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Comparing Teaching Approaches About Maxwell's Displacement Current

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Abstract Due to its fundamental role for the consolidation of Maxwell's equations, the displacement current is one of the most important topics of any introductory course on electromagnetism. Moreover, this episode is widely used by historians and philosophers of science as a case study to investigate several issues (e.g. the theory–experiment relationship). Despite the consensus among physics educators concerning the relevance of the topic, there are many possible ways to interpret and justify the need for the displacement current term. With the goal of understanding the didactical transposition of this topic more deeply, we investigate three of its domains: (1) The historical development of Maxwell's reasoning; (2) Different approaches to justify the term insertion in physics textbooks; and (3) Four lectures devoted to introduce the topic in undergraduate level given by four different professors. By reflecting on the differences between these three domains, significant evidence for the knowledge transformation caused by the didactization of this episode is provided. The main purpose of this comparative analysis is to assist physics educators in developing an epistemological surveillance regarding the teaching and learning of the displacement current.

1 Introduction

Maxwell's insertion of the displacement current term in Ampère's law is among the greatest achievements of the human mind. It was a crucial step for the prediction of electromagnetic waves that led to the unification of electromagnetism and optics. It can also be seen as a major innovation in physics' methods, since the term is deduced within a

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theoretical reasoning—instead of being the mathematical representation of observable phenomena—and its empirical detection came 20 years later than its proposition.

Consequently, this episode constitutes an unlimited source for debates (many of them controversial) among philosophers and historians of science, especially concerning the relationship between observation and theory. According to Poincaré (1958), Maxwell's triumph was due to his profound sense of mathematical symmetry. Buchwald (1996) uses this episode to highlight the intimate and mutual interplay between mathematics and physics in the nineteenth century. The displacement current insertion is the first example mentioned by Steiner (1998) to emphasize the crucial role played by "Pythagorean analogies"—i.e. the analogies inexpressible in any other language but that of pure mathematics—in the "cardinal discoveries of contemporary physics". Duhem (1991) cites the displacement current episode to underline some of the differences between English and Continental scientists concerning the role given to algebra in logical systems; just to mention a few examples.

Due to its essential role for the consolidation of Maxwell's equations, the teaching of the displacement current is one of the most important topics of any course on electromagnetism. However, taking the educational perspective into account, there are many possibilities for justifying its insertion, which emphasize different aspects and encompass different views. Solving the charging capacitor problem, evoking symmetry arguments or highlighting inconsistencies with the continuity equation are some of the possibilities to justify the need for a change in Ampère's law. In this work, we draw attention to some implications of these different didactic approaches according to criteria like historical accuracy, philosophical position, mathematical pre-requisites and learning goals.

This paper is divided in three sections that address three different spheres of the didactization of this episode. It begins with a historical overview of Maxwell's path to the displacement current proposition. Then, several physics textbooks and articles are analyzed according to the way the term is presented and the reasons for its insertion. With the goal of investigating these differences more deeply and in authentic didactic situations, we analyze four physics lectures on the displacement current term, which were given by four different lecturers in undergraduate introductory level courses. By adopting a qualitative research approach, we intend to provide a substantial analysis of the didactical transposition of this episode with the purpose of informing physics educators in their task of teaching about the displacement current term.

2 Historical Overview

In this section we present a brief overview of Maxwell's reasoning path that led to the displacement current term. We follow the traditional analysis of his work in three major papers, namely On Faraday's Lines of Force (I), On Physical Lines of Force (II) and A Dynamical Theory of the Electromagnetic Field (III),[1] and his final synthesis A Treatise on Electricity and Magnetism (IV). This review will help us to compare the didactic approaches from a historical perspective and identify some problems related to the inadvertent mention to Maxwell's thought in the textbooks and lectures analyzed. Due to the fact that several studies have already investigated these works in detail (e.g. Whittaker 1910; Bork 1963; Bromberg 1968; Chalmers 1975; Buchwald 1988; Siegel 1991; Darrigol 2000), a succinct and focused presentation shall be enough for the purposes of this work.

[1] All the citations from these papers (*I*, *II* and *III*) refer to *The Scientific Papers of James Clerk Maxwell,* edited by W. D. Niven (1890).

2.1 On Faraday's Lines of Force (1855)

Maxwell's main goal in *I* is to give a mathematical formulation of Faraday's field conception (*I*, p. 157–158). In order to accomplish that, he makes an extensive use of analogies, which is a major trait of Maxwell's reasoning. In this particular work, the analogy between electric/magnetic phenomena and the motion of an incompressible fluid, previously investigated by Thomson, is used as a powerful heuristic guide to describe Faraday's notion of *lines of force* mathematically. Instead of seeing the model as a faithful causal description of reality, Maxwell stresses its precision and descriptive role:

> By referring everything to the purely geometrical idea of the motion of an imaginary fluid, I hope to attain generality and precision, and to avoid the dangers arising from a premature theory professing to explain the cause of the phenomena. (*I*, p. 159).

The displacement current term is not mentioned in *I*. When expressing the magnetic effect of an electric current, Maxwell presents the relation we know today as Ampère's law ($\boldsymbol{J} = \nabla \times \boldsymbol{H}$) as follows:

> [...] if we define the measure of an electric current to be the total intensity of magnetizing force in a closed curve embracing it, we shall have

$$\mathrm{a} = \frac{\mathrm{d}\beta}{\mathrm{d}z} - \frac{\mathrm{d}\gamma}{\mathrm{d}y} \qquad \mathrm{b} = \frac{\mathrm{d}\gamma}{\mathrm{d}x} - \frac{\mathrm{d}\alpha}{\mathrm{d}z} \qquad \mathrm{c} = \frac{\mathrm{d}\alpha}{\mathrm{d}y} - \frac{\mathrm{d}\beta}{\mathrm{d}x}$$

These equations[2] enable us to deduce the distribution of the currents of electricity whenever we know the values of α, β, γ, the magnetic intensities (*I*, p. 194).

Thus, in *I* the only possible way to relate magnetic forces with electric effects is through conduction currents. Nevertheless, a remark pointing an apparent limitation of these equations is mentioned right afterwards:

> We may observe that the above equations give by differentiation[3]

$$\frac{da}{dx} + \frac{db}{dy} + \frac{dc}{dz} = 0$$

which is the equation of continuity for closed currents. **Our investigations are therefore for the present limited to closed currents; and we know little of the magnetic effects of any currents which are not closed** (*I*, p. 194–195, our emphasis).

For incompressible fluids, this result implies that there exist neither sources nor sinks in any point in space. Using our modern concept of electricity that relates sources/sinks of electric field lines with charges, this equation is incompatible with the principle of charge conservation. Despite the difficulty in identifying Maxwell's reasons for making this comment, we notice that he was aware of the limitations of associating magnetic effects only to conduction currents. In his following papers, he works on changing these equations (Ampère's law and continuity equation) by adding a term that we now call displacement current. In *II* this term is presented by means of a mechanical interpretation.

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[2] Where *a*, *b* and *c* are the Cartesian components of the electric current (today we think in terms of the current density vector) and α, β, γ the Cartesian components of the magnetic force (magnetic field in actual terms). When compared with the actual representation of the components of the curl of a vector field, the different signs are due to a different orientation of the axes (x–west, z–south and y–upwards).

[3] This is equivalent to our current notion of the divergence of a vector field.

2.2 On Physical Lines of Force (1861)

The displacement current term appears for the first time in *II*. Maxwell's goal in this paper is to investigate the mechanical results of some states of tension and motion in an elastic medium in order to, *analogically*, compare these results with the electric and magnetic phenomena. Following Thomson's work on the vortical nature of magnetism, Maxwell expanded it to include electrostatics and optics, which enabled him to express the velocity of light in terms of electromagnetic quantities (Darrigol 2000). Once again, without regarding the mechanical model as a direct representation of reality, he stresses the heuristic role of this formal analogy:

> The author of this method of representation does not attempt to explain the origin of the observed forces by the effects due to these strains in the elastic solid, **but makes use of the mathematical analogies of the two problems to assist the imagination in the study of both** (*II*, p. 453, our emphasis).

The origin of the displacement current term comes from a correlation between electric *conduction* (conductors) and electric *displacement* (insulators) as it is explained in Maxwell's following quotation:

> Bodies which do not permit a current of electricity to flow through them are called insulators. But though electricity does not flow through them, **the electrical effects are propagated through them** […] Here then we have **two independent qualities of bodies**, one by which they allow of the passage of electricity through them, and the other by which they **allow of electrical action being transmitted through them without any electricity being allowed to pass**. […] The effect of this action on the whole dielectric mass is **to produce a general displacement of the electricity in a certain direction**. This displacement does not amount to a current, because when it has attained a certain value it remains constant, **but it is the commencement of a current, and its variations constitute currents in the positive or negative direction, according as the displacement is increasing or diminishing** (*II*, p. 490–491, our emphasis).

