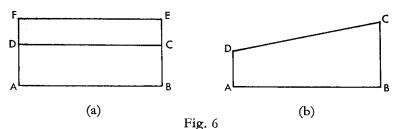
Therefore, it will be proportional to any rectangle constructed on AB, because all such rectangles are of proportional, although unequal, altitude. Therefore, by chapter seven, this quality is imaginable by rectangle ABCD and similarly by rectangle ABEF which is greater and also by one that is less. Moreover, any such quality is said to be "uniform" or "of equal intensity" in all of its parts.



Figures in MSS BLSCG. Letters C and D in figure (a) are interchanged in MS L.

Again it ought to be known that some quality is imaginable by a quadrangle having two right angles on the base and the other two angles unequal, e.g., by quadrangle ABCD [see Fig. 6(b)] and by every quadrangle constructed on base AB which is of proportional altitude, whether it be greater or less, as is clear in chapter seven. Moreover, any such quality is spoken of as "uniformly difform terminated in both extremes at some degree," so that the more intense extreme is designated in the acute angle C and the more remiss in the obtuse angle D. The superior line, e.g., line CD, is called "the line of summit," or in relation to quality it can be called "the line of intensity" because the intensity varies according to its variation

#### I.xi On uniform and difform quality

And so every uniform quality is imagined by a rectangle and every quality uniformly difform terminated at no degree is imaginable by a right triangle. Further, every quality uniformly difform terminated in both extremes at some degree is to be imagined by a quadrangle having right angles on its base and the other two angles unequal. Now every other linear quality is said to be "difformly difform" and is imaginable by means of figures otherwise disposed according to manifold variation. Some modes of the "difformly difform" will be examined later. The

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lum 11<sup>m</sup> [N] 11 mg. [SA] / Capitulum 11<sup>m</sup> tr. P

2 itaque: igitur P[A]

2-3 rectangulum om. L

4 est om. F[MAC] / Omnis vero: et omnis F[M] omnis ergo [C]

4-5 uniformiter difformis om. L[N]

5 utrinque ad BF tr. [N] utrique ad P[G]
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- utrimque ad [SE] utriusque ad [A] utrumque ad [C] utramque ad V ad utrumque L[M] / gradum: non gradum B
- 7 qualitas linearis: linealis qualitas P linearis qualitas [E] lignealis qualitas [A] / ymaginabilis: ymaginanda LP[EANC]
- 8 variationem: varietatem P

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videbuntur. Predicte vero differentie intensionum non melius nec clarius neque facilius notificari possunt quam per tales ymaginationes et relationes ad figuras, quamvis quedam alie descriptiones seu notificationes possint dari que etiam per huiusmodi figurarum ymaginationes fiunt note: Ut si diceretur qualitas uniformis est que in omnibus partibus subiecti equaliter est intensa. Qualitas vero uniformiter difformis est cuius omnium trium punctorum proportio distantie inter primum et 2<sup>m</sup> ad distantiam inter 2<sup>m</sup> et 3<sup>m</sup> est sicut proportio excessus primi supra 2<sup>m</sup> ad excessum 2<sup>1</sup> supra 3<sup>m</sup> in intensione, ita quod punctum intensiorem illorum trium voco primum.

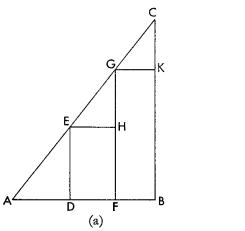
Istud primo declaratur de ea qualitate uniformiter difformi que terminatur ad non gradum que signetur seu ymaginetur per triangulum ABC [Fig. 7(a)]. Erectis itaque tribus perpendicularibus lineis BC, FG et DE, protrahatur HE equedistans linee DF et similiter GK equedistans linee FB. Fient ergo duo parvi trianguli CKG et GHE qui sunt equianguli; ergo per  $4^{am}$   $6^{i}$  Euclidis proportio GK ad EH est sicut proportio CK excessus ad GH excessum. Et quoniam GK est equalis FB et similiter EH est equalis DF, erit proportio FB ad DF, que quidem linee sunt distantie trium punctorum ipsius basis, sicut proportio CK ad GH, qui sunt excessus altitudinis proportionalis intensioni eorundum punctorum. Cum igitur qualitas linee AB sit talis quod proportio punctorum linee in intensione est sicut proportio linearum in altitudine super eadem puncta perpendiculariter erectarum, patet evidenter propositum, scilicet quod que est proportio excessus primi puncti supra secundum ad excessum secundi supra tertium in intensione, eadem est proportio distantie inter primum punctum et secundum ad distan-

- 11 ad: et P / possint dari BV[LSG] dari possunt P possent dari F[M] possunt dari [EANC]
- 12 que: qui F / ymaginationes: notificationes F[M]
- 13 est<sup>1</sup> om. F[M] autem [N] / equaliter est tr. LP[AC] inequaliter [N] est qualiter [G]
- 14 vero om. F[M]
- 15 est: est proportio P[EA]
- 16 supra¹ BFL[SMCG] super P[EAN] ad V / supra²: super FP[ENG]
- 18 primo BV[SG] postea F[M] ergo L[A] igitur P[EN] vero [C] / declaratur: de<sup>2</sup> B
- 19 signetur om. [EG] significetur F[M] / seu om. [EG] vel B

- 20 Erectis om. [N] erectum LP[A] / BC: BC et BV BE P AB et [G]
- 21 HE: EH B[SCG] / GK: KG P[E] / equedistans linee: equedistanter LP[EN]
- 22 Fiunt L[N] / parvi om. PL[EN] / CKG et GHE: GHE et CKG L[N] scilicet GHE et CKG P[EA] / qui: que P / eque anguli FP[MA]
- 23 6 Euclidis om. P[A] 6 L[EN]
- 24 excessum om. V
- 24-25 et2...FB om. B[A]
- 26 que V
- 26–27 proportionalis intensioni om. L[C]
- 27 intensioni BV intensionum F[SMNG] intensionis P[EA]
- 31 supra<sup>1</sup>: super V ad [N] / supra<sup>2</sup>: super FP[EN] / in intensione om. L[C]
- 32 primum punctum tr. P[E] punctum [A]

aforesaid differences of intensities cannot be known any better, more clearly, or more easily than by such mental images and relations to figures, although certain other descriptions or points of knowledge could be given which also become known by imagining figures of this sort: as if it were said that a uniform quality is one which is equally intense in all parts of the subject, while a quality uniformly difform is one in which if any three points [of the subject line] are taken, the ratio of the distance between the first and the second to the distance between the second and the third is as the ratio of the excess in intensity of the first point over that of the second point to the excess of that of the second point over that of the third point, calling the first of those three points the one of greatest intensity.

