Memoir of John Jacob Abel 1857–1938," in National Academy of Sciences, U.S.A., *Biographical Memoirs,* **24** (1946), 231–257. Several of Abel's own papers provide valuable synthetic and expository accounts of his work. See *Experimental and Chemical Studies of the Blood With an Appeal for More Extended Chemical Training for the Biological and Medical Investigator,* the first Mellon lecture, given at the University of Pittsburgh under the auspices of the Society for Biological Research (Pittsburgh, 1915), also in *Science,* **42** (1915), 135–147, 165–178; "Some Recent Advances in Our Knowledge of the Ductless Glands," in *Bulletin of the Johns Hopkins Hospital,* **38** (January 1926), 1–32; and "Chemistry in Relation to Biology and Medicine With Especial Reference to Insulin and Other Hormones," in *Science,* **66** (1927), 307–319, 337–346.

The most important source for Abel's life and scientific career is his papers, deposited at the Welch Medical Library, the Johns Hopkins University. This extensive collection, including correspondence, notebooks, and other memoranda, constitutes an important source of information on the development of American biochemistry and pharmacology as well as for the history of the specific research areas that concerned Abel.

II. Secondary Literature. There is no full-length biography of Abel available, but among the most useful of numerous biographical sketches are Carl Voegtlin, "John Jacob Abel. 1857–1938," in *Journal of Pharmacology and Experimental Therapeutics,* **67** (1939), 373–406, a detailed account of Abel's scientific work; H. H. Swain, E. M. K. Geiling, and A. Heingartner, "John Jacob Abel at Michigan. The Introduction of Pharmacology Into the Medical Curriculum," in *University of Michigan Medical Bulletin,* **29** (1963), 1–14; E. K. Marshall, Jr., "Abel the Prophet," in *The Johns Hopkins Magazine,* **1** (1950), 11–14; Paul D. Lamson, "John Jacob Abel—A Portrait," in *Bulletin of the Johns Hopkins Hospital,* **68** (1941), 119–157, an engagingly detailed personal portrait; McNider's "Biographical Memoir," cited above; H. H. Dale, "John Jacob Abel. 1857–1938," in *Obituary Notices of Fellows of the Royal Society,* **2** (1939), 577–581; and E. M. K. Geiling, "John Jacob Abel," in *Dictionary of American Biography,* XXII, Supp. 2 (New York, 1958), 4–5. *John Jacob Abel, M.D., Investigator, Teacher, Prophet. 1857–1938* (Baltimore, 1957) is a useful commemorative volume that includes the Lamson and Marshall sketches cited above, as well as a number of Abel's most important papers. In 1957 the Johns Hopkins University celebrated the centenary of Abel's birth with a symposium to which contributions were made by Torald Sollman, Samuel Amberg, Carl Voegtlin, L. G. Rowntree, E. K. Marshall, Jr., E. M. K. Geiling, and Warfield M. Firor; the proceedings appeared in the *Bulletin of the Johns Hopkins Hospital,* **101** (1957), 297–328. An excellent recent study of Abel's insulin work by Jane Murnaghan and Paul Talalay that succeeds in placing this research in the broader context of twentieth-century biochemistry is "John Jacob Abel and the Crystallization of Insulin," in *Perspectives in Biology and Medicine,* **10** (1967), 334–380.

Charles E. Rosenberg

ABEL

ABEL, NIELS HENRIK (*b.* Finnöy, an island near Stavanger, Norway, 5 August 1802; *d.* Froland, Norway, 6 April 1829), *mathematics.*

Abel's father, Sören Georg Abel, was a Lutheran minister and himself the son of a minister. He was a gifted and highly ambitious theologian, educated at the University of Copenhagen, which was at that time the only such institution in the united kingdom of Denmark-Norway. He had married Ane Marie Simonson, the daughter of a wealthy merchant and shipowner in the town of Risör, on the southern coast. Finnöy was the first parish for pastor Abel; it was small and toilsome, comprising several islands. The couple had seven children, six sons and a daughter; Niels Henrik was their second child.

In 1804 Sören Georg Abel was appointed successor to his father in the parish of Gjerstad, near Risör. The political situation in Norway was tense. Because of its alliance with Denmark the country had been thrown into the Napoleonic Wars on the side of France, and a British blockade of the coast created widespread famine. Pastor Abel was prominent in the nationalistic movement, working for the creation of separate Norwegian institutions—particularly a university and a national bank—if not for outright independence. At the conclusion of the peace treaty of Kiel, Denmark ceded Norway to Sweden. The Norwegians revolted and wrote their own constitution, but after a brief and futile war against the Swedes under Bernadotte, they were compelled to seek an armistice. A union with Sweden was accepted, and Abel's father became one of the members of the extraordinary Storting called in the fall of 1814 to write the necessary revision of the new constitution.

