Publications and Honors. Richter's productivity was prodigious; he published approximately 291 titles, encompassing a wide variety of subjects, including ichnology, trilobite research, Devonian paleogeography, biostratigraphy, micropaleontology, taxonomy and systematics, paleoecology, and constructive morphology. His work was instrumental in establishing the Senckenberg laboratory as the center for actuogeologic and actuopaleontologic studies. Richter was an advocate and interpreter of the international rules of taxonomy and in 1930 was voted onto the International Commission for Zoological Nomenclature. For the next twenty years he worked tirelessly in the verification of types, the development of archival techniques, and the production of collection catalogs.

Richter received many honors. He became an external or corresponding member of the National Research Council of the United States (1929), the Institut royal des Sciences naturelles de Belgique (1930), the Geological Society of London (1950), the Academica delle Scienze in Bologna (1953), and the Instituto de Investigaciones Geológicas "Lucas Mallada" in Madrid (1953). Richter was served as president of the International Union of Paleontology from 1933 to 1937. He was named an honorary member of the Palaeontological Society of America (1926), the International Congress of Geologists in Moscow (1937), and the Société belge de Géologie, de Paléontologie, et d'Hydrologie (1938). He received medals of honor from the Senckenberg Naturalist Society (1951) and also received the Gold Medal of the Paläontologische Gesellschaft (1951) and the Hans Stille Medal given by the Deutsche Geologische Gesellschaft (1951). To celebrate his seventieth birthday, former students and friends put together a special Festschrift volume, published by the Senckenberg Naturalist Society. Rudolf Richter passed away at Frankfurt on 5 January 1957, a few weeks after his wife Emma. At the end he was totally paralyzed but still managed to work with the help of his students and colleagues.

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S. George Pemberton

RIEMANN, GEORG FRIEDRICH BERNHARD

(*b.* Breselenz, near Dannenberg, Germany, 17 September 1826; *d.* Selasca, Italy, 20 July 1866). For the original article on Riemann see *DSB,* vol. 11.

Whereas Georg Friedrich Riemann's scientific results themselves have almost completely been analyzed within the original article in the *DSB,* this postscript emphasizes the mathematical tradition in which he stood, his reception by his contemporaries and in the time shortly after his death, and the shaping influence that he had on mathematics as a scientific discipline.

Academic Teachers. Among mathematicians that influenced Riemann, perhaps the most important was Carl Friedrich Gauss. In fact, the only references in Riemann's doctoral dissertation are to two papers of Gauss; Riemann had studied the writings of Gauss in the university libraries of Göttingen and Berlin. However, Riemann did not write his thesis under what in the early twenty-first century would be called supervision: He apparently informed Gauss about the topic of the dissertation only after he had already finished it. Still, this was the usual procedure at German universities at that time.

Moritz Abraham Stern, Riemann's second academic teacher at Göttingen, is often termed a second-rate mathematician, sometimes even without noting that one is comparing him to Gauss. In any case, Riemann received a profound knowledge of the state of the art of analysis as taught in Germany at that time from Stern's lectures on calculus.

More important was the influence of Peter Gustav Lejeune Dirichlet. Shortly after Riemann arrived at Berlin, Dirichlet recognized his talents and guided his studies of the literature. In particular, he introduced Riemann to the modern techniques in analysis, which had been developed in France in the beginning of the nineteenth century by mathematicians such as Augustin-Louis Cauchy, Siméon-Denis Poisson, and others, and of which many contemporary German mathematicians were unaware. This support with respect to the literature continued even when Riemann was preparing his Habilitation thesis on Fourier series at Göttingen, when Dirichlet sent him the necessary material for the introductory historical section.

One can even speculate that Ferdinand Gotthold Max Eisenstein had a decisive influence on Riemann's mathematics, because Eisenstein found the functional equation for the *L*-series modulo 4 while Riemann was in Berlin, but Riemann was not interested in closer personal contact with Eisenstein. And he had other opportunities for the inspiration for the functional equation of the zeta function and its proof in his 1859 paper on the distribution of primes.

