Wirbeltier-Evolution und Faunengeschichte im Kaenozoikum (Berlin, 1983).

EMIL KUHN-SCHNYDER

DIRAC, PAUL ADRIEN MAURICE (*b.* Bristol, England, 8 August 1902; *d.* Miami, Florida, 20 October 1984), *quantum mechanics, relativity, cosmology*.

Dirac was one of the greatest theoretical physicists in the twentieth century. He is best known for his important and elegant contributions to the formulation of quantum mechanics; for his quantum theory of the emission and absorption of radiation, which inaugurated quantum electrodynamics; for his relativistic equation of the electron; for his "prediction" of the positron and of antimatter; and for his "large number hypothesis" in cosmology. Present expositions of quantum mechanics largely rely on his masterpiece *The Principles of Quantum Mechanics* (1930), and a great part of the basic theoretical framework of modern particle physics originated in his early attempts at combining quanta and relativity. Not only his results but also his methods influenced the way much of theoretical physics is done today, extending or improving the mathematical formalism before looking for its systematic interpretation.

Dirac spent most of his academic career at Cambridge and received all the honors to which a British physicist may reasonably aspire. He became a fellow of St. John's College at the age of twenty-five, a fellow of the Royal Society in 1930, Lucasian professor of mathematics in 1932, a Nobel laureate in 1933 for his "discovery of new fertile forms of the theory of atoms and for its applications," a Royal Medalist in 1939, and a Copley Medalist in 1952. He was frequently invited to lecture or to do research abroad. For instance, he traveled around the world in 1929, visited the Soviet Union several times in the 1930's, and was a fellow at the Institute for Advanced Studies, Princeton, in the years 1947–1948 and 1958–1959. In 1973 he was made a member of the Order of Merit. Dirac retired in 1969 but resumed his scientific career in 1971 at Florida State University. In January 1937 Dirac married Margit Wigner, the sister of Eugene Wigner; they had two daughters.

Dirac made his mark through his scientific writings. He had few students: the fundamental problems that he tackled were not for beginners. Unlike many of his colleagues, he was little involved in war projects.

Bristol. Dirac's mother, Florence Hannah Holten, was British; his father, Charles Adrien Ladislas Dirac, was an émigré from French Switzerland. His father did not receive friends at home and forced Paul to silence by imposing French as the language spoken at the dinner table. From childhood Dirac was a loner, enjoying the contemplation of nature, long walks, or gardening more than social life. He was not much inclined to collaboration and did his best thinking by himself. At the Merchant Venturer's Technical College, where his father taught French, he excelled in science and mathematics, and neglected literary and artistic subjects.

From 1918 to 1921 Dirac trained to be an electrical engineer at Bristol University. This background, he explained later, strongly influenced his way of doing physics: he learned how to tolerate approximations when trying to describe the physical world and how to solve problems step by step. He also developed a nonrigorous constructive conception of mathematics, beautifully articulating symbols before precisely defining them, very much as the British physicist Oliver Heaviside did in his calculus.

In 1921 the postwar economic depression prevented Dirac from finding a job, so he accepted two years of free tuition from the mathematics department at Bristol. During this period he was influenced by an outstanding professor of mathematics, Peter Fraser, who convinced him that rigor was sometimes useful and imparted to him his love for projective geometry, with its derivations of complicated theorems by means of simple one-to-one correspondences.

At Bristol, Dirac also attended Charlie Dunbar Broad's philosophy course for students of science, in which Broad criticized the fundamental concepts of science on the basis of Alfred North Whitehead's principle of extensive abstraction and argued that the ideal objects of mathematics must be constructed from the mutual relations—not the inner structure—of the roughly perceived objects of nature. This genesis was supposed to explain the relevance of geometrical concepts when they were applied to the physical world, particularly the success of Einstein's theory of relativity. For Broad, theorists were best when they were their own philosophers. Dirac also read John Stuart Mill's *System of Logic* (1843), but derived the opposite conclusion: that philosophy was "just a way to think about discoveries already made."

Broad's lectures included a serious account of the theory of relativity, which immediately fascinated Dirac. Arthur S. Eddington's *Space, Time and Gravitation* (1920), written in the euphoric period after the British eclipse expedition confirming Albert Einstein's theory in 1919, made a further impression on Dirac. Evidence of epistemological comments by "the fountainhead of relativity in England" can be found in several places in Dirac's work.

Cambridge. In the fall of 1923, Dirac entered St. John's College, Cambridge, as a research student, thanks to an 1851 Exhibition studentship and a grant from the department of scientific and industrial research for work in advanced mathematics. He hoped to study relativity with Ebenezer Cunningham, but was assigned Ralph Fowler as his adviser. Fowler was not only a preeminent specialist in statistical mechanics but also the enthusiastic leader of quantum theoretical research at Cambridge. As a correspondent of Niels Bohr, he regularly got information about the latest advances or failures in atomic theory. As the son-in-law of Ernest Rutherford, he took a strong interest in the experimental work at the Cavendish Laboratory (at Cambridge, theoretical physics was part of the Faculty of Mathematics).

Because of his retiring personality and the relative isolation of the various colleges, Dirac did not have any regular scientific interlocutor but Fowler. To compensate, he joined two physicists' clubs, the $\nabla^2 V$ Club and the more casual Kapitza Club, where theorists and experimenters discussed recent problems and welcomed foreign visitors. He also attended the colloquia at the Cavendish and, to keep up with developments in fundamental mathematics, took part in the tea parties of the distinguished Cambridge mathematician Henry Frederick Baker, who was concerned primarily with projective geometry.

