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Newton's absolute space, Mach's principle and the possible reality of fictitious forces

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Abstract. Newton's defence of absolute space and the critique it suffered from Berkeley, Leibniz and Mach are presented from an historical point of view. Mach's ideas and their influence on Einstein are discussed in order to argue that to contemplate inertial forces as arising from real interactions, rather than being only useful fictions, is not as far-fetched as suggested by the standard approach to the teaching of classical mechanics.

Résumé. L'article propose une revue historique de l'idée d'espace absolu, soutenue par Newton et critiquée par Berkeley, Leibniz et Mach. Les idées de Mach et leur influence sur Einstein, sont discutées dans le but de contempler la vision des forces fictives comme issues des interactions réelles, et non seulement fictions utiles. Ceci peut montrer qui cette vision n'est pas de si absurde comme le suggère l'approche traditionelle de l'enseignement de la méchanique classique.

1. Introduction

In the teaching of classical mechanics it is usual to draw a distinction between forces due to interactions and inertial forces. In the standard approach to the subject those forces are presented as having different ontological status: whereas the former are seen as real, true forces, the intensity of which can be calculated by using, for instance, Newton's law of gravitation or Coulomb's law, the latter are not associated with any of the fundamental interactions, do not obey the action-reaction law and are regarded as bearing a fiction-like character, hence they are dubbed fictitious forces.

In the standard approach, the use of fictitious forces is a useful trick which allows one to extend the application of Newton's first and second laws in non-inertial frames of reference, in which they *are not, strictly speaking, valid*. Simple examples, sometimes considered in elementary physics, are of a mass and string suspended from the ceiling of an accelerated train car, and of a satellite orbiting the Earth, when treated from the point of view of a frame of reference fixed to the car and to the satellite. *Although in these simpler cases the use of inertial* forces are not of great help, 'centrifugal' and the much more complicated 'Coriolis forces' appear almost inevitably in situations where the effects of, for instance, the rotating Earth cannot be neglected, as in the case of long-range projectiles and movements of the atmosphere.

The study of movements in non-inertial frames of reference is a conceptual minefield, as testified by the recurrent literature on the topic (Cooper 1969, Rothman 1970, Bartlett 1972, Ziauddin 1973, Taylor 1974, Bauman 1980, Savage and Williams 1989, Swartz 1989) and also by exchanges of letters in physics teaching journals. As any experienced teacher must have noticed, learners tend to consider inertial forces as real and to get confused about their use, the most common case being the misuse of the centripetal and centrifugal forces. These conceptual difficulties have led respected authors (Rogers 1977; Warren 1979) to recommend that preference should be given to inertial frames and to stress the imaginary nature of the inertial forces. The advice is sound, and certainly helps to prevent many misunderstandings, as far as one remains restricted to the standard approach, and accept its all important presupposition that the local inertial properties of a body are not significantly affected by the distant distribution of matter in universe.

On the other hand, the possibility, seldom mentioned in textbooks, that this presupposition may not be true, was given serious consideration by physicists influenced by Mach's Principle, an expression coined by Einstein, after the Austrian physicist and philosopher Ernst Mach (1838–1916). Although the acceptance of Mach's Principle is a controversial issue in contemporary physics it can be argued that, whatever one's opinion about its validity, the con*sideration of its history and implications is of educa-*

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tional value, since it allows for a critical examination of ideas often taken for granted in the teaching of mechanics. The most influential of Mach's writings is his *The Science of Mechanics—a Critical and Historical Account of its Development*, first edited in German in 1883, in which a critique of Newton's presentation of mechanics is undertaken. Of particular interest to us is his critique of the role of absolute space in Newton's theory (a point glossed over in most presentations of classical mechanics) and, in order to frame the matter in its due historical perspective and context, we start by considering Newton's ideas about space and their early critics.

