

NEWTONIAN RELATIVITY

A Neglected Manuscript, an Understressed Corollary

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GALILEAN-HUYGENSIAN RELATIVITY

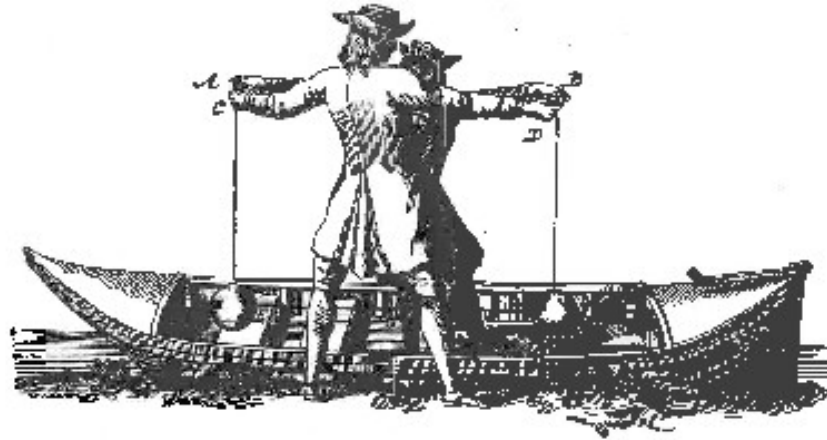
- “The motion of bodies and their equal and unequal speeds are to be understood, respectively, in relation to other bodies which are considered as at rest, even though perhaps both the former and the latter are involved in a common motion. **And accordingly when two bodies collide with one another, even if both together are further subject to another uniform motion, they will move each other with respect to a body that is carried by the same common motion no differently than if this motion coming from outside were absent to all.**”

[Huygens, manuscript at Royal Society, 1669]

- “A wonderful law of nature (which I can verify for spherical bodies, and which seems to be general for all whether the collision be direct or oblique and whether the bodies are hard or soft) is that **the common center of gravity of two, three, or more bodies always moves uniformly in the same direction in the same straight line before and after their collision.**”

[Huygens, *Phil. Trans. of the R.S.*, 1669]

Huygens, 1669 Manuscript



“Thus, if someone conveyed on a boat that is moving with a uniform motion were to cause equal balls to strike one another at equal speeds with respect to himself and the parts of the boat, we say that both should rebound also at equal speeds with respect to the same passenger, just as would clearly happen if he were to cause the same balls to collide at equal speeds in a boat at rest or while standing on the ground.”

NEWTON in “Augmented” De Motu Tract

Law 3: The motions of bodies in a given space are the same among themselves [Lat., *inter se*] whether that space is at rest or moves perpetually and uniformly straight forward without circular motion.

Law 4: By the mutual actions among bodies the common center of gravity does not change its state of motion or rest.

Scholium: ... Moreover, the whole space of the planetary heavens is either at rest (as is commonly believed) or moves uniformly straight forward, and similarly the common center of gravity of the planets (by Law 4) is either at rest or moves the same way. In either case the motions of the planets among themselves [Lat., *inter se*] (by Law 3) take place in the same manner and their common center of gravity is at rest with respect to the whole space, and so it ought to be considered the immobile center of the whole planetary system. Thence indeed the Copernican system is proved *a priori*. For if a common center of gravity is computed for any position of the planets, it either lies in the body of the Sun or will always be very near it....

[manuscript Dec. 1684?; first published, Rouse Ball, 1893]

Corollaries to Laws of Motion, *Principia* 1687

Corol. 3: The quantity of motion that is determined by adding the motions made in the same direction and subtracting the motions made in the opposite direction is not changed by the actions of bodies among themselves **[inter se]**.

Corol. 4: By the actions of bodies among themselves **[inter se]** their common center of gravity does not change its state of motion or rest; and therefore the common center of gravity of all bodies mutually putting one another into motion **(excluding external actions or impediments)** either is at rest or moves uniformly straight forward.

Corol. 5: The motions of bodies **enclosed in a given space** are the same among themselves **[inter se]** whether that space is at rest or moves uniformly straight forward without circular motion.

Corol. 6: If bodies are moving in any way whatsoever among themselves **[inter se]** and are urged by equal accelerative forces along parallel lines, they will all continue to move in the same way among themselves **[inter se]** as if they were not urged forward by those forces.

