Fitzgerald, 'A model', 142–51, quoted passage on 151. Cf. Fitzgerald, 'Electromagnetic radiation', 270–1.

58 Thomson, 'Propagation of laminar motion', quoted passage on 352 (Thomson's italics); Thompson, *Kelvin,* 1043, 1046–7, quoted passage on 1043.

59 Thompson, *Kelvin,* 1044–6, quoted passages on 1044–5. See also W. Thomson, 'On a gyrostatic adynamic constitution for "ether" ' (1889–90), in Thomson, *Mathematical and physical papers,* 3:466–72; W. Thomson, 'Ether, electricity, and ponderable matter' (1889), in ibid., 484–511, esp. 500–11; W. Thomson, 'Steps toward a kinetic theory of matter' (1889), in W. Thomson, *Popular lectures and addresses,* 3 vols. (London and New York, 1889–91), 1:225–59, esp. 242–50; Whittaker (1951), 145; Schaffner (1972), 68–75, 194–203.

60 A. E. Woodruff, article "Joseph Larmor" in Dictionary of scientific biography; J. Larmor, 'A dynamical theory of the electric and luminiferous medium (*abstract of memoir following: and general discussion*)' (1893), in Larmor, *Papers,* 1:389–413, quoted passages on 389. Cf. J. Larmor, 'A dynamical theory of the electric and luminiferous medium: part I' (1894), in ibid., 414–535, on 414–15. For a somewhat different view of Larmor in particular and British ether theory in general, see Doran (1975).

61 Larmor, 'Dynamical theory (*abstract*)', 389–90. Cf. Larmor, 'Dynamical theory: part I', 417.

62 Larmor, 'Dynamical theory: part I', 514 ff.

63 Cf., e.g., Doran (1975); Schofield (1970); Hesse (1961), 126–225; Buchwald (1977), 134–6; B. Stewart and P. G. Tait, *The unseen universe; or, physical speculations on a future state,* 7th ed., (London, 1886); Heimann (1972); Wilson (1971), esp. 34–41.

9

German concepts of force, energy, and the electromagnetic ether: 1845–1880

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The goal of unity in nature

The notion that there should exist a single pervasive ether uniting all natural phenomena was not new in the middle of the nineteenth century. Descartes, Leibniz, and Kant long before had provided respectability for the idea of primitive matter, *Urstoff,* or the *Weltäther* as a possible ground for systematic natural philosophy. Their views may be taken to stand as well for the ultimate goals of some practising scientists of the eighteenth and early nineteenth centuries, as indicated for the British in Chapter 1 of this book. Nevertheless, precise description of empirically distinguishable phenomena, such as electricity, magnetism, and heat, had generated more specific referents than simply 'ether'. Many ethereal media – electric, magnetic, caloric, luminiferous, and gravitational – populated different regions of specialised investigation. Not until the late 1840s did the general philosophical and the specialised claims find a common foundation acceptable across the broad spectrum of physical scientists. And only then did 'ether' turn into a generally recognisable object of research.

No doubt a major factor in the acceptance of a single *Weltäther* was the explanatory power of the wave theory of light, which seemed to require a medium throughout space. By itself, however, the wave theory was too limited in its range of applications to compel general assent to, or even general concern with, a role for ether throughout nature. That situation changed rapidly around midcentury when the wave theory of light was coupled to the theory of conservation of force and the mechanical theory of heat. This newly interrelated set of ideas formed a qualitatively different context for physical theory.

Conservation of force arose from and further motivated the search for unity in nature, for the ultimate identity and interconvertibility of all natural powers. And this search had by 1845 been legitimised by manifold empirical interconversions.[1] Yet the only adequate foundation available for a physical theory of conservation was mechanics: conservation of *vis viva* (or, as the concept acquired independence from force, conservation of kinetic plus potential 'energy') and the mechanical theory of heat. The mechanical theory of heat proposed that heat in a body consists in internal *vis viva* and potential energy, and usually that heat radiated between bodies consists in some form of transmitted motion. The latter view presupposed an ether throughout space – already required by the wave theory of light – that could transmit the motions of heat as well as the closely related motions of light. As an ideal mechanical medium it would conserve *vis viva,* again a basic tenet of the wave theory. By postulating an ether, physicists could incorporate heat and light into mechanics under conservation of *vis viva*. Reversing the argument, if ether did not exist, then the mechanical foundation of light, heat, and conservation was destroyed, and with it the satisfying vision of a unified physical science. Many investigators considered the existence of a unifying medium throughout space a necessity for the rational comprehension of science as it stood at midcentury.

General assent to the existence of a single ether, however, did not imply agreement on its nature. Those concerned primarily with one set of phenomena – for example, heat, electricity, physical optics, or philosophy – constructed ether primarily to satisfy the demands of their own area, often without paying close attention to other areas but hoping eventually to subsume them. A comprehensive history of unifying ether theories after midcentury would have to begin by considering this wide variety of bases for constructing ether and then show how most of them lost significance; for by about 1880, electromagnetism, including the electromagnetic theory of light and radiant heat, had emerged as dominant. I shall not attempt that complex analysis here. I shall consider only the electromagnetic bases themselves and only as they developed in the German-language community, in which I include authors whose work was disseminated in German-language journals or who received their training in German universities. Restricting discussion of the search for unity in this way will allow a focus, first, on the reception and reformulation in Germany of French mathematical action-at-a-distance theories of electricity and magnetism and, second, on the development of mathematical field theories as alternatives to action at a distance. Both topics will serve to elucidate concepts of force that were unique to German natural philosophy. These general concepts will form the third, and overriding, theme of my discussion. A few introductory remarks on fields and on German physics at midcentury are necessary.

Fields defined

A distinction between action at a distance and field action, as it came to be generally understood in the nineteenth century, is basic to much that follows. The distinction, in the first instance, separates action directly over finite distances from action only between contiguous elements, that is, immediate (*unmittelbar*) from mediate (*mittelbar*) action between separated objects (Heilbron, this volume). In a field view, local action depends directly on local conditions. It is related to changes in conditions at a distance indirectly, through the mediating action of the field existing in the intervening space. The field, moreover, has an existence in space independent of its sources. It carries in itself the power to effect action, that is, quantity of force or energy, and propagates that power in time from point to point. A space in which force is defined at every point merely as a resultant of sources acting from all distances, such as in Laplace's gravitational theory or Poisson's electrostatics, does not qualify as a field. More stringently, an action between two objects that is merely modified (rather than mediated) by an intervening substance, where every point of the substance interacts directly with the objects, is not a field action. This stipulation is necessary in order that descriptions of polarised media in terms of forces acting directly at a distance should be distinguished from polarisation in a field by contiguous action. Examples of the former are Poisson's 1822 theory of induced magnetism,[2] William Thomson's 1845 theory of dielectric inductive capacity (Wise, 1977; Siegel, this volume), and Helmholtz's 1870 electromagnetic ether theory of light (described in a later section of this chapter). All of these authors presented their theories in opposition to contiguous action or field theories.

Fields in the nineteenth century were basically of two kinds: force fields and ether fields, depending on whether force itself was taken to be a power distributed in space (as in modern electromagnetic theory) or whether the power was carried in the state of a medium, ether. Faraday's mature theory of lines of force provides a classic example of a pure force field. Such theories, however, were less acceptable around midcentury than they had been earlier or would be later. This was, if anything, more true in Germany than elsewhere, partly for reasons that form an integral part of my story and that lead to a few general considerations on German concepts of force and ether.

Natural philosophy in German physics

German physicists[3] at midcentury continued to express intellectual concerns traditional in German Idealist philosophy while vehemently rejecting its excesses in speculative *Naturphilosophie*. The significance of this love–hate relationship with tradition is difficult to specify with precision. In an effort to sharpen its relevance for concepts of force and ether, I shall employ as a foil a man who expressed many of the values of physical science but who was not a physicist and whose work remained unacceptable to practising physicists. J. R. Mayer, a medical doctor, self-educated in physics, became well known after 1850 for his pioneering analysis of the mechanical equivalent of heat and of conservation of force.[4] His first paper on conservation was published in 1842 in Liebig's *Annalen der Chemie und Pharmacie,* following rejection by Poggendorf's physics-oriented *Annalen der Physik und Chemie.* Along with several subsequent papers, it remained largely unknown until Helmholtz began to announce widely the priority of Mayer's paper over his own classic conservation paper of 1847. In treating Mayer as a foil I shall consider first concepts that he and the practising physicists continued to employ, then aspects of their common rejection of *Naturphilosophie,* and finally the physicists' rejection of Mayer's theoretical style.

In company with many German scientists of the nineteenth century, Mayer concerned himself not merely with the physical coherence of material nature but also with the relation between matter and mind, or better, between *Natur* and *Geist,* where *Geist* implies both mind and soul. From Spinoza and Leibniz in the seventeenth century, through Kant and Hegel in the nineteenth, most German natural philosophers would have agreed with Mayer's rationalist judgement: 'What subjectively is correctly thought, is also objectively true'.[5] Matter occupied the world of nature as ideas occupied the world of mind. And where logic governed the rational relations of ideas, forces governed the interactions of matter. In the wide variety of constructions available by the mid-nineteenth century for these ideas, no single aspect is more common than the notion of force as an entity occupying a middle position between inert matter and *Geist,* between nonliving nature and the region of purposiveness and beauty, progress and freedom. Mayer posited 'three categories of existence: 1. matter, 2. force, and 3. soul, or the *geistig* principle'. Although he was among those physically oriented physiologists who separated sharply the conserved forces of nonliving nature from *Geist,* force was for him still a stepping-stone between matter and *Geist.* 'Having once attained the insight that there are not merely material objects, that there are also forces, forces in the narrow sense of the new science, just as indestructible as the matter of the chemists, then it is only a further step to the assumption and recognition of *geistig* existence'.[6]

Not all physical scientists, or even biological scientists, were willing to admit the existence of a separate spiritual realm; some, like Helmholtz, sidestepped the issue; others, like the materialist Büchner, explicitly attempted to reduce the realm of mind to that of matter and force.[7] Even such agnostic and reductionist theorists, however, carried in Germany the weight of the tradition that Mayer more directly represented. Forces, they generally agreed, played the role in nature that relations of ideas played in mind. Forces expressed the rationality of nature. They expressed causality (Mayer, Riemann, Helmholtz) or law (Weber, Fechner, Helmholtz) or, generally, relation.

As relations, forces were often seen as something beyond the things related; they were taken not as seated in isolated matter but as arising only in the interrelations of matter. The significance of this begins to emerge when one considers that it refers force more nearly to chemical affinity – *Verwandschaft* = relationship – than to mechanical push or pull. Mayer provides an illustration for falling objects grounded in the principle of sufficient reason. 'A cause, which effects the raising of a weight, is a force; its effect, *the raised weight,* is therefore likewise *a force.* More generally expressed this is: *spatial separation [Differenz] of ponderable objects is a force'.*[8] The concept of force as a relation (and relationship) will provide the overriding theme in my discussion of German electromagnetic ether theories.

