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Journal Title: Thomas of Bradwardine, his Tractatus de proportionibus; its significance for the development of mathematical physics

Trans. #: 1102283



Article Author:

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Article Title: PLEASE SCAN PAGES: 87-117 (from chapters 2 & 3) Page numbers given for the English. In addition to those, please scan the corresponding Latin (if they're on other pages not included in the page ranges I gave)

Location: Science-Engineering Library IN LIBRARY

Item #:

Volume:

CUSTOMER INFORMATION:

Issue:

Alan G Aversa
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Month/Year: 1955

Pages: 87-117 (scan notes and title/copyright pages for chapter requests)

STATUS: Graduate
DEPT: NDSNDG

Imprint:

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CHAPTER TWO

Having looked into these introductory matters, let us now proceed with the undertaking which was proposed at the outset. And first, after the manner of Aristotle, let us criticize erroneous theories, so that the truth may be the more apparent.

There are four false theories to be proposed as relevant to our investigation, the first of which holds that: the proportion between the speeds with which motions take place varies as the difference whereby the power of the mover exceeds the resistance offered by the thing moved.

This theory claims in its favor that passage from Book I of the De Caelo et Mundo (in the chapter on the "infinite") in the text which reads: "It is necessary that proportionally as the mover is in excess, etc.," together with Averroes' Comment 71 on Book IV of the Physics, in which he says: "Every motion takes place in accordance with the excess of the power of the mover over that of the thing moved." In Comment 35, on Book VII of the Physics, he further states that: "The speed proper to any given motion varies with the excess of the power of the mover over that of the thing moved," and in the final Comment, Comment 39, he says that: "The speed of alteration and the quantity of time will vary in accordance with the amount whereby the power of that which is causing the alteration exceeds the resistance offered by what undergoes the alteration." Many other passages afford similar remarks.

The present theory may, however, be torn down in several ways:

First, according to this theory, it would follow that, if a given mover moved a given mobile through a given distance in a given time, half of that mover would not move half of the mobile through the same distance in an equal time, but only through half the distance. The consequence is clear, because, if the whole mover exceeds the whole mobile by the whole excess, then half the mover exceeds half the mobile by only half the former amount; for, just as 4 exceeds 2 by 2, half of 4 (namely, 2) exceeds half of itself (that is, 1) by 1, which is only half of the former excess.

That such a consequence is false is apparent from the fact that Aristotle proves, at the close of Book VII of the Physics, that: "If a given power moves a given mobile through a given distance in a given time, half that power will move half the mobile through an equal distance in an equal time." Aristotle's reasoning is quite sound, for, since the half is related to the half by the same proportion as the whole is to the whole, the two motions will, therefore, be of equal speed.

Secondly, it follows from this theory that, given two movers moving two mobilia through equal distances in equal times, the two movers, conjoined, would not move the two mobilia, conjoined, through an exactly equal distance in an equal time, but, instead, through double that distance. This consequence follows necessarily because the excess of the two movers, taken together, over the two mobilia, taken together, is twice the excess of each of them over its own mobile; for, just as

excedit unitatem per unitatem, duo autem binarii (qui quarternarium faciunt) excedunt duas unitates (quae dualitatem constituunt) per dualitatem, quae est dupla ad excessum binarii super unitatem. Et ita est in omnibus aliis ubi duo excessa a duobus excedentibus aequaliter exceduntur. Falsitas consequentis patet per Aristotelem, septimo Physicorum, ut prius, ubi probat istam conclusionem: "Si duae potentiae divisim moveant duo mobilia per aequalia spatia in aequali tempore, illae duae potentiae coniunctae movebunt illa duo mobilia coniuncta per aequale spatium in aequali tempore cum priori." Et haec ratio Aristotelis satis probat hoc, scilicet: Proportionaliter se habet motor compositus ad motum compositum et motor simplex ad motum simplex. 40 45

Tertio sic: Tunc ex proportione geometrica, scilicet similitudine proportionem motorum ad sua mota, non sequitur aequalis velocitas motuum, quia nec excessuum; quoniam eadem est proportio duorum ad unum et sex ad tres, excessus tamen unius est unitas, alius autem ternarius. Consequens autem ad quod deducitur est falsum, et contra Aristotelem septimo Physicorum, in fine et multis locis, ubi semper, ex aequalitate proportionum motorum ad sua mota, arguit aequalitatem velocitatum in motibus. Idem vult Averroes super loca praedicta, et similiter quarto Physicorum, commento 71, et super primum De caelo et mundo, commento 63, et aliis multis locis. 50 55 60

Nec potest dici quod Aristoteles et Averroes intelligunt, in locis praedictis, per proportionem seu analogiam, proportionalitatem arithmeticae seu aequalitatem excessuum, ut dicunt quidam; quia, septimo Physicorum, probat Aristoteles istam conclusionem: "Si aliqua potentia moveat aliquod mobile per aliquod spatium in aliquo tempore, medietas motoris movebit medietatem moti per aequale spatium in aequali tempore, quoniam similiter secundum analogiam sicut se habet medietas motoris ad medietatem moti et totus motor ad totum motum," quod tamen de proportionalitate arithmetica, quae significat aequalitatem excessuum, dignoscitur esse falsum (ut in primo argumento contra hanc opinionem sufficienter est ostensum). Et Averroes ibidem dicit quod sic erit eadem proportio, "sicut universaliter demonstrant geometri." Et potest hoc demonstrari geometricè per hunc modum: Sicut totus motor ad medietatem motoris, ita totum motum ad medietatem moti. Igitur permutatim, per septimam suppositionem primi capituli, sicut totus motor ad totum motum, ita medietas motoris ad medietatem moti; et hoc erat probandum. 65 70 75

Glosa etiam praedicta stare non potest, quia Aristoteles, septimo Physicorum, probat hanc conclusionem: "Si duo motores divisim moveant duo mobilia per aequalia spatia in aequali tempore, illi duo motores coniunctim movebunt illa duo mobilia coniuncta per 80

anything having a value of 2 exceeds unity by 1, so two such "2's" (which make 4) exceed two "1's" (which make 2) by 2, which is twice the excess of 2 over 1. The foregoing holds in all cases in which two subtrahends are equally exceeded by two minuends.

That the above consequence is false is evident from the foregoing argument of Aristotle, in Book VII of his Physics, where he demonstrates the following conclusion: "If two powers move two mobilia, separately, through equal distances in equal times, those powers conjoined, will move the two mobilia conjoined, through an equal distance in a time equal to the former one." This argument of Aristotle is sufficient proof that the relation between a single mover and its mobile, and a compound mover and its mobile, is a proportional one.

In the third place, it would follow that a geometric proportion (that is, a similarity of proportions) of movers to their mobilia would not produce equal speeds, since it does not represent an equality of excesses; for, although the proportions of 2 to 1 and 6 to 3 are the same, the excess of the one term over the other is 1 in the first case and 3 in the second case.

The consequence to which we are thus led is, however, false and opposed to Aristotle's opinion, as expressed at the close of Book VII of the Physics and in many other places, where, from an equality of proportions of movers to their mobilia, he always argues equal speeds. Averroes supports the same view in his remarks on the passages just mentioned and also in his Comment 71 on Physics IV, Comment 63 on De Caelo et Mundo I, and in many other places.

Nor can it be legitimately maintained that, in the passages cited, Aristotle and Averroes understand, by the words "proportion" and "analogy," arithmetic proportionality (that is, equality of differences), as some have claimed. Indeed, in Book VII of the Physics, Aristotle proves this conclusion: "If a given power moves a given mobile through a given distance in a given time, half that power will move half the mobile through an equal distance in an equal time, because, "analogically," the relation of half the mover to half the mobile is similar to that of the whole mover to the whole mobile." Such a statement, interpreted as referring to arithmetic proportionality, is discernibly false (as has already been made sufficiently clear in the first argument raised against the present theory). Moreover, regarding this same passage, Averroes says that the proportion will be the same "in the sense that geometricians universally employ in demonstrations."

The above thesis of Aristotle may be demonstrated geometrically as follows: As is the whole mover to half the mover, so is the whole mobile to half the mobile. Therefore, permutatively, (by Axiom 7 of Chapter I): As is the whole mover to the whole mobile, so is half the mover to half the mobile. And this is what was to have been proved.

The reading of Aristotle proposed by the present theory does not stand up, moreover, because, in Book VII of the Physics, Aristotle also proves this conclusion: "If two movers, separately, move two mobilia through equal distances in equal times, those two movers, conjointly, will move those two mobilia, conjoined, through a distance and in a time

aequale spatium in aequali tempore cum priori," per idem medium. "Analogum enim est" (et per "analogum" intelligit proportionale). Sed non proportionalitate arithmetica, quia non secundum aequalem 85 excessum excedit simplex motor simplex motum et compositus compositum, ut ex secundo argumento contra hanc opinionem apparet. Et Averroes ibidem probat quod "erit eadem proportio, non aequalis excessus, per primam quinti Elementorum Euclidis dicentis: 'Si fuerint quarumlibet quantitatum aliarum totidem 90 aequae multiplicitates aut aequae maiores aut aequae minores aut singulae singulis aequales, necesse est quemadmodum una illarum ad sui comparem, totum quoque ex his aggregatum ad omnes istas pariter acceptas similiter se habere.'" Igitur glosa praedicta non poterit esse vera. 95

