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Dark energy, gravitation and the Copernican principle

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1.1 Cosmological models and their hypotheses

1.1.1 Introduction

The progresses of physical cosmology during the past ten years have led to a "standard" cosmological model in agreement with all available data. Its parameters are measured with increasing precision but it requires the introduction of a dark sector, including both dark matter and dark energy, attracting the attention of both observers and theoreticians.

Among all the observational conclusions, the existence of a recent acceleration phase of the cosmic expansion is more and more robust. The quest for the understanding of its physical origin is however just starting (Peebles and Ratra, 2003; Peter and Uzan, 2005; Copeland et al. , 2006; Uzan, 2007). Models and speculations are flourishing and we may wonder to which extent the observations of our local Universe may reveal the physical nature of the dark energy. In particular, there exist limitations to this quest intrinsic to cosmology, related to the fact that most observations are located on our past light-cone (Ellis, 1975), and to finite volume effects (Bernardeau and Uzan, 2004) that can make many physically acceptable possibilities undistinguishable in practice.

This text aims at discussing the relations between the cosmic acceleration and the theory of gravitation and more generally with the hypotheses underlying the construction of our cosmological model, such as the validity of general relativity on astrophysical scales and the Copernican principle. We hope to illustrate that cosmological data have now the potential of testing these hypotheses, which go beyond the measurements of its parameters.

1.1.2 Cosmology, physics and astronomy

Cosmology seats at the cross-road between theoretical physics and astronomy.

Theoretical physics, based on physical laws, tries to describe the fundamental components of nature and their interactions. These laws can be probed locally by experiments. These laws need to be extrapolated to construct cosmological models. Hence any new idea or discovery concerning these laws can naturally call for an extension of our cosmological model (e.g. introducing massive neutrinos in cosmology is now mandatory).

Astronomy confronts us with phenomena that we have to understand and explain consistently. This often requires the introduction of hypotheses beyond those of the physical theories (§ 1.1.3) in order to "*save the phenomena*" (Duhem, 1908), as is actually the case with the dark sector of our cosmological model. Needless to remind that even if a cosmological model is in agreement with all observations, whatever their accuracy, it does not prove that it is the "correct" model of the Universe, in the sense that it is the correct cosmological extrapolation and solution of the local physical laws.

Dark energy confronts us with a compatibility problem since, in order to "save the phenomena" of the observations, we have to include new ingredients (constant, matter fields or interactions) beyond those of our established physical theories. However the required value for the simplest dark energy model, i.e. the cosmological constant, is more than 60 order of magnitude smaller to what is expected from theoretical grounds (§ 1.1.6). This tension between what is required by astronomy and what is expected from physics reminds us of the twenty centuries long debate between Aristotelians and Ptolemeans (Duhem, 1913), that was resolved not only by the Copernican model but more important by a better understanding of the physics since

Newton gravity was compatible only with one of these three models that, at the time, could not be distinguished observationally.

1.1.3 hypotheses of our cosmological model

The construction of any cosmological model relies on 4 main hypotheses,

(H1) a theory of gravity,
(H2) a description of the matter contained in the Universe and their non-gravitational interactions,
(H3) symmetry hypotheses, and
(H4) an hypothesis on the global structure, i.e. the topology, of the Universe.

These hypotheses are not on the same footing since H1 and H2 refer to the physical theories. These two hypotheses are however not sufficient to solve the field equations and we must make an assumption on the symmetries (H3) of the solutions describing our Universe on large scales while H4 is an assumption on some global properties of these cosmological solutions, with same local geometry.

Our reference cosmological model is the ΛCDM model. It assumes that gravity is described by general relativity (H1), that the Universe contains the fields of the standard model of particle physics plus some dark matter and a cosmological constant, the latter two having no physical explanation at the moment. Note that in the cosmological context this involves an extra-assumption since what will be required by the Einstein equations is the effective stress-energy tensor averaged on large scales. It thus implicitly refers to a, usually not explicited, averaging procedure (Ellis and Buchert, 2005). It also deeply involves the Copernican principle as a symmetry hypotheses (H3), without which the Einstein equations usually can not been solved, and usually assumes that the spatial sections are simply connected (H4). H2 and H3 imply that the description of standard matter reduces to a mixture of a pressureless and a radiation perfect fluids.

1.1.4 Copernican principle

The *cosmological principle* supposes that the Universe is spatially isotropic and homogeneous. In particular, this implies that there exists a privileged class of observers, called fundamental observers, who all see an isotropic universe around them. It implies the existence of a cosmic time and states that all the properties of the universe are the same everywhere at the same cosmic time. It is supposed to hold for the smooth-out structure of the Universe on large scales. Indeed, this principle has to be applied in a statistical sense since there exist structures in the universe.

We can distinguish it from the *Copernican principle* which merely states that we do not live in a special place (the center) of the Universe. As long as isotropy around the observer holds, the principle actually leads to the same conclusion than the cosmological principle.

The cosmological principle makes definite predictions about all unobservable regions beyond the observable universe. It completely determines the entire structure of the Universe, even for regions that cannot be observed. From this point of view, this hypothesis, which cannot be tested, is very strong. On the other hand it leads to a complete model of the universe. The Copernican principle has more modest consequences and leads to the same conclusions but only for the observable universe where isotropy has been verified. It does not make any prediction on the structure of the Universe for unobserved regions (in particular, space could be homogeneous and non isotropic on scales larger than the observable Universe). We refer to Bondi (1960), North (1965) and Ellis (1975) for further discussions on the definition of these two principles.

We emphasized that, as shall be discussed in the next section, our reference cosmological model includes a primordial phase of inflation in order to explain the origin of the large scale structures of the Universe. Inflation gives a theoretical prejudice in favor of the Copernican principle since it predicts that all classical (i.e. non-quantum) inhomogeneities (curvature, shear, ...) have been washed-out during this phase. If it is sufficiently long, we expect the principle to hold on scales much larger than those of the observable universe, hence backing-up the cosmological principle, since unobservable regions today arise from the same causal process that affected the conditions in our local Universe. While the standard predictions of inflation are in agreement with all astronomical data, we should not forget it is only a theoretical argument on which we shall come back in the case we find observable evidences against isotropy (Pereira et al., 2007; Pitrou et al., 2008), curvature (Uzan et al., 2003) and homogeneity (e.g. such as a spatial topology of the Universe).

This principles leads to a Robertson-Walker (RW) geometry with metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a^2(t)\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j, \tag{1.1}$$

where t is the cosmic time and γ_{ij} is the spatial metric on the constant time hypersurfaces, which are homogeneous and isotropic, and thus of constant curvature. It follows that the metric is reduced to a single function of time, the scale factor. This implies that there is a one-to-one mapping between the cosmic time and the redshift

$$1+z=\frac{a_0}{a(t)}, \tag{1.2}$$

if the expansion is monotonous.

1.1.5 ΛCDM reference model

The dynamics of the scale factor can be determined from the Einstein equations which reduce for the metric (1.1) to the Friedmann equations

$$H^2 = \frac{8\pi G}{3}\rho - \frac{K}{a^2} + \frac{\Lambda}{3}, \tag{1.3}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho+3P) + \frac{\Lambda}{3}. \tag{1.4}$$

$H \equiv \dot{a}/a$ is the Hubble function and $K = 0, \pm 1$ is the curvature of the spatial sections. G and Λ are the Newton and cosmological constants. ρ and P are respectively the energy density and pressure of the cosmic fluids and are related by

$$\dot{\rho} + 3H(\rho+P) = 0.$$

Defining the dimensionless density parameters as

$$\Omega = \frac{8\pi G\rho}{3H^2}\ , \quad \Omega_\Lambda = \frac{\Lambda}{3H^2}\ , \quad \Omega_K = -\frac{K}{H^2a^2}, \tag{1.5}$$

respectively for the matter, the cosmological constant and the curvature, the first Friedmann equation can be rewritten as

$$\begin{aligned} E^2(z) &\equiv \left(\frac{H}{H_0}\right)^2 \\ &= \Omega_{\rm rad0}(1+z)^4 + \Omega_{\rm mat0}(1+z)^3 + \Omega_{K0}(1+z)^2 + \Omega_{\Lambda 0}\ , \end{aligned} \tag{1.6}$$

with $\Omega_{K0} = 1 - \Omega_{\rm rad0} - \Omega_{\rm mat0} - \Omega_{\Lambda 0}$. All background observables, such as the luminosity distance, the angular distance,..., are functions of $E(z)$ and are thus not independent.

Besides this background description, the ΛCDM also accounts for an understanding of the large scale structure of our universe (galaxy distribution, cosmic microwave background anisotropy) by using the theory of cosmological perturbations at linear order. In particular, in the sub-Hubble regime, the growth rate of the density perturbation is also a function of $E(z)$.

One must, however, extend this minimal description by a primordial phase in order to solve the standard cosmological problems (flatness, horizon...).

In our reference model, we assume that this phase is described by an inflationary period during which the expansion of the universe is almost-exponentially accelerated. In such a case, the initial conditions for the gravitational dynamics that will lead to the large scale structure are also determined so that our model is completely predictive. We refer to the chapter 8 of Peter and Uzan (2005) for a detailed description of these issues that are part of our cosmological model but not directly related to our actual discussion.

In this framework, the dark energy is well defined and reduces to a single number equivalent to a fluid with equation of state $w = P/\rho = -1$. This model is compatible with all astronomical data which roughly indicates that

$$\Omega_{\Lambda 0} \simeq 0.73, \qquad \Omega_{\mathrm{mat}0} \simeq 0.27, \qquad \Omega_{\Lambda 0} \simeq 0.$$

1.1.6 The cosmological constant problem

This model is theoretically well-defined, observationally acceptable, phenomenologically simple and economical. From the perspective of general relativity the value of Λ is completely free and there is no argument allowing us to fix it, or equivalently, the length scale $\ell_\Lambda = |\Lambda_0|^{-1/2}$, where Λ_0 is the astronomically deduced value of the cosmological constant. Cosmology roughly imposes that

$$|\Lambda_0| \leq H_0^2 \iff \ell_\Lambda \leq H_0^{-1} \sim 10^{26}\,\mathrm{m} \sim 10^{41}\,\mathrm{GeV}^{-1}\ .$$

In itself this value is no problem, as long as we only consider classical physics. Notice however that it is disproportionately large compared to the natural scale fixed by the Planck length

$$\ell_\Lambda > 10^{60}\ell_{\mathrm{P}} \iff \frac{\Lambda_0}{M_{\mathrm{Pl}}^2} < 10^{-120} \iff \rho_{\Lambda_0} < 10^{-120} M_{\mathrm{Pl}}^4 \sim 10^{-47}\,\mathrm{GeV}^4\ , \tag{1.7}$$

when expressed in terms of energy density.

The main problem arises from the interpretation of the cosmological constant. The local Lorentz invariance of the vacuum implies that its energy-momentum tensor must take the form (Zel'dovich, 1988) $\langle T_{\mu\nu}^{\mathrm{vac}}\rangle = -\langle\rho\rangle g_{\mu\nu}$, that is equivalent to the one of a cosmological constant. From the quantum point of view, the vacuum energy receives a contribution of the order of

$$\langle\rho\rangle_{\mathrm{vac}}^{\mathrm{EW}} \sim (200\,\mathrm{GeV})^4\ , \qquad \langle\rho\rangle_{\mathrm{vac}}^{\mathrm{Pl}} \sim (10^{18}\,\mathrm{GeV})^4\ , \tag{1.8}$$

arising from the zero point energy, respectively fixing the cutoff frequency of the theory to the electroweak scale or to the Planck scale. This contribution implies a disagreement of respectively 60 to 120 orders of magnitude with astronomical observations!

This is the cosmological constant problem (Weinberg, 1989). It amounts to understanding why

$$|\rho_{\Lambda_0}| = |\rho_\Lambda + \langle\rho\rangle_{\rm vac}| < 10^{-47}\,\mathrm{GeV}^4 \tag{1.9}$$

or equivalently,

$$|\Lambda_0| = |\Lambda + 8\pi G\langle\rho\rangle_{\rm vac}| < 10^{-120} M_{\rm Pl}^2\ , \tag{1.10}$$

i.e. why ρ_{Λ_0} is so small today, but non-zero.

Today, there is no known solution to this problem and two approaches have been designed. One the one hand one sticks to this model and extend the cosmological model in order to explain why we observe a so small value of the cosmological constant (Garriga and Vilenkin, 2004 ; Carr and Ellis, 2008). We shall come back on this approach later. On the other hand, one hopes that there should exist a physical mechanism to exactly cancel the cosmological constant and looks for another mechanism to explain the observed acceleration of the Universe.

