

conflict with members of the Académie des Sciences delayed his membership until 1929, near the end of his life. In his later years he devoted himself chiefly to paleontological and philosophical studies and to botanical work in his extensive garden.

#### BIBLIOGRAPHY

I. ORIGINAL WORKS. Le Bel's works have been reprinted as *Vie et oeuvres de J.-A. Le Bel*, Marcel Delépine, ed. (Paris, 1949). His theory was explained in "Sur les relations qui existent entre les formules atomiques des corps organiques et le pouvoir rotatoire de leurs dissolutions," in *Bulletin de la Société chimique de Paris*, **22** (1874), 337-347.

II. SECONDARY LITERATURE. Aside from the biography by Delépine listed above, there are short obituaries by W. J. Pope in *Journal of the Chemical Society* (1930), 2789-2791; and by E. Wedekind in *Zeitschrift für angewandte Chemie*, **43** (1930), 985-986.

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**LEBESGUE, HENRI LÉON** (*b.* Beauvais, France, 28 June 1875; *d.* Paris, France, 26 July 1941), *mathematics*.

Lebesgue studied at the École Normale Supérieure from 1894 to 1897. His first university positions were at Rennes (1902-1906) and Poitiers (1906-1910). At the Sorbonne, he became *maître de conférences* (in mathematical analysis, 1910-1919) and then *professeur d'application de la géométrie à l'analyse*. In 1921 he was named professor at the Collège de France and the following year was elected to the Académie des Sciences.

Lebesgue's outstanding contribution to mathematics was the theory of integration that bears his name and that became the foundation for subsequent work in integration theory and its applications. After completing his studies at the École Normale Supérieure, Lebesgue spent the next two years working in its library, where he became acquainted with the work of another recent graduate, René Baire. Baire's unprecedented and successful researches on the theory of discontinuous functions of a real variable indicated to Lebesgue what could be achieved in this area. From 1899 to 1902, while teaching at the Lycée Centrale in Nancy, Lebesgue developed the ideas that he presented in 1902 as his doctoral thesis at the Sorbonne. In this work Lebesgue began to develop his theory of integration which, as he showed, includes within its scope all the bounded discontinuous functions introduced by Baire.

The Lebesgue integral is a generalization of the integral introduced by Riemann in 1854. As Riemann's

theory of integration was developed during the 1870's and 1880's, a measure-theoretic viewpoint was gradually introduced. This viewpoint was made especially prominent in Camille Jordan's treatment of the Riemann integral in his *Cours d'analyse* (1893) and strongly influenced Lebesgue's outlook on these matters. The significance of placing integration theory within a measure-theoretic context was that it made it possible to see that a generalization of the notions of measure and measurability carries with it corresponding generalizations of the notions of the integral and integrability. In 1898 Émile Borel was led through his work on complex function theory to introduce radically different notions of measure and measurability. Some mathematicians found Borel's ideas lacking in appeal and relevance, especially since they involved assigning measure zero to some dense sets. Lebesgue, however, accepted them. He completed Borel's definitions of measure and measurability so that they represented generalizations of Jordan's definitions and then used them to obtain his generalization of the Riemann integral.

After the work of Jordan and Borel, Lebesgue's generalizations were somewhat inevitable. Thus, W. H. Young and G. Vitali, independently of Lebesgue and of each other, introduced the same generalization of Jordan's theory of measure; in Young's case, it led to a generalization of the integral that was essentially the same as Lebesgue's. In Lebesgue's work, however, the generalized definition of the integral was simply the starting point of his contributions to integration theory. What made the new definition important was that Lebesgue was able to recognize in it an analytical tool capable of dealing with—and to a large extent overcoming—the numerous theoretical difficulties that had arisen in connection with Riemann's theory of integration. In fact, the problems posed by these difficulties motivated all of Lebesgue's major results.

The first such problem had been raised unwittingly by Fourier early in the nineteenth century: (1) If a bounded function can be represented by a trigonometric series, is that series the Fourier series of the function? Closely related to (1) is (2): When is the term-by-term integration of an infinite series permissible? Fourier had assumed that for bounded functions the answer to (2) is Always, and he had used this assumption to prove that the answer to (1) is Yes.