Quarto sic: Tunc sequeretur quod aliquod mixtum habens resistantiam intrinsecam velocius movetur in pleno quam in vacuo. Sit enim A grave mixtum habens potentiam motivam et potentiam resistivam in se, et descendat ex se in aliquo medio (quod sit B) et sit C terra pura, minoris potentiae quam excessus A super 100 totam suam resistantiam. Tunc A haberet ex se motum certae velocitatis in vacuo. Subtilietur igitur B medium donec C moveatur aequali velocitate in B cum A in vacuo. Et tunc ponatur A in illo medio cum C, et movebitur velocius illo; habet enim maiorem excessum. Et C movetur in illo medio aequali velocitate 105 cum A in vacuo. Igitur movetur velocius in illo medio pleno quam in vacuo. Quod autem B possit tantum subtiliari donec C movetur in illo velocitate praedicta, apparet; quia ad quamtamcumque velocitatem datam potest motus localis velocitari per subtiliationem medii, ut patet quarto Physicorum, capitulo de vacuo, ubi 110 supponitur quod per subtiliationem medii (manente eodem motore), possibile est devenire ad quamtamcumque velocitatem motus localis datam. Et per hoc concluditur quod, posito motu locali in vacuo, idem movetur localiter aequavelociter in vacuo et in pleno. 115

Quinto sic: Tunc sequeretur quod, si aliquis motor excederet suam resistantiam per minorem excessum quam alius suam, tardius moveret illam. Descendat igitur terra magna cum aliqua resistantia, quam secundum excessum magnum excedat, et descendat aliqua terra minoris potentiae excessu maioris terrae 120 ad suam resistantiam. Et maneat maior terra et sua resistantia non alterata et subtilietur medium in quo movetur minor terra, donec moveatur aequavelociter cum maiori. Tunc minor terra

equal to the former;" if the medium through which the motions take place remains the same, "for the motions are 'analogous'." By "analogous" he means a proportional, but not in the sense of arithmetic proportionality, for the simple mover does not exceed the simple mobile by the same amount that the compound mover exceeds the compound mobile, (as was made clear in the second argument against this theory).

Averroes, commenting on this passage, proves that, "although the proportion will be the same, the excess will not." He does this by employing Theorem I of Book V of Euclid's Elements, which states that: "If the members of a given set of quantities are either equal multiples of, equally greater or less than, or exact equals of the members of a corresponding set, it follows that the relations between corresponding individual terms of the two sets will be the same as the aggregate relation of the two sets." Therefore, the above-mentioned reading of Aristotle cannot be taken as valid.

The fourth criticism is as follows: It would follow, on the basis of the present theory, that a mixed body, possessing internal resistance, could move faster through a medium than through a vacuum. Let A, for example, be a heavy mixed body (possessing within itself both motive and resistive power), and imagine it to descend of itself through some medium, B. Let C represent a quantity of pure earth, possessing less power than the excess of motive power over resistance in A. A will, of itself, move at a determinate speed in a vacuum. Now let the medium, B, be rarified to the point at which C moves in it with a speed equal to that of A in the vacuum. If A is now placed in the same medium with C, it should move faster than C (for it possesses a greater excess of motive power over resistance). C will move in that medium with a speed equal to that of A, moving in the vacuum. Therefore A will move faster through the medium than through the vacuum.

That it is, in fact, possible to rarify B to a point at which C would move in it with the speed just specified is evidently true, for, by rarefaction of the medium, local motion can be accelerated to any desired degree. This is shown to be the case in Book IV of the Physics (in the chapter on the "void"), where it is stated that, with the moving power remaining constant, it is possible to arrive at any given speed of local motion by rarifying the medium.

Thus (positing that a local motion could take place in a void), we find that the same body could move at the same velocity in both a medium and a vacuum.

Our fifth criticism is that it would also follow that, if a given mover exceeded its resistance by a lesser amount than another mover exceeded its resistance, the former motion would be the slower one. Let a large quantity of earth, possessing a given resistance which its downward force greatly exceeds, be supposed to fall. Let also another quantity of earth, possessing a lesser such excess of power, be supposed to fall. Letting the larger quantity of earth and the resistance associated with it remain constant, let the medium in which the smaller quantity of earth moves now be rarified to the point at which its speed becomes equal to that of the larger quantity. The smaller quantity now moves its

aequevelociter movet suam resistantiam sicut maior terra suam, et tamen illam per excessum minorem excedit. 125

Sexto sic: Tunc sequeretur quod, si terra pura movetur in aliquo medio quod in dupla proportione excederet vel maiori, non posset moveri in duplo velocius in aliquo medio alio. Non enim posset excedere aliquod medium per duplum excessum, quoniam tunc totum esset excessus. Et tunc, manente eodem motore, non in infinitum per subtiliationem medii posset velocitas motus generari (quod ex prioribus constat esse falsum). 130

Septimo sic: Tunc sequeretur quod, si aliquis motor excederet suam resistantiam per maiorem excessum quam alius suam, velocius moveret illam; et tunc quodcumque mobile fortis homo per maiorem excessum excederet quam debilior motor (sicut puer vel musca vel aliquid huiusmodi) excederet suum mobile, moveret et illud velocius. Et cum homo fortis per maiorem excessum excedit quodcumque mobile cum quo potest moveri maius quam debilior motor excedit aliquod mobile, sequitur quod fortis homo moveret quodcumque mobile cum quo possit moveri velocius quam debilior motor moveret aliquod mobile cum quo posset moveri, cuius contrarium declarat experimentum. Videmus enim quod musca portando aliquod modicum velociter multum volat, et puer aliquod modicum velociter satis movet, et homo fortis unum magnum mobile (quod vix potest movere) movet valde tarde. Et licet illi mobili apponatur maius quam musca vel puer posset movere, movet totum non multum tardius tunc quam prius. 140 145 150

Ex his igitur omnibus sufficienter ostenditur ista conclusio negativa: Proportio velocitatum in motibus non sequitur excessum potentiae motoris super potentiam rei motae.

Contraria autem his non difficile est solvere, quoniam Aristoteles et Averroes commentator, quando dicunt quod velocitas in motu sequitur excellentiam sive excessum potentiae motoris super rem motam, vel aliquid huiusmodi, intelligunt per excellentiam sive excessum proportionem maioris inaequalitatis qua potentia motoris excellit sive excedit potentiam rei motae. 155

SECUNDA PARS CAPITULI SECUNDI

Sequitur de secunda opinione erronea ponente proportionem velocitatum in motibus sequi proportionem excessus potentiae motoris super potentiam rei motae. Et hoc videtur fundari in dicto Averrois super septimo Physicorum, commento 36: ibi enim dicit quod "velocitas motus est secundum proportionem excessus potentiae motoris super potentiam rei motae." 160 165

Haec autem opinio debet redargui tamquam falsa. Ponatur enim quod excessus potentiae motoris super potentiam rei motae sit

resistance with the same speed that the larger moves its own, and yet exceeds it by a smaller amount.

Sixthly, it would also follow that, if a bit of pure earth were moving in some medium whose resistance its power exceeded by a ratio of two to one, or more, it could not move at double that speed in any other medium. It could not exceed any medium by double the first excess, for, in that case, the entire moving power would be excess, and, with the moving power remaining constant, it would consequently not be possible to increase the speed of the motion indefinitely by rarefaction of the medium. Such a consequence has already been established as false.

In the seventh place, another consequence would be that, if a given mover were to exceed its resistance by a greater amount than another mover exceeded its own resistance, the former motion would be the faster. Then, since a strong man exceeds anything he moves by a greater excess of power than a weaker mover (such as a boy, or a fly, or something of that sort) exceeds what it moves, he should move it more rapidly.

Experience, however, teaches us the contrary, for we see that a fly carrying some small particle flies very rapidly, and that a boy also moves a small object rather rapidly. A strong man, on the other hand, moving some large object which he can scarcely budge, moves it very slowly, and even if there were added to what he moves a quantity larger than either the fly or the boy can move, the man will then move the whole not much more slowly than he did before.

From all these considerations, therefore, the following negative conclusion is sufficiently well established:

The proportion of speeds in motions does not vary with the amount whereby the power of the mover exceeds that of the thing moved.

Objections to this conclusion are not difficult to dissolve, for Aristotle and Averroes, when they say that the speed of a motion varies in accordance with the amount by which the power of the mover excels or exceeds that of the thing moved, understand by "excellence," or "excess," a proportion of greater inequality whereby the power of the mover excels, or exceeds, the power of the thing moved.

CHAPTER TWO, PART TWO [THEORY II]

Let us now turn to the second erroneous theory, which supposes the proportion of the speeds of motions to vary in accordance with the proportion of the excesses whereby the moving powers exceed the resisting powers. This idea is evidently based on Averroes' Comment 36 on Book VII of the Physics, for he there states that the speed of a motion is determined by the proportion whereby the power of the mover exceeds that of the thing moved.

This theory should, however, be refuted as false. For just imagine the case in which the excess of the power of the mover over that of the

aequalis potentiae rei motae. Tunc nullus motor potest movere aliquod mobile velocius nec tardius illo motu; quia nullius motoris proportio excessus suae potentiae ad potentiam rei motae potest esse maior vel minor (ut per septimam conclusionem primi capituli patet). 170

Secundo sic: Movens primo movet totum per totam suam potentiam et non per excessum suae potentiae. Igitur motus et velocitas eius primo et essentialiter sequuntur habitudinem vel proportionem totius potentiae motoris ad potentiam moti, et non proportionem excessus, nisi fuerit accidentaliter et ex consequenti. 175

Sic igitur patet ista conclusio negativa: Proportio velocitatum in motibus non sequitur proportionem excessus potentiae motoris super potentiam rei motae. 180

Dictum autem Averrois, si quis glosare voluerit, potest sicut alias auctoritates pro prima opinione adductas.

