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Yes, the Sun is located near the corotation circle

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Abstract. The total component velocity field of Cepheids was analysed in terms of a disk galaxy model perturbed by spiral density waves. The main result is: the Sun is situated very close to the corotation resonance where the rotation velocities of the disk and of the spiral pattern coincide. The displacement ΔR of the Sun from the corotation circle is: $\Delta R \approx 0.1$ kpc.

Key words: Galaxy: kinematics and dynamics – Galaxy: structure

1. Introduction

The corotation circle is the circle in a spiral galaxy were the rotation velocity of the galactic disc coincides with that of the spiral pattern (Lin, Yuan & Shu, 1969, hereafter LYS). The question where it is located is not yet solved. LYS suggest that the corotation is situated at the very end of the Galaxy. Marochnik, Mishurov & Suchkov (1972, hereafter MMS) proposed a model in which it lies close to the Sun circle. Independently of MMS, Creze & Mennessier (1973, hereafter CM) analysing the stellar kinematics came to the same conclusion.

In the past years, numerous papers were written to solve this problem. Some of them (Yuan, 1969; Burton, 1971; Lin et al. 1978; Comeron & Torra, 1990 etc) support the model of Lin and his collaborators. However, other investigations (Nelson & Matsuda, 1977; Mishurov et al. 1979, 1997; Fridman et al. 1994 etc) lead to the result of MMS and CM.

In our previous paper (Mishurov et al. 1997, hereafter Paper 1), we considered this task using the statistical method of stellar motion analysis proposed by CM and the new line-of-sight velocities of the classical Cepheids measured by Pont et al (1994) and Gorynya et al (1992, 1996).

It is well known that it is impossible to determine the galactic rotation velocity at the solar circle using only the line-of-sight velocities. Therefore in order to derive the value of the corotation radius in Paper 1, we had to adopt the value of galactic rotation velocity to be equal to the standard one (Kerr & Lynden-Bell, 1986).

The new precise stellar proper motions in the extended solar neighbourhood obtained from HIPPARCOS enable us to self-consistently determine all the basic parameters required to solve the problem under consideration and to answer the question formulated in the title of Paper 1.

2. Statistical method of estimating the parameters

Taking into account the gravitational field of the spiral galactic density waves (GDW) the radial (along the galactocentric radius) and the azimuthal components of the systematic velocities of any star can be represented as $\tilde{v}_R$ and $\Omega R + \tilde{v}_\vartheta$, where Ω is the rotation angular velocity of the disk, $\tilde{v}_R$ and $\tilde{v}_\vartheta$ are perturbations under the GDW, R is the galactocentric distance of the star. Following CM, the line-of-sight velocity of the star (v_r) relative to the Sun and the relative transversal velocity along the galactic longitude (v_l) can be written as:

$$\begin{aligned} v_r \;=\; & \{[-2A + 0.5R_\odot\Omega''_\odot(R-R_\odot)](R-R_\odot)\sin(l) \\ & +\tilde{v}_\vartheta\sin(l+\vartheta) - \tilde{v}_R\cos(l+\vartheta) \\ & +u_\odot\cos(l) - v_\odot\sin(l)\}\cos(b) - w_\odot\sin(b), \qquad (1) \\ v_l \;=\; & -\Omega_\odot r\cos(b) + [-2A + 0.5R_\odot\Omega''_\odot(R-R_\odot)] \\ & \times(R-R_\odot)\cos(l) + \tilde{v}_\vartheta\cos(l+\vartheta) + \tilde{v}_R\sin(l+\vartheta) \\ & -u_\odot\sin(l) - v_\odot\cos(l), \qquad (2) \end{aligned}$$

where $\Omega_\odot$ is the rotation velocity of the Galaxy at the Sun distance (hereafter the subscript "$_\odot$" denotes the values corresponding to the solar coordinates), $A = -0.5R_\odot\Omega'_\odot$ is Oort's A-constant (the prime denotes a derivative with respect to R), R, ϑ, z is the cylindrical coordinate system with the origin at the Galactic center, the z-axis being directed along the axis of the Galactic rotation and $\vartheta_\odot = 0$, l and b are the galactic longitude and latitude of the star, r is the distance from the Sun, $u_\odot$, $v_\odot$ and $w_\odot$ are the components of the solar peculiar velocity.

