origin (theology, grammar, state administration, poetry, and others); and the second analyzes those imported from the Hellenic world. In the latter part much information on the history of the sciences can be found, for it contains chapters on philosophy, logic, medicine, arithmetic, geometry, astronomy, music, mechanics, and alchemy. Of special note is the author's interest in giving the correct etymologies of the terms he defines, their equivalents in Persian or Greek, and, in some cases, numerical examples (for example, when speaking of *jabr* and *muqābala*) that avoid misunderstandings.

Al-Khuwārizmī rarely states his sources and, when they do appear, they are not, in the case of scientific subjects, the best ones. Yet the latter were undoubtedly known to him; otherwise the good information that he does transmit could not be explained. On the other hand, he seems to have some points of contact with the *Rasā'il* ("Epistles") of the Ikhwān al-Ṣafā'.

BIBLIOGRAPHY

An inventory of al-Khuwārizmī's MSS is in C. Brockelmann, *Geschichte der arabischen Literatur*, I (Weimar, 1898), 282, and supp. I (Leiden, 1944), 434. The text has been published by B. Carra de Vaux under the title *Liber Mafātīḥ al-olum explicans vocabula technica scientiarum* (Leiden, 1895; repr. 1968). Many chs. of the second part have been trans. by E. Wiedemann and others. Details of these trans. can be found in C. E. Bosworth, "A Pioneer Arabic Encyclopedia of the Sciences: Al-Khuwārizmī, Keys of the Sciences," in *Isis*, **54** (1963), 97–111. See also G. Sarton, *Introduction to the History of Science*, I (Baltimore, 1927), 659–660; and E. Wiedemann, "Al-Khuwārizmī," in *Encyclopédie de l'Islam*, II (Leiden–Paris, 1927), 965.

J. VERNET

AL-KHWĀRIZMĪ, ABŪ JA'FAR MUḤAMMAD IBN MŪSĀ (*b.* before 800; *d.* after 847), *mathematics, astronomy, geography*.

Only a few details of al-Khwārizmī's life can be gleaned from the brief notices in Islamic bibliographical works and occasional remarks by Islamic historians and geographers. The epithet "al-Khwārizmī" would normally indicate that he came from Khwārizm (Khorezm, corresponding to the modern Khiva and the district surrounding it, south of the Aral Sea in central Asia). But the historian al-Ṭabarī gives him the additional epithet "al-Quṭrubbullī," indicating that he came from Quṭrubbull, a district between the Tigris and Euphrates not far from Baghdad,[1] so perhaps his ancestors, rather than he himself, came from Khwārizm; this interpretation is confirmed by some sources which state that his "stock" (*aṣl*) was from Khwārizm.[2] Another epithet given to him by al-Ṭabarī, "al-Majūsī," would seem to indicate that he was an adherent of the old Zoroastrian religion. This would still have been possible at that time for a man of Iranian origin, but the pious preface to al-Khwārizmī's *Algebra* shows that he was an orthodox Muslim, so al-Ṭabarī's epithet could mean no more than that his forebears, and perhaps he in his youth, had been Zoroastrians.

Under the Caliph al-Ma'mūn (reigned 813–833) al-Khwārizmī became a member of the "House of Wisdom" (Dār al-Ḥikma), a kind of academy of scientists set up at Baghdad, probably by Caliph Harūn al-Rashīd, but owing its preeminence to the interest of al-Ma'mūn, a great patron of learning and scientific investigation. It was for al-Ma'mūn that al-Khwārizmī composed his astronomical treatise, and his *Algebra* also is dedicated to that ruler. We are told that in the first year of his reign (842) Caliph al-Wāthiq sent al-Khwārizmī on a mission to the chief of the Khazars, who lived in the northern Caucasus.[3] But there may be some confusion in the source here with another "Muḥammad ibn Mūsā the astronomer," namely, one of the three Banū Mūsā ibn Shākir. It is almost certain that it was the latter, and not al-Khwārizmī, who was sent, also by al-Wāthiq, to the Byzantine empire to investigate the tomb of the Seven Sleepers at Ephesus.[4] But al-Khwārizmī survived al-Wāthiq (*d.* 847), if we can believe the story of al-Ṭabarī that he was one of a group of astronomers, summoned to al-Wāthiq's sickbed, who predicted on the basis of the caliph's horoscope that he would live another fifty years and were confounded by his dying in ten days.

All that can be said concerning the date and order of composition of al-Khwārizmī's works is the following. The *Algebra* and the astronomical work, as we have seen, were composed under al-Ma'mūn, in the earlier part of al-Khwārizmī's career. The treatise on Hindu numerals was composed after the *Algebra*, to which it refers. The treatise on the Jewish calendar is dated by an internal calculation to 823–824. The *Geography* has been tentatively dated by Nallino ("al-Khuwārizmī," p. 487) to soon after 816–817, since one of the localities it mentions is Qiman, an Egyptian village of no importance whatever except that a battle was fought there in that year; but the inference is far from secure. The *Chronicle* was composed after 826, since al-Ṭabarī quotes it as an authority for an event in that year.[5]

The *Algebra* is a work of elementary practical mathematics, whose purpose is explained by the author

AL-KHWĀRIZMĪ

(Rosen trans., p. 3) as providing "what is easiest and most useful in arithmetic, such as men constantly require in cases of inheritance, legacies, partition, lawsuits, and trade, and in all their dealings with one another, or where the measuring of lands, the digging of canals, geometrical computations, and other objects of various sorts and kinds are concerned." Indeed, only the first part of the work treats of algebra in the modern sense. The second part deals with practical mensuration, and the third and longest with problems arising out of legacies. The first part (the algebra proper) discusses only equations of the first and second degrees. According to al-Khwārizmī, all problems of the type he proposes can be reduced to one of six standard forms. These are (here, as throughout, we use modern notation, although al-Khwārizmī's exposition is always rhetorical) the following:

(1) $ax^2 = bx$

(2) $ax^2 = b$

(3) $ax = b$

(4) $ax^2 + bx = c$

(5) $ax^2 + c = bx$

(6) $ax^2 = bx + c,$

where a, b, and c are positive integers. Such an elaboration of cases is necessary because he does not recognize the existence of negative numbers or zero as a coefficient. He gives rules for the solution of each of the six forms—for instance, form (6) is solved by

$$x^2 = (b/a)\,x + c/a,$$

$$x = \sqrt{\left[\frac{1}{2}\left(\frac{b}{a}\right)\right]^2 + \frac{c}{a}} + \frac{1}{2}\left(\frac{b}{a}\right).$$

He also explains how to reduce any given problem to one of these standard forms. This is done by means of the two operations *al-jabr* and *al-muqābala*. *Al-jabr*, which we may translate as "restoration" or "completion," refers to the process of eliminating negative quantities. For instance, in the problem illustrating standard form (1) (Rosen trans., p. 36), we have

$$x^2 = 40x - 4x^2.$$

By "completion" this is transformed to

$$5x^2 = 40x.$$

Al-muqābala, which we may translate as "balancing," refers to the process of reducing positive quantities of the same power on both sides of the equation. Thus, in the problem illustrating standard form (5) (Rosen trans., p. 40), we have

$$50 + x^2 = 29 + 10x.$$

By *al-muqābala* this is reduced to

$$21 + x^2 = 10x.$$

These two operations, combined with the arithmetical operations of addition, subtraction, multiplication, and division (which al-Khwārizmī also explains in their application to the various powers), are sufficient to solve all types of problems propounded in the *Algebra*. Hence they are used to characterize the work, whose full title is *al-Kitāb al-mukhtaṣar fī ḥisāb al-jabr waʾl-muqābala* ("The Compendious Book on Calculation by Completion and Balancing"). The appellation *al-jabr waʾl-muqābala*, or *al-jabr* alone, was commonly applied to later works in Arabic on the same topic; and thence (via medieval Latin translations from the Arabic) is derived the English "algebra."

In his *Algebra* al-Khwārizmī employs no symbols (even for numerals) but expresses everything in words. For the unknown quantity he employs the word *shayʾ* ("thing" or "something"). For the second power of a quantity he employs *māl* ("wealth," "property"), which is also used to mean only "quantity." For the first power, when contrasted with the second power, he uses *jidhr* ("root"). For the unit he uses *dirham* (a unit of coinage). Thus the problem

$$(x/3 + 1)(x/4 + 1) = 20$$

and the first stage in its resolution,

$$x^2/12 + x/3 + x/4 + 1 = 20,$$

appear, in literal translation, as follows:

> A quantity: I multiplied a third of it and a *dirham* by a fourth of it and a *dirham:* it becomes twenty. Its computation is that you multiply a third of something by a fourth of something: it comes to a half of a sixth of a square (*māl*). And you multiply a *dirham* by a third of something: it comes to a third of something; and [you multiply] a *dirham* by a fourth of something to get a fourth of something; and [you multiply] a *dirham* by a *dirham* to get a *dirham*. Thus its total, [namely] a half of a sixth of a square and a third of something and a quarter of something and a *dirham*, is equal to twenty *dirhams*.[6]

After illustrating the rules he has expounded for solving problems by a number of worked examples, al-Khwārizmī, in a short section headed "On Business Transactions," expounds the "rule of three," or how to determine the fourth member in a proportion sum where two quantities and one price, or two prices and one quantity, are given. The next part concerns practical mensuration. He gives rules for finding the area of various plane figures, including the circle, and for finding the volume of a number of solids, including cone, pyramid, and truncated pyramid. The third part, on legacies, consists entirely of solved problems. These involve only arithmetic or simple linear equations but require considerable knowledge of the complicated Islamic law of inheritance.

We are told that al-Khwārizmī's work on algebra was the first written in Arabic.[7] In modern times considerable dispute has arisen over the question of whether the author derived his knowledge of algebraic techniques from Greek or Hindu sources. Both Greek and Hindu algebra had advanced well beyond the elementary stage of al-Khwārizmī's work, and none of the known works in either culture shows much resemblance in presentation to al-Khwārizmī's. But, in favor of the "Hindu hypothesis," we may note first that in his astronomical work al-Khwārizmī was far more heavily indebted to a Hindu work than to Greek sources; second, that his exposition is completely rhetorical, like Sanskrit algebraic works and unlike the one surviving Greek algebraic treatise, that of Diophantus, which has already developed quite far toward a symbolic representation; third that the "rule of three" is commonly enunciated in Hindu works but not explicitly in any ancient Greek work; and fourth that in the part on mensuration two of the methods he gives for finding the circumference of the circle from its diameter are specifically Hindu.[8]

On the other hand, in his introductory section al-Khwārizmī uses geometrical figures to explain equations, which surely argues for a familiarity with Book II of Euclid's *Elements*. We must recognize that he was a competent enough mathematician to select and adapt material from quite disparate sources in order to achieve his purpose of producing a popular handbook. The question of his sources is further complicated by the existence of a Hebrew treatise, the *Mishnat ha-Middot*, which is closely related in content and arrangement to the part of al-Khwārizmī's work dealing with mensuration. If we adopt the conclusion of Gandz, the last editor of the *Mishnat ha-Middot*, that it was composed about A.D. 150,[9] then al-Khwārizmī must be the borrower, either through an intermediary work or even directly—his treatise on the Jewish calendar (see below) shows that he must have been in contact with learned Jews. But the Hebrew treatise may be a later adaptation of al-Khwārizmī's work. Gad Sarfatti (*Mathematical Terminology in Hebrew Scientific Literature of the Middle Ages*, Jerusalem, 1968, 58–60) argues on linguistic grounds that the *Mishnat ha-Middat* belongs to an earlier Islamic period.

