679; and Pierre Larousse, ed., *Grand dictionnaire universel,* 15 vols. (Paris, 1866–1876), VIII, 1086.

ALEX BERMAN

GAUSS, CARL FRIEDRICH (*b.* Brunswick, Germany, 30 April 1777; *d.* Göttingen, Germany, 23 February 1855), *mathematical sciences.*

The life of Gauss was very simple in external form. During an austere childhood in a poor and unlettered family he showed extraordinary precocity. Beginning when he was fourteen, a stipend from the duke of Brunswick permitted him to concentrate on intellectual interests for sixteen years. Before the age of twenty-five he was famous as a mathematician and astronomer. At thirty he went to Göttingen as director of the observatory. There he worked for forty-seven years, seldom leaving the city except on scientific business, until his death at almost seventy-eight.

In marked contrast to this external simplicity, Gauss's personal life was complicated and tragic. He suffered from the political turmoil and financial insecurity associated with the French Revolution, the Napoleonic period, and the democratic revolutions in Germany. He found no mathematical collaborators and worked alone most of his life. An unsympathetic father, the early death of his first wife, the poor health of his second wife, and unsatisfactory relations with his sons denied him a family sanctuary until late in life.

In this difficult context Gauss maintained an amazingly rich scientific activity. An early passion for numbers and calculations extended first to the theory of numbers and then to algebra, analysis, geometry, probability, and the theory of errors. Concurrently he carried on intensive empirical and theoretical research in many branches of science, including observational astronomy, celestial mechanics, surveying, geodesy, capillarity, geomagnetism, electromagnetism, mechanics, optics, the design of scientific equipment, and actuarial science. His publications, voluminous correspondence, notes, and manuscripts show him to have been one of the greatest scientific virtuosos of all time.

Early Years. Gauss was born into a family of town workers striving on the hard road from peasant to lower middle-class status. His mother, a highly intelligent but only semiliterate daughter of a peasant stonemason, worked as a maid before becoming the second wife of Gauss's father, a gardener, laborer at various trades, foreman ("master of waterworks"), assistant to a merchant, and treasurer of a small insurance fund. The only relative known to have even modest intellectual gifts was the mother's brother, a master weaver. Gauss described his father as "worthy of esteem" but "domineering, uncouth, and unrefined." His mother kept her cheerful disposition in spite of an unhappy marriage, was always her only son's devoted support, and died at ninety-seven, after living in his house for twenty-two years.

Without the help or knowledge of others, Gauss learned to calculate before he could talk. At the age of three, according to a well-authenticated story, he corrected an error in his father's wage calculations. He taught himself to read and must have continued arithmetical experimentation intensively, because in his first arithmetic class at the age of eight he astonished his teacher by instantly solving a busy-work problem: to find the sum of the first hundred integers. Fortunately, his father did not see the possibility of commercially exploiting the calculating prodigy, and his teacher had the insight to supply the boy with books and to encourage his continued intellectual development.

During his eleventh year, Gauss studied with Martin Bartels, then an assistant in the school and later a teacher of Lobachevsky at Kazan. The father was persuaded to allow Carl Friedrich to enter the Gymnasium in 1788 and to study after school instead of spinning to help support the family. At the Gymnasium, Gauss made very rapid progress in all subjects, especially classics and mathematics, largely on his own. E. A. W. Zimmermann, then professor at the local Collegium Carolinum and later privy councillor to the duke of Brunswick, offered friendship, encouragement, and good offices at court. In 1792 Duke Carl Wilhelm Ferdinand began the stipend that made Gauss independent.

When Gauss entered the Brunswick Collegium Carolinum in 1792, he possessed a scientific and classical education far beyond that usual for his age at the time. He was familiar with elementary geometry, algebra, and analysis (often having discovered important theorems before reaching them in his studies), but in addition he possessed a wealth of arithmetical information and many number-theoretic insights. Extensive calculations and observation of the results, often recorded in tables, had led him to an intimate acquaintance with individual numbers and to generalizations that he used to extend his calculating ability. Already his lifelong heuristic pattern had been set: extensive empirical investigation leading to conjectures and new insights that guided further experiment and observation. By such means he had already independently discovered Bode's law of planetary distances, the binomial theorem for rational exponents, and the arithmetic-geometric mean.

During his three years at the Collegium, Gauss continued his empirical arithmetic, on one occasion finding a square root in two different ways to fifty decimal places by ingenious expansions and interpolations. He formulated the principle of least squares, apparently while adjusting unequal approximations and searching for regularity in the distribution of prime numbers. Before entering the University of Göttingen in 1795 he had rediscovered the law of quadratic reciprocity (conjectured by Lagrange in 1785), related the arithmetic-geometric mean to infinite series expansions, conjectured the prime number theorem (first proved by J. Hadamard in 1896), and found some results that would hold if "Euclidean geometry were not the true one."

In Brunswick, Gauss had read Newton's *Principia* and Bernoulli's *Ars conjectandi,* but most mathematical classics were unavailable. At Göttingen, he devoured masterworks and back files of journals, often finding that his own discoveries were not new. Attracted more by the brilliant classicist G. Heyne than by the mediocre mathematician A. G. Kästner, Gauss planned to be a philologist. But in 1796 came a dramatic discovery that marked him as a mathematician. As a by-product of a systematic investigation of the cyclotomic equation (whose solution has the geometric counterpart of dividing a circle into equal arcs), Gauss obtained conditions for the constructibility by ruler and compass of regular polygons and was able to announce that the regular 17-gon was constructible by ruler and compasses, the first advance in this matter in two millennia.

The logical component of Gauss's method matured at Göttingen. His heroes were Archimedes and Newton. But Gauss adopted the spirit of Greek rigor (insistence on precise definition, explicit assumption, and complete proof) without the classical geometric form. He thought numerically and algebraically, after the manner of Euler, and personified the extension of Euclidean rigor to analysis. By his twentieth year, Gauss was driving ahead with incredible speed according to the pattern he was to continue in many contexts—massive empirical investigations in close interaction with intensive meditation and rigorous theory construction.

During the five years from 1796 to 1800, mathematical ideas came so fast that Gauss could hardly write them down. In reviewing one of his seven proofs of the law of quadratic reciprocity in the *Göttingische gelehrte Anzeigen* for March 1817, he wrote autobiographically:

> It is characteristic of higher arithmetic that many of its most beautiful theorems can be discovered by induction with the greatest of ease but have proofs that lie anywhere but near at hand and are often found only after many fruitless investigations with the aid of deep analysis and lucky combinations. This significant phenomenon arises from the wonderful concatenation of different teachings of this branch of mathematics, and from this it often happens that many theorems, whose proof for years was sought in vain, are later proved in many different ways. As soon as a new result is discovered by induction, one must consider as the first requirement the finding of a proof by *any possible* means. But after such good fortune, one must not in higher arithmetic consider the investigation closed or view the search for other proofs as a superfluous luxury. For sometimes one does not at first come upon the most beautiful and simplest proof, and then it is just the insight into the wonderful concatenation of truth in higher arithmetic that is the chief attraction for study and often leads to the discovery of new truths. For these reasons the finding of new proofs for known truths is often at least as important as the discovery itself [*Werke,* II, 159–160].

The Triumphal Decade. In 1798 Gauss returned to Brunswick, where he lived alone and continued his intensive work. The next year, with the first of his four proofs of the fundamental theorem of algebra, he earned the doctorate from the University of Helmstedt under the rather nominal supervision of J. F. Pfaff. In 1801 the creativity of the previous years was reflected in two extraordinary achievements, the *Disquisitiones arithmeticae* and the calculation of the orbit of the newly discovered planet Ceres.

Number theory ("higher arithmetic") is a branch of mathematics that seems least amenable to generalities, although it was cultivated from the earliest times. In the late eighteenth century it consisted of a large collection of isolated results. In his *Disquisitiones* Gauss summarized previous work in a systematic way, solved some of the most difficult outstanding questions, and formulated concepts and questions that set the pattern of research for a century and still have significance today. He introduced congruence of integers with respect to a modulus ($a \equiv b \pmod{c}$ if c divides a-b), the first significant algebraic example of the now ubiquitous concept of equivalence relation. He proved the law of quadratic reciprocity, developed the theory of composition of quadratic forms, and completely analyzed the cyclotomic equation. The *Disquisitiones* almost instantly won Gauss recognition by mathematicians as their prince, but readership was small and the full understanding required for further development came only through the less austere exposition in Dirichlet's *Vorlesungen über Zahlentheorie* of 1863.

In January 1801 G. Piazzi had briefly observed and lost a new planet. During the rest of that year the astronomers vainly tried to relocate it. In September, as his *Disquisitiones* was coming off the press, Gauss decided to take up the challenge. To it he applied both a more accurate orbit theory (based on the ellipse rather than the usual circular approximation) and improved numerical methods (based on least squares). By December the task was done, and Ceres was soon found in the predicted position. This extraordinary feat of locating a tiny, distant heavenly body from seemingly insufficient information appeared to be almost superhuman, especially since Gauss did not reveal his methods. With the *Disquisitiones* it established his reputation as a mathematical and scientific genius of the first order.

The decade that began so auspiciously with the *Disquisitiones* and Ceres was decisive for Gauss. Scientifically it was mainly a period of exploiting the ideas piled up from the previous decade (see Figure 1). It ended with *Theoria motus corporum coelestium in sectionibus conicis solem ambientium* (1809), in which Gauss systematically developed his methods of orbit calculation, including the theory and use of least squares.

Professionally this was a decade of transition from mathematician to astronomer and physical scientist. Although Gauss continued to enjoy the patronage of the duke, who increased his stipend from time to time (especially when Gauss began to receive attractive offers from elsewhere), subsidized publication of the *Disquisitiones,* promised to build an observatory, and treated him like a tenured and highly valued civil servant, Gauss felt insecure and wanted to settle in a more established post. The most obvious course, to become a teacher of mathematics, repelled him because at this time it meant drilling ill-prepared and unmotivated students in the most elementary manipulations. Moreover, he felt that mathematics itself might not be sufficiently useful. When the duke raised

