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CMB observations in LTB universes: Part I. Matching peak positions in the CMB spectrum

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Abstract. Acoustic peaks in the spectrum of the cosmic microwave background in spherically symmetric inhomogeneous cosmological models are studied. At the photon-baryon decoupling epoch, the universe may be assumed to be dominated by non-relativistic matter, and thus we may treat radiation as a test field in the universe filled with dust which is described by the Lemaitre-Tolman-Bondi (LTB) solution. First, we give an LTB model whose distance-redshift relation agrees with that of the concordance Λ CDM model in the whole redshift domain and which is well approximated by the Einstein-de Sitter universe at and before decoupling. We determine the decoupling epoch in this LTB universe by Gamow's criterion and then calculate the positions of acoustic peaks. Thus obtained results are not consistent with the WMAP data. However, we find that one can fit the peak positions by appropriately modifying the LTB model, namely, by allowing the deviation of the distance-redshift relation from that of the concordance Λ CDM model at $z > 2$ where no observational data are available at present. Thus there is still a possibility of explaining the apparent accelerated expansion of the universe by inhomogeneity without resorting to dark energy if we abandon the Copernican principle. Even if we do not take this extreme attitude, it also suggests that local, isotropic inhomogeneities around us may seriously affect the determination of the density contents of the universe unless the possible existence of such inhomogeneities is properly taken into account.

Keywords: supernova type Ia - standard candles, CMBR theory

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1 Introduction

In the study of cosmology, we usually assume that we do not live in a special position in the universe, the so-called Copernican principle. However, although this principle is natural, it should not be blindly assumed but should be justified observationally. Conventionally observational data are interpreted under the assumption of a homogeneous and isotropic universe on average. Therefore it is not clear at all how big the systematic errors would be in the determination of the cosmological parameters if this assumption were abandoned. In other words, it is important to investigate possible “anti-Copernican” models of the universe and test if such models can be observationally excluded. In this paper, as one of such attempts, we try to construct a cosmological model without dark energy which is consistent with the observed distance-redshift relation as well as with the WMAP data.

In anti-Copernican models, we are assumed to be located at a special place in the universe, usually at the center of a spherically symmetric inhomogeneous universe [1–5]. In recent years, such models have attracted much attention [6–24], and various ways to observationally test these models have been proposed by many authors [25–48].

One of the simplest ways to construct an anti-Copernican model is to solve the inverse problem of the distance-redshift relation. Although the isotropy of the universe around us has been confirmed with high accuracy by the observation of the cosmic microwave background (CMB), this does not automatically imply homogeneity of the universe. Thus, in solving the inverse problem, we may assume that the universe is spherically symmetric around us. In addition, we usually assume that the universe is dominated by cold dark matter, that is, by dust. The spherically symmetric dust filled spacetime is described by the Lemaître-Tolman-Bondi (LTB) solution. The LTB solution has three arbitrary functions of the radial coordinate approximately corresponding to the density profile, the spatial curvature and the big-bang time perturbation, with one of them being a gauge degree of freedom representing the choice

of the radial coordinate. These arbitrary functions may be determined by requiring that the resulting LTB universe be consistent with selected important observational data (e.g., the distance-redshift relation). However, we should note that it is not apparent at all if these three functions have enough degrees of freedom to fit all of the important observational data.

In 1999, C el erier solved the inverse problem analytically at small redshifts $z \ll 1$ in the form of the Maclaurin series [5]. Then, in 2002, Iguchi, Nakamura and Nakao constructed numerically an LTB model whose distance redshift relation agrees with that of the Λ CDM model at $z \lesssim 1.6$ [49].¹ However, they could not go beyond $z \sim 1.6$ due to a technical problem. Since then the inverse problem has been discussed by various authors [51–57]. In 2008, Yoo, Kai and Nakao succeeded in constructing an LTB model whose distance-redshift relation agrees with that of the concordance Λ CDM model in the whole redshift domain and which is homogeneous in the early stage of the universe, that is, the big-bang time perturbation being taken to be zero [57]. Note that because of the existence of a gauge degree of freedom in the choice of the radial coordinate, there remains only one functional degree of freedom. In [57], this is represented by the function that determines the spatial curvature.

Not only the distance-redshift relation but also CMB observations in inhomogeneous cosmology have been widely discussed [6, 7, 9, 10, 14–16, 21–24, 33, 36, 37, 58, 59]. In many of these works, it is assumed that the universe is homogeneous in the spacelike asymptotic region from which the CMB photons come, hence the CMB photon distribution is assumed to be the same as that in the homogeneous and isotropic universe models at the time of decoupling. Several authors proposed parametrized LTB models and gave constraints on the parameters with observational data [6, 9–11, 14–16, 21].

Recently, Bolejko and Wyithe [10] suggested that it is possible to construct an LTB model which is consistent not only with the distance-redshift relation but also with the acoustic peak positions of the WMAP data [60] by choosing appropriately the arbitrary functions of the LTB solution. In this paper, by modifying the LTB model given in ref. [57] we show explicitly that this is indeed possible. For simplicity, we use Gamow’s criterion to determine the decoupling epoch instead of invoking a precise numerical calculation. This is the same procedure adopted in ref. [22]. An advantage of this simplified prescription is that it makes it easy to understand the physical degrees of freedom in the LTB universe that determine the CMB anisotropy spectrum. We note that, in our construction, only the asymptotic homogeneity on the past light cone of the observer at the symmetry center was necessary in contrast to a much stronger asymptotic spatial homogeneity condition (see a related discussion in ref. [61]).

This paper is organized as follows. In section 2, we give a brief review of the LTB solution and give a fitting function for the spatial curvature of the LTB model that has the exactly same distance-redshift relation as the concordance Λ CDM model [57]. In section 3, we discuss Gamow’s criterion and the physical degrees of freedom at decoupling. The position of the first acoustic peak in the CMB spectrum in an LTB universe is discussed in section 4. In section 5, we construct an LTB model which is consistent with the acoustic peak positions of the WMAP data by modifying the asymptotic structure of the LTB model obtained in [57]. section 6 is devoted to summary and discussion.

