - y)^2$ is always spacelike, i.e., $(x - y)^2 < 0$. If one denotes by $n^\mu(x)$ the unit normal at the point x , then for a spacelike surface $n^\mu(x)n_\mu(x) = 1$ for all x on σ . A plane $t = \text{constant}$ is a special case for which all $xn^\mu(x) = (1, 0, 0, 0)$.

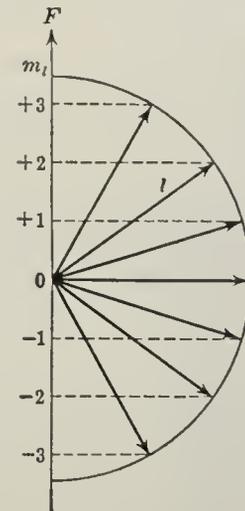
SPACE, LOCALLY EUCLIDEAN. See **locally Euclidean**.

SPACE QUANTIZATION. According to classical theory, an atom brought into an electric or magnetic field \mathbf{F} will carry out a precession such that the angular momentum \mathbf{l} describes a cone with the field direction as axis and with a constant component m_l of the angular momentum in the field direction. While classically m_l can have any value between $+|\mathbf{l}|$ and $-|\mathbf{l}|$ (that is any angle between \mathbf{l} and the field direction is possible), in quantum theory the magnitude of m_l can have only the

discrete values $m_l(h/2\pi)$, where m_l , the magnetic quantum number, has the values

$$m_l = l, l - 1, l - 2, \dots, -l$$

that is only certain angles of \mathbf{l} with respect to the field direction are possible. (See the figure.)



Space quantization of l in a field \mathbf{F} for $l = 3$. From G. Herzberg, *Molecular Spectra and Molecular Structure. I. Spectra of Diatomic Molecules*. 2nd ed., D. Van Nostrand Company, Inc., Princeton, 1950.

Since m_l can at most have the magnitude $l(h/2\pi)$ which is smaller than $\sqrt{l(l+1)}(h/2\pi)$, the magnitude of \mathbf{l} , the latter can never point exactly in the field direction. Space quantization persists and m_l remains defined even for vanishingly small field ($2l + 1$ -fold degeneracy). In atomic and molecular physics space quantization applies to all angular momentum vectors (electronic angular momentum, angular momentum of molecular rotation, etc.).

SPACE TENSOR. See **tensor, space**.

SPACE-TIME, FLAT. See **flat space-time**.

SPACE, VECTOR. See **vector space**.

SPAN. The term span as a noun is used to denote both a structure or portion of a structure between supports, and the distance between supports. (See **truss**.)

SPARK-IGNITION ENGINE (ALSO CALLED EXPLOSION ENGINE). A reciprocating engine which induces a mixture of air and the vapor of a volatile liquid fuel, compresses it to a temperature below the ignition point of the mixture, and ignites the mixture with the aid of an electric spark produced by a spark plug. The engine produces useful

power by expanding the resulting high-pressure gas against the piston.

A spark-ignition engine can operate as a four-stroke or two-stroke engine. In a four-

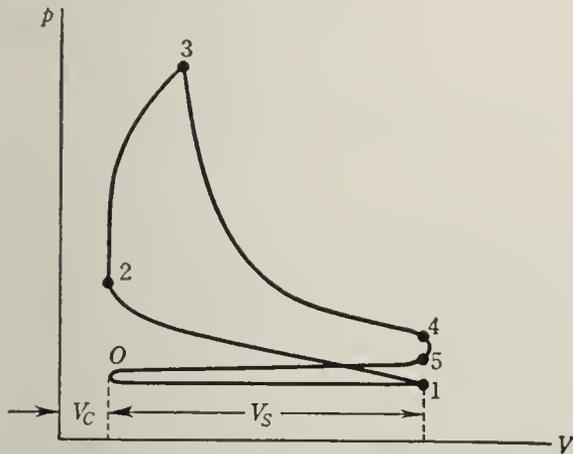


Fig. 1. Cycle of four-stroke, spark-ignition engine.

stroke spark-ignition engine, Figure 1, the sequence of operations is as follows:

- 0-1. Introduction of combustible mixture. Inlet valve open.
- 1-2. Compression of combustible mixture. Valves closed.
- 2-3. Combustion from spark ignition nearly at constant volume ("Explosion"). Valves closed.
- 3-4. Expansion; working stroke. Valves closed.
- 4-5. Release of burnt gases. Exhaust valve opens at 4.
- 5-0. Expulsion of burnt gases. Exhaust valve open.

When the engine operates as a **two-stroke engine**, the only difference in the mode of working results from the different way of inducing the mixture and of expelling the burnt gases. (See **two-stroke engine**.)

It is customary to analyze the operation of a spark-ignition engine under the following simplifying assumptions:

- (1) The cylinder contains a constant quantity of gas throughout the cycle, and the chemical composition of the gas is also assumed constant.
- (2) The generation of heat resulting from internal combustion is replaced by heat transfer from an outside source.
- (3) The process of rejection of heat which occurs from the combustion gas to the atmosphere outside the engine is replaced by cooling at constant volume.

- (4) The specific heats are assumed constant, and the gas is assumed perfect.

These assumptions result in the Otto cycle; Figure 2, which is common to the two-stroke

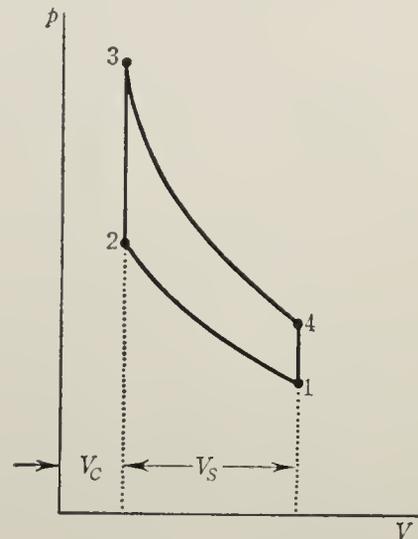


Fig. 2. Otto cycle.

and the four-stroke spark-ignition engine. In it compression and expansion are assumed **isentropic**. The resulting thermal efficiency is

$$\eta_{th} = 1 - \frac{1}{r^{\gamma-1}}$$

where $r = (V_c + V_s)/V_c$ is the **compression ratio**, and γ is the ratio of specific heats c_p/c_v .

Most modern automobiles are powered by high-speed spark-ignition engines. Until the advent of jet-engines and gas turbines, the spark-ignition engine had the lowest power-to-weight ratio of all engines and was, therefore the most suitable engine for aircraft propulsion.

SPARK SPECTRUM. The spectra of the singly, doubly, triply, etc., ionized atoms are referred to as first, second, third, etc., spark spectra. (See also **arc spectrum**.)

SPECIAL FUNCTIONS. This name is usually given to certain classes of functions some of which are described, e.g., in this book under the following headings: **special polynomials; Legendre functions and polynomials, Hermite, hypergeometric, Jacobi, Laguerre, trigonometric, Chebyshev, etc.** (Certain other functions, having little in common with these, e.g., the **gamma function**, are sometimes called special functions.)

These functions can be defined in several ways. If we wish, e.g., to define them by their

orthogonality properties, we proceed as follows. Let a fundamental interval $a \leq x \leq b$, which may also be infinite, be chosen. Let $p(x) \geq 0$ be a non-negative function, to be called a "weighting function." The class of special functions in question is to be the class that arises from orthonormalization (by the **Gram-Schmidt process**) of the sequence of functions $\sqrt{p(x)}$, $x\sqrt{p(x)}$, $x^2\sqrt{p(x)}$, \dots . Usually these functions can also be defined (and it is to this fact that they owe their great importance in mathematical physics) as the eigenfunctions of an **eigenvalue** problem for an ordinary differential equation, by a **generating function**, by a **recursion formula**, by a **Rodrigues formula**, or often by some minimum property. For illustration see under the respective headings mentioned above, and for the simplest possible illustration see under **Legendre polynomials**. These special functions owe their great importance in applications to the possibility, because of their orthogonality properties, of expanding an arbitrary function in terms of them in a generalized **Fourier series**. (See also entry following.)

SPECIAL POLYNOMIALS. Special functions which are polynomials. Among the most important examples are the following (see also under the respective names).

(1) **Legendre polynomials**

$$(1 - 2xy + y^2)^{-1/2} = \sum_{n=0}^{\infty} P_n(x)y^n.$$

(2) **Associated Legendre polynomials**

$$\frac{(2m)!(1 - x^2)^{m/2}y^m}{2^m m!(1 - 2xy + y^2)^{m+1/2}} = \sum_{n=m}^{\infty} P_n^m(x)y^n.$$

(3) **Bessel function of integral order**

$$\exp \left[\frac{x}{2} \left(u - 1/u \right) \right] = \sum_{n=0}^{\infty} J_n(x)u^n.$$

(4) **Hermite polynomials**

$$\exp [x^2 - (z - x)^2] = \sum_{n=0}^{\infty} \frac{H_n(x)z^n}{n!}.$$

The Hermite polynomials find applications in **statistics** in the **Gram-Charlier Type A** series and are also useful in certain applications in physics to problems of heat and quantum mechanics.

(5) **Laguerre polynomials**

$$(1 - z)^{-1} \exp \left(\frac{-xz}{1 - z} \right) = \sum_{n=0}^{\infty} \frac{L_n(x)z^n}{n!}.$$

(6) **Associated Laguerre polynomials**

$$(-1)^k (1 - z)^{-1} \left(\frac{z}{1 - z} \right)^k \exp \left(\frac{-xz}{1 - z} \right) = \sum_{n=k}^{\infty} \frac{L_n^k(x)z^n}{n!}.$$

(7) **Chebyshev polynomials**

$$\frac{1 - xy}{1 - 2xy + y^2} = \sum_{n=0}^{\infty} T_n(x)y^n.$$

SPECIES. The species (or symmetry type) of an eigenfunction, normal vibration, etc., of a molecule indicates its behavior with respect to the symmetry operations possible for the system. The species correspond to the irreducible representations of the point group.

SPECIFIC ACTIVITY. (1) The activity of a radioisotope of an element per unit mass of element present in the sample. (2) The activity per unit mass of a pure radionuclide. (3) The activity per unit mass of any sample of radioactive material. Specific activity is commonly given in a wide variety of units (e.g., millicuries per gram, disintegrations per second per milligram, counts per minute per milligram, etc.).

SPECIFIC CURVATURE. See curvature of a surface, second.

SPECIFIC DAMPING CAPACITY. If a specimen is taken round a stress cycle and the energy loss is ΔW , the ratio $\Delta W/W$ is called the specific damping capacity, where W is the maximum elastic energy stored in the specimen during the cycle. For sinusoidal cycles, it can be related to other measures of internal friction (see **friction, internal**). The specific damping capacity is sometimes referred to as the *specific loss*.

SPECIFIC FUEL CONSUMPTION. See specific heat consumption.

SPECIFIC HEAT CONSUMPTION. Amount of heat Q_{sp} required to produce one unit of work, W , in a heat engine. Except for the fact that Q_{sp} is usually expressed in heat units whereas W is expressed in work units, the

specific heat consumption is the reciprocal of the **brake thermal efficiency** of the plant, η_o

$$Q_{sp} \sim \frac{1}{\eta_o}$$

Evidently

$$Q_{sp} = \frac{2544}{\eta_o} \text{ BTU/HP hr} = \frac{3413}{\eta_o} \text{ BTU/KW hr.}$$

Often the specific heat consumption is calculated per unit of indicated or theoretical work rather than per unit work at the main coupling. In such case it is necessary to replace the boiler thermal efficiency η_o , by the **thermal efficiency** of the cycle η_{th} , the relative efficiency, η_r , etc.

In steam plants some time ago it was customary to indicate the specific steam consumption, D_{sp} instead of the specific heat consumption:

$$D_{sp} = \frac{2544}{W} \text{ lbm/HP hr} = \frac{3413}{W} \text{ lbm/KW hr}$$

where W stands for the work measured at the brake, indicated work, etc., as the case may be. The specific steam consumption, as opposed to the specific heat consumption, does not give a good idea of the efficiency of the plant, because of the wide range of entrance conditions encountered in practice; it is useful only when comparisons are made between engines with identical entrance conditions.

In internal combustion engines it is usual to indicate the *specific fuel consumption*, B_{sp} , instead of the specific heat consumption. The specific heat consumption is then

$$Q_{sp} = B_{sp}H_n$$

where H_n is the net (lower) calorific value of the fuel. In practice, the range of values of H_n used in internal combustion engines is very narrow, consequently B_{sp} gives a good enough idea of the efficiency of the engine. For more precise comparisons it is necessary to take into account the heat supplied with the cylinder lubricant and to augment B_{sp} by the specific consumption of lubricating oil, b_{sp} .

SPECIFIC HEAT, ELECTRONIC. See **electronic specific heat**.

SPECIFIC HEAT FUNCTION, EINSTEIN. See **Einstein specific heat function**.

SPECIFIC HEAT, LATTICE VIBRATION CONTRIBUTION. See **lattice vibration contribution to specific heat**.

SPECIFIC HEAT OF HYDROGEN. The vibrational specific heat of hydrogen gas is frozen out, and the translational specific heat has its classical value $\frac{3}{2}kN$ (k is Boltzmann's constant, N is number of molecules in the system). The rotational specific heat c_r at high temperatures is equal to the classical value of kN (2 degrees of freedom), but at room temperature and below follows the equation

$$c_r = \frac{3}{4}c_r^{ortho} + \frac{1}{4}c_r^{para},$$

where "ortho" and "para" refer to the ortho- and para-states of the hydrogen molecule. This expression for the rotational specific heat is found experimentally, but differs from the theoretical expression. The difference is due to the fact that the ortho-para ratio is frozen in at its high temperature value 3.

SPECIFIC HEATS OF GASES. These values may be expressed by terms corresponding to the various independent degrees of freedom. If the **partition function** Z is given by the equation

$$Z = Z_a Z_b Z_c,$$

the specific heat c is given by the equation

$$c = c_a + c_b + c_c,$$

where each c_i follows from the corresponding Z_i through the formula

$$c_i = k\beta^2 \frac{\partial^2 \ln Z_i}{\partial \beta^2}$$

where k is Boltzmann's constant, and $\beta = 1/kT$ with T the absolute temperature. In actual cases the terms c_a, c_b, \dots correspond to the vibrational, rotational, and translational degrees of freedom.

SPECIFIC IMPULSE (I_{sp}). In rocket terminology, a parameter indicative of efficiency: a property of the propellant combination and the mixture ratio. Specific impulse is equal to pounds of thrust developed per pound of propellants consumed (fuel plus oxidizer) per second, or the ratio of thrust to propellant mass flow.

SPECIFIC IMPULSE, IDEAL OR THEORETICAL (I'_{sp}). The maximum realizable

impulse obtainable from a given combination of propellants; calculated from thermo-chemical relations. The difference between the ideal and measured specific impulse (I_{sp}) is due to heat losses, incomplete combustion of propellants, flow losses and variations in back pressure.

SPECIFIC INDUCTIVE CAPACITY. See dielectric constant; permeability.

SPECIFIC LOSS. See specific damping capacity.

SPECIFIC MAGNETIC ROTATORY POWER OF A MEDIUM. The Verdet constant of the medium.

SPECIFIC PROPELLANT CONSUMPTION. In rocket propulsion, a term expressing the rate of burning of propellants per unit of thrust. Its units are reciprocal time. Mathematically:

$$SPC = \frac{dw/dt}{F} = \frac{1}{I_{sp}} \text{ sec}^{-1} = \frac{3600}{I_{sp}} \text{ hour}^{-1}.$$

The relation gives a method of comparing the efficiency of different motors.

SPECIFIC PROPERTY. See thermodynamic property.

SPECIFIC REFRACTION. A relationship between the refractive index of a medium at any definite wavelength and its density, of the form

$$r = \left(\frac{n^2 - 1}{n^2 + 2} \right) \left(\frac{1}{\rho} \right)$$

in which r is the specific refraction of the medium, n is its index of refraction at any definite wavelength, and ρ is its density. The relation does not always give a constant value of r as the density is varied, and hence must be considered as an approximation.

SPECIFIC ROTATION. See rotation, specific; molecular rotation.

SPECIFIC STEAM CONSUMPTION. See specific heat consumption.

SPECTRA. See following entries and also those under spectrum.

SPECTRA, ATOMIC. See atomic spectra.

SPECTRA, INVERSION. See microwave spectra of molecules.

SPECTRA, ISOTOPE EFFECT IN MOLECULAR. See isotope effect in molecular spectra.

SPECTRAL ABSORPTANCE. The ratio of the spectral concentration of the absorbed luminous flux to that of the incident flux, for a given wavelength.

SPECTRAL CENTROID. An average wavelength, computed especially for filters and other light-transmitting devices, by taking a weighted average, for each wavelength, of the spectral distribution of the incident light, the transmittance of the device, and the luminosity data for the eye.

SPECTRAL CHARACTERISTIC. A relation, usually shown by a graph, between wavelength and some other variable. (1) In the case of a luminescent screen, the spectral characteristic is the relation between wavelength and emitted radiant power per unit wavelength interval. (2) In a photoelectric device, it is the relation between wavelength and sensitivity per unit wavelength interval.

SPECTRAL CLASSIFICATION OF STARS. In the latter quarter of the 19th century Edward C. Pickering began obtaining the spectra of the stars using an objective prism. The work was carried on for many years under the direction of Miss Annie J. Cannon and in 1924 the Henry Draper Catalogue in nine volumes was published. This catalogue gives the approximate coordinates, the approximate magnitudes, and the spectral type of 225,300 stars. During the years since the appearance of this catalogue the work has been carried on by numerous observers and at the present time we have the spectral types of over half a millions stars brighter than 13th magnitude.

In spite of the great range in luminosities of the stars the spectra of over 99% of them may be classified in a relatively small number of classes, which arrange themselves in a continuous sequence. The order is as follows:

R—N
O—B—A—F—G—K—M

S

The sequence O to M is continuous, while R and N-types form a side branch going off between G and K, and the S group branches off between K and M. In the sequence a sort of decimal notation is used with B5 indicating a spectra half way from B to A, F2 indicating a spectra closer to F than to G, etc.

In classes O and B we find that emission-line and absorption-line objects are found. In the O types the lines of hydrogen are very prominent, together with lines of ionized helium and doubly ionized oxygen, nitrogen and carbon. In the B type stars the lines of helium and hydrogen increase in prominence while ionized helium disappears. Singly-ionized helium, carbon, nitrogen and oxygen are important, while doubly-ionized iron and ionized magnesium make their appearance. As we move along to the A-type stars we find the hydrogen lines at their greatest prominence in A2, with the helium lines weakening and the lines of the metals making their appearance. Throughout the sequence from A through F the hydrogen lines continually weaken and the lines characteristic of the metals strengthen. When class G is reached, the most prominent lines are the H and K lines of calcium, together with a mixture of the atomic and molecular bands of the common metals. In class K the low temperature lines of the metals become more and more important until in class M we find the molecular bands, particularly those of titanium oxide, dominating the spectrum. The spectra of the stars on the R—N branch contain the bands of the carbon compounds, while the S-class stars have the bands of zirconium oxide and lanthanum oxide.

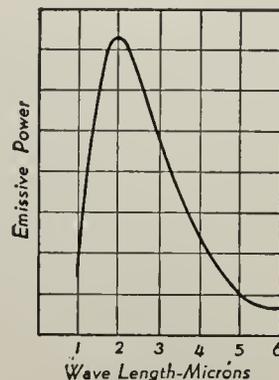
The spectral sequence of the stars is closely correlated with the apparent colors, temperature, and other physical characteristics.

In color we find the O and B stars intrinsically blue, the A—F group are white, the K stars are orange, and the M stars are distinctly red. This immediately suggests a decreasing scale of stellar temperatures. (Cf. **spectrum-luminosity relation; temperature calibration of the spectral types.**)

SPECTRAL CONCENTRATION. Spectral concentration (of a radiometric quantity such as radiant flux, radiant intensity, etc.) is the quotient of this quantity, taken over an infinitesimal range on either side of a given wavelength (or frequency) by the range.

SPECTRAL EMISSIVITY (OF A THERMAL RADIATOR). The ratio of the spectral concentration of radiant emittance of the thermal radiator to that of a **full radiator** at the same temperature.

SPECTRAL ENERGY DISTRIBUTION. When radiation exhibiting a continuous spectrum is quantitatively analyzed, it is found that quite different amounts of power are represented by the radiation within equal ranges of wavelength or of frequency having different limits. The proportion in any such range depends upon the character of the source. If one divides the spectrum into small intervals of wavelength, say 10 angstroms, and plots the power output for each range as ordinate with the mean wavelength of the interval as abscissa, the result is a curve showing the distribution of power through the spectrum.



(See Planck law; relative spectral energy distribution.)

SPECTRAL FUNCTION. In the theory of stationary time-series, and more generally of stationary stochastic processes, a function defined as

$$F(w) = \frac{2}{\pi} \int_0^{\infty} \rho(x) \frac{\sin xw}{x} dx, \quad 0 \leq w \leq \infty,$$

where $\rho(x)$ is the **autocorrelation function** of the series $u(t)$,

$$\rho(x) = \frac{\text{cov} \{u(t), u(t+x)\}}{\text{var} \{u(t)\}}.$$

The spectral density

$$f(w) = \frac{dF}{dw} = \frac{2}{\pi} \int_0^{\infty} \rho(x) \text{cov} xw dx$$

may be regarded as a Fourier transform of the autocorrelation function and *vice versa*.

The definition given relates to the continuous series. Analogous definitions can be framed for discontinuous series.

The graph of $f(w)$ as ordinate against w as abscissa is called the spectrum; that of $F(w)$ against w is called the integrated spectrum. The graph of a certain constant multiple of $f(w)$ against $2\pi/w$ is the periodogram.

SPECTRAL LINE, DOPPLER WIDTH OF. See Doppler width of spectral line.

SPECTRAL LINE, EQUIVALENT WIDTH OF. See equivalent width of spectral line.

SPECTRAL LINE, INTENSITY OF. See intensity of a spectral line.

SPECTRAL LINES, INTERCOMBINATION. See intercombination lines.

SPECTRAL LINE WIDTH, NATURAL. See natural line width.

SPECTRAL LUMINANCE FACTOR. Of a non-self-luminous body, for stated conditions of illumination and of observation, the ratio, at any wavelength λ , of the **spectral concentration** of luminance of the body considered, when illuminated and observed under these conditions, to the spectral density of luminance of a perfect diffuser receiving the same illumination.

SPECTRAL NORM. This term is sometimes used for the **spectral radius**, but this function does not possess the **norm** properties. Also, and more appropriately, applied to the function $\|A\|$ defined by

$$\|A\|^2 = \rho(A^*A),$$

which is a **norm**.

SPECTRAL RADIUS OF A MATRIX A. The quantity $\rho(A) = \max_i |\lambda_i(A)|$ for all eigenvalues of A . (See **norm**; **spectral norm**.)

SPECTRAL REFLECTANCE. The ratio of the **spectral concentration** of the reflected luminous flux to that of the incident flux for a given wavelength.

SPECTRAL TERM. See **term**.

SPECTRAL TRANSMITTANCE. The ratio of the **spectral concentration** of the transmitted luminous flux to that of the incident flux, at the same wavelength.

SPECTRA, MICROWAVE. See microwave spectra of molecules.

SPECTRA, MOLECULAR. See molecular spectra.

SPECTRA, MOLECULAR, COMBINATION DIFFERENCES AND COMBINATION SUMS IN. See combination differences and combination sums.

SPECTRA, MOLECULAR, PERTURBATIONS. See perturbations in molecular spectra.

SPECTRA OF MOLECULES, ELECTRONIC. See electronic spectra of molecules.

SPECTRA OF MOLECULES, VIBRATIONAL. See vibrational spectra of molecules.

SPECTRA, QUANTUM THEORY OF. See quantum theory of spectra.

SPECTRA, ROTATIONAL. See rotational spectra of molecules.

SPECTRA, SPIN-REORIENTATION. See microwave spectra of molecules.

SPECTROSCOPIC BINARY STARS. (Cf. **orbit**, **elliptic motion**.) In the course of programs for the determination of the radial velocities of the stars it occasionally happens that, after correcting for the components of the earth's orbital and rotational motions in the line of sight, the radial velocities of some stars show periodic changes. On the assumption that this variable motion is due to the orbital motion of one star of a pair that are revolving about a common center of gravity, we refer to the star as a spectroscopic binary. Figure 1 is a diagram of such a system. The center of gravity of the system is at G . The line from G to the sun intersects the sphere centered on G at T , with the other pole at K . The plane HLM may be called the plane of the apparent orbit. S is the position of one of the stars. GS is a radius vector, expressed in kilometers, of one of the stars whose spectra is observed, and the radial velocity is found to be variable.

GB is the radius vector when the star is closest to G and we have BGS , or APG as the true **anomaly**, θ . We shall assume that LSP is the direction of motion, in which case L is the

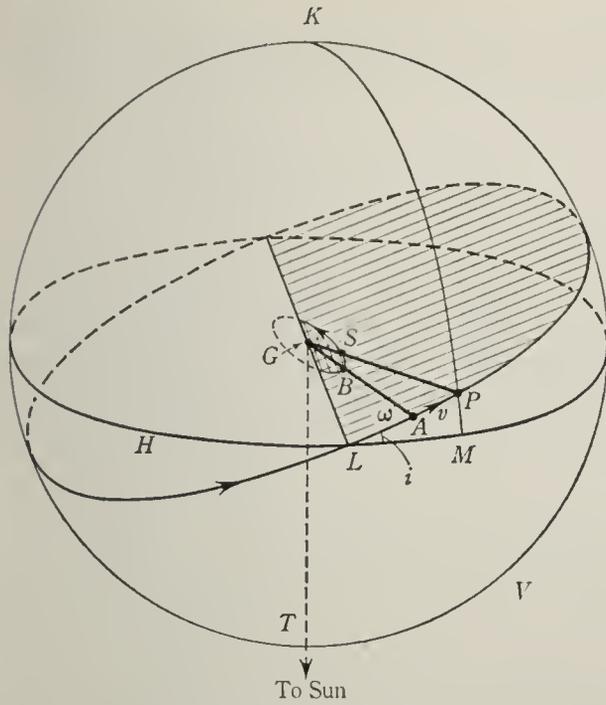


Fig. 1. Diagram for motion of spectroscopic binary star.

ascending node. LA is the orbital element ω . (See **orbit, astronomical elements.**) Call the distance of S from the plane HLM , z , with z positive when the star is on the side of the plane HLM opposite to the sun. This means that $z = r \sin PM$. We also have $\sin PM = \sin (v + \omega) \sin i$. Hence

$$z = r \sin (v + \omega) \sin i \quad (1)$$

where z is expressed in kilometers and $\frac{dz}{dt}$ is the radial velocity that is observed for the star. As the star moves in the orbit, this radial ve-

locity will apparently vary in the direction of the sun. Since the system may have a radial velocity V , the radial velocity at any instant will be

$$R = \frac{dz}{dt} + V. \quad (2)$$

Once the approximate period of variation of the radial velocity has been determined, a **least squares** solution will give an accurate period. Then all observations are reduced to one epoch and a mean velocity curve is obtained. See Figure 2.

Many methods have been developed for the determination of the orbit of a spectroscopic binary. The orbit is actually the orbit of one of the stars about the center of gravity of the system. In case the velocity curve for both stars can be obtained, the two velocity curves are treated separately. The methods are long and detailed; therefore the treatment below is restricted to part of the determination of an orbit by the method of Lehmann-Filhes, which has probably been used more often than the others.

Equation 2 above may be written as

$$\frac{dz}{dt} = K[\cos (v + \omega) + e \cos \omega] \quad (3)$$

where

$$K = \frac{na \sin i}{\sqrt{1 - e^2}}.$$

$\frac{dz}{dt}$ is a maximum when $(v + \omega)$ is 0° , and the maximum ordinate YC , which is designated by α , is given by

$$\alpha = K(1 + e \cos \omega).$$

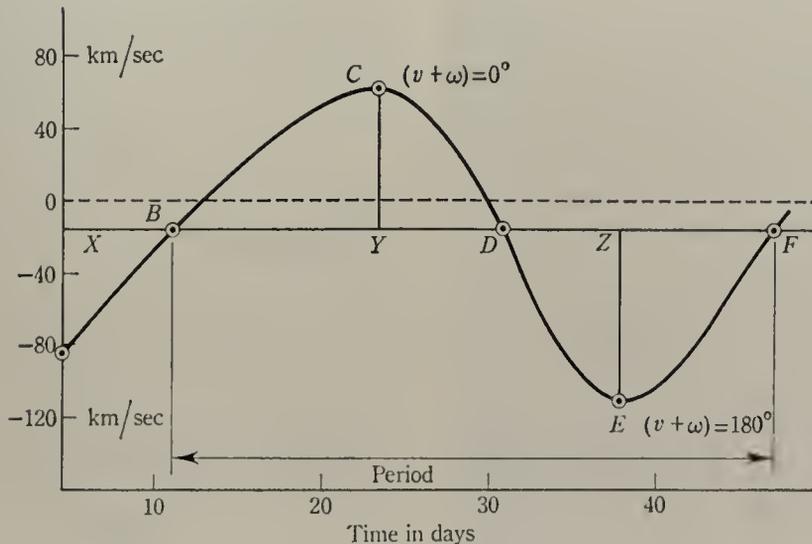


Fig. 2. Mean velocity curve of spectroscopic binary star.

The radial velocity will be a minimum when $(v + \omega) = 180^\circ$

$$\beta = K(1 - e \cos \omega).$$

From these two relations,

$$K = \frac{1}{2}(\alpha + \beta)$$

and

$$e \cos \omega = \frac{\alpha - \beta}{\alpha + \beta}. \quad (4)$$

The measurement of α and β permits the values of K and $e \cos \omega$ to be derived.

The V -axis of the spectroscopic binary may now be determined.

The two areas BCD and DEF between the velocity curve and the time axis must be equal. The value of the area BCD is simply $q(z_D - z_B)$ where q depends upon the constants in which BCD is measured. By the same reasoning the area DEF is $q(z_F - z_D)$. Now z_F and z_B are equal and the numerical values of the two areas must be the same. The establishment of the $BYDZF$ axis is accomplished by trial and error methods using a planimeter, with the restriction that the axis must be parallel to the axis of apparent zero velocity. In the case shown on Figure 2, the BF line is about 15 km/sec below the zero velocity axis, which indicates that the binary system as a whole has a velocity of -15 km/sec.

The area CYD , which is denoted by Δ_1 , is given by $\Delta_1 = z_D - z_C$. Now at C , the value of $(v + \omega)$ is 0° and from (1) we have $z_C = 0$ and $\Delta_1 = z_D$. By exactly the same sort of reasoning we can show that DZE area is equal to that of CYD . This means that the areas BCY and ZEF are equal.

If v_1 and r_1 are the values of the true **anomaly** and radius vector in the true orbit at the point corresponding to D , then, by (1) and $\Delta_1 = z_D$, we have $\Delta_1 = r \sin(v_1 + \omega) \sin i$.

At D , dz/dt is zero and hence by (3) and (4)

$$\cos(v_1 + \omega) = -e \cos \omega = -\frac{\alpha - \beta}{\alpha + \beta}.$$

Hence

$$\sin(v_1 + \omega) = \pm \frac{2\sqrt{\alpha\beta}}{\alpha + \beta}. \quad (5)$$

In passing along the velocity-curve from C toward E , we find that the velocity changes from positive to negative at D . Referring to Figure 2, we see that point D on the velocity curve must correspond to the point of

the true orbit for which z has a maximum positive value. Hence, $\sin(v_1 + \omega)$ is positive and equal to

$$\sin(v_1 + \omega) = \pm \frac{2\sqrt{\alpha\beta}}{\alpha + \beta}.$$

Next, consider the area of the velocity-curve ZEF , which is denoted by $\Delta_2 = z_F - z_E$ or, since $z_E = 0$, by $\Delta_2 = z_F$. If v_2 and r_2 refer to the point on the true orbit corresponding to point F on the velocity-curve, we have $\Delta_2 = r_2 \sin(v_2 + \omega) \sin i$. But at F dz/dt is 0, hence

$$\left. \begin{aligned} \cos(v_2 + \omega) &= -e \cos \omega = -\frac{\alpha - \beta}{\alpha + \beta} \\ \text{and} \\ \sin(v_2 + \omega) &= -\frac{2\sqrt{\alpha\beta}}{\alpha + \beta}. \end{aligned} \right\} (6)$$

We also find

$$\frac{\Delta_1}{\Delta_2} = -\frac{r_1}{r_2} \quad (7)$$

and further,

$$e \sin \omega = \frac{2\sqrt{\alpha\beta}}{\alpha + \beta} \cdot \frac{\Delta_2 - \Delta_1}{\Delta_2 + \Delta_1} \quad (8)$$

hence, since we had $e \cos \omega = \frac{\alpha - \beta}{\alpha + \beta}$, both e and ω may be found. The time of perihelion, Σ , may be found since at that point $v = 0$ and if the corresponding value of dz/dt is designated as $(dz/dt)_1$ and given by Equation (3) we then have

$$(dz/dt)_1 = K(1 + e) \cos \omega.$$

This quantity may be found since K , e , and ω are all supposed to have been previously found.

From Equation (3) we have

$$K = \frac{na \sin i}{\sqrt{1 - e^2}} = \frac{1}{2}(\alpha + \beta).$$

The period, T , is known, and $n = \frac{2\pi}{T}$ hence

$$a \sin i = \frac{T(\alpha + \beta)\sqrt{1 - e^2}}{4\pi}.$$

With the period stated in days, and velocities measured in kilometers per mean solar second,

$$a \sin i = \frac{21,600T}{\pi} (\alpha + \beta) \sqrt{1 - e^2}.$$

This relation gives ($a \sin i$) in kilometers, T being expressed in days. Unless the inclination, i , can be derived by other means, the semi-major axis, a , with respect to the center of gravity of the system, cannot be determined.

SPECTROSCOPIC PARALLAX. See **stellar luminosities**.

SPECTROSCOPIC TERM. See **term**.

SPECTRUM. (1) A visual display, a photographic record, or a plot of the distribution of the intensity (and sometimes the phase) of radiation of a given kind as a function of its wavelength, energy, frequency, momentum, mass or any related quantity. (2) A continuous range of frequencies, usually wide in extent, within which waves have some specified common characteristic, e.g., audio-frequency spectrum, radio-frequency spectrum, etc. (3) The resolution pattern of a group of masses over a region according to increasing mass, in which particles of a given mass are physically isolated from those of neighboring masses. (4) (For mathematical usage, see **spectral function**.)

SPECTRUM, ABSORPTION. The spectrum resulting when the source is continuous radiation passed through an absorbing medium, commonly dark at some of those wavelengths for which the **emission spectrum** of the medium would be bright.

SPECTRUM, ALPHA PARTICLE. The distribution in energy or momentum of the α -particles emitted by a pure **radionuclide**, or, less commonly, by a mixture of radionuclides. Each α -emitting nuclide yields a characteristic spectrum consisting of one or more sharp lines, each line being due to a particular group of monoenergetic particles. When more than one group is present, the distribution is said to have fine structure; this results from transitions to more than one nuclear energy state of the product nuclide, the group of highest energy coming from the ground-state transition. In exceptional cases (RaC' and ThC'), lines are observed due to groups that have very low intensities (10^{-6} to 10^{-4}) relative to those for the main groups. The particles producing such lines are called long-range α -particles. They result when the emitting nuclei are formed in excited states in the preceding **β -disintegration** (of RaC or ThC)

and emit α -particles directly from the excited states, instead of becoming de-excited by the more usual gamma emission. The normal **α -disintegration energy** is then augmented by the **excitation energy**.

SPECTRUM, ARC. See **arc spectrum**.

SPECTRUM, BAND. The spectra of molecules consist of groups or bands of closely spaced lines. These spectra were initially unresolved, and hence called bands in distinction to the sharp spectral lines of atoms. The bands of most molecules have now been resolved into their separate lines. Band spectra is the customary name for the spectra of molecules.

In nuclear physics, band spectra are useful in determining **nuclear spin** and **statistics**, and **isotopic abundances**.

SPECTRUM, BETA-RAY. (1) The distribution in energy or momentum of the β -particles (not including conversion electrons) emitted in a β -decay process. The β -ray spectrum is always continuous up to a maximum energy. Its shape depends upon the nature of the particular β -decay process. (2) Sometimes (loosely) the energy spectrum of the electrons emitted by a radioactive source, irrespective of their origin. In addition to the continuous spectrum of definition (1), it may show lines due to internal conversion or to Auger electrons.

SPECTRUM, CHARGE-TRANSFER. A term applied by Mullikan to spectra resulting from the transitions of an electron between a bonding and an antibonding orbital.

SPECTRUM, CONTINUOUS. (1) Light or any other radiation may have such composition that, when analyzed with a **spectroscope**, it presents apparently an unbroken continuity of wavelength over a wide range. Such, for example, is the light from an ordinary lamp filament. Any incandescent solid, liquid, or gas under high pressure will radiate with a continuous spectrum. Sunlight appears to have a continuous spectrum until analyzed carefully, when the continuous spectrum is found to be crossed by a multitude of dark **Fraunhofer lines**. A continuous spectrum may extend without interruption from the extreme infrared to the extreme ultraviolet. (2) A **spectral function** in which wavelengths (and

wave numbers and frequencies) are represented by the continuum of real numbers (or a portion thereof) rather than by a discrete sequence of numbers. A continuous function on an infinite interval, even though the function is non-zero over only a finite interval, must be represented by the **Fourier transform** rather than by **Fourier series**, and the resulting spectrum will be continuous. (See also **discrete spectrum**.)

SPECTRUM, CONTINUOUS, FOR PARTICLES. A spectrum exhibiting a continuous variation in energy or momentum.

SPECTRUM, DARK-LINE. A spectrum which contains some lines darker than others, or than a continuous spectrum background. (See **Fraunhofer lines**.)

SPECTRUM, DIFFRACTION. The spectrum produced by **diffraction**, as may be produced by a **diffraction grating**. (See also **spectrum, normal**.)

SPECTRUM, DISCONTINUOUS. A term sometimes applied specifically to a combined band and line spectrum. (See **spectrum, band** and **spectrum, line**.)

SPECTRUM, DISCRETE. (1) A **spectral function** in which the component wavelengths (and wave numbers and frequencies) constitute a discrete sequence of values (finite or infinite in number) rather than a continuum of values.

A **Fourier analysis** of a function will yield a discrete spectrum only if the function is periodic, or is assumed to be so, or if the function is represented by a finite sample of its values. **Fourier series** may be used for the analysis. (2) The same as **spectrum, line**. (Cf. **spectrum, continuous**.)

SPECTRUM, EMISSION. The spectrum (continuous, line or band) produced by radiation from any emitting source, as distinguished from absorption spectra. (See **spectrum, absorption**.)

SPECTRUM, FINE. The resolution of lines in atomic emission spectra, by high-power spectroscopes, into two or more fine lines situated close together. The fine lines in atomic spectra arise from so-called term **multiplicities**, i.e., from transitions between groups of higher, to groups of lower, levels, the levels

comprising each group lying close together. Groups of close-lying levels are obtained by coupling the orbital angular momentum vector with the spin angular momentum vector of the atom in a variety of possible combinations.

SPECTRUM, FLASH. See **flash spectrum**.

SPECTRUM, HYPERFINE. In the case of atomic emission spectra, there is, in addition to the fine structure (see **spectrum, fine**) of the spectral lines attributable to term multiplicities a hyperfine structure (fine lines very close together), which is attributable to a number of causes, including among others the isotope effect and the effect of the **spin** of the atomic nucleus. This spin couples with the total **angular momentum** or **orbital angular momentum** of the electrons and yields a series of resulting vectors representative of closely-spaced energy levels.

SPECTRUM LEVEL OF ROOM NOISE. See **noise, spectrum level of room**.

SPECTRUM LINE. The image (generally of a slit) produced, in a dispersing system, by a monochromatic radiation. Also used in the sense of: the nearly monochromatic radiation emitted or absorbed in a transition between two atomic or molecular levels.

SPECTRUM LOCUS. The representation of the spectral color stimuli on a **chromaticity diagram**.

SPECTRUM-LUMINOSITY RELATIONS, STELLAR. See **stellar spectrum-luminosity relations**.

SPECTRUM, MAGNETIC. A term sometimes applied to particles (such as iron filings) spread out in a **magnetic field**, and distributed so as to show **lines of force**.

SPECTRUM, MAGNETIC RESONANCE. Spectra produced by absorption or emission of energy, commonly in the radio frequency region, by molecules which change their magnetic quantum numbers on absorption or emission of quanta of radiowaves. These spectra may be produced in a method developed by Rabi in which a molecular beam is sent through two inhomogeneous magnetic fields at right angles to it; between the two fields is a constant field, superimposed by a radio frequency field, the resonance between these last

two fields determining the magnitude of splitting of the energy levels. Other methods of observing magnetic resonance are similar in that they involve the superposition of a radio frequency field on a steady or slowly varying magnetic field, about which the atoms precess. (See **Larmor precession**.)

SPECTRUM, MASS. A spectrum showing the distribution in mass or in mass-to-charge ratio of ionized atoms, molecules, or molecular fragments. The mass spectrum of an element shows the relative abundances of the isotopes of the element. The mass spectrum of a compound is quite complicated and not easily related to the relative abundances of the various kinds of atoms and groups present; however, under given experimental conditions it is characteristic of the compound and often useful for identification and assay.

SPECTRUM, NEUTRON. In the theory of neutron transport, the distribution-in-energy of neutrons in a small volume of material surrounding a point. Often the term refers to the portion of the distribution having energies between zero and 1.44 electron volts.

SPECTRUM, NORMAL. A term applied to the diffraction **spectrum**, because its dispersion is linear with wavelength, which is not the case in a spectrum obtained from a prism.

SPECTRUM (OF A RADIATION). (1) The image produced by the dispersion of a radiation into its monochromatic components. (2) The composition of a complex radiation.

SPECTRUM OF TURBULENCE. A representation of the various scales of motion constituting a field of **turbulence**. Most useful is an energy frequency spectrum defined by

$$F(n) = \frac{2}{\pi} \int_0^{\infty} R_{\xi} \cos n\xi d\xi$$

where R_{ξ} is the Lagrangian **autocorrelation coefficient**. $F(n)dn$ then represents the fraction of the total turbulent energy in eddies of frequency between n and $n + dn$.

If the fluctuations are measured at a fixed point in a stream moving with mean velocity \bar{u} we replace R_{ξ} by r_{τ} or r_x and assume* that τ is x/\bar{u} to give

*This assumption is generally valid if \bar{u} is large compared with the velocity fluctuations.

$$F(n) = \frac{2}{\pi \bar{u}} \int_0^{\infty} r_x \cos \frac{nx}{\bar{u}} dx.$$

An energy wave number (or eddy size) spectrum may be defined by

$$E(k) = \frac{2}{\pi} \int_0^{\infty} r_x \cos kx dk$$

where $E(k)dk$ now gives the fraction of the total turbulent energy in wave numbers between k and $k + dk$. r_x is the Eulerian autocorrelation coefficient.

The total turbulent energy per unit mass is

$$\int_0^{\infty} \bar{u}'^2 F(n) dn = \int_0^{\infty} \bar{u}'^2 E(k) dk.$$

$F(n)$ and $E(k)$ are called energy spectral density functions.

SPECTRUM, PHONON. The spectrum of thermal vibrations in a solid. The simplest approximation is that of Debye, who treated the crystal as a continuum, the atomicity being introduced by "cutting off" the distribution at a point where the number of modes equalled the number of degrees of freedom, $3N$. This was improved by Born and von Kármán, who showed that there should be a cut off at a definite **wave-number**, the same for each of the different modes of polarization, and hence corresponding to different maximum frequencies for longitudinal and transverse modes. More detailed calculations, based on assumed interatomic force constants, have confirmed this general form of spectrum but no simple representation can be found. The spectrum fixes the **specific heat** as a function of temperature, and also such properties as **thermal conductivity**, **infrared** and **Raman spectra**, etc.

SPECTRUM, PRIMARY. The first-order spectrum produced by a **diffraction grating**.

SPECTRUM, RAMAN. A spectrum obtained by illuminating a substance with radiation, and obtaining from the radiations scattered at right angles a spectrum of those frequencies differing from the incident radiation.

SPECTRUM, RESONANCE. A spectrum excited by the interaction with a substance (usually a molecular gas) of radiation of a definite frequency or frequencies.

SPECULAR REFLECTION OR REFRACTION. When the interface between two media is smooth, i.e., belongs to class C_1 , then the resulting reflection or refraction is specular or regular. If the interface is of class C_0 but not C_1 , the resulting reflection is **diffuse**.

SPEED. The magnitude of the velocity vector \mathbf{v} is called the speed, v . $\mathbf{v} = v\mathbf{T}$ where \mathbf{T} is the unit vector in the direction of the tangent to the path.

SPEED, CRITICAL. A resonant speed of rotation. (See **resonance**.)

SPEED OF A CAMERA LENS. See **f-number**.

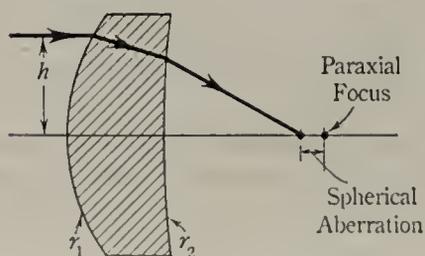
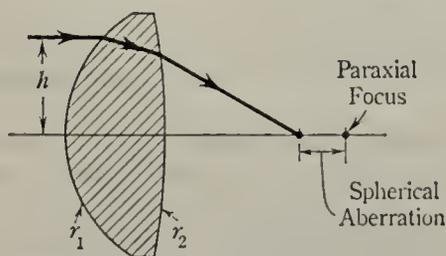
SPEED OF SOUND. See **velocity of sound**.

SPENCER-FANO METHOD. A powerful analytical method in the theory of the penetration of gamma-rays into matter and in the space-energy distribution of neutrons slowing down in hydrogen. The central idea is to expand the space-dependent parts of the **spherical harmonics** moments in terms of suitably chosen polynomials, with a well-chosen weight function, rather than to make a **Fourier analysis** of the space dependence.

SPHERE OF CURVATURE (OF A CURVE AT A POINT, P). The sphere passing through four points of the curve in the limiting case, if this exists, as they coincide at P . Also called *osculating sphere*.

SPHERE, RELATIVISTIC FIELD OF. See **relativity, general**.

SPHERICAL ABERRATION. (1) Qualitatively, spherical aberration is the image error



due to the improper union, near the **Gaussian image point**, of rays that originate from an object point. There is third order and higher order spherical aberration according to the theory used to trace the rays.

(2) Quantitatively, the first of the five **Seidel aberration** functions is the spherical aberration function.

SPHERICAL COORDINATES. See **curvilinear orthogonal coordinates**.

SPHERICAL CURVATURE, CENTER OF (OF A CURVE AT A POINT). The center of the sphere of curvature of the curve at the point.

SPHERICAL CURVATURE, RADIUS OF (OF A CURVE AT A POINT). The radius of the sphere of curvature of the curve at the point.

SPHERICAL HARMONICS. In analogy to **harmonic functions** in the plane, the solutions of the **Laplace equation** in **spherical coordinates**. Spherical surface harmonics are special sets taken over the surface of a sphere, therefore, the harmonic components are restricted to an integral number of waves over the sphere. Spherical harmonics have been applied in the study of the large-scale oscillations of the atmosphere.

SPHERICAL HARMONICS METHOD. In transport theory, a method of analyzing the transport equation by expanding the flux in terms of a series of appropriate spherical harmonics in the angle variable. The differential-integral equation is thus reduced to a system of differential equations, and the dependent variables are the spherical harmonic moments of the angular distribution. In plane geometry, the spherical harmonic functions are **Legendre polynomials** in the variable $\mu = \cos(\Omega, x)$. (See P_N approximation.)

SPHERICAL IMAGE. See **spherical representation of a surface**.

SPHERICAL INDICATRIX OF BINORMAL TO A CURVE. The locus of a point whose position vector is equal to the unit binormal of the curve.

SPHERICAL INDICATRIX OF TANGENT TO A CURVE. The locus of a point whose position vector is equal to the unit tangent of the curve.

SPHERICAL POLAR COORDINATE SYSTEM. An orthogonal, curvilinear coordinate system, in which the coordinates r , θ and ϕ specify the position of a point in space. In this system the unit vectors are e_r , e_θ , and e_ϕ . The directions of these unit vectors vary from point to point in space. The vector line element is

$$ds = e_r dr + e_\theta r d\theta + e_\phi r \sin \theta d\phi$$

and its magnitude is

$$ds = [(dr)^2 + (r d\theta)^2 + (r \sin \theta d\phi)^2]^{1/2}.$$

In the common right-handed system, the unit vectors are orthogonal, i.e., $e_r \cdot e_\theta = e_r \cdot e_\phi = e_\theta \cdot e_\phi = 0$ and satisfy the relations $e_r \times e_\theta = e_\phi$, $e_\theta \times e_\phi = e_r$, $e_\phi \times e_r = e_\theta$. The volume element is

$$d\tau = r^2 \sin \theta dr d\theta d\phi.$$

The transformation equations connecting this system with rectangular Cartesian coordinates are:

$$x = r \sin \theta \cos \phi,$$

$$y = r \sin \theta \sin \phi,$$

$$z = r \cos \theta.$$

SPHERICAL REPRESENTATION OF A RECTILINEAR CONGRUENCE.

Draw through the center of a unit sphere a radius parallel to each ray of the congruence. The points of intersection of these radii with the sphere then provide a representation of the **rectilinear congruence** which is called a spherical representation of the congruence.

SPHERICAL REPRESENTATION OF A SURFACE.

A representation of the surface on a unit sphere. Also called *Gaussian representation*. The point on the unit sphere corresponding to a point on the surface is the *spherical image* of that point.

SPHERICAL SOUND WAVES.

See **sound waves, spherical**.

SPHERICAL SURFACE. A surface all points of which are at a fixed distance, the *radius*, from a fixed point, the *center*. The term sphere is frequently used for this surface but it more properly means a solid bounded by a spherical surface.

In **rectangular coordinates** its general equation is

$$x^2 + y^2 + z^2 + Gx + Hy + Kz + L = 0$$

but if the center is taken at the origin of the coordinate system, the equation becomes $x^2 + y^2 + z^2 = r^2$, where r is the radius of the spherical surface.

The surface is measured in terms of the following parts: (1) *Zone*, a portion of the surface included between parallel planes. The bases of the zones are circumferences made by the planes but if one of the bounding planes is tangent to the surface, it is a zone of one base. The distance between the planes is the *altitude* of the zone. (2) *Lune*, a part of the surface bounded by the circumferences of two great circles. (3) *Spherical polygon*, a portion of a spherical surface bounded by three or more arcs of great circles. (In the case of three arcs only, the figure is a *spherical triangle*.)

If A is surface area, the various cases give for it: sphere, $4\pi r^2$; zone, $2\pi rh$; lune, $\pi r^2 a/90$; triangle or polygon, $\pi r^2 E/180$, where r is the radius of the surface, h is the altitude of the zone, a is the number of degrees in the angle, and E is the spherical excess, defined by $E = T - 180(n - 2)$, where T is the sum of the angles and there are n sides to the polygon.

SPHERICAL TOP MOLECULE.

See **polyatomic molecules, types of**.

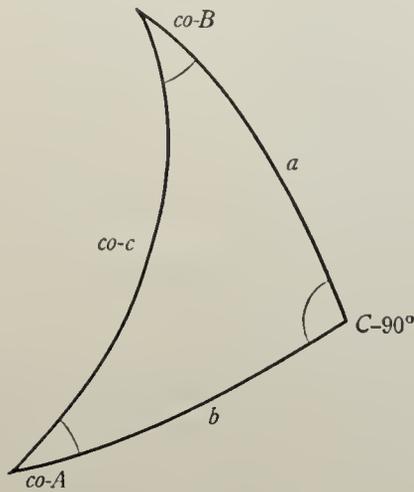
SPHERICAL TRIGONOMETRY. A generalization of plane **trigonometry**, spherical trigonometry is primarily concerned with the solution of spherical triangles. It is used in many navigation and astronomical problems, as well as in the construction of certain kinds of maps.

Solution of a spherical triangle means the finding of unknown sides and angles from given values for other sides and angles. The right spherical triangle is the simplest case, but it, unlike the plane right triangle, can have two or even three right angles. If, however, it has two right angles the sides opposite them are quadrants and the third angle has the same measure as its opposite side. If all three angles are right angles, the measure of each side is 90° . These cases are all relatively simple, hence we consider a triangle which has only one right angle, its opposite side in general not being 90° .

Let a , b , c be the sides of a right spherical triangle, measured by the angle subtended at the center of a sphere, and with opposite angles A , B , C , where $A = 90^\circ$. Then 10 relations exist as follows:

$$\begin{aligned} \sin a &= \sin A \sin c & \sin b &= \sin B \sin c \\ \sin a &= \tan b \cot B & \sin b &= \tan a \cot A \\ \cos A &= \cos a \sin B & \cos B &= \cos b \sin A \\ \cos A &= \tan b \cot c & \cos B &= \tan a \cot c \\ \cos c &= \cot A \cot B & \cos c &= \cos a \cos b. \end{aligned}$$

Since these equations are rather awkward, Napier's rules are convenient in actual use. They can be stated as follows: let $co-A$, $co-B$, $co-c$, mean the complements of the angles A and B , and the hypotenuse c , and arrange the parts as shown. Then calling any angle a middle part, it will have two parts adjacent to it and two parts opposite to it. Napier's rules are then: (i) the sine of any part equals the product of the tangents of the adjacent parts; (ii) the sine of any part equals the product of the cosines of opposite parts.



Application of Napier's rules.

In the case of an oblique spherical triangle, the following relations may be obtained. (1) Law of sines.

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c}.$$

(2) Law of cosines.

$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

and

$$\cos A = -\cos B \cos C + \sin B \sin C \cos a.$$

(3) Haversine law.

$$\text{hav } a = \text{hav } (b - c) + \sin b \sin c \text{hav } A.$$

(4) Half-angle formulas.

$$\sin A/2 = \sqrt{\frac{\sin(s-b)\sin(s-c)}{\sin b \sin c}}$$

and

$$\tan A/2 = \frac{r}{\sin(s-a)},$$

where $2s = (a + b + c)$ and $r^2 \sin s = \sin(s-a)\sin(s-b)\sin(s-c)$. (5) Napier's analogies.

$$\frac{\tan(a+b)/2}{\tan c/2} = \frac{\cos(A-B)/2}{\cos(A+B)/2}.$$

(6) Gauss formulas.

$$\sin(A-B)/2 = \frac{\sin(a-b)/2 \cos C/2}{\sin c/2}.$$

(7) Rule of quadrants. In any spherical triangle, one-half the sum of two angles is in the same quadrant as one-half the sum of the sides opposite. There are additional relations in several of these cases (for example, 3, 4, 5) obtainable by cyclic permutation of A, B, C , and a, b, c .

In solving a spherical triangle there are six possible cases, depending on the parts given, as follows: (I) a, b, C ; (II) A, B, c ; (III) b, c, C ; (IV) A, B, a ; (V) a, b, c ; (VI) A, B, C . Case (III) is an ambiguous case since there may be two solutions or only one. Each of the six cases may be solved by several combinations of the relations given in the preceding paragraph.

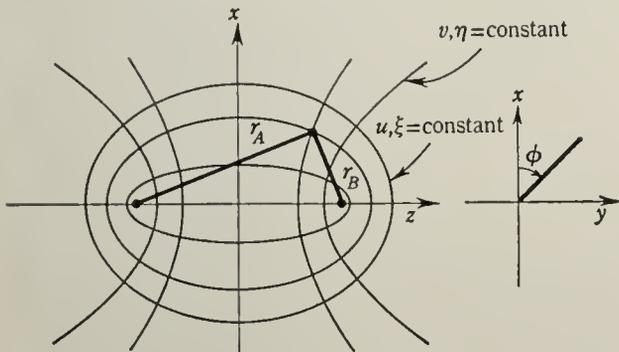
SPHERICAL VECTOR WAVE FUNCTIONS. To find the angular distribution and polarization of the light scattered by a small particle (small compared to the wavelength) is an easy problem. The solution was the basis of Rayleigh's famous explanation of the blue color of the sky. At the other extreme, a large particle can be treated easily by geometrical optics.

For many applications: the scattering of radar by mists, particle size determination in aerosols and protein suspensions (e.g., blood), etc., the exact solution, due to G. Mie, of Maxwell's equations for arbitrary spherical particles must be used. This solution is expressed in terms of spherical vector wave functions (alternatively, light scattering functions for spherical particles or Mie functions) that are extensively tabulated. (Pangonis, Heller, and Jacobson, *Tables of light scattering functions for spherical particles*, Wayne State University Press, Detroit, 1957, contains Mie's results and references to previous tables.)

SPHERICAL VORTEX. A type of vortex usually associated with the name of M. J. M. Hill. It is a vortex whose **substance** is spherical, the exterior fluid passing round it as in the irrotational motion round a sphere. There is no **accompanying fluid**.

SPHEROIDAL (OBLATE AND PROLATE) COORDINATES. Degenerate cases of the ellipsoidal coordinate system, which arise when two of the three axes become equal.

In the prolate spheroidal system the surfaces are (1) prolate spheroids ($u = \text{constant}$) obtained by rotating ellipses about their major



axis (z -axis), (2) hyperboloids of two sheets ($v = \text{constant}$) obtained by rotating hyperbolas about their axis (z -axis), and (3) planes passing through the z -axis ($\phi = \text{constant}$). The transformation equations are (with the foci at $\pm a$):

$$\begin{aligned}x &= a \sinh u \sin v \cos \phi \\y &= a \sinh u \sin v \sin \phi \\z &= a \cosh u \cos v.\end{aligned}$$

It is often useful (in two center problems) to use the coordinates ξ and η in place of u and v , where $\xi = \cosh u$ and $\eta = \cos v$. These two coordinates may alternatively be defined by:

$$\xi = \frac{r_A + r_B}{2a} \quad \text{and} \quad \eta = \frac{r_A - r_B}{2a}.$$

The limits of the variables are $-1 \leq \eta \leq 1$, $1 \leq \xi \leq \infty$, $0 \leq \phi \leq \pi$; or $0 \leq u \leq \infty$, $0 \leq v \leq \pi$.

In the oblate spheroidal case, the ellipses are rotated about their minor axis to form oblate spheroids ($u = \text{constant}$), and the hyperbolas are rotated about this same axis to give hyperbolas of one sheet ($v = \text{constant}$). Again taking the z -axis to be the axis of revolution, the transformation equations are:

$$\begin{aligned}x &= a \cosh u \sin v \cos \phi \\y &= a \cosh u \sin v \sin \phi \\z &= a \sinh u \cos v.\end{aligned}$$

The same figure as that for the prolate spheroidal case applies if the x - and z -axes are interchanged.

SPIN. (1) The intrinsic angular momentum of an elementary particle, equal in magnitude to $n\hbar/2$, where n is a small integer and $2\pi\hbar$ is Planck's constant. Particles with even n obey **Bose-Einstein statistics**; those with odd n , **Fermi-Dirac statistics**. Thus, electrons, positrons, neutrons, protons, neutrinos and μ -mesons have spin $\frac{1}{2}\hbar$ and are fermions, while π -mesons have spin 0 and photons spin \hbar , and are bosons. (See **electron spin**.)

(2) The total angular momentum of a complex system, particularly an atomic nucleus, again equal to $n\hbar/2$ where n is an integer. For a nucleus of mass number A (and therefore containing A nucleons) n is even or odd according as A is even or odd. (See **nuclear spin**.)

SPIN-DEPENDENT FORCE. The force between two particles which depends on their relative spin orientations and possibly on their spin directions relative to the line joining the particles. Physical bases could be the interaction between the magnetic moments of the particles, or in the case of nuclear forces, the exchange of π -mesons between the nucleons.

SPIN, ELECTRON. See **electron spin**.

SPIN, NUCLEAR. See **nuclear spin**.

SPINODAL CURVE. The curve which separates, inside the saturation curve (see **binodals**), the unstable phases from the stable (or metastable) ones. (See **stability of phases**.)

SPIN OPERATORS, PAULI. See **Pauli spin operators**.

SPIN-ORBIT COUPLING. See **Russell-Saunders coupling**.

SPINOR CALCULUS. The coordinates will be denoted by $x^1 = x$; $x^2 = y$; $x^3 = z$; $x^4 = ct$. The metric tensor is then $\eta^{11} = \eta^{22} = \eta^{33} = \eta_{11} = \eta_{22} = \eta_{33} = 1$.

$$\eta^{44} = \eta_{44} = -1.$$

A spinor of rank one is a 2-dimensional vector $g = (g_1, g_2)$, whose components obey the transformation law

$${}^*g_t = \sum_{s=1}^2 \alpha_{ts} g_s \quad t = 1, 2$$

where

$$\begin{vmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{vmatrix} = 1.$$

A "dotted" spinor with the components (f_1, f_2) obeys the transformation law

$${}^*f_1 = \bar{\alpha}_{11} f_1 + \bar{\alpha}_{12} f_2$$

$${}^*f_2 = \bar{\alpha}_{21} f_1 + \bar{\alpha}_{22} f_2.$$

(The bars denote conjugate complex.)

Spinors of higher ranks are transformed like products of spinors of first rank. A "mixed" spinor a_{ki} is transformed like the product $a_k b_i$.

Spin indices are raised and lowered by the metric spin tensor ϵ_{kl} , where

$$\epsilon_{kl} = \epsilon_{lk} = \epsilon_{k\dot{l}} = \epsilon_{\dot{k}l} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

and

$$\epsilon^{kl} = \epsilon^{lk} = \epsilon^{\dot{k}\dot{l}} = \epsilon^{\dot{l}k} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$a^k = \epsilon^{kl} a_l$$

$$a_k = \epsilon_{kl} a^l$$

$$\epsilon^{kl} \epsilon_{lm} = \delta_m^k.$$

Summations are performed only over undotted or dotted indices, but not over mixed indices. Furthermore

$$a^1 = a_2 \quad b^{\dot{1}} = b_{\dot{2}}$$

$$a^2 = -a_1 \quad b^{\dot{2}} = -b_{\dot{1}}$$

$$a^l b_l = -a_l b^l$$

$$a_l a^l = 0.$$

Connection between Spinors and World Tensors. Let A^p be a contravariant tensor and a_{rs} a mixed spinor, then

$$A^1 = A_1 = \frac{1}{2}(a_{21} + a_{12})$$

$$A^2 = A_2 = \frac{1}{2}i(a_{21} - a_{12})$$

$$A^3 = A_3 = \frac{1}{2}(a_{11} - a_{22})$$

$$A^4 = -A_4 = \frac{1}{2}i(a_{11} + a_{22})$$

and conversely

$$a_{21} = -a^{\dot{1}2} = A^1 + iA^2 = A_1 + iA_2$$

$$a_{12} = -a^{\dot{2}1} = A^1 - iA^2 = A_1 - iA_2$$

$$a_{11} = +a^{\dot{2}2} = A^3 + iA^4 = A_3 - iA_4$$

$$-a_{21} = -a^{\dot{1}1} = A^3 - iA^4 = A_3 + iA_4$$

$$AA^p = -\frac{1}{2}a_{rs} a^{\dot{r}t}.$$

Spinor differential operators:

$$\partial^{\dot{1}}_1 = \partial_{21} = \frac{\partial}{\partial x^1} + i \frac{\partial}{\partial x^2}$$

$$-\partial^{\dot{2}}_2 = \partial_{12} = \frac{\partial}{\partial x^1} - i \frac{\partial}{\partial x^2}$$

$$\partial^{\dot{2}}_1 = \partial_{11} = \frac{\partial}{\partial x^3} - \frac{\partial}{\partial x^4}$$

$$\partial^{\dot{1}}_2 = -\partial_{22} = \frac{\partial}{\partial x^3} + \frac{\partial}{\partial x^4}.$$

If ϕ^p corresponds to the spinor ϕ_{rs} , then

$$\text{div } \Phi \equiv \phi^p_{,p} = -\frac{1}{2} \partial_{rs} \phi^{\dot{r}s}$$

and if the ordinary scalar S belongs to the spinor scalars, then

$$\square S = \frac{\partial^2 S}{\partial x_1^2} + \frac{\partial^2 S}{\partial x_2^2} + \frac{\partial^2 S}{\partial x_3^2} + \frac{\partial^2 S}{\partial x_4^2} = -\frac{1}{2} \partial_{rs} \partial^{\dot{r}t} S.$$

Let $\alpha = \beta + i\gamma$ (β and γ real numbers), and $R(\alpha) \equiv \beta$ and $I(\alpha) \equiv \gamma$ and $|\alpha| = +\sqrt{\beta^2 + \gamma^2}$ and $\gamma^{\lambda \cdot \tau}$ the Lorentz-transformation matrix

$$\gamma^{\lambda \cdot \tau} = \begin{bmatrix} 1 & & & -v \\ \sqrt{1 - v^2/c^2} & 0 & 0 & \sqrt{1 - v^2/c^2} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -v/c^2 & 0 & 0 & 1 \\ \sqrt{1 - v^2/c^2} & & & \sqrt{1 - v^2/c^2} \end{bmatrix}.$$

If g_{rs} is a mixed spinor of rank 2, and ${}^*g_{rs}$ the corresponding spinor in the new coordinate system, then

$${}^*g_{11} = |\alpha_{11}|^2 g_{11} + \bar{\alpha}_{11} \alpha_{12} g_{12} + \bar{\alpha}_{12} \alpha_{11} g_{21} + |\alpha_{12}|^2 g_{22}$$

$${}^*g_{12} = \bar{\alpha}_{11} \alpha_{21} g_{11} + \bar{\alpha}_{11} \alpha_{22} g_{12} + \bar{\alpha}_{12} \alpha_{21} g_{21} + \bar{\alpha}_{12} \alpha_{22} g_{22}$$

$$\begin{aligned}
 g_{\dot{2}1}^* &= \bar{\alpha}_{21}\alpha_{11}g_{i11} + \bar{\alpha}_{21}\alpha_{12}g_{i12} + \bar{\alpha}_{22}\alpha_{11}g_{\dot{2}1} \\
 &\quad + \alpha_{22}\alpha_{12}g_{\dot{2}2} \\
 g_{\dot{2}2}^* &= |\alpha_{21}|^2g_{i11} + \bar{\alpha}_{21}\alpha_{22}g_{i12} + \bar{\alpha}_{22}\alpha_{21}g_{\dot{2}1} \\
 &\quad + |\alpha_{22}|^2g_{\dot{2}2}
 \end{aligned}$$

where

$$\begin{aligned}
 \gamma^{1\cdot 1} &= R(\bar{\alpha}_{11}\bar{\alpha}_{22}) + R(\bar{\alpha}_{12}\alpha_{21}) \\
 \gamma^{2\cdot 1} &= I(\bar{\alpha}_{11}\alpha_{22}) + I(\alpha_{12}\bar{\alpha}_{21}) \\
 \gamma^{3\cdot 1} &= R(\alpha_{11}\bar{\alpha}_{21}) - R(\bar{\alpha}_{12}\alpha_{22}) \\
 \gamma^{4\cdot 1} &= -R(\alpha_{11}\bar{\alpha}_{21}) - R(\bar{\alpha}_{12}\alpha_{22}) \\
 \gamma^{1\cdot 2} &= I(\bar{\alpha}_{22}\alpha_{11}) + I(\bar{\alpha}_{21}\alpha_{12}) \\
 \gamma^{2\cdot 2} &= R(\bar{\alpha}_{11}\alpha_{22}) - R(\bar{\alpha}_{12}\alpha_{21}) \\
 \gamma^{3\cdot 2} &= I(\bar{\alpha}_{21}\alpha_{11}) + I(\bar{\alpha}_{12}\alpha_{22}) \\
 \gamma^{4\cdot 2} &= I(\bar{\alpha}_{11}\alpha_{21}) + I(\bar{\alpha}_{12}\alpha_{22}) \\
 \gamma^{1\cdot 3} &= R(\bar{\alpha}_{11}\alpha_{12}) - R(\bar{\alpha}_{21}\alpha_{22}) \\
 \gamma^{2\cdot 3} &= I(\bar{\alpha}_{11}\alpha_{12}) + I(\bar{\alpha}_{22}\alpha_{21}) \\
 \gamma^{3\cdot 3} &= \frac{1}{2}(|\alpha_{11}|^2 + |\alpha_{22}|^2 - |\alpha_{12}|^2 - |\alpha_{21}|^2) \\
 \gamma^{4\cdot 3} &= \frac{1}{2}(|\alpha_{21}|^2 + |\alpha_{22}|^2 - |\alpha_{11}|^2 - |\alpha_{12}|^2) \\
 \gamma^{1\cdot 4} &= -R(\alpha_{11}\bar{\alpha}_{12}) - R(\bar{\alpha}_{21}\alpha_{22}) \\
 \gamma^{2\cdot 4} &= -I(\alpha_{11}\bar{\alpha}_{12}) + I(\alpha_{21}\bar{\alpha}_{22}) \\
 \gamma^{3\cdot 4} &= \frac{1}{2}(|\alpha_{12}|^2 + |\alpha_{22}|^2 - |\alpha_{11}|^2 - |\alpha_{21}|^2) \\
 \gamma^{4\cdot 4} &= \frac{1}{2}(|\alpha_{11}|^2 + |\alpha_{12}|^2 + |\alpha_{21}|^2 + |\alpha_{22}|^2).
 \end{aligned}$$

Dual Tensors. The dual of the antisymmetric contravariant tensor $F^{\alpha\beta}$, denoted by $\overset{\Delta}{F}^{\alpha\beta}$, is defined by

$$\overset{\Delta}{F}^{\alpha\beta} = \eta^{\alpha\rho}\eta^{\beta\sigma}\overset{\Delta}{F}_{\rho\sigma}$$

where

$$\overset{\Delta}{F}_{\rho\sigma} = \frac{i}{2}\delta_{\rho\sigma\lambda\tau}F^{\lambda\tau}.$$

It is assumed that all components $F^{\alpha\beta}$ are real. $\delta_{\rho\sigma\lambda\tau}$ is the Kronecker symbol. A tensor $F^{\alpha\beta}$ is self dual if

$$F^{\rho\sigma} = \overset{\Delta}{F}^{\rho\sigma}.$$

Let $G^{\alpha\beta}$ be a self dual antisymmetric tensor. The components of $G^{\alpha\beta}$ can be expressed in terms of a three-dimensional, complex-valued vector $\mathbf{k} = (k_1, k_2, k_3)$ where

$$k_r = a_r + ib_r \quad (r = 1, 2, 3)$$

$$G^{r4} = b_r - ia_r \quad (r = 1, 2, 3)$$

$$((i^{23}, i^{31}, i^{12}) = (k_1, k_2, k_3).$$

The associated symmetric spinor g_{rs} has the components:

$$g_{11} = 2[a_2 - b_1 - i(b_2 + a_1)]$$

$$g_{22} = 2[a_2 + b_1 - i(b_2 - a_1)]$$

$$g_{12} = g_{21} = 2(b_3 + ia_3)$$

$$g_{i1} = 2[a_2 - b_1 + i(b_2 + a_1)]$$

$$g_{i2} = 2[a_2 + b_1 + i(b_2 - a_1)]$$

$$g_{i3} = g_{3i} = 2(b_3 - ia_3).$$

If

$$\mathbf{k} = \mathbf{H} - i\mathbf{E} \quad (\mathbf{H} = \text{magnetic field} \\ \mathbf{E} = \text{electric field})$$

then

$$g_{ii} = 2[H_2 + E_1 + i(H_1 - E_2)]$$

$$g_{i2} = 2[H_2 - E_1 + i(H_1 + E_2)]$$

$$g_{i3} = g_{3i} = 2[-E_3 - iH_1].$$

One can also introduce a current-density spinor S_{rs} corresponding to the contravariant current density world-vector $S^\lambda = (I_1, I_2, I_3, \rho)$:

$$S_{11} = I_3 + \rho$$

$$S_{12} = I_1 - iI_2$$

$$S_{21} = I_1 + iI_2$$

$$S_{22} = -I_3 + \rho.$$

Maxwell's equations in spinor language are

$$\partial^{\dot{r}} \cdot ig_{\dot{r}m} = 2\pi S_{lm}.$$

Principal reference: O. Laporte and G. E. Uhlenbeck, *Application of Spinor Analysis to the Maxwell and Dirac Equations*, *Phys. Rev.* **37**, 1,380 (1931).

SPIN PARAMAGNETISM OF CONDUCTION ELECTRONS. In the presence of a magnetic field, a nonferromagnetic metal will exhibit paramagnetism as a result of the difference between the number of conduction electrons which have their spins parallel to the direction of the field and the number whose spins are antiparallel. Pauli has shown that the resulting susceptibility χ is given by the expression

$$\chi = \frac{12m\beta^2}{h^2} \left(\frac{N\pi^2}{9} \right)^{1/2}$$

where N is the number of conduction electrons per unit volume and $\beta = eh/2mc$ and is the Bohr magneton. This susceptibility is three

times as great as the diamagnetic susceptibility produced by changes in the orbital wave functions as given by Landau's formula (see **orbital diamagnetism of conduction electrons**). Thus, when both effects are taken into account, the value of the total paramagnetic susceptibility is two-thirds that shown above.

SPIN-REORIENTATION SPECTRA. See **microwave spectra of molecules**.

SPIN TEMPERATURE. In a spin system with spin-lattice interactions, the lattice acts as a thermostat for the spin system. In equilibrium, a temperature equal to the temperature of the lattice can be assigned to the spins, and the populations of the energy levels, E_i , of the spin system will be given by the Boltzmann factor $e^{-E_i/kT}$, where T is the temperature of the lattice. If the spin populations are inverted (as in maser action), the spin temperature will no longer be that of the lattice (it may be negative, see **temperature, negative**). If the spin lattice relaxation time, τ , is long, experiments of duration less than τ may be performed on the inverted spin system.

SPIN-VALENCE THEORY. See **Heitler-London theory of valence**.

SPIRAL LOXODROMIC. See **loxodrome**.

SPLIT-PLOT METHOD. An experimental design in which each plot is divided into two, and two additional treatments allocated one to each half.

SPREADING PRESSURE. Call A^{film} the Helmholtz free energy of a film adsorbed at the surface of a liquid or a solid, and Ω the area occupied by this film. The derivative

$$\Pi = - \left(\frac{\partial A^{\text{film}}}{\partial \Omega} \right)_{T,n} \quad (1)$$

is called the spreading pressure or surface pressure of the film.

If the film is adsorbed on liquid, Π may be related to its surface tension γ by

$$\Pi = \gamma - \gamma_0 \quad (2)$$

γ_0 is the surface tension of the liquid in the absence of the film.

Dilute films have often been considered as *two-dimensional gases*. Equations of state analogous to those valid for ordinary gases

have been applied therefore to the spreading pressure.

SPRING CONSTANT. Designating the tensile or compressive force applied to the spring by F , and the extension or contraction of the spring by x , the spring constant is dF/dx . If the spring is linear so that the spring constant is, in fact, constant, it is given by F/x .

SPUR. Synonym of **trace**.

SQUARE CONTINGENCY. See **contingency**.

SQUARE WAVE. A wave which alternately assumes two fixed values for equal lengths of time, the time of transition being negligible in comparison with the duration of each fixed value.

A square wave requires a considerable number of sinusoidal frequencies to express it. These components are not mere mathematical fictions but are true electrical components in the case of an electric wave. They may be separated and examined by means of proper filter circuits. Since a square wave will contain a long series of frequencies it may be used for rapidly determining the frequency response of a piece of equipment by applying the wave to the input and noting the distortion of the output wave. The distortion is due to certain frequencies of the original wave being attenuated or amplified out of proportion in passing through the circuit. Thus the necessity of making a laborious series of tests at various frequencies using sine waves is avoided.

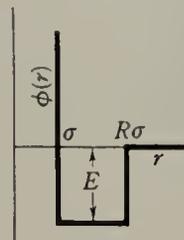
SQUARE WELL POTENTIAL. The intermolecular potential

$$\phi(r) = \infty \quad r < \sigma \quad (1)$$

$$= -E \quad \sigma < r < R\sigma \quad (2)$$

$$= 0 \quad r > R\sigma \quad (3)$$

represents **rigid spheres** surrounded by an attractive core of strength E which extends to separations $R\sigma$ (see figure).



Square well potential.

The square well potential takes account in a crude way of both the attractive and repulsive parts of the intermolecular forces.

S-STATE. A state of zero orbital angular momentum.

STABILITY. See **buckling; lateral buckling; critical load;** and entries immediately following.

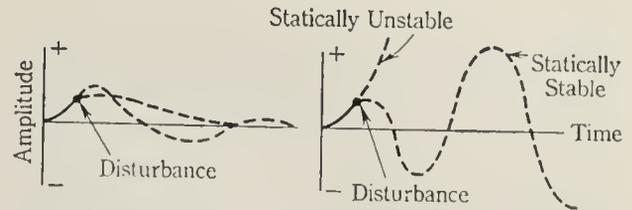
STABILITY, DIFFUSION. See **diffusion stability.**

STABILITY FACTOR. In a transistor circuit, the ratio of the change in collector current to the change in I_{co} (d-c collector current for zero emitter current). In many circuits, this ratio is considerably greater than one, causing large changes in collector current as a function of temperature, due to the large temperature coefficient of I_{co} (11% per °C for germanium).

STABILITY, MECHANICAL. Mechanical stability is that property of a body which causes it to develop forces opposing any position or motion disturbing influence. The subject may be divided into *static stability* and *dynamic stability*. The former is concerned with the production of the restoring forces, the latter with the oscillations that are set up in the system as a result of the restoring forces.

Another classification is into (1) *positive stability* when the displaced object returns to an initial state of equilibrium after a temporary disturbance, (2) *neutral stability* when the object tends to remain in a definite position but when disturbed may come to rest in a new position, (3) *negative stability* (i.e., instability), when the object assumes an entirely new position when disturbed from its initial state. A simple damped pendulum illustrates the first; a sphere on a horizontal plane, the second; while a slender cylinder standing vertically on end is a case of negative stability.

Let it be assumed that an object at rest or in a state of uniform motion receives a disturbing force. Depending on the kind of stability possessed, it might react with one of the motions shown in the accompanying figure. If it is dynamically stable as well as statically stable, its motion-time history may be one of diminishing oscillation or of simple subsidence, depending on the magnitude of damping, and



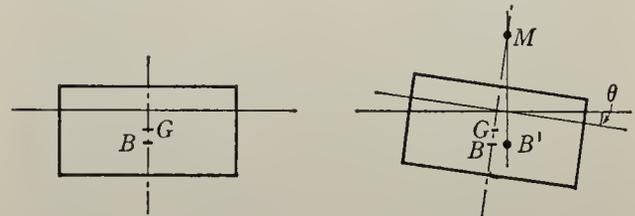
Left; Positive stability (both statically and dynamically stable). Right; Negative stability (both dynamically unstable).

inertial effects. Dynamic instability may occur with either static stability or static instability. These lead to divergent oscillation or to complete divergence.

For simple linear systems, in which the time dependence can be expressed as $e^{i\omega t}$, the system is stable when ω is real, and unstable when the coefficient of i in the imaginary part of ω is less than zero. If the coefficient is greater than zero, the system is overstable; when $\omega = 0$, it is neutrally stable. For a fuller discussion, see R. von Mises, *The Theory of Flight*, Chapter XIX (McGraw-Hill, 1945).

STABILITY, NUMERICAL. See **numerical stability.**

STABILITY OF A FLOATING BODY. Referring to the left-hand sketch in the figure, the weight of a floating body, acting at the



Stability of a floating body.

center of gravity G , is balanced by the force of buoyancy, acting at the center of buoyancy B which is at the centroid of the displaced liquid.

When the body is given a small angular displacement θ , as in the right-hand sketch, the center of buoyancy moves to a new position B' . The point M , where the vertical line through B' intersects the line BG , is called the *metacenter*. If the weight of the body is W , the restoring moment is $W\theta(GM)$, where the distance GM , known as the *metacentric height*, is considered positive when M is above G . Thus the condition for stability is that M should be above G .

The distance from the metacenter M to the center of buoyancy B is I/V , where V is the

volume of liquid displaced by the body and I is the moment of inertia of the horizontal section of the body at the free liquid surface about the axis of tilt. (See also **Archimedes' principle**.)

STABILITY OF LAMINAR FLOW. It is known from experiments that fluid flow may be either laminar or turbulent, the latter form being more usual at high **Reynolds numbers**. In conditions in which turbulent flow is usually observed it is reasonable to assume that the laminar form of flow is unstable, and there is therefore considerable interest in determining the conditions for instability of laminar flow. It should be emphasized, however, that transition from laminar to turbulent flow does not occur immediately after the laminar flow becomes unstable; if the only disturbances present are very small and if the rate of amplification of these disturbances is also small, transition to turbulent flow may be considerably delayed and in some cases may not occur at all within the region considered.

Three basic types of instability will be considered, as follows: (1) Instability of curved flows, due to centrifugal forces. (2) Instability caused by vertical density gradients. (3) Instability of flow in straight ducts and in boundary layers.

In *curved flow*, if viscosity is neglected and if the streamlines are circular, with velocity v at radius r , it can be shown that the flow is stable if vr increases with r , and *vice versa*. This shows, for example, that the flow between concentric cylinders, with the outer one at rest and the inner one rotating, would always be unstable if there were no influence of viscosity on the stability. Another example is the flow in a boundary layer along a concave wall; in the inner part of the layer vr increases as r decreases, so that if viscosity had no effect on the stability the flow would always be unstable.

With real fluids viscosity has a stabilizing influence on these curved flows, so that instability only develops when the Reynolds number exceeds a certain critical value. For the two cases already mentioned, the conditions for instability in a viscous fluid have been determined theoretically by considering a small three-dimensional disturbance and the results have been found to agree well with experiments.

The effect of a vertical density gradient on

the stability of a horizontal laminar flow is in some respects similar to the effect of curvature, since buoyancy forces in the former case have an effect similar to that of centrifugal forces in the latter case. An example is the flow of air over a flat horizontal surface with a vertical gradient of temperature. The stability of such a flow depends on the *Richardson number*

$$R_i = \frac{-\frac{g}{\rho} \cdot \frac{d\rho}{dy}}{\left(\frac{du}{dy}\right)^2},$$

in addition to the usual dependence on Reynolds number. Here y is measured vertically upward and u is the velocity of flow. If $R_i < 0$, the density gradient has a destabilizing effect and the flow is always unstable. For sufficiently large positive values of R_i the flow is stable. The critical value of R_i for stability depends on the velocity profile and on the Reynolds numbers; for a **Blasius** laminar boundary-layer profile the flow is stable for $R_i > 1/24$ even at infinite Reynolds number. At lower Reynolds numbers the critical value of R_i is smaller.

The third type of instability to be considered is the type occurring in a two-dimensional boundary layer or duct, with uniform density and without appreciable curvature. The theory is based on the superposition on the mean flow of a *small* two-dimensional disturbance which is a function of time as well as space. The use of a two-dimensional disturbance is justified, because it can be shown that a given two-dimensional flow remains stable up to a higher Reynolds number when the disturbance is assumed to be three-dimensional than when it is two-dimensional. Thus the critical Reynolds number for complete stability is determined correctly by considering two-dimensional disturbances.

The undisturbed flow is assumed in the theory to be *parallel* to a plane wall, so that the velocity depends only on distance measured normal to the wall and there is no velocity component normal to the wall. This assumption is correct for flow far from the inlet in a channel with parallel walls, but only approximately correct for a boundary layer. Nevertheless, the results of experiments on boundary layers agree well with the theory.

It is convenient to introduce a reference velocity U_o and a reference length l , so that the equations can be expressed in non-dimensional terms. Distances parallel and normal to the flow are denoted by xl and yl , the corresponding velocity components are uU_o and vU_o , and time is $\frac{l}{U_o} \cdot t$. The dimensionless velocity u may be expressed as

$$u = U + u',$$

where U refers to the mean flow and u' to the disturbance.

The stream function representing a single oscillation of the disturbance is assumed to be

$$\psi = \phi(y)e^{i\alpha(x-ct)}, \quad (1)$$

where only the real part has physical meaning. The quantity c is complex, and its imaginary part c_i represents the amplification or damping of the oscillation. If c_i is positive the disturbance is amplified and is unstable, while if it is negative the disturbance decays and is stable.

By substituting Equation (1) into the equations of motion and eliminating the pressure, the following equation is obtained:

$$(U - c)(\phi'' - \alpha^2\phi) - U''\phi = -\frac{i}{\alpha R}(\phi'''' - 2\alpha^2\phi'' + \alpha^4\phi). \quad (2)$$

In this equation primes denote differentiation with respect to y and $R = \frac{U_o l}{\nu}$ is the Reynolds number.

Equation (2), known as the *Orr-Sommerfeld equation*, is the basis of all theoretical work on stability of two-dimensional parallel flows. The general treatment of the equation is difficult, but since experiments show that instability usually occurs only at high Reynolds numbers it is natural to consider first the effect of letting R tend to infinity. This leads to the *frictionless stability equation* (or *inviscid Orr-Sommerfeld equation*)

$$(U - c)(\phi'' - \alpha^2\phi) - U''\phi = 0. \quad (3)$$

It is important to note, however, that even in Equation (3) the influence of viscosity on the mean flow has been taken into account, since Equation (2) is derived from the **Navier-Stokes equations**. The only difference between Equations (2) and (3) is that in the latter the influence of viscosity on the fluctuations is assumed to tend to zero.

An important result that can be obtained from the frictionless stability Equation (3) is that the existence of a point of inflection in the mean velocity profile is a necessary and sufficient condition for instability. This type of instability is known as *frictionless* or *inviscid instability*, since it appears as a consequence of Equation (3). The effect of viscosity on instability of this type is to *reduce* the instability. In contrast, there is another type of instability, known as *viscous instability*, which depends essentially on viscosity and does not appear as a consequence of Equation (3). An example of such a case is the laminar boundary layer on a flat plate, which is considered later in this entry.

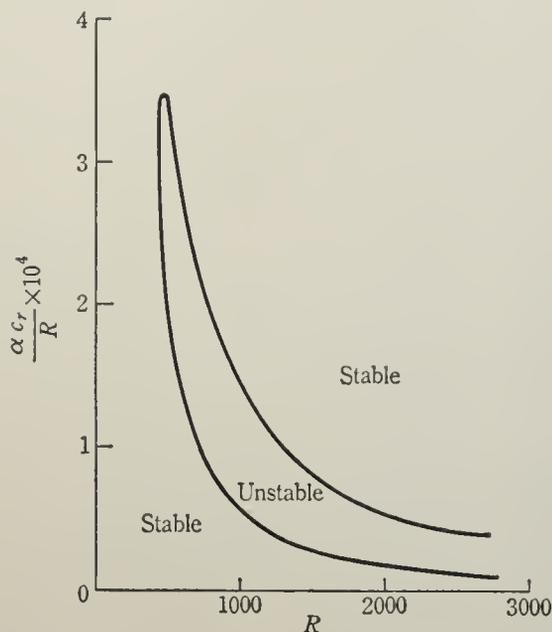
It has been stated that frictionless instability occurs when there is a point of inflection in the mean velocity profile. In a laminar boundary layer such a point of inflection occurs when there is a positive pressure gradient, but not otherwise, so that a positive pressure gradient leads to instability at high Reynolds numbers. This is in agreement with experiments on streamline bodies at high Reynolds numbers, which show that transition from laminar to turbulent flow in the boundary layer usually occurs soon after the pressure minimum.

To evaluate the conditions for stability when the Reynolds number is not infinite, some account must be taken of the viscous terms in Equation (2). This has been done for a number of different cases; the type of result obtained will be illustrated by considering one of the simplest cases of viscous instability, the laminar boundary layer on a flat plate with zero pressure gradient.

The method used is to find two solutions ϕ_1 and ϕ_2 from the frictionless Equation (3), and a second pair of solutions ϕ_3 and ϕ_4 from a new subsidiary equation. This subsidiary equation is obtained from Equation (2) by retaining only the most important viscous term. The retention of this term enables all the boundary conditions for viscous flow to be satisfied; the solutions ϕ_1 and ϕ_2 of Equation (3) are capable of satisfying only two of the four boundary conditions.

The results of these calculations for the boundary layer on a flat plate are shown in the figure, where $\frac{\alpha c_r}{R}$ is plotted against R . The Reynolds number R is defined as $U_o \delta^*/\nu$, where

U_o is the velocity outside the boundary layer and δ^* is the **displacement thickness**. c_r is the real part of c , so that the frequency of the disturbance is $\frac{\alpha c_r}{2\pi}$. The curve shown defines the conditions for neutral stability ($c_i = 0$). For $R < 420$ oscillations of any frequency are damped and the flow is completely stable. For greater Reynolds numbers there is a range of frequencies for which the oscillations are ampli-



Neutral stability curve for laminar boundary layer on flat plate.

fied. Experiments using a hot-wire anemometer have confirmed the results shown in the figure.

The Reynolds number $\frac{U_o \delta^*}{\nu}$ at which transition

to turbulence occurs in the boundary layer is about 3000 for a flat plate in a stream of very low turbulence. This is considerably greater than the value suggested at first sight by the figure, because a considerable length of plate is required for amplification of the disturbances and the process of transition to turbulent flow.

The two branches of the neutral "stability curve" shown in the figure tend towards zero at very large Reynolds numbers, because there is no point of inflection in the velocity profile and the only instability is of the viscous type.

The stability of a laminar boundary layer on a smooth wall may be increased greatly either by sucking fluid from the boundary layer into the wall (which must then be porous) or, in the case of a gas, by heat transfer from the gas to the wall. Conversely, the laminar boundary layer in a gas is made less

stable by heat transfer from the wall to the gas. The effect of heat transfer is due mainly to variation of viscosity with temperature which leads to a change of velocity profile. (See C. C. Lin, *The Theory of Hydrodynamic Stability*, Cambridge University Press, 1955.)

STABILITY OF PHASES. (See **thermodynamic stability conditions**.) Consider an unperturbed system P which consists of a single phase, while the perturbed system P' contains in addition a small amount of a new phase. The phase P is *stable* in respect to P' if the transformation $P' \rightarrow P$ is a spontaneous irreversible change, while the transformation $P \rightarrow P'$ can only occur as the result of an external action or of molecular fluctuations.

One has to specify the nature of the new phase. Either its intensive properties (molar volume, composition, etc.) differ but infinitesimally from those of the original phase, or they differ from them by a finite, non-zero amount.

The following possibilities exist: (1) The initial phase is *stable* with respect to all phases, whether infinitesimally different or not; (2) The initial phase is *stable* with respect to all phases infinitesimally different from it, but there is at least one other phase with respect to which it is not stable; (3) The initial phase is *unstable* with respect to phases infinitesimally different from it.

In the first case one says that the phase is *stable*, in the second *metastable* and in the third *unstable*.

If a phase is unstable it will disappear as a result of molecular fluctuations. On the contrary in the case of *metastable* phases, the system may remain indefinitely in equilibrium. On the other hand, if nuclei of a new, more stable phase are introduced the system changes over into this phase. (See also **thermal stability; stability, mechanical; diffusion stability**.)

STABILITY OF SYSTEM.

(a) *General Definitions.* (i) a system is stable if and only if all its **normal modes** tend to zero as $t \rightarrow \infty$. (ii) A system is critically stable if its normal modes are all bounded but one or more do not tend to zero as $t \rightarrow \infty$. (iii) A system is unstable if one or more of its normal modes is unbounded.

These are equivalent to saying that a system is stable if the roots of its **characteristic equation** all lie to the left of the imaginary axis of

the s -plane; that a system is critically stable if its characteristic equation has any simple roots on the imaginary axis, all others lying to the left of that axis; that the system is unstable if its characteristic equation has either multiple roots on the imaginary axis or roots to the right of that axis.

An alternative definition sometimes used is that a system is stable if and only if it never gives rise to an unbounded output when energized by a bounded input. The essential difference between this definition and the above is that a system which is critically stable by the first definition is unstable by the second.

(b) *Algebraic Criteria.* (i) Routh Criterion. If, as is the case with all lumped parameter systems, the characteristic equation can be written in polynomial form

$$a_0 + a_1s + a_2s^2 + \dots + a_ns^n = 0$$

the array

a_0	a_2	a_4	\dots
a_1	a_3	a_5	\dots
A_1	A_2	A_3	\dots
B_1	B_2	\dots	\dots
C_1	\dots	\dots	\dots

is set up, in which $A_1 = (a_1a_2 - a_0a_3)/a_1$, $A_2 = (a_1a_4 - a_0a_5)/a_1$, $A_3 = (a_1a_6 - a_0a_7)/a_1$ etc., and succeeding rows are obtained from the preceding two rows in the same manner as the third is obtained from the first two. The rows will be found to get shorter by one element every two rows and to be $(n + 1)$ in number.

The system is stable if and only if all the elements of the first column have the same sign. (In fact, the number of changes of sign on going down the first column is the number of roots of the characteristic equation in the right half-plane.)

If a zero is formed in the first column before the array is complete (which prohibits the calculation of further rows) it may indicate either instability or critical stability. If the zero is preceded by two rows consisting of the same number of elements and such that the ratio of corresponding elements in the two rows is constant, roots on the imaginary axis are indicated; moreover, in this case, the elements of either of these rows, read from right to left, successively multiplied by s^0 , s , s^2 , etc., added and equated to zero will give the equa-

tion whose roots are the roots of the characteristic equation lying (in conjugate pairs) on the imaginary axis. If on the other hand the zero is not preceded by two rows with proportional elements then there is at least one root in the right-half plane and the system is unstable.

A necessary, but, except when $n = 1$ or 2 , an insufficient condition for stability is that all the polynomial coefficients shall be non-zero and all have the same sign. This condition is always implicitly contained in the Routh criterion.

(ii) Hurwitz Criterion. Using the same polynomial form of the characteristic equation as before, the array

a_1	a_0	0	0	0	\dots	\dots
a_3	a_2	a_1	a_0	0	0	\dots
a_5	a_4	a_3	a_2	a_1	a_0	\dots
a_7	a_6	\dots	\dots	\dots	\dots	\dots

is drawn up, extending for $(n-1)$ rows and columns, absent coefficients being replaced by zeros.

The system is stable if and only if

(1) All the coefficients of the polynomial are of the same sign, which must be assumed positive, and

(2) The values of the determinants of order 2 to $(n-1)$ and having the top left array element as their top left element are also positive.

(c) *Topographical or Nyquist Criterion.* (i) For a Control System. It is assumed that if $G(s)$ is the output to error transfer function then as $s \rightarrow \infty$, $G(s) \rightarrow 0$. This is normally the case in any practical system.

If the system is stable on closed loop, the vector locus of $G(i\omega)$ drawn from $\omega \rightarrow -\infty$ to $\omega \rightarrow +\infty$ in the sense of increasing ω (i.e., the vector locus of $G(s)$ corresponding to following the imaginary axis in the s -plane) encircles the point $(-1,0)$ P times counterclockwise, where P is the number of poles of $G(s)$ to the right of the imaginary axis, multiple poles counting according to their order.

The following points should be noted:

(a) If $P \neq 0$ the system is unstable on open loop, i.e., with the (-1) feedback from output to error broken.

(b) The value of P may be found, in lumped parameter systems, by writing $G(s) = N(s)/D(s)$ where N and D are polynomials;

P is then the number of zeros of $D(s)$ to the right of the imaginary axis, which may be found most simply by the Routh criterion.

(c) If $G(s)$ has poles on the imaginary axis these must be circumvented in the s -plane by infinitesimal semicircles in the right half-plane having these poles as centers. These indentations will correspond in the plane of $G(s)$ to a clockwise rotation through 180° at indefinitely large radius for every simple pole so encountered, multiple poles again counting according to their order. These infinite semicircles must be taken into account in assessing the number of encirclements of the point $(-1,0)$.

(d) Since $G(-i\omega)$ is the complex conjugate of $G(i\omega)$, the locus of $G(s)$ for negative real frequencies is the mirror image, in the real axis of the $G(s)$ plane, of the locus for corresponding positive real frequencies.

(e) If and only if $P = 0$, an adequate simpler criterion is that the locus of $G(i\omega)$ drawn in the sense of increasing ω , shall leave the point $(-1,0)$ to its left.

(ii) For a General Feedback System. It is of course always possible to reduce the basic equations of the system

$$a_{1j}(s)Q_1(s) + a_{2j}(s)Q_2(s) + \dots + a_{nj}(s)Q_n(s) = f_j(s); j = 1, 2, \dots n$$

(see **system equations**), by eliminating all but two of the Q 's, to a pair of equations relating these two quantities only, which can be put in the form

$$Q_1(s) - \beta(s)Q_2(s) = F_1(s)$$

$$Q_2(s) - \mu(s)Q_1(s) = F_2(s).$$

This in effect reduces the system to a single loop system of loop transfer function $\mu(s)\beta(s)$. Since the characteristic equation is

$$\Delta(s) = 1 - \mu(s)\beta(s) = 0$$

the above formulation of the Nyquist criterion may be applied, replacing $G(s)$ in the preceding section by $-\mu(s)\beta(s)$. Thus the system will be stable provided

(a) The locus of $-\mu(i\omega)\beta(i\omega)$ encircles the point $(-1,0)$ P times counterclockwise, or

(b) The locus of $+\mu(i\omega)\beta(i\omega)$ encircles the point $(+1,0)$ P times counterclockwise, or

(c) The locus of $1 - \mu(i\omega)\beta(i\omega)$ encircles the origin P times counterclockwise, P being in each case the number of simple poles of $\mu(s)\beta(s)$

lying in the right-hand half-plane, any poles on the imaginary axis being circumvented as explained above.

In a multi-loop system however, μ or β or both may comprise subsidiary, possibly unstable loops, so that in general $P \neq 0$. To avoid the reduction of the equations to the above form and the subsequent finding of the value of P by the Routh Criterion, the Nyquist method may be extended as follows.

We start from the premise that a passive system is necessarily stable. In other words if, in the given system, the active elements are made inactive, we have a stable system. We then imagine each of the active elements to be activated one at a time until the system is back to its normal state, investigating at each stage, the behavior of the **return difference** for the re-activated element. (The order in which the elements are reactivated is entirely a matter of convenience.)

Rendering an element inactive has the effect, in the set of general system equations, of making some parameter of that element zero. In the case of an electron tube, the amplification factor or the mutual conductance vanish; in the case of a rotary machine amplifier the armature voltage per ampere-turn vanishes, and analogously for other types of active elements. Further, it can be shown that the return difference for any parameter is the ratio of the values assumed by the system determinant when the parameter has its normal value and when this value is made zero, provided only that the determinant is a linear function of the parameter.

We suppose then that there are m active elements and, having placed these in some arbitrary order, we denote by Δ_r the value of the system determinant when the first r of these elements are activated, the remainder being inactive. Starting from the completely inactive system, the return difference for the first element is Δ_1/Δ_0 . The return difference for the second element (the first one remaining active) is Δ_2/Δ_1 . The return difference for the third (the first two remaining active) is Δ_3/Δ_2 and so on until finally the return difference for the m th element (all other remaining active) is Δ_m/Δ_{m-1} in which furthermore $\Delta_m \equiv \Delta$, the normal system determinant. Now the number of counterclockwise encirclements of the origin made by the Nyquist locus of the return difference of the r th element, Δ_r/Δ_{r-1} is $(z_{r-1} - z_r)$

where z_r denotes the number of zeros of Δ_r in the right half-plane. Hence the sum of such encirclements made by all the successive return difference loci is $(z_0 - z_1) + (z_1 - z_2) + \dots + (z_{m-1} - z_m) = z_0 - z_m$. But z_0 is zero since the system is then passive. For stability, moreover, z_m must be zero. Hence *for stability of the normal system the sum of the encirclements of the origin made by the various return difference Nyquist loci must be zero.*

STABILITY, SLOPE. See slope stability.

STABILITY, STATIC. In incompressible fluids, a quantity equal to

$$\beta = -\frac{1}{\rho} \frac{\partial \rho}{\partial z}$$

where z is measured vertically upwards, and ρ is the density. In fluids compressed under their own weight it is equal to $\frac{1}{\theta} \frac{\partial \theta}{\partial z}$, where θ is the potential temperature, this being a measure of the departure of the lapse rate from the adiabatic value. (See **adiabatic lapse rate.**) The vertical buoyancy force on unit mass of fluid displaced a distance ζ in a horizontally stratified fluid is $-g\beta\zeta$.

STABILIZATION OF VARIANCE. The transformation of a variate by some functional relation to a new variate, carried out with the object of making the error variance of the new variate more nearly constant under the various conditions of the experiment.

STABLE LINEAR NETWORK FUNCTION. See network function, stable linear.

STABLE PERIOD. See period, reactor.

STACK LOSS. See heat carried away.

STAEBLE-LIHOTZKY CONDITION. In geometric optics, the Staebble-Lihotzky condition or isoplanasic condition is necessary and sufficient for an oblique bundle of rays to be symmetric with respect to a given principal ray. It states that an optical system is corrected for coma error if and only if the spherical aberration and sine condition are proportional. Thus it is one of the most valuable criteria in optical design.

STAGE EFFICIENCY. In a turbine or compressor stage, the ratio of the useful work per

unit mass flow to the isentropic enthalpy drop h_s across the stage, or

$$\eta_s = \frac{u_1 \mathcal{U}_{1c} - u_2 \mathcal{U}_{2c}}{h_c}$$

(For explanation of symbols, see velocity diagram.)

STAGNATION POINT. A point at which the fluid velocity is zero, in particular a point on the boundary at which the velocity is zero in an inviscid fluid or at which there is no tangential stress in a viscous fluid. It is a point at which the **stagnation pressure** can be measured.

STAGNATION PRESSURE. The pressure at a point of stagnation (zero velocity) on a body around which fluid flows.

STAGNATION TEMPERATURE. The temperature acquired by a (compressible) fluid at a **stagnation point** in adiabatic flow. The **potential temperature** defined using the stagnation pressure as standard.

STALLING OF AN AIRFOIL. For small or moderate incidence the lift coefficient of an airfoil increases approximately linearly with incidence, as predicted by potential-flow theory. With increasing incidence, however, the positive pressure gradient on the upper surface increases, and a stage is reached at which this pressure gradient causes separation of the boundary layer. The airfoil then *stalls*, and further increases of incidence gives little or no increase of lift coefficient and may sometimes cause a rapid decrease.

The nature of the stall depends on the shape of the airfoil section, particularly on the thickness/chord ratio and the radius of curvature at the leading edge. On a thick airfoil with a well rounded leading edge, the stall occurs when the boundary layer separates in the *turbulent* state towards the rear of the airfoil. With increasing incidence the separation point moves gradually forward.

On thinner airfoils, especially those with relatively sharp leading edges, the boundary layer separates in the laminar state near the leading edge. This often occurs at a low or moderate incidence, and at high Reynolds numbers the separation is followed almost immediately by transition to turbulent flow and reattachment. The effect of the separation on the pressure distribution is then very small and

the airfoil is not usually regarded as being stalled. With further increase of incidence a stage is reached at which either the boundary-layer fails to re-attach after separation or the point of re-attachment moves back towards the rear of the airfoil. The former case occurs on airfoils of moderate thickness with rounded leading edges and leads to an abrupt decrease of lift when the boundary-layer fails to re-attach. In the latter case, which occurs on very thin airfoils, or airfoils having sharp leading edges, the lift coefficient remains roughly constant with increasing incidence.

The stalling behavior of a wing of finite aspect ratio depends not only on the sections but also on the planform. With a strongly tapered wing, without twist, the effective incidence is higher at the tips than at the center, so that with increasing incidence stalling occurs first at the tips. Conversely, an untwisted wing of rectangular planform stalls first at the center. Sweepback also has important effects on stalling; the outer part of a swept back wing usually stalls first, giving a nose-up change of pitching moment at the stall.

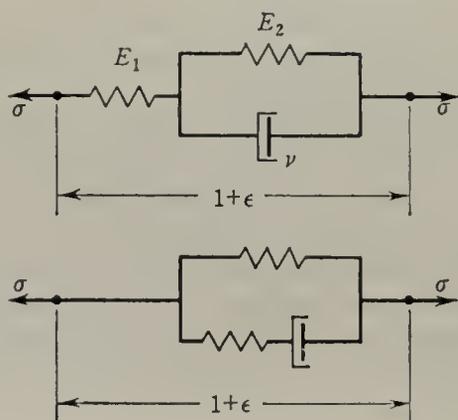
STANCHION. British term for column.

STANDARD COLORIMETRIC OBSERVER, CIE. See CIE standard colorimetric observer.

STANDARD DEVIATION. The standard deviation of a frequency distribution is the square root of its variance. It is the most useful measure of dispersion.

STANDARD ERROR. A name often given to the standard deviation of a sampling distribution.

STANDARD LINEAR SOLID. Viscoelastic material exhibiting instantaneous and delayed elasticity only. It is represented by models of



the form shown in the figure, which are equivalent for appropriate choice of constants.

The constitutive equation in differential form is of first order in derivatives of both stress and strain, and takes the form:

$$\frac{\nu}{E_1} \dot{\sigma} + \left(1 + \frac{E_2}{E_1}\right) \sigma = \nu \dot{\epsilon} + E_2 \epsilon.$$

STANDARD MEASURE. A variate may be changed to standard units by the transformation

$$t = \frac{x - \bar{x}}{\sigma_x},$$

where t is in standard units, x is the variate, \bar{x} is the mean, and σ_x is the standard deviation of the distribution. In such cases t has a mean zero and a standard deviation of 1, and is said to be in standard measure.

STANDARD REFRACTION. The refraction which would occur in an idealized atmosphere in which the refractive index decreases uniformly with height at the rate of 39×10^{-6} per kilometer. Standard refraction may be included in ground wave calculations by use of an effective earth radius of 8.5×10^6 meters, or $\frac{1}{3}$ the geometrical radius of the earth.

STANDARD TEMPERATURE AND PRESSURE. See normal temperature and pressure.

STANDING WAVE. A wave disturbance which is not progressive, i.e., one in which any component of the field can be specified as a function of position multiplied by a sinusoidal function of time. (See also standing waves (in fluid).)

STANDING-WAVE RATIO. Any transmission line such as a waveguide or an acoustic transmission system, unless terminated by its characteristic impedance, will exhibit a superposition of standing and progressive waves. The standing wave ratio is a measure of the relative amplitudes of the two types of wave and is defined as the ratio of the maximum amplitude of pressure (or voltage) to the minimum amplitude of pressure (or voltage) measured along the path of the waves. Thus, at a given frequency in a uniform waveguide the standing-wave ratio is the ratio of the maximum to the minimum amplitudes of corresponding components of the field (or the voltage or current) along the waveguide in the

direction of propagation. Alternatively, the standing wave ratio may be expressed as the reciprocal of the ratio defined above.

STANDING WAVES (IN FLUID). Waves in a moving stream of fluid whose pattern is fixed relative to the bed. The motion is steady, the fluid passing along fixed streamlines.

The velocity of the waves relative to the fluid is equal and opposite to the velocity of the fluid relative to the bed.

Standing waves are commonly produced on rivers and streams where the cross section area varies, and in air currents over uneven country. In the latter case stationary clouds are often produced in the wavecrests with the air passing through them. (Cf. **stationary waves (in fluid).**)

STAR. (1) The set of coefficients associated with any particular approximation to derivatives by differences used in the numerical solution of **partial differential equations.** (2) When a high energy nuclear particle, such as a proton of many Bev energy, collides with a nucleus, or a meson is absorbed, a large number of secondary particles may be knocked out in all directions. Many of these particles are electrically charged and so give rise to ionization in a closed chamber or a photographic plate. The general appearance of the event is thus one of a set of rays emanating from a center, and to it has been given the descriptive term "star." For a long time the only source of particles of such energy was the cosmic radiation.

STAR CLUSTERS. The stars are not scattered at random over the sky. Here and there obvious star clusters appear, some of which are conspicuous to the naked eye and have been appropriately named, e.g., the Pleiades, the Hyades, Coma Berinices, etc. In contrast to these open clusters, the telescope reveals a number of clusters of stars gathered together in a more or less compact spherical array.

Since most of the open clusters are found in the general region of the milky way, they are referred to as *galactic clusters* while the more compact clusters are known as *globular clusters*.

(1) *The Galactic Clusters.* About 500 galactic clusters have been identified, containing

from about twenty to a few hundred stars with at least one, the Perseus Cluster, with over one thousand stars. Those that have been studied have all been within 20,000 light years of the sun, but there is no reason to assume that they are not equally numerous in other regions of our galactic system.

(a) *Moving Clusters.* Usually the proper motions of stars in the same constellation are so varied in amount and direction that no connection between them can be found. However, in several cases, notably Ursa Major and the Hyades, the proper motions are in such close agreement that we may say with confidence that they are moving together. In the constellation of Ursa Major six of the eight stars forming the "dipper" have the same proper motions. The determination of the radial velocities removes all doubt. In this case we have a cluster that is moving relative to the sun but is so close to the sun that the members of the moving cluster are scattered over a considerable area of the sky.

In the smaller group, known as the Hyades, we find a group of at least 150 members, moving together (Aldebaran is not one of the group). In this case the apparent paths converge, which indicates that the cluster is moving away from the vicinity of the sun and in the general direction of the present apparent position of Betelgeuse. The determination of the radial velocity of one or more members of the cluster may be combined with the observed proper motion to determine the space velocity of the cluster as about 44 km/sec. The distance is about 40 parsecs. We can also determine that 800,000 years ago the cluster was closest to the sun and at a distance of about 20 parsecs. From a study of the magnitudes and colors of the members of the cluster (cf. **spectrum-luminosity relations**), the absolute magnitudes, M , of the stars may be found. Combining this with the observed magnitudes, m , of the same objects, the distance r may be determined by

$$\log_{10} r = 1 + \frac{m - M}{5}$$

in which r is the distance in parsecs, due regard having been made for possible absorption of light in space. Distances of a number of other galactic clusters have been determined by this and other methods. They range from 40 to 2250 parsecs.

(2) *Globular Clusters.* A globular cluster is a considerable number of stars that are apparently gathered together in a relatively small volume of space. Several of them may be recognized with the unaided eye, e.g., the globular cluster in the constellation of Hercules, and an even brighter one in the constellation of Centaurus. The apparent distances between the individual stars seem to decrease toward the center. In fact the central portions cannot be resolved into their individual stars with the telescopes of highest resolving power.

The majority of the globular clusters are on the side of the sun toward the center of the galactic plane with fully one-third of them in the direction of the constellation of Sagittarius. Variable stars have been found in many of the globular clusters and most of them are of the RR Lyrae Type (cf. **variable stars**). The radial velocity of some of these have been determined and the results indicate that the variables are actually in the clusters. Since these RR Lyrae variables have the same medium absolute magnitude, the distances of the clusters may be determined by measuring the median apparent magnitude of the variables, due allowance being made for the absorption of light in space.

The globular clusters differ in size and luminosity over a considerable range. Their diameters average about 130 light years and the absolute magnitudes average about -7 which means that they have an average brightness of about 100,000 that of the sun. They are relatively scarce in space with none closer to the sun than 20,000 light years.

Detailed studies of some of the globular clusters yield the **color index** of some of the stars as well as the brightness of the same stars. From these data we are in a position to compare the spectrum-luminosity relations for the globular clusters with that of the spectrum-luminosity relations of stars relatively closer to the sun. This has been done for several clusters and the star populations in the clusters have been found to be different from that in the vicinity of the sun.

STARK EFFECT. In an electric field of strength E an atomic or molecular energy level corresponding to a total angular momentum J splits into $J + 1$ or $J + \frac{1}{2}$ component levels, depending on whether J is integral or half integral, with the quantum numbers

$$|M| = J, J - 1, \dots, 0 \text{ or } \frac{1}{2}.$$

The energies of these levels are given by

$$W = W_0 - \bar{\mu}_E E$$

where W_0 is the energy in zero field and $\bar{\mu}_E$ is the mean component of the electric dipole moment in the field direction. (See also **space quantization**.) Different from the situation in a magnetic field (**Zeeman effect**) the energy of the Stark effect levels depends only on the absolute value of M , that is, pairs of levels which differ only by the sign of the magnetic quantum number ($+M$ and $-M$) have the same energy.

If the energy of the component levels is expressed in the form

$$W = W_0 + AE + BE^2 + CE^3 + \dots$$

where A , B , and C are constants for the energy level (see below), the second term gives the first order or *linear Stark effect*, the third term the second order or *quadratic Stark effect*.

The **selection rules** for the quantum number M are

$$\Delta M = 0, 1 \quad (M = 0 \leftarrow \rightarrow M = 0 \text{ for } \Delta J = 0).$$

Lines corresponding to transitions with $\Delta M = 0$ are plane polarized with the electric vector parallel to the field direction (π -components), those with $\Delta M = 1$, plane polarized with the electric vector perpendicular to the field direction (σ -components).

I. *Atomic Spectra.* For hydrogen the constants A , B , and C in the above equation for the energy W of the Stark components are given by

$$A = \frac{3h^2}{8\pi^2 m e} n(n_2 - n_1),$$

$$B = \frac{h^6}{2^{10} \pi^6 m^3 e^6} n^4$$

$$\times [17n^2 - 3(n_2 - n_1) - 9m_l^2 + 19]$$

$$C = \frac{3h^{10}}{2^{15} \pi^{10} m^5 e^{11}} n^7$$

$$\times [23n^2 - (n_2 - n_1)^2 + 11m_l^2 + 39].$$

Here h is Planck's constant, e and m , charge and mass, respectively, of the electron, and n , the principal quantum number. n_1 , n_2 , and

m_l are electric quantum numbers, subject to the condition

$$m_l = n - n_2 - n_1 - 1$$

where

$$n_1 = 0, 1, 2, \dots, n - 1;$$

$$n_2 = 0, 1, 2, \dots, n - 1;$$

$$m_l = 0, \pm 1, \pm 2, \dots, \pm(n - 1);$$

$$n = 1, 2, \dots, \infty.$$

For fields $E < 100000$ volts the lower states of hydrogen show only the first order effect with a symmetrical splitting of the levels about the field-free position. The second order effect becomes large for higher states and higher fields, and results in a unidirectional displacement.

For non-hydrogenlike atoms the second order Stark effect predominates quite generally over the first order effect. (For more detail see H. E. White, *Introduction to Atomic Spectra*, McGraw-Hill Book Co., 1934, p. 401 ff.)

II. *Molecular Spectra*. For molecules without permanent dipole moment

$$\bar{\mu}_E = a_{J|M}|E$$

where $a_{J|M}$ is a constant depending on J and M .

For linear or symmetric top molecules with a permanent dipole moment, μ ,

$$\bar{\mu}_E = + \frac{\mu M k}{J(J+1)} - \frac{4\pi^2 I_o \mu^2 E}{h^2} \left[\frac{(J^2 - M^2)}{J^3(2J-1)(2J+1)} - \frac{[(J+1)^2 - M^2][(J+1)^2 - K^2]}{(J+1)^3(2J+1)(2J+3)} \right].$$

Here h is Planck's constant, I_o is the molecular moment of inertia about an axis perpendicular to the figure axis, and $K = \pm k$, is the quantum number corresponding to the component of J in the direction of the figure axis. For diatomic molecules K must be replaced by Λ , for linear polyatomic molecules by l (see G. Herzberg, *Infrared and Raman Spectra of Polyatomic Molecules*, D. Van Nostrand Co., Inc., 1945, p. 380). If K (or Λ or l) is zero the preceding expression simplifies to

$$\bar{\mu}_E = - \frac{4\pi^2 I_o \mu^2 E}{h^2} \left[\frac{J(J+1) - 3M^2}{J(J+1)(2J-1)(2J+3)} \right],$$

that is, only a quadratic, and no linear Stark effect occurs.

STAR MASSES AND MASS FUNCTION.

See stellar masses and mass function.

STAR MASS-LUMINOSITY RELATION.

See stellar mass-luminosity relation.

STARS, BINARY SPECTROSCOPIC. See spectroscopic binary stars.

STAR SPECTRUM-LUMINOSITY RELATIONS.

See stellar spectrum-luminosity relations.

STARS, SPECTRAL CLASSIFICATION.

See spectral classification of stars.

STARS, VARIABLE. See variable stars.

STARTING VORTEX. See airfoil started from rest.

STATE. The state of a thermodynamic system is described by the set of its macroscopic properties. The state is fixed when its macroscopic properties have fixed numerical values. It is known from experiment that only a limited number of properties are independent for any given system. When the independent properties are given fixed values, all the properties, and hence the state, become fixed. It is immaterial which particular properties are fixed, provided that they are mutually independent, and that the requisite number of them is chosen. For example, the state of a given quantity of a gas is fixed when two independent properties are fixed; of the three properties, temperature T , pressure p , specific volume v , any two can be chosen as independent, but specific volume and density ρ , cannot be so chosen, because they are related by the equation $\rho v = 1$, and are not, therefore, mutually independent. (See entries immediately following and stationary state.)

STATE, EQUATION OF. See equation of state.

STATE OF STRAIN. The strain field in a body is often referred to as the *state of strain in the body*, or the *strain in the body* and the strain tensor at a point of the body as the *state of strain at the point*, or the *strain at the point*.

STATE OF STRESS. The stress field in a body is often referred to as the *state of stress*

in the body, or the stress in the body and the stress tensor at a point of the body as the state of stress at the point, or the stress at the point.

STATE SUM. See partition function.

STATICALLY ADMISSIBLE. A state of stress throughout a body which satisfies the equations of equilibrium and the boundary conditions on surface traction is termed statically admissible. When employed in connection with the lower bound theorem of limit analysis, a further restriction is that the stresses everywhere be below yield values.

STATICALLY DETERMINATE. See determinate structure.

STATICALLY INDETERMINATE. See indeterminate structure.

STATIC FRICTION. See friction, coefficient of.

STATIC MOMENT. See moment, static.

STATIC PRESSURE. The pressure at a point in a fluid, defined as (1) the normal stress on unit area moving with the fluid, or (2) the mean of the normal components of stress on three mutually perpendicular elements of surfaces at that point, at rest relative to the fluid. In fluids of small viscosity, the differences between these three normal components of stress are usually very small, so that any one of them is nearly equal to the pressure.

The word "static" is often included to distinguish the pressure, as defined above, from other quantities such as total pressure and dynamic pressure.

STATICS. The study of systems at rest or moving with uniform velocity with respect to an inertial frame.

STATIC STABILITY. See stability, static.

STATIONARY STATE. (1) In a quantum-mechanical system, a state corresponding to a definite value of the energy. (2) A thermodynamic system is in a stationary state when its intensive variables (see also intensive properties) are time-independent. The definition includes two classes of situations:

(1) *Equilibrium states:* the system is in thermodynamic equilibrium; no irreversible processes occur in it and there are no exchanges with the outside world. The entropy produc-

tion (see also thermodynamics, second law of) vanishes.

(2) *Stationary non-equilibrium states:* in this case irreversible processes proceed in the system. The time-independence of the intensive properties is due to a compensation of the effect of the irreversible processes by exchanges with the outside world.

An isolated piece of metal initially at a non-uniform temperature will, of course, reach a state of thermodynamic equilibrium. But if we cool it at one end and heat it at the other, it will reach a stationary non-equilibrium state. Another example is afforded by a system which receives a component M from the outside environment and transforms it through a certain number of intermediate states into a final product F which is returned to the external environment. After some time a stationary state arises when the concentration of the intermediate components no longer vary with time.

One may say that stationary non-equilibrium states are reached when there exist some constraints which prevent the system from reaching true thermodynamic equilibrium. In the first example it is the difference of temperature at both ends, in the second, it is the value of the concentrations of the initial and final products in the external environment.

In non-equilibrium stationary states the entropy production does not vanish. (See minimum entropy production; time variation of the entropy production.)

STATIONARY STOCHASTIC PROCESS. A stochastic process $\{x(t)\}$ is strictly stationary if the multivariate distribution of any consecutive set $x(t), x(t+1), \dots, x(t+k)$ is the same for any value of t and for all k . A process is weakly stationary if the mean value $E(x_t) = m_t$ is independent of t and the covariance, $\text{cov}(x_t, x_\tau)$, depends only on the difference $t - \tau$.

STATIONARY TIME SERIES. A time series having stable statistical properties in the following sense. Let $x(t)$ denote the value of the variable at time t . Hold t fixed and imagine an indefinite series of repetitions of essentially the same generating process, giving rise to a population (ensemble) of values of $x(t)$. For a stationary time series, the ensemble probability distribution of $x(t)$ is independent of t . When the probability distribution changes

very gradually with t , the time series is called *quasi-stationary*.

STATIONARY WAVES (IN FLUID). Waves with nodes fixed in the fluid, the particles between the nodes being in oscillation, the displacement of the particles relative to the mean position, being proportional to $\cos kx \cos \sigma t$.

Stationary waves on the surface of a fluid under gravity are produced where progressive waves are reflected at a vertical wall which is parallel to their crests. The center of gravity of the whole fluid oscillates vertically (unlike in progressive waves) so that the pressure on the bed of the fluid fluctuates with period π/σ . When a train of ocean waves is reflected normally at a coastline seismic waves are produced in the sea bed by this pressure fluctuation, which are known as microseisms and can be detected far inland. (Cf. **standing waves (in fluid)**.)

STATISTIC. A statistic is a function of the observations in a **sample** usually designed to estimate a **parameter** of the **population** from which the sample was drawn.

STATISTICAL HYPOTHESIS. See **hypothesis, statistical**.

STATISTICAL INDEPENDENCE. The relationship between two or more random variables when their joint **probability density** function can be expressed as the product of the individual functions,

$$F(x,y) = H(x)G(y);$$

and the ranges of variation of x and y are independent. Otherwise, variates are statistically dependent.

STATISTICAL MATRIX. See **density matrix**.

STATISTICAL MECHANICS. That part of physics where one derives the properties of matter in bulk from a knowledge of the behavior of its constituent parts. It is developed from **kinetic theory** but is distinguished from it by considering **ensembles** of systems rather than single systems. Statistical methods are needed because physical systems contain so many particles that the equations of classical mechanics become so many that—even if we had sufficient knowledge about the system to determine the initial conditions—their solution is impossible.

In **classical** (or **Boltzmann**) statistics one considers distributions in **phase space**. Each system corresponds to a point (the representative point) in phase space, and an ensemble corresponds to a collection of points. Averages are then taken over the **ensemble**, and it is proved that the average behavior of a system in a macrocanonical ensemble, for instance, is the same as the behavior of a system in temperature equilibrium.

In **quantum statistics** instead of using points in phase space one uses energy levels, and instead of **ensemble densities** one uses the **density matrix**. Depending on the spin of the constituent particles one uses **Bose-Einstein** or **Fermi-Dirac statistics**.

It is possible to derive from statistical mechanics many results of thermodynamics, and in particular Boltzmann's **H-theorem** is the statistical counterpart of the second law of thermodynamics.

STATISTICAL METHOD OF ENTROPIES.

A method using **partition functions** from assumed molecular models or spectroscopic data. The relation used is

$$S = R \ln Z - R \frac{\partial(\ln Z)}{\partial(\ln \beta)}$$

where S is the entropy, R the gas constant, Z the partition function, and $\beta = 1/kT$ (k being the Boltzmann constant, T , the absolute temperature).

STATISTICAL QUALITY CONTROL. A method of controlling the quality of a manufactured product. It aims at tracing and eliminating systematic variations, the remaining variation being then of the nature of a distribution of error; and the properties of the latter can be measured and used to guarantee the average quality of the product. Such "guarantee" may be expressed in terms of risk of exceeding certain tolerances or of other statistical criteria.

STATISTICAL WEIGHT. (1) Degree of degeneracy of quantum state; (2) **statistical weight factor**; (3) sometimes **statistical weight factor** multiplied by **Boltzmann factor**. (See also entries following.)

STATISTICAL WEIGHT FACTOR. In the statistical investigation of a given quantity, a **statistical weight** can be assigned to each value or range of values of the quantity, which

is the number of times this value or range of values occurs. In statistical mechanics, it is defined as the number of microstates corresponding to a given macrostate.

STATISTICAL WEIGHT, NUCLEAR REACTOR. In the theory of neutron chain reactors, the weight or importance associated with a small volume ΔV of the system in the calculation is the effect of a change in material properties at ΔV upon the overall multiplication of the system. In the case of a one-velocity model of the reactor, the

$$\text{Statistical Weight} = \frac{\psi^2(\mathbf{r})dV}{\int \psi^2(\mathbf{r})dV}$$

where ψ is the unperturbed neutron flux, and the integration is extended over the entire system.

STATISTICAL WEIGHTS OF ATOMIC ENERGY LEVELS. The statistical weight g of an atomic energy level is given by

$$g = g_J \times g_I$$

where g_J and g_I , respectively, are equal to the number of possible orientations of the total electronic angular momentum \mathbf{J} and the nuclear spin \mathbf{I} in a magnetic field. Therefore

$$g = (2J + 1) \times (2I + 1).$$

STATISTICAL WEIGHTS OF MOLECULAR ENERGY LEVELS. The statistical weight g of a molecular energy level is given by

$$g = d \times g_J \times g_I.$$

Here d stands for the degeneracy of the electronic and the vibrational state, g_J (in the following also called g_{JK}) is equal to the number of possible orientations of the total angular momentum, apart from nuclear spin, \mathbf{J} of the molecule in a magnetic field. g_I depends on the nuclear spins I_i , and, for certain point groups, on the statistics of the nuclei.

The statistical weights of the rotational energy levels, apart from the factor d , are as follows:

(a) *Diatomic and Linear Polyatomic Molecules.* The statistical weight due to end over end rotation of the molecule is given by

$$g_J = 2J + 1.$$

For molecules without a center of symmetry we have

$$g_I = (2I_1 + 1)(2I_2 + 1) \cdots$$

where I_1, I_2, \cdots are the spins of the individual nuclei. Since in this case g_I is the same for all rotational levels, it can, for most purposes, be omitted.

If the molecule has a center of symmetry and if the number of pairs of identical nuclei following Fermi statistics is odd (while the number of pairs of identical nuclei following Bose statistics is even or odd), the statistical weight due to the nuclear spins is

$$g_I^s = \frac{1}{2}[(2I_X + 1)^2(2I_Y + 1)^2(2I_Z + 1)^2 - (2I_X + 1)(2I_Y + 1)(2I_Z + 1) \cdots]$$

for the symmetric rotational levels, and

$$g_I^a = \frac{1}{2}[(2I_X + 1)^2(2I_Y + 1)^2(2I_Z + 1)^2 + (2I_X + 1)(2I_Y + 1)(2I_Z + 1) \cdots]$$

for the antisymmetric rotational levels. If the number of pairs of identical nuclei following Fermi statistics is even, the situation is reversed. Here X, Y, Z, \cdots refer to the different pairs of nuclei.

If only one pair of identical nuclei has a nonzero nuclear spin I , the ratio of the statistical weights of the symmetric to the antisymmetric rotational levels becomes simply $(I + 1)/I$ or $I/(I + 1)$, depending on whether the nuclei follow Bose or Fermi statistics.

(b) *Symmetric Top Molecules.* The statistical weight of a rotational state due to the over-all rotation is

$$g_{JK} = 2J + 1, \quad \text{for } K = 0$$

$$g_{JK} = 2(2J + 1), \quad \text{for } K > 0.$$

For molecules without symmetry the statistical weight due to the nuclear spin is

$$g_I = (2I_1 + 1)(2I_2 + 1)(2I_3 + 1) \cdots$$

In this case g_I contributes only a constant factor to the total statistical weight

$$g = g_{JK} \times g_I$$

and can usually be omitted.

If the molecule has symmetry, rotational levels of different species have different statistical weights depending on spin and statistics of the identical nuclei. If, for instance, the figure axis of the molecule is a threefold axis of rotation, in a totally symmetric vibrational

and electronic state the levels with $K = 0, 3, 6, 9, \dots$ (A) have

$$g_I = \frac{1}{3}(2I + 1)(4I^2 + 4I + 3)$$

while those with $K = 1, 2, 4, 5, 7, 8, \dots$ (E) have

$$g_I = \frac{1}{3}(2I + 1)(4I^2 + 4I).$$

If the spin of all the identical nuclei is zero, the levels with $K = 1, 2, 4, 5, 7, 8, \dots$ are entirely missing.

(c) *Spherical Top Molecules.* The statistical weight of a given rotational level due to the over-all rotation is

$$g_J = (2J + 1)^2$$

and that due to the nuclear spin is

$$g_I = (2I_1 + 1)(2I_2 + 1)(2I_3 + 1) \dots$$

For molecules that are spherical tops on account of their symmetry there are additional symmetry properties, e.g., A , E , and F for tetrahedral molecules. The over-all statistical weight is then a product of $(2J + 1)$ times a factor that depends in a complicated way both on J and the spin of the identical nuclei.

(d) *Asymmetric Top Molecules.* The statistical weight of a rotational level J_r due to the over-all rotation of the molecule is

$$g_J = 2J + 1.$$

If the molecule has no axis of symmetry, the statistical weight due to the nuclear spins is

$$g_I = (2I_1 + 1)(2I_2 + 1)(2I_3 + 1) \dots$$

If the molecule has one twofold axis of symmetry, the dependence of the statistical weights of the symmetric and antisymmetric rotational levels on the spins and statistics of the identical nuclei is the same as in linear molecules. If there are three axes of symmetry, more complex relations hold.

(See G. Herzberg, *Molecular Spectra and Molecular Structure. II. Infrared and Raman Spectra of Polyatomic Molecules*, D. Van Nostrand Company, Inc., Princeton, 1945.)

STATISTICAL WEIGHT THEOREM. In the theory of neutron chain reactors, a formula derived from perturbation theory in which the change in the multiplication (or criticality) of a system caused by a local perturbation is expressed in terms of unperturbed fluxes and adjoint fluxes as well as the

change in capturing or scattering properties itself.

STATISTIC, ANCILLARY. See *ancillary statistic*.

STATISTICS, BOLTZMANN. See *Boltzmann statistics*.

STATISTICS, BOSE-EINSTEIN. See *Bose-Einstein statistics*.

STATISTICS, FERMI-DIRAC. See *Fermi-Dirac statistics*.

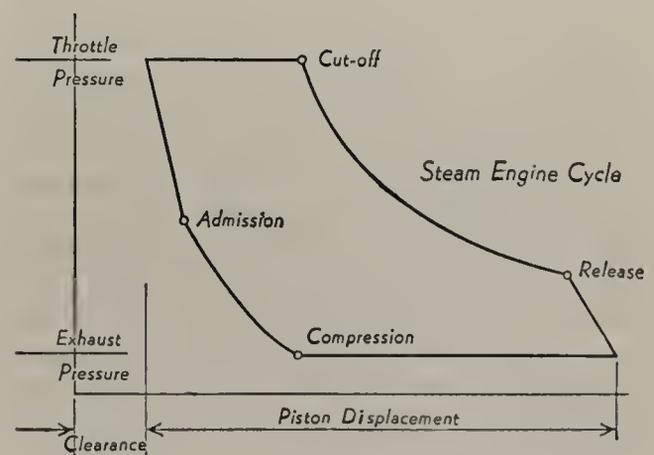
STATISTICS, GENTILE. See *Gentile statistics*.

STEADY PERIOD. See *period, reactor*.

STEADY STATE. A process is said to occur in steady state (steady-state) when the (open or closed) system used to describe it is invariant with respect to time. (See *thermodynamic system*.)

STEADY-STATE OSCILLATION. See *oscillation, steady-state*.

STEAM ENGINE CYCLE. The cycle upon which a steam engine operates, described as follows: Slightly before the piston reaches the dead-center position corresponding to minimum cylinder volume, the valve connects the cylinder with the steam line so that as the piston starts on its outward travel, the full steam pressure is acting on it. The beginning of this action is known as the event of *admission*. When some 20 to 30% of the stroke has been completed, the valve closes the port on the event known as *cut-off*, and during the remainder of the stroke, the steam is expanded *adiabatically* to the accompaniment of de-



Ideal steam engine cycle.

creasing pressure. Near the end of its stroke, the valve again opens the port, this time connecting the cylinder with the exhaust line. This event is known as *release*. The cylinder remains connected with the exhaust during the return stroke of the piston, and the steam is expelled until approximately $\frac{2}{3}$ of the return stroke has been completed. The valve then closes the port, and the remaining steam is trapped in the cylinder and compressed. The beginning of this process is known as the event of *compression*. The four events just described govern the form of the steam engine cycle. The engine using it can develop a horsepower hour by using from 10 to 25 lbs of steam, depending upon the expansion permitted by the terminal conditions of the steam.

STEAM POINT. Temperature of equilibrium between pure liquid water and its vapor at the pressure of one standard atmosphere. The correction to be applied when the pressure p is 760 millimeters of mercury exactly is given by

$$t_p = t_{760} + 0.0367(p - 760) - 0.000023(p - 760)^2$$

where t is expressed in degrees C or K, and p is given in millimeters of mercury.

STEAM RATE. The rate at which a steam engine or turbine consumes steam per unit power output is its steam rate. Originally, this quantity was called water rate, but since an engine operates on steam, the term steam rate seems more appropriate. It is usually given in units of pounds of steam per horsepower hour, or per kilowatt hour in the case of a direct-connected or turbo-generator unit. The steam rate varies with the load, being minimum at a load known as the most economical load. This is the rated load in the turbine, and somewhat less than rated in the engine. Steam rate is affected by initial pressure and degree of superheat; also, by exhaust pressure. The higher the initial pressure, or temperature, and the lower the final pressure, the smaller the steam rate will be. Thermal efficiency is inversely proportional to steam rate. Steam rate, multiplied by load, is steam consumption, and when this product is plotted against load, the result is often nearly a straight line, up to the most economical load. This is the well-known *Willans Line*.

STEEPEST DESCENT METHOD. (1) A term sometimes applied to the saddle point method. (2) However the term is also applied to a method of solving simultaneous equations and of minimizing (or maximizing) a function. Given a function of several variables

$$\Phi(\xi_1, \xi_2, \dots, \xi_n)$$

to be minimized, form $\text{grad } \Phi$, the vector whose elements are $\partial\Phi/\partial\xi_i = \gamma_i$. The function

$$\phi(\lambda) = \Phi(\xi_1 - \lambda\gamma_1, \dots, \xi_n - \lambda\gamma_n)$$

is a function of only a single variable. Let $\xi_1^{(0)}, \dots, \xi_n^{(0)}$ represent a point sufficiently close to the minimum, let $\gamma_i^{(0)}$ represent the value of γ_i at that point; substitute $\xi_i^{(0)} - \lambda\gamma_i^{(0)}$ in Φ and minimize the resulting ϕ as a function of λ . If this minimum occurs at $\lambda^{(0)}$, then $\xi_i^{(1)} = \xi_i^{(0)} - \lambda^{(0)}\gamma_i^{(0)}$ will be, in general, a closer approximation to the minimum.

(3) In a related form, the method used to evaluate integrals of the kind

$$I = \int \chi(z)e^{t f(z)} dz,$$

where $f(z)$ and $\chi(z)$ are analytical functions of z , and where t is large, real and positive. Integrals of this type occur in many branches of physics, notably in statistical mechanics.

STEFAN-BOLTZMANN LAW. The Helmholtz free energy of a radiation field is given by (see thermodynamics, characteristic functions of)

$$A = -\frac{1}{3}aT^4V \tag{1}$$

where T is the absolute temperature, V , the volume and a , the universal constant

$$a = \frac{8\pi^5 k^4}{15c^3 h^3} \tag{2}$$

where k is Boltzmann's constant, c , the speed of light, and h , Planck's constant.

From (1) it follows immediately that the radiation pressure, the energy and the heat capacity are given by

$$p = \frac{1}{3}aT^4 \tag{3}$$

$$U = aT^4V \tag{4}$$

$$C = 4aT^3V. \tag{5}$$

Formula (4) is the Stefan-Boltzmann law.

From (4) we see that aT^4 is the radiation energy per unit volume in an enclosure. If

a small hole is made in such an enclosure the radiation emitted through the hole per unit area and unit time is σT^4 where σ is given by

$$\sigma = \frac{1}{4}ac \quad (6)$$

(c being the speed of light). This constant σ is called the *Stefan-Boltzmann constant*.

STELLAR DYNAMIC PARALLAXES. If the individual masses of the two components of a binary star, m_1 and m_2 , are determined from the mass-luminosity relation, it is possible to determine the parallax, p'' , of the system. From **Kepler's Laws** we can write

$$m_1 + m_2 = \frac{a^3}{P^2 p''^3}$$

in which p'' is the parallax of the system in seconds of arc and P is the period, given in sidereal years. In a visual binary a is the separation of the components, which may be measured in seconds of arc. p'' , the parallax, is the only remaining unknown, and for this we have, therefore

$$p'' = \frac{a}{\sqrt[3]{P^2(m_1 + m_2)}}$$

(Cf. **stellar masses**.)

STELLAR LUMINOSITIES. 1. *Absolute Magnitude.* The *apparent magnitude* of a star is related to the brightness of the star as it is observed from the earth. The absolute magnitude is the apparent magnitude that the star would appear to have if it were at a standard distance from the earth. By agreement the standard distance has been set at 10 parsecs or 32.6 light years. (See **astronomic distance units**.) As a first approximation we assume that space is transparent.

When the apparent magnitude, m , of a star is known, together with the parallax, p'' , the absolute magnitude, M , is given by

$$M = m + 5 + 5 \log_{10} p''$$

or

$$M = m + 5 - 5 \log_{10} r$$

where r is the distance in parsecs.

On this basis we find the sun to have an absolute magnitude of $+4.8$; which means that if the sun were at a distance of 10 parsecs from the earth it would appear as an easily-observed, but by no means a conspicuous, star.

2. *Relative Luminosity.* When the absolute magnitudes, M_1 and M_2 , of two stars are

known, the ratio of their luminosities, L_1 and L_2 , is given by

$$\log_{10}(L_2/L_1) = 0.4(M_1 - M_2).$$

3. *Luminosity.* The luminosity of a star is expressed in terms of the sun as a standard. The absolute magnitude of the sun is 4.8 and we have

$$\begin{aligned} \log_{10}(\text{luminosity}) \\ = 0.4(4.8 - \text{absolute magnitude}). \end{aligned}$$

4. *Absorption of Light in Space.* In many regions of space there is distinct evidence of the absorption of light by interstellar matter. Some of the methods for determining the absorption of light in space are discussed elsewhere (cf. **interstellar material**). However it has a distinct bearing in the determination of magnitudes of stars in certain regions of the sky.

5. *Effect of Non-Transparent "Space."* In regions where space absorption is suspected the absolute magnitudes would be calculated by

$$M = m + 5 - 5 \log_{10} r - A(r)$$

in which $A(r)$ is the absorption of light in the direction in which the object is found. This effect is very important in certain problems relative to the structure of the universe. As a case in point let us consider its effect on the apparent magnitudes of the stars and their distances. Let us assume that we have determined the apparent magnitude, m , of a star and from its spectral characteristics we have a good estimate of its absolute magnitude, M . We wish to determine the distance of this star. If we express the space absorption over distance, r , as $A(r)$ we have

$$(1) \quad m = M - 5 + 5 \log_{10} r + A(r).$$

However, if we did not know of space absorption we would have

$$(2) \quad m = M - 5 + 5 \log_{10} D.$$

Subtracting (2) from (1) and dividing by 5,

$$\log_{10} D = \log_{10} r + 0.2A(r)$$

or

$$(3) \quad D = r \cdot 10^{(0.2A(r))}.$$

The importance of space absorption in determining the structure of the sidereal system may be realized by an examination of (3).

Say we have a region where the space absorption is 5 magnitudes, i.e., $A(r) = 5$. If we neglect this factor, the distance of the star will be ten times as great as that obtained by correcting for space absorption.

STELLAR MAGNITUDE. In 1850 Pogson proposed a system for classifying the stars based upon the relative brightnesses as they appear to the human eye. This was based upon an earlier system of Hipparchus and Ptolemy in which the brightest stars were considered as being of magnitude 1 and of the faintest visible stars as of magnitude 6. Adopting the announcement of Herschel that a typical star of magnitude 1 was actually about 100 times as bright as a star of magnitude 6, Pogson proposed that the brightness ratio between successive magnitudes should be $100^{1/5}$, or approximately 2.512.

This leads to a logarithmic scale of brightness and magnitude

$$B_1/B_2 = 2.512^{(m_2 - m_1)}$$

in which m_1 and m_2 are the stellar magnitudes corresponding to brightnesses B_1 and B_2 . This magnitude scale is one of relative brightness and a system of standard magnitudes must be established. The stars in the immediate vicinity of the north pole of rotation of the celestial sphere were selected and a system of primary standards, known as the North Polar Sequence, has been adopted by the International Astronomic Union. Secondary standards have been established by many different observatories covering the entire celestial sphere.

This visual magnitude scale is based upon the sensitivity of the human eye, including the color sensitivity. With the application of photography to astronomical observing it was immediately evident that the color sensitivity of the photographic plate was very different from that of the human eye. If we consider two stars of the same visual brightness one of which is green in color and the other red a photograph will show the green star to be brighter than the red. Furthermore different types of photographic emulsions have different color characteristics. Both visual and photographic magnitudes are in use. For the sake of uniformity the zero points of the two scales are so adjusted that the mean photographic magnitudes of stars of spectral class AO between magnitudes 5.5 and 6.5 shall be

the same as the Harvard visual magnitudes for these stars. This is far from an ideal method for classifying the brightnesses of the stars but is the best available at the present time and is widely used. The difference between the photographic magnitude and the visual magnitude for the same star is known as the color index of the star

$$m_{ptg} - m_{vis} = C.I.$$

Color index is correlated with the spectral type of the star.

STELLAR MASSES. Measures of binary stars, whether visual, spectroscopic, or eclipsing in type, provide knowledge of the masses of the stars. When the period of revolution of a binary star is established we have, from **Kepler's Laws**

$$T = \frac{2\pi a^3}{G\sqrt{m_1 + m_2}}$$

in which G is the constant of gravitation. If T , the period, is expressed in years, a , the mean distance between the components, is given in astronomical units. If the above equation is applied to the sun-earth system, we have $1 = \frac{2\pi}{G}$ neglecting the small mass of the earth in comparison with that of the sun. Therefore

$$m_1 + m_2 = \frac{a^3}{T^2}$$

The observations give a'' (in seconds of arc). To convert this to astronomical units the parallax, p'' , must be known and $a = \frac{a''}{p''}$ astronomical units. Hence, for a binary of measured parallax, the mass of the system in terms of the mass of the sun is given by

$$m_1 + m_2 = \frac{a''^3}{p''^3 T^2}$$

If the individual motions of either or both of the components, relative to neighboring stars, can be determined, the ratio of the individual masses may be determined. (Cf. **stellar dynamic parallaxes.**)

STELLAR MASSES AND MASS FUNCTION. (1) *From Visual Binaries.* Two elements of the orbit of a **binary star** are of importance in determining the masses of the stars: the period T (in years) and the semi-

major axis a (in seconds of arc). The parallax p'' of the system is also important.

From the period, T , we determine the mean angular motion $n = 2\pi/T$. The micrometric measures of the apparent distance between the stars are in seconds of arc. Therefore the semi-major axis, a , is determined in seconds of arc. If the parallax, p'' , is known, the semi-major axis (in astronomical units) $a_1 = \frac{a}{p''}$.

Now the relation between the mean angular velocity, n , and the linear semi-major axis is

$$n^2 a_1^3 = G(m_1 + m_2)$$

in which G is the constant of gravitation and m_1 and m_2 are the masses of the stars. This may be written as

$$\frac{a_1^3}{T^2} = \frac{G}{4\pi^2} (m_1 + m_2)$$

or also

$$m_1 + m_2 = \frac{a^3}{p''^3 T^2} \quad (1)$$

in terms of the sun's mass as unity.

(2) *From Spectroscopic Binaries.* When both spectra are measurable for radial velocities we have the semi-major axes of both stars relative to the center of mass of the system and have at once $a_1 + a_2 = a_0$ expressed in astronomical units. We then have

$$m_1 + m_2 = a_0^3 \left(\frac{365\frac{1}{4}}{T} \right)^2$$

Since one astronomical unit is 149,500,000 km,

$$(m_1 + m_2) \sin^3 i = \left(\frac{a_1 \sin i + a_2 \sin i}{149,500,000} \right)^3 \left(\frac{365\frac{1}{4}}{T} \right)^2$$

in which a_1 and a_2 are expressed in kilometers and T in days.

(3) *The Mass Function.* We now turn to the more common situation in which only one spectrum is useable. From the solution of such a system no determination of the individual masses is possible. However, we can determine two equations containing three unknowns: the two masses m_1 and m_2 and the major axis of the system a_0 .

$$m_1 + m_2 = \left\{ \frac{a_0}{149,500,000} \right\}^3 \left(\frac{365.25}{T} \right)^2$$

$$m_1 + m_2 = 3.993 \times 10^{-20} \frac{a_0^3}{T^2}$$

eliminating a_0 we have

$$\frac{m_2 \sin^3 i}{(m_1 + m_2)^2} = \frac{3.993 \times 10^{-20} (a_1 \sin i)^3}{T^2}$$

The quantity on the right can be evaluated from the orbit determined for the binary. The function on the left is called the mass-function. It is usually given by computers of orbits of spectroscopic binaries.

STELLAR MASS-LUMINOSITY RELATION. The masses of the stars differ much less among themselves than do most of their other properties. Nevertheless as far back as 1924, Eddington discovered that plotting mass against absolute magnitude indicated a definite correlation. With the increase of available data the curve is now well established, although not thoroughly justified by theoretical considerations. However, by the use of this relation one can determine the distance of a **binary star** if the distance between the components can be measured. (Cf. **dynamical parallax.**)

STELLAR MOTIONS. (1) *Proper Motion.* If the position of a star, i.e., right ascension and declination, is accurately determined and then compared with the position determined a number of years ago, a difference in position is found. Most of this difference may be accounted for by **precession, nutation, and aberration**, which is shared in common with all stars in the region. After allowances have been made for these common motions, a residual motion may be found and this is known as the proper motion of the star.

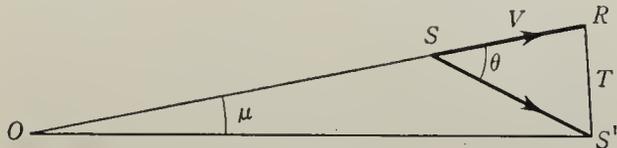
With the application of photography to astronomical research, the proper motions of many thousand stars have been determined by the use of the blink microscope. Two plates, taken with the same astrographic telescope and with the same scale, a number of years apart, are placed under two similar microscope objectives of the same focal length. The images are brought to focus in one ocular. The observer may shift rapidly from one plate to the other by a "blinking" device. If there has been any proper motion of any star in the interval between the taking of the plates a jumping effect is noticed by the observer. The direction and magnitude of the "jump" is the amount of proper motion in the interval. The common motions of precession, nutation, and

aberration are common to all stars in the regions and produce no jumping effect.

Strictly speaking, proper motion should be defined as the motion as it would be observed from the sun, and thus be cleared of the effects of annual parallax. Proper motion is expressed in seconds of arc per year and frequently symbolized by μ .

(2) *Radial Velocity.* The speed with which a star is approaching or receding from the earth may be determined by the application of the **Doppler principle**. The accuracy of determination of this speed may be as small as ± 0.5 km/sec for high dispersion spectrograms of multilined stellar spectra. When this speed is corrected for the component of the earth's rotation in the line of sight, we have what is known as the radial velocity of the star. These radial velocities show a seasonal variation due to the motion of the earth about the sun. A statistical study of these variations provides a method for the determination of the solar parallax.

(3) *Space Motion.* In the figure let O be the position of the observer (in view of the magnitude of the distance from the sun to the star we may also assume that O is the position of the sun).



S is a position of a star and S' is position of the same star a year later. μ is the proper motion of the star and SR is the radial velocity V . Both of these quantities are assumed to be known as well as the parallax p'' of the star.

To find RS' (or T), the tangential velocity, we have μ/p'' which is expressed in astronomic units per year. To express this in linear units we have

$$\frac{1 \text{ ast. unit}}{1 \text{ year}} = \frac{149,450,000 \text{ km}}{31,556,926 \text{ sec}} = 4.738 \text{ km/sec.}$$

Hence

$$T = 4.74\mu/p'' \text{ km/sec.} \quad (1)$$

The radial velocity is determined directly in km/sec and the space velocity is given by

$$v = (V^2 + T^2)^{1/2} \text{ km/sec.} \quad (2)$$

Calling the angle $RSS' = \theta$ we have

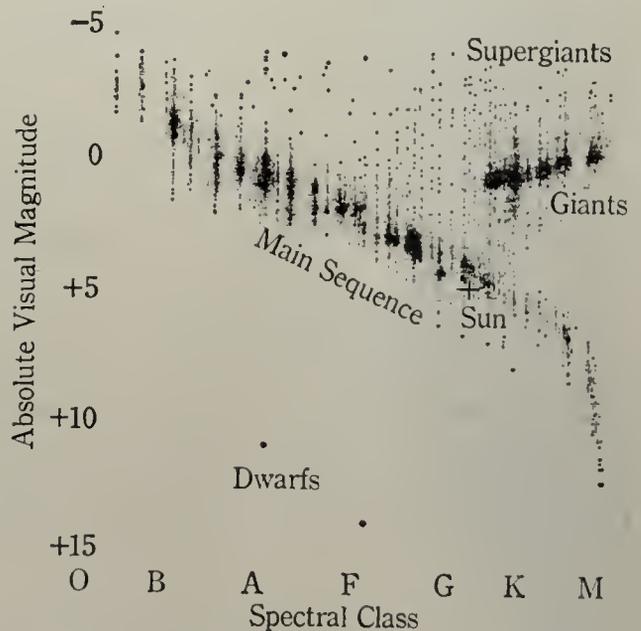
$$V = v \cos \theta; \quad T = v \sin \theta. \quad (3)$$

(Cf. solar motion; moving clusters.)

STELLAR PARALLAX. The angle subtended at a star by the average distance of the earth from the sun (one astronomical unit). Stellar parallax thus bears an inverse relationship the distance of the star from the sun. The parallax of a star may be determined by observing its change of position during the year relative to stars visually close to it, which are really so much further away that their position is not greatly affected (with respect to still other stars) by the earth's annual revolution. (See also **astronomical distance units; parallax.**)

STELLAR SPECTRUM-LUMINOSITY RELATIONS. Not long after the spectral classification of the stars was well under way, it was noted that, while the early type stars, e.g., A and B spectral types, all seemed to have about the same absolute magnitudes, nevertheless the later types, i.e., the K and M types, showed a distinct separation into bright and faint stars. These two classes of brightness became known as giant for the very bright and dwarf for the relatively faint.

As more and more stars were classified as to absolute magnitude and spectral type, the sep-



Absolute magnitudes of stars of the type 1 population arrayed with respect to their spectral classes. The point at magnitude +4.8 and G2 represents the sun.

aration into giant and dwarf stars remained evident. However, the diagram began to show super-giant red stars and also relatively faint early types which were classified as dwarfs.

The color index of a star is defined as the difference between the magnitude as determined photographically, m_{ptg} , and the magnitude determined visually, m_{vis} . Thus we define color index, $C.I. = m_{ptg} - m_{vis}$. A diagram plotting spectral type against color index shows much the same characteristics as the spectrum-luminosity diagram. A vast amount of research is in progress in this general field but, while progress is being made, no definite conclusions should be drawn at this time. (Cf. **stellar spectrum-temperature relations.**)

STELLAR SPECTRUM-TEMPERATURE RELATION. The temperatures of stars of various spectral types have been determined from spectrophotometric observations employing five or six colors. Extensive programs by Kuiper (*Astrophysical Journal* **88** (1938)) and Stebbins & Whitford (*ibid.*, **102** (1945)) indicate the following results:

Spectral Type	Kuiper	Stebbins & Whitford
B0	25,100°K	21,000°K
B5	15,500	16,000
A0	10,700	11,300
F0	7500	7700
F5	6470	6800
G0	6000	5980
G5	5360	5600
K0	4910	5280
M0	3700	3590

Beginning with the F5 type we find differing temperatures between the main-sequence, the sub-giants, and the super-giants. As an example it is found that K2 stars show the following temperatures: main sequence, 4760°K, sub-giants, 4280°K, giants, 3810°K, and super-giants, 3590°K. While these figures are of comparable value, the differences are significant and indicate that we have much to learn about the stars as radiating masses of gas.

STEP FUNCTION. A function $f(x)$ defined over the interval $[a, b]$ is a step function if $[a, b]$ can be divided into a finite number of subintervals in each of which $f(x)$ is constant. The Heaviside unit function is an example.

STEPHAN-BOLTZMANN LAW. See Stefan-Boltzmann law.

STERADIAN. A unit solid angle, which intersects a sphere with center at the vertex of the angle in a region with area equal to the square of the radius. The total solid angle about a point equals 4π steradians since the area of a sphere of unit radius equals 4π .

STEREOGRAPHIC PROJECTION OF A SPHERE ON A PLANE. For a given point P , called the pole, on the surface of a sphere S , and for a given plane M not passing through P , and perpendicular to a diameter through P , the line joining P with a variable point Q on M intersects S in a second point R . This mapping of the points R of the sphere S on the points Q of M is called a stereographic projection of S on M .

STEREO POWER OF A BINOCULAR OPTICAL INSTRUMENT. The magnifying power multiplied by the ratio of the distance between the objective axes to the distance between the axes of the eyepieces.

STEREOSPECTROGRAM. A method of representing spectral data in which the three variables, concentration of solute, optical density, and wavelength of light, are plotted in three dimensions to produce a three-dimensional figure; or else in two dimensions by choosing an oblique axis in addition to the customary x -axis and y -axis.

STERIC FACTOR. See frequency factor; chemical interactions.

STIELTJES INTEGRAL. See Riemann-Stieltjes integral and Lebesgue-Stieltjes integral.

STIFFNESS COEFFICIENT, FACTOR. (1) In slope-deflection and moment-distribution techniques for solving continuous frames, the ratio $k = EI/L$ of flexural rigidity to length of beam or column is called the stiffness factor or coefficient. If the far end of the member is hinged, $\frac{3}{4}k$ is substituted for k . When the flexural rigidity varies along the member, the stiffness coefficient is determined as $\frac{1}{4}$ the ratio of end moment to end rotation with the far end fixed. (2) See oscillation, damped harmonic.

STIGMATIC. (1) An optical instrument is stigmatic if it has the same focal length in all meridians. (See astigmatism.)

(2) A bundle of rays is stigmatic if it is homocentric.

STILB. A unit of brightness of a surface equal to 1 candle/cm².

STIMULI. See color stimulus specifications; matching stimuli; reference stimuli; basic stimulus; cardinal stimuli.)

STIRLING FORMULA. An asymptotic expansion of $\ln \Gamma(z)$, namely,

$$\ln \Gamma(z) = (z - \frac{1}{2}) \ln z - z + \frac{1}{2} \ln 2\pi + \sum_{r=1}^{\infty} \frac{(-1)^{r-1} B_r}{2r(2r-1)z^{2r-1}},$$

where the B_r are the **Bernoulli numbers**. The formula is often given in the equivalent forms

$$n! = n^n e^{-n} (2\pi n)^{\frac{1}{2}} \left(1 + \frac{1}{12n} + 0(n^{-2}) \right).$$

If n is a large number

$$\log n! = (n + \frac{1}{2}) \log n - n + \frac{1}{2} \log 2\pi + \frac{1}{12n} - 0(1/n^3),$$

where the last term is of the order of $1/n^3$. The last form is approximated by:

$$\log n! = n \log n - n.$$

STIRLING INTERPOLATION FORMULA. In the notation of **difference operators**

$$E^u = 1 + u\mu\delta + \frac{u^2}{2!} \delta^2 + \frac{u(u^2 - 1^2)}{3!} \mu\delta^3 + \frac{u^2(u^2 - 1^2)}{4!} \delta^4 + \frac{u(u^2 - 1^2)(u^2 - 2^2)}{5!} \mu\delta^5 + \dots$$

STIRLING NUMBERS. The coefficients $S_\nu^{(n)}$ in the expansion (see **difference operators**)

$$u^{(n)} = S_n^{(n)} u^n + S_{n-1}^{(n)} u^{n-1} + \dots + S_1^{(n)} u.$$

For $\nu > n$, $S_\nu^{(n)} = 0$. These are also called Stirling numbers of the first kind, those of the second kind being the coefficients $s_r^{(n)}$ of the reverse expansion

$$u^n = S_n^{(n)} u^{(n)} + S_{n-1}^{(n)} n^{(n-1)} + \dots + S_1^{(n)} u^{(1)}.$$

It follows from these definitions that

$$S_\nu^{(n)} = \frac{d^\nu u^{(n)} / du^\nu}{\nu!} \Big|_{u=0}, \quad S_\nu^{(n)} = \frac{\Delta^\nu u^n}{\nu!} \Big|_{u=0}.$$

Their importance is due in part to their occurrence in the following **differentiation formulas**:

$$\theta^\nu = S_\nu^{(\nu)} \Delta^\nu + \frac{S_\nu^{(\nu+1)} \Delta^{\nu+1}}{\nu + 1} + \frac{S_\nu^{(\nu+2)} \Delta^{\nu+2}}{(\nu + 1)(\nu + 2)} + \dots$$

$$\Delta^\nu = S_\nu^{(\nu)} \theta^\nu + \frac{S_\nu^{(\nu+1)} \theta^{\nu+1}}{\nu + 1} + \frac{S_\nu^{(\nu+2)} \theta^{\nu+2}}{(\nu + 1)(\nu + 2)} + \dots$$

STOCHASTIC. A word implying the presence of a **random variable**, e.g., stochastic variation is equivalent to the variation of a set of quantities under probability distributions, stochastic equations contain some variables which are random, and so forth.

Stochastic convergence means convergence in probability, e.g., x_n converges to a stochastically if, for any positive ϵ , $P\{|x_n - a| > \epsilon\}$ tends to zero as n tends to infinity. Two variables are stochastically independent if they have a joint probability distribution that is the product of the marginal distribution.

STOCHASTIC MATRIX. See **matrix, stochastic**.

STOCHASTIC PROCESS. A family of random variables $\{x(t)\}$, depending on some parameters t , often a time variable.

STOCHASTIC PROCESS, STATIONARY. See **stationary stochastic process**.

STOCKMAYER POTENTIAL. This intermolecular potential is of the form

$$\phi(r, \theta_a, \theta_b, \phi_b - \phi_a) = 4\epsilon \left[\left(\frac{\sigma}{r} \right)^{12} - \left(\frac{\sigma}{r} \right)^6 \right] - \frac{\mu_a \mu_b}{r^3} g(\theta_a, \theta_b, \phi_b - \phi_a). \quad (1)$$

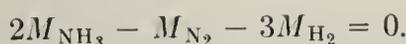
This function is a superposition of a **Lennard-Jones (6, 12) potential** and the interaction of two point dipoles of strengths μ_a, μ_b . The meaning of the function g is the same as for the **Keesom potential**.

STOICHIOMETRIC COEFFICIENT. A chemical reaction can be written (see **conservation of mass in closed systems**) in the form

$$\sum_{\gamma} \nu_{\gamma} M_{\gamma} = 0.$$

M_{γ} is the molecular mass of the component γ . The coefficients ν_{γ} which can usually be expressed as a set of whole numbers, either positive or negative, are the stoichiometric coefficients of the reactants and products. They are positive if the substance appears in the reaction, and negative if the substance disappears.

For example the NH_3 synthesis can be written:



The stoichiometric coefficients of NH_3 , N_2 and H_2 are respectively 2, -1 and -3 .

STOICHIOMETRIC EQUATION. See **conservation of mass in closed systems**.

STOKES FLOW. Flow at small **Reynolds number**, i.e., flow in which the inertia forces are negligible compared with the viscous forces, which is not turbulent, and which is in effect steady. Examples are

(1) The motion with speed U of a small sphere of radius a in liquid of viscosity μ . The drag force is $6\pi\mu aU$, e.g., cloud droplets falling through air. The formula is valid for Reynolds numbers less than about 5.

(2) **Convection cells** in a shallow layer of viscous liquid which is heated below just sufficiently to produce steady overturning motion.

(3) The **Hele-Shaw cell**.

The equations governing the motion are written

$$\text{grad } p = \mu \nabla^2 \mathbf{v}$$

or

$$\frac{\partial p}{\partial x_i} = \mu \frac{\partial u_i}{\partial x_j \partial x_j},$$

according to the notation used.

STOKES LAWS. (1) See **Stokes flow** (1).

(2) The wavelength of **luminescence** excited by radiation is always greater than that of the exciting radiation. (See **antistokes lines**.)

STOKES LINES. Raman lines (see **Raman effect**) displaced towards longer wavelengths with respect to the incident line.

STOKES POLARIZATION THEOREM. A theorem which establishes that the following seven polarization states are mutually exclusive and that any complex radiation belongs to one of the seven states.

- (1) Unpolarized light
- (2) Plane-polarized light
- (3) Circularly-polarized light
- (4) Elliptically polarized light
- (5) Unpolarized plus plane polarized light
- (6) Unpolarized plus circularly polarized light
- (7) Unpolarized plus elliptically polarized light

For example, a mixture of plane-polarized, plus elliptically-polarized, plus unpolarized light belongs to state 7.

STOKES THEOREM. The surface integral of the **curl** of a **vector function** equals the **line integral** of that function around a closed curve bounding the surface

$$\int_S \nabla \times \mathbf{V} \cdot d\mathbf{S} = \oint \mathbf{V} \cdot d\mathbf{s}.$$

This statement is valid only if the contour is traversed in taking the line integral in the direction of turning of a right-handed screw that progresses in the positive $d\mathbf{s}$ direction (assuming a right-handed coordinate system). When the surface S is part of a closed surface, it is conventional to take the positive $d\mathbf{s}$ direction as outward. If the components of \mathbf{V} in rectangular Cartesian coordinates are u, v, w and the **direction cosines** of the normal to $d\mathbf{S}$ are λ, μ, ν the theorem may also be given as

$$\int_S \left[\lambda \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z} \right) + \mu \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial x} \right) + \nu \left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \right] dS = \oint (u dx + v dy + w dz).$$

STOKES THEOREM, FOUR DIMENSIONAL. See **Minkowski world**.

STOP. A component of an optical system functioning as an aperture.

STOPPING POWER. A measure of the effect of a substance upon the kinetic energy of a charged particle passing through it. The *linear stopping power* S_l is the energy loss per unit distance, and is given by $S_l = -dE/dx$, where E is the kinetic energy of the particle

and x is the distance traversed in the medium. The *mass stopping power* S_m is the energy loss per unit surface density traversed, and is given by $S_m = S_l/\rho$, where ρ is the density of the substance. The *atomic stopping power* S_a of an element is the energy loss per atom, per unit area normal to the particle's motion, and is given by $S_a = S_l/n = S_m A/N$; where n is the number of atoms per unit volume, N is the Avogadro number, and A is the atomic weight. The *molecular stopping power* of a compound is similarly defined in terms of molecules; it is very nearly if not exactly equal to the sum of the atomic stopping powers of the constituent atoms. The *relative stopping power* is the ratio of the stopping power of a given substance to that of a standard substance, commonly aluminum, oxygen or air. The *stopping equivalent* for a given thickness of a substance is that thickness of a standard substance capable of producing the same energy loss. The *air equivalent* is the stopping equivalent in terms of air at 15°C and 1 atm as the standard substance. The term *equivalent stopping power* is not clearly defined, but sometimes is used synonymously with relative stopping power and sometimes with stopping equivalent.

STORAGE COMPLIANCE. See **compliance, storage.**

STORAGE MODULUS. See **modulus, storage.**

STORAGE UNIT. That part of a **stored-program computer** which stores the **code** or the numerical data or both. In most current machines the storage unit is divided into cells, usually of between 30 and 50 binary elements, these cells are ordered and the ordinal number of the cell is its address. Generally, too, a given cell can be used to store either data or commands, a command including an operational code symbol and one or more address symbols for locating the operands. Operation can begin with any cell, the contents being interpreted as one or more commands, and will usually proceed in sequence until a transfer or jump command interrupts, although in some machines every command specifies also the location of the next command to be executed.

STORED-PROGRAM COMPUTER. A computer that is capable of storing instructions as well as data and of carrying out a sequence of

operations as prescribed in advance. The modern high-speed general-purpose computers are **digital computers** of this type. Any such machine will possess a basic repertory of certain arithmetic and logical operations, each designated by a certain symbol, basically a binary sequence. A sequence of such symbols constitutes a **code** which, introduced into the **storage unit** along with the necessary data will direct the machine accordingly.

STÖRMER METHOD (FOR THE NUMERICAL SOLUTION OF AN ORDINARY DIFFERENTIAL EQUATION OF SECOND ORDER). For the general equation

$$y'' = f(x, y, y'),$$

this method uses the **predictor**

$$y_{\nu+1} = 2y_{\nu} - y_{\nu-1} + h^2(f_{\nu} + \frac{1}{12}\nabla^2 f_{\nu} + \frac{1}{240}\nabla^4 f_{\nu} + \frac{1}{40}\nabla^6 f_{\nu} + \dots).$$

It is useful in particular when y' does not occur explicitly in f . The equations apply also when y and f are vectors. See **difference operators** for the definition of ∇ . (See Murphy, *Ordinary Differential Equations and Their Solution*, D. Van Nostrand Co., Inc., Princeton, 1960.)

STOSSZAHLANSATZ. An assumption made by Boltzmann to prove his **H-theorem**. The assumption is that there is no correlation between velocities and positions of different particles so that the number of collisions undergone by a particle is independent of its position or velocity and only depends on the cross section for collisions. (For further discussion, see **molecular chaos**.)

STP. See **normal temperature and pressure; standard temperature and pressure.**

STRAGGLING. The random variation or fluctuation of a property associated with ions of a given kind in passing through matter. *Range straggling* is the variation in the range of particles that are all of the same initial energy. *Angle straggling* is the variation in the direction of motion of particles after passing through a certain thickness of matter, the paths of the particles initially being parallel. *Statistical straggling* is that variation in range, ionization or direction which is due to fluctuations in the distance between collisions in the stopping medium and in the energy loss and deflection angle per collision. *Instrumental*

straggling is that additional straggling which is due to such instrumental effects as noise, gain instability, source thickness, and poor geometry.

STRAIN. The deformation produced in a solid as the result of **stress**. (See entries following.)