Before arriving in Cambridge, Dirac did not know about the Bohr atom. This gap in his knowledge was quickly and excellently filled by Fowler's detailed lectures. Dirac also read Arnold Sommerfeld's textbook *Atomic Structure and Spectral Lines* (English ed., 1923), Bohr's *On the Application of the Quantum Theory to Atomic Structure* (1923), and Max Born's *Vorlesungen über Atommechanik* (1925). These three fundamental texts involved advanced techniques of Hamiltonian dynamics (to derive the most general expression of the rules of quantization), which Dirac learned from Edmund T. Whittaker's standard text, *A Treatise on the Analytical Dynamics of Particles and Rigid Bodies* (1904). Perhaps more than anything in quantum theory he enjoyed reading Eddington's *Mathematical Theory of Relativity* (1923), which developed the tensor apparatus of Einstein's and Hermann Weyl's theories of gravitation. They became his models of beauty in mathematical physics.

Fowler was quick to detect the qualities of his new student and began to encourage his originality. Only six months after arriving in Cambridge, Dirac started to publish substantial research papers. Whenever his subject had not been imposed by Fowler, he tried to clarify and to generalize in a relativistic way points that he had found obscure in his readings—for instance, the definition of a particle's speed according to Eddington, or the covariance of Bohr's frequency condition, or the expression of the collision probability in the then-fashionable "detailed balancing" calculations. The main characteristics of Dirac's style showed through in this early work: directness, economy in mathematical notation, and little reference to past work.

At the end of 1924, following suggestions by Fowler and Darwin, Dirac focused on the more fundamental problem of generalizing the application of Paul Ehrenfest's adiabatic principle in quantum theory. According to this principle, the quantum conditions for a complicated system could be obtained by infinitely slow ("adiabatic") deformation of a simpler system for which one knew to which variables q the Bohr-Sommerfeld rule $\int pdq = nh$ applied.

Another method, introduced by Karl Schwartzschild and systematized by Johannes Burgers, applied to the so-called multiperiodic systems, the configuration of which can be expressed in terms of s periodic functions with s incommensurable frequencies $\omega_1, \omega_2, \ldots \omega_\alpha, \ldots \omega_s$. One had only to introduce the "angle" variables $w_\alpha = \omega_\alpha t$ and the corresponding Hamiltonian conjugates, the "action" variables J_α. In the nondegenerate case for which s is also the number of degrees of freedom, the quantum conditions can simply be written $J_\alpha = n_\alpha \hbar$, where $2\pi\hbar$ is Planck's constant. Burgers showed that this procedure was equivalent to the adiabatic principle because the J's are adiabatic invariants. Dirac increased both the rigor of the demonstration and its scope, including magnetic fields and degeneracy. He also tried to remove the restriction of multiperiodicity and to calculate the energy levels of the helium atom, but he failed. Presumably he believed that a good part of the difficulties of quantum theory could be solved by extension of the adiabatic principle without facing the basic paradoxes emphasized by Bohr and the Göttingen school. Bohr's correspondence principle did not trigger Dirac's interest as a hint toward a fundamentally new quantum mechanics. His only consideration of it was purely operational, as a set of rules to derive intensities of emitted radiation in the action-angle formalism.

Commutators and Poisson Brackets. In 1925 Bohr and Werner Heisenberg both brought their revolutionary spirit to Cambridge. Bohr lectured in May after being distressed by the results of Walther Bothe and Hans Geiger's experiment confirming the light-quantum explanation of the Compton effect and making the paradoxical features of light more obvious than ever. According to Bohr, Pauli, and Born, the crisis in quantum theory had reached its climax. The world needed a new mechanics that would preserve the quantum postulates and agree asymptotically with classical mechanics. Heisenberg came to Cambridge in July 1925 with what soon proved to meet this expectation. He lectured at the Kapitza Club, on "term zoology and Zeeman botanics"—that is, on his latest theory of spectral multiplets and anomalous Zeeman effects. It is not known how much of this talk dealt with more recent ideas, nor if Dirac in fact attended it. Fowler certainly heard of Heisenberg's brand-new "quantum kinematics" in private conversations, and asked to be kept informed.

In late August or early September, Fowler gave Dirac the proof sheets of Heisenberg's fundamental paper, "A Quantum-Theoretical Reinterpretation (*Umdeutung*) of Kinematics and Mechanical Relations." Heisenberg had replaced the position x of an electron by an array $x_{nm}e^{i(E_n - E_m)t/\hbar}$ representing the amplitudes of virtual oscillators directly giving the observable properties of scattered or emitted radiation corresponding to the energy levels E_m and E_n. To keep the new kinematics as analogous as possible to the classical one, he guessed the multiplication law of two arrays x_{nm} and y_{nm} from the corresponding rule for the Fourier coefficients of x and y, and obtained $(xy)_{nl} = \sum_m x_{nm}y_{ml}$. In the same way he guessed the quantum version of the quantization rule $\int pdq = nh$ as $\sum_m |q_{nm}|^2(E_m - E_n) = \hbar^2/2\mu$ (μ being the electron mass). The dynamics—the equation of evolution for x—was taken over from classical dynamics. At that point the most advanced quantum problem that Heisenberg could solve was the weakly anharmonic oscillator. The "essential difficulty," he noticed, was the fact that, according to the new multiplication rule, $xy \neq yx$.

Since there was no familiar Hamitonian formalism in Heisenberg's paper, it was about ten days before Dirac realized that the new multiplication law might solve the difficulties of quantum theory. He first looked for a relativistic generalization of Heisenberg's scheme, but this proved premature. More successfully, he tried to connect it to a Hamiltonian formalism. The difference $xy - yx$, once evaluated for high quantum numbers and in terms of action-angle variables J and w, gave $i\hbar \sum_\alpha \frac{\partial x}{\partial w_\alpha}\frac{\partial y}{\partial J_\alpha} - \frac{\partial y}{\partial w_\alpha}\frac{\partial x}{\partial J_\alpha}$, that is, the classical Poisson bracket $\{x, y\}$ times $i\hbar$. In other words, Heisenberg's strange noncommutativity had a classical counterpart in the Poisson-bracket algebra of Hamiltonian mechanics. Dirac then assumed that the relation $xy - yx = i\hbar\{x, y\}$ held in general (far from the classical limit and for nonmultiperiodic systems) and provided the proper quantum conditions. For canonically conjugate variables p and q it reduced to $qp - pq = i\hbar$, containing Heisenberg's quantization rule.

