movement). His study of the moon was inspired by Barnaba Oriani, director of the Brera Observatory in Milan. Oriani had suggested that he and Francesco Carlini, who had done geodetic work with Plana, should attempt to compile reasonably precise lunar tables solely on the basis of the law of universal gravity—that is, using only the observational data essential to determine the arbitrary constants of the problem. Plana soon quarreled with Carlini, who withdrew in disgust; and Plana succeeded alone, after almost twenty years. The results were presented in the three-volume *Théorie du mouvement de la lune* (Turin, 1832). The work was not widely read and received criticism that was not always unfounded; but it is of notable scientific and philosophical value, and as such it was well received.

In 1827 Plana was named astronomer royal; in 1844 he became a hereditary baron; in 1848, a senator; and in 1860, *associé étranger* of the Paris Academy of Sciences.

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F. G. TRICOMI

PLANCK, MAX KARL ERNST LUDWIG (*b*. Kiel, Germany, 23 April 1858; *d*. Göttingen, Germany, 4 October 1947), *theoretical physics, philosophy of physics.*

Planck was the fourth child of Johann Julius Wilhelm von Planck of Göttingen, professor of civil law at Kiel, and Emma Patzig of Greifswald; the family also included two children of Wilhelm von Planck by his first wife, Mathilde Voigt of Jena, who had died in Greifswald. Max Planck's ancestors on his father's side included clergymen and lawyers.

In the spring of 1867 the family moved from Kiel, where Max had completed the first classes of elementary school, to Munich. There he entered the classical Königliche Maximilian-Gymnasium in May 1867. His mathematical talents emerged early, and thus he gratefully recalled his teacher Hermann Müller, who also taught him astronomy and mechanics. Müller's explanation of the principle of conservation of energy made a strong impression on him, and this principle became one of the foundations of Max Planck's later work.

When Planck graduated from the Gymnasium in July 1874, he had not yet decided in what subject he wanted to continue his studies. As a child he had already displayed considerable ability in music; he was an excellent performer on the piano and organ. The encouragement of this talent was one result of an educational tradition that has largely been lost in Germany today: the many-sided stimulation provided by home and school, which was known as *Bildung* ("education," "cultivation," "culture"), and which encompassed religious training just as much as, say, mountain climbing with one's family.[1] From such a background Planck emerged prepared in principle to study the mathematical disciplines, or music, or even classical philology, which also attracted him because of its grammatical and moral harmony. It is not known why he did not choose philology. He probably gave up his musical career when, on seeking advice from a professional musician, he was told: "If you have to *ask*, you'd better study something else!" Nevertheless music remained a lifelong hobby for Planck.

Planck matriculated at the University of Munich on 21 October 1874 and initially decided to study mainly mathematics, influenced by the lectures of Gustav Bauer, who also taught the calculus of variations and probability theory, as well as other subjects.[2] He was soon attracted to physics, although Philipp von Jolly tried to persuade him that nothing essentially new remained to be discovered in this branch of learning.[3] But Planck stood by his rejection of pure mathematics because of his deep interest in questions concerning the nature of the universe[4] ("Weltanschauung"). Student notes[5] show that he attended Gustav Bauer's lectures on "Analytische Geometrie," Ludwig Seidel's course on "Höhere Algebra," Jolly's "Mechanische Wärmetheorie," and the physics lectures of Wilhelm Beetz. Such lectures were predominantly concerned with experimental physics, although lectures titled "Mathematical Physics" can be traced back to the beginning of the nineteenth century.[6] In any case this was the only time in Planck's life when he carried out experiments (for example, on the osmosis of gases).

Because of illness, he had to interrupt his studies during the summer term of 1875. He went to the University of Berlin for the winter semester of 1877–1878 and the summer of 1878. There he heard, in addition to the lectures of Weierstrass, those of Kirchhoff and Helmholtz, although he was not convinced that he learned very much from the latter two. By himself he studied Clausius' *Mechanische Wärmetheorie* in detail and later remarked that this private study was what had finally drawn him into physics.[7] His attempt to master thermodynamics as independently as possible he labeled "Nur nach eigener Überzeugung" ("Only when I have convinced myself"). These investigations led him to the preparation of his doctoral dissertation on the second law of thermodynamics, for which he was awarded the Ph.D. degree at the University of Munich on 28 July 1879. As customary then, he had already, in October 1878, passed the *Staatsexamen für das Höhere Lehramt* for a teaching certificate in mathematics and physics; and he taught these subjects for a few weeks at the Maximilian-Gymnasium.

On 14 June 1880 Planck was given the *venia legendi* at the University of Munich for his paper *Gleichgewichtszustände isotroper Körper in verschiedenen Temperaturen*. In this paper he extended the mechanical theory of heat, using the entropy concept, to treat elastic forces acting on bodies at different temperatures. It may be noted, however, that his habilitation lecture in the same year was "Über die Prinzipien der mechanischen Gastheorie," in accord with the lectures given by his colleagues at Munich. Later, when he was at Kiel and Berlin, he enjoyed a stimulating correspondence on the current problems of thermodynamics with his friend Leo Graetz, then *Privatdozent* at Munich. At Munich he also made friends with Carl Runge,[8] who in later years gave him valuable mathematical assistance.