STRAIN DEVIATOR. The shear strain components of the strain tensor ϵ_{ij} , given by:

$$e_{ij} = \epsilon_{ij} - \frac{1}{3}\epsilon_{kk}\delta_{ij}.$$

STRAIN ELLIPSOID. Using the **indicial notation**, suppose that in the deformation of a body, a generic particle initially at X_i in a rectangular Cartesian coordinate system x moves to x_i in the same system. Let dS and ds be the lengths in the undeformed and deformed states of the body of a linear element located at this particle. Using the **summation convention** and the notation

$$G_{ij} = \frac{\partial x_k}{\partial X_i} \frac{\partial x_k}{\partial X_j},$$

$$(ds)^2 = G_{ij}dX_i dX_j.$$

If dX_i are regarded as the current coordinates in a rectangular Cartesian coordinate system with axes parallel to those of the system x and origin at X_i , then any ellipsoid obtained by taking $(ds)^2$ constant is called a *reciprocal strain ellipsoid*. Again, using the summation convention and the notation

$$g_{ij} = \frac{\partial X_k}{\partial x_i} \frac{\partial X_k}{\partial x_j},$$

$$(dS)^2 = g_{ij}dx_i dx_j.$$

If dx_i are regarded as the current coordinates in a rectangular Cartesian coordinate system with axes parallel to those of the system x and origin at x_i , then any ellipsoid obtained by taking $(dS)^2$ constant is called a strain ellipsoid. For infinitesimal deformations,

$$\frac{1}{2}[(ds)^2 - (dS)^2] = e_{ij}dX_i dX_j,$$

where e_{ij} are the infinitesimal strain components in the coordinate system x . The quadric obtained by taking $\frac{1}{2}[(ds)^2 - (dS)^2]$ constant and regarding dX_i as current rectangular Cartesian coordinates, as above, is called the *strain quadric* or *elongation quadric*.

STRAIN ENERGY, DISTORTIONAL. See **distortional strain energy**.

STRAIN-ENERGY FUNCTION. For quasi-static deformations of an elastic material, the work done by the external forces may be expressed as a volume integral of the type

$$\int W dV,$$

where dV is a generic element of volume of the undeformed body and W is a single-valued function of the displacement gradients, the integration being carried out over the volume of the body in its undeformed state. For any deformation of a perfectly-elastic material, the excess of the work done in the deformation by the external forces, over the increase of kinetic energy, may be similarly expressed, W is called the *strain-energy function for the material*. It may be regarded physically as the mechanical energy stored per unit volume (measured in the undeformed state) of the material. Also called *strain-energy density* or *strain-energy density function*.

STRAIN ENERGY, MAGNETIC. See **magnetic strain energy**.

STRAIN FIELD (IN A BODY). The **tensor field** defined by the **strain tensor** throughout the body.

STRAIN FOR FINITE DEFORMATIONS.

A number of symmetric, second-order tensors, the components of which are defined in terms of the displacement gradients, have been called the *strain tensor*, or more loosely *strain*, and no particular one has gained general acceptance. If in the deformation of a body, a generic particle initially at X_i ($i = 1, 2, 3$) in the rectangular Cartesian coordinate system x moves to x_i in the same system, then the tensor with com-

ponents $\frac{1}{2}\left(\frac{\partial x_k}{\partial X_i} \frac{\partial x_k}{\partial X_j} - \delta_{ij}\right)$ in the system x ,

where δ_{ij} denotes the **Kronecker delta**, is called the *Lagrangian strain tensor*. Here the **indicial notation** and **summation convention** is used. The components of the tensor in any coordinate system are called the *Lagrangian strain components* in that coordinate system. A variety of other tensors, which have the same principal directions and eigenvalues which are simple functions of the eigenvalues of this tensor, have been called *Lagrangian strain tensor*. The tensor with components

$\frac{1}{2}\left(\frac{\partial X_k}{\partial x_i} \frac{\partial X_k}{\partial x_j} - \delta_{ij}\right)$ in the system x is called

the *Eulerian strain tensor* and its components in any coordinate system are called the *Eulerian strain components* in that system. Again, a variety of other tensors, which have the same principal directions and eigenvalues which are simple functions of the eigenvalues of this tensor have been called *Eulerian strain tensor*.

STRAIN FOR INFINITESIMAL DEFORMATIONS. Using the **indicial notation** and **summation convention**, suppose a generic point of a body initially at x_i ($i = 1, 2, 3$) in a rectangular Cartesian coordinate system x undergoes a displacement with components u_i in this system. Then the relative displacement du_i of two neighboring points of the body initially at $x_i + dx_i$ and x_i is given by

$$du_i = \frac{\partial u_i}{\partial x_j} dx_j$$

$$= \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) dx_j + \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right) dx_j.$$

With the notation

$$du'_i = e_{ij} dx_j, \quad e_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

and

$$du''_i = \tilde{\omega}_{ij} dx_j, \quad \tilde{\omega}_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right),$$

where du'_i is the relative displacement of the neighboring points in a pure homogeneous deformation and, provided we make the assumption that the displacement gradients are negligibly small compared with unity, du''_i denotes the relative displacement of the neighboring points in a rigid rotation. e_{ij} are called the *strain components* or *components of strain* in the coordinate system x at the point x_i of the body. The components e_{ij} ($i = j$) are called the *normal components* of strain, and the components e_{ij} ($i \neq j$) are called the *tangential* or *shear components* of strain. $\tilde{\omega}_{ij}$ are called the *components of rotation*. [In early works, e_{11} , e_{22} , e_{33} , $2e_{23}$, $2e_{31}$ and $2e_{12}$ are often referred to as the strain components, e_{11} , e_{22} , e_{33} then being called the *normal* and $2e_{23}$, $2e_{31}$, $2e_{12}$ the *tangential*, or *shear*, components of strain.] e_{ij} are the components in the coordinate system x of a symmetric second-order Cartesian tensor called the *infinitesimal strain tensor* or, more loosely, the *strain tensor*. $\tilde{\omega}_{ij}$ are the components in the coordinate system x of a skew-symmetric second-order tensor called the rotation tensor.

The second-order covariant, contravariant and mixed tensors, which have as their components in the rectangular Cartesian coordinate system x the quantities e_{ij} , are called the *covariant*, *contravariant* and *mixed infinitesimal strain tensors*, respectively or, more loosely, the *covariant*, *contravariant* and *mixed strain tensors*, respectively.

STRAIN GENERALIZED. A quantity or set of quantities measuring deformation such that the generalized stress (see **stress, generalized**) multiplied by the generalized strain gives work. Sometimes a distinction is made between **generalized displacement** and strain, restricting the latter to deformations per unit of length, area or volume, as for example, curvature of plates, beams, or shells, or angle of twist per unit of length.

STRAIN HARDENING. See **work hardening**.

STRAIN INVARIANT. An invariant under some transformation group, usually the orthogonal group, of the **strain tensor**. The term strain invariants is often used to describe three basic invariants of the strain tensor under the orthogonal transformation group, in terms of which any invariant of the strain tensor can be expressed as a single-valued function. The choice of these three invariants of the strain tensor is to some extent ambiguous. For the infinitesimal strain tensor, they are usually taken as e_{ii} , $\frac{1}{2}(e_{ii}e_{jj} - e_{ij}e_{ji})$ and $|e_{ij}|$, where e_{ij} denotes the components of the infinitesimal strain tensor in a rectangular Cartesian coordinate system and the **indicial notation** and **summation convention** are used.

STRAIN, MOHR CIRCLE FOR. See **Mohr circle for strain**.

STRAIN, PHYSICAL COMPONENTS OF. See **physical components of strain**.

STRAIN, PLANE. See **plane strain**.

STRAIN, STATE OF. See **state of strain**.

STRANGENESS NUMBER. A quantum number describing some, as yet not understood (1960), property of fundamental particles. The strangeness number of a particle = 2 (charge - Isotopic spin component) - Number of nucleons represented by the mass of the particle. The neutron, proton, and π -mesons have a strangeness number of zero. Other particles have numbers from +1 to -2.

STRATIFICATION. A device used in sampling to increase representativity and reduce sampling error of estimation. The population under inquiry is divided into groups or strata according to some known criterion and the sample allocated over the strata by some convenient rule. Stratification may be geographical but is not necessarily so, e.g., a population may be stratified by age.

STREAM FUNCTION. (1) If the flow is in planes parallel to the (x,y) plane the equation of continuity for incompressible flow is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

and

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}$$

where ψ is called *Earnshaw's stream function*. *Lagrange's stream function* is equal to $-\psi$ as defined here. In irrotational motion $\nabla^2 \psi = 0$ and if ϕ is the velocity potential

$$\phi + i\psi = f(x + iy).$$

If the boundary is cylindrical and is rotating with angular velocity Ω about the origin and has velocity of translation (U, V) then on the boundary

$$\psi = Uy - Vx - \frac{1}{2}\Omega r^2 + \text{const.}$$

(2) If the flow has an axis of symmetry, in spherical polar coordinates (r, θ, λ) with corresponding components of velocity (u, v, w) the equation of continuity for incompressible flow independent of λ is

$$\frac{\partial}{\partial r}(r^2 u \sin \theta) + \frac{\partial}{\partial \theta}(rv \sin \theta) = 0$$

and

$$u = -\frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta}, \quad v = \frac{1}{r \sin \theta} \frac{\partial \psi}{\partial r}$$

where ψ is called *Stokes's stream function*.

More generally in any system in which the equation of continuity has only two terms there exists a stream function which is constant along any streamline.

STREAMING, NEUTRON. The rectilinear motion of neutrons through voids or channels in a chain-reacting system.

STREAMLINE. In steady motion the **lines of flow** are also the paths of the fluid particles and are then called streamlines.

Streamline flow is steady non-turbulent flow, and is characteristic of flow at low **Reynolds numbers**, of irrotational flow, and of many inviscid flows.

In two dimensional flow the **stream function** is constant along a streamline.

Free streamlines are the boundaries of the wake of a body, when the wake is occupied by fluid at rest relative to the body. Free streamlines are isobars and are sections of the **vortex sheet** which bounds the wake. The flow outside the wake is usually assumed to be irrotational. The theory of free streamlines is unrealistic in that the vortex sheet is unstable.

STREAM TERMS. See **Boltzmann transport equation**.

STREAM TUBE, FLOW OF GAS IN. See **nozzle or duct, flow of gas in**.

STREHL DEFINITION. To deal with the effect of small focusing errors in the diffraction pattern of a point source, Zernike introduced the notion of Strehl definition. If P_o is the intersection of the principal ray with a smooth receiving surface S in the image layer of an optical system; if $p/4\pi$ is the number of fringes of defocusing of S and q/π the number of fringes of lateral displacement of a point P relative to P_o then $I(p,q)$, the normalized intensity at P , is the intensity at P as a fraction of that at the center of an aberration-free image. The maximum of I under the constraint $p = 0$ is the Strehl definition of the image in the surface S .

STRENGTH, ULTIMATE. See **ultimate strength**.

STRESS. A stress is the quantitative expression of a condition within an elastic material due to deformation, or strain, brought about by external forces, inequalities of temperature, or otherwise. Its measure is always the ratio of a force to an area. By some, stress is interpreted as a force distributed over an area, and the above ratio is called the "unit of stress." (See following entries.)

STRESS, ALLOWABLE OR WORKING. Most structural and machine elements are designed so that at their working or design load a nominal shear or normal stress calculated on

the basis of elastic behavior does not exceed a specified value. These specified or allowable or working stresses are usually determined as some fraction of the **yield** or **ultimate strength** of the material. Although appropriate for fatigue, in general the allowable stress method leads to a wide variation in the true **factor of safety**.

STRESS(ES), ASSEMBLY. See **assembly stresses**.

STRESS, CIRCUMFERENTIAL OR HOOP. The circumferential or hoop stress in a thin pressurized sphere is $pD/4t$ where p is the interior pressure, D , the diameter and t , the wall thickness. In a circular cylinder the hoop stress is $pD/2t$. (See **autofrettage**.)

STRESS, COMBINED. A more complicated state of stress than uniaxial tension or compression. When one principal stress is zero and the other two are non-zero, the term **biaxial stress** is used; when all three are non-zero, the term **tri-axial** is employed.

STRESS DEVIATOR. The shear stress components of the stress tensor σ_{ij} , given by:

$$s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma_{kk}\delta_{ij}.$$

Plastic flow is essentially governed by deviatoric stress components.

STRESS, DIRECT. Another term for **normal stress**. In application to trusses, the word stress may be used for the axial force in a member.

STRESS EFFECT, NORMAL. See **normal stress effect**.

STRESSES, RESIDUAL. See **residual stresses**.

STRESS FIELD (IN A BODY). The tensor field defined by the stress tensor throughout the body.

STRESS FUNCTION. In certain types of problem in the mechanics of continua, it is possible to solve the equations of equilibrium, or of motion, for the stress components in terms of a smaller number of functions of position, called stress functions, which satisfy an appropriate number of differential equations. As examples, there are the **Airy stress functions**, **Maxwell stress functions**, **Morrera stress**

functions, **Papkovich-Neuber stress functions** and **Galerkin stress functions**.

STRESS, GENERALIZED. Quantities such as membrane force per unit length of middle surface, or plate moment per unit length are called generalized stresses. Multiplied by generalized strains, they give work per unit of length, area, or volume.

STRESS, HYDROSTATIC. A state of positive or negative pressure without shear stress on any plane.

STRESS, MOHR CIRCLE FOR. See **Mohr circle for stress**.

STRESS INVARIANT. An invariant under some transformation group, usually the orthogonal group, of the stress tensor. The term stress invariants is often used to describe three basic invariants of the stress tensor under the orthogonal group, in terms of which any invariant of the stress tensor can be expressed as a single-valued function. The choice of these three invariants of the stress tensor is to some extent ambiguous, but they are usually taken as σ_{ii} , $\frac{1}{2}(\sigma_{ii}\sigma_{jj} - \sigma_{ij}\sigma_{ji})$ and $|\sigma_{ij}|$, where σ_{ij} denotes the components of the stress tensor in a rectangular Cartesian coordinate system and the **indicial notation** and **summation convention** are used.

STRESS, NORMAL. See **normal stress**.

STRESS, OCTAHEDRAL. The shear stress on the octahedral plane which is equally inclined to the three principal axes of stress. A limitation on its value as proposed by Nadai is equivalent to a limitation on the shear strain or **distortional energy**, and so is the equivalent of the Maxwell-Huber-Mises criterion for the onset of plastic deformation.

STRESS, PHYSICAL COMPONENTS OF. See **physical components of stress**.

STRESS, PLANE. See **plane stress**; **generalized plane stress**.

STRESS(ES), PRINCIPAL. At any point of a body in which a stress field exists, a rectangular Cartesian coordinate system can be chosen in which the tangential components of the stress (shearing stresses) at that point vanish. The normal components of the stress in this system are the principal stresses, or *principal components of stress*, at the point. One of

them is the maximum normal stress for any plane at the point and one is the minimum. They are the **eigenvalues** of the **stress tensor** at the point. The directions of the axes of this rectangular Cartesian coordinate system are the *principal directions of stress* or *principal axes of stress*, at the point, and its coordinate planes are the *principal planes of stress* at the point. (See **Mohr circle**.)

STRESS QUADRIC. The quadric equation in a rectangular Cartesian coordinate system x

$$\sigma_{ij}x_i x_j = \text{constant},$$

where σ_{ij} are the stress components in the system x and the **indicial notation** and **summation convention** are used.

STRESS RAISER. A notch, hole, inclusion, or other defect is called a stress raiser because it causes stress concentration or increase.

STRESS, RESIDUAL. See **residual stresses**.

STRESS, SECONDARY. See **secondary stresses**.

STRESS, SHEAR. See **stress tensor**.

STRESS-STRAIN CURVE. A stress-strain curve is a graphical representation of the relation between unit **stress** and unit **deformation** in a stressed body as a gradually increasing load is applied. (See **modulus, secant**.)

STRESS-STRAIN CURVES, ISOCHRONOUS. A set of pseudo **stress-strain curves** obtained from creep data at constant stress. At a given time each creep curve shows a total strain at the test stress, and this strain plotted against the corresponding stress gives an isochronous stress-strain curve.

STRESS TENSOR. The three stress **vectors** acting on three elements of area at a point perpendicular to the axes of an arbitrary rectangular Cartesian coordinate system have nine components which may be regarded as the components of a second-order Cartesian tensor which is called the *Cartesian stress tensor*. Unless an applied body couple is acting at the point, this tensor is symmetric. Its components in any rectangular Cartesian coordinate system are called the *components of stress*, or *stress components*, in that system. Using the **indicial notation**, suppose the components of the stress vector acting on the element of area normal to the x_j -axis of a rec-

tangular Cartesian coordinate system x are σ_{ij} ($i = 1, 2, 3$). Then σ_{11} , σ_{22} , σ_{33} are called the *normal* components of stress. The components σ_{ij} ($i \neq j$) are called the *tangential*, *shearing* or *shear* components.

The second-order covariant and contravariant tensors, which have these quantities as their components in a rectangular Cartesian coordinate system, are called the *covariant stress tensor* and *contravariant stress tensor*, respectively. Similarly, the mixed tensor, covariant of order 1 and contravariant of order 1, which has as its components in a rectangular Cartesian coordinate system the components of the Cartesian stress tensor in that coordinate system, is called the *mixed stress tensor*.

STRESS TRAJECTORY. A line whose tangent at each point is in the direction of a principal stress. (See **stress, principal**.)

STRESS VECTOR. The force, per unit area, with which the material on one side of an element of area is a body acts on that on the other side.

STRICTION, LINE OF. See **ruled surface**.

STRING FUNICULAR POLYGON. See **funicular polygon**.

STRONG AXIS. See **axis, strong**.

STRONG ELECTROLYTES. When certain substances such as common salt are dissolved in water, the solute is composed largely of free ions, such as Na^+ and Cl^- in the case of common salt. Such substances are called strong electrolytes. On the contrary substances which contain in solution an important fraction of undissociated neutral molecules are called *weak electrolytes*. (See **limiting law for strong electrolytes**.)

STROUHAL NUMBER. A dimensionless parameter relating frequency of shedding of **vortices** to the wind velocity and characteristic dimension,

$$\text{Strouhal Number} = \frac{\omega d}{2\pi V},$$

where ω is the frequency of vortex shedding in radians per second, d is the diameter of the missile or structure in feet and V is the velocity of air flow in feet per second.

STRUCTURE, DETERMINATE. See **determinate structure**.

STRUCTURE FACTOR (STRUCTURE AMPLITUDE). A factor which depends upon the effect of crystal structure on the intensity of diffracted radiation, and is defined as the ratio of the amplitudes of the radiation scattered by a single unit cell to that scattered by a free electron. It is sometimes considered as the product of a geometrical structure factor s and the combined atomic scattering factors Σf . If, for a given direction of scattering, the phase difference of the i^{th} atom in the unit cell is ϕ_i , and the **atomic scattering factor** for that atom is f_i , then the structure factor F is given by the expression

$$F^2 = (\Sigma f_i \cos \phi_i)^2 + (\Sigma f_i \sin \phi_i)^2.$$

STRUT. A structural member subjected to direct compression. Although often used interchangeably with the word column, strut denotes most often either a short column or a secondary or bracing member.

STUDENT'S DISTRIBUTION. If x is an observation from a normal population with mean zero, and if s^2 is an efficient estimate of the population variance based on n degrees of freedom, the quantity $t = x/s$ has the distribution

$$\frac{\Gamma(\frac{1}{2}n)}{(n\pi)^{\frac{1}{2}}\Gamma(\frac{1}{2}n - 1)} \left(1 + \frac{t^2}{n}\right)^{-\frac{1}{2}(n+1)}, \quad -\infty < t < \infty.$$

The t distribution is the basis for significance tests and confidence or fiducial statements referring to means, regression coefficients, etc.

STURM-LIOUVILLE EQUATION. A differential equation, conveniently written in operator form as

$$L(u) + \lambda wu = 0$$

where the differential operator $L(u) = (pu) - qu$ and p, q are functions of the independent variable x . Those values of the parameter λ for which solutions of the equation exist satisfying certain boundary conditions are called **eigenvalues** of the equation (together with its **boundary conditions**).

The function $w(x)$ is called a weighting function, since the eigenfunctions u_n, u_m, \dots (solutions corresponding to various eigenvalues) are orthogonal with $w(x)$ for weighting function, thus:

$$\int w(x)u_n(x)u_m(x)dx = 0,$$

where the integration is taken over the interval determined by the boundary conditions. Many eigenvalue problems in mathematical physics are of this form. The above equation is a particular example, the most important and the first considered, of a Sturm-Liouville equation. (See **Sturm-Liouville problem**.)

STURM-LIOUVILLE PROBLEM. A differential equation

$$\sum_{i=0}^n (-1)^i [p_i y^{(i)}]^{(i)} = \lambda \rho y$$

of order $2n$ in y as an unknown function of x , together with $2n$ boundary conditions of the form

$$\sum_{i=0}^{2n-1} \{c_{ij}y^{(i)}(a) + d_{ij}y^{(i)}(b)\} = 0, \quad j = 1, 2, \dots, 2n,$$

where the coefficients ρ and p_i are functions of x , while the a and b are the end points of the interval $[a, b]$ and the c_{ij}, d_{ij} are constants. It is customary to write Ly for the left side of the given differential equation, so that L is a linear homogeneous operator. A function $u(x)$ which satisfies the boundary conditions is called admissible (if it has a continuous $2n^{\text{th}}$ derivative) and if, for every pair u, v of admissible functions, we have $(u, Lv) = (Lu, v)$, where the parentheses denote scalar product, that is $(u, v) = \int_a^b uv dx$, then the given problem is said to be self-adjoint. Since, by integration by parts we see that

$$(u, Lv) - (Lu, v) = \sum_{i=0}^n \sum_{j=0}^n (-1)^{i+j} \times \{u^{(j)}[p_i v^{(i)}]^{(i-1-j)} - v^{(j)}[p_i u^{(i)}]^{(i-1-j)}\} \Big|_a^b,$$

it is clear that the problem will be self-adjoint if the boundary conditions are such as to guarantee the vanishing of this expression, which, being bilinear in the derivatives of u and v , is called the bilinear concomitant of the problem. An admissible non-identically-vanishing function u which satisfies $Lu = \lambda \rho u$ is called an **eigenfunction** of the problem and a value of λ for which such a function exists is the corresponding **eigenvalue**. The fundamental importance of the self-adjointness of a problem lies in the fact that any two of its eigen-

functions u_m, u_n (corresponding to distinct eigenvalues λ_m, λ_n) are orthogonal with weighting function ρ ; that is,

$$\int_a^b u_m u_n dx = 0,$$

so that the coefficients in the (generalized) **Fourier expansion** of an arbitrary function can be readily calculated.

STURM SEQUENCE. A sequence of polynomials associated with a given polynomial possessing the properties required for applying the **Sturm theorem**, that no consecutive pair vanish simultaneously, and that when one vanishes the adjacent ones are opposite in sign.

STURM THEOREM (FOR LOCATING ROOTS OF AN ALGEBRAIC EQUATION $P(x) = 0$). Taking $P = P_0, P' = P_1$, apply the **Euclidean algorithm** for the highest common factor of P_0 and P_1 . In the **Sturm sequence** $P_0, P_1, P_2 \dots$ evaluated at any r , let V_r represent the number of variations in sign. Then the equation has exactly V_r distinct roots exceeding r . While the application of **Budan's theorem** is simpler, it provides less information in general. Moreover, if P_i is not a constant, P_i being the last term in the sequence, all multiple roots satisfy $P_i = 0$. (See **algebraic equations**.)

SUBGRAPH. A **graph** containing a subset of the **edges** of the original graph.

SUBGRAPH COMPLEMENT. The complement of a **subgraph** is the set of all **elements** of the graph not in the subgraph.

SUBGRAPH, MAXIMAL CONNECTED. A **connected graph** is essentially one that is in "one piece." An unconnected graph must therefore be decomposable into several **connected** "pieces." This notion can be made precise by using the notion of a maximal connected subgraph.

Let two **vertices** β_1 and β_2 of a graph G be defined to be equivalent if there exists a **path** between them and denote this relation symbolically by $\beta_1 \doteq \beta_2$. Clearly if

- (a) $\beta_1 \doteq \beta_2$ and $\beta_2 \doteq \beta_3, \beta_1 \doteq \beta_3$;
- (b) $\beta_1 \doteq \beta_1$;
- (c) If $\beta_1 \doteq \beta_2, \beta_2 \doteq \beta_1$.

Thus the relation is an equivalence relation in the strict mathematical sense because it is (a) transitive, (b) reflexive and (c) symmetric. The collection of all vertices of G is partitioned into disjoint equivalence classes such that two vertices belong to the same class if and only if they are equivalent. If G is finite, the number of these classes is also finite and can be enumerated S_1, S_2, \dots, S_p . A little thought reveals that the set of all edges whose two endpoints are vertices in S_r ($r = 1, 2, \dots, P$) is a **connected subgraph** G_r of G . Moreover G_r ($r = 1, 2, \dots, P$) is a maximal connected subgraph of G in the sense that the addition of any more vertices to G_r renders it unconnected. Evidently $P = 1$ if and only if G is connected.

SUBGRAPH, PROPER. A subgraph which does not contain all the **edges** of the **graph**.

SUBGRAPHS, DISJOINT. Subgraphs of a **graph** which have no **vertices** in common.

SUBGROUP. Elements of a **group**, in number less than or equal to the order of the group, which satisfy the group postulates. Every group possesses at least two subgroups: the unit element and the group itself. (For various types of subgroup, conjugate, normal, etc., see **group**.)

SUBHARMONIC. A sinusoidal quantity having a frequency which is an integral submultiple of the fundamental frequency (see **frequency, fundamental**) of a periodic quantity to which it is related. For example, a wave, the frequency of which is half the fundamental frequency of another wave, is called the second subharmonic of that wave.

SUBLIMATION. The process of change of **phase** from solid to gaseous. The opposite process has no special name.

SUBLIMATION CURVE. The graphical representation of the variation with temperature of the **vapor pressure** of a solid.

SUBMERGED SURFACE, FORCE ON A. See **force on a submerged surface**.

SUBROUTINE. A portion of a routine for a **stored-program computer**, e.g., a part which evaluates a trigonometric or other standard function, which has been so coded that it can be made an integral part of any routine that calls for this particular computation.

SUBSIDIARY VALENCE FORCES. A term used to distinguish between the principal (skeleton) valence forces in polymer molecules, and other forces, mainly hydrogen bonds and Van der Waals interactions, which often play an important part in the structure of these molecules.

SUBSONIC AND SUPERSONIC FLOW, LINEAR EQUATIONS. For a body in a uniform stream, the equations of motion can be linearized by assuming that the perturbation velocity components are all small compared with the stream velocity U . Taking the x -axis parallel to the undisturbed stream, the velocity components in the directions of the axes x , y and z may be written as $(U + u)$, v and w , where u , v and w are the *perturbation* velocity components. With this assumption, and excluding the case of transonic flow where $M \rightarrow 1$, the linear equation

$$(1 - M^2) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0 \quad (1)$$

is obtained, where M is the Mach number of the undisturbed stream.

For irrotational flow, a perturbation velocity potential ϕ may be introduced, so that

$$u = \frac{\partial \phi}{\partial x}, \quad v = \frac{\partial \phi}{\partial y}, \quad w = \frac{\partial \phi}{\partial z}.$$

Equation (1) may then be expressed as

$$(1 - M^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0. \quad (2)$$

For subsonic flow ($M < 1$) this equation is used to relate the flow past a given body, at Mach number M , to the *incompressible* flow past another body obtained by an affine transformation of the first body. This leads to the **Prandtl-Glauert** and **Göthert rules** for subsonic flow.

For supersonic flow ($M > 1$) the equation is of hyperbolic type. It is then not possible to relate the flow past a given body to any incompressible flow, but similarity rules can be formulated, relating the flows at two different supersonic Mach numbers. For many cases solutions of the equation can be obtained; for two-dimensional flow these are particularly simple, leading to the **Ackeret theory** of supersonic airfoils.

In **slender-body theory** the flow near the

body is approximated by neglecting the first term in Equation (2).

At hypersonic speeds ($M \gg 1$) the equation is no longer valid, because certain terms involving products of the perturbation velocity components and M^2 are no longer negligible. (See G. N. Ward, *Linearized theory of steady high-speed flow*, Cambridge, 1955.)

SUBSTANCE OF A VORTEX. The fluid containing the **vorticity**. A **line vortex** has no substance. A **vortex ring** has a finite volume of substance surrounded by fluid in irrotational motion.

In Hill's **spherical vortex** the substance is spherical.

SUBSTANTIAL DERIVATIVE. Also called *material derivative* and *total derivative*. (See **total derivative**.)

SUB-STELLAR POINT. The point on the surface of the earth at which a line from the center of the earth to a celestial object cuts the surface of the earth is known as the sub-stellar point of the object. If the celestial object is the moon, the point may be called the sub-lunar point; if the object is the sun, the point is the sub-solar point, if a star, the sub-astral point, etc.

SUBSTITUTIONAL ALLOYS. Alloys in which atoms of one element have been replaced by atoms of another, without changing the basic crystal structure. (See also **cooperative phenomena**.)

SUCTION OF LAMINAR BOUNDARY LAYER. If fluid is sucked into a porous wall, the thickness of the boundary layer is decreased and the stability of laminar flow is increased. Thus transition from laminar to turbulent flow is prevented or delayed. Although suction increases the skin friction in laminar flow, the overall effect in many cases is to cause the flow to be laminar when it would otherwise be turbulent, so that the skin friction is considerably reduced. Suction also reduces the tendency to separation.

A particularly simple case is that of a laminar boundary layer on a flat plate, with zero pressure gradient and uniform suction. With increasing distance downstream from the leading edge of the plate, the boundary layer tends asymptotically towards a state in which the thickness and skin friction are constant.

In this asymptotic state the rate of growth of the boundary layer due to outward diffusion of vorticity may be considered to be exactly balanced by the rate of removal of fluid by suction. The displacement thickness and shear stress at the wall are given by

$$\delta^* = \frac{\nu}{v_o} \quad \text{and} \quad \tau_o = \rho U v_o,$$

where ρ and ν are the density and kinematic viscosity of the fluid, U is the free-stream velocity, and v_o is the average normal velocity of suction through the wall.

SUFFICIENCY. A statistic t is said to be sufficient for a parameter θ if the distribution of a sample conditional on t does not depend on θ . There are other definitions of an equivalent kind. The basic idea is that t contains all the information in the sample about θ ; if t is known any other knowledge concerning the sample values adds nothing to inferences about θ .

SUMMATONAL INVARIANT. Molecular properties are called summational invariants when their sums for the molecules participating in a collision are not modified by the encounter. Typical summational (or collisional) invariants are the mass, the momentum, and the energy of the particles.

SUMMATION CONVENTION. The convention, often named after Einstein, that whenever the same index, called a dummy or umbral index, appears twice in a given expression, either as subscript or superscript, then the single expression in which the letter so occurs stands for a sum of similar expressions, the letter in question taking on the values, usually 1, 2, \dots , n , of a range agreed upon in advance. Thus, e.g.,

$$\frac{\partial u^i}{\partial x^j} dx^j \text{ denotes } \frac{\partial u^i}{\partial x^1} dx^1$$

$$+ \frac{\partial u^i}{\partial x^2} dx^2 + \dots + \frac{\partial u^i}{\partial x^n} dx^n,$$

and $\gamma_{ij}x_{ij}$ is the sum of n^2 terms. (See **indicial notation**.)

SUMMATION OF SERIES. In general, methods of approximate computation are based upon the use of infinite series and sequences whose limits are the desired quanti-

ties, and in practice one must truncate the series or select a particular term in the sequence. It is advantageous, therefore, that convergence be as rapid as possible. In some instances a series may not converge and yet may be regarded as defining the required quantity in some sense. Thus the series $1 + x + x^2 + x^3 + \dots$ fails to converge when $|x| > 1$; on the other hand it is the formal development of $(1 - x)^{-1}$ and may be considered to define this quantity whether or not the series converges. Moreover, if one applies the **delta-square process** to the sequence of partial sums of this series one obtains precisely $(1 - x)^{-1}$. Thus the process, in this instance, transforms the sequence of partial sums into a new sequence of which every term is the value of the quantity formally defined by the series. It can be regarded as a device for accelerating convergence, when convergence occurs, or of converting from divergence to convergence otherwise, in either event a method for summing the series.

Linear methods of summation have been most exhaustively studied, and the **Euler transformation** is one of the simplest and most useful. The delta-square process is among the simpler and more powerful of known non-linear methods. The elements u_i of an infinite sequence can always be regarded as the partial sums of a series, hence one can direct attention to a series or to a sequence, according to convenience. The terms u_n of a sequence can always be regarded as functional values $u(x)$ of a function for which $x = n^{-1}$. Hence finding the limit of a sequence of terms u_n can be regarded as equivalent to the problem of extrapolating the function $u(x)$ to $x = 0$, when functional values are known for $x = n^{-1}$. From this point of view one can use **interpolation** polynomials, **continued fractions** (e.g., formed with **reciprocal differences**) or any other convenient method of **approximation**.

SUMNER LINE. Same as **line of position**.

SUM OF TENSORS. If $a_{j_1 j_2 \dots j_N}^{i_1 i_2 \dots i_M}$ and $b_{j_1 j_2 \dots j_N}^{i_1 i_2 \dots i_M}$ are the components of two tensors (tensor fields) of equal weights, each contravariant of order M and covariant of order N , in a generic coordinate system in n -dimensional space, then the quantities $a_{j_1 j_2 \dots j_N}^{i_1 i_2 \dots i_M} + b_{j_1 j_2 \dots j_N}^{i_1 i_2 \dots i_M}$ are the components in the coordinate system of a tensor (tensor field), which is also contravariant of order M and covariant of order N ,

and is called the sum of the two tensors (tensor fields).

SUM OVER STATES. See **partition function**.

SUMPTNER PRINCIPLE. When a light source is placed at a point inside a sphere with a perfectly diffusing surface, then every part of the surface appears equally illuminated when viewed through an opening in the surface. This principle is the basis of the *Ulbricht sphere*.

SUM RULE FOR ATOMIC SPECTRA, BURGER - DORGELO - ORNSTEIN. See **Burger-Dorgelo-Ornstein sum rule for atomic spectra**.

SUM RULES FOR ELECTRONIC TRANSITIONS, VIBRATIONAL. See **vibrational sum rules for electronic transitions**.

SUN, MOTION OF. See **solar motion**.

SUPERAERODYNAMICS. See **rarefied gas dynamics**.

SUPERCONDUCTING TRANSITION. The change from the normal to the superconducting state is a reversible phase transition of the second kind, taking place at a temperature which depends on the magnetic field as well as on the material. The entropy of the normal state is always greater than that of the superconducting state by an amount.

$$-\frac{H_c}{2\pi} \frac{dH_c}{dT},$$

where H_c is the **critical field** at temperature T .

SUPERCONDUCTIVITY ENERGY EQUATION. When the London theory (see **London superconductivity equations**) is applied to any isothermal process taking place in a superconductor, the following relation is obtained:

$$\frac{\partial}{\partial t} \left[\frac{\mathbf{E} \cdot \mathbf{D} + \mathbf{H} \cdot \mathbf{B}}{8\pi} + \frac{\Lambda j^{(s)2}}{2} \right] + \sigma E^2 + \frac{c}{4\pi} \nabla \cdot (\mathbf{E} \times \mathbf{H}) = 0 \quad (1)$$

\mathbf{E} and \mathbf{B} are, respectively, the electric field in electrostatic units, and the magnetic induction in electromagnetic units.

$$\mathbf{D} = \epsilon \mathbf{E} \quad \text{and} \quad \mathbf{H} = \mathbf{B}/\mu$$

where for an isotropic material \mathbf{D} is the displacement current, \mathbf{H} is the magnetic field, and ϵ and μ are the dielectric constant and magnetic permeability respectively. Λ is a temperature-dependent constant of the material, $j^{(s)}$ is the superconducting part of the current density, and c is the velocity of light.

In equation (1), the quantity in the square bracket plays the role of a free-energy density.

SUPERCONDUCTIVITY EQUATIONS, LONDON. See **London superconductivity equations**.

SUPERCONDUCTIVITY, ISOTOPE EFFECT IN. See **isotope effect in superconductivity**.

SUPERELEVATION (I.E., BANKING OF A TRACK OR ROAD). The vertical distance the outer track of a railway is raised above the inner track on a curve in order to supply the centripetal force needed for motion around the curve.

SUPEREXCHANGE. A mechanism invoked to explain strong antiferromagnetic coupling such as that, for example, between two Mn^{++} ions separated by an O^{--} ion in MnO . A form of **configuration** interaction is involved—one of the excess electrons on the oxygen ion goes on to one of the Mn^{++} ions, where it couples with its spin. The other electron on the oxygen ion must be of opposite spin by the **Pauli exclusion principle**, and couples by exchange with the spin of the remaining Mn^{++} ion. The overall effect is thus a coupling between the spins of the Mn^{++} ions, even though these are much too far apart to have a direct **exchange** interaction.

SUPERHEATED STEAM. See **superheated vapor**.

SUPERHEATED VAPOR. Vapor at a state which does not correspond to two-phase (vapor-liquid or vapor-solid) equilibrium, i.e., at a state when its pressure is not determined by its temperature, the two constituting independent variables. For all substances, the pressure p of superheated vapor is higher than the saturation pressure p_s , which corresponds to its temperature T . Conversely, the temperature T of a superheated vapor is higher than the saturation temperature T_s , corresponding to its pressure p .

Sometimes (particularly in steam engineer-

ing), the state of a superheated vapor is specified by its pressure p and the degree of superheat (or the number of degrees of superheat) $T - T_s$. All gases are highly superheated vapors.

SUPERLATTICE. An arrangement of atoms of two or more substances in which the atoms of one substance occupy regular positions in the lattice of the other, without any chemical combination occurring between them. (See also **cooperative phenomena**.)

SUPERMULTIPLY. (1) A supermultiplet comprises the spectral lines corresponding to all the possible transitions between two *polyads*. A polyad is the group of terms of the same **multiplicity**, arising from the same **electron configuration**. A *pp* configuration of **non-equivalent electrons**, for instance, gives rise to two triads, $^1S, ^1P, ^1D$, and $^3S, ^3P, ^3D$, respectively. (2) A term used in the classification of the energy levels of light atomic nuclei, having to do with the symmetry of the wave function under permutation of the spatial coordinates alone. If nuclear forces are charge-independent and central, nuclear states may be classified according to their total orbital angular momentum L , their total spin angular momentum S , and their total isotopic spin T . States of different T, S, L will in general have different energies. However, if in addition the forces depend solely upon the spatial coordinates of the nucleus (i.e., **Wigner and Majorana forces**) then certain states with the same value of L but different values of T and S will be degenerate. Such a grouping of degenerate states is called a supermultiplet, and the concept was first introduced by Wigner in 1937. The supermultiplet classification has proved very useful in studies of the structure of light nuclei. It breaks down for heavier nuclei (atomic mass number >50) owing to the increasing importance of non-central forces and of the electrostatic repulsion between protons which violates charge-independence.

SUPERPOSITION PRINCIPLE (OR THEOREM). If a physical system is acted on by a number of independent influences, the resultant influence is the sum (vector or algebraic as circumstances dictate) of the individual influences. The principle takes on many specific forms depending on the nature of the system and the influence in question.

For example, when two forces act simultaneously on a particle the resultant force is the vector sum of the two. Another example is provided by the small oscillations of a system about a state of equilibrium. Thus the total displacement of a vibrating string is the algebraic sum of all its various harmonic modes of **oscillation** which add without interfering with each other. The principle is validated in this case by the fact that the wave equation governing the oscillations is linear. Superposition does not apply to non-linear systems.

In mechanics of solids the validity of superposition requires both linearity in the **stress-strain** relation and negligible effect of change in geometry on the conditions of equilibrium.

The principle can also be applied to **quantum mechanics**. Here it is exemplified by the postulate that any **state function** of a given quantum mechanical system corresponding to a given observable (e.g., the energy) can be expressed as a linear expansion of the **eigenstates** of the system for the same observable. (See also **Laplace transform theorems**.)

SUPERSELECTION RULES. It is a consequence of the usual postulates of quantum mechanics that to every realizable state of a physical system there corresponds a *unique* ray in **Hilbert space**. It may, however, be the case that there exist rays in Hilbert space which do not correspond to any physically realizable state. This situation occurs, for example, in relativistic field theories. There the Hilbert space of rays can be decomposed into orthogonal subspaces $\mathcal{H}_A, \mathcal{H}_B, \mathcal{H}_C \dots$ such that the relative phase of the components of a vector in each of the subspaces is arbitrary and not measurable. The phenomenon responsible for the breakup of the Hilbert space into several incoherent orthogonal subspaces is called a superselection rule. Examples of such superselection rules are the laws of electric charge conservation and baryon conservation. Mathematically, a superselection rule corresponds to the existence of an operator x' (which is not a multiple of the identity) and which commutes with *all* observables. If there exists such an operator x' commuting with *all* observables, then the Hilbert space can be decomposed into incoherent orthogonal subspaces. All rays within a single subspace are realizable, but a ray which has components in two or more subspaces is not. No observables will have matrix elements between different subspaces,

since the eigenfunctions of observables must, by definition, correspond to realizable states.

SUPERSTRUCTURE. See *superlattice*; also *cooperative phenomena*.

SUPERSTRUCTURE LINES. X-ray lines occurring because of the existence of *superstructure* in a *substitutional alloy*.

SUPER-UNDAL FLOW. Flow at speeds greater than the speed of any gravity waves, sometimes called *super-critical*, or *shooting flow*. In a single layer of fluid the **Froude number** exceeds unity, and stationary hydraulic **jumps**, but no **standing waves**, can occur. The flow is analogous to *super-sonic flow* in the sense that no disturbances can travel upstream, but there is no similar analogy to *sub-sonic flow* because gravity waves are usually highly dispersive with no minimum wave speed.

SUPPLEMENTARY CONDITION. Any condition imposed on the state vector Φ in a quantized field theory (see *field theory, quantized*) in order that this vector should correspond to an actual state; e.g., in *quantum electrodynamics*, the **Lorentz condition**

$$\frac{\partial A_\mu}{\partial x_\mu} \Phi = 0.$$

SURFACE, ANTICLASTIC. See *anticlastic surface*.

SURFACE, CHARACTERISTIC OF (FOR PARAMETER VALUE A). The curve defined by the equations $f(x, y, z, a) = 0$, $\partial f(x, y, z, a) / \partial a = 0$, for a particular value of a , where $f(x, y, z, a) = 0$ is a one-parameter family of surfaces. As a varies, the characteristics trace out a surface called the *envelope of the one-parameter family of surfaces*.

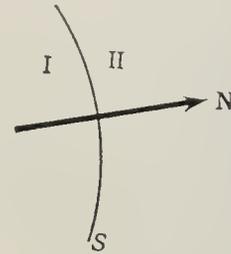
SURFACE COORDINATES OF A POINT. See *surface*.

SURFACE CURL OF A VECTOR. See *surface divergence of a vector*.

SURFACE, DIRECTOR. See *director surface*.

SURFACE DIVERGENCE OF A VECTOR. Assume that a surface S divides the space into two regions I and II and let the positive unit normal be directed from I and II. Let A_1 be

the surface value of the vector A on the side I of S and similarly A_2 be the value on the side II. Then $N \cdot (A_2 - A_1)$ is called the *surface divergence of A* and $N \times (A_2 - A_1)$ is



called the *surface curl of A* (sometimes written *DIV A* and *CURL A*). If the normal component of A changes continuously, then the *surface divergence of A* is zero, and if the tangential component changes continuously, then the *surface curl of A* is zero.

SURFACE FORCE. Force acting on an element of surface area at a point on the surface of a body which is proportional to the area of the surface element. The area may be calculated in any specified state of deformation of the body, e.g., the undeformed state, or the state of deformation at the instant under consideration. Hence, *surface force per unit area measured in the undeformed state* and *surface force per unit area measured in the deformed state*. Also called *surface traction*.

SURFACE, FORCE ON A SUBMERGED. See *force on a submerged surface*.

SURFACE-FRICTION DRAG. See *drag*.

SURFACE INTEGRAL. See *integral, multiple*.

SURFACE IN THREE-DIMENSIONAL SPACE. The locus of a point whose space coordinates are functions of two independent parameters, e.g., u and v . The equations expressing the dependence of the space coordinates on u and v are called *parametric equations of the surface*. The values of u and v corresponding to a point on the surface are called *surface coordinates of the point*. The curves $u = \text{constant}$ and $v = \text{constant}$ on the surface are called *parametric lines*, or *parametric curves*, or *coordinate lines on the surface*.

SURFACE INTRINSIC DERIVATIVE OF A SURFACE TENSOR. $t_{\lambda\mu \dots \nu}^{\alpha\beta \dots \gamma}$ are the components in a generic coordinate system u on a surface of a surface tensor, which is defined

along a curve lying in the surface by its dependence on a parameter τ . The surface intrinsic derivative of the surface tensor with respect to τ is the tensor with components $\delta l_{\lambda\mu\cdots\nu}^{\alpha\beta\cdots\gamma} / \delta\tau$ in the coordinate system u defined by

$$\frac{\delta l_{\lambda\mu\cdots\nu}^{\alpha\beta\cdots\gamma}}{\delta\tau} = l_{\lambda\mu\cdots\nu,\sigma}^{\alpha\beta\cdots\gamma} \frac{du^\sigma}{d\tau},$$

where $l_{\lambda\mu\cdots\nu,\sigma}^{\alpha\beta\cdots\gamma}$ are the components in the system u of the covariant derivative of the tensor.

SURFACE, INTRINSIC PROPERTIES OF. See *intrinsic properties of a surface*.

SURFACE, LIOUVILLE. See *Liouville surface*.

SURFACE, MINIMAL. See *minimal surface*.

SURFACE, MONGE FORM FOR EQUATION OF. See *Monge form for equation of surface*.

SURFACE OF CENTERS (OR CENTRO-SURFACE) OF A SURFACE. The locus of the centers of curvature (see *curvature, center of*) of the surface. It usually consists of two sheets, corresponding to the two systems of lines of curvature. Also called the *evolute of the surface*. (See also *involute of a surface*.)

SURFACE OF LEAST CONFUSION. The locus of centers of *circles of least confusion* in optical imagery. For a thin lens the surface is a sphere centered on the center of the lens.

SURFACE OF NEGATIVE TOTAL CURVATURE. See *antielastic surface*.

SURFACE OF POSITIVE TOTAL CURVATURE. See *synclastic surface*.

SURFACE OF REVOLUTION. A surface which may be generated by rotating a plane curve about an axis in its plane. Sections of the surface perpendicular to the axis are circles called *parallel circles*, or *parallels*, or *lines of latitude*. Sections of the surface by planes containing the axis are called *meridian sections*, or *meridians*, or *lines of longitude*.

SURFACE RECOMBINATION RATE. See *recombination rate, surface*.

SURFACE, RULED. See *ruled surface*.

SURFACES, APPLICABLE. See *applicable surfaces*.

SURFACES, FAMILY OF (IN THREE-DIMENSIONAL SPACE). The equation $f(x,y,z,a,b,\cdots) = 0$ in which a,b,\cdots are constants represents a surface in three-dimensional space. The infinitude of surfaces which are obtained by assigning all possible values to a,b,\cdots is called a family of surfaces. It is called a one-parameter, two-parameter, \cdots family of surfaces accordingly as there are one, two, \cdots parameters a,b,\cdots in the equation.

SURFACES, INVERSE. See *inverse surfaces*.

SURFACE, SKEW. See *skew surface*.

SURFACES, PARALLEL. See *parallel surfaces*.

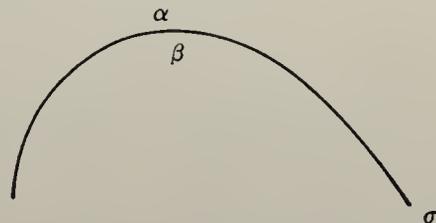
SURFACE, SPHERICAL. See *spherical surface*.

SURFACE, SYNCLASTIC. See *synclastic surface*.

SURFACE TENSION. See *capillarity*.

SURFACE TENSION OF A SOLID. See *Gibbs-Wulff relations*.

SURFACE TENSION OF CURVED INTERFACE. Consider a system consisting of two homogeneous bulk phases α and β connected by a curved surface layer σ . The pressures inside the phases α and β are p^α and p^β (see figure).



Curved interface between two phases due to surface tension.

The **surface tension** γ is related to the pressures p^α, p^β by the *Laplace formula*

$$p^\beta - p^\alpha = \gamma \left(\frac{1}{\rho_1} + \frac{1}{\rho_2} \right) \quad (1)$$

where ρ_1, ρ_2 are the principal radii of curvature of the interface.

For a spherical surface of radius r we have the simpler expression

$$p^\beta - p^\alpha = \frac{2\gamma}{r} \quad (2)$$

The expressions (1) or (2) form the basis of the experimental determinations of γ by the capillary rise method.

SURFACE TENSOR. See **tensor, surface**.

SURFACE TENSOR, INTRINSIC DERIVATIVE OF. See **surface intrinsic derivative**.

SURFACE WAVE. In the wide sense a wave propagated without change of form in a medium with a single, or several parallel plane boundaries, with its wave front always perpendicular to these boundaries is a surface wave. This statement may be extended to concentric cylindrical boundaries. This definition is wide enough to include within its range the case of perfectly related waves traveling along a perfect conductor. If we narrow this definition to exclude the reflection effect, then we also exclude from surface wave the disturbance within a dielectric slab or rod to which its name surface wave has been given.

In the narrow sense, therefore, we must examine the dimension of each medium present transverse to the boundary surface (and hence to the direction of propagation). If this dimension is infinite, then a surface wave must be exponentially damped at large distances from the guiding surface. If it is not infinite there is no such requirement, but we can insist that one medium at least shall extend to infinity in the transverse direction before a surface wave can be said to occur.

The same definition may be used both in the theory of **elastic waves** and in the theory of **electromagnetic waves**.

In fluids, surface waves are gravity waves in which the amplitude of the motion at the surface of the fluid is large compared with that at the bottom; waves whose length is small compared with the depth of the fluid concerned. Their properties are not affected by a change of depth of the fluid so long as it remains relatively deep. In inviscid fluids with a free surface at constant (atmospheric) pressure, the gas above being of negligible density, the velocity of surface waves is $(g/k)^{1/2}$ where $2\pi/k$ is the wavelength.

SURFACE, YIELD. A surface in stress space which is the locus of all stress points that represent states of stress at the **yield limit**.

SUSCEPTANCE. The imaginary part of **admittance**. Note that if $Z = R + jX$, where Z is impedance, R , resistance, and X , reactance,

$$A = \frac{1}{R + jX} = \frac{R - jX}{R^2 + X^2}$$

and the susceptance is

$$S = \frac{-X}{R^2 + X^2}$$

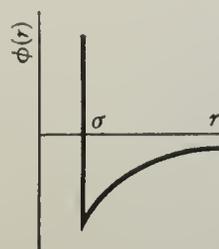
SUSCEPTIBILITY. See **magnetic susceptibility**; **electric susceptibility**.

SUTHERLAND MODEL. The intermolecular potential

$$\phi(r) = \infty, \quad r < \sigma \quad (1)$$

$$= -cr^{-\gamma}, \quad r > \sigma \quad (2)$$

represents **rigid spheres** which attract one another according to an inverse power law (see figure). It is generally called the *Sutherland potential*.



Sutherland model.

This model gives reasonable results in calculation of both thermodynamic properties and transport properties.

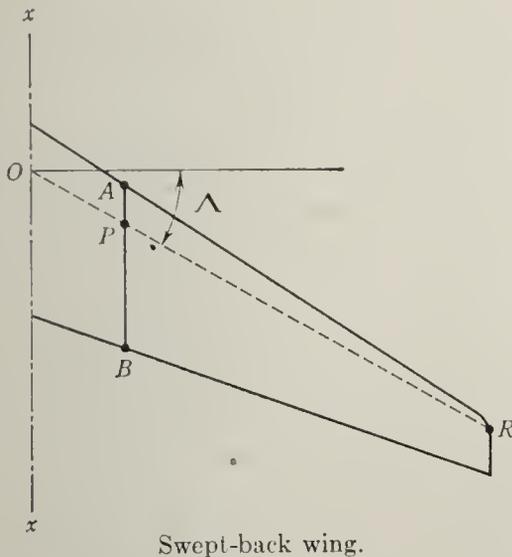
S-WAVE(S). Seismological term for a shear wave. (See **wave(s)**, **shear**.)

SWEPT-BACK WING. The figure shows one half of a wing which is symmetrical about the center line xx . AB is drawn parallel to xx and the point P on AB is defined by

$$\frac{AP}{AB} = f.$$

OR is the locus of all points such as P , for constant f . Then the angle Λ between OR and a normal to xx defines the sweep-back of the wing. The value of f commonly used in defining sweep-back is $1/4$; OR is then the quarter-chord line, and Λ is described as the sweep-back of the quarter-chord line. In gen-

eral, OR may not be a straight line, so that the angle of sweep-back may vary along the span of the wing.



Swept-back wings are used mainly for high-speed flight, in order to delay to higher Mach numbers the increase of drag coefficient and other changes that occur on an unswept wing at high subsonic speeds. A qualitative understanding of the effects of sweep-back may be obtained by considering the relatively simple case of an **infinite yawed wing**. Because of effects at the tip and center of a finite swept wing, however, the favorable effects of sweep-back, in raising the Mach number at which there is a rapid increase of drag coefficient, are considerably less than for an infinite yawed wing.

Lifting-line theory can be used only for wings with small or zero sweep, and for calculating the pressure distribution and loading on a wing with considerable sweep-back it is necessary to use **lifting-surface theory**. When the sweep-back is large and the aspect ratio small, the theory of **wings of small aspect ratio** may be used. The flow in the boundary layer on a swept-back wing is three-dimensional, and this often leads to important effects which are not observed in two-dimensional flow. Sweep-back tends to make the flow in a laminar boundary layer *unstable*, so that transition to turbulent flow occurs more readily than on an upswept wing. Also, there is a strong tendency for the boundary layer to separate on the outer part of a swept-back wing, at only a moderately high angle of incidence. This leads to important changes in the pressure distribution on the wing and, in particular,

usually causes a loss of lift and an increase of (nose-up) pitching moment.

SWEPT VOLUME. The volume V_s displaced ("swept") by the piston of a reciprocating engine during the whole of the stroke s . Hence

$$V_s = \frac{\pi D^2}{4} s$$

where D is the diameter of the cylinder bore. In automobile engineering, the sum of the swept volumes of all cylinders is sometimes referred to as *engine capacity*.

SYLVESTER DIALYTIC METHOD. See **dialytic**.

SYMMETRICAL WAVE FUNCTION. See under **exchange energy**.

SYMMETRIC BINARY RELATION. A binary relation R is symmetric if $aRb = bRa$ for all pairs a, b for which R is defined.

SYMMETRIC GROUP. A group consisting of all possible $n!$ permutations of n elements.

SYMMETRIC FUNCTION. A function whose value remains unchanged under any permutation of its independent variables. That is, any function $f(x_1, x_2, \dots, x_n)$ not affected by an interchange of any x_i and x_j . If these are roots of an algebraic equation

$$x^n - c_1 x^{n-1} + c_2 x^{n-2} - \dots \pm c_n = 0,$$

then

$$c_1 = \sum x_i, \quad c_2 = \sum x_i x_j, \quad c_3 = \sum x_i x_j x_k, \dots$$

are the elementary symmetric functions, where the summations are extended over all distinct products of distinct factors, the x_i being themselves considered independently varying. Any rational symmetric function of the x_i is a rational function of the c_i .

SYMMETRIC TENSOR. See **tensor, symmetric**.

SYMMETRIC TOP MOLECULE. See **polyatomic molecules, types of**.

SYMMETRIC WAVE FUNCTION. A wave function that is invariant with respect to the interchange of the variables of two particles:

$$\psi(x_1, x_2, \dots, x_n) = \psi(x_2, x_1, \dots, x_n),$$

where x_i stands for all the variables of the i^{th} particle. The state of a system of N bosons

can be described by such a symmetric wave function.

SYMMETRY AND QUANTUM MECHANICS. The possibility of abstracting laws of motion from the chaotic set of events which surround us stems from the following circumstances.

(1) Given a physical system it is possible to isolate a (finite) manageable set of relevant initial conditions, and

(2) Given the same set of initial conditions the resulting motion of the system will be the same no matter where and when these conditions are realized (at least in our neighborhood of the universe). In the language of symmetry principles (2) is the statement that the laws of nature are independent of the position of the observer as well as of the time at which he performs his experiment, or equivalently in mathematical terminology, that the laws of motion are covariant with respect to displacements in space and time, i.e., with respect to the transformations

$$\begin{aligned} \mathbf{x} &\rightarrow \mathbf{x} + \mathbf{a} \\ t &\rightarrow t + \tau. \end{aligned} \tag{a}$$

Experiments have also yielded the fact that the orientation in space of an event is likewise an irrelevant initial condition (isotropy of space) and this principle can be translated into the statement that the laws of motion are covariant under spatial rotations. The principle of Galilean invariance (based on the success of Newton's laws of motion) further asserted that the laws of nature are independent of the state of motion of the observer as long as it is uniform with constant velocity.

These symmetry principles are usually stated in terms of two observers, O and O' , who are in a definite relation to each other. For example, observer O may be moving with constant velocity, \mathbf{v} , relative to O' in such a way that the relation of the labels of the spatial points and the reading of their clocks in their respective coordinate systems are given by the following equations:

$$\begin{aligned} \mathbf{x}' &= \mathbf{x} - \mathbf{v}t \\ t' &= t. \end{aligned} \tag{b}$$

The principle of Galilean invariance then asserts that the "laws of nature" are the same

for the two observers. The form of the equations of motion is the same for both observers, i.e., the equations of motion are covariant with respect to the transformation (b). Two observers using inertial coordinate systems (i.e., ones in which the laws of motion are the same) are said to be equivalent.

The aforementioned invariance principles were experimentally established and may have only limited applicability. Thus, **Lorentz invariance** has replaced the principle of Galilean invariance. The recent discovery that the law of parity conservation (invariance under interchange of left-handed and right-handed coordinate systems) does not hold in weak interactions has reemphasized that an invariance principle and its consequences must be experimentally verified.

At the macroscopic level, the notion of an invariance (or symmetry) principle can be made precise with the help of the concept of the complete description of a physical system. By the latter is meant a specification of the trajectories of all particles together with a full description of all fields at all points of space for all time. The equation of motion then allows one to determine whether the system could in fact, have evolved in the way specified by the complete description. The existence of an invariance principle then requires that the following three postulates be satisfied:

(1) It should be possible to translate a complete description of a physical system from one coordinate system into every equivalent coordinate system;

(2) The translation of a dynamically possible description should again be dynamically possible;

(3) The criteria for the dynamic possibility of a complete description should be identical for equivalent observers.

Postulate 2 is equivalent to the statement that a possible motion to one observer must also appear possible to any other observer and postulate 3 to the statement of the form invariance of the equation of motion.

In a quantum mechanical framework postulate 1 remains as stated. It implies that there exists a well defined connection and correspondence between the labels attributed to the space-time points by each observer, between the vectors each observer attributes to a given physical system and between the operators (observables) used by the two observers. Pos-

tulate 2 is usually formulated in terms of **transition probabilities** and states that the transition probability is independent of the frame of reference. Different equivalent observers make the same prediction as to the outcome of an experiment carried out on a system. Note that this system will be in a different relation to each of the observers. Observer O will attribute the vector $|\psi\rangle$ to the state of the system, whereas observer O' will describe the state of this same system by a vector $|\psi'\rangle$. It is, however, assumed that given two systems, S_0 and $S_{0'}$, which are in the same relation to each of the two observers (i.e., the values of the observables of the system S_0 as measured by observer O are the same as the values of the observables of $S_{0'}$ as measured by O') the observers will describe the state of their respective systems by the same vector. One calls $|\psi'\rangle$ the translation of the vector $|\psi\rangle$. Stated mathematically, postulate 2 asserts that if $|\psi\rangle$ and $|\phi\rangle$ are two states and $|\psi'\rangle$ and $|\phi'\rangle$ their translations, then

$$|(\psi',\phi')|^2 = |(\psi,\phi)|^2 \quad (c)$$

If all rays in Hilbert space are distinguishable, it is then a mathematical theorem that the correspondence $|\psi\rangle \rightarrow |\psi'\rangle$, as a result of (c), is effected by a **unitary** or **anti-unitary** operator, i.e.,

$$|\psi\rangle = U(O,O')|\psi'\rangle \quad (d)$$

where U depends on the coordinate systems between which it affects the correspondence and $U(O,O') = I$ (the identity) if $O' = O$. Postulate 3 next asserts that U can only depend on the relation of the two coordinate systems and not on the intrinsic properties of either one. For example, for **Lorentz transformation**, $U(O',O)$ must be identical with $U(O''',O'')$ if O''' is in the same relation to O'' as O' is to O , i.e., if O''' arises from O'' by the same Lorentz transformation, L , by which O' arises from O . If this were not so, there would be an intrinsic difference between the frames O',O and O'',O''' . The operator $U(O,O')$ is completely determined up to a factor of modulus unity by the transformation, L , which carries O into O' , and one writes $U = U(L)$ and $|\psi'\rangle = U(L) \cdot |\psi\rangle$. If one considers three equivalent frames, then one must obtain the same state by going from the first frame O to the second $O' = L_1O$ and then to the third $O'' = L_2O'$, as by going directly from the first to the third frame $O'' = L_3O$, with $L_3 = L_2L_1$, hence

$$\begin{aligned} |\psi''\rangle &= U(L_2)U(L_1)|\psi\rangle \\ &= U(L_3)|\psi\rangle \end{aligned} \quad (e)$$

from which it follows that

$$U(L_3) = \omega(L_2, L_1)U(L_2)U(L_1) \quad (f)$$

$$U(I) = I \quad (g)$$

where ω is a number of modulus one which can depend on L_1 and L_2 and arises because of the indeterminate factor of modulus one in the state vectors. The U 's as a result of Equations (f) and (g), are said to form a unitary or (anti-unitary) *representation up to a factor of the group of transformations* under which the observers are equivalent. For special relativity this group is the group of inhomogeneous Lorentz transformations.

It now follows from postulate 2 and the fact that all frames which can be reached by the symmetry transformations are equivalent for the description of the system, that together with $|\psi\rangle$, $U(L)|\psi\rangle$ must also be a possible state of the system as described by observer O . Thus a relativity invariance requires the vector space, V , describing the possible states of a quantum mechanical system to be invariant under all relativity transformations, i.e., V must contain together with every $|\psi\rangle$ all transformations $U(L)|\psi\rangle$, where L is any relativity transformation. This is the active view of formulating relativistic invariance and deals only with the transformed states of a single observer. This active interpretation is to be contrasted to the passive viewpoint outlined above, involving two observers. For example, a three dimensional rotation may be interpreted passively as a rotation of the co-ordinate system or actively as a rotation of the system S under observation.

SYMMETRY ARGUMENT. A logical treatment based on the idea that when there is no apparent reason that either of two possible behaviors should be preferred to the other, neither will occur. Examples are the proof that a spherically symmetric body resting on a horizontal surface is stable in all positions, and the proof that the electrostatic field around a long, straight charged conductor must be radial. In the first case, it is argued that a rotation of the body about the point of support in any direction leads to a system indistinguishable from rotation in any other direction. In the second, there is no reason

that the lines of force should circulate either clockwise or counter-clockwise about the conductor when viewed from either end; therefore the field can have no tangential component.

SYMMETRY, AXIS OF. A line drawn within a body or within a set of points in such a location and direction that a rotation of the body through an angle ($2\pi/n$) radians about the line as an axis, n being an integer greater than unity, results in a configuration indistinguishable from the original configuration. A body or set having such an axis is said to have n -fold symmetry, and the line is said to be an n -fold axis. Thus a line through the center of a cube and parallel to a face is a four-fold axis of symmetry, while a body diagonal of the cube is a two-fold axis.

SYMMETRY AXIS OF SECOND KIND OR ROTATION-REFLECTION AXIS. A symmetry element by which the crystal is brought into self-coincidence by a combined rotation and reflection in a plane perpendicular to the axis of rotation. In Schoenflies notation, S_n for an n -fold axis. (See symmetry classes.)

SYMMETRY, CENTER OF. A symmetry element such that any line through it will intersect the crystal at equal distances on either side. (See symmetry classes.)

SYMMETRY CLASSES. Every crystal belongs to one of the 32 different classes of symmetry, or **point groups**. The standard notation for describing these classes is that of Schoenflies, who assigned symbols to the various possible **symmetry elements** according to the following rules:

<i>Symbol</i>	<i>Symmetry Element Possessed by Group</i>
C_n	n -fold axis
D_n	n -fold axis, and n two-fold axes perpendicular to it
S_n	n -fold axis of rotary-reflection
V	three mutually perpendicular two-fold axes
T (tetrahedral)	four three-fold axes towards vertices of regular tetrahedron
O	three mutually perpendicular four-fold axes (cubic group)
subscript v	(vertical) reflection plane containing symmetry axis
subscript h	(horizontal) reflection plane perpendicular to symmetry axis

<i>Symbol</i>	<i>Symmetry Element Possessed by Group</i>
subscript d	(dihedral) reflection plane bisecting angle between two two-fold axes
subscript i	inversion
subscript s	reflection plane

Another system of notation is that of the **Hermann-Mauguin symbols**.

SYMMETRY COORDINATES. If a polyatomic molecule has symmetry the determination of the **normal vibrations** is simplified if instead of $3N$ Cartesian displacement coordinates $3N - 6$ (or $3N - 5$) new internal coordinates, "symmetry coordinates," are introduced, such that the center of mass is at rest and the angular momentum equal to zero. To a given value of one of these coordinates correspond displacements of the nuclei in agreement with one of the symmetry types (see **symmetry properties of normal vibrations of polyatomic molecules**). If this is done the secular determinant can be factored into as many smaller determinants as there are different species. The degree of each of these sub-determinants, that is, the number f_j of vibrations of the particular species j can be readily obtained from the number of the various atoms in the molecule. The number of force constants belonging to each species is $\frac{1}{2}f_j(f_j + 1)$ and therefore the total number of force constants is $\sum \frac{1}{2}f_j(f_j + 1)$ which is smaller, often much smaller, than $\frac{1}{2}n(n + 1)$. But even then the number of force constants is in general larger than the number of normal frequencies.

SYMMETRY, DYAD. A two-fold symmetry axis.

SYMMETRY ELEMENT. An operation which brings a crystal into a position that is indistinguishable from its original position. The symmetry elements are: rotation axes, reflection planes, inversion centers and rotation-reflection axes.

SYMMETRY GROUP. A group whose elements are those operations which transform regular bodies into themselves, i.e., the tetrahedral group.

SYMMETRY, HEXAD. A six-fold symmetry axis.

SYMMETRY, PLANE OF. A plane passed through a body or through a set of points in

such a location and direction that the reflection of all points in the plane results in a configuration indistinguishable from the original configuration. Thus, a cube has many planes of symmetry through its center, including those parallel to the faces and those passing through face diagonals.

SYMMETRY PROPERTIES OF MOLECULAR EIGENFUNCTIONS. The behavior of a molecular eigenfunction with respect to a symmetry operation is equal to the "product" of the behavior of the electronic, vibrational, and rotational eigenfunctions separately.

The behavior of the electronic and vibrational eigenfunctions of a molecule with respect to a given symmetry operation is analogous to that of the normal vibrations. (See **symmetry properties of normal vibrations in molecules.**) The symmetry properties of the rotational eigenfunctions of the different types of molecules are as follows:

(1) *Diatom and Linear Polyatomic Molecules.* A rotational level is called positive or negative (+ or -) depending on whether the total eigenfunction ψ remains unaltered or changes its sign by reflection of all the particles (electrons and nuclei) at the origin (inversion). In addition, if the molecule has a center of symmetry, a rotational level is symmetric (*s*) or antisymmetric (*a*) depending on whether or not the total eigenfunction ψ of the system (apart from the nuclear spin function) remains unchanged or changes sign, when all the nuclei on one side of the center are simultaneously exchanged with the corresponding ones on the other side.

(2) *Symmetric Top Molecules.* In the non-planar symmetric top molecule a reflection of all particles at the origin (inversion) leads to a configuration which cannot also be obtained by rotation of the molecule. Corresponding to these two configurations of the molecule, each rotational level J, K is doubly degenerate as long as the potential hill separating the two configurations is infinitely high. For a finite potential hill, a splitting occurs into two sublevels which have opposite symmetry with respect to an inversion. The eigenfunctions of these sublevels contain equal contributions from the positive and negative "original" levels (inversion doubling).

For $K > 0$ the K degeneracy exists in addition to the inversion doubling, so that each

level with a given J and $K (>0)$ consists of four sublevels.

In the planar symmetric top molecule no inversion doubling occurs. Each rotational level $J, K (>0)$ consists of two sublevels either both positive or both negative with respect to inversion.

For molecules which are symmetrical tops due to their symmetry, additional symmetry properties arise corresponding to the property symmetric-antisymmetric in linear molecules. Levels belonging to different species are distinguished by symbols *A, E*, etc.

(3) *Spherical Top Molecules.* In the case of a spherical top, the distinction between positive and negative rotational levels can be ignored since they always occur in close pairs, and in no case of spherical top molecules has the inversion doubling been resolved.

For molecules which are spherical tops due to their symmetry, additional symmetry properties arise, analogous to those for the symmetric top molecules.

(4) *Asymmetric Top Molecules.* Apart from the symmetry property positive-negative with respect to inversion, which is unimportant for asymmetric top molecules, the rotational levels are distinguished by the behavior of their eigenfunctions with respect to rotations by 180° about the axes of largest and smallest moment of inertia (C_2^c and C_2^a). There are thus four different types (species) of rotational levels, briefly described by ++, +-, -+, and --, where the first sign refers to the behavior of the rotational eigenfunction with respect to C_2^c , the second to the behavior with respect to C_2^a .

The highest level (J_{+J}) of each set with a given J is always + with respect to C_2^c , the two next are -, the two next +, and so on. The lowest level (J_{-J}) of each set with a given J is always + with respect to C_2^a , the two next -, the two next +, and so on.

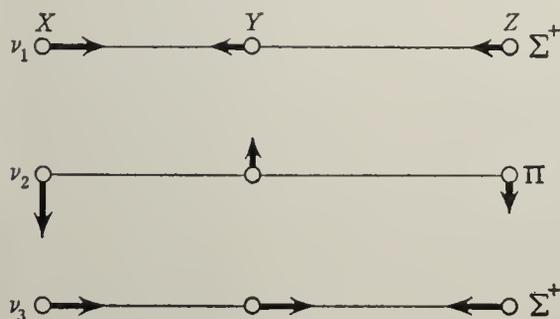
If an asymmetric top molecule has elements of symmetry, the eigenfunctions have additional symmetry properties corresponding to the exchange of identical nuclei, e.g., *A* and *B* (similar to *a* and *s* of linear molecules) for molecules with one twofold axis and *A, B₁, B₂, B₃* for molecules with three twofold axes.

SYMMETRY PROPERTIES OF NORMAL VIBRATIONS IN MOLECULES. The nuclear frame of a molecule may have one or several symmetry elements, such as a plane of

perpendicular) two-fold axes of symmetry C_2 , three (mutually perpendicular) planes of symmetry σ , and a center of symmetry i (in addition to the symmetry element I , corresponding to the identity operation). Examples are plane C_2H_4 , plane O_4 , plane N_2O_4 .

Point group $C_{\infty v}$ (Table 2) is appropriate to molecules having one ∞ -fold axis of symmetry, and an infinite number of planes of symmetry σ_v through this axis, that is, to heteropolar diatomic molecules, for instance CN, and linear polyatomic molecules without center of symmetry.

The figure shows as an example the normal vibrations of a linear XYZ molecule. For any



Normal vibrations of a linear XYZ molecule. (From G. Herzberg, *Molecular Spectra and Molecular Structure. II. Infrared and Raman Spectra of Polyatomic Molecules*. D. Van Nostrand Company, Inc., Princeton, 1945.)

linear molecule of point group $C_{\infty v}$ the normal vibrations can only belong to the species Σ^+ and Π . However the eigenfunctions of the higher vibrational levels of the perpendicular vibrations (ν_2 in XYZ) may have species Σ^- , Δ , Φ , \dots .

SYMMETRY REQUIREMENTS ON COUPLING OF IRREVERSIBLE PROCESSES.

Symmetry considerations limit the possibility of interference between irreversible processes. Let us, for example, write the **phenomenological relations** for a system with a heat flow along the geometrical coordinate α and subject at the same time to a chemical reaction.

The phenomenological relations are

$$\begin{aligned} J_{th} &= L_{11}X_{th} + L_{12}X_{ch} \\ J_{ch} &= L_{21}X_{th} + L_{22}X_{ch}. \end{aligned} \quad (1)$$

Here J_{th} is the heat flow, J_{ch} the rate of the chemical reaction, X_{th} the temperature gradient and X_{ch} the chemical **affinity**. A first reduction of the number of phenomenolog-

ical coefficients is obtained by the Onsager **reciprocity relations** which give

$$L_{12} = L_{21}. \quad (2)$$

In this case

$$L_{12} = L_{21} = 0. \quad (3)$$

Suppose $X_{th} = 0$; (1) then gives

$$J_{th} = L_{12}X_{ch} \quad (4)$$

but X_{ch} is a scalar and J_{th} a vector (heat flow) so that a scalar "cause" would produce a vectorial "effect."

This would be contrary to the general requirements of symmetry principles (often called Curie's symmetry principles) according to which macroscopic causes have always fewer elements of symmetry than the effects they produce. This implies in the present example the relation (3).

SYMMETRY, TETRAD. A four-fold symmetry axis.

SYMMETRY, TRIAD. A three-fold symmetry axis.

SYMMETRY TYPE. See *species*.

SYNCLASTIC SURFACE. A surface, or portion of a surface, on which the two principal radii of curvature at each point have the same sign. Also called *surface of positive total curvature*.

SYNODIC PERIOD. The synodic period of any member of the solar system is the time required for the object to go from some particular position relative to the sun as seen from the earth back to the same position again. In the case of the moon the synodic period is the time required for the moon to go from conjunction, or new moon, back to conjunction again. This period of approximately 29.5 days is the original month as used by ancient astronomers in the construction of the calendar.

Since a planet is best observed at opposition, the synodic period of the planet gives the interval of time between successive positions of favorable observation. The synodic period is related to the sidereal period, i.e., the actual period of revolution of an object about the sun, by a simple relationship:

Let P be the sidereal period of the object.

S the synodic period of the same object.

E the sidereal period of the earth (approximately 365.25 days).

Then

$1/S = 1/P - 1/E$ for planets with orbits inside that of the earth.

$1/S = 1/E - 1/P$ for planets with orbits outside that of the earth.

SYNTHETIC DIVISION. To divide the polynomial

$$P(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \dots$$

by $x - r$, write

$$\begin{array}{cccccc|c} a_0 & a_1 & a_2 & \cdots & a_n & & r \\ & a_0r & b_1r & \cdots & b_{n-1}r & & \\ \hline a_0 & b_1 & b_2 & \cdots & & & R \end{array}$$

where each number below the line is the sum of those above. Then

$$Q(x) = a_0x^{n-1} + b_1x^{n-2} + b_2x^{n-3} + \dots,$$

where

$$P(x) = (x - r)Q(x) + R.$$

SYNTHETIC KERNEL. See kernel, synthetic.

SYSTEM. See indicial notation; system equations; thermodynamic system; transfer function.

SYSTEMATIC SAMPLE. A sample obtained by a systematic method, as opposed to random choice. For example, a sample of persons may be chosen from a list by selecting one at constant intervals, e.g., the tenth, twentieth, thirtieth \dots .

SYSTEM, COMPONENTS OF. See indicial notation.

SYSTEM, DOUBLE. See indicial notation.

SYSTEM, ELEMENTS OF. See indicial notation.

SYSTEM EQUATIONS. The set of differential or integro-differential equations expressing the inter-relations between the various time-varying quantities of which the system is comprised and the external inputs fed into the system, if any.

In a linear system of n quantities $q_1(t), q_2(t) \dots q_n(t)$ with, for simplicity, only a single input $\theta(t)$ these equations can always be written in the form

$$\begin{aligned} a_{11}q_1 + a_{21}q_2 + \dots + a_{n1}q_n &= -\theta \\ \dots & \\ a_{1m}q_1 + a_{2m}q_2 + \dots + a_{nm}q_n &= 0, \end{aligned} \quad (m = 2, 3, \dots n)$$

where the a 's are in general functions of the operator $\frac{d}{dt}$. (The case of multiple inputs can be treated by superposition or by considering the right-hand side elements to be $-\theta_1(t), -\theta_2(t)$, etc.)

Taking **Laplace transforms** of these equations under general initial conditions gives

$$\begin{aligned} a_{11}Q_1 + \dots + a_{n1}Q_n &= f_1(s) - \Theta(s) \\ \dots & \\ a_{1m}Q_1 + \dots + a_{nm}Q_n &= f_m(s), \end{aligned} \quad (m = 2, 3, \dots n)$$

in which the a 's are now the same functions of s as they were of $\frac{d}{dt}$, $\Theta(s)$ and $Q_r(s)$ are the

transforms of $\theta(t)$ and $q_r(t)$ and the $f(s)$ functions are in general determined by the initial values of the q 's and possibly their time-derivatives and, in the case of $f_1(s)$, possibly also by the initial values of $\theta(t)$ and its derivatives. If the system is initially quiescent, therefore, all the f 's vanish.

This set of simultaneous algebraic equations give the solutions

$$Q_r(s) = + \frac{\Delta_r(s)}{\Delta(s)}$$

where $\Delta(s)$ is the system determinant

$$\begin{vmatrix} a_{11} & \dots & a_{n1} \\ \dots & \dots & \dots \\ a_{1m} & \dots & a_{nm} \end{vmatrix}$$

formed by the array of a 's, and $\Delta_r(s)$ is the determinant formed from Δ by substituting the column of right-hand-side elements for the r th column of Δ . If the system is initially quiescent, $\Delta_r(s)$ simplifies, in the above form of the equations, to $-\Delta_{r1}(s)\Theta(s)$, $\Delta_{r1}(s)$ being the cofactor of a_{r1} in $\Delta(s)$. This solution of course assumes that the equations are independent, that is that $\Delta(s) \neq 0$.

It must be noted that the set of system equations is not a unique set, since any number of new equations can be formed by linear combinations of a given set. We may for instance multiply the first equation above by $k_1(s)$, the second by $k_2(s)$ and add the results to give a

new equation which may be substituted, say, for the first equation. Whereas this process does not alter the solution for $Q_r(s)$ (since both $\Delta_r(s)$ and $\Delta(s)$ will be multiplied by $k_1(s)$) just because $\Delta(s)$ is multiplied by $k_1(s)$, the zeros of $\Delta(s)$, which are the roots of the system **characteristic equation**, will now contain also the zeros of the arbitrary function $k_1(s)$ and one may therefore be led to fallacious conclusions regarding, for instance, the stability of the system. It is important from this point of view therefore that $\Delta(s)$ be assessed from the original set of system equations, which correspond to existing physical relations between the quantities, and not from an arbitrarily deduced, modified set of equations.

Methods of Obtaining System Equations.

In purely electrical systems probably the simplest way of writing down the set of equations is either by mesh analysis or by nodal analysis. The first of these is essentially a statement of voltage balance in any mesh of a system, the second a statement of current balance at any node. In mixed systems or non-electrical systems, particularly in control systems, the equations are normally set up by considering the value of any quantity in the system as con-

sisting of the sum of the contributions made directly to it by the other quantities of the system and by the input, if any, directly applied to the quantity in question. This generally leads to equations of the form given above but with all a 's of the form a_{jj} equal to (-1) , e.g.,

$$q_1 = a_{21}q_2 + a_{31}q_3 + \cdots + a_{n1}q_n + \theta.$$

The form of the equations, however, is the same. This approach to the system equations is the basis of **signal flow diagrams**.

SYSTEM, INDICIAL. See **indicial notation**.

SYSTEM SIGNAL FLOW DIAGRAMS. See **signal flow diagrams**.

SYSTEM, SIMPLE. See **indicial notation**.

SYSTEM, STABILITY OF. See **stability (of system)**.

SYSTEM, THERMODYNAMIC. See **thermodynamic system**.

SYSTEM TRANSFER FUNCTION. See **transfer function**.

T

TALBOT. The M.K.S. unit of luminous energy. One talbot is equal to 10^7 lumergs. One joule of radiant energy having a luminous energy efficiency of x lumens per watt is x talbots of luminous energy. One lumen is a luminous flux of one talbot per second.

TALBOT LAW. The apparent brightness of an object viewed through a slotted-disk, rotating above a critical frequency, is proportional to the ratio of the angular aperture of the open to the opaque sectors.

TAMM-DANCOFF METHOD. Technique for approximating to the wave-function of a system of interacting particles, especially nucleons and mesons, by describing it as a superposition of a certain number of possible states. This number is decided upon *a priori* and determines the order of the approximation. No explicit assumption is made about the smallness of the interaction and indeed in contrast to the usual perturbation theory, the theory of the motion is developed non-adiabatically, i.e., the equation of motion of the interacting particles is developed at the same time as the term describing their interaction.

TANGENT. See tangent to a curve at point P .

TANGENT, GEODESIC. See geodesic tangent.

TANGENTIAL ACCELERATION. See acceleration.

TANGENTIAL FOCAL LINE. See focal lines.

TANGENTIAL FOCUS (PRIMARY FOCUS). The first order focus of a bundle in the tangential plane of a meridional ray.

TANGENTIAL PLANE. The meridional plane containing a meridional ray of an optical system.

TANGENT LINE. See tangent plane to surface.

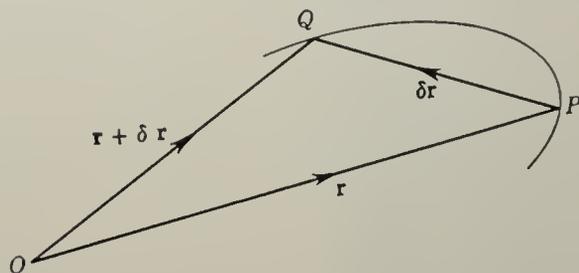
TANGENT, LOSS. The ratio of the imaginary part to the real part of the complex modulus or compliance:

$$\tan \theta = \frac{m_2}{m_1} = \frac{k_2}{k_1}.$$

TANGENT MODULUS. See modulus, tangent and modulus, secant.

TANGENT PLANE TO SURFACE. The tangent to any curve drawn on a surface is called a *tangent line* to the surface. All tangent lines to a surface at a point lie in a plane called the tangent plane to the surface at the point. The straight line through this point perpendicular to the tangent plane is called the *normal to the surface* at the point. Two surfaces are said to *touch* at a common point if their tangent planes at that point coincide.

TANGENT TO A CURVE AT A POINT P . Let \mathbf{r} and $\mathbf{r} + \delta\mathbf{r}$ be the vectors drawn from an origin O to the point P and a neighboring point



Q on the curve. Let δs be the distance measured along the curve from P to Q . Denote the limit $\lim_{\delta s \rightarrow 0} \delta\mathbf{r}/\delta s$, if it exists, by \mathbf{t} . Then \mathbf{t} is the *unit tangent* to the curve at P . A line through P in the direction of \mathbf{t} is called the *tangent* to the curve at P .

TANTOCHROME. A curve in a vertical plane such that a particle sliding frictionless along it under the sole influence of gravity will vibrate with the same period for all amplitudes. This curve is a cycloid. The period of vibration is

$$T = 4\pi \sqrt{\frac{r}{g}}.$$

Such a pendulum is also known as *Huygen's pendulum* after its discoverer.

TAYLOR SERIES. A convergent power series is generally useful as a representation of a function. Let us assume that $f(x)$ can be so given as an infinite series in powers of $(x - a)$, expecting the series to be valid for values of x near a . Then,

$$f(x) = b_0 + b_1(x - a) + b_2(x - a)^2 + \dots + b_k(x - a)^k + \dots$$

and the coefficients can be obtained by successive differentiations to give

$$b_0 = f(a); \quad b_1 = f'(a); \quad b_2 = f''(a)/2!; \quad \dots; \\ b_k = f^{(k)}(a)/k!; \quad \dots$$

The result, which is known as the Taylor series (Brook Taylor, 1685–1741, was an English mathematician), is

$$f(x) = f(a) + f'(a)(x - a) + f''(a) \frac{(x - a)^2}{2!} + \dots + f^{(k)}(a) \frac{(x - a)^k}{k!} + \dots$$

The procedure, so far, has been purely formal for it has only been assumed that such an expansion is possible. One can, however, apply the extended **mean value theorem** of differential calculus which shows that the Taylor series expansion and the extended mean value theorem are identical, if $x = b$ and $a < z < x$, provided we stop after n terms. Let the first n terms in the Taylor series (the n th term involves $f^{(n-1)}$ or $(n - 1) = k$) be called S_n and the last term in the extended mean value theorem be called the remainder,

$$R = f^{(n)}(z)(x - a)^n/n!,$$

so that $f(x) - S_n = R$. Now if R approaches zero as n approaches infinity, then the Taylor series does indeed converge to give $f(x)$. Whether indeed $R \rightarrow 0$ as $n \rightarrow \infty$ must be tested in each case.

Another useful form of the Taylor series is obtained if we let $h = (x - a)$ and replace a by x_0 . Finally dropping the subscript on x_0 , the result is

$$f(x + h) = f(x) + f'(x)h + f''(x) \frac{h^2}{2!} + \dots + f^{(k)}(x) \frac{h^k}{k!} + \dots$$

The remainder after n terms can be given in several equivalent forms. Some of them are

$$R_n(x) = \frac{(x - a)^n f^{(n)}}{n!} \{a + \phi(x - a)\}, \\ 0 < \phi < 1.$$

$$R_n(x) = \frac{1}{(n - 1)!} \int_0^{x-a} f^{(n)}(x - t)t^{n-1} dt.$$

In the second form of the series

$$R_n(x) = \frac{h^n f^{(n)}}{n!} (x + \phi h);$$

$$R_n(x) = \frac{1}{(n - 1)!} \int_0^h f^{(n)}(x + h - t)t^{n-1} dt.$$

Use of the Taylor series requires that the function and its derivatives of all orders must exist at $x = a$. The **binomial series** is a special case. (See also **Maclaurin series**.)

The Taylor series may be extensively generalized. Thus, in several variables

$$f(x + h, y + k, \dots) = f(x, y, \dots) + df(x, y, \dots) + \frac{1}{2!} d^2 f(x, y, \dots) + \dots + \frac{1}{k!} d^k f(x, y, \dots) + \dots + R_n.$$

It can also be used for functions of a **complex variable**. In that case the boundary of the region of the complex plane in which a Taylor series converges is called the circle of convergence. For a given function, the radius of this circle depends on the point (center of the circle) about which the series is developed. Outside the boundary, and also in at least one point on it, the power series diverges, but the function may frequently be calculated.

TAYLOR SPIRAL. The meteorological analogy of the **Ekman spiral**; the spiral traced out by a point whose position vector is the horizontal wind vector at height z in air in which the eddy viscosity and horizontal pressure gradient are independent of height, the wind being steady at all heights and so approximating to the **geostrophic wind** at great height. The wind vector is

$$u + iv = V_G - 2^{1/2} V_G \sin \alpha e^{-Bz - i(Bz - \alpha + \frac{\pi}{4})}$$

where V_G is the geostrophic wind (assumed to be in the x -direction), α is the angle between the isobars and the surface wind, and

$$B^2 = f/2K$$

where f is twice the vertical component of the earth's angular velocity, and K is the eddy viscosity.

TCHEBYCHEV. See **Chebyshev.**

t -DISTRIBUTION. See **student's distribution.**

TECHNICAL WORK. See **flow work.**

TELECENRIC SYSTEM. An optical system having either the **entrance pupil** or the **exit pupil** at infinity.

TELEGRAPHER'S EQUATION. A second order partial differential equation with constant coefficients, of the form

$$\frac{\partial^2 u}{\partial x^2} = a^2 \frac{\partial^2 u}{\partial t^2} + b \frac{\partial u}{\partial t} + cu.$$

The equation appears, for example, in the analysis of the time-dependent **Boltzmann equation** in the theory of neutron transport. It appears if a correct P_1 approximation to the Boltzmann equation is made, and the second derivative insures that the proper retardation effects (based on the finite velocity of neutrons) appear in time-dependent solutions.

TELLER-REDLICH PRODUCT RULE. See **isotope effect in molecular spectra.**

TEMPERATURE. That property of systems which determines whether they are in thermodynamic equilibrium. Two systems are in equilibrium when their temperatures (measured on the same temperature scale) are equal. The existence of the property defined as temperature is a consequence of the zeroth law of **thermodynamics**. The zeroth law of thermodynamics leads to the conclusion that in the case of all systems there exist functions of their independent properties x_i such that at equilibrium

$$\phi_a(x_{ia}) = \phi_b(x_{ib}) = \theta \quad (a)$$

where subscripts a and b refer to two systems a and b each described by n_a properties x_{ia} and n_b properties x_{ib} respectively. The hyper-surface

$$\theta = \phi(x_i) = \text{constant}$$

is called an *isotherm*, and the pairs of hyper-surfaces (a) at equilibrium are called *corresponding isotherms*.

In order to establish a *temperature scale* it is necessary to assign numerical values θ to these corresponding isotherms in an arbitrary manner, subject only to the condition that the resulting function shall be single-valued. A temperature scale is established by taking the following steps:

(1) An arbitrary system is chosen (*thermometer*).

(2) It is agreed to maintain $n - 1$ properties of the system constant, and to use the n -th property (*thermometric property* $x_n = X$) as a measure of temperature θ .

(3) A single-valued *thermometric function* is assumed. Usually the function is simple, for example

$$\theta = aX \quad (b)$$

or

$$\theta = AX + B. \quad (c)$$

The function usually contains one or several constants (a, A, B , etc.).

(4) The values of the constants in the thermometric function are determined with reference to fixed thermometric points whose temperatures are arbitrarily assumed. The *fixed thermometric points* most frequently employed are: the *ice point*, *steam point* and *triple point* of water.

It is not surprising that there exist many different scales of temperature, so-called *empirical temperature scales*, because of the large amount of arbitrariness inherent in the choice.

A *Celsius* (or *Centigrade*) *temperature scale* is obtained by choosing the thermometric function (c) and assigning the following arbitrary values of temperature, θ , to the ice point (θ_i) and steam point (θ_s) respectively

$$\theta_i = 0^\circ\text{C}$$

$$\theta_s = 100^\circ\text{C}.$$

Hence on a Celsius scale

$$\theta = 100 \frac{X - X_i}{X_s - X_i} \quad (d)$$

(X_s measured at θ_s , X_i measured at θ_i).

A *Fahrenheit* temperature scale is obtained by using the function (e) but with

$$\theta_i = 32^\circ\text{F}$$

$$\theta_s = 212^\circ\text{F}.$$

Hence

$$\theta = 32^\circ + 180^\circ \frac{X - X_i}{X_s - X_i} \quad (\text{e})$$

An *absolute scale* is obtained by choosing Equation (b). Depending on the value assigned to a , we obtain the *Kelvin* or *Rankine scale*. The *Kelvin absolute temperature scale* assigns to the triple point of water the value

$$\theta_3 = 273.16^\circ\text{K} \quad (\text{f})$$

which thus becomes a *universal constant of physics*. Hence for the Kelvin scale

$$\theta = 273.16^\circ \frac{X}{X_3} \quad (\text{g})$$

where X_3 is measured at θ_3 . For the *Rankine absolute temperature scale* we assume

$$\theta_3 = 273.16 \times 1.8^\circ\text{R} = 491.69^\circ\text{R}. \quad (\text{h})$$

It is clear that by definition

$$1^\circ\text{K} = 1.8^\circ\text{R}, \quad (\text{i})$$

and that on the Rankine scale

$$\theta = 491.69^\circ \frac{X}{X_3} \quad (\text{j})$$

Different empirical temperature scales will naturally differ from each other except at the respective fixed thermometric points. Even different scales of the same type (say different Celsius scales) will differ at all temperatures, except the steam point and ice point, depending on the fortuitous properties of the system chosen as a thermometer. It is, therefore, necessary to remove these differences and to obtain a more universal scale. This has been achieved in two ways. The practical way of achieving uniformity is to lay down detailed rules concerning the thermometer (actually different thermometers depending on the range of temperatures to be measured). Such rules have been agreed on internationally and constitute the *international temperature scale*. Another way is to derive a universal scale from the principles of **thermodynamics**. The latter is called a *thermodynamic temperature scale*. Some authors refer to it as the *absolute temperature scale* which may be a source of confusion with the Kelvin and Rankine scales described earlier.

The *thermodynamic temperature scale* T is defined by the second law of **thermodynamics**

(**Carnot theorem**). It can be shown that the *thermodynamic temperature scale* is identical with the *perfect-gas temperature scale* defined as follows:

(1) The system is a gas thermometer (filled with a *real gas*).

(2) The thermometric property is the product pV *extrapolated to zero pressure*, i.e.,

$$r = \lim_{p \rightarrow 0} (pV). \quad (\text{k})$$

Hence the *thermodynamic Kelvin scale* is given by

$$T = 273.16 \frac{r}{r_3} \text{ (degrees K abs.)}, \quad (\text{l})$$

the *thermodynamic Celsius scale* is given by

$$t = 100 \frac{r - r_i}{r_s - r_i} \text{ (degrees C)}, \quad (\text{m})$$

the *thermodynamic Rankine scale* is given by

$$T = 491.69 \frac{r}{r_3} \text{ (degrees R abs.)}, \quad (\text{n})$$

and the *thermodynamic Fahrenheit scale* is given by

$$t = 32 + 180 \frac{r - r_i}{r_s - r_i} \text{ (degrees F)}. \quad (\text{o})$$

The zeros on the Kelvin and Rankine scales coincide and are termed the *absolute zero of temperature*. The absolute zero of temperature cannot be achieved by any finite process, as proved by the third law of **thermodynamics**.

The relation between the Celsius and Rankine thermodynamic scales is determined by

$$T_i = 273.16^\circ\text{K} = 0^\circ\text{C} \quad (\text{p})$$

and that between the Fahrenheit and Rankine scales is determined by

$$T_i = 491.69^\circ\text{R} = 32^\circ\text{F}. \quad (\text{q})$$

Hence the absolute zero of temperature is at

$$-273.16^\circ\text{C} \text{ or } -459.69^\circ\text{F}. \quad (\text{r})$$

The relation between the *international temperature scale* and the thermodynamic temperature scale must be determined empirically with the aid of careful measurements involving gas thermometers. Work on this comparison is now in progress at the National Bureau of Standards, at the Physikalisches-technische Bun-

desanstalt in Germany, in the USSR and in Japan.

TEMPERATURE, ABSOLUTE. See **temperature**.

TEMPERATURE COEFFICIENT OF CHEMICAL EQUILIBRIUM. See **Van't Hoff theorem**.

TEMPERATURE COEFFICIENT OF REACTIVITY. See **reactivity, coefficient of**.

TEMPERATURE GRADIENT. In a spatial temperature (scalar) field, $T(x,y,z)$, the associated vector field ∇T . This can often be expressed as

$$\lim_{\Delta s \rightarrow 0} \frac{t_2 - t_1}{\Delta s},$$

where t_1 and t_2 are the temperatures on two isothermal surfaces separated by a distance Δs . In many cases of linear heat flow, the value of $(t_2 - t_1)/\Delta s$ is independent of Δs and the average temperature gradient is identical at any point.

TEMPERATURE, NEGATIVE. A collection of atomic or molecular systems is said to be in a negative temperature state when more of the atoms (or molecules) are in high energy states than in low energy states. For example, consider a collection of quantum mechanical systems with only two energy levels (e.g., an atomic system with only two spin states). In equilibrium at a temperature T , the number in the higher energy state (2) relative to the number in the lower energy state (1) is given by the Boltzmann factor

$$N_2/N_1 = e^{-(E_2 - E_1)/kT},$$

where T is the absolute temperature. Suppose that T is defined by this equation, i.e.

$$T = - \frac{(E_2 - E_1)}{k \log_e (N_2/N_1)}.$$

Under normal conditions N_2 is less than N_1 and T is positive. If $N_2 = N_1$, T is infinite. If N_2 is greater than N_1 , then T is negative. As N_1 approaches zero, T approaches zero. Thus, if all of the atomic systems in a two level system were in the upper energy state (E_2), the absolute temperature would be zero, but it would have been approached from negative temperatures. On the other hand if all of the systems are in the lower energy state (E_1), the absolute

temperature would again be zero, but it would have been approached from positive temperatures.

TEMPERATURE, NEUTRAL. The temperature at which the electromotive force produced by heating one junction of a **thermocouple** the other junction being constant at 0°C , reaches a maximum. Above this temperature the electromotive force steadily decreases until it reaches a zero value. The latter point is called the temperature of inversion.

TEMPERATURE, NEUTRON. In the spectrum or distribution-in-energy of neutrons present in a chain reacting system, it is often possible to identify a "Maxwellian" component, of form $\phi_M \sim E \exp(-\beta E)$ in the low energy flux. In this case an effective neutron temperature, which is generally different from the moderator temperature, may be defined through the relation $\beta = (kT_{eff})^{-1}$. k is Boltzmann's constant.

TEMPERATURE RECOVERY FACTOR. For a body in a uniform gas stream of velocity V and temperature T_1 , let T_0 be the temperature at the stagnation point, with zero heat transfer. If the temperature at any other point on the body is T_w , with zero heat transfer, the temperature recovery factor is defined as

$$r = \frac{T_w - T_1}{T_0 - T_1}. \quad (1)$$

Alternatively, since $T_0 - T_1 = \frac{V^2}{2C_p}$, Equation 1 may be written

$$T_w = T_1 + r \frac{V^2}{2C_p}. \quad (2)$$

For a flat plate at zero incidence, with a **laminar boundary layer**, the temperature recovery factor is very nearly equal to the square root of the **Prandtl number**. For a flat plate with a **turbulent boundary layer** the recovery factor is rather greater, but still less than unity.

TEMPERATURE SCALES. See **temperature**.

TENDENCY (OF BAROMETRIC PRESSURE). The change in atmospheric pressure during the last three hours; a convenient approximation to the local rate of change of pressure for use on weather charts.

The thickness tendency is the change in

thickness (see **thickness pattern**) between two standard isobaric surfaces in a specified time.

TENSION, COEFFICIENT OF. See **coefficient of tension**.

TENSION DIAGONAL. See **diagonal, tension**.

TENSION FIELD. When the shear in the web of a beam exceeds the buckling load, the web wrinkles and, if the web is thin, the shear is carried primarily by tension parallel to the corrugations as the load is increased. Tension fields are depended upon extensively in the design of subsonic aircraft, because the buckling load of thin sheets is well within the elastic range and deformations are recoverable.

TENSOR. See **tensor field**.

TENSOR, ABSOLUTE (TENSOR FIELD). Tensor (**tensor field**) of weight zero. Often called tensor when context admits no confusion.

TENSOR, ALTERNATING. An absolute tensor the components of which in each coordinate system are the elements of an ϵ -system (**epsilon-system**).

TENSOR ANALYSIS. The theory of the mathematical operations which may be performed with tensors.

TENSOR, ANTI-SYMMETRIC. See **tensor, skew-symmetric**.

TENSOR CALCULUS. See **absolute differential calculus**.

TENSOR, CARTESIAN. See **tensor field, Cartesian**.

TENSOR, CONTRAVARIANT. See **tensor field**.

TENSOR, CONTRAVARIANT METRIC. See **metric tensor**.

TENSOR, COVARIANT. See **tensor field**.

TENSOR, COVARIANT METRIC. See **metric tensor**.

TENSOR, CURVATURE. See **curvature tensor**.

TENSOR, DENSITY. Relative tensor (**tensor field**) of weight unity.

TENSOR DERIVATIVE, INTRINSIC. See **intrinsic derivative of tensor field**.

TENSOR DERIVATIVE OF A TENSOR FIELD. A tensor field may be a space tensor field with respect to transformation on some of its indices (e.g., Latin indices) and a surface tensor field with respect to transformation on others (e.g., Greek indices). As an example, consider the tensor field which has components $t^i_{\alpha\beta}$ in the space coordinate system x and the surface coordinate system u . The tensor derivative of the tensor field with respect to u^γ is denoted $t^i_{\alpha\beta,\gamma}$ where

$$t^i_{\alpha\beta,\gamma} = \frac{\partial t^i_{\alpha\beta}}{\partial u^\gamma} + (g)\Gamma^i_{jk}t^j_{\alpha\beta}x^k_{,\gamma} - (a)\Gamma^\delta_{\alpha\gamma}t^i_{\delta\beta} - (a)\Gamma^\delta_{\beta\gamma}t^i_{\alpha\delta}.$$

$(g)\Gamma^i_{jk}$ and $(a)\Gamma^\alpha_{\beta\gamma}$ are the **Christoffel symbols** of the second kind formed from the covariant **metric tensors** for the coordinate systems x and u respectively. Similar definitions apply to the tensor derivatives of tensor fields of different orders.

TENSOR, DOUBLE. Tensor of total order two.

TENSOR, EIGENVALUE EQUATION OF. See **eigenvalue equation of tensor**.

TENSOR, ENERGY-MOMENTUM. See **energy-momentum tensor**.

TENSOR FIELD. In each n -dimensional curvilinear coordinate system n^{M+N} functions of position are specified. Using the **indicial notation** let us denote those defined in the coordinate system x by $t^{i_1 i_2 \dots i_M}_{k_1 k_2 \dots k_N}$ and those defined in the coordinate system \bar{x} by $\bar{t}^{i_1 i_2 \dots i_M}_{k_1 k_2 \dots k_N}$. In each case the M superscripts and N subscripts take values ranging over 1, 2, \dots , n independently. The functions are related by

$$\bar{t}^{i_1 i_2 \dots i_M}_{k_1 k_2 \dots k_N} = \left| \frac{\partial x}{\partial \bar{x}} \right|^W \frac{\partial \bar{x}^{j_1}}{\partial x^{p_1}} \frac{\partial \bar{x}^{j_2}}{\partial x^{p_2}} \dots \frac{\partial \bar{x}^{j_M}}{\partial x^{p_M}} \frac{\partial x^{q_1}}{\partial \bar{x}^{k_1}} \frac{\partial x^{q_2}}{\partial \bar{x}^{k_2}} \dots \frac{\partial x^{q_N}}{\partial \bar{x}^{k_N}} t^{p_1 p_2 \dots p_M}_{q_1 q_2 \dots q_N},$$

in which the **summation convention** is employed, $|\partial x/\partial \bar{x}|$ denotes the Jacobian determinant $\partial(x^1, x^2, \dots, x^n)/\partial(\bar{x}^1, \bar{x}^2, \dots, \bar{x}^n)$, W is an integer (positive or negative) and the summation convention is employed. The aggregate of sets of functions defined in this manner in every curvilinear coordinate system is called an n -di-

mensional *tensor field* of weight W . The tensor field is said to be *contravariant* of order M and *covariant* of order N . It is said to have *total order* or *order* $M + N$. If $M = 0$ the tensor field is said to be a *covariant tensor field*. If $N = 0$, it is said to be a *contravariant tensor field*. If neither M nor N is zero, it is said to be a *mixed tensor field*.

The set of functions $t_{k_1 k_2 \dots k_N}^{j_1 j_2 \dots j_M}$ are called the *components of the tensor field* in the coordinate system x ; similarly the set of functions $\bar{t}_{k_1 k_2 \dots k_N}^{j_1 j_2 \dots j_M}$ are called the components of the tensor field in the coordinate system \bar{x} . The superscripts are called *contravariant indices* and the subscripts *covariant indices*.

At each point of space at which the tensor field is defined, it determines a set of n^{M+N} quantities in each curvilinear coordinate system. The aggregate of these sets of quantities is called an n -dimensional tensor at the point considered. The set of n^{M+N} quantities determined in the coordinate system x are the components of the tensor in the coordinate system x . The definitions of weight, contravariance, covariance, and order of a tensor are analogous to those for a tensor field.

A tensor field is often referred to as a *tensor*, when no confusion is likely to arise.

TENSOR FIELD, CARTESIAN. In each n -dimensional rectangular Cartesian coordinate system n^M functions of position are specified. Let us denote the functions defined in the rectangular Cartesian coordinate systems x and \bar{x} by $t_{i_1 i_2 \dots i_M}$ and $\bar{t}_{i_1 i_2 \dots i_M}$ respectively. In each case the M subscripts i_1, i_2, \dots, i_M take values ranging over $1, 2, \dots, M$ independently. The functions are related by

$$\bar{t}_{i_1 i_2 \dots i_M} = \frac{\partial \bar{x}_{i_1}}{\partial x_{j_1}} \frac{\partial \bar{x}_{i_2}}{\partial x_{j_2}} \dots \frac{\partial \bar{x}_{i_M}}{\partial x_{j_M}} t_{j_1 j_2 \dots j_M}$$

where the summation convention is used and the relation is valid for all choices of the rectangular Cartesian coordinate systems x and \bar{x} . The aggregate of sets of n^M functions so defined is called an n -dimensional Cartesian tensor field. M is called the *order* of the Cartesian tensor field. The set of functions $t_{i_1 i_2 \dots i_M}$ are called the components of the Cartesian tensor field in the coordinate system x ; similarly the set of functions $\bar{t}_{i_1 i_2 \dots i_M}$ are called the components of the Cartesian tensor field in the coordinate system \bar{x} .

At each point of space at which the Cartesian tensor field is defined, it determines a

set of n^M quantities in each rectangular Cartesian coordinate system. The aggregate of these sets of quantities is called an n -dimensional *Cartesian tensor* at the point considered. The set of n^M quantities determined in the coordinate system x are called the *components of the tensor* in the coordinate system x .

A Cartesian tensor field is often referred to as a *Cartesian tensor* and, particularly in applied mathematical contexts, as a *tensor*.

TENSOR FIELD, COVARIANT DERIVATIVE OF. See *covariant derivative of tensor field*.

TENSOR, FUNDAMENTAL. See *metric tensor*.

TENSOR, ISOTROPIC. A tensor which has the same components in all rectangular Cartesian coordinate systems which are related by a rigid rotation.

TENSOR, LOWERING INDICES ON. See *raising and lowering indices on a tensor*.

TENSOR, METRIC. See *metric tensor*.

TENSOR, MIXED. See *tensor field*.

TENSOR MULTIPLICATION, INNER. See *tensors, inner product of*.

TENSOR, PRINCIPAL DIRECTIONS OF SYMMETRIC SECOND-ORDER. See *principal directions of symmetric second-order tensor*.

TENSOR PRODUCT. See *product of tensors*.

TENSOR PRODUCT, INNER. See *tensors, inner product of*.

TENSOR, RAISING INDICES ON. See *raising and lowering indices on a tensor*.

TENSOR, RELATIVE. Tensor (tensor field) of non-zero weight.

TENSOR, RICCI. See *Ricci tensor*.

TENSOR, RIEMANN-CHRISTOFFEL. See *Riemann-Christoffel tensor*.

TENSORS, ADDITION OF. See *sum of tensors*.

TENSORS, AFFINE. See *affine tensors and free vectors*.

TENSORS, ASSOCIATED. Two tensors (see **tensor field**) are said to be associated tensors (tensor fields) if one can be obtained from the other by a series of inner multiplications by the covariant or contravariant **metric tensor**.

TENSORS, DUAL, RELATION TO SPINORS. See **spinor calculus**.

TENSORS, FOUR VECTORS AND. See **four vectors and tensors**.

TENSORS (TENSOR FIELDS), INNER PRODUCT OF. The tensor (**tensor field**) formed from the outer **product of two tensors** (tensor fields) by setting a covariant index on one of the tensors (tensor fields) equal to a contravariant index on the other tensor (tensor field) and running over the range of values appropriate to the repeated index in accordance with the **summation convention**. In Cartesian tensor analysis, the Cartesian tensor (tensor field) formed from the outer product of two Cartesian tensors (tensor fields) by setting a subscript on one of the Cartesian tensors (tensor fields) equal to a subscript on the other Cartesian tensor (tensor field) and summarize in accordance with the summation convention.

TENSOR, SKEW-SYMMETRIC. If the interchange of two covariant or of two contravariant indices in the components of a tensor changes the signs of the components but not their absolute values, the tensor is said to be skew-symmetric (or *anti-symmetric*) with respect to those indices. A covariant or contravariant tensor of order two is said to be skew-symmetric (or anti-symmetric) if its components are changed in sign but not in absolute value by interchange of the indices.

TENSORS, MULTIPLICATION OF. See **product of tensors**.

TENSORS, MULTIPLICATION OF (INNER). See **tensors, inner product of**.

TENSORS, OUTER PRODUCT OF. See **product of tensors**.

TENSOR, SPACE. Tensor defined for transformations of space coordinates.

TENSORS, PRODUCT OF. See **product of tensors**.

TENSORS, QUOTIENT LAW FOR. See **quotient law for tensors**.

TENSORS, SUM OF. See **sum of tensors**.

TENSOR, SURFACE. Tensor defined for transformations of surface coordinates. A tensor may be a space tensor (see **tensor, space**) with respect to transformation on some of its indices and a surface tensor with respect to transformation on others.

TENSOR, SURFACE, INTRINSIC DERIVATIVE OF. See **surface intrinsic derivative**.

TENSOR, SYMMETRIC. If the interchange of two covariant or of two contravariant indices in the components of a tensor does not alter the values of the components, then the tensor is said to be symmetric with respect to those indices. A covariant or contravariant tensor of order two is said to be symmetric if its components are unaltered by interchange of the indices. (See **indicial notation**.)

TENSOR, TYPE OF. If two tensors (see **tensor field**) have the same order of covariance, the same order of contravariance and the same weight, they are said to be of the same type.

TENSOR, WEIGHTED (TENSOR FIELD). A relative tensor (**tensor field**).

TENSOR, WEIGHT OF. See **tensor field**.

TERM, ENERGY STATE. See **energy state term**.

TERM (IN SPECTROSCOPY). The quantity

$$T = \frac{E}{hc}$$

is referred to as "term" or "term value." Here E stands for the energy of an atomic or molecular state (relative to a given zero value), h , for Planck's constant, and c , for the velocity of light. The **wavenumber** ν of a spectral line is equal to the difference between the term values of the energy levels involved in the transition. (See **Bohr frequency condition; combination principle**.)

TERMINAL VERTEX. See **vertex, terminal**.

TERMINAL VELOCITY. See **velocity, terminal**.

TERM SYMBOLS, ATOMIC. See **atomic term symbols**.

TERM SYMBOLS, MOLECULAR. See molecular term symbols.

TERNARY SYSTEMS. As first pointed out by Gibbs, the composition of a solution containing three components may be represented by a point in an equilateral triangle whose vertices represent the three pure components. If the side of the triangle is taken as unity, then the mole fractions x_A, x_B, x_C in the solution under consideration are given by the distances, measured along lines parallel to the sides of the triangle, of the point P at distances from the sides of the triangle, A, B and C , respectively. This representation insures automatically that

$$x_A + x_B + x_C = 1.$$

TEST FUNCTION. A complex valued function $\phi(x_1, \dots, x_n)$ of n real variables that are m times continuously differentiable ("of class C^m ") with $1 \leq n < \infty$ and $0 \leq m \leq \infty$. In most cases the test functions are taken to be indefinitely differentiable (C^∞) functions. The variables $x_1 \dots x_n$ range over n -dimensional Euclidean Space R^n . The test functions are defined on all R^n . The complement of the largest open set where $\phi(x_1 \dots x_n)$ is zero is called the support of $\phi(x)$.

TETRACHORIC CORRELATION. If a bivariate normal distribution is dichotomized in two directions, the value of ρ , the **correlation coefficient**, is uniquely determined by the proportions falling into the four resulting categories. If an observed 2×2 contingency table is regarded as having arisen in this way, an estimate of ρ can be obtained, using tables of the bivariate normal distribution. Such an estimate is called a tetrachoric correlation.

TETRAGONAL SYSTEM. One of the seven crystal systems. In this system, the three axes are mutually perpendicular, and two are of equal length.

TETRALITY PRINCIPLE. The optical differential invariant

$$s_u a_v - s_v a_u = s'_u a'_v - s'_v a'_u$$

is invariant under the transformatives

$$(1) \quad \mathbf{a} \rightarrow -\mathbf{s}, \quad \mathbf{s} \rightarrow \mathbf{a},$$

$$(2) \quad \mathbf{a}' \rightarrow -\mathbf{s}', \quad \mathbf{s}' \rightarrow \mathbf{a}',$$

so the optical properties of a system are preserved under the group of transformations generated by (1) and (2).

TETRATOHEDRAL CRYSTALS. That class of crystal symmetry which has only one quarter of the maximum number of faces allowed by the crystal system to which the class belongs.

THEODORSEN METHOD. See airfoil theory (two-dimensional).

THEOREM OF MINIMUM ENTROPY PRODUCTION. See time variation of the entropy production.

THEOREM OF MINIMUM POTENTIAL ENERGY. See potential energy, theorem of minimum.

THEOREMS OF CONSTRAINT. See Chatelier (Le)-Braun principle.

THEOREMS OF MODERATION. See Chatelier (Le)-Braun principle.

THEORETICAL TEMPERATURE OF COMBUSTION. See combustion.

THEORY OF ESTIMATION. See estimation, theory of.

THEORY OF GAMES. See games theory.

THERMAL. (As a noun.) A mass of buoyant (usually warm) fluid rising through denser surroundings. A term first used by glider pilots to refer to the aerial convection currents in which they were able to soar over hot ground.

The motion in an isolated thermal rising in uniform, otherwise undisturbed, surroundings consists of a circulation rather like that of a vortex ring, but with a region of turbulent mixing in about the upper third of the volume.

The velocity decreases like z^{-1} and the concentration (or temperature excess) like z^{-3} , z being measured from the vertex of the cone of semi-angle about 15° which envelops the path of the thermal. Of the exterior fluid entrained, about half enters into the mixing region on the upper half of the surface while the rest enters as a laminar stream at the rear (as with a vortex ring) and becomes mixed when it reaches the upper part of the thermal.

The circulation round the thermal measured in a circuit up the axis and closed outside it is constant.

THERMAL BOUNDARY LAYER. See boundary layer in a compressible fluid.

THERMAL COEFFICIENTS. The principle of conservation of energy (see **energy conservation in closed systems**) called also the first law of thermodynamics gives for the heat dQ received during the time dt

$$dQ = dU + pdV. \quad (1)$$

In variables T, V, ζ , where ζ is the extent of reaction (see **extent of reaction**) this gives

$$dQ = C_{V,\xi}dT + l_{T,\xi}dV + u_{T,V}d\xi. \quad (2)$$

where

$$C_{V,\xi} = \left(\frac{\partial U}{\partial T}\right)_{V,\xi}, \quad l_{T,\xi} = p + \left(\frac{\partial U}{\partial V}\right)_{T,\xi},$$

$$u_{T,V} = \left(\frac{\partial U}{\partial \xi}\right)_{T,V}. \quad (3)$$

$C_{V,\xi}, l_{T,\xi}, u_{T,V}$ are called the thermal coefficients for the variables T, V , and ξ .

At constant V and ξ , we have

$$dQ = C_{V,\xi}dT \quad (4)$$

so that $C_{V,\xi}$ is the amount of heat required to raise the temperature of the system one degree, while both the composition and volume of the system are unchanged. $C_{V,\xi}$ is called the heat capacity at constant volume and composition. When referred to a mole of substance, it is called the molar heat capacity or specific heat at constant volume and composition.

At constant T and ξ , we have

$$dQ = l_{T,\xi}dV \quad (5)$$

and $l_{T,\xi}$ is thus the heat which must be supplied to the system to maintain a constant temperature when the volume is increased by a unit amount in the absence of chemical reactions or phase change. It is called the latent heat of volume change.

For a perfect gas

$$l_{T,\xi} = p. \quad (6)$$

At constant T and V

$$dQ = u_{T,V}d\xi. \quad (7)$$

$u_{T,V}$ is the heat received by the system when the reaction proceeds an extent $d\xi$ at constant T and V .

Thus if $u_{T,V}$ is negative the reaction proceeds with the evolution of heat (exothermic reaction) while if $u_{T,V}$ is positive, heat is ab-

sorbed when the reaction proceeds (endothermic reaction *).

We have similar relations for the thermal coefficients in variables p, T, ξ . Instead of (2) we have now

$$dQ = C_{p,\xi}dT + h_{T,\xi}dp + h_{T,p}d\xi \quad (8)$$

where the thermal coefficients $C_{p,\xi}, h_{T,\xi}, h_{T,p}$ may be easily expressed in terms of the **enthalpy** H

$$C_{p,\xi} = \left(\frac{\partial H}{\partial T}\right)_{p,\xi}, \quad h_{T,\xi} = \left(\frac{\partial H}{\partial p}\right)_{T,\xi} - V,$$

$$h_{T,p} = \left(\frac{\partial H}{\partial \xi}\right)_{T,p}. \quad (9)$$

$C_{p,\xi}$ is the heat capacity of the system at constant pressure and composition; $h_{T,\xi}$ is the latent heat of pressure change at constant temperature and composition. For a **perfect gas**

$$h_{T,\xi} = -V. \quad (10)$$

Finally $h_{T,p}$ is the heat of reaction at constant T and p ; it is the amount of heat absorbed by the system for unit reaction at constant T and p . From (9) it can be deduced that the heat of reaction is the difference between the sum of the partial molar enthalpies of the products of reaction (see **partial molar quantities**) each multiplied by its stoichiometric coefficient and the corresponding sum for the reactants

$$h_{T,p} = \left(\frac{\partial H}{\partial \xi}\right)_{T,p} = \sum_i \nu_i h_i. \quad (11)$$

There exist simple relations between the thermal coefficients in variables V, T, ξ and p, T, ξ . (See also **Clausius equations** for the relations between thermal coefficients.)

THERMAL CONDUCTIVITY. See Fourier law.

THERMAL CONDUCTIVITY COEFFICIENT. See transport coefficient.

THERMAL CONDUCTIVITY, KINETIC THEORY OF. In a gas in which there exists

* The concept of endothermic (and exothermic) reactions has been extended to nuclear reactions to describe those reactions in which there is a net absorption (or evolution) of energy, that is, a reaction in which the total kinetic energy of the reactants is greater (or less) than that of the products. More precisely it is one in which the Q -value is negative (or positive). (See **reaction energy**.)

a temperature gradient, there will be a flow of energy from the hotter to the cooler regions. The thermal conductivity coefficient κ is defined by the equation

$$w = -\kappa \frac{dT}{dx},$$

where w is the heat (energy) current in the x -direction and dT/dx the temperature gradient.

In kinetic theory one assumes that the heat current is due to particles of higher energy moving towards colder regions, and vice versa. By considering the net flow of energy through a unit area perpendicular to the x -direction one finds

$$\kappa = c\eta c_v,$$

where c is a numerical constant of order unity, c_v the specific heat per unit mass, and η the coefficient of viscosity.

THERMAL DIFFUSION COEFFICIENT. See **transport coefficient**.

THERMAL EFFICIENCY OF CYCLE. The thermal or thermodynamic efficiency of a cycle is defined as the ratio of the work of the cycle to the sum of all positive quantities of heat absorbed by the cycle:

$$\eta = \frac{W}{Q}.$$

Since the first law of **thermodynamics** asserts that $W = Q - |Q_o|$ where $|Q_o|$ is the sum of all negative quantities of heat, we also have

$$\eta = 1 - \frac{|Q_o|}{Q}.$$

The latter form is found to be more convenient in practice when the thermal efficiency of particular cycles is evaluated.

The concept of thermal efficiency is applied only to cycles for which $W > 0$ (power cycles) and not to ones for which $W < 0$ (reversed cycles: refrigerator or heat pump). The thermal efficiency ($W > 0$) is a measure of the amount of heat ($Q - |Q_o|$) converted into work by the cycle in relation to the heat absorbed, Q , whose provision involves expense. The excellence of reversed cycles is judged in a different manner. (See **coefficient of performance**, **heat pump effectiveness**.)

THERMAL EQUATION OF STATE. See **caloric equation of state**.

THERMAL EQUILIBRIUM. The boundary separating two systems is said to be *thermally conducting* if it has the following property: If any two separate systems each in complete internal equilibrium are brought together so as to be in contact through a thermally-conducting wall, the two systems will be found in general not to be in mutual equilibrium but they will adjust themselves till a new mutual equilibrium is reached. The two systems are then said to have reached a state of thermal equilibrium.

If two systems are both in thermal equilibrium with a third system, then they are in thermal equilibrium with each other.

This proposition is sometimes called the *zeroth law of thermodynamics*.

THERMAL EXPANSION OF SOLID. As a consequence of the **anharmonic terms** in the potential energy of a solid, when the atoms have large amplitudes of thermal vibration their average positions tend to move apart. It can be shown that the expansion coefficient is nearly constant, being expressed as

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p$$

at high temperatures and constant pressure (see **Grüneisen's constant**), while at low temperatures a relation of the form

$$\delta V \propto \delta U$$

holds, where δV is the change in volume from the absolute zero and δU is the thermal energy of the lattice.

THERMALIZATION OF NEUTRONS. The study of the manner in which fast neutrons, being moderated in a material system, reach a steady-state distribution in space and energy, in the thermal neutron region of the spectrum, as well as a study of the thermal distribution itself.

THERMAL MOTIONS IN A LATTICE. Rising temperature causes an increase in the oscillations of particles in a lattice, and hence weakens the intensity of the Laue diffraction pattern. The effective scattering factor, f (at wavelength λ) is then related to the scattering

factor at rest, f_0 , by the Debye-Waller equation

$$f = f_0 e^{-M}$$

where

$$M = \frac{6h^2 T}{mk\Theta_M^2} \frac{\sin^2 \theta}{\lambda} \left\{ \frac{1}{x} \int_0^\infty \frac{\xi d\xi}{e^\xi - 1} + \frac{x}{4} \right\}.$$

h and k are Planck's and Boltzmann's constants, m is the mean atomic mass, $x = h\nu_0/kT$ where ν_0 is the **Debye frequency** and Θ_M is a parameter a few per cent greater than the Debye temperature and is given by

$$\Theta_M = (\sqrt{3} h/K)(3N/4\pi)^{1/3} K^{1/2} W^{1/3} \rho^{1/3} f(\sigma)^{1/3}.$$

K is cubic compressibility, W is atomic weight, ρ is density, σ is **Poisson's ratio**, and N is Avogadro's number, and

$$f(\sigma) = [(1 + \sigma)/3(1 - \sigma)]^{3/2} + 2[2(1 + \sigma)/3(1 - 2\sigma)]^{3/2}.$$

THERMAL NEUTRON. See **neutron, thermal**.

THERMAL POTENTIAL. Also called **Gibbs function**, **Gibbs free energy**, or **thermodynamic potential**, it is defined by the equation

$$G = U - TS + pV,$$

where G is the thermal potential, U is the internal energy, T is the absolute temperature, S is the entropy, p is the pressure, and V is the volume.

THERMAL RADIATION. Emission of radiation in which the radiant energy originates in the thermal excitation of atoms or of molecules.

THERMAL ROSSBY NUMBER. The non-dimensional ratio of the **inertial force** due to the **thermal wind** and the **Coriolis force** in the flow of a fluid which is heated from below.

$$R_{oT} = \frac{U_T}{fL},$$

where f is the **Coriolis parameter**, L a characteristic length, and U_T a characteristic thermal wind. The characteristic thermal wind is

$$U_T = \frac{g\epsilon\delta}{f} \frac{\Delta_r\theta}{\Delta r}$$

where g is the acceleration of gravity, ϵ the **coefficient of thermal expansion**, $\Delta_r\theta/\Delta r$ a char-

acteristic radial temperature gradient, and δ depth of the fluid. Compare **Rosby number**. (See **thermal instability**.)

THERMAL SHOCK. The situation in which very large thermal stresses are produced by a steep transient gradient of temperature. Brittle materials may fracture, while ductile materials can absorb the "shock" by small plastic deformations.

THERMAL STABILITY. (See **thermodynamic stability conditions**; **stability of phases**.) A phase must satisfy certain conditions if it is to be stable (or metastable). For systems consisting of a single component the necessary and sufficient conditions are the conditions of thermal stability and of **mechanical stability**. The condition of thermal stability is

$$C_V > 0.$$

The heat capacity at constant volume of all stable (or metastable) phases is positive.

THERMAL UNITS. Thermal measurements involve, in addition to **mechanical units**, the specification of temperature. For a discussion of **temperature scales**, see that entry.

Since heat is a form of energy, any of the mechanical units of energy, such as the erg, joule, or foot pound, may be used to measure quantity of heat. Other units, based on the thermal properties of water, had become well established before the first law of **thermodynamics** was enunciated. The use of these units persists, and several of them are used interchangeably with the mechanical units in this dictionary. The most widely accepted is the **calorie** (cal) which was originally defined as that quantity of heat which is required to raise the temperature of 1 gram of water by 1°C under atmospheric pressure. Since the specific heat of water varies with temperature, this definition was changed to require the heating to be performed from 14.5°C to 15.5°C, and resulting unit is called the 15-degree calorie (1 cal_{15°}). In most cases 1 cal is too small a unit and its 1000 multiple, the kilocalorie (kcal, kcal_{15°}) is used. In order to provide a simple link with mechanical or electrical measurements, the International Steam Tables Conference adopted the IT-kcal by the definition

$$860 \text{ kcal}_{IT} = 1 \text{ kw hr}_{\text{int}} = 3.6 \times 10^6 \text{ joule}_{\text{int}}.$$

In chemical measurements use is made of the thermochemical calorie:

$$1 \text{ kcal}_{15^\circ} = 0.99968 \text{ kcal}_{IT}$$

$$= 1.00036 \text{ kcal}_{th \text{ chem.}}$$

In order to avoid all inconvenience, it is now recommended to measure quantities of heat directly in mechanical units, i.e., in **joules**, where

$$1 \text{ J} = 1 \text{ Nm} = 1 \frac{\text{kg m}^2}{\text{sec}^2}$$

The equivalent unit in the English system is the British thermal unit (B.Th.U. or Btu) originally defined as $\frac{1}{180}$ of the quantity of heat required to raise the temperature of water from its freezing point to its boiling point at a pressure of one standard atmosphere. At present it is preferred to link the unit directly with the IT-kcal by the definition

$$1 \text{ kcal}_{IT}/\text{kg} = 1.8 \text{ Btu}/\text{lb m.}$$

Hence

$$1 \text{ Btu} = 0.252074 \text{ kcal}_{15^\circ} = 0.251996 \text{ kcal}_{IT}$$

$$= 1055.056 \text{ J} = 1,075.857 \text{ kpm}$$

$$= 2.93071 \times 10^{-4} \text{ kw hr}_{int.}$$

Some authors prefer to use an alternative unit to the Btu which is based on the Celsius scale, namely the Centigrade heat unit (C.H.U. or Chu), originally defined as $\frac{1}{100}$ of the quantity of heat required to raise the temperature of water from its freezing point to its boiling point at a pressure of one standard atmosphere. It is preferable to define this unit by the equation

$$1 \text{ kcal}_{IT}/\text{kg} = 1 \text{ Chu}/\text{lb m.}$$

The centigrade heat unit is now largely obsolete.

The four basic systems of thermal units em-

ployed in this volume are summarized in the table. Relations among derived units, such as those of specific heat, entropy, etc., may be obtained by methods identical with that outlined under **mechanical units**. For example:

$$\frac{0.0235 \text{ Btu}}{^\circ\text{F}} = \frac{0.0235 \text{ Btu}}{^\circ\text{F}}$$

$$\times \frac{1 \text{ joule}}{9.478(10)^{-4} \text{ Btu}} \times \frac{1.80^\circ\text{F}}{1^\circ\text{K}}$$

$$= \frac{44.6 \text{ joule}}{^\circ\text{K}}$$

THERMAL UTILIZATION. In a neutron chain reactor of infinite extent, the thermal utilization, denoted f , is the ratio of the number of thermal neutrons absorbed by fuel material per unit time to the total number of thermal neutrons absorbed per unit time. In a reactor of finite size, f is no longer an intensive quantity and depends slightly upon neutron leakage.

THERMAL WIND. The rate of change with height (z) of the **geostrophic wind**, so-called because it is proportional to the horizontal temperature gradient. Thus

$$f \times \frac{\partial v_G}{\partial z} = \frac{g}{T} \text{ grad}_H T$$

where T is the absolute temperature. If the x -axis is taken horizontally at right angles to the isotherms the thermal wind strength is

$$\frac{\partial v_G}{\partial z} = \frac{g}{fT} \frac{\partial T}{\partial x},$$

in a direction along the isotherms.

RELATIONS AMONG THERMAL UNITS

Quantity	MKS °C System	Equivalents in Other Systems			
		MKS °K System	cgs °K System	f lbm s °F System	f lbm s °R System
Temperature difference	1°C	1°K	1°K	1.80°F	1.80°R
Temperature	$x^\circ\text{C}$	$(273.16 + x)^\circ\text{K}$	$(273.16 + x)^\circ\text{K}$	$(32 + 9x/5)^\circ\text{F}$	$(491.7 + 9x/5)^\circ\text{F}$
Energy	1 joule = 0.2390 cal	1 joule	$(10)^7 \text{ erg}$	9.478(10) ⁻⁴ Btu 0.7153 ft lbf	9.478(10) ⁻⁴ Btu

The thermal wind is the best practical approximation to the vertical gradient of the horizontal wind above the friction layer derivable from temperature and pressure observations. (See also **thickness pattern**.)

THERMIONIC EMISSION. See **Richardson-Dushman equation**; **Schottky effect**.

THERMODYNAMIC COORDINATE. See **thermodynamic property**.

THERMODYNAMIC EFFICIENCY OF CYCLE. See **thermal efficiency of cycle**.

THERMODYNAMIC EQUILIBRIUM. The general condition for thermodynamic equilibrium is that the entropy production vanishes (see **thermodynamics, second law of**; **entropy production**). This implies that all rates of irreversible processes vanish.

The conditions of thermodynamic equilibrium may also be expressed in terms of the characteristic functions (see **thermodynamics, characteristic functions of**). They are:

For given S and V , U is a minimum.

For given S and p , H is a minimum.

For given T and V , A is a minimum.

For given T and p , G is a minimum.

The minima of U , H , A , G correspond to *stable equilibria* (see **thermodynamic stability conditions**). A maximum would correspond to *unstable situations*. (See also **chemical potentials**; **chemical affinity**; **law of mass action**; **vapor pressure**; **thermal equilibrium**.)

THERMODYNAMIC EQUILIBRIUM IN GRAVITATIONAL FIELD. The conditions of thermodynamic equilibrium as regards the species i between two phases α and β defined not only by their temperature, pressure and composition but also by their gravitational potentials are

$$\mu_i^\alpha + M_i\phi^\alpha = \mu_i^\beta + M_i\phi^\beta \quad (1)$$

where ϕ is the gravitational potential, μ_i , the **chemical potential** and M_i , the **molar mass**.

THERMODYNAMIC FUNCTIONS OF MIXING. In order to separate the effects of mixing from the effects of changes in temperature or pressure, it is useful to compare the mixture to the pure components taken at

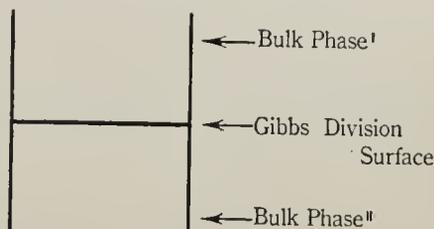
the same pressure and temperature. For example in the case of a binary system,

$$G^M = G^0(p, T, n_1, n_2) - G_1^0(p, T, n_1) - G_2^0(p, T, n_2) \quad (1)$$

is the change of the **Gibbs function** due to mixing, or the *Gibbs free energy of mixing*.

The enthalpy of mixing has a specially simple physical meaning. Since the mixing process is taking place at constant pressure and temperature the enthalpy of mixing is equal to the heat received by the system during the mixing. For this reason it is also called the *heat of mixing*.

THERMODYNAMIC FUNCTIONS OF SURFACE PHASES. Thermodynamic functions of surface phases may be easily defined in terms of **Gibbs division surface**. For example, the **Helmholtz function** (**Helmholtz free energy**) A^σ of a surface phase is defined in terms of the total free energy of the system A and



Thermodynamic functions of surface phases.

the free energies of the bulk phases prime and second prime (see figure) by

$$A^\sigma = A - A' - A'' \quad (1)$$

One may also introduce thermodynamic functions of surface phases per *unit area*, for example

$$A^\sigma = \frac{A}{\Omega} \quad (2)$$

is the **Helmholtz free energy per unit area**, where Ω is the area of the surface phase.

THERMODYNAMIC POTENTIAL. See **thermal potential**.

THERMODYNAMIC PROPERTY. (Also called *property* for short. Other synonyms are *parameter of state*, *thermodynamic coordinate*.) Any measurable, macroscopic characteristic of a (thermodynamic) system in terms of which its behavior can be studied quantitatively. Whenever a system is in (thermodynamic) equilibrium, all its properties assume

fixed values. The set of values of the properties of a system constitute its *state*. The particular quantities which are considered in a given case are determined empirically and through an understanding of the behavior of a given system undergoing a specified process. Examples of properties are pressure, temperature, specific volume, refractive index, viscosity, magnetic induction, magnetic field intensity, concentration of a chemical species in a reacting mixture, etc.

In the case of homogeneous systems, a distinction is made between *intensive properties* and *extensive properties*. An intensive property has the same value for the whole system, as for any part thereof. Examples are pressure, temperature, and magnetic field intensity. The value of an extensive property is proportional to the mass of the homogeneous system. Examples are volume and magnetization. Often extensive properties are referred to unit mass of the homogeneous system. They are then called *specific properties*. Specific properties, like intensive properties, are independent of the mass of the system. If X is any extensive property of a homogeneous system (e.g., volume V) and m is the mass of the system, then

$$x = X/m$$

is the corresponding specific property (e.g., the specific volume $v = V/m$).

The mathematical nature of a thermodynamic potential, such as **internal energy**, **enthalpy**, **Helmholtz function**, **Gibbs function**, etc., is akin to that of a property. If the thermodynamic potentials are measured from a fixed (but arbitrary) reference state, the resulting values are fixed and unique, whenever the state of the system is fixed. It must, however, be realized that the thermodynamic potentials (like, e.g., electric or gravitational potentials) have no natural zero. Consequently, no meaning can be attached to the *difference* in their values for two parts of one system, because this will depend on the value ascribed to the potential at the reference state. For example, if the internal energy of a homogeneous system of mass m at a given state 1 is U_{o1} with respect to an arbitrary state 0, and if at the latter it is assumed that the internal energy is U_o , we can write for state 1

$$U_1 = U_{1o} + U_o.$$

If we now consider a mass m' of the same system, at the same state (as characterized by the intensive and specific properties), we have

$$U'_1 = U'_{1o} + U'_o.$$

We can assert that

$$\frac{U'_1}{U_1} = \frac{m'u_{1o}}{mu_{1o}} = \frac{m'}{m}, \quad (u_{1o} = U_{1o}/m = U'_{1o}/m')$$

if we also choose

$$\frac{U'_o}{U_o} = \frac{m'}{m}.$$

But

$$U'_1 - U_1 = (m' - m)u_{1o} + (m' - m)u_o, \\ (u_o = U_o/m = U'_o/m')$$

cannot be made independent of the arbitrary choice of the value of u_o and hence has no physical significance.

Contrariwise, the difference in volumes

$$V'_1 - V_1 = (m' - m)v_1, \\ (v_1 = V_1/m = V'_1/m')$$

has a definite physical significance.

THERMODYNAMICS. (More precisely classical thermodynamics as distinct from statistical thermodynamics.) That branch of physics which describes phenomena in which changes in **temperature** play an important part. Such processes involve the transformation of energy from one form to another and thermodynamics deals with the laws which govern such transformations. Thermodynamics makes a clear and systematic distinction between real *irreversible* phenomena and idealized *reversible* processes.

Thermodynamics is firmly based on experiment. It is developed principally from four fundamental, so-called *laws of thermodynamics*: the zeroth law, the first law, the second law, and the third law (see entries following). Each of these laws constitutes an axiomatic generalization obtained on the basis of experience. However, the generalizations from experience required for the full development of thermodynamics are not confined to the above four laws. Hence thermodynamics is not an axiomatic science.

The laws of thermodynamics cannot be directly and exhaustively verified by suitable experiments. They represent bold generaliza-

tions made on the basis of restricted experiments and their truth is ascertained from the fact that none of the consequences derived from them have failed to be verified experimentally.

Thermodynamics deals with real processes and requires the knowledge of the **thermodynamic properties** of real substances. These must be either measured directly or derived by methods used in other branches of physics. Thermodynamics does not provide methods whereby such properties can be evaluated theoretically. However, certain combinations of properties could not exist in one particular substance if its behavior is not to be in conflict with the basic laws of thermodynamics. Thus thermodynamics is able to provide tests for the consistency of measurements relating to particular substances, or methods of evaluating some of them if some others have been measured. It also provides a general framework for the other branches of physics (**kinetic theory of gases, statistical mechanics, quantum mechanics**) which deal systematically with the evaluation of thermodynamic properties.

THERMODYNAMICS, CHARACTERISTIC FUNCTIONS IN. The fundamental Gibbs formula for the total differential of the entropy may be written

$$dU = TdS - pdV + \sum_i \mu_i dn_i \quad (1)$$

where μ_i is the chemical potential of component i . Formula (1) expresses the dependence of the energy on the set of independent variables S, V, n_i . It is generally preferable to use other sets of independent variables. In order to obtain easily the basic thermodynamic relations in different variables it is useful to introduce the new thermodynamic functions

$$H = U + pV \quad (2)$$

$$A = U - TS \quad (3)$$

$$G = U - TS + pV = H - TS. \quad (4)$$

H is the **enthalpy**, A , the **Helmholtz function**, G , the **Gibbs function**.

Other names for A are the *force function for constant temperature*, the *Helmholtz free energy*, the *work function*, and for G , the *total thermodynamic potential* or the *Gibbs free energy*.

If now we differentiate the relations (2)–(4) and replace in (1), we obtain

$$dH = TdS + Vdp + \sum_i \mu_i dn_i \quad (5)$$

$$dA = -SdT - pdV + \sum_i \mu_i dn_i \quad (6)$$

$$dG = -SdT + Vdp + \sum_i \mu_i dn_i. \quad (7)$$

Each of these relations relates a *characteristic function* to its appropriate independent variables. Gibbs has called these relations the *fundamental equations*. Their importance lies in the basic property that all the thermodynamic functions can be expressed in terms of the chosen characteristic functions and the derivatives with respect to the corresponding independent variables.

For this reason the characteristic functions and the fundamental equations also play a central role in relating thermodynamics to **statistical mechanics**.

Sometimes one also uses the functions

$$J = -\frac{A}{T} \quad (8)$$

and

$$Y = -\frac{G}{T} \quad (9)$$

called the *Massieu function* and the *Planck function*.

THERMODYNAMICS, ENERGY CONSERVATION IN. See **energy conservation in thermodynamics**.

THERMODYNAMICS, FIRST LAW OF.

There are in existence several alternative, but entirely equivalent statements of the first law. The statements differ in that the hypotheses of one may be the conclusions of another and conversely. Irrespectively of formulation, it is possible to state that the most essential statements implied in the first law are:

(1) To each **closed system** whose behavior can be described by an equation of state containing a finite number of independent variables, and to each **continuous system** whose elements can be treated in the limit as a closed system, it is possible to ascribe a property called *internal energy* (some authors use here the rather misleading designation of *energy*) which, like a potential, is a function of state, except that it contains an arbitrary constant.

(2) When a system undergoes a **cycle**, whether reversible or irreversible, so that it returns periodically to a fixed equilibrium state, the sum of all the elements of work dW done around the cycle is exactly equal to the sum of all the elements of heat dQ , or

$$\oint dW = \oint dQ. \quad (\text{a})$$

(3) In a reversible process work and heat are expressed by line integrals, their integrands dW or dQ being **Pfaffian linear forms**. In a small reversible step

$$dQ = dU + dW \quad (\text{b})$$

where dU is the elementary change in internal energy, in general, neither dQ nor dW are perfect differentials, but dU is a **perfect differential**. During an irreversible or reversible finite process between states 1 and 2

$$Q_{12} = U_2 - U_1 + W_{12}. \quad (\text{c})$$

Mathematically, the most satisfactory exposition of the first law was given by Born and **Carathéodory**. The exposition assumes that the concept of work is known (say from mechanics) but that the concept of heat is not known. Generalizing known experimental facts (Joule's paddle-wheel experiment, Rumford's cannon-boring experiments, etc.) it is asserted that:

The work performed *adiabatically* in any reversible or irreversible process between two states of equilibrium 1 and 2 of any **closed system** depends on the end states 1 and 2 alone and is independent of the particular details of the process $1 \rightarrow 2$.

Mathematically this means that the integral $\int_1^2 dW_{ad}$ is that of a perfect differential (since it is independent of the path 1-2), and consequently we can write

$$\int_1^2 dW_{ad} = U_1 - U_2. \quad (\text{d})$$

We write $U_1 - U_2$ rather than $U_2 - U_1$ in order to follow the generally-accepted convention that work done *by* a system is positive and that in the process the energy of the system *decreases*.

Equation (d) constitutes the formulation of the first law of thermodynamics for adiabatic processes. When a process is *not* adiabatic,

Equation (d) does not hold, but we can generalize it to

$$\int_1^2 dQ = U_2 - U_1 + \int_1^2 dW \quad (\text{e})$$

by introducing a new physical quantity, the heat Q , to balance Equation (e). It is easy to show that this new quantity possesses all the attributes which we intuitively ascribe to heat, namely:

(1) The addition of heat to a system changes its state.

(2) In calorimetric mixing involving no work, heat is conserved if the calorimeter is enclosed in adiabatic walls.

In writing Equation (e) it has been assumed that heat *added to the system* is positive and that the addition of heat *increases* the internal energy of the system. Equation (e) can also be written

$$Q_{12} = \Delta U + W_{12}. \quad (\text{f})$$

This is the most general statement of the first law. It is seen that statement (a) follows from it. Statement (a) shows that an engine operating cyclically, such as a **perpetual motion engine of the first kind** cannot produce work $\oint dW > 0$ unless an equivalent amount of heat has been consumed, or unless an equivalent amount of work in another form has been added. Indeed, using Equation (f) with $Q_{12} = 0$ and $\Delta U = 0$ (for a cycle) we can put

$$W_{12} = W - W' = 0.$$

If W is the work done, then work $W' = W$ must have been consumed.

It is evident that the first law can be interpreted as the law of **conservation of energy**.

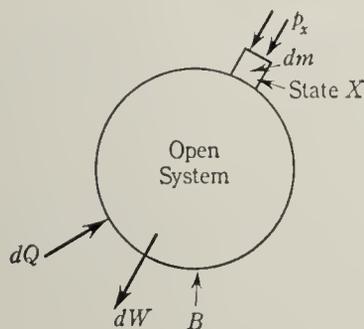
It should be noted that only differences in the internal energy of closed systems can be measured. This can be done either by measuring the adiabatic work according to (d), or the difference between heat and work, according to Equations (c) or (e). If two homogeneous systems a and b undergo identical changes of state 1-2 (expressed in terms of their intensive properties) then

$$\frac{U_{2a} - U_{1a}}{U_{2b} - U_{1b}} = \frac{\Delta U_a}{\Delta U_b} = \frac{m_a}{m_b}, \quad (\text{g})$$

m denoting mass. Owing to the existence of an arbitrary additive constant (as in any potential), the question as to the value of *differ-*

ence between the internal energies of mass m_a of a homogeneous system, and mass m_b of an identical homogeneous system at the same state (as expressed by intensive properties) has no meaning.

In practical applications it is very convenient to reformulate the first law, Equation (c) to a form applicable to an **open system**. Assuming that an elementary mass dm crosses the boundary B of the open system infinitely



slowly, and only under the action of the normal pressure p_x , there being no shearing stresses involved, and assuming that the process is so slow that the state throughout the open system is homogeneous, we would obtain

$$dQ - dW = d(mu) - h_x dm. \quad (h)$$

It is implied that the chemical properties of mass dm are the same as those of the fluid inside the system. Equation (h) can also be written as a *rate equation*

$$\dot{Q} - \dot{W} = \frac{d(mu)}{dt} - h_x \dot{m} \quad (i)$$

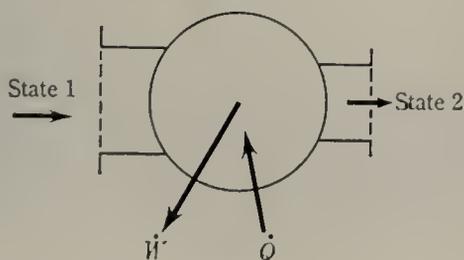
where the dots denote differentiation with respect to time t .

When the processes in the open system are steady (*steady-state system* or *steady-flow process*), Equation (h) becomes

$$dQ - dW = -\sum h_{x_i} dm_i \quad (j)$$

when the summation extends over all the paths along which matter crosses the boundary. Since mass must be conserved, we also have

$$\sum \dot{m}_i = 0.$$



In the most common case with two (which is the least number) paths for matter crossing the boundary, we have

$$\dot{Q} - \dot{W} = \dot{m}(h_2 - h_1). \quad (k)$$

Equation (k) is sometimes referred to as the *enthalpy theorem*.

THERMODYNAMICS, FUNDAMENTAL EQUATIONS OF. See *fundamental equations of thermodynamics*.

THERMODYNAMICS OF IRREVERSIBLE PROCESSES. Extension of classical thermodynamics to non-equilibrium situations (see *thermodynamics; second law; stationary state; entropy production; time variation of entropy production; reciprocity relations*). The main feature of thermodynamics of irreversible processes consists in the evaluation of the entropy production and the entropy flow starting from the fundamental Gibbs formula for the total differential of the entropy. Its use for non-equilibrium conditions is a new postulate. The physical justification of this postulate is that near equilibrium the entropy depends on the same independent variable as for equilibrium processes. A comparison between this postulate and kinetic theory of gases shows that it is valid in the range of applicability of the usual phenomenological laws, such as the **Fourier law** (see *phenomenological relations*). In the case of chemical reactions the reaction rates must be sufficiently slow so as not to perturb the **Maxwell equilibrium distribution of velocities** to an appreciable extent.

THERMODYNAMICS, SECOND LAW OF. The gist of the second law of thermodynamics is contained in two statements: (1) The **Carnot theorem**. The quantity of heat dQ exchanged by any (closed) thermodynamic system during an infinitesimal reversible process and which is, in general given by a **Pfaffian differential form** in the independent properties of the system, can be represented as a product $dQ = TdS$ of the thermodynamic **temperature** T which is a function of the empirical temperature θ only and of the perfect **differential** dS of a potential S called **entropy**, which is a function of the independent properties of the system only, except for an arbitrary additive constant. In other words the Pfaffian of heat, dQ , is integrable, its integrating factor T

being a function of the **empirical temperature** θ only. The Carnot theorem, therefore, proves the existence of thermodynamic temperature and entropy. (2) The principle of entropy increase. In an **isolated system**, the entropy of all the bodies contained in it remains constant during a *reversible* process, increases during an *irreversible* (natural) process and can never decrease:

$$\Delta S \geq 0. \quad (1)$$

It is seen from (1) that the difference in entropy associated with two equilibrium states 1 and 2 is measured (or calculated) by the Clausius integral

$$S_2 - S_1 = \int_1^2 \frac{dQ}{T} \quad (2)$$

along *any reversible path* connecting the two equilibrium states.

An alternative and equivalent formulation asserts that in any **closed system** (not necessarily isolated), the change in entropy dS is composed of two terms

$$dS = d_e S + d_i S. \quad (3)$$

The term due to an exchange of heat with the surroundings

$$d_e S = \frac{dQ}{T} \quad (4)$$

can be positive, negative or zero, depending on dQ . The term $d_i S$ is due to the process taking place within the system. *The term $d_i S$ can never be negative.* It is *positive* when the process is *irreversible* and *zero* when the process is *reversible*.

The preceding two statements are derived from a formulation of the second law which constitutes a generalization of experimental fact. In the last analysis all formulations assert that *natural processes are irreversible and occur spontaneously in one direction only.* There are in existence several formulations of the second law, all of them entirely equivalent. These are due to **Clausius**, **Planck**, **Kelvin**, **Ostwald**, **Carathéodory** and others. The mode of derivation of statements 1 and 2 above depends on the *formulation* adopted. All formulations except Carathéodory's assert the impossibility of reversing some particular irreversible process and lead to derivations which are lengthy, sometimes tedious, and mathematically inelegant. The **Carathéodory for-**

mulation of the second law generalizes the common feature of all irreversible processes, namely the existence of *inaccessible states* in adiabatic processes. Indeed, if an adiabatic process from state 1 to 2 is irreversible, then the initial state 1 cannot be reached adiabatically from state 2, however close state 2 is to state 1, and is therefore *inaccessible* from it. The Carnot theorem quoted above is an almost immediate consequence of the first part of Carathéodory's formulation of the second law and of **Carathéodory's mathematical theorem.** The principle of entropy increase is also easily derived from the second part of **Carathéodory's formulation.**

By the use of the principle of entropy increase it is easy to prove that the **Carnot cycle** is the most efficient way of converting heat into work given two reservoirs of temperatures T and T_o and to show that

$$\eta = \frac{W}{Q} = \frac{T - T_o}{T_o}. \quad (5)$$

The impossibility of building a **perpetual motion engine of the second kind** as well as the truth of the **Clausius** and **Planck-Kelvin formulations of the second law** also follow from it.

In the case of continuous systems (see entry following) (i.e., systems in which the state varies from point to point), undergoing near-equilibrium irreversible processes (i.e., processes which do not lead to large local gradients of properties), the second law can be written in the forms

$$\left. \begin{aligned} \rho \frac{\partial}{\partial t} \int s dV + \oint \mathbf{s}_n dA &= \int \theta dV \\ \text{or} \quad \rho \frac{\partial s}{\partial t} + \text{div } \mathbf{s} &= \theta. \end{aligned} \right\} \quad (6)$$

Here s denotes the entropy per unit volume, \mathbf{s} is the entropy flux vector

$$\mathbf{s} = \frac{\mathbf{q}}{T} \quad (7)$$

where \mathbf{q} is the heat flux vector, and θ is the rate of local entropy production, also called *entropy generated locally* (V is the volume, A is the area). The principle of entropy increase asserts that

$$\theta \geq 0. \quad (8)$$

The principle of entropy increase provides a criterion for the determination of the direction of natural processes: they proceed in a direction which causes $\int d_i S$ to increase, as well as a criterion for equilibrium: it is reached when $\int d_i S$ is a maximum. In an **isolated system**

$$\int d_e S = 0,$$

and hence the entropy increases and reaches a maximum at equilibrium (see **thermodynamic equilibrium**).

Some authors argue that the universe is an isolated system and that, therefore, all processes in the universe cause its entropy to increase. The entropy of the universe tends to a maximum and when that will have been reached, no processes will be possible. The universe will die an "entropy death." It is doubtful whether the universe can in fact be regarded as an isolated system and the remaining conclusions are doubtful, and at least highly speculative.

THERMODYNAMICS, SECOND LAW OF, IN MULTICOMPONENT CLOSED AND OPEN SYSTEMS. In one-component closed systems the total differential entropy is given by (see **thermodynamics, second law of, in one-component closed systems**)

$$dS = \frac{dU}{T} + \frac{p}{T} dV. \quad (1)$$

If one considers multicomponent systems one can generalize (1) by writing

$$dS = \frac{dU}{T} + \frac{p}{T} dV - \sum_i \frac{\mu_i}{T} dn_i. \quad (2)$$

The quantities μ_i are called **chemical potentials** and are defined by

$$\mu_i = -T \left(\frac{\partial S}{\partial n_i} \right)_{V,n}. \quad (3)$$

which are intensive variables.

Formula (2) as well as the definition of the chemical potential, is due to Gibbs and plays a fundamental role in the evaluation of the **entropy production** and **entropy flow**.

For example, using the relations expressing the conservation of mass and of energy in

closed systems (see **mass conservation in closed systems**; **energy conservation in closed systems**) one may in the presence of a chemical reaction put (2) into the form

$$dS = \frac{dQ}{T} - \sum_i \frac{\nu_i \mu_i}{T} d\xi \quad (4)$$

where ξ is the **extent of reaction**.

In agreement with the general formulation of the second law, the entropy change is composed of two terms:

(a) The entropy change due to interactions with the exterior. This gives the entropy flow

$$d_e S = \frac{dQ}{T}. \quad (5)$$

(b) The entropy production inside the system due to the chemical reaction

$$d_i S = \frac{A}{T} d\xi \geq 0 \quad (6)$$

where A is the **chemical affinity**

$$A = - \sum_i \nu_i \mu_i. \quad (7)$$

This can be easily extended to *open* systems. The entropy flow contains then supplementary contributions related to the exchange of matter with the outside world.

THERMODYNAMICS, SECOND LAW OF, IN ONE-COMPONENT CLOSED SYSTEMS.

If one considers a closed system which contains a single component, all irreversible processes being excluded, the entropy production vanishes (see **thermodynamics, second law of**)

$$d_i S = 0, \quad dS = d_e S \quad (1)$$

we then define the entropy by the formula

$$dS = \frac{dQ}{T} \quad (2)$$

where dQ is the heat received by the system and T a positive quantity called the *absolute temperature* which satisfies the following requirements:

- (a) T is positive;
- (b) T is a universal function of the temperature of the system as recorded by measuring some arbitrary property like electrical resistance; T is an increasing function of the "empirical" temperature of the system.

Formula (2) still remains valid when S is multiplied by some constant a and T by a^{-1} . Thus to complete the definition of S and T we have to assign an arbitrary value to T for a well defined point in the empirical scale. One defines the triple point of water as exactly 273.16 degrees. The scale of temperature with the size of the degree so fixed is called the *absolute scale of temperature* or the *Kelvin scale* denoted by $^{\circ}\text{K}$. Thus the triple point of water is by definition 273.16°K .

Formula (2) expresses that $1/T$ is the *integrating factor* of dQ (which is *not* a total differential). This is the starting point of **Carathéodory's formulation of the second law**.

Using the principle of conservation of energy (see **energy conservation in closed systems**) we may also write (2) in the form

$$dS = \frac{dU}{T} + \frac{pdV}{T}. \quad (3)$$

This is the total differential of the entropy in the variables U and V .

THERMODYNAMICS, THIRD LAW OF.

This principle due to Nernst can be enunciated in different forms. The formulation used here which is due essentially to F. Simon, Fowler and Guggenheim, has the advantage of being of universal validity. It states: *It is impossible by any procedure no matter how idealized, to reach the absolute zero in a finite number of operations.* As a consequence it follows that: *For any isothermal process involving only phases in internal equilibrium or alternatively, if any phase is in frozen metastable equilibrium, provided the process does not disturb this frozen equilibrium:*

$$\lim_{T \rightarrow 0} \Delta S = 0 \quad (1)$$

where $\lim_{T \rightarrow 0} \Delta S$ means the value of the entropy change obtained by a smooth extrapolation from the lowest temperature at which measurements have been made.

As a simple example we may consider the solidification of liquid helium (isotope of mass four). The coexistence curve is given by the **Clapeyron-Clausius equation**. Because of (1) we see that

$$\left(\frac{dp}{dT}\right)_{T \rightarrow 0} \rightarrow 0. \quad (2)$$

The melting pressure should have, therefore, in the limit of $T \rightarrow 0$, a zero slope, which has been confirmed experimentally.

THERMODYNAMIC STABILITY CONDITIONS. (See also **thermodynamic equilibrium**.) Suppose we have a system in a state P which is perturbed to a neighboring state P' . This perturbation may be due to an external action or to molecular fluctuations.

The equilibrium state P is said to be stable with respect to this transformation if the production of entropy accompanying it is negative. (See **thermodynamics, second law of; entropy production**.)

$$(\Delta_i S)_{PP'} < 0. \quad (1)$$

The inverse process, that is the change from P' to P is then a spontaneous irreversible process accompanied by a positive production of entropy.

The perturbation is characterized in the first place by the nature of the change PP' and secondly by the conditions under which it is carried out. One may for example consider a change in which T and p are maintained constant or which occurs under adiabatic conditions. (See **adiabatic changes**.)

In the most important cases the stability conditions may be directly connected with the behavior of the characteristic functions (see **thermodynamics, characteristic functions of**). For example at constant V and T

$$(\Delta_i S)_{V,T} = -(\Delta A)_{V,T} \quad (2)$$

and the condition (1) will be satisfied if

$$A \text{ is a minimum at } P. \quad (3)$$

Such minimum conditions permit to calculate explicitly the thermodynamic stability conditions. (See **stability of phases; thermal stability; mechanical stability; diffusion stability**.)

THERMODYNAMIC SYSTEM. Any collection of matter enclosed in a clearly-defined boundary in terms of which a given analysis is performed. The choice of a system for the description of a particular thermodynamic process is arbitrary; it does not affect the conclusions reached but a judicious choice may considerably simplify the description. The boundary of the system may be drawn in any manner whatsoever; it need not be rigid, and

the system need not contain the same material objects at all times.

A system whose walls are impermeable to matter is called a *closed system*. A closed system contains the same collection of material objects of all times. If the boundary is crossed by matter the system is called an *open system* and its boundary is called a **control surface**. If the **state** at any given point within a control surface is independent of time, the open system is said to be in *steady state*. In particular the mass and energy within the control surface of an open system in steady state must remain constant. The *law of conservation of mass* for such a system can be written

$$\Sigma \dot{m}_i = 0$$

where \dot{m}_i denotes the rates of mean flow through the k discrete points i through which matter enters or leaves the control surface. (For the law of conservation of energy see **thermodynamics, first law of**).

A closed system whose boundary is also impermeable to the passage of heat and work is called an *isolated system*. Alternatively, the designation is applied also to closed systems whose boundaries are **adiabatic** and impermeable to the passage of heat only.

THERMODYNAMICS, ZEROTH LAW OF.

If two systems are in thermodynamic equilibrium with a third across a **diathermal wall** then they are in equilibrium with each other. It follows, as can be shown in detail, that in the case of all systems there exist functions of their n independent properties x_i such that at equilibrium

$$\theta = \phi(x_i)$$

assumes the same value for all systems. The hypersurfaces

$$\theta = \text{constant}$$

are called *isotherms* (or *isothermals*), those of the same temperature for two different systems being called corresponding isotherms. (See also **temperature**.)

THERMOELASTICITY, THE THEORY OF.

A theory which investigates the effect of the thermal state of an elastic solid upon the stress distribution, and the influence of the stresses in an elastic body on the conduction of the heat through it. The subject was investigated by Duhamel (1838). Recent work

has concerned both dynamical (time-dependent) problems in which the stress distribution and the temperature distribution cannot be treated separately but are governed by a set of cross-linked equations, and also the simpler problems of steady thermal stresses in elastic bodies. The latter can be formulated as follows: for an elastic body whose boundaries are parallel to the plane $z = 0$ and in which a temperature field $T \{1 + \theta(x, y, z)\}$ exists, T is the temperature at zero stress and strain. Assume no body forces are active and no heat sources are present, and introduce cylindrical coordinates

$$r = \rho l, \phi, z = \zeta l$$

where l is a characteristic length. If the rigidity modulus μ is taken as the unit of stress, l the unit of length and T as the unit of temperature, the formulation of those problems which exhibit symmetry about the z -axis is as follows:

$$\begin{aligned} & \left(\frac{\partial^2 u}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial u}{\partial \rho} - \frac{u}{\rho^2} \right) + \frac{\partial^2 u}{\partial \zeta^2} \\ & + (\beta^2 - 1) \frac{\partial^2 w}{\partial \rho \partial \zeta} = b \frac{\partial \theta}{\partial \rho} \\ & \frac{\partial^2 w}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial w}{\partial \rho} + \beta^2 \frac{\partial^2 w}{\partial \zeta^2} \\ & + (\beta^2 - 1) \frac{\partial}{\partial \zeta} \left(\frac{\partial u}{\partial \rho} + \frac{u}{\rho} \right) = b \frac{\partial \theta}{\partial \zeta} \end{aligned}$$

where

$$\beta^2 = \frac{2(1 - \nu)}{1 - 2\nu}, \quad b = \frac{2(1 + \nu)}{1 - 2\nu} \alpha T.$$

where ν is **Poisson's ratio**, α is the coefficient of linear expansion, ul is the displacement in the r -direction and wl in the z -direction. The variation of θ throughout the elastic solid is governed by **Laplace's equation**

$$\frac{\partial^2 \theta}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \theta}{\partial \rho} + \frac{\partial^2 \theta}{\partial \zeta^2} = 0.$$

These equations can be solved by **Hankel transform** techniques (reference: I. N. Sneddon, *Boundary Value Problems in Thermoelasticity*; in *Boundary Problems in Differential Equations*, ed. by R. E. Langer, University of Wisconsin Press, 1960).

THERMOELECTRIC POWER. The thermoelectric force per degree, dE_{AB}/dT , where E_{AB} is the emf generated at the junction of

metals A and B at temperature T . From the definition

$$S_A = \frac{dE_A}{dT} = \int_0^T \frac{\sigma_A dT}{T}$$

the absolute thermoelectric power is obtained from the values of the **Thomson coefficient**, σ_A , at all temperatures, measured at low temperatures against a superconductor whose Thomson effect vanishes. The Peltier coefficient of two metals is then given by

$$\pi_{AB} = -T(S_B - S_A).$$

THERMOMECHANICAL EFFECT. See thermomolecular pressure.

THERMOMETRIC EFFECTS. See galvanometric and thermometric effects.

THERMOMOLECULAR PRESSURE DIFFERENCE. The interaction of heat flow and the flow of matter gives rise to certain cross phenomena of which the most important are the thermomolecular pressure difference and the thermomechanical effect.

In the first case a temperature difference between two vessels connected by a capillary or a membrane causes matter to flow and sets up a pressure difference. The ratio of corresponding pressure and temperature differences is called the "*thermomolecular pressure difference*."

In the second case the pressure difference and temperature are held constant. Then a heat flow is observed which is proportional to the flow of matter.

The proportionality coefficient which corresponds to the heat transferred with the unit of mass, is called the "*heat of transfer*" of the thermomechanical effect. The **phenomenological relations** are here

$$\begin{aligned} J_{th} &= -\frac{L_{11}}{T^2} \Delta T - \frac{L_{12}}{T} V \Delta p \\ J_m &= -\frac{L_{21}}{T^2} \Delta T - \frac{L_{22}}{T} V \Delta p \end{aligned} \quad (1)$$

where J_m is the flow of matter, J_{th} the flow of heat, L_{11} , L_{12} , L_{21} , L_{22} the phenomenological coefficients, T the absolute temperature, p the pressure, V the **molar volume**, Δt , Δp the differences of temperature and of pressure between the two vessels.

According to (1) the thermomolecular pressure is given by

$$\left(\frac{\Delta p}{\Delta T}\right)_{J_m=0} = -\frac{L_{21}}{L_{22}VT}. \quad (2)$$

Similarly the thermomechanical effect is given by

$$\left(\frac{J_{th}}{J_m}\right)_{T=0} = \frac{L_{12}}{L_{22}}. \quad (3)$$

Therefore the heat of transfer Q^x may be easily expressed in terms of the phenomenological coefficients

$$Q^x = \frac{L_{12}}{L_{22}}. \quad (4)$$

Onsager's **reciprocity relations** establish the relation between the thermomolecular pressure difference and the thermomechanical effect

$$\left(\frac{\Delta p}{\Delta T}\right)_{J_m=0} = -\frac{1}{VT} \left(\frac{J_{th}}{J_m}\right)_{\Delta T=0}. \quad (5)$$

It is thus clear that both effects will appear in the same system.

In the case of two gases which communicate by a hole the diameter of which is small compared to the mean free path a simple application of **kinetic theory of gases** gives

$$\frac{p_I}{p_{II}} = \left(\frac{T_I}{T_{II}}\right)^{1/2}. \quad (6)$$

From this one can easily calculate the thermomolecular pressure as well as the heat of transfer (4).

THERMORHEOLOGICALLY SIMPLE MATERIAL. A **viscoelastic material**, the properties of which at different temperatures are represented by shifts along the log (time) axis of all compliance, modulus and spectral functions.

THERMOSTATICS. Application of thermodynamics to equilibrium states. (See also **thermodynamics of irreversible processes; thermodynamics, second law of**.)

THERMOSTATISTICS. A term introduced by Kramers for statistical mechanics to indicate its relation to and difference from thermodynamics.

THERMOTROPIC MODEL. A model atmosphere used in numerical forecasting in

which the parameters to be forecast are the height of one constant-pressure surface (usually 500 mb) and one temperature (usually the mean temperature between 1000 mb and 500 mb). Thus, a surface prognostic chart can also be constructed. The **quasi-geostrophic approximation** is employed and the **thermal wind** is assumed constant with height.

THÉVENIN ACOUSTICAL THEOREM.

If an acoustical impedance z_A be connected at any point in an acoustical system, the volume current \dot{X} in this acoustical impedance is the pressure p at the point prior to the connection divided by the sum of the acoustical impedance z_A and the acoustical impedance z'_A , where z'_A is the acoustical impedance at the point prior to connecting z_A .

THÉVENIN MECHANICAL RECTILINEAL THEOREM.

If a mechanical rectilinear impedance z_M be connected at any point in a mechanical rectilinear system, the resultant velocity of this mechanical rectilinear impedance is the product of the velocity and mechanical rectilinear impedance z'_M of the system both measured at the point prior to the connection divided by the sum of the mechanical rectilinear impedances z_M and z'_M .

THÉVENIN MECHANICAL ROTATIONAL THEOREM.

If a mechanical rotational impedance z_R be connected at any point in a mechanical rotational system, the resultant angular velocity of this mechanical rotational impedance is the product of the angular velocity and mechanical rotational impedance z'_R of the system both measured at the point prior to the connection divided by the sum of the mechanical rotational impedances z_R and z'_R .

THÉVENIN THEOREM. The current in any terminating impedance Z_T connected to any network is the same as if Z_T were connected to a generator whose voltage is the open circuit voltage of the network, and whose internal impedance Z_R is the impedance looking back into the network from the terminals of Z_T , with all generators replaced by impedances equal to the internal impedance of these generators. (See also **Norton theorem**.)

THICKNESS. See **thickness pattern**.

THICKNESS PATTERN. The *thickness* is the vertical distance between two specified isobaric surfaces in the atmosphere. It is a

measure of the mean temperature between the surfaces and therefore its horizontal gradient is a measure of the mean **thermal wind** between these levels. The thickness pattern on a weather chart therefore displays the field of wind shear (horizontal vorticity).