Dirac was very pleased with this close analogy between classical and quantum mechanics because it allowed him to retain the "beauty" of classical mechanics and to transfer Hamiltonian techniques to quantum mechanics. Hence he could develop very quickly a version of quantum mechanics more elegant than that developed at Göttingen.

q-Numbers. The identity between commutator and Poisson brackets led to the fundamental equations $i\hbar g = gH - Hg$ (for any dynamical variable g evolving with the Hamiltonian H) and $qp - pq = i\hbar$ (for any canonical couple), determining the formalism of quantum mechanics. Dirac thought that Heisenberg's interpretation of the quantum variables in terms of matrices giving the observable properties of radiation was provisional and too restrictive; he preferred a symbolic approach, developing the algebra of abstract undefined "q-numbers" and looking only later for those numbers' representation in terms of observable (ordinary) "c-numbers." The domain of q-numbers had to be extensible, adapting to the further progress of the theory. Some of the axiomatic properties that Dirac imposed on them—for instance, the unicity of the square root and no divisor of zero—had to be dropped later because they cannot be realized in an algebra of operators. Dirac's idea of q-numbers and his axioms for them most probably originated at Baker's tea parties. In Baker's *Principles of Geometry* there is an abstract noncommutative algebra of coefficients for linear combinations of points, which permitted elegant and condensed proofs of theorems in projective geometry (where noncommutativity means dropping Pappus's theorem).

In the case of multiperiodic systems, Dirac could show that his fundamental equations were satisfied by an algebra of matrices with rows and columns corresponding to integral values (times h) of the action variables J. In this representation the energy matrix is diagonal, which suggests that the diagonal elements represent the spectrum of the system. Through a correspondence argument Dirac identified the matrix element $x_{J'J''}$ of the electric polarization with the amplitude of the corresponding transition $J' \rightarrow J''$, in accordance with Heisenberg's original definition of the position matrix. In this representation Dirac could solve the hydrogen atom in early 1926 (a little later than Wolfgang Pauli, but independently). Within a few months he also found the basic commutation and composition rules for angular momentum in multielectron atoms, and he made the first relativistic quantum-mechanical calculation giving the characteristics of Compton scattering. Physicists in Copenhagen were impressed by this achievement, the more so because Dirac treated the field classically, without light quanta.

Dirac assembled all these bright results in his doctoral dissertation, completed in June 1926. At that time he had solved by himself about as many quantum problems as the entire Göttingen group together. In principle his q-numbers were more general and more flexible than the Göttingen matrices, which were rigidly connected to a priori observable quantities. But Dirac had been able to solve the quantum equations only insofar as action-angle variables could be introduced into the corresponding classical problem. To proceed further, he needed a new method of finding representations of q-numbers. That is exactly what Erwin Schrödinger made available in a series of papers submitted for publication between January and June 1926.

The Impact of Schrödinger's Equation. Dirac's first reaction to Schrödinger's equation was negative: Why a second quantum mechanics, since there already was one? Why propose that matter waves were analogous to light waves, since the properties of light waves were already so paradoxical? In a letter written on 26 May 1926, Heisenberg convinced him that Schrödinger's equation $H\left(q, -i\hbar\frac{\partial}{\partial q}\right)\psi_n = E_n\psi_n$ (for one degree of freedom) provided a simple and general method to calculate the matrix elements of a general function F of p and q just by forming the integrals $F_{mn} = \int\psi_m^*(q)F\left(q, -i\hbar\frac{\partial}{\partial q}\right)\psi_n(q)\ dq$. Then, in an astonishingly short time, Dirac accumulated new essential results. The time dependence of the matrix elements could be supplied by the equation $H\psi = i\hbar\partial\psi/\partial t$, suggested by the relativistic substitution $p_\mu \rightarrow i\hbar\partial/\partial x_\mu$. A set of identical particles, following Heisenberg's idea of eliminating unobservable differences from the formalism, had to be represented by either symmetric or antisymmetric wave functions in configuration space, the first corresponding to the Bose-Einstein statistics and the second to Pauli's exclusion principle. Finally, Dirac developed the time-dependent perturbation theory to calculate Einstein's B coefficients of absorption and stimulated emission. He also improved his calculation of the Compton effect. To reach these physical results he did not subscribe to Schrödinger's picture of $|\psi|^2$ as a density of electricity; instead he relied on Heisenberg's interpretation of the polarization matrix or on Born's statistical interpretation of the ψ function.

Interpretation of Quantum Dynamics. Dirac was not satisfied by the provisional and parochial assumptions made to interpret q-numbers and the quantum formalism: according to Heisenberg, the diagonal elements of H and the elements of the polarization matrix had an immediate meaning; according to a paper by Born (June 1926), the coefficients c_n in the development $\psi = \sum_n c_n\psi_n$ over the set of eigenfunctions ψ_n gave the probability $|c_n|^2$ for the system to be in the state n; and, according to Schrödinger's fourth memoir (June 1926), $|\psi|^2$ was "a sort of weight function in configuration space." In Dirac's view, a general interpretation should be based on a transformation theory, as in the theory of relativity (and as emphasized by Eddington).

