Foundations of Physics, Vol. 26, No. 5, 1996

Noninvariant One-Way Velocity of Light

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Received October 25, 1995

After discussing in the first five sections the meaning and the difficulties of the principle of relativity we present a new set of spacetime transformations between inertial systems ("inertial" transformations), based on three assumptions: (1) The two-way velocity of light is c in all inertial systems and in all directions; (2) Time dilation effects take place with the usual relativistic factor; (3) Clocks are synchronized in the way chosen by nature itself, e.g., in the Sagnac effect. We show that our new transformation laws can explain the available experimental evidence in spite of the implied noninvariance of the one-way velocity of light.

1. THE RELATIVITY PRINCIPLE AND CLOCK SYNCHRONIZATION

The one-way velocity of light in moving inertial systems (e.g., on Earth) has never been measured accurately. Often people stress that in order to measure it one needs synchronized clocks, but that in order to synchronize clocks one must know the one-way velocity of light, so that the logical situation becomes circular. All the laboratory experiments (from Fizeau, Foucault, Michelson, to the recent ones) measured instead the *two-way* velocity of light. Since such measurements are obviously possible with just one clock, the synchronization problem did not arise. When Einstein[1] formulated his theory of relativity he *postulated* that the velocity of light has always the same value c.

There is now some agreement among physicists at least on the conclusion that the constancy of the one-way velocity of light is a useful *convention* and that it must not be considered as something dictated by objective

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properties of the natural world. It has been observed[2] that: "When clocks are synchronized according to the Einstein procedure the equality of the velocity of light in two opposite directions is trivial and cannot be the subject of an experiment." The history of this idea is, however, very long.

Already in 1898 Poincaré,[3] discussing the independence of the velocity of light of its direction of propagation stated: "This is a postulate without which it would be impossible to start any measurement of this velocity. It will always be impossible to verify experimentally the said postulate." Similarly, in 1916 Einstein[4] wrote: "that light requires the same time to traverse the path *AM* ... as the path *BM* [*M* being the mid-point of the line *AB*] is in reality *neither a supposition nor a hypothesis* about the physical nature of light, but a *stipulation* which I can make of my own free will." Reichenbach[5] considered the following situation: In an inertial system *S*, a flash of light leaves point *A* at time t_1, is reflected back in point *B* at time t_2, and arrives again in *A* at time t_3. The problem is how to synchronize the clock near point *B* with the clock near point *A*. In the theory of relativity, it is *assumed* that the one-way velocity of light has the same value from *A* to *B* as from *B* to *A*, so that $t_3 - t_2 = t_2 - t_1$, whence the clock-*B* time t_2 can be written in terms of the two clock-*A* times t_1 and t_3 as follows:

$$t_2 = t_1 + \tfrac{1}{2}(t_3 - t_1) \tag{1}$$

Reichenbach commented:

"This definition is essential for the special theory of relativity, but it is not epistemologically necessary. If we were to follow an arbitrary rule restricted only to the form

$$t_2 = t_1 + \varepsilon(t_3 - t_1) \qquad 0 < \varepsilon < 1$$

it would likewise be adequate and could not be called false. If the special theory of relativity prefers the first definition, i.e., sets ε equal to 1/2, it does so on the grounds that this definition leads to simpler relations."

In 1979, Jammer,[6] discussing the Reichenbach coefficient ε, stressed that: "One of the most fundamental ideas underlying the conceptual edifice of relativity, as repeatedly stressed by Hans Reichenbach and Adolf Grünbaum, is the conventionality ingredience of intrasystemic distant simultaneity." Later he added: "The "thesis of the conventionality of intrasystemic distant simultaneity" or briefly, "conventionality thesis" consists in the statement that the numerical value of ε need not necessarily be 1/2, but may be any number in the open interval between 0 and 1, i. e., $0 < \varepsilon < 1$, without ever leading to any conflict with experience."

Clearly, different values of ε correspond to different values of the one-way velocity of light. In fact, one can write

$$t_2 - t_1 = \frac{l}{\tilde{c}(\theta)} \quad \text{and} \quad t_3 - t_2 = \frac{l}{\tilde{c}(\theta + \pi)} \tag{2}$$

where l is the AB distance and $\tilde{c}(\theta)$ is the one-way velocity of light from A to B in the considered inertial frame S. In general $\tilde{c}(\theta)$ will depend on the angle θ between the direction AB and the absolute velocity of S. Of course $\tilde{c}(\theta + \pi)$ is the one-way velocity from B to A. By adding the previous relations one gets

$$t_3 - t_1 = \frac{l}{\tilde{c}(\theta)} + \frac{l}{\tilde{c}(\theta + \pi)} = \frac{2l}{c} \tag{3}$$

the last step being necessary, because the *two-way* velocity of light has been measured with great precision and always found to be c. From (2) and (3) one easily gets

$$\varepsilon = \frac{t_2 - t_1}{t_3 - t_1} = \frac{c}{2\tilde{c}(\theta)} \tag{4}$$

Therefore freedom of choice of ε means freedom of choice of the one-way velocity of light!

Einstein's synchronizations method is not only compatible with the relativity principle, but is rather its most important and direct consequence. In fact Maxwell's equations outside electric charges imply the validity of d'Alembert's wave equation for electric and magnetic fields, which describes the propagation of electromagnetic waves having velocity c independently of the state of motion of the sources. If Maxwell's equations must hold in every inertial frame as required by the relativity principle, also the numerical value of the velocity of light must necessarily remain the same. Given this argument, it is not clear why Einstein felt the need of postulating separately the relativity principle and the constancy of the velocity of light. Probably he had doubts about the actual validity of Maxwell's equations and did not want to rely too heavily on them.