The difference in **geostrophic wind** between two levels is given by

$$|\mathbf{v}_{G2} - \mathbf{v}_{G1}| = \frac{g}{f} |\text{grad}_H h'|$$

where h' is the thickness. The direction is along lines of constant thickness (i.e., along the mean isotherms of the layer).

THIN-AIRFOIL THEORY. In thin-airfoil theory the airfoil is considered as a symmetrical fairing superposed on a curved "mean line," as shown in Figure 1. The chord line

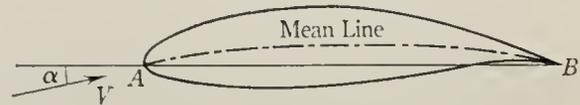


Fig. 1.

is the straight line AB joining the ends of the mean line, and the angle of incidence α is the inclination of the stream to the chord line. For the special case of a symmetrical airfoil the mean line is straight and coincides with the chord line.

It is assumed, in thin-airfoil theory, that the angle of incidence, thickness ratio and camber are all small. The problem is then a linear one and it is permissible to consider separately the pressure distributions for (1) the symmetrical fairing at zero incidence, (2) the mean line (with zero thickness) at the "ideal" angle of incidence, (3) the loading, additional to (2), associated with the actual angle of incidence.

Since the problem is linear, these three pressure distributions can be *added* to give the pressure distribution on the airfoil. Except near the leading edge, the results obtained for airfoils of moderate thickness at small incidences usually agree fairly well with more exact theories. If only the lift and pitching moment are required, it is only necessary to consider the contributions (2) and (3) to the pressure distribution. (Since the theory assumes an inviscid fluid, the drag cannot be estimated.)

The pressure distribution on the symmetrical

fairing at zero incidence (Figure 2) may be calculated by assuming an appropriate distribution of sources and sinks along the axis of x . It



Fig. 2.

is easily shown that the required source strength per unit length is $2V \frac{dy}{dx}$, where V is the velocity of the stream.

To calculate the pressure distribution on an airfoil of zero thickness given by the mean line, a distribution of vortices is assumed as shown in Figure 3. The distribution of vortex strength

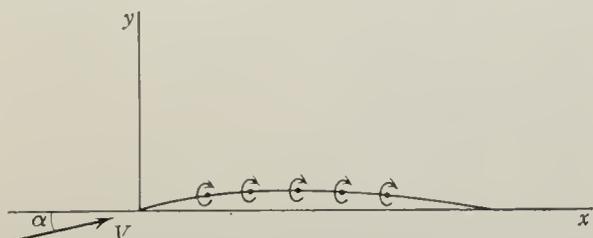


Fig. 3.

must satisfy the condition that the thin airfoil is a stream line. This condition is

$$\frac{dy}{dx} - \alpha = \frac{v}{V},$$

where v is the induced velocity component in the y direction due to all the vortices.

The *ideal angle of incidence* α_i is the value of α giving finite velocity at the leading edge. The corresponding lift coefficient is usually known as the *design lift coefficient*. The usual procedure is to calculate, for the mean line, the ideal angle of incidence α_i and the pressure distribution associated with it. Then the pressure distribution to be added, for the airfoil at incidence α , is as calculated for a thin flat plate at incidence $(\alpha - \alpha_i)$. This may be found as a special case of the transformation giving a **Joukowski airfoil**, the circulation being chosen, of course, to satisfy the **Joukowski condition** that the velocity should be finite at the trailing edge. The theory gives an infinite velocity at the leading edge of the flat plate, for any non-zero angle of incidence. Thus thin-airfoil theory cannot be expected to predict correctly the pressure distribution close

to the leading edge of a real airfoil. Nevertheless, the predicted values of lift and pitching moment are finite, and agree fairly well with experiments.

Thin-airfoil theory shows that, for any thin airfoil, $\frac{dC_L}{d\alpha} = 2\pi$ and the aerodynamic center is at the quarter-chord point, i.e., at a distance $\frac{c}{4}$ from the leading edge, where c is the chord.

The *incidence for zero lift* is

$$\alpha_0 = \frac{1}{\pi} \int_0^\pi \frac{dy}{dx} (1 - \cos \theta) d\theta,$$

where $\frac{x}{c} = \frac{1}{2} (1 - \cos \theta)$ and $\frac{dy}{dx}$ refers to the mean line as in Figure 3.

The pitching-moment coefficient at zero lift is

$$C_{m_0} = \mu + \frac{\pi}{2} \cdot \alpha_0,$$

where

$$\mu = - \int_0^\pi \frac{dy}{dx} \cdot \sin^2 \theta d\theta.$$

The pitching moment coefficient about an axis through the quarter-chord point (the aerodynamic center) is equal to C_{m_0} for any incidence. (See H. Glauert, *The Elements of Aerofoil and Airscrew Theory*, 2nd edition, Cambridge University Press, 1947.)

THIRD ORDER OPTICS. See **Gaussian optics**.

THIRTEEN-MOMENT APPROXIMATION. The solution of the Boltzmann non-linear integro-differential equation has proved to be quite difficult. The Enskog-Chapmann method of solution has yielded a first approximation which is in good agreement with experiment, as long as the macroscopic quantities do not vary appreciably over a distance of one mean free path in the gas. The second approximation (see **Burnett equations**), however, has not turned out to be an adequate tool for larger gradients, and the rate of convergence of this series is not well established.

H. Grad has suggested an alternate method for the solution of the Boltzmann equation: he sets up a distribution function which is a product of a Maxwellian distribution and a

series of multidimensional **Hermite polynomials**. The coefficients of these polynomials are related to the first velocity moments of the distribution function. Grad showed that, by increasing the number of terms, the series converged rapidly to a true solution of the Boltzmann equation. The first approximation is equivalent to the first approximation in the Enskog-Chapmann method; both lead to the **Navier-Stokes equations** of change. But the second approximation differs.

The 13 moments approximation, with 13 scalar parameters, corresponding to the moments of order 0, 1, 2 and a contraction of the 3rd moment, has been used extensively; it allows one to predict the properties of gases at low densities, when wall effects can become important, and yields qualitative information under such extreme conditions as shock waves, when the distribution function varies appreciably over a distance of one mean free path.

THIXOTROPY. Property of a material which softens when strained.

THOMAS PRECESSION. See **relativity, precession**.

THOMSON SCATTERING. The scattering of electromagnetic radiation by essentially free electrons in the limit of long wavelengths (i.e., wavelengths for which the **Compton Effect** can be ignored). The differential scattering cross section (giving the cross section as a function of scattering angle θ) is $d\sigma/d\Omega = \frac{1}{2}r_o^2(1 + \cos^2 \theta)$, where $r_o = e^2/mc^2$ is the classical electron radius. Here e and m are the charge and rest mass of the electron, and c the velocity of light. The total cross section is therefore $\sigma_o = (8\pi/3)r_o^2$, independent of wavelength. (See **scattering**.)

THREE-BODY PROBLEM. Even though no general solution of the problem is available, nevertheless, there are several practical computational methods for determining the positions of planets and other members of the solar system, taking into account the gravitational attraction of all effective members. Such solutions are all made by successive approximations and various methods of computing **perturbations**, rather than by the application of any general solution.

A number of particular solutions of the three-body problem have been made by mathematicians, notable among them being the solu-

tion by Lagrange. He showed that it is possible for an asteroid to be stable in a position such that it is equidistant from both the sun and Jupiter. In this case the three objects would be on the vertices of an equilateral triangle and the asteroid **orbit** would have the same period as that of Jupiter. This case is illustrated in nature by the members of the so-called Trojan group. (See **two-body problem; planetary motion**.)

THREE-EIGHTHS RULE. The *quadrature* formula

$$\int_{x_0}^{x_3} f(x)dx = 3h(f_0 + 3f_1 + 3f_2 + f_3)/8,$$

where

$$f_i = f(x_i) = f(x_0 + ih).$$

The **remainder** is

$$R = -3h^5 f^{IV}(\xi)/80,$$

hence the formula is in general somewhat less exact than the simpler **Simpson's rule**.

THREE-INDEX SYMBOLS. See **Christoffel three-index symbols**.

THREE-MOMENT THEOREM. See **Clapeyron theorem**.

THRESHOLD, ABSOLUTE LUMINANCE. The minimum **luminance** which can be discriminated by the fully dark-adapted eye.

THRESHOLD, ABSOLUTE PURITY. The minimum **purity** perceptible in contrast with white, by the light-adapted eye. The absolute purity threshold is a function of wavelength.

THRESHOLD ENERGY (THRESHOLD). The minimum energy of an incident particle in a reaction below which the reaction cannot take place. For example, in the photoelectric effect, the energy $h\nu$ of the photon must exceed the threshold energy $h\nu_o = \epsilon$, where ϵ is the binding energy of the emitted electron. In pair production, the energy $h\nu$ must exceed $2mc^2$, which is the threshold for this process. In a nuclear reaction, the lowest energy at which an endothermic reaction can take place.

THRESHOLD OF AUDIBILITY. The threshold of audibility for a specified sound signal is the minimum effective **sound pressure** of the signal that is capable of evoking an

auditory sensation. The ambient noise reaching the ear is assumed to be negligible. The threshold of audibility as a function of the frequency for the frequency range 30 to 15,000 cycles is given by the empirical equation,

$$p = \frac{160 + 4f^3 \times 10^{-8}}{f^2}$$

where p is the sound pressure, in dynes per square centimeter, and f is the frequency, in dynes per square centimeter.

THROTTLING EXPERIMENT. This experiment provides most direct information concerning the dependence of **enthalpy** on pressure and temperature. This experiment was first performed by Joule and Lord Kelvin (William Thomson); for this reason it is also often called the *Joule-Thomson experiment*. In this experiment a stream of gas in a thermally insulated container is forced through a plug, the pressure being greater on the near side than on the far side. The system is in a steady state such that in a given time a certain mass of gas is pushed in at a pressure p_1 and during the same time an equal mass of gas streams away at pressure p_2 .

In this experiment the enthalpy is conserved, that is the condition of the gas after and before throttling are related by

$$H(T_2, p_2) = H(T_1, p_1). \quad (1)$$

Provided the pressure drop is not too great, this may be replaced by

$$dH = \left(\frac{\partial H}{\partial T}\right)_p dT + \left(\frac{\partial H}{\partial p}\right)_T dp = 0. \quad (2)$$

The *Joule-Thomson coefficient* measures the ratio of the temperature fall to the pressure drop

$$\left(\frac{\partial T}{\partial p}\right)_H = - \left(\frac{\partial H}{\partial p}\right)_T / \left(\frac{\partial H}{\partial T}\right)_p. \quad (3)$$

THRUST. (1) In general, a force exerted by one member of a structure upon another, generally an outward-directed force. (See **arch**.) (2) The force exerted in any direction by a powered screw. (3) In jet propulsion, the force, in the direction of motion, resulting from the components of the pressure forces (in excess of ambient atmospheric pressure) acting on all inner surfaces of the vehicle or missile and which are parallel to the direction

of motion. Thrust less drag equals accelerating force. (4) The force acting on a rocket motor due to the reaction caused by the expulsion of matter at high velocity. This force is given by the sum of all static pressures acting axially on the area elements of the solid wall surrounding the gas in the motor, taking into account also the forces acting on the exit area.

THRUST COEFFICIENT. In rocket motor design, a parameter which can be determined experimentally. It is a term by which the chamber pressure and throat area must be multiplied in order to obtain the thrust of the motor. Thus:

$$F = C_F p_c A_t,$$

where F is the thrust, C_F is the thrust coefficient, p_c is the chamber pressure and A_t is the throat area.

THRUST LINE. See **pressure line**.

THRUST RATIO, AUGMENTED. See **augmented thrust ratio**.

TIDAL WAVES. Gravity waves in which the velocity depends both on the wavelength and the depth of the fluid. In a more restrictive sense, waves produced by the gravitation of a heavenly body.

TIE. See **bar**.

TIED ARCH. When the supporting foundation is unable to take the outward thrust of an arch, the supports can be tied by tension members so that the foundation need supply vertical forces only. (See **arch**.)

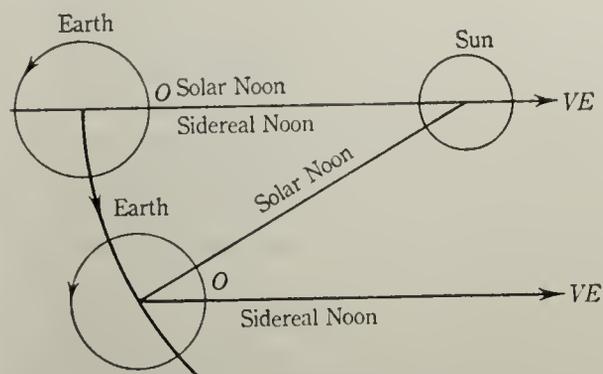
TIE ROD. A very slender structural bar used as a tension tie. It buckles under very small compressive force and is usually assumed incapable of taking any compression.

TIGHT BINDING APPROXIMATION. One of two alternative approaches to the problem of calculating the energy of an electron in a solid. It is assumed that the electron is effectively in an atomic **orbital** centered on a particular atom of the lattice, with a small perturbation allowing it to jump to neighboring sites. The assumed wave function is thus a **Bloch function** in which the periodic function is a pure atomic **wave function**. This approximation works well for narrow deep-lying **bands**, but must be augmented with the free-

electron type of approximation for **conduction electrons**.

TIME AND TIME-KEEPING. Time may be defined as a measured duration. There are two motions of the earth that provide natural units of time: the rotation of the earth about its axis, which provides us with the day; and the revolution of the earth about the sun, which provides us with the year. However, one cannot determine the period of rotation or of revolution without reference to external points. Two such points are available, the vernal equinox and the sun. The period of time between successive passages of the sun across an observer's meridian is known as the *solar day*, and the period between successive passages of the vernal equinox across the observer's meridian is known as the *sidereal period*.

From the figure, in which the directions, but not the magnitudes of the motions are



correctly shown, it is seen that the solar day is longer than the sidereal day. The earth moves about the sun in an elliptical orbit with the axis of rotation inclined to the orbit plane by about $23^{\circ}5'$. Because of the Keplerian laws of orbital motion, the length of a "day" when the earth is at perihelion is shorter than when the earth is at aphelion. Furthermore, the plane of the earth's orbit is inclined to the plane of the earth's equator, and this introduces further variations in the length of the day.

To avoid these irregularities in the length of the "day" (the interval between successive passages of the sun across the observer's meridian) one defines a **mean sun** that moves eastward in the celestial equator with constant velocity, completing the circuit in the same period as that of the apparent sun in the ecliptic. A *mean solar day* is the period between successive passages of the mean sun

across a terrestrial meridian. The *mean solar second* which has been the international standard time unit for many years is $1/86,400$ part of the mean solar day.

1. *Mean solar time* is reckoned from the beginning of the day at midnight through twenty-four hours. The *local mean time* for any particular place is 12 hrs plus the local hour angle of the mean sun. In the United States mean time is commonly expressed as two twelve-hour periods, the first beginning with the day at midnight and designated as "A.M." and the second starting at noon as "P.M." Astronomers, navigators, scientists, and most Europeans reckon time from midnight through 24 hours. The local mean solar time for the meridian of Greenwich is known as universal time.

1a. *Zone time* is the local mean solar time for the meridians at fifteen degree (one hour) intervals from Greenwich and the zone boundaries are $7^{\circ}5'$ either side of the zone meridians. They are numbered +1, +2, etc., west of Greenwich and -1, -2, etc., east. Standard time zones correspond to zone times except that the boundaries are established for economic convenience.

2. *Apparent time* is the local hour angle of the true sun measured from the local meridian. It is the time that is indicated by a properly adjusted sun-dial. The difference between mean and apparent time is known as the equation of time and is published in publications such as the *American Ephemeris*, almanacs, etc.

3. The *sidereal time* for any station is the hour angle of the vernal equinox. Since the mean sun is moving relative to the vernal equinox at a constant rate, the time divisions on the mean and sidereal systems are not the same. Since the tropical year contains 365.2422 mean solar days and exactly one more sidereal day, it follows that the number of sidereal seconds is equal to the number of mean solar seconds multiplied by $\frac{366.2422}{365.2422}$ or by 1.00273791. A clock keeping sidereal time gains upon mean time by 9.8565 sidereal seconds in one mean hour, or by 9.8296 sidereal seconds in one sidereal hour.

4. *Ephemeris time*. For many years it has been realized that there are slight "irregularities" in the rotation period of the earth. These can be smoothed out by clocks for these are

adjusted by observations of the stars to compensate for slight gains or losses. However, for long range predictions of the positions of the planets, a unit of time must be established to be invariable by definition. This is known as *ephemeris time*. The *ephemeris day* is a purely arbitrary unit consisting of 86,400 ephemeris seconds and is only approximately equal to a mean time day. •The *ephemeris second* is not exactly $1/86,400$ of a day but, by definition is $1/31,556,925.9747$ of a tropical year. The reduction, universal time to ephemeris time, is determined from observational material and cannot be exactly determined for several years. For example in 1901.5 the reduction is -2.54^s or $-0^r.000029$. The reduction table for recent years indicates the trend:

TO OBTAIN EPHEMERIS TIME
ADD TO UNIVERSAL TIME

Year	Seconds	Days
1951.5	29.59	0.000342
1952.5	30.21	0.000350
1955.5	31.5	0.00036
1957.5	33	0.00038
1958.5	33	0.00038
1959.5	34	0.00039
1960.5	35	0.00040

The 1960 *American Ephemeris and Nautical Almanac* for 1960 is the first edition to use Ephemeris Time and further details may be found there.

5. *Atomic time*. For many years the standard clocks have been pendulum-controlled. Within recent years oscillating electronic circuits, with frequency controlled by a crystal, have been found to be remarkably stable over long periods of time. With such atomic clocks several minute changes in the rotation period of the earth have been detected. In other words the crystal-controlled atomic clock is steadier than the earth! It seems highly probable that within not too many years these ammonia clocks, cesium clocks, etc. will supplant the rotating earth as the fundamental clocks. (See **calendar**; **month**.)

TIME, DELAY. See **delay time**.

TIME DILATATION. See **Lorentz transformation**.

TIME, GENERATION. See **generation time**.

TIME INTERVAL. The number of times that a specified periodic process is observed to take place between two events. The time taken for each repetition of the periodic process is used as the unit of time. The interval from event *A* to event *B* is usually taken as positive when *B* occurs after *A*; negative when *B* occurs before *A*; zero when the two are simultaneous. This is the only sense in which time has meaning in physical science, because the older concept of absolute time has been shown by **relativity** theory to be unsound—two events that are simultaneous for one observer are not necessarily simultaneous for another.

TIME REVERSAL OPERATION. By time-reversibility is meant the invariance of the laws of a system under transformation from the four-dimensional coordinate system xt to $x't'$ where

$$x = x' \quad t' = -t.$$

The time reversed state of the state which contains particles at positions x_i with momenta p_i and spin σ_i is a state of the same particles at x'_i , but with momenta $-p_i$ and spins $-\sigma_i$. Furthermore the direction of time is reversed (incoming particles are transformed into outgoing ones). The operator T which generates this correspondence is **anti-unitary** (so as to preserve commutation rules), and satisfies $T^2 = 1$ (for spinless particles) and $T^2 = -1$ for systems containing an odd number of spin $\frac{1}{2}$ particles. The transformation of the field operators for several field systems are listed below:

(a) Electromagnetic field:

$$T^{-1}\mathcal{E}_j(\mathbf{x}, t)T = \mathcal{E}_j(\mathbf{x}, -t)$$

$$T^{-1}\mathcal{H}_j(\mathbf{x}, t)T = -\mathcal{H}_j(\mathbf{x}, -t).$$

T transforms a one-photon state into a one-photon state of opposite linear momentum without interchanging right circular and left circular polarization.

(b) Dirac particles:

$$T^{-1}\psi(\mathbf{x}, t)T = \eta^T C^{-1} \gamma_5 \psi(+\mathbf{x}, -t)$$

where $|\eta^T| = 1$. These definitions leave the free-field Hamiltonian invariant.

TIME-SHIFT THEOREM. See **Laplace transform theorems**.

TIME-TEMPERATURE SHIFT. For certain polymers the effect of an increase in temperature is to speed up all **viscoelastic** processes by

a constant factor. This represents a shift of all spectra along the log (time) axis by an amount dependent on the temperature, the time-temperature shift.

TIME VARIATION OF THE ENTROPY PRODUCTION. The central quantity of thermodynamics of irreversible processes (see also thermodynamics, second law of) is the entropy production per unit time

$$\frac{d_i S}{dt} = \sum_k J_k X_k \geq 0. \quad (1)$$

The X_k are the generalized forces or affinities of the irreversible processes and J_k the corresponding rates (see thermodynamics of irreversible processes; rates of irreversible processes.)

The time variation of the entropy production may be split into two parts

$$d\mathcal{P} = d_X \mathcal{P} + d_J \mathcal{P} = \sum_k J_k dX_k + \sum_k X_k dJ_k. \quad (2)$$

$d_X \mathcal{P}$ is the part due to the changes of the forces and $d_J \mathcal{P}$ the part due to changes of the rates.

It should be noticed that neither $d_X \mathcal{P}$ nor $d_J \mathcal{P}$ are total differentials.

One has then the following two basic theorems:

Theorem (a) Under the following restrictions:

- (1) Linear phenomenological relations.
- (2) Validity of Onsager's reciprocity relations.
- (3) Phenomenological coefficients are treated as constants

$$d_X \mathcal{P} = d_J \mathcal{P} = \frac{1}{2} d\mathcal{P}. \quad (3)$$

Theorem (b) For time independent boundary conditions (or constraints) one has in the whole domain of validity of the thermodynamics of irreversible processes

$$d_X \mathcal{P} = \sum_k J_k dX_k \leq 0. \quad (4)$$

This theorem is based on the thermodynamic stability conditions.

As a consequence of (3) and (4) the entropy production itself decreases with time in the region stated under (a) and is minimum in stationary states. This is Prigogine's theorem of minimum entropy production, valid in the neighborhood of equilibrium. It gives a varia-

tional principle for non-equilibrium stationary states.

This theorem gives a simple meaning of the stability of non-equilibrium stationary states (to a minimum entropy production corresponds a stable state and to a maximum, an unstable state). It permits therefore an extension of the Le Chatelier-Braun principle to non-equilibrium situations (see Chatelier (Le)-Braun principle).

For situations in which (3) is not satisfied an extension of the theorem of minimum entropy production is not always possible.

TIP. See blade.

TOMONAGA-SCHWINGER EQUATION.

The covariant differential equation governing the time evolution of a quantized field system in the interaction picture.

Consider the space-like surface σ . To each point \mathbf{x} ($= x^0, \mathbf{x}$) on this surface, we may assign a time $x^0 = t(\mathbf{x})$. In the limit of a plane surface, each point has the same time t , the coordinate of the plane $t = \text{constant}$. A natural generalization of the interaction picture state vector can now be made, namely $\Psi[t(\mathbf{x})]$. Thus one can consider the interaction picture equation

$$i\hbar \partial_t \Psi(t) = H_I(t) \Psi(t) \quad (1)$$

as a limiting equation in the case the space-like surface σ becomes a plane $t = \text{constant}$. For a field system

$$H_I(t) = \int_t \mathcal{H}_I(x) d^3x \quad (2a)$$

$$\approx \sum_{\text{all cells}} \mathcal{H}_I(x) \Delta V \quad (2b)$$

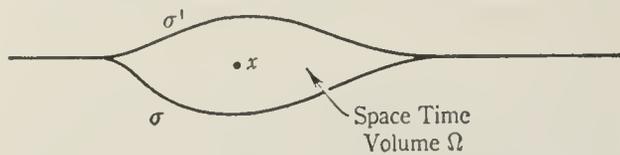
where in (2b) we have approximated the integral as a sum over three dimensional cells of volume ΔV . The equation which governs the evolution of the system about the point \mathbf{x} is then taken to be

$$i\hbar \frac{\partial \Psi[t(\mathbf{x})]}{\partial t(\mathbf{x})} = \mathcal{H}_I(x) \Delta V \Psi[t(x)]. \quad (3)$$

Let us denote $\Psi[t(\mathbf{x})]$ by $\Psi(\sigma)$. We next define the invariant operation $\delta/\delta\sigma(x)$ by

$$\frac{\delta}{\delta\sigma(x)} \Psi(\sigma) = \lim_{\Omega \rightarrow 0} \frac{\Psi(\sigma') - \Psi(\sigma)}{\Omega(x)} \quad (4)$$

where σ' and σ are two space-like surfaces which differ from each other by an infinitesimal



amount in the neighborhood of the space-time point x , and $\Omega(x) \left(\approx \int dt(x) d^3x \right)$ is the space-time volume enclosed between σ and σ' . The covariant generalization is thus

$$i\hbar c \frac{\delta\Psi(\sigma)}{\delta\sigma(x)} = \mathcal{H}_I(x)\Psi(\sigma) \quad (5)$$

which is called the Tomonaga-Schwinger equation. In order that solutions to (5) exist, the following integrability conditions must be satisfied

$$\left[\frac{\delta^2}{\delta\sigma(x)\delta\sigma(x')} - \frac{\delta^2}{\delta\sigma(x')\delta\sigma(x)} \right]_{x,x' \text{ on } \sigma} \Psi(\sigma) = 0 \quad (6)$$

which in turn imply that

$$[\mathcal{H}_I(x), \mathcal{H}_I(x')] = 0, \quad \text{for } x, x' \text{ on } \sigma. \quad (7)$$

TOPOGRAPHICAL CRITERION FOR A CONTROL SYSTEM. See stability (of system).

TOPOLOGICAL GRAPH. See graph, planar.

TOPOLOGICAL (OR CONTINUOUS) GROUP. A group for whose elements a topology is defined in such a way that the group operations (formation of product and of inverse) are continuous. Thus, the set of real numbers forms a topological group if the group operation is given by addition and the distance between two numbers by the absolute value of their difference. Consider, e.g., the (one-parameter) group of rotations of the Euclidean plane given by the elementary formulas

$$X = x \cos \theta - y \sin \theta, \quad Y = x \sin \theta + y \cos \theta,$$

corresponding to the group of matrices

$$\begin{pmatrix} \cos \theta, & -\sin \theta \\ \sin \theta, & \cos \theta \end{pmatrix}$$

depending on the parameter θ . If we now define two matrices as being close together if every element of one is close to the corresponding element of the other (see under topology), then it is clear that multiplication of matrices is continuous, i.e., that if $A = A(\theta_1)$ is a fixed

matrix and $B = B(\theta_2)$ is a variable one, then small changes in B will produce only small changes in AB . For applications, especially to physics, the important topological groups are the "full linear group," the "unimodular group," etc. (See under group.)

TOPOLOGICAL SPACE. The numerous ways (the commonest being given below) of defining an abstract topological space T , whose elements are usually called points, although in applications they may, for example, be real numbers, or solutions of differential equations, depend on the fact that topology is essentially a study of the operation of passing to a limit. Such an operation on a subset M of T may be thought of as adjoining to M the set of all its limit points; that is, of forming the closure \bar{M} of M .

Thus, a topological space is a set T of arbitrary elements to each subset M of which is assigned a subset \bar{M} , called the closure of M , in such a way that, calling a set closed if it is identical with its own closure:

- (1) each single element is a closed set,
- (2) the closure of any set is closed,
- (3) the closure of the union of two sets is the union of their closures.

If we now define an open set as the complement in T of a closed set, and a neighborhood of a set of points as any open set containing it, we may introduce five types of topological spaces, each contained in its predecessor, in terms of their separation properties. Thus, given any two distinct points, a space is called a T_0 space if at least one of the points has a neighborhood not containing the other, a T_1 space if each of the points has a neighborhood not containing the other point, a T_2 (or Hausdorff) space if they have a pair of discrete neighborhoods, a T_3 (or regular) space if each of the points has a neighborhood disjoint from some neighborhood of any closed set containing the other point, and a T_4 (or normal) space if any two closed sets have disjoint neighborhoods.

If S is any set of elements of any kind, i.e., not necessarily part of a topological space, we can define a metric in S by assigning to every two points x, y of S a non-negative real number $\rho(x, y)$, called the distance between x and y , such that $\rho(x, x) = 0$, $\rho(x, y) > 0$ if $x \neq y$, $\rho(x, y) = \rho(y, x)$ and $\rho(x, y) + \rho(y, z) \geq \rho(x, z)$. Then for any two subsets P and Q of S we define the distance between P and Q as the least upper bound of the distances from any point in

P to any point in Q and make S into a topological space by defining the closure \bar{P} of P as the set of points in S whose distance from P is zero. Such a space, which is easily seen to be normal, is called a metric space. Most of the important topological spaces of applied mathematics, e.g., **Hilbert space**, are metric. An example, important in analysis, of a non-metrizable space is given by the Banach space of continuous functions from $[0,1]$, a δ -neighborhood of a given function f being defined as the set of functions g with $\max |f(x) - g(x)| < \delta$.

A topological transformation from one topological space to another (which may be identical with the first) is a one-to-one transformation which is continuous in both directions (a transformation being continuous if, supposing that any point P is transformed into P' , then for any neighborhood N' of P' there exists a neighborhood N of P such that every point of N is transformed into a point of N').

TOPOLOGY. The study of topological transformations. (See **topological space**.)

TOP, "SLEEPING." A top is said to "sleep" when it rotates with constant angular speed about its axis in a vertical position with neither **precession** nor **nutation**.

TOROIDAL COORDINATES. A curvilinear system closely related to **bipolar coordinates**. If the traces of the surfaces in that system are taken in the XY -plane as two families of mutually orthogonal circles, then rotation of the circles about the Z -axis forms a family of **spherical surfaces** and a family of tori. The toroidal coordinate surfaces are then taken as: (1) spherical, with centers on the Z -axis at a distance of $\pm a \cot \xi$ from the origin and with radii of $a \csc \xi$, $\xi = \text{const.}$; (2) anchor rings with radii of $a \coth \eta$ for the axial circles and circular cross sections of radii $a \operatorname{csch} \eta$, $\eta = \text{const.}$; (3) planes through the Z -axis, $\psi = \text{const.}$, where $\psi = \tan^{-1} y/x$. These coordinates are related to rectangular coordinates by the equations

$$x = r \cos \psi, \quad y = r \sin \psi$$

$$r = \frac{a \sinh \eta}{\cosh \eta - \cos \xi}; \quad z = \frac{a \sinh \xi}{\cosh \eta - \cos \xi};$$

$$0 \leq \xi \leq 2\pi; \quad 0 \leq \psi \leq 2\pi; \quad 0 \leq \eta \leq \infty.$$

TORQUE. (1) For a single particle the torque is the moment of the resultant force on the

particle with respect to a particular origin. This is expressed by the vector relation $\mathbf{L} = \mathbf{r} \times \mathbf{F}$, where \mathbf{L} is the torque, \mathbf{r} is the position vector with respect to the origin, and \mathbf{F} is the resultant force. The torque is equal to the time rate of change of the moment of momentum. (2) For a rigid body the torque with respect to a set of axes is expressed by the relation

$$\mathbf{L} = \int_v \mathbf{r} \times \mathbf{F}_r dv,$$

where \mathbf{F}_r is the resultant force per unit volume due to external forces on the element dv and \mathbf{r} is the position vector of the volume element. (See **moment of force**.) For a rigid body undergoing free rotation about a single axis, the torque $\mathbf{L} = I\alpha$, where I is the moment of inertia and α is the angular acceleration. (3) In engineering mechanics usage, the moment of all forces to one side of a cross section of a bar about the axis of the bar is called the torque or twisting moment at the section. (See **torsion**.)

TORSE. See **developable (surface)**.

TORSION. (Also called simple torsion.) (See also **curvature**.) A state of deformation of a right cylindrical (or prismatic) body with plane ends in which each plane cross section normal to the generators rotates through an angle proportional to its distance from some reference cross section. The cross section may in addition undergo other displacements which are identical for corresponding points on all cross sections. The constant of proportionality between the angle of rotation and the distance along the cylinder is called the *twist, amount of twist, amount of torsion, or twist per unit length*. Most commonly, torsion refers to a deformation of the type described, such that the cylindrical surface of the body is force-free, so that the deformation is maintained by forces applied to the ends of the cylinder only. If the forces applied to each end are statically equivalent to a couple about a direction parallel to the generators of the cylinder, possibly together with a force, the couple is called the *torsional couple or twisting couple*.

In **classical elasticity theory** for isotropic materials, if τ is the amount of torsion, then the displacement components u, v, w in a rectangular Cartesian coordinate system x, y, z , resulting from the torsion of a cylinder with

its generators parallel to the z -axis, may be written, provided the z -axis is suitably chosen,

$$u = -\tau yz, \quad v = \tau zx \quad \text{and} \quad w = \tau\phi(x,y),$$

where ϕ is a function of position on the cross section called the *torsion function* or *warping function*. If the z -axis is chosen so that the displacement field can be expressed in this way, its position is called the *axis of torsion* or *axis of twist*. ϕ must be a single-valued **harmonic function** and must satisfy the boundary condition

$$\frac{\partial\phi}{\partial n} = yl - xm$$

on the cylindrical surface, where l and m are the cosines of the angles between the outward-drawn normal to the surface, and the axes x and y , respectively, and $\partial/\partial n$ denotes differentiation along the outward-drawn normal. The problem of determining a two-dimensional harmonic function satisfying these conditions is called the *torsion problem*. The forces which must be applied in order to maintain the torsion in the cylinder are statically equivalent to equal and opposite couples applied to the two ends of the cylinder and directed parallel to the generators of the cylinder. They are given in terms of ϕ by

$$\pm\mu\tau \iint \left(x^2 + y^2 + x \frac{\partial\phi}{\partial y} - y \frac{\partial\phi}{\partial x} \right) dx dy,$$

where μ is the rigidity modulus for the material and the integration is carried out over the cross section of the cylinder. The positive ratio, torsional couple/amount of twist is called the *torsional rigidity* of the cylinder.

Since the torsion function is harmonic, there exists a function $\psi(x,y)$ such that $\phi + i\psi$ is a function of $x + iy$, where $i = \sqrt{-1}$. ψ is a harmonic function of x and y and satisfies the boundary condition

$$\psi - \frac{1}{2}(x^2 + y^2) = \text{constant}$$

on the cylindrical boundary. It is called the *conjugate of the torsion function*, or *conjugate torsion function*.

TORSIONAL BUCKLING. See **buckling, torsional**.

TORSIONAL STIFFNESS. See **torsion**.

TORSIONAL WAVES IN A ROD. A rod may be twisted about an axis of the rod in

such a manner that each transverse section remains in its own plane. If the section is not circular there will be motion parallel to the axis of the bar. For a circular cross section and a homogeneous bar the equations of motion are analogous to those of longitudinal waves in the rod. The velocity of propagation c , in centimeters per second, of torsional waves in a rod, is

$$c = \sqrt{\frac{E}{2\rho(\sigma + 1)}} \tag{1}$$

where E is Young's modulus, in dynes per square centimeter, ρ is the density, in grams per cubic centimeter, and σ is Poisson's ratio.

TORSION, MEMBRANE ANALOGY. See **membrane analogy, torsion**.

TORSION (OF A CURVE AT A POINT P).

Let \mathbf{b} be the unit binormal to a curve and s the distance measured along the curve from a fixed point. The vector $d\mathbf{b}/ds$ at P is parallel to the unit normal \mathbf{n} at P . The torsion τ is then defined by the relation

$$d\mathbf{b}/ds = -\tau\mathbf{n}.$$

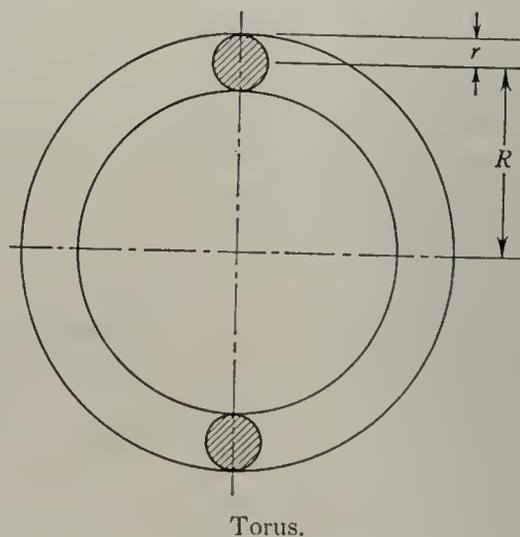
TORSION, RADIUS (OF A CURVE AT A POINT). Reciprocal of the **torsion**.

TORUS. The surface having the shape of a doughnut with a hole in it, generated as a surface of revolution by rotating the circle

$$(y - b)^2 + z^2 = a^2$$

around the z -axis. Its equation, when rationalized, is of the fourth degree

$$(x^2 + y^2 + z^2 + b^2 - a^2)^2 = 4b^2(x^2 + y^2).$$



With dimensions as shown in the figure, its volume, $V = 2\pi^2 Rr^2$ and its surface area, $A = 4\pi^2 Rr$.

TOTAL CURVATURE. See *curvature of a surface, second*.

TOTAL DERIVATIVE (ALSO CALLED MATERIAL OR SUBSTANTIAL DERIVATIVE). The time rate of change of a scalar or vector property of a moving particle as observed by an observer moving with the particle. If \mathbf{v} is the instantaneous velocity of the particle, the total derivative for a scalar U is

$$\begin{aligned} \frac{dU}{dt} &= \frac{\partial U}{\partial t} + \frac{\partial U}{\partial x} \frac{dx}{dt} + \frac{\partial U}{\partial y} \frac{dy}{dt} + \frac{\partial U}{\partial z} \frac{dz}{dt} \\ &= \frac{\partial U}{\partial t} + (\nabla U) \cdot \mathbf{v} \end{aligned}$$

and for a vector \mathbf{A}

$$\begin{aligned} \frac{d\mathbf{A}}{dt} &= \frac{\partial \mathbf{A}}{\partial t} + \frac{\partial \mathbf{A}}{\partial x} \frac{dx}{dt} + \frac{\partial \mathbf{A}}{\partial y} \frac{dy}{dt} + \frac{\partial \mathbf{A}}{\partial z} \frac{dz}{dt} \\ &= \frac{\partial \mathbf{A}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{A}. \end{aligned}$$

TOTAL DETERMINATION. In regression analysis, the square of the multiple correlation R^2 .

TOTAL DIFFERENTIAL. See *differential*.

TOTAL EMISSIVITY (OF A THERMAL RADIATOR). The ratio of the spectral concentration of radiant emittance of the thermal radiator to that of a full radiator at the same temperature.

TOTAL HEAT. See *enthalpy*.

TOTAL NEUTRON FLUX. See *flux, neutron, total*.

TOTAL PRESSURE. The pressure that would be attained at a point in a flowing fluid if the fluid were brought to rest at that point isentropically. For an incompressible fluid the total pressure is

$$p_o = p + \frac{1}{2} \rho V^2,$$

where p is the static pressure, ρ is the density of the fluid and V is the velocity.

For a compressible fluid the total pressure is

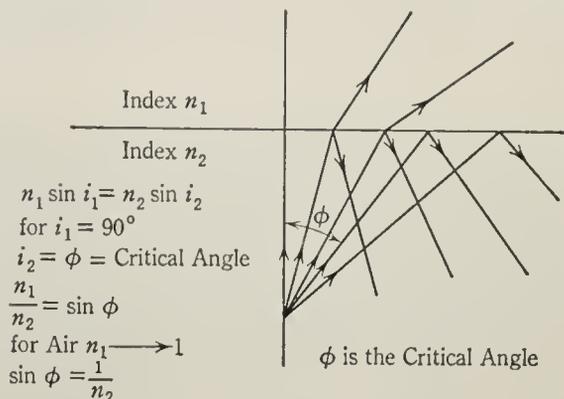
$$p_o = p \left[1 + \left(\frac{V^2}{2} \right) M^2 \right]^{\frac{\gamma}{\gamma-1}}$$

where M is the Mach number.

TOTAL RADIATION TEMPERATURE.

The temperature of a full radiator at which it has the same (total) radiant emittance as the body considered.

TOTAL REFLECTION. No conventional mirror will ever reflect all of the radiation striking it. (See *reflection coefficient*.) However, radiation in a more dense medium, meet-



ing the boundary of a less dense medium at an angle greater than the critical angle, will be totally reflected back into the more dense medium. The critical angle is

$$\phi_c = \sin^{-1} \frac{n_1}{n_2}$$

where n_2 is the index of refraction of the medium in which the radiation is incident and n_1 that of the second medium.

TOTAL THEORY OF PLASTICITY. See *deformation theory*.

TOTAL THERMODYNAMIC POTENTIAL. See *thermodynamics, characteristic functions of*.

TOTAL TRANSMITTANCE. The ratio of the luminous flux transmitted by a body to that which it receives. In *mixed transmission*, the total transmission factor is the sum of two components, the direct transmission factor (τ_r) and the diffuse transmission factor (τ_d).

TPC THEOREM. The statement that a local field theory which is invariant under orthochronous proper Lorentz transformations is automatically invariant under the combined operation *TPC*, where *T* is the *time-inversion operation*, *P*, the *parity operation* and *C*, the *charge conjugation operation*.

Yost has established the equivalence between the *TPC* theorem and the **weak local commutativity** of field operators. A set of field operators $\psi_0, \psi_1 \cdots \psi_n$ has the weak local commutativity property at a set of real 4-vectors $\xi_1 \cdots \xi_n$ if the following relation:

$$\begin{aligned} &(\Psi_0, \psi_0(y_0) \cdots \psi_n(y_n)\Psi_0) \\ &= \epsilon(\Psi_0, \psi_n(y_n) \cdots \psi_1(y_1)\psi_0(y_0)\Psi_0) \end{aligned}$$

holds for all $(y_0, \cdots y_n)$ such that the differences

$$\eta_j = y_{j-1} - y_j$$

lie in some real neighborhood of the ξ_j . Here Ψ_0 is the vacuum state and $\epsilon = \pm 1$, the sign depending on whether the permutation of fermion fields between left and right hand sides of Equation (1) is even or odd.

If we denote the **Wightman functions** (the vacuum expectation of these operators) by

$$W(\eta_1 \cdots \eta_n) = (\Psi_0, \psi_0(y_0) \cdots \psi_n(y_n)\Psi_0)$$

then the weak local commutativity property of the field operators $\psi_0(y_0) \cdots \psi_n(y_n)$ asserts that

$$W(\eta_1 \cdots \eta_n) = V(-\eta_1 \cdots -\eta_n)$$

where

$$V(\eta_1, \cdots \eta_n) = (\Psi_0, \psi_n(-y_n) \cdots \psi_0(-y_0)\Psi_0).$$

The theorem of Yost states that for *TCP* invariance of the Wightman function, which is the statement that

$$W(\eta_1, \cdots \eta_n) = V(\eta_1, \cdots \eta_n)$$

it is necessary and sufficient that the fields $(\psi_0 \cdots \psi_n)$ have the weak local commutativity property at one set of vectors $\xi_1 \cdots \xi_n$ in a special domain D . The domain D consists of those real points $\xi_1 \cdots \xi_n$ for which

$$\left(\sum_1^n \lambda_j \xi_j \right)^2 < 0$$

when the λ_j are any set of real non-negative numbers not all zero, such that $\sum_{j=1}^n \lambda_j = 1$.

TRACE. The sum of the diagonal elements of a **matrix**, indicated by $Tr \mathbf{A} = \sum A_{ii}$. Its properties include $Tr \mathbf{AB} = Tr \mathbf{BA}$; $Tr \mathbf{C} = Tr \mathbf{A} \cdot Tr \mathbf{B}$, where $\mathbf{C} = \mathbf{A} \times \mathbf{B}$, the direct **product**. It is also called *Spur* from the German word. When a matrix is a **representation of a group**, its trace is called the *character* of the representation.

TRACK LENGTH, NEUTRON. Since the path traversed in time dt by a neutron with speed v is vdt , the quantity

$$\Phi(r,t) = \int_0^\infty dE \int d\hat{\Omega} N(r,e,\hat{\Omega},t)v$$

defined elsewhere as **flux, neutron, total**, may also be considered the total track length per unit time per unit volume.

TRAILING-VORTEX DRAG. The drag of a wing or other lifting system associated with the kinetic energy that is being continually added to the trailing vortices. For a wing having given values of the aspect ratio A and lift coefficient C_L , the coefficient of trailing vortex drag has the minimum value $C_L^2/\pi A$, when the load distribution across the span is *elliptic*.

Trailing-vortex drag has usually been called induced drag in the past, but as the later term is now often used with another meaning, trailing-vortex drag is to be preferred. (See also **lifting-line theory**.)

TRAIN, EDGE. See **edge train**.

TRAJECTORY. The path followed by a particle or projectile.

TRAJECTORY STRESS. See **stress, trajectory**.

TRANSADMITTANCE, INTERNAL. See **internal transadmittance**.

TRANSCENDENTAL. A term applied to numbers, equations, or functions which are not algebraic. The word is intended to suggest only that transcendental operations cannot be defined by elementary methods (*Quod algebrae vires transcendit*).

A transcendental number is not a root of a polynomial in one unknown, with integral coefficients. Typical examples are: e , the base of natural logarithms and π , the area of a circle with unit radius. A function y of x is called an algebraic function if y can be defined in terms of x by an algebraic equation; that is, by setting a polynomial in x and y equal to zero. A function which is not algebraic is called transcendental. It is clear that an algebraic function can have only finitely many zeros; thus, e.g., $\sin x$ is a transcendental function.

TRANSFERABLE QUANTITY. A quantity or property possessed by a fluid which is transferred by turbulent motion without affecting the motion. Examples are water vapor and smoke concentration.

Heat is a transferable quantity when the buoyancy forces it produces are negligible compared with the inertia forces, that is when the **Reynolds number** of the turbulence is sufficiently large. But, clearly, in thermal convection, which is a form of turbulence, the turbulence itself is produced by the gradient of the quantity being transferred, and it is then regarded as a non-transferable quantity.

Momentum, likewise, is a non-transferable quantity in turbulence produced in zones of large shear. But momentum has also to be regarded as a non-transferable quantity because it can be communicated from one fluid element to another without mixing (or molecular transfer) through the agency of the pressure forces. The turbulence may therefore have a different **mixing length** for momentum from the length for other properties such as heat, pollution concentration, etc.

On the other hand, it has been argued that **vorticity** is a transferable quantity provided that the motion is two-dimensional in parallel planes; but such motion is special in that there is no stretching of vortex lines and is therefore not turbulent in the proper sense.

TRANSFER FUNCTION. If a system, determined by the inter-relations between the n quantities $q_1(t) \cdots q_n(t)$ which the system is considered as containing, is excited by a single input $\theta_1(t)$, it is always possible to write the relations existing between θ_1 and the q 's in the form of n equations of the form

$$a_{11}q_1 + a_{21}q_2 + \cdots + a_{n1}q_n + \theta_1 = 0$$

.....

$$a_{1m}q_1 + a_{2m}q_2 + \cdots + a_{nm}q_n = 0, \quad (1 < m \leq n)$$

where the a 's are in general functions of the operator $\frac{d}{dt}$. If **Laplace transforms** of these equations are written down, subject to the condition that the system is initially quiescent, that is, that θ and all the q 's (and hence their derivatives) are zero for negative time, we obtain

$$a_{11}Q_1 + a_{21}Q_2 + \cdots + a_{n1}Q_n + \Theta_1 = 0$$

.....

$$a_{1m}Q_1 + a_{2m}Q_2 + \cdots + a_{nm}Q_n = 0$$

where $\Theta_1(s)$, $Q_r(s)$ are respectively the transforms of $\theta_1(t)$, $q_r(t)$ and where the a 's are now the same functions of s as they were of $\frac{d}{dt}$.

These equations give the general determinantal solution

$$\frac{Q_r}{\Theta_1} = - \frac{\Delta_{r1}}{\Delta}$$

where Δ is the system determinant formed by the a 's and Δ_{r1} is the co-factor of a_{r1} in Δ .

The above ratio is defined as the **transfer function** between $Q_r(s)$ and $\Theta_1(s)$. It is therefore the ratio of the Laplace transform of the selected quantity to that of the specified input, subject to quiescent initial conditions.

If $q_n(t)$ is the output quantity of the system, the ratio

$$\frac{Q_n}{\Theta_1} = - \frac{\Delta_{n1}}{\Delta}$$

is defined to be the *system transfer function* for the particular input.

By an extended use of the term, the ratio

$$\frac{Q_r}{Q_p} = \frac{\Delta_{r1}}{\Delta_{p1}}$$

is defined to be the *transfer function* between $Q_r(s)$ and $Q_p(s)$.

It must be noted however that these definitions are only unique when the point of application of the input is specified. This is easily seen by reference to the signal flow diagram for



a simple two-quantity system, in which the input is applied first to Q_1 and then to Q_2 .

In the first case,

$$Q_1 = \Theta + a_{21}Q_2, \quad Q_2 = a_{12}Q_1$$

which give

$$Q_1 = \frac{Q_2}{a_{12}} = \frac{\Theta}{1 - a_{12}a_{21}}$$

whereas in the second case,

$$Q_2 = \Theta + a_{12}Q_1, \quad Q_1 = a_{21}Q_2$$

which give

$$\frac{Q_1}{a_{21}} = Q_2 = \frac{\Theta}{1 - a_{12}a_{21}}.$$

It will be seen that none of the ratios $\frac{Q_1}{\Theta}$, $\frac{Q_2}{\Theta}$, $\frac{Q_1}{Q_2}$ have the same values in the two cases, though, as might be expected, the value of $\frac{Q_1}{\Theta}$ in the first case is equal to that of $\frac{Q_2}{\Theta}$ in the second.

The system transfer function is the Laplace transform of its **normal response** (as a time function) to unit **impulse function** (of the first order) at $t = 0$, this response being termed the system **weighting function**.

Any transfer function, with $s = i\omega$, gives the corresponding frequency-function, the modulus and phase of which are respectively equal to the amplitude ratio and the relative phase of the two time-quantities involved when they both vary sinusoidally at angular frequency ω .

TRANSFER FUNCTION, REACTOR. See **reactor, transfer function**.

TRANSFER IMPEDANCE. See **impedance, transfer**.

TRANSFER OF CONTROL. In the usual stored-program computers, **commands** are executed in the sequence in which they occur in storage unless a command of a specific type (a transfer command) interrupts the sequence and causes the machine to look elsewhere for its next instruction. The command may be *conditional*, to be executed only in case a certain condition is satisfied, or *unconditional*.

TRANSFINITE NUMBER. See **cardinal**.

TRANSFORM. 1. If **A**, **B**, **X** are three **matrices**, or more generally three elements of any **group** then $\mathbf{B} = \mathbf{X}^{-1}\mathbf{A}\mathbf{X}$ is the transform of **A** by **X** and **A**, **B** are conjugate to each other. The complete set of group elements which are conjugate to each other form a class of the group. 2. An **integral equation** of the first kind,

$$f(y) = \int K(x, y)F(x)dx$$

is called the transform of $F(x)$. (See **integral transform**; **Fourier transform**; **impedance transform**.)

TRANSFORMATION. The word "transformation" is essentially synonymous with such words as **operator**, **function**, **correspondence**, **mapping**, etc. For historical reasons, however, the different words are usually used in different settings, the word "transformation" most often perhaps in connection with changes of coordinate systems, e.g., in the plane. Thus, consider a point (x, y) in a **plane**, referred to a **rectangular coordinate system**, XOY and let another coordinate system $X'O'Y'$ have its origin at the point (h, k) . Then the coordinates of the point in the second system will be (x', y') , where $x = x' + h$, $y = y' + k$. Similarly, if another system $X''O''Y''$ has the same origin as XOY but the coordinate axes are rotated by the angle θ , the relations are $x = x'' \cos \theta - y'' \sin \theta$, $y = x'' \sin \theta + y'' \cos \theta$. A more general case is an affine transformation, $x' = a_1x + b_1y + c_1$, $y' = a_2x + b_2y + c_2$. If the **determinant** of the coefficients, $\Delta = a_1b_2 - a_2b_1 \neq 0$, special cases of it are translations, rotations, reflections in the axes, strains, elongations, and compressions.

If $\Delta = 0$, the transformation is **singular**.

Matrix notation is frequently useful for discussing transformations. Consider now rectangular coordinate systems in space, $OXYZ$ and $OX'Y'Z'$. The same **vector** could be described in either system by the relation $\mathbf{x}' = \mathbf{B}\mathbf{x}$, where the elements of the matrix **B** are **direction cosines** for the various pairs of coordinate axes. Alternatively, one may have one coordinate system and rotate the vector \mathbf{x} through the angle ϕ to obtain a new vector \mathbf{y} . A similar matrix equation results, $\mathbf{x} = \mathbf{R}\mathbf{y}$, but the elements of **B** and **R** are not identical.

Given a transforming matrix like **B** or **R**, it is often the case that still another coordinate system is desirable so that simpler relations exist between the two vectors. We then seek a transformation of the form $\mathbf{B} = \mathbf{P}\mathbf{A}\mathbf{Q}$, in which **A** and **B** are said to be equivalent. Important special transformations are: (a) $\mathbf{P} = \mathbf{Q}^{-1}$, collineatory or similarity transformation (German, *Ähnlichkeitstransformation*); (b) $\mathbf{P} = \mathbf{Q}$, congruent; (c) $\mathbf{P} = \mathbf{Q}^\dagger$, conjunctive; (d) $\mathbf{P} = \mathbf{Q} = \mathbf{Q}^{-1}$, real orthogonal; (e) $\mathbf{P} = \mathbf{Q}^\dagger = \mathbf{Q}^{-1}$, unitary. Case (c) is identical with (b) if the matrices are real; (d) is also case (b) and case (e) is also (c). (For the meaning of the symbols designating the properties of **P**, see **matrix**.)

A second common use of the word transformation occurs in the theory of functions of a

complex variable, thus if $w = f(z)$, the linear fractional transformation is

$$w = (az + b)/(cz + d); \quad (ad - bc) \neq 0,$$

where a, b, c, d are constants, real or complex. It is a mapping or transformation in the complex plane. A circle in the z -plane is transformed into a circle in the w -plane. A straight line transforms in the same way, for it may be regarded as a special case of a circle. Since there are only three independent constants in the transformation, any three points z_1, z_2, z_3 are images of $w_1, w_2,$ and w_3 . Thus the points in the z -plane become $0, 1, \infty$ by the transformation,

$$w = (z_2 - z_3)(z - z_1)/(z_2 - z_1)(z - z_3).$$

Other terms used for this type of transformation are Möbius, general bilinear, and homographic transformation. (See also **conformal representation**.)

TRANSFORMATION, AFFINE. See **affine transformation**.

TRANSFORMATION, CANONICAL. See **canonical transformation**.

TRANSFORMATION, CONTACT. See **contact transformation**.

TRANSFORMATION, GALILEAN. See **Galilean transformation**.

TRANSFORMATION GROUPS, GENERAL.

(A) The orthogonal linear transformation(s) (Lorentz) group is defined as the group of transformations that leaves the infinitesimal quantity

$$(ds)^2 = dx_1^2 + dx_2^2 + dx_3^2 + dx_4^2, \text{ invariant.}$$

It consists of $(\infty)^6$ transformations (i.e., contains 6 arbitrary parameters). A proper orthogonal transformation is a subgroup with the functional determinant $+1$. It can be interpreted as a rigid rotation of the axes. The transformation equations are

$$x'_i = \sum_{j=1}^n a_{ij}x_j \quad (i = 1, \dots, n).$$

For a proper orthogonal transformation the determinant $(a_{ij}) = +1$, for an improper transformation it is -1 .

(B) The affine group contains all linear transformations. A special case of it is that transformation which transforms the equation of the

light cone $x_1^2 + x_2^2 + x_3^2 + x_4^2 = 0$ into itself, so that

$$x'_1{}^2 + x'_2{}^2 + x'_3{}^2 + x'_4{}^2 = \rho(x_1^2 + x_2^2 + x_3^2 + x_4^2)$$

where ρ is an arbitrary function of the coordinates.

(C) The projective group is the group of linear fractional transformations

$$x'_i = \frac{\sum_{m=1}^n a_{im}x_m + b_i}{\sum_{m=1}^n c_mx_m + d} \quad (i = 1, 2, 3, \dots, n).$$

A special case of it plays a great role in optics, viz., the collinear mapping

$$x' = \frac{a_1x + b_1}{a_0x + d_0}; \quad y' = \frac{b_3y}{a_0x + d_0}.$$

It maps the y -axis onto itself and the x -axis is also a symmetry axis.

(D) The group of all continuous point transformations with non-vanishing Jacobian, i.e.,

$$x'_i = f_i(x_1x_2x_3x_4) \quad (i = 1, 2, 3, 4)$$

with

$$\left| \frac{\partial x_i}{\partial x'_k} \right| \neq 0.$$

All tensor relations remain invariant under this transformation.

(E) Weyl's group. The parallel displacement of a contravariant vector ξ^i is again defined by

$$d\xi^i = -\Gamma_{rs}^i \xi^r dx^s$$

with

$$\Gamma_{rs}^i = \Gamma_{sr}^i$$

where

$$\Gamma_{\alpha\kappa}^\lambda = \frac{1}{2}g^{\lambda\sigma}(g_{i\sigma,\kappa} + g_{\kappa\sigma,i} - g_{i\kappa,\sigma}) + \frac{1}{4}g^{\lambda\sigma}(g_{i\sigma}\phi_\kappa + g_{\kappa\sigma}\phi_i - g_{i\kappa}\phi_\sigma).$$

The ϕ_λ are four arbitrary functions. The $g_{\mu\nu}$ represent the gravitational field and the ϕ_μ are the components of the world vector potential in Weyl's unified theory. Under ordinary coordinate transformations the ϕ_μ are postulated to transform like vectors, but under "gauge

transformations," i.e., $\bar{g}_{ik} = \lambda g_{ik}$, the ϕ_μ transform according to the law

$$\bar{\phi}_i = \phi_i - \frac{1}{\lambda} \frac{\partial \lambda}{\partial x^i},$$

where λ is an arbitrary function of the x^μ .

TRANSFORMATION LAWS FOR FIELD STRENGTHS.

$$E'_1 = E_1$$

$$E'_2 = \frac{1}{\sqrt{1 - \beta^2}} (E_2 - \beta B_3)$$

$$E'_3 = \frac{1}{\sqrt{1 - \beta^2}} (E_3 + \beta B_2)$$

$$D'_1 = D_1$$

$$D'_2 = \frac{1}{\sqrt{1 - \beta^2}} (D_2 - \beta H_3)$$

$$D'_3 = \frac{1}{\sqrt{1 - \beta^2}} (D_3 + \beta H_2)$$

$$B'_1 = B_1$$

$$B'_2 = \frac{1}{\sqrt{1 - \beta^2}} (B_2 + \beta E_3)$$

$$B'_3 = \frac{1}{\sqrt{1 - \beta^2}} (B_3 - \beta E_2)$$

$$H'_1 = H_1$$

$$H'_2 = \frac{1}{\sqrt{1 - \beta^2}} (H_2 + \beta D_3)$$

$$H'_3 = \frac{1}{\sqrt{1 - \beta^2}} (H_3 - \beta D_2).$$

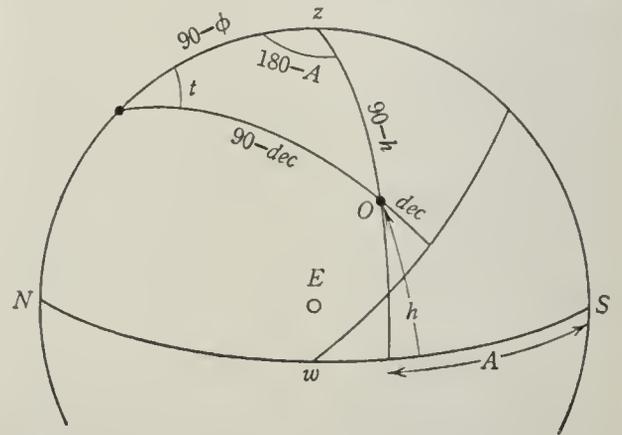
$\beta = \frac{v}{c}$, where v is the velocity of the system Σ' in the positive x -direction of the system Σ . (See also under **Minkowski's electrodynamics for moving systems.**)

TRANSFORMATION, LORENTZ. See Lorentz transformation.

TRANSFORMATION OF ASTRONOMIC COORDINATES. The astronomic coordinates of a celestial object are considered to be spherical in character. The transformation from one system to another is simply an exercise in spherical trigonometry. We shall dis-

cuss only one case here to indicate the general method. Other transformations will be found in the article on **navigational astronomy.**

The figure is drawn for an observer in the northern hemisphere in north latitude ϕ and



with the object O west of the observer's meridian. The equatorial coordinates of O are the declination, dec , and the local hour angle, t . These are represented as north and west, respectively. The horizontal coordinates of O are the altitude h and the azimuth A .

The spherical triangle of the figure is frequently referred to as the astronomer's triangle or the navigator's triangle. The sides of the triangle are $(90^\circ - \phi)$, $(90^\circ - h)$ and $(90^\circ - dec)$. The vertex angles are t , $(180^\circ - A)$ and the angle at O which is known as the parallactic angle.

Applying the general principles of spherical trigonometry we have

$$\begin{aligned} \sin dec &= \sin h \sin \phi - \cos h \cos \phi \cos A \\ \cos dec \cos t &= \sin h \cos \phi + \cos h \sin \phi \cos A \\ \cos dec \sin t &= \cos h \sin A. \end{aligned}$$

Transformations to ecliptic and galactic coordinates can be handled in similar manner. (See also **navigational astronomy.**)

TRANSFORMATION, ORDER-DISORDER. See order-disorder transformation.

TRANSFORMATION, ORTHOGONAL. (1) A linear transformation from one rectangular Cartesian coordinate system (x_1, x_2, x_3) to another (x'_1, x'_2, x'_3) can be written:

$$\begin{aligned} x'_1 &= a_{11}x_1 + a_{12}x_2 + a_{13}x_3, \\ x'_2 &= a_{21}x_1 + a_{22}x_2 + a_{23}x_3, \\ x'_3 &= a_{31}x_1 + a_{32}x_2 + a_{33}x_3. \end{aligned}$$

If x_1, x_2, x_3 specify a radius vector, then the radius vector specified by x'_1, x'_2, x'_3 must be of the same magnitude (see **invariance principle**). This requires that $\sum_i a_{ij}a_{ik} = \delta_{jk}$, where $j, k = 1, 2, 3$, and δ_{jk} is the **Kronecker delta**. A transformation that satisfies these relations is called orthogonal.

As an example, consider the transformation from the two-dimensional system x_1, x_2 to the rotated system x'_1, x'_2 . The transformation equations are

$$\begin{aligned}x'_1 &= x_1 \cos \phi + x_2 \sin \phi, \\x'_2 &= -x_1 \sin \phi + x_2 \cos \phi.\end{aligned}$$

It is easily shown that the orthogonality relations are satisfied for this transformation.

A linear orthogonal transformation can be expressed in terms of a transformation **matrix**,

$$\mathbf{A} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}.$$

The transformation equations can then be written as $\mathbf{A}\mathbf{r} = \mathbf{r}'$ where \mathbf{A} is a matrix operator, operating on the vector \mathbf{r} and transforming it into the vector \mathbf{r}' . For the two dimensional example the transformation matrix is

$$\mathbf{A} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix}.$$

(2) The above ideas can be generalized to multidimensional spaces such as **Hilbert space**. (See also **eigenfunction**.)

TRANSFORMATION, PROPER LORENTZ. See **proper Lorentz transformation**.

TRANSFORMATION THEORY OF THERMODYNAMICS OF IRREVERSIBLE PROCESSES. The **entropy production** can be written as the sum of the products of generalized forces or affinities and the corresponding rates (or generalized fluxes) of the irreversible processes (see **thermodynamics, second law of**)

$$\frac{d_i S}{dt} = \sum_k J_k X_k \geq 0. \quad (1)$$

Instead of using in (1) a given set of forces X_k and rates J_k , one may introduce a new set of forces X'_k which are linear combinations of the old ones and choose a new set of rates J'_k

in such a way that the entropy production is invariant

$$\sum_k J_k X_k = \sum_k J'_k X'_k. \quad (2)$$

The description in terms of (J_k, X_k) is macroscopically equivalent to that in terms of (J'_k, X'_k) .

An alternative procedure would be to introduce first a new set of rates, J'_k , which are linear combinations of the old ones and then to choose the new X'_k in order to satisfy (2).

Systems described by (J_k, X_k) as by (J'_k, X'_k) are called *equivalent systems*.

TRANSFORMED SECTION. In the analysis of the bending of beams of more than one material or of beams bent into the plastic range, it is often convenient to reduce the problem to an equivalent beam of one elastic material of modulus E_t . A transformed section is employed which must supply the same force per unit of depth. Therefore the cross-sectional width b at each level is altered to b_t so that the product $bE = b_t E_t$ is the same for the transformed section and the original beam.

TRANSFORMER, IDEAL. (1) A hypothetical **transformer** which neither stores nor dissipates energy but only transmits it. Its self-inductances have a finite ratio, and unity coefficient of coupling. Its self- and mutual-impedances are pure inductances of infinitely great value. A two-winding ideal transformer satisfies the following relations:

$$\begin{aligned}E_1 &= nE_2 \\I_1 &= I_2/n\end{aligned}$$

and transforms **load impedances** by the factor n^2 . (2) An ideal transformer may also be defined as a transformer employing an ideal core or cores operating in the unsaturated region of the core characteristic, having perfect coupling between windings and further characterized by absence of winding resistances, losses, and capacitances.

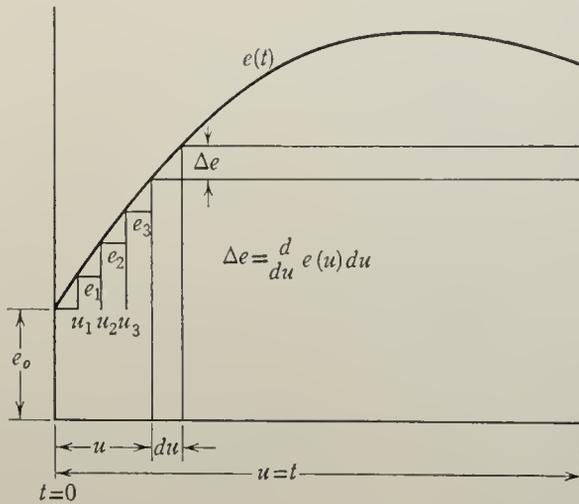
TRANSFORM, FOURIER. See **Fourier transform**.

TRANSFORM, IMPEDANCE. See **impedance transform**.

TRANSIENT RESPONSE OF A DYNAMICAL SYSTEM TO AN ARBITRARY FORCE (DUHAMEL'S INTEGRAL). The Heaviside

operational calculus is used to obtain the response of various combinations of elements in electrical, mechanical rectilinear, mechanical rotational, and acoustical dynamical systems to a unit force excitation. The value of the unit force excitation is that the response to any arbitrary force may be obtained from the unit force solution by a single integration of Duhamel's integral. The proof and use of Duhamel's integral for obtaining the transient response of a dynamical system to any arbitrary force is carried out below.

The discussion is confined to the electrical system. This proof, as in the case of other such systems, can be extended to apply to mechanical and acoustical systems. Let the arbitrary electromotive force be represented by the curve of the figure. The curve can be as-



Step function approximation.

sumed to be made up of a series of unit type electromotive forces, as shown in the figure. At $t = 0$ an electromotive force e_0 is impressed upon the system. A time u_1 later, an electromotive force e_1 is added, a time u_2 later, an electromotive force e_2 is added, etc., all being of the unit type. The current at a time t is then the sum of the currents due to e_0 at $t = 0$, e_1 at $t = u_1$, etc. The current due to e_0 is $e_0 A_E(t)$, where $A_E(t)$ is the indicial electrical admittance. The current due to the electromotive force Δe , which begins at time u , is obviously $\Delta e A_E(t - u)$, $t - u$ being the time elapsed since the unit electromotive force Δe was turned on. Therefore, the total current at the time $u = t$ is

$$i = e_0 A_E(t) + \sum_{u=0}^{u=t} \Delta e A_E(t - u) \tag{1}$$

$$e = \frac{d}{du} e(u) du$$

$$i = e_0 A_E(t) + \int_0^t A_E(t - u) \frac{d}{du} e(u) du. \tag{2}$$

The above expression may be transformed into different forms. The integral may be transformed by integrating as follows:

$$\int_0^t U dV = UV \Big|_0^t - \int_0^t V dU \tag{3}$$

$$U = A(t - u)$$

$$dV = de(u)$$

$$V = e(u).$$

Making the above substitutions, a new expression for the current follows:

$$i = e(t) A_E(0) + \int_0^t e(u) \frac{\partial}{\partial u} A_E(t - u) du. \tag{4}$$

Equation 4 is a fundamental formula which shows the mathematical relation between the current and the type of electromotive force and the constants of the system.

The important conclusions regarding Duhamel's integral are as follows: The indicial admittance of an electrical network determines within a single integration the behavior of a network to any type of electromotive force. In other words a knowledge of the indicial admittance is the only information necessary to completely predict the performance of a system, including the steady state.

The velocity, angular velocity and volume current in the mechanical rectilinear, mechanical rotational and acoustical systems are analogous to the equation for the current in the electrical system. Therefore Duhamel's integrals in the mechanical rectilinear, mechanical rotational and acoustical systems are as follows:

$$v = f_M(t) A_M(0) + \int_0^t f_M(u) \frac{\partial}{\partial u} A_M(t - u) du \tag{5}$$

$$\theta = f_R(t) A_R(0) + \int_0^t f_R(u) \frac{\partial}{\partial u} A_R(t - u) du \tag{6}$$

$$U = p(t) A_A(0) + \int_0^t p(u) \frac{\partial}{\partial u} A_A(t - u) du. \tag{7}$$

The following general conclusion can be stated as follows: The indicial admittance of any vibrating system determines within a single integration the behavior of the system to any type of applied force.

The Heaviside calculus then becomes an important tool in the solution of transient problems in mechanical and acoustical systems. Since a great number of problems in these fields involve impulsive forces, the use of analogies makes it possible to use the tremendous storehouse of information on electrical systems for the solution of problems in mechanical and acoustical systems.

As an example illustrating the use of Duhamel's integral, consider an electromotive force $E\epsilon^{-\beta t}$ impressed on an electrical circuit consisting of an electrical resistance and electrical capacitance in series. (In this discussion, ϵ is used for the base of natural logarithms.)

The solution of the indicial electrical admittance equation from Equation 5 is

$$A_E(t) = \frac{1}{r_E} \epsilon^{-\frac{1}{C_E r_E} t} \quad (8)$$

It follows from Equation 8 that

$$\begin{aligned} A_E(t-u) &= \frac{1}{r_E} \epsilon^{-\frac{1}{C_E r_E} (t-u)} \\ \frac{\partial}{\partial t} A_E(t-u) &= -\frac{1}{C_E r_E^2} \epsilon^{-\frac{1}{C_E r_E} (t-u)} \\ e(u) &= E\epsilon^{-\beta u} \\ e(t) &= E\epsilon^{-\beta t} \\ A_E(0) &= \frac{1}{r_E} \end{aligned}$$

Substituting the above expression in Equation 4, the current is given by

$$i = E \frac{\epsilon^{-\beta t}}{r_E} + \int_0^t -E \frac{\epsilon^{-\beta u}}{C_E r_E^2} \epsilon^{-\frac{1}{C_E r_E} (t-u)} du \quad (9)$$

$$i = \frac{EC_E}{1 - C_E r_E \beta} \left[\frac{\epsilon^{-\frac{1}{C_E r_E} t}}{C_E r_E} - \beta \epsilon^{-\beta t} \right] \quad (10)$$

Similar analysis and analogous equations may be obtained for a combination of a me-

chanical rectilinear resistance and a compliance, a mechanical rotational resistance and a rotational compliance, and an acoustical resistance and an acoustical capacitance.

TRANSIENT RESPONSE OF A MASS AND A MECHANICAL RECTILINEAL RESISTANCE. The differential equation of a force

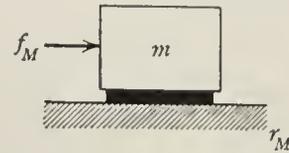


Fig. 1. A mass, m , and mechanical rectilinear resistance, r_M , driven by a force, f_M .

driving a mechanical rectilinear resistance and mass, as shown in Figure 1, is

$$m \frac{dv}{dt} + r_M v = f_M \quad (1)$$

where m is the mass, in grams, r_M is the mechanical rectilinear resistance, in mechanical ohms, v is the linear velocity, in centimeters per second, and f_M is the driving force, in dynes.

In Equation 1 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$m p v + r_M v = f_M \quad (2)$$

The mechanical rectilinear admittance is

$$\frac{v}{f_M} = \frac{1}{r_M + m p} \quad (3)$$

If $f_M = 0$ for $t = 0$ and unity for $t \geq 0$ then the ratio v/f_M is called the mechanical rectilinear indicial admittance designated as $A_M(t)$. The mechanical rectilinear indicial admittance is

$$A_M(t) = \frac{1}{r_M + m p} \mathbf{1} \quad (4)$$

Equation 4 may be written

$$A_M(t) = \frac{1}{(\alpha_M + p)m} \mathbf{1} \quad (5)$$

where $\alpha_M = r_M/m$.

The solution is

$$A_M(t) = \frac{1}{m \alpha_M} (1 - \epsilon^{\alpha_M t}) \quad (6)$$

or

$$A_M(t) = \frac{1}{r_M} (1 - e^{-\frac{r_M}{m}t}). \quad (7)$$

The response characteristic is shown in Figure 2. The velocity is zero for $t = 0$. The velocity increases for values of $t > 0$ and approaches the value $1/r_M$.

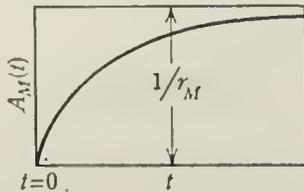


Fig. 2. In the system directly above, the linear velocity as a function of the time for unit excitation.

TRANSIENT RESPONSE OF A MECHANICAL RECTILINEAL RESISTANCE AND A COMPLIANCE. The differential equation of a force driving a mechanical rectilinear re-

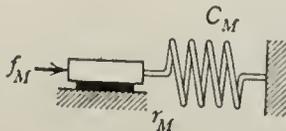


Fig. 1. A mechanical rectilinear resistance, r_M , and a compliance, C_M , driven by a force, f_M .

sistance and compliance, as shown in Figure 1, is

$$f_M = r_M v + \frac{1}{C_M} \int v dt \quad (1)$$

where C_M is the compliance, in centimeters per dyne, r_M is the mechanical rectilinear resistance, in mechanical ohms, v is the linear velocity, in centimeters, and f_M is the force, in dynes.

In Equation 1 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$f_M = r_M v + \frac{v}{C_M p}. \quad (2)$$

The mechanical rectilinear admittance is

$$\frac{v}{f_M} = \frac{p C_M}{1 + p r_M C_M}. \quad (3)$$

If $f_M = 0$ for $t < 0$ and unity for $t \geq 0$, then the ratio v/f_M is called the mechanical rectilinear indicial admittance designated as $A_M(t)$.

The mechanical rectilinear indicial admittance is

$$A_M(t) = \frac{p C_M}{1 + p r_M C_M} \mathbf{1} = \frac{p}{(\alpha_M + p) r_M} \mathbf{1} \quad (4)$$

where $\alpha_M = 1/r_M C_M$.

The solution of Equation 4 is

$$A_M(t) = \frac{e^{-\alpha_M t}}{r_M} = \frac{e^{-\frac{t}{r_M C_M}}}{r_M}. \quad (5)$$

The response characteristic is shown in Figure 2. The linear velocity is $1/r_M$ for $t = 0$. The velocity decreases for values of $t > 0$ and approaches the value zero as a limit.

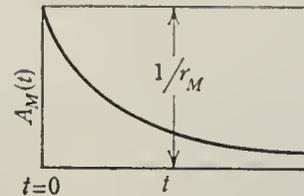


Fig. 2. In the system directly above, the linear velocity as a function of the time for unit excitation.

TRANSIENT RESPONSE OF A MECHANICAL RECTILINEAL RESISTANCE, MASS AND COMPLIANCE. The differential equation of a force driving a mass, mechanical

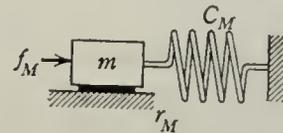


Fig. 1. A mass, m , mechanical rectilinear resistance, r_M , and compliance, C_M , driven by a force, f_M .

rectilinear resistance and compliance is shown in Figure 1 as

$$m \ddot{x} + r_M \dot{x} + \frac{x}{C_M} = f_M \quad (1)$$

where m is the mass, in grams, r_M is the mechanical rectilinear resistance, in mechanical ohms, C_M is the compliance, in centimeters per dyne, \ddot{x} is the acceleration, in centimeters per second per second, \dot{x} is the velocity, in centimeters per second, x is the displacement, in centimeters, and f_M is the driving force, in dynes.

Substituting v for \dot{x} , Equation 1 may be written

$$m \frac{dv}{dt} + r_M v + \frac{1}{C_M} \int v dt = f_M. \quad (2)$$

In Equation 2 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$mpv + r_M v + \frac{v}{C_M P} = f_M. \quad (3)$$

The mechanical rectilinear admittance is

$$\frac{v}{f_M} = \frac{P}{mp^2 + r_M P + \frac{1}{C_M}}. \quad (4)$$

If $f_M = 0$ for $t < 0$ and unity for $t \geq 0$, then the ratio v/f_M is called the mechanical rectilinear indicial admittance, designated as $A_M(t)$.

The mechanical rectilinear indicial admittance is

$$A_M(t) = \frac{P}{mp^2 + r_M P + \frac{1}{C_M}} 1. \quad (5)$$

Let

$$\alpha_M = \frac{r_M}{m}$$

$$\omega_M = \sqrt{\frac{1}{mC_M} - \alpha_M^2}.$$

The mechanical rectilinear indicial admittance is

$$A_M(t) = \frac{1}{m\omega_M} \frac{P\omega_M}{(p + \alpha_M)^2 + \omega_M^2} 1. \quad (6)$$

The solution is

$$A_M(t) = \frac{1}{m\omega_M} \epsilon^{-\alpha_M t} \sin \omega_M t. \quad (7)$$

The response for $r_M^2 < 4m/C_M$ is shown in Figure 2A. It is a damped sinusoid.

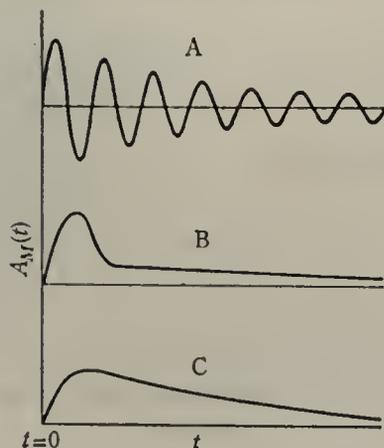


Fig. 2. In the system directly above, the linear velocity as a function of the time for unit excitation.

If $r_M^2 > 4m/C_M$, then the solution becomes

$$A_M(t) = \frac{1}{m\beta_M} \epsilon^{-\alpha_M t} \sinh \beta_M t \quad (8)$$

where

$$\beta_M = \sqrt{\alpha_M^2 - \frac{1}{mC_M}}.$$

The response of this condition is shown in Figure 2B.

If $r_M^2 = 4m/C_M$ then $\sin \omega_M t$ approaches $\omega_M t$ and the solution is

$$A_M(t) = \frac{t}{m} \epsilon^{-\alpha_M t}. \quad (9)$$

The response for this condition is shown in Figure 2C.

TRANSIENT RESPONSE OF A MECHANICAL ROTATIONAL RESISTANCE AND A ROTATIONAL COMPLIANCE. The differential equation of a torque driving a mechan-

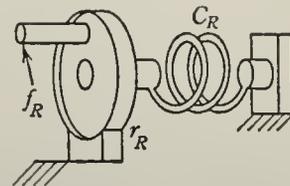


Fig. 1. A mechanical rotational resistance, r_R , and a rotational compliance, C_R , driven by a torque, f_R .

ical rotational resistance and rotational compliance, as shown in Figure 1, is

$$f_R = r_R \theta + \frac{1}{C_R} \int \theta dt \quad (1)$$

where C_R is the rotational compliance, in radians per dyne per centimeter, r_R is the mechanical rotational resistance, in rotational ohms, θ is the angular velocity, in radians per second, and f_R is the torque, in dyne centimeters.

In Equation 1 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$f_R = r_R \theta + \frac{\theta}{C_R P}. \quad (2)$$

The mechanical rotational admittance is

$$\frac{\theta}{f_R} = \frac{pC_R}{1 + p r_R C_R}. \quad (3)$$

If $f_R = 0$ for $t < 0$ and unity for $t \geq 0$, then the ratio θ/f_R is called the mechanical rotational indicial admittance, designated as $A_R(t)$.

The mechanical rotational indicial admittance is

$$A_R(t) = \frac{pC_R}{1 + pr_R C_R} \mathbf{1} = \frac{p}{(\alpha_R + p)r_R} \mathbf{1} \quad (4)$$

where $\alpha_R = 1/r_R C_R$.

The solution of Equation 4 is

$$A_R(t) = \frac{\epsilon^{-\alpha_R t}}{r_R} = \frac{\epsilon^{-\frac{t}{r_R C_R}}}{r_R} \quad (5)$$

The response characteristic is shown in Figure 2. The angular velocity is $1/r_R$ for $t = 0$. The angular velocity decreases for values of $t > 0$ and approaches the value zero as a limit.

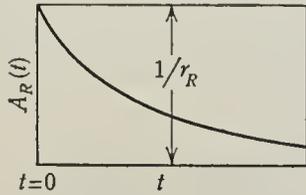


Fig. 2. In the system directly above, the angular velocity as a function of the time for unit excitation.

TRANSIENT RESPONSE OF A MECHANICAL ROTATIONAL RESISTANCE, MOMENT OF INERTIA AND ROTATIONAL COMPLIANCE. The differential equation of a torque driving a moment of inertia, meehan-

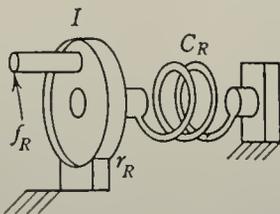


Fig. 1. A moment of inertia, I , mechanical rotational resistance, r_R , and rotational compliance, C_R , driven by a torque, f_R .

ical rotational resistance and rotational compliance, as shown in Figure 1, is

$$I\ddot{\phi} + r_R\dot{\phi} + \frac{\phi}{C_R} = f_R \quad (1)$$

where I is the moment of inertia, in gram-(centimeter)², r_R is the mechanical rotational resistance, in rotational ohms, C_R is the rotational compliance, in radians per dyne per centimeter, $\ddot{\phi}$ is the angular acceleration, in radians per second per second, $\dot{\phi}$ is the angular velocity, in

radians per second, ϕ is the angular displacement, in radians, and f_R is the driving torque, in dyne centimeters.

Substituting θ for $\dot{\phi}$, Equation 1 may be written

$$\frac{d\theta}{dt} + r_R\theta + \frac{1}{C_R} \int \theta dt = f_R. \quad (2)$$

In Equation 2 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes,

$$Ip\theta + r_R\theta + \frac{\theta}{C_R p} = f_R. \quad (3)$$

The mechanical rotational admittance is

$$\frac{\theta}{f_R} = \frac{p}{Ip^2 + r_R p + \frac{1}{C_R}} \quad (4)$$

If $f_R = 0$ for $t < 0$ and unity for $t \geq 0$, then the ratio θ/f_R is called the mechanical rotational indicial admittance designated as $A_R(t)$.

The mechanical rotational indicial admittance is

$$A_R(t) = \frac{p}{Ip^2 + r_R p + \frac{1}{C_R}} \mathbf{1}. \quad (5)$$

Let

$$\alpha_R = \frac{r_R}{I}$$

$$\omega_R = \sqrt{\frac{1}{IC_R} + \alpha_R^2}.$$

The mechanical rotational indicial admittance is

$$A_R(t) = \frac{1}{I\omega_R} \frac{p\omega_R}{(p + \alpha_R)^2 + \omega_R^2} \mathbf{1}. \quad (6)$$

The solution is

$$A_R(t) = \frac{1}{I\omega_R} \epsilon^{-\alpha_R t} \sin \omega_R t. \quad (7)$$

The response for $r_R^2 < 4I/C_R$ is shown in Figure 2A. It is a damped sinusoid.

If $r_R^2 > 4I/C_R$, then the solution becomes

$$A_R(t) = \frac{1}{I\beta_R} \epsilon^{-\alpha_R t} \sinh \beta_R t \quad (8)$$

where

$$\beta_R = \sqrt{\alpha_R^2 - \frac{1}{IC_R}}$$

The response for this condition is shown in Figure 2B.

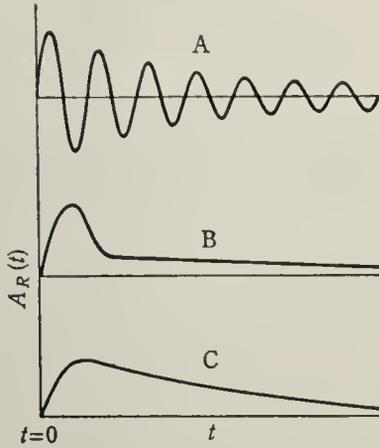


Fig. 2. In the system directly above, the angular velocity as a function of the time for unit excitation.

If $r_R^2 = 4I/C_R$, then $\sin \omega_R t$ approaches $\omega_R t$ and the solution is

$$A_R(t) = \frac{t}{I} \epsilon^{-\alpha_R t} \tag{9}$$

The response for this condition is shown in Figure 2C.

TRANSIENT RESPONSE OF A MOMENT OF INERTIA AND MECHANICAL ROTATIONAL RESISTANCE. The differential equation of a torque driving a mechanical ro-

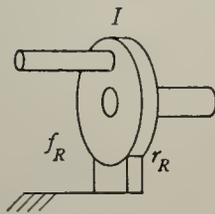


Fig. 1. A moment of inertia, I , and a mechanical rotational resistance, r_R , driven by a torque, f_R .

tational resistance and moment of inertia, as shown in Figure 1, is

$$I \frac{d\theta}{dt} + r_R \theta = f_R \tag{1}$$

where I is the moment of inertia, in gram (centimeter)², r_R is the mechanical rotational resistance, in rotational ohms, θ is the angular velocity, in radians per second, and f_R is the torque, in dyne centimeters.

In Equation 1 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$I p \theta + r_R \theta = f_R \tag{2}$$

The mechanical rotational admittance is

$$\frac{\theta}{f_R} = \frac{1}{r_R + I p} \tag{3}$$

If $f_R = 0$ for $t < 0$ and unity for $t \geq 0$ then the ratio θ/f_R is called the mechanical rotational indicial admittance, designated as $A_R(t)$.

The mechanical rotational indicial admittance is

$$A_R(t) = \frac{1}{r_R + I p} \mathbf{1} \tag{4}$$

Equation 4 may be written

$$A_R(t) = \frac{1}{(\alpha_R + p) I} \mathbf{1} \tag{5}$$

where $\alpha_R = r_R/I$.

The solution of Equation 5 is

$$A_R(t) = \frac{1}{I \alpha_R} (1 - \epsilon^{-\alpha_R t}) \tag{6}$$

or

$$A_R(t) = \frac{1}{r_R} (1 - \epsilon^{-\frac{r_R}{I} t}) \tag{7}$$

The response characteristic is shown in Figure 2. The angular velocity is zero for $t = 0$. The angular velocity increases for values of $t > 0$ and approaches the value $1/r_R$.

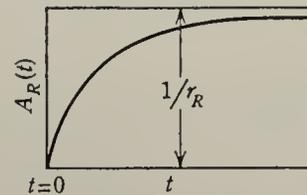


Fig. 2. In the system directly above, the angular velocity as a function of the time for unit excitation.

TRANSIENT RESPONSE OF AN ACOUSTICAL RESISTANCE AND AN ACOUSTICAL CAPACITANCE. The differential equation of a sound pressure driving an acoustical resistance and an acoustical capacitance, as shown in Figure 1, is

$$p = r_A U + \frac{1}{C_A} \int U dt \tag{1}$$

where C_A is the acoustical capacitance, in (centimeter)⁵ per dyne, r_A is the acoustical resistance, in acoustical ohms, U is the volume cur-

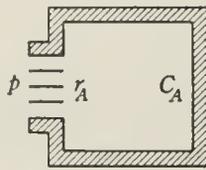


Fig. 1. An acoustical resistance, r_A , and an acoustical capacitance, C_A , driven by a sound pressure, p .

rent, in cubic centimeters per second, and p is the sound pressure, in dynes per square centimeter.

In Equation 1 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$p = r_A U + \frac{U}{C_A p} \quad (2)$$

The acoustical admittance is

$$\frac{U}{p} = \frac{p C_A}{1 + p r_A C_A} \quad (3)$$

If $p = 0$ for $t < 0$ and unity for $t \geq 0$, then the ratio U/p is called the acoustical indicial admittance designated as $A_A(t)$.

The acoustical indicial admittance is

$$A_A(t) = \frac{p C_A}{1 + p r_A C_A} \mathbf{1} = \frac{p}{(\alpha + p) r_A} \mathbf{1} \quad (4)$$

where $\alpha_A = 1/r_A C_A$.

The solution of Equation 4 is

$$A_A(t) = \frac{\epsilon^{-\alpha_A t}}{r_A} = \frac{\epsilon^{-\frac{t}{r_A C_A}}}{r_A} \quad (5)$$

The response characteristic is shown in Figure 2. The volume current is $1/r_A$ for $t = 0$. The volume current decreases for values of $t > 0$ and approaches the value zero as a limit.

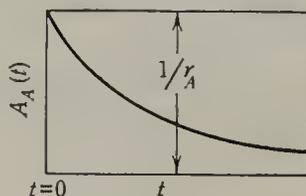


Fig. 2. In the system directly above, the volume current as a function of the time for unit excitation.

TRANSIENT RESPONSE OF AN ACOUSTICAL RESISTANCE, INERTANCE AND ACOUSTICAL CAPACITANCE. The differential equation of a sound pressure driving an acoustical resistance and an inertance con-

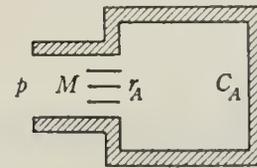


Fig. 1. An inertance, M , acoustical resistance, r_A , and acoustical capacitance, C_A , driven by pressure, p .

needed to an acoustical capacitance, as shown in Figure 1, is

$$M \ddot{X} + r_A \dot{X} + \frac{X}{C_A} = p \quad (1)$$

where M is the inertance, in grams per (centimeter)⁴, r_A is the acoustical resistance, in acoustical ohms, C_A is the acoustical capacitance, in (centimeter)⁵ per dyne, \dot{X} is the volume current, in cubic centimeters per second, and p is the pressure, in dynes per square centimeter.

Substituting U for \dot{X} , Equation 1 may be written

$$M \frac{dU}{dt} + r_A U + \frac{1}{C_A} \int U dt = p \quad (2)$$

In Equation 2 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$M p U + r_A U + \frac{U}{p C_A} = p \quad (3)$$

The acoustical admittance is

$$\frac{U}{p} = \frac{p}{M p^2 + r_A p + \frac{1}{C_A}} \quad (4)$$

If $p = 0$ for $t < 0$ and unity for $t \geq 0$, then the ratio U/p is called the acoustical indicial admittance designated as $A_A(t)$,

The acoustical indicial admittance is

$$A_A(t) = \frac{p}{M p^2 + r_A p + \frac{1}{C_A}} \mathbf{1} \quad (5)$$

Let

$$\alpha_A = \frac{r_A}{2M}$$

$$\omega_A = \sqrt{\frac{1}{MC_A} - \alpha_A^2}$$

The acoustical indicial admittance is

$$A_A(t) = \frac{1}{M\omega_A} \frac{p\omega_A}{(p + \alpha_A)^2 + \omega_A^2} \mathbf{1}. \quad (6)$$

The solution of Equation 6 is

$$A_A(t) = \frac{1}{M\omega_A} \epsilon^{-\alpha_A t} \sin \omega_A t. \quad (7)$$

The response for $r_A^2 < 4M/C_A$ is shown in Figure 2A. It is damped sinusoid.

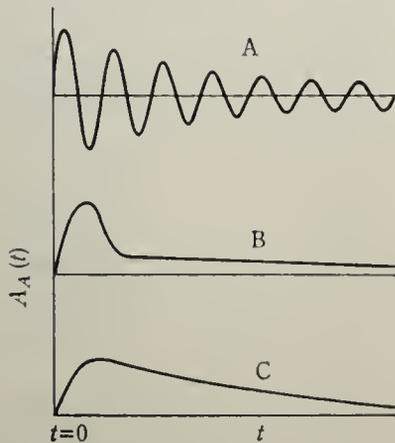


Fig. 2. In the system directly above, the volume current as a function of the time for unit excitation.

If $r_A^2 > 4M/C_A$, then the solution becomes

$$A_A(t) = \frac{1}{L\beta_A} \epsilon^{-\alpha_A t} \sinh \beta_A t \quad (8)$$

where

$$\beta_A = \sqrt{\alpha_A^2 - \frac{1}{MC_A}}$$

The response for this condition is shown in Figure 2B.

If $r_A^2 = 4M/C_A$, then $\sin \omega_A t$ approaches $\omega_A t$ and the solution is

$$A_A(t) = \frac{t}{M} \epsilon^{-\alpha_A t}. \quad (9)$$

The response for this condition is shown in Figure 2C.

TRANSIENT RESPONSE OF AN ELECTRICAL RESISTANCE AND AN ELECTRICAL CAPACITANCE IN SERIES. The differential equation of an electromotive force,

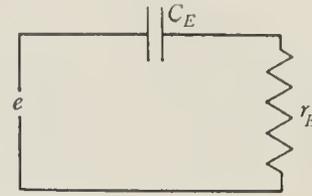


Fig. 1. An electrical capacitance, C_E , and electrical resistance, r_E , driven by an electromotive force, e .

electrical resistance and electrical capacitance connected in series, as shown in Figure 1, is

$$e = r_E i + \frac{1}{C_E} \int i dt \quad (1)$$

where C_E is the electrical capacitance, in abfarads, r_E is the electrical resistance, in abohms, i is the current, in abamperes, and e is the electromotive force, in abvolts.

In Equation 1 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$e = r_E i + \frac{i}{C_E p}. \quad (2)$$

The electrical admittance is

$$\frac{i}{e} = \frac{p C_E}{1 + p r_E C_E}. \quad (3)$$

If $e = 0$ for $t < 0$ and unity for $t \geq 0$, then the ratio i/e is called the electrical indicial admittance designated as $A_E(t)$.

The electrical indicial admittance is

$$A_E(t) = \frac{p C_E}{1 + p r_E C_E} \mathbf{1} = \frac{p}{(\alpha_E + p) r_E} \mathbf{1} \quad (4)$$

where $\alpha_E = 1/r_E C_E$.

The solution of Equation 4 is

$$A_E(t) = \frac{\epsilon^{-\alpha_E t}}{r_E} = \frac{\epsilon^{-\frac{t}{r_E C_E}}}{r_E}. \quad (5)$$

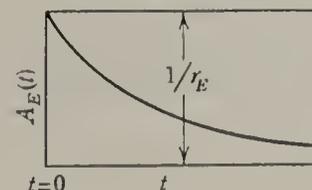


Fig. 2. In the system directly above, the current as a function of time for unit excitation.

The response characteristic is shown in Figure 2. The current is $1/r_E$ for $t = 0$. The current decreases for values of $t > 0$ and approaches the value zero as a limit.

TRANSIENT RESPONSE OF AN ELECTRICAL RESISTANCE, INDUCTANCE AND ELECTRICAL CAPACITANCE IN SERIES. The differential equation of an electromotive force, electrical resistance, induct-

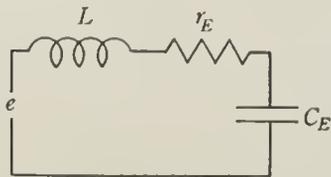


Fig. 1. An inductance, L , electrical resistance, r_E , and electrical capacitance, C_E , driven by an electromotive force, e .

ance and electrical capacitance connected in series, as shown in Figure 1, is

$$L \frac{di}{dt} + r_E i + \frac{1}{C_E} \int i dt = e \quad (1)$$

where L is the inductance, in abhenries, r_E is the electrical resistance, in abohms, C_E is the electrical capacitance, in abohms, i is the current, in abamperes, and e is the electromotive force, in abvolts.

In Equation 1 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$Lp i + r_E i + \frac{i}{C_E p} = e. \quad (2)$$

The electrical admittance is

$$\frac{i}{e} = \frac{p}{Lp^2 + r_E p + \frac{1}{C_E}}. \quad (3)$$

If $e = 0$ for $t < 0$ and unity for $t \geq 0$, then the ratio i/e is called the electrical indicial admittance designated as $A_E(t)$.

The electrical indicial admittance is

$$A_E(t) = \frac{p}{Lp^2 + r_E p + \frac{1}{C_E}} \mathbf{1}. \quad (4)$$

Let

$$\alpha_E = \frac{r_E}{2L}$$

$$\omega_E = \sqrt{\frac{1}{LC_E} - \alpha_E^2}.$$

The electrical indicial admittance is

$$A_E(t) = \frac{1}{L\omega_E} \frac{p\omega_E}{(p + \alpha_E)^2 + \omega_E^2} \mathbf{1}. \quad (5)$$

The solution of Equation 5 is

$$A_E(t) = \frac{1}{L\omega_E} \epsilon^{-\alpha_E t} \sin \omega_E t. \quad (6)$$

The response for $r_E^2 < 4L/C_E$ is shown in Figure 2A. It is a damped sinusoid.

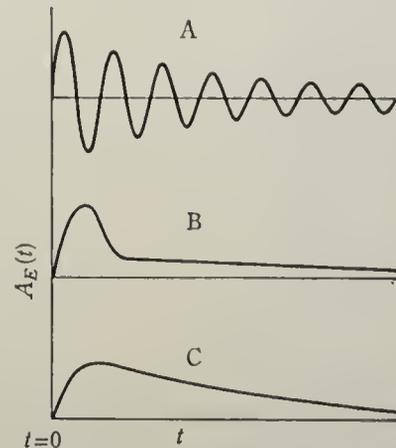


Fig. 2. In the system directly above, the current as a function of the time of unit excitation.

If $r_E^2 > 4L/C_E$, then the solution becomes

$$A_E(t) = \frac{1}{L\beta_E} \epsilon^{-\alpha_E t} \sinh \beta_E t \quad (7)$$

where

$$\beta_E = \sqrt{\alpha_E^2 - \frac{1}{LC_E}}.$$

The response for this condition is shown in Figure 2B.

If $r_E^2 = 4L/C_E$, then $\sin \omega_E t$ approaches $\omega_E t$ and the solution is

$$A_E(t) = \frac{t}{L} \epsilon^{-\alpha_E t}. \quad (8)$$

The response for this condition is shown in Figure 2C.

TRANSIENT RESPONSE OF AN INDUCTANCE AND AN ELECTRICAL RESISTANCE IN SERIES. The differential equation of an electromotive force, electrical resistance

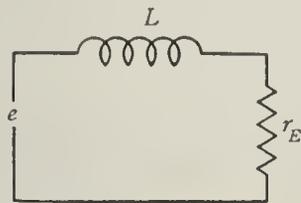


Fig. 1. An inductance, L , and electrical resistance, r_E , in a series, driven by an electromotive force, e .

and inductance connected in series, as shown in Figure 1, is

$$L \frac{di}{dt} + r_E i = e \tag{1}$$

where L is the inductance, in abhenries, r_E is the electrical resistance, in abohms, i is the current, in abamperes, and e is the electromotive force, in abvolts.

In Equation 1 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$L p i + r_E i = e. \tag{2}$$

The electrical admittance is

$$\frac{i}{e} = \frac{1}{r_E + L p}. \tag{3}$$

If $e = 0$ for $t < 0$ and unity for $t \geq 0$, then the ratio i/e is called the electrical indicial admittance designated as $A_E(t)$. The electrical indicial admittance is

$$A_E(t) = \frac{1}{r_E + L p} \mathbf{1}. \tag{4}$$

Equation 4 may be written

$$A_E(t) = \frac{1}{(\alpha_E + p)L} \mathbf{1} \tag{5}$$

where $\alpha_E = r_E/L$.

The solution of Equation 5 is

$$A_E(t) = \frac{1}{L \alpha_E} (1 - \epsilon^{-\alpha_E t}) \tag{6}$$

or

$$A_E(t) = \frac{1}{r_E} (1 - \epsilon^{-\frac{r_E}{L} t}). \tag{7}$$

The response characteristic is shown in Figure 2. The current is zero for $t = 0$. The current increases for values of $t > 0$ and approaches the value $1/r_E$.

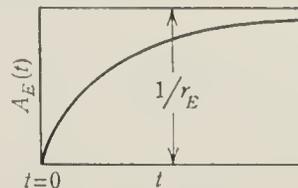


Fig. 2. In the system directly above, the current as a function of the time of unit excitation.

TRANSIENT RESPONSE OF AN INERTANCE AND ACOUSTICAL RESISTANCE. The differential equation of a sound pressure

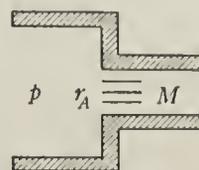


Fig. 1. An inertance, M , and acoustical resistance, r_A , driven by a sound pressure, p .

driving an acoustical resistance and inertance, as shown in Figure 1, is

$$M \frac{dU}{dt} + r_A U = p \tag{1}$$

where M is the inertance, in grams per (centimeter)⁴, r_A is the acoustical resistance, in acoustical ohms, U is the volume current, in cubic centimeters, and p is the sound pressure, in dynes per square centimeter.

In Equation 1 let the symbol p stand for the operator $\frac{d}{dt}$, then the Heaviside operational equation becomes

$$M p U + r_A U = p. \tag{2}$$

The acoustical admittance is

$$\frac{U}{p} = \frac{1}{r_A + M p}. \tag{3}$$

If $p = 0$ for $t < 0$ and unity for $t \geq 0$, then the ratio U/p is called the acoustical indicial admittance designated as $A_A(t)$.

The acoustical indicial admittance is

$$A_A(t) = \frac{1}{r_A + M p} \mathbf{1}. \tag{4}$$

Equation 4 may be written,

$$A_A(t) = \frac{1}{(\alpha_A + \rho)M} \mathbf{1} \quad (5)$$

where $\alpha_A = r_A/M$.

The solution of Equation 5 is

$$A_A(t) = \frac{1}{M\alpha_A} (1 - e^{-\alpha_A t}) \quad (6)$$

or

$$A_A(t) = \frac{1}{r_A} (1 - e^{-\frac{r_A}{M} t}). \quad (7)$$

The response characteristic is shown in Figure 2. The volume current is zero for $t = 0$. The volume current increases for values of $t > 0$ and approaches the value $1/r_A$.

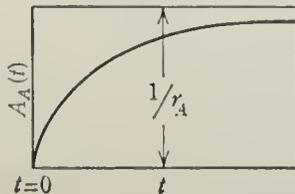


Fig. 2. In the system directly above, the volume current as a function of the time for unit excitation.

TRANSITION. (1) A change of form or external appearance. (2) A change of **state**. (3) A change of **phase**. (4) The process by which a quantum mechanical system, such as an atom or a nucleus, changes from one energy **eigenstate** to another, with the energy difference being balanced by changes in interacting systems or by the emission or absorption of particles or electromagnetic radiation.

TRANSITION, ALLOWED. A transition between two states of a quantum-mechanical system marked by a comparative ease with which the change in quantum numbers involved can be accomplished. For example, transitions in which the total angular momentum quantum number changes by one in units of \hbar , are frequently allowed; and, other things being equal, a transition involving a change of one will take place with greater probability than a competing transition involving a change in this quantum number of two or more. (See **selection rules**; **forbidden transition**.)

TRANSITION, FORBIDDEN. See **forbidden transition**.

TRANSITION, FORCED. A transition between two energy **eigenstates** of a quantum mechanical system which is brought about by the interaction of the system with an external field originating from some source other than the vacuum polarization.

TRANSITION LOSS. In acoustic measurements, the transition loss is defined as follows: At any point in a transmission system, the ratio of the available power from that part of the system ahead of the point under consideration to the power delivered to that part of the system beyond the point under consideration. If the input and/or output power consists of more than one component, such as multi-frequency signal or noise, then the particular components used and their weighting should be specified. This loss is usually expressed in **decibels**.

In wave-propagation usage, transition loss is defined in one of two ways, as follows: (1) At a transition or discontinuity between two transmission media, the difference between the power incident upon the discontinuity and the power transmitted beyond the discontinuity which would be observed if the medium beyond the discontinuity were match-terminated. (2) The ratio in decibels of the power incident upon the discontinuity to the power transmitted beyond the discontinuity which would be observed if the medium beyond the discontinuity were match-terminated.

TRANSITION POINT FROM LAMINAR TO TURBULENT FLOW. See **transition to turbulent flow in a boundary layer**.

TRANSITION PROBABILITY. A term usually applied to the description of a discrete-state **Markov chain**. This term is the conditional probability P_{jk} that the system, governing the evolution of such a chain, is in the state E_k , given that it was in the state E_j at the previous time-point. The transition probabilities P_{jk} are usually arranged in a matrix $P = (p_{jk})$, which is a **stochastic matrix**. For example, in quantum mechanics, the probability that at time t an atom will undergo a transition from state 1 to state k when the atom has been exposed to monochromatic light of frequency $\omega/2\pi$ for t seconds. It is denoted by $|c_k|^2$ which is given by

$$|c_k|^2 = \frac{e^2 F_o^2}{2\hbar^2} |x_{k1}|^2 \frac{1 - \cos(\omega_k - \omega)t}{(\omega_k - \omega)^2}$$

$$= \frac{e^2 F_o^2}{4\hbar^2} |x_{k1}|^2 \frac{\sin^2(\omega_k - \omega)t}{\left(\frac{\omega_k - \omega}{2}\right)^2}$$

where $\omega_k \equiv \frac{E_k - E_1}{\hbar}$, E_1 being the lowest energy state of the system, and where

$$-eF_o x \sin \omega t$$

is the perturbing energy added to the atom by the light wave of amplitude F_o , its electric vector being taken in the x -direction, and its monochromatic frequency being $\omega/2\pi$. This formulation neglects the energy due to the magnetic vector of the light wave.

TRANSITION PROBABILITY, EINSTEIN.

Einstein's transition probabilities A_{nm} for spontaneous emission of radiation, and B_{mn} for absorption, according to wave mechanics are given by

$$A_{nm} = \frac{64\pi^4 \nu_{nm}^3 \sum |R^{nmk}|^2}{3h d_n}$$

and

$$B_{mn} = \frac{8\pi^3 \sum |R^{nmk}|^2}{3h^2 c d_m}$$

Here n and m number the degenerate sublevels of the upper state n and the lower state m , of degeneracy d_n and d_m , respectively, ν_{nm} is the **wave number** of the emitted photon, h is Planck's constant, c is the velocity of light, and R^{nmk} the matrix element of the transition.

TRANSITION, RADIATIVE. A transition between two energy **eigenstates** of a quantum mechanical system, accompanied by the emission of electromagnetic radiation.

TRANSITION RÉGIME. See roughness, effect on skin friction.

TRANSITIONS OF HIGHER ORDER. In an ordinary phase change which one may also call a transition of the *first order*, one has

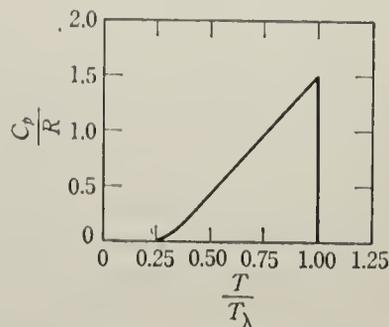
$$\left. \begin{array}{l} G \text{ continuous} \\ S = -\left(\frac{\partial G}{\partial T}\right)_p, \quad V = \left(\frac{\partial G}{\partial p}\right)_T \end{array} \right\} \begin{array}{l} \text{1st order} \\ \text{transitions} \\ \text{discontinuous} \end{array}$$

where G is the Gibbs free energy (see **thermodynamics, characteristic functions of**). A transition is called of the *second order* if

$$\left. \begin{array}{l} G, \left(\frac{\partial G}{\partial T}\right)_p, \left(\frac{\partial G}{\partial p}\right)_T \\ C_p = -T \frac{\partial^2 G}{\partial T^2}, \frac{\partial V}{\partial T} = \frac{\partial^2 G}{\partial p \partial T} \end{array} \right\} \begin{array}{l} \text{continuous} \\ \text{2d order} \\ \text{transitions} \\ \text{discontinuous} \end{array}$$

The definition of higher order transitions (third order ...) is obvious.

This terminology is due to Ehrenfest (see also **Ehrenfest relations**). In a second order transition the specific heat has the temperature dependence represented schematically in the figure.



Specific heat in a second order transition.

The point at which the specific heat presents a discontinuity is called the λ -point.

The question of whether second order transitions really exist in nature is still open.

Independently of the existence of a second order transition, one may call a λ -point the temperature at which the specific heat has a singularity whatever its analytical nature, provided one has not an ordinary first order transition with latent heat.

Lambda points of particular interest are those occurring in order-disorder transitions, at the **Curie point** of ferroelectric or ferromagnetic materials, and at the transition temperature of helium I to helium II. The transition was called lambda by Ehrenfest because of the shape of the specific heat curve. Under orthobaric conditions, liquid helium undergoes this change at 2.19°K; but the temperature at which this change occurs decreases with increasing external pressure.

TRANSITION, SPONTANEOUS. A transition between two energy **eigenstates** of a quan-

tum mechanical system which is brought about by the interaction of the system with the field due to vacuum polarization. (See **transition, forced**.)

TRANSITION, SUPERCONDUCTING. See **superconducting transition**.

TRANSITION TEMPERATURE. See **brittle fracture**.

TRANSITION TEMPERATURE, GLASS. See **glass transition temperature**.

TRANSITION TEMPERATURE, SECOND ORDER. See **second order transition temperature**.

TRANSITION TO TURBULENT FLOW IN A BOUNDARY LAYER. The boundary layer on a body in a stream of fluid is always laminar in the vicinity of the front stagnation point, but if the Reynolds number is high the boundary layer usually becomes turbulent further downstream. The position on the body where the boundary layer changes from the laminar to the turbulent state is often known as the *transition point*, although it is now known that the transition does not occur at a fixed point. The simplified concept of transition at a fixed point is useful, however, in calculating the development of a boundary layer.

In the absence of large disturbances, the necessary first stage in the succession of events leading to transition is the *instability* of the laminar flow. The stability or instability of the laminar boundary layer depends on the Reynolds number, the curvature of the surface and the velocity profile of the boundary layer. Heat transfer and suction also have important effects. Any change leading to greater instability of the laminar boundary layer tends to promote transition, but transition may not actually occur until a point is reached a considerable distance downstream of that at which instability first appears. On a smooth streamline body, free from surface waviness, the position of transition at high Reynolds numbers is usually a short distance downstream of the pressure minimum. This is because the velocity profile of the boundary layer contains a point of inflexion when the pressure gradient is positive, and this leads to instability. On streamline bodies as used in practice, transition often occurs far upstream of the pressure minimum; this is usually be-

cause of the important effects of slight waviness or roughness of the surface.

Instability of the laminar boundary layer, as understood here, means that *small* disturbances in the flow become amplified as they proceed downstream. In order to explain the phenomenon of transition it is necessary to assume that some disturbances are present in the initial laminar flow, even if these are extremely small. In practice, disturbances may be caused by very slight roughness of the surface, turbulence of the main stream, or sound waves.

When the laminar flow is unstable, disturbances are amplified as they proceed downstream and eventually "spots" of turbulent flow appear. The mechanism of the formation of these spots of turbulence is not yet clear, but it seems probable that at some stage vorticity becomes concentrated along discrete lines, so that vortex loops appear in the flow. These vortex loops then apparently become distorted and extended so that spots of turbulence are formed.

As the spots of turbulence proceed downstream they grow in size, so that for points in the boundary layer further downstream the flow is turbulent for a greater proportion of the total time. Eventually a point is reached, sufficiently far downstream, where the flow is turbulent all the time and the process of transition is complete.

An interesting feature of the flow with turbulent spots is that during the passage of each spot the mean velocity near the wall increases, so that the velocity profile becomes more like that found in a turbulent boundary layer. In the ensuing laminar flow, after the passage of the spot, this modified velocity profile makes the laminar boundary layer *more stable* than usual. Thus there is a period of *calm*, after the passage of each spot, in which the disturbances in the laminar boundary layer are damped. During this period of calm, however, the mean velocity profile reverts to its usual form for the laminar boundary layer, so that disturbances are amplified once again.

When the Reynolds number is not very high, the laminar boundary layer often *separates* before transition occurs. In some cases the laminar shear layer becomes turbulent soon after separation and reattaches to the surface as a turbulent boundary layer. This type of

flow is known as a *laminar separation bubble*. (See also **stability of laminar flow**.)

TRANSITION TO TURBULENT FLOW IN A PIPE OR CHANNEL. The flow at a given section in a pipe or channel may be either laminar or turbulent, depending on the entry conditions, the Reynolds number and the distance of the section from the entry. In most cases of practical interest there are fairly large disturbances at entry; then if the Reynolds number exceeds a certain value R_{crit} the flow eventually becomes turbulent. For Reynolds numbers less than R_{crit} the flow remains laminar indefinitely, and even very large disturbances at entry are eventually damped out.

Basing the Reynolds number on the mean velocity, R_{crit} is about 2000 for a pipe of circular section and 1400 for a channel of rectangular section with large ratio of height to width. The lengths used in defining these Reynolds numbers are the diameter of the circular pipe and the width of the rectangular channel.

When only very small disturbances are present at entry the flow remains laminar up to considerably higher Reynolds numbers, even at larger distances from the entry. In a pipe of circular section, if very great care is taken to eliminate disturbances at the entry, laminar flow can be obtained at Reynolds numbers of 20,000 or more.

In cases where the flow eventually becomes turbulent, the distance x from the entry to the start of turbulent flow depends on the entry disturbances and the Reynolds number. For a pipe of diameter d , $\frac{x}{d}$ decreases as the Reynolds number or the entry disturbance is increased. (See also **entry length, for pipe or channel**.)

TRANSLATION. A motion in which all points of a system have identical **displacements**. If each point moves on a straight line the translation is **rectilinear**, if on a curve it is **curvilinear**. The general motion of a **rigid body** is a combination of translation and rotation.

TRANSLATION GROUP. The totality of operations by which a **crystal lattice** may be transformed into itself by bodily displacements without rotation.

TRANSLATION OPERATION. The geometrical process of displacing a body along a

straight line, keeping lines fixed in the body always parallel to themselves.

TRANSLATION, PRIMITIVE. See **primitive translation**.

TRANSMISSIBILITY OF FORCE. The term transmissibility of force refers to the fact that sliding forces along their lines of action does not affect the resultant. The effect of a force on a rigid body depends upon its *magnitude, sense, and line of action* but not on its *point of application*.

TRANSMISSION. The passage of radiation through a medium. In some cases the term is restricted by the requirement that the wavelength of the monochromatic components of the radiation be invariant under transmission, thus excluding the phenomena of the Raman effect. (See **mixed transmission; uniform diffuse transmission**.)

TRANSMISSION COEFFICIENT. See **absolute reaction rate theory; scattering coefficient**.

TRANSMISSION LINE. A system of conductors which guides electromagnetic energy from one point to another. Usually used in the restricted sense for a **wave guide** whose transverse dimensions are small.

TRANSMISSION MATRIX. An array of quantities which relate the voltage and current at the input of a two-terminal pair network to the output voltage and current.

TRANSMISSION PLANE. The **plane of vibration** of polarized light which will pass through a given polarizer.

TRANSMISSIVITY. The **internal transmission** factor of unit thickness of a transmitting material under conditions in which the boundary of the material has no influence. (See **absorptive power**.)

TRANSMITTANCE. See **total transmittance; internal transmittance; direct transmittance; diffuse transmittance**.

TRANSMITTANCE, DIFFUSE. See **diffuse transmittance**.

TRANSMITTANCE, DIRECT. See **direct transmittance**.

TRANSMITTANCE, DIRECTIONAL LUMINOUS. The ratio of the **luminance** of the

surface at which light leaves a diffusing object to the **illuminance** of the surface at which the light is incident upon the object; provided that (1) the luminance of the emergent surface is expressed in **lamberts** and the illuminance of the incident surface in lumens/sq cm, or (2) the luminance of the emergent surface is expressed in **foot-lamberts** and the illuminance of the incident surface in **foot-candles**.

TRANSMITTANCY. The ratio of the **transmittance** of a solution to that of the pure solvent in equivalent thickness.

TRANSMITTING POWER. See **absorptive power**.

TRANSMITTIVITY. The internal transmittance (see **transmittance**) for unit thickness of a non-diffusing substance.

TRANSONIC AREA RULE. This rule can be derived from **slender-body theory**, but when first stated by R. T. Whitcomb it was based on a study of experimental results at transonic speeds. The rule states that, near the speed of sound, zero-lift drag rise of thin low-aspect ratio wing-body combinations is primarily dependent on the axial distribution of cross-sectional area normal to the air stream. The rule is of great practical importance; for example, if the area of cross section of an aircraft fuselage is reduced by “waisting” in the vicinity of the wing, the drag at transonic speeds is considerably reduced. (See also **slender-body theory**.)

TRANSONIC HODOGRAPH EQUATIONS. The non-linear equations of two-dimensional transonic flow may be transformed into linear equations by formulating them in the hodograph plane.

There are several ways of formulating these equations, but in the simplest approach it is assumed that the perturbation velocities are small. The equation of motion in the physical plane is then

$$(1 - M^2) \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = \frac{(\gamma + 1)M^2}{U} \cdot u \frac{\partial u}{\partial x}, \quad (1)$$

and in this form the equation is valid at transonic speeds, i.e., when $M \rightarrow 1$.

Introducing the condition of irrotationality and transforming to the hodograph plane, the equations obtained are

$$(1 - M^2) \frac{\partial y}{\partial v} + \frac{\partial x}{\partial u} = \frac{(\gamma + 1)M^2}{U} \cdot u \frac{\partial y}{\partial v} \quad (2)$$

and

$$\frac{\partial x}{\partial v} - \frac{\partial y}{\partial u} = 0. \quad (3)$$

These equations are linear in the dependent variables x and y . In solving the equations, difficulties arise in satisfying the boundary conditions, because these are of course given in the physical plane, in terms of x and y . Nevertheless, useful solutions for transonic flow have been obtained by use of the hodograph equations. (See L. Howarth (Ed.), *Modern Developments in Fluid Dynamics—High Speed Flow*, Chapter VII, Oxford, 1953.)

TRANSONIC SIMILARITY LAW. The various forms of the transonic similarity law are based on the small-perturbation equation of motion. If $u = \frac{\partial \phi}{\partial x}$, $v = \frac{\partial \phi}{\partial y}$ and $w = \frac{\partial \phi}{\partial z}$ are the perturbation velocity components in the directions of the axes x , y , z , and the stream has a velocity U and a Mach number M , this equation is

$$(1 - M^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = \frac{M^2(\gamma + 1)}{U} \cdot \frac{\partial \phi}{\partial x} \cdot \frac{\partial^2 \phi}{\partial x^2}. \quad (1)$$

With the term on the right-hand side included, this equation is valid at transonic speeds.

For two-dimensional flow a transonic similarity parameter K is introduced, defined by

$$K = \frac{1 - M^2}{[\tau(\gamma + 1)M^2]^{3/4}} \quad (2)$$

where τ is the “thickness ratio” of the body, i.e., the ratio of characteristic dimensions in the y and x directions, respectively. It may then be shown that for corresponding points on a series of bodies related by an **affine transformation**, the pressure coefficient C_p is given by

$$\frac{C_p[(\gamma + 1)M^2]^{3/4}}{\tau^{3/4}} = P(K) \quad (3)$$

where P is a function depending on the shape of the body, when τ has been specified. Similarly, expressions for lift and drag coefficients of airfoils may be derived.

Equation (3) is not valid at hypersonic speeds, but except for this restriction it is valid at all speeds, both subsonic and supersonic. It is usually called the transonic similarity law because it was first derived (by von Kármán, in a slightly different form) for transonic speeds.

For three-dimensional bodies other forms of the transonic similarity law have been derived. (See H. W. Liepmann and A. Roshko, *Elements of Gasdynamics*, John Wiley, 1957.)

TRANSPORTATION PROBLEM. A **linear programming** problem of special structure, describing the following situation. One wishes to ship a number of units of an item from several warehouses to a number of retail stores. Each store requires a certain number of units of the item, while each warehouse can supply up to a certain amount. One knows the unit shipping cost between each warehouse and each store, and assumes that the cost relationship is linear. The objective is to determine how many units should be sent from each warehouse to each store so that the total shipping cost is a minimum.

TRANSPORT EQUATION, BOLTZMANN. Consider the evolution of the velocity distribution function f . This evolution proceeds through mechanical changes (flow) and collision.

In gases at low densities, the effect of collisions involving more than two molecules is negligible. If the collisions with the walls can also be neglected, the evolution of the molecular distribution function $f_i(\mathbf{r}, \mathbf{v}, t)$ is given by the Boltzmann integro-differential equation:

$$\frac{\partial f_i(\mathbf{r}, \mathbf{v}, t)}{\partial t} + \mathbf{v}_i \frac{\partial f_i(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{r}_i} + \mathbf{X}_i \frac{\partial f_i(\mathbf{r}, \mathbf{v}, t)}{\partial \mathbf{v}_i} = 2\pi \iint (f'_i f'_j - f_i f_j) g_{ij} b db d\mathbf{v}_j \quad (1)$$

b is the **impact parameter**, g_{ij} is the relative velocity. $\partial f_i / \partial t$ is the local variation of the distribution function. Equation (1) results from three causes: (1) $\mathbf{v}_i (\partial f_i / \partial \mathbf{r}_i)$, often called the *Streaming Term*, is the variation of the distribution function caused by the movement of the particles in and out of the volume element under consideration; (2) $\mathbf{X}_i (\partial f_i / \partial \mathbf{v}_i)$ is the variation of the distribution function resulting from the external forces \mathbf{X}_i acting on the molecules; (3) finally, the term on the right-hand side of

(1) expresses the variation of f_i which results from the binary collisions between the molecules. f_i increases when a collision between two molecules having initial velocities \mathbf{v}'_i and \mathbf{v}'_j creates a molecule of the proper velocity \mathbf{v}_i . It decreases when a molecule of velocity \mathbf{v}_i collides with a molecule of velocity \mathbf{v}_j . The term on the right-hand side of (1) is called the *Collision Integral* of the Boltzmann equation.

The Boltzmann equation has raised two major problems: the first is the justification of this equation, which was first proposed by Boltzmann from purely intuitive arguments. It is an equation of the *first order in time*, i.e., an "irreversible" equation whereas the equations of classical or quantum mechanics for the movement of the individual particles are reversible. The apparent paradox of deriving an irreversible behavior from a reversible microscopic equation has received considerable attention, at the hands of the Ehrenfest and Smoluehowski. Recently it has been possible to derive, under similar conditions, a transport equation without the introduction of probabilistic arguments. Such derivations start necessarily with the N -body problem and are therefore of rather high mathematical complexity.

The second problem has been the solution of this non-linear integro-differential equation. This has been accomplished first by Enskog and Chapman (see **Enskog series**). Other methods have been suggested since; the most powerful of these seems presently to be the 13-moments approximation suggested by Grad (see **thirteen-moments approximation**).

An extension of (1) to *denser gases* composed of hard spheres has been suggested by Enskog: at high densities, the mean free path is short, comparable with a collision diameter.

The instantaneous **collisional transfer** of momentum and kinetic energy from the center of one molecule to that of another is not negligible. If the center of the first molecule is at \mathbf{r} , that of the other is at $\mathbf{r} - \sigma \mathbf{k}$ where σ is the collision diameter, and \mathbf{k} is the unit vector along the **apse line**. Moreover, since an appreciable fraction of the available space is occupied by the molecules of finite volume, the collision frequency is larger than the value computed at low densities by a factor Y , which can be deduced from the equation of state, and whose value must be evaluated at the half distance between the centers of the two collid-

ing molecules. Three-body collisions and collisions involving a larger number of molecules can again be neglected because of the hard sphere hypothesis.

The collision integrals in Equation (1) should therefore be of the form:

$$2\pi \sum_j \iint [f'_i(\mathbf{r}, \mathbf{v}, t) f'_j(\mathbf{r} + \sigma \mathbf{k}, \mathbf{v}'_i, t) Y(\mathbf{r} + \frac{1}{2} \sigma \mathbf{k}) - f_i(\mathbf{r}, \mathbf{v}, t) f_j(\mathbf{r} - \sigma \mathbf{k}, \mathbf{v}_i, t) Y(\mathbf{r} - \frac{1}{2} \sigma \mathbf{k})] g_{ij} b db d\mathbf{v}_i \quad (2)$$

The transport properties calculated from this modified Boltzmann equation are in surprisingly good agreement with experiment, as long as the effect of the attractive forces is small.

TRANSPORT EQUATION, INTEGRAL. See integral transport equation.

TRANSPORT KERNEL. See kernel, transport.

TRANSPORT MEAN FREE PATH. See mean free path, transport.

TRANSPORT NUMBER. The fraction of the total electrical current carried by the ionic species i is called the transport number t_i of the species i .

TRANSPORT PROCESSES. See Eyring theory.

TRANSPORT THEORY BOUNDARY CONDITIONS. See boundary conditions, transport theory.

TRANSPORT THEORY, NEUTRON. See neutron transport theory.

TRANSPOSE (OF A MATRIX). The matrix obtained from the given matrix by interchanging its rows and columns; that is, by replacing the element a_{ij} by a_{ji} .

TRANSPOSITION. A permutation consisting solely of the interchange of two objects.

TRANSVERSE LOAD. See lateral load.

TRANSVERSE SHEAR. See shear.

TRANSVERSE WAVE (ELECTRIC OR MAGNETIC). A wave which has no electric field component in its direction of travel is a transverse electric (TE) wave. Similarly one can describe transverse magnetic (TM) and transverse electromagnetic (TEM) waves.

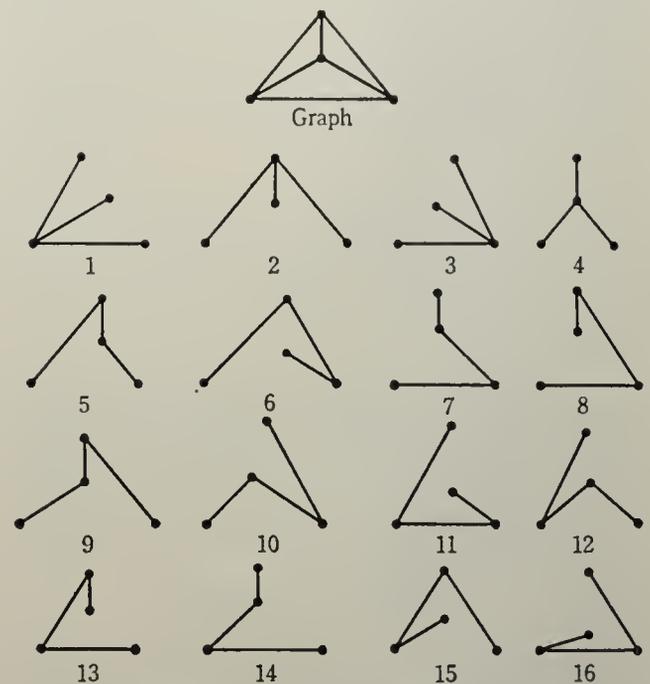
TRAPEZOIDAL RULE. (For numerical quadrature.)

$$\int_a^b f(x) dx = (b - a)[f(a) + f(b)]/2 + R,$$

$$R = -h^3 f''(\xi)/12, \quad a \leq \xi \leq b, \quad h = b - a.$$

This is one of the simplest of the numerical integration rules, but it can often give quite satisfactory results.

TREE. A connected subgraph of a connected graph which contains all the vertices of the graph but is free of circuits. Occasionally the phrase "complete" tree is used but the definition given here is more in accord with modern engineering terminology. A graph can possess many trees. In the accompanying figure is an example of a graph with four vertices, six edges and sixteen trees. See also the discussion under digraphs.



It can be easily shown that every finite connected graph G contains at least one tree. A tree contains v vertices and $v - 1$ branches where v is the number of vertices in G .

TRESCA YIELD CONDITION. Plastic flow occurs when the difference between the maximum and minimum principal stresses equals twice the yield stress in shear, k :

$$\sigma_{max} - \sigma_{min} = 2k.$$

The corresponding yield surface is a hexagonal cylinder. This law is equivalent to the

statement that plastic flow takes place when the maximum shear stress occurring reaches the value k .

TRIANGLE, GEODESIC. See **geodesic triangle**.

TRIANGLE, SPHERICAL. See **spherical trigonometry**.

TRIANGULAR MATRIX. A matrix in which all elements are zero above the diagonal (a lower triangular matrix), or else below the diagonal (an upper triangular matrix). It is a unit upper or lower triangular matrix when also every diagonal element = 1. The product of two triangular matrices of the same type is again of that type, i.e., (unit) upper or lower; and the reciprocal of a triangular matrix (when it exists) is also of the same type. A triangular matrix is properly triangular if the diagonal is null.

TRIAxIALITY OR TRIAXIAL STRESS. See **stress, combined**.

TRICHROMATIC COEFFICIENTS. Three coefficients, based on the response of the standard eye to the spectral distribution of light from a standard source, which may be used to describe the **chromaticity** of a source.

TRICHROMATIC SYSTEM. Any system of color specification based on the possibility of matching colors by the additive mixture of three suitably chosen standard stimuli.

TRICHROMATIC UNITS. Relative units of stimulus quantity, applicable to stimuli of any color and such that the quantity of any stimulus, when expressed in these units, is equal to the sum of the **tristimulus values**.

TRICLINIC SYSTEM. One of the seven **crystal systems**. In this system, the axes are unequal, and not at right angles to each other.

TRIGONOMETRIC FUNCTION. If ϕ is one of the acute angles in a right-angle triangle, x is the side of the triangle nearest to ϕ , y the side opposite the angle, and r the hypotenuse, the trigonometric functions are:

$$\sin \phi = y/r; \quad \cos \phi = x/r;$$

$$\tan \phi = y/x; \quad \csc \phi = 1/\sin \phi;$$

$$\sec \phi = 1/\cos \phi; \quad \cot \phi = 1/\tan \phi.$$

The names of these functions are, respectively: sine, cosine, tangent, cosecant, secant, cotangent.

In the general case, where ϕ is not an acute angle in a right-angle triangle, the same definitions, with slight modification, can be used. Place the angle ϕ in a right-handed **rectangular coordinate system** with its vertex at the origin and its initial side on the positive OX -axis. The terminal side of the angle may lie in any one of the four quadrants but select any point on it (x, y) and draw a radius vector r to the point from the origin. The three letters x, y, r will then determine the trigonometric functions by the previous definitions.

TRIGONOMETRIC FUNCTION, INVERSE.

The inverse function to $y = \sin z$ is the angle whose sine is y , or symbolically, $z = \arcsin y = \sin^{-1} y$. Other inverse trigonometric functions are indicated in a similar way. If $y^2 < 1$, the following series expansions may be used:

$$\begin{aligned} \sin^{-1} y &= y + \frac{y^3}{6} + \frac{1 \cdot 3}{2 \cdot 4 \cdot 5} y^5 \\ &\quad + \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7} y^7 + \dots \\ &= \frac{\pi}{2} - \cos^{-1} y \end{aligned}$$

$$\begin{aligned} \tan^{-1} y &= y - \frac{1}{3}y^3 + \frac{1}{5}y^5 - \frac{1}{7}y^7 + \dots \\ &= \frac{\pi}{2} - \cot^{-1} y \end{aligned}$$

and, for $y^2 > 1$,

$$\begin{aligned} \tan^{-1} y &= \frac{\pi}{2} - \frac{1}{y} + \frac{1}{3y^3} - \frac{1}{5y^5} + \dots \\ \sec^{-1} y &= \frac{\pi}{2} - \frac{1}{y} - \frac{1}{6y^3} - \frac{1 \cdot 3}{2 \cdot 4 \cdot 5y^5} \\ &\quad - \frac{1 \cdot 3 \cdot 5}{2 \cdot 4 \cdot 6 \cdot 7y^7} + \dots \\ &= \frac{\pi}{2} - \operatorname{csc}^{-1} y. \end{aligned}$$

TRIGONOMETRIC INTERPOLATION.

Formulas analogous to the **Lagrange formula** for polynomial interpolation can be obtained. They all have the form

$$f(x) \doteq \sum f_i T_i(x) / T_i(x_i).$$

Possible choices of $T_i(x)$ are the following:

$$T_i(x) = \prod_{j \neq i} \sin(x - x_j); 2;$$

$$T_i(x) = \prod_{j \neq i} (\cos x - \cos x_j);$$

$$T_i(x) = \sin x \prod_{j \neq i} (\cos x - \cos x_j);$$

$$T_i(x) = \prod_{j \neq i} (\sin x - \sin x_j).$$

TRIGONOMETRIC SERIES. See **Fourier series**.

TRIGONOMETRY. The mathematical study of the **triangle**, normally the plane triangle. The main problem of trigonometry is the finding of unknown parts of a triangle from given data. For a right triangle, two parts must be given in addition to the right angle and one of them must be a side. Let A, B, C be the angles and a, b, c the sides opposite them, respectively. Then if the right angle is C , the remaining parts of the triangle are found from the relation $A + B = 90^\circ$, and the Pythagorean theorem, $a^2 + b^2 = c^2$. Alternatively, suitable combinations of the **trigonometric functions** may be used.

For an oblique triangle, three of the six parts can be found if three others are given, including one side. One possibility is the division of the oblique triangle into two right triangles and the solution of each of these as described in the preceding paragraph. One could also use:

(1) the law of sines, $a/\sin A = b/\sin B = c/\sin C$; (2) the law of cosines, $a^2 = b^2 + c^2 - 2bc \cos A$; (3) the law of tangents, $(a - b)/(a + b) = \frac{\tan(A - B)/2}{\tan(A + B)/2}$; (4) the half-angle

formula, $\tan A/2 = r/(s - a)$, where the perimeter of the triangle, $2s = (a + b + c)$ and r is the radius of a circle inscribed in it, $sr^2 = (s - a)(s - b)(s - c)$. Additional relations can be obtained for (2) - (4) by cyclic permutation of the letters a, b, c and A, B, C .

The solution of an oblique triangle is conveniently checked by the Mollweide formulas: $(a + b)/c = \frac{\cos(A - B)/2}{\sin C/2}$; $(a - b)/2 = \frac{\sin(A - B)/2}{\cos C/2}$ and two other equations, found

by cyclic permutation of the letters, a, b, c and A, B, C .

TRIGONOMETRY, SPHERICAL. See **spherical trigonometry**.

TRIPLE-DIAGONAL MATRIX. A matrix $\mathbf{A} = (a_{ij})$ for which $a_{ij} = 0$ when $|i - j| > 1$. Thus nonnull elements occur only along, just above, or just below the diagonal. These occur in the theory of **continued fractions**. Some methods for computing **eigenvalues** and **eigenvectors** of a matrix aim first at transforming the matrix to triple-diagonal form.

TRIPLE POINT. The equilibrium state involving the coexistence of three different phases of a particular system. The triple point of one chemical component is of particular importance in thermodynamics, since then the **variance** of the system involving three phases is zero. (See **phase rule**.) Thus the intensive variables, in particular the *pressure and temperature* are fixed. A triple point can be formed by the liquid, solid and gaseous phases of a single independent component. A triple point can also exist when, for example, two solid phases coexist with a single liquid or gaseous phase. (A system can have only one gaseous phase because gases mix in all proportions.) When reference is made to the triple point of water, the system involving ice, liquid water and water vapor is usually meant, although triple points between allotropic modification of ice and, say, liquid water can also be produced. At the (ice-liquid-vapor) triple point of water the temperature is

$$T_3 = 273.16^\circ\text{K}$$

(as fixed by convention, see **temperature**) and the pressure is

$$p_3 = 0.00623 \text{ kp/cm}^2 = 0.08854 \text{ lbf/in}^2.$$

TRIPLE PRODUCT OF VECTORS. If $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are three vectors, they may be combined to form products with meaning as follows: (1) $\mathbf{A}(\mathbf{B} \cdot \mathbf{C})$, a vector with the same direction as \mathbf{A} and magnitude $ABC \cos \theta$, where θ is the angle between \mathbf{B} and \mathbf{C} . (2) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$, the scalar triple product, giving the volume of a parallelepiped with edges $\mathbf{A}, \mathbf{B}, \mathbf{C}$. It is frequently indicated by the symbol $[\mathbf{ABC}]$ and if the three vectors all lie in the same plane $[\mathbf{ABC}] = 0$. It may be written in terms of its components as a **determinant**

$$[\mathbf{ABC}] = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}.$$

Its properties include

$$\begin{aligned} [ABC] &= [BCA] = [CAB] = -[ACB] \\ &= -[BAC] = -[CBA]. \end{aligned}$$

(3) The vector triple product, $\mathbf{V} = \mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ is perpendicular to both \mathbf{A} and the vector $\mathbf{B} \times \mathbf{C}$. It therefore lies in the plane determined by \mathbf{B} and \mathbf{C} . Its properties include: $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}) = -\mathbf{A} \times (\mathbf{C} \times \mathbf{B}) = (\mathbf{C} \times \mathbf{B}) \times \mathbf{A} = -(\mathbf{B} \times \mathbf{C}) \times \mathbf{A}$.

TRISTIMULUS VALUES (OF A LIGHT).

The amounts of the three reference or matching stimuli required to give a match with the light considered, in a given trichromatic system. In the CIE (1931) colorimetric system the symbols X , Y , Z are recommended for the tristimulus values. These values may be obtained by multiplying the **spectral concentration** of the radiation at each wavelength by the **distribution coefficients** and integrating these products over the whole spectrum.

TROLAND. A unit of retinal illuminance, being the visual stimulation resulting from an illuminance of 1 candle/sq m when the apparent area of the **entrance pupil** of the eye is 1 sq m.

TROPICAL YEAR. See **year**.

TROUGH (OF LOW PRESSURE). A form seen on a chart of isobars or chart of contours of an isobaric surface. A trough may be U-shaped with rounded contours, or V-shaped when a marked front lies in it. A trough in the thickness pattern denotes a wedge of cold air, and usually denotes cold air in the upper air contour pattern.

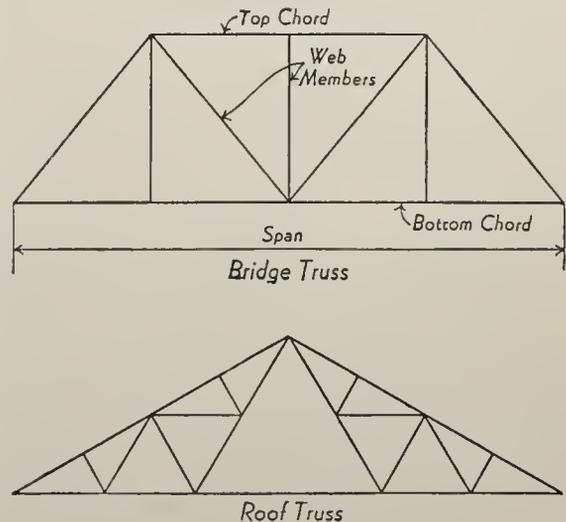
TRUNCATED SAMPLE. See **censoring**.

TRUNCATION. Approximating the sum of an infinite series by the sum of a finite number of terms in that series. As applied to the dropping of digits in a number, the term contrasts with **rounding**.

TRUNCATION ERROR. Strictly the error due to dropping all but a finite number of terms from a possibly infinite series, but often applied also to the error due to representing the limit of a sequence by one of its terms, or to representing a function by an interpolation polynomial. (See **error**.)

TRUSS. A truss is a triangulated structure composed of straight bars which act primarily as tension or compression members. In the analysis for primary stress the bars are supposed to be hinged or pin-connected at their common point of intersection, called a joint or panel point. Loads are applied at the panel points only. In an actual truss the joints are more likely to be riveted or welded than pinned, but the axes of the bars of a joint do all intersect at a common point. Because the joint can transmit moment, bending stresses are developed due to deflection. (See **secondary stress**.)

The top line of members of a truss is called the **top chord** and the lower line is known as



the lower or bottom chord. The members connecting the top and bottom chords are called web members. The span of the truss is the horizontal distance center to center of end bearings. (See also **Hennenberg method**; **Hoff convergence method**.)

TRUSS, COMPLEX. A statically determinate truss which is neither a simple truss nor a compound truss.

TRUSS, COMPOUND. A truss composed of two or more simple trusses joined together in a **statically determinate** manner.

TRUSS, SIMPLE. A *simple plane truss* is a **statically determinate** truss formed by starting with a triangle of three bars or a foundation and establishing successive points of the truss by two bars, not in the same straight line, attached to existing points or the foundation.

A *simple space truss* is a statically determinate truss formed by starting with a pyramid

of four bars or a foundation and establishing successive points by three non-coplanar bars attached to existing points or the foundation.

TSCHEBYSHEFF. See entries under **Chebyshev**.

TUBES, CURVED. When a curved tube is bent, the cross section flattens and the change in geometry has a significant effect on the stress distribution and the stiffness. It is not satisfactory to use ordinary beam theory. Kármán has shown that for a tube of circular cross section with center line radius R , thickness t , and radius a , the parameter tR/a^2 is significant.

TURBINE, BACK-PRESSURE. See **back-pressure turbine**.

TURBINE, EXHAUST GAS. See **exhaust gas turbine**.

TURBOCOMPRESSOR. See **rotary compressor**.

TURBULENCE. A condition of fluid motion in which properties of the fluid, such as heat content, water vapor content, etc., are diffused at a rate large compared with the rate of diffusion by molecular motions. Turbulence also diffuses momentum, i.e., it produces stresses in shearing motion. (See **Reynolds stresses**.)

To be regarded as turbulent the motion must be viewed on a sufficiently large scale. Theoretically the motion at any moment might be described exactly, but it is not always either practicable or useful to attempt this. The motion may then be regarded as a smooth mean motion with superimposed eddies or fluctuations of velocity which are only considered in large numbers, their effects being measured by taking averages of the quantity affected over a sufficiently large space or time for many of the eddies to be included.

In many theories the effects of the eddies have been thought of as analogous to molecular motions, and have been represented in equations by **turbulent, or eddy, transfer coefficients**.

For motion to be regarded as turbulent, it is not sufficient that it should consist of fluctuations, however irregular. Thus sound waves and gravity waves do not constitute turbulence. Turbulence is essentially a motion in which stirring and mixing take place, in which marked volumes of fluid become stretched into

tortuous shapes of progressively increasing complexity. For this to happen the motion must consist of an irregular pattern of **vorticity** in which the length scale of the eddies is continuously being reduced on the average by the stretching of the **vortex lines**.

The scale of the smallest eddies which can occur is determined by the dynamic viscosity of the fluid. The larger eddies are practically unaffected by viscosity.

Turbulence may be produced essentially in three ways: by mechanical forces at the boundary of the fluid, by inertia forces, or by body forces. In each case the main feature is the production of vorticity and its subsequent degeneration into eddies of ever-decreasing size which are ultimately brought to rest by viscosity.

The mechanical forces at the boundary produce boundary layers in which the shear becomes so large that the motion becomes unstable and the orderly vorticity of the shearing layer becomes a chaotic pattern of eddies. Or the boundary layer may separate from the surface as a **vortex sheet** or layer of large vorticity and the degeneration then takes place away from the boundary, generally in the wake of a solid body.

Turbulence may be produced by inertia forces when a jet or isolated mass of non-turbulent fluid is injected into non-turbulent surroundings. Turbulence may also be produced by causing fluid to rotate in surroundings in which there is a smaller circulation: the centrifugal forces are then unstably stratified and the rotating fluid mixes outwards into the surroundings.

The production of turbulence by centrifugal (inertia) forces is very similar to its production by static instability (unstable density gradient under gravity). In this case it may be called thermal turbulence because the density gradients are usually produced by temperature gradients.

(See also **Reynolds stresses; mixing length; transfer coefficients; transferable quantity; jets; free convection; turbulent boundary layers; wakes**.)

TURBULENCE, DECAY OF. Turbulence is envisaged as a complicated pattern of **vorticity** with motions present on a wide range of length (or frequency) scales, called eddies. The small eddies produce **Reynolds stresses** in the

large ones so that energy passes from the large to the small eddies. The dissipation of the energy of the large eddies by viscosity is negligible in comparison with the energy transferred to the smaller eddies. This transfer is often referred to as the "cascade" of energy down the scale of eddy sizes.

The energy of turbulence originates when large eddies are produced in some way, or when the Reynolds stresses due to the eddies transfer energy from shearing motion in the mean flow. The rate of dissipation by viscosity is proportional to the square of the shear and to the viscosity, and is greatest in the smallest eddies. The greater the rate at which energy is passed down from the larger eddies, the smaller is the size of the smallest eddies. The nature of the motion at this end of the eddy size-spectrum is determined only by the mean rate of dissipation of energy per unit mass, ϵ , and the dynamic viscosity ν , which between them determine the order of magnitude of the smallest eddy size. This, by dimensional argument, is seen to be proportional to $(\nu^3/\epsilon)^{1/4}$.

TURBULENCE, ENERGY OF. Turbulence presents problems of considerable difficulty primarily because its intensity is determined by a great variety of influences, so that its effects cannot generally be represented by transfer coefficients which are independent of the motion on which they have an effect.

The energy of a state of turbulent motion is always being reduced by the decay process. It is being increased by the transfer of energy from the mean motion if it produces any **Reynolds stresses**.

Turbulent energy may at the same time be increased by the creation of eddies of an intermediate size by ancillary processes such as wake formation behind obstacles and the release of energy by buoyant convection. In the latter case there is no process equivalent to the cascade down the scale of eddy sizes because individual eddies composed of buoyant fluid (**thermals**) increase in size as they ascend. In certain special cases such as the atmosphere eddies above a certain size may release new sources of turbulent energy as, for example, the latent heat of condensation of clouds. The large eddies may generate regions in which the flow is unstable, so that smaller eddies are generated directly and not by the degeneration of larger ones.

The destruction of turbulent energy can, in addition to the ordinary process of decay into small eddies which are destroyed by viscosity, take place in a variety of ways. Work may be done by the pressure fluctuations on flexible boundaries; the fluctuations of particle position may be accompanied by variations of temperature which are damped by radiation or thermal conductivity, though this is usually negligible; in a fluid of variable density (decreasing upwards) work may be done against buoyancy forces on all scales of motion because the center of gravity of the fluid is raised when weight is transferred upwards; and in a medium capable of transmitting energy in the form of waves, the turbulent energy is dispersed as waves. Thus gravity waves may be produced at a free surface or surface of density discontinuity or internally in a stably stratified fluid, and sound waves may be generated by turbulence of sufficient intensity as, for example, in the region of large shear round a high speed jet.

The successful analytical treatment of the dynamics of any particular form of turbulence depends upon there being a dominant **source** or **sink** of energy. Generally, simplifying assumptions about the energy are made, and then dimensional analysis reveals relationships in which the constants are determined by experiment. Outstanding examples of this procedure are the **mixing length** theory and the similarity theory, and these can be combined with statistical treatment of the dynamics in the manner of **Reynolds equations**.

TURBULENCE, GENERATION OF. See **grid**, as a means of generating turbulence.

TURBULENCE, HOMOGENEOUS. Turbulence is said to be homogeneous when there is no spatial variation of the mean properties of the turbulent component of the motion, although the local properties may be directional, as, for example, when there is large, but uniform, shear in the mean motion.

TURBULENCE IN WIND TUNNELS. One of the aims in designing a wind tunnel is usually that the turbulence of the stream in the working section should be as low as possible. In any experiment where a turbulent stream is required, a grid can then be used to introduce turbulence of the required intensity.

Reduction of turbulence in the working section of a wind tunnel is achieved by introducing (1) a large contraction before the working section and (2) several screens of woven wire gauze before contraction. (See also **gauze, flow through; grid, as means of generating turbulence.**)

TURBULENCE, ISOTROPIC. Isotropic turbulence is turbulent motion which in the mean possesses no directional properties. **Turbulence** which is created anisotropic tends towards isotropy because the fluctuations of pressure gradient produced by components of motion in one direction tend to transfer energy into motion in a direction at right angles.

Turbulence is not generally isotropic when there is large shear in the mean motion.

TURBULENCE, MICROSCALE OF. A measure of the size of the eddies present in turbulent motion, denoted by the length λ . The autocorrelation coefficient r_x falls off more rapidly with increasing x when there are smaller eddies present. $r_0 = 1$ and $(dr_x/dx)_{x=0} = 0$, so that the curvature of r_x at $x = 0$ provides the relevant measure. λ is defined by

$$\frac{1}{\lambda^2} = \lim_{x \rightarrow 0} \frac{1 - r_x}{x^2}.$$

The mean rate of dissipation of turbulent energy per unit mass of fluid, denoted by ϵ , in isotropic turbulence is related to λ according to

$$\epsilon = 15 \nu \overline{(u'^2)^{1/2}} / \lambda^2,$$

$\overline{(u'^2)^{1/2}} / \lambda^2$ being a measure of the shearing rate and ν the dynamic viscosity.

Another measure of the size of the eddies in which the dissipation by viscosity matches the supply of energy extracted from motions on a larger scale is provided by the length $(\nu^3/\epsilon)^{1/4}$. This length turns out to be much smaller than λ because $\overline{u'^2}$ is the intensity of the turbulence as a whole and not of the smallest eddies which are being destroyed by viscosity. In using this measure it must be assumed that the motion (including u'^2) is entirely determined by the two quantities ν and ϵ .

TURBULENCE, REYNOLDS NUMBER OF. The most general definition of the **Reynolds number** of turbulence is

$$E^{1/2} L / \nu$$

where E is the kinetic energy per unit mass and L is a length scale of the eddies containing most of the energy. Usually L is taken as the microscale of turbulence (see **turbulence, microscale of**).

In the smallest eddies, controlled by the viscosity, and in which the intensity of motion is determined by the rate of supply of energy ϵ , the appropriate length is $(\nu^3/\epsilon)^{1/4}$, and time $(\nu/\epsilon)^{1/2}$. The Reynolds number is thus an absolute constant (no nondimensional numbers can be derived from a combination of the data ϵ and ν alone).

TURBULENCE, SIMILARITY OF. The spectrum of eddy sizes ranges from the largest eddies which are created by some external influence to the smallest in which the energy is dissipated into heat by viscosity. In between there may be a range of eddy sizes in which no eddies are created by an external influence and the dissipation by viscosity is negligible compared with the cascade of energy down the scale of eddy sizes. In this range, called the inertial range, the motion is similar on all scales and the intensity is determined only by ϵ , the rate at which energy is extracted from the motion on a larger scale and is handed down to the smaller eddies.

For an inertial range to exist the turbulence on that scale must be strictly homogeneous and isotropic, but it is generally supposed, in making use of the idea of similarity, that the turbulence is locally homogeneous and isotropic.

The energy spectrum functions in the inertial range are of the form given by

$$\overline{u'^2} F(n) \propto \epsilon n^{-2} \text{ (frequency spectrum)}$$

$$\overline{u'^2} E(k) \propto \epsilon^{2/3} k^{-5/3} \text{ (wave number spectrum).}$$

If the motion is similar on all scales the diffusion coefficient must depend on the scale considered. Thus the mean rate of separation of two particles a distance l apart is determined only by the intensity of the turbulence (i.e., by ϵ) and l itself. Thus the appropriate coefficient for Fickian diffusion is

$$K \propto \epsilon^{1/2} l^{3/2}.$$

The same form is necessarily assumed by any transfer coefficient of the turbulence, but it is only significant in turbulence possessing

similarity because only then is ϵ independent of the relevant length scale.

Other deductions may be made about the nature and effects of turbulence assuming the motion to be similar on all scales, but they can only be applied with confidence to actual turbulence on a scale small compared with the eddies which are created and large compared with those dissipated by viscosity.

TURBULENT BOUNDARY LAYER. In a turbulent boundary layer, turbulent motion of the fluid is superimposed on the mean flow, and this gives rise to Reynolds stresses which have an important effect. In particular, over the greater part of the boundary layer, the Reynolds shear stress is large compared with the viscous shear stress. Thus in turbulent flow the boundary-layer equations are

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{dp}{dx} + \nu \frac{\partial^2 u}{\partial y^2} - \frac{\partial}{\partial y} (\overline{u'v'}), \quad (1)$$

and the equation of continuity.

Equation (1) is the same as for a laminar boundary layer, except for the addition of the last term on the right-hand side, presenting the Reynolds shear stress. In this term, u' and v' are the instantaneous components of fluctuating velocity and the bar denotes a mean value with respect to time.

Analytical treatment of turbulent boundary layers is very difficult, mainly because it is not possible to express the Reynolds shear stress in terms of the mean-flow parameters. Moreover the conditions at a given section of the boundary layer depend not only on the local conditions, but also to a large extent on the previous history of the layer.

Three main regions may be distinguished in a turbulent boundary layer. First there is an outer region, covering most of the boundary layer, in which large-scale eddies predominate and the shear stresses are comparatively small. Fluid from outside the boundary layer is swept into the layer by the large eddies in this region. The instantaneous position of the outer edge of the layer is wavy and is continually changing, so that at a given fixed point near the edge of the layer turbulence occurs intermittently.

The second region to be considered is the inner part of the boundary layer where the flow is determined only by the properties of the fluid and the shear stress at the wall. In this

region the velocity gradient is considerably greater than in the outer region and the velocity profile is logarithmic. This is the region in which energy is transferred from the mean flow to the turbulence. The transfer of energy from the mean flow starts with the larger eddies; energy is transferred from these to successively smaller eddies until finally the energy is dissipated by the action of viscosity in very small eddies.

The third region to be considered is a very thin layer next to the wall in which the viscous shear stress is large compared with the Reynolds shear stress. The thickness of this layer is usually less than $\frac{1}{100}$ of the total boundary-layer thickness. The layer is often known as the *laminar sub-layer*, although the flow in it is not strictly laminar. Except for the laminar sub-layer, and a thin region just outside it, the viscous shear stress is negligible over the whole of a turbulent boundary layer.

Most of the approximate methods for calculating the development of a turbulent boundary layer in a pressure gradient make use of the **momentum equation**. In these methods the momentum equation is integrated by making use of empirical data giving the skin-friction coefficient in terms of the Reynolds number and a *form parameter*. Various form parameters have been used, but a common one is H , the ratio of displacement thickness to momentum thickness. It is assumed that the velocity profile, expressed non-dimensionally, depends only on this one parameter, and further empirical data are used to give the variation of the form parameter itself.

These methods, based on empirical data, are reasonably satisfactory for predicting skin-friction drag and the thickness of the boundary layer, but there is always considerable uncertainty in any attempt to predict the position of separation. (See also **form parameter for turbulent boundary layer; logarithmic profile; momentum equation for boundary layer**.) (See H. Schlichting, *Boundary Layer Theory* (English Translation by J. Kestin), Pergamon, 1955.)

TURBULENT BOUNDARY LAYER, FORM PARAMETER FOR. See **form parameter for turbulent boundary layer**.

TURBULENT CONVECTION. The transport or transfer by turbulent motion of a

property which is carried by a fluid. Equivalent to diffusion by turbulence.

TURBULENT FLOW IN A BOUNDARY LAYER, TRANSITION TO. See transition to turbulent flow in a boundary layer.

TURBULENT FLOW IN A PIPE OR CHANNEL, TRANSITION TO. See transition to turbulent flow in a pipe or channel.

TURBULENT FLOW IN PIPE. See pipe, turbulent flow in.

TURBULENT, OR EDDY, TRANSFER COEFFICIENTS. (Ger. *Austausch*.) A coefficient which is equal to the rate of transfer, by turbulent motion, of a quantity possessed by the fluid, across unit area in a direction down the mean gradient of the quantity, divided by the mean gradient.

The coefficient is sometimes represented by K with a suffix to denote the quantity being transferred, e.g., K_M for momentum (to represent the shearing stresses due to the turbulence) and K_H to represent the transfer of heat down the mean temperature gradient.

The representation of the effect of turbulence in this way assumes there to be a complete analogy with molecular transfer, and is sometimes referred to as "K-theory." The coefficients appear in the relevant equations (e.g., Navier-Stokes equations) in the same way as the corresponding molecular coefficients. The limitation of the theory is that the coefficient is a property of the turbulence and may vary from place to place and with time unlike the molecular coefficients which represent physical properties of the fluid substance. In the case of momentum which may be imparted from one mass of fluid to another without mixing, and heat which may have important buoyancy forces accompanying it, the quantity being transferred may itself influence the turbulence, and may therefore not be a transferable quantity.

The turbulent transfer coefficients are called the eddy viscosity, eddy conductivity, or eddy diffusivity according to the property being transferred.

TURNOVER FREQUENCY. See Nyquist frequency.

TWIG. See branch.

TWIST BUCKLING. See buckling, twist.

TWIST, CENTER OF. See shear center.

TWISTED CURVE (IN THREE-DIMENSIONAL SPACE). A space curve that does not lie in a plane. Also called *skew curve*.

TWO-BODY PROBLEM. The so-called two-body problem is the foundation of celestial mechanics. The solution of the problem requires two fundamental assumptions: (1) that two and only two objects exist in the universe, and (2) that some law of force between the two objects is given. With these assumptions admitted, the two-body problem may briefly be stated as follows: given the relative positions of two objects at any instant, together with their motions and masses at that instant, to predict the positions and motions of the objects at any subsequent instant. (See planetary motion; three-body problem.)

TWO-COMPONENT EQUATION OF THE NEUTRINO. This equation was first proposed by Weyl in 1929 to describe a particle of mass zero and spin $\frac{1}{2}$, but rejected because of its non-invariance under space reflections. When recently experiment revealed that parity is not conserved in β decay, Lee and Yang (1957) (who were responsible for the original suggestion that parity might not be conserved in weak interactions and indicated ways of testing this hypothesis) proposed that neutrinos obey the Weyl equation to account for the observed non-conservation of parity. The Weyl equation, the equation obeyed by the two-component amplitude ϕ describing neutrinos is

$$i\hbar\partial_t\phi(x) = i\hbar c\sigma \cdot \nabla\phi(x) \quad (a)$$

where $\sigma = (\sigma_1, \sigma_2, \sigma_3)$ are the 2×2 Pauli matrices. This equation is form invariant under restricted inhomogeneous Lorentz transformations, $x' = \Lambda x + a$, if

$$\phi'(x') = S(\Lambda)\phi(x) \quad (b)$$

where $S(\Lambda)$ is a 2×2 matrix which satisfies the equation

$$S(\Lambda)\sigma^\mu S(\Lambda)^{-1} = \sum_{\nu=0}^3 \Lambda_\nu^\mu \sigma^\nu \quad (c)$$

where $\sigma^\mu = (1, \sigma_1, \sigma_2, \sigma_3)$; i.e., $\phi(x)$ is a 2-component spinor which under homogeneous Lorentz transformations transforms according to the $D(0, \frac{1}{2})$ representation. Equation (a) is not covariant under space inversion, since under a space inversion a quantity transform-

ing under the $D(0, \frac{1}{2})$ representation is taken into one transforming according to the $D(\frac{1}{2}, 0)$ representation.

The Weyl Equation admits of plane wave solutions of the form $\phi = e^{-ipx}u(p)$ if $p_0 = \pm|p|$, so that the particles described by the equation travel with the speed of light (as required for mass zero particles). Furthermore, each solution of a given sign of the energy corresponds to a definite orientation of the spin of the particle with respect to the direction of motion. Thus for $p_0 = +c|p|$ the spin is anti-parallel to \mathbf{p} and can be represented by a left-hand screw. The solution with $p_0 = -c|p|$ has the spin parallel to \mathbf{p} and can be represented by a right-hand screw.

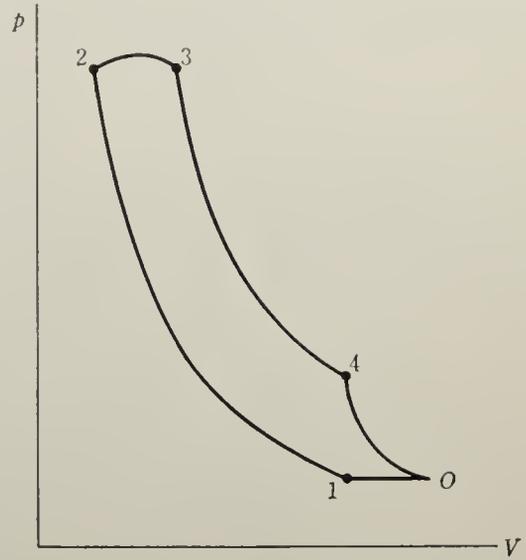
In a **hole theory** interpretation of the negative energy states the anti-neutrino will have a momentum opposite to that of the negative energy state which is vacated. Hence, the relation of the spin direction to the momentum of the anti-neutrino is represented by a right-hand screw.

The non-invariance under space inversion of the two component theory can now be differently stated. Under space-inversion ($\mathbf{p} \rightarrow -\mathbf{p}, \mathbf{x} \rightarrow -\mathbf{x}, \sigma \rightarrow \sigma$) the state of energy $p_0 = +c|\mathbf{p}|$ of momentum \mathbf{p} with spin and momentum parallel (helicity +1) is carried into a state of momentum \mathbf{p} , energy $p_0 = +c|\mathbf{p}|$ and helicity -1, i.e., spin and momentum anti-parallel; but no such state exists for the two-component theory.

TWO-STROKE ENGINE. A reciprocating engine on which one working stroke is executed per one revolution (two strokes). A two-stroke engine is equipped with inlet ports and exhaust ports which are covered and uncovered by the piston. It is also provided with a separate scavenger pump. The sequence of operations is as follows:

- 0-1. Scavenging and introduction of working fluid (air in diesel engine, combustible mixture in gasoline engine). Slits uncovered.
- 1-2. Compression. Slits covered.
- 2-3. Combustion (at nearly constant volume in gasoline engine, at nearly constant pressure in diesel engine). Slits covered.
- 3-4. Expansion. Slits covered.
- 4-0. Release. Slits uncovered.

Operations 0-1 and 1-2 are performed during one stroke, operations 2-3, 3-4 and 4-0, during a following stroke, and this sequence continues.



Cycle of two-stroke engine.

Practical engines differ in details from the preceding idealized description. There exist engines which incorporate an exhaust valve to render scavenging more efficient and which, therefore, resemble somewhat a **four-stroke engine**.

TYPE NUMBER OF CONTROL SYSTEM. See control system, type number.

U

UEHLING EFFECT. See vacuum polarization.

ULBRICHT SPHERE. See Sumptner principle.

ULTIMATE STRENGTH. The maximum load reached in a tension test divided by the original cross-sectional area of the bar is called the ultimate tensile strength. For a ductile material it is simply a measure of instability or necking, and is much lower than the breaking load divided by the area at fracture. Nevertheless the **working stress** is often chosen as a fraction of the ultimate strength.

ULTIMATE STRENGTH THEORY. A method of analysis and design of reinforced concrete structures in which elastic behavior is assumed, but failure is assumed to occur when the **limit moment** is reached somewhere in the structure. This compromise between plastic limit design and elastic behavior is an attempt to take the brittle behavior of concrete into account without too much sacrifice of economy.

ULTRAVIOLET SPECTRUM. See spectrum, ultraviolet.

UMBRAL INDEX. See summation convention.

UMKLAPP PROCESSES. A German term meaning "flop-over" processes. A type of collision between phonons, or between phonons and electrons, where **crystal momentum** is not conserved. No principle is violated, because the magnitude of the crystal momentum or wave vector is arbitrary up to the addition of any multiple of a vector of the **reciprocal lattice**. These processes provide the major part of thermal resistance in dielectric solids, and are important in electrical conduction in metals.

U.M.P. (UNIFORMLY MOST POWERFUL) TEST. A test of a statistical hypothesis which has greater power than any other test for a family of alternative hypotheses.

UNATTAINABILITY OF ABSOLUTE ZERO. See thermodynamics, third law of.

UNCERTAINTY. General term for the estimated amount by which the observed or calculated value of a quantity may depart from the "true" value. The uncertainty is often expressed as the **average deviation**, the **probable error**, or the **standard deviation**.

UNCERTAINTY PRINCIPLE. See indeterminacy principle.

UNCOUPLING PHENOMENA IN SPECTROSCOPY. Change in the coupling conditions in the atom or molecule with consequent changes in the spectrum. For example, uncoupling of the electron spin from the electronic orbital angular momentum (Paschen-Back effect) or the uncoupling of the electron spin from the internuclear axis of a diatomic molecule (transition from Hund's case (a) to (b)). For more detail see the literature quoted with the entries **Paschen-Back effect** and **Hund's coupling cases**.

UNDETERMINED COEFFICIENTS, METHOD OF. A method which may be used in several different situations. For example, if we wish to find that solution of the differential equation

$$(1 + x^2)y'' + 2xy' - 2y = 0$$

for which $y(0) = 0$, $y'(0) = 1$, we may set

$$y = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n + \cdots,$$

with undetermined coefficients a_0, a_1, \cdots , which may be successively obtained by substitution in the equation.

UNIFORM CIRCULAR MOTION. In order that a particle of mass m be maintained in motion with constant **angular velocity** ω and at constant radius r from a fixed center, it is necessary that a centripetal force be acting. This force is

$$\mathbf{f} = m\boldsymbol{\omega} \times \boldsymbol{\omega} \times \mathbf{r},$$

where \mathbf{r} is the instantaneous radius vector from the center to the particle. (See **triple vector product**.) This force is always directed toward the center, and has a magnitude

$$f = m\omega^2 r = mv^2/r,$$

where v is the linear speed of the particle. The particle must be continually accelerated inward with an acceleration of magnitude $\omega^2 r$, because the velocity is changing in direction, even though it is constant in magnitude.

UNIFORM DIFFUSE REFLECTION. Diffusion by reflection, such that the luminance is the same in all directions.

UNIFORM DIFFUSE TRANSMISSION. Diffuse transmission in which the spatial distribution of transmitted flux is such that the luminance is the same in all forward directions.

UNIFORM MOTION. Uniform motion is motion at constant speed. If the motion is rectilinear, the velocity is constant and the acceleration therefore is zero.

UNIFORM POINT SOURCE. A point source which emits radiation uniformly in all directions.

UNIMODULAR GROUP. See **Lie group**.