To arrive at the interpretation, Dirac first worked out the transformations connecting the various matrix representations of his fundamental equations $qp - pq = i\hbar$ and $i\hbar\dot{g} = gH - Hg$. He called ξ and α two maximal sets of commuting q-numbers; ξ' and α', corresponding eigenvalues; and (ξ'/α'), the transformation from the representation where ξ is diagonal to the one where α is diagonal, acting on the representation $g_{\xi'\xi''}$ of g according to $g_{\alpha'\alpha''} = \int(\alpha'/\xi')g_{\xi'\xi''}(\xi''/\alpha'')\ d\xi'\ d\xi''$. In this framework the solutions of the (time-independent) Schrödinger equation were nothing but a particular transformation for which α contains H and ξ contains the position. The notations, introduced for the sake of economy and in obvious analogy to tensor notation, proved to be extremely convenient and spread widely, especially after their later improvement (1939) into the "bra-ket" (or "bra" and "ket") notation. In fact, the symbolic rules were better defined than the mathematical substratum, which was made clear only much later by mathematicians. For instance, the treatment of continuous spectra on the same footing as the discrete ones necessitated singular "δ-functions" (as in $(x'/x'') = \delta(x' - x'')$, perceived by Dirac as limits of sharply peaked functions but raised today to the rank of Schwartz distributions.

To interpret his transformations, Dirac needed only a minimal assumption suggested by the correspondence principle: that for an arbitrary physical quantity g expressed in terms of ξ and the canonical conjugate η, $g_{\xi'\xi'}$ signifies the average of the corresponding classical g for $\xi = \xi'$ and η uniformly distributed. From $\delta(g - g')|_{\xi'\xi'} = |(\xi'/g')|^2$ it follows that $|(\xi'/g')|^2 \, dg'$ is proportional to the probability that g is equal to g' within dg' when $\xi = \xi'$. Dirac finished this transformation theory in November 1926 at Copenhagen.

In Göttingen, Pascual Jordan obtained roughly the same results at the same time, though from a different point of view. He defined axiomatically a concept of canonical conjugation at the quantum level and looked for the transformations $(\xi, \eta) \rightarrow (\alpha, \beta)$ from one canonical couple to another. In this more general framework the quantum variables did not necessarily have a classical counterpart, and conjugation did not necessarily correspond to Poisson-bracket conjugation. In other words, Dirac's transformation theory was more constraining than Jordan's, and gave more precise directions for the future extensions of quantum mechanics.

Dirac was also original in his conception of the role of probability in quantum mechanics. He thought that probabilities entered into the description of quantum phenomena only in the determination of the initial state (still described in terms of p's and q's), and not necessarily in the behavior of an isolated system. But, as Bohr had said at the Solvay Conference in 1927, isolated systems were unobservable. Dirac then assumed that the state of the world was represented by its wave function ψ and that it changed abruptly during a measurement, whereupon "nature made a choice."

Dirac retained his basic machinery of transformations in his subsequent lectures on quantum mechanics, but he introduced a substantial change in his fundamental textbook, *The Principles of Quantum Mechanics* (1930). In the original exposition of transformation theory, he had carefully avoided the concept of quantum state, presumably to depart from Schrödinger's idea of ψ as a state. In his *Principles*, however, he presented the principle of superposition and the related concept of space of states as capturing the most essential feature of quantum theory: the interference of probabilities. It seems plausible that this move was inspired by Bohr's insistence on the superposition principle and by John von Neumann's and Hermann Weyl's formulations of quantum mechanics, in which Hilbert spaces played a central role. From this perspective transformations were just a change of base in the space of states. The correspondence with Hamiltonian formalism appeared only in a later chapter of the book.

A New Radiation Theory. Dirac liked his transformation theory because it was the outcome of a planned line of research and not a fortuitous discovery. He forced his future investigations to fit it. The first results of this strategy were almost miraculous. First came his new radiation theory, in February 1927, which quantized for the first time James Clerk Maxwell's radiation in interaction with atoms. Previous quantum-mechanical studies of radiation problems, except for Jordan's unpopular attempt, retained purely classical fields. In late 1925 Jordan had applied Heisenberg's rules of quantization to continuous free fields and obtained a light-quantum structure with the expected statistics (Bose-Einstein) and dual fluctuation properties. Dirac further demonstrated that spontaneous emission and its characteristics—previously taken into account only by special postulates—followed from the interaction between atoms and the quantum field. Essential to this success was the fact that Dirac's transformation theory eliminated from the interpretation of the quantum formalism every reference to classical emitted radiation, contrary to Heisenberg's original point of view and also to Schrödinger's concept of ψ as a classical source of field.

This work was done during Dirac's visit to Copenhagen in the winter of 1927. Presumably to please Bohr, who insisted on wave-particle duality and equality, Dirac opposed the "corpuscular point of view" to the quantized electromagnetic "wave point of view." He started with a set of massless Bose particles described by symmetric ψ waves in configuration space. As he discovered by "playing with the equations," this description was equivalent to a quantized Schrödinger equation in the space of one particle; this "second quantization" was already known to Jordan, who during 1927 extended it into the basic modern quantum field representation of matter. Dirac limited his use of second quantization electromagnetic to radiation: to establish that the corpuscular point of view, once brought into this form, was equivalent to the wave point of view.

The Dirac Equation. An even more astonishing fruit of Dirac's transformation theory was his relativistic equation of the electron. He and many other theorists had already made use of the most obvious candidate for such an equation—$(\hbar^2\partial_\mu\partial^\mu + m^2)\psi = 0$ (Klein-Gordon)—but it did not include the spin effects necessary to explain atomic spectra. More crucially for Dirac, it could not fit into the transformation theory because it could not be rewritten under the form $i\hbar\partial\psi/\partial t = H\psi$. To be both explicitly relativistic and linear in $\partial/\partial t$, the new equation had to take the form $(i\hbar\gamma^\mu\partial_\mu - m)\psi = 0$ or, more explicitly, $i\hbar\partial\psi/\partial t = \vec{\alpha} \cdot \vec{p} + \beta m$. For the spectrum to be limited to values satisfying Einstein's relation $E^2 = p^2 + m^2$, the coefficients $\vec{\alpha}$ and β had to be such that $(\vec{\alpha} \cdot \vec{p} + \beta m)^2 = p^2 + m^2$—that is, $\beta^2 = 1$, $\alpha_i\alpha_j + \alpha_j\alpha_i = 2\,\delta_{ij}$, and $\alpha_i\beta + \beta\alpha_i = 0$.