Now, if a scientific statement is true it cannot be refused, but if it is only conventional it becomes immediately interesting to look for alternatives, which will be based on conventions different from the usually accepted one. In particular, if the constancy of the velocity of light is conventional it is obviously legitimate to study theories in which this constancy is not true. But by so doing, the principle of relativity will also be violated. This can only mean that the relativity principle itself is only a useful human convention and not a fact of nature (a conventional truth cannot be a necessary consequence of an objective truth). These considerations seem to weaken the *necessity* to accept the strong formulation of the relativity principle: it is possible and interesting to explore alternative paths.

2. MATHEMATICAL INSTABILITY OF SPECIAL RELATIVITY

It is important to stress that the Lorentz transformations are necessary consequences of the relativity principle. In fact, one can always choose two Cartesian coordinate systems in the inertial reference frames S and S_0 by assuming:

1. that space is homogeneous and isotropic, and that time is homogeneous;
2. that in S_0 the velocity of light is the same in all directions, so that Einstein's sychronization can be applied and velocities relative to S_0 can be measured; that the origin of S (equation $x=0$) is seen from S_0 moving with velocity v parallel to the $+x_0$ axis, that is, satisfying $x_0 = vt_0$;
3. that the observer in S sees his origin ($x=y=z=0$) coincident with that of S_0 at $t=0$, and vice-versa that the observer in S_0 sees his origin ($x_0=y_0=z_0=0$) coincident with that of S at $t_0=0$;
4. that planes (x_0, y_0) and (x, y) coincide at all times t_0; that also planes (x_0, z_0) and (x, z) coincide for all times t_0;
5. that planes (y_0, z_0) and (y, z) coincide at time $t_0=0$.

It was shown in Ref. 7 that the previous conditions reduce necessarily the transformation law from S_0 to S to the form

$$\left.\begin{aligned} x &= f_1(x_0 - vt_0) \\ y &= g_2 y_0 \\ z &= g_2 z_0 \\ t &= e_1 x_0 + e_2(y_0 + z_0) + e_4 t_0 \end{aligned}\right\} \tag{5}$$

where the five coefficients f_1, g_2, e_4, e_1, and e_2 can depend on v. If at this point one assumes the validity of the relativity principle (including invariance of light velocity) the previous transformations reduce *necessarily* to the Lorentz ones.

In other words, if one considers a five-dimensional space in which the coefficients f_1, g_2, e_4, e_1, and e_2 are represented as Cartesian coordinates, one can say that for a given value of v all coefficients are completely fixed by the relativity postulate, and therefore represented by a geometrical point. In this space there is no finite area representing relativity, only a structureless unprotected point. All other points lead to the logical negation of the relativity principle. One can obviously conclude that any eventual violation of the Lorentz transformations found at any time in the future, however small, will imply that relativity itself does not hold as a description of nature.

It can therefore be said that the special theory of relativity (STR) is mathematically "unstable," in the sense that any shift, however small, of any one of the five coefficients f_1, g_2, e_4, e_1, and e_2 away from its relativistic value implies necessarily the existence of a privileged frame. In other words, either Lorentz has given mankind a final truth with his transformations, or the existence of a privileged frame shall be accepted in the future.

3. RELATIVITY AND PHYSICAL REALITY

The theory of relativity leads to peculiar consequences if used to understand how objective reality has to be described. To begin with, *what one sees* cannot be considered real in the present, because by looking at distant objects one does not see them as they are now, but as they were when the light now entering our instruments left them. Also, it is not reasonable to attribute reality to the future, because common sense tells us that it does not yet exist and that it is at least partly undetermined. For these reasons a reasonable definition of reality seems to be the following: *all that exists now, here and elsewhere.* A different choice would define to be real either things that do not exist anymore, or things that do not yet exist. The light from a distant galaxy can take hundreds of millions of years to reach our instruments, and in this long time the object that emitted it could have dissolved, have collided with another cosmic object, or have exploded (there are pictures of galaxies devastated by huge explosions). It will, anyway, have evolved, and could *now* differ significantly from what is observed.

Let us adopt the relativistic description, with a Minkowski diagram having space in abscissae (only one dimension for graphical reasons!) and time in ordinates. At time $t = 0$ an observer U_0 located in the origin of an inertial system S must regard as being objectively real down to the smallest detail all events in space. In a bidimensional diagram space is represented as x axis, whose equation is $t = 0$, and which therefore contains all events simultaneous with the instantaneous presence of U_0 in the origin at $t = 0$.

If we consider, however, another inertial reference frame S' its axes x' and t' are represented in the Minkowski diagram as straight lines in the plane (x, ct) because of the linearity of the Lorentz transformations. The observer U'_0 at rest in S' must attribute reality to all events happening at his present time $t' = 0$. These events are of course different from those constituting the reality of U_0. According to the relativity principle it does not make any sense to ask which of the two observers is right. Given the complete symmetry between inertial systems, they are both right. So all the events on the x' axis, whose equation is $t' = 0$ and whose inclination depends on the velocity of S' relative to S, can be considered just as real as those on the x-axis. The reality line of the observer U'_0 has an inclination in time with respect to that of U_0 and also passes through the origin of the Minkowski diagram (see Fig. 1). Therefore U'_0 will attribute reality to events in U_0's future, which are therefore not part of his reality. In the previous example, however, these future events are elsewhere and do not belong to the personal future of U_0, who is assumed at rest in the origin $x = 0$.