Let us clarify this first with respect to a quality uniformly difform which is ter-



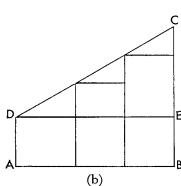


Fig. 7

Figure (a) in MSS BLEDGSC, with letters B and C interchanged in MS B. Figure (a) is reversed in orientation in MS E. Figure (b) in MSS BLDCSG.

minated at no degree and which is designated or imagined by  $\triangle ABC$  [see Fig. 7 (a)]. With the three perpendicular lines BC, FG, and DE erected, then let HE be drawn parallel to line DF and similarly GK parallel to line FB. Therefore, the two small triangles CKG and GHE are formed and they are equiangular. Hence, by [proposition] VI.4 of [the Elements of] Euclid<sup>1</sup>, GK/EH = CK/GH, CK and GH being excesses. And since GK = FB and similarly EH = DF, so FB/DF = CK/GH, FB and DF being the distances on the base of the three points and CK and CH being the excesses of altitude proportional to the intensity of these same points. Since, therefore, the quality of line AB is such that the ratio of the intensities of the points of the line is as the ratio of the altitudes of the lines perpendicularly erected on those same points, that which has been proposed is evidently clear, namely that the ratio of the excess in intensity of the first point over the second to the excess of the second over the third is the same as the ratio of the distance between the first and second points to the distance between the second and the third, and similarly for

I.xi

<sup>&</sup>lt;sup>1</sup> See the Commentary, I.viii, lines 25, 27.

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tiam inter secundum et tertium, et ita de quibuscunque tribus aliis punctis. Igitur qualitati sic difformi recte convenit quod premittebatur, et ita per talem triangulum bene designabatur.

Per eundem modum predicta descriptio sive proprietas potest ostendi de qualitate uniformiter difformi terminata utrobique ad gradum, et sic una que ymaginetur per quadrangulum ABCD in quo protrahatur linea DE equedistans basi AB et fiet triangulus DEC [Fig. 7(b)]; deinde protrahantur linee altitudinis in quadrangulo et alie transversales equedistantes basi in isto triangulo faciendo parvos triangulos. Et tunc faciliter poterit argui de illis excessibus et distantiis in isto triangulo sicut superius arguebatur in alio, prout intuenti potest leviter apparere.

Omnis autem qualitas se habens alio modo a predictis dicitur difformiter difformis et potest describi negative, scilicet qualitas que non est in omnibus partibus subiecti equaliter intensa nec omnium trium punctorum ipsius proportio excessus primi supra secundum ad excessum secundi supra tertium est sicut proportio distantiarum eorum.

#### [I.xii] Capitulum 12<sup>m</sup> de eisdem aliter

Rursum in notitiam premissarum differentiarum possumus duci ex ymaginatione motus, ymaginetur enim punctus d regulariter moveri super lineam AB et sit ita quod quicunque punctus linea AB super quem venerit punctus d sit eidem puncto d similis et equalis in intensione [Fig. 8]. Si igitur in principio motus ipse punctus d habeat aliquem gradum vel aliquam intensionem et continue sine sui alteratione maneat in eodem gradu durante illo motu, tunc describet in linea AB qualitatem uniformem. Si vero in principio motus punctus d nichil habeat illius qualitatis et durante motu. ipse punctus d continue alteretur et regulariter intendatur, tunc describet qualitatem uniformiter difformem terminatam ad non gradum. Si autem d

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33 tribus aliis punctis BV[SG] aliis tribus
  punctis FL[EMC] punctis tribus P[AN]
34 sic: sicut (?) F / ita FL[EMANC] illud
  [S] illa BV[?G] ita recte P
35 bene om. P recte [EA]
36 sive: seu F[A] vel [M]
37 sic om. [C] sit [AES]
38 ymaginetur BVL[GC] ymaginatur FP
   [ANMES]
39 fiat P[C] fiant [S] / protrahatur V[SEC] /
  linea [ES]
                                              1 Capitulum...aliter om. [AMS] capitulum
40-41 et...triangulos om. F
41-42 Et1...triangulo om. L[C]
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43 potest leviter tr. P[A] leviter poterit [M]
   patent \lceil G \rceil
44 dicatur LP[EAD]
45 negative scilicet om. F[M] negative [S] /
   non: nec F/M
46 omnium om. F[M]
47 primi om. P / supra <sup>1,2</sup>: super P[EAN]
48 distantie P[EA]
I.xii: BVFLP
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12<sup>m</sup> [N] 12 mg. [SA] / Capitulum 12<sup>m</sup>

tr.F[G] post aliter / de...aliter om. P / eis-

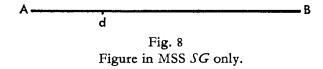
any other three points. Hence what we have premised in regard to a quality difform in this way is quite fitting, and so it (this quality) was well designated by such a triangle.

By the same method the aforesaid description or property can be demonstrated for a quality uniformly difform terminated in both extremes at [some] degree, and thus for one which we let be imagined by quadrangle ABCD in which line DE is drawn parallel to base AB forming  $\Delta DEC$  [see Fig. 7(b)]. Then let lines of altitude be drawn in the quadrangle and also transversals parallel to the base in this triangle, thus forming small triangles. And then one can easily argue concerning the excesses and the distances in this triangle just as was argued in the other one. This will be easily apparent to one who is observant.

Further, every quality which is disposed in [any] other way than those described earlier is said to be "difformly difform." It can be described negatively as a quality which is not equally intense in all parts of the subject nor in which, when any three points of it are taken, the ratio of the excess of the first over the second to the excess of the second over the third is equal to the ratio of their distances.

