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A Critical Analysis of Helmholtz's Argument against Weber's Electrodynamics

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We present Helmholtz's argument against Weber's electrodynamics. It is related with a fixed charged nonconducting spherical shell and a charged particle moving inside it. Then we utilize Weber's electrodynamics plus Schrödinger's expression for gravitational interactions in order to obtain the equation of motion and to study this situation. We show that this approach avoids the problems pointed out by Helmholtz. Moreover, it indicates that the effective inertial mass of the charged particle will depend not only on the electrostatic potential of the shell but also on its velocity. This is a relevant aspect of Weber's theory.

1. INTRODUCTION

There has been a renewed interest in Weber's electrodynamics recently; see Refs. 1–5, Ref. 6, Chap. 6, Refs. 7–9 etc. Some of these works discuss Helmholtz's argument against Weber's theory.(10) In this paper we present a solution of the paradox pointed out by Helmholtz utilizing Schrödinger's potential energy for gravitational interactions(11) (English translation in Ref. 12). This has never been done before.

Weber's potential energy between two point charges q_1 and q_2 is given by

$$U_W = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r} \left(1 - \frac{\dot{r}^2}{2c^2}\right) \tag{1}$$

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Here $\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}$ is the vacuum permittivity, r is the distance between the charges, $\dot{r} = dr/dt$, and $c = 3 \times 10^8$ ms^{-1}.

Helmholtz's argument was briefly summarized by Maxwell as follows (Ref. 13, Vol. 2, Chapter 23, p. 485):

> A fixed nonconducting spherical surface, of radius a, is uniformly charged with electricity to the surface-density σ. A particle, of mass m and carrying a charge e of electricity, moves within the sphere with velocity v. The electrodynamic potential calculated from the formula
>
> $$\psi = \frac{ee'}{r}\left[1 - \frac{1}{2c^2}\left(\frac{\partial r}{\partial t}\right)^2\right]$$
>
> is
>
> $$4\pi a \sigma e \left(1 - \frac{v^2}{6c^2}\right)$$
>
> and is independent of the position of the particle within the sphere. Adding to this V, the remainder of the potential energy arising from the action of other forces and $mv^2/2$, the kinetic energy of the particle, we find as the equation of energy
>
> $$\frac{1}{2}\left(m - \frac{4}{3}\frac{\pi a \sigma e}{c^2}\right) v^2 + 4\pi a \sigma e + V = \text{const}$$
>
> Since the second term of the coefficient of v^2 may be increased indefinitely by increasing a, the radius of the sphere, while the surface density σ remains constant, the coefficient of v^2 my be made negative. Acceleration of the motion of the particle would then correspond to diminution of its *vis viva*, and a body moving in a closed path and acted on by a forcelike friction, always opposite in direction to its motion, would continually increase in velocity, and that without limit. This impossible result is a necessary consequence of assuming any formula for the potential which introduces negative terms into the coefficient of v^2.

We discussed in detail this criticism by Helmholtz in (Ref. 2, Sect. 7.3: charged spherical shell). Before considering this a failure of Weber's electrodynamics (as was the point of view of Helmholtz and Maxwell), this prediction should be tested experimentally. Although this negative mass behavior is unusual, nature may behave like this. We don't know any experiments devised to test this prediction. Even supposing that nature will not behave like this, there are ways out of this paradox as pointed out by Phipps.[7–9] He supposed Weber's potential energy as an approximation valid up to second order in $\dot{r}/c$. For high velocities he proposed:

$$U_P = \frac{q_1 q_2}{4\pi\varepsilon_0}\frac{1}{r}\sqrt{1 - \frac{\dot{r}^2}{c^2}} \qquad (2)$$

Expanding the square root up to second order in $\dot{r}/c$ yields Weber's potential energy. With this new electromagnetic potential energy Phipps succeeded in overcoming Helmholtz's criticism.

In this work we present an alternative approach. Instead of changing Weber's potential energy, we consider a modification of Newton's law of gravitation and of his second law of motion, as first suggested by Schrödinger.