UNIMOLECULAR REACTIONS. A polyatomic molecule with a sufficient amount of energy distributed over its degrees of freedom will eventually decompose if left alone long enough. Such molecules may be called *active molecules*. Actually, active molecules are continually losing their excess energy through collisions and are reconverted to normal molecules. This is the process called *deactivation*. It is easily seen that for this reason the rate of such a process will be of the first order at sufficiently high pressures (see **order of chemical reactions**).

The usual description of the decomposition of active molecules is based on the two following steps:

(1) Establishment of the thermal equilibrium inside the molecule corresponding to the given energy;

(2) Fluctuations in the distribution of the energy among the internal degrees of freedom until sufficient energy is concentrated on a vulnerable bond. This bond is then broken and the molecule dissociates.

UNIQUE FACTORIZATION THEOREM.

A theorem of this kind may be stated in several connections, but most commonly as follows (often called the fundamental theorem of arithmetic): a positive integer can be expressed in one and only one way, apart from the order of the factors, as a product of prime numbers.

UNITARY GROUP. See **Lie group**.

UNITARY MATRIX. See **matrix**.

UNITARY MODULAR GROUP. See **Lie group**.

UNITARY OPERATOR. An operator U is said to be unitary if for every pair of vectors, f, g in the Hilbert space \mathcal{H}

$$(Uf, Ug) = (f, g) \quad (\text{a})$$

and every vector g in \mathcal{H} can be written in the form $g = Uh, h$ in \mathcal{H} . It follows from (a) and the properties of the scalar product that U is linear and has an inverse U^{-1} which is equal to its hermitian adjoint, $U^{-1} = U^*$.

UNIT BINORMAL. See **binormal**.

UNIT CELL. The basic unit of a **crystal structure**, being the minimum volume from which the crystal may be constructed by **translation operations** only.

UNITED ATOM. See **building-up principle**.

UNIT ELEMENT. That member often called E (or I), of a group, or of a field, such that $EA = AE = A$ for every element A .

UNIT, FUNDAMENTAL. Any one of the basic set of arbitrarily defined units on which a system of units is based. All other units of the system may be derived from the set of fundamental units, and every physical quantity expressed in the system has dimensions that may be expressed as combinations of the fundamental units. (See **units, system of**.)

UNIT IMPULSE FUNCTION. See **impulse function**.

UNIT MATRIX. A **matrix** with unity in each place on the main diagonal and zero elsewhere.

UNIT NORMAL (AT A POINT OF A SURFACE). A unit vector in the direction of the normal to the surface. If the equation of the

surface is taken in parametric form, u and v being the parameters, then the positive direction of the unit normal is usually taken in the direction of the vectors $\frac{\partial \mathbf{r}}{\partial u} \times \frac{\partial \mathbf{r}}{\partial v}$, where \mathbf{r} is the radius vector of the point on the surface whose parametric coordinates are u, v .

UNIT OPERATOR. An operator, usually denoted by I , which leaves unchanged every element in its domain. Also called the identity operator. As a consequence, if A is any other operator which can be multiplied by I , we have $IA = AI = A$.

UNIT PLANES. Alternative term for the principal planes of Gaussian optics.

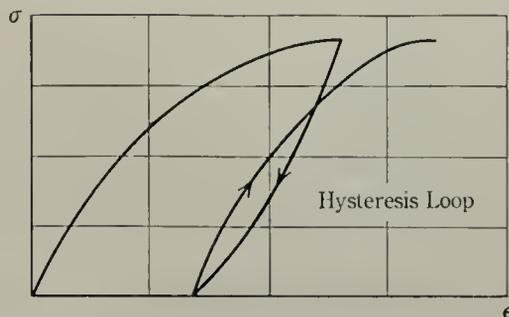
UNIT PRINCIPAL NORMAL. See normal, principal.

UNITS, SYSTEM OF. A set of definitions of standard physical quantities, in terms of which similar quantities may be measured or expressed, each quantity being specified in terms of some arbitrary standard or by a defining equation, and all of the definitions being mutually consistent. (See **acoustical units; electromagnetic units; mechanical units; thermal units.**)

UNIT TANGENT. See tangent to a curve at point P .

UNIVERSAL GAS CONSTANT. See gas constant.

UNLOADING. To a first approximation, the strains associated with reduction of the stress below the **yield limit** after plastic flow are purely elastic. More detailed measurements



Stress-strain diagram showing unloading and loading.

reveal a narrow hysteresis loop associated with unloading and loading after plastic flow as shown in the figure.

UNNATURAL PROCESSES. See natural and unnatural processes.

UNIVERSAL FERMI INTERACTION. See Fermi theory of beta decay.

UNRESISTED EXPANSION. See adiabatic process.

UNSTABLE EQUILIBRIUM. See equilibrium, unstable.

UNSTABLE PHASES. See stability of phases.

UNSYMMETRICAL BENDING. The bending of a beam about a non-principal axis is called unsymmetrical bending because the **neutral axis** does not coincide with the axis of the bending moment (see **lateral deflection**). For any two perpendicular axes x, y , in the cross section with origin at the centroid, the elastic distribution of normal stress is given by

$$\sigma = \frac{(M_y I_x + M_x I_{xy})x}{I_x I_y - I_{xy}^2} + \frac{(M_x I_y + M_y I_{xy})y}{I_x I_y - I_{xy}^2}$$

where M_y is the component of the moment vector in the positive y -direction, M_x , in the positive x -direction, I_x and I_y are the moments of inertia about x and y , respectively, and I_{xy} is the product of inertia. The inclination of the neutral axis β is given by the ratio of y and x for $\sigma = 0$

$$\tan \beta = \frac{M_y I_x + M_x I_{xy}}{M_x I_y + M_y I_{xy}}$$

UPPER BOUND THEOREM FOR PLASTICITY. See limit theorems.

UPPER CALORIFIC VALUE. See combustion.

UPPER IGNITION LIMIT. See ignition limit.

V

VACUUM CORRECTION (OPTICAL).
See **correction to vacuum.**

VACUUM POLARIZATION EFFECTS.
The physical consequences of the interaction of charges with the zero point fluctuations of the electron-positron field and other charged matter fields.

According to Dirac's **Hole theory**, the vacuum is characterized by the fact that all negative energy electronic states are occupied. If now a positive energy charged particle is present, the electronic field which accompanies the charge distorts the charge distribution of the "vacuum" and the vacuum appears polarized. This effect is proportional to the charge which produces it and is always and unavoidably present. The charge which one observes experimentally is not the same as the bare charge, but corresponds in fact to bare plus induced charge. The phenomenon may be viewed as follows: a charge Q_0 (the bare charge) as a result of interaction with the electron-positron field surrounds itself by a cloud of electrons and positrons. Some of these, with net charge δQ of the same sign as Q escape to infinity leaving a net charge $-\delta Q$ in the part of the cloud which is within a distance \hbar/mc of the charge Q_0 . At a distance which is large compared with \hbar/mc one therefore observes an effective charge $Q = Q_0 - \delta Q$, the renormalized charge which is identified with the experimental charge of the body. The effect of this polarization is thus to make the observed charge less than the bare charge.

The problematical aspect of the above stems from the fact that the induced charge δQ is infinite when calculated according to hole theory or its more modern formulation. This difficulty is circumvented by the process of **charge renormalization**, which identifies the quantity $Q_0 + \delta Q$ (even though divergent) with observed finite (experimentally measurable) charge of the test body, Q_{exp} .

The presence of these vacuum polarization effects gives rise to deviations from Coulomb's law between two static test charges. This is known as the *Uehling effect*.

The Uehling effect gives rise to a correction of -27 megacycles to the **Lamb shift** of $n = 2$ level of atomic hydrogen. The presence of this correction term has been verified experimentally. For mesic atoms, the contribution to the level shift due to the vacuum polarization by the field of the nucleus is much larger than that due to the mesic self-energy (**radiative shifts**). Thus for the 1 S level of the π^- -p atom the level shift is about -10 ev and in the π^- -Mg atom the 1 S level is lowered about 1.4 kev. That level shifts of this order of magnitude do occur in mesic atoms has been verified experimentally.

VALENCE BAND. The range of energy levels in a crystal which are normally occupied by the valence electrons which bind the crystal together. In a semiconductor or insulator, this band is separated from the **conduction band** by an **energy gap**.

VALENCE BOND METHOD. The solution of the **Schrödinger equation** for a molecule is a very difficult wave-mechanical many-body problem. Except in the case of the H_2 molecule and the H_2^+ molecular ion, the bond energy and the wave function can only be evaluated by rough approximations. The oldest of these is the valence bond method introduced by Heitler and London, and further developed by Slater and Pauling; it is therefore sometimes referred to as the *HLSP method*.

There is a close similarity between this method and the old chemical picture of the bond. The valency electrons in a molecule are assumed, in a first approximation, to be paired in doublets which are localized between the atoms. There is, however, an important difference which can best be illustrated by considering a molecule AB, where both atoms have 1 valency electron each. The older chemical representation of AB was any one of three structures:



The first is a covalent bond, the next two are ionic structures.

In the valence bond approximation, these structures are retained, but the molecule is not assumed to be representable by any structure *alone*: the wave function of AB is built up by a linear combination of partial (and fictitious) wave functions corresponding to the three structures. Use of the **variational principle** allows one to determine the coefficients of these partial wave functions in the linear combinations.

Such a linear combination of wave functions is always a better (but not necessarily the best) description of the molecule. The molecule AB can thus never be represented accurately by one alone of the structures (1), but one must take into account the “*resonance*” (i.e., the necessity of building up linear combinations) of the various structures.

One can associate a fictitious energy E with any of the approximation ψ of the wave function

$$E = \frac{\int \psi^* H \psi d\tau}{\int \psi^2 d\tau} \quad (2)$$

where H is the Hamiltonian.

The difference between the energy calculated from the linear combination of two partial wave functions and the lowest of the energies deduced from the partial wave functions alone is called the *resonance energy*. It depends on the structures chosen initially to describe the molecule.

For polyatomic molecules, the valence bond approximation still retains the idea of molecular structures in which the electrons are grouped in pairs, but the number of structures which must be included in the linear combination of wave functions in order to have a fair representation of the molecule can be quite large.

VALENCE FORCES IN POLYATOMIC MOLECULES. In order to reduce the number of unknown force constants, often simplifying assumptions are made about the restoring forces in the molecule. The assumption most often used is that of valence forces, that is, of a strong restoring force in the line of every valence bond and a weaker one opposing a change of the angle between two valence bonds connecting one atom with two others.

Thus, if in nonlinear symmetric XY_2 molecules k_1 is the force constant of the XY bond and k_δ the force constant of the $Y-X-Y$ angle,

the following simple relations between the frequencies and force constants are obtained by solving the corresponding secular equations (see **normal vibrations and normal coordinates in polyatomic molecules**):

$$\begin{aligned} 4\pi^2(\nu_1^2 + \nu_2^2) &= \left(1 + \frac{2m_Y}{m_X} \cos^2 \alpha\right) \frac{k_1}{m_Y} \\ &\quad + \frac{2}{m_Y} \left(1 + \frac{2m_Y}{m_X} \sin^2 \alpha\right) \frac{k_\delta}{l^2} \\ 16\pi^4 \nu_1^2 \nu_2^2 &= 2 \left(1 + \frac{2m_Y}{m_X}\right) \frac{k_1}{m_Y^2} \cdot \frac{k_\delta}{l^2} \\ 4\pi^2 \nu_3^2 &= \left(1 + \frac{2m_Y}{m_X} \sin^2 \alpha\right) \frac{k_1}{m_Y}. \end{aligned}$$

Here m_X and m_Y are the masses of the atoms X and Y , α is half the $Y-X-Y$ angle, and l is the XY distance.

For linear symmetric XY_2 , one finds

$$\begin{aligned} 4\pi^2 \nu_1^2 &= \frac{k_1}{m_Y} \\ 4\pi^2 \nu_2^2 &= \frac{2}{m_Y} \left(1 + \frac{2m_Y}{m_X}\right) \frac{k_\delta}{l^2} \\ 4\pi^2 \nu_3^2 &= \left(1 + \frac{2m_Y}{m_X}\right) \frac{k_1}{m_Y}. \end{aligned}$$

For linear XYZ molecules, if k_1 and k_2 are the force constants, l_1 and l_2 the lengths of the XY and YZ bonds, one finds

$$\begin{aligned} 4\pi^2(\nu_1^2 + \nu_3^2) &= k_1 \left(\frac{1}{m_X} + \frac{1}{m_Y}\right) \\ &\quad + k_2 \left(\frac{1}{m_Y} + \frac{1}{m_Z}\right) \\ 16\pi^4 \nu_1^2 \nu_3^2 &= \frac{m_X + m_Y + m_Z}{m_X m_Y m_Z} k_1 k_2 \\ 4\pi^2 \nu_2^2 &= \frac{1}{l_1^2 l_2^2} \left(\frac{l_1^2}{m_Z} + \frac{l_2^2}{m_X} \right. \\ &\quad \left. + \frac{(l_1 + l_2)^2}{m_Y}\right) k_\delta. \end{aligned}$$

VALENCE, THEORY OF. See Heitler-London theory of valence; molecular orbital theory of valence.

VALENCY, DIRECTED. See directed valency.

VAN DER WAALS EQUATION. A form of the equation of state applicable to a (dilute) real gas. Its form is

$$\left(p + \frac{a}{V^2}\right)(V - b) = RT,$$

where p is the pressure, V is the volume, T is the absolute temperature, R is the gas constant, if we are dealing with one mole of the gas, and a and b are constants.

VAN DER WAALS FORCES. Interatomic and intermolecular forces of attraction, other than valence forces. The potential energy due to van der Waals forces varies inversely with the sixth power of the interatomic or intermolecular distance r . For atoms, the only van der Waals forces are the **London forces**. In the case of molecules additional forces of attraction are provided by the mutual interaction of induced dipole moments (induction effect), and, for molecules with a permanent dipole moment, by the interaction of the latter (orientation effect). The potential energy due to the induction effect is given by

$$U(r) = -\frac{2\alpha\mu^2}{r^6}$$

and the potential energy due to the orientation effect by

$$U(r) = -\frac{2\mu^4}{3r^6kT}.$$

Here α stands for the polarizability, μ , for the permanent dipole moment, k , for Boltzmann's constant and T , for the absolute temperature.

Van der Waals forces are of importance as weak binding forces between inert atoms and between saturated molecules. (See also **intermolecular forces** and **bond types**.)

VAN't HOFF FORMULA OR THE OSMOTIC PRESSURE. See **Osmotic pressure**.

VAN't HOFF THEOREM. If a reaction is exothermic (see **thermal coefficients**) an increase in temperature moves the equilibrium position of the reaction back. On the other hand, if the reaction is endothermic a rise in temperature advances the equilibrium.

Quantitatively this theorem may be expressed by

$$\left(\frac{d\xi}{dT}\right)_p = -\frac{h_{T,p}}{Tg_{T,p}} \quad (1)$$

where ξ is the **extent of the reaction**, $h_{T,p}$ is the heat of reaction, and

$$g_{T,p} = -\left(\frac{\partial^2 G}{\partial \xi^2}\right)_{p,T} \quad (2)$$

is the second derivative of the Gibbs free energy with respect to the extent of reaction. (See **thermodynamics, characteristic functions of**.)

The **chemical stability condition** is

$$g_{T,p} < 0. \quad (3)$$

This theorem is also clearly related to the Le Chatelier-Braun principle (see **Chatelier (Le)-Braun principle**).

VAPOR PRESSURE EQUATION. Equation for the vapor pressure p of a solid obtained by integration of the Clausius-Clapeyron law and using the **third law of thermodynamics**,

$$\ln p = -\beta\chi + \frac{5}{2} \ln T - \frac{1}{k} \int_{\beta}^{\infty} d\beta \int_{\beta}^{\infty} c_s d\beta + i,$$

where $\beta = 1/kT$ (k is the Boltzmann constant, T is absolute temperatures), χ is the latent heat of vaporization, c_s is the specific heat of the solid, and i is the **chemical constant**.

VAPOR PRESSURE, INFLUENCE OF CURVATURE ON. Let us consider a small spherical drop of curvature r . Its vapor pressure p is related to the vapor pressure p_o of a bulk liquid phase at the same temperature (for the bulk phase $1/r = 0$) by the **Kelvin formula**

$$\ln \frac{p}{p_o} = \frac{2\gamma V}{r RT}$$

where V is the molar volume of the drop and γ the surface tension. In this formula V has been considered as independent of pressure and the vapor as a perfect gas.

This formula shows that the vapor pressure increases when the drop becomes smaller.

VAPOR PRESSURE IN PERFECT SOLUTIONS. In **perfect solutions** the vapor pressure of each component is proportional to its mole fraction

$$p_i = p_i^0 x_i \quad (1)$$

where p_i^0 is the vapor pressure of pure i . This is the **Raoult law**.

If the vapor does not behave as a perfect gas mixture, the **fugacity** must be used in (1)

instead of the partial pressure. Inversely, if (1) is satisfied in a range of temperature, the solution is perfect.

VAPOR PRESSURE OF IDEAL SOLUTIONS. If one considers a solution which is not perfect but which is ideal when sufficiently dilute (see **ideal solutions, perfect solutions, vapor pressure of perfect solutions**) one has for the vapor pressures of the solutes

$$p_s = k_s x_s \quad (s = 2 \cdots c) \quad (1)$$

where x_s is the mole fraction of the solute s . The difference from perfect solutions is that k_s is not the vapor pressure of pure component s . Formula (1) is called the *Henry law*.

VARIABLES, CANONICALLY CONJUGATE. See **canonical equation of motion**.

VARIABLE STARS. Many of the stars vary in brightness and/or other characteristics. In this article are discussed only those variable stars in which the brightness varies due to causes inherent in the star itself, e.g., pulsation. Stars whose brightness varies due to external causes are discussed in articles on **eclipsing binaries**; and spectroscopic binaries.

(1) *Periodic Variables (Pulsating Stars)*. Observations of the brightness of a variable star, extending over a considerable period of time, frequently show a periodic rise to a maximum and a decline to a minimum. Once the period is approximately determined, a **least squares** solution will give a more precise period and all of the observations may be reduced to a particular epoch. From this a mean light curve is obtained. Figure 1 shows the mean light curve for the star Eta Aquilae. With the period established the elements of the variable

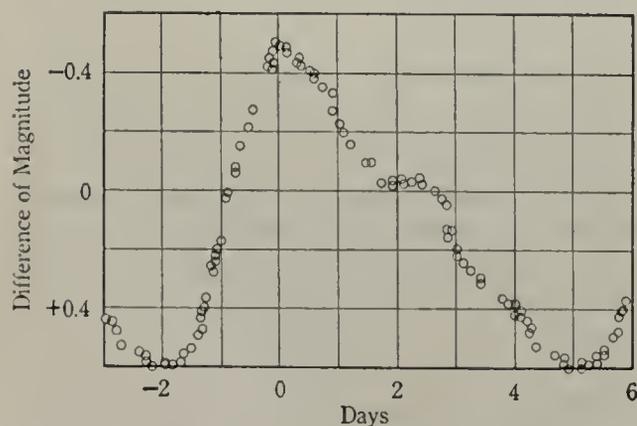


Fig. 1. Light curve of Eta Aquilae. A classical Cepheid having a period of about 7 days. (Photoelectric light curve by C. C. Wylie.)

may be determined. These are: The *epoch*, which is the date of some well defined feature of the curve (e.g., the maximum) and the *period of variation*. These are usually expressed in **Julian Days** and decimal fractions. The elements of Eta Aquilae are: Maximum Brightness $J.D. 2,414,827.15 + 7^d.1767.E$ (E is the number of the epoch counting $J.D. 2,414,827.15$ as $E = 0$).

(1a). *Cepheid Variables*. This class of objects gets its name from the type star δ Cephei, the first of this class to be investigated. The light curves of the Cepheids are characterized by a more rapid rise to maximum than the decline to minimum, and also by the more irregular shape of the curve on the declining side.

Two types of Cepheid variables are now recognized. The type discussed above is generally known as the classical Cepheid and about five hundred of them are known at present. They show a definite concentration toward the plane of the milky way. The periods run from one to fifty days, and the range of brightness from maximum to minimum is about one magnitude. Studies of their colors indicates that they are yellow supergiants (cf., **spectrum-luminosity relations**). The color changes during the cycle of light variation but they are not redder than GO at maximum. These are frequently referred to as Type I Cepheids.

Type II Cepheids have a different distribution in space than those of Type I. They are found more frequently in the globular clusters (cf., **star clusters**) and also near the center of our galaxy (cf., **galactic system**).

(1b). *RR Lyrae Variables*. The RR Lyrae variables have a number of characteristics in common with the Cepheids, and they are classed as pulsating stars. They were first found in the globular clusters and were known as *cluster variables*. We now know many of them outside of the clusters. Their light curves have characteristics similar to those of the Cepheids, such as rapid rise to maximum and slow decline to minimum. However in the RR Lyrae stars the rise is extremely rapid, in some cases rising from a minimum to a maximum one magnitude brighter in less than an hour. The periods range from a little over one hour to about a day, and they are slowly changing. The RR Lyrae variables differ from the Cepheids in spectral class, ranging from

A0 to F5, therefore being in the blue giant class.

Radial velocities have been determined for both Cepheids and RR Lyrae variables. The radial velocity is found to be variable indicating that at maximum brightness the star is approaching the earth more rapidly than at minimum. Furthermore, the spectral types of these variables change in synchronism with the light curve. For example, in one case we have a maximum magnitude of 4.35 when the apparent radial velocity is -10 km/sec and at minimum magnitude 5.6 the radial velocity is 25 km/sec. It has been established for δ Cephei that the ultraviolet radiation changes through a greater range than does the green, and the green has a larger amplitude than the infrared. Furthermore, there is a slight shift in phase with the ultraviolet coming to maximum slightly before the green and the green in turn slightly before the infrared. All of these changes; magnitude, radial velocity, spectral type and color are in such close synchronism that it seems certain that we are dealing with truly pulsating stars.

These pulsating stars have still another very important correlation in what is known as the **period-luminosity relation**. This is shown

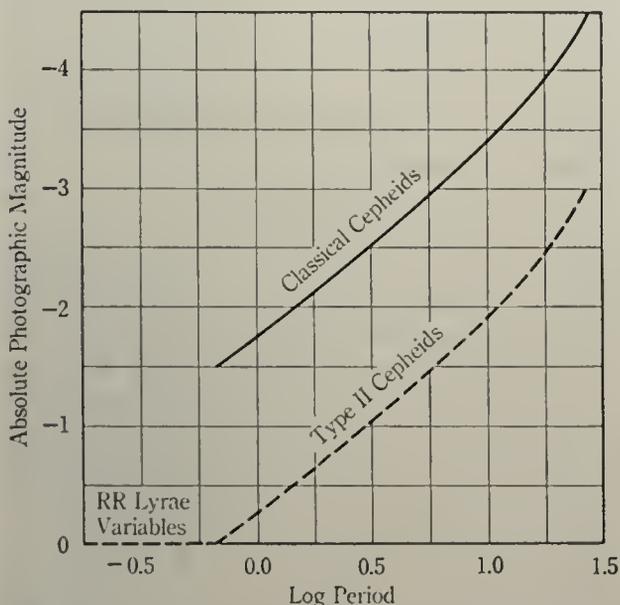


Fig. 2. Period-luminosity curves for Cepheids. The absolute median photographic magnitude of a Cepheid can be read from these curves when the period of the light variation is known. The median magnitudes of all RR Lyrae variables are zero.

graphically in Figure 2. When the period of a Type I Cepheid, a Type II Cepheid, or a RR Lyrae Type star is determined the median

absolute magnitude may be determined by reference to the curves. The upper curve is for the Type I Cepheid, the lower curve is for the Type II Cepheid, and the RR Lyrae stars have median absolute photographic magnitudes of zero. The median apparent magnitude is determined by observation. Calling this m , and the value from the curve, M , we have the distance, r , given by

$$\log_{10} r = \frac{(m - M + 5)}{5}$$

from which the distance r is given in parsecs and must be corrected for any known absorption of light in the intervening space.

(2) *The RV Tauri Stars*. This is a classification of variable stars with periods between those of the Cepheids and those of the long period variables. This embraces stars with periods from 19 to 150 days. In this group we find periods that are cyclic in character rather than strictly periodic. Many of them show a tendency to alternation of deep and shallow minima. The physical characteristics of this group are less well defined than those of the cepheids. Mean light curves are difficult, if not impossible, to determine because of variations in period and amplitude. About 75% of this group have median periods between 60 and 100 days. The spectral class at maximum ranges from F0 to G without any apparent correlation with period. Many of them show bright lines, notably of hydrogen, at some stage of their cycle. Many of them develop the bands of titanium oxide at their deep minima, although the spectral class at minimum is rarely later than K0. The RV Tauri group is not a physically homogeneous group. For example, RV Tauri and AC Hercules are of nearly the same period, 78 days, but the former has a spectral range from G4 to K4, while the latter ranges from F0 to G8.

(3) *Long-Period Variables*. Here we have another well-defined group of over 500 stars with periods ranging from 90 to 700 days. They are distinguished from the RV Tauri group by their larger amplitudes and their more definite spectral features. The periods of the 500 brightest long-period variables are quite symmetrically the limiting periods for this class, with two equally prominent frequency maxima at about 260 and 340 days. When all known long-period variables are

considered, 260 days is by far the most prominent period. There is considerable, but not conclusive, evidence of the existence of at least two groups that blend into each other.

The amplitude of variation of the long period variables is large. However, care must be taken in defining this since the magnitudes at successive maxima may be quite different, and there is the same variability of the minima. The most representative range is from median maximum to median minima and this averages about five magnitudes for the M-type long period variables and a bit less for the N-type.

A number of studies of the distribution of the long period variables in space have been undertaken. Nothing of any considerable importance has been established.

The spectra of the long-period variables show the molecular bands that are characteristic of the M, N, S, and R spectral classes. This indicates relatively low temperature stars. However, bright lines of hydrogen, iron (neutral and once or twice-ionized) together with bright lines of magnesium, silicon, and indium are found in the spectrum of the typical, and first discovered, long period variable Mira (O Ceti). Other anomalies in the Balmer lines have been noted, but nothing like a complete explanation is available. (cf. *novae*.)

VARIANCE. In statistics, the variance of a population is the second **moment** about the mean. The same definition applies to a sample of value but some writers define the variance of x_1, x_2, \dots, x_N as

$$\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2.$$

The grounds for this are that such a quantity has an **expectation** equal to the parent variance and thus is an unbiased estimator of that variance. In mechanics, the term refers to the number of **degrees of freedom** of a system. In physics and chemistry, the variance is the number of degrees of freedom of a system, or the degrees of freedom themselves.

VARIANCE-COVARIANCE MATRIX. See **dispersion**.

VARIANCE OF A THERMODYNAMICAL SYSTEM. See **phase rule**.

VARIANCE, STABILIZATION OF. See **stabilization of variance**.

VARIATE DIFFERENCE METHOD. If the non-random part of a time-series can be represented locally by a **polynomial**, this part can be removed by taking successive differences of the series. If the variances of the successive difference series are calculated, and the r th variance divided by $(2r)!/(r!)^2$, the resulting numbers should decrease until the trend has been eliminated and then remain at a roughly constant value which estimates the variance of the random part of the original series. A similar technique can be used to estimate the **correlation** between the random parts of two series. (See **weighting**.)

VARIATIONAL METHOD. See **Rayleigh-Ritz method**.

VARIATIONAL PRINCIPLES FOR NON-EQUILIBRIUM STATES. See **time variation of the entropy production**.

VARIATION, BOUNDED. A function $f(x)$ is said to be of bounded variation over a closed interval $[a, b]$ in its domain of definition if there exists a constant M such that

$$|f(x_1) - f(x_0)| + |f(x_2) - f(x_1)| + \dots \\ + |f(x_n) - f(x_{n-1})| < M$$

for every subdivision $x_0 = a, x_1, x_2, \dots, x_r$ of $[a, b]$.

VARIATION, COEFFICIENT OF. See **coefficient of variation**.

VARIATION OF LATITUDE. In the latter part of the 18th century the mathematician Euler predicted, from purely theoretical considerations, that the **latitude** of every point on the surface of the earth should be varying. The amount of this variation depends upon the shape, the elasticity, and other physical characteristics of the earth, together with the attractions of external objects, such as the moon, for our rotating planet. The first actual observations of variation of latitude were made in 1888, nearly a century after Euler's prediction, by the careful observations of the astronomer Kustner.

VARIATION PRINCIPLE AND BOND ENERGIES. The variation principle has often been used as a tool to solve wave-mechanical problems: Let ψ be a trial wave function; it may itself be a linear combination of simpler

functions. The variation principle states that one has the relationship:

$$\epsilon = \int \psi H \psi^* d\tau \geq E$$

H is the Hamiltonian, and $d\tau$ is a volume element. E is the true energy of the system. One may thus choose between different possible wave functions: the best approximation to the true wave function is that function ψ from which one calculates the least energy. This principle is also called the *minimum energy principle*.

VARIATIONS, CALCULUS OF. The problem of the calculus of variations is to determine functions in such a way that a definite integral depending upon them and their derivatives may assume an extreme value. In the simplest case the problem is to minimize the value of the integral

$$I[y] = \int_a^b F(x, y, y') dx$$

with given a , b , $y(a)$, $y(b)$. Thus, if $y = f(x)$ is the desired minimizing function and if we consider neighboring functions $y + \epsilon\eta(x)$, where $\eta(x)$ is any fixed function of x with $\eta(a) = \eta(b) = 0$ and ϵ is a real number, we may regard $I[y + \epsilon\eta]$ as a function of ϵ alone with a minimum at $\epsilon = 0$, so that

$$\left. \frac{dI}{d\epsilon} \right|_{\epsilon=0} = 0.$$

But

$$\frac{dI}{d\epsilon} = \int_a^b \left(\frac{\partial F}{\partial y} \eta + \frac{\partial F}{\partial y'} \eta' \right) dx,$$

so that, by integration by parts

$$\int_a^b \eta \left\{ \frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) \right\} dx = 0,$$

and from the fundamental lemma (see below)

$$\frac{\partial F}{\partial y} - \frac{d}{dx} \left(\frac{\partial F}{\partial y'} \right) = 0,$$

which is called the *Euler-Lagrange equation* of the original problem. It represents a necessary condition for an extreme value of the definite integral.

The fundamental lemma referred to above may be stated thus. If $\phi(x)$ is continuous in the closed interval $[a, b]$ and $\int_a^b \eta(x)\phi(x)dx = 0$ for all $\eta(x)$ with continuous first derivatives and such that $\eta(a) = \eta(b) = 0$, then $\phi(x)$ is identically zero.

The above remarks can be generalized directly to deal with integrands involving derivatives of higher than first order and with the case where the integrand depends upon more than one function. The function $\epsilon\eta(x)$ is called the variation of y and $\epsilon \frac{dI}{d\epsilon}$ is the variation of I . (Compare the definition of the **differential** of a dependent variable.)

VARIGNON THEOREM. The algebraic sum of the moments of two coplanar concurrent forces with respect to a point in their plane is equal to the moment of the resultant with respect to the same point. (See also **moments, theorem of.**)

VDI FORMULA. Semi-empirical expression for the calculation of the net (lower) **calorific value** of coal from its ultimate composition. In English units

$$H_n = [14,580c + 50,400(h - \frac{1}{8}o) + 4500s - 1080w] \text{ Btu/lbm.}$$

(c , h , o , s and w denote the masses of carbon, hydrogen, oxygen, sulfur and water, respectively, per unit mass of coal.)

VECTOR. A **matrix** of n rows and a single column (or of n columns and a single row). Geometrically or physically, it can represent directional magnitudes such as displacements, velocities, accelerations, forces, etc.

A vector is often indicated graphically by means of an arrow (technically called a *stroke*). The length of the arrow is proportional to the scalar magnitude of the vector and the direction in which the arrow points is the direction of the vector. The tail or initial point of the arrow is its *origin*; the head or final point is its *terminus*.

A vector of unit length, drawn in the positive direction and tangential to a coordinate system, is a unit vector. It is not necessary that the system be orthogonal. In the common case, a rectangular Cartesian coordinate system is used and the unit vectors along OX , OY , OZ axes are called **i**, **j**, **k**, respectively.

For an arbitrary vector, scalar quantities called the components are required to determine it numerically. In three dimensions, they are directed lines, parallel to the axes of a coordinate system. Thus, if a rectangular Cartesian system is used, with unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$, any vector may be written as $\mathbf{A} = \mathbf{i}A_x + \mathbf{j}A_y + \mathbf{k}A_z$, where (A_x, A_y, A_z) are its three components. In the more general case of an n -dimensional vector the components of the vector are the n **matrix** elements of a column or row matrix. If, in a rectangular coordinate system, a point has coordinates (x, y, z) then its *position vector* is one drawn from the coordinate origin to the point. It may be written as $\mathbf{R} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$. In polar coordinates or in spherical polar coordinates, a vector drawn from the origin of the coordinate system to a point is a *radius vector*.

As previously seen, a vector is commonly indicated by a bold-face letter such as \mathbf{A} , which stands for its three scalar components (A_1, A_2, A_3) referred to some coordinate system. In the Gibbs notation, scalar and vector products are shown with dots and crosses, respectively. Thus, if C is a scalar and $\mathbf{V}, \mathbf{A}, \mathbf{B}$ are vectors, then $C = \mathbf{A} \cdot \mathbf{B}$ and $\mathbf{V} = \mathbf{A} \times \mathbf{B}$. Less commonly used symbols have been proposed by Hamilton, Grassmann, Heaviside, and others. They include: TA (T for tensor), $|A|$ for the magnitude of a vector; $SAB, (\mathbf{AB}), \mathbf{A} \cdot \mathbf{B}$ for the scalar product; $VAB, \mathbf{A} \times \mathbf{B}$, and $[\mathbf{AB}]$ for the vector product.

A more precise definition of a vector is often required. Suppose a point located in a rectangular coordinate system has components (x_1, x_2, x_3) . The same point, however, could also be described in other coordinate systems, obtained from the first one by translation of the origin and rotations about the coordinate axes. If the components of the point in the second system are (x'_1, x'_2, x'_3) , assumed for convenience to have the same origin as that of the first system, then the relation between the components, called a linear **transformation**, is

$$x'_i = \sum_{j=1}^3 c_{ij}x'_j; \quad i = 1, 2, 3$$

where the c_{ij} are the nine **direction cosines** between the various coordinate-axis pairs. Matrix notation may also be used to write $\mathbf{x}' = \mathbf{R}\mathbf{x}$, where \mathbf{x}' and \mathbf{x} are column vectors; \mathbf{R} is the orthogonal matrix of the direction cosines. If this transformation law holds, one

speaks of a *polar*, or *localized vector*; if the law does not hold, a **pseudovector** or *axial vector*.

The concept of vector may be generalized extensively. A *four-vector* has four components. One type is called a **quaternion**, another, used principally in relativity theory, has for its components (x, y, z, ict) , where x, y, z are positional coordinates, $i = \sqrt{-1}$, c is the velocity of light, and t is the time. The components of such a vector in one coordinate system are related to the components in another system by a *Lorentz transformation*. (See also **tensor**.)

If (x', y', z') are functions of (x, y, z) , then the vector $\mathbf{V}' = \mathbf{i}x' + \mathbf{j}y' + \mathbf{k}z'$ is a vector function of the vector $\mathbf{V} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$, where $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ are unit vectors. The function f is a linear vector function if $f(\mathbf{A} + \mathbf{B}) = f(\mathbf{A}) + f(\mathbf{B})$, for all vectors \mathbf{A}, \mathbf{B} and $f(k\mathbf{V}) = kf(\mathbf{V})$, where k is a scalar. More generally, suppose the components of \mathbf{V} and \mathbf{V}' are (V_1, V_2, V_3) and (V'_1, V'_2, V'_3) respectively and that the relation between the two vectors in matrix form is $\mathbf{V} = \mathbf{M}\mathbf{V}'$, where \mathbf{M} is a (3×3) -matrix with elements M_{ij} . Then the function is a linear vector function, for $f(k\mathbf{V}) = kf(\mathbf{V})$ and $f(\mathbf{V} + \mathbf{V}') = f(\mathbf{V}) + f(\mathbf{V}')$, as before.

The vector \mathbf{V} can be written as the sum of a symmetric and an antisymmetric linear vector function, $\mathbf{V} = \mathbf{S} + \mathbf{A}$. If $\mathbf{S} = \mathbf{Q}\mathbf{V}'$, then $Q_{ii} = M_{ii}$, $Q_{ij} = Q_{ji} = (M_{ij} + M_{ji})/2$, $i \neq j$. Similarly, $\mathbf{A} = \mathbf{T}\mathbf{V}'$, $T_{ii} = 0$, $T_{ij} = (M_{ij} - M_{ji})/2$. The antisymmetric function can also be written as a vector **product**, $\mathbf{A} = \mathbf{T} \times \mathbf{V}'$.

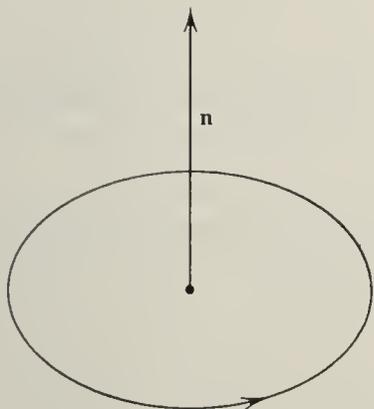
VECTOR ADDITION. If \mathbf{A}, \mathbf{B} are vectors with components A_x, A_y, A_z and B_x, B_y, B_z , respectively, their sum is a new vector $\mathbf{C} = \mathbf{A} + \mathbf{B}$, with components $A_x + B_x, A_y + B_y, A_z + B_z$. Vector addition obeys the commutative and associative laws of algebra: $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$; $(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C})$. To subtract a vector \mathbf{B} from a vector \mathbf{A} , take the negative of \mathbf{B} and add $-\mathbf{B}$ to \mathbf{A} .

VECTOR ANALYSIS, FOUR DIMENSIONAL. See Minkowski world.

VECTOR ANALYSIS, INTEGRAL THEOREMS OF. See integral theorems of vector analysis.

VECTOR AREA. The vector area of a surface element dS , denoted by $d\mathbf{S}$, is $n dS$, where \mathbf{n} is a unit normal vector to the surface ele-

ment and the sense of \mathbf{n} is given by the right-hand rule (see figure). An arbitrary sense of circulation is chosen along the boundary of of



dS , if the fingers of the right hand point in the sense of the circulation, the thumb points in the direction of \mathbf{n} . dS is independent of the shape of dS . For any closed surface, the total vector surface is zero.

VECTOR, AXIAL. See pseudovector.

VECTOR, BASE. See linearly independent vectors.

VECTOR, BRA. See bra vector.

VECTOR, COMPONENT OF. See component of a vector.

VECTOR COMPOSITION. See vector addition.

VECTOR, CONTRAVARIANT. A vector whose components a^l transform according to the law

$$a'^l = \frac{\partial x'^l}{\partial x^i} a^i.$$

Typical contravariant vectors are the infinitesimal displacement or velocity vector. (Cf. covariant vector.)

A contravariant vector is a contravariant tensor of order one. (See tensor field.)

VECTOR, COVARIANT. A vector whose components b_l transform according to the law

$$b'_l = \frac{\partial x^i}{\partial x'^l} b_i.$$

A typical covariant vector is the gradient of a scalar. (See also vector, contravariant.)

The inner product of a contravariant vector a^l and a covariant vector b_l is an invariant, i.e.,

$$\sum_{l=1}^N a^l b_l = \sum_{l=1}^N a'^l b'_l \quad (N = \text{dimensions of space}).$$

In orthogonal Cartesian coordinates there is no distinction between a covariant or contravariant vector.

A covariant vector is a covariant tensor of order one. (See tensor field.)

VECTOR CURVATURE (OF A CURVE AT A POINT). A vector whose direction is that of the principal normal to the curve at the point and whose magnitude is equal to the curvature of the curve.

VECTOR, DARBOUX. See Darboux vector.

VECTOR DECOMPOSITION. See decomposition of a vector.

VECTOR DERIVATIVE. If a vector \mathbf{R} is a function of a single scalar variable t , there are three possible ways in which \mathbf{R} may vary with t , for if \mathbf{R}_1 and \mathbf{R}_2 refer to t_1 and t_2 , respectively, then \mathbf{R}_2 may differ from \mathbf{R}_1 : in magnitude only; in direction only; in both magnitude and direction. Since even the general case is relatively simple, assume that a curve is traced by the terminus of the continuously varying vector \mathbf{R} , the origin of the vector being kept fixed at the origin of a coordinate system. Let A and B be two neighboring points on this curve and let \mathbf{R}_1 and \mathbf{R}_2 be their position vectors, then the vector $\Delta\mathbf{R} = \mathbf{R}_2 - \mathbf{R}_1$ has the direction of the secant AB , which approaches the tangent to the curve at A as $\Delta t = t_2 - t_1$ approaches zero. The quotient $\Delta\mathbf{R}/\Delta t$ is the average rate of change of \mathbf{R} in the interval between t_1 and t_2 . The derivative is defined as

$$\lim_{t \rightarrow 0} \Delta\mathbf{R}/\Delta t = d\mathbf{R}/dt.$$

In terms of unit vectors, and with the use of primes for differentiation, $\mathbf{R} = \mathbf{i}R_x + \mathbf{j}R_y + \mathbf{k}R_z$, $\mathbf{R}' = \mathbf{i}R'_x + \mathbf{j}R'_y + \mathbf{k}R'_z$, $\mathbf{R}'' = \mathbf{i}R''_x + \mathbf{j}R''_y + \mathbf{k}R''_z$. For a composite function of two or more vectors, each depending on a single scalar t , the usual rules of differentiation hold except that the order of the vectors must be retained if vector products are involved.

There are also several differential vector operators. (See del, gradient, divergence, curl, Laplacian, d'Alembertian.)

VECTOR DIFFERENTIAL IDENTITIES.

Let ϕ be a scalar and \mathbf{U} and \mathbf{V} vector functions of x, y, z . The following identities hold:

$$\nabla \cdot \phi \mathbf{U} = \phi \nabla \cdot \mathbf{U} + \mathbf{U} \cdot \nabla \phi$$

$$\nabla \times \phi \mathbf{U} = \phi \nabla \times \mathbf{U} + \nabla \phi \times \mathbf{U}$$

$$\nabla \cdot \mathbf{U} \times \mathbf{V} = \mathbf{V} \cdot \nabla \times \mathbf{U} - \mathbf{U} \cdot \nabla \times \mathbf{V}$$

$$\begin{aligned} \nabla \times (\mathbf{U} \times \mathbf{V}) &= \mathbf{U}(\mathbf{V} \cdot \nabla) - \mathbf{V}(\nabla \cdot \mathbf{U}) \\ &\quad + \mathbf{U}(\nabla \cdot \mathbf{V}) - \mathbf{V}(\nabla \cdot \mathbf{U}) \end{aligned}$$

$$\begin{aligned} \nabla(\mathbf{U} \cdot \mathbf{V}) &= (\mathbf{U} \cdot \nabla)\mathbf{V} + (\mathbf{V} \cdot \nabla)\mathbf{U} \\ &\quad + \mathbf{U} \times (\nabla \times \mathbf{V}) + \mathbf{V} \\ &\quad \times (\nabla \times \mathbf{U}) \end{aligned}$$

$$\nabla \times (\nabla \phi) = 0$$

$$\nabla \cdot (\nabla \times \mathbf{U}) = 0$$

$$\nabla \times (\nabla \times \mathbf{U}) = \nabla(\nabla \cdot \mathbf{U}) - \nabla^2 \mathbf{U}.$$

VECTOR DIVERGENCE. See **divergence**.

VECTOR FIELD. A region of space, each point of which is described by a **vector**. Thus, in three dimensions, each point is described by three quantities, the **components** of the vector along the coordinate axes. Examples are wind velocities in the atmosphere, electrostatic or electromagnetic fields. (See also **scalar field**.)

VECTOR FIELD, CARTESIAN. Cartesian tensor field (see **tensor field, Cartesian**) of order unity. Sometimes referred to as **vector field**.

VECTOR FIELD, LAMELLAR. See **lamellar vector field**.

VECTOR FLUX. If \mathbf{V} describes a **vector field**, for example the velocity of an incompressible fluid, then the total **flux** through a surface S in the field is given by

$$\iint_S \mathbf{v} \cdot d\mathbf{S}.$$

The vector \mathbf{V} may refer to electric, magnetic, or gravitational force; heat or a fluid, etc. The surface integral may be converted to a volume integral by **Gauss's theorem**.

VECTOR, HERTZ. See **Hertz vector**.

VECTOR, IRROTATIONAL. If the **curl** of a vector function of position vanishes everywhere in a region, the function is said to be an irrotational vector (or a **lamellar vector**) in

this region. It follows that if \mathbf{V} is an irrotational vector so that its **curl** is zero, i.e., $\nabla \times \mathbf{V} = 0$, then $\mathbf{V} = \nabla \phi$, where ϕ is some **scalar function** of position.

VECTOR, KET. See **ket vector**.

VECTOR(S), LINEARLY INDEPENDENT. See **linearly independent vectors**.

VECTOR MULTIPLICATION. There are two distinct kinds of products of two vectors: the scalar product and the vector product (but see **pseudovector**). They are also sometimes called inner and outer products but these terms more commonly refer to **tensor products**. There are also several possibilities for the product of three or four vectors, as the subsequent discussion will show.

(1) *Scalar Product.* If \mathbf{A} and \mathbf{B} are two vectors, of magnitude A, B , respectively, their scalar product is $\mathbf{A} \cdot \mathbf{B} = AB \cos \theta$, where θ is the angle between the two vectors. This product, which is a scalar quantity, is also known as the dot product. In Cartesian coordinates $\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$. If the vectors are complex, the result of multiplication is the Hermitian scalar product (see **vector space**).

The scalar product of two vectors obeys the commutative and distributive laws: $\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$; $\mathbf{A} \cdot (\mathbf{B} + \mathbf{C}) = \mathbf{A} \cdot \mathbf{B} + \mathbf{A} \cdot \mathbf{C}$. If \mathbf{A} is perpendicular to \mathbf{B} , then $\mathbf{A} \cdot \mathbf{B} = 0$, and consequently if $\mathbf{A} \cdot \mathbf{B} = 0$, then \mathbf{A} is perpendicular to \mathbf{B} and the two vectors are said to be orthogonal. If \mathbf{A} is parallel to \mathbf{B} , then $\mathbf{A} \cdot \mathbf{B} = AB$. Consequently, $\mathbf{A} \cdot \mathbf{A} = A^2$, the square of the length of \mathbf{A} .

(2) *Vector Product.* The vector product of \mathbf{A} and \mathbf{B} (also called skew or cross product) is of length $C = AB \sin \theta$ and its direction is perpendicular to the plane determined by \mathbf{A} and \mathbf{B} . The sense of the vectors $\mathbf{V} = \mathbf{A} \times \mathbf{B}$ is such that a right-handed rotation about \mathbf{V} carries \mathbf{A} into \mathbf{B} through an angle θ that is not greater than 180° . In Cartesian coordinates, with unit vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ one has

$$\begin{aligned} \mathbf{C} &= \mathbf{A} \times \mathbf{B} \\ &= (A_y B_z - A_z B_y)\mathbf{i} + (A_z B_x - A_x B_z)\mathbf{j} \\ &\quad + (A_x B_y - A_y B_x)\mathbf{k} \end{aligned}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}.$$

SOME TYPES OF VECTOR FIELDS AND THE RELATED PHYSICAL PROBLEMS

Characterization	Mathematical Consequences	Some Physical Problems Characterized by Such Fields
1. $\nabla \times \mathbf{V} = 0$ Irrotational $\nabla \cdot \mathbf{V} = 0$ Solenoidal	$\mathbf{V} = -\nabla\Phi$ $\nabla^2\Phi = 0$ Knowledge of Φ will determine the field	Steady state heat conduction Irrotational motion of incompressible ideal fluid Electrostatic field Gravitational field
2. $\nabla \times \mathbf{V} = 0$ Irrotational $\nabla \cdot \mathbf{V} \neq 0$ Not solenoidal	$\mathbf{V} = -\nabla\Phi$ $\nabla^2\Phi \neq 0$	$\nabla^2\Phi = -f(P)$ (Poisson's equation) Gravitational field inside a mass Electric field within a volume distribution of charge $\nabla^2\Phi = -f(P, \Phi)$ Schrödinger's equation with time dependence removed: $\nabla^2\Phi = \frac{8\pi^2}{h^2} [V(P) - E]\Phi$ where E is a constant $\nabla^2\Phi = -f(P, t, \Phi)$ Three dimensional wave equation $\nabla^2\Phi = \frac{1}{c^2} \frac{\partial^2\Phi}{\partial t^2}$ Diffusion equation $K\nabla^2\Phi = \frac{\partial\Phi}{\partial t}$ Conduction of electricity $c^2\nabla^2E = K \frac{\partial^2E}{\partial t^2} + \sigma\mu \frac{\partial E}{\partial t}$
3. $\nabla \times \mathbf{V} \neq 0$ Rotational $\nabla \cdot \mathbf{V} = 0$ Solenoidal	$\mathbf{V} = \nabla \times \mathbf{A}$ where \mathbf{A} is called vector potential Since $\nabla \cdot \nabla \times \mathbf{A} = 0$, $\nabla \times \mathbf{V} = \nabla \times \nabla \times \mathbf{A}$ $= \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$ Frequently choose $\nabla \cdot \mathbf{A} = 0$; hence $\nabla^2\mathbf{A} \neq 0$ (but known) $\nabla \cdot \mathbf{A} = 0$ and determine \mathbf{A} from these equations	Magnetic field due to steady currents $\nabla \times \mathbf{V} = -\nabla^2\mathbf{A} = -4\pi c$ Incompressible fluid with vorticity
4. $\nabla \times \mathbf{V} \neq 0$ Rotational $\nabla \cdot \mathbf{V} \neq 0$ Not solenoidal	Assume $\mathbf{V} = \nabla S + \nabla \times \mathbf{A}$ Then $\nabla \cdot \mathbf{V} = \nabla^2 S + \nabla \cdot \nabla \times \mathbf{A}$ But $\nabla \cdot \nabla \times \mathbf{A} = 0$; hence we determine $\nabla^2 S$ from known value of $\nabla \cdot \mathbf{V}$ $\nabla \times \mathbf{V} = \nabla \times \nabla S + \nabla \times \nabla \times \mathbf{A}$ $= \nabla(\nabla \cdot \mathbf{A}) - \nabla^2\mathbf{A}$ Assume $\nabla \cdot \mathbf{A} = 0$ and determine \mathbf{A} from known expression for $\nabla \times \mathbf{V}$ We have three equations to solve: $\nabla^2 S = \nabla \cdot \mathbf{V}$ (a known function) $\nabla \cdot \mathbf{A} = 0$ $\nabla^2\mathbf{A} = -\nabla \times \mathbf{V}$ (a known function) Having determined S and \mathbf{A} $\mathbf{V} = \nabla S + \nabla \times \mathbf{A}$	Maxwell's equations within matter $\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$ $\nabla \cdot \mathbf{D} = \rho$ $\nabla \times \frac{\mathbf{B}}{\mu} = \mathbf{i} + \frac{\partial \mathbf{D}}{\partial t}$ $\nabla \cdot \mathbf{B} = 0$

Vector multiplication is not commutative, in this case, for $\mathbf{A} \times \mathbf{B} = -\mathbf{B} \times \mathbf{A}$ but the distributive law of multiplication still holds.

(3) *Products of Unit Vectors.* For the two kinds of vector multiplication described in (1) and (2), the results for unit vectors are: $\mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{i} = \mathbf{i} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{k} = \mathbf{k} \cdot \mathbf{j} = 0$; $\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = \mathbf{k} \cdot \mathbf{k} = i^2 = j^2 = k^2 = 1$; $\mathbf{i} \times \mathbf{j} = -\mathbf{j} \times \mathbf{i} = \mathbf{k}$; $\mathbf{j} \times \mathbf{k} = -\mathbf{k} \times \mathbf{j} = \mathbf{i}$; $\mathbf{k} \times \mathbf{i} = -\mathbf{i} \times \mathbf{k} = \mathbf{j}$; $\mathbf{i} \times \mathbf{i} = \mathbf{j} \times \mathbf{j} = \mathbf{k} \times \mathbf{k} = 0$.

(4) *Triple Products of Vectors.* Three vectors $\mathbf{A}, \mathbf{B}, \mathbf{C}$ may be combined to form products with meaning in several ways: (a) $\mathbf{A}(\mathbf{B} \cdot \mathbf{C})$, a vector with the same direction as \mathbf{A} and magnitude $ABC \cos \theta$, where θ is the angle between \mathbf{B} and \mathbf{C} . (b) $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})$, the scalar triple product, giving the volume of a parallelepiped with edges $\mathbf{A}, \mathbf{B}, \mathbf{C}$. It is frequently indicated by the symbol $[\mathbf{ABC}]$ and if the three vectors all lie in the same plane $[\mathbf{ABC}] = 0$. It may be written in terms of its components as a **determinant**

$$[\mathbf{ABC}] = \begin{vmatrix} A_x & A_y & A_z \\ B_x & B_y & B_z \\ C_x & C_y & C_z \end{vmatrix}.$$

Its properties include $[\mathbf{ABC}] = [\mathbf{BCA}] = [\mathbf{CAB}] = -[\mathbf{ACB}] = -[\mathbf{BAC}] = -[\mathbf{CBA}]$. (c) The vector triple product, $\mathbf{V} = \mathbf{A} \times (\mathbf{B} \times \mathbf{C})$ is perpendicular to both \mathbf{A} and the vector $(\mathbf{B} \times \mathbf{C})$. It therefore lies in the plane determined by \mathbf{B} and \mathbf{C} . Its properties include: $\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = \mathbf{B}(\mathbf{A} \cdot \mathbf{C}) - \mathbf{C}(\mathbf{A} \cdot \mathbf{B}) = -\mathbf{A} \times (\mathbf{C} \times \mathbf{B}) = (\mathbf{C} \times \mathbf{B}) \times \mathbf{A} = -(\mathbf{B} \times \mathbf{C}) \times \mathbf{A}$.

Since the vector product changes its sign when the order of multiplication is changed, the sign of the triple vector product depends on the order of the parentheses and their factors.

IP5. *Quadruple Products of Vectors.* If $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$ are any four vectors, two types of quadruple products can occur: (a) $(\mathbf{A} \times \mathbf{B}) \cdot (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C})(\mathbf{B} \cdot \mathbf{D}) - (\mathbf{A} \cdot \mathbf{D})(\mathbf{B} \cdot \mathbf{C})$; (b) $(\mathbf{A} \times \mathbf{B}) \times (\mathbf{C} \times \mathbf{D}) = (\mathbf{A} \cdot \mathbf{C} \times \mathbf{D})\mathbf{B} - (\mathbf{B} \cdot \mathbf{C} \times \mathbf{D})\mathbf{A}$.

Products of more than four vectors can always be reduced to combinations of one or more of the preceding types.

VECTOR, NULL. See **null vector**.

VECTOR OPERATOR. A symbolic operator containing vector quantities. Those frequently used include **del**, the **Laplacian operator**, the **d'Alembertian operator**.

VECTOR(S), ORTHOGONAL. Two vectors whose scalar product is zero. For vectors rep-

resented by directed line segments in the plane or three-dimensional space, this is equivalent to the vectors (or lines) being perpendicular. (See **vector multiplication**.)

VECTOR, PARALLEL DISPLACEMENT OF. See **parallel displacement of a vector**.

VECTOR, POLAR. If its components in one Cartesian coordinate system are given by the column vector \mathbf{x} , then its components in another such system are $\mathbf{x}' = \mathbf{R}\mathbf{x}$, where \mathbf{R} is an **orthogonal matrix**. A directed quantity which cannot satisfy this requirement is a **pseudo-vector**.

VECTOR, POSITION. The vector from the origin to a point is called the position vector of the point. If, in a rectangular coordinate system, a point has coordinates (x, y, z) then its position vector is a **vector** drawn from the coordinate origin to the point. It may be written as $\mathbf{R} = ix + jy + kz$, where $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ are unit vectors.

VECTOR POTENTIAL. (1) Three functions of position and time, forming a vector $\mathbf{A}(\mathbf{r}, t)$ in ordinary space, used together with the scalar **potential** ϕ (scalar potential of an electric charge distribution as a function of (\mathbf{r}, t) of form, $\phi = \int \frac{\rho}{r} dv$, where ρ is the charge density) to specify an electromagnetic field. In **Minkowski space**, $A_x, A_y, A_z, i\phi$ form the components of a four-vector. In Gaussian units the electromagnetic field is given by

$$\mathbf{B} = \nabla \times \mathbf{A}$$

$$\mathbf{E} = -\nabla\phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}.$$

(2) A solenoidal **vector field**, such as magnetic induction, is one whose **divergence** vanishes everywhere: $\nabla \cdot \mathbf{B} = 0$. Such a vector is derivable from a vector potential; i.e., we can write $\mathbf{B} = \nabla \times \mathbf{A}$. The vector potential due to a distribution of current density is

$$\mathbf{A} = \mu \int \mathbf{J} \frac{1}{r} dv$$

where r is the distance between the point of observation and dv , the volume element. For a closed linear current loop, this becomes

$$\mathbf{A} = \mu I \int \frac{1}{r} d\mathbf{l}$$

so that

$$\begin{aligned} \mathbf{H} &= \mathbf{B}/\mu = \nabla \times \mathbf{A}/\mu \\ &= I \int \nabla \left(\frac{1}{r} \right) \times d\mathbf{l} = I \int \frac{d\mathbf{l} \times \mathbf{r}}{r^3}. \end{aligned}$$

This result is often expressed by the (non-unique) resolution into differential elements:

$$d\mathbf{H} = I \frac{d\mathbf{l} \times \mathbf{r}}{r^3}$$

which is known as the **Ampere law**, or the **Biot-Savart law**.

VECTOR PRODUCT. See **vector multiplication**.

VECTOR PRODUCT, QUADRUPLE. See **quadruple vector product**.

VECTOR, PSEUDO-. See **pseudovector**.

VECTOR, RADIUS. In polar coordinates or in spherical polar coordinates (see **curvilinear orthogonal coordinates**) a **vector** drawn from the origin of the coordinate system to a point. It is thus one of the two or three quantities required to describe the position of the point in the coordinate system.

VECTOR RESOLUTION INTO A SOLENOIDAL AND IRROTATIONAL VECTOR.

A vector \mathbf{B} can be decomposed into a sum of two vectors \mathbf{H} and \mathbf{K} where \mathbf{H} is solenoidal and \mathbf{K} is irrotational, i.e.,

$$\mathbf{B} = \mathbf{H} + \mathbf{K}$$

where $\mathbf{H} = \text{curl } \mathbf{D}$ and $\mathbf{K} = \text{grad } f$ and

$$\mathbf{D} = \frac{1}{4\pi} \iiint_V \frac{\text{curl } \mathbf{B}}{r} dV + \frac{1}{4\pi} \iint_S \frac{\mathbf{B} \times d\mathbf{S}}{r}$$

and

$$f = -\frac{1}{4\pi} \iiint_V \frac{\text{div } \mathbf{B}}{r} dV + \frac{1}{4\pi} \iint_S \frac{\mathbf{B} \cdot d\mathbf{S}}{r}.$$

V is a volume surrounding the point P at which the vector \mathbf{B} is considered. S is the closed surface of V . r is the distance from the point P to an arbitrary point of V .

VECTORS, ASSOCIATED (VECTOR FIELDS). The contravariant vector (vector field v^i) and the covariant vector (vector field)

v_i are said to be associated vectors (vector fields) if

$$v_i = g_{ij}v^j$$

where g_{ij} is the covariant **metric tensor**.

VECTORS, BASE (FOR A COORDINATE SYSTEM). See **base vectors (for a coordinate system)**.

VECTOR, SENSE OF. See **sense of a vector**.

VECTORS, FOUR. See **four vectors and tensors**.

VECTORS, FREE. See **affine tensors and free vectors**.

VECTOR, SOLENOIDAL, IN A REGION. A vector \mathbf{F} such that its integral over every reducible surface S in the region is zero; i.e.,

$$\int_S \mathbf{F} \cdot \mathbf{n} dS = 0,$$

where \mathbf{n} is the unit vector in the direction of the outer normal to the element of area dS . The **divergence** of a vector is zero at every point in a region if the vector is solenoidal in the region, or if the vector is the **curl** of some vector function. (See **vector potential**.)

VECTOR SPACE. A generalization of ordinary three-dimensional space to n dimensions. A **vector** in such space, if its components are real numbers x_1, x_2, \dots, x_n may be considered as a row or column matrix \mathbf{x} . The **scalar product** of two vectors \mathbf{x} and \mathbf{y} is a **scalar**

$$\tilde{\mathbf{x}}\mathbf{y} = x_1y_1 + x_2y_2 + \dots + x_ny_n$$

where $\tilde{\mathbf{x}}$ is the transpose of \mathbf{x} and two vectors are orthogonal in such a space if their scalar product vanishes, $\tilde{\mathbf{x}}\mathbf{y} = 0$. The square of the length of the vector (or its **norm** squared) is also a scalar

$$N^2 = \tilde{\mathbf{x}}\mathbf{x} = x_1^2 + x_2^2 + \dots + x_n^2.$$

If $N = 1$, the vector is *normalized*.

If the components of the vector are **complex**, the space is a *Hermitian (or unitary) space*. The *Hermitian scalar product* is

$$\mathbf{x}^\dagger\mathbf{y} = x_1^*y_1 + x_2^*y_2 + \dots + x_n^*y_n$$

where \mathbf{x}^\dagger is the associate matrix to \mathbf{x} and x_i^* is the complex conjugate to x_i . The norm is defined by $N^*\mathbf{N} = \mathbf{x}^\dagger\mathbf{x}$. The condition for orthogonality is $\mathbf{x}^\dagger\mathbf{y} = \mathbf{y}^\dagger\mathbf{x} = 0$ and the vector is normalized if $\mathbf{x}^\dagger\mathbf{x} = 1$.

VECTORS, RECIPROCAL BASE. See base vectors, reciprocal.

VECTORS, SIX. See four vectors and tensors.

VECTOR, STATE. See state vector.

VECTORS, TRIPLE PRODUCT OF. See triple product of vectors.

VECTOR SYSTEM, RECIPROCAL. See reciprocal vector system.

VELOCITIES, GENERALIZED. In particle mechanics, the time rates of change of the generalized coordinates. (See coordinates and momenta, generalized.)

VELOCITY. (1) The time rate of change of position. Velocity is a **vector** quantity; a statement of a velocity therefore includes both a magnitude, expressed in units of length divided by time, and a direction relative to some frame of reference. The defining equation for instantaneous velocity is $\mathbf{v} = d\mathbf{r}/dt$, where \mathbf{r} is the vector specifying position relative to an origin and t is the time. (Cf. **velocity, average**.) (2) Sometimes loosely used to express magnitude only, i.e., synonymous with speed.

VELOCITY, ABSOLUTE (IN A TURBINE). See absolute velocity.

VELOCITY, ANGULAR. See angular velocity.

VELOCITY, AREAL. Suppose a point to move on a plane. The rate at which a radius vector to the point sweeps out area is called areal velocity. When a particle is acted upon by a force directed along the radius, **angular momentum** is conserved and the areal velocity is constant.

VELOCITY, AVERAGE. Referring to a single material particle, the ratio of the change in the position vector to the time interval involved in the change. It is a vector quantity whose magnitude is not generally the average speed.

VELOCITY, CHARACTERISTIC. In propulsion, the ratio of the exhaust jet velocity to the thrust coefficient, which is a measure of the effectiveness with which the chemical reactants of the propellants in the rocket motor produce the high-temperature, high-

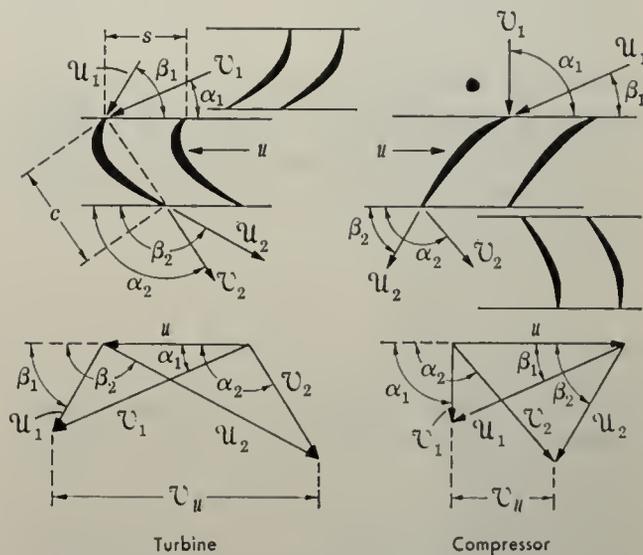
pressure gases. It is expressed by the relationships:

$$c^* = \frac{V_j}{C_f} = \frac{g}{dw/dt} p_c A_t = \frac{g}{c_w}$$

where c^* is the characteristic velocity, V_j is the exhaust jet velocity, C_f is the thrust coefficient, g is the acceleration of gravity, dw/dt is the flow rate of propellant by weight, p_c is the chamber pressure, A_t is the nozzle throat area, and c_w is the weight flow coefficient.

VELOCITY DENSITY. See continuous system, Lagrangian and Hamiltonian formalism for.

VELOCITY DIAGRAM. Hodograph of velocities at inlet and exit of a turbine or compressor stage. It is assumed that the velocities are uniform in the inlet and exit cross section (one-dimensional approximation) and no allowance is made for the development of a boundary layer of secondary flows due to three-dimensional effects. Losses are taken into account by attributing their effect to changes in the uniform velocities and in their deviation from ideal velocities. The velocity diagrams are shown in the figure.



The following terminology is used: α is the angle of absolute velocities; β is the angle of relative velocities; suffix 1 denotes entering flow; suffix 2 denotes flow at exit; u is the peripheral velocity of buckets (blades); V are the absolute velocities; u are velocities relative to buckets (blades); suffix a denotes axial components of velocities; suffix c denotes circulating (tangential or whirl) components of velocities.

The force F in the direction of motion is then

$$\mathbf{F} = \dot{m}(\mathcal{V}_{1c} - \mathcal{V}_{2c}) = \dot{m}\mathcal{V}_w \quad (1)$$

where \dot{m} is the rate of mass flow, and $\mathcal{V}_w = \mathcal{V}_{1c} - \mathcal{V}_{2c} = u_{1c} - u_{2c}$ is the *whirl velocity*. The power of the stage is

$$\text{IP} = \dot{m}(u_1\mathcal{V}_{1c} - u_2\mathcal{V}_{2c}) \quad (2)$$

which is known as the **Euler (turbine) formula**. Here, in contrast with the figure and Equation (1) it has *not* been assumed that the blade velocity u is the same at inlet and exit.

VELOCITY, EFFECTIVE. The effective velocity at a point is the root mean square value of the instantaneous **velocity** over a complete cycle at that point. The unit is the centimeter per second.

VELOCITY LEVEL. The velocity level, in decibels, of a sound is 20 times the logarithm to the base 10 of the ratio of the particle velocity of the sound to the reference particle velocity (see **velocity, particle**). The reference particle velocity should be stated explicitly. In many sound fields the particle velocity ratios are not proportional to the square root of corresponding power ratios and hence cannot be expressed in decibels in the strict sense; however, it is common practice to extend the use of the decibel to these cases.

VELOCITY OF SLIP. See **rarefied gas dynamics**.

VELOCITY OF SOUND. The velocity with which the phase of a sound wave is propagated. In fluids, the velocity is given by $(\beta_a \rho)^{-1/2}$, where β_a is the adiabatic compressibility and ρ the density of the fluid. In gases at moderately low pressures, β_a can be taken as equal to $1/\gamma p$ where p is the pressure and γ is the ratio of the specific heat at constant pressure to that at constant volume. For dry air at moderate pressures, the velocity of sound in meters per second is given by the expression $331.45\sqrt{(T/273.16)}$, where T is the absolute temperature.

While only one type of wave can be propagated through a fluid, in solids a number of different types of elastic wave can be propagated (see **waves, elastic in solids**.) Thus, in an isotropic unbounded solid medium, longitudinal waves travel with the velocity $\sqrt{((K + \frac{4}{3}G)/\rho)}$ where K is the bulk modulus, G is the shear modulus and ρ is the density,

while transverse waves travel with the velocity $\sqrt{(G/\rho)}$. In bounded solids, the velocity of propagation depends in general on the wavelength, but where the wavelength is large compared with the cross section, dispersion can be neglected. Thus, for thin rods the velocity of longitudinal waves is $\sqrt{(E/\rho)}$, where E is Young's modulus, whilst for thin plates the velocity is $\sqrt{((E/\rho)(1 - \sigma^2))}$ where σ is **Poisson's ratio**. (See **wave propagation in rods**.)

VELOCITY, PARTICLE. In a sound wave, the velocity of a given infinitesimal part of the medium, with reference to the medium as a whole, due to the sound wave. The commonly used unit is the centimeter per second. The terms "instantaneous particle velocity," "effective particle velocity," "maximum particle velocity," and "peak particle velocity" have meanings which correspond with those of the related terms used for sound pressure.

VELOCITY POTENTIAL OF FLUID. See **potential flow; slender-body theory**.

VELOCITY POTENTIAL OF SOUND. A scalar point function whose **gradient** gives the particle velocity at any point (see **velocity, particle**). Some authors, in analogy with the electric scalar potential, define the velocity potential function so that its negative gradient gives the particle velocity at any point.

VELOCITY PROFILE. The profile obtained when velocity is plotted against distance across a fluid stream.

VELOCITY, TERMINAL (FREE FALL IN AIR). When a particle falls in air or some other fluid medium which resists its motion with a force varying as some power of the speed, analysis shows that it approaches a limiting speed which can be calculated by equating the magnitude of the resisting force to the force of gravity.

VENA CONTRACTA. The phenomenon whereby a non-turbulent jet of fluid issuing from an orifice or suitable nozzle, into a region of constant pressure, narrows as it emerges so that its sectional area becomes less than that of the orifice.

VERDE AND WICK, METHOD OF. In the theory of the space and energy distribution of neutrons slowing down from a source, this

method is one of the effective analytical methods in the case of an energy-dependent **mean free path** which approaches zero as the neutron energy approaches zero. It is based upon an analysis of the **Fourier-Laplace transforms** of the spherical harmonic moments of the neutron distribution, involving, in particular, the distribution of poles in the energy (Laplace) transform plane.

VERDET CONSTANT. A proportionality factor in an equation of the **Faraday effect**, the rotation of the plane of polarization of light by transparent substances in a magnetic field. In the relationship:

$$\alpha = \omega l H$$

α is the angle of rotation, l is the depth of the medium transversed by the light, H is the intensity of the magnetic field, and ω is the Verdet constant.

VERNAL EQUINOX. The line of intersection of the plane of the ecliptic and the plane of the equator intersects the celestial sphere in two points, the vernal equinox and the autumnal equinox. The vernal equinox is the zero point for the spherical coordinates of **right ascension** (measured along the celestial equator) and of celestial longitude (measured along the ecliptic). This point gets its name from the fact that the sun is close to the vernal equinox in the spring of the year.

VERSOR. A **unit vector**.

VERTEX. (1) An endpoint of an **edge**. Point, 0-cell and node are three other names also used in the literature. As a rule an isolated point is not considered a vertex. (2) The point common to the two straight lines forming an angle.

VERTEX, CUT. Let G be a connected separable graph. Then, by definition there exists at least one **subgraph** G_s which has only one vertex β_c in common with its complement; β_c is a cut vertex.

A necessary and sufficient condition for a graph G to be **nonseparable** is that it have no cut vertex.

VERTEX DEGREE. The degree of a **vertex** is the number of edges **incident** at the vertex.

VERTEX, FINAL. The **vertex** of the last **edge** of an **edge sequence** not shared with the previous edge.

VERTEX, INITIAL. The **vertex** of the first **edge** of an **edge sequence** which is not common to the second edge.

VERTEX, INTERNAL. A **vertex** of an **edge sequence** which is not terminal.

VERTEX MATRIX. See **matrix, vertex**.

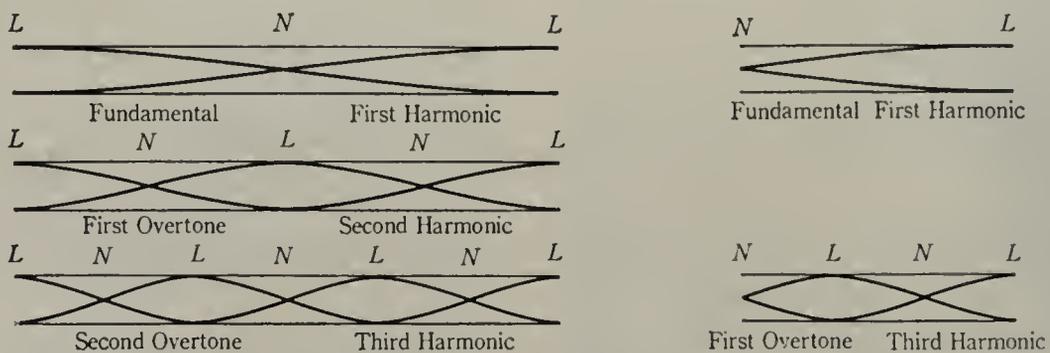
VERTEX, TERMINAL. The initial and final **edges** of an **edge sequence**.

VERTICAL CIRCLE. Any great circle on the celestial sphere that passes through the zenith and the nadir is known as a vertical circle. The vertical circle that passes through the poles of rotation is known as the *meridian*. The vertical circle that passes through the east and west points of the horizon is known as the *prime vertical*.

VERTICAL IONIZATION POTENTIAL. See **ionization potential**.

VERTICES, ADJACENT. Two vertices of a **graph** G are adjacent if they are endpoints of an **edge** in G .

VIBRATING AIR COLUMN. The fundamental resonant frequency f , in cycles per



Modes of vibration of the air column in a pipe open at both ends and in a pipe closed at one end and open at the other end. The velocity nodes and loops are indicated by N and L .

second, of a pipe, open at both ends, as in the figure, is

$$f = \frac{c}{\lambda} = \frac{c}{2l} \tag{1}$$

where l is the length of the pipe, in centimeters, c is the velocity of sound, in centimeters per second, and λ is the wavelength, in centimeters.

The overtones of an open pipe are harmonics of the fundamental. That is, $f_2 = 2f_1, f_3 = 3f_1, f_4 = 4f_1$, etc.

The fundamental resonant frequency f , in cycles per second, of a pipe closed at one end and open at the other end (see figure), is

$$f = \frac{c}{\lambda} = \frac{c}{4l} \tag{2}$$

The overtones of the pipe closed at one end are the odd harmonics. That is $f_2 = 3f_1, f_3 = 5f_1$, etc.

In the above examples the end connection has been omitted.

The added length at the open end has been shown to be $.82R$ where R is the radius of the pipe.

VIBRATING CIRCULAR MEMBRANE.

The fundamental frequency f_{01} , in cycles per second, of a circular stretched membrane is given by

$$f_{01} = \frac{.382}{R} \sqrt{\frac{T}{m}} \tag{1}$$

where m is the mass, in grams per square centimeter of area, R is the radius of the membrane, in centimeters, and T is the tension, in dynes per centimeter.

The fundamental vibration is with the circumference as a node and a maximum displacement at the center of the circle (Figure 1A). The frequencies of the next two overtones with nodal circles are

$$f_{02} = 2.30f_{01} \tag{2}$$

$$f_{03} = 3.60f_{01} \tag{3}$$

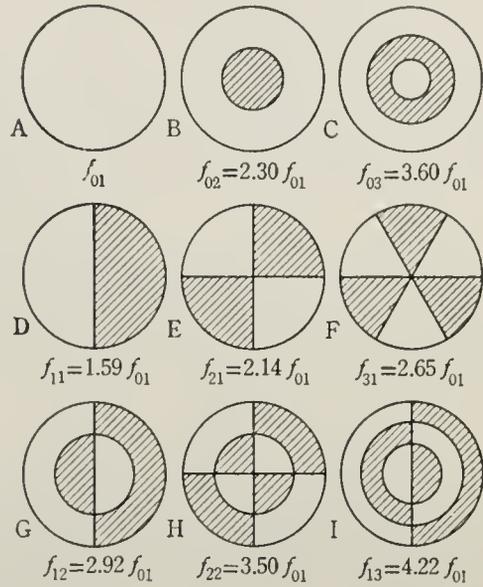
and are shown in Figures 1B and 1C. The frequencies of the first, second and third overtones with nodal diameters are

$$f_{11} = 1.59f_{01} \tag{4}$$

$$f_{21} = 2.14f_{01} \tag{5}$$

$$f_{31} = 2.65f_{01}. \tag{6}$$

These nodes are shown in Figures 1D, 1E, and 1F. Following these simpler forms of vibration are combinations of nodal circles and nodal



Modes of vibration of a stretched circular membrane. Shaded segments are displaced in opposite phase to unshaded.

diameters. The frequency of one nodal circle and one nodal diameter, Figure 1G is

$$f_{12} = 2.92f_{01}. \tag{7}$$

The frequency of one nodal circle and two nodal diameters, Figure 1H, is

$$f_{22} = 3.50f. \tag{8}$$

The frequency of two nodal circles and one nodal diameter, Figure 1I, is

$$f_{13} = 4.22f_{01}. \tag{9}$$

VIBRATING CIRCULAR PLATE CLAMPED AT THE EDGE.

A circular clamped plate is shown in Figure 1. The

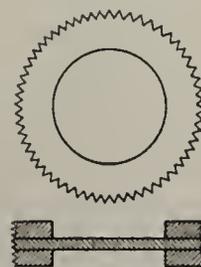


Fig. 1. A circular plate clamped at the edge.

fundamental frequency f_{01} , in cycles per second, is given by

$$f_{01} = \frac{.467t}{R^2} \sqrt{\frac{E}{\rho(1 - \sigma^2)}} \tag{1}$$

where t is the thickness of the plate, in centimeters, R is the radius of the plate up to the clamping boundary, in centimeters, ρ is the density, in grams per cubic centimeter, σ is the Poisson's ratio, and E is Young's modulus, in dynes per square centimeter.

The fundamental frequency is with the circumference as a node and a maximum displacement at the center (Figure 2A).

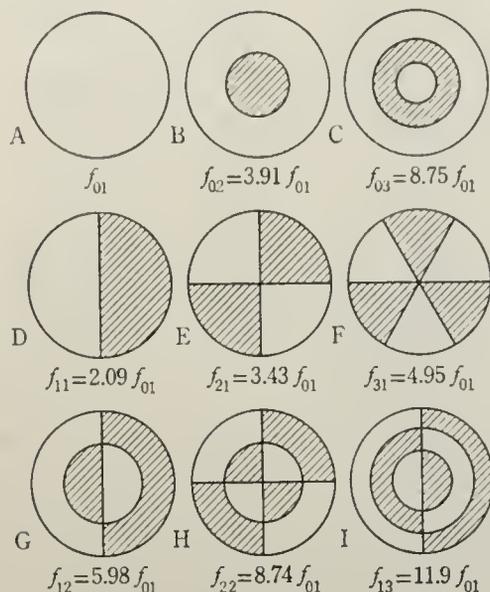


Fig. 2. Modes of vibration of a clamped circular plate. Shaded segments are displaced in opposite phase to unshaded.

The frequency of the next two overtones with nodal circles, Figures 2B and 2C, are,

$$f_{02} = 3.91f_{01} \tag{2}$$

$$f_{03} = 8.75f_{01}. \tag{3}$$

The frequencies of the first, second, and third overtones with nodal diameters are

$$f_{11} = 2.09f_{01} \tag{4}$$

$$f_{21} = 3.43f_{01} \tag{5}$$

$$f_{31} = 4.95f_{01}. \tag{6}$$

These nodes are shown in Figures 2D, 2E, and 2F.

Following these simpler forms of vibration are combinations of nodal circles and nodal diameters. The frequency of one nodal circle and one nodal diameter, Figure 2G, is

$$f_{12} = 5.98f_{01}. \tag{7}$$

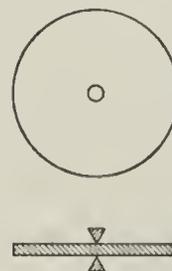
The frequency of one nodal circle and two nodal diameters, Figure 2H, is

$$f_{22} = 8.74f_{01}. \tag{8}$$

The frequency of two nodal circles and one nodal diameter, Figure 2I, is

$$f_{13} = 11.9f_{01}. \tag{9}$$

VIBRATING CIRCULAR PLATE SUPPORTED AT THE CENTER. A circular plate under no tension, uniform in cross section, edges perfectly free and supported at the



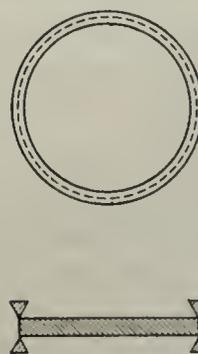
A circular plate supported at the center.

center is shown in the figure. The frequency f , in cycles per second, for the umbrella mode, is

$$f = \frac{.172t}{R^2} \sqrt{\frac{E}{\rho(1 - \sigma^2)}}$$

where t is the thickness of the plate, in centimeters, R is the radius of the plate, in centimeters, ρ is the density, in grams per cubic centimeter, σ is Poisson's ratio, and E is Young's modulus, in dynes per square centimeter.

VIBRATING CIRCULAR PLATE SUPPORTED AT THE PERIPHERY. A plate under no tension, uniform in cross section,



A circular plate supported at the periphery.

edges simply supported at the periphery is shown in the figure. The fundamental frequency is

$$f = \frac{.233t}{R^2} \sqrt{\frac{E}{\rho(1 - \sigma^2)}}$$

where t is the thickness of the plate, in centimeters, R is the radius of the plate, in centi-

meters, ρ is the density, in grams per cubic centimeter, σ is Poisson's ratio, and E is Young's modulus, in dynes per square centimeter.

VIBRATING FREE CIRCULAR PLATE.

A circular plate under no tension, uniform in cross section and perfectly free is shown in Figure 1. For a vibration with nodal circle,

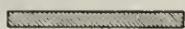
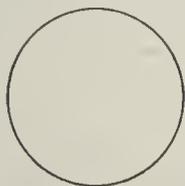
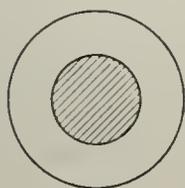


Fig. 1. A free circular plate.

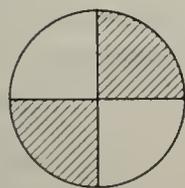
as depicted in Figure 2A, the frequency f , in cycles per second, is

$$f = \frac{.412t}{R^2} \sqrt{\frac{E}{\rho(1 - \sigma^2)}} \quad (1)$$

where t is the thickness of the plate, in centimeters, R is the radius of the plate, in centimeters, ρ is the density, in grams per cubic cen-



A



B

Fig. 2. Modes of vibration of the free circular plate.

timeter, σ is Poisson's ratio, and E is Young's modulus, in dynes per square centimeter.

For a vibration with two nodal diameters, as depicted in Figure 2B, the frequency is

$$f = \frac{.193t}{R^2} \sqrt{\frac{E}{\rho(1 - \sigma^2)}} \quad (2)$$

VIBRATING RECTANGULAR MEMBRANE. The fundamental frequency f , in cycles per second, of a rectangular stretched

membrane with the sides in the ratio of 1 to 2 is given by

$$f = \frac{.792}{\sqrt{ab}} \sqrt{\frac{T}{m}}$$

where m is the mass, in grams per square centimeter, $a = 2b$, the length of the long side, in centimeters, b is the length of the short side in centimeters, and T is the tension, in dynes per centimeter.

VIBRATING SQUARE MEMBRANE. The fundamental frequency f , in cycles per second, of a square stretched membrane is given by

$$f = \frac{.705}{a} \sqrt{\frac{T}{m}}$$

where m is the mass, in grams per square centimeter of area, a is the length of a side, in centimeters, and T is the tension, in dynes per centimeter.

VIBRATION. Vibration commonly refers to a to-and-fro motion; its meaning is often broadened to include any periodic physical process, such, for example, as a cyclic variation in electric or magnetic field intensity. When an elastic body is deformed and released, it is in general set into oscillation such that the displacement of any particle from its equilibrium position is a more or less complicated harmonic function of the time. The vibration may or may not be symmetrical with respect to the neutral position; in any case the maximum displacement is called the amplitude of the vibration. By analogy, the same terms and the same analysis are applied to vibrations of any type.

VIBRATIONAL EIGENFUNCTIONS. See molecular eigenfunctions.

VIBRATIONAL ENERGY LEVELS OF A MOLECULE. The vibrational energy levels of a diatomic molecule can be represented by the formula

$$G(v) = \omega_e(v + \frac{1}{2}) - \omega_e x_e(v + \frac{1}{2})^2 + \omega_e y_e(v + \frac{1}{2})^3 + \omega_e z_e(v + \frac{1}{2})^4 + \dots$$

where v is the vibrational quantum number which assumes the values 0, 1, 2, \dots , and where ω_e is, apart from a factor c , the vibrational frequency for infinitesimal amplitude. One has in general $\omega_e z_e \ll \omega_e y_e \ll \omega_e x_e \ll \omega_e$ and frequently

$\omega_e z_e \approx 0$ and $\omega_e y_e \approx 0$. The frequency ω_e is related to the force constant k_e in the equilibrium position by the relation

$$\omega_e = \frac{\nu_{\text{osc}}}{c} = \frac{1}{2\pi c} \sqrt{\frac{k_e}{\mu}}$$

or

$$\begin{aligned} k_e &= 4\pi^2 \mu c^2 \omega_e^2 \\ &= 5.8883 \times 10^{-2} \mu_A \omega_e^2 \text{ dyne/cm} \end{aligned}$$

where μ and $\mu_A (= \mu N_A)$ stand for the **reduced mass** in grams and in atomic weight units ($0^{16} = 16$), respectively.

The zero-point vibrational energy of a diatomic molecule is

$$G(0) = \frac{1}{2}\omega_e - \frac{1}{4}\omega_e x_e + \frac{1}{8}\omega_e y_e + \frac{1}{16}\omega_e z_e + \dots$$

For polyatomic molecules in the approximation in which the **normal vibrations** are well defined, that is, when the potential energy contains only quadratic terms, the vibrational energy is simply

$$G(v_1, v_2, v_3, \dots) = \sum \omega_i \left(v_i + \frac{d_i}{2} \right)$$

where v_i and d_i are the vibrational quantum number and the degree of degeneracy of the i th normal vibration and $\omega_i = \nu_i/c$. If the potential energy contains higher powers, that is

$$\begin{aligned} 2V &= \sum_i \sum_j k_{ij} q_i q_j + \sum \sum \sum f_{ijk} q_i q_j q_k \\ &\quad + \sum \sum \sum \sum g_{ijkl} q_i q_j q_k q_l + \dots \end{aligned}$$

the vibrational energy becomes

$$\begin{aligned} G(v_1, v_2, \dots) &= \sum_i \omega_i \left(v_i + \frac{d_i}{2} \right) \\ &\quad + \sum_i \sum_{k \geq i} x_{ik} \left(v_i + \frac{d_i}{2} \right) \left(v_k + \frac{d_k}{2} \right) \\ &\quad + \sum_i \sum_{k \geq i} g_{ikl} l_k + \dots \end{aligned}$$

Here the anharmonicity constants x_{ik} are small compared to the ω_i if the deviations from a quadratic potential are small; the ω_i are now the (classical) vibrational frequencies in cm^{-1} for infinitesimal amplitudes (so-called zero-order frequencies). The l_i are integral numbers which assume the values

$$l_i = v_i, \quad v_i - 2, \quad v_i - 4, \quad \dots \quad 1 \text{ or } 0$$

and the g_{ik} are constants of the order of the x_{ik} (not to be confused with the potential constants g_{ijkl}).

The zero-point energy is

$$G(0, 0, \dots) = \sum_i \omega_i \frac{d_i}{2} + \sum_i \sum_{k \geq i} x_{ik} \frac{d_i d_k}{4} \dots$$

VIBRATIONAL PARTITION FUNCTION.

The contribution to the total **partition function** of molecules associated with their vibrational energy.

VIBRATIONAL SPECTRA OF MOLECULES.

Vibrational spectra correspond to transitions between two vibrational levels belonging to the same electronic state of a non-rotating molecule.

(a) *Diatomic Molecules.* An infrared vibration spectrum can occur only if the molecule has no center of symmetry, that is, if it does not consist of two like nuclei. A vibrational Raman spectrum occurs for both symmetrical and asymmetrical molecules. For the harmonic oscillator the selection rule for the vibrational quantum number is (both in the Raman spectrum and the infrared)

$$\Delta v = \pm 1$$

For the anharmonic oscillator no strict selection rule exists, but transitions with $\Delta v = 1$ are much stronger than those with $\Delta v = 2$, those with $\Delta v = 2$ much stronger than those with $\Delta v = 3$, and so on.

Absorption of light by the molecule in the ground state produces a series of bands whose wave numbers correspond to the energies of successive vibrational levels

$$\begin{aligned} \nu_{\text{abs}} &= G(v) - G(0) \\ &= \omega_e \left[\left(v + \frac{1}{2} \right) - \frac{1}{2} \right] - \omega_e x_e \left[\left(v + \frac{1}{2} \right)^2 - \frac{1}{4} \right] \\ &\quad + \omega_e y_e \left[\left(v + \frac{1}{2} \right)^3 - \frac{1}{8} \right] + \dots \\ &= G_0(v) = \omega_0 v - \omega_0 x_0 v^2 + \omega_0 y_0 v^3 + \dots \end{aligned}$$

The same formula holds for the displacements observed in the Raman spectrum.

(b) *Polyatomic Molecules.* In the harmonic oscillator approximation, the only allowed transitions are those in which one vibration changes its vibrational quantum by one unit, i.e.,

$$\Delta v_i = \pm 1, \quad \Delta v_k = 0$$

If the anharmonicity of the vibrations is taken into account, also transitions in which v_i changes by several units or in which several v_i change

will occur. But they are in general less intense than the fundamentals.

If the molecule has symmetry, certain rigorous selection rules for vibrational transitions hold irrespective of the degree of anharmonicity. Quite generally a vibrational transition $v' \longleftrightarrow v''$ is allowed in the infrared when there is at least one component of the dipole moment M that has the same species (i.e., the same behavior with respect to the symmetry operations permitted by the symmetry of the molecule) as the product $\psi'_{v'}\psi''_{v''}$. A vibrational transition $v' \longleftrightarrow v''$ is allowed in the Raman effect if at least one component of the polarizability tensor has the same species as the product $\psi'_{v'}\psi''_{v''}$.

As a result, for example, for molecules with a center of symmetry, transitions that are allowed in the infrared are forbidden in the Raman spectrum, and those allowed in the Raman spectrum are forbidden in the infrared.

A table of the species of the components of the dipole moment and of the polarizability of the more important point groups are given in G. Herzberg, *Molecular Spectra and Molecular Structure, II. Infrared and Raman Spectra of Polyatomic Molecules*, D. Van Nostrand Co., Inc., Princeton, 1945.

For molecules for which the inversion doubling is not negligible the additional selection rule has to be taken into account that in the infrared only sublevels of opposite parity can combine with one another ($+ \longleftrightarrow -$), whereas in the Raman effect only sublevels of the same parity can combine with one another ($+ \longleftrightarrow +$, $- \longleftrightarrow -$).

VIBRATIONAL SUM RULE FOR ELECTRONIC TRANSITIONS. In a diatomic molecule the sums of the band strengths (emission intensity divided by ν^4 or absorption intensity divided by ν) of all bands with the same upper or the same lower state are proportional to the number of molecules in the upper or lower state, respectively, i.e.

$$\sum_{v''} \frac{I_{em. v'v''}}{\nu^4} \propto N_{v'}$$

and

$$\sum_{v'} \frac{I_{abs. v'v''}}{\nu} \propto N_{v''}$$

Here v' and v'' stand for the vibrational quantum numbers of the upper and lower state, respectively. (The vibrational sum rule is valid only if the electronic transition moment R_e is a

constant for all vibrational transitions giving an appreciable contribution.)

VIBRATION, COMBINATION. See combination vibration.

VIBRATION, NORMAL MODE. See normal modes.

VIBRATION-ROTATION SPECTRA OF MOLECULES. See rotation-vibration spectra of molecules.

VIBRATIONS, DAMPED. A damped vibration is the stable motion of a vibrating particle under the influence of a restoring force (see vibrations, free) and a resistive or frictional force. This frictional force is opposed to the direction of the velocity. For small velocities it may often be taken as proportional to the first power of the velocity, just as the spring force is taken as proportional to displacement from equilibrium. In this case the equation for one-dimensional motion in the x -direction is

$$m \frac{d^2x}{dt^2} = -kx - R \frac{dx}{dt},$$

where R is the friction constant. The resulting motion is oscillatory with gradually diminishing amplitude (= under-critically damped motion). If the friction force is very large the motion becomes over-critically damped. In this case the displaced particle will gradually creep back to its neutral position, but not vibrate.

VIBRATIONS, FORCED. The motion of a physical system under the combined influence of its restoring force, resistive force and an applied external force. The most common situation is that of an applied periodic force. If the resistive force is proportional to the velocity, and the spring force to the displacement, the one-dimensional equation of motion is

$$m \frac{d^2x}{dt^2} = -kx - R \frac{dx}{dt} + F \sin \omega t,$$

where F is the amplitude and ω is the frequency of the applied force. The resulting motion is the superposition of the damped vibration (transient vibration) and the steady-state or stationary motion due to the applied force $F \sin \omega t$. The steady-state motion realizes its greatest amplitude when the applied

frequency coincides with the natural frequency (resonance). (See also **vibrations, free** and **vibrations, damped**.)

VIBRATIONS, FREE, OF UNDAMPED SYSTEM. The to-and-fro-motion of the constitutive part of a physical system. More specifically, the resulting periodic motion under the influence of a restoring force which in the linear case is proportional to the displacement from the neutral or equilibrium position. In the case of a single mass point m , and restoring force kx , the equation of motion of the free vibration is

$$m \frac{d^2x}{dt^2} = -kx$$

where k is the so-called spring constant and x is the displacement from equilibrium. The resulting motion is sinusoidal. The angular frequency is $\omega_0 = \sqrt{\frac{k}{m}}$ (also called the resonance,

or natural, frequency of the system). A typical example is the motion of a weight suspended from a spring with very small damping.

VIBRATIONS, NON-PERIODIC. A vibration is non-periodic if it does not repeat itself in all details after a certain period of time, T . Strictly speaking, a damped vibration is non-periodic. Also an undamped system may vibrate non-periodically if it consists of several degrees of freedom, where different periods have an irrational ratio to each other. (See also **vibrations, free**; **vibrations, damped** and **vibrations, forced**.)

VIBRATIONS OF POLYATOMIC MOLECULES, NORMAL. See **normal vibrations** and **normal coordinates in polyatomic molecules**.

VIBRATION, SPECIES OF. See **species**.

VIERENDEEL TRUSS OR GIRDER. An open web structure which acts as a continuous or **rigid frame** with vertical web members and horizontal or inclined chord members. The applied forces are carried by a combination of direct stress and bending in the members, with the bending stress as well as the direct stress an essential or primary stress. (See **rigid frame**.)

VIETH-MÜLLER CIRCLE (CIRCUMHOPTER). A circle in the horizontal plane

and passing through the nodal points of the eyes of an observer. The points of a Vieth-Müller circle are apparently equidistant from the observer and so are apparent circles centered on the egocenter in the **Luneburg geometry** of binocular vision.

VIETH-MÜLLER TORUS. If a **Vieth-Müller circle** rotates about the line joining the nodal points of the eyes, a Vieth-Müller torus is generated. This is an apparently equidistant surface in the **Luneburg geometry** of binocular vision. The angle of convergence is constant for points on the Vieth-Müller torus.

VIGNETTING. The gradual diminishing of the bundle of rays, coming from an object point and allowed to pass through an axially symmetrical optical system, as the point moves off-axis.

VIGNETTING DIAGRAM. The intersection with the **entrance pupil** of the maximal bundle of rays that pass through an optical system and an object point P is the vignetting diagram of P in the system.

VIRIAL COEFFICIENTS. See **virial equation of state**.

VIRIAL EQUATION OF STATE. By the use of the virial equation of state it is possible adequately to represent the p - v - T relation by adopting a power series for the product pv in terms of either volume or pressure. Consequently, two alternative forms of the virial equation of state are in existence:

$$pv = A \left(1 + \frac{B}{v} + \frac{C}{v^2} + \frac{D}{v^3} + \dots \right) \quad (1)$$

$$pv = \bar{A} + \bar{B}p + \bar{C}p^2 + \bar{D}p^3 + \dots \quad (2)$$

The coefficients $A, B, \dots, \bar{A}, \bar{B}, \dots$ are all functions of temperature only; they are called **virial coefficients**, A, \bar{A} being the first virial coefficient, B, \bar{B} being the second virial coefficient, etc. It is clear that for $p \rightarrow 0, v \rightarrow \infty$, either equation must reduce to the **perfect gas law** $pV = RT$, hence

$$\bar{A} = A = RT. \quad (3)$$

It is clear that the corresponding virial coefficients in (1) and (2) must be related. In fact, we have

$$\bar{B} = B \quad (4)$$

$$\bar{C} = \frac{C - B^2}{A} \quad \text{or} \quad C = A\bar{C} + \bar{B}^2, \text{ etc.} \quad (5)$$

VIRIAL OF A SYSTEM. If in a system of n particles confined to a finite region of space whose position vectors with respect to a given origin are \mathbf{r}_i the resultant force on the i th particle is \mathbf{F}_i , the virial of the system is defined to be

$$-\frac{1}{2} \overline{\sum_{i=1}^n \mathbf{r}_i \cdot \mathbf{F}_i}$$

The bar over the sum refers to a time average over a time interval long compared with the time taken by a particle to traverse the region in which the particles are confined. It is understood that not only are the magnitudes of the position vectors \mathbf{r}_i bounded, but that the same is true of the velocities of the particles. According to the virial theorem the virial of the system is equal to the average kinetic energy of the system. (See also **energy of n -particle system**.)

VIRTUAL DISPLACEMENT. See **displacement, virtual**.

VIRTUAL MASS (OR APPARENT ADDITIONAL MASS). The force (due to the inertia of the fluid) required to produce unit acceleration in a body immersed in a fluid, in addition to that due to the inertia of the body.

The virtual mass of a sphere immersed in inviscid fluid is half the mass of the fluid displaced.

VIRTUAL PROCESS. Process which may be pictured as the emission of a particle or quantum followed so quickly by its absorption or further interaction that the energy and momentum of the particle in this intermediate state are ill-defined. Each such process corresponds to a term in the series expansion of the **Hamiltonian** for the whole process and may be represented on a **Feynman diagram**.

VIRTUAL QUANTUM. In second and higher order **perturbation theory**, a **matrix element** connecting an initial state with a final state involves intermediate states in which energy is not conserved. A quantum or photon in such an intermediate state is termed a virtual quantum. Thus the self-energy of an elec-

tron (due to the electromagnetic field) is pictured as arising from the continual emission and reabsorption by the electron of virtual quanta. Similarly the **Coulomb energy** between two electrons may be pictured as arising from the emission of virtual quanta by one of the electrons and their absorption by the other.

VIRTUAL STATE. An intermediate state in a quantum mechanical process which enters only into the calculations.

VIRTUAL TEMPERATURE. The temperature T_v of dry air having the same density as the given sample whose dry bulb temperature is T . The difference $T_v - T$ is a measure of the buoyancy (excess temperature) given to air by mixing it with water vapor whose relative density ϵ is 0.6221.

$$T_v = T \frac{1 + x/\epsilon}{1 + x}$$

where x is the **humidity mixing ratio**, and the temperatures are absolute. For temperatures in the neighborhood of 0°C an approximate formula is

$$T_v - T = Tx(1/\epsilon - 1) \approx \frac{1}{8}x^\circ\text{C}.$$

if x is measured in gm of water vapor per kg of dry air.

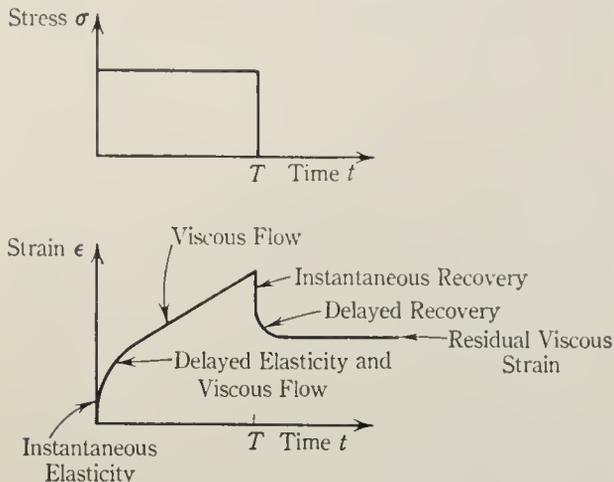
VIRTUAL WORK, PRINCIPLE OF. In the mechanics of solids undergoing continuous deformations

$$\int_A T'_i u_i^* dA + \int_V F'_i u_i^* dV = \int_V \sigma'_{ij} \epsilon_{ij}^* dV$$

where the surface tractions T'_i , the body forces F'_i and the stresses σ'_{ij} form an equilibrium system. The displacement u_i^* and the strains ϵ_{ij}^* are compatible, but need bear no relation to the equilibrium system. The principle is the starting point for the derivation of the basic theorems of structures and mechanics of solids.

VISCOELASTICITY. Mechanical behavior of material which exhibits viscous and delayed elastic response to stress in addition to instantaneous elasticity. Such properties can be considered to be associated with rate effects—time derivatives of arbitrary order of both stress and strain appearing in the constitutive equation—or hereditary or memory influences which include the history of the

stress and strain variation from the undisturbed state. The typical response of a viscoelastic material to a pulse of constant stress is shown in the figure.



(See viscoelastic models; Maxwell material; Kelvin material; standard linear solid; relaxation time; viscoelastic operator.)

VISCOELASTICITY, LINEAR. Viscoelasticity in which linear operators relate stress and strain components. (See viscoelastic operator.)

VISCOELASTIC MODELS. The relation between the variation of a stress component, σ , and the corresponding strain component, ϵ , produced in a viscoelastic material can sometimes be represented by a mechanical analogue of the relation between force and extension for a network or linkage of springs and dash pots. These are termed viscoelastic models, and are often convenient in visualizing the response of such materials. Common simple models are designated by the nomenclature: Maxwell, Kelvin or Voigt, standard linear solid. Figure 1 shows the simplest

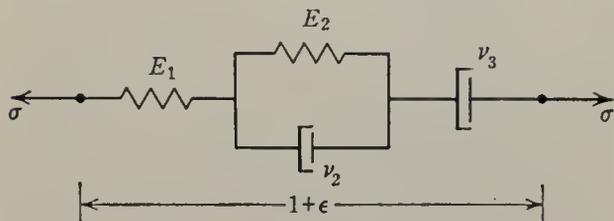


Fig. 1.

model which exhibits a combination of all three types of viscoelastic response: instantaneous elasticity, delayed elasticity and viscous flow. This is commonly termed the four-element model. For appropriate choice of element con-

stants, the second model (Figure 2) shown can correspond to identical material behavior.

In order to represent more complex material behavior, friction elements and inertia ele-

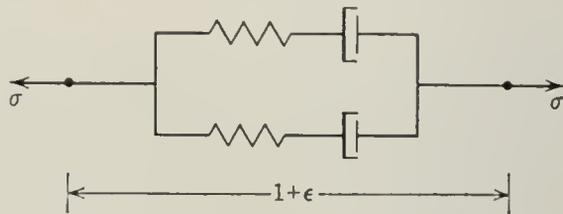


Fig. 2.

ments are sometimes included in addition to the springs and dashpots.

VISCOELASTIC OPERATOR. An operator which occurs in the stress-strain relation for a viscoelastic material. Depending on the mathematical form of relation adopted, this can be a differential or integral operator, or a complex function of frequency. Different operator forms can be chosen to represent the same material. Two possible integral operator forms have been given under compliance, creep and modulus, relaxation. The usual differential operator form is of the type:

$$\sum_0^p p_v \left(\frac{\partial^v}{\partial t^v} \right) \sigma = \sum_0^q q_v \left(\frac{\partial^v}{\partial t^v} \right) \epsilon$$

where p_v and q_v are material constants. The Maxwell material and standard linear solid expressed in this way involve first order operators as exhibited under these headings.

VISCOSITY. The property whereby a strain of a fluid produces a stress while the strain is taking place.

The coefficient of viscosity, or kinematic viscosity, μ , is the tangential (or shearing) stress per unit area in a fluid undergoing unit shear. Thus if the velocity is of magnitude u and parallel to the x -axis, and varies only in the y direction, the stress, denoted by τ_{xy} or p_{xy} , is

$$\tau_{xy} = \mu \frac{\partial u}{\partial y} \quad \text{or} \quad \tau_{ij} = \mu \frac{\partial u_i}{\partial x_j}$$

according to the notation used.

In an isotropic fluid

$$\tau_{xy} = \tau_{yx}.$$

The dynamic viscosity, ν , is defined by

$$\nu = \mu/\rho.$$

ν is the coefficient of conduction of vorticity in a fluid.

The cgs unit of viscosity is sometimes called the poise (after Poiseuille).

VISCOSITY, DYNAMIC. (1) See **viscosity**. (2) An equivalent viscosity pertaining to general viscoelastic behavior, and is given by the **complex modulus** divided by the frequency: m_2/ω .

VISCOSITY, KINETIC THEORY OF. In a gas in which there exists a velocity gradient, there will be a shearing force exerted by the faster particles upon the slower ones. As stated under **viscosity**, this force is

$$\mu \frac{\partial u}{\partial y},$$

where μ is the coefficient of viscosity (the notation η for this coefficient, and $-\eta \partial u/\partial y$ for the force, is also used) and

$$\frac{\partial u}{\partial y}$$

is the gradient (in the y -direction) of the velocity (in the x -direction), due to a force in the x -direction perpendicular to the y -direction.

Considering the force to equal the rate of change of momentum, which in the present case is equivalent to the net momentum (in the x -direction) transported in the y -direction through a unit area, one finds

$$\mu = c\rho\bar{u}\lambda,$$

where c is a numerical constant of order unity, ρ is the density, \bar{u} is the mean velocity, and λ is the mean free path.

VISCOSITY, NEWTONIAN. See **Newtonian viscosity**.

VISCOUS DAMPING. See **damping, viscous**.

VISCOUS FLUID. A fluid in which internal stresses are produced by a straining motion: the stresses are usually assumed to be proportional to the rate of strain.

VISIBILITY FACTOR. For radiation of a given wavelength, the ratio of the **luminous flux** at that wavelength to the corresponding **radiant flux**.

VISIBILITY OF FRINGES. The visibility of fringes is defined as

$$\frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

where I_{\max} is the maximum intensity of the fringe system and I_{\min} is the minimum intensity.

VISUAL FIELD. The visual field is all points of space at which an object can be perceived when the head and eye are kept fixed. The field may be monocular or binocular.

VISUAL SPACE. See **Lunenburg geometry**.

VOIGT MATERIAL. Synonymous with **Kelvin material**.

VOIGHT MODEL. Synonymous with **Kelvin model**.

VOLT. A unit of electrical potential difference, abbreviation V or v. (1) The absolute volt is the steady potential difference which must exist across a conductor which carries a steady current of one absolute ampere and which dissipates thermal energy at the rate of one watt. The absolute volt has been the legal standard of potential difference since 1950. (2) The International volt, the legal standard prior to 1950, is the steady potential difference which must be maintained across a conductor which has a resistance of one International ohm and which carries a steady current of one International ampere.

$$1 \text{ Int. volt} = 1.000330 \text{ Abs. volts.}$$

(See **electrical units**.)

VOLTAGE SOURCE, IDEAL. An energy source that provides at a given terminal-pair (port), a voltage function that is independent of the current at that terminal-pair (port). An ideal voltage source is idle when its environment is an open circuit.

VOLTERRA EQUATION. The **integral equation** of the form

$$\phi(x) = f(x) - \lambda \int_{x_0}^x K(x,y)\phi(y)dy$$

in which the upper limit of integration is the variable x . If $f(x) = 0$, the equation is **homogeneous**. When one or both limits become infinite, or when the kernel becomes infinite at one or points within the range x_0 to x , the equation is **singular**.

VOLUME, CENTER OF. See **centroid** (of a geometrical figure).

VOLUME FRACTIONS. Consider a binary mixture. If ρ denotes the ratio of the volume of molecules of type B to that of type A , the volume fractions of molecules A and B are defined by

$$\phi_A = \frac{x_A}{x_A + \rho x_B} \quad \phi_B = \frac{x_B}{x_A + \rho x_B} \quad (1)$$

where x_A, x_B are the **mole fractions**.

VOLUME INTEGRAL. See **integral, multiple**.

VOLUME RECOMBINATION RATE. See **recombination rate, volume**.

VOLUME VELOCITY. The rate of flow of the medium through a specified area due to a sound wave. The terms "instantaneous volume velocity," "effective volume velocity," "maximum volume velocity," and "peak volume velocity" have meanings which correspond with those of the related terms used for **sound pressure**.

VON NEUMANN MATRIX. See **density matrix**.

VORTEX. A vortex tube, or vortex, is the region bounded by the **vortex lines** which intersect a closed curve. The strength of a vortex is the same at any section, and is equal to the circulation round the boundary of the section of it by any surface which completely traverses it.

VORTEX LINE. A line whose tangent at every point is in the direction of the **vorticity**, given by

$$\frac{dx}{\xi} = \frac{dy}{\eta} = \frac{dz}{\zeta},$$

where (ξ, η, ζ) is the vorticity.

Vortex lines are either closed curves or begin and end on the boundaries of the fluid (or in regions of infinite vorticity such as vortex sheets and line vortices).

In an inviscid fluid of uniform density the vortex lines are advected with the fluid and are composed always of the same fluid particles. This is also true of an inviscid "well stirred" fluid (i.e., one in which the density is a unique function of the pressure only).

VORTEX PAIR. Two parallel line vortices of opposite sign which travel through the surrounding fluid in the direction of the motion between them. The accompanying fluid is a cylinder of oval section. If the vortices are of strength $\pm\kappa$, their velocity of translation is $\kappa/2\pi d$, where d is the distance between them.

VORTEX RING. A field of motion produced by a closed circular **vortex tube**. The tube travels through the surrounding fluid in the direction of the velocity on its axis. The *substance* of a vortex ring is the fluid which possesses vorticity. The surrounding motion is irrotational. The substance is surrounded by the *accompanying fluid* which is composed of particles which repeatedly circulate round the substance and therefore travel along with it. The flow round the accompanying fluid is the same as the irrotational flow round a solid body of the same shape. If one vortex ring is contained within the accompanying fluid of another, the two rings continue to pass through one another, otherwise one passes through the other only once.

VORTEX SHEET. A surface of discontinuity of velocity, at which the **vorticity** is infinite. A vortex sheet is either unstable and becomes rolled up into a number of eddies each rather like a line vortex, or is diffused by viscosity and becomes a thick layer of large vorticity.

VORTEX, STARTING. See **airfoil started from rest**.

VORTEX STREET. Two parallel rows of equally spaced line vortices. If the vortices of one row are opposite the points midway between those of the other it is called a Kármán vortex street. If the strength of each vortex is κ in one row and $-\kappa$ in the other, the spacing being a , and b being the distance between the rows, the vortices move in a direction parallel to the street with speed U_s given by

$$U_s = \frac{\kappa}{2a} \tanh \frac{\pi b}{a}.$$

Vortex streets are found in the wake of a cylinder at moderate **Reynolds numbers**. According to Lamb the configuration is stable for small sinusoidal disturbances only if

$$\cosh^2 \frac{\pi b}{a} = 2, \quad \text{or} \quad b = 0.281a.$$

The street forms first as an instability in the boundary of the wake, but at higher Reynolds numbers is produced by the shedding of vortices alternately from each side of the cylinder. This produces a periodic sideways force on the cylinder which is probably responsible for the oscillation of wires in a wind.

VORTEX TUBE. See **vortex**.

VORTICITY. The **vector** obtained by taking the **curl** of the flow velocity. It is also the antisymmetric part of the velocity gradient tensor $\partial u_i / \partial x_j$ (u_i is the component of the velocity parallel to Ox_i). It measures the rate of rotation of a fluid, e.g., in a uniformly rotating fluid the vorticity is twice the angular velocity. In rectangular cartesian coordinates it is given by

$$\begin{aligned} \nabla \times \mathbf{v} &= \text{curl } \mathbf{v} = \boldsymbol{\omega} = (\xi, \eta, \zeta) \\ &= \left(\frac{\partial w}{\partial y} - \frac{\partial v}{\partial z}, \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}, \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right), \end{aligned}$$

or

$$\omega_k = \epsilon_{ijk} \frac{\partial u_i}{\partial x_j}.$$

If a small fluid element were suddenly "frozen" so that the distortion by the motion were eliminated, its angular momentum per unit mass would be $\frac{1}{2}\boldsymbol{\omega}$.

The vorticity of a fluid element is changed according to the equation

$$\begin{aligned} \frac{D\boldsymbol{\omega}}{Dt} &= (\boldsymbol{\omega} \cdot \text{grad})\mathbf{v} + \text{grad } \log \rho \\ &\quad \times \left(\mathbf{F} + \mathbf{g} - \frac{D\mathbf{v}}{Dt} \right) + \nu \nabla^2 \boldsymbol{\omega}. \end{aligned}$$

The right hand side shows that vorticity is generated according to the first term, by the turning and stretching of the vortex tubes, and according to the second when the direction of the density gradient is different from the resultant of the inertia forces $\left(\frac{D\mathbf{v}}{Dt}\right)$, gravity (\mathbf{g}), and other body forces (\mathbf{F}), the rotation produced being such as to make these directions coincide. According to the third term the vorticity is diffused, or conducted, with ν , the dynamic viscosity, being the coefficient of conductivity.

VOUSSOIR ARCH. An **arch** composed of uncemented blocks with joints or interfaces normal to the axis of the arch. As such an arch lacks ability to take tension at the joints, the **pressure line** must lie within the arch at the joints. The common assumption in design is that the pressure line must lie within the middle third or **kern** so that no tension is produced in the blocks.

W

WAGNER BEAM. A beam in which a **tension field** is set up in the web at the working load.

WAGNER EFFECT. See **airfoil started from rest**.

WAGNER STRESS. See **stress, Wagner**.

WAIDNER - BURGESS STANDARD. A standard of **luminous intensity** designed to supplant the Bureau of Standards **candle**. Defined as the luminous intensity of 1 cm² of a **black body** at the melting point of platinum.

WAKE. The region of disturbed fluid behind a moving body. Usually defined as the region containing **vorticity**, generally turbulent, or the region bounded by a **vortex sheet** originating at the body. The wake may contain eddies moving or stationary relative to the body, or a region of motionless fluid as supposed in the theory of free streamlines.

WALL ENERGY. The **energy** (per unit area) of the boundary between oppositely oriented ferromagnetic domains.

WALLIS FORMULA. The formula

$$\int_0^{\pi/2} \sin^n \theta d\theta = \int_0^{\pi/2} \cos^n \theta d\theta$$

$$= \frac{\Gamma\left(\frac{n+1}{2}\right)}{\Gamma\left(\frac{n}{2}+1\right)} \cdot \frac{\sqrt{\pi}}{2}, \quad n > -1,$$

where Γ denotes the **gamma function**.

WALLIS PRODUCT. The infinite product

$$\frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdots \frac{2n}{2n-1} \cdot \frac{2n}{2n+1} \cdots$$

Its value is $\pi/2$.

WALL LOSSES. In a reciprocating engine or compressor, the working fluid is periodically induced, expanded (or compressed) and dis-

charged. Consequently, the temperature of the working fluid inside the cylinder varies between given limits. Since the thermal capacity of the cylinder walls is usually large compared with that of the working fluid, and since heat is transferred between them and the working fluid at a relatively slow rate, the walls assume a temperature intermediate between the highest and lowest temperatures of the working fluid, except for a small depth. Consequently, during part of the cycle, heat flows from the wall to the fluid, and in the opposite direction during the remainder of the cycle. This flow is composed of a net flow in one direction (heat loss) and a fluctuation giving no net flow of heat. However, the fluctuating heat flow causes **entropy** increases which result in a loss of work. In a prime mover (e.g., steam engine) because heat is exchanged at a finite temperature difference in either direction, the work obtained from it is less than it would be if this periodic heat flow did not exist. In a compressor, the work required to drive it becomes larger than it would otherwise be. The difference between the two is termed wall loss.

The flow of heat associated with the wall loss prevents processes in engines or compressors from being **isentropic** (no heat flow) or **isothermal** (flow of heat in one direction only.)

WARPING. See **torsion**.

WATT SPECTRUM. In the physics of fission, an analytic fit of the experimentally determined fission spectrums, due to B. E. Watt. It is

$$f(E) = 0.484e^{-E} \sinh \sqrt{2E} \quad (E \text{ in mev}).$$

(See **fission spectrum**.)

WAVE. A disturbance which is propagated in a medium in such a manner that at any point in the medium the displacement is a function of the time, while at any instant the displacement at a point is a function of the position of the point. Any physical quantity

which has the same relationship to some independent variable (usually time) that a propagated disturbance has, at a particular instant, with respect to space, may be called a wave. In this definition, displacement is used as a general term, indicating not only mechanical displacement, but also electric displacement, etc. In short, a wave is a time-varying quantity which is also a function of position; for example, any time-varying voltage or current in a **network** is often called a wave. Other examples of waves are: (1) wave on the surface of a liquid, in which the disturbance is the displacement of any particle in the surface from its equilibrium position; (2) acoustic wave, in which the disturbance is the change in pressure from its equilibrium value at any point in a material medium (fluid or solid); (3) electromagnetic wave, in which the disturbance is the change in the electric and magnetic field intensities from their equilibrium values in space. The first two types are known as mechanical waves, since the propagated disturbance involves motion of a medium.

WAVE AMPLITUDE. The magnitude of the maximum change from equilibrium of the disturbance characterizing the wave.

WAVE(S), BULK. See **wave(s), dilatational**.

WAVE, CAPILLARY. See **capillary wave**.

WAVE, CIRCULATING. A wave of which the **equiphase surfaces** are half-planes issuing from an axis, called the axis of circulation. Such waves are possible only in non-dissipative media.

WAVE, COMPRESSION. See **compression wave**.

WAVE, CONVERGING. A spherical wave (see **wave, spherical**) which is traveling in the direction of decreasing radius.

WAVE CREST. The position of maximum positive disturbance in a progressive wave. (See **wave, progressive**.)

WAVE, CYLINDRICAL. A wave whose **equiphase surfaces** form a family of coaxial or confocal cylinders.

WAVE(S), DILATATIONAL. Dilatational waves are elastic waves (see **waves, elastic, in solids**), which are propagated through a solid medium and are characterized by longitudinal particle motion, i.e., the particle displacements are in the direction of propagation. For such waves the **curl** of the particle velocity field is zero, and in an isotropic solid such waves travel with the velocity $\sqrt{[(K + \frac{4}{3}G)/\rho]}$, where K is the bulk modulus, G is the shear modulus, and ρ is the density of the medium. Dilatational waves are also called *irrotational waves, bulk waves, longitudinal waves*, and, in seismology, *P waves*.

WAVE(S), DISTORTIONAL. See **wave(s), shear**.

WAVE DRAG. See **drag**.

WAVE, ELASTIC. See **waves, elastic**.

WAVE(S), ELECTROMAGNETIC. See **electromagnetic waves**.

WAVE(S), ELECTROMAGNETIC PLANE, BASIC TYPES. The three basic types of electromagnetic plane wave are: (1) Transverse electromagnetic (TEM)—Both the **E** and **H** components of the field lie in the plane that is transverse to the direction of propagation. (2) Transverse electric (TE)—The **E** field lies wholly in the plane that is transverse to the direction of propagation, whereas the **H** field does not. (3) Transverse magnetic (TM)—The **H** field lies wholly in the plane that is transverse to the direction of propagation, where the **E** field does not.

WAVE EQUATION. The most important special case of **hyperbolic partial differential equations**. For two independent variables, it takes the form

$$\frac{\partial^2 \phi}{\partial y^2} - c^2 \frac{\partial^2 \phi}{\partial x^2} = g(x, y). \quad (1)$$

y is often taken as the time-coordinate, whence (1) is referred to as the one-dimensional wave equation. The **Riemann function** for (1) has the constant value $\psi(x, y; X, Y) = \frac{1}{c}$. The solution of the **Cauchy problem** for (1) with data prescribed on the x -axis becomes at a point (X, Y) , with $Y > 0$.

$$\begin{aligned} \phi(X, Y) = & \frac{1}{2} \left[\phi(X + cY, 0) \right. \\ & + \phi(X - cY, 0) + \frac{1}{2c} \int_{X-cY}^{X+cY} \phi_y(\xi, 0) d\xi \\ & \left. + \frac{1}{2c} \int_0^Y \int_{X-c(Y-y)}^{X+c(Y-y)} g(x, y) dx dy \right]. \quad (2) \end{aligned}$$

It follows from (2) that the general solution of the *homogeneous* one-dimensional wave equation is

$$\phi(x, y) = f(x + cy) + g(x - cy)$$

with suitable functions f and g of one variable.

The three-dimensional wave equation has the form

$$\phi_{tt} = c^2(\phi_{xx} + \phi_{yy} + \phi_{zz}) + g(x, y, z, t). \quad (3)$$

For the initial values $\phi(x, y, z, 0) = f(x, y, z)$, $\phi_t(x, y, z, 0) = g(x, y, z)$, the solution can be obtained by **Fourier analysis**; it is

$$\begin{aligned} & 4\pi c^2 \phi(x, y, z, t) \\ & = \frac{\partial}{\partial t} \left[\frac{1}{t} \iint_{r=ct} f(x + \xi, y + \eta, z + \zeta) dS \right. \\ & \quad \left. + \frac{1}{t} \iint_{r=ct} g(x + \xi, y + \eta, z + \zeta) dS \right] \\ & + \iiint_{r < ct} \frac{1}{r} v \left(x + \xi, y + \eta, z + \zeta, t - \frac{r}{c} \right) d\xi d\eta d\zeta \quad (4) \end{aligned}$$

where ξ, η, ζ denote the variables of integration and $r^2 = \xi^2 + \eta^2 + \zeta^2$. Equation (4) shows that the solution at (x, y, z, t) depends only on the values of the initial data and of their derivatives on the sphere of radius ct about the point (x, y, z) . The wave equation (3) thus has the feature that the domain of dependence is a surface in the initial sphere (*strong form of Huygen's principle*).

WAVE EQUATION, RELATIVISTICALLY-INVARIANT. See *relativistically invariant wave equations*.

WAVE(S), EQUIVOLUMINAL. See *wave(s), shear*.

WAVE, EXPANSION. See *expansion wave*.

WAVE, FREE. See *free wave*.

WAVE FRONT. A surface at all points of which the *phase* of the wave has the same

value at a given instant. Wave propagation can be thought of as the motion of a wave front through a medium.

WAVE FRONT ABERRATIONS. H. H. Hopkins, Linfoot, Maréchal and others have developed the wave theory of aberration. A characteristic function S is introduced as an optical path length from a wave front to an ideal spherical one in image space. S is a solution of the differential equation of optics

$$S_x^2 + S_y^2 + S_z^2 = n^2$$

and may be expanded in terms of constituent aberration functions, $S = \Sigma S_i$. The aberrations S_i can be identified as the classical ray-theory aberrations, but the point of view developed from the wave theory leads to valuable new insight into aberration theory.

WAVE FRONT, PLANE. If the loci of points of constant phase of a beam of radiation are parallel planes, the wave fronts are called "plane" and the radiation is spoken of as "parallel."

WAVE FRONT, SPHERICAL. As radiation from a point source passes through a uniform medium, the loci of points with the same phase will be a series of concentric spheres. The radiation travels always normal to these spheres or wave fronts.

WAVE FUNCTION. A function of the dynamical variables of a particle or a system of particles which describes the state of the system in *wave mechanics*.

WAVE FUNCTION, ADJOINT. See *adjoint wave function*.

WAVE FUNCTION, ANTISYMMETRIC. See *antisymmetric wave function*.

WAVE FUNCTION, COULOMB. See *Coulomb wave function*.

WAVE FUNCTION RENORMALIZATION. See *divergences*.

WAVE FUNCTION, SIGMA, PI (σ, π). The electrons of a molecule can be classified according to the quantum number of the component of their orbital momentum about the internuclear (bond) axis. Depending on whether this quantum number is 0, 1, 2, \dots , the electrons (and their wave functions) are called $\sigma, \pi, \delta, \dots$ electrons (or wave functions).

For diatomic molecules, a σ wave-function is symmetrical around the axis of the molecule; a π wave function is anti-symmetrical with respect to this axis. For planar molecules, the σ and π wave functions are respectively symmetrical and antisymmetrical with respect to a reflection in the plane of the molecule. The π wave function is perpendicular to the plane, and possesses a node in this plane. (See **hindered rotation**.)

WAVE FUNCTION, SYMMETRICAL. See **exchange energy**; also preceding entry.

WAVE, GRAVITY. See **gravity wave**.

WAVE GROUP, GAUSSIAN. See **Gaussian wave group**.

WAVE(S), GROUP VELOCITY OF. The velocity with which the envelope of a group of waves of neighboring frequencies travels in a medium in which the **phase velocity** is a function of frequency. It is

$$G = c - \lambda \frac{dc}{d\lambda},$$

and is usually identical with the velocity of energy propagation (c is wave velocity at wavelength λ).

WAVEGUIDE. A system, generally bounded by surfaces of high conductivity, which may be used to guide electromagnetic waves, and hence electromagnetic energy from one point to another. Commonly refers to hollow metal pipes so used.

If z denotes the coordinate in the direction of the waveguide and t time, three kinds of solutions of **Maxwell's equations** can be found in which both the electric field vector \mathbf{E} and the magnetic field vector \mathbf{H} depend only on t and z through the factor $\exp i(\omega t - kz)$. These three types: (1) TEM, (2) TE and (3) TM are defined in the entry **wave(s), electromagnetic plane, basic types**. No TEM wave can be propagated down the inside of a hollow conductor.

WAVE, GUIDED. See **guided wave**.

WAVE, HARMONIC. A wave in which the disturbance at any point in space through which the wave is propagated varies sinusoidally with the time. Specifically if the wave is a plane harmonic wave in the x -direction, the disturbance ξ is given by

$$\xi = A \sin(\omega t - kx)$$

where A is the wave amplitude, $\omega = 2\pi f$ (where f is the frequency) and $k = \omega/V$ (where V is the wave velocity). (Cf. **wave propagation**.)

WAVE(S), IRROTATIONAL. See **wave(s), dilatational**.

WAVE, KINETIC ENERGY DENSITY. The kinetic energy per unit volume in a medium traversed by a mechanical wave. It is a function of time and space.

WAVE, LEE. See **lee wave**.

WAVELENGTH ASSOCIATED WITH ONE ELECTRON-VOLT. By combining the basic relation of **quantum theory** that the energy of a photon is the **Planck constant** multiplied by the frequency, with the energy which an electron receives in falling through one volt of potential difference, there results a wavelength associated with one electron-volt of $12,395 \times 10^{-8}$ cm (Birge, 1941).

WAVELENGTH CONSTANT. The **wave equation**:

$$q = a \sin 2\pi \left(\frac{t}{T} - \frac{x}{\lambda} + \delta \right)$$

is sometimes written as

$$q = a \sin \left(\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x + \epsilon \right).$$

$\frac{2\pi}{T} = \omega$, the "angular velocity" or "angular

frequency." $\frac{2\pi}{\lambda} = k$ (or ν) is called the wavelength constant or, sometimes, the wave number or wave parameter.

WAVELENGTH, DOMINANT. See **dominant wavelength**.

WAVELENGTH, EFFECTIVE. See **effective wavelength**.

WAVELENGTH OF YAW. A term used in rocket aerodynamics referring to the fact that when a rocket yaws its period of oscillation is inversely proportional to the velocity. As a result, a rocket traverses the same distance while performing an oscillation, regardless of velocity. This distance for a complete cycle of oscillation, is called the wavelength of yaw. It is given by:

$$\sigma = 2\pi \sqrt{\frac{Mk^2\rho}{K_M d^3}}$$

where σ is the wavelength of yaw, M is the mass of the rocket, k is the radius of gyration (about a transverse axis through the center of gravity), ρ is the air density, K_M is the coefficient of moment, and d is the diameter of the rocket.

WAVE, LONGITUDINAL. See **wave(s)**, **dilatational**.

WAVE MECHANICS. That form of quantum mechanics in which the emphasis is on the wave aspects of the quantum mechanical system. Historically, it was discovered on a different line from that of quantum mechanics, but the two expressions are now used as meaning the same subject. (For some of the ideas of wave mechanics, see **quantum mechanics**.)

WAVE, NEUTRAL. See **neutral wave**.

WAVE NUMBER. The reciprocal of the **wavelength** in a harmonic wave. Some authors use $2\pi/\lambda$ instead of $1/\lambda$ in this sense. The wave number ν of a spectral line is defined as the number of waves per centimeter

$$\nu = 1/\lambda = \nu'/c$$

where ν' and λ are, respectively, the frequency and wavelength (*in vacuo*), and c , the velocity of light.

WAVE NUMBERS, HICKS FORMULA FOR. See **Hicks formula**.

WAVE PARAMETER. The quantity $2\pi/\lambda$, commonly symbolized by k . (See also **wave number** and **wave constant**.)

WAVE PERIOD. In a harmonic wave the time between attainment of successive maximum disturbances at the same place. The reciprocal of the frequency.

WAVE, PERIODIC. A wave in which the disturbance repeats itself in equal intervals of time at each point of space and at equal intervals of space at each given instant of time.

WAVE, PERMANENT. See **permanent wave**.

WAVE PHASE. The argument in the **wave function**. Thus in the general arbitrary pro-

gressive wave function $f(x - Vt)$ the phase is $x - Vt$. For a harmonic plane wave in which the disturbance has the form $\sin(\omega t - kx)$, the phase is $\omega t - kx$. Here ω is the angular frequency = $2\pi f$, where f is the actual frequency, $k = 2\pi/\lambda$, where λ is the wavelength.

WAVE(S), PHASE VELOCITY OF. The velocity with which a point of constant phase is propagated in a progressive sinusoidal wave.

WAVE(S), PLASTIC. Stress waves which produce plastic deformation as they are propagated through solids (especially metals). They generally travel at velocities lower than that of the corresponding elastic waves (see **waves, elastic, in solids**), and appreciable energy dissipation takes place.

WAVE(S), PROGRESSIVE. Traveling waves, as opposed to stationary waves. (See **wave, standing**.)

WAVE(S), RAYLEIGH SURFACE. Elastic waves which can be propagated in the neighborhood of the free surface of an elastic solid and travel parallel to it. Their effect decreases rapidly with depth, and their velocity of propagation is less than that of either longitudinal or shear waves in the body of a solid. (See **velocity of sound**.) The velocity of propagation can be expressed as $k\sqrt{(G/\rho)}$ where k is a numerical constant slightly less than unity, which depends on Poisson's ratio, σ . For $\sigma = 0.25$, $k = 0.9194$, and for $\sigma = 0.5$, $k = 0.9554$. The particle motion in Rayleigh surface waves is in a plane perpendicular to the free surface and parallel to the direction of propagation. For simple harmonic Rayleigh surface waves the particle trajectory is an ellipse.

WAVE, REFLECTED. When a wave in a medium of certain propagation characteristics is incident upon a discontinuity or a second medium, the wave component that results in the first medium in addition to the incident wave. (See **wave, incident**.)

WAVE, REFRACTED. That part of an incident wave (see **wave, incident**) which travels from one medium into a second medium.

WAVE(S), ROTATIONAL. See **wave(s)**, **shear**.

WAVES, ELASTIC, IN PLATES. A longitudinal elastic wave traveling in a thin plate

(i.e., one whose thickness is small compared with the wavelength) is propagated at a velocity given by $\sqrt{[(E/\rho)(1 - \sigma^2)]}$, where E is **Young's modulus**, σ is **Poisson's ratio**, and ρ is the density. At wavelengths comparable with the plate thickness, the phase velocity decreases and waves of very short wavelength travel with the velocity of Rayleigh surface waves. (See **wave(s)**, **Rayleigh surface**.)

WAVES, ELASTIC, IN RODS. Three types of elastic wave can be propagated along rods. These are longitudinal (or extensional), torsional, and flexural. When the wavelength is large compared with the lateral dimensions of the rod, longitudinal waves travel with the velocity $\sqrt{(E/\rho)}$ where E is Young's modulus and ρ is the density of the rod. Torsional waves travel with the velocity $\sqrt{(G/\rho)}$ where G is the shear modulus, and flexural waves are dispersed and travel with the velocity $(2\pi k/\Lambda)\sqrt{(E/\rho)}$, where Λ is the wavelength and k is the radius of gyration of a cross-section of the rod about the neutral axis. When the wavelength becomes comparable with the lateral dimensions of the rod, the expressions for the velocity of longitudinal and flexural waves become extremely complicated. For wavelengths very small compared with the lateral dimensions, the phase velocity of both these types of waves approaches that of Rayleigh surface waves (see **waves**, **Rayleigh surface**). Torsional waves are not dispersed, and travel with the velocity (G/ρ) for all wavelengths.

WAVES, ELASTIC, IN SOLIDS. Through an isotropic unbounded elastic solid, two types of elastic wave can be propagated. The particle motion in these two types of wave is parallel to the direction of propagation and perpendicular to it respectively. A wave in which the particle motion is parallel to the direction of propagation is termed longitudinal, dilatational, bulk or irrotational, and in seismology is called a P wave. A wave in which the particle velocity is perpendicular to the direction of propagation is called transverse, distortional, shear, rotational or equivoluminal, and in seismology is called an S wave. Longitudinal waves travel with the velocity $\sqrt{[(K + \frac{4}{3}G)/\rho]}$ where K is the bulk modulus, G the shear modulus, and ρ the density of the solid medium. Transverse waves travel with the velocity $\sqrt{(G/\rho)}$.

On the surface of a semi-infinite solid, Ray-

leigh surface waves (see **wave(s)**, **Rayleigh surface**) can be propagated. The particle motion of these is in a plane perpendicular to the surface and parallel to the direction of propagation. In bounded solids, a number of different types of elastic wave can be propagated, and in general their velocity of propagation depends on the wavelength. (See **waves**, **elastic**, **in rods**; **waves**, **elastic**, **in plates**.)

WAVE, SHEAR. See **shear wave**; **Helmholtz wave**.

WAVE, SHEAR-GRAVITY. See **shear-gravity wave**.

WAVE, SHOCK. See **shock wave**.

WAVE, SIMPLE. See **simple wave**.

WAVES, LONG. See **long wave**.

WAVE, SOLENOIDAL. A wave in which the divergence of the vector quantity representing the disturbance vanishes identically. A plane solenoidal elastic wave is a transverse wave. (See **wave**, **transverse**.)

WAVE, SOLITARY. See **solitary wave**.

WAVE, SPHERICAL. A wave whose equi-phase surfaces form a family of concentric spheres.

WAVE, SQUARE. See **square wave**.

WAVE, STANDING. A wave disturbance which is not progressive, i.e., one in which the energy flux is zero at all points. Such waves can exist only in completely non-dissipative media, and therefore can only be approximated in real situations. In such waves, any component of the field can be specified as a function of position multiplied by a sinusoidal function of time. See **standing waves** (**in fluids**).

WAVE, STATIONARY. See **stationary waves** (**in fluid**).

WAVE(S), STRESS. A generic term used to describe mechanical waves propagated through materials, especially solids. Elastic waves, viscoelastic waves, plastic waves and shock waves are particular examples of stress waves. (See **waves**, **elastic**, **in solids**; **waves**, **viscoelastic**; **waves**, **plastic**, and **waves**, **shock**.)

WAVE SURFACE. See **normal congruence**.

WAVE, SURFACE. See surface wave.

WAVE TROUGH. The position where the disturbance in a progressive wave has its minimum (algebraic) value. It is the opposite of wave crest. (See wave, progressive.)

WAVE VECTOR. A vector, usually represented as \mathbf{k} , which, at each position in space, points in the direction of propagation of the wave under consideration. The magnitude assigned to the wave vector associated with a wave of wavelength λ is not unique, but with present usage it is almost certain to be either unity, $1/\lambda$, or $2\pi/\lambda$. The magnitude of the wave vector is called the **wave number** when $|k| = 1/\lambda$ and sometimes when $|k| = 2\pi/\lambda$.

WAVE VELOCITY. See phase velocity.

WAVE(S), VISCOELASTIC. When elastic waves (see waves, elastic, in solids) are propagated through solid media, some dissipation always takes place. For most elastic solids, however, the effect is small, though for some materials, especially rubbers and plastics, the attenuation is large. The phase velocity is then found to depend on the frequency of the disturbance. The entire nature of wave propagation is here modified. Waves propagated under such conditions are called viscoelastic.

WEAK AXIS. See axis, weak.

WEAK LOCAL COMMUTATIVITY. A set of field operators $\psi_o(y_o), \psi_1(y_1) \cdots, \psi_n(y_n)$ is said to have the weak local commutativity property at a set of real points $(\xi_1 \cdots \xi_n)$ if the relation

$$(\Psi_o, \psi_o(y_o) \cdots \psi_n(y_n)\Psi_o) = \epsilon(\Psi_o, \psi_n(y_n) \cdots \psi_1(y_1)\psi_o(y_o)\Psi_o) \quad (1)$$

holds for all (y_o, \cdots, y_n) such that the difference

$$\eta_j = y_{j-1} - y_j \quad (2)$$

lie within some real neighborhood of the ξ_j . Here Ψ_o is the invariant unique vacuum state and ϵ is ± 1 , the sign depending on whether the permutation of fermion field operators between the left and right side of Equation (1) is even or odd.

WEBER. The unit of flux is the rationalized MKS system. (See electrical units.)

WEBER DIFFERENTIAL EQUATION. The second-order equation

$$y'' + (n + \frac{1}{2} - \frac{1}{4}x^2)y = 0.$$

It has an irregular singular point at ∞ and occurs in solving the wave equation in parabolic cylindrical coordinates. It is a special case of the confluent hypergeometric equation and its solutions are confluent hypergeometric series.

WEBER NUMBER. In fluid mechanics, a dimensionless product:

$$W = \frac{\rho v^2 L}{\gamma}$$

where W is the Weber number, ρ is the mass density, v is the velocity, L is the length, and γ is the surface tension.

WEDDLE RULE (FOR NUMERICAL QUADRATURE).

$$\int_{x_0}^{x_6} f(x)dx = 3(f_0 + 5f_1 + f_2 + 6f_3 + f_4 + 5f_5 + f_6) + R,$$

$$R = -h^7[10f^{(VI)}(\xi_1) + 9h^2f^{(VII)}(\xi_2)]/1400.$$

WEIERSTRASS APPROXIMATION THEOREM. If $f(x)$ is continuous on a closed interval, then for every $\epsilon > 0$ there exists a polynomial $P(x)$ such that $|f(x) - P(x)| < \epsilon$ for every x in the interval.

WEIERSTRASS FUNCTION. Weierstrass' function $\wp(z) = \wp(z/w, w')$ is an elliptic function of periods $2\omega, 2\omega'$ which: (1) is of order two; (2) has a double pole at $z = 0$, the principal part of the function at the pole being z^{-2} ; (3) makes $\wp(z) - z^{-2}$ analytic in a neighborhood of, and vanish at $z = 0$. In this definition, the *order* of an elliptic function is its number of poles, each counted according to its multiplicity, in the parallelogram formed for fixed z_0 by the points

$$z = z_0 + 2\xi\omega + 2\eta\omega', \quad 0 \leq \xi < 1; 0 \leq \eta < 1.$$

Weierstrass' zeta function is a certain meromorphic function with simple poles, derivable from Weierstrass' function, but not itself doubly-periodic and hence not an elliptic function.

WEIERSTRASS M-TEST (FOR UNIFORM CONVERGENCE). A series of functions $u_1(x) + u_2(x) + \cdots$ is uniformly and absolutely convergent in a closed interval $[a, b]$ in its domain of definition if there exists a conver-

gent series of positive constants $M_1 + M_2 + \dots + M_n + \dots$ such that $|u_n(x)| \leq M_n$ for every n and all x in $[a, b]$.

WEIGHT. The weight of a body is the resultant of the forces the Earth's gravitational field exerts upon it. The resultant passes through a point called the **center of gravity** which is clearly defined when the dimensions of the body are negligible compared with the distance of the body from the center of the Earth. (For statistical weight, see **weighting; statistical weight.**)

WEIGHTED MEAN. See **arithmetic mean.**

WEIGHT FRACTIONS. Concentrations in a mixture are sometimes expressed in terms of the weight fractions

$$w_i = \frac{m_i}{m} \quad (1)$$

where m_i is the mass of component i , and m the total mass. Clearly

$$\Sigma w_i = 1. \quad (2)$$

WEIGHTING. (1) The artificial adjustment of measurements in order to account for factors which, in the normal use of the device, would otherwise be different from the conditions during measurement. For example, background noise measurements may be weighted by applying factors or by introducing networks to reduce measured values in inverse ratio to their interfering effects. (2) When several different determinations of the same physical quantity have been made, the more accurate of the measurements should be counted more heavily than the less accurate in estimating the most probable value of the quantity. Thus, if q_1, q_2 , etc., are the individual values, the most probable value is

$$q = \frac{w_1 q_1 + w_2 q_2 + \dots}{w_1 + w_2 + \dots},$$

where w_1, w_2 , etc., are weighting factors. The theory of the propagation of errors shows that the weighting factor for any individual determination is inversely proportional to the square of the **uncertainty** of that determination. The quantity q is called the **weighted mean**. (3) In **statistical mechanics** different states will have different *a priori* probabilities, and in computing average values, these states must be weighted with factors proportional to

their probabilities, the computation then proceeding as in (2) above. (4) See **statistical weight**.

WEIGHTING FUNCTION. A function $w(x) \geq 0$ defined over an interval $[a, b]$ is a weighting function for a set of functions $u_1(x)u_2(x), \dots$, if

$$\int_a^b u_m u_n w dx = 0, \quad \text{for } m \neq n,$$

the functions $u_i(x)$ being then said to form a (generalized) orthogonal set. The concept of weighting function is similarly defined in the theory of approximation. (See **approximation in the mean.**)

WEIGHTING FUNCTION OF A SYSTEM.

A function of time, often denoted by $W(t)$, and defined as the **normal response** of a system to an input in the form of unit **impulse function** (of the first order) at $t = 0$, namely $\delta(t)$. It is the inverse **Laplace transform** of the output to input **transfer function** of the system.

The name arises from the fact that if the system has an arbitrary input $\theta_i(t)$, this may be considered over any infinitesimal time interval dt , as an impulse of amplitude $\theta_i(t)dt$. It follows that at time T the normal response of the system to this input will be

$$\theta_o(T) = \int_{-\infty}^T W(T-t)\theta_i(t)dt$$

and is thus made up of the component impulses of the input each weighted by the factor $W(T-t)$.

For any physical system $W(t) = 0$ when $t < 0$.

WEIGHT LOADING FACTOR. In rocket aerodynamics, a term which approximates the ratio of **thrust** and **drag**. It is given by:

$$\mu = \frac{W_o}{A}.$$

Where μ is the weight loading factor, W_o is the takeoff weight of the missile, and A is the reference (drag) area. A high value of μ means that only a small fraction of the thrust need be diverted from lifting the missile to overcome drag, and better performance is to be expected.

WEIGHT OF TENSOR (TENSOR FIELD). See **tensor field**.

WEIGHT, STATISTICAL. See *statistical weight*.

WEINGARTEN FORMULAS. Formulas which provide expressions for the tensor derivatives of the unit normal n^i ($i = 1, 2, 3$) to the surface in terms of the fundamental magnitudes of the first and second orders, $a_{\alpha\beta}$ and $b_{\alpha\beta}$. They are

$$n^i{}_{,\alpha} = -b_{\alpha\beta}a^{\beta\gamma}x_{\gamma}{}^i.$$

WEISSENBERG EFFECT. The climbing of a **viscoelastic** liquid up a stirrer rod and analogous effects associated with the **normal stress** or **cross effects**.

WELL-STIRRED FLUID. A fluid in which any two elements are interchangeable without altering its structure. If the fluid is incompressible its density must be uniform. In a compressible fluid such as the atmosphere the vertical temperature gradient is the **adiabatic lapse rate**, and gravity forces produce no **vorticity**.

WET-BULB POTENTIAL TEMPERATURE. (Also called pseudo wet-bulb potential temperature.) The temperature an air parcel would have if cooled from its initial state adiabatically to saturation, and thence brought to 1000 mb by a **saturation-adiabatic process**. This temperature is conservative with respect to reversible adiabatic changes.

WET-BULB TEMPERATURE. (1) **Isobaric wet-bulb temperature:** The temperature an air parcel would have if cooled adiabatically to saturation at constant pressure by evaporation of water into it, all **latent heat** being supplied by the parcel.

(2) **Adiabatic wet-bulb temperature** (or pseudo wet-bulb temperature): The temperature an air parcel would have if cooled adiabatically to saturation and then compressed adiabatically to the original pressure in a **saturation-adiabatic process**. This is the wet-bulb temperature as read off the thermodynamic diagram and is always less than the isobaric wet-bulb temperature, usually by a fraction of a degree centigrade.

(3) The temperature read from the wet-bulb thermometer. The thermodynamics of this thermometer is still under discussion, but for practical purposes the temperature so obtained is identified with the isobaric wet-bulb temperature.

WET COMBUSTION GASES. See *combustion*.

WET GASES. See *combustion*.

WET STEAM. A system consisting of saturated water vapor and water. In all thermodynamic changes, the pressure of wet steam is uniquely related to its temperature, the relation between them being known as the vapor-pressure curve.

The state of wet steam is usually described by its **dryness fraction** and one additional parameter. (See also **Clapeyron equation**.)

WEYL EQUATION. The wave equation obeyed by the 2 component spinor $\phi(x)$ which describes the neutrino:

$$i\hbar\partial_t\phi(x) = i\hbar c\sigma\cdot\nabla\phi(x)$$

where $\sigma_1, \sigma_2, \sigma_3$, are the 2×2 Pauli matrices. (See **two component equation of the neutrino**.)

WHIRL VELOCITY. See *velocity diagram*.

WHITE BODY. A hypothetical "body" whose surface absorbs no **electromagnetic radiation** of any wavelength, i.e., one which exhibits zero **absorptivity** for all wavelengths; an idealization exactly opposite to that of the **black body**.

WHITTAKER DIFFERENTIAL EQUATION. A second-order equation

$$y'' + \left(\frac{\frac{1}{4} - n^2}{x^2} + \frac{k}{x} - \frac{1}{4} \right) y = 0$$

with **singular points** at 0 and ∞ . It is a canonical form of the confluent **Gauss hypergeometric equation** found by removing the term in y' , the first **derivative**. Its solutions, sometimes called the Whittaker functions, are special cases of the confluent hypergeometric series. (See **confluent hypergeometric equation**.)

WICK-CHANDRASEKHAR METHOD. In neutron transport theory, a method of analysis of the transport equation whereby the integral term describing scattering is replaced by a formula of numerical integration. Thus, in the one-velocity formulation of transport theory, the differential-integral equation becomes a system of differential equations. This method, sometimes called the method of dis-

crete ordinates, is most powerful when it is applied to one-velocity problems in systems having plane geometry.

WICK METHOD. In the theory of the slowing down of neutrons at large distances from a source, one of the effective analytical methods. It is based upon the fact that at very large distances from a source, the angular distribution of neutrons of energy E is essentially a delta function, peaked in the "out-going" direction. All of the angular moments are of comparable magnitude and vary slowly with their index, n . One can therefore replace the difference expressions in n by simpler differential expressions, in the analysis of the transport equation.

WIDTH, EFFECTIVE. The non-uniform distribution of normal stress associated with shear lag or buckling lag can be taken into account by considering only a fraction of the actual width of a flange, plate or stiffener as effective.

WIDTH OF A SPECTRAL LINE. See natural line width, equivalent width of a spectral line, half width of a spectral line, Doppler width of a spectral line.

WIDTH, PRACTICAL. See practical width.

WIEN LAWS. From a study of the spectral energy distribution of thermal radiation, W. Wien, in 1896, arrived at three laws relating to the radiation from a black body.

(1) The wavelength λ_m of the spectral distribution, for which the radiation has greatest intensity, is inversely proportional to the absolute temperature T of the black body:

$$\lambda_m T = c_1.$$

Thus as the temperature rises, the "peak" of the distribution curve is displaced or shifted toward the short-wavelength end of the spectrum. This is commonly called Wien's "displacement law." The value of the "displacement constant" c_1 is about 0.2897 centimeter-degree.

(2) The emissive power of the black body within the maximum-intensity wavelength interval $d\lambda$ is proportional to the fifth power of the absolute temperature:

$$dE_m = c_2 T^5 d\lambda.$$

Subsequent work by Planck and others gives the value of the constant c_2 as about 1.288×10^{-4} erg/cm.³ sec. deg.⁵

(3) Wien's third law is an attempt to express the spectral energy distribution of the radiation from the black body at temperature T , as follows:

$$dE_\lambda = A\lambda^{-5}e^{-B/\lambda T}d\lambda,$$

in which dE_λ is the emissive power within the wavelength interval $d\lambda$ and A and B are constants to be empirically determined.

The first and second laws are in accord with thermodynamic theory and with Planck's equation, and also agree very accurately with experiment. The third law is empirical, but is almost identical in form with Planck's equation. However, Wien's distribution law fails at longer wavelengths.

WIGHTMAN FUNCTIONS. Consider a relativistic quantum field theory. For simplicity we will consider a neutral scalar field which is described by an operator $\phi^*(x) = \phi(x)$ which under the inhomogeneous Lorentz group transforms as

$$U(a, \Lambda)\phi(x)U(a, \Lambda)^{-1} = \phi(\Lambda x + a) \quad (1)$$

It is assumed there is a Hilbert space of vectors, \mathfrak{H} , corresponding to the physical states.

In particular it is assumed that there exists a unique Lorentz invariant state represented by Ψ_0 , with

$$U(a, \Lambda)\Psi_0 = \Psi_0 \quad (2)$$

where $U(a, \Lambda)$ is a representation of the inhomogeneous Lorentz group. ($U(a, \Lambda)$ is unitary when Λ is orthochronous, antiunitary otherwise), a is a space-time translation and Λ is a homogeneous Lorentz transformation. Since $U(a, 1) = \exp iP_\mu \cdot a^\mu$, where P_μ is the energy momentum four vector of the field, Ψ_0 is the unique (proper) state of zero energy and momentum ($P_\mu\Psi_0 = 0$). It is furthermore assumed that there are no vectors in the theory representing negative energy states, or states with space-like momenta.

The proper operator

$$\phi(f) = \int \phi(x)f(x)d^4x \quad (3)$$

where $f(x)$ is a test function in the space of infinitely differentiable functions C^∞ with compact support, are applicable to the vacuum state Ψ_0 and to dense sets in \mathfrak{H} . Polynomials

in $\phi(f)$ are applicable to the vacuum and the vectors obtained in this way are dense in \mathfrak{H} .

The vacuum expectation value of products of field operators

$$W^{(n)}(x_1 \cdots x_n) = (\Psi_0, \phi(x_1) \cdots \phi(x_n)\Psi_0) \tag{4a}$$

are called Wightman functions. The multilinear functional

$$W^{(n)}(f_1 \cdots f_n) = (\Psi_0, \phi(f_1) \cdots \phi(f_n)\Psi_0) \tag{4b}$$

is a distribution in each variable separately. A theorem by L. Schwartz asserts that it has a unique extension to become a distribution on $4n$ space giving precise meaning to Wightman function $W^{(n)}(x_1 \cdots x_n)$. The hermiticity of the operator ϕ implies that

$$\overline{W^{(n)}(f_1 \cdots f_n)} = W^{(n)}(\bar{f}_n \cdots \bar{f}_1). \tag{5}$$

The Lorentz invariance of the theory implies that the distributions are Lorentz invariant

$$W^{(n)}(x_1 \cdots x_n) = W^{(n)}(\Lambda x_1 + a, \cdots \Lambda x_n + a) \tag{6a}$$

(no time inversion)

$$= \bar{W}^{(n)}(\Lambda x_1 + a_1, \cdots \Lambda x_n + a_n) \tag{6b}$$

(with time inversion).

By translation invariance $W^{(n)}(x_1, x_2, \cdots x_n)$ is only a function of $x_i - x_{i+1}$ for $i = 1, \cdots, n - 1$. The Fourier transformed distribution

$$G^{(n)}(p_1, p_2, \cdots p_n) = \int d^4x_1 \cdots \int d^4x_n \tag{7}$$

$\times \exp(+i\sum p_i x_i) W(x_1, x_2, \cdots x_n)$

is assumed to exist. (This is equivalent to the assumption that both $W^{(n)}$ and $G^{(n)}$ are in the space S' of tempered distributions.) By virtue of translation invariance, a δ function factor corresponding to energy momentum conservation, can be separated from $G^{(n)}$.

$$G^{(n)}(p_1, \cdots p_n) = \delta(p_1 + \cdots + p_n) H^{(n)}(p_1 \cdots p_n). \tag{8}$$

Because of the absence of negative energy states $p_1, p_1 + p_2, \cdots p_1 + p_2 \cdots + p_{n-1}$ are all in or on the forward light cone. This has the consequence that the Wightman function

$$W^{(n)} = W^{(n)}(\xi_1, \cdots \xi_{n-1}) \tag{9}$$

$\xi_i = x_i - x_{i+1}$

are boundary values of analytic functions. If we denote by $\bar{W}^{(n)}$ the Fourier transform of $W^{(n)}(\xi_1 \cdots \xi_{n-1})$

$$\int \exp(-i \sum_j^{n-1} p_j \xi_j) W^{(n)}(p_1 \cdots p_{n-1}) \tag{10}$$

$\times d^4p_1 \cdots d^4p_{n-1} = \bar{W}^{(n)}(\xi_1 \cdots \xi_{n-1})$

then the absence of states with negative energy or space like momenta implies

$$\bar{W}^{(n)}(p_1 \cdots p_{n-1}) = 0$$

unless $p_j^2 \geq 0$ and $p_{j0} \geq 0$. If we write $\xi_i = \xi_i - i\eta_i$ where η_i lies in the future light cone, then the analytic properties of $W^{(n)}$ are displayed and proved by the formula

$$W^{(n)}(\xi_1, \xi_2, \cdots \xi_{n-1}) = \int \cdots \int \bar{W}^{(n)}(p_1 \cdots p_{n-1}) \tag{11}$$

$\times \exp(-i \sum_j^{n-1} p_j \xi_j) d^4p_1 \cdots d^4p_{n-1}$

If the operators are local so that $[\phi(x), \phi(y)] = 0$ for $(x - y)^2 < 0$, then

$$W^{(n)}(x_1, \cdots x_i, x_{i+1} \cdots x_n) = W^{(n)}(x_1, \cdots x_{i+1}, x_i \cdots x_n) \tag{12}$$

if $(x_i - x_{j+1})^2 < 0$.

Finally by virtue of the positive definite character of the norm of any vector Ψ in \mathfrak{H} , $\|\Psi\| > 0$, with the choice

$$\Psi = \{ \alpha_0 f_0 + \alpha_1 \phi(f) + \cdots + \alpha_n \int d^4x \cdots \int d^4x_n f_n(x_1 \cdots x_n) \times \phi(x_1) \cdots \phi(x_n) + \cdots \} \Psi_0$$

one has an infinite sequence of inequalities

$$\sum_{ij} \bar{\alpha}_i \alpha_j \int d^4x_1 \cdots \int d^4x_i \int d^4y_1 \cdots \int d^4y_j \tag{13}$$

$\times \bar{f}_i(x_1 \cdots x_i) W^{(i+j)}(x_1 \cdots x_i, y_1 \cdots y_j)$
 $f_j(y_1 \cdots y_j) \geq 0$.

Wightman has shown that if $W^{(n)}(x_1 \cdots x_n)$ ($n = 0, 1, 2, \cdots$) is a sequence of distributions on $4n$ space satisfying Equation (6) (relativistic invariance), Equation (5) (hermiticity), Equation (13) positive definiteness and Equation (12) (local commutativity), then there ex-

ists a Hilbert space \mathcal{H} , a representation of the inhomogeneous Lorentz group, a vacuum state Ψ_0 and a neutral scalar field ϕ , such that the n -fold vacuum expectation value of ϕ is $W^{(n)}$.

This theorem has the consequence that one can as well study vacuum expectation values as fields. Bargmann, Hall and Wightman have further shown that $W^{(n)}$ is in fact only a function of the $n(n-1)/2$ scalar products $\xi_i \xi_j$. (See also **Lehmann representation**.)

WIGNER COEFFICIENT. A coefficient which occurs in the quantum mechanical theory of angular momentum. Let $\phi(jm)$ denote a normalized set of orthogonal eigenfunctions of the angular momentum operator \mathbf{J}^2 whose eigenvalues are $j(j+1)\hbar^2$, where j is integral or half-integral, and $2\pi\hbar$ is Planck's constant, and of its z -component M whose eigenvalues are $m\hbar$, where m takes any of the values $-j, -j+1, \dots, j-1, j$. Let \mathbf{J}_1 and \mathbf{J}_2 be any two commuting angular momenta, and let their resultant be $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2$, with $M = M_1 + M_2$. Then the eigenfunctions corresponding to the resultant angular momentum are

$$\phi(jm) = \sum_{m_1+m_2=m} C_{m_1 m_2 m}^{j_1 j_2 j} \phi(j_1 m_1) \phi(j_2 m_2)$$

where the coefficients C are the Wigner coefficients. As usually defined, the coefficients are real numbers which have been tabulated for most of the values of the suffices of practical interest.

WIGNER FORCE. A central force between two nucleons which is derivable from a potential determined entirely by the distance between the two particles.

WIGNER-WILKINS MODEL. In elucidating the subject of neutron thermalization—the manner in which fast neutrons interacting with matter reach a steady state distribution in the thermal energy range—Wigner and Wilkins have studied the interaction of neutrons with a thermally agitated gas of hydrogen atoms. A “ $1/v$ ” absorption cross section is ascribed to the hydrogen atoms. The Boltzmann transport equation describing this process is tractable and Wigner and Wilkins have solved it.

WILCOXON TEST. A distribution-free test of the homogeneity of two samples.

WILKINS MODEL. See **Wigner-Wilkins model**.

WILKS CRITERION. A criterion used in multivariate analysis, due to S. S. Wilks. It depends on the ratio of two dispersion determinants and is used to test hypotheses concerning the homogeneity of sets of means and variances. It can thus be regarded as the multivariate extension of the variance-ratio. (See **F-test**.)

WILLIOT DIAGRAM (ALSO WILLIOT-MOHR DIAGRAM). A graphical representation drawn to scale of the relative movements of the panel points of a plane truss under load. One of the joints is taken as fixed in position and one of the bars as fixed in direction. A **Mohr correction diagram** therefore is necessary for any unsymmetrical truss or unsymmetrical loading.

WIND (CLASSIFICATION AND EQUATIONS). The wind usually refers to the horizontal component of the air motion relative to the earth. The forces on an element of air moving almost horizontally over the earth's surface with velocity \mathbf{v} are: (1) the inertia force, represented by the acceleration $D\mathbf{v}/Dt$; (2) the pressure gradient force, represented by $-\frac{1}{\rho} \text{grad } p$; (3) the deviating force due to the earth's rotation, represented by $2\mathbf{v}\Omega \sin \phi$ at right angles to the velocity or more generally by $\mathbf{v} \times \mathbf{f}$; (4) the shearing stresses produced by the relative motion of the layers above and below, represented by \mathbf{F} ; so that the equation of motion is

$$\frac{D\mathbf{v}}{Dt} + \frac{1}{\rho} \text{grad } p - \mathbf{v} \times \mathbf{f} - \mathbf{F} = 0.$$

(1) (2) (3) (4)

Winds are classified according to the relative importance of various terms, as follows: (see also these items separately):

Geostrophic wind: The flow is unaccelerated and frictionless; only terms (2) and (3) remain. The wind is expressed in terms of the horizontal gradient of pressure, and is along the isobars.

Gradient wind: Here term (1) is assumed to be approximately equal to \mathbf{v}^2/R where R is the radius of curvature of the isobars (not of the streamlines) and acceleration in the direction of motion is ignored. The wind is given in terms of the pressure gradient by a quadratic equation. Term (4) is omitted.

Antitriptic wind: A wind in which terms (2)

and (4) are dominant and (4) represents friction at the ground and is therefore a force in the opposite direction to the motion, and the wind is towards low pressure.

Ageostrophic wind: The actual wind less the geostrophic wind. Departure from the geostrophic wind may be produced in a variety of ways (see **cyclostrophic force**, **isalobaric wind**, **geostrophic acceleration**) in frictionless motion; shearing stresses which make term (4) important, may be due to the shear of the **thermal wind** and eddy stresses usually caused by buoyant convection or to drag at the earth's surface. In the latter case the wind is always caused to blow with a component across the isobars towards low pressure, but if the stress is between two layers whose difference of velocity is due to the thermal wind, one of the layers will usually have a component towards high pressure.

In addition to the above considerations the velocity at a given level may be influenced by the descent (or ascent) of air from adjacent levels at which the velocity is different.

WIND LOAD. Building codes specify the minimum horizontal pressure which the structure must be designed to withstand. Pressures vary with height above ground and are generally in the range of 20 to 40 lbs per square foot. The pressure on the windward side is accompanied by a suction on the lee side. This suction or uplift is of special importance in the design of roofs.

WINDMILL. See **airscrew**.

WIND TUNNEL CORRECTIONS. See **wind tunnel interference**.

WIND-TUNNEL INTERFERENCE. The object of most wind-tunnel experiments is the investigation of the flow past a given body, when the body is placed in a uniform stream of infinite extent. Since the working section of the wind tunnel is of finite size, the flow is not the same as it would be in an infinite stream; corrections must therefore be applied to the measurements to allow for the effects of the working-section boundaries. The conditions imposed at the boundaries are simple; if the working section is closed, the velocity component normal to the wall must be zero at the wall, while if the working section is an open jet

the pressure must be constant along the boundary of the jet, and equal to that in the surrounding chamber.

For estimating the magnitude of the corrections the model in the tunnel is usually represented by an appropriate combination of vortices, sources and sinks, and the method of **images** is applied. For a wind tunnel with a rectangular working section, a doubly infinite array of images is used to satisfy the required condition at the boundary.

The problem of estimating the corrections is a complex one, usually approached by subdividing the total effect of the images into a number of independent components which are assumed to be additive. The more important of these components are considered below.

With a closed working section, one effect of the walls is to prevent the streamlines in the flow past the body from bulging out as they would in an infinite stream. Thus the streamlines near the body are forced closer together and the velocity is increased, an effect known as *blockage*.

With an open jet the condition of constant pressure at the boundary means that the pressures near the body are not reduced as much as they would be in an infinite stream. Thus the proximity of the jet boundary causes a *decrease* of velocity or negative "blockage" effect.

For a non-lifting body the blockage correction is usually estimated by replacing the body by a doublet, and calculating the velocity induced at the body by all the image doublets. For a lifting airfoil it is usually assumed that the blockage correction is independent of the angle of incidence and the calculation is made for the airfoil at zero lift.

In a closed tunnel there is an additional blockage correction due to the wake of the body. Within the wake the velocity is reduced, so that to satisfy the continuity condition the velocity outside the wake must be increased. The required correction is calculated by using a source at the body to represent the wake; the correction is then obtained as a function of the **drag coefficient** and the relevant dimensions of the body and tunnel. For an open jet wind tunnel it is usually assumed that the wake blockage correction is zero.

For a lifting airfoil there are further corrections to be applied, depending on the lift. In

calculating these corrections, the airfoil is usually replaced by a horse-shoe vortex if it is of finite span, or by a single bound vortex if it spans the wind tunnel so that it is effectively two-dimensional.

For a wing of finite span the chord is usually small compared with the height of the working section. The images of the bound vortex then have a negligible effect, and only the images of the trailing vortices need to be considered. The effect of these images (in a closed tunnel) is to induce an upwash at the position of the wing, tending to increase the incidence. Thus for a given lift coefficient, the measured values of the incidence and drag coefficient must be *increased* in order to obtain the value for an infinite stream.

For a two-dimensional airfoil spanning the working section, the images of the boundary vortex induce a velocity at the airfoil which has a component normal to the direction of the undisturbed stream. This transverse velocity component varies along the tunnel axis, so that in addition to the effects of the images on incidence and drag, there is an induced *curvature* of the stream. A given airfoil in this curved stream is equivalent to an airfoil of slightly different camber in a rectilinear stream. Thus corrections must be applied to the angle of incidence and drag and also to the camber of the airfoil. For convenience, the correction to the camber is often converted into equivalent corrections to lift and pitching moment, using results obtained from **thin-airfoil theory** for a circular-arc airfoil.

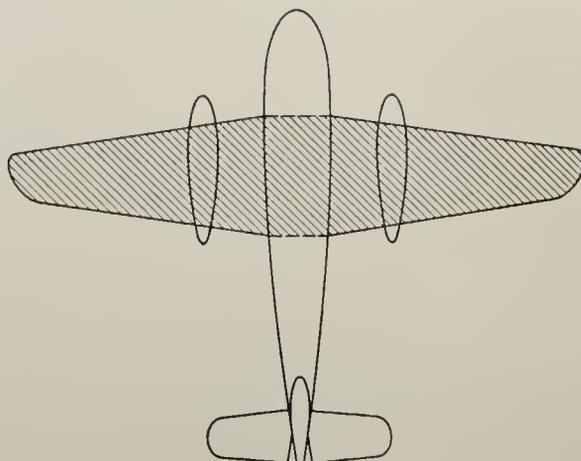
In most wind tunnels there is a small gradient of static pressure along the axis of the working section. This gradient affects the drag of a body in the wind tunnel in two ways; first there is a horizontal "buoyancy" force, and secondly there is an additional component of drag due to the acceleration of the flow. Both these effects are usually small unless the volume of the body is large, but they can be important in experiments on large streamline bodies of revolution.

If the Mach number of the stream is not small, the corrections discussed earlier must be modified to allow for the effects of compressibility. For subsonic speeds these modifications are calculated from the linearized equation, assuming that all the perturbation velocity components are small. It is found that the usual low-speed corrections have to

be multiplied by an appropriate power of $(1 - M^2)^{-1/2}$, the power depending on the particular correction involved and on whether the body is two- or three-dimensional. At supersonic speeds it is usually possible to choose the size of body, in relation to the tunnel size, so that all Mach waves and shock waves reflected from the tunnel walls pass well downstream of the body; there are then no effects of the walls and the flow is the same as in an infinite stream. (See R. C. Pankhurst and D. W. Holder, *Wind-Tunnel Technique* (Pitman, 1952).)