2. Absolute and relative space in Newton's *Principia*

In the opening pages of his major work, *Mathematical Principles of Natural Philosophy*, Newton defined the basic concepts (quantity of matter, quantity of motion, innate force, impressed force and centripetal force) which will be used in the statement of his three laws and in their application to the study of moving bodies. Following these definitions an extended 'scholium' is presented for the discussion of time, space, place and motion, which he considered as ideas well known to all and therefore not needing to be defined. Nevertheless, he observed that, since common people conceived those quantities only from their relation to sensible objects, it was convenient to distinguish what he believed to be the absolute, true and mathematical character of such notions from their relative, apparent and common sense one. As far as space was concerned (a parallel discussion was made for time) the distinction was presented as:

> 'Absolute space, in its own nature, without relation to anything external, remains always similar and immovable. Relative space is some movable dimension or measure of the absolute spaces: which our senses determine by its position to bodies; and which is commonly taken for the immovable space; such is the dimension of a subterraneous, an aerial, or celestial space, determined by its position with respect to the earth. Absolute and relative space are the same in figure and magnitude; but they do not remain always numerically the same. For if the earth, for instance, moves, a space of our air, which relatively and in respect of the earth remains always the same, will at one time be one part of the absolute space into which the air passes; at another time it will be another part of the same, and so, absolutely understood, it will be continually changed.' (Newton 1952a, pp 8–9)

The motion of a body—its translation from one place to another—could, accordingly, be absolute or relative, depending in what sort of space the places occupied by the body were being considered. Newton illustrated the point with the example of a sailor walking on a ship: the true, absolute motion of the sailor across the absolute immovable space would arise partly from his relative motion with regard to the ship, partly from the relative motion of the ship with regard to the Earth, and partly from the motion of the Earth through absolute space. Nowadays, when the idea that motion should always be described in relation to a reference frame is part of introductory secondary school mechanics, it may appear that Newton was overstating an obvious point, but the discussion coming next in the 'scholium', makes clear that the real issue for him was to justify the idea of absolute space. For he first needed to stress the differences between relative and absolute motion.

Newton was conscious of the difficulties posed by the conception of absolute space, conceived by him as infinite, homogeneous and isotropic. Since the parts of such a space are indistinguishible from one another by our senses, the distances, positions, places and, consequently, motion, are normally determined with reference to visible bodies, considered as immovable:

> 'And so, instead of absolute places and motions, we use relative ones; and that without any inconvenience in common affairs; but in philosophical disquisitions, we ought to abstract from our senses, and consider things themselves, distinct from what are only sensible measures of them. For it may be that there is no body really at rest, to which the places and motions of others may be referred.' (Newton 1952a, p 10)

In the Newtonian framework, an impressed force always generates a change in the state of absolute rest or absolute motion of a body, i.e. an acceleration relative to absolute space. On the other hand a relative acceleration cannot be taken as indicating the action of an impressed force, since that acceleration can be generated by the movement of the reference body. Newton had it clear, therefore, that it was not possible to ascertain, from relative kinematics only, absolute motion. Was then absolute motion, and consequently absolute space, to remain an abstract hypothesis in his theory? Certainly not for him, because he believed he had found in the centrifugal effects, shown in circular motions, the answer to his efforts for enhancing the credibility of absolute space.

This point was illustrated by the famous rotating vessel example, in which Newton asks us to consider a vessel hung by a long strongly twisted cord and then filled with water. At first the system is at rest and the water surface is plane. The vessel then is whirled in a way that the cord is untwisted rapidly. In the beginning of the movement the water surface will remain plane, but as the vessel gradually communicates, by friction, its motion to the water, the liquid will begin to revolve and recede from the middle, ascending to the sides of the receptacle, with its surface acquiring a concave shape that becomes more pronounced the swifter the motion. After some time the vessel and the water will have the same rotation rate, being in a state of relative rest, and in this situation the water surface will be concave. Newton argued that the endeavour of the water to recede from the axis of motion, as shown by its ascent, is evidence of its true and absolute circular motion, that may be known and measured by this endeavour. The reasoning is based on the fact that the receding effect increases as the relative motion between water and vessel decreases and could not, therefore, depend on this relation.

In another example, a thought experiment this time, Newton supposed two globes connected by a cord, revolving around their common centre of gravity. He remarked that if one only observed the motion of the globes relative to external remote bodies, such as the fixed stars, then it was impossible to ascertain whether the motion belonged to the globes or to the stars. On the other hand, the circular motion of the globes could be found and quantified by observing the tension experienced by the cord, even in an immense vacuum, with no other external body with which the globes could be compared.

Newton's effort in defending the existence of absolute space can be understood when one considers that it had a logical function in his theory of motion, by establishing a conceptual requisite for the validity of the First Law: 'Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed on it'. We have only to ask 'rest, or uniform motion in a right line, in relation to what?' to appreciate the reason for Newton's postulation of absolute space, that was for him the ultimate, as we call it today, inertial frame of reference. In his framework, absolute space provided the ideal condition in which his laws of motion could be applied in an absolutely rigorous form.