By the end of the nineteenth century it was recognized—and emphasized—that term-by-term integration is not always valid even for uniformly bounded series of Riemann-integrable functions, precisely because the function represented by the series need not be Riemann-integrable. These developments,

however, paved the way for Lebesgue's elegant proof that term-by-term integration is permissible for any uniformly bounded series of Lebesgue-integrable functions. By applying this result to (1), Lebesgue was able to affirm Fourier's conclusion that the answer is Yes.

Another source of difficulties was the fundamental theorem of the calculus,

$$\int_a^b f'(x) dx = f(b) - f(a).$$

The work of Dini and Volterra in the period 1878–1881 made it clear that functions exist which have bounded derivatives that are not integrable in Riemann's sense, so that the fundamental theorem becomes meaningless for these functions. Later further classes of these functions were discovered; and additional problems arose in connection with Harnack's extension of the Riemann integral to unbounded functions because continuous functions with densely distributed intervals of invariability were discovered. These functions provided examples of Harnack-integrable derivatives for which the fundamental theorem is false. Lebesgue showed that for bounded derivatives these difficulties disappear entirely when integrals are taken in his sense. He also showed that the fundamental theorem is true for an unbounded, finite-valued derivative  $f'$  that is Lebesgue-integrable and that this is the case if, and only if,  $f$  is of bounded variation. Furthermore, Lebesgue's suggestive observations concerning the case in which  $f'$  is finite-valued but not Lebesgue-integrable were successfully developed by Arnaud Denjoy, starting in 1912, using the transfinite methods developed by Baire.

The discovery of continuous monotonic functions with densely distributed intervals of invariability also raised the question: When is a continuous function an integral? The question prompted Harnack to introduce the property that has since been termed absolute continuity. During the 1890's absolute continuity came to be regarded as the characteristic property of absolutely convergent integrals, although no one was actually able to show that every absolutely continuous function is an integral. Lebesgue, however, perceived that this is precisely the case when integrals are taken in his sense.

A deeper familiarity with infinite sets of points had led to the discovery of the problems connected with the fundamental theorem. The nascent theory of infinite sets also stimulated an interest in the meaningfulness of the customary formula

$$L = \int_a^b [1 + (f')^2]^{1/2}$$

for the length of the curve  $y = f(x)$ . Paul du Bois-Reymond, who initiated an interest in the problem in 1879, was convinced that the theory of integration is indispensable for the treatment of the concepts of rectifiability and curve length within the general context of the modern function concept. But by the end of the nineteenth century this view appeared untenable, particularly because of the criticism and counterexamples given by Ludwig Scheeffer. Lebesgue was quite interested in this matter and was able to use the methods and results of his theory of integration to reinstate the credibility of du Bois-Reymond's assertion that the concepts of curve length and integral are closely related.

Lebesgue's work on the fundamental theorem and on the theory of curve rectification played an important role in his discovery that a continuous function of bounded variation possesses a finite derivative except possibly on a set of Lebesgue measure zero. This theorem gains in significance when viewed against the background of the century-long discussion of the differentiability properties of continuous functions. During roughly the first half of the nineteenth century, it was generally thought that continuous functions are differentiable at "most" points, although continuous functions were frequently assumed to be "piecewise" monotonic. (Thus, differentiability and monotonicity were linked together, albeit tenuously.) By the end of the century this view was discredited, and no less a mathematician than Weierstrass felt that there must exist continuous monotonic functions that are nowhere differentiable. Thus, in a sense, Lebesgue's theorem substantiated the intuitions of an earlier generation of mathematicians.

Riemann's definition of the integral also raised problems in connection with the traditional theorem positing the identity of double and iterated integrals of a function of two variables. Examples were discovered for which the theorem fails to hold. As a result, the traditional formulation of the theorem had to be modified, and the modifications became drastic when Riemann's definition was extended to unbounded functions. Although Lebesgue himself did not resolve this infelicity, it was his treatment of the problem that formed the foundation for Fubini's proof (1907) that the Lebesgue integral does make it possible to restore to the theorem the simplicity of its traditional formulation.

During the academic years 1902–1903 and 1904–1905, Lebesgue was given the honor of presenting the Cours Peccot at the Collège de France. His lectures, published as the monographs *Leçons sur l'intégration . . .* (1904) and *Leçons sur les séries trigonométriques* (1906), served to make his ideas better known. By

1910 the number of mathematicians engaged in work involving the Lebesgue integral began to increase rapidly. Lebesgue's own work—particularly his highly successful applications of his integral in the theory of trigonometric series—was the chief reason for this increase, but the pioneering research of others, notably Fatou, F. Riesz, and Fischer, also contributed substantially to the trend. In particular, Riesz's work on  $L^p$  spaces secured a permanent place for the Lebesgue integral in the theory of integral equations and function spaces.

