cepted the presence of minerals in the plant as incidental, however, and thought them the result of the presence of minerals in the soil. Bertrand's work in 1897, and especially his later claim that a lack of manganese caused an interruption of growth, forced a change in thinking on this matter. He concluded that the metal formed an essential part of the enzyme, and, more generally, that a metal might be a necessary functioning part of the oxidative enzyme. From this and similar researches he developed his concept of the trace element, essential for proper metabolism.

During his career Bertrand published hundreds of papers on the organic effects of various metals. In 1911 he showed that the development of the mold *Aspergillis niger* was greatly influenced by the presence of minute amounts of manganese. For such researches Bertrand was forced to develop more precise methods of organic analysis, many of which later came into widespread use.

Bertrand's researches were immediately applied to the elimination of previously undiagnosable pathological conditions, thereafter recognized as the result of deficiencies of trace elements. His work also provided the basis for further elaboration of the enzymatic systems involved in respiration and metabolic processes.

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With M. Javiller, Bertrand wrote "Influence du manganèse sur le développement de l'*Aspergillis niger,*" "Influence combinée du zinc et du manganèse sur le développement de l'*Aspergillis niger,*" and "Influence du zinc et du manganèse sur la composition minérale de l'*Aspergillis niger,*" all of which appeared in *Comptes rendus de l'Académie des sciences* (Paris), **152** (1911), 225–228, 900–902, and 1337–1340, respectively.

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ALAN S. KAY

BERTRAND, JOSEPH LOUIS FRANÇOIS (*b.* Paris, France, 11 March 1822; *d.* Paris, 5 April 1900), *mathematics.*

Bertrand's father was Alexandre Bertrand, a writer of popular scientific articles and books. Alexandre had attended the École Polytechnique in Paris with Auguste Comte and Jean Marie Constant Duhamel, and the latter married his sister. When his father died, young Bertrand went to live with the Duhamels. A well-known professor of mathematics at the École Polytechnique, Duhamel was the right man to guide his precocious nephew. At the age of eleven the boy was allowed to attend classes at the École Polytechnique. In 1838, at sixteen, Bertrand took the degrees of bachelor of arts and bachelor of science, and at seventeen he received the doctor of science degree with a thesis in thermomechanics. The same year (1839) he officially entered the École Polytechnique, and in 1841 he entered the École des Mines. Bertrand's first publications date from this period, the first being "Note sur quelques points de la théorie de l'électricité" (1839), which deals with Poisson's equation, $\Delta V = -4\pi\rho$, and the law of Coulomb.

In 1841 Bertrand became a professor of elementary mathematics at the Collège Saint-Louis, a position that he filled until 1848. In May 1842 he and his brother, returning to Paris from a visit to their friends the Aclocques at Versailles, were nearly killed in a railroad accident which left a scar on Bertrand's face. Bertrand married Mlle. Aclocque in 1844, in which year he also became *répétiteur d'analyse* at the École Polytechnique. Three years later he became *examinateur d'admission* at this school and *suppléant* of the physicist Jean-Baptiste Biot at the Collège de France. In 1848, during the revolution, Bertrand served as a captain in the national guard. He published much during these years—in mathematical physics, in mathematical analysis, and in differential geometry. The first of Bertrand's many textbooks, the *Traité d'arithmétique*, appeared in Paris in 1849 and was followed by the *Traité élémentaire d'algèbre* (1850); both were written for secondary schools. They were followed by textbooks for college instruction. Bertrand always knew how to fascinate his readers and his lecture audiences, and his books had a wide appeal because of content and style. In 1853 he edited and annotated the third edition of J. L. Lagrange's *Mécanique analytique.* From the many publications in this period, one, "Mémoire sur le nombre de valeurs . . .," introduces the so-called problem of Bertrand: to find the subgroups of the symmetric groups of lowest possible index. Another publication, "Mémoire sur la théorie des courbes à double courbure" (1850), discusses curves with the property that a linear relation exists between first and second curvature; these are known as curves of Bertrand.

In 1852 Bertrand became professor of special mathematics at the Lycée Henry IV (then Lycée Napoléon). He also taught at the École Normale Supérieure. In 1856 he replaced Jacques Charles François Sturm as professor of analysis at the École Polytechnique, where he became the colleague of Duhamel. He then left secondary education to pursue his academic career. In 1862 he succeeded Biot at the Collège de France. Bertrand held his position at the École Polytechnique until 1895, that at the Collège de France until his death.

In 1856 Bertrand was elected to the Académie des Sciences, where in 1874 he succeeded the geologist Élie de Beaumont as *secrétaire perpétuel.* In 1884 he replaced the chemist Jean-Baptiste Dumas in the Académie Française. These high academic positions, combined with his erudition, his eloquence, and his natural charm, gave him a position of national prominence in the cultural field.