#### I.xii On these same [qualities considered] in another way

Again, we can be led to a knowledge of the differences which have been premised by the imagery of motion. For let point d be imagined as moving regularly on line AB and in such a way that any point of line AB over which d comes will be equal and similar in intensity to that same point d [see Fig. 8]. If, therefore, in the beginning of the motion the point d has a certain degree or some intensity and it con-



tinually remains in that same degree without alteration throughout the motion, then it will describe in line  $\mathcal{A}B$  a uniform quality. But if in the beginning of the motion point d has none of the quality and during the motion point d is continually altered and regularly increased in intensity, then it will describe a quality uniformly difform terminated at no degree. If, moreover, d is regularly increased in intensity,

dem: eodem L

2 Rursum corr. B ex Sursum (et sursum habet

A) / possimus L

5 sit...d om. FP[A] / Si: Tunc F[M]

8 describet: describeret P[A] describeretur

[C]

9-10 nichil...d om. V

10 ipse: tunc ipse L / punctus d tr. P punctus

L / regulariter: continue [SG]

10-12 tunc....intendatur om. V

11-13 ad...terminatam om. B

and if it is joined to the base in neither extreme the quality or difformity is terminated in both extremes at [some] degree. And since such a line cannot be joined to the base in both extremes—for it is a straight line and thus would form a single line with the base which also is a straight line—it is clear that there cannot be a quality uniformly difform terminated in both extremes at no degree. Further, if the line of intensity or summit line is a curve or is composed of several lines rather than one, then the quality imaginable by that figure will be difformly difform, and it can be that it is terminated in both extremes at some degree, or in both extremes at no degree, or at some degree in one extreme and at no degree in the other.

#### I.xiv On simple difform difformity

We now treat of difform difformity; there are two modes of such difformity: simple and composite. We must first talk of the simple mode. Simple difform difformity is that which can be designated by a figure whose line of summit or line of intensity is a single line, i.e. not composed of several lines. It is necessary, therefore, that the line be a curve; because if it were straight, then it would be simply a uniformity or uniform difformity, as is clear from the preceding chapter. Furthermore, it is necessary that the curvature of the summit line does not attain that of a circular segment greater than a semicircle so that the angle<sup>1</sup> on the base is greater than a right angle, as was clear in chapter five. However, it can happen that the angle on the base is less than a right angle by any amount you please.

Therefore, for example, let there be line AB, whose quality can be designated by semicircle ACB [see Fig. 10]. This is possible, as is evident from chapter seven. And so I now say that the same quality of line AB is imaginable or can be designated by a figure having an altitude greater or less than that of the semicircle by any amount you please.<sup>3</sup> For let line CD be drawn as a perpendicular to center D and

I.xiv

<sup>&</sup>lt;sup>1</sup> In this case the angle would be a mixed angle composed of the curve and the straight base line.

<sup>&</sup>lt;sup>2</sup> All the manuscripts have "chapter 4," but this is a clear reference to the penultimate sentence in I.v.

<sup>&</sup>lt;sup>3</sup> See the Commentary, I.xiv, lines 14-54.

<sup>5</sup> sive: seu BF[EMC] / pluribus: partibus L partibus pluribus [N]

<sup>6</sup> quia: que P[EA] / foret: esset linea F[M] foret linea [E] esset [C] / esset: foret P[E] forent [A]

<sup>9</sup> ut¹ BVF[ACGS] quod LP[EMN] / patet L / ex om. P[A] / 4º capitulo BVF[MS] tr. LP[EANGC]

<sup>10</sup> etiam: et P[G]

<sup>13</sup> ACB BV[FM] ABC PL[AENGCS] /
patuit V / 7º capitulo tr. P[E] 4º capitulo
F[M] capitulo alio [A]

<sup>14</sup> linea B

<sup>15</sup> maioris om. L / ac: aut P[A] / ac etiam om. L

<sup>16</sup> supra LP[M]

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et iterum protrahatur una alia linea perpendicularis, que sit EF, super lineam AB. Cum igitur sit possibile duas lineas minores istis duabus super eadem puncta perpendiculariter stare se habentes invicem in eadem proportione sicut et iste due, que sunt CD et EF, et conformiter possint fieri linee maiores aut minores super omnia puncta linee AB stante semper eadem proportione inter eas que est inter lineas perpendiculares super AB in semicirculo ACB, sequitur quod super AB basim poterit erigi figura minus al.a et tamen erit proportionalis altitudinis huic semicirculo ACB et pari ratione magis alta etiam quantumlibet. Igitur per capitulum  $7^m$  per quamlibet istarum figurarum potest qualitas linee AB recte ymaginari indifferenter.

Unde et nisi ita esset quod qualitas linee  $\mathcal{A}B$  ymaginabilis per semicirculum posset ymaginari per figuram maiorem aut minorem et alteri proportionari, sequeretur quod intensio puncti D non posset recte designari per maiorem vel minorem lineam quam sit linea DC et sic de aliis punctis nisi intensio variaretur et ita quelibet intensio determinaret sibi lineam certe quantitatis per quam esset ymaginabilis et tunc intensio esset equalis et comparabilis linee vel extensioni in quantitate et per consequens motus localis comparabilis alterationi in velocitate, que omnia videntur nimis absurda.

Quelibet tamen figura per quam est ymaginabilis ista qualitas linee AB est curva. Utrum autem figura minor quam semicirculus per quam ista qualitas potest ymaginari sit portio circuli discutiendum relinquo. Sed dico

- 17 una alia linea P[AESC] om. L una alia B[NG] una linea V linea alia F[M] / que...EF BVF[AMSCG] que sit linea FE L[MEN] et tr. L[N] post AB / supra I
- 19 perpendiculariter stare tr. F[M]
- 20 iste: ille F[M] / et<sup>3</sup> BVF[MN] vel LP[EC] ut [SG] aut [A] / possint BVL[G] possunt PF[EMANSC]
- 21-22 stante...AB om. F[M]
- 22 lineas om. P[A] / perpendiculares: proportionales B perpendiculariter erectas [E] / AB: AB erectas P[A]
- 23 ACB LP[EN] om. BVF[MSCG] ABC [A] | post ACB add. L[N] erectas
- 24 altitudinis om. F[M] / ACB BVLF[C] ABC P[ASEMNG]

- 25 magis: etiam magis L[N] / capitulum  $7^m$  tr. P[EA]
- 28 figuram: circulum P / aut: et L sive [A]
- 29 sequeretur BPL[EMG] sequitur VF[SANC] / intensio puncti: punctus F[M]
- 30 vel: aut *P[A]*
- 31 lineam: certam lineam F[M] / certe: certe scilicet F[M]
- 32 esset equalis tr. F[M] est equalis [C] / equalis: ymaginabilis P[A]
- 33 in quantitate: quantumlibet P
- 34 videntur nimis tr. F[M] / nimis om. [N] valde P[A] / nimis absurda tr. L
- 35 tamen om. V / est ymaginabilis FLP-[MANC] tr. BV[ESG]

