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F. SZABADVÁRY

PEANO, GIUSEPPE (*b.* Spinetta, near Cuneo, Italy, 27 August 1858; *d.* Turin, Italy, 20 April 1932), *mathematics, logic.*

Giuseppe Peano was the second of the five children of Bartolomeo Peano and Rosa Cavallo. His brother Michele was seven years older. There were two younger brothers, Francesco and Bartolomeo, and a sister, Rosa. Peano's first home was the farm Tetto Galant, near the village of Spinetta, three miles from Cuneo, the capital of Cuneo province, in Piedmont. When Peano entered school, both he and his brother walked the distance to Cuneo each day. The family later moved to Cuneo so that the children would not have so far to walk. The older brother became a successful surveyor and remained in Cuneo. In 1974 Tetto Galant was still in the possession of the Peano family.

Peano's maternal uncle, Michele Cavallo, a priest and lawyer, lived in Turin. On this uncle's invitation Peano moved to Turin when he was twelve or thirteen. There he received private lessons (some from his uncle) and studied on his own, so that in 1873 he was able to pass the lower secondary examination of the Cavour School. He then attended the school as a regular pupil and in 1876 completed the upper secondary program. His performance won him a room-and-board scholarship at the Collegio delle Provincie, which was established to assist students from the provinces to attend the University of Turin.

Peano's professors of mathematics at the University of Turin included Enrico D'Ovidio, Angelo Genocchi, Francesco Siacci, Giuseppe Basso, Francesco Faà di Bruno, and Giuseppe Erba. On 16 July 1880 he completed his final examination "with high honors." For the academic year 1880–1881 he was assistant to D'Ovidio. From the fall of 1881 he was assistant and later substitute for Genocchi until the latter's death in 1889. On 21 July 1887 Peano married Carola Crosio, whose father, Luigi Crosio (1835–1915), was a genre painter.

On 1 December 1890, after regular competition, Peano was named extraordinary professor of infinitesimal calculus at the University of Turin. He was promoted to ordinary professor in 1895. In 1886 he had been named professor at the military academy, which was close to the university. In 1901 he gave up his position at the military academy but retained his professorship at the university until his death in 1932, having transferred in 1931 to the chair of complementary mathematics. He was elected to a number of scientific societies, among them the Academy of Sciences of Turin, in which he played a very active role. He was also a knight of the Order of the Crown of Italy and of the Order of Saint Maurizio and Saint Lazzaro. Although he was not active politically, his views tended toward socialism; and he once invited a group of striking textile workers to a party at his home. During World War I he advocated a closer federation of the allied countries, to better prosecute the war and, after the peace, to form the nucleus of a world federation. Peano was a nonpracticing Roman Catholic.

Peano's father died in 1888; his mother, in 1910. Although he was rather frail as a child, Peano's health was generally good. His most serious illness was an attack of smallpox in August 1889. After having taught his regular class the previous afternoon, Peano died of a heart attack the morning of 20 April 1932. At his request the funeral was very simple, and he was buried in the Turin General Cemetery. Peano was survived by his wife (who died in Turin on 9 April 1940), his sister, and a brother. He had no children. In 1963 his remains were transferred to the family tomb in Spinetta.

Peano is perhaps most widely known as a pioneer of symbolic logic and a promoter of the axiomatic method, but he considered his work in analysis most important. In 1915 he printed a list of his publications, adding: "My works refer especially to infinitesimal calculus, and they have not been entirely useless, seeing that, in the judgment of competent persons, they contributed to the constitution of this science as we have it today." This "judgment of competent persons" refers in part to the *Encyklopädie der mathematischen Wissenschaften*, in which Alfred Pringsheim lists two of Peano's books among nineteen important calculus texts since the time of Euler and Cauchy. The first of these books was Peano's first major publication and is something of an oddity in the history of mathematics, since the title page gives the author as Angelo Genocchi, not Peano: *Angelo Genocchi, Calcolo differenziale e principii di calcolo integrale, publicato con aggiunte dal D.r Giuseppe Peano*. The origin of the book is that Bocca Brothers wished to publish a calculus text based on Genocchi's lectures. Genocchi did not wish to write such a text but gave Peano permission to do so. After its publication Genocchi, thinking Peano lacked regard for him, publicly disclaimed all credit for the book, for which Peano then assumed full responsibility.

