Foundations of Physics Letters, Vol. 10, No. 1, 1997

TIME ON A ROTATING PLATFORM

F. Goy

Università di Bari, Dipartimento di Fisica
Via Amendola 173
1-70126 Bari, Italy
goy@axpba0.ba.infn.it

F. Selleri

Università di Bari, Dipartimento di Fisica
INFN - Sezione di Bari
Via Amendola 173
1-70126 Bari, Italy

Received September 24, 1996

Traditional clock synchronisation on a rotating platform is shown to be incompatible with the experimentally established transformation of time. The latter transformation leads directly to solve this problem through noninvariant one-way speed of light. The conventionality of some features of relativity theory allows full compatibility with existing experimental evidence.

Key words: relativity (special and general), synchronisation, Sagnac effect.

1. MUON STORAGE-RING EXPERIMENT

We start by recalling a well known experiment in which the relativistic approach works perfectly, and take from it two lessons concerning the transformation of time between the laboratory and a rotating platform.

Lifetimes of positive and negative muons were measured in CERN Storage-Ring experiment [1] for muon speed 0.9994 c, corresponding to a γ factor of 29.33. Muons circulated in a 14 m diameter ring with an acceleration of 10^{18}g. Excellent agreement was found

0894-9875/97/0200-0017$12.50/0 © 1997 Plenum Publishing Corporation

with the relativistic formula

$$\tau_0 = \frac{\tau_{\text{rest}}}{\sqrt{1-\beta^2}}\,, \tag{1}$$

where τ_0 is the observed muon lifetime, τ_{rest} is the lifetime of muons at rest, and $\beta = v/c$, v being the laboratory speed of the muon on its circular orbit.

Consider an ideal platform rotating with the same angular velocity as the muon in the e.m. field (with respect to such a platform the muon is at rest). Consider also four different observers:

1. O_L is the observer in the laboratory reference frame S_L, assumed to be an inertial frame. Thus O_L could be the CERN storage ring experimenter.
2. O_a is the accelerated but localized observer who lives on the rim of the platform S_a, very near the muon which looks constantly at rest to him; O_a has a local knowledge of the platform and of its physical properties extending only to the immediate surroundings of his position;
3. $\tilde{O}_a$ is a second accelerated observer. He knows everything about the platform (the accelerated frame S_a) through which he can freely move;
4. O_T is an observer living in an inertial frame S_T in which at a certain time O_a and the muon are instantaneously at rest. S_T will be called the "tangent" inertial frame.

We give now the description of the muon lifetime from the point of view of these four observers.

D1. According to O_L the muon lifetime τ_0 is greatly enhanced with respect to that (τ_{rest}) of muons at rest in S_L. His observations are expressed by Eq. (1).
D2. According to O_a, who knows only the time marked by his local clock, the muon lifetime is τ_{rest}. Of course O_a is under the action of a large acceleration (10^{18}g), which he detects as a radial gravitational field, but nevertheless his lifetime measurements give just τ_{rest}, as for muons at rest in S_L observed by O_L.
D3. According to the accelerated observer $\tilde{O}_a$ the clocks on the platform have a pace dependent on their position, the fastest going one being that in the centre; in agreement with the equivalence principle he attributes this phenomenon to the presence of a position-dependent radial graviational field of cosmic origin. He can check that the lifetime of muons near the rim of the platform is either τ_0 or τ_{rest} depending on the clock chosen (in the centre or near the rim, respectively) for measuring it. Therefore he explains the value

τ_{rest} found by O_a as a consequence of the cosmic gravitational field delaying in the same way muon decay and the clock used by O_a for lifetime measurements.

D4. According to the observer O_T belonging to the tangent inertial frame S_T the lifetime is τ_{rest} (measured of course for muons at rest in his frame).