1.1.7 The equation of state of dark energy

The equation of state of the dark energy is obtained from the expansion history, assuming the standard Friedmann equation. It is thus given by the general expression (Martin et al. , 2006)

$$3\Omega_{\rm de} w_{\rm de} = -1 + \Omega_K + 2q, \tag{1.11}$$

q being the deceleration parameter,

$$q \equiv -\frac{a\ddot{a}}{\dot{a}^2} = -1 + \frac{1}{2}(1+z)\frac{\mathrm{d}\ln H^2}{\mathrm{d}z}. \tag{1.12}$$

This expression (1.11) does not assume the validity of general relativity or any theory of gravity but gives the relation between the dynamics of the expansion history and the property of the matter that would lead to this acceleration if general relativity described gravity. Thus, the equation of state, as defined in Eq. (1.11), reduces to the ratio of the pressure, $P_{\rm de}$, to the energy density $\rho_{\rm de}$ of an effective dark energy fluid under this assumption only, that is if

$$H^2 = \frac{8\pi G}{3}(\rho + \rho_{\rm de}) - \frac{K}{a^2}, \tag{1.13}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + \rho_{\rm de} + 3P + 3P_{\rm de}). \tag{1.14}$$

All the background information about dark energy is thus encapsulated in the single function $w_{\rm de}(z)$. Most observational constraints on the dark energy equation of states refer to this definition.

1.2 Modifying the minimal ΛCDM

The *Copernican principle* implies that the spacetime metric reduces to a single function, the scale factor $a(t)$ that can be Taylor expanded as $a(t) = a_0 + H_0(t-t_0) - \frac{1}{2}q_0H_0^2(t-t_0)^2 + \ldots$. It follows that the conclusions that the cosmic expansion is accelerating ($q_0 < 0$) does not involve any hypothesis about the theory of gravity (other than the one that the spacetime geometry can be described by a metric) or the matter content, as long as this principle holds.

The assumption that the Copernican principle holds, and the fact that it is so central in drawing our conclusion on the acceleration of the expansion, splits our investigation into two avenues. Either we assume that the Copernican principle holds and we have to modify the laws of fundamental physics or we abandon the Copernican principle, hoping to explain dark energy without any new physics but at the expense of living in a particular place in the Universe. While the first solution is more orthodox from a cosmological point of view, the second is indeed more conservative from a physical point of view. It will be addressed in § 1.2.4. We are thus in front of a choice between "simple" cosmological solutions with new physics and more involved cosmological solutions of standard physics.

This section focuses on the first approach. If general relativity holds then Eq. (1.4) tells us that the dynamics has to be dominated by a dark energy fluid with $w_{\rm de} < -\frac{1}{3}$ for the expansion to be accelerated. The simplest solution is indeed the cosmological constant Λ for which $w_{\rm de} = -1$ and which is the only model not introducing new degrees of freedom.

1.2.1 General classification of physical models

1.2.1.1 General Relativity

Einstein's theory of gravity relies on two independent hypotheses.

First, the theory rests on the Einstein equivalence principle, which includes the universality of free fall, the local position and local Lorentz invariances in its weak form (as other metric theories) and is conjectured to satisfy it in its strong form. We refer to Will (1981) for a detailed explanation of these principles and their implications. The weak equivalence principle can be mathematically implemented by assuming that all matter fields are minimally coupled to a single metric tensor $g_{\mu\nu}$. This metric defines the length and times measured by laboratory clocks and rods so that it can be called the *physical metric*. This implies that the action for any matter field, ψ say, can be written as $S_{\text{matter}}[\psi, g_{\mu\nu}]$. This so-called *metric coupling* ensures in particular the validity of the universality of free-fall.

The action for the gravitational sector is given by the Einstein-Hilbert action

$$S_{\text{gravity}} = \frac{c^3}{16\pi G} \int \mathrm{d}^4x \sqrt{-g_*} R_*, \tag{1.15}$$

where $g^*_{\mu\nu}$ is a massless spin-2 field called the *Einstein metric*. The second hypothesis states that both metrics coincide

$$g_{\mu\nu} = g^*_{\mu\nu}.$$

The underlying physics of our reference cosmological model (i.e. hypotheses H1 and H2) is thus described by the action

$$S_{\text{gravity}} = \frac{c^3}{16\pi G} \int \mathrm{d}^4x \sqrt{-g}(R - 2\mathbf{\Lambda}) + \sum_{\text{standard model}+\mathbf{CDM}} S_{\text{matter}}[\psi_i, g_{\mu\nu}], \tag{1.16}$$

which includes all known matter fields plus two unknown components (in bold face).

1.2.1.2 Local experimental constraints

The assumption of a metric coupling is well tested in the Solar system. First it implies that all non-gravitational constants are spacetime independent, which have been tested to a very high accuracy in many physical systems and for various fundamental constants (Uzan, 2003; Uzan, 2004; Uzan and Leclercq, 2008), e.g. at the 10^{-7} level for the fine structure constant on time scales ranging to 2-4 Gyrs. Second, the isotropy has been tested from the constraint on the possible quadrupolar shift of nuclear energy levels (Prestage et al., 1985; Chupp et al., 1989; Lamoreaux et al., 1986) proving that matter couples to a unique metric tensor at the 10^{-27} level. Third, the universality of free fall of test bodies in an external gravitational field at the 10^{-13} level in the laboratory (Baessler et al., 1999; Adelberger, et al. 2001). The Lunar Laser ranging experiment (Williams et al., 2004), which compares the relative acceleration of the Earth and Moon in the gravitational field of the Sun, also probe the strong equivalence principle at the 10^{-4} level. Fourth, the Einstein effect (or gravitational redshift) states that two identical clocks located at two different positions in a static Newton potential U and compared by means of electromagnetic signals shall exhibit a difference in clock rates of $1 + [U_1 - U_2]/c^2$, where U is the gravitational potential. This effect has been measured at the 2×10^{-4} level (Vessot and Levine, 1978).

The parameterized post-Newtonian formalism (PPN) is a general formalism that introduces 10 phenomenological parameters to describe any possible deviation from general relativity at the first post-Newtonian order (Will, 1981). The formalism assumes that gravity is described by a metric and that it does not involve any characteristic scale. In its simplest form, it reduces to the two Eddington parameters entering the metric of the Schwartzschild metric in isotropic coordinates

$$g_{00} = -1 + \frac{2Gm}{rc^2} - 2\beta^{\rm PPN}\left(\frac{2Gm}{rc^2}\right)^2, \qquad g_{ij} = \left(1 + 2\gamma^{\rm PPN}\frac{2Gm}{rc^2}\right)\delta_{ij}.$$

Indeed, general relativity predicts $\beta^{\rm PPN} = \gamma^{\rm PPN} = 1$. These two phenomenological parameters are constrained (1) by the shift of the Mercury perihelion (Shapiro et al., 1990) which implies that $|2\gamma^{\rm PPN} - \beta^{\rm PPN} - 1| < 3 \times 10^{-3}$, (2) the Lunar laser ranging experiments (Williams et al., 2004) which implies $|4\beta^{\rm PPN} - \gamma^{\rm PPN} - 3| = (4.4 \pm 4.5) \times 10^{-4}$ and (3) by the deflection of electromagnetic signals which are all controlled by $\gamma^{\rm PPN}$. For instance the very long baseline interferometry (Shapiro et al., 2004) implies that $|\gamma^{\rm PPN} - 1| = 4 \times 10^{-4}$ while the measurement of the time delay variation to the Cassini spacecraft (Bertotti et al., 2003) sets $\gamma^{\rm PPN} - 1 = (2.1 \pm 2.3) \times 10^{-5}$.

The PPN formalism does not allow to test finite range effects that could be caused e.g. by a massive degree of freedom. In that case one expects a Yukawa-type deviation from the Newton potential,

$$V = \frac{Gm}{r}\left(1 + \alpha \mathrm{e}^{-r/\lambda}\right),$$

that can be probed by "fifth force" experimental searches. λ characterizes the range of the Yukawa deviation while its strength α may also include a composition-dependence (Uzan, 2003). The constraints on (λ, α) are summarized in Hoyle et al. (2004) which typically shows that $\alpha < 10^{-2}$ on scales ranging from the millimeter to the Solar system size.

In general relativity, the graviton is massless. One can however give it a mass, but this is very constrained. In particular, around a Minkowski background, the mass term must have the very specific form of the Pauli-Fierz type in order to avoid ghosts (see below for a more precise definition) to be excited. This mass term is however inconsistent with Solar system constraints because there exists a discontinuity (van Dam and Veltman, 1970; Zakharov, 1970) between the case of a strictly massless graviton and a very light one. In particular, such a term can be ruled out from the Mercury perihelion shift.

General relativity is also tested with pulsars (Damour, and Esposito-Farèse, 1998; Esposito-Farèse, 2005) and in the strong field regime (Psaltis, 2008). For more details we refer to Will (1981), Damour and Lilley (2008) and Turyshev (2008). Needless to say that any extension of general relativity has to pass these constraints. However, deviations from general relativity can be larger in the past, as we shall see, which makes cosmology an interesting physical system to extend these constraints.

1.2.1.3 Universality classes

There are many possibilities to extend this minimal physical framework. Let us start by defining universality classes (Uzan, 2007) by restricting our discussion to field theories. This has the advantage to identify the new degrees of freedom and their couplings.

The first two classes assume that gravitation is well described by general relativity and introduce new degrees of freedom beyond those of the standard model of particle physics. This means that one adds a new term $S_{\rm de}[\psi; g_{\mu\nu}]$ in the action (1.16) while keeping the Einstein-Hilbert action and the coupling of all the fields (standard matter and dark matter) unchanged. They are:

1. Class A consists of models in which the acceleration is driven by the gravitational effect of the new fields. They thus must have an equation of state smaller than $-\frac{1}{3}$. They are not coupled to the standard matter fields or to dark matter so that one is adding a new sector

$$S_{\rm de}[\phi; g_{\mu\nu}]$$

to the action (1.16), where ϕ stands for the dark energy field (not necessarily a scalar field). Standard examples include *quintessence* models (Wetterich, 1988; Ratra and Peebles, 1988) which invoke a canonical scalar field slow-rolling today, *solid dark matter* models (Battye et al., 1999) induced by frustrated topological defects networks, *tachyon* models (Sen, 1999), *Chaplygin gas* (Kamenshchik et al., 2001) and *K-essence* (Armendariz-Picon et al., 2000; Chiba et al., 2000) models invoking scalar fields with a non-canonical kinetic term.

2. Class B introduces new fields which do not dominate the matter content so that they do not change the expansion rate of the universe. They are thus not required to have an equation of state smaller than

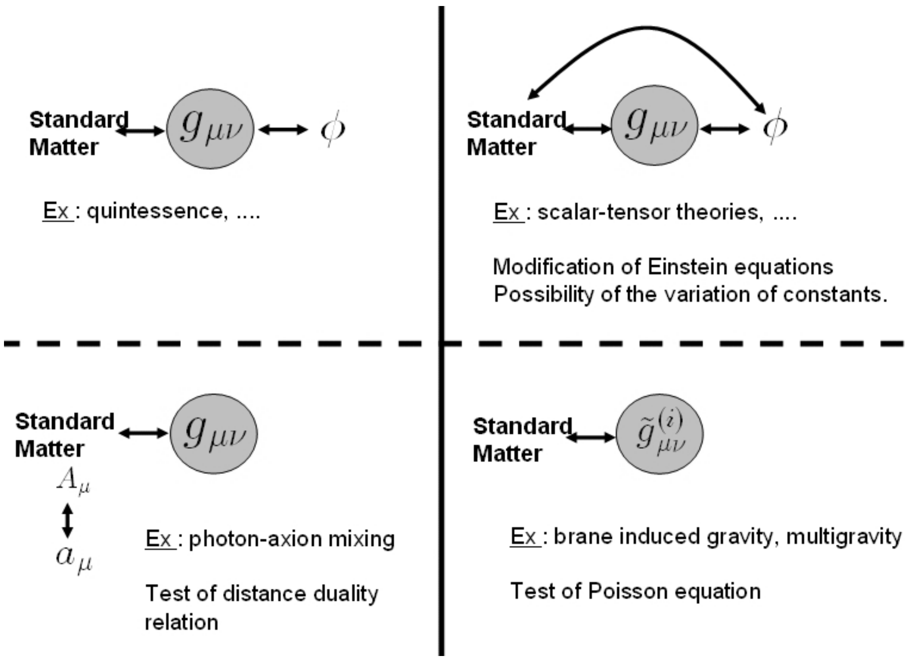

Fig. 1.1. Summary of the different classes of physical dark energy models. As discussed in the text, various tests can be designed to distinguish between them. The classes differ according to the nature of the new degrees of freedom and their couplings. Left column accounts for models where gravitation is described by general relativity while right column models describe a modification of general relativity. In the upper line classes, the new fields dominate the matter content of the universe at low redshift. Upper-left models (class A) consist of models in which a new kind of gravitating matter is introduced. In the upper-right models (class C), a light field induces a long-range force so that gravity is not described by a massless spin-2 graviton only. In this class, Einstein equations are modified and there may be a variation of the fundamental constants. The lower-right models (class D) correspond to models in which there may exist an infinite number of new degrees of freedom, such as in some class of braneworld scenarios. These models predict a modification of the Poisson equation on large scales. In the last class (lower-left, class B), the distance duality relation may be violated. From Uzan (2007).