In his understanding of conservation of force, Mayer expressed another traditional idea common among his contemporaries: *'There is in truth only a single force'*,[9] a force conserved through all the transformations of nature. In Germany before midcentury, however, this idea had its home in the speculative systems of *Naturphilosophie,* which Mayer, as well as most practising scientists, was at pains to combat. *Naturphilosophie* is often associated in its negative connotations with the natural philosophies of Schelling and Hegel, for whom nature did not exist independent of *Geist* and who therefore animated all nature with *Geist.*[10] But seen as the attempt to construct a priori a system of nature that would reflect the operations of mind, *Naturphilosophie* should include as well the less subjective philosophies of Kant and Herbart, who preserved an external – though unknowable – *Ding an sich* as a constraining ground for conceptualisations. As their means for construction of a system of nature the *Naturphilosophen* sought to employ an ultimate logic of mental activity, often a dialectical logic that would allow the mind to construct its concepts through irony, that is, through a mutual conflict of primitive opposites, leading to a synthesis at a higher level. The fundamental opposites in the realm of nature were powers or forces, typically attractive and repulsive, and there was given, as both product and ground of the original conflict of attraction and repulsion, a primordial ether filling space as a continuum. It served as the ground of all other forms of matter and force. In Kant's words:

> The elementary system of the moving forces of matter depends upon the existence of a *substance* which is the basis (the primordi-

ally originating moving force) of all moving forces of matter, and of which it can be said as a postulate (not as an hypothesis): There exists a *universally distributed all-penetrating* matter within the space it *occupies* or fills through repulsion, which *agitates* itself uniformly in all its parts and endlessly *persists* in this motion.[11]

The secondary forms of matter and moving force were typically conceived either as modifications of the primitive ether motions or as higher powers of the original conflict of forces. Thus they arose either dynamically or dialectically, with no clear separation between the two modes. Also, since matter and moving force were the grounds for each other, they too were not clearly distinct. Here then is a striking difference from action-at-a-distance theories. Primitive matter, like force, is distributed continuously – as matter in action-at-a-distance theories was not. Concomitantly, force, like matter, occupies space – as action-at-a-distance forces did not. One sees immediately the conceptual precedent for both continuous ether fields, filling space with energy of motion, and pure force fields, filling space with energy of attraction and repulsion.

In all systems of *Naturphilosophie,* finally, the dynamical processes continued uninterruptedly, as reflections of the rational operations of *Geist*. Kant, for example, claimed that 'the primordial forces of motion, as originally agitating, cannot bring themselves to rest, for a state of rest itself presupposes a counteraction of the agitating forces in actuality, not merely in potentiality, so that the hindrance of these motions in universal rest is self-contradictory'.[12] Ideas of this sort, widespread in early nineteenth-century Germany, form the basis in Idealist traditions for attempts such as Mayer's to enunciate conservation more precisely.

While continuing to seek the unity of nature through the unity and interconvertibility of forces, and while continuing to believe in a close parallel between rational relations of ideas and physical relations, Mayer and many of his contemporaries nevertheless rejected the notion that one could construct the true system of nature by merely developing the structure of thought. One could not know enough about *Geist*. They rejected in particular the dialectical process, with its conflict of opposites leading to a higher synthesis. And they required that all legitimate science be closely tied to empirical observations. Of particular relevance here, through an emphasis on the empirical materiality of objects, physicists separated matter from force and, like Mayer, required their independent conservation.

Though Mayer thus represents, in the traditions he rejected as well as in those he continued, a variety of common goals, he did not reject enough to suit the dominant mood of practising physicists. In its objectionable aspects

his work remained explicitly metaphysical. He employed empirical results primarily as confirming instances and attempted to establish conservation of force a priori, relying on the principle of sufficient reason, and without a rigorous treatment of *vis viva*. Helmholtz expressed the common attitude when he called this a 'metaphysically formulated pseudoproof'.[13]

Mayer fell outside the mainstream of physics in other ways. As noted earlier, most physical scientists sought to unify nature by constructing realistic physical models, usually on a mechanical basis; but even though Mayer had himself discovered the mechanical equivalent of heat, and even though he accepted a mechanical wave theory of light, he did not accept the theory that heat was mechanical. He argued that, although force in the form of heat could be interconverted with mechanical forces – with *vis viva* or a raised weight – the fact of interconversion did not justify taking any particular form of force as the fundamental one or the basis of transformations. The obvious advantage of a completely mechanical theory was that it provided just such an explanatory foundation for conversion and conservation of forces. In refusing, furthermore, to reduce heat (and electricity) to matter, motion, and the forces acting between parts of matter, Mayer maintained the *naturphilosophisch* notion of force as a sort of substance, now independent of matter but still having the same status as matter. To describe both light and heat transmitted through space, for example, he required not only a material ether to carry light waves but an independently transmitted and apparently immaterial force of heat.[14] That the rejection of *Naturphilosophie* meant precisely the rejection of such 'metaphysical' force substances in favour of the mechanics of matter helps to explain why space-occupying force fields did not receive serious consideration in Germany much before 1880. A mechanical ether provided the only legitimate basis for unity. In the last decades of the century, however, as electromagnetic fields and energy relations, described on a positivist basis, came increasingly to be considered the proper foundation for physical theory, Mayer's views were resurrected as precursors of the new trends, particularly by so-called Energeticists who made energy the basis of all reality.[15]

The following discussion of electromagnetic ether theories covers the period from 1845 to 1880, from enunciation of conservation of energy to the period just preceding full incorporation of the energy idea into pure force fields. I attempt to show how general philosophical concerns both conditioned and were conditioned by several specific electromagnetic ether theories. The result is less a coherent history of such theories in the period than a small collection of conceptual histories taken as representative of the full story.

The great questions of electromagnetism as they arose in the German context were two: How were electromagnetic forces, which seemed to require a velocity-dependent relation between portions of electrical matter, to be comprehended; and how were electromagnetism and light, apparently quite closely related, to be unified? Could ether actually perform its role as the unifying medium? Three primary examples serve to develop these questions and the major sorts of answers proposed for them. The questions arose through Wilhelm Weber's velocity- and acceleration-dependent law of electrical forces; and Weber attempted to resolve them through a comprehensive ether theory based on an action-at-a-distance interpretation of the force. Bernhard Riemann outlined, preliminarily, an equally comprehensive reinterpretation utilising a variety of ether field pictures and emphasising propagation of force in time. Hermann von Helmholtz, finally, attempted to explain apparent propagation of force within an action-at-a-distance framework, but his ether theory served instead only to mediate between the demise of action-at-a-distance theories and the rise of mature field theories. All three of these examples point up the significance in German physics of the concept of force as relation.

Weber's ether and Fechner's metaphysics

In the historiography of nineteenth-century physics we have become used to the notion of a Continental action-at-a-distance tradition formalised by early nineteenth-century Frenchmen, typically Laplace and Poisson, and adopted somewhat later by Germans such as Gauss, Wilhelm Weber, and Helmholtz (Woodruff, 1962). As the culmination of this tradition we are likely to think of Weber's law for forces acting at a distance between particles of electrical fluids. This is the law announced by Weber in 1846 that made the force between two electrical particles, e and e', depend not only on the inverse square of the distance r between them but also on their relative velocity v and relative acceleration a:

$$F = \frac{ee'}{r^2}\left(1 - \frac{1}{c^2}v^2 + \frac{2r}{c^2}a\right) \qquad (9.1)$$

(Here c is a constant that Weber and Kohlrausch showed in 1857 to be approximately $\sqrt{2}$ times the velocity of light.)[16] The action-at-a-distance tradition indeed explains much about Weber's law, but in travelling from France to Germany the concept of action at a distance was transformed. Whereas Laplace and his associates described force as though it emanated from one particle of matter and acted on another particle at a distance – a description that implicitly tied force to a particle as its source – Weber insisted that force existed only as a pairwise relation between particles. The pair of particles, therefore, formed his fundamental unit of analysis. I shall begin to develop

the implications of this idea through a description of Weber's programme for unity in physics, proceeding then to metaphysical foundations of the programme as presented by Gustav Theodor Fechner.[17]

Weber's electrical ether

From the first presentation of his force law in 1846, Weber conceived it as the core of an incomplete theory of an electrical ether that might eventually unify many or all natural phenomena, in the sense that it would reduce many forces to a single law of force. By 1848 he had shown that the force law could be derived from a potential, a function describing a system of particles that always acquires the same value when the particles acquire their initial position and velocities.[18] Existence of such a function guaranteed conservation of *vis viva* in the system and, therefore, conservation of all those natural powers that could be subsumed under the law of force. Thus unity and conservation were both early elements of Weber's programme. But for Weber and other empirically minded physicists unity under a specific law of force was the primary goal, conservation somewhat secondary. Only gradually did the conserved and generalised quantity of force, measured by *vis viva* and work, acquire conceptual independence as kinetic and potential 'energy'.

This view of Weber's project is consistent with the French tradition that he partially adopted. Laplace, among others, suggested that chemical affinities might possibly be explained as modifications of the inverse square force of gravity.[19] (As will appear later in this section, Weber's force law may be seen as such a modification.) Apparently following Boscovich in the attempt to make Newton agree with Leibniz, Laplace also reduced Newton's finite atoms of matter to material points so that they could never collide and so that only attractive and repulsive forces could act between them. That guaranteed conservation of *vis viva,* implying that the universe could never run down and that equality of cause and effect, or the principle of sufficient reason, would always be observed.[20] Ampère carried the argument considerably further when he attempted to construct chemical elements from geometrical arrangements of material points interacting through attractive and repulsive forces. After adopting the wave theory, Ampère also incorporated light and radiant heat as vibrations in a self-repulsive ether.[21] This ether formed an atmosphere around every point atom by attraction and extended through all space. It therefore served to transmit, as waves of heat and light, the *vis viva* of vibrating atoms, which he thought might constitute the internal heat of bodies. The same neutral ether could decompose to form positive and negative electrical fluids that, when flowing in opposite directions, would constitute an electrical current. Such double currents when flowing around an atom or molecule would form a little magnet or electrodynamic molecule, thereby accounting for magnetic materials.[22] In this physical picture, Ampère assumed that all of the ultimate forces were forces of material points that would conserve *vis viva,* but he did not attempt to formulate a theory based on interchanges of conserved quantities. His best-known law of force – the ponderomotive force between any two abstracted elements, or short sections, of current – he presented as a purely positive description of empirical observations.

Weber and his friend Fechner followed this French tradition when they reduced Ampère's abstract law of force between two currents to a physical action between point atoms of the two electrical fluids supposed to constitute both currents. The net force resulted from four interactions, all governed by the same law of force,[23] namely, the velocity- and acceleration-dependent law of force displayed in equation 9.1. The first term described Coulomb's inverse square electrostatic force between two separated electrical particles, attractive between unlike particles and repulsive between like particles. The remaining two terms modified the static law for electrodynamics, or electricity in motion, including both electromagnetic moving forces between constant velocity currents (Ampère's law) and electromotive forces induced between accelerating currents (Faraday's law). All major phenomena of electricity and magnetism were thus included under Weber's fundamental law of attraction and repulsion.

Again following Ampère, Weber believed in 1846 that the two electrical fluids in their normal unseparated state formed a neutral ether surrounding ponderable molecules and extending through all space. He hoped to be able to explain the wave theory of light on the basis of oscillations in this ether governed by the electrodynamic force law.[24] His project gained considerable credibility through association with Michael Faraday's discovery of diamagnetism in 1845 and Faraday's related discovery that diamagnetic bodies, when placed in a strong magnetic field, would rotate the plane of polarisation of light transmitted through them. Diamagnetism is the induction in normally nonmagnetic material, when it is subjected to a strong magnetic force, of a magnetic polarity opposite to that induced in normally magnetic substances, or paramagnetics. Weber explained the phenomenon successfully as induction of Ampèrian double currents in the neutral electrical atmosphere surrounding ponderable molecules.[25] These induced diamagnetic currents were similar, but opposite in direction, to the permanent currents around paramagnetic molecules. They offered a natural ground for explanation of magnetic rotation of light, particularly when the luminiferous ether was identified with the neutral electrical ether.