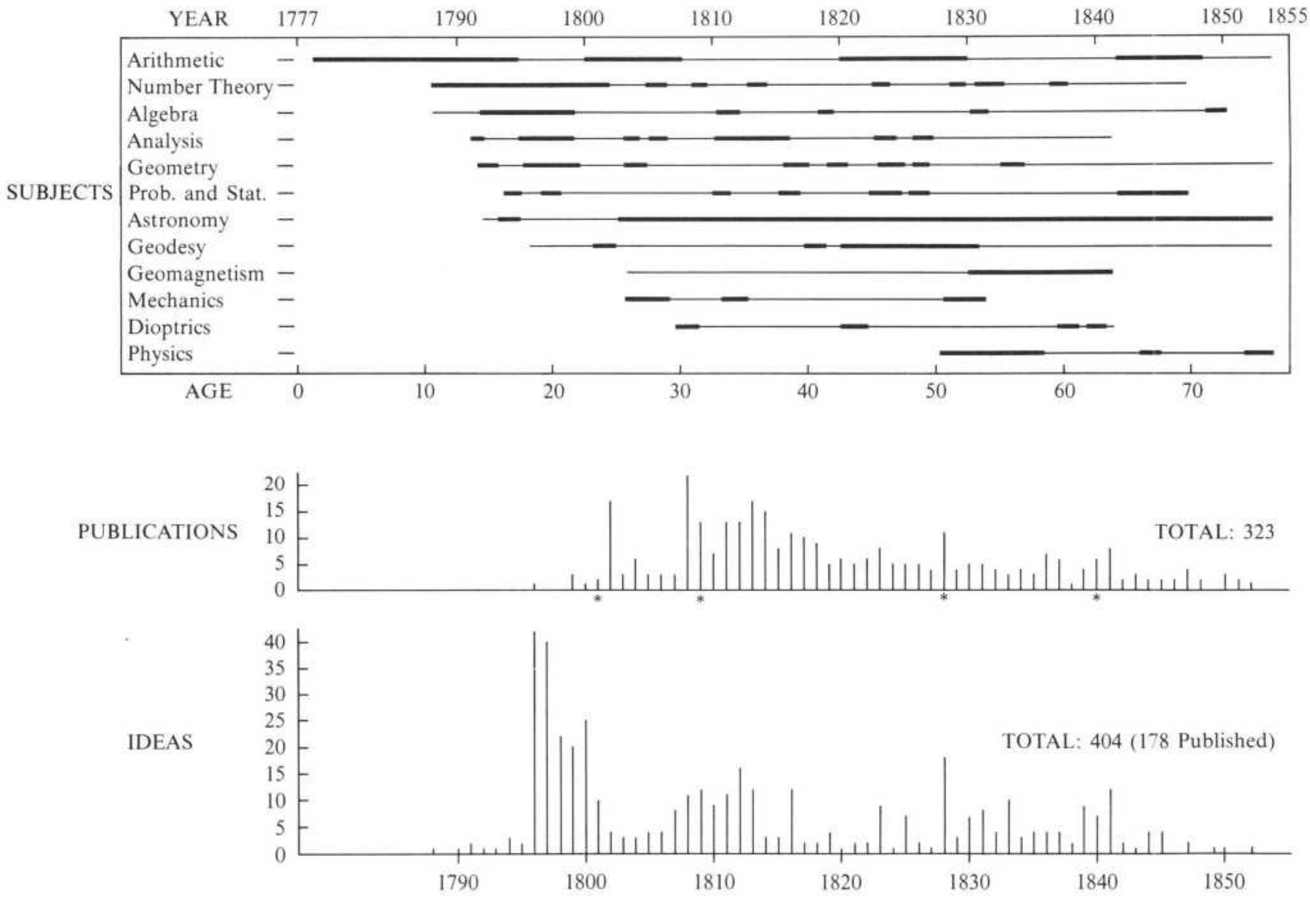

FIGURE 1. Interests, ideas, and publications. The horizontal lines show time spans of Gauss's interests in different subjects. Heavy lines indicate periods of intensive activity. The annual counts of recorded ideas include published and unpublished "results" (conjectures, theorems, proofs, concepts, hypotheses, theories), significant observations, experimental findings, and inventions. They are based on an examination of published materials, including correspondence and notebooks published after his death. Because of intrinsic ambiguities in dating, identification, and evaluation, this chart gives only an approximate picture of creative flux. The graph of publications shows the number of titles published in each year, including reviews. A count of pages would be similar except for surges (marked by *): 1801 (*Disquisitiones*), 1809 (*Theoria motus*), 1828 (least squares, surfaces, astronomy, biquadratic residues), and 1840 (geomagnetism).

his stipend in 1801, Gauss told Zimmermann: "But I have not earned it. I haven't yet done anything for the nation."

Astronomy offered an attractive alternative. A strong interest in celestial mechanics dated from reading Newton, and Gauss had begun observing while a student at Göttingen. The tour de force on Ceres demonstrated both his ability and the public interest, the latter being far greater than he could expect in mathematical achievements. Moreover, the professional astronomer had light teaching duties and, he hoped, more time for research. Gauss decided on a career in astronomy and began to groom himself for the directorship of the Göttingen observatory. A systematic program of theoretical and observational work, including calculation of the orbits of new planets as they were discovered, soon made him the most obvious candidate. When he accepted the position in 1807, he was already well established professionally, as evidenced by a job offer from St. Petersburg (1802) and by affiliations with the London Royal Society and the Russian and French academies.

During this decisive decade Gauss also established personal and professional ties that were to last his lifetime. As a student at Göttingen he had enjoyed a romantic friendship with Wolfgang Bolyai, and the two discussed the foundations of geometry. But Bolyai returned to Hungary to spend his life vainly trying to prove Euclid's parallel postulate. Their correspondence soon practically ceased, to be revived again briefly only when Bolyai sent Gauss his son's work on non-Euclidean geometry. Pfaff was the only German mathematician with whom Gauss could converse, and even then hardly on an equal basis. From 1804 to 1807 Gauss exchanged a few letters on a high mathematical level with Sophie Germain in Paris, and a handful of letters passed between him and the mathematical giants in Paris, but he never visited France or collaborated with them. Gauss remained as isolated in mathematics as he had been since boyhood. By the time mathematicians of stature appeared in Germany (e.g., Jacobi, Plücker, Dirichlet), the uncommunicative habit was too ingrained to change. Gauss inspired Dirichlet, Riemann, and others, but he never had a collaborator, correspondent, or student working closely with him in mathematics.

In other scientific and technical fields things were quite different. There he had students, collaborators, and friends. Over 7,000 letters to and from Gauss are known to be extant, and they undoubtedly represent only a fraction of the total. His most important astronomical collaborators, friends, and correspondents were F. W. Bessel, C. L. Gerling, M. Olbers, J. G. Repsold, H. C. Schumacher. His friendship and correspondence with A. von Humboldt and B. von Lindenau played an important part in his professional life and in the development of science in Germany. These relations were established during the period 1801–1810 and lasted until death. Always Gauss wrote fewer letters, gave more information, and was less cordial than his colleagues, although he often gave practical assistance to his friends and to deserving young scientists.

Also in this decade was established the pattern of working simultaneously on many problems in different fields. Although he never had a second burst of ideas equal to his first, Gauss always had more ideas than he had time to develop. His hopes for leisure were soon dashed by his responsibilities, and he acquired the habit of doing mathematics and other theoretical investigations in the odd hours (sometimes, happily, days) that could be spared. Hence his ideas matured rather slowly, in some cases merely later than they might have with increased leisure, in others more felicitously with increased knowledge and meditation.

This period also saw the fixation of his political and philosophical views. Napoleon seemed to Gauss the personification of the dangers of revolution. The duke of Brunswick, to whom Gauss owed his golden years of freedom, personified the merits of enlightened monarchy. When the duke was humiliated and killed while leading the Prussian armies against Napoleon in 1806, Gauss's conservative tendencies were reinforced. In the struggles for democracy and national unity in Germany, which continued throughout his lifetime, Gauss remained a staunch nationalist and royalist. (He published in Latin not from internationalist sentiments but at the demands of his publishers. He knew French but refused to publish in it and pretended ignorance when speaking to Frenchmen he did not know.) In seeming contradiction, his religious and philosophical views leaned toward those of his political opponents. He was an uncompromising believer in the priority of empiricism in science. He did not adhere to the views of Kant, Hegel, and other idealist philosophers of the day. He was not a churchman and kept his religious views to himself. Moral rectitude and the advancement of scientific knowledge were his avowed principles.

Finally, this decade provided Gauss his one period of personal happiness. In 1805 he married a young woman of similar family background, Johanna Osthoff, who bore him a son and daughter and created around him a cheerful family life. But in 1809 she died soon after bearing a third child, which did not long survive her. Gauss "closed the angel eyes in which for five years I have found a heaven" and was plunged into a loneliness from which he never fully recovered. Less than a year later he married Minna Waldeck, his deceased wife's best friend. She bore him two sons and a daughter, but she was seldom well or happy. Gauss dominated his daughters and quarreled with his younger sons, who immigrated to the United States. He did not achieve a peaceful home life until the younger daughter, Therese, took over the household after her mother's death (1831) and became the intimate companion of his last twenty-four years.

Early Göttingen Years. In his first years at Göttingen, Gauss experienced a second upsurge of ideas and publications in various fields of mathematics. Among the latter were several notable papers inspired by his work on the tiny planet Pallas, perturbed by Jupiter: *Disquisitiones generales circa seriem infinitam* (1813), an early rigorous treatment of series and the introduction of the hypergeometric functions, ancestors of the "special functions" of physics; *Methodus nova integralium valores per approximationem inveniendi* (1816), an important contribution to approximate integration; *Bestimmung der Genauigkeit der Beobachtungen* (1816), an early analysis of the efficiency of statistical estimators; and *Determinatio attractionis quam in punctum quodvis positionis datae exerceret planeta si eius massa per totam orbitam ratione temporis quo singulae partes describuntur uniformiter esset dispertita* (1818), which showed that the perturbation caused by a planet is the same as that of an equal mass distributed along its orbit in proportion to the time spent on an arc. At the same time Gauss continued thinking about unsolved mathematical problems. In 1813 on a single sheet appear notes relating to parallel lines, declinations of stars, number theory, imaginaries, the theory of colors, and prisms (*Werke,* VIII, 166).

Astronomical chores soon dominated Gauss's life. He began with the makeshift observatory in an abandoned tower of the old city walls. A vast amount of time and energy went into equipping the new observatory, which was completed in 1816 and not properly furnished until 1821. In 1816 Gauss, accompanied by his ten-year-old son and one of his students, took a five-week trip to Bavaria, where he met the optical instrument makers G. von Reichenbach, T. L. Ertel (owner of Reichenbach's firm), J. von Fraunhofer, and J. von Utzschneider (Fraunhofer's partner), from whom his best instruments were purchased. As Figure 1 shows, astronomy was the only field in which Gauss worked steadily for the rest of his life. He ended his theoretical astronomical work in 1817 but continued positional observing, calculating, and reporting his results until his final illness. Although assisted by students and colleagues, he observed regularly and was involved in every detail of instrumentation.

It was during these early Göttingen years that Gauss matured his conception of non-Euclidean geometry. He had experimented with the consequences of denying the parallel postulate more than twenty years before, and during his student days he saw the fallaciousness of the proofs of the parallel postulate that were the rage at Göttingen; but he came only very slowly and reluctantly to the idea of a different geometric theory that might be "true." He seems to have been pushed forward by his clear understanding of the weaknesses of previous efforts to prove the parallel postulate and by his successes in finding non-Euclidean results. He was slowed by his deep conservatism, the identification of Euclidean geometry with his beloved old order, and by his fully justified fear of the ridicule of the philistines. Over the years in his correspondence we find him cautiously, but more and more clearly, stating his growing belief that the fifth postulate was unprovable. He privately encouraged others thinking along similar lines but advised secrecy. Only once, in a book review of 1816 (*Werke,* IV, 364–368; VIII, 170–174), did he hint at his views publicly. His ideas were "besmirched with mud" by critics (as he wrote to Schumacher on 15 January 1827), and his caution was confirmed.

But Gauss continued to find results in the new geometry and was again considering writing them up, possibly to be published after his death, when in 1831 came news of the work of János Bolyai. Gauss wrote to Wolfgang Bolyai endorsing the discovery, but he also asserted his own priority, thereby causing the volatile János to suspect a conspiracy to steal his ideas. When Gauss became familiar with Lobachevsky's work a decade later, he acted more positively with a letter of praise and by arranging a corresponding membership in the Göttingen Academy. But he stubbornly refused the public support that would have made the new ideas mathematically respectable. Although the friendships of Gauss with Bartels and W. Bolyai suggest the contrary, careful study of the plentiful documentary evidence has established that Gauss did not inspire the two founders of non-Euclidean geometry. Indeed, he played at best a neutral, and on balance a negative, role, since his silence was considered as agreement with the public ridicule and neglect that continued for several decades and were only gradually overcome, partly by the revelation, beginning in the 1860's, that the prince of mathematicians had been an underground non-Euclidean.

Geodesist. By 1817 Gauss was ready to move toward geodesy, which was to be his preoccupation for the next eight years and a burden for the next thirty.

