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A. Albert Baker, Jr.

CURTZE, E. L. W. MAXIMILIAN (*b.* Ballenstedt, Harz, Germany, 4 August 1837; *d.* Thorn, Germany [now Torun, Poland], 3 January 1903), *mathematics.*

Curtze was the son of Eduard Curtze, a physician, and of Johanna Nicolai-Curtze. He attended the Gymnasium in Bernburg and from 1857 to 1860 studied in Greifswald, primarily under Johann August Grunert. After he passed the teaching examination in 1861, he taught at the Gymnasium in Thorn, where one of his colleagues was the Copernicus scholar Leopold Prove.

Curtze had an excellent knowledge of the current mathematical literature and an unusual talent for languages; he translated many valuable mathematical works from Italian into German, an outstanding example being Schiaparelli's *Precursori di Copernico nell'antichità* (1876). He did not publish a comprehensive work on his main field—the editing of medieval manuscripts, especially those in the rich collection of the library of Thorn—but he did publish valuable reports on the treasures of this library (1871; 1873–1878). He was also responsible for editions of Oresme's *Algorismus proportionum* (1868) and of his mathematical writings (1870), the *Liber trium fratrum de geometria* (1885), Peter of Dacia's commentary on Sacrobosco's *Algorisms* (1897), and Anaritius' commentaries on Euclid's *Elements* (1899). Other publications are the collection *Urkunden zur Geschichte der Mathematik im Mittelalter und der Renaissance* (1902) and a carefully researched biography of Copernicus (1899).

Curtze contributed many essays to mathematical journals and journals of the history of science. His work was greatly aided by his skill in deciphering hard-to-read handwriting and by visits to libraries in Uppsala and Stockholm (1873) and central Germany (1896). He began corresponding with Moritz Cantor in 1865 but did not meet him until 1896.

In his time Curtze was the outstanding expert on medieval mathematical texts. Through his careful editions he pointed out new paths in a field that was then little investigated.

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J. E. Hofmann

CUSA, NICHOLAS, also known as **Nikolaus von Cusa, Nicolaus Cusanus** (*b.* Kues, Moselle, Germany, *ca.* 1401; *d.* Todi, Umbria, Italy, 11 August 1464), *philosophy, mathematics.*

Nicholas was the son of a well-to-do Moselle fisherman, Johann Cryffts (or Krypffs, or Krebs), and his wife Katharina Römer. He may have received his early education at Deventer, in the Netherlands, in the school kept by the Brothers of the Common Life; he would thus have been early influenced by the *devotio moderna* movement. (This reform movement, which strove for a practical, Christocentric religious practice, was spread throughout the low countries and the Rhineland at that period.) Nicholas then entered the University of Heidelberg (he went as companion to Count Ulrich von Manderscheid) and presumably took the introductory course in philosophy there.

In 1417 Nicholas went on to the University of Padua, where he studied canon law with Prosdocimo

de Comitibus and others; in 1423 he earned the degree *doctor decretalium.* At Padua, Nicholas met the physician Toscanelli—with whom he attended the lectures on astrology of Prosdocimo Beldomandi—and the humanist educators Guarino da Verona and Vittorino da Feltre. He also attended the penitential sermons of the Franciscan Bernardino of Siena.

In 1425 Nicholas entered the service of Elector Otto von Ziegenhain, archbishop of Trier. In 1426, he was in Cologne, where he had presumably come as a teacher of canon law (although he may have studied theology there in 1425). In Cologne, Nicholas began his independent researches into original source materials, probing deep into the annals of Germanic law; he was thus able to prove that the Donation of Constantine was in fact an eighth-century forgery. He also found copies of Latin writings long believed to be lost; the most important of these were the *Natural History* of Pliny the Elder and twelve comedies of Plautus. His fame spread; in 1428 and again in 1435 he was offered a post as teacher of canon law at the newly established University of Louvain, but he turned it down in each instance.