Following such early explanatory successes, Weber began to develop in the

1850s an increasingly comprehensive picture of the interaction of ponderable molecules with ether, a picture that became by 1880 a nearly complete electrical theory of matter, including even gravitation. It was in the course of these extensions that he departed markedly from French traditions, when he isolated and reified into a physical structure a notion that had been central even to his original force doctrine: the pairwise relation. This reified relation was the atompair, to which all actions were to be reduced. But the pair itself in Weber's conception could not be reduced; it was more than a sum of two atoms. A single 'physical point, or atom', Weber later insisted, could possess only mass and motion. More complex properties of matter, even extension, had to be considered as arising, not from properties attributed to individual atoms, but from the independent properties of pairs, existing only in their relation.[26] In 1846, Weber had thought that the acceleration-dependent term in his law of force might indicate a mediating action of ether between two atoms or even the existence of irreducible three-particle relations, because the relative acceleration of two atoms would depend directly, through Newton's second law of motion, on the action of any third particle on the first two.[27] In chemical reactions, Berzelius had named such forces 'catalytic forces', and their appearance in the electrical law of force heightened Weber's awareness of the possibility that his law might contain the reduction, long sought by Faraday and others, of chemical to electrical action. If three-particle and higher-order relations were fundamental, however, nature would be infinitely complex. Weber soon found relief from this complexity in his potential function, from which the force law could be derived in the conventional way as a gradient. The potential contained purely pairwise relations: relative position r and relative velocity v,

$$V = \frac{ee'}{r}\left(\frac{v^2}{c^2} - 1\right)$$

Believing already that the whole of a two-particle system was greater than the sum of its parts, Weber considered it extremely important that according to this potential function the 'totality of many bodies' would not give rise to new forces and new properties that could not be reduced to the properties of single particles and pairs.[28]

The simplicity alone of Weber's expression for potential probably argued for its precedence over the force law, but it is indicative of the changes taking place in physical theory around midcentury that he expressed that precedence as he assimilated his unifying force law to the general unity of energy conservation. Under the energy doctrine, potential was not only a simplifying expression for the force between two atoms; it was potential energy, the work expended against those forces in assembling the pair with given relative position and velocity. Total energy, thought of as contained in the pair, was this potential energy plus the *vis viva* (kinetic energy) of the pair. As energy was separated from moving force within the general concept of force or power, Weber came to recognise potential energy as the extra physical something in the abstract relation of a pair. In 1871 he quoted August Beer approvingly: 'In many respects one can speak with more justification of the *physical existence of work, expressed through the potential,* than of the *physical existence of a force,* of which one can only say that it *seeks to change physical relations of bodies'*.[29] A brief summary of the evolution of Weber's programme for ether will help clarify both this changing perspective and his grand plan for the unity of nature based on the atompair.

In 1852, Weber developed an explanatory model for resistance to electrical conduction in which he first described atompairs.[30] The model raised concrete problems in the relation between electrical and ponderable particles that ultimately suggested their unity. If free ether consisted of neutral pairs of positive and negative particles, Weber reasoned, then the particles of each pair should orbit about each other under the action of their attractive force. When subjected to an applied electromotive force, as inside a wire connected to a battery, such orbiting pairs would undergo successive breakup and recombination into new pairs, resulting in opposite motions of positive and negative particles along the wire. Resistance to this double current would derive from the force required to divide electrical pairs. (He did not yet discuss how the expended force disappeared.) Since all pairs were identical, the number of divisions per unit time would distinguish resistances in different materials. Conductors, nonconductors, and free ether, supposedly, would decrease in resistance with decreasing ether density. Though heuristically interesting, this model of double currents and resistance was clearly inadequate. It provided no ground for explaining different densities of ether in different materials. More problematically, it required that the double currents about ponderable molecules, those responsible for magnetism, have completely independent positive and negative components, for otherwise the component currents would resist each other and stop. Weber could only suggest that the positive and negative molecular currents moved in circles of different radii. He offered no explanation of the difference nor even an account of why electrical particles would orbit about ponderable molecules in the first place.

He did not stop long to ponder these obvious difficulties. By 1855 he had outlined the more profound problems that would occupy the remainder of his career.[31] Electrical resistance, he thought, must derive somehow from the connection of electrical particles to ponderable particles: What was the connection? Electrolysis showed a close relation between specific chemical elements and specific quantities of electrical charge: What then was the relation between chemical affinity and charge? One could add the velocity- and acceleration-dependent terms of the electrical force law to the gravitational force law without altering observable results (because $1/c^2$ was small): How close was the analogy thereby revealed between gravitational and electrical forces? By 1862, Weber had added to his unifying list the problem of converting work to heat, specifically, of converting the work done to produce electrical motions in a current into the heat of ponderable molecules, thereby conserving energy. He had also seen that this relation, as well as many other relations between ponderable molecules and electricity, might be discovered in a modified version of molecular currents.[32]

Rather than being surrounded by two currents in smaller and larger circles, molecules might have one kind of electricity, negative say, adhering to their central mass, while only positive electricity circulated about this negative nucleus and also constituted ether. That would eliminate the problem of resistance to molecular currents while simultaneously explaining resistance to conduction currents. To move positive particles from one molecule to another would require work from an applied electromotive force. Motion in conduction currents, furthermore, could be transferred to motions in the molecular currents, appearing as the heat of molecules or the heat of a ponderable body. Any disturbances of the molecular currents would cause oscillations in them at the frequency of the molecular orbits, and the oscillations would produce waves in the surrounding ether, or heat radiation. Similar wave disturbances would constitute light.

This was the model for which Weber, during the sixties and seventies, sought to build a mathematical foundation. Responding directly to the claims of energy conservation, he now made energy and energy exchange the basis of his analysis. By reexamining the states of motion of electrical pairs under the action of his fundamental force law, he found that not only two unlike particles, but also two like particles, could form a stable atompair.[33] In fact, any system of two like particles would possess two different states between which no transitions could occur except under the action of an external force. In a bound or attractive state (*Molekularbewegung*) the two particles remained always closer together than a small limiting value r_0, whereas in an alternate unbound or repulsive state (*Fernbewegung*) they moved between r_0 and infinity. Similar states existed no matter how large the masses of the particles. If, therefore, negative electrical particles were somehow united with ponderable atoms, pairs of the resulting atoms could form stable massive configurations (*Molekularbewegung*) under electrical forces alone. Similarly, positive pairs of negligible mass in the state of *Fernbewegung* could constitute ether.

With that rigorous result, Weber had broken open his set of problems, for now he could envision constructing all ponderable matter out of identical negative atoms, thereby uniting chemical atomic theory with electrical theory. The chemical identity of elements would depend on the number of negative atoms united in a massive nucleus and on their organization; magnetic, electrical, and thermal properties would depend on the states of binding between the nucleus and the light, positive atoms surrounding it.[34] Thermal radiation and light, as before, would interact with matter as waves in the positive ether. Weber conceived this ether first as a gas and later as a stable mass, but in either case it consisted of only positive pairs in the repulsive unbound state.[35] All phenomena of nature seemed to be subsumable under the new mathematical results; all, that is, but one – gravitation.

In 1875, Weber wrote the law of gravitational force in a form corresponding to his chemical atomic theory, where the mass m and charge e of ponderable atoms were proportional, $m = \alpha e$, so that

$$f = \frac{mm'}{r^2} = \alpha^2 \frac{ee'}{r^2}$$

This proportionality between the electrostatic and gravitational forces acting between all ponderable bodies suggested that the two might be integrally related, especially since the usual velocity- and acceleration-dependent terms could be added without affecting appreciably the long-range interactions of gravity. To reduce gravity to electrical forces, however, a force was required that at large distances would act attractively between *neutral* molecules, that at smaller distances would become repulsive (as in a gas), and that at yet smaller distances would again become attractive (as in chemical bonding). At the suggestion of a close associate at Leipzig, Karl Friedrich Zöllner, Weber attempted to encompass all of these factors within his force law, by employing two hypotheses: (1) that each of the identical ponderable atoms combining to form chemical elements was a neutral system consisting of a negative, but highly massive, central particle with a positive satellite of much smaller mass; and (2) that the attractive force between equal but opposite electrical particles was *greater than* the repulsive force between like particles, by a factor $(1 + \alpha)$.[36] On these assumptions the net force between two neutral ponderable atoms (four interactions between two negative–positive pairs) would appear as a net attractive force between the negative component e of each pair and the positive component e' of the other:

$$F = 2\alpha \frac{ee'}{r^2}\left(1 - \frac{1}{c^2}v^2 + \frac{2r}{c^2}a\right)$$

This could be the 'gravitational' force at large distances r if α were the ratio between the large mass of a negative particle and the small mass of a positive particle.

Weber had here transformed his original neutral electrical ether of 1846 into a system consisting still of only positive and negative atoms but encompassing now all of the properties of both ether and matter. He was tantalisingly close to a single force law uniting all nature – with one qualification. The final scheme is coherent only with the understanding that force is a relation of two atoms, and its intensity depends on the character of both, for otherwise there is no basis for hypothesis (2).

Fechner's philosophical basis for action at a distance

The preceding discussion of Wilhelm Weber's evolving ether theory has displayed the coherence of form that marked his theorising throughout at least forty years of research. This coherence cannot be fully appreciated, however, without recognition of the philosophical goals he intended the theory to fulfil. Weber wrote with the traditions of German Idealism in mind. He apparently considered it a prerequisite of any valid theory that it make provision for a close connection between *Geist* and *Natur* but that it not degenerate into Materialism by reducing *Geist* to the mechanics of matter. The pairwise relation, which contributed properties to the whole beyond those of its parts, fulfilled this criterion.

> If [two] material essences, which are spatially and temporally separated, interact, then the ground of this interaction lies in the essence of both *as a whole*. The interdependent parts of this *whole* exist in different spatial and temporal points. If there are *material* essences which as *wholes* cannot be reduced to a point in space and time, that applies even more to *spiritual* [*geistig*] essences.[37]

Weber did not publish many of his thoughts on this connection between *Geist* and *Natur,* but throughout his life he associated himself most closely with people who made it their central concern. In the cases of his brother Ernst Heinrich Weber and of Fechner, both celebrated professors at Leipzig, it led to research in psychophysics. For Zöllner, also at Leipzig as professor of astronomy, it led eventually to experiments in spiritualism, experiments in which the Weber brothers and Fechner took part.[38] Weber's philosophical concerns seem to be reflected most closely in the writings of Fechner, with whom Weber first developed his ideas of an electrical double current and neutral ether, and whose 1855 defence of atomism, *Ueber die physikalische und philosophische Atomenlehre,* relied heavily on interaction with Weber.[39]

In the *Atomenlehre,* Fechner sought to found metaphysical atomism on contemporary experimental and mathematical physics, on the 'presentable connections of objects [*Weltdinge*], directly and compactly summarised in their ultimate points and knots [*letzte Spitzen und Knoten*]'.[40] The ultimate points and knots were, to Fechner, physical atoms and forces. Arrayed as enemies against this atomism of the physicists he saw the dynamicists and dialecticians described earlier: Kant, Schelling, Hegel, and Herbart. To them matter was a constructed concept. Even space was phenomenal, an appearance deriving from the process of perception, and time too they ascribed to perception. In that way, *Natur* had been filled with *Geist,* in contrast to the crude, materialistic, and spiritless conceptions of atomism.