Al-Khwārizmī wrote a work on the use of the Hindu numerals, which has not survived in Arabic but has reached us in the form of a Latin translation (probably much altered from the original). The Arabic title is uncertain; it may have been something like *Kitāb ḥisāb al-ʿadad al-hindī* ("Treatise on Calculation With the Hindu Numerals"),[10] or possibly *Kitāb al-jamʿ waʾl-tafrīq bi ḥisāb al-hind* ("Book of Addition and Subtraction by the Method of Calculation of the Hindus").[11] The treatise, as we have it, expounds the use of the Hindu (or, as they are misnamed, "Arabic") numerals 1 to 9 and 0 and the place-value system, then explains various applications. Besides the four basic operations of addition, subtraction, multiplication, and division, it deals with both common and sexagesimal fractions and the extraction of the square root (the latter is missing in the unique manuscript but is treated in other medieval works derived from it). In other words, it is an elementary arithmetical treatise using the Hindu numerals. Documentary evidence (eighth-century Arabic papyri from Egypt) shows that the Arabs were already using an alphabetic numeral system similar to the Greek (in which 1, 2, 3, . . . 9, 10, 20, 30, . . . 90, 100, 200, . . . 900 are each represented by a different letter). The sexagesimal modified place-value system used in Greek astronomy must also have been familiar, at least to learned men, from the works such as Ptolemy's *Almagest* which were available in Arabic before 800. But it is likely enough that the decimal place-value system was a fairly recent arrival from India and that al-Khwārizmī's work was the first to expound it systematically. Thus, although elementary, it was of seminal importance.

The title of al-Khwārizmī's astronomical work was *Zīj al-sindhind*.[12] This was appropriate, since it is based ultimately on a Sanskrit astronomical work brought to the court of Caliph al-Manṣūr at Baghdad soon after 770[13] by a member of an Indian political mission. That work was related to, although not identical with, the *Brāhmasphuṭasiddhānta* of Brahmagupta. It was translated into Arabic under al-Manṣūr (probably by al-Fazārī), and the translation was given the name *Zīj al-sindhind*. *Zīj* means "set of astronomical tables"; and *sindhind* is a corruption of the Sanskrit *siddhānta*, which presumably was part of the title of the Hindu source work. This translation formed the basis of astronomical works (also called *Zīj al-sindhind*) by al-Fazārī and Yaʿqūb ibn Ṭāriq in the late eighth century. Yet these astronomers also used other sources for their work, notably the *Zīj al-shāh*, a translation of a Pahlavi work composed for the Sassanid ruler Khosrau I (Anūshirwān) about 550, which was also based on Hindu sources.

Al-Khwārizmī's work is another "revision" of the *Zīj al-sindhind*. Its chief importance today is that it is the first Arabic astronomical work to survive in anything like entirety. We are told that there were two editions of it; but we know nothing of the differences between them, for it is available only in a Latin translation made by Adelard of Bath in the early twelfth century. This translation was made not from the original but from a revision executed by the Spanish Islamic astronomer al-Majrīṭī (*d.* 1007–1008) and perhaps further revised by al-Majrīṭī's pupil Ibn al-Ṣaffār (*d.* 1035).[14] We can, however, get some notions of the original form of the work from extracts and commentaries made by earlier writers.[15] Thus from the tenth-century commentary of Ibn al-Muthannā we learn that al-Khwārizmī constructed his table of sines to base 150 (a common Hindu parameter), whereas in the extant tables base 60 (more usual in Islamic sine tables) is employed. From the same source we learn that the epoch of the original tables was era Yazdegerd (16 June 632) and not the era Hijra (14 July 622) of al-Majrīṭī's revision.[16]

The work as we have it consists of instructions for computation and use of the tables, followed by a set of tables whose form closely resembles that made standard by Ptolemy. The sun, the moon, and each of the five planets known in antiquity have a table of mean motion(s) and a table of equations. In addition there are tables for computing eclipses, solar declination and right ascension, and various trigonometrical tables. It is certain that Ptolemy's tables, in their revision by Theon of Alexandria, were already known to some Islamic astronomers; and it is highly likely that they influenced, directly or through intermediaries, the form in which al-Khwārizmī's tables were cast.

But most of the basic parameters in al-Khwārizmī's tables are derived from Hindu astronomy. For all seven bodies the mean motions, the mean positions at epoch, and the positions of the apogee and the node all agree well with what can be derived from the *Brāhmasphuṭasiddhānta*. The maximum equations are taken from the *Zīj al-shāh*. Furthermore, the method of computing the true longitude of a planet by "halving the equation" prescribed in the instructions is purely Hindu and quite alien to Ptolemaic astronomy.[17] This is only the most notable of several Hindu procedures found in the instructions. The only tables (among those that can plausibly be assigned to the original *Zīj*) whose content seems to derive from Ptolemy are the tables of solar declination, of planetary stations, of right ascension, and of equation of time. Nowhere in the work is there any trace of original observation or of more than trivial computation by the author. This appears strange when we learn that in the original introduction (the present one must be much altered) al-Khwārizmī discussed observations made at Baghdad under al-Ma'mūn to determine the obliquity of the ecliptic.[18] The value found, 23° 33′, was fairly accurate. Yet in the tables al-Khwārizmī adopts the much worse value of 23° 51′ from Theon. Even more inexplicable is why, if he had the Ptolemaic tables available, he preferred to adopt the less accurate parameters and obscure methods of Hindu astronomy.