Up to now, we did not define concretely the nature of perturbations and the analytical representation of $\tilde{v}_R$ and $\tilde{v}_\vartheta$. If one adopts the formalism of the linear density wave theory for spiral arms, the following expressions may be written according to LYS:

$$\tilde{v}_R = f_R\cos(\chi)\;; \qquad \tilde{v}_\vartheta = f_\vartheta\sin(\chi)\;, \qquad (3)$$

where f_R and f_ϑ are the amplitudes of perturbed velocities,

$$\chi = m\big[\cot(i)\ \ln(R/R_\odot) - \vartheta\big] + \chi_\odot \qquad (4)$$

the wave phase, m the number of arms ($m > 0$), and i the pitch angle (for trailing arms $i < 0$). $\chi_\odot$ is the wave phase at the Sun position (the "Sun phase").

For tightly wound spirals ($|i| \ll 1$), f_R, f_ϑ and i are slowly-varying functions of R compared to χ, and we can consider them to be constants.

In the following, the problem breaks into two steps. In the first step by means of statistical analysis of the observed stellar velocity field, one can derive 9 parameters of the model: $\Omega_\odot, A, R_\odot\Omega''_\odot, f_R, f_\vartheta, i, \chi_\odot, u_\odot$ and $v_\odot$ ($w_\odot$ cannot be accurately obtained over the distant stars. We adopted the standard value $7\,\mathrm{km\,s^{-1}}$, CM; Pont et al. 1994). At the second step using the results of the density wave theory (LYS), we compute the dimensionless wave frequency ν, the difference $\Delta\Omega$ between the angular rotation velocity of the spiral pattern Ω_p and $\Omega_\odot$ ($\Delta\Omega = \Omega_p - \Omega_\odot$) and by equating $\Omega(R_c) = \Omega_p$ the displacement ΔR of the Sun relative to the corotation radius R_c ($\Delta R = R_\odot - R_c$; see details in Paper 1 and below).

To solve the statistical part of the task, the weighted least squares method was used (Draper & Smith, 1981; Lewis, 1990). The unknown quantities can be found by minimization of the residual δ^2, where:

$$\delta^2 = \frac{1}{N_r + N_l - p}\left\{\sum^{N_r}(\widehat{v}_r - \widehat{v}^o_r)^2 + \beta^2\sum^{N_l}(\widehat{v}_l - \widehat{v}^o_l)^2\right\}. \tag{5}$$

Here $\widehat{v}_{r,l}$ and $\widehat{v}^o_{r,l}$ are weighted values of theoretical and observed velocity components, the weights being adopted inversely proportionally to the individual errors of measured velocities ε_{v_r}, ε_{v_l} , i.e. $\widehat{v}_{r,l} = v_{r,l}\,/\varepsilon_{v_{r,l}}$, $\widehat{v}^o_{r,l} = v^o_{r,l}\,/\varepsilon_{v_{r,l}}$; $\beta = \sigma_r\,/\sigma_l$, $\sigma_{r,l}$ are dispersions of the weighted velocity components, $N_{r,l}$ are the numbers of data on line-of-sight velocities and proper motions, p is the number of parameters to be derived.

Because σ_r and σ_l are not hitherto known, the parameter β in Eq. (5) is unknown as well. So, the procedure to search a minimum of δ^2 is somewhat changed in comparison with that of Paper 1. Let us consider it briefly. As one can see from Eqs. (1–4), four parameters (f_R, f_ϑ, i and $\chi_\odot$) enter nonlinearly. However, if two of them (i and $\chi_\odot$) are fixed, then for given β over the other 7 parameters δ^2 is linear, and these 7 quantities can be found by means of standard least squares. Hence the strategy of search for the minimum of δ^2 is as follows. For fixed β and any given i and $\chi_\odot$, we find $min\,\delta^2$ over $\Omega_\odot, A, R_\odot\Omega''_\odot$, $f_R, f_\vartheta, u_\odot$ and $v_\odot$. Then for this fixed value of β, we construct the surface $min\,\delta^2$ as a function of two arguments (i, $\chi_\odot$) and look for (i^o, $\chi^o_\odot$) corresponding to the global minimum of δ^2. For this minimum, we calculate σ_r and σ_l and derive the new value for β (see also Lewis, 1990). Further, for new β we again construct the surface $min\,\delta^2$ as a function of i and $\chi_\odot$, look for the global minimum and so on. The procedure is repeated until the values of β converge.