His interest was of long standing. As early as 1796 he worked on a surveying problem, and in 1799–1800 he advised Lt. K. L. E. von Lecoq, who was engaged in military mapping in Westphalia. Gauss's first publication was a letter on surveying in the *Allgemeine geographische Ephemeriden* of October 1799. In 1802 he participated in surveying with F. X. G. von Zach. From his arrival in Göttingen he was concerned with accurately locating the observatory, and in 1812 his interest in more general problems was stimulated by a discussion of sea levels during a visit to the Seeberg observatory. He began discussing with Schumacher the possibility of extending into Hannover the latter's survey of Denmark. Gauss had many motives for this project. It involved interesting mathematical problems, gave a new field for his calculating abilities, complemented his positional astronomy, competed with the French efforts to calculate the arc length of one degree on the meridian, offered an opportunity to do something useful for the kingdom, provided escape from petty annoyances of his job and family problems, and promised additional income. The last was a nontrivial matter, since Gauss had increasing family responsibilities to meet on a salary that remained fixed from 1807 to 1824.

The triangulation of Hannover was not officially approved until 1820, but already in 1818 Gauss began an arduous program of summer surveying in the field followed by data reduction during the winter. Plagued by poor transportation, uncomfortable living conditions, bad weather, uncooperative officials, accidents, poor health, and inadequate assistance and financial support, Gauss did the fieldwork himself with only minimal help for eight years. After 1825 he confined himself to supervision and calculation, which continued to completion of the triangulation of Hannover in 1847. By then he had handled more than a million numbers without assistance.

An early by-product of fieldwork was the invention of the heliotrope, an instrument for reflecting the sun's rays in a measured direction. It was motivated by dissatisfaction with the existing unsatisfactory methods of observing distant points by using lamps or powder flares at night. Meditating on the need for a beacon bright enough to be observed by day, Gauss hit on the idea of using reflected sunlight. After working out the optical theory, he designed the instrument and had the first model built in 1821. It proved to be very successful in practical work, having the brightness of a first-magnitude star at a distance of fifteen miles. Although heliostats had been described in the literature as early as 1742 (apparently unknown to Gauss), the heliotrope added greater precision by coupling mirrors with a small telescope. It became standard equipment for large-scale triangulation until superseded by improved models from 1840 and by aerial surveying in the twentieth century. Gauss remarked that for the first time there existed a practical method of communicating with the moon.

Almost from the beginning of his surveying work Gauss had misgivings, which proved to be well founded. A variety of practical difficulties made it impossible to achieve the accuracy he had expected, even with his improvements in instrumentation and the skillful use of least squares in data reduction. The hoped-for measurement of an arc of the meridian required linking his work with other surveys that were never made. Too hasty planning resulted in badly laid out base lines and an unsatisfactory network of triangles. He never ceased trying to overcome these faults, but his virtuosity as a mathematician and surveyor could not balance the factors beyond his control. His results were used in making rough geographic and military maps, but they were unsuitable for precise land surveys and for measurement of the earth. Within a generation, the markers were difficult to locate precisely or had disappeared altogether. As he was finishing his fieldwork in July 1825, Gauss wrote to Olbers that he wondered whether other activities might have been more fruitful. Not only did the results seem questionable but he felt during these years, even more than usual, that he was prevented from working out many ideas that still crowded his mind. As he wrote to Bessel on 28 June 1820, "I feel the difficulty of the life of a practical astronomer, without help; and the worst of it is that I can hardly do any connected significant theoretical work."

In spite of these failures and dissatisfactions, the period of preoccupation with geodesy was in fact one of the most scientifically creative of Gauss's long career. Already in 1813 geodesic problems had inspired his *Theoria attractionis corporum sphaeroidicorum ellipticorum homogeneorum methodus nova tractata,* a significant early work on potential theory. The difficulties of mapping the terrestrial ellipsoid on a sphere and plane led him in 1816 to formulate and solve in outline the general problem of mapping one surface on another so that the two are "similar in their smallest parts." In 1822 a prize offered by the Copenhagen Academy stimulated him to write up these ideas in a paper that won first place and was published in 1825 as the *Allgemeine Auflösung der Aufgabe die Theile einer gegebenen Fläche auf einer anderen gegebenen Fläche so auszubilden dass die Abbildung dem Abgebildeten in den kleinsten Theilen ähnlich wird.* This paper, his more detailed *Untersuchungen über Gegenstände der höhern Geodäsie* (1844–1847), and geodesic manuscripts later published in the *Werke* were further developed by

German geodesists and led to the Gauss-Krueger projection (1912), a generalization of the transverse Mercator projection, which attained a secure position as a basis for topographic grids taking into account the spheroidal shape of the earth.

Surveying problems also motivated Gauss to develop his ideas on least squares and more general problems of what is now called mathematical statistics. The result was the definitive exposition of his mature ideas in the *Theoria combinationis observationum erroribus minimis obnoxiae* (1823, with supplement in 1828). In the *Bestimmung des Breitenunterschiedes zwischen den Sternwarten von Göttingen und Altona durch Beobachtungen am Ramsdenschen Zenithsector* of 1828 he summed up his ideas on the figure of the earth, instrumental errors, and the calculus of observations. However, the crowning contribution of the period, and his last breakthrough in a major new direction of mathematical research, was *Disquisitiones generales circa superficies curvas* (1828), which grew out of his geodesic meditations of three decades and was the seed of more than a century of work on differential geometry. Of course, in these years as always, Gauss produced a stream of reviews, reports on observations, and solutions of old and new mathematical problems of varying importance that brought the number of his publications during the decade 1818–1828 to sixty-nine. (See Figure 1.)

Physicist. After the mid-1820's, there were increasing signs that Gauss wished to strike out in a new direction. Financial pressures had been eased by a substantial salary increase in 1824 and by a bonus for the surveying work in 1825. His other motivations for geodesic work were also weakened, and a new negative factor emerged—heart trouble. A fundamentally strong constitution and unbounded energy were essential to the unrelenting pace of work that Gauss maintained in his early years, but in the 1820's the strain began to show. In 1821, family letters show Gauss constantly worried, often very tired, and seriously considering a move to the leisure and financial security promised by Berlin. The hard physical work of surveying in the humid summers brought on symptoms that would now be diagnosed as asthma and heart disease. In the fall of 1825, Gauss took his ailing wife on a health trip to spas in southern Germany; but the travel and the hot weather had a very bad effect on his own health, and he was sick most of the winter. Distrusting doctors and never consulting one until the last few months of his life, he treated himself very sensibly by a very simple life, regular habits, and the avoidance of travel, for which he had never cared anyway. He resolved to drop direct participation in summer surveying and to spend the rest of his life "undisturbed in my study," as he had written Pfaff on 21 March 1825.

Apparently Gauss thought first of returning to a concentration on mathematics. He completed his work on least squares, geodesy, and curved surfaces as mentioned above, found new results on biquadratic reciprocity (1825), and began to pull together his long-standing ideas on elliptic functions and non-Euclidean geometry. But at forty-eight he found that satisfactory results came harder than before. In a letter to Olbers of 19 February 1826, he spoke of never having worked so hard with so little success and of being almost convinced that he should go into another field. Moreover, his most original ideas were being developed independently by men of a new generation. Gauss did not respond when Abel sent him his proof of the impossibility of solving the quintic equation in 1825, and the two never met, although Gauss praised him in private letters. When Dirichlet wrote Gauss in May 1826, enclosing his first work on number theory and asking for guidance, Gauss did not reply until 13 September and then only with general encouragement and advice to find a job that left time for research. As indicated in a letter to Encke of 8 July, Gauss was much impressed by Dirichlet's "eminent talent," but he did not seem inclined to become mathematically involved with him. When Crelle in 1828 asked Gauss for a paper on elliptic functions, he replied that Jacobi had covered his work "with so much sagacity, penetration and elegance, that I believe that I am relieved of publishing my own research." Harassed, overworked, distracted, and frustrated during these years, Gauss undoubtedly underestimated the value of his achievements, something he had never done before. But he was correct in sensing the need of a new source of inspiration. In turning toward intensive investigations in physics, he was following a pattern that had proved richly productive in the past.

In 1828 Alexander von Humboldt persuaded Gauss to attend the only scientific convention of his career, the Naturforscherversammlung in Berlin. Since first hearing of Gauss from the leading mathematicians in Paris in 1802, Humboldt had been trying to bring him to Berlin as the leading figure of a great academy he hoped to build there. At times negotiations had seemed near success, but bureaucratic inflexibilities in Berlin or personal factors in Göttingen always intervened. Humboldt still had not abandoned these hopes, but he had other motives as well. He wished to draw Gauss into the German scientific upsurge whose beginnings were reflected in the meeting; and especially he wished to involve Gauss in his own efforts, already extending over two decades, to organize worldwide geomagnetic observations. Humboldt had no success in luring Gauss from his Göttingen hermitage. He was repelled by the Berlin convention, which included a "little celebration" to which Humboldt invited 600 guests. Nevertheless, the visit was a turning point. Living quietly for three weeks in Humboldt's house with a private garden and his host's scientific equipment, Gauss had both leisure and stimulation for making a choice. When Humboldt later wrote of his satisfaction at having interested him in magnetism, Gauss replied tactlessly that he had been interested in it for nearly thirty years. Correspondence and manuscripts show this to be true; they indicate that Gauss delayed serious work on the subject partly because means of measurement were not available. Nevertheless, the Berlin visit was the occasion for the decision and also provided the means for implementing it, since in Berlin Gauss met Wilhelm Weber, a young and brilliant experimental physicist whose collaboration was essential.

In September 1829 Quetelet visited Göttingen and found Gauss very interested in terrestrial magnetism but with little experience in measuring it. The new field had evidently been selected, but systematic work awaited Weber's arrival in 1831. Meanwhile, Gauss extended his long-standing knowledge of the physical literature and began to work on problems in theoretical physics, and especially in mechanics, capillarity, acoustics, optics, and crystallography. The first fruit of this research was *Über ein neues allgemeines Grundgesetz der Mechanik* (1829). In it Gauss stated the law of least constraint: the motion of a system departs as little as possible from free motion, where departure, or constraint, is measured by the sum of products of the masses times the squares of their deviations from the path of free motion. He presented it merely as a new formulation equivalent to the well-known principle of d'Alembert. This work seems obviously related to the old meditations on least squares, but Gauss wrote to Olbers on 31 January 1829 that it was inspired by studies of capillarity and other physical problems. In 1830 appeared *Principia generalia theoriae figurae fluidorum in statu aequilibrii,* his one contribution to capillarity and an important paper in the calculus of variations, since it was the first solution of a variational problem involving double integrals, boundary conditions, and variable limits.

The years 1830–1831 were the most trying of Gauss's life. His wife was very ill, having suffered since 1818 from gradually worsening tuberculosis and hysterical neurosis. Her older son left in a huff and immigrated to the United States after quarreling with his father over youthful profligacies. The country was in a revolutionary turmoil of which Gauss thoroughly disapproved. Amid all these vexations, Gauss continued work on biquadratic residues, arduous geodesic calculations, and many other tasks. On 13 September 1831 his wife died. Two days later Weber arrived.

As Gauss and Weber began their close collaboration and intimate friendship, the younger man was just half the age of the older. Gauss took a fatherly attitude. Though he shared fully in experimental work, and though Weber showed high theoretical competence and originality during the collaboration and later, the older man led on the theoretical and the younger on the experimental side. Their joint efforts soon produced results. In 1832 Gauss presented to the Academy the *Intensitas vis magneticae terrestris ad mensuram absolutam revocata* (1833), in which appeared the first systematic use of absolute units (distance, mass, time) to measure a nonmechanical quantity. Here Gauss typically acknowledged the help of Weber but did not include him as joint author. Stimulated by Faraday's discovery of induced current in 1831, the pair energetically investigated electrical phenomena. They arrived at Kirchhoff's laws in 1833 and anticipated various discoveries in static, thermal, and frictional electricity but did not publish, presumably because their interest centered on terrestrial magnetism.