An appointment as professor extraordinarius at the University of Kiel on 2 May 1885 gave Planck greater scientific independence. Positions of this kind were then rather new in Germany and were restricted primarily to theoretical physics, which did not have a very high status compared to experimental physics.[9] It seems that, as a result, Planck had relatively few students and so had correspondingly more time available for research in his new subject. Yet it is remarkable that in the winter semester of 1887–1888 he announced simultaneously four lecture courses: "Vorträge und Übungen aus der Electricitätslehre," "Theoretische Optik," "Mechanische Wärmetheorie," and "Besprechung wichtiger Literaturerscheinungen auf dem Gebiete der Wärmelehre." These topics, combined with the report of Hertz's recently

performed experiments on and simplification of Maxwell's electromagnetic theory, point toward Planck's combination of these fields in his radiation theory in the 1890's.

The appointment at Kiel also gave him some personal security, with an adequate annual salary (2,000 marks), so that he was able to marry his fiancée from Munich, Marie Merck, and establish a household.

In his publications during this period, Planck still concentrated, as he had done in Munich, on applications of his ideas to physical (or "general"[10]) chemistry. After completing his prize essay on *Das Princip der Erhaltung der Energie* (1887), which included a ninety-one-page historical introduction, he turned again to the "second principle" and in three papers tried to generalize it to cover the theory of dilute solutions and thermoelectricity. These studies later culminated in his monograph *Grundriss der allgemeinen Thermochemie* (1893), which had a thirty-one-page historical introduction, and in his *Vorlesungen über Thermodynamik* (1897).

On the basis of these successful researches in thermodynamics (or *Thermomechanik* as it was then called), Planck was appointed on 29 November 1888 to be the successor of Kirchhoff, as assistant professor at the University of Berlin and director of the Institute for Theoretical Physics (newly founded for him). He served as professor ordinarius in Berlin from 23 May 1892 to 1 October 1926. He quickly attained professional recognition; he was at once made a member of the Physikalische Gesellschaft zu Berlin and was elected to the Königlich-Preussische Akademie der Wissenschaften zu Berlin on 11 June 1894. Before 1900 he participated demonstrably in the meetings of the Gesellschaft Deutscher Naturforscher und Ärzte, namely in 1891, 1898, and 1899, on which occasions he took the opportunity to engage in scientific exchanges with Boltzmann. His circle of colleagues included such men as Emil du Bois-Reymond, Hermann von Helmholtz, Ernst Pringsheim, Wilhelm Wien, Max B. Weinstein (who in 1883 had edited a translation of Maxwell's *Treatise on Electricity and Magnetism*), the physicist Carl A. Paalzow of the Technische Hochschule in Berlin-Charlottenburg, August Kundt, Werner von Siemens, and also the theologian Adolph von Harnack, the historian Theodor Mommsen, and the Germanic philologist Wilhelm Scherer. He was, in particular, closely connected with the experimental physicists at the Physikalisch-Technische Reichsanstalt (founded in 1887)—Otto Lummer, W. Wien, Ludwig Holborn, Ferdinand Kurlbaum, and others. An enormous correspondence began to develop with scientists

outside of Berlin—H. Hertz, Ernst Lecher, Leo Koenigsberger, A. Sommerfeld, P. Ehrenfest, Albert Schweitzer, and others.

As an admirer of Helmholtz it was appropriate for Planck to combine his physics with music, but in contrast to Helmholtz, tempered scale he preferred the natural scale, and commissioned the construction of a harmonium with 104 tones in each octave. Such interests went hand in hand with private home concerts, in which the violinist Joseph Joachim and Maria Scherer participated.

In his scientific work at Berlin, Planck endeavored to give an independent character to "mathematical physics." An indication of this was the lecture "System der gesammten Physik," in which he followed an approach somewhat similar to that of Kirchhoff.[11] Planck moreover was drawn into discussions on more general ideas of his time. This was a consequence of his striving for generalization. Thus, in 1895, he defended Clausius' form of the second law of thermodynamics against the gross oversimplifications of the "new energetics" of G. Helm, W. Ostwald, and later E. Mach. Planck also was an indefatigable advocate of the absolute validity of laws of nature, a position that places him in the mainstream of the search for absolute constants in the second half of the nineteenth century. His views on both these points already hint at his later interests in the philosophical foundations of science.

The bulk of his systematic work may be divided into thermodynamics, radiation theory, relativity, and philosophy of science. The first culminated in his *Vorlesungen über Thermodynamik*, already mentioned. By 1895 he was fully occupied with irreversible processes, especially in electrodynamics. He connected his early studies on thermodynamic irreversibility with Maxwell's electromagnetic theory of light, in the form given it by Helmholtz, O. Heaviside, and H. Hertz. The theory was at that time the subject of fundamental experimental and theoretical investigations by Berlin scientists, who showed in many lectures on the history of the exact sciences that they considered this new theory of light to be in keeping with the most up-to-date physics. Planck combined Clausius' phenomenological method with Kirchhoff's theorem that light and heat radiation in thermal equilibrium are independent of the nature of the substance, a theorem that filled Planck with enthusiasm. This combination, along with the statistical methods of calculation, was what would lead him in 1900 to the energy elements of the new radiation law. His third major interest, relativity, arose in the winter of 1905–1906 from the publications of Lorentz and Einstein, but not without knowledge of the work

of Poincaré. It is characteristic of Planck that, in 1907, he connected the "principle of relativity" with his quantum of action h.

Planck had always been inclined toward generalization. Encouraged by finding himself in the spotlight of publicity, he now attacked even more general questions in some twenty published popular lectures (as well as in unpublished letters) devoted to "developing and explaining" his "scientific views." This pursuit of general ideas going back to the 1890's really started with his lecture "Einheit des physikalischen Weltbildes" given at Leiden in 1908, and continued in following years with numerous reflections on the relations of science to philosophy, religion, and human nature.