TERTIA PARS CAPITULI SECUNDI

Sequitur de tertia opinione erronea, quae ponit proportionem velocitatum in motibus (manente eodem motore vel aequali) sequi proportionem passorum, et (manente eodem passo vel aequali) sequi proportionem motoris. 185

Et hoc videtur fundari quantum ad primam partem, in textu Aristotelis et in multis locis. Nam quarto Physicorum, capitulo de vacuo, textu commenti 71, sic dicit: "Sit enim B quidem aqua, D vero aer; quanto vero subtilior aer aqua et incorporalior, tanto citius A," id est mobile, "per D movebitur quam per B. Habet igitur eandem rationem," id est proportionem, "secundum quam distat aer ab aqua, velocitas ad velocitatem; quare si duplex subtile est aer, duplici tempore quod est ipsum B pertransibit quam D." Et textus sequens manifeste supponit quod (manente eodem motore et medio variato) proportio velocitatum in motibus sequitur proportionem mediorum, et etiam proportio temporum mensurantium illos motus sequitur proportionem mediorum econverso: scilicet quod maius tempus correspondet motui per medium densius, et minus tempus motui per subtilius medium correspondet. 190 200

Et primo De caelo et mundo, capitulo de infinito, sic dicit: "Ab eodem," scilicet agente, "enim supponatur in pluri et minori tempore maius et minus pati, quaecumque proportionaliter tempore divisa sunt."

thing moved is equal to the power of the thing moved. No mover will be able to move any mobile either faster or slower than the speed produced by this proportion, because no other proportion can be either greater or smaller than that whereby the excess of power of this mover over its mobile is related to the power of the mobile as a whole (as is demonstrable by Theorem VII, Chapter I).

In the second place, a moving power moves a whole mobile primarily by means of its total strength, and not by means of a residuum of its strength. A motion and its speed vary primarily and essentially, therefore, with the relation, or proportion, between the entire power of the mover and that of the thing moved, and not (except accidentally and secondarily) according to a proportion of excess.

This negative conclusion is therefore evident: The proportion of the speeds of motions does not vary in accordance with the proportion whereby the excess of the moving power over its mobile is related to the power of that mobile. The above-mentioned statement by Averroes may, if anyone were so to desire, be interpreted in the same way as were the other authorities cited in support of Theory I.

CHAPTER TWO, PART THREE
[THEORY III]

There follows the third erroneous theory, which claims that: (with the moving power remaining constant) the proportion of the speeds of motions varies in accordance with the proportion of resistances, and (with the resistance remaining constant) that it varies in accordance with the proportion of moving powers.

With respect to its first part, this theory is seen to be founded on many passages of Aristotle's writings. In Book IV of the Physics (in the chapter on the "void") he speaks as follows: "Let B represent a given quantity of water and D a given quantity of air. Now, by however much air is thinner and more incorporeal than water, by so much will A (that is, the moving body) move faster through D than through B. Let the one speed bear the same ratio, or proportion, to the other as that whereby air differs from water, and then, if air is twice as thin, the body will traverse B in twice the time required to traverse D." Furthermore, the text immediately following manifestly makes the supposition that, with the moving power remaining constant and the medium being varied, the proportion of the speeds of motions varies in accordance with the proportion of media, and that, conversely, the proportion of the times measuring those motions varies also in accordance with the proportion of media (namely, that the longer time corresponds to the motion through the denser medium and the shorter time to the motion through the rarer medium).

Further, in Book I of the De caelo (in the chapter on the "infinite") he speaks as follows: "It is held to undergo a greater and less effect by the action of the same agent, in a longer and shorter time, any such effects are divided proportionally to the time."

Et septimo Physicorum, in fine, vult Aristoteles quod si aliqua 205
 potentia moveat aliquod mobile per aliquod spatium in aliquo
 tempore, eadem potentia movebit medietatem eiusdem mobilis
 per duplum spatium in aequali tempore, et per idem spatium in
 medietate temporis.

Tantum pro prima parte huius opinionis. Et pro secunda parte 210
 huius opinionis supponit Aristoteles, quarto Physicorum, capitulo
 de vacuo, quod gravia et levia diversa in quantitate (si alia simil-
 iter se habeant) ferantur per aequale spatium in eodem medio
 velocius et tardius secundum proportionem gravium et levium
 adinvicem. 215

Et in septimo Physicorum, in fine (secundum expositionem
 Averrois) vult Aristoteles quod "si aliqua potentia moveat aliquod
 mobile per aliquod spatium in aliquo tempore, dupla potentia
 movebit illud mobile per duplum spatium in aequali tempore"

Et octavo Physicorum, versus finem, vult Aristoteles quod 220
 potentia motiva dupla ad aliam moveat idem mobile in medietate
 temporis quantum potentia minor in toto tempore; et universaliter
 quod potentia motiva maior alia moveat idem mobile quantum
 potentia minor in minori tempore proportionaliter secundum con-
 versionem proportionis (scilicet quod maiori potentiae debetur 225
 tempus minus et minori tempus maius).

Et idem vult Aristoteles primo De caelo et mundo, capitulo de
 infinito, ubi loquens de gravibus quae debent descendere per
 aequale spatium in eodem medio, sic dicit: "Analogiam," id est
 proportionem, "quam gravitates habent et tempora econtrario 230
 habebunt; puta si media gravitas in hoc, dupla in medietate eius"

Et tertio De caelo (ubi probat Aristoteles omne corpus motum
 recte gravitatem aut levitatem habere) supponit quod gravia inae-
 qualia in virtute pertranseunt spatia eis proportionalia in eodem
 medio, in eodem tempore. 235

Idem patet per primam conclusionem De ponderibus, quae sic
 dicit: "Inter quaelibet gravia est velocitatis in descendendo et
 ponderis eodem ordine sumpta proportio"

Idem per rationem poterit sic ostendi: Si unus motor sit prae-
 cise in duplo maioris potentiae quam alius, praecise in duplo 240
 plus potest movere idem mobile vel aequaliter duplum mobile.
 Nam si sit praecise duplae potentiae, potest praecise duplum
 facere (quia si posset plus quam duplum, esset potentiae maioris
 quam duplae; et si non posset duplum, sed minus tantum, esset
 potentiae minoris quam duplae). 245

Haec pro secunda parte positionis istius; et sic ista positio
 quantum ad utramque partem videtur esse fundata.

Ista tamen positio est dupliciter arguenda: primo super insuf-
 ficientia, secundo super mendacio consequentis.

Est autem insufficientis quia non docet proportionem velocita- 250

And, at the end of Book VII of the Physics, Aristotle wishes it to be understood that, if a given power moves a given mobile through a given distance in a given time, the same power will move half the same mobile through twice the distance in an equal time, and through the same distance in half the time.

Thus much in favor of the first half of the present theory.

In support of the second part of this position, Aristotle holds, in Book IV of the Physics (in the chapter concerning the "void") that, other conditions remaining constant, heavy and light bodies differing in quantity will move through a given distance in the same medium more swiftly and more slowly in accordance with the proportion of the heavy and light bodies to each other.

According to Averroes' exposition, at the close of Book VII of the Physics, Aristotle intends that, if a given power moves a given mobile through a given distance in a given time, double the power will move that mobile through double the distance in an equal time.

At the close of Book VIII of the Physics, Aristotle maintains that a motive power which is double another such power will move the same mobile in half the time required by the lesser power, and that, universally, a motive power greater than another will move the same mobile which the smaller power moves in a time that is less by converse proportion (that is, that less time is required by the larger power and more time by the lesser power).

Moreover, Aristotle intends the same thing in Book I of the De Caelo (in the chapter on the "infinite"), where, speaking of heavy bodies that fall equal distances in the same medium, he writes as follows: "The 'analogy' (that is, the proportion) between the weights will be the contrary of that between the times. For example, if the whole weight in a given time, then double the weight in half that time."

Further, in Book III of the De Caelo, where Aristotle proves every body to possess a rectilinear gravity or levity, he states that heavy bodies unequal in power will traverse, in the same medium and the same time, distances proportional to those powers.

The same is evident from Theorem I of the De Ponderibus, which states the following: "The proportion between the speeds of descent of any given heavy bodies is the same as that between their respective weights."

The theory may also be set forth by the following reasoning: If one mover has exactly twice the power possessed by another, it can move the same mobile exactly twice as much, or move twice the mobile the same amount, for, if it is exactly twice the power, it can accomplish exactly twice as much. If it could accomplish more than twice as much, it would be of more than twice the power; and if it were not capable of twice, but only of less, it would be of less than twice the power.

So much for the second part of this position, and thus we have seen what are the foundations of both parts of the theory.

The theory is, however, refutable on two grounds: first, on that of insufficiency, second, because it yields false consequences.

It is insufficient, because it does not determine the proportion of the

tum in motibus nisi in quibus est idem motor vel aequalis, seu idem mobile vel aequale. De motibus autem ubi diversantur tam moventia quam mota, penitus nihil dicit.