¹Before refs. [5, 49], Mustapha et.al discussed an inverse problem using the LTB solution [50].

2 LTB model from the inverse problem

As mentioned in the introduction, we consider a spherically symmetric inhomogeneous universe filled with dust. This universe is described by an exact solution of the Einstein equations, known as the Lemaître-Tolman-Bondi (LTB) solution. The metric of the LTB solution is given by

$$ds^2 = -c^2 dt^2 + \frac{(\partial_r R(t, r))^2}{1 - k(r)r^2} dr^2 + R^2(t, r) d\Omega^2, \quad (2.1)$$

where $k(r)$ is an arbitrary function of the radial coordinate r . The matter is dust whose stress-energy tensor is given by

$$T^{\mu\nu} = \rho u^\mu u^\nu, \quad (2.2)$$

where $\rho = \rho(t, r)$ is the mass density, and u^a is the four-velocity of the fluid element. The coordinate system in eq. (2.1) is chosen in such a way that $u^\mu = (1, 0, 0, 0)$.

The area radius $R(t, r)$ satisfies one of the Einstein equations,

$$\left(\frac{\partial R}{\partial t}\right)^2 = \frac{2GM(r)}{R} - c^2 k(r)r^2, \quad (2.3)$$

where $M(r)$ is an arbitrary function related to the mass density ρ by

$$\rho(t, r) = \frac{1}{4\pi R^2(t, r)} \frac{dM(r)}{dr}. \quad (2.4)$$

Following ref. [62], we write the solution of eq. (2.3) in the form,

$$R(t, r) = (6GM(r))^{1/3} (t - t_B(r))^{2/3} S(x), \quad (2.5)$$

$$x = c^2 k(r) r^2 \left(\frac{t - t_B(r)}{6GM(r)}\right)^{2/3}, \quad (2.6)$$

where $t_B(r)$ is an arbitrary function which determines the big bang time, and $S(x)$ is a function defined implicitly as

$$S(x) = \begin{cases} \frac{\cosh \sqrt{-\eta} - 1}{6^{1/3} (\sinh \sqrt{-\eta} - \sqrt{-\eta})^{2/3}}; & x = \frac{-(\sinh \sqrt{-\eta} - \sqrt{-\eta})^{2/3}}{6^{2/3}} \text{ for } x < 0, \\ \frac{1 - \cos \sqrt{\eta}}{6^{1/3} (\sqrt{\eta} - \sin \sqrt{\eta})^{2/3}}; & x = \frac{(\sqrt{\eta} - \sin \sqrt{\eta})^{2/3}}{6^{2/3}} \text{ for } x > 0, \end{cases} \quad (2.7)$$

and $S(0) = (3/4)^{1/3}$. The function $S(x)$ is analytic for $x < (\pi/3)^{2/3}$. Some characteristics of the function $S(x)$ are given in refs. [57] and [62].

As shown in the above, the LTB solution has three arbitrary functions, $k(r)$, $M(r)$ and $t_B(r)$. One of them is a gauge degree of freedom for rescaling of the radial coordinate r . In this paper, we fix this by setting

$$M(r) = \frac{4}{3} \pi \rho_0 r^3, \quad (2.8)$$

where ρ_0 is the energy density at the symmetry center at present $\rho_0 = \rho(t_0, 0)$. As in the case of the homogeneous and isotropic universe, the present Hubble parameter H_0 is related to ρ_0 as

$$H_0^2 + k(0)c^2 = \frac{8}{3} \pi G \rho_0. \quad (2.9)$$

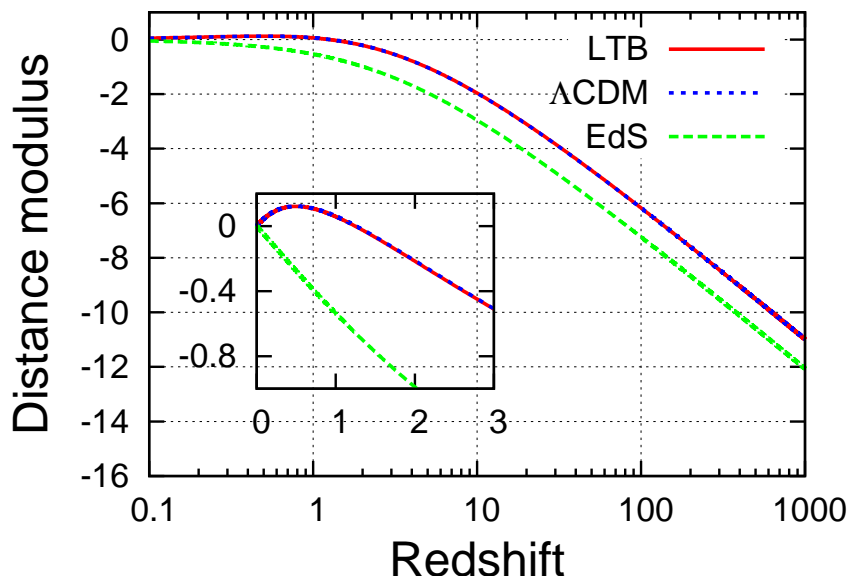


Figure 1. Distance modulus for our LTB model and the concordance Λ CDM model.

As in ref. [57], we assume the simultaneous big bang, i.e.,

$$t_B(r) = 0. \quad (2.10)$$

For notational simplicity, we introduce dimensionless quantities,

$$\tilde{r} := \frac{H_0 r}{c}, \quad \tilde{k}(\tilde{r}) := \frac{k(r)c^2}{H_0^2}.$$

The observed distance-redshift relation is consistent with the homogeneous and isotropic universe model with $(\Omega_{m0}, \Omega_{\Lambda0}) = (0.3, 0.7)$, where Ω_{m0} is the density parameter of the total non-relativistic matter (i.e., cold dark matter plus baryons) and $\Omega_{\Lambda0}$ is that of the cosmological constant, the so-called concordance Λ CDM model. We determine $\tilde{k}(\tilde{r})$ so that the distance-redshift relation of our LTB model agrees with that of the concordance Λ CDM model. In ref. [57], the inverse problem was solved numerically for $\tilde{k}(\tilde{r})$. A fitting function to the numerical result obtained in ref. [57] is given by²

$$\tilde{k}_{\text{fit}}(\tilde{r}) = \frac{0.545745}{0.211472 + \sqrt{0.026176 + \tilde{r}}} - \frac{2.22881}{(0.807782 + \sqrt{0.026176 + \tilde{r}})^2}. \quad (2.11)$$

As shown in figure 1, the distance in our LTB universe model with $\tilde{k}(\tilde{r}) = \tilde{k}_{\text{fit}}(\tilde{r})$ agrees with that in the concordance Λ CDM model in the whole redshift domain.