The formal association between electric conduction and displacement is justified since they both represent some kind of "electric movement". Dynamically speaking, this electric displacement must be related to an electric force, and this is mathematically expressed by the equation $R = -4\pi E^2 h$, where R is the electromotive force parallel to the z axis, E a coefficient depending on the nature of the dielectric and h the electric displacement.

This more general view of electric movements, that considers the *variation* of electric displacements $\left(\frac{dh}{dt} = -4\pi E^2 \frac{dR}{dt}\right)$ as currents, must then appear in the equations that represent the relation between currents and magnetic forces. This is done (within the context the elasticity theory) by adding a term (what we now call displacement current) in Ampère's law as follows:

> […] a variation of displacement is equivalent to a current, and this current must be taken into account in equations (9) [Ampère's law] and added to r. The three equations then become
>
> $$\mathrm{p} = \frac{1}{4\pi}\left(\frac{d\gamma}{dy} - \frac{d\beta}{dz} - \frac{1}{E^2}\frac{dP}{dt}\right) \quad \mathrm{q} = \frac{1}{4\pi}\left(\frac{d\alpha}{dy} - \frac{d\gamma}{dx} - \frac{1}{E^2}\frac{dQ}{dt}\right) \quad \mathrm{r} = \frac{1}{4\pi}\left(\frac{d\beta}{dx} - \frac{d\alpha}{dy} - \frac{1}{E^2}\frac{dR}{dt}\right)$$
>
> where p, q, r are the electric currents in the directions of x, y, and z; α, β, γ are the components of magnetic intensity; and P, Q, R are the electromotive forces [in the three directions respectively[4]] (*II*, p. 496–497).

[4] Note that now the axes are oriented according to the actual convention.

These equations could be expressed with the actual compact notation by $\boldsymbol{J} = \nabla \times \boldsymbol{H} - \frac{\partial \boldsymbol{E}}{\partial t}$. When comparing with *I*, it is evident that a term has been added and it is possible to associate a variation of the electric field[5] with magnetic effects. In *III*, we will see that due to another way of presenting these equations, this interpretation is not as clear.

The limitation of closed currents pointed out in *I* is then eliminated by adding a term for the time variation of the free electricity quantity *e* (*II*, p. 496):

> Now if *e* be the quantity of free electricity in unit of volume, then the equation of continuity will be
>
> $$\frac{dp}{dx} + \frac{dq}{dy} + \frac{dr}{dz} + \frac{de}{dt} = 0$$

In fact, this continuity equation is very similar to the one we use today to express the charge conservation ($\nabla . \boldsymbol{J} + \frac{\partial \rho}{\partial t} = 0$).

Maxwell's most known achievement in this work is to suggest that "light consists in the transverse undulations of the same medium which is the cause of electric and magnetic phenomena" (II, p. 500). However, his reasons and methods to derive this conclusion are sources for a rather controversial debate among historians of science. Nevertheless, the importance of the electric displacement for this mechanical derivation of light lies in its relation to the elasticity of the medium:

> […] when we find electromotive force producing electric displacement in a dielectric, and when we find the dielectric recovering from its state of electric displacement with an equal electromotive force, **we cannot help regarding the phenomena as those of an elastic body, yielding to a pressure, and recovering its form when the pressure is removed** (*II*, p. 491–492, our emphasis).

In this sense, Maxwell supposed that a vibration of the medium could represent light. More specifically, he considered transverse waves in the elastic medium, related its coefficients k (elasticity) and m (density) with the electromagnetic constants ε and μ, and showed that the velocity of these waves coincides with the ratio between the electromagnetic and electrostatic charge units. The value of this ratio, c, was already known from experiments performed by Weber and Kohlrausch and seemed to agree with Fizeau's value for the speed of light, supporting Maxwell's conclusion. However, as Duhem (1902) pointed out, Maxwell had overlooked a factor of 2 in his deduction, which gave reasons for historians to infer that he adjusted his calculations to make the velocity of the waves in the elastic medium come out close to the observed velocity of light.[6]

2.3 A Dynamical Theory of the Electromagnetic Field (1864)

In general, this paper is written in a more precise and abstract way, since the mechanical models found in *I* and *II* are replaced by the notion of electromagnetic field. In this sense, the concrete images previously represented by drawings do not appear and the concise/abstract language of equations prevails. Nevertheless, concerning his view on the way the electric and magnetic effects propagate in space, Maxwell insists that one of the main character of his investigations is to propose a *field theory*, as opposed to a theory that conceives *action at distance* (*III*, p. 527).

[5] The term field is not to be found in this work. In *II* Maxwell is talking about forces.

[6] Different views can be found, for example, in Chalmers (1975) and Siegel (1991). A deeper discussion about Maxwell's theory of light is not among the goals of this work.

Regarding the displacement current term, in this work Maxwell gives a broader definition of current, which encompasses the conduction current and the electric displacement, being therefore related to the total motion of electricity. When presenting the twenty general equations of the electromagnetic field containing twenty variables, the first is exactly the relation between variations of the electrical displacement, the true conduction and the total current (*III*, p. 554):

> The variations of the electrical displacement [each component represented by a time derivative] must be added to the currents p, q, r to get the total motion of electricity, which we may call p′,q′, r′, so that

$$p' = p + \frac{df}{dt} \quad q' = q + \frac{dg}{dt} \quad r' = r + \frac{dh}{dt}$$

Then, this new concept of total current is incorporated in the equations that describe the relation between electric movement (total current) and the circulation of magnetic field (*III*, p. 557):

$$\frac{d\gamma}{dy} - \frac{d\beta}{dz} = 4\pi p' \quad \frac{d\alpha}{dz} - \frac{d\gamma}{dx} = 4\pi q' \quad \frac{d\beta}{dx} - \frac{d\alpha}{dy} = 4\pi r'$$

Maxwell called these the *equations of Total Currents* and it is worth noticing that, in this form, the relation between a changing electric field and a circulating magnetic field (using today's terms) is no longer evident as it was in *II*.

In a chapter called *Electromagnetic theory of light*, Maxwell combined his field equations to obtain a wave equation, in a similar way we are used to in today's physics lectures. As opposed to *II*, in *III* he reached a truly electromagnetic optics in which light became a waving electromagnetic field (Darrigol 2000).

2.4 Treatise on Electromagnetism and Electricity (1873)[7]

In the *Treatise* Maxwell presents a comprehensive version of the theoretical investigations on electromagnetism in an extremely didactic style, with its structure resembling a classic physics textbook. When the general equations of the electromagnetic field are given (Chapter IX, volume 2) Maxwell refers to the historical development of the displacement current and presents his reasons for its proposal:

> This equation [Ampère's law] is true only if we take *u*, *v*, and *w* [current's Cartesian components] as the components of that electric flow which is due to the variation of electric displacement as well as to true conduction. **We have very little experimental evidence relating to the direct electromagnetic action of currents due to the variation of electric displacement in dielectrics, but the extreme difficulty of reconciling the laws of electromagnetism with the existence of electric currents which are not closed is one reason among many why we must admit the existence of transient currents due to the variation of displacement. Their importance will be seen when we come to the electromagnetic theory of light** (*IV*, v. 2, p. 231, our emphasis).

This passage confirms that the displacement current term is added mainly due to theoretical (instead of empirical) considerations and that its crucial role is due to the possibility of unifying optics and electromagnetism. This may explain why the displacement current has been such a fruitful case study for both historians and philosophers of science. Evidently, Maxwell has not only formalized the electromagnetic knowledge available in his time—in the sense of merely translating what was already known into a mathematical language—but he also gave original and crucial contributions to the theory. This involves

[7] The page numbers for *IV* refer to Maxwell, J. C. (1878).

the proposal of a broader notion of current encompassing the displacement current term. Maxwell's recognition of his own contribution concerning such amplification is made clear in the following quotation (*IV*, p. 232–233, our emphasis):

> We have now determined the relations of the principal quantities concerned in the phenomena discovered by Ørsted, Ampère and Faraday. To connect these with the phenomena described in the former parts of this treatise, **some additional relations are necessary**. [...] **One of the chief peculiarities of this treatise** is the doctrine which it asserts, that the **true electric current** $\mathfrak{G}$, that on which the electromagnetic phenomena depend, is not the same thing as $\mathfrak{K}$, the current of conduction, but that the time-variation of $\mathfrak{D}$, the electric displacement, must be taken into account in estimating the total movement of electricity, so that we must write,

$$\mathfrak{G} = \mathfrak{K} + \dot{\mathfrak{D}} \text{ (Equation of True Currents)}$$

In *IV* we find Maxwell expressing the electromagnetic equations not only in terms of its components, but also with the synthetic language of Hamilton's quaternions (in this work, vectors are denoted by German capital letters). What we learn today as the Ampère-Maxwell law $\left(\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}\right)$ was in the *Treatise* written as $4\pi\mathfrak{C} = V\nabla\mathfrak{H}$, where $\mathfrak{C}$ represent the true currents (conduction $\mathfrak{K}$ + displacement $\dot{\mathfrak{D}}$), $\mathfrak{H}$ is the magnetic force and $V\nabla$ indicates that only the vector part of the result of the operation ∇ is considered, which represents our actual curl operator.