During the Commune of 1871 Bertrand's Paris house was burned, and many of his manuscripts were lost, among them those of the third volume of his textbook on calculus and his book on thermodynamics. He was able to rewrite and publish the latter as *Thermodynamique.* Afterward he lived at Sèvres and then at Viroflay. At his home Bertrand enjoyed being the center of a lively intellectual circle. Many of his pupils became well-known scientists—for instance, Gaston Darboux, who succeeded him as *secrétaire perpétuel.* In his *Leçons sur la théorie générale des surfaces,* Darboux elaborated many results of Bertrand and his mathematical circle.

Bertrand's publications, apart from his textbooks, cover many fields of mathematics. Although his work lacks the fundamental character of that of the great mathematicians of his period, his often elegant studies on the theory of curves and surfaces, of differential equations and their application to analytical mechanics, of probability, and of the theory of errors were widely read. Many of his articles are devoted to subjects in theoretical physics, including capillarity, theory of sound, electricity, hydrodynamics, and even the flight of birds. In his *Calcul des probabilités,* written, like all his books, in an easy and pleasant style, there is a problem in continuous probabilities known as Bertrand's paradox. It deals with the probability that a stick of length $a > 2l$, placed blindly on a circle of radius l, will be cut by the circle in a chord of less than a given length $b < 2l$. It turns out that this probability is undetermined unless specific assumptions are made about what constitute equally likely cases (i.e., what is meant by "placed blindly").

From 1865 until his death Bertrand edited the *Journal des savants.* For this periodical, as for the *Revue des deux mondes,* he wrote articles of a popular nature, many dealing with the history of science. This interest in history of science appears also in the many *éloges* he wrote as *secrétaire perpétuel* of the Academy, among which are biographies of Poncelet, Élie de Beaumont, Lamé, Leverrier, Charles Dupin, Foucault, Poinsot, Chasles, Cauchy, and F. F. Tisserand. He also wrote papers on Viète, Fresnel, Lavoisier, and Comte, and books on d'Alembert and Pascal.

Bertrand spent the later part of his life in the midst of his large family, surrounded by his friends, who were many and distinguished. His son Marcel and his nephews Émile Picard and Paul Appell were his fellow members in the Académie des Sciences. In 1895 his pupils gave him a medal in commemoration of his fifty years of teaching at the École Polytechnique. The influence of Bertrand's work, however, is hardly comparable to that of several of his contemporaries and pupils. Lest it be judged ephemeral, it must be viewed in the context of nineteenth-century Paris and of Bertrand's brilliant academic career, his exalted social position, and the love and respect given him by his many pupils.

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BERTRAND, MARCEL-ALEXANDRE (*b.* Paris, France, 2 July 1847; *d.* Paris, 13 February 1907), *geotectonics, stratigraphy, general geology.*

Bertrand's father was the mathematician Joseph Bertrand. Marcel studied at the École Polytechnique and the École des Mines in Paris. After graduation he worked in the Geological Survey of France, and in 1886 he succeeded his teacher Béguyer de Chancourtois at the École des Mines. In 1896 the Académie des Sciences elected him to the chair Pasteur had held.

Inspired by the writings of Eduard Suess, Bertrand always maintained a concern for what he called the grand problems of general geology. Early in his career he devoted his attention to the general problems of mountain structure while producing a dozen sheets of the geologic map of France. He solved the anomaly of le Beausset (and was awarded the Prix Fontannes by the Geological Society of France for it in 1889) by discovering that the islands of Triassic sediments resting on Cretaceous formations are the eroded remains of an enormous overturned fold. His conception of very large-scale overturned folds and overthrusts related the geological structure of Provence to that of the Alps. Bertrand was the first to conceive of the overthrust structure of the Alps, and by this theory of *grandes nappes* he attempted to connect the structures of the Pyrenees, Provence, and the Alps. His analysis of horizontal crustal compression and the displacements resulting from it won the Prix Vaillant of the Académie des Sciences of the Institut de France in 1890, but the essay was not published until 1908.

Bertrand developed an orogenic wave concept that he used to separate earth history into natural divisions on the basis of successive periods of intense folding and orogeny, each division identified with a chain of mountains. Working from Suess's brilliant synthesis, Bertrand demonstrated in 1887 that the Caledonian, Hercynian, and Alpine deformation produced consecutively those three mountain chains, thus building up the European continent gradually from north to south.

In 1894, at Zurich, Bertrand offered his very original conception of the complete sedimentary cycle with its recurring facies; each cycle represented one of the fundamental deformations. He showed that four kinds of facies are repeated in the different mountain chains, typically gneiss, followed by schistous flysch, then coarse flysch and coarse sandstone. At this time he also added the Huronian orogeny of Precambrian time to the other three deformations. In essaying a mechanism for these orogenies, Bertrand revived, then abandoned, the tetrahedral plan of the earth of Lowthian Green and Michel-Lévy.

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