was Einstein's program. He never lost his hope that a field theory of the right kind might eventually reach this goal.

That Einstein, without whom twentieth-century physics would be unthinkable, should have chosen to follow a separate path was a source of great regret to his colleagues. In Max Born's words: "Many of us regard this as a tragedy—for him, as he gropes his way in loneliness, and for us who miss our leader and standard-bearer."[19] But to Einstein himself his choice was inevitable; it was the natural outgrowth of all his years of striving to find a unified foundation for physics. This was what he meant when he ended his scientific autobiography by writing that he had tried to show "how the efforts of a lifetime hang together and why they have led to expectations of a definite form."[20]

MARTIN J. KLEIN

EINSTEIN: Theory of Relativity.

Einstein first wrote on radiation in his statistical mechanical discussion of Wien's law in 1904. He had already long thought about the fundamental problems of radiation; at age sixteen he had puzzled deeply over the question of what light would look like to an observer moving with it. As a student, one of the extramechanical applications of mechanics that had most fascinated him was the theory of light as a wave motion in a quasi-rigid elastic ether. In 1901 he was absorbed in unpublished, independent investigations of the critical problem of the motion of matter through the light ether.

At the turn of the century, the focus of discussion of the light ether was the electron theory. The electron theory had not been taught at the Zurich Polytechnic, and Einstein had had to instruct himself in it. Early in 1903 he began an intensive study of the theory, especially H. A. Lorentz' formulation of it. Lorentz' theory was founded on the concept of an absolutely stationary light ether. The ether completely permeated matter, with the consequence that bodies moving through it were not impeded and did not drag the ether with them. The ether was a dynamical substance but clearly not a mechanical one. Its dynamical properties were described precisely by Maxwell's electromagnetic field equations. The sole connection between the ether and matter occurred through the electrons that Lorentz assumed were contained in all ponderable molecules. The two kinds of physical entities in Lorentz' theory were the continuous ether and the discrete electrons; Maxwell's partial differential equations for the continuous field described the state of ether, and the ordinary differential equations of Newtonian mechanics described the motion of the electrons.

The two entities together with their respective formalisms—continuous field theory and particle mechanics—constituted the characteristic dualism of Lorentz' theory. This dualism, which pervaded late nineteenth-century physics, was most clearly defined and confronted in Lorentz' theory. From 1900 on there was increasing concern to eliminate the dualism by recognizing the mass concept and the laws of Newtonian mechanics as consequences of the more fundamental laws of electron dynamics. This reduction was the program of the electromagnetic view of nature, which was advocated by W. Wien, M. Abraham, and others.

Like the more influential of his contemporaries, Einstein regarded the separateness of the concepts of electromagnetism and particle mechanics as the outstanding fault of physical theory. He did not, however, subscribe to the electromagnetic program but originated new strategies for unifying the parts of physics; his 1905 light-quantum hypothesis and relativity theory were the fruits of such strategies. In his light-quantum study, he attacked the dualism of field and particle concepts by showing reasons to conceive radiation not as a continuous wave phenomenon, but as a finite collection of discrete, independent energy particles, or quanta. Quanta were foreign to the Maxwell-Lorentz theory, and Einstein was convinced that that theory had to be changed—one reason why he could not accept the electromagnetic program, since it posited the existing electromagnetic theory as exact.

Einstein wrote his theory of relativity in full awareness of its relation to his work on light quanta earlier that year; relativity did not depend on the exactness of Maxwell's theory, a fact that was important to Einstein. He recognized that electromagnetism, no less than mechanics, had to be reformed; and he retained certain concepts of both sciences in seeking the synthesis that removed the dualism from physical theory. He introduced the particle concept from mechanics into the theory of light in his light-quantum study, and he introduced the mechanical concept of relativity into field theory in his relativity study. In his light-quantum study he concluded that light is discontinuous; in his relativity study he rejected the concept of the ether outright as being superfluous in a consistent electromagnetic theory. He saw the stationary, continuous ether of the electron theory as the chief impediment to a unified physics, and in 1905 he put forward two distinct arguments against its admissibility.

Special Relativity. The stated purpose of Einstein's first paper on relativity in 1905, "Zur Elektrodynamik bewegter Körper"[21] ("On the Electrodynamics of Moving Bodies"), was to produce a "simple and consistent theory of the electrodynamics of moving bodies based on Maxwell's theory for stationary bodies." Until the publication of his paper, the current theory had been neither simple nor consistent. As an example of what struck Einstein as undesirable complications, there was the sharp theoretical distinction made between the two ways in which the interaction between a magnet and a conductor was supposed to produce a current in the latter. In one case the magnet was assumed to be at rest, with the conductor in motion; in the other case the conductor was assumed to be at rest, with the magnet in motion. Although the resulting current was the same in both cases, the respective explanations differed from one another and invoked different concepts. Given Einstein's strong conviction that the logical simplicity of a scientific theory was an important token of its validity, the foregoing example (a commonplace experiment in elementary physics) suggested the desirability of finding a point of view from which the phenomena could be accounted for more simply. It suggested to him the possibility that, as was already the case in mechanics, a theory of the electrodynamics of moving bodies should specify only relative motions, there being no phenomenological basis for defining absolute motions.

The validity of this point of view was further confirmed for Einstein by the failure of various "ether-drift" experiments designed to detect the "absolute motion" of the earth, through variations in the velocity of light or other optical or electromagnetic effects of such motion. These experiments were undertaken in the expectation that the laws of electrodynamics and optics for a stationary reference system must take different forms in a moving system.

A variety of attempts were made by contemporary physicists to remove the conflicts with accepted theory produced by such experiments. One of the most famous of these expedients was the hypothesis of the Fitzgerald contraction proposed by G. F. Fitzgerald—and, independently, by Lorentz—to account for the failure of the Michelson-Morley experiment. The interferometer employed in this particular attempt to measure the absolute speed, v, of the earth's motion through the ether, was of sufficient sensitivity to detect variations in the speed of light, c, to the second order of the magnitude v/c. The conclusively negative result was explained by Fitzgerald as having been caused by a contraction of the arm of the interferometer in the direction of its motion through the ether, by the factor $(1 - v^2/c^2)^{1/2}$, this contraction being just sufficient to offset the expected variation in the velocity of light.

This and other similar supplementary hypotheses added further complications to a theory of electrodynamics which in Einstein's opinion was already unnecessarily complex. By his own testimony the failure of the ether-drift experiments did not play a determinative role in his thinking but merely provided additional evidence in favor of his belief that inasmuch as the phenomena of electrodynamics were "relativistic," the theory would have to be reconstructed accordingly.

Another critic of "arbitrary hypotheses" such as the Fitzgerald contraction was the notable mathematician Henri Poincaré, who as early as 1895 had perceived the operation of a general law in the repeated failures of experiments designed to detect the absolute motion of the earth. Poincaré complained about there being certain explanations for the absence of first-order effects and other explanations for the absence of second-order effects,[22] and he was prepared to postulate that no physical experiment, regardless of its degree of accuracy, could detect the earth's absolute motion. He called this postulate the principle of relativity,[23] perhaps borrowing from Maxwell, who had referred to the "doctrine of the relativity of all physical phenomena" in his *Matter and Motion.*

Poincaré anticipated Einstein in asserting that all laws of nature, optical and electrodynamical as well as mechanical, should be brought within the scope of the principle of relativity. But their points of view and their programs were not identical. Poincaré remained in many respects committed to the traditional theory of electrodynamics. He adhered to the concept of the ether; and, while he appealed to the principle of relativity in deducing important results in electrodynamics, he appears to have imagined that the principle of relativity itself might be accounted for by an appropriate modification of the ether theory.

Einstein's approach was both more radical and more consistent. In his "Autobiographical Notes," Einstein recalled that he had long attempted to correct the dualistic fault of Lorentz' electron theory by direct, constructive approaches. But by the middle of 1905 he had come to see that to succeed he must proceed indirectly, by means of some universal principle. The model he had before him was thermodynamics, the science that had already guided his thought in statistical mechanics and radiation theory. He characterized thermodynamics as a theory of principle, one based on statements such as that of the impossibility of perpetual motion. He contrasted thermodynamics with the more common constructive theory built up from hypothetical statements, notably the theory of the continuous ether and the kinetic-molecular theory of gases.