The described procedure enables one to see the topography of the surface $min\,\delta^2$, and this increases the relability of calculations.

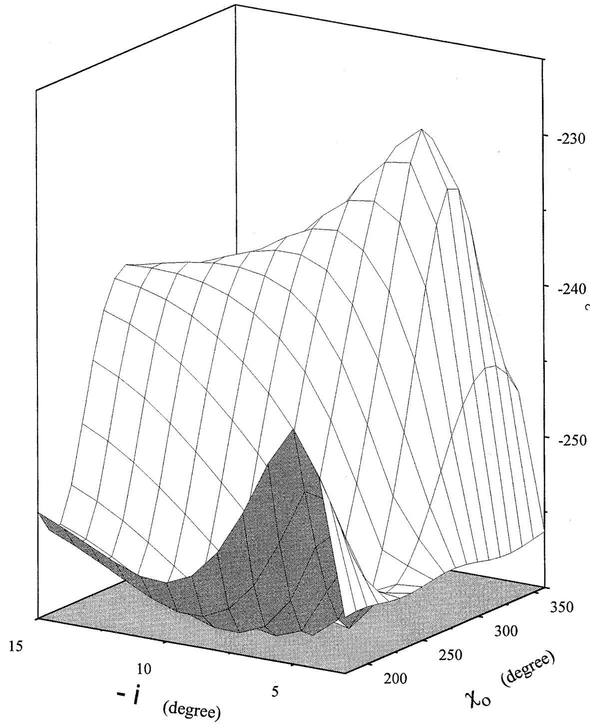

Fig. 1. The surface $-\min\delta^2$ versus pitch angle i and Sun phase $\chi_\odot$ (for data of run 3).

After the global minimum of δ^2 is localised the values of all 10 parameters are defined more exactly by an iterative procedure (Draper & Smith, 1981).

3. Observational data

The classical Cepheids are the most convenient objects for our aims. Because of high luminosities they, are seen at large distances from the Sun, comparable with the interarm distance, and have the most accurate distance scale. This is why the Cepheids attract our attention.

As observational material, we used the line-of-sight velocities from Pont et al (1994), Gorynya et al (1996), Caldwell & Coulson (1987), the proper motions from Hipparcos catalogue (ESA, 1997) and the distances according to Berdnikov & Efremov (1985; see also Dambis et al. 1995). As in Paper 1, we also adopt the limitations on the z-coordinate of a star $|z| \leq 0.5\,\mathrm{kpc}$ (Pont et al. 1994; Lewis 1990) and period pulsation $P < 9^d$. The binary systems and stars with proper motions more than 200 km s^{-1} were excluded from our sample as well. In all samples, there are 131 values for v_r and 117 for v_l. Stars mainly appear to be situated in the region $r \leq 4\,\mathrm{kpc}$ (10 stars are at distances between $4\,\mathrm{kpc}$ and $5\,\mathrm{kpc}$).

$R_\odot \approx 7.5 \pm 1\,\mathrm{kpc}$ was used as a standard (Nikiforov & Petrovskaya, 1994). The results change slightly with variation of $R_\odot$.

Table 1. The parameters and their errors (the bottom lines) derived by means of statistical analysis.

