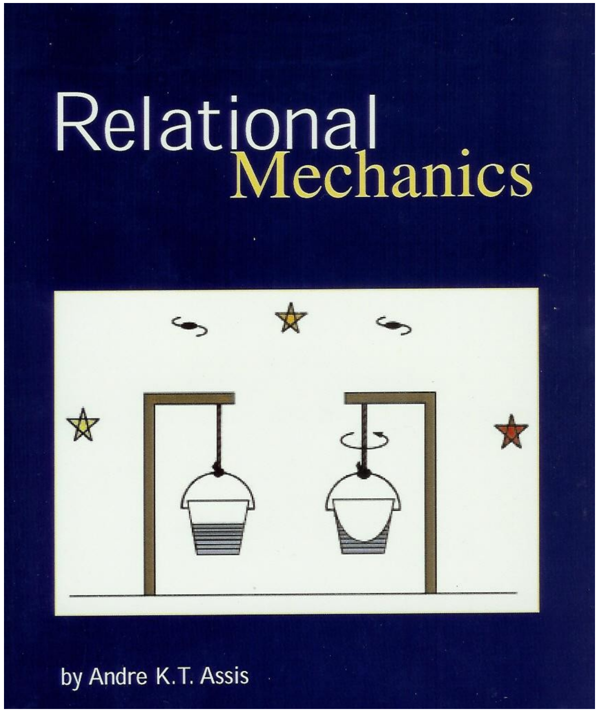
Relational Mechanics

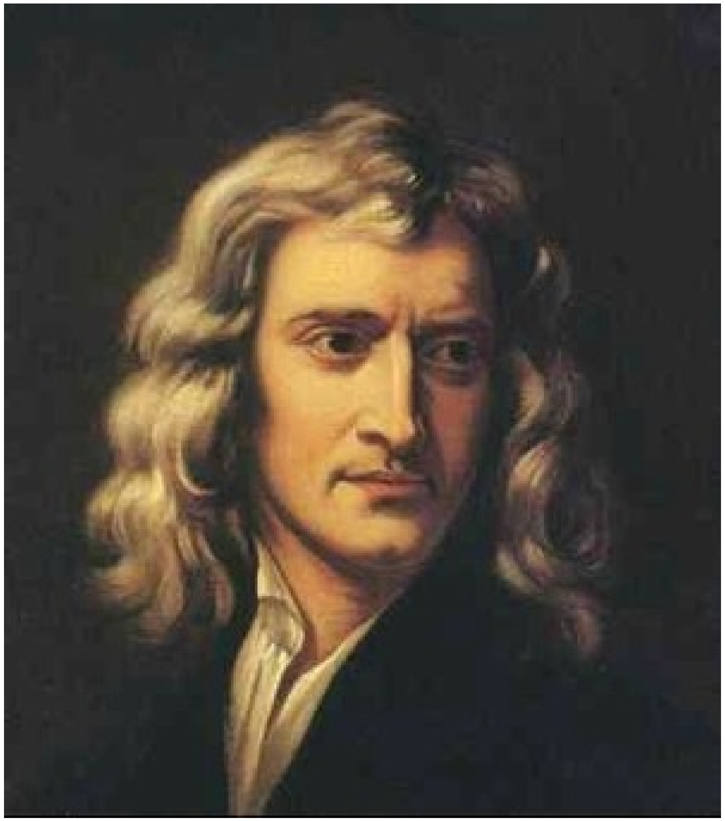
A. K. T. Assis

University of Campinas, Brazil



www.ifi.unicamp.br/~assis

Isaac Newton (1642 – 1727)



1687: Principia

PHILOSOPHIÆ

NATURALIS

PRINCIPIA

MATHEMATICA.

Autore *J S. NEWTON*, *Trin. Coll. Cantab. Soc.* Matheſeos Profeſſore *Lucaſiano*, & Societatis Regalis Sodali.

IMPRIMATUR.

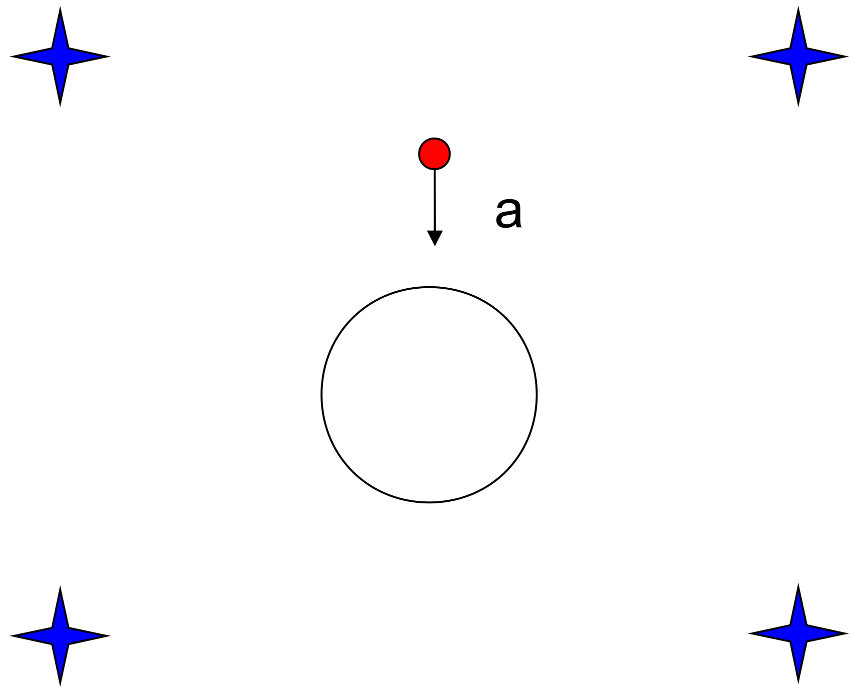
S. PEPYS, *Reg. Soc.* PRÆSES.

Julii 5. 1686.

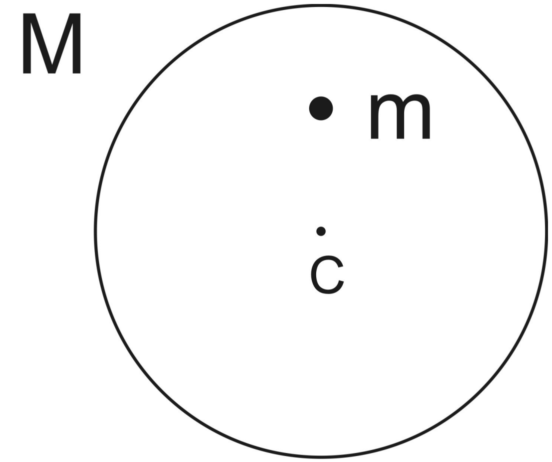
LONDINI,

Juſſu *Societatis Regiæ* ac Typis *Joſephi Streater*. Proſtat apud plures Bibliopolas. *Anno* MDCLXXXVII.