The first lesson to be learned from the previous conclusions concerns the transformation of time given by Eq. (1): *The laboratory time interval Δt_0 between two events taking place in a fixed position on the rotating disk* (muon injection and decay, in the previous example) *is seen dilated by the usual relativistic factor compared with the corresponding time interval Δt measured by the accelerated observer O_a:*

$$\Delta t_0 = \frac{\Delta t}{\sqrt{1-\beta^2}} \,. \tag{2}$$

The second lesson is that the observers O_a *and* O_T *agree on the laws of nature, for example on the decay rate of muons at rest, even though* O_a *feels the presence of a radial gravitational field (of cosmic origin) while* O_T *does not.* Of course this conclusion is not new. For example Einstein [2], Møller [3], and Vigier [4] assumed that the acceleration of a clock C_a relative to an inertial system has no influence on the rate of C_a, and that the increase in the proper time of C_a at any time is the same as that of the standard clocks of the inertial system in which C_a is momentarily at rest. Of course the situation is different for an observer considering acceleration due to a gravitational field, as shown above. The identical conclusions of O_a and O_T imply that the speed of light found locally in the accelerated system should be the same as that observed in the "tangent" inertial frame. But in special relativity the latter speed is always c and we are so brought to conclude that the speed of light relative to an accelerated system should also be c. However this conclusion gives rise to endless trouble.

2. THE TRADITIONAL CLOCK SYNCHRONIZATION PROCEDURE

If a disk is rotating with constant angular velocity with respect to an inertial frame, one can obtain the metric on the disk as follows: in the inertial system the invariant squared space-time distance ds^2 in cartesian coordinates is:

$$ds^2 = c^2 dt_0^2 - dx_0^2 - dy_0^2 - dz_0^2. \tag{3}$$

In general relativity one is free to adopt any set of coordinates useful for solving a given problem, independently of their physical meaning.

In the case of the rotating disk it is simpler to use the coordinates in the right-hand side of the following transformations, as done for example by Langevin [5] and by some textbooks [6]:

$$\begin{aligned} t_0 &= t, \\ x_0 &= r\cos(\varphi + \omega t), \\ x_0 &= r\sin(\varphi + \omega t), \\ z_0 &= z. \end{aligned} \tag{4}$$

The variables t, r, φ, z give *a possible* (although not the best) description of physical events for an observer at rest on the disk. In (4) simplicity is attained at the price of provisionally neglecting time dilation and length contraction. By substituting (4) in (3) one can easily obtain:

$$ds^2 = (1 - \omega^2 r^2/c^2)(c\,dt)^2 - \frac{2\omega r^2}{c} d\varphi(c\,dt) - dz^2 - dr^2 - r^2 d\varphi^2. \tag{5}$$

Equation (5) defines a metric g_{ij} which is stationary, but not static. If $x^0 = ct,\ x^1 = r,\ x^2 = \varphi,\ x^3 = z$, its elements are

$$\begin{aligned} g_{00} &= 1 - \omega^2 r^2/c^2, & g_{11} &= g_{33} = -1, \\ g_{02} &= g_{20} = -r^2\omega/c, & g_{22} &= -r^2, \end{aligned} \tag{6}$$

all other elements being zero. Note that the space-time described by (6) is flat because $R_{ijkl}(t', x', y', z') = 0 \Rightarrow R_{ijkl}(t, r, \varphi, z) = 0\,[i, j, k, l = 0, 1, 2, 3]$, where R_{ijkl} is the Riemann tensor. For the same reason of covariance the metric defined in (6) is necessarily a solution of the Einstein equations in empty space $R_{ij} = 0\,[i, j = 0, 1, 2, 3]$, where R_{ij} is the Ricci tensor.