No run	r_m kpc	m	number of data v_r/v_l	$\Omega_\odot$ $(\frac{km}{s\,kpc})$	A $(\frac{km}{s\,kpc})$	$R_\odot\Omega''_\odot$ $(\frac{km}{s\,kpc^2})$	$u_\odot$ $(\frac{km}{s})$	$v_\odot$ $(\frac{km}{s})$	$\|i\|$ $(^\circ)$	$\chi_\odot$ $(^\circ)$	f_R $(\frac{km}{s})$	f_ϑ $(\frac{km}{s})$	β	min δ^2
1	1.	2	126/20	27.0	21.0	15.3	−12.2	15.7	6.8	317.	5.0	−9.1	2.42	168.
				±5.4	±1.2	±2.7	±1.5	±1.5	6.2 ÷ 7.6	±8.	±1.9	±2.0		
2	1.5	2	126/38	27.0	20.2	13.4	−11.0	15.1	6.2	319.	3.6	−8.8	3.89	176.
				±2.8	±1.2	±2.6	±1.4	±1.4	5.6 ÷ 6.9	±8.	±1.8	±1.9		
3	2.	2	128/63	27.2	18.8	10.8	−7.8	13.6	6.0	322.	3.3	−7.9	6.58	227.
				±1.7	±1.3	±2.8	±1.3	±1.4	5.3 ÷ 6.8	±9.	±1.6	±2.0		
4	2.5	2	128/74	27.3	18.5	10.6	−7.8	13.3	5.8	321.	3.0	−7.7	7.04	229.
				±1.5	±1.2	±2.7	±1.3	±1.3	5.2 ÷ 6.6	±9.	±1.6	±2.0		
5	3.	2	128/85	27.7	18.3	10.6	−7.8	13.1	5.7	321.	2.8	−7.4	7.30	233.
				±1.4	±1.2	±2.7	±1.3	±1.3	5.1 ÷ 6.5	±9.	±1.6	±1.9		
6	3.5	2	128/90	27.7	18.3	10.5	−7.8	13.1	5.7	321.	2.8	−7.4	7.46	229.
				±1.4	±1.2	±2.6	±1.2	±1.3	5.1 ÷ 6.4	±9.	±1.6	±1.9		
7	2.	pure	128/63	27.9	17.3	6.9	−9.0	11.6	−	−	−	−	7.38	260.
		rotation		±1.7	±0.8	±1.8	±1.2	±1.1						
8	2.	4	128/63	27.3	19.0	10.5	−8.2	12.8	11.4	340.	3.5	−7.5	6.67	221.
				±1.7	±1.2	±2.5	±1.3	±1.2	9.3 ÷ 14.7	±9.	±1.7	±1.8		

Table 2. The correlation matrix

	A	$R_\odot\Omega''_\odot$	f_R	f_ϑ	cot i	$\chi_\odot$	$\Omega_\odot$	$R_\odot$
A	1.00	.82	−.12	−.60	.67	.17	.17	.00
$R_\odot\Omega''_\odot$	.82	1.00	−.09	−.59	.66	.00	.06	.00
f_R	−.12	−.09	1.00	.07	.17	.17	−.16	.00
f_ϑ	−.60	−.59	.07	1.00	−.33	−.18	.06	.00
cot i	.67	.66	.17	−.33	1.00	.18	−.07	.00
$\chi_\odot$	.17	.00	.17	−.18	.18	1.00	−.17	.00
$\Omega_\odot$	.17	.06	−.16	.06	−.07	−.17	1.00	.00
$R_\odot$	.00	.00	.00	.00	.00	.00	.00	1.00

4. Results

Let us first suppose that the Galaxy has a two-armed pattern ($m = 2$) and consider the statistical part of our task. Table 1 gives the results of computation of the unknown parameters for this case[1]. To analyse the effects of systematic errors in proper motions, stars were split into 6 groups. Each group includes proper motions only for those stars whose distances from the Sun satisfy the condition $r \le r_m$, where r_m was chosen as in Table 1 (runs No 1-6). However, we did not split the data on line-of-sight velocities, and in all runs we used all these data. One can see from Table 1 that results change slightly with increasing r_m. As optimal, we adopt run 3 ($r_m = 2$ kpc). Fig 1 show the surface of residual $min\delta^2$ as a function of two arguments i and $\chi_\odot$, computed over stellar samples of run 3. Here one can see the clear global minimum, which enables to derive the unknown quantities reliably.

[1] Notice here that during calculations we checked the deviations of stellar velocities from the model by means of the "3D-rule":stars with $|\,\widehat{v} - \widehat{v}^o| \ge 3\,min\delta$ were excluded from the sample.

Table 2 gives the correlation matrix of errors for the sought-for parameters. We need this matrix in the following.