2.5 Educational Implications

This brief historical overview highlights the complexity of Maxwell's reasoning and the difficulty to trace the path that led him to the proposal of the displacement current. Following Maxwell's thought becomes even more difficult considering that we are trained in the actual vector calculus notation (div, curl, grad) and that the abstract idea of a field (electric and magnetic) propagating through empty space is introduced in the very beginning of every course on electromagnetism.

Exactly due to the lack of clarity in defining and justifying the displacement current term, many critical remarks are addressed to this topic. Among them, Larmor (cited in Bromberg 1968) accuses Maxwell of ignoring the theory of dielectric polarization of Poisson and Kelvin. Chalmers (1975, p. 49) even claims that Maxwell was driven by the possibility of deducing a truly electromagnetic theory of light and "juggled with his theory until he found the form of the displacement current that would enable him to derive a wave equation". Duhem (1991, p. 79) also strongly criticizes Maxwell's approach by asking: "How can we explain this almost complete absence of definition even when it is a question of the most novel and most important elements, and this indifference to setting up the equations for a physical theory?"

Other authors defend Maxwell's approach. Among them, Bromberg (1968) remarks that the main difficulty for Maxwell's successors to understand his thought lies in the fact that he first introduced the displacement in the sense of Faraday's dielectric polarization, but then changed its original meaning in the course of his work, creating a totally new quantity. Similarly, Siegel (1991) suggests that it is necessary to consider Maxwell's work as a whole in its British context instead of giving attention mainly to the signs in the equations.

The non-triviality of this episode—emphasized both in the historical overview and in the controversial debates between historians and philosophers of science—highlights some of the challenges faced by those who intend to teach about the displacement current in a historically accurate way. In fact, it seems impossible to focus on the theoretical implications of this insertion and at the same time give a precise view of its historical development. From a didactical/pedagogical perspective, the choices made by lecturers are strongly dependent on their learning goals. In the next section, we investigate the arguments used by several textbooks and articles to justify the displacement current insertion. Both the historical overview and the philosophical debates presented in this section will provide us with a critical lens to analyze these different approaches.

3 Didactic Transpositions in textbooks

Although textbooks usually have an important role in all educational areas, they play a *distinguished* one in science. Since the early nineteenth century, collections of consensual knowledge were organized with the purpose of training future scientists. The dissemination of these collections was responsible for a gradual extinction of the educational use of original texts. Kuhn remarks that the emergence of science textbooks is actually a consequence of epistemological peculiarities of the scientific knowledge. According to him, the very existence of textbooks reinforces the paradigmatic character of the experimental sciences. In Kuhn's words:

> Perhaps the most striking feature of scientific education is that, to an extent quite unknown in other creative fields, it is conducted through textbooks, works written especially for students. […] Apparently scientists agree about what it is that every student of the field must know. That is why, in the design of a pre-professional curriculum, they can use textbooks instead of eclectic samples of research. (Kuhn 1963, p. 350–51)

This consensus among scientists (materialized in textbooks) does not allow, however, to regard textbooks as faithful representatives of their research field. As much as they try to remain faithful to the original context, these books are didactic productions inserted in a specific educational project. This means that particular features and requirements of this education system must be taken into account when compared with the original epistemological context of the scientific knowledge. Chevallard (1991) clarifies this point by arguing that "school knowledge" is the result of a process he calls "didactic transposition", which *transforms* the academic/scientific knowledge (*savoir savant*) in a didactized one (*savoir a enseignant*).

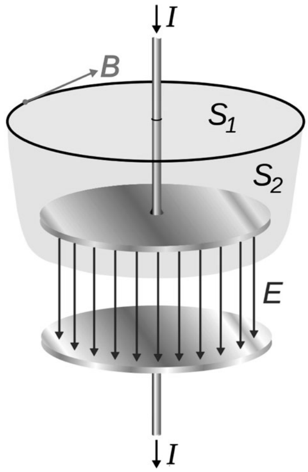
Far from being an absolute critic, the differentiation between school and scientific knowledge should be seen as a warning against a robust consensus.[8] Unsurprisingly, this transformation is often responsible for mistakes, over simplifications and distortions. This becomes especially problematic when textbooks make "attempts" to contextualize their approach historically. The inevitable tension between introducing the scientific paradigm and inserting historical remarks leads not just to a *superficial*, but mainly to an *incorrect* historical presentation. Whitaker calls this kind of approach *quasi-history*[9] and describes it as follows:

> [quasi-history is] a result of the large numbers of books by authors who have felt the need to enliven their account of this [as well as others] episode[s] with a little historical background, but have in fact

[8] According to Chevallard (2007, p. 11), the admission of this very transformation of knowledge can be a traumatic process for the ones involved in teaching. In his words: "Knowledge is not a given […] it is built up, and transformed, and—such was the keyword—*transposed*. The wound was twofold. For some people, especially for teachers, the statement was a threat to the unconscious belief that the world of knowledge was homogeneous, isotropic and indefinitely unblemished—therefore unquestionable. To others, […] who regarded themselves as the true masters of knowledge [scholars], to them, the transposition principle came as a repudiation of their as yet unchallenged authority.

[9] See also the term *pseudo-history* in Allchin (2004).

Fig. 1 Charging capacitor (public domain)



> **rewritten the history so that it fits in step by step with the physics**. Because the description of the physics is logical and orderly, the impression is necessarily given that this was also the way in which the ideas emerged historically (Whitaker 1979, p. 109, our emphasis).

The literature is vast with analysis of similar problems in textbooks and there is no need for a review here. For the following analysis, it is important to be aware not only of the process, but also of some of the consequences of a didactic transposition. As we will see, in the *didactization* (transformation) of the displacement current episode, several historical aspects (presented in Sect. 2) are not considered. Based on the analysis of textbooks and articles,[10] in this section we present a synthesis of the different lines of reasoning used to justify the need for the displacement current term and discuss some of their educational implications.

3.1 The Charging Capacitor Problem

The most common way to justify the need for the displacement current insertion is the charging (or discharging) capacitor problem (see Fig. 1). The problem is normally approached to show a contradiction in Ampère's law, since the application of the law gives different results for the circulation of magnetic field ($\oint \boldsymbol{B} \cdot d\boldsymbol{l}$), depending on the choice of the surface bounded by the same path ($\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 I$ for S_1 and $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = 0$ for S_2). The situation can also be seen as in disagreement with Kirchhoff's current law, since there would be current flowing before (and after) the capacitor plates, but not between them. In a more general way, the charging capacitor problem is used as an exemplar to highlight the incompleteness of Ampère's law in dynamic situations.

In introductory level textbooks, Maxwell's equations are commonly presented within the integral formalism, which also seems to constrain the possibilities of explaining the need for the displacement current term. Possibly related to this limitation, references to charge conservation, when found, are not fully exploited. Another possible approach is to

[10] The sample of the textbooks consists mainly of traditional and well established collections (e.g. Halliday et al. 2011; Tipler 1982), the ones used as references in some of the lectures analyzed (Giancoli 2000; Serway and Jerwey 2008) as well as other famous "unorthodox" approaches such as Feynman et al. (1964) and Purcell (1985). The articles mentioned are based on a literature review we conducted (mainly in the AJP) of papers where alternative ways to justify the displacement current insertion are presented.

insert the displacement current as an *ad hoc* hypothesis and treat the charging capacitor as an application. In this case, its validity is established by its success in giving agreement to previously observed electromagnetic phenomena and in predicting new phenomena subsequently checked by experimental observation.

Due to the possibility of justifying the insertion of the displacement current term in a clear and rational way (and also within the integral formalism), the charging capacitor problem has proved irresistible[11] for many physics textbooks and is strongly established in the didactic transpositions of the displacement current. Nevertheless, when we look for this example in Maxwell's work, we do not find such an important role in his argumentation, but a more speculative result of an empirical situation where the displacement current would be applied. In the *Treatise* we found the following remark:

> The current produces magnetic phenomena in its neighborhood. If any closed curve be drawn, and the line-integral of the magnetic force taken completely round it, then, if the closed curve is not linked with the circuit, the line-integral is zero, but if it is linked with the circuit, so that the current i flows through the closed curve, the line-integral is $4\pi i$ […] Note—The line-integral $4\pi i$ depends solely on the quantity of the current, and not on any other thing whatever. It does not depend on the nature of the conductor through which the current is passing, as, for instance, whether it be a metal or an electrolyte, or an imperfect conductor. **We have reason for believing that even when there is no proper conduction, but merely a variation of electric displacement, as in the glass of a Leyden jar during charge or discharge, the magnetic effect of the electric movement is precisely the same.** (*IV*, v. 2, p. 142–144, our emphasis).