Although Lebesgue was primarily concerned with his own theory of integration, he played a role in bringing about the development of the abstract theories of measure and integration that predominate in contemporary mathematical research. In 1910 he published an important memoir, "Sur l'intégration des fonctions discontinues," in which he extended the theory of integration and differentiation to  $n$ -dimensional space. Here Lebesgue introduced, and made fundamental, the notion of a countably additive set function (defined on Lebesgue-measurable sets) and observed in passing that such functions are of bounded variation on sets on which they take a finite value. By thus linking the notions of bounded variation and additivity, Lebesgue's observation suggested to Radon a definition of the integral that would include both the definitions of Lebesgue and Stieltjes as special cases. Radon's paper (1913), which soon led to further abstractions, indicated the viability of Lebesgue's ideas in a much more general setting.

By the time of his election to the Académie des Sciences in 1922, Lebesgue had produced nearly ninety books and papers. Much of this output was concerned with his theory of integration, but he also did significant work on the structure of sets and functions (later carried further by Lusin and others), the calculus of variations, the theory of surface area, and dimension theory.

For his contributions to mathematics Lebesgue received the Prix Houllevigue (1912), the Prix Poncelet (1914), the Prix Saintour (1917), and the Prix Petit d'Ormoy (1919). During the last twenty years of his life, he remained very active, but his writings reflected a broadening of interests and were largely concerned with pedagogical and historical questions and with elementary geometry.

#### BIBLIOGRAPHY

Lebesgue's most important writings are "Intégrale, longueur, aire," in *Annali di matematica pura ed applicata*, 3rd ser., 7 (1902), 231–359, his thesis; *Leçons sur l'intégration et la recherche des fonctions primitives* (Paris, 1904;

2nd ed., 1928); *Leçons sur les séries trigonométriques* (Paris, 1906); and "Sur l'intégration des fonctions discontinues," in *Annales scientifiques de l'École normale supérieure*, 3rd ser., 27 (1910), 361–450. A complete list of his publications to 1922 and an analysis of their contents is in Lebesgue's *Notice sur les travaux scientifiques de M. Henri Lebesgue* (Toulouse, 1922). Lebesgue's complete works are being prepared for publication by l'Enseignement Mathématique (Geneva).

Biographical information and references on Lebesgue can be obtained from K. O. May, "Biographical Sketch of Henri Lebesgue," in H. Lebesgue, *Measure and the Integral*, K. O. May, ed., (San Francisco, 1966), 1–7. For a discussion of Lebesgue's work on integration theory and its historical background, see T. Hawkins, *Lebesgue's Theory of Integration: Its Origins and Development* (Madison, Wis., 1970).

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**LE BLANC, MAX JULIUS LOUIS** (b. Barten, East Prussia [now Poland], 26 May 1865; d. Leipzig, Germany, 31 July 1943), *chemistry*.

Le Blanc received his chemical education at the universities of Tübingen, Munich, and Berlin, where in 1888 he became privatdocent with A. W. Hofmann in organic chemistry. In 1891 he went to Wilhelm Ostwald's institute at Leipzig and wrote his inaugural essay, "On the Electromotive Force of Polarization."

In 1896 he left his position as extraordinary professor at Leipzig to become the director of the electrochemical division of the Höchster Farbenfabriken. Five years later he accepted a call to the chair of physical chemistry at the technical institute at Karlsruhe, and in 1906 he became director of the physical chemical institute at Leipzig. He was personally responsible for much of the building and equipping of these two institutions. After his retirement in 1934 he remained a participant in scientific affairs mainly through his service to the Sächsische Akademie der Wissenschaften zu Leipzig.

Most of Le Blanc's chemical studies involved the phenomenon of electrochemical polarization: the nature of the decomposition voltage of aqueous solutions of electrolytes and the groundwork for the understanding of the processes occurring at the electrodes in electrolysis. Noting that the decomposition potentials of many aqueous solutions of salts were nearly the same (1.7 volts), he concluded that the electrode processes were identical and that hydrogen and oxygen gas were discharged at the electrodes.

In 1891 Le Blanc introduced the concept of over-voltage, the amount by which the decomposition