It was at this period that Nicholas became associated with Heimeric von Campen, who led him into closer acquaintance with the Scholastics as well as introducing him to Proclus' commentary on Plato's *Parmenides* and to the writings of Ramón Lull (whose work Nicholas annotated and, around 1428, copied out in part with his own hand).

Nicholas was ordained priest *ca.* 1430. In this year, too, the archbishopric of Trier was claimed by Ulrich von Manderscheid. In the contest that followed, Nicholas acted as Ulrich's representative in pleading his case before the Council of Basel, convened in 1431. (It was later reconvened in Ferrara and then in Florence.) He did not succeed in getting the council to grant Ulrich's request, but he was recognized as a participant and became one of the leading spokesmen for the conciliar faction. Nicholas dedicated his book *De concordantia catholica* (1433) to his fellow councillors; in it he based his arguments on the thesis that the authority of an ecumenical council was superior to that of a pope and elaborated a comprehensive system of government and society. He stressed the church as the supreme earthly society, and found a divine pattern for priestly concord. (That he could be more realistic as a politician is shown by his 1433 proposal for negotiations with the Hussites.) While in Basel, Nicholas met the humanist Piccolomini, who in 1440 became secretary to Pope Felix V and in 1458 himself became Pope Pius II.

In 1436 the council received an appeal from Christian Byzantium, then sorely pressed by the Turks. The prospect of union of the eastern and western churches triggered stormy arguments in Basel, in which Nicholas sided with the pope's party. In 1437 he took part in an embassy sent to Constantinople to bring the princes of the Byzantine church and the emperor John VIII Palaeologus to the west. Nicholas knew some Greek; he also brought from Constantinople manuscripts containing reports on the councils of Constantinople and Nicaea which shed new light on the doctrinal dispute that centered on the text of the Creed. (Although formal unification of the two churches was achieved in 1437 it was not recognized by Constantinople.)

Through his new alliance with the papal minority Nicholas was given other diplomatic missions. During 1438 and 1439 he was engaged in constant negotiations with the German princes, most of whom either supported the majority conciliar position or wished to remain neutral. It was in part through Nicholas' efforts that these princes agreed to recognize the authority of Pope Eugene IV and his successor Nicholas V. Coincident with his diplomatic work, Nicholas had begun his major philosophical treatise, *De docta ignorantia,* finished in 1440 (by his own account he had struck upon its key notion, the *coincidentia oppositorum,* on his way back from Constantinople).

Nicholas' work puts mathematics and experimental science at the service of philosophy in his attempt to describe the limits of human knowledge. In the *De docta ignorantia* he made new interpretations of those philosophically oriented introductory books of mathematics with which he was acquainted, most notably Boethius' *Institutio geometrica* and Thomas Bradwardine's *Geometria speculativa.*

Trained in the methods of Aristotelian logic, Nicholas found them inadequate to his purpose, since he considered them applicable only to finite phenomena. The Divine, being infinite, is inaccessible to the mind of man, but may be approached through a method of symbolic visualization which resolves apparent antitheses; necessarily, however, at the end of the process man must acknowledge his ignorance. Thus, if the truth is probably inaccessible to the mind of man, man can intellectually get closer and closer to it through the sum of his private knowledge—without ever quite reaching it, since the truth represents an absolute, unchanging maximum limit beyond the scope of man's understanding.

Nicholas made extensive use of geometric figures in his *visio intellectualis;* he chose them because the rational language of demonstration was not suited to explain *intellectus*—the power of knowing that is superior to human reason. Thus one could, for example, increase the number of vertices of a regular

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polygon until, in infinity, it was transformed into a circle; and while triangles, squares, and circles differed from each other on a finite level, they were resolved beyond it on the infinite scale—although man, of course, could never know such ultimate resolution, being bound by finity to approximations only. Likewise, the contradiction of opposites of a straight line and a circle may be resolved in infinity, since a circle of infinitely long radius has a straight line for its circumference.

