Newtonian–Machian analysis of the neo-Tychonian model of planetary motions

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Corrigendum: Newtonian–Machian analysis of neo-Tychonian model of planetary motions

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Abstract
Two typographical errors in Popov 2013 *Eur. J. Phys.* **34** 383 are corrected.

Equation (4.4) had the wrong overall sign and should instead read:

$$U_{\rm ps}(\mathbf{r}) = \frac{GmM_S}{r_{SE}^2}\hat{\mathbf{r}}_{SE}\cdot\mathbf{r}. \tag{4.4}$$

The second term of the right-hand side of equation (4.7) had the wrong sign, and it should instead read:

$$L_{ME} = \frac{1}{2}m_M\dot{\mathbf{r}}_{ME}^2 + \frac{Gm_MM_S}{|\mathbf{r}_{ME}-\mathbf{r}_{SE}|} - \frac{Gm_MM_S}{r_{SE}^2}\hat{\mathbf{r}}_{SE}\cdot\mathbf{r}_{ME}. \tag{4.7}$$

These errors are merely typographical and they do not influence the results and conclusions in the original paper.

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Newtonian–Machian analysis of the neo-Tychonian model of planetary motions

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Abstract

The calculation of the trajectories in the Sun–Earth–Mars system is performed using two different models, both in the framework of Newtonian mechanics. The first model is the well-known Copernican system, which assumes that the Sun is at rest and that all the planets orbit around it. The second is a less well-known model, developed by Tycho Brahe (1546–1601), according to which the Earth stands still, the Sun orbits around the Earth, and the other planets orbit around the Sun. The term 'neo-Tychonian system' refers to the assumption that orbits of distant masses around the Earth are synchronized with the Sun's orbit. It is the aim of this paper to show the kinematical and dynamical equivalence of these systems, under the assumption of Mach's principle.

(Some figures may appear in colour only in the online journal)

1. Introduction

Discussion of the motion of celestial bodies is one of the most interesting episodes in the history of science. There were two diametrically opposite schools of thought: one assumed that the Sun stands still, and the Earth and other planets orbit around it; while the other assumed that the Earth stands still, and the Sun and the other planets in some manner orbit around the Earth. The first school of thought comes from Aristarchus (310–230 BC) and is generally addressed as *heliocentrism*, while the other originated with Ptolemy (90–168 BC) and is generally known as *geocentrism*. Since Aristotle, the ultimate authority in science for more than two millennia, accepted the geocentric assumption, it became the dominant viewpoint among scientists of the time. The changeover came with Copernicus (the so-called 'Copernican revolution') who in his work *De Revolutionibus* proposed a hypothesis that the Sun stands in the middle of the known Universe, and that the Earth orbits around it, together with other planets. Copernicus's system was merely better than Ptolemy's, because Copernicus assumed that the trajectories of the planets are perfect circles, and required the same number of epicycles (sometimes even more) as Ptolemy's model [1]. The accuracy of Ptolomy's model is still a subject of lively debate among historians of science [2].

The next episode in this controversy was Kepler's system, with elliptical orbits of planets around the Sun. This system did not require epicycles, it was precise and elegant. It is therefore the general view that Kepler's work finally settled the question of whether it is the Sun or the Earth that moves. But what is less well-known is that Tycho Brahe, Kepler's tutor, developed a geostatic system that was just as accurate and elegant as Kepler's: the Sun orbits around the Earth, and all the other planets orbit around the Sun. The trajectories are ellipses, and all Kepler's laws are satisfied. At that time, Kepler's and Brahe's models were completely equivalent and equally elegant, since neither of them could explain the mechanism behind and the reasons for the orbits being the way they are. This had to wait for Newton.

Sir Isaac Newton, as is generally accepted, gave the ultimate explanation for planetary motion that accorded with Kepler's model, and excluded Brahe's. The laws of motion and the inverse square law of gravity were able to reproduce all the observed data only with the assumption that the Sun (i.e. the center of mass of the system, which can be very well approximated by the center of the Sun) stands still, and all the planets move around it. According to Newton's laws, it is impossible for the small Earth to keep the big Sun in its orbit: the gravitational pull is just too weak. This argument is very strong, and it seemed to settle the question for good.