In 1905 Einstein refounded the Maxwell-Lorentz theory on a new kinematics based on two universal postulates. The first postulate, or the "principle of relativity," pointed directly to Einstein's goal of unifying mechanics and electromagnetism; the postulate stipulated that the "same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good." The second postulate stipulated that light always moves with the same velocity in free space, regardless of the motion of the source. The second postulate was later described by W. Pauli as "the true essence of the old ether point of view."[24]

Both of Einstein's postulates, taken separately, had considerable experimental support, but in the ordinary view they appeared to be irreconcilable. For if the second postulate held true in one inertial system K (as it was assumed to hold true in the ether), then by virtue of the first postulate it would have to hold true in all reference systems in uniform translatory motion relative to K. But if the velocity of light were measured as c in terms of the space and time coordinates x, y, z, t of the reference system K, then it could not in current theory take the same value c in terms of the coordinates x', y', z', t' of another reference system K' in uniform motion v relative to K. For it was taken for granted that the coordinates of the two systems K and K' had to be related by the transformation equations:

$$\begin{aligned} x' &= x - vt, \\ y' &= y, \\ z' &= z, \\ t' &= t. \end{aligned} \qquad (1)$$

The correctness of the transformation (1) had been held virtually above suspicion throughout the history of modern science. It was with respect to this transformation that the laws of classical mechanics remained invariant for all inertial systems. The equations of (1) were also obviously true in the common-sense view of space and time.

It was Einstein's fundamental insight into the problems of electrodynamics to perceive that the transformation (1) could not be assumed to be true a priori. Their form was an assumption made plausible by the invariance properties of Newtonian dynamics, but it was by no means necessary. Indeed, the transformation equations relating the space and time measurements between two coordinate systems are part of a physical theory and have to be consistent with experience. Accordingly Einstein was led to undertake a profound analysis of the appropriate procedures by which space and time coordinates are established within a reference system. His object was to provide a physically meaningful and justifiable basis for the derivation of an alternative set of transformation equations consistent with the joint validity of his two postulates.

One step in that direction had been taken ten years earlier by Lorentz. In a treatise on "electrical and optical phenomena in moving bodies," Lorentz introduced a new concept which he called *Ortszeit* ("local time").[25] He used this primarily as a mathematical shortcut to simplify the form of Maxwell's equations in a system K' assumed to be in uniform motion relative to the unique stationary system K in which these equations held true exactly. Local time involved a departure from the transformation equation $t' = t$. Although Lorentz appears to have viewed local time as a mathematical artifice, it represented in embryo a concept of time that Einstein would later justify adopting for the whole of physics.

The "Kinematical Part" of Einstein's paper "On the Electrodynamics of Moving Bodies" begins with an analysis of the meaning of time in physics. This had been a subject relatively exempt from fundamental scrutiny because of the extraordinary strength of traditional intuitive beliefs. Einstein later acknowledged that his familiarity with the writings of David Hume and Ernst Mach had fostered the kind of critical reasoning underlying this part of his work.

The formidable psychological obstacle to revising the transformation equations (1) had been the concept of absolute simultaneity "rooted in the unconscious."[26] But to be physically meaningful, the synchronization of spatially separated clocks must be defined in terms of an actual physical process. Although, as Einstein later emphasized, one is not in principle restricted to light signals as the standard process for coordinating the clock settings, he chose that particular method on the grounds that the propagation of electromagnetic waves was a process about which most was known.[27] Accordingly, he proposed that within an inertial system the clocks at any two points A and B could be synchronized by stipulating that the time interval in which light travels from A to B is the same as from B to A.

Einstein then proceeded to deduce on the basis of his two postulates the transformation equations relating the four coordinates x', y', z', t' in the system K' to the coordinates x, y, z, t in the inertial system K, with respect to which K' was in uniform translatory velocity v along the x-axis. (For simplicity the two x-axes were assumed to coincide, the other pairs of

EINSTEIN

axes remaining parallel.) Fundamental to the derivation and meaning of this transformation was its dependence upon the synchronizing operation he had already defined for establishing the time coordinates in each system.

The following equations were deduced:

$$\begin{aligned} x' &= \frac{x - vt}{(1 - v^2/c^2)^{1/2}} \\ y' &= y \\ z' &= z \\ t' &= \frac{t - vx/c^2}{(1 - v^2/c^2)^{1/2}} \end{aligned} \qquad (2)$$

It should be noted here that, unknown to Einstein, the transformation equations (2) had already appeared in a paper published in 1904 by Lorentz, "Electromagnetic Phenomena in a System Moving With Any Velocity Smaller Than That of Light."[28] Therefore, these equations were called the Lorentz transformation by Poincaré. (Einstein did not use this name, either in 1905 or in his 1907 review paper.)

In Einstein's theory the transformation equations (2) express the kinematical content of his two postulates. Solving all four equations (2) for x, y, z, t in terms of x', y', z', t', their symmetry becomes perspicuous, the only change between the two sets being the sign (i.e., the direction) of v. As Einstein perceived, the new transformation presented a revolutionary theory of space and time. Consider any two inertial systems K and K' moving with relative velocities $\pm v$, respectively. By application of the equations (2), it follows that a rigid body of length l as measured in K, in which it is at rest, measures $l(1 - v^2/c^2)^{1/2}$ in K'; and a rigid body of length l as measured in K', in which it is at rest, measures $l(1 - v^2/c^2)^{1/2}$ in K. A clock at rest in K runs slow by $1 - (1 - v^2/c^2)^{1/2}$ seconds per second when timed by the clocks in K'; and reciprocally an identical retardation occurs for the rate of a clock at rest in K' as measured in K. Thus, lengths and time intervals are magnitudes relative to the inertial systems in which they are measured. The reciprocity of length contraction and time dilation between any two inertial systems renders physically meaningless questions as to whether such effects are "apparent" in one system and "real" in the other, or vice versa. For those contemporaries of Einstein who were committed to the ether theory or the concept of absolute simultaneity this conclusion was difficult to accept.

Another important kinematical theorem of "On the Electrodynamics of Moving Bodies" was Einstein's revised law for the addition of velocities. For the simplest case (in which the velocities v and w are in the direction of the x-axis of the inertial system in which they are composed) this law takes the form:

$$V = \frac{v + w}{1 + vw/c^2}. \qquad (3)$$

From this equation the limiting value of the velocity of light, c, can also be deduced. The composition of no two velocities v and w, each of which is less than c, can equal c; and the composition of any velocity less than c with c equals c.

Because of the complete generality of Einstein's first postulate, it follows that the fundamental principle of the special theory of relativity can be expressed by stating that the laws of physics are invariant with respect to the Lorentz transformation. The imposition of this formal restriction on all possible laws immediately facilitated the development of a greatly simplified theory of electrodynamics for both stationary and moving bodies. That, of course, had been Einstein's original objective.

Applications of Special Relativity. In the concluding or "Electrodynamical Part" of Einstein's original paper on relativity, he presented applications of his theory to various phenomena of electrodynamics. He proved that the Maxwell-Hertz equations for the electromagnetic field, both in empty space and when convection currents are taken into account, were invariant under the Lorentz transformation. He also showed how the force acting upon a point charge in motion in an electromagnetic field could be calculated simply by a transformation of the field to a system of coordinates at rest relative to the charge. In the new theory, electric and magnetic forces did not exist independently of the motion of the system of coordinates. From this point of view, the explanation of the currents produced by the relative motion of a magnet and a conductor was not complicated by theoretical distinctions based on their absolute motions.

Einstein derived several other theorems in the optics and electrodynamics of moving bodies on the basis of the theory for stationary bodies. His method was to choose the appropriate coordinate systems in each case and then apply the transformation equations (2). In this way he deduced relativistic (i.e., Lorentz-invariant) laws for Doppler's principle, for aberration, for the energy of light, and for the pressure of radiation on perfect reflectors. In what was a more difficult problem, he derived the three relativistic laws describing the motion of an electron in an electromagnetic field.