Riemann and the Dirichlet Principle. Riemann's use of the Dirichlet principle was decisive for the advancement of mathematics because of the results that he obtained by its ingenious, sometimes even bold, use in his doctoral dissertation (1851) and in his paper on Abelian functions (1857). Furthermore, it is one of the landmarks for the change from an algorithmically to a conceptually oriented view of mathematics: The function one is in search of is not given by an explicit formula but implicitly as the solution of a variational problem, even if one can deduce some of the properties that it will necessarily have, for example, that it is harmonic. Resorting to a pure existence statement was clearly ahead of the way of thinking among most mathematicians in the middle of the nineteenth century, and many of Riemann's contemporaries felt uncomfortable with his method.

Sometimes the story of the criticism of the Dirichlet principle is depicted as a battle between the German mathematical centers at Berlin and Göttingen on the highly prestigious theory of Abelian functions in the following way: With his 1857 paper, Riemann took the lead ahead of the Berlin representative Karl Weierstrass. Weierstrass struck back after Riemann's death by showing that variational principles need not have a solution, so that Riemann's proofs were incomplete. But, finally, the Göttingen school under the impetus of Felix Klein vindicated Riemann's visions.

Nevertheless, the situation was a bit more complicated: To be sure, Weierstrass read a note to the Berlin Academy in 1870 that contained a counterexample to a naive use of variational principles. In the terminology of fairy tales, however, Weierstrass was not the stepmother who poisoned Snow White but rather the child who openly said what (almost) everybody knew about the emperor's clothes. In fact, rumors against the liberal use of variational principles had been around as early as the late 1850s and they did not come only from Weierstrass or even Berlin mathematicians—as one learns, for example, from the notes of Felice Casorati from his conversations with Leopold Kronecker in 1864 and from a letter of the Russian mathematician Georg August Thieme to Richard Dedekind on a visit to Riemann at Göttingen in the summer of 1862.

Additionally, because Weierstrass's note was published only in the second volume of his *Mathematische Werke* in 1895, the first *published* counterexample to the Dirichlet principle was in an 1871 article by one of Riemann's own students, Friedrich Emil Prym. Furthermore, Prym directly attacked the specific Dirichlet principle as it had been presented in Dirichlet's lectures, whereas Weierstrass commented on variational principles in general. Also David Hilbert fully acknowledged the justification of Weierstrass's criticism (which was all the easier for him because in 1901 he had contributed to the proof of the Dirichlet principle as used by Riemann). Even Riemann himself knew about the gaps in his reasoning but argued

Mathematician Georg Friedrich Riemann.

in a discussion with Weierstrass in 1859 that he had made use of Dirichlet's principle only as an easy resort that was just at hand. Even if that tool was faulty, his existence theorems still were true.

In fact, not the least influence that Riemann had on mathematics was that he set out a research agenda that was pursued by other mathematicians, most prominently from the Göttingen school: It took more than half a century before Hermann Weyl would be able to transfer Riemann's vision of a "Fläche" (which came to be known as "Riemann surface") into a detailed definition in the language of set theory. At about the same time Richard Courant in his doctoral dissertation also completely vindicated Riemann's use of the Dirichlet principle. But the Berlin school around Weierstrass was also active in this research beginning with Weierstrass himself, who studied Riemann's writings and tried to translate the results into his more algorithmically oriented way of thinking even if he had problems with the Riemann's way of reasoning. Furthermore, Weierstrass's favorite disciple, Hermann Amandus Schwarz, proved explicit formulas that solve the mapping problem of Riemann's doctoral dissertation for the case of polygons and also attacked the general case.

The Distribution of Riemann's Ideas. Riemann exerted his influence mainly through written sources: his own publications; books by his students, themselves prominent, such as Carl Neumann's *Vorlesungen über Riemann's Theorie der Abel'schen Integrale;* and also by handwritten copies of the notes taken during his lectures. These were distributed throughout Europe, in Germany, of course, but to a great extent also in Italy and even in Russia with the Italian mathematicians being extremely receptive to Riemann's ideas. This does not mean that his mathematical standing was not well regarded elsewhere: He was a member of the Gesellschaft der Wissenschaften in Göttingen, of the Preussische Akademie der Wissenschaften (Berlin), of the Bayerische Akademie der Wissenschaften (Munich), of the Académie des Sciences (Paris), and of the Royal Society (London).