Corollaries to Laws of Motion, *Principia* 1687

Corol. 3: *Quantitas motus quae colligitur capiendo summan motuum factorum ad eandem partem differentiam factorum ad contrarias, non mutatur ab actione corporum inter se.*

Corol. 4: *Commune gravitatis centrum ab actionibus corporum inter se non mutat statum suum vel motus vel quietis, & propterea corporum omnium in se mutuo agentium (exclusis actionibus & impedimentis externis) commune centrum gravitatis vel quiescit vel movetur uniformiter in directum.*

Corol. 5: *Corporum dato spatio inclusorum iidem sunt motus inter se, sive spatium illud quiescat, sive moveatur idem uniformiter in directum absque motu circulari.*

Corol. 6: *Si corpora moveantur quomodocunque inter se & a viribus acceleratricibus aequalibus secundum lineas parallelas urgeuntur; pergunt omnia eodem modo moveri inter se ac si viribus illis non essent incitata.*

The Understressed Corollary

***Corol. 6:* If bodies are moving in any way whatsoever among themselves and are urged by equal accelerative forces along parallel lines, they will all continue to move in the same way among themselves as if they were not urged forward by those forces.**

For those forces, by acting equally (in proportion to the quantities [i.e. masses] of the bodies to be moved) and along parallel lines, will (by law 2) move all the bodies equally (with respect to velocity), and so **will never change their positions and motions among themselves.**

Newton, “System of the World,” 1685

8. The force which governs the superior planets is directed not toward the Earth. It is directed toward the Sun.

... It is possible to imagine that the Sun and planets are urged equally and along parallel lines by some other forces. **But (by Corol. 6 of the Laws) the situation of the planets among themselves will not be changed by such a force, and no sensible effect will be produced. But we are dealing with the causes of sensible effects.** Therefore, let every force of this kind be set aside as uncertain [Lat. *præcarius*] and having nothing to do with the phenomena of the heavens; then all the remaining force by which the planet Jupiter is urged will tend (by Prop. 3, Corol. 1) toward the center of the Sun.

Newton, “System of the World,” 1685

27. All the planets revolve around the Sun.

... Further, since the planets (Venus, Mars, Jupiter, and the others) do, by radii drawn to the Sun, describe regular orbits, and (as has been shown) areas proportional to the times so far as our senses can tell, it follows (by Prop. 3 and Prop. 35, Corol. 3) that the Sun is urged by **no noteworthy force unless one by which all the planets are urged equally in proportion to the quantities of their bodies and along parallel lines, and hence the whole System is transferred straight forward. Let that translation of the whole System be set aside,** and the Sun will be nearly at rest in its center. If the Sun revolved about the Earth and carried the remaining planets around itself, the Earth would have to attract the Sun with a great force and would moreover have to attract the circumsolar planets with no force having a sensible effect (totally contrary to Prop. 35, Corol. 3).

Corollary 6 Evolves During 1685

- Draft of Corollary 6, prior to initial version of Bk. 1, Prop. 3
- **Initial version of Bk. 1: at least to Prop. 43 (subseq. 74)**
 - Props. 3 and “35” (subseq. 66 on 3-body problem) cite Corollary 6
- Insert a Prop. 35, add 21 corol. to Prop. 35–36 (**subseq. 65 and 66**)
- **Initial version of “System of the World” (“Liber Secundus”)**
- **Subsequent revisions of “System of the World”**
 - **First use of “accelerative force” -- replacing “local force”**
- **New “Definitions”:** **“accelerative quantity of force”, “mass”**
- **Corol. 6 revised to final form: “accelerative force”**
- **Bk. 1 expanded (late 1685, early 1686)**
 - **Insert 30 Props. before Prop. 34, others at the end**

Excerpt from Proposition 65 on the 3-Body Problem

Case 2. Let us now imagine a system of lesser bodies revolving in the way just described around a much greater one, or any other system of two bodies revolving around each other, to be moving uniformly straight forward and at the same time to be urged sideways by the force of another very much greater body, situated at a great distance. Then, since the equal accelerative forces by which the bodies are urged along parallel lines do not change the situations of the bodies among one another, but cause the whole system to be transferred simultaneously, while the motions of the parts among themselves are maintained; it is manifest that no change whatsoever of the motion of the bodies attracted among themselves will result from their attractions toward the greater body, **unless such change comes either from the inequality of the accelerative attractions or from the inclination to one another of the lines along which the attraction takes place....**