The requirement of Lorentz invariance for the laws of physics led Einstein and fellow physicists, including Planck, to the revision of a number of the laws of classical mechanics. This revision had already been initiated by Einstein in his laws of motion for electrons. In their relativistic formulations, masses and, correspondingly, forces could no longer have absolute magnitudes independent of the coordinate systems in which they were measured. Thus the expressions for momentum and energy also took new relativistic forms. Einstein later claimed that one of the foremost achievements of the special theory was its unification of the conservation laws for momentum and energy.[29]

Another demonstration of the heuristic power of the principle of Lorentz invariance was provided by a second paper published by Einstein in 1905, "Ist die Trägheit eines Körpers von seinem Energiegehalt abhängig?" ("Does the Inertia of a Body Depend Upon Its Energy-Content?").[30] In calculating, by means of the Lorentz transformation, the loss of kinetic energy $(K_0 - K_1)$ for a body emitting radiation energy in the amount L, Einstein was able to deduce the equation

$$K_0 - K_1 = Lv^2/2c^2.$$

This expression revealed (in view of the definition of kinetic energy: $mv^2/2$) that as the result of its radiation the mass of the body had been diminished by L/c^2. Arguing that the particular form in which the body lost some of its energy did not affect the calculated diminishment of its mass, Einstein concluded that the mass of a body is a measurement of its energy content. Differences in its energy content equal differences in its mass in accordance with the equation

$$\Delta E = \Delta mc^2.$$

In accordance with his frequent practice of suggesting appropriate experimental research, Einstein proposed that this law might be tested by experiments with radium salts. He observed that the exchange of radiation between bodies should involve an exchange of mass; light quanta have mass exactly as do ordinary molecules, and thus a bridge was established between the concepts of electromagnetism and mechanics. The mechanical concept of mass lost its isolation, becoming a form of energy, as characteristic of radiation as of ordinary matter. The chief value of the mass-energy law for Einstein lay in its contribution to the problem of the dualism in physical theory. In 1907 he carried this viewpoint one step further, assuming that mass and energy are completely equivalent concepts, the rest mass of a body being a measure of its "latent" energy content in accordance with the famous equation[31]

$$E = mc^2.$$

The electromagnetic mass question and other questions central to the work of Lorentz and his contemporaries remained the focus of German electromagnetic research for several years after Einstein's 1905 relativity paper. The universal significance of relativity was not generally recognized at first; Einstein's theory was regarded as merely another statement of Lorentz' electron theory, not as an important statement in its own right.

Geometric Significance of Relativity. The universal implications of Einstein's theory were first clearly revealed by the Göttingen mathematician Hermann Minkowski in 1907 and 1908. Minkowski argued that relativity implied a complete revision of our conception of space and time, and that this revision applied throughout physics and not just to electrodynamics where it originated. His four-dimensional formulation of the theory, his application of it to mechanics, and his advocacy generally had a decisive historical importance in winning physicists to the new theory and in clarifying its revolutionary significance. By 1910 relativity was fairly well understood in its full generality, and it began to be widely accepted, especially in Germany.

Minkowski recast the special theory of relativity in a form which had a decisive influence in the geometrization of physics. (The memoir was published in 1908.) As David Hilbert expressed it in his memorial lecture, Minkowski, in the formalism which he developed, "was able to reveal the inner simplicity and the true essence of the Laws of Nature." Minkowski considered the world as described by the special theory of relativity to be a four-dimensional flat space-time in which the events are points, the histories of particles represented by curves (world lines), and the inertial frames correspond to Cartesian coordinates spanning this space-time. The history of a particle moving in the absence of an external force is the straightest possible, it is a geodesic. This space-time is flat. Given a geodesic it is always possible to find another which does not intersect the first one. (The Euclidean space of elementary geometry is also flat. There the geodesics are straight lines, and parallel straight lines do not intersect.) Going from one inertial frame to another and using the transcription of data as given by the Lorentz transformations (2) corresponds to relabeling the events by changing the coordinate system in space-time. This very strongly geometrical point of view exhibited the fundamental features of the special theory clearly, and ultimately led to Einstein's belief that all laws of nature should be geometrical propositions concerning space-time.

Gravitational Theory. Einstein understood the universal significance of his relativity theory from the start. In the years immediately following 1905, he continued to work and publish on problems in relativistic electron theory; at the same time he tried to frame a relativistic theory of gravitation, a branch of physics that belonged to mechanics, not electromagnetism. In 1905 he had already freed gravitation from its exclusively mechanical context by his law of mass-energy equivalence; radiation too has mass and should gravitate. He tried to revise the Newtonian gravitational law so that it agreed with the demands of relativistic kinematics.

Any theory of gravitation contains three major parts: (*a*) the field equations relating the gravitational field to its sources (in Newtonian theory, there is just one equation, Poisson's equation for the gravitational potential using the mass density as the source); (*b*) the equations of motion of material bodies in this gravitational field (the Newtonian equation of motion in the Newtonian theory); and (*c*) the equations of motion of the electromagnetic field in the presence of gravitational fields (in the classical theory these are Maxwell's equations *uninfluenced* by the gravitational field). (See Table I for an outline of the development of the theory in these terms.)

Einstein, in his first attempt, retained the scalar potential of classical gravitation theory, generalizing the potential equation by adding a second time-derivative term. The field equation of gravitation then transforms correctly, and gravitation becomes, like electromagnetism, a finitely propagated action. He did not get far with this approach and did not publish it. The difficulty was that, according to the mass-energy law, the inertial mass of a body varies with its internal and kinetic energies, so that the acceleration of free fall might depend on these energies; this would contradict the notion, suggested by experience and adopted by Einstein as a premise, that all bodies have the same gravitational acceleration regardless of their velocities and internal states. This persuaded him that his 1905 principle of relativity was an inadequate basis for a gravitational theory and, hence, for a unified, nondualistic physics.

There was another way to approach the gravitational problem, one based on the recognition that the

Date	Equations of Motion in the Presence of Gravitation	Field Equations for Gravitation
1907	Equivalence principle using uniformly accelerated frames; Maxwell's equations in the presence of gravitation	
1911	Equivalence principle rediscussed	Scalar field theory
1912		Nonlinearity of field equations Gravitational induction (leading to Mach's principle of 1918)
1913	Equivalence of all frames in which conservation laws hold History of particle as a geodesic in space-time	Introduction of metric in space-time as seat of gravitation and tensorial formulation of laws
1914	Maxwell's equations correctly given in presence of gravitation	
1915	Final formulation of general theory (announced 25 March)	
1916	Equations of motions of particles and of the electromagnetic field	Field equations of gravitation expressed through curvature of space-time

TABLE I. Evolution of the General Theory free fall of a body is independent of its energy if its gravitational mass varies with energy in the same way as its inertial mass. Although there was no theoretical reason why the two kinds of mass should behave in the same way, Einstein did not doubt that they did so. He made the strict equivalence of inertial and gravitational mass the key to a proper understanding of gravitation, and he developed this understanding in his first published statement on gravitational theory. In the same survey article on relativity, which contained the energy-mass equation, he elevated the equality of inertial and gravitational mass, or, equivalently, the equality of the acceleration of the free fall of all bodies, to the status of an equivalence principle.

Einstein explained that the principle of relativity must be extended to accelerated coordinate systems; a coordinate system accelerated relative to an inertial frame is equivalent, in a sufficiently small spatial region, to a frame which is not accelerated relative to an inertial one but in which a gravitational field is present. This comes about in the following way: In an inertial frame let bodies be at rest, or move with a uniform motion. From the point of view of an observer at rest in the accelerated frame, these bodies appear to have an acceleration; this acceleration is the same for all bodies independent of their mass (being equal and opposite to the acceleration of the frame accelerated relative to the inertial frame). This acceleration naturally can be transformed away by simply using the original inertial frame as the frame of reference.