As understood by Nicholas, the infinite could take two forms, the infinitely large and the infinitely small. No contradiction was here implied, however, since the infinitely large and the infinitely small could both be contained in the concept of maximum; the largest possible thing was of maximum largeness, while the smallest possible thing was of maximum smallness. The concept of maximum admitted one absolute maximum (God) which could also be seen as unifying absolute maximum and absolute minimum, being infinite and therefore without degree.

Such geometrical examples are the essence of the *coincidentia oppositorum,* by means of which Nicholas hoped to resolve all problems formerly considered insoluble. Since apparent contradictions are united in infinity, the largest possible number must coincide with the smallest possible—one—and since numbers are discrete entities, all are contained in the ultimate unity and can be produced from that unity, which is also the measure of all intermediary quantities. Pursuant to this reasoning Nicholas referred to Anaxagoras and assumed that each entity is present in every other entity. In geometry, continuous forms correspond to numbers; the point stands alone as the smallest initial unit and generates lines, surfaces, and solid forms. The most perfect geometric forms—the infinitely large circle and the infinitely large sphere—are at the same time coincident with their generating point.

In cosmology, the application of the *coincidentia oppositorum* led Nicholas to determine that there could be no cosmic mechanism or center point for the motions of the heavens, since such a point of necessity included the whole universe. The universe mirrors God and is a relative maximum, since it contains all things except God, in Whom all is contained; therefore, the universe has no fixed center and no circumference, being relatively infinite. Therefore, the earth is not the center of the universe, nor is it stationary; it moves, as do all other bodies in space, with a motion that is not absolute but relative to the beholder. Nicholas further suggests by analogy that the earth may not be the only body that supports life. Moreover, the earth is not completely round, there is no maximum movement of the other heavenly bodies as opposed to the fixity of the earth, and no possible movement of bodies in diametrically opposite directions (such as up and down). Nicholas' cosmological reasoning, although garbed in theological language, here anticipates scientific discovery; his later treatise on the subject, *De figura mundi* (1462) is unfortunately lost.

Also lost is an earlier work, *De conjecturis,* of which only a later, much revised version of some time prior to 1444 exists. In the *De docta ignorantia,* Nicholas makes several specific corrections to the earlier version, from which some indication of its contents may be gained; he draws upon earlier notions of what is knowable, which he restates. It is also clear that the earlier book made the same extensive use of symbolic reasoning as the later, although its purport was more clearly metaphysical. Nicholas' earlier cosmology drew upon the Neoplatonic notions of the hierarchy of God–angel–soul–body and united it with the four elements to produce a metaphysics of unity, to which he assigned analogous mathematical values—i.e., he assigned the numbers 1, 10, 100, and 1,000 to the four elements and maintained the Pythagorean relation $1 + 2 + 3 + 4 = 10$ as a symbol of the *arithmetica universalis.* He paid special attention to the relationship of oneness and otherness, which he represented as two opposed quadratic pyramids, conjoined so that each had its vertex at the center point of the other's base plane. One pyramid stood for light, the other for darkness, one for the male principle, the other for the female, and so on. In the *De docta ignorantia* Nicholas was able to use his newly formulated doctrine of the *coincidentia oppositorum* to resolve these contradictions and to develop his theory of a unity in which each form partakes of and mirrors every other form.

Although the *De docta ignorantia* was both respected and influential, such speculations left Nicholas open to attack from his political enemies for pantheistic teachings and other damnable heresies. Johannes Wenck acted as spokesman for Nicholas' detractors in publishing *De ignota litteratura,* against which Nicholas defended himself with his *Apologia doctae ignorantiae* (1449), quoting at length from Dionysius and Areopagite and the Church Fathers.

Following the publication of the *Apologia doctae ignorantiae* Nicholas undertook further diplomatic missions for the Curia and did not expound or develop his system in more extensive writings. He did, however, dictate a number of short treatises which were copied down and circulated among his friends. In addition, some 300 of his sermons, dating from 1431 on, were recorded in the form of brief notes.