Planck received the high distinction of the 1918 Nobel Prize in physics, but his personal life was clouded by misfortune. His wife died on 17 October 1909, his son Karl during World War I (1916), and his two daughters Margarete and Emma during childbirth (1917 and 1919). His older son from this marriage was executed in 1944 on suspicion of conspiracy to assassinate Adolf Hitler. On 14 March 1911 Planck married the niece of his first wife, Marga von Hoesslin; they had one son, Hermann.

Planck lived through two world wars, and his correspondence with Lorentz, Schweitzer, and others shows that he maintained an uncorrupted independent viewpoint and a positive attitude toward life. In 1944 almost all his manuscripts and books in Berlin were destroyed during an air raid. From 1943 to 1945 he lived in Rogätz, near Magdeburg, and then for the last two and a half years of his life he was in Göttingen, where he witnessed the founding of the Max Planck Gesellschaft zur Förderung der Wissenschaften, successor to the Kaiser Wilhelm Gesellschaft founded in 1911. (He had been its president from 1930 to 1937.)

Thermodynamics. Several principal features characterize Planck's treatment of this, his earliest field of research. Aside from his extremely consistent procedure for extending the theory of cyclic processes to an arbitrary number of arbitrary bodies in 1879, Planck at that time preferred to base his analysis on the distinction between neutral and natural (irreversible) processes. Although he was trying to continue the work of Clausius, he deviated from his predecessor's calculation of "equivalence values," which he eliminated by including all heat reservoirs, and simplified the statement of the second law by requiring the compensation of all losses such as those by friction and heat conduction during a complete change of state, so as to make the process reversible. Planck was still implicitly using the concept of

"state" to mean a function of condition depending on temperature, pressure, and volume; in the twentieth century, under the influence of J. Willard Gibbs, this concept would acquire another meaning arising from the theory of elementary regions in phase space. In 1882 Planck did cite Gibbs's first paper of 1873, but he later admitted in his autobiography that Gibbs had anticipated most of his results on this topic.

In 1880, and also at the Versammlung Deutscher Naturforscher und Ärzte at Halle in 1891, Planck conformed closely to Clausius' phenomenological method. He even argued that problems could be solved "without the help of special assumptions about the molecular constitution of bodies," then a basis of his project to determine the effect of temperature in the theory of elasticity. This attitude brought him into protracted internal conflict over the corpuscular hypothesis and the statistical interpretation of nature, which was not yet really established by experiment in the nineteenth century, a conflict that only occasionally came to the surface in his discussions with Boltzmann.

Finally, in 1897, Planck abandoned also the "second" view[12] of the nature of heat—that heat consists of some kind of motion of particles, the precise nature of which was not specified (cf. Clausius 1850)—and turned to the "third" method, in which one "abstains completely from any definite assumption about the nature of heat." He retained that viewpoint even in the 1905 edition of his *Vorlesungen über Thermodynamik*, although he noted that the "principle of increase of entropy" has "no independent significance" but comes rather from "known theorems of the probability calculus." Even after incorporating the Nernst heat theorem (1906), he stated in 1910 that he was leaving the atomic theory "completely out of the picture." In 1912 he still saw both the quantum hypothesis[13] and the Nernst theorem simply as "recent thermodynamic theories."

Planck's *Vorlesungen* was effective for more than thirty years as an exceptionally clear, systematic, and skillful presentation of thermodynamics.[14]

Heat Radiation and Electrodynamics. Planck's contribution to the theory of heat radiation comprised the adroit combination of his studies on irreversibility with the new electrodynamics. A recently discovered manuscript reveals that his interest in heat radiation may have been stimulated by John Tyndall's book *Heat Considered as a Mode of Motion*,[15] on which he made critical notes in 1878.[5] In place of the older concepts "accord" and "discord" for heat absorption, adopted by Tyndall, Planck introduced, particularly in the case of gases, the principle of energy conservation to explain the equilibrium between ether motion and the heated body (later to become his "resonator"). This exchange process was mathematically formulated in the 1890's with the inclusion of radiation damping. In particular, Tyndall's "calorescence"—the opposite of fluorescence—perplexed Planck because of the old question of whether an individual atom has one or more vibration frequencies. He reduced light absorption by solid bodies to the conduction of heat from atom to atom by oscillations. Thus, for Planck in the following years, heat conduction problems were closely connected with radiation phenomena as they had been in the first half of the nineteenth century. He put a question mark against Tyndall's remark that the period of vibration of an atom that corresponds to its maximum amplitude is the same for all bodies; this involved the question of the displacement of maximum wavelength with temperature, about which Planck also made a note on page 569 of Kirchhoff's *Gesammelte Abhandlungen*, which had appeared in 1882.

