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George F R Ellis

ACGC and Department of Mathematics and Applied Mathematics, University of Cape Town, Rondebosch 7701, Cape Town, South Africa

E-mail: george.ellis@uct.ac.za

Abstract
This paper looks at how inhomogeneous spacetime models may be significant for cosmology. First it addresses how the averaging process may affect large-scale dynamics, with backreaction effects leading to effective contributions to the averaged energy–momentum tensor. Second, it considers how local inhomogeneities may affect cosmological observations in cosmology, possibly significantly affecting the concordance model parameters. Third, it presents the possibility that the universe is spatially inhomogeneous on Hubble scales, with a violation of the Copernican principle leading to an apparent acceleration of the universe. This could perhaps even remove the need for the postulate of dark energy.

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1. Introduction

The standard models of present day cosmology are perturbed FLRW (Friedmann–Lemaître–Robertson–Walker) models. These models, developed by Einstein, de Sitter, Friedmann, Lemaître, Robertson, and Walker in the period from 1917 to 1935, are exactly spatially homogeneous and isotropic, with an implied smooth fluid approximation; an early standard reference on their properties is by Robertson [111]. The main developments since then are, first, consideration of much more complex matter content than considered at that time, in particular considering inclusion of background radiation interacting with multiple matter components and scalar fields, allowing in particular an inflationary early epoch; second, and consequent on this, a sophisticated history of the physical evolution of the contents of the universe, including in particular nucleosynthesis and matter–radiation decoupling; and third, following the pioneering work of Lifschitz, the extension of these models to perturbed models, where linearized structure formation and its effects on the background radiation can be studied. Observational relations can be calculated in these models and compared with astronomical data, confirming that they give good physical models that account satisfactorily for these

observations. Summaries are given in many texts, e.g. Dodelson [45], Peters and Uzan [106], Ellis, Maartens and MacCallum [61].

The basic model is very successful, but has major mysteries: particularly the nature of dark matter on the one hand, and the nature of the dark energy causing acceleration of the universe at recent times on the other. However, like all models, it is an idealization: it represents the background model and linear perturbations around it very well, but the real universe has a nonlinear structure and voids at scales smaller than the Hubble scale [66], which are not well represented by these models.

The FLRW model is a large-scale approximation to these nonlinear structures that is supposed to represent the result of global averaging of inhomogeneities. There are three key issues here:

- local inhomogeneities may affect the averaged large-scale dynamics,
- local inhomogeneities affect photon propagation, and so may affect cosmological observations,
- maybe the universe is after all not spatially homogeneous on the largest scales and is better represented at late times by a Lemaître–Tolman–Bondi (LTB) spherically symmetric model, where we are situated near the centre of a Hubble scale void.

These concerns, which are not mutually exclusive, gain traction because of the mysterious issue of dark energy, whose nature is completely unknown. So the question is not just whether inhomogeneities may significantly affect the interpretation of observations in cosmology; it is whether they can affect the need for dark energy, or at least significantly affect the concordance model parameter values. In brief: is inhomogeneity important for cosmology itself, apart from being central to the study of structure growth?

These are the issues I shall introduce here. There is a large literature on these topics, so I can only refer to representative publications on them in the following sections; most of the relevant papers will be mentioned in the further articles in this focus section. Note that this is not a paper on the use of inhomogeneous models to explore structure formation in the expanding universe: that is a separate, though related, issue.

1.1. Preliminaries

The Einstein field equations (EFE) algebraically determine R_{ab} from the matter tensor T_{ab}:[1]

$$R_{ab} = T_{ab} - \tfrac{1}{2}\, T\, g_{ab} + \Lambda\, g_{ab} \quad \Rightarrow \quad R = -T + 4\,\Lambda\, . \tag{1}$$

When the matter takes a 'perfect fluid' form

$$T_{ab} = (\mu + p)\, u_a\, u_b + p\, g_{ab} \quad \Rightarrow \quad T = -(\mu - 3p) \tag{2}$$

with μ the total energy density and p the isotropic pressure, the Ricci tensor expression is

$$R_{ab} = (\mu + p)\, u_a\, u_b + \tfrac{1}{2}\, (\mu - p + 2\Lambda)\, g_{ab} \quad \Rightarrow \quad R = (\mu - 3p) + 4\,\Lambda\, . \tag{3}$$

This is necessarily the case in a FLRW model. The cosmological constant Λ is equivalent to a Ricci tensor contribution (3) with $\mu_\Lambda + p_\Lambda = 0$. That is, one can represent Λ either on the left-hand side of the EFE as in (1) or on the right-hand side of the EFE as a fluid (2) with the equation of state $p = -\mu$.

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[1] Geometrized units, characterized by $c = 1 = 8\pi G/c^2$, are used throughout.

2. Backreaction effects

2.1. The basic idea

The concept of backreaction from smaller to larger scales was developed in a paper by Brill and Hartle [17] in the context of John Wheeler's idea of geons. It was extended to the case of gravitational radiation in two beautiful papers by Isaacson [77, 78]. He envisaged high-frequency waves superimposed on a slowly varying background:

$$g_{\mu\nu} = \gamma_{\mu\nu} + \epsilon h_{\mu\nu}, \qquad \partial\gamma \simeq \gamma/L, \qquad \partial h \simeq h/\lambda, \tag{4}$$

where L is the lengthscale of variation of the background metric and λ that of the gravitational waves superimposed on the background. Then

$$R_{\mu\nu}(\gamma + \epsilon h) = R^{(0)}_{\mu\nu} + \epsilon R^{(1)}_{\mu\nu} + \epsilon^2 R^{(2)}_{\mu\nu} + \epsilon^3 R^{(3+)}_{\mu\nu} \tag{5}$$

where $R^{(0)}_{\mu\nu} = R^{(0)}_{\mu\nu}(\gamma)$ and the others are functions of $h_{\mu\nu}$. Thus, if the actual spacetime is empty: $R_{\mu\nu}(g) = 0$, the background metric is not that of an empty spacetime: $R^{(0)}_{\mu\nu} \neq 0$, and there is an effective matter term on the right-hand side of the EFE. One finds $R^{(1)}_{\mu\nu} = 0$ and

$$R^{(0)}_{\mu\nu} - \tfrac{1}{2}R^{(0)}\gamma_{\mu\nu} = -8\pi T^{\text{eff}}_{\mu\nu}, \tag{6}$$

$$T^{\text{eff}}_{\mu\nu} = \frac{\epsilon^2}{8\pi}\left(R^{(2)}_{\mu\nu} - \frac{1}{2}R^{(2)}\gamma_{\mu\nu}\right) \tag{7}$$

so the gravitational wave appears as a source of the background. This is backreaction from the small-scale structure to the large-scale structure.

