*TIM MAUDLIN**

SUBSTANCES AND SPACE-TIME: WHAT ARISTOTLE WOULD HAVE SAID TO EINSTEIN

Abstract—This essay consists of two parts. The first is an exegetical analysis of the "stripping" argument of *Metaphysics* Z.3. I contend that the passage is not in *propria persona* and that the resolution of the aporia depends upon a careful consideration of the metaphysical relationship between essential properties and the subjects of which they are predicated. The second part applies this conclusion to a problem recently raised by John Earman and John Norton about whether the general theory of relativity is compatible with both determinism and a substantivalist interpretation of space-time. I argue that their difficulty can be avoided by an Aristotelian account of the essential properties of space-time.

Introduction

LIKE EMPEDOCLES' βουγενῆ ἀνδρόπρῳρα ("man-faced ox-progeny") this essay will doubtless appear to be an ungainly, if not monstrous, concatenation of diverse topics. To conjoin exegesis of Aristotle's *Metaphysics* with an examination of the general theory of relativity must seem an act both ahistorical and philosophically perverse. And indeed, the marriage of these subjects is not an entirely happy one. Detail in the textual analysis has been suppressed in favor of a concise statement of the metaphysical moral. Still, the advantages afforded by this peculiar conjunction outweigh the drawbacks, for it illustrates the relevance of long-standing philosophical analyses to modern problems, problems which often present themselves in a gaudy technical garb. Although Aristotle had no notion of anything like modern physics, some of the puzzles he grappled with are exactly those which we face in trying to interpret our scientific theories. I hope to show that some of the recent debates concerning the status of space-time in general relativity afford a case in point.

Aristotle's *Metaphysics* is a tract concerned with being, and the central books focus on those entities which most unqualifiedly partake in being: substances. Book Z, in particular, is devoted to the articulation of criteria by which to distinguish substances from non-substances; to the identification of substances; and to examinations both of the relationship between substances

*Rutgers University, Department of Philosophy, Davison Hall, Douglass Campus, New Brunswick, NJ 08903, U.S.A.

Received 25 *March* 1989; *in revised form* 6 *October* 1989.

Stud. Hist. Phil. Sci., Vol. 21, No. 4, pp. 531–561, 1990. Printed in Great Britain.
0039–3681/90 $3.00 + 0.00

and their properties and of that between form and matter. This last topic is particularly pressing for Aristotle, for having supplemented the doctrine of the *Categories* with a hylomorphic analysis of substance he must now determine whether and how form and matter can combine to produce a truly unified substance. He finds that assimilating the relationship of matter and form to that of substance and accident has disastrous consequences, so he must struggle to find a new model for their inter-relationship. The first section of this essay examines one of the central passages in the treatment of matter and form, the famous "stripping" argument of Z.3. I shall contend that this passage contains a premise, commonly overlooked in discussions, which identifies the argument as non-Aristotelian and whose denial provides Aristotle with a means of avoiding some absurd consequences. Resolution of this puzzle demands an appreciation of the unique relationship which exists between substances and their essential properties, a relationship more intimate than that between substance and accident. This moral shall be employed in the analysis of the ontological status of space-time in the later section of the paper.

The notion of substance has played a leading role in the ongoing debate about the nature of space and time. Although neither Isaac Newton nor Samuel Clarke claimed that space is a substance, the positions taken in the famous debate between Leibniz and Clarke have come to be known, respectively, as relationism and substantivalism. Various philosophical views asserting and denying that space and time are real entities distinct from physical objects have been refined, and the pendulum swings between them have often been guided by technical results of current physical theory. One such argument has recently been presented by John Earman and John Norton, who maintain that a substantivalist must be willing to accept a radical indeterminacy in physical law as the price for that ontology.[1] The second part of this paper will examine their argument and the strictures on substances that underlie it. I shall argue that the moral found in Aristotle has application here, since an Aristotelian account of the relationship between space-time points and their properties may provide the substantivalist an escape from Earman and Norton's dilemma. I shall also argue that the structure of the mathematical entities used to represent space-time naturally suggests a misleading analysis of the ontology of space-time itself, which analysis underpins the *prima facie* indeterminism in the substantival interpretation of the General Theory of Relativity. But let us begin with our passage from The Philosopher.

