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THE SCIENTIFIC WORK OF GEORGES LEMAITRE

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At the time when LEMAÎTRE was beginning his career, science and philosophy had been overwhelmed by the principle of relativity, which had swept away long-cherished modes of thought and provided an entirely new outlook to dominate man's view of nature. LEMAÎTRE entered into the new discipline and soon began making contributions to it.

One of the main points of discussion at that time was simultaneity, which had lost its absolute significance. LEMAÎTRE considered a rigid solid, moving generally in curved space-time, and examined the problem of introducing a standard of simultaneity inside the solid, referring to its motion [1]. Two neighbouring points of space-time are defined to be simultaneous if the line joining them is orthogonal to the world-

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line of the matter. This local simultaneity can then be integrated along a line, giving a *line of simultaneity*, any two points of which are simultaneous with respect to this line joining them.

Now it may be that all lines joining two points give the same standard of simultaneity. There would then be a general meaning for simultaneity with respect to the solid. In such a case LEMAÎTRE calls the motion of class A. When this condition is not fulfilled the solid may still be a rigid one, with all distances measured along lines of simultaneity remaining constant, but any point can be simultaneous to any other by a suitable choice of the line of simultaneity joining them. LEMAÎTRE then calls the motion of class B. He made a study of all the possible kinds of motion of class B, elliptic, hyperbolic, loxodromic and parabolic.

In 1928 I introduced a relativistic wave equation for the electron having different transformation properties from the tensor equations that one had previously been confined to in relativity. Two papers of LEMAÎTRE are devoted to studying these transformation properties [2, 3].

There are various methods that one may use for building up the theory. EDDINGTON had previously developed one method, based on his system of pentads. LEMAÎTRE developed another, showing up some new transformation features of the equations.

LEMAÎTRE begins with a general antisymmetric 4×4 matrix $T_{\mu\nu} = -T_{\nu\mu}$. It can be expressed linearly in terms of six basic antisymmetric matrices, forming two sets A_1, A_2, A_3 and B_1, B_2, B_3. They have the following properties, which LEMAÎTRE enumerates. (1) The square of each of them is unity. (2) The A's anticommute and the B's anticommute. (3) $A_1A_2A_3 = B_1B_2B_3 = i$. (4) Each A commutes with each B. (5) Each product A_jB_k is symmetric. (6) The square of A_jB_k is unity. (7) All the matrices A_jB_k are linearly independent. (8) Two matrices A_jB_k anticommute or commute according to whether they have or have not a common factor. A consequence of (7) is that the nine symmetric A_jB_k together with the six antisymmetric A_j and B_k and the unit matrix form sixteen independent 4×4 matrices.

In terms of these matrices the wave equation may be written

$$\{A_1p_1 + A_2p_2 + A_3p_3 + B_1p_0 - imc\, B_3\}\ \Psi = 0\ .$$

The operator here is a special case of the general antisymmetrical 4×4 matrix

$$T = A_1p_1 + A_2p_2 + A_3p_3 + B_1q_1 + B_2q_2 + B_3q_3\ .$$

If we apply any transformation to the four Ψ's, say $\overline{\Psi}^{\mu} = K^{\mu}{}_{\sigma}\, \Psi^{\sigma}$, or, in matrix notation, $\overline{\Psi} = K\Psi$, it corresponds to a transformation of T,

$$\overline{T} = \breve{k}'Tk'\ ,$$

where k' denotes the reciprocal matrix to k and $\breve{k}$ denotes the transpose of k. If we restrict the matrix k to be of determinant unity, then the determinant of T is invariant. The square root of this determinant is found to be

$$\sqrt{|T|} = p_1^2 + p_2^2 + p_3^2 - q_1^2 - q_2^2 - q_3^2\ .$$

The resulting transformation of the p's and q's is thus an orthogonal transformation in $3+3$ dimensions. The group of these transformations has fifteen independent rotations, corresponding to the fifteen independent 4×4 matrices apart from the unit matrix.