This illustrates the basic backreaction proposal: coarse-graining microstructure results in effective matter components at macro scales that can influence the macro (coarse-grained) dynamics. The issue was taken up *inter alia* by Szekeres [118], who showed that this averaging effect could be expressed in a weak-field polarization formalism in analogy with the electromagnetic case, by MacCallum and Taub [97] who derived Isaacson's results using a two-time Lagrangian formalism and by Noonan [105] who extended Isaacson's formulation to include matter (an astronomical 'medium').

2.2. Non-commutativity of EFE and averaging

The basic point is that averaging the geometry and calculating the field equations do not commute [57, 59]. We use angle brackets to denote averaging over a suitable volume V, so $\overline{g}_{ab} \equiv \langle g_{ab} \rangle$ is the background metric with inverse $\overline{g}^{ab}$ given by $\overline{g}^{ab}\overline{g}_{bc} = \delta^a_c$, and indices should be raised and lowered using the full metric g^{ab}, g_{ab}. Then

$$g_{ab} = \overline{g}_{ab} + \delta g_{ab} \ \ g^{ab}g_{bc} = \delta^a_c, \qquad g^{ab} = \overline{g}^{ab} + h^{ab} \tag{8}$$

shows that

$$(\overline{g}^{ab} + h^{ab})(\overline{g}_{bc} + \delta g_{bc}) = \delta^a_c \tag{9}$$

so $\overline{g}^{ab} \neq \langle g^{ab} \rangle$ and $h^{ab} \neq \delta g^{ab} \equiv g^{ae}g^{bf}(\delta g_{ef})$. Consequently, the Christoffel symbols gain extra terms relative to the averaged Christoffel symbols: $\Gamma^a{}_{bc} = \overline{\Gamma}^a{}_{bc} + \delta\Gamma^a{}_{bc}$ and the Ricci and Einstein tensors in turn gain extra terms:

$$R_{ab} = \overline{R}_{ab} + \delta R_{ab} \ \Rightarrow G_{ab} = \overline{G}_{ab} + \delta G_{ab}; \tag{10}$$

hence, the averaged EFE gain an extra term:

$$G_{ab} = T_{ab} + \Lambda\, g_{ab} \Rightarrow \overline{G}_{ab} + \delta G_{ab} = \overline{T}_{ab} + \Lambda\, g_{ab}. \tag{11}$$

These extra terms are effective matter terms in the large-scale field equations, consequent on the coarse-graining of small-scale inhomogeneities; this is called the backreaction from the smaller to the larger scales, and is consequent on the fact that coarse-graining (or averaging) does not commute with calculating the EFE from the metric tensor:

$$\overline{G}_{ab} = \widetilde{T}_{ab} + \Lambda\, g_{ab}, \quad \widetilde{T}_{ab} := \overline{T}_{ab} - \delta G_{ab}, \tag{12}$$

where $\widetilde{T}_{ab}$ is the effective coarse-grained source term, the second term on the right being the effect of matter averaging and the third term the geometric backreaction effect. The Isaacson gravitational radiation calculation summarized above is a specific example (a vacuum spacetime with a rapidly varying gravitational wave appears to have an effective matter content when viewed on larger scales).

In principle, carrying out that calculation is straightforward but very complex. However, to be certain of the result, one needs to average in the real universe, not the background spacetime. The basic problem then is that averaging involves integration of tensor quantities over a spacetime volume, and so is not a well-defined tensorial operation: changing the coordinates will change the result in an arbitrary way. One can try to handle this by:

1. defining a covariant averaging of tensors via bitensors, or
2. using only field equations involving averaged scalars, perhaps involving a convolution rather than simple averaging, or
3. carrying out the calculation in a weak-field approximation where the integrals can be well defined in a highly symmetric background spacetime, and the difference between the integral in the background spacetime and the real spacetime is negligible, or
4. choosing a uniquely defined physically motivated coordinate system in the fully nonlinear spacetime.

   All have been tried. There are problems in each case:

1. there is no uniquely defined usable bitensor, as the Synge parallel transport bitensor does not work (it leaves the metric tensor invariant);
2. it is not easy to find well-defined scalars that fully define the geometry and dynamics in a generic case;
3. the linearized procedure may not accurately reflect the needed integral in the real spacetime;
4. one is breaking general covariance in this procedure; one has to motivate that the result is physically meaningful.

2.3. Cosmological applications: fitting and averaging

The application of the idea of backreaction to cosmology was raised in [57], see also [64], and then taken up *inter alia* by Futamase [8, 67, 68], Stoeger *et al* [116, 136], and particularly by Buchert and collaborators, first in the Newtonian case [18] and then in the GR case [19]. The implications for cosmology have been discussed more recently by Kolb, Matarrese, Wiltshire, Räsänen, Sussman, and others.

The key point from the discussion above is that backreaction from small-scale inhomogeneity to the large-scale geometry can generate a dynamic effect in the effective Friedmann equation for the cosmology, allowing an acceleration contribution due to backreaction from 'small-scale' inhomogeneities. This has a potential effect on cosmological

parameters [20, 22]; the question is whether it is large enough to give a significant contribution to dark energy. Kolb and Wiltshire propose that it can provide a sufficient source of all the effective dark energy, leading to the possibility of concordance cosmology without Λ. In contrast, as discussed below, many others deny that the effect is important.