However, at the end of 19th century, the famous physicist and philosopher Ernst Mach (1839–1916) came up with the principle that states the equivalence of non-inertial frames. Using the famous 'Newton's bucket' argument, Mach argues that all so-called pseudo-forces (forces which result from accelerated motion of the reference frame) are in fact *real* forces originating from the accelerated motion of distant masses in the Universe, as seen by the observer in the non-inertial frame. Some go even further, stating that 'every single physical property and behavioral aspect of isolated systems is determined by the whole Universe' [3]. According to Mach's principle, the Earth could be considered as the 'pivot point' of the Universe: the fact that the Universe is orbiting around the Earth will create the exact same forces that we usually ascribe to the motion of the Earth.

Mach's principle played a major role in the development of Einstein's general theory of relativity [4], as well as other developments in gravitational theory, and has inspired some interesting experiments [5]. This principle still serves as a guide for some physicists who attempt to reformulate ('Machianize') Newtonian dynamics [6, 7], or try to construct new theories of mechanics [8]. Some arguments against and critiques of Mach's principle have also been raised [9]. Since the time of its original appearance [10–12], Mach's principle has been reformulated in a number of different ways [13, 14]. For the purpose of this paper, we will focus on only one of the consequences of Mach's principle: that the inertial forces can be seen as resulting from real interactions with distant matter in the Universe, as was for example shown by Zylbersztajn [15].

The only question remains: *are these forces by themselves enough to explain all translational motions that we observe from the Earth, and can they reproduce Brahe's model?* The discussion in this paper will show that the answer to this question is positive. In order to demonstrate it, we will consider the Sun–Earth–Mars system.

The paper is organized as follows. In section 2 an overview is given for the two-body problem in the central potential and Kepler's problem. In section 3 the calculations for the Earth's and Mars's trajectories are performed in the heliocentric system, both analytically (by applying the results from the previous section) and numerically. In section 4 the calculations of the Sun's and Mars's trajectories are performed in the geocentric system, due to the presence of the pseudo-potential originating from the fact of the accelerated motion of the Universe. Finally, the conclusion of the analysis is given.

2. The two-body problem in the central potential

2.1. General overview

We start with the overview of the two-body problem in Newtonian mechanics. Although there are alternative and simpler ways to solve this problem [16, 17], we follow the usual textbook approach [18, 19]. The Lagrangian of the system reads

$$L = \tfrac{1}{2}m_1\dot{\mathbf{r}}_1^2 + \tfrac{1}{2}m_2\dot{\mathbf{r}}_2^2 - U(|\mathbf{r}_1 - \mathbf{r}_2|), \tag{2.1}$$

where U is potential energy that depends only on the magnitude of the difference of radii vectors (so-called *central potential*). We can easily rewrite this equation in terms of relative position vector $\mathbf{r} \equiv \mathbf{r}_1 - \mathbf{r}_2$, and let the origin be at the centre of mass, i.e. $m_1\mathbf{r}_1 + m_2\mathbf{r}_2 \equiv 0$. The solution to these equations is

$$\mathbf{r}_1 = \frac{m_2}{m_1 + m_2}\mathbf{r}, \qquad \mathbf{r}_2 = -\frac{m_1}{m_1 + m_2}\mathbf{r}. \tag{2.2}$$

So the Lagrangian (2.1) becomes

$$L = \tfrac{1}{2}\mu\dot{\mathbf{r}}^2 - U(r), \tag{2.3}$$

where $r \equiv |\mathbf{r}|$ and μ is *the reduced mass*

$$\frac{1}{\mu} = \frac{1}{m_1} + \frac{1}{m_2}. \tag{2.4}$$

In that manner, the two-body problem is reduced to a one-body problem of a particle with coordinates $\mathbf{r}$ and mass μ in the potential $U(r)$.

Using polar coordinates, the Lagrangian (2.3) can be written as:

$$L = \tfrac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2) - U(r). \tag{2.5}$$

One can immediately see that variable ϕ is cyclic (it does not appear in the Lagrangian explicitly). The consequence of that fact is the momentum conservation law, since $(\partial/\partial t)(\partial L/\partial\dot{\phi}) = \partial L/\partial\phi = 0$. Therefore,

$$\ell \equiv \frac{\partial L}{\partial\dot{\phi}} = \mu r^2\dot{\phi} = \text{const.} \tag{2.6}$$

is the integral of motion.

