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New non-zero photon mass interpretation of the Sagnac effect as direct experimental justification of the Langevin paradox

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Abstract

The experimental discovery by Dufour and Prunier that the Sagnac fringe displacement does not change when the source and fringe detector are locally transferred from the rotating platform to the laboratory frame is interpreted as a natural result of general relativity theory when one introduces non-zero mass photons into the theory of light. It also yields a proof of the reality of Langevin's paradox as a natural consequence of assuming that non-zero mass photons (1) follow real space-time paths, (2) are associated with real physical internal clock-like motions, (3) imply the existence of an absolute local inertial frame Σ_0 first associated by Lorentz with Maxwell's equations. © Published by Elsevier Science B.V.

1. Introduction

In a recent paper Kelly [1] has suggested that the Dufour–Prunier experiments [2] (which confirm with a different set-up Sagnac's discovery of fringe shifts displacements in rotating interferometers [3]) contradict one of the two basic requirements of special relativity theory, i.e., (1) that the speed of light is independent of the speed of its source, (2) that its value c is a constant for all observers in inertial frames travelling at uniform speed with respect to each other, since the second can be shown to clash with the observed existence of the same fringe displacement on the rotating disk and the enclosing laboratory.

The aim of the present Letter is to show that the introduction of a small photon rest mass $m_\gamma \neq 0$ into the theory of light explains this result within the frame of general relativity theory itself i.e., that it can be considered (a) as a particular experimental proof of the real existence of Langevin's twin (clock)) paradox, (b) as evidence for the existence of an absolute local inertial frame, Σ_0, recently revived in the literature [4], since one can show that, in that case, as a consequence of Lorentz's real contraction and time slowing the Sagnac fringe shift only depends on the invariant phase shift (proper time difference) of the two interfering light signals which travel clockwise and anticlockwise on the rotating disk.

2. Sagnac effect

The fact that a light signal, that is sent both clockwise and anticlockwise around a path on a rotating disc, takes different times to return to the source was discovered by Sagnac over eighty years

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ago [3] (i.e. the Sagnac effect) is an unsolved fundamental problem in physics. For example, in a recent review paper Hasselbach and Nicklaus [5] list many explanations of the Sagnac effect proposed by various authors over the intervening years. They sum up the situation as follows: "This great variety (if not disparity) in the derivation of the phase shift constitutes one of the several controversies that have been surrounding the Sagnac phase shift since the earliest days of studying interference in rotating frames of reference." Several references to each suggested explanation are listed in their paper. In all of these references one finds attempts to explain the effect by assuming that the movement of the disc affects, in some way, the behaviour of the light. Kelly's main new point is to show that the movement of the disc has no influence whatever on the behaviour of the light and that the appearance of the disk's angular velocity is essentially related to delayed intervention of the measuring device only.

A schematic representation [1] of the test done by Sagnac is shown in Fig. 1. A light source at point S emits light to a beam splitter at point C. Some of the light traverses the path SCDEFC, and is then reflected to an "observer" at O. Some of the light goes the other way, around SCFEDCO. The whole apparatus can rotate with an angular velocity ω. The light source S and the "observer" O (in reality a photographic plate) are both fixed to the rotating apparatus, and rotate with it.

When the disc is stationary, light sent around in opposite directions will arrive back at the same instant at point C. The beam splitter at C acts as an interferometer and is used to display this static situation and to determine whether any change occurs

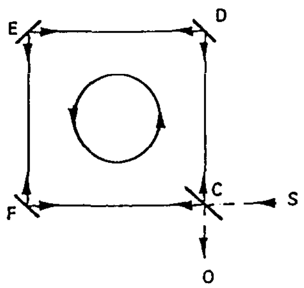

Fig. 1. Sagnac test.

when the disc is set in motion [6]. In the static case, the interferometer produces fringes (dark and bright bands) where the light recombines, following its traversing of a circuit.

When the disc is spinning, the observer detects a shift in the fringes to one side, indicating that the two light signals are out of phase and do not return to point C at the same instant. The shift is of the same magnitude, but in the opposite direction, when the direction of rotation is reversed.

Sagnac derived the difference in time, $\mathrm{d}t$, between the times taken by the light to traverse the path in opposite directions, as

$$\mathrm{d}t = 4A\omega/c^2, \tag{1}$$

where terms of the order of ω^2 and less are ignored.

In Eq. (1) A is the area enclosed by the light path, and c is the speed of light. Note that the interferometer that displays this time difference is on board the rotating disc.

Sagnac [3] showed experimentally that the centre of rotation can be away from the geometric centre of the apparatus, without affecting the above result. He also showed that, although the mirrors move as the disc rotates and as the light moves around the circuit, this movement has a negligible effect on the magnitude of the fringe shift.