WIND TUNNELS, TURBULENCE IN. See **turbulence in wind tunnels.**

WING AREA (OF AIRPLANE). The wing area used for defining coefficients of force and moment for an airplane is usually the *gross wing area*, shown as the shaded area in the figure. This includes the area formed by join-



ing the wing roots with straight lines across the fuselage, and also includes any parts of the wing covered by engine nacelles.

WING DRAG. See **slender-body theory.**

WING, ELLIPTICALLY LOADED. See **elliptically loaded wing.**

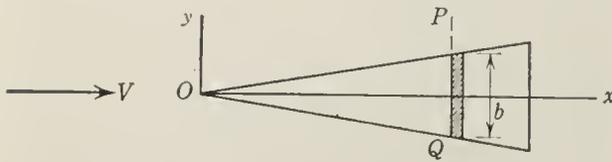
WING, INFINITE YAWED. See **infinite yawed wing.**

WING, PROFILE DRAG OF A. See **profile drag of a wing.**

WINGS OF SMALL ASPECT RATIO. A simple theory for thin wings of very small aspect ratio has been developed by R. T. Jones, following earlier work by Munk on airship hulls. The theory is really a special case of **slender-body theory** and is most suitable for

wings with swept-back leading edges ending in straight trailing edges normal to the direction of the stream.

The figure shows a simple example of a wing of this type; the axis of x is parallel to the direc-



tion of the stream and the wing is assumed to have zero thickness. The incidence of the wing is α and the velocity of the stream is V . The flow in a plane PQ , normal to the x -axis and at rest relative to the undisturbed fluid, is the same as that due to a two-dimensional flat plate moving through stationary fluid with a velocity $V\alpha$, normal to its surface. The assumption made in obtaining this result is essentially that $\frac{db}{dx}$ is small.

It can easily be shown that the lift on an element of the wing at PQ , of width b and length dx is

$$l = \frac{1}{2}\rho V^2 \pi \alpha b \cdot \frac{db}{dx} \cdot dx, \quad (1)$$

so that the *local* lift coefficient at PQ is

$$C_l = \pi \alpha \frac{db}{dx}. \quad (2)$$

Also, the pressure difference Δp between the two faces of the wing is given by

$$\frac{\Delta p}{\frac{1}{2}\rho V^2} = \frac{2\alpha}{\sin \theta} \cdot \frac{db}{dx}, \quad (3)$$

where $y = \frac{b}{2} \cos \theta$.

Thus Δp becomes infinite at the side of the wing (unless $\frac{db}{dx} = 0$). This corresponds to the infinite suction found theoretically at the leading edge of a thin two-dimensional airfoil.

If the wing has regions where $\frac{db}{dx}$ is negative, Equation (3) should be replaced in these regions by the **Joukowski condition**. A consequence of this is that sections of the wing behind the section of maximum width develop no lift.

It is found that the lift distribution across

the span of the wing is elliptical, and the overall lift coefficient of the wing is

$$C_L = \frac{\pi}{2} \cdot A \alpha, \quad (4)$$

where A is the aspect ratio.

The drag coefficient, associated with the energy in the trailing vortices, is

$$C_D = \frac{C_L^2}{\pi A} = C_L \frac{\alpha}{2}. \quad (5)$$

It is interesting to note that **lifting line theory** shows that the trailing-vortex drag coefficient of an unswept wing of high aspect ratio, with elliptic spanwise loading, is also equal to $\frac{C_L^2}{\pi A}$, this being the minimum value for given values of C_L and A .

Equation (5) also shows that the resultant force on the wing is in a direction half-way between the normal to the wing and the normal to the stream. For a wing of zero thickness this is only possible with an infinite suction at the leading edge. If it is assumed that this leading-edge suction is not developed, but that the other features of the flow are unchanged, the resultant force is normal to the wing and the drag coefficient is doubled. That is,

$$C_D = \frac{2C_L^2}{\pi A} = C_L \alpha. \quad (6)$$

In practice, with a thin uncambered wing of the kind shown in the figure, separation occurs at the leading edges. This prevents the development of leading-edge suction, and also leads to the formation of vortex sheets which coil up above the wing surface and cause an increase of lift, for a given incidence. (This extra lift is non-linear, being approximately proportional to α^2 .) Hence, even if the resultant force is still normal to the wing surface, so that $C_D = \alpha C_L$, C_D is less than the value $\frac{2C_L^2}{\pi A}$ given by Equation (6).

For the special case of the triangular (Delta) wing shown in the figure, the pressure is uniform along any straight line passing through the apex O , except in the vicinity of the trailing edge where the theory becomes invalid. Also, the center of pressure of the wing is at the centroid and the pressure distribution is nearly the same for all Mach numbers, at both subsonic and supersonic speeds, provided the half-

angle at the apex of the wing is always small compared with the Mach angle.

The theory given here cannot be applied satisfactorily to wings with straight leading edges normal to the stream. It does give reasonably accurate results, however, for wings of elliptical planform with aspect ratios up to about 1. (See also **delta wing**, **slender-body theory**.) (See A. Robinson and J. A. Laurmann, *Wing Theory*, Cambridge, 1956.)

WINKLER-BACH FORMULA. The stresses produced by bending of initially curved beams, in a plane of symmetry, with radius of center line or axis R a small multiple of the depth of the beam, can be derived easily by the assumption of linear distribution of fiber elongation and by taking stress as modulus times strain. The result is a hyperbolic distribution of stress, the Winkler-Bach formula.

$$\sigma = \frac{M}{AR} \left(1 + \frac{1}{B} \frac{y}{R + y} \right),$$

$$B = - \frac{1}{A} \int_A \frac{y}{R + y} dA$$

where A is the cross-sectional area, M , the bending moment, and y , the distance from the centroidal axis, positive outward from the center of curvature. The displacement of the neutral axis from the centroid is given by setting $\sigma = 0$, $y_0 = -BR/(B + 1)$.

The maximum stresses are given with high accuracy by the Winkler-Bach formula but the radial or σ_y stresses, which are quite large near the neutral axis, do not appear at all in the simplified theory.

WISHART DISTRIBUTION. In multivariate analysis, the distribution of **variances** and **covariances** in samples from a normal population.

WOOD EFFECT. The alkali metals are found to be transparent to light in the **ultraviolet**. According to the free electron theory of metals, the effect should occur for light of wavelengths less than

$$\lambda_0 = 2\pi(mc^2/4\pi Ne^2)^{1/2}$$

where e and m are the charge and mass of the free electrons, of number density N , and c is the velocity of light.

WORD. In the terminology of **digital computers**, a set of characters occupying one stor-

age cell and treated as a unit by the machine, e.g., in being transferred to and from registers.

WORK. The work done by a force on a particle during a given displacement is the space integral of the force over the path taken by the particle. Specifically if the force is \mathbf{F} and the elementary displacement of the particle $d\mathbf{r}$, where \mathbf{r} is its position vector, the work done is

$$\int_A^B \mathbf{F} \cdot d\mathbf{r}$$

where the path extends from point A to point B . At every point $d\mathbf{r}$ is directed along the path. (See **conservative system**; **work-kinetic energy theorem**.)

WORK, FLOW. See **flow work**.

WORK FUNCTION, ELECTRONIC. The energy (usually measured in electron-volts) needed to remove an electron from the **Fermi level** in a metal to a point an infinite distance away outside the surface. The work function is important in the theory of thermionic emission. In that case, as for example, that of an electron escaping from the heated, negatively-charged filament of a vacuum tube, the work function may be called the thermionic work function. Photoelectric emission has a corresponding work function.

WORK FUNCTIONS, THERMODYNAMIC. See **thermodynamics**, **characteristic functions** of.

WORK-KINETIC ENERGY THEOREM. The work done by the resultant force on a particle during a given displacement is equal to the change in kinetic energy experienced by the particle during this displacement. It also holds for a rigid body and for incompressible fluids.

WORK HARDENING. A metal is said to work harden or strain-harden if at a given state of stress and strain the additional stresses required for each change in strain do positive work on the change in strain. In simple tension, or more generally for proportional or radial loading, the meaning of work-hardening is simply positive slope of the stress-strain curve. (See **modulus**, **secant**.)

WORKING LOAD. The load for which a structure is designed.

WORKING STRESS. The maximum stress produced in an elastic body at the working load is called the working stress. The term is also used for allowable stress in design.

WORK, PRINCIPLE OF VIRTUAL. See virtual work, principle of.

WRENCH. A force and a couple whose moment vector is parallel to the **line of action** of the force constitute a wrench. The **resultant** of any force system can always be reduced to a wrench which is the simplest form conceptually, although usually not very helpful in the solution of problems. (See also **couple, theorems about.**)

WRINKLING. A term employed to designate the buckling pattern of thin **sheets**.

WRONSKIAN. Let y_1, y_2, \dots, y_n be functions of x , then the Wronskian of the functions is the **determinant**

$$W = \begin{vmatrix} y_1 & y_2 & \cdots & y_n \\ y'_1 & y'_2 & \cdots & y'_n \\ \cdot & \cdot & \cdot & \cdot \\ y_1^{(n-1)} & y_2^{(n-1)} & \cdots & y_n^{(n-1)} \end{vmatrix}.$$

If $W = 0$, the n functions are linearly dependent; if $W \neq 0$, they are linearly independent. When the y_i are solutions of a linear differential equation of n th order, evaluation of the Wronskian is a simple means for deciding whether or not the n functions give the complete solution of the differential equation.

X

XI (ξ). In the theory of slowing down of neutrons through collisions in matter, the average lethargy loss experienced by a neutron colliding with a free stationary atom of mass, M .

$$\xi = 1 + \frac{(M - 1)^2}{2M} \ln \frac{M + 1}{M - 1}.$$

X-RAY LEVELS, MOSELEY DIAGRAM FOR. See Moseley diagram for x-ray levels.

X-RAY REFLECTION, INTEGRATED. A measure of the intensity of the beam of x-rays reflected by a given atomic plane, obtained by rotating the crystal through a small angle about the general direction of the beam, and averaging the intensity. This is necessary because the beam is never quite sharply defined, owing to mosaic structure in the crystal. It is usually written

$$\rho = E\omega/I$$

where E is the total reflected energy, ω the angular velocity of rotation, and I the total incident radiation energy per second.

X-RAY SPECTRA, INTENSITY OF. The relation between the integrated reflection

$\rho(hkl)$ and the **structure amplitude** $F(hkl)$ for the (hkl) plane is given by

$$\rho(hkl) = \frac{N^2 e^4}{m^2 c^4} |F(hkl)|^2 \frac{\lambda^2}{2\mu \sin^2 \theta} \cdot \frac{1 + \cos^2 2\theta}{2}$$

where N is the number of unit cells per cm^3 of crystal, m is the mass of the atom, λ the wavelength of the x-rays, μ their absorption coefficient in the crystalline material, and θ the **glancing angle**.

X-RAY SPECTRUM, CHARACTERISTIC.

In an atomic spectrum the series of lines corresponding to transitions from or to one of the inner **electron shells** (K, L, \dots). (See also **Moseley's law; Moseley diagram for x-ray levels.**)

x-UNIT (XU) (SIEGBAHN x-UNIT). The unit of wavelength used in the X-ray and γ -ray region. It is 0.202% larger than 10^{-11} em (10^{-3} Å), the unit it was originally intended to represent. The exact relation between the two units is

$$1000 \text{ XU} = (1.002039 \pm 0.000014) \text{ \AA}.$$

Y

YANG-FELDMAN FORMALISM AND THE S-MATRIX. Suppose we are given a quantized field theory described by the following equation in the **Heisenberg picture**:

$$(\square + m^2)A(x) = -j(x)$$

where $j(x)$ is the source for the field $A(x)$. One can then with Yang and Feldman define the in-and-out field as follows:

$$\begin{aligned} A(x) &= A_{in}(x) - \int_{-\infty}^{+\infty} \Delta_R(x-x')j(x')d^4x' \\ &= A_{out}(x) - \int_{-\infty}^{+\infty} \Delta_A(x-x')j(x')d^4x' \end{aligned}$$

where Δ_A, Δ_R are the advanced and retarded singular functions, respectively. The in-and-out field thus satisfies free-field equations

$$(\square + m^2) A_{in/out}(x) = 0$$

since

$$(\square + m^2)\Delta_R(x-x') = -\delta(x-x').$$

On the assumption that

$$A_{in}^{(+)}(x)\Psi_0 = 0 = A_{out}^{(+)}(x)\Psi_0$$

where

$$\begin{aligned} A_{out}^{(+)}(x) &= i \int d\sigma^\mu(x) \Delta^{(\pm)}(x-x') \overleftrightarrow{\partial}_\mu A(x) \\ \overleftrightarrow{f \partial}_\mu g &= f \partial_\mu g - (\partial_\mu f) g \end{aligned}$$

and Ψ_0 is the (invariant) unique vacuum state of the theory, the in- and out-fields which satisfy the free-field commutation rules

$$[A_{in/out}(x), A_{in/out}(y)] = i\hbar c \Delta(x-y)$$

belong to the same irreducible representation of the free-field commutation rules. There therefore exists a unitary matrix, the **S-matrix**, such that

$$S^{-1}A_{in}(x)S = A_{out}(x).$$

Yang and Feldman further proved that this matrix is the same as the one customarily defined in the interaction representation.

From a consideration of the properties of the singular functions

$$\begin{aligned} \Delta(x) &= \frac{-i}{(2\pi)^3} \int_{-\infty}^{+\infty} d^4k \exp(-ik \cdot x) \delta(k^2 - \mu^2) \epsilon(k) \\ &= -\frac{1}{(2\pi)^4} \int_C \frac{\exp(-ik \cdot x)}{k^2 - \mu^2} d^4k \end{aligned}$$

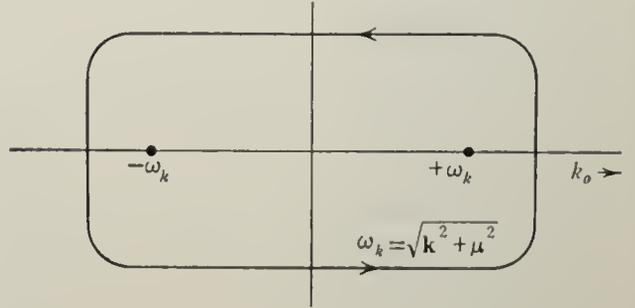


Fig. 1.

where C denotes the contour indicated in Figure 1 and

$$\Delta_R^A(x) = \frac{-1}{(2\pi)^4} \int_{C_R} \frac{\exp(-ik \cdot x)}{k^2 - \mu^2} d^4k$$

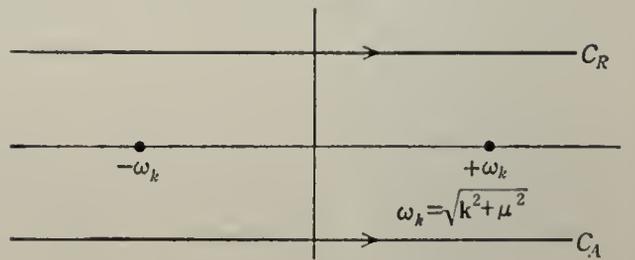


Fig. 2.

where the contour C_A and C_R is indicated in Figure 2, namely that

$$\begin{aligned} \Delta_R(x) &= -\Delta(x) & \text{for } x_0 > 0 \\ &= 0 & \text{for } x_0 < 0 \\ \Delta_A(x) &= \Delta(x) & \text{for } x_0 > 0 \\ &= 0 & \text{for } x_0 < 0 \end{aligned}$$

$$\Delta(x) = \Delta_A(x) - \Delta_R(x)$$

$$\Delta_A(x) = \Delta_R(-x).$$

Yang and Feldman tried to argue that since $\Delta_R(x) = 0$ for $x_o < 0$

$$\lim_{x_o \rightarrow -\infty} A(x) = A_{in}(x)$$

$$\lim_{x_o \rightarrow +\infty} A(x) = A_{out}(x).$$

If there were no divergences in the theory these definitions would be satisfactory if interpreted in the sense of weak operator convergence, i.e.,

$$\lim_{t \rightarrow \pm\infty} (\Phi, A^f(t)\Psi) = (\Phi, A_{in/out}^f\Psi)$$

where

$$A^f(t) = i \int_{\sigma} \left(\frac{\partial \bar{f}}{\partial x^\mu} A(x) - \bar{f} \frac{\partial A}{\partial x^\mu} \right) d\sigma^\mu(x)$$

and f is any solution of the Klein-Gordon equation

$$(\square^2 + m^2)f = 0 \quad (f, f) \geq 0.$$

Note that $A_{in/out}^f$ is time-independent. These are the **asymptotic conditions** for the field $A(x)$.

Whenever the theory is such as to require an (infinite) **wave function renormalization**, the above definition breaks down and one must in addition consider a smoothing in time procedure for $A^f(t)$ to define it rigorously.

YANG THEOREM ON ANGULAR DISTRIBUTIONS. In 1948, C. M. Yang proved a theorem which states that if only incoming waves of orbital angular momentum L contribute appreciably to a reaction, the angular distribution of the outgoing particles in the **center of mass system** is an even polynomial of cosine θ with maximum exponent not higher than $2L$. The angle θ is measured between the incoming and outgoing particles in the center of mass system.

YATES CORRECTION. An adjustment to the ordinary **chi-square** for a 2×2 table, proposed by F. Yates to correct for the effect of discontinuous variation in such tables when frequencies are small.

YAWED WING, INFINITE. See **infinite yawed wing**.

YEAR. The year may be considered as the time required for the sun to move apparently from one point back to the same point again. This apparent motion is due to the actual motion of the earth about the sun.

(1) The *sidereal year* is the time required for the sun to move apparently from a given star back to the same star again. From the purely mechanical point of view this is the true year. The length of the sidereal year is 365.25636 **mean solar days**.

(2) The *tropical year* is the time required for the sun to move apparently from the vernal equinox back to that point again. Because of precession of the equinox the tropical year is shorter than the sidereal, its length being 365.24220 mean solar days. This is the type of year that is used in calendar building for civil life.

(3) The *anomalous year* is the period between two successive passages of the sun through perihelion. This is 365.25964 mean solar days and is seldom used.

YIELD CONDITION. The stress level at which significant plastic deformation becomes possible.

YIELD CONDITION, GUEST. See **Guest yield condition**.

YIELD CONDITION, MISES. See **Mises yield condition**.

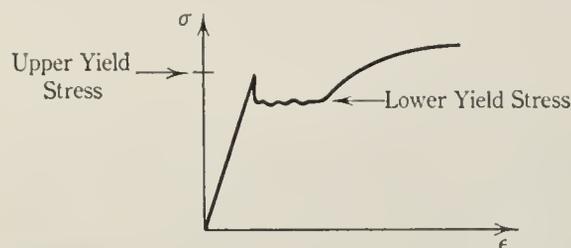
YIELD CONDITION, TRESCA. See **Tresca yield condition**.

YIELD LIMIT. See **limit, yield**.

YIELD POINT, STRESS, STRENGTH. A yield point or yield stress usually means the lowest stress at which appreciable plastic deformation is found. At the lower yield point of structural steel, plastic deformation of the order of one per cent occurs without increase of stress, but this is the gross effect of inhomogeneous plastic deformation, in which locally the successive elements of the material jump from an elastic upper yield point state to the strain representing the beginning of **work hardening**. In most metals the yield stress is difficult to define experimentally so that a yield strength is used. Offset yield strengths are established by lines parallel to the elastic beginning portion of the **stress-strain curves**; the **permanent set** value of 0.002 is common.

YIELD STRESS, LOWER. The yield stress required to maintain continued plastic flow after it has been initiated. (See **yield stress, upper**.)

YIELD STRESS, UPPER. Some materials, for example mild steel, demand a greater yield stress to initiate plastic flow than to maintain



Stress-strain curve showing upper and lower yield stresses.

it. The initiating stress is called the upper yield stress. A typical stress-strain curve for such a material is shown in the diagram.

YIELD SURFACE. See **surface, yield**.

YOUNG'S MODULUS. The ratio of the tensile force per unit cross-sectional area (stress)

to the extension per unit length for simple extension (strain) in a longitudinal direction, in a rod of isotropic material. (See **elastic constants**.) Young's modulus is often denoted by the letter E and, when the **generalized Hook's law** is applicable to the material, given in terms of the Lamé constants by

$$E = \frac{\mu(3\lambda + 2\mu)}{\lambda + \mu}.$$

YULE SCHEME. See **autoregression**.

YVON METHOD. In the analysis of the Boltzmann **transport equation**, this method, a generalization of the spherical harmonics method, is of particular use in the determination of flux near the boundary in systems having plane geometry. It consists in expanding the flux $\Phi(x, \mu, E)$ in terms of one set of polynomials for $\mu > 0$ and in terms of another set for $\mu < 0$. (See **spherical harmonics method**.)

Z

ZEEMAN EFFECT. In a magnetic field of strength H an atomic or molecular energy level corresponding to a total angular momentum \mathbf{J} is split into $2J+1$ component levels characterized by the magnetic quantum number

$$M = J, J - 1, J - 2, \dots, -J$$

and having the energy

$$W = W_0 - \bar{\mu}_H H$$

Here W_0 is the energy without field and $\bar{\mu}_H$ the mean value of the component of the magnetic moment of the atom or molecule in the field direction. (See also **space quantization**.)

The different contributions to the magnetic moment are: (1) the magnetic moment associated with the orbital and spin angular momenta of the electrons, (2) the magnetic moment associated with the nuclear spin, and (3) in molecules, the magnetic moment produced by the rotational motion. The first contribution is of the order of a **Bohr magneton**, the second and third are of the order of a **nuclear magneton**, that is of the order of $1/1836$ of the first. (For more detail on the effect of the nuclear magnetic moment see **hyperfine structure**.)

The **selection rules** for M are

$$\Delta M = 0, \pm 1 \quad \text{and} \quad M = 0 \leftarrow \rightarrow M = 0$$

for $\Delta J = 0$.

Lines corresponding to transitions with $\Delta M = 0$ are plane polarized with the electric vector parallel to the field direction (π -components), those with $\Delta M = \pm 1$ are plane polarized with the electric vector perpendicular to the field direction (σ -components).

I. Atomic Spectra. In a weak or moderately strong magnetic field (for very strong fields see **Paschen-Back effect**)

$$\bar{\mu}_H = -\mu_0 M g.$$

Here μ_0 is the Bohr magneton

$$\mu_0 = \frac{e h}{2mc 2\pi}$$

(e and m , respectively, are the charge and mass of the electron, h , Planck's constant, c , velocity of light), and g , the **Landé splitting factor**

$$g = 1 + \frac{J(J + 1) + S(S + 1) - L(L + 1)}{2J(J + 1)}.$$

(S and L , respectively, are quantum numbers corresponding to the resultant electron spin and the resultant electronic orbital angular momentum.)

The energy of the component levels is given by

$$W = W_0 - \bar{\mu}_H H = W_0 + h o M g$$

where

$$o = \frac{1}{2\pi} \frac{eH}{2mc}$$

is the **Larmor frequency**.

For $S = 0$ the g -factor is equal to 1, that is the Zeeman splitting is the same for all singlet terms. The Zeeman pattern of singlet lines (*normal Zeeman effect*) therefore consists of three equally spaced lines with $\Delta M = 0, +1,$

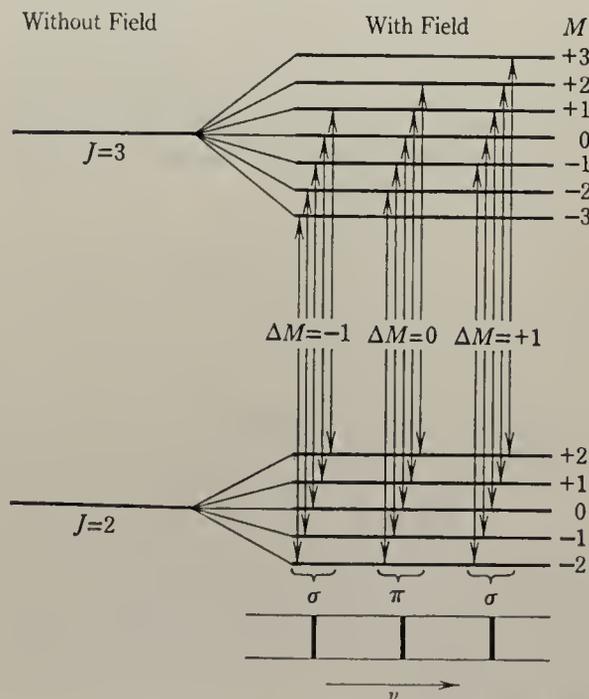


Fig. 1. Normal Zeeman effect for a combination $J = 3 \rightarrow J = 2$.

and -1 , respectively, and a separation (in cm^{-1}) $\Delta\nu = o/c = 4.6699 \times 10^{-5}H$ (see Figure 1).

For $S \neq 0$ (multiplet terms) the g -factor, and, as a consequence, the Zeeman splitting is different for different terms (*anomalous Zeeman effect*). Multiplet lines show a rather complicated, though always symmetrical Zeeman pat-

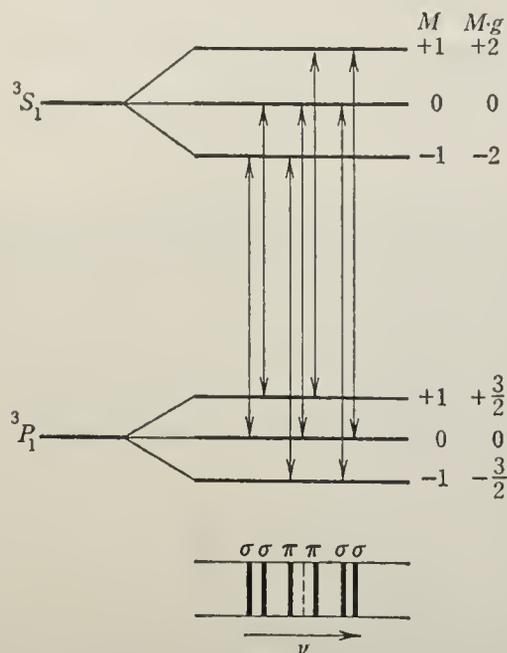


Fig. 2. Anomalous Zeeman effect, splitting of ${}^3S_1 \rightarrow {}^3P_1$ transition.

tern (see Figure 2). For more details see H. E. White, *Introduction to Atomic Spectra*, McGraw-Hill, Inc., New York, 1934, p. 217 ff.

II. *Molecular Spectra*. (a) $S = 0$; $\Lambda \neq 0$ (S and Λ , respectively, quantum numbers of the electron spin and the electronic orbital angular momentum about the internuclear axis of linear and diatomic molecules):

$$\bar{\mu}_H = \frac{\Lambda^2}{J(J+1)} M \mu_o.$$

(b) $S \neq 0$; $\Lambda = 0$:

$$\bar{\mu}_H = 2M_S \mu_o$$

where

$$M_S = S, S-1, \dots, -S$$

is the quantum number of the component of \mathbf{S} in the field direction.

(c) $S = 0$; $\Lambda = 0$ (electronic ground states of most molecules):

$$\bar{\mu}_H = g_r M \mu_{on}$$

where μ_{on} is the nuclear magneton

$$\mu_{on} = \frac{e}{2m_p c} \frac{h}{2\pi}$$

(m_p , mass of the proton), and g_r , the rotational g -factor, a number of the order 1, characteristic of the particular molecular state.

(d) $S \neq 0$; $\Lambda \neq 0$.

In this case complicated formulas hold. (For details see G. Herzberg, *Molecular Spectra and Molecular Structure, Vol. 1, Spectra of Diatomic Molecules*, p. 300 ff., D. Van Nostrand Co., Inc., Princeton, N. J., 1950.)

ZERNIKE'S ORTHOGONAL POLYNOMIALS. See circle polynomials.

ZERO GAP. See zero line of a band.

ZERO LINE OF A BAND. The line of wave number ν_o corresponding to the pure vibrational transition with $J' = J'' = 0$ (J' and J'' , rotational quantum numbers of the upper and lower state). This line is always forbidden, and therefore forms a gap in the regular series of lines (zero-gap). The wave number ν_o is frequently also referred to as "band origin."

ZERO-LOAD TEST. In a statically determinate truss under a given set of applied loads there is a unique state of bar forces. Therefore, with zero applied load the bars must each have zero force. A test for static determinacy is to assume a force in one of the bars despite the absence of load on a truss. The equations of equilibrium then are applied joint by joint. If it is possible to satisfy them, the truss is statically indeterminate.

ZERO PHASE OF LOOP FREQUENCY-FUNCTION. See gain margin.

ZERO POINT ENERGY. The energy still present in a system when it is in its ground state. This zero-point energy would not be present in a classical case, and is thus a typical quantum effect. For a harmonic oscillator it is equal to one-half of the energy of the oscillator quanta. In solids and liquids with their large number of degrees of freedom it will contribute an appreciable term to their binding energy. It is responsible for the fact that helium does not solidify under its own vapor pressure.

ZERO POINT FIELD. In quantum electrodynamics, the electromagnetic field is quan-

tized and, like any other quantum mechanical system, has a zero point energy, of one-half quantum per state of the field. The field associated with these zero point energies is known as the zero point field. (See also **vacuum polarization**.)

ZERO POINT FLUCTUATIONS. Fluctuations in the **zero point field**. (See also **vacuum polarization**.)

ZEROTH LAW OF THERMODYNAMICS. See **thermodynamics**, **zeroth law of**.

ZETA FUNCTION. Discussed under **Riemann zeta function**.

ZITTERBEWEGUNG. In the current due to the motion of a wave packet describing the state of a single Dirac electron, or in the ex-

pectation value of the coordinates, there is an oscillatory term due to an interference of those parts of the wave packet which belong to positive and negative energy states. This oscillation was called by Schrödinger "*Zitterbewegung*." There is no *Zitterbewegung* related to the motion of a wave packet which corresponds entirely to positive or to negative values of the energy.

ZONAL MAGNIFICATION. If (p_o, q_o) and (p_1, q_1) are the **direction cosines** of a ray of an axial bundle before and after refraction in an axial symmetric optical system and n_o, n_1 the corresponding indices, the $n_o p_o / n_1 p_1 = n_o q_o / n_1 q_1$ is the zonal magnification of the bundle.

ZUSTANDSSUMME. See **partition function**.

French—English Index

- Aberration de la lumière. Aberration of light
 Aberrations de fronts d'ondes. Wave front aberrations
 Aberrations de Seidel. Seidel aberrations
 Aberration de sphéricité. Spherical aberration
 Aberration de sphéricité latérale. Lateral spherical aberration
 Aberration de sphéricité longitudinale. Longitudinal spherical aberration
 Aberrations d'un système optique. Aberrations of an optical system
 Absorptance. Absorption factor
 Absorptance. Absorptance
 Absorption. Stopping power
 Absorption d'énergie radiante. Absorption of radiant energy
 Absorptivité. Absorptivity
 Accélération. Acceleration
 Accélération absolue. Absolute acceleration
 Accélération angulaire. Angular acceleration
 Accélération centripète. Centripetal acceleration
 Accélération de Coriolis. Coriolis acceleration
 Accélération de gravité. Acceleration of gravity
 Accélération géostrophique. Geostrophic acceleration
 Accélération tangentielle. Acceleration, tangential
 Acoustique par rayons. Ray acoustics
 Action. Action
 Activité absolue. Absolute activity
 Activité spécifique. Specific activity
 Addition de tenseurs. Addition of tensors
 Addition vectorielle. Vector addition
 Adjoint de matrice. Adjoint of a matrix
 Adjoint d'opérateur. Adjoint of an operator
 Advection. Advection
 Affinité électronique. Electron affinity
 Aile à charge elliptique. Elliptically-loaded wing
 Aile en delta. Delta wing
 Ailes de petit rapport de largeur à hauteur. Wings of small aspect ratio
 Aimantation. Magnetization
 Albédo. Albedo
 Algèbre. Algebra
 Algèbre booléenne. Boolean algebra
 Algèbre d'un groupe. Algebra of a group
 Algorithme. Algorithm
 Algorithme euclidien. Euclidean algorithm
 Aliénation. Alienation
 Allongement ou extension. Elongation or extension
 Alternance des multiplicités. Multiplicities, alternation of
 Alternatif. Alternating
 Amortissement. Damping
 Amortissement magnétomécanique. Magnetomechanical damping
 Amplification. Gain
 Amplification. Magnification
 Amplification angulaire. Angular magnification
 Amplification de conversion. Conversion gain
 Amplification latérale. Lateral magnification
 Amplification longitudinale. Longitudinal magnification
 Amplification normale. Normal magnification
 Amplification radiale. Radial magnification
 Amplitude de courbure normale. Curvature, amplitude of normal
 Amplitude de dispersion. Scattering amplitude
 Amplitude d'impulsion. Pulse amplitude
 Amplitude d'onde. Wave amplitude
 Amplitude d'oscillation. Amplitude of oscillation
 Analogie de position. Column analogy
 Analogie de Reynolds. Reynolds analogy
 Analogie de tas de sable. Sand heap analogy
 Analogie hydrodynamique. Hydrodynamical analogy
 Analogies dynamiques classiques. Dynamical analogies, classical
 Analogies dynamiques de mobilité. Dynamical analogies, mobility
 Analyse. Analysis
 Analyse de composant. Component analysis
 Analyse de covariance. Analysis of covariance
 Analyse des facteurs. Factor analysis
 Analyse de variance. Analysis of variance
 Analyse du lieu des racines. Root locus analysis
 Analyse numérique. Numerical analysis
 Analyse plastique. Plastic analysis
 Analyse successive. Sequential analysis
 Analyse tensorielle. Tensor analysis
 Analyseur différentiel. Differential analyzer
 Angle azimutal. Azimuth angle
 Angle d'aberration. Aberration angle
 Angle d'azimut principal. Angle of principal azimuth
 Angle d'éboulement. Repose, angle of
 Angle de Brewster. Brewster's Angle
 Angle de convergence. Angle of convergence
 Angle de déviation. Angle of deviation
 Angle de déviation. Lateral deflection
 Angle de diffraction. Angle of diffraction
 Angle de diffusion. Scattering angle
 Angle de frottement. Friction, angle of
 Angle de Hall. Hall angle
 Angle de liaison. Bond angle
 Angle de Mach. Mach angle
 Angle de phase. Phase angle
 Angle de projection. Projection angle
 Angle de réflexion. Glancing angle
 Angle de réflexion. Angle of reflection
 Angle dièdre. Dihedral angle
 Angle d'incidence. Incidence, angle of
 Angle d'incidence principale. Angle of principal incidence
 Angle de réfraction. Angle of refraction
 Angles de Russell. Russell angles
 Angle d'ouverture. Aperture angle
 Angles d'un cristal. Crystal angles
 Angle minimum de déviation. Deviation, minimum angle of
 Anharmonique. Anharmonicity
 Anisotrope. Anisotropic
 Anneau. Ring
 Anneau circulaire. Annulus
 Anneau de lie. Lic ring
 Anneau infinitésimal. Infinitesimal ring

- Anticommutation. Anticommutation
 Antiferromagnétisme. Antiferromagnetism
 Antisymétrique. Antisymmetric
 Aplati aux pôles. Oblate
 Achromatique. Achromatic
 Approche à la limite différé. Deferred approach to the limit
 Approximation dans la moyenne. Approximation in the mean
 Approximation de Born. Born approximation
 Approximation de champ moléculaire. Molecular field approximation
 Approximation de Goertzel-Greuling. Goertzel-Greuling approximation
 Approximation de Hartree. Hartree approximation
 Approximation de Kirkwood. Kirkwood's approximation
 Approximation de liaison étroite. Tight binding approximation
 Approximation quasi-chimique. Quasi-chemical approximation
 Arche. Arch
 Arête. Edge
 Arête de rebroussement. Edge of regression
 Armature. Truss
 Armature complexe. Truss, complex
 Armature composée. Truss, compound
 Armature ou support de Vierendeel. Vierendeel truss or girder
 Armature simple. Truss, simple
 Arrangement atomique. Configuration, atomic
 Arrangement électronique. Configuration, electronic
 Arrangement par interaction. Configuration, interaction
 Arrondissement. Rounding
 Assemblage. Assembly
 Association. Association
 Astigmatisme. Astigmatism
 Astigmatisme d'une surface. Astigmatism of a surface
 Asymptote. Asymptote
 Athermane. Athermanous
 Atténuation. Attenuation
 Atténuation prédéterminée. Weighting
 Autocorrélation. Autocorrelation
 Autofrettage. Autofretting
 Audiogramme. Audiogram
 Autorégression. Autoregression
 Axe. Shaft
 Axe de rotation. Rotation axis
 Axe de rotation avec translation. Screw axis
 Axe de rotation-inversion. Rotation-inversion axis
 Axe de rotation-réflexion. Rotation-reflection axis
 Axe de symétrie. Symmetry, axis of
 Axe de symétrie double. Symmetry, dyad
 Axe de symétrie quadruple. Symmetry, tetrad
 Axe de symétrie sextuple. Symmetry, hexad
 Axe de symétrie triple. Symmetry, triad
 Axe élastique. Elastic axis
 Axe faible. Axis, weak
 Axe fort. Axis, strong
 Axe instantané de rotation. Instantaneous axis of rotation
 Axe neutre. Neutral axis
 Axe optique. Axis, optic
 Axe principal. Axis, principal
 Axe s'appliquant au centre de gravité. Centroidal axis
 Axes cristallographiques. Crystallographic axes
 Axes mobiles. Moving axes
 Bande. Band
 Bande de conduction. Conduction band
 Bande de conduction dégénérée. Degenerate conduction band
 Bandes fondamentales. Fundamental bands
 Bar. Bar
 Barn. Barn
 Baroclinique. Baroclinic
 Barotrope. Barotropic
 Barre d'accouplement. Tie rod
 Barrière de Coulomb. Coulomb barrier
 Barrière de potentiel. Potential barrier
 Baryon. Baryon
 Base d'un système de nombres. Base of a system of numbers
 Battements. Beats
 Batterie. Battery
 Binaire. Binary
 Binormale. Binormal
 Biprisme. Biprism
 Biréfringence. Double refraction
 Boson. Boson
 Bougie-mètre. Meter-candle
 Bougie-mètre-seconde. Meter-candle-second
 Bougie-pied. Foot-candle
 Brachistochrone. Brachistochrone
 Branche. Branch
 Brilliance. Luminance
 Bruit. Noise
 Cadre. Loop
 Cadre d'inertie. Inertial frame
 Cadre. Frame
 Cadre rigide. Rigid frame
 Calcul. Calculus
 Calculateur. Computer
 Calculateur chiffre. Digital computer
 Calcul de rotationnels. Spinor calculus
 Calcul des variations. Variations, calculus of
 Calcul différentiel absolu. Absolute differential calculus
 Calcul opérationnel de Heaviside. Heaviside operational calculus
 Canal de réaction. Reaction channel
 Candela. Candela
 Candela par mètre carré. Candela per square meter
 Canonique. Canonical
 Cantilever. Cantilever
 Capacité de rotation. Rotation capacity
 Capacité d'amortissement spécifique. Specific damping capacity
 Capacité électrique. Electrical capacitance
 Capacité limite (charge). Carrying capacity (load)
 Capillarité. Capillarity
 Caractère. Character
 Caractéristique. Characteristic
 Caractéristique de fonctionnement. Operating characteristic
 Caractéristique de Hamilton. Hamilton's characteristic
 Caractéristique de surface. Surface, characteristic of
 Caractéristique directionnelle. Directional characteristic
 Caractéristique spectrale. Spectral characteristic
 Cardinal. Cardinal
 Carré intégrable. Integrable square
 Carte. Mapping

- Carte conforme. Conformal mapping
 Carte conforme du plan complexe. Conformal mapping of the complex plane
 Carte de Patterson. Patterson map
 Cas de couplage de Hund. Hund's coupling cases
 Catacaustique. Catacaustic
 Catastrophe de polarisabilité. Polarizability catastrophe
 Caténaire. Catenary
 Caustique. Caustic
 Caustique par réflexion. Diacaustic
 Cavitation. Cavitation
 Cellules de Bénard. Bénard cells
 Cellule de Hele-Shaw. Hele-Shaw cell
 Centre. Center
 Centre Aérodynamique. Aerodynamic center
 Centre de collinéation. Center of collineation
 Centre de courbure sphérique. Spherical curvature, center of
 Centre de cisaillement. Shear center
 Centre de courbure. Curvature, center of
 Centre de gravité. Center of gravity
 Centre de gravité d'une figure géométrique. Centroid of a geometrical figure
 Centre de masse. Center of mass
 Centre de moments. Center of moments
 Centre de percussion. Percussion, center of
 Centre de pression. Center of pressure
 Centre de symétrie. Symmetry, center of
 Centre d'inversion. Center of inversion
 Centre d'oscillation. Center of oscillation
 Centre d'un groupe. Center of a group
 Centre élastique. Elastic center
 Centre instantané. Centrode
 Centre instantané. Instantaneous center
 Centre optique. Optical center
 Centres ponctuels de répulsion. Point centers of repulsion
 Cercle de contrainte de Mohr. Mohr's circle for strain
 Cercle de courbure. Curvature, circle of
 Cercle de moindre diffusion. Circle of least confusion
 Cercle de Vieth-Müller. Vieth-Müller circle
 Cercle d'effort de Mohr. Mohr's circle for stress
 Cercle d'inertie de Mohr. Mohr's circle for inertia
 Cercle géodésique sur une surface. Geodesic circle on a surface
 Chaîne de Markov. Markov chain
 Chaleur. Thermal radiation
 Chaleur de dilution. Heat of dilution
 Chaleur de formation atomique. Atomic heat of formation
 Chaleur non compensée. Uncompensated heat
 Chaleurs spécifiques des gaz. Specific heats of gases
 Chaleur spécifique électronique. Electronic specific heat
 Champ. Field
 Champ critique. Critical field
 Champ de charge variable dans l'espace. Field of moving charge in space
 Champ de Coulomb. Coulomb field
 Champ de force conservatif. Force field, conservative
 Champ de Lorentz. Lorentz field
 Champ de rayonnement. Radiation field
 Champ de vecteurs. Vector field
 Champ de vue. Field of view
 Champ libre. Field, free
 Champ scalaire. Scalar field
 Champs internes dans les diélectriques. Internal fields in dielectrics
 Champ tensoriel. Tensor field
 Champ tensoriel. Tension field
 Champ tensoriel cartésien. Tensor field, cartesian
 Champ vectoriel aperiodique. Lamellar vector field
 Changement adiabatique. Adiabatic change
 Charge admissible. Working load
 Charge axiale. Axial load
 Charge composée. Combined loading
 Charge critique. Critical load
 Charge dynamique. Dynamic loading
 Charge excentrique. Eccentric loading
 Charge latérale. Lateral load
 Charge limite. Limit load
 Charge permanente. Dead load
 Charge proportionnelle. Proportional loading
 Charge radiale. Radial loading
 Charpente. Framework
 Chaudière. Boiler
 Chi-carré. Chi-square
 Chiffres significatifs. Digits, significant
 Chiffres significatifs. Significant figures
 Choc. Impact
 Choc élastique. Impact, elastic
 Choc non-élastique. Impact, inelastic
 Choc thermique. Thermal shock
 Ciment précontraint. Prestressed concrete
 Ciment renforcé. Concrete, reinforced
 Cinématique. Kinematics
 Cinématiquement admissible. Kinematically admissible
 Cinétique. Kinetics
 Cinétique de réaction. Kinetics, reactor
 Circuit-C. C-Circuit
 Circuit. Circuit
 Circuit orienté. Circuit, oriented
 Circuits fondamentaux. Circuits, fundamental
 Circulation. Circulation
 Cisaillement. Shear
 Cisaillement horizontal. Shear, horizontal
 Cisaillement longitudinal. Shear, longitudinal
 Classes de symétrie. Symmetry classes
 Clausius. Clausius
 Code. Code
 Coefficient auger. Auger coefficient
 Coefficient binôme. Binomial coefficient
 Coefficient d'absorption. Absorption coefficient
 Coefficient d'absorption acoustique des surfaces. Sound absorption coefficient of surfaces
 Coefficient d'absorption interne. Internal absorptance
 Coefficient d'absorption spectrale. Spectral absorptance
 Coefficient d'accommodation. Accommodation coefficient
 Coefficient d'Ångström. Ångström coefficient
 Coefficient d'Émission. Emission coefficient
 Coefficient d'extinction. Extinction coefficient
 Coefficient d'isolement du bruit. Noise insulation factor
 Coefficient de Callier. Callier coefficient.
 Coefficient de collision. Collision coefficient
 Coefficient de compressibilité. Compressibility factor
 Coefficient de condensation. Condensation, coefficient of
 Coefficient de contrainte. Restraint coefficient
 Coefficient de détermination. Determination, coefficient of
 Coefficient de diffusion. Diffusion factor
 Coefficient de dilatation. Coefficient of expansion
 Coefficient de dispersion. Scattering coefficient
 Coefficient de dispersion atomique. Atomic scattering factor
 Coefficients de factorielle. Factorial coefficients

- Coefficient de frottement. Friction, coefficient of
 Coefficient de non-détermination. Non-determination, coefficient of
 Coefficient de plasticité. Plastic modulus
 Coefficient de poussée. Lift coefficient
 Coefficient de pression. Pressure coefficient
 Coefficient de probabilité. Coefficient of probability
 Coefficient de Racah. Racah coefficient
 Coefficient de réchauffement. Reheat factor
 Coefficient de recombinaison. Recombination, coefficient of
 Coefficient de réflexion. Back-scattering coefficient
 Coefficient de réflexion acoustique. Sound reflection coefficient
 Coefficient de restitution. Restitution, coefficient of
 Coefficients de rigidité de facteur. Stiffness coefficients, factor
 Coefficient de sécurité. Safety factor
 Coefficient de surtension. Q factor
 Coefficient de tension. Coefficient of tension
 Coefficient de trainage. Drag coefficient
 Coefficients de transmission de turbulence. Turbulent transfer coefficients
 Coefficient de transmission interne. Internal transmittance coefficient
 Coefficient de variation. Coefficient of variation
 Coefficient de viriel. Virial coefficient
 Coefficient de Wigner. Wigner coefficient
 Coefficient lumineux. Luminous coefficient
 Coefficient osmotique. Osmotic coefficient
 Coefficient stoechiométrique. Stoichiometric coefficient
 Coefficients d'influence. Influence coefficients
 Coefficients de luminosité. Luminosity coefficients
 Coefficients de répartition (optiques). Distribution coefficients (optical)
 Coefficients thermiques. Thermal coefficients
 Cofacteur. Cofactor
 Cohésion partielle. Partial coherence
 Coïncidence rectiligne. Rectilinear congruence
 Collision directe. Direct collision
 Collision inverse. Inverse collision
 Cologarithme. Cologarithm
 Cologarithme d'un nombre. Antilogarithm of a number
 Colonne. Column
 Colonne modale d'une matrice. Modal column of a matrix
 Coma. Coma
 Combinaison. Combination
 Commutateur. Commutator
 Compact. Compact
 Complément. Complement
 Complémentarité. Complementarity
 Complément de sous-diagramme. Subgraph complement
 Complet. Complete
 Complexe d'un groupe. Complex of a group
 Compliance acoustique. Acoustical capacitance
 Compliance complexe. Compliance, complex
 Compliance de perte. Compliance, loss
 Compliance de verre. Compliance, glass
 Compliance rectiligne. Rectilineal compliance
 Compliance rotatoire. Rotational compliance
 Comportement d'avion. Airplane performance
 Composante de diagramme. Graph component
 Composante d'un vecteur. Component of a vector
 Composants indépendants. Independent components
 Composition de deux tenseurs. Composition of two tensors
 Composition de vecteurs. Composition of vectors
 Composition des forces. Forces, composition of
 Compresseur. Compressor
 Compressibilité. Compressibility
 Compression. Compression
 Compteur. Calculator
 Concentration. Concentration
 Concentration spectrale. Spectral concentration
 Concentrique. Homocentric
 Condensateur. Condenser
 Condensation d'Einstein. Einstein condensation
 Condensé. Condensate
 Condition asymptotique. Asymptotic condition
 Condition aux limites. Boundary condition
 Condition de Clausius. Clausius' condition
 Condition de fréquence de Bohr. Bohr frequency condition
 Condition de Herschel. Herschel's condition
 Condition de Hölder. Holder condition
 Condition de Joukowski. Joukowski condition
 Condition de Lipschitz. Lipschitz condition
 Condition de Lorentz. Lorentz condition
 Condition de Petzval. Petzval's condition
 Condition de rendement. Yield condition
 Condition de sinus de Abbe. Abbe sine condition
 Condition de stabilité chimique. Chemical stability condition
 Condition de Staebble-Lihotzky. Staebble-Lihotzky condition
 Condition élastique de Mohr. Mohr's yield condition
 Conditions aux limites de Neumann. Neumann boundary conditions
 Conditions de stabilité thermodynamique. Thermodynamic stability conditions
 Condition supplémentaire. Supplementary condition
 Cône dans écoulement supersonique. Cone in supersonic flow
 Cône de frottement. Friction, cone of
 Cône de Mach. Mach cone
 Cônes visuels. Cones, visual
 Confluence. Confluence
 Congru. Congruent
 Congruence de courbes. Congruence of curves
 Congruence normale. Normal congruence
 Conique. Conic
 Conjugation de liens doubles. Conjugation of double bonds
 Conjugée d'une matrice Hermitique. Hermitian conjugate of a matrix
 Connexion. Branch point
 Conservation de la masse. Conservation of mass
 Conservation de l'énergie. Energy conservation
 Conservation de l'énergie. Conservation of energy
 Conservation de spin. Conservation of angular momentum
 Conservation du moment. Conservation of momentum
 Consommation de chalur spécifique. Specific heat consumption
 Constant(e). Constant
 Constante chimique. Chemical constant
 Constante cryoscopique. Cryoscopic constant
 Constante de Boltzmann. Boltzmann constant
 Constante de chambre. Room constant
 Constante de densité de rayonnement. Radiation density constant
 Constante de désintégration. Decay coefficient
 Constante de Euler. Euler constant

- Constante de Euler-Mascheroni. Euler-Mascheroni constant
 Constante de Fermi. Fermi constant
 Constante de gaz. Gas constant
 Constante de Gruneisen. Gruneisen constant.
 Constante de Hubble. Hubble constant
 Constante de Karman. Karman constant
 Constante de Kundt. Kundt constant
 Constante de Lamb. Lamb's constant
 Constante de Madelung. Madelung constant
 Constante d'entropie. Entropy constant
 Constante de phase. Phase factor
 Constante de phase acoustique. Acoustical phase constant
 Constante de Planck. Planck constant
 Constante de propagation. Propagation constant
 Constante de propagation acoustique. Acoustical propagation constant
 Constante de radioscopie. Screening constant
 Constante de ressort. Spring constant
 Constante de longueur d'onde. Wavelength constant
 Constante de Rydberg. Rydberg constant
 Constante de structure fine de Sommerfeld. Sommerfeld's fine structure constant
 Constante de Verdet. Verdet constant
 Constante diélectrique. Dielectric constant
 Constante d'intégration. Constant of integration
 Constante électromagnétique. Electromagnetic constant
 Constante photoélectrique. Photoelectric constant
 Constantes d'élasticité. Elastic constants
 Constantes de symétrie de révolution d'une molécule. Rotational constants of a molecule
 Constantes piézoélectriques. Piezoelectric constants
 Contingence. Contingency
 Continu. Connected
 Continuation analytique. Analytic continuation
 Continuité. Continuity
 Continuité absolue. Absolute continuity
 Continuité d'état. Continuity of state
 Contraction. Contraction
 Contrainte. Constraint
 Contrainte de fibre. Fiber stress
 Contrainte généralisée. Strain, generalized
 Contrainte normale. Normal stress
 Contrainte superflue. Redundant constraint
 Contraste. Contrast
 Contrôle proportionnel. Control, proportional
 Contrôle statistique de qualité. Statistical quality control
 Convection. Convection
 Convection de turbulence. Turbulence convection
 Convention de sommation. Summation convention
 Convergence. Convergence
 Convergence conditionnelle. Conditional convergence
 Coordonnée. Coordinate
 Coordonnée bipolaire. Bipolar coordinate
 Coordonnée cachée. Hidden coordinate
 Coordonnées canoniques. Canonical coordinates
 Coordonnées coniques. Conical coordinates
 Coordonnées cylindriques. Cylindrical coordinates
 Coordonnées cylindriques elliptiques. Elliptic cylindrical coordinates
 Coordonnées cylindriques paraboliques. Parabolic cylindrical coordinates
 Coordonnées de chromaticité. Chromaticity coordinates
 Coordonnées de symétrie. Symmetry coordinates
 Coordonnées ellipsoïdales. Ellipsoidal coordinates
 Coordonnées et moments généralisés. Coordinates and momenta, generalized
 Coordonnées géodésiques. Geodesic coordinates
 Coordonnées orthogonales curvilignes. Curvilinear orthogonal coordinates
 Coordonnées polaires géodésiques. Geodesic polar coordinates
 Coordonnées paraboliques. Parabolic coordinates
 Coordonnées paraboloidales. Paraboloidal coordinates
 Coordonnées polaires. Polar coordinates
 Coordonnées polaires sphériques. Spherical polar coordinates
 Coordonnées rectangulaires. Rectangular coordinates
 Coordonnées sphéroïdales. Spheroidal coordinates
 Coordonnées sphéroïdales aplaties. Oblate spheroidal coordinates
 Coordonnées toroïdales. Toroidal coordinates
 Corde. Chord
 Corde aérodynamique moyenne. Aerodynamic mean chord
 Corde d'aile portante. Chord of airfoil
 Corps gris. Gray body
 Corps noir. Black body
 Corps rigide. Rigid body
 Correcteur. Corrector
 Correction. Correction
 Correction de Eucken. Eucken correction
 Correction de Rydberg. Rydberg correction
 Correction de Yates. Yates correction
 Correction par rapport au vide. Correction to vacuum
 Correction du gamma. Gamma correction
 Correction radiative. Radiative correction
 Corrections de Dancoff. Dancoff corrections
 Corrections de Sheppard. Sheppard's corrections
 Corrélacion. Correlation
 Corrélacion angulaire. Angular correlation
 Corrélacion bisériale. Biserial correlation
 Corrélacion canonique. Canonical correlation
 Corrélacion de degré. Rank correlation
 Corrélacion tétrachorique. Tetrachoric correlation
 Corrélogramme. Correlogram
 Cosinus de direction optique. Optical direction cosines
 Couche de valeur moyenne. Half-thickness
 Couche électronique. Shell
 Couche limite. Boundary layer
 Couche limite de turbulence. Turbulent boundary layer
 Couche limite laminaire. Laminar boundary-layer
 Couches électroniques dans un atome. Electron shells in an atom
 Couleur. Color
 Couleur de Planck (corps noir). Planckian color
 Couplage. Coupling
 Couplage dans les structures atomiques et moléculaires. Coupling, in atomic and molecular structures
 Couplage d'échange. j, j coupling
 Couplage gradient. Gradient coupling
 Couplage pseudoscalaire. Pseudoscalar coupling
 Couplage pseudovectoriel. Pseudovector coupling.
 Couplage Russell-Saunders. Russell-Saunders coupling
 Couple. Couple
 Coupleur. Link
 Constante de couplage. Coupling constant
 Couple. Torque
 Coupure. Cut-off
 Courant de redressement de contact. Contact rectification current

- Courant de rotation. Shear flow
 Courant humide. Wet stream
 Courant naturel. Streamline
 Courant neutronique. Current, neutron
 Courant neutronique. Streaming, neutron
 Courant plastique. Plastic flow
 Courants naturels libres. Free streamlines
 Courbe. Curve
 Courbé. Bent
 Courbe analytique. Analytic curve
 Courbe d'accroissement. Growth curve
 Courbe d'ébullition et courbe de condensation. Boiling curve and condensation curve
 Courbe de cristallisation. Crystallization curve
 Courbe de dispersion expérimentale. Scattering curve, experimental
 Courbe de distribution d'intensité. Intensity distribution, curve of
 Courbe de Jordan. Jordan curve
 Courbe de Morse. Morse curve
 Courbe d'espace. Space curve
 Courbe de sublimation. Sublimation curve
 Courbe d'excitation. Excitation curve
 Courbe d'interaction. Interaction curve
 Courbe effort-déformation. Stress-strain curve
 Courbe élastique. Elastic curve
 Courbe isobougie. Isocandela curve
 Courbe logistique. Logistic curve
 Courbe ou câble caténaire. Catenary curve or cable
 Courbes de Bertrand. Bertrand curves
 Courbes de potentiel de répulsion dans une molécule. Repulsive potential curves in a molecule
 Courbes de Sargent. Sargent curves
 Courbes d'intensité sonore. Loudness contours
 Courbe torse. Twisted curve
 Courbure. Bending
 Courbure. Kurtosis
 Courbure. Flexure
 Courbure. Curvature
 Courbure de champ. Curvature of field
 Courbe de déviation. Deflection curve
 Courbe de rendement de fission. Fission yield curve
 Courbure de rayon. Curvature of beam
 Courbure de vis. Curvature, screw
 Courbure d'une courbe en un point. Curvature of a curve at a point
 Courbure d'une surface. Curvature of a surface
 Courbure normale moyenne. Curvature, mean normal
 Courbe du spectre. Spectrum locus
 Courbure géodésique. Geodesic curvature
 Courbure normale. Normal curvature
 Courbure scalaire. Scalar curvature
 Courbure totale de lentille. Curvature of lens, total
 Courbure vectorielle. Vector curvature
 Covariance. Covariance
 Covolume. Covolume
 Cracoviens. Cracovians
 Crête d'onde. Wave crest
 Creux d'onde. Wave trough
 Cristallogramme. Crystallogram
 Cristallographie. Crystallography
 Cristaux tétraoédres. Tetrahedral crystals
 Critère de Mach. Mach criterion
 Critère série de Lundquist. Lundquist's series criterion
 Critérium de Rayleigh du pouvoir de résolution. Rayleigh criterion for resolution
 Critérium de Wilks. Wilks' criterion
 Crochet de Lagrange. Lagrange bracket
 Crochet de Poisson. Poisson bracket
 Crochets de Gauss. Gaussian brackets
 Cumulateurs. Cumulants
 Curie. Curie
 Cycle. Cycle
 Cycle de Born-Haber. Born-Haber cycle
 Cycle de Carnot. Carnot cycle
 Cycle d'effort. Cycle of stress
 Cycle de Rankine. Rankine cycle
 Cycle de Rankine inverse. Reversed Rankine cycle
 Cycloïde. Cycloid
 Cylindre rigide. Rigid cylinder
 D'Alembertien. D'Alembertian
 Décalage. Angular displacement
 Décibel. Decibel
 Décimal. Decimal
 Décomposition des forces. Forces, resolution of
 Décomposition d'un vecteur. Decomposition of a vector
 Décomposition en facteurs polynômes. Polynomial factorisation
 Découverte d'erreurs tabulaires. Detection of tabular errors
 Décrément logarithmique. Logarithmic decrement
 Défaut de masse. Mass defect
 Défaut de Schottky. Schottky defect
 Définition de Strchl. Strehl definition
 Déflation. Deflation
 Déformation. Deformation
 Déformation permanente. Set permanent
 Dégénération d'échange. Exchange degeneracy
 Dégradation de l'énergie. Degradation of energy
 Degré de liberté. Degree of freedom
 Degré de microréciprocité. Microreciprocal degree
 Degré de répétition d'impulsions. Pulse repetition rate
 Degré. Grade
 Delta de Kronecker. Kronecker delta
 Demi-largeur d'une ligne spectrale. Half-width of a spectral line
 Dénombrable. Countable
 Dense. Dense
 Densité de charge des noyaux. Charge density of nuclei
 Densité. Density
 Densité de Lagrange. Lagrange density
 Densité de charge magnétique. Magnetic charge density
 Densité de courant magnétique. Magnetic current density
 Densité d'énergie acoustique. Sound-energy density
 Densité de probabilité. Probability density
 Densité de ralentissement. Slowing-down density
 Densité des niveaux d'énergie dans une bande de conduction. Density of quantum states in conduction band
 Densité lumineuse. Density, luminous
 Densité neutronique. Density, neutron
 Densité optique. Optical density
 Densité optique externe. External optical density
 Densité scalaire. Scalar density
 Densité spéculaire. Density, specular
 Densité tensorielle. Tensor density
 Dérivée. Derivative
 Déphasage pour dispersion. Scattering phase shift
 Déplacement de Lamb. Lamb shift
 Déplacement électrique dans les diélectriques. Electrical displacement in dielectrics

- Déplacement parallèle d'un vecteur. Parallel displacement of a vector
- Déplacement temps-température. Time-temperature shift
- Déplacement. Displacement
- Dérivée covariante d'un champ tensoriel. Covariant derivative of tensor field
- Dérivée de Helmholtz. Helmholtz derivative
- Dérivée de vecteur. Vector derivative
- Dérivée d'un tenseur. Tensor derivative
- Dérivée intrinsèque d'un champ tensoriel. Intrinsic derivative of tensor field
- Dérivée totale. Total derivative
- Désaimantation adiabatique. Adiabatic demagnetization
- Déterminant. Determinant
- Déterminant de Fredholm. Fredholm determinant
- Déterminant de Gram. Gram determinant
- Déterminant de Slater. Slater determinant
- Déterminant séculaire. Secular determinant
- Détermination de courbes empiriques. Curve fitting
- Détermination totale. Total determination
- Développable. Developable
- Développante d'une courbe. Involute of a curve
- Développante d'une surface. Involute of a surface
- Développante osculatrice. Developable, osculating
- Développante polaire. Developable, polar
- Développante rectificative. Developable, rectifying
- Développée. Evolute
- Déviation. Deflection
- Déviation. Straggling
- Déviation de courbure. Deflection, bending
- Déviation moyenne. Mean deviation
- Déviation totale d'un prisme. Prism, total deviation of
- Diagonal(e). Diagonal
- Diagramme complet. Graph, complete
- Diagramme continu. Graph, connected
- Diagramme de Boulvin. Boulvin diagram
- Diagramme de chromaticité. Chromaticity diagram
- Diagramme de corrélation. Correlation diagram
- Diagramme de dispersion. Scatter diagram
- Diagramme de Mollier. Mollier chart
- Diagramme d'épaisseur. Thickness pattern
- Diagramme de rayonnement. Radiation pattern
- Diagramme de Rousseau. Rousseau diagram
- Diagramme des moments. Moment diagram
- Diagramme des niveaux d'énergie. Energy level diagram
- Diagramme de tirage en dégradé. Vignetting diagram
- Diagramme de Williot. Williot diagram
- Diagramme dirigé. Graph, directed
- Diagramme double. Graph, dual
- Diagramme en losange. Lozenge diagram
- Diagramme d'entropie. Entropy chart
- Diagramme planaire. Graph, planar
- Diagrammes de régime de signal. Signal flow diagrams
- Diagrammes homeomorphes. Graphs, homeomorphic
- Diagrammes isomorphes. Graphs, isomorphic
- Diagrammes moléculaires. Molecular diagrams
- Diagramme topologique. Graph, topological
- Dialytique. Dialytic
- Diamagnétisme. Diamagnetism
- Diamètre apparent. Apparent diameter
- Diapoint. Diapoint
- Diathermane. Diathermanous
- Dichroïsme circulaire. Circular dichroism.
- Différence astigmat. Astigmatic difference
- Différence de pression thermomoléculaire. Thermomolecular pressure difference
- Différence de retour. Return difference
- Différence moyenne. Mean difference
- Différences de combinaison. Combination differences
- Différences divisées. Divided differences
- Différences réciproques. Reciprocal differences
- Différentiation covariante. Covariant differentiation
- Différentiation. Differentiation
- Différentiation logarithmique. Logarithmic differentiation
- Différentiation sous le signe intégral. Differentiation under the integral sign
- Différentiel. Differential
- Diffluence. Diffluence
- Diffraction. Diffraction
- Diffraction de neutrons. Diffraction of neutrons
- Diffuseur. Diffuser
- Diffusion. Angular distribution
- Diffusion de la lumière. Diffusion of light
- Diffusion de Rayleigh. Rayleigh scatter
- Diffusion des solides. Diffusion of solids
- Diffusion de Thomson. Thomson scattering
- Diffusion potentielle. Potential scattering
- Digramme. Digraph
- Dilatation. Dilatation
- Dilatation. Expansion
- Dimension. Dimension
- Dimensions de réseau. Lattice dimensions
- Diminution de turbulence. Turbulence, decay of
- Dioptrie. Diopter
- Dipôle. Dipole
- Dipôle. Doublet
- Direction. Direction
- Directionnel. Directional
- Directions conjuguées. Conjugate directions
- Directions principales. Principal directions
- Directions privilégiées. Privileged directions
- Directrice d'une surface réglée. Directrix of a ruled surface
- Discontinuité. Discontinuity
- Discriminant. Discriminant
- Dislocation. Dislocation
- Dispersion. Dispersion
- Dispersion. Scattering
- Dispersion anormale. Anomalous dispersion
- Dispersion de rotation. Dispersion of rotation
- Dispersion de rotation. Rotatory dispersion
- Dispersion linéaire réciproque. Dispersion, reciprocal linear
- Dispersion partielle. Dispersion, partial
- Dispersion sonore. Scattering, sound
- Dispersivité de réfraction. Refractive dispersivity
- Dispersivité moléculaire. Dispersivity, molar
- Dispersivité spécifique. Dispersivity, specific
- Disque de Rayleigh. Rayleigh disc
- Dissipation. Dissipation
- Distance de Mahalanobi. Mahalanobi's distance
- Distance focale. Focal length
- Distance focale réduite. Reduced focal length
- Distorsion. Distortion
- Distorsion géométrique. Geometric distortion
- Distribution arc-sinus. Arc-sine distribution
- Distribution bêta. Beta distribution
- Distribution binôme. Binomial distribution
- Distribution de Cauchy. Cauchy distribution
- Distribution d'échantillonnage. Sampling distribution
- Distribution de Fisher. Fisher's distribution
- Distribution de Gauss. Gaussian distribution
- Distribution d'énergie radiante par bande de longueurs d'onde. Spectral energy distribution

- Distribution d'énergie spectrale relative. Relative spectral energy distribution
- Distribution de Porter-Thomas. Porter-Thomas distribution
- Distribution de Pascal. Pascal distribution
- Distribution de Poisson. Poisson distribution
- Distribution de probabilité. Probability distribution
- Distribution de "Student". Student's distribution
- Distribution de Wishart. Wishart distribution
- Distribution en énergie. Distribution in energy
- Distribution en forme de J. J-shaped distribution
- Distribution en moment. Distribution in momentum
- Distribution en phase. Distribution in phase
- Distribution exponentielle. Exponential distribution
- Distribution gamma. Gamma distribution
- Distribution hypergéométrique. Hypergeometric distribution
- Distribution logarithmique. Logarithmic distribution
- Distribution normale. Normal distribution
- Distribution rectangulaire. Rectangular distribution
- Distributions de Pearson. Pearson distributions
- Distributions non-centrales. Non-central distributions
- Divergence. Divergence
- Divergence de surface d'un vecteur. Surface divergence of a vector
- Division synthétique. Synthetic division
- Domaine. Domain
- Doublage d'inversion. Inversion doubling
- Ductilité. Ductility
- Durcissement dû au fonctionnement. Work hardening
- Durée d'affaiblissement d'impulsion. Pulse decay time
- Durée de diffusion neutronique. Diffusion time, neutron
- Durée de réaction. Period, reactor
- Durée d'établissement d'impulsion. Pulse rise time
- Durée d'établissement d'impulsion. Pulse time, leading-edge
- Durée de vie moyenne. Life, mean
- Durée d'impulsion moyenne. Pulse time, mean
- Durée d'impulsions. Pulse duration
- Dureté. Hardness
- Dyadiques. Dyadics
- Dynamique. Dynamics
- Dynamique des gaz raréfiés. Rarefied gas dynamics
- Ébullition. Boiling
- Écart d'énergie. Energy gap
- Échantillon. Sample
- Échantillon systématique. Systematic sample
- Échantillonnage des quotas. Quota sampling
- Échelle centigrade. Centigrade scale
- Échelle de Pythagore. Pythagorean scale
- Échelle juste. Scale, just
- Échelon. Echelon
- Éclairement. Illumination
- Économies dues à réflecteur. Reflector savings
- Écoulement de gaz dans tuyère. Nozzle, flow of gas in
- Écoulement de glissement. Slip flow
- Écoulement hypersonique. Hypersonic flow
- Écoulement laminaire dans un canal. Channel, laminar flow in
- Écoulement non-rotationnel. Irrotational flow
- Écoulements subsonique et supersonique. Subsonic and supersonic flow
- Effet Auger. Auger effect
- Effet Bauschinger. Bauschinger effect.
- Effet Debye. Debye effect
- Effet de Coanda. Coanda effect
- Effort de cohésion. Bond stress
- Effet de Compton. Compton effect
- Effet de contrainte normale. Normal stress effect
- Effet de glissement. Slip effect
- Effet de phase binaural. Binaural phase effect
- Effet de taille. Size effect
- Effet de transmission du bruit. Noise transmission effect
- Effet d'intermittence. Intermittency effect
- Effet Doppler-Fizeau pour le son. Doppler effect for sound
- Effet Faraday. Faraday effect
- Effet inductif et réactivité. Inductive effect and reactivity
- Effet isotope dans les spectres moléculaires. Isotope effect in molecular spectra
- Effet magnétocalorique. Magnetocaloric effect
- Effet Paschen-Back. Paschen-Back effect
- Effet photoélectrique. Photoelectric work function
- Effet Raman. Raman effect
- Effet rapide d'interaction. Fast effect, interaction
- Effets Doppler. Doppler effects
- Effets galvanomagnétiques et thermomagnétiques. Galvanomagnetic and thermomagnetic effects
- Effet Slutsky-Yule. Slutsky-Yule effect
- Effet Stark. Stark effect
- Effet volta. Contact potential
- Effet Weissenberg. Weissenberg effect
- Effet Wood. Wood effect
- Effet Zeeman. Zeeman effect
- Efficace. Root-mean-square
- Efficacité lumineuse. Luminous efficiency
- Efficacité lumineuse relative. Relative luminous efficiency
- Efficacité neutronique. Collision density, neutron
- Efficacité radiante. Radiant efficiency
- Effort admissible. Stress, allowable
- Effort circulaire. Stress, circumferential
- Effort composé. Stress, combined
- Effort de service. Working stress
- Effort direct. Stress, direct
- Effort généralisé. Stress, generalized
- Effort hydrostatique. Stress, hydrostatic
- Effort octaédrique. Stress, octahedral
- Effort principal. Stress, principal
- Efforts d'assemblage. Assembly stresses
- Efforts de réaction. Reaction stresses
- Efforts de Reynolds. Reynolds stresses
- Efforts résiduels. Residual stresses
- Eikonal. Eikonal
- Élasticité retardée. Elasticity, delayed
- Électrodynamique de Minkowski pour corps en mouvement. Minkowski's electrodynamics for moving bodies
- Électrodynamique quantique. Quantum electrodynamics
- Électroneutralité. Electroneutrality
- Électrons équivalents. Equivalent electrons
- Électrons non-équivalents. Non-equivalent electrons
- Électron-volt. Electron-volt
- Élémentaire. Elementary
- Élément de circuit. Element, circuit
- Élément de symétrie. Symmetry element
- Élément orienté. Element, oriented
- Élément. Element
- Éléments conjugués d'un groupe. Conjugate elements of a group
- Élément unité. Unit element
- Élimination de Gauss. Gaussian elimination

- Élimination. Elimination
 Ellipse. Ellipse
 Ellipse géodésique. Geodesic ellipse
 Ellipsoïde. Ellipsoid
 Ellipsoïde de moment. Ellipsoid, momental
 Ellipsoïde de Poincot. Ellipsoid of Poincot
 Ellipsoïde d'inertie. Inertia ellipsoid
 Émetteur sélectif. Selective emitter
 Émittance. Emittance
 Émittance. Radiant emittance
 Énergie. Energy
 Énergie au point zéro. Zero point energy
 Énergie cinétique. Energy, kinetic
 Énergie cinétique d'un mouvement non-rotatoire. Kinetic energy of irrotational motion
 Énergie complémentaire. Complementary energy
 Énergie critique. Threshold energy
 Énergie d'activation. Activation energy
 Énergie d'anisotropie. Anisotropy energy
 Énergie d'arête de bande. Band edge energy
 Énergie d'échange. Exchange energy
 Énergie de cohésion. Energy, cohesion
 Énergie de contrainte magnétique. Magnetic strain energy
 Énergie de corrélation. Correlation energy
 Énergie de Coulomb. Coulomb energy
 Énergie de Debye. Debye energy
 Énergie de désintégration alpha. Alpha disintegration energy
 Énergie de désintégration bêta. Beta disintegration energy
 Énergie de désintégration d'une réaction nucléaire. Reaction energy, nuclear
 Énergie de dissociation. Dissociation energy
 Énergie de dissociation d'une molécule. Dissociation energy of a molecule
 Énergie de distorsion. Distortion energy
 Énergie de Fermi. Fermi energy
 Énergie de Keesom. Keesom energy
 Énergie de liaison. Binding energy
 Énergie de liaison atomique. Atomic bond energy
 Énergie de paroi. Wall energy
 Énergie de réseau des cristaux. Lattice energy of crystals
 Énergie de résonance. Delocalization energy
 Énergie de résonance. Resonance energy
 Énergie de turbulence. Turbulence, energy of
 Énergie d'excitation. Excitation energy
 Énergie d'ionisation. Ionization energy
 Énergie d'un système à n -particules. Energy of n -particle system
 Énergie disponible. Available energy
 Énergie électronique (classique). Electron energy (classical)
 Énergie libérée. Q -value
 Énergie potentielle. Energy, potential
 Énergie potentielle dans un diélectrique. Self energy in a dielectric
 Énergies de liaison. Bond energies
 Ensemble. Ensemble
 Ensemble. Manifold
 Ensemble canonique. Canonical ensemble
 Ensemble macrocanonique. Macrocanonical ensemble
 Ensemble microcanonique. Microcanonical ensemble
 Ensemble secondaire. Coset
 Enthalpie. Enthalpy
 Entropie. Entropy
 Entropie de gaz monoatomique. Entropy of monatomic gas
 Enveloppe. Envelope
 Épaisseur d'énergie de couche limite. Energy thickness of boundary layer
 Équation. Equation
 Équation à deux composantes du neutrino. Two-component equation of the neutrino
 Équation adiabatique d'état. Adiabatic equation of state
 Équation à différences. Difference equation
 Équation adjointe. Adjoint equation
 Équation à quatre moments. Four-moment equation
 Équation auxiliaire. Auxiliary equation
 Équation bicarrée. Biquadratic equation
 Équation biharmonique. Biharmonic equation
 Équation canonique de mouvement. Canonical equation of motion
 Équation caractéristique. Characteristic equation
 Équation caractéristique de matrice. Characteristic equation of a matrix
 Équation critique. Critical equation
 Équation d'Abel. Abel equation
 Équation de Benedict-Webb-Rubin. Benedict-Webb-Rubin equation
 Équation de Bernoulli. Bernoulli equation
 Équation de Bethe-Salpeter. Bethe-Salpeter equation
 Équation de Blasius. Blasius equation
 Équation de Boltzmann. Boltzmann transport equation
 Équation de Boltzmann-Planck. Boltzmann-Planck equation
 Équation de Born-Mayer. Born-Mayer equation
 Équations de Burnett. Burnett equations
 Équation de Callendar. Callendar equation
 Équation de capacité de chaleur de Debye. Debye heat capacity equation
 Équations de changement de Enskog-Maxwell. Enskog-Maxwell equations of change
 Équation de Clairaut. Clairaut equation
 Équation de Clausius-Clapeyron. Clausius-Clapeyron equation
 Équation de Clausius-Mossotti. Clausius-Mossotti equation
 Équation de continuité. Equation of continuity
 Équation de couleur. Color equation
 Équation de Curie. Curie's equation
 Équation de Debye. Debye equation
 Équation de Debye-Hückel. Debye-Hückel equation
 Équation de déplacement. Transport equation
 Équation de déplacement intégral. Integral transport equation
 Équation de Dirac. Dirac equation
 Équation de Drude. Drude equation
 Équation de Duhem-Margules. Duhem-Margules equation
 Équation de Euler. Euler equation
 Équations de Euler-Lagrange. Euler-Lagrange equations
 Équation de Franklin. Franklin equation
 Équation de Fokker-Planck. Fokker-Planck equation
 Équation de Fredholm. Fredholm equation
 Équation de Gibbs-Duhem. Gibbs-Duhem equation
 Équation de Helmholtz-Lagrange. Helmholtz equation
 Équation de Hermite. Hermite equation
 Équation de Horn. Horn equation
 Équation d'eikonal. Eikonal equation
 Équation d'Einstein pour la capacité de chaleur. Einstein heat capacity equation
 Équations de Karman-Friedrich. Karman-Friedrich's equations

- Équation de Keyes-Smith-Gerry. Keyes-Smith-Gerry equation
- Équation de Kirkwood pour la constante diélectrique. Kirkwood's equation for the dielectric constant
- Équation de Klein-Gordon. Klein-Gordon equation
- Équation de Koch. Koch's equation
- Équation de la constante diélectrique de Onsager. Onsager's equation for the dielectric constant
- Équation de l'âge. Age equation, Fermi
- Équation de Laguerre. Laguerre equation
- Équation de Lamé. Lamé equation
- Équation de Laplace. Laplace equation
- Équation de lentille électrooptique. Electron-optical lens equation
- Équation de Mathieu. Mathieu equation
- Équation de Mollier. Mollier equation
- Équation de mouvement de Heisenberg. Heisenberg equation of motion
- Équation d'énergie d'avion. Airplane energy equation
- Équation d'énergie de supraconductibilité. Superconductivity energy equation
- Équation d'énergie pour couche limite. Energy equation for boundary layer
- Équation d'énergie pour un flux adiabatique permanent. Energy equation for steady adiabatic flow
- Équation de Newton. Newton's equation
- Équation de Nutting. Nutting's equation
- Équation de Pauli-Weisskopf. Pauli-Weisskopf equation
- Équation de Pell. Pell's equation
- Équation de Poisson. Poisson equation
- Équation de quantité de mouvement pour couche limite. Momentum equation for boundary layer
- Équation de Rayleigh-Jeans. Rayleigh-Jeans equation
- Équation de Reynolds. Reynolds equation
- Équation de Riccati. Riccati equation
- Équation de Richardson-Dushman. Richardson-Dushman equation
- Équation de Riemann-Papperitz. Riemann-Papperitz equation
- Équation de Rydberg. Rydberg equation
- Équation de Saekur-Tetrode. Saekur-Tetrode equation
- Équation de Schrödinger. Schrödinger equation
- Équation de son de courte durée. Impulsive sound equation
- Équation de Sturm-Liouville. Sturm-Liouville equation
- Équation d'état. Equation of state
- Équation d'état de Clausius. Clausius' equation of state
- Équation d'état de Mie-Grüneisen. Mie-Grüneisen equation of state
- Équation d'état de viriel. Virial equation of state
- Équation d'état thermique. Caloric equation of state
- Équation de tenseur à valeur propre. Eigenvalue equation of tensor
- Équation de tension de vapeur. Vapor pressure equation
- Équation de Tomonaga-Schrödinger. Tomonaga-Schrödinger equation
- Équation de Tomonaga-Schwinger. Tomonaga-Schwinger equation
- Équation de Van der Waal. Van der Waal's equation
- Équation de Volterra. Volterra equation
- Équation de Weiss. Weiss's equation
- Équation de Weyl. Weyl equation
- Équation différentielle. Differential equation
- Équation différentielle de Clausius. Clausius' differential equation
- Équation différentielle de Laplace. Laplace differential equation
- Équation différentielle de Whittaker. Whittaker differential equation
- Équation différentielle partielle. Partial differential equation
- Équation différentielle partielle de Hamilton-Jacobi. Hamilton-Jacobi partial differential equation
- Équation différentielle partielle elliptique. Elliptic partial differential equation
- Équation différentielle ordinaire. Ordinary differential equation
- Équation d'onde. Wave equation
- Équation elliptique. Elliptic equation
- Équation hyperbolique. Hyperbolic equation
- Équation hypergéométrique de Gauss. Gauss' hypergeometric equation
- Équation indiciaire. Indicial equation
- Équation intégrale. Integral equation
- Équation parabolique. Parabolic equation
- Équation photoélectrique d'Einstein. Einstein photoelectric equation
- Équations algébriques. Algebraic equations
- Équations d'amplitude finie (acoustiques). Finite amplitude equations (acoustic)
- Équations de Cauchy-Reimann. Cauchy-Reimann equations
- Équations de déviation de pente. Slope-deflection equations
- Équations de Fresnel. Fresnel equations
- Équations de Gauss et Codazzi. Equations of Gauss and Codazzi
- Équations de Gibbs-Helmholtz. Gibbs-Helmholtz equations
- Équations de Lagrange. Lagrange equations
- Équations de Laue. Laue equations
- Équations de Legendre. Legendre equations
- Équations de lentille de Gauss. Gaussian lens equations
- Équations de Maxwell. Maxwell's equations
- Équations de mouvement. Motion, equations of
- Équations de mouvement de Euler. Euler equations of motion
- Équations de mouvement fluide. Equations of fluid motion
- Équations de Navier-Stokes. Navier-Stokes equations
- Équations de physique mathématique. Mathematical physics, equations of
- Équations de Proca. Proca equations
- Équations de supraconductibilité de London. London superconductivity equations
- Équations de tracé de rayon paraxial. Paraxial ray tracing equations
- Équations d'hodographe transonique. Hodograph equations, transonic
- Équations d'optique canoniques. Canonical equations of optics
- Équations dynamiques des rayons lumineux. Dynamical equations of light rays
- Équations diophantiennes. Diophantine equations
- Équations intrinsèques d'une courbe. Curve, intrinsic equations of
- Équations collinéaires d'optique. Collineation equations of optics
- Équations des vents. Wind equations
- Équations du taux de cristallisation. Crystallization rate equations
- Équations linéaires. Linear equations
- Équations normales. Normal equations

- Équations simultanées. Simultaneous equations
 Équicontinu. Equicontinuous
 Équilibre. Balance
 Équilibre. Equilibrium
 Équilibre chimique. Chemical equilibrium
 Équilibre de forces sur un corps rigide. Equilibrium of forces on a rigid body
 Équilibre d'une particule. Equilibrium of a particle
 Équilibre dynamique. Equilibrium, dynamic
 Équilibre gelé. Frozen equilibrium
 Équilibre instable. Equilibrium, unstable
 Équilibre neutre. Equilibrium, neutral
 Équilibre thermique. Thermal equilibrium
 Équilibre thermodynamique. Thermodynamic equilibrium
 Équipartition de l'énergie. Equipartition of energy
 Équivalence. Equivalence
 Équivalent mécanique de la lumière. Mechanical equivalent of light
 Ergodicité. Ergodicity
 Erreur. Error
 Erreur d'arrondissement. Rounding error, roundoff
 Erreur de troncature. Truncation error
 Erreur normale. Standard error
 Erreur probable. Probable error
 Erreur relative. Relative error
 Espace d'arrangement. Configuration space
 Espace de Banach. Banach space
 Espace de couleur. Color space
 Espace de Hilbert. Hilbert space
 Espace de moment. k -space
 Espace de phase. Phase space
 Espace Euclidien. Euclidean space
 Espace gamma. Gamma space
 Espace mu. Mu space
 Espace-temps plat. Flat space-time
 Espace topologique. Topological space
 Espace vectoriel. Vector space
 Espèce. Species
 Essais. Runs
 Établissement de supports. Settlement of supports
 Étalon de Waidner-Burgess. Waidner-Burgess standard
 État de caoutchouc. Rubbery state
 État de contre-réaction. Degenerate state
 État de moment angulaire orbital zéro. S -state
 État fondamental. Fundamental state
 État métastable. Metastable state
 État normal. Normal state
 État propre. Eigenstate
 État vitreux. Glassy state
 États correspondants. Corresponding states
 États d'énergie négative. Negative energy states
 États localisés. Localized states
 État stationnaire. Stationary state
 État virtuel. Virtual state
 Étendue de la réaction. Extent of reaction
 Étranger. Extraneous
 Étranglement. Choking
 Évaluation de matrice S dans théorie de champ quantifié. S -matrix in quantified field theory; evaluation of
 Évolutive de diffraction. Diffraction evolutive
 Exact. Exact
 Exaltation. Exaltation
 Excédent. Redundancy
 Excédent neutronique. Neutron excess
 Expansion de Chebyshev. Chebyshev expansion
 Expansion de fraction partielle. Partial fraction expansion
 Expansion de Laplace. Laplace expansion
 Expansion de Prandtl-Meyer. Prandtl-Meyer expansion
 Expansion thermique. Thermal expansion
 Expectative. Expectation
 Expérience de factorielle. Factorial experiment
 Exposant. Exponent
 Expression à intégrer. Integrand
 Extension en phase. Extension in phase
 Extinction. Extinction
 Extrapolation. Extrapolation
 Facteur d'affaiblissement. Attenuation factor
 Facteur d'affaiblissement sonore. Sound attenuation factor
 Facteur de Boltzmann. Boltzmann factor
 Facteur de conversion de masse atomique. Atomic mass conversion factor
 Facteur de charge. Load factor
 Facteur d'écoulement. Carryover factor
 Facteur de crête de train porteur. Pulse carrier, crest factor of
 Facteur de dissymétrie ou d'anisotropie. Dissymmetry factor or anisotropy factor
 Facteur de fission rapide. Fast fission factor
 Facteur de forme. Shape factor
 Facteur de fréquence. Frequency factor
 Facteur de géométrie. Geometry factor
 Facteur de luminance spectrale. Spectral luminance factor
 Facteur de normalisation. Normalizing factor
 Facteur de perte diélectrique. Loss factor, dielectric
 Facteur de réflexion. Luminous reflectance
 Facteur de réflexion réflectance. Reflectivity
 Facteur de réflectance spectrale. Spectral reflectance
 Facteur de répartition. Distribution factor
 Facteur de rétablissement de température. Temperature recovery factor
 Facteur de stabilité. Stability factor
 Facteur de structure. Structure factor
 Facteur de synthèse. Build-up factor
 Facteur de transmittance spectrale. Spectral transmittance
 Facteur g de Landé. Landé's g -factor
 Facteur-groupe. Factor-group
 Facteur intégrant. Integrating factor
 Facteurs de forme et de position de Coddington. Coddington shape and position factors
 Factorielle. Factorial
 Faisceau. Beam; Cluster
 Famille. Family
 Famille de surfaces. Surfaces, family of
 Fatigue. Fatigue
 Fente. Slit
 Fermé. Closed
 Fermeture. Closure
 Fermi. Fermi
 Fermion. Fermion
 Feuille. Sheet
 Feuille de tourbillon. Vortex sheet
 Fictif. Dummy
 Filtrage. Smoothing
 Flèche. Sagitta
 Flexion asymétrique. Unsymmetrical bending
 Fluage. Creep
 Fluctuations. Fluctuations
 Fluide bien remué. Well-stirred fluid
 Fluide de Newton. Newtonian fluid

- Fluide non-visqueux. Inviscid fluid
 Fluide visqueux. Viscous fluid
 Flux adjoint. Flux, adjoint
 Flux de Helmholtz. Helmholtz flow
 Flux d'énergie acoustique. Sound energy flux
 Flux d'énergie géométrique. Geometrical energy flux
 Flux de Stokes. Stokes flow
 Flux lumineux. Luminous flux
 Flux neutronique angulaire. Flux, neutron, angular
 Flux neutronique total. Flux, neutron, total
 Flux vectoriel. Vector flux
 Fonction. Function
 Fonction adjointe d'onde. Adjoint wave function
 Fonction analytique. Analytic function
 Fonction analytique d'une variable complexe. Regular function of a complex variable
 Fonction bêta. Beta function
 Fonction bêta incomplète. Incomplete beta function
 Fonction caractéristique. Characteristic function
 Fonction caractéristique d'angle. Angle characteristic function
 Fonction caractéristique d'un ensemble. Characteristic function of a set
 Fonction caractéristique mixte. Mixed characteristic function
 Fonctions caractéristiques en thermodynamique. Characteristic functions in thermodynamics
 Fonction complémentaire d'une équation linéaire différentielle. Complementary function of a linear differential equation
 Fonction continue. Continuous function
 Fonction de Bessel. Bessel function
 Fonction de Bloch. Bloch function
 Fonction de Brillouin. Brillouin function
 Fonction de chaleur spécifique d'Einstein. Einstein specific heat function
 Fonction de décision. Decision function
 Fonction de densité de probabilité. Probability density function
 Fonction de dissipation. Dissipation function
 Fonction de distribution radiale. Radial distribution function
 Fonction de Flux. Stream function
 Fonction de Gibbs. Gibbs function
 Fonction de Green. Green's function
 Fonction de Gegenbauer. Gegenbauer function
 Fonction de Helmholtz. Helmholtz function
 Fonction de Lagrange. Lagrangian function
 Fonction de Langevin. Langevin function
 Fonction de l'importance nucléaire. Importance function
 Fonction de luminosité. Luminosity function
 Fonction de mesure. Test function
 Fonction de mesure. Measure function
 Fonction de Placzek. Placzek function
 Fonction de Planck. Planck function
 Fonction de Neumann. Neumann function
 Fonction de perte. Loss function
 Fonction de répartition de Fermi-Dirac. Fermi-Dirac distribution function
 Fonction d'erreur. Error function
 Fonction de séparation. Partition function
 Fonction de séparation d'antenne orientable. Rotator, partition function of
 Fonction de séparation des molécules diatomiques. Diatomic molecules, partition function of
 Fonction de séparation des molécules polyatomiques. Polyatomic molecules, partition function of
 Fonction de séparation rotatoire. Rotational partition function
 Fonction de séparation vibratoire. Vibrational partition function
 Fonction de transmission. Transfer function
 Fonction de transmission de réacteur. Reactor transfer function
 Fonction d'excitation atomique. Excitation function, atomic
 Fonction d'excitation nucléaire. Excitation function, nuclear
 Fonction d'onde. Wave function
 Fonction d'onde antisymétrique. Antisymmetric wave function
 Fonction d'onde de Coulomb. Coulomb wave function
 Fonction d'onde sigma-pi. Wave function, Sigma, Pi
 Fonction d'onde symétrique. Symmetric wave function
 Fondation élastique. Elastic foundation
 Fonction elliptique. Elliptic function
 Fonction en escalier. Step function
 Fonction entière. Entire function
 Fonction gamma. Gamma function
 Fonction gamma incomplète. Incomplete gamma function
 Fonction génératrice. Generating function
 Fonction Hamiltonienne d'optique. Hamiltonian function of optics
 Fonction Hamiltonienne d'un système. Hamiltonian function of a system
 Fonction harmonique. Harmonic function
 Fonction holomorphe. Holomorphic function
 Fonction hyperbolique. Hyperbolic function
 Fonction hypergéométrique. Hypergeometric function
 Fonction implicite. Implicit function
 Fonction inverse. Inverse function
 Fonction méromorphe. Meromorphic function
 Fonctionnel. Functional
 Fonctionnement avec relaxation. Relaxation behavior
 Fonction orthogonale. Orthogonal function
 Fonction propre. Eigenfunction
 Fonction paire. Even function
 Fonctions d'aberration de Nijboer-Zernike. Nijboer-Zernike aberration functions
 Fonctions de diffusion de la lumière. Light scattering functions
 Fonctions de Legendre. Legendre functions
 Fonctions de phase minimum. Minimum phase functions
 Fonctions de Wightman. Wightman functions
 Fonctions d'excès. Excess functions
 Fonctions d'impulsions. Impulse functions
 Fonctions d'onde de vecteur sphérique. Spherical vector wave functions
 Fonctions spéciales. Special functions
 Fonction spectrale. Spectral function
 Fonctions potentielles des molécules. Potential functions of molecules
 Fonctions propres moléculaires. Molecular eigenfunctions
 Fonction symétrique. Symmetric function
 Fonctions thermodynamiques de mélange. Thermodynamic functions of mixing
 Fonctions thermodynamiques de phases de surface. Thermodynamic functions of surface phases
 Fonction trigonométrique. Trigonometric function
 Fonction unité de Heaviside. Heaviside unit function
 Fonction zêta de Riemann. Riemann zeta function
 Force. Force

- Force appliquée. Force, applied
 Force centrale. Force, central
 Force centrale. Central force
 Force centrifuge. Centrifugal force
 Force centripète. Centripetal force
 Force conservatrice. Conservative force
 Force cyclostrophique. Cyclostrophic force
 Force de Bartlett. Bartlett force
 Force de Coriolis. Coriolis force
 Force de faible portée. Short-range force
 Force de Heisenberg. Heisenberg force
 Force de ligne. Line strength
 Force de Lorentz. Lorentz force
 Force de Majorana. Majorana force
 Force dépendante des spins. Spin-dependent force
 Force de Wigner. Wigner force
 Force dissipative. Dissipative force
 Force distribuée. Force, distributed
 Force d'oscillateur. Oscillator strength
 Force efficace. Force, effective
 Force électromotrice. Electromotive force
 Force externe ou active. Force, external or active
 Force généralisée. Force, generalized
 Force géostrophique. Geostrophic force
 Force interne. Force, internal
 Force ionique. Ionic strength
 Force magnétisante. Magnetizing force
 Force magnétomotrice. Magnetomotive force
 Force mécanomotrice efficace. Effective mechanomotive force
 Force "perdue". Force, "lost"
 Forces concourantes. Forces, concurrent
 Forces coplanaires. Forces, coplanar
 Forces d'échange. Exchange forces
 Forces de cohésion. Cohesion, forces of
 Forces de dispersion. Dispersion forces
 Forces de London. London forces
 Forces de valence dans les molécules polyatomiques. Valence forces in polyatomic molecules
 Forces de Van der Waal. Van der Waal's forces
 Forces d'inertie. Inertia forces
 Forces d'induction. Induction forces
 Forces en thermodynamique de procédés irréversibles. Forces in thermodynamics of irreversible processes
 Forces intermoléculaires. Intermolecular forces
 Forces nucléaires. Nuclear forces
 Forces subsidiaires de valence. Subsidiary valence forces
 Force sur une surface submergée. Force on a submerged surface
 Forces tensorielles. Non-central forces
 Force tangentielle. Force, tangential
 Forêt. Bit
 Forêt. Forest
 Formalisme de Lagrange et de Hamilton pour systèmes continus. Continuous systems, Lagrangian and Hamiltonian formalism for
 Formalisme de Lagrange pour systèmes de champ. Lagrangian formalism for field systems
 Formalisme de Yang-Feldman et la matrice S . Yang-Feldman formalism and the S -matrix
 Forme. Form
 Forme bilinéaire. Form, bilinear
 Forme canonique du principe de Fermat. Canonical form of Fermat's principle
 Forme différentielle linéaire. Linear differential form
 Forme différentielle du second degré. Quadratic differential form
 Forme indéterminé. Indeterminate form
 Forme normale de Hess. Hess's normal form
 Forme pour équation de surface de Monge. Monge's form for equation of surface
 Formes d'équations d'ondes. Wave equation, forms of
 Formule à quatre facteurs. Four-factor formula
 Formule de baromètre. Barometer formula
 Formule de Batho. Batho's formula
 Formule de Breit-Wigner. Breit-Wigner formula
 Formule de Cauchy pour indice de réfraction. Cauchy formula for refractive index
 Formule de Conwell-Weisskopf. Conwell-Weisskopf formula
 Formule de dispersion. Dispersion formula
 Formule de dispersion de Hartmann. Hartmann dispersion formula
 Formule de dispersion de Herzberger. Herzberger's dispersion formula
 Formule de Euler-Maclaurin. Euler-Maclaurin formula
 Formule de Euler sur les charges de gauchissement de colonnes. Euler buckling loads for columns
 Formule de Eykman. Eykman formula
 Formule de Gibbs pour la tension superficielle. Gibbs formula for the surface tension
 Formule de Gregory. Gregory formula
 Formule de Gregory-Newton. Gregory-Newton formula
 Formule de Gruneisen. Gruneisen formula
 Formule de Kelvin. Kelvin formula
 Formule de Klein-Nishina. Klein-Nishina formula
 Formule de Johnson et Lark-Horowitz. Johnson and Lark-Horowitz formula
 Formule d'équilibre de Saha. Saha equilibrium formula
 Formule de Heine. Heine formula
 Formule de Helmholtz-Ketteler. Helmholtz-Ketteler formula
 Formule de Hicks. Hicks formula
 Formule de Landau. Landau's formula
 Formule de Masse. Mass formula
 Formule de perturbation de Rayleigh-Schrödinger. Rayleigh-Schrödinger perturbation formula
 Formule de quadrature de Chebyshev. Chebyshev quadrature formula
 Formule de quadrature de Gauss. Gaussian quadrature formula
 Formule de quadrature fermée. Closed quadrature formula
 Formule de quadrature ouverte. Open quadrature formula
 Formule de radiation de Planck. Planck radiation formula
 Formule de récursion. Recursion formula
 Formule de Ritz. Ritz formula
 Formule de Rodrigues. Rodrigues formula
 Formule de sécante pour colonnes. Secant formula for columns
 Formule de sommation de Euler. Euler summation formula
 Formule de Schläfli. Schläfli formula
 Formule de Wallis. Wallis formula
 Formule de Winkler-Bach. Winckler-Bach formula
 Formule d'interpolation de Bessel. Bessel interpolation formula
 Formule d'interpolation de Everett. Everett interpolation formula
 Formule d'interpolation de Hermite. Hermite interpolation formula
 Formule d'interpolation de Lagrange. Lagrange interpolation formula.

- Formule d'interpolation de Stirling. Stirling interpolation formula
 Formule du diamagnétisme de Langevin-Pauli. Diamagnetism, Langevin-Pauli formula
 Formule du point central. Midpoint formula
 Formule intégrale de Cauchy. Cauchy integral formula
 Formules de Bredt. Bredt formulas
 Formules de différentiation numérique. Differentiation formulas numerical
 Formules de Gauss. Gauss' formulas
 Formules de quadrature de Newton-Cotes. Newton-Cotes quadrature formulas
 Formules de Serret-Frenet. Serret-Frenet formulae
 Formules de reste. Remainder formulas
 Formules de Schwarzschild-Kohlschutter. Schwarzschild-Kohlschutter formulas
 Formules de Weingarten. Weingarten's formulas
 Formules d'interpolation de Gauss. Gaussian interpolation formulas
 Foyers. Focal points
 Foyer primaire. Tangential focus
 Foyer secondaire. Sagittal focus
 Foyer secondaire. Second focal point
 Fraction continue. Fraction, continued
 Fraction de branchement. Branching fraction
 Fraction de Fechner. Fechner fraction
 Fraction de tassement. Packing fraction
 Fraction impropre. Improper fraction
 Fraction interne. Fraction, internal
 Fraction partielle. Fraction, partial
 Fraction propre. Proper fraction
 Fractions continues. Continued fractions
 Fractions de poids. Weight fractions
 Fractions de volume. Volume fractions
 Fractions moléculaires. Mole fractions
 Frange. Bleeding
 Frange. Fringe
 Frein. Drag
 Fréquence atomique. Atomic frequency
 Fréquence de battement. Beat frequency
 Fréquence de collision. Collision frequency
 Fréquence de Debye. Debye frequency
 Fréquence de relaxation. Relaxation frequency
 Fréquence de répétition d'impulsions. Pulse repetition frequency
 Fréquence de résonance. Resonant frequency
 Fréquence de résonance d'un cristal. Crystal, resonance frequency of
 Fréquence relative. Relative frequency
 Front d'onde. Wave front
 Front d'onde plane. Wave front, plane
 Front d'onde sphérique. Wave front, spherical
 Frottement. Rolling resistance
 Frottement de Peau. Skin friction
 Fugacité. Fugacity
- Gain de régénération. Breeding gain
 Gamma. Gamma
 Gamme des gris. Gray scale
 Gamme tempérée. Scale, equally tempered
 Gaz de Bose-Einstein. Bose-Einstein gas
 Gaz dégénéré. Degenerate gas
 Gaz électronique dégénéré. Degenerate electron gas
 Gaz imparfait. Imperfect gas
 Gaz réels. Real gases
 Géodésique nulle. Null-Geodesic
 Géométrie. Geometry
- Géométrie de Luneburg. Luneburg geometry
 Géométrie descriptive. Descriptive geometry
 Gond. Hinge
 Gradient. Gradient
 Gradient de température. Temperature gradient
 Grande déviation. Deflection, large
 Graphique d'automorphisme. Graph, automorphism
 Graphique illimité. Graph, infinite
 Graphique linéaire. Graph, linear
 Graphique non-orienté. Graph, non-oriented
 Graphique non-séparable. Graph, non-separable
 Graphique orienté. Graph, oriented
 Graphique séparable. Graph, separable
 Grossissement. Magnifying power
 Grossissement axial. Axial magnification
 Grossissement linéaire. Linear magnification
 Grossissement zonal. Zonal magnification
 Groupe. Group
 Groupe abélien. Abelian group
 Groupe cristallographique. Crystallographic group
 Groupe de Lie. Lie group
 Groupe de permutation. Permutation group
 Groupe de points. Point group
 Groupe de rotation. Rotation group
 Groupe de signaux. Word
 Groupe d'espaces. Space group
 Groupes de transformations générales. Transformation groups, general
 Groupe de translation. Translation group
 Groupe dièdre. Dihedral group
 Groupe d'ondes de Gauss. Gaussian wave group
 Groupe linéaire complet. Full linear group
 Groupe topologique. Topological group
 Guide d'ondes. Waveguide
- Harmonique. Wave, harmonic
 Haute élasticité. Elasticity, high
 Hélice. Airscrew
 Hélice. Helix
 Herpolhodie. Herpolhode
 Hertz. Hertz
 Hessien. Hessian
 Holonome. Holonomic
 Homogène. Homogeneous
 Homomorphisme. Homomorphism
 Humidité. Humidity
 Hybridation des orbités électroniques. Hybridization of electron orbitals
 Hydrodynamique relativiste. Relativistic hydrodynamics
 Hydrostatique. Hydrostatics
 Hyperbole. Hyperbola
 Hyperbole géodésique. Geodesic hyperbola
 Hyperboloïde. Hyperboloid
 Hyperconjugaison. Hyperconjugation
 Hypothèses ergodiques. Ergodic hypotheses
 Hypothèse linéaire. Linear hypothesis
 Hypothèse statistique. Hypothesis, statistical
- Idéalement plastique. Ideally plastic
 Idempotent. Idempotent
 Identité. Identity
 Identité d'Abel. Abel identity
 Identité de Bianchi. Bianchi identity
 Identité de de Moivre. De Moivre identity
 Identité de Ricci. Ricci identity

- Identités différentielles de vecteurs. Vector differential identities
- Image de Heisenberg. Heisenberg picture
- Image d'interaction. Interaction picture
- Impédance acoustique. Impedance, acoustical
- Impédance acoustique caractéristique. Impedance, characteristic acoustical
- Impédance acoustique d'entrée. Impedance, throat acoustical
- Impédance acoustique spécifique. Impedance, specific acoustical
- Impédance mécanique. Mechanical impedance
- Imperfection de Frenkel. Frenkel defect
- Impulsion. Pulse
- Impulsion. Impulse
- Impulsion angulaire. Angular impulse
- Incident. Incident
- Incommensurable. Incommensurable
- Indépendance. Independence
- Indépendance de charge. Charge-independence
- Indépendant. Independent
- Indicatrice. Indicatrix
- Indicatrice de diffusion. Indicatrix of diffusion
- Indicatrice de réfraction. Indicatrix of refraction
- Indicatrice sphérique de tangente à une courbe. Spherical indicatrix of tangent to a curve
- Indice de cétane. Cetane number
- Indice de coordination. Coordination number
- Indice de déplacement. Transport number
- Indice de probabilité. Index of probability
- Indice de réfraction. Index of refraction
- Indice de valence libre. Free valence index
- Indice libre. Index, free
- Indices de Bravais-Miller. Bravais-Miller indices
- Indices de cristaux de Miller. Crystal indices, Miller
- Inductance. Inductance
- Induction magnétique. Magnetic induction
- Inélastique. Anelastic
- Inégalité d'Abel. Abel inequality
- Inégalité de Bessel. Bessel's inequality
- Inégalité de Bienayme-Chebyshev. Bienayme-Chebyshev inequality
- Inégalité de Chebyshev. Chebyshev inequality
- Inégalité de Clausius. Clausius' inequality
- Inégalité de Hölder. Hölder inequality
- Inégalité de Minkowski. Minkowski inequality
- Inégalité de Schwartz. Schwartz inequality
- Inégalités linéaires. Linear inequalities
- Inertance. Inertance
- Inertie. Inertia
- Inférence non-paramétrique. Non-parametric inference
- Inférence sans répartition. Distribution-free inference
- Infinité. Infinity
- Infinitésimal. Infinitesimal
- Information. Information
- Intégrale. Integral
- Intégrale circulatoire. Integral, circulatory
- Intégrale d'échange. Exchange integral
- Intégrale de chevauchement. Overlap integral
- Intégrale de Clausius. Clausius' integral
- Intégrale de composition. Convolution
- Intégrale de composition. Convolution integral
- Intégrale de Coulomb. Coulomb integral
- Intégrale de Dirichlet. Dirichlet integral
- Intégrale de Duhamel. Duhamel's integral
- Intégrale d'énergie. Energy integral
- Intégrale de Lebesgue. Lebesgue integral
- Intégrale de phase. Phase integral
- Intégrale de Poisson. Poisson integral
- Intégrale de résonance. Resonance integral
- Intégrale de Riemann. Riemann integral
- Intégrale de Stieltjes. Stieltjes integral
- Intégrale elliptique. Elliptic integral
- Intégrale impropre. Improper integral
- Intégrale indéfinie d'une fonction. Antiderivative of a function
- Intégrale multiple. Multiple integral
- Intégration de contour. Contour integration
- Intégration de série. Series integration
- Intensité de champ magnétique. Magnetic field strength
- Intensité de radioactivité. Intensity of radioactivity
- Intensité de radiation. Intensity of radiation
- Intensité des sons. Pitch level
- Intensité d'un bruit. Loudness of a sound
- Intensité d'une ligne spectrale. Intensity of a spectral line
- Intensité d'une onde sonore sphérique. Intensity of a spherical sound wave
- Intensité d'une source de particules. Intensity of a source of particles
- Intensité horizontale moyenne. Mean horizontal intensity
- Intensité lumineuse. Candle power
- Intensité lumineuse. Luminous intensity
- Intensité radiante. Radiant intensity
- Intensité moyenne sphérique. Mean spherical intensity
- Intensité sonore. Sound intensity
- Interaction de Fermi universelle. Universal Fermi interaction
- Interaction directe. Direct interaction
- Interaction onde de choc-couche limite. Shock-wave boundary-layer interaction
- Interaction phonon-phonon. Phonon-phonon interaction
- Interférence. Interference
- Interpolation. Interpolation
- Interpolation d'intervalle optimum. Optimum-interval interpolation
- Interpolation inverse. Inverse interpolation
- Interpolation multiple. Multivariate interpolation
- Interpolation trigonométrique. Trigonometric interpolation
- Interprétation de particule. Particle interpretation
- Intervalle de collision. Collision interval
- Intervalle d'impulsions. Pulse interval
- Intervalle d'impulsions. Pulse spacing
- Invariance de jauge. Gauge invariance
- Invariance de jauge et lois de la conservation. Gauge invariance and conservation laws
- Invariance relativiste d'une théorie physique. Relativistic invariance of a physical theory
- Invariance relativiste et mécanique quantique. Relativistic invariance and quantum mechanics
- Invariant. Invariant
- Invariant de Poincaré. Poincaré's invariant
- Invariant de sommation. Summational invariant
- Invariant différentiel optique. Optical differential invariant
- Inversion. Inversion
- Inversion de matrice. Matrix inversion
- Inversion du temps. Time reversal
- Ion complexe. Complexion
- Irradiation. Irradiation
- Isentropique. Isentropic
- Isallobare. Isallobar
- Isobare. Isobar

- Isolux. Isolux
 Isolux. Isophot
 Isotherme. Isothermal
 Isotherme d'adsorption. Adsorption isotherm
 Isotherme d'adsorption de Langmuir. Langmuir ad-
 sorption isotherm
 Isothermes. Isotherms
 Isotrope. Isotropic
- Jacobien. Jacobian
 Joint de grains. Grain boundary
- Lame. Blade
 Lame torsadée. Blade, twisted
 Lambert. Lambert
 Laplacien. Laplacian
 Largeur. Span
 Largeur de bande d'impulsion. Pulse bandwidth
 Largeur de bande efficace (acoustique). Effective band
 width (acoustic)
 Largeur de Doppler. Doppler width
 Largeur effective. Width, effective
 Largeur de ligne naturelle. Natural line width
 Largeur des lignes de résonance magnétique. Magnetic
 resonance, line width
 Largeur de niveau. Level width
 Largeur pratique. Practical width
 La plus petite différence de hauteur de son perceptible.
 Pitch difference, minimum perceptible
 Le Nombre e . e , the number
 Lenteur de phase. Phase slowness
 Lentille de Luneburg. Luneburg lens
 Le plus grand commun diviseur. Highest common
 factor
 Le plus petit cercle d'aberration. Least circle of aberra-
 tion
 Léthargie. Lethargy
 Liaison affine fondamentale. Fundamental affine con-
 nection
 Liaison coordonnée. Coordinate bond
 Liaison hétéropolaire. Heteropolar bond
 Liaison homopolaire. Homopolar bond
 Liaison de l'hydrogène. Hydrogen bond
 Libre parcours moléculaire. Molecular free path
 Libre parcours moyen. Mean free path
 Libre parcours moyen de déplacement. Transport mean
 free path
 Libre parcours moyen d'un phonon. Phonon mean free
 path
 Libre temps moyen. Mean free time
 Lié. Bound
 Lieu de Planck (Corps noir) Planckian locus
 Ligne asymptotique sur une surface. Asymptotic line
 on a surface
 Ligne d'action de force. Force, line of action of
 Ligne de courbure. Line of curvature
 Ligne de glissement. Slip line
 Ligne de Mach. Mach line
 Ligne de pression. Pressure line
 Ligne des apsides. Apse line
 Ligne de transmission. Transmission line
 Ligne de vortacité. Vortex line
 Ligne de zéro de bande. Zero line of a band
 Ligne d'influence. Influence line
 Ligne géodésique. Geodesic line
 Ligne polaire. Polar line
- Lignes d'écoulement. Lines of flow
 Lignes de Luder. Luder's lines
 Lignes de Stokes. Stokes lines
 Lignes de superstructure. Superstructure lines
 Lignes de température de couleur égale. Isotemperature
 loci
 Lignes d'intercombinaison. Intercombination lines
 Lignes focales. Focal lines
 Lignes isométriques sur une surface. Isometric lines on
 a surface
 Limité. Bounded
 Limite d'élasticité. Elastic limit
 Limité élastique. Yield point
 Limite de Rayleigh pour aberration de sphéricité.
 Rayleigh limit for spherical aberration
 Limite élastique. Limit, yield
 Limite proportionnelle. Limit, proportional
 Limites de certitude. Confidence limits
 Linéaire. Linear
 Liquide de Bose-Einstein. Bose-Einstein liquid
 Localement Euclidien. Locally Euclidean
 Logarithme. Logarithm
 Logarithme hyperbolique. Hyperbolic logarithm
 Loi adiabatique de Ehrenfest. Ehrenfest adiabatic law
 Loi adiabatique pour états quantifiés. Adiabatic law
 for quantized states
 Loi associative. Associative law
 Loi commutative. Commutative law
 Loi d'action de masse. Law of mass action
 Loi d'Amagat. Amagat's law
 Loi de Barba. Barba's law
 Loi de Beer. Beer's law
 Loi de Bouguer. Bouguer law
 Loi de Boyle. Boyle's law
 Loi de Brewster. Brewster's law
 Loi de Charles. Charles' law
 Loi de composition de l'accélération. Acceleration,
 composition law of
 Loi de Cotton-Mouton. Cotton-Mouton law
 Loi de Curie-Weiss. Curie-Weiss law
 Loi de Dalton. Dalton's law
 Loi de Darcy. Darcy's Law
 Loi de déperdition de chaleur de Newton. Newton's
 law for heat loss
 Loi de déplacement de Sommerfeld-Kossel. Sommer-
 feld-Kossel displacement law.
 Loi de diffusion de Fick. Fick's law of diffusion
 Loi de dilution de Ostwald. Ostwald's dilution law
 Loi de distribution de Maxwell. Maxwell distribution
 law
 Loi de distribution de Maxwell-Boltzmann. Maxwell-
 Boltzmann distribution law
 Loi de distribution de Nernst. Nernst distribution law
 Loi de Duane et Hunt. Duane and Hunt law
 Loi de force gravitationnelle normale de Gauss. Gauss'
 law of normal gravitational force
 Loi de Fourier. Fourier's law
 Loi de Gladstone-Dale. Gladstone-Dale law
 Loi de Goldschmidt. Goldschmidt law
 Loi de Haüy. Haüy law
 Loi de Hess. Hess Law
 Loi de Hooke. Hooke's law
 Loi de Kirchhoff. Kirchhoff's law
 Loi de Kutta-Joukowski. Kutta-Joukowski law
 Loi de Lambert. Cosine emission law
 Loi de Lenz. Lenz law
 Loi de limite pour électrolytes forts. Limiting law for
 strong electrolytes

- Loi de logarithme itéré. Iterated logarithm, law of
 Loi de Lorentz. Lorentz law
 Loi de Lorenz-Lorentz. Lorenz-Lorentz law
 Loi de Moseley. Moseley's law
 Loi de Moutier. Moutier law
 Loi de Ohm. Ohm's law
 Loi de Planck. Planck law
 Loi de probabilité composée. Probability, law of compound
 Loi de radiation de Kirchoff. Kirchoff's radiation law
 Loi de réciprocité photographique. Reciprocity law, photographie
 Loi de résistance fluide de Newton. Newton's law of fluid resistance
 Loi de Sabin. Sabin law
 Loi des cosinus. Law of cosines
 Loi des états correspondants. Corresponding states, law of
 Loi des gaz parfaits. Perfect gas law
 Loi des grands nombres. Law of large numbers
 Loi de similarité hypersonique. Hypersonic similarity law
 Loi de similitude transonique. Transonic similarity law
 Loi des sinus. Law of sines
 Loi des tangentes. Law of tangents
 Loi de Stefan-Boltzmann. Stefan-Boltzmann law
 Loi de Talbot. Talbot law
 Loi de thermodynamique de Zeroth. Zeroth law of thermodynamics
 Loi distributive. Distributive law
 Loi du carré du cosinus de Malus. Malus cosine-squared law
 Loi du cosinus de Knudsen. Knudsen cosine law
 Loi du quotient pour tenseurs. Quotient law for tensors
 Lois de Kepler. Kepler's laws
 Lois de la réflexion. Reflection, laws of
 Lois de la réfraction. Refraction, laws of
 Lois de radiation de corps noir. Black body radiation laws
 Loi(s) de Snell. Snell's law(s)
 Lois de Stokes. Stokes laws
 Lois de transformation pour intensités de champ. Transformation laws for field strengths
 Lois de Wien. Wien laws
 Lois du mouvement de Newton. Newton's laws of motion
 Longueur de diffusion. Diffusion length
 Longueur de liaison. Bond length
 Longueur de mélange. Mixing length
 Longueur de migration des neutrons. Migration length for neutrons
 Longueur d'entrée pour tuyau ou canal. Entry length for pipe or channel
 Longueur de ralentissement. Slowing-down length
 Longueur de rayonnement. Radiation length
 Longueur de relaxation. Relaxation length
 Longueur d'extrapolation linéaire. Linear extrapolation length
 Longueur d'onde dominante. Dominant wavelength
 Longueur d'onde efficace. Effective wavelength
 Longueur d'une courbe. Length of a curve
 Longueur équivalente de pendule. Pendulum, equivalent length of
 Lumen. Lumen
 Lumen-heure. Lumen-hour
 Lumière de référence. Basic stimulus
 Lux-seconde. Lux-second
 Machine à analogie. Analog computer
 Machine à calculer à programme emmagasiné. Stored-program computer
 Magnéton Bohr. Bohr magneton
 Magnéton nucléaire. Nuclear magneton
 Magnétostatique. Magnetostatics
 Magnétostriction. Magnetostriction
 Magnétostriction à saturation d'un cristal. Crystal saturation magnetostriction
 Maille. Unit cell
 Mantisse. Mantissa
 Marge d'amplification. Gain margin
 Marge de phase. Phase margin
 Martingale. Martingale
 Masse. Mass
 Masse atomique. Atomic mass
 Masse réduite. Reduced mass
 Masse virtuelle. Virtual mass
 Matière de Bingham. Bingham material
 Matière de Burgers. Burgers material
 Matière de Levy-Mises. Levy-Mises material
 Matière de Mises. Mises material
 Matière de Prandtl-Reuss. Prandtl-Reuss material
 Matière de Saint Venant-Mises. Saint Venant-Mises material
 Matière de Voigt. Voigt material
 Matrice. Matrix
 Matrice de densité. Density matrix
 Matrice de diffusion. Matrix scattering
 Matrice de Jordan. Jordan matrix
 Matrice de transmission. Transmission matrix
 Matrice diagonale triple. Triple-diagonal matrix
 Matrice d'incidence. Matrix, incidence
 Matrice hermitique. Hermitian matrix
 Matrice inverse. Inverse matrix
 Matrice nulle. Null matrix
 Matrice positive définie. Positive definite matrix
 Matrice S . S -matrix
 Matrices de Duffin-Kemmer. Duffin-Kemmer matrices
 Matrice triangulaire. Triangular matrix
 Matrice unité. Unit matrix
 Maxwell. Maxwell
 Mécanique ondulatoire. Wave mechanics
 Mécanique quantique. Quantum mechanics
 Mécanique statistique. Statistical mechanics
 Médian. Median
 Mélanges de gaz parfait. Perfect gas mixtures
 Mélanges polymères. Polymer mixtures
 Mesure. Measure
 Mesure de caustique de Gaviola. Gaviola's caustic test
 Mesure de convergence de Abel. Abel test for convergence
 Mesure de convergence de Cauchy. Cauchy convergence test
 Mesure de D'Alembert. D'Alembert test
 Mesure de Fisher-Yates. Fisher-Yates test
 Mesure de lieu. Measure of location
 Mesure-Étalon. Standard measure
 Mesure intégrale de convergence de Cauchy. Cauchy integral convergence test
 Mesure par M de Weierstrass. Weierstrass M -test
 Méthode Adams-Bashford. Adams-Bashford method
 Méthode Bairstow. Bairstow method
 Méthode d'analyse de Euler. Eulerian method of analysis
 Méthode d'analyse de Lagrange. Lagrangian method of analysis

- Méthode de Bernoulli. Bernoulli method
Méthode de Bêthe. Bêthe's method
Méthode de biorthogonalisation de Lanezo. Lanczo's method of biorthogonalization
Méthode de Born-Oppenheimer. Born-Oppenheimer method
Méthode de charge élastique. Elastic load method
Méthode de charge unité fictive. Dummy unit load method
Méthode de Chio. Chio's method
Méthode de Choleski. Choleski's method
Méthode de convergence de Hoff. Hoff's convergence method
Méthode de Crout. Crout's method
Méthode de Dandelin. Dandelin's method
Méthode de Doolittle. Doolittle method
Méthode de Euler pour résoudre une équation différentielle ordinaire. Euler method for solving an ordinary differential equation
Méthode de Evjen. Evjen method
Méthode de Feynman. Feynman's method
Méthode de Frobenius. Frobenius method
Méthode de Gauss-Seidel. Gauss-Seidel method
Méthode de Givens. Givens method
Méthode de Goertzel-Selengut. Goertzel-Selengut method
Méthode de Graeffe. Graeffe method
Méthode de Gram-Schmidt. Gram-Schmidt process
Méthode de Hartree-Fock. Hartree-Fock method
Méthode de Henneberg. Henneberg's method
Méthode de Hitchcock. Hitchcock method
Méthode de Holte. Holte's method
Méthode de Horner. Horner method
Méthode de Jacobi. Jacobi method
Méthode de Jezek. Jezek's method
Méthode de Jordan. Jordan method
Méthode de Klein-Rydberg. Klein-Rydberg method
Méthode de l'escalier roulant. Escalator method
Méthode de Liebmann. Liebmann method
Méthode de Lobatchevsky. Lobachevskii method
Méthode de Macauley. Macauley's method
Méthode de Maxwell-Mohr. Maxwell-Mohr method
Méthode de Milne. Milne's method
Méthode de Monte-Carlo. Monte Carlo method
Méthode de Newton. Newton's method
Méthode de Oseen. Oseen's method
Méthode de Picard. Picard method
Méthode de point de selle. Saddle point method
Méthode de Rayleigh-Ritz. Rayleigh-Ritz method
Méthode de Ritter. Ritter's method
Méthode de Ritz. Ritz method
Méthode de Runge-Kutta. Runge-Kutta method
Méthode des approximations de polynômes. Method of polynomial approximations
Méthode des caractéristiques. Characteristics, method of
Méthode des charges fictives. Loads, method of fictitious
Méthode de Schmidt-Hilbert. Schmidt-Hilbert method
Méthode des coefficients indéterminés. Undetermined coefficients, method of
Méthode des cordes. Chord method
Méthode des différences de variates. Variate difference method
Méthode de Seidel. Seidel method
Méthode de Serber-Wilson. Serber-Wilson method
Méthode des gradients conjugués. Conjugate gradients, method of
Méthode des harmoniques sphériques. Spherical harmonics method
Méthode des images. Method of images
Méthode des inégalités. Inequalities, method of
Méthode des jonctions. Joints, method of
Méthode des liaisons de valence. Valence bond method
Méthode des moments. Moments method
Méthode des orbitales moléculaires. Molecular orbitals method
Méthode des ordonnées discrètes. Discrete ordinates method
Méthode de Spencer-Fano. Spencer-Fano method
Méthodes des racines carrées. Root-squaring methods
Méthode de Störmer. Störmer method
Méthode de Tamm-Dancoff. Tamm-Dancoff method
Méthode de Verde et Wick. Method of Verde and Wick
Méthode de Wick. Wick method
Méthode de Wick-Chandrasekhar. Wick-Chandrasekhar method
Méthode de Yvon. Yvon method
Méthode d'itération. Method of iteration
Méthode d'interpolation d'Aitken. Aitken method of interpolation
Méthodes d'agrandissement. Enlargement, methods of
Méthodes directes. Direct methods
Méthode simple. Simplex method
Méthode S_N . S_N method
Méthodes opérationnelles. Operational methods
Méthode statistique d'entropies. Statistical method of entropies
Mineur. Minor
Mobilité. Mobility
Mobilité de Hall. Hall mobility
Mode. Mode
Mode d'impulsions. Pulse mode
Mode fondamentale. Mode, fundamental
Modèle cellulaire de l'état liquide. Cell model of the liquid state
Modèle d'absorbant infini. Infinite absorber model
Modèle de Kelvin. Kelvin model
Modèle de Kronig-Penney. Kronig-Penney model
Modèle de Maxwell. Maxwell model
Modèle de noyaux de Schmidt. Schmidt model of nuclei
Modèle de Réseau. Lattice model
Modèle de résonance étroit. Narrow resonance model
Modèle de Sutherland. Sutherland model
Modèle de Voigt. Voigt model
Modèle de Wigner-Wilkins. Wigner-Wilkins model
Modèle de Wilkins. Wilkins model
Modèle optique du noyau. Optical model of the nucleus
Modèle plastique. Plastic design
Modèles de diffraction. Diffraction patterns
Modèles de liaison. Bond types
Mode optique. Optical mode
Modes d'oscillation. Oscillation, modes of
Modes normaux. Normal modes
Module complexe. Modulus, complex
Module de cisaillement. Modulus, shear
Module de compression. Modulus, bulk
Module de continuité. Modulus of continuity
Module de fondation. Foundation modulus
Module d'élasticité. Modulus of elasticity
Module d'emmagasinage. Modulus, storage
Module de perte. Modulus, loss
Module d'équilibre. Modulus, equilibrium
Module de relaxation. Modulus, relaxation
Module de rupture. Modulus of rupture

- Module de section. Section modulus
 Module de verre. Modulus, glass
 Module de Young. Young's modulus
 Module initial. Initial modulus
 Module réduit ou double. Modulus, reduced or double
 Moindres carrés. Least squares
 Molarité. Molality
 Molécules polyatomiques. Polyatomic molecules
 Moment. Moment
 Moment cinétique. Kinetic momentum
 Moment de cristal. Crystal momentum
 Moment de flexion. Bending moment
 Moment de force. Moment of force
 Moment de produit. Product moment
 Moment des quantités de mouvement. Moment of momentum
 Moment d'inertie. Moment of inertia
 Moment d'une distribution. Moment of a distribution
 Moment limite ou plastique. Moment, limit or plastic
 Moment magnétique. Magnetic moment
 Moment magnétique de neutron. Neutron magnetic moment
 Moment magnétique nucléaire. Nuclear magnetic moment
 Moment quadripolaire. Quadrupole moment
 Moments et produits d'inertie. Inertia, moments and products of
 Moment statique. Moment, static
 Monde de Minkowski. Minkowski world
 Monocouches localisées. Localized monolayers
 Momogramme. Nonomgram or nomogram
 Moteur à réaction. Jet engine
 Mouvement. Motion
 Mouvement curviligne. Curvilinear motion
 Mouvement cyclonique. Cyclonic motion
 Mouvement de grimpeur. Creeping motion
 Mouvement d'impulsion. Impulsive motion
 Mouvement planaire. Plane motion
 Mouvement planétaire. Planetary motion
 Mouvement rectiligne. Rectilinear motion
 Mouvements d'origine thermique dans un réseau. Thermal motions in a lattice
 Mouvement sinusoïdal. Harmonic motion
 Mouvement sinusoïdal simple. Simple harmonic motion
 Mouvement uniforme. Uniform motion
 Mouvement uniforme dans un cercle. Uniform motion in a circle
 Moyenne arithmétique. Arithmetic mean
 Moyenne des différences. Standard deviation
 Moyenne géométrique. Geometric mean
 Moyenne harmonique. Harmonic mean
 Moyenne mobile. Moving average
 Multicollinéarité. Multicollinearity
 Multiplicateur de Lagrange. Lagrange multiplier
 Multiplication de tenseurs. Multiplication of tensors
 Multiplication effective. Multiplication, effective
 Multiplication infinie. Multiplication, infinite
 Multiplication vectorielle. Vector multiplication
 Multiplicité. Multiplicity
 Mur adiabatique. Adiabatic wall
 Nabra. Nabla
 Néper. Neper
 Netteté de résonance. Resonance, sharpness of
 Newton. Newton
 Nilpotent. Nilpotent
 Niveau de Bruit. Noise level
 Niveau de Fermi. Fermi level
 Niveau de pression de bande. Band pressure level
 Niveau de pression de bande sonore. Sound band pressure level
 Niveau de pression sonore. Sound pressure level
 Niveau de pression sonore d'une octave. Sound octave-band pressure level
 Niveau de spectre de pression sonore. Sound pressure spectrum level
 Niveau des signaux. Signal level
 Niveau d'intensité sonore. Loudness level
 Niveau d'intensité sonore. Sound intensity level
 Niveau d'intensité sonore d'une source. Source level
 Niveau sonore. Sound level
 Niveaux dégénérés. Degenerate levels
 Niveaux d'énergie de vibration d'une molécule. Vibrational energy levels of a molecule
 Niveaux d'énergie moléculaire. Molecular energy levels
 Niveaux d'énergie nucléaire. Nuclear energy levels
 Niveaux d'énergie rotatoire d'une molécule. Rotational energy levels of a molecule
 Nodeus partiels. Nodes, partial
 Nombre. Number
 Nombre atomique effectif. Atomic number, effective
 Nombre complexe. Complex number
 Nombre cyclomatique. Cyclomatic number
 Nombre d'Avogadro. Avogadro constant
 Nombre de Abbe. Abbe number
 Nombre de Froude. Froude number
 Nombre de Grashof. Grashof number
 Nombre de Knudsen. Knudsen number
 Nombre de mach. Mach number
 Nombre de mach critique. Critical mach number
 Nombre de masse. Atomic mass unit
 Nombre de masse. Mass number
 Nombre de Nusselt. Nusselt number
 Nombre de Prandtl. Prandtl number
 Nombre de Rayleigh. Rayleigh number
 Nombre de Reynolds. Reynolds number
 Nombre de Reynolds de turbulence. Turbulence, Reynolds number of
 Nombre de Richardson. Richardson number
 Nombre de Rossby. Rossby number
 Nombre d'onde. Wave number
 Nombre rationnel. Rational number
 Nombres conjugués. Conjugate numbers
 Nombres de Bernoulli. Bernoulli numbers
 Nombres de Stirling. Stirling numbers
 Nombres magiques. Magic numbers
 Non-holonôme. Non-holonomic
 Normale. Normal
 Normale à une courbe. Normal to a curve
 Normale principale. Principal normal
 Normale unité. Unit normal
 Normalisation. Normalization
 Norme. Norm
 Norme spectrale. Spectral norm
 Notation de Bow. Bow notation
 Notation de position. Positional notation
 Notation indiciaire. Indicial notation
 Noyau. Kernel
 Noyau de déplacement. Kernel, transport
 Noyau de diffusion. Kernel, diffusion
 Noyau de dispersion. Kernel, scattering
 Noyau gaussien. Kernel, gaussian
 Noyau synthétique. Kernel, synthetic

- Noyaux conjugués. Conjugate nuclei
 Numéro atomique. Atomic number
 Nutation. Nutation
- Obliquité. Skewness
 Onde. Wave
 Onde convergente. Wave, converging
 Onde cylindrique. Wave, cylindrical
 Onde de choc. Wave, shock
 Onde de circulation. Wave, circulating
 Onde de densité d'énergie cinétique. Wave, kinetic energy density
 Onde de rotation. Wave, shear
 Onde longitudinale. Wave, longitudinal
 Onde *P*. *P* wave
 Onde périodique. Wave, periodic
 Onde rectangulaire. Square wave
 Onde réfléchi. Wave, reflected
 Onde réfractée. Wave, refracted
 Onde simple. Simple wave
 Onde solénoïdale. Wave, solenoidal
 Onde sphérique. Wave, spherical
 Onde stationnaire. Wave, standing
 Ondes capillaires. Capillary waves
 Ondes de dilatation. Waves, dilatational
 Ondes d'effort. Waves, stress
 Ondes de Helmholtz. Helmholtz waves
 Ondes de Rossby. Rossby waves
 Ondes de sol. *S* waves
 Ondes de sol. Surface waves
 Ondes de surface de Rayleigh. Waves, Rayleigh surface
 Ondes de torsion dans une tige. Torsional waves in a rod
 Ondes élastiques dans les plaques. Waves, elastic, in plates
 Ondes élastiques dans les solides. Waves, elastic, in solids
 Ondes élastiques dans les tiges. Waves, elastic, in rods
 Ondes longitudinales (électriques ou magnétiques). Transverse waves (electric or magnetic)
 Ondes longitudinales dans une tige. Longitudinal waves in a rod
 Ondes non-dispersives. Non-dispersive waves
 Ondes plastiques. Waves, plastic
 Ondes progressives. Waves, progressive
 Onde(s) sonore(s) stationnaire(s). Sound wave(s), stationary
 Ondes stationnaires. Waves, stationary
 Ondes viscoélastiques. Waves, viscoelastic
- Opérateur. Operator
 Opérateur antilinéaire. Anti-linear operator
 Opérateur antiunitaire. Anti-unitary operator
 Opérateur chronologique. Chronological operator
 Opérateur de création. Creation operator
 Opérateur de désintégration. Annihilation operator
 Opérateur de destruction. Destruction operator
 Opérateur de parité. Parity operator
 Opérateur dyadique. Dyadic operator
 Opérateur Hermitique. Hermitian operator
 Opérateur intégral. Integral operator
 Opérateur inverse. Inverse operator
 Opérateur Laplacien. Laplacian operator
 Opérateur potentiel. Potential operator
 Opérateurs à différences. Difference operators
 Opérateurs de Dirac. Dirac operators
 Opérateurs du spin de Pauli. Pauli spin operators
 Opérateur unitaire. Unitary operator
- Opérateur unité. Unit operator
 Opérateur vectoriel. Vector operator
 Opérateur viscoélastique. Viscoelastic operator
 Opération de conjugation de charge. Charge conjugation operation
 Opération de translation. Translation operation
 Optique de Gauss. Gaussian optics
 Optique géométrique. Geometrical optics
 Orbitale. Orbital
 Orbitale d'anti-liaison. Orbital, antibonding
 Orbitales de liaison. Bonding orbitals
 Orbitales moléculaires non-localisées. Nonlocalized molecular orbitals
 Orbites de Bohr. Bohr orbits
 Ordo-symbole. Order-symbol
 Ordre à grande portée. Long-range order
 Ordre de faible portée. Short-range order
 Ordre des réactions chimiques. Order of chemical reactions
 Orthogonalisation. Orthogonalization
 Orthonormal. Orthonormal
 Oscillateur harmonique. Harmonic oscillator
 Oscillateur non-harmonique. Oscillator, anharmonic
 Oscillation. Oscillation
 Oscillation de régime permanent. Oscillation, steady-state
 Oscillations amorties. Vibrations, damped
 Oscillations aperiodiques. Vibrations, non-periodic
 Oscillations forcées. Vibrations, forced
 Oscillations libres d'un système non-amorti. Free vibrations of undamped system
 Oscillations libres ou propres. Oscillations, free or natural
 Ouverture. Aperture
 Ouverture circulaire. Circular aperture
 Ouverture numérique. Numerical aperture
 Ouverture rectangulaire. Rectangular aperture
 Ouverture relative. *f*-number
- Paires homométriques. Homometric pairs
 Panne. Failure
 Parabole. Parabola
 Parabole de Condon. Condon parabola
 Paraboloid. Paraboloid
 Paradoxe de Gibbs. Gibbs paradox
 Paradoxe de récurrence. Recurrence paradox
 Paradoxe d'horloge. Clock paradox
 Parallaxe. Parallax
 Parallélépipède. Parallelepiped
 Parallèles géodésiques. Geodesic parallels
 Parallélogramme. Parallelogram
 Parallélogramme de forces. Forces, parallelogram of
 Paramagnétisme. Paramagnetism
 Paramagnétisme de spin des électrons de conduction. Spin paramagnetism of conduction electrons
 Paramètre. Parameter
 Paramètre de Cayley-Klein. Cayley-Klein parameter
 Paramètre de Coriolis. Coriolis parameter
 Paramètre de Euler-Rodrigues. Euler-Rodrigues parameter
 Paramètre de forme pour couche limite à turbulence. Form parameter for turbulent boundary layer
 Paramètre de réseau. Lattice constant
 Paramètre d'impact. Impact parameter
 Paramètre d'onde. Wave parameter
 Paramètres cristallographiques. Crystallographic parameters
 Paramètres de cristal. Crystal parameters

- Paramètres de Ramberg-Osgood. Ramberg-Osgood parameters
- Paramètres différentiels de Beltrami. Beltrami's differential parameters
- Parcours. Path
- Parcours optique. Optical path
- Parfaitement plastique. Perfectly plastic
- Parfocal. Parfocal
- Parité. Parity
- Paroi diathermane. Diathermal wall
- Particularités. Singularities
- Particule de matière. Particle, material
- Particulier. Particular
- Pas de vis. Pitch of screw
- Pavillon conique fini. Horn, finite conical
- Pavillon conique infini. Horn, infinite conical
- Pavillon cylindrique fini. Horn, finite cylindrical
- Pavillon cylindrique infini (tuyau infini). Horn, infinite cylindrical (infinite pipe)
- Pavillon exponentiel fini. Horn, finite exponential
- Pavillon exponentiel infini. Horn, infinite exponential
- Pavillon parabolique infini. Horn, infinite parabolic
- Pendule. Pendulum
- Pendule balistique. Pendulum, ballistic
- Pendule composé. Pendulum, physical
- Pendule composé. Pendulum, compound
- Pendule conique. Pendulum, conical
- Pendule de Foucault. Pendulum, Foucault's
- Pendule de Kater. Pendulum, Kater
- Pendule de torsion. Pendulum, torsion
- Pendule réversible. Pendulum, reversible
- Pendule simple. Pendulum, simple
- Pendule sphérique. Pendulum, spherical
- Pente. Slope
- Période de déformation. Period of deformation
- Période de polarisation. Polarization cycle
- Période de radioactivité. Half-life
- Période de répétition d'impulsions. Pulse repetition period
- Période de restitution. Period of restitution
- Période de retour. Return period
- Période d'onde. Wave period
- Périodogramme. Periodogram
- Période. Period
- Période propre. Period, natural
- Perméabilité. Permeability
- Permutation. Permutation
- Permutation cyclique. Permutation, cyclic
- Permutation paire ou impaire. Permutation, even or odd
- Perpendiculaire principale. Normal, principal
- Perte par diffraction (son). Divergence loss (sound)
- Perte par dispersion. Scattering loss
- Perte par transfert. Transition loss
- Pertes de paroi. Wall losses
- Perturbations dans les spectres moléculaires. Perturbations in molecular spectra
- Pervéance. Pervance
- Petites ondes de Huygens. Huygens' wavelets
- Phase. Phase
- Phase d'onde. Phase of the wave
- Phase d'une grandeur périodique. Phase of a periodic quantity
- Phases. Phases
- Phases de cristal. Crystal phases
- Phénomène de coopération. Cooperative phenomenon
- Phénomène de découplage en spectroscopie. Uncoupling phenomena in spectroscopy
- Phénomène de Gibbs. Gibbs phenomenon
- Phénomènes de relaxation. Relaxation phenomena
- Phone. Phon
- Phonon. Phonon
- Plan d'Argand. Argand plane
- Plan de courbure. Curvature, plane of
- Plan de Gauss. Gauss plane
- Plan de guidance. Guide plane
- Plan de propagation. Transmission plane
- Plan de réflexion. Reflection plane
- Plan de symétrie. Symmetry, plane of
- Plan de vibration. Plane of vibration
- Plan focal. Focal plane
- Planimètre. Planimeter
- Plan méridien. Meridion(al) plane
- Plan oseulateur. Oseulating plane
- Plans aérodynamiques de Joukowski. Joukowski airfoils
- Plan sagittal. Sagittal plane
- Plans principaux. Principal planes
- Plans principaux. Unit planes
- Plan tangent à une surface. Tangent plane to surface
- Plan tangentiel. Tangential plane
- Plaques circulaires. Plates, circular
- Plaques rectangulaires. Plates, rectangular
- Plasticité. Plasticity
- Plastique-rigide. Plastic-rigid
- Plume. Plume
- Poids. Weight
- Poids atomique. Atomic weight unit
- Poids élastiques. Elastic weights
- Poids statistique. Statistical weight
- Poids statistiques des niveaux d'énergie atomique. Statistical weights of atomic energy levels
- Poids statistiques des niveaux d'énergie moléculaire. Statistical weights of molecular energy levels
- Point circulaire sur une surface. Circular point on a surface
- Point critique. Critical point
- Point d'appui. Bearing
- Point d'ébullition. Boiling point
- Point de chromatéité de l'échantillon. Sample point
- Point de Curie. Curie temperature
- Point de stagnation. Stagnation point
- Point de vapeur. Steam point
- Point d'inflexion. Inflection point
- Point d'objet. Object point
- Point elliptique. Elliptic point
- Point hyperbolique. Hyperbolic point
- Point lambda. Lambda point
- Point ordinaire. Ordinary point
- Point parabolique sur une surface. Parabolic point on a surface
- Point particulier. Address
- Point planaire. Planar point
- Points aplanétiques d'une sphère. Aplanic points of a sphere
- Points cardinaux d'un système optique. Cardinal points of an optical system
- Points conjugués dans un système optique. Conjugates in an optical system
- Points conjugués d'un pendule composé. Conjugate points of a compound pendulum
- Points de Bravais. Bravais points
- Points fixes. Fixed points
- Point singulier d'une fonction. Singular point of a function
- Points principaux. Principal points

- Points nodaux. Nodal points(s)
 Point terminal extrapolé. Extrapolated end-point
 Poise. Poise
 ●Polarisabilité. Polarizability
 Polarisation. Polarization
 Polarisation circulaire. Circular polarization
 Polarisation dans un plan. Plane polarization
 Pôle. Pole
 Pôle d'une fonction analytique. Pole of an analytic function
 Polhodie. Polhode
 Polyèdre. Polyhedron
 Polygone. Polygon
 Polygone de force. Force Polygon
 Polygone funiculaire. Funicular polygon
 Polymère. Polymer
 Polynôme. Polynomial
 Polynôme de Bernstein. Bernstein polynomial
 Polynôme de Hurwitz. Hurwitz polynomial
 Polynômes de Bernoulli. Bernoulli polynomials
 Polynômes de cercle. Circle polynomials
 Polynômes de Chebyshev. Chebyshev polynomials
 Polynômes de Hermite. Hermite polynomials
 Polynômes de Jacobi. Jacobi polynomials
 Polynômes de Laguerre. Laguerre polynomials
 Polynômes de Legendre. Legendre polynomials
 Polynômes orthogonaux. Orthogonal polynomials
 Polynômes spéciaux. Special polynomials
 Portique de butée. Portal frame
 Pose. Exposure
 Postulat. Postulate
 Potentiel cinétique. Kinetic potential
 Potentiel complexe dans un flux hydrodynamique. Complex potential in hydrodynamic flow
 Potentiel critique. Critical potential
 Potentiel dans champ inverse du carré. Potential in inverse square field
 Potentiel d'arrêt. Potential, stopping
 Potentiel de Buckingham. Buckingham potential
 Potentiel de convection. Convective potential
 Potentiel de Coulomb. Coulomb potential
 Potentiel de déformation. Deformation potential
 Potentiel de Keesom. Keesom potential
 Potentiel de Lennard-Jones. Lennard-Jones potential
 Potentiel de Morse. Morse potential
 Potentiel de Stockmayer. Stockmayer potential
 Potentiel de vitesse du son. Velocity potential of sound
 Potentiel d'excitation. Excitation potential
 Potentiel d'ionisation. Ionization potential
 Potentiel électrique. Potential, electric
 Potentiel électrochimique. Electrochemical potential
 Potentiel gravitationnel. Gravitational potential
 Potentiel interatomique. Interatomic potential
 Potentiel nucléaire. Nuclear potential
 Potentiel scalaire magnétique. Magnetic scalar potential
 Potentiels chimiques. Chemical potentials
 Potentiel thermique. Thermal potential
 Potentiel vectoriel. Vector potential
 Potentiel vectoriel magnétique. Magnetic vector potential
 Poussée axiale. Axial thrust
 Poussée et moment sur un plan aérodynamique. Lift and moment on an airfoil
 Poutre cantilever. Beam, cantilever
 Poutre composite. Beam, composite
 Poutre continu. Beam, continuous
 Poutre courbe. Beam, curved
 Poutre de colonne. Beam, column
 Poutre en porte à faux. Beam, overhanging
 Poutre fixe. Beam, fixed-ended or encastre
 Poutre sur fondation élastique. Beam on elastic foundation
 Pouvoir d'absorption. Absorptive power
 Pouvoir de dispersion. Dispersive power
 Pouvoir de résolution. Resolving power
 Pouvoir de résolution d'un prisme. Prism, resolving power of
 Pouvoir de rotation. Rotatory power
 Pouvoir émissif. Emissive power
 Pouvoir émissif spectral. Spectral emissivity
 Pouvoir émissif total. Total emissivity
 Précession de Larmor. Larmor precession
 Précession de relativité. Relativity precession
 Précision. Precision
 Précision. Accuracy
 Prédissociation. Predissociation
 Premier foyer. First focal point
 Première loi de thermodynamique. First law of thermodynamics
 Premier nombre de Beitti. First Beitti number
 Premier théorème fondamental. First fundamental theorem
 Pression de rayonnement. Radiation pressure
 Pression de stagnation. Stagnation pressure
 Pression d'étalement. Spreading pressure
 Pression dynamique. Dynamic pressure
 Pression normale. Normal pressure
 Pression osmotique. Osmotic pressure
 Pression partielle. Partial pressure
 Pression statique. Static pressure
 Pression thermomoléculaire. Thermomolecular pressure
 Pression totale. Total pressure
 Primitive. Primitive
 Principe d'action. Action principle
 Principe d'Archimède. Archimedes' principle
 Principe de Babinet. Babinet's principle
 Principe de Carathéodory. Carathéodory's principle
 Principe de causalité. Causality principle
 Principe de combinaison. Combination principle
 Principe de correspondance. Principle of correspondence
 Principe de d'Alembert. D'Alembert's principle
 Principe de Dirichlet. Dirichlet principle
 Principe de Franck-Condon. Franck-Condon principle
 Principe de Gauss de moindre contrainte. Least constraint, Gauss principle of
 Principe de Hamilton. Hamilton principle
 Principe de Huygens. Huygens' principle
 Principe de l'augmentation d'entropie. Principle of entropy increase
 Principe de Le Châtelier-Braun. Chatelier (Le)-Braun principle
 Principe de moindre action. Least action, principle of
 Principe de moindre énergie. Least-energy principle
 Principe de Neumann. Neumann principle
 Principe d'équilibrage. Detailed balancing, principle of
 Principe des états correspondants. Corresponding states, principle of
 Principe des travaux virtuels. Work, principle of virtual
 Principe de Sumptner. Sumptner's principle
 Principe de superposition. Superposition principle
 Principe des variations et énergies de liaison. Variation principle and bond energies
 Principe de synthèse. Building-up principle
 Principe d'exclusion de Pauli. Pauli exclusion principle

- Principe d'incertitude. Indeterminacy principle
 Principe impulsion-moment. Impulse-momentum principle
 Principe isoélectronique. Isoelectronic principle
 Principes des variations pour états permanents. Variational principles for non-equilibrium states
 Prisme. Prism
 Prisme de Glan-Thompson. Glan-Thompson prism
 Probabilité. Likelihood
 Probabilité. Probability
 Probabilité a posteriori. Posterior probability
 Probabilité d'échappement à la résonance. Resonance escape probability
 Probabilité de collision. Collision probability
 Probabilité de courant. Current probability
 Probabilité de pénétration (pénétrabilité). Penetration probability (penetrability)
 Probabilité de transition. Transition probability
 Probabilité fondamentale. Fundamental probability
 Probabilités de transition d'Einstein. Einstein transition probabilities
 Problème Behrens-Fisher. Behrens-Fisher test
 Problème de Cauchy. Cauchy problem
 Problème de Dido. Dido's problem
 Problème de la masse critique minimum. Minimum critical mass problem
 Problème de Milne. Milne's problem
 Problème de Pfaff. Pfaff problem
 Problème de Sturm-Liouville. Sturm-Liouville problem
 Problème isopérimétrique. Isoperimetric problem
 Problèmes de valeur initiale. Initial-value problems
 Procédé additif. Additive process
 Procédé de double intégration. Double integration procedure
 Procédé delta-carré. Delta-square process
 Procédés naturels et non-naturels. Natural and unnatural processes
 Procédé Umklapp. Umklapp process
 Processus stochastique stationnaire. Stationary stochastic process
 Produit. Product
 Produit de tenseurs. Product of tensors
 Produit de Wallis. Wallis product
 Produit direct (de sous-groupes). Direct product (of subgroups)
 Produit interne. Inner product
 Produit interne de tenseurs. Inner product of tensors
 Produit normal. Normal product
 Produit scalaire. Scalar product
 Profil de vitesse. Velocity profile
 Profil logarithmique de vitesse. Logarithmic profile of velocity
 Profondeur de champ. Depth of field
 Profondeur de pénétration. Penetration depth
 Profondeur effective. Depth, effective
 Profondeur optique. Optical depth
 Programme. Program
 Progression. Progression
 Progression arithmétique. Arithmetic progression
 Progression harmonique. Harmonic progression
 Projection stéréographique d'une sphère sur un plan. Stereographic projection of a sphere on a plane
 Propagation de l'erreur. Error, propagation
 Proportion de polarisation. Proportion of polarization
 Propriétés de symétrie des fonctions propres moléculaires. Symmetry properties of molecular eigenfunctions
 Propriétés de symétrie ou vibrations normales dans les molécules. Symmetry properties or normal vibrations in molecules
 Propriétés intrinsèques d'une surface. Intrinsic properties of a surface
 Propriétés molaires. Molar properties
 Propriété thermodynamique. Thermodynamic property
 Pseudoscalaire. Pseudoscalar
 Pseudovecteur. Pseudovector
 Puissance. Power
 Puissance au frein. Brake horsepower
 Puissance d'une surface. Power of a surface
 Puissance effective d'une lentille. Effective power of a lens
 Puissance sonore d'une source. Sound power of a source
 Puissance spécifique de rotation d'un milieu. Specific rotatory power of a medium
 Pulsation. Angular velocity
 Pulsation. Pulsation
 Pureté d'excitation. Excitation purity
 Quadrature numérique. Quadrature, numerical
 Quadripolaire. Quadrupole
 Quadrique centrale. Central quadric
 Quadrivecteurs et quadritenseurs. Four-vectors and tensors
 Quadruple produit de vecteurs. Quadruple product of vectors
 Quantification. Quantization
 Quantification de champ. Field quantization
 Quantification des signaux. Quantization of signals
 Quantification d'espace. Space quantization
 Quantification d'un champ électromagnétique. Quantization of electromagnetic field
 Quantifier. Quantize
 Quantile. Quantile
 Quantique. Quantic
 Quantité de mouvement. Momentum
 Quantités moléculaires partielles. Partial molar quantities
 Quantité transmissible. Transferable quantity
 Quantum. Quantum
 Quantum virtuel. Virtual quantum
 Quasi-niveaux de Fermi. Fermi levels, quasi
 Quaternion. Quaternion
 Racine. Radix
 Racine. Binary point
 Racine principale. Principal root
 Radiance. Radiance
 Radiance. Luminous emittance
 Radiance. Irradiance
 Radiateur. Radiator
 Radiateur sélectif. Selective radiator
 Radiation complexe. Complex radiation
 Rang d'une matrice. Rank of a matrix
 Rapport. Ratio
 Rapport air-combustible. Air-fuel ratio
 Rapport d'amortissement. Damping ratio
 Rapport d'amplitude. Standing-wave ratio
 Rapport de corrélation. Correlation ratio
 Rapport de la largeur à la hauteur de l'image. Aspect ratio
 Rapport de Poisson. Poisson's ratio
 Rapport de régénération. Breeding ratio
 Rapport gyromagnétique. Gyromagnetic ratio

- Rapport magnétomécanique. Magnetomechanical ratio
- Rapports de commutation. Commutation relations
- Rapports de dispersion. Dispersion relations
- Rapports d'énergie dans un champ de force uniforme. Energy relations in uniform force field
- Rapports de réciprocity. Reciprocity relations
- Rattachement de couche limite après séparation. Re-attachment of boundary layer after separation
- Rayl. Rayl
- Rayon atomique. Atomic radius
- Rayon axial. Axial ray
- Rayon court. Beam, short
- Rayon de Bohr. Bohr radius
- Rayon de champ. Field ray
- Rayon de courbure. Curvature, radius of
- Rayon de courbure sphérique. Spherical curvature, radius of
- Rayon de giration. Radius of gyration
- Rayon de torsion. Torsion, radius of
- Rayon de Wagner. Wagner beam
- Rayon efficace de barreau d'essai. Radius, effective, of control rod
- Rayon électronique (classique). Electron radius (classical)
- Rayon gravitationnel. Gravitational radius
- Rayon infrarouge. Infrared radiation
- Rayonnement cohérent. Coherent radiation
- Rayonnement quadripolaire. Quadrupole radiation
- Rayon oblique. Skew ray
- Rayon principal. Principal ray
- Rayon simple. Beam, simple
- Rayon spectral d'une matrice. Spectral radius of a matrix
- Rayon vecteur. Vector, radius
- Réactance. Reactance
- Réaction. Reaction
- Réaction. Feedback
- Réaction endothermique. Endothermic reaction
- Réaction exothermique. Exothermic reaction
- Réaction perpendiculaire. Normal reaction
- Réaction thermique. Heat regeneration
- Réactions chimiques couplées. Coupled chemical reactions
- Réactions indépendantes. Independent reactions
- Réactivité. Reactivity
- Réalisation. Realization
- Recirculation. Recirculation
- Réflectance diffuse. Diffuse reflectance
- Réflectance directe. Direct reflectance
- Réflexion diffuse. Diffuse reflection
- Réflexion intégrée de rayons X. X-ray reflection, integrated
- Réflexion réflex. Reflex reflection
- Réflexion spéculaire. Specular reflection
- Réflexion totale. Total reflection
- Réfraction atomique. Atomic refraction
- Réfraction conique. Conical refraction
- Réfraction idéale. Standard refraction
- Réfraction spécifique. Specific refraction
- Réfringence. Refractivity
- Refroidissement de diffusion. Diffusion cooling
- Région critique. Critical region
- Région de vitesse réciproque. Reciprocal velocity region
- Région jointe simplement. Simply connected region
- Régime laminaire. Laminar flow
- Régime laminaire dans un tuyau. Pipe, laminar flow in
- Régime potentiel. Potential flow
- Régime secondaire. Secondary flow
- Régime turbulent dans un tuyau. Pipe, turbulent flow in
- Règle de Bragg. Bragg rule
- Règle de Cramer. Cramer's rule
- Règle de fausse position. Regula falsi
- Règle de flux. Flow rule
- Règle de Gothert. Gothert's rule
- Règle de Laporte. Laporte rule
- Règle de Leibnitz. Leibnitz rule
- Règle de L'Hôpital. Hospital rule
- Règle de non-croisement. Non-crossing rule
- Règle de permanence g . g -permanence rule
- Règle de phase. Phase rule
- Règle de Prandtl-Glauert. Prandtl-Glauert rule.
- Règle de Silsbee. Silsbee rule
- Règle de Simpson. Simpson's rule
- Règle des nombres pairs et impairs de stabilité nucléaire. Odd-even rule of nuclear stability
- Règle des signes de Descartes. Descartes rule of signs
- Règle des sommes de vibrations pour transitions électroniques. Vibrational sum rule for electronic transitions
- Règle de surface transsonique. Transonic area rule
- Règle de trapézoïde. Trapezoidal rule
- Règle de Weddle. Weddle rule
- Règle d'inertie de Routh. Routh's rule of inertia
- Règle d'intervalle de Landé. Landé's interval rule
- Règles d'anticommutation. Anti-commutation rules
- Règles de commutation. Commutation rules
- Règles de commutation covariante. Covariant commutation rules
- Règles de Hume-Rothery. Hume-Rothery rules
- Règles de Hund pour multiplets atomiques. Hund's rules for atomic multiplets
- Règles de sélection. Selection rules
- Règles de sélection nucléaire. Selection rules, nuclear
- Règles des sommes de rotation. Rotational sum rules
- Règles de super-sélection. Superselection rules
- Régression. Regression
- Relation de Geiger-Nutall. Geiger-Nutall relation
- Relation exposition-densité. Exposure-density relationship
- Relations de Cauchy. Cauchy relations
- Relations de Ehrenfest. Ehrenfest's relations
- Relations de Lamé. Lamé relations
- Relations de Maxwell. Maxwell's relations
- Relations de Onsager. Onsager relations
- Relativité générale. Relativity, general
- Relativité spéciale. Relativity, special
- Relaxation. Relaxation
- Relaxation de groupe. Group relaxation
- Rendement. Efficiency
- Rendement. Gain ratio
- Rendement collectif. Joint efficiency
- Rendement de fluorescence. Fluorescence yield
- Rendement d'étage. Stage efficiency
- Rendement indiqué. Indicated efficiency
- Rendement lumineux. Visibility factor
- Rendement mécanique. Mechanical efficiency
- Rendement quantique. Quantum efficiency
- Rendement quantique. Quantum yield
- Rendement thermique de cycle. Thermal efficiency of cycle
- Renormalisation de masse. Mass renormalization
- Répartition. Distribution
- Réponse de fréquence. Frequency response
- Réponse indiciaire. Indicial response

- Réponse normale. Normal response
 Réponse transitoire d'un système dynamique. Transient response of a dynamical system
 Représentation. Representation
 Représentation de Dyson. Dyson representation
 Représentation de Foldy-Wouthuysen. Foldy-Wouthuysen representation
 Représentation de groupes et mécanique quantique. Representation of groups and quantum mechanics
 Représentation de Heisenberg. Heisenberg representation
 Représentation de Lehmann. Lehmann representation
 Représentation de réponse de fréquence. Frequency response representation
 Représentation de Schrödinger. Schrödinger representation
 Représentation d'interaction. Interaction representation
 Représentation d'un groupe. Representation of a group
 Représentation sphérique d'une congruence rectiligne. Spherical representation of a rectilinear congruence
 Représentation sphérique d'une surface. Spherical representation of a surface
 Représentations du groupe hétérogène de Lorentz. Representations of the inhomogeneous Lorentz group
 Représentations équivalentes de groupes. Equivalent representations of groups
 Répulsion de Born. Born repulsion
 Réseau. Lattice
 Réseau. Space lattice
 Réseau actif. Network, active
 Réseau cristallin réciproque. Reciprocal lattice
 Réseau non maillé. Tree
 Résidu. Residue
 Résilience. Resilience
 Résistance. Resistance
 Résistance acoustique de courant continu. Resistance flow
 Résistance à la rupture. Ultimate strength
 Résistance de rayonnement. Radiation resistance
 Résistance électrique. Electrical resistance
 Résistivité acoustique. Resistance, specific acoustic
 Résonance de Fermi. Fermi resonance
 Résonance ferromagnétique. Ferromagnetic resonance
 Résonance magnétique nucléaire. Nuclear magnetic resonance
 Résonance mécanique quantique. Resonance, quantum mechanical
 Ressort(s). Spring(s)
 Reste. Remainder
 Résultant. Resultant
 Retard. Delay time
 Retard de cisaillement. Shear lag
 Retenue. Carry
 Réticulation. Wrinkling
 Réversibilité microscopique. Microscopic reversibility
 Réversion de série. Reversion of series
 Rigidité à la flexion. Rigidity, flexural
 Risque du consommateur. Consumer's risk
 Rosette. Rosette
 Rotation. Curl
 Rotation. Rotation
 Rotation moléculaire. Molecular rotation
 Rotation spécifique. Rotation, specific
 Rotor. Rotor
 Routine. Routine
 Rutherford. Rutherford
 Sabin. Sabin
 Scalaire. Scalar
 Scalaire absolu. Scalar, absolute
 Scalaire relatif. Scalar relative
 Scalène. Scalene
 Seconde loi de thermodynamique. Second law of thermodynamics
 Seconde quantification. Quantization, second
 Second théorème de limite. Second limit theorem
 Second théorème fondamental. Second fundamental theorem
 Section. Bay
 Section efficace. Cross-section
 Section principale d'un cristal. Principal section of a crystal
 Section transformée. Transformed section
 Sécurité de fonctionnement. Reliability
 Sens d'un vecteur. Sense of a vector
 Sensibilité. Responsivity
 Sensibilité acoustique. Responsiveness, acoustic
 Sensibilité de résolution. Resolution sensitivity
 Sensibilité photoélectrique. Photoelectric sensitivity
 Sensibilité. Sensitivity
 Séparable. Separable
 Séparation. Partition
 Séparation de couche limite. Separation of boundary layer
 Séparation de variables. Separation of variables
 Séparation d'impulsions. Pulse separation
 Série. Series
 Série asymptotique. Asymptotic series
 Série de Brackett. Brackett series
 Série de composition d'un groupe. Composition series of a group
 Série de Dirichlet. Dirichlet series
 Série de Edgeworth. Edgeworth's series
 Série de Fourier. Fourier series
 Série de Gram-Charlier. Gram-Charlier series
 Série de Laurent. Laurent series
 Série de Liouville-Neumann. Liouville-Neumann series
 Série de Lyman. Lyman series
 Série de Maclaurin. Maclaurin series
 Série de Neumann. Neumann series
 Série de Paschen. Paschen series
 Série de Pfund. Pfund series
 Série de Rydberg. Rydberg series
 Série de Taylor. Taylor series
 Série diffuse. Diffuse series
 Série exponentielle. Power series
 Série harmonique de sons. Harmonic series of sounds
 Série multinôme. Multinomial series
 Série précise. Sharp series
 Série principale. Principal series
 Séries de Bergmann. Bergmann series
 Séries de Enskog pour les solutions de l'équation de Boltzmann. Enskog's series for the solutions of the Boltzmann equation
 Seuil d'audibilité. Threshold of audibility
 Seuil de sensibilité photoélectrique. Photoelectric threshold
 Signum. Signum
 Sillage. Wake
 Similitude de turbulence. Turbulence, similarity of
 Similitude dynamique. Dynamical similarity
 Simultanément mesurable. Simultaneously measurable
 Singulier. Singular

- Singularité analytique d'une équation différentielle.
Regular singularity of a differential equation
- Singularité essentielle d'une fonction analytique. Essential singularity of an analytic function
- Solide colorimétrique. Color solid
- Solution de Bayes. Bayes' solution
- Solution singulière. Singular solution
- Solutions analytiques. Regular solutions
- Solutions athermiques. Athermal solutions
- Solutions conformes. Conformal solutions
- Solutions parfaites. Perfect solutions
- Sommation d'une série. Summation of series
- Somme de Petzval. Petzval sum
- Somme de réseau. Lattice sum
- Somme de Slater. Slater sum
- Somme de tenseurs. Sum of tensors
- Somme directe. Direct sum
- Sommes de combinaison. Combination sums
- Sommet. Vertex
- Sommet "endormi". Top, "sleeping"
- Sommet final. Vertex, final
- Sommet initial. Vertex, initial
- Sommet interne. Vertex, internal
- Sommet terminal. Vertex, terminal
- Sommets adjacents. Vertices, adjacent
- Sone. Sone
- Soufflerie. Blower
- Source. Source
- Source de son de surface rectangulaire plane. Plane rectangular surface sound source
- Source idéale de tension. Voltage source, ideal
- Source négative. Sink
- Source ponctuelle. Point source
- Source ponctuelle uniforme. Uniform point source
- Source sonore à variation linéaire. Straight line sound source
- Source sonore circulaire. Circular ring sound source
- Sources de l'image. Image sources
- Sous-diagramme. Subgraph
- Sous-diagramme propre. Subgraph, proper
- Sous-groupe. Subgroup
- Sous-harmonique. Subharmonic
- Spectre. Spectrum
- Spectre caractéristique de rayons X. X-ray spectrum, characteristic
- Spectre continu. Continuum
- Spectre de fission. Fission spectrum
- Spectre d'égale énergie. Equi-energy spectrum
- Spectre de puissance. Power spectrum
- Spectre de turbulence. Spectrum of turbulence
- Spectre de Watt. Watt spectrum
- Spectre neutronique. Spectrum, neutron
- Spectres atomiques. Atomic spectra
- Spectres de rotation dans les noyaux. Rotational spectra in nuclei
- Spectres de rotation des molécules. Rotation spectra of molecules
- Spectres de rotation-vibration des molécules. Rotation vibration spectra of molecules
- Spectres électroniques des molécules. Electronic spectra of molecules
- Spectres moléculaires. Molecular spectra
- Spectres vibratoires des molécules. Vibrational spectra of molecules
- Sphère de courbure. Sphere of curvature
- Sphère de Riemann. Riemann sphere
- Sphères rigides. Rigid spheres
- Sphéromètre. Spherometer
- Spin. Spin
- Spin. Angular momentum
- Spin électronique. Electron spin
- Spin isobare. Isobaric spin
- Spin isotope. Isotope spin
- Spin nucléaire. Nuclear spin
- Spirale de Cornu. Cornu spiral
- Spirale d'Ekman. Ekman spiral
- Spirale de Taylor. Taylor spiral
- Stabilisation de variation. Stabilization of variance
- Stabilité. Stability
- Stabilité d'avion. Airplane stability
- Stabilité de diffusion. Diffusion stability
- Stabilité de pente. Slope stability
- Stabilité de phases. Stability of phases
- Stabilité de régime laminaire. Stability of laminar flow
- Stabilité d'un corps flottant. Stability of a floating body
- Stabilité mécanique. Mechanical stability
- Stabilité numérique. Numerical stability
- Stabilité statique. Static stability
- Stabilité thermique. Thermal stability
- Statique. Statics
- Statique graphique. Graphical statics
- Statiquement admissible. Statically admissible
- Statistique. Statistics
- Statistique Bose-Einstein. Bose-Einstein statistics
- Statistique de Boltzmann. Boltzmann statistics
- Statistique de Gentile. Gentile statistics
- Statistique Fermi-Dirac. Fermi-Dirac statistics
- Statistique quantique. Quantum statistics
- Stéradian. Steradian
- Stéréospectrogramme. Stereospectrogram
- Stigmatique. Stigmatic
- Stilb. Stilb
- Stochastique. Stochastic
- Stratification. Stratification
- Striction. Reduction in area
- Structure des cristaux. Crystal structure
- Structure déterminée. Determinate structure
- Structure fine. Fine structure
- Structure fine de l'hydrogène. Hydrogen fine structure
- Structure hyperfine. Hyperfine structure
- Structure indéterminée. Indeterminate structure
- Structure nucléaire. Nuclear structure
- Structures de la benzène. Benzene, structures of
- Substance d'un tourbillon. Substance of a vortex
- Suction de couche limite laminaire. Suction of laminar boundary layer
- Suffisance. Sufficiency
- Suite. Sequence
- Suite de Cauchy. Cauchy sequence
- Suite dérivée. Derived set
- Suite de Sturm. Sturm sequence
- Suite nulle. Null sequence
- Super-réseau. Superlattice
- Support. Strut
- Suréchange. Superexchange
- Surface aplanétique. Aplanatic surface
- Surface d'aile d'avion. Wing area of airplane
- Surface de centres. Surface of centers
- Surface de chauffage. Heating surface
- Surface de diffusion neutronique. Diffusion area, neutron
- Surface de distribution d'intensité. Intensity distribution, surface of
- Surface de division de Gibbs. Gibbs division surface
- Surface de Liouville. Liouville surface

