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On the Interaction of Elementary Particles

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At the present stage of the quantum theory little is known about the nature of interaction of elementary particles, Heisenberg considered the interaction of "Platzwechsel" between the neutron and the proton to be of importance to the nuclear structure.

Recently Fermi treated the problem of β-disintegration on the hypothesis of "neutrino". According to this theory, the neutron and the proton can interact by emitting and absorbing a pair of neutrino and electron. Unfortunately the interaction energy calculated on such assumption is much too small to account for the binding energies of neutrons and protons in the nucleus.

To remove this defect, it seems natural to modify the theory of Heisenberg and Fermi in the following way. The transition of a heavy particle from neutron state to proton state is not always accompanied by the emission of light particles, i.e., a neutrino and an electron, but the energy liberated by the transition is taken up sometimes by another heavy particle, which in turn will be transformed from proton state into neutron state. If the probability of occurrence of the latter process is much larger than that of the former, the interaction between the neutron and the proton will be much larger than in the case of Fermi, whereas the probability of emission of light particles is not affected essentially.

Now such interaction between the elementary particles can be described by means of a field of force, just as the interaction between the charged particles is described by the electromagnetic field. The above considerations show that the interaction of heavy particles with this field is much larger than that of light particles with it.

In the quantum theory this field should be accompanied by a new sort of quantum, just as the electromagnetic field is accompanied by the photon.

In this paper the possible natures of this field and the quantum accompanying it will be discussed briefly and also their bearing on the nuclear structure will be considered.

Besides such an exchange force and the ordinary electric and magnetic forces there may be other forces between the elementary particles, but we disregards the latter for the moment.

Fuller account will be made in the next paper.

Field Describing the Interaction

In analogy with the scalar potential of the electromagnetic field, a function $U(x, y, z, t)$ is introduced to describe the field between the neutron and the proton. This function will satisfy an equation similar to the wave equation for the electromagnetic potential.

Now the equation

$$\left\{\Delta - \frac{1}{c^2}\,\frac{\partial^2}{\partial t^2}\right\}\; U = 0 \tag{1}$$

has only static solution with central symmetry $\frac{1}{r}$, except the additive and the multiplicative constants. The potential of force between the neutron and proton should, however, not be of Coulomb type, but decrease more rapidly with distance. It can expressed, for example by

$$+\text{or} - g^2 \frac{e^{-\lambda r}}{r}, \tag{2}$$

where g is a constant with the dimension of electric charge, i.e., cm.$^{3/2}$ sec.$^{-1}$ gr.$^{1/2}$ and λ with the dimension cm.$^{-1}$

Since this function is a static with central symmetry of the wave equation

$$\left\{\Delta - \frac{1}{c^2}\,\frac{\partial^2}{\partial t^2} - \lambda^2\right\}\; U = 0, \tag{3}$$

let this equation be assumed to be the correct equation for U in vacuum. In the presence of the heavy particles, the U–field interacts with them and causes the transition from neutron state to proton state.

Now, if we introduce the matrices

$$\tau_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

and denote the neutron state and the proton state by $\tau_3 = 1$ and $\tau_3 = -1$ respectively, the wave equation is given by

$$\left\{\Delta - \frac{1}{c^2}\,\frac{\partial^2}{\partial t^2} - \lambda^2\right\}\; U = -4\pi g \tilde{\Psi}\;\frac{\tau_1 - i\tau_2}{2}\;\Psi, \tag{4}$$

where Ψ denoted the wave function of the heavy particles, being a function of time, position, spin as well as τ_3', which takes the value either 1 or -1.

Next, the conjugate complex function $\tilde{U}(x,\ y,\ z,\ t)$, satisfying the equation

$$\left\{\Delta - \frac{1}{c^2}\ \frac{\partial^2}{\partial t^2} - \lambda^2\right\}\ \tilde{U} = -4\pi g \tilde{\Psi}\ \frac{\tau_1 + i\tau_2}{2}\ \Psi, \tag{5}$$

is introduced, corresponding to the inverse transition from proton to neutron state.