Soon afterward Planck was influenced by the increasing German interest in Maxwell's theory. It is understandable that, as a theoretical physicist, he would seek to unite his earlier thermodynamic studies with the new theory, an attempt in the spirit of his lecture "System der gesammten Physik." Thus, in his inaugural lecture at the Berlin Academy in 1894, he clearly expressed the hope "that we can also explain those processes which are directly dependent on temperature, as manifested especially in heat radiation, without first having to make a laborious detour through the mechanical interpretation of electricity." Indeed, he had already asserted more clearly in 1891 that "the principle of entropy increase must extend to all forces of nature . . . not only thermal and chemical, but also electrical and other processes."[16] Consequently, in 1895, Planck began just in electrodynamics to prepare for this undertaking by treating the irreversibility of heat radiation, before he introduced entropy into his equations in 1898. Planck's ultimate goal was the investigation of irreversible processes through the study of conservative effects (that is, conservative or radiation damping).[17]

Starting from a concrete Hertzian "secondary conductor" (receiver), he confirmed his first mathematical steps by Vilhelm Bjerknes' experiments with oscillators, also called resonators.[18] Planck then gradually eliminated all their special properties in his calculation, since he knew nothing about their actual nature. Consequently, as soon as the Berlin experimenters succeeded in constructing Kirchhoff's second blackbody, the *bloss Hohlraum*, Planck equated the effect of certain real radiating bodies to that of the "radiation state of the vacuum,"[19] but continued to use the expression "resonator" in his calculations.

He also referred explicitly to W. Wien's cavity radiation of 1894, which Wien partly abandoned in 1896. Like Wien, Planck felt compelled to include certain centers of radiation in addition to the radiation itself, following the approach of W. A. Michelson, who had published the first application of statistical-molecular theory to radiation in 1887. According to Planck's intention in 1897 the resonator was to produce by its vibrations an energy exchange between absorbed and emitted radiation. L. Boltzmann, who had been debating with Loschmidt and others the interpretation of irreversibility, now attacked the equilibrating role of the resonator and so made it central to Planck's considerations.[20] Consequently in 1898 Planck replaced it by a "spectral-analyzing resonator."

He also introduced the concept of natural radiation, the analogue of Boltzmann's assumption of molecular chaos in the kinetic theory of gases. This assumption of randomness in the radiation allowed Planck to establish a causal relation between the energy U of a resonator of frequency v_0 and the intensity of the surrounding radiation field of the same frequency. Planck derived the equation

$$U = I_0(t) - \frac{1}{2\sigma v_0} \frac{dI_0(t)}{dt}.$$

Here σ is the damping constant (logarithmic decrement) of the resonator and $I_0(t)$ measures the energy of the radiation field at the frequency v_0. Planck had not introduced any molecular statistics at this stage of his work.

In the following year, 1899, Planck completed a derivation of the spectral distribution law for heat radiation, obtaining the form first given by Wien in 1896. For his purpose Planck had to introduce the entropy S of the resonator by means of a definition, writing

$$S = - \left(\frac{U}{av}\right) \ln \left(\frac{U}{ebv}\right). \qquad (1)$$

U and v are the energy and frequency of the resonator as before, e is the base of the natural logarithms, and a and b are "two universal positive constants" which appeared here for the first time. The resulting expression for the energy U of the resonator had the form

$$U = bv \exp \left(-\frac{av}{\theta}\right), \qquad (2)$$

where θ is the absolute temperature. Making use of the relationship between the resonator energy and the spectral intensity of the radiation at equilibrium, a

relationship he derived in this same article, Planck obtained Wien's form of the distribution law,

$$E_\lambda = \frac{2c^2b}{\lambda^5} \exp \left(-\frac{ac}{\lambda\theta}\right),$$

where c is the velocity of light, λ is the wavelength, and E_λ is the radiant energy per unit volume in a unit interval of wavelength.

Planck evaluated the constants a and b numerically, obtaining for b the value 6.885×10^{-27} erg sec, but he made no attempt to give either constant a physical interpretation.

In this regard there has arisen the myth to which Planck himself in 1901 gave voice: comparing the constants a and b of equation (2) with those in his 1900 energy equation, which is only slightly different from (1):

$$U = hv \left\{\exp \left(\frac{hv}{kT}\right) - 1\right\}^{-1}, \qquad (3)$$

he set h equal to b, k equal to b/a. But these equalities are theoretically (and experimentally) wrong.[21] It is characteristic that until the end of his life Planck continued in this error—apparently because of his bias in favor of theories of a supposedly timeless nature, in contrast to those that come and go in the course of historical change. Thus he made the experiments responsible for the differences between the two sets of constants, writing that "the divergence of the figures corresponds to the deviations in the measurements of the various observers. . . ."

With equation (2), Planck had established Wien's distribution law, which was confirmed by experiment over a wide range of frequencies. By arguing backward from (2), he could even conclude that his definition of the entropy (1) was confirmed. He was clearly not satisfied with this result.

Planck obtained a little-known additional result in 1899: the difference between the absorbed and the emitted energy is given by the equation

$$dU_0 = Z \frac{df}{dt} dt - \frac{8\pi^2 v_0^2}{3c^3} \left(\frac{df}{dt}\right)^2 dt$$

where Z is the intensity of the exciting wave, and f is the electric moment. Planck concluded that "the absorbed energy would in some circumstances be negative. . . . In this case the 'exciting wave' [Z] would extract energy from the resonator."[22] Einstein treated this situation on the basis of the quantum theory in 1916, when he gave a new derivation of Planck's radiation law. In 1921 Planck gave it the name "negative Einstrahlung." This effect is now known as

stimulated emission and is the basis of the "MASER" (microwave amplification by stimulated emission of radiation), invented in 1954.