"Langevin, in 1921 [7], commented on the practical tests done by Sagnac [3], and claimed that the effect, i.e., the observed time difference $\mathrm{d}t$, had to be in accord with the theory of relativity. He said that because that theory fitted the "whole of the known experimental facts" of physics in general, the tests had to be explicable by that theory. In 1935, however, Prunier [8] published a note questioning Langevin's reasoning and argued that the practical tests were not explained by relativity theory. There followed a series of papers, by Dufour [9] and Langevin [7] in which was debated the question whether, or not, the effect was in accord with the theory of special relativity and whether an apparatus could be constructed to settle the question. This first debate ended in stalemate" [1].

Dufour and Prunier [10] then collaborated in a series of dissimilar Sagnac-type practical tests. First, in 1937, they rigorously repeated the original Sagnac tests. They then repeated the method used by Pogany

[11], who had the light emitter fixed in the laboratory, but had the photographic recorder on the disc. They then carried out the experiment with both the light emitter and the photographic recorder taken off the disc, and set up fixed in the laboratory [11], i.e., the set-up adopted by Harress [12].

It should be noted that, in all these cases, the *interference of the light signals occurs on board the spinning disc,* i.e., the interferometer (fringe detector) is always fixed to the disc; the photographic recorder, which is either on or off the disc, then captures the image of the fringe shift. The experiment where the photographic equipment is off the disc is the more complicated of the two. Two extra lenses are required to send the image out from the disc and on to the photographic plate fixed in the laboratory, consequently, the spread of the readings widens from $\pm 5\%$ in the case where the record is made on board the disc to $\pm 15\%$ off the disc [12].

In 1939, Dufour and Prunier carried out their final experiment [10]. They did a test with both the beginning and end of the light path on the spinning disc, but with the middle portion of the path reflected off mirrors fixed in the laboratory (directly above the disc). In this test, they had both the light emitter and the photographic recorder fixed in the laboratory. "On the spinning disc" means that the light is confined to a path by a set of mirrors which are fixed to, and rotate with the disc.

The fringe shifts resulting from all the above Dufour and Prunier tests were the same as in their original Sagnac-type tests. This fact is of critical significance in understanding what is occurring, as will be discussed later.

In 1942 Dufour and Prunier published a composite paper [10] reviewing their total experimental work to date. At the end of this paper they state that the relativity theory seems to be in complete disagreement with the result which was garnered from the experiment. For a time this was the end of the debate [1].

In order to get an idea of the magnitude of the Sagnac effect, it is helpful to calculate the disc-rotation speeds necessary to obtain significant fringe shifts. Consider Fig. 2, where the light path is confined to a circle of radius r. The equation which expresses the relationship between interference fringes and time differences is $F = \mathrm{d}t\ (c/\lambda)$, where F is the number of fringe shifts detected and λ is the wavelength of the light used. From Eq. (1) and, since $v = r\omega$ for circular motion (where v is the tangential velocity of a point on the circle), one has

$$F = \frac{4A\omega}{c\lambda} = \frac{4\pi r v}{c\lambda}. \tag{2}$$

In order to obtain a fringe shift of one fringe, using a disc of 1 m radius, the velocity around the perimeter of the circuit has to be only about [1] 13 m/s.

The first known Sagnac-type test performed was carried out in 1911 by Harress [12]. His apparatus was similar to Sagnac's, consisting of a rotating disc carrying a light emitter and photographic recorder (both fixed in the laboratory); light signals were sent around the disc in opposite directions. Von Laue, in 1920, showed that the Sagnac effect could be detected in Harress's numerical results [14].

Pogany repeated the Sagnac tests [11]. By using more sturdy apparatus and higher speeds of rotation he obtained a fringe shift 25 times greater than that achieved by Sagnac ($F = 1.8$ versus $F = 0.07$ fringe), thus reducing the experimental error and allowing the fringe shift to be measured with greater accuracy.

To indicate the accuracy of more modern Sagnac-type tests, Macek and Davis [13] give the accuracy of the laser equipment used as 1 in 10^{12}. In 1913, when Sagnac carried out his tests, the accuracy was about [2] 1 in 10^2.

With $m_\gamma = 0$ the Sagnac effect is evidently not compatible with the initial assumption of restricted relativity theory where the velocity of light is constant. This is only to be expected of course since the corresponding motions of light are not along straight lines with constant velocity if $m_\gamma \neq 0$. As one knows, following Einstein himself, motions along curved world lines imply the introduction of accelerations, i.e., the utilization of the mathematical formalism of

[1] That this is so can be seen by setting $F = 1$, $r = 1$, $\lambda = 5500 \times 10^{-10}$ cm (a typical figure) and $c = 3 \times 10^8$ m/s in the above equation [1].