Free fall in Newtonian mechanics



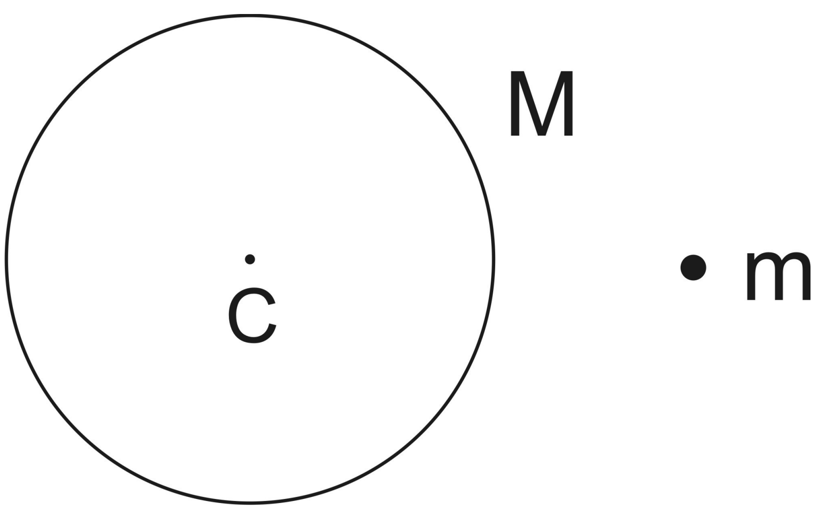
$$F = G\frac{m_1 m_2}{r^2}$$



"Theor. 30: If to every point of a spherical surface there tend equal centripetal forces decreasing as the square of the distances from these points, I say, that a corpuscle placed within that surface will not be attracted by these forces any way."

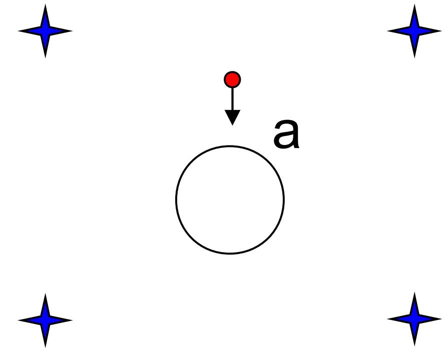
$$F = 0$$

“Theor. 31: The same things supposed as above, I say, that a corpuscle placed without the spherical surface is attracted towards the centre of the sphere with a force inversely proportional to the square of its distance from that centre.”



$$F = G\frac{M_g m_g}{r^2}$$

Free fall in Newtonian mechanics



$$F_E + F_* = m_i a$$

$$G\frac{m_g M_{gE}}{R_E^2} + 0 = m_i a \quad \Rightarrow \quad a = \frac{m_g}{m_i}\frac{GM_{gE}}{R_E^2}$$

$$a = \frac{GM_{gE}}{R_E^2} = g = 9.8\frac{m}{s^2}$$

Galileo: cork and lead fall together.

$$\frac{m_{g\,C}}{m_{i\,C}} = \frac{m_{g\,L}}{m_{i\,L}} = 1$$

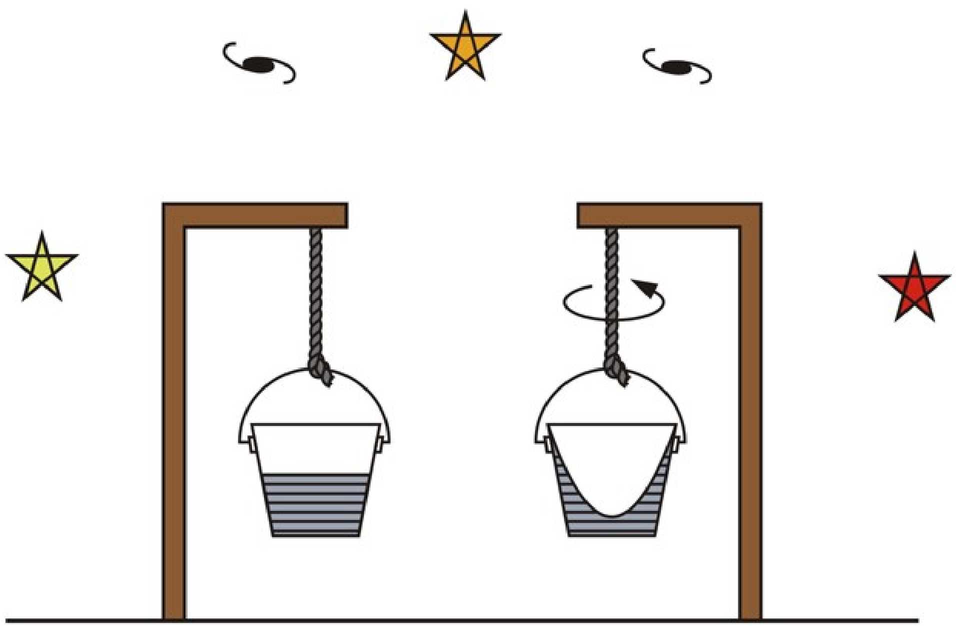
Newton’s 2nd law of motion:

$$F = \frac{d\left(m_i v\right)}{dt} = m_i a$$

Newton, Principia:

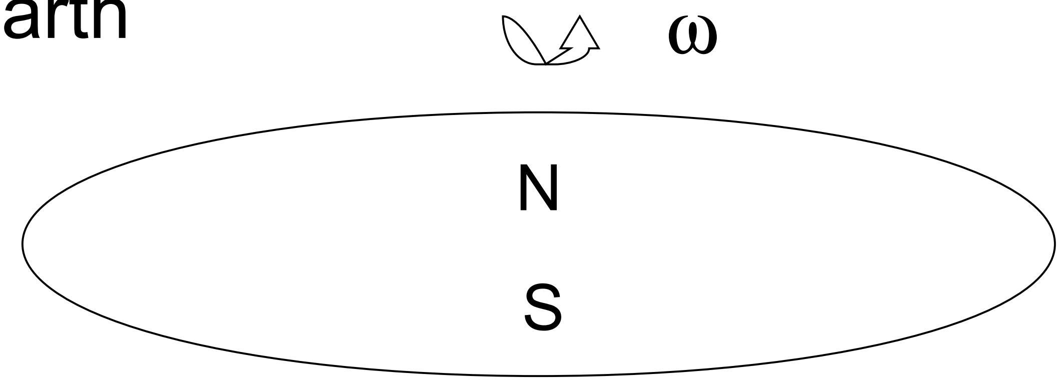
“Absolute space, <u>without relation to anything external</u>, remains always similar and immovable.”