The meaning of the given argument can easily be understood by assuming that in S there are different observers $U_1, U_2, \ldots, U_n$ placed in different points $x_1, x_2, \ldots, x_n$ of the x axis, all provided with clocks synchronized according to the Einstein procedure. These observers are all equivalent in their description of reality, since time $t = 0$ is the same for all

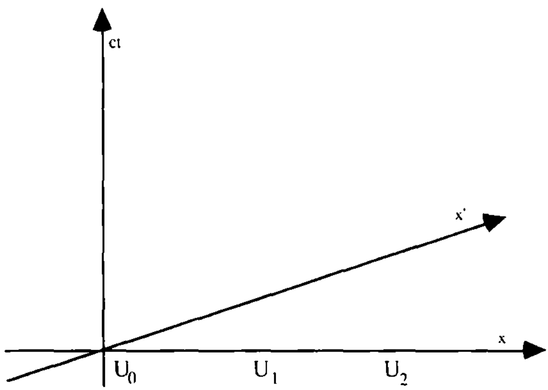

Fig. 1. In a Minkowski diagram the reality line of a moving observer is the x' axis which includes events belonging to the future of the observers U_1, U_2,... placed on the x axis of a different inertial frame.

of them, and reality consists of the events placed on the x axis; naturally they are also equivalent to the observer U_0 in $x=0$ considered before. It is now clear that the reality line of U'_0 passes through the personal future of some of the observers at rest in S [those placed in points having positive (negative) x if S' moves with velocity in direction $+x(-x)$].

Reality has so far only been attributed to a single instant of the future but the argument can easily be generalized. Indeed, infinitely many reality lines pass through every point above the x axis of the diagram (ct, x), each such line representing the (relativistic) reality of some legitimate inertial observer. The only restriction is the inclination of these reality lines in a diagram (ct, x): it can never exceed 45°, since all velocities are subluminal. Nevertheless past, present, and future are completely real, that is, pre-established in the minutest detail. Passing from two to four dimensions we can conclude that all of spacetime (ct, x, y, z) is real, despite the different perception humans have of it. In other words, my future should be real, i.e., fixed in the tiniest details, despite its looking to me as largely undetermined, unshaped, presently unreal.

Thus, relativity leads to a very strange conception of the universe, in which a single reality fills uniformly past, present, and future: at my present other observers no less legitimate than I consider my personal future as given in all detail. According to them there is not the slightest freedom which I can use in order to influence the course of events. The impression I have of a reality evolving sometimes in a casual (nondeterministic) way would therefore be entirely subjective, a limitation (due to my poor means of observation) to a fixed time section of the complete fourdimensional reality. Relativity therefore leads one to accept a hyperdeterministic universe in which the whole future is completely pre-established in the minutest details and in which all sensations of individual freedom (even those limited to very simple events, like the choice between holding and dropping a stone) are pure illusions.

The previous argument is founded on the idea that every observer is right in considering real all that exists around him and elsewhere at *his* present time. There is of course another possibility compatible with relativism, based on the idea that all observers are wrong and that no reality exists outside the thinking subjects. In such a case the plane (ct, x) would become a "*tabula rasa*" in which nothing exists, and the corresponding philosophy would be that of the purest idealism. Such a "solution" is obviously even less interesting than the last one. It does not seem possible to escape from this vicious circle (hyperdeterminism, idealism) without abandoning the principle of relativity.

Karl Popper, in his autobiography[8] is critical of hyperdeterminism. He recalls a discussion with Einstein in Princeton (1950): "I tried to persuade him to give up his determinism, which amounted to the view that the world was a four-dimensional Parmenidean block universe in which change was a human illusion, or very nearly so. (He agreed that this had been his view, and while discussing it I called him "Parmenides.")" Popper's identification is justified, since for both Einstein and Parmenides the subjective impression of evolution is pure appearence. Popper found this description of reality unacceptable, and it is difficult to disagree with him.

4. RELATIVITY PRINCIPLE AND EQUIVALENCE PRINCIPLE

Coming now to general relativity one can observe that also in its case the validity of the relativity principle is far from obvious. Newton was convinced that an "absolute" space exists and had produced the nice example of the water in the rotating bucket for showing that absolute effects can be produced. In his 1916 paper on general relativity Einstein[9] started by repeating Newton's reasoning and showing that he had deeply understood its meaning. He gave the example of two deformable spheres A and B placed in the interstellar space "at so great distance from each other and from all other masses that only those gravitational forces need to be taken into account which arise from the interaction of different parts of the same body." At a certain moment one of them (let us say A) is set in rapid rotation around the line joining the centers of A and B. From a strictly rotational point of view one could say that relativity holds, because the observer in B says "A is rotating," but also the observer on A could say that he sees B rotating. The situation changes, however, if one considers the deformation of A due to the centrifugal forces, because in this case both observers must agree that it is due to the rotational motion of A with respect to empty space. Since no deformation arises for B, it must be concluded that not all aspects of rotation can be considered to be relative, and thus that some are absolute. By the way, there are well known methods with which the inhabitants of an ellipsoid can measure the degree of squeezing of their planet: remember Maupertuis' 1736 expedition in Lapland for choosing between Newton's and Descartes' theories.

At this point Einstein adds the doubtful notion that the previously described situation of the two spheres can still be compatible with the relativity principle in spite of the obvious asymmetry due to deformation. One reason he gives is that inertial forces and gravitational forces both give rise to accelerations which are exactly the same for all bodies: therefore, he says, fictitious forces must be of gravitational origin. This is of course the equivalence principle, Einstein's physical reformulation of the more abstract Mach's principle. I hasten to add that the equivalence principle is a beautiful and fully acceptable physical idea. To pull it on the side of relativity seems very doubtful, however. It is not the same to have a local mass generating a well-defined static gravitational field, and to have a dynamical reaction of the cosmic gravitational fields on the accelerated bodies. The two situations present enough differences to be immediately distinguishable: in practice no physical observer will ever be in doubt as to the origin of the forces felt in his rest frame. Probably Einstein had in mind also the need to get rid of Newton's mystical interpretation of absolute space, and on this point it is easy to agree once more with him. Taken for granted that everything is physically concrete and that the manifestations of space have a basis in the cosmic gravitational fields, there remains the fact that not all phenomena appear to be the same in all reference frames.