- Surface de migration des neutrons. Migration area for neutrons
- Surface de moindre confusion. Surface of least confusion
- Surface de Petzval. Petzval surface
- Surface de ralentissement. Slowing-down area
- Surface de révolution. Surface of revolution
- Surface de Riemann. Riemann surface
- Surface développable. Developable surface
- Surface isobare. Isobaric surface
- Surface minimale. Minimal surface
- Surface oblique. Skew surface
- Surface quadrique. Quadric surface
- Surface réglée. Ruled surface
- Surfaces applicables. Applicable surfaces
- Surfaces cartésiennes. Cartesian surfaces
- Surfaces figurées. Figured surfaces
- Surfaces inverses. Inverse surfaces
- Surfaces parallèles. Parallel surfaces
- Surface vectorielle. Vector area
- Surhaussement. Superelevation
- Susceptibilité paramagnétique. Paramagnetic susceptibility
- Symbole de permutation. Permutation symbol
- Symboles de Christoffel. Christoffel (3-index) symbols
- Symboles de Hermann-Manguin. Hermann-Manguin symbols
- Symboles de Schönflies pour description des cristaux. Schönflies crystal symbols
- Symétrie des cristaux. Crystal symmetry
- Symétrie et mécanique quantique. Symmetry and quantum mechanics
- Système acoustique. Acoustical system
- Système afocal. Afocal system
- Système anamorphique. Anamorphic system
- Système catoptrique. Catoptric system
- Système centre de masse. Center-of-mass system
- Système condensé. Condensed system
- Système conjugué de courbes on a surface. Conjugate system of curves on a surface
- Système conservateur. Conservative system
- Système cubique. Cubic system
- Système de bandes. Band system
- Système de Chebyshev. Chebyshev system
- Système de commande par magnétostriktion. Driving system, magnetostriction
- Système de contrôle. Control system
- Système de coordonnées. Frame of reference
- Système de coordonnées rotatives. Rotating coordinate system
- Système de laboratoire et système centre de masse. Laboratory system and center-of-mass system
- Système de mobilité mécanique. Mechanical mobility system
- Système d'équations. System equations
- Système d'oscillation dégénéré. Degenerate oscillating system
- Système d'unités. Units, system of
- Système-*e*. *e*-System
- Système électrique. Electrical system
- Système électrodynamique de commande. Driving system, dynamic
- Système électromagnétique de commande. Driving system, electromagnetic
- Système électrostatique de commande. Driving system, electrostatic
- Système élémentaire. Elementary system
- Système-epsilon. Epsilon-system
- Système fermé. Closed system
- Système hexagonal. Hexagonal system
- Système L et système C. *L*-system and *C*-system
- Système mécanique rectiligne. Mechanical rectilinear system
- Système mécanique rotatoire. Mechanical rotational system
- Système orthonormal. Orthonormal system
- Système orthorhombique. Orthorhombic system
- Système orthoscopique. Orthoscopic system
- Système orthotomique. Orthotomic system
- Système ouvert. Open system
- Système rhomboédrique. Rhombohedral system
- Systèmes azéotropes. Azeotropic systems
- Systèmes azéotropiques. Azeotropic systems
- Systèmes continus. Continuous systems
- Systèmes cristallins. Crystal systems
- Systèmes de force équivalents. Equivalent force systems
- Systèmes discontinus. Discontinuous systems
- Systèmes idéals. Ideal systems
- Systèmes inversibles et irréversibles. Reversible and irreversible processes
- Systèmes isolés. Isolated systems
- Systèmes linéaires. Linear systems
- Systèmes optiques parfaits. Perfect optical systems
- Systèmes ternaires. Ternary systems
- Système télécentrique. Telecentric system
- Système tétragone. Tetragonal system
- Système thermodynamique. Thermodynamic system
- Système trichromatique. Trichromatic system
- Système triclinique. Triclinic system
- Système vectoriel réciproque. Reciprocal vector system
- Table de Pade. Pade table
- Taille critique. Critical size
- Talbot. Talbot
- Tangente à une courbe. Tangent to a curve
- Tangente géodésique. Geodesic tangent
- Tantochrone. Tantochrone
- Taux d'activation. Activation rate
- Taux d'amplification d'un projecteur. Magnification ratio of a projector
- Taux d'avancement. Degree of advancement
- Taux de changement local. Local rate of change
- Taux de conversion. Conversion ratio
- Taux de minceur. Slenderness ratio
- Taux de recombinaison. Recombination rate
- Taux de rendement. Efficiency ratio
- Taux de retour. Return ratio
- Taux des fonctionnements irréversibles. Rates of irreversible processes
- Taux d'impulsions. Pulse duty factor
- Température. Temperature
- Température absolue. Absolute temperature
- Température de Boyle. Boyle temperature
- Température de brillance. Luminance temperature
- Température de couleur. Color temperature
- Température de Debye. Debye temperature
- Température de dégénération. Degeneracy temperature
- Température de Fermi. Fermi temperature
- Température de rayonnement totale. Total radiation temperature
- Température de stagnation. Stagnation temperature
- Température de transition de seconde espèce. Second order transition temperature

- Température d'inversion. Inversion temperature
 Température et pression normales. Normal temperature and pressure
 Température neutronique. Temperature, neutron
 Température potentielle. Potential temperature
 Température pour la théorie cinétique. Kinetic theory temperature
 Température virtuelle. Virtual temperature
 Temps de ralentissement. Slowing-down time
 Temps de relaxation. Relaxation time
 Temps de réponse. Response time
 Tendance de pression barométrique. Tendency of barometric pressure
 Tenseur absolu. Tensor, absolute
 Tenseur alterné. Tensor, alternating
 Tenseur antisymétrique. Tensor, skew-symmetric
 Tenseur covariant fondamental. Fundamental covariant tensor
 Tenseur de courbure. Curvature tensor
 Tenseur de Ricci. Ricci tensor
 Tenseur de Riemann-Christoffel. Riemann-Christoffel tensor
 Tenseur d'espace. Tensor, space
 Tenseur de surface. Tensor, surface
 Tenseur double. Tensor, double
 Tenseur isotrope. Tensor, isotropic
 Tenseur métrique. Metric tensor
 Tenseur relatif. Tensor, relative
 Tenseurs associés (champs tensoriels). Tensors, associated (tensor fields)
 Tenseurs affines et vecteurs libres. Affine tensors and free vectors
 Tenseur symétrique. Tensor, symmetric
 Tension de surface dynamique. Dynamical surface tension
 Tension de vapeur dans les solutions parfaites. Vapor pressure in perfect solutions
 Tension de vapeur de solutions idéales. Vapor pressure of ideal solutions
 Tension diagonale. Diagonal tension
 Tensions secondaires. Secondary stresses
 Tension superficielle de surface limite courbe. Surface tension of curved interface
 Tension thermoélectrique. Thermoelectric power
 Terme de Pauli. Pauli term
 Terme d'état d'énergie. Energy state, term
 Terme impair d'un atome. Odd term of an atom
 Terme pair d'un atome. Even term of an atom
 Termes anharmoniques. Anharmonic terms
 Termes Balmer. Balmer terms
 Terme spectroscopique. Term, spectroscopic
 Théorème acoustique de Thévenin. Thévenin's acoustic theorem
 Théorème alternatif. Alternative theorem
 Théorème binôme. Binomial theorem
 Théorème d'Abel sur les séries exponentielles. Abel theorem on power series
 Théorème d'approximation de Weierstrass. Weierstrass approximation theorem
 Théorème d'Ascoli. Ascoli's theorem
 Théorème d'axe perpendiculaire. Perpendicular axis theorem
 Théorème de Abbe-Maxwell. Abbe-Maxwell theorem
 Théorème de Bayes. Bayes' theorem
 Théorème de Bernoulli. Bernoulli theorem
 Théorème de Beltrami-Enneper. Beltrami-Enneper theorem
 Théorème de Bloch. Bloch theorem
 Théorème de Bolzano-Weierstrass. Bolzano-Weierstrass theorem
 Théorème de Budan. Budan theorem
 Théorème de Carathéodory. Carathéodory's theorem
 Théorème de Carnot. Carnot's theorem
 Théorème de Castigliano. Castigliano's theorem
 Théorème de Clapeyron. Clapeyron's theorem
 Théorème de Cochran. Cochran's theorem
 Théorème de comparaison pour convergence. Comparison theorem for convergence
 Théorème de Crocco. Crocco's theorem
 Théorème de décomposition en facteurs unique. Unique factorization theorem
 Théorème de De Moivre-Laplace. De Moivre-Laplace theorem
 Théorème de Duhem. Duhem's theorem
 Théorème de Earnshaw. Earnshaw theorem
 Théorème de Ehrenfest. Ehrenfest theorem
 Théorème de Euler sur la courbure normale. Euler theorem on normal curvature
 Théorème de Euler sur les fonctions homogènes. Euler theorem for homogeneous functions
 Théorème de facteur algébrique. Factor theorem of algebra
 Théorème de Floquet. Floquet theorem
 Théorème de fluctuation-dissipation. Fluctuation-dissipation theorem
 Théorème de Fredholm pour équations intégrales. Fredholm theorem for integral equations
 Théorème de Fuchs. Fuchs theorem
 Théorème de Gauss-Markov. Gauss-Markov theorem
 Théorème de Gauss sur la courbure. Gauss' theorem on curvature
 Théorème de Haag. Haag's theorem
 Théorème de Hamilton. Hamilton's theorem
 Théorème de Hamilton-Cayley. Hamilton-Cayley theorem
 Théorème de Heine-Borel. Heine-Borel theorem
 Théorème de Jacobi en optique. Jacob's theorem in optics
 Théorème de Jordan-Hölder. Jordan-Hölder theorem
 Théorème de Kelvin. Circulation theorem (Kelvin)
 Théorème de la chaleur de Nernst. Nernst heat theorem
 Théorème de Lagrange. Lagrange theorem
 Théorème de Le Châtelier. Chatelier (Le) theorem
 Théorème de l'enthalpie. Enthalpy theorem
 Théorème de limite centrale. Central limit theorem
 Théorème de limite sinusoidale. Sinusoidal limit theorem
 Théorème de Liouville. Liouville theorem
 Théorème de Maupertuis. Maupertuis' theorem
 Théorème de Maxwell pour isothermes. Maxwell's theorem for isotherms
 Théorème de Meunier. Meunier's theorem
 Théorème de modération. Moderation theorem
 Théorème de Morera. Morera theorem
 Théorème d'énergie potentielle minimum. Potential energy, theorem of minimum
 Théorème de Norton. Norton's theorem
 Théorème de Nyquist. Nyquist theorem
 Théorème de poids statistique. Statistical weight theorem
 Théorème de polarisation de Stokes. Stokes polarization theorem
 Théorème de Poynting. Poynting theorem
 Théorème de Réciprocité. Reciprocity theorem
 Théorème de réciprocity acoustique. Acoustical reciprocity theorem

- Théorème de réciprocity électroacoustique. Reciprocity theorem, electroacoustical
- Théorème de réciprocity électromagnétique. Reciprocity theorem, electromagnetic
- Théorème de réciprocity (électromagnétique) Reciprocity theorem (electromagnetic)
- Théorème de réciprocity électromécanique. Electrical-mechanical reciprocity theorem
- Théorème de réciprocity électrique. Electrical reciprocity theorem
- Théorème de réciprocity mécanique-acoustique. Mechanical-acoustical reciprocity theorem
- Théorème de réciprocity mécanique rectiligne. Mechanical rectilinear reciprocity theorem
- Théorème de réciprocity mécanique rotatoire. Mechanical rotational reciprocity theorem
- Théorème de résidu. Residue theorem
- Théorème de Ricci. Ricci's theorem
- Théorème de Rolle. Rolle theorem
- Théorème de Saurel. Saurel's theorem
- Théorème de similitude. Similarity theorem
- Théorème de Sturm. Sturm theorem
- Théorème de superposition. Superposition theorem
- Théorème de transfert pour moment d'inertie. Moment of inertia, transfer theorem for
- Théorème de travail-énergie cinétique. Work-kinetic energy theorem
- Théorème de Van't Hoff. Van't Hoff theorem
- Théorème de Varignon. Varignon theorem
- Théorème d'optique géométrique de Kirchoff. Kirchoff's geometric optics theorem
- Théorème du moindre travail. Least work, theorem of
- Théorème du point fixe de Brouwer. Brouwer fixed point theorem
- Théorème électrique de Thévenin. Thévenin's electrical theorem
- Théorème-H. H-theorem
- Théorème intégral de Cauchy. Cauchy integral theorem
- Théorème mécanique rectiligne de Thévenin. Thévenin's mechanical rectilinear theorem
- Théorème mécanique rotatif de Thévenin. Thévenin's mechanical rotational theorem
- Théorème optique. Optical theorem
- Théorème réciproque. Reciprocal theorem
- Théorèmes de Gibbs-Konovalov. Gibbs-Konovalov theorems
- Théorèmes de la valeur moyenne. Mean value theorems
- Théorèmes de limite. Limit theorems
- Théorèmes intégraux d'analyse vectorielle. Integral theorems of vector analysis
- Théorie cinétique. Kinetic theory
- Théorie cinétique de conductibilité thermique. Thermal conductivity, kinetic theory of
- Théorie cinétique de diffusion. Diffusion, kinetic theory of
- Théorie cinétique de viscosité. Viscosity, kinetic theory of
- Théorie classique de l'électron. Classical electron theory
- Théorie de bande des solides. Band theory of solids
- Théorie de Born et Von Karman. Born and von Karman theory
- Théorie de Born-Infeld. Born-Infeld theory
- Théorie de champ. Field theory
- Théorie de champ quantifié relativiste. Relativistic quantum field theory
- Théorie de collision de cinétique chimique. Collision theory of chemical kinetics
- Théorie de déformation plastique. Deformation theory of plasticity
- Théorie de désintégration bêta de Fermi. Fermi theory of beta decay
- Théorie de déviation. Deflection theory
- Théorie de diffusion neutronique. Neutron diffusion theory
- Théorie de distribution. Distribution theory
- Théorie de Enskog. Enskog's theory
- Théorie de Eyring sur les procédés de déplacement. Eyring theory of transport processes
- Théorie de grande déviation de Marguerre. Marguerre's large deflection theory
- Théorie de Heitler-London sur la molécule d'hydrogène. Hydrogen molecule, Heitler-London theory of
- Théorie de la ligne de sustentation. Lifting-line theory
- Théorie de la perturbation. Perturbation theory
- Théorie de la surface de sustentation des ailes. Lifting-surface theory of wings
- Théorie de la valence de Heitler-London. Heitler-London theory of valence
- Théorie de ligne de rupture. Rupture line theory
- Théorie de matrice S et théorie de champ quantifié. S -matrix theory and quantum field theory
- Théorie de Neymann-Pearson. Neymann-Pearson theory
- Théorie de perturbation covariante. Covariant perturbation theory
- Théorie de plan aérodynamique. Airfoil theory
- Théorie d'équilibre. Equilibrium theory
- Théorie de renouvellement. Renewal theory
- Théorie de réseau des coefficients d'élasticité. Elastic coefficients, lattice theory of
- Théorie des jeux. Games theory
- Théorie des moments de Froude. Froude momentum theory
- Théorie des positrons de Feynman. Feynman's positron theory
- Théorie d'estimation. Estimation, theory of
- Théorie des "trous". Hole theory
- Théorie des quanta de capacité de chaleur. Quantum theory of heat capacity
- Théorie des quanta de dispersion. Quantum theory of dispersion
- Théorie des quanta des spectres. Quantum theory of spectra
- Théorie des quanta du magnétisme. Magnetism, quantum theory of
- Théorie des queues. Queues, theory of
- Théorie de valence des orbitales moléculaires. Molecular orbital theory of valence
- Théorie des vibrations de cristaux de Debye. Debye theory of crystal vibrations
- Théorie différentielle de plasticité. Incremental theory of plasticity
- Théorie d'orbite planétaire. Planetary orbit theory
- Théorie du déplacement neutronique. Neutron transport theory
- Théorie du diamagnétisme de Langevin. Langevin theory of diamagnetism
- Théorie du ferromagnétisme de Heisenberg. Heisenberg theory of ferromagnetism
- Théorie du second ordre de Busemann. Busemann's second-order theory
- Théorie électronique de Dirac. Dirac electron theory
- Théorie électronique des métaux. Electron theory of metals
- Thermicité de neutrons. Thermalization of neutrons

- Thermodynamique des systèmes irréversibles. Thermodynamics of irreversible processes
- Thermostatique. Thermostaties
- Thermostatistique. Thermostatistics
- Thixotropie. Thixotropy
- Topologie. Topology
- Tore. Torus
- Tore de Vieth-Müller. Vieth-Müller torus
- Torsion. Torsion
- Torsion. Wrench
- Tourbillon. Vortex
- Tourbillon de Rankine. Rankine vortex
- Tourbillon sphérique. Spherical vortex
- Trace. Trace
- Tracé de Kurie. Kurie plot
- Tracé de Richardson. Richardson plot
- Tracé d'une matrice. Trace of a matrix
- Tracé d'un rayon. Ray tracing
- Traitement homogène. Homogeneous process
- Trajectoire. Trajectory
- Trajectoire de contrainte. Stress trajectory
- Trajectoire dirigée. Path, directed
- Trajectoires électroniques dans un atome. Electron orbits in an atom
- Transcendant. Transcendental
- Transformation. Transformation
- Transformation affine. Affine transformation
- Transformation canonique. Canonical transformation
- Transformation canonique. Contact transformation
- Transformation de Euler. Euler transformation
- Transformation de Galilée. Galilean transformation
- Transformation de Lorentz. Lorentz transformation
- Transformation de Lorentz propre. Proper Lorentz transformation
- Transformation de Schwartz-Christoffel. Schwartz-Christoffel transformation
- Transformation hétérogène de Lorentz. Inhomogeneous Lorentz transformation
- Transformation hétérogène orthochrone de Lorentz. Orthochronous inhomogeneous Lorentz transformation
- Transformation homogène de Lorentz. Homogeneous Lorentz transformation
- Transformation homogène orthochrone de Lorentz. Orthochronous homogeneous Lorentz transformation
- Transformation homographique. Homographic transformation
- Transformation ordre-désordre. Order-disorder transformation
- Transformations homogènes limitées de Lorentz. Restricted homogeneous Lorentz transformations
- Transforme. Transform
- Transforme de Fourier-Bessel. Fourier-Bessel transform
- Transforme de Fourier. Fourier transform
- Transforme de Hankel. Hankel transform
- Transforme de Hilbert. Hilbert transform
- Transforme de Laplace. Laplace transform
- Transforme de Mellin. Mellin transform
- Transforme intégrale. Integral transform
- Transition de régime turbulent. Transition of turbulent flow
- Transition interdite. Forbidden transition
- Transition permise. Allowed transition
- Transition de supraconductibilité. Superconducting transition
- Transitions d'ordre plus élevé. Transitions of higher order
- Translation. Translation
- Translation primitive. Primitive translation
- Transmissibilité d'une force. Transmissibility of force
- Transmission. Transmissivity
- Transmission. Transmission
- Transmission du contrôle. Transfer of control
- Transmission sonore. Sound transmission
- Transmittance. Transmittance
- Transmittance diffuse. Diffuse transmittance
- Transmittance directe. Direct transmittance
- Transmittance totale. Total transmittance
- Transmittivité. Transmittivity
- Transpose d'une matrice. Transpose of a matrix
- Transposition. Transposition
- Travail. Work
- Travaux de sortie électroniques. Work functions, electronic
- Triangle de couleur. Color triangle
- Triangle de forces. Forces, triangle of
- Triangle de Pascal. Pascal triangle
- Triangle géodésique. Geodesic triangle
- Trigonométrie. Trigonometry
- Trigonométrie sphérique. Spherical trigonometry
- Triple point. Triple point
- Triple produit de vecteurs. Triple product of vectors
- Troisième loi de thermodynamique. Third law of thermodynamics
- Troncature. Truncation
- "Trou". "Hole"
- Tube de jonction à pente variable. Exponential connector
- Tubes courbes. Tubes, curved
- Turbulence. Turbulence
- Turbulence dans tunnels aérodynamiques. Turbulence in wind tunnels
- Turbulence homogène. Turbulence, homogeneous
- Turbulence isotrope. Turbulence, isotropic
- Type de tenseur. Tensor, type of
- Unité Amagat. Amagat unit
- Unité Ångström. Ångström unit
- Unité arithmétique. Arithmetic unit
- Unité d'accumulation. Storage unit
- Unité Debye. Debye unit
- Unité de chaleur centigrade. Centigrade heat unit
- Unité de gerbe. Shower unit
- Unité de mèche. Mache unit
- Unité de quantité de lumière. Lumerg
- Unité de Siegbahn (unité x). Siegbahn x -unit
- Unité fondamentale. Unit, fundamental
- Unités acoustiques. Acoustical units
- Unités d'acoustique. Acoustic units
- Unités de chaleur. Heat, units of
- Unités électromagnétiques. Electromagnetic units
- Unités mécaniques. Mechanical units
- Unités thermiques. Thermal units
- Unités trichromatiques. Trichromatic units
- Unité x . x -unit
- Utilisation thermique. Thermal utilization
- Valence dirigée. Directed valency
- Valeur calorifique. Calorific value
- Valeur de référence. Reference value
- Valeur d'expectative. Expectation value
- Valeur gamma. Gamma-value
- Valeur moyenne d'une fonction. Mean value of a function

- Valeur propre. Eigenvalue
 Valeurs propres. Eigenvalues
 Valeurs singulières. Singular values
 Valeurs trichromatiques. Tristimulus values
 Vapeur sèche. Dry vapor
 Variable d'action. Action variable
 Variable d'angle. Angle variable
 Variables de Lode. Lode's variables
 Variables extensives. Extensive variables
 Variance. Variance
 Variation de temps de la production d'entropie. Time variation of the entropy production
 Variation limitée. Variation, bounded
 Vecteur. Vector
 Vecteur bra. Bra vector
 Vecteur contravariant. Contravariant vector
 Vecteur covariant. Covariant vector
 Vecteur de Burgers. Burgers vector
 Vecteur de Darboux. Darboux vector
 Vecteur de position. Position vector
 Vecteur de Poynting. Hertz vector
 Vecteur de Poynting. Poynting vector
 Vecteur d'état. State vector
 Vecteur ket. Ket vector
 Vecteur non-rotatif. Vector, irrotational
 Vecteur nul. Null vector
 Vecteur polaire. Vector, polar
 Vecteur principal d'une matrice. Principal vector of a matrix
 Vecteur propre. Eigenvector
 Vecteurs associés (champs de vecteurs). Vectors, associated (vector fields)
 Vecteurs de base. Base vectors
 Vecteurs linéairement indépendants. Linearly independent vectors
 Vecteurs de flux. Flux vectors
 Vecteurs orthogonaux. Vector(s), orthogonal
 Vent. Wind
 Vent géostrophique. Geostrophic wind
 Ventres. Anti-nodal points
 Vérification de preuve. Proof test
 Vérification de Wilcoxon. Wilcoxon's test
 Vérifications d'hypothèse. Significance tests
 Vibration. Flutter
 Vibration de combinaison. Combination vibration
 Vibrations normales dans les molécules polyatomiques. Normal vibrations in polyatomic molecules
 Vie. Lifetime
 Vie moyenne d'un état atomique. Mean lifetime of an atomic state
 Viriel. Virial
 Viriel d'un système. Virial of a system
 Viscoélasticité linéaire. Viscoelasticity, linear
 Viscosité. Viscosity
 Viscosité de Newton. Newtonian viscosity
 Viscosité dynamique. Viscosity, dynamic
 Visible. Observable
 Vitesse. Velocity
 Vitesse corpusculaire. Velocity, particle
 Vitesse critique. Speed, critical
 Vitesse de diffusion. Diffusion velocity
 Vitesse de diminution de l'intensité sonore. Rate of decay of sound
 Vitesse d'échappement. Escape velocity
 Vitesse de frottement. Friction velocity
 Vitesse de groupe. Group velocity
 Vitesse de groupe d'ondes. Waves, group velocity of
 Vitesse de phase. Phase velocity
 Vitesse de phase d'onde(s). Wave(s), phase velocity of
 Vitesse de réaction chimique. Chemical reaction rate
 Vitesse de recombinaison. Recombination velocity
 Vitesse de volume. Volume velocity
 Vitesse effective. Velocity, effective
 Vitesse moléculaire. Molecular velocity
 Vitesse moyenne. Velocity, average
 Vitesses généralisées. Velocities, generalized
 Vitesse surfacique. Velocity, areal
 Voisin le plus proche. Nearest neighbor
 Vortex. Vortex ring
 Vorticité. Vorticity
 Vorticité anticyclonique. Anticyclonic vorticity
 Vorticité cyclonique. Cyclonic vorticity
 Vorticité géostrophique. Geostrophic vorticity
 Voûte en Voussoir. Voussoir arch
 Weber. Weber
 Wronskien. Wronskian

German—English Index

- Abbe-Maxwell-Lehrsatz. Abbe-Maxwell theorem
 Abbesche Sinusbedingung. Abbe sine condition
 Abbesche Zahl. Abbe number
 Abbeugung. Diffraction
 Abbeugungswinkel. Diffraction angle
 Abbilden. Mapping
 Abbildungskonstante. Madelung constant
 Abbildungsmethode. Images, method of
 Abbildungsoberflächen. Figured surfaces
 Abdrosseln. Choking
 Abelsche Gleichheit. Abel identity
 Abelsche Gleichung. Abel equation
 Abelsche Gruppe. Abelian group
 Abelscher Konvergenzversuch. Abel test for convergence
 Abelscher Satz der Potenzreihe. Abel theorem on power series
 Abelsche Ungleichheit. Abel inequality
 Abfall. Slope
 Abfallstetigkeit. Slope stability
 Abfallumlenkgleichungen. Slope-deflection equations
 Abgeleitete Reihe. Derived set
 Abgerundeter Rundungsfehler. Rounding error, round-off
 Abgesonderte Systeme. Isolated systems
 Abirrungen eines optischen Systems. Aberrations of an optical system
 Abirrwinkel. Aberration angle
 Ablenkungsdeviate. Deviate, deviation
 Ablenkungstheorie. Deflection theory
 Ablenkungswinkel. Angle of deviation
 Abschaltung. Cut-off
 Abschätzung der S -Matrize in der gequantelten Feldtheorie. S -matrix in quantized field theory: evaluation of
 Abschirmungskonstante. Screening constant
 Abschnittsmodul. Section modulus
 Abschütteln. Shakedown
 Absolutbeschleunigung. Absolute acceleration
 Absolutskalar. Scalar, absolute
 Absolutspanner. Tensor, absolute
 Absoluttätigkeit. Absolute activity
 Absolute Differentialrechnung. Absolute differential calculus
 Absolute Kontinuität. Absolute continuity
 Absolute Rückstoßverhältnistheorie. Absolute reaction rate theory
 Absolute Temperatur. Absolute temperature
 Absorption der Strahlungsenergie. Absorption of radiant energy
 Abstumpfung. Truncation
 Abstumpfungsfehler. Truncation error
 Abweichung. Divergence
 Abwicklungslinie. Developable
 Abzweigung. Branch
 Achsenbündel. Axial bundle
 Achsenvergrößerung. Axial magnification
 Adams-Bashford-Methode. Adams-Bashford method
 Aderlaß. Bleeding
 Adiabatische Entmagnetisierung. Adiabatic demagnetization
 Adiabatischer Gleichungszustand. Adiabatic equation of state
 Adiabatisches Ablaufverhältnis. Adiabatic lapse rate
 Adiabatisches Gesetz für Quantisierungszustände. Adiabatic law for quantized states
 Adiabatische Wand. Adiabatic wall
 Adiabatische Wechselung. Adiabatic change
 Aerodynamische Mittelsehne. Aerodynamic mean chord
 Affine Grundverbindung. Fundamental affine connection
 Affine Spanner und freie Vektoren. Affine tensors and free vectors
 Affine Umwandlung. Affine transformation
 Afokalsystem. Afocal system
 Ähnlichkeitslehrsatz. Similarity theorem
 Aitken-Methode der Interpolation. Aitken method of interpolation
 Aktivierungsenergie. Activation energy
 Aktivierungsverhältnis. Activation rate
 Aktives Netzwerk. Network, active
 Akustikansprechung. Responsiveness, acoustic
 Akustische Ausbreitungskonstante. Acoustical propagation constant
 Akustische Einheiten. Acoustic units
 Akustische Impedanz. Impedance, acoustical
 Akustische Phasenkonstante. Acoustical phase constant
 Akustischer Wechselseitigkeitslehrsatz. Reciprocity theorem, acoustical
 Akustisches Reziprokalgesetz. Acoustical reciprocity theorem
 Albedo. Albedo
 Algebra, Buchstabenrechnung. Algebra
 Algebra einer Gruppe. Algebra of a group
 Algebraische Gleichungen. Algebraic equations
 Algorithmus. Algorithm
 Allgemeine Relativität. Relativity, general
 Allgemeine Umwandlungsgruppen. Transformation groups, general
 Alpha-Zerfallenergie. Alpha disintegration energy
 Altersannäherung. Age approximation
 Amagatseinheiten. Amagat unit
 Amagats Gesetz. Amagat's law
 Ampere. Ampere
 Amperewindung. Ampere turn
 Ampersche Regel. Ampere law
 Amperscher Lehrsatz. Ampere theorem
 Analogenrechner. Analog computer
 Analyse. Analysis
 Analytische Fortsetzung. Analytic continuation
 Analytische Funktion. Analytic function
 Analytische Kurve. Analytic curve
 Anamorphisches System. Anamorphic system
 Änderung. Variance
 Anelastisch. Anelastic
 Anfangsmodul. Initial modulus
 Anfangwertigkeitsproblem. Initial-value problems

- Anfügungsmatrize. Matrix, adjacency
 Angeschlossen. Connected
 Angewandte Kraft. Force, applied
 Angewiesene Darstellung. Graph, directed
 Angezeigte Wirksamkeit. Indicated efficiency
 Angleichungsbeiwert. Accomodation coefficient
 Ängströmbeiwert. Ångstrom coefficient
 Ängströmeinheit. Ångstrom unit
 Anharmonie. Anharmonicity
 Anhäufner. Cumulants
 Anisotrop. Anisotropic
 Anisotroper Nichtleiter. Dielectric, anisotropic
 Anisotropenergie. Anisotropy energy
 Anlenkung. Hinge
 Annäherungsmittel. Approximation in the mean
 Anomale Zerstreuung. Anomalous dispersion
 Anordnung. Assembly
 Anordnungsmaß. Measure of location
 Anregungsenergie. Excitation energy
 Anregungsreinheit. Excitation purity
 Anregungsspannung. Excitation potential
 Ansaugungsisotherme. Adsorption isotherm
 Anschmiegungebene. Osculating plane
 Anschnitt. Bearing
 Anschrift. Address
 Anstiegungsfunktion. Ramp function
 Anstellmatrize. Matrix, incidence
 Anstellwinkel. Incidence, angle of
 Anstellwinkl. Angle of incidence
 Anstrengung. Stragging
 Anteilprobenahme. Quota sampling
 Antiklastische Oberfläche. Anticlastic surface
 Antitryptischer Wind. Antitriptic wind
 Antriebsbewegung. Impulsive motion
 Anweisend. Directional
 Anwendbare Oberflächen. Applicable surfaces
 Anwendbare Weite. Practical width
 Aplanatische Kugelpunkte. Aplanatic points of a sphere
 Aplanatische Oberfläche. Aplanatic surface
 Achromatisch. Achromatic
 Apodisation. Apodization
 Apsis. Apse line
 Arbeit. Work
 Arbeit-Bewegungsenergielehrsatz. Work-kinetic energy theorem
 Arbeitsbetonung. Working stress
 Arbeitslast. Working load
 Archimedesprinzip. Archimedes' principle
 Arcus-Sinus-Verteilung. Arc-sine distribution
 Argandebene. Argand plane
 Arithmetische Einheit. Arithmetic unit
 Arithmetische Reihe. Arithmetic progression
 Arithmetisches Mittel. Arithmetic mean
 Ascolis Lehrsatz. Ascoli's theorem
 Assoziationsgesetz. Associative law
 Assoziationsspanner (Spannerfelder). Tensors, associated (tensor fields)
 Assoziationsvektoren (Vektorfelder). Vectors, associated (vector fields)
 Astigmatismus. Astigmatism
 Asymptotische Linie auf einer Oberfläche. Asymptotic line on a surface
 Asymptotische Reaktortheorie. Asymptotic reactor theory
 Asymptotischer Umstand. Asymptotic condition
 Asymptotische Reihe. Asymptotic series
 Atherman. Athermanous
 Atmosphärische Drucklinie. Isobar
 Atomanregungsfunktion. Excitation function, atomic
 Atomare Gliedsymbole. Atomic term symbols
 Atomarer Streufaktor. Atomic scattering factor
 Atombildungswärme. Atomic heat of formation
 Atombindungsenergie. Atomic bond energy
 Atomfrequenz. Atomic frequency
 Atomgestalt. Configuration, atomic
 Atomgewichtseinheit. Atomic weight unit
 Atomhalbmesser. Atomic radius
 Atommasse. Atomic mass
 Atommasseeinheit. Atomic mass unit
 Atomrefraktion. Atomic refraction
 Atomspektra. Atomic spectra
 Atomzahl. Atomic number
 Attrappe. Dummy
 Ätzend. Caustic
 Auflastpaarungstätigkeit. Charge conjugation operation
 Auflastwiedernormalisierung. Charge renormalization
 Aufbaufaktor. Build-up factor
 Aufbauprinzip. Building-up principle
 Auflösend. Dialytic
 Auflösung der Kräfte. Forces, resolution of
 Auflösungsempfindsamkeit. Resolution sensitivity
 Auflösungskraft. Resolving power
 Auflösungskraft der Kantensäule. Prism, resolving power of
 Aufnahme. Absorptance
 Aufnahmefähigkeit. Absorptive power
 Aufnahmebeiwert. Absorption coefficient
 Aufnahmefaktor. Absorption factor
 Aufnahmevermögen. Absorptivity
 Aufprall. Impact
 Aufprallnebenveränderliche. Impact parameter
 Aufspaltungsspektrum. Fission spectrum
 Auftriebslinientheorie. Lifting-line theory
 Auftriebsbeiwert. Lift coefficient
 Auftrieboberflächenflügeltheorie. Lifting-surface theory of wings
 Auftrieb-und-Moment auf einem Tragflügel. Lift and moment on an airfoil
 Auf-und-Niederverzeichnisse eines Spanners. Raising and lowering indices on a tensor
 Augenblicksachse der Umdrehung. Instantaneous axis of rotation
 Augenblicksquantisierung. Quantization, second
 Augenblickszentrum. Instantaneous center
 Augereffekt. Auger effect
 Augerertrag. Auger yield
 Augerbeiwert. Auger coefficient
 Ausbreiter. Diffuser
 Ausdehnung. Dilatation
 Ausdehnung. Expansion
 Ausdehnungswellen. Waves, dilatational
 Auseinanderfließen. Diffluence
 Ausfallwinkel. Angle of reflection
 Ausgesandtes. Emittance
 Ausgleichender Winkelwechsl. Angle change, balancing
 Ausgleicher. Equilibrant
 Auskunft. Information
 Auslegerarm. Cantilever
 Auslegerbalken. Beam, cantilever
 Auslöschung. Extinction
 Auslöschungsbeiwert. Extinction coefficient
 Ausrenken. Wrench
 Ausrenkungen. Dislocations

- Ausschaltung. Elimination
 Außenoptikdichte. External optical density
 Äußere oder aktive Kraft. Force, external or active
 Ausstrahlen. Irradiance
 Ausstreuungskern. Kernel, scattering
 Ausströmungsbeiwert. Emission coefficient
 Ausströmungsvermögen. Emissive power
 Austauschenergie. Exchange energy
 Austauschentartung. Exchange degeneracy
 Austauschintegral. Exchange integral
 Austauschkräfte. Exchange forces
 Austauschlinearstreuung. Dispersion, reciprocal linear
 Austauschunterschiede. Reciprocal differences
 Auswahlregeln. Selection rules
 Auswertgerät. Computer
 Automorphismusdarstellung. Graph, automorphism
 Avogadrokonstante. Avogadro constant
 Axialbelastung. Axial load
 Axiallängsdruck. Axial thrust
 Axialstrahlung. Axial ray
 Azcothropische Systeme. Azcothropic systems
 Azeotropische Systeme. Azeotropic systems
 Azyklisches Gebiet. Simply connected region
- Babinetsches Prinzip. Babinet's principle
 Bairstowsche Methode. Bairstow method
 Balken. Beam
 Balken auf elastischer Grundlage. Beam on elastic foundation
 Balkenbewegungsgleichungen. Beam equations of motion
 Balkenkrümmung. Curvature of beam
 Balmerglieder. Balmer terms
 Banacherraum. Banach space
 Banddruckebene. Band pressure level
 Bande. Band
 Bandkörpertheorie. Band theory of solids
 Bandrandenergie. Band edge energy
 Bandsystem. Band system
 Barbas Gesetz. Barba's law
 Baroklinik. Baroclinic
 Barometerformel. Barometer formula
 Barotropisch. Barotropic
 Bartlettsche Kraft. Bartlett force
 Baryon. Baryon
 Basis eines Zahlensystems. Base of a system of numbers
 Basisvektoren. Base vectors
 Bathos Formel. Batho's formula
 Baum. Tree
 Bauschingerwirkung. Bauschinger effect
 Bayesche Lösung. Bayes' solution
 Bayescher Lehrsatz. Bayes' theorem
 Beanspruchungsumlenkung. Strain deviator
 Bedeutsame Dezimalstellen. Digits, significant
 Bedeutsamkeitsprüfungen. Significance tests
 Bedienung. Operator
 Bedienungseinheit. Unit operator
 Bedingungskonvergenz. Conditional convergence
 Bedingungsahl. Condition number
 Beers Gesetz. Beer's law
 Beförderungsgrad. Degree of advancement
 Begrenzt. Bounded
 Beharrungsvermögen. Inertia
 Behauptung. Postulate
 Behinderte Umdrehung um ein Einzelbindeglied. Hindered rotation around a single bond
 Behrens-Fischer-Prüfung. Behrens-Fisher test
 Beigeordnete einer Matrix. Adjoint of a matrix
 Beigeordnete eines Betriebsmittels. Adjoint of an operator
 Beigeordnete Gleichung. Adjoint equation
 Beigeordneten eines optischen Systems. Conjugates in an optical system
 Beigeordneter Schmelzfluß. Flux, adjoint
 Beigeordnete Wellenfunktion. Adjoint wave function
 Beiwert der thermischen Ausdehnung. Coefficient of (thermal) expansion
 Belästigungen in Molekularspektra. Perturbations in molecular spectra
 Belästigungstheorie. Perturbation theory
 Belastungsgrenze. Critical load
 Beleuchtung. Illumination
 Belichtungsebenen. Focal planes
 Beltrami-Enneper-Lehrsatz. Beltrami-Enneper theorem
 Beltrami's Differentialnebenveränderlichen. Beltrami's differential parameters
 Bemerkbar. Observable
 Benardzellen. Bénard cells
 Benedict-Webb-Rubin-Gleichung. Benedict-Webb-Rubin equation
 Benzolgefüge. Benzene, structures of
 Bereich. Domain
 Bereich. Range
 Bereich-Energie-Beziehung. Range-energy relation
 Bergmannreihen. Bergmann series
 Berichtigung zum Leerraum. Correction to vacuum
 Bernoulligleichung. Bernoulli equation
 Bernoullilehrsatz. Bernoulli theorem
 Bernoullimethode. Bernoulli method
 Bernoullinumern. Bernoulli numbers
 Bernoullipolynome. Bernoulli polynomials
 Bernsteinpolynom. Bernstein polynomial
 Bertrandkurven. Bertrand curves
 Berührungsspannung. Contact potential
 Beschaffenheitsgesetz der Beschleunigung. Acceleration, composition law of
 Beschaffenheitsreihe einer Gruppe. Composition series of a group
 Beschaffenheit zweier Spanner. Composition of two tensors
 Beschäftigungsnummern. Occupation numbers
 Beschleunigung. Acceleration
 Beschränkungsbeiwert. Restraint coefficient
 Beschreibungsraumlehre. Descriptive geometry
 Beschwerung. Weighting
 Beschwerungsfunktion. Weighting function
 Besondere Relativität. Relativity, special
 Besonders. Particular
 Besselfunktion. Bessel function
 Besselsche Ungleichheit. Bessel's inequality
 Bessels Interpolationsformel. Bessel interpolation formula
 Beständigkeit. Stability
 Beständigkeitsfaktor. Stability factor
 Bestandteilenanalyse. Component analysis
 Bestandteil eines Kurvenschaubilds. Graph component
 Bestimmungsbeiwert. Determination, coefficient of
 Bestimmungszahl. Determinant
 Bestrahlung. Irradiation
 Betafunktion. Beta function
 Betätigungsmerkmal. Operating characteristic
 Betätigungsmethoden. Operational methods
 Betaverteilung. Beta distribution
 Betazerfallenergie. Beta disintegration energy

- Bethe-Salpeter-Gleichung. Bethe-Salpeter equation
 Bethes Methode. Bethe's method
 Betonung-Beanspruchung-Kurve. Stress-strain curve
 Betonungsablenkung. Stress deviator
 Betonungssteigerung. Stress raiser
 Betonungswellen. Waves, stress
 Betonungswurfbahn. Stress trajectory
 Beugungswinkel. Angle of diffraction
 Bewegliche Bindungsordnung. Mobile bond order
 Beweglichkeit. Mobility
 Bewegung. Motion
 Bewegungsachsen. Moving axes
 Bewegungsdurchschnitt. Moving average
 Bewegungsenergie. Energy, kinetic
 Bewegungsgleichungen. Motion, equations of
 Bewegungsgröße. Impulse
 Bewegungsgröße. Momentum
 Bewegungsgröße. Kinetics
 Bewegungstheorie der Zerstreuung. Diffusion, kinetic theory of
 Bezeichnende Akustikimpedanz. Impedance, characteristic acoustical
 Bezeichnende Winkelfunktion. Angle characteristic function
 Bezeichnungskreis. Definition circle.
 Bezeichnungsröntgenrückstrahlungsspektrum. X-ray spectrum, characteristic
 Bianchigleichheit. Bianchi identity
 Biegung. Bending
 Biegungsablenkung. Deflection, bending
 Biegungsmembranenanalogue. Membrane analogy, bending
 Biegungsmoment. Bending moment
 Biegungsröhre. Tubes, curved
 Bienayme-Chebyshev-Ungleichheit. Bienayme-Chebyshev inequality
 Bigit. Bigit
 Bildermethode. Method of images
 Bildquellen. Image sources
 Binär. Binary
 Binärpunkt. Binary point
 Bindearten. Bond types
 Bindebetonung. Bond stress
 Bindegliedeigenschaft. Bond character
 Bindekreisbahnen. Bonding orbitals
 Bindelänge. Bond length
 Bindungsdarstellung. Graph, connected
 Bindungsenergie. Binding energy
 Bindungsenergien. Bond energies
 Bindungswinkel. Bond angle
 Binghamstoff. Bingham material
 Binormal. Binormal
 Biquadratisch. Biquinary
 Biquadratische Gleichung. Biquadratic equation
 Bißchen. Bit
 Blasius-Gleichung. Blasius equation
 Blatt. Blade
 Blatt. Sheet
 Blattgeschwindigkeitsverhältnis. Blade-speed ratio
 Blattverwindung. Blade, twisted
 Bleibende Verformung. Set permanent
 Blindwiderstand. Reactance
 Blinkebene. Signal level
 Blinkströmungsdarstellung. Signal flow diagrams
 Blochfunktion. Bloch function
 Blochs Lehrsatz. Bloch theorem
 Bodenformen einer Oberfläche. Ground forms of a surface
 Bodenzustand. Ground state
 Bogen. Arch
 Bogenanmerkung. Bow notation
 Bogenbalken. Beam, curved
 Bohrsche Kreisbahnen. Bohr orbits
 Bohrsche Frequenzbedingung. Bohr frequency condition
 Bohrscher Halbmesser. Bohr radius
 Bohrsches Magnetron. Bohr magneton
 Boltzmannfaktor. Boltzmann factor
 Boltzmann-Planck-Gleichung. Boltzmann-Planck equation
 Boltzmanns Konstante. Boltzmann constant
 Boltzmannstatistik. Boltzmann statistics
 Boltzmannüberführungsgleichung. Boltzmann transport equation
 Bolzano-Weierstraß-Lehrsatz. Bolzano-Weierstrass theorem
 Bolzen. Slug
 Booleanalgebra. Boolean algebra
 Born-Haber-Kreislauf. Born-Haber cycle
 Born-Infeld-Theorie. Born-Infeld theory
 Born-Mayer-Gleichung. Born-Mayer equation
 Born-Oppenheimer-Methode. Born-Oppenheimer method
 Bornsche Abstoßung. Born repulsion
 Bornsche Annäherung. Born approximation
 Born-Von Karman-Theorie. Born and von Karman theory
 Bose-Einstein-Flüssigkeit. Bose-Einstein liquid
 Bose-Einstein-Gas. Bose-Einstein gas
 Bose-Einstein-Statistik. Bose-Einstein statistics
 Bose-Einstein-Teilchen. Boson
 Bouguets Gesetz. Bouguer law
 Boulvins Darstellung. Boulvin diagram
 Boylegesetz. Boyle's law
 Boyletemperatur. Boyle temperature
 Brachistochron. Brachistochrone
 Braggregel. Bragg rule
 Bramme. Slab
 Bravais-Miller-Verzeichnisse. Bravais-Miller indices
 Bravaispunkte. Bravais points
 Bravektor. Bra vector
 Brechungseinwickelnde. Diffraction evolute
 Brechungsgesetze. Refraction, laws of
 Brechungsindikatrix. Indicatrix of refraction
 Brechungskennziffer. Index of refraction
 Brechungsmuster. Diffraction patterns
 Brechungsstreuungsvermögen. Refractive dispersivity
 Brechungsvermögen. Refractivity
 Brechungswelle. Wave, refracted
 Brechungswinkel. Angle of refraction
 Bredtformeln. Bredt formulas
 Breit-Wigner-Formel. Breit-Wigner formula
 Bremspferdestärke. Brake horsepower
 Bremspotential. Potential, stopping
 Bremsthermische Wirksamkeit. Brake thermal efficiency
 Bremsvermögen. Stopping power
 Brennlinien. Focal lines
 Brennpunkt Abstand. Focal length
 Brennpunkte. Focal points
 Brewsters Gesetz. Brewster's law
 Brewsters Winkel. Brewster's angle
 Brillouinfunktion. Brillouin function
 Brouwers Festpunktlehrsatz. Brouwer fixed point theorem
 Bruchabstammungsbeiwert. Fractional parentage coefficient

- Bruchlinientheorie. Rupture line theory
 Bruchmodul. Modulus of rupture
 Buchstabenrechnungsfaktorenlehrsatz. Factor theorem of algebra
 Bucht. Bay
 Buckinghampotential. Buckingham potential
 Budanlehrsatz. Budan theorem
 Burgers-Material. Burgers material
 Burgersvektor. Burgers vector
 Burnettgleichungen. Burnett equations
 Busemanns Nebenordnungstheorie. Busemann's second-order theory
- Callier-Beiwert. Callier coefficient
 Caratheodorys Lehrsatz. Carathéodory's theorem
 Caratheodorys Prinzip. Carathéodory's principle
 Carnotisation. Carnotization
 Carnotscher Kreis. Carnot cycle
 Carnots Lehrsatz. Carnot's theorem
 Castiglianos Lehrsatz. Castigliano's theorem
 Cauchy-Beziehungen. Cauchy relations
 Cauchy-Formel für Refraktionsverzeichnis. Cauchy formula for refractive index
 Cauchy-Integrallehrsatz. Cauchy integral theorem
 Cauchy-Integralformel. Cauchy integral formula
 Cauchy-Integralkonvergenzprüfung. Cauchy integral convergence test
 Cauchy-Konvergenzprüfung. Cauchy convergence test
 Cauchy-Problem. Cauchy problem
 Cauchy-Reihenfolge. Cauchy sequence
 Cauchy-Reimann-Gleichungen. Cauchy-Reimann equations
 Cauchy-Verteilung. Cauchy distribution
 Cayley-Klein-Zustandsgröße. Cayley-Klein parameter
 Cetanzahl. Cetane number
 Charakter. Character
 Charles-Gesetz. Charles' law
 Chebyshevausdehnung. Chebyshev expansion
 Chebyshev-Polynome. Chebyshev polynomials
 Chebyshev-Quadratformel. Chebyshev quadrature formula
 Chebyshev-System. Chebyshev system
 Chebyshevungleichheit. Chebyshev inequality
 Chemische Konstante. Chemical constant
 Chemische Potentiale. Chemical potentials
 Chemisches Gleichgewicht. Chemical equilibrium
 Chemischer Stetigkeitszustand. Chemical stability condition
 Chios Methode. Chio's method
 Chiquadrat. Chi-square
 Choleskis Methode. Choleski's method
 Christoffelzeichen. Christoffel (3-index) symbols
 Chromatikdarstellung. Chromaticity diagram
 Chromatiknebengeordneten. Chromaticity coordinates
 Chronologwirkender. Chronological operator
 C-Kreislauf. C-circuit
 Clairaut-Gleichung. Clairaut equation
 Clapeyrons Lehrsatz. Clapeyron's theorem
 Clausius. Clausius
 Clausiusbedingung. Clausius' condition
 Clausius-Clapeyron-Gleichung. Clausius-Clapeyron equation
 Clausius-Differentialgleichung. Clausius' differential equation
 Clausius-Integral. Clausius' integral
 Clausius-Mossotti-Gleichung. Clausius-Mossotti equation
- Clausius-Ungleichheit. Clausius' inequality
 Clausius-Zustandsgleichung. Clausius' equation of state
 Coanda-Wirkung. Coanda effect
 Cochrans Lehrsatz. Cochran's theorem
 Coddingtons Gestalt- und Stellungsfaktoren. Coddington shape and position factors
 Comptoneffekt. Compton effect
 Condonparabel. Condon parabola
 Conwell-Weisskopf-Formel. Conwell-Weisskopf formula
 Coriolis Beschleunigung. Coriolis acceleration
 Corioliskraft. Coriolis force
 Coriolis Nebenveränderliche. Coriolis parameter
 Cornu-Spirale. Cornu spiral
 Cotton-Mouton-Gesetz. Cotton-Mouton law
 Cramers Regel. Cramer's rule
 Croccos Lehrsatz. Crocco's theorem
 Crouts Methode. Crout's method
 Curie. Curie
 Curiegleichung. Curie's equation
 Curietemperatur. Curie temperature
 Curie-Weiss-Gesetz. Curie-Weiss law
- D'Alembertsche. D'Alembertian
 D'Alembertsche Prüfung. D'Alembert test
 D'Alembertsches Prinzip. D'Alembert's principle
 Daltons Gesetz. Dalton's law
 Dampfdruck der idealen Lösungen. Vapor pressure of ideal solutions
 Dampfdruckgleichung. Vapor pressure equation
 Dampfdruck in vollkommenen Lösungen. Vapor pressure in perfect solutions
 Dämpfung. Damping
 Dämpfungsfaktor. Attenuation factor
 Dämpfungsverhältnis. Damping ratio
 Dancoffsche Verbesserungen. Dancoff corrections
 Dandelins Methode. Dandelin's method
 Darbouxvektor. Darboux vector
 Darcys Gesetz. Darcy's law
 Darstellungsfaktor. Diagram factor
 Darstellungsrang. Graph rank
 Darstellungsstatistik. Graphical statics
 Darstellungsverbindung. Graph connectivity
 Dauerbalken. Beam, continuous
 Dauerfunktion. Continuous function
 Daucrsysteme. Continuous systems
 Debye-Hückel-Gleichung. Debye-Hückel equation
 Debysche Einheit. Debye unit
 Debysche Energie. Debye energy
 Debysche Frequenz. Debye frequency
 Debysche Gleichung. Debye equation
 Debysche Gleichung für Wärmekapazität. Debye heat capacity equation
 Debysche Theorie der Kristallschwingungen. Debye theory of crystal vibrations
 Debysche Wirkung. Debye effect
 Debye-Temperatur. Debye temperature
 Deckwort. Code
 Dedekindschnitt. Dedekind cut
 Deformierungspotential. Deformation potential
 Deformierungstheorie der Plastizität. Deformation theory of plasticity
 Dehnbarkeitsmodul. Modulus of elasticity
 Deltaflügel. Delta wing
 Deltaquadratsverfahren. Delta-square process
 De Moivre-Gleichheit. De moivre identity

- De Moivre-Laplace-Lehrsatz. De Moivre-Laplace theorem
- Descartes-Vorzeichenregeln. Descartes rule of signs
- Dezibel. Decibel
- Dezimalrechner. Digital computer
- Dezimalstelle. Decimal
- Dezimalstellen. Significant figures
- Diagonalspannung. Diagonal tension
- Diakaustisch. Diaacoustic
- Diamagnetismus. Diamagnetism
- Diapunkt. Diapoint
- Diatherman. Diathermanous
- Diatherme Wand. Diathermal wall
- Dicht. Dense
- Dichtbindende Annäherung. Tight binding approximation
- Dichtheit. Density
- Dichtheitsmatrize. Density matrix
- Dickmuster. Thickness pattern
- Dickflüssigkeit. Viscous fluid
- Didos Problem. Dido's problem
- Dielektrischer Verlustfaktor. Loss factor, dielectric
- Dienstlich. Functional
- Differential. Differential
- Differentialgleichung. Differential equation
- Differentialzergliederer. Differential analyzer
- Differenzierung. Differentiation
- Differenzierung unter dem Integralvorzeichen. Differentiation under the integral sign
- Diffusionsabkühlung. Diffusion cooling
- Diffusionsgeschwindigkeit. Diffusion velocity
- Diophantgleichungen. Diophantine equations
- Dirac-Elektronentheorie. Dirac electron theory
- Diracgleichung. Dirac equation
- Diractäter. Dirac operators
- Direkte Methoden. Direct methods
- Direkter Spiegelungsfaktor. Direct reflectance
- Direktprodukt (von Untergruppen). Direct product (of subgroups)
- Direktrix einer Linearoberfläche. Directrix of a ruled surface
- Direktsumme. Direct sum
- Dirichletintegral. Dirichlet integral
- Dirichletprinzip. Dirichlet principle
- Dirichletreihe. Dirichlet series
- Diskriminante. Discriminant
- Divergenzverlust (Schall). Divergence loss (sound)
- Doolittle-Methode. Doolittle method
- Doppelbindungspaarung. Conjugation of double bonds
- Doppeldarstellungen. Graphs, dual
- Doppelgliedbeiwert. Binomial coefficient
- Doppelgliedlehrensatz. Binomial theorem
- Doppelgliedrige Verteilung. Binomial distribution
- Dopplerintegrationsverfahren. Double integration procedure
- Doppelkantensäule. Biprism
- Doppellinearform. Form, bilinear
- Doppellinie. Doublet
- Doppelpolige Nebengeordneten. Bipolar coordinate
- Dopplerbreite. Doppler width
- Dopplerschallwirkung. Doppler effect for sound
- Dopplerwirkungen. Doppler effects
- Doppelspanner. Tensor, double
- Doppelstrahlungsbrechung. Double refraction
- D -Quadratstatistik. D^2 -statistic
- Drallkurve. Twisted curve
- Drallmoment. Torque
- Drehungsfreier Vektor. Vector, irrotational
- Drehungsfreie Strömung. Irrotational flow
- Drei-achtel-Regel. Three-eighths rule
- Dreianregungswerte. Tristimulus values
- Dreieckmatrize. Triangular matrix
- Dreieckslehre. Trigonometry
- Dreieckslehrefunktion. Trigonometric function
- Dreifachpunkt. Triple point
- Dreifachschrägmatrize. Triple-diagonal matrix
- Dreifachvektorenprodukt. Triple product of vectors
- Dreifarbeneinheiten. Trichromatic units
- Dreifarbensystem. Trichromatic system
- Dreiklinisches System. Triclinic system
- Dreizehnmomentannäherung. Thirteen moment approximation
- Drittes Gesetz der Thermodynamik. Third law of thermodynamics
- Druckbeiwert. Pressure coefficient
- Drucklinie. Pressure line
- Druckmittelpunkt. Center of pressure
- Drude-Gleichung. Drude equation
- Duane-und-Hunt-Gesetz. Duane and Hunt law
- Duffin-Kemmer-Matrizen. Duffin-Kemmer matrices
- Duhamelsintegral. Duhamel's integral
- Duhem-Margules-Gleichung. Duhem-Margules equation
- Duhems Lehrsatz. Duhem's theorem
- Dünn-Tragflügel-Theorie. Thin-airfoil theory
- Durchbiegung. Deflection, flexure
- Durchbiegungskurve. Deflection curve
- Durchdringbarkeit. Permeability
- Durchdringungsfähigkeit. Perveance
- Durchgangserwiderung eines dynamischen Systems. Transient response of a dynamical system
- Durchgangssteifheit. Rigidity, flexural
- Durch Parallelröhre geleitet. Parallelepiped
- Durchschallkurvengleichungen. Hodograph equations, transonic
- Durchschnittsgeschwindigkeit. Velocity, average
- Durchschnittsleben. Life, mean
- Durchströmungsgaze. Gauze, flow through
- Durchwackeln. Wave through
- Dürftigkeitsverhältnis. Slenderness ratio
- Dyadik. Dyadics
- Dyadiktäter. Dyadic operator
- Dyadisches Ebenmass. Symmetry, dyad
- Dynamik. Dynamics
- Dynamikähnlichkeit. Dynamical similarity
- Dynamikdruck. Dynamic pressure
- Dynamikgleichungen von Lichtstrahlungen. Dynamical equations of light rays
- Dynamikladung. Dynamic loading
- Dynamische Oberflächenspannung. Dynamical surface tension
- Dynamisches Gleichgewicht. Equilibrium, dynamic
- Dynamisches Treibsystem. Driving system, dynamic
- Dyson-Vertretung. Dyson representation
- Earnshaw-Lehrsatz. Earnshaw theorem
- Ebenenbewegung. Plane motion
- Ebenenmaßachse. Symmetry, axis of
- Ebenenmaßeigenschaften der Molekulareigenfunktionen. Symmetry properties of molecular eigenfunctions
- Ebenenmaßeigenschaften oder Normalschwingungen in Molekülen. Symmetry properties or normal vibrations in molecules
- Ebenenmaßelement. Symmetry element
- Ebenenmaßklassen. Symmetry classes

- Ebenmaß- und Quantenmechanik. Symmetry and Quantum mechanics
- Ebcnmaßzentrum. Symmetry, center of
- Ebcnverlegung in wasserstoffähnlichen Atomen. Level displacements in hydrogen-like atoms
- Ebenweite. Level width
- Echter Bruchteil. Proper fraction
- Edgeworth-Reihen. Edgeworth's series
- Effektive Kraft. Force, effective
- Effektive Linsenkraft. Effective power of a lens
- Effektiver Halbmesser eines Bedienungsgestänges. Radius, effective, of control rod
- Effektive Wellenlänge. Effective wavelength
- Effektivgeschwindigkeit. Velocity, effective
- Effektivtiefe. Depth, effective
- Effektivweite. Width, effective
- Ehrenfests adiabatisches Gesetz. Ehrenfest adiabatic law
- Ehrenfests Beziehungen. Ehrenfest's relations
- Ehrenfests Lehrsatz. Ehrenfest theorem
- Eigenfunktion. Eigenfunction
- Eigenfunktion. Characteristic function
- Eigentliche Masse. Virtual mass
- Eigentlicher Zustand. Virtual state
- Eigentliches Quantum. Virtual quantum
- Eigentliche Temperatur. Virtual temperature
- Eigenunterdarstellung. Subgraph, proper
- Eigenvektoren. Eigenvectors
- Eigenwärme der Gase. Specific heats of gases
- Eigenwert. Eigenvalue
- Eigenwerte. Eigenvalues
- Eigenwertgleichung eines Spanners. Eigenvalue equation of tensor
- Eigenzustand. Eigenstate
- Eikonal. Eikonal
- Eikonalgleichung. Eikonal equation
- Einbiegungspunkt. Inflection point
- Eindringliche Veränderlichen. Intensive variables
- Eindringungstiefe. Penetration depth
- Eindringungswahrscheinlichkeit (Durchdringungsvermögen). Penetration probability (penetrability)
- Einfache harmonische Bewegung. Simple harmonic motion
- Einfacher Balken. Beam, simple
- Einfaches Pendel. Pendulum, simple
- Einfache Welle. Simple wave
- Einfachgitterträger. Truss, simple
- Eingerichteter Kreislauf. Circuit, oriented
- Einheitensystem. Units, system of
- Einheitlich. Homogeneous
- Einheitliche Lorentztransformation. Homogeneous Lorentz transformation
- Einheitliche Verwirbelung. Turbulence, homogeneous
- Einhüllende. Evolute
- Einlaß. Admittance
- Einschließliche Funktion. Implicit function
- Einsteins Eigenwärmefunktion. Einstein specific heat function
- Einsteins photoelektrische Gleichung. Einstein photoelectric equation
- Einsteins Übergangswahrscheinlichkeiten. Einstein transition probabilities
- Einsteins Wärmekapazitätsgleichung. Einstein heat capacity equation
- Einsteinverdichtung. Einstein condensation
- Einwertige Bewegung. Uniform motion
- Einwertige Bewegung in einem Kreis. Uniform motion in a circle
- Einwertige Punktoberfläche. Uniform point source
- Einwirkungsbeiwerte. Influence coefficients
- Einwirkungslinie. Influence line
- Einzelausgleichsprinzip. Detailed balancing, principle of
- Einzelfaktorisationslehrsatz. Unique factorization theorem
- Einzigartig. Singular
- Einzigtäter. Unitary operator
- Eisenbeton. Concrete, reinforced
- Eisenmagnetischer Widerhall. Ferromagnetic resonance
- Ekmanspirale. Ekman spiral
- Elastische Achse. Elastic axis
- Elastische Nachwirkung. After effect, elastic
- Elastischer Aufprall. Impact, elastic
- Elastische Wellen in Körpern. Waves, elastic, in solids
- Elastische Wellen in Platten. Waves, elastic, in plates
- Elastische Wellen in Stangen. Waves, elastic, in rods
- Elastisches Zentrum. Elastic center
- Elektrische und magnetische Induktanz. Induction, electric and magnetic
- Elastizitätsgewichte. Elastic weights
- Elastizitätsgrenze. Elastic limit
- Elastizitätsgrundlage. Elastic foundation
- Elastizitätskonstanten. Elastic constants
- Elastizitätskurve. Elastic curve
- Elastizitätsladungsmethode. Elastic load method
- Elektrische Dielektrikverschiebung. Electrical displacement in dielectrics
- Elektrische Kapazität. Electrical capacitance
- Elektrische oder magnetische Scherungswellen. Transverse waves (electric or magnetic)
- Elektrischer Strom. Current, electric
- Elektrisches Potential. Potential, electric
- Elektrischer Zweipol. Dipole (electrical)
- Elektrisch-mechanischer Austauschlehrsatz. Electrical-mechanical reciprocity theorem
- Elektrizitätsaustauschlehrsatz. Electrical reciprocity theorem
- Elektrizitätssystem. Electrical system
- Elektrizitätswiderstand. Electrical resistance
- Elektroakustischer Wechselseitigkeitslehrsatz. Reciprocity theorem, electroacoustical
- Elektrochemisches Potential. Electrochemical potential
- Elektromagnetische Konstante. Electromagnetic constant
- Elektromagnetischer Wechselseitigkeitslehrsatz. Reciprocity theorem, electromagnetic
- Elektromagnetisches Treibsystem. Driving system, electromagnetic
- Elektromagnetische Einheiten. Electromagnetic units
- Elektronische Arbeitsfunktionen. Work functions, electronic
- Elektromotorische Kraft. Electromotive force
- Elektronenenergie (klassische) Electron energy (classical)
- Elektronengestalt. Configuration, electronic
- Elektronenkonfiguration. Electron configuration
- Elektronenmetalltheorie. Electron theory of metals
- Elektronenverwandtschaft. Electron affinity
- Elektronenspin. Electron spin
- Elektroneutralität. Electroneutrality
- Elektronische Eigenwärme. Electronic specific heat
- Elektronische Molekülspektren. Electronic spectra of molecules
- Elektronkreisbahnen in einem Atom. Electron orbits in an atom
- Elektron-Optik-Linsengleichung. Electron-optical lens equation

- Elektronenschalen in einem Atom. Electron shells in an atom
- Elektronvolt. Electron-volt
- Elektrostatisches Treibsystem. Driving system, electrostatic
- Element. Element
- Elementar. Elementary
- Elementarsystem. Elementary system
- Elementeeinheit. Unit element
- Ellipsenbelasteter Flügel. Elliptically-loaded wing
- Ellipsenintegral. Elliptic integral
- Ellipsenpunkt. Elliptic point
- Ellipsenteildifferentialgleichung. Elliptic partial differential equation
- Ellipsenträgheit. Inertia ellipsoid
- Ellipsenzylindernebengeordnete. Elliptic cylindrical coordinate
- Ellipsoidalnebengeordnete. Ellipsoidal coordinate
- Ellipsoide. Ellipsoid
- Elliptische Funktion. Elliptic function
- Elliptische Gleichung. Elliptic equation
- Empfindsamkeit. Sensitivity
- Endliche Amplitudengleichungen. Finite amplitude equations (acoustic)
- Endliches Kegelhorn. Horn, finite conical
- Endliches Potenzhorn. Horn, finite exponential
- Endliches Zylinderhorn. Horn, finite cylindrical
- Endmoment. Moment, end
- Endpunktmethod. End-point method
- Endseitelpunkt. Vertex, terminal
- Engwiderschallmuster. Narrow resonance model
- Energie. Energy
- Energie des N-Teilchensystems. Energy of n-Particle system
- Energiegleichung zum gleichmäßigen adiabatischen Strömungsverlauf. Energy equation for steady adiabatic flow
- Energielücke. Energy gap
- Energiendicke einer Grenzschicht. Energy thickness of boundary layer
- Energierhaltung. Conservation of energy
- Energierhaltung. Energy conservation
- Energiengleichverteilung. Equipartition of energy
- Energieintegral. Energy integral
- Energiemomentenspanner. Energy-momentum tensor
- Energieniveaudarstellung. Energy level diagram
- Energierrückversetzung. Degradation of energy
- Energieverhältnisse in einem einwertigen Kraftfeld. Energy relations in uniform force field
- Energieverteilung. Distribution in energy
- Ensemble. Ensemble
- Ensembledichte. Ensemble density
- Enskog-Maxwell-Änderungsgleichungen. Enskog-Maxwell equations of change
- Enskogreihen zur Lösung der Boltzmann-Gleichung. Enskog's series for the solutions of the Boltzmann equation
- Enskogtheorie. Enskog's theory
- Entartetes Elektronengas. Degenerate electron gas
- Entartetes Leitungsband. Degenerate conduction band
- Entartetes Schwingungssystem. Degenerate oscillating system
- Entartungsebenen. Degenerate levels
- Entartungsgas. Degenerate gas
- Entartungstemperatur. Degeneracy temperature
- Entartungszustand. Degenerate state
- Entfremdung. Alienation
- Enthüllung. Exposure
- Enthüllung-Dichte-Verhältnis. Exposure-density relationship
- Enthaltbarkeit. Constraint
- Entleerung. Deflation
- Entlokalisierungsenergie. Delocalization energy
- Entropie eines Monatomgases. Entropy of monatomic gas
- Entropiekonstante. Entropy constant
- Entscheidungsfunktion. Decision function
- Entspannung. Relaxation
- Entspannungsbenehmen. Relaxation behavior
- Entspannungsfrequenz. Relaxation frequency
- Entspannungslänge. Relaxation length
- Entspannungsmodul. Modulus, relaxation
- Entspannungsphänomene. Relaxation phenomena
- Entspannungszeit. Relaxation time
- Entsprechende Zustände. Corresponding states
- Entsprechungsprinzip. Correspondence principle
- Entweichungsgeschwindigkeit. Escape velocity
- Epsilon-system. Epsilon-system
- Erdachsenschwankung. Nutation
- Erdbebenskalen. Earthquake scales
- Erdbeschleunigung. Acceleration of gravity
- Erdwirkung auf ein Flugzeug. Ground effect on airplane
- Erfrieren in Freiheitsgraden. Freezing in of degrees of freedom
- Ergänzung. Complement
- Ergänzungsenergie. Complementary energy
- Ergänzungsfunktion einer Lineardifferentialgleichung. Complementary function of a linear differential equation
- Ergiebigkeitsgrenze. Limit, yield
- Ergodische Hypothese. Ergodic hypotheses
- Ergodizität. Ergodicity
- Erhaltungskraft. Conservative force
- Erhaltungskraftgebiet. Force field, conservative
- Erhellung. Illuminance
- Erlaubter Übergang. Transition, allowed
- Ermüdung. Fatigue
- Erneuerungstheorie. Renewal theory
- Errechnungsgefüge. Determinate structure
- Erregungskurve. Excitation curve
- Ersatzelektronen. Equivalent electrons
- Ersatzgruppenvertretungen. Equivalent representations of groups
- Ersatzkraftsysteme. Equivalent force systems
- Erschütterung-Ausdehnung-Theorie. Shock-expansion theory
- Erschütterungspunkt. Percussion, center of
- Erschütterungswelle. Wave, shock
- Erschütterungswellengrenzschichtzwischenwirkung. Shock-wave boundary-layer interaction
- Erste Beitzahl. First beitti number
- Erster Brennpunkt. First focal point
- Erster Fundamentalsatz. First fundamental theorem
- Erster Grenzsatz. First limit theorem
- Erster Scheitelpunkt. Vertex, initial
- Erstes Gesetz der Thermodynamik. First law of thermodynamics
- Erstflugentweichungswahrscheinlichkeit. Escape probability, first flight
- Ertragsbedingung. Yield condition
- Ertragsoberfläche. Surface, yield
- Erwartung. Expectation
- Erwartungswert. Expectation value
- Erwiderungsvermögen. Responsivity
- Erwiderungszeit. Response time

- Erzeugungsfunktion. Generating function
 Erzeugungsverhältnis. Generation rate
 Erzeugungswirkender. Creation operator
 Erzeugungszeit. Generation time
 E-System. *e*-System
 Euckenverbesserung. Eucken correction
 Euklidischer Algorithmus. Euclidean algorithm
 Euklidischer Raum. Euclidean space
 Euler-Lagrange-Gleichungen. Euler-Lagrange equations
 Eulerbewegungsgleichungen. Euler equations of motion
 Eulergleichung. Euler equation
 Eulerknicklasten für Säulen. Euler Buckling loads for columns
 Eulerkonstante. Euler constant
 Eulerlehrsatz für einheitliche Funktionen. Euler theorem for homogeneous functions
 Eulerlehrsatz über Normalkrümmung. Euler theorem on normal curvature
 Euler-Maclaurin-Formel. Euler-Maclaurin formula
 Euler-Mascheroni-Konstante. Euler-Mascheroni constant
 Eulermethode zur Lösung einer gewöhnlichen Differentialgleichung. Euler method for solving an ordinary differential equation
 Euler-Rodrigues-Nebenveränderliche. Euler-Rodrigues parameter
 Eulersche Analysenmethode. Eulerian method of analysis
 Eulersche Summenformel. Euler Summation formula
 Eulerumwandlung. Euler transformation
 Everett-Interpolationsformel. Everett interpolation formula
 Evjen-Methode. Evjen method
 Evolvente einer Kurve. Involute of a curve
 Evolvente einer Oberfläche. Involute of a surface
 Exothermrückwirkung. Exothermic reaction
 Experimentenentwurf. Design of experiments
 Exponentenschluß. Exponential connector
 Exponentverteilung. Exponential distribution
 Extrapolierung. Extrapolation
 Exzenterladung. Eccentric loading
 Eykmanformel. Eykman formula
 Eyringtheorie der Beförderungsvorgänge. Eyring theory of transport processes
E-Zahl. *e*, the number
- Fagazität. Fagacity
 Faktorenanalyse. Factor analysis
 Faktorenbeiwerte. Factorial coefficients
 Faktorengruppe. Factor-group
 Faktorensteifheitsbeiwerte. Stiffness coefficients, factor
 Fanglänge. Mixing length
 Faradaywirkung. Faraday effect
 Faserbetonung. Fiber stress
 Farbdreieck. Color triangle
 Farbe. Color
 Farbgleichung. Color equation
 Farbkörper. Color solid
 Farbraum. Color space
 Farbtemperatur. Color temperature
 Farbanreizfunktion. Color stimulus function
 Fechnerbruchteil. Fechner fraction
 Feder. Spring(s)
 Federkonstante. Spring constant
 Fedrigkeit. Resilience
 Fehler. Error
- Feingefüge. Fine structure
 Fehlgriff. Blunder
 Feinverteilung. Dispersion
 Feinverteilungsformel. Dispersion formula
 Feinverteilungskräfte. Dispersion forces
 Feldkrümmung. Curvature of field
 Feldquantisierung. Field quantization
 Feldstrahlung. Field ray
 Feldtheorie. Field theory
 Feldtiefe. Depth of field
 Fermi. Fermi
 Fermialtersgleichung. Age equation, fermi
 Fermi-Betazerfalltheorie. Fermi theory of beta decay
 Fermi-Dirac-Statistik. Fermi-Dirac statistics
 Fermi-Dirac-Verteilungsfunktion. Fermi-Dirac distribution function
 Fermiebene. Fermi level
 Fermienergie. Fermi energy
 Fermikonstante. Fermi constant
 Fermion. Fermion
 Fermitemperatur. Fermi temperature
 Fermiwiderschall. Fermi resonance
 Feste Endmomente. Fixed-end moments
 Festpunkte. Fixed points
 Feststehender oder Encastre-Balken. Beam, fixed-ended or encastre
 Feuchtigkeit. Humidity
 Feynmans Methode. Feynman's method
 Feynmans Positrontheorie. Feynman's positron theory
 Ficksches Zerstreuungsgesetz. Diffusion, Fick law
 Fisherverteilung. Fisher's distribution
 Fisher-Yates-Prüfung. Fisher-Yates test
 Flächenmesser. Planimeter
 Flachraumzeit. Flat space-time
 Flachrund. Oblate
 Flachrunde Kugelkoordinaten. Oblate spheroidal coordinates
 Flattern. Flutter
 Floquet-Lehrsatz. Floquet theorem
 Fluchtlinietafel oder Nomogramm. Nomograph or nomogram
 Flügel des Kleinaspektverhältnisses. Wings of small aspect ratio
 Flügelgebiet eines Flugzeugs. Wing area of airplane
 Flugzeugaufführung. Airplane performance
 Flugzeugbeständigkeit. Airplane stability
 Flugzeugenergiegleichung. Airplane energy equation
F-Nummer. *f*-Number
 Fokker-Planck-Gleichung. Fokker-Planck equation
 Foldy-Wouthuysen-Vertretung. Foldy-Wouthuysen representation
 Form. Form
 Formen der Wellengleichung. Wave equation, forms of
 Formparameter zur Wirbelgrenzschicht. Form parameter for turbulent boundary layer
 Fortführer. Continuant
 Fortgesetzte Bruchteile. Continued fractions
 Fortgetragene Wärme. Heat carried away
 Fortpflanzung. Convection
 Fortschrittswellen. Waves, progressive
 Fortsetzungsgleichung. Equation of continuity
 Foucaults Pendel. Pendulum, Foucault's
 Fourier-Bessel-Umwandlung. Fourier-Bessel transform
 Fouriergesetz. Fourier's law
 Fourierreihen. Fourier series
 Fourierumwandlung. Fourier transform
F-Prüfung. *F*-test
 Franck-Condon-Prinzip. Franck-Condon principle

- Franklin-Gleichung. Franklin equation
 Fredholm-Bestimmungszahl. Fredholm determinant
 Fredholm-Gleichung. Fredholm equation
 Fredholmlehrsatz für Integralgleichungen. Fredholm theorem for integral equations
 Freiernergie der Keimbildung. Free energy of germ formation
 Freie oder natürliche Schwingungen. Oscillations, free or natural
 Freie Stromlinien. Free streamlines
 Freies Wertigkeitsverzeichnis. Free valence index
 Freigebiet. Field, free
 Freiheitsgrade. Degrees of freedom
 Freikennziffer. Index, free
 Freikörperdarstellung. Free-body diagram
 Freimassenpunktdynamik. Dynamics of a free mass point
 Freimolekülströmung. Free-molecule flow
 Freischwingungen von ungedämpften Systemen. Free vibrations of undamped systems
 Frenkelschaden. Frenkel defect
 Frequenzerwiderung. Frequency response
 Frequenzerwiderungsvertretung. Frequency response representation
 Frequenzfaktor. Frequency factor
 Fresnelgleichungen. Fresnel equations
 Frobeniusmethode. Frobenius method
 Froudesche Momentumtheorie. Froude momentum theory
 Froudesche Zahl. Froude number
 Fuchslehrsatz. Fuchs theorem
 Führungsrandpulszeit. Pulse time, leading-edge
 Fundamentaleinheit. Unit, fundamental
 Fundamentaler Kovariantspanner. Fundamental covariant tensor
 Fundamentalkreisläufe. Circuits, fundamental
 Funktion. Function
 Funktion der Wahrscheinlichkeitsdichte. Probability density function
 Fuß-Kerze. Foot-candle
- Gabelreihe. Brackett series
 Galileanumwandlung. Galilean transformation
 Galvanomagnetische und thermomagnetische Wirkungen. Galvanomagnetic and thermomagnetic effects
 Gamma. Gamma
 Gammafunktion. Gamma function
 Gammaraum. Gamma space
 Gammaverbesserung. Gamma correction
 Gammaverteilung. Gamma distribution
 Gammawert. Gamma-value
 Gaskonstante. Gas constant
 Gasströmung in einem Mundstück. Nozzle, flow of gas in
 Gattung. Species
 Gausebene. Gauss plane
 Gauss-Markov-Lehrsatz. Gauss-Markov theorem
 Gauss'sche Ausschaltung. Gaussian elimination
 Gauss'sche Eingabeln. Gaussian brackets
 Gauss'sche Formeln. Gauss' formulas
 Gauss'sche Hypergeometriegleichung. Gauss' hypergeometric equation
 Gauss'sche Interpolationsformeln. Gaussian interpolation formulas
 Gauss'sche Linsengleichungen. Gaussian lens equations
 Gauss'sche Optik. Gaussian optics
- Gauss'sche Quadraturformel. Gaussian quadrature formula
 Gauss'scher Kern. Kernel, Gaussian
 Gauss'scher Wölbungslehrsatz. Gauss' theorem on curvature
 Gauss'sches Normalschwerkraftgesetz. Gauss' law of normal gravitational force
 Gauss'sches Prinzip der geringsten Enthaltsamkeit. Least constraint, Gauss principle of
 Gauss'sche Wellengruppe. Gaussian wave group
 Gauss-Seidel-Methode. Gauss-Seidel method
 Gauss- und Codazzi-Gleichungen. Equations of Gauss and Codazzi
 Gaussverteilung. Gaussian distribution
 Gaviolas Ätzprüfung. Gaviola's caustic test
 G-Dauerregel. *g*-Permanence rule
 Gebiet. Field
 Gebiet der bewegenden Zwischenraumladung. Field of moving charge in space
 Gebietmoment der Trägheit. Inertia, area moment of
 Gebietsgeschwindigkeit. Velocity, areal
 Gebietsprodukt der Trägheit. Inertia, area product of
 Gebietsverminderung. Reduction in area
 Gebläse. Blower
 Gebogen. Bent
 Gebunden. Bound
 Gebundene Veränderung. Variation, bounded
 Gebundener Bogen. Tied arch
 Geburt- und Tod-Vorgang. Birth-and-death process
 Gedämpfte Schwingungen. Vibrations, damped
 Gedrängt. Compact
 Geeignete Lorentz-Umwandlung. Proper Lorentz transformation
 Gee-Pfund. Gee-pound
 Gefällekupplung. Gradient coupling
 Gefieder. Plume
 Gefügefaktor. Structure factor
 Gegenableitung einer Funktion. Antiderivative of a function
 Gegenbauerfunktion. Gegenbauer function
 Gegenbindendes Kreisbahnglied. Orbital, antibonding
 Gegeneinheitlicher Täter. Anti-unitary operator
 Gegeneisenmagnetismus. Antiferromagnetism
 Gegenhauptpunkte. Anti-principal points
 Gegenlineartäter. Anti-linear operator
 Gegenlogarithmus einer Nummer. Antilogarithm of a number
 Gegenmitteilung. Anticommutation
 Gegenresonanz. Antiresonance
 Gegensatz. Contrast
 Gegenstrombauchpunkte. Anti-nodal points
 Gegensymmetrisch. Antisymmetric
 Gegensymmetrische Wellenfunktion. Antisymmetric wave function
 Gegenumschaltungsregeln. Anti-commutation rules
 Gegenveränderungsvektor. Vector, contravariant
 Gegenzykloneverwirbelung. Anticyclonic vorticity
 Gehörmesserschrieb. Audiogram
 Geiger-Nutall-Verhältnis. Geiger-Nutall relation
 Gelenkmethode. Joints, method of
 Gemeinschaftliche Wirksamkeit. Joint efficiency
 Gemischte Kennfunktion. Mixed characteristic function
 Genau. Exact
 Genauigkeit. Accuracy, precision
 Genüge. Sufficiency
 Geodätische Ellipse. Geodesic ellipse
 Geodätische Hyperbel. Geodesic hyperbola

- Geodätische Linie. Geodesic line
 Geodätische Nebengeordneten. Geodesic coordinates
 Geodätische Parallelen. Geodesic parallels
 Geodätische Polarkoordinaten. Geodesic polar coordinates
 Geodätischer Kreis auf einer Oberfläche. Geodesic circle on a surface
 Geodätischer Tangens. Geodesic tangent
 Geodätisches Dreieck. Geodesic triangle
 Geodätische Wölbung. Geodesic curvature
 Geometrie. Geometry
 Geometriefaktor. Geometry factor
 Geometricoptik. Geometrical optics
 Geometrischer Energienfluß. Geometrical energy flux
 Geometrieverzerrung. Geometric distortion
 Geometrisches Mittel. Geometric mean
 Geostrophische Beschleunigung. Geostrophic acceleration
 Geostrophische Kraft. Geostrophic force
 Geostrophischer Wind. Geostrophic wind
 Geostrophische Wirbelung. Geostrophic vorticity
 Geradefunktionen. Even function
 Geradeglied eines Atoms. Even term of an atom
 Gerade oder ungerade Vertauschung. Permutation, even or odd
 Geradlinig. Linear
 Geradlinige Bewegung. Rectilinear motion
 Geradlinige Schallquelle. Straight line sound source
 Geradlinige Übereinstimmung. Rectilinear compliance
 Geradlinige Unabhängigkeitsvektoren. Linearly independent vectors
 Gerechte Skala. Scale, just
 Geringstpotentialenergielehre. Potential energy, theorem or minimum
 Gesamtabkömmling. Total derivative
 Gesamtablenkung der Kantensäule. Prism, total deviation of
 Gesamtaussendung. Total emissivity
 Gesamtbalken. Beam, composite
 Gesamtbestimmung. Total determination
 Gesamtfunktion. Entire function
 Gesamtneutronenschmelzfluß. Flux, neutron, total
 Gesamtstrahlungstemperatur. Total radiation temperature
 Gesamtübertragung. Total transmittance
 Gesamtwölbung einer Linse. Curvature of lens, total
 Geschlossen. Closed
 Geschlossene Quadraturformel. Closed quadrature formula
 Geschlossener Randzug. Edge train, closed
 Geschlossene Schalen. Closed shells
 Geschlossene Systeme. Closed systems
 Geschmeidigkeit. Ductility
 Geschchnittener Scheitelpunkt. Vertex, cut
 Geschosstreibendes Pendel. Pendulum, ballistic
 Geschwindigkeit. Velocity
 Geschwindigkeitsprofil. Velocity profile
 Geschwindigkeitsschallpotential. Velocity potential of sound
 Gesetz der entsprechenden Zustände. Corresponding states, law of
 Gesetz der Verbundwahrscheinlichkeit. Probability, law of compound
 Gesetz des wiederholten Logarithmus. Iterated logarithm, law of
 Gesetzmäßige Einzigartigkeit einer Differentialgleichung
 Regular singularity of a differential equation
 Gesetzmäßige Funktion einer Komplexveränderlichen.
 Regular function of a complex variable
 Gesetzmäßige Lösungen. Regular solutions
 Gesonderte Ordinatenmethoden. Discrete ordinates method
 Gesonderte Veränderliche. Discrete variate
 Gestaltänderung. Deformation
 Gestaltfaktor. Shape factor
 Gestalthitzekapazität. Configurational heat capacity
 Gestaltraum. Configuration space
 Getreidegrenze. Grain boundary
 Getrennte Unterschiede. Divided differences
 Gewandtheit. Routine
 Gewicht. Weight
 Gewichtbruchteile. Weight fractions
 Gewichtsspanner. Tensor, weighted
 Gewinn. Gain
 Gewöhnliche Differentialgleichung. Ordinary differential equation
 Gewöhnlicher Punkt. Ordinary point
 Gezwungene Schwingungen. Vibrations, forced
 Gibbs-Duhem-Gleichung. Gibbs-Duhem equation
 Gibbs-Helmholtz-Gleichungen. Gibbs-Helmholtz equations
 Gibbs-Konovalov-Lehrsätze. Gibbs-Konovalov theorems
 Gibbs-Phänomen. Gibbs phenomenon
 Gibbs'sche Formel zur Oberflächenspannung. Gibbs formula for the surface tension
 Gibbs'sche Funktion. Gibbs function
 Gibbs-Teilungsoberfläche. Gibbs division surface
 Gibbs-Widerspruch. Gibbs paradox
 Gitter. Lattice
 Gittergrößen. Lattice dimensions
 Gitterkonstante. Lattice constant
 Gitterkristallenergie. Lattice energy of crystals
 Gittermuster. Lattice model
 Gittersumme. Lattice sum
 Gittertheorie der Elastizitätsbeiwerte. Elastic coefficients, lattice theory of
 Gitterträger. Truss
 Givens-Methode. Givens method
 Gladstone-Dale-Gesetz. Gladstone-Dale law
 Glan-Thompson-Kantensäule. Glan-Thompson prism
 Glaseinwilligung. Compliance, glass
 Glasglänzender Zustand. Glassy state
 Glas-Kennzahl. Modulus, glass
 Glatteinwilligung. Compliance, shear
 Glätten. Smoothing
 Gleichdrucklinie. Isobar
 Gleichebige Kräfte. Forces, coplanar
 Gleichentfernte Alleen. Equidistant alleys
 Gleichenergisches Spektrum. Equi-energy spectrum
 Gleichfokal. Parfocal
 Gleichfortlaufend. Equicontinuous
 Gleichgewicht. Equilibrium
 Gleichgewichtskennzahl. Modulus, equilibrium
 Gleichgewichtstheorie. Equilibrium theory
 Gleichgültige Umstände. Indifferent states
 Gleichheit. Identity
 Gleichläufige Alleen. Parallel alleys
 Gleichmaßkontrolle. Control, proportional
 Gleichtemperierte Skala. Scale, equally tempered
 Gleichung. Equation
 Gleichungen der flüssigen Bewegungen. Equations of fluid motion
 Gleichungen der mathematischen Physik. Mathematical physics, equations of

- Gleichwertige Pendellänge. Pendulum, equivalent length of
- Gleichwertigkeit. Equivalence, parity
- Gleichzeitig meßbar. Simultaneously measurable
- Gleichzeitige Kräfte. Forces, concurrent
- Gleichzustandsschwingung. Oscillation, steady-state
- Gleiseck. Parallelogram
- Gleiseck der Kräfte. Forces, parallelogram of
- Glied. Link
- Gliedenergiezustand. Energy state, term
- Goertzel-Greuling-Annäherung. Goertzel-Greuling approximation
- Goertzel-Selengut-Methode. Goertzel-Selengut method
- Goldschmidts Gesetz. Goldschmidt's law
- Gother's Regel. Gother's rule
- Goudsmit-und-Uhlenbeck-Voraussetzung. Goudsmit and Uhlenbeck assumption
- Grad. Grade
- Gradmatrize. Matrix, degree
- Graeffe-Methode. Graeffe method
- Gram-Bestimmungszahl. Gram determinant
- Gram-Charlier-Reihen. Gram-Charlier series
- Gram-Schmidt-Verfahren. Gram-Schmidt process
- Grashofzahl. Grashof number
- Graukörper. Grey body
- Grauskala. Gray scale
- Greens Funktion. Green's function
- Gregory-Formel. Gregory formula
- Gregory-Newton-Formel. Gregory-Newton formula
- Grenzanalyse- und Entwurf. Limit analysis and design
- Grenzfeld. Critical field
- Grenzgebiet. Critical region
- Grenzesetz für Starkelektrolyte. Limiting law for strong electrolytes
- Grenzgeschwindigkeit. Speed, critical
- Grenzgröße. Critical size
- Grenzlast. Limit load
- Grenzlehrsätze. Limit theorems
- Grenzmachzahl. Critical mach number
- Grenz- oder Plastizitätsmoment. Moment, limit or plastic
- Grenzschicht. Boundary layer
- Grenzschichtenergiegleichung. Energy equation for boundary layer
- Grenzschichttrennung. Separation of boundary layer
- Grenzspannung. Critical potential
- Grenzstärke. Ultimate strength
- Grenzwertproblem. Boundary value problem
- Großdurchbiegung. Deflection, large
- Größenwirkung. Size effect
- Großkanonischensemble. Grand canonical ensemble
- Großkennzahlgesetz. Law of large numbers
- Großteilungsfunktion. Grand partition function
- Grundreiz. Basic stimulus
- Grundbanden. Fundamental bands
- Grundlegungsmodul. Foundation modulus
- Grundmodus. Mode, fundamental
- Grundsatz der Entropiezunahme. Principle of entropy increase
- Grundschnidevorrichtungen. Cut sets, fundamental
- Grundwahrscheinlichkeitssatz. Fundamental probability set
- Grundzustand. Fundamental state
- Gruneisen-Formel. Gruneisen formula
- Gruneisen-Konstante. Gruneisen constant
- Gruppe. Cluster, group
- Gruppenförmige Phenomene in Gasen. Clustering phenomena in gases
- Gruppenentspannung. Group relaxation
- Gruppengeschwindigkeit. Group velocity
- Gruppengeschwindigkeit der Wellen. Waves, group velocity of
- Gruppenintegrale. Cluster integrals
- Gruppenkomplex. Complex of a group
- Gruppenvertretung. Representation of a group
- Gruppenvertretung und Quantenmechanik. Representation of groups and quantum mechanics
- Gruppenzentrum. Center of a group
- G-Summenregel. g -Sum rule.
- Guests Ergiebigkeitsbedingung. Guest's yield condition
- Gummiähnlicher Zustand. Rubbery state
- Haags Lehrsatz. Haag's theorem
- Haarröhrenwirkung. Capillarity
- Haftvermögensenergie. Energy, cohesion
- Haftvermögenskräfte. Cohesion, forces of
- Halbdicke. Half-thickness
- Halbleben. Half-life
- Halbmesservektor. Vector, radius
- Halbweite einer Spektrallinie. Half-width of a spectral line
- Hallbeweglichkeit. Hall mobility
- Hallwinkel. Hall angle
- Hamilton-Bezeichnung. Hamilton's characteristic
- Hamilton-Cayley-Lehrsatz. Hamilton-Cayley theorem
- Hamiltonische Optikfunktion. Hamiltonian function of optics
- Hamiltonische Systemfunktion. Hamiltonian function of a system
- Hamilton-Jacobi-Teildifferentialgleichung. Hamilton-Jacobi partial differential equation
- Hamilton-Lehrsatz. Hamilton's theorem
- Hamilton-Prinzip. Hamilton principle
- Handel-Umwandlung. Handel transform
- Hänge-Wirbelwiderstand. Trailing-vortex drag
- Harmonische Bewegung. Harmonic motion
- Harmonische Funktion. Harmonic function
- Harmonische Progression. Harmonic progression
- Harmonischer Oszillator. Oscillator, harmonic
- Harmonisches Mittel. Harmonic mean
- Harmonische Welle. Wave, harmonic
- Härte. Hardness
- Hartkugelgas. Hard sphere gas
- Hartmann-Streuungsformel. Hartmann dispersion formula
- Hartree-Annäherung. Hartree approximation
- Hartree-Fock-Methode. Hartree-Fock method
- Hauptachse. Axis, principal
- Hauptanstellwinkel. Angle of principal incidence
- Hauptbetonung. Stress, principle
- Hauptebenen. Principal planes
- Hauptkristallabschnitt. Principal section of a crystal
- Hauptnormale. Normal, principal
- Hauptpunkte. Principal points
- Hauptpunkte eines optischen Systems. Cardinal points of an optical system
- Hauptreihe. Principal series
- Hauptrichtungen. Principal directions
- Hauptseitenwinkel. Angle of principal azimuth
- Hauptstrahlung. Principal ray
- Hauptvektor einer Matrize. Principal vector of a matrix
- Hauptwurzel. Principal root
- Hautreibung. Skin friction

- Hauygesetz. Hauy law
 Heavisides Betriebsrechnung. Heaviside operational calculus
 Heavisides Einheitsfunktion. Heaviside unit function
 Heidenstatistik. Gentile statistics
 Heine-Borel-Lehrsatz. Heine-Borel theorem
 Heine-Formel. Heine formula
 Heisenberg-Bild. Heisenberg picture
 Heisenberg-Kraft. Heisenberg force
 Heitler-London-Wertigkeitstheorie. Heitler-London theory of valence
 Heisenbergs Bewegungsgleichung. Heisenberg equation of motion
 Heisenbergs Eisenmagnetismustheorie. Heisenberg theory of ferromagnetism
 Heisenberg-Vertretung. Heisenberg representation
 Heitler-London-Wasserstoffmolekültheorie. Hydrogen molecule, Heitler-London theory of
 Heizoberfläche. Heating surface
 Hele-Shaw-Zelle. Hele-Shaw cell
 Helizität. Helicity
 Helmholtzabkömmling. Helmholtz derivative
 Helmholtzfunktion. Helmholtz function
 Helmholtzgleichung. Helmholtz equation
 Helmholtz-Ketteler-Formel. Helmholtz-Ketteler formula
 Helmholtzströmung. Helmholtz flow
 Helmholtzwellen. Helmholtz waves
 Hencky-Prandtl-Netz. Hencky Prandtl net
 Hennebergs Methode. Henneberg's method
 Herbeigeführte Polarisierung. Induced polarization
 Herleitung. Derivative
 Hermann-Manguin-Symbole. Hermann-Manguin symbols
 Hermitebeigeordnete einer Matrize. Hermitian conjugate of a matrix
 Hermitegleichung. Hermite equation
 Hermiteinterpolationsformel. Hermite interpolation formula
 Hermitepolynome. Hermite polynomials
 Hermitsche Matrize. Hermitian matrix
 Hermitscher Täter. Hermitian operator
 Herpolhode. Herpolhode
 Herschels Bedingung. Herschel's condition
 Hertz. Hertz
 Hertzvektor. Hertz vector
 Herzbergers Streuungsformel. Herzberger's dispersion formula
 Hessesch. Hessian
 Hessische Normalform. Hesse's normal form
 Hessgesetz. Hess law
 Heteropolarband. Heteropolar bond
 Hexadebenmaß. Symmetry, hexad
 Hicks Formel. Hicks formula
 Hilbertraum. Hilbert space
 Hilbertumwandlung. Hilbert transform
 Hingewiesene Wertigkeit. Directed valency
 Hinterwahrscheinlichkeit. Posterior probability
 Hinweiswertigkeit. Reference value
 Hitchcock-Methode. Hitchcock method
H-Lehrsatz. *H*-Theorem
 Hochelastizität. Elasticity, high
 Hochrangübergänge. Transitions of higher order
 Höchster Gemeinfaktor. Highest common factor
 Hochzahl. Exponent
 Hochzüchtung der Elektronenkreisbahnen. Hybridisation of electron orbitals
 Hoffsche Konvergenzmethode. Hoff's convergence method
 Hohraumbildung. Cavitation
 Höhlungsfeld in einem Nichtleiter. Cavity field in a dielectric
 Hölderbedingung. Hölder condition
 Hölderungleichheit. Hölder inequality
 Holonomisch. Holonomic
 Holomorphe Funktion. Holomorphic function
 Holtes Methode. Holte's method
 Holzwirkung. Wood effect
 Homogenverfahren. Homogeneous process
 Homographische Umwandlung. Homographic transformation
 Homometrische Paare. Homometric pairs
 Homomorphdarstellungen. Graphs, homomorphic
 Homomorphismus. Homomorphism
 Homopolarbindeglied. Homopolar bond
 Homozentrisch. Homocentric
 Hooke'sches Gesetz. Hooke's law
 Hornermethode. Horner method
 Horngleichung. Horn equation
 Horoptos. Horopter
 Hörschwelle. Audibility, threshold of
 Hotellings *T*-Glieder. Hotellings *T*
 Hubblekonstante. Hubble constant
 Hubvolumen. Swept volume
 Hume-Rothery-Regeln. Hume-Rothery rules
 Hund's Kupplungsfälle. Hund's coupling cases
 Hund's Regeln für atomische Vervielfacher. Hund's rules for atomic multiplets
 Hurwitzpolynom. Hurwitz polynomial
 Huygensches Prinzip. Huygens' principle
 Huygensche Wellchen. Huygens' Wavelets
 Hydrauliksprung. Hydraulic jump
 Hydrodynamikanalogie. Hydrodynamical analogy
 Hydrostatik. Hydrostatics
 Hydrostatikbetonung. Stress, hydrostatic
 Hyperbel. Hyperbola
 Hyperbelfunktion. Hyperbolic function
 Hyperbelgleichung. Hyperbolic equation
 Hyperbellogarithmus. Hyperbolic logarithm
 Hyperbelpunkt. Hyperbolic point
 Hyperboloid. Hyperboloid
 Hyperfeinstruktur. Hyperfine structure
 Hypergeometrische Funktion. Hypergeometric function
 Hypergeometrische Verteilung. Hypergeometric distribution
 Hyperkonjugation. Hyperconjugation
 Hypermultipléterm. Hypermultiplet
 Ideale Gasmischungen. Perfect gas mixtures
 Idealplastisch. Ideally plastic
 Ideales Gasgesetz. Perfect gas law
 Ideale Spannungsquelle. Voltage source, ideal
 Ideale Stromquelle. Current source, ideal
 Idealplastisch. Perfectly plastic
 Idealsysteme. Ideal systems
 Idempotenzmatrize. Idempotent
 Ignorierbare Nebengeordnete. Ignorable coordinate
 Impedanz. Impedance
 Impedanzumwandlung. Impedance transform
 Impulsdicke der Grenzschicht. Momentum thickness of boundary layer
 Impulsfunktionen. Impulse functions

- Impulsleichung zur Grenzschicht. Momentum equation for boundary layer
- Impulsive Schallgleichung. Impulsive sound equation
- Impulsmoment. Moment of momentum
- Impuls-Momentum-Prinzip. Impulse-momentum principle
- Indikatrix. Indicatrix
- Indizialanmerkung. Indicial notation
- Indizialerwiderung. Indicial response
- Indizialgleichung. Indicial equation
- Induktanz. Inductance
- Induktionsapparat. Inductor
- Induktionseffekt-und-Rückwirksamkeit. Inductive effect and reactivity
- Induktionskräfte. Induction forces
- Induktionslehrsatz. Induction theorem
- Inkremententheorie der Formbarkeit. Incremental theory of plasticity
- Inkrementzusammenfall. Collapse, incremental
- Innenaufnahme. Internal absorptance
- Inneneigenschaften einer Oberfläche. Intrinsic properties of a surface
- Innenfelder in Nichtleitern. Internal fields in dielectrics
- Innenkraft. Force, internal
- Innenprodukt. Inner product
- Innenprodukt eines Spanners. Inner product of tensors
- Innenreibung. Friction, internal
- Innenscheitelpunkt. Vertex, internal
- Innenübertragung. Internal transmittance
- Innerlicher Abkömmling eines Spannerfeldes. Intrinsic derivative of tensor field
- Integral. Integral
- Integralbedienung. Integral operator
- Integralgleichung. Integral equation
- Integrallehrsätze der Vektorenanalyse. Integral theorems of vector analysis
- Integralübertragungsgleichung. Integral transport equation
- Integralumwandlung. Integral transform
- Integrationskonstante. Constant of integration
- Integrierausdruck. Integrand
- Integrierbares Quadrat. Integrable square
- Integrierfaktor. Integrating factor
- Integrierte Röntgenstrahlungsbrechung. X-ray reflection, integrated
- Intermolekularkräfte. Intermolecular forces
- Interpolation. Interpolation
- Interpolation der Dreieckslehre. Trigonometric interpolation
- Ionenbindungsmerkmal. Ionic bond character
- Ionische Kraft. Ionic strength
- Ionisierungsenergie. Ionization energy
- Ionisierungspotential. Ionization potential
- Irrungenausbreitung. Error, propagation of
- Irrungensfunktion. Error function
- Isentropisch. Isentropic
- Isobarcnoberfläche. Isobaric surface
- Isobarcntrudeln. Isobaric spin
- Isoelektronisches Prinzip. Isoelectronic principle
- Isokerzenkurve. Isocandela curve
- Isolux. Isolux
- Isometrische Linien auf einer Oberfläche. Isometric lines on a surface
- Isomorphdarstellungen. Graphs, isomorphic
- Isoperimetrisches Problem. Isoperimetric problem
- Isophoto. Isophot
- Isotemperaturorte. Isotemperature loci
- Isothermen. Isotherms
- Isothermisch. Isothermal
- Isotoptrudeln. Isotope spin
- Isotrop. Isotropic
- Isotopwirkung in Molekülspektren. Isotope effect in molecular spectra
- Isotropnichteiter. Dielectric, isotropic
- Isotropspanner. Tensor, isotropic
- Isotropverwirbelung. Turbulence, isotropic
- Jacobimethode. Jacobi method
- Jacobipolynome. Jacobi polynomials
- Jakobisch. Jacobian
- Jacoblehrsatz in der Optik. Jacob's theorem in optics
- Jagdbares Wild. Fair game
- Jähster Abstieg. Steepest decent
- Jezeks Methode. Jezek's method
- J-gestalt Verteilung. J-shaped distribution
- J-J-Kupplung. j, j Coupling
- Johnson-und Lark-Horowitz-Formel. Johnson and Lark-Horowitz formula
- Jordan-Hölder-Lehrsatz. Jordan-Hölder theorem
- Jordankurve. Jordan curve
- Jordan-Matrize. Jordan matrix
- Jordan-Methode. Jordan method
- Joukowski-Bedingung. Joukowski condition
- Joukowski-Tragflügel. Joukowski airfoils
- Kalendergleichung. Callendar equation
- Kalkulus. Calculus
- Kalorienwert. Calorific value
- Kanonisch. Canonical
- Kanonische Bewegungsgleichung. Canonical equation of motion
- Kanonische Form des Fermatprinzips. Canonical form of Fermat's principle
- Kanonische Korrelation. Canonical correlation
- Kanonische Nebengeordneten. Canonical coordinates
- Kanonische Optikgleichungen. Canonical equations of optics
- Kanonisches Ensemble. Canonical ensemble
- Kanonische Umwandlung. Canonical transformation
- Kantenreihenfolge. Edge sequence
- Kantenrückgriff. Edge of regression
- Kantensäule. Prism
- Kantenvielfältigkeit. Edge multiplicity
- Kapazität. Capacitance
- Kapillarwellen. Capillary waves
- Kardinal. Cardinal
- Karman-Friedrich-Gleichungen. Karman-Friedrich's equations
- Karmankonstante. Karman constant
- Kartesische Oberflächen. Cartesian surfaces
- Kartesisches Spannerfeld. Tensor field, cartesian
- Katakaustisch. Catacaustic
- Katerpendel. Pendulum, kater
- Katoptrisches System. Catoptric system
- Keesomenergie. Keesom energy
- Keesompotential. Keesom potential
- Kegelbrechung. Conical refraction
- Kegelförmig. Conic
- Kegelnebengeordnete. Conical coordinate
- Kegelpendel. Pendulum, conical
- Kegelschnitt. Ellipse
- Kegelschnittdrehkörper. Paraboloid
- Kehlkustikimpedanz. Impedance, throat acoustical
- Kehrgitterwerk. Reciprocal lattice

- Kelvinformel. Kelvin formula
 Kelvinmuster. Kelvin model
 Kennfunktion einer Reihe. Characteristic function of a set
 Kennfunktionen in der Thermodynamik. Characteristic functions in thermodynamics
 Kenngleichung. Characteristic equation
 Kenngleichung einer Matrize. Characteristic equation of a matrix
 Kenngröße. Characteristic
 Kernladungsdichte. Charge density of nuclei
 Kennwertmethoden. Characteristics, method of
 Keplers Gesetze. Kepler's laws
 Kern. Kernel
 Kernanregungsfunktion. Excitation function, nuclear
 Kernauswahlregeln. Selection rules, nuclear
 Kerndrehung. Nuclear spin
 Kernenergieebenen. Nuclear energy levels
 Kerngefüge. Nuclear structure
 Kernkräfte. Nuclear forces
 Kernmagnetenmoment. Nuclear magnetic moment
 Kernmagnetenresonanz. Nuclear magnetic resonance
 Kernmagneton. Nuclear magneton
 Kernrückwirkungsenergie. Reaction energy, nuclear
 Kernspannung. Nuclear potential
 Kerzenstärke. Candle power
 Kettenkurve auf einem Drahtseil. Catenary curve on cable
 Kettenlinie. Catenary
 Kettenregel der Differenzierung. Chain rule of differentiation
 Ketvektor. Ket vector
 Keyes-Smith-Gerry-Gleichung. Keyes-Smith-Gerry equation
 Kinematik. Kinematics
 Kinematisch zulässig. Kinematically admissible
 Kinetikenergiedichtewelle. Wave, kinetic energy density
 Kinetiktheorie. Kinetic theory
 Kinetiktheorie der Thermalleitungsfähigkeit. Thermal conductivity, kinetic theory of
 Kinetiktheorietemperatur. Kinetic theory temperature
 Kinetische Energie der drehungsfreien Bewegung. Kinetic energy of irrotational motion
 Kinetisches Momentum. Kinetic momentum
 Kinetisches Potential. Kinetic potential
 Kinetische Theorie der Zähflüssigkeit. Viscosity, kinetic theory of
 Kirchhoffs Gesetz. Kirchhoff's law
 Kirchhoffs Lehrsatz der geometrischen Optik. Kirchhoff's geometric optics theorem
 Kirchhoffs Strahlungsgesetz. Kirchhoff's radiation law
 Kirkwoods Annäherung. Kirkwood's approximation
 Kirkwoods Gleichung für die Konstante eines Nichtleiters. Kirkwood's equation for the dielectric constant
 Klassische Dynamikvergleiche. Dynamical analogies, classical
 Klassische Elektronentheorie. Classical electron theory
 Klassischer Elektronenhalbmesser. Electron radius (classical)
 Klein-Gordon-Gleichung. Klein-Gordon equation
 Klein-Nishina-Formel. Klein-Nishina formula
 Klein-Rydberg-Methode. Klein-Rydberg method
 Kleinste Quadrate. Least squares
 Kleinster Abirringkreis. Least circle of aberration
 Kleinstring. Infinitesimal ring
 Knickauprall. Buckling, impact
 Knicken. Buckling
 Knickverzug. Buckling lag
 Knotenpunkte. Nodal points(s)
 Knudsen-Kosinusgesetz. Knudsen cosine law
 Knudsenzahl. Knudsen number
 Kochsche Gleichung. Koch's equation
 Koma. Coma
 Kolumne. Column
 Kolumnaranalogie. Column analogy
 Komatische Kreise. Comatic circles
 Kombinationsschwingung. Combination vibration
 Kombinationssummen. Combination sums
 Komplexeinwilligung. Compliance, complex
 Komplexgitterträger. Truss, complex
 Komplexion. Complexion
 Komplexkennzahl. Modulus, complex
 Komplexpotential in der Hydrodynamikströmung. Complex potential in hydrodynamic flow
 Komplexstrahlung. Complex radiation
 Komplexzahl. Complex number
 Kondensator. Capacitor
 Kondensieren. Condensate
 Konforme Abbildung. Conformal mapping
 Konforme Abbildung der Komplexebene. Conformal mapping of the complex plane
 Konforme Lösungen. Conformal solutions
 Konsequentes Abschätzungsgerät. Consistent estimator
 Konservatives System. Conservative system
 Kontaktberichtigungsstrom. Contact rectification current
 Kontaktumwandlung. Contact transformation
 Kontinuität. Continuity
 Kontrolleübertragung. Transfer of control
 Kontrollsystem. Control system
 Konvektionänderungsverhältnis. Convective rate of change
 Konvergenz. Convergence
 Konvergenzwelle. Wave, converging
 Konvergenzwinkel. Angle of convergence
 Konzentration. Concentration
 Koordinate. Coordinate
 Koordinationsbindung. Coordinate bond
 Koordinationszahl. Coordination number
 Körperkräfte. Body forces
 Korrespondenzgrundsatz. Principle of correspondence
 Körpereigengleichungen einer Kurve. Curve, intrinsic equations of
 Körperdurchwandern. Diffusion of solids
 Korrelation. Correlation
 Korrelationsdarstellung. Correlation diagram
 Korrelationsenergie. Correlation energy
 Korrelationsverhältnis. Correlation ratio
 Korrelogramm. Correlogram
 Kosinusausstrahlungsgesetz. Cosine emission law
 Kosinusgesetz. Law of cosines
 Kovarianz. Covariance
 Kovariantabkömmling des Spannungsfeldes. Covariant derivative of tensor field
 Kovariantbestörungstheorie. Covariant perturbation theory
 Kovariantumschaltungsregeln. Covariant commutation rules
 Kovariantunterscheidung. Covariant differentiation
 Kovariantvektor. Covariant vector
 Kovarianzanalyse. Analysis of covariance
 Kovolumen. Covolume
 Kraft. Force, power
 Kraft auf einer Unterwasser Oberfläche. Force on a submerged surface

- Kraftbehinderungstabelle. Entropy chart
 Kraftbehinderung. Entropy
 Kräftebeschaffenheit. Forces, composition of
 Krätedreieck. Forces, triangle of
 Kraftdurchlässigkeit. Transmissibility of force
 Kräftegleichgewicht auf einem Steifkörper. Equilibrium of forces on a rigid body
 Kraftsmoment. Moment of force
 Kraftsspektrum. Power spectrum
 Kraftvieleck. Force polygon
 Krakowianer. Cracovians
 Kräusel. Curl
 Kreisbahnglied. Orbital
 Kreis der geringsten Verwirrung. Circle of least confusion
 Kreisdichromismus. Circular dichroism
 Kreiselmagnetisches Verhältnis. Gyromagnetic ratio
 Kreiselnhalbmesser. Radius of gyration
 Kreisende Öffnung. Circular aperture
 Kreislauf. Circulation, cycle
 Kreislaufelement. Element, circuit
 Kreislaufintegral. Integral, circulatory
 Kreislauflehrsatz. Circulation theorem (Kelvin)
 Kreislaufmatrize. Matrix, circuit
 Kreisplatten. Plates, circular
 Kreispolarisierung. Circular polarization
 Kreispolynome. Circle polynomials
 Kreispunkt auf einer Oberfläche. Circular Point on a surface
 Kreisringschallquelle. Circular ring sound source
 Kristallelektrische Konstanten. Piezoelectric constants
 Kreisvertauschung. Permutation, cyclic
 Kreiswelle. Wave, circulating
 Kreuzglied. Cross-link
 Kreuzmethode. Cross method
 Kriechbewegung. Creeping motion
 Kriechenwilligung. Compliance, creep
 Kristallbildungskurve. Crystallization curve
 Kristallbildungsverhältnisgleichungen. Crystallization rate equations
 Kristallebenmaß. Crystal symmetry
 Kristallgefüge. Crystal structure
 Kristallehre. Crystallography
 Kristallmomentum. Crystal momentum
 Kristallnebenveränderlichen. Crystal parameters
 Kristallogramm. Crystallogram
 Kristallographische Achsenverhältnisse. Crystallographic axial ratios
 Kristallographische Gruppe. Crystallographic group
 Kristallographische Mittellinien. Crystallographic axes
 Kristallographische Nebenveränderlichen. Crystallographic parameters
 Kristallphasen. Crystal phases
 Kristallsysteme. Crystal systems
 Kristallwinkel. Crystal angles
 Kritikfaktor. Criticality factor
 Kritische Gleichung. Critical equation
 Kritischer Punkt. Critical point
 Kroneckerdelta. Kronecker delta
 Kronig-Penney-Modell. Kronig-Penney model
 k -Raum. k -space
 Krummlinige Bewegung. Curvilinear motion
 Krummlinige Orthogonalnebengeordneten. Curvilinear orthogonal coordinates
 Krümmungslinie. Line of curvature
 Krümmungsradius. Curvature, radius of
 Krümmungsebene. Curvature, plane of
 Krümmungskreis. Curvature, circle of
 Krümmungsspanner. Curvature tensor
 Krümmungsweite einer Normale. Curvature, amplitude of normal
 Kryoskopische Unveränderliche. Cryoscopic constant
 K -Statistik. k -statistics
 Kubiksystem. Cubic system
 Kugelabirring. Spherical aberration
 Kugelabirringshalbmesser. Spherical curvature, radius of
 Kugelabirringzentrum. Spherical curvature, center of
 Kugelähnliche Koordinate. Spheroidal coordinate
 Kugeltangensindikator einer Kurve. Spherical indicatrix of tangent to a curve
 Kugelanmessungsgerät. Spherometer
 Kugeldreieckslehre. Spherical trigonometry
 Kugelharmoniemethode. Spherical harmonics method
 Kugelpendel. Pendulum, spherical
 Kugelpolarkoordinaten. Spherical polar coordinates
 Kugelvektorwellenfunktionen. Spherical vector wave functions
 Kugelvertretung einer Oberfläche. Spherical representation of a surface
 Kugelvertretung einer rechteckigen Kongruenz. Spherical representation of a rectilinear congruence
 Kugelwelle. Wave, spherical
 Kugelwellenstirn. Wave front, spherical
 Kugelwirbel. Spherical vortex
 Kundtkonstante. Kundt constant
 Kupplung. Coupling
 Kupplung im Atom- und Molekülgefüge. Coupling, in atomic and molecular structure
 Kupplungskonstante. Coupling constant
 Kurie-Schaubild. Curie plot
 Kurtose. Kurtosis
 Kurve. Curve
 Kurvenanpassung. Curve fitting
 Kurve der Spaltungsergiebigkeit. Fission yield curve
 Kurvenkongruenz. Congruence of curves
 Kurvenlänge. Length of a curve
 Kurvenlinieschallquelle. Curved line sound source
 Kurvenwölbung an einem Punkt. Curvature of a curve at a point
 Kurzbalken. Beam, short
 Kutta-Joukowski-Gesetz. Kutta-Joukowski law
 Laboratoriumssystem und Massenpunktsystem. Laboratory system and center-of-mass system
 Ladungsunabhängigkeit. Charge-independence
 Lagereinwilligung. Compliance, storage
 Lagrangesche Analysenmethode. Lagrangian method of analysis
 Lagrangesche Dichte. Lagrange density
 Lagrangesche Etage. Lagrange bracket
 Lagrangesche Funktion. Lagrangian function
 Lagrangesche Gleichungen. Lagrange equations
 Lagrangesche Interpolationsformel. Lagrange interpolation formula
 Lagrangescher Formalismus für Feldsysteme. Lagrangian formalism for field systems
 Lagrangescher Lehrsatz. Lagrange theorem
 Lagrangescher Vervielfacher. Lagrange multiplier
 Lagrangesches und Hamiltonisches Formenwesen für Dauersysteme. Continuous systems, Lagrangian and Hamiltonian formalism for
 Laguerre-Gleichung. Laguerre equation
 Laguerre-Polynome. Laguerre polynomials
 Lambdapunkt. Lambda point

- Lambert. Lambert
 Lambsche Konstante. Lamb's constant
 Lambsche Verschiebung. Lamb shift
 Lamé-Beziehungen. Lamé relations
 Lamé-Gleichung. Lamé equation
 Lamellar-Vektorenfeld. Lamellar vector field
 Laminareinfließender Kanal. Channel, laminar flow in
 Laminargrenzschicht. Laminar boundary-layer
 Laminarrohrströmung. Pipe, laminar flow in
 Laminarströmung. Laminar flow
 Laminarströmungbeständigkeit. Stability of laminar flow
 Lanczos-Methode der Biorthogonalisation. Lanczos method of biorthogonalization
 Landaus Formel. Landau's formula
 Landés G -Faktor. Landé's g -factor
 Landés Pausenregel. Landé's interval rule
 Langevin Diamagnetismustheorie. Langevin theory of diamagnetism
 Langevin-Funktion. Langevin function
 Langevin-Pauli-Formel für Diamagnetismus. Diamagnetism, Langevin-Pauli formula
 Längenverhältnis. Aspect ratio
 Langmuir Adsorptionsisotherme. Langmuir adsorption isotherm
 Längskugelaberrung. Longitudinal spherical aberration
 Längsscherung. Shear, longitudinal
 Längsstabwellen. Longitudinal waves in a rod
 Langstreckenordnung. Long range order
 Längsvergrößerung. Longitudinal magnification
 Laplace-Ausdehnung. Laplace expansion
 Laplace-Differentialgleichung. Laplace differential equation
 Laplace-Gleichung. Laplace equation
 Laplace'sche. Laplacian
 Laplace'sches Betriebsmittel. Laplacian operator
 Laplace-Umwandlung. Laplace transform
 Laporte-Regel. Laporte rule
 Lärm. Noise
 Lärmebene. Noise level
 Lärmisolationfaktor. Noise insulation factor
 Larmor-Vorausgehung. Larmor precession
 Lärmübertragungseffekt. Noise transmission effect
 Lasteinheitattrappenmethode. Dummy unit load method
 Lastvielfaches. Load factor
 Lastwechsel. Cycle of stress
 Latein-Quadrat. Latin square
 Lathargie. Lathargy
 Laue-Gleichungen. Laue equations
 Läufe. Runs
 Läufer. Rotor
 Laurent-Reihe. Laurent series
 Lautstärke. Sound intensity
 Lautstärkeniveau. Sound intensity level
 Lautstärkeniveau. Loudness level
 Lautstärkeumrisse. Loudness contours
 L'hospital-Regel. L'hospital rule
 Lebensdauer. Lifetime
 Lebesgue-Integral. Lebesgue integral
 Le Chatelier-Braun-Prinzip. Chatelier (Le)-Braun principle
 Le Chatelier-Lehrsatz. Chatelier (Le) theorem
 Legendre-Funktionen. Legendre functions
 Legendre-Gleichung. Legendre equation
 Legendre-Polynome. Legendre polynomials
 Lehmann-Vertretung. Lehmann representation
 Lehrsatz der geringsten Arbeit. Least work, theorem of
 Leibnitzregel. Leibnitz rule
 Leitebene. Guide plane
 Leitungsband. Conduction band
 Lennard-Jones-Potential. Lennard-Jones potential
 Lenz-Gesetz. Lenz law
 Letzter Scheitelpunkt. Vertex, final
 Leuchtbeiwert. Luminous coefficient
 Leuchtdichte. Density, luminous
 Leuchtenausbeute. Fluorescence yield
 Leuchtenbeiwerte. Luminosity coefficients
 Leuchtfuß. Luminous flux
 Leuchtkraftfunktion. Luminosity function
 Leuchtkraftwirksamkeit. Luminous efficiency
 Leuchtspiegelungsfaktor. Luminous reflectance
 Leuchtstärke. Luminous intensity
 Leuchtstärkeinheit. Candela
 Leuchtstärkeinheit pro Quadratmeter. Candela per square meter
 Leuchttemperatur. Luminance temperature
 Levy-Mises-Stoff. Levy-Mises material
 Lichtaberrung. Aberration of light
 Lichtaussendung. Luminous emittance
 Lichtersehen. Luminance
 Lichtzerstreuende Funktionen. Light scattering functions
 Lichtzerstreuung. Diffusion of light
 Liebmann-Methode. Liebmann method
 Lie-Gruppe. Lie group
 Lie-Ring. Lie ring
 Lineardarstellung. Graph, linear
 Lineardifferentialform. Linear differential form
 Linearextrapolationslänge. Linear extrapolation length
 Lineargleichungen. Linear equations
 Linearhypothese. Linear hypothesis
 Linearplanen. Linear programming
 Linearsysteme. Linear systems
 Linearungleichheiten. Linear inequalities
 Linearvergrößerung. Linear magnification
 Linearviskoelastizität. Viscoelasticity, linear
 Linie der Tätigkeitskraft. Force, line of action of
 Liniensehen einer Oberfläche. Astigmatism of a surface
 Linienstärke. Line strength
 Linienwirbel. Line vortices
 Linierte Oberfläche. Ruled surface
 Liouville-Oberfläche. Liouville surface
 Liouville-Lehrsatz. Liouville theorem
 Liouville-Lehrreihe. Liouville-Neumann series
 Lipschitz-Bedingung. Lipschitz condition
 Lobachevskii-Methode. Lobachevskii method
 „Loch“. „Hole“
 Loetheorie. Hole theory
 Lodesche Veränderlichen. Lode's variables
 Logarithmische Differenzierung. Logarithmic differentiation
 Logarithmisches Geschwindigkeitsprofil. Logarithmic profile of velocity
 Logarithmische Verminderung. Logarithmic decrement
 Logarithmische Verteilung. Logarithmic distribution
 Logarithmus. Logarithm
 Logarithmuswechselbeziehung. Log correlation
 Logikskurve. Logistic curve
 Logit. Logit
 Londonkräfte. London forces
 London-Überleitungsfähigkeitsgleichungen. London superconductivity equations
 Lorentzbedingung. Lorentz condition
 Lorentzfeld. Lorentz field
 Lorentzgesetz. Lorentz law

- Lorentzkraft. Lorentz force.
 Lorentz-Lorentz-Gesetz. Lorentz-Lorentz law
 Lorentzumschaltung. Lorentz transformation
 Lozeng-Darstellung. Lozenge diagram
L-R-Umwandlung. *L R transformation*
L-System und C-System. *L-system and C-system*
 Luderlinien. Luder's lines
 Luftdruckneigung. Tendency of barometric pressure
 Luftfunktionen. Airy functions
 Luftleiter. Antenna
 Luftschraube. Airscrew
 Luftstandardwirksamkeit. Air-standard efficiency
 Lufthorizontalebewegung. Advection
 Lufttreibstoffverhältnis. Air-fuel ratio
 Lumerg. Lumerg
 Lumen. Lumen
 Lumenstunde. Lumen-hour
 Lundquist-Reihenahaltungspunkt. Lundquist's series
 criterion
 Luneburg-Geometrie. Luneburg geometry
 Luneburglinse. Luneburg lens
 Luxnachkömmling. Lux-second
 Lyman-Reihe. Lyman series
- Macauleys Methode. Macauley's method
 Machanhaltungspunkt. Mach criterion
 Macheeinheit. Mache unit
 Machkegel. Mach cone
 Machlinie. Mach line
 Machnummer. Mach number
 Maclaurianreihe. Maclaurin series
 Machwinkel. Mach angle
 Magneteninduktion. Magnetic induction
 Magnetische Abspannungsenergie. Magnetic strain
 energy
 Magnetische Einberufung. Magnetic induction
 Magnetische Feldstärke. Magnetic field strength
 Magnetische Ladungsdichte. Magnetic charge density
 Magnetische Resonanzlinienbreite. Magnetic reson-
 ance, line width
 Magnetisches Moment. Magnetic moment
 Magnetisches Skalarpotential. Magnetic scalar poten-
 tial
 Magnetische Strömungsdichte. Magnetic current den-
 sity
 Magnetisches Vektorenpotential. Magnetic vector
 potential
 Magnetisierung. Magnetization
 Magnetisierungskraft. Magnetizing force
 Magnetobewegungskraft. Magnetomotive force
 Magnetokalorieneffekt. Magnetocaloric effect
 Magnetomechanische Dämpfung. Magnetomechanical
 damping
 Magnetomechanisches Verhältnis. Magnetomechanical
 ratio
 Magnetostatik. Magnetostatics
 Magnetostraktion. Magnetostriction
 Magnetostraktion der Kristallsättigung. Crystal satura-
 tion magnetostriction
 Magnetostraktionstreibsystem. Driving system, mag-
 netostriction
 Mahalanobische Entfernung. Mahalanobis's distance
 Majoranakraft. Majorana force
 Makrokanonisches Ensemble. Macrocanonical en-
 semble
 Makrozustand. Macrostate
- Malus-Kosinusquadratgesetz. Malus cosine-squared
 law
 Manigfaltig. Manifold
 Mantisse. Mantissa
 Marguerres Großablenkungstheorie. Marguerre's large
 deflection theory
 Markovkette. Markov chain
 Martingale. Martingale
 Masse. Mass
 Massendefekt. Mass defect
 Massenerhaltung. Conservation of mass
 Massenformel. Mass formula
 Massenpunkt. Center of mass
 Massenpunktsystem. Center-of-mass system
 Massenwiederenergie. Mass renormalization
 Massenwirkungsgesetz. Law of mass action
 Massenzahl. Mass number
 Maßfunktion. Measure function
 Mäßigkeitslehrsatz. Moderation theorem
 Materialdifferenzierung. Material differentiation
 Mathieugleichung. Mathieu equation
 Matrize. Matrix
 Matrizeinheit. Unit matrix
 Matrizenrang. Rank of a matrix
 Matrizenspur. Trace of a matrix
 Matrizenstreuung. Matrix scattering
 Matrizenumkehrung. Matrix inversion
 Matrizenumsetzung. Transpose of a matrix
 Maupertuilehrsatz. Maupertui theorem
 Maximalverbundene Unterdarstellung. Subgraph,
 maximal connected
 Maxwell. Maxwell
 Maxwell-Boltzmann-Verteilungsgesetz. Maxwell-Boltz-
 mann distribution law
 Maxwell-Mohr-Methode. Maxwell-Mohr method
 Maxwellmuster. Maxwell model
 Maxwells Beziehungen. Maxwell's relations
 Maxwells Fischaue. Maxwell's fisheye
 Maxwells Gleichungen. Maxwell's equations
 Maxwells Isothermenlehrsatz. Maxwell's theorem for
 isotherms
 Maxwellverteilungsgesetz. Maxwell distribution law
 Mechanik-Akustikwechselstetigkeits-Lehrsatz. Mecha-
 nical-acoustical reciprocity theorem
 Mechanische Einheiten. Mechanical units
 Mechanische Impedanz. Mechanical impedance
 Mechanischer Lichtgegenwert. Mechanical equivalent
 of light
 Mechanisches Beweglichkeitssystem. Mechanical mobi-
 lity system
 Mechanisches geradliniges System. Mechanical recti-
 lineal system
 Mechanisches geradliniges Wechselseitigkeitssystem.
 Mechanical rectilinear reciprocity theorem
 Mechanische Stetigkeit. Stability, mechanical
 Mechanisches Umdrehungssystem. Mechanical rota-
 tional system
 Mechanischer Umdrehungswechselseitigkeitslehrsatz.
 Mechanical rotational reciprocity theorem
 Mechanische Wirksamkeit. Mechanical efficiency
 Mechanismenmethode der Rahmenanalyse. Mechanism
 method of analysis of frames
 Mehrfachintegral. Multiple integral
 Mehrfachpendel. Compound pendulum
 Mehrfachumänderungsinterpolation. Multivariate inter-
 polation
 Mehrkörperkraft. Many-body force
 Mehrkollinearität. Multicollinearity

- Mehrzahlreihe. Multinomial series
 Mellin-Umwandlung. Mellin transform
 Meridianebene. Meridion(al) plane
 Meromorphe Funktion. Meromorphic function
 Messen. Measure
 Messungsunveränderlichkeit. Gauge invariance
 Messungsunveränderlichkeit und Erhaltungsgesetze.
 Gauge invariance and conservation laws
 Metastabiler Zustand. Metastable state
 Meterkerze. Meter-candle
 Meterkerzensekunde. Meter-candle-second
 Methode der Polynomannäherungen. Method of Poly-
 nomial approximations
 Metrischer Spanner. Metric tensor
 Meuniers Lehrsatz. Meunier's theorem
 Mie-Grüncisen-Zustandsgleichung. Mie-Grüncisen
 equation of state
 Mikrokanonisches Ensemble. Microcanonical ensemble
 Mikroumkehrungsprinzip. Microscopic reversibility,
 principle of
 Mikrowechelseitigergrad. Microreciprocal degree
 Mikrozustand. Microstate
 Miller-Kristallkennziffer. Crystal indices, Miller
 Milnes Methode. Milne's method
 Milnes Problem. Milne's problem
 Minderwert (oder eine Matrize) Minor (or a matrix)
 Mindestablenkungswinkel. Deviation, minimum angle
 of
 Mindestwahrnehmbarer Steigungsunterschied. Pitch
 difference, minimum perceptible
 Minimaloberfläche. Minimal surface
 Minimabschätzungsmethode. Minimax method of
 estimation
 Minimannäherung. Minimax approximation
 Minimumgewichtentwurf. Minimum weight design
 Minimumgrenzmassenproblem. Minimum critical mass
 problem
 Minimumphasenfunktionen. Minimum phase functions
 Minkowskis Elektrodynamik für bewegende Körper.
 Minkowski's electrodynamics for moving bodies
 Minkowski-Ungleichheit. Minkowski inequality
 Minkowski Welt. Minkowski world
 Mises-Ergebnigkeitszustand. Mises yield condition
 Mises-Material. Mises material
 Mißerfolg. Failure
 Mitführungspotential. Convective potential
 Mitlogarithmus. Cologarithm
 Mittelablenkung. Mean deviation
 Mittelkraft. Resultant
 Mittel Lebensdauer eines atomischen Zustandes. Mean
 lifetime of an atomic state
 Mittel Lebensdauer eines Elektrons in einer Falle. Mean
 lifetime of electron in trap
 Mittelkugelhärte. Mean spherical intensity
 Mittellinie. Median
 Mittelnormalkrümmung. Curvature, mean normal
 Mittelpulszeit. Pulse time, mean
 Mittelpunkt. Center
 Mittelpunktformel. Midpoint formula
 Mittelpunkt einer geometrischen Figur. Centroid of a
 geometrical figure
 Mittelpunktsachse. Centroidal axis
 Mittelpunktsbahn. Centrode
 Mittelunterschied. Mean difference
 Mittelwert einer Funktion. Mean value of a function
 Mittlere freie Weglänge. Mean free path
 Mittlere freie Zeit. Mean free time
 Mittlere Waagrechtstärke. Mean horizontal intensity
 Mittlere Wertigkeitslehre. Mean value theorems
 Mitwirkendes Phänomen. Cooperative phenomena
 Mobiländerungsdichte in einem Halbleiter. Density of
 mobile charges in semiconductor
 Modalspalte einer Matrize. Modal column of a matrix
 Modus. Mode
 Mohrs Betonungskreis. Mohr's circle for stress
 Mohrs Ergiebigkeitsbedingung. Mohr's yield condition
 Mohrs Spannungskreis. Mohr's circle for strain
 Mohrs Trägheitskreis. Mohr's circle for inertia
 Molalität. Molality
 Molareigenschaften. Molar properties
 Molarverteilung. Dispersivity, molar
 Molbruchteile. Mole fractions
 Molekularenergieebenen. Molecular energy levels
 Molekularfeldannäherung. Molecular field approxima-
 tion
 Molekularfreie Weglänge. Molecular free path
 Molekulargeschwindigkeit. Molecular velocity
 Moleküldarstellungen. Molecular diagrams
 Moleküleigenfunktionen. Molecular eigenfunctions
 Molekülkreisbahnmethode. Molecular orbitals method
 Molekularkreisbahnwertigkeitstheorie. Molecular orbital
 theory of valence
 Molekularumdrehung. Molecular rotation
 Molekularzerstörungsenergie. Dissociation energy of a
 molecule
 Molekülspektren. Molecular spectra
 Molekültermensymbole. Molecular term symbols
 Molliergleichung. Mollier equation
 Molliertabelle. Mollier chart
 Moment. Moment
 Momentendarstellung. Moment diagram
 Momentenmethode. Moment area method
 Momentenmethode. Moments method
 Momenten- und Produktträger. Inertia, moments and
 products of
 Momentpunkt. Center of moments
 Momentumerhaltung. Conservation of momentum
 Momentumverteilung. Distribution in momentum
 Momentalellipsoide. Ellipsoid, momental
 Monge'sche Form zur Oberflächengleichung. Monge's
 form for equation of surface
 Montagebetonungen. Assembly stresses
 Monte-Carlo-Methode. Monte Carlo method
 Morera-Lehrsatz. Morera theorem
 Morse-Kurve. Morse curve
 Morse-Spannung. Morse potential
 Moseley's Gesetz. Moseley's law
 Moutiergesetz. Moutier law
 Multiplett. Multiplet
 Multiplettstärkeregel. Intensity rules for multiplets
 Mu-Raum. Mu space
 Mutmaßliche Irrung. Probable error
 Nabla. Nabla
 Nachgiebigkeitspunkt. Yield point
 Nachlaß. Residue
 Nachlaßlehrsatz. Residue theorem
 Nächster Nachbar. Nearest neighbor
 Nachteilfaktor. Disadvantage factor
 Nachziehen. Drag
 Nahbestellung. Short-range order
 Näherungslinie. Asymptote
 Nahkraft. Short-range force
 Naßstrom. Wet stream
 Natürliche Linienweite. Natural line width

- Natürliche Periode. Period, natural
 Natürliche und unnatürliche Vorgänge. Natural and unnatural processes
 Navier-Stokes-Gleichungen. Navier-Stokes equations
 Nebenbetonungen. Secondary stresses
 Nebenfaktor. Coefactor
 Nebenfluß. Secondary flow
 Nebengeordnete. Coordinate
 Nebengleichung. Auxiliary equation
 Nebengruppe. Coset
 Nebenscheitelpunkte. Vertices, adjacent
 Nebenstatistik. Ancillary statistic
 Nebenwertigkeitskräfte. Subsidiary valence forces
 Negative Energiezustände. Negative energy states
 Neper. Neper
 Nernstverteilungsgesetz. Nernst distribution law
 Nernstwärmelehre. Nernst heat theorem
 Neumannfunktion. Neumann function
 Neumanngrenzbedingungen. Neumann boundary conditions
 Neumannprinzip. Neumann principle
 Neumannreihen. Neumann series
 Neutralachse. Neutral axis
 Neutrales Gleichgewicht. Equilibrium, neutral
 Neutralpunkt. Aerodynamic center
 Neutronenabbeugung. Diffraction of neutrons
 Neutronenalter. Age, neutron
 Neutronenanstoßdichte. Collision density, neutron
 Neutronenbahnlänge. Track length, neutron
 Neutronenübermaß. Neutron excess
 Neutronendichte. Density, neutron
 Neutronenspektrum. Spectrum, neutron
 Neutronenströmung. Current, neutron
 Neutronenströmung. Streaming, neutron
 Neutronentemperatur. Temperature, neutron
 Neutronenthalmisation. Thermalization of neutrons
 Neutronenwanderungsgebiet. Migration area for neutrons
 Neutronenwanderungslänge. Migration length for neutrons
 Neutronenzerstreuungsgebiet. Diffusion area, neutron
 Neutronenübertragungstheorie. Neutron transport theory
 Neutronenzerstreuungszeit. Diffusion time, neutron
 Neutronenzerstreuungstheorie. Neutron diffusion theory
 Neutronmagnetischer Moment. Neutron magnetic moment
 Newton. Newton
 Newton-Cotes-Quadraturformeln. Newton-Cotes quadrature formulas
 Newtonische Flüssigkeit. Newtonian fluid
 Newtonisches Gesetz des Flüssigkeitswiderstandes. Newton's law of fluid resistance
 Newtonisches Gesetz des Wärmeverlustes. Newton's law for heat loss
 Newtonische Zähflüssigkeit. Newtonian viscosity
 Newtons Bewegungsgesetze. Newton's laws of motion
 Newtons Gleichung. Newton's equation
 Newtons Methode. Newton's method
 Neyman-Pearson-Theorie. Neyman Pearson theory
 Nichtbestimmtheitsbeiwert. Non-determination, coefficient of
 Nichtgleichwertige Elektronen. Non-equivalent electrons
 Nichtholonisch. Non-holonomic
 Nichtigkeitssdarstellung. Graph, nullity
 Nichtkreislaufelement. Element, non-circuit
 Nichtleitergrenzformeln. Dielectric boundary formulas
 Nichtleiterhysterese. Dielectric hysteresis
 Nichtleiterkonstante. Dielectric constant
 Nichtlokalisierte Molekülkreisbahnen. Nonlocalized molecular orbitals
 Nichtorientierte Darstellung. Graph, non-oriented
 Nichtparametrische Einmischung. Non-parametric interference
 Nichtperiodische Schwingungen. Vibrations, non-periodic
 Nichtüberquerende Regel. Non-crossing rule
 Nichtverbindende Moleküländerungen. Non-combining modifications of molecules
 Nichtverbundene Unterdarstellung. Subgraphs, disjoint
 Nichtzentrale Kräfte. Non-central forces
 Nichtzentrale Verteilungen. Non-central distributions
 Nichtzerstreuende Wellen. Non-dispersive waves
 Niederertragbetonung. Yield stress, lower
 Nijboer-Zernike-Abirrungsfunktionen. Nijboer-Zernike aberration functions
 Niveaueinheiten. Unit planes
 Niveaustellenstirn. Wave front, plane
 Norm. Norm
 Normal. Normal
 Normalbetonung. Normal stress
 Normaldruck. Normal pressure
 Normalbetonungswirkung. Normal stress effect
 Normale an einer Kurve. Normal to a curve
 Normaleinheit. Unit normal
 Normalerscheinungsformen. Normal modes
 Normalerwiderung. Normal response
 Normalfehler. Standard error
 Normalgleichungen. Normal equations
 Normalisierung. Normalization
 Normalisierungsfaktor. Normalizing factor
 Normalmaß. Standard measure
 Normalprodukt. Normal product
 Normalrückwirkung. Normal reaction
 Normalschwingungen in polyatomischen Molekülen. Normal vibrations in polyatomic molecules
 Normaltemperatur und -druck. Normal temperature and pressure
 Normalübereinstimmung. Normal congruence
 Normalvergrößerung. Normal magnification
 Normalverteilung. Normal distribution
 Normalwölbung. Normal curvature
 Normalzustand. Normal state
 Nortons Lehrsatz. Norton's theorem
 Null-geodätisch. Null-geodesic
 Nullgesetz der Thermodynamik. Zeroth law of thermodynamics
 Nulllinie eines Bandes. Zero line of a band
 Null-Ladungsprüfung. Zero-load test
 Nullmatrize. Null matrix
 Nullpunktenergie. Zero point energy
 Nullreihen. Null sequence
 Nullvektor. Null vector
 Nusseltzahl. Nusselt number
 Nuttings Gleichung. Nutting's equation
 Nyquist-Lehrsatz. Nyquist theorem
 Oberertragbetonung. Yield stress, upper
 Oberfläche der geringsten Verwirrung. Surface of least confusion
 Oberflächenabwicklungslinie. Developable surface

- Oberflächendivergenz eines Vektors. Surface divergence of a vector
- Oberflächeneigenschaft. Surface, characteristic of
- Oberflächenfamilie. Surfaces, family of
- Oberflächenkraft. Power of a surface
- Oberflächenkrümmung. Curvature of a surface
- Oberflächenspanner. Tensor, surface
- Oberflächenspannung eines gewölbten Zwischengesichts. Surface tension of curved interface
- Oberflächenübertragungsimpedanz. Impedance, surface transfer
- Oberflächenwellen. Surface waves
- Oberhängerbalken. Beam, overhanging
- Oberschallreihen. Harmonic series of sounds
- Oberschwinger. Harmonic oscillator
- Objektpunkt. Object point
- Öffnung. Aperture
- Öffnungswinkel. Aperture angle
- Offene Quadraturformel. Open quadrature formula
- Offener Randzug. Edge train, open
- Offene Systeme. Open systems
- Ohmsches Gesetz. Ohm's law
- Oktahedralbetonung. Stress, octahedral
- Onsager-Beziehungen. Onsager relations
- Onsagers Gleichung zur Nichtleiterkonstante. Onsager's equation for the dielectric constant
- Optikdichte. Optical density
- Optiktiefe. Optical depth
- Optikzentrum. Optical center
- Optimum-Zwischenraum-Interpolation. Optimum interval interpolation
- Optische Achse. Optic axis
- Optische Differentialunveränderliche. Optical differential invariant
- Optische Entfernung. Optical distance
- Optische Richtungskosinus. Optical direction cosines
- Optischer Lehrsatz. Optical theorem
- Optischer Modus. Optical mode
- Optischer Pfad. Optical path
- Optisches Kernmuster. Optical model of the nucleus
- Ordnung der chemischen Rückwirkungen. Order of chemical reactions
- Ordnung-Unordnung-Umwandlung. Order-disorder transformation
- Ordo-Symbol. Ordo-symbol
- Orientierte Darstellung. Graph, oriented
- Orientierte Schneidevorrichtung. Cut set, oriented
- Orientiertes Element. Element, oriented
- Ortbegrenzte Einzelschichten. Localized monolayers
- Ortbeschränkte Zustände. Localized states
- Orthochroneinheitliche Lorentztransformation. Orthochronous homogeneous Lorentz transformation
- Orthochronuneinheitliche Lorentztransformation. Orthochronous inhomogeneous Lorentz transformation
- Orthogonalfunktion. Orthogonal function
- Orthogonalisierung. Orthogonalization
- Orthogonalpolynome. Orthogonal polynomials
- Orthogonalvektor(en) Vector(s), orthogonal
- Orthonormal. Orthonormal
- Orthonormales System. Orthonormal system
- Orthorhombensystem. Orthorhombic system
- Orthoskopisches System. Orthoscopic system
- Orthotomisches System. Orthotomic system
- Örtliches Wechselverhältnis. Local rate of change
- Örtlich euklidisch. Locally euclidean
- Ortslehredarstellung. Graph, topological
- Oseens Methode. Oseen's method
- Osmotischer Beiwert. Osmotic coefficient
- Osmotischer Druck. Osmotic pressure
- Ostwalds Verdünnungsgesetz. Ostwald's dilution law
- Oszillatorkraft. Oscillator strength
- Paar. Couple
- Paarungsanweisungen. Conjugate directions
- Paarungselemente einer Gruppe. Conjugate elements of a group
- Paarungsgefällmethode. Conjugate gradients, method of
- Paarungskerne. Conjugate nuclei
- Paarungspunkte eines Mehrfachpendels. Conjugate points of a compound pendulum
- Paarungssystem der Kurven auf einer Oberfläche. Conjugate system of curves on a surface
- Paarungszahlen. Conjugate numbers
- Padetisch. Pade table
- Parabel. Parabola
- Parabelgleichung. Parabolic equation
- Parabelnebengeordneten. Parabolic coordinates
- Parabelpunkt auf einer Oberfläche. Parabolic point on a surface
- Parabelzylindernebengeordneten. Parabolic cylindrical coordinates
- Paraboloidnebengeordneten. Paraboloidal coordinates
- Parallelliniengehaltbestimmung. Parallel line assay
- Paralleloberflächen. Parallel surfaces
- Parallelverschiebung eines Vektors. Parallel displacement of a vector
- Paramagnetische Empfänglichkeit. Paramagnetic susceptibility
- Paramagnetismus. Paramagnetism
- Paraxiale Einzeloberflächengleichung. Paraxial single-surface equation
- Paraxiale Strahlungsnachspurgleichungen. Paraxial ray tracing equations
- Parexische Analyse. Parexic analysis
- Paritätsbedingung. Parity operator
- Pascaldreieck. Pascal triangle
- Pascalverteilung. Pascal distribution
- Paschen-Back-Wirkung. Paschen-Back effect
- Paschenreihe. Paschen series
- Packungsanteil. Packing fraction
- Pattersonkarte. Patterson map
- Pauli-Ausschluß-Prinzip. Pauli exclusion principle
- Pauli-Drehungstäter. Pauli spin operators
- Pauli-Glied. Pauli term
- Pauli-Weisskopf-Gleichung. Pauli-Weisskopf equation
- Pearson-Verteilungen. Pearson distributions
- Pells Gleichung. Pell's equation
- Pendel. Pendulum
- Periode. Period
- Periodenreaktor. Period, reactor
- Periodische Wellen. Wave, periodic
- Periodogramm. Periodogram
- Petzval-Oberfläche. Petzval surface
- Petzvals Bedingung. Petzval's condition
- Petzval-Summe. Petzval sum
- Pfad. Path
- Pfaffproblem. Pfaff problem
- Pfeilflügel. Swept-back wing
- Pfortrahmen. Portal frame
- Pfundreihe. Pfund series
- Phase. Phase
- Phase einer Periodengröße. Phase of a periodic quantity
- Phasen. Phases

- Phasenausdehnung. Extension in phase
 Phasenberandung. Phase margin
 Phasenbeständigkeit. Stability of phases
 Phasenfaktor. Phase factor
 Phasengeschwindigkeit. Phase velocity
 Phasengeschwindigkeit der Wellen. Waves, phase velocity of
 Phasenintegral. Phase integral
 Phasenlangsamkeit. Phase slowness
 Phasenraum. Phase space
 Phasenregel. Phase rule
 Phasenverteilung. Distribution in phase
 Phasenwinkel. Phase angle
 Phon. Phon
 Phonon. Phonon
 Phononische mittlere freie Weglänge. Phonon mean free path
 Phonon-Phonon-Zwischenwirkung. Phonon-Phonon interaction
 Photoelektrische Arbeitsfunktion. Photoelectric work function
 Photoelektrische Empfindsamkeit. Photoelectric sensitivity
 Photoelektrische Konstante. Photoelectric constant
 Photoelektrische Schwelle. Photoelectric threshold
 Photographisches Wechselseitigkeitsgesetz. Reciprocity law, photographic
 Physisches Pendel. Pendulum, physical
 Picard-Methode. Picard method
 Placzekfunktion. Placzek function
 Planardarstellung. Graph, planar
 Planarpunkt. Planar point
 Planetenbewegung. Planetary motion
 Planetenkreisbahntheorie. Planetary orbit theory
 Plastische Analyse. Plastic analysis
 Plastischer Entwurf. Plastic design
 Plastische Störung. Plastic flow
 Planckkonstante. Planck constant
 Plancksche Farbe. Planckian color
 Plancksche Funktion. Planck function
 Planckscher Locus. Planckian locus
 Plancksches Gesetz. Planck law
 Plancksche Strahlungsformel. Planck radiation formula
 Plastischer Modul. Plastic modulus
 Plastische Wellen. Plastic waves
 Plastisch-steif. Plastic-rigid
 P_n -Annäherung. P_n approximation
 Poincares Unveränderliche. Poincare's invariant
 Poincots Ellipsoide. Ellipsoid of Poincot
 Poissonsche Gabel. Poisson bracket
 Poissonsche Gleichung. Poisson equation
 Poissonsches integral. Poisson integral
 Poissonsches Verhältnis. Poisson's ratio
 Poissonsche Verteilung. Poisson distribution
 Pol. Pole
 Polarabwicklungslinie. Developable, polar
 Polarisation. Polarization
 Polarisierbarkeit. Polarizability
 Polarisierbarkeitskatastrophe. Polarizability catastrophe
 Polarisierungsebene. Plane polarization
 Polarisierungskreis. Polarization cycle
 Polarisierungsverhältnis. Proportion of polarization
 Polarlinie. Polar line
 Polarkoordinaten. Polar coordinates
 Polarvektor. Vector, polar
 Pol einer analytischen Funktion. Pole of an analytic function
 Polhode. Polhode
 Polyatomische Moleküle. Polyatomic molecules
 Polymer. Polymer
 Polymerische Mischungen. Polymer mixtures
 Polynomfaktorisierung. Polynomial factorization
 Polynomial. Polynomial
 Porter-Thomas-Verteilung. Porter-Thomas distribution
 Positivbestimmte Matrize. Positive definite matrix
 Positiver Pol. Anode
 Positronium. Positronium
 Positronvektor. Positron vector
 Potentialenergie. Energy, potential
 Potentialmolekülfunktionen. Potential functions of molecules
 Potentialschranke. Potential barrier
 Potentialstreuung. Potential scattering
 Potentialströmung. Potential flow
 Potenzreihe. Power series
 Plastizität. Plasticity
 Poyntinglehrsatz. Poynting theorem
 Poyntingvektor. Poynting vector
 Prandtl-Glauert-Regel. Prandtl-Glauert rule
 Prandtl-Meyer-Ausdehnung. Prandtl-Meyer expansion
 Prandtl-Reuss-Material. Prandtl-Reuss material
 Prandtlzahl. Prandtl number
 Prinzip der eigentlichen Arbeit. Work, principle of virtual
 Prinzip der entsprechenden Zustände. Corresponding states, principle of
 Prinzip der geringsten Energie. Least-energy principle
 Prinzip der geringsten Wirkung. Least action, principle of
 Probenahmeverteilung. Sampling distribution
 Probepfung. Proof test
 Probepunkt. Sample point
 Probit. Probit
 Proca-Gleichungen. Proca equations
 Produkt. Product
 Produktmoment. Product moment
 Profilwiderstand eines Flügels. Profile drag of a wing
 Programm. Program
 Progression. Progression
 Projektionswinkel. Projection angle
 Prüfungsfunktion. Test function
 Pseudoskalar. Pseudoscalar
 Pseudoskalarkupplung. Pseudoscalar coupling
 Pseudovektor. Pseudovector
 Pseudovektorkupplung. Pseudovector coupling
 Puls. Pulse
 Pulsamplitude. Pulse amplitude
 Pulsbandweite. Pulse bandwidth
 Pulsdauer. Pulse duration
 Pulszerfallzeit. Pulse decay time
 Punktgruppe. Point group
 Pulsicrung. Pulsatance
 Pulsmodus. Pulse mode
 Pulspause. Pulse interval
 Pulspflichtfaktor. Pulse duty factor
 Punktquelle. Point source
 Pulssteigerungszeit. Pulse rise time
 Pulstrennung. Pulse separation
 Pulsübergehen. Pulse spacing
 Pulswiederholungsfrequenz. Pulse repetition frequency
 Pulswiederholungsperiode. Pulse repetition period
 Pulswiederholungsverhältnis. Pulse repetition rate
 P -Welle. P wave
 Pythagorasskala. Pythagorean scale

- Q-Faktor. *Q* factor
 Quadratdifferentialform. Quadratic differential form
 Quadratwelle. Square wave
 Quadratwellepotential. Square well potential
 Quadratische Oberfläche. Quadric surface
 Quadrupelvektorenprodukt. Quadruple product of vectors
 Quant. Quantum
 Quantenelektrodynamik. Quantum electrodynamics
 Quantenertrag. Quantum yield
 Quantenhaft. Quantic
 Quantenmagnetismustheorie. Magnetism, quantum theory of
 Quantenmechanik. Quantum mechanics
 Quantenmechanikresonanz. Resonance, quantum mechanical
 Quantenstatistik. Quantum statistics
 Quantentheorie der Spektra. Quantum theory of spectra
 Quantentheorie einer Wärmekapazität. Quantum theory of heat capacity
 Quantentheorie einer Zerstreuung. Quantum theory of a dispersion
 Quantenwirksamkeit. Quantum efficiency
 Quantenzustandsdichte in einem Leitungsband. Density of quantum states in conduction band
 Quantil. Quantile
 Quantisieren. Quantize
 Quantisierung. Quantization
 Quantisierung des elektromagnetischen Feldes. Quantization of electromagnetic field
 Quasi-chemische Annäherung. Quasi-chemical approximation
 Quasiferminiveaus. Fermi levels, quasi
 Quaternion. Quaternion
 Quelle. Source
 Quellenebene. Source level
 Quellsinkenmethode. Source-sink method
 Querschnitt. Cross-section
 Querunterschiedsgleichheit. Cross-differentiation identity
 Q-Wert. *Q*-value

 Racah-Bewert. Racah coefficient
 Radialladung. Radial loading
 Radialvergrößerung. Radial magnification
 Radialverteilungsfunktion. Radial distribution function
 Rahmen. Frame
 Rahmenwerk. Framework
 Ramanwirkung. Raman effect
 Ramberg-Osgood-Nebenveränderlichen. Ramberg-Osgood parameters
 Rand. Edge
 Randbedingung. Boundary condition
 Randzug. Edge train
 Rang. Rank
 Rankine-Kreislauf. Rankine cycle
 Rankine-Wirbel. Rankine vortex
 Rankkorrelation. Rank correlation
 Rationalnummer. Rational number
 Raumähnliche Oberfläche. Space-like surface
 Raumgitter. Space lattice
 Raumgruppe. Space group
 Raumkurve. Space curve
 Raumquantisierung. Space quantization
 Raumspanner. Tensor, space
 Rayl. Rayl
 Rayleigh-Anhaltspunkt zur Auflösung. Rayleigh criterion for resolution
 Rayleigh-Grenze zur Kugelaberrung. Rayleigh limit for spherical aberration
 Rayleigh-Jeans-Gleichung. Rayleigh-Jeans equation
 Rayleighoberflächewellen. Waves, Rayleigh surface
 Rayleigh-Ritz-Methode. Rayleigh-Ritz method
 Rayleigh-Scheibe. Rayleigh disk
 Rayleigh-Schrodinger-Belastigungsformel. Rayleigh-Schrodinger perturbation formula
 Rayleigh-Streuung. Rayleigh scatter
 Rayleigh-Zahl. Rayleigh number
 Reaktorübertragungsfunktion. Reactor transfer function
 Realgase. Real gases
 Rechenmaschine. Calculator
 Rechteckebeneoberflächenschallquelle. Plane rectangular surface sound source
 Rechteckige Kongruenz. Rectilinear congruence
 Rechteckige Nebengeordneten. Rectangular coordinates
 Rechteckige Öffnung. Rectangular aperture
 Rechteckige Platten. Plates, rectangular
 Reaktionsfähigkeit. Reactivity
 Rechteckige Verteilung. Rectangular distribution
 Regula falsi. Regula falsi
 Reduzierte Brennpunktlänge. Reduced focal length
 Reduzierte Masse. Reduced mass
 Reduzierte oder Doppelmodul. Modulus, reduced or double
 Reflexabglanz. Reflex reflection
 Reibungsbeiwert. Friction, coefficient of
 Reibungsgeschwindigkeit. Friction velocity
 Reibungskegel. Friction, cone of
 Reibungswinkel. Friction, angle of
 Reihe. Series
 Reihenfolge. Sequence
 Reihenfolgenanalyse. Sequential analysis
 Reihenintegration. Series integration
 Reihensumme. Summation of series
 Reihenumkehrung. Reversion of series
 Reinbiegung. Bending, pure
 Rektifizierungsabwicklungslinie. Developable, rectifying
 Rektifizierungslinie. Rectifying line
 Rekursionsformel. Recursion formula
 Relativfehler. Relative error
 Relativfrequenz. Relative frequency
 Relativistische Hydrodynamik. Relativistic hydrodynamics
 Relativistische Quantenfeldtheorie. Relativistic quantum field theory
 Relativistische Unveränderlichkeit einer physischen Theorie. Relativistic invariance of a physical theory
 Relativistische Unveränderlichkeits- und Quantenmechanik. Relativistic invariance and quantum mechanics
 Relativistische Unveränderlichkeitswellengleichungen. Relativistically invariant wave equations
 Relativitätsvoraussetzung. Relativity precession
 Relativleuchtwirksamkeit. Relative luminous efficiency
 Relativspanner. Tensor, relative
 Resonanzenergie. Resonance energy
 Resonanzfluchtwahrscheinlichkeit. Resonance escape probability
 Resonanzfrequenz. Resonant frequency
 Resonanzintegral. Resonance integral

- Resonanzkristallfrequenz. Crystal, resonance frequency of
- Resonanzschärfe. Resonance, sharpness of
- Restbetonungen. Residual stresses
- Reynolds-Analogie. Reynolds analogy
- Reynolds-Betonungen. Reynolds stresses
- Reynoldsgleichung. Reynolds equation
- Reynoldszahl. Reynolds number
- Reynoldszahl der Verwirbelung. Turbulence, Reynolds number of
- Rhombohedralsystem. Rhombohedral system
- Riceatigleichung. Riccati equation
- Riccigleichheit. Ricci identity
- Ricei-Lehrsatz. Ricci's theorem
- Riccispanner. Ricci tensor
- Richardson-Auftragung. Richardson plot
- Richardson-Dushman-Gleichung. Richardson-Dushman equation
- Richardson-Zahl. Richardson number
- Richtigsteller. Corrector
- Richtigstellung. Correction
- Richtung. Direction
- Richtungsmerkmal. Directional characteristic
- Richtungswinkel. Azimuth angle
- Riemann-Abbildungslehrensatz. Riemann mapping theorem
- Riemann-Cristoffel-Spanner. Riemann-Cristoffel tensor
- Riemann-Papperitz-Gleichung. Riemann-Papperitz equation
- Riemannintegral. Riemann integral
- Riemannoberfläche. Riemann surface
- Riemannsphäre. Riemann sphere
- Riemannzetafunktion. Riemann zeta function
- Ring. Ring
- Ringraum. Annulus
- Ringwulst. Torus
- Ritters Methode. Ritter's method
- Ritzformel. Ritz formula
- Ritzmethode. Ritz method
- Rodriguesformel. Rodrigues formula
- Rohr- oder Kanaleingangslänge. Entry length for pipe or channel
- Rollelehrensatz. Rolle theorem
- Rollenwiderstand. Rolling resistance
- Rolltreppemethode. Escalator method
- Rosette. Rosette
- Rosbysehe Zahl. Rossby number
- Rosby-Wellen. Rossby waves
- Rotationsvermögen. Rotatory power
- Rotierungszerstreuung. Rotatory dispersion
- Rousseaudarstellung. Rousseau diagram
- Rouths Trägheitsregel. Routh's rule of inertia
- Rückfall. Regression
- Rückfluß. Creep
- Rückfokalweite. Back focal length
- Rückkopplung. Feedback
- Rückstrahlungsgesetze. Reflection, laws of
- Rückführungsbeiwert. Restitution, coefficient of
- Rückführungsperiode. Period of restitution
- Rückführungsperiode. Return period
- Rückführungsunterschied. Return difference
- Rückführungsverhältnis. Return ratio
- Rückstrahlvermögen. Reflectivity
- Rückstreuungsbeiwert. Back-scattering coefficient
- Rückwirkung. Reaction
- Rückwirkungsumfang. Extent of reaction
- Rückwirkungskinetik. Kinetics, reactor
- Rückwirkungsbetonungen. Reaction stresses
- Rückwirkungskanal. Reaction channel
- Ruhemoment. Moment, static
- Ruhewinkel. Repose, angle of
- Ruhebeständigkeit. Static stability
- Rundung. Rounding
- Runge-Kutta-Methode. Runge-Kutta method
- Runzeln. Wrinkling
- Russel-Saunders-Kupplung. Russell-Saunders coupling
- Russelwinkel. Russell angles
- Rutherford. Rutherford
- Rydbergkonstante. Rydberg constant
- Rydberggleichung. Rydberg equation
- Rydbergkorrigierung. Rydberg correction
- Rydbergreihe. Rydberg series
- Sabin. Sabin
- Sabingesetz. Sabin law
- Sackur-Tetrodegleichung. Sackur-tetrode equation
- Sagittalbrennpunkt. Sagittal focus
- Sagittalebene. Sagittal plane
- Sagitte. Sagitta
- Saha-Gleichgewichtformel. Saha equilibrium formula
- Säkularbestimmungszahl. Secular determinant
- Sammelgleichungen der Optik. Collineation equations of optics
- Sammelpunkt. Center of collineation
- Sandhäufenanalogie. Sand heap analogy
- Sankt-Venant-Mises-Material. Saint Venant-Mises material
- Sargentkurven. Sargent curves
- Sattelpunktmethode. Saddle point method
- Saugung der Laminargrenzschicht. Suction of laminar boundary layer
- Säulebalken. Beam, column
- Saurels Lehrsatz. Saurel's theorem
- Schäffli-Formel. Schäffli formula
- Schale. Shell
- Schallaufnahmebeiwert der Oberflächen. Sound absorption coefficient of surfaces
- Schalllautstärke. Loudness of a sound
- Schallbanddruckebene. Sound band pressure level
- Schalldämpfungsfaktor. Sound attenuation factor
- Schalldruckebene. Sound pressure level
- Schalldruckspektrumebene. Sound pressure spectrum level
- Schallebene. Sound level
- Schalleinheiten. Acoustical units
- Schalleistungsfähigkeit. Acoustical capacitance
- Schallenergiedichte. Sound-energy density
- Schallenergiefluß. Sound energy flux
- Schallkraft einer Quelle. Sound power of a source
- Schalloktavenbanddruckebene. Sound octave-band pressure level
- Schallspiegelungsbeiwert. Sound reflection coefficient
- Schallstreuung. Scattering, sound
- Schallsystem. Acoustical system
- Schallübertragung. Sound transmission
- Schallzerfallverhältnis. Rate of decay of sound
- Schar. Family
- Scharfreihe. Sharp series
- Schätzungstheorie. Estimation, theory of
- Schauereinheit. Shower unit
- Scheindurchmesser. Apparent diameter
- Scheinlastmethode. Loads, method of fictitious
- Scheitelmatrix. Matrix, vertex
- Scheitelpunkt. Vertex
- Scheitelpunktgrad. Vertex degree

- ScherAuslenkung. Deflection, shear
 Scherkraftsverzug. Shear lag
 Schermodul. Modulus, shear
 Scherströmung. Shear flow
 Scherung. Shear
 Scherungswellen. Waves, shear
 Scherungszentrum. Shear center
 Schcune. Barn
 Schichtintegration. Contour intcgration
 Schichtung. Stratification
 Schiefoberfläche. Skew surface
 Schiefstrahlung. Skew ray
 Schiefwinkligkeit. Skewness
 Schlafoberschicht. Top, "sleeping"
 Schläge. Beats
 Schlangentheorie. Queues, theory of
 Schlankkörpertheorie. Slender-body theory
 Schlegelbalken. Beam, haunched
 Schleifenflug. Loop
 Schlupflinie. Slip line
 Schlupfströmung. Slip flow
 Schlupfwirkung. Slip effect
 Schmelzmittelvektoren. Flux vectors
 Schmidt-Hilbert-Methode. Schmidt-Hilbert method
 Schmidt-Kernmuster. Schmidt model of nuclei
 Schneidevorrichtung. Cut set
 Schneidevorrichtungsmatrize. Cut set matrix
 Schnellspaltungsfaktor. Fast fission factor
 Schönflies-Kristallsymbole. Schönflies crystal symbols
 Schottky-Mangel. Schottky defect
 Schräglinie. Diagonal
 Schräg-symmetrischer Spanner. Tensor, skew-symmetric
 Schraubenachse. Screw axis
 Schraubengang. Convolution
 Schraubengrad. Pitch of screw
 Schraubenlinie. Helix
 Schraubenwölbung. Curvature, screw
 Schrödingerbild. Schrödinger picture
 Schrödingergleichung. Schrödinger equation
 Schrödingervertretung. Schrödinger representation
 Schüttemperatur. Bulk temperature
 Schüttkennzahl. Modulus, bulk
 Schüttmodul. Bulk modulus
 Schwache Achsc. Axis, weak
 Schwache Ortskommutativität. Weak local commutativity
 Schwächung. Attenuation
 Schwachwirkend. Nilpotent
 Schwankung-Vergeudungslehrsatz. Fluctuation-dissipation theorem
 Schwankungen. Fluctuations
 Schwartz-Cristoffel-Umwandlung. Schwartz-Christoffel transformation
 Schwartz-Ungleichheit. Schwartz inequality
 Schwarzkörper. Black body
 Schwarzkörper Strahlungsgesetze. Black body radiation laws
 Schwarzschild-Kohlschutter-Formeln. Schwarzschild-Kohlschutter formulas
 Schwabe. Poise
 Schwebefrequenz. Beat frequency
 Schwellenergie. Threshold energy
 Schwerbewegliche Gleichung. Pondermotive equation
 Schwerenhalbmesser. Gravitational radius
 Schwerenspannung. Gravitational potential
 Schwerpunkt. Center of gravity
 Schwimmkörperbeständigkeit. Stability of a floating body
 Schwingende Abwicklungslinie. Developable, osculating
 Schwingung. Oscillation
 Schwingungsamplitude. Amplitude of oscillation
 Schwingungsarten. Oscillation, modes of
 Schwingungsebene. Plane of vibration
 Schwingungsenergieebene eines Moleküls. Vibrational energy levels of a molecule
 Schwingungspunkt. Center of Oscillation
 Schwingungsspektra der Moleküle. Vibrational spectra of molecules
 Schwingungssummregel für elektronische Übergänge. Vibrational sum rule for electronic transitions
 Schwingungsteilfunktionen. Vibrational partition function
 Secantspaltenformel. Secant formula for columns
 Sechskantensystem. Hexagonal system
 Sehachse. Axis, optic
 Sehkegel. Cones, visual
 Sehne. Chord
 Sehnenmethode. Chord method
 Sehspalte. Diopter
 Sehwinkelabweichung. Parallax
 Seidel-Abirrungen. Seidel aberrations
 Seidelmethode. Seidel method
 Seitenausbiegung. Lateral deflection
 Seitenknicken. Lateral buckling
 Seitenkugelabirring. Lateral spherical aberration
 Seitenladung. Lateral load
 Seitenvergrößerung. Lateral magnification
 Seitwärts. Sidesway
 Selbstenergie in einem Nichtleiter. Self energy in a dielectric
 Selbstgefrett. Autofrettage
 Selbstkonsequentes Feld. Self-consistent field
 Selbstkorrelation. Autocorrelation
 Selbstzurückdrehung. Autoregression
 Selektivaussender. Selective emitter
 Selektivstrahler. Selective radiator
 Sendungsvermögen. Transmittivity
 Senkrechachsenlehrsatz. Perpendicular axis theorem
 Senkrechtwelle. Wave, longitudinal
 Serber-Wilson-Methode. Serber-Wilson method
 Serret-Frenet-Formeln. Serret-Frenet formulae
 Sheppards Verbesserungen. Sheppard's corrections
 Sicherheitsfaktor. Safety factor
 Sichtbarkeitsfaktor. Visibility factor
 Sichtfeld. Field of view
 Siedekurve und Verdichtungskurve. Boiling curve and condensation curve
 Siedend. Boiling
 Siedepunkt. Boiling point
 Sieder. Boiler
 Siegbahn-X-Einheit. Siegbahn x-unit
 Sigma-Pi-Wellenfunktion. Wave function, sigma-pi
 Signum. Signum
 Silsbec-Regel. Silsbec rule
 Simplexmethode. Simplex method
 Simpsons Regel. Simpson's rule
 Simultangleichungen. Simultaneous equations
 Singularlösung. Singular solution
 Singularpunkt einer Funktion. Singular point of a function
 Singularwerte. Singular values
 Sinken. Sink
 Sinusförmigkeitsgrenzlehrsatz. Sinusoidal limit theorem