Newton’s bucket experiment



$$z = \frac{\omega^2}{2g} r^2$$

Flattening of the Earth



Newton, Principia: “The diameter of the earth at the equator is to its diameter from pole to pole as 230 to 229.”

$$\frac{D_E}{D_P} = 1 + \frac{15}{16\pi G} \frac{\omega^2_{EAS}}{\rho_E} = \frac{230}{229} = 1.004$$

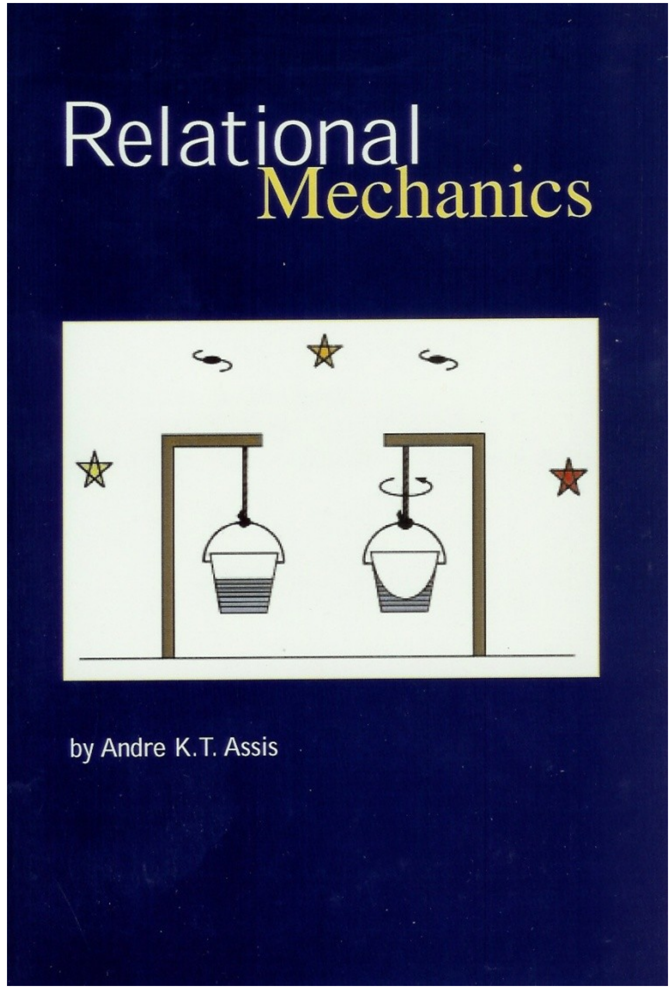
Mach in The Science of Mechanics, 1883:

- “The principles of mechanics can be so conceived, that even for relative rotations centrifugal forces arise.”
- “Try to fix Newton’s bucket and rotate the heaven of fixed stars, and then prove the absence of centrifugal forces.”

Einstein, The Meaning of Relativity, 1922:

"What is to be expected along the line of Mach's thought?

A rotating hollow body must generate inside of itself a Coriolis field, and a radial centrifugal field as well."



Relational Mechanics, A. K. T. Assis (Apeiron, Montreal, 1999)

Relational Mechanics

The sum of all forces acting on any body is always zero in all frames of reference.

$$\vec{F} = -H_g m_{1g} m_{2g} \frac{\hat{r}}{r^2}\left(1 - 3\frac{\dot{r}^2}{c^2} + 6\frac{r\,\ddot{r}}{c^2}\right)$$

Weber (1804-1891) in 1846:

Coulomb (1785): $$\vec{F} = \frac{q_1 q_2}{4\pi \varepsilon_0} \frac{\hat{r}}{r^2}$$

Ampère (1826): $$\vec{F} = \frac{\mu_0}{4\pi} I_1 I_2 \frac{\hat{r}}{r^2} f(\alpha, \beta, \gamma)$$

Faraday (1831): $$emf = -M \frac{dI}{dt}$$

Idea: $$I d\vec{\ell} \Leftrightarrow q\vec{v}$$

$$\vec{F} = \frac{q_1 q_2}{4\pi \varepsilon_0} \frac{\hat{r}}{r^2} \left[1 + k_1 v_1 v_2 + k_2 a_{12} \right]$$

Properties of Weber's Electrodynamics

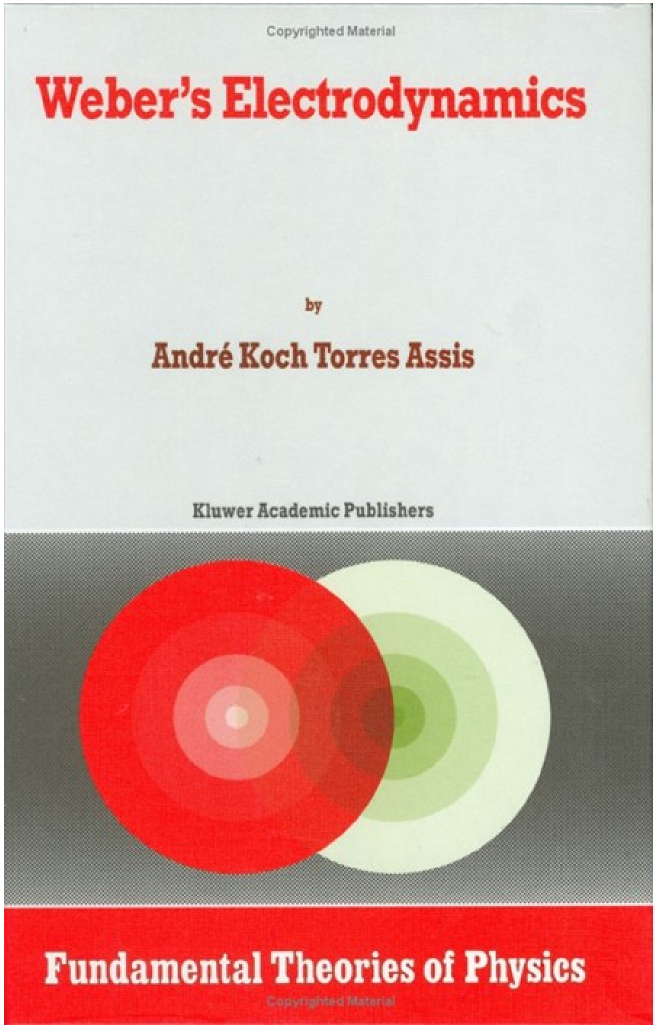
- In the static situation ($dr/dt = 0$ and $d^2r/dt^2 = 0$) we recover the laws of Coulomb and Gauss.
- Action and reaction, conservation of linear momentum.
- Central force, conservation of angular momentum.
- It can be derived from a velocity dependent potential energy:

$$U = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{1}{r} \left(1 - \frac{\dot{r}^2}{2c^2} \right)$$

- Conservation of energy:

$$\frac{d(K + U)}{dt} = 0$$

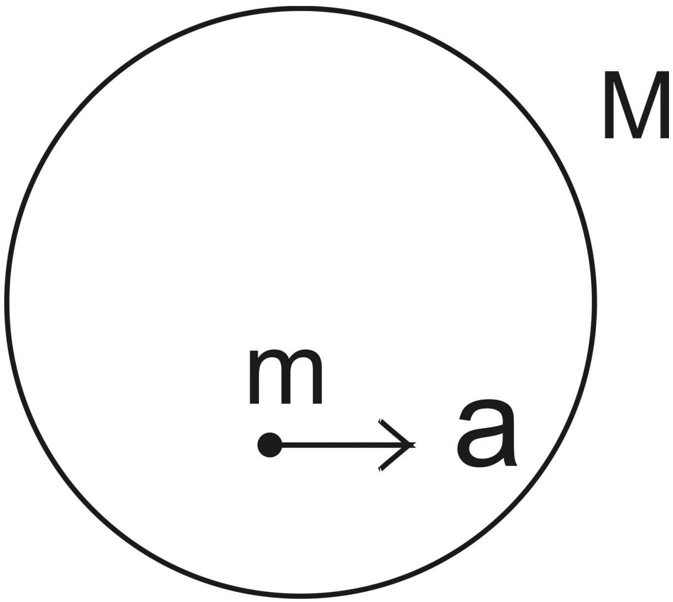
- Ampère’s circuital law can be derived from Weber’s force.
- Faraday’s law of induction can be derived from Weber’s electrodynamics (see Maxwell, *Treatise*).
- It is completely **relational**. That is, it depends only upon r, dr/dt and d^2r/dt^2. Therefore, it has the same value for all observers and in all reference frames. It depends only upon intrinsic magnitudes of the system, that is, upon the relations between the interacting bodies.