Niels Henrik Abel and his brothers received their first instruction from their father, but in 1815 Abel and his older brother were sent to the Cathedral School in Christiania (Oslo). This was an old school to which many public officials in the province sent their children; some fellowships were available. The Cathedral School had been excellent, but was then at a low ebb, because most of its good teachers had accepted positions at the new university, which began instruction in 1813.

Abel was only thirteen years old when he left home, and it seems probable that deteriorating family life expedited his departure. During the first couple of years his marks were only satisfactory; then the quality of his work declined. His brother fared even worse; he began to show signs of mental illness and finally had to be sent home.

In 1817 an event took place at the school that was destined to change Abel's life. The mathematics teacher mistreated one of the pupils, who died shortly

ABEL

afterward, possibly as a consequence of the punishment. The teacher was summarily dismissed and his place was taken by Bernt Michael Holmboe, who was only seven years older than Abel. Holmboe also served as an assistant to Christoffer Hansteen, professor of astronomy and the leading scientist at the university.

It did not take Holmboe long to discover young Abel's extraordinary ability in mathematics. He began by giving him special problems and recommending books outside the school curriculum. The two then started to study together the calculus texts of Euler, and later the works of the French mathematicians, particularly Lagrange and Laplace. So rapid was Abel's progress that he soon became the real teacher. From notebooks preserved in the library of the University of Oslo one sees that even in these early days he was already particularly interested in algebraic equation theory. By the time he finished school, he was familiar with most of the important mathematical literature. Holmboe was so delighted by the mathematical genius he had discovered that the rector of the school made him moderate his statements about Abel in the record book. But the professors at the university were well informed by Holmboe about the promising young man and made his personal acquaintance. Besides Hansteen, who also taught applied mathematics, there was only one professor of mathematics, Sören Rasmussen, a former teacher at the Cathedral School. Rasmussen, a kindly man, was not a productive scholar; his time was largely taken up by tasks assigned to him by government, particularly in his post as an administrator of the new Bank of Norway.

During his last year at school Abel, with the vigor and immodesty of youth, attacked the problem of the solution of the quintic equation. This problem had been outstanding since the days of del Ferro, Tartaglia, Cardano, and Ferrari in the first half of the sixteenth century. Abel believed that he had succeeded in finding the form of the solution, but in Norway there was no one capable of understanding his arguments, nor was there any scientific journal in which they could be published. Hansteen forwarded the paper to the Danish mathematician Ferdinand Degen, requesting its publication by the Danish Academy.

Degen could not discover any fault in the arguments, but requested that Abel illustrate his method by an example. Degen also found the topic somewhat sterile and suggested that Abel turn his attention to a topic "whose development would have the greatest consequences for analysis and mechanics. I refer to the elliptic transcendentals [elliptic integrals]. A serious investigator with suitable qualifications for research of this kind would by no means be restricted to the many beautiful properties of these most remarkable functions, but could discover a Strait of Magellan leading into wide expanses of a tremendous analytic ocean" (letter to Hansteen).

Abel began constructing his examples for the solution of the fifth-degree equation, but discovered to his dismay that his method was not correct. He also followed Degen's suggestion about the elliptic transcendentals, and it is probable that within a couple of years he had in the main completed his theory of the elliptic functions.

In 1818 pastor Abel was reelected to the Storting, after an unsuccessful bid in 1816. But his political career ended in tragedy. He made violent unfounded charges against other representatives and was threatened with impeachment. This, together with his drunkenness, made him the butt of the press. He returned home in disgrace, a disillusioned man. Both he and his wife suffered from alcoholism, and the conditions at the vicarage and in the parish became scandalous. It was generally considered a relief when he died in 1820. His widow was left in very straitened circumstances, with a small pension barely sufficient to support her and her many children.

The penniless Abel entered the university in the fall of 1821. He was granted a free room at the university dormitory and received permission to share it with his younger brother Peder. But the new institution had no fellowship funds, and some of the professors took the unusual measure of supporting the young mathematician out of their own salaries. He was a guest in their houses and became particularly attracted to the Hansteen home, and to Mrs. Hansteen and her sisters.