The closest approach of the dialecticians to a physicist's atomism appeared in Herbart's 'monadology',[41] a metaphysics somewhat like Leibniz's, based on a hierarchy of discrete, simple essences, called reals or monads. A high-level monad would be the soul of an organism, whereas a low-level one would appear in the physical world as an atom. All of our knowledge, for Herbart, both of external (physical) and of internal (psychological) phenomena derived from the relations between monads. Modelling his view of these relations after a notion of force as chemical affinity (*Verwandschaft,* implying kinship or sympathy in a relationship), Herbart asserted that the relations arose as a process of conflict between monads, each striving to preserve itself. The dialectic of nature, therefore, occurred not between forces but between monads, and one could have knowledge of this conflict through the conflict between one's own soul and external monads, which gave rise to discrete presentations (*Vorstellungen*) in the mind. The relations between presentations mirrored the relations of external monads and produced one's perceptions of the objective world. Space and time appeared as general aspects of the process of relation, whereas the properties of bulk matter arose from more specific relations.

Into this, Fechner proposed to inject some commonsense reality, the reality of empirical appearances reduced to their essence. Most real of all, he argued, were material objects in space and time, which could be tasted, smelled, touched, and heard, not merely once but repeatedly, and not merely by one person but by many.[42] Matter, then, was an empirical reality and should lie at the foundation of any philosophy of nature. But additionally, physical research in its modern state required that matter be conceived as atomic. The successes of chemical atomic theory, particularly, established this view for ponderable matter; and the wave theory of light, to the degree that it explained on the basis of a particulate ether such phenomena as polarisation and differential diffraction of different wavelengths, established the atomic constitution of ether. These claims could hardly have been considered the consensus of physical scientists, but Fechner had in mind specifically as representatives of

the 'new mathematical physics' Weber and the French analysts: Laplace, Poisson, Fresnel, and Cauchy. Their work in particle mechanics was only beginning to receive a serious challenge from mathematically developed continuum mechanics, as Jed Buchwald (this volume) has shown for physical optics.

Modern physics established for Fechner the practical necessity of atomism, both for normal matter and for ether. But he had still to construct for the atomistic world view a satisfactory philosophical foundation in order to present it as a coherent idea, and ideal, in the context of German traditions. Most important, he wished to establish, contrary to the dialecticians and dynamicists, that atomism was neither materialistic nor spiritless. The materialism of traditional atomistic mechanics, Fechner reasoned, resided in two notions: that atoms consisted fundamentally of extended gross matter and that forces inhered in this matter, or that forces were properties of matter.[43] On that view all phenomena of nature would derive from independently acting atoms of matter. There would be no unifying tissue tying all of nature together as an organic whole, such that the whole was more than the parts; and there would be no room for the interrelation of mind and matter.

To elevate atomism towards the realm of *Geist,* Fechner began by supposing that material atoms stripped of their relations to other atoms were actually only physical points, or real physical monads, existing in real space and time. These atoms possessed no properties of their own other than mobility. All motions of atoms and all properties of assemblies of atoms, or matter, derived from relations between atoms.[44] To the degree, then, that the concept of matter referred to properties of matter, matter was constructed from immaterial relations of point atoms. That concept allowed an immediate translation from the realm of matter to the realm of mind. Relations between atoms in inorganic nature were laws, physical laws of force; these same laws, however, represented the rule of *Geist: 'Geist* steps up and asks, what have I to do with you? And the atoms say: we spread our individualities under your unity; the law [of force] is the commander of our band, but you are the king in whose service he leads'.[45]

In the new ordering of concepts – space, time, atoms, laws, *Geist,* and ultimately God – one moved from what one could know objectively to what one could know subjectively. At the interface between the external world and the perceiving subject, objective and subjective were only two references to the same thing.[46] That was the basis of Fechner's and Ernst Weber's well-known pioneering work in empirical psychology, labelled psychophysics by Fechner. Their investigations rested on the assumption that a determinate increase in objective stimulus should produce a corresponding intensity of subjective response (e.g., pain). The entire line of argument involved a simple inversion, similar to Herbart's, of the usual dialectic of conflicting attractive and repulsive forces, which had produced continuous matter throughout space. Fechner supposed instead that the 'dialectic' of discrete atoms gave rise to forces, that forces were the objective relations between atoms, and that this relation either constituted or measured subjective perception.[47]

Fechner's world view provided not so much a complete philosophy of nature as a programme for quantitative research, the programme that he and Ernst Weber pursued primarily in psychophysics and that Wilhelm Weber pursued in physics. Beginning with an empirically and theoretically derived law of pairwise atomic interactions, the force law or the law of potential energy, one would attempt to construct the empirical properties of matter, of ether, and of their interactions. Fechner believed that a single kind of atom, or monad, would serve to explain all of these phenomena; qualitative differences would depend only on different groupings of the atoms. Already in 1855 one could conceive of reducing magnetism, heat, and light to motions and relations of electrical matter; but were both attractive and repulsive forces, and both positive and negative electricities, necessary? What of gravitation; and what of mass itself as a quality of atoms? Fechner took hope in the increasing range of Weber's pairwise force law and attempted to uncover its ultimate implications. If the pairwise law alone proved insufficient, however, he was prepared, like Weber, to consider higher-order relations of three or more particles and higher-order relations of relations.[48]

I have described the Weber–Fechner action-at-a-distance theory of matter and ether at some length in order to show, first, the broad range of applications of the theory envisioned by Weber and, second, its metaphysical significance as presented by Fechner. Both considerations should indicate, third, that when critics of action at a distance charged that such action was incomprehensible, because one could not imagine how the action could reach from one body to another, they were missing much of what action at a distance meant to its German adherents. One body did not simply act on another – they interacted in a relationship – and the force existed as the law of relation. Nevertheless, Weber's law of force severely strained the plausibility of transforming an abstract relation into a real entity. It made of force a relation dependent not only on the distance between two atoms but also on the rate of change of that distance (velocity) and on the rate of change of the rate (acceleration). It is one thing to imagine a relation between two atoms at an instant; but it is quite another to imagine that relation depending implicitly on just prior and just following instants, as Weber's law required. Weber and Fechner found little

difficulty in this, apparently because they so closely identified force with a logical relation in which space and time had similar status. A number of other physicists, however, sought a more physical explanation for time dependence. It indicated to them that force did not act instantaneously between two bodies but took time to propagate from one to the other.

Already in 1846, upon reading Weber's first electrodynamics paper, Carl Friedrich Gauss, Weber's close associate at Göttingen, remarked that he had earlier made a similar investigation himself but had lacked what he regarded as the keystone: 'namely, *the derivation* of the additional forces [added to electrostatic force] from *non-instantaneous* actions, actions propagating in time (in a manner similar to that of light)'.[49] This association between electricity and light became all the more pressing with realisation that the constant *c* in Weber's law was close to the velocity of light. Alternatives to Weber's theory of the electrical ether, therefore, often proposed that electrical action propagated at the speed of light and, in fact, constituted light.

Propagation of force and early field theories

For force to propagate from one place to another, on analogy with the wave theory of light, it must be describable not simply as an abstract relation but as something existing objectively in space and distributed throughout space. That was the judgement of those who saw in Weber's implicitly time-dependent law of action at a distance the denial altogether of such action. Their alternatives were what soon came to be called field theories: both ether fields, in which force actually consisted in a form of motion in ether, and pure force fields, in which force itself propagated independently through space. Although physically quite different alternatives, ether fields and force fields arose in the same context and emphasised the same descriptive foundation, namely direct descriptions, dynamic relations, and partial differential equations.

A partial differential equation in space and time may be seen as describing the behaviour in time of any and all infinitesimal elements of space, elements taken to be characterised by one or more properties. Because the equation describes the change of these properties across an element it also describes their change between elements, and it traces that change in time. It therefore expresses naturally any continuous process of propagation from element to element through a field of properties, which might be properties of ether or properties of force. Partial differential equations, then, served as a broad path for describing directly the processes of nature while ostensibly avoiding speculation. When taken also as the most fundamental description, they served as the ground for continuum theories of ether and force. And they sometimes served as a mathematical rationale for maintaining the emphasis of *Naturphilosophie* on nature as continuous dynamic process.

Field theory was not established independently as an ongoing research tradition in Germany much before 1880; it existed more as an undercurrent that surfaced along with the notion of energy as an independent, conserved entity and in response to an action-at-a-distance interpretation by Helmholtz in 1870 of Maxwell's field theory. For that reason I present here only a cursory summary of several early field theories in order to establish their character. I consider Helmholtz's theory in a following section as a critical articulation that helped to motivate mature field theory.

Ether fields

The ether field for electromagnetism arose first, and with enduring characteristics, directly under the gaze of Weber himself in the work of his young student, assistant, and friend at Göttingen, Bernhard Riemann.[50] Riemann was also a student of Gauss and a friend of one of the most luminous exponents of differential equations applied to physics, Lejeune Dirichlet. Riemann shared Fechner's taste for metaphysics and for unifying all natural phenomena on a common physical basis, though from a different perspective. Basing his position on the epistemology and psychology of Herbart – as opposed to Herbart's *Naturphilosophie* of monads, which had been important for Fechner – Riemann argued that only relations or states could serve as causes of action, or forces, for only something subject to change of degree could itself be the cause of such change: 'What an agent strives to effect must be determined through the concept of the agent'.[51] Being (of things) was not subject to change of degree; therefore things could not be causes, only relations or states could be.

Some perspective on Riemann's view can be obtained by observing that Herbart had considered perception to consist of reception by the mind of discrete presentations, followed by 'sinking' of the presentations below the threshold of consciousness. This sinking derived from mutual suppression of opposed presentations through the forces they exerted on one another. Applying to perception the notion of force that he employed in his discussion of monads, Herbart regarded the forces between presentations as affinities, grounded not in the presentations but in their relations. He supposed also that the relations established themselves in time, approaching only asymptotically a static balance of forces. Perception was thus a dynamic process, and Herbart attempted to describe it through differential equations in time. Riemann seized on this aspect of Herbart's psychology and made continuity and differential equations the starting point for both psychology and physics. 'We observe', noted Riemann in 1853, 'a continuous activity of our soul'. This activity consisted in presentations continuously disappearing from consciousness and yet remaining as part of the substance of the soul. By analogy, he reasoned, gravitation might consist in a continuous flow of imponderable space-filling *Stoff* (substance or ether) into ponderable atoms. In fact, 'both hypotheses may be replaced by the one, that in all ponderable atoms substance perpetually enters the spiritual world from the world of body'.[52] In this way action at a distance would not exist; gravitation might be thought of as dependent on the pressure of ether immediately surrounding ponderable atoms, where pressure in turn depended on velocity of the ether. The same ether could serve to propagate the oscillations that we perceive as light and heat.