$-\frac{1}{3}$. These fields are however coupled to photons and thus affect the observations. An example (Csaki et al. , 2002; Deffayet et al. , 2002) is provided by *photon-axion oscillations* which aims at explaining the dimming of supernovae not by an accelerated expansion but by the fact that part of the photons has oscillated into invisible axions. In that particular case, the electromagnetic sector is modified according to

$$S_{\rm em}[A_\mu; g_{\mu\nu}] \to S_{\rm em}[A_\mu, a_\mu; g_{\mu\nu}].$$

A specific signature of these models would be a violation of the distance duality relation (see § 1.3.3.1).

Then come models with a modification of general relativity. Once such a possibility is considered, many new models arise (Will, 1981). They are:

3. Class C includes models in which a finite number of new fields are introduced. These fields couple to the standard model fields and some of them dominate the matter content (at least at late time). This is the case in particular of scalar-tensor theories in which a scalar field couples universally and leads to the class of extended quintessence models, chameleon models or $f(R)$ models depending on the choice of the coupling function and potential (see § 1.2.3). For these models, one has a new sector

$$S_{\varphi}[\varphi; g_{\mu\nu}]$$

and the couplings of the matter fields will be modified according to

$$S_{\rm matter}[\psi_i; g_{\mu\nu}] \rightarrow S_{\rm matter}[\psi_i; A_i^2(\varphi) g_{\mu\nu}].$$

If the coupling is not universal, a signature may be the variation of fundamental constants and a violation of the universality of free fall. This class also offers the possibility to enjoy $w_{\rm de} < -1$ with a well-defined field theory and includes models in which a scalar field couples differently to standard matter field and dark matter.

4. Class D includes more drastic modifications of general relativity with e.g. the possibility to have more types of gravitons (massive or not and most probably an infinite number of them). This is for instance the case of models involving extra-dimensions such as e.g. multi-brane models (Gregory et al., 2000), multigravity (Kogan et al., 2000), brane induced gravity (Dvali et al., 2000) or simulated gravity (Carter et al., 2001). In these cases, the new fields modified the gravitational interaction on large scale but do not necessarily dominate the matter content of the universe. Some of these models may also offer the possibility to mimic an equation of state $w_{\rm de} < -1$.

These various modifications, summarized on Fig. 1.1 can indeed be combined to get more exotic models.

1.2.1.4 "Modified gravity" vs new matter

The different models in the literature are often categorized as "modified gravity" or "new matter". This distinction may however be subtle.

First, we shall define *gravity* as the long range force that cannot be screened. We are used to describe this interaction by general relativity so that it is associated with a massless spin-2 graviton. In our view, gravity cannot be modified but only its description, i.e. general relativity. As an example, scalar-tensor theories (see § 1.2.3) extend general relativity by a spin-0 interaction which can be long-range according to the mass of the scalar field. In this case, the interaction is even universal so that it does not imply any violation of the weak Einstein equivalence principle.

Note also that whatever the model, it requires the introduction of new fields beyond those of the standard model. The crucial difference is that in models with "new matter" (e.g. class A), the amount of dark energy is imposed by initial conditions and its gravitational effect induces the acceleration of the Universe. In a "modified gravity" model (e.g. classes C and D) the standard matter and cold dark matter generate an effective dark energy component. The acceleration may thus be a consequence of the fact that the gravitational interaction is weaker than expected on large scales. But, it may be that the energy density of the new field also dominates the dynamics but still be determined by the energy density of the standard field.

1.2.2 Modifying General Relativity

1.2.2.1 In which regime?

Before investigating gravity beyond general relativity, let us try to sketch the regimes in which these modifications may (or shall) appear. We can distinguish the following regimes.

- *Weak-strong field regimes* can be characterized by the amplitude of the gravitational potential. For a spherical static spacetime, $\Phi = GM/rc^2$. It is of order of $\Phi_\odot \sim 2 \times 10^{-6}$ at the surface of the Sun and equal to $\frac{1}{2}$ for a black-hole.
- *Small-large distances*. Such modifications can be induced by a massive degree of freedom that will induce a Yukawa like coupling. While constrained on the size of the solar system, we have no constraints on scales larger that $10h^{-1}$ Mpc.
- *Low-high acceleration regimes* are of importance to discuss galaxy rotation curves and (galactic) dark matter, as suggested by the MOND phenomenology (Milgrom, 1983). In particular the kind of modification of the gravitation theory that could account for the dark matter cannot occur at a characteristic distance because of the Tully-Fischer law.
- *Low-high curvature regimes* will distinguish the possible extensions of the Einstein-Hilbert action. For instance a quadratic term of the form αR^2

becomes significant compared to R when $GM/r^3c^2 \gg \alpha^{-1}$ even if Φ remains small. In the Solar system $R_\odot \sim 4 \times 10^{-28}\mathrm{cm}^{-2}$.

In cosmology, we can suspect various possible regimes in which to modify general relativity. The dark matter problem can be accounted for by a modification of Newton gravity below the typical acceleration $a_0 \sim 10^{-8}\mathrm{cm.s}^{-2}$. It follows that the regime for which a dark matter component is required can be characterized by

$$\Phi R < a_0^2 \sim 3 \times 10^{-31} R_\odot. \tag{1.17}$$

Concerning the homogeneous universe, one can sort out from the Friedmann equations that

$$R_{\mathrm{FL}}(z) = 3H_0^2[\Omega_{\mathrm{m}0}(1+z)^3 + 4\Omega_{\Lambda 0}], \tag{1.18}$$

from which we deduce that $R_{\mathrm{FL}} \sim 10^{-5} R_\odot$ at the time of nucleosynthesis, $R_{\mathrm{FL}} \sim 10^{-20} R_\odot$ at the time of decoupling and $R_{\mathrm{FL}} \sim 10^{-28} R_\odot$ at $z = 1$. The curvature scale associated to a cosmological constant is $R_\Lambda = \frac{1}{6}\Lambda$ and the cosmological constant (or dark energy) problem corresponds to a low curvature regime,

$$R < R_\Lambda \sim 1.2 \times 10^{-30} R_\odot. \tag{1.19}$$

The fact that the limits (1.17) and (1.19) intersect illustrates the coincidence problem, that is $a_o \sim cH_0$ and $\Omega_{\mathrm{m}0} \sim \Omega_{\Lambda 0}$. Note that both arise on curvature scales much smaller than those probed in the solar system.

Let us now turn to the cosmological perturbations. The gravitational potential at the time of the decoupling ($z \sim 10^3$) is of the order of $\Phi \sim 10^{-5}$. During the matter era, the Poisson equation imposes that $\Delta\Phi \propto \delta\rho_{\mathrm{m}} a^2$ which is almost constant. It follows that we never expect a potential larger than $\Phi \sim 10^{-5}$ on cosmological scales. We are thus always in a weak field regime. The characteristic distance scale is fixed by the Hubble radius c/H_0. The curvature perturbation associated with the large scale structures is, in the linear theory, of the order

$$\delta R = \frac{6}{a^2}\Delta\Phi \sim 3H_0^2\Omega_{\mathrm{m}0}(1+z)^3\delta_{\mathrm{m}}(z).$$

Since at redshift zero, $\langle\delta_{\mathrm{m}}^2\rangle = \sigma_8 \sim 1$ in a ball of radius of 8 Mpc, we conclude that $\langle\delta R^2\rangle^{1/2} \sim 3H_0^2\Omega_{\mathrm{m}0}\sigma_8$ while $R_{\mathrm{FL}} = 3H_0^2\Omega_{\mathrm{m}0}$ if $\Lambda = 0$. This means that the curvature perturbation becomes of the order of the background curvature at a redshift $z \sim 0$, even if we are still in the weak field limit. This implies that the effect of the large scale structures on the background dynamics may be non-negligible. This effect has been argued to be at the origin of the acceleration of the universe (Ellis and Buchert, 2005; Ellis, 2008) but no convincing formalism to describe this backreaction has been constructed yet. Note that in this picture, the onset of the acceleration phase will be determined by the amplitude of the initial power spectrum.

In conclusion, to address the dark energy or dark matter problem by a modification of general relativity, we are interested in modifications on large scales (typically Hubble scales), low acceleration (below a_0) or small curvature (typically R_Λ).

1.2.2.2 General constraints

In modifying general relativity, we shall demand that the new theory

- *does not contain ghosts*, i.e. degrees of freedom with negative kinetic energy. The problem with such a ghost is that the theory would be unstable. In particular, the vacuum can decay in an arbitrary amount of positive energy (standard) gravitons whose energy would be balanced by negative energy ghosts.
- *has a Hamiltonian bounded from below.* Otherwise, the theory would be unstable, even if one cannot explicitly identify a ghost degree of freedom.
- the new degrees of freedom are not *tachyon*, i.e. do not have a negative mass.
- is *compatible with local tests* of deviation from general relativity, in particular in the Solar system described in § 1.2.1.2.

Then, starting from the action (1.16), we see that we can either modify the Einstein-Hilbert action while letting the coupling of all matter fields to the metric unchanged or modify the coupling(s) in the matter action. The possibilities are numerous (Will, 1981; Esposito-Farèse and Bruneton, 2007; Uzan, 2007) and we cannot start an extensive review of the models here. We shall thus consider some examples that will illustrate the constraints cited above, but with no goal of exhaustivity.

1.2.2.3 Modifying the Einstein-Hilbert action

Let us start with the example of higher order gravity models based on the quadratic action (here we follow the very clear analysis of Esposito-Farèse and Bruneton (2007) for our discussion)

$$S_{\rm gravity} = \frac{c^3}{16\pi G}\int \mathrm{d}^4x\sqrt{-g}\left[R + \alpha C^2_{\mu\nu\rho\sigma} + \beta R^2 + \gamma \mathrm{GB}\right], \tag{1.20}$$

where $C_{\mu\nu\rho\sigma}$ is the Weyl tensor and $\mathrm{GB} \equiv R^2_{\mu\nu\rho\sigma} - 4R^2_{\mu\nu} + R^2$ is the Gauss-Bonnet term. α, β and γ are three constants with dimension of an inverse mass square. Since GB does not contribute to the local field equations of motion, we will not consider it further. The action (1.20) gives a renormalisable theory of quantum gravity at all order provided α and β are non-vanishing (Stelle, 1978). However, such theories contain ghosts. This can be seen from the graviton propagator which takes the form $1/(p^2+\alpha p^4)$. It can indeed be decomposed in irreducible fractions as

$$\frac{1}{p^2+\alpha p^4} = \frac{1}{p^2} - \frac{1}{p^2+\frac{1}{\alpha}}.$$

The first term is nothing but the standard propagator of the usual massless graviton. The second term correspond to an extra-massive degree of freedom with mass α^{-1} and its negative sign indicates that it carries negative energy: it is a ghost. Moreover if α is negative, this ghost is also a tachyon! The only viable such modification arises from βR^2, which introduces a massive spin-0 degree of freedom.

These considerations can be extended to more general theories involving an arbitrary function of the metric invariants, $f(R, R_{\mu\nu}, R_{\mu\nu\rho\sigma})$, which also generically (Hindawi et al. , 1996; Tomboulis, 1996) contain a massive spin-2 ghost. They are thus not stable theories with the exception of $f(R)$ theories, discussed in § 1.2.3.3.

A possibility may seem to consider models designed such that their second-order expansion never shows any negative energy kinetic term. As recalled in Esposito-Farèse and Bruneton (2007) and Woodard (2006), these models still exhibit instabilities the origin of which can be related to a theorem by Ostrogradsky (1850) showing that their Hamiltonian is generically not bounded from below.

We summarized this theorem following the presentation by Woodard (2006). Consider a Lagrangian depending on a variable q and its first two time derivatives $\mathcal{L}(q,\dot{q},\ddot{q})$ and assume that it is not degenerate, i.e. that $\ddot{q}$ cannot be eliminated by an integration by part. Then the definition $p_2 \equiv \partial\mathcal{L}/\partial\ddot{q}$ can be inverted to get $\ddot{q}$ as a function q, $\dot{q}$ and p_2, $\ddot{q}[q,\dot{q},p_2]$, and the initial data must be specified by two pairs of conjugate momenta defined by $(q_1,p_1) \equiv (q, \partial\mathcal{L}/\partial\dot{q} - \mathrm{d}(\partial\mathcal{L}/\partial\ddot{q})/\mathrm{d}t)$ and $(q_2,p_2) \equiv (\dot{q}, \partial\mathcal{L}/\partial\ddot{q})$. The Hamiltonian defined as $\mathcal{H} = p_1\dot{q}_1 + p_2\dot{q}_2 - \mathcal{L}$ can be shown to be the generator of time translations and the Hamilton equations which derive from $\mathcal{H}$ are indeed equivalent to the Euler-Lagrange equations derived from $\mathcal{L}$. In terms of q_i and p_i, the Hamiltonian takes the form

$$\mathcal{H} = p_1 q_2 + p_2\ddot{q}[q_1,q_2,p_2] - \mathcal{L}(q_1,q_2,\ddot{q}[q_1,q_2,p_2]).$$

This expression is however linear in p_1 so that the Hamiltonian is not bounded from below and the theory is necessarily unstable. Let us note that this constraint can be avoided by non-local theories, that is if the Lagrangian depends on an infinite number of derivatives, as e.g. string theory, even though its expansion may look pathological.