A simple way for obtaining a critical understanding of Einstein's point of view is the following: suppose that in some inertial reference frame S there is an electric charge at rest. In S one will observe an electrostatic field, while in other inertial reference frames in motion with respect to S one will also observe the presence of a magnetic field due to the time variation of the electric field. Is this a proof that the relativity principle does not hold? Obviously not, because no physical law stops us from putting a similar charge at rest in a different inertial frame. A little asymmetrical *fact* cannot imply a breakdown of the relativity principle: if a gentleman is singing in S we cannot say that the principle of relativity is violated; it would be so only if it were physically impossible for him to go singing in all other inertial frames. Einstein implies essentially the following: if the Universe had been at rest in a reference frame different from the one in which it actually stays, then the fictitious forces would behave differently, and the reference frames which today we call inertial would not be such anymore, while other frames that today we consider accelerated would become inertial. In other words, the fact that the Universe stays in a frame rather than in another one, according to Einstein, does not imply a breakdown of the relativity principle. Here it is difficult to follow him, because his reasoning gives clear priority to the reference frames (which are useful human constructions) over the concrete reality of the whole Universe. Our whole material world is given a role similar to that of the singing gentleman, and must mentally be shifted from a frame to another. It can well seem dangerous to reason in such a way. But if something has to be modified there remains only the idea that privileged frames do exist.

5. THE RELATIVITY PRINCIPLE AS A BROKEN SYMMETRY

"Enclose yourself, with a friend, in the largest covered room of any big ship, and introduce flies, butterflies and other such flying animals; there should also be a great tank full of water, with little fish in it; furthermore, suspend some bucket up high, from which drops fall into a vase with a narrow mouth, placed right below it: and, while the ship is still, carefully observe how the little flying animals travel toward all parts of the room with equal speed; the fish will be seen to swim indifferently in all directions; the drops will continue to fall into the vase underneath; and you, tossing something to your friend, will not have to throw it any more vigorously this way or that, provided the distances are the same; and jumping, as one says, with your feet together, you will go equally far in all directions. Once you have carefully observed all these things, which are hardly surprising as long as the ship is still, set it in motion, with any speed (provided the motion is uniform, and not fluctuating this way and that), and you will notice no change in any of the effects mentioned, nor from any of them will you be able to tell whether the ship is moving or not."

With this famous sentence Galilei gave the first modern formulation of the relativity principle. He would certainly not have written it if every object had been invested by a very intense flux of radiation coming from the direction of motion of the ship and if, as a consequence, every form of life had been instantaneously wiped out. Fortunately our ships are not exposed to such effects, but there exist conceptually similar situations.

In fact there are perfectly conceivable frames of reference in which the light coming from stars and galaxies toward which these frames are travelling is Doppler shifted toward violet to such an extent that it is composed of ultrahard gamma rays: in these frames all forms of life and all material aggregations (instruments) would immediately be wiped out. In what sense, then, should these frames be considered equivalent to those moving with small cosmic velocity, if not even in principle it is possible to admit that physical experiments can be carried out with an apparatus at rest in them?

In the 2.7 K cosmic background radiation an anisotropy has been detected which is probably due to the motion of the solar system in space (toward the constellation Leo with a velocity of about 300 km/s). This gives rise to a very weak net flux of radiation, because the Doppler effect makes the radiation coming from the direction of the Leo a bit more intense than that coming from the opposite direction. In every ordinary situation the consequences of this anisotropy are negligibly small. Nevertheless one cannot pretend that no important matter of principle exists here, and there have been authors who considered the anisotropy of the background radiation as a proof of lack of validity in nature of the relativity principle, for example, Bondi,[10] who discussed a "clear conflict between cosmology and ordinary physics," or Bergmann,[11] who stated: "The principle of relativity would hold only for certain types of experiments (those excluding interaction with the background radiation, for instance), or provided experiments are not refined beyond a certain degree of accuracy or sensitivity."

These difficulties can be overcome by admitting that the principle of relativity is some kind of broken symmetry: true with a very good precision in every ordinary situation, and nevertheless not anymore true in the case both of experiments carried out in inertial frames moving with large cosmic velocity, and of very precise experiments in normal inertial frames.

6. NATURE'S CHOICE OF SYNCHRONIZATION

It was shown in Ref. 12 that the most general transformation laws of the general type (5) between two inertial frames S_0 and S satisfying the conditions of constant two-way velocity of light and of time dilation according to the usual relativistic factor are

$$\left.\begin{aligned} x &= \frac{x_0 - \beta c t_0}{R(\beta)} \\ y &= y_0 \\ z &= z_0 \\ t &= R(\beta)\, t_0 + e_1(x_0 - \beta c t_0) + e_2(y_0 + z_0) \end{aligned}\right\} \tag{6}$$

where e_1, e_2 are two undetermined functions of velocity v, $\beta = v/c$, and

$$R(\beta) = \sqrt{1-\beta^2} \tag{7}$$

Length contraction by the usual factor $\sqrt{1-\beta^2}$ is also a consequence of (6). Rotational invariance around the x axis allows one to take $e_2 = 0$. The velocity of light compatible with all this was shown to be

$$\frac{1}{\tilde{c}(\theta)} = \frac{1}{c} + \left[\frac{\beta}{c} + e_1 R(\beta)\right] \cos\theta \tag{8}$$

where θ is the angle between the direction of propagation of light and the absolute velocity v of S. The transformations (6) with $e_2 = 0$ represent the complete set of theories equivalent to STR: if e_1 is varied, different elements of this set are obtained, which are all equivalent as far as the explanation of experimental results is concerned. The Lorentz transformation is recovered as a particular case with

$$e_1 = -\frac{1}{c}\frac{\beta}{R(\beta)} \tag{9}$$

Different values of e_1 are obtained from different clock-synchronization *conventions*. In all cases but that of STR such theories do not imply the validity of the relativity principle.

The simplest possibility in (6) is obviously $e_1 = 0$. A simple way of justifying this choice is to apply the so-called absolute synchronization,[2] by setting all clocks of S to time $t = 0$ when the passing clock at rest in the privileged system S_0 shows the time $t_0 = 0$.