Riemann attempted initially to analyse the processes of gravitation and light in terms of resistance of a homogeneous ether to change of volume (gravitation) and to change of shape (light), the latter reducing to resistance to change of length in any physical line. He hoped also to be able to include electricity and electromagnetism in the schema, with electrostatic inverse square force depending on change of volume and electrodynamic force on change of length. All actions, therefore, were to be actions only between 'neighbouring' or 'immediately surrounding' elements. Although Riemann did not carry much further this first glimmer of a field theory, he preserved its essential characteristics in several later attempts. He always sought to unify nature on the basis of a geometrically conceived system of continuous dynamic processes in ether, founding his description on differential equations that described the processes of relation that one could perceive directly, that is, forces and interconversions of forces. His project was probably the first attempt at a mathematically founded unified field theory, much in the spirit of Einstein's later attempts, and he assigned it more weight even than his now-famous efforts in pure mathematics.[53]

Riemann's more sophisticated field descriptions in the late 1850s rested on his general assumption that the cause of both motion and change of motion (inertia and accelerating force) of a body at any point should be sought in 'the form of motion of a substance spread continuously through the entire infinite space . . . This substance can therefore be conceived as a physical space whose points move in the geometrical one'.[54] Neglecting now any explicit correlation with the sinking of presentations in perception, Riemann proposed certain motions of a homogeneous ether that would reproduce the partial differential equations of gravitation and of light propagation. The first of these equations was a continuity equation, with a nonrotational velocity system $\mathbf{u}$ standing for force:

Divergence $\mathbf{u} = -4\pi\rho$

It stated that the net flux of ether into any volume element should be given by the ponderable mass ρ per unit volume of the element. The second equation was a wave equation for transverse oscillations in velocity w (not oscillations in displacement) propagating at the speed of light c:

$$\frac{\partial^2 w}{\partial t^2} = c^2\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2}\right)$$

Combining the two motions for gravity and light produced a well-behaved velocity function, which confirmed the possibility of uniting the two processes.

Riemann had difficulty incorporating electrostatic and electromagnetic effects into the system, but in a series of lectures delivered in 1861 he indicated that variations of density in ether might be the key. If V were the usual electrostatic potential and $\mathbf{u}$ an electrodynamic potential (vector potential), the latter could be chosen to satisfy the equation

$$\frac{\partial V}{\partial t} = \text{Divergence } \mathbf{u}$$

In that case, 'V may be regarded as the density, $\mathbf{u}$ as the flux of this ether'.[55] A stable gradient in density would then correspond to electrostatic force, whereas a time rate of change in density, and associated fluxes, would correspond to electrodynamic effects. Actually, only the rotational part of the flux system in ether was required for electrodynamics, so that the nonrotational part was presumably still left over for gravitation.

Riemann never published his speculations on the unified ether field, no doubt because he did not succeed in fully integrating the various effects of gravity, light, electricity, and magnetism. He did, however, present a paper to the Göttingen Society of Science in 1858 (published only in 1867, after his death), which helps to complete this schema. Here he proposed an electromagnetic theory of light. It returns us to Weber's electrodynamic force law and to the problem of explaining velocity- and acceleration-dependence in a relation at a distance.

'I have found', Riemann announced,

> that the electrodynamic effects of galvanic currents may be explained if one assumes that the action of one electrical mass on the rest does not occur instantaneously but propagates to them with a constant velocity (equal to the velocity of light within the limits of observational error). The differential equation for the propagation of electrical force, according to this assumption, will be the same as that for the propagation of light and radiant heat.[56]

His assumption meant that one electrical mass experienced the action of another always as an electrostatic force (inverse square force or inverse first power potential), but it experienced at time t an action produced at an earlier time t', where the time lag was the propagation time from the position of the acting particle to that of the affected particle. The potential for such a 'retarded' force obeyed a wave equation rather than the usual continuity equation of electrostatics. And Riemann provided a proof that the retarded potential, when referred only to the time of action t, would correspond to the potential of Weber's force law. There are certain inadequacies in the proof, however, and perhaps for that reason Riemann withdrew the paper from publication.[57]

In summarising the general characteristics of Riemann's programme for unifying physics, we note that he sought to replace Weber's discrete relations at a distance with continuous action between neighbouring elements, which circumvented the problem of time-dependent forces. More important, he replaced the abstract relation, force, with states and processes in ether, providing a physical basis for the relation. Electrostatic potential, for example, he ascribed to density in ether. This made variations of density the reality of force, a shift of considerable importance for the energy concept, for it spread force throughout space as potential energy. Most profoundly, by reducing causality (forces or dynamics) in the observable world to the problem of describing motions in ether (kinematics), Riemann reduced physics to descriptive mathematics, or to the geometry of motions in a 'physical space'. That pushed the problem of causes one step farther back into metaphysics, and into the dynamical principles of ether. Riemann recognised this explicitly in regard to his idea of gravitation and light as two forms of motion in ether:

> The further development of this hypothesis divides into two parts insofar as one seeks:
>
> 1. The laws of ether motion [*Stoffbewegungen*] which must be assumed for explanation of phenomena,
> 2. The causes from which these motions can be explained.
>
> The first task is a mathematical one, the second a metaphysical.[58]

He employed as a foundation for the metaphysical dynamics of ether the principle of continuity of motion and a maximum–minimum condition on the velocity potential integrated over all space (the latter exhibited only mathematically). These ideas should be compared with the very similar ideas of William Thomson, Maxwell, and other midcentury British physicists. They formed the core of what the British called 'dynamical theory' (Moyer, 1978; Wise, 1980).

In this regard, Riemann's emphasis on differential equations as the basis for positive description, coupled with energy as the basis for force, is particularly significant. Both ideas made the study of fluid motion, or hydrodynamics, a pressing research topic, even when a theorist did not set out to reduce all forces to ethereal states and motions. Helmholtz, for example, established in 1858 several important theorems in hydrodynamics while rejecting such a reduction.[59] Nevertheless, his proof that a vortex in a frictionless fluid would last forever became the basis of William Thomson's ether theory of vortex atoms (Siegel, this volume). G. R. Kirchhoff developed hydrodynamics considerably further, apparently with the intention of ultimately ridding physics of forces. Adopting basic theorems of Thomson's and Tait's *Principles of natural philosophy,* he made maximum and minimum conditions on fluid motion the foundation of his hydrodynamics.[60] This suggested a quite general approach to the dynamics of an ether that would explain observable forces. At the same time, however, Kirchhoff spread the new positivism of differential equations, denying the need for specific hypotheses on the reality of nature.[61] He did not pursue directly a generalised ether theory.

Perhaps the most sustained programme of hydrodynamic reduction was carried on by a Norwegian mathematical physicist working in near isolation at the University of Oslo. Carl Anton Bjerknes attended Dirichlet's lectures at Göttingen in 1855–6 and studied there also with Riemann.[62] Impressed with Dirichlet's proof that a solid sphere in a uniform stream of frictionless, imponderable fluid would not be dragged along by the flow, Bjerknes set out to discover whether oscillations and pulsations of such spheres, producing oscillations and pulsations in the fluid, might be affected by forces of the type usually ascribed to action at a distance. In a series of mathematical papers stretching from 1863 to the end of the century, but largely unknown outside Scandinavia, he gradually extended the scope of his theory until by 1880 he could reproduce hydrodynamically the basic forces of magnetism (including induced diamagnetism and paramagnetism), electrostatics, and electromagnetism.[63] Isolated magnetic poles and electric charges appeared as pulsations in volume, polar magnets as linear oscillations, and electric currents as rotational oscillations about the axis of the current. One troublesome aspect of this description was that it produced forces opposite to those observed in nature, giving always repulsions where attractions should have appeared. More important, however, Bjerknes's description did not keep up with the demands of electromagnetic theory as it developed after about 1870, when Maxwell's electromagnetic theory of light became widely known on the Continent.

Although hydrodynamics offered a ready avenue for reducing forces to propagated motions in an ether field and, thereby, for unifying all forces, one could equally well begin, not with the nature of ether, but with the differential

equations one wished to represent, and then construct whatever kinds of ether motions the equations seemed to require. That was the approach of Maxwell in Britain and of his nearest competitor on the Continent, Ludwig Lorenz. A professor of engineering at Copenhagen and, like Bjerknes, relatively unknown, Lorenz developed during the 1860s sophisticated mathematical descriptions of processes in physical optics, particularly of reflection and refraction at boundaries. He conceived his optical equations originally on the basis of an elastic ether, but he soon became convinced that elasticity theory was incompatible mathematically with the boundary requirements of optics. Abandoning the elastic ether, and ostensibly all ethers, he developed phenomenologically a set of wave equations for the propagation of light that behaved correctly at boundaries. But Lorenz was not content with a mathematical theory of optics; like many others he sought the physical unity of nature.

'As is well known', Lorenz began a paper in 1867, 'science in our century has succeeded in demonstrating so many connections between the different forces, between electricity and magnetism, between heat, light, molecular, and chemical forces, that one is led with a certain necessity to regard them all as *manifestations of one and the same force*'. Despite this necessity, unity still seemed distant: 'Generally, the two electricities are still viewed as electrical *fluids,* light as vibrations of the *ether,* and heat as motions of the *molecules of bodies*'.[64] Lorenz presented his own first step towards remedy with a demonstration that the motions of light waves were actually motions of electric currents and that both were propagated by contiguous action.

Beginning his analysis with general equations directly relating electric currents and electromotive forces (equations of conduction, previously derived by Kirchhoff from Weber's law), Lorenz argued that no change in observable consequences would follow if these equations, representing instantaneous action at a distance, were replaced by a similar set incorporating the assumption that electrical action traveled at the velocity of light. That reproduced Kirchhoff's equations in terms of retarded potentials, like Riemann's. From the latter equations he then derived a set of partial differential equations of the same form as his phenomenological wave equations for light, but more comprehensive. Reversing the derivation, he obtained the conduction equations by integration from the wave equations, assuming boundary conditions that he had previously found necessary for light at an interface. This proof of mathematical identity Lorenz took to be sufficient evidence for identifying light physically with electric currents and also for assuming the differential wave equations for electric currents to be the 'original and generally valid ones'. With the characteristic emphasis of field theory, he argued on the basis of this priority of the differential description for the further priority of continuously propagated action over action at a distance: 'Every action of electricity and of electrical currents in reality depends only on the electrical condition of the *immediately surrounding* elements'.[65]

The form of the interconvertible equations of light and of currents, Lorenz continued, showed that light could be regarded as '*rotational* oscillations [of current] in the interior of bodies around axes whose direction is the same as that which we regard as the direction of vibration according to the elasticity theory',[66] that is, around the axis of linear polarisation, transverse to the direction of propagation. Whereas a steady current would be a steady rotation continued along its own axis, oscillations in the current would be propagated by electromagnetic induction perpendicular to the current axis, constituting light. One sees immediately the family resemblance of this hypothesis to Maxwell's picture, apparently unknown to Lorenz, of magnetic vortices in the ether. That raises the question of what medium Lorenz imagined to be rotating. He preempted the question unambiguously: 'This conception gives scarcely any basis for maintaining the hypothesis of an ether, since one may very probably assume that in so-called empty space there is contained so much material substance that it offers sufficient substratum for the motion'.[67] But of course this is just another ether hypothesis, one in which ether and normal matter differ only in aggregation or density and not in substance.