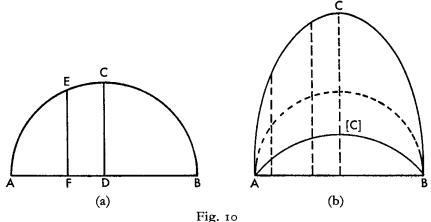


Figure (a) in MSS BLDSCJG. In MS J, C is written over G, and E is replaced by F and F by E. In MS L, the arc is greater than a semicircle. Figure (b) is in MSS BLSCG. I have added the broken lines and have made the curves resemble ellipses, which they do not in the MSS. In MS B, both curves are drawn lower than a semicircle. In S, the top curve is a semicircle. In C, the figure is very crudely drawn. In L and G, the curves are lower and higher than a semicircle but are certainly not elliptical. See commentary.

again let another line EF be drawn as a perpendicular to line AB. Therefore, since it is possible to construct on the same points two other perpendiculars less than CD and EF but having the same ratio between them as do CD and EF and in the same way to construct on all the points of line AB perpendiculars which are greater or less than the corresponding perpendiculars in semicircle ACB constructed on those points of AB and having between any two of them the same ratio as the corresponding perpendiculars on AB in semicircle ACB, it follows that there can be erected on base AB a figure of less height but which will be proportional in altitude to this semicircle and with equal reason a figure of greater height by any amount you wish. Therefore, by chapter seven the quality of line AB can be correctly imagined by any of these figures without it making any difference [which figure is used].

For if it were not so that the quality of line AB imaginable by the semicircle could be imagined by a figure greater or less than the semicircle which is proportional [in altitude to the semicircle] it would follow (1) that the intensity of point D could not be correctly designated by a greater or lesser line than DC, and similarly for all the points, unless the intensity were varied, and thus (2) that any intensity would in itself determine the definite length of the line by which it would be imaginable, and then (3) an intensity would be equivalent and comparable to a line or to quantitative extension, and as a consequence (4) local motion would be comparable in velocity to [qualitative] alteration, all of which seems excessively absurd.

However, any figure by which this quality of line AB is imaginable is curved. But whether the figure less than a semicircle by which this quality can be imagined is a segment of a circle, I leave aside as a matter to be discussed. But I do say that it

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quod per nullam maiorem potest designari que sit portio circuli, per nullam enim figuram potest ista qualitas designari cuius AB non sit basis seu corda; sed AB non potest esse corda in circulo minore quam circulus ACB si esset completus, cuius ipsa est dyameter. Ergo qualitas ista non potest ymaginari per maiorem figuram que sit portio circuli minoris quam circulus ACB; sed neque circuli maioris, quia aut illa portio esset maior medietate sui circuli, ergo per eam nulla qualitas posset designari, ut patet ex 4º [/5º?] capitulo; aut esset minor medietate sui circuli. Ergo, cum ista portio minor medietate maioris circuli haberet eandem cordam cum semicirculo ACB, illa portio esset minor et esset pars illius semicirculi, ut faciliter patet et potest probari per ultimam sexti Euclidis. Igitur ista qualitas non potest designari per figuram que sit portio circuli et que sit maior quam semicircuculus ACB et tamen potest designari per maiorem figuram curvam, ut ante probatum est. Igitur illius maioris figure curvitas non erit circularis et tamen terminabit altitudinem figure proportionalem ei quam terminat curvitas circularis; erunt itaque in altitudine proportionales figure, quarum una est curvitas circularis et alia curvitas non circularis.

### [I.xv] Capitulum 15<sup>m</sup> de quatuor generibus simplicis difformitatis difformis

Omnis igitur simplex difformis difformitas aut est ymaginabilis per figuram que non est portio circuli nec proportionalis altitudinis alicui circuli portioni, sed eius summitas determinatur curvitate irrationali, aut ymaginabilis est per figuram cuius summitas determinatur curvitate rationali, scilicet, circulari vel ei proportionali in altitudine, et utroque modo dupliciter:

- 38 per om. V / designari: considerari F ymaginari fM
- 39 non om. F[M]
- 40 circulus: triangulus F / ACB: ABC [EAS]
- 40-41 si...dyameter om. F[M]
- 41 ipse P
- 42 circuli minoris tr. F[MNG]
- 43 sed om. [M] / neque: ut F[M] / aut: vel P[EA]
- 44 ergo...eam: et per talem [M]
- 44-45 ergo...circuli om. F
- 44 potest P[MAG] / designari: ymaginari [M] / ex BV[SMG] in LP[EANC]
- 5 sui circuli tr. LP

- 45-46 minor medietate LP[EAN] minor BF[SMCG] minoris et V
- 47 pars illius: portio F[M] / faciliter om. [N] / patet et om. [SNC] etiam L etiam patet et P[A]
- 47-48 potest BLF[AG] posset VP[SEM-NC]
- 48 probari: probari seu posset F / ultimam: penultimam [JG] / Euclidis om. BV[S] / Igitur: ergo LP[AN] / ista qualitas tr. B
- 49 designari: ymaginari F[M]
- 51 Igitur: ergo B[M] / illius: istius F[M] nullius L / figure curvitas tr. V / erit: est [N]

cannot be designated by a greater figure which is at the same time a segment of a circle. For this quality can be designated by no figure of which AB is not the base or chord. But AB cannot be the chord in a circle smaller than circle ACB if that circle were completed, for AB is the diameter of that circle. Therefore, this quality cannot be imagined by a greater figure which is a segment of a smaller circle than circle ACB; nor also of a greater figure which is a segment of a larger circle. [This last is evident, for then that segment would either (1) be greater than half of its own circle, and therefore no quality could be designated by it, as is clear from chapter five, or (2) it would be less than half its own circle. [But in the case of the second possibility, since the segment which is less than half of a larger circle would have the same chord as semicircle ACB, the segment would be less [in area] and would be a part of semicircle ACB, as is easily evident and can be proved by the last<sup>4</sup> [proposition] of the sixth [book] of [the *Elements* of] Euclid. Therefore, this quality cannot be designated by a figure which is a segment of a circle and is [at the same time] greater [in altitude and area] than semicircle ACB, and yet it can be designated by [some] greater curved figure, as was proved before. Therefore, the curvature of the greater [curved ]figure will not be circular but will bound a figure which in altitude is proportional to that which the circular curvature bounds; and so there will be two figures proportional in altitude, the curvature of one being circular and that of the other being non-circular.