1.2.2.4 Modifying the matter action

Many other possibilities, known as bi-metric theories of gravity, arise if one assumes that $g_{\mu\nu} \neq g^*_{\mu\nu}$. Instead one can postulate that the physical metric is a combination of various fields, e.g.

$$g_{\mu\nu}[g^*_{\mu\nu}, \varphi, A_\mu, B_{\mu\nu}, \ldots] = A^2(\varphi)\left[g^*_{\mu\nu} + \alpha_1 A_\mu A_\nu + \alpha_2 g^*_{\mu\nu} g^{\alpha\beta}_* A_\alpha A_\beta + \ldots\right].$$

As long as these new fields enter quadratically, their field equation is generically of the form $(\nabla_\mu \nabla^\mu)A = AT$ where T is the matter source. It follows that matter cannot generate them if their background value vanishes. On the other hand, if their background value does not vanish then these fields define a preferred frame and Lorentz invariance is violated.

Such modifications have however drawn some attention, in particular in the attempts of constructing a field theory reproducing the MOND phenomenology (Milgrom, 1983). In particular, in order to increase light deflection in scalar-tensor theories of gravity, a *disformal coupling* (Bekenstein, 1993), $g_{\mu\nu} = A^2(\varphi)g^*_{\mu\nu} + B(\varphi)\partial_\mu\varphi\partial_\nu\varphi$, was introduced. It was generalized to *stratified theory* (Sanders, 1997). by replacing the gradient of the scalar field by a dynamical unit vector field ($g^*_{\mu\nu}A^\mu A^\nu = -1$), $g_{\mu\nu} = A^2(\varphi)g^*_{\mu\nu} + B(\varphi)A_\mu A_\nu$. This is at the basis of the TeVeS theory proposed by Bekenstein (Bekenstein, 2004). The mathematical consistency and the stability of these field theories were investigated in depth in the excellent analysis of Esposito-Farèse and Bruneton (2007). It was shown that no present theory passes all available experimental constraints while being stable and admitting a well-posed Cauchy problem.

Esposito-Farèse and Bruneton (2007) also notice that while couplings of the form $g_{\mu\nu}[g^*_{\mu\nu}, R^*_{\mu\nu}, R^*_{\mu\nu\alpha\beta}, \ldots]$ seem to lead to well-defined theories in vacuum (in particular) when linearizing around a Minkowsky background, they are unstable inside matter, because the Ostrogradsky theorem strikes back.

The case in which only a scalar partner, $g_{\mu\nu} = A^2(\varphi)g^*_{\mu\nu}$, is introduced leads to consistent field theories and is the safest way to modify the matter coupling. We shall discuss these scalar-tensor theories of gravity in § 1.2.3.

1.2.2.5 Higher-dimensional theories

Higher-dimensional models of gravity, among which string theory (see e.g. Damour and Lilley (2008)) predict non-metric coupling as those discussed in the previous section. Many scalar fields, known as *moduli*, appear in the dimensional reduction to four dimensions.

As a simple example, let us consider a five-dimensional spacetime and assume that gravity is described by the Einstein-Hilbert action

$$S = \frac{1}{12\pi^2 G_5} \int \bar{R}\sqrt{|\bar{g}|}\,\mathrm{d}^5 x, \tag{1.21}$$

where we denote by a bar quantities in 5 dimensions to distinguish them with the analogous quantities with no bar in 4 dimensions. The aim is to determine the independent elements of the metric g_{AB}, which are 15 in five dimensions. We decompose the metric into a symmetric tensor part $g_{\mu\nu}$, with 10 independent components, a vector part, A_α, with four components and finally a scalar field, ϕ, to complete the counting of the number of degrees of freedom ($15 = 10 + 4 + 1$). The metric is thus decomposed as

$$\bar{g}_{AB} = \begin{pmatrix} g_{\mu\nu} + \frac{1}{M^2}\phi^2 A_\mu A_\nu & \frac{1}{M}\phi^2 A_\mu \\ \frac{1}{M}\phi^2 A_\nu & \phi^2 \end{pmatrix}, \tag{1.22}$$

where the different components depend *a priori* both on the usual space-time coordinates x^α and the coordinate in the extra-dimension y. The constant M has dimensions of a mass, so that A_α also has dimensions of mass, whereas the scalar field ϕ is here dimensionless. Finally, while capital latin indices vary in the entire 5-dimensional space-time, $A, B = 0, \cdots, 4$, greek indices span the 4-dimensional space-time, namely $\mu, \nu = 0, \cdots, 3$. Compactifying on a circle and assuming that none of the variables depends on the transverse direction y (*cylinder condition*), the action (1.21) reduces to the four-dimensional action

$$S = \frac{1}{16\pi G} \int \mathrm{d}^4 x \sqrt{-g}\phi \left(R - \frac{\phi^2}{4M^2} F_{\alpha\beta}F^{\alpha\beta} \right), \tag{1.23}$$

where $F_{\alpha\beta} \equiv \partial_\alpha A_\beta - \partial_\beta A_\alpha$ and where we have set

$$G = \frac{3\pi \bar{G}_5}{4V_{(5)}},$$

and factored out the finite volume of the fifth dimension, $V_{(5)} = \int \mathrm{d}y$. The scalar field couples explicitly to the kinetic term of the vector field. It can be checked that this coupling cannot be eliminated by a redefinition of the metric, whatever the function $A(\phi)$: this is the well-known conformal invariance of electromagnetism in four dimensions. Such a term induces a variation of the fine structure constant as well as a violation of the universality of free-fall (Uzan, 2003). Such dependencies of the masses and couplings are generic for higher-dimensional theories and in particular string theory.

The cylinder condition is justified as long as we consider the fifth dimension to be topologically compact with the topology of a circle. In this case, all the fields which are defined in this space, i.e. the four-dimensional metric $g_{\mu\nu}$, the vector A_α and the dilaton ϕ, and any additional matter fields that the theory should describe, are periodic functions of the extra-dimension and can therefore be expanded into Fourier modes. The radius R of this dimension then turns out to be naturally $R \sim M^{-1}$. For large enough M, the radius is too small to have observable consequences: to be sensitive to the fifth dimension, the energies involved must be comparable to M. Decomposing all the fields in Fourier modes a e.g.

$$\phi\left(x_\mu, y\right) = \sum_{n=-\infty}^{+\infty} \phi_n\left(x_\mu\right) \mathrm{e}^{inMy}, \qquad \text{with} \qquad \phi_{-n} = \phi_n^\star \tag{1.24}$$

(ϕ real), we conclude that the four-dimensional theory will also contain a infinite tower of modes of increasing mass.

While these tree-level predictions of string theory are in contradiction with experimental constraints, many mechanisms can reconcile it with experiment. In particular, it has been claimed that quantum loop corrections to the tree-level action may modify the coupling in such a way that it has a minimum (Damour and Polyakov, 1994). The scalar field can thus be attracted toward this minimum during the cosmological evolution so that the theory is attracted toward general relativity. Another possibility is to invoke an environmental dependence, as can be implemented in scalar-tensor theories by the chameleon mechanism (Khoury and Weltman, 2004) which invokes a potential with a minimum not coinciding with the one of the coupling function.

In higher dimensions, the Einstein-Hilbert action can also be modified by adding the Gauss-Bonnet term GB since it does not enter the field equations only in four dimensions. The D-dimensional Einstein-Hilbert action can then be modified to include a term of the form αGB. In particular, it is the case in the low-energy limit of heterotic string theory (Gross and Sloan, 1987). In various configurations, in particular with branes, it has been argued that the Gauss-Bonnet invariant can also couple to a scalar field (Amendola et al., 2006), i.e. $\alpha(\varphi)$GB. As long as the modification is linear in GB, it is ghost-free.

In the context of braneworld, it was shown that some models with infinite volume extra-dimension can produce a modification of general relativity leading to an acceleration of the expansion. In the DGP model (Dvali et al., 2000), one considers beside the 5-dimensional Einstein-Hilbert a 4-dimensional term induced on the brane

$$S = \frac{M_5^2}{2}\int \bar{R}_5\sqrt{|\bar{g}_5|}\,\mathrm{d}^5x + \frac{M_4^2}{2}\int R_4\sqrt{|g_4|}\,\mathrm{d}^4x. \tag{1.25}$$

There is a competition between these two terms and the five-dimensional term dominates on scales larger than $r_c = M_4^2/2M_5^3$. The existence or absence of ghost in this class of models is still under debate. Some of these models (Deffayet, 2005) have also been claimed to describe massive gravitons without being plagued by the van Dam-Veltman-Zakharov discontinuity (see § 1.2.1.2).

As a conclusion, higher-dimensional models offer a rich variety of possibilities among which some may be relevant to describe a modification of general relativity on large scales.

1.2.3 Example: scalar-tensor theories

As discussed in § 1.2.2.4, the case in which only a scalar partner to the graviton is introduced leads to consistent field theories and is the safest way to modify the matter coupling.

1.2.3.1 Formulation

In scalar-tensor theories, gravity is mediated not only by a massless spin-2 graviton but also by a spin-0 scalar field that couples universally to matter fields (this ensures the universality of free fall). In the Jordan frame, the action of the theory takes the form

$$\begin{aligned} S &= \int \frac{\mathrm{d}^4x}{16\pi G_*}\sqrt{-g}\,[F(\varphi)R - g^{\mu\nu}Z(\varphi)\varphi_{,\mu}\varphi_{,\nu} - 2U(\varphi)] \\ &\qquad + S_{\mathrm{matter}}[\psi; g_{\mu\nu}] \end{aligned} \tag{1.26}$$

where G_* is the bare gravitational constant. This action involves three arbitrary functions (F, Z and U) but only two are physical since there is still the possibility to redefine the scalar field. F needs to be positive to ensure that the graviton carries positive energy. S_{matter} is the action of the matter fields that are coupled minimally to the metric $g_{\mu\nu}$. In the Jordan frame, the matter is universally coupled to the metric so that the length and time as measured by laboratory apparatus are defined in this frame.

It is useful to define an Einstein frame action through a conformal transformation of the metric

$$g^*_{\mu\nu} = F(\varphi) g_{\mu\nu}. \tag{1.27}$$

In the following all quantities labelled by a star (*) will refer to Einstein frame. Defining the field φ_* and the two functions $A(\varphi_*)$ and $V(\varphi_*)$ (see e.g. Esposito-Farèse and Polarski, 2001) by

$$\begin{aligned}
\left(\frac{\mathrm{d}\varphi_*}{\mathrm{d}\varphi}\right)^2 &= \frac{3}{4}\left(\frac{\mathrm{d}\ln F(\varphi)}{\mathrm{d}\varphi}\right)^2 + \frac{1}{2F(\varphi)} && (1.28)\\
A(\varphi_*) &= F^{-1/2}(\varphi) && (1.29)\\
2V(\varphi_*) &= U(\varphi)F^{-2}(\varphi), && (1.30)
\end{aligned}$$

the action (1.26) reads as

$$\begin{aligned}
S &= \frac{1}{16\pi G_*}\int \mathrm{d}^4x\sqrt{-g_*}\left[R_* - 2g_*^{\mu\nu}\partial_\mu\varphi_*\partial_\nu\varphi_* - 4V(\varphi_*)\right]\\
&\quad + S_{\mathrm{matter}}[A^2(\varphi_*)g^*_{\mu\nu};\psi]. && (1.31)
\end{aligned}$$

The kinetic terms have been diagonalised so that the spin-2 and spin-0 degrees of freedom of the theory are perturbations of $g^*_{\mu\nu}$ and φ_* respectively.