The simplest entities satisfying these relations are 4×4 matrices, as Dirac noted with the help of Pauli's $\vec{\sigma}$ matrices (such that $[\vec{\sigma} \cdot \vec{p}]^2 = p^2$). Surprisingly, the new equation included spin effects, the value 2 of the gyromagnetic factor, and the correct fine structure formula (Sommerfeld's), as worked out approximately by Dirac and exactly by Darwin and Walter Gordon. Other theorists (Pauli, Darwin, Jordan, Hendrik, Kramers) had been searching for a wave equation integrating spin and relativistic effects, but they all started by assuming the existence of spin, either as an intrinsic particle rotation or as a wave polarization. In contrast, the key to Dirac's success was his persistent adherence to the simplest classical model, the point-electron, as a basis for quantization. Spin effects, as might have been expected from their involving h, were a consequence of relativistic quantization.

Antimatter, Monopoles. The Dirac equation played an essential role not only in atomic physics but also in high-energy physics, through the Klein-Nishina and Møller formulas describing the absorption of relativistic particles in matter. Nevertheless, it presented several strange features that enhanced the "magic" of Dirac's work: a new type of relativistic covariance involving the spinor representations of the Lorentz group, soon elucidated by Göttingen mathematicians; the trembling of the electron imagined by Schrödinger to harmonize the observed electron speed and the expectation value c of the speed operator from Dirac's equation; and, above all, the negative-energy difficulty.

The equation $E^2 = p^2 + m^2$, applying to the spectrum of free Dirac electrons, has two roots: $E = \pm(p^2 + m^2)^{1/2}$; therefore a Dirac electron with an initially positive energy should fall indefinitely by spontaneous emission toward states of lower and lower energy. To avoid this, Dirac imagined in late 1929 that the states of negative energy were normally filled up according to the exclusion principle and that holes in this "sea" would represent protons. If this were true, Dirac had in hand a grandiose unification of the particle physics of his time. But he still had to explain the ratio m_p/m_e between the proton mass and the electron mass. He thought that the disparity in mass might originate in the mutual interaction between the "sea" electrons. The precise numerical value of the ratio would perhaps appear at the same time as the other dimensionless constant, $e^2/4\pi\hbar c$, as suggested by Eddington in a 1928 paper containing a mysterious derivation of this remarkable number.

Eddington believed that electromagnetic interactions could be reduced to the "exchange" interactions, the change of sign of a wave function owing to the permutation of two fermions or to a full rotation being of the same nature as the change of phase following an electromagnetic gauge transformation. At the end of his speculation, he got $e^2/4\pi\hbar c = 1/136$. In his search for a theoretical derivation of m_p/m_e and $e^2/4\pi\hbar c$, Dirac also concentrated on the phase of the wave function.

It is usually assumed that the phase of a wave function is unambiguously defined in space (for a given gauge). But a multivalued phase is also admissible, Dirac noted, as long as the variation of phase around a closed loop is the same for any wave function (to preserve the regular statistical interpretation of ψ based on quantities such as $|\int\psi_1^*(\vec{r})\psi_2(\vec{r})\,d^3r|$). To ensure the continuity of ψ, the variation of phase around an infinitesimal closed loop can only be a multiple of 2π. This determines lines of singularities starting from (gauge) invariant singular points. Now, following the relation between electromagnetic potential and phase implied by gauge invariance, the singular points must be identified with magnetic monopoles carrying the charge $g = n\hbar c/2e$. If there is only one monopole g in nature, every electric charge must be a multiple of $\hbar c/2g$. Dirac always considered this explanation of the quantization of charge in nature as the strongest argument in favor of monopoles.

Unfortunately, no other restriction on e followed from this line of reasoning and Dirac missed his targets, the derivation of $e^2/4\pi\hbar c$ and a subsequent determination of m_p/m_e. But the latter was no longer needed: in 1931 he learned from Weyl that, due to charge conjugation symmetry, the holes in his "sea" theory necessarily carried the charge $-e$. In the same year and in a single paper, he proclaimed the necessity of antielectrons (and also antiprotons) and the possibility of monopoles, and pondered the most efficient method of advance in theoretical physics. As in his quantum-theoretical work, he had first to work out the formalism in terms of abstract symbols denoting states and observables, and next to investigate the symbols' interpretation. This was, Dirac said, "like Eddington's principle of identification," according to which the interpretation of the fundamental tensors of general relativity came after their mathematical justification.

Dirac gave a full quantum-mechanical treatment of his monopoles in 1948 with the help of "nonphysical strings" allowing a Hamiltonian formulation. More recently monopoles have been shown to be necessary in any non-Abelian gauge theory, including electromagnetic interactions. But no experimental evidence has yet been found. On the other hand, the antielectron (or positron) was discovered by Carl Anderson and Patrick Blackett in the years 1932–1933, much earlier than foreseen by Dirac, although its concept faced the general prejudice against a charge-symmetric nature.

The Multitime Theory. After the discovery of the positron, most theorists agreed that the negative-energy difficulty was solved by the "sea" concept. Another fundamental difficulty, also rooted in one of Dirac's early works, his radiation theory, lasted much longer. In 1929, when working out their version of quantum electrodynamics, Heisenberg and Pauli discovered that the second order of approximation involved infinite terms, even when it was related to physical phenomena such as level shifts in atoms. The difficulty looked so serious that in the first edition of his *Principles*, Dirac omitted the quantization of the electromagnetic field and presented only the light-quantum configuration-space approach.