The proper time differential $d\tau$ of a clock located in a fixed point of the disk of space coordinates (r, φ, z) is obtained by equalling all space differentials to 0 and taking into account that $d\tau = ds/c$, so that

$$d\tau = \sqrt{g_{00}}\,dt = \sqrt{1 - \omega^2 r^2/c^2}\,dt. \tag{7}$$

The length element dl between a point A of the three dimensional space with coordinates $x^\alpha + dx^\alpha [\alpha = 1, 2, 3]$ and an infinitesimally near point B of coordinates x^α is found by sending a light signal from B to A and back, and assuming that the *two-way* velocity of light on the disk is c in all directions. The proper time in B

needed for this operation (multiplied by c) is by definition twice the length dl between A and B. It is found that:

$$dl^2 = \left(-g_{\alpha\beta} + \frac{g_{0\alpha} + g_{0\beta}}{g_{00}}\right) dx^\alpha dx^\beta = dr^2 + dz^2 + \frac{r^2 d\varphi^2}{1 - \omega^2 r^2/c^2} \quad (8)$$

Note the r-dependent coefficient of $r^2 d\varphi^2$: *space* is not flat.

In relativity an observer on a rotating platform O_a can synchronise clocks placed in different points *by assuming that the one-way velocity is c in all directions of his noninertial frame.* This assumption is in agreement with the "second lesson" taken from the muon experiment, but becomes the source of a big problem which makes experts conclude that "the rotating platform in relativity is a mystery." By applying it, the "time" t_B in B is called synchronous with the "time" t_A in an infinitesimally near point A when

$$t_B = t_A + \Delta t = t_A - \frac{1}{c}\frac{g_{0\alpha} dx^\alpha}{g_{00}} = t_A + \frac{\omega r^2 d\varphi}{c^2(1 - \omega^2 r^2/c^2)}. \quad (9)$$

This definition is equivalent to the standard synchronisation procedure of SRT and is obtained by assuming that the point B receives a signal from A exactly at midtime between the times of light departure from A and return in A. The assumption is that the one-way velocity of light is c.

Notice that the points A and B chosen for defining the synchronisation (9) are totally arbitrary: in general they are not at the same distance r from the centre. The consequence is that Δt is not a total differential. In fact for all functions $f(r, \varphi)$ one has

$$\Delta t = \frac{\omega r^2 d\varphi}{c^2(1 - \omega^2 r^2/c^2)} \neq df(r, \varphi). \quad (10)$$

The proof is very simple. In fact, Δt is proportional to $d\varphi$ and does not contain dr. The coefficient of $d\varphi$ however does contain r. The two statements are incompatible for the total differential of any $f(r, \varphi)$. A diffeomorphism of the type

$$T: \begin{cases} t \to \tilde{t} = f_1(r, \varphi)t + f_2(r, \varphi), \\ r \to \tilde{r} = f_3(r, \varphi), \\ \varphi \to \tilde{\varphi} = f_4(r, \varphi) \end{cases}$$

is unable to transform Δt in a total differential (proof available by the authors upon request). [Physical meaning of T: the spatial part is chosen time independent, so that we remain on the disk; time undergoes the most general linear transformation; z plays no role]. The latter result agrees with Landau and Lifschitz [6] who stated that the inequality (10) is not dependent on the choice (4) of transformations, but is of general validity since

$$\Delta t = -\frac{1}{c}\frac{g_{0\alpha}dx^{\alpha}}{g_{00}} \tag{11}$$

can be a total differential only in very special and physically uninteresting cases. Møller [3] and Landau and Lifshtiz [6] say that it is possible to define the standard synchronisation along a non closed curve, but that it is impossible along a closed curve when the metric is not static. In fact, given (10), this synchronisation procedure is path dependent, so that one will generally not obtain the same result when synchronising a clock B with a clock A using two different paths. This means also that if a clock B is synchronised with A and a clock C is synchronised with B, C will generally not be synchronised with A. This matter was investigated by Anandan [7] who admitted the existence of a "time lag" in synchronising clocks around the circle and found for it a rather abstract interpretation, and by Ashtekar and Magnon [8] who limited themselves to a formal approach.