The further issue that arises is that while some form of averaging process is in principle what one should do to arrive at the large-scale geometry of the universe on the basis of observations, in practice what is normally done is the inverse. One assumes *a priori* a FLRW model as a background model, and then uses some form of observationally based fitting process to determine its basic parameters [64]. This in effect defines a mapping from the smooth background model into the perturbed more realistic space time, which then *defines* the specific perturbations that occur about the background model [60], for if you change the fitting—what is often called the —you change the perturbations.

Now there are many ways one can conceive of to perform such a fitting, and indeed averaging is one of them: in principle one can average energy densities, pressures, expansion rates, etc, to arrive at a FLRW model from a more accurate representation. However, in practice, fitting is done via astronomical observations down the past null cone, leading to fitting procedures for the FLRW parameters as set out in the paradigmatic paper by Sandage [112], updated by all the myriad other data now used to determine the best-fit FLRW model [4]. Once one has fitted a specific FLRW model to the observable region of the universe, one can then try to determine the specific local deviations from the background model—as for example in all the studies trying to identify the great attractor [6, 90, 95]. Ideally, what one would do is show that both a coarse-graining procedure and a suitable fitting procedure for a realistic lumpy universe model—depicting all the great walls, voids, etc—would give the same result. No one has so far shown how this might work.

An interesting question here is whether (i) there is a scale above which the universe is exactly FLRW, or (ii) at all scales the universe is only ever approximately FLRW. In fact while averaging can in principle lead to an almost homogeneous model to any degree of approximation, it can never lead to exact homogeneity, if the initial model is not homogeneous [116]: there will always remain residual traces of those inhomogeneities. Fitting of course starts off with such an exactly homogeneous model. Thus, in this sense the two cannot be exact inverses of each other, and there cannot be any scale where the universe is exactly FLRW—but it can be very closely so.

Whether these effects are sufficient to significantly alter the cosmological parameters determined from supernova observations [71] is an important ongoing debate involving interesting modelling and general relativity issues, and particularly how one models a universe with genuinely large-scale voids, as well as the nature of the Newtonian limit in cosmology (see the papers by Buchert, Clarkson, Kolb, Räsänen, and Wiltshire in this special section [21, 39, 86, 108, 130]). In this section we consider various approaches to averaging and determining backreaction.

2.3.1. The Zalaletdinov approach. The problem with employing a tensorial averaging procedure is that the result is not covariant: one obtains coordinate-dependent results unless one uses bitensors to define covariant averaging in a local domain, as proposed by Zalaletdinov [132, 133]. This can be done for curvature and matter, but is difficult to do in a unique way for metric itself, because the metric is invariant under parallel propagation, so the Synge bitensor will not work. In any case this approach leads to complex equations that have not yet been productive in terms of the cosmological backreaction problem, despite some valiant attempts [44].

2.3.2. The Buchert approach. Alternatively, one can avoid this problem by only averaging scalars, as Buchert [19, 20] does. He shows this can in principle provide an effective acceleration term in the averaged equations.

The key point is that expansion and averaging do not commute: in any domain *D*, for any field ψ,

$$\partial_t \langle \psi \rangle_D - \langle \partial_t \psi \rangle_D = \langle \Theta \psi \rangle_D - \langle \theta \rangle_D \langle \psi \rangle_D, \tag{13}$$

where Θ is the expansion rate. This leads to Buchert's modified Friedmann and Raychaudhuri equations: e.g.

$$\partial_t \langle \Theta \rangle_D = \Lambda - 4\pi G \rho_D + 2\langle II \rangle_D - \langle I \rangle_D^2, \tag{14}$$

where $II = \Theta^2/3 - \sigma^2$ and $I = \Theta$, σ being the shear. This in principle allows acceleration terms to arise from the averaging process, through the term $\langle II \rangle_D$. To complete the dynamical equations, one needs the shear evolution, but this cannot easily be obtained from the full set of 1+3 dynamic equations through such averaging of scalars. Hence, Buchert's analysis relies on an ansatz for this evolution, which is not fully justified from the underlying dynamics. There are integrability conditions linking the shear to the curvature that give a combined conservation law for curvature plus fluctuations. This forms the basis for the assumed closure conditions, leading to exact classes of solutions where the evolution of the averaged shear is determined. The closure condition replaces what in Friedmannian cosmology would be the equation of state for the sources; here it is the equation of state for the effective sources.

The Buchert equations indicate the broad nature of the effect and are widely used as the basis of further studies, for instance by Kolb *et al*, Wiltshire, and Räsänen. Buchert presents his approach in his article in this focus section. The use of scalars more generally is proposed by Coley [43].

2.3.3. The renormalization group approach. Carfora and Piotrkowska have developed a sophisticated geodesic-ball based averaging approach, *inter alia* using the ideas of the renormalization group [30]. This has led to intriguing analyses of the effects of such averaging on cosmology [23–25, 27], giving formula for averaged effects on cosmic parameters. This is a very sophisticated extension of the basic Buchert programme; indeed it is something of a technical *tour de force* [29]. Its relation to practical cosmological observations is still to be developed.

2.4. Nonlinear models

The previous approaches are not tied in to specific geometric models of the universe. The key issue however is how good the linear models are at representing the nonlinear inhomogeneities in the real universe, with gigantic voids, walls, and so on at larger scales, and mainly empty space at smaller scales [125].

- Voids have been known as a feature of the Megaparsec universe since the first galaxy redshift surveys were compiled. Voids are enormous regions with sizes in the range of $20–50/h$ Mpc that are practically devoid of any galaxy, usually roundish in shape and occupying the major share of space in the universe [38]. Forming an essential ingredient of the cosmic web, they are surrounded by elongated filaments, sheetlike walls, and dense compact clusters.