Now Einstein observed that a gravitational field locally generates a physical situation which is identical to the one described by the accelerated observer. For all bodies undergo the same acceleration in a gravitational field independent of their mass, and the acceleration of a body can be transformed away using a frame of reference which falls freely with the body whose acceleration we wish to transform away. If this be the case, one can immediately discuss, at least in a heuristic fashion, the influence of gravitational fields on phenomena, by solving another problem, the description of the same phenomena in the absence of a gravitational field but viewed from an accelerated frame. In this way Einstein showed that gravitational fields influence the motion of clocks. Since the frequency of an emitted spectral line can be used as a clock, it follows that there is a frequency shift between an emitted and an observed spectral line, if the gravitational potential at the location of the observer is different from that at the emitter. He also showed that all electromagnetic phenomena are influenced by a gravitational field; for example, the light rays are bent if they pass in the vicinity of gravitating bodies. It had long been suspected that there might be an interaction between electromagnetic and gravitational fields (see, for example, Faraday's experiments); however, this was the first concrete suggestion as to what this interaction should be, how it arises, and what the order of the magnitude of these effects are.

Program for General Relativity. The year 1907 was a turning point in relativity. From then on Einstein (then twenty-eight years old) was interested less in the special theory of relativity and more in its possible generalization. From the point of view of the special theory, space-time was a given framework, within which the natural phenomena took place. These were still described as in classical physics; there were fields of force and material bodies which acted on each other. The next generalization consisted in eliminating the gravitational fields of force by allowing the structure of space-time to change in such a way that the free motion in this altered space-time should in some way correspond to the motion under the influence of gravitational fields in the space-time of the special theory. The final generalization, which was never successfully attained, would have consisted in eliminating the electromagnetic fields of force as well by altering the geometry of space-time in a suitable way. The first generalization, the geometrization of gravitation, led eventually to the general theory of relativity; the additional geometrization of the electromagnetic fields of force led to the invention of the unified field theories.

In 1907 the details of this program were still obscure. The equivalence principle, relating the gravitational field and accelerated frames of reference, was already there. This helped Einstein to discuss some of the effects of a gravitational field on the electromagnetic field. The geometrization of this principle, the mathematical characterization of the gravitational field, its sources, and the relation between the field and its sources, i.e., the gravitational field equations, were still missing. In the next twelve years, Einstein was occupied with building a complete theory of gravitation rising out of his heuristic principle.

Between 1907 and 1911 Einstein published nothing more on gravitation. He was preoccupied with finding a reformed electron theory that incorporated both electric charges and light quanta; his chief heuristic guides in this search were thermodynamics, his fluctuation method in statistical mechanics, and his special theory of relativity. In the context of this largely unpublished work, he came to understand that the solution to the dualism problem was to write physics in terms of continuous field quantities and nonlinear partial differential equations that yield singularity-free particle solutions. The field equations were to account for particles and their interaction; they were to contain the laws of motion of particles, eliminating the dualistic need for particle mechanics in addition to field theory. He was soon to find additional support for this understanding in his general relativity theory, which required that field equations have just those mathematical properties he had decided upon on physical grounds.

For Einstein the connection of the particle and the field was the central problem of physics, and he saw the whole of the problem as contained in the connection of the electron and the electromagnetic field. A cardinal point of his unification objective in physics was to deduce the electron and its motion from the field equations. The whole difficulty was that it proved impossible to find the proper modification of Maxwell's equations that would permit such a deduction; any modification seemed arbitrary without a universal principle to determine its selection. In the years prior to 1911, he believed that the special relativity principle, together with statistical mechanical considerations, was an adequate guide for finding the new electromagnetic equations capable of describing particles. After several years of arduous effort, he recognized that he had been mistaken and shifted his expectations to the more powerful universal principle—the postulate of general relativity—as offering the possibility of avoiding arbitrariness in constructing field theories with particle solutions. The new postulate restricted much more severely the mathematical form the field equations could take.

By 1911 Einstein had become deeply pessimistic over the prospect of soon finding a new electron theory that incorporated quanta in a natural way. That year he returned to his 1907 gravitational theory, and for the next several years he looked to gravitation rather than electromagnetism as the starting point for the reform of physical theory. In 1911 the first Solvay Congress met to discuss the crisis in physics signaled by the quantum theory. Two years later, in 1913, Niels Bohr published his quantum theory of atoms and molecules. Just when the physics community began seriously to reorient itself toward quantum problems, Einstein seemed to move away from them. By 1911 he had come to regard the particle or quantum aspect of nature as secondary to the field aspect. He thought it was futile to attempt a fundamental understanding of the microscopic structure of nature until the macroscopic structure of the field was understood; when it was, quantum phenomena would be deduced from it. He never ceased to struggle with quanta, but his concern was less obvious than it had once been. His direct contribution to the later development of the quantum theory tended to be in the nature of criticism and suggestion; all the while he struggled to vindicate this way of developing physics by seeking a theory of the total field that would finally clarify the quantum problem.

Following a 1911 paper on the influence of gravitation on the propagation of light,[32] Einstein published two remarkable memoirs in 1912[33] which were efforts to construct a complete theory of gravitation incorporating the equivalence principle. In these memoirs Einstein supposed that the gravitational field can be characterized completely by one function, the local speed of light, analogous to the Newtonian description, where only the gravitational potential appears. The equivalence principle gives no clue to how the field equation describing the gravitational field should be constructed. By an extraordinary argument he extended the potential equation of Newton, which determines in that theory the gravitational potential, and came to the conclusion (*a*) that the equation must be nonlinear and (*b*) that this nonlinearity can be interpreted to show that the source of the gravitational field is not only the energy associated with the rest mass of bodies but also depends on the energy residing in the gravitational field itself. This was the first appearance of a nonlinear field equation for gravitation. In the first of his 1912 memoirs, Einstein wrote the differential equation of the static gravitational field as

$$\Delta c = kc\rho,$$

where c is the local velocity of light, k is a universal gravitational constant, and ρ is the density of matter. Next he derived the equations of motion of a body, using the equivalence principle: the force on a material point of mass m at rest in a gravitational field is

$$-m \operatorname{grad} c.$$

In his second memoir in 1912, he used the equivalence principle to show the influence of a static gravitational field on electromagnetic and thermal processes. Also in the same year, Einstein pointed out that from the energy expression which followed from his theory as it then was, one could conclude that if a body is enclosed in an envelope of gravitating matter its mass might be expected to increase. This he considered as a suggestion that perhaps the whole mass of a body could be conceived as arising from the gravitational interaction of this body with all the other bodies in the universe. He further pointed out that a similar point of view, without any theory, had already been advocated by Mach. This suggestion, that the mass of a body is the manifestation of the presence of other bodies in the universe, Einstein later (1918) called Mach's principle; the first appearance of an idea that a theory of gravitation should discuss the problem raised by Mach was in 1912. During this same period, others were more reluctant to take these far-reaching steps and there were several attempts to invent a gravitational theory in which there are preferred frames of reference such that the velocity of light is constant and has the same value as in empty space. Such a theory is, however, in conflict with the equivalence principle. The most comprehensive attempt was G. Nordström's in 1913.

Einstein himself came to the opposite conclusion. Instead of abandoning the investigation of uniformly accelerated frames of reference, he decided that the approach was too narrow and that in the generalization of the special theory not only these special transformations should be permitted, but more general ones. He was as yet unwilling to introduce general coordinate transformations. In his 1913 review article and in his memoir with Grossman, he insisted that only those frames of reference are admissible in which the conservation laws hold true. In fact these conservation laws were used by him to invent the field equations for gravitation. The use of more general frames of reference brought two important aspects into the theory. One was the use of more general mathematical tools which practically forced Einstein toward the final answer; the other was the observation that if more general transformations are permitted the gravitational field must be characterized not by one function but by ten functions. Moreover these ten functions have a simple geometric significance; they characterize the metric properties of space-time at every point.