The LORENTZ transformations form a subgroup in which q_2 and q_3 remain invariant. LEMAÎTRE gives various examples of these transformations and connects up his work with EDDINGTON's. He also works out the LORENTZ coefficients in terms of the elements of k, thereby throwing new light on the LORENTZ transformation.

In his second paper LEMAÎTRE takes a slightly different starting point, building up his theory in terms of quadriquaternions, which are products of two independent kinds of quaternions. The basic elements are now similar to the previous A's and B's, all multiplied by i, so they satisfy $A_1 = A_2A_3 = -A_3A_2$, $A_1^2 = -1$, etc., and similarly for the B's.

LEMAÎTRE introduces the quantities

$$D_1 = A_1B_1 , \qquad D_2 = A_2B_2 , \qquad D_3 = A_3B_3 ,$$

satisfying $$D_1 = D_2D_3 = D_3D_2 , \qquad D_1^2 = 1, \text{ etc.},$$

and also the quantities

$$S_1 = A_3B_2 , \qquad S_2 = A_1B_3 , \qquad S_3 = A_2B_1 ,$$

satisfying $$S_1 = S_2S_3 = S_3S_2 , \qquad S_1^2 = 1, \text{ etc.}$$

One can express the A's and B's in terms of the D's and S's. LEMAÎTRE further introduces the quantity

$$C = \tfrac{1}{4}\ (1 + D_1 + D_2 + D_3) .$$

This has the property of absorbing the D's, according to

$$CD_1 = D_1C = C , \qquad C^2 = C .$$

Any quadriquaternion T can be expressed in term of S's and D's, and so TC can be written as

$$TC = (\Psi_0 + \Psi_1S_1 + \Psi_2S_2 + \Psi_3S_3)\ C = \Psi , \quad \text{say.}$$

This is a special kind of quadriquaternion, involving just four numbers Ψ_0, Ψ_1, Ψ_2, Ψ_3. LEMAÎTRE calls it a final vector. Similarly $CT = \chi^*$ is an initial vector. A final vector and an initial vector have two kinds of product, an inner product $\chi^*\Psi = C\chi^*\Psi C$ and an outer product $\Psi\chi^* = \Psi C\chi^*$. Any quadriquaternion multiplied on the left into a final vector gives another final vector, and this allows it to be represented by a matrix. However there is no necessity to do so, and one can develop the whole theory as a symbolic calculus. LEMAÎTRE made further developments of his theory and again showed how to connect it with EDDINGTON's pentads.

Many new versions of the theory have been formulated since the early work of EDDINGTON and LEMAÎTRE. In particular, there is VAN DER WAERDEN's spinor analysis, which is extremely powerful.

COSMOLOGY

LEMAÎTRE's main work lies in the realm of cosmology, the study of the structure of the universe as a whole. This is where his great contribution to science lies.

EINSTEIN started the modern development of cosmology by pointing out that his field equations, in their general form with the cosmological constant, allow the possibility of a finite universe which is closed and has no boundary. EINSTEIN's universe is static. It has a constant size. Now observations by HUBBLE showed that very distant objects, the spiral nebulae, are receding from us with velocities proportional to their distance. This leads one to believe that the universe is expanding and not static.

DE SITTER proposed another model for the universe, one that is very interesting mathematically and that does provide the expansion needed by HUBBLE's observations. But DE SIT-

TER's universe has zero density for matter everywhere. It is empty. For this reason it is not physically acceptable.

A more powerful attack on the problem was made by FRIEDMAN. Instead of assuming a particular model, he considered generally a universe that is spatially homogeneous and isotropic, with a finite radius that varies with the time, and worked out all the possibilities that Einstein's equations allow, obtaining various solutions for the way in which the radius can vary with the time. FRIEDMAN kept to the mathematics and did not discuss whether any of his solutions could apply to the actual universe. For this reason his work did not arouse much immediate interest and was neglected by other cosmologists for many years.