Various nonlinear models have been developed that try to approximate this kind of situation without using a linearization procedure; they are discussed in the articles by Bolejko, Célérier, and Krasinski (and see [12, 75, 91] for discussions of exact inhomogeneous models). The

original such models were the 'Swiss cheese' models of Einstein and Straus [52, 53], where spherical 'vacuoles' with a spherical mass at the centre are cut out of an expanding FLRW universe model. This gives an exact solution of the EFE with static voids embedded in an expanding universe. However, there is no dynamical backreaction from the inhomogeneities in these models, because the matching conditions between the voids and the expanding universe require that the mass at the centre of each vacuole is the same as would have been there if there were no vacuole.

Models with voids have been developed in depth by Wiltshire [126–128] who has emphasized that time runs differently in the voids, potentially leading to a substantial effect when integrated over long times. Furthermore voids expand while clusters collapse or stay the same size, so the universe becomes void dominated, and the region we live in is increasingly not representative or 'average'. These models can potentially lead to apparent acceleration of the universe [113]. However, the degree to which the models represent the real universe is not clear. These models are discussed in the article by Wiltshire.

A completely different approach is to construct the expanding model from an aggregation of local spherical vacuum regions, joined together at boundary surfaces, as developed first by Wheeler and Lindquist [94]. These models are radically different from all the others in that here one does not start with a FLRW model and then perturb it or excise regions from it: rather a FLRW-like structure emerges at large scales as an approximation to the small-scale vacuum domains with embedded static masses. Thus, there is no backreaction to a large-scale model because there was no such model to begin with. Rather the junctions between local inhomogeneities underlie the large-scale dynamics, which is emergent rather than the result of averaging. This approach has been developed interestingly at recent times by Clifton and Fereira [40, 42]. These models are discussed in the article by Clifton (and see also [121], discussed further below).

2.5. Perturbative approach

In contrast to these attempts at nonlinear models, there is a large literature studying backreaction effects on the basis of linearly perturbed FLRW models. Differing views are held as to the result, reviewed recently by Clarkson and Maartens [37] (and see also [36]). Some workers claim that the weak-field approximation is adequate to describe the nonlinear structures, because the gravitational potential is very small even though the density contrasts are very large, and consequently the backreaction effect is negligible (see [5] for this view). Counter claim by Kolb, Wiltshire, Matarrese and others (see e.g. [87, 129]) emphasize that as there are major voids in the expanding universe, a weak-field kind of approximation to a spatially homogeneous model is not adequate: you have to properly model (possibly quasi-static) voids and their junctions to the expanding external universe, and the linear models are not adequate for this purpose. An in-between view is given by Clarkson, Ananda, and Larena [33].

A recent contribution from the skeptical side is by Green and Wald [74], using an ultra-local averaging procedure to show—in direct contradiction of Buchert's claim—that no acceleration can result from backreaction associated with such averaging, because the effect is trace free. The limiting process embodied in this elegant work probably does not adequately represent the results of averaging over finite volumes, as represented by the other methods discussed here, because it does not in fact involve any such averaging, so this method does not disprove Buchert's results. Indeed it is unlikely that this 'trace-free' result is true for models that genuinely represent averaging over finite volumes, as their short wavelength limit is not obviously related to smoothing over finite size volumes.

There are many workers skeptical of any significant effect, with strong arguments based on the perturbed FLRW approach: the gravitational potentials involved are so small that a quasi-Newtonian analysis is adequate, and the backreaction effect does indeed occur but is negligible. However, others suggest it may be at least large enough to affect the cosmic relation between energy densities and expansion that leads us to deduce that the spatial curvature is almost flat. Greater conceptual clarity on the modelling issues involved is required; the issue is discussed in the articles by Kolb and Clarkson. Three specific issues arise that suggest caution is advisable before accepting the pessimistic view.

2.5.1. The averaging process.   In the weak-field case, the perturbed quantities can be averaged in the background unperturbed Robertson–Walker geometry: a linearized calculations in the background spacetime. This is central to the weak-field approach. But that procedure is inadequate for truly nonlinear cases, where the integral needs to be done over a generic lumpy (nonlinearly perturbed) spacetime that is not 'perturbations' of a high-symmetry background. It is precisely in these cases that the most interesting effects will occur.

2.5.2. Global coordinates: models with genuine voids.   The response often given is that even though the density may be highly nonlinear, in a suitable non-comoving quasi-Newtonian frame the gravitational potential remains very small. Then one has $\delta\rho/\rho \simeq 10^{+28}$ but $\delta\phi/\phi \simeq 10^{-5}$. This is possible because the second derivatives of the potential are not small, and they are what enter the field equations to balance the very large density perturbations [26]; so a suitable linearized approach is acceptable.

Underlying this is the issue of global existence of the quasi-Newtonian coordinates in situations of real inhomogeneity with locally static almost empty spacetimes joining together to form an expanding universe, as envisaged by Lindquist and Wheeler. The case for global validity of these coordinates is put for example by Ishibashi and Wald [80] and by Baumann *et al* [5]. In the Poisson gauge to second order in scalar perturbations the metric is

$$\mathrm{d}s^2 = -(1 + 2\Phi + \Phi^{(2)})\mathrm{d}t^2 - a(t)V_i\,\mathrm{d}x^i\,\mathrm{d}t + a^2(t)[(1 - 2\Psi - \Psi^{(2)})\delta_{ij} + h_{ij}]\mathrm{d}x^i\,\mathrm{d}x^j. \quad (15)$$

The first-order scalar perturbations are given by Φ, Ψ, and the second-order ones by $\Phi^{(2)}$, $\Psi^{(2)}$, which are needed for a consistent analysis of backreaction, as are the vector perturbation V_i and trace-free tensor perturbation h_{ij}.

But the fact that such coordinates can on the one hand be used globally in an asymptotically flat inhomogeneous region, such as the solar system, and on the other in a linearly perturbed FLRW model does not mean it can be used globally for a genuinely inhomogeneous expanding universe model including both kinds of domains, as claimed by Wald and Ishibashi. For example Lindquist and Wheeler [94] do not give a global coordinate system: they match local coordinates to each other across a boundary. But this is not done exactly, because the geometry is too complex to do so. The one case where one can do the job exactly is an expanding two-mass solution with locally static voids joined to create an expanding universe with compact space sections [121]. The surprising result is that the join can only be done across a null surface (a 'horizon'), with intermediate spatially homogeneous anisotropically expanding vacuum regions—it is the existence of these regions that allows the universe to expand. It is not possible to find global coordinates of the form (15) in such a spacetime, as posited by Wald and Ishibashi. Thus, in that case the weak-field arguments do not apply because the coordinate system on which they rely does not exist globally. They may however be possible in Swiss Cheese models, where it is the intervening fluid domains that allow the static vacuum domains to move way from each other, but these are not the kind of situation

we consider here, with galaxies everywhere embedded in genuinely vacuum regions and no fluid-filled domains acting as buffers between them.