[2] The reader is referred to Ref. [14] for a historical review of the Sagnac effect.

general relativity. If one assumes with him [15] (a) that any curved world-line path can be approximated by a succession of small inertial straight lines with different specific constant velocities, i.e., can be analysed in terms of a succession of Lorentz transformations and (b) that the change in direction between successive small segments (equivalent to point-like accelerations) does not influence the time evolution of observable physical quantities such as rod length, time, wave phase etc. measured with respect to other inertial frames [3], i.e., that one can thus approximate all non-inertial motions in terms of a succession of small inertial motions [4], we shall now show that one can interpret the Sagnac effect with a non-zero photon mass.

3. Sagnac effect and non-zero photon mass

The development of a new interpretation of the Sagnac effect with a non-zero mass photon within the frame of general relativity theory rests on the following assumptions:

The existence of a non-zero photon mass $m_\gamma \approx 10^{-65}$ g first developed by de Broglie, Schrödinger, and in that case Einstein, has been recently reintroduced into the literature [17]. As one knows

(i) Light corresponds to spin 1 fields described by a four-vector density

$$A_\mu = R_\mu(x_\alpha)\exp[\mathrm{i}S(x_\alpha)/\hbar],$$

where R_μ is a real four vector density $\rho^{1/2}a_\mu$ with $a_\mu a^\mu = \text{const}$, $a_\mu \partial^\mu S = \partial^\mu A_\mu = 0$ and $\Box A_\mu = (m_\gamma^2 c^4/\hbar^2) A_\mu$ in vacuum.

(ii) This field carries (and pilots) particle or soliton-like photons which move along average drift-lines tangent to the four-momenta $p_\mu = \partial^\mu S$ with an average probability distribution $\rho = R_\mu R^\mu$ and $p_\mu a^\mu = 0$.

(iii) These photons (and the corresponding wave constitutive elements) are real extended clock-like structures, "piloted" by their surrounding elements, which move within time-like tubes with a subluminal velocity $c_\nu < c$ and have $c_\nu V = c^2$ where c_ν represents the particle velocity corresponding to a given frequency ν (i.e. $E = h\nu = m_\gamma^0 c^2(1 - c_\nu^2/c^2)^{-1/2}$ and V denotes the associated superluminal phase velocity. At the photon's positions the phase of the photon's pilot wave equals for a small time the phase (frequency) of the photon's internal oscillations (so that $m_\gamma^0 c^2 = h\nu_0$ in their rest frame), i.e., it coincides with the external phase of the piloting vector field A_μ: a basic assumption in quantum mechanics. Photons thus follow curved world paths L as a consequence of the influence of the quantum potential generated by A_μ and behave like extended clocks moving with velocities $< c$.

(iv) In general relativity the observed interference of light originating from the same sources arriving at the same world point O through different curved paths, corresponds to the phase difference generated by the corresponding propagation, when one utilizes the corresponding proper times as evolution parameters. If one now considers photons (and wave elements) as real extended clocks with a rest frame frequency $h\nu^0 = m_\gamma^0 c^2$ this implies, if one limits oneself to the consideration of two paths L_a and L_b, between S and O, that the fringe shift $S_a - S_b$ is proportional to the difference of proper times $\tau_a - \tau_b$ taken to pass from S to O along L_a and L_b: or, in other terms, to the difference of age of two Langevin twins travelling along L_a and L_b. *In this model, Sagnac-type interference experiments thus correspond to a physical test (proof) of the Langevin paradox: where the rotating disk only determines the origin and final positions of the interfering clocks (i.e., photons or twins) following different world lines with different velocities.* As stated by Kelly [1] the Sagnac effect is a measure of the difference in proper times of the two light signals while they are away on their travel in the two opposite directions.

As has been shown in the literature, $m_\gamma \neq 0$ implies the real existence of an absolute space-time frame Σ_0 as a consequence of restricted relativity theory itself. In this frame Σ_0, light, with frequency ν, moves in all directions with the same velocity. When applied to clock retardation, in the case of two

[3] This assumption (b) has been experimentally confirmed, even in the case of light, by experiments made by Majorana, Beckman and Mandis, Michelson and Morley which showed that the instantaneous acceleration of light by rotating mirrors does not modify the fringe shifts (i.e. phase or time delay) in interference set-ups (for a detailed justification see Ref. [16]).

independent isolated clocks or photons (denoted A and B) moving in a region of the Universe in such a way that they coincide on at least two occasions at times t_1 and t_2 in an inertial frame H_0, then as Hafele and Keating have confirmed, this theory predicts that in general one clock will become retarded with respect to the other as a consequence of the difference of their motions along different paths in Σ_0. As shown by Builder [19] in a remarkable paper according to the restricted and general theory of relativity this retardation (a) cannot result from their individual accelerations (1) because in general relativity the rate of a clock is not a function of its acceleration and (2) because one can consider any path as a succession of different inertial motions in vanishingly small time intervals, (b) cannot depend on the velocity of a clock relative to the other. Indeed if we denote by $\boldsymbol{u}$ and $\boldsymbol{v}$ the velocities (in Σ_0) of A and B at any time t and their coincidence occurs at times t_1 and t_2, then the rates of the clocks at the instant t_0 are respectively