[1]The arguments are presented in J. Earman and J. Norton, 'What Price Spacetime Substantivalism? The Hole Story', *British Journal for the Philosophy of Science* **38** (1987), 515–525; in J. Norton, 'Einstein, the Hole Argument, and the Objectivity of Space', in J. Forge (ed.), *Measurement, Realism, and Objectivity* (Dordrecht; D. Reidel, 1987), pp. 153–188; and in J. Earman, 'Why Space is Not a Substance', typescript.

Part 1: The Z.3 Puzzle[2]

In *Metaphysics* Z.3 Aristotle begins his investigation into substance. Four candidates for "the substance of each thing" are put forward: the essence, the universal, the genus, and the subject, the last of which is the topic of Z.3 (1028b34–36). The subject (*τὸ ὑποκείμενον*, literally "what underlies") is identified as that of which everything else is predicated, and which is itself predicated of nothing else. This characterization immediately rules out all entities from the categories other than substance from being the ultimate (*πρῶτον*) subject since qualities, quantities, places, &c, are always qualities, quantities and places *of* something. Yet an individual such as Socrates, of whom qualities such as musicality may be predicated, and who would be a paradigm substance in the analysis Aristotle gives in the *Categories*, can also be analyzed into form and matter, and so it is not yet clear whether the ultimate subject is the form, the matter, or the compound of the two (1029a2–9). We must therefore examine which of these has the best claim to be that of which everything else is predicated. At 1029a7–30 Aristotle presents one way of understanding what a subject is, a way which entails that the identification of substance with the ultimate subject must be mistaken. The argument runs:[3]

> So it has now been stated in outline what substance is, *viz.*, it is that which is not said of a subject but everything else is said of it. Yet we must not only define it thus, for this is not sufficient: it is unclear, and moreover matter becomes substance. For if it is not substance, whatever else it might be escapes us; for since everything else has been stripped off nothing seems to remain. For all the other things are, on the one hand, affections and products and powers of bodies, and again the length and breadth and depth are sorts of quantities and not substances (for quantity is not a substance); but, rather, that to which these ultimately belong, *that* is substance. But since length and breadth and depth have been stripped off, we don't see anything left unless that which is bounded [or defined] by these is something, so that matter alone must seem to be substance to those who investigate it in this way. I mean by "matter" that which in itself is said to be neither something (*τι*)[4] nor so-much nor

[2]I am indebted for very useful comments on the section to Rob Bolton, Michael Rohr, and especially to Mary Louise Gill, whose influence permeates the whole. This section is an abbreviated version of 'Keeping Body and Soul Together: The Z.3 Puzzle and the Unity of Substances', *The University of Dayton Review* **19**, no. 3 (1988–1989), 121–134. Readers interested in the full range of textual arguments for this interpretation are directed to that paper.

[3]All translations are my own, from the text of Jaeger. I have aspired to literalness rather than elegance, and have tried to maintain consistent translations of the central technical terms.

[4]The translation of the term "*τι*" throughout this passage cannot but be tendentious. Aristotle says that according to this argument "*τι*" cannot be predicated of the ultimate subject, but then goes on to use that term in referring to it. This might suggest that the term is being used ambiguously; Rob Bolton has suggested to me that whereas here it means "a particular thing", later it just means "something" (albeit not a member of any of the categories). This interpretation can remove some self-contradictoriness from the passage, but to so *translate* it would hide the (at least) surface paradoxicality of the view. I choose to translate it uniformly as "something" because I believe that paradoxicality to be intended. An exactly parallel, and evidently intentional, rhetorical flourish is used by Plato in a text which I believe to be one of the sources of this argument: *Theaetetus* 202a4. More on that passage anon.

any of the other things by which being is defined. For there is something of which each of these is predicated, something (τι) whose being is distinct from the being of the things predicated (ᾧ τὸ εἶναι ἕτερον καὶ τῶν κατηγοριῶν ἑκάστῃ). For all the other things [than substance] are predicated of substance, and this itself is predicated of matter, so that the final [subject] in itself is neither something (τι) nor so-much nor anything else at all. Nor is it even the negations of these, for they too will belong to it accidentally. It follows for those considering it by these arguments that matter is substance, yet this is impossible. For separability and "thisness" are taken to belong most of all to substance, wherefore the form and the compound of both would seem to be substance rather than matter.