In 1932 Dirac tried to start a new revolution by giving up (for electrodynamics) the most basic requirement of his quantum-mechanical work: the Hamiltonian structure of dynamical equations. Imitating Heisenberg's revolutionary breakthrough, he declared that the new theory should eliminate unobservable things like the electromagnetic fields during the interaction process, and focus on their asymptotic values before and after the interaction. The electromagnetic field, he said, was nothing but a means of observation, and therefore should not be submitted to Hamiltonian treatment. On these lines he derived a set of equations that apparently were quite new; in fact, as Leon Rosenfeld soon pointed out, it differed from the theory of Heisenberg and Pauli only in the use of the interaction representation (for which the quantum fields evolve as free fields) and of a multitime configuration space for electrons (instead of Jordan's quantized waves). Nonetheless, once it had been improved with the help of Vladïmir A. Fock and Boris Podolsky, Dirac's formulation had the great advantage of being explicitly covariant, a feature particularly attractive to the Japanese quantum-field theorists Hideki Yukawa and Sin-Itiro Tomonaga.

The Large-Number Hypothesis. The infinities were still there. The discovery of the positron in 1932 gave some hope that the deformations of Dirac's "sea," the "vacuum polarization," would cure them. But such was not the case (although Wendell Furry and Victor Weisskopf made the infinities "smaller"), and Dirac himself judged the "sea" theory ugly. In 1936, depressed by this state of affairs, he hastily concluded from some experimental results of Robert S. Shankland that the energy principle should be given up in relativistic quantum theory. Needing some diversion, he turned to cosmological speculation following Eddington, who believed in a grand unification of atomic physics and cosmology. Dirac also knew Edward A. Milne, the other famous Cambridge cosmologist, who had been his supervisor for a term in 1925, and he had made friends with the American astronomer Howard P. Robertson, who believed in the expansion of the universe, during a short stay at Göttingen in 1927.

Like Eddington, Dirac focused on dimensionless numbers built from the fundamental constants of both atomic and cosmic phenomena; and he observed that there was a cluster of these numbers around 10^{39}, including the age of the universe in atomic time units and the ratio of electric forces to gravitational ones inside atoms. In 1937 he proposed the "large-number hypothesis," according to which numbers in the same cluster should be simply related. Consequently, the gravitation constant had to vary in time, as in Milne's cosmology and contrary to general relativity.

Milne believed in an "extended principle of relativity," which stipulated that the universe should look the same from wherever it is observed, and completed it by the stricture that the cosmological theory should not include any constant having dimensions. To elaborate his own cosmology further, Dirac provisionally adopted Milne's first principle (he rejected it later, in 1939) but replaced the second hypothesis—which conflicted with Eddington's idea that atomic constants should play a role in cosmology—with his large-number hypothesis. As a result the spiral nebulas (then the furthest objects known, from whose behavior Edwin Hubble had deduced his recession law) had to recede in time according to $t^{1/3}$, and the curvature of the three-dimensional space had to be zero. Relativity did not enter these reasonings; Dirac expected it to play only a subsidiary role in cosmology, since Hubble's law provided a natural speed at any point of space—and therefore a natural time axis. To reconcile this position with his admiration for Einstein's theory of gravitation, Dirac introduced two different metrics for atomic and cosmic phenomena. Only the second one was ruled by Einstein's theory; the first one varied in time according to the large-number hypothesis.

Cosmology was not just a hobby for Dirac. Rather, as he explained in 1939, it embodied his notion of progress in physics—an ever increasing mathematization of the world. In the old mechanistic conception, the equations of motion were mathematical but the initial conditions were given by observation. In the new cosmology the state preceding the initial explosion (posited by Georges Lemaître) was so simple that any complexity in nature pertained to the mathematical evolution. In this context Dirac even expressed the hope that the history of the universe would be only a history of the properties of numbers from 1 to 10^{39}. From the 1970's to the end of his life he often came back to his cosmological ideas. His large-number hypothesis has been seriously considered by several astrophysicists in spite of its speculative character.

Classical Point Electron, Indefinite Metrics. The rest of Dirac's work, from the 1930's on, centered on quantum electrodynamics. Dirac remained true to the research method that he had developed in his early work. He never reached his ultimate aim, a mathematically clean theory, but left interesting by-products of his quest. All the creators of quantum mechanics attempted to deal with the disease of infinite self-energy. One possibility they discussed was a revision of the correspondence basis, the classical theory of electrodynamics, which already involved either ambiguities (dependence on the structure of a finite electron) or infinite self-energy (for point electrons). In 1938 Dirac created a finite theory of point electrons by a convenient "reinterpretation" of the Maxwell-Lorentz equations that canceled the infinite self-mass. In spite of its formal beauty, this theory involved unphysical "runaway" solutions (spontaneously accelerating electrons) that could be eliminated (at the classical level) only at the price of making supraluminal signals possible. Not fully conscious of the latter difficulty, Dirac brought his equations to the Hamiltonian form and quantized them. Unfortunately, only half the divergent integrals of quantum electrodynamics were cured by this procedure. To take care of the other half, Dirac imagined in 1942 a nonpositive (he called it "indefinite") metric in Hilbert space that allowed a new natural representation of the field commutation rules but implied negative probabilities difficult to interpret physically. Pauli admired the new formalism but criticized Dirac's artificial interpretation of it, which involved a "hypothetical world" initially (before collisions occur in the real world) empty of photons and filled up with positrons (to dry out the sea).