LEMAÎTRE took up the subject [4] without knowing about FRIEDMAN's work. LEMAÎTRE continually had the physical conditions in mind and was striving for a theory of the real universe. He first considered the suitability of a homogeneous model. The real universe is far from homogeneous, the matter being mainly concentrated in stars, which are themselves concentrated in galaxies. But the galaxies seem to be distributed uniformly over the whole of space as far as we can see, apart from clustering, and are extremely numerous. LEMAÎTRE likens the galaxies to the molecules of a gas. Collisions are infrequent, so it is a rarified gas. One thus gets a homogeneous picture of the universe, allowing one to speak of a density of matter.

There will also be a uniform pressure, arising partly from the motion of the matter and partly from electromagnetic radiation. The part that arises from the motion of matter is very small compared with the energy density of the matter and LEMAÎTRE neglects it. The part that arises from electromagnetic radiation is also very small for the actual universe at the present time, but LEMAÎTRE retains it in his general theory, and uses it later on when he deals with stagnation.

Let p denote the electromagnetic pressure, so the electromagnetic energy density is $3p$. The total energy density is then $\rho = \delta + 3p$, where δ denotes the contribution of the matter. Let R denote the radius of the universe, varying with the time, so that the line element is

$$ds^2 = -R^2\, d\sigma^2 + dt^2\ , \tag{1}$$

where $R d\sigma$ is the elementary distance in three-dimensional space. LEMAÎTRE deduces from EINSTEIN's field equations

$$3\,\frac{R'^2}{R^2} + \frac{3}{R^2} = \lambda + k\rho \tag{2}$$

and

$$2\,\frac{R''}{R} + \frac{R'^2}{R^2} + \frac{1}{R^2} = \lambda - kp\ , \tag{3}$$

where $R' = dR/dt$ and λ is the cosmological constant, whose value is unknown. k is a known constant, whose value is 8π in natural units. Equation (2) is the same as FRIEDMAN's, and (3) is the same as FRIEDMAN's except for the pressure term, which FRIEDMAN did not take into account. These equations lead to the equation of conservation of energy

$$\frac{d\rho}{dt} + \frac{3R'}{R}(\rho + p) = 0\ . \tag{4}$$

The relation between R and t can now be worked out,

$$t = \int \frac{dR}{\sqrt{\left(\frac{\lambda R^2}{3} - 1 + \frac{\alpha}{3R} + \frac{\beta}{R^2}\right)}}\ . \tag{5}$$

Here α is a constant connected with the total mass M,

$$\alpha = k\,\delta\,R^3 = k\,M/\pi^2\;,$$

and β is a constant of integration associated with the pressure; $\beta = 0$ if $p = 0$.

For EINSTEIN's static universe there is a connection between the constants α and λ, namely

$$(6) \qquad \alpha = 2/\sqrt{\lambda}\;.$$

LEMAÎTRE assumes this relation between the constants to hold also for the general case (5). This gives, if we take $\beta = 0$ and put $\lambda = R_0^{-2}$,

$$(7) \qquad t = R_0\sqrt{3}\int\frac{dR}{R - R_0}\sqrt{\frac{R}{R + 2R_0}}\;.$$

which can be integrated to give

$$(8) \qquad t = R_0\sqrt{3}\log\frac{1 + x}{1 - x} + R_0\log\frac{\sqrt{3}\,x - 1}{\sqrt{3}\,x + 1} + C\;,$$

where $x^2 = R/(R + 2R_0)$. This solution makes R increase steadily as t increases. Its initial value, occurring at $t = -\infty$, is R_0. There is thus a steady expansion, with an infinite time since the start.