This step was immense in its implications: (*a*) it forced the abandonment of the Newtonian notion that the gravitational field could be characterized by one scalar function, the gravitational potential; (*b*) it forced on Einstein the notion that gravitation is explicitly related to the geometrical structure of space-time. In his 1913 paper with Grossman,[34] Einstein adopted as the fundamental invariant of this theory the generalization of the four-dimensional line element ds originally introduced by Minkowski for a flat space-time. If space-time is not flat, ds must be expressed in terms of a general coordinate frame. Then two events, labeled

$$x^1, x^2, x^3, x^4$$

and

$$x^1 + dx^1, \quad x^2 + dx^2, \quad x^3 + dx^3, \quad x^4 + dx^4,$$

have a separation ds whose square is given by

$$ds^2 = \sum_{\mu=1}^{4} \sum_{\nu=1}^{4} g_{\mu\nu}\, dx^\mu\, dx^\nu.$$

The sixteen functions $g_{\mu\nu}$ form a symmetric tensor field (thus ten $g_{\mu\nu}$ are independent). Einstein considered these functions as the basic objects of his theory describing the manifestations of gravitation. In particular he assumed that as in the special theory the history of a body will be a geodesic; thus the history will be that curve in space-time for which $\int ds$ is a minimum, $\delta \int ds = 0$. Since the $g_{\mu\nu}$ appear in ds this principle will now determine the motion of a body influenced by gravity. Einstein regarded the $g_{\mu\nu}$ as the gravitational potentials, which replaced the single scalar c of his 1912 theory. The principal problem of his gravitational theory was, then, to determine the $g_{\mu\nu}$. For this purpose, Einstein sought an extension of Poisson's equation for the gravitational potential ϕ,

$$\Delta\phi = 4\pi\rho,$$

which he wrote

$$\Gamma_{\mu\nu} = \chi\Theta_{\mu\nu}.$$

$\Gamma_{\mu\nu}$ is constructed from derivatives of the $g_{\mu\nu}$ and is the analogue of $\Delta\phi$; $\Theta_{\mu\nu}$ contains the material sources of the field and is the analogue of ρ; χ is a gravitational constant. In any given physical situation, the $\Theta_{\mu\nu}$ may be assumed known. The problem was to determine the $\Gamma_{\mu\nu}$; Einstein believed that the principle of the conservation of energy and momentum is sufficient for this purpose.

Enunciation of General Relativity. Thus in 1913 the situation was as follows: The equation of motion of a particle in a gravitational field had been given; the equations of motion of the electromagnetic field in the presence of a gravitational field were incomplete; the field equations of gravity were incomplete; Newtonian concepts had been retained in order to save the conservation laws together with preferred frames of reference, to wit, precisely those where the conservation laws still held true as laws. This last stage was soon passed. In 1914 the field equations for the electromagnetic field (Maxwell's equations) were given correctly in the presence of a gravitational field. On 25 March 1915 Einstein announced in the Prussian Academy of Sciences that he also had the field equations of gravitation in hand. The results were published in several articles later in the same year,[35] and on 20 November, David Hilbert, in Göttingen, independently found the same field equations. A greatly expanded and detailed memoir appeared in 1916 which, so to speak, became the Authorized Version.[36] Einstein's new understanding in 1915 and 1916 was that the gravitational field can be characterized by Riemann's curvature tensor $G_{\mu\nu}$, a tensor obtained from the $g_{\mu\nu}$ by differentiation. He wrote the gravitational field equations as

$$G_{\mu\nu} = -K(T_{\mu\nu} - \tfrac{1}{2}g_{\mu\nu}\, T),$$

where T is the scalar of the material energy tensor $T_{\mu\nu}$ and K is a gravitational constant. This was his new analogue of Poisson's equation. The $g_{\mu\nu}$ that are determined by the field equations determine, through the separate equations of motion, the history of a body in the gravitational field.

The final theory now presented was of immense sweep and great conceptual simplicity. All frames of reference are equally good; the classical conservation laws fade away—they are no longer laws but mere identities and lose the significance they had before. There are no gravitational forces present in the sense that the theory contains electromagnetic ones, or elastic ones. Gravitation appears in a different way. The fixed, given space-time of the special theory has gone; what before had erroneously been labeled as the influence of a body by gravitation on the motion of another is now given as the influence of one body on the geometry of space-time in which the free motion of the other body occurs. This free motion in the altered space-time is what was mistaken as the forced motion (forced by a gravitational field) in an unaltered space-time. The laws of nature are now geometrical propositions concerning space-time. Space-time is a metric space, which means that we have a rule how to compute the separation of any two points in the space. This can be done if we know ten numbers at any point and thus have ten functions in space-time. Once these ten functions are given, everything that can be known can be computed. As stated before, three questions should be answered by the theory:

(1) *What corresponds to the field equations of gravitation?* The presence of matter alters the metric properties of space-time; in particular, the curvature of space-time at a point is determined by the amount of matter and electromagnetic field and their motion at that point. This alteration of the metric properties has an effect on the history of the motion of a body and on the history of the development of the electromagnetic field. These are embodied in the equations of motion.

(2) *What are the equations of motion of a body?* The equation of motion of a body is given by the statement that the history of a body always be a geodesic. But, of course, a geodesic in a curved space is quite different from a flat one; altering the curvature alters the motion.

(3) *What are the equations of motion of the electromagnetic field?* The equations of motion of the electromagnetic field are not geometrical propositions. Here we take over Maxwell's equations from the special theory where it was specified for a flat space-time, and simply transcribe them for a curved one. (The fact that these equations do not refer to anything geometrical sent Einstein in search of a more general theory, a unified field theory, where all the laws would have a geometrical significance.)

The equivalence principle is contained in the theory in the following fashion. According to this principle, one can make the effects of a gravitational field, on the motion of a particle or on an electromagnetic field, disappear locally by a transformation to a frame of reference which has the same acceleration as the gravitational field at that point. In this frame one can describe locally all events as in the special theory. Since the effects of the gravitational field are represented in the general theory by the fact that space-time is curved, this must mean that the equations of motion, on the one hand, and space-time, on the other, must be of such nature that the effects of this curvature can be transformed away locally. This arises because (*a*) the equations of motion of a particle, and of the electromagnetic field, are such that the curvature does not appear in them explicitly, and (*b*) space-time can be approximated locally by its flat tangential space, in the same way that a curved surface can be approximated by a flat tangential plane in the immediate vicinity of the point of contact.

Experimental Predictions of General Relativity. What were the experimental predictions? The equivalence principle had already predicted that the gravitational field must influence the electromagnetic field, and this result was also obtained in the developed theory. In addition Einstein in 1915 had already pointed out that according to Newtonian theory a solitary planet moving around the sun describes an elliptic orbit. Because of the presence of the other planets, this motion is perturbed and the axis of the ellipse slowly rotates relative to the fixed stars; i.e., it precesses. If these effects are computed on a Newtonian basis for the planet Mercury, it will be found that the experimentally observed precession is larger than the computed one. Einstein showed that if the motion of the single planet around the sun is calculated from the general theory, there is already a small precession. If this value is added to the Newtonian value of the precession caused by the other planets, the resulting total precisely fits the experimental results. This was the first unexpected success of the theory. The two other predictions concerned the effect of the gravity on light. In testing the first, the sun is treated as the body causing the bending. In order that the effect on the light coming from other stars might be observed, an eclipse was requisite. Although Einstein suggested such an experiment before the 1914 eclipse, the first observations were made in 1919. The values observed scatter around the theoretically predicted value with probable errors large enough to include it. The second prediction concerned the shift of spectral lines in the presence of a gravitating body and, for some time, the astrophysical observations were inaccurate. Not until 1960 was the prediction accurately verified in the laboratory.

Subsequent Investigations. For the rest of his life Einstein's investigations on relativity centered on the following points: (1) mathematical investigations into the structure of the theory; (2) approximate solutions of the general theory and their physical implications; (3) the application of the theory to the universe as a whole, to cosmology, and to Mach's principle; (4) general discussions of relativity and popular expositions; and (5) efforts to incorporate the electromagnetic field into the geometry of space-time. We shall take up the discussion of each point in chronological order.

Mathematical Investigations. As far as the mathematical structure of general relativity was concerned there were three classes of problems which concerned Einstein particularly: the role and meaning of conservation laws; the relation of the equations of motion of bodies to the field equations of gravitation; and the role and nature of singularities in the theory.