Abel's first task at the university was to satisfy the requirements for the preliminary degree, *Candidatus Philosophiae.* Once this was achieved, after a year, Abel was entirely on his own in his studies. There were no advanced courses in mathematics and the physical sciences, but this does not seem to have been a handicap; in a letter from Paris a little later he stated that he had read practically everything in mathematics, important or unimportant.

He devoted his time to advanced research and his efforts received a strong impetus when Hansteen started a scientific periodical, *Magazin for Naturvidenskaben.* In 1823 this journal published Abel's first article, in Norwegian, a study of functional equations. Mathematically it was not important, nor was his second little paper. The subscribers to the magazine had been promised a popular review, however, and Hansteen, probably after criticism, felt obliged to apologize for the character of these papers: "Thus

ABEL

I believe that the *Magazin* in addition to scientific materials should also further the tools serving for their analysis. It will be reckoned to our credit that we have given the learned public an opportunity to become acquainted with a work from the pen of this talented and skillful author" (*Magazin,* **1**). Abel's next paper, "Opläsning afet Par Opgaver ved bjoelp af bestemte Integraler" ("Solution of Some Problems by Means of Definite Integrals"), is of importance in the history of mathematics, since it contains the first solution of an integral equation. The paper, which went unnoticed at the time, in part because it was in Norwegian, deals with the mechanical problem of the motion of a mass point on a curve under the influence of gravitation. During the winter of 1822–1823 Abel also composed a longer work on the integration of functional expressions. The paper was submitted to the university Collegium in the hope that that body would assist in its publication. The manuscript has disappeared, but it seems likely that some of the results obtained in it are included in some of Abel's later papers.

Early in the summer of 1823 Abel received a gift of 100 daler from Professor Rasmussen to finance a trip to Copenhagen to meet Degen and the other Danish mathematicians. His letters to Holmboe reveal the mathematical inspiration that he received. He stayed in the house of his uncle and here made the acquaintance of his future fiancée, Christine Kemp.

Upon his return to Oslo, Abel again took up the question of the solution of the quintic equation. This time he took the reverse view and succeeded in solving the centuries-old problem by proving the impossibility of a radical expression that represents a solution of the general fifth- or higher-degree equation. Abel fully realized the importance of his result, so he had it published, at his own expense, by a local printer. To reach a larger audience, he wrote it in French: "Mémoire sur les équations algébriques où on démontre l'impossibilité de la résolution de l'équation générale du cinquième degré." To save expense the whole pamphlet was compressed to six pages. The resulting brevity probably made it difficult to understand; at any rate, there was no reaction from any of the foreign mathematicians—including the great C. F. Gauss, to whom a copy was sent.

It had become clear that Abel could no longer live on the support of the professors. His financial problems had been increased by his engagement to Christine Kemp, who had come to Norway as a governess for the children of a family living near Oslo.

Abel applied for a travel grant, and after some delays the government decided that Abel should receive a small stipend to study languages at the university to prepare him for travel abroad. He was then to receive a grant of 600 daler for two years of foreign study.

Abel was disappointed at the delay but dutifully studied languages, particularly French, and used his time to prepare a considerable number of papers to be presented to foreign mathematicians. During the summer of 1825 he departed, together with four friends, all of whom also intended to prepare themselves for future scientific careers; one of them later became professor of medicine, and the three others became geologists. Abel's friends all planned to go to Berlin, while Abel, upon Hansteen's advice, was to spend his time in Paris, then the world's principal center of mathematics. Abel feared being lonely, however, and also decided to go to Berlin, although he well knew that he would incur the displeasure of his protector.

Abel's change of mind turned out to be a most fortunate decision. On passing through Copenhagen, Abel learned that Degen had died, but he secured a letter of recommendation from one of the other Danish mathematicians to Privy Councilor August Leopold Crelle. Crelle was a very influential engineer, intensely interested in mathematics although not himself a strong mathematician.

When Abel first called upon Crelle, he had some difficulty in making himself understood, but after a while Crelle recognized the unusual qualities of his young visitor. The two became lifelong friends. Abel presented him with a copy of his pamphlet on the quintic equation, but Crelle confessed that it was unintelligible to him and recommended that Abel write an expanded version of it. They talked about the poor state of mathematics in Germany. In a letter to Hansteen, dated from Berlin, 5 December 1825, Abel wrote:

> When I expressed surprise over the fact that there existed no mathematical journal, as in France, he said that he had long intended to edit one, and would presently bring his plan to execution. This project is now organized, and that to my great joy, for I shall have a place where I can get some of my articles printed. I have already prepared four of them, which will appear in the first number.