The hypothetical character of Maxwell's statement about the charging capacitor problem—stressed in the sentence "we have reason to believe"—shows that this conclusion is plausible within the theoretical framework of the *Treatise*, but implies that an experimental confirmation of this magnetic effect should be pursuit.

Experiments that aimed at providing evidence for the existence of the displacement current by measuring the magnetic effects between the plates of a charging (or discharging) capacitor were indeed made.[12] The technical difficulties related to the measurement of such a negligible effect are normally surmounted by using materials with high dielectric constants and inserting a toroidal coil between the capacitor plates.

Although the results of these experiments have been successfully reproduced, some authors criticized their interpretation as an evidence for the existence of the displacement current. The main problem lies on the causal assertion that the displacement current is responsible for the magnetic field. French (2000) defends that this interpretation is incorrect, since the magnetic field inside the plates could be ascribed entirely to conduction currents if one applies Biot-Savart's law and takes the fringing field effects into account.[13] The situation is much more complex if a dielectric is inserted between the plates due to the oscillatory movement of charges under the application of an alternating electric field.

3.2 Charge Conservation

Using the differential formulation of Maxwell's equations, some textbooks (e.g. Feynman et al. 1964; Demtröder 2009) relate the insertion of the displacement current term with an

[11] This expression was used by Holton (1969) to justify the didactic use of the Micheslon-Morley experiment in the teaching of Special Relativity, despite its dispensable role for Einstein's theory.

[12] The first record of such experiment goes back to 1899 in the report "*On the Magnetic Action of Displacement-currents in a Dielectric*" written by Silvanus P. Thomson. Similar experiments are found in (Meissner 1964; Carver and Rajhel 1974; Rizotto 1999).

[13] The situation is discussed in detail in Purcell (1985, pp. 328–330) and in French and Tessman (1963).

attempt to make Ampère's law coherent with charge conservation (expressed in the continuity equation) and with Gauss's law. This line of argumentation is used in Feynman's lectures as follows:

> He [Maxwell] then noticed that there was something strange about Eq. (18.1) $\left[\nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\varepsilon_0 c^2}\right]$. If one takes the divergence of this equation, the left-hand side will be zero, because the divergence of a curl is always zero. So this equation requires that the divergence of $\boldsymbol{j}$ also be zero. But if the divergence of $\boldsymbol{j}$ is zero, then the total flux of current out of any closed surface is also zero.
> The flux of current from a closed surface is the decrease of the charge inside the surface. This certainly cannot in general be zero because we know that charges can be moved from one place to another. The equation $\nabla \cdot \boldsymbol{j} = -\frac{\partial \rho}{\partial t}$ has, in fact, been almost our definition of $\boldsymbol{j}$. This equation expresses the very fundamental law that electric charge is conserved—any flow of charge must come from some supply. Maxwell appreciated this difficulty and proposed that it could be avoided by adding the term $\frac{\partial \boldsymbol{E}}{\partial t}$ to the right-hand side of Eq. (18.1); he then got $c^2 \nabla \times \boldsymbol{B} = \frac{\boldsymbol{j}}{\varepsilon_0} + \frac{\partial \boldsymbol{E}}{\partial t}$ (Feynman et al. 1964, 18-1).[14]

In other words, the main argument is that since the term $\nabla \times \boldsymbol{B}$ is "divergence free" by virtue of its mathematical structure (the divergence of the *curl* is always zero), this means that Ampère's law is only applicable to closed circuits. It seems easier to make this inconsistency plausible within the vector differential formalism.[15] This highlights the difficulties faced by didactic approaches that present only the integral formulation if they intend to mention Maxwell's concern with the continuity equation. According to Siegel (1991) this problem is indeed in the center of Maxwell's reasons for inserting the displacement current:

> Maxwell did basically what the standard account says he did: He modified Ampère's law in order to generalize it to the open circuit, in a manner consistent with the equation of continuity and Coulomb's law [we have referred to $\nabla \cdot \boldsymbol{E} = \frac{\rho}{\varepsilon_0}$ as Gauss's law]. His goal in that, however, was not a complete and consistent set of electromagnetic equations for its own sake, but rather a complete and consistent mechanical model of the electromagnetic field. (Siegel 1991, p. 97).

If consistency with the continuity equation (charge conservation) is such a fundamental principle, then it seems plausible to consider the possibility of deriving the displacement current through an inverted reasoning, i.e. by imposing its validity and obtaining the $\partial \boldsymbol{E}/\partial t$ term as a consequence. This is the line of reasoning used by Mello (1972) to obtain the displacement current term from Biot-Savart's law. Although the formalism needed to follow this derivation is normally beyond introductory level courses, the main idea behind it seems to be pedagogically relevant also for the students' first contact with the electromagnetic theory.

Many elementary textbooks begin the study of magnetostatics with Biot-Savart's law, similarly to Coulomb's law in electrostatics. There is indeed an analogy between these laws (Coulomb and Biot-Savart), since the fields are inversely proportional to r^2 and directly proportional to their sources (Weber and Macomb 1989). Moreover, in electrostatics chapters it is very common to find Coulomb's law being derived from Gauss's law (or the contrary, Gauss's law from Coulomb's), which stresses the equivalence between

[14] It is important to mention that Feynman clearly states that Maxwell did not reason this way. The sequence of this passage shows Feynman's careful position concerning assertions about Maxwell's thought: "It was not yet customary in Maxwell's time to think in terms of abstract fields. Maxwell discussed his ideas in terms of a model in which the vacuum was like an elastic solid. He also tried to explain the meaning of his new theory in terms of the mechanical model".

[15] Gauthier (1983) proposes a method that exploits the general implications of electric charge conservation and is formulated in the integral representation of Maxwell's equations. However, we were not able to find textbooks using this method.

these two laws (at least for static situations). Preserving this analogical reasoning, it is plausible to expect some sort of equivalence between Ampère's and Biot-Savart's law. Considering Biot-Savart's law as[16]:

$$\boldsymbol{B} = \frac{\mu_0}{4\pi}\int_C \frac{Id\boldsymbol{l}' \times \hat{r}}{r^2} = \frac{\mu_0}{4\pi}\int_V \frac{\boldsymbol{J}(\vec{r}') \times \hat{r}}{r^2} dV'.$$

The first term is applied to a steady line current whereas the second one to a volume current. In order to obtain Ampère's law from it, the *curl* operator is applied on both sides of the equation (volume integral) and, after some manipulation,[17] the following result is obtained:

$$\nabla \times \boldsymbol{B} = \frac{\mu_0}{4\pi}\nabla \times \int_V \frac{\boldsymbol{J}(\vec{r}') \times \hat{r}}{r^2} dV' = \mu_0 \boldsymbol{J}(\vec{r})$$

which is the expression of Ampère's law, the analog to Gauss' law in electrostatics. In the process of this derivation, the physical assumption of steady-state magnetic phenomena ($\nabla \cdot \boldsymbol{J} = 0$) has to be made. However, what Mello (1972) shows in his paper is that if this assumption is *not* made and instead the charge conservation is imposed ($\nabla \cdot \boldsymbol{J} = -\partial\rho/\partial t$), then the displacement current term arises from the derivation.

The other way around, i.e. the derivation of Biot-Savart's from Ampère's law, is made possible by a mathematical way of representing the magnetic field using the vector potential formalism. Considering the non-existence of magnetic poles ($\nabla \cdot \boldsymbol{B} = 0$) one can write

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}$$

where **A** is the vector potential. Then, substituting in Ampère's law (differential form)

$$\nabla \times \nabla \times \boldsymbol{A} = \nabla(\nabla \cdot \boldsymbol{A}) - \nabla^2 \boldsymbol{A} = \mu_0 \boldsymbol{J}.$$

To solve this differential equation one needs to choose a $\nabla \cdot \boldsymbol{A}$ dependence, the simplest choice being $\nabla \cdot \boldsymbol{A} = 0$.[18] Then, the solution becomes

$$\boldsymbol{A} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{J}}{r} dV'$$

whose rotational is exactly Biot-Savart's expression. Most textbooks that introduce the vector potential contain this demonstration.

In another attempt to underline the relation between Biot-Savart's and Ampère's laws, Weber and Macomb (1989) investigated different solutions for the following problem:

> Suppose we have two point charges of equal magnitude but opposite charge placed on the z axis at $\pm L/2$. Find the value of magnetic field $\boldsymbol{B}$ at any point in the midplane defined by $z = 0$ due to a steady current between the two point charges (Tipler 1982, p. 750–751).

[16] $\vec{r}$ represents the distance between the wire element and the point at which the field is calculated (given that $\vec{r}' = 0$) and $d\boldsymbol{l}'$ is a vector whose magnitude is the length of the differential element of the wire and whose direction is equal to the conventional current.