There are, however, much stronger reasons for adopting a space-independent transformation of time. The assumed indifference of the physical reality concerning clock synchronization exists only insofar as one neglects accelerations: when these come into play, every inertial system exists, so to say, only for a vanishingly small time interval and it is physically impossible in the accelerated frame to adopt any time-consuming procedure for the synchronization of distant clocks, such as Einstein's procedure. Yet physical events take place and synchronization is somehow fixed by nature itself: we will see how this happens next.

A simple accelerated system is a rotating disk, and the Sagnac effect[13] is well known to take place in such conditions: a monochromatic light source placed on the disk emits two coherent beams of light in opposite directions. These travel along a circumference concentric with the disk, until they reunite in a point A and interfere, after a 2π propagation. The result can be achieved by forcing the light to propagate tangentially to the internal surface of a cylindrical mirror. The positioning of the interference figure depends on the disk rotational velocity (Sagnac effect). Most textbooks deduce the Sagnac formula (our Eq. (14) below) in the laboratory, but say nothing about the description of the phenomenon given by an observer placed on the rotating platform: we will see that SRT predicts a null effect on the platform, while our approach based on the inertial transformations gives the right answer. For simplicity we will assume that the laboratory is at rest in the privileged frame.

Sagnac Effect in the Laboratory

Light propagating in the rotational direction of the disk must cover a distance larger than the disk circumference length L by a quantity $x = vt_{01}$

equaling the shift of A during the time t_{01} taken by light to reach the interference region. Therefore

$$L + x = ct_{01} \qquad x = vt_{01} \tag{10}$$

From these equations it is easy to get

$$t_{01} = \frac{L}{c - v} \tag{11}$$

Light propagating in the direction opposite to that of rotation must instead cover a distance smaller than the disk circumference length L by a quantity $y = vt_{02}$ equaling the shift of A during the time t_{02} taken by light to reach the interference region. Therefore

$$L - y = ct_{02} \qquad y = vt_{02} \tag{12}$$

One now gets

$$t_{02} = \frac{L}{c + v} \tag{13}$$

The time difference Δt_0 between the two propagations is the parameter fixing the phase difference in the considered interference point. From (11) and (13) it follows that

$$\Delta t_0 = t_{01} - t_{02} = \frac{2L}{c} \frac{\beta}{1 - \beta^2} = \frac{2L_0}{c} \frac{\beta}{R(\beta)} \tag{14}$$

Obviously $L = L_0 R(\beta)$ is the disk circumference length reduced in the laboratory by the usual relativistic factor (7) if L_0 is the rest length of the same disk. The consistency of Eq. (14) with experimental data has been checked in many experiments.

Sagnac Effect on the Disk

Every small portion of the circumference of the rotating platform can be considered to be instantaneously at rest in a moving inertial frame of reference locally "tangent" to the disk. Therefore Eq. (8) applies for the velocity of light on the disk. Only the cases of light moving parallel and antiparallel to the local absolute velocity must be considered. It follows from (8) that the inverse velocity of light for these two cases is respectively given by

$$\left.\begin{aligned}\frac{1}{\tilde{c}(0)}=\frac{1}{c}+\left[\frac{\beta}{c}+e_1 R(\beta)\right]\\ \frac{1}{\tilde{c}(\pi)}=\frac{1}{c}-\left[\frac{\beta}{c}+e_1 R(\beta)\}\right]\end{aligned}\right\} \tag{15}$$

The time difference on the disk is given by

$$\Delta t=t_1-t_2=\frac{L_0}{\tilde{c}(0)}-\frac{L_0}{\tilde{c}(\pi)} \tag{16}$$

Substituting (15) in (16) one gets

$$\Delta t=\Delta t_0 R(\beta)\left[1+\frac{ce_1 R(\beta)}{\beta}\right] \tag{17}$$

where Δt_0 is given by (14) and $R(\beta)$ is the usual factor describing the dilation of time intervals in a moving frame. Given that (6) implies the usual time-dilation phenomenon, one can see that only the value $e_1=0$ leads to physical agreement with (14), while prediction (9) of SRT gives instead $\Delta t=0$. Therefore we reach the conclusion that of all theories having different values of e_1 only one ($e_1=0$) gives a rational description of the Sagnac effect on the rotating platform. In the case of $e_1 \neq 0$ the calculated time difference on the platform disagrees with the prediction (14) in the laboratory, a prediction which is of course the same for all theories of type (6) (SRT included), since in the laboratory (assumed to be at rest in the privileged frame) Einstein's synchronization was used.

Finally we review some arguments of Ref. 7 which, like the Sagnac effect, point to the superiority of the inertial transformations. Two identical spaceships **A** and **B** are initially at rest on the x_0 axis of the (privileged) inertial system S_0 at a distance d_0 from one another. Their clocks are synchronized with those of S_0. At time $t_0=0$ they start accelerating in the $+x_0$ direction, and they do so in the same identical way having the same velocity $v(t_0)$ at all times t_0 of S_0, until at $t_0=\bar{t}_0$ they reach a preassigned velocity parallel to $+x_0$. For all $t_0 \geqslant \bar{t}_0$ the spaceships can be considered at rest in a different inertial system S, which they concretely constitute.

It will now be shown that the transformation relating S_0 and S is necessarily the inertial one, if no clock synchronization is applied correcting what nature itself generated during the acceleration of the two spaceships. Since **A** and **B** accelerate exactly in the same way, their clocks will accumulate exactly the same delay with respect to those at rest in S_0. Motion is the same for **A** and **B** and all effects of motion will necessarily coincide, in particular time delay. Therefore two events simultaneous in S_0 will be such also in S, even if they take place in different points of space. Clearly we have a case of absolute simultaneity and the condition $e_1 = 0$ must hold in (6), reducing these transformations to their inertial form, given below in Eq. (9).