The thought that a magnetometer might also serve as a galvanometer almost immediately suggested its use to induce a current that might send a message. Working alone, Weber connected the astronomical observatory and the physics laboratory with a mile-long double wire that broke "uncountable" times as he strung it over houses and two towers. Early in 1833 the first words were sent, then whole sentences. This first operating electric telegraph was mentioned briefly by Gauss in a notice in the *Göttingische gelehrte Anzeigen* (9 August 1834; *Werke,* V, 424–425), but it seems to have been unknown to other inventors. Gauss soon realized the military and economic importance of the invention and tried unsuccessfully to promote its use by government and industry on a large scale. Over the years, the wire was replaced twice by one of better quality, and various improvements were made in the terminals. In 1845 a bolt of lightning fragmented the wire, but by this time it was no longer in use. Other inventors (Steinheil in Munich in 1837, Morse in the United States in 1838) had independently developed more efficient and exploitable methods, and the Gauss–Weber priority was forgotten.

The new magnetic observatory, free of all metal that might affect magnetic forces, was part of a network that Humboldt hoped would make coordinated measurements of geographical and temporal variations. In 1834 there were already twenty-three magnetic observatories in Europe, and the comparison of data from them showed the existence of magnetic storms. Gauss and Weber organized the Magnetische Verein, which united a worldwide network of observatories. Its *Resultate aus den Beobachtungen des magnetischen Vereins* appeared in six volumes (1836–1841) and included fifteen papers by Gauss, twenty-three by Weber, and the joint *Atlas des Erdmagnetismus* (1840). These and other publications elsewhere dealt with problems of instrumentation (including one of several inventions of the bifilar magnetometer), reported observations of the horizontal and vertical components of magnetic force, and attempted to explain the observations in mathematical terms.

The most important publication in the last category was the *Allgemeine Theorie des Erdmagnetismus* (1839). Here Gauss broke the tradition of armchair theorizing about the earth as a fairly neutral carrier of one or more magnets and based his mathematics on data. Using ideas first considered by him in 1806, well formulated by 1822, but lacking empirical foundation until 1838, Gauss expressed the magnetic potential at any point on the earth's surface by an infinite series of spherical functions and used the data collected by the world network to evaluate the first twenty-four coefficients. This was a superb interpolation, but Gauss hoped later to explain the results by a physical theory about the magnetic composition of the earth. Felix Klein has pointed out that this can indeed be done (*Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert* [Berlin, 1926], pt. 1, p. 22), but that little is thereby added to the effective explanation offered by the Gaussian formulas. During these years Gauss found time to continue his geodesic data reduction, assist in revising the weights and measures of Hannover, make a number of electric discoveries jointly with Weber, and take an increasing part in university affairs.

This happy and productive collaboration was suddenly upset in 1837 by a disaster that soon effectively terminated Gauss's experimental work. In September, at the celebration of the 100th anniversary of the university (at which Gauss presented Humboldt with plans for his bifilar magnetometer), it was rumored that the new King Ernst August of Hannover might abrogate the hard-won constitution of 1833 and demand that all public servants swear a personal oath of allegiance to himself. When he did so in November, seven Göttingen professors, including Weber and the orientalist G. H. A. von Ewald, the husband of Gauss's older daughter, Minna, sent a private protest to the cabinet, asserting that they were bound by their previous oath to the constitution of 1833. The "Göttingen Seven" were unceremoniously fired, three to be banished and the rest (including Weber and Ewald) permitted to remain in the town. Some thought that Gauss might resign, but he took no public action; and his private efforts, like the public protest of six additional professors, were ignored. Why did Gauss not act more energetically? At age sixty he was too set in his ways, his mother was too old to move, and he hated anything politically radical and disapproved of the protest. The seven eventually found jobs elsewhere. Ewald moved to Tübingen, and Gauss was deprived of the company of his most beloved daughter, who had been ill for some years and died of consumption in 1840. Weber was supported by colleagues for a time, then drifted away and accepted a job at Leipzig. The collaboration petered out, and Gauss abandoned further physical research. In 1848, when Weber recovered his position at Göttingen, it was too late to renew collaboration and Weber continued his brilliant career alone.

As Gauss was ending his physical research, he published *Allgemeine Lehrsätze in Beziehung auf die im verkehrten Verhältnisse des Quadrats der Entfernung wirkenden Anziehungs- und Abstossungskräfte* (1840). Growing directly out of his magnetic work but linked also to his *Theoria attractionis* of 1813, it was the first systematic treatment of potential theory as a mathematical topic, recognized the necessity of existence theorems in that field, and reached a standard of rigor that remained unsurpassed for more than a century, even though the main theorem of the paper was false, according to C. J. de la Vallée Poussin (see *Revue des questions scientifiques,* **133** [1962], 314–330, esp. 324). In the same year he finished *Dioptrische Untersuchungen* (1841), in which he analyzed the path of light through a system of lenses and showed, among other things, that any system is equivalent to a properly chosen single lens. Although Gauss said that he had possessed the theory forty years before and considered it too elementary to publish, it has been labeled his greatest work by one of his scientific biographers (Clemens Schäfer, in *Werke,* XI, pt. 2, sec. 2, 189 ff.). In any case, it was his last significant scientific contribution.

Later Years. From the early 1840's the intensity of Gauss's activity gradually decreased. Further publications were either variations on old themes, reviews, reports, or solutions of minor problems. His reclusion is illustrated by his lack of response in 1845 to Kummer's invention of ideals (to restore unique factorization) and in 1846 to the discovery of Neptune by Adams, Le Verrier, and Galle. But the end of magnetic research and the decreased rate of publication did not mean that Gauss was inactive. He continued astronomical observing. He served several times as dean of the Göttingen faculty. He was busy during the 1840's in finishing many old projects, such as the last calculations on the Hannover survey. In 1847 he eloquently praised number theory and G. Eisenstein in the preface to the collected works of this ill-fated young man who had been one of the few to tell Gauss anything he did not already know. He spent several years putting the university widows' fund on a sound actuarial basis, calculating the necessary tables. He learned to read and speak Russian fluently, apparently first attracted by Lobachevsky but soon extending his reading as widely as permitted by the limited material available. His notebooks and correspondence show that he continued to work on a variety of mathematical problems. Teaching became less distasteful, perhaps because his students were better prepared and included some, such as Dedekind and Riemann, who were worthy of his efforts.

During the Revolution of 1848 Gauss stood guard with the royalists (whose defeat permitted the return of his son-in-law and Weber). He joined the Literary Museum, an organization whose library provided conservative literature for students and faculty, and made a daily visit there. He carefully followed political, economic, and technological events as reported in the press. The fiftieth anniversary celebration of his doctorate in 1849 brought him many messages and formal honors, but the world of mathematics was represented only by Jacobi and Dirichlet. The paper that Gauss delivered was his fourth proof of the fundamental theorem of algebra, appropriately a variation of the first in his thesis of 1799. After this celebration, Gauss continued his interests at a slower pace and became more than ever a legendary figure unapproachable by those outside his personal circle. Perhaps stimulated by his actuarial work, he fell into the habit of collecting all sorts of statistics from the newspapers, books, and daily observations. Undoubtedly some of these data helped him with financial speculations shrewd enough to create an estate equal to nearly 200 times his annual salary. The "star gazer," as his father called him, had, as an afterthought, achieved the financial status denied his more "practical" relatives.

Due to his careful regimen, no serious illnesses had troubled Gauss since his surveying days. Over the years he treated himself for insomnia, stomach discomfort, congestion, bronchitis, painful corns, shortness of breath, heart flutter, and the usual signs of aging without suffering any acute attacks. He had been less successful in resisting chronic hypochondria and melancholia which increasingly plagued him after the death of his first wife. In the midst of some undated scientific notes from his later years there suddenly appears the sentence "Death would be preferable to such a life," and at fifty-six he wrote Gerling (8 February 1834) that he felt like a stranger in the world.

After 1850, troubled by developing heart disease, Gauss gradually limited his activity further. He made his last astronomical observation in 1851, at the age of seventy-four, and later the same year approved Riemann's doctoral thesis on the foundations of complex analysis. The following year he was still working on minor mathematical problems and on an improved Foucault pendulum. During 1853–1854 Riemann wrote his great *Habilitationsschrift* on the foundations of geometry, a topic chosen by Gauss. In June 1854 Gauss, who had been under a doctor's care for several months, had the pleasure of hearing Riemann's probationary lecture, symbolic of the presence in Germany at last of talents capable of continuing his work. A few days later he left Göttingen for the last time to observe construction of the railway from Kassel. By autumn his illness was much worse. Although gradually more bedridden, he kept up his reading, correspondence, and trading in securities until he died in his sleep late in February 1855.

Mathematical Scientist. Gauss the man of genius stands in the way of evaluating the role of Gauss as a scientist. His mathematical abilities and exploits caused his contemporaries to dub him *princeps,* and biographers customarily place him on a par with Archimedes and Newton. This traditional judgment is as reasonable as any outcome of the ranking game, but an assessment of his impact is more problematic because of the wide gap between the quality of his personal accomplishments and their effectiveness as contributions to the scientific enterprise. Gauss published only about half his recorded innovative ideas (see Figure 1) and in a style so austere that his readers were few. The unpublished results appear in notes, correspondence, and reports to official bodies, which became accessible only many years later. Still other methods and discoveries are only hinted at in letters or incomplete notes. It is therefore necessary to reexamine Gauss as a participant in the scientific community and to look at his achievements in terms of their scientific consequences.

The personality traits that most markedly inhibited the effectiveness of Gauss as a participant in scientific activity were his intellectual isolation, personal ambition, deep conservatism and nationalism, and rather narrow cultural outlook. It is hard to appreciate fully the isolation to which Gauss was condemned in childhood by thoughts that he could share with no one. He must soon have learned that attempts to communicate led, at best, to no response; at worst, to the ridicule and estrangement that children find so hard to bear. But unlike most precocious children, who eventually find intellectual comrades, Gauss during his whole life found no one with whom to share his most valued thoughts. Kästner was not interested when Gauss told him of his first great discovery, the constructibility of the regular 17-gon. Bolyai, his most promising friend at Göttingen, could not appreciate his thinking. These and many other experiences must have convinced Gauss that there was little to be gained from trying to interchange theoretical ideas. He drew on the great mathematicians of the past and on contemporaries in France (whom he treated as from another world); but he remained outside the mathematical activity of his day, almost as if he were actually no longer living and his publications were being discovered in the archives. He found it easier and more useful to communicate with empirical scientists and technicians, because in those areas he was among peers; but even there he remained a solitary worker, with the exception of the collaboration with Weber.

Those who admired Gauss most and knew him best found him cold and uncommunicative. After the Berlin visit, Humboldt wrote Schumacher (18 October 1828) that Gauss was "glacially cold" to unknowns and unconcerned with things outside his immediate circle. To Bessel, Humboldt wrote (12 October 1837) of Gauss's "intentional isolation," his habit of suddenly taking possession of a small area of work, considering all previous results as part of it, and refusing to consider anything else. C. G. J. Jacobi complained in a letter to his brother (21 September 1849) that in twenty years Gauss had not cited any publication by him or by Dirichlet. Schumacher, the closest of Gauss's friends and one who gave him much personal counsel and support, wrote to Bessel (21 December 1842) that Gauss was "a queer sort of fellow" with whom it is better to stay "in the limits of conventional politeness, without trying to do anything uncalled for."

Like Newton, Gauss had an intense dislike of controversy. There is no record of a traumatic experience that might account for this, but none is required to explain a desire to avoid emotional involvements that interfered with contemplation. With equal rationality, Gauss avoided all noncompulsory ceremonies and formalities, making an exception only when royalty was to be present. In these matters, as in his defensive attitude toward possible wasters of his time, Gauss was acting rationally to maximize his scientific output; but the result was to prevent some interchanges that might have been as beneficial to him as to others.

