The dynamical description of the geocentric Universe

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Abstract. Using Mach's principle, we will show that the observed diurnal and annual motion of the Earth can just as well be accounted as the diurnal rotation and annual revolution of the Universe around the fixed and centered Earth. This can be performed by postulating the existence of vector and scalar potentials caused by the simultaneous motion of the masses in the Universe, including the distant stars.

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1. Introduction

The modern day use of the word *relativity* in physics is usually connected with Galilean and special relativity, i.e. the equivalence of the systems performing the uniform rectlinear motion, so-called *inertial frames*. Nevertheless, the physicists and philosophers never ceased to debate the various topics under the heading of *Mach's principle*, which essentially claims the equivalence of all co-moving frames, including non-intertial frames as well.

Historically, this issue was first brought out by Sir Isaac Newton in his famous rotating bucket argument. As Newton saw it, the bucket is rotating in the absolute space and that motion produces the centrifugal forces manifested by the concave shape of the surface of the water in the bucket. The motion of the water is therefore to be considered as "true and absolute", clearly distinguished from the relative motion of the water with respect to the vessel [1].

Mach, on the other hand, called the concept of absolute space a "monstrous conception" [2], and claimed that the centrifugal force in the bucket is the result only of the relative motion of the water with respect to the masses in the Universe. Mach argued that if one could rotate the whole Universe around the bucket, the centrifugal forces would be generated, and the concave-shaped surface of the water in the bucket would be identical as in the case of rotating bucket in the fixed Universe. Mach extended this principle to the once famous debate between geocentrists and heliocenstrists, claiming that both systems can equally be considered correct [3].

His arguments, however, remained of mostly philosophical nature. Since he was convinced empiricist, he believed that science should be operating only with observable facts, and the only thing we can observe are relative motions. Therefore, every notion of absolute motion or a preferred inertial frame, whether inertial or non-inertial, is not a scientific one but rather a mathematical or philosophical preference.

As Hartman and Nissim-Sabat [4] correctly point out, Mach never formulated the mathematical model or an alternative set of physical laws which can explain the motions of the stars, the planets, the Sun and the Moon in a Tychonian or Ptolemaic geocentric systems. For that reason, some physicist in the modern days have tried to "Machianize" the Newtonian mechanics in various ways [5, 6], or even try to construct new theories of mechanics [7]. There have also been attempts to reconcile Mach's principle with the General Theory of Relativity, some of which were profoundly analyzed in the paper by Raine [8].

In the recent paper [9] we have used the concept of the so-called pseudo-force and derived the expression for the potential which is responsible for it. This potential can be considered as a real potential (as shown by Zylbersztajn [10]), which can easily explain the annual motion of the Sun and planets in the Neo-Tychonian system. In the same manner, one can explain the annual motion of the stars and the observation of the stellar parallax [11].

It is the aim of this paper to use the same approach to give the dynamical explanation of the diurnal motion of the celestial bodies as seen from the Earth, and thus give the mathematical justification for the validity of Mach's arguments regarding the equivalence of the Copernican and geocentric systems.

The paper is organized as follows. In section the vector potential is introduces in general terms. This formalism is then applied to analyze the motions of the celestial bodies as seen from the Earth in section . Finally, the conclusion of the analysis is given.

2. Vector potential formalism

Following Mach's line of thought, one can say that the simultaneously rotating Universe generates some kind of gravito-magnetic vector potential, $\mathbf{A}$. By the analogy with the classical theory of fields [12] one can write down the Lagrangian which includes the vector potential,

$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 + m\,\dot{\mathbf{r}}\cdot\mathbf{A} + \frac{1}{2}m\mathbf{A}^2 - mU_{\text{ext}}\,, \tag{2.1}$$

where m is the mass of the particle under consideration, and U_{ext} is some external scalar potential imposed on the particle, for example the gravitational interaction.

We know, as an observed fact, that every body in the rotational frame of reference undergoes the equations of motion given by [13]

$$m\ddot{\mathbf{r}} = \mathbf{F}_{\text{ext}} - 2m(\boldsymbol{\omega}_{\text{rel}} \times \dot{\mathbf{r}}) - m\left[\boldsymbol{\omega}_{\text{rel}} \times (\boldsymbol{\omega}_{\text{rel}} \times \mathbf{r})\right]\,, \tag{2.2}$$

where $\boldsymbol{\omega}_{\text{rel}}$ is relative angular velocity between the given frame of reference and the distant masses in the Universe, and $\mathbf{F}_{\text{ext}} = -\nabla U_{\text{ext}}$ some external force acting on a particle.

It can be easily demonstrated that one can derive Equation (2.2) by applying the Euler-Lagrange equations on the following "observed" Langrangian

$$L_{\text{obs}} = \frac{1}{2}m\dot{\mathbf{r}}^2 + m\,\dot{\mathbf{r}}\cdot(\boldsymbol{\omega}_{\text{rel}} \times \mathbf{r}) + \frac{1}{2}m(\boldsymbol{\omega}_{\text{rel}} \times \mathbf{r})^2 - mU_{\text{ext}}\,. \tag{2.3}$$

By comparison of the general Lagrangian (2.1) and the "observed" Lagrangian (2.3) one can write down the expression for the vector potential $\mathbf{A}$,

$$\mathbf{A} = \boldsymbol{\omega}_{\text{rel}} \times \mathbf{r}\,. \tag{2.4}$$

It is important to notice that there is no notion of the absolute rotation in this formalism. The observer sitting on the edge of the Newton's rotating bucket can only observe and measure the relative angular velocity between him or her and the distant stars $\boldsymbol{\omega}_{\text{rel}}$, incapable of determine whether it is the bucket or the stars that is rotating.