Of the many notable things in this book, the *Encyklopädie der mathematischen Wissenschaften* cites theorems and remarks on limits of indeterminate expressions, pointing out errors in the better texts then in use; a generalization of the mean-value theorem for derivatives; a theorem on uniform continuity of functions of several variables; theorems on the existence and differentiability of implicit functions; an example of a function the partial derivatives of which do not commute; conditions for expressing a function of several variables with Taylor's formula; a counterexample to the current theory of minima; and rules for integrating rational functions when roots of the denominator are not known. The other text of Peano cited in the *Encyklopädie* was the two-volume *Lezioni di analisi infinitesimale* of 1893. This work contains fewer new results but is notable for its rigor and clarity of exposition.

Peano began publication in 1881 with articles on the theory of connectivity and of algebraic forms. They were along the lines of work done by D'Ovidio and Faà di Bruno. Peano's work in analysis began in 1883 with an article on the integrability of functions. The article of 1890 contains original notions of integrals and areas. Peano was the first to show that the first-order differential equation $y' = f(x, y)$ is solvable on the sole assumption that f is continuous. His first proof dates from 1886, but its rigor leaves something to be desired. In 1890 this result was generalized to systems of differential equations using a different method of proof. This work is also notable for containing the first explicit statement of the axiom of choice. Peano rejected the axiom of choice as being outside the ordinary logic used in mathematical proofs. In the *Calcolo geometrico* of 1884 Peano had already given many counterexamples to commonly accepted notions in mathematics, but his most famous example was the space-filling curve that was published in 1890. This curve is given by continuous parametric functions and goes through every point in a square as the parameter ranges over some interval. Some of Peano's work in analysis was quite original, and he has not always been given credit for his priority; but much of his publication was designed to clarify and to make rigorous the current definitions and theories. In this regard we may mention his clarification of the notion of area of a surface (1882, independently discovered by H. A. Schwarz); his work with Wronskians, Jacobians, and other special determinants, and with Taylor's formula; and his generalizations of quadrature formulas.

Peano's work in logic and in the foundations of mathematics may be considered together, although he never subscribed to Bertrand Russell's reduction of mathematics to logic. Peano's first publication in logic was a twenty-page preliminary section on the operations of deductive logic in *Calcolo geometrico secondo l'Ausdehnungslehre di H. Grassmann* (1888). This section, which has almost no connection with the rest of the text, is a synthesis of, and improvement on, some of the work of Boole, Schröder, Peirce, and McColl. The following year, with the publication of *Arithmetices principia, nova methodo exposita*, Peano not only improved his logical symbolism but also used his new method to achieve important new results in mathematics; this short booklet contains Peano's first statement of his famous postulates for the natural numbers, perhaps the best known of all his creations. His research was done independently of the work of Dedekind, who the previous year had published an analysis of the natural numbers, which was essentially that of Peano but without the clarity of Peano. (This was the only work Peano wrote in Latin.) *Arithmetices principia* made important innovations in logical notation, such as $\in$ for set membership and a new notation for universal quantification. Indeed, much of Peano's notation found its way, either directly or in a somewhat modified form, into mid-twentieth-century logic.

In the 1890's he continued his development of logic, and he presented an exposition of his system to the First International Congress of Mathematicians (Zurich, 1897). At the Paris Philosophical Congress of 1900, Peano and his collaborators—Burali-Forti, Padoa, and Pieri—dominated the discussion. Bertrand Russell later wrote, "The Congress was a turning point in my intellectual life, because I there met Peano."