Comparing the data of Table 1 with those derived in Paper 1, note that the Sun phase $\chi_\odot$ has increased about 30°. So the Sun happens to be closer to the potential well bottom of the spiral galactic density wave gravitational field. Generally speaking, this does not mean that the Sun is situated in the center of a "visible" arm, since it is known that the tracers of the spiral structure are displaced relative to $min\varphi_S$ (φ_S is the spiral gravitational potential, Roberts, 1969). However, such value for $\chi_\odot$ seems to be too large. Apparently it is biased due to closeness of the Sun to the corotation.

In order to understand the effects of perturbation due to spiral arms, we also give in Table 1 the parameters of galactic rotation derived in approximation of pure rotation (run 7) determined over the same stars as in run 3. The second derivative ($R_\odot\Omega''_\odot$) appreciably was changed whereas the rotation velocity and Oort's A-constant changed only slightly.

The perturbation from the spiral wave happens to be statistically significant. Indeed, the F-statistic of the zero hypothesis

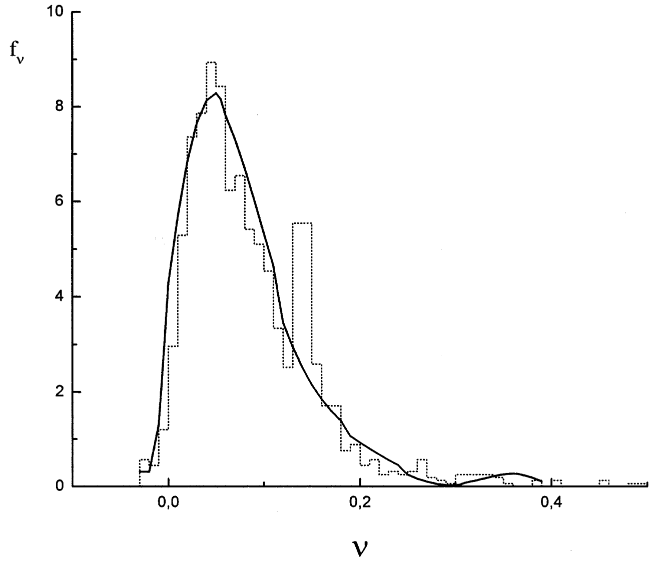

Fig. 2. The "experimental" distribution function for $N = 1500$ (dotted line – histogram) and its approximation by polgnomial (solid line) of dimensionless frequence ν.

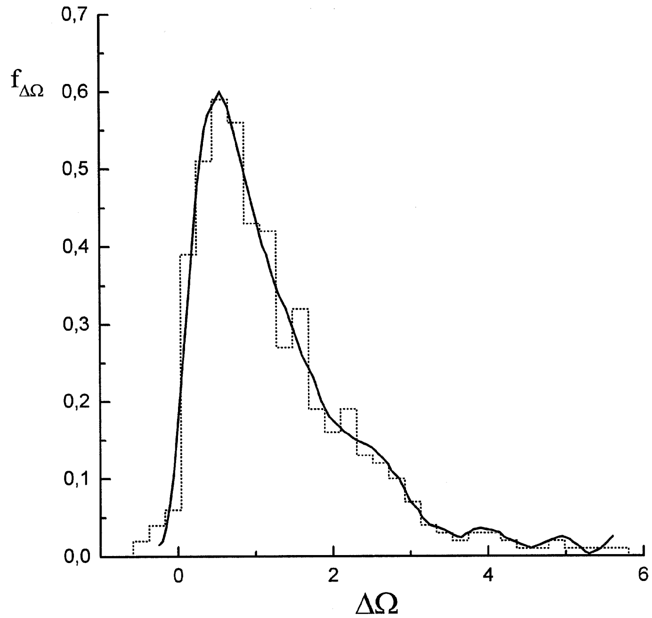

Fig. 3. The same as Fig. 2, but for difference $\Delta\Omega = \Omega_p - \Omega_\odot$.