In this frame, the field equations take the form

$$\begin{aligned}
G^*_{\mu\nu} &= 8\pi G_* T^*_{\mu\nu}\\
&+ 2\partial_\mu\varphi_*\partial_\nu\varphi_* - g^*_{\mu\nu}\left(\partial_\alpha\varphi_*\right)^2 - 2g^*_{\mu\nu}V && (1.32)\\
(\nabla_\mu\nabla^\mu)_*\varphi_* &= V_{,\varphi_*} - 4\pi G_*\alpha(\varphi_*)T^*_{\mu\nu}g_*^{\mu\nu} && (1.33)\\
\nabla_\mu T_*^{\mu\nu} &= \alpha(\varphi_*)T^*_{\sigma\rho}g_*^{\sigma\rho}\partial^\nu\varphi_* && (1.34)
\end{aligned}$$

where we have defined the Einstein frame stress-energy tensor

$$T_*^{\mu\nu} \equiv \frac{2}{\sqrt{-g_*}}\frac{\delta S_{\mathrm{matter}}}{\delta g^*_{\mu\nu}},$$

related to the Jordan frame stress-energy tensor by $T^*_{\mu\nu} = A^2 T_{\mu\nu}$. The function

$$\alpha(\varphi_*) \equiv \frac{\mathrm{d}\ln A}{\mathrm{d}\varphi_*}. \tag{1.35}$$

characterizes the coupling of the scalar field to matter (we recover general relativity with a minimally coupled scalar field when it vanishes). For completeness, we also introduce

$$\beta(\varphi_*) \equiv \frac{\mathrm{d}\alpha}{\mathrm{d}\varphi_*}. \tag{1.36}$$

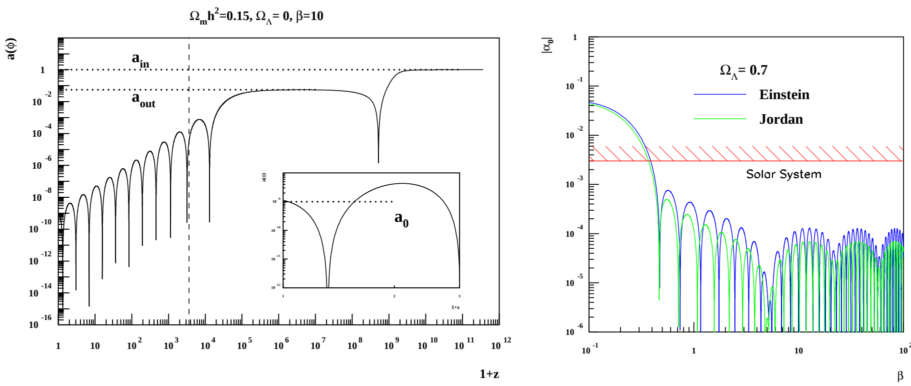

Fig. 1.2. *Left*: Evolution of the dilaton as a function of redshift. In the radiation era the dilaton freezes to a constant value and is then driven toward the minimum of the coupling function during the matter era. *Right*: constraints on scalar-tensor theories of gravity with a massless dilaton with quadratic coupling in the (α_0, β) plane. At large β the primordial nucleosynthesis sets more stringent constraints than the Solar system. From Coc et al. (2006).

Note that in Einstein frame the Einstein equations (1.32) are the same as those obtained in general relativity with a minimally coupled scalar field.

The action (1.26) defines an effective gravitational constant $G_{\text{eff}} = G_*/F = G_*A^2$. This constant does not correspond to the gravitational constant effectively measured in a Cavendish experiment. The Newton constant measured in this experiment is

$$G_{\text{cav}} = G_* A_0^2 (1 + \alpha_0^2) \tag{1.37}$$

where the first term, $G_* A_0^2$ corresponds to the exchange of a graviton while the second term $G_* A_0^2 \alpha_0^2$ is related to the long range scalar force.

1.2.3.2 Cosmological signatures

The post-Newtonian parameters can be expressed in terms of the values of α and β today as

$$\gamma^{\text{PPN}} - 1 = -\frac{2\alpha_0^2}{1+\alpha_0^2}, \qquad \beta^{\text{PPN}} - 1 = \frac{1}{2}\frac{\beta_0 \alpha_0^2}{(1+\alpha_0^2)^2}. \tag{1.38}$$

The Solar system constraints discussed in § 1.2.1.2 imply α_0 to be very small, typically $\alpha_0^2 < 10^{-5}$ while β_0 can still be large. Binary pulsar observations (Esposito-Farèse, 2005) impose that $\beta_0 > -4.5$.

The previous constraints can be satisfied even if the scalar-tensor theory was far from general relativity in the past. The reason is that these theories can be attracted toward general relativity (Damour and Nordtvedt, 1993) if their coupling function or potential has a minimum. This can be illustrated in the case of a massless ($V = 0$) dilaton with quadratic coupling ($a \equiv \ln A = \frac{1}{2}\beta\varphi_*^2$). The Klein-Gordon equation (1.33) can be rewritten in terms of the number of e-folds in Einstein frame as

$$\frac{2}{3-\varphi_*'^2}\varphi_*'' + (1-w)\varphi_*' = -\alpha(\varphi_*)(1-3w). \tag{1.39}$$

As emphasized by Damour and Nordtvedt (1993), this is the equation of motion of a point particle with a velocity dependent inertial mass, $m(\varphi_*) = 2/(3-\varphi_*'^2)$ evolving in a potential $\alpha(\varphi_*)(1-3w)$ and subject to a damping force, $-(1-w)\varphi_*'$. During the cosmological evolution the field is driven toward the minimum of the coupling function. If $\beta > 0$, it drives φ_* toward 0, that is $\alpha \to 0$, so that the scalar-tensor theory becomes closer and closer to general relativity. When $\beta < 0$, the theory is driven away from general relativity and is likely to be incompatible with local tests unless φ_* was initially arbitrarily close to 0.

During the radiation era, $w = \frac{1}{3}$ and the coupling is not efficient so that φ_* freezes to a constant value. Then, during the matter era, the coupling acts as a potential with a minimum in zero, hence driving φ_* towards zero and the theory towards general relativity (see Fig. 1.2).

This offers a rich phenomenology for cosmology and in particular for the dark energy question. It was shown that quintessence models can be extended to scalar-tensor theory of gravity (Uzan, 1999; Bartolo and Pietroni, 2000) and that it offers the possibility to have an equation of state smaller than -1 with a well-defined theory (Martin et al., 2006). The constraints on the deviations from general relativity can also be sharpened by the use of cosmological observations such as cosmic microwave background anisotropies (Riazuelo and Uzan, 2002), weak gravitational lensing (Schimd et al., 2005) and big-bang nucleosynthesis (Coc et al., 2006). Fig. 1.2 summarizes the constraints that can be obtained from primordial nucleosynthesis.

1.2.3.3 Note on $f(R)$ models

As discussed in § 1.2.2.3, the only higher order modifications of the Einstein-Hilbert leading to a well-defined theory are

$$S = \frac{1}{16\pi G_*}\int f(R)\sqrt{-g}\mathrm{d}^4x + S_{\mathrm{matter}}[g_{\mu\nu};\mathrm{matter}]. \tag{1.40}$$

Such a theory leads to the field equations

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} - \nabla_\mu\partial_\nu f'(R) + g_{\mu\nu}(\nabla_\mu\nabla^\mu)f'(R) = 8\pi G_* T_{\mu\nu}, \tag{1.41}$$

where a prime indicates a derivative of the function with respect to its argument, i.e. $f'(R) \equiv \mathrm{d}f/\mathrm{d}R$.

Interestingly, one can show that these theories reduce to a scalar-tensor theory (Gottlöber et al., 1990; Teyssandier and Tourrenc, 1993; Mangano and Sokolowski, 1994; Wands, 1994). To show this, let us introduce an auxiliary field φ and consider the action

$$\begin{aligned} S &= \frac{1}{16\pi G_*}\int \left[f'(\varphi)R + f(\varphi) - \varphi f'(\varphi)\right]\sqrt{-g}\mathrm{d}^4x \\ &\quad + S_{\mathrm{matter}}[g_{\mu\nu};\mathrm{matter}]. \end{aligned} \tag{1.42}$$

The variation of this action with respect to the scalar field indeed implies, if $f''(\varphi) \neq 0$ (The case $f'' = 0$ is equivalent to general relativity with a cosmological constant), that

$$R - \varphi = 0. \tag{1.43}$$

This constraint permits to rewrite Eq. (1.41) in the form

$$f'(\varphi)G_{\mu\nu} - \nabla_\mu\partial_\nu f'(\varphi) + g_{\mu\nu}(\nabla_\mu\nabla^\mu)f'(\varphi) + \frac{1}{2}[\varphi f'(\varphi) - f(\varphi)]g_{\mu\nu} = 8\pi G_* T_{\mu\nu}, \tag{1.44}$$

which then reduces to Eq. (1.32) after the field redeifinitions necessary to shift to the Jordan frame. Note that, even if the action (1.42) does not possess a kinetic term for the scalar field, the theory is well defined since the true spin-0 degree of freedom clearly appears in the Einstein frame, and with a positive energy.

The change of variable (1.28) implies that we can choose $\varphi_* = \frac{\sqrt{3}}{2}\ln f'(\varphi)$ so that the theory in the Einstein frame is defined by

$$A^2 \propto \mathrm{e}^{-\frac{4\varphi_*}{\sqrt{3}}}, \qquad V = \frac{1}{4}\left\{\varphi(\varphi_*)\mathrm{e}^{\frac{2\varphi_*}{\sqrt{3}}} - f[\varphi(\varphi_*)]\right\}\mathrm{e}^{-\frac{4\varphi_*}{\sqrt{3}}}. \tag{1.45}$$

Note that α_0 cannot be made arbitrarily small since the form of the coupling function A arises from the function f. In order to make these models compatible with Solar system constraints, the potential should be such that the scalar field is massive enough, while still being bounded from below.

This example highlights the importance of looking for the true degrees of freedom of the theory. A field redefinition can be a useful tool to show that two theories are actually equivalent. This result was generalized (Wands, 1994) to theories involving $f[R, (\nabla_\mu\nabla^\mu)R, \ldots, (\nabla_\mu\nabla^\mu)^n R]$ which were shown to be equivalent to $(n+1)$ scalar-tensor theories.

This equivalence between $f(R)$ and scalar-tensor theories assumes that the Ricci scalar is a function of the metric and its first derivatives. There is a difference when one considers $f(R)$ theories in the Palatini formalism (Flanagan, 2004), in which the metric and the connections are assumed to be independent fields, since while still being equivalent to scalar-tensor theories, the scalar field does not propagate because it has no kinetic term in the Einstein frame. It thus reduces to a Lagrange parameter whose field equation sets a constraint.

1.2.3.4 Extensions

The previous set-up can easily be extended to include n scalar fields (Damour and Esposito-Farèse, 1992) in which case the kinetic term will contain a $n\times n$ symmetric matrix, $g_*^{\mu\nu}\gamma_{ab}(\varphi_c)\partial_\mu\varphi^a\partial_\nu\varphi^b$.

Another class of models arises when one considers more general kinetic terms of the form $f(s,\varphi)$ where $s = g_*^{\mu\nu}\partial_\mu\varphi\partial_\nu\varphi$. When the coupling function reduces to $A = 1$, these models are known as K-essence (Armendariz-Picon et al., 2000; Chiba et al., 2000). We refer to Esposito-Farèse and Bruneton (2007) and Bruneton (2006) for a discussion of the conditions to be imposed on f in order for such a theory to be well-defined.

1.2.3.5 Reconstructing theories

This section has illustrated the difficulty of modifying consistently general relativity. Let us emphasize that most of the models we discussed contain several free functions and general relativity in some continuous limit. It is clear that most of them cannot be excluded observationally.

It is important to remember that we hope these theories to go beyond a pure description of the data. In particular, it is obvious that the function $E(z)$ defined in Eq. (1.6) for a ΛCDM model can be reproduced by many different models. In particular, one can always design a scalar field model inducing an energy density $\rho_{\rm de}(z)$, obtained from the observed function $H^2(z)$ by subtracting the contributions of the matter we know (i.e. pressureless matter and radiation). Its potential is given by (Uzan, 2007)

$$\begin{aligned} V(a) &= \frac{H(1-X)}{16\pi G}\left(6H + 2aH' - \frac{aHX'}{1-X}\right), \\ Q(a) &= \int \frac{\mathrm{d}\ln a}{\sqrt{8\pi G}}\left[aX' - 2(1-X)a\frac{H'}{H}\right], \end{aligned} \tag{1.46}$$

with $X(a) \equiv 8\pi G\rho_{\rm de}(a)/3H^2(a)$ in order to reproduce$\{H(a), \rho_{\rm de}(a)\}$.

The background dynamics provides only one observable function, namely $H(z)$, so that it can be reproduced by many theories having at least one free function. To go further, we must add independent information, which can be provided e.g. by the growth rate of the large scale structure. An illustrative game was presented in Uzan (2007) in which it was shown that while the background dynamics of the DGP model (Dvali et al. , 2000) can be reproduced by a quintessence model, both models did not share the same growth rate and can be distinguished, in principle, at this level. However, both the background and sub-Hubble perturbation dynamics of the DGP model can be reproduced by a well-defined scalar-tensor theory, which has two arbitrary functions. The only way to distinguish the two models is then to add local information since the scalar-tensor theory that reproduces the cosmological dynamics of the DGP model would induce a time variation of the gravitational constant above acceptable experimental limits.

This shows the limit of the model-dependent approach in which a reconstructed theory could simply be seen as a description of a set of data if its number of free functions is larger than the observable relations provided by the data. The reconstruction method can however lead to interesting conclusions and to the construction of counter-examples. For instance, it was shown (Esposito-Farèse and Polarski, 2001) that a scalar-tensor theory with $V = 0$ cannot reproduce the background dynamics of the ΛCDM.