[17] A detailed presentation can be found in Greiner (1998).

[18] The differential equation describing a propagation of the vector potential was derived by Maxwell, who defined $\boldsymbol{A}$ as electrokinetic momentum and his choice was $\nabla \cdot \boldsymbol{A} = 0$, called afterwards Coulomb's gauge (Buchwald 1988, p. 103).

First, the authors applied Ampère-Maxwell's law (i.e. with the displacement current term) together with Coulomb's law to carry out the calculation of $\boldsymbol{B}$. Next, they used Biot-Savart's law for the same purpose and noted that nothing but conduction current was needed to obtain the same solution. In their conclusion, the authors highlight the importance of emphasizing the equivalence between the two laws at the introductory level, to assure students that both laws embody the same physics.

Before we present another common way of justifying the displacement current, an important remark is needed. The overall argument used in the previous subsections imply the idea that Maxwell modified Ampère's law to accommodate the charging capacitor problem (3.1) and/or to make it coherent with the continuity equation (3.2). From a historical perspective, this is however untrue, since Ampère's law (in the way proposed by Ampère) was in fact a *force law* between two circuits transporting electric currents (Whittaker 1910). In other words, Ampère's theory was developed within a complete different theoretical framework—based on the idea of action at a distance—whereas Maxwell follows Faraday's tradition and proposes a *field theory*. According to Buchwald (1988), this is in fact Maxwell's greatest contribution to modern physics:

> Modern theory seeks unified explanations in an unmodifiable set of field equations coupled through electron motion to intricate microphysical models. Maxwellian theory sought unity through a highly plastic set of field equations coupled to Hamilton's principle […]. In modern theory, charge is the source of the electric field, and current is a source of the magnetic field. In Maxwellian theory, charge is produced by electric field; current, in the usual sense of rate of change of charge over time, is only indirectly related to the magnetic field (Buchwald 1988, p. 23).

The fundamental step toward the displacement current was taken in the context of Maxwell's attempt to link the equations related to electric currents with the electrostatics ones, through the continuity equation. Maxwell's interpretation of electromagnetic phenomena established a chain of parallel linkages between the mechanical and electromagnetic levels that provided a connection, in the theory, between magnetic and electrical fields, which were understood as fundamental. Charges and currents belonged to another sort of tradition, the Continental one, which regarded these concepts as primary. According to Siegel (1991), Lorentz constructed a dualistic theory giving charges, currents and fields the same importance.

3.3 Symmetry Arguments

In the search for plausible reasons for Maxwell's proposal of the displacement current term, some authors claim the need for symmetry between $\boldsymbol{E}$ and $\boldsymbol{B}$ in the curl equations. The argument is presented in two textbooks as follows:

> In Chapter 30 you saw that a changing magnetic flux induces an electric field, and we ended up with Faraday's law of induction in the form
>
> $$\oint \boldsymbol{E} \cdot d\boldsymbol{s} = -\frac{d\phi_{\boldsymbol{B}}}{dt} \text{ (Faraday's law of induction)}$$
>
> Here, $\boldsymbol{E}$ is the electric field induced along a closed loop by the changing magnetic flux $\phi_{\boldsymbol{B}}$ encircled by that loop. **Because symmetry is often so powerful in physics**, we should be tempted to ask whether induction can occur in the opposite sense; that is, **can a changing electric flux induce a magnetic field?** (Halliday et al. 2011, p. 863, our emphasis).

> Is it possible that magnetic fields could be produced in another way as well? For if a changing magnetic field produces an electric field, as discussed in Section 29–7, then **perhaps the reverse**

Table 1 Symmetries in physics quantities of electromagnetism (Chaves 2001)

	Parity	Time reversal
ρ, $\nabla \cdot \mathbf{E}$	Scalar	Symmetric
$\nabla \cdot \mathbf{B}$	Pseudoscalar	Symmetric
$\nabla \times \mathbf{E}$, $\partial \mathbf{B}/\partial t$	Pseudovector	Symmetric
$\nabla \times \mathbf{B}$, $\partial \mathbf{E}/\partial t$, $\mathbf{J}$	Vector	Anti-symmetric
$\nabla \cdot \mathbf{J}$, $\partial \rho/\partial t$	Scalar	Anti-symmetric
$\nabla \rho$, $\mathbf{E}$, $\partial \mathbf{J}/\partial t$	Vector	Symmetric
$\nabla \times \mathbf{J}$, $\mathbf{B}$	Pseudovector	Anti-symmetric

> **might be true as well**: that a changing electric field will produce a magnetic field. If this were true, **it would signify a beautiful symmetry in nature**. (Giancoli 2000, p. 788, our emphasis).

Despite the eventual pedagogical usefulness of this argument, it is very unlikely that symmetry considerations played an important role in Maxwell's insertion of the displacement current. According to Bork (1963), it was Oliver Heaviside the first physicist to explicitly refer to the symmetry of Maxwell's equations. Moreover, if symmetry were to play such an important role, the same claim would have to be demanded from Gauss's electric and magnetic laws. However, the textbook authors who emphasize the need for symmetry in the curl equations, do not seem to interpret or even to mention the lack of magnetic charge as an asymmetry in Maxwell's equations.

Nevertheless, it is indeed possible to justify the displacement current term using more convincible arguments that are related to deeper symmetries connecting the whole theoretical framework described by Maxwell's equations. One possibility[19] is to consider all the quantities involved in the equations (charge density ρ, current density $\boldsymbol{J}$, the electric and magnetic fields $\boldsymbol{E}$ and $\boldsymbol{B}$) as well as their derivatives both in space and in time ($\nabla\rho$, $\partial\rho/\partial t$, $\nabla \cdot \boldsymbol{J}$, $\partial \boldsymbol{J}/\partial t$, $\nabla \cdot \boldsymbol{E}$, $\nabla \times \boldsymbol{E}$, $\nabla \cdot \boldsymbol{B}$, $\nabla \times \boldsymbol{B}$). Assuming the Newtonian paradigm that *the basic laws of Nature are differential equations in space and time,* one can arrange the quantities according to its tensorial character (scalar, pseudoscalar, vector or pseudovector) to preserve parity. Charge is considered to be a scalar due to space isotropy. $\boldsymbol{E}$ is a vector, since its direction is parallel to the electrical force, whereas $\boldsymbol{B}$ is a pseudovector, since the magnetic force is perpendicular to it. We still need to consider time invariance, the symmetry under time reversal.[20] Table 1 summarizes these considerations and displays the result of this clustering.

Arranging the terms according to these criteria, the easiest choice is linearly independent combination resulting in first order differential equations. Besides being the simplest choice, it also satisfies the superposition principle, which is actually one of the core assumptions of the electromagnetic theory. We obtain Maxwell's equations in the first four lines of Table 1 (complementing them with suitable coefficients[21]). The fifth line is linearly dependent from the previous ones and corresponds to the continuity equation. The sixth possible equation is not considered because it is nonlinear (the coefficients to arrange the equation are speed dependent). The linear combination in the last line of Table 1

[19] We found this line of reasoning in Chaves (2001). A detailed and very didactic presentation can also be found in Diener et al. (2013).

[20] Time reversal symmetry is considered when the variable t is substituted by $-t$ and the quantity remains invariant or not (symmetric or anti-symmetric, respectively).

[21] See Diener et al. (2013) for the process of obtaining these coefficients.

cannot be always valid, since it is restrict to superconductors. Electromagnetism (Maxwell's equations) is the unique manner to obtain a force originated by two fields (a vector and a pseudovector), which depends on a scalar conservative charge and its currents, and satisfies the superposition principle, parity and time (and space) symmetries (Chaves 2001). Following this procedure, the displacement current term arises straightforwardly in the fourth line.

Another possibility of deriving the displacement current term from profound symmetry arguments[22] is proposed by Yano (1968) and is based on the approach used in Purcell's famous textbook on electromagnetism (Purcell 1985).[23] The starting point for the derivation is a set of five postulates, fundamentally derived from experiment: (1) Gauss's law; (2) Superposition principle; (3) Charge invariance; (4) Special relativity; (5) No magnetic poles.

According to Yano (1968) these postulates are sufficient to derive the displacement current without any reference to charge conservation. Consider two inertial frames (S and S'), where S' moves relatively to S with velocity $\mathbf{v}$. In S, there is only an electric field independent of time, but due the relative movement between the frames, a magnetic field will be detected in S', which is expressed by $\boldsymbol{B}' = (1/c)\mathbf{v} \times \boldsymbol{E}'$ (Yano 1968, p. 599).