$$\mathrm{d}t_a/\mathrm{d}t_0 = \left(1 - u^2/c^2\right)^{1/2},$$

$$\mathrm{d}t_b/\mathrm{d}t_0 = \left(1 - v^2/c^3\right)^{1/3} \tag{3}$$

and the proper times of the clocks between their coincidence are given respectively by

$$t_a = \int_{t_1}^{t_2} \left(1 - u^2/c^2\right)^{1/2} \mathrm{d}t_0,$$

$$t_b = \int_{t_1}^{t_2} \left(1 - v^2/c^2\right) \mathrm{d}t_0. \tag{4}$$

Thus, in the interval between their coincidences clocks become retarded relative to clock B by the amount

$$t_a - t_b = \int_{t_1}^{t_2} \left(1 - v^2/c^2\right)^{1/2} \mathrm{d}t_0 - \int_{t_1}^{t_2} \left(1 - v^2/c^2\right)^{1/2} \mathrm{d}t_0, \tag{5}$$

an invariant value which does not depend on the difference $\boldsymbol{u} - \boldsymbol{v}$ of their respective velocities [4].

It follows from this "that any physical explanation of the phenomena described in Eqs. (3), (4) must be sought in the form of these equations rather than in their numerical content as determined by the measures of the system Σ. The fact that the form of the equations is independent of the choice of the inertial reference system implies that the existence in nature of the phenomena described by the equations is independent of the existence of any such inertial reference systems, hypothetical or physical."

Yet the fact that the clocks do behave differently when their speeds are different requires that they interact physically with something, in a manner which depends on their speeds. For the context requires that the two clocks be ideal standard clocks which behave identically in all respects when subject to the same conditions. Thus any difference in their behaviour must be ascribed to a difference in their physical interaction with their environment.

Since the context requires that the clocks be isolated from interaction with other actual bodies or physical systems in their vicinity, we are forced to conclude that they must interact with something universal; in our case the local Dirac "aether" which carries real physical waves of quantum theory [20].

As also noted by Builder [19]:

"The only hypothesis that is tenable, and that is compatible with the foregoing considerations, is that

[4] As also remarked by Builder [19] in this situation "any physical explanation of these causal relations must obviously be independent of the inertial reference system chosen for measurement (or calculation). This is clearly required by the fact that Eqs. (1)–(3) take precisely the same form when expressed in the measures of any inertial reference system whatsoever. It is also required by the context, for this precludes any physical interaction of the clocks with any such reference system. This may be illustrated as follows. We could, if we wished, regard u and v in Eq. (3) as the speeds of the clocks relative to the system Σ_0 as measured in Σ. Yet we could not ascribe direct causal significance to these speeds relative to Σ_0, because any corresponding interaction, between the clocks and Σ is precluded by the context. Indeed, the equations hold even if the system Σ_0 is purely hypothetical and if the quantities in the equations are merely postulated, or if they are calculated from measurements made in some other system." One must add that, since then [21] absolute motion has been shown to be detectable, as a consequence of Maxwell's equations.

there exists a unique absolute inertial system Σ_0 which interacts with, and affects, the behaviour of the clocks in a manner dependent on their speeds relative to it, i.e., their absolute speeds.

This hypothesis is clearly sufficient. A reference system Σ_0 at rest relative to this postulated absolute inertial system would be one of the reference systems to which the restricted theory is applicable. We can therefore write for the rates of the clocks A and B, as measured in Σ_0,

$$\mathrm{d}t_a/\mathrm{d}t_0 = \left(1 - u_0^2/c^2\right)^{1/2},$$
$$\mathrm{d}t_b/\mathrm{d}t_0 = \left(1 - v_0^2/c^2\right)^{1/2}, \qquad (6)$$

and for the relative retardation

$$t_a - t_b = \int_{t_{01}}^{t_{02}} \left(1 - u_0^2/c^2\right)^{1/2} \mathrm{d}t_0 - \int_{t_{01}}^{t_{02}} \left(1 - v_0^2/c^2\right)^{1/2} \mathrm{d}t_0, \qquad (7)$$

where u_0 and v_0 are the absolute speeds of the clocks at each absolute time t_0."

We thus have in Eqs. (6) and (7) a causal account of the behaviour of the clocks given explicitly in terms of their absolute speed u_0 and v_0. All the observable consequences of (6) and (7) can be verified by measurements made in any inertial system Σ and by calculations using Eqs. (3) and (5). In other words, although Eqs. (1) and (3) do not contain u_0 and v_0 explicitly, they do express, in terms of the speeds u and v, all the observable consequences of Eqs. (6) and (7).

Thus we conclude that the relative retardation of clocks predicted by the restricted theory does indeed compel us to recognize the causal significance of absolute velocities and that this recognition is compatible with the fact that these absolute velocities do not appear explicitly in the relativistic expression for the relative retardation."