Similar equation will hold for the vector function, which is the analogue of the vector potential of the electromagnetic field. However, we disregard it for the moment, as there's no correct relativistic theory for the heavy particles. Hence simple non–relativistic wave equation neglecting spin will be used for the heavy particle, it the following way

$$\begin{aligned}\Bigg\{\frac{h^2}{4}\ \left(\frac{1+\tau_3}{M_N} + \frac{1-\tau_3}{M_P}\right)\ \Delta \quad &+ ih\ \frac{\partial}{\partial t} - \frac{1+\tau_3}{2}\ M_N c^2 - \frac{1-\tau_3}{2}\ M_P c^2 \\ &-g\ \left(\tilde{U}\ \frac{\tau_1 - i\tau_2}{2} + U\ \frac{\tau_1 + i\tau_2}{2}\right)\Bigg\}\ \Psi = 0,\end{aligned} \tag{6}$$

where h is Planck's constant divided by 2π and M_N, M_P are the masses of the neutron and the proton respectively. The reason for taking the negative sign in front of g will be mentioned later.

The equation (6) corresponds to the Hamiltonian

$$\begin{aligned}H = \left(\frac{1+\tau_3}{4M_N} + \frac{1-\tau_3}{4M_P}\right)\ \overrightarrow{p}^2 \quad &+ \frac{1+\tau_3}{2}\ M_N c^2 + \frac{1-\tau_3}{2}\ M_P c^2 \\ &+g\ \left(\tilde{U}\ \frac{\tau_1 - i\tau_2}{2} + U\ \frac{\tau_1 + i\tau_2}{2}\right)\end{aligned} \tag{7}$$

where $\overrightarrow{p}$ is the momentum of the particle. If we put $M_N c^2 - M_P c^2 = D$ and $M_N + M_P = 2M$, the equation (7) becomes approximately

$$H = \frac{\overrightarrow{p}^2}{2M} + \frac{g}{2}\ \left\{\tilde{U}\ \ (\tau_1 - i\tau_2) + U\ \ (\tau_1 + i\tau_2)\right\} + \frac{D}{2}\ \tau_3, \tag{8}$$

where the constant term Mc^2 omitted.

Now consider two heavy particles at point (x_1, y_1, z_1) and (x_2, y_2, z_2) respectively and assume their relative velocity to be small. The field at (x_1, y_1, z_1) due to the particle at (x_2, y_2, z_2) are, from (4) and (5),

$$\left.\begin{aligned} &U\,(x_1,\ y_1,\ z_1) = g\ \frac{e^{-\lambda\tau_{12}}}{\tau_{12}}\ \frac{\left(\tau_1^{(2)} - i\tau_2^{(2)}\right)}{2} \\ \text{and} \\ &\tilde{U}\,(x_1,\ y_1,\ z_1) = \ g\frac{e^{-\lambda\tau_{12}}}{\tau_{12}}\ \frac{\left(\tau_1^{(2)} - i\tau_2^{(2)}\right)}{2}, \end{aligned}\right\} \tag{9}$$

where $(\tau_1^{(1)},\ \tau_2^{(1)},\ \tau_3^{(1)})$ and $(\tau_1^{(2)},\ \tau_2^{(2)},\ \tau_3^{(2)})$ are the matrices relating to the first and the second particles respectively, and τ_{12} is the distance between them.

Hence the Hamiltonian for the system is given, in the absence of the external fields by,

$$\begin{aligned} H = \; & \frac{\overrightarrow{p}_1^2}{2M} + \frac{\overrightarrow{p}_2^2}{2M} + \frac{g^2}{4} \left\{ \left(\tau_1^{(1)} - i\tau_2^{(1)}\right) \left(\tau_1^{(2)} + i\tau_2^{(2)}\right) \right. \\ & \left. + \left(\tau_1^{(1)} + i\tau_2^{(1)}\right) \left(\tau_1^{(2)} - i\tau_2^{(2)}\right) \right\} \frac{e^{-\lambda \tau_{12}}}{\tau_{12}} + \left(\tau_3^{(1)} + \tau_3^{(2)}\right) D = \\ & \frac{\overrightarrow{p}_1^2}{2M} + \frac{\overrightarrow{p}_2^2}{2M} + \frac{g^2}{2} \left(\tau_1^{(1)}\tau_1^{(2)} + \tau_2^{(1)}\tau_2^{(2)}\right) \frac{e^{-\lambda \tau_{12}}}{\tau_{12}} + \left(\tau_3^{(1)} + \tau_3^{(2)}\right) D, \end{aligned} \tag{10}$$

where $\overrightarrow{p}_1, \overrightarrow{p}_2$ are the momenta of the particles.