Force fields

Ether fields provided a means for spreading force in space and thereby for escaping the problems of Weber's velocity- and acceleration-dependent law of force. But they also transformed push–pull notions of force at a point into energy states in ether, reinforcing a more general recognition of energy as an independent, conserved, entity. Coupled with emphasis on direct, positive description, this suggested an alternative field conception: Treat energy as the reality and ignore the unobservable ether. As discussed previously, such views only gradually overcame the stigma of *Naturphilosophie*, but characteristics of the emergent transition appear in a very short paper presented by Carl Neumann in 1868 to the Göttingen Society of Science.[68]

Neumann developed in a new way Riemann's earlier theory of propagated forces and retarded potentials. 'I take the liberty', Neumann asserted, 'of regarding potential [potential energy] as primary, as the characteristic motive power (*Bewegungs-Antrieb*), while conceiving forces as secondary, as the form in which that power manifests itself'. He then demonstrated that if potential propagated with the velocity of light, then the effective action of an electrostatic 'emissive potential' of one electrical mass m on another m', given by mm'/r, would be changed when the two were in relative motion into

a 'receptive potential', corresponding exactly to Weber's law of force. All of the known laws of electromagnetic action (known only for *closed* currents) likewise followed. As an indication of the generality of his approach, Neumann showed that the results were independent of whether currents were considered from a 'unitary' perspective – Weber's view, with the positive electrical fluid in motion and the negative tied to the ponderable mass of the conductor – or from a 'dualistic' perspective, with both fluids in motion. This generality would not pertain, he noted, for open circuits, a consideration soon to be emphasised by Helmholtz.

Neumann's intent seems to have been not so much to replace Weber's atomistic electrical ether with a pure force field as to defend Weber and his law against a charge by Helmholtz that velocity-dependent potentials were unacceptable from the perspective of energy conservation. He had recently taken a position at Leipzig, where he came into close association with Weber's circle of friends, and in several later publications he fortified Weber's defences. Nevertheless, the thrust of Neumann's defence ran quite counter both to Weber's premises and to his style of analysis. By taking potential (energy) as fundamental and making it propagate through space he gave to it the independent status of a force field, somewhat like J. R. Mayer's force of heat. With good reason the later Energeticists, who took energy to be the only ultimate reality, looked back to Neumann as well as to Mayer as pioneers of their viewpoint.[69] In typical fashion also for all field theorists, Neumann usually based his descriptions on differential equations rather than force laws, justifying that approach as the most positive and nonhypothetical, and looked to maximum and minimum principles for the foundation of dynamics. Here that meant that he took Hamilton's principle in mechanics to have 'unlimited validity' and used it both to derive Weber's force law from the 'receptive potential' and to show that this potential would obey, approximately, conservation of *vis viva*. Even though Neumann employed his field description only to describe the force between particles of Weber's electrical ether, he made that ether largely superfluous for the propagation of light; and even though he may have had in mind a material basis, or ether field, for his propagating potential, he made no reference to it and thereby undermined the necessity of such a material foundation for a force field.

Helmholtz: energy and electrodynamics

Field theories of electromagnetism, as stressed previously, emerged to replace action at a distance for the general community of German physicists when energy was recognised as an entity different from and more fundamental than moving force. Field theories also emerged only when it seemed apparent that no action-at-a-distance theory could avoid satisfactorily the velocity- and acceleration-dependence of Weber's law. The problems of both conceptual shifts can be observed in two famous papers by Helmholtz, his 1847 formulation of conservation of force and his 1870 alternative to Maxwell's electromagnetic field theory.

In 1847, Helmholtz set out quite consciously to build a bridge between German metaphysics (Kant's) and French physics, much as Weber and Fechner were doing at the same time.[70] Helmholtz, however, sought to exclude from nature all considerations of *Geist,* in the sense of soul and purpose, which signalled to him the excesses of *Naturphilosophie*. He wished to establish the physics of point atoms and attractive and repulsive forces as necessary, on both rational and empirical grounds, and in so doing to unite all causes in nature under the determinism of conservation of *vis viva*. Helmholtz's primary conceptual tools in this effort, I shall argue, were the concept of force as relation and a distinction between quantity and intensity.

To Helmholtz, as to Weber and Fechner, forces existed as relations: 'Motive force, as the cause of change, can be predicated only in cases involving at least two bodies spatially related to one another and is then to be defined as the effort of the two bodies to change their relative positions' (cf. the discussion of Weber in the section 'Weber's electrical ether'). To Helmholtz again these relations were rational relations, but in a restricted sense. Following Kant in the view that our knowledge of nature is scientific only to the degree that we understand its laws as necessary, he argued that forces must be invariable: 'As ultimate causes, forces which do not vary in time should be found'.[71] Even implicit time dependence, he supposed, would vitiate the comprehensibility of nature. If force did not always return to the same value for the same spatial relation, *vis viva* would not necessarily be conserved, but might be produced continuously from nothing. That would violate not only the principle of sufficient reason but also, from the empirical side, the impossibility of a perpetual motion machine. The same sort of argument applied to extended atoms, for which continuously acting rotational forces could be imagined, so that only attractive and repulsive spatial forces were allowable.

To formulate his argument verbally and mathematically, Helmholtz had continually to move back and forth between concepts of force as moving force and as *vis viva* and to reconcile the two ideas. This he accomplished with a distinction, drawn in the Kantian tradition, between quantity and intensity as two different categories of understanding. We know concepts as quantities or intensities depending on whether they are considered to add up extensively – that is, side by side (*nebeneinander*), like space – or intensively – on top of one another (*aneinander*), like density. In his construction

of continuous matter from forces, Kant applied this distinction in order to demonstrate how a given quantity of matter could completely fill different spaces with different intensities, in contrast to the full-or-empty conception of Newton and Descartes.[72] Helmholtz, conceiving forces already as rational relations, took the logical distinction over into the realm of forces acting between discrete, point atoms, making Newtonian moving force the measure of intensity of force and, under various circumstances, either *vis viva,* potential, or work the measure of the conserved quantity of force.

The intriguing aspect of Helmholtz's description is that he ascribed two aspects to the single notion of a tensional force between atoms at a given spatial separation. The relation possessed at any instant an intensity of tension producing changes in spatial relation, but it also possessed a quantity of tension connecting the past history of the relation to its future, to which the present action either added or subtracted.[73] This quantity was potential. Helmholtz described it as the quantity of tensional force, or work, available in a spatial relation for future consumption, thinking of the quantity consumed between two separations R and r as 'the sum total of the intensities of all the forces which act at all distances between R and r'.[74] Tensional force consumed became *vis viva,* and conversely, *vis viva* lost became a quantity of tensional force available again for consumption.

Helmholtz was able to maintain mathematically his dual conception of the single entity force only through a nonrigorous understanding of an integral. Development of the independent energy concept removed this problem while separating intensity and quantity into force and energy. The point here, however, is that already in Helmholtz's conception potential was a real physical quantity that became increasingly real through myriad theoretical and experimental applications of the conservation law. And as it became an entity the rational notion of force as an abstract relation of atoms became less acceptable. Weber's notion of velocity- and acceleration-dependent action at a distance was unacceptable to Helmholtz on rational grounds and Helmholtz's own notion of action at a distance proved unacceptable on physical grounds.

When, in 1870, Helmholtz turned his attention to electromagnetism he faced two successful challenges to his conception of force, each of which offered a conceivable physical basis for the unity of nature (Woodruff, 1968; Hirosige, 1969). On the one hand, implacably, sat Weber's law for forces acting at a distance, which Helmholtz had finally to admit could conserve *vis viva* formally in spite of its implicit time dependence. He set as his first task uncovering its deeper flaws. On the other hand loomed the propagation theories, particularly Maxwell's elegant mathematical system, but also the several less extensive field theories already discussed. Helmholtz could not accept ether fields like Maxwell's and Riemann's, which replaced force altogether with motions in ethereal matter, apparently because he considered that matter could act only through force;[75] but neither could he accept pure force fields, because a propagating pure force, taking time to act, would be again a time-dependent force like Weber's. Yet field theories of both kinds did succeed in explaining light propagation electromagnetically, as by now seemed necessary. The retarded potential theories reproduced correct empirical laws of electrodynamics from the assumption that inverse square forces travelled at the velocity of light, and Maxwell had derived the correct velocity of light from purely electrodynamic laws. Helmholtz's second and major task, therefore, was to provide an action-at-a-distance alternative to Weber's law that would rely on non–time-dependent forces and still provide an electromagnetic theory of light propagation.

Helmholtz began his search for flaws in Weber's conception by comparing Weber's potential function with a well-known phenomenological potential of Franz Neumann (Carl Neumann's father), describing interactions between closed circuits, and with Maxwell's theory. For this purpose he put Carl Neumann's propagating potential on an equal footing with Weber's mathematically equivalent, direct one. The differences among Weber's, Franz Neumann's, and Maxwell's expressions had not previously been easy to define, but Helmholtz showed in a lucid analysis that all three could be subsumed under a single potential describing the interaction between any two current elements of lengths ds and $d\sigma$, carrying currents i and j, and separated by a distance r:[76]

$$-\frac{ij}{2c^2r}[(1+k)\cos(d\sigma, ds)+(1-k)\cos(r, ds)\cos(r, d\sigma)]\,ds\,d\sigma$$

Here k is a variable parameter. When $k=-1$, Weber's potential results, assuming that a current consists of electrical particles in motion. Franz Neumann's potential corresponds to $k=+1$ when the expression is integrated over closed circuits; and $k=0$ leads to results similar to Maxwell's. When applied only to closed circuits, k vanishes from the general expression and all three cases reduce to Neumann's. Only an investigation of open circuits could distinguish among them.

As mentioned previously, Kirchhoff had derived from Weber's law general equations for currents in extended conductors. Helmholtz now found in the same way equations of electricity in motion corresponding to his generalised potential function. From the new equations he showed that Weber's potential, and only that one among the three choices, would in certain cases of open circuits lead to unstable motions of electricity. For example, if two like electrical particles approached each other at a very high speed, their relative velocity would become infinite in a finite distance. Weber and his supporters soon suggested ways of circumventing the difficulty, and a bitter dispute followed, but Helmholtz found no reason to doubt his original judgement that 'the inadequacy of the Weberian law here brought to light is founded deep in its nature'.[77]

Helmholtz still attributed the inadequacy of Weber's law to velocity dependence in the potential between any two electrical atoms. He attempted to establish his own potential solely on the basis of spatial separation between any two points where currents existed (phenomenologically rather than as moving atoms of electricity), this being 'the only spatial magnitude which is completely determined by two points'.[78] Current velocity, nevertheless, remained hidden in the strength of the current at any point. Helmholtz simply ignored it, even though he himself considered currents to be electricity in motion.

Such mathematical manoeuvres could not impress physical theorists like Weber; indeed, Helmholtz later stated that his law was 'no elementary expression of the ultimate acting forces'.[79] It did allow him, however, to employ an instantaneous action-at-a-distance force between currents while avoiding either explicit velocity-dependence or propagation of force.

The latter issue brings us to Helmholtz's second task, explaining propagation of light electromagnetically. For that he appropriated Maxwell's theory to his own ends. Maxwell had shown in 1863 that an elastic displacement in a medium possessing the electric and magnetic properties of free space would propagate at the velocity of light, through a process of mutual induction between electric and magnetic polarisations (Siegel, this volume). The polarisations were attributed by Maxwell to motions in ether; they constituted electric and magnetic forces, and no actions except between contiguous elements of ether occurred. Few on the Continent pretended to grasp how this contiguous action was supposed to function, nor what electricity might be, and Maxwell offered little aid. But his equations provided a thorough description of electromagnetic effects and of electromagnetic waves propagating at the speed of light. The critical element in those equations was the assumption that electric displacement, or polarisation, acted during the displacement exactly like an electric conduction current.