Another convincing argument showing that the condition $e_1 = 0$ gives the most natural description of physical reality is the following. Suppose that our spaceships have passengers P_A and P_B, who are twins. Of course in principle nothing can stop them from resynchronizing their clocks once they have finished accelerating and the two spaceships are at rest in S. If they do so, however, they find in general that they have different biological ages at the same (resynchronised) S time, even if they started the space trip at exactly the same S_0 time and with the same velocity, as stipulated above. Everything is regular, instead, if they do not operate any asymmetrical modification of the time shown by their clocks.

7. THE INERTIAL TRANSFORMATIONS

In the previous section we showed quite generally that the condition $e_1 = 0$ is the right choice of synchronization. This implies that from all positions in S_0 the time in S will be seen to be the same, and therefore that no positions dependent time-lag factor will be present in the transformation of time. This gives rise to a transformation different from the Lorentz one, but nevertheless particularly simple[7]:

$$\left.\begin{aligned} x &= \frac{x_0 - \beta c t_0}{R(\beta)} \\ y &= y_0 \\ z &= z_0 \\ t &= R(\beta)\, t_0 \end{aligned}\right\} \tag{18}$$

The velocity of light relevant to a theory based on (18) can easily be found by putting $e_1 = 0$ in (8):

$$\frac{1}{\tilde{c}(\theta)} = \frac{1 + \beta \cos\theta}{c} \tag{19}$$

The transformation (18) can be inverted and gives

$$\left.\begin{aligned} x_0 &= R(\beta)\left[x + \frac{\beta c}{R^2(\beta)} t\right] \\ y_0 &= y \\ z_0 &= z \\ t_0 &= \frac{1}{R(\beta)} t \end{aligned}\right\} \tag{20}$$

Note that there is a formal difference between (18) and (20). The latter implies, for example, that the origin of S_0 (satisfying $x_0 = y_0 = z_0 = 0$) is described in S by $y = z = 0$ and by

$$x = -\frac{\beta c}{1-\beta^2} t$$

This origin is thus seen to move with speed $\beta c/(1-\beta^2)$, which can exceed c, but cannot be superluminal. In fact a light pulse seen from S to propagate in the same direction as S_0 has $\theta = \pi$, and thus [using (18)] has speed $\tilde{c}(\pi) = c/(1-\beta)$, which can easily be checked to satisfy

$$\frac{c}{1-\beta} \geqslant \frac{c\beta}{1-\beta^2}$$

One of the typical features of these transformations is of course the presence of velocities which can grow without limit *when they are relative to moving systems* having absolute velocities βc near to c. Absolute velocities can instead never exceed c.[14] In STR one is used to relative velocities that are always equal and opposite, but this symmetry is a consequence of the particular synchronization used and cannot be expected to hold more generally.[15]

Consider now a third inertial system S' moving with velocity $\beta' c$ and its transformation from S_0, which of course is

$$\left.\begin{aligned} x' &= \frac{x_0 - \beta' c t_0}{R(\beta')} \\ y' &= y_0 \\ z' &= z_0 \\ t' &= R(\beta') t_0 \end{aligned}\right\} \tag{21}$$

where $R(\beta')$ is given by (7) with β' replacing β. By eliminating the S_0 variables from (21) and (20) one obtains the transformation between the two moving systems S and S':

$$\left.\begin{aligned} x' &= \frac{R(\beta)}{R(\beta')}\left[x - \frac{\beta' - \beta}{R^2(\beta)}\, ct\right] \\ y' &= y \\ z' &= z \\ t' &= \frac{R(\beta')}{R(\beta)}\, t \end{aligned}\right\} \tag{22}$$

A transformation having the form (18) was once written by Tangherlini,[16] while (20) and (22) do not seem to exist in the scientific literature: a possible name for (18)–(20)–(22) is "inertial transformation." In its most general form (22) the inertial transformation depends on two velocities (v and v'). When one of them is zero, either S or S' coincide with the privileged system S_0 and the transformation (22) becomes either (18) or (20).

A feature characterizing the transformations (18)–(20)–(22) is the existence of *absolute simultaneity*: two events taking place in different geometrical points of S but at the same t are judged to be simultaneous also in S' (and vice versa), this property being a consequence of the absence of space variables in the transformation of time. Of course the existence of absolute simultaneity does not imply that time is absolute: on the contrary, the β-dependent factor in the transformation of time gives rise to time-dilation phenomena similar to those of STR. *Time dilation* in another sense is, however, also absolute: a clock at rest in S is seen from S_0 to run slower, but a clock at rest in S_0 is seen from S to run *faster* so that both observers will agree that motion relative to S_0 slows down the pace of clocks. Quantitatively one has for both situations

$$\Delta t = \sqrt{1-\beta^2}\,\Delta t_0 \tag{23}$$

where Δt and Δt_0 are the time intervals between any two given events measured with clocks at rest in S and in S_0, respectively. The difference with respect to STR is, however, more apparent than real: a meaningful comparison of rates implies that a clock T_0 at rest in S_0 must be compared with clocks at rest in different points of S, and the result is therefore dependent on the convention adopted for synchronizing the latter clocks.

Absolute length contraction can also be deduced from (18)–(20). A rod at rest on the x axis of S between the points with coordinates x_2 and x_1

is seen in S_0, to have end points x_{02} and x_{01} at a common time t_0, where from (18)

$$x_2 = \frac{x_{02} - vt_0}{\sqrt{1-\beta^2}} \qquad x_1 = \frac{x_{01} - vt_0}{\sqrt{1-\beta^2}} \tag{24}$$

From this one obtains

$$x_2 - x_1 = \frac{1}{\sqrt{1-\beta^2}} (x_{02} - x_{01}) \tag{25}$$

The reasoning can be inverted by considering the rod at rest in S and observed from S_0, and using the transformation (20). One gets then, after a few simple steps

$$x_{02} - x_{01} = \sqrt{1-\beta^2}\,(x_2 - x_1) \tag{26}$$

which could also be obtained by inverting (25). The two results are thus mathematically equivalent and lead to the conclusion (with which both observers agree) that motion relative to S_0 leads to contraction. This is obviously an absolute effect, but again the discrepancy with the STR is due to the different conventions concerning clock synchronization: the length of a moving rod can only be obtained by marking the simultaneous positions of its end points, and is therefore dependent on the very definition of simultaneity of distant events.