This should encourage us to consider the simplest possible extension, namely with the minimum number of new degrees of freedom and arbitrary functions. In that sense the ΛCDM model is very economical since it reproduces all observations at the expense of a single new constant.

1.2.4 Beyond the Copernican principle

As explained above, the conclusion that the cosmic expansion is accelerated is deeply related to the Copernican principle. Without such a uniformity principle, the reconstruction of the geometry of our spacetime becomes much more involved.

Indeed, most low redshift observations provide the measurements of some physical quantities (luminosity, size, shape...) as a function of the position on the celestial sphere and the redshift. In any spacetime, the redshift is defined as

$$1 + z = \frac{(u^\mu k_\mu)_{\rm emission}}{(u^\mu k_\mu)_{\rm observation}}, \tag{1.47}$$

where u^μ is the 4-velocity of the cosmic fluid and k^μ the tangent vector to the null geodesic relating the emission and the observation (see Fig. 1.3). The redshift depends on the structure of the past light-cone and thus on the symmetries of the spacetime. It reduces to the simple expression (1.2) only for a Roberston-Walker spacetime. Indeed, it is almost impossible to prove

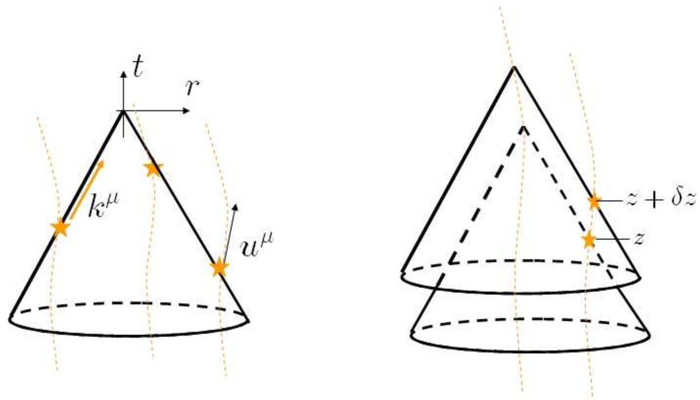

Fig. 1.3. *Left*: Most low-redshift data are localized on our past light-cone. In a non-homogeneous spacetime there is no direct relation between the redshift that is observed and the cosmic time, needed to reconstruct the expansion history. *Right*: The time drift of the redshift allows to extract information about two infinitely close past light-cones. δz depends on the proper motions of the observer and the sources as well as the spacetime geometry.

that a given observational relation, such as the magnitude-redshift relation, is not compatible with an other spacetime geometry.

While isotropy around us seems well established observationally (see e.g. Ruiz-Lapuente, 2007), homogeneity is more difficult to test. The possibility, that we may be living close to the center of a large under-dense region has sparked considerable interest, because such models can successfully match the magnitude-redshift relation of type Ia supernovae without the need to modify general relativity or add dark energy.

In particular, the low redshift (background) observations such as the magnitude-redshift relation can be matched (Célérier, 2000; Tomita, 2001; Iguchi et al. , 2002; Ellis, 2008) by a non-homogeneous spacetime of the Lemaître-Tolman-Bondi (LTB) family, i.e. a spherically symmetric solution of Einstein equations sourced by pressureless matter and no cosmological constant.

The geometry of a LTB spacetime (Lemaître, 1933; Tolman, 1934; Bondi, 1947) is described by the metric

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + S^2(r,t)\mathrm{d}r^2 + R^2(r,t)\mathrm{d}\Omega^2$$

where $S(r,t) = R'/\sqrt{1+2E(r)}$ and $\dot{R}^2 = 2M(r)/R(r,t) + 2E(r)$, using a dot and prime to refer to derivatives with respect to t and r respectively.

The Einstein equations can be solved parametrically as

$$\{R(r,\eta), t(r,\eta)\} = \left\{\frac{M(r)}{\mathcal{E}(r)}\Phi'(\eta), T_0(r) + \frac{M(r)}{[\mathcal{E}(r)]^{3/2}}\,\Phi(\eta)\right\} \tag{1.48}$$

where Φ is defined by $\Phi(\eta) = \big(\sinh\eta - \eta, \eta^3/6, \eta - \sin\eta\big)$, and $\mathcal{E}(r) = (2E, 2, -2E)$ according to whether E is positive, null or negative.

This solution depends on 3 arbitrary functions of r only, $E(r)$, $M(r)$ and $T_0(r)$. Their choice determines the model completely. For instance $(E, M, T_0) = (-K_0 r^2, M_0 r^3, 0)$ corresponds to a Robertson-Walker space-time. One can further use the freedom in the choice of the radial coordinate to fix one of the three functions at will so that one effectively has only 2 arbitrary independent functions.

Let us sketch the reconstruction and use r as the integration coordinate, instead of z. Our past light-cone is defined as $t = \hat{t}(r)$ and we set $\mathcal{R}(r) \equiv R[\hat{t}(r), r]$. The time derivative of R is given by $\dot{R}[\hat{t}(r), r] \equiv \mathcal{R}_1 = \sqrt{2M_0 r^3/\mathcal{R}(r) + 2E(r)}$. Then we get $R'[\hat{t}(r), r] \equiv \mathcal{R}_2(r) = -[\mathcal{R}(r) - 3(\hat{t}(r) - T_0(r))\mathcal{R}_1(r)/2]E'/E - \mathcal{R}_1(r)T_0'(r) + \mathcal{R}(r)/r$. Finally, more algebra leads to $\dot{R}'[\hat{t}(r), r] \equiv \mathcal{R}_3(r) = [\mathcal{R}_1(r) - 3M_0 r^3(\hat{t}(r) - T_0(r))/\mathcal{R}^2(r)]E'(r)/2E(r) + M_0 r^3 T_0'(r)/\mathcal{R}^2 + \mathcal{R}_1(r)/r$. Thus, $\dot{R}$, R' and $\dot{R}'$ evaluated on the light cone are just functions of $\mathcal{R}(r)$, $E(r)$, $T_0(r)$ and their first derivatives. Now, the null geodesic equation gives that

$$\frac{\mathrm{d}\hat{t}}{\mathrm{d}r} = -\frac{\mathcal{R}_2(r)}{\sqrt{1+2E(r)}}, \quad \frac{\mathrm{d}z}{\mathrm{d}r} = \frac{1+z}{\sqrt{1+2E(r)}}\mathcal{R}_3(r),$$

and

$$\frac{\mathrm{d}\mathcal{R}}{\mathrm{d}r} = \left[1 - \frac{\mathcal{R}_1(r)}{\sqrt{1+2E(r)}}\right]\mathcal{R}_2(r).$$

These are 3 first order differential equations relating 5 functions $\mathcal{R}(r)$, $\hat{t}(r)$, $z(r)$ $E(r)$ and $T_0(r)$. To reconstruct the free functions we thus need 2 observational relations. The reconstruction from background data alone is under-determined and one must fix one function by hand. The angular distance-redshift relation, $\mathcal{R}(z) = D_A(z)$, is the obvious choice. This explains why the magnitude-redshift relation can be matched (Célérier, 2000; Tomita, 2001; Iguchi et al. , 2002) by a LTB geometry? Indeed the geometry is not fully reconstructed.

It follows that many issues are left open. First, can we use more observational data to close the reconstruction of the LTB geometry. Indeed the knowledge of the growth rate of the large scale structure could be used, as for the reconstruction of the two arbitrary functions of a scalar-tensor theory but no full investigation of the perturbation theory around a LTB spacetime has been performed (Zibin, 2008; Dunsby and Uzan, 2009). Second, can we construct the model-independent test of the Copernican principle avoiding the necessity to restrict to a given geometry since we may have to consider more complex spacetimes that the LTB one. Third, we would have to understand how these models reproduce the predictions of the standard cosmological model on large scales and at early times, e.g. how are the cosmic microwave background anisotropies and the big-bang nucleosynthesis dependent on these spacetime structures.

1.2.5 Conclusions

This section has investigated two different ways to modify our reference cosmological model by either extending the description of the laws of nature or by extending the complexity of the geometry of our spacetime by relaxing the Copernican principle.

Whatever the choice, we see that many possibilities are left open. All of them introduce new degrees of freedom, either as physical fields or new geometrical freedom, and free functions. They also contain the standard ΛCDM as a continuous limit (e.g. the potential can become flat, the arbitrary functions of a LTB can reduce to their FLRW form etc.) These extensions are thus almost non-excludable by cosmological observations alone and as we have seen, they can reduce to pure descriptions of the data. Again, we must be guided by some principles.

The advantages of the model-dependent approaches is that we know whether we are dealing with well-defined theories or spacetime structures. All cosmological observables can be consistently computed so that these models can be safely compared to observations to quantify how close from a pure ΛCDM the model of our Universe should be. They can also forecast the ability of coming surveys to constrain them.

The drawback is that we cannot test all the possibilities which are too numerous. An alternative is to design parameterizations which have the advantage, we hope, to encompass many models. The problem is then the physical interpretation of the new parameters that are measured from the observations.

Another route, that we shall now investigate, is to design null tests of the ΛCDM model in order to indicate what kind of modifications, if any, are required by the observations.

1.3 Testing the underlying hypotheses

Let us first clarify what we mean by a *null test*. Once the physical theory and the properties of its cosmological solution have been fixed, there exist rigidities between different observable quantities. They reflect the set of assumptions of our reference cosmological models. By testing these rigidities we can strengthen our confidence in the principles on which our model lies. In case we can prove that some of them are violated, it will just give us a hint in the way to extend our cosmological model and on which principle has to be questioned.

Let us take a few examples that will be developed below.

- The equation of state of the dark energy must be $w_{\rm de} = -1$ and constant in time.
- The luminosity and angular distances must be related by the distance duality relation stating that $D_L(z) = (1+z)^2 D_A(z)$.
- On sub-Hubble scales, the gravitational potential and the perturbation of the matter energy density must be related by the Poisson equation, $\Delta\Phi = 4\pi G\rho_{\rm m} a^2\delta_{\rm m}$, which derives from the Einstein equation in the weak field limit.
- On sub-Hubble scales, the background dynamics and the growth of structure are not independent.
- The constants of nature must be strictly constant.

These rigidities are related to different hypotheses, such as the validity of general relativity or Maxwell theory. We shall now describe them and see how they can be implemented with cosmological data.

1.3.1 Testing the Copernican principle

The main difficulty in testing the Copernican principle, as discussed in § 1.2.4, lies in the fact that all observations are located on our past light-cone and that many four-dimensional spacetimes may be compatible with the same three-dimensional light-like slice (Ellis, 1975).

Recently, it was realized that cosmological observations may however provide a test of the Copernican principle (Uzan et al. , 2008b). This test exploits the time drift of the redshift that occurs in any expanding spacetime, as first pointed out in the particular case of Robertson-Walker spacetimes for which it takes the form (Sandage, 1962; McVittie, 1962)

$$\dot{z} = (1+z)H_0 - H(z) \ . \qquad (1.49)$$

Such an observation gives informations on the dynamics outside the past light-cone since it compares the redshift of a given source at two times and thus on two infinitely close past light-cones (see Fig. 1.3-right). It follows that it contains an information about the spacetime structure along the worldlines of the observed sources that must be compatible with the one derived from the data along the past light-cone.

For instance, in a spherically symmetric spacetime, the expression (1.49) depends on the shear, $\sigma(z)$, of the congruence of the wordlines of the comoving observers evaluated along our past light-cone,

$$\dot{z} = (1+z)H_0 - H(z) - \frac{1}{\sqrt{3}}\sigma(z) \ .$$

It follows that, when combined with other distance data, it allows to determine the shear on our past light-cone and we can check whether it is compatible with zero, as expected for any Robertson-Walker spacetime.

In a RW universe, we can go further and determine a consistency relation between several observables. From the metric (1.1), one deduces that $H^{-1}(z) = D'(z)\left[1+\Omega_{K0}H_0^2D^2(z)\right]^{-1/2}$, where a prime stands for ∂_z and $D(z) = D_L(z)/(1+z)$; this relation being independent of the Friedmann equations. It follows that in any Robertson-Walker spacetime the *consistency relation*,

$$1 + \Omega_{K0}H_0^2\left(\frac{D_L(z)}{1+z}\right)^2 - [H_0(1+z) - \dot{z}(z)]^2\left[\frac{\mathrm{d}}{\mathrm{d}z}\left(\frac{D_L(z)}{1+z}\right)\right]^2 = 0,$$

between observables must hold whatever the matter content and the field equations, since it derives from pure kinematical relations that do not rely on the dynamics (a similar analysis is provided in Clarkson et al. , 2008). The measurement of $\dot{z}(z)$ will also allow (Uzan et al. , 2008b) to close the reconstruction of the local geometry of such an under-dense region (as discussed in § 1.2.4).