- Sinusgesetz. Law of sines
 Skalar. Scalar
 Skalardichte. Scalar density
 Skalarfeld. Scalar field
 Skalarprodukt. Scalar product
 Skalarverwandte. Scalar relative
 Skalarwölbung. Scalar curvature
 Skalen. Scalene
 Slater-Bestimmungszahl. Slater determinant
 Slater-Summe. Slater sum
 Slutsky-Yule-Wirkung. Slutsky-Yule effect
 S-Matrize. S-matrix
 S-Matrizetheorie und Quantenfeldtheorie. S-matrix theory and quantum field theory
 Snells Gesetz(e). Snell's law(s)
 S_N-Methode. S_N method
 Sommerfelds Feingefügekonstante. Sommerfeld's fine structure constant
 Sommerfeld-Kossel-Verschiebungsgesetz. Sommerfeld-Kossel displacement law
 Sonderfunktionen. Special functions
 Sonderpolynome. Special polynomials
 Sonus. Sone
 Spalte. Slit
 Spalte-Auftragung-Methode. Split-plot method
 Spannerabkömmling. Tensor derivative
 Spanneranalyse. Tensor analysis
 Spannerart. Tensor, type of
 Spannerbeifügung. Addition of tensors
 Spannerdichte. Tensor density
 Spannerfeld. Tensor field
 Spannerprodukt. Product of tensors
 Spannersumme. Sum of tensors
 Spannervervielfältigung. Multiplication of tensors
 Spannweite. Span
 Spannung in einem Umkehrquadratfeld. Potential in inverse square field
 Spannungsbedienung. Potential operator
 Spannungsbeiwert. Coefficient of tension
 Spannungsfeld. Tension field
 Spannungskern. Potential core
 Spannungstemperatur. Potential temperature
 Speicherungseinheit. Storage unit
 Speicherungsprogrammrechner. Stored-program computer
 Spektralanhäufung. Spectral concentration
 Spektralaufnahmevermögen. Spectral absorptance
 Spektralaussendungsvermögen. Spectral emissivity
 Spektralebene des Zimmerlärms. Noise, spectrum level of room
 Spektraleigenschaft. Spectral characteristic
 Spektralenergieverteilung. Spectral energy distribution
 Spektralfunktion. Spectral function
 Spektralhalbmesser einer Matrize. Spectral radius of a matrix
 Spektralleuchtfaktor. Spectral luminance factor
 Spektralnorm. Spectral norm
 Spektralspiegelungsvermögen. Spectral reflectance
 Spektralübertragung. Spectral transmittance
 Spektroskopischer Spanner. Term, spectroscopic
 Spektrum. Spectrum
 Spektrumloкус. Spectrum locus
 Spencer-Fano-Methode. Spencer-Fano method
 Spenderbindung. Donor bond
 Sperrentspannung. Block relaxation
 Spezifische Akustikimpedanz. Impedance, specific acoustical
 Spezifische Brechung. Specific refraction
 Spezifische Dämpfungskapazität. Specific damping capacity
 Spezifischer Akustikwiderstand. Resistance, specific acoustic
 Spezifischer Wärmeverbrauch. Specific heat consumption
 Spezifisches Umdrehungsvermögen eines Mittels. Specific rotatory power of a medium
 Spezifische Tätigkeit. Specific activity
 Spezifische Umdrehung. Rotation, specific
 Spezifische Zerstreuung. Dispersivity, specific
 Spiegeldichte. Density, specular
 Spiegelerparnisse. Reflector savings
 Spiegelnückstrahlung. Specular reflection
 Spiegelungsebene. reflection plane
 Spiegelungswelle. Wave, reflected
 Spieltheorie. Games theory
 Spinabhängigkeitskraft. Spin-dependent force
 Spinnen. Spin
 Spinoidalkurve. Spinodal curve
 Spinorkalkulus. Spinor calculus
 Spinparamagnetismus der Leitungselektronen. Spin paramagnetism of conduction electrons
 Spitzenfaktor der Pulsträger. Pulse carrier, crest factor of
 Spur. Trace
 Stabsichtiger Unterschied. Astigmatic difference
 Staebble-Lihotzky-Bedingung. Staebble-Lihotzky condition
 Staffelung. Echelon
 Stampfmomentbeiwert. Pitching moment coefficient
 Standardbrechung. Standard refraction
 Standardablenkung. Standard deviation
 Standschallwelle(n). Sound wave(s), stationary
 Standstochastikvorgang. Stationary stochastic process
 Standwellen. Stationary waves
 Standwelle. Wave, standing
 Standwelleverhältnis. Standing-wave ratio
 Standzustände. Stationary states
 Stange. Bar
 Starke Achse. Axis, strong
 Starkeffekt. Stark effect
 Stärke einer Kugelschallwelle. Intensity of a spherical sound wave
 Stärke einer Spektrallinie. Intensity of a spectral line
 Stärke einer Teilchenquelle. Intensity of a source of particles
 Stärkeverteilungskurve. Intensity distribution, curve of
 Stärkeverteilungsoberfläche. Intensity distribution, surface of
 Statik. Statics
 Statischer Druck. Static pressure
 Statisch zulässig. Statically admissible
 Statistisch. Statistic
 Statistische Entropiemethode. Statistical method of entropies
 Statistische Gewichte der atomischen Energieebenen. Statistical weights of atomic energy levels
 Statistische Gewichte der Molekularenergieebenen. Statistical weights of molecular energy levels
 Statistische Hypothese. Hypothesis, statistical
 Statistische Mechanik. Statistical mechanics
 Statistische Qualitätskontrolle. Statistical quality control
 Statistischer Gewichtlehrsatz. Statistical weight theorem
 Statistisches Gewicht. Statistical weight
 Staudruck. Stagnation pressure
 Staupunkt. Stagnation point

- Stautemperatur. Stagnation temperature
 Steckerströmung. Plug flow
 Stefan-Boltzmann-Gesetz. Stefan-Boltzmann law
 Steifrahmen. Rigid frame
 Steifsphären. Rigid spheres
 Steifwinkel. Glancing angle
 Steifzylinder. Rigid cylinder
 Steifkörper. Rigid body
 Steigung. Gradient
 Stetigkeitsmodul. Modulus of continuity
 Steigungsniveau. Pitch level
 Stellungsanmerkung. Positional notation
 Stellungsvektor. Vector, position
 Steradian. Steradian
 Stereographische Kugeldarstellung auf einer Ebene
 Stereographic projection of a sphere on a plane
 Stereospektrogramm. Stereospectrogram
 Stetig. Constant
 Stetiges. Continuum
 Stichprobe. Sample
 Stiel. Strut
 Stieltjesintegral. Stieltjes integral
 Stigmatisch. Stigmatic
 Stilb. Stilb
 Stirling-Interpolationsformel. Stirling interpolation formula
 Stirlingnummern. Stirling numbers
 Stochastisch. Stochastic
 Stockmayerpotential. Stockmayer potential
 Stoffteilchen. Particle, material
 Stoichiometrikbeiwert. Stoichiometric coefficient
 Stokes-Gesetze. Stokes laws
 Stokes-Linien. Stokes lines
 Stokes-Polarisationslehrsatz. Stokes polarization theorem
 Stokes-Strömung. Stokes flow
 Störmer-Methode. Störmer method
 Störung. Interference
 Störungsnebenveränderlichen. Nuisance parameters
 Stoßbeiwert. Collision coefficient
 Stoßfrequenz. Collision frequency
 Stoßpotentialkurven in einem Molekül. Repulsive potential curves in a molecule
 Stoßtheorie der chemischen Kinetik. Collision theory of chemical kinetics
 Stoßüberweisung. Collisional transfer
 Stoßwahrscheinlichkeit. Collision probability
 Stoßzwischenraum. Collision interval
 Strahlenakustik. Ray acoustics
 Strahlennachspüren. Ray tracing
 Strahlentätigkeitsstärke. Intensity of radioactivity
 Strahler. Radiator
 Strahltriebwerk. Jet engine
 Strahlung. Radiance
 Strahlungsdichtekostante. Radiation density constant
 Strahlungsdruck. Radiation pressure
 Strahlungsemittierung. Radiant emittance
 Strahlungsfeld. Radiation field
 Strahlungslänge. Radiation length
 Strahlungsmuster. Radiation pattern
 Strahlungsstärke. Intensity of radiation
 Strahlungsverbesserung. Radiative correction
 Strahlungswiderstand. Radiation resistance
 Strahlungswirksamkeit. Radiant efficiency
 Strangvieleck. Funicular polygon
 Strehl-Definierung. Strehl definition
 Streifen. Fringe
 Streuung. Scattering
 Streuungsamplitude. Scattering amplitude
 Streuungsbeiwerte. Scattering coefficients
 Streuungsdarstellung. Scatter diagram
 Streuungsdruck. Spreading pressure
 Streuungsphasenschiebung. Scattering phase shift
 Streuungsverhältnisse. Dispersion relations
 Streuungsverlust. Scattering loss
 Streuungsvermögen. Dispersive power
 Streuungswinkel. Scattering angle
 Stromblatt. Current sheet
 Stromdichte. Current density
 Stromlinie. Streamline
 Strömungsfunktion. Stream function
 Strömungslinien. Lines of flow
 Strömungspunkt. Steam point
 Strömungsregel. Flow rule
 Strömungstheorie der Plastizität. Flow theory of plasticity
 Stromwahrscheinlichkeit. Current probability
 Studentenverteilung. Student's distribution
 Stufenfunktion. Step function
 Stufenwirksamkeit. Stage efficiency
 Stürmische Rohrströmung. Pipe, turbulent flow in
 Sturmlehrsatz. Sturm theorem
 Sturm-Liouville-Gleichung. Sturm-Liouville equation
 Sturm-Liouville-Problem. Sturm-Liouville problem
 Sturmreihenfolge. Sturm sequence
 Sublimierungskurve. Sublimation curve
 Summenübereinkunft. Summation convention
 Summenunveränderliche. Summational invariant
 Sumptners Prinzip. Sumptner's principle
 Sutherlandmodell. Sutherland model
 S-Wellen. S waves
 Symmetrieebene. Symmetry, plane of
 Symmetrische Funktion. Symmetric function
 Symmetrischer Spanner. Tensor, symmetric
 Symmetrische Wellenfunktion. Symmetric wave function
 Synklastikoberfläche. Synclastic surface
 Synthetische Einteilung. Synthetic division
 Synthetischer Kern. Kernel, synthetic
 Systematische Probenahme. Systematic sample
 Systemgleichungen. System equations
 S-Zustand. S-state
 Tafelfehlererkennung. Detection of tabular errors
 Talbot. Talbot
 Talbotgesetz. Talbot law
 Tamm-Dancoff-Methode. Tamm-Dancoff method
 Tangens an einer Kurve. Tangent to a curve
 Tangensbeschleunigung. Acceleration, tangential
 Tangensbrennpunkt. Tangential focus
 Tangensebene. Tangential plane
 Tangensebene an einer Oberfläche. Tangent plane to surface
 Tangenskraft. Force, tangential
 Tangensmodul. Modulus, tangent
 Tangentengesetz. Law of tangents
 Tantochron. Tantochrone
 Tätigkeit. Action
 Tätigkeitsprinzip. Action principle
 Tätigkeitsveränderliche. Action variable
 Taylorreihen. Taylor series
 Taylorspirale. Taylor spiral
 Teilbruchstück. Fraction, partial
 Teilbruchstückausdehnung. Partial fraction expansion
 Teilchenauslegung. Particle interpretation

- Teilchengeschwindigkeit. Velocity, particle
 Teilchengleichgewicht. Equilibrium of a particle
 Teildruck. Partial pressure
 Teildifferentialgleichung. Partial differential equation
 Teilknoten. Nodes, partial
 Teilmolargrößen. Partial molar quantities
 Teilung. Partition
 Teilungsfunktion. Partition function
 Teilungsfunktion der diatomischen Moleküle. Diatomic molecules, partition function of
 Teilfunktion der polyatomischen Moleküle. Polyatomic molecules, partition function of
 Teilungsfunktion des Umdrehers. Rotator, partition function of
 Teilverbreitung. Dispersion, partial
 Teilzahlgesetz für einen Spanner. Quotient law for tensor
 Teilzahlunterschiedsalgorithmus. Quotient-difference algorithm
 Teilzusammenhang. Partial coherence
 Telezentrisches System. Telecentric system
 Temperatur. Temperature
 Temperaturerholungsfaktor. Temperature recovery factor
 Temperatursteigung. Temperature gradient
 Tensor. Tensor
 Ternärsysteme. Ternary systems
 Tetrachlorische Korrelation. Tetrachloric correlation
 Tetradebenmaß. Symmetry, tetrad
 Tetragonalsystem. Tetragonal system
 Tetrahedrankristalle. Tetrahedral crystals
 Tetralitätsprinzip. Tetrality principle
 Thermodynamik der unumkehrbaren Vorgänge. Thermodynamics of irreversible processes
 Thermodynamikeigenschaft. Thermodynamic property
 Thermodynamikfunktionen der Oberflächenphasen. Thermodynamic functions of surface phases
 Thermodynamikgleichgewicht. Thermodynamic equilibrium
 Thermodynamikkräfte der unumkehrbaren Vorgänge. Forces in thermodynamics of irreversible processes
 Thermodynamikmischungsfunktionen. Thermodynamic functions of mixing
 Thermodynamikstetigkeitszustände. Thermodynamic stability conditions
 Thermodynamiksystem. Thermodynamic system
 Thermoelektrische Kraft. Thermoelectric power
 Thermomolekulardruck. Thermomolecular pressure
 Thermomolekulardrucksunterschied. Thermomolecular pressure difference
 Thermostatik. Thermostatitics
 Thermostatistik. Thermostatistics
 Thévenins akustischer Lehrsatz. Thévenin's acoustical theorem
 Thévenins elektrischer Lehrsatz. Thévenin's electrical theorem
 Thévenins mechanischer geradliniger Lehrsatz. Thévenin's mechanical rectilinear theorem
 Thévenins mechanischer Umdrehungslehrsatz. Thévenin's mechanical rotational theorem
 Thixotropie. Thixotropy
 Thomson-Streuung. Thomson scattering
 Tomonaga-Schrödinger-Gleichung. Tomonaga-Schrödinger equation
 Tomonaga-Schwinger-Gleichung. Tomonaga-Schwinger equation
 Topologie. Topology
 Topologische Gruppe. Topological group
 Topologischer Raum. Topological space
 Toroidalnebengeordneten. Toroidal coordinates
 Torsionmembranenanalgie. Membrane analogy, torsion
 Totaldruck. Total pressure
 Totalspiegelung. Total reflection
 Totlast. Dead load
 Tragen. Carry
 Trägerdichte. Carrier density
 Trägerkapazität. Carrying capacity (load)
 Tragfähige Düse. Jet, buoyant
 Tragflächensehne. Chord of airfoil
 Tragflächentheorie. Airfoil theory
 Tragflügelkaskade. Cascade of airfoils
 Tragflügelversagung. Stalling of an airfoil
 Trägheit. Inertance
 Trägheitskräfte. Inertia forces
 Trägheitsmoment. Moment of inertia
 Trägheitsrahmen. Inertial frame
 Traidebenmaß. Symmetry, traid
 Transponierung. Transposition
 Trapezregel. Trapezoidal rule
 Trennbar. Separable
 Trennbare Darstellung. Graph, separable
 Trennung der Veränderlichen. Separation of variables
 Tresca-Ergiebigkeitsbedingung. Tresca's yield condition
 Trockendampf. Dry vapor
 Tröpfchenverdichtung. Drop-wise condensation
 Überaustausch. Superexchange
 Überauswahlregeln. Superselection rules
 Überdeckungsintegral. Overlap integral
 Übereinstimmend. Congruent
 Überentspannung. Overrelaxation
 Überführungsgleichung. Transport equation
 Überführungs-mittlere-freie-Weglänge. Transport mean free path
 Überführungszahl. Transport number
 Überfülle. Redundancy
 Überfüllezwang. Redundant constraint
 Überfunktionen. Excess functions
 Übergangsverlust. Transition loss
 Übergangswahrscheinlichkeit. Transition probability
 Übergitter. Superlattice
 Überlagerungslehrsatz. Superposition theorem
 Überlagerungsprinzip. Superposition principle
 Überleitungsenergiegleichung. Superconductivity energy equation
 Überleitungsübergang. Superconducting transition
 Übermultipllett. Supermultiplet
 Überquerungsentwurf. Cross-over design
 Überragend. Transcendental
 Überrest. Remainder
 Überrestformeln. Remainder formulas
 Überschallähnlichkeitsgesetz. Hypersonic similarity law
 Überschallströmung. Hypersonic flow
 Überschallströmungskegel. Cone in supersonic flow
 Übersetzung. Translation
 Übersetzungsgruppe. Translation group
 Übersetzungstätigkeit. Translation operation
 Überspanntheit. Exaltation
 Überstrukturlinien. Superstructure lines
 Übertragbare Größe. Transferable quantity
 Übertragung. Transmission
 Übertragungsebene. Transmission plane
 Übertragungsfähigkeit. Transmittance

- Übertragungsfaktor. Carryover factor
 Übertragungsfunktion. Transfer function
 Übertragungskern. Kernel, transport
 Übertragungslehrsatz zum Trägheitsmoment. Moment of inertia, transfer theorem for
 Übertragungslinie. Transmission line
 Übertragungsmatrize. Transmission matrix
 Übertragungsvermögen. Transmissivity
 Überwellenströmung. Super-undal flow
 Uhrparadoxie. Clock paradox
 Ultrarote Rationation. Infrared radiation
 Umdrehung. Rotation
 Umdrehungsenergieebene eines Moleküls. Rotational energy levels of a molecule
 Umdrehungsgruppe. Rotation group
 Umdrehungskapazität. Rotation capacity
 Umdrehungskernspektra. Rotational spectra in nuclei
 Umdrehungskonstanten eines Moleküls. Rotational constants of a molecule
 Umdrehungskoordinatensystem. Rotating coordinate system
 Umdrehungsoberfläche. Surface of revolution
 Umdrehungsachse. Rotation axis
 Umdrehungsschwingungsspektren der Moleküle. Rotation vibration spectra of molecules
 Umdrehungsspektren der Moleküle. Rotation spectra of molecules
 Umdrehungs-Spiegelungs-Achse. Rotation-reflection axis
 Umdrehungssummenregeln. Rotational sum rules
 Umdrehungsteilungsfunktion. Rotational partition function
 Umdrehungsübereinstimmung. Rotational compliance
 Umdrehungs-Umkehrungs-Achse. Rotation-inversion axis
 Umdrehungsverbreitung. Dispersion of rotation
 Umdrehungsverhältnis. Conversion ratio
 Umfang. Dimension
 Umfangreiche Veränderlichen. Extensive variables
 Umfangsbetonung. Stress, circumferential
 Umfangseinwilligung. Compliance, bulk
 Umgekehrter Rankine-Kreislauf. Reversed Rankine cycle
 Umgerechneter Endpunkt. Extrapolated end-point
 Umhüllung. Envelope
 Umkehrstoß. Inverse collision
 Umkehrbare und unumkehrbare Vorgänge. Reversible and irreversible processes
 Umkehrbares Pendel. Pendulum, reversible
 Umkehrfunktion. Inverse function
 Umkehrinterpolation. Inverse interpolation
 Umkehrmatrize. Inverse matrix
 Umkehrerflächen. Inverse surfaces
 Umkehrpunkt. Center of inversion
 Umkehrstundformel. Inverse hour formula
 Umkehrtreibungsmittel. Inverse operator
 Umkehrung. Inversion
 Umkehrungstemperatur. Inversion temperature
 Umkehrungsverdoppelung. Inversion doubling
 Umklappvorgänge. Umklapp processes
 Umrechnungswert der Atommassen. Atomic mass conversion factor
 Umrechnungszunahme. Conversion gain
 Umschalter. Commutator
 Umschaltungsbeziehungen. Commutation relations
 Umschaltungsregeln. Commutation rules
 Umwandelter Abschnitt. Transformed section
 Umwandeln. Transform
 Umwandlung. Transformation
 Umwandlungsgesetze für Feldstärken. Transformation laws for field strengths
 Unabhängig. Independent
 Unabhängige Bestandteile. Independent components
 Unabhängige Rückwirkungen. Independent reactions
 Unabhängigkeit. Independence
 Unausgeglichene Wärme. Uncompensated heat
 Unbestimmbare Form. Indeterminate form
 Unbestimmbarkeitsprinzip. Indeterminacy principle
 Unbestimmtes Gefüge. Indeterminate structure
 Unbestimmungsbeiwertemethode. Undetermined coefficients, method of
 Unechte Normalschwingungen der polyatomischen Moleküle. Non-genuine normal vibrations of polyatomic molecules
 Unechter Bruchteil. Improper fraction
 Uneinheitliche Lorentzumschaltung. Inhomogeneous Lorentz transformation
 Unelastischer Aufprall. Impact, inelastic
 Unendliche Darstellung. Graph, infinite
 Unendlicher Gierflügel. Infinite yawed wing
 Unendliches Aufnahmegerät. Infinite absorber model
 Unendliches Kegelhorn. Horn, infinite conical
 Unendliches Parabelhorn. Horn, infinite parabolic
 Unendliches Potzhorn. Horn, infinite exponential
 Unendliches Zylinderhorn (unendliches Rohr). Horn, infinite cylindrical (infinite pipe)
 Unendliche Vervielfältigung. Multiplication, infinite
 Unendlichkeit. Infinity
 Unermeßbar. Incommensurable
 Ungeeignetes Integral. Improper integral
 Ungerade Funktion. Odd function
 Ungerade-gerade Regel der Kernstetigkeit. Odd-even rule of nuclear stability
 Ungerades Atomglied. Odd term of an atom
 Ungleichförmigkeitsfaktor oder Anisotropiefaktor. Dissymmetry factor or anisotropy factor
 Ungleichheitenmethode. Inequalities, method of
 Unharmonische Glieder. Anharmonic terms
 Unharmonischer Oszillator. Oscillator, anharmonic
 Universalfermizwischenwirkung. Universal fermi interaction
 Unklebrige Flüssigkeit. Inviscid fluid
 Unkupplungsphänomene in der Spektroskopie. Uncoupling phenomena in spectroscopy
 Unmittelbare Betonung. Stress, direct
 Unmittelbarer Anstoß. Direct collision
 Unmittelbare Übertragung. Direct transmittance
 Unmittelbare Wechselwirkung. Direct interaction
 Unnützbare Gleichgewichte. Frozen equilibrium
 Unschärfe Übertragung. Diffuse transmittance
 Unstetiges Gleichgewicht. Equilibrium, unstable
 Unstetigkeit. Discontinuity
 Unstetigkeitssysteme. Discontinuous systems
 Unsymmetrische Biegung. Unsymmetrical bending
 Unsystematische Veränderliche. Random variable
 Unterdarstellung. Subgraph
 Unterdarstellungskomplement. Subgraph complement
 Untergruppe. Subgroup
 Unterharmonisch. Subharmonic
 Unterordnung. Subroutine
 Unterschall- und Überschall-Strömung. Subsonic and supersonic flow
 Unterschiedsgleichung. Difference equation
 Unterschiedstäter. Difference operators
 Unterstützungssiedlung. Settlement of supports

- Untrennbare Darstellung. Graph, non-separable
 Unveränderliche. Invariant
 Unvollkommenes Gas. Imperfect gas
 Unvollständige Betafunktion. Incomplete beta function
 Unvollständige Gammafunktion. Incomplete gamma function
 Unvollständige Sperre. Incomplete block
 Unwesentlich. Extraneous
 Ursächlichkeitsprinzip. Causality principle
 Ursprünglich. Primitive
 Ursprüngliche Übersetzung. Primitive translation
- Van der Waals Gleichung. Van der Waals' equation
 Van der Waals Kräfte. Van der Waals' forces
 Van't Hoff Lehrsatz. Van't Hoff theorem
 Varignon-Lehrsatz. Varignon theorem
 Vektor. Vector
 Vektorabkömmling. Vector derivative
 Vektorbedienug. Vector operator
 Vektorbestandteil. Component of a vector
 Vektordifferentialgleichheiten. Vector differential identities
 Vektorenbeschaffenheit. Composition of vectors
 Vektorenfeld. Vector field
 Vektorenfluß. Vector flux
 Vektorengbiet. Vector area
 Vektorenraum. Vector space
 Vektorensinn. Sense of a vector
 Vektorenerfall. Decomposition of a vector
 Vektorenzusatz. Vector addition
 Vektorenpotential. Vector potential
 Vektorvermehrung. Vector multiplication
 Vektorenwölbung. Vector curvature
 Vena contracta. Vena contracta
 Verallgemeinerte Beanspruchung. Strain, generalized
 Verallgemeinerte Betonung. Stress, generalized
 Verallgemeinerte Geschwindigkeiten. Velocities, generalized
 Verallgemeinerte Kraft. Force, generalized
 Verallgemeinerte Nebengeordneten und Momenta. Coordinates and momenta, generalized
 Veränderte Unterschiedsmethode. Variate difference method
 Veränderungenkalkulus. Variations, calculus of
 Veränderungsanalyse. Analysis of variance
 Veränderungsbeiwert. Coefficient of variation
 Veränderungsbeständigkeit. Stabilization of variance
 Veränderungsprinzipien für Nichtgleichgewichtszustände. Variational principles for non-equilibrium states
 Veränderungsprinzip und Bindungsenergien. Variation principle and bond energies
 Verbindung. Combination
 Verbindungsunterschiede. Combination differences
 Verborgene Koordinate. Hidden coordinate
 Verbotener Übergang. Forbidden transition
 Verbundbetonung. Stress, combined
 Verbundgitterträger. Truss, compound
 Verbundpendel. Pendulum, compound
 Verdetkonstante. Verdet constant
 Verde-und-Wick-Methode. Method of Verde and Wick
 Verdichtbarkeit. Compressibility
 Verdichter. Compressor, condenser
 Verdichtungsbeiwert. Condensation, coefficient of
 Verdichtungsraum. Clearance volume
 Verdichtungssystem. Condensed system
- Verdrehung. Torsion
 Verdrehungshalbmesser. Torsion, radius of
 Verdrehungs- oder Verzwirnungsknicken. Buckling, torsional or twist
 Verdrehungspendel. Pendulum, torsion
 Verdrehungswellen in einem Stab. Torsional waves in a rod
 Verdünnte Gasdynamik. Rarefied gas dynamics
 Verdünnungswärme. Heat of dilution
 Vereinigung. Association
 Vereinigungsgrundsatz. Combination principle
 Verfestigung. Work hardening
 Verformungsperiode. Period of deformation
 Verfrachtung. Entrainment
 Verfügbare Energie. Available energy
 Vergeudung. Dissipation
 Vergeudungsfunktion. Dissipation function
 Vergeudungskraft. Dissipative force
 Vergleiche der Bewegungsdynamik. Dynamical analogies, mobility
 Vergleichsfolgerung. Fiducial inference
 Vergleichslehrsatz zur Konvergenz. Comparison theorem for convergence
 Vergrößerung. Magnification
 Vergrößerungsmethoden. Enlargement, methods of
 Vergrößerungsprojektorverhältnis. Magnification ratio of a projector
 Vergrößerungsvermögen. Magnifying power
 Verhältnis. Ratio
 Verhältnisse der unumkehrbaren Vorgänge. Rates of irreversible processes
 Verhältnisgrenze. Limit, proportional
 Verhältnisladung. Proportional loading
 Verkuppelte chemische Gegenwirkungen. Coupled chemical reactions
 Verlängertes Sinusverhältnis. Extended sine relationship
 Verlängerung oder Ausdehnung. Elongation or extension
 Verlangsamungsdichte. Slowing-down density
 Verlangsamungsgebiet. Slowing-down area
 Verlangsamungslänge. Slowing-down length
 Verlangsamungszeit. Slowing-down time
 Verlorene Kraft. Force, "lost"
 Verlusteinwilligung. Compliance, loss
 Verlustfunktion. Loss function
 Verlustkennzahl. Modulus, loss
 Verlusttangens. Tangent, loss
 Vermehrungskonstante. Propagation constant
 Vermengung. Confounding
 Vernehmbarkeitsschwelle. Threshold of audibility
 Vernichtungstäter. Destruction operator
 Vernichtungswirkender. Annihilation operator
 Verschiebung. Displacement
 Verschuß. Closure
 Verschobene Grenzannäherung. Deferred approach to the limit
 Versor. Versor
 Verstärker. Amplifier
 Verstoß gegen die Sinusbedingung. Offense against the sine condition
 Versuchstreuungskurve. Scattering curve, experimental
 Vertauschung. Permutation
 Vertauschungsgesetz. Commutative law
 Vertauschungsgruppe. Permutation group
 Vertauschungssymbol. Permutation symbol
 Verteilte Kraft. Force, distributed
 Verteilung. Distribution

- Verteilung der Relativspektralenergie. Relative spectral energy distribution
- Verteilungsbeiwerte (optische). Distribution coefficients (optical)
- Verteilungsfaktor. Distribution factor
- Verteilungsfreie Folgerung. Distribution-free inference
- Verteilungsgesetz. Distributive law
- Verteilungstheorie. Distribution theory
- Verteilungsmoment. Moment of a distribution
- Vertrauensgrenzen. Confidence limits
- Vertretung. Representation
- Vertretungen der uneinheitlichen Lorentzgruppe. Representations of the inhomogeneous Lorentz group
- Vervielfältig. Factorial
- Vervielfältigkeit. Multiplicity
- Vervielfältigkeitswechselung. Multiplicities, alternation of
- Vervielfältigungsexperiment. Factorial experiment
- Verweisungsrahmen. Frame of reference
- Verwirbelte Düse. Jet, turbulent
- Verwirbelung. Turbulence
- Verwirbelung in Windtunneln. Turbulence in wind tunnels
- Verwirbelungsähnlichkeit. Turbulence, similarity of
- Verwirbelungsenergie. Turbulence, energy of
- Verwirbelungsgrenzschicht. Turbulent boundary layer
- Verwirbelungskonvektion. Turbulence convection
- Verwirbelungsmikroskala. Turbulence, microscale of
- Verwirbelungsspektrum. Spectrum of turbulence
- Verwirbelungsübertragungsbeiwerte. Turbulent transfer coefficients
- Verwirbelungszerfall. Turbulence, decay of
- Verwirklichung. Realization
- Verzehrers Risiko. Consumer's risk
- Verzerrung. Distortion
- Verzerrungsenergie. Distortion energy
- Verzierungsdarstellung. Vignetting diagram
- Verzögerte Elastizität. Elasticity, delayed
- Verzögerungslinie. Delay line
- Verzögerungszeit. Delay time
- Verzweigungsbruchstück. Branching fraction
- Verzweigungspunkt. Branch point
- V-Gruppe. Dihedral group
- Vieleck. Polygon
- Vierendeelgitterträger. Vierendeel truss or girder
- Vierfaktorenformel. Four-factor formula
- Vielflächner. Polyhedron
- Vierkraft. Four-force
- Viermomentengleichung. Four-moment equation
- Vierpol. Quadrupole
- Vierpolmoment. Quadrupole moment
- Vierpolstrahlung. Quadrupole radiation
- Viertelsehnenpunkt. Quarter-chord point
- Viervektoren-und-Spanner. Four-vectors and tensors
- Vieth-Müller-Kreis. Vieth-Müller circle
- Vieth-Müller-Ringwulst. Vieth-Müller torus
- Virial. Virial
- Virialbeiwert. Virial coefficient
- Virial eines Systems. Virial of a system
- Virialzustandsgleichung. Virial equation of state
- Viskoelastischer Bediener. Viscoelastic operator
- Viskoelastische Wellen. Waves, viscoelastic
- Voigtmaterial. Voigt material
- Voigtmodell. Voigt model
- Volldarstellung. Graph, complete
- Vollkommene Lösungen. Perfect solutions
- Vollkommene Optiksysteine. Perfect optical systems
- Volllineargruppe. Full linear group
- Vollständig. Complete
- Volterraequation. Volterra equation
- Volumenbruchteile. Volume fractions
- Volumengeschwindigkeit. Volume velocity
- Voraussager. Predictor
- Vorbeanspruchter Beton. Prestressed concrete
- Vorbehaltene einheitliche Lorentzumlagerungen. Restricted homogeneous Lorentz transformations
- Vorbeifließender Stumpfkörper. Bluff body, flow past
- Vorbeifließender Umdrehungszylinder. Rotating cylinder, flow past
- Vorentmischung. Predissociation
- Vorherrschende Wellenlänge. Dominant wavelength
- Vorratsmodul. Modulus, storage
- Vorrechtsrichtungen. Privileged directions
- Vousoir-Gewölbe. Voussoir arch
- V-Winkel. Dihedral angle
- Waagerechtscherung. Shear, horizontal
- Wache. Wake
- Wachstumskurve. Growth curve
- Wagner-Balken. Wagner beam
- Wahllehre. Alternative theorem
- Wahllosigkeit. Randomness
- Wahrscheinlichkeit. Probability
- Wahrscheinlichkeitsbeiwert. Coefficient of probability
- Wahrscheinlichkeitsdichte. Probability density
- Wahrscheinlichkeitskennziffer. Index of probability
- Wahrscheinlichkeitsprobenahme. Probability sampling
- Wahrscheinlichkeitsverteilung. Probability distribution
- Waidner-Burgess-Standard. Waidner-Burgess standard
- Wald. Forest
- Wallisformel. Wallis formula
- Wallisprodukt. Wallis product
- Wandenergie. Wall energy
- Wandverluste. Wall losses
- Wärmeaufnahme rückwirkung. Endothermic reaction
- Wärmeausdehnung. Thermal expansion
- Wärmebeständigkeit. Thermal stability
- Wärmebewegungen in einem Gitter. Thermal motions in a lattice
- Wärmebeiwerte. Thermal coefficients
- Wärmeinhalt. Enthalpy
- Wärmeinhaltslehre. Enthalpy theorem
- Wärmeeinheiten. Thermal units
- Wärmegebrauch. Thermal utilization
- Wärme Gleichgewicht. Thermal equilibrium
- Wärmeerschütterung. Thermal shock
- Wärmeton. Heat tone
- Wärme Gleichgewichtszustand. Caloric equation of state
- Wärmepotential. Thermal potential
- Wärmestrahlung. Thermal radiation
- Wärmeundurchlässige Lösungen. Athermal solutions
- Wärmewirksamkeit eines Kreislaufes. Thermal efficiency of cycle
- Wasserstoffbindeglied. Hydrogen bond
- Wasserstoffeingefüge. Hydrogen fine structure
- Wattspektrum. Watt spectrum
- Weber. Coulomb, Weber
- Weberentartung. Coulomb degeneracy
- Weberenergie. Coulomb energy
- Weberfeld. Coulomb field
- Webergesetz. Coulomb law
- Weberintegral. Coulomb integral
- Weberpotential. Coulomb potential
- Weberschranke. Coulomb barrier
- Weberwellenfunktion. Coulomb wave function

- Wechselnd. Alternating
 Wechselseitigkeitsbeziehungen. Reciprocity relations
 Wechselseitigkeitsgeschwindigkeitsgebiet. Reciprocal velocity region
 Wechselseitigkeitslehrsatz. Reciprocity theorem
 Wechselseitigkeitsvektorensystem. Vector system, reciprocal
 Wechselseitigkeitsvektorensystem. Reciprocal vector system
 Wechselspanner. Tensor, alternating
 Wechselströme. Alternating currents
 Wechselstromkreise. Alternating current circuits
 Wechselwirkende Methoden zur Gleichungslösung. Iterative methods for solving equations
 Wechselwirkungsbild. Interaction picture
 Wechselwirkungsgestalt. Configuration, interaction
 Wechselwirkungskurve. Interaction curve
 Wechselwirkungsschnelleffekt. Fast effect, interaction
 Wechselwirkungsvertretung. Interaction representation
 Weddle-Regel. Weddle rule
 Weierstrass-Annäherungslehrsatz. Weierstrass approximation theorem
 Weierstrass M-Prüfung. Weierstrass M-test
 Weingartens Formeln. Weingarten's Formulas
 Weissenbergwirkung. Weissenberg effect
 Weiss'sche Gleichung. Weiss's equation
 Weitläufige Rückstrahlung. Diffuse reflection
 Welle. Shaft, wave
 Wellenamplitude. Wave amplitude
 Wellenfunktion. Wave function
 Wellengleichung. Wave equation
 Wellenlängekonstante. Wavelength constant
 Wellenleiter. Waveguide
 Wellenmechanik. Wave mechanics
 Wellennebenveränderliche. Wave parameter
 Wellennummer. Wave number
 Wellenperiode. Wave period
 Wellenphase. Phase of the wave
 Wellenspitze. Wave crest
 Wellenstand. Waves stationary
 Wellenstirn. Wave front
 Wellenstirnabirrungen. Wave front aberrations
 Wertigkeitsbindungsmethode. Valence bond method
 Wertigkeitskräfte in polyatomischen Molekülen. Valence forces in polyatomic molecules
 Wesentliche Eigenheit einer analytischen Funktion. Essential singularity of an analytic function
 Weyl-Gleichung. Weyl equation
 Whittaker Differentialgleichung. Whittaker differential equation
 Wichtigkeitsfunktion. Importance function
 Wick-Chandrasekhar-Methode. Wick-Chandrasekhar method
 Wick-Methode. Wick method
 Widerspruchsrekurrenz. Recurrence paradox
 Widerstand. Resistance
 Widerstandsbeiwert. Drag coefficient
 Widerstandsströmung. Resistance flow
 Wiederanhangung der Grenzschicht nach der Trennung. Reattachment of boundary layer after separation
 Wiederdurchzug. Recirculation
 Wiedererwärmungsfaktor. Reheat factor
 Wiedererzeugungswärme. Heat regeneration
 Wiederholbarkeit. Repeatability
 Wiederholungsmethode. Method of iteration
 Wiedernormalisierung der Masse. Renormalization of mass
 Wiederverbindungsbeiwert. Recombination, coefficient of
 Wiederverbindungsgeschwindigkeit. Recombination velocity
 Wiederverbindungsverhältnis. Recombination rate
 Wiengesetze. Wien laws
 Wightmanfunktionen. Wightman functions
 Wigner-Beiwert. Wigner coefficient
 Wigner-Kraft. Wigner force
 Wigner-Wilkins-Modell. Wigner-Wilkins model
 Wilcoxon's Prüfung. Wilcoxon's test
 Wilkins Modell. Wilkins model
 Wilks Anhaltspunkt. Wilks' criterion
 Williotdarstellung. Williot diagram
 Wind. Wind.
 Windladung. Wind load
 Windgleichungen. Wind equations
 Windungsintegral. Convolution integral
 Winkelbeschleunigung. Angular acceleration
 Winkelgeschwindigkeit. Angular velocity
 Winkelimpuls. Angular momentum
 Winkelkorrelation. Angular correlation
 Winkelmomentenerhaltung. Conservation of angular momentum
 Winkelneutronenschmelzfluß. Flux, neutron, angular
 Winkelverschiebung. Angular displacement
 Winkelveränderliche. Angle variable
 Winkelvergrößerung. Angular magnification
 Winkelverteilung. Angular distribution
 Winkler-Bach-Formel. Winkler-Bach formula
 Winzig. Infinitesimal
 Wirbel. Vortex
 Wirbelblatt. Vortex sheet
 Wirbellinie. Vortex line
 Wirbelpaar. Vortex pair
 Wirbelring. Vortex ring
 Wirbelstraße. Vortex street
 Wirbelströmungsübergang. Transition of turbulent flow
 Wirbelsubstanz. Substance of a vortex
 Wirbelung. Vorticity
 Wirksame Atomzahl. Atomic number, effective
 Wirksame Mechanomotivkraft. Effective mechanomotive force
 Wirksame Schallbandweite. Effective band width (acoustic)
 Wirksame Vervielfältigung. Multiplication, effective
 Wirksamkeit. Efficiency
 Wirksamkeitsverhältnis. Efficiency ratio
 Wirkung auf Hautreibungsrauheit. Roughness, effect on skin friction
 Wishart-Verteilung. Wishart distribution
 Wohlgerührte Flüssigkeit. Well-stirred fluid
 Wölbung. Curvature
 Wölbungskugel. Sphere of curvature
 Wölbungshalbmesser. Radius of curvature
 Wölbungspunkt. Curvature, center of
 Wort. Word
 Wronskian. Wronskian
 Wurfbahn. Trajectory
 Wurzel. Radix
 Wurzellokusanalyse. Root locus analysis
 Wurzelmittelquadrat. Root-mean-square
 Wurzelinbläser. Roots blower
 Wurzelquadratmethoden. Root-squaring methods
 X-Einheit. x-Unit

- Yang-Feldman-Formalismus und die S-Matrize. Yang-Feldman formalism and the S-matrix
 Yates-Korrigierung. Yates correction
 Young-Modul. Young's modulus
 Yvon-Methode. Yvon method

 Zähflüssigkeit. Viscosity
 Zahl. Number
 Zählbar. Countable
 Zahlenanalyse. Numerical analysis
 Zahlenmäßige Differenzierungsformeln. Differentiation formulas, numerical
 Zahlenmäßige Quadratur. Quadrature, numerical
 Zahlenöffnung. Numerical aperture
 Zahlenstetigkeit. Numerical stability
 Zaubersahlen. Magic numbers
 Zeemaneffekt. Zeeman effect
 Zeichenähnlichkeiten. singularities
 Zeichenquantisierung. Quantization of signals
 Zeitänderung der Entropieproduktion. Time variation of the entropy production
 Zeittemperaturverschiebung. Time-temperature shift
 Zeitumkehrung. Time reversal
 Zelle. Cell
 Zelleneinheit. Unit cell
 Zellmuster des Flüssigkeitszustands. Cell model of the liquid state
 Zensierend. Censoring
 Zentigradwärmeeinheit. Centigrade heat unit
 Zentigradskala. Centigrade scale
 Zentralkraft. Force, central
 Zentrumoberfläche. Surface of centers
 Zentralgrenze-Lehrsatz. Central limit theorem
 Zentralkräfte. Central forces
 Zentralquadrat. Central quadric
 Zentrierbeschleunigung. Centripetal acceleration
 Zentrifugalkraft. Centrifugal force
 Zerfallsbeiwert. Decay coefficient
 Zerhackter Effekt. Intermittency effect
 Zerstreuter Spiegelungsfaktor. Diffuse reflectance
 Zerstreuungsreihe. Diffuse series
 Zerstreuungsfaktor. Diffusion factor
 Zerstreuungslänge. Diffusion length
 Zerstreuungsstetigkeit. Diffusion stability
 Zersetzungsenergie. Dissociation energy
 Zerstreuungsindikatrix. Indicatrix of diffusion
 Zerstreuungskern. Kernel, diffusion
 Ziellose Sperrern. Randomized blocks
 Zimmerkonstante. Room constant
 Zitterbewegung. Zitterbewegung

 Zonenvergrößerung. Zonal magnification
 Zuchtgewinn. Breeding gain
 Zuchtverhältnis. Breeding ratio
 Zufälligkeit. Contingency
 Zugstange. Tie rod
 Zulässige Betonung. Stress, allowable
 Zunahmerand. Gain margin
 Zunahmeverhältnis. Gain ratio
 Zurückweisungspunkte. Point centers of repulsion
 Zusammendrückung. Compression
 Zusammenfluß. Confluence
 Zusammenhängende Strahlung. Coherent radiation
 Zusammenladung. Combined loading
 Zusammenziehung. Contraction
 Zusätzlichkeit. Complementarity
 Zusätzlichkeitsbedingung. Supplementary condition
 Zusatzvorgang. Additive process
 Zustandsgleichung. Equation of state
 Zustandsgröße. Parameter
 Zustandskontinuität. Continuity of state
 Zustandsvektor. State vector
 Zustrebekraft. Centripetal force
 Zuverlässigkeit. Reliability
 Zwei-Bestandteilgleichung der Neutrino. Two-component equation of the neutrino
 Zweiharmonische Gleichung. Biharmonic equation
 Zweiknoten. Binodals
 Zweiohrige Phasenwirkung. Binaural phase effect
 Zweireihige Korrelation. Biserial correlation
 Zweischrieb. Digraph
 Zweiter Brennpunkt. Second focal point
 Zweiter Grenzsatz. Second limit theorem
 Zweiter Grundsatz. Second fundamental theorem
 Zweite Rang-Übergangstemperatur. Second order transition temperature
 Zweites Gesetz der Thermodynamik. Second law of thermodynamics
 Zwischenfall. Incident
 Zwischenatompotential. Interatomic potential
 Zwischenschallgebietsregel. Transonic area rule
 Zwischenschallähnlichkeitsgesetz. Transonic similarity law
 Zwischenschallhodographgleichungen. Transonic hodograph equations
 Zwischenverbindungslien. Intercombination lines
 Zwischenviertelbereich. Interquartile range
 Zykloide. Cycloid
 Zyklomatische Nummer. Cyclomatic number
 Zyklonenbewegung. Cyclonic motion
 Zyklonenwirbeligkeit. Cyclonic vorticity
 Zyklostrophische Kraft. Cyclostrophic force
 Zylinderwelle. Wave, cylindrical

Spanish—English Index

- Aberración de la luz. Aberration of light
 Aberración esférica. Spherical aberration
 Aberración esférica lateral. Lateral spherical aberration
 Aberración esférica longitudinal. Longitudinal spherical aberration
 Aberraciones del frente de onda. Wave front aberrations
 Aberraciones de Seidel. Seidel aberrations
 Aberraciones de un sistema óptico. Aberrations of an optical system
 Abertura. Aperture
 Abertura circular. Circular aperture
 Abertura numérica. Numerical aperture
 Abertura rectangular. Rectangular aperture
 Absorción de energía radiante. Absorption of radiant energy
 Acción. Action
 Aceleración. Acceleration
 Aceleración absoluta. Absolute acceleration
 Aceleración angular. Angular acceleration
 Aceleración centripeta. Centripetal acceleration
 Aceleración complementaria. Coriolis acceleration
 Aceleración de la gravedad. Acceleration of gravity
 Aceleración geostrófica. Geostrophic acceleration
 Aceleración tangencial. Acceleration, tangential
 Acoplamiento. Coupling
 Acoplamiento de Russell-Saunders. Russell-Saunders coupling
 Acoplamiento en las estructuras atómica y molecular. Coupling, in atomic and molecular structure
 Acoplamiento gradiente. Gradient coupling
 Acoplamiento inductivo. Flux linkage
 Acoplamiento j, j . j, j coupling
 Acotado. Bounded
 Actividad absoluta. Absolute activity
 Actividad específica. Specific activity
 A cuadrado integrable. Integrable square
 Acumulación. Cluster
 Acústica rectilínea. Ray acoustics
 Adición de tensores. Addition of tensors
 Adición vectorial. Vector addition
 Adjunto de una matriz. Adjoint of a matrix
 Adjunto de un operador. Adjoint of an operator
 Afinidad electrónica. Electron affinity
 Agudeza de resonancia. Resonance, sharpness of
 Ajuste de curvas. Curve fitting
 Alabeo. Skewness
 Ala Elípticamente cargada. Elliptically-loaded wing
 Albedo. Albedo
 Álgebra. Algebra
 Álgebra de Boole. Boolean algebra
 Álgebra de grupo. Algebra of a group
 Algoritmo. Algorithm
 Algoritmo cociente-diferencia. Quotient-difference algorithm
 Algoritmo de Euclides. Euclidean algorithm
 Alisadura. Smoothing
 Alternación de multiplicidades. Multiplicities, alternation of
 Alternativo. Alternating
 Altura Efectiva de una antena. Effective height (antenna)
 Amortiguamiento. Damping
 Amortiguamiento magnetomecánico. Magneto-mechanical damping
 Amplitud de banda efectiva (acústica). Effective band width (acoustic)
 Amplitud de dispersión. Scattering amplitude
 Amplitud de impulso. Pulse amplitude
 Amplitud de la curvatura normal. Curvature, amplitude of normal
 Amplitud de onda. Wave amplitude
 Amplitud de oscilación. Amplitude of oscillation
 Análisis. Analysis
 Análisis de la covariancia. Analysis of covariance
 Análisis de las componentes. Component analysis
 Análisis de la variancia. Analysis of variance
 Análisis factorial. Factor analysis
 Análisis límite y diseño. Limit analysis and design
 Análisis numérico. Numerical analysis
 Análisis plástico. Plastic analysis
 Análisis sucesional. Sequential analysis
 Análisis tensorial. Tensor analysis
 Analizador diferencial. Differential analyzer
 Analogía de Reynolds. Reynolds analogy
 Analogía hidrodinámica. Hydrodynamical analogy
 Analogías dinámicas clásicas. Dynamical analogies, classical
 Analogías dinámicas de movilidad. Dynamical analogies, mobility
 Anarmonicidad. Anharmonicity
 Ancho de banda del impulso. Pulse bandwidth
 Ancho del nivel (de energía). Level width
 Anchura de las líneas de absorción por resonancia magnética. Magnetic resonance, line width
 Anchura Doppler. Doppler width
 Anchura efectiva. Width, effective
 Anchura natural de líneas. Natural line width
 Anchura práctica. Practical width
 Anelástico. Anelastic
 Ángulo de aberración. Aberration angle
 Ángulo de abertura. Aperture angle
 Ángulo de azimut principal. Angle of principal azimuth
 Ángulo de Brewster. Brewster's angle
 Ángulo de convergencia. Angle of convergence
 Ángulo de desfasamiento. Phase angle
 Ángulo de desviación. Angle of deviation
 Ángulo de desviación. Glancing angle
 Ángulo de difracción. Diffraction angle
 Ángulo de difracción. Angle of diffraction
 Ángulo de dispersión. Scattering angle
 Ángulo de enlace. Bond angle
 Ángulo de fricción. Friction, angle of
 Ángulo de Hall. Hall angle
 Ángulo de incidencia. Incidence, angle of

- Ángulo de incidencia. Angle of incidence
 Ángulo de incidencia principal. Angle of principal incidence
 Ángulo de mach. Mach angle
 Ángulo de proyección. Projection angle
 Ángulo de reflexión. Angle of reflection
 Ángulo de refracción. Angle of refraction
 Ángulo de reposo. Repose, angle of
 Ángulo diedro. Dihedral angle
 Ángulo mínimo de desviación. Deviation, minimum angle of
 Ángulos cristalinos. Crystal angles
 Ángulos de Russell. Russell angles
 Anholónimo. Non-holonomic
 Anillo. Ring
 Anillo de lie. Lie ring
 Anillo infinitesimal. Infinitesimal ring
 Anisotropía. Anisotropic
 Anticonmutación. Anticommutation
 Antiderivada de una función. Antiderivative of a function
 Antiferromagnetismo. Antiferromagnetism
 Antilogaritmo de un número. Antilogarithm of a number
 Antisimétrico. Antisymmetric
 Aplicación. Mapping
 Apocromático. Apochromatic
 Aproximación al punto de ensilladura. Minimax approximation
 Aproximación cuasi-química. Quasi-chemical approximation
 Aproximación de Born. Born approximation
 Aproximación de Goertzel-Greuling. Goertzel-Greuling approximation
 Aproximación de Hartree. Hartree approximation
 Aproximación de Kirkwood. Kirkwood's approximation
 Aproximación de la edad. Age approximation
 Aproximación en media. Approximation in the mean
 Aproximación P_n . P_n approximation
 Árbol. Tree
 Arco. Arch
 Arco Voussoir. Voussoir arch
 Area de difusión de neutrones. Diffusion area, neutron
 Area de migración de neutrones. Migration area for neutrons
 Area de retardación. Slowing-down area
 Area efectiva (antena). Effective area (antenna)
 Area vectorial. Vector area
 Arista. Edge
 Armadura. Truss
 Armadura. Framework
 Armadura compleja. Truss, complex
 Armadura compuesta. Truss, compound
 Armadura simple. Truss, simple
 Arrastre. Drag
 Ascenso y descenso de los índices de un tensor. Raising and lowering indices on a tensor
 Asíntota. Asymptote
 Asociación. Association
 Astigmatismo. Astigmatism
 Astigmatismo de una superficie. Astigmatism of a surface
 Atenuación. Attenuation
 Atérmico. Athermanous
 Audiograma. Audiogram
 Aumento. Magnification
 Aumento angular. Angular magnification
 Aumento axial. Axial magnification
 Aumento de fase. Extension in phase
 Aumento lateral. Lateral magnification
 Aumento lineal. Linear magnification
 Aumento longitudinal. Longitudinal magnification
 Aumento normal. Normal magnification
 Aumento radial. Radial magnification
 Autocorrelación. Autocorrelation
 Autoestado. Eigenstate
 Autofunción. Eigenfunction
 Autofunciones moleculares. Molecular eigenfunctions
 Automorfismo de un gráfico. Graph, automorphism
 Autoregresión. Autoregression
 Autovalor. Eigenvalue
 Autovalores. Eigenvalues
 Autovectores. Eigenvectors
 Avalúo de matrices S en teoría de campos cuantificados. S -matrix in quantized field theory: evaluation of
 Azimut. Azimuth angle

 Balanza. Balance
 Banda. Band
 Banda de conducción. Conduction band
 Banda de conducción degenerada. Degenerate conduction band
 Bandas fundamentales. Fundamental bands
 Barión. Baryon
 Baroclínico. Baroclinic
 Barotrópico. Barotropic
 Barn. Barn
 Barra. Bar
 Barrera de Coulomb. Coulomb barrier
 Barrera de potencial. Potential barrier
 Base de un sistema de números. Base of a system of numbers
 Biquinario. Biquinary
 Binario. Binary
 Binodales. Binodals
 Binormal. Binormal
 Bloque imcompleto. Incomplete block
 Bloques aleatorizados. Randomized blocks
 Bocina cilíndrica finita. Horn, finite cylindrical
 Bocina cilíndrica infinita (tubo infinito). Horn, infinite cylindrical (infinite pipe)
 Bocina cónica finita. Horn, finite conical
 Bocina cónica infinita. Horn, infinite conical
 Bocina exponencial finita. Horn, finite exponential
 Bocina exponencial infinita. Horn, infinite exponential
 Bocina parabólica infinita. Horn, infinite parabolic
 Borde de regresión. Edge of regression
 Bosón. Boson
 Braquistócrona. Brachistochrone
 Brillo. Luminance
 Bujía. Candle
 Bujía por metro cuadrado. Candle per square meter

 Cadena de Markov. Markov chain
 Calculador. Computer
 Calculador. Calculator
 Calculadora con programa acumulado. Stored-program computer
 Calculador de analogía. Analog computer
 Calculador digital. Digital computer
 Cálculo. Calculus
 Cálculo de espinores. Spinor calculus
 Cálculo de variaciones. Variations, calculus of

- Cálculo diferencial absoluto. Absolute differential calculus
- Cálculo operacional de heaviside. Heaviside operational calculus
- Caldera. Boiler
- Calor atómico de formación. Atomic heat of formation
- Calor de dilución. Heat of dilution
- Calor específico de gases. Specific heat of gases
- Calor específico electrónico. Electronic specific heat
- Calor nocompensado. Uncompensated heat
- Cambio adiabático. Adiabatic change
- Cambio de fase por dispersión. Scattering phase shift
- Camino. Path
- Camino dirigido. Path, directed
- Camino óptico. Optical path
- Campo auto-consistente. Self-consistent field
- Campo crítico. Critical field
- Campo de cavidad en un dieléctrico. Cavity field in a dielectric
- Campo de Coulomb. Coulomb field
- Campo de fuerzas. Field of force
- Campo de fuerzas conservativo. Force field, conservative
- Campo de Lorentz. Lorentz field
- Campo de radiación. Radiation field
- Campo de tensiones. Tension field
- Campo de una partícula en movimiento en el espacio. Field of moving charge in space
- Campo de visión. Field of view
- Campo eléctrico. Electric field
- Campo electromagnético. Electromagnetic field
- Campo electrostático. Electrostatic field
- Campo escalar. Scalar field
- Campos internos en dieléctricos. Internal fields in dielectrics
- Campo libre. Field, free
- Campo tensorial. Tensor field
- Campo tensorial cartesiano. Tensor field, cartesian
- Campo vectorial. Vector field
- Campo vectorial laminar. Lamellar vector field
- Canal de reacción. Reaction channel
- Canónico. Canonical
- Cantidad de movimiento angular. Angular momentum
- Cantidad de movimiento cinética. Kinetic momentum
- Cantidad de movimiento cristalina. Crystal momentum
- Cantidades molares parciales. Partial molar quantities
- Cantidad transferible. Transferable quantity
- Capa. Shell
- Capacidad de amortiguamiento específica. Specific damping capacity
- Capacidad de rotación. Rotation capacity
- Capacidad térmica configuracional. Configurational heat capacity
- Capacitancia acústica. Acoustical capacitance
- Capacitancia eléctrica. Electrical capacitance
- Capa límite. Boundary layer
- Capa límite laminar. Laminar boundary-layer
- Capa límite turbulenta. Turbulent boundary layer
- Capa semireductora. Half-thickness
- Capas cerradas. Closed shells
- Capas electrónicas en un átomo. Electron shells in an atom
- Capas monomoleculares localizadas. Localized monolayers
- Capilaridad. Capillarity
- Carácter. Character
- Carácter del enlace. Bond character
- Carácter del enlace iónico. Ionic bond character
- Característica. Characteristic
- Característica de Hamilton. Hamilton's characteristic
- Característica direccional. Directional characteristic
- Característica espectral. Spectral characteristic
- Características de trabajo. Operating characteristic
- Características de una superficie. Surface, characteristic of
- Cardinal. Cardinal
- Carga axial. Axial load
- Carga combinada. Combined loading
- Carga crítica. Critical load
- Carga de trabajo. Working load
- Carga dinámica. Dynamic loading
- Carga excéntrica. Eccentric loading
- Carga lateral. Lateral load
- Carga límite. Limit load
- Carga muerta. Dead load
- Carga proporcional. Proportional loading
- Carga radial. Radial loading
- Carga útil. Carrying capacity (load)
- Carnotización. Carnotization
- Cascada de planos aerodinámicos. Cascade of airfoils
- Casos de acoplamiento de Hund. Hund's coupling cases
- Catacústica. Catacaustic
- Catástrofe de polarizabilidad. Polarizability catastrophe
- Catenaria. Catenary
- Cáustica. Caustic
- Cavitación. Cavitation
- Cel das de convección de Benard. Bénard cells
- Célula elemental. Unit cell
- Centro. Center
- Centro aerodinámico. Aerodynamic center
- Centro de colineación. Center of collineation
- Centro de curvatura. Curvature, center of
- Centro de curvatura esférica. Spherical curvature, center of
- Centro de deslizamiento. Shear center
- Centro de Gravedad. Center of gravity
- Centro de inversión. Center of inversion
- Centro de masa. Center of mass
- Centro de momentos. Center of moments
- Centro de oscilación. Center of oscillation
- Centro de percusión. Percussion, center of
- Centro de presión. Center of pressure
- Centro de simetría. Symmetry, center of
- Centro de un grupo. Center of a group
- Centro elástico. elastic center
- Centroide de una figura geométrica. Centroid of a geometrical figure
- Centro instantáneo. Instantaneous center
- Centro óptico. Optical center
- Centros puntuales de repulsión. Point centers of repulsion
- Cerrado. Closed
- Chi-cuadrado. Chi-square
- Choque clástico. Impact, elastic
- Choque de ondas. Wave, shock
- Choque directo. Direct collision
- Choque inelástico. Impact, inelastic
- Choque inverso. Inverse collision
- Choque térmico. Thermal shock
- Chorro turbulento. Jet, turbulent
- Ciclo. Cycle
- Ciclo de Born-Haber. Born-Haber cycle
- Ciclo de Carnot. Carnot cycle
- Ciclo de polarización. Polarization cycle
- Ciclo de Rankine. Rankine cycle
- Ciclo de Rankine inverso. Reversed Rankine cycle

- Ciclo de tensión. Cycle of stress
 Cicloide. Cycloid
 Cilindro de adaptación. Slug
 Cilindro rígido. Rigid cylinder
 Cimicento elástico. Elastic foundation
 Cinemática. Kinematics
 Cinemáticamente admisible. Kinematically admissible
 Cinética. Kinetics
 Cinética del reactor. Kinetics, reactor
 Circuito. Circuit
 Circuito-C. C-circuit
 Circuito orientado. Circuit oriented
 Circuitos básicos. Circuits, fundamental
 Circulación. Circulation
 Círculo de aberración mínima. Least circle of aberration
 Círculo de confusión mínima. Circle of least confusion
 Círculo de curvatura. Curvatura, circle of
 Círculo de deformación de Mohr. Mohr's circle for strain
 Círculo de esfuerzo de Mohr. Mohr's circle for stress
 Círculo de inercia de Mohr. Mohr's circle for inertia
 Círculo de Vieth-Müller. Vieth-Müller circle
 Círculos comáticos. Comatic circles
 Circunferencia geodésica sobre una superficie. Geodesic circle on a surface
 Clase lateral. Coset
 Clases de simetría. Symmetry classes
 Clausius. Clausius
 Clausura. Closure
 Coeficiente Ångstrom. Ångstrom coefficient
 Coeficiente binomial. Binomial coefficient
 Coeficiente de absorción. Absorption coefficient
 Coeficiente de absorción espectral. Spectral absorptance
 Coeficiente de absorción sonora de superficies. Sound absorption coefficient of surfaces
 Coeficiente de acomodación. Accommodation coefficient
 Coeficiente de Auger. Auger coefficient
 Coeficiente de Callier. Callier coefficient
 Coeficiente de choque. Collision coefficient
 Coeficiente de condensación. Condensation, coefficient of
 Coeficiente de desintegración. Decay coefficient
 Coeficiente de determinación. Determination, coefficient of
 Coeficiente de dilatación (térmica). Coefficient of (thermal) expansion
 Coeficiente de emisión. Emission coefficient
 Coeficiente de emisión espectral. Spectral emissivity
 Coeficiente de emisión total. Total emissivity
 Coeficiente de extinción. Extinction coefficient
 Coeficiente de fijación. Restraint coefficient
 Coeficiente de fricción. Friction, coefficient of
 Coeficiente de indeterminación. Non-determination, coefficient of
 Coeficiente de presión. Pressure coefficient
 Coeficiente de probabilidad. Coefficient of probability
 Coeficiente de Racah. Racah coefficient
 Coeficiente de recombinación. Recombination, coefficient of
 Coeficiente de reflexión espectral. Spectral reflectance
 Coeficiente de reflexión sonora. Sound reflection coefficient
 Coeficiente de restitución. Restitution, coefficient of
 Coeficiente de retrodispersión. Back-scattering coefficient
 Coeficiente de tensión. Coefficient of tension
 Coeficiente de variación. Coefficient of variation
 Coeficiente estequiométrico. Stoichiometric coefficient
 Coeficiente luminoso. Luminous coefficient
 Coeficiente osmótico. Osmotic coefficient
 Coeficientes de dispersión. Scattering coefficients
 Coeficientes de distribución. Distribution coefficients (optical)
 Coeficientes de influencia. Influence coefficients
 Coeficientes de luminosidad. Luminosity coefficients
 Coeficientes de transferencia turbulenta. Turbulent transfer coefficients
 Coeficientes de Wigner. Wigner coefficient
 Coeficientes factoriales. Factorial coefficients
 Coeficientes térmicos. Thermal coefficients
 Coeficiente virial. Virial coefficient
 Código. Code
 Cofactor. Cofactor
 Coherencia parcial. Partial coherence
 Cojinete. Bearing
 Cologaritmo. Cologarithm
 Color. Color
 Color Planckiano. Planckian color
 Columna. Column
 Columna modal de una matriz. Modal column of a matrix
 Coma. Coma
 Combinación. Combination
 Compacto. Compact
 Complejo de un grupo. Complex of a group
 Complementaridad. Complementarity
 Complemento. Complement
 Complemento de un subgráfico. Subgraph complement
 Completo. Complete
 Componente de un vector. Component of a vector
 Componente de un Gráfico. Graph component
 Componentes independientes. Independent components
 Comportamiento de relajación. Relaxation behavior
 Composición de dos tensores. Composition of two tensors
 Composición de fuerzas. Forces, composition of
 Composición de vectores. Composition of vectors
 Compresibilidad. Compressibility
 Compresión. Compression
 Compresor. Compressor
 Concentración. Concentration
 Concentración espectral. Spectral concentration
 Condensación de Einstein. Einstein condensation
 Condensado. Condensate
 Condensador. Condenser
 Condición asintótica. Asymptotic condition
 Condición de borde. Edge condition
 Condición de Clausius. Clausius' condition
 Condición de contorno. Boundary condition
 Condición de estabilidad química. Chemical stability condition
 Condición de Herschel. Herschel's condition
 Condición de Hölder. Hölder condition
 Condición de Joukowski. Joukowski condition
 Condición de la frecuencia de Bohr. Bohr frequency condition
 Condición de Lipschitz. Lipschitz condition
 Condición de Lorentz. Lorentz condition
 Condición del seno de Abbe. Abbe sine condition
 Condición de Petzval. Petzval's condition
 Condición de rendimiento de Tresca. Tresca's yield condition
 Condición de Staebli-Lihotzky. Staebli-Lihotzky condition

- Condiciones de contorno de Neumann. Neumann boundary conditions
- Condiciones de estabilidad termodinámica. Thermodynamic stability conditions
- Condición suplementaria. Supplementary condition
- Conexión de un gráfico. Graph connectivity
- Conector exponencial. Exponential connector
- Conexión afin fundamental. Fundamental affine connection
- Conexo. Connected
- Configuración atómica. Configuration, atomic
- Configuración de interacción. Configuration, interaction
- Configuración electrónica. Configuration, electronic
- Configuración electrónica. Electron configuration
- Confluencia. Confluence
- Confundido. Confounding
- Congruencia de curvas. Congruence of curves
- Congruencia normal. Normal congruence
- Congruencia rectilínea. Rectilinear congruence
- Congruente. Congruent
- Cónica. Conic
- Conjugación de enlaces dobles. Conjugation of double bonds
- Conjugada Hermitiana de una matriz. Hermitian conjugate of a matrix
- Conjugados en un sistema óptico. Conjugates in an optical system
- Conjunto. Ensemble
- Conjunto canónico. Canonical ensemble
- Conjunto de corte. Cut set
- Conjunto de corte orientado. Cut set, oriented
- Conjunto de probabilidad fundamental. Fundamental probability set
- Conjunto derivado. Derived set
- Conjunto macrocanónico. Macrocanonical ensemble
- Conjunto microcanónico. Microcanonical ensemble
- Conjuntos de corte fundamentales. Cut sets, fundamental
- Conmutador. Commutator
- Conmutatividad local débil. Weak local commutativity
- Cono de fricción. Friction, cone of
- Cono de Mach. Mach cone
- Cono en flujo supersónico. Cone in supersonic flow
- Conos de visión. Cones, visual
- Conservación de la cantidad de movimiento. Conservation of momentum
- Conservación de la cantidad de movimiento angular. Conservation of angular momentum
- Conservación de la energía. Energy conservation
- Conservación de la energía. Conservation of energy
- Conservación de la masa. Conservation of mass
- Constante. Constant
- Constante ambiente. Room constant
- Constante crioscópica. Cryoscopic constant
- Constante de acoplamiento. Coupling constant
- Constante de apantallado. Screening constant
- Constante de Avogadro. Avogadro constant
- Constante de Boltzmann. Boltzmann constant
- Constante de densidad de radiación. Radiation density constant
- Constante de estructura fina de Sommerfeld. Sommerfeld's fine structure constant
- Constante de entropía. Entropy constant
- Constante de Euler. Euler constant
- Constante de Euler-Mascheroni. Euler-Mascheroni constant
- Constante de fase acústica. Acoustical phase constant
- Constante de Fermi. Fermi constant
- Constante de Gruneisen. Gruneisen constant
- Constante de Hubble. Hubble constant
- Constante de integración. Constant of integration
- Constante de Karman. Karman constant
- Constante de Kundt. Kundt constant
- Constante de Lamb. Lamb's constant
- Constante de longitud de onda. Wavelength constant
- Constante de los gases. Gas constant
- Constante del resorte. Spring constant
- Constante de Madelung. Madelung constant
- Constante de Planck. Planck constant
- Constante de propagación. Propagation constant
- Constante de propagación acústica. Acoustical propagation constant
- Constante de reticulado. Lattice constant
- Constante de Rydberg. Rydberg constant
- Constante de Verdet. Verdet constant
- Constante dieléctrica. Dielectric constant
- Constante electromagnética. Electromagnetic constant
- Constante fotoeléctrica. Photoelectric constant
- Constante química. Chemical constant
- Constantes elásticas. Elastic constants
- Constantes piezoeléctricas. Piezoelectric constants
- Constantes rotacionales de una molécula. Rotational constants of a molecule
- Consumo térmico específico. Specific heat consumption
- Continuante. Continuant
- Continuidad. Continuity
- Continuidad absoluta. Absolute continuity
- Continuidad de estado. Continuity of state
- Continuo. Continuum
- Contornos sonoros. Loudness contours
- Contracción. Deflation
- Contracción. Contraction
- Contracción de la vena fluida. Vena contracta
- Contraste. Contrast
- Control estadístico de la calidad. Statistical quality control
- Convección. Convection
- Convección turbulenta. Turbulence convection
- Convención de la suma. Summation convention
- Convergencia. Convergence
- Convergencia condicional. Conditional convergence
- Convolución. Convolution
- Coordenada. Coordinate
- Coordenadas bipolares. Bipolar coordinates
- Coordenadas canónicas. Canonical coordinates
- Coordenadas cilíndricas. Cylindrical coordinates
- Coordenadas cilíndrico parabólicas. Parabolic cylindrical coordinates
- Coordenadas cónicas. Conical coordinates
- Coordenadas cromáticas. Chromaticity coordinates
- Coordenadas curvilíneas ortogonales. Curvilinear orthogonal coordinates
- Coordenadas elipsoidales. Ellipsoidal coordinates
- Coordenadas elíptico-cilíndricas. Elliptic cylindrical coordinates
- Coordenadas esferoidales. Spheroidal coordinates
- Coordenadas geodésicas. Geodesic coordinates
- Coordenadas parabólicas. Parabolic coordinates
- Coordenadas paraboloidales. Paraboloidal coordinates
- Coordenadas polares. Polar coordinates
- Coordenadas polares esféricas. Spherical polar coordinates
- Coordenadas polares geodésicas. Geodesic polar coordinates
- Coordenadas rectangulares. Rectangular coordinates

- Coordenadas simétricas. Symmetry coordinates
 Coordenadas toroidales. Toroidal coordinates
 Coordenadas y momentos generalizados. Coordinates and momenta, generalized
 Corchete de Lagrange. Lagrange bracket
 Corchete de Poisson. Poisson bracket
 Corchetes de Gauss. Gaussian brackets
 Corona circular. Annulus
 Corrección. Correction
 Corrección al vacío. Correction to vacuum
 Corrección de Eucken. Eucken correction
 Corrección de radiación. Radiative correction
 Corrección de Rydberg. Rydberg correction
 Corrección de Yates. Yates correction
 Correcciones de Dancoff. Dancoff corrections
 Correcciones de Sheppard. Sheppard's corrections
 Corrección gamma. Gamma correction
 Corrector. Corrector
 Correlación. Correlation
 Correlación angular. Angular correlation
 Correlación biserial. Biserial correlation
 Correlación canónica. Canonical correlation
 Correlación desplazada. Lag correlation
 Correlación ordinal. Rank correlation
 Correlación tetracórica. Tetrachoric correlation
 Correlogramo. Correlogram
 Corriente de rectificación de contacto. Contact rectification current
 Corriente húmeda. Wet stream
 Corrientes de neutrones. Current, neutron
 Corrimiento de Lamb. Lamb shift
 Cortadura de Dedekind. Dedekind cut
 Corte. Shear
 Cosenos directores ópticos. Optical direction cosines
 Cota. Bound
 Covolumen. Covolume
 Covariancia. Covariance
 Cresta de Onda. Wave crest
 Cristalografía. Crystallography
 Cristalograma. Crystallogram
 Criterio de convergencia de Abel. Abel test for convergence
 Criterio de convergencia de Cauchy. Cauchy convergence test
 Criterio de convergencia de Cauchy mediante la integral impropia. Cauchy integral convergence test
 Criterio de D'Alembert. D'Alembert test
 Criterio de la serie de Lundquist. Lundquist's series criterion
 Criterio de mach. Mach criterion
 Criterio de resolución de Rayleigh. Rayleigh criterion for resolution
 Criterio de Wilks. Wilks' criterion
 Criterio *M* de Weierstrass. Weierstrass *M*-test
 Cuadrado latino. Latin square
 Cuadrados mínimos. Least squares
 Cuadratura numérica. Quadrature, numerical
 Cuádrica. Quadric surface
 Cuádrica central. Central quadric
 Cuadripolo. Quadrupole
 Cuadro. Bay
 Cuadro de entropía. Entropy chart
 Cuadro de Heisenberg. Heisenberg picture
 Cuadro de Mollier. Mollier chart
 Cuadro de Schrödinger. Schrödinger picture
 Cuadro rígido. Rigid frame
 Cuanta. Quantum
 Cuanta virtual. Virtual quantum
 Cuántico. Quantic
 Cuantificación. Quantization
 Cuantificación de campos. Field quantization
 Cuantificación de campos electromagnéticos. Quantization of electromagnetic field
 Cuantificación de Señales. Quantization of signals
 Cuantificación espacial. Space quantization
 Cuantificar. Quantize
 Cuantilo. Quantile
 Cuaternión. Quaternion
 Cuerda. Chord
 Cuerda aerodinámica media. Aerodynamic mean chord
 Cuerda de plano aerodinámico. Chord of airfoil
 Cuerpo. Field
 Cuerpo gris. Gray body
 Cuerpo negro. Black body
 Cuerpo rígido. Rigid body
 Cumulantes. Cumulants
 Curie. Curie
 Curtosis. Kurtosis
 Curva. Curve
 Curva alabeada. Twisted curve
 Curva analítica. Analytic curve
 Curva catenaria. Catenary curve on cable
 Curva Cuspidal. Spinodal curve
 Curva de crecimiento. Growth curve
 Curva de cristalización. Crystallization curve
 Curva de desviación. Deflection curve
 Curva de dispersión experimental. Scattering curve, experimental
 Curva de distribución de intensidades. Intensity distribution, curve of
 Curva de ebullición y curva de condensación. Boiling curve and condensation curve
 Curva de excitación. Excitation curve
 Curva de interacción. Interaction curve
 Curva de Jordan. Jordan curve
 Curva de Morse. Morse curve
 Curva de rendimiento de fisión. Fission yield curve
 Curva de sublimación. Sublimation curve
 Curvado. Bent
 Curva elástica. Elastic curve
 Curva espacial. Space curve
 Curva logística. Logistic curve
 Curvas de Bertrand. Bertrand curves
 Curvas de potencial repulsivo en una molécula. Repulsive potential curves in a molecule
 Curvas Sargent. Sargent curves
 Curva tensión-deformación. Stress-strain curve
 Curvatura. Curvature
 Curvatura de una curva en un punto. Curvature of a curve at a point
 Curvatura de una superficie. Curvature of a surface
 Curvatura de una viga. Curvature of beam
 Curvatura de un campo. Curvature of field
 Curvatura escalar. Scalar curvature
 Curvatura geodésica. Geodesic curvature
 Curvatura media. Curvature, mean normal
 Curvatura normal. Normal curvature
 Curvatura total de una lente. Curvature of lens, total
 D'Alembertiano. D'Alembertian
 Debilitamiento de turbulencia. Turbulence, decay of
 Decibel. Decibel
 Decimal. Decimal
 Decremento logarítmico. Logarithmic decrement
 Defecto de masa. Mass defect

- Defecto Frenkel. Frenkel defect
 Deformación. Deformation
 Deformación. Buckling
 Deformación generalizada. Strain, generalized
 Degeneración de Coulomb. Coulomb degeneracy
 Degeneración por intercambio. Exchange degeneracy
 Degradación de la energía. Degradation of energy
 Delineamiento de rayos. Ray tracing
 Delta de Kronecker. Kronecker delta
 Densidad. Density
 Densidad de carga de los núcleos. Charge density of nuclei
 Densidad de carga magnética. Magnetic charge density
 Densidad de conjunto. Ensemble density
 Densidad de corriente magnética. Magnetic current density
 Densidad de energía cinética ondulatoria. Wave, kinetic energy density
 Densidad de energía sonora. Sound-energy density
 Densidad de estados cuánticos en bandas de conducción. Density of quantum states in conduction band
 Densidad de impacto de neutrones. Collision density, neutron
 Densidad de Lagrange. Lagrange density
 Densidad de neutrones. Density, neutron
 Densidad de probabilidad. Probability density
 Densidad de retardación. Slowing-down density
 Densidad escalar. Scalar density
 Densidad especular. Density, specular
 Densidad luminosa. Density, luminous
 Densidad óptica. Optical density
 Densidad óptica externa. External optical density
 Densidad portadora. Carrier density
 Densidad tensorial. Tensor density
 Denso. Dense
 Derivada. Derivative
 Derivada covariante de un campo tensorial. Covariant derivative of tensor field
 Derivada de Helmholtz. Helmholtz derivative
 Derivada intrínseca de un campo tensorial. Intrinsic derivative of tensor field
 Derivada tensorial. Tensor derivative
 Derivada total. Total derivative
 Derivada vectorial. Vector derivative
 Desarrollable. Developable
 Desarrollable polar. Developable, polar
 Desarrollable rectificante. Developable, rectifying
 Desarrollo de Chebyshev. Chebyshev expansion
 Desarrollo de Laplace. Laplace expansion
 Desarrollo de Prandtl-Meyer. Prandtl-Meyer expansion
 Desarrollo en fracciones parciales. Partial fraction expansion
 Descomposición de un vector. Decomposition of a vector
 Desigualdad de Abel. Abel inequality
 Desigualdad de Bessel. Bessel's inequality
 Desigualdad de Bienayme-Chebyshev. Bienayme-Chebyshev inequality
 Desigualdad de Chebyshev. Chebyshev inequality
 Desigualdad de Clausius. Clausius' inequality
 Desigualdad de Hölder. Hölder inequality
 Desigualdad de Minkowski. Minkowski inequality
 Desigualdad de Schwartz. Schwartz inequality
 Desigualdades lineales. Linear inequalities
 Deslizamiento horizontal. Shear, horizontal
 Deslizamiento longitudinal. Shear, longitudinal
 Desplazamiento. Displacement
 Desmagnetización adiabática. Adiabatic demagnetization
 Desplazamiento angular. Angular displacement
 Desplazamiento de niveles en átomos hidrogenoides. Level displacements in hydrogen-like atoms
 Desplazamiento eléctrico en dieléctricos. Electrical displacement in dielectrics
 Desplazamiento paralelo de un vector. Parallel displacement of a vector
 Desviación. Deflection
 Desviación estándar. Standard deviation
 Desviación lateral. Lateral deflection
 Desviación media. Mean deviation
 Desviación total de un prisma. Prism, total deviation of
 Detección de errores tabulares. Detection of tabular errors
 Determinación total. Total determination
 Determinante. Determinant
 Determinante de Fredholm. Fredholm determinant
 Determinante de Gram. Gram determinant
 Determinante de Slater. Slater determinant
 Determinante secular. Secular determinant
 Diacústica. Diacoustic
 Diádicos. Dyadics
 Diagonal. Diagonal
 Diagrama cromático. Chromaticity diagram
 Diagrama de Boulvin. Boulvin diagram
 Diagrama de correlación. Correlation diagram
 Diagrama de Lozenge. Lozenge diagram
 Diagrama de momentos. Moment diagram
 Diagrama de nivel de energía. Energy level diagram
 Diagrama de Rousseau. Rousseau diagram
 Diagramas de flujo con señales. Signal flow diagrams
 Diagramas moleculares. Molecular diagrams
 Diagrama Williot. Williot diagram
 Dialítico. Dialytic
 Diámetro aparente. Apparent diameter
 Diamagnetismo. Diamagnetism
 Diatérmico. Diathermanous
 Dicroísmo circular. Circular dichroism
 Dígitos significativos. Digits, significant
 Digráfico. Digraph
 Diferencia astigmática. Astigmatic difference
 Diferenciación. Differentiation
 Diferenciación bajo el signo de integral. Differentiation under the integral sign
 Diferenciación covariante. Covariant differentiation
 Diferenciación logarítmica. Logarithmic differentiation
 Diferencia de presión termomolecular. Thermomolecular pressure difference
 Diferencia de retorno. Return difference
 Diferencial. Differential
 Diferencia media. Mean difference
 Diferencias de combinación. Combination differences
 Diferencias divididas. Divided differences
 Diferencias recíprocas. Reciprocal differences
 Difracción. Diffraction
 Difracción de neutrones. Diffraction of neutrons
 Difusión de la luz. Diffusion of light
 Difusión de sólidos. Diffusion of solids
 Difusor. Diffuser
 Dilatación. Dilatation
 Dimensión. Dimension
 Dimensiones reticulares. Lattice dimensions
 Dinámica. Dynamics
 Dinámica de una masa puntual libre. Dynamics of a free mass point
 Dinámica de un gas enrarecido. Rarefied gas dynamics

- Dioptria. Diopter
 Dipolo (eléctrico). Dipole (electrical)
 Direccional. Directional
 Dirección. Address
 Dirección. Direction
 Dirección de propagación. Direction of propagation
 Direcciones conjugadas. Conjugate directions
 Direcciones principales. Principal directions
 Direcciones privilegiadas. Privileged directions
 Directriz de una superficie reglada. Directrix of a ruled surface
 Disco de Rayleigh. Rayleigh disk
 Discontinuidad. Discontinuity
 Discriminante. Discriminant
 Diseño de experimentos. Design of experiments
 Diseño plástico. Plastic design
 Disimétrico. Symmetry, dyad
 Disipación. Dissipation
 Dislocaciones. Dislocations
 Dispersión. Scattering
 Dispersión. Dispersion
 Dispersión anómala. Anomalous dispersion
 Dispersión de Rayleigh. Rayleigh scattering
 Dispersión de rotación. Dispersion of rotation
 Dispersión de Thomson. Thomson scattering
 Dispersión lineal recíproca. Dispersion, reciprocal linear
 Dispersión parcial. Dispersion, partial
 Dispersión potencial. Potential scattering
 Dispersión rotatoria. Rotatory dispersion
 Dispersión sonora. Scattering, sound
 Dispersividad específica. Dispersivity, specific
 Dispersividad molar. Dispersivity, molar
 Dispersividad refractiva. Refractive dispersivity
 Distancia de Mahalanobis. Mahalanobis' distance
 Distancia focal posterior. Back focal length
 Distancia óptica. Optical distance
 Distribución. Distribution
 Distribución angular. Angular distribution
 Distribución arco seno. Arc-sine distribution
 Distribución beta. Beta distribution
 Distribución binomial. Binomial distribution
 Distribución de Cauchy. Cauchy distribution
 Distribución de energía espectral relativa. Relative spectral energy distribution
 Distribución de Fisher. Fisher's distribution
 Distribución de Gauss. Gaussian distribution
 Distribución de Pascal. Pascal distribution
 Distribución de Poisson. Poisson distribution
 Distribución de Porter-Thomas. Porter-Thomas distribution
 Distribución de probabilidad. Probability distribution
 Distribución de student. Student's distribution
 Distribución de Wishart. Wishart distribution
 Distribución en cantidad de movimiento. Distribution in momentum
 Distribución en energía. Distribution in energy
 Distribución en fase. Distribution in phase
 Distribuciones de Pearson. Pearson distributions
 Distribuciones no centrales. Non-central distributions
 Distribución espectral de energía. Spectral energy distribution
 Distribución exponencial. Exponential distribution
 Distribución gamma. Gamma distribution
 Distribución hipergeométrica. Hypergeometric distribution
 Distribución jotaforme. J-shaped distribution
 Distribución logarítmica. Logarithmic distribution
 Distribución de muestreo. Sampling distribution
 Distribución normal. Normal distribution
 Distribución rectangular. Rectangular distribution
 Distorsión. Distortion
 Distorsión geométrica. Geometric distortion
 Divergencia. Divergence
 División sintética. Synthetic division
 Doblete. Doublet
 Dominio. Domain
 Dual de un gráfico. Graph, dual
 Ductilidad. Ductility
 Duración del impulso. Pulse duration
 Dureza. Hardness
 Ebullición. Boiling
 Economía por uso de reflector. Reflector savings
 Ecuación. Equation
 Ecuación adjunta. Adjoint equation
 Ecuación a dos componentes del neutrino. Two-component equation of the neutrino
 Ecuación auxiliar. Auxiliary equation
 Ecuación biarmónica. Biharmonic equation
 Ecuación bicuadrática. Biquadratic equation
 Ecuación canónica del movimiento. Canonical equation of motion
 Ecuación característica. Characteristic equation
 Ecuación característica de una matriz. Characteristic equation of a matrix
 Ecuación crítica. Critical equation
 Ecuación de Abel. Abel equation
 Ecuación de Benedict-Webb-Rubin. Benedict-Webb-Rubin equation
 Ecuación de Bernoulli. Bernoulli equation
 Ecuación de Bethe-Salpeter. Bethe-Salpeter equation
 Ecuación de Blasius. Blasius equation
 Ecuación de Boltzmann-Planck. Boltzmann-Planck equation
 Ecuación de Born-Mayer. Born-Mayer equation
 Ecuación de Callendar. Callendar equation
 Ecuación de color. Color equation
 Ecuación de Clairaut. Clairaut equation
 Ecuación de Clausius-Clapeyron. Clausius-Clapeyron equation
 Ecuación de Clausius-Mossotti. Clausius-Mossotti equation
 Ecuación de continuidad. Equation of continuity
 Ecuación de Curie. Curie's equation
 Ecuación de Debye. Debye equation
 Ecuación de Debye-Hückel. Debye-Hückel equation
 Ecuación de diferencias. Difference equation
 Ecuación de Dirac. Dirac equation
 Ecuación de Drude. Drude equation
 Ecuación de Duhem-Margules. Duhem-margules equation
 Ecuación de energía del aeroplano. Airplane energy equation
 Ecuación de energía de superconductividad. Superconductivity energy equation
 Ecuación de energía para la capa límite. Energy equation for boundary layer
 Ecuación de Energía de un flujo adiabático estacionario. Energy equation for steady adiabatic flow
 Ecuación de estado. Equation of state
 Ecuación de estado adiabático. Adiabatic equation of state
 Ecuación de estado de Clausius. Clausius' equation of state

- Ecuación de estado de Mie-Grüneisen. Mie-Grüneisen equation of state
- Ecuación de estado, térmico. Caloric equation of state
- Ecuación de Euler. Euler equation
- Ecuación de Fokker-Planck. Fokker-Planck equation
- Ecuación de Franklin. Franklin equation
- Ecuación de Fredholm. Fredholm equation
- Ecuación de Gibbs-Duhén. Gibbs-Duhén equation
- Ecuación de Helmholtz. Helmholtz equation
- Ecuación de Hermite. Hermite equation
- Ecuación de Keyes-Smith-Gerry. Keyes-Smith-Gerry equation
- Ecuación de Kirkwood para la constante dieléctrica. Kirkwood's equation for the dielectric constant
- Ecuación de Klein-Gordon. Klein-Gordon equation
- Ecuación de Koch. Koch's equation
- Ecuación de la bocina. Horn equation
- Ecuación de la capacidad térmica de Debye. Debye heat capacity equation
- Ecuación de la capacidad térmica de Einstein. Einstein heat capacity equation
- Ecuación de la edad de Fermi. Age equation, Fermi
- Ecuación de Laguerre. Laguerre equation
- Ecuación de Lamé. Lamé equation
- Ecuación de Laplace. Laplace equation
- Ecuación de la presión de vapor. Vapor pressure equation
- Ecuación de Legendre. Legendre equation
- Ecuación del movimiento de Heisenberg. Heisenberg equation of motion
- Ecuación de Mathieu. Mathieu equation
- Ecuación de Mollier. Mollier equation
- Ecuación de momento cuadrimensional. Four-moment equation
- Ecuación de Newton. Newton's equation
- Ecuación de Nutting. Nutting's equation
- Ecuación de ondas. Wave equation
- Ecuación de Onsager de la constante dieléctrica. Onsager's equation for the dielectric constant
- Ecuación de Pauli-Weisskopf. Pauli-Weisskopf equation
- Ecuación de Pell. Pell's equation
- Ecuación de Poisson. Poisson equation
- Ecuación de autovalores de un tensor. Eigenvalue equation of tensor
- Ecuación de Rayleigh-Jeans. Rayleigh-Jeans equation
- Ecuación de Reynolds. Reynolds equation
- Ecuación de Riccati. Riccati equation
- Ecuación de Richardson-Dushman. Richardson-Dushman equation
- Ecuación de Riemann-Papperitz. Riemann-Papperitz equation
- Ecuación de Rydberg. Rydberg equation
- Ecuación de Sackur-Tetrode. Sackur-Tetrode equation
- Ecuación de Schrödinger. Schrödinger equation
- Ecuación de Sturm-Liouville. Sturm-Liouville equation
- Ecuación de Tomonaga-Schrödinger. Tomonaga-Schrödinger equation
- Ecuación de Tomonaga-Schwinger. Tomonaga-Schwinger equation
- Ecuación de transporte. Transport equation
- Ecuación de transporte de Boltzmann. Boltzmann transport equation
- Ecuación de Van Der Waals. Van der Waals' equation
- Ecuación de Volterra. Volterra equation
- Ecuación de Weiss. Weiss's equation
- Ecuación de Weyl. Weyl equation
- Ecuación diferencial. Differential equation
- Ecuación diferencial de Clausius. Clausius' differential equation
- Ecuación diferencial de Laplace. Laplace differential equation
- Ecuación diferencial de Whittaker. Whittaker differential equation
- Ecuación diferencial ordinaria. Ordinary differential equation
- Ecuación diferencial parcial. Partial differential equation
- Ecuación diferencial parcial de Hamilton-Jacobi. Hamilton-Jacobi partial differential equation
- Ecuación diferencial parcial elíptica. Elliptic partial differential equation
- Ecuación Eikonal. Eikonal equation
- Ecuación elíptica. Elliptic equation
- Ecuaciones algebraicas. Algebraic equations
- Ecuaciones canónicas de la óptica. Canonical equations of optics
- Ecuaciones de amplitud finita (acústica). Finite amplitude equations (acoustic)
- Ecuaciones de Burnett. Burnett equations
- Ecuaciones de cambio de Enskog-Maxwell. Enskog-Maxwell equations of change
- Ecuaciones de Cauchy-Reimann. Cauchy-Reimann equations
- Ecuaciones de Colineación de la óptica. Collineation equations of optics
- Ecuaciones de delineamiento de rayos paraxiales. Paraxial ray tracing equations
- Ecuaciones de Euler del movimiento. Euler equations of motion
- Ecuaciones de Euler-Lagrange. Euler-Lagrange equations
- Ecuaciones de Fresnel. Fresnel equations
- Ecuaciones de Gauss y Codazzi. Equations of Gauss and Codazzi
- Ecuaciones de Gibbs-Helmholtz. Gibbs-Helmholtz equations
- Ecuaciones de Karman-Friedrich. Karman-Friedrich's equations
- Ecuaciones de la física matemática. Mathematical physics, equations of
- Ecuaciones de las lentes de Gauss. Gaussian lens equations
- Ecuaciones de Laue. Laue equations
- Ecuaciones de lentes en óptica-electrónica. Electron-optical lens equation
- Ecuaciones del movimiento fluido. Equations of fluid motion
- Ecuaciones de Maxwell. Maxwell's equations
- Ecuaciones de movimiento. Motion, equations of
- Ecuaciones de Navier-Stokes. Navier-Stokes equations
- Ecuaciones de onda relativísticamente invariantes. Relativistically invariant wave equations
- Ecuaciones de Proca. Proca equations
- Ecuaciones de superconductividad de London. London superconductivity equations
- Ecuaciones de velocidad de cristalización. Crystallization rate equations
- Ecuaciones de viento. Wind equations
- Ecuaciones diofánticas. Diophantine equations
- Ecuaciones dinámicas del rayo luminoso. Dynamical equations of light rays
- Ecuaciones intrínsecas de una curva. Curve, intrinsic equations of
- Ecuaciones lineales. Linear equations

- Ecuaciones normales. Normal equations
 Ecuaciones simultáneas. Simultaneous equations
 Ecuación fotoeléctrica de Einstein. Einstein photoelectric equation
 Ecuación hiperbólica. Hyperbolic equation
 Ecuación hipergeométrica de Gauss. Gauss' hypergeometric equation
 Ecuación indicial. Indicial equation
 Ecuación integral. Integral equation
 Ecuación integral de transporte. Integral transport equation
 Ecuación parabólica. Parabolic equation
 Ecuación ponderomotriz. Ponderomotive equation
 Ecuación sonora impulsiva. Impulsive sound equation
 Ecuación virial de estado. Virial equation of state
 Edad de neutrones. Age, neutron
 Efecto Auger. Auger effect
 Efecto Bauschinger. Bauschinger effect
 Efecto binaural de fase. Binaural phase effect
 Efecto Compton. Compton effect
 Efecto Debye. Debye effect
 Efecto de deslizamiento. Slip effect
 Efecto de fisión rápida. Fast effect, interaction
 Efecto de inducción y reactividad. Inductive effect and reactivity
 Efecto de intermitencia. Intermittency effect
 Efecto del Esfuerzo normal. Normal stress effect
 Efecto de tamaño. Size effect
 Efecto de transmisión del ruido. Noise transmission effect
 Efecto Doppler en acústica. Doppler effect for sound
 Efecto elástico secundario. After effect, elastic
 Efecto Faraday. Faraday effect
 Efecto isotópico en espectro moleculares. Isotope effect in molecular spectra
 Efecto magnetocalórico. Magnetocaloric effect
 Efecto ondulatorio. Flutter
 Efecto Paschen-Back. Paschen-Back effect
 Efecto Raman. Raman effect
 Efecto Schottky. Schottky effect
 Efectos Doppler. Doppler effects
 Efectos galvanomagnético y termomagnético. Galvanomagnetic and thermomagnetic effects
 Efecto Slutsky-Yle. Slutsky-Yle effect
 Efecto Stark. Stark effect
 Efecto terrestre sobre un aeroplano. Ground effect on airplane
 Efecto Weissenberg. Weissenberg effect
 Efecto Zeeman. Zeeman effect
 Eficiencia luminosa. Luminous efficiency
 Eikonal. Eikonal
 Eje centroidal. Centroidal axis
 Eje débil. Axis, weak
 Eje del tornillo. Screw axis
 Eje de rotación. Rotation axis
 Eje de rotación-inversión. Rotation-inversion axis
 Eje de rotación-reflexión. Rotation-reflection axis
 Eje de simetría. Symmetry, axis of
 Eje elástico. Elastic axis
 Eje fuerte. Axis, strong
 Eje instantáneo de rotación. Instantaneous axis of rotation
 Eje neutro. Neutral axis
 Eje óptico. Optic axis
 Eje principal. Axis, principal
 Ejes cristalográficos. Crystallographic axes
 Ejes móviles. Moving axes
 Elastancia. Elastance
 Electricidad. Electricity
 Electrodinámica cuántica. Quantum electrodynamics
 Electrodinámica de cuerpos en movimientos de Minkowski. Minkowski's electrodynamics for moving bodies
 Electromagnetismo. Electromagnetism
 Electrones equivalentes. Equivalent electrons
 Electrones no-equivalentes. Non-equivalent electrons
 Electroneutralidad. Electroneutrality
 Electrón-volt. Electron-volt
 Elemental. Elementary
 Elemento. Element
 Elemento de montaje. Element, circuit
 Elemento de simetría. Symmetry element
 Elemento orientado. Element, oriented
 Elementos conjugados de un grupo. Conjugate elements of a group
 Elemento unidad. Unit element
 Elevador de tensión. Stress raiser
 Eliminación. Elimination
 Eliminación de Gauss. Gaussian elimination
 Elipse. Ellipse
 Elipse geodésica. Geodesic ellipse
 Elipsoide. Ellipsoid
 Elipsoide de inercia. Inertia ellipsoid
 Elipsoide de Poinso. Ellipsoid of Poinso
 Elipsoide de Poinso. Ellipsoid, momental
 El número e . e , the number
 Elongación ó extensión. Elongation or extension
 Emisor selectivo. Selective emitter
 Emitancia. Emittance
 Emitancia luminosa. Luminous emittance
 Emitancia radiante. Radiant emittance
 Empuje axial. Axial thrust
 Energía. Energy
 Energía al cero absoluto. Zero point energy
 Energía cinética. Energy, kinetic
 Energía cinética del movimiento irrotacional. Kinetic energy of irrotational motion
 Energía complementaria. Complementary energy
 Energía de activación. Activation energy
 Energía de anisotropía. Anisotropy energy
 Energía de cohesión. Energy, cohesion
 Energía de correlación. Correlation energy
 Energía de Coulomb. Coulomb energy
 Energía de Debye. Debye energy
 Energía de deformación elástica. Resilience
 Energía de deformación magnética. Magnetic strain energy
 Energía de desintegración alfa. Alpha disintegration energy
 Energía de desintegración beta. Beta disintegration energy
 Energía de disociación. Dissociation energy
 Energía de disociación de una molécula. Dissociation energy of a molecule
 Energía de distorsión. Distortion energy
 Energías de enlace. Bond energies
 Energía de excitación. Excitation energy
 Energía de Fermi. Fermi energy
 Energía de intercambio. Exchange energy
 Energía de ionización. Ionization energy
 Energía de Keesom. Keesom energy
 Energía de la unión atómica. Atomic bond energy
 Energía del borde de banda. Band edge energy
 Energía de pared. Wall energy
 Energía de reacción nuclear. Reaction energy, nuclear
 Energía de resonancia. Resonance energy

- Energía de resonancia. Delocalization energy
 Energía de Turbulencia. Turbulence, energy of
 Energía de umbral. Threshold energy
 Energía de unión. Binding energy
 Energía disponible. Available energy
 Energía electrónica (clásica). Electron energy (classical)
 Energía potencial. Energy, potential
 Energía propia de un dieléctrico. Self energy in a dielectric
 Energía reticular de cristales. Lattice energy of crystals
 Enfriamiento por difusión. Diffusion cooling
 Enlace coordinado. Coordinate bond
 Enlace dador. Donor bond
 Enlace de hidrógeno. Hydrogen bond
 Entalpía. Enthalpy
 Entropía. Entropy
 Entropía de un gas monoatómico. Entropy of monatomic gas
 Envolvente. Envelope
 Ecuaciones de Lagrange. Lagrange equations
 Equicontinuo(a). Equicontinuous
 Equilibrante. Equilibrant
 Equilibrio. Equilibrium
 Equilibrio de fuerzas en un cuerpo rígido. Equilibrium of forces on a rigid body
 Equilibrio de una partícula. Equilibrium of a particle
 Equilibrio dinámico. Equilibrium, dynamic
 Equilibrio inestable. Equilibrium, unstable
 Equilibrio neutro. Equilibrium, neutral
 Equilibrio químico. Chemical equilibrium
 Equilibrio térmico. Thermal equilibrium
 Equilibrio termodinámico. Thermodynamic equilibrium
 Equipartición de la energía. Equipartition of energy
 Equivalencia. Equivalence
 Equivalente mecánico de la luz. Mechanical equivalent of light
 Ergodicidad. Ergodicity
 Error. Error
 Error de truncamiento. Truncation error
 Error estándar. Standard error
 Error por redondeo. Rounding error, roundoff
 Error probable. Probable error
 Error relativo. Relative error
 Escala centígrada. Centigrade scale
 Escala gris. Gray scale
 Escala igualmente temperada. Scale, equally tempered
 Escala Pitagórica. Pythagorean scale
 Escalar. Scalar
 Escalar absoluto. Scalar, absolute
 Escalas Sísmicas. Earthquake scales
 Escaleno. Scalene
 Escalonada. Echelon
 Esfera de curvatura. Sphere of curvature
 Esfera de Riemann. Riemann sphere
 Esferas rígidas. Rigid spheres
 Esferómetro. Spherometer
 Esfuerzo de trabajo. Working stress
 Esfuerzo normal. Normal stress
 Eslabón. Link
 Espacio de Banach. Banach space
 Espacio de colores. Color space
 Espacio de configuración. Configuration space
 Espacio de Hilbert. Hilbert space
 Espacio de las fases. Phase space
 Espacio Euclidiano. Euclidean space
 Espacio gamma. Gamma space
 Espacio mu. Mu space
 Espacio tensorial. Tensor, space
 Espacio-tiempo plano. Flat space-time
 Espacio topológico. Topological space
 Espacio vectorial. Vector space
 Especies. Species
 Espectro. Spectrum
 Espectro atómico. Atomic spectra
 Espectro de fisión. Fission spectrum
 Espectro de neutrones. Spectrum, neutron
 Espectro de nivel de presión sonora. Sound pressure spectrum level
 Espectro de potencia. Power spectrum
 Espectro de rayos X característicos. X-ray spectrum, characteristic
 Espectro de rotación de moléculas. Rotation spectra of molecules
 Espectro de turbulencia. Spectrum of turbulence
 Espectro de vibración de moléculas. Vibrational spectra of molecules
 Espectro de Watt. Watt spectrum
 Espectro electrónico de moléculas. Electronic spectra of molecules
 Espectro molecular. Molecular spectra
 Espectro molecular de vibración y rotación. Rotation vibration spectra of molecules
 Espectro rotacional en núcleos. Rotational spectra in nuclei
 Esperanza. Expectation
 Espesor de energía de la capa límite. Energy thickness of boundary layer
 Espesor del campo. Depth of field
 Espesor efectivo. Depth, effective
 Espín. Spin
 Espín del electrón. Electron spin
 Espín isobárico. Isobaric spin
 Espín isotópico. Isotope spin
 Espín nuclear. Nuclear spin
 Espiral de Cornu. Cornu spiral
 Espiral de Ekman. Ekman spiral
 Espiral de Taylor. Taylor spiral
 Esquema de interacción. Interaction picture
 Esquema de radiación. Radiation pattern
 Esquemas de difracción. Diffraction patterns
 Estabilidad. Stability
 Estabilidad de difusión. Diffusion stability
 Estabilidad de fases. Stability of phases
 Estabilidad del aeroplano. Airplane stability
 Estabilidad de un cuerpo flotante. Stability of a floating body
 Estabilidad de un flujo laminar. Stability of laminar flow
 Estabilidad estática. Static stability
 Estabilidad mecánica. Mechanical stability
 Estabilidad mecánica. Stability, mechanical
 Estabilidad numérica. Numerical stability
 Estabilidad térmica. Thermal stability
 Estabilización de variancia. Stabilization of variance
 Estadística. Statistic
 Estadística auxiliar. Auxiliary statistic
 Estadística cuántica. Quantum statistics
 Estadística D^2 . D^2 -statistic
 Estadística de Boltzmann. Boltzmann statistics
 Estadística de Bose-Einstein. Bose-Einstein statistics
 Estadística de Fermi-Dirac. Fermi-dirac statistics
 Estadística de Gentile. Gentile statistics
 Estadística- k . k -statistics
 Estado degenerado. Degenerate state
 Estado fundamental. Fundamental state

- Estado fundamental. Ground state
 Estado metastable. Metastable state
 Estado normal. Normal state
 Estado virtual. Virtual state
 Estado vítreo. Glassy state
 Estado *S*. *S*-state
 Estados correspondientes. Corresponding states
 Estados de energía negativa. Negative energy states
 Estados estacionarios. Stationary states
 Estados indiferentes. Indifferent states
 Estados localizados. Localized states
 Estática. Statics
 Estática gráfica. Graphical statics
 Estáticamente admisible. Statically admissible
 Estereoespectrograma. Stereospectrogram
 Esterracián. Steradian
 Estimador compatible. Consistent estimator
 Estímulo básico. Basic stimulus
 Estocástico. Stochastic
 Estratificación. Stratification
 Estructura cristalina. Crystal structure
 Estructura determinada. Determinate structure
 Estructura fina. Fine structure
 Estructura fina del hidrógeno. Hydrogen fine structure
 Estructura indeterminada. Indeterminate structure
 Estructura nuclear. Nuclear structure
 Estructuras del benceno. Benzene, structures of
 Estructura superfina. Hyperfine structure
 Eventualidad. Contingency
 Evoluta. Evolute
 Evoluta de difracción. Diffraction evolute
 Exactitud. Accuracy
 Exacto(a). Exact
 Exaltación. Exaltation
 Exasimétrico. Symmetry, hexad
 Exceso neutrónico. Neutron excess
 Expansión. Expansion
 Expansión térmica. Thermal expansion
 Experimento factorial. Factorial experiment
 Exponente. Exponent
 Exposición. Exposure
 Extinción. Extinction
 Extraño. Extraneous
 Extrapolación. Extrapolation

 Facetas cristalinas. Crystal phases
 Factor crítico. Criticality factor
 Factor de absorción. Absorptance
 Factor de absorción. Absorption factor
 Factor de absorción interna. Internal absorptance
 Factor de aislamiento del ruido. Noise insulation factor
 Factor de arrastre. Drag coefficient
 Factor de asimetría. Dissymmetry factor or anisotropy factor
 Factor de atenuación. Attenuation factor
 Factor de atenuación sonora. Sound attenuation factor
 Factor de Boltzmann. Boltzmann factor
 Factor de carga. Load factor
 Factor de compresibilidad. Compressibility factor
 Factor de conversión de la masa atómica. Atomic mass conversion factor
 Factor de cresta de un portador de impulsos. Pulse carrier, crest factor of
 Factor de desventaja. Disadvantage factor
 Factor de difusión. Diffusion factor
 Factor de disipación. Dissipation factor
 Factor de dispersión atómica. Atomic scattering factor
 Factor de distribución. Distribution factor
 Factor de estabilidad. Stability factor
 Factor de estructura. Structure factor
 Factor de fase. Phase factor
 Factor de fisión rápida. Fast fission factor
 Factor de forma. Shape factor
 Factor de frecuencia. Frequency factor
 Factor de geometría. Geometry factor
 Factor de pérdidas en un dieléctrico. Loss factor, dielectric
 Factor de recalentamiento. Reheat factor
 Factor de reconstrucción. Build-up factor
 Factor de recuperación térmica. Temperature recovery factor
 Factor de seguridad. Safety factor
 Factor de sobrecarga. Carryover factor
 Factor de trabajo de impulsos. Pulse duty factor
 Factor de visibilidad. Visibility factor
 Factores de forma y posición de Coddington. Coddington shape and position factors
 Factor espectral de brillo. Spectral luminance factor
 Factor *g* de Landé. Landé's *g*-factor
 Factorial. Factorial
 Factor integrante. Integrating factor
 Factorización de polinomios. Polynomial factorization
 Factor normalizante. Normalizing factor
 Factor *Q*. *Q* factor
 Falla. Failure
 Familia. Family
 Familia de superficies. Surfaces, family of
 Fase. Phase
 Fase de la onda. Phase of the wave
 Fase de onda. Wave phase
 Fase de una magnitud periódica. Phase of a periodic quantity
 Fases. Phases
 Fatiga. Fatigue
 Fenómeno cooperativo. Cooperative phenomena
 Fenómeno de agrupación en gases. Clustering phenomena in gases
 Fenómeno de desdoblamiento en espectroscopia. Uncoupling phenomena in spectroscopy
 Fenómeno de Gibbs. Gibbs phenomenon
 Fenómeno de relajación. Relaxation phenomena
 Fermi. Fermi
 Fermión. Fermion
 Fiabilidad. Reliability
 Filtro. Filter
 Flecha. Sagitta
 Flexión. Flexure
 Flexión. Bending
 Flexión asimétrica. Unsymmetrical bending
 Flexión pura. Bending, pure
 Fluctuación. Straggling
 Fluctuaciones. Fluctuations
 Fluencia. Creep
 Fluido Newtoniano. Newtonian fluid
 Fluido viscoso. Viscous fluid
 Flujo. Flux
 Flujo adjunto. Flux, adjoint
 Flujo angular. Flux, neutron, angular
 Flujo de deslizamiento. Slip flow
 Flujo de deslizamiento. Shear flow
 Flujo de energía geométrica. Geometrical energy flux
 Flujo de energía sonora. Sound energy flux
 Flujo de gas en toberas. Nozzle, flow of gas in
 Flujo de Helmholtz. Helmholtz flow
 Flujo de moléculas libres. Free-molecule flow

- Flujo de neutrones. Streaming, neutron
 Flujo de resistencia. Resistance flow
 Flujo de Stokes. Stokes flow
 Flujo irrotacional. Irrotational flow
 Flujo laminar. Laminar flow
 Flujo laminar en canales. Channel, laminar flow in
 Flujo laminar en tubos. Pipe, laminar flow in
 Flujo luminoso. Luminous flux
 Flujo plástico. Plastic flow
 Flujo potencial. Potential flow
 Flujo secundario. Secondary flow
 Flujo subsónico y supersónico. Subsonic and supersonic flow
 Flujo supersónico. Hypersonic flow
 Flujo total. Flux, neutron, total
 Flujo turbulento en tubos. Pipe, turbulent flow in
 Flujo vectorial. Vector flux
 Foco Planckiano. Planckian locus
 Foco sagital. Sagittal focus
 Foco tangencial. Tangential focus
 Fon. Phon
 Fonón. Phonon
 Forma. Form
 Forma canónica del principio de Fermat. Canonical form of Fermat's principle
 Forma diferencial cuadrática. Quadratic differential form
 Forma diferencial lineal. Linear differential form
 Forma indeterminada. Indeterminate form
 Formalismo de Lagrange y Hamilton para sistemas continuos. Continuous systems, Lagrangian and Hamiltonian formalism for
 Formalismo de Yang-Feldman y la matriz S . Yang-Feldman formalism and the S -matrix
 Forma normal de Hesse. Hesse's normal form
 Formas de la ecuación de ondas. Wave equation, forms of
 Fórmula abierta de la cuadratura. Open quadrature formula
 Fórmula barométrica. Barometer formula
 Forma bilinear. Form, bilinear
 Fórmula cerrado de la cuadratura. Closed quadrature formula
 Fórmula de Batho. Batho's formula
 Fórmula de Breit-Wigner. Breit-Wigner formula
 Fórmula de Conwell-Weisskopf. Conwell-Weisskopf formula
 Fórmula de dispersión. Dispersion formula
 Fórmula de equilibrio de Saha. Saha equilibrium formula
 Fórmula de Euler-Maclaurin. Euler-Maclaurin formula
 Fórmula de Eykman. Eykman formula
 Fórmulas de Gauss. Gauss' formulas
 Fórmula de Gregory. Gregory formula
 Fórmula de Gregory-Newton. Gregory-Newton formula
 Fórmula de Gruneisen. Gruneisen formula
 Fórmula de Heine. Heine formula
 Fórmula de Helmholtz-Ketteler. Helmholtz-Ketteler formula
 Fórmula de Hicks. Hicks formula
 Fórmula de interpolación de Bessel. Bessel interpolation formula
 Fórmula de interpolación de Everett. Everett interpolation formula
 Fórmula de interpolación de Hermite. Hermite interpolation formula
 Fórmula de interpolación de Lagrange. Lagrange interpolation formula
 Fórmula de interpolación de Stirling. Stirling interpolation formula
 Fórmula de Johnson y Lark-Horowitz. Johnson and Lark-Horowitz formula
 Fórmula de Kelvin. Kelvin formula
 Fórmula de Klein-Nishina. Klein-Nishina formula
 Fórmula de la cuadratura de Chebyshev. Chebyshev quadrature formula
 Fórmula de la cuadratura de Gauss. Gaussian quadrature formula
 Fórmula de la dispersión de Hartmann. Hartmann dispersion formula
 Fórmula de la dispersión de Herzberger. Herzberger's dispersion formula
 Fórmula de la integral de Cauchy. Cauchy integral formula
 Fórmula de la masa. Mass formula
 Fórmula de Landau. Landau's formula
 Fórmula de la radiación de Planck. Planck radiation formula
 Fórmula de la suma de Euler. Euler summation formula
 Fórmula de la tensión superficial de Gibbs. Gibbs formula for the surface tension
 Fórmula del diamagnetismo de Langevin-Pauli. Diamagnetism, Langevin-Pauli formula
 Fórmula del índice de refracción de Cauchy. Cauchy formula for refractive index
 Fórmula de los cuatro factores. Four-factor formula
 Fórmulas de los restos. Remainder formulas
 Fórmula del punto medio. Midpoint formula
 Forma de Monge de la ecuación de una superficie. Monge's form for equation of surface
 Fórmula de perturbación de Rayleigh-Schrodinger. Rayleigh-Schrodinger perturbation formula
 Fórmula de recursión. Recursion formula
 Fórmula de resistencia al pandeo de Euler. Euler buckling loads for columns
 Fórmula de Ritz. Ritz formula
 Fórmula de Rodrigues. Rodrigues formula
 Fórmula de Schläfli. Schläfli formula
 Fórmula de Wallis. Wallis formula
 Fórmula de Winkler-Bach. Winkler-Bach formula
 Fórmulas de Bredt. Bredt formulas
 Fórmulas de cuadraturas de Newton-Cotes. Newton-Cotes quadrature formulas
 Fórmulas de diferenciación numérica. Differentiation formulas numerical
 Fórmulas de interpolación de Gauss. Gaussian interpolation formulas
 Fórmulas de Serret-Frenet. Serret-Frenet formulas
 Fórmulas de Schwarzschild-Kohlschutter. Schwarzschild-Kohlschutter formulas
 Fórmulas de Weingarten. Weingarten's formulas
 Fracción continua. Fraction, continued
 Fracción de Bifurcación. Branching fraction
 Fracción de empaquetamiento. Packing fraction
 Fracción de Fechner. Fechner fraction
 Fracción de regeneración. Breeding ratio
 Fracciones continuas. Continued fractions
 Fracciones molares. Mole fractions
 Fracciones volumétricas. Volume fractions
 Fracción impropia. Improper fraction
 Fracción parcial. Fraction, partial
 Fracción propia. Proper fraction
 Franja. Fringe
 Frecuencia atómica. Atomic frequency

- Frecuencia de Choque. Collision frequency
 Frecuencia de Debye. Debye frequency
 Frecuencia de pulsación. Beat frequency
 Frecuencia de relajación. Relaxation frequency
 Frecuencia de repetición de impulsos. Pulse repetition frequency
 Frecuencia de resonancia de un cristal. Crystal, resonance frequency of
 Frecuencia de ruptura. Cut-off
 Frecuencia instantánea. Frequency, instantaneous
 Frecuencia relativa. Relative frequency
 Frecuencia resonante. Resonant frequency
 Frente de onda. Wave front
 Frente de onda esférico. Wave front, spherical
 Frente de onda plano. Wave front, plane
 Fricción interna. Friction, internal
 Fricción superficial. Skin friction
 Frontera reticular. Grain boundary
 Fuente. Source
 Fuente de tensión ideal. Voltage source, ideal
 Fuente puntual. Point source
 Fuente puntual uniforme. Uniform point source
 Fuente sonora anular. Circular ring sound source
 Fuente sonora curva. Curved line sound source
 Fuente sonora plana rectangular. Plane rectangular surface sound source
 Fuente sonora rectilínea. Straight line sound source
 Fuentes virtuales. Image sources
 Fuerza. Force
 Fuerza aplicada. Force, applied
 Fuerza central. Force, central
 Fuerza centrífuga. Centrifugal force
 Fuerza centripeta. Centripetal force
 Fuerza ciclostrófica. Cyclostrophic force
 Fuerza conservativa. Conservative force
 Fuerza cuatridimensional. Four-force
 Fuerza de Bartlett. Bartlett force
 Fuerza de Coriolis. Coriolis force
 Fuerza de enlace. Bond stress
 Fuerza de Heisenberg. Heisenberg force
 Fuerza de Lorentz. Lorentz force
 Fuerza de Majorana. Majorana force
 Fuerza dependiente del espín. Spin-dependent force
 Fuerza de rango corto. Short-range force
 Fuerza de Wigner. Wigner force
 Fuerza disipativa. Dissipative force
 Fuerza distribuida. Force, distributed
 Fuerza efectiva. Force, effective
 Fuerza electromotriz. Electromotive force
 Fuerza exterior ó activa. Force, external or active
 Fuerza generalizada. Force, generalized
 Fuerza geostrófica. Geostrophic force
 Fuerza interior. Force, internal
 Fuerza magnetizante. Magnetizing force
 Fuerza magnetomotriz. Magnetomotive force
 Fuerza mecanomotriz efectiva. Effective mechano-motive force
 Fuerza perdida. Force, "lost"
 Fuerza sobre una superficie sumergida. Force on a submerged surface
 Fuerza sobre un conductor. Force on conductor
 Fuerza tangencial. Force, tangential
 Fuerzas centrales. Central forces
 Fuerzas concurrentes. Forces, concurrent
 Fuerzas coplanares. Forces, coplanar
 Fuerzas de cohesión. Cohesion, forces of
 Fuerzas de dispersión. Dispersion forces
 Fuerzas de inducción. Induction forces
 Fuerzas de inercia. Inertia forces
 Fuerzas de intercambio. Exchange forces
 Fuerzas de London. London forces
 Fuerzas de valencia en moléculas poliatómicas. Valence forces in polyatomic molecules
 Fuerzas de Van Der Waals. Van Der Waals' forces
 Fuerzas en termodinámica de procesos irreversibles. Forces in thermodynamics of irreversible processes
 Fuerzas intermoleculares. Intermolecular forces
 Fuerzas no centrales. Non-central forces
 Fuerzas nucleares. Nuclear forces
 Fuerzas pluricorpusculares. Many-body force
 Fuerzas secundarias de valencia. Subsidiary valence forces
 Fugacidad. Fugacity
 Función. Function
 Funcional. Functional
 Función analítica. Analytic function
 Función armónica. Harmonic function
 Función beta. Beta function
 Función beta-incompleta. Incomplete beta function
 Función característica. Characteristic function
 Función característica de ángulo. Angle characteristic function
 Función característica de un conjunto. Characteristic function of a set
 Función característica mixta. Mixed characteristic function
 Función complementaria de una ecuación diferencial lineal. Complementary function of a linear differential equation
 Función continua. Continuous function
 Función de Bessel. Bessel function
 Función de Bloch. Bloch function
 Función de Brillouin. Brillouin function
 Función de corriente. Stream function
 Función de decisión. Decision function
 Función de densidad de probabilidad. Probability density function
 Función de disipación. Dissipation function
 Función de distribución de Fermi-Dirac. Fermi-Dirac distribution function
 Función de distribución radial. Radial distribution function
 Función de error. Error function
 Función de estímulo colorimétrica. Color stimulus function
 Función de excitación atómica. Excitation function, atomic
 Función de excitación nuclear. Excitation function, nuclear
 Función de Gegenbauer. Gegenbauer function
 Función de Gibbs. Gibbs function
 Función de Green. Green's function
 Función de Helmholtz. Helmholtz function
 Función de importancia. Importance function
 Función de Langevin. Langevin function
 Función del calor específico de Einstein. Einstein specific heat function
 Función de luminosidad. Luminosity function
 Función de medida. Measure function
 Función de Neumann. Neumann function
 Función de onda. Wave function
 Función de onda adjunta. Adjoint wave function
 Función de onda antisimétrica. Antisymmetric wave function

- Función de onda de Coulomb. Coulomb wave function
 Función de onda, sigma-Pi. Wave function, sigma-pi
 Función de onda simétrica. Symmetric wave function
 Función de partición. Partition function
 Función de partición de moléculas diatómicas. Diatomic molecules, partition function of
 Función de partición de moléculas poliatómicas. Polyatomic molecules, partition function of
 Función de partición de un rotador. Rotator, partition function of
 Función de partición de vibración. Vibrational partition function
 Función de partición rotacional. Rotational partition function
 Función de pérdidas. Loss function
 Función de Placzek. Placzek function
 Función de Planck. Planck function
 Función de prueba. Test function
 Función de trabajo fotoeléctrico. Photoelectric work function
 Función de transferencia. Transfer function
 Función de transferencia del reactor. Reactor transfer function
 Función elíptica. Elliptic function
 Función entera. Entire function
 Función escalonada. Step function
 Funciones características en termodinámica. Characteristic functions in thermodynamics
 Funciones de aberración de Nijboer-Zernike. Nijboer-Zernike aberration functions
 Funciones de Airy. Airy functions
 Funciones de dispersión luminosa. Light scattering functions
 Funciones de exceso. Excess functions
 Funciones de fase minimal. Minimum phase functions
 Funciones de impulso. Impulse functions
 Funciones de Legendre. Legendre functions
 Funciones de Wightman. Wightman functions
 Funciones especiales. Special functions
 Función espectral. Spectral function
 Funciones potenciales de moléculas. Potential functions of molecules
 Funciones termodinámicas de fases superficiales. Thermodynamic functions of surface phases
 Funciones vectoriales de ondas esféricas. Spherical vector wave functions
 Función gamma. Gamma function
 Función gamma incompleta. Incomplete gamma function
 Función generatriz. Generating function
 Función Hamiltoniana de la óptica. Hamiltonian function of optics
 Función Hamiltoniana de un sistema. Hamiltonian function of a system
 Función hiperbólica. Hyperbolic function
 Función hypergeométrica. Hypergeometric function
 Función holomorfa. Holomorphic function
 Función impar. Odd function
 Función implícita. Implicit function
 Función inversa. Inverse function
 Función Lagrangiana. Lagrangian function
 Función meromorfa. Meromorphic function
 Función ortogonal. Orthogonal function
 Función par. Even function
 Función ponderal. Weighting function
 Función regular de una variable compleja. Regular function of a complex variable
 Función simétrica. Symmetric function
 Función trigonométrica. Trigonometric function
 Función unidad de Heaviside. Heaviside unit function
 Función zeta de Riemann. Riemann zeta function
 Gamma. Gamma
 Ganancia. Gain
 Ganancia de conversión. Conversion gain
 Ganancia de regeneración. Breeding gain
 Gas de Einstein-Bose. Bose-Einstein gas
 Gas degenerado. Degenerate gas
 Gas electrónico degenerado. Degenerate electron gas
 Gases reales. Real gases
 Gas imperfecto. Imperfect gas
 Geodésica nula. Null-geodesic
 Geometría. Geometry
 Geometría de Luneburg. Luneburg geometry
 Geometría descriptiva. Descriptive geometry
 Gradiente. Gradient
 Gradiente de temperatura. Temperature gradient
 Gradiente vertical adiabático de temperatura. Adiabatic lapse rate
 Grado. Grade
 Grado de adelantamiento. Degree of advancement
 Grado del vértice. Vertex degree
 Grado de reacción. Extent of reaction
 Grado microrecíproco. Microreciprocal degree
 Grados de libertad. Degrees of freedom
 Gráfico completo. Graph, complete
 Gráfico conexo. Graph, connected
 Gráfico de Kurie. Kurie plot
 Gráfico de Richardson. Richardson plot
 Gráfico dirigido. Graph, directed
 Gráfico infinito. Graph, infinite
 Gráfico inseparable. Graph, non-separable
 Gráfico lineal. Graph, linear
 Gráfico no-orientado. Graph, non-oriented
 Gráfico orientado. Graph, oriented
 Gráfico planar. Graph, planar
 Gráfico separable. Graph, separable
 Gráficos homeomórficos. Graphs, homeomorphic
 Gráficos isomórficos. Graphs, isomorphic
 Gráfico topológico. Graph, topological
 Grupo. Group
 Grupo Abeliano. Abelian group
 Grupo cristalográfico. Crystallographic group
 Grupo de Lie. Lie group
 Grupo de ondas de Gauss. Gaussian wave group
 Grupo de permutaciones. Permutation group
 Grupo de rotaciones. Rotation group
 Grupo de traslaciones. Translation group
 Grupo diédrico. Dihedral group
 Grupo espacial. Space group
 Grupo factor. Factor-group
 Grupo lineal general. Full linear group
 Grupo puntual. Point group
 Grupo topológico. Topological group
 Grupos de transformaciones. Transformation groups, general
 Guía de ondas. Waveguide
 Haz axial. Axial bundle
 Hélice. Helix
 Hélice aérea. Airscrew
 Hertz. Hertz
 Hessiano. Hessian

- Hibridación de órbitas electrónicas. Hybridisation of electron orbitals
- Hidrodinámica relativista. Relativistic hydrodynamics
- Hidrostática. Hydrostatics
- Hipérbola. Hyperbola
- Hipérbola geodésica. Geodesic hyperbola
- Hiperboloide. Hyperboloid
- Hipótesis de Goudsmit y Uhlenbeck. Goudsmit and Uhlenbeck assumption
- Hipótesis Ergódica. Ergodic hypotheses
- Hipótesis estadística. Hypothesis, statistical
- Hipótesis lineal. Linear hypothesis
- Hoja. Sheet
- Holónimo. Holomic
- Homocéntrico. Homocentric
- Homogéneo. Homogeneous
- Homomorfismo. Homomorphism
- Hormigón armado. Concrete, reinforced
- Hormigón pretensado. Prestressed concrete
- Hueco. "Hole"
- Humedad. Humidity
- Idealmente plástico. Ideally plastic
- Idempotente. Idempotent
- Identidad. Identity
- Identidad de Abel. Abel identity
- Iluminación. Illumination
- Identidad de Bianchi. Bianchi identity
- Identidad de De Moivre. De Moivre identity
- Identidad de Ricci. Ricci identity
- Identidades diferenciales vectoriales. Vector differential identities
- Iluminancia. Illuminance
- Impacto, choque. Impact
- Impedancia acústica. Impedance, acoustical
- Impedancia acústica característica. Impedance, characteristic acoustical
- Impedancia acústica de entrada. Impedance, throat acoustical
- Impedancia acústica específica. Impedance, specific acoustical
- Impedancia mecánica. Mechanical impedance
- Impulso. Pulse
- Impulso. Impulse
- Impulso. Momentum
- Impulso angular. Angular impulse
- Incidente. Incident
- Inconmensurable. Incommensurable
- Independencia. Independence
- Independencia de la carga. Charge-independence
- Independiente. Independent
- Indicatriz. Indicatrix
- Indicatriz de difusión. Indicatrix of diffusion
- Indicatriz de refracción. Indicatrix of refraction
- Indicatriz esférica de la tangente a una curva. Spherical indicatrix of tangent to a curve
- Índice de probabilidad. Index of probability
- Índice de refracción. Index of refraction
- Índice de valencia libre. Free valence index
- Índice libre. Index, free
- Índices cristalinos de Miller. Crystal indices, Miller
- Índices de Bravais-Miller. Bravais-Miller indices
- Inducción magnética. Magnetic induction
- Inductancia. Inductance
- Inercia. Inertia
- Inertancia. Inertance
- Inferencia a libre distribución. Distribution-free inference
- Inferencia aparamétrica. Non-parametric inference
- Inferencia fiducial. Fiducial inference
- Infinitesimal. Infinitesimal
- Infinito. Infinity
- Información. Information
- Integración de contorno. Contour integration
- Integración de series. Series integration
- Integral. Integral
- Integral circulatoria. Integral, circulatory
- Integral de Clausius. Clausius' integral
- Integral de convolución. Convolution integral
- Integral de Coulomb. Coulomb integral
- Integral de Dirichlet. Dirichlet integral
- Integral de Duhamel. Duhamel's integral
- Integral de fase. Phase integral
- Integral de intercambio. Exchange integral
- Integral de la energía. Energy integral
- Integral de Lebesgue. Lebesgue integral
- Integral de Poisson. Poisson integral
- Integral de resonancia. Resonance integral
- Integral de Riemann. Riemann integral
- Integral de Stieltjes. Stieltjes integral
- Integral elíptica. Elliptic integral
- Integrales rampantes. Overlap integral
- Integral impropia. Improper integral
- Integrale múltiple. Multiple integral
- Integrando. Integrand
- Intensidad de campo magnético. Magnetic field strength
- Intensidad del campo. Field strength
- Intensidad del campo eléctrico. Electric field strength
- Intensidad del oscilador. Oscillator strength
- Intensidad de radiación. Intensity of radiation
- Intensidad de una fuente de partículas. Intensity of a source of particles
- Intensidad de una línea espectral. Intensity of a spectral line
- Intensidad de una onda sonora esférica. Intensity of a spherical sound wave
- Intensidad esférica media. Mean spherical intensity
- Intensidad horizontal media. Mean horizontal intensity
- Intensidad iónica. Ionic strength
- Intensidad luminosa. Candle power
- Intensidad luminosa. Luminous intensity
- Intensidad radiante. Radiant intensity
- Intensidad radioactiva. Intensity of radioactivity
- Intensidad sonora. Loudness of a sound
- Intensidad sonora. Sound intensity
- Interacción directa. Direct interaction
- Interacción fonón-fonón. Phonon-phonon interaction
- Interacción seudoescalar. Pseudoscalar coupling
- Interacción seudovectorial. Pseudovector coupling
- Interacción universal de Fermi. Universal Fermi interaction
- Interferencia. Interference
- Interpolación. Interpolation
- Interpolación de intervalo óptimo. Optimum-interval interpolation
- Interpolación inversa. Inverse interpolation
- Interpolación multivariada. Multivariate interpolation
- Interpolación trigonométrica. Trigonometric interpolation
- Intervalo de choque. Collision interval
- Intervalo de impulsos. Pulse interval
- Invariancia relativista de una teoría física. Relativistic invariance of a physical theory

- Invariancia relativista y mecánica cuántica. Relativistic invariance and quantum mechanics
- Invariante. invariant
- Invariante de Poincaré. Poincaré's invariant
- Invariante de suma. Summational invariant
- Invariante diferencial óptico. Optical differential invariant
- Inversión. Inversion
- Inversión del tiempo. Time reversal
- Inversión de una matriz. Matrix inversion
- Inversión de una serie. Reversion of series
- Involta de una curva. Involute of a curve
- Involuta de una superficie. Involute of a surface
- Ion complejo. Complex ion
- Irradiación. Irradiation
- Irradiancia. Irradiance
- Isobara. Isobar
- Isoentrópico. Isentropic
- Isofotométrica. Isolux
- Isofotométrica. Isophot
- Isoterma de adsorción. Adsorption isotherm
- Isoterma de adsorción de Langmuir. Langmuir adsorption isotherm
- Isotermas. Isotherms
- Isotérmico. Isothermal
- Isotrópico. Isotropic
- Jacobiano. Jacobian
- Juego equitativo. Fair game
- k*-espacio. *k*-space
- Lambert. Lambert
- Laplaciano. Laplacian
- Lazo. Loop
- Lente de Luneburg. Luneburg lens
- Lentitud de fase. Phase slowness
- Ley adiabática de Ehrenfest. Ehrenfest adiabatic law
- Ley adiabática para estados cuantificados. Adiabatic law for quantized states
- Ley asociativa. Associative law
- Ley cero de la termodinámica. Zeroth law of thermodynamics
- Ley cociente de tensores. Quotient law for tensors
- Ley conmutativa. Commutative law
- Ley de acción de masas. Law of mass action
- Ley de Amagat. Amagat's law
- Ley de Barba. Barba's law
- Ley de Beer. Beer's law
- Ley de Bouguer. Bouguer law
- Ley de Boyle. Boyle's law
- Ley de Brewster. Brewster's law
- Ley de Charles. Charles' law
- Ley de composición de aceleraciones. Acceleration, composition law of
- Ley de Cotton-Mouton. Cotton-Mouton law
- Ley de Curie-Weiss. Curie-Weiss law
- Ley de Dalton. Dalton's law
- Ley de Darcy. Darcy's law
- Ley de desplazamiento de Sommerfeld-Kossel. Sommerfeld-Kossel displacement law
- Ley de distribución de Maxwell. Maxwell distribution law
- Ley de distribución de Maxwell-Boltzmann. Maxwell-Boltzmann distribution law
- Ley de distribución de Nernst. Nernst distribution law
- Ley de Duane y Hunt. Duane and Hunt law
- Ley de emisión del coseno. Cosine emission law
- Ley de estados correspondientes. Corresponding states, law of
- Ley de Faraday de la inducción electromagnética. Faraday law of electromagnetic induction
- Ley de Fick de la difusión. Fick's law of diffusion
- Ley de Fourier. Fourier's law
- Ley de Gladstone-Dale. Gladstone-Dale law
- Ley de Goldschmidt. Goldschmidt law
- Ley de Hauy. Hauy law
- Ley de Hess. Hess law
- Ley de Hooke. Hooke's law
- Ley de imposibilidad de cruzamiento. Non-crossing rule
- Ley de Kirchhoff. Kirchhoff's law
- Ley de Kutta-Joukowski. Kutta-Joukowski law
- Ley de la difusión de Fick. Diffusion, Fick law
- Ley de la dilución de Ostwald. Ostwald's dilution law
- Ley de la fuerza gravitacional normal de Gauss. Gauss' law of normal gravitational force
- Ley de la pérdida térmica de Newton. Newton's law for heat loss
- Ley de la probabilidad, compuesta. Probability, law of compound
- Ley de la radiación de Kirchhoff. Kirchhoff's radiation law
- Ley de la resistencia fluída de Newton. Newton's law of fluid resistance
- Ley del coseno cuadrado de Malus. Malus cosine-squared law
- Ley del coseno de Knudsen. Knudsen cosine law
- Ley de Lenz. Lenz law
- Ley del logaritmo iterado. Iterated logarithm, law of
- Ley de Lorentz. Lorentz law
- Ley de Lorentz-Lorentz. Lorentz-Lorentz law
- Ley de los gases perfectos. Perfect gas law
- Ley de los grandes números. Law of large numbers
- Ley de Moseley. Moseley's law
- Ley de Moutier. Moutier law
- Ley de Ohm. Ohm's law
- Ley de paridad de la estabilidad nuclear. Odd-even rule of nuclear stability
- Ley de Planck. Planck law
- Ley de reciprocidad fotográfica. Reciprocity law, photographic
- Ley de Sabin. Sabin law
- Ley de semejanza supersónica. Hypersonic similarity law
- Ley de Stefan-Boltzmann. Stefan-Boltzmann law
- Ley(es) de Snell. Snell's law(s)
- Ley de Talbot. Talbot law
- Ley distributiva. Distributive law
- Leyes de Kepler. Kepler's laws
- Leyes del movimiento de Newton. Newton's laws of motion
- Leyes de radiación del cuerpo negro. Black body radiation laws
- Leyes de reflexión. Reflection, laws of
- Leyes de refracción. Refraction, laws of
- Leyes de Stokes. Stokes laws
- Leyes de transformación de intensidades de campo. Transformation laws for field strengths
- Leyes de Wien. Wien laws
- Ley límite para electrolitos fuertes. Limiting law for strong electrolytes
- Libre camino medio. Mean free path

- Libre camino medio del fonón. Phonon mean free path
 Libre camino medio de transporte. Transport mean free path
 Libre camino molecular. Molecular free path
 Límite de Rayleigh de la aberración esférica. Rayleigh limit for spherical aberration
 Límite de rotura. Ultimate strength
 Límite elástico. Elastic limit
 Límite elástico aparente. Yield point
 Límites de confianza. Confidence limits
 Línea asintótica sobre una superficie. Asymptotic line on a surface
 Línea cero de una banda. Zero line of a band
 Línea de acción de una fuerza. Force, line of action of
 Línea de corriente. Streamline
 Línea de curvatura. Line of curvature
 Línea de deslizamiento. Slip line
 Línea de influencia. Influence line
 Línea de Mach. Mach line
 Línea de presión. Pressure line
 Línea de rectificación. Rectifying line
 Línea de transmisión. Transmission line
 Línea de vórtice. Vortex line
 Línea geodésica. Geodesic line
 Lineal. Linear
 Línea polar. Polar line
 Líneas de corriente libres. Free streamlines
 Líneas de flujo. Lines of flow
 Líneas de intercombinación. Intercombination lines
 Líneas de Luder. Luder's lines
 Líneas de Stokes. Stokes lines
 Líneas de superestructura. Superstructure lines
 Líneas focales. Focal lines
 Líneas isométricas sobre una superficie. Isometric lines on a surface
 Líquido de Bose-Einstein. Bose-Einstein liquid
 Llevar. Carry
 Localmente Euclidiano. Locally Euclidean
 Logaritmo. Logarithm
 Logaritmo hiperbólico. Hyperbolic logarithm
 Longitud de difusión. Diffusion length
 Longitud de extrapolación lineal. Linear extrapolation length
 Longitud del enlace. Bond length
 Longitud del trazo del neutrón. Track length, neutron
 Longitud de migración de neutrones. Migration length for neutrons
 Longitud de onda efectiva. Effective wavelength
 Longitud de onda predominante. Dominant wavelength
 Longitud de radiación. Radiation length
 Longitud de relajación. Relaxation length
 Longitud de retardación. Slowing-down length
 Longitud de una curva. Length of a curve
 Longitud equivalente del péndulo. Pendulum, equivalent length of
 Longitud focal. Focal length
 Longitud focal reducida. Reduced focal length
 Lugar espectral. Spectrum locus
 Lumen. Lumen
 Lumen hora. Lumen-hour
 Lumerg. Lumerg
 Lux-segundo. Lux-second
 Macroestado. Macrostate
 Magnetización. Magnetization
 Magnetón de Bohr. Bohr magneton
 Magnetón nuclear. Nuclear magneton
 Magnetostática. Magnetostatics
 Magnetostricción. Magnetostriction
 Mantisita. Mantissa
 Mapa de Patterson. Patterson map
 Marco. Frame
 Margen de fase. Phase margin
 Margen de ganancia. Gain margin
 Martingala. Martingale
 Masa. Mass
 Masa atómica. Atomic mass
 Masa electromagnética. Electromagnetic mass
 Masa reducida. Reduced mass
 Masa virtual. Virtual mass
 Material de Bingham. Bingham material
 Material de Burgers. Burgers material
 Material de Levy-Mises. Levy-Mises material
 Material de Mises. Mises material
 Material de Prandtl-Reuss. Prandtl-Reuss material
 Material de Saint Venant-Mises. Saint Venant-Mises material
 Material de Voigt. Voigt material
 Matrices de Duffin-Kemmer. Duffin-Kemmer matrices
 Matriz de densidad. Density matrix
 Matriz. Matrix
 Matriz de incidencia. Matrix, incidence
 Matriz de Jordan. Jordan matrix
 Matriz de transmisión. Transmission matrix
 Matriz Hermitiana. Hermitian matrix
 Matriz inversa. Inverse matrix
 Matriz nula. Null matrix
 Matriz positiva definida. Positive definite matrix
 Matriz S. S-matrix
 Matriz triangular. Triangular matrix
 Matriz triple diagonal. Triple-diagonal matrix
 Matriz unidad. Unit matrix
 Máximo común divisor. Highest common factor
 Maxwell. Maxwell
 Mecánica cuántica. Quantum mechanics
 Mecánica de ondas. Wave mechanics
 Mecánica estadística. Statistical mechanics
 Media aritmética. Arithmetic mean
 Media armónica. Harmonic mean
 Media geométrica. Geometric mean
 Mediana. Median
 Medida. Measure
 Medida de posición. Measure of location
 Medida estándar. Standard measure
 Menor de una matriz. Minor (of a matrix)
 Método Bairstow. Bairstow method
 Método de Adams-Bashford. Adams-Bashford method
 Método de aproximaciones polinomiales. Method of polynomial approximations
 Método de Bernoulli. Bernoulli method
 Método de Bethe. Bethe's method
 Método de biortogonalización de Lanczos. Lanczos method of biorthogonalization
 Método de Born-Oppenheimer. Born-Oppenheimer method
 Método de Chio. Chio's method
 Método de Choleski. Choleski's method
 Método de Crout. Crout's method
 Método de Dandelin. Dandelin's method
 Método de Euler de resolución de una ecuación diferencial ordinaria. Euler method for solving an ordinary differential equation
 Método de Evjen. Evjen method
 Método de Feynman. Feynman's method

- Método de Frobenius. Frobenius method
 Método de Gauss-Seidel. Gauss-Seidel method
 Método de Goertzel-Selengut. Goertzel-Selengut method
 Método de Graeffe. Graeffe method
 Método de Hartree-Fock. Hartree-Fock method
 Método de Henneberg. Henneberg's method
 Método de Hitchcock. Hitchcock method
 Método de Holte. Holte's method
 Método de Horner. Horner method
 Método de interpolación de Aitken. Aitken method of interpolation
 Método de iteración. Method of iteration
 Método de Jacobi. Jacobi method
 Método de Jesek. Jesek's method
 Método de Jordan. Jordan method
 Método de Klein-Rydberg. Klein-Rydberg method
 Método de la carga elástica. Elastic load method
 Método de la convergencia de Hoff. Hoff's convergence method
 Método de la cuerda. Chord method
 Método de las armónicas esféricas. Spherical harmonics method
 Método de las características. Characteristics, method of
 Método de las cargas ficticias. Loads, method of fictitious
 Método de las desigualdades. Inequalities, method of
 Método de las diferencias de las variables. Variate difference method
 Método de las imágenes. Method of images
 Método de las órbitas moleculares. Molecular orbitals method
 Método de las ordenadas discretas. Discrete ordinates method
 Método de del delta-cuadrado. Delta-square process
 Método de Liebmann. Liebmann method
 Método del momento areal. Moment area method
 Método de Lobachevskii. Lobachevskii method
 Método de los coeficientes indeterminados. Undetermined coefficients, method of
 Método de los gradientes conjugados. Conjugate gradients, method of
 Método de los momentos. Moments method
 Método del punto de ensilladura. Saddle point method
 Método de Macaulay. Macaulay's method
 Método de Maxwell-Mohr. Maxwell-Mohr method
 Método de Milne. Milne's method
 Método de Monte Carlo. Monte Carlo method
 Método de Newton. Newton's method
 Método de Oseen. Oseen's method
 Método de Picard. Picard method
 Método de Rayleigh-Ritz. Rayleigh-Ritz method
 Método de Ritter. Ritter's method
 Método de Ritz. Ritz method
 Método de Runge-Kutta. Runge-Kutta method
 Método de Schmidt-Hilbert. Schmidt-Hilbert method
 Método de Seidel. Seidel method
 Método de Spencer-Fano. Spencer-Fano method
 Método de Serber-Wilson. Serber-Wilson method
 Método de Tamm-Dancoff. Tamm-Dancoff method
 Método de Wick. Wick method
 Método de Wick-Chandrasekhar. Wick-Chandrasekhar method
 Método estadístico de entropías. Statistical method of entropies
 Método Euleriano de análisis. Eulerian method of analysis
 Método Lagrangiano de análisis. Lagrangian method of analysis
 Método Minimax de estimación. Minimax method of estimation
 Método simple. Simplex method
 Método S_N . S_N method
 Métodos de Ampliación. Enlargement, methods of
 Métodos directos. Direct methods
 Métodos iterados de resolución de ecuaciones. Iterative methods for solving equations
 Métodos operacionales. Operational methods
 Metro-bujía. Meter-candle
 Metro-bujía-segundo. Meter-candle-second
 Mezclas de gases perfectos. Perfect gas mixtures
 Mezclas de polímeros. Polymer mixtures
 Microescala de turbulencia. Turbulence, microscale of
 Microestado. Microstate
 Minimum diferencia de tono perceptible. Pitch difference, minimum perceptible
 Modelo absorbente infinito. Infinite absorber model
 Modelo celular del estado líquido. Cell model of the liquid state
 Modelo de Kelvin. Kelvin model
 Modelo de Kronig-Penney. Kronig-Penney model
 Modelo de los núcleos de Schmidt. Schmidt model of nuclei
 Modelo de Maxwell. Maxwell model
 Modelo de Sutherland. Sutherland model
 Modelo de resonancia estrecho. Narrow resonance model
 Modelo de Voigt. Voigt model
 Modelo de Wigner-Wilkins. Wigner-Wilkins model
 Modelo de Wilkins. Wilkins model
 Modelo óptico del núcleo. Optical model of the nucleus
 Modelo reticulado. Lattice model
 Modo. Mode
 Modo de impulsos. Pulse mode
 Modo fundamental. Mode, fundamental
 Modo óptico. Optical mode
 Modo predominante. Dominant mode
 Modos de oscilación. Oscillation, modes of
 Modos normales. Normal modes
 Módulo de compresibilidad. Modulus, bulk
 Módulo de continuidad. Modulus of continuity
 Módulo de corte. Modulus, shear
 Módulo de elasticidad. Modulus of elasticity
 Módulo de elasticidad de volumen. Bulk modulus
 Módulo de equilibrio. Modulus, equilibrium
 Módulo de relajación. Modulus, relaxation
 Módulo de ruptura. Modulus of rupture
 Módulo de sección. Section modulus
 Módulo de Young. Young's modulus
 Módulo inicial. Initial modulus
 Módulo plástico. Plastic modulus
 Molaridad. Molality
 Moléculas poliatómicas. Polyatomic molecules
 Momento. Moment
 Momento areal de inercia. Inertia, area moment of
 Momento cuádrupolar. Quadrupole moment
 Momento de fuerzas. Moment of force
 Momento de inercia. Moment of inertia
 Momento de la cantidad de movimiento. Moment of momentum
 Momento de una distribución. Moment of a distribution
 Momento de una fuerza. Torque
 Momento estático. Moment, static
 Momento flexor. Bending moment
 Momento magnético. Magnetic moment

- Momento magnético del neutrón. Neutron magnetic moment
 Momento magnético nuclear. Nuclear magnetic moment
 Momento producto. Product moment
 Momentos y productos de inercia. Inertia, moments and products of
 Motor a reacción. Jet engine
 Movilidad. Mobility
 Movilidad de Hall. Hall mobility
 Movimiento. Motion
 Movimiento armónico. Harmonic motion
 Movimiento armónico simple. Simple harmonic motion
 Movimiento ciclónico. Cyclonic motion
 Movimiento curvilíneo. Curvilinear motion
 Movimiento impulsivo. Impulsive motion
 Movimiento planetario. Planetary motion
 Movimiento plano. Plane motion
 Movimiento rectilíneo. Rectilinear motion
 Movimientos térmicos en una red. Thermal motions in a lattice
 Movimiento uniforme. Uniform motion
 Movimiento uniforme sobre una circunferencia. Uniform motion in a circle
 Muestra. Sample
 Muestra puntual. Sample point
 Muestra sistemática. Systematic sample
 Muestreo de probabilidad. Probability sampling
 Muestreo por cuotas. Quota sampling
 Multicolinealidad. Multicollinearity
 Multiplete. Multiplet
 Multiplicación de tensores. Multiplication of tensors
 Multiplicación efectiva. Multiplication, effective
 Multiplicación vectorial. Vector multiplication
 Multiplicador de Lagrange. Lagrange multiplier
 Multiplicidad. Multiplicity
 Multiplicidad de borde. Edge multiplicity
 Multipolos eléctricos. Electric multipoles
- Nabla. Nabla
 Neper. Neper
 Newton. Newton
 Nilpotente. Nilpotent
 Nivel de Fermi. Fermi level
 Nivel de intensidad sonora. Sound intensity level
 Nivel de la fuente. Source level
 Nivel de presión de una banda sonora. Sound band pressure level
 Nivel de presión acústica. Sound pressure level
 Nivel de presión de banda. Band pressure level
 Nivel de presión de octava. Sound octave-band pressure level
 Nivel de ruido. Noise level
 Nivel de señal. Signal level
 Nivel de tono. Pitch level
 Niveles de energía molecular. Molecular energy levels
 Niveles de energía nuclear. Nuclear energy levels
 Niveles de energía de vibración de una molécula. Vibrational energy levels of a molecule
 Niveles degenerados. Degenerate levels
 Niveles energéticos rotacionales de una molécula. Rotational energy levels of a molecule
 Nivel espectral de ruido ambiente. Noise, spectrum level of room
 Nivel sonoro. Loudness level
 Nivel sonoro. Sound level
 Nomograma. Nomogram or nomogram
- Norma. Norm
 Norma espectral. Spectral norm
 Normal. Normal
 Normal a una curva. Normal to a curve
 Normalización. Normalization
 Normal principal. Normal, principal
 Normal unidad. Unit normal
 Notación de Bow. Bow notation
 Notación indicial. Indicial notation
 Notación posicional. Positional notation
 Núcleo. Kernel
 Núcleo de difusión. Kernel, diffusion
 Núcleo de dispersión. Kernel, scattering
 Núcleo de transporte. Kernel, transport
 Núcleo Gaussiano. Kernel, Gaussian
 Núcleo potencial. Potential core
 Núcleos conjugados. Conjugate nuclei
 Núcleo sintético. Kernel, synthetic
 Nulidad de un Gráfico. Graph, nullity
 Numerable. Countable
 Número. Number
 Número atómico. Atomic number
 Número atómico efectivo. Atomic number, effective
 Número ciclomático. Cyclomatic number
 Número complejo. Complex number
 Número crítico de Mach. Critical mach number
 Número de Abbe. Abbe number
 Número de condición. Condition number
 Número de coordinación. Coordination number
 Número de Froude. Froude number
 Número de Grashof. Grashof number
 Número de Knudsen. Knudsen number
 Número de Mach. Mach number
 Número de masa. Mass number
 Número de Nusselt. Nusselt number
 Número de onda. Wave number
 Número de Prandtl. Prandtl number
 Número de Rayleigh. Rayleigh number
 Número de Reynolds. Reynolds number
 Número de Reynolds de Turbulencia. Turbulence, Reynolds number of
 Número de Richardson. Richardson number
 Número de transporte. Transport number
 Número *f*. *f*-number
 Número rotacional. Rotational number
 Números conjugados. Conjugate numbers
 Números de Bernoulli. Bernoulli numbers
 Números de ocupación. Occupation numbers
 Números de Rossby. Rossby number
 Números de Stirling. Stirling numbers
 Números mágicos. Magic numbers
 Números significativos. Significant figures
 Nutación. Nutation
- Observable. Observable
 Onda. Wave
 Onda armónica. Wave, harmonic
 Onda cilíndrica. Wave, cylindrical
 Onda circulante. Wave, circulating
 Onda convergente. Wave, converging
 Onda cuadrada. Square wave
 Onda esférica. Wave, spherical
 Onda estacionaria. Wave, standing
 Onda guiada. Guided wave
 Onda longitudinal. Wave, longitudinal
 Onda *P*. *P* wave
 Onda periódica. Wave, periodic

- Onda reflejada. Wave, reflected
 Onda refractada. Wave, refracted
 Ondas capilares. Capillary waves
 Onda solenoidal. Wave, solenoidal
 Ondas de Helmholtz. Helmholtz waves
 Ondas de Rossby. Rossby waves
 Ondas de superficie. Surface waves
 Ondas de tensión. Waves, stress
 Ondas dilatacionales. Waves, dilatational
 Ondas elásticas en barras. Waves, elastic, in rods
 Ondas elásticas en placas. Waves, elastic, in plates
 Ondas elásticas en sólidos. Waves, elastic, in solids
 Ondas estacionarias. Waves, stationary
 Ondas longitudinales en una barra. Longitudinal waves in a rod
 Ondas no dispersivas. Non-dispersive waves
 Ondas plásticas. Plastic waves
 Ondas plásticas. Waves, plastic
 Ondas progresivas. Waves, progressive
 Ondas *S*. *S* waves
 Onda simple. Simple wave
 Ondas sonoras estacionarias. Sound wave(s), stationary
 Ondas torsionales en una barra. Torsional waves in a rod
 Ondas transversales (eléctricas o magnéticas). Transverse waves (electric or magnetic)
 Ondas viscoelásticas. Waves, viscoelastic
 Operación de conjugación de la carga. Charge conjugation operation
 Operación de traslación. Translation operation
 Operador. Operator
 Operador antiunitario. Anti-unitary operator
 Operador antilineal. Anti-linear operator
 Operador anulación. Annihilation operator
 Operador cronológico. Chronological operator
 Operador de creación. Creation operator
 Operador de destrucción. Destruction operator
 Operador de paridad. Parity operator
 Operador diádico. Dyadic operator
 Operadores de diferencias. Difference operators
 Operadores de Dirac. Dirac operators
 Operador Hermitiano. Hermitian operator
 Operador integral. Integral operator
 Operador inverso. Inverse operator
 Operador Laplaciano. Laplacian operator
 Operador potencial. Potential operator
 Operador unidad. Unit operator
 Operador unitario. Unitary operator
 Operador vectorial. Vector operator
 Operador viscoelástico. Viscoelastic operator
 Óptica de Gauss. Gaussian optics
 Óptica geométrica. Geometrical optics
 Órbita. Orbital
 Órbitas de Bohr. Bohr orbits
 Órbitas de enlaces. Bonding orbitals
 Órbitas electrónicas en un átomo. Electron orbits in an atom
 Órbitas moleculares no localizadas. Nonlocalized molecular orbitals
 Orden de alcance corto. Short-range order
 Orden de largo alcance. Long range order
 Orden de reacciones químicas. Order of chemical reactions
 Ortogonalización. Orthogonalization
 Ortonormal. Orthonormal
 Oscilación. Oscillation
 Oscilador anarmónico. Oscillator, anharmonic
 Oscilador armónico. Oscillator, harmonic
 Oscilador armónico. Harmonic oscillator
 Oscilaciones libres o naturales. Oscillations, free or natural
 Palabra. Word
 Par. Couple
 Parábola. Parabola
 Parábola de Condon. Condon parabola
 Paraboloide. Paraboloid
 Paradoja de Gibbs. Gibbs paradox
 Paradoja de la recurrencia. Recurrence paradox
 Paradoja de los relojes. Clock paradox
 Paralaje. Parallax
 Paralelas geodésicas. Geodesic parallels
 Paralelepipedo. Parallelepiped
 Paralelogramo. Parallelogram
 Paralelogramo de fuerzas. Forces, parallelogram of
 Paramagnetismo. Paramagnetism
 Paramagnetismo debido al espín de los electrones de conducción. Spin paramagnetism of conduction electrons
 Parámetro. Parameter
 Parámetro de Cayley-Klein. Cayley-Klein parameter
 Parámetro de choque. Impact parameter
 Parámetro de Coriolis. Coriolis parameter
 Parámetro de Euler-Rodrigues. Euler-Rodrigues parameter
 Parámetro de onda. Wave parameter
 Parámetros cristalinos. Crystal parameters
 Parámetros cristalográficos. Crystallographic parameters
 Parámetros de Ramberg-osgood. Ramberg-osgood parameters
 Parámetros diferenciales de Beltrami. Beltrami's differential parameters
 Parámetros enojosos. Nuisance parameters
 Pared adiabática. Adiabatic wall
 Pared diatérmica. Diathermal wall
 Pares isométricos. Homometric pairs
 Parfocal. Parfocal
 Paridad. Parity
 Partición. Partition
 Partícula material. Particle, material
 Particular. Particular
 Paso de un tornillo. Pitch of screw
 Patrón de Waidner-Burgess. Waidner-Burgess standard
 Pendiente. Slope
 Péndulo. Pendulum
 Péndulo balístico. Pendulum, ballistic
 Péndulo compuesto. Pendulum, compound
 Péndulo cónico. Pendulum, conical
 Péndulo de Foucault. Pendulum, Foucault's
 Péndulo de Kater. Pendulum, Kater
 Péndulo de torsión. Pendulum, torsion
 Péndulo esférico. Pendulum, spherical
 Péndulo físico. Pendulum, physical
 Péndulo reversible. Pendulum, reversible
 Péndulo simple. Pendulum, simple
 Pérdida por dispersión. Scattering loss
 Pérdida por divergencia. Divergence loss (sound)
 Pérdida por transición. Transition loss
 Pérdida tangencial. Tangent, loss
 Perfectamente plástico. Perfectly plastic
 Perfil de arrastre de un ala. Profile drag of a wing
 Perfil de velocidad. Velocity profile
 Perfiles de Joukowski. Joukowski airfoils

- Perfil logarítmico de velocidad. Logarithmic profile of velocity
- Performance del aeroplano. Airplane performance
- Periodo. Period
- Período de extinción del impulso. Pulse decay time
- Periodo de deformación. Period of deformation
- Periodo del reactor. Period, reactor
- Periodo de onda. Wave period
- Periodo de restitución. Period of restitution
- Periodo de retorno. Return period
- Periodograma. Periodogram
- Período natural. Period, natural
- Periodos de repetición de impulsos. Pulse repetition period
- Permeabilidad. Permeability
- Permutación. Permutation
- Permutación cíclica. Permutation, cyclic
- Permutación, par ó impar. Permutation, even or odd
- Perturbaciones en el espectro molecular. Perturbations in molecular spectra
- Perveancia. Perveance
- Peso. Weight
- Peso estadístico. Statistical weight
- Pesos estadísticos de niveles energéticos atómicos. Statistical weights of atomic energy levels
- Pesos estadísticos de niveles energéticos moleculares. Statistical weights of molecular energy levels
- Pie-bujía. Foot-candle
- Placas circulares. Plates, circular
- Placas rectangulares. Plates, rectangular
- Planimetro. Planimeter
- Plano de Argand. Argand plane
- Plano de curvatura. Curvature, plane of
- Plano de Gauss. Gauss plane
- Plano de polarización. Plane polarization
- Plano de reflexión. Reflection plane
- Plano de simetría. Symmetry, plane of
- Plano de transmisión. Transmission plane
- Plano de vibración. Plane of vibration
- Plano guía. Guide plane
- Plano meridional. Meridion(al) plane
- Plano osculador. Osculating plane
- Plano sagital. Sagittal plane
- Planos focales. Focal planes
- Planos principales. Principal planes
- Planos principales. Unit planes
- Plano tangencial. Tangential plane
- Plano tangente a una superficie. Tangent plane to surface
- Plasticidad. Plasticity
- Poder absorbente. Absorptive power
- Poder absorbente. Absorptivity
- Poder de detención. Stopping power
- Poder de resolución de un prisma. Prism, resolving power of
- Poder de resolución. Resolving power
- Poder dispersivo. Dispersive power
- Poder efectivo de una lente. Effective power of a lens
- Poder emisivo. Emissive power
- Poder rotatorio. Rotatory power
- Poder rotatorio específico de un medio. Specific rotatory power of a medium
- Poder termoeléctrico. Thermoelectric power
- Poise. Poise
- Polarizabilidad. Polarizability
- Polarización. Polarization
- Polarización circular. Circular polarization
- Poliedro. Polyhedron
- Polígono. Polygon
- Polígono de fuerzas. Force polygon
- Polígono funicular. Funicular polygon
- Polímero. Polymer
- Polinomio. Polynomial
- Polinomio de Bernstein. Bernstein polynomial
- Polinomio de Hurwitz. Hurwitz polynomial
- Polinomios circulares. Circle polynomials
- Polinomios de Bernoulli. Bernoulli polynomials
- Polinomios de Chebyshev. Chebyshev polynomials
- Polinomios de Hermite. Hermite polynomials
- Polinomios de Jacobi. Jacobi polynomials
- Polinomios de Laguerre. Laguerre polynomials
- Polinomios de Legendre. Legendre polynomials
- Polinomios especiales. Special polynomials
- Polinomios ortogonales. Orthogonal polynomials
- Polo. Pole
- Polo de una función analítica. Pole of an analytic function
- Ponderación. Weighting
- Positrón. Positronium
- Postulado. Postulate
- Potencia. Power
- Potencia al freno. Brake horsepower
- Potencia de amplificación. Magnifying power
- Potencia de reflexión. Reflectivity
- Potencial cinético. Kinetic potential
- Potencial complejo en flujo hidrodinámico. Complex potential in hydrodynamic flow
- Potencial crítico. Critical potential
- Potencial de Buckingham. Buckingham potential
- Potencial de contacto. Contact potential
- Potencial de convección. Convective potential
- Potencial de Coulomb. Coulomb potential
- Potencial de deformación. Deformation potential
- Potencial de detención. Potential, stopping
- Potencial de excitación. Excitation potential
- Potencial de ionización. Ionization potential
- Potencial de Keesom. Keesom potential
- Potencial de Lennard-Jones. Lennard-Jones potential
- Potencial de Morse. Morse potential
- Potencial de Stockmayer. Stockmayer potential
- Potencial de velocidad del sonido. Velocity potential of sound
- Potencial eléctrico. Potential, electric
- Potencial electroquímico. Electrochemical potential
- Potencial escalar magnético. Magnetic scalar potential
- Potenciales químicos. Chemical potentials
- Potencial gravitacional. Gravitational potential
- Potencial interatómico. Interatomic potential
- Potencial nuclear. Nuclear potential
- Potencial térmico. Thermal potential
- Potencial vectorial. Vector potential
- Potencial vectorial magnético. Magnetic vector potential
- Potencia sonora de una fuente. Sound power of a source
- Precisión. Precision
- Predictor. Predictor
- Predisociación. Predissociation
- Presión de dispersión. Spreading pressure
- Presión de estancamiento. Stagnation pressure
- Precesión de Larmor. Larmor precession
- Presión de radiación. Radiation pressure
- Presión de vapor en soluciones ideales. Vapor pressure of ideal solutions
- Presión de vapor en soluciones perfectas. Vapor pressure in perfect solutions

- Presión dinámica. Dynamic pressure
 Presión estática. Static pressure
 Presión osmótica. Osmotic pressure
 Presión normal. Normal pressure
 Presión parcial. Partial pressure
 Precesión relativista. Relativity precession
 Presión termonuclear. Thermomolecular pressure
 Presión total. Total pressure
 Primera ley de la termodinámica. First law of thermodynamics
 Primer número de Beitti. First Beitti number
 Primer punto focal. First focal point
 Primer teorema fundamental. First fundamental theorem
 Primer teorema límite. First limit theorem
 Primitivo(a). Primitive
 Principio de acción. Action principle
 Principio de acción mínima. Least action, principle of
 Principio de Arquímedes. Archimedes' principle
 Principio de Babinet. Babinet's principle
 Principio de Carathéodory. Carathéodory's principle
 Principio de causalidad. Causality principle
 Principio de combinación. Combination principle
 Principio de correspondencia. Principle of correspondence
 Principio de D'Alembert. D'Alembert's principle
 Principio de Dirichlet. Dirichlet principle
 Principio de energía mínima. Least-energy principle
 Principio de estados correspondientes. Corresponding states, principle of
 Principio de exclusión de Pauli. Pauli exclusion principle
 Principio de Franck-Condon. Franck-Condon principle
 Principio de Gauss del vínculo mínimo. Least constraint, Gauss principle of
 Principio de Hamilton. Hamilton principle
 Principio de Huygens. Huygens' principle
 Principio de incertidumbre. Indeterminacy principle
 Principio de Le Chatelier-Braun. Chatelier (Le)-Braun principle
 Principio de incremento de la entropía. Principle of entropy increase
 Principio del impulso-cantidad de movimiento. Impulse-momentum principle
 Principio de los trabajos virtuales. Work, principle of virtual
 Principio de Neumann. Neumann principle
 Principio de reconstrucción. Building-up principle
 Principio de reversibilidad microscópica. Microscopic reversibility, principle of
 Principio de Sumptner. Sumptner's principle
 Principio de superposición. Superposition principle
 Principio isoelectrónica. Isoelectronic principle
 Principios variacionales en estados en desequilibrio. Variational principles for non-equilibrium states
 Principio variacional y energía de enlace. Variation principle and bond energies
 Prisma. Prism
 Prisma de Glan-Thompson. Glan-Thompson prism
 Prisma doble. Biprism
 Probabilidad. Probability
 Probabilidad a posteriori. Posterior probability
 Probabilidad de choque. Collision probability
 Probabilidad de corriente. Current probability
 Probabilidad de escape a la captura por resonancia. Resonance escape probability
 Probabilidad de penetración (penetrabilidad). Penetration probability (penetrability)
 Probabilidades de transición de Einstein. Einstein transition probabilities
 Probabilidad de transición. Transition probability
 Probit. Probit
 Problema a valores de contorno. Boundary value problem
 Problema de Cauchy. Cauchy problem
 Problema de Dido. Dido's problem
 Problema de la masa crítica mínima. Minimum critical mass problem
 Problema de Milne. Milne's problem
 Problema de Pfaff. Pfaff problem
 Problema de Sturm-Liouville. Sturm-Liouville problem
 Problema isoperimétrico. Isoperimetric problem
 Problemas a valores iniciales. Initial-value problems
 Procedimiento de integración doble. Double integration procedure
 Proceso aditivo. Additive process
 Proceso de Gram-Schmidt. Gram-Schmidt process
 Proceso estocástico estacionario. Stationary stochastic process
 Proceso homogéneo. Homogeneous process
 Procesos de nacimiento y muerte. Birth-and-death process
 Procesos naturales e innaturales. Natural and unnatural processes
 Procesos reversibles e irreversibles. Reversible and irreversible processes
 Procesos Umklapp. Umklapp processes
 Producto. Product
 Producto amplificación-ancho de banda. Gain-bandwidth product
 Producto areal de inercia. Inertia, area product of
 Producto cuádruple de vectores. Quadruple product of vectors
 Producto de tensores. Product of tensors
 Producto de Wallis. Wallis product
 Producto directo (de subgrupos). Direct product (of subgroups)
 Producto escalar o interior. Inner product
 Producto escalar. Scalar product
 Producto infinito. Multiplication, infinite
 Producto interior de tensores. Inner product of tensors
 Producto normal. Normal product
 Producto triple de vectores. Triple product of vectors
 Profundidad de penetración. Penetration depth
 Profundidad óptica. Optical depth
 Programa. Program
 Programación lineal. Linear programming
 Progresión. Progression
 Progresión aritmética. Arithmetic progression
 Progresión armónica. Harmonic progression
 Prolongación analítica. Analytic continuation
 Promedio móvil. Moving average
 Propagación de errores. Error, propagation of
 Propiedades de simetría de autofunciones moleculares. Symmetry properties of molecular eigenfunctions
 Propiedades intrínsecas de una superficie. Intrinsic properties of a surface
 Propiedades molares. Molar properties
 Propiedad termodinámica. Thermodynamic property
 Proporción de polarización. Proportion of polarization
 Proyección estereográfica de una esfera sobre un plano. Stereographic projection of a sphere on a plane
 Prueba de Behrens-Fisher. Behrens-Fisher test
 Prueba de Fisher-Yates. Fisher-Yates test
 Prueba de la f . f -test
 Prueba de la carga nula. Zero-load test

