

## Straightforward Derivation of $E = mc^2$

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The equivalence of mass and energy is a direct consequence of the variation of mass with velocity. That mass varies with velocity according to the Lorentz factor  $\gamma$  is a consequence of special relativity, but it does not depend on the bizarre notions of space and time associated with that theory. Relational optics also infers such a relationship under the assumption that the transformation law for momentum is the same for matter as for light [See *Universe without Space and Time*, p. 112]. The famous equation  $E = mc^2$  can be derived simply by using Newton's second law with the Lorentz factor to calculate the kinetic energy imparted to a mass. The kinetic energy so calculated manifests itself as an increase in inertial mass. The procedure for calculating the change in kinetic energy is outlined on pages 139-140 of *Universe without Space and Time*. The mathematical details are filled in below.

$$p = \frac{m_0 v}{\sqrt{1 - \frac{v^2}{c^2}}}; \quad v = \frac{p}{\sqrt{m_0^2 + \frac{p^2}{c^2}}}$$

$$F = \frac{dp}{dt} = \frac{d(\gamma m_0 v)}{dt}; \quad \gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

$$dE = F dx = \frac{dp}{dt} dx = v dp = \frac{p dp}{\sqrt{m_0^2 + \frac{p^2}{c^2}}} = \frac{\frac{1}{2} d(p^2)}{\sqrt{m_0^2 + \frac{p^2}{c^2}}} = \frac{\frac{c^2}{2} dz}{\sqrt{m_0^2 + z}}; \quad z = \frac{p^2}{c^2}$$

$$E = \int_0^{KE} dE = \int_0^{\frac{p^2}{c^2}} \frac{\frac{c^2}{2} dz}{\sqrt{m_0^2 + z}} = c^2 \sqrt{m_0^2 + \frac{p^2}{c^2}} - m_0 c^2 = c^2 \sqrt{m_0^2 + \frac{\gamma^2 m_0^2 v^2}{c^2}} - m_0 c^2$$

$$= \gamma m_0 c^2 - m_0 c^2 = m_0 c^2 \left( 1 + \frac{1}{2} \frac{v^2}{c^2} + \dots \right) - m_0 c^2 = \frac{1}{2} m_0 v^2 + \dots$$