8. THE "QUASIGROUP" OF INERTIAL TRANSFORMATIONS

In the present section we will check whether the inertial transformations form a group and give a negative answer. In fact let $\Omega(\beta, \beta')$ be the transformation (22), dependent on the two dimensionless absolute velocities β and β', and let $I = \{\Omega(\beta, \beta')\}$ be the set of all such transformations. Two elements of I differ from one another only for the value of one or both velocities β and β'. The Tangherlini transformation (18) is $\Omega(0, \beta)$; its inverse (20) is $\Omega(\beta, 0)$, so that they both belong to I. It follows that:

[1] The **identical transformation** is an element of I because for $\beta = \beta' = 0$ (22) becomes

$$\left.\begin{aligned} x' &= x \\ y' &= y \\ z' &= z \\ t' &= t \end{aligned}\right\}$$

which can be written as $\Omega(0, 0) \in I$.

[2] The inverse transformation of $\Omega(\beta, \beta')$ is obtained by inverting (22):

$$\left.\begin{aligned} x &= \frac{R(\beta')}{R(\beta)}\left[x' - \frac{\beta-\beta'}{R^2(\beta')}ct'\right] \\ y &= y' \\ z &= z' \\ t &= \frac{R(\beta)}{R(\beta')}t' \end{aligned}\right\} \tag{27}$$

Obviously the inverse of $\Omega(\beta, \beta')$ is $\Omega(\beta', \beta) \in I$. Quite generally the inverse of a transformation is obtained by interchanging the two absolute velocities β and β'.

[3] The **product of two inertial transformations** is obtained as follows: consider the inertial transformation $\Omega(\beta', \beta'')$ from S' to S'':

$$\left.\begin{aligned} x'' &= \frac{R(\beta')}{R(\beta'')}\left[x' - \frac{\beta''-\beta'}{R^2(\beta')}ct'\right] \\ y'' &= y' \\ z'' &= z' \\ t'' &= \frac{R(\beta'')}{R(\beta')}t' \end{aligned}\right\} \tag{28}$$

By inserting (22) in it one obtains

$$\left.\begin{aligned} x'' &= \frac{R(\beta)}{R(\beta'')}\left[x - \frac{\beta''-\beta}{R^2(\beta)}ct\right] \\ y'' &= y \\ z'' &= z \\ t'' &= \frac{R(\beta'')}{R(\beta)}t \end{aligned}\right\} \tag{29}$$

which is $\Omega(\beta, \beta'') \in I$. The previous result can also be written

$$\Omega(\beta, \beta')\,\Omega(\beta', \beta'') = \Omega(\beta, \beta'') \tag{30}$$

This is the multiplication law of inertial transformations: as one can see, the common velocity disappears from the product. Notice however that it is not possible to multiply any two transformations of the set, but only two such that the second velocity of the first one coincides with the first velocity of the second one. For this reason the inertial transformations do not form a group.

[4] The **associative law** of the multiplication of inertial transformations can now be established. Consider four inertial frames S, S', S'', and S''' and the following transformation:

$$\Omega(\beta, \beta'): S \Rightarrow S'$$
$$\Omega(\beta', \beta''): S' \Rightarrow S''$$
$$\Omega(\beta'', \beta'''): S'' \Rightarrow S'''$$

By applying (30) one easily gets

$$[\Omega(\beta, \beta')\, \Omega(\beta', \beta'')]\, \Omega(\beta'', \beta''') = \Omega(\beta, \beta'')\, \Omega(\beta'', \beta''') = \Omega(\beta, \beta''')$$

and

$$\Omega(\beta, \beta')[\Omega(\beta', \beta'')\, \Omega(\beta'', \beta''')] = \Omega(\beta, \beta')\, \Omega(\beta', \beta''') = \Omega(\beta, \beta''')$$

so that the associative law is satisfied.

9. MICHELSON-TYPE EXPERIMENTS

Consider a laboratory at rest in an inertial frame of reference S moving with absolute velocity $\vec{v}$ and suppose that an experiment on the interference of light is performed with instruments at rest in this laboratory. A light ray is divided in two (coherent) parts by a semitransparent mirror placed in point P. The first part propagates along the broken path $P - A_1 - A_2 \cdots A_{m-1} - Q$, where suitably oriented reflecting mirrors are placed in the intermediate points, the second one along the similar path $P - B_1 - B_2 \cdots B_{n-1} - Q$. Finally the two parts come to overlap in Q where they interfere. The point Q can be any point of an extended interference figure (see Fig. 2).

On the first path define the vectors $\vec{l}_{ai}$, (having modulus l_{ai}), with $i = 1, 2, \ldots, m$, coinciding with the rectilinear segments described by light and all oriented in the direction of propagation, i.e., from P towards Q; and similarly on the second path define the vectors $\vec{l}_{bj}$ (having modulus l_{bj}), with $j = 1, 2, \ldots, n$. The interference in Q is determined by the time delay ΔT between the two rays. The prediction of SRT is easy to obtain, given that light propagates in all directions with the constant velocity c. One has

$$\Delta T = T_B - T_A = \frac{L_B - L_A}{c} \qquad (31)$$

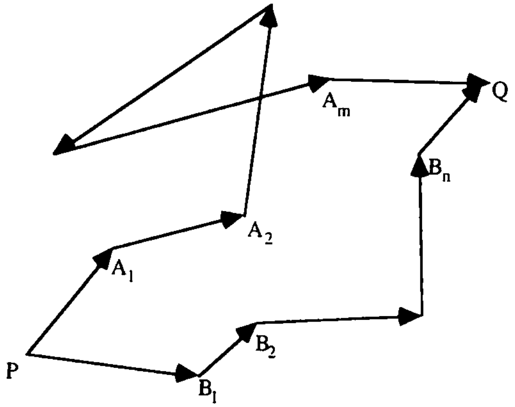

Fig. 2. A beamsplitter in P generates two coherent beams of light which propagate along different paths until reaching Q where they interfere.