During the development of the general theory, Einstein had intended to hold fast to the conservation of energy and momentum in the usual (special relativistic) sense as far as possible. At the same time he was driven by other considerations toward the idea that the laws should be generally covariant, i.e., that the laws should have the same form for all observers in space-time, irrespective of their states of motion. These two desires, the maintenance of these conservation laws and of general covariance, proved mutually incompatible. The final theory is generally covariant; it has conservation identities and not conservation laws in the usual sense, although certain covariant laws do exist for special cases in general relativity.

The problem of the equation of motion of bodies is the following. The 1916 theory had a classical structure in the sense that there were both field equations (the curvature of space-time is determined by the mass and motion of bodies in space-time) and equations of motion of bodies (the world line of small mass is a geodesic). Are these two statements really separate? If the field equations were linear, they indeed would be. They are not linear, however, and Einstein showed that if matter is represented by a point singularity of the metric field, these singularities are located on world lines that are geodesics of space-time, provided its metric satisfies the equation of general relativity.[37]

The role and nature of singularities in the solutions greatly troubled Einstein both for mathematical and for physical reasons, and the question influenced his thinking on Mach's principle and on the necessity of unified field theories (the latter are discussed below).

Approximate Solutions. Experimentally, we know that there are observers such that the effects of gravitation are quite small in extended portions of space-time. This enables one to solve the equations of the general theory approximately. These approximate solutions show that, contrary to the Newtonian theory, there are gravitational disturbances that travel as waves. Recent experimental observations by Joseph Weber suggest that these waves may occur in nature. If these waves are compared with electromagnetic waves in empty space, significant differences are noted. Their polarization properties are different. They are associated with more complicated modes of motion of the source. They are thus less efficient in carrying away the kinetic energy of the motion of the source. Both types of waves propagate with the same velocity, however—that of light.

Application of the Theory. If we consider the universe as a whole, we cannot consider gravitation as a small effect, a small deformation of an otherwise flat space-time. In particular the following predicament arises. If inertia and gravitation are inseparable, how is it possible to have a situation in which the effects of gravitation are small? After all, that would mean that in the absence of any such small effect, we still would have a flat space-time with an inertial motion possible in it, while gravitation would be entirely absent. Previously (1912),[38] Einstein had considered the possibility that there should be a gravitational induction effect, according to which the presence of other masses alters the value of the mass of a given body. From this consideration he erected the hypothesis that, conceivably, the whole mass of a body is generated by the presence of other masses.[39] If this be true within the general theory, two things should follow:

(*a*) There should be no solution of the field equations applicable to the whole universe which can describe an empty space-time (since then in this empty space-time geodesics would exist, which could be taken as giving the history of the inertial motion of a particle).

(*b*) The value of the mass of a body should be determined by the presence and amount of other masses in the universe. (It is now also believed that

Mach's principle should contain an explanation of why the gravitational interaction is always attractive.)

It was the notion that the existence of inertia and gravitation must be explained along these lines that Einstein called Mach's principle (1918). In 1917 he grappled with the first half of this problem.[40] If matter in the universe is generally distributed uniformly, is it possible to find a time-independent solution of the field equations that describe a spatially finite (closed) space-time, and will this space-time vanish if the total mass of the universe is spatially infinite? He found that no such solution existed unless he modified the field equations, adding to them the so-called cosmological term. This term has no observable effect on any of the local solutions used in the experimental tests but alters the solution as a whole. With this modification, however, a solution does exist. Thereby, the first aspect of Mach's principle would be satisfied. But Einstein was dissatisfied with this answer because an arbitrary modification had had to be introduced into the field equations.

In 1922 A. Friedmann found that even without the cosmological term there are still solutions of the field equations where matter has a finite density everywhere in space, provided this density is not time-independent. In 1929 Hubble announced his discovery that the red shift of spectral lines coming from distant sources increases uniformly with the distance. This phenomenon can be interpreted as evidence for a uniform expansion of the universe as can be described by the Friedmann solutions.

General Discussions and Expositions. During these years Einstein was also concerned to clarify misconceptions about the theory of relativity and to present his views on natural sciences on a less abstract level. Among his efforts in this direction, one particularly beautiful lecture must be mentioned. In 1921, at the Prussian Academy's commemorative session honoring Frederick the Great, Einstein delivered a lecture on geometry and experience in which he summed up his views on the geometrization of physics and relativity and the relation of mathematics to the external world.[41] Here he gave his famous answer to the puzzling question of why mathematics should be so well adapted to describing the external world: "Insofar as the Laws of Mathematics refer to the external world, they are not certain; and insofar as they are certain, they do not refer to reality."

The Electromagnetic Field and the Geometry of Space-Time. There were two main reasons for Einstein's dissatisfaction with the general theory. One was the seemingly still-inadequate geometrization of physics. He felt that not only gravitational but also electromagnetic effects should be manifestations of the geometry of space-time. The other interactions, such as nuclear forces and the forces responsible for beta decay, were not yet known. Einstein never considered the geometrization of the other interactions, although in the 1940's E. Schrödinger made an attempt to invent a unified field theory incorporating gravitational, electromagnetic, and nuclear interactions.

The other problem was the relation of matter to the singularities of the gravitational field. Einstein felt that a complete and correct field theory should be without singularities while in the general theory the field equations are in general singular.[42] This, he believed, is due to the inadequate description of matter as handled in the general theory. Thus the stage was set for a search for a more extended theory. This new theory should have two basic features. It should enlarge the geometry of space-time in such a manner that new geometrical objects could be introduced which can be associated with the electromagnetic field; the physically relevant solutions (whatever that may mean) should be nonsingular. Although the initial steps in this direction were not taken by Einstein himself, he did become more and more preoccupied with this problem and in his later years the construction of such a theory was his main concern. This was Einstein's ultimate response to the mechanical-electromagnetic crisis in physical theory he had first talked about in the opening of his 1905 light-quantum paper. (In 1953 Einstein said to the author that although it is doubtful that a unified field theory of the type he was seeking could exist, even its nonexistence would be of sufficient interest to be worth establishing it. If he did not do it, Einstein said, perhaps nobody ever would.)

How might the geometry of space-time be enriched with new geometrical objects which then could be considered as candidates for the description of electromagnetic phenomena? Since no clear guiding principle stemming from physics existed (or exists even today), we must rely on geometrical intuition, which necessity would also serve as a motivation.

Practically all the work in this direction fell into one of two categories. Either the dimensionality of space-time would be preserved and the geometry altered in a formal fashion, or the dimensionality of space-time would be enlarged in a formal fashion and the metric geometry preserved. Hermann Weyl initiated the first line of thought in 1918; Kaluza the second in 1921.

Weyl's unified field theory considered space-time to be endowed with a more general geometry. This approach enabled him to introduce four extra functions in space-time, in terms of which the electromagnetic field can be expressed. Einstein immediately noticed (1918), however, that if the same physical interpretation for the geometry be maintained as in the general theory, Weyl's theory leads in its original form to results that contradict experience. In 1921 A. S. Eddington observed that Weyl's geometry of space-time is a special case of a much more general class of geometries, usually called affine geometries, which depend on a profound generalization of the notion of parallelism. Einstein's first investigation of these ideas (1923) introduced the notion of distant parallelism. In 1930, however, he found that the new theory admitted solutions that describe gravitating masses represented by singularities at rest relative to each other under the sole influence of their gravitational interaction. Experience clearly contradicts this consequence. Einstein originally rejected these solutions on the grounds that they contain singularities, and later he rejected the theory itself.

In 1931 Einstein and Walter Mayer reformulated Kaluza's five-dimensional theory retaining a four-dimensional space-time. In 1938 and 1941 Einstein again discussed theories of this type, before returning to the notion that space-time may be endowed with an affine geometry. Several different geometries were envisaged in papers written in collaboration with V. Bargmann (1944), E. G. Strauss (1945–1946), and Bruria Kaufmann (1955). The last was his final published memoir.