The *Geography*, *Kitāb ṣūrat al-arḍ* ("Book of the Form of the Earth"), consists almost entirely of lists of longitudes and latitudes of cities and localities. In each section the places are arranged according to the "seven climata" (in many ancient Greek geographical works the known world was divided latitudinally into seven strips known as "climata," each clima being supposed to enjoy the same length of daylight on its longest day), and within each clima the arrangement is by increasing longitude. Longitudes are counted from an extreme west meridian, the "shore of the western ocean." The first section lists cities; the second, mountains (giving the coordinates of their extreme points and their orientation); the third, seas (giving the coordinates of salient points on their coastlines and a rough description of their outlines); the fourth, islands (giving the coordinates of their centers, and their length and breadth); the fifth, the central points of various geographical regions; and the sixth, rivers (giving their salient points and the towns on them).

It is clear that there is some relationship between this work and Ptolemy's *Geography*, which is a description of a world map and a list of the coordinates of the principal places on it, arranged by regions. Many of the places listed in Ptolemy's work also occur in al-Khwārizmī's, with coordinates that are nearly the same or systematically different. Yet it is very far from being a mere translation or adaptation of Ptolemy's treatise. The arrangement is radically different, and the outline of the map which emerges from it diverges greatly from Ptolemy's in several regions. Nallino is surely right in his conjecture that it was derived by reading off the coordinates of a map or set of maps based on Ptolemy's but was carefully revised in many respects. Nallino's principal argument is that al-Khwārizmī describes the colors of the mountains in a way which could not possibly represent their physical appearance but might well represent their depiction on a map. To this we may add that in those areas where al-Khwārizmī agrees, in general, with Ptolemy, the coordinates of the two frequently differ by 10, 15, 20, or more minutes, up to one degree of arc; such discrepancies cannot be explained by scribal errors but are plausible by supposing a map as intermediary. The few maps accompanying the sole manuscript of al-Khwārizmī's *Geography* are crude things; but we know that al-Ma'mūn had had constructed a world map, on which many savants worked. According to al-Mas'ūdī, the source of this information, al-Ma'mūn's map was superior to Ptolemy's.[19] Nallino makes the plausible suggestion that al-Khwārizmī's *Geography* is based on al-Ma'mūn's world map (on which al-Khwārizmī himself had probably worked), which in turn was based on Ptolemy's *Geography*, which had been considerably revised.

The map which emerges from al-Khwārizmī's text is in several respects more accurate than Ptolemy's, particularly in the areas ruled by Islam. Its most notable improvement is to shorten the grossly exaggerated length of the Mediterranean imagined by Ptolemy. It also corrects some of the distortions applied by Ptolemy to Africa and the Far East (no doubt reflecting the knowledge of these areas brought back by Arab merchants). But for Europe it could do little more than reproduce Ptolemy; and it introduces errors of its own, notably the notion that the Atlantic is an inland sea enclosed by a western continent joined to Europe in the north.

The only other surviving work of al-Khwārizmī is a short treatise on the Jewish calendar, *Istikhrāj ta'rīkh al-yahūd* ("Extraction of the Jewish Era"). His interest in the subject is natural in a practicing astronomer. The treatise describes the Jewish calendar, the 19-year intercalation cycle, and the rules for determining on what day of the week the first day of the month Tishrī shall fall; calculates the interval between the Jewish era (creation of Adam) and the Seleucid era; and gives rules for determining the mean longitude of the sun and moon using the Jewish calendar. Although a slight work, it is accurate, well informed, and of importance as evidence for the antiquity of the present Jewish calendar.

Al-Khwārizmī wrote two works on the astrolabe, *Kitāb 'amal al-asṭurlāb* ("Book on the Construction of the Astrolabe") and *Kitāb al-'amal bi'l-asṭurlāb* ("Book on the Operation of the Astrolabe"). Probably from the latter is drawn the extract found in a Berlin manuscript of a work of the ninth-century astronomer al-Farghānī. This extract deals with the solution of various astronomical problems by means of the astrolabe—for instance, determination of the sun's altitude, of the ascendant, and of one's terrestrial latitude. There is nothing surprising in the content, and it is probable that al-Khwārizmī derived it all from earlier works on the subject. The astrolabe was a Greek invention, and we know that there were once ancient Greek treatises on it. Astrolabe treatises predating al-Khwārizmī survive in Syriac (by Severus Sebokht, seventh century) and in Arabic (now only in Latin translation, by Māshā'llāh, late eighth century).

The *Kitāb al-ta'rīkh* ("Chronicle") of al-Khwārizmī does not survive, but several historians quote it as an authority for events in the Islamic period. It is possible that in it al-Khwārizmī (like his contemporary Abū Ma'shar) exhibited an interest in interpreting history as fulfilling the principles of astrology.[20] In that case it may be the ultimate source of the report of Ḥamza al-Iṣfahānī about how al-Khwārizmī cast the horoscope of the Prophet and showed at what hour Muḥammad must have been born by astrological deduction from the events of his life.[21] Of a book entitled *Kitāb al-rukhāma* ("On the Sundial") we know only the title, but the subject is consonant with his other interests.