²It should be noted that the fitting function has a non-vanishing first derivative at the center, corresponding to the existence of a spike in the density profile. In our model the central region has a void-like structure with $\Omega_{m0} \sim 0.09$ at the center. However, the density quickly rises to $\Omega_{m0} \sim 0.2$ at $z = 0.3$ in agreement with large scale structure observations, as shown in ref. [57]. The longitudinal Hubble parameter $H_L := \partial_t \partial_r R / \partial_r R$ in our LTB model is about 10% larger than that in the concordance Λ CDM model at $z \sim 2$. Although we need more careful investigations, considering large error bars and uncertainties in the local Hubble measurements [63], there seems no apparent conflict between our model and observation.

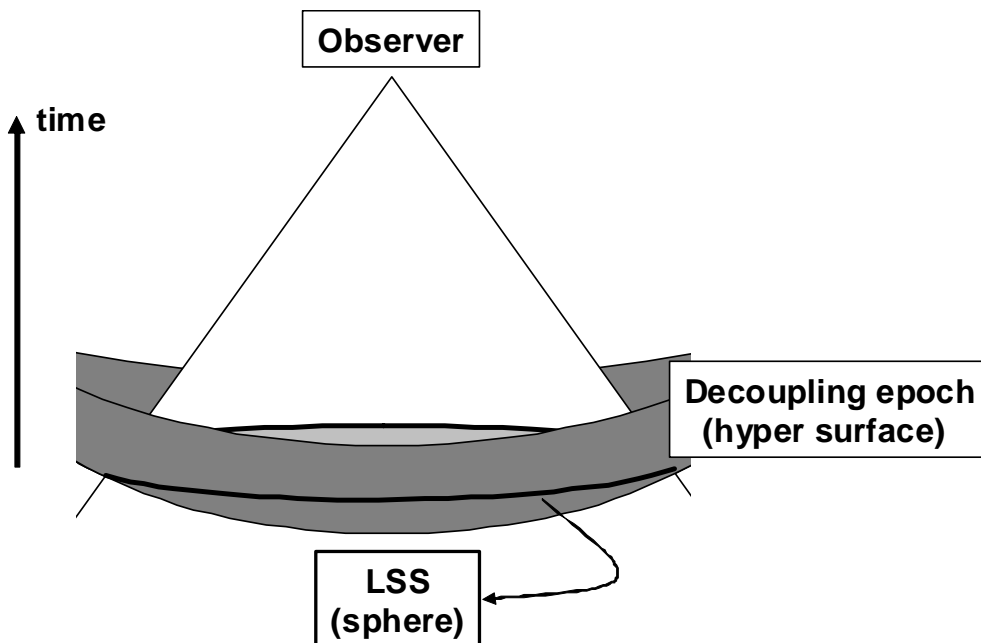


Figure 2. Schematic figure for the hypersurface of the decoupling epoch in an LTB universe.

3 Decoupling epoch in the LTB universe

The decoupling between photons and baryons occurs in an inhomogeneous universe just as in the case of a homogeneous universe. That is, it occurs when the mean free path of photons becomes effectively infinite due to almost complete recombination of electrons to protons.

Since there is no radiation component in our LTB model, we cannot treat the decoupling in a rigorous manner. However, as in the concordance model, we expect the energy density of the radiation to be only a small fraction of the total density at decoupling, hence its effect on the spacetime geometry is small, if not negligible. In fact, the radiation energy density estimated in our LTB model turns out to be about 20% of the total energy density. This means that treating the radiation as a test field in our model is consistent to a first approximation.

Another approximation we adopt is the instantaneous decoupling. Namely, we assume decoupling to occur on a single spacelike hypersurface. Since our LTB universe model is inhomogeneous but spherically symmetric, it is natural to assume that the decoupling hypersurface is also inhomogeneous but spherically symmetric. Thus it is specified by the form,

$$t = t_D(r). \quad (3.1)$$

The cross section of this hypersurface with the past directed null cone from the observer at the center constitutes the last scattering surface (LSS) of CMB photons (see figure 2). The LSS is a spacelike 2-dimensional sphere by the assumed symmetry. We use the subscript $*$ to express physical quantities on the LSS. In our approximation, ignoring secondary effects, the CMB anisotropy is essentially determined by the distribution of photons on the LSS.

The geodesic equations to determine the past light cone from the observer at the center are written in the form,

$$(1+z)\frac{dt}{dz} = -\frac{\partial_r R}{\partial_t \partial_r R}, \quad (3.2)$$

$$(1+z)\frac{dr}{dz} = \frac{c\sqrt{1-k(r)r^2}}{\partial_t \partial_r R}, \quad (3.3)$$

where the past directed radial null geodesics have been parametrized by the cosmological redshift z . We denote the solution of the above equations by

$$t = t_{lc}(z), \quad r = r_{lc}(z). \quad (3.4)$$

Now to discuss the decoupling condition, we consider the matter contents of the universe around the time of decoupling. For simplicity, we consider a universe consists of cold dark matter, protons and electrons, and neutral hydrogen atoms. In particular, we neglect helium. Since the contribution of these other components is not large, this simplification should not lead to a serious error in our analysis. We also assume that, until the decoupling time, photons, electrons and protons are in thermal equilibrium. The energy density of the electrons and photons is negligible, and hence the constituents of our LTB model are cold dark matter and baryons, the latter of which consist of protons and hydrogen atoms. Thus the baryon number density n_b is equal to the total number density of protons and hydrogen atoms, and the electron number density n_e is equal to the proton number density.