The existence of a synchronisation problem is physically strange because if the whole disk is initially at rest in the laboratory (inertial) frame S_L, with clocks near its rim synchronised with the regular procedure used for all clocks of S_L, then when the disk moves, accelerates, and attains a constant angular velocity, the clocks must slow their rates but cannot desynchronise for symmetry reasons, since they have at all times the same speed. From such a point of view it is difficult to see why there should be any difficulty in defining time on the rotating platform. The necessity to distinguish sharply between questions of clock phase (distant simultaneity) from those of clock rate was stressed by Phipps [9] with whom we fully agree on this point.

3. NONINVARIANT SPEED OF LIGHT

The laboratory is assumed to be an inertial frame in which clocks have been synchronised with the standard relativistic method.

We consider only clocks on the uniformly rotating platform having radius R and angular velocity ω that are near its rim. We assume them to be synchronised as follows: When the clocks of the laboratory show the time $t_0 = 0$ then also the clocks on the platform are all set at the time $t = 0$. By symmetry reasons the clocks on the platform will share the following property during the uniform rotation: any observer at rest in the laboratory near the rim of the platform whose clock marks the time t_0 will see the clock on the platform passing by in that very moment marking the time

$$t = t_0\sqrt{1-\beta^2}. \tag{12}$$

with $\beta = \omega R/c$.

Near the rim of the platform besides clocks there are also (i) A light source Σ placed in a fixed position; near Σ there is a clock C_Σ; (ii) A backward reflecting mirror M placed in diametrically opposite position with respect to Σ; near M there is a clock C_M. At time t_i of $C_\Sigma \Sigma$ emits a flash of light that propagates circularly and (we assume) in the direction of rotation of the disk with respect to the laboratory, until it arrives at M at time t_2 of C_M. The flash is reflected back, propagates circularly in the opposite direction, arrives back at Σ at time t_3 of C_Σ.

In the theory of relativity it is *assumed* that the one-way velocity of light has the same value from Σ to M as from M to Σ, so that $t_3 - t_2 = t_2 - t_1$, whence the C_M time t_2, can be written in terms of the two C_Σ times t_1 and t_3 as follows:

$$t_2 = t_1 + \frac{1}{2}(t_3 - t_1) \tag{13}$$

Reichenbach commented [10]: *"This definition is essential for the special theory of relativity, but it is not epistemologically necessary. If we were to follow an arbitrary rule restricted only to the form*

$$t_2 = t_1 + \varepsilon(t_3 - t_1), \quad 0 < \varepsilon < 1 \tag{14}$$

it would likewise be adequate and could not be called false. If the special theory of relativity prefers the first definition, i.e., sets ε equal to *1/2, it does so on the ground that this definition leads to simpler relations."* On the possibility to choose freely ε according to (14) agreed, among others, Grünbaum [11], Jammer [12], Mansouri and Sexl [13], Sjödin [14], Cavalleri [15], and Ungar [16].

Clearly, different values of ε correspond to different values of the one-way speed of light. In fact, one can write

$$t_2 - t_1 = \frac{L}{2\tilde{c}(0)} \quad \text{and} \quad t_3 - t_2 = \frac{L}{2\tilde{c}(\pi)}, \tag{15}$$

where $L/2$ is the $\Sigma - M$ distance measured along the rim of the disk, $\tilde{c}(0)$ is the one-way velocity of light from Σ to M and $\tilde{c}(\pi)$ is the one-way velocity from M to Σ. By adding the previous relations, one gets

$$t_3 - t_1 = \frac{L}{2\tilde{c}(0)} + \frac{L}{2\tilde{c}(\pi)} = \frac{L}{c}, \tag{16}$$

the last step being necessary, because the *two-way* velocity of light has been measured with great precision and always found to be c. From (14), (15) and (16) one easily gets

$$\varepsilon = \frac{t_2 - t_1}{t_3 - t_1} = \frac{c}{2\tilde{c}(0)}. \tag{17}$$

Therefore freedom of choice of ε means freedom of choice of the one-way velocity of light! We believe that it is necessary to exploit the free choice of the one-way speed of light, which has never been measured, given that the standard assumption $\tilde{c}(0) = \tilde{c}(\pi) = c$ leads to contradictions as we saw, and as will become even clearer by the following considerations.