“NEWTONIAN RELATIVITY” – First Pass

If all changes of position and motions of a system of bodies among themselves depend only on the actions of those bodies on one another, and not on any action external to the system, then the true motions of those bodies **among themselves** are the motions as referred to their common center of gravity, **without regard to any motion whatsoever that the entire system of bodies may or may not have in common with one another.**

“A QUASI-INSULAR SYSTEM”

A Qualifier

- A prominent use of Corollary 6 in the *Principia* is to justify treating orbiting subsystems – in particular, Jupiter and its satellites – as quasi-insular systems.
- Contrary to the conditions of Corollary 6, however, the accelerative force of the Sun on the bodies within this subsystem is neither exactly equal ($\delta \leq 0.5\%$) on each of the bodies, nor exactly along parallel lines ($\delta \leq 0.14^\circ$).
- Therefore, Newton does not appear to require that the conditions of Corollary 6 hold exactly, but only to a level of precision appropriate in relation to the predominant changes of motions of the bodies among themselves.

“NEWTONIAN RELATIVITY” – Second Pass

If the principal changes of position and motion of a system of bodies among themselves depend only on the actions of those bodies on one another, and not on any action external to the system, then the principal true motions of the bodies **among themselves** are the motions that stand out once all motions in the system are referred to the common center of gravity of its bodies, **without regard to any motion whatsoever that the entire system of bodies may or may not have *quam proxime* in common with one another.**

“A NEARLY QUASI-INSULAR SYSTEM”

DURATION

“Astronomers correct this inequality [in solar days] in order to measure celestial motions on the basis of a truer time. **It is possible that there is no uniform motion by which time may have an exact measure.** All motions can be accelerated and retarded, but the flow of absolute [i.e. true] time cannot be changed.... Accordingly, duration is rightly distinguished from its sensible measures and is gathered [Lat., *colligitur*] from them by means of an Astronomical Equation. Moreover, the need for using this equation in determining when phenomena occur is proved by experience with a pendulum clock and also by eclipses of the satellites of Jupiter.”

PLACE

“But since these parts of space [i.e. individual places] cannot be seen and cannot be distinguished from one another by our senses, we use sensible measures in their stead. For we define all places on the basis of the positions and distances of things from some body that we regard as immovable, and then we reckon [Lat. *aestimamus*] all motions with respect to these places, insofar as we conceive of bodies as being changed in position with respect to them. Thus, instead of absolute [i.e. true] places and motions we use relative ones.... **For it is possible that there is no body truly at rest to which places and motions may be referred....** [And even if there is such a body at some far remove, since] it cannot be known from the situation of bodies among one another in our regions whether or not any of these maintains a given position with relation to that distant body, true rest cannot be defined on the basis of the situation of these among themselves.”

An Asymmetry in What Poses a Problem

=

Δ

TIME

Same Time

Duration

SPACE

Same Place

Distance

To determine simultaneity, correct for finite speed of light

To determine distances, proceed as in Euclidean geometry

Orbital Motion

Specifying any orbital motion, even in a quasi-insular system, requires

- A means of determining if the body returns, if not to the same place, then at least to the **same angular location** – i.e. θ and ξ in $\langle r, \theta, \xi \rangle$ coordinates
- A measure of **duration** enabling specification of the time – i.e. the orbital **period** – required for the body to return, if not to the same place, then at least to the same angular location.

NEWTONIAN RELATIVITY

- The system of the planets is an at least nearly quasi-insular system – i.e. a system in which, if not all, then at least all of the principal changes of position and motions of its bodies **among themselves** result entirely from the actions of those bodies on one another.
- Sidereal time – i.e. the time for any one star to cross the meridian at any location on Earth – provides an at least provisional measure of duration within the system.
- The “fixed” stars provide an at least provisional reference, external to the system, with respect to which “true” angular locations within the system – same and different – can be specified.

Three Newtonian “Working Hypotheses”

- 1) Within our planetary system, changes in position and motions of the bodies among themselves **(at least to some high level of approximation)** result from the actions of those bodies on one another, and not from actions on them from outside the system.
- 2) The rotation of the Earth about its axis **(at least to some high level of approximation)** is uniform.
- 3) The “fixed” stars **(at least to some high level of approximation)** do not change angular position with respect to the center of gravity of our planetary system.