In homogeneous and isotropic cosmology, the decoupling time is well determined by Gamow's criterion,

$$H = \Gamma, \quad (3.5)$$

where H is the Hubble parameter and Γ is the rate of collisions of a photon with electrons. Using the Thomson scattering cross section σ_T and the electron number density n_e , Γ is written as

$$\Gamma = cn_e\sigma_T. \quad (3.6)$$

In our LTB model, we also adopt this Gamow's criterion, with the identification of the "Hubble parameter H " with

$$H^2 = \frac{8\pi G}{3}\rho. \quad (3.7)$$

Since, by virtue of eq. (2.10), our LTB universe model will be very similar to the Einstein-de Sitter universe near the decoupling time, the above definition should be accurate enough for the present purpose.

In order to estimate the electron number density, let us consider the ionization rate $X_e := n_e/n_b$. In the thermal equilibrium, the ionization rate X_e satisfies Saha's equation,

$$\frac{1-X_e}{X_e^2} = \frac{4\sqrt{2}\zeta(3)}{\sqrt{\pi}}\eta \left(\frac{k_B T}{m_e c^2}\right)^{3/2} \exp\left(\frac{13.59\text{eV}}{k_B T}\right), \quad (3.8)$$

where $\zeta(x)$ is the zeta function, and T , η , k_B and m_e are the temperature, the baryon-to-photon ratio, the Boltzmann constant and the electron mass, respectively. Since the ionization rate at decoupling drops down to $X_e \sim 10^{-5}$, we approximate the above equation by

$$X_e^2 \simeq \frac{\sqrt{\pi}}{4\sqrt{2}\zeta(3)}\frac{1}{\eta} \left(\frac{k_B T}{m_e c^2}\right)^{-3/2} \exp\left(-\frac{13.59\text{eV}}{k_B T}\right). \quad (3.9)$$

Using this equation, Gamow's criterion (3.5) is rewritten in the form,

$$\eta = \frac{32\sqrt{2\pi}\zeta(3)}{3} \frac{G\rho}{(cn_{\gamma 0}\sigma_T)^2} \left(\frac{k_B T_0}{m_e c^2}\right)^{3/2} \left(\frac{T_0}{T}\right)^{9/2} \exp\left(\frac{13.59\text{eV}}{k_B T}\right), \quad (3.10)$$

where $n_{\gamma 0}$ and $T_0 \simeq 2.725\text{K}$ are the present photon number density and observed CMB temperature, respectively.

Since we assume thermal equilibrium of electrons, protons and photons until the decoupling time $t = t_D(r)$, the physical state of the decoupling hypersurface, which is spherically symmetric, is determined by the distributions of the temperature $T = T_D(r)$, the baryon-to-photon ratio $\eta = \eta_D(r)$, and the matter energy density $\rho = \rho(t_D(r), r)$. For convenience, in place of $\rho(t_D(r), r)$, we introduce the following quantity:

$$\alpha_D(r) := \frac{\rho(t_D(r), r)}{\rho_0} \left(\frac{T_0}{T_D(r)}\right)^3. \quad (3.11)$$

The quantity $\alpha_D(r)$ is proportional to the ratio of the matter density and the photon number density. Then, from eq. (3.10), we obtain

$$\eta_D(r) = \frac{32\sqrt{2\pi}\zeta(3)}{3} \frac{G\rho_0}{(cn_{\gamma 0}\sigma_T)^2} \left(\frac{k_B T_0}{m_e c^2}\right)^{3/2} \alpha_D(r) \left(\frac{T_0}{T_D(r)}\right)^{3/2} \exp\left(\frac{13.59\text{eV}}{k_B T_D(r)}\right). \quad (3.12)$$

We note that once $T_D(r)$ and $\alpha_D(r)$ are given, the hypersurface $t = t_D(r)$ can be obtained from eq. (3.11).

On the LSS, we must have

$$\frac{T_D(r_{1c}(z_*))}{T_0} = \frac{T_*}{T_0} = 1 + z_*. \quad (3.13)$$

Then, if we regard $T_D(r)$, $\eta_D(r)$ and $\alpha_D(r)$ as mutually independent functions, we have one functional condition (3.12) and one boundary condition at $z = z_*$ to constrain these three functions. But these are not enough to determine the decoupling hypersurface, $t = t_D(r)$, through eq. (3.11). We would need two more functional conditions. However, for the present purpose, we do not need to determine the total shape of the decoupling hypersurface, but we only have to know the location of the LSS. Therefore we only need a single condition on the LSS in addition to eqs. (3.12) and (3.13). In this paper, for simplicity, we impose

$$\eta_* = 6.2 \times 10^{-10}. \quad (3.14)$$

We numerically solve the radial null geodesic equations (3.2) and (3.3). At each redshift z , we define T and α by

$$T = (1 + z)T_0, \quad (3.15)$$

$$\alpha = \frac{\rho(t_{1c}(z), r_{1c}(z))}{\rho_0(1 + z)^3}. \quad (3.16)$$

During the numerical integration, we check at each redshift z whether eq. (3.12) is satisfied by $T_D = T$, $\alpha_D = \alpha$ and $\eta_D = \eta_*$. If eq. (3.12) is satisfied, we stop integrating eqs. (3.2)

and (3.3), and identify z , T and α at this moment with z_* , T_* and α_* , respectively. The result we have obtained is

$$\frac{T_*}{T_0} = 1 + z_* \simeq 1129, \quad (3.17)$$

$$\alpha_* \simeq 4.856, \quad (3.18)$$

for our LTB universe model.

In the above discussion, we have focused on only α_* and η_* , and we have given no further constraint on $\alpha_D(r)$ and $\eta_D(r)$. Therefore, we can assume $\alpha_D(r) = \alpha_*$ and $\eta_D(r) = \eta_*$ over the whole decoupling hypersurface. Inhomogeneities of α_D and η_D correspond to an isocurvature perturbation of the matter density and a baryon isocurvature perturbation, respectively. However this assumption might lead to a contradiction to the observational data of the kinematic Sunyaev-Zel'dovich effects as will be discussed in a forthcoming paper. It is also worthy to notice that the anomalous primordial Lithium abundances reported in ref. [64] can be explained by introducing the baryon isocurvature perturbation [22].