Considering Special relativity, Gauss's law and Charge invariance, the transformation properties between the frames are $E'_{\perp} = \gamma E_{\perp}$ and $E'_{\parallel} = E_{\parallel}$, where $\gamma = \sqrt{1 - \frac{\mathbf{v}^2}{c^2}}$ and the symbols $\parallel$ $\perp$ represent the parallel and perpendicular directions of the field components compared to the velocity direction, respectively. Differentiating these equations in respect to time, the relation $\frac{\partial \boldsymbol{E}'}{\partial t} + (\mathbf{v} \cdot \nabla_{\mathbf{r}'})E' = 0$ is obtained. Taking the curl of $\boldsymbol{B}'$ in the equation $\boldsymbol{B}' = (1/c)\mathbf{v} \times \boldsymbol{E}'$, and combining it with the result of the time differentiation, Yano (1968) was able to obtain the Ampère-Maxwell law[24] and the displacement current arose naturally in this derivation.

In this section we presented different ways to justify the insertion of the displacement current term found in textbooks and articles (*knowledge to be taught*). Within the three main approaches (charging capacitor, charge conservation and symmetry arguments) we not only contrasted them with the historical development of Maxwell's reasoning (Sect. 2), but also highlighted several nuances concerning issues related to the interpretation of the term, experimental evidence of its existence, implications of the used notation, among others. Much evidence for the knowledge transformation through didactization were found and commented. The analysis of these didactical approaches, as well as their controversies and implications, adds another level of complexity to the apparent routine task of an informed professor preparing his/her lecture to present the displacement current term in introductory level. If trying to remain faithful to the historical development of Maxwell's work seemed to be an impossible (even undesirable) task, now the professor is confronted with a broad spectrum of teaching approaches, which encompass different formalisms and worldviews. In the next section, we analyze four lectures given by four different physics

[22] Here symmetry is seen as invariance of an object or system to a set of changes/transformations (Lederman and Hill 2004).

[23] In his book, Purcell focuses on relativistic discussions from the very beginning. When analyzing the way the fields (electric and magnetic) transform from one frame of reference to another, a deep symmetry between these fields is highlighted. See Purcell (1985, pp. 235-240) for a detailed approach.

[24] $\nabla_{\mathbf{r}'} \times \boldsymbol{B}' = \frac{1}{c}\left[\mathbf{v}'\left(\nabla_{\mathbf{r}'} \cdot \boldsymbol{E}\right) - \left(\mathbf{v}' \cdot \nabla_{\mathbf{r}'}\right)\boldsymbol{E}'\right] = \frac{1}{c}\left[\mathbf{v}'(4\pi\rho) - \frac{\partial}{\partial t}\boldsymbol{E}'\right] = \frac{1}{c}\left[4\pi\boldsymbol{J} - \frac{\partial \boldsymbol{E}'}{\partial t}\right]$. Different constants are due to unit systems choice.

professors in introductory electromagnetism courses and evaluate how these different aspects come into play in authentic classroom situations.

4 Teaching Approaches in Lectures

Another significant transformation takes place when the didactized knowledge (*to be taught*) reaches the classroom (*knowledge taught*), a process called *internal transposition* by Chevallard (1991). In this second transformation, other factors like educational goals, teachers' conceptions/beliefs/expertise, students' previous knowledge and conceptions (among many others) are subject to exert influence.

In order to investigate the didactic transposition of the displacement current episode in this dimension, four different lectures are examined. Considering that the displacement current is a consolidated topic of every electromagnetism course, there are numerous lectures about it available in the Internet, which are provided by physics professors from reputable universities. Due to our purpose of correlating these three dimensions (historical, didactical materials and actual teaching approaches), we do not aim at deriving any kind of representative/general conclusion about "the way" this topic is taught in university level. Our main interest is to understand these cases (4 lectures) more deeply in the light of what was discussed in the two previous sections. The important thing is, of course, to maintain the same context (first course on electromagnetism in undergraduate level) and to define clearly the beginning and end points of the analysis.

The excerpts of the four lectures examined in this section were selected according to the following criterion: the analysis *begins* when the displacement current term is introduced for the first time and *ends* when the focus is changed (another problem/topic/experiment is approached or when the lecture simply ends). In other words, our focus is to understand how the term is *introduced* and its insertion *justified*. The previous section shall guide our analysis, since it enables us to perceive how the broad variety of possibilities to introduce the displacement current (*knowledge to be taught*) influences the professors' didactic choices (*knowledge taught*). It may also be fruitful to compare the professors' teaching approaches with the ones found in the textbooks indicated as references to the students.

4.1 Lectures' Description and Analysis

4.1.1 Lecture A

Duration: 24 min
Students: 1st year (2nd semester)—Engineering and Science majors
Textbook: Giancoli (2000) + Lecture notes
Lecturer[25]: Prof. Walter Lewin-MIT

The charging capacitor problem is presented in the very beginning of the lecture to introduce the topic. The changing electric field between the circular plates (radius R) of the capacitor (see Fig. 2) is mentioned and a relation between dE/dt is derived as follows:

[25] When asking for permission for using his lectures with research purposes, Prof. Walter Lewin explicitly demanded that his name should be mentioned. The names of the other lecturers will not be mentioned to preserve confidentiality according to ethic standards in qualitative educational research.

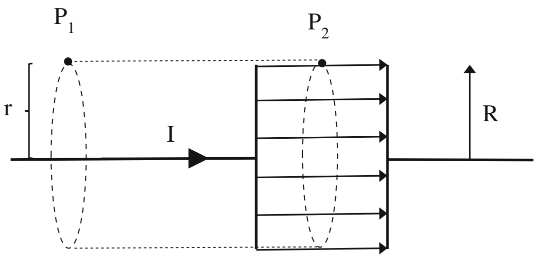


Fig. 2 Charging capacitor problem—Lecture A

$$E = \frac{\tau_{free}}{\kappa\varepsilon_0} = \frac{Q_{free}}{\pi R^2 \kappa\varepsilon_0}$$

$$\text{since } I = \frac{dQ_{free}}{dt}, \text{ then } \frac{dE}{dt} = \frac{I}{\pi R^2 \kappa\varepsilon_0}$$

The problem to be solved is then formulated: "How can we calculate **B** here (P_1) and here (P_2) now that we have this opening in the wire?" (A, 3:00). Biot-Savart's law is considered as a possible solution, but quickly disregarded due its technical difficulty. Then, Ampère's law (integral form) is applied to calculate the magnetic field ($\boldsymbol{B}$) in P_1 and P_2, which results $\boldsymbol{B}_{P_1} = \frac{\mu_0 I}{2\pi r}$ and $\boldsymbol{B}_{P_2} = 0$. This last result (no magnetic field at P_2) is presented as problematic with sentences like "This is absurd. It cannot be" (A, 5:20). Afterwards, another surface is used to determine $\boldsymbol{B}_{P_1}$and the traditional paradoxical situation (different values for $\boldsymbol{B}_{P_1}$ depending on the chosen open surface to apply Ampere's law) is emphasized.
The insertion of the displacement current term by Maxwell is justified by the *symmetry argument* as the following quotation (A, 7:30–7:45) shows:

> What is so special about in between the capacitor plates is that there is a changing electric field. And Maxwell reasoned: Gee, Faraday's law tells me that a changing magnetic flux gives rise to an electric field. So, he says, maybe a changing electric flux gives rise to a magnetic field.

The displacement current term is then inserted and the amended law is presented as $\oint \boldsymbol{B} \cdot \boldsymbol{dl} = \mu_0 \left(I + \kappa\varepsilon_0 \frac{d}{dt} \int \boldsymbol{E} \cdot \boldsymbol{dA} \right)$, which is then applied to the charging capacitor problem again to show that the previous inconsistency (different values for $\boldsymbol{B}_{P_1}$ depending on the chosen open surface) are solved. The assumption of neglecting fringe fields is clearly stated.

The charging capacitor problem is finished with the calculation of the magnetic field between the plates, resulting in the expression $\boldsymbol{B}_{P_2} = \frac{\mu_0 I r}{2\pi R^2}$. A diagram relating $\boldsymbol{B}_{P_2}$and $\boldsymbol{r}$ is plotted and once again the fringe effects neglect is discussed in the transition from $r < R$ to $r > R$.

The presentation of the displacement current ends with the remark that with this insertion Maxwell was able to predict the existence of electromagnetic waves, which were later detected experimentally by Hertz. The name "displacement" is also justified by Maxwell's original idea of dielectric polarization.

In Lecture A, we notice a combination of symmetry arguments and the charging capacitor problem (motivation) to introduce the displacement current. Biot-Savart's law is briefly mentioned, but not deeply explored. When compared with the reference textbook (Giancoli 2000), it is possible to identify several similarities. This does not mean, of course, a passive textbook reproduction, but the line of argument and the problems solved are indeed very similar. In a general way, we can say that the professor made "modulations" of the textbook knowledge in the displacement current introduction. In fact, we perceive a more careful and detailed presentation in what concerns the limits/simplifications of the charging capacitor problem (fringe fields). It is also worth noticing that expressions referring to Maxwell's reasoning, although historically inadequate, are commonly found.