Prop. 35 (subseq, 65): *More than two bodies whose [centripetal] forces decrease as the square of the distances from their centers are able to move among themselves in ellipses and, by radii drawn to the foci, are able to describe areas proportional to the times quam proxime.*

Corol. 3: Hence, if the parts of this system – without any significant perturbation – move in ellipses or circles, it is manifest that these parts are either not urged, except most lightly, by accelerative forces tending toward other bodies or they are all urged equally and along parallel lines quam proxime.

A Continuing Test of the Working Hypotheses

Can a robust physical source *within the system* be established for every increasingly subtle small change of position and motions of the bodies among themselves in our planetary system?

“Robust”: having multiple consequences within the system

“Subtle”: identifiable only after more prominent changes of position and motions, masking smaller ones, have been taken into account

NEWTON in “Augmented” De Motu Tract

“By reason of the deviation of the Sun from the center of gravity, the centripetal force does not always tend to that immobile center, and hence the planets neither move exactly in ellipses nor revolve twice in the same orbit. Each time a planet revolves it traces a fresh orbit, as in the motion of the Moon, and each orbit depends on the combined motions of all the planets, not to mention the actions of all these on each other. But to consider simul-taneously all the causes of motion and to define these motions by exact laws admitting of easy calculation exceeds, if I am not mistaken, the force of any human mind.”

[manuscript Dec. 1684?; first published, Rouse Ball, 1893]

Newton's Continuing Test

Prop. 14. *The aphelia and nodes of the [planetary] orbits are at rest.*

The aphelia are at rest, by book 1, prop. 11, as are the planes of the orbits, by prop. 1 of the same book; and if these planes are at rest, the nodes are also at rest. But yet from the actions of the revolving planets and comets among themselves some inequalities will arise, which, however, are so small that they can be ignored here.

Corol. 1: *The fixed stars also are at rest, for they maintain given positions with respect to the aphelia and nodes.*

Corol. 2: *And so, as the fixed stars have no sensible parallax arising from the annual motion of the Earth, their forces, owing to the immense distance of those bodies from us, produce no sensible effects in the region of our System.*

A Continuing Test of the Working Hypotheses

Would the aphelia remain in the same angular position with respect to one another and to the fixed stars were it not for actions of the bodies within our planetary system among themselves?

i.e., confirm that any motion of any of the planetary aphelia results from sources within the system, thereby confirming that no action from outside the system affects the aphelia – thus nor too the position and motions of the bodies among themselves.

The Unnoticed Manuscript, Add. 3965, fol. 341r

in spatia inmersa ultra orbem Saturni.

Ceterum Trajectoriam quam Cometa descripsit, et viam
 ejus eandem quam singulis in locis proiecit --- a pag 497 ad
 pag. 510 --- tempus periodicum Cometa. Q. E. S. 311

Prop XLIII. Theor. XXII

Ut Terra quiescat in centro systematis ~~Solis~~ Solis & ~~Cometae~~ Planetae, & Cometae, requiritur et gravitas universalis, et alia incupit
 vis quae tendit eam agit in omnia corpora aequaliter pro quantitate materiae
 in ipsis ~~et dirigat secundum~~ ~~linas parallelas in plagam eandem cum~~
~~linea quae ducitur a centro Solis per eandem centrum Terra~~ et aequalis
 est gravitati acceleratrici qua Terra tendit in Solem, eiq; contraria, ~~hinc~~
 tendens ~~secundum~~ ~~linas parallelas in plagam eandem cum~~ linea quae ducitur a
 centro Solis ad centrum Terra
 Nam ~~hinc~~ ~~vis in corpora omnia aequaliter et secundum lineas parallelas agendo~~
~~in corpora omnia hinc vis aequaliter accelerat~~
~~igitur secundum lineas parallelas et perpendicularis solum, inter se non movetur~~
~~sed sicut corpora eodem modo per vis gravitatis, inter se movetur, ac si non agerent in eandem~~
 Terra vero, ~~si~~ cum hac vis gravitatis ejus in Solem aequalis sit
 et contraria, in equilibrio inter has duas vires manere potest
 et quiescere. Et sic corpora caelestia circa Terram quiescant
 moveri possunt ut in Systemate Tychonico.