4 Acoustic peaks in the CMB spectrum

4.1 Acoustic peaks in the homogeneous and isotropic background

Let us first briefly review the positions of the acoustic peaks in the CMB spectrum in the homogeneous and isotropic background universe. The acoustic peak positions in the CMB spectrum can be written as [65]

$$\ell_m = (m - \phi_m)\pi \frac{d_A}{r_{s*}}, \quad (4.1)$$

where d_A is the angular diameter distance from the LSS to the observer at present, r_{s*} is the radius of the sound horizon at the time of decoupling, and ϕ_m is a small correction to the position of the m -th peak. As explicitly shown in the appendix A, r_{s*} and ϕ_m (for $m = 1, 2, 3$) can be expressed in terms of $\omega_m := \Omega_{m0}h^2$, $\omega_b := \Omega_{b0}h^2$, $\omega_\gamma := \Omega_{\gamma0}h^2$, z_* and the spectral index n_s which characterizes the spectrum of the initial density perturbation, where Ω_{m0} , Ω_{b0} , $\Omega_{\gamma0}$ are the density parameters of the total non-relativistic matter, of baryons and of radiation, respectively, and $h = H_0/(100\text{km/s/Mpc})$.

In order to compare the first acoustic peak in the CMB spectrum predicted by our LTB universe model to the observational results, it is convenient to use the quantity \mathcal{S} defined by [6]

$$\mathcal{S} := \frac{\ell_1}{\ell_1^{\text{WMAP}}}, \quad (4.2)$$

where ℓ_1 is the first acoustic peak in the CMB spectrum and $\ell_1^{\text{WMAP}} = 220.8$ is the mean value of the first peak in the WMAP data [60] (see table 1). Since the 1σ error in the WMAP data is $\Delta\ell_1 = 0.7$, models with $|\mathcal{S} - 1| > 0.0063$ are ruled out at 67% CL.

Before closing this subsection, it may be worthwhile to note the role of the instantaneous decoupling approximation. Its role is to identify a homogeneous universe model which is a good approximation to our universe model around the time of decoupling. Once the parameters of a homogeneous universe model are identified, the positions of the acoustic peaks are calculated from the fitting formula (30) derived from numerical simulations for homogeneous universe models. Hence except for the effect of errors in the estimation of the cosmological parameters, there will be no deviation of the acoustic peaks due to the instantaneous decoupling approximation.

	1st	2nd	3rd
WMAP	220.8 ± 0.7	530.9 ± 3.8	700-1000
Our model	220.8	529.3	782.5

Table 1. second and third peak positions in WMAP data and our LTB model with $A = -1.069$. We have assumed $n_s = 0.963$.

4.2 Position of the first acoustic peak in the LTB model

Because the inhomogeneity in our LTB model consists of only growing modes, the inhomogeneity decreases as we go back to the early stage. Actually, our LTB model can be well approximated by the Einstein-de Sitter (EdS) universe at the decoupling time. In this subsection, we therefore approximate our LTB model at the decoupling epoch by an EdS universe model. Hereafter, we place a bar over a quantity in this EdS universe. We determine it by setting the decoupling temperature \bar{T}_D and the baryon-to-photon ratio $\bar{\eta}$ in this EdS universe model equal to T_* on the LSS in our LTB model and η_* given by eq. (3.14), respectively. Further, we set the present CMB temperature to be equal to T_0 also in the EdS model. These three conditions determine $\bar{\omega}_\gamma$, $\bar{\omega}_b$ and $\bar{\omega}_m$, and the Hubble parameter \bar{H}_0 at present,

$$\bar{H}_0^2 = H_*^2 \left(\frac{T_0}{T_*} \right)^3 = \frac{8\pi G}{3} \rho_* \left(\frac{T_0}{T_*} \right)^3. \quad (4.3)$$

Using the definition of α , we have

$$\bar{H}_0^2 = \frac{8\pi G}{3} \rho_0 \alpha_* = [1 + \tilde{k}(0)] H_0^2 \alpha_* \simeq 0.09120 H_0^2 \alpha_*. \quad (4.4)$$

Since the present CMB temperature is T_0 , we have

$$\bar{\omega}_\gamma = 4.2 \times 10^{-5} \left(\frac{T_0}{2.725\text{K}} \right)^4. \quad (4.5)$$

The quantity $\bar{\omega}_b$ is expressed in terms of the baryon-to-photon ratio as

$$\bar{\omega}_b = \frac{8\pi G}{3\bar{H}_0^2} \eta_* \bar{n}_{\gamma 0} m_p \bar{h}^2 = 3.658 \times 10^7 \eta_* \left(\frac{T_0}{2.725\text{K}} \right)^3, \quad (4.6)$$

where m_p is the proton mass. As for the matter density, we have $\bar{\Omega}_{m0} = 1$ by assumption, hence

$$\bar{\omega}_m = \bar{\Omega}_{m0} \bar{h}^2 = \bar{h}^2 = 0.09120 \alpha_* h^2, \quad (4.7)$$

where we have used eq. (4.4).

By using eqs. (3.14) and (3.18), we can evaluate $\bar{\omega}_\gamma$, $\bar{\omega}_b$ and $\bar{\omega}_m$. Then, substituting $\omega_\gamma = \bar{\omega}_\gamma$, $\omega_b = \bar{\omega}_b$ and $\omega_m = \bar{\omega}_m$ for eqs. (A.11), (A.12), (A.15) and (A.16), we find

$$|\bar{\mathcal{S}} - 1| \simeq 0.075 > 0.0063, \quad (4.8)$$

where we have used eq. (3.17) and set $h = 0.71$ and $T_0 = 2.725\text{K}$. This result implies that our LTB model cannot explain the observed first peak position in the CMB spectrum.