The description of the light circulating along the rim of the disk given by the laboratory observers will be the following: At time t_{01} the source emits a light flash that propagates circularly and arrives at M at time t_{02}, is reflected back, propagates circularly, arrives back at Σ at time t_{03}. These laboratory times are related to the corresponding platform times by

$$t_{0i} = \frac{t_i}{\sqrt{1-\beta^2}}, \quad i = 1, 2, 3, \tag{18}$$

as a consequence of (12).

If L_0 is the disk circumference length measured in the laboratory, light propagating in the rotational direction of the disk must cover a distance larger than $L_0/2$ by a quantity $x = \omega R(t_{02} - t_{01})$ equalling the shift of M during the time $t_{02} - t_{01}$ taken by light to reach M. Therefore

$$\frac{L_0}{2} + x = c(t_{02} - t_{01}), \quad x = \omega R(t_{02} - t_{01}) \tag{19}$$

From these equations it is easy to get

$$t_{02} - t_{01} = \frac{L_0}{2c(1-\beta)}. \tag{20}$$

After reflection light propagates in the direction opposite to that of rotation and must now cover a distance smaller than the disk semi-circumference length $L_0/2$ by a quantity $y = \omega R(t_{03} - t_{02})$ equalling the shift of Σ during the time $t_{03} - t_{02}$ taken by light to reach Σ. Therefore,

$$\frac{L_0}{2} - y = c(t_{03} - t_{02}), \quad y = \omega R(t_{03} - t_{02}). \tag{21}$$

One now gets

$$t_{03} - t_{02} = \frac{L_0}{2c(1+\beta)}. \tag{22}$$

Summing together (20) and (22), it follows

$$t_{03} - t_{01} = \frac{L_0}{c}\frac{1}{1-\beta^2}. \tag{23}$$

We show next that these relations fix the synchronisation on the disk. In fact (18) applied to (20) and (23) gives

$$t_2 - t_1 = \frac{L_0\sqrt{1-\beta^2}}{2c(1-\beta)}, \quad t_3 - t_1 = \frac{L_0}{c\sqrt{1-\beta^2}}, \tag{24}$$

so that

$$\varepsilon = \frac{t_2 - t_1}{t_3 - t_1} = \frac{1+\beta}{2}. \tag{25}$$

Comparing with (17), we get

$$\tilde{c}(0) = \frac{c}{1+\beta}. \tag{26}$$

An analogous reasoning made for light emitted by Σ in the direction opposite to disk rotation leads to

$$\tilde{c}(\pi) = \frac{c}{1-\beta}. \tag{27}$$

Equations (26)-(27) give the one-way speed of light on the platform. They are particular cases of the formula $\tilde{c}(\theta) = c/(1 + \beta\cos\theta)$ discussed at length in Ref. [17] and shown to be compatible with the experimental evidence at the special relativistic level (no accelerations).

4. THE SAGNAC EFFECT

The reasoning of the previous section was made under the assumption that the platform clocks near the rim (the only ones we considered) were all synchronised with the laboratory clocks at the same (laboratory) time. This procedure clearly amounts also to a synchronisation of the platform clocks with respect to one another. It might lead to the incorrect idea that the obtained one-way velocity of light different from c is only a consequence of the chosen synchronisation. In order to dispel this impression we repeat here the reasoning by using only the clock C_Σ on the platform.