The expansion leads to a red-shift for distant matter, which one can easily work out. One finds that, for matter at a distance r, the velocity of recession is

$$v/c = R'r/R\ .$$

From (5) and (6) with $\beta = 0$, one gets

$$\frac{R'}{R} = \frac{1}{R_0\sqrt{3}}\sqrt{(1 - 3y^2 + 2y^3)}\ ,$$

where $y = R_0/R$. Using HUBBLE's value for the red-shift and a value for R_0 determined from an estimate of the average density of matter in the universe, LEMAÎTRE calculated $y = 0.0465$, giving $R/R_0 = 21.5$.

LEMAÎTRE thus set up his first model, according to which the universe started off in the infinite past from some asymptotic radius R_0, about $1/20$ of the present radius, and expanded, at first very slowly, the rate of expansion tending to zero logarithmically as one goes back into the infinite past, then at an accelerating rate.

The model was obtained with the assumption of Einstein's relation (6) between the constants. One might try departing from this relation. LEMAÎTRE found that if the departure is at all considerable, one would have the expansion starting at a time of the order 10^9 years ago, which is the time provided by HUBBLE's constant if the expansion were approximately uniform. Now one of the big difficulties then confronting cosmologists was that stellar evolution required a much longer time than this. LEMAÎTRE therefore favoured his model, which pushed back the start of the expansion into the infinite past.

He subsequently modified this view, stating that it does not really help with the discrepancy to push back the start of the universe in this way, because in the early stages of the expansion all physical processes would take place extremely slowly, the rate slowing down logarithmically as one goes back to the infinite past. Thus the extra time in the early stages would not be available for much stellar evolution to occur. This kind of argument showed up LEMAÎTRE's appreciation of the need for understanding the physical significance of one's equations.

INITIATION OF THE EXPANSION

LEMAÎTRE was interested in the question of what originally caused the expansion of a universe in equilibrium [5]. EDDINGTON had suggested that the formation of condensations might be the cause. LEMAÎTRE found that condensations have no direct effect on the equilibrium. However, they lead to another effect, namely a diminution of the exchange of energy between distant parts of the universe, owing to kinetic energy no longer being able to wander freely, but having a chance of being captured by a condensation and then remaining bound to it. LEMAÎTRE calls this process *stagnation*.

To calculate the effect, LEMAÎTRE considers one particular condensation of spherical symmetry and averages over the other condensations so that they also have spherical symmetry. There is then a neutral zone around the particular condensation, where its gravitation is balanced by the gravitational influence of the outside condensations. He calculates the expansion of this neutral zone, which gives a measure of the expansion of the universe as a whole. He gets the same equation as for the expansion of a homogeneous universe, except for having a different expression P replacing the pressure p.

Thus the condensation does not affect the expansion. However if there is stagnation, the expression P gets altered, and this does affect the expansion. Stagnation may thus start an expansion from a state previously in equilibrium.

In a further elaboration of his theory of perturbations in the homogeneous universe [6], LEMAÎTRE finds the possibility of a region ceasing to expand before the gravitation is completely balanced by the cosmical repulsion, so that the expansion is followed by a contraction. Thus collapsing regions can appear in a generally expanding space. Further, there is the possibility of equilibrium regions, which are unstable and divide into collapsing regions that remain approximately at the same distance from one another.

LEMAÎTRE suggests that the collapsing regions are the spiral nebulae and the equilibrium regions are the clusters of spiral nebulae. According to this hypothesis the mean density of all clusters should be the same and should be connected with Hubble's constant. LEMAÎTRE checks these results with observation and finds them to be approximately true.

THE PRIMEVAL ATOM

The cosmological work discussed up to the present all follows according to established methods based on Einstein's theory of gravitation. LEMAÎTRE was not satisfied with this orthodox approach and made a bold new departure, breaking away from accepted principles. He put forward the idea that the universe started from a single super-radioactive atom with an extremely large mass. This idea was presented at a discussion on the evolution of the universe at a meeting of the British

Association for the Advancement of Science in London in 1931 [7].