Weber’s Electrodynamics, A. K. T. Assis (Kluwer, 1994)

$$\vec{F} = -H_g m_{1g} m_{2g} \frac{\hat{r}}{r^2}\left(1 - 3\frac{\dot{r}^2}{c^2} + 6\frac{r\,\ddot{r}}{c^2}\right)$$

Integrated force of the shell of mass *M* upon a test body *m* accelerated inside it:

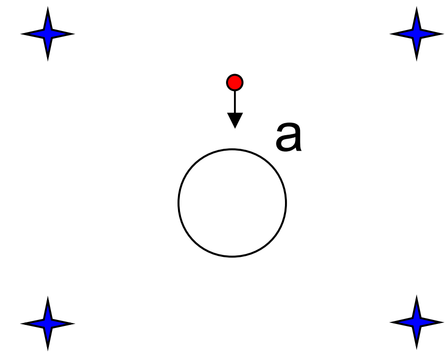


$$\vec{F} = -\phi\, m_g \vec{a}$$

$$\phi = \frac{2H_g M_g}{Rc^2}$$

Free fall in Relational Mechanics

$$F_E + F_* = 0$$



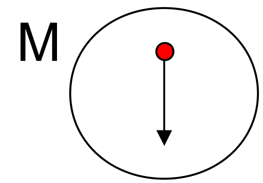
$$H_g \frac{m_g M_{gE}}{r^2} - \Phi\, m_g\, a = 0$$

$$a = \frac{H_g}{\Phi} \frac{M_{gE}}{r^2}$$

with $$\frac{H_g}{\Phi} = \frac{H_o^2}{4\pi \rho_*} \approx 6.7 \times 10^{-11} \frac{Nm^2}{kg^2} = G$$

Experimental test

a = 9.8 m/s^2



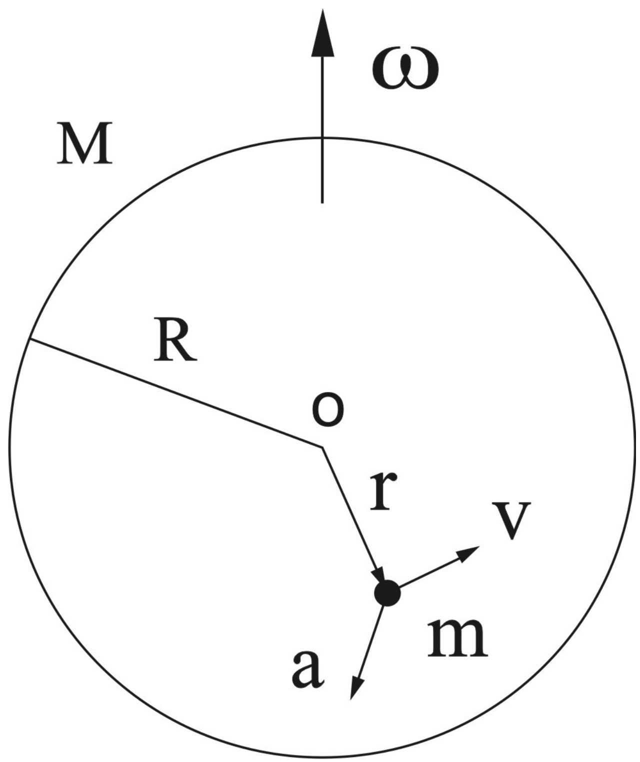
a = ?

$$a_N = a_E = 9.8\, m/s^2$$

$$a_{RM} = g\left(1 - \frac{2GM}{Rc^2}\right)$$

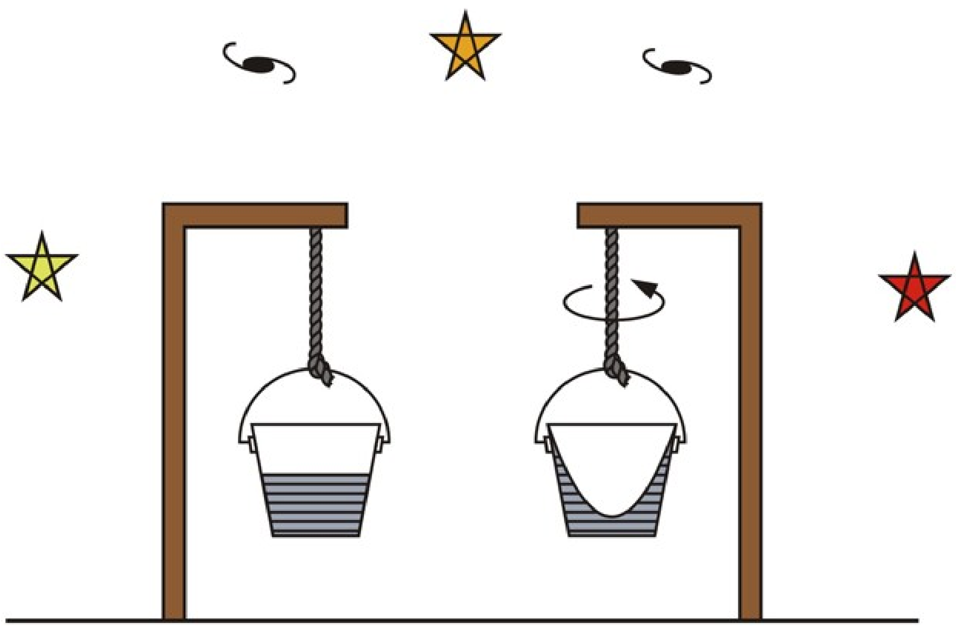
$$R = 1m, \quad M = 10^3 kg, \quad \frac{2GM}{Rc^2} = 10^{-24}$$

Spinning shell:



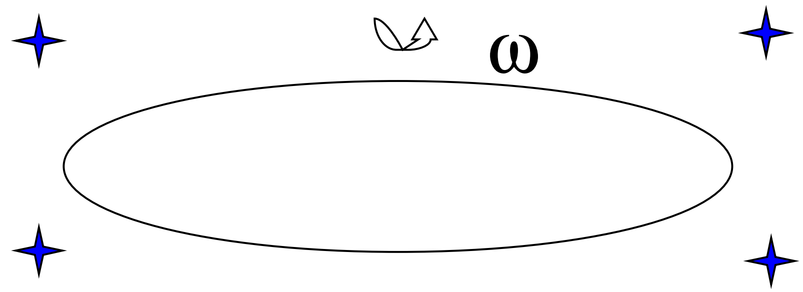
$$\vec{F} = -\frac{2H_gM_g}{Rc^2}m_g\left(\vec{a} + \vec{\omega} \times \vec{\omega} \times \vec{r} + 2\vec{v} \times \vec{\omega}\right)$$

Newton's bucket experiment



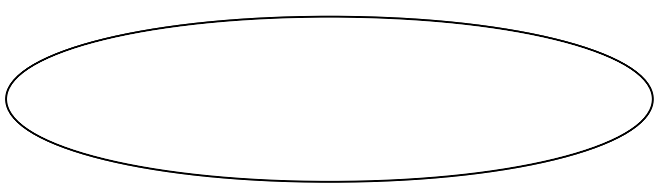
$$z_N = \frac{\omega_{WAS}^2}{2g} r^2$$

$$z_{RM} = \frac{\omega_{W*}^2}{2g} r^2$$

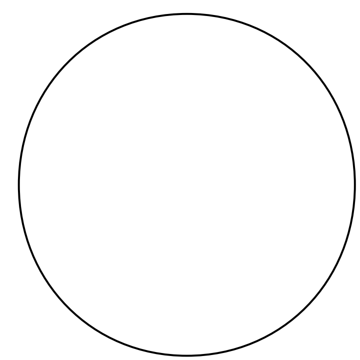


Carl Neumann (1869): “What would be the shape of the earth if all other astronomical bodies were annihilated?”