Est autem ista positio ex mendacio arguenda, quia aliqua potentia motiva localiter potest movere aliquod mobile aliqua tarditate, et potest movere dupla tarditate. Ergo (per istam positionem) potest movere duplum mobile. Et potest movere quadrupla tarditate: igitur quadruplum mobile, et sic in infinitum. Igitur quaelibet potentia motiva localiter esset infinita. 255

Similiter autem potest argui de quolibet mobili. Nam quodlibet mobile potest moveri aliqua tarditate et dupla et quadrupla et sic sine statu: igitur ab aliquo motore, et a subduplo, et a subquadruplo et sic sine fine. Igitur quodlibet mobile a quolibet motore potest moveri. 260

Nec potest dici quod tarditas motus non potest in infinitum duplari: quia si sic, sit A tarditas mobilis quae duplari non potest. Volvatur igitur sphaera, seu corpus columnare, super axem quiescentem. Tunc in aliqua parte iuxta polum sphaerae, seu axem corporis columnaris, est tarditas dupla ad A ut est satis notum et facile demonstrare. Tunc cum ista parte colligetur corda fortis et longa, in cuius extremo alligetur aliquod ponderosum, quod sit B. Tunc tarditas motus B est dupla ad A tarditatem; et hoc est quod volumus demonstrare. 270

Nec potest dicere cavillator quod motus B est motus per accidens, et in potentia tantum, ideo non facit ad propositum, quia iste motus habet motorem in actu et motum seu mobile in actu, terminos a quo et ad quem in actu, tempus in actu, et spatium seu locum pertransitum in actu. Igitur est motus in actu. 275

Nec potest dici quod motor non est in actu sed in potentia quia est pars sphaerae, vel corporis columnaris; quia totum movet primo, et pars ex consequenti. Et etiam tunc, si homo traheret illud ponderosum per cordam cum manu, illud moveret per accidens, quia per partem hominis. Et etiam quia tunc nullus motus ab extrinseco esset motus per se vel in actu, quia nullum movens extrinsecum potest per se totum applicari moto, sed secundum partem tantum. 285

Tertio est ista positio super mendacio arguenda, quoniam experimentum sensibile docet huius positionis contrarium. Videmus enim quod, uno homine movente aliquod ponderosum (quod potest vix solus movere motu valde tardo) si alius sibi adiungatur, illi duo movent illud multo plus quam in duplo velocius. 290

Et patet manifeste de pondere suspenso ad axem circumvolvibilem, quod per suum descensum movet insensibiliter et volvit axem seu rotam motu insensibili (sicut accidit in horologio) ad quod si suspendatur tantum pondus, totum descendet et circumvolveth axem seu rotam multum plus quam in duplo velocius (ut sensui sufficienter constat). 295

speeds of motions except in cases where either the mover or the mobile are constant. Concerning motions in which the moving forces, as well as the mobilia, are varied, it tells us almost nothing.

The theory is, on the other hand, to be refuted on the ground of falsity, for the reason that a given motive power can move a given mobile with a given degree of slowness and can also cause a motion of twice that slowness. According to this theory, therefore, it can move double the mobile. And, since it can move with four times the slowness, it can move four times the mobile, and so on ad infinitum. Therefore, any motive power would be of infinite capacity.

A similar argument may be made from the standpoint of the mobile. For any mobile may be moved with a given degree of slowness, with twice that degree, four times, and so on without end; and, therefore, by the given mover, and by half of it, one fourth of it, and so on, without end. Any mobile could, therefore, be moved by any mover.

Nor is it legitimate to object that slowness of motion cannot be doubled indefinitely, for, supposing this to be true, let A represent some slowness of a mobile that cannot be doubled. Now imagine a sphere or cylinder, revolving about a fixed axis; then, at some point near the pole of the sphere, or the axis of the cylinder, there is a degree of slowness double that of A, as is quite clear and easy to demonstrate. Now, at this point let there be attached a strong, long cord, at whose end is affixed a given weight, B. Then the slowness of the motion of B is twice that of A, and this is what we wished to demonstrate.

Nor can some quibbler properly claim that the motion of B is accidental motion, merely motion in potentia, and that it really has no relevance to the question; for this motion possesses a mover, a thing moved, initial and final limits, a time and a space traversed...all in actu. Therefore, the motion is a real one. Nor can it be maintained that the mover is not in actu, but only in potentia, inasmuch as it is part of the sphere, or cylinder, and because it is the whole that moves primarily and the part by consequence. In that case, if a man were to pull that weight by hand, by means of the cord, it would move accidentally, because by virtue of a part of the man; it would then follow that no motion, extrinsically caused, could be a "real" motion, (one which is in actu), since no mover can apply itself wholly to the thing moved, but can only do so by means of a part.

Thirdly, the present theory is to be refuted on the ground of falsity, because sense experience teaches us the opposite. We see, indeed, that if, to a single man who is moving some weight which he can scarcely manage with a very slow motion, a second man joins himself, the two together can move it much more than twice as fast. The same principle is quite manifest in the case of a weight suspended from a revolving axle, which it moves insensibly during the course of its own insensible downward movement (as is the case with clocks). If an equal clock weight is added to the first, the whole descends and the axle, or wheel, turns much more than twice as rapidly (as is sufficiently evident to sight).

Consimiliter autem accidit de tarditate (manente eodem moto et diminuto motore, et etiam econtrario).

Et sic patet ista conclusio negativa: Proportio velocitatum in motibus (manente eodem motore vel aequali) non sequitur proportionem passorum; nec (manente eodem passo vel aequali) sequitur proportionem motorum. 300

Pro rationibus autem quae istam positionem fundare videntur, dicendum quod omnes auctoritates volentes quod (existente eodem motore) proportio velocitatum in motibus sequitur proportionem passorum, intelligunt quod sequitur proportionem passorum ad sua agentia. 305

Et ideo pro prima auctoritate, quarti Physicorum, dicendum quod Aristoteles intelligit quod quanto est minor proportio aeris (propter maiorem subtilitatem et incorporeitatem) quam aqua, ad illud quod debet utrumque dividere, tanto illud citius movebitur per aerem quam per aquam; quoniam quanto proportio aeris ad aliquid est minor proportione aquae ad illud, tanto proportio illius ad aerem est maior quam ad aquam; et proportio velocitatum in motibus sequitur proportionem moventium ad res motas (ut posterius ostendetur). Et auctoritas alia allegata similiter est glosanda; et illa primi De caelo et mundo quae dicit: "Ab eodem enim supponatur in pluri et minori tempore maius et minus pati quaecumque proportionaliter tempori divisa sunt," id est, quorumcumque proportionem ad illud idem agens, proportionaliter tempori sunt divisae. 310 315 320

Conclusio autem allegata ex septimo Physicorum, quae dicit, "si aliqua potentia moveat aliquod mobile per aliquod spatium in aliquo tempore, eadem potentia movebit medietatem illius per duplum spatium in aequali tempore," intelligit per, "medietatem mobilis," partem mobilis habentem ad illam potentiam motivam medietatem proportionis totius mobilis ad eandem. Et hoc bene patet per Averroem, qui ibi probat conclusionem praedictam hoc modo: "Cum diviserimus motum," id est, rem motam, "contingit necessario ut proportio potentiae motoris ad motum," id est, rem motam, "sit dupla illius proportionis." Quod tamen esset falsum nisi intelligeret modo dicto, quoniam licet aliquod totum habeat aliquam proportionem ad aliud totum, non sequitur quod habeat duplam proportionem primae proportionis ad medietatem illius (ut in sequentibus ostendetur). Et per ista auctoritatum sequentium glosa patet. 325 330 335

Conclusio autem De ponderibus allegata similiter debet intellegi:

Since the situation regarding retardation is closely similar, whether the thing moved be constant and the mover be diminished or the reverse, we arrive at this negative conclusion: With the mover remaining constant, the proportion of the speeds of motions does not vary in accordance with the proportion of resistances, nor, with the resistance remaining constant, does it vary in accordance with the proportion of movers.

As for the reasons which seemed to support this theory, it should be pointed out that all authorities claiming that, with the mover remaining constant, the proportion of the speeds of motions varies in accordance with the proportion of resistances, really mean that the proportion of speeds varies with the proportion of the things affected to the things affecting them.

As a matter of fact, in the case of the first authority cited (that of Book IV of the Physics) it should be realized that what Aristotle means is that, to whatever extent the proportion of air to a given body which moves through it is smaller than that of water to that same body (due to the greater thinness and incorporeality of air), to that extent the body will move faster through air than through water; for to whatever extent the proportion of air to a given body is smaller than the proportion of water to that body, to the same extent is the proportion of the body to air larger than its proportion to water, and, as will later be shown, the proportion of the speeds of motions varies in accordance with the proportion of movers to things moved.

The second authority cited should also be construed in the same sense, together with that passage, from Book I of the De Caelo et Mundo, which reads: "It is held to undergo a greater and less effect, by the action of the same agent, in a longer and shorter time, and any such effects are divided proportionally to the time." In other words, the proportions of any given effects are divided proportionally to the time.

In the case, moreover, of the theorem drawn from Book VII of the Physics (which states that, if a given power moves a given mobile through a given distance in a given time, the same power will move half of it through double the distance in an equal time), Aristotle understands, by "half the mobile," a part of the mobile possessing a proportional relation to the given moving power which is half that of the whole mobile to that power.

This quite clearly appears to be Averroes' interpretation, for, regarding the passage in question, he proves the above-mentioned theorem as follows: "When we divide the motion (or the thing moved), it follows necessarily that the proportion of the power of the mover to the motion (or thing moved) is twice the former proportion. This would, nevertheless, be untrue, unless understood in the sense previously indicated, for, although a given whole may bear a given proportion to some other whole, it does not follow that it bears a proportion to half of the latter whole that is, half the original proportion (as will be shown in what follows)." It is in this sense that the following authorities should be interpreted.