On the face of it, this passage presents a *reductio ad absurdum* of the definition of substance as that of which everything is predicated while it is predicated of nothing else. For the definition seems to pick out as substance some sort of matter which of itself is entirely characterless, indefinite and amorphous; but substances are pre-eminently individual *things* (cf 1082a11–20). But, as with every aporia which Aristotle presents, we must try to determine how much of the argument is in *propria persona*, whether Aristotle is committed to each of the premisses, and whether there is any escape from the conclusion. In this instance such an inquiry is especially urgent, for if the argument is in *propria persona*, not only will it show that Aristotle believed the ultimate subject of predication not to be substance, it will also indicate that he regarded that ultimate subject as entirely characterless. That is, this passage might seem to prove Aristotle to be committed to the doctrine of prime matter.[5]

I shall maintain that the argument of Z.3 is not Aristotle's own, that he is not committed by it to the doctrine of prime matter, and that it employs what is for Aristotle a mistaken account of the relationship between form and matter in a compound substance. I shall begin by presenting some circumstantial clues that suggest this reading. But the major burden of this part shall be an exposition and analysis of the premisses employed in the argument. Among these we shall find one that runs counter to the ensuing discussions in the *Metaphysics*, which allows us to see just why Aristotle need not accept the conclusion of Z.3.

There are clear indications in this passage that Aristotle was following out a line of thought already extant in the philosophical literature. He notes that various things follow "for those who investigate it in this way" (1029a19, cf. also ἐκ μὲν οὖν τούτων θεωροῦσι at a26). This may only refer to those who accept that the ultimate subject is substance, but it may also indicate that other

[5]Whether the passage is in Aristotle's own voice is already a matter of some debate. W. Charleton, for example, takes the main line of argument to be that of certain opponents of Aristotle who are identified in *Physics* II (*Aristotle's Physics* I, II, translated with notes by W. Charleton (Oxford: Clarendon, 1970) p. 138). H. M. Robinson, in contrast, states that he finds no plausible grounds for supposing a change in voice ('Prime Matter in Aristotle', *Phronesis* **19** (1974), 184). Much of this part of the essay will be devoted to defending the view that the argument is not Aristotle's.

peculiar premisses are used in the deduction. The conclusion of the passage seems stronger than the announced intention and also conflicts with Aristotle's later pronouncements. For although he begins by suggesting that the characterization of substance as subject is obscure and in need of clarification, he seems to end by rejecting it altogether since it implies that a totally indeterminate matter is substance. Yet the synopsis and review of Z which begins chapter H still recognizes the subject as substance (1042a13) and identifies this underlying substance as matter (1042b9). So either Aristotle has simply forgotten the result of Z.3 or he feels that those objections to the subject-criterion of substance have been overcome. Let us take the latter possibility as a working hypothesis and seek a premise of the Z.3 argument which may have come to be rejected.

If the foremost point of the Z.3 passage is to identify the ultimate subject of predication, one line alone seems sufficient to solve that problem: " For all the other things are predicated of substance, and this itself is predicated of matter" (1029a23). Aristotle nowhere denies this assertion, and it corresponds with the identification of substance-qua-subject with matter in H.1 and H.2. But it is not simply the result that matter becomes substance which generates the difficulties of Z.3, but that the matter should be so entirely characterless, possessing none of the usual marks of substance. This reduction of matter to the totally indefinite does not follow simply from the claim that all things are predicated of substance and it of matter. Abstraction of all the predicates from the subject is involved, this being the stripping process so central to the text. What is involved in this abstraction? What conditions must hold for the separation of predicate and subject to be possible?

The stripping process of Z.3 is a matter of logical abstraction, of disregarding properties. Just as we construct a proof about triangle *qua* triangle not by considering a triangle which is neither scalene nor isosceles but by allowing the proof only to employ those features mentioned in the definition of a triangle, so too can we more generally disregard the inessential features of an object. We do not abstract color, say, from Socrates by imagining him to have no color, or by considering processes for removing all color from him, but by recognizing that the characterization of Socrates *qua* human being need not advert to his color.[6] If the stripping process is such a disregarding of properties, then the success of the procedure critically depends upon one condition: *the predicate disregarded cannot be part of the defining conditions of the subject*

[6]Russell Dancy, in 'On Some of Aristotle's Second Thoughts About Substances: Matter', *Philosophical Review* **LXXXVII** (1978). 372–413, assimilates the abstraction process to one of *imagining the property physically removed from the substance*, thus obscuring the vital difference between this logical argument and the empirical arguments concerning substantial change. Dancy suggests, for example, that we "strip off" a statue's color using turpentine (p. 398). Such appeal to physical procedures is clearly too weak for Aristotle's purpose: how could one strip off color generally, or be assured of an actual process which would eliminate all of the particular affections, products, and powers of bodies?

of which it is predicated. The essence of the predicate and that of the subject must be logically distinct, else the result of disregarding the predicate will not be to leave one with the subject as such.