As finally remarked by Builder [19]:

"The relative retardation of clocks is an effect which seems to be unique in that its measure is an invariant for all observers, whatever their state of motion. However, it is important to realize that this unique character arises solely from the fact that each of the clocks considered in this context incorporates an integrating device which provides an observable record of the accumulated effects of variation in its rate. Were we considering the periodic processes in a single atom, we would be without such a cumulative record; but, as has been indicated above, Eq. (7) would still require us to postulate some absolute systems Σ_0 which would affect the rate of these periodic processes in accordance with the absolute speed of the atom."

4. Sagnac effect with non-zero photon mass

We first directly justify relation (1) in the particular set-up of Fig. 2. In the absolute local inertial frame Σ_0 one can analyze the Sagnac-type experiment of Fig. 1 as follows.

(1) We first assume that the center of the rotating disk is at rest in S_0 then as seen by an inertial observer S locally at rest on the circumference (which turns in Σ_0 with respect to an observer at rest at C with the velocity $v = r\omega$) its local time is contracted with respect to an observer at rest at C with the velocity $v = r\omega$ by the factor $\gamma = (1 - v^2/c^2)^{1/2}$. In Σ_0 the velocity of light c_ν^0 for a given frequency ν_0 in Σ_0 is always isotropic and takes in Σ the values

$$c_\nu^\pm = \frac{c_\nu^0 \pm r\omega}{1 \pm r\omega c_\nu^0/c^2}$$

in the clockwise and anticlockwise directions where Σ is the instantaneous inertial rest frame of S.

For most experiments of course we can write $c_\nu^\pm \equiv \mathrm{const} \cong c$.

(2) We then analyze the result of observations made in Σ_0. The light source and interferometer are at S

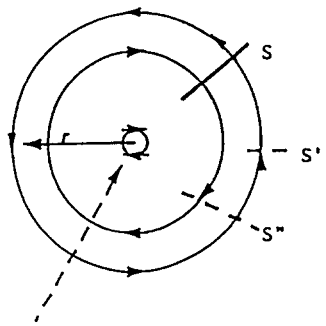

Fig. 2. Circular Sagnac test.

(i.e., at rest in Σ or moving in Σ_0) and both are fixed on the rotating disk. Let t_0 be the time taken by a light signal to traverse the circumference of the circle and to return to the source interferometer, when both the disk and the observer are stationary. Thus t_0 is the path-length divided by the speed of light. If we assume that light travels at a constant velocity c_ν^0 in all directions we get $t_0 = 2\pi r/c_\nu^0$. When a light signal is emitted from the light source a portion of the signal goes clockwise (denoted by the inner line of Fig. 2) and the rest goes anticlockwise. Following Kelly [1], let us then "consider" first the situation as observed by an observer stationary in the laboratory. The anticlockwise signal is going against the rotation of the equipment and will return to the light source when the source and interferometer are now at S′. The signal travelling clockwise, with the direction of rotation of the equipment, will return to the interferometer at S.

Let $\mathrm{d}s'$ be the distance SS′ and $\mathrm{d}s''$ the distance SS″. Let t' be the time measured by an observer situated in the stationary laboratory for the light to go from S to S′ in the anticlockwise direction.The time measured by that observer is

$$t = \frac{2\pi r - \mathrm{d}s}{c_\nu^+}. \qquad (8)$$

But t' is also the time taken by the disc to move a distance $\mathrm{d}s'$ in the clockwise direction. Therefore $t' = \mathrm{d}s'/v$ with $v = r\omega$, $\mathrm{d}s' = t'v$ and, from (8)

$$\mathrm{d}s = \frac{(2\pi r - \mathrm{d}s)\nu}{c_\nu^+}, \quad \frac{\mathrm{d}s}{\nu} = \frac{2\pi r}{c_0^+ + \nu},$$

or

$$t = \frac{2\pi r}{c_0^2 + \nu}. \qquad (9)$$

Note that Eqs. (8) and (9) both give the time recorded by a stationary observer; the equations simply state this time in different mathematical terms.

Following similar calculations one gets for t'', the time measured by a stationary observer for the light to go from S to S″ in a clockwise direction

$$t'' = \frac{2\pi r}{c_\nu^- + \nu}. \qquad (10)$$

Substracting Eq. (9) from (10), the difference between the time for the light to go clockwise and anticlockwise is given by (assuming $c_\nu^+ \cong c_\nu^-$)

$$\mathrm{d}t = \frac{2\pi r}{c_\mu^+ - v} - \frac{2\pi r}{c_\mu^+ + v} = \frac{4\pi v r}{c^2 - v^2}, \qquad (11a)$$

and, since $v = r\omega$, and A is the area πr^2 of a circle, one has (with $c_v \cong c$)

$$\mathrm{d}t = 4A\omega(c^2 - v^2)^{-1}. \qquad (11b)$$

The v^2 term is negligible for practical tests, and may be ignored, thus justifying Eq. (1) when the paths of light are on the disk. Evidently this is not true in general [5] and relation (11) does not apply to the case where the light path is not in the plane of the spinning disc: since the time spent by a light signal on its journey has to be calculated. This time will be different from the result obtained by using the projected-area method.