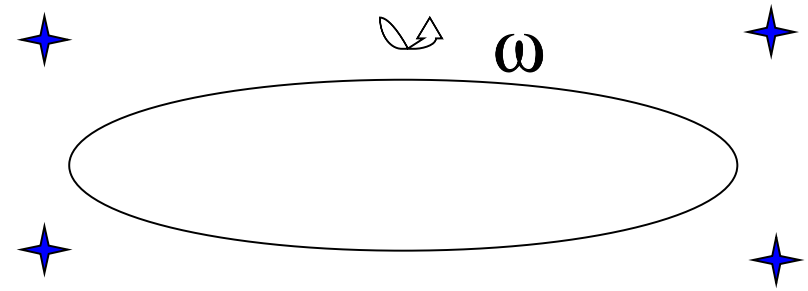
Newton, Einstein



Mach



Flattening of the Earth



$$\frac{D_E}{D_P} = 1 + \frac{15}{16\pi G}\frac{\omega_{EAS}^2}{\rho_E} = \frac{230}{229} = 1.004$$

$$\frac{D_E}{D_P} = 1 + 4\frac{\rho_*}{\rho_E}\frac{\omega_{E*}^2}{H_o^2} = \frac{230}{229} = 1.004$$

Conclusion

Relational Mechanics:

$$\sum \vec{F} = 0$$

$$\vec{F} = -H_g m_{1g} m_{2g} \frac{\hat{r}}{r^2}\left(1 - 3\frac{\dot{r}^2}{c^2} + 6\frac{r\ddot{r}}{c^2}\right)$$

Main Results

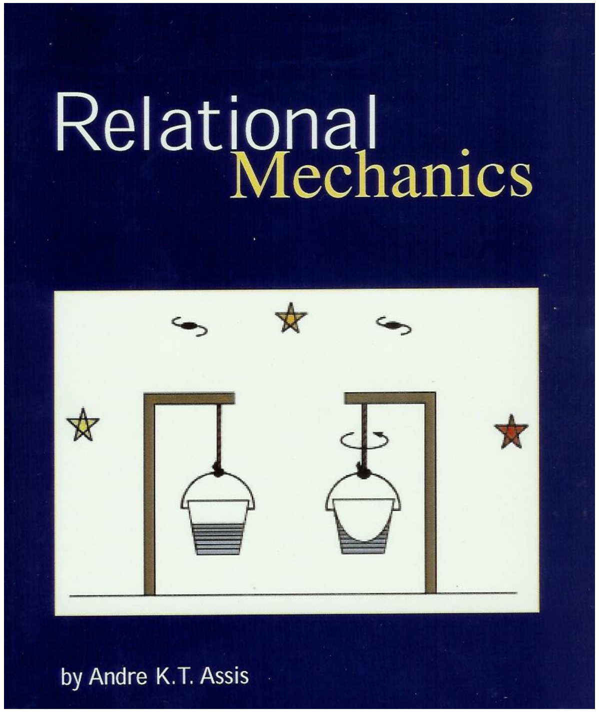
- Derivation of Newton's 2nd law: $F = m\,a$
- Derivation of equivalence principle: $m_i = m_g$
- Centrifugal and Coriolis forces as real forces of gravitational origin.
- Quantitative implementation of Mach's principle.

www.ifi.unicamp.br/~assis

Extras - Relational Mechanics

A. K. T. Assis

University of Campinas, Brazil



www.ifi.unicamp.br/~assis

Newton – 1687 – Principia – Definitions

- 1 - "The quantity of matter is the measure of the same, arising from the density and bulk conjointly."

$$m_i = \rho V$$

- 2 – "The quantity of motion is the measure of the same, arising from the velocity and quantity of matter conjointly."

$$\vec{p} = m_i \vec{v}$$

Newton – 1687 – Principia

"Axioms or Laws of Motion"

- "Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it."
- "The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed."
- "To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

“The power of gravity operates according to the quantity of the solid matter which they contain, and propagates its virtue on all sides to immense distances, decreasing always as the inverse square of the distances.”

$$\vec{F} = G\frac{m_{g1}m_{g2}}{r^2}\hat{r}$$

“Absolute, true and mathematical time, flows equably without relation to anything external.”

Leibniz (1646 – 1716)

- “Space is something merely relative, the order of coexistences.”
- “Time is something merely relative, the order of successions.”

Berkeley (1685 – 1753)

- “There is only relative motion, to conceive motion there must be at least two bodies.”

- “To determine motion it would be enough to bring in, instead of absolute space, relative space as confined to the heavens of the fixed stars.”

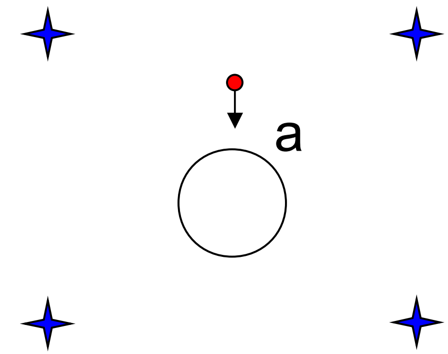
Leibniz-Clarke controversy (1715-1716):

Clarke: “It is affirmed (by Leibniz), that motion necessarily implies a relative change of situation in one body, with regard to other bodies: and yet no way is shown to avoid this absurd consequence, that then the mobility of one body depends on the existence of other bodies; and that any single body existing alone, would be incapable of motion; or that the parts of a circulating body, (suppose the sun,) would lose the *vis centrifuga* arising from their circular motion, if all the extrinsic matter around them were annihilated.”