- Prueba de Wilcoxon. Wilcoxon's test
 Pruebas de significación. Significance tests
 Pulsaciones. Beats
 Punto binario. Binary point
 Punto circular sobre una superficie. Circular point on a surface
 Punto crítico. Critical point
 Punto de ebullición. Boiling point
 Punto de estancamiento. Stagnation point
 Punto de inflexión. Inflection point
 Punto de ramificación. Branch point
 Punto de vapor. Steam point
 Punto elíptico. Elliptic point
 Punto hiperbólico. Hyperbolic point
 Punto lambda. Lambda point
 Punto objeto. Object point
 Punto ordinario. Ordinary point
 Punto parabólico sobre una superficie. Parabolic point on a surface
 Punto planar. Planar point
 Puntos antinodales. Anti-nodal points
 Puntos antiprincipales. Anti-principal points
 Puntos aplanáticos de una esfera. Aplanatic points of a sphere
 Puntos cardinales de un sistema óptico. Cardinal points of an optical system
 Puntos conjugados de un péndulo compuesto. Conjugate points of a compound pendulum
 Puntos de Bravais. Bravais points
 Puntos fijos. Fixed points
 Puntos focales. Focal points
 Punto singular de una función. Singular point of a function
 Puntos nodales. Nodal points
 Puntos principales. Principal points
 Punto triple. Triple point
 Pureza de excitación. Excitation purity
- Quasi-niveles de Fermi. Fermi levels, quasi
- Radiación coherente. Coherent radiation
 Radiación compleja. Complex radiation
 Radiación de campo. Field, radiation
 Radiación de un cuadrupolo. Quadrupole radiation
 Radiación electromagnética. Electromagnetic radiation
 Radiación infraroja. Infrared radiation
 Radiación térmica. Thermal radiation
 Radiador. Radiator
 Radiador selectivo. Selective radiator
 Radiancia. Radiance
 Radio atómico. Atomic radius
 Radio de Bohr. Bohr radius
 Radio de giro. Radius of gyration
 Radio de curvatura. Curvature, radius of
 Radio de curvatura esférica. Spherical curvature, radius of
 Radio del electrón (clásico) Electron radius (classical)
 Radio de torsión. Torsion, radius of
 Radio efectivo de la barra de regulación. Radius, effective, of control rod
 Radio espectral de una matriz. Spectral radius of a matrix
 Radio gravitacional. Gravitational radius
 Radio vector. Vector, radius
 Raíz. Radix
 Raíz media cuadrática. Root-mean-square
- Raíz principal. Principal root
 Rama. Branch
 Rango. Range
 Rango. Rank
 Rango de una matriz. Rank of a matrix
 Rango de un gráfico. Graph rank
 Rango intercuartil. Interquartile range
 Ranura. Slit
 Rayo alabeado. Skew ray
 Rayo axial. Axial ray
 Rayo principal. Principal ray
 Razón de amortiguamiento. Damping ratio
 Razón de correlación. Correlation ratio
 Razón de delgadez. Slenderness ratio
 Razón de ganancia. Gain ratio
 Razón de rendimiento. Efficiency ratio
 Razón giromagnética. Gyromagnetic ratio
 Reacción. Reaction
 Reacción endotérmica. Endothermic reaction
 Reacciones independientes. Independent reactions
 Reacciones químicas acopladas. Coupled chemical reactions
 Reacción exotérmica. Exothermic reaction
 Reacción normal. Normal reaction
 Reactancia. Reactance
 Reactividad. Reactivity
 Realización. Realization
 Recirculación. Recirculation
 Red activa. Network, active
 Redes duales. Dual networks
 Redondeo. Rounding
 Reducción en área. Reduction in area
 Redundancia. Redundancy
 Reflectancia difusa. Diffuse reflectance
 Reflectancia dirigida. Direct reflectance
 Reflectancia luminosa. Luminous reflectance
 Reflexión de rayos X integrada. X-ray reflection, integrated
 Reflexión difusa. Diffuse reflection
 Reflexión especular. Specular reflection
 Reflexión total. Total reflection
 Refracción atómica. Atomic refraction
 Refracción cónica. Conical refraction
 Refracción doble. Double refraction
 Refracción específica. Specific refraction
 Refracción estándar. Standard refraction
 Refractividad. Refractivity
 Regeneración térmica. Heat regeneration
 Regla de Bragg. Bragg rule
 Regla de Cramer. Cramer's rule
 Regla de diferenciación de funciones compuestas. Chain rule of differentiation
 Regla de Gothert. Gothert's rule
 Regla de inercia de Routh. Routh's rule of inertia
 Regla del área transónica. Transonic area rule
 Regla de las fases. Phase rule
 Regla de la suma-g. g-Sum rule
 Regla de la suma rotacional. Rotational sum rules
 Regla de la suma vibracional para transiciones electrónicas. Vibrational sum rule for electronic transitions
 Regla de Leibnitz. Leibnitz rule
 Regla del flujo. Flow rule
 Regla de L'Hospital. L'Hospital rule
 Regla del intervalo de Landé. Landé's interval rule
 Regla de los signos de Descartes. Descartes rule of signs
 Regla de Prandtl-Glauert. Prandtl-Glauert rule
 Regla de Silsbee. Silsbee rule

- Regla de Simpson. Simpson's rule
 Regla de Weddle. Weddle rule
 Regla trapezoidal. Trapezoidal rule
 Reglas de anticonmutación. Anti-commutation rules
 Reglas de conmutación. Commutation rules
 Reglas de conmutación covariante. Covariant commutation rules
 Reglas de Hume-Rothery. Hume-Rothery rules
 Reglas de Hund para multipletes atómicos. Hund's rules for atomic multiplets
 Reglas de intensidad para multipletes. Intensity rules for multiplets
 Reglas de selección. Selection rules
 Reglas de selección nuclear. Selection rules, nuclear
 Reglas de superselección. Superselection rules
 Región crítica. Critical region
 Región de velocidad recíproca. Reciprocal velocity region
 Región equipotencial. Equipotential region
 Región simplemente conexa. Simply connected region
 Regresión. Regression
 Regula falsi. Regula falsi
 Relación. Ratio
 Relación aire-combustible. Air-fuel ratio
 Relación de aspecto. Aspect ratio
 Relación de aumento de un proyector. Magnification ratio of a projector
 Relación de conversión. Conversion ratio
 Relación de Geiger-Nutall. Geiger-Nutall relation
 Relación de onda estacionaria. Standing-wave ratio
 Relación de Poisson. Poisson's ratio
 Relación de retorno. Return ratio
 Relaciones de Cauchy. Cauchy relations
 Relaciones de conmutación. Commutation relations
 Relaciones de dispersión. Dispersion relations
 Relaciones de Ehrenfest. Ehrenfest's relations
 Relaciones de Maxwell. Maxwell's relations
 Relaciones de Lamé. Lamé relations
 Relaciones de Onsager. Onsager relations
 Relaciones de reciprocidad. Reciprocity relations
 Relaciones energéticas en un campo de fuerzas uniforme. Energy relations in uniform force field
 Relación exposición-densidad. Exposure-density relationship
 Relación magnetomecánica. Magnetomechanical ratio
 Relación rango-energía. Range-energy relation
 Relajación. Relaxation
 Relajación en bloque. Block relaxation
 Relatividad especial. Relativity, special
 Relatividad general. Relativity, general
 Remanencia. Set permanent
 Rendimiento. Efficiency
 Rendimiento cuántico. Quantum yield
 Rendimiento cuántico. Quantum efficiency
 Rendimiento de Auger. Auger yield
 Rendimiento de etapa. Stage efficiency
 Rendimiento de fluorescencia. Fluorescence yield
 Rendimiento indicado. Indicated efficiency
 Rendimiento límite. Limit, yield
 Rendimiento luminoso relativo. Relative luminous efficiency
 Rendimiento mecánico. Mechanical efficiency
 Rendimiento radiante. Radiant efficiency
 Rendimiento superficial. Surface, yield
 Rendimiento térmico al freno. Brake thermal efficiency
 Rendimiento térmico de un ciclo. Thermal efficiency of cycle
 Renormalización de la carga. Charge renormalization
 Renormalización de la masa. Mass renormalization
 Repetibilidad. Repeatability
 Representación. Representation
 Representación conforme del plano complejo. Conformal mapping of the complex plane
 Representación conforme. Conformal mapping
 Representación de Dyson. Dyson representation
 Representación de Foldy-Wouthuysen. Foldy-Wouthuysen representation
 Representación de grupos y mecánica cuántica. Representation of groups and quantum mechanics
 Representación de Heisenberg. Heisenberg representation
 Representación de interacción. Interaction representation
 Representación de Lehmann. Lehmann representation
 Representación de respuestas de frecuencia. Frequency response representation
 Representación de Schrödinger. Schrödinger representation
 Representación de un grupo. Representation of a group
 Representaciones del grupo inhomogéneo de Lorentz. Representations of the inhomogeneous Lorentz group
 Representaciones equivalentes de grupos. Equivalent representations of groups
 Representación esférica de una congruencia rectilínea. Spherical representation of a rectilinear congruence
 Representación esférica de una superficie. Spherical representation of a surface
 Repulsión de Born. Born repulsion
 Residuo. Residue
 Resistencia. Resistance
 Resistencia acústica específica. Resistance, specific acoustic
 Resistencia de radiación. Radiation resistance
 Resistencia eléctrica. Electrical resistance
 Resistencia por rodadura. Rolling resistance
 Resolución de fuerzas. Forces, resolution of
 Resonancia de Fermi. Fermi resonance
 Resonancia ferromagnética. Ferromagnetic resonance
 Resonancia magnética nuclear. Nuclear magnetic resonance
 Resonancia mecánico cuántica. Resonance, quantum mechanical
 Resorte(s). Spring(s)
 Respuesta acústica. Responsiveness, acoustic
 Respuesta de frecuencias. Frequency response
 Respuesta indicial. Indicial response
 Respuesta normal. Normal response
 Respuesta transitoria de un sistema dinámico. Transient response of a dynamical system
 Resto. Remainder
 Resultante. Resultant
 Reticulado. Lattice
 Reticulado espacial. Space lattice
 Reticulado recíproco. Reciprocal lattice
 Retroalimentación. Feedback
 Riesgo del consumidor. Consumer's risk
 Roseta. Rosette
 Rotación. Rotation
 Rotación específica. Rotation, specific
 Rotación molecular. Molecular rotation
 Rotor. Rotor
 Rotor. Curl
 Ruido. Noise
 Runflas. Runs

- Rutherford. Rutherford
Rutina. Routine
- Sabin. Sabin
Salto hidráulico. Hydraulic jump
Sección principal de un cristal. Principal section of a crystal
Sección transformada. Transformed section
Sección transversal. Cross-section
Segunda cuantificación. Quantization, second
Segunda ley de la termodinámica. Second law of thermodynamics
Segundo punto focal. Second focal point
Segundo teorema fundamental. Second fundamental theorem
Segundo teorema límite. Second limit theorem
Semejanza de turbulencia. Turbulence, similarity of
Semejanza dinámica. Dynamical similarity
Semianchura de una línea espectral. Half-width of a spectral line
Sensibilidad. Sensitivity
Sensibilidad de resolución. Resolution sensitivity
Sensibilidad fotoeléctrica. Photoelectric sensitivity
Sentido de un vector. Sense of a vector
Separable. Separable
Separación de variables. Separation of variables
Separación de impulsos. Pulse separation
Separación de impulsos. Pulse spacing
Separación de la capa límite. Separation of boundary layer
Separación energética entre dos bandas. Energy gap
Serie. Series
Serie armónica de sonidos. Harmonic series of sounds
Serie de Bergmann. Bergmann series
Serie de Brackett. Brackett series
Serie de Dirichlet. Dirichlet series
Serie de Enskog para las soluciones de la ecuación de Boltzmann. Enskog's series for the solutions of the Boltzmann equation
Serie de Lyman. Lyman series
Serie de Paschen. Paschen series
Serie de Pfund. Pfund series
Serie de Taylor. Taylor series
Serie difusa. Diffuse series
Serie nitida. Sharp series
Serie principal. Principal series
Series asintóticas. Asymptotic series
Series de composición de un grupo. Composition series of a group
Series de Edgeworth. Edgeworth's series
Series de Fourier. Fourier series
Series de Gram-Charlier. Gram-Charlier series
Series de Laurent. Laurent series
Series de Liouville-Neumann. Liouville-Neumann series
Series de Maclaurin. Maclaurin series
Series de Neumann. Neumann series
Series de potencias. Power series
Series de Rydberg. Rydberg series
Series multinomiales. Multinomial series
Seudoescalar. Pseudoscalar
Seudovector. Pseudovector
Signo. Signum
Símbolo de permutación. Permutation symbol
Símbolos cristalinos de Schönflies. Schönflies crystal symbols
Símbolos de Christoffel. Christoffel (3-index) symbols
Símbolos de Hermann-Mauguin. Hermann-Mauguin symbols
Símbolos de términos atómicos. Atomic term symbols
Símbolos de términos moleculares. Molecular term symbols
Simetría cristalina. Crystal symmetry
Simetría y mecánica cuántica. Symmetry and quantum mechanics
Simultáneamente medible. Simultaneously measurable
Singular. Singular
Singularidades. Singularities
Singularidad esencial de una función analítica. Essential singularity of an analytic function
Singularidad regular de una ecuación diferencial. Regular singularity of a differential equation
Sistema acústico. Acoustical system
Sistema afocal. Afocal system
Sistema anamórfico. Anamorphic system
Sistema de bandas. Band system
Sistema catóptrico. Catoptric system
Sistema centroidal. Center-of-mass system
Sistema condensado. Condensed system
Sistema conjugado de curvas sobre una superficie. Conjugate system of curves on a surface
Sistema conservativo. Conservative system
Sistema cúbico. Cubic system
Sistema de Chebyshev. Chebyshev system
Sistema de control. Control system
Sistema de coordenadas rotativo. Rotating coordinate system
Sistema de ecuaciones. System, equations
Sistema de movilidad mecánica. Mechanical mobility system
Sistema de referencia. Frame of reference
Sistema de unidades. Units, system of
Sistema-*e*. *e*-System
Sistema eléctrico. Electrical system
Sistema elemental. Elementary system
Sistema epsilon. Epsilon-system
Sistema exagonal. Hexagonal system
Sistema inercial. Inertial frame
Sistema *L* y Sistema *C*. *L*-system and *C*-system
Sistema mecánico rectilíneo. Mechanical rectilinear system
Sistema mecánico rotacional. Mechanical rotational system
Sistema motriz dinámico. Driving system, dynamic
Sistema motriz electromagnético. Driving system, electromagnetic
Sistema motriz electrostático. Driving system, electrostatic
Sistema motriz por magnetostricción. Driving system, magnetostriction
Sistema ortonormal. Orthonormal system
Sistema ortorrómbico. Orthorhombic system
Sistema ortoscópico. Orthoscopic system
Sistema oscilante degenerado. Degenerate oscillating system
Sistema recíproco de vectores. Vector system, reciprocal
Sistema romboédrico. Rhombohedral system
Sistemas abiertos. Open systems
Sistemas aislados. Isolated systems
Sistemas azeotrópicos. Azeotropic systems
Sistemas azeotrópicos. Azeotropic systems
Sistemas cerrados. Closed systems
Sistemas continuos. Continuous systems
Sistemas cristalinos. Crystal systems

- Sistemas de fuerzas equivalentes. Equivalent force systems
- Sistemas discontinuos. Discontinuous systems
- Sistemas ideales. Ideal systems
- Sistemas lineales. Linear systems
- Sistemas ópticos perfectos. Perfect optical systems
- Sistemas ternarios. Ternary systems
- Sistema telecéntrico. Telecentric system
- Sistema termodinámico. Thermodynamic system
- Sistema tetragonal. Tetragonal system
- Sistema triclinico. Triclinic system
- Sistema tricromático. Trichromatic system
- Sobrerrelajación. Overrelaxation
- Solución de Bayes. Bayes' solution
- Soluciones atérmicas. Athermal solutions
- Soluciones conformes. Conformal solutions
- Soluciones perfectas. Perfect solutions
- Soluciones regulares. Regular solutions
- Solución singular. Singular solution
- Son. Sone
- Stilb. Stilb
- Störmer method. Störmer method
- Subarmónico. Subharmonic
- Subgráfico. Subgraph
- Subgráfico conexo maximal. Subgraph, maximal connected
- Subgráfico propio. Subgraph, proper
- Subgráficos disjuntos. Subgraphs, disjoint
- Subgrupo. Subgroup
- Subrutina. Subroutine
- Substancia de un vórtice. Substance of a vortex
- Succión de una capa límite laminar. Suction of laminar boundary layer
- Suceptibilidad paramagnética. Paramagnetic susceptibility
- Sucesión. Sequence
- Sucesión de Cauchy. Cauchy sequence
- Sucesión de Sturm. Sturm sequence
- Sucesión marginal. Edge sequence
- Sucesión nula. Null sequence
- Suficiencia. Sufficiency
- Suma de Petzval. Petzval sum
- Suma de series. Summation of series
- Suma de Slater. Slater sum
- Suma de tensores. Sum of tensors
- Suma directa. Direct sum
- Suma reticular. Lattice sum
- Sumas de combinación. Combination sums
- Sumidero. Sink
- Superconjugación. Hyperconjugation
- Superficie alabeada. Skew surface
- Superficie anticlástica. Anticlastic surface
- Superficie aplanática. Aplanatic surface
- Superficie de calefacción. Heating surface
- Superficie de revolución. Surface of revolution
- Superficie de Riemann. Riemann surface
- Superficie de centros. Surface of centers
- Superficie de confusión mínima. Surface of least confusion
- Superficie de distribución de intensidades. Intensity distribution, surface of
- Superficie de división de Gibbs. Gibbs division surface
- Superficie de equifase. Equiphasic surface
- Superficie de Liouville. Liouville surface
- Superficie de ondas de Rayleigh. Waves, Rayleigh surface
- Superficie de Petzval. Petzval surface
- Superficie desarrollable. Developable surface
- Superficie equipotencial. Equipotential surface
- Superficie isobárica. Isobaric surface
- Superficie minimal. Minimal surface
- Superficie reglada. Ruled surface
- Superficies aplicables. Applicable surfaces
- Superficies Cartesianas. Cartesian surfaces
- Superficies figuradas. Figured surfaces
- Superficies inversas. Inverse surfaces
- Superficies paralelas. Parallel surfaces
- Superintercambio. Superexchange
- Supermultiplete. Hypermultiplet
- Supermultiplete. Supermultiplet
- Superreticulado. Superlattice
- Talbot. Talbot
- Tamaño crítico. Critical size
- Tangente a una curva. Tangent to a curve
- Tangente geodésica. Geodesic tangent
- Teorema acústico de Thévenin. Thévenin's acoustical theorem
- Teorema alternativo. Alternative theorem
- Teorema binomial. Binomial theorem
- Teorema de Abbe-Maxwell. Abbe-Maxwell theorem
- Teorema de aproximación de Weierstrass. Weierstrass approximation theorem
- Teorema de Ascoli. Ascoli's theorem
- Teorema de Bayes. Bayes' theorem
- Teorema de Beltrami-Enneper. Beltrami-Enneper theorem
- Teorema de Bernoulli. Bernoulli theorem
- Teorema de Bloch. Bloch theorem
- Teorema de Bolzano-Weierstrass. Bolzano-Weierstrass theorem
- Teorema de Budan. Budan theorem
- Teorema de Carathéodory. Carathéodory's theorem
- Teorema de Carnot. Carnot's theorem
- Teorema de Castiglione. Castiglione's theorem
- Teorema de Clapeyron. Clapeyron's theorem
- Teorema de Cochran. Cochran's theorem
- Teorema de convergencia por mayoramiento. Comparison theorem for convergence
- Teorema de Crocco. Crocco's theorem
- Teorema de De Moivre-Laplace. De Moivre-Laplace theorem
- Teorema de Duhem. Duhem's theorem
- Teorema de Earnshaw. Earnshaw theorem
- Teorema de Ehrenfest. Ehrenfest theorem
- Teorema de equivalencia. Equivalence theorem
- Teorema de Euler sobre funciones homogéneas. Euler theorem for homogeneous functions
- Teorema de factorización única. Unique factorization theorem
- Teorema de Floquet. Floquet theorem
- Teorema de fluctuación-disipación. Fluctuation-dissipation theorem
- Teorema de Fredholm sobre ecuaciones integrales. Fredholm theorem for integral equations
- Teorema de Fuchs. Fuchs theorem
- Teorema de Gauss-Markov. Gauss-Markov theorem
- Teorema de Hamilton. Hamilton's theorem
- Teorema de Hamilton-Cayley. Hamilton-Cayley theorem
- Teorema de Heine-Borel. Heine-Borel theorem
- Teorema de Jacobi en óptica. Jacob's theorem in optics
- Teorema de Jordan-Hölder. Jordan-Hölder theorem
- Teorema de la circulación (Kelvin). Circulation theorem (Kelvin)

- Teorema de la curvatura de Gauss. Gauss' theorem on curvature
- Teorema de la curvatura normal de Euler. Euler theorem on normal curvature
- Teorema de la entalpía. Enthalpy theorem
- Teorema de Lagrange. Lagrange theorem
- Teorema de la integral de Cauchy. Cauchy integral theorem
- Teorema de la óptica geométrica de Kirchhoff. Kirchhoff's geometric optics theorem
- Teorema de la tangente. Law of tangents
- Teorema del calor de Nernst. Nernst heat theorem
- Teorema del coseno. Law of cosines
- Teorema de Le Chatelier. Chatelier (Le) theorem
- Teorema de Liouville. Liouville theorem
- Teorema del mínimo de energía potencial. Potential energy, theorem of minimum
- Teorema de los ejes perpendiculares. Perpendicular axis theorem
- Teorema de los residuos. Residue theorem
- Teorema del peso estadístico. Statistical weight theorem
- Teorema del punto fijo de Brouwer. Brouwer fixed point theorem
- Teorema del seno. Law of sines
- Teorema del trabajo-energía cinética. Work-kinetic energy theorem
- Teorema del trabajo mínimo. Least work, theorem of
- Teorema de Maupertui. Maupertui theorem
- Teorema de Maxwell para isoterms. Maxwell's theorem for isotherms
- Teorema de Meunier. Meunier's theorem
- Teorema de moderación. Moderation theorem
- Teorema de Morera. Morera theorem
- Teorema de Norton. Norton's theorem
- Teorema de Nyquist. Nyquist theorem
- Teorema de polarización de Stokes. Stokes polarization theorem
- Teorema de Poynting. Poynting theorem
- Teorema de reciprocidad. Reciprocity theorem
- Teorema de reciprocidad acústica. Reciprocity theorem, acoustical
- Teorema de reciprocidad eléctrica. Electrical reciprocity theorem
- Teorema de reciprocidad electroacústica. Reciprocity theorem, electroacoustical
- Teorema de reciprocidad electromagnética. Reciprocity theorem, electromagnetic
- Teorema de reciprocidad mecánico-acústico. Mechanical-acoustical reciprocity theorem
- Teorema de reciprocidad mecánico rectilínea. Mechanical rectilinear reciprocity theorem
- Teorema de reciprocidad mecánico rotacional. Mechanical rotational reciprocity theorem
- Teorema de representación de Riemann. Riemann mapping theorem
- Teorema de Ricci. Ricci's theorem
- Teorema de Rolle. Rolle theorem
- Teorema de Saurel. Saurel's theorem
- Teorema de semejanza. Similarity theorem
- Teorema de reciprocidad eléctrico-mecánica. Electrical-mechanical reciprocity theorem
- Teorema de Sturm. Sturm theorem
- Teorema de superposición. Superposition theorem
- Teorema de transferencia para el momento de inercia. Moment of inertia, transfer theorem for
- Teorema de Van't Hoff. Van't Hoff theorem
- Teorema de varignon. Varignon theorem
- Teorema eléctrico de Thévenin. Thévenin's electrical theorem
- Teorema factorial del algebra. Factor theorem of algebra
- Teorema-*H*. *H*-theorem
- Teorema Límite central. Central limit theorem
- Teorema mecánico rectilíneo de Thévenin. Thévenin's mechanical rectilinear theorem
- Teorema mecánico rotacional de Thévenin. Thévenin's mechanical rotational theorem
- Teorema óptico. Optical theorem
- Teorema recíproco. Reciprocal theorem
- Teoremas integrales del análisis vectorial. Integral theorems of vector analysis
- Teorema límite sinusoidal. Sinusoidal limit theorem
- Teoremas de Gibbs-Konovalov. Gibbs-Konovalov theorems
- Teoremas del valor medio. Mean value theorems
- Teoremas límites. Limit theorems
- Teorema sobre las series de potencias de Abel. Abel theorem on power series
- Teorema de Haag. Haag's theorem
- Teoría absoluta de la velocidad de reacción. Absolute reaction rate theory
- Teoría cinética. Kinetic theory
- Teoría cinética de la conductividad térmica. Thermal conductivity, kinetic theory of
- Teoría cinética de la difusión. Diffusion, kinetic theory of
- Teoría cinética de la viscosidad. Viscosity, kinetic theory of
- Teoría clásica del electrón. Classical electron theory
- Teoría cuántica de la capacidad térmica. Quantum theory of heat capacity
- Teoría cuántica de la dispersión. Quantum theory of dispersion
- Teoría cuántica del espectro. Quantum theory of spectra
- Teoría de bandas de energía en sólidos. Band theory of solids
- Teoría de Born-Infeld. Born-Infeld theory
- Teoría de Born y Von Karman. Born and Von Karman theory
- Teoría de choque en cinética química. Collision theory of chemical kinetics
- Teoría de difusión de neutrones. Neutron diffusion theory
- Teoría de distribución. Distribution theory
- Teoría de Enskog. Enskog's theory
- Teoría de estimación. Estimation, theory of
- Teoría de Eyring de procesos de transporte. Eyring theory of transport processes
- Teoría de Heitler-London de la molécula de hidrógeno. Hydrogen molecule, Heitler-London theory of
- Teoría de Huecos. Hole theory
- Teoría de Juegos. Games theory
- Teoría de la desintegración beta de Fermi. Fermi theory of beta decay
- Teoría de la desviación. Deflection theory
- Teoría de la línea de ruptura. Rupture line theory
- Teoría de las perturbaciones. Perturbation theory
- Teoría de las vibraciones cristalinas de Debye. Debye theory of crystal vibrations
- Teoría de la valencia de Heitler-London. Heitler-London theory of valence
- Teoría del cuerpo delgado. Slender-body theory
- Teoría del diamagnetismo de Langevin. Langevin theory of diamagnetism
- Teoría del electrón de Dirac. Dirac electron theory

- Teoría del equilibrio. Equilibrium theory
- Teoría del ferromagnetismo de Heisenberg. Heisenberg theory of ferromagnetism
- Teoría de los campos cuánticos relativistas. Relativistic quantum field theory
- Teoría del perfil de ala delgado. Thin-airfoil theory
- Teoría del positrón de Feynman. Feynman's positron theory
- Teoría del reactor asintótico. Asymptotic reactor theory
- Teoría del transporte de neutrones. Neutron transport theory
- Teoría de matrices S y teoría cuántica de campos. S -Matrix theory and quantum field theory
- Teoría de Neyman-Pearson. Neyman Pearson theory
- Teoría de órbitas planetarias. Planetary orbit theory
- Teoría de perturbación covariante. Covariant perturbation theory
- Teoría de planos aerodinámicos. Airfoil theory
- Teoría de renovación. Renewal theory
- Teoría de segundo orden de Busemann. Busemann's second-order theory
- Teoría electrónica de metales. Electron theory of metals
- Teoría cuántica del magnetismo. Magnetism, quantum theory of
- Teoría incremental de la plasticidad. Incremental theory of plasticity
- Teoría orbital molecular de la valencia. Molecular orbital theory of valence
- Teoría reticular de coeficientes elásticos. Elastic coefficients, lattice theory of
- Temperatura. Temperature
- Temperatura absoluta. Absolute temperature
- Temperatura de Boyle. Boyle temperature
- Temperatura de brillo. Luminance temperature
- Temperatura de Debye. Debye temperature
- Temperatura de color. Color temperature
- Temperatura de Curie. Curie temperature
- Temperatura de degeneración. Degeneracy temperature
- Temperatura de estancamiento. Stagnation temperature
- Temperatura de Fermi. Fermi temperature
- Temperatura de inversión. Inversion temperature
- Temperatura de neutrones. Temperature, neutron
- Temperatura de radiación total. Total radiation temperature
- Temperatura de teoría cinética. Kinetic theory temperature
- Temperatura de transición de segundo orden. Second order transition temperature
- Temperatura de volumen. Bulk temperature
- Temperatura potencial. Potential temperature
- Temperatura virtual. Virtual temperature
- Temperatura y presión normales. Normal temperature and pressure
- Tendencia de la presión barométrica. Tendency of barometric pressure
- Tensión admisible. Stress, allowable
- Tensión combinada. Stress, combined
- Tensión diagonal. Diagonal tension
- Tensión directa. Stress, direct
- Tensión electromagnética. Electromagnetic stress
- Tensiones de reacción. Reaction stresses
- Tensiones residuales. Residual stresses
- Tensiones de Reynolds. Reynolds stresses
- Tensiones secundarias. Secondary stresses
- Tensión generalizada. Stress, generalized
- Tensión hidrostática. Stress, hydrostatic
- Tensión octaédrica. Stress, octahedral
- Tensión principal. Stress, principal
- Tensión superficial de una interfase curvada. Surface tension of curved interface
- Tensión superficial dinámica. Dynamical surface tension
- Tensor absoluto. Tensor, absolute
- Tensor alternado. Tensor, alternating
- Tensor antisimétrico. Tensor, skew-symmetric
- Tensor covariante fundamental. Fundamental covariant tensor
- Tensor de campo electromagnético. Electromagnetic field tensor
- Tensor de curvatura. Curvature tensor
- Tensor de Ricci. Ricci tensor
- Tensor de Riemann-Cristoffel. Riemann-Cristoffel tensor
- Tensor de superficie. Tensor, surface
- Tensor doble. Tensor, double
- Tensor energía-impulso. Energy-momentum tensor
- Tensor isotrópico. Tensor, isotropic
- Tensor métrico. Metric tensor
- Tensor ponderado. Tensor, weighted
- Tensor relativo. Tensor, relative
- Tensor simétrico. Tensor, symmetric
- Tensores afines y vectores libres. Affine tensors and free vectors
- Tensores asociados (campos tensoriales). Tensors, associated (tensor fields)
- Tercera ley de la termodinámica. Third law of thermodynamics
- Terminización de neutrones. Thermalization of neutrons
- Térmico atómico impar. Odd term of an atom
- Término de Pauli. Pauli term
- Término espectroscópico. Term, spectroscopic
- Término par de un átomo. Even term of an atom
- Términos anarmónicos. Anharmonic terms
- Términos de Balmer. Balmer terms
- Términos de estados de energía. Energy state, term
- Termodinámica de procesos irreversibles. Thermodynamics of irreversible processes
- Termostadística. Thermostatistics
- Termostática. Thermostatics
- Tetrasimétrico. Symmetry, tetrad
- Tiempo de difusión de neutrones. Diffusion time, neutron
- Tiempo de generación. Generation time
- Tiempo de relajación. Relaxation time
- Tiempo de respuesta. Response time
- Tiempo de retardación. Slowing-down time
- Tiempo de retraso. Delay time
- Tiempo libre medio. Mean free time
- Tipo de tensor. Tensor, type of
- Tipos de enlace. Bond types
- Tono térmico. Heat tone
- Topología. Topology
- Toro. Torus
- Toro de Vieth-Müller. Vieth-Müller torus
- Torsión. Torsion
- Trabajao. Work
- Trabajo de extracción. Work functions, electronic
- Transferencia de control. Transfer of control
- Transferencia por impacto. Collisional transfer
- Transformación. Transformation
- Transformación afin. Affine transformation
- Transformación canónica. Canonical transformation
- Transformación de contacto. Contact transformation
- Transformación de Euler. Euler transformation

- Transformación de Fourier-Bessel. Fourier-Bessel transform
- Transformación de Galileo. Galilean transformation
- Transformación de Handel. Handel transform
- Transformación de Hilbert. Hilbert transform
- Transformación de Laplace. Laplace transform
- Transformación de Lorentz. Lorentz transformation
- Transformación de Lorentz homogénea. Homogeneous Lorentz transformation
- Transformación de Lorentz no-homogénea. Inhomogeneous Lorentz transformation
- Transformación de Lorentz propia. Proper Lorentz transformation
- Transformación de Mellin. Mellin transform
- Transformación de Schwartz-Christoffel. Schwartz-Christoffel transformation
- Transformaciones homogéneas restringidas de Lorentz. Restricted homogeneous Lorentz transformations
- Transformación homográfica. Homographic transformation
- Transformación integral. Integral transform
- Transformación $L R$. $L R$ transformation
- Transformación ortócrona homogénea de Lorentz. Orthochronous homogeneous Lorentz transformation
- Transformación ortócrona inhomogénea de Lorentz. Orthochronous inhomogeneous Lorentz transformation
- Transformada. Transform
- Transición admisible. Transition, allowed
- Transición del flujo turbulento. Transition of turbulent flow
- Transición de superconducción. Superconducting transition
- Transiciones de orden superior. Transitions of higher order
- Transición permitida. Allowed transition
- Transición prohibida. Forbidden transition
- Transmisibilidad de fuerzas. Transmissibility of force
- Transmisión. Transmission
- Transmisión sonora. Sound transmission
- Transmisividad. Transmissivity
- Transmitancia. Transmittance
- Transmitancia difusa. Diffuse transmittance
- Transmitancia directa. Direct transmittance
- Transmitancia espectral. Spectral transmittance
- Transmitancia interna. Internal transmittance
- Transmitancia total. Total transmittance
- Transmitividad. Transmittivity
- Transposición. Transposition
- Transpuesta de una matriz. Transpose of a matrix
- Trascendente. Transcendental
- Traslación. Translation
- Traslación primitiva. Primitive translation
- Trayectoria. Trajectory
- Trayectoria de tensión. Stress trajectory
- Trayectoria media de chaparrón. Shower unit
- Traza. Trace
- Traza de una matriz. Trace of a matrix
- Tren marginal. Edge train
- Tren marginal abierto. Edge train, open
- Tren marginal cerrado. Edge train, closed
- Triángulo de colores. Color triangle
- Triángulo de fuerzas. Forces, triangle of
- Triángulo de Pascal. Pascal triangle
- Triángulo geodésico. Geodesic triangle
- Trigonometría. Trigonometry
- Trigonometría esférica. Spherical trigonometry
- Trisimétrico. Symmetry, triad
- Trompo dormido. Top, "sleeping"
- Truncamiento. Truncation
- Turbulencia. Turbulence
- Turbulencia en túneles de viento. Turbulence in wind tunnels
- Turbulencia homogénea. Turbulence, homogeneous
- Turbulencia isotrópica. Turbulence, isotropic
- Umbral de Audibilidad. Audibility, threshold of
- Umbral fotoeléctrico. Photoelectric threshold
- Unidad Ångstrom. Ångstrom unit
- Unidad aritmética. Arithmetic unit
- Unidad de almacenamiento. Storage unit
- Unidad de Amagat. Amagat unit
- Unidad Debye. Debye unit
- Unidad de masa atómica. Atomic mass unit
- Unidad de peso atómico. Atomic weight unit
- Unidad de Siegbahn. Siegbahn x -Unit
- Unidades acústicas. Acoustical units
- Unidades acústicas. Acoustic units
- Unidades de calor. Heat, units of
- Unidades electromagnéticas. Electromagnetic units
- Unidades mecánicas. Mechanical units
- Unidades térmicas. Thermal units
- Unidades tricromáticas. Trichromatic units
- Unidad fundamental. Unit, fundamental
- Unidad mache. Mache unit
- Unidad térmica centígrada. Centigrade heat unit
- Unidad x . x -Unit
- Unión heteropolar. Heteropolar bond
- Unión homopolar. Homopolar bond
- Universo de Minkowski. Minkowski world
- Utilización térmica. Thermal utilization
- Valencia dirigida. Directed valency
- Valle de onda. Wave through
- Valor calorífico. Calorific value
- Valor de esperanza. Expectation value
- Valor de referencia. Reference value
- Valores singulares. Singular values
- Valor gamma. Gamma-value
- Valor medio de una función. Mean value of a function
- Valor- Q . Q -Value
- Vapor seco. Dry vapor
- Variable aleatoria. Random variable
- Variable aleatoria discreta. Discrete variate
- Variable de acción. Action variable
- Variable de ángulo. Angle variable
- Variabes de Lode. Lode's variables
- Variabes extensivas. Extensive variables
- Variabes intensivas. Intensive variables
- Variación acotada. Variation, bounded
- Variación temporal de la producción de entropía. Time variation of the entropy production
- Variación. Variance
- Variedad. Manifold
- Vector. Vector
- Vector Bra. Bra vector
- Vector campo eléctrico. Electric field vector
- Vector contravariante. Vector, contravariant
- Vector covariante. Vector, covariant
- Vector de Burgers. Burgers vector
- Vector de Darboux. Darboux vector
- Vector de desplazamiento eléctrico. Electric displacement vector
- Vector de Hertz. Hertz vector

- Vectores asociados (campos vectoriales) Vectors, associated (vector fields)
 Vectores cuatridimensionales y tensores. Four-vectors and tensors
 Vectores de base. Base vectors
 Vectores de flujo. Flux vectors
 Vectores linealmente independientes. Linearly independent vectors
 Vectores ortogonales. Vector(s), orthogonal
 Vector estado. State vector
 Vector irrotacional. Vector, irrotational
 Vector Ket. Ket vector
 Vector nulo. Null vector
 Vector polar. Vector, polar
 Vector posición. Vector, position
 Vector Poynting. Poynting vector
 Vector principal de una matriz. Principal vector of a matrix
 Velocidad. Velocity
 Velocidad angular. Angular velocity
 Velocidad areal. Velocity, areal
 Velocidad crítica. Speed, critical
 Velocidad de activación. Activation rate
 Velocidad de atenuación del sonido. Rate of decay of sound
 Velocidad de convección. Convective rate of change
 Velocidad de difusión. Diffusion velocity
 Velocidad de escape. Escape velocity
 Velocidad de fase. Phase velocity
 Velocidad de fase de las ondas. Waves, phase velocity of
 Velocidad de fricción. Friction velocity
 Velocidad de generación. Generation rate
 Velocidad de grupo. Group velocity
 Velocidad de grupo, de ondas. Waves, group velocity of
 Velocidad de partícula. Velocity, particle
 Velocidad de recombinación. Recombination rate
 Velocidad de recombinación. Recombination velocity
 Velocidad de repetición de impulsos. Pulse repetition rate
 Velocidad de volumen. Volume velocity
 Velocidad efectiva. Velocity, effective
 Velocidades de procesos irreversibles. Rates of irreversible processes
 Velocidades generalizadas. Velocities, generalized
 Velocidad local. Local rate of change
 Velocidad media. Velocity, average
 Velocidad molecular. Molecular velocity
 Ventilador. Blower
 Verosimilitud. Likelihood
 Versor. Versor
 Vértice. Vertex
 Vértice final. Vertex, final
 Vértice inicial. Vertex, initial
 Vértice interno. Vertex, internal
 Vertices adyacentes. Vertices, adjacent
 Vértice terminal. Vertex, terminal
 Vibración de combinación. Combination vibration
 Vibraciones amortiguadas. Vibrations, damped
 Vibraciones aperiódicas. Vibrations, non-periodic
 Vibraciones forzadas. Vibrations, forced
 Vibraciones libres de sistemas no amortiguados. Free vibrations of undamped system
 Vibraciones normales en moléculas poliatómicas. Normal vibrations in polyatomic molecules
 Vida media. Half-life
 Vida media. Life, mean
 Vida media. Lifetime
 Vida media de un estado atómico. Mean lifetime of an atomic state
 Viento. Wind
 Viento geostrófico. Geostrophic wind
 Viga. Beam
 Viga empotrada. Beam, fixed-ended or encastre
 Viga columna. Beam, column
 Viga compuesta. Beam, composite
 Viga continua. Beam, continuous
 Viga corta. Beam, short
 Viga curvada. Beam, curved
 Viga saliente. Beam, overhanging
 Viga simple. Beam, simple
 Viga sobre cimiento elástico. Beam on elastic foundation
 Viga voladiza. Beam, cantilever
 Viga voladiza. Cantilever
 Vínculo. Constraint
 Vínculo redundante. Redundant constraint
 Virial. Virial
 Virial de un sistema. Virial of a system
 Viscoelasticidad lineal. Viscoelasticity, linear
 Viscosidad. Viscosity
 Viscosidad dinámica. Viscosity, dynamic
 Viscosidad Newtoniana. Newtonian viscosity
 Volumen barrido. Swept volume
 Vórtice. Vortex
 Vórtice anular. Vortex ring
 Vórtice de Rankine. Rankine vortex
 Vórtice esférico. Spherical vortex
 Vorticidad. Vorticity
 Vorticidad anticiclónica. Anticyclonic vorticity
 Vorticidad ciclónica. Cyclonic vorticity
 Vorticidad geostrófica. Geostrophic vorticity
 Weber. Weber
 Wronskiano. Wronskian

Russian—English Index

- Аббе-Максвелла теорема. Abbe-Maxwell theorem
Аббе условие синусов. Abbe sine condition
Абелева группа. Abelian group
Абелево уравнение. Abel equation
Абеля неравенство. Abel inequality
Аберрации оптической системы. Aberrations of an optical system
Аберрация света. Aberration of light
Абсолютная активность. Absolute activity
Абсолютная непрерывность. Absolute continuity
Абсолютная температура. Absolute temperature
Абсолютное дифференциальное исчисление. Absolute differential calculus
Абсолютное ускорение. Absolute acceleration
Абсолютный скаляр. Absolute scalar
Абсолютный тензор. Absolute tensor
Авария. Failure
Авогадро постоянная. Avogadro constant
Автокорреляция. Autocorrelation
Авторегрессия. Autoregression
Адамса-Башфорда метод. Adams-Bashford method
Адвекция. Advection
Аддитивный процесс. Additive process
Адиабатическая стенка. Adiabatic wall
Адиабатический закон квантованных состояний. Adiabatic law for quantized states
Адиабатическое изменение. Adiabatic change
Адиабатическое размагничивание. Adiabatic demagnetization
Адиабатическое убывание температуры с высотой. Adiabatic lapse rate
Адиабатическое уравнение состояния. Adiabatic equation of state
Адрес инструкции. Address
Адсорбционная изотерма. Adsorption isotherm
Адьюнкт. Cofactor
Азимут. Azimuth angle
Аэродинамическая подъёмная сила. Aerodynamic lift
Аксиальное увеличение. Axial magnification
Аксиальный луч. Axial ray
Аксиальный пучок. Axial bundle
Активная сила. Active force
Активная сеть. Active network
Акустическая ёмкость. Acoustical capacitance
Акустическая постоянная распространения. Acoustical propagation constant
Акустическая система. Acoustical system
Акустическая теорема взаимности. Acoustical reciprocity theorem
Акустическая фазовая постоянная. Acoustical phase constant
Акустические единицы. Acoustical units
Акустический импеданс. Acoustical impedance
Акустический импеданс горловины. Throat acoustical impedance
Алгебра. Algebra
Алгебра группы. Algebra of a group
Алгебраические уравнения. Algebraic equations
Алгебраическое дополнение. Cofactor
Алгоритм. Algorithm, program
Альбедо. Albedo
Амага единица. Amagat unit
Ампер. Ampere
Ампера теорема. Ampere's theorem
Ампервиток. Ampere turn
Амплитуда волны. Wave amplitude
Амплитуда импульса. Pulse amplitude
Амплитуда колебания. Amplitude of oscillation
Амплитуда нормальной кривизны. Amplitude of normal curvature
Амплитуда рассеяния. Scattering amplitude
Анализ. Analysis
Анализ ковариантности. Analysis of covariance
Анализ составляющих. Component analysis
Аналитическая кривая. Analytic curve
Анаморфная система. Anamorphic system
Аналитическая функция. Analytic function
Аналитическое продолжение. Analytic continuation
Ангармонический осциллятор. Anharmonic oscillator
Ангармоничность. Anharmonicity
Ангстрем. Angstrom unit
Анизотропный. Anisotropic
Анизотропный диэлектрик. Anisotropic dielectric
Анод. Anode
Аномальная дисперсия. Anomalous dispersion
Ансамбль. Ensemble
Антенна. Antenna
Антиглавные точки. Anti-principal points
Антиклиналь. Arch
Антилинейный оператор. Anti-linear operator
Антилогарифм. Antilogarithm
Антипроизводная функции. Antiderivative of a function
Антирезонанс. Antiresonance
Антисимметричная волновая функция. Antisymmetric wave function
Антисимметричный. Antisymmetric
Антиунитарный оператор. Anti-unitary operator
Антиферромагнетизм. Antiferromagnetism
Антициклональная завихренность. Anticyclonic vorticity
Анериодические колебания. Non-periodic vibrations
Апертура. Aperture
Апланатическая поверхность. Aplanatic surface
Апланатические точки шара. Aplanatic points of a sphere
Апостериорная вероятность. A posteriori probability
Апохроматический. Achromatic
Аппроксимация молекулярного поля. Molecular field approximation
Аппроксимация сильной связи. Strong coupling approximation
Арганда плоскость. Argand plane
Арифметическая прогрессия. Arithmetic progression

- Арифметический блок. Arithmetic unit
 Арка с затяжкой. Tied arch
 Архимеда закон. Archimedes' principle
 Асимптота. Asymptote
 Асимптотическая линия. Asymptotic line
 Асимптотическая теория рескторов. Asymptotic re-actor theory
 Асимптотический ряд. Asymptotic series
 Асимптотическое условие. Asymptotic condition
 Асколи теорема. Ascoli's theorem
 Ассоциация. Association
 Астигматизм. Astigmatism
 Астигматизм поверхности. Astigmatism of a surface
 Астигматическая разность. Astigmatic difference
 Атомная единица массы. Atomic mass unit
 Атомная конфигурация. Atomic configuration
 Атомная рефракция. Atomic refraction
 Атомная теплоёмкость. Atomic heat of formation
 Атомная частота. Atomic frequency
 Атомное число. Atomic number
 Атомные спектры. Atomic spectra
 Аудиограмма. Audiogram
 Аффинное преобразование. Affine transformation
 Аффинный тензор. Affine tensor
 Аэродинамическая труба. Wind tunnel
 Аэродинамическая хорда. Aerodynamic chord
 Аэродинамический центр. Aerodynamic center
- Бабинз принцип. Babinet's principle
 Базис системы номеров. Base of a system of numbers
 Байеса теорема. Bayes' theorem
 Балка. Beam
 Баллистический маятник. Ballistic pendulum
 Бальмера серия. Balmer series
 Банаха пространство. Banach space
 Бар. Bar
 Барион. Baryon
 Барн. Barn
 Бароклинический. Baroclinic
 Баротропический. Barotropic
 Батарея. Battery
 Баушингера эффект. Bauschinger effect
 Беера закон. Beer's law
 Бегущая волна. Progressive wave
 Безвихревое течение. Potential flow, irrotational flow
 Безвихревый вектор. Irrotational vector
 Безвихревый поток. Potential flow, irrotational flow
 Безразличное состояние. Indifferent state
 Бернуллиевы полиномы. Bernoulli polynomials
 Бернулли метод. Bernoulli method
 Бернштейна полином. Bernstein polynomial
 Бесконечная труба. Infinite pipe
 Бесконечно малое кольцо. Infinitesimal ring
 Бесконечно малый. Infinitesimal
 Бесконечное размножение. Infinite multiplication
 Бесконечность. Infinity
 Бесконечный график. Infinite graph
 Бесконечный рупор. Infinite horn
 Бесселева функция. Bessel function
 Бесселя неравенство. Bessel inequality
 Бета функция. Beta function
 Бете метод. Bethe's method
 Бете-Сальпетер уравнение. Bethe-Salpeter equation
 Бетон, армированный. Concrete, reinforced
 Бигармоническое уравнение. Biharmonic equation
- Биение. Combination vibration
 Биения. Beats
 Биквадратное уравнение. Biquadratic equation
 Биквиарный. Biquinary
 Билинейная форма. Bilinear form
 Бинормаль. Binormal
 Биномальная теорема. Binomial theorem
 Биномиальное распределение. Binomial distribution
 Биномиальный коэффициент. Binomial coefficient
 Биполярная координата. Bipolar coordinate
 Ближайший соседний. Nearest-neighbor
 Ближний порядок. Short-range order
 Близкодействующая сила. Short-range force
 Блок памяти. Storage unit
 Блоха теорема. Bloch theorem
 Бозе-Эйнштейна газ. Bose-Einstein gas
 Бойля закон. Boyle's law
 Боковая кривизна. Lateral buckling
 Боковая сферическая аберрация. Lateral spherical aberration
 Боковое смещение. Sidesway
 Боковой прогиб. Lateral deflection
 Больцано-Вейерштрасса теорема. Bolzano-Weierstrass theorem
 Больцманна постоянная. Boltzmann constant
 Бомбардирующий. Incident
 Боровская орбита. Bohr orbit
 Боровский радиус. Bohr radius
 Борна приближение. Born approximation
 Борна-Инфельда теория. Born-Infeld theory
 Борна-Оппенгеймера метод. Born-Oppenheimer method
 Борна-Фон Кармана теория. Born and Von Karman theory
 Борновское отталкивание. Born repulsion
 Босон. Boson
 Бра вектор. Bra vector
 Браве точки. Bravais points
 Браве-Миллера индексы. Bravais-Miller indices
 Брахистохрона. Brachistochrone
 Брейт-Вигнера формула. Breit-Wigner formula
 Бриллюэна функция. Brillouin function
 Брэгга правило. Bragg rule
 Брюстера закон. Brewster's law
 Бугера закон. Bouguer law
 Булева алгебра. Boolean algebra
 Буняковского неравенство. Schwartz inequality
 Бургерса вектор. Burgers vector
- Вакуумная поправка. Correction to vacuum
 Вал. Shaft
 Валентная связь. Valence bond
 Валентная сила. Valence force
 Ван-дер-Ваальса уравнение. Van der Waals' equation
 Вариационное исчисление. Calculus of variations
 Вариационный принцип. Variational principle
 Вебер. Weber
 Ведомая волна. Guided wave
 Вейерштрасса теорема. Weierstrass theorem
 Вейля уравнение. Weyl equation
 Вековой определитель. Secular determinant
 Вектор. Vector
 Вектор напряженности электрического поля. Electric field vector
 Вектор площади. Area vector
 Вектор положения. Position vector

- Вектор потока. Flux vector
 Вектор состояния. State vector
 Вектор электрического смещения. Electric displacement vector
 Векторная производная. Vector derivative
 Векторное поле. Vector field
 Векторное пространство. Vector space
 Векторное сложение. Vector addition
 Векторные дифференциальные тождества. Vector differential identities
 Векторный оператор. Vector operator
 Векторный потенциал. Vector potential
 Вентилятор. Blower
 Верёвочный многоугольник. Funicular polygon
 Вероятная ошибка. Probable error
 Вероятность. Probability
 Вероятность выхода (в первом полете). Escape probability (first flight)
 Вероятность избежания резонансного захвата. Resonance escape probability
 Вероятность перехода. Transition probability
 Вероятность прохождения. Penetration probability (penetrability)
 Вероятность столкновения. Collision probability
 Вероятностный коэффициент. Coefficient of probability
 Вероятностный отбор. Probability sampling
 Вершина. Vertex
 Вершина волны. Wave crest
 Вес. Weight
 Весовая функция. Weighting function
 Весы. Balance
 Ветвь. Branch
 Ветер. Wind
 Ветровая нагрузка. Wind load
 Взаимность. Reciprocity
 Взаимодействие. Interaction
 Взаимозамещения закон фотографич. Reciprocity law, photographic
 Взвешивание. Weighting
 Взвешиванный тензор. Weighted tensor
 Вигнера коэффициент. Wigner coefficient
 Вид. Species
 Вид колебания. Mode of oscillation, mode
 Вина закон. Wien law
 Винтовая кривизна. Screw curvature
 Вириал. Virial
 Вириал системы. Virial of a system
 Вириальное уравнение состояния. Virial equation of state
 Вириальный коэффициент. Virial coefficient
 Виртуальная масса. Virtual mass
 Виртуальная температура. Virtual temperature
 Виртуальное состояние. Virtual state
 Виртуальный квант. Virtual quantum
 Вискоэластичная волна. Viscoelastic waves
 Виток. Loop, turn
 Вихрь. Vortex, curl
 Вихревая форсунка. Turbulent jet
 Вихревое кольцо. Vortex ring
 Влага. Humidity
 Влажность. Humidity
 Влажный пар. Wet steam
 Влияние отражателя реактора. Reflector savings
 Внецентренная нагрузка. Eccentric loading
 Внешняя оптическая плотность. External optical density
 Внешняя сила. External force
 Внешний. Extraneous
 Внутренняя поглощающая способность. Internal absorptance
 Внутренняя прозрачность. Internal transmittance
 Внутренняя сила. Internal force
 Внутреннее поле в диэлектрике. Internal field in dielectric
 Внутреннее трение. Internal friction,
 Водородная связь. Hydrogen bond
 Водородная тонкая структура. Hydrogen fine structure
 Возвратная разность. Return difference
 Воздушный винт. Airscrew
 Возмущение. Perturbation
 Возраст нейтронов. Age, neutron
 Возрастное приближение. Age approximation
 Волна. Wave
 Волна напряжения. Stress wave
 Волна расширения. Dilatational wave
 Волна сдвига. Shear wave
 Волновая механика. Wave mechanics
 Волновая функция. Wave function
 Волновод. Waveguide
 Волновое уравнение. Wave equation
 Волновое число. Wave number
 Вольтерра уравнение. Volterra equation
 Воображаемый источник. Image source
 Воспроизводимость. Repeatability
 Вращательная дисперсия. Rotational dispersion
 Вращательная мощность. Rotary power
 Вращательная податливость. Rotational compliance
 Вращательная постоянная. Rotational constant
 Вращательная сумма состояний. Rotational partition function
 Вращательно-колебательный спектр. Rotation-vibration spectrum
 Вращательный уровень. Rotational level
 Вращающаяся система координат. Rotating coordinate system
 Вращающий момент. Torque
 Вращение. Rotation
 Время диффузии нейтронов. Neutron diffusion time
 Время жизни. Lifetime
 Время жизни, среднее. Mean life
 Время задержки. Delay time
 Время замедления. Slowing-down time
 Время затухания. Decay time
 Время нарастания импульса. Pulse rise time
 Время обратного хода. Return period
 Время набегающего фронта. Pulse leading-edge time
 Время релаксации. Relaxation time
 Время срабатывания. Response time
 Вронского определитель. Wronskian
 Всасывание. Suction
 Вспомогательное уравнение. Auxiliary equation
 Вторая основная теорема. Second fundamental theorem
 Вторая предельная теорема. Second limit theorem
 Вторая производная лапласана. Buckling
 Вторая фокусная точка. Second focal point
 Второй закон термодинамики. Second law of thermodynamics
 Вторичное квантование. Second quantization
 Вторичное течение. Secondary flow
 Входная длина (трубы или канала). Entry length (for pipe or channel)
 Выдержка. Exposure
 Выборочная точка. Sample point

- Выборочное распределение. Sampling distribution
 Выигрыш. Gain
 Выкачивание. Deflation
 Вынужденные колебание. Forced vibrations
 Выпрямительная линия. Rectifying line
 Вырождение. Degeneracy, confluence
 Вырожденная зона проводимости. Degenerate conduction band
 Вырожденная колебательная система. Degenerate oscillating system
 Вырожденное состояние. Degenerate state
 Вырожденные уровни. Degenerate levels
 Вырожденный газ. Degenerate gas
 Вырожденный электронный газ. Degenerate electron gas
 Высота тона. Pitch
 Выход флуоресценции. Fluorescence yield
 Вычет. Residue
 Вычислительная машина. Calculator, computer
 Вязкая жидкость. Viscous fluid
 Вычислительная машина непрерывного действия. Analog computer
 Вязкость. Viscosity
- Габаритный объем. Clearance volume
 Гаечный ключ. Wrench
 Газ жестких шариков. Hard-sphere gas
 Газовая постоянная. Gas constant
 Галилеевое преобразование. Galilean transformation
 Гальваномагнетический эффект. Galvanomagnetic effect
 Гамильтона функция оптики. Hamiltonian function of optics
 Гамильтониан системы. Hamiltonian function of a system
 Гамма. Gamma
 Гамма функция. Gamma function
 Гармоническая волна. Harmonic wave
 Гармоническая прогрессия. Harmonic progression
 Гармоническая функция. Harmonic function
 Гармонический осциллятор. Harmonic oscillator
 Гармонический ряд звуков. Harmonic series of sounds
 Гармоническое движение. Harmonic motion
 Гармоническое среднее. Harmonic mean
 Гаусса-Маркова теорема. Gauss-Markov theorem
 Наименьшая связь. Least constraint
 Гаусса-Сейделя метод. Gauss-Seidel method
 Гауссова оптика. Gaussian optics
 Гауссовое распределение. Gaussian distribution
 Гауссовы скобки. Gaussian brackets
 Гашение. Extinction
 Гейгера-Нуталла соотношение. Geiger-Nuttall relation
 Гексагональная система. Hexagonal system
 Генеалогический коэффициент. Fractional parentage coefficient
 Геодезическая гипербола. Geodesic hyperbola
 Геодезическая касательная. Geodesic tangent
 Геодезическая кривизна. Geodesic curvature
 Геодезическая линия. Geodesic line
 Геодезические полярные координаты. Geodesic polar coordinates
 Геодезические координаты. Geodesic coordinates
 Геодезические параллели. Geodesic parallels
 Геодезический круг. Geodesic circle
- Геодезический треугольник. Geodesic triangle
 Геодезический эллипс. Geodesic ellipse
 Геометрическая дисторсия. Geometric distortion
 Геометрическая оптика. Geometrical optics
 Геометрический множитель. Geometry factor
 Геометрический поток энергии. Geometrical energy flux
 Геометрическое искажение. Geometric distortion
 Геометрическое место. Locus
 Геометрия. Geometry
 Геострофическая завихренность. Geostrophic vorticity
 Геострофическая сила. Geostrophic force
 Геострофический ветер. Geostrophic wind
 Геострофическое ускорение. Geostrophic acceleration
 Герц. Hertz, cycle per second
 Герца вектор. Hertz vector
 Гетерополярная связь. Heteropolar bond
 Гиббса-Гельмгольца уравнения. Gibbs-Helmholtz equations
 Гиббса-Дюгема уравнение. Gibbs-Duhem equation
 Гиббса-Коновалова теоремы. Gibbs-Konovalov theorems
 Гиббса функция. Gibbs function
 Гибкость. Ductility
 Гибридизация электронных орбиталей. Hybridization of electron orbitals
 Гивенса метод. Givens method
 Гидравлический скачок. Hydraulic jump
 Гидродинамическая аналогия. Hydrodynamical analogy
 Гидростатика. Hydrostatics
 Гильберта пространство. Hilbert space
 Гипербола. Hyperbola
 Гиперболическая точка. Hyperbolic point
 Гиперболическая функция. Hyperbolic function
 Гиперболическое уравнение. Hyperbolic equation
 Гиперболической логарифм. Hyperbolic logarithm
 Гиперболоид. Hyperboloid
 Гипергеометрическая функция. Hypergeometric function
 Гипергеометрическое распределение. Hypergeometric distribution
 Гипергеометрическое уравнение. Hypergeometric equation
 Гипермультиплет. Hypermultiplet
 Гиромагнитное соотношение. Gyromagnetic ratio
 Главная нормаль. Principal normal
 Главная ось. Principal axis
 Главная плоскость. Principal plane
 Главное напряжение. Principal stress
 Главное сечение кристалла. Principal section of a crystal
 Главные направления. Principal directions
 Главные плоскости. Principal planes
 Главные румбы. Principal compass points
 Главный азимутальный угол. Angle of principal azimuth
 Главный корень. Principal root
 Главный луч. Principal ray
 Глубина поля. Depth of field
 Глубина проникновения. Penetration depth
 Голографические уравнения. Hodograph equations
 Голоморфная функция. Holomorphic function
 Голономический. Holonomic
 Гомеографическое преобразование. Homographic transformation

- Гомеометрические пары. Homometric pairs
 Гомеоморфизм. Homomorphism
 Гомеоморфные графики. Homeomorphic graphs
 Гомеополлярная связь. Homopolar bond
 Гомогенный. Homogeneous
 Гомоцентрический. Homocentric
 Горизонтальный сдвиг. Horizontal shear
 Гравитационная сила. Gravitational force
 Гравитационный потенциал. Gravitational potential
 Гравитационный радиус. Gravitational radius
 Градиент. Gradient
 Градиент температуры. Temperature gradient
 Градиентная инвариантность. Gauge invariance
 Градус. Grade
 Граница. Bound
 Граница зерна. Grain boundary
 Граничная задача. Boundary-value problem
 Граничное условие. Boundary condition
 Граничные формулы диэлектриков. Dielectric boundary formulas
 Грань. Edge
 Грассхофа число. Grashof number
 График автоморфизма. Automorphism graph
 Графическая статика. Graphical statics
 Грина функция. Green's function
 Громкость звука. Loudness of a sound
 Группа. Group
 Группа вращений. Rotation group
 Группа Ли. Lie group
 Групповая релаксация. Group relaxation
 Групповая скорость. Group velocity
 Гука закон. Hooke's law
 Гурвица полином. Hurwitz polynomial
 Густой. Dense
 Густота. Density
 Гюйгенса принцип. Huygens' principle
- Давление излучения. Radiation pressure
 Даламбера принцип. D'Alembert's principle
 Даламбертиан. D'Alembertian
 Дальность. Range
 Дальний порядок. Long-range order
 Дальтона закон. Dalton's law
 Данкова поправки. Dancoff corrections
 Дарбу вектор. Darboux vector
 Движение. Motion
 Движущееся среднее. Moving average
 Двойное интегрирование. Double integration
 Двойное лучепреломление. Double refraction
 Двойный тензор. Double tensor
 Двойственная схема. Dual network
 Двойственный график. Dual graph
 Двоичная точка. Binary point
 Двоичное число. Bit
 Двоичный. Binary
 Двухатомная молекула. Diatomic molecule
 Двухкомпонентное уравнение нейтрино. Two-component equation of the neutrino
 Дебаевская температура. Debye temperature
 Дебая-Гюккеля уравнение. Debye-Hückel equation
 Девнация. Deviation
 Деградирование энергии. Degradation of energy
 Дедекинда сечение. Dedekind cut
 Действие. Action
 Действующая высота (антенны). Effective height (antenna)
 Действующая длина волны. Effective wavelength
- Действующая механическая движущая сила. Effective mechanomotive force
 Действующая площадь (антенны). Effective area (antenna)
 Действующая сила. Effective force,
 Действующая ширина полосы. Effective band width
 Действующие увеличение линзы. Effective power of a lens
 Декарта правило знаков. Descartes rule of signs
 Декартовы координаты. Rectangular coordinates
 Демпфирование. Damping
 Дерево. Tree
 Десятичный. Decimal
 Дефект массы. Mass defect
 Дефляция. Deflation
 Деформационная теория пластичности. Deformation theory of plasticity
 Деформация. Deformation
 Децибел. Decibel
 Джонсона и Ларк-Горовица формула. Johnson and Lark-Horowitz formula
 Диадное исчисление. Dyadics
 Диадный оператор. Dyadic operator
 Диагональное растяжение. Diagonal tension
 Диагональный. Diagonal
 Диаграмма излучения. Radiation pattern
 Диаграмма корреляции. Correlation diagram
 Диаграмма моментов. Moment diagram
 Диаграмма разброса. Scatter diagram
 Диалитичный. Dialytic
 Диамагнетизм. Diamagnetism
 Диапазон. Band, span
 Динамика. Dynamics
 Динамика свободной точечной массы. Dynamics of a free mass point
 Динамическая вязкость. Dynamic viscosity
 Динамическая модель подвижности. Dynamical analogy of mobility
 Динамическая нагрузка. Dynamic loading
 Динамические модели. Dynamical analogies
 Динамические уравнения. Dynamical equations
 Динамический возбудитель. Dynamic driving system
 Динамический привод. Dynamic driving system
 Динамическое давление. Dynamic pressure
 Динамическое поверхностное натяжение. Dynamical surface tension
 Динамическое подобие. Dynamical similarity
 Динамическое равновесие. Dynamic equilibrium
 Диоптр. Diopter
 Диполь (электрический). Dipole (electrical)
 Дирака операторы. Dirac operators
 Дирихле принцип. Dirichlet principle
 Дискриминант. Discriminant
 Дислокация. Dislocation
 Диспергирующая способность. Dispersive power
 Дисперсия. Dispersion, variance
 Дисперсия вращения. Dispersion of rotation
 Дисперсионная сила. Dispersion force
 Дисперсионная формула. Dispersion formula
 Дисперсионные соотношения. Dispersion relations
 Дисперсионный анализ. Analysis of variance
 Диссипация. Dissipation
 Дифференциал. Differential
 Дифференциальное уравнение. Differential equation
 Дифференциальное уравнение в частных производных. Partial differential equation

- Дифференциальный анализатор. Differential analyzer
 Дифференцирование. Differentiation
 Дифференцирование под знаком интеграла. Differentiation under the integral sign
 Диффракция. Diffraction
 Диффракция нейтронов. Diffraction of neutrons
 Диффракционная картина. Diffraction patterns
 Диффракционная эволюта. Diffraction evolute
 Диффузионная длина. Diffusion length
 Диффузионная устойчивость. Diffusion stability
 Диффузионная функция влияния. Diffusion kernel
 Диффузионное охлаждение. Diffusion cooling
 Диффузия света. Diffusion of light
 Диффузия твердых тел. Diffusion of solids
 Диффузионная площадь нейтронов. Neutron diffusion area
 Диффузная отражательность. Diffuse reflectance
 Диффузная прозрачность. Diffuse transmittance
 Диффузная серия. Diffuse series
 Диффузное отражение. Diffuse reflection
 Диффузор. Diffuser
 Диэдрическая группа. Dihedral group
 Диэдрический угол. Dihedral angle
 Диэлектрический гистерезис. Dielectric hysteresis
 Диэлектрическая проницаемость. Dielectric constant
 Длина замедления. Slowing-down length
 Длина кривой. Length of a curve
 Длина линейной интерполяции. Linear extrapolation length
 Длина миграции. Migration length
 Длина релаксации. Relaxation length
 Длина связи. Bond length
 Длина следа нейтрона. Neutron track length
 Длина смешивания. Mixing length
 Длительность импульса. Pulse duration
 Добротность. Q factor
 Доверительный интервал. Confidence limits
 Долина волны. Wave trough
 Домен. Domain
 Доминирующая волна. Dominant wavelength
 Доминирующий режим. Dominant mode
 Донорная связь. Donor bond
 Дополнение. Complement
 Дополнимость. Complementarity
 Дополнительное напряжение. Secondary stresses
 Дополнительное условие. Supplementary condition
 Допплера эффект. Doppler effect
 Допплеровская ширина. Doppler width
 Допустимое напряжение. Allowable stress
 Достаточность. Sufficiency
 Дрожание. Flutter
 Дросселирование. Choking
 Дублет. Doublet
 Дуга. Arch
 Дырка. "Hole"
 Дырочная теория. Hole theory
 Дюгема-Маргулеса соотношение. Duhem-Margules equation
 Дюгема теорема. Duhem's theorem
 Евклида алгоритм. Euclidean algorithm
 Евклидово пространство. Euclidean space
 Единица атомного веса. Atomic weight unit
 Единицы тепла. Units of heat
 Единичная матрица. Unit matrix
 Единичная нормаль. Unit normal
 Единичная плоскость. Unit plane
 Единичная функция. Step function
 Единичная ячейка. Unit cell
 Единичный оператор. Unit operator
 Единичный элемент. Unit element
 Емкость. Capacitance
 Жесткая рама. Rigid frame
 Жесткий. Rigid
 Жесткий цилиндр. Rigid cylinder
 Жесткий шар. Rigid sphere
 Жесткое регулирование. Proportional control
 Жесткость. Hardness
 Жесткость при изгибе. Flexural rigidity
 Жордана метод. Jordan method
 Жордана-Хелдера теорема. Jordan-Hölder theorem
 Жуковского аэродинамическая поверхность. Joukowski airfoil
 Зависящий от спина. Spin-dependent
 Завихренность. Vorticity
 Заглушение. Damping
 Заделанная балка. Beam, fixed-ended or encastre
 Задний фокус. Back focal length
 Задняя фокальная длина. Back focal length
 Закалка степени свободы. Freezing-in of degree of freedom
 Закон ассоциативности. Associative law
 Закон больших чисел. Law of large numbers
 Закон действующих масс. Law of mass action
 Закон идеального газа. Perfect gas law
 Закон излучения. Emission law
 Закон итерированного логарифма. Law of iterated logarithm
 Закон коммутативности. Commutative law
 Закон косинусов. Law of cosines
 Закон перестановочности. Commutative law
 Закон распределительности. Distributive law
 Закон синусов. Law of sines
 Закон сложения ускорений. Composition law of acceleration
 Закон соответственных состояний. Law of corresponding states
 Закон тангенсов. Law of tangents
 Законы преобразования. Transformation laws
 Закрытая система. Closed system
 Закрытый. Closed
 Замедление. Moderation
 Замедление нейтронов. Thermalization of neutrons
 Замедленное вращение. Hindered rotation
 Замедленное приближение к пределу. Deferred approach to the limit
 Замыкание. Closure
 Запас по усилению. Gain margin
 Запас по фазе. Phase margin
 Запас усиления. Gain margin
 Запертый. Closed
 Заполненная оболочка. Closed shell
 Запрещенный переход. Forbidden transition
 Зарядная инвариантность. Charge-invariance
 Зарядовая плотность нуклеонов. Charge density of nuclei
 Зарядовая ренормализация. Charge renormalization
 Зарядовая сопряженность. Charge conjugation

- Зарядовое сопряжение. Charge conjugation
 Затухание. Attenuation, damping
 Затухание звука. Sound attenuation
 Затухание турбулентности. Decay of turbulence
 Затухающие колебания. Damped vibrations
 Затяжка. Tie rod
 Звено. Link
 Звуковая мощность. Sound power
 Зеемана эффект. Zeeman effect
 Зейделя aberrации. Seidel aberrations
 Зеркально-поворотная ось. Rotation-reflection axis
 Зеркальное отражение. Specular reflection
 Значащая цифра. Significant digit
 Значущая цифра. Significant figure
 Значение энергии ядерной реакции. Q-Value
 Зоммерфельда-Косселя закон смещения. Sommerfeld-Kossel displacement law
 Зона проводимости. Conduction band
 Зонная теория твердого тела. Band theory of solids
 Зональное увеличение. Zonal magnification
- Идеальная оптическая система. Perfect optical system
 Идеально пластический. Ideally plastic
 Идеальные системы. Ideal systems
 Идеальный генератор напряжения. Ideal voltage source
 Идеальный генератор тока. Ideal current source
 Идемпотентный. Idempotent
 Идентичность. Identity
 Изаллобара. Isallobar
 Избирательный. Selective
 Избирательный излучатель. Selective radiator
 Избыток нейтронов. Neutron excess
 Избыток фазы. Phase margin
 Избыточность. Redundancy
 Избыточный коэффициент воспроизводства ядерного горючего. Breeding gain
 Извращение. Inversion
 Изгиб. Bending, bending deflection, flexure
 Изгиб балки. Curvature of beam
 Изетропный. Isentropic
 Излишняя связь. Redundant constraint
 Излишние функции. Excess functions
 Излучаемая мощность. Emissive power
 Излучатель. Radiator
 Излучение. Emittance
 Излучение абсолютно чёрного тела. Black body radiation law
 Изменение. Fluctuation, change
 Изобара. Isobar
 Изобарная поверхность. Isobaric surface
 Изобарный спин. Isobaric spin
 Изображение частотной характеристики. Frequency response representation
 Изолиния громкости. Loudness contour
 Изолированные системы. Isolated systems
 Изолюкс. Isolux
 Изометрические линии на поверхности. Isometric lines on a surface
 Изоморфные графики. Isomorphic graphs
 Изопериметрическая проблема. Isoperimetric problem
 Изотерма. Isotherm
 Изотермический. Isothermal
 Изотопический спин. Isotope spin
 Изотопный эффект. Isotope effect
 Изотропный тензор. Isotropic tensor
 Изотропная турбулентность. Isotropic turbulence
 Изотропный. Isotropic
 Изотропный диэлектрик. Isotropic dielectric
 Изофот. Isophot
 Изозлектронный принцип. Isoelectronic principle
 Изоэнергетический спектр. Equal-energy spectrum
 Импеданс. Impedance
 Импульс. Impact, impulse, pulse, momentum
 Импульсное движение. Impulsive motion
 Импульсные функции. Impulse functions
 Импульсный режим. Pulse mode
 Инвариант. Invariant
 Индикаторная мощность. Indicated efficiency
 Индикатрисса. Indicatrix
 Индикатрисса диффузии. Indicatrix of diffusion
 Индикатрисса преломления. Indicatrix of refraction
 Индифферентное состояние. Indifferent state
 Индициальное уравнение. Indicial equation
 Индуктивность. Inductance
 Индуктивные силы. Induction forces
 Индуктор. Inductor
 Индукция. Induction
 Инерциальная система отсчета. Inertial frame
 Инерция. Inertia
 Инкрементальная теория пластичности. Incremental theory of plasticity
 Интеграл. Integral
 Интеграл свертки. Convolution integral
 Интеграл энергии. Energy integral
 Интегральное отражение рентгеновских лучей. Integrated X-ray reflection
 Интегральное преобразование. Integral transform
 Интегральное уравнение. Integral equation
 Интегральное уравнение переноса. Integral transport equation
 Интегральные теоремы векторного исчисления. Integral theorems of vector analysis
 Интегральный оператор. Integral operator
 Интегратор. Differential analyzer, integrator
 Интегрирующий множитель. Integrating factor
 Интенсивность звука. Sound intensity
 Интенсивность излучения. Intensity of radiation
 Интенсивность источника частиц. Intensity of a source of particles
 Интенсивность линии. Line strength
 Интенсивность радиоактивности. Intensity of radioactivity
 Интенсивность света. Luminous intensity
 Интенсивность спектральной линии. Intensity of a spectral line
 Интенсивные переменные. Intensive variables
 Интерполяция. Interpolation
 Интерполяция по оптимальном интервале. Optimum-interval interpolation
 Интерполяционная формула. Interpolation formula
 Интерференция. Interference
 Информация. Information
 Инфракрасное излучение. Infrared radiation
 Ионичность. Ionic strength
 Ионная связь. Ionic bond
 Иррадиация. Irradiation
 Искажение. Distortion
 Исключение. Elimination, exclusion
 Исправление. Correction
 Испытания на холостом ходе. No-load test

Источник. Source
 Печеление. Calculus
 Итеративные методы. Iterative methods

Капитация. Cavitation
 Калибровочная инвариантность. Gauge invariance
 Канал реакции. Reaction channel
 Каноническая корреляция. Canonical correlation
 Канонические координаты. Canonical coordinates
 Канонические уравнения оптики. Canonical equations of optics
 Канонический. Canonical
 Канонический ансамбль. Canonical ensemble
 Каноническое преобразование. Canonical transformation
 Каноническое уравнение движения. Canonical equation of motion
 Капельная конденсация. Drop-wise condensation
 Капиллярность. Capillarity
 Капиллярные волны. Capillary waves
 Кардинальный. Cardinal
 Кардинальные точки оптической системы. Cardinal points of an optical system
 Каркас. Framework
 Кармана постоянная. Karman constant
 Кармана-Фридрихеа уравнения. Karman-Friedrich's equations
 Карно цикл. Carnot cycle
 Касательная. Tangent
 Касательная плоскость. Tangent plane
 Касательная сила. Tangential force
 Касательное ускорение. Tangential acceleration
 Каскад аэродинамических профилей. Cascade of airfoils
 Катера маятник. Kater pendulum
 Катушка индуктивности. Inductor
 Качание. Oscillation, rocking
 Квадратичная поверхность. Quadric surface
 Квадратная дифференциальная форма. Quadratic differential form
 Квадратно интегрируемый. Square integrable
 Квадруполь. Quadrupole
 Квадрупольное излучение. Quadrupole radiation
 Квадрупольный момент. Quadrupole moment
 Квазихимическое приближение. Quasi-chemical approximation
 Квант. Quantum
 Квантование. Quantization
 Квантование поля. Field quantization
 Квантовать. Quantize
 Квантовая механика. Quantum mechanics
 Квантовая статистика. Quantum statistics
 Квантовая теория дисперсии. Quantum theory of dispersion
 Квантовая теория магнетизма. Quantum theory of magnetism
 Квантовая теория спектров. Quantum theory of spectra
 Квантовая теория теплоемкости. Quantum theory of heat capacity
 Квантовая электродинамика. Quantum electrodynamics
 Квантовой выход. Quantum efficiency, quantum yield
 Квантовомеханический резонанс. Quantum-mechanical resonance
 Кватернион. Quaternion

Кельвина теорема циркуляции. Circulation theorem (Kelvin)
 Кеплера законы. Kepler's laws
 Кет-вектор. Ket vector
 Кизома потенциал. Keesom potential
 Кинематика. Kinematics
 Кинематически разрешимо. Kinematically admissible
 Кинетика. Kinetics
 Кинетика реактора. Reactor kinetics
 Кинетическая теория. Kinetic theory
 Кинетическая теория вязкости. Kinetic theory of viscosity
 Кинетическая теория диффузии. Kinetic theory of diffusion.
 Кинетическая энергия. Kinetic energy
 Кинетическая энергия безвихревого движения. Kinetic energy of irrotational motion
 Кинетический импульс. Kinetic momentum
 Кинетический потенциал. Kinetic potential
 Кипение. Boiling
 Кирквуда приближения. Kirkwood's approximation
 Кирхгофа закон. Kirchhoff's law
 Клапейрона уравнение. Clapeyron's theorem
 Класс симметрии. Symmetry class
 Классическая теория электрона. Classical electron theory
 Клаузиуса дифференциальное уравнение. Clausius' differential equation
 Клаузиуса-Клапейрона уравнение. Clausius-Clapeyron equation
 Клаузиуса-Моссоти уравнение. Clausius-Mossotti equation
 Клейна-Гордона уравнение. Klein-Gordon equation
 Клейна-Нишины формула. Klein-Nishina formula
 Клейна-Ридберга метод. Klein-Rydberg method
 Клеро уравнение. Clairaut equation
 Кнудсена закон косинусов. Knudsen cosine law
 Ковариантная производная. Covariant derivative
 Ковариантная теория возмущений. Covariant perturbation theory
 Ковариантное дифференцирование. Covariant differentiation
 Ковариантность. Covariance
 Ковариантные правила перестановочности. Covariant commutation rules
 Ковариантный. Covariant
 Ковариантный вектор. Covariant vector
 Ковкость. Ductility
 Когерентное излучение. Coherent radiation
 Код. Code
 Колбочка сетчатки. Cone, visual
 Колебание. Oscillation, vibration, fluctuation
 Колебательный уровень. Vibrational level
 Колебательный спектр. Vibrational spectrum
 Коллинеарности уравнения оптики. Collineation equations of optics
 Количество движения. Momentum
 Кологарифм. Cologarithm
 Колонка. Column
 Колонна. Beam, column
 Кольцевой источник звука. Circular ring sound source
 Кольцо. Ring
 Кольцо ли. Lie ring
 Кома. Coma
 Комбинационное рассеяние. Raman effect

- Комбиноционные разности. Combination differences
- Комбинация. Combination
- Коммутативность. Commutativity
- Коммутатор. Commutator
- Компактный. Compact
- Компенсация гаммы. Gamma correction
- Комплекс группы. Complex of a group
- Комплексная податливость. Complex compliance
- Комплексное излучение. Complex radiation
- Комплексное число. Complex number
- Комплексный модуль. Complex modulus
- Комплексный потенциал. Complex potential
- Композиция двух тензоров. Composition of two tensors
- Компонента графика. Graph component
- Компрессор. Compressor
- Комптона-эффект. Compton effect
- Конвекционная скорость. Convective rate of change
- Конвекционный потенциал. Convective potential
- Конвекция. Convection
- Конгруэнтность кривых. Congruence of curves
- Конгруэнтный. Congruent
- Конденсат. Condensate
- Конденсатор. Capacitor, condenser (elec.)
- Конденсор. Condenser (steam)
- Конденсированная система. Condensed system
- Конечные амплитудные уравнения (акуст.) Finite amplitude equations (acoustic)
- Конечный график. Finite graph
- Конечный рупор. Finite horn
- Конические координаты. Conical coordinates
- Конический. Conic
- Конический маятник. Conical pendulum
- Конический рупор. Conical horn
- Коническое преломление. Conical refraction
- Консервативная сила. Conservative force
- Консервативная система. Conservative system
- Консервативное силовое поле. Conservative force field
- Консоль. Cantilever
- Консольная балка. Cantilever beam
- Константа. Constant
- Константа взаимодействия Ферми. Fermi constant
- Контактное преобразование. Contact transformation
- Контактный потенциал. Contact potential
- Контактный ток выпрямления. Contact rectification current
- Континуум. Continuum
- Контравариантный. Contravariant
- Контравариантный вектор. Contravariant vector
- Контраст. Contrast
- Контур. Circuit, resonant circuit, contour
- Контурное интегрирование. Contour integration
- Конус в сверхзвуковом потоке. Cone in supersonic flow
- Конус трения. Cone of friction
- Конфигурационная теплоемкость. Configurational heat capacity
- Конфигурационное пространство. Configuration space
- Конфигурация взаимодействия. Interaction configuration
- Конформное изображение. Conformal mapping
- Конформное решение. Conformal solution
- Концевой момент. End moment
- Концентрация. Concentration
- Координата. Coordinate
- Координата эллиптического цилиндра. Elliptic cylindrical coordinate
- Координатная связь. Coordinate bond
- Координаты симметрии. Symmetry coordinates
- Координационное число. Coordination number
- Копланарные силы. Coplanar forces
- Корневой годограф. Root locus
- Кориолиса параметер. Coriolis parameter
- Кориолиса сила. Coriolis force
- Кориолиса ускорение. Coriolis acceleration
- Корну спираль. Cornu spiral
- Корпус. Frame, framework
- Корреляция. Correlation
- Косая поверхность. Skew surface
- Косвенный вывод. Fiducial inference
- Космическая скорость (ракеты). Escape velocity
- Косой луч. Skew ray
- Кососимметричный тензор. Tensor, skew-symmetric
- Коттона-мутона закон. Cotton-Mouton law
- Коха уравнение. Koch's equation
- Коши критерий сходимости. Cauchy convergence test
- Коэффициент аккомодации. Accomodation coefficient
- Коэффициент акустической изоляции. Noise insulation factor
- Коэффициент амплитуды импульса. Crest factor of a pulse
- Коэффициент Ангстрема. Ångstrom coefficient
- Коэффициент анизотропии (асимметрии). Dissymmetry factor (anisotropy factor)
- Коэффициент атомного рассеяния. Atomic scattering factor
- Коэффициент азродинамической силы. Lift coefficient
- Коэффициент безопасности. Safety factor
- Коэффициент видимости. Visibility factor
- Коэффициент воспроизводства ядерного горючего. Breeding ratio
- Коэффициент восстановления. Coefficient of restitution
- Коэффициент восстановления температуры. Temperature recovery factor
- Коэффициент гашения. Extinction coefficient
- Коэффициент давления. Pressure coefficient
- Коэффициент диффузии. Diffusion factor
- Коэффициент диэлектрических потерь. Dielectric loss factor
- Коэффициент жесткости. Stiffness coefficient
- Коэффициент заполнения. Pulse duty factor
- Коэффициент затухания. Attenuation factor, damping ratio
- Коэффициент заземления. Restraint coefficient
- Коэффициент излучения. Emission coefficient
- Коэффициент изменения. Coefficient of variation
- Коэффициент конденсации. Coefficient of condensation
- Коэффициент критичности. Criticality factor
- Коэффициент лобового сопротивления. Drag coefficient
- Коэффициент нагрузки. Load factor
- Коэффициент определения. Coefficient of determination
- Коэффициент отражения. Reflectivity
- Коэффициент отраженного рассеяния. Back-scattering coefficient
- Коэффициент переноса. Carryover factor

- Коэффициент поглощения. Absorption coefficient
 Коэффициент поглощения слоя единичной толщины. Absorptivity, absorption power
 Коэффициент подогрева. Reheat factor
 Коэффициент полезного действия (к.п.д.). Efficiency
 К.п.д. воздушного цикла. Air-standard efficiency
 К.п.д. каскада. Stage efficiency
 Коэффициент пропускания. Transmissivity
 Коэффициент размножения на быстрых нейтронах. Fast fission factor
 Коэффициент распада. Decay coefficient
 Коэффициенты распределения (оптические). Distribution coefficients (optical)
 Коэффициент рассеяния. Scattering coefficient
 Коэффициент растяжения. Coefficient of tension
 Коэффициент рекомбинации. Coefficient of recombination
 Коэффициент самоиндукции. Inductance
 Коэффициент светлоты. Luminous coefficient
 Коэффициент сжатия. Compressibility factor
 Коэффициент стоячей волны (к.с.в.). Standing-wave ratio
 Коэффициент столкновения. Collision coefficient
 Коэффициент теплового расширения. Coefficient of (thermal) expansion
 Коэффициент трения. Coefficient friction of
 Коэффициент устойчивости. Stability factor
 Коэффициент фазы. Phase factor
 Коэффициент эмиссии. Emission coefficient
 Коэффициент энергопотери. Dissipation factor
 Коэффициенты влияния. Influence coefficients
 Краевая задача. Boundary value problem
 Краевые условия. Edge condition
 Край. Edge, boundary
 Крамера теорема. Cramer's rule
 Кратность. Multiplicity
 Кривая. Curve
 Кривая возбуждения. Excitation curve
 Кривая возгонки. Sublimation curve
 Кривая взаимодействия. Interaction curve
 Кривая выхода продуктов деления. Fission yield curve
 Кривая кипения. Boiling curve
 Кривая конденсации. Condensation curve
 Кривая кристаллизации. Crystallization curve
 Кривая напряжение-деформация. Stress-strain curve
 Кривая отклонения. Deflection curve
 Кривая распределения интенсивности. Intensity-distribution curve
 Кривая роста. Growth curve
 Кривая упругости. Elastic curve
 Кривизна. Curvature, bending, flexure, buckling
 Кривизна кривой в точке. Curvature of a curve at a point
 Кривизна поверхности. Curvature of a surface
 Кривизна поля. Curvature of field
 Криволинейное движение. Curvilinear motion
 Криволинейные ортогональные координаты. Curvilinear orthogonal coordinates
 Криволинейный источник звука. Curved line sound source
 Кристаллические системы. Crystal systems
 Кристаллическое строение. Crystal structure
 Кристаллограмма. Crystallogram
 Кристаллографическая группа. Crystallographic group
 Кристаллографические координатные оси. Crystallographic axes
 Кристаллографические параметры. Crystallographic parameters
 Кристаллография. Crystallography
 Критерий сходимости. Test for convergence
 Критическая масса. Critical mass
 Критическая нагрузка. Critical load
 Критическая область. Critical region
 Критическая скорость. Critical speed
 Критическая точка. Critical point
 Критические размеры. Critical size
 Критический потенциал. Critical potential
 Критическое поле. Critical field
 Критическое уравнение. Critical equation
 Критическое число Маха. Critical Mach number
 Крокко теорема. Crocco's theorem
 Кромка. Edge
 Кронекера символ. Kronecker delta
 Круг кривизны. Circle of curvature
 Круглое отверстие. Circular aperture
 Круговая перестановка. Cyclic permutation
 Круговая поляризация. Circular polarization
 Круговая частота. Pulsatance
 Круговой дихроизм. Circular dichroism
 Кружок определенности. Definition circle
 Крутильные волны. Torsional waves
 Крутильный маятник. Torsion pendulum
 Кручение. Torsional buckling, twisting, torsion
 Крыло с положительной стреловидностью. Swept-back wing
 Крыло с эллиптической нагрузкой. Elliptically-loaded wing
 Крюченная кривая. Twisted curve
 Кубическая система. Cubic system
 Кулон. Coulomb
 Кулона закон. Coulomb's law
 Кулоновское поле. Coulomb's field
 Кумулянт. Cumulant
 Кундта постоянная. Kundt constant
 Кутта-Жуковского закон. Kutta-Joukowski law
 Кюри. Curie
 Кюри-Вейсса закон. Curie-Weiss law
 Кюри диаграмма. Curie plot
 Кюри температура. Curie temperature
 Лабораторная система координат. Laboratory system
 Лагерра полиномы. Laguerre polynomials
 Лагранжа теорема. Lagrange theorem
 Лагранжиан. Lagrangian
 Лаймана серия. Lyman series
 Лэмба постоянная. Lamb's constant
 Ламбда-точка. Lambda point
 Ламберт. Lambert
 Ламе соотношения. Lamé relations
 Ламинарное векторное поле. Lamellar vector field
 Ламинарное течение. Laminar flow
 Ламинарный пограничный слой. Laminar boundary-layer
 Ламинарный поток. Laminar flow
 Ландау формула. Landau's formula
 Ланде множитель. Landé's g -factor
 Ланжевена теория диамагнетизма. Langevin theory of diamagnetism
 Ланцюса метод биортогонализации. Lanczos method of biorthogonalization

- Лапласа преобразование. Laplace transform
 Лапласиан. Laplacian
 Лапорта правило. Laporte rule
 Ларморова прецессия. Larmor precession
 Латеральная сферическая аберрация. Lateral spherical aberration
 Лауэ уравнения. Laue equations
 Лебега интеграл. Lebesgue integral
 Лезвие. Blade, edge
 Лежандра полиномы. Legendre polynomials
 Лейбница правило. Leibnitz rule
 Лемана изображение. Lehmann representation
 Леннарда-Джонса потенциал. Lennard-Jones potential
 Ленца правило. Lenz law
 Лес. Forest
 Летаргия. Lethargy
 Летное качество самолета. Airplane performance
 Летучесть. Fugacity
 Ливневая душа. Shower unit
 Линейная вискоэластичность. Linear viscoelasticity
 Линейная гипотеза. Linear hypothesis
 Линейная дифференциальная форма. Linear differential form
 Линейная система. Linear system
 Линейно независимые векторы. Linearly independent vectors
 Линейное неравенство. Linear inequality
 Линейное программирование. Linear programming
 Линейное увеличение. Linear magnification
 Линейное уравнение. Linear equation
 Линейный. Linear
 Линейный вихрь. Line vortex
 Линейный график. Linear graph
 Линейный источник. Straight line source
 Линия влияния. Influence line
 Линия давления. Pressure line
 Линия действия силы. Line of action of force
 Линия задержки. Delay line
 Линия кривизны. Line of curvature
 Линия обтекания. Streamline
 Линия передачи. Transmission line
 Линия скольжения. Slip line
 Линия тока. Line of flow
 Линованная поверхность. Ruled surface
 Липшица условие. Lipschitz condition
 Лист. Sheet
 Лиувилля-Нейманна ряд. Liouville-Neumann series
 Лиувилля теорема. Liouville theorem
 Лобачевского метод. Lobacevskii method
 Ловушка. Trap
 Логарифм. Logarithm
 Логарифмическая корреляция. Log correlation
 Логистическая кривая. Logistic curve
 Логарифмический декремент. Logarithmic decrement
 Логарифмический профиль скоростей. Logarithmic profile of velocity
 Логарифмическое дифференцирование. Logarithmic differentiation
 Логарифмическое распределение. Logarithmic distribution
 Лодэ переменные. Lode's variables
 Локализованные состояния. Localized states
 Локальная скорость изменения. Local rate of change
 Локально евклидовой. Locally Euclidean
 Локальные единичные слои. Localized monolayers
 Лондона уравнения сверхпроводимости. London superconductivity equations
 Лондоновская сила. London force
 Лопатка. Blade
 Лопиталья правило. L'hospital rule
 Лорана ряд. Laurent series
 Лоренца преобразование. Lorentz transformation
 Луч. Beam, ray
 Лучевая акустика. Ray acoustics
 Лучистость. Radiance
 Лэмбовское смещение. Lamb shift
 Лэнгмюира изотерма. Langmuir adsorption isotherm
 Людера линии. Luder's lines
 Люкс-секунда. Lux-second
 Люмен. Lumen
 Люмен-час. Lumen-hour
 Люнебурга линза. Luneburg lens
 Магическое число. Magic number
 Магнетон Бора. Bohr magneton
 Магнитная деформация. Magnetic strain
 Магнитная индукция. Magnetic induction
 Магнитная плотность заряда. Magnetic charge density
 Магнитная плотность тока. Magnetic current density
 Магнитная проницаемость. Permeability
 Магнитный вектор-потенциал. Magnetic vector potential
 Магнитный момент. Magnetic moment
 Магнитный момент нейтрона. Neutron magnetic moment
 Магнитный момент ядра. Nuclear magnetic moment
 Магнитный резонанс. Magnetic resonance
 Магнитный скалярный потенциал. Magnetic scalar potential
 Магнитодвижущая сила. Magnetomotive force
 Магнитокалорический эффект. Magnetocaloric effect
 Магнитомеханическое соотношение. Magnetomechanical ratio
 Магнитомеханическое демпфирование. Magnetomechanical damping
 Магнитостатика. Magnetostatics
 Магнитострикционный возбудитель. Magnetostriction driving system
 Магнитострикция. Magnetostriction
 Маделунга постоянная. Madelung constant
 Майорана сила. Majorana force
 Макет. Dummy
 Маклорэна ряд. Maclaurin series
 Макроканонический ансамбль. Macrocanonical ensemble
 Макросостояние. Macrostate
 Максвелл. Maxwell
 Максвелла-Больцманна распределение. Maxwell-Boltzmann distribution
 Максвелла-Мора метод. Maxwell-Mohr method
 Максвелла уравнения. Maxwell's equations
 Максвелловское распределение. Maxwell distribution
 Малюса закон. Malus law
 Мантисса. Mantissa
 Маркова цепь. Markov chain
 Масса. Mass
 Масса атома. Atomic mass

- Массовое число. Mass number
 Массовые силы. Body forces
 Математическое ожидание. Expectation value
 Материальная частица. Material particle
 Матрица. Matrix
 Матрица падения. Incidence matrix
 Матрица плотности. Density matrix
 Матрица пропускания. Transmission matrix
 Матрица рассеяния. Scattering matrix
 Матрица электрической цепи. Matrix, circuit
 Матъе уравнение. Mathieu equation
 Маха число. Mach number
 Маятник. Pendulum
 Мгновенная ось вращения. Instantaneous axis of rotation
 Мгновенный центр. Instantaneous center
 Медиан. Median
 Межатомный потенциал. Interatomic potential
 Межмолекулярные силы. Intermolecular forces
 Меллина преобразование. Mellin transform
 Мера. Measure
 Мера местоположения. Measure of location
 Мера функции. Measure of a function
 Меридиональный луч. Meridional ray
 Мероморфная функция. Meromorphic function
 Метастабильное состояние. Metastable state
 Метод аппроксимации полиномами. Method of polynomial approximations
 Метод воображаемых нагрузок. Method of fictitious loads
 Метод граничной точки. End-point method
 Метод изображений. Method of images
 Метод итерации. Method of iteration
 Метод моментной площади. Moment area method
 Метод моментов. Moments method
 Метод неопределенных коэффициентов. Method of undetermined coefficients
 Метод неравенств. Method of inequalities,
 Метод отдельных ординат. Discrete ordinates method
 Метод седловой точки. Saddle point method
 Метод случайных испытаний. Monte Carlo method
 Метод сопряженных градиентов. Method of conjugate gradients
 Метод упругой нагрузки. Elastic load method
 Метод характеристик. Method of characteristics
 Метод хорд. Chord method
 Метод искусственной нагрузки. Dummy unit load method
 Метр-свеча. Meter-candle
 Метр-свеча-секунда. Meter-candle-second
 Метрический тензор. Metric tensor
 Механизмовой анализ ферм. Mechanism analysis of frames
 Механическая вращательная взаимность. Mechanical rotational reciprocity
 Механическая вращательная система. Mechanical rotational system
 Механическая единица. Mechanical unit
 Механическая линейная взаимность. Mechanical rectilinear reciprocity
 Механическая линейная система. Mechanical rectilinear system
 Механическая устойчивость. Mechanical stability
 Механический импеданс. Mechanical impedance
 Механический к.п.д. Mechanical efficiency
 Механический эквивалент света. Mechanical equivalent of light
 Механоакустическая взаимность. Mechanical-acoustical reciprocity
 Ми-Грюнейзена уравнение состояния. Mic-Grüneisen equation of state
 Мизеса условие выхода. Mises yield condition
 Микроканонический ансамбль. Microcanonical ensemble
 Микроскопическая обратимость. Microscopic reversibility
 Микросостояние. Microstate
 Миллера индексы. Miller crystal indices
 Милне метод. Milne's method
 Минимальная поверхность. Minimal surface
 Минимум угол отклонения. Minimum angle of deviation
 Минковского мир. Minkowski world
 Миnor. Minor
 Мгновенная частота. Instantaneous frequency
 Многоатомные молекулы. Polyatomic molecules
 Многоатомный. Polyatomic
 Многократный интеграл. Multiple integral
 Многообразие. Manifold
 Многоугольник. Polygon
 Многочленный ряд. Multinomial series
 Множественность. Multiplicity
 Моделирующая машина. Analog computer
 Модель бесконечного поглотителя. Infinite absorber model
 Модель узкого резонанса. Narrow resonance model
 Модуль упругости. Modulus of elasticity
 Модуль пластичности. Plastic modulus
 Модуль разрыва. Modulus of rupture
 Модуль релаксации. Relaxation modulus
 Модуль сдвига. Shear modulus
 Модуль упругости. Young's modulus
 Мозли закон. Moseley's law
 Молекулярная орбиталь. Molecular orbital
 Молекулярная свободная длина. Molecular free path
 Молекулярная скорость. Molecular velocity
 Молекулярное вращение. Molecular rotation
 Молекулярная диаграмма. Molecular diagram
 Молекулярные собственные функции. Molecular eigenfunctions
 Молекулярные энергетические уровни. Molecular energy levels
 Молекулярный спектр. Molecular spectrum
 Молярная дисперсность. Molar dispersivity
 Молярная доля. Mole fraction
 Молярные свойства. Molar properties
 Момент. Moment
 Момент заземленного конца. Fixed-end moment
 Момент изгиба. Bending moment
 Момент инерции. Moment of inertia
 Момент инерции площади. Area moment of inertia
 Момент количества движения. Angular momentum, moment of momentum
 Момент распределения. Moment of a distribution
 Момент силы. Moment of force
 Момент сопротивления. Section modulus
 Момент тангажа. Pitching moment
 Моменты и продукты инерции. Moments and products of inertia
 Морера теорема. Morera theorem
 Морза кривая. Morse curve
 Мост. Arch, bridge
 Мощность. Power
 Мощность излучения. Emissive power

- Муавра-Лапласа теорема. De Moivre-Laplace theorem
Муавра формула. De Moivre identity
Мультиплет. Multiplet
- Набла. Nabla
Наблюдаемый. Observable
Наведенная поляризация. Induced polarization
Навье-Стокса уравнение. Navier-Stokes equation
Нагревательная поверхность. Heating surface
Надежность. Reliability
Надстройка. Superstructure
Найквиста теорема. Nyquist theorem
Наименьшие квадраты. Least squares
Наименьший кружок аберрации. Least circle of aberration
Наклеп. Work hardening
Наклон. Slope
Наличная энергия. Available energy
Наложение. Superposition
Наложенное поле. Impressed field
Намагниченность. Magnetization
Намагничивающая сила. Magnetizing force
Направление. Direction
Направление вектора. Sense of a vector
Направление распространения. Direction of propagation
Направленная путь. Directed path
Направленный. Directional
Направленный график. Directed graph
Направляющая. Directrix of a ruled surface
Направляющие косинусы. Direction cosines
Напряжение волокна. Fiber stress
Напряжение обратной силы. Reaction stress
Напряжение при пределе текучести. Yield stress
Напряженность магнитного поля. Magnetic field strength
Напряженность поля. Field strength
Напряженность электрического поля. Electric field strength
Нарушение условия синусов. Violation of sine condition
Насыщенная магнитоstriction кристалла. Crystal saturation magnetostriction
Натуральная ширина линии. Natural line width
Натуральный процесс. Natural processes
Начальный модуль. Initial modulus
Начертательная геометрия. Descriptive geometry
Незаимный элемент. Oriented element
Невязкий. Inviscid
Неголономный. Non-holonomic
Недиспергирующая волна. Non-dispersive wave
Независимость. Independence
Независимые реакции. Independent reactions
Независимые слагающие. Independent components
Независимый. Independent
Незамкнутая квадратурная формула. Open quadrature formula
Незамкнутая система. Open system
Неидеальный газ. Imperfect gas
Неизменный. Invariant
Неймана граничные условия. Neumann boundary conditions
Нейтральное равновесие. Neutral equilibrium
Нейтронная диффузия. Neutron diffusion
Нейтральная линия. Neutral axis
- Некомбинирующие модификации молекул. Non-combining modifications of molecules
Нелокализованный. Nonlocalized
Необратимый процесс. Irreversible process
Неоднородное преобразование. Inhomogeneous transformation
Неопределенная структура. Indeterminate structure
Неопределенная форма. Indeterminate form
Неориентированный график. Non-oriented graph
Неподлинный. Non-genuine
Непосредственные методы. Direct methods
Непосредственное соударение. Direct collision
Непрерывная дробь. Continued fraction
Непрерывная система. Continuous system
Непрерывная функция. Continuous function
Непрерывность. Continuity
Непрерывность состояния. Continuity of state
Неравносторонний. Scalene
Неразделимый график. Non-separable graph,
Неразрезная балка. Continuous beam
Непер. Neper
Нернста закон распределения. Nernst distribution law
Несимметричный изгиб. Unsymmetrical bending
Несобственная дробь. Improper fraction
Несобственный интеграл. Improper integral
Несовершенная функция. Incomplete function
Несовершенный блок. Incomplete block
Несоизмеримый. Incommensurate
Неупругое столкновение. Inelastic impact
Неустойчивое равновесие. Unstable equilibrium
Нецентральная сила. Non-central force
Нецентральное распределение. Non-central distribution
Нечетная функция. Odd function
Нечетно-четное правило. Odd-even rule
Нечетный терм в атоме. Odd term in an atom
Неэквивалентные электроны. Non-equivalent electrons
Неявная функция. Implicit function
Нильпотентный. Nilpotent
Номер накопителя. Address
Номер ячейки памяти. Address
Номограмма. Nomograph or nomogram
Норма. Norm
Нормаль к кривой. Normal to a curve
Нормальная конгруэнтность. Normal congruence
Нормальная кривизна. Normal curvature
Нормальная реакция. Normal reaction
Нормальная характеристика. Normal response
Нормальное давление. Normal pressure
Нормальное напряжение. Normal stress
Нормальное произведение. Normal product
Нормальное распределение. Normal distribution
Нормальное состояние. Normal state
Нормальное увеличение. Normal magnification
Нормальное уравнение. Normal equation
Нормальные колебания. Normal vibrations
Нормальный вид колебаний. Normal mode
Нормальный. Normal
Нормирование. Normalization
Нормировочный множитель. Normalizing factor
Нортон теорема. Norton's theorem
Нулевая линия полосы. Zero line of a band
Нулевая матрица. Null matrix
Нулевая последовательность. Null sequence
Нулевая энергия. Zero point energy
Нулевой вектор. Null vector

- Нулевой граф. Nullity graph
 Нульгеодезический. Null-geodesic
 Нуссельта критерий. Nusselt number
 Нутация. Nutation
 Ньютон. Newton
 Ньютона закон. Newton's law
 Ньютона-Котеса квадратурные формулы. Newton-Cotes quadrature formulas
 Ньютонова вязкость. Newtonian viscosity
- Область. Domain, region
 Область обратной скорости. Reciprocal velocity region
 Облучение. Exposure, irradiation, bombardment
 Обменная сила. Exchange force
 Обменное вырождение. Exchange degeneracy
 Обменный интеграл. Exchange integral
 Обнажение. Exposure
 Обнаружение табличных погрешностей. Detection of tabular errors
 Обобщенная деформация. Generalized strain
 Обобщенная сила. Generalized force
 Обобщенная скорость. Generalized velocity
 Обобщенное напряжение. Stress, generalized
 Обобщенные координаты. Generalized coordinates
 Обозначение регистра. Address
 Оболочка. Shell
 Образец. Sample
 Обратимый маятник. Reversible pendulum
 Обратимый процесс. Reversible process
 Обратная векторная система. Reciprocal vector system
 Обратная интерполяция. Inverse interpolation
 Обратная матрица. Inverse matrix
 Обратная решетка. Reciprocal lattice
 Обратная связь. Feedback
 Обратная теорема. Reciprocal theorem
 Обратная функция. Inverse function
 Обратное столкновение. Inverse collision
 Обратные поверхности. Inverse surfaces
 Обратные разницы. Reciprocal differences
 Обратный оператор. Inverse operator
 Обратный цикл. Reversed cycle
 Обращение времени. Time reversal
 Обращение матрицы. Matrix inversion
 Обращения ряда. Reversion of series
 Обрезание. Truncation
 Обтекание. Flow past
 Обтекание крутово тела. Flow past bluff body
 Общая группа преобразований. General transformation group
 Общая кривизна линзы. Total curvature of lens
 Общая относительность. General relativity
 Общее решение (однородного линейного дифференциального уравнения). Complementary function (of a linear differential equation)
 Общий наибольший делитель. Highest common factor
 Обыкновенная точка. Ordinary point
 Обыкновенное дифференциальное уравнение. Ordinary differential equation
 Объемная податливость. Bulk compliance
 Объемная скорость. Volume velocity
 Объемная упругость. Bulk modulus
 Обменная энергия. Exchange energy
 Объемные силы. Body forces
- Огибающая. Envelope
 Ограниченная вариация. Bounded variation
 Ограниченный. Bounded, restricted
 Одновременно измеримый. Simultaneously measurable
 Однородная турбулентность. Homogeneous turbulence
 Однородное преобразование лоренца. Homogenous Lorentz transformation
 Однородный. Homogeneous
 Однородный процесс. Homogeneous process
 Оже эффект. Auger effect
 Ожидание. Expectation
 Озена метод. Oseen's method
 Окологзвукое подобие. Transonic similarity
 Окологзвуковой. Transonic
 Окологзвуковой годограф. Transonic hodograph
 Округление. Rounding
 Октаэдральное напряжение. Octahedral stress
 Ома закон. Ohm's law
 Онсагера соотношения. Onsager relations
 Оператор. Operator
 Оператор аннигиляции. Annihilation operator
 Оператор вискозластичности. Viscoelastic operator
 Оператор потенциала. Potential operator
 Оператор рождения. Creation operator
 Оператор уничтожения. Destruction operator
 Оператор четности. Parity operator
 Операторные методы. Operational methods
 Опорное значение. Reference value
 Определенная структура. Determinate structure
 Определитель. Determinant
 Оптическая ветвь. Optical mode
 Оптическая глубина. Optical depth
 Оптическая модель. Optical model
 Оптическая ось. Optical axis
 Оптическая плотность. Optical density
 Оптическая путь. Optical path
 Оптическая теорема. Optical theorem
 Оптический дифференциальный инвариант. Optical differential invariant
 Оптический куб. Biprism
 Оптический центр. Optical center
 Оптическое расстояние. Optical distance
 Орбиталь. Orbital
 Ориентированный график. Oriented graph
 Ортогонализация. Orthogonalization
 Ортогональная функция. Orthogonal function
 Ортогональные векторы. Orthogonal vectors
 Ортогональные полиномы. Orthogonal polynomials
 Ортонормальная система. Orthonormal system
 Орторомбическая система. Orthorhombic system
 Ортоскопическая система. Orthoscopic system
 Ортохронный. Orthochronous
 Осадка опор. Settlement of supports
 Освещение. Illumination
 Освещенность. Illuminance
 Осевая нагрузка. Axial load
 Осевая тяга. Axial thrust
 Ослабление. Attenuation
 Осмотический коэффициент. Osmotic coefficient
 Осмотическое давление. Osmotic pressure
 Основание системы счисления. Radix
 Основная аффинная связь. Fundamental affine connection
 Основная вероятность. Fundamental probability
 Основная волна. Fundamental mode
 Основная единица. Fundamental unit

- Основное состояние. Fundamental state, ground state
- Основные полосы. Fundamental bands
- Основной ковариантный тензор. Fundamental covariant tensor
- Особенная точка. Singular point
- Особенное значение. Singular value
- Особенное решение. Singular solution
- Особенность. Singularity
- Особенный. Particular, singular
- Останавливающий потенциал. Stopping potential
- Остаток. Remainder
- Остаточная деформация. Permanent set
- Остаточное напряжение. Residual stress
- Остаточные формулы. Remainder formulas
- Оствальда закон. Ostwald's dilution law
- Остов. Frame, framework
- Остроградского-Гаусса теорема. Gauss theorem
- Острота резонанса. Sharpness of resonance
- Осцилляция. Oscillation
- Осуществление. Realization
- Ось винта. Screw axis
- Ось вращения. Rotation axis
- Ось симметрии. Symmetry, axis
- Ось упругости. Elastic axis
- Отбор пара. Bleeding
- Отверстие. Aperture
- Отзывчивость. Responsiveness
- Отклонение. Deflection, deviation
- Отклоняться. Deviate
- Относительная погрешность. Relative error
- Относительная световая отдача. Relative luminous efficiency
- Относительная частота. Relative frequency
- Относительное распределение. Relative distribution
- Относительное удлинение. Aspect ratio
- Относительный тензор. Relative tensor
- Отношение. Ratio
- Отношение вероятностей распада. Branching fraction
- Отношение возврата. Return ratio
- Отношение выдержки к почернению. Exposure-density relationship
- Отношение дальности к энергии. Range-energy relationship
- Отношение корреляции. Correlation ratio
- Отношение преобразования. Conversion ratio
- Отношение усиления. Gain ratio
- Отношение эффективностей. Efficiency ratio
- Отображение. Mapping
- Отражение. Reflection
- Отражение звука. Sound reflection
- Отраженная волна. Reflected wave
- Отрыв пограничного слоя. Separation of boundary layer
- Отсечка. Cut-off
- Отталкивающий потенциал. Repulsion potential
- Ошибка округления. Rounding error, roundoff
- Ошибка. Error
- Падающий. Incident
- Пара. Couple
- Парабола. Parabola
- Параболические координаты. Parabolic coordinates
- Параболические цилиндрические координаты. Parabolic cylindrical coordinates
- Параболический рупор. Parabolic horn
- Параболическое уравнение. Parabolic equation
- Параболоид. Paraboloid
- Параболоидные координаты. Paraboloidal coordinates
- Парадокс рекуррентности. Recurrence paradox
- Парадокс часов. Clock paradox
- Параксимальный луч. Paraxial ray
- Параллакс. Parallax
- Параллелепипед. Parallelepiped
- Параллелограмм. Parallelogram
- Параллелограмм сил. Parallelogram of forces
- Параллельное перемещение вектора. Parallel displacement of a vector
- Параллельные поверхности. Parallel surfaces
- Парамагнетизм. Paramagnetism
- Парамагнитная восприимчивость. Paramagnetic susceptibility
- Параметр. Parameter
- Параметр волны. Wave parameter
- Параметр столкновения. Impact parameter
- Параметр удара. Impact parameter
- Параметр формы. Form parameter
- Параметры кристалла. Crystal parameters
- Паровой котел. Boiler
- Парциальная когерентность. Partial coherence
- Парциальное давление. Partial pressure
- Парциальные молярные величины. Partial molar quantities
- Паскаля треугольник. Pascal triangle
- Паули-Вайскопфа уравнение. Pauli-Weisskopf equation
- Паули терм. Pauli term
- Пашена-Бака эффект. Paschen-Back effect
- Пашена ряд. Paschen series
- Пеленг. Bearing
- Первая предельная теорема. First limit theorem
- Первая основная теорема. First fundamental theorem
- Первая фокальная точка. First focal point
- Первый закон термодинамики. First law of thermodynamics
- Перевал. Steepest descent
- Переводный коэффициент. Conversion factor
- Перегородка. Partition
- Передаточная функция. Transfer function
- Передача управления. Transfer of control
- Передача столкновением. Collisional transfer
- Передача шума. Noise transmission
- Перекрытие. Overlap
- Переломление. Refraction
- Переломленная волна. Refracted wave,
- Переломляющая способность. Refractivity
- Переменчивость множественностей. Alternation of multiplicities
- Переменная действия. Action variable
- Переменный. Alternating
- Переменный ток. Alternating current
- Переменный тензор. Alternating tensor
- Перемещение. Translation
- Перенормировка массы. Renormalization of mass
- Перенос. Carry
- Перенос нейтронов. Neutron transport
- Переносимая величина. Transferable quantity
- Перерелаксация. Overrelaxation
- Перестановка. Permutation
- Перестановочная группа. Permutation group

- Перестановочные соотношения. Commutation relations
- Переход. Transition
- Переход второго порядка. Second order transition
- Переход высшего порядка. Transition of higher order
- Переходная потеря. Transition loss
- Переходная характеристика. Transient response
- Перечислимый. Countable
- Период. Period
- Период волны. Wave period
- Период восстановления. Period of restitution
- Период деформации. Period of deformation
- Период импульса. Pulse interval, pulse spacing
- Период повторения импульсов. Pulse repetition period
- Период реактора. Reactor period
- Периодическая волна. Periodic wave
- Периодограмма. Periodogram
- Периферальное напряжение. Circumferential stress
- Петля. Loop
- Петцваля поверхность. Petzval surface
- Пикара метод. Picard method
- Пирсона распределение. Pearson distribution
- Пифагорова шкала. Pythagorean scale
- Плавающее тело. Floating body
- Планетарное движение. Planetary motion
- Планиметр. Planimeter
- Планка закон. Planck law
- Пластина. Slab, plate
- Пластическая деформация. Plastic flow
- Пластичность. Plasticity
- Пластичные волны. Plastic waves
- Плита. Plate, slab
- Пловучая струя. Buoyant jet
- Плоская волна. Plane wave
- Плоская поляризация. Plane polarization
- Плоское движение. Plane motion
- Плоскостной график. Planar graph
- Плоскость колебания. Plane of vibration
- Плоскость кривизны. Curvature, plane of
- Плоскость меридиана. Meridional plane
- Плоскость наведения. Guide plane
- Плоскость отражения. Reflection plane
- Плоскость пропускания. Transmission plane
- Плоскость симметрии. Symmetry, plane of
- Плотность ансамблей. Ensemble density
- Плотность вероятности. Probability density
- Плотность замедления. Slowing-down density
- Плотность звуковой энергии. Sound-energy density
- Плотность излучения. Radiation density
- Плотность квантовых состояний. Density of quantum states
- Плотность лучистого потока. Irradiance
- Плотность нейтронов. Neutron density
- Плотность носителей тока. Carrier density
- Плотность потока облучения. Radiant emittance
- Плотность подвижных носителей тока. Density of mobile charges
- Плотность распределения вероятности. Probability density function
- Плотность соударений нейтронов. Neutron collision density
- Плотность тока. Current density
- Площадь замедления. Slowing-down area
- Площадь крыла. Wing area
- Площадь миграции. Migration area
- Поверхностная волна. Surface wave
- Поверхностная расходимость. Surface divergence
- Поверхностная фаза. Surface phase
- Поверхностное натяжение. Surface tension
- Поверхностный тензор. Surface tensor
- Поверхность вращения. Surface of revolution
- Поверхность распределения интенсивности. Surface of intensity distribution
- Поверхность центров. Surface of centers
- Повреждение. Failure
- Погасание. Extinction
- Погашение. Extinction
- Поглотитель. Sink, absorber
- Поглощающая способность. Absorptance
- Поглощение звука. Sound absorption
- Поглощение лучистой энергии. Absorption of radiant energy
- Пограничный слой. Boundary layer
- Погрешность. Error
- Подавление. Elimination, suppression
- Податливость к ползучести. Creep compliance
- Податливость к сдвигу. Shear compliance
- Подвижная ось. Moving axis
- Подвижность. Mobility
- Подгруппа. Subgroup
- Подзвуковой. Subsonic
- Подинтегральная функция. Integrand
- Подкос. Strut
- Поднятие и опускание индексов тензора. Raising and lowering indices of a tensor
- Подобие. Similarity
- Подпрограмма. Subroutine
- Позитроний. Positronium
- Показатель вероятности. Index of probability
- Показатель преломления. Index of refraction
- Показатель степени. Exponent
- Поле. Field
- Поле зрения. Field of view
- Поле излучения. Radiation field
- Полевой луч. Field ray
- Полезная мощность. Brake horsepower
- Ползучесть. Creep
- Ползучие движение. Creeping motion
- Полимер. Polymer
- Полимерные смеси. Polymer mixtures
- Полином. Polynomial
- Полиэдр. Polyhedron
- Полная линейная группа. Full linear group
- Полная прозрачность. Total transmittance
- Полная производная. Total derivative
- Полное давление. Total pressure
- Полное определение. Total determination
- Полное отражение. Total reflection
- Полный. Complete
- Полный график. Complete graph
- Полный коэффициент излучения. Total emissivity
- Полный поток нейтронов. Total neutron flux
- Положительная определенная матрица. Positive definite matrix
- Полоса. Band, fringe (interference)
- Полупериод распада. Half-life
- Полуширина спектральной линии. Half-width of a spectral line
- Полос. Pole
- Полос аналитической функции. Pole of an analytic function
- Полюсная линия. Polar line
- Поляризация. Polarization

- Поляризационная катастрофа. Polarizability catastrophe
- Поляризуемость. Polarizability
- Полярные координаты. Polar coordinates
- Полярный вектор. Polar vector
- Помехи. Noise, interference
- Пондеромотивный. Ponderomotive
- Поперечная нагрузка. Lateral load
- Поперечная волна. Transverse wave
- Поперечное сечение. Cross section
- Поперечное увеличение. Lateral magnification
- Поправка. Correction
- Порог. Threshold, cutoff
- Порог слышимости. Threshold of audibility
- Порог фотоэффекта. Photoelectric threshold
- Пороговая энергия. Threshold energy
- Порода. Species
- Портальная рама. Portal frame
- Портера-Томаса распределение. Porter-Thomas distribution
- Порядковый номер атома. Atomic number
- Порядок химических реакций. Order of chemical reactions
- Последовательность. Sequence
- Последовательный анализ. Sequential analysis
- Постановка экспериментов. Design of experiments
- Посторонний. Extraneous
- Постоянная. Constant
- Постоянная интегрирования. Constant of integration
- Постоянная пружины. Spring constant
- Постоянная распространения. Propagation constant
- Постоянная решетки. Lattice constant
- Постоянная связи. Coupling constant
- Постоянная энтропии. Entropy constant
- Построение лучевых диаграмм. Ray tracing
- Постулат. Postulate
- Потенциал возбуждения. Excitation potential
- Потенциал деформации. Deformation potential
- Потенциал ионизации. Ionization potential
- Потенциал скоростей. Velocity potential
- Потенциальная температура. Potential temperature
- Потенциальная энергия. Potential energy
- Потенциальное рассеяние. Potential scattering
- Потенциальный барьер. Potential barrier, Coulomb barrier
- Потенциальный остов. Potential core
- Потенциальный поток. Potential flow
- Потери в стенках. Wall losses
- Потеря от расхождения (звука). Divergence loss (sound)
- Потеря от рассеяния. Scattering loss
- Поток. Flux
- Поток вектора. Vector flux
- Поток звуковой энергии. Sound energy flux
- Поток нейтронов. Neutron current
- Потокоцепление. Flux linkage
- Правдоподобие. Likelihood
- Правила интенсивности для мультиплетов. Intensity rules for multiplets
- Правила коммутативности. Commutation rules
- Правила отбора. Selection rules
- Правила сверхотбора. Superselection rules
- Правило ложности. Regula falsi
- Правило потока. Flow rule
- Правило трапеции. Trapezoidal rule
- Правило фаз. Phase rule
- Правило частот Бора. Bohr frequency condition
- Практическая ширина. Practical width
- Прандтля-Глауэрта правило. Prandtl-Glauert rule
- Прандтля-Мейера разложение. Prandtl-Meyer expansion
- Прандтля число. Prandtl number
- Предварительно напряжённый бетон. Prestressed concrete
- Преддиссоциация. Predissociation
- Предел. Bound
- Предел прочности. Ultimate strength
- Предел текучести. Yield point
- Предел упругости. Elastic limit
- Предельная нагрузка. Limit load
- Предельная теорема. Limit theorems
- Предельный закон. Limiting law
- Прикосновение. Impact, contact
- Предельный момент. Limit moment
- Представление. Representation
- Представление взаимодействия. Interaction representation
- Представление группы. Representation of a group
- Преимущественное направление. Preferred direction
- Преобразование. Transform, transformation
- Преобразование сопротивления. Impedance transform
- Преобразованное сечение. Transformed section
- Прерывистые системы. Discontinuous systems
- Приближение в среднем. Approximation in the mean
- Приведенная масса. Reduced mass
- Приведенная фокусная длина. Reduced focal length
- Призма. Prism
- Приложенная сила. Force, applied
- Примитивное перемещение. Primitive translation
- Примитивный. Primitive
- Принуждение. Constraint
- Принцип виртуальной работы. Principle of virtual work
- Принцип детального баланса. Principle of detailed balancing
- Принцип импульса-момента. Impulse-momentum principle
- Принцип исключения. Exclusion principle
- Принцип комбинации. Combination principle
- Принцип наименьшего действия. Least-action principle
- Принцип наименьшей энергии. Least-energy principle
- Принцип накопления. Building-up principle
- Принципы неопределенности. Indeterminacy principle
- Принципы причинности. Causality principle
- Принцип соответствия. Correspondence principle
- Принцип соответственных состояний. Principle of corresponding states
- Принципальная серия. Principal series
- Принципальные схемы. Fundamental circuits
- Принципальный вектор матрицы. Principal vector of a matrix
- Приосевой луч. Paraxial ray
- Приращение изгиба. Incremental collapse
- Проба. Sample, test
- Пробег. Range, path
- Проблемы с начальными значениями. Initial-value problems
- Пробная функция. Test function
- Пробное испытание. Proof test

- Проводимость. Admittance
 Проводимость лампы. Perveance
 Программа. Routine, program
 Прогрессия. Progression
 Продольная волна. Longitudinal wave
 Продольная сферическая аберрация. Longitudinal spherical aberration
 Продольное увеличение. Longitudinal Magnification
 Продольные волны. Longitudinal waves
 Продольный сдвиг. Longitudinal shear
 Продукт инерции площади. Area product of inertia
 Проектор. Projector
 Прозрачность. Transmittance
 Произведение. Product
 Произведение тензоров. Product of tensors
 Произведение усиления и ширины полосы. Gain-bandwidth product
 Произведение четырех векторов. Quadruple product of vectors
 Произведенное множество. Derived set
 Производная. Derivative
 Производящая функция. Generating function
 Пролет. Span, bay
 Промежуток столкновений. Collision interval
 Пропеллер. Airscrew, propeller
 Пропорциональная нагрузка. Proportional loading
 Пропорциональный контроль. Proportional control
 Пропорциональный предел. Proportional limit
 Пропорция поляризации. Proportion of polarization
 Пропускание. Transmission
 Пропускная способность. Carrying capacity (load)
 Простая волна. Simple wave
 Простая ферма. Simple truss
 Просто связанная область. Simply connected region
 Простой маятник. Simple pendulum
 Пространственная группа. Space group
 Пространственная кривая. Space curve
 Пространственная поверхность. Space-like surface
 Пространственная решётка. Space lattice
 Пространственно-временной континуум. Space-time
 Пространственное квантование. Space quantization
 Пространственный тензор. Space tensor
 Профиль скоростей. Velocity profile
 Профильное сопротивление. Profile drag
 Прохождение. Transmission
 Процесс переброса. Umklapp process
 Пружина. Spring
 Прямая отражательная способность. Direct reflectance
 Прямая прозрачность. Direct transmittance
 Прямая сумма. Direct sum
 Прямое взаимодействие. Direct interaction
 Прямое напряжение. Direct stress
 Прямое произведение (подгрупп). Direct product (of subgroups)
 Прямолинейная конгруэнтность. Rectilinear congruence
 Прямолинейная податливость. Rectilinear compliance
 Прямолинейное движение. Rectilinear motion
 Прямоугольная волна. Square wave
 Прямоугольная яма. Square well
 Прямоугольное отверстие. Rectangular aperture
 Прямоугольное распределение. Rectangular distribution
 Псевдовектор. Pseudovector
 Псевдовекторная связь. Pseudovector coupling
 Псевдоскаляр. Pseudoscalar
 Псевдоскалярная связь. Pseudoscalar coupling
 Пуаз. Poise
 Пуанкаре инвариант. Poincare's invariant
 Пуассона интеграл. Poisson integral
 Пульсация. Flutter, pulsation
 Плотность. Density
 Путь. Path
 Пучность. Anti-nodal point, loop
 Пучок. Beam
 Пфафа проблема. Pfaff problem
 Пфунда ряд. Pfund series
 Пьезоэлектрические постоянные. Piezoelectric constants
 Работа. Work
 Работа выхода. Work function
 Рабочая нагрузка. Working load
 Рабочая характеристика. Operating characteristic
 Рабочее напряжение. Working stress
 Рабочий объем. Swept volume
 Равновесие. Equilibrium, balance
 Равновесие сил. Equilibrium of forces
 Равновесие частицы. Equilibrium of a particle
 Равномерное движение. Uniform motion
 Равномерное распределение энергии. Equipartition of energy
 Равнофазная поверхность. Equiphase surface
 Равноценность. Equivalence
 Радиальная нагрузка. Radial loading
 Радиальное распределение. Radial distribution
 Радиальное увеличение. Radial magnification
 Радиационная длина. Radiation length
 Радиационная поправка. Radiative correction
 Радиус атома. Atomic radius
 Радиус вектор. Radius vector
 Радиус инерции. Radius of gyration
 Радиус кривизны. Radius of curvature
 Радиус кручения. Radius of torsion
 Разброс. Straggling
 Развертка кривой. Involute of a curve
 Развертка поверхности. Involute of a surface
 Развертывающийся. Developable
 Развертывающаяся поверхность. Developable surface
 Развертывающийся, выпрямительный. Developable, rectifying
 Развертывающийся, полярный. Developable, polar
 Развертывающийся, соприкасающийся. Developable, osculating
 Разделение. Partition, separation
 Разделение переменных. Separation of variables
 Разделенные разности. Divided differences
 Разделимый. Separable
 Разделимый график. Separable graph
 Разложение вектора. Decomposition of a vector
 Разложение на множители. Factorization
 Разложение на элементарные дроби. Partial fraction expansion
 Разложение полиномов. Polynomial factorization
 Разложение сил. Forces, resolution of
 Размер. Dimension
 Размеры решетки. Lattice dimensions
 Разностное уравнение. Difference equation
 Разностный оператор. Difference operator
 Разрежение. Dilation, expansion
 Разрешающая способность. Resolving power

- Разреженный газ. Rarefied gas
 Разрешённый переход. Allowed transition
 Разрыв непрерывности. Discontinuity
 Рака коэффициент. Raca coefficient
 Рама. Frame, framework
 Рамная опора. Bent
 Ранг. Rank
 Ранг матрицы. Rank of a matrix
 Ранкина цикл. Rankine cycle
 Раскрыв. Aperture
 Распределение. Distribution
 Распределение вероятности. Probability distribution
 Распространение. Propagation, transmission
 Распространение звука. Sound transmission
 Распределение по импульсам. Distribution in momentum
 Распределение по фазам. Distribution in phase
 Распределение по энергиям. Distribution in energy
 Распределённая сила. Distributed force
 Распространение ошибок. Propagation of error
 Рассела-Сондерса связь. Russell-Saunders coupling
 Рассеяние. Scattering, dissipation
 Рассеяние звука. Scattering of sound
 Рассеянная отражательность. Diffuse reflectance
 Рассеянное отражение. Diffuse reflection
 Рассеяющая сила. Dissipative force
 Раствор. Aperture
 Расслоение. Stratification
 Растяжимость. Ductility
 Расходимость. Divergence
 Расхождение. Divergence
 Расчёт по минимальному весу. Minimum weight design
 Растяжение. Expansion, tension
 Расширение. Dilatation, expansion, broadening
 Расширение соотношения синусов. Extension of sine relationship
 Раута правило. Routh's rule
 Рациональное число. Rational number
 Реактивное сопротивление. Reactance
 Реактивность. Reactivity
 Реальный газ. Real gas
 Реактивный двигатель. Jet engine
 Реакция. Reaction
 Ребро. Edge
 Регрессия. Regression
 Регулярная особенность. Regular singularity
 Регулярная функция. Regular function
 Регулярное решение. Regular solution
 Регулирующий стержень. Control rod
 Режим. Mode
 Резерфорд. Rutherford
 Резиновое состояние. Rubbery state
 Резкая серия. Sharp series
 Резонансная частота. Resonant frequency
 Резонансная частота кристалла. Resonance frequency of crystal
 Резонансная энергия. Resonance energy
 Резонансный интеграл. Resonance integral
 Результат. Resultant
 Рекуперация тепла. Heat regeneration
 Рекуррентная формула. Recursion formula
 Релаксационное поведение. Relaxation behavior
 Релаксационные явления. Relaxation phenomena
 Релаксация. Relaxation
 Релятивистская гидродинамика. Relativistic hydrodynamics
 Релятивистский инвариант. Relativistic invariant
 Релятивистская инвариантность. Relativistic invariance
 Релятивистская прецессия. Relativity precession
 Релятивистская теория. Relativistic theory
 Рентгеновский спектр. X-ray spectrum
 Репер. Fixed point, reference point
 Рефлексное отражение. Reflex reflection
 Рефрактивная дисперсивность. Refractive dispersivity
 Рециркуляция. Recirculation
 Решетка. Lattice, frame, framework
 Решетчатая модель. Lattice model
 Ридберга постоянная. Rydberg constant
 Риккати уравнение. Riccati equation
 Римана поверхность. Riemann surface
 Ритца формула. Ritz formula
 Ричардсона-Душмана уравнение. Richardson-Dushman equation
 Ричардсона критерий. Richardson number
 Риччи тождество. Ricci identity
 Родрига формула. Rodrigues formula
 Розетка. Rosette
 Ролля теорема. Rolle theorem
 Ромбоздральная система. Rhombohedral system
 Россби число. Rossby number
 Ротатор. Rotator
 Ротационные правила суммирования. Rotational sum rules
 Ротационный спектр. Rotational spectrum
 Ротор. Rotor, curl
 Рунге-Кутта метод. Runge-Kutta method
 Рупор. Horn
 Рэйнолдса критерий. Reynolds number
 Рэлея поверхностная волна. Rayleigh surface waves
 Рэлея-Джинса уравнение. Rayleigh-Jeans equation
 Рэлея диск. Rayleigh disk
 Ряд. Series
 Сабина. Sabin
 Сабина закон. Sabin law
 Сагитта. Sagitta
 Сагиттальная плоскость. Sagittal plane
 Сагиттальный фокус. Sagittal focus
 Сакура-Тетрода уравнение. Sackur-Tetrode equation
 Самосогласованное поле. Self-consistent field
 Саха формула. Saha formula
 Свертывание. Convolution
 Свертка. Convolution
 Сверхзвуковой. Supersonic, hypersonic
 Сверхзвуковое течение. Hypersonic flow
 Сверхзвуковой закон подобия. Hypersonic similarity law
 Сверхмультиплет. Supermultiplet
 Сверхпроводимость. Superconductivity
 Сверхпроводящий переход. Superconducting transition
 Сверхрешетка. Superlattice
 Сверхсопряжение. Hyperconjugation
 Сверхтонкая структура. Hyperfine structure
 Световая отражательная способность. Luminous reflectance
 Световое излучение. Luminous emittance
 Световой выход. Luminous efficiency
 Световой поток. Luminous flux
 Свечность. Candle power

- Свободно опертая балка. Simple beam
 Свободное поле. Free field
 Свободное течение молекул. Free flow of molecules
 Свободные колебания. Free or natural oscillations
 Свободные колебания незатухающей системы. Free vibrations of undamped system
 Свободные линии обтекания. Free streamlines
 Свободный вектор. Free vector
 Свободный индекс. Free index
 Свойства симметрии. Symmetry properties
 Связанный график. Connected graph
 Связность графика. Graph connectivity
 Связь. Coupling
 Сглаживание. Smoothing
 Сгусток. Cluster
 Сдвиг. Shear
 Сдвиги уровней. Level displacements
 Семейство. Family
 Семейство поверхностей. Family of surfaces
 Сербера-Вильсона метод. Serber-Wilson method
 Сеть. Network, net, circuit
 Сжатое сечение. Vena contracta
 Сжимаемость. Compressibility
 Сжатие. Compression
 Сигбана X-единица. Siegbahn x-unit
 Сигнум. Signum
 Сила. Force
 Сила на погруженной поверхности. Force on a submerged surface
 Сила на проводник. Force on conductor
 Сила осциллятора. Oscillator strength
 Сила сцепления. Force of cohesion
 Силовое поле. Field of force
 Силовой полигон. Force polygon
 Силовой треугольник. Triangle of forces
 Силы инерции. Inertia forces
 Силы между многими телами. Many-body forces
 Силы пересекающие в одной точке. Concurrent forces
 Сильная особая точка. Essential singularity
 Символ перестановки. Permutation symbol
 Символы молекулярных термов. Molecular term symbols
 Симметричная волновая функция. Symmetric wave function
 Симметричная функция. Symmetric function
 Симметрично-поворотная ось. Rotation-inversion axis
 Симметричный тензор. Symmetric tensor
 Симметрия кристалла. Crystal symmetry
 Симпсона формула. Simpson's rule
 Синтетическое ядро. Synthetic kernel
 Синусоидальная предельная теорема. Sinusoidal limit theorem
 Система единиц. System of units
 Система обратных векторов. Reciprocal vector system
 Система отсчета. Frame of reference
 Система регулирования. Control system
 Система уравнений. Simultaneous equations
 Система центра инерции. Center-of-mass system
 Систематическая проба. Systematic sample
 Скаляр. Scalar
 Скалярная кривизна. Scalar curvature
 Скалярная плотность. Scalar density
 Скалярное поле. Scalar field
 Скалярное произведение. Inner product, scalar product
 Складка. Wrinkling
 Скольжение. Slip
 Скорость. Velocity
 Скорость активации. Activation rate
 Скорость выхода. Escape velocity
 Скорость диффузии. Diffusion velocity
 Скорость затухания звука. Rate of decay of sound
 Скорость рекомбинации. Recombination rate
 Скорость частицы. Particle velocity
 Скорость химической реакции. Chemical reaction rate
 Скосность. Skewness
 Скручивание. Twisting, torsional buckling
 Скрытая координата. Hidden coordinate
 Слагание векторов. Composition of vectors
 След. Trace, wake
 Слияние. Confluence
 Слово. Word
 Сложение сил. Composition of forces
 Сложение тензоров. Addition of tensors
 Сложная вероятность. Compound probability
 Сложная ферма. Complex truss
 Сложное напряжение. Combined stress
 Случайная переменная. Random variable
 Смесь идеальных газов. Perfect-gas mixture
 Смешанная нагрузка. Combined loading
 Смешанная характеристическая функция. Mixed characteristic function
 Смешанное произведение трех векторов. Triple product of vectors
 Смешивание. Confounding
 Смещение. Translation, displacement
 Снеллиуса закон. Snell's law
 Собственная дробь. Proper fraction
 Собственная производная тензорного поля. Intrinsic derivative of tensor field
 Собственная функция. Eigenfunction
 Собственная энергия. Self energy
 Собственное значение. Eigenvalue
 Собственное преобразование. Proper transformation
 Собственное состояние. Eigenstate
 Собственное уравнение тензора. Eigenvalue equation of tensor
 Собственные значения. Eigenvalues
 Собственные свойства поверхности. Intrinsic properties of a surface
 Собственный вектор. Eigenvector
 Собственный вес. Dead load
 Собственный период. Natural period
 Совершенно пластический. Perfectly plastic
 Совершенное решение. Perfect solution
 Соединённый. Connected
 Сокращение. Contraction
 Соответственные состояния. Corresponding states
 Соотношения кристаллографических осей. Crystallographic axial ratios
 Сопло. Nozzle
 Соприкасающаяся плоскость. Osculating plane
 Сопротивление. Resistance, impedance, drag
 Сопротивление протого рода. Rolling resistance
 Сопротивление жидкости. Fluid resistance
 Сопротивление излучению. Radiation resistance
 Сопротивление к поверхностному переносу. Surface transfer impedance
 Сопротивление сжатию. Bulk modulus
 Сопряжение двойной связи. Conjugation of double bonds

- Сопряженная волновая функция. Adjoint wave function
- Сопряженная матрица. Adjoint of a matrix
- Сопряженная система кривых. Conjugate system of curves
- Сопряженная точка. Conjugate point
- Сопряженное уравнение. Adjoint equation
- Сопряженные направления. Conjugate directions
- Сопряженные точки оптической системы. Conjugates in an optical system
- Сопряженные химические реакции. Coupled chemical reactions
- Сопряженные числа. Conjugate numbers
- Сопряженные элементы группы. Conjugate elements of a group
- Сопряженные ядра. Conjugate nuclei
- Сопряженный оператор. Adjoint of an operator
- Сопряженный поток. Adjoint flux
- Сореля теорема. Saurel's theorem
- Сорт. Grade
- Состав горючей смеси. Air-fuel ratio
- Составляющая вектора. Component of a vector
- Составная балка. Composite beam
- Составной маятник. Compound pendulum
- Собственные уравнения кривой. Intrinsic equations of curve
- Составная ферма. Compound truss
- Состояние с отрицательной энергией. Negative energy state
- Сохранение импульса. Conservation of momentum
- Сохранение массы. Conservation of mass
- Сохранение момента количества движения. Conservation of angular momentum
- Сохранение энергии. Conservation of energy
- Спектр. Spectrum
- Спектр деления. Fission spectrum
- Спектр мощности. Power spectrum
- Спектр нейтронов. Spectrum, neutron
- Спектр турбулентности. Spectrum of turbulence
- Спектральная концентрация. Spectral concentration
- Спектральная лучеиспускательность. Spectral emissivity
- Спектральная норма. Spectral norm
- Спектральная отражательная способность. Spectral reflectance
- Спектральная поглощающая способность. Spectral absorptance
- Спектральная прозрачность. Spectral transmittance
- Спектральная функция. Spectral function
- Спектральная характеристика. Spectral characteristic
- Спектральная яркость. Spectral luminance
- Спектральное распределение. Spectral distribution
- Спектральный радиус. Spectral radius
- Спектроскопический терм. Spectroscopic term
- Специальная относительность. Special relativity
- Специальная функция. Special function
- Специальный полином. Special polynomial
- Спин. Spin
- Спин электрона. Electron spin
- Спин ядра. Nuclear spin
- Спиновые операторы. Spin operators
- Спиновой парамагнетизм. Spin paramagnetism
- Спинорное исчисление. Spinor calculus
- Спираль. Helix
- Спиральность. Helicity
- Сплющенный. Oblate
- Сплющенный сфероид. Oblate spheroid
- Спутная струя. Wake
- Средняя свободная линия переноса. Transport mean free path
- Средняя скорость. Average velocity
- Среднее арифметическое. Arithmetic mean
- Среднее время жизни. Mean lifetime
- Среднее геометрическое. Geometric mean
- Среднее значение. Mean value
- Среднее отклонение. Mean deviation
- Среднее свободное время. Mean free time
- Среднеквадратичное. Root-mean-square
- Средний. Average, median
- Средний свободный пробег фонона. Phonon mean free path
- Средняя горизонтальная интенсивность. Mean horizontal intensity
- Средняя нормальная кривизна. Mean normal curvature,
- Средняя разность. Mean difference
- Средняя длина свободного пробега. Mean free path
- Средняя сферическая интенсивность. Mean spherical intensity
- Средняя температура массы. Bulk temperature
- Средняя точка. Midpoint
- Сродство с электроном. Electron affinity
- Стабилизация дисперсий. Stabilization of variance
- Стандартная мера. Standard measure
- Стандартная ошибка. Standard error
- Стандартное отклонение. Standard deviation
- Стандартное преломление. Standard refraction
- Статика. Statics
- Статистика. Statistic
- Статистический контроль качества. Statistical quality control
- Статистическая гипотеза. Statistical hypothesis
- Статистическая механика. Statistical mechanics
- Статистически разрешенный. Statistically admissible
- Статистический вес. Statistical weight
- Статическая устойчивость. Static stability
- Статический момент. Static moment
- Статическое давление. Static pressure
- Стационарная волна. Stationary wave
- Стационарная звуковая волна. Stationary sound wave
- Стационарное колебание. Steady-state oscillation
- Стационарное состояние. Stationary state
- Стеклообразное состояние. Glassy state
- Степенный ряд. Power series
- Степень матрицы. Degree of a matrix
- Степень поверхности. Power of a surface
- Степень продвижения. Degree of advancement
- Степень реакции. Extent of reaction
- Степень свободы. Degree of freedom
- Стерadian. Steradian
- Стереографическая проекция. Stereographic projection
- Стереоспектрограмма. Stereospectrogram
- Стереофонический фазовой эффект. Binaural phase effect
- Стефана-Больцмана закон. Stefan-Boltzmann law
- Стехиометрический. Stoichiometric
- Стигматичный. Stigmatic
- Стильб. Stilb
- Стилтьеса интеграл. Stieltjes integral
- Стирлинга интерполяционная формула. Stirling interpolation formula
- Стойка. Strut, column

- Стокса теорема. Stokes theorem
 Столб. Column
 Стохастический. Stochastic
 Стохастический процесс. Stochastic process
 Стоячая волна. Standing wave, stationary wave
 Строение ядра. Nuclear structure
 Структура. Structure, frame, framework
 Структурный фактор. Structure factor
 Структуры бензола. Structures of benzene
 Студента распределение. Student's distribution
 Субгармоника. Subharmonic
 Сумма колебательных состояний. Vibrational partition function
 Сумма состояний. Partition function
 Сумма тензоров. Sum of tensors
 Сумма ряда. Summation of series
 Суммарный коэффициент полезного действия. Joint efficiency
 Сухой пар. Dry vapor
 Сфера кривизны. Sphere of curvature
 Сферическая абберация. Spherical aberration
 Сферическая волна. Spherical wave
 Сферическая индикатрисса. Spherical indicatrix
 Сферическая кривизна. Spherical curvature
 Сферическая тригонометрия. Spherical trigonometry
 Сферические гармоники. Spherical harmonics
 Сферические полярные координаты. Spherical polar coordinates
 Сферический вихрь. Spherical vortex
 Сферический маятник. Spherical pendulum
 Сферическое изображение. Spherical representation
 Сфероидальные координаты. Spheroidal coordinates
 Сферометр. Spherometer
 Схема. Circuit, scheme, diagram
 Схема уровней энергии. Energy level diagram
 Сходимость. Convergence
 Сходящаяся волна. Converging wave
 Счетный. Countable, enumerable
 Сырость. Humidity
- Тамма-Данкова метод. Tamm-Dancoff method
 Тангенс угла потерь. Loss tangent
 Тангенциальная плоскость. Tangential plane
 Тангенциальная сила. Tangential force
 Тангенциальный фокус. Tangential focus
 Твёрдость. Hardness
 Тевенина теорема. Thévenin's theorem
 Температура. Temperature
 Температура вырождения. Degeneracy temperature
 Температура заторможенного слоя. Stagnation temperature
 Температура инверсии. Inversion temperature
 Температура нейтронов. Neutron temperature
 Температура по кинетической теории. Kinetic theory temperature
 Тенденция барометрического давления. Tendency of barometric pressure
 Тензор кривизны. Curvature tensor
 Тензор энергии и импульса. Energy-momentum tensor
 Тензор электромагнитного поля. Electromagnetic field tensor
 Тензорная плотность. Tensor density
 Тензорная производная. Tensor derivative
 Тензорное поле. Tensor field
- Тензорный анализ. Tensor analysis
 Теорема взаимности. Reciprocity theorem
 Теорема вычетов. Residue theorem
 Теорема диссипационного колебания. Fluctuation-dissipation theorem
 Теорема индукции. Induction theorem
 Теорема минимума потенциальной энергии. Theorem of minimum potential energy
 Теорема наименьшей работы. Least work, theorem of
 Теорема наложения. Superposition theorem
 Теорема о среднем. Mean value theorem
 Теорема перпендикулярной оси. Perpendicular axis theorem
 Теорема сопоставления для сходимости. Comparison theorem for convergence
 Теорема статистического веса. Statistical weight theorem
 Теорема эквивалентного генератора. Thévenin's theorem
 Теорема эквивалентности. Equivalence theorem
 Теория абсолютной скорости реакции. Absolute reaction rate theory
 Теория аэродинамических профилей. Airfoil theory
 Теория бета-распада. Theory of beta decay
 Теория возмущения. Perturbation theory
 Теория дырок. Hole theory
 Теория игр. Games theory
 Теория отклонений. Deflection theory
 Теория оценок. Theory of estimation
 Теория планетарных орбит. Planetary orbit theory
 Теория пластического течения. Flow theory of plasticity
 Теория позитрона. Positron theory
 Теория поля. Field theory
 Теория равновесия. Equilibrium theory
 Теория распределения. Distribution theory
 Теория соударений. Collision theory
 Тепло разбавления и разведения. Heat of dilution
 Тепловая единица. Thermal unit
 Тепловая устойчивость. Thermal stability
 Тепловое движение. Thermal agitation
 Тепловое излучение. Thermal radiation
 Тепловое равновесие. Thermal equilibrium
 Тепловое расширение. Thermal expansion
 Тепловое уравнение состояния. Caloric equation of state
 Тепловой коэффициент. Thermal coefficient
 Тепловой к.п.д. Thermal efficiency
 Тепловой потенциал. Thermal potential
 Тепловой удар. Thermal shock
 Теплоемкость. Specific heat
 Теплопроводящая стена. Diathermal wall
 Теплопроизводительность. Calorific value
 Терм энергетического состояния. Energy-state term
 Термический скачок. Thermal shock
 Термодинамика необратимых процессов. Thermodynamics of irreversible processes
 Термодинамическая система. Thermodynamic system
 Термодинамическая статистика. Thermostatistics
 Термодинамическая устойчивость. Thermodynamic stability
 Термодинамическая функция. Thermodynamic function
 Терромагнитный эффект. Thermomagnetic effect
 Термодинамическое равновесие. Thermodynamic equilibrium

Термодинамическое свойство.	Thermodynamic property	Угловое распределение.	Angular distribution
Термомолекулярное давление.	Thermomolecular pressure	Угловое смещение.	Angular displacement
Термостатика.	Thermostatics	Угловое увеличение.	Angular magnification
Термоэлектрическая мощность.	Thermoelectric power	Угловое ускорение.	Angular acceleration
Тетрагональная система.	Tetragonal system	Угловой импульс.	Angular impulse
Толчок.	Impact, jar	Угловой поток нейтронов.	Angular neutron flux
Томсона рассеяние.	Thomson scattering	Углы кристалла.	Crystal angles
Тонкая структура.	Fine structure	Угол абберации.	Aberration angle
Тонкое тело.	Slender body	Угол диффракции.	Angle of diffraction
Топологическая группа.	Topological group	Угол естественного откоса.	Angle of repose
Топологический график.	Topological graph	Угол отклонения.	Angle of deviation
Топологическое пространство.	Topological space	Угол отражения.	Angle of reflection
Топология.	Topology	Угол падения.	Angle of incidence
Тор.	Torus, annulus	Угол преломления.	Angle of refraction
Торможение.	Drag, braking	Угол проекции.	Projection angle
Тормозная мощность.	Brake horsepower	Угол рассеяния.	Scattering angle
Тормозная способность.	Stopping power	Угол раствора.	Aperture angle
Тороидальные координаты.	Toroidal coordinates	Угол связи.	Bond angle
Точечная группа.	Point group	Уголь скольжения.	Glancing angle
Точечный источник.	Point source	Угол сходимости.	Angle of convergence
Точка ветвления.	Branch point	Угол трения.	Angle of friction
Точка кипения.	Boiling point	Угол фазы.	Phase angle
Точка пара.	Steam point	Угол эффекта Холла.	Hall angle
Точка перегиба.	Inflection point	Удар.	Impact, shock
Точность.	Accuracy, precision	Ударная волна.	Shock wave
Точный.	Exact, accurate	Ударная вязкость.	Resilience
Траектория.	Trajectory	Ударное расширение.	Shock expansion
Траектория напряжения.	Stress trajectory	Ударный прогиб.	Impact buckling
Трансляционная группа.	Translation group	Удельная активность.	Specific activity
Трансляция.	Translation	Удельная дисперсность.	Specific dispersivity
Транспонирование.	Transposition	Удельная затухающая способность.	Specific damping capacity
Транспонированная матрица.	Transpose of a matrix	Удельная прозрачность.	Transmittivity
Трансцендентный.	Transcendental	Удельная теплопроводность.	Thermal conductivity
Трение обшивки.	Skin friction	Удельное вращение.	Specific rotation
Треугольная матрица.	Triangular matrix	Удельное преломление.	Specific refraction
Треугольник цветов.	Color triangle	Удельный акустический импеданс.	Specific acoustical impedance
Треугольное крыло.	Delta wing	Удельный расход тепла.	Specific heat consumption
Тригонометрическая интерполяция.	Trigonometric interpolation	Удлинение.	Elongation or extension
Тригонометрическая функция.	Trigonometric function	Узловая точка.	Nodal point
Тригонометрия.	Trigonometry	Уклон.	Grade
Триклинная система.	Triclinic system	Уклонение.	Error, deviation
Трихроматическая система.	Trichromatic system	Уменьшение площади.	Reduction in area
Трихроматические единицы.	Trichromatic units	Умножение векторов.	Vector multiplication
Тройная система.	Ternary system	Умножение тензоров.	Multiplication of tensors
Тройная точка.	Triple point	Умова-Пуинтинга вектор.	Poynting vector
Труба.	Tube	Универсальное взаимодействие.	Universal interaction
Турбулентная конвекция.	Turbulence convection	Унитарный оператор.	Unitary operator
Турбулентное течение.	Turbulent flow	Уносимое тепло.	Heat carried away
Турбулентность.	Turbulence	Упаковочный множитель.	Packing fraction
Турбулентный пограничный слой.	Turbulent boundary layer	Упорядочение.	Order-disorder transformation
Тэйлора ряд.	Taylor series	Упругая волна.	Elastic waves
Тягучесть.	Ductility	Упругие коэффициенты.	Elastic coefficients
		Упругие постоянные.	Elastic constants
		Упругое основание.	Elastic foundation
		Упругое последствие.	Elastic after effect
		Упругое столкновение.	Impact, elastic
		Упругость пара.	Vapor pressure
Увеличение.	Magnification, gain, enlargement	Уравнение.	Equation
Увеличительная сила.	Magnifying power	Уравнение возраста.	Age equation
Увлечение.	Entrainment	Уравнение непрерывности.	Equation of continuity
Угловая корреляция.	Angular correlation	Уравнение переноса.	Transport equation
Угловая переменная.	Angle variable	Уравнение составляющих ветра.	Wind equation
Угловая скорость.	Angular velocity	Уравнение состояния.	Equation of state
Угловая характеристическая функция.	Angle characteristic function	Уравнение теплоёмкости.	Heat capacity equation

- Уравнение энергии для пограничного слоя. Energy equation for boundary layer
 Уравнение энергии для устойчивого адиабатического потока. Energy equation for steady adiabatic flow
 Уравнения движения. Equations of motion
 Уравнения движения жидкости. Equations of fluid motion
 Уравнения Диофанта. Diophantine equations
 Уравнения математической физики. Equations of mathematical physics
 Уравнения системы. System equations
 Уравнения скорости кристаллизации. Crystallization rate equations
 Уравнения угловой деформации. Slope-deflection equations
 Уравновешивающий. Equilibrant
 Уравновешивание. Balance
 Уровень громкости. Loudness level
 Уровень давления звука. Sound pressure level
 Уровень давления звуковой полосы. Sound band pressure level
 Уровень звука. Sound level
 Уровень значимости. Significance level
 Уровень интенсивности звука. Sound intensity level
 Уровень источника. Source level
 Уровень сигнала. Signal level
 Уровень шума. Noise level
 Усекание. Truncation
 Усиление при преобразовании. Conversion gain
 Усиление. Gain
 Усилитель. Amplifier
 Ускорение. Acceleration
 Ускорение силы тяготения. Acceleration of gravity
 Условие выхода. Yield condition
 Условная сходимость. Conditional convergence
 Условие химического равновесия. Chemical stability condition
 Успокоение. Damping
 Усталость. Fatigue
 Устойчивость. Stability
 Устойчивость ламинарного потока. Stability of laminar flow
 Устойчивость самолета. Airplane stability
 Устойчивость фаз. Stability of phases
 Устранение. Elimination
- Фаза. Phase
 Фаза волны. Phase of a wave
 Фаза рассеяния. Scattering phase shift
 Фазовая скорость. Phase velocity
 Фазовое пространство. Phase space
 Фазы кристалла. Crystal phases
 Фактор группа. Factor-group
 Фактор накопления. Build-up factor
 Факториал. Factorial
 Факториальный коэффициент. Factorial coefficient
 Факториальный эксперимент. Factorial experiment
 Фарадея эффект. Faraday effect
 Фасонные поверхности. Figured surfaces
 Фейнмана метод. Feynman's method
 Ферма. Truss
 Ферми. Fermi
 Фермион. Fermion
 Ферромагнитный резонанс. Ferromagnetic resonance
 Физический маятник. Physical pendulum
 Фика закон диффузии. Fick's law of diffusion
- Фильтр. Filter
 Фогта модель. Voigt model
 Фокальная плоскость. Focal plane
 Фокальные линии. Focal lines
 Фокальные точки. Focal points
 Фокусное расстояние. Focal length
 Фон. Background
 Фонон. Phonon
 Форма. Form
 Формат изображения. Aspect ratio
 Формула барометра. Barometer formula
 Формула гауссовой кривизны. Gaussian curvature formula
 Формула масс. Mass formula
 Формула четырёх множителей. Four-factor formula
 Формфактор. Shape factor, form factor
 Фотозлектрическая постоянная. Photoelectric constant
 Фотозлектронная работа выхода. Photoelectric work function
 Фотозлектрическая чувствительность. Photoelectric sensitivity
 Фредгольма определитель. Fredholm determinant
 Франка-Кондона принцип. Franck-Condon principle
 Френе формулы. Frenet formulae
 Фрикционная скорость. Friction velocity
 Фронт волны. Wave front
 Фугативность. Fugacity
 Фуко маятник. Foucault's pendulum
 Функции минимального сдвига фаз. Minimum phase functions
 Функции рассеяния света. Light scattering functions
 Функционал. Functional.
 Функциональный, Functional
 Функция. Function
 Функция возбуждения. Excitation function
 Функция диссипации. Dissipation function
 Функция заключения. Decision function
 Функция ошибок. Error function
 Функция потерь. Loss function
 Функция распределения. Distribution function
 Функция светлоты. Luminosity function
 Функция теплоемкости. Specific heat function
 Функция тока. Stream function
 Фурье преобразование. Fourier transform
 Фурье-Бесселя преобразование. Fourier-Bessel transform
 Футосвеча. Foot-candle
- Хаотичность. Randomness
 Характер связи. Bond character
 Характеристика. Characteristic
 Характеристика направленности. Directional characteristic
 Характеристика поверхности. Characteristic of surface
 Характеристическая функция. Characteristic function
 Характеристическая функция группы. Characteristic function of a set
 Характеристический акустический импеданс. Characteristic acoustical impedance
 Характеристическое уравнение. Characteristic equation
 Характеристическое уравнение матрицы. Characteristic equation of a matrix

Хартриг приближение. Hartree approximation
 Химическая константа. Chemical constant
 Химические потенциалы. Chemical potentials
 Химическое равновесие. Chemical equilibrium
 Хол. Rap. root (of stem), course
 Холлы подвижность. Hall mobility
 Хорда. Chord
 Христовфеля трехиндексные символы. Christoffel 3-index symbols
 Хромотичности координаты. Chromaticity coordinates
 Хронологический оператор. Chronological operator
 Хэвисайда операционное исчисление. Heaviside operational calculus

Цвет. Color

Цветовая диаграмма. Chromaticity diagram
 Цветовая температура. Color temperature
 Цветная функция. Color function
 Цветная шкала. Colorgrade scale
 Центр. Center
 Центр группы. Center of a group
 Центр давления. Center of pressure
 Центр инерции. Center of mass
 Центр колебания. Center of oscillation
 Центр комбинации. Center of combination
 Центр кривизны. Center of curvature
 Центр моментов. Center of moments
 Центр сдвига. Shear center
 Центр симметрии. Center of symmetry
 Центр сотрясения. Center of percussion
 Центр тяжести. Center of gravity, centroid
 Центр упругости. Elastic center
 Центральная предельная теорема. Central limit theorem
 Центральные силы. Central forces
 Центробежная сила. Centrifugal force
 Центроидальная ось. Centroidal axis
 Центробежностное ускорение. Centripetal acceleration
 Центрипетальная сила. Centripetal force
 Цепи переменного тока. Alternating current circuits
 Цепная кривая. Saddle curve
 Цепная линия. Saddle
 Цепное правило дифференцирования. Chain rule of differentiation

Цепь. Circuit, chain

Цикл. Cycle

Цикл напряжения. Cycle of stress

Цикл полурезонанса. Resonance cycle

Циклоиды. Cycloid

Циклоидная зависимость. Cycloidal velocity

Циклоидное движение. Cycloidal motion

Циклоидальная сила. Cycloidal force

Цилиндрические волны. Cylindrical wave

Цилиндрические координаты. Cylindrical coordinates

Цилиндрический групп. Cylindrical group

Циркуляционный интеграл. Circulatory integral

Циркуляция. Circulation

Цифровые вычислительные машины. Digital computer

Частичная дробь. Partial fraction

Частичные узлы. Partial nodes

Частный. Partial

Частота беккинг. Beat frequency

Частота повторения импульсов. Pulse repetition frequency

Частота релаксации. Relaxation frequency

Частота столкновений. Collision frequency

Частотная характеристика. Frequency response

Частичная дисперсия. Partial dispersion

Частичная когеренция. Partial coherence

Частотный коэффициент. Frequency factor

Челышева квадратная формула. Chebyshev quadratic formula

Черное тело. Black body

Четная перестановка. Even permutation

Четная функция. Even function

Четность. Parity

Четный ток атома. Even term of an atom

Четырехмерная сила. Four-force

Четырехмерный вектор. Four-vector

Четырехкомпонентные уравнения. Four-moment equations

Численная квадратура. Numerical quadrature

Численная устойчивость. Numerical stability

Численные дифференцирование. Numerical differentiation

Численный анализ. Numerical analysis

Число. Number

Число заполнения. Occupation number

Чувствительность. Sensitivity

Чувствительность по разрешению. Resolution sensitivity

Чистота возбуждения. Excitation purity

Чистый изгиб. Pure bending

Шар закона. Poise

Шарль закон. Charles' law

Шарнир. Hinge

Шварц-Христовфеля преобразование. Schwarz-Christoffel transformation

Шварц неравенство. Schwarz inequality

Шварцшильда-Кобальшпетера формула. Schwarzschild-Kobalshpetter formulas

Шварцшильда обозначения. Schwarzschild symbols

Шероховатость. Roughness

Ширина запрещенной зоны. Energy gap

Ширина импульса. Pulse duration, pulse width

Ширин полосы. Bandwidth

Ширин уровня. Level width

Шкалы нелинейности. Earthquake scales

Шкалы-Гильберта метод. Schmidt-Hilbert method

Шкоуэ эффект. Schottky effect

Шкур матрицы. Trace of a matrix

Шредингера уравнение. Schrödinger equation

Штурм эффект. Stark effect

Штурма метод. Störmer method

Штурм-Лувилля уравнение. Sturm-Liouville equation

Шум. Noise

Шумовой спектр. Noise spectrum

Шель. Sit, gap

Эверетта интерполяционные формулы. Everett interpolation formula

Эволюция. Evolution

Эволюция. Evolution

Эволюция движения. Evolution of motion

Эйконал. Eikonal

- Эйлера критическая нагрузка колонны. Euler buckling loads for columns
- Эйлера-Лагранжа уравнения. Euler-Lagrange equations
- Эйлера-Маклорэна формула. Euler-Maclaurin formula
- Эйлера-Родрига параметер. Euler-Rodrigues parameter
- Эйнштейна вероятности перехода. Einstein transition probabilities
- Эйри функции. Airy functions
- Эйринга теория. Eyring theory
- Эйткена метод интерполяции. Aitken method of interpolation
- Эквивалентная длина маятника. Pendulum, equivalent length of
- Эквивалентное изображение групп. Equivalent representations of groups
- Эквивалентность. Equivalence
- Эквивалентные системы сил. Equivalent force systems
- Эквивалентные электроны. Equivalent electrons
- Эквипотенциальная область. Equipotential region
- Эквипотенциальная поверхность. Equipotential surface
- Экзалтация. Exaltation
- Экзотермическая реакция. Exothermic reaction
- Экранирование. Screening
- Экспериментальная кривая рассеяния. Experimental scattering curve
- Экспозиция. Exposure
- Экспоненциальное распределение. Exponential distribution
- Экспоненциальный рупор. Exponential horn
- Экспоненциальный соединитель. Exponential connector
- Экстраполированная граничная точка. Extrapolated end-point
- Экстраполяция. Extrapolation
- Электрическая емкость. Electrical capacitance
- Электрическая жесткость. Elastance
- Электрическая система. Electrical system
- Электрическая теорема взаимности. Electrical reciprocity theorem
- Электрические мультиполи. Electric multipoles
- Электрический потенциал. Electric potential
- Электрический ток. Electric current
- Электрическое поле. Electric field
- Электрическое смещение в диэлектриках. Electrical displacement in dielectrics
- Электрическое сопротивление. Electrical resistance
- Электричество. Electricity
- Электродвижущая сила. Electromotive force
- Электромагнетизм. Electromagnetism
- Электромагнитная масса. Electromagnetic mass
- Электромагнитная постоянная. Electromagnetic constant
- Электромагнитное излучение. Electromagnetic radiation
- Электромагнитное напряжение. Electromagnetic stress
- Электромагнитное поле. Electromagnetic field
- Электромагнитные единицы. Electromagnetic units
- Электромагнитный возбудитель. Electromagnetic driving system
- Электромеханическая теорема взаимности. Electrical-mechanical reciprocity theorem
- Электрон-вольт. Electron-volt
- Электронейтральность. Electroneutrality
- Электронная конфигурация. Electron configuration
- Электронная оболочка. Electron shell
- Электронная орбита. Electron orbit
- Электронная теория металлов. Electron theory of metals
- Электронная теплоемкость. Electronic specific heat
- Электронные спектры молекул. Electronic spectra of molecules
- Электронный радиус (классический). Electron radius (classical)
- Электрооптическая линза. Electron-optical lens
- Электростатический возбудитель. Electrostatic driving system
- Электростатическое поле. Electrostatic field
- Электрохимический потенциал. Electrochemical potential
- Элемент. Element, cell
- Элемент симметрии. Symmetry element
- Элементарная система. Elementary system
- Элемент схемы. Circuit element
- Элементарный. Elementary
- Эллипс. Ellipse
- Эллипсоид. Ellipsoid
- Эллипсоид инерции. Inertia ellipsoid
- Эллипсоид моментов. Momental ellipsoid
- Эллипсоидная координата. Ellipsoidal coordinate
- Эллиптическая функция. Elliptic function
- Эллиптический интеграл. Elliptic integral
- Эллиптическое уравнение. Elliptic equation
- Эллиптическое уравнение в частных производных. Elliptic partial differential equation
- Эндотермичная реакция. Endothermic reaction
- Энергетическая мощность пограничного слоя. Energy thickness of boundary layer
- Энергетические соотношения в однородном поле. Energy relations in uniform force field
- Энергетический к.п.д. источника. Radiant efficiency
- Энергии связи. Bond energies
- Энергия. Energy
- Энергия активации. Activation energy
- Энергия альфа-распада. Alpha disintegration energy
- Энергия анизотропии. Anisotropy energy
- Энергия атомной связи. Atomic bond energy
- Энергия бета-распада. Beta disintegration energy
- Энергия возбуждения. Excitation energy
- Энергия диссоциации. Dissociation energy
- Энергия ионизации. Ionization energy
- Энергия искажения. Distortion energy
- Энергия решетки кристалла. Lattice energy of crystals
- Энергия связи. Binding energy
- Энергия системы n частиц. Energy of n -particle system
- Энергия стенки. Wall energy
- Энергия сцепления. Energy, cohesion
- Энергия турбулентности. Energy of turbulence
- Энскога-Максвелла уравнения. Enskog-Maxwell equations
- Энскаго ряд. Enskog's series
- Энергия электрона (классическая). Electron energy (classical)
- Энтальпия. Enthalpy
- Энтронпия. Entropy
- Энтронпия одноатомного газа. Entropy of monatomic gas
- Эргодическая гипотеза. Ergodic hypothesis
- Эргодичность. Ergodicity

- Эренфеста соотношения. Ehrenfest's relations
 Эрмита уравнение. Hermite equation
 Эрмитовская сопряжённая матрицы. Hermitian conjugate of a matrix
 Эрмитовский оператор. Hermitian operator
 Эффект быстрых нейтронов. Fast effect, interaction
 Эффект оже. Auger effect
 Эффект прерывистости. Intermittency effect
 Эффект размеров. Size effect
 Эффективная высота (антенны). Effective height (antenna)
 Эффективная глубина. Effective depth
 Эффективная площадь (антенны). Effective area (antenna)
 Эффективная скорость. Effective velocity
 Эффективная ширина. Effective width
 Эффективная ширина полосы. Effective band width
 Эффективное размножение. Effective multiplication
 Эффективность. Effectiveness, efficiency
 Эффективный атомный номер. Effective atomic number,
 Эшелон. Echelon
- Якоба теорема в оптике. Jacob's theorem in optics
 Явления сгущения в газах. Clustering phenomena in gases
 Ядерная реакция. Nuclear reaction
 Ядерные силы. Nuclear forces
 Ядерный магнетон. Nuclear magneton
 Ядерный магнитный резонанс. Nuclear magnetic resonance
 Ядерный потенциал. Nuclear potential
 Ядерный уровень. Nuclear energy level
 Ядро. Kernel, nucleus
 Ядро рассеяния. Scattering kernel
 Ядро переноса. Transport kernel
 Якоби метод. Jacobi method
 Якобиан. Jacobian
 Яркостная температура. Luminance temperature
 Яркость. Luminance
 Ячейка. Cell

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