A practical example of a case where the signal is not solely in the plane of the disc is the 1939 Dufour and Prunier [10] test mentioned above. They had the path of the light partly on the spinning disc, and partly in the fixed laboratory. The light firstly traversed a path on the spinning disc, was then reflected vertically up to a mirror fixed overhead in the laboratory light, then traversed linear horizontal paths and came vertically back down to the disc: whereupon it completed the horizontal trajectory on the disc. Because the overhead horizontal path was directly above the path adopted in the earlier test, done entirely on the spinning disc, the projected area of this circuit on to the plane of the disc was the same as before.

The extra portions of interest are the two vertical connections, between that part of the circuit on the disc and that part overhead in the laboratory. The 22 tests done gave an average fringe shift of $F = 0.056$, with the individual results varying from 0.046 to 0.078. Using the projected area in Eq. (11b), one arrives at the theoretical result 0.053 against which to compare the test results. If the time for the light to

[5] To date, it has been assumed that, if the plane of the light path in a Sagnac-type test does not coincide with the plane of the rotating disc, the "area" A to be used in Eq. (1) is the projection of the area enclosed by the light path on to the plane of the disc [14]. It is here contended that this method is not correct, because the experimental results contradict the use of the projected area.

travel the vertical connections is taken into consideration (two lengths of 10 cm each), the result would be about 0.059 [1].

Incontrovertible proof that the projected area is not the correct criterion in general is provided in a paper by Dufour and Prunier [10] published in 1941. They showed how they had, in the 1939 tests described above (but not published at the time), varied the path of the light in the fixed laboratory, to give a much reduced projected area on to the plane of the rotating disc. In one case, they changed the projected area by a factor of 2.5 without any observable change in fringe shift. Using the projection of the enclosed area onto the disc beneath, for area A in Eq. (1), could not be reconciled with the fact that the projected area had changed so much, while the Sagnac effect had not altered. However, the path length for the light in both the in-plane and out-of-plane experiments was approximately the same (so that no change in the fringe shift is predicted by the "path-length" definition) [1].

5. Description of the Sagnac effect within general relativity with non-zero photon mass

We first show, by applying our preceding analysis to the particular case of various Sagnac-type set ups, that its particular validity in the inertial set-up of Fig. 1 implies the existence of a non-zero photon rest mass $m_\gamma \neq 0$ i.e., that one can thus also justify relation (1) within relativity theory. We start from two coaxial flat disks interferometers S_0 and S where S rotates uniformly with respect to S_0 with an uniform constant angular velocity ω. An observer instantaneously at rest in S_0 (the laboratory frame for example) which sees units of length l_0 and time t_0 will see different units l and t in S. Since S_0 and S are instantaneously inertial frames the transitions $\mathrm{d}l_0 \to \mathrm{d}l$ and $\mathrm{d}t_0 \to \mathrm{d}t_0$ are (following Einstein's argument) given by Lorentz transformations. In a polar system of coordinates (r, θ) in S the distance between two neighbouring points (r, θ) and $(r + \mathrm{d}r, \theta + \mathrm{d}\theta)$, measured with rods and clocks of S_0 is always

$$\mathrm{d}\sigma^2 = \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 \qquad (12)$$

for an observer in S_0.

For such an observer the measure of $\mathrm{d}l$ in S along a radial direction ($v = 0$) does not change but it contracts when perpendicular to the radius (when $v = \omega r$) so that $\mathrm{d}l_0 \to \mathrm{d}l_0(1 - \omega^2 r^2/c^2)^{1/2}$. For an observer in S_0 the distance between (r, θ) and $(r + \mathrm{d}r, \theta + \mathrm{d}\theta)$ measured in the accelerated system S becomes

$$\mathrm{d}\sigma^2 = \mathrm{d}r^2 + \frac{\mathrm{d}\theta}{1 - \omega^2 r^2/c^2}, \qquad (13)$$

so that for example a circle of circumference S ($v = \text{const}$) measured with the Galilean measures of S_0 ($\omega = 0$), i.e., $S_0 = 2\pi r$, changes when measured with the Galilean measures of S, i.e., $S = S_0(1 - r^2\omega^2/c^2)^{-1}$ and the *geometry tied to the "natural" units of a measure in S is no longer an Euclidean geometry*. For example if we compare time measurement in S and S_0 using two clocks H and H_0 (synchronised at the initial instant where S and S_0 spatially coincide) one sees that the ratio of the measured times t and t_0, in H and H_0, at a subsequent time, is (following Einstein's argument) given by the corresponding indications of two clocks H′ and H_0: where H′ is an inertial clock in a system which coincides locally with A at this moment. This implies that

$$t = t_0\left(1 - r^2\omega/c^2\right)^{1/2}, \qquad (14)$$

so that if the clock H comes back on H_0, the observers in S and S_0 will both conclude that the clock H is retarted: exactly as predicted by Langevin's clock paradox.