Two light flashes leave Σ at time t_1. The first one propagates circularly in the sense opposite to the platform rotation and comes back to Σ after a 2π rotation at time t_2. The second one propagates circularly in the same rotational sense of the platform and comes back to Σ after a 2π rotation at time t_3. Quite generally we can write

$$t_2 - t_1 = \frac{L}{\tilde{c}(\pi)}, \quad t_3 - t_1 = \frac{L}{\tilde{c}(0)}. \tag{28}$$

It follows that

$$t_3 - t_2 = L\left[\frac{1}{\tilde{c}(0)} - \frac{1}{\tilde{c}(\pi)}\right]. \tag{29}$$

Describing the experiment from the point of view of the laboratory observer, one must give a treatment strictly analogous to that of the previous section. It results in

$$t_{03} - t_{01} = \frac{L_0}{c(1-\beta)}, \quad t_{02} - t_{01} = \frac{L_0}{c(1+\beta)}. \tag{30}$$

The time delay between the two arrivals back in Σ is therefore observed in the laboratory to be

$$t_{03} - t_{02} = \frac{2L_0\beta}{c(1-\beta^2)}, \tag{31}$$

which is the standard formula for the Sagnac effect. Noticing that (31) is the time difference between two events taking place in the same point Σ on the disk we can apply what we called the first lesson from the muon experiment [Eq. (2)] to the laboratory time interval $t_{03} - t_{02}$ and also use (29) to get

$$\frac{1}{\tilde{c}(0)} - \frac{1}{\tilde{c}(\pi)} = \frac{t_3 - t_2}{L} = 2\frac{L_0}{Lc}\frac{\beta}{\sqrt{1-\beta^2}}. \tag{32}$$

Of course one has $L_0 = L\sqrt{1-\beta^2}$: The rotating disk circumference length appears contracted in the laboratory. Therefore

$$\frac{1}{\tilde{c}(0)} - \frac{1}{\tilde{c}(\pi)} = \frac{2\beta}{c}. \tag{33}$$

It is enough to add to (33) the condition that the two-way velocity of light is c,

$$\frac{1}{\tilde{c}(0)} + \frac{1}{\tilde{c}(\pi)} = \frac{2L}{c}, \tag{34}$$

to arrive again at the results (26) and (27). There is absolutely no way of obtaining the relativistic condition $\tilde{c}(0) = \tilde{c}(\pi) = c$. By accepting (26)-(27) we find instead a perfectly rational description of the Sagnac effect on the rotating platform and overcome the long-standing "mystery" of the rotating platform.

5. CONCLUSIONS

The Sagnac effect [18] is essentially the observation of a phase shift between two coherent beams travelling on opposite paths in an interferometer placed on a rotating disk. Nowadays the Sagnac effect is observed with light (in ring lasers and in fiber optics interferometers [19]) and in interferometers built for electrons [20], neutrons [21], atoms [22] and superconducting Cooper pairs [23]. The phase shift in the interferometers is a consequence of the time delay between the arrivals of two beams, so a Sagnac effect is also measured directly with atomic clocks timing light beams sent around the earth via satellites [24]. In the typical experiment for the study of the effect a monochromatic light source placed on the disk emits two coherent beams of light in opposite directions along the disk circumference until they reunite in a small region and interfere, after a 2π propagation. The positioning of the interference figure depends on the disk rotational velocity. Textbooks deduce the Sagnac formula in the laboratory (essentially our Eq. (31) above), but say nothing about the description of the phenomenon on the rotating platform. Exceptions to this trend are Langevin [5], Anandan [7], Dieks and Nienhuis [25], and Post [26], but dissatisfaction remains widespread, because none of these treatments is free of ambiguities. For example Langevin's approach leads to all the difficulties we discussed in the second section [however in his 1937 paper he recognized the possibility of a nonstandard velocity of light on the rotating platform and gave formulae which agree to first order with our results (26)-(27)]. As a second example, Post's relativistic formula is not generally valid, but limited to the arbitrary case where the origin of the "tangent" inertial frame coincides with the centre of the rotating disk.