The theorem drawn from the De Ponderibus must be read in the

"Inter quaelibet gravia," et cetera, id est, inter quaelibet gravia est velocitatis in descendendo et proportionis ponderis ad suam resistantiam eodem ordine sumpta proportio. Et hoc debet intellegi, resistantia existente aequali hinc inde. Auctor tamen non ponit aliqua principia illam conclusionem probantia. 340

Potest autem obici contra glosam praedictam quod nullus commentator probat illam conclusionem ad intellectum praedictum, sed ad alium quem habet ista positio: quod nulla est proportio nec aliquis excessus potentiae motivae ad potentiam resistivam (et ideo quem verba illius conclusionis praetendunt). 345

Dicendum quod nullus commentator quem nos vidimus probat istam conclusionem ad intellectum praedictum, nec aliquem alium. Nam unus commentator capit duo pondera inaequalia et duas lineas inaequales istorum descensus designantes, et capit primo tamquam datum ab adversario quod proportio maioris ponderis ad minus est maior proportione maioris lineae ad minorem. Et ex hoc arguit proportionem minoris ponderis ad maius esse minorem proportionem minoris lineae ad maiorem: ex quo concludit minorem esse proportionem ponderum quam descensuum, cuius oppositum erat datum. Istud autem non obviat, quoniam primo erat datum maiorem esse proportionem maioris ponderis ad minus proportionem maioris descensus ad minorem, et huic non repugnat sed sequitur conversim esse proportionem minorem (scilicet minoris ponderis ad maius) quam minoris descensus seu lineae ad maiorem. 350 355 360

Alius autem commentator similiter cum primo capit duo pondera inaequalia et eorum descensus inaequales, et apponit minori ponderi aliud pondus ut ex ambobus compositum sit aequale maiori. Et supponit quod descensus ponderis appositi per se, per tempus aequale tempori priorum descensuum, appositus minori descensui, maiori descensui, adaequatur. Et illud non est prius probatum, nec per se notum, nec sequens, nec est universaliter verum, sed in pluribus casibus est falsum quam verum (ut ex sequentibus apparebit). Et ex isto supposito concludit quod minor est proportio maioris ponderis ad illud pondus appositum quam maioris descensus ad descensum illius ponderis appositi; cuius contrarium asserit esse datum. Sed non est ita, quia non erat prius datum universaliter: "quorumlibet ponderum inaequalium esse maiorem proportionem maioris ad minus, proportionem suorum descensuum eodem ordine acceptorum," sed specialiter, "istorum duorum ponderum prius acceptorum et suorum descensuum." Et ideo illi non repugnat quod aliquorum aliorum ponderum inaequalium sit minor proportio maioris ad minus quam suorum descensuum, eodem ordine acceptorum. Aliquorum enim ponderum inaequalium est aequalis proportio maioris ad minus et suorum 365 370 375 380

same way. "Between whatever heavy bodies, etc. ...," that is, between whatever heavy bodies the proportion of speed of descent and that of the proportion of weight to resistance are taken in the same order. (And with the proviso that the resistance remains equal.)

The author of this work, however, proposes no principle in proof of this theorem, and it may well be objected against the above interpretation, that neither does any commentator prove the theorem in this sense, but rather in another, which the present theory supports (namely, that there is neither any proportion nor excess of motive power over resistive power) and which is, in fact, what the words of this theorem really mean.

The fact is that no commentator whom we have seen either proves this theorem according to our interpretation, or according to any other. One commentator, for example, takes two unequal weights and two unequal lines representing their descents, and then, first taking it as granted by his adversary that the proportion of the larger weight to the smaller is greater than the proportion of the longer line to the shorter, he argues from this that the proportion of the smaller weight to the larger is less than the proportion of the shorter line to the longer. From this he concludes that the proportion of weights is less than that of descents (the opposite of which had been stipulated).

This, however, presents no obstacle, for it was admitted that the proportion between the weights was greater than that between the descents, in the first place, and it not only does not invalidate the theorem, but it follows (conversely) that the proportion of the smaller weight to the larger will be smaller than that of the shorter descent, or line, to the longer.

Another commentator, also taking two unequal weights and their unequal descents, adds to the smaller weight a second, such that the two together are equal to the larger. He now posits that the descent of the added weight, by itself, through a time equal to that of the previous descents, when added to the descent of the lesser weight, is equal to the descent of the larger weight.

This had neither been previously proved, nor is independently known, nor follows logically, nor is universally true. In many cases it is, in fact, false (as will appear from what follows).

From this supposition, at any rate, he concludes that the proportion of the larger weight to the weight that was added is less than that of the descent of the larger weight to the descent of the additional weight (the opposite of what he states to have been given). Yet this is not the case, for it was not previously laid down as universally true that, "of any given unequal weights, the proportion of the larger to the smaller is in the proportion of their descents, taken in the same order;" but only specifically that, "of these two weights, which have been chosen, and of their descents." This is, therefore, not incompatible with the thesis that, in the case of certain other weights, the proportion of the larger to the smaller should be less than that of their respective descents; for, in the case of some weights, the proportion of the larger to the smaller is equal to that of their respective descents, in other

descensuum, eodem ordine acceptorum, aliquorum autem maior, et aliquorum minor (ut in sequentibus erit lucide demonstratum). 385

Ad rationem quae auctoritatibus plus movet, dicendum quod prima consequentia non valet. Et principia causalis adducta ad probationem illius est vera, quia semper dupla potentia minoris potentiae potest movere duplum mobile mobili minoris potentiae per aequale spatium in aequali tempore. Sed ex hoc non sequitur quod posset movere idem mobile in duplo velocius, sed bene sequitur quod posset movere illud mobile tanta velocitate maiori quanta est duplae difficultatis ad velocitatem priorem, et duplam virtutem requirit. Et illa velocitas aliquando erit praecise dupla velocitatis prioris, aliquando autem maior quam dupla, quandoque vero minor (ut ex sequentibus erit clarum). 395

QUARTA PARS CAPITULI SECUNDI

Quarta vero opinio ponit quod nulla est proportio nec aliquis excessus potentiae motivae ad potentiam resistivam, et ideo proportio velocitatum in motibus non sequitur aliquam proportionem nec excessum potentiae motivae ad potentiam mobilis, sed quoddam dominium et habitudinem naturalem motoris ad motum. 400

Et haec positio videtur fundari super auctoritatem Averrois, octavo Physicorum, commento 79, ubi in solutione eiusdem conclusionis dicit quod: "potentia non corporea non dicitur finita nec infinita, quia finitum et infinitum tantum de corporibus dicuntur, nec etiam una potentia incorporea potest dici maior alia." Maius enim et minus solius est quantitatis. Nec etiam potentiae separatae a corpore sunt proportionales, nec habent proportionem, quoniam proportio solius est magnitudinis ad magnitudinem. Ex istis videtur sequi quod nulla potentia motiva est finita vel infinita nec maior vel minor, nec aliquo modo proportionalis potentiae rei motae, quia omnis potentia motiva non est corpus, sed forma extensa in corpore vel a corpore separata. 405

Et haec positio, una cum dicto Averrois, per definitiones proportionis poterunt confirmari. 415

Primo: Proportio est comparatio rerum eiusdem generis (ut patet per definitionem proportionis primo capitulo assignatam). Sed potentia activa et passiva non sunt eiusdem generis, ut videtur.

Praeterea, si potentia activa et passiva haberent proportionem adinvicem, tunc essent comparabiles. Igitur essent eiusdem speciei specialissimae: Igitur haberent subjectum vel substantiam eiusdem speciei specialissimae. Consequens est falsum, quia potentia dividitur per activum et passivum, ut genus per differentias repugnantes. Et consequentia patet per Aristotelem septimo 420

cases it is greater, and in yet other cases less (as will be clearly demonstrated, later on).

As for the argument which is most convincing to writers on this subject, it should be replied that, although the causal principle adduced in its proof is true, the first consequence drawn from it does not hold. Fundamentally, a power which is double another power can move a mobile which is twice that moved by the lesser power through an equal distance and time. Instead of it following from this that the greater power can move the lesser mobile twice as fast, it rather follows that the greater power can move the lesser mobile at a speed which is as much greater as that expressed by the proportion of double the resistance to the former speed, and that it requires twice the power to do it. That speed will, in some cases, be exactly double the former speed, in some cases more than double, and in some cases less than double (as will be clear from later portions of this work).

CHAPTER TWO, PART FOUR
[THEORY IV]

A fourth theory declares there to be neither any proportion nor any relation of excess between motive and resistive powers. It holds that, instead of there being some proportion or excess of motive power over thing moved, motions take place in accordance with some sort of "natural dominance" of mover over moved.

This contention may be seen as founded on the authority of Averroes, who (Comment 79, Physics VIII) in solution of the same problem, says that an incorporeal power is neither to be called finite nor infinite, because only bodies may be referred to in this way. Furthermore, one incorporeal power may not be referred to as greater than another, since the terms, greater and less, may be applied only to quantities. Moreover, powers separate from body can neither be proportionals nor possess proportional relations, since a proportion can only be between one magnitude and another.

From the above it is seen to follow that no motive power can be either finite or infinite, greater or lesser, or in any way proportional to the power of the thing it moves, because no motive power is a body, but is rather a form (either extensive within the body or separate from it).

This theory, together with Averroes' opinion, may be further confirmed by the definition of proportion, for a proportion is a comparison of two things of the same kind (as is evident on the basis of the definition of proportion set forth in Chapter I). It is obvious, however, that active and passive powers are not of the same kind.

Furthermore, if active and passive powers were to bear a proportion to each other, they would then be comparables. They would, therefore, be of exactly the same species and would consequently, have a subject or substance of exactly the same species. This consequence is false, because powers are divided into active and passive, as is a genus, by incompatible differences. That it nevertheless follows is shown by Aristotle, at the end of Book VII of the Physics, where he expresses

Physicorum, versus finem, ubi vult quod omnia quae debent ad 425
invicem comparari, tam subiectum sive substantia comparationis,
quam illud seu illa in quo vel in quibus est comparatio, erunt
eiusdem speciei individualis et differentiam nullam habentes.

Praeterea, si potentiae motivae esset aliqua proportio ad poten- 430
tiam rei motae, illa esset proportio inaequalitatis maioris quia
deberet excedere potentiam rei motae. Et cum omne excedens aliud
dividitur in excellentiam et in illud quod excellitur (ut patet quarto
Physicorum, capitulo de vacuo) sequitur quod quaelibet potentia
motiva posset dividi isto modo. Quod falsum est, quia omnes poten- 435
tiae motivae incorporeae sunt indivisibiles simpliciter, et aliqua
potentia corporea est minor secundum extensionem quam potentia
rei motae. Nec potest dici quod Aristoteles loquatur ibi tantum de
excellencia proprie (quae tantum in quantitatibus reperitur) quia
loquitur de excessu subtilitatis ad subtilitatem.

Ad idem, primo Rhetoricae Aristotelis, capitulo septimo (ubi 440
determinat de maiori bono et magis conferente) sic scribitur:
"Sit itaque excellens quidem tantum et amplius, excessum autem
quod inexistent." Non igitur dictum Aristotelis tantum de eccellente
proprie est verum.