For an observer in S_0 this retardation is just an effect resulting from the acceleration of H along its motion. For an observer tied to S, where H has been constantly at rest, this results from the existence of a gravitational field in S (which is motionless for him) resulting from a potential $U = -r^2\omega^2/c^2$ so that

$$t = t_0\left(1 - 2U/c^2\right)^{1/2} \qquad (15)$$

as predicted by the general theory of relativity.

The time t is thus variable in S and depends on the position of the clock H. All clocks located at the same distance r from the common center of S_0 and S thus observe the same "local time" t in S.

This prediction of relativity theory is very remarkable since it implies (contrarily to one of its basic postulates) that if one has $m_\gamma = 0$ and one defines with c the velocity of light in S (i.e. utilizes the associated local time) this velocity is not constant in the inertial frame S. This "contradiction" (as mentioned before) has been known (but swept under the rug) for a long time. For example to maintain the velocity c for light Langevin proposed the introduction, in S, of a new time definition (called "natural time"), unrelated with Lorentz transformations, defined by the relation

$$\mathrm{d}\tau = \left(1 - r^2\omega^2/c^2\right)^{1/2} \times \left[\mathrm{d}t_0 - r^2\omega \,\mathrm{d}\theta c^2\left(1 - r^2\omega^2 c^{-2}\right)\right],$$

which varies with r and θ and has never been justified physically.

This contradiction can be illustrated as follows: In the inertial system S_0 we have by definition

$$\mathrm{d}s_0^2 = -\mathrm{d}x_0^2 - \mathrm{d}y_0^2 - \mathrm{d}z_0^2 + c^2\,\mathrm{d}t_0^2 = -\mathrm{d}r_0^2 - t_0^2\,\mathrm{d}\theta_0^2 - \mathrm{d}z_0^2 + c^2\,\mathrm{d}t_0^2 = 0. \quad (16)$$

Since S_0 rotates with respect to S with the constant angular velocity $-\omega$ we have

$$r = r_0, \quad \theta = \theta_0 - \omega t_0, \quad z = z_0,$$

so that

$$-\mathrm{d}r^2 - r^2\,\mathrm{d}\theta^2 - \mathrm{d}z^2 \mp \frac{2r^2\omega\,\mathrm{d}\theta\,\mathrm{d}t}{\left(1 - r^2\omega^2/c^2\right)^{1/2}} + c^2\,\mathrm{d}t^2 = 0. \quad (17)$$

Introducing the usual spatial Euclidean element $\mathrm{d}\sigma_1 = \mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2 + \mathrm{d}z^2$ this becomes

$$d\sigma_1^2 \pm \frac{2r^2\omega^2}{\left(1 - r^2\omega^2/c^2\right)^{1/2}}\,\mathrm{d}\theta\,\mathrm{d}t - c^2\,\mathrm{d}t^2 = 0, \quad (18)$$

which implies that in S the velocity of light V defined by $V = \mathrm{d}\sigma_e/\mathrm{d}t$ can be written

$$V^2 = c^2 \mp \frac{2r^2\omega}{\left(1 - r^2\omega^2/c^2\right)^{1/2}}\frac{\mathrm{d}\theta}{\mathrm{d}t} \neq c^2.$$

Evidently this result is in contradiction with relativity theory but this contradiction is bypassed [20] (i) if one drops the assumption $m_\gamma = 0$, (ii) if one somehow modifies relativity theory.

In this Letter we choose assumption (i) and will

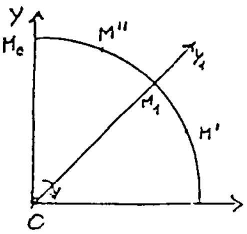

Fig. 3.

now show that if $m_\gamma \neq 0$ we can interpret all known Sagnac-type effects and justify relations (1) and (3).

To justify relation (1) with $m_\gamma \neq 0$ let us assume that the disk S_0 is at rest in the absolute inertial frame Σ_0 with origin O at the disk center. In its neighbourhood, for a given frequency ν_0 light travels with the same constant velocity v_0 in all directions. Of course any result observed in $V = \mathrm{d}L/\mathrm{d}\tau$ can be transposed in another inertial frame Σ by the Lorentz transformation $\Sigma \to \Sigma_0$. The corresponding photons (and plane wave constitutive elements) can be compared to bilocal clocks with an intrinsic real time, i.e., frequency ν with