This Hamiltonian is equivalent to Heisenberg's Hamiltonian, if we take for "Platzwechselintegral"

$$J(\tau) = -g^2 \, \frac{e^{-\lambda r}}{r}, \tag{11}$$

except that the interaction between the neutrons and the electrostatic repulsion between the protons are not taken into account. Heisenberg took the positive sign for $J(r)$, so that the spin of the lowest energy state of H^2 was 0, whereas in our case, owing to the negative sign in front of g^2, the lowest energy state has the spin 1, which is required from the experiment.

Two constants g and λ appearing in the above equations should be determined by comparison with experiment. For example, using the Hamiltonian (10) for heavy particles, we can calculate the mass defect of H^2 and the probability of scattering of a neutron by a proton provided that the relative velocity is small compared with the light velocity.

Rough estimation shows that the calculated values agree with the experimental results, if we take for λ the value between 10^{12} cm^{-1}. and 10^{13} cm^{-1}. and for g a few times of the elementary charge e, although no direct relation between g and e was suggested in the above considerations.

Nature of the Quanta Accompanying the Field

The U–field above considered should be quantized according to the general method of the quantum theory. since the neutron and the proton both obey fermi's statistics, the quanta accompanying the U–field should obey Bose's statistics and the quantization can be carried out the line similar to that of the electromagnetic field.

The low of conservation of the electric charge demands that the quantum should have charge either $+e$ or $-e$. The field quantity U corresponds to the operator which increases the number of negatively charged quanta and decreases the number of positively charged quanta by one respectively. $\tilde{U}$, which is the complex conjugate of U, corresponds to the inverse operator.

Next, denoting

$$p_x = -ih\,\frac{\partial}{\partial x}, \quad \text{etc.,} \quad \mathrm{W} = \mathrm{ih}\,\frac{\partial}{\partial \mathrm{t}},$$

$$m_U c = \lambda h,$$

the wave equation for U in free space can be written in the form

$$\left\{p_x^2 + p_y^2 + p_z^2 - \frac{W^2}{c^2} + m_U c^2\right\}\ U = 0, \tag{12}$$

so that the quantum accompanying the field has the proper mass $m_U = \frac{\lambda h}{c}$.

Assuming $\lambda = 5 \times 10^{12}$ cm^{-1}., we obtain for m_U a value 2×10^2 times as large as the electron mass. as such a quantum with large mass and positive or negative charge has never been found by the experiment, the above theory seems to be on a wrong line. We can show, however, that, in the ordinary nuclear transformation, such a quantum can not be emitted into outer space.

Let us consider, for example, the transition from a neutron state of energy W_N to a proton state of energy W_P, both of which include the proper energies. These states can be expressed by the wave function

$$\Psi_N(x,y,z,t,\ 1) = u(x,y,z)e^{-iW_N t/h}, \quad \Psi_N(x,y,z,t,\ -1) = 0$$

and

$$\Psi_P(x,y,z,t,\ 1) = 0, \quad \Psi_P(x,y,z,t\ -1) = \nu(x,y,z)e^{-iW_P t/h},$$

so that, on the right hand side of the equation (4), the term

$$-4\pi g\tilde{\nu} u e^{-it(W_N - W_P)/h}$$

appears.

Putting $U = U'(x,y,z)e^{i\omega t}$, we have from (4)

$$\left\{\Delta - \left(\lambda^2 - \frac{\omega^2}{c^2}\right)\right\}\ U' = -4\pi g\tilde{\nu} u, \tag{13}$$

where $\omega = \frac{W_N - W_P}{h}$. Integrating this, we obtain a solution

$$U'(\overrightarrow{r}) = g\int\int\int\ \frac{e^{-\mu|r-r'|}}{|\ \overrightarrow{r} - \overrightarrow{r}'\ |}\ \tilde{\nu}(\overrightarrow{r}')u(\overrightarrow{r}')d\nu', \tag{14}$$

$$\text{where}\ \ \mu = \sqrt{\lambda^2 - \frac{\omega^2}{\mathrm{c}^2}}.$$

If $\lambda > \frac{|\omega|}{c}$ or $m_U c^2 >|\ W_N - W_P\ |$, μ is real and the function $J(r)$ of Heisenberg has the form $-g^2\ \frac{e^{-\mu r}}{r}$, in which μ, however, depends on $|\ W_N - W_P\ |$, becoming smaller and smaller as the latter approaches $m_U c^2$. This means that the range of interaction between a neutron and a proton increases as $|\ W_N - W_P\ |$ increases.