Previous speakers had brought up the difficulties arising from the usual theories of cosmology. The main difficulty was emphasized by DE SITTER, namely the discrepancy in the time scale required by the evolution of the stars and that provided by the expansion of the universe. DE SITTER concluded that there was not much connection between evolution and the expansion. Evolution must have started long before the expansion.

LEMAÎTRE pointed out how unsatisfactory such a dichotomy would be. He referred to his calculations showing that, as soon as condensations are formed in the uniform primeval gas, they will lead to stagnation, which must start the expansion. Thus evolution and the expansion must have started at the same time. This requires a complete revision of the accepted ideas of stellar evolution. In LEMAÎTRE words « *We need a fireworks theory of evolution. The last two thousand million years are slow evolution: they are ashes and smoke of bright but very rapid fireworks.* »

He then brought up the subject of cosmic rays. He pointed out that their energy is comparable with the total energy of matter, their ratio being perhaps 10^{-3} and certainly not less than 10^{-5}. If they originated before the expansion of space, their energy was then still larger. LEMAÎTRE considered that such high-energy cosmic rays could originate only in the stars, as only the stars would have had enough energy. But the stars could not then have had an atmosphere, as the passage of the cosmic rays through an atmosphere would have degraded them. Thus the stars were born without atmosphere, probably some 10^{10} years ago, and their atmosphere evolved after the escape of the cosmic rays.

How can such stars have been formed? LEMAÎTRE proposed that the stars were each originally a single atom of very high atomic mass, comparable to the mass of the star. Such an atom would undergo super-radioactive disintegration, the cosmic rays being then emitted. The greater part of the products of disintegration would be kept back by the gravitation of such a massive atom, although a small part would escape before the products of disintegration were numerous enough to form an atmosphere. So LEMAÎTRE states « *Cosmic rays are glimpses of the primeval fireworks of the formation of a star from an atom, coming to us after their long journey through space.* »

The energy of cosmic rays produced by such a tremendous disintegration must be extremely high, and one might wonder whether the observed energy of the cosmic rays is high enough to fit this hypothesis. One must take into account that the observed energy will not be the same as the original energy, but will be reduced in the ratio of the expansion of space. LEMAÎTRE estimated this ratio to be at least 20 and considered this adequate to avoid a discrepancy.

How does this explanation of the origin of cosmic rays appear in the light of present-day knowledge? On the basis of his theory LEMAÎTRE predicted that cosmic rays would not consist entirely of photons (as was believed in those days), but would contain fast β-rays and α-particles, and even new rays of greater masses and charges. This was later confirmed by observation. For the charged particles, we now know about the process of the synchroton, which can accelerate them to very high energies. It is quite likely that suitable electric and magnetic fields exist in the galaxies to operate this process and thus generate some of the cosmic rays. But the energy attainable is not sufficient to account for the most energetic of the observed cosmic rays, and it may well be that not all the less energetic ones are produced in this way. There is thus a residue for which there is no known origin except a cosmical one, and presumably this would have to be somewhat on the lines proposed by LEMAÎTRE, the rays originating at a time close to the origin of the universe when conditions were very different from what they are now.

The stars in LEMAÎTRE's theory would have various masses determined by the masses of the atoms from which they came. The luminosity would also be determined by the original mass, so there should be a connection between mass and luminosity. LEMAÎTRE did not work out this idea, but he hoped that it would provide an alternative explanation of the mass-luminosity relation (the Russell diagram) not requiring such a slow evolutionary process as the usual theory.

From this discussion of stars and cosmic rays LEMAÎTRE proceeded to the climax of his theory — that the universe started with an extremely small radius, with all the mass concentrated in the form of a single atom. The whole activity and development of the universe comes from the radioactive disintegration of a single primeval atom. This was the natural conclusion to the argument that he had so painstakingly built up.