Al-Khwārizmī's scientific achievements were at best mediocre, but they were uncommonly influential. He lived at a time and in a place highly favorable to the success of his works: encouraged by the patronage of the caliphs, Islamic civilization was beginning to assimilate Greek and Hindu science. The great achievements of Islamic science lay in the future, but these early works which transmitted the new knowledge ensured their author's lasting fame. Between the ninth and twelfth centuries algebra was developed to a far more sophisticated level in Islamic lands, aided by the spread of knowledge of Diophantus' work. But even such advanced algebraists as al-Karajī (*d.* 1029) and 'Umar al-Khayyāmī (*d.* 1123–1124) still used the rhetorical exposition popularized by al-Khwārizmī.

Al-Khwārizmī's *Algebra* continued to be used as a textbook and praised highly (see, for instance, the quotations in Hajī Khalfa, V, 67–69). The algebraic part proper was twice translated into Latin in the twelfth century (by Robert of Chester and by Gerard of Cremona) and was the chief influence on medieval European algebra, determining its rhetorical form and some of its vocabulary (the medieval *cossa* is a literal translation of Arabic *shay'*, and *census* of *māl*). The treatise on Hindu numerals, although undoubtedly important in introducing those useful symbols into more general use in Islamic lands, achieved its greatest success only when introduced to the West through Latin translation in the early twelfth century (occasional examples of the numerals appeared in the West more than a century earlier, but only as isolated curiosities). The work quickly spawned a number of adaptations and offshoots, such as the *Liber alghoarismi* of John of Seville (*ca.* 1135), the *Algorismus* of John of Sacrobosco (*ca.* 1250), and the *Liber ysagogarum Alchorizmi* (twelfth century). In fact, al-Khwārizmī's name became so closely asso-

AL-KHWĀRIZMĪ

ciated with the "new arithmetic" using the Hindu numerals that the Latin form of his name, *algorismus*, was given to any treatise on that topic. Hence, by a devious path, is derived the Middle English "augrim" and the modern "algorism" (corrupted by false etymology to "algorithm").

The other works did not achieve success of such magnitude; but the *Zīj* continued to be used, studied, and commented on long after it deserved to be superseded. About 900 al-Battānī published his great astronomical work, based on the *Almagest* and tables of Ptolemy and on his own observations. This is greatly superior to al-Khwārizmī's astronomical work in nearly every respect, yet neither al-Battānī's opus nor the other results of the prodigious astronomical activity in Islamic lands during the ninth and tenth centuries drove al-Khwārizmī's *Zīj* from the classroom. In fact it was the first such work to reach the West, in Latin translation by Adelard of Bath in the early twelfth century. Knowledge of this translation was probably confined to England (all surviving manuscripts appear to be English), but many of al-Khwārizmī's tables reached a wide audience in the West via another work, the *Toledan Tables*, a miscellaneous assembly of astronomical tables from the works of al-Khwārizmī, al-Battānī, and al-Zarqāl which was translated into Latin, probably by Gerard of Cremona, in the late twelfth century and which, for all its deficiencies, enjoyed immense popularity throughout Europe for at least 100 years.

The *Geography* too was much used and imitated in Islamic lands, even after the appearance of good Arabic translations of Ptolemy's *Geography* in the later ninth century caused something of a reaction in favor of that work. For reasons rather obscure the medieval translators of Arabic scientific works into Latin appear to have avoided purely geographical treatises, so al-Khwārizmī's *Geography* was unknown in Europe until the late nineteenth century. But some of the data in it reached medieval Europe via the lists of longitudes and latitudes of principal cities, which were commonly incorporated into ancient and medieval astronomical tables.[22]

NOTES

1. Al-Ṭabarī, de Goeje ed., III, 2, 1364.
2. E.g., *Fihrist*, Flügel ed., I, 274; followed by Ibn al-Qifṭī, Lippert, ed., p. 286.
3. Al-Muqaddasī, de Goeje ed., p. 362.
4. The story is found in several sources, all of which call the envoy "Muḥammad ibn Mūsā the astronomer." Only one, al-Mas'ūdī, *Kitāb al-tanbīh*, de Goeje ed., p. 134, adds "ibn Shākir." For the full story see Nallino, "Al-Khuwārizmī," pp. 465–466.
5. Al-Ṭabarī, de Goeje ed., III, 2, 1085.
6. Rosen, text, p. 28, somewhat emended. The translation is mine.
7. E.g., Hajī Khalfa, Flügel ed., V, 67, no. 10012.
8. These are $c = \sqrt{10d^2}$ and $c = 62832d/20000$. See Rosen's note on pp. 198–199 of his ed. The second value, which is very accurate, is attested for the later *Pauliśasiddhānta* and also, significantly, for Ya'qūb ibn Ṭāriq, al-Khwārizmī's immediate predecessor, by al-Bīrūnī, *India*, Sachau trans., I, 168–169. Pauliśa presumably derived it from the *Āryabhaṭīya* (see the *Āryabhaṭīya*, Clark ed., p. 28).
9. See Gandz's ed. of the *Mishnat ha-Middot*, pp. 6–12.
10. Some such title seems to be implied by Ibn al-Qifṭī, Lippert ed., pp. 266–267.
11. As conjectured by Ruska, "Zur ältesten arabischen Algebra," pp. 18–19.
12. *Fihrist*, Flügel ed., I, 274.
13. See, e.g., al-Bīrūnī, *India*, Sachau trans., II, 15.
14. For the latter revision see Ibn Ezra, *Libro de los fundamentos*, p. 109.
15. For a list of these see Pingree and Kennedy, commentary on al-Hāshimī's *Book of the Reasons Behind Astronomical Tables*, sec. 11; see also in biblio. (below).
16. For the value 150 see, e.g., Goldstein, *Ibn al-Muthannâ*, p. 178. For the epoch, *ibid.*, p. 18.
17. On "halving the equation" see Neugebauer, *al-Khwārizmī*, pp. 23–29.
18. Ibn Yūnus, quoted by Nallino, "Al-Khuwārizmī," p. 469.
19. Al-Mas'ūdī, *Kitāb al-tanbīh*, de Goeje ed., p. 33.
20. On Abū Ma'shar see especially Pingree, *The Thousands of Abū Ma'shar*.
21. Ḥamza, *Ta'rīkh*, Beirut ed., p. 126. However, Ḥamza quotes this not directly from the *Chronicle* (which he uses elsewhere, *ibid.*, p. 144) but from Shādhān's book of Abū Ma'shar's table talk, so the ultimate source might be a conversation between al-Khwārizmī and Abū Ma'shar.
22. The list in the *Toledan Tables*, which is certainly in part related to al-Khwārizmī's *Geography*, is printed with commentary in Toomer, "Toledan Tables," pp. 134–139.