Free fall in Relational Mechanics

$$F_E + F_* = 0$$

$$H_g \frac{m_g M_{gE}}{r^2} - \Phi\, m_g\, a = 0$$



$$a = \frac{H_g}{\Phi} \frac{M_{gE}}{r^2}$$ with $$\frac{H_g}{\Phi} = \frac{H_o^2}{4\pi \rho_*} \approx 6.7 \times 10^{-11} \frac{Nm^2}{kg^2} = G$$

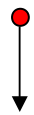
$$\frac{a_{mU}}{a_*} = \frac{1}{3} \frac{M_{gE}}{M_{g*}} \frac{R_*^2}{r^2}$$ with

$$R_* = c / H_o \approx 10^{26} m$$

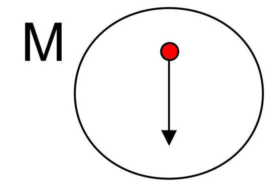
$$M_* = 4\pi R_*^3 \rho_* / 3 \approx 10^{52} kg$$

$$a_* = R_* H_0^2 \approx 6 \times 10^{-10} m / s^2$$

Experimental test



a = 9.8 m/s^2



a = ?

$$a_N = a_E = 9.8\, m/s^2$$

$$a_{RM} = g\left(1 - \frac{2GM}{Rc^2}\right)$$

$$R = 1m,\ M = 10^3\, kg,\ \frac{2GM}{Rc^2} = 10^{-24}$$

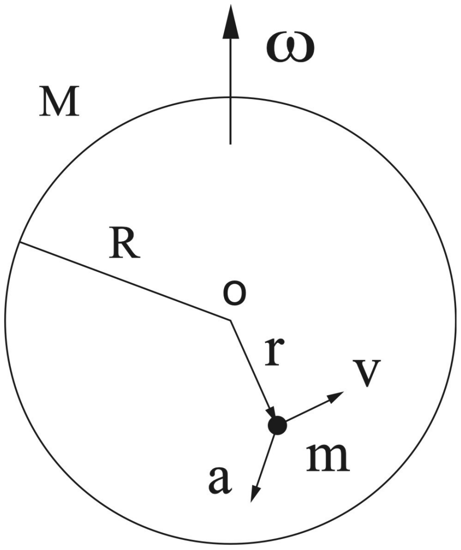
Einstein in *The Meaning of Relativity* ***(1922)***:

A model of interaction satisfying Mach's principle should lead to some consequences, namely

1) The inertial mass of a body should increase with the agglomeration of masses in its neighborhood.
2) A body in an otherwise empty universe should have no inertia.
3) A body should experience an acceleration if nearby bodies are accelerated. The accelerating force should be in the same direction as the acceleration of the later.
4) A rotating body should generate inside it a Coriolis force.

Relational Mechanics X General Relativity

Mass m inside a spinning shell:



$$\vec{F}_{RM} = -\frac{2H_g M_g}{Rc^2} m_g \left(\vec{a} + \vec{\omega} \times \vec{\omega} \times \vec{r} + 2\vec{v} \times \vec{\omega}\right)$$

$$\vec{F}_{GR} = -\frac{4GM_g}{15Rc^2} m_g \left(\vec{\omega} \times \vec{\omega} \times \vec{r} + 10\vec{v} \times \vec{\omega} + 2(\vec{r} \cdot \vec{\omega})\vec{\omega}\right)$$

Weber's force

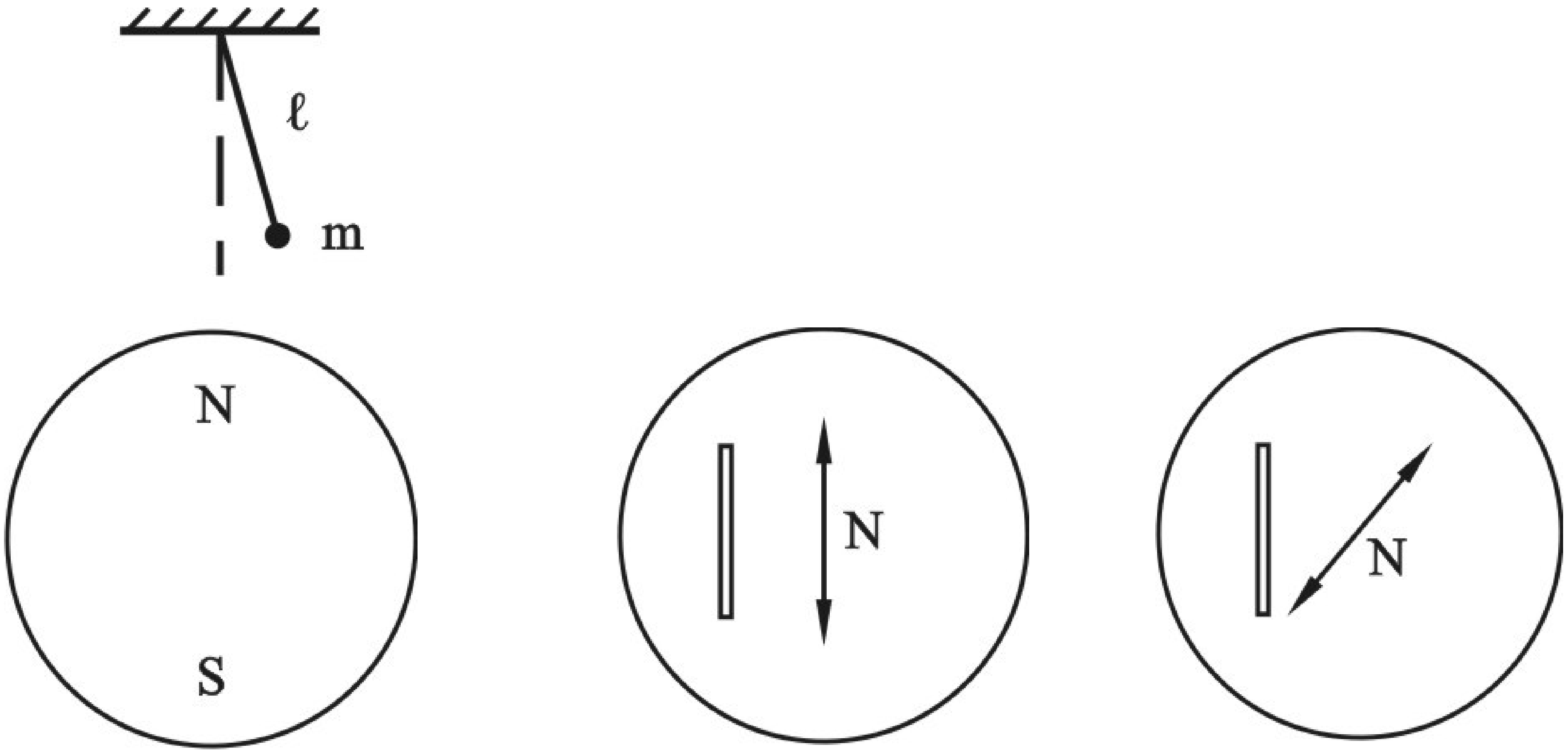
$$\vec{F} = \frac{q_1 q_2}{4\pi\varepsilon_0} \frac{\hat{r}}{r^2} \left(1 - \frac{\dot{r}^2}{2c^2} + \frac{r\,\ddot{r}}{c^2} \right)$$