In order to find a solution for the trajectory of a particle, it is not necessary to write the Euler–Lagrange equations explicitly. Instead, one can use the energy conservation law,

$$E = \frac{1}{2}\mu(\dot{r}^2 + r^2\dot{\phi}^2) + U(r) = \frac{1}{2}\mu\dot{r}^2 + \frac{\ell^2}{2\mu r^2} + U(r). \tag{2.7}$$

Straightforward integration of (2.7) gives the equation for the trajectory,

$$\phi(r) = \int \frac{\ell\ \mathrm{d}r/r^2}{\sqrt{2\mu\,[E - U(r)] - \ell^2/r^2}}. \tag{2.8}$$

2.2. Kepler's problem

Let us now consider the particle in the potential

$$U(r) = -\frac{k}{r}, \tag{2.9}$$

generally known as *Kepler's problem*. Since our primary interest is in the planetary motions under the influence of gravity, we will take $k > 0$. Integration of equation (2.8) for that potential gives

$$\frac{p}{r} = 1 + e\cos\phi, \tag{2.10}$$

where $2p$ is called *lactus rectum* of the orbit, and e is *eccentricity*. These quantities are given by

$$p = \frac{\ell^2}{\mu k}, \qquad e = \sqrt{1 + \frac{2E\ell^2}{\mu k^2}}. \tag{2.11}$$

Expression (2.10) is the equation of a conic section with one focus in the origin. For $E < 0$ and $e < 1$ the orbit is an ellipse.

One can also determine minimal and maximal distances from the source of the potential, called *perihelion* and *aphelion*, respectively

$$r_{\min} = \frac{p}{1+e}, \qquad r_{\max} = \frac{p}{1-e}. \tag{2.12}$$

These parameters can be observed directly, and often are used to test a model or a theory regarding planetary motions.

3. The Earth and Mars in the heliocentric perspective

According to Newton's law of gravity, the force between two massive objects reads

$$\mathbf{F} = -\frac{Gm_1m_2}{|\mathbf{r}_1 - \mathbf{r}_2|^3}(\mathbf{r}_1 - \mathbf{r}_2), \tag{3.1}$$

which leads to a potential ($\mathbf{F} = -\nabla U$)

$$U(|\mathbf{r}_1 - \mathbf{r}_2|) = -\frac{Gm_1m_2}{|\mathbf{r}_1 - \mathbf{r}_2|}. \tag{3.2}$$

This is obviously Kepler's potential (2.9) with $k = Gm_1m_2$, where G is Newton's gravitational constant.

Since the Sun is more than five orders of magnitude more massive than the Earth and Mars, we will in all future analysis use the approximation

$$\mu \approx m_i, \tag{3.3}$$

where m_i is mass of the observed planet. For the same reason, gravitational interaction between the Earth and Mars can be neglected, since it is negligible compared to the interaction between the Earth/Mars and the Sun.

Using these assumptions, we can write down corresponding Lagrangians,

$$\begin{aligned} L_{ES} &= \frac{1}{2}m_E\dot{\mathbf{r}}_{ES}^2 + \frac{Gm_EM_S}{r_{ES}}, \\ L_{MS} &= \frac{1}{2}m_M\dot{\mathbf{r}}_{MS}^2 + \frac{Gm_MM_S}{r_{MS}}, \end{aligned} \tag{3.4}$$

where m_E and m_M are masses of the Earth and Mars, respectively. Subscripts ES (MS) correspond to the motion of the Earth (Mars) with respect to the Sun. These trajectories

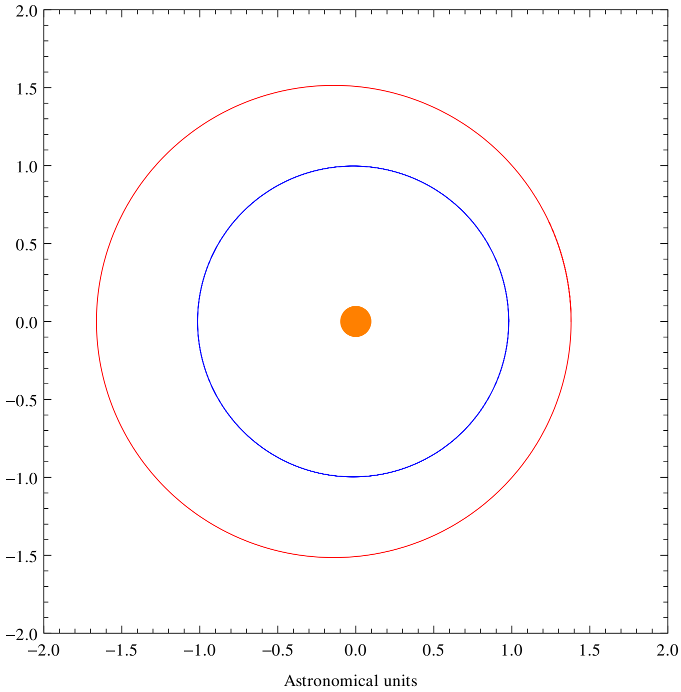

Figure 1. Trajectories of the Earth and Mars in the heliocentric system over the period of two years. Blue and red lines represent the Earth's and Mars's orbits, respectively.