Ista autem positio poterit reprobari, quia si inter potentias non 445
esset proportio (eo quod non sunt quantitates) eadem ratione nec
inter voces. Et tunc totius musicae modulatio deperiret. Nam
epogdous seu tonus in sesquioctava proportione consistit: diates-
saron autem in sesquitertia: diapente in sesquialtera: diapason
(quae ex diatessaron et diapente componitur) in duplo; diapason 450
cum diapente in tripla: et bis diapason in quadrupla proportione
fundatur (ut ex diversis locis Musicae satis patet).

Praeterea, Averroes, super septimum Physicorum, commento 455
36 et commento 38, probat quasdam conclusiones de proportione
velocitatum in motibus per quasdam geometricas conclusiones (ut
in tertio argumento contra primam opinionem apparet). Et primo
De caelo et mundo, commento 65, probat istam conclusionem in
textu: "Nullum infinitum potest movere finitum" (capiendo ab ad-
versario quod infinitum posset movere finitum in tempore finito,
et quod agens finitum potest eodem tempore movere partem illius 460
passi finiti). Et tunc capit aliquod movens finitum, quod se habet
ad primum movens finitum acceptum sicut totum passum finitum
ad istam partem: et tunc arguit permutatim (scilicet, per duodec-
imam quinti Elementorum Euclidis) quod sic se habet maius
movens finitum ad totum passum sicut minus movens finitum ad 465
istam partem. Ex quo concludit maius movens finitum movere
illud passum totum in aequali tempore quo minus movens finitum
moveret istam partem, et quo agens infinitum etiam moveret illud
totum.

the opinion that all things which are to be compared must be of the same individual species and entirely without difference with respect to the subject or substance of comparison, as well as to that regard or those regards in which the comparison is made.

Further, if there were some proportion of a motive power to the thing it moves, it would have to be one of greater inequality, since it would have to exceed the power of the thing moved. And since everything which exceeds something else is divisible into what exceeds and what is exceeded (as appears from Book IV of the Physics, in the chapter on the "void") it follows that any motive power may be so divided. This is false, for all incorporeal motive powers are fundamentally indivisible, and an embodied motive power is smaller in extension than the power of the thing moved.

Nor can it be claimed that Aristotle is here speaking of excesses only in the strictest sense (the sense in which they are found among quantities), for he is actually referring to the excess of rarity of one medium over another. To the same effect, in Book I, Chapter 7 of Aristotle's Rhetoric, where relative good and relative utility are under discussion, there appears the following: "So let the thing that surpasses be as much as and more than the exceeded thing contained within it." Aristotle's dictum is, therefore, not true merely of the strict usage of "excellence."

This theory is, however, capable of disproof, for if there were no proportion between powers, for the reason that they are not quantities of the same kind, neither could there be such a proportion between musical pitches, and the entire science of harmonics would collapse, accordingly. For the epogdoös or "tone" is constituted in the proportion of nine to eight, the diatessaron in the proportion of four to three, the diapente in the proportion of three to two, the diapason (composed of diatessaron and diapente) in the proportion of two to one, the diapason and diapente combined in the proportion of three to one, and the double diapason in the proportion of four to one. This is sufficiently evident from various passages of the Music.

Furthermore, Averroes, in Comments 36 and 38 on Book VII of the Physics, proves certain theorems concerning the proportion of the speeds of motions by means of geometric theorems, as has already appeared in the third argument against Theory I. In Comment 65 on Book I of the De caelo, he also proves this theorem: It being granted that the infinite can move the finite in a finite time and that a finite agent can, in the same time, move part of this finite resistance, no infinite can move a finite. What he does is take a second finite mover which bears the same relation to the former as the whole finite resistance bears to this part. He then argues permutatively, from Definition xii of Book V of Euclid's Elements, that the proportion of the larger finite mover to the whole resistance is equal to that of the lesser mover to the part. And from this he concludes that the larger finite mover moves the whole resistance in a time equal to that in which the lesser finite mover moves the part and also equal to that in which an infinite mover moves the whole resistance.

Praeterea, secundum istam positionem et secundum veritatem, 470
 potentia motiva dominatur super potentiam rei motae. Et secundum
 Averroem, in multis locis, potentia motiva excedit potentiam rei
 motae, et movens est maioris potentiae quam res mota. Igitur sicut
 dominatur et excedit et est maioris potentiae, sic oportet quod hoc
 sit secundum aliquam proportionem proprie communiterve accept- 475
 am. Aristoteles etiam et Averroes supponunt, multis locis, ali-
 quam esse proportionem potentiae motoris ad potentiam rei motae.

Et sic patet haec affirmativa conclusio: Cuiuslibet potentiae
motivae ad potentiam resistivam rei motae aliqua proportio reper-
itur. 480

Primae autem rationes pro opinione adductae faciliter solvun-
 tur per definitionem proportionis primo capitulo praelibatam. Nam
 inter potentiam motivam et potentiam resistivam proportio non
 proprie dicta, sed communiter reperitur.

Alia autem ratio de comparatione solvitur per distinctionem 485
 comparationis consimilem. Auctoritas autem allegata intelligitur
 de comparatione propriissime et non communiter dicta. Notum
 enim est quod in genere sit comparatio (ut virtuosior, et scientior
 et similibus) et in genere generalissimo. Nam forma est magis
 substantia quam materia, vel compositum ex ambobus; et etiam 490
 in transcendente omne genus, quoniam substantia est magis ens
 quam accidens.

Pro ultima ratione dicendum verum esse potentiae motivae ad
 potentiam rei motae aliquam proportionem et excessum, commun- 495
 iter loquendo, reperiri. Et ad auctoritatem dicentem quod omne
 excellens dividitur in excellentiam et in illud quod excellitur,
 dicendum quod sicut excellens est duplex, ita et dividi in excellen-
 tiam et in illud quod excellitur est duplex (scilicet communiter et
 proprie). Omne igitur excellens proprie dividitur proprie isto
 modo. 500

Excellentes vero communiter sic partitur. Omne enim excellens
 communiter (quantum est de ratione excellentis) potest remitti
 seu minorari ad similitudinem seu aequalitatem illius quod excelli-
 tur. Et sic potest capi tota latitudo qua excellit et similitudo seu
 aequalitas quae in eccellente virtualiter et in potentia continetur. 505

Vel sic: Omne excellens aliud, communiter dividitur in excellen-
 tiam et in illud quod excellitur (verum est non in se, sed in com-
 paratione ad aliud extrinsecum, puta actionem vel passionem seu
 resistantiam). Et sic potentiae moventis et moti et resistantiae
 cuiuslibet possunt omnibus modis secundum excedens et excess- 510
 um adinvicem comparari. Et ideo si capiatur potentia motiva
 aequalis potentiae resistivae, eadem potentia motiva est dupla
 medietatis istius potentiae resistivae (non quia duplum potest
 movere, sed quia duplum istius est praecise tantae virtutis in

Further, according to the present theory and (as a matter of fact) in reality, the power of the mover "dominates" that of the thing moved. In many passages Averroes says that the power of the mover exceeds that of the thing moved, and that the mover is of greater power than the thing moved. If, therefore, it "dominates" and exceeds and is of greater power, this must necessarily take place according to some proportion, whether strictly or generally understood, and both Aristotle and Averroes, in many passages, suppose there to be some proportion of the power of the mover to that of the thing moved.

We thus arrive at this affirmative conclusion: A proportion is found to exist between any motive power and the resistive power of the thing it moves.

The first reasons which were brought forward in favor of the present theory are easily countered by means of the definition of proportion already given in Chapter I, for the proportion found to exist between motive and resistive power is not a strict, but only a general one.

The next argument concerning comparison is overthrown by a similar distinction in the meaning of the word, "comparison." The authority which was cited is to be understood as speaking of strict rather than general comparison. It is to be noted, indeed, that comparison is made: (1) within a genus (as, for example, indicated by the terms: "more virtuous," "wiser," etc.), (2) within the most general genus (for example, "form is substance rather than matter or a compound of the two"), and (3) also in transcendence of every genus (for example, "substance is being rather than accident").

As for the final argument, it is to be replied that it is true that there is, in the general sense, a proportion of excess between motive and resistive power. To the authority cited as saying that everything that exceeds is divided into what exceeds and what is exceeded, it must be replied that, just as "what exceeds" may be taken in two senses, so also "to be divided into what exceeds and what is exceeded" may be taken in two senses (i.e., generally and strictly). Everything that exceeds is, therefore, strictly divisible in this manner.

In the general sense, on the other hand, what exceeds may be divided as follows: In a general sense everything that exceeds may be reduced or lessened until equal to what was exceeded; and thus may be understood the entire latitude whereby it exceeds and likewise the similiarity or equality which is contained virtually and potentially in the thing that exceeds.

Or it may be carried out thus: Everything exceeding something else is divided, in the general sense, into excess and what is exceeded, not, of course, in itself, but in comparison to something else outside itself (for example, action, and passion or resistance). In this sense, the powers of mover, moved and resistance can be compared to each other in every way in terms of excess and what exceeds. And if one were to take the example of a motive power equal to a resistive power, that motive power is twice half of the resistive power, not because it can produce twice the motion, but because twice the halved resistance has precisely the power of resistance that the motive power has of moving.

resistendo sicut ista potentia motiva in movendo). Et de omni 515
alia proportione motoris ad motum proportionaliter est ponendum.

Haec igitur opinio, una cum prioribus, erronea dicitur.