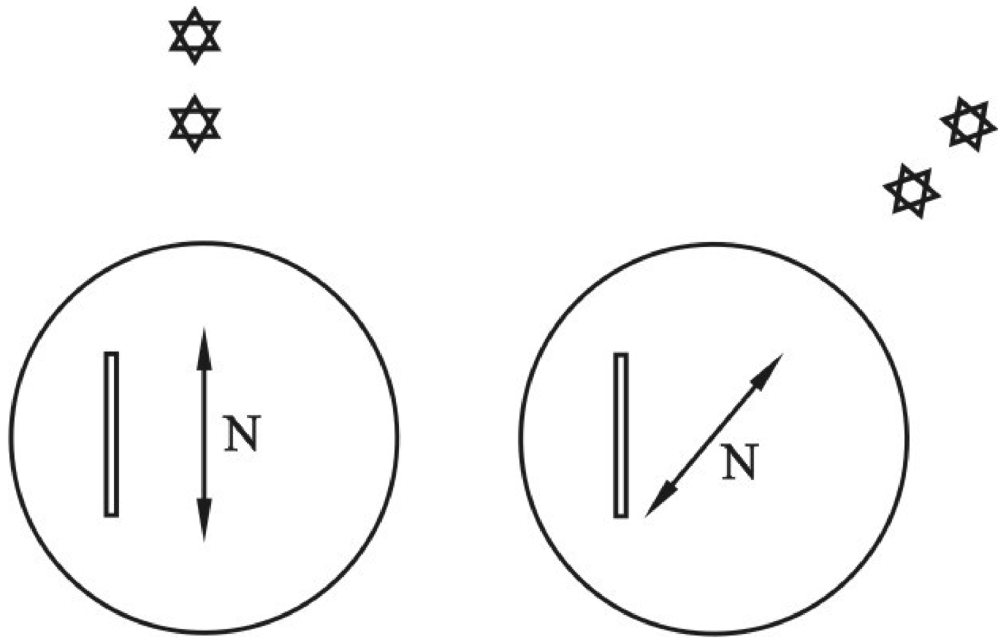
$$\dot{r} = \frac{dr}{dt}$$

$$\ddot{r} = \frac{d^2 r}{dt^2}$$

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 3 \times 10^8 \frac{m}{s}$$

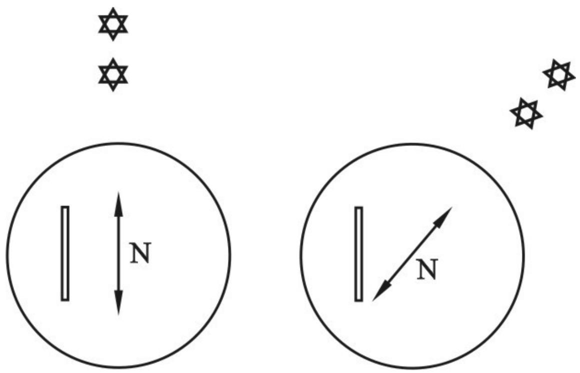
Foucault’s pendulum (1851)





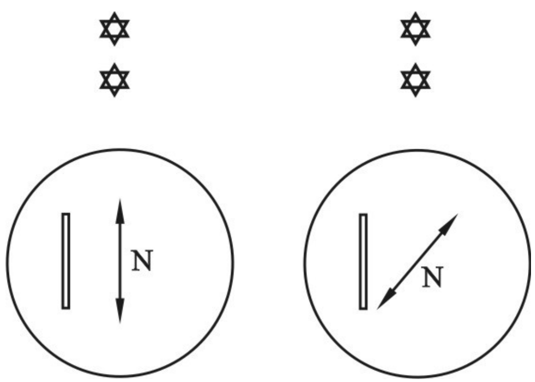
$$T_P = T_* = 24 \text{ hours}$$

Coincidence?



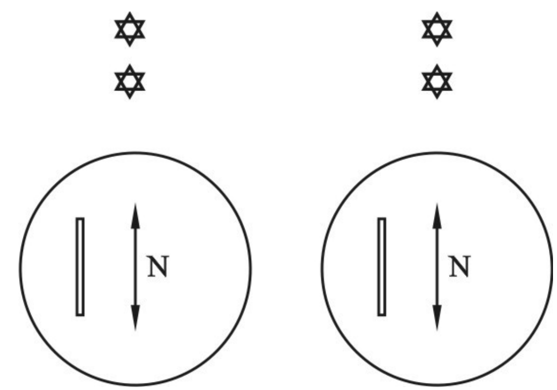
What would happen to the pendulum if we stopped all the stars?

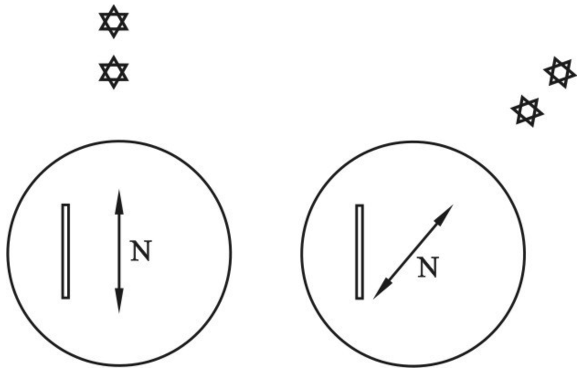
Newton



T_P = 24 hours

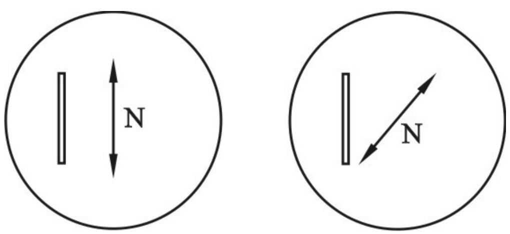
Mach





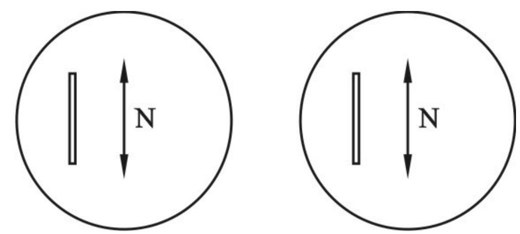
What would happen to the pendulum if all the stars were annihilated?