It is also of interest to note that Planck, like Wien before him, treated the temperature of radiation.[23]

By the end of 1899 Planck noted that the experimental results published by Rubens, Lummer, and E. Pringsheim in September 1899 showed deviations from Wien's law and thus from the predictions of his own theory of oscillators, which he still connected with ponderable atoms.[24] He attempted to save the phenomena (as Rubens had done in 1898) and in March 1900 introduced the second derivation with respect to the resonator energy U:

$$\frac{d^2 S}{dU^2} = - \frac{\alpha}{U}.$$

By integrating this expression and applying the definition

$$\frac{dS}{dU} = \frac{1}{T},$$

he arrived again at Wien's law.

We can only offer external reasons as to why in October 1900 Planck made the modification

$$\frac{d^2 S}{dU^2} = \frac{\alpha}{U(\beta + U)} \qquad (4)$$

and thereby arrived at the new radiation formula. Planck's inference from the behavior of an individual oscillator to the collective behavior of n oscillators was criticized by Lummer and Wien at the Congrès International de Physique at Paris in August 1900, and by E. Pringsheim at the Versammlung Deutscher Naturforscher und Ärzte at Aachen in September 1900. (Unfortunately nothing is known about Planck's conversation with Boltzmann at Munich in September 1899, where he learned from the experimentalists about more significant experimental deviations.) The decisive proof for *curved* "isochromatics" (lines of the temperature function for constant wavelength) against those of Wien's law (straight lines) encouraged the experimenters, who reported it orally in February 1900, although only at the end of September did they publish their results. Stimulated by a visit of Rubens on 7 October, Planck on the very same day wrote down equation (4) together with his new radiation equation. He was already predisposed in October to associate a logarithmic function of U with a probability calculation, presumably influenced by Boltzmann. In any case, Planck admitted in his first paper on the new theory, in October 1900, that the

so-called n-resonator problem disturbed him. He was not able to resolve part of this problem until the end of 1906. In the last instance he was guided by the old principle of greatest simplicity.[25]

Planck, in December 1900, relied on Boltzmann for the statistical basis of his formula for the resonator energy S, and proposed

$$S = k \ln R_0,$$

where R_0 is the maximum number of his "complexions" of a group of resonators with definite frequency. Boltzmann in turn had used the device of approximating a continuum by finite intervals, a tradition going back to the seventeenth century.[26] For the energy U_N of each group of N resonators, Planck introduced a finite number of equal energy elements, $\epsilon = h\nu$, and as his accompanying table shows, these elements were associated with each individual resonator.[27] Or, as he wrote three weeks later, let U_N be conceived as a "discrete quantity [Grösse], compounded of a whole number of equal finite parts." With the help of a combinatorial argument he computed R as

$$\frac{(N + P - 1)!}{(N - 1)! \, P!},$$

where $U_N = NU = P\epsilon$. After simplification by Stirling's formula, and with

$$\frac{dS}{dU} = \frac{1}{T}$$

he obtained U as the function of frequency and temperature already given above in equation (3). The corresponding equation for the spatial density of the spectral distribution of the radiation, Planck's distribution law, had the form

$$u = \frac{8\pi h\nu^3}{c^3} \left\{ \exp\left(\frac{h\nu}{kT}\right) - 1 \right\}^{-1},$$

where u is the energy per unit volume in a unit frequency interval.

At least in 1905 Planck felt that the finite "energy quantum" was "a new hypothesis alien to the resonator theory [of classical electrodynamics]." Thus, in 1910, he abandoned the hypothesis of discrete absorption, and in 1914 he even gave up discontinuous emission. From the calculation in 1910 arose Planck's concept of zero point energy, $h\nu/2$.

In his book *Vorlesungen über die Theorie der Wärmestrahlung*, published in 1906, Planck introduced a new interpretation of his constant, h. He examined the resonator's states with the help of its

phase plane, whose axes represent coordinate and momentum. The locus of phase points corresponding to a fixed energy U for a resonator of frequency ν is an ellipse enclosing an area equal to (U/ν). Planck considered a series of concentric ellipses, each having an area exceeding that of its predecessor by the amount h. The energy difference ΔU of successive ellipses would then be given by the equation

$$\frac{\Delta U}{\nu} = h.$$

The total area enclosed by successive ellipses would be h, $2h$, $3h$, $\cdots$. The number of resonators having a definite amount of energy would now become, in the new language, resonators "falling in a *definite* energy region," the size of which depends on h. Within an elementary region h (elliptic ring surface) of the state space, oscillators of different frequencies ν are distributed according to the assumption of elementary disorder, that is, "an almost uniform" distribution, prevails.[28] Henceforth Planck preferred this "quantity of action" to Einstein's "energy quantum." One difficulty was that Planck, in 1906, no longer used the maximum energy but, rather, only the average energy in his calculation.[29]

A supplemental result that should be mentioned is the first proof of the applicability of Maxwell's theory in the infrared region, furnished in 1903 by Planck and his friend the experimentalist Rubens.[30] The attempt at assimilation into the classical theory led Planck in 1911–1912 to the application of a method of identification of parameters in the quantum and classical theories, which Bohr cited in 1913 and called "correspondence" in 1920.

When, in the course of time, more serious consideration was given to rotating dipoles, Planck turned to Adriaan D. Fokker's generalization of the Einstein fluctuation theory and in 1917 proved the basic equation of Fokker's theory.

Relativity Theory. In 1906 Planck was one of the first scientists to take up what he called "the principle of relativity introduced by H. A. Lorentz and stated more generally by A. Einstein," and to extend this theory (to which H. Poincaré had also contributed) from electrodynamics to mechanics.