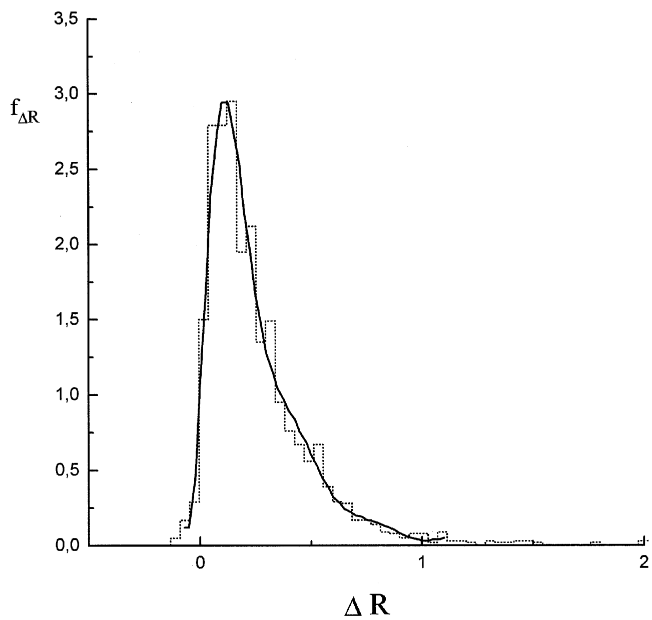

Fig. 4. The same as Fig. 2, but for the displacement of the Sun from the corotation circle $\Delta R = R_\odot - R_c$.

$f_R = f_\vartheta = 0$ against the hypothesis of the inclusion of motions caused by a spiral density wave is equal to $F \approx 17.2$ whereas the critical value of F with 1%-risk level is $F_{cr} \approx 4.7$ (Draper & Smith, 1981). Because $F > F_{cr}$, the hypothesis of the pure rotation should be rejected.

A number of authors (e.g. Georgelin & Georgelin, 1976, Malahova & Petrovskaya, 1992, Vallee, 1995 etc) assume that the Galaxy has a 4-arm pattern[2]. We tried to analyse whether we can make a choice between the 2-arm or the 4-arm models. The results for a 4-arm pattern derived over stars from run 3 are given in Table 1 (run 8). One can see that the value of residual varied insignificantly. Hence we cannot make a choce between these two models. Other parameters varied slightly as well, excepting for the pitch angle: $|i|$ increased approximately twice.

This result seems to be quite clear. Indeed, from Eqs. (3, 4) one could hope to determine m and $\cot\ i$ separately. But two circumstances prevent this. First, because of the tight wind of spirals ($|\cot\ i| \gg 1$), the wave phase varies mainly across an arm, i.e. approximately along R and slowly along ϑ. Second, stars of the sample are confined to a narrow sector $\vartheta : |\vartheta| \leq 26°$. Therefore, the most sensitive changes in χ are connected with the first term in Eq. (4). Hence, dealing with these stars, we can hope to reliably derive only the product $m \cot\ i$. So a double increase of m will lead to a decrease of $\cot\ i$ approximatly the same times.

The procedure of calculation $\Delta\Omega$ and ΔR was described in detail in Paper 1. Unlike Paper 1, we can now estimate both $\Omega_\odot$ and the dispersion of radial (galactocentric) velocity for this sample of Cepheids. For this aim, we select the stars from run 3 which all have measured velocity components (in all 59 stars). The dispersion happens to be $c_\mathrm{R} \approx 15.4\,\mathrm{km\,s^{-1}}$.

The displacement of the Sun from the corotation radius ΔR depends nonlinearly on 8 parameters: $\Omega_\odot, A, R_\odot\Omega''_\odot$, f_R, $f_\vartheta, i, R_\odot$, and c_R. There is no problem to compute it for any set of these parameters. However some problem arises when one tries to evaluate the errors of ΔR because extremely difficult formulas. Therefore to estimate $\nu, \Delta\Omega, \Delta R$ and the bounds of the above quantities variations (in other words, the errors), we use a method of numerical experiments. Briefly the idea is as follows. By means of a random number generator, we construct a

[2] It is interesting to note that Bash (1981) took into account the perturbations from the spiral arms and for Georgelin's data derived the 2-arm pattern.