Newton

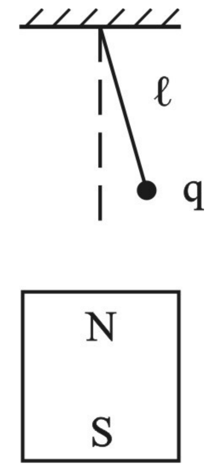


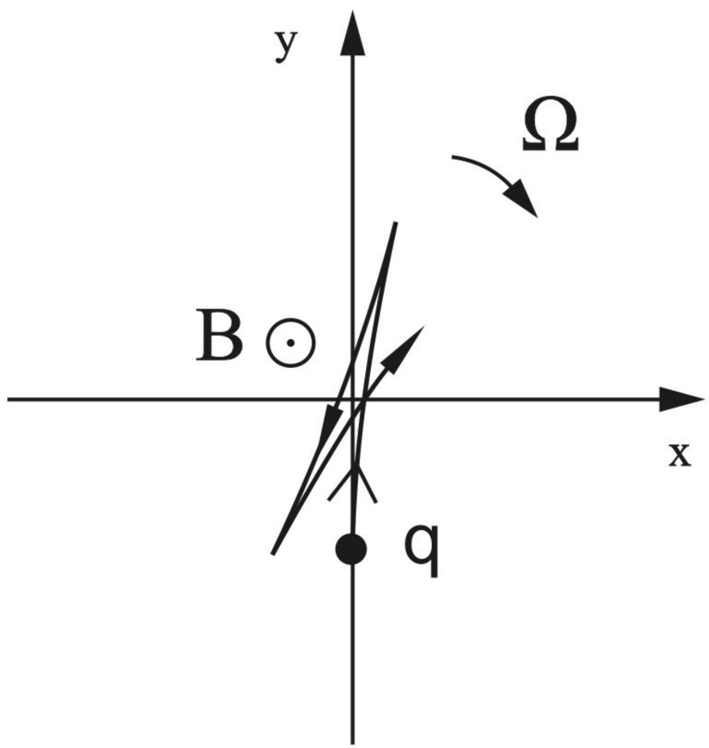
T_P = 24 hours

Mach



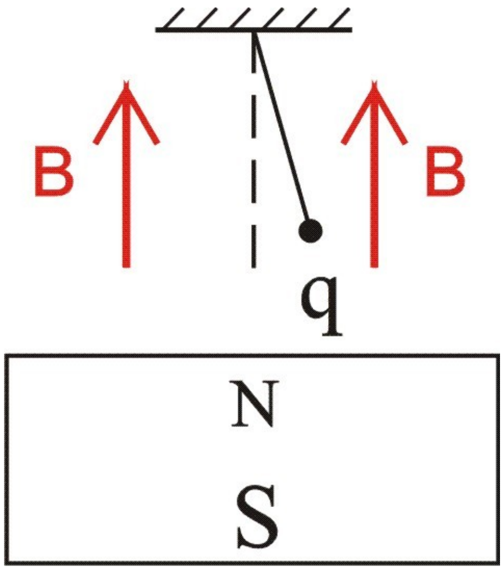
Charged pendulum in a magnetic field:

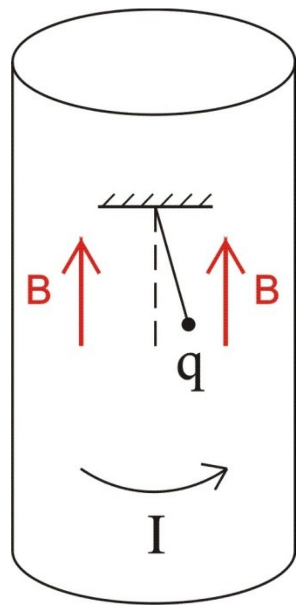


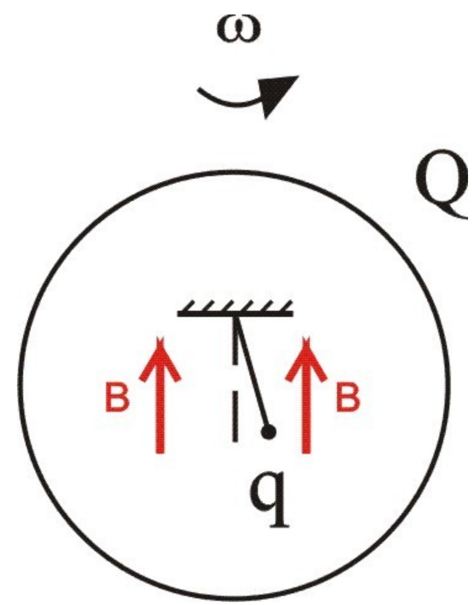


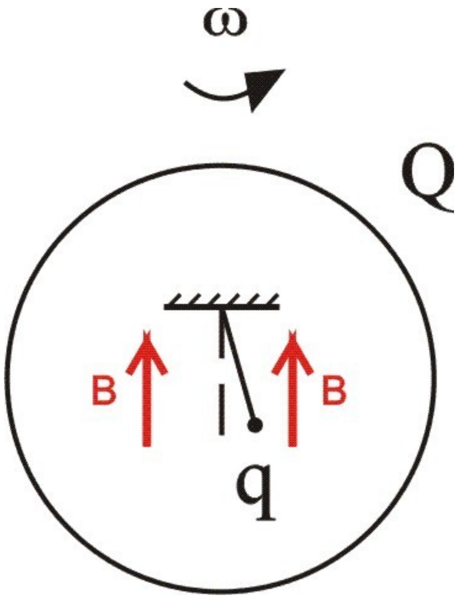
$$\vec{F} = q\vec{v} \times \vec{B}$$

$$\Omega = \frac{qB}{2m}$$

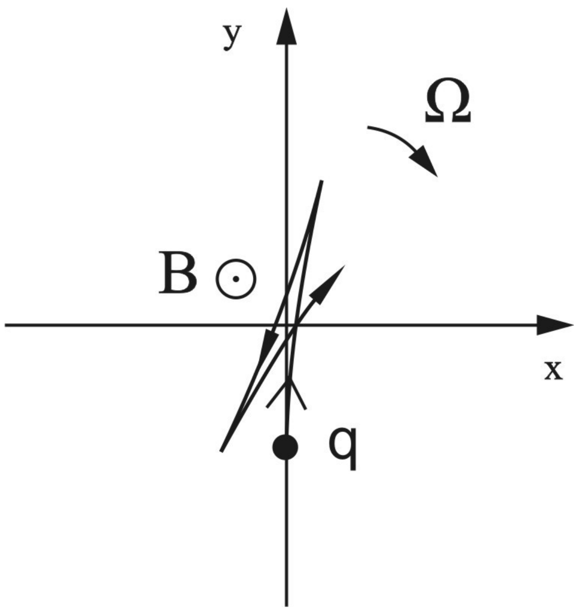

B
B
q
N
S


B
B
q
I


ω
Q
B
B
q



$$B = \frac{\mu_o Q\omega}{6R}$$



$$\Omega = \frac{qB}{2m} = \frac{q}{2m}\frac{\mu_o Q\omega}{6R}$$

Newton's bucket in Relational Mechanics:

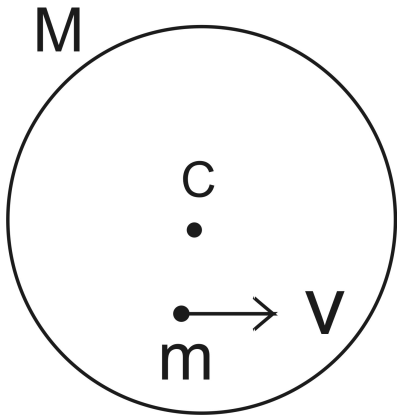
$$z_{RM} = \frac{\omega_{W*}^2}{2g} r^2 = \frac{4\pi \rho_*}{H_o^2} \frac{R_E^2 \omega_{W*}^2}{2M_E} r^2$$

$$z_N = \frac{\omega_{WAS}^2}{2g} r^2 = \frac{R_E^2 \omega_{WAS}^2}{2GM_E} r^2$$

E. Schrödinger, Ann. Phys. **77**, 325 (1925)

$$V = -\frac{H_g m_1 m_2}{r}\left(1 - \frac{3\dot{r}^2}{c^2}\right)$$

After integration for a test body inside a shell:



$$V = -\frac{H_g M m}{R}\left(1 - \frac{v^2}{c^2}\right)$$

After integration over the known Universe:

$$V = \Phi\left(\frac{mv^2}{2} - \frac{mc^2}{2}\right)$$

$$V = -\frac{H_g m_1 m_2}{r}\left(1 - \frac{2}{3}\frac{1}{\left(1 - \dot{r}^2/c^2\right)^{3/2}}\right)$$

After integration over the known Universe:

$$V = -\frac{2\pi \; H_g \rho_*}{H_o^2}\left(\frac{mc^2}{\sqrt{1 - v^2/c^2}} - \frac{3}{2}mc^2\right)$$