5 Matching peaks in the CMB spectrum

Let us consider if we can resolve the inconsistency of our LTB model with the observed first acoustic peak position in the CMB spectrum. Since SNe observations of the distance-redshift relation are still restricted to relatively low redshifts $z < 2$, the distance-redshift relation may not necessarily agree with that in the concordance Λ CDM model at $z > 2$. Therefore we may modify the curvature function $\tilde{k}(\tilde{r})$ in the domain of large r so that the position of the first peak in an LTB model agrees with the observed one without affecting the distance-redshift relation at $z < 2$.

As soon as we allow modifications of our LTB model at high redshifts, we have too much freedom to fix the model uniquely. Therefore, for simplicity, we consider a single-parameter modification. Namely, we adopt the curvature function,

$$\tilde{k}(\tilde{r}; A) = \tilde{k}_{\text{fit}}(\tilde{r}) \times f(\tilde{r}; A), \quad (5.1)$$

where the function $f(\tilde{r}; A)$ is defined by

$$f(\tilde{r}; A) = \begin{cases} 1 & \text{for } \tilde{r} < 2, \\ 1 + \frac{16A(\tilde{r} - 2)^3(323 - 123\tilde{r} + 12\tilde{r}^2)}{3125} & \text{for } 2 \leq \tilde{r} < 9/2, \\ 1 + A & \text{for } 9/2 \leq \tilde{r}. \end{cases} \quad (5.2)$$

The functions $\tilde{k}(\tilde{r}; A)$ and $f(\tilde{r}; A)$ are depicted in figure 3 for several values of A .

As in the previous section, we adopt the value $\eta_* = 6.2 \times 10^{-10}$. Then we can evaluate $\mathcal{S} - 1$ for each value of A . $\mathcal{S} - 1$ is depicted as a function of A in figure 4. As shown in this figure, \mathcal{S} almost vanishes at $A = -1.069$. This fact means that the modified LTB model with $t_B = 0$, $\tilde{k} = \tilde{k}(\tilde{r}; -1.069)$, $\eta_* = 6.2 \times 10^{-10}$ and $h = 0.71$ is consistent with the observed distance-redshift relation and the position of the first acoustic peak in the CMB spectrum simultaneously. Then we may check if the positions of the second and third peaks are also consistent. As shown in the table 1, they indeed turn out to be consistent with the WMAP data [60]. The distance modulus in this LTB universe model with $A = -1.069$ is shown in figure 5.

The resultant LTB universe model is approximated by an EdS universe model with the Hubble parameter $h \simeq 0.49$ in the spatially asymptotic region. This result is consistent with previous work in the literature [6, 9, 14, 66] in which the possibility for lower Hubble universes to explain the CMB spectrum without dark energy has been pointed out.

6 Summary and discussion

In this paper, we investigated if an LTB universe model can account not only for the observed distance-redshift relation but also for the observed peak positions in the CMB spectrum, assuming that we are at the symmetry center of the universe.

First, we presented an LTB model whose distance-redshift relation is equal to that of the concordance Λ CDM model in the whole redshift domain. This model is expressed in terms of a fitting function for the curvature numerically obtained in ref. [57]. Then, we determined the last scattering surface for photons in this model by numerically integrating the past-directed radial null geodesics emanating from the central observer until Gamow's criterion is satisfied.

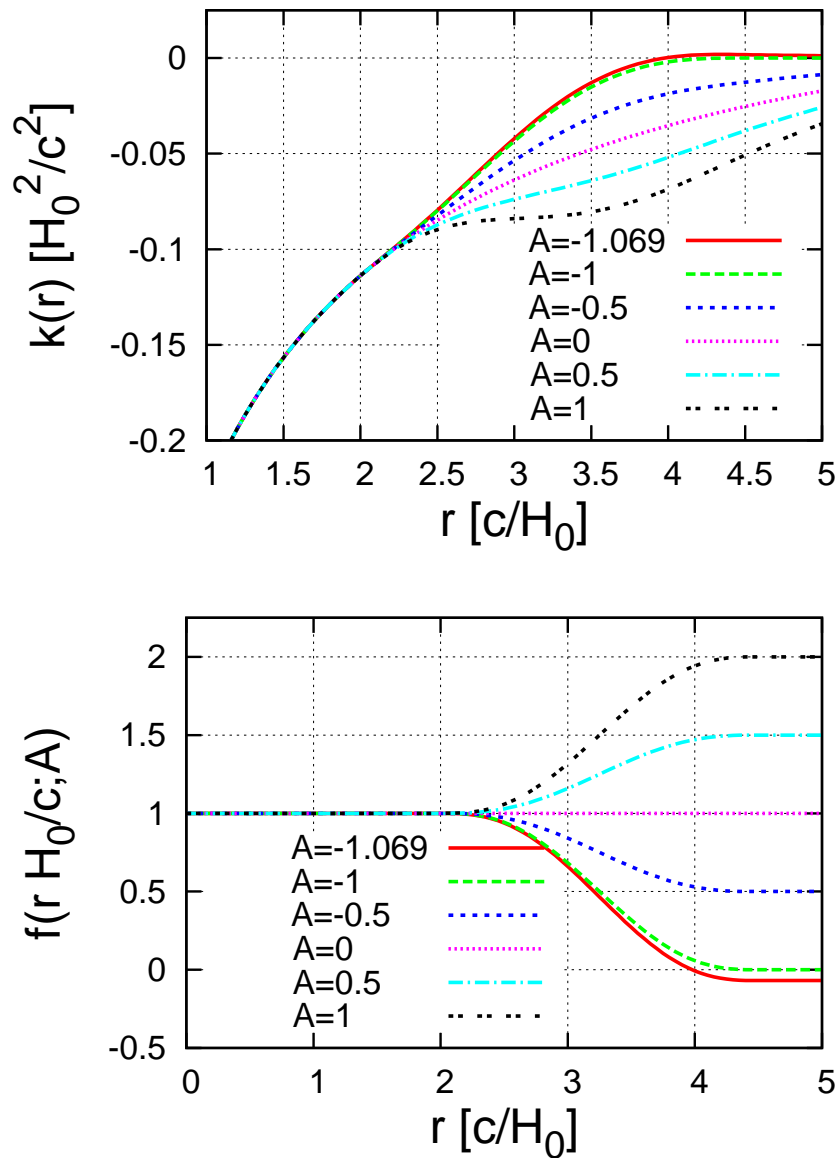


Figure 3. $\tilde{k}(\tilde{r}; A)$ and $f(\tilde{r}; A)$ as functions of \tilde{r} for several values of A .