It is well known, especially after the works of Reichenbach [10] and of Mansouri and Sexl [13], that clock synchronisation is a purely conventional procedure when only inertial frames are involved. In other words one is free to choose either the standard synchronisation, or a nonstandard one leading to a noninvariant one-way velocity of light. Either choice will allow full agreement with experimental facts. However we have shown that the conventionality of the synchronisation procedure is not preserved in accelerated systems, and that a theory free of logical contradictions *must* choose a one-way velocity of light which is nonstandard when measured in the accelerated frame. By the way, this is exactly what is already done in practice by physicists synchronising clocks around the earth by means of light signals. "Thus one discards Einstein synchronisation in the rotating frame" said Ashby in the opening talk of the 1993 International Frequency Control Symposium [27].

ACKNOWLEDGEMENT

One of us (F.G.) would like to thank the Physics Department of Bari University for the hospitality extended to him during the preparation of the present paper.

REFERENCES

1. J. Bailey et al., *Nature* **268** (1977) 301.
2. A. Einstein, *Ann. Phys.* **35** (1911) 898.
3. C. Møller, *The Theory of Relativity*, 2nd edn. (Clarendon, Oxford, 1972).
4. J.-P. Vigier, *Found. Phys.* **25** (1995) 1461.
5. P. Langevin, *C. R. Acad. Sc. (Paris)* **173** (1921) 831; *C. R. Acad. Sc. (Paris)* **205** (1937) 304.
6. L. Landau and E. M. Lifschitz, *The Classical Theory of Field* (Pergamon, London, 1959), p. 281. V. Fock, *The Theory of Space and Time and Gravitation* (Pergamon, London, 1959), p. 107. R. Adler, M. Bazin, and M. Schiffer, *Introduction to General Relativity* (McGraw Hill, New York, 1965), p. 453.
7. J. Anandan, *Phys. Rev. D* 24 (1981) 338.
8. A. Ashtekar and A. Magnon, *J. Math. Phys.* **16** (1975) 341.
9. T. E. Phipps, Jr., *Galilean Electrodynamics* **6** (1995) 1.
10. H. Reichenbach, *The Philosophy of Space & Time* (Dover, New York, 1958).
11. A. Grünbaum, *Philosophical Problems of Space and Time* (Reidel, Dordrecht, 1973).
12. M. Jammer, "Some fundamental problems in the special theory of

relativity," in *Problems in the Foundations of Physics*, G. Toraldo di Francia, ed. (North Holland, Amsterdam, 1979).
13. R. Mansouri and R. U. Sexl, *Gen. Rel. Grav.* **8** (1977) 497.
14. T. Sjödin, *Nuovo Cimento B* **51** (1979) 229.
15. G. Cavalleri, *Nuovo Cimento B* **104** (1989) 545.
16. A. A. Ungar, *Found. Phys.* **21** (1991) 691.
17. F. Selleri, *Found. Phys.* **26** (1996) 641; *Found. Phys. Lett.* **9** (1996) 43.
18. G. Sagnac, *C. R. Acad. Sc. (Paris)* **157** (1913) 708; ibid. 1410.
19. E. Udd, *Laser Focus/Electro Optics*, December 1965, p. 64.
20. H. Hasselbach and M. Nicklaus, *Phys. Rev. A* **48** (1993) 143.
21. R. Colella, A. W. Overhauser, and S. A. Werner, *Phys. Rev. Lett.* **34** (1975) 1472.
22. P. Storey and C. Cohen-Tannoudji, *J. Phys. II France* **4** (1994) 1999.
23. J. E. Zimmerman and J. E. Mercereau, *Phys. Rev. Lett.* **14** (1965) 887.
24. D. W. Allan et al., *IEEE Trans. Instr. Meas. IM-34* (1985) 118; *Science* **228** (1985) 69.
25. D. Dieks and G. Nienhuis, *Am. J. Phys.* **58** (1990) 650.
26. E. J. Post *Rev. Mod. Phys.* **39** (1967) 475.
27. N. Ashby, *Proceedings, IEEE International Frequency Control Symposium* (1993), p. 2.