Considering these features from his own perspective, Helmholtz assumed ether to be an electrically and magnetically polarisable medium in which all forces – electrostatic, electromagnetic, and magnetic – acted directly at a distance between its parts (presumably atoms, but not explicitly). The polarisation at any point, an elastic response, was proportional to the sum of the polarising forces acting on that point from all distances. Helmholtz's basic equations, therefore, were two elasticity equations for electric and magnetic polarisations, both of the form $\mathbf{F} = a\mathbf{P}$, where **F** is force, **P** polarisation, and a an elastic constant. The forces, however, were complex and interconnected between the two equations. Electric forces resulted from free electricity, electricity of polarisation, changing conduction currents, changing polarisation currents, and changing magnetic polarisations, the last three being sources of electromagnetic induction. Magnetic forces, similarly, resulted from magnetic polarisation and from electric conduction currents and polarisation currents.

Employing his own electromagnetic potential, but including both conduction and polarisation currents, Helmholtz derived from the elasticity equations partial differential equations for purely electric and for purely magnetic polarisations, having eliminated cross terms. These new equations were wave equations with solutions describing both transverse and longitudinal waves. The choice $k = 0$ in the potential made the longitudinal waves vanish (infinite velocity), leaving only transverse waves of electric and magnetic polarisation with perpendicular oscillations. Assuming the electric polarisability of ether to be very large, Helmholtz showed that the transverse waves would spread – *propagate* is not quite right – with the velocity of light. In the limit, therefore, and with the choice $k = 0$, his theory reduced mathematically to Maxwell's theory of light.[80]

In this derivation the original assumption of instantaneous action at a distance disappeared, to be replaced by equations apparently describing successive action between contiguous elements of a medium. The result, however, was only mathematical and not physical; it did not imply that polarisations actually propagated, but only that they spread in time. In each original elasticity equation – through differentiations, transformations, and substitutions – Helmholtz had replaced the summed forces of currents and polarisations, acting *from all distances* on a local region, by differentials of their effects (polarisations) *in the local region*. Spreading of polarisation, or the wave equation, arose from the replacement process as a coincidental result of the relation between spatial and temporal derivatives in the laws of electromagnetic action. No approximations were necessary, not even neglect of long-range actions over short-range ones.

Helmholtz was fully aware of the coincidental nature of his agreement with Maxwell and turned the coincidence to support his own view: 'The remarkable analogy between the motion of electricity in a dielectric and that of the luminiferous ether does not depend on the particular form of Maxwell's hypothesis, but follows in essentially the same way if we maintain the older view of electrical action at a distance'.[81] But though he used his own formulation of spatial action at a distance against Maxwell's ether field, Helmholtz also turned Maxwell's field against Weber's time-dependent action at a distance. 'Maxwell's hypothesis appears to me to be very important', he said in 1872, 'because it furnishes proof that nothing is implied in electrodynamical phenomena that forces us to reduce them to an anomalous [*ganz abweichend*] kind of natural forces, to forces that depend, not merely on the positions of the corresponding masses but also on their motions'.[82] With strong physical theories on either hand, both alien, Helmholtz stood on the neutral territory of uninterpreted mathematics and argued that neither alternative possessed the force of necessity, the necessity he had attempted to establish in 1847 for direct spatial action between point atoms.

Conclusion

What was the historical significance of Helmholtz's ether theory? This question has sometimes been answered with the observation that Helmholtz translated Maxwell's theory into terms comprehensible to Continental physicists; and that is certainly correct so far as it goes. By providing a double-ended theory that displayed both poles from which previous electromagnetic theories of light propagation had been sought – atoms and forces *versus* differential field equations – Helmholtz made clear what propagation would have to mean from the perspective of forces acting at a distance. But in supplying this illumination, I suggest, the theory also illuminated the inadequacies of any action-at-a-distance theory, especially for energy considerations.

By 1870 propagation meant propagation of energy. Energy had become the symbol of unity in nature and the quantity that had to be followed through any series of conversions and transmissions: for example, from a chemical battery, to an electric current, to polarisation of the ether, to induced currents, to heat. Any acceptable electromagnetic theory of light, therefore, had to supply a direct explanation of the process of propagation of the energy of light. Ironically, Helmholtz, the most notable author of energy concepts on the Continent, could not meet this demand in his own theory, primarily because it did not distribute energy independently through space; it was not a field theory.

Wave theorists of light ever since Fresnel had considered light to be *vis viva* in ether. In the usual action-at-a-distance theories, which treated ether as an elastic solid, one supposed that direct actions over several molecular distances were negligible in comparison with actions between adjacent molecules. Even here, therefore, *vis viva* was effectively propagated by successive action. Time for transmission derived from inertia of the molecules, each taking time to respond to the force exerted by its neighbours, and the process was conceived mechanically under Newton's laws of motion. In Helmholtz's phenomenological ether theory, however, actions occurred at all distances, there were no masses to carry *vis viva,* all energies were potential energies, and no mechanics was supplied. Propagation of light could almost be conceived as a spreading of potential energy of polarisation, but one had to regard the energy of polarisation in any local element of ether as a temporary receptacle for energy of abstract spatial relation between this element and all other elements at all distances. The local polarisation, upon relaxation, was again transmitted to all distances. Energy did not propagate between contiguous elements of ether as seemed necessary for phenomena of light, particularly when the wave equation was considered fundamental.

Problems of this sort are not merely retrospective evaluations of Helmholtz's theory. They were recognised and discussed immediately and formed the background for the reception and development of Maxwell's field theory in Germany. Joseph Stefan, for example, published in 1874 a detailed analysis of the energy relations in Helmholtz's theory, intending to illuminate the question

> whether one does not in general have to conceive magnetic and electric phenomena as conditions of a medium, perhaps the luminiferous ether, and whether particularly one does not have to assume that magnetic and electric forces are only apparent actions at a distance, being in fact immediate actions of the medium, dependent on its momentary states and therefore also propagated, just as these states, with finite velocity.[83]

Although Stefan preferred the latter theory he recognised that it was not yet a necessity but only more coherent. He limited his critique to a development of the energy relations that would be required for translating between the two views. He obtained in this way a clear perception of a problem soon widely recognised as basic to the development of Maxwell's, or any other, field theory of electromagnetism: What was the relation between energy in the ether field and the source of that field? His own preference was to treat energy in ether as the independent reality and to make source strength dependent on the state of the ether.

Although Stefan's specific views were apparently not influential, the problems that he recognised in Helmholtz's interpretation of Maxwell's theory were also appreciated by the acknowledged giants of Continental electromagnetic theory, H. A. Lorentz and Heinrich Hertz. Lorentz originally followed Helmholtz's own path and tried to generate a more detailed mechanical description of the role of electricity in matter that would allow an explanation of phenomena of physical optics in terms of the interaction of an electromagnetic

wave of polarisation with electricity bound to molecules (Hirosige, 1969). Hertz, however, represented the new generation of physicists who believed that differential equations presented the most perspicuous view of reality and that theories should be based directly on them. After bemoaning the incomprehensibility of the relation in Maxwell's theory between sources and propagating effects he produced in 1890 his purely mathematical theory of an electromagnetic field.[84] He eliminated the distinction present in both Maxwell's and Helmholtz's theories between polarising forces and polarisation – thereby eliminating the distinction between forces and ether – and made of the field an entity unto itself. By employing this independent field and relating it to sources, Lorentz created his electron theory in the 1890s (Schaffner, 1969). Lorentz believed that some sort of ether had to be imagined as the basis of the field, but others, such as Wien and Abraham, actively sought a pure force field that would reduce even matter and mechanics to electromagnetic processes (McCormmach, 1970). This electromagnetic view of nature matched the goal of the Energeticists – Wilhelm Ostwald and Georg Helm are notable – who sought to reduce all of nature to energy alone. Although their programme foundered on the second law of thermodynamics, their rejection of atomism and belief in the ultimate continuity of nature was widely shared, even by critics such as Max Planck, to the degree that atomists like Ludwig Boltzmann despaired of having any impact in Germany at all.

Developments in the electromagnetic theory of light only partly caused this shift to continuity in nature, but they well exemplify general trends. The atomistic ethers of Weber and Helmholtz had given way to the electromagnetic ether field, which came increasingly to mean a field of electromagnetic energy. Energy and continuous flux seem best to symbolise late nineteenth-century views of nature in Germany.

Notes

1 T. S. Kuhn, 'Energy conservation as an example of simultaneous discovery', in *Critical problems in the history of science,* ed. M. Clagett (Madison, Wis., 1959), 321–56; P. M. Heimann, 'Conversion of forces and the conservation of energy', *Centaurus 18* (1974), 147–61.

2 S. D. Poisson, 'Mémoire sur la théorie du magnétisme', *Mémoire de l'Académie 5* (1820–2), 247–338, 488–533.

3 *Physicists* will refer here to practising scientists holding institutional positions in physics, mathematics, astronomy, or related fields when they devoted significant effort to *Physik,* as recognised in the abstracting journal *Fortschritte der Physik.*

4 R. S. Turner, 'Julius Robert Mayer', in *Dictionary of scientific biography,* vol. 9 (1974), 235–40.

5 'Ueber notwendige Konsequenzen und Inkonsequenzen der Wärmemechanik', a lecture delivered in 1869 to the Versammlung deutscher Naturforscher und Aerzte, in

Die Mechanik der Wärme in gesammelten Schriften von Robert Mayer, ed. J. J. Weyrauch, 3rd rev. ed. (Stuttgart, 1893), 357.

6 Ibid., 356.

7 H. von Helmholtz, 'The aim and progress of physical science', opening address in 1869 to the Versammlung deutscher Naturforscher und Aerzte, in H. von Helmholtz, *Selected writings of Hermann von Helmholtz*, ed. and trans. Russell Kahl (Middletown, Conn., 1971), 223–45; F. Gregory, *Scientific materialism in nineteenth-century Germany* (Boston, 1977), 100–21.

8 'Bemerkungen über die Kräfte der unbelebten Natur', *Annalen der Chemie und Pharmcie 42* (1842); reprinted in Weyrauch, *Mechanik der Wärme*, 24.

9 *Die organische Bewegung in ihrem Zusammenhange mit dem Stoffwechsel: ein Beitrag zur Naturkunde* (Heilbronn, 1845); reprinted in Weyrauch, *Mechanik der Wärme*, 48.

10 B. Gower, 'Speculation in physics: the history and practice of *Naturphilosophie*', *Studies in History and Philosophy of Science 3* (1973), 301–56.

11 I. Kant, *Kant's gesammelte Schriften*, ed. Preussischen Akademie der Wissenschaften, vols. 21–2, *Opus postumum* (Berlin and Leipzig, 1936), 21:593, lines 7–15, quoted in W. K. Werkmeister, 'The Critique of Pure Reason and Physics', *Kant Studien 68* (1977), 41.

12 *Opus postumum*, 22:583, lines 23–7, quoted in Werkmeister, 'The Critique and Physics', 45.

13 In *Zusatz* 5 (1881) to 'Ueber die Erhaltung der Kraft: eine physikalische Abhandlung' (1847), in H. von Helmholtz, *Wissenschaftliche Abhandlungen von Hermann von Helmholtz* (Leipzig, 1882), 1:73; reprinted in Helmholtz, *Selected writings*, 53.

14 *Beiträge zur Dynamik des Himmels: in populärer Darstellung* (Heilbronn, 1848); reprinted in Weyrauch, *Mechanik der Wärme*, 162, 176.