In 1891 Peano founded the journal *Rivista di matematica*, which continued publication until 1906. In the journal were published the results of his research and that of his followers, in logic and the foundations of mathematics. In 1892 he announced in the *Rivista* the *Formulario* project, which was to take much of his mathematical and editorial energies for the next sixteen years. He hoped that the result of this project would be the publication of a collection of all known theorems in the various branches of mathematics. The notations of his mathematical logic were to be used, and proofs of the theorems were to be given. There were five editions of the *Formulario*. The first appeared in 1895; the last was completed in 1908, and contained some 4,200 theorems. But Peano was less interested in logic as a science per se than in logic as used in mathematics. (For this reason he called his system "mathematical logic.") Thus the last two editions of the *Formulario* introduce sections on logic only as it is needed in the proofs of mathematical theorems. The editions through 1901 do contain separate, well-organized sections on logic.

The postulates for the natural numbers received minor modifications after 1889 and assumed their definitive form in 1898. Peano was aware that the postulates do not characterize the natural numbers and, therefore, do not furnish a definition of "number." Nor did he use his mathematical logic for the reduction of mathematical concepts to logical concepts. Indeed, he denied the validity of such a reduction. In a letter to Felix Klein (19 September 1894) he wrote: "The purpose of mathematical logic is to analyze the ideas and reasoning that especially figure in the mathematical sciences." Peano was neither a logicist nor a formalist. He believed rather that mathematical ideas are ultimately derived from our experience of the material world.

In addition to his research in logic and arithmetic, Peano also applied the axiomatic method to other fields, notably geometry, for which he gave several axiom systems. His first axiomatic treatment of elementary geometry appeared in 1889 and was extended in 1894. His work was based on that of Pasch but reduced the number of undefined terms from four to three: point and segment, for the geometry of position (1889), and motion, also necessary for metric geometry (1894). (This number was reduced to two by Pieri in 1899.)

The treatise *Applicazioni geometriche del calcolo infinitesimale* (1887) was based on a course Peano began teaching at the University of Turin in 1885 and contains the beginnings of his "geometrical calculus" (here still influenced by Bellavitis' method of equipollences), new forms of remainders in quadrature formulas, new definitions of length of an arc of a curve and of area of a surface, the notion of a figure tangent to a curve, a determination of the error term in Simpson's formula, and the notion of the limit of a variable figure. There is also a discussion of the measure of a point set, of additive functions of sets, and of integration applied to sets. Peano here generalized the notion of measure that he had introduced in 1883. Peano's popularization of the vectorial methods of H. Grassmann—beginning with the publication in 1888 of the *Calcolo geometrico secondo l'Ausdehnungslehre di H. Grassmann*—was of more importance in geometry. Grassmann's own publications have been criticized for their abstruseness. Nothing could be clearer than Peano's presentation, and he gave great impetus to the Italian school of vector analysis.

Peano's interest in numerical calculation led him to give formulas for the error terms in many commonly used quadrature formulas and to develop a theory of "gradual operations," which gave a new method for the resolution of numerical equations. From 1901 until 1906 he also contributed to actuarial mathematics, when as a member of a state commission he was asked to review a pension fund.

Peano also wrote articles on rational mechanics (1895–1896). Several of these articles dealt with the motion of the earth's axis and had their origin in the famous "falling cat" experiment of the Paris Academy of Sciences in the session of 29 October 1894. This experiment raised the question: "Can the earth change its own orientation in space, using only internal actions as animals do?" Peano took the occasion to apply his geometrical calculus in order to show that, for example, the Gulf Stream alone was able to alter the orientation of the earth's axis. This topic was the occasion of a brief polemic with Volterra over both priority and substance.

By 1900 Peano was already interested in an international auxiliary language, especially for science. On 26 December 1908 he was elected president of the Akademi Internasional de Lingu Universal, a continuation of the Kadem Volapüka, which had been organized in 1887 by the Reverend Johann Martin Schleyer in order to promote Volapük, the artificial language first published by Schleyer in 1879. Under Peano's guidance the Academy was transformed into a free discussion association, symbolized by the change of its name to Academia pro Interlingua in 1910. (The term "interlingua" was understood to represent the emerging language of the future.) Peano remained president of the Academia until his death. During these years Peano's role as interlinguist eclipsed his role as professor of mathematics.