$\dot{z}(z)$ has a typical amplitude of order $\delta z \sim -5 \times 10^{-10}$ on a time scale of $\delta t = 10$ yr, for a source at redshift $z = 4$. This measurement is challenging, and impossible with present-day facilities. However, it was recently revisited in the context of Extremely Large Telescopes (ELT), arguing they could measure velocity shifts of order $\delta v \sim 1 - 10$ cm/s over a 10 years period from the observation of the Lyman-α forest. It is one of the science drivers in design of the CODEX spectrograph (Pasquini et al. , 2005) for the future European ELT. Indeed, many effects, such as proper motion of the sources, local gravitational potential, or acceleration of the Sun may contribute to the time drift of the redshift. It was shown (Liske et al. , 2008; Uzan et al. ,

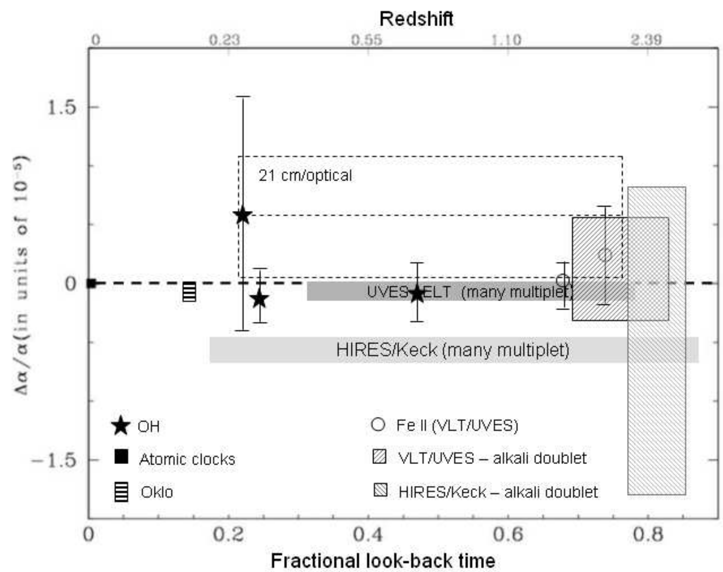

Fig. 1.4. Constraints on the time variation of the fine structure constant α from the observations of quasar absorption spectra.

2008), however, that these contributions can be brought to a 0.1% level so that the cosmological redshift is actually measured.

Let us also stress that another idea was also recently proposed (Goodman, 1995; Caldwell and Stebbins, 2008). It is based on the distortion of the Planck spectrum of the cosmic microwave background.

1.3.2 Testing General relativity on astrophysical scales

1.3.2.1 Test of local position invariance

The local position invariance is one aspect of the Einstein equivalence principle which is at the basis of the hypothesis of metric coupling. It implies that all constants of nature must be strictly constant. The indication that the numerical value of any constant has drifted during the cosmological evolution would be a sign in favor of models of the classes C and D.

The test of the constancy of the fundamental constants has seen a very intense activity in the past decade. In particular the observations from quasar absorption spectra have relaunched a debate on the possible variation of the fine structure constant. Recently it was also argued (Coc et al. , 2007) that a time variation of the Yukawa couplings may allow to solve the lithium-7 problem that, at the moment, has no other physical explanation.

Constraints can be obtained from many physical systems such as atomic clocks ($z = 0$), the Oklo phenomenon ($z \sim 0.14$), the lifetime of unstable nuclei and meteorite data ($z \sim 0.2$), quasar absorption spectra ($z = 0.2-3$), cosmic microwave background ($z \sim 10^3$) and primordial nucleosynthesis ($z \sim 10^8$). The time variation of fundamental constants is also deeply related to the universality of free fall. We refer to Uzan (2003) and (2004) for extensive reviews on the methods and the constraints, which are summarized on Figure 1.4.

In conclusion, we have no compelling evidence for any time variation of a constant, which sets strong constraints on the couplings between the dark energy degrees of freedom and ordinary matter. We can conclude the local position invariance holds in our observable universe and that metric couplings are favored.

1.3.2.2 Test of the Poisson equation

Extracting constraints on deviations from GR is difficult because large scale structures entangle the properties of matter and gravity. On sub-Hubble scales, one can, however, construct tests reproducing those in the Solar system. For instance, light deflection is a test of GR because we can measure independently the deflection angle and the mass of the Sun.

On sub-Hubble scales, relevant for the study of the large-scale structure, the Einstein equations reduce to the Poisson equation

$$\Delta\Psi = 4\pi G\rho_{\mathrm{m}} a^2 \delta_{\mathrm{m}} = \frac{3}{2}\Omega_{\mathrm{m}} H^2 a^2 \delta_{\mathrm{m}}, \tag{1.50}$$

relating the gravitational potential and the matter density contrast.

As first pointed out by Uzan and Bernardeau (2001), this relation can be tested on astrophysical scales, since the gravitational potential and the matter density perturbation can be measured independently from the use of cosmic shear measurements and galaxy catalogs. The test was recently implemented with the CFHTLS-weak lensing data and the SDSS data to conclude that the Poisson equation holds observationally to about 10 Mpc (Doré et al. , 2007).

As an example, Fig. 1.5 depicts the expected modifications of the matter power spectrum and of the gravitational potential power spectrum in the case of a theory in which gravity switches from a standard four-dimensional gravity to a DGP-like five-dimensional gravity above a crossover scale of $r_s = 50h^{-1}$ Mpc. Since gravity becomes weaker on large scales, density fluctuations stop growing, exactly as when the cosmological constant starts dominating. It implies that the density contrast power spectrum differs

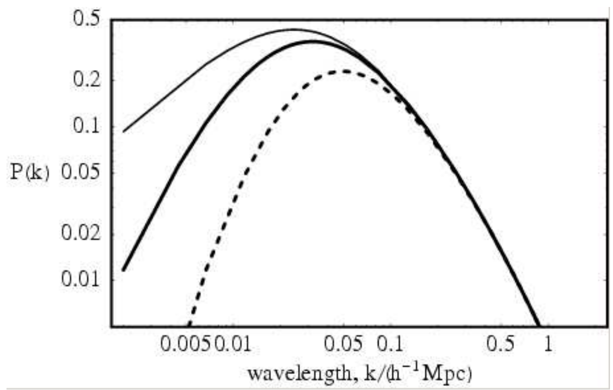

Fig. 1.5. In a theory in which gravity switches from a standard four-dimensional gravity to a DGP-like five-dimensional gravity above a crossover scale of $r_s = 50h^{-1}$ Mpc, there are different cosmological implications concerning the growth of cosmological perturbations. Since gravity becomes weaker on large scales, fluctuations stop growing. It implies that the density contrast power spectrum (thick line) differs from the standard one (thin line) but, more important, from the gravitational potential power spectrum (dash line). From Uzan and Bernardeau (2001).

from the standard one but, more important, from the gravitational potential power spectrum.

Let us emphasize that, the deviation from the standard behavior of the matter power spectrum is model dependent (it depends in particular on the cosmological parameters), but that the discrepancy between the matter and gravitational potential Laplacian power spectra is a direct signature of a modification of general relativity.

The main limitation in the applicability of this test is due to the biasing mechanisms (i.e. the fact that galaxies do not necessarily trace faithfully the matter field) even if it is thought to have no significant scale dependence at such scales.

1.3.2.3 Toward a post-ΛCDM formalism

The former test of the Poisson equation exploits one rigidity of the field equations on sub-Hubble scales. It can be improved by considering the full set of equations.

Assuming that the metric of spacetime takes the form

$$\mathrm{d}s^2 = -(1+2\Phi)\mathrm{d}t^2 + (1-2\Psi)a^2\gamma_{ij}\mathrm{d}x^i\mathrm{d}x^j \tag{1.51}$$

on sub-Hubble scales, the equation of evolution reduces to the continuity equation

$$\delta'_{\mathrm{m}} + \theta_{\mathrm{m}} = 0, \tag{1.52}$$

where θ is the divergence of the velocity perturbation and a prime denotes a derivative with respect to the conformal time, the Euler equation

$$\theta'_{\mathrm{m}} + \mathcal{H}\theta_{\mathrm{m}} = -\Delta\Phi, \tag{1.53}$$

where $\mathcal{H}$ is the comoving Hubble parameter,the Poisson equation (1.50) and

$$\Phi = \Psi. \tag{1.54}$$

These equations imply many relation between the cosmological observables. For instance, decomposing δ_{m} as $D(t)\epsilon(x)$ where ϵ encodes the initial conditions, the growth rate $D(t)$ evolves as

$$\ddot{D} + 2H\dot{D} - 4\pi G\rho_{\mathrm{m}} D = 0.$$

This equation can be rewritten in terms of $p = \ln a$ as time variable (Peter and Uzan, 2005) and considered not as a second order equation for $D(t)$ but as a first order equation for $H^2(a)$

$$(H^2)' + 2\left(\frac{3}{a} + \frac{D''}{D'}\right)H^2 = 3\frac{\Omega_{\mathrm{m}0}H_0^2 D}{a^2 D'}$$

where a prime denotes a derivative with respect to p. It can be integrated as (Chiba and Nakamura, 2007)

$$\frac{H^2(z)}{H_0^2} = 3\Omega_{\mathrm{m}0}\left(\frac{1+z}{D'(z)}\right)^2 \int \frac{D}{1+z}(-D')\mathrm{d}z. \tag{1.55}$$

This exhibits a rigidity between the growth function and the Hubble parameter. In particular the Hubble parameter determined from background data and from perturbation data using Eq. (1.55) must agree. This was used in the analysis of Wang et al. (2007).

Another relation exist between θ_{m} and δ_{m}. The Euler equation implies that

$$\theta_{\mathrm{m}} = -\beta(\Omega_{\mathrm{m}0}, \Omega_{\Lambda 0})\delta_{\mathrm{m}}, \tag{1.56}$$

with

$$\beta(\Omega_{\mathrm{m}0}, \Omega_{\Lambda 0}) \equiv \frac{\mathrm{d}\ln D(a)}{\mathrm{d}\ln a}. \tag{1.57}$$

We conclude that the perturbation variables are not independent and the relation between them are inherited from some assumptions on the dark energy. Phenomenologically, we can generalize Eqs. (1.52-1.54) to

$$\delta'_{\rm m} + \theta_{\rm m} = 0, \tag{1.58}$$

$$\theta'_{\rm m} + \mathcal{H}\theta_{\rm m} = -\Delta\Phi + S_{\rm de}, \tag{1.59}$$

$$-k^2\Phi = 4\pi G F(k,H)\delta_{\rm m} + \Delta_{\rm de}, \tag{1.60}$$

$$\Delta(\Phi - \Psi) = \pi_{\rm de}. \tag{1.61}$$

We assume that there is no production of baryonic matter so that the continuity equation is left unchanged. $S_{\rm de}$ describes the interaction between dark energy and standard matter. $\Delta_{\rm de}$ characterizes the clustering of dark energy, F accounts for a scale dependence of the gravitational interaction and $\pi_{\rm de}$ is an effective anisotropic stress. It is clear that the ΛCDM corresponds to $(F, \pi_{\rm de}, \Delta_{\rm de}, S_{\rm de}) = (1, 0, 0, 0)$. The expression of $(F, \pi_{\rm de}, \Delta_{\rm de}, S_{\rm de})$ for quintessence, scalar-tensor, $f(R)$ and DGP models and more generally for models of the classes A-D can be found in Uzan (2007).

From an observational point of view, weak lensing survey gives access to $\Phi+\Psi$, galaxy maps allow to reconstruct $\delta_g = b\delta_{\rm m}$ where b is the bias, velocity fields give access to θ. In a ΛCDM, the correlations between these observable are not independent since, for instance $\langle\delta_g\delta_g\rangle = b^2\langle\delta^2_{\rm m}\rangle$, $\langle\delta_g\theta_m\rangle = -b\beta\langle\delta^2_{\rm m}\rangle$ and $\langle\delta_g\kappa\rangle = 8\pi G\rho_{\rm m}a^2 b\langle\delta^2_{\rm m}\rangle$.

Various ways of combining these observables have been proposed, construction of efficient estimators and forecast for possible future space mission designed to make these tests as well as the possible limitations (arising e.g. from non-linear bias, the effect of massive neutrinos or the dependence on the initial conditions) are now being extensively studied (Zhang et al. , 2007; Jain and Zhang, 2007; Amendola et al. , 2008; Song and Koyama, 2008).

To finish let us also mention that the analysis of the weakly non-linear dynamics allows to develop complementary tests of the Poisson equation (Bernardeau, 2004) but no full investigation in the framework presented here has been performed yet.

1.3.3 Other possible tests

1.3.3.1 Distance duality

As long as photons travel along null geodesics and the geodesic deviation equation holds, the source angular distance, $r_{\rm s}$, and the observer area distance, $r_{\rm o}$, must be related by the *reciprocity relation* (Ellis, 1971), $r^2_{\rm s} = r^2_{\rm o}(1+z)^2$ regardless of the metric and matter content of the spacetime.