CAPITULUM TERTIUM

His igitur ignorantiae nebulis demonstrationum flatibus effuga- 1
tis, superest ut lumen scientiae resplendeat veritatis. Scientia
autem veritatis ponit quintam opinionem, dicentem quod proportio
velocitatum in motibus sequitur proportionem potentiae motoris
ad potentiam rei motae. 5

Et hoc est quod vult Averroes, super quarto Physicorum, com-
mento 71, sic dicens: "Universaliter manifestum est quod causa
diversitatis et aequalitatis motuum est aequalitas et diversitas
proportionis motoris ad rem motam. Cum igitur fuerint duo mo-
tores et duo mota, et proportio alterius motoris ad alterum motum 10
fuerit sicut proportio reliqui motoris ad reliquum motum, tunc
duo motus erunt aequales in velocitate; et cum diversatur propor-
tio, diversabitur motus secundum istam proportionem."

Et infra, in eodem: "Diversitas motuum in velocitate et tardi-
tate est secundum hanc proportionem quae est inter duas poten- 15
tias" (scilicet motivas et resistivas).

Et secundo De caelo, commento 36, sic dicit: "Velocitas enim
et tarditas non fit nisi secundum proportionem potentiae motoris
ad potentiam rei motae; quanto igitur fuerit proportio maior, tanto
magis motus erit velocior; et quanto proportio minor, tanto motus 20
erit tardior."

Et septimo Physicorum, commento 35, ex duplicatione propor-
tionis potentiae motoris ad motum, arguit duplicationem velocitatis
in motu, sic dicens: "Cum dividerimus motum contingit necessario
ut proportio potentiae motoris ad motum sit dupla istius propor- 25
tionis, et sic velocitas dupla ad istam velocitatem."

Et post, commento ultimo, dicit: "Haec duo," scilicet velocitas
alterationis et quantitas temporis, "sequuntur proportionem inter
alterans et alteratum. Si igitur proportio fuerit magna, velocitas
erit magna et tempus breve, et econtrario" 30

Ad idem Aristoteles et Averroes (ut patet tertio argumento
contra primam opinionem) volunt multis locis quod aequalitatem
proportionis motoris ad motum sequitur aequalis velocitas motuum.
Aequalitas igitur proportionis motorum ad mota est causa, qua
primo posita, ponitur primo aequalis velocitas motuum, et qua 35
primo remota, primo removetur. Ergo aequalitas proportionis
motorum ad mota est prima et praecisa causa aequalitatis veloci-

Concerning every other proportion of mover to moved this is proportionally true.

This theory, therefore, together with the former ones, is pronounced false.

CHAPTER THREE

Now that these fogs of ignorance, these winds of demonstration, have been put to flight, it remains for the light of knowledge and of truth to shine forth. For true knowledge proposes a fifth theory which states that the proportion of the speeds of motions varies in accordance with the proportion of the power of the mover to the power of the thing moved.

This is what Averroes intends when he says, in Comment 71 on Book IV of the Physics: "It is manifest that, universally, the cause of the diversity and equality of motion is the equality and diversity of the proportion of mover to thing moved. If, therefore, there are two movers and two things moved, and the proportions between these movers and the things which they respectively move are equal, then the two motions are of equal speed. If the proportion is varied, the motion is also varied in that proportion"

Further on in the same comment, he also says: "The difference between motions with respect to slowness and fastness varies in accordance with the proportion between the two powers (namely, motive and resistive)"

In Comment 36, on Book II of the De caelo, he says: "Fastness and slowness do not occur otherwise than in accordance with the proportion of the power of the mover to that of the thing moved. By however much, therefore, the proportion is greater, by so much will the motion be faster; and by however much the proportion is less, by so much will the motion be slower."

In Comment 35, on Book VII of the Physics, from a doubling of the proportion of mover to moved he argues a doubling of the speed of the motion, as follows: "If we divide the mobile in two, it necessarily comes about that the proportion of the power of the mover to the thing moved becomes double the former proportion, and thus the speed will be twice what it was before."

Further on, in the final comment, he remarks that: "These two (that is, the speed of alteration and the quantity of time) vary in accordance with the proportion between that which causes the alteration and that which undergoes it. If, therefore, the proportion is great, the speed will be great and the time short, and conversely"

Concerning this same problem, both Aristotle and Averroes (as is evident in the third argument against Theory I) express, in many passages, the opinion that, from an equality of proportion between mover and moved, there follows equality of speed. Equality of the proportion of movers to mobilia is, therefore, the causal condition which, when fulfilled, posits an equal speed of motions and which, when not fulfilled,

tatem in motibus; igitur ad variationem istius causae primo sequitur variatio proportionis in motibus.

Praeterea, non videtur aliqua positio qua potest rationabiliter 40
salvari proportio velocitatum in motibus nisi aliqua iam dictarum.
Sed quattuor primae sunt destructae; tantum igitur remanet quinta
vera.

Sic igitur patet ista conclusio: Proportio velocitatum in motibus
sequitur proportionem potentiaram moventium ad potentias resis- 45
tivas, et etiam econtrario. Vel sic sub aliis verbis, eadem sententia
remanente: Proportiones potentiaram moventium ad potentias re-
sistivas, et velocitates in motibus, eodem ordine proportionales
existunt, et similiter econtrario. Et hoc de geometrica proportion-
alitate intelligas. 50

Secunda conclusio: Si potentiae moventis ad potentiam sui moti
sit dupla proportio, potentia motiva geminata movebit idem motum
praecise in duplo velocius.

Hanc ostensive demonstras. Sit enim A potentia motiva dupla
ipsius B potentiae resistivae, et sit C potentia motiva dupla 55
ipsius A. Tunc (per primam conclusionem primi capituli) proportio
C ad B est praecise dupla ad proportionem A ad B. Igitur (per
proximam) C movebit B praecise in duplo velocius quam A moveat
B. Et hoc est propositum.

Tertia conclusio: Si potentiae moventis ad potentiam sui moti 60
sit dupla proportio, eadem potentia movebit medietatem eiusdem
moti velocitate praecise duplata. Hanc, ut proximam, rationaliter
demonstrabis.

Quarta conclusio: Si potentiae moventis ad potentiam sui moti
sit maior quam dupla proportio, potentia motiva geminata motus 65
eiusdem duplam velocitatem nequaquam attinget. Hoc per quartam
primi capituli et per primam tertii concluditur ostensive.

Quinta conclusio: Si potentiae moventis ad potentiam sui moti
sit maior quam dupla proportio, eadem potentia movente medie-
tatem eiusdem moti, velocitas motus nullatenus fiet dupla. Hoc 70
per tertiam primi et primam tertii concludetur ostensive.

Sexta conclusio: Si potentiae moventis ad potentiam sui moti
sit minor quam dupla proportio, dupla potentia movente idem
motum, motus ultra duplum velocitatem excrescet. Istam per sex-
tam primi et primam tertii ostensione faciliter patebit. 75

Septima conclusio: Si fuerit potentiae moventis ad potentiam
sui moti minor quam dupla proportio, eadem potentia movente
medietatem eiusdem moti, motus ultra duplam velocitatem trans-
ibit. Hoc per quintam primi et primam tertii ostensive patebit.

makes impossible an equality of speeds. Equality of the proportion of movers to mobilia is, thus, the primary and precise cause of equality of the speeds of motions, and to the variation of this cause there directly corresponds the variation of proportion between different motions.

Furthermore, there does not seem to be any theory whereby the proportion of the speeds of motions may be rationally defended, unless it is one of those already mentioned. Since, however, the first four have been discredited, therefore the fifth must be the true one.

We, therefore, arrive at the following theorem:

Theorem I. The proportion of the speeds of motions varies in accordance with the proportion of motive to resistive forces, and conversely. Or, to put it in another way, which means the same thing: The proportion of the proportions of motive to resistive powers is equal to the proportion of their respective speeds of motion, and conversely. This is to be understood in the sense of geometric proportionality.

Theorem II. If the proportion of the power of the mover to that of its mobile is that of two to one, double the motive power will move the same mobile exactly twice as fast. This may be demonstrated by means of an example. Let A be a motive power that is twice B (its resistance), and let C be a motive power that is twice A. Then, (by Theorem I, Chapter I) the proportion of C to B is exactly double that of A to B. Therefore, (by the immediately preceding theorem) C will move B exactly twice as fast as A does. This is what was to be proved.

Theorem III. If the proportion of the power of the mover to that of its mobile is two to one, the same power will move half the mobile with exactly twice the speed. This you may demonstrate by an argument like that used for Theorem II.

Theorem IV. If the proportion of the power of the mover to that of its mobile is greater than two to one, when the motive power is doubled the motion will never attain twice the speed. This may be demonstrated by means of Theorem IV, Chapter I; and Theorem I, Chapter III.

Theorem V. If the proportion of the power of the mover to that of its mobile is less than two to one, when the resistance of the mobile is halved the motion will never attain twice the speed. This may be demonstrated by means of Theorem III, Chapter I and Theorem I, Chapter III.

Theorem VI. If the proportion of the power of the mover to that of its mobile is less than two to one, when the power moving this mobile is doubled it will increase the speed to more than twice what it was. This is likewise easily demonstrable, from Theorem VI, Chapter I and Theorem I, Chapter III.

Theorem VII. If the proportion of the power of the mover to that of its mobile is less than two to one, when the same mover moves half that mobile the speed of the motion will be more than doubled. This may be demonstrated clearly, from Theorem V, Chapter I and Theorem I, Chapter III.

Ex nulla proportione aequalitatis vel minoris inaequalitatis motoris ad motum sequitur ullus motus. 80

Hanc per primam tertii, et septimam et octavam primi, demonstrative concludes (adiuncta hac suppositione, per se nota: "Omnis motus eiusdem speciei secundum velox vel tardum possunt adinvicem comparari"). 85

Omnis motus ex proportione maioris inaequalitatis producitur, et ex omni proportione maioris inaequalitatis fieri potest motus.