The logical independence of the predicate and subject, then, is an essential premise of the Z.3 analysis. Further, it is not employed by Aristotle as a tacit background assumption, but is quite explicitly stated: "For there is something (τι) of which each of these is predicated, *something whose being is distinct from the being of the things predicated*" (1029a21–23). Indeed, this premise in conjunction with the assertion that "all the other things are predicated of substance, and this itself is predicated of matter" immediately yields the conclusion that matter cannot be substance. For whatever characteristics make something a substance are predicated of matter, which in itself must have a distinct, and hence non-substantial, essence. So our quarry is nearly cornered: we now have a simple two-premise argument whose conclusion conflicts with Aristotle's retention of matter or the subject as a sort of substance in book H. If he is to be consistent he must either reject that substance is predicated of matter or that subject and predicate always have logically distinct beings.

The distinctness of the being of the predicate from the being of the subject is *prima facie* a very questionable premise. After all, one would suppose that anything of which substance is predicated thereby *is* a substance, rather than the opposite. And, to take a mundane example, although we all agree that there is something of which the hardness, rectangularity, color, &c. of the table are predicated, we would balk at the suggestion that the subject is something entirely distinct from these properties. After all, it is the *table* which is colored, hard, rectangular, &c. and we have no reason to believe that the table could be defined (or could exist) independently of these qualities. The distinctness thesis immediately implies that *nothing* can be affirmed of the ultimate subject of predication as such (καθ' αὑτὸ), for to do so would be to predicate something of it. Indeed, despite his sanguine locutions early in the passage, Aristotle is forced to forgo calling the ultimate matter even *something* (τι) (1029a20).

As the reader will doubtless have anticipated, it is the premise which asserts the universal distinctness of the being of the subject from that of the predicate which I believe not to be accepted by Aristotle. If he has imported this premise from elsewhere, we should expect him to develop an account of predication which denies it. And should this premise fall, with it would fall both the general argument for prime matter in Z.3 and the objections to the characterization of substance as the ultimate subject of predication.

The stripping procedure of Z.3 does not immediately strike one as non-Aristotelian because the process of abstraction is one Aristotle often uses. One commonly can disregard various predicates of a subject, and must do so to properly understand *qua* what the subject has certain of its properties. And the stripping process clearly *is* legitimate when the being of the subject and the predicate are distinct. But this holds just when the predicate is an *accident* of the subject, rather than part of its *essence*. The Z.3 thesis, then, is that *every* predicate is an accident of its ultimate subject, prime matter, and hence can be logically stripped from it. But Aristotle denies this very thesis in *Metaphysics* Γ.4:

> And those who say this generally destroy the substance and the essence. For it is necessary for them to say that everything is accidental... To indicate the essence [of something] is to say that its being (τὸ εἶναι αὐτῷ) is nothing else... So that it is necessary for them to say that there will not be such an account [i.e. of the essence] of anything, but all attributes are accidental. For substance and accident are distinguished by this: the white is accidental to the man because although he is white, white is not in his essence.[7] But if everything is predicated accidentally, there will not be any ultimate thing of which they are predicated, if the accident always indicates a predicate of some subject. For it [i.e. the identification of new subjects] must go on *ad infinitum*, but this is impossible. (1007a20–b1).

The argument refuted in this passage bears a strong resemblance to that of Z.3. Someone has taken a position which implies that the predicate is always accidentally predicated of the subject, that it is never part of the essence of the subject, so that the beings of the predicate and of the subject are always distinct. But then the being of the subject itself must be predicated of yet another distinct subject, and so on indefinitely. The Z.3 passage bites off the regress by positing an ultimate subject which *has* no essence, nothing true of it as such. But Aristotle would surely conclude that the Z.3 argument, as much as this, destroys the substance and the essence since according to it nothing is ever essentially predicated of a subject. If Aristotle here rejects the notion that all predication is accidental, it is difficult to imagine that he would accept in Z.3 that the ultimate subject of predication is prime matter, all of whose positive attributes at a time are accidents.