Journal für die reine und angewandte Mathematik, or *Crelle's Journal,* as it is commonly known, was the leading German mathematical periodical during the nineteenth century. The first volume alone contains seven papers by Abel and the following volumes contain many more, most of them of preeminent importance in the history of mathematics. Among the first is the expanded version of the proof of the

ABEL

impossibility of the solution of the general quintic equation by radicals. Here Abel develops the necessary algebraic background, including a discussion of algebraic field extensions. Abel was at this time not aware that he had a precursor, the Italian mathematician Paolo Ruffini. But in a posthumous paper on the equations which are solvable by radicals Abel states: "The only one before me, if I am not mistaken, who has tried to prove the impossibility of the algebraic [radical] solution of the general equations is the mathematician Ruffini, but his paper is so complicated that it is very difficult to judge on the correctness of his arguments. It seems to me that it is not always satisfactory." The result is usually referred to as the Abel-Ruffini theorem.

After Abel's departure from Oslo an event took place that caused him much concern. Rasmussen had found his professorship in mathematics too burdensome when combined with his public duties. He resigned, and shortly afterward the faculty voted to recommend that Holmboe be appointed to fill the vacancy. Abel's Norwegian friends found the action highly unjust, and Abel himself probably felt the same way. Nevertheless, he wrote a warm letter of congratulation to his former teacher, and they remained good friends. But it is evident that from this moment Abel worried about his future and his impending marriage; there was no scientific position in sight for him in his home country.

During the winter in Berlin, Abel contributed to *Crelle's Journal*; among the notable papers are one on the generalization of the binomial formula and another on the integration of square root expressions. But one of his main mathematical concerns was the lack of stringency in contemporary mathematics. He mentioned it repeatedly in letters to Holmboe. In one of these, dated 16 January 1826, he wrote:

> My eyes have been opened in the most surprising manner. If you disregard the very simplest cases, there is in all of mathematics not a single infinite series whose sum has been stringently determined. In other words, the most important parts of mathematics stand without foundation. It is true that most of it is valid, but that is very surprising. I struggle to find the reason for it, an exceedingly interesting problem.

A result of this struggle was his classic paper on power series which contains many general theorems and also, as an application, the stringent determination of the sum of the binomial series for arbitrary real or complex exponents.

During the early spring of 1826, Abel felt obliged to proceed to his original destination, Paris. Crelle had promised to accompany him, and on the way they intended to stop in Göttingen to visit Gauss. Unfortunately, pressure of business prevented Crelle from leaving Berlin. At the same time, Abel's Norwegian friends were planning a geological excursion through central Europe, and, again reluctant to be separated from them, he joined the group. They traveled by coach through Bohemia, Austria, northern Italy, and the Alps. Abel did not reach Paris until July, low on funds after the expensive trip.

The visit to Paris was to prove disappointing. The university vacations had just begun when Abel arrived, and the mathematicians had left town. When they returned, he found that they were aloof and difficult to approach; it was only in passing that he met Legendre, whose main interest in his old age was elliptic integrals, Abel's own specialty. For presentation to the French Academy of Sciences Abel had reserved a paper that he considered his masterpiece. It dealt with the sum of integrals of a given algebraic function. Abel's theorem states that any such sum can be expressed as a fixed number p of these integrals, with integration arguments that are algebraic functions of the original arguments. The minimal number p is the genus of the algebraic function, and this is the first occurrence of this fundamental quantity. Abel's theorem is a vast generalization of Euler's relation for elliptic integrals.

Abel spent his first months in Paris completing his great memoir; it is one of his longest papers and includes a broad theory with applications. It was presented to the Academy of Sciences on 30 October 1826, under the title "Mémoire sur une propriété générale d'une classe très-étendue de fonctions transcendantes." Cauchy and Legendre were appointed referees, Cauchy being chairman. A number of young men had gained quick distinction upon having their works accepted by the Academy, and Abel awaited the referees' report. No report was forthcoming, however; indeed, it was not issued until Abel's death forced its appearance. Cauchy seems to have been to blame; he claimed later that the manuscript was illegible.

Abel's next two months in Paris were gloomy; he had little money and few acquaintances. He met P. G. L. Dirichlet, his junior by three years and already a well-known mathematician, through a paper in the Academy sponsored by Legendre. Another acquaintance was Frédéric Saigey, editor of the scientific revue *Ferrusac's Bulletin,* for whom Abel wrote a few articles, particularly about his own papers in *Crelle's Journal.* After Christmas he spent his last resources to pay his fare to Berlin.