BIBLIOGRAPHY

The principal medieval Arabic sources for al-Khwārizmī's life and works are Ibn al-Nadīm, *Kitāb al-fihrist*, Gustav Flügel, ed., 2 vols. (Leipzig, 1872; repr. Beirut, 1964), I, 274—trans. by Heinrich Suter, "Das Mathematiker-Verzeichniss im Fihrist des Ibn Abî Ja'ḳûb an-Nadîm," which is Abhandlungen zur Geschichte der Mathematik, VI, 29, supp. to *Zeitschrift für Mathematik und Physik*, 37 (1892); Ibn al-Qifṭī, *Ta'rīkh al-ḥukamā'*, Julius Lippert, ed. (Leipzig, 1903; repr. Baghdad, n.d.), p. 286, a mere repetition of the *Fihrist* but with more information under the entry "Kanka," p. 266; Ṣâ'id al-Andalusî, *Kitâb ṭabaḳât al-umam* (*Livre des catégories des nations*), which is Publications de l'Institut des Hautes Études Marocaines, XXVIII, Régis Blachère, trans. (Paris, 1935), pp. 47–48, 130; Hajī Khalfa, *Lexicon bibliographicum*, G. Flügel, ed., V (London, 1850; repr. London–New York, 1964), 67–69, no. 10012; *Annales quos scripsit Abu Djafar Mohammed ibn Djarir at-Tabari*, M. J. de Goeje, ed., III, 2 (Leiden, 1881; repr. Leiden, 1964), 1364; *Descriptio imperii moslemici auctore al-Mokaddasi*, M. J. de Goeje, ed. (Leipzig, 1876–1877), p. 362; al-Mas'ūdī, *Kitāb al-tanbīh wa'l-ishrāf*, which is Bibliotheca Geographorum Arabicorum, VIII, M. J. de Goeje, ed. (Leiden,

1894; repr. 1967), pp. 33, 134. The best modern account of his life is C. A. Nallino, "Al-Khuwārizmī e il suo rifacimento della Geografia di Tolomeo," in his *Raccolta di scritti editi e inediti*, V (Rome, 1944), 458–532 (an amended repr. of his article in *Atti dell'Accademia nazionale dei Lincei. Memorie*, Classe di scienze morali, storiche e filologiche, 5th ser., II, pt. 1), and sec. 2, 463–475, where references to further source material may be found.

The Arabic text of the *Algebra* was edited with English trans. by Frederic Rosen as *The Algebra of Mohammed ben Musa* (London, 1831; repr. New York, 1969). Editing and trans. are careless. A somewhat better Arabic text is provided by the ed. of ʿAlī Muṣṭafā Masharrafa and Muḥammad Mursī Aḥmad (Cairo, 1939), which is Publications of the Faculty of Science, no. 2. Both eds. are based only on the MS Oxford Bodleian Library, I 918, 1, but other MSS are known to exist. I owe the refs. to the following to Adel Anbouba: Berlin 5955 no. 6, ff. 60r–95v; also a MS at Shibin el-Kom (Egypt) mentioned in *Majalla Maʿhad al-Makhṭūṭāt al-ʿArabiyya* (Cairo, 1950), no. 19. The section of the *Algebra* concerning mensuration is published with an English trans. by Solomon Gandz, together with his ed. of the *Mishnat ha-Middot*, which is *Quellen und Studien zur Geschichte der Mathematik, Astronomie und Physik*, Abt. A, **2** (1932). A useful discussion of the *Algebra* is given by Julius Ruska, "Zur ältesten arabischen Algebra und Rechenkunst," in *Sitzungsberichte der Heidelberger Akademie der Wissenschaften*, Phil.-hist. Kl. (1917), sec. 2, where further bibliography will be found. On the section dealing with legacies, see S. Gandz, "The Algebra of Inheritance," in *Osiris*, **5** (1938), 319–391. The Latin trans. by Robert of Chester was edited with English trans. by Louis Charles Karpinski, *Robert of Chester's Latin Translation of the Algebra of al-Khowarizmi* (Ann Arbor, 1915); repr. as pt. I of Louis Charles Karpinski and John Garrett Winter, *Contributions to the History of Science* (Ann Arbor, 1930). The editor perversely chose to print a sixteenth-century reworking rather than Robert's original translation, but his introduction and commentary are occasionally useful. The anonymous Latin version printed by G. Libri in his *Histoire des sciences mathématiques en Italie*, I (Paris, 1858), 253–297, is probably that of Gerard of Cremona, but the problem is complicated by the existence of another Latin text which is a free adaptation of al-Khwārizmī's *Algebra*, whose translation is expressly ascribed to Gerard of Cremona. This is printed by Baldassarre Boncompagni in *Atti dell'Accademia pontificia dei Nuovi Lincei*, **4** (1851), 412–435. A. A. Björnbo, "Gerhard von Cremonas Übersetzung von Alkwarizmis Algebra und von Euklids Elementen," in *Bibliotheca mathematica*, 3rd ser., **6** (1905), 239–241, argues that the version printed by Libri is the real Gerard translation. On al-Karaji's *Algebra* see Adel Anbouba, *L'algèbre al-Badīʿ d'al-Karagī* (Beirut, 1964), which is Publications de l'Université Libanaise, Section des Études Mathématiques, II. On ʿUmar al-Khayyāmī's *Algebra* see F. Woepcke, *L'algèbre d'Omar al-Khayyâmî* (Paris, 1851); and, for discussion and further bibliography, Hâmit Dilgan, *Büyük matematikci Ömer Hayyâm* (Istanbul, 1959), in the series Istanbul Technical University Publications. On Hindu values of π see *Alberuni's India*, Edward C. Sachau, trans., I (London, 1910), 168–169; and *The Āryabhaṭīya of Āryabhaṭa*, Walter Eugene Clark, trans. (Chicago, 1930), p. 28.