Prop. XLIII. Theor. XXII

Ut Terra quiescat in centro Systematis Solis Planetarum & Cometarum, requiritur et gravitas universalis, et alia insuper vis quae agit in omnia corpora aequaliter pro quantitate materiae in ipsis et aequalis est gravitati acceleratrici qua Terra tendit in Solem, eique contraria est, tendendo secundum lineas parallelas in plagam eandem cum linea quae ducitur a centro Solis ad centrum Terrae.

Nam talis vis in corpora omnia aequaliter & secundum lineas parallelas agendo situm eorum inter se non mutat sed sinit corpora eodem modo per vim gravitatis universalis inter se moveri, ac si non ageret in eadem. Terra vero, cum haec vis gravitati ejus in Solem aequalis sit & contraria, in aequilibrio inter has duas vires manere potest et quiescere. Et sic corpora caelestia circa Terram quiescentem moveri possunt ut in Systemate Tychonico.

Prop. 43, Theor. 22: *In order for the Earth to be at rest in the center of the system of the Sun, Planets, and Comets, there is required both universal gravity and another force in addition that acts on all bodies equally according to the quantity of matter in each of them and is equal and opposite to the accelerative gravity with which the Earth tends to the Sun, tending along parallel lines on the same flat surface with the line drawn from the center of the Sun to the center of the Earth.*

For, such a force, acting on all bodies equally and along parallel lines, does not change their position among themselves, and permits bodies to move among themselves through the force of universal gravity in the same way as if it [the external accelerative force] were not acting on them. Since this force is equal and opposite to its gravity toward the Sun, the Earth can truly remain in equilibrium between these two forces and be at rest. And thus celestial bodies can move around the Earth at rest, as in the Tychonic System.

For example, if two balls, at a given distance from each other with a cord connecting them, were revolving about a common center of gravity, the endeavor (*conatus*) of the balls to recede from the axis of motion could be known from the tension of the cord, and thus the quantity of circular motion could be computed.... Now if some distant bodies were set in that space and maintained given positions with respect to one another, as the fixed stars do in our region, it could not, of course, be known from the relative change of position of the balls among the bodies whether the motion was to be attributed to the bodies or to the balls. **But if the cord was examined and its tension was found to be the very one which the motion of the balls required, it would be legitimate (*liceret*) to conclude that the motion belonged to the balls and that the bodies were at rest.**

Newton, Scholium on Space and Time, all editions

... that you believe it *to be discordant that no real motion is given, but only relative*. Yet I hold this to be very sure, and am not checked by the argument and experiments of **Mr. Newton in his Principles of Philosophy, which I know to be in error; and I am eager to see whether he will not make a retraction in the new edition of this book**, which David Gregorius is to procure.

Huygens to Leibniz, 29 May 1694, tr. Howard Stein

Now in the circulation of 2 bodies bound by the thread AB one knows that they have received impulsion which has produced their mutual relative motion of direction. **But one cannot know, by considering them alone, whether they were pushed equally, or whether only one was pushed. For if A alone had been pushed, the circular motion and the tension in the thread would have followed all the same**, although the circle would then have a progressive motion with respect to the other bodies at rest.

That I have therefore shown how in circular motion just as well as in free and straight motion there is nothing but what is relative.

Huygens, Notebooks, 1688 or later, tr. Howard Stein

External or Internal?

- ***External:*** no equal and opposite action and reaction with any of the bodies acting on one another in the planetary system or any other known body outside of it
- ***Internal:*** accelerative force revolves, not quite uniformly, (or the whole system revolves about a “black-hole), always in line with the Earth-Sun axis with a 365.25 day period
- ***Regardless:*** the force would alter positions and motions of bodies among themselves within our planetary system insofar as the center of gravity of the system would revolve in orbit around the Earth; hence **the system would cease having claim to being even nearly quasi-insular**

Could the Force Have Been Ruled Out Empirically?

- Not detectable from *any* motions within our planetary system, save from the accelerative force not acting exactly equally or exactly in parallel on all bodies in the system

Not any motion of planetary apheia, nor even the tides or the precession of the equinox

- Motion of the Earth *relative to* specific “fixed” stars detectable from two post Newton discoveries:

Aberration of light (Bradley 1729, confirmed 1748)

Annual stellar parallax (Bessel 1839)

Newton Says

“For the whole difficulty of [natural] philosophy seems to be to discover the forces of nature from the phenomena of motions and then to demonstrate the other phenomena from those forces.”