Summary. If we turn to summarize Einstein's achievements in relativity what we see is a new point of view and innumerable consequences. The basic new point of view was the explicit recognition that the invariance properties of the laws of physics are of fundamental importance and that these invariance properties stem from immediate physical facts and are required by them. The physical notion of invariance arises from the experimental fact that the descriptions change in a specific way if the arrangement of measuring devices is altered in a specific manner. This alteration may be a simple spatial rotation, or a simple transfer of the origin of the coordinate system, or something more complicated that endows the whole laboratory with a uniform motion, or with a motion with a uniform acceleration. That the results so obtained can be linked together implies that these observations, and hence the physical quantities they describe, transform in a given way as the measuring devices are altered.

The special theory concerned itself with that transformation of labeling of space-time points which corresponds physically to uniform translation of the whole laboratory. Einstein's great achievement was the explicit realization that this transformation of labels cannot be specified without a specific assumption about the operational meaning of simultaneity with respect to events that are spatially separate. From this realization Einstein was led to the only consistent definition of simultaneity, and thus to the correct understanding of the Lorentz transformations and to an appreciation of their great generality.

If invariance properties are of such importance then a strong geometrical interpretation of the laws of nature becomes highly desirable, since a thing that has no definite invariance properties cannot be even thought of geometrically. This consideration led Einstein to accept Minkowski's point of view that space and time should be considered as forming one geometrical object, space-time, a four-dimensional flat space.

The next step was to analyze the relations between the descriptions of phenomena in frames accelerated relative to each other. From these relations Einstein drew the conclusion that this four-dimensional space-time cannot be flat, and that gravitation is the name given to those phenomena that appear because space-time is not flat. The curvature of space-time is due to its energy and mass content. The remarkable success of this theory derived from its automatic explanation of two features of gravitation: Why is the inertial mass equal to the gravitational mass? And why is gravitation a universal property acting on everything in the universe? The answer is that the two masses are equal because they are one and the same, since they appear in the theory uniquely as the cause of the curvature of space-time. Gravitation is a universal manifestation because it is the property of space-time, and hence everything that is in space-time (which is, literally, every thing) must experience it.

The last efforts of Einstein on unified field theories were a logical continuation of his previous efforts. The chain of argument may be said to run as follows. If invariance properties are of utmost importance, physics should be thought of as a geometry, because thought is then occupied only with objects that have invariant properties. This led to a theory according to which the structure of space-time is the seat of gravitation interaction. Is this structure perhaps so rich that not only gravitational interaction but other interactions are also determined by it? That investigation proved to be unsatisfactory, perhaps because only gravitation is a universal interaction. Nevertheless even this effort turned out to be prophetic; in modern physics it is more and more the practice to proceed with a formal guessing at the laws of nature, the guess being based on formal simplicity and on invariance. The interpretation of the theory

EINSTEIN

often emerges only after the structure of the equations guessed at are better understood. In this, Einstein was a forerunner.

When Einstein's total work in physics is considered, it can be said that his achievements are rivaled only by those of Isaac Newton. Both scientists were guided in their work by unique insights into the nature of physical reality and both represent the utmost fulfillment of the creative imagination in science.

NANDOR L. BALAZS

NOTES

1. A. Einstein, "H. A. Lorentz, His Creative Genius and His Personality," in G. L. de Haas-Lorentz, ed., *H. A. Lorentz. Impressions of His Life and Work* (Amsterdam, 1957), p. 5.
2. A. Einstein, *The World As I See It* (New York, 1934), p. 250.
3. Ernst Straus, "Assistant bei Albert Einstein," in C. Seelig, ed., *Helle Zeit–Dunkle Zeit* (Zurich, 1956), p. 71.
4. A. Einstein, "Autobiographical Notes," in P. A. Schilpp, ed., *Albert Einstein: Philosopher-Scientist* (Evanston, Ill., 1949, 1951), p. 5.
5. *Ibid.*, p. 16.
6. A. Einstein to Johannes Stark (7 Dec. 1907); quoted in A. Hermann, "Albert Einstein und Johannes Stark," in *Sudhoffs Archiv*, **50** (1966), 272.
Einstein's papers can be readily located by consulting the years of publication in the bibliographies listed below.
7. Walther Nernst, "Über neuere Probleme der Wärmetheorie," in *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin* (1911), p. 86.
8. A. Einstein, "Über die Entwicklung unserer Anschauungen über das Wesen und die Konstitution der Strahlung," in *Physikalische Zeitschrift*, **10** (1909), 817.
9. *Ibid.*, p. 824.
10. A. Einstein to Michele Besso (13 May 1911).
11. A. Einstein, "Beiträge zur Quantentheorie," in *Verhandlungen der Deutschen physikalischen Gesellschaft*, **16** (1914), 823.
12. A. Einstein to M. Besso (11 Aug. 1916).
13. Erwin Schrödinger, "Über das Verhältnis der Heisenberg-Born-Jordanschen Quantenmechanik zu der meinen," in *Annalen der Physik*, **79** (1926), 735.
14. A. Einstein to Max Born (4 Dec. 1926).
15. A. Einstein to E. Schrödinger (26 Apr. 1926).
16. Niels Bohr, *Atomic Physics and Human Knowledge* (New York, 1958), pp. 66, 71.
17. A. Einstein to M. Born (4 Dec. 1926).
18. A. Einstein, "Einleitende Bemerkungen über Grundbegriffe," in *Louis de Broglie, Physicien et penseur* (Paris, 1953), p. 6.
19. M. Born, "Einstein's Statistical Theories," in Schilpp, p. 163.
20. A. Einstein, "Autobiographical Notes," in Schilpp, p. 94.
21. A. Einstein, *Annalen der Physik*, **17** (1905), 891–921.
22. H. Poincaré, *Rapports présentés au Congrès International de Physique de 1900* (Paris, 1900), I, 22–23.
23. H. Poincaré, "L'état actuel et l'avenir de la physique mathématique," in *Bulletin des sciences mathématiques*, **28** (1904), 306; repr. in English as "The Principles of Mathematical Physics," in *Monist*, **15** (1905), 5.
24. W. Pauli, *Theory of Relativity*, G. Field, trans. (New York, 1958), p. 5.
25. H. A. Lorentz, *Versuch einer Theorie der electrischen und optischen Erscheinungen in bewegten Körpen* (Leiden, 1895), pp. 49–50.
26. A. Einstein, "Autobiographical Notes," in Schilpp, p. 52.
27. A. Einstein, *The Meaning of Relativity* (Princeton, N. J., 1922), p. 28.
28. H. A. Lorentz, in *Proceedings of the Royal Academy of Sciences of Amsterdam*, **6** (1904), 809.
29. A. Einstein, "Autobiographical Notes," in Schilpp, p. 60.
30. A. Einstein, in *Annalen der Physik*, **18** (1905), 639–641.
31. A. Einstein, "Über das Relativitätsprinzip und die aus demselben gezogenen Folgerungen," in *Jahrbuch der Radioaktivität*, **4** (1907), 442.
32. A. Einstein, "Einfluss der Schwerkraft auf die Ausbreitung des Lichtes," in *Annalen der Physik*, 4th ser., **35** (1911), 898.
33. A. Einstein, "Lichtgeschwindigkeit und Statik des Gravitationsfeldes," *ibid.*, **38** (1912), 355; "Theorie des statischen Gravitationsfeldes," *ibid.*, p. 443.
34. A. Einstein, "Entwurf einer verallgemeinerten Relativitätstheorie und eine Theorie der Gravitation," in *Zeitschrift für Mathematik und Physik*, **62** (1913), 225, pt. 2 written with M. Grossman.
35. A. Einstein, "Zur allgemeinen Relativitätstheorie," in *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin*, pt. 2 (1915), pp. 778, 799; "Erklärung der Perihelbewegung des Merkur aus der allgemeinen Relativitätstheorie," *ibid.*, p. 831; "Feldgleichungen der Gravitation," *ibid.*, p. 844.
36. A. Einstein, "Grundlagen der allgemeinen Relativitätstheorie," in *Annalen der Physik*, 4th ser., **49** (1916), 769.
37. A. Einstein, "Allgemeine Relativitätstheorie und Bewegungsgesetz," in *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin*, Phys.-math. Kl. (1927), pp. 2, 235, pt. 1 written with J. Grommer; "Gravitational Equations and the Problems of Motion," in *Annals of Mathematics*, 2nd ser., **39** (1938), 65, written with L. Infeld and B. Hoffmann; "Gravitational Equations and the Problems of Motion. II," *ibid.*, **41** (1940), 455, written with L. Infeld.
38. A. Einstein, "Gibt es eine Gravitationswirkung die der elektrodynamischen Induktionswirkung analog ist?," in *Vierteljahrschrift für gerichtliche Medizin und öffentliches Sanitätswesen*, 3rd ser., **44** (1912), 37.
39. A. Einstein, "Prinzipielles zur allgemeinen Relativitätstheorie," in *Annalen der Physik*, 4th ser., **55** (1918), 241.
40. A. Einstein, "Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie," in *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin*, pt. 1 (1917), p. 142.
41. A. Einstein, *Geometrie und Erfahrung* (Berlin, 1921).
42. A. Einstein, "Demonstration of the Non-existence of Gravitational Fields With a Non-vanishing Total Mass Free of Singularities," in *Revista. Instituto de física, Universidad nacional de Tucumán*, **2A** (1941), 11.