2. SCHRÖDINGER'S MECHANICS

Helmholtz and Maxwell assumed that the classical kinetic energy was valid to slow and high velocities. Nowadays we know that a better expression in agreement with experiments involving electrons moving near light velocity is given by

$$E_k = \frac{mc^2}{\sqrt{1 - v^2/c^2}} + \text{const} \tag{3}$$

Schrödinger has an extremely important work of 1925 where he could derive this expression from a modified Newton's law of gravitation.[11, 12] Essentially, he proposed an interaction gravitational potential energy U_S between two gravitational point masses m_1 and m_2 given by

$$U_S = -G\frac{m_1 m_2}{r}\left[3 - \frac{2}{(1 - \dot{r}^2/c^2)}\right] \tag{4}$$

An analogous expression has been obtained independently by Wesley.[5] When Schrödinger integrated this expression for a test mass m interacting gravitationally with a homogeneous and isotropic universe, he was able to derive Eq. (3) with a gravitational mass m and with the velocity v being the velocity of the test particle relative to the distant universe. He also derived the following Lagrangian describing the gravitational interaction of this test mass with the distant universe: $L = -mc^2\sqrt{1 - v^2/c^2} + \text{const}$.

It must be remarked that this choice of the relativistic-like kinetic energy makes both (electrodynamic energy and kinetic energy) to be basically of the same level or degree from the point of view of a relativistic consideration. This is a reasonable reason to adopt Schrödinger's approach.

Weber's Lagrangian for the test charge q interacting with the charged spherical shell is given by $L = -(q\sigma a/\varepsilon_0)(1 + v^2/6c^2)$ (Ref. 2, Secs. 3.5: Langrangian and Hamiltonian Formulations of Weber's Electrodynamics and 7.3: Charged Spherical Shell). The Lagrangian for the test particle interacting with the charged spherical shell and the distant universe is then given by

$$L = -mc^2\sqrt{1-v^2/c^2} - \frac{q\sigma a}{\varepsilon_0}\left(1+\frac{v^2}{6c^2}\right) + \text{const} \tag{5}$$

Helmholtz mentioned in his analysis that the test body is acted on by a forcelike friction, always opposite in direction to its motion. For this reason we include in the analysis a frictional force of the form $-b\vec{v}$, where $b>0$ is the coefficient of friction. The equation of motion then becomes

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_j}\right) - \frac{\partial L}{\partial q_j} + \frac{\partial F}{\partial \dot{q}_j} = 0 \tag{6}$$

Here L is given by Eq. (5) and F is Rayleigh's dissipation function given by (Ref. 14, p. 24):

$$F = \frac{bv^2}{2} \tag{7}$$

We will analyze here the situation in which the charged particle moves in a circle of constant radius ρ_0 due to a radial constraint. Its velocity is then given by $\vec{v} = \rho_0\dot{\theta}\hat{\theta}$. With this condition and Eqs. (5) and (7) we have

$$\left[\frac{1}{(1-\dot{x}^2)^{3/2}} - \alpha\right]\ddot{x} + \frac{b\dot{x}}{m} = 0 \tag{8}$$

where $\dot{x} = \rho_0\dot{\theta}/c$ is the normalized velocity, $\ddot{x} = \rho_0\ddot{\theta}/c$, and $\alpha = q\sigma a/3\varepsilon_0 mc^2$ is a dimensionless parameter. The solution of this equation is given by (with $\dot{x}_0$ being the normalized initial velocity)

$$\frac{bt}{m} = \ln\left(\frac{1+\sqrt{1-\dot{x}^2}}{1+\sqrt{1-\dot{x}_0^2}}\right) - (1-\alpha)\ln\frac{\dot{x}}{\dot{x}_0} - \frac{1}{\sqrt{1-\dot{x}^2}} + \frac{1}{\sqrt{1-\dot{x}_0^2}} \tag{9}$$

There are two situations to analyze: $\alpha < 1$ and $\alpha \geqslant 1$. For $\alpha < 1$ we see that $1-\alpha(1-\dot{x}^2)^{3/2} > 0$. In this case the acceleration has the opposite sign of the velocity. Figure 1 shows a qualitative analysis of Eq. (8) for $\alpha < 1$.