If, from a logical point of view, absolute space constituted the background stage upon which the laws of mechanics could be applied, there remained the question of how it might be used in practice, since it was undetectable to our senses. Newton was not very explicit himself about how to handle the question. When discussing the example of the revolving globes he mentions the remote fixed stars that keep always a given position one to another, as a reference for the motion of the globes. It was, maybe, this passage that led to the belief that Newton assumed for fixed stars to be a practical substitute for absolute space, but he never said this explicitly. Yet, in Book III of the *Principia* (*The System of the World*), in which Newton applied the quite abstract theoretical framework developed in Book I. (*The Motion of Bodies*) to the concrete movement of celestial bodies, he assumed the centre of the system of the world (which he identified with the common centre of gravity of the earth, the sun, and all the planets) to be immovable, defining in this way a possible inertial frame (Jammer 1970).

Newton's strong conviction concerning the existence of absolute space (and for that matter, absolute time) can be related to his theology. Influenced by the ideas of cabalist and neoplatonic scholar Henry More, and of his former teacher at Cambridge, the mathematician Isaac Barrow, he endowed absolute space and time with religious meaning, the omnipresence and eternity of God respectively (Burtt 1954; Jammer 1970). In the General Scholium he added to the second edition of the Principia, made public in 1713, he stated:

> 'And from his true dominion it follows that God is a living, intelligent, and powerful Being; and, from his other perfections, that he is supreme or most perfect. He is eternal and infinite, omnipotent and omniscient; that is his duration reaches from eternity to eternity; his presence from infinity to infinity; he governs all things and knows all things that are or can be done. He is not eternity and infinity, but eternal and infinite; he is not duration or space, but he endures and is present. *He endures forever, and is everywhere present; and, by existing always and everywhere, he constitutes duration and space.*' (Newton 1952a, p 370; emphasis added).

And in Query 28 of his *Optics*, first edited in 1704, he asked:

> 'And these things being rightly dispatched, does it not appear from phenomena that there is a Being incorporeal, living, intelligent, omnipresent who in infinite space (as it were in his sensory) sees the things themselves intimately, and throughly perceives them, and comprehends them wholly by their immediate presence to himself.' (Newton 1952b, p 529)

3. Early critics of Newton's absolute space

The first criticism of absolute space in a major classical work appeared in 1710, with the publication of the *Treatise Concerning the Principles of Human Knowledge* by Irish philosopher George Berkeley. Although an admirer of Newton's work, Berkeley had strong feelings about conceiving motion as anything but relative and his rejection of absolute space, and therefore absolute motion, followed from his belief that no physical world exists behind the apparent elementary sense impressions subjected to the reflection of the mind. The notion of absolute space, not accessible to our senses, and independent of the existence of mind should not, accordingly, be granted any form of reality (Jammer 1970). He admitted that Newton's theory led to correct results, and the principles proved by experience (the ones describing the observed regularities in the motion of bodies) were true, but concepts such as absolute space and absolute motion, force, gravity and attraction, were to be regarded as mathematical hypotheses and not as true physical qualities. Some of them, such as force, gravity and attraction should not be rejected because they function correctly, being useful for reasoning and performing calculations about moving bodies. Absolute space and absolute motion, which did not function properly, should be disposed of and nothing would be lost in the theory, pointed Berkeley, if absolute space is substituted by the system of the fixed stars and absolute motion by motion relative to them (Popper 1953; Jammer 1957).

Being a man of the church (in the last eighteen years of his life he was a Bishop) the association of space with God could not be passed without consideration. Having denied absolute space, his position was antagonistic to the ones held by More, Barrow and Newton. For him, the chief advantage arising from the disposal of absolute space was:

> 'that we are freed from that dangerous dilemma, to which several who have employed their thoughts on that subject imagine themselves reduced, to wit, of thinking either that Real Space is God, or else that there is something beside God which is eternal, uncreated, infinite, indivisible, immutable. Both of which may justly be thought pernicious and absurd notions'. (Berkeley 1952, p 436)

Newton's religious remarks also became the central focus for the critical standpoint assumed by Leibniz in the correspondence he kept with theologian and natural philosopher Dr Samual Clarke. The debate was sparked, in November 1715, by a letter from the German philosopher directed to Princess Caroline of Wales (who was interested in the philosophies of both Newton and Leibniz, and corresponded with the latter over several years) expressing the view that religion was becoming extremely weak in England, and blaming the ideas of Newton and Locke for it (Erlichson 1967). Newton's image of space as the sensory of God was particularly castigated.