Indeed, the solid angle from the source cannot be measured so that $r_{\rm s}$

is not an observable quantity. But, it can be shown that, if the number of photons is conserved, the source angular distance is related to the luminosity distance, D_L, by the relation $D_L = r_s(1+z)$. It follows that there exist a *distance duality relation* between the luminosity and angular distances,

$$D_L = D_A(1+z)^2. \tag{1.62}$$

This distance duality relation must hold if the reciprocity relation is valid and if the number of photons is conserved. In fact, one can show that in a metric theory of gravitation, if Maxwell equations are valid, then both the reciprocity relation and the area law are satisfied and so is the distance duality relation.

There are many possibilities for one of these conditions to be violated. For instance the non-conservation of the number of photons can arise from absorption by dust, but more exotic models involving photon-axion oscillation in an external magnetic field (Csaki et al., 2002; Deffayet et al., 2002) (class B) can also be a source of violation. More drastic violations would arise from theories in which gravity is not described by a metric theory and in which photons do not follow null geodesic.

A test of this distance duality relies on the X-ray observations and Sunyaev-Zel'dovich (SZ) effect of galaxy clusters (Uzan et al., 2004).

Galaxy clusters are known as the largest gravitationally bound systems in the universe. They contain large quantities of hot and ionized gas which temperatures are typically 10^{7-8} K. The spectral properties of intra-cluster gas show that it radiates through bremsstrahlung in the X-ray domain. Therefore, this gas can modify the cosmic microwave background spectral energy distribution through inverse Compton interaction of photons with free electrons. This is the so-called SZ effect. It induces a decrement in the cosmic microwave background brightness at low frequencies and an increment at high frequencies.

In brief, the method is based on the fact that the cosmic microwave background temperature (i.e. brightness) decrement due to the SZ effect is given by $\Delta T_{\rm SZ} \sim L\overline{n_e T_e}$ where the bar refers to an average over the line of sight and L is the typical size of the line of sight in the cluster. T_e is the electron temperature and n_e the electron density. Besides, the total X-ray surface brightness is given by $S_X \sim \frac{V}{4\pi D_L^2}\overline{n_e n_p T_e^{1/2}}$ where the volume V of the cluster is given in terms of its angular diameter by $V = D_A^2\theta^2 L$. It follows that

$$S_X \sim \frac{\theta^2}{4\pi}\frac{D_A^2}{D_L^2}L\overline{n_e n_p T_e^{1/2}}. \tag{1.63}$$

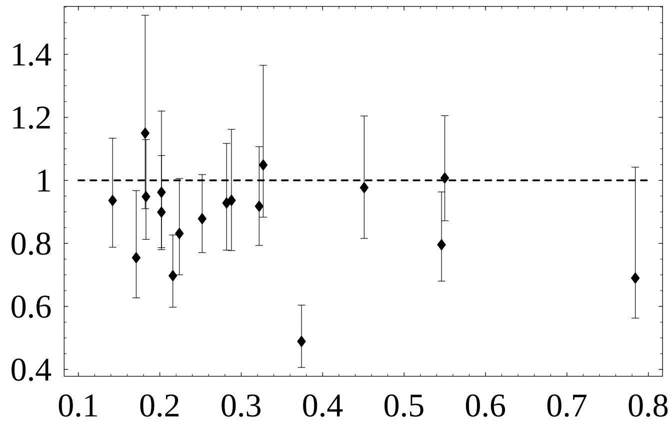

Fig. 1.6. Test of distance duality. The constraint of η defined in Eq. (1.64) at different redshift are obtained by combining SZ and X-ray measurements from 18 clusters. No sign of violation of the distance duality relation is seen. From Uzan et al. (2004).

The usual approach is to assume the distance duality relation so that forming the ratio $\Delta T^2_{\mathrm{SZ}}/S_X$ eliminates n_e.

We can however use these observation to measure

$$\eta(z) = \frac{D_L(z)}{(1+z)^2 D_A(z)} \tag{1.64}$$

and thus test whether $\eta = 1$. Fig. 1.6 summarizes the constraints that have been obtained from the analysis of 18 clusters. No sign of violation of the distance duality relation is seen, contrary to an early claim by Bassett and Kunz (2004).

1.3.3.2 Gravity waves

In models involving two metrics, gravitons and standard matter are coupled to different metrics. It follows that the propagation of gravity waves and light may be different. As a consequence the arrival times of gravity wave and light should not be equal.

An estimation (Kahya and Woodard, 2007) in the case of the TeVeS theory for the supernovae 1987A indicates that light shall arrive days before the gravity waves, which should be easily detectable.

We also emphasize that models in which gravity waves propagate slower than electromagnetic waves are also very constrained by the observations of cosmic rays (Moore and Nelson, 2001) because particles propagating faster than the gravity waves emit gravi-Cerenkov radiation.

These two examples highlight that the cosmological tests of general relativity *do not reduce* to the study of the large scale structures.

1.3.3.3 A word on topology

The debate concerning the topology of our Universe is the continuation of the historically long discussed question on whether our Universe is finite or infinite.

The hypothesis on the global topology does no influence local physics and let most of the theoretical and observational conclusions unchanged. Local geometry has however a deep impact since it sets the topologies that are acceptable. In our cosmological model, the Copernican principle implies that we are dealing with 3-dimensional spaces of constant curvature. Besides, almost spatial flatness limits those topologies that would be detectable (Weeks et al., 2003).

A non-trivial topology would violate global isotropy and let signatures mainly on the statistical isotropy of the cosmic microwave background anisotropies (Gaussman et al., 2001; Luminet et al., 2003, Riazuelo et al., 2004; Riazuelo et al., 2004b; Uzan et al., 2004). The current constraints imply (Shapiro et al., 2007) that the size of the Universe has to be larger than 0.91 times the diameter of the last scattering surface, that is 24 Gpc.

Even though the cosmological constant can be related to a characteristic size of the order of $\Lambda^{-1/2} \sim H_0^{-1}$, no mechanism relating the size of the Universe and the cosmological constant has been constructed (Calder and Lahav (2008) however suggests a possible relation to the Mach principle).

1.4 Conclusion

The acceleration of the cosmic expansion and the understanding of its origin drives us to reconsider the construction of our reference cosmological models.

Three possibilities seem open to us.

- *Stick to the* ΛCDM. The model is well-defined, does not require to extend the low-energy version of the law of nature, and is compatible with all existing data. However, in order to make sense of the cosmological constant and avoid the cosmological constant problem one needs to invoke a very large structure (Weinberg, 1989; Garriga and Vilenkin, 2004; Carr and Ellis, 2008), the multiverse, a collection of universes in which the value of the cosmological constant, as well as those of other physical constants, are randomized in different regions. Such a structure, while advocated on the basis of the string landscape (Suskind, 2006), has no clear

mathematical definition (Ellis et al., 2004) but it aims at suppressing the contingence of our physical models (such as their symmetry groups, value of constants,...that, by construction, cannot be explained by these models) at the price of an anthropic approach which may appear as half-way between pure anthropocentrism, fixing us at the center of the universe, and the cosmological principle, stating that no place can be favored in any way.
In such a situation, it is clear that the Copernican principle holds on the size of the observable universe and even much beyond. However on the scales of the multiverse, it has to be abandoned since, according to this view, we can only live in regions of the multiverse where the value of the cosmological constant is small enough for observers to exist (see Fig. 1.7).

  The alternative would be to better understand the computation of the energy density of the vacuum.

- *Assume* $\Lambda = 0$ and then
  - *Assume no new physics.* In such a case, we must abandon the Copernican principle on the size of the observable universe. This leads us to consider more involved solutions of known and established physical theories. Indeed, the main objection would be to understand why we shall live in such a particular place.
    Note however that the Copernican principle can be restored on much larger scales (i.e. super-Hubble but without the need to invoke a structure like the multiverse). On these scales, one can argue that there shall exist a distribution of over- and under-dense regions of all sizes and density profiles. In this sense, we are just living in one of them, in the same sense that stars are more likely to be in galaxies (see Fig. 1.7) and the Copernican principle seems to be violated on Hubble scales, just because we live in such a structure which happens to have a size comparable to the one of the observable universe.
  - *Invoke new physics.* This an be achieved in numerous ways. The main constraint is to construct a well-defined theory. In such a case the Copernican principle can hold both on the size of the observable universe but also on much larger scales.

At the moment, none of these three possibilities is satisfactory, mainly because it forces us to speculate on scales much beyond those of the observable universe. A last possibility, that was alluded to in § 1.2.2.1, is the possibility that the acceleration is induced by the backreaction of the large scale structures but this still needs in depth investigation. We have argued that future cosmological observations can shed some light on the way to modify our reference cosmological model and extend the tests of the fundamental laws of physics, such as general relativity, as well as some extra-hypotheses such as the Copernican principle and the topology of space. In this sense, we follow the most standard physical approach in which any null test that can be done must be done in order to extend our understanding of the domain of validity of the description of the physical laws we are using.

From an observational point of view, demonstrating a violation of the Copernican principle on the size of the observable universe will indicate that the second solution is the most likely, but nothing forces us to accept the associated larger spacetime described in Fig. 1.7. If any of the tests presented here, or other to be designed, is positive then we will have an indication that the dark energy is not the cosmological constant and on the kind of extension required. The question of why the cosmological constant strictly vanishes will still have to be understood, either from physical ground or by invoking a multiverse-like structure. If all the tests are negative, whatever the precision of the observation, then the ΛCDM will remain a cosmological model on which we will have no handle.

Constructing a cosmological model which will make sense both from physics and the observations still require a lot of work. Many models can save the phenomena but none are based on firm physical grounds. The fact that we have to invoke structures on scales much larger than those than can be probed to make sense of the acceleration of the cosmic expansion may indicate that we may be reaching a limit of what physical cosmology can explain.

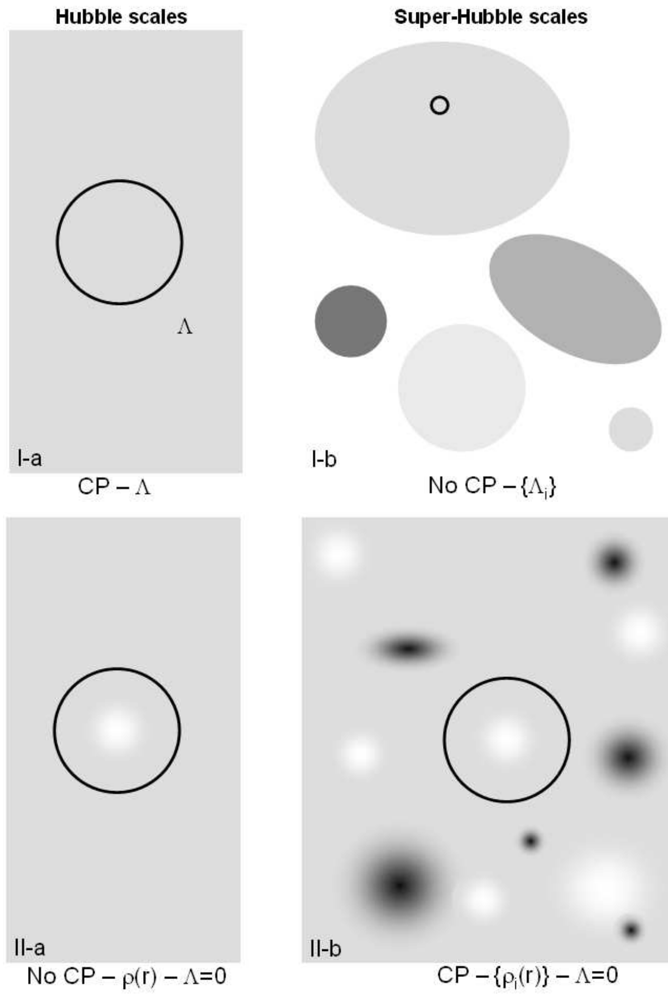

Fig. 1.7. On the scales of the observable universe (circle), the acceleration of the universe can be explained by a cosmological constant (or more generally a dark energy component) in which case the construction of the cosmological model relies on the Copernican principle (upper-left). To make sense of a cosmological constant, one introduces a large structure known as the multiverse (upper-right) which can be seen as a collection of universes of all sizes and in which the values of the cosmological constant, as well as other constants, are randomized. The anthropic principle then states that we observe only those universes where the value of these constants are such that observers can exist. In this sense we have to abandon the Copernican principle on the scales of the multiverse. An alternative is to assume that there is no need for a cosmological constant or new physics, in which case we have to abandon the Copernican principle and assume e.g. that we are living in an under-dense region (lower-left). However, we may recover the Copernican principle on larger scales if there exist a distribution of over- and under-dense regions of all sizes and densities on super-Hubble scales, without the need for a multiverse. In such a view, the Copernican principle will be violated on Hubble scale, just because we live in such a structure which happens to have a size comparable to the one of the observable universe.

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