4.1.2 Lecture B

Duration: 35 min
Students: 2nd year—Engineering and Science majors
Textbook: Serway and Jewett (2008) + Lecture notes

What we traditionally call Ampère-Maxwell law is presented in the beginning of the lecture without any further justification (also without reference to Maxwell). After writing the expression $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 i + \mu_0 \varepsilon_0 \frac{d\Phi_E}{dt}$ on the blackboard, the lecturer stresses that the first part (conduction current) was already studied and that now they were going to focus on the new element (displacement current). The meaning of the formula is then explained (B, 1:20)

> So what this [expression] is telling you is: besides current producing a magnetic field, if the electric field is changing as a function of time, then it creates a magnetic field as well.

By multiplying the values of μ_0 and ε_0 the professor highlights the numerical difference between the two terms and emphasizes how small the displacement current term is compared with the conduction current term. This fact is used to justify the assumption that in most cases when there is current in a wire, the effect (***B***) caused by the conduction current "dominates" the one caused by the displacement current.

Next, the focus is driven to the units of $\varepsilon_0 \frac{d\Phi_E}{dt}$. Since it adds to a current in the expression, this term should also have current units. This is the reason why we call it also a current, "whatever this thing means" (B, 3:15). In other words, when the electric field changes "it acts as a current" (B, 3:40).

The topic of the next chapter (Faraday's law) is then briefly introduced with the goal of showing the formal similarity between Faraday's and Ampère-Maxwell laws. The *symmetry argument* (flux variation implies field generation) is given and analogical relations (both similarities and differences) between the two laws are highlighted as follows:

> You can see how close this [Faraday] is to that [Ampère-Maxwell]. It is almost like they are brother and sister. This one [Faraday] is saying that if you change the magnetic flux as a function of time, it creates an electric field that circulates around it […] So in order to create an electric field, instead of just putting two charges together, I can simply get a magnet and move it in [gestures] and out of a coil, then I will have an electric field circulating in that coil […] Now this one [Ampère-Maxwell] is the counterpart of this [Faraday]. But the reason this term [displacement current] is not as noticeable and as applicable is because: Firstly the ε_0 makes it too small […] and secondly because usually if you want to create a magnetic field you either use a magnet or you run a current in a wire, we don't really need to do this [points at the displacement current] in order to create a magnetic field. (B, 5:00–7:05)

The resolution and discussion of two numerical problems are conducted in the following 27 min. The first problem presents a simple AC circuit (AC source $V = 120\sqrt{2}.sin2\pi ft$ and a resistor R) and the main goal is to determine if there is a magnetic field in the exact instant when there is no current flowing in the circuit.

The solution and discussion of this problem takes 17 min. The professor argues that when *i(t)* is not zero, then the magnetic field at a distance *r* from the wire is easily calculated using only the first part of the expression as follows: $\oint \boldsymbol{B} \cdot d\boldsymbol{l} = \mu_0 i$, since $i = \frac{V}{R}$ and due to the symmetry of the problem the magnetic field at a distance *r* from the wire is equal to $\boldsymbol{B} = \frac{4\pi.10^{-7}}{2\pi r}.\frac{120\sqrt{2}}{R}.sin2\pi ft$. However, when $i(t) = 0$, then it is necessary to use the displacement current term in order to calculate the magnetic field. After some numerical calculations (values for the resistance of the wire and its dimensions are given) a function *B(r)* is obtained and its extremely small magnitude is highlighted.

The second problem, which is solved in 10 min, is the charging capacitor and once again numerical values for the main variables involved are given (C = 2 μF, R = 10Ω, V = $12.e^{-t/RC}$). The goal of the problem is to calculate the magnetic field between the capacitor plates during the charging process. Since there is no conduction current between the plates, it is the displacement current (i_d) the one responsible for the magnetic field. After calculating the expression for the displacement current ($i_d = \frac{12}{R}e^{-t/RC}$) the lecturer highlights that this is the exact same expression of the conduction current, which reinforces his argument that a changing electric field is equivalent to a current. It is worth noticing that the charging capacitor problem was not used to show a limitation of the application of Ampère's law (which was the case in Lecture A).

Lecture B is focused on dimensional and numeric analysis. The didactic approach, although not completely opposite to the one found in the textbook used as reference, does differ from it in several aspects. The charging capacitor problem is solved as an application of a given relation (Ampère-Maxwell law)—instead of being used to show a limitation of Ampère's law (textbook presentation)—and much attention is given to technical/technological aspects. Additionally, the symmetry arguments found in the lecture are not present in the textbook, neither is the strong emphasis on the numerical difference between the two terms (conduction and displacement currents). It is also worth mentioning that no reference to Maxwell (or his reasoning) is made during the lecture.

4.1.3 Lecture C

Duration: 80 min
Students: 2nd year—Physics majors
Textbook: Lecture notes

The lecture begins with a summary of what has been studied so far in the course. On the left side of the blackboard, a table is presented, which contains the 4 Maxwell's equations represented in different ways (words, drawings, equations in the integral and differential forms), the expressions of the Lorentz force and the continuity equation. The professor announces that the goal of the lecture is to show an inconsistency in the whole theoretical system expressed by the table and to solve it. During this 80-min excerpt, much time is spent with philosophical discussions about several issues concerning the development of a physical theory, as it is exemplified in the following quotations:

> A physical theory has to be coherent. […] This is related to a big conviction we have that the world is coherent. Therefore, in many difficult situations we look for coherence, we even impose it, and when we do that our theory moves forward (C, 1:00–3:30)

The mentioned incoherence is that Ampère's law does not satisfy the continuity equation. The problem is presented as follows:

$$\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} \quad \text{Ampère's law}$$

$$\nabla \cdot (\nabla \times \boldsymbol{B}) = \mu_0 (\nabla \cdot \boldsymbol{J}) = 0, \text{ therefore } \quad \nabla \cdot \mathbf{J} = 0$$

$$\text{which is inconsistent with the continuity equation } \nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t}$$

In order to solve this problem, the term $\left(\mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}\right)$ is added to the expression. Then, the professor shows that this new expression (Ampère-Maxwell law $\nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \varepsilon_0 \frac{\partial \boldsymbol{E}}{\partial t}$) is coherent with the continuity equation.

This apparent arbitrary insertion gives rise again to a provocative philosophical discussion, which was conducted with an intense participation of the students:

> Is it ugly what I just did here? Simply insert the term and show that now the problem is solved? […] Well, you can find this ugly or beautiful, but this is what we do all the time. This is what gives Nobel Prizes to people. They take something that is not working and make it work! In a sense, Maxwell made it work. (C, 14:40–17:00)

This remark motivates a 10-minute discussion with the students about the relation between theory and experiment, the legitimacy of the mathematization of physics, and other issues. Although we identify explicit references to Maxwell during the lecture, it is important to notice that when talking about Maxwell's achievements and methods, the professor seems to take a careful position, which is illustrated by the following statement:

> Maxwell did not insert the term this way. His reasoning was different. It is hard for us to deeply understand Maxwell's thought taking into account what we know and learn today. Maxwell believed, for example, that the space was filled with matter. […] What we are learning in this course is a post Einstein version of the electromagnetic theory, so the concepts are somehow different. (C, 9:00–11:20)

In the last 45 min of the lecture, the charging capacitor problem is approached. The inconsistency originated from the application of Ampère's law (different values for the magnetic field depending on the chosen open surface) is highlighted and the new expression (with the displacement current term) is used to show how such inconsistency is solved. Next, Gauss's law and the current definition are used to show the equivalence between the changing electric field and the current, in other words, to show that the displacement current term indeed corresponds formally to a current. Then, the magnetic field between the plates is determined and a function *B(r)* is plotted. The consequences of choosing different open surfaces are extensively discussed and the fringe effect neglect is briefly mentioned.

In Lecture C the displacement current term is included as an *ad hoc* hypothesis, within the framework of an epistemological discussion about the construction of physics theories. The use of the differential formalism allows the professor to present and discuss the charge conservation problem (inconsistency between Ampère's law and the continuity equation) and use it to motivate the need for the displacement current. Moreover, the formalism also enables a solution of the capacitor problem that highlights the principle of charge conservation. The professor adopted a careful position concerning historical information and explicitly referred to differences between Maxwell's reasoning/formalism and the modern interpretations. No textbook is formally adopted in this course, but lecture notes made by the professor, which leads to a different kind of transposition that diverges from the more traditional ones, especially in what concerns philosophical discussions. This may suggest that the textbooks available do not satisfy the professor completely. It is also worth mentioning that this lecture is by far the most dialogical one (in terms of interactions between the lecturer and the students).