Dr Samuel Clarke, the foremost disciple of Newton, was asked by the Princess to answer the letter and, in the ensuing exchange of papers (that lasted until Leibniz's death in November 1716 and which were published by Clarke) issues such as the relationship between God and space, the nature of space and time, the principle of sufficient reason, the principle of the identity of indiscernibles, the nature of gravity, the possibility of void and atoms and the measurement of force were discussed (Erlichson 1967). The disciple defended the master's views, quite certainly under his guidance, or at least in consultation with him. For instance, he claimed that Newton used the word *sensorium* as an illustration by similitude, in order to make his point more intelligible, but that God does not need any organ whatsoever to perceive things. For more details about the religious overtones of the controversy see Burtt (1954) and Jammer (1970).

In various points of the correspondence Leibniz presented his views on space, as being no more than a system of relations without real existence and, knowing that the relational conception of space held by Leibniz could not be contested on kinematic grounds, Clarke resorted to a dynamical example, generalized from the sensations experienced in cases of sudden changes in the state of rest or movement:

> 'If space was nothing but the order of things coexisting; it would follow, that if God should remove in a straight line the whole material world entire, with any swiftness whatsoever; yet it would still always continue in the same place: and that nothing could receive any shock upon the most sudden stopping of that motion.' (Quoted in Erlichson 1967, p 92)

In his answer, Leibniz brought back the issue to kinematics by arguing for the indiscernibility of identical parts of an absolute space independent of matter:

> 'The fiction of a material universe, moving forward in an infinite empty space, cannot be admitted. It is altogether unreasonable and impracticable. For, besides that there is no real space out of the material universe; such an action would be without any design in it: it would be working without doing anything, *agendo nihil agere*. There would happen no change, which could be observed by any person whatsoever.' (Quoted in Erlichson 1967, p 95)

But, as Erlichson (1967) points out, whether or not any change would be observable was the point at stake and Clarke was justified in criticizing his opponent for begging the question, and not being given a proper answer to his argument that a sudden increase or stoppage of the motion of whole universe would produce a sensible shock to its parts. Jammer (1970) also stresses that, as far the dispute remained on kinematical grounds, Leibniz can be considered the winner, but he was not as successful in dealing with the dynamical questions. The death of Leibniz interrupted the correspondence after Clarke's fifth letter, but it is unlikely that either of the two would have conceded any ground. The Leibniz–Clarke controversy stands out as a clear example of a polemic that served to cristallize the opponent's points of view rather than fostering any agreement.

The achievements of Newtonian mechanics (it was a physics that worked, as instanced by its success in astronomy) had the implication that the criticisms advanced by Berkeley and Leibniz found little resonance for more than 150 years. In the mid 18th century Euler wrote that absolute space warranted the validity of the Principle of Inertia, MacLaurin stated that the Principle could only be intelligible by admitting absolute space and, in the French school, authors such as Lagrange, Laplace and Poisson assumed the idea of absolute space as a working hypotheses and did not devote efforts to its theoretical justification. During most of the 19th century space was accepted as a logical necessity for the formulation of mechanics while, pragmatically, ignored in practical applications of the theory (Jammer 1970).

On the other hand, the attempt to find an absolute medium for the movement of light waves, to some extent, transferred the problem to the field of optics without giving up the ideal of establishing an absolute frame of reference for mechanics. For, if detected, the luminiferous ether could then, possibly, be identified with absolute space. The negative results of optical experiments designed to detect the ether led to its dismissal at the beginning of the 20th century.

4. Mach's critique of absolute space

While the luminiferous ether evaded attempts of experimental detection, the question of absolute space was brought again to the field of mechanics by Ernst Mach who, in the preface to the first German edition of *The Science of Mechanics*, published when he was a professor in Prague, stated his aim as 'to clear up ideas, expose the real significance of the matter, and get rid of metaphysical obscurities'. Among those were parts of Newton's formulation, such as his conceptions of force, mass (that Mach will substitute by the operational definition presented in most physics textbooks today) and what he will later call, in the preface to the German seventh edition of the book, 'the monstrous conceptions of absolute space and absolute time'.