Attendance at his lectures personally would have influenced only few mathematicians, the maximal number of thirteen students at a lecture being documented in the Göttingen archives (whereas a few years later Ernst Eduard Kummer (1810–1893) and Weierstrass would have an auditorium of about two hundred students at Berlin).

One of the results that was attributed by hearsay to Riemann is the example $\sum_{n=1}^{\infty} (\sin n^2 x)/n^2$ of a continuous but nondifferentiable function. Neither this example nor any concrete statement about it can be found in Riemann's writings. Although the function is not differentiable at the points of a dense set, there are some points at which it is differentiable, and so it is not nowhere-differentiable. A complete analysis of its differentiability was only given around 1970. What the sources do reveal, however, is that both Kronecker and Weierstrass were interested in the function, that Weierstrass claimed to have a proof that this function is not differentiable at the points of a dense set, and that Riemann himself had studied the boundary behavior of theta functions to such an extent that only an interchange of limit processes was necessary to obtain this result. (In fact, from his lecture notes one learns that Riemann did not care too much about such technical questions.)

Development of Mathematics. Riemann's results and mathematical ideas have had far-reaching consequences, even on the present view of the universe: In his Habilitation lecture on the hypotheses on which geometry is founded Riemann had defined the (differential) geometric structure of space by means of a positive definite differential form, which locally induces a Euclidean structure. Already Hermann Minkowski had studied the generalization to a no longer positive definite but still nondegenerate differential form. So for his theory of general relativity Albert Einstein only had to generalize Riemann's concept from a locally Euclidean to a locally Minkowskian structure. (In fact, Einstein had studied Riemann's Habilitation lecture as early as in his student days.)

If one considers mathematics as a scientific discipline, Riemann's influence on the way in which mathematical objects are conceived in the early 2000s is perhaps even more important. Before his doctoral thesis a function of a complex variable was given as an analytic term that could be used to calculate the values of this function. (Even after the thesis Riemann's older but longer living contemporary Weierstrass followed this approach in the form of power series both successfully and influentially.) Riemann, by contrast, would rather look for a characterization of such a function by its properties, in this case, the Cauchy-Riemann differential equations. As another example, he studied the hypergeometric function not mainly by means of the series by which Gauss had defined it but by means of the differential equation that it fulfills, which would spare him long and tedious calculations. Still more impressive is the advantage that Riemann's approach had in the theory of elliptic and Abelian functions when one compares his results with the pages of long lists of formulas that are typical for the writings of Niels Henrik Abel and, especially, Carl Gustav Jacob Jacobi.

Minkowski attributed to Dirichlet a second mathematical principle besides the one mentioned above, namely to minimize blind calculation and to maximize thoughts led by visions. His close friend Hilbert even brought forth the principle that one should lead proofs by thoughts and not by calculations, in direct connection to Riemann. In this respect, the latter was a turning point in the history of mathematics, one of the lesser examples being the fact that he was the first mathematician who not only defined the notion of an integral but also explicitly defined what it means that a function is integrable. (It is worth noting that Dedekind, Riemann's fellow student and colleague at Göttingen, also deeply influenced by Dirichlet, approached algebra in much the same way as Riemann approached analysis and geometry. And from Dedekind a line of influence runs via Emmy Noether, Emil Artin, and Bartel Leendert van der Waerden to the structuralistic Bourbaki school of the second half of the twentieth century.)

One should stress, however, that Riemann himself was by no means averse to or even afraid of calculations: One learns from the notes of his introductory lectures on complex analysis that he would not hesitate to use besides the nonconstructive Dirichlet principle the more algorithmic and "hands-on" means of power series expansions and even the idea of analytic continuation. Furthermore, his statement that it is probable ("wahrscheinlich") that all zeros of the zeta function have real part 1/2 was based on long explicit calculations of zeros.