LEMAÎTRE gave us a fascinating exciting new picture of the universe in which the dominating theme is evolution. The universe had a definite beginning, an extremely active one, and conditions have gradually settled down to what we see today. There has been continual evolution from the fiery beginning. There are prospects for continuing evolution into the future, as far ahead as we can see. One may expect evolution on earth to proceed hand-in hand with cosmic evolution and to lead to a better and brighter future for all mankind.

Once when I was talking with LEMAÎTRE about this subject and feeling stimulated by the grandeur of the picture that he has given us, I told him that I thought cosmology was the branch of science that lies closest to religion. However LEMAÎTRE did not agree with me. After thinking it over he suggested psychology as lying closest to religion.

Some time after this exciting work of LEMAÎTRE, a rival and more prosaic view of the universe was put forward and received much support — the steady state cosmology. According to this view the universe had no origin, but has always existed much the same as it is now, and will continue so into the infinite future. The continual moving away of distant matter, as shown by the red-shift of the galaxies, is compensated by the continual creation of new matter. With this theory there can be no cosmic evolution. No essential change is taking place. I have never liked this view because of its denial of evolution, but still it enjoyed great popularity at one time, largely because it could be made to fit the observations and there was some interesting mathematics connected with it.

The development of radio-astronomy within the last few years has opened up new vistas and vastly increased our knowledge of distant parts of the universe. As a result, the steady state theory no longer fits the observations and has been abandoned. LEMAÎTRE's hypothesis of a violent beginning of the universe is now generally accepted. It is called big bang theory, by contrast to the steady state theory.

LATER WORK

LEMAÎTRE has subsequently done a great deal of work on clusters of nebulae, and this has led him to a somewhat more detailed picture of the early stages of the universe [8]. He points out the difficulty of understanding the origin of clusters because of the large relative velocity of the nebulae in a cluster. Presumably the nebulae condensed from clouds of gas with similarly large relative velocities. At a sufficiently remote time in the past, when the radius of space was much smaller, these clouds would have been in contact. How could they then have acquired such large relative velocities?

To escape from the dilemma, we must suppose that at that time the clouds did not exist. In the remote past, according to LEMAÎTRE, there would only have been cosmic rays. If a small mass of gas were then formed in some way, it would absorb cosmic rays and would grow and form a cloud. Clouds that evolved like this would have enormous velocities, the velocities of the cosmic rays that happened to encounter one another sufficiently gently to make the first steps in the establishment of the clouds. Nearly all the cosmic rays would get absorbed by the clouds, only a small fraction, say 10^{-4} of them, escaping.

The early cosmic rays contained all kinds of atomic particles. There is a problem why the matter now observed in the stars is almost entirely hydrogen and helium. LEMAÎTRE supposed that there might have been some process of materialization which turned the very large kinetic energy of the early cosmic rays trapped by the clouds into hydrogen, or possibly helium. Such a process of materialization is not known in present-day physics, where we have conservation of barions, but still I think it is worth considering, because there is no certainty that conditions have not changed greatly since those early days.

With an evolutionary universe of the type proposed by LEMAÎTRE it is possible that the laws of nature are also evolving and are not so immutable as is usually supposed. Laws which are usually regarded as permanent may be changing at a rate too slow to show up in the laboratory, but large enough to require vast changes in our understanding of the early development of the universe. For example, it may be that the constant of gravitation is not really constant, but varies with the absolute time. This idea was first put forward by MILNE and I have favoured it because, with a suitable law for the variation, it provides an explanation of some of the very large numbers met with in nature.

There is not yet any reliable observational evidence to guide us in these questions, and for the present we must just try to find the most satisfying theory. Until recently the popularity of the steady state theory has deterred people from working on such lines. But we may expect renewed activity here in the future with the resurgence of the big bang theory.

The measure of greatness in a scientific idea is the extent to which it stimulates thought and opens up new lines of research. In these respects we must rate LEMAÎTRE's cosmology of the highest calibre.

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