Mach's criticisms were deeply rooted in his philosophy of science, which assumed that only sensations can be known and are real (phenomenalism) and that the purpose of science is to describe and to relate appearances in the simplest way possible (economy of thought); accordingly, he recommended that theoretical entities should be avoided and, when unavoidable, may be used only as provisional aids. The dislike for theoretical, not directly observable, constructs led him to be an opponent of atomic theory to the end of his life (Blackmore 1972). Mach's ideas on the nature of science became a reference for members of the Vienna Circle that, in the twenties, developed logical-positivism, a philosophy of science grounded on radical empiricism. Although influential for decades, logical-positivism is, today, considered to have been superceded as a philosophy of science (Suppe 1977). The rejection by Mach of absolute space follows from his desire to eliminate from science those notions which do not have a sensorial counterpart and, in this respect, there are points of contact between his epistemology and Berkeley's, who also anticipated some of Mach's criticism of absolute space (Popper 1953). According to Karl Menger, who wrote the introduction to the sixth American edition of *The Science of Mechanics*, the fact that Berkeley was not quoted as a source by Mach could be due to his fear of being associated with Berkeley's spiritualism.

In his critique of absolute space, Mach's efforts were directed at defusing Newton's argument based on the rotating bucket, according to which the concave shape acquired by the surface of the water is due to its rotation in absolute space. But, argued Mach, the only thing one can really ascertain is that the water is rotating in relation to the Earth and the fixed stars and, therefore, all that can be soundly concluded is that this relative motion is the reason for the centrifugal effects. He stressed his point by defying his readers to fix Newton's bucket and rotate the heaven of fixed stars, and then to try to prove the absence of centrifugal forces.

Similarly, phenomena that were explained as effects of an absolute rotation of the Earth about its axis, as for instance its oblate form, the weakning of the acceleration of gravity at the equator and the rotation of the plane of oscillation of Foucault's pendulum, could be reinterpreted from a relational point of view. From an absolute space perspective, all these phenomena would not exist if the Earth was at rest and the other heavenly bodies were endowed with absolute motion around it, so that the same relative rotation is produced. This is indeed the case, points out Mach, only:

> '... if we start *ab initio* from the idea of absolute space. But if we take our stand on the basis of facts, we shall find we have knowledge only of *relative* space and motions. *Relatively*, not considering the unknown and neglected medium of space, the motions of the universe are the same whether we adopt the Ptolemaic or Copernican mode of view. Both views are, indeed, equally *correct*; only the latter is more simple and more *practical*. The universe is not *twice* given, with one earth at rest and an earth in motion; but only *once*, with its *relative* motions, alone determinable. It is, accordingly, not permitted us to say how things would be if the earth did not rotate. We may interpret the one case that is given us, in different ways. If, however, we so interpret it that we come into conflict with experience, our interpretation is simply wrong. The principles of mechanics can, indeed, be so conceived, that even for relative rotations centrifugal forces arise.
>
> Newton's experiment with the rotating vessel of water simply inform us, that the relative rotation of the water with respect to the sides of the vessel produces *no noticeable centrifugal forces*, but that such forces *are* produced by its relative rotation with respect to the mass of the earth and other celestial bodies. No one is competent to say how the experiment would turn out if the sides of the vessel increased in thickness and mass till they were ultimately several leagues thick. The one experiment only lies before us, and our business is, to bring it into accord with the other facts known to us, and not to the arbitrary fictions of our imagination.' (Mach 1960, pp 283–4)

The 'arbitrary fictions of our imagination', absolute space and absolute motion, were to be disposed of by resorting to the heaven of 'fixed stars', considered by Mach to be, in his time, the only practically usable system of reference, and the best approximation to an inertial system. Ideally, the ultimate inertial system would be one non-accelerated with regard to the centre of mass of the universe but, because only a limited number of masses was within the reach of knowledge, he reckoned its determination could not be fully carried out. Today, when the rotation of our galaxy and the expansion of the universe are both acknowledged, the 'fixed stars' might be substituted for by that rigid frame in which all other galaxies appear to be receding radially (Rindler 1977). According to Mach's interpretation, any role played by absolute space in mechanics would be undistinguishable from the role played by the 'fixed stars' (the best approximation he had for the rest of the universe) and, consequently, the local inertial properties of bodies can be imagined as influenced by the rest of the universe.