Peano's mathematical logic and his ideography for mathematics were his response to Leibniz' dream of a "universal characteristic," whereas Interlingua was to be the modern substitute for medieval Latin, that is, an international language for scholars, especially scientists. Peano's proposal for an "interlingua" was *latino sine flexione* ("Latin without grammar"), which he published in 1903. He believed that there already existed an international scientific vocabulary, principally of Latin origin; and he tried to select the form of each word which would be most readily recognized by those whose native language was either English or a Romance language. He thought that the best grammar was no grammar, and he demonstrated how easily grammatical structure may be eliminated. His research led him to two areas: one was the algebra of grammar, and the other was philology. The latter preoccupation resulted most notably in *Vocabulario commune ad latino-italiano-français-english-deutsch* (1915), a greatly expanded version of an earlier publication (1909). This second edition contains some 14,000 entries and gives for each the form to be adopted in Interlingua, the classic Latin form, and its version in Italian, French, English, and German (and sometimes in other languages), with indications of synonyms, derivatives, and other items of information.

In his early years Peano was an inspiring teacher; but with the publication of the various editions of the *Formulario*, he adopted it as his text, and his lectures suffered from an excess of formalism. Because of objections to this method of teaching, he resigned from the military academy in 1901 and a few years later stopped lecturing at the Polytechnic. His interest in pedagogy was strong, and his influence was positive. He was active in the Mathesis Society of school teachers of mathematics (founded in 1895); and in 1914 he organized a series of conferences for secondary teachers of mathematics in Turin, which continued through 1919. Peano constantly sought to promote clarity, rigor, and simplicity in the teaching of mathematics. "Mathematical rigor," he wrote, "is very simple. It consists in affirming true statements and in not affirming what we know is not true. It does not consist in affirming every truth possible."

As historian of mathematics Peano contributed many precise indications of origins of mathematical terms and identified the first appearance of certain symbols and theorems. In his teaching of mathematics he recommended the study of original sources, and he always tried to see in his own work a continuation of the ideas of Leibniz, Newton, and others.

The influence of Peano on his contemporaries was great, most notably in the instance of Bertrand Russell. There was also a school of Peano: the collaborators on the *Formulario* project and others who were proud to call themselves his disciples. Pieri, for example, had great success with the axiomatic method, Burali-Forti applied Peano's mathematical logic, and Burali-Forti and Marcolongo developed Peano's geometrical calculus into a form of vector analysis. A largely different group was attracted to Peano after his shift of interest to the promotion of an international auxiliary language. This group was even more devoted; and those such as Ugo Cassina, who shared both the mathematical and philological interests of Peano, felt the closest of all.

It has been said that the apostle in Peano impeded the work of the mathematician. This is no doubt true, especially of his later years; but there can be no question of his very real influence on the development of mathematics. He contributed in great measure to the popularity of the axiomatic method, and his discovery of the space-filling curve must be considered remarkable. While many of his notions, such as area and integral, were "in the air," his originality is undeniable. He was not an imposing person, and his gruff voice with its high degree of lallation could hardly have been attractive; but his gentle personality commanded respect, and his keen intellect inspired disciples. Much of Peano's mathematics is now of historical interest; but his summons to clarity and rigor in mathematics and its teaching continues to be relevant, and few have expressed this call more forcefully.

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HUBERT C. KENNEDY

PEARL, RAYMOND (*b*. Farmington, New Hampshire, 3 June 1879; *d*. Hershey, Pennsylvania, 17 November 1940), *biology, genetics*.

Pearl was the only child of Frank Pearl and Ida May McDuffee. He attended public schools in Farmington and nearby Rochester. In 1899 he earned the A.B. in biology at Dartmouth College and the Ph.D. in 1902 at the University of Michigan, where his dissertation was on the behavior of a flatworm (*Planaria*). He also studied at the University of Leipzig in 1905 and at University College, London, from 1905–1906. In London he studied under Karl Pearson, whose influence led Pearl to apply statistics to population studies.

Pearl was an instructor in zoology at the University of Pennsylvania (1906–1907) until he became chairman of the department of biology at the Maine Agricultural Experiment Station (1907–1918), where he studied the heredity and reproduction of poultry and cattle. As chief of the statistical division of the U.S. Food Administration from 1917 to 1919, he studied the relationship of food to population. Pearl's long association with the Johns Hopkins University began