So real inhomogeneities have properties that are not the same as perturbed FLRW models that are fluid filled everywhere. The key issue underlying the two-mass result is the rigidity of local spherical vacuum regions that is embodied in Birkhoff's theorem. So a criticism might be that Birkhoff's theorem applies only to exact spherically symmetry vacuum solutions; the argument will not apply to more realistic solutions with almost spherically symmetric vacuum domains. However, this argument is invalid: an 'almost Birkhoff' theorem shows that the Birkhoff result is stable [73]. On this view, the issue is whether (on appropriate averaging scales) the real universe is globally filled with an intergalactic medium that can serve as the substratum allowing expansion to take place in a way compatible with the weak-field view (because there are then in fact no vacuum regions, such as those represented in the Lindquist–Wheeler-type models). This may or may not be the case.

In [26], it is shown that the second derivatives can be of order 1 in the situation given by the other numbers for metrical perturbations. Curvature is thus important and not a perturbation of a flat model, but it is the curvature that drives the backreaction effect. The degree to which a suitable linearized approach is acceptable as a model of genuinely inhomogeneous regions thus remains open to debate, particularly in the case of a linearized treatment on a flat background, where the curvature remains small.

2.5.3. The gauge issue.   Finally, underlying this all is the gauge issue: to what degree are the results dependent on the choice of how the background metric is mapped into the more realistic model? One can after all always find a gauge where the density perturbation $\delta\rho$ is zero [60]. The key is to find a gauge-invariant formalism to tackle the problem—if that is possible [62]. The major attempt to tackle this so far is by Gasperini, Marozzi, and Venziano [69, 70]. This has not yet however led to specific conclusions about cosmological acceleration. This issue is related to the complexities of appropriately defining the background spacetime [64, 88].

The overall conclusion is that while it may at first seem rather unlikely that dynamical backreaction is of significance in the late universe, there are some unresolved questions, so that one should keep an open mind. The issue is debated in some of the following papers in this focus section. Furthermore, it may be important in the early universe: for example Mukhanov *et al* have shown that the backreaction of cosmological perturbations on the background can become important already at energies below the self-reproduction scale in inflationary universe scenarios [102]. However, I will not discuss that context here.

3. Optical effects of local inhomogeneity

Small-scale inhomogeneity can have significant effects on the propagation of photons in a lumpy universe, with potentially important effects on observations. There are three issues here.

3.1. Redshift effects

Firstly, inhomogeneities can affect redshifts, as for example in the Rees–Sciama effect [109] where CMB photons falling into a time-dependent potential well experience an overall change in redshift because they climb out of a different shaped well than when they fell in. Also if light is emitted from a source within a potential well, it will be redshifted as it climbs out; this effect lies behind the 'timescape cosmology' proposal of Wiltshire [127] who points out that the associated time dilation effect is cumulative over the history of the source.

3.2. Area distance effects

Secondly inhomogeneities can affect area distances, which underlie the apparent angular diameter, and hence apparent luminosity, of images [54]. The key point is the difference between Ricci focusing and Weyl focusing, as emphasized by Bertotti [7]. The focussing of an irrotational bundle of null geodesics with tangent vector K^a is given by

$$\mathrm{d}\hat{\theta}/\mathrm{d}v = -R_{ab}K^aK^b - 2\hat{\sigma}^2 - \hat{\theta}^2 \tag{16}$$

$$\mathrm{d}\hat{\sigma}_{mn}/\mathrm{d}v = -E_{mn}, \tag{17}$$

where $\hat{\theta}$ is the expansion and $\hat{\sigma}$ the shear of the null rays, R_{ab} is the Ricci tensor, determined pointwise by the matter distribution, and E_{ab} the electric part of the Weyl tensor, determined non-locally by matter elsewhere.

In the case of Robertson–Walker observations, there is zero Weyl tensor and a non-zero Ricci tensor, so (16), (17) become

$$\mathrm{d}\hat{\theta}/\mathrm{d}v = -R_{ab}K^aK^b - \hat{\theta}^2 \tag{18}$$

$$\mathrm{d}\hat{\sigma}_{mn}/\mathrm{d}v = 0 \tag{19}$$

which are the standard equations underlying observations in a FLRW model. Actual observations however are the opposite: photons travel through empty space (on small scales), described by the zero Ricci tensor and non-zero Weyl tensor: so (16), (17) become

$$\mathrm{d}\hat{\theta}/\mathrm{d}v = -2\hat{\sigma}^2 - \hat{\theta}^2 \tag{20}$$

$$\mathrm{d}\hat{\sigma}_{mn}/\mathrm{d}v = -E_{mn}. \tag{21}$$

This averages out to FLRW equations when averaged over whole sky, which is not obvious! This does not follow from energy conservation *per se*, but rather depends on how area distances average out over the sky. But supernova observations are preferentially made in directions where there is no matter in between to interfere with the observations; hence, area distances, and so cosmological observations, will be different in this case.

The usual way of handling this is to use the Dyer–Roeder (DR) equation [47, 48, 51] that takes matter into account but not shear, because the shear enters the focusing equation quadratically, and so is negligible if shear is small. Thus, the DR equation takes into account only the Ricci focusing due to a specified fraction $f(v)$ of the uniform density of matter in the universe:

$$\mathrm{d}\hat{\theta}/\mathrm{d}v = -f(v)R_{ab}K^aK^b - \hat{\theta}^2. \tag{22}$$

When $f = 1$ one has the FLRW result; when $f = 0$ one has photons travelling through vacuum regions in the clumpy universe.