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(These sermons, given in both Latin and vernacular German, were at first devotional exercises preached on holidays, but later Nicholas began to introduce his own philosophical tenets into them, and by 1444 the mystical influence of Meister Eckhart is apparent in them.) The sermons provided the bases for Nicholas' later tracts, including *De quaerendo Deum* (1445), an orthodox devotional guide; and *De filiatione Dei* (1445), *De ultimis diebus* (1446), *De genesi* (1447), and *De Deo abscondito* (1450[?]), which derive in part from ideas set forth in *De conjecturis* and *De docta ignorantia.* His *De dato patris tris luminum* (1445 or 1446) and *De visione Dei* (1458) both argue against his alleged pantheism.

Nicholas' services to the Curia were rewarded in 1446 when Pope Eugene IV appointed him a cardinal *in petto;* in 1448 Pope Nicholas V made him a full cardinal, with the titular see of St. Peter in Vincoli. Nicholas received the red hat in 1450 and was named bishop of Brixen, where the cathedral chapter recognized him only reluctantly. At the end of 1450 he was appointed legate for Germany; as such he undertook a journey through Germany, Belgium, and the Netherlands where he preached reform and worked for compromise and conciliation in secular and ecclesiastical disputes. He soon became embroiled in violent quarrels with the nobility; Duke Sigismund of Tirol intervened and threatened to use force.

In 1458 Nicholas went to Rome, where his friend Piccolomini had been elected Pope Pius II. On his return to Germany in 1460, he was immediately locked up in Bruneck castle by Duke Sigismund's mercenaries; eventually Nicholas capitulated to the secular forces and returned to Rome. He was there appointed a papal representative.

During this period, in addition to his sermons, Nicholas undertook brief works on mathematics, to which he sought to apply some of the new insights that he had reached philosophically. His principal aim was to transform a circle into a straight line and a square; he confined himself to approximations. In *Transmutationes geometricae* (1445) he displays familiarity with simpler Euclidean theorems and appears to rely heavily on Bradwardine's *Geometria speculativa,* which he never mentions, however. He further refers to a fragment of the writings of Pappus, and shows considerable knowledge of the practical geometries available to him, drawing upon them for his many examples of applied geometry.

Nicholas' *Complementa arithmetica* may also date from 1445, although the exact year of its first publication is not known and it now exists only in a corrupted form. In it, Nicholas first expressed the idea that the difference between the radii of a series of circles inscribed in a series of regular polygons is proportional to the difference in area between the inscribed circle and its corresponding regular polygon.

In his *De circuli quadratura* (1450) Nicholas used a computational approach similar to that employed in his *Transmutationes geometricae;* it is uncertain what earlier authors may have influenced this work. The *De circuli* is remarkable for its discussion of the disputed intermediate value theorem, derived from Aristotle, and of the angle-of-contingence problem and the problem of exactly squaring a circle. Two of Nicholas' *Idiota* dialogues, *De sapientia* and *De mente* (both 1450) are the philosophical synthesis of his mathematical work at this time: in them, a layman who recognizes God's work in nature is given an opportunity, in the form of Platonic dialogues, to explain Cusan philosophy—especially its mathematicizing tendencies—to an Aristotelian scholar. (A third such dialogue, *Idiota de staticis experimentis,* of the same year, has a more practical bias, and contains numerous methods for determining physical parameters through the use of such apparatus as scales and a water clock—for example, the work tells in detail how to determine the humidity of air by measuring the weight of wool.)