sample of normally-distributed 7 quantities (in all $7 \times N$, where N is the volume of the sample for any of these 7 quantities, N being a large number) $\alpha_{\Omega_\odot}$, α_A, $\alpha_{R_\odot \Omega''_\odot}$, α_{f_R}, α_{f_ϑ}, $\alpha_{\cot\, i}$, $\alpha_{R_\odot}$ with corresponding average values $\Omega_\odot$, A, $R_\odot \Omega''_\odot$, f_R, f_ϑ, $\cot\, i$, $R_\odot$ from Table 1 and the correlation matrix from Table 2 (we assume that correlation of $R_\odot$ with other quantities equals to zero because as it was mentioned above the results depend slightly on $R_\odot$). Then for each realisation of this 7-dimensional vector α, we compute $\nu, \Delta\Omega$ and ΔR and construct the distribution functions of the above quantities. The corresponding histograms are shown in Figs. 2–4. Approximation of these histograms allows to derive the most probable values and errors. The most probable values are: $\nu \approx 0.048$; $\Delta\Omega \approx 0.5\,\mathrm{km\,s^{-1}kpc^{-1}}$, $\Delta R \approx 0.1$ kpc. The bound of variations are: for $\nu : 0.024 \div 0.134$; for $\Delta\Omega : 0.4 \div 2.2\,\mathrm{km\,s^{-1}kpc^{-1}}$, for $\Delta R : 0\,.07 \div 0.4$ kpc.

Hence the Sun is very close to the corotation circle.

5. Conclusions

The complete component velocity field of the Cepheids was analysed in terms of a disk galaxy model perturbed by spiral density waves. The kinematics of short period Cepheids (pulsation period $P < 9^d$) supports the model of Marochnik et al (1972) and Creze & Mennessier (1973) that the Sun is very close to the corotation resonance: the displacement of the Sun from the corotation circle $\Delta R = R_\odot - R_c \approx 0.1$ kpc. This is the main result of the paper.

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References

Berdnikov L.N., Efremov Yu.N., 1985, Astron. Zirc.(Soviet) N1388, 1
Burton W.B., 1971, A&A 10, 76
Bash F.N., 1981, ApJ 250, 551
Comeron F., Torra J., 1990, A&A 241, 57
Caldwell J.A.R., Coulson I.M., 1987, AJ 93, 1090
Creze M., Mennessier M.O. 1973, A&A 27, 281
Dambis A.K., Mel’nik A.M., Rastorguev A.S., 1995, Astron. Letters (Soviet) 21, 291
Draper N.R., Smith H., 1981, Applied Regression Analysis. John Wiley and Sons. Inc., New York – Chichester – Brisbane – Toronto – Singapore
Fridman A.M., Khoruzhii O.V., Lyakhovich V.V., Avedisova V.S. Are There Giant Vortices in Solar Vicinity? In: van Woerden H. (ed.) Abstracts of Poster papers for the XXII General Assembly of IAU. Den Haag, the Netherlands, 15-27 August 1994, p. 167
ESA, 1997, The Hipparcos Catalogue. ESA, SP-1200
Georgelin Y.M., Georgelin Y.P., 1976, A&A 49, 57
Gorynya N.A., Samus N.N., Rastorguev A.S., Sachkov M.E., 1996, Astron. Letters (Soviet) 22, 198.
Kerr F.J., Linden-Bell D., 1986, MNRAS 221, 1023
Lewis J.R., 1990, MNRAS 244, 247
Lin C.C., Yuan C., Shu F.H., 1969, ApJ 155, 721
Lin C.C., Yuan C., Roberts W.W., 1978, A&A 69, 181
Malahova Yu.N., Petrovskaya I.V., 1992, A&A Transact 1, 221
Marochnik L.S., Mishurov Yu.N., Suchkov A.A., 1972, Ap&SS 19, 285
Mishurov Yu.N., Zenina I.A., Dambis A.K., Mel’nik A.M., Rastorguev A.S., 1997, A&A 323, 775
Mishurov Yu.N., Pavlovskaya E.D., Suchkov A.A., 1979, Astron. Zhurnal (Soviet) 56, 268
Nelson A.H., Matsuda T., 1977, MNRAS 179, 663
Nikiforov I.I., Petrovskaya I.V., 1994, Astron. Zhurnal (Soviet) 71, 725
Pont F., Mayor M., Burki G., 1994, A&A 285, 415
Roberts W.W., 1969, ApJ 158, 123
Vallee J.P., 1995, ApJ 454, 119
Yuan C., 1969, ApJ 158, 889