How this works out depends on how dark matter is clustered, which differs on different scales. The Dyer–Roeder approximation is good if the Weyl focusing term (causing gravitational lensing) can always be neglected in this way; this needs investigation in the light of the expected clustering pattern; many examples are given by Mortsell [100], showing that the effect is potentially significant, and analytic forms by Kantowski [84]. When this approximation is valid, the outcome depends crucially on what fraction of the overall cosmic density (baryonic and non-baryonic) occurs in a smooth form along the line of sight on different scales. Note that on some angular scales the clumping experienced along the line of sight will be partially compensated, in that each void (a low density region on the line of sight) will be matched by a wall (a high-density region) so that the overall density is the same as the

background density. However, they will not exactly compensate because three-dimensionally compensated voids do not reduce to a one-dimensionally compensated distribution of matter along the line of sight [10]. It will also have some impact on the CMB observations [11].

One can investigate these effects in nonlinear models. How it works out in Swiss cheese models is investigated *inter alia* in [83, 85], confirming that there can indeed be a significant effect. The case of observations in a Wheeler–Lindquist-type model is investigated in [40, 42].

The key issue is how empty the voids really are, from supergalactic scales down to the 'vacuum' regions in the solar system. There are some galaxies in the large-scale voids, but are they embedded in an intergalactic gas of baryons and CDM? If so what fraction is its density of the global average density of the model (when smoothed on the largest scales)? The answer does not seem to be known: but the outcome depends crucially on these figures. On the small scales relevant to the supernova observations, one may expect mostly empty space, except perhaps for CDM left over from structure formation, but it is unclear what the relevant fractional density is on these scales.

3.3. Affine parameter effects

Finally, there are effects that arise through altering the relation $z(v)$ between the affine parameter z and the redshift z. These effects have been little studied. However, it is worth noting that it is only through this relation that the cosmological constant can affect observation relations such as the area distance redshift relation (Λ does not explicitly enter the null Raychaudhuri relation (16)). Thus, this may well be interesting to investigate.

Overall these effects are indeed likely to be significant: that is, they may be significant enough to appreciably affect the parameter values of the concordance model of cosmology [81, 105]. How this works out is crucially dependent on how matter is distributed on small scales, and how empty the voids really are. This is an important area for investigation and is discussed by Mattsson.

4. Spatial homogeneity?

So far, I have considered the effect of local inhomogeneities on global dynamics and observations, where 'local' means sufficiently small that we can claim that overall the Copernican principle—the claim that the universe is the same everywhere—still holds when we coarse-grain on large enough scales. The further issue of interest is whether this is in fact the case: might it be that the Copernican principle does not hold, so the FLRW models are in fact misleading models of the large-scale geometry of the visible region of the universe?

The cosmological principle was introduced by Milne in the 1930s, and then formalized in a technical sense by Robertson and Walker. It was the foundation of cosmology in the 1960s to 1980s, see Bondi [15] and Weinberg [124]. But it is an *a priori* philosophical principle. It produces world models that work—namely the standard models of cosmology. But is it true? Can it be tested? Maybe there are inhomogeneous models that would fit the observations as well—or even better.

4.1. The argument for homogeneity

It is not obvious that the universe is spatially homogeneous [55, 56]. We can directly observe isotropy, but not homogeneity, firstly because we effectively observe the universe from one spacetime point, and secondly because when undertaking astronomical observations, the finite speed of light inextricably mixes spatial distance with time.

Arguments for homogeneity are discussed in [58]. Direct determination of homogeneity from number counts is in principle possible, but fails in practice because of the look-back time necessarily associated with all cosmological observations: we cannot uniquely separate out spatial inhomogeneity from a time evolution of sources [55, 103]. Similarly in principle an observational verification of the Mattig magnitude-redshift relation for galaxies in FLRW models [54, 112] (or its generalization to non-zero Λ) would suffice [63]. This in-principle direct determination of homogeneity depends on being precisely fit by FLRW data functions, and does not depend on observations by other fundamental observers. But again this is not practicable. So how can one proceed?

The high degree of isotropy of astronomical observations (averaged on a large enough scale) suggests an observational basis for the assumption of spatial homogeneity. Indeed a universe which is isotropic everywhere is necessarily a FLRW model (Walker [123], Ehlers [49]). But we cannot check if this is true or not: it is an assumption, because we can only test isotropy where we are. However, we can attain a weaker version of the Walker result: Ehlers, Geren and Sachs [50] proved the EGS theorem that isotropy everywhere of the CBR only is sufficient to prove a FLRW geometry, if the universe is expanding. This result has been strengthened even further through generalizations of the EGS theorem to almost isotropy and to models with matter and dark energy [37, 117]. This provides a stronger motivation for spatial homogeneity, but until recently still relied on an untested philosophical assumption: addition of a Copernican principle, assuming that we are not in a special position in the universe, so everyone else will also see isotropic background radiation. The result then follows. However, it is now known that this assumption is indeed at least partly testable via measurements of CMB spectrum distortions, as will be discussed below.

There are a number of other observational tests of the Copernican principle that are now possible, because of observational improvements in the past decade. Before coming to them, I will first discuss the inhomogeneous models that make this an interesting possibility.