The Latin text of the treatise on Hindu numerals was first published, carelessly, *Algoritmi de numero indorum* (Rome, 1857), which is Trattati d'aritmetica, B. Boncompagni, ed., I. A facs. text of the unique MS was published by Kurt Vogel, *Mohammed ibn Musa Alchwarizmi's Algorismus* (Aalen, 1963), which is Milliaria, III. Vogel provides a transcription as inaccurate as his predecessor's and some useful historical information. Of the numerous medieval Latin works named *Algorismus* the following have been published: John of Seville's *Alghoarismi de practica arismetrice*, B. Boncompagni, ed. (Rome, 1857), which is Trattati d'aritmetica, II; John of Sacrobosco's *Algorismus*, edited by J. O. Halliwell as "Joannis de Sacro-Bosco tractatus de arte numerandi," in his *Rara mathematica*, 2nd ed. (London, 1841), pp. 1–31; and Alexander of Villa Dei (*ca.* 1225), "Carmen de algorismo," *ibid.*, pp. 73–83. See also M. Curtze, "Über eine Algorismus-Schrift des XII Jahrhunderts," in *Abhandlungen zur Geschichte der Mathematik*, **8** (1898), 1–27.

The Latin version of al-Khwārizmī's *Zīj* was edited by H. Suter, *Die astronomischen Tafeln des Muḥammed ibn Mūsā al-Khwārizmī* (Copenhagen, 1914), which is Kongelige Danske Videnskabernes Selskabs Skrifter, 7. Raekke, Historisk og filosofisk Afd., III, 1. Suter has a useful commentary, but an indispensable supplement is O. Neugebauer, *The Astronomical Tables of al-Khwārizmī* (Copenhagen, 1962), Kongelige Danske Videnskabernes Selskabs, Historisk-filosofiske Skrifter, IV, 2, which provides a trans. of the introductory chapters and an explanation of the basis and use of the tables. Important information on al-Khwārizmī's *Zīj* will be found in the forthcoming ed. of al-Hāshimī's *Book of the Reasons Behind Astronomical Tables* (*Kitāb fī ʿIlal al-Zījāt*), ed. and trans. by Fuad I. Haddad and E. S. Kennedy, with a commentary by David Pingree and E. S. Kennedy. The Arabic text of Ibn al-Muthannā's commentary is lost, but one Latin and two Hebrew versions are preserved. The Latin version has been miserably edited by E. Millás Vendrell, *El comentario de Ibn al-Mutannā a las Tablas astronómicas de al-Jwārizmī* (Madrid–Barcelona, 1963). It is preferable to consult Bernard R. Goldstein's excellent ed., with English trans. and commentary, of the Hebrew versions, *Ibn al-Muthannā's Commentary on the Astronomical Tables of al-Khwârizmî* (New Haven–London, 1967). On the origin of the *Sindhind* and early versions of it, see David Pingree, "The Fragments of the Works of al-Fazārī," in *Journal of Near Eastern Studies*, **29** (1970), 103–123; "The Fragments of the Works of Yaʿqūb ibn Ṭāriq," *ibid.*, **26** (1968), 97–125; and *The Thousands of Abū Maʿshar* (London, 1968). On Maslama and Ibn al-Ṣaffār's revision of al-Khwārizmī's *Zīj* see Ibn Ezra, *El libro de los fundamentos de las tablas astronómicas*, J. M. Millás Vallicrosa, ed. (Madrid-Barcelona, 1947), pp. 75, 109–110. The relationship of the mean motions in al-Khwārizmī's *Zīj* to the *Brāhmasphuṭasid-*

dhānta was demonstrated by J. J. Burckhardt, "Die mittleren Bewegungen der Planeten im Tafelwerk des Khwârizmî," in *Vierteljahrsschrift der Naturforschenden Gesellschaft in Zürich*, **106** (1961), 213–231; and by G. J. Toomer, review of O. Neugebauer's *The Astronomical Tables of al-Khwārizmī*, in *Centaurus*, **10** (1964), 203–212. Al-Battānī's *Zīj* was edited magisterially by C. A. Nallino, *Al-Battānī sive Albatenii opus astronomicum*, 3 vols. (Milan, 1899–1907), which is Pubblicazioni del Reale Osservatorio di Brera in Milano, XL (vols. I and II repr. Frankfurt, 1969; vol. III repr. Baghdad [?], 1970 [?] [n.p., n.d.]). The *Toledan Tables* have never been printed in their entirety, but they are extensively analyzed by G. J. Toomer, "A Survey of the Toledan Tables," in *Osiris*, **15** (1968), 5–174.