#### I.xv On four kinds of simple difform difformity

Therefore, every simple difform difformity either (1) is imaginable by a figure which is not a segment of a circle nor proportional in altitude to some segment of a circle but whose summit is determined by an irrational curvature, 1 or (2) is imaginable by a figure whose summit is determined by a rational curvature, namely, by a circular figure or one proportional to it in altitude. And each of these two kinds

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I.xv

I See the Commentary, I.xv, line 5.

plicis om. P

4 alicui: alicuius [FM]

5 irrationabili L

5-6 aut...curvitate iter. V et post curvitate add. V irrationali

I Capitulum I5<sup>m</sup> tr. P | quatuor om. L | sim-

6 curvitate om. P
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or by good or bad angels, or immediately by God, of whom it has been written in the book of Daniel<sup>2</sup> that "He revealeth deep and hidden things and knoweth what is in darkness."

# Here begins the second part of this tract and it treats of the difformity of successive things

#### II.i On the double difformity of motion

Every successive motion of a divisible subject has parts and is divisible in one way according to the division and extension or continuity of the mobile, in another way according to the divisibility and duration or continuity of time, and in a third way—at least in imagination—according to the degree and intensity of velocity. From its first continuity motion is said to be "great" or "small"; from its second, "short" or "long," and from its third, "swift" or "slow." And so motion has two extensions, one that pertains to the subject and the other that pertains to time, and one intensity. Now the two extensions can be imagined in a certain way as mutually intersecting at right angles in the manner of a cross, 1 so that the extension of duration ought to be said to be "longitude" and the extension in subject ought to be called "latitude," while the intensity could be called the "altitude" of this motion or velocity. But according to what was premised in the third chapter of the first part, if intensity of velocity were to be called its "latitude," then each of the extensions in relationship to intensity could be called "longitude," and so velocity will have a double longitude just as it has a double extension, and in each of these extensions the intensity of velocity can be varied in multiple ways. And since dif-

2 Daniel 2:22.

II.i

See the Commentary, II.i, lines 12-13.

<sup>5</sup> subjecti: sive L

<sup>6</sup> extensionem et divisionem L / continuationem V

<sup>7</sup> et: temporis et L

<sup>7-8</sup> temporis: eiusdem L

<sup>10</sup>  $\operatorname{aut}^{1,2}$ :  $\operatorname{vel} L$ 

<sup>12</sup> Due: que (?) L

<sup>13</sup> ortogonaliter B[SG] / se-: seu L / seinvicem  $tr.\ V$  post crucis

<sup>15</sup> posset BV[AFCM] potest L[ENSG] possit[P]

<sup>16</sup> intensio: in tempore [PFM]

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#### II.iii On the quantity of the intensity of velocity

Since each uniformity of motion posited in the first chapter consists in equality of intensity and each difformity arises from inequality [of intensity] we ought to set out first [the measure of gradual intensity, i.e. we ought to specify] with what the gradual intensity of the velocity is measured. However, in the matter of velocity three closely related ideas can be considered. One is the total quantity of the velocity taking into account both intensity and extension. I shall speak of this in the third part of this tract, which will be concerned with the measures of qualities and velocities. Another thing to be considered in connection with velocity is the denomination in terms of which a subject is said to become such a kind more quickly or more slowly. I shall also speak of this in the following chapter. Third, there is the gradual intensity [of velocity]. This is the subject which must now be considered. Therefore, I say universally that that degree of velocity is absolutely more intense or greater by means of which in an equal time more is acquired or lost of that perfection according to which the motion takes place. For example, in local motion that degree of velocity is greater and more intense by means of which more space or distance would2 be traversed. In alteration, similarly, that degree of velocity is greater by means of which more intensity of quality would be acquired or lost; and so in augmentation, by means of which more quantity is acquired, and in diminution, by means of which more quantity or extension is lost. And so generally [our definition would hold] wherever motion would be found.

#### II.iv On diverse ways of [considering] velocity

We must not overlook the fact that the same motion or flux is called by many names that connote a variety of things, and, according to the denomination, velocity is attended or measured in a variety of ways, so that the quantity of gradual intensity is assigned in diverse ways, with which, however, the definition stated earlier in the preceding chapter is in accord.

For example, first, in circular motion a mobile is said "to be moved" and it is

II.iii

<sup>1</sup> See the Commentary, II.iii, lines 11-14.

<sup>2</sup> Ibid., line 15.

C] reperiuntur L

II.iv: BVL

2 idem: idem est L

2-3 nominibus diversimode tr.LV

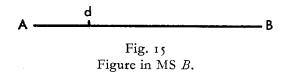
3-4 denominans: denominata L[E] denominata nominans [N]

7 movere V

<sup>4</sup> post quod add. B quantitas intensionis sive mensuratur ita quod

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these however are reducible to those premised. For sometimes in motion there is succession according to inception. For example, in local motion it is possible that the whole of some body begins to be moved at the same time, but it is possible [at another time] that one part begins to be moved after another. A case in point is if point d[see Fig. 15] is imagined to flow over mobile AB so that the part of mobile



AB traversed by point d is moved and the part not yet traversed remains at rest until traversed by point d, as in a pliable rod which would begin to be moved in this way. But this is precisely what happens in the motion of alteration where succession is found according to quantitative parts of the subject and even in the generation of substantial, material form. In such generation there is succession according to quantitative parts, or according to extension, but not however according to gradual parts and intensity. An example is in the generation of fire. Succession of this kind is assimilated in a certain way to local motion, as is evident in the example just posited of the motion of point d, and it is [also] assimilated in a certain way to augmentation since there [i.e., in augmentation] continually more and more, or a greater and greater portion, of the subject is moved or more form is generated. But every succession which is found in this [type] is (1) according to parts of the subject, or (2) according to parts of the time, or (3) according to the velocity of the succession—as, in the example posited, point d would be imagined to be moved more quickly or more slowly. And everything thus is reduced to the divisions specified in the first chapter. Accordingly, every uniformity or difformity which could be found in this kind of succession is to be reduced to, and is contained in, the two kinds of uniformity and difformity already posited in the first chapter.