Preface to first edition of *Principia*

“It is certainly very difficult to find out the true motions of individual bodies and actually to differentiate them from apparent motions, because the parts of immovable space in which the bodies move make no impression on the senses. Nevertheless, the case is not utterly hopeless. For evidence [*argumenta*] is at hand, partly from apparent motions, which are the differences of true motions, and partly from the forces that are the causes and effects of true motions.”

Principia, last paragraph of Scholium on space and time, 1687, 1713

“Causal Analysis”

- **Which details of the system make a difference, and what differences do they make?**
- **Which variables describe those details, and how do differences in the values of any one such variable affect the values of the others?**
- **Which variables require intervention from outside the system in order for their values to change?**

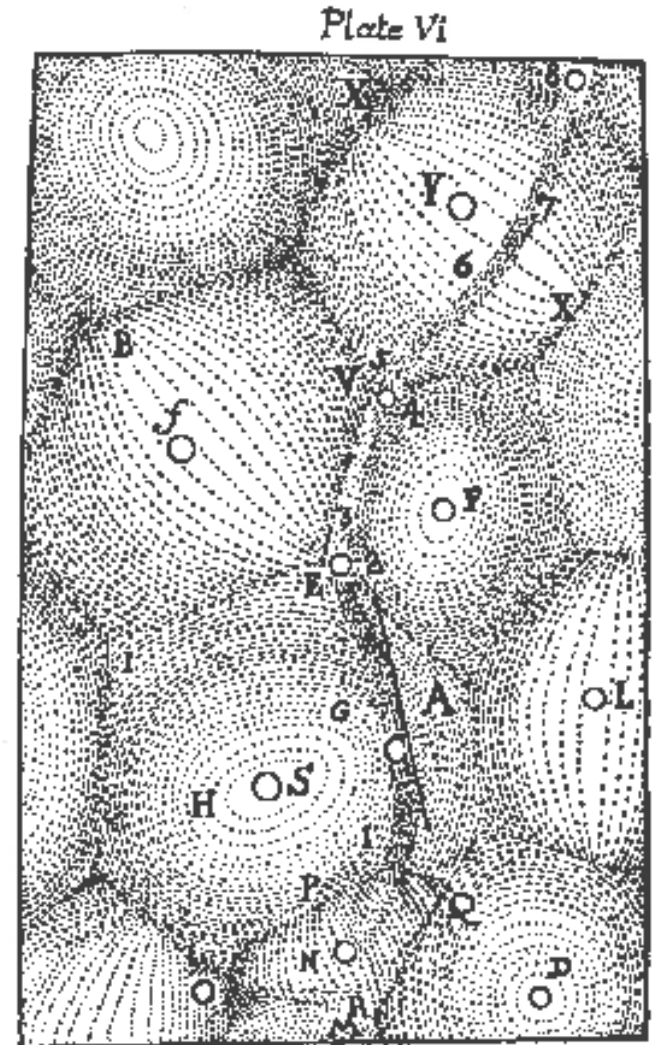
Newtonian Relativity vs. Cartesian Vortices

“They [i.e. planet trajectories] are continuously changed with the passing of the ages.”

“But a few centuries from now, all these things [in particular, positions of aphelia] will be observed to have changed from the way in which they are now.”

“For, **inasmuch as all the bodies in the universe are contiguous and act on one another**, the movement of each is affected by the movements of all the others and therefore varies in innumerable ways.”

Descartes, *Principia*, III, 34, 36, 157



THE **TWO** HYPOTHESES IN THE *PRINCIPIA*'S ARGUMENT FOR COPERNICANISM

Hypothesis 1. *The center of the system of the world is at rest.*

Proposition 11. *The common center of gravity of the Earth, the Sun, and all the planets is at rest.*

For that center (**by Corol. 4 of the Laws**) either will be at rest or will move uniformly straight forward. But if that center always moves forward, the center of the universe will also move, contrary to the hypothesis.

Corollary 4. *By the actions of bodies **among themselves [inter se]** their common center of gravity does not change its state of motion or rest; and therefore the common center of gravity of all bodies mutually putting one another into motion (**excluding external actions or impediments**) either is at rest or moves uniformly straight forward.*