In 1946 Dirac realized that his new equations allowed a finite nonperturbative solution; in addition, they could be connected with the regular formalism (with only positive probabilities) by a change of representation, that is, a unitary transformation in Hilbert space. Although not able to explicate this transformation (which presumably would reintroduce infinities), Dirac concluded in 1946 that the difficulties of quantum electrodynamics were purely mathematical. During the next year other theorists realized that the difficulties were connected instead with a proper definition of physical parameters like charge and mass. Nonetheless, the indefinite metric proved to be indispensable in quantum field theory for another reason: a covariant quantization of Maxwell's field requires the introduction of (unobservable) states of negative probability. In the 1960's several theorists, including Heisenberg, also developed Dirac's idea of a finite quantum electrodynamics with indefinite metrics.

Relativistic Ether, Strings. Developed by other physicists in 1947, renormalization, a way to absorb infinities in a proper redefinition of mass and charge, allowed very successful calculations of higher-order corrections to atomic and electrodynamical processes. From this resulted the best numerical agreement ever encountered between a fundamental theory and experiment. Always more concerned with internal beauty than with experimental verdict, Dirac called it a "fluke" and kept searching for a closed quantum electrodynamics purged of infinities at every stage of calculation. His point of view quickly became heterodox as more and more theorists thought that quantum electrodynamics did not have to exist by itself, but only as a part of a more general theory encompassing other types of interactions. As if to stress his originality, Dirac did not show any interest in the growing but messy field of nuclear and particle physics.

Some of Dirac's late attempts at a new quantum electrodynamics brought fundamentally new ideas. For instance, in 1951 he resurrected ether, arguing that quantum theory allowed a Lorentz invariant notion of ether for which all drift speeds at a given point of space-time are equiprobable, in analogy with the *S* states of the hydrogen atom, which are invariant by rotation although the underlying classical model is not. The idea had come to him after the proposal of a new electrodynamics for which the potential is restricted by $A_\mu A^\mu = k^2$, which suggests a natural ether speed $v_\mu = k^{-1}A_\mu$, even in the absence of matter.

In 1955 Dirac proposed strings as the basic representation of quantum electrodynamics, a photon corresponding to a closed string and an electron corresponding to the extremity of an open string. Originally suggested by a manifestly gauge-invariant formulation of quantum electrodynamics in which the electron is explicitly dragging an electromagnetic field with it, this picture "made inconceivable the things we do not want to have," for instance, a physically meaningless "bare" electron.

The Lagrangian in Quantum Mechanics. None of the above-mentioned attempts questioned the basic frame of quantum mechanics that Dirac had established in his younger years. But all through his scientific career he looked for alternative or more general formulations of quantum mechanics that might be more suitable for relativistic applications. Some of the products of this kind of exploration proved to be of essential importance. For instance, in 1933, exploiting a relation discovered by Jordan between quantum canonical transformations and the corresponding classical generating functions, he found that the transformation (q_{t+T}/q_t) from q taken at time t to q taken at time $t + T$ "corresponded" to $\exp i\int_t^{t+T} L(q, \dot{q})\, dt$, where L denotes the Lagrangian and $q(t)$, the classical motion between q_t and q_{t+T}. In the same paper he introduced the "generalized transformation functions," substituting the covariant motion of timelike surface of measurement for the usual hyperplanes "t = constant" in four-dimensional space. The remark about the Lagrangian, generalized by Dirac himself in 1945 to provide the amplitude of probability of a trajectory, inspired Richard Feynman in his discovery of the "Feynman-integrals," now the most efficient method of quantization. The "general transformation function" was adopted by the Japanese school to suggest, in combination with Dirac's multitime theory, Tomonaga's manifestly covariant formulation of quantum electrodynamics (1943).

The Role of Mathematics. Dirac believed in a "mathematical quality of nature." In the ideal physical theory, the whole of the description of the universe would have its mathematical counterpart. Conversely, he claimed around 1924, at one of Baker's tea parties, that any really interesting mathematical theory should find an application in the physical world. After sufficient progress, the field of mathematics would be purified and reduced to applied mathematics, that is, theoretical physics. The foundation of this belief in an asymptotic convergence of mathematics and physics is not easy to trace in Dirac's writings, since he generally avoided philosophical discussion. What could be said mathematically was clear enough to him, and he did not require, as most philosopher-physicists would, a recourse to common language to improve understanding. When circumstances compelled him to epistemological statements—for instance, in the foreword to his *Principles*—he simply borrowed them from physicist-philosophers who were "right by definition": Bohr and Eddington. From both these masters he took the rejection of mental pictures in space-time of the old physics. From Eddington he had the idea of a "nonpicturable substratum" and the recognition, through the development and justification of transformation theories, of "the part played by the observer in himself introducing the regularity that appears in his observations, and the lack of arbitrariness in the ways of nature."

It is doubtful that Dirac regarded these statements as really meaningful. When expressing his personal feelings on the role of mathematics, his leitmotiv was the idea of "mathematical beauty." For him the main reason for the successful appearance of groups of transformations in modern theories was their mathematical beauty, something no more subject to definition than beauty in art, but obvious to the connoisseur. In this perspective the mathematical quality of nature could be just the expression of its beauty. More significantly, Dirac's requirement of beauty materialized into a methodology: one had first to select the most beautiful mathematics and then, following Eddington's "principle of identification," try to connect it to the physical world.

To implement the first stage of this methodology, a more definite notion of beauty is needed. Dirac constantly refers to the museum of his early beautiful mathematical experiences. First comes the magic of projective geometry, exemplifying the power to find surprising relations between picturable mathematical objects through simple, invisible manipulations. Then follows general relativity with the appearance of symmetry transformations, and tensor calculus perceived as a symphony of symbols. At the moment of its introduction, beauty excludes rigor. Exact mathematical meaning comes after a heuristic symbolic stage, as in the introduction of the δ-function or of the q-numbers. It is less difficult, according to Dirac, to find beautiful mathematics than to interpret it in physical terms. Here is perhaps the most creative part of his work, invoking subtle analogies and correspondence with older bits of theories.

On the whole, Dirac's method sounds highly a priori, but he occasionally insisted on the necessity of a proper balance between inductive and deductive methods. A more detailed analysis would also show that where he was the most successful, he always remained securely tied to the empirically solid parts of existing theories.