Thus he showed that one could write for the x component, X, of the force acting on a particle of mass m,

$$X = \frac{d}{dt} \left\{ m\dot{x} \Big/ \sqrt{1 - \frac{q^2}{c^2}} \right\},$$

where $\dot{x}$ is the x component of the particle's velocity and $q = \sqrt{\dot{x}^2 + \dot{y}^2 + \dot{z}^2}$ is the magnitude of that velocity. Planck also showed how these relativistic equations of motion of a particle could be put into Lagrangian and Hamiltonian form by a proper choice of the Lagrangian function, H ("kinetische Potential").

In 1907 Planck clearly stated that the classical separation of the energy into an internal energy of state, independent of the velocity of the body, and an external part that depends only on velocity, could no longer be maintained. He made a connection with his radiation researches by investigating the dynamics of moving blackbody radiation and its relation to the quantity of action, W. He found that this quantity,

$$W = \int_1^2 H \, dt,$$

remains invariant under Lorentz transformations: "to each change in nature there corresponds a definite number of elements of action, independent of the choice of coordinate system."

Planck felt that the relativity principle was experimentally confirmed only by the negative results of the experiment of Michelson and Morley. On the other hand, the measurements of simultaneous magnetic and electric deviations of electron beams by Walter Kaufmann, which involved the dependence of the mass of the electron on its velocity, gave some difficulties. Kaufmann, in January 1906, asserted that his "results are not compatible with the Lorentz-Einstein basic assumption. The equations of Abraham [for a rigid spherical electron, 1903] and of Bucherer [deformable electron of fixed volume, 1904] represent the results of observations equally well." Nine months later, Planck recognized that because of the disagreement of Kaufmann's values with the new Lorentz-Einstein theory, "there is still an essential gap in the theoretical interpretation of the measured quantities," especially since the calculated electron velocity was higher than the speed of light. Toward the middle of the year 1907 Planck changed his calculation of the "apparatus constants" and, with the help of the new value of Adolf Bestelmeyer for the charge-to-mass ratio of the electron, succeeded in arriving at the conclusion that "the chances of relativity theory are somewhat better." This historic episode is yet another demonstration of Planck's close attachment to experiment—in this case, the only positive one available at the time to test relativity.

Philosophy, Religion. Planck's writings on general subjects, published between 1908 and 1937, have received scarcely any historical appreciation.[31] These writings emerged from his occupation with the basis of physical theories. Just as he had found a generalizing

synthesis of electrodynamic and thermodynamic principles in the theory of heat radiation, so he was now concerned to comprehend the character of physics as a whole. Having in 1891 ascribed to an ideal process "the role of a pathfinder whose statements have very great generality" even though they lack "probative force"—a role just like that assigned by W. Wien to the thought experiment[32]—Planck suggested in 1894 that "the time is past when one person can deal with both, specialized knowledge [physics] and general knowledge [theory]." He took every opportunity to exhibit the "role of the theoretician in scientific progress." At that time Planck believed that in principle all natural phenomena can be reduced to mechanics, yet he conceded that thermal phenomena could be described by only two non-mechanical laws, and that the connection between electrodynamics and optics, and perhaps also heat radiation, did not depend on mechanics. He postulated "the attainment of a permanently inalterable goal, which rests on the establishment of a single grand connection among all forces of nature"—a foreshadowing of what he called in his 1908 Leiden lecture "the unity of the physical picture of the world" (Einheit des physikalischen Weltbildes).

Along the same line was his search for "natural units" independent of particular bodies, which would "retain their meaning for all times and for all cultures, including extraterrestrial and nonhuman ones."[33] Consequently he turned against Mach's positivism in 1908, and, in his lectures at Columbia University in 1909 ("Das gegenwärtige System der theoretischen Physik"), Planck stressed the path away from observation and anthropomorphism in physics toward a "constant world picture" (Weltbild). He rejected pure subjectivity on the grounds that it would allow any two physicists to maintain two equally valid but different interpretations of a phenomenon, from the standpoint of their different world views. It is remarkable that, by referring to historical examples, Planck supported his ideas on definitions and theorems (1908) and, in 1909, on the unification of empirical knowledge and practice by theory. The characteristic of the theoretician, and especially of one with conservative attitudes like Planck, was frequent reference to history.[34]

On the other hand, Planck warned against overestimating the value of physical theories by applying them to the life of the spirit. In a letter to the theologian Adolph von Harnack in 1914, he clearly separated Weltanschauung, that is, "to grasp the whole in its totality," from Wissenschaft. While "philosophical systems succeed one another, the later one being not necessarily the better . . .," he wrote, "there

is only one unique science, and this is binding on all mankind . . . it marches forward though it never will and never can attain its ideal goal."

In the rewarding article "Die Stellung der neueren Physik zur mechanischen Naturanschauung," on which he lectured in 1910 at Königsberg, Planck said that theories have tottered under the impact of new experimental techniques and therefore one needs a "working hypothesis" that can "be generated only from an appropriate world view." Since the mechanical world view is no longer acceptable in all areas, for example, in the case of the "aether" (Planck mentioned Nernst's neutrons), one must look for a new world view. Having renounced the requirement of intuitive clarity [Anschaulichkeit], Planck saw that the new physical system of the world would have to be based on the constants of nature as cornerstones. Of course Planck also thought that "if a hypothesis has once proved to be fruitful, one becomes accustomed to it and then little by little it acquires a certain intuitive clarity quite on its own." In 1913 Planck added the ever valid physical principles as invariants in nature, although he admitted that, for example, the principle of immutability of atoms had not remained valid. He now equated the world view to an unprovable hypothesis and recommended that physics also should adopt "faith, at least the faith in a certain reality outside us." This was to be the kernel of Planck's subsequent philosophy, in contrast to his positivistic attitude during the Kiel period, which he confessed had been the basis for his earlier phenomenological methods.