Now the scattering (elastic or inelastic) of a neutron by a nucleus can be considered as the result of the following double process: the neutron falls into a proton level in the nucleus and a proton in the latter jumps to a neutron state of positive kinetic energy, the total energy being conserved throughout the process. The above argument, then shows that the probability of scattering may in some cases increase with the velocity of the neutron.

According to the experiment of Bonner, the collision cross section of the neutron increases, in fact, with the velocity in the case of lead whereas it decreases in the case of carbon and hydrogen, the rate of decrease being slower in the former than the latter. The origin of this effect is not clear, but the above considerations do not, at least, contradict it. For, if the binding energy of the proton in the nucleus becomes comparable with $m_U c^2$, the range of interaction of the neutron with the former will increase considerable with the velocity of the neutron, so that the cross section will decrease slower in such case than in the case of hydrogen, i.e., free proton. Now the binding energy of the proton in C^{12}, which is estimated from the difference of masses of C^{12} and B^{11}, is

$$12,0036 - 11,0110 = 0,9926.$$

This corresponds to a binding energy 0,0152 in mass unit, being thirty times the electron mass. Thus in the case of carbon we can expect the effect observed by Bonner. The arguments are only tentative, other explanations being, of course, not excluded.

Next if $\lambda < \frac{|\omega|}{c}$ or $m_U c^2 <| W_N - W_P |$, μ becomes pure imaginary and U expresses spherical undamped wave, implying that a quantum with energy greater than $m_U c^2$ can be emitted in outer space by the transition of the heavy particle from neutron state to proton state, provided
that $| W_N - W_P |> m_U c^2$.

The velocity of U–wave is greater but the group velocity is smaller than the light velocity c, as in the case of the electron wave.

The reason why such massive quanta, if they ever exist, are not yet discovered may be ascribed to the fact that the mass m_U is so large that condition $| W_N - W_P |> m_U c^2$ is not fulfilled in ordinary nuclear transformation.

§ 4. Theory of $\beta -$ Disintegration

Hitherto we have considered only the interaction of U–quanta with heavy particles. Now, according to our theory, the quantum emitted when a heavy particle jumps from a neutron state to a proton state can be absorbed by a light particle which will then in consequence of energy absorption rise from a neutrino state of negative energy to an electron state of positive energy. thus an anti–neutrino and an electron are emitted simultaneously from the nucleus. Such intervention of a massive quantum does not alter essentially the probability of β-disintegration, which has been calculated on the hypothesis of direct coupling of a heavy particle and a light particle, just as, in the theory of internal conversion of γ-ray, the intervention of the proton does not affect the final result. Our theory, therefore, does not differ essentially from Fermi's theory.

Fermi considered that an electron and a neutrino are emitted simultaneously from the radioactive nucleus, but this is formally equivalent to the assumption that a light particle jumps from a neutrino state of negative energy to an electron state of positive energy.

For, if the eigenfunctions of the electron and the neutrino be Ψ_k, ϕ_k respectively, where $k = 1, 2, 3, 4$, a term of the form

$$-4\pi g' \sum_{k=1}^{4} \tilde{\psi}_k \phi_k \tag{15}$$

should be added to the right hand side of the equation (5) for $\tilde{U}$, where g' is a new constant with the same dimension as g.