This famous Riemann hypothesis was still open in 2007, even though in 2000 the Clay Foundation offered a million-dollar prize for its solution. So Riemann not only opened new, far-leading doors in science, in particular mathematics, but his work continued to show the way to new frontiers.

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Peter Ullrich

RIKER, WILLIAM HARRISON (*b.* Des Moines, Iowa, 22 September 1920; *d.* Rochester, New York, 26 June 1993), *political science, rational choice, positive political theory, federalism.*

Riker did nothing less than revolutionize the study of political science. He introduced the study of game theory and decision theory into the study of politics, moving it away from purely normative and ad hoc descriptions. He coined the term *positive political theory* to describe the effort to develop individual-level, descriptive generalizations about political behavior, often based on axiomatic propositions and always on the supposition that people behave rationally. The term, and the approach, are now accepted parts of political science. A brilliant teacher and a clever administrator, he oversaw the development of the first graduate program in political science devoted to high-level training in quantitative methods, including game theory, decision theory, and econometrics, at the University of Rochester.

Riker's father Ben, a book seller, and his mother Alice, moved his family to Battle Creek and Detroit, Michigan, before settling in Indianapolis, where Riker graduated from high school in 1938. Four years later he graduated from DePauw University and, after a brief hiatus, went to graduate school at Harvard University, where he received a PhD in government 1948.

Research. Riker began his teaching career at Lawrence College (now University) in Wisconsin in 1948. He taught courses in American politics, but he was also called upon to teach other subjects, including an extraordinary course on political philosophy. His early work was traditional. He authored a textbook on American politics that was well received but hardly pathbreaking. He also wrote a book on the National Guard, presaging another topic—federalism—that he studied throughout his career.

In the mid-1950s his thinking and writing changed dramatically. He read Duncan Black's *Theory of Committees and Elections* and was impressed with the insights that the so-called median voter theorem might yield for understanding political behavior. He devoured John von Neumann and Oskar Morgenstern's *The Theory of Games and Economic Behavior*, also thinking that it could provide insights into political behavior. He began thinking of applications of a logical approach to the study of politics. In 1958 he published a paper on the "paradox of voting" (a "cyclical" result, in which voters who have individually transitive preferences over three alternatives, *a, b,* and *c,* might nonetheless vote by majority rule for alternative *a* over alternative *b,* for *b* over *c,* and yet for *c* over *a*) in the U.S. Congress. His first major work was *The Theory of Political Coalitions* (1962). In it he developed what he called the size principle, the idea that political entrepreneurs tend to build support coalitions that are only as large as needed to win (typically a bare majority). To attract a larger coalition is "wasteful" in that the leadership has to make policy or other concessions when they already have enough support to win.

Another signal work was a paper published in 1968 on voter turnout. He and his then-student, Peter Ordeshook, inquired how it was that rational individuals would go to the polls, knowing that the likelihood of their making a difference (being the decisive vote between two candidates) was infinitesimally small. Their formulation—adding consideration of another factor, generally called "citizen duty"—did not solve the problem, but it led to numerous subsequent papers and is still used today to frame the question. A few years later, and also with Ordeshook, he published a text that explained the positive approach to the study of politics and amply demonstrated the kinds of individual and collective behavior to which it could be applied.

Another of Riker's lifelong interests was the role of institutions in politics. This was manifested early on in his work on federalism and in his fascination with "Duverger's law" (the idea that plurality elections in single-member districts promulgate a political system with only two parties). Institutions were at the heart of much of his later work, especially in *Liberalism against Populism,* which dealt with the difficult subject of how to justify democracy in light of the numerous problems and paradoxes associated with the making of social choices. In his later works, he was intrigued with agenda setting broadly conceived, coining the term *heresthetics* to refer to the art of manipulating issue agendas for political advantage and applying it to a study of the campaign to ratify the U.S. Constitution.

Teaching: Building a Graduate Program. On the strength of his new approach, which he called positive political theory, Riker was hired by the University of Rochester to begin a graduate program to complement its newly developed strength in economics. His department-building skills were on par with his teaching skills. He sought out faculty who were sympathetic with his view of political science—sometimes practitioners themselves, but always willing to work with students who would push the bounds of the rational-choice perspective as far as they could take