Shortly after his return to Berlin, Abel fell ill; he seems to have then suffered the first attack of the

ABEL

tuberculosis that was later to claim his life. He borrowed some money from Holmboe, and Crelle probably helped him. Abel longed to return to Norway but felt compelled to remain abroad until his fellowship term had expired. Crelle tried to keep him in Berlin until he could find a position for him at a German university; in the meantime he offered him the editorship of his *Journal.*

Abel worked assiduously on a new paper: "Recherches sur les fonctions elliptiques," his most extensive publication (125 pages in the *Oeuvres complètes*). In this work he radically transformed the theory of elliptic integrals to the theory of elliptic functions by using their inverse functions corresponding in the most elementary case to the duality

$$y = \arcsin x = \int \frac{dx}{\sqrt{1 - x^2}} \quad x = \sin y.$$

The elliptic functions thereby become a vast and natural generalization of the trigonometric functions; in the wake of Abel's work they were to constitute one of the favorite research topics in mathematics during the nineteenth century. Abel had already developed most of the theory as a student in Oslo, so he was able to present the theory of elliptic functions with a great richness of detail, including double periodicity, expansions in infinite series and products, and addition theorems. The theory led to the expressions for functions of a multiple of the argument with the concomitant determination of the equations for fractional arguments and their solution by radicals, much in the way that Gauss had treated the cyclotomic equations; Abel's letters to Holmboe (from Paris in December 1826 and from Berlin on 4 March 1827) indicate that he was particularly fascinated by a determination of the condition for a lemniscate to be divisible into equal parts by means of compass and ruler, analogous to Gauss's construction of regular polygons. The last part deals with the so-called theory of complex multiplication, later so important in algebraic number theory.

Abel returned to Oslo on 20 May 1827, to find that the situation at home was as gloomy as he had feared. He had no position in prospect, no fellowship, and an abundance of debts. His application to have his fellowship prolonged was turned down by the Department of Finance, but the university courageously awarded him a small stipend out of its meager funds. This action was criticized by the department, which reserved the right to have the amount deducted from any future salary he might receive.

Abel's fiancée found a new position with friends of Abel's family, the family of the owner of an ironworks at Froland, near Arendal. During the fall Abel eked out a living in Oslo by tutoring schoolboys and probably with the help of friends. At the new year the situation became brighter. Hansteen, a pioneer in geomagnetic studies, received a large grant for two years to examine the earth's magnetic field in unexplored Siberia. In the meantime Abel became his substitute both at the university and at the Norwegian Military Academy.

The first part of the "Recherches" was published in *Crelle's Journal* in September 1827, and Abel completed the second part during the winter. He lived in isolation at Oslo; there was no package mail during the winter, and he had no inkling of the interest his memoir had created among European mathematicians. Nor did he know that a competitor had appeared in the field of elliptic functions until early in 1828, when Hansteen showed him the September issue of the *Astronomische Nachrichten.* In this journal a young German mathematician, K. G. J. Jacobi, announced without proofs some results concerning the transformation theory of elliptic integrals. Abel hurriedly added a note to the manuscript of the second part of the "Recherches," showing how Jacobi's results were the consequence of his own.

Abel was keenly aware that a race was at hand. He interrupted a large paper on the theory of equations that was to contain the determination of all equations that can be solved by radicals; the part that was published contained the theory of those equations that are now known as Abelian. He then wrote, in rapid succession, a series of papers on elliptic functions. The first was "Solution d'un problème général concernant la transformation des fonctions elliptiques." This, his direct response to Jacobi, was published in *Astronomische Nachrichten*; the others appeared in *Crelle's Journal.* In addition, Abel prepared a book-length memoir, "Précis d'une théorie des fonctions elliptiques," which was published after his death. Jacobi, on the other hand, wrote only brief notices which did not reveal his methods; these were reserved for his book, *Fundamenta nova theoriae functionum ellipticarum* (1829).

Much has been written about the early theory of elliptic functions. There seems to be little doubt that Abel was in possession of the ideas several years before Jacobi. On the other hand, it is also an established fact that Gauss, although publishing nothing, had discovered the principles of elliptic functions long before either Abel or Jacobi.