Insatiable drive, a characteristic of persistent high achievers, could hardly in itself inhibit participation; but conditioned by other motivations it did so for Gauss. Having experienced bitter poverty, he worked toward a security that was for a long time denied him. But he had absorbed the habitual frugality of the striving poor and did not want or ever adopt luxuries of the parvenu. He had no confidence in the democratic state and looked to the ruling aristocracy for security. The drive for financial security was accompanied by a stronger ambition, toward great achievement and lasting fame in science. While still an adolescent Gauss realized that he might join the tiny superaristocracy of science that seldom has more than one member in a generation. He wished to be worthy of his heroes and to deserve the esteem of future peers. His sons reported that he discouraged them from going into science on the ground that he did not want any second-rate work associated with his name. He had little hope of being understood by his contemporaries; it was sufficient to impress and to avoid offending them. In the light of his ambitions for security and lasting fame, with success in each seemingly required for the other, his choice of career and his purposeful isolation were rational. He did achieve his twin ambitions. More effective communication and participation might have speeded the development of mathematics by several decades, but it would not have added to Gauss's reputation then or now. Gauss probably understood this well enough. He demonstrated in some of his writings, correspondence, lectures, and organizational activities that he could be an effective teacher, expositor, popularizer, diplomat, and promoter when he wished. He simply did not wish.

Gauss's conservatism has been described above, but it should be added here that it extended to all his thinking. He looked nostalgically back to the eighteenth century with its enlightened monarchs supporting scientific aristocrats in academies where they were relieved of teaching. He was anxious to find "new truths" that did not disturb established ideas. Nationalism was important for Gauss. As we have seen, it impelled him toward geodesy and other work that he considered useful to the state. But its most important effect was to deny him easy communication with the French. Only in Paris, during his most productive years, were men with whom he could have enjoyed a mutually stimulating mathematical collaboration.

It seems strange to call culturally narrow a man with a solid classical education, wide knowledge, and voracious reading habits. Yet outside of science Gauss did not rise above petit bourgeois banality. Sir Walter Scott was his favorite British author, but he did not care for Byron or Shakespeare. Among German writers he liked Jean Paul, the best-selling humorist of the day, but disliked Goethe and disapproved of Schiller. In music he preferred light songs and in drama, comedies. In short, his genius stopped short at the boundaries of science and technology, outside of which he had little more taste or insight than his neighbors.

The contrast between knowledge and impact is now understandable. Gauss arrived at the two most revolutionary mathematical ideas of the nineteenth century: non-Euclidean geometry and noncommutative algebra. The first he disliked and suppressed. The second appears as quaternion calculations in a notebook of about 1819 (*Werke,* VIII, 357–362) without having stimulated any further activity. Neither the barycentric calculus of his own student Moebius (1827), nor Grassmann's *Ausdenunglehre* (1844), nor Hamilton's work on quaternions (beginning in 1843) interested him, although they sparked a fundamental shift in mathematical thought. He seemed unaware of the outburst of analytic and synthetic projective geometry, in which C. von Staudt, one of his former students, was a leading participant. Apparently Gauss was as hostile or indifferent to radical ideas in mathematics as in politics.

Hostility to new ideas, however, does not explain Gauss's failure to communicate many significant mathematical results that he did approve. Felix Klein (*Vorlesungen über die Entwicklung der Mathematik im 19. Jahrhundert,* pt. 1, 11–12) points to a combination of factors—personal worries, distractions, lack of encouragement, and overproduction of ideas. The last might alone have been decisive. Ideas came so quickly that each one inhibited the development of the preceding. Still another factor was the advantage that Gauss gained from withholding information, although he hotly denied this motive when Bessel suggested it. In fact, the Ceres calculation that won Gauss fame was based on methods unknown to others. By delaying publication of least squares and by never publishing his calculating methods, he maintained an advantage that materially contributed to his reputation. The same applies to the careful and conscious removal from his writings of all trace of his heuristic methods. The failure to publish was certainly not based on disdain for priority. Gauss cared a great deal for priority and frequently asserted it publicly and privately with scrupulous honesty. But to him this meant being first to discover, not first to publish; and he was satisfied to establish his dates by private records, correspondence, cryptic remarks in publications, and in one case by publishing a cipher. (See bibliography under "Miscellaneous.") Whether he intended it so or not, in this way he maintained the advantage of secrecy without losing his priority in the eyes of later generations. The common claim that Gauss failed to publish because of his high standards is not convincing. He did have high standards, but he had no trouble achieving excellence once the mathematical results were in hand; and he did publish all that was ready for publication by normal standards.

In the light of the above discussion one might expect the Gaussian impact to be far smaller than his reputation—and indeed this is the case. His inventions, including several not listed here for lack of space, redound to his fame but were minor improvements of temporary importance or, like the telegraph, uninfluential anticipations. In theoretical astronomy he perfected classical methods in orbit calculation but otherwise did only fairly routine observations. His personal involvement in calculating orbits saved others trouble and served to increase his fame but were of little long-run scientific importance. His work in geodesy was influential only in its mathematical by-products. From his collaboration with Weber arose only two achievements of significant impact. The use of absolute units set a pattern that became standard, and the Magnetische Verein established a precedent for international scientific cooperation. His work in dioptrics may have been of the highest quality, but it seems to have had little influence; and the same may be said of his other works in physics.

When we come to mathematics proper, the picture is different. Isolated as Gauss was, seemingly hardly aware of the work of other mathematicians and not caring to communicate with them, nevertheless his influence was powerful. His prestige was such that young mathematicians especially studied him. Jacobi and Abel testified that their work on elliptic functions was triggered by a hint in the *Disquisitiones arithmeticae.* Galois, on the eve of his death, asked that his rough notes be sent to Gauss. Thus, in mathematics, in spite of delays, Gauss did reach and inspire mathematicians. Although he was more of a systematizer and solver of old problems than an opener of new paths, the very completeness of his results laid the basis for new departures—especially in number theory, differential geometry, and statistics. Although his mathematical thinking was always concrete in the sense that he was dealing with structures based on the real numbers, his work contained the seeds of many highly abstract ideas that came later. Gauss, like Archimedes, pushed the methods of his time to the limit of their possibilities. But unlike his other ability peer, Newton, he did not initiate a profound new development, nor did he have the revolutionary impact of a number of his contemporaries of perhaps lesser ability but greater imagination and daring.

Gauss is best described as a mathematical scientist, or, in the terms common in his day, as a pure and applied mathematician. Ranging easily, competently, and productively over the whole of science and technology, he always did so as a mathematician, motivated by mathematics, utilizing every experience for mathematical inspiration. (Figure 2 shows some of the interrelations of his interests.) Clemens Schäfer, one of his scientific biographers, wrote in *Nature* (**128** [1931], 341): "He was not really a physicist in the sense of searching for new phenomena, but rather

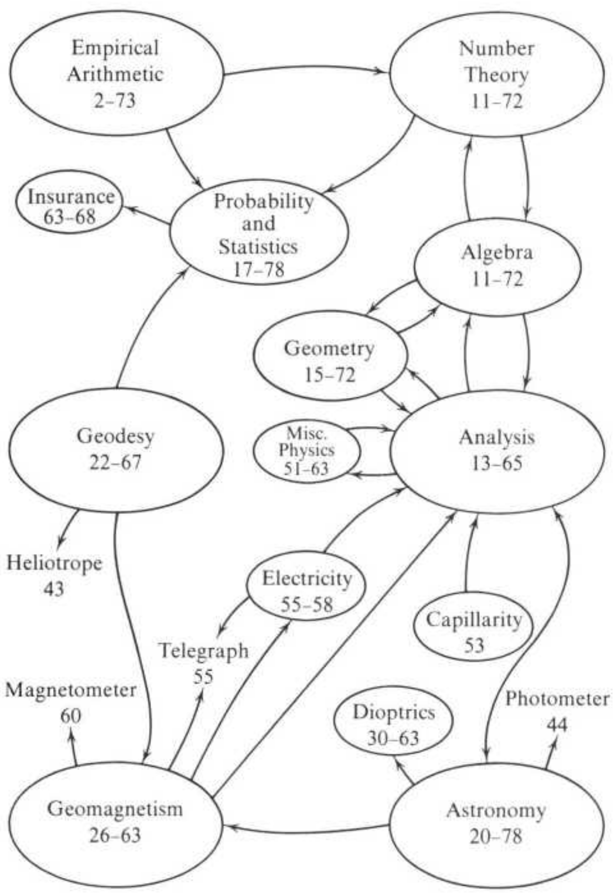

FIGURE 2. Main lines of development of Gauss's scientific ideas. Arrows suggest the most important directions of motivation and inspiration. Numerals indicate ages. His four most important inventions are given outside of any enclosing curves. The sizes of the ellipses suggest the weight of each field in his total effort, and the year span is indicative also of the number and variety of activities in each field. This figure should be compared with Figure 1.

always a mathematician who attempted to formulate in exact mathematical terms the experimental results obtained by others." Leaving aside his personal failures, whose scientific importance was transitory, Gauss appears as the ideal mathematician, displaying in heroic proportions in one person the capabilities attributed collectively to the community of professional mathematicians.

BIBLIOGRAPHY

A complete Gauss bibliography would be far too large to include here, and the following is highly selective. Abbreviations used throughout are the following: *AMM: American Mathematical Monthly. AN: Astronomische Nachrichten. BA: Abhandlungen der (Königlichen) Bayerischen Akademie der Wissenschaften,* Mathematisch-naturwissenschaftliche Abteilung, II Klasse. *BAMS: Bulletin of the American Mathematical Society. BB: Bullettino (Bollettino) di bibliografia e di storia delle scienze matematiche (e fisiche)* (Boncompagni). *BSM: Bulletin des sciences mathématiques et astronomiques* (Darboux). *Crelle: Journal für die reine und angewandte Mathematik. DMV: Jahresbericht der Deutschen Mathematiker-vereinigung. FF: Forschungen und Fortschritte. GA: Abhandlungen der Akademie (K. Gesellschaft) der Wissenschaften zu Göttingen,* Mathematisch-naturwissenschaftliche Klasse. *GGM: Gauss-Gesellschaft Mitteilungen. GN: Nachrichten (Jahrbuch, Jahresbericht) der Gesellschaft der Wissenschaften zu Göttingen. HUB: Wissenschaftliche Zeitschrift der Humboldt-Universität Berlin,* Mathematisch-naturwissenschaftliche Reihe. *IINT: Trudy (Arkhiv) Instituta istorii nauki i tekhniki. IMI: Istoriko-matematicheskie issledovaniya. JMPA: Journal de mathématiques pures et appliquées* (Liouville). *LB: Berichte über die Verhandlungen der (Königlichen) Sächsischen Gesellschaft der Wissenschaften zu Leipzig. MA: Mathematische Annalen. MDA: Monatsberichte der Deutschen Akademie der Wissenschaften zu Berlin. NA: Nouvelles annales de mathématiques. NMM: National Mathematics Magazine.* OK: Ostwalds Klassiker der exacten Wissenschaften (Leipzig). *SM: Scripta mathematica. TSM: Scientific Memoirs, Selected from the Transactions of Foreign Academies and Learned Societies and From Foreign Journals* by Richard Taylor. *VIET: Voprosy istorii estestvoznaniya i tekhniki. Zach: Monatliche Correspondenz zur Beförderung der Erd- und Himmelskunde* (Zach). *ZV: Zeitschrift für Vermessungswesen.*

I. Original Works. All of Gauss's publications (including his fine reviews of his own papers) are reprinted in the *Werke,* published in 12 vols. by the Königliche Gesellschaft der Wissenschaften zu Göttingen (Leipzig–Berlin, 1863–1933). The *Werke* contains also a generous selection of his unpublished notes and papers, related correspondence, commentaries, and extensive analyses of his work in each field. The first 7 vols., edited by Ernst C. J. Schering, who came to Göttingen as a student in 1852 and taught mathematics there from 1858 until his death

in 1897, contain Gauss's publications arranged by subject, as follows: I. *Disquisitiones arithmeticae* (1863; 2nd ed., with commentary, 1870). II. Number Theory (1863; 2nd ed., with the unpublished sec. 8 of the *Disquisitiones*, minor additions, and revisions, 1876). III. Analysis (1866; 2nd ed., with minor changes, 1876). IV. Probability, Geometry, and Geodesy (1873; 2nd ed., almost unchanged, 1880). V. Mathematical Physics (1867; unchanged 2nd ed., 1877). VI. Astronomy (1873). VII. *Theoria motus* (1871; 2nd ed., with new commentary by Martin Brendel and previously unpublished Gauss MSS, 1906).