$$h\nu = m_\gamma c^2\left(1 - v_0^2/c^2\right)^{-1/2}. \quad (19)$$

One now considers a photon M which starts in M_0 and moves on the circle of radius r with a constant velocity v_0 with respect to S_0 (Σ_0), i.e., V with respect to S. After a time $\mathrm{d}t_0$ the axis OY of S becomes OY_1 and the photon is in M′ or M″ through clockwise or anticlockwise notion. In S_0 it travels, if $\mathrm{d}L_0$ denotes in S_0 the lengths M_1M (or M_1M), a distance

$$\mathrm{d}\sigma_0 = V_0\,\mathrm{d}t_0 = \mathrm{d}L_0 \pm r_0\omega\,\mathrm{d}t_0,$$

since photon motion and disk rotation are independent physical processes. This yields

$$V_0 = \frac{\mathrm{d}L_0}{\mathrm{d}t_0} \pm r_0\omega.$$

In the system S this velocity becomes

$$V = \frac{\mathrm{d}L}{\mathrm{d}\tau}$$

and Einstein's law and addition of velocities (only valid when $m_\gamma \neq 0$) yields

$$V = \frac{V_0 \mp r\omega}{1 \mp r\omega V_0/c^2}, \quad V_0 = \frac{V \pm r\omega}{1 \pm r\omega V/c^2}. \quad (20)$$

In S_0 when two photons start together from M_0 with uniform motions $+V_0$ and $-V_0$ they return to the same point after a time $t_0 = 2\pi r/V_0$.

In S_0, if we call dL_0 and dL_0'' the distances M_0M and M_0M'' travelled by photons moving in the anticlockwise and clockwise directions, then the time t_0 to pass from M_0 to M_0' is

$$t_0 = \frac{2\pi r - dL_0}{V_0},$$

but t_0 is also the time taken by the disk to move a distance dL_0 in the clockwise direction so that $t_0 = dL_0/r\omega$ and $dL_0 = (2\pi r - dL_0)\, r\omega/V_0$, i.e.,

$$\frac{dL_0}{r\omega} = \frac{2\pi r}{V_0 + r\omega}, \quad t_0 = \frac{2\pi r}{V_0 + r\omega}.$$

An identical argument yields for t_0'' (i.e., the time, measured in S_0, for light (photons) to travel from M_0 to M'' in a clockwise direction) the values

$$t_0'' = \frac{2\pi r}{V_0 - r\omega},$$

so that we finally obtain for the difference between the times for the light to move clockwise and anticlockwise the expression

$$dt_0 = \frac{2\pi}{V_0 - r\omega} - \frac{2\pi r}{V_0 + r\omega} \cong \frac{4\pi r^2\omega}{V_0 - r^2\omega^2},$$

since $A = r^2\omega^2$ is the area of a circle S_0 in Σ_0 when we have $r^2\omega^2/c^2 = 0$.

If we have $r^2\omega^2/c^2 \cong 0$,

$$dt_0 = \frac{4\pi A}{V_0 - r^2\omega^2} \cong \frac{4A\omega}{c^2}. \tag{21}$$

We shall now show that as experimentally established by Dufour and Prunier (C.R.Acad.Sci. Paris 204 (1937) 1925), the introduction of $m_\gamma \neq 0$ implies that *the fringe displacement is the same when the light source and observer are located on S or in S_0, i.e., at neighbouring points on the disk's periphery.*

At first sight of course this appears very surprising since the time t_0 of an inertial observer tied to S_0 in Σ_0 located at the center O of the interferometer (called "central time" by Langevin [7]) is different from the local time t of a tangent instantaneous inertial observer carried by the rotating disk at a distance r, since we have

$$t = t_0\left(1 - r^2\omega^2/c^2\right)^{1/2}$$

in the restricted theory of relativity. We have shown, however, that in his rest frame Σ the corresponding invariant element can be written in the form

$$ds^2 = (c^2 - r^2\omega^2)\, dt^2 - 2\omega r^2\, d\theta\, dt - (dr^2 + r^2\, d\theta) \tag{22}$$

and thus see directly that the rectangular term $d\theta\, dt$ implies an anisotropy in the velocity of light (given by the relations (20)) tied to a non-Euclidean metric.

With this metric Langevin has shown that one recovers relation (1) so that the phase shift is a scalar invariant under Lorentz transformations.

This is only to be expected since in S the times of motion of the two interfering paths (clockwise and anticlockwise) are different since the associated light velocities are different. On S the wavelengths are not equal and the period equal: contrarily to the S_0 case. This result thus confirms (1) the scalar invariant character of the phase of lights and (2) the existence of $m_\gamma \neq 0$.

We conclude with the remark that within this interpretation Sagnac-type interferometers can be considered as measuring devices to detect absolute space-time motions with respect to the local inertial frame Σ_0 where the 2.7°K microwave radiation is isotropic for all light velocities [3]; a property shared with one piece Faraday generators [21] within the frame of Maxwell's theory of light. The utilization of the Sagnac interferometer to detect locally absolute space time motions will be discussed in detail in a forthcoming paper.

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