The metric of this LTB model can be well approximated by the EdS universe at the decoupling epoch. Thus, assuming that our model universe experiences the same history as that of a homogeneous and isotropic universe before decoupling, we evaluated the positions of the acoustic peaks in the CMB spectrum. It was found that thus obtained position of the first acoustic peak deviates from the observed position more than 7%, implying that our LTB universe model whose distance-redshift relation agrees with that of the concordance Λ CDM model for the entire redshift domain is ruled out.

In order to resolve this inconsistency, we considered modifications of the LTB model in the region far from the symmetry center, corresponding to the redshift domain of $z > 2$. Since the observation of Type Ia supernovae are limited within $z < 2$, this modification does

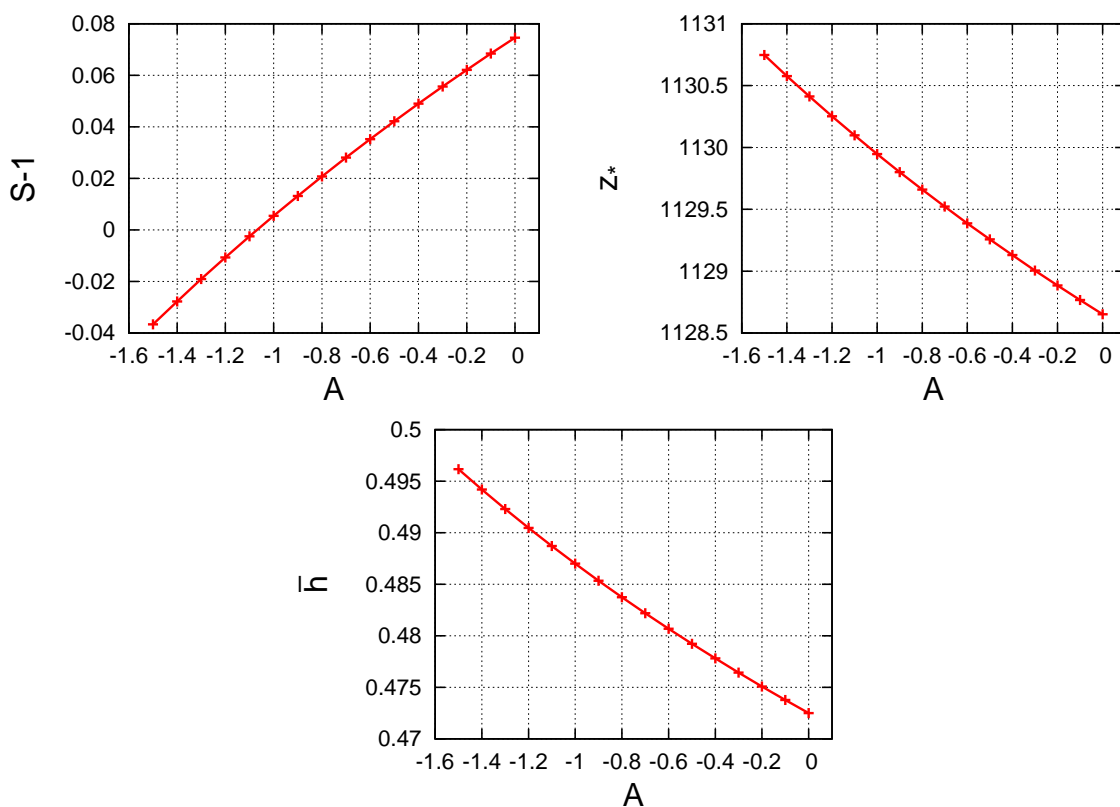


Figure 4. $S - 1$ (left), z_* (center) and \bar{h} (right) as a function of A .

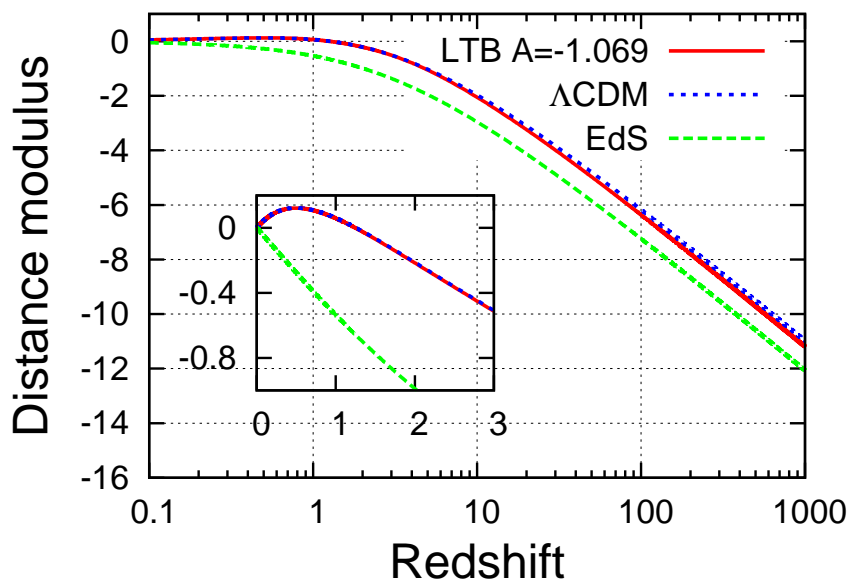


Figure 5. The distance modulus for the LTB universe model with $t_B = 0$, $\tilde{k}(\tilde{r}) = \tilde{k}_{\text{fit}}(\tilde{r})f(\tilde{r}; -1.069)$.

not contradict the current observational data. Then we considered a one-parameter family of LTB models whose distance-redshift relation matches the observational data at $z < 2$, and found a parameter that gives the first acoustic peak position consistent with the observed peak position. Then we checked the second and third peak positions and found that they are also consistent with the WMAP data [60].