After the death of Schering, work was continued under the aggressive leadership of Felix Klein, who organized a campaign to collect materials and enlisted experts in special fields to study them. From 1898 until 1922 he rallied support with fourteen reports, published under the title "Bericht über den Stand der Herausgabe von Gauss' Werken," in the *Nachrichten* of the Göttingen Academy and reprinted in *MA* and *BSM*. The fruits of this effort were a much enlarged Gauss Archive at Göttingen, many individual publications, and vols. VIII–XII of the *Werke*, as follows: VIII. Supp. to vols. I–IV (1900), papers and correspondence on mathematics (the paper on pp. 36–64 is spurious. See *Werke*, X, pt. 1, 137). IX. Geodesy (1903). Supp. to vol. IV, including some overlooked Gauss publications. X, pt. 1. Supp. on pure mathematics (1917), including the famous *Tagebuch* in which Gauss from 1796 to 1814 recorded mathematical results. Found in 1898 by P. Stäckel and first published by F. Klein in the *Festschrift zur Feier des hundertfünfzigjährigen Bestehens der Königlichen Gesellschaft der Wissenschaften zu Göttingen* (Berlin, 1901) and in *MA*, **57** (1903), 1–34, it was here reprinted with very extensive commentary and also in facsimile. A French trans. with commentary by P. Eymard and J. P. Lafon appeared in *Revue d'histoire des sciences et de leurs applications*, **9** (1956), 21–51. See also G. Herglotz, in *LB*, **73** (1921), 271–277. X, pt. 2. Biographical essays described below (1922–1933). XI, pt. 1. Supp. on Physics, Chronology, and Astronomy (1927). XI, pt. 2. Biographical essays described below (1924–1929). XII. Varia. *Atlas des Erdmagnetismus* (1929). A final volume, XIII, planned to contain further biographical material (especially on Gauss as professor), bibliography, and index, was nearly completed by H. Geppert and E. Bessel-Hagen but not published.

A. *Translations and Reprints.* The *Demonstratio nova* of 1799 together with the three subsequent proofs of the fundamental theorem (1815, 1816, 1849) were published in German with commentary by E. Netto under the title *Die vier Gauss'schen Beweise* . . . in OK, no. 14 (1890). The *Disquisitiones* (1801) is available in French (1807), German, with other works on number theory (1889; repr. New York, 1965), Russian (1959), and English (1966). Gauss's third published proof of the law of quadratic reciprocity (1808) is translated in D. E. Smith, *Source Book in Mathematics*, I (New York, 1929), 112–118. All his published proofs of this theorem are collected in *Sechs Beweise des Fundamentaltheorems über quadratische Reste*, E. Netto, ed., in OK, no. 122 (1901).

The *Theoria motus* (1809) was translated into English (1857), Russian (1861), French (1864), and German (1865). *Disquisitiones generales circa seriem* (1813) appeared in a German translation by H. Simon in 1888, and *Theoria attractionis* (1813) was translated in *Zach*, **28** (1813), 37–57, 125–234, and reprinted in OK, 19 (1890). The *Determinatio attractionis* (1818) was translated in OK, 225 (1927). The *Allgemeine Auflösung* (1825) was reprinted with related works of Lagrange in OK, 55 (1894). *Theoria combinationis* and supps. of 1823 appeared in French (by J. Bertrand, 1855), German (1887), and with other related work in *Abhandlungen zur Methode der kleinsten Quadrate*, translated by A. Börsch and P. Simon (Berlin, 1887), and in *Gauss's Work (1803–1826) on the Theory of Least Squares*, translated from French by H. F. Trotter (Princeton, N.J., 1957). The *Allgemeine Auflösung* of 1825 appeared in *Philosophical Magazine*, **4** (1828), 104–113, 206–215. *Disquisitiones generales circa superficies curvas* (1828) was translated into French in *NA*, **11** (1852), 195–252, and with notes by E. Roger (Grenoble, 1855); into German by O. Böklen in his *Analytische Geometrie des Raumes* (1884), and by Wangerin in OK, 5 (1889); into Russian (1895), Hungarian (1897); and English (1902). *Über ein neues allgemeines Grundgesetz* (1829) was translated in *NA*, **4** (1845), 477–479.

The *Intensitas vis magneticae* (1833) appears in the *Effemeridi astronomiche di Milano, 1839* (Milan, 1838); in OK, 53 (1894); and in W. F. Magie, *Source Book in Physics* (New York–London, 1935; repr., Cambridge, Mass., 1963), pp. 519–524. The *Allgemeine Theorie des Erdmagnetismus* of 1839 was promptly published in English in *TSM*, **2** (1841), 184–251, 313–316. The *Allgemeine Lehrsätze* (1840) was translated in *JMPA*, **7** (1842), 273–324, and reprinted in OK, 2 (1889). *Dioptrische Untersuchungen* (1841) appeared in English in *TSM*, **3** (1843), 490–498 (see also *Ferrari's Dioptric Instruments* [London, 1919]); and in French in *Annales de chimie*, **33** (1851), 259–294, and in *JMPA*, **1** (1856), 9–43. The *Untersuchungen über Gegenstände der höheren Geodäsie* (1844, 1847) was reprinted as OK, 177 (Leipzig, 1910).

Very little material from the *Nachlass* first printed in the *Werke* has been reprinted or translated. Parts of *Werke*, XI, pt. 1, on the arithmetic-geometric mean and modular functions appear in the OK, 255 (1927), translation of the *Determinatio attractionis* (1818). Some Gauss MSS and editor's commentary are translated from *Werke*, XII, by Dunnington in *Carl Friedrich Gauss, Inaugural Lecture on Astronomy and Papers on the Foundations of Mathematics* (Baton Rouge, La., 1937). Notes on Gauss's astronomy lectures by A. T. Kupffer are printed in A. N. Krylov, *Sobranie trudy* (Moscow–Leningrad, 1936), VI. The following selecta have appeared in Russian: *Geodezicheskie issledovania Gaussa* . . . (St. Petersburg, 1866); *Izbrannye trudy po zemnomu magnetizmu* (Leningrad, 1952); *Izbrannye geodezicheskie sochinenia* (Moscow, 1957).

B. *Correspondence.* Only the major collections are listed here. Many other letters have been published in journal articles and in bibliographies. G. F. J. A. von Auwers, *Briefwechsel zwischen Gauss und Bessel* (Leipzig, 1880). E. Schönberg and T. Gerardy, "Die Briefe des Herrn P. H.

L. von Bogulawski . . .," in *BA,* **110** (1963), 3-44. F. Schmidt and P. Stäckel, *Briefwechsel zwischen C. F. Gauss and W. Bolyai* (Leipzig, 1899). P. G. L. Dirichlet, *Werke,* II (Berlin, 1897), 373-387. C. Schäfer, *Briefwechsel zwischen Carl Friedrich Gauss und Christian Ludwig Gerling* (Berlin, 1927). T. Gerardy, *Christian Ludwig Gerling und Carl Friedrich Gauss. Sechzig bisher unveröffentlichte Briefe* (Göttingen, 1964). H. Stupuy, ed., *Oeuvres philosophiques de Sophie Germain* (Paris, 1879), pp. 298 ff.; and 2nd ed., pp. 254 ff. K. Bruhns, *Briefe zwischen A. v. Humboldt und Gauss* (Leipzig, 1877) (see also K.-R. Bierman, in *FF,* **36** [1962], 41-44, also in *GGM,* **4** [1967], 5-18). T. Gerardy, "Der Briefwechsel zwischen C. F. Gauss und C. L. Lecoq," in *GN* (1959), 37-63. W. Gresky, "Aus Bernard von Lindenaus Briefen an C. F. Gauss," in *GGM,* **5** (1968), 12-46. W. Valentiner, *Briefe von C. F. Gauss an B. Nicolai* (Karlsruhe, 1877). C. Schilling and I. Kramer, *Briefwechsel zwischen Olbers und Gauss,* 2 vols. (Berlin, 1900-1909). C. Pfaff, *Sammlung von Briefen, gewechselt zwischen Johann Friedrich Pfaff und . . . anderen* (Leipzig, 1853). P. Riebesell, "Briefwechsel zwischen C. F. Gauss und J. C. Repsold," in *Mitteilungen der mathematischen Gesellschaft in Hamburg,* **6** (1928), 398-431. C. A. Peters, *Briefwechsel zwischen C. F. Gauss und H. C. Schumacher,* 6 vols. (Altona, 1860-1865). T. Gerardy, *Nachtrage zum Briefwechsel zwischen Carl Friedrich Gauss und Heinrich Christian Schumacher* (Göttingen, 1969).

C. *Archives.* The MSS, letters, notebooks, and library of Gauss have been well preserved. The bulk of the scientific *Nachlass* is collected in the Gauss Archiv of the Handschriftenabteilung of the Niedersächsischen Staats- und Universitätsbibliothek, Göttingen, and fills 200 boxes. (See W. Meyer, *Die Handschriften in Göttingen* [Berlin, 1894], III, 101-113.) Theo Gerardy has for many years been working to arrange and catalog these materials. (See T. Gerardy, "Der Stand der Gaussforschung," in *GGM,* **1** [1964], 5-11.) Personal materials are concentrated in the municipal library of Brunswick. These include the contents of the Gauss Museum, removed from Gauss's birthplace before its destruction during World War II. (See H. Mack, "Das Gaussmuseum in Braunschweig," in *Museumskunde,* n.s. **1** [1930], 122-125.) Gauss's personal library forms a special collection in the Göttingen University Library. His scientific library was merged with the observatory library. There are also minor deposits of MSS, letters, and mementos scattered in the libraries of universities, observatories, and private collectors throughout the world. The best published sources on the Gauss archival material are Felix Klein's reports on the progress of the *Werke* mentioned above and in the yearly *Mitteilungen* of the Gauss-Gesellschaft (GGM), founded in Göttingen in 1962.

II. SECONDARY LITERATURE. There is no full-scale biography of the man and his work as a whole, although there are many personal biographies and excellent studies of his work in particular fields.

A. *Bibliography.* No complete Gauss bibliography has been published. The best ones are in Poggendorff, VII A, supp., Lieferung 2 (1970), 223-238; and in Dunnington's biography (see below).

B. *Biography.* The year after Gauss's death, Sartorius von Waltershausen, a close friend of his last years, published *Gauss zum Gedächtniss* (Leipzig, 1856). An English trans. by his great-granddaughter, Helen W. Gauss, was published as *Gauss, a Memorial* (Colorado Springs, Colo., 1966).