4.1.4 Lecture D

Duration: 13 min
Students: 2nd year—Engineering and science majors
Textbook: Lecture notes and PowerPoint presentations

Lecture D is dedicated to an overall presentation of Maxwell's equations. The last 13 min are devoted to the Ampère-Maxwell law, which is presented as follows:

> The circulation of the magnetic field vector around any amperian loop is proportional to the sum of the total conduction current and the displacement current through any surface bounded by the path. (D, 0:00-1:40)

The law is presented without further justification $\left(\oint \boldsymbol{B}.\boldsymbol{dl} = \mu_0\left(I + \varepsilon_0 \frac{d\Phi_E}{dt}\right)\right)$ and the professor emphasizes that up until that moment in the course only the conduction current has been discussed. Then, the difference between conduction and displacement current is explained:

> Conduction current is an electric current which is associated with the motion of charged particles. If charge is transferred from one place to another place, this is conduction current. For example current in a wire is an example of conduction current. Now, the second term here $\left[\varepsilon_0 \frac{d\Phi_E}{dt}\right]$ is called displacement current. It is also an electric current, but this current is not associated with the transfer of charge. It is associated only with the change of electric field. (D, 2:20-3:10)

The Ampère-Maxwell law is explained in a very general manner. A surface, considered to be somewhere in space, is crossed by a current (charged particles) and electric field lines. If the electric field is changing in time, this means that a displacement current is also flowing through the surface. The conventions (right-hand rule) are mentioned in order to get the direction of the magnetic field created by both currents.

The charging capacitor problem is approached in the last 8 min of the lecture (fringe fields neglect is briefly mentioned). However, instead of using it to show any paradoxical situation when applying Ampère's law, the main focus is to derive the formal equivalence between displacement current and conduction current. Analyzing the situation where the plates of a capacitor are being charged by a current *I*, the lecturer calls attention for the fact that there is no conduction current flowing between the plates (briefly mentions that there would be conduction current if the electric field were strong enough to overcome the dielectric strength of the medium between the plates). Then, the equivalence between conduction current and displacement current is presented as follows:

$$I_d = \varepsilon_0 \frac{d\Phi_E}{dt} = \varepsilon_0 \frac{d\left(\frac{Q}{\varepsilon_0}\right)}{dt} = \frac{dQ}{dt} = I$$

> The displacement current flowing through this surface is equal to the rate at which the capacitor is being charged. And this rate actually is related to how fast charged particles are delivered here. […] In fact, they are equal! My conclusion is that the displacement current which flows between the two plates is equal to the conduction current in the leads to the conductor. So right now we closed a loop for the current. Current now flows everywhere. It happens that at some places of the circuit current flows as the conduction current and in some places it flows as the displacement current (D, 11:20-13:00)

The lecture ends with the general idea that "current flows everywhere", which allows us to understand how the electric signals are transmitted from broadcasting stations, i.e. due to the "displacement currents flowing between the antennas".

Table 2 Summary of significant differences in the four lectures

Lec.	Reason for the insertion	Main focus	Reference to Maxwell	Charging capacitor
A	Limitation of Ampère's law to solve the charging capacitor problem. Displacement current (via symmetry arguments with Faraday's law) solves the problem	Find the magnetic field **B** (r) in the region between the plates (also when $r > R$)	Maxwell's reasoning: Symmetry with Faraday's law; Calculates the speed of light "out of nothing"; Reason for the name displacement (polarization in a dielectric)	Solved in the beginning as a motivation for the displacement current insertion. Highlights the "no fringe fields" idealization
B	No actual reason (in the sense of some problem with Ampère's law). Formal similarity with Faraday's law is used to make the equation plausible	Numerical problem solving and difference in magnitude between the displacement and conduction currents	No mention to Maxwell	Solved as a numerical problem to show the equivalence between conduction and displacement current
C	Inconsistency of Ampère's law with the continuity equation. Differential formalism	Philosophical discussion (theory–experiment relation and need for theoretical coherence)	Mentions that Maxwell's reasoning was different from the actual (Post-Einstein) version of the Electromagnetism	Solved to show the limitation of Ampère's law and the coherence between Ampère-Maxwell law and the continuity equation
D	No reason for the new term is given; Ampère-Maxwell law is presented	Two different kinds of current. Displacement current propagates through space; Technological applications (E-M waves)	Names the equation as Ampère-Maxwell law, but no reference to Maxwell's reasoning is made	Solved to show the formal equivalence between conduction and displacement current. Fringe fields mentioned

In Lecture D, the displacement current is introduced within the theoretical framework of Maxwell's equations. Even though the professor uses self developed teaching materials, the presentation is indeed relatively similar to what can be found in traditional textbooks. Similarly to Lecture B, in D the capacitor problem is treated as an application of a given law (no justifications for the displacement current term). However, the problem is solved in an analytical (not numerical) way and has the goal of showing a formal equivalence between conduction and displacement current (instead of relating it to limitations of Ampère's law). The difference between conduction and displacement current is strongly emphasized and technological applications are mentioned. No direct remark about Maxwell's reasoning was found.

It is not among the goals of this work to make any kind of judgment concerning the quality of the lectures, but rather to highlight some significant differences in their presentations, which confirms the assumption that several factors (e.g. learning goals, teachers' conceptions, students' previous knowledge, among many others) come into play in the level of internal transposition (i.e., the knowledge actually taught in classroom). A summary of some of these differences is presented in Table 2.

5 Concluding remarks

In this work we analyzed the didactic transposition process of one core episode in the history of electromagnetism—the displacement current term insertion—in two stages: from Maxwell's original work (Sect. 2) to textbooks (Sect. 3) and from them to the classroom (Sect. 4). We were able to highlight several differences between these spheres, which testify the knowledge *transformation* produced by the didactization of this topic (Chevallard 1991).

Taking into account the relation between the historical dimension and textbooks' presentations, we notice that many aspects are lost (and, of course, others are added) in didactic discourses. The development of Maxwell's reasoning, with its different models and conceptualizations of the displacement current, is not found in textbooks, neither are the philosophical debates about the interpretation of this term and the validity of the methods utilized in its proposition. What we usually observe are logical/rational presentations that display straightforward lines of reasoning. This is in accordance with Kuhn's view of science (especially physics) textbooks as representatives of the paradigmatic nature of the scientific knowledge.

Nevertheless, we do find considerable differences in the way the displacement current term is justified in the textbooks and articles analyzed. We identified three main arguments (charging capacitor problem, continuity equation and symmetry considerations) used to introduce the topic and discussed both their advantages and shortcomings. One important aspect is the chosen formalism, which seems to constrain the possibilities of justifying the term insertion. Furthermore, we notice that the (dis)charging capacitor problem is highly established in the didactic transpositions of the displacement current. It was solved, either as an introduction or as an application, in all textbooks and lectures analyzed. Despite its secondary importance in Maxwell's work, it has become "pedagogically irresistible" probably due to the fact that it enables a rational introduction of the displacement current within the integral formalism.

The analysis of four introductory lectures on the displacement current enabled us to identify several peculiarities of the *internal transposition*, i.e. the knowledge transformation from textbooks to the classroom. We clearly see that different messages are implicitly given and different aspects of this episode are stressed, which are probably related to different learning goals set by the lecturers. Although we may identify an overall similarity between their presentations and the ones found in textbooks, Lecture C appears to be an exception, mainly due to the professor's use of this episode to conduct epistemological discussions about the theory–experiment relation as well as other aspects related to the sociological character of physics, which are not commonly addressed in typical textbooks.

When comparing the teaching approaches (Sects. 3 and 4) with the nuances of the historical development of this concept (Sect. 2), it is quite clear that focusing on the history of the displacement current would not fulfill the traditional goals of a course designed to teach engineers and science majors the products of science (context of the lectures analyzed). However, we might wonder what kind of influence would this historical knowledge have on the didactic discourse of a lecturer. In order to speculate in this direction, the notion of *epistemological surveillance* (Chevallard 1991) seems to be quite useful. Chevallard remarks that the didactical transposition is for the teacher:

> […] a tool that allows revised, take away, to question evidences, doubt about the simple ideas, abandon familiarity, hence misleading its object of study. In a word, is what enables exercising its **epistemological surveillance**. (Chevallard, 1991, p. 16)

In this sense, the *epistemological surveillance* involves one adopting a careful posture when speaking about the historical development of the knowledge that is being taught. This ability should allow professors/teachers not only to recognize oversimplifications, mythifications, distortions and mistakes in textbooks, but also to avoid incurring in them during their lectures. In the displacement current case, such posture would prevent oversimplified or inadvertent reference to Maxwell's reasoning.

This work has pointed out the enormous complexity involved in the teaching of a crucial topic of the electromagnetic theory. It presents a broad variety of possibilities to justify the need for its insertion—which is associated with Shulman's notion of Pedagogical Content Knowledge (Shulman, 1986)—and gives an overview of some of the consequences of its didactization process—which should foster the development of an important ability called *epistemological surveillance* (Chevallard, 1991). These are both fundamental conditions for the development of an effective and well-informed physics education.

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