can be calculated using the exact solution (2.10) with appropriate strength constants k and initial conditions which determine E and ℓ. Another way is to solve the Euler–Lagrange equations numerically, using astronomical parameters [20] (e.g. aphelion and perihelion of the Earth/Mars) to choose the initial conditions that fit the observed data. The former has been done using the *Wolfram Mathematica* package. The result is shown in figure 1.

For the latter comparison, one could write the expressions for the e and p parameters for the Earth. Putting the expressions for energy (2.7) and momentum (2.6) into equation (2.11) it is straightforward to obtain

$$p = \frac{\dot{\phi}^2 r^4}{GM_S},$$

$$e = \sqrt{1 - \frac{2GM_S\dot{\phi}^2 r^3 - \dot{r}^2\dot{\phi}^2 r^4 - \dot{\phi}^4 r^6}{G^2 M_S^2}}, \qquad (3.5)$$

where $\dot{\phi}$, $\dot{r}$ and r are angular velocity, radial velocity and distance respectively, taken in the same moment of time (e.g. in $t = 0$).

Figure 2 displays motion of Mars as viewed from the Earth, gained by trivial coordinate transformation

$$\mathbf{r}_{ME}(t) = -\mathbf{r}_{ES}(t) + \mathbf{r}_{MS}(t), \qquad (3.6)$$

where $\mathbf{r}_{ES}(t)$ and $\mathbf{r}_{MS}(t)$ are solutions of the Euler–Lagrange equations for the Lagrangians (3.4). Equation (3.6) is just the mathematical expression of Brahe's claim. Retrograde motion

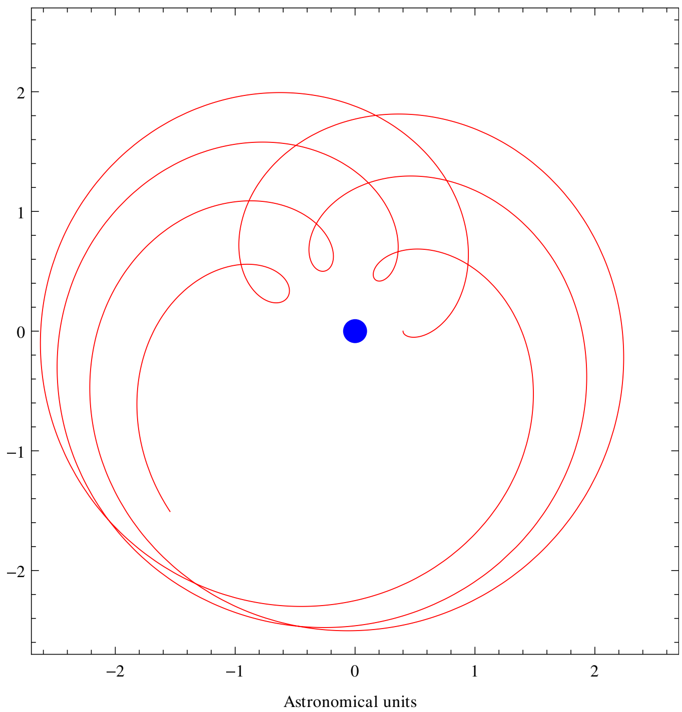

Figure 2. Trajectory of Mars as seen from the Earth over the period of seven years. Calculation of this trajectory is done numerically in the heliocentric system.

of Mars can be useful in the attempt to understand and determine orbital parameters, as was shown qualitatively and quantitatively by Thompson [21].