There can be imagined one further succession, for every velocity is capable of being increased in intensity and decreased in intensity. Now its continuous increase in intensity is called acceleration, and indeed this acceleration or augmentation of velocity can take place more quickly or more slowly. Whence it sometimes happens that velocity is increasing and acceleration is decreasing, while sometimes both are simultaneously increasing. Similarly acceleration of this sort sometimes takes place uniformly and sometimes non-uniformly and in diverse ways. But since every divisibility or succession which is found in acceleration of this sort is according to parts of the subject, or according to parts of the time, or ac-

II.v

<sup>&</sup>lt;sup>1</sup> See the Commentary, II.v, lines 27-32.

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poris aut secundum intensionem gradualem, ex qua trina divisibilitate duplex oritur uniformitas sive difformitas, ut ostensum est in capitulo primo, ideo sicut prius omnis uniformitas atque difformitas que potest ex hoc oriri reducitur ad duo genera supradicta, scilicet ad uniformitatem secundum partes subiecti aut ad uniformitatem que est secundum partes temporis, et ita de difformitate. De ea itaque que est secundum partes subiecti mobilis primo dicatur.

### [II.vi] Capitulum 6<sup>m</sup> de difformitate velocitatis secundum partes quantitativas subiecti

De uniformitate subiectiva ac difformitate velocitatum quantum ad earum configurationem et figurarum variationem penitus dicendum est sicut dictum est in prima parte huius tractatus de uniformitate et difformitate permanentium qualitatum, quoniam difformitas velocitatis eodem modo potest ymaginari et eodem modo proportionatur et configuratur et totidem ac eisdem modis potest diversificari quot et quibus modis prius fuit ostensa variari difformitas qualitatum, sicut per terminari ad gradum et ad non gradum, per hoc quod quedam est difformitas simplex, alia composita, et quod composita multipliciter distinguitur, et sic de omnibus differentiis supradictis.

Verbi gratia [Fig. 16], si linea AB moveatur, possibile est quod quilibet punctus eius alteri comparatus equali velocitate moveatur, scilicet omnia equevelociter, et hoc vel motu locali vel alteratione, et tunc erit velocitas secundum partes subiecti uniformis. Similiter possibile est quod velocitas puncti A sit duplo maior quam velocitas puncti C dividentis C duplo maior quam velocitas puncti C dividentis reliquam medietatem per medium, et sic proportionaliter de aliis punctis, et quod nulla sit velocitas in puncto C terminante; et in isto casu erit velocitas

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36 seu L / in L[ENFMPCS] om. BV[AG]
                                                 liter L
37 atque: sive V
                                               8 prius...ostensa: ostensa fuit prius V
38-39 secundum...uniformitatem om.
                                               9 qualitatis L
   [FMPSC]
                                              10 alia: et alia L et quedam \lceil NS \rceil
39 aut BV[G] et L[EN] sive [A]
                                              11 distinguuntur L/G
41 dicam L
                                              13 post gratia desinit E / si...AB: AB linea L
                                              14 punctus BV[ASG] ACDB punctus L[N]
II.vi: BVL
                                                 punctus ACDB [FMPC]
 3 ac BV[FMSCG] et L[ENP] sive [A]
                                              16 subjecti uniformis tr. L / est om. V
 4 configurationem: assignationem L figura-
                                              19 proportionabiliter L
                                              20 isto casu BV[ACG] tr. L[N] illo casu
   tionem [A]
 5 tractatus om. V / et: ac L
                                                 [FM] io casu [S] alio casu [P]
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7 modo om. L / proportionaliter et configura-

cording to gradual intensity, and from such threefold divisibility arises twofold uniformity or difformity, as was demonstrated in the first chapter, therefore, as before, every uniformity and difformity which can so arise is reduced to the two above-mentioned kinds, that is, to uniformity and difformity according to parts of the subject or uniformity and difformity according to parts of the time. And so let us first speak of that which is according to parts of the subject.

## II.vi On difformity of velocity according to the quantitative parts of the subject

In regard to the configuration and variation in figures representing uniformity and difformity of velocities with respect to subject, one should speak completely in the same way as we spoke before in the first part of the tract where the uniformity and difformity of permanent qualities were discussed. This is clear since the difformity of velocity can be imagined in the same way, can be proportioned and figured in the same way, and can be diversified in as many and in the same ways, as it was demonstrated before that the difformity of qualities is varied. These variations are, for example: (1) in being terminated at [some] degree or at no degree, and (2) that some difformity is simple and some composite, and (3) that the composite difformity is distinguishable in many ways, and so on for all the differences mentioned above.

For example [see Fig. 16], if line AB is moved, it is possible that any point of it is moved with the same velocity as any other point, namely that they are all moved

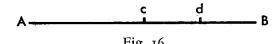


Figure in MSS BLSG. MS L also has a right triangle with base AB marked.

equally fast. This is so in either local motion or alteration. And we have then velocity uniform according to the parts of the subject. Similarly, it is possible that the velocity of point A is twice that of point c (dividing line AB in half) and the velocity of c is twice that of point d (dividing the remaining half in half), and so on proportionally for the other points, with there being no velocity in terminal point B. In this case the velocity will be uniformly difform terminated at no degree in

<sup>&</sup>lt;sup>1</sup> See the Commentary, II.vi, lines 13-23.

of God in eternity... certainly there will be no greater joy in that city than this song to the glory of the grace of Christ."5

# Here begins the third part [of this treatise]: On the Acquisition and Measure of Qualities and Velocities

III.i How the acquisition of quality is to be imagined

Succession in the acquisition of quality can take place in two ways: (1) according to extension, (2) according to intensity, as was stated in the fourth chapter of the second part. And so extensive acquisition of a linear quality ought to be imagined by the motion of a point flowing over the subject line in such a way that the part

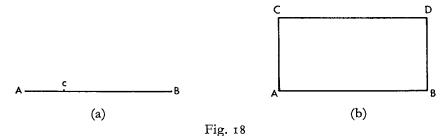


Figure (a) in MSS BG. Figure (b) in MSS SJGL. MS L merely has two parallel lines AB and CD close together.

traversed has received the quality and the part not yet traversed has not received the quality. An example of this occurs if point c were moved over line AB so that any part traversed by it would be white and any part not yet traversed would not yet be white [see Fig. 18(a)]. Further the extensive acquisition of a surface quality ought to be imagined by the motion of a line dividing that part of the surface that

cantico in gloriam gratiae Christi, cuius sanguine liberati sumus, nihil erit profecto illi iucundus civitati." Augustine is referring to *Psalms* 88:2.