4.2. Large-scale inhomogeneity?

The proposal that inhomogeneous models can explain the supernova observations without any dark energy is discussed by Célérier [31] and Tanimoto and Nambu [119]. The idea is that there is a large-scale inhomogeneity of the observable universe such as that described by the Lemaître–Tolman–Bondi (LTB) pressure-free spherically symmetric models ([14], see also see [12, 75, 91]), and we are near the centre of a void. The LTB models have comoving coordinates

$$ds^2 = -dt^2 + B^2(r, t) + A^2(r, t)(d\theta^2 + \sin^2 \theta \, d\phi^2),$$

where

$$B^2(r, t) = A'(r, t)^2 (1 - k(r))^{-1}$$

and the evolution equation is

$$(\dot{A}/A)^2 = F(r)/A^3 + 8pG\rho_\Lambda/3 - k(r)/A^2$$

with the energy density given by $F'(A'A^2)^{-1} = 8pG\rho_M$. There are two arbitrary functions of the spatial coordinate r: namely $k(r)$ (curvature) and $F(r)$ (matter). That this freedom enables us to fit the supernova observations with no dark energy or other exotic physics is a consequence of a theorem proved by Mustapha *et al* [103], updated in [96, 98]. One can also fit the basic nucleosynthesis data and CBR observations because they refer to much larger values of r, see e.g. Alexander *et al* [2]. The key point is that different scales are probed by different astronomical observations and can in principle all be fitted by adjusting the free

spatial functions at different distances. One can also use baryon acoustic oscillation (BAO) measurements to estimate distances [38], but note that to calculate the CBR and BAO results with precision, one must use the LTB perturbation theory [35], not the theory of FLRW perturbations.

A typical observationally viable model is one in which we live roughly centrally (within 10% of the central position) in a large void with a compensated underdense region stretching to $z \simeq 0.08$ with $\delta\rho/\rho \simeq -0.4$ and size $160/h$ Mpc to $250/h$ Mpc, a jump in the Hubble constant of about 1.20 at that distance, and no dark energy or quintessence field [3, 9, 131]. Actually you do not need a void to explain the observations; more general models can do the job [32, 75]. One can also use the more complex Szekeres universes to obtain observationally viable models [79].

One ends up with a degeneracy: both FLRW and LTB models can explain the basic cosmological observations, as was confirmed for example by the SDSS team [114]. One needs more detailed modelling to distinguish which is the better model when precision cosmological observations are taken into account. Before I address these tests, some theoretical objections must be faced.

4.3. Dynamical origins and probability

Given that we can fit the observations by such a model, is there a plausible dynamic scenario for them? Because evolution along individual world lines in such models is governed by the Friedmann equation, inflation followed by a Hot Big Bang era can have the same basic dynamics as in the standard model, but with position-dependent parameters. One argument for homogeneity is that inflation creates a high degree of uniformity, and in the subsequent cosmic evolution, perturbations can only grow to a certain size. Above that scale, we should have the inflation-created uniformity. But that depends on the details supposed for the inflationary epoch. If there are multiple inflaton fields and appropriate inflationary potential and initial conditions, then it should certainly be possible to arrive at an inhomogeneous situation, for example, multi-stream inflation [1] gives such a mechanism. This involves two inflaton fields, a hill in the potential, and tunnelling between different paths from initial to final states, resulting in different numbers of e-foldings in different places. This mechanism can create large over- or under-densities of the kind envisaged here.

Many dismiss these models on probability grounds: it is improbable that we are near the centre of such a model. But there is always improbability in cosmology: we can shift it around, but it is always there. Three comments are in order. First, there simply is no proof that the universe is probable, that is, a philosophical assumption, which may not be true. Second, a study by Linde *et al* [93] shows that (given a particular choice of measure) this kind of inhomogeneity actually is a probable outcome of inflationary theory, with ourselves being located near the centre. And third, Boljeko and Sussman argue [13] that the problem of improbability is ameliorated if one has for example a Szekeres rather than LTB solution.

Overall, one cannot simply dismiss such models out of hand. Philosophical opinions and probability arguments will have to give way to the results of observational testing of these models.

4.4. Observational tests of spatial homogeneity

Given that we can find both inhomogeneous models to reproduce the observations without any exotic energy and homogeneous models with some form of dark energy that explain the same observations, can we distinguish between the two? Ideally, we need a model-independent

test: is a RW geometry the correct metric for the observed universe region, irrespective of assumptions about the dynamics and matter content? Four kinds of tests are possible.

4.4.1. Behaviour near the origin.   The universe must not have a geometric cusp at the origin, as this implies a singularity there. Thus it has been claimed that there are centrality conditions that must be fulfilled in the inhomogeneous models (Vanderveld *et al* [122]). The distance modulus behaves as $\Delta \mathrm{d}m(z) = -(5/2)q_0 z$ in standard ΛCDM models, but if this were true in a LTB void model without Λ this has been said to imply a singularity (Clifton *et al* [41]); observational tests of this requirement will be available from intermediate redshift supernovae in the future. However, [65] and [92] show that this is not a real issue.

4.4.2. Area distance versus Hubble parameter.   Measures of the area distance and Hubble parameter as a function of redshift can give a direct test of spatial homogeneity. There are two geometric effects on distance measurements: curvature Ω_k bends null geodesics and expansion $H(z)$ changes radial distances. In RW geometries, we can combine the Hubble rate and distance data to find the curvature today:

$$\Omega_k = \frac{[H(z)D'(z)]^2 - 1}{[H_0 D(z)]^2}.$$

This relation is independent of all other cosmological parameters, including dark energy model and theory of gravity. It can be used at a single redshift to determine Ω_k, but must give the same result for all redshifts. The important result of Clarkson *et al* [34] is that since Ω_k is independent of z, we can differentiate to get the consistency relation

$$C(z) := 1 + H^2(DD'' - D'^2) + HH'DD' = 0,$$

which depends only on a RW geometry: it is independent of curvature, dark energy, nature of matter, and theory of gravity. Thus, it gives the desired consistency test for spatial homogeneity. In realistic models we should expect $C(z) \simeq 10^{-5}$, reflecting perturbations about the RW model related to structure formation. Errors may be estimated from a series expansion

$$C(z) = \left[q_0^{(D)} - q_0^{(H)}\right]z + O(z^2),$$

where $q_0^{(D)}$ is measured from distance data and $q_0^{(H)}$ from the Hubble parameter. It is simplest to measure $H(z)$ from BAO data. It is only as difficult carrying out this test as carrying out dark energy measurements of $w(z)$ from Hubble data, which requires $H'(z)$ from distance measurements or the second derivative $D''(z)$. Another promising approach is to use the time drift of cosmological redshifts as a way of determining these functions [120]. An analysis of how well the time drift of redshift $\dot{z}$ can distinguish an LTB model from a FLRW model is given in [46].