The European mathematicians watched with fascination the competition between the two young mathematicians. Legendre noticed Jacobi's announcements and also received a letter from him. In a

meeting of the French Academy in November 1827, he praised the new mathematical star; the speech was reproduced in the newspapers and Legendre sent the clipping to Jacobi. In his reply Jacobi, after expressing his thanks, pointed out Abel's "Recherches" and its general results. Legendre responded: "Through these works you two will be placed in the class of the foremost analysts of our times." He also expressed his disappointment over Jacobi's method of publication and was irritated when Jacobi confessed that in order to derive some of his results he had had to rely on Abel's paper. About this time also, Abel began a correspondence with Legendre and poured out his ideas to him.

All that the European mathematicians knew about Abel's condition in Norway was that he had only a temporary position and had recently been compelled to tutor schoolboys to make a living. The main source of their information was Crelle, who constantly used his influence to try to obtain an appointment for Abel at a new scientific institute to be created in Berlin. Progress was very slow, however. In September 1828 four prominent members of the French Academy of Sciences took the extraordinary step of addressing a petition directly to Bernadotte, now Charles XIV of Norway-Sweden, calling attention to Abel and urging that a suitable scientific position be created for him. In a meeting of the Academy, on 25 February 1829, Legendre also paid tribute to Abel and his discoveries, particularly to his results in the theory of equations.

In the meantime Abel, in spite of his deteriorating health, wrote new papers frantically. He spent the summer vacation of 1828 on the Froland estate with his fiancée. At Christmas he insisted on visiting her again, notwithstanding that it required several days' travel in intense cold. He was feverish when he arrived, but enjoyed the family Christmas celebration. He may have had a premonition that his days were numbered, however, and he now feared that the great paper submitted to the French Academy had been lost forever. He therefore wrote a brief note, "Demonstration d'une propriété générale d'une certaine classe de fonctions transcendantes," in which he gave a proof of the main theorem. He mailed it to Crelle on 6 January 1829.

While waiting for the sled that was to return him to Oslo, Abel suffered a violent hemorrhage; the doctor diagnosed his illness as tuberculosis and ordered prolonged bed rest. He died in April, at the age of twenty-six, and was buried at the neighboring Froland church during a blizzard. The grave is marked by a monument erected by his friends. One of them, Baltazar Keilhau, wrote to Christine Kemp, without ever having seen her, and made her an offer of marriage which she accepted. Two days after Abel's death Crelle wrote jubilantly to inform him that his appointment in Berlin had been secured.

On 28 June 1830, the French Academy of Sciences awarded its Grand Prix to Abel and Jacobi for their outstanding mathematical discoveries. After an intensive search in Paris the manuscript of Abel's great memoir was rediscovered. It was published in 1841, fifteen years after it had been submitted. During the printing it again disappeared, not to reappear until 1952 in Florence.

Crelle wrote an extensive eulogy of Abel in his *Journal* (**4** [1829], 402):

> All of Abel's works carry the imprint of an ingenuity and force of thought which is unusual and sometimes amazing, even if the youth of the author is not taken into consideration. One may say that he was able to penetrate all obstacles down to the very foundations of the problems, with a force which appeared irresistible; he attacked the problems with an extraordinary energy; he regarded them from above and was able to soar so high over their present state that all the difficulties seemed to vanish under the victorious onslaught of his genius. . . . But it was not only his great talent which created the respect for Abel and made his loss infinitely regrettable. He distinguished himself equally by the purity and nobility of his character and by a rare modesty which made his person cherished to the same unusual degree as was his genius.

BIBLIOGRAPHY

I. Original Works. Abel's complete works are published in two editions, *Oeuvres complètes de N. H. Abel, mathématicien,* ed. and annotated by B. Holmboe (Oslo, 1839), and *Nouvelle édition,* M. M. L. Sylow and S. Lie, eds., 2 vols. (Oslo, 1881).

II. Secondary Literature. Materials on Abel's life include *Niels Henrik Abel: Mémorial publié à l'occasion du centenaire de sa naissance* (Oslo, 1902) which comprises all letters cited in the text; and O. Ore, *Niels Henrik Abel; Mathematician Extraordinary* (Minneapolis, Minn., 1957).

Oystein Ore

ABEL, OTHENIO (*b.* Vienna, Austria, 20 June 1875; *d.* Pichl am Mondsee, Austria, 4 July 1946), *paleontology, paleobiology.*

Abel's greatest scientific achievement, the founding of paleobiology, undoubtedly grew out of his background. For several generations his ancestors on his father's side had been gardeners. His grandfather had taken his examination in botany under Nicolaus Jaquin and had received *summa cum eminentia;* his