The text of the *Geography* was published from the unique MS by Hans von Mžik, *Das Kitāb Ṣūrat al-Arḍ des Abū Ǧaʿfar Muḥammad ibn Mūsā al-Ḫuwārizmī* (Leipzig, 1926). The classic study of the work is that by C. A. Nallino mentioned above. See also Hans von Mžik, "Afrika nach der arabischen Bearbeitung der Γεωγραφικὴ ὑφήγησις des Claudius Ptolemaeus von Muḥammad ibn Mūsā al-Ḫwārizmī," which is *Denkschriften der K. Akademie der Wissenschaften* (Vienna), Phil.-hist. Kl., **59**, no. 4 (1916); and "Osteuropa nach der arabischen Bearbeitung der Γεωγραφικὴ ὑφήγησις des Klaudios Ptolemaios von Muḥammad ibn Mūsā al-Ḫuwārizmī," in *Wiener Zeitschrift für die Kunde des Morgenlandes*, **43** (1936), 161–193; and Hubert Daunicht, *Der Osten nach der Erdkarte al-Ḫuwārizmīs* (Bonn, 1968), with further bibliography.

The treatise on the Jewish calendar is printed as the first item in *al-Rasāʾil al-mutafarriqa fiʾl-hayʾa* (Hyderabad [Deccan], 1948). See E. S. Kennedy, "Al-Khwārizmī on the Jewish Calendar," in *Scripta mathematica*, **27** (1964), 55–59. The extract from the treatise on the astrolabe survives in MSS Berlin, Arab. 5790 and 5793. A German trans. and commentary was given by Josef Frank, *Die Verwendung des Astrolabs nach al Chwârizmî* (Erlangen, 1922), which is Abhandlungen zur Geschichte der Naturwissenschaften und der Medizin, no. 3. Severus Sabokht's treatise was edited by F. Nau, "Le traité sur l'astrolabe plan de Sévère Sabokt," in *Journal asiatique*, 9th ser., **13** (1899), 56–101, 238–303, also printed separately (Paris, 1899). The Latin trans. of Māshā'llāh's treatise was printed several times in the sixteenth century; a modern ed. is in R. T. Gunther, *Chaucer and Messehalla on the Astrolabe*, which is *Early Science in Oxford*, V (Oxford, 1929), 133–232. For the *Chronicle* the principal excerptor is Elias of Nisibis, in his *Chronography*, written in Syrian and Arabic. See the ed. of the latter, with trans. and commentary, by Friedrich Baethgen, *Fragmente syrischer und arabischer Historiker*, which is Abhandlungen für die Kunde des Morgenlandes, VIII, 3 (Leipzig, 1884; repr. Nendeln, Lichtenstein, 1966), esp. pp. 4–5. A fuller ed., with Latin trans., is given in E. W. Brooks and J.-B. Chabot, *Eliae metropolitae Nisibeni opus chronologicum*, 2 vols. (Louvain, 1910), which is Corpus Scriptorum Christianorum Orientalium, Scriptores Syri, vols. XXIII and XXIV. See also the French trans. by L.-J. Delaporte, *La chronographie d'Élie bar-Šinaya* (Paris, 1910). See also Ḥamza al-Ḥasan al-Iṣfahānī, *Taʾrikh sinī mulūk al-arḍ wa l-anbiyāʾ* (Beirut, 1961), pp. 126, 144. Other excerptors are listed by Nallino, "Al-Khuwārizmī," pp. 471–472.

G. J. TOOMER

KIDD, JOHN (*b.* London, England, 10 September 1775; *d.* Oxford, England, 17 September 1851), *chemistry, anatomy.*

John Kidd was the son of John Kidd, a captain of a merchant ship; and his mother was the daughter of Samuel Burslem, vicar of Etwall, near Derby. He married Fanny Savery, daughter of the chaplain of St. Thomas's Hospital, London, and they had four daughters.

In 1789 Kidd entered Westminster School, London, with a king's scholarship. He was elected to a studentship at Christ Church, Oxford, in 1793, and graduated B.A. in 1797, M.A. in 1800. He then studied at Guy's Hospital, London, from 1797 to 1801, and took his medical degrees at Oxford, M.B. in 1801 and M.D. in 1804. In 1801 he returned to Oxford as reader in chemistry, and in 1803 he became the first Aldrichian professor of chemistry.

Kidd began his teaching career just when the reformed system of examinations, introduced at Oxford in 1800, was requiring greater concentration by students on the classical and mathematical syllabus. Undergraduates, especially those aiming at an honors degree, were thus discouraged by their college tutors from attending lectures on scientific subjects. This system, which did not allow a formal study of the sciences, was attacked from many quarters, but the science teachers at Oxford tacitly acquiesced during the first three decades of the nineteenth century.

In his pamphlet *An Answer to a Charge Against the English Universities* (1818), Kidd discussed to what extent chemistry and other sciences ought to be taught in Oxford, where the training of students was almost exclusively for the church, the law, and the diplomatic service. He concluded that it would be unreasonable to introduce any general requirement regarding the study of science. Nevertheless, Kidd noted that his chemistry course, consisting of thirty lectures a year, was equal in length to that at Guy's Hospital, which was regarded as "the best school in London for Physical Sciences." Kidd also lectured on mineralogy and geology and published the textbooks *Outlines of Mineralogy* (1809) and *A Geological Essay* (1815). Among his pupils was William Buckland, who took over his teaching in these subjects when the readership in mineralogy was instituted in 1813.

After his election to the readership in anatomy in