This is the simplest direct test of spatial homogeneity, and its implementation should be regarded as a high priority: for if it confirms spatial homogeneity, that reinforces the evidence for the standard view in a satisfying way, but if it does not, it has the possibility of undermining the entire project of searching for a physical form of dark energy.

In the future, the same measurements can potentially be carried out by gravitational wave observations of black hole binary mergers [76, 82, 107].

4.4.3. The CMB spectrum: verifying the EGS theorem conditions.   The peaks in the CMB anisotropy power spectrum can be adequately accommodated in the LTB family of models [38]. The key further point is that one can use scattered CMB photons to check CMB isotropy

at points away from the origin (Goodman [72]; Caldwell and Stebbins [28]), thus checking some of the conditions required by the EGS theorem.

If the CMB radiation is anisotropic around distant observers (as will be true in inhomogeneous models), then the Sunyaev–Zeldovich scattered photons will cause a distorted CMB spectrum, as anisotropy of the CMB out there will cause a mixing of temperatures in the scattered photons. Such anisotropy can arise in two ways [28]. First, the kinematic SZ effect occurs due to relative motion between matter and the CMB at distant points. Gradients in the void gravitational potential causes gas to move relative to CMB frame; hence, there will be a CMB dipole out there. This violates the EGS conditions, and scattering mixes these temperatures, causing a spectral distortion. Second, potential wells cause anisotropy due to gravitational redshift effects. If some photons originate inside the void and others outside, this again causes a locally anisotropic CMB out there, and SZ scattering compares potentials at the two locations.

It has recently been claimed by two groups that such CMB observations disprove inhomogeneity [101, 134], but counter claims [38] give specific models where the CMB observations are acceptably accounted for.

The problem seems to be first that the papers [101, 134] refer to restricted families of LTB models, which have to be generalized to include radiation effects in order to handle the CMB observations; the radiation and the matter may not be comoving [38]. Also if one only considers LTB models with fixed bang time, one has removed half the freedom of the LTB models; it is then hardly surprising if fitting the observations is difficult. Generic analysis should allow varying the bang time. Second, these are not self-consistent studies, as they use FLRW perturbation theory to study structure formation in LTB models. One needs to use LTB perturbation theory [35] to get consistent results.

Future work of interest here will be to check to what degree such tests can verify the full requirements of the extended versions of the EGS theorem discussed by Clarkson and Maartens [37]. Can they fully test the needed anisotropy requirements for one of the extended versions of the EGS results, or do they only serve as partial checks of the needed conditions, because they only check mixing of lower order CMB multipoles?

4.4.4. Thermal history-based tests.   If the kinds of structures that occur in distant regions are similar to those nearby, this indicates that the thermal histories leading to the existence of those structures must have been the same, and this suggests that the universe must have been spatially homogeneous at the relevant early times—which will imply that it is homogeneous today. This is the *postulate of uniform thermal histories* (PUTH) [16]. Conversely, if the kinds of objects that have come into being far away look different from those nearby, this indicates spatial inhomogeneity.

In principle this can be applied for example to studies of galaxies and and large-scale structure; this has not yet been formally done. However, a present application is to element abundances. There are now claims of some anomalies in the abundance of lithium with distance (see e.g. [89]). Regis and Clarkson [110] show that this can be taken as indirect evidence for spatial inhomogeneity.

Observations in inhomogeneous models are discussed in the papers by Zibin and Moss [135], and by Marra and Notari [99].

5. Conclusion

An implicit averaging is effectively at the foundation of how the standard model deals with matter and structure formation, while being uniform on large scales. The problem

of averaging is far from being solved—but it is a problem that will not go away. Small-scale inhomogeneities may possibly cause observable effects through dynamical backreaction, but this is a controversial suggestion. Ultimately, we probably need a general relativistic simulation of structure formation to resolve the issue of averaging. However, such inhomogeneities certainly can significantly affect the observational determination of the parameters of the concordance cosmological model. Whether this is the case or not depends on the detailed nature of clustering of dark matter, on small scales, in the universe, which is not known at present.

Additionally, we must take seriously the idea that the acceleration apparently indicated by supernova data could be due to large-scale inhomogeneity with no dark energy. Observational tests of the latter possibility are as important as pursuing the dark energy (exotic physics) option in a homogeneous universe. Theoretical prejudices as to the universe's geometry, and our place in it, must bow to such observational tests. Precisely because of the foundational nature of the Copernican principle for standard cosmology, we need to fully check this foundation. And one must emphasize here that standard CMB anisotropy studies do not *prove* the Copernican principle: they *assume* it at the start.

Whatever the outcome of these studies, the point remains that inhomogeneity is a critical topic in cosmology. Simplified models of inhomogeneity such as LTB models, where we can actually calculate dynamics and predict observational relations, are an important part of the necessity to probe every aspect of the standard model, as are studies of the nature of the backreaction effect and the effects of inhomogeneities on observations.

5.1. To be done

To complete our understanding of this issue *inter alia* we need to

- develop a general relativistic simulation of structure formation,
- develop perturbation studies of the LTB models, and hence CMB anisotropies and LSS observations, in a self-consistent way,
- develop the PUTH approach [16] for galaxies and LSS,
- use observations and simulations to characterize in detail the DM inhomogeneity on small scales, and find out to what degree it clusters with baryons on these scales,
- hence to characterize in detail the DM and baryonic IGM (inter-galactic medium) that may permeate the 'voids' in the cosmic web, at different scales,
- along with determining homogeneity, we really need to determine the smallest length scale on which the universe is almost FLRW, if indeed it is almost FLRW on large scales. This is related to the possibility that some of the data we use for determining the cosmology may not be probing almost FLRW scales of the universe—e.g. may not be probing the Hubble flow.

Finally on should remember that the issues mentioned here are not mutually exclusive. If we do live in a Hubble scale inhomogeneity, the universe is additionally inhomogeneous on smaller scales. Hence, the eventual aim must be to investigate the combination of all these effects.

Acknowledgments

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