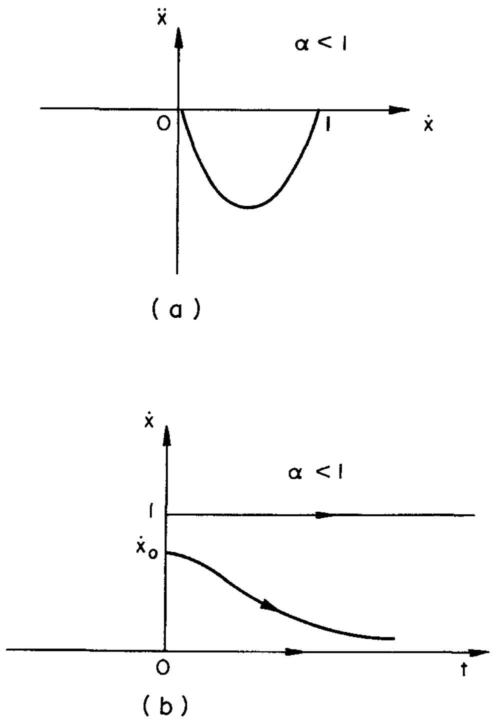

Fig. 1. (a) Behavior of Eq. (8) for $\alpha < 1$. The acceleration is zero for $\dot{x} = 1$ and for $\dot{x} = 0$. (b) The normalized velocity of the charged particle moving inside a charged spherical shell for $\alpha < 1$. We can see that the velocity goes to zero when $t \to \infty$. Here $\dot{x}_0$ is the normalized initial velocity.

From Eq. (8) we can see that $1 - \alpha(1 - \dot{x}^2)^{3/2} = 0$ for $\alpha \geqslant 1$ when

$$\dot{x}_c = \sqrt{1 - \frac{1}{\alpha^{2/3}}} \tag{10}$$

Here $\dot{x}_c$ is the normalized critical velocity. In these cases the acceleration diverges when $\dot{x} \to \dot{x}_c$. Figure 2 presents a qualitative graphical analysis of Eq. (8) for $\alpha \geqslant 1$. When $\dot{x} \to \dot{x}_c$ the acceleration goes to infinity, $\ddot{x} \pm \infty$. When the normalized initial velocity, $\dot{x}_0$, is greater than the critical velocity, the velocity decreases, tending to the critical velocity; see Fig. 2. When $\dot{x}_0 < \dot{x}_c$, the velocity increases, tending also to the critical velocity. After reaching the critical velocity, the charged particle remains at this velocity.

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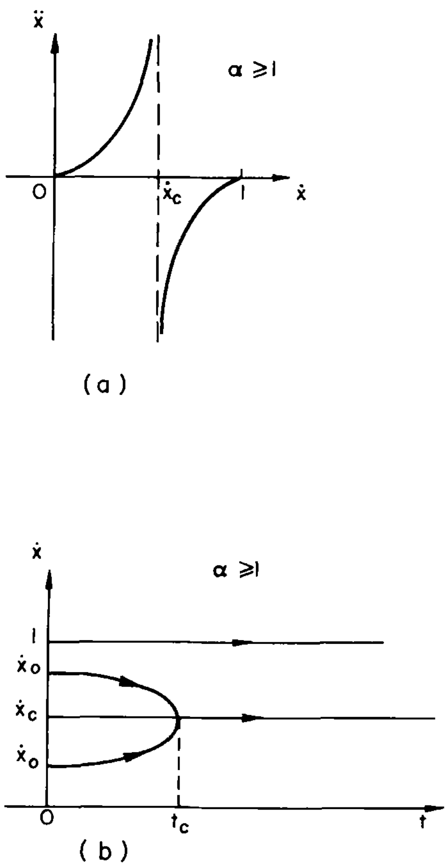

Fig. 2. (a) Behavior of Eq. (8) for $\alpha \geqslant 1$. The acceleration goes to infinity when the velocity tends to the critical velocity, $\dot{x}_c$, given by Eq. (10). For $\alpha = 1$ the critical velocity is zero, $\dot{x}_c = 0$. (b) Situation for $\alpha \geqslant 1$.

3 DISCUSSION AND CONCLUSION

Helmholtz and Maxwell concluded from their analysis that the velocity of the charged particle would increase indefinitely by increasing the radius of the charged sphere. Their conclusion was based not only on Weber's electrodynamics but also on Newton's mechanics. Here we have shown that this does not happen anymore even maintaining Weber's electrodynamics, if we utilize the appropriate kinetic energy as obtained by Schrödinger and

Wesley. In this case for any initial velocity smaller than c, the final velocity goes to zero if $\alpha < 1$, as in the classical or relativistic theories.