Prima pars huius per primam et octavam tertii (suppositione proxima coadiuncta) patebit. Secunda pars huius apparet eo quod omnis excessus motoris ad motum sufficit producendum (ut erit alibi demonstratum). 90

Quocumque motu dato, potest motus in duplo velocior et motus in duplo tardior inveniri.

Hoc per primam tertii et secundam partem nonae eiusdem concluditur (auxiliante ista suppositione, per se nota: "Proportio maioris inaequalitatis motoris ad motum in additione et diminutione duplari poterit in infinitum"). 95

Quantumcumque gravius alio in eodem medio tardius et velocius illo et aequali velocitate potest descendere.

Sit enim A grave mixtum ex gravi et levi quantumlibet habens pondus, et B grave simplex ita parvi ponderis ut desideras. Et subtilietur aliquod medium donec B habeat ad illud aequalem proportionem proportioni gravitatis A ad levitatem in eo, vel maiorem; et ponantur ambo in illo medio. Tunc gravitas A habet minorem proportionem ad totam suam resistentiam intrinsecam et extrinsecam quam B habeat ad suam resistentiam. Igitur (per primam tertii) A tardius movetur quam B. 100 105

Iterum econtra condensetur medium donec proportio B ad illud sit minor proportione gravitatis A ad totam suam resistentiam intrinsecam et extrinsecam. Tunc (per primam huius) A movetur velocius quam B. 110

Tertio, adaptetur sic medium quod proportio B ad illud sit aequalis proportioni gravitatis A ad suam resistentiam intrinsecam et extrinsecam. Tunc (per primam huius) A et B aequavelociter movebuntur. 115

Vel sic: A haberet ex se in vacuo motum certae velocitatis, quae sit C. Et subtilietur aliquod medium donec B descendat in illo velocitate C, vel maiori. Tunc A, positum in eodem medio, tardius B descendet. Rursus condensetur medium sicut oportet, et reliqua duo provenient. 120

Unde manifestum est quod cuiuslibet gravis simplicis quaelibet velocitas et tarditas, et cuiuslibet gravis mixti quaelibet tarditas, per subtiliationem et condensationem medii, in infinitum poterit duplicari; necnon quod cuiuslibet gravis mixti quaelibet velocitas per subtiliationem medii geminari non potest. (Istud correlarium ex praedictis sufficienter constabit.) 125

Theorem VIII. No motion follows from either a proportion of equality of one of lesser inequality, between mover and moved. With the addition of the following axiom, independently known:

Axiom 1. All motions of the same species may be compared to each other with regard to slowness and fastness; this theorem may be proved by means of Theorems VII and VIII of Chapter I and Theorem I, Chapter III.

Theorem IX. Every motion is produced by a proportion of a greater inequality, and from every proportion of greater inequality a motion may arise. The first part of this may be proved by Theorems I and VIII of Chapter III and the axiom just given. The second part is demonstrable from the fact that every excess of mover over moved suffices to produce motion, as will be shown elsewhere.

Theorem X. Given any motion, one twice as fast and one twice as slow can be determined. This may be proved by Theorem I and Theorem IX (Part 2) of Chapter III, with the help of the following axiom, independently known:

Axiom 2. A proportion of greater inequality of mover to moved may be halved or doubled indefinitely.

Theorem XI. An object may fall in the same medium both faster, slower, and equally with some other object that is lighter than itself.

Let, for example, A represent a heavy mixed body composed of heavy and light and having a certain weight, and let B represent some pure heavy body, as small as you please. Now let a given medium be rarified to the point at which B bears to it a proportion equal to, or greater than, that of the heaviness to the lightness in A. Then let both bodies be placed in the same medium. The heaviness of A will now be in a lesser proportion to its total intrinsic and extrinsic resistance than B is to its resistance. Therefore, by Theorem I, Chapter III, A moves more slowly than B.

Conversely, let the medium be condensed to the point at which the proportion of B to it is less than the proportion of the heaviness of A to its entire intrinsic and extrinsic resistance. Then, by Theorem I of this chapter, A moves faster than B.

Thirdly, let the medium be so determined that the proportion of B to it is equal to the proportion of the heaviness of A to its entire intrinsic and extrinsic resistance. Then, by Theorem I of the present chapter, A and B will move at equal speeds.

Alternately, let A be supposed to have a determinate speed, C, in a vacuum, and let some medium be rarified until B falls in it with speed C or faster; then A, placed in the same medium, will fall more slowly than B. Conversely, let the medium now be condensed as required, and the remaining two consequences will follow.

Corollary 1. It is manifest, from the foregoing, that the fastness and the slowness of any pure body and the slowness of any mixed body may be doubled indefinitely, but that the fastness of a mixed body may not be so doubled by rarefaction of the medium. This corollary is sufficiently well established on the basis of what has been said above.

Omnia mixta compositionis consimilis aequali velocitate in vacuo movebuntur. Nam in omnibus talibus motores sunt proportionales suis resistentiis. Igitur (per primam huius) omnia talia aequivelociter movebuntur. 130

Ex hoc quoque scies quod, si duo gravia mixta inaequalia, compositionis consimilis, in aequilibri in vacuo suspendantur, gravius declinabit. Sint enim A et B duo talia gravia, A maius, B vero minus, et sit C gravitas A, D vero levitas similiter eiusdem. E autem sit gravitas B, F vero levitas sit eiusdem. Tunc C, D, E, F 135 sunt quattuor proportionalia, et C est maximum, F vero minimum. Igitur (per octavam suppositionem primi huius) C et F pariter congregata excedunt D et E pariter adiuncta. Et C et F nituntur elevare B, et tantum D et E resistunt. Igitur (per secundam partem nonae conclusionis huius) B ascendet, A vero descendet. 140

SECUNDA PARS CAPITULI TERTII

Nunc, in secunda parte huius capituli, superest quaedam praedictorum disputatione rimari.

Primo, contra istam positionem et contra huius primam conclusionem sunt omnia ista quae pro alia opinionum praedictarum sunt adducta; de quibus est hic silendum, quia de eis prius sufficienter est dictum. 145

Secundo, ex ista positione videntur sequi tria inconvenientia: videlicet, quod ex aequali proportione motorum ad mobilia quandoque sequitur inaequalitas velocitatum in motibus, et quod ex minori proportione unius motoris ad suum mobile quam alterius ad suum quandoque sequitur aequalis velocitas in motibus, et maior quandoque. 150

Sint enim A et B duae terrae purae inaequales, A maior, B minor; et sint C et D duo aeres inaequales uniformes proportionales A et B, et C maior, D minor. Et A, per suum descensum, dividit C, B vero D. Vel ergo istae divisiones sunt aequales secundum tempus, vel non. Si sic, igitur A dividit et movetur velocius B; pertransit enim maius spatium in aequali tempore. Et sic patet primum. Iterum, manente proportione B ad D, minoretur proportio A ad C donec movetur aequivelociter cum B: et tunc patet secundum. Rursus, maioretur etiam econtra modicum proportio A ad C, non tamen ad aequalitatem proportionis B ad D. Tunc A dividit et movetur velocius quam in secundo casu. Igitur velocius quam B; et sic sequitur tertium. 160

Ideo dicendum quod, stante primo casu, illae divisiones totales non sunt aequales secundum tempus, sed A et B (propter aequali- 165

Theorem XII. All mixed bodies of similar composition will move at equal speeds in a vacuum. In all such cases the moving powers bear the same proportion to their resistances. Therefore, by Theorem I of this chapter, all such bodies move at the same speed. From this you must also understand that:

Corollary 2: If two heavy mixed bodies of unequal weight, but similar composition, were balanced on a scale within a vacuum, the heavier would descend.

Let A and B represent two such heavy bodies, A greater and B less; let C and D represent the heaviness and lightness of A, respectively, and let E and F represent the heaviness and lightness of B, respectively. Then C, D, E and F are four proportionals, C being the greatest and F the smallest. Therefore (by Theorem VIII, Chapter I) C and F, combined, exceed D and E, combined. Since C and F tend to raise B and only D and E resist, therefore (by Part ii of Theorem IX of the present chapter) B ascends and A descends.

CHAPTER THREE, PART TWO

Now, in the second half of this chapter, it remains to explore further, by disputation, some of the things which have just been set forth.

In the first place, against the present theory, and especially against Theorem I, may be alleged everything that was cited in favor of the preceding theories. Since enough has already been said about these objections, we will omit further discussion of them here.

Secondly, it may be objected that three inconsistencies follow from the present theory: namely, that (1) from equal proportions of movers to mobilia there may, on occasion, result unequal speeds, that (2) when the proportion of one mover to its mobile is less than that of another mover to its mobile, the speeds may, on occasion, be equal, and that (3) this may also happen when the proportion is greater.

For example, let A and B represent two unequal quantities of pure earth, A being the larger and B the smaller, and let C and D represent two unequal but uniform quantities of air equally proportional to A and B, C being the greater and D the lesser. In its fall A divides C and B divides D.

Now, these descents either occupy equal times, or they do not. If they do, then A falls faster than B, since it traverses a greater distance in an equal time. This is the first inconsistency.

On the other hand, with the proportion of B to D remaining constant, let the proportion of A to C be diminished until A moves at the same speed as B. Here is the second inconsistency.

In the other direction, let the proportion of A to C be increased a little, but not enough to become equal to the proportion of B to D. Now A falls faster than in the second case, and therefore faster than B. Here is the third inconsistency.

It should be pointed out in reply that while, in the first case, the total descents are not accomplished in equal times, A and B do nevertheless

THOMAS OF BRADWARDINE HIS TRAC-
TATUS DE PROPORTIONIBUS

Published by the University of Wisconsin Press
430 Sterling Court, Madison 6, Wisconsin

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of Wisconsin

Second printing, 1961

Printed in the United States of America
by Cushing-Malloy, Inc., Ann Arbor, Michigan

Library of Congress Catalog Card Number 54-6740