<sup>&</sup>lt;sup>5</sup> De civitate Dei, XXII, 30 (ed. of Dombart, Vol. 2, 633-34): "Alioquin si se fuisse miseros nescituri sunt, quo modo, sicut ait psalmus, misericordias Domini in aeternum cantabunt? Quo

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alteratam a parte nundum alterata. Et acquisitio extensiva corporee qualitatis conformiter ymaginanda est per motum superficiei dividentis partem alteratam a parte nundum alterata.

Acquisitio autem intensiva qualitatis punctualis ymaginanda est per motum puncti continue ascendentis super punctum subiectivum et motu suo describentis perpendicularem lineam ymaginatam super eundem punctum subiectivum. Acquisitio vero intensiva qualitatis linearis ymaginanda est per motum linee perpendiculariter ascendentis super lineam subiectivam et suo fluxu vel ascensu derelinquentis superficiem per quam designatur qualitas acquisita. Verbi gratia [Fig. 18(b)]: Sit AB linea subiectiva. Dico igitur quod intensio puncti A ymaginatur per motum vel per ascensum perpendicularem puncti C et intensio linee AB vel acquisitio intensionis ymaginatur per ascensum linee CD. Acquisitio autem intensiva qualitatis superficialis conformiter ymaginanda est per ascensum superficiei motu suo ymaginato derelinquentis corpus per quod illa qualitas designatur. Et similiter acquisitio intensiva corporee qualitatis ymaginatur per motum superficiei quia superficies fluxu suo ymaginato derelinquit corpus et non contingit dare quartam dimensionem sicut dictum fuit  $4^0$  capitulo prime partis.

Et sicut nunc dictum est de acquisitione qualitatis ita conformiter dicendum est et ymaginandum de deperditione, sive deperdatur extensio sive etiam intensio, ymaginatur enim talis deperditio per motus oppositos motibus prius dictis. Sicut etiam nunc dictum est de acquisitione aut deperditione qualitatis ita conformiter ymaginandum est de acquisitione aut deperditione velocitatis tam in extensione quam in intensione.

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15 alteratam om. L alterate [FMP] / a: ab altera L[N]
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- 15-17 alterata....alterata om. [C] / Et...alterata om. [FMP]
- 15–16 qualitatis om. L[AN]
- 17 alteratam: alteram L
- 19 subjectum L/C
- 19-21 et...subjectivum om. L[N]
- 21 subjectivum om. L subjectum [C]
- 22 subjectivam: summam L
- 24 subjecta L
- 25 per<sup>2</sup> BL[FMP] om. [AVNSCG] / ascensum BL[VANS] ascensionem [FMPCG]
- 27 autem om. L[N] vero [F]
- 29 designatur: ymaginatur [V]
- 30 acquisitio om. L[N]
- 30-31 superficiei: superficiei qualitatis L/N
- 32 fuit: est  $L/4^{\circ}$  capitulo  $tr. \lceil VAC \rceil$

- 34 deperditione BL[VNSG] perditione
  [AFMPC] / deperdatur: deperditioL/extensio L[VNG] etiam extensio B extensive [FMP] intensio [AS] intensive [C]
- 35 etiam intensio B[V] om. L intensio [NG] etiam intensive [F] etiam extensive [C] etiam extensio [A] intensive [MP] extensio [S]
- 36 etiam om. [G] enim L[N]
- 36-37 aut deperditione BL[V] om. [NFMP C] de perditione [A] et de perditione [G] perditione [S]
- 37–38 dependitione: de perditione [AFP] perditione [V]
- 38 in extensione quam in intensione B[VFM PG] in (om. C) intensione quam in (om. S) extensione L[SC] in intensum quam extensum [N] extensive quam intensive [A]

has been altered from the part not yet altered. And the extensive acquisition of a corporeal quality in a similar way is to be imagined by the motion of the surface dividing the part altered from the part not yet altered.<sup>1</sup>

The intensive acquisition of punctual quality is to be imagined by the motion of a point continually ascending over a subject point and by its motion describing a perpendicular line imagined [as erected] on that same subject point. But the intensive acquisition of a linear quality is to be imagined by the motion of a line perpendicularly ascending over the subject line and in its flux or ascent leaving behind a surface by which the acquired quality is designated. For example [see Fig. 18(b)], let AB be the subject line. I say, therefore, that the intension of point A is imagined by the motion, or by the perpendicular ascent, of point C, and the intension of line AB, or the acquisition of the intensity, is imagined by the ascent of line CD. Further, the intensive acquisition of a surface quality is in a similar way to be imagined by the ascent of a surface, which (by its motion) leaves behind a body by means of which that quality is designated. And similarly the intensive acquisition of a corporeal quality is imagined by the motion of a surface because a surface by its imagined flux leaves behind a body, and one does not have to pose a fourth dimension, as has been said in the fourth chapter of the first part.

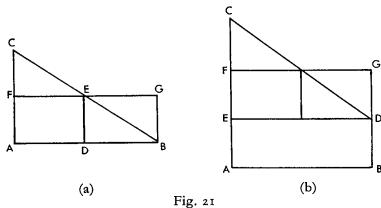
One should speak and conceive of the loss of quality in the same way that we have now spoken of its acquisition, whether that loss is of extension or intensity. For such loss is imagined by movements which are the opposite of the movements described before. Furthermore, one ought to speak of the acquisition or loss of velocity, both in extension and intensity, in the same way we have just spoken of the acquisition or loss of quality.

<sup>&</sup>lt;sup>1</sup> See the Commentary, III.i, lines 13-17.

if it is imagined to be a successive entity. Whence it is said in Isaias: "And the light of the moon shall be as the light of the sun, and the light of the sun shall be sevenfold as the light of seven days," for evidently the light of one day increased intensively by sevenfold is as the light which would be extended through a space of seven days.