Planck had the whole of the human condition in his purview. Thus, in 1922, shortly after the revolution, he emphasized clearly that in science disputed questions "can not be settled by joint manifestos or even by majority votes"; "the whole of science . . . is an inseparable unity." He drew a contrast between science in itself and the discussion of controversial scientific issues. In the last years of life he wrote again on scientific controversies. He approved of them in principle but warned against personal interest in "dogmatically attempting to defend one's own opinion," to which he attributed "the great majority of scientific controversies."

Closely linked to Planck's conception of an external world are his statements concerning Kausalgesetz and Willensfreiheit (1923). He argued that the contradiction between the two concepts is only apparent. Causality is not subordinate to logic but, rather, is a category of reason (Vernunft). In agreement with Kant, Planck associated causality with metaphysics[35] and assumed that it is valid in nature as well as in mental life; moreover, that it is

unprovable. As in statistics, it does not even need to be recognized unequivocally; indeed one cannot get along without the products of the "power of imagination" *(Einbildungskraft)*, which cannot be reduced to causality (for example, concepts such as shortest light-path, virtual motions). Causality itself must be given its appropriate meaning in each individual field of intellectual interest; thus philosophy cannot be placed above the special sciences. Planck's assumption of the lawfulness of nature is presumed, namely, that accuracy and simplicity dominate natural law. In history and psychology Planck attributed to causality the "motive of action"—excepting the "I" since one cannot predict one's own actions on the basis of causality. Within this gap reigns "freedom of the will," including belief in miracles. God alone has insight into man's own causality. Planck supported the moral law, ethical obligation, and the categorical imperative. Such causality demands that men remain responsible to their consciences, even those "whose excessive involvement in immature social theories has disturbed their impartiality and removed their natural inhibitions." Thus each religion is compatible with a rigorous scientific point of view, if it neither comes into contradiction with itself nor with the law of causal dependence of all external processes. Each complements the other. Science also brings to light ethical values, it teaches us veracity *(Wahrhaftigkeit)* and reverence *(Ehrfurcht)*—by the "glance at the divine secret in one's own breast."

In 1930 Planck declared that youthful yearning for a comprehensive world view need not decay into the extremes of mysticism and superstition. A science that is not conceived merely rationally invites a faith in the future upon entering into it. Planck elaborated this theme in *Die Physik im Kampf um die Weltanschauung* (1935), where he placed "abstraction" alongside such faith or working hypothesis and emphasized the utility as well as the ideal character of thought experiments. It was precisely in the inseparability of knowledge from the scholar that Planck saw the favorable influence of science. He addressed himself to "science, religion, and art [including music]" as a whole. Given the abstractions required, he declared that neither science nor ethics can be considered ideally complete.

In his 1937 lecture "Religion und Naturwissenschaft," Planck expressed the view that God is omnipresent and held that "the holiness of the unintelligible Godhead is conveyed by the holiness of intelligible symbols." Atheists attach too much importance merely to the symbols. On the other hand, "understanding without symbols would be impossible." Planck, who from 1920 until his death was

a churchwarden at Berlin-Grunewald, professed his belief in an almighty, omniscient, beneficent God, although he did not personify him. The Godhead is "identical in character with the power of natural law." Both science and religion, although starting from different standpoints, wage a "tireless battle against skepticism and dogmatism, against unbelief and superstition," with the goal: "toward God!"

In his last lecture (1946), "Scheinprobleme der Wissenschaft," Planck held that there are more pseudoproblems [*Scheinprobleme*] "than one commonly assumes." They arise when assumptions are wrong (as in the problem of perpetual motion) or unclear (nature of the electron) or when there is no connection at all between things (such as body and soul). At the end Planck returned to the confusion of viewpoints (for example, pain and wound), his concern of the preceding four decades; but he denied that everything is just a matter of different viewpoints. To such "shallow relativism" he contrasted absolute values: in the exact sciences the absolute constants, in the religious domain truth *(Wahrhaftigkeit)*. Striving toward them was for Planck the task of practical life, the value of which should be recognized by the fruits.

NOTES

PAV = Abhandlungen und Vorträge
PVW = Vorlesungen über die Theorie der Wärmestrahlung

1. Planck participated in this favorite pastime with his family throughout his entire life. It is not surprising, therefore, that his son Karl was coauthor of *Führer durch die Mieminger Berge*, with R. Burmeister (Munich, 1920).
2. *Curriculum vitae*, handwritten by Planck on 22 July 1922. I thank Dr. G. Ross of Hildesheim, who made the MS available.
3. Jolly may have remembered the advice once given to him by Ettinghausen in Vienna, who dissuaded him from puzzling his brain over a problem other people had failed to solve; see G. Böhm, *Philipp von Jolly* (Munich, 1886), 12.
4. A. Hermann, *Max Planck* (Hamburg, 1973), 11.
5. In the office of the Physikalische Gesellschaft, Magnus Haus, Am Kupfergraben, Berlin, D.D.R.
6. Lectures on mathematical physics had been given long before this at the University of Berlin. In early 1875 G. R. Kirchhoff was appointed professor of this subject there, and at the University of Innsbruck, Ferdinand Peche already had a similar position in 1868.
7. Planck tried without success to contact Clausius; he did not appear to derive much benefit from his correspondence with the mathematician Carl Neumann at Leipzig (son of Franz Neumann).
8. See article in *Dictionary of Scientific Biography* by Paul Forman.
9. The famous chemist Adolph von Baeyer let Planck know, in his 1879 examination, that he did not think much of theoretical physics (Planck, 1949, 4).
10. See J. R. Partington, *A History of Chemistry*, IV (London, 1964), 595–596.