From this perspective, the changing orientation of the plane of oscillation of Foucault's pendulum or the bulging of the Earth's equator does not indicate an absolute rotation of our planet in space, but only its relative motion with respect to the ensemble of distant material bodies and, therefore, an interaction of some sort must be assumed between the bodies suffering the inertial effects and the distant matter of the universe such as stars and galaxies. Mach himself did not try to specify more precisely the nature of this interaction, and the idea that local motion could be affected by distant stars offended the physical intuition of men like Eddington, Russell, Whittaker, Whitehead and Weyl (Bridgman 1961). But the concept did appeal, among others, to a young student.

5. Mach's principle and modern physics

Albert Einstein first read *The Science of Mechanics* around 1897 while a student at Zurich Technical University and, on different occasions, acknowledged the strong impression the work exerted upon him 'owing to its physical orientation toward fundamental concepts and fundamental laws'. He also praised Mach's 'incorruptible scepticism and independence' and pointed out that the book had shaken his faith in 'mechanics as the final basis of all physical thinking'. In his younger years he was also influenced by Mach's epistemology, which shows up more evidently in the operationalist views of measurement and of the concepts of space and time adopted in his 1905 paper on special relativity (Holton 1973; quotes above on p 223).

Mach's relational views and his assumption about the connection between the overall distribution of matter in the Universe and local inertial properties, which Einstein named Mach's Principle in a 1918 paper (Paty 1993), were of heuristic value in the development of his general theory of relativity, by means of which he expected to explain the link between matter there, inertia here. In a letter to Mach, sent around the New Year of 1911–1912, he complained about Max Planck's negative views on Mach's efforts and on his own general theory, and referred to Mach's Principle as the only single epistemological argument he could bring forward in favour of the theory (Holton 1973). In another letter to Mach, dated 25 June 1913, he was enthusiastic about the perspective that the observation of the solar eclipse in 1914 would confirm their views (due to the outbreak of war he had to wait until the solar eclipse of 1919):

> 'Recently you have probably received my new publication on relativity and gravitation which I have at last finished after unending labour and painful doubt. Next year at the solar eclipse it will turn out whether the light rays are bent by the Sun, in other words whether the basic and fundamental assumptions of the equivalence of the acceleration of the reference frame and of the gravitational field really holds. If so, then your inspired investigations into the foundations of mechanics—despite Planck's unjust criticism—will receive a splendid confirmation. For it is a necessary consequence that inertia has its origin in a kind of mutual interaction of bodies, fully in the sense of your critique of Newton's bucket experiment.' (Quoted in Holton 1973, p 228)

In spite of Einstein's remarks, being associated to relativity was an honour that Mach (suspicious of the strong speculative theoretical trend and the lack of certainty and contact with sensory experience taken by the new theory) explicitly declined in the preface to his *The Principles of Physical Optics*, written in 1913 but posthumously published in 1921.

Einstein, in turn, would become vocal against Mach's narrow empiricism and positivism. Lecturing in Paris, in 1922, he would call Mach 'un bon mécanicien' but 'déplorable philosophe' and comment that there is more to science than the study of the existing relations between the data of experience. For Holton (1973) Einstein changed gradually his epistemological commitments, from an early acceptance of Mach's ideas, towards a rationalist–realist conception of a world existing behind phenomena, that can only be grasped with the help of intuitive and speculative theoretical constructions transcending sense experience. A recent critique of this view is provided by Paty (1993), who stresses non-positivistic trends at earlier stages of Einstein's thinking.

Einstein's rejection of Mach's epistemology did not prevent him from trying to implement Mach's Principle in the framework of his general theory, and the inclusion of the cosmological constant (that he later gave up) in the 1917 version of the field equations served, although not exclusively for that purpose. The attempt failed because it was soon shown by de Sitter that these field equations admitted a solution in the absence of matter, i.e., a completely empty universe could be endowed with a space and time structure, which is incompatible with Mach's Principle (Jammer 1970). Einstein never managed to successfully incorporate in the equations of general relativity the same idea that played such an important heuristic role in the development of the theory itself, and in a letter of February 1954 would explicitly renounce it:

> 'One shouldn't talk any longer of Mach's principle, in my opinion. It arose at a time when one thought that 'ponderable bodies' were the only physical reality and that in a theory all elements that are not fully determined by them should be conscientiously avoided. I am quite aware of the fact that for a long time, I, too, was influenced by this fixed idea.' (Quoted in Holton 1973, p 251)

The fact that Mach's Principle did not find its way into in the equations of general relativity led both to 'anti-Machian' solutions of the equations as Gödel's model and Kerr's metric (Rindler 1977) for instance, and to modifications incorporating the Principle, as Brans–Dicke and Hoyle–Narlikar theories (Narlikar 1977). Others, such as Hönl and Wheeler, suggested considering Mach's Principle as a supplementary cosmological principle that could function as a selection rule amid the many solutions proposed for cosmological problems, but not incorporated in the formalism of general relativity (Tonnelat 1972; Paty 1993).

One can also find in the literature attempts to implement Mach's Principle within the framework of classic Euclidean three-dimensional space. The interest of this exercise, as pointed out by one of its first proponents, is that, while lacking the range of validity of Einstein's theory, those approximations are sufficient for one to work out implications concerning the inertial properties of matter (Sciama 1957). A recent example of this sort of procedure is provided by Assis (1989), who derived the inertial forces that appear in a frame rotating with respect to the 'fixed stars'. He assumed a gravitational interaction (calculated by using an analogue to Weber's law of force between relatively moving electric charges) between the particle and the rest of the universe, and a basic postulate, which replaces Newton's first and second laws, stating that the resulting force acting on any body is always zero.

The proposition sounds less strange when one considers the familiar example of a satellite orbiting around the Earth. Considered from a frame of reference revolving with it, the satellite is at rest and, following the standard approach. Newton's second law can be applied by introducing a fictitious centrifugal force counteracting the gravitational pull of the Earth; considered from the frame of reference fixed to the Earth (but not rotating with it, so that it can be better approximated to an inertial frame) the satellite is accelerated and only the gravitational pull of the Earth is considered. But, from a Machian point of view, the centrifugal force is the result of a real interaction with the rest of the universe and cannot be disposed of because the frame of reference was changed.

6. Conclusion

The aim of this paper was to show that, following Mach's Principle, inertial forces can be seen as resulting from real interactions with distant matter in the universe. Although not consensual (see Bunge 1966, for an epistemological critique), this possibility played a major heuristic role in the development of Einstein's general relativity, and is entertained by a number of physicists. Nevertheless, it is difficult not to feel a grain of dogmatism and a missing sense of history in the standard presentation of classical mechanics, both at university and pre-university levels (advanced books on general relativity tend to discuss the point). Teachers and textbooks alike do, of course, make passing remarks about the limitations of the classical theory as exposed by relativity and quantum mechanics, but the Newtonian point of view is usually presented as unproblematic and internally consistent: absolute space is not mentioned, but is implictly assumed; the 'fixed stars' are said to provide a very good approximation to a inertial frame of reference, but the question of why this happens to be so is not raised; inertial forces are considered useful fictions but no other possibility presented.

On the other hand, some problems related to the teaching of the issues raised in this paper must be recognized. To start with, research in the field of alternative conceptions has shown how difficult it is for most learners to acquire a conceptual understanding of the Newtonian inertial view of motion, and one is entitled to ask if the presentation of a critique of this view would not compound the difficulty. An historical analogy may be appropriate here.

In his foreword to Max Jammer's *Concepts of Space*, Einstein pointed out that it required a severe struggle to arrive at the concept of absolute space, indispensable for the development of classical mechanics and that Newton's decision was, in his times, the only possible and fruitful one; and that no less strenuous efforts were needed for overcome this concept lately. In the same way, it can be suggested that a critical discussion about absolute space and of the possible reality of inertial forces should be postponed until after the Newtonian inertial view (its assumption of absolute included) is learned, a condition necessary to understand the criticism it suffered.

Accepted curricular science by its turn, enshrined as it is in curricular guides, textbooks and examination boards, follows the standard approach in the teaching of inertial forces and it is easy to foresee a student being failed in a public examination because he or she considered as real the centrifugal force acting on a satellite orbiting the Earth. This constraint would not be so strong if physics (and science in general) were taught as a way of viewing nature, allowing for different perspectives to be considered and for a sense of history to pervade the curriculum. If so, themes such as the possible reality of ficticious forces could be discussed more naturally in school classrooms. The tradition has not been such, but there is nothing sacred about tradition.

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