Other sources based on personal acquaintance and/or more or less reliable contemporary evidence are the following: L. Hänselmann, *K. F. Gauss. Zwölf Capital aus seinem Leben* (Leipzig, 1878); I. M. Simonov, *Zapiski i vospominaniya o puteshestvii po Anglii, Frantsii, Belgii i Germanii v 1842 godu* (Kazan, 1844); A. Quetelet, in *Correspondance mathématique et physique,* **6** (1830), 126-148, 161-178, 225-239, repr. in A. Quetelet, *Sciences mathématiques et physiques chez les Belges* (Brussels, 1866); Ernst C. J. Schering, *Carl Friedrich Gauss' Geburtstag nach hundertjähriger Wiederkehr, Festrede* (Göttingen, 1877); M. A. Stern, *Denkrede . . . zur Feier seines hundertjährigen Geburtstages* (Göttingen, 1877); F. A. T. Winnecke, *Gauss. Ein Umriss seines Lebens und Wirkens* (Brunswick, 1877); Theodor Wittstein, *Gedächtnissrede auf C. F. Gauss zur Feier des 30 April 1877* (Hannover, 1877); R. Dedekind, *Gauss in seiner Vorlesungen über die Methode der kleinsten Quadrate. Festschrift . . . Göttingen* (Berlin, 1901), repr. in Dedekind, *Gesammelte mathematische Werke,* II (1931), 293-306; Moritz Cantor lecture of 14 November 1899, in *Neue Heidelberger Jahrbucher,* **9** (1899), 234-255; and Rudolf Borch, "Ahnentafel des . . . Gauss," in *Ahnentafeln berühmter Deutscher,* I (Leipzig, 1929), 63-65.

Most of the personal biographical literature is derivative from the above sources and is of the "beatification forever" type, in which fact and tradition are freely mixed. Only a few works of special interest are mentioned here. Heinrich Mack, *Carl Friedrich Gauss und die Seinen* (Brunswick, 1927), contains substantial excerpts from family correspondence and a table of ancestors and descendants. F. Cajori published family letters in *Science,* n.s. **9** (19 May 1899), 697-704, and in *Popular Science Monthly,* **81** (1912), 105-114. Other studies based on documents are T. Gerardy, "C. F. Gauss und seine Söhne," in *GGM,* **3** (1966), 25-35; W. Lorey, in *Mathematisch-physikalische Semesterberichte* (Göttingen), **3** (1953), 179-192; and Hans Salié, in the collection edited by Reichardt described below. The most complete biography to date is G. W. Dunnington, *Carl Friedrich Gauss, Titan of Science* (New York, 1955), a useful derivative compendium of personal information and tradition, including translations from Sartorius, Hänselmann, and Mack, the largest bibliography yet published, and much useful data on genealogy, friends, students, honors, books borrowed at college, courses taught, etc.

During the Third Reich two rather feeble efforts—L. Bieberbach, *C. F. Gauss, ein deutsches Gelehrtenleben* (Berlin, 1938); and E. A. Roloff, *Carl Friedrich Gauss* (Osnabrück, 1942)—were made to claim Gauss as a hero, but it is clear that Gauss would have loathed the fascists as the final realization of his worst fears about bourgeois politics. Neither author mentions that Gauss's favorite mathematician, whom he praised extravagantly, was Gotthold Eisenstein.

Erich Worbs, *Carl Friedrich Gauss, Ein Lebensbild* (Leipzig, 1955), makes an effort to relate Gauss realistically to his times. W. L. Schaaf, *Carl Friedrich Gauss, Prince of Mathematicians* (New York, 1964), is a popularization addressed to juveniles.

C. *Scientific Work.* The literature analyzing Gauss's scientific work is expert and comprehensive, although its fragmentation by subject matter gives the impression of dealing with several different men. Beginning in 1911, F. Klein, M. Brendel, and L. Schlesinger edited a series of eight studies under the title *Materialien für eine wissenschaftliche Biographie von Gauss* (Leipzig, 1911–1920), most of which were later incorporated in the *Werke.* On the occasion of the hundredth anniversary of Gauss's death, there appeared *C. G. Gauss Gedenkband,* Hans Reichardt, ed. (Leipzig, 1957), republished as *C. F. Gauss, Leben und Werk* (Berlin, 1960); and I. M. Vinogradov, ed., *Karl Friedrich Gauss, 100 let so dnya smerti, sbornik statei* (Moscow, 1956). These collections will be abbreviated as Klein, Reichardt, and Vinogradov, respectively, when individual articles are listed below.

Brief anniversary evaluations by mathematicians are the following: R. Courant and R. W. Pohl, *Carl Friedrich Gauss, Zwei Vorträge* (Göttingen, 1955)—Courant's lecture also appeared in *Carl Friedrich Gauss . . . Gedenkfeier der Akademie der Wissenschaften . . . Göttingen anlässlich seines 100ten Todestages* (Göttingen, 1955) and was translated in T. L. Saaty and J. F. Weyl, eds., *The Spirit and the Uses of the Mathematical Sciences* (New York, 1969), pp. 141–155; J. Dieudonné, *L'oeuvre mathématique de C. F. Gauss* (Paris, 1962), a talk at the Palais de la Découverte, 2 December 1961; R. Oblath, "Megemlékezés halálának 100-ik évfordulóján," in *Matematikai lapok,* **6** (1955), 221–240; and K. A. Rybnikov, in *VIET,* **1** (1956), 44–53.

The following selected titles are arranged by topic.

Algebra. A. Fraenkel, "Zahlbegriff und Algebra bei Gauss," (Klein, VIII), in *GN,* supp. (1920); "Der Zusammenhang zwischen dem ersten und dem dritten Gauss'schen Beweis des Fundamentalsatzes der Algebra," in *DMV,* **31** (1922), 234–238; A. Ostrowski, "Über den ersten und vierten Gauss'schen Beweis des Fundamentalsatzes der Algebra," in *Werke,* X, pt. 2, sec. 3 (1933), 3–18 (an enlarged revision of Klein, VIII [1920], 50–58); R. Kochendörfer, in Reichardt, pp. 80–91; and M. Bocher, "Gauss's Third Proof of the Fundamental Theorem of Algebra," in *BAMS,* **1** (1895), 205–209.

Analysis. A. I. Markushevich, "Raboty Gaussa po matematicheskomu analizu," in Vinogradov, pp. 145–216, German trans. in Reichardt, pp. 151–182; K. Schröder, "C. F. Gauss und die reelle Analysis," in Reichardt, pp. 184–191; O. Bolza, "Gauss und die Variationsrechnung," in *Werke,* X, pt. 2, sec. 5 (1922), 3–93; L. Schlesinger, "Fragment zur Theorie des arithmetisch-geometrischen Mittels" (Klein, II), in *GN* (1912), 513–543; *Über Gauss' Arbeiten zur Funktionentheorie* (Berlin, 1933), also in *Werke,* X, pt. 2, sec. 2 (1933), 3–210—an enlarged revision of Klein II which appeared in *GN* (1912), 1–140; H. Geppert, "Wie Gauss zur elliptischen Modul-funktion kam," in *Deutsche Mathematik,* **5** (1940), 158–175; E. Göllnitz, "Über die Gauss'sche Darstellung der Funktionen sinlemn x und coslemn x als Quotienten unendlicher Produkte," in *Deutsche Mathematik,* **2** (1937), 417–420; P. Gunther, "Die Untersuchungen von Gauss in der Theorie der elliptischen Funktionen," in *GN* (1894), 92–105, and in trans. in *JMPA,* 5th ser., **3** (1897), 95–111; H. Hattendorff, *Die elliptischen Funktionen in dem Nachlasse von Gauss* (Berlin, 1869); A. Pringsheim, "Kritisch-historische Bemerkungen zur Funktionentheorie," in *BA* (1932), 193–200; (1933), 61–70; L. Schlesinger, "Über die Gauss'sche Theorie des arithmetisch-geometrischen Mittels . . .," in *Sitzungsberichte der Preussischen Akademie der Wissenschaften zu Berlin,* **28** (1898), 346–360; and "Über Gauss Jugendarbeiten zum arithmetisch-geometrischen Mittel," in *DMV,* **20** (1911), 396–403.

Astronomy. M. Brendel, "Über die astronomischen Arbeiten von Gauss," in *Werke,* XI, pt. 2, sec. 3 (1929), 3–254, enlarged revision of Klein, vol. VII, pt. 1 (Leipzig, 1919); M. F. Subbotin, "Astronomicheskie i geodesicheskie raboty Gaussa," in Vinogradov, pp. 241–310; and O. Volk, "Astronomie und Geodäsie bei C. F. Gauss," in Reichardt, pp. 206–229.

Geodesy and Surveying. A. Galle, "Über die geodätischen Arbeiten von Gauss," in *Werke,* XI, pt. 2, sec. 1 (1924), 3–161; W. Gronwald *et al., C. F. Gauss und die Landesvermessung in Niedersachsen* (Hannover, 1955); T. Gerardy, *Die Gauss'sche Triangulation des Königreichs Hannover (1821 bis 1844) und die Preussischen Grundsteuermessungen (1868 bis 1873)* (Hannover, 1952); G. V. Bagratuni, *K. F. Gauss, kratky ocherk geodezicheskikh issledovanii* (Moscow, 1955); M. F. Subbotin, in Vinogradov (see under Astronomy); W. Gäde, "Beiträge zur Kenntniss von Gauss' praktisch-geodätischen Arbeiten," in *ZV,* **14** (1885), 53–113; T. Gerardy, "Episoden aus der Gauss'schen Triangulation des Königreichs Hannover," in *ZV,* **80** (1955), 54–62; H. Michling, *Erläuterungsbericht zur Neuberechnung der Gauss-Kruegerischen Koordinaten der Dreiecks- und Polygonpunkte der Katasterurmessung* (Hannover, 1947); "Der Gauss'sche Vizeheliotrop," in *GGM,* **4** (1967), 27–30; K. Nickul, "Über die Herleitung der Abbildungsgleichung der Gauss'schen Konformen Abbildung des Erdellipsoids in der Ebene," in *ZV,* **55** (1926), 493–496; and O. Volk, in Reichardt (see under Astronomy).

Geomagnetism. Ernst Schering, "Carl Friedrich Gauss und die Erforschung des Erdmagnetismus," in *GA,* **34** (1887), 1–79; T. N. Roze and I. M. Simonov, in *K. F. Gauss, Izbrannye trudy po zemnomu magnitizmu* (Leningrad, 1952), pp. 237–336; H.-G. Körber, "Alexander von Humboldts und Carl Friedrich Gauss' organisatorisches Wirken auf geomagnetischen Gebiet," in *FF,* **32** (1958), 1–8; and K.-R. Biermann, "Aus der Vorgeschichte der Aufforderung A. v. Humboldts an der Präsidenten der Royal Society . . .," in *HUB,* **12** (1963), 209–227.

Geometry. P. Stäckel, "C. F. Gauss als Geometer," in *Werke,* X, pt. 2, sec. 4 (1923), 3–121, repr. with note by L. Schlesinger from Klein, V (1917), which appeared also in *GN,* **4** (1917), 25–140; A. P. Norden, "Geometricheskie raboty Gaussa," in Vinogradov, pp. 113–144; R. C. Archi-

bald, "Gauss and the Regular Polygon of Seventeen Sides," in *AMM,* **27** (1920), 323-326; H. Carslaw, "Gauss and Non-Euclidean Geometry," in *Nature,* **84,** no. 2134 (1910), 362; G. B. Halsted, "Gauss and non-Euclidean Geometry," in *AMM,* **7** (1900), 247, and on the same subject, in *AMM,* **11** (1904), 85-86, and in *Science,* **9,** no. 232 (1904), 813-817; and E. Hoppe, "C. F. Gauss und der Euklidische Raum," in *Naturwissenschaften,* **13** (1925), 743-744, and in trans. by Dunnington in *Scripta mathematica,* **20** (1954), 108-109 (Hoppe objects to the story that Gauss measured a large geodesic triangle in order to test whether Euclidean geometry was the "true" one, apparently under the impression that this would have been contrary to Gauss's ideas. Actually, Gauss considered geometry to have an empirical base and to be testable by experience.); V. F. Kagan, "Stroenie neevklidovoi geometrii u Lobachevskogo, Gaussa i Boliai," in *Trudy Instituta istorii estestvoznaniya,* **2** (1948), 323-389, repr. in his *Lobachevskii i ego geometriya* (Moscow, 1955), pp. 193-294; N. D. Kazarinoff, "On Who First Proved the Impossibility of Constructing Certain Regular Polygons . . .," in *AMM,* **75** (1968), 647; P. Mansion, "Über eine Stelle bei Gauss, welche sich auf nichteuklidische Metrik bezieht," in *DMV,* **7** (1899), 156; A. P. Norden, "Gauss i Lobachevskii," in *IMI,* **9** (1956), 145-168; A. V. Pogorelov, "Raboty K. F. Gaussa po geometrii poverkhnostei," in *VIET,* **1** (1956), 61-63; and P. Stäckel and F. Engel, *Die Theorie der Parallelinien* (Leipzig, 1895); "Gauss, die beiden Bolyai und die nichteuklidische Geometrie," in *MA,* **49** (1897), 149-206, translated in *BSM,* 2nd ser., **21** (1897), 206-228.