where

$$L_A = \sum_{i=1}^{m} l_{ai} \qquad L_B = \sum_{j=1}^{n} l_{bj} \tag{32}$$

Consider next the same quantity ΔT by starting from the inertial transformations, according to which the inverse velocity of a light ray propagating in S in a direction forming an angle θ with the absolute velocity $\vec{v}$ of S is given by

$$\frac{1}{\tilde{c}(\theta)} = \frac{1 + \beta \cos \theta}{c} \tag{33}$$

In such a case the time delay between the light rays that followed the two different paths is

$$\Delta T = \sum_{j=1}^{n} \frac{l_{bj}}{\tilde{c}(\theta_{bj})} - \sum_{i=1}^{m} \frac{l_{ai}}{\tilde{c}(\theta_{ai})} \tag{34}$$

where $\theta_{ai}(\theta_{bj})$ is the angle between $\vec{l}_{ai}$ and $\vec{v}$ ($\vec{l}_{bj}$ and $\vec{v}$). By inserting (33) in (34) one gets

$$\Delta T=\frac{L_B-L_A}{c}+\frac{1}{c^2}\sum_{j=1}^{n} l_{bj}v\cos\theta_{bj}-\frac{1}{c^2}\sum_{i=1}^{m} l_{ai}v\cos\theta_{ai}$$
$$=\frac{L_B-L_A}{c}+\frac{1}{c^2}\sum_{j=1}^{n}\vec{l}_{bj}\cdot\vec{v}-\frac{1}{c^2}\sum_{i=1}^{m}\vec{l}_{ai}\cdot\vec{v}$$
$$=\frac{L_B-L_A}{c} \tag{35}$$

The last step is a consequence of

$$\sum_{j=1}^{n}\vec{l}_{bj}=\sum_{i=1}^{m}\vec{l}_{ai} \tag{36}$$

because the two sides of (36) are separately equal to the vector joining points P and Q. As one can see, the results (31) and (35) coincide. Therefore a theory based on the inertial transformations can explain the results of all the interferometric experiments carried out up to now (Michelson–Morley,[17] Kennedy–Thorndike,[18] Majorana,[19] etc.) as well as the SRT.

The previous result can easily be extended to a closed trajectory: it is enough to assume that point P coincides with Q and to ignore one of the two paths. The total time T_A required by light to cover the now closed path A is then

$$T_A=\frac{L_A}{c}+\frac{1}{c^2}\sum_{i=1}^{m} l_{ai}v\cos\theta_{ai}=\frac{L_A}{c}+\frac{1}{c^2}\sum_{i=1d}^{m}\vec{l}_{ai}\cdot\vec{v} \tag{37}$$

again coincident with the time L_A/c predicted by SRT, since

$$\sum_{i=1}^{m}\vec{l}_{ai}=0 \tag{38}$$

given that PQ is now a closed path. Therefore the inertial transformations predict that every measurement of the velocity of light made on closed paths in every possible inertial frame will necessarily give the value c predicted by SRT. The latter result generalizes the idea (which for us was an assumption) that the *two-way* velocity of light is always c.

10. CONCLUSIONS

Our choice of synchronization ("absolute," according to Mansouri and Sexl[2]) has been made by considering accelerations. This would perhaps not please a purist of special relativity, but it is worth stressing that the normally accepted relationship between SRT and accelerated systems is far from negligible. Accelerations are, for instance, essential in the so-called twin paradox, which is a prediction of SRT made before the general theory was even conceived. Very large accelerations enter in the experiment on the lifetime of muons circulating in the CERN Muon Storage Ring[20] which is considered the most accurate quantitative test of time dilation. The acceleration of Earth is essential for perceiving the retardation/anticipation of the eclipses of Jupiter's satellites. The same can be said about the possibility of detecting stellar aberration. And so on. Accelerations are instead avoided when they seem to generate difficulties within the existing theory. This is perhaps an understandable initial reaction, but, it is not acceptable in the long run that a scientific community keeps hiding its head in the sand instead of facing the real difficulties. Fortunately in recent times several new and interesting ideas are being developed[21].

There remain at least three fundamental questions to solve, before a theory based on the inertial transformations can be coinsidered reasonably complete:

1. Maxwell's equations must be reformulated. They will take a more general form, dependent on the absolute velocity of the inertial frame with respect to which they are considered, and will assume the usual form only in the privileged frame. This work is under way and will probably generate a good agreement with experiments, given the results of the present paper concerning optical interference experiments.

2. The kinematics of high-energy particle interactions must be reconsidered. This problem was solved in Ref. 14, where it was shown that a totally general equivalence exists between our predictions and those of the SRT. In fact, energy and momentum are defined in such a way as to coincide *numerically* (not analytically) with those of the SRT for all particles and in all inertial frames, once they coincide in the fundamental frame. Therefore the kinematics of high-energy processess, the determination of particle masses, and so on, do not require a different analysis from the one successfully carried out by particle physicists up to the present time.

3. Also the general theory needs modifications because the ds^2 cannot be considered invariant anymore, given that the inertial transformations predict a frame-dependent one-way velocity of light. It will be necessary to show that the modified theory makes the same successful predictions as the general theory of relativity. A lot of interesting work thus remains to be done.

ACKNOWLEDGMENTS

I wish to thank Alessandro Afriat, Vito Di Bari, Majid Rahnama, and Huseyin Yilmaz for their kind suggestions and criticisms.

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