The acceleration that the Earth experiences due to the gravitational force of the Sun is usually referred as *centripetal acceleration* and is given by

$$\mathbf{a}_{\mathrm{cp}} = \frac{\mathbf{F}_{\mathrm{cp}}}{m_E} = -\frac{GM_S}{r_{ES}^2}\hat{\mathbf{r}}_{ES}, \tag{3.7}$$

where $\hat{\mathbf{r}}$ is the unit vector in the direction of vector $\mathbf{r}$, $\mathbf{r}_{ES}(t)$ is the radius vector describing motion of the Earth with respect to the Sun, and $\mathbf{F}_{\mathrm{cp}}$ is centripetal force, i.e. the force that causes the motion.

4. The Sun and Mars in the geocentric perspective

4.1. The pseudo-potential

From the heliocentric perspective, the fact that the Earth moves around the Sun results in centrifugal pseudo-force, observed only by the observer on the Earth. But if we apply Mach's principle to the geocentric viewpoint, one is obliged to speak about the *real* forces resulting from the fact that the Universe as a whole moves around the observer sitting on the stationary Earth. Although these forces will be further considered as the real forces, we well keep the usual terminology and call them pseudo-forces, for the sake of convenience. Our focus here is on the annual orbits, not on diurnal rotation which requires some additional physical assumptions [8, 22] that are beyond the scope of this paper.

The Universe is regarded as an $(N+1)$-particle system (N celestial bodies plus the planet Earth). From the point of a stationary Earth, one can write down the Lagrangian that describes the motions of celestial bodies:

$$L = \frac{1}{2}\sum_{i=1}^{N} m_i \dot{\mathbf{r}}_i^2 - \frac{1}{2}\sum_{i=1}^{N} \frac{Gm_i m_j}{r_{ij}} - \sum_{i=1}^{N} \frac{Gm_E m_i}{r_i} - U_{\mathrm{ps}}, \tag{4.1}$$

where $r_{ij} \equiv |\mathbf{r}_i - \mathbf{r}_j|$, U_{ps} stands for pseudo-potential, satisfying $\mathbf{F}_{\mathrm{ps}} = -\nabla U_{\mathrm{ps}}$. $\mathbf{F}_{\mathrm{ps}}$ is the pseudo-force given by

$$\mathbf{F}_{\mathrm{ps}} = -m \sum_{i=1}^{N} \mathbf{a}_{\mathrm{cp},i}, \tag{4.2}$$

where $\mathbf{a}_{\mathrm{cp},i}$ is centripetal acceleration for given celestial body (with respect to the Earth) and m is a mass of the object that is subjected to this force. It is easy to see that the dominant contribution in these sums comes from the Sun. The close objects (planets, moons, etc) are much less massive than the Sun, and the massive objects are much further away. The same approximation is used implicitly in section 3.

In the Machian picture, the centripetal acceleration is a mere relative quantity, describing the rate of change of relative velocity. Therefore, centripetal acceleration of the Sun with respect to the Earth is given by equation (3.7), with $\mathbf{r}_{ES} = -\mathbf{r}_{SE}$. Considering all that, equation (4.2) becomes

$$\mathbf{F}_{\mathrm{ps}} = -\frac{GmM_S}{r_{SE}^2}\hat{\mathbf{r}}_{SE}, \tag{4.3}$$

where $\mathbf{r}_{SE}(t)$ describes the motion of the Sun around the Earth, and m is the mass of the body under consideration.

We can now finally write down the pseudo-potential that influences every body observed by the still observer on Earth:

$$U_{\mathrm{ps}}(\mathbf{r}) = -\frac{GmM_S}{r_{SE}^2}\hat{\mathbf{r}}_{SE}\cdot\mathbf{r}, \tag{4.4}$$

where $\mathbf{r}(t)$ describes the motion of the particle of mass m with respect to the Earth. Note that this is not a central potential.

4.2. The Sun in the Earth's pseudo-potential

In order to determine the Sun's orbit in Earth's pseudo-potential, one needs to take dominant contributions of the Lagrangian (4.1), as was explained earlier. Taking into account the expression for pseudo-potential given in equation (4.4), one ends up with

$$L_{SE} = \frac{1}{2}M_S\dot{\mathbf{r}}_{SE}^2 - \frac{GM_S^2}{r_{SE}}. \tag{4.5}$$

This Lagrangian has the exact same form as the reduced Lagrangian (2.3). That means that we can immediately determine the orbit by means of equation (2.11) by substituting $\mu = M_S$ and $k = GM_S^2$. This leads to the following result (subscript SE will be omitted):

$$p = \frac{\dot{\phi}^2 r^4}{GM_S},$$

$$e = \sqrt{1 - \frac{2GM_S\dot{\phi}^2 r^3 - \dot{r}^2\dot{\phi}^2 r^4 - \dot{\phi}^4 r^6}{G^2M_S^2}}, \tag{4.6}$$