15 G. Helm, *Die Energetik: Nach ihrer geschichtlichen Entwicklung* (Leipzig, 1898), 16–27.

16 R. Kohlrausch and W. Weber, 'Elektrodynamische Maassbestimmungen insbesondere Zurückführung der Stromintensitäts – Messungen auf mechanisches Maass', *Abhandlungen der Königlichen Sächsischen Gesellschaft der Wissenschaften zu Leipzig 3* (1857); reprinted in *Wilhelm Weber's Werke*, 6 vols. (Berlin, 1892–4), 3:652. See also the 'Vorwort' to Kohlrausch and Weber's paper, *Berichte über die Verhandlungen der Königlichen Sächischen Gesellschaft der Wissenschaften zu Leipzig 17* (1855); reprinted in *Werke*, 3:594ff.

17 My description may be taken as complementary to Kenneth Caneva's presentation of Weber's and Fechner's methodology as hypothetico-deductive. K. Caneva, 'From galvanism to electrodynamics: the transformation of German physics and its social context', *Historical Studies in the Physical Sciences 9* (1978), 63–159. See also K. H. Wiederkehr, *Wilhelm Eduard Weber: Erforscher der Wellenbewegung und der Elektrizität, 1804–1891* (Stuttgart, 1967); A. P. Molella, 'Philosophy and nineteenth-century German electrodynamics: the problem of atomic action at a distance', unpublished doctoral dissertation, Cornell University, 1972.

18 'Ueber ein allgemeines Grundgesetz der elektrischen Wirkung', *Abhandlungen bei Begründung der Königlichen Sächsischen Gesellschaft der Wissenschaften zu* (Leipzig 1, 1846), reprinted in Weber, *Werke*, 3:157; 'Auszug', *Annalen der Physik und Chemie 73* (1848), reprinted in ibid., 245.

19 'Reflections on the law of universal gravitation', in P. de Laplace, *The system of the world*, trans. H. H. Harte (Dublin, 1830), bk. 4, chap. 15.

20 'Of the motion of a system of bodies', in Laplace, *System*, bk. 3, chap. 5; *Essai philosophique sur les probabilités*, 6th ed. (Paris, 1840), 2–4.

21 'Lettre de M. Ampère à M. le comte Berthollet, sur la détermination des proportions dans lesquelles les corps se combinent d'après le nombre et la disposition respective des molécules dont leurs particules intégrantes sont composées', *Annales de chimie 90* (1814), 45–86; 'Note de M. Ampère sur la chaleur et sur la lumiere considérées comme résultant de mouvemens vibratoires', *Annales de chimie 58* (1835), 432–44.

22 Summarised by L. Pearce Williams, *Michael Faraday, a biography* (New York, 1971), 142–51.

23 G. T. Fechner, 'Ueber die Verknupfung der Faraday'schen Inductions-Erscheinungen mit den Ampère'schen elektro-dynamischen Erscheinungen', *Annalen der Physik und Chemie 64* (1845), 337–45.

24 'Allgemeines Grundgesetz', 3:213 ff.

25 'Ueber die Erregung und Wirkung des Diamagnetismus nach den Gesetzen inducirter Ströme', *Annalen der Physik und Chemie 73* (1848); reprinted in Weber, *Werke*, 3:255–68.

26 'Ueber das Aequivalent lebendiger Kraft', *Annalen der Physik und Chemie 152* (1874); reprinted in Weber, *Werke*, 4:302. See also fragment of letter from Weber to Fechner in G. T. Fechner, *Ueber die physikalische und philosophische Atomenlehre* (Leipzig, 1855), 73.

27 'Allgemeines Grundgesetz', 3:212 ff.

28 'Aequivalent lebendiger Kraft', 4:303.

29 'Electrodynamische Maassbestimmungen insbesondere über das Princip der Erhaltung der Energie', *Abhandlungen der Königlichen Sächsischen Gesellschaft der Wissenschaften zu Leipzig 10* (1871); reprinted in Weber, *Werke*, 4:255 n.

30 'Elektrodynamische Maassbestimmungen insbesondere Widerstandsmessungen', *Abhandlungen der Königlichen Sächsischen Gesellschaft der Wissenschaften zu Leipzig 1* (1852); reprinted in Weber, *Werke*, 3: esp. 400–5.

31 'Vorwort', 3:595 ff.; Kohlrausch and Weber, 'Zurückführung der Stromintensitäts-Messungen auf mechanisches Maass', 3:652–67.

32 'Zur Galvanometrie', *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen 10* (1862); reprinted in Weber, *Werke*, 4:91–6.

33 'Princip der Erhaltung der Energie', 4:268–78.

34 Ibid., 278 ff.; 'Ueber die Bewegungen der Elektricität in Körpern von molekularer Konstitution', *Annalen der Physik und Chemie 156* (1875); reprinted in Weber, *Werke*, 4:334–57.

35 'Elektrodynamische Maassbestimmungen insbesondere über die Energie der Wechselwirkung', *Abhandlungen der Königlichen Sächsischen Gesellschaft der Wissenschaften zu Leipzig 11* (1878), reprinted in Weber, *Werke*, 4:389–95; 'Elektrodynamische Maassbestimmungen insbesondere über den Zusammenhang des elektrischen Grundgesetzes mit dem Gravitationsgesetze', first published in ibid., 4:524 ff.

36 See Zöllner's editorial contributions to *Abhandlungen zur atomistischen Theorie der Elektrodynamik von Wilhelm Weber*, vol. 1, bk. 1, of J. C. F. Zöllner, *Principien einer elektrodynamischen Theorie der Materie* (Leipzig, 1876); Weber, 'Zusammenhang des elektrischen Grundgesetzes mit dem Gravitationsgesetze', 4:481–5.

37 'Aphorismen', in Weber, *Werke*, 4:630 ff.

38 J. C. F. Zöllner, *Transcendental physics*, trans. C. C. Massey (London, 1880).

39 *Atomenlehre*, 23, 53, 73, 187, 206–9.

40 Ibid., ix.

41 See J. F. Herbart, *Johann Friedrich Herbart's sämmtliche Werke* (Leipzig, 1850–2), ed. G. Hartenstein, vol. 5, *Lehrbuch zur Psychologie* (1816), and vols. 3–4, *Allgemeine Metaphysik, nebst den Anfängen der philosophischen Naturlehre* (Köngsberg, 1828–9), esp. pt. 5, 'Umrisse der Naturphilosophie'.

42 *Atomenlehre*, 90–1, 95.

43 Ibid., 63–73.
44 Ibid., basic viewpoint developed 106–18.
45 Ibid., 65.
46 Ibid., 95.
47 Ibid., 113. Cf. G. T. Fechner, *Elemente der Psychophysik* (Leipzig, 1860), esp. chaps. 1, 5.
48 *Atomenlehre,* 181–210.
49 C. F. Gauss, *Werke* (Göttingen, 1877), 5:629.
50 Riemann's associations and interests at Göttingen are described by R. Dedekind in 'Lebenslauf', *Bernhard Riemann's gesammelte mathematische Werke,* ed. H. Weber, 2nd ed. (Leipzig, 1892), 539–58.
51 'Fragmente philosophischen Inhalts', in H. Weber, *Riemann's Werke,* 524.
52 Ibid., 528 ff.
53 Ibid., 503.
54 Ibid., 533.
55 *Schwere, Elektricität, und Magnetismus: nach den Vorlesungen von Bernhard Riemann,* ed. K. Hattendorff (Hannover, 1876), 330.
56 'Ein Beitrag zur Elektrodynamik', *Annalen der Physik und Chemie 131* (1867); reprinted in H. Weber, *Riemann's Werke,* 288.
57 See note by H. Weber in *Riemann's Werke,* 293.
58 Ibid., 533 ff.
59 'Ueber Integrale der hydrodynamischen Gleichungen, welche den Wirbelbewegungen entsprechen', *Journal für die reine und angewandte Mathematik 55* (1858); reprinted in Helmholtz, *Wissenschaftliche Abhandlungen,* 1:101–34.
60 *Vorlesungen über mathematische Physik: Mechanik* (Leipzig, 1876), lectures 15–26.
61 Ibid., 'Vorrede'.
62 Bjerknes's career is described by V. Bjerknes in C. A. Bjerknes, *Hydrodynamische Fernkräfte: fünf Abhandlungen über die Bewegung kugelförmiger Körper in einer inkompressiblen Flüssigkeit (1863–1880),* trans. A. Korn, ed. A. Korn and V. Bjerknes (Leipzig, 1915), 212–23. See also Dedekind, 'Lebenslauf', *Riemann's Werke,* 551.
63 'Hydrodynamische Analogien zu den elektrostatischen und magnetischen Kräften', *Naturen* (1880), reprinted in Bjerknes, *Hydrodynamische Fernkräfte,* 176–211; G. Forbes, 'Hydrodynamic analogies to electricity and magnetism', *Nature 24* (1881), 360 ff.
64 'Ueber die Identität der Schwingungen des Lichts mit den electrischen Strömen', *Annalen der Physik und Chemie 131* (1867), 243 ff.
65 Ibid., 261–2.
66 Ibid., 262.
67 Ibid., 263.
68 'Resultate einer Untersuchungen über die Principien der Elektrodynamik', *Nachrichten von der Königlichen Gesellschaft der Wissenschaften und der Georg Augustus Universität zu Göttingen* (1868), 223–35. For Neumann's earlier attempt to derive the magnetic rotation of light from Weber's law of force see Knudsen (1976).
69 E.g., Helm, *Energetik,* 229–31.
70 P. M. Heimann, 'Helmholtz and Kant: the metaphysical foundations of *"Ueber die Erhaltung der Kraft"', Studies in History and Philosophy of Science 5* (1974), 205–38; Y. Elkana, 'Helmholtz's "Kraft": an illustration of concepts in flux', *Historical Studies in the Physical Sciences 2* (1970), 263–98.
71 'Erhaltung der Kraft', in Helmholtz, *Wissenschaftliche Abhandlungen,* 1:14; reprinted in Helmholtz, *Selected writings,* 5.
72 *Metaphysical foundations of natural science,* trans. J. Ellington (Indianapolis and New York, 1970), esp. chap. 2.

73 'Erhaltung der Kraft', in Helmholtz, *Wissenschaftliche Abhandlungen,* 1:17–19; reprinted in Helmholtz, *Selected writings,* 6–8.
74 Ibid., in Helmholtz, *Wissenschaftliche Abhandlungen,* 1:22; reprinted in Helmholtz, *Selected writings,* 11.
75 Ibid., in Helmholtz, *Wissenschaftliche Abhandlungen,* 1:14; reprinted in Helmholtz, *Selected writings,* 4.
76 'Ueber die Bewegungsgleichungen der Elektricität für ruhende leitende Körper', *Journal für die reine und angewandte Mathematik 72* (1870); reprinted in Helmholtz, *Wissenschaftliche Abhandlungen,* 1:567.
77 Ibid., 553.
78 Ibid., 565.
79 'Ueber die Theorie der Elektrodynamik', *Monatsberichte der Berliner Akademie* (1872); reprinted in Helmholtz, *Wissenschaftliche Abhandlungen,* 1:637.
80 'Bewegungsgleichungen der Elektricität', 1:625–8.
81 Ibid., 558.
82 'Theorie der Elektrodynamik', 1:639.
83 'Ueber die Gesetze der magnetischen und elektrischen Kräfte in magnetischen und dielektrischen Medien und ihre Beziehung zur Theorie des Lichtes', *Sitzungsberichte der Kaiserlichen Königlichen Akademie der Wissenschaften zu Wien 70* (1874), 596.
84 R. McCormmach, 'Heinrich Hertz', in *Dictionary of scientific biography,* vol. 6 (1972), 340–50.