Miscellaneous. K.-R. Biermann, "Einige Episoden aus den russischen Sprachstudien des Mathematikers C. F. Gauss," in *FF,* **38** (1964), 44-46; E. Göllnitz, "Einige Rechenfehler in Gauss' Werken," in *DMV,* **46** (1936), 19-21; and S. C. Van Veen, "Een conflict tusschen Gauss en een Hollandsch mathematicus," in *Wiskunstig Tijdschrift,* **15** (1918), 140-146. The following four papers deal with the ciphers in which Gauss recorded some discoveries: K.-R. Biermann, in *MDA,* **5** (1963), 241-244; **11** (1969), 526-530; T. L. MacDonald, in *AN,* **241** (1931), 31; P. Männchen, in *Unterrichtsblätter für Mathematik und Naturwissenschaften,* **40** (1934), 104-106; and A. Wietzke, in *AN,* **240** (1930), 403-406.

Number Theory. P. Bachmann, "Über Gauss' Zahlentheoretische Arbeiten" (Klein, I), in *GN* (1911), pp. 455-508, and in *Werke,* X, pt. 2, sec. 1 (1922), 3-69; B. N. Delone, "Raboty Gaussa po teorii chisel," in Vinogradov, pp. 11-112; G. J. Rieger, "Die Zahlentheorie bei C. F. Gauss," in Reichardt, pp. 37-77; E. T. Bell, "The Class Number Relations Implicit in the *Disquisitiones arithmeticae,*" in *BAMS,* **30** (1924), 236-238; "Certain Class Number Relations Implied in the *Nachlass* of Gauss," *ibid.,* **34** (1928), 490-494; "Gauss and the Early Development of Algebraic Numbers," in *NMM,* **18** (1944), 188-204, 219-223; L. E. Dickson, *History of the Theory of Numbers,* 3 vols. (Washington, D.C., 1919)—the indexes are a fairly complete guide to Gauss's extraordinary achievements in this field; J. Ginsburg, "Gauss' Arithmetization of the Problem of 8 Queens," in *SM,* **5** (1938), 63-66; F. Van der Blij, "Sommen van Gauss," in *Euclides* (Groningen), **30** (1954), 293-298; and B. A. Venkov, "Trudy K. F. Gaussa po teorii chisel i algebra," in *VIET,* **1** (1956), 54-60. The following papers concern an erroneous story, apparently started by W. W. R. Ball, that the Paris mathematicians rejected the *Disquisitiones arithmeticae:* R. C. Archibald, "Gauss's *Disquisitiones arithmeticae* and the French Academy of Sciences," in *SM,* **3** (1935), 193-196; H. Geppert and R. C. Archibald, "Gauss's *Disquisitiones Arithmeticae* and the French Academy of Sciences," *ibid.,* 285-286; G. W. Dunnington, "Gauss, His Disquisitiones Arithmeticae and His Contemporaries in the Institut de France," in *NMM,* **9** (1935), 187-192; A. Emch, "Gauss and the French Academy of Science," in *AMM,* **42** (1935), 382-383. See also G. Heglotz, "Zur letzten Eintragung im Gauss'schen Tagebuch," in *LB,* **73** (1921), 271-277.

Numerical Calculations. P. Männchen, "Die Wechselwirkung zwischen Zahlenrechnung und Zahlentheorie bei C. F. Gauss" (Klein, VI), in *GN,* supp. **7** (1918), 1-47, and in *Werke,* X, pt. 2, sec. 6 (1930), 3-75; and A. Galle, "C. F. Gauss als Zahlenrechner" (Klein, IV), in *GN,* supp. **4** (1917), 1-24.

Philosophy. A. Galle, "Gauss und Kant," in *Weltall,* **24** (1925), 194-200, 230, repr. in *GGM,* **6** (1969), 8-15; P. Mansion, "Gauss contre Kant sur la géométrie non-Euclidienne," in *Mathesis,* 3rd ser., **8,** supp. (Dec. 1908), 1-16, in *Revue néoscolastique,* **15** (1908), 441-453, and in *Proceedings of the Third (1908) International Congress of Philosophy in Heidelberg* (Leipzig, 1910), pp. 438-447; and H. E. Timerding, "Kant und Gauss," in *Kant-Studien,* **28** (1923), 16-40.

Physics. H. Falkenhagen, "Die wesentlichsten Beiträge von C. F. Gauss aus der Physik," in Reichardt, pp. 232-251; H. Geppert, "Über Gauss' Arbeiten zur Mechanik und Potentialtheorie," in *Werke,* X, pt. 2, sec. 7 (1933), 3-60; and C. Schäfer, "Gauss physikalische Arbeiten (Magnetismus, Elektrodynamik, Optik)," in *Werke,* XI, pt. 2, sec. 2 (1929), 2-211; "Gauss's Investigations on Electrodynamics," in *Nature,* **128** (1931), 339-341.

Probability and Statistics (Including Least Squares). B. V. Gnedenko, "O raboty Gaussa po teorii veroyatnostei," in Vinogradov, pp. 217-240; A. Galle, "Über die geodätischen Arbeiten von Gauss," in *Werke,* XI, pt. 2, sec. 6 (1924), 3-161; C. Eisenhart, "Gauss," in *International Encyclopedia of the Social Sciences,* VI (New York, 1968), 74-81; P. Männchen, "Über ein Interpolationsverfahren des jugendlichen Gauss," in *DMV,* **28** (1919), 80-84; H. L. Seal, "The Historical Development of the Gauss Linear Model," in *Biometrika,* **54** (1967), 1-24; T. Sofonea, "Gauss und die Versicherung," in *Verzekerings-Archive,* **32** (Aktuar Bijv, 1955), 57-69; and Helen M. Walker, *Studies in the History of Statistical Method* (Baltimore, 1931).

Telegraph. Ernst Feyerabend, *Der Telegraph von Gauss und Weber im Werden der elektrischen Telegraphie* (Berlin, 1933); and R. W. Pohl, "Jahrhundertfeier des elektromagnetischen Telegraphen von Gauss und Weber," in *GN* (1934), pp. 48-56, repr. in *Carl Friedrich Gauss, Zwei Vorträge* (Göttingen, 1955), pp. 5-12.

The author gratefully acknowledges many helpful suggestions and comments from Kurt-R. Biermann. Thanks are due also to the library staff at the University of Toronto for many services. The author claims undivided credit only for errors of fact and judgment.

KENNETH O. MAY

GAUTIER, ARMAND E.-J. (*b.* Narbonne, France, 23 September 1837; *d.* Cannes, France, 27 July 1920), *chemistry.*

Gautier, the son of Louis Gautier, a physician and landowner, studied chemistry at Montpellier under J. E. Bérard (a former assistant to Berthollet) and J. A. Béchamp. In contrast with his teachers, he favored the use of atomic representations, rather than equivalents, in chemical notation. He especially supported the ideas of Charles Gerhardt, who had taught at Montpellier until 1851. After receiving a medical degree in 1862, Gautier left Montpellier for the Paris laboratory of Adolphe Wurtz. His isolation in 1866 of the isonitriles (isomers of the nitriles), or carbylamines, as he called them, was an important contribution to the new chemical theories.

Gautier prepared his carbylamines in a double decomposition reaction between silver cyanide and a simple or compound ether. Despite the penetrating odor of the carbylamines, their formation had escaped earlier workers on cyanohydric ethers, since cyanogen compounds were believed to be completely analogous to halogen compounds; the production of isomers in a simple reaction of the cyanogen radical was unimaginable. Influenced, however, by Wurtz's researches on the "ammonia type," Gautier demonstrated that the isonitriles existed and were indeed amines, whereas ordinary nitriles were more like salts. He noted that the cyanogen carbon atom in the carbylamine permits polymerization into explosives as well as a direct union with sulfur or oxygen. Gautier's explanation illuminated analogous relations between the cyanic ethers and their isomers, and between the cyanates and fulminates. The use of silver cyanide with methyl or ethyl iodide to form the carbylamine suggested to Victor Meyer the reaction of these iodides with silver nitrate to produce nitrated aliphatics (nitromethane, nitroethane, etc.).

Gautier incorporated his researches on the carbylamines into his doctoral thesis ("Des nitriles des acides gras") in 1869 and so impressed Henri Sainte-Claire Deville that the young chemist was quickly appointed to the chemical laboratory of the École Pratique des Hautes-Études. In 1874 Gautier became director of the new laboratory of biological medicine at the Faculté de Médecine, and in 1884 he succeeded Wurtz in the chair of medical chemistry, a post which he held until 1912.

Gautier's researches in these years were prodigious. In 1873 he noticed the release of small quantities of volatile alkaloids during the bacterial fermentation of albuminous material. He demonstrated in 1882 that such alkaloids are constant products of the normal life of animal tissues and are eliminated from the healthy body in urine and saliva. He developed methods for the quantitative analysis of trace amounts of arsenic and demonstrated that such traces exist in healthy animals, especially in the skin. He further established the therapeutic value of arsenic compounds. Gautier analyzed iodine and free hydrogen in the air, iodine and fluorine in organic substances, the coloring matter of grapes, the composition of mineral waters, and chemical reactions related to volcanic phenomena.

He published numerous textbooks, the most significant of which was the three-volume *Cours de chimie minérale, organique et biologique* (1887–1892).

BIBLIOGRAPHY

I. ORIGINAL WORKS. In addition to the *Cours de chimie* (2nd ed., 1895–1897; Vol. II, 3rd ed., 1906), Gautier's texts include *Chimie appliquée à la physiologie, à la pathologie, et à l'hygiène,* 2 vols. (Paris, 1874); *La chimie de la cellule vivante,* 2 vols. (Paris, 1894–1898); and *L'alimentation et les régimes chez l'homme sain et chez les malades* (Paris, 1904; 3rd ed., 1908). An almost complete bibliography of Gautier's scientific and popular writings may be found in Lebon (see below).

II. SECONDARY LITERATURE. Ernest Lebon's biography, *Armand Gautier. Biographie, bibliographie analytique des écrits* (Paris, 1912), is the best single source of information about Gautier. An obituary notice in *Nature* (16 Sept. 1920), 85–86, is reprinted in *Journal of the Chemical Society* (London), **119**, pt. 1 (1921), 537–539. An *éloge* by Henri Deslandres, read at the Académie des Sciences (2 Aug. 1920), is reprinted in *Revue scientifique,* **58** (1920), 471–472.

MARY JO NYE

GAUTIER, PAUL FERDINAND (*b.* Paris, France, 12 October 1842; *d.* Paris, 7 December 1909), *astronomical instrumentation.*

Born into a family of modest means, Gautier was obliged to begin working at the age of thirteen. From the age of eighteen—when he was employed by M. L. F. Secrétan—until his death, he was occupied with construction of astronomical instruments.

Gautier's instruments were closely linked to the strides made by late nineteenth-century astronomy: many of the major refracting telescopes, astrographs,