BIBLIOGRAPHY

I. There are three principal bibliographies of Einstein's writings. The first, compiled by Margaret C. Shields, covers his writings to May 1951 and includes general works as well as scientific articles and books. It is to be found in P. A. Schilpp, ed., *Albert Einstein: Philosopher–Scientist* (Evanston, Illinois, 1949, 1951), pp. 689–760. This important book also contains Einstein's "Autobiographical Notes," a series of essays on his work by physicists, mathematicians, and philosophers, and Einstein's "Remarks" concerning these essays. The second bibliography, containing only the scientific writings, is E. Weil, *Albert Einstein. A Bibliography of His Scientific Papers 1901–1954* (London, 1960). The third is Nell Boni, Monique Ross, and Dan H. Laurence, *A Bibliographical Checklist and Index to the Published Writings of Albert Einstein* (New York, 1960).

No regular edition of Einstein's scientific papers has

appeared yet. There is a microfilm edition (Readex Microprint, New York).

II. Einstein's books include *Relativity, the Special and the General Theory: A Popular Exposition* (London, 1920); *The Meaning of Relativity* (Princeton, N. J., 1921); *Investigations on the Theory of the Brownian Movement*, R. Fürth, ed. (London, 1926); *The World As I See It* (New York, 1934); *The Evolution of Physics* (New York, 1938), written with Leopold Infeld; *Out of My Later Years* (New York, 1950); and *Ideas and Opinions* (New York, 1954).

III. Portions of Einstein's correspondence have been published: *Albert Einstein, Hedwig und Max Born. Briefwechsel 1916–1955*, M. Born, ed. (Munich, 1969); *Albert Einstein/Arnold Sommerfeld Briefwechsel*, A. Hermann, ed. (Basel, 1968); Albert Einstein, Erwin Schrödinger, Max Planck, H. A. Lorentz, *Letters on Wave Mechanics*, K. Przibram, ed., M. J. Klein, trans. (New York, 1967); Albert Einstein, *Lettres à Maurice Solovine* (Paris, 1956).

Also see O. Nathan and H. Norden, eds., *Einstein on Peace* (New York, 1960), and M. J. Klein, *Paul Ehrenfest. Volume 1. The Making of a Theoretical Physicist* (Amsterdam, 1970). Both books quote extensively from Einstein's correspondence.

IV. Many biographies of Einstein have appeared, but nothing like a definitive study of either the man or his work yet exists. Philipp Frank, *Einstein. His Life and Times*, G. Rosen, trans. (New York, 1947), written by a physicist and philosopher of science who knew Einstein for over forty years, is the most thorough work. It does not, however, discuss Einstein's work in any detail. It suffers from having been written during Einstein's lifetime and without the use of manuscript sources. Carl Seelig, *Albert Einstein. A Documentary Biography*, M. Savill, trans. (London, 1956), quotes extensively from Einstein's correspondence and is particularly good on the earlier part of his life.

Another biography of particular interest is that by Rudolf Kayser, Einstein's son-in-law, *Albert Einstein. A Biographical Portrait* (New York, 1930); this was actually written under the pseudonym Anton Reiser.

A recent biography that presents interesting ideas on Einstein's thought is Boris Kuznetsov, *Einstein*, V. Talmy, trans. (Moscow, 1965).

V. Some articles of particular interest are Robert S. Shankland, "Conversations With Albert Einstein," in *American Journal of Physics*, **31** (1963), 37–47; Gerald Holton, "On the Origins of the Special Theory of Relativity," *ibid.*, **28** (1960), 627–636; "Influences on Einstein's Early Work in Relativity Theory," in *American Scholar*, **37** (winter 1968), 59–79; "Mach, Einstein, and the Search for Reality," in *Daedalus*, **97** (1968), 636–673; and "Einstein, Michelson, and the 'Crucial' Experiment," in *Isis*, **60** (1969), 133–197; Tetu Hirosige, "Theory of Relativity and the Ether," in *Japanese Studies in the History of Science* (1968), pp. 37–53; Martin J. Klein, "Einstein's First Paper on Quanta," in *The Natural Philosopher*, **2** (1963), 57–86; "Einstein and the Wave-Particle Duality," *ibid.*, **3** (1964), 1–49; "Einstein, Specific Heats, and the Early Quantum Theory," in *Science*, **148** (1965), 173–180; "Thermodynamics in Einstein's Thought," *ibid.*, **157** (1967), 509–516; and "The First Phase of the Bohr-Einstein Dialogue," in *Historical Studies in the Physical Sciences*, **2** (1970), 1–39; R. McCormmach, "Einstein, Lorentz, and the Electron Theory," *ibid.*, 41–87.

VI. Einstein's manuscripts, notes, and correspondence have been collected by the estate of Albert Einstein and are kept at present at the Institute for Advanced Study, Princeton, N. J. Information on certain other Einstein manuscripts may be found in T. S. Kuhn, J. L. Heilbron, P. L. Forman, and L. Allen, *Sources for History of Quantum Physics* (Philadelphia, 1967).

EINTHOVEN, WILLEM (*b.* Semarang, Java, 21 May 1860; *d.* Leiden, Netherlands, 28 September 1927), *physiology.*

Einthoven's father was municipal physician of Semarang; he married Louise M. M. C. de Vogel. He died in 1866, and four years later his widow settled in Utrecht with their six children. There Willem Einthoven graduated from high school and registered as a medical student in 1879. In 1886 he married his cousin Frédérique Jeanne Louise de Vogel; they had three daughters and a son.

While a student, Einthoven was active in sports; when he broke his wrist in a fall, he made it the occasion to publish a study on the pronation and supination of the forearm (1882). On 4 July 1885 he received the Ph.D in medicine *cum laude* with a thesis on stereoscopy through color differentiation. The following December he was appointed professor of physiology at Leiden.

In 1895, after the London physiologist A. D. Waller had published the curve for the action current of the heart as deduced from the body surface and had announced that he was unable to calculate its true shape (as recorded with Lippmann's capillary electrometer), Einthoven repeated this experiment. He defined the physical constants of the capillary electrometer and calculated the true curve, which he called the electrocardiogram. Einthoven considered direct registration of the curve's true shape a necessity. Starting from the mirror galvanometer of Deprez-d'Arsonval, he arrived at his brilliant conception of the string galvanometer. In 1896, while working on the construction of this instrument and developing the necessary photographic equipment, he registered electrocardiograms with the capillary electrometer as well as heart sounds of humans and animals.

For making electrocardiograms Einthoven chose the ordinate and abscissa in such a way that all details of the electrocardiogram would appear as clearly as possible. In 1903 he defined the standard measures for general use—one centimeter movement of the ordinate for one millivolt tension difference and a shutter speed of twenty-five millimeters per