BIBLIOGRAPHY

I. ORIGINAL WORKS. A list of Dirac's publications is in the biography by Dalitz and Peierls (see below). His main works are "The Fundamental Equations of Quantum Mechanics," in *Proceedings of the Royal Society of London*, **A109** (1925), 642–653; "On the Theory of Quantum Mechanics," *ibid.*, **A112** (1926), 661–677; "The Physical Interpretation of the Quantum Dynamics," *ibid.*, **A113** (1927), 621–641; "The Quantum Theory of Emission and Absorption of Radiation," *ibid.*, **A114** (1927), 243–265; "The Quantum Theory of the Electron, I," *ibid.*, **A117** (1928), 610–624; *The Principles of Quantum Mechanics* (Oxford, 1930); "A Theory of Electrons and Protons," in *Proceedings of the Royal Society of London*, **A126** (1930), 360–365; "Quantized Singularities in the Electromagnetic Field," *ibid.*, **A133** (1931), 60–72; and "The Cosmological Constants," in *Nature*, **139** (1937), 323. Nontechnical writings include "The Relation Between Mathematics and Physics," in *Royal Society of Edinburgh, Proceedings*, **59** (1939), 122–129; "The Evolution of the Physicist's Picture of Nature," in *Scientific American*, **208**, no. 5 (1963), 45–53; *The Development of Quantum Theory* (New York, 1971); and "Recollections of an Exciting Era," in Charles Weiner, ed., *History of Twentieth-Century Physics* (New York, 1977), 109–146.

Some of Dirac's papers have been deposited at the Churchill College Archive, Cambridge. Photocopies of manuscripts and letters, and an interview by Thomas S. Kuhn, are available in the Archive for the History of Quantum Physics (Berkeley, Copenhagen, London, New York, Rome).

II. SECONDARY LITERATURE. Joan Bromberg, "The Concept of Particle Creation Before and After Quantum Mechanics," in *Historical Studies in the Physical Sciences*, **7** (1976), 161–183, and "Dirac's Quantum Electrodynamics and the Wave-Particle Equivalence," in Charles Weiner, ed., *History of Twentieth-Century Physics* (New York, 1977), 147–157; Hendrik Casimir, "Paul Dirac, 1902–1984," in *Naturwissenschaftliche Rundschau*, **38** (1985), 219–223; R. H. Dalitz and Sir Rudolf Peierls, "Paul Adrien Maurice Dirac," in *Biographical Memoirs of Fellows of the Royal Society*, **32** (1986), 137–185; Olivier Darrigol, "La genèse du concept de champ quantique," in *Annales de physique*, **9** (1984), 433–501, and "The Origins of Quantized Matter Waves," in *Historical Studies in the Physical and Biological Sciences*, **16**, no. 2 (1986), 197–253; Michelangelo de Maria and Francesco La Teana, "Schrödinger's and Dirac's Unorthodoxy in Quantum Mechanics," in *Fundamenta scientiae*, **3** (1982), 129–148; Norwood R. Hanson, *The Concept of the Positron* (Cambridge, 1963); Max Jammer, *The Conceptual Development of Quantum Mechanics* (New York, 1966); Helge Kragh, "The Genesis of Dirac's Relativistic Theory of Electrons," in *Archive for the History of Exact Sciences*, **24** (1981), 31–67, "The Concept of the Monopole," in *Studies in History and Philosophy of Science*, **12** (1981), 141–172, "Cosmo-physics in the Thirties: Towards a History of Dirac's Cosmology," in *Historical Studies in the Physical Sciences*, **13**, no. 1 (1982), 69–108, and, as editor, *Methodology and Philosophy of Science in Paul Dirac's Physics*, University of Roskilde text no. 27 (Roskilde, 1979); B. N. Kursunoglu and E. P. Wigner, eds., *Reminiscences About a Great Physicist* (Cambridge, 1987); Jagdish Mehra and Helmut Rechenberg, *The Historical Development of Quantum Theory*, IV, *The Fundamental Equations of Quantum Mechanics* (New York, 1982); Donald F. Moyer, "Origins of Dirac's Electron, 1925–1928," in *American Journal of Physics*, **49** (1981), 944–949, "Evaluation of Dirac's Electron," *ibid.*, 1055–1062, and "Vindication of Dirac's Electron," *ibid.*, 1120–1135; Abdus Salam and Eugene P. Wigner, eds., *Aspects of Quantum Theory* (Cambridge, 1972); and J. G. Taylor, ed., *Tribute to Paul Dirac* (Bristol, 1987).

OLIVIER DARRIGOL

DOBZHANSKY, THEODOSIUS (*b.* Nemirov, Ukraine, Russia, 25 January 1900; *d.* Davis, California, 18 December 1975), *genetics, evolution.*

Dobzhansky was the only child of Sophia Voinarsky and of Grigory Dobrzhansky (the precise transliteration of the Russian family name), a teacher of high school mathematics. In 1910 the family moved to the outskirts of Kiev. During his early gymnasium years, Dobzhansky became an avid butterfly collector. In the winter of 1915–1916, he met Victor Luchnik, a twenty-five-year-old college dropout who was a dedicated entomologist specializing in Coccinellidae beetles. Luchnik convinced Dobzhansky that butterfly collecting would not lead anywhere, that he should become a specialist. Dobzhansky chose to work with ladybugs, which were the subject of his first scientific publication (1918).

Before Dobzhansky graduated in biology from the University of Kiev in 1921, he was hired as an instructor in zoology at the Polytechnic Institute in Kiev. He taught there until 1924, when he became an assistant to Yuri Filipchenko, head of the new department of genetics at the University of Leningrad. Filipchenko had started research with *Drosophila* fruitflies, and Dobzhansky was encouraged