Now the eigenfunctions of the neutrino state with energy and momentum just opposite to those of the state ϕ_k is given by $\phi'_k = -\delta_{kl\tilde{\phi}l}$ and conversely $\phi_k = \delta_{kl\tilde{\phi}l'}$, where

$$\delta = \begin{pmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{pmatrix},$$

so that (15) becomes

$$-4\pi g' \sum_{k,l=1}^{4} \tilde{\psi}_k \delta_{kl} \tilde{\phi}'_l. \tag{16}$$

From equations (13) and (15), we obtain for the matrix element of the interaction energy of the heavy particle and the light particle an expression

$$gg' \int \ldots \int \tilde{\nu}(\overrightarrow{r}_1) u(\overrightarrow{r}_1) \sum_{k=1}^{4} \tilde{\psi}_k(\overrightarrow{r}_2) \phi_k(\overrightarrow{r}_2) \frac{e^{-\lambda r_{12}}}{r_{12}} \, d\nu_1 d\nu_2, \tag{17}$$

corresponding to the following double process: a heavy particle falls from the neutron state with the eigenfunction $u(\overrightarrow{r})$ into the proton state with the eigenfunction $\nu(\overrightarrow{r})$ and simultaneously a light particle jumps from the neutrino state $\phi_k(\overrightarrow{r})$ of negative energy to the electron state $\psi_k(\overrightarrow{r})$ of positive energy. In (17) λ is taken instead of μ, since the difference of energies of the neutron state and the proton state, which is equal to the sum of the upper limit of the energy spectrum of β-rays and the proper energies of the electron and the neutrino, is always small compared with $m_U c^2$.

As λ is much larger than the wave numbers of the electron state and the neutrino state, the function $\frac{e^{-\lambda r_{12}}}{r_{12}}$ can be regards approximately as a δ-function multiplied by $\frac{4\pi}{\lambda^2}$ for the integrations with respect to $x_2, \ y_2, \ z_2$.

The factor $\frac{4\pi}{\lambda^2}$ comes from

$$\int\int\int \frac{e^{-\lambda r_{12}}}{r_{12}}\, d\nu_2 = \frac{4\pi}{\lambda^2}.$$

Hence (17) becomes

$$\frac{4\pi g g'}{\lambda^2}\int\int\int \tilde{\nu}(\overrightarrow{r})u(\overrightarrow{r})\ \sum_k\ \tilde{\psi}_k(\overrightarrow{r})\phi_k(\overrightarrow{r})d\nu \tag{18}$$

or by (16)

$$\frac{4\pi g g'}{\lambda^2}\ \int\int\int \tilde{\nu}(\overrightarrow{r})u(\overrightarrow{r})\ \sum_{k,l}\ \tilde{\psi}(\overrightarrow{r})\delta_{kl'}\tilde{\phi}'_l(\overrightarrow{r})d\nu, \tag{19}$$

which is the same as the expression (21) of Fermi, corresponding to the emission of a neutrino and an electron of positive energy states $\phi'_k(\overrightarrow{r})$ and $\psi_k(\overrightarrow{r})$, except that the factor $\frac{4\pi g g'}{\lambda^2}$ is substituted for Fermi's g.

Thus the result is the same as that of Fermi's theory, in this approximation, if we take

$$\frac{4\pi g g'}{\lambda^2} = 4 \times 10^{-50}\mathrm{cm}^3.\mathrm{erg},$$

from which the constant g' can be determined. Taking, for example, $\lambda = 5\times 10^{12}$ and $g = 2\times 10^{-9}$, we obtain $g' \cong 4\times 10^{-17}$, which is about 10^{-8} times as small as g.

This means that the interaction between the neutrino and the electron is much smaller than between the neutron and the proton so that the neutrino will be far more penetrating than the neutron and consequently more difficult to observe. The difference of g and g' may be due to the difference of masses of heavy and light particles.

Summary

The interactions of elementary particles are described by considering a hypothetical quantum which has the elementary charge and the proper mass and which obeys Bose's statistics. The interaction of such a quantum with the heavy particle should be far greater than that with the light particle in order to account for the large interaction of the neutron and the proton as well as the small probability of β-disintegration.

Such quanta, if they ever exist and approach the matter close enough to be absorbed, will deliver their charge and energy to the latter. If, then, the quanta with negative charge come out in excess, The matter will be charged to a negative potential.

These arguments, of course, of merely speculative character, agree with the view that the high speed positive particles in the cosmic rays are generated by the electrostatic field of the earth, which is charged to a negative potential.

The massive quanta may also have some bearing on the shower produced by cosmic rays.