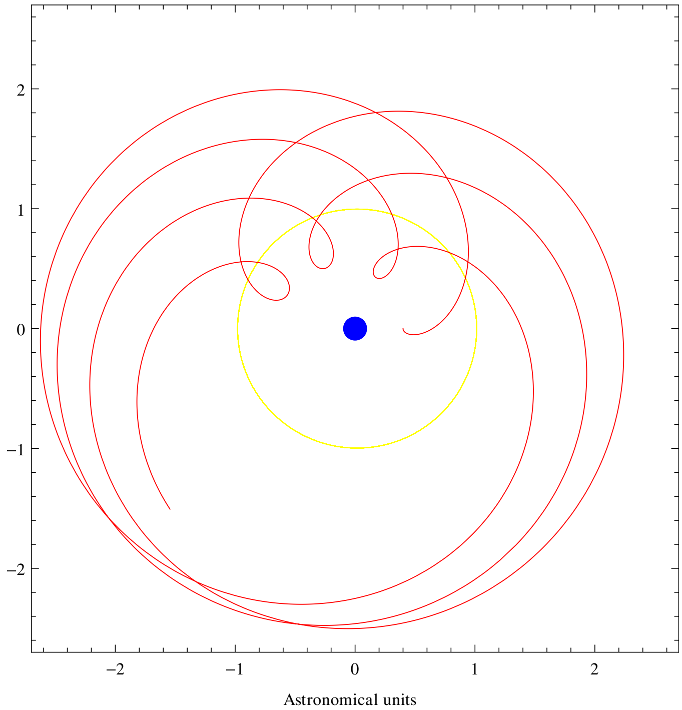

Figure 3. Trajectories of the Sun (yellow) and Mars (red) moving in the Earth's pseudo-potential over the period of seven years. Calculation of this trajectory is performed numerically in the geocentric system.

which is the exact equivalent to the previous result given in equation (3.5), since $\dot{\phi}$, $\dot{r}$ and r are relative quantities, by definition equivalent in both models. We can therefore conclude that the Sun's orbit in the Earth's pseudo-potential is equivalent to that observed from the Earth in the heliocentric system.

It remains to show the same thing for Mars's orbit.

4.3. Mars in the Earth's pseudo-potential

In the similar way to before, we take the dominant contributions of Lagrangian (4.1) together with equation (4.4) and form the following Lagrangian:

$$L_{ME} = \frac{1}{2} m_M \dot{\mathbf{r}}_{ME}^2 - \frac{G m_M M_S}{|\mathbf{r}_{ME} - \mathbf{r}_{SE}|} - \frac{G m_M M_S}{r_{SE}^2} \hat{\mathbf{r}}_{SE} \cdot \mathbf{r}_{ME}, \tag{4.7}$$

where subscript ME refers to the motion of Mars with respect to the Earth, and $\mathbf{r}_{SE}(t)$ is the solution of the Euler–Lagrange equations for the Lagrangian (4.5).

The Euler–Lagrange equations for $\mathbf{r}_{ME}(t)$ using Lagrangian (4.7) are too complicated to be solved analytically, but they can easily be solved numerically. The numerical solutions for the equations of motion for both the Sun and Mars are displayed in figure 3. The equivalence of the trajectories gained in the two different ways is obvious, justifying the model proposed by Brahe.

5. Conclusion

The analysis of planetary motions has been performed in the Newtonian framework with the assumption of Mach's principle. The kinematical equivalence of the Copernican (heliocentric) and the neo-Tychonian (geocentric) systems is shown to be a consequence of the presence of pseudo-potential (4.4) in the geocentric system, which, according to Mach, must be regarded as the real potential originating from the fact of the simultaneous acceleration of the Universe. This analysis can be performed on any other celestial body observed from the Earth. Since the Sun and Mars are chosen arbitrarily, and there is nothing special about Mars, one can expect to obtain the same general conclusion.

There is another interesting remark that follows from this analysis. If one could put the whole Universe in accelerated motion around the Earth, the pseudo-potential corresponding to pseudo-force (4.2) will immediately be generated. That same pseudo-potential then causes the Universe to stay in that very state of motion, without any need for exterior forces acting upon it.

Acknowledgments

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