3. Trajectories of the celestial bodies around the fixed Earth

3.1. Diurnal motion

It is one thing to postulate that rotating masses in the Universe generate the vector potential given by (2.4), but quite another to claim that this same potential can be used to explain and understand the very motion of these distant masses. We will now demonstrate that this is indeed the case.

The observer sitting on the surface of the Earth makes several observations. First, he or she notices that there is a preferred axes (say z) around which all Universe rotates with the period of approximately 24 h. Then, according to the formalism given in Section 2, he or she concludes that the Earth must be immersed in the vector potential given by

$$\mathbf{A} = \Omega\, \hat{\mathbf{z}} \times \mathbf{r}\,, \tag{3.1}$$

where $\Omega \approx (2\pi/24\ \mathrm{h})$ is the observed angular velocity of the celestial bodies ‡.

One can now re-write the Lagrangian (2.1) together with the Equation (3.1) and focus only on the contributions coming from the vector potential $\mathbf{A}$,

$$L_{\mathrm{rot}} = \frac{1}{2} m \dot{\mathbf{r}}^2 + m\,\Omega\,\dot{\mathbf{r}} \cdot (\hat{\mathbf{z}} \times \mathbf{r}) + \frac{1}{2} m\,\Omega^2\,(\hat{\mathbf{z}} \times \mathbf{r})^2\,. \tag{3.2}$$

The Euler-Lagrange equations for this Lagrangian, written for each component of the Cartesian coordinates, are given by

$$\begin{aligned} \ddot{x} &= -\,2\,\Omega\,\dot{y} + \Omega^2\,x \\ \ddot{y} &= 2\,\Omega\,\dot{x} + \Omega^2\,y \\ \ddot{z} &= 0\,. \end{aligned} \tag{3.3}$$

The solution of this system of differential equations reads

$$\begin{aligned} x(t) &= r\,\cos\Omega t \\ y(t) &= r\,\sin\Omega t \\ z(t) &= 0\,, \end{aligned} \tag{3.4}$$

where r is the initial distance of the star from the z axes. The observer can therefore conclude that the celestial bodies perform real circular orbits around the static Earth due to the existence of the vector potential $\mathbf{A}$ given by Equation (3.1). This conclusion is equivalent to the one that claims that the Earth rotates around the z axes and the celestial bodies don't.

3.2. Annual motion

The second thing the observer on the Earth notices is the periodical annual motion of the celestial bodies around the z' axes which is inclined form the axes of diurnal rotation z by the angle of approximately 23.5°. This motion can be explained if one assumes that the Earth is immersed in the so-called pseudo-potential

$$U_{\mathrm{ps}}(\mathbf{r}) = \frac{G M_S}{r_{SE}^2} \hat{\mathbf{r}}_{SE} \cdot \mathbf{r}\,. \tag{3.5}$$

‡ The period of the relative rotation between the Earth and the distant stars is called *sidreal day* and it equals 23 h 56' 4.0916". Common time on a typical clock measures a slightly longer cycle, accounting not only for the Sun's diurnal rotation but also for the Sun's annual revolution around the Earth (as seen from the geocentric perspective) of slightly less than 1 degree per day [14].

Here G stands for Newton's constant, M_S stands for the mass of the Sun and $\mathbf{r}_{SE}(t)$ describes the motion of the Sun as seen form the Earth. The Sun's trajectory $\mathbf{r}_{SE}(t)$ is shown to be an ellipse in x'-y' plane (defined by the z' axes from the above). Using this potential alone one can reproduce the observed retrograde motion of the Mars or explain the effect of the stellar parallax as the real motion of the distant stars in the x'-y' plane. All this was demonstrated in the previous communications [9, 11].

3.3. Total account

One can finally conclude that all celestial bodies in the Universe perform the twofold motion around the Earth:

(i) circular motion in the x-y plane due to the vector potential $\mathbf{A}$ (3.1) with the period of approximately 24 hours and

(ii) elliptical orbital motion in the x'-y' plane due to the scalar potential $U_{\rm ps}$ (3.5) with the period of approximately one year.

Using Equations (2.1), (3.1) and (3.5) one can write down the complete classical Lagrangian of the geocentric Universe,

$$L = \frac{1}{2}m\dot{\mathbf{r}}^2 + m\,\Omega\,\dot{\mathbf{r}}\cdot(\hat{\mathbf{z}}\times\mathbf{r}) + \frac{1}{2}m\,\Omega^2\,(\hat{\mathbf{z}}\times\mathbf{r})^2 - m\frac{GM_S}{r_{SE}^2}\hat{\mathbf{r}}_{SE}\cdot\mathbf{r} - mU_{\rm loc}\,, \tag{3.6}$$

where $U_{\rm loc}$ describes some local interaction, e.g. between the planet and its moon.

It is a matter of trivial exercise to show that these potentials can easily account for the popular "proofs" of Earth's rotation like the Faucault's pendulum or the existence of the geostationary orbits.

4. Conclusion

We have presented the mathematical formalism which can justify Mach's statement that both geocentric and Copernican modes of view are "equally actual" and "equally correct" [3]. This is performed by introducing two potentials, (1) vector potential that accounts for the diurnal rotations and (2) scalar potential that accounts for the annual revolutions of the celestial bodies around the fixed Earth. These motions can be seen as real and self-sustained. If one could put the whole Universe in accelerated motion around the Earth, the potentials (3.1) and (3.5) would immediately be generated and would keep the Universe in that very same state of motion *ad infinitum.*

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