## III.vii On the measure of difform qualities and velocities

Every quality, if it is uniformly difform, is of the same quantity as would be the quality of the same or equal subject that is uniform according to the degree of the middle point of the same subject. I understand this to hold if the quality is linear. If it is a surface quality, [then its quantity is equal to that of a quality of the same subject which is uniform] according to the degree of the middle line; if corporeal, according to the degree of the middle surface, always understanding [these concepts] in a conformable way. This will be demonstrated first for a linear quality. Hence let there be a quality imaginable by  $\triangle ABC$ , the quality being uniformly difform and terminated at no degree in point B [see Fig. 21(a)]. And let D be the middle point of the subject line. The degree of this point, or its intensity, is imag-



Figures in *BLSJG*. Figures are rotated through 90° in MS G. In figure (b) in MS L, there is no center perpendicular. In MS J, line ED is missing and the center perpendicular is marked KH.

Both figures are reversed in MS J.

ined by line DE. Therefore, the quality which would be uniform throughout the whole subject at degree DE is imaginable by rectangle AFGB, as is evident by the tenth chapter of the first part. Therefore, it is evident by the 26th [proposition] of [Book] I [of the *Elements*] of Euclid<sup>2</sup> that the two small triangles EFC and EGB

III.vii

<sup>3</sup> Isaias 30:26.

<sup>&</sup>lt;sup>1</sup> See the Commentary, III.vii, lines 3-5.

<sup>&</sup>lt;sup>2</sup> Ibid., line 14.

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equales. Ergo maior triangulus BAC qui designat qualitatem uniformiter 15 difformem et quadrangulus AFGB qui designaret qualitatem uniformem secundum gradum puncti medii sunt equales. Ergo qualitates per huiusmodi triangulum et quadrangulum ymaginabiles sunt equales. Et hoc est propositum.

Eodem modo potest argui de qualitate uniformiter difformi terminata utrinque ad certum gradum, sicut esset qualitas ymaginabilis per quadrangulum ABCD [Fig. 21(b)]. Protrahatur enim linea DE equedistans basi subiecte et fieret triangulus CED. Deinde protrahatur per gradum puncti medii linea FG equalis et equedistans basi subiecte, et protrahatur etiam linea GD. Tunc sicut prius probabitur quod triangulus CED et quadrangulus EFGD sunt equales. Ergo addito utrobique quadrangulo communi AEDBfient duo tota equalia, scilicet quadrangulus ACDB qui designat qualitatem uniformiter difformem et quadrangulus AFGB qui designaret qualitatem uniformem secundum gradum puncti medii ipsius subiecti AB. Igitur per capitulum 10m prime partis qualitates per huiusmodi quadrangulos designabiles sunt equales.

Conformiter potest argui de qualitate superficiali ac etiam de corporali. De velocitate vero omnino dicendum est sicut de qualitate lineari, dum tamen loco puncti medii capiatur instans medium temporis velocitatem huiusmodi mensurantis. Sic itaque patet cui qualitati aut velocitati uniformi adequatur qualitas sive velocitas uniformiter difformis. Proportio autem qualitatum et velocitatum uniformiter difformium est sicut proportio qualitatum et velocitatum simpliciter uniformium quibus adequantur. Et de mensura et proportione illarum uniformium dictum est in capitulo precedenti.

Si autem qualitas seu velocitas fuerit difformiter difformis, tunc, si componatur ex partibus uniformibus aut uniformiter difformibus, ipsa poterit mensurari per suas partes, de quarum mensura dictum est ante. Si vero qualitas fuerit alio modo difformis, sicut difformitate illa que per curvitatem designatur, tunc oporteret recurrere ad mensurationem figurarum curvarum inter se aut earum cum rectis figuris; et hoc est alterius speculationis. Sufficiant ergo que dicta sunt.

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21 utrinque B[SG] om. [C] utriusque L[A]
  utrobique [VN] uterque [FMP]
23-24 et ... et2 om. L
26 sunt B[VANSG] fient L erunt [FMPC]
                                             [FMP]
26-27 equales...fient B[VANSG] om. [FM
  PC] equales quare L
27-28 ACDB...quadrangulus om. L
                                          38 Et om. L[AN]
28 designaret B[AVS] designat L[FNM]
   ?P, ?C, ?G]
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- 33  $De^{I}$  om. L[N]
- 34 capiatur B[VSG] om. [FMP], tr. L[AN] post temporis / instans: instantis L infert
- 36 qualitatis aut velocitatis L
- 37 et: aut L[N] sive [A]
- 39 et: et de L[VMP] / uniformium om. L[N]
- 41 qualitas...velocitas: velocitas seu quali-

are equal. Therefore, the larger  $\Delta B A C$ , which designates the uniformly difform quality, and the rectangle A F G B, which designates the quality uniform in the degree of the middle point, are equal. Therefore the qualities imaginable by a triangle and a rectangle of this kind are equal. And this is what has been proposed.

In the same way it can be argued for a quality uniformly difform terminated in both extremes at a certain degree, as would be the quality imaginable by quadrangle ABCD [see Fig. 21(b)]. For let line DE be drawn parallel to the subject base and  $\Delta CED$  would be formed. Then let line FG be drawn through the degree of the middle point which is equal and parallel to the subject base. Also, let line GD be drawn. Then, as before, it will be proved that  $\Delta CED = EFGD$ . Therefore, with the common rectangle AEDB added to both of them, the two total areas are equal, namely quadrangle ACDB, which designates the uniformly difform quality, and the rectangle AFGB, which would designate the quality uniform at the degree of the middle point of the subject AB. Therefore, by chapter ten of the first part, the qualities designatable by quadrangles of this kind are equal.

It can be argued in the same way regarding a surface quality and also regarding a corporeal quality. Now one should speak of velocity in completely the same fashion as linear quality, so long as the middle instant of the time measuring a velocity of this kind is taken in place of the middle point [of the subject].<sup>3</sup> And so it is clear to which uniform quality or velocity a quality or velocity uniformly difform is equated. Moreover, the ratio of uniformly difform qualities and velocities is as the ratio of the simply uniform qualities or velocities to which they are equated. And we have spoken of the measure and ratio of these uniform [qualities and velocities] in the preceding chapter.

Further, if a quality or velocity is difformly difform, and if it is composed of uniform or uniformly difform parts, it can be measured by its parts, whose measure has been discussed before. Now, if the quality is difform in some other way, e.g. with the difformity designated by a curve, then it is necessary to have recourse to the mutual mensuration of the curved figures, or to [the mensuration of] these [curved figures] with rectilinear figures; and this is another kind of speculation. 4 Therefore what has been stated is sufficient.