We have not discussed relative heights of acoustic peaks in this paper. At the moment, we have no idea if one can also fit them by an LTB model. There is, however, a hint in the literature. In ref. [66], it was reported that the observed CMB spectrum can be regarded as that in the EdS universe with a low value of the Hubble constant if one tunes the primordial power spectrum of the density perturbation. If this is the case, then we may be able to find an LTB model which can account for the observed relative heights of the acoustic peaks. This issue is left for future work.

In this paper, we have not introduced isocurvature components in the inhomogeneity on the last scattering surface. In a forthcoming paper, we plan to investigate if we may construct an LTB model consistent with the observation of the kinematic Sunyaev-Zel'dovich effect. Our preliminary analysis indicates that isocurvature perturbations between the non-relativistic matter and photons are necessary in order to explain it. However, if we introduce large radial isocurvature perturbations on the decoupling hypersurface, the history of the inhomogeneous universe model before decoupling would differ substantially from the conventional adiabatic perturbation scenario in the homogeneous and isotropic background. In such a case, our analysis of the acoustic peaks in the CMB spectrum may be invalidated. However, since we have considered only a single-parameter family of LTB models, while there still remains a functional degree of freedom, it is premature to make any definite statement at the moment. Apparently much more work seems necessary to exclude or select the LTB universe as a viable alternative model of our universe.

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A Peak positions in homogeneous universes

A.1 Expression for the sound horizon

Since the universe considered in this paper is well approximated by the homogeneous and isotropic universe with vanishing spatial curvature near the decoupling epoch, we consider the only such a universe model here. The infinitesimal world interval is given by

$$ds^2 = -c^2 dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2). \quad (\text{A.1})$$

The scale factor $a(t)$ is normalized so that its present value a_0 is unity.

The sound horizon r_s is defined by

$$r_s = \int_0^t \frac{c_s}{a} dt = \int_0^a \frac{c_s}{a^2 H} da, \quad (\text{A.2})$$

where c_s is the sound speed given by

$$c_s = \frac{c}{\sqrt{3(1+R)}}, \quad (\text{A.3})$$

$$R := \frac{3\rho_b}{4\rho_\gamma} = \frac{3\omega_b T_0}{4\omega_\gamma T} = \frac{3\omega_b}{4\omega_\gamma} \frac{1}{1+z}. \quad (\text{A.4})$$

At the matter-radiation equality time, we have

$$R_{\text{eq}} := \frac{3\omega_b T_0}{4\omega_\gamma T_{\text{eq}}} = \frac{3\omega_b}{4\omega_m}, \quad (\text{A.5})$$

where we have used that the temperature T_{eq} at the equality time can be written as

$$T_{\text{eq}} = \frac{\omega_m}{\omega_\gamma} T_0. \quad (\text{A.6})$$

The denominator of the integrand in (A.2) can be written as

$$a^2 H = \sqrt{\Omega_{\text{m}0} a_{\text{eq}} \left(\frac{R}{R_{\text{eq}}} + 1 \right)}, \quad (\text{A.7})$$

where a_{eq} is the scale factor at the equality time, $\Omega_{\text{m}0}$ is the density parameter of the total non-relativistic matter and we have used the relation $R = aR_{\text{eq}}/a_{\text{eq}}$. Using the expressions (A.3) and (A.7), we can perform the integration in (A.2) as

$$r_{s*} = \frac{2}{k_{\text{eq}}} \sqrt{\frac{2}{3R_{\text{eq}}}} \ln \left(\frac{\sqrt{1+R_*} + \sqrt{R_* + R_{\text{eq}}}}{1 + \sqrt{R_{\text{eq}}}} \right), \quad (\text{A.8})$$

where

$$k_{\text{eq}} = \frac{H_0}{c} \sqrt{\frac{2\Omega_{\text{m}0}}{a_{\text{eq}}}} = \frac{H_0}{c} \sqrt{2\Omega_{\text{m}0} \frac{T_{\text{eq}}}{T_0}} = 4.714 \times 10^{-4} \frac{\omega_m}{\sqrt{\omega_\gamma}} h^{1/2} \text{Mpc}^{-1}, \quad (\text{A.9})$$

and

$$R_* = \frac{3\omega_b}{4\omega_\gamma} \frac{1}{1+z_*}. \quad (\text{A.10})$$

A.2 Small shift parameters for the first, second and third peaks

We define the ratio r_* of the radiation density to the matter density at decoupling as

$$r_* = \frac{\rho_{\gamma*}}{\rho_{\text{m}*}} = \frac{\omega_\gamma T_*}{\omega_m T_0} = \frac{\omega_\gamma}{\omega_m} (1+z_*). \quad (\text{A.11})$$

Using this ratio, a fitting formula given in ref. [67] gives

$$\phi_1 = a_1 r_*^{a_2}, \quad (\text{A.12})$$

$$\phi_2 = \phi_1 + \delta\phi_2 = \phi_1 + c_0 - c_1 r_* - c_2 r_*^{-c_3} + 0.05(n_s - 1), \quad (\text{A.13})$$

$$\phi_3 = e_1 r_*^{e_2} - 0.037(n_s - 1), \quad (\text{A.14})$$

where

$$a_1 = 0.286 + 0.626\omega_b, \quad (\text{A.15})$$

$$a_2 = 0.1786 - 6.308\omega_b + 174.9\omega_b^2 - 1168\omega_b, \quad (\text{A.16})$$

$$c_0 = -0.1 + 0.213 \exp(-52\omega_b), \quad (\text{A.17})$$

$$c_1 = 0.063 \exp(-3500\omega_b^2) + 0.015, \quad (\text{A.18})$$

$$c_2 = 6 \times 10^{-6} + 0.137(\omega_b - 0.07)^2, \quad (\text{A.19})$$

$$c_3 = 0.8 + 70\omega_b, \quad (\text{A.20})$$

$$e_1 = 0.302 - 2.112\omega_b + 0.15 \exp(-384\omega_b), \quad (\text{A.21})$$

$$e_2 = -0.04 - 4.5\omega_b. \quad (\text{A.22})$$

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