11. G. R. Kirchhoff, *Untersuchungen über das Sonnenspectrum und die Spectren der chemischen Elemente*, H. Kangro ed., (Osnabrück, 1972), II, 9.

12. The *first* view was the *statistical*. Cf. Planck, *Vorlesungen über Thermodynamik* (Leipzig, 1897), forewords.

13. Although Planck first created this concept only after 1906, he retained it—among other expressions for it—at least until 1915; but in 1909 he also used "quantum theory," in accord with his philosophical views.

14. Translations: Russian (1900), English (1903), French (1913), Spanish (1922), Japanese (1932); the 11th German ed. appeared in 1966.

15. Planck followed the German translation, Helmholtz and Wiedemann, eds. (Brunswick, 1867), based on the second English ed. (London, 1865).

16. PAV III, 3, and PAV I, 382, respectively.

17. See L. Rosenfeld, "La première phase de l'évolution de la théorie des Quanta," in *Osiris*, **2** (1936), 149–196; M. J. Klein, "Planck, Entropy and Quanta 1901–1906," in *The Natural Philosopher*, **1** (1963), 87–99; and "Thermodynamics and Quanta in Planck's work," in *Physics Today*, **19** (Nov. 1966), 23–27.

18. PAV I, 452 and 484.

19. G. R. Kirchhoff (1972), [1], 42 (300), [2], 16.

20. On "The Development of Boltzmann's Statistical Ideas," see M. J. Klein, in *Acta physica austriaca*, supp. **10** (1973), 53–106; Planck did not support Zermelo's views in all respects.

21. H. Kangro (1970), 144–148. (Cited in Bibliography, below.)

22. PAV I, 562; PVW (1906), 113 and 186; (1913), 156; (1921), 174.

23. PAV I, 594 and 684; II, 756–757; PVW (1906), 127 and 167–168; (1913), 170; (1921), 186; see H. Kangro (1970), 100, 143.

24. H. Kangro (1970), 180.

25. A systematic treatment of the principle was given by Joachim Jungius in the seventeenth century; see H. Kangro, "Organon Joachimi Jungii ad demonstrationem Copernici hypotheseos Keppleri conclusionibus suppositae," in *Organon*, **9** (1973), 169–183.

26. Stephen G. Brush, *Kinetic Theory*, II (Oxford, 1966), 119; *Planck's Original Papers in Quantum Physics*, annotated by H. Kangro, translated by D. ter Haar and S. G. Brush, edited by H. Kangro (London, 1972), n. 32.

27. In 1973 T. S. Kuhn announced to me a different interpretation and his plan to write a book on it, which I have not yet seen. Our discussion stimulates me to cite for the reader's consideration the following passages of sources in the original language: PAV 700–701, 720–721; PVW (1906) 153. PAV II, 244–247 and 452–454. A. E. Haas, J. W. Nicholson, N. Bohr, and W. Wien, among others, followed Planck's view of discrete resonator energies from 1900 on: see *Sitzungsberichte der Kaiserlichen Akademie der Wissenschaften*, Mathematisch-naturwissenschaftliche, Abt. IIa, **119** (1910), 125; *Monthly Notices of the Royal Astronomical Society*, **72** (1912), 677; *Philosophical Magazine*, 6th ser., **26** (1913), 4, and letters of Planck to Wien. Einstein supported this view, which Planck held up to 1909, in *Physikalische Zeitschrift*, **10** (1909), 822, although Einstein himself tried to shift the accent to independent light quanta.

28. PVW (1906), 151; (1913), 136–139.

29. On the question of constructing the entropy maximum, see H. Kangro (1970), chs. 8, 9.

30. H. Kangro, "Ultrarotstrahlung bis zur Grenze elektrisch erzeugter Wellen, das Lebenswerk von Heinrich Rubens," in *Annals of Science*, **26** (1970), 235–259.

31. Pertinent information will be found in H. Hartmann, *Max Planck als Mensch und Denker* (Basel–Thun–Düsseldorf, 1953).

32. *Gedankenexperiment* presumably coined by H. C. Ørsted: see *Berichte über die 9. Versammlung Deutscher Naturforscher und Ärzte* (Hamburg, 1830), 18.

33. M. Planck, in *Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften* (1899), 479–480; M. J. Klein, in *Physics Today*, **19** (November 1966), 26.

34. M. Planck, in *Kultur der Gegenwart*, I (Leipzig–Berlin, 1915), pt. 3, sec. 3, pp. 692–702, also 714–731; see also Planck's addresses from 1919 to 1930 commemorating the founding of the Academy by Leibniz, in *Max Planck in seinen Akademie-Ansprachen*, Deutsche Akademie der Wissenschaften zu Berlin, ed. (Berlin, 1948). Instructive examples are also in Planck's "Das Wesen des Lichts" (1919) and "Theoretische Physik" (1930), in PAV III, 108–120, 209–218.

35. In 1929 Planck added *axiomatists* to positivists and metaphysicians as "workers on the physical world-picture" but deplored their tendency toward formalism without content; see PAV III, 183.

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