Even for $\alpha \geqslant 1$ the velocity does not increase indefinitely anymore. With $0 < v_0 < c$ the final velocity will be given by $v_c = c\sqrt{1 - 1/\alpha^{2/3}}$. It is easily seen that ultimate velocity will always be in the range $0 \leqslant v_c \leqslant c$, no matter the value of $\alpha \geqslant 1$.

It must be remarked that Eq. (8) can be written as

$$m_{ei}\ddot{x} + b\dot{x} = 0 \tag{11}$$

This equation is similar to the equation of motion of classical mechanics but now with an effective inertial mass given by

$$m_{ei} = m\left[\frac{1}{(1-\dot{x}^2)^{3/2}} - \alpha\right] \tag{12}$$

The relativistic equation of motion would be the same as Eqs. (11) and (12), but with $\alpha = 0$.

Figure 3 presents a qualitative graphical analysis of Eq. (12) for $\alpha \geqslant 1$. When $0 < \dot{x} < \dot{x}_c$ the effective inertial mass is negative. This means that the velocity increases in the presence of friction up to $\dot{x}_c$. For $\dot{x}_c < \dot{x}_0 < 1$ the effective inertial mass is positive, so that the velocity decreases toward $\dot{x}_c$.

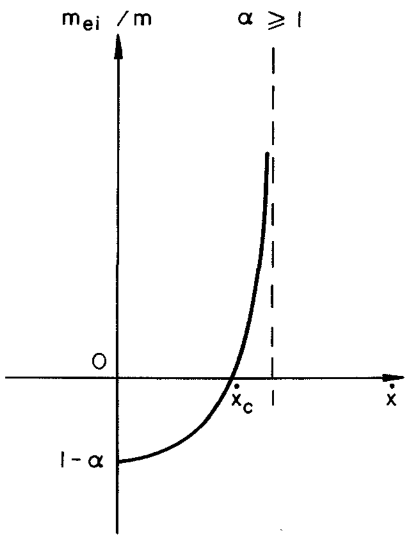

Fig. 3. The normalized effective inertial mass as a function of the normalized velocity for a given $\alpha \geqslant 1$.

With this concept of an effective inertial mass depending on the electrostatic energy and on the velocity of the particle, it is easy to understand the behavior of the test charge.

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REFERENCES

1. A. K. T. Assis, "On Mach's principle," *Found. Phys. Lett.* **2**, 301–318 (1989).
2. A. K. T. Assis, *Weber's Electrodynamics* (Kluwer Academic Dordrecht, 1994).
3. J. P. Wesley, "Weber electrodynamics, Part I. General theory, steady current effects," *Found. Phys. Lett.* **3**, 443–469 (1990).
4. J. P. Wesley, "Weber electrodynamics, Part II. Unipolar induction, Z-antenna," *Found. Phys. Lett.* **3**, 471–490 (1990).
5. J. P. Wesley, "Weber electrodynamics, Part III. Mechanics, gravitation," *Found. Phys. Lett.* **3**, 581–605 (1990).
6. J. P. Wesley *Selected Topics in Advanced Fundamental Physics* (Benjamin Wesley, Blumberg, Germany, 1991).
7. T. E. Phipps, Jr., "Toward modernization of Weber's force law," *Phys. Essays* **3**, 414–420 (1990).
8. T. E. Phipps, Jr., "Weber-type laws of action-at-a-distance in modern physics," *Apeiron* **8**, 8–14 (1990).
9. T. E. Phipps, Jr., "Derivation of a modernized Weber force law," *Phys. Essays* **5**, 425–428 (1992).
10. H. von Helmholtz, "On the theory of electrodynamics," *Philos. Mag.* **44**, 530–537 (1872).
11. E. Schrödinger, "Die Erfüllbarkeit der Relativitätsforderung in der klassischen Mechanik," *Ann. Phys. (Leipzig)* **77**, 325–336 (1925).
12. E. Schrödinger, "The possibility of fulfillment of the relativity requirement in classical mechanics," in J. B. Barbour and H. Pfister, eds., *Mach's Principle—From Newton's Bucket to Quantum Gravity* (Birkhäuser, Boston, 1995), pp. 147–158 [translated by J. B. Barbour].
13. J. C. Maxwell, *A Treatise on Electricity and Magnetism.* (Dover, New York, 1954).
14. H. Goldstein, *Classical Mechanics*, 2nd edn. (Addison-Wesley, Reading, 1980).