In 1452 Nicholas read Jacob of Cremona's translation of the works of Archimedes, and was much impressed with the latter's indirect method of deduction, which he erroneously associated with his own *coincidentia oppositorum.* He had by now thought through his earlier ideas, and set them down systematically in a series of works, beginning with the *Complementa mathematica* (1453; expanded in 1454), which contained many more approximations than the earlier books (although these dealt largely with special cases, and provided no new insights), and the *Complementum theologicum,* a continuation that pertained primarily to symbolic interpretations. Nicholas presented an important variation of the approximation proportion in the *Perfectio mathematica* (1458)—which, more importantly, also contained the notion that the method of *visio intellectualis* could yield an infinitely small arc of a circle and its corresponding chord (although inadequately expressed by Nicholas, this anticipated an infinitesimal concept of great significance). The *Aurea propositio in mathematicis* (1459) contains the final refinement of these ideas.

During the 1450's, Nicholas continued to apply his doctrine of *coincidentia oppositorum* to religious, as well as mathematical, problems. The shattering event of the year 1453—the fall of the Byzantine empire—led him to write *De pace fidei,* published in that year. In it, he presents a dialogue among seventeen articulate representatives of different nations and faiths and

calls for mutual tolerance. Although he maintains the superiority of Christianity, he proposes that the differences among faiths are largely those of ritual and stresses the unifying factor of monotheism. He further undertook a thorough and critical investigation of Islam; his *Cribatio Alkoran* (published in 1461) is, however, based upon a poor translation of the Koran.

Nicholas' later works also include a number of relatively short treatises in which simple physical or mathematical examples are expanded into philosophical symbols. In *De beryllo* (1458) he compares the *visio intellectualis* to the effect of a magnifying glass; and in *De possest* (1460) he uses the example of a circular disc rotating at infinite speed within a stationary ring to show that all points of the circumference of the disc are at all points of the interior of the ring simultaneously—hence, motion and rest are identical, and thus time unfolds from the present and thus the single instant and eternity are the same in infinity. In *De ludo globi* (1463) he moves on to discuss the spiral motion of a partially concave sphere, and in *De non aliud* (1462) he reverts to dialogue form for a critical conversation with Aristotle, Plato, and Proclus. *De venatione sapientiae* (1463) stands in relation to his philosophy as the *Aurea proposito in mathematicis* does to his mathematics; it is a retrospective summation of his earlier attempts to illuminate for others his private intellectual world.

As a philosopher, Nicholas was chiefly concerned with knowing the ways of God; his mystical and symbolic approach was calculated to encourage man to seek the unity of all things and the end of antitheses in light of his insight that the Divine could not be known directly. He rejected the rationalism of the Schoolmen, and revitalized Neoplatonism in his time. That he failed always to be understood and appreciated by his contemporaries is due in part to the peculiarity of his language, neither medieval nor humanistic and flawed by inadequately defined words and concepts.

That he failed also to reach the ideal of conciliation that he preached may be attributed, too, to his character, since he was frequently hot-headed, temperamental, and inclined to arbitrary decisions. Despite his demonstrated skill as an imaginative diplomat, Nicholas spent the last years of his ecclesiastical career in Rome in a series of squabbles with the Italian clique and in ineffectual attempts to reform the clergy, the orders, and the Curia.

Only in the nineteenth century did the importance of Cusan thought begin to become clear, and only in the twentieth century was any thorough study of it begun. Precise study has been made possible through the happy circumstance that the home for the aged in Kues, generously endowed by Nicholas, has survived the ravages of time and war; its library, a chief source for Cusan scholars, has, except for minor losses, remained intact.

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J. E. Hofmann

CUSHING, HARVEY WILLIAMS (*b.* Cleveland, Ohio, 8 April 1869; *d.* New Haven, Connecticut, 7 October 1939), *neurosurgery, neurophysiology.*

Cushing was the sixth son and the tenth child (seven lived to maturity) of Henry Kirke and Betsey Maria Williams Cushing. Henry Kirke Cushing, a third-generation physician, combined a large practice with the professorship of midwifery, diseases of women, and medical jurisprudence at Cleveland Medical College and was also for many years a trustee of Western Reserve University. Reserved with his children, he left much responsibility for their upbringing to his wife, a gracious, highly intelligent woman quite capa-