If, then, the Z.3 argument is not in Aristotle's own voice, and if the unacceptable premise is not the identification of substance with the ultimate subject but rather the thesis that the being of the predicate and that of the subject are always logically distinct, then we should expect Aristotle to develop an account of the relationship between substantial form and the matter of which it is predicated which contradicts that thesis. This is just what he does.

Let us begin by considering the most clear and, *prima facie*, undeniable example of a substance: a living animal. An animal, such as a human being, has a form: its soul. That soul is predicated of a certain particular kind of matter: an organic body (cf. *de Anima* 412a28, *Metaphysics* 1037a5 ff., 1041b6). The immediate matter of a human is a complete body composed of

[7]The phrase here, οὐχ ὅπερ λευκόν, does not explicitly mention the essence, but is best rendered as saying that white is not in the essence of man. Cf. Jonathan Barnes' commentary in his translation of the *Posterior Analytics* (Oxford: Clarendon, 1975), p. 168.

non-uniform parts such as the face, hand, heart, &c. The matter of these in turn are the organic uniform parts such as blood and bone.[8] What is the logical relationship between the being of the form and the being of the matter in this case?

Aristotle is quite clear and emphatic on this point. The non-uniform parts, and even their uniform constituents, only are what they are when ensouled. A dead hand is a hand in name only, like the stone hand of a statue. A finger cannot survive separation from the body and remain a finger. Even flesh and bone, although their activities are less immediately evident, are truly flesh and bone only when part of a functioning animal (cf. *Metaphysics* 1035b22–27, 1036b27–32, 1040b5–16, *De Generatione Animalium* 734b25 ff.). So in the case of ensouled beings, the matter of which the soul is predicated is what it is only *when ensouled.* This dependence is reflected in the *definitions* of the parts: the parts are *logically* (and hence ontologically) posterior to the whole since their definitions must make reference to the whole ensouled organism:

> And the account of the right angle does not resolve into the account of the acute, but that of the acute into that of the right. For someone defining the acute must make use of the right since "acute" is "less than a right angle". And the circle is similarly related to the semi-circle, for the semi-circle is defined by the circle, and also the finger by the whole; for a finger is such-and-such a part of a man. So whatever are parts in the sense of matter, into which a thing is resolved as into matter, are posterior. . . (1035b6–12).

Similarly, the entire organic body of which the soul is predicated must be logically posterior to the soul since without the soul *it wouldn't be the sort of body it is.* Definition of the body must make mention of the soul and its functions.

This dependence of the subject on its predicate in the case of predicating a substantial form of matter permits Aristotle to escape the Z.3 aporia. Since the being of the matter is not logically distinct from the being of the form,[9] the abstraction process cannot be carried out. Form and matter, in the case of composites such as animals, manifest the sort of *unity* which is the hallmark of true substances. Much of *Metaphysics* Z, H, and Θ is devoted to the explication of how such a unity is possible. In the case of the unity of matter and form in a composite, the problem is originally motivated by the Z.3 puzzle.

The problem of the unity of form and matter in a substance, or of subject and predicate, has led by different routes to the twin monstrosities of metaphysics: prime matter and bare particulars. One need only begin with the

[8]The non-uniform parts are those whose own parts are not of the same sort. Thus, a part of a face such as a nose is not itself a face. In contrast, a part of flesh is itself flesh and a part of bone, bone.

[9]In saying that the being of the matter and that of the form are not logically distinct, I do not mean to imply that they are *identical*, just that one must make reference to the form in defining the matter. It is this degree of logical interdependence that thwarts the stripping process.

assumption that any concrete particular can be successively analyzed into a distinct form and matter, or predicate and subject, of which it is the logical sum. If one then asks, at each step of the analysis, from which part the *particularity* of the particular comes, one is ultimately faced with a dilemma. On the one hand, since the form or predicate is always a *universal*, the particularity would seem to always be inherited from the subject or matter. Following this line of thought one ends with particulars which have no characteristics in common with any others, and which can only be named, not defined and not known. That is, one arrives at a sort of bare particular or *haecceity*. Aristotle, in Z.3, examines the other path. He first strips off all of the universals and then notes that what remains *cannot* be a separable individual since it has no identifiable properties. But the moral in either case is the same. Subjects must not always be metaphysically separable from their predicates. Some properties must be essential properties if absurdities are to be avoided. In the next section we shall see how denying this fundamental principle has led to surprising conclusions about the compatibility of determinism and substantivalism about space-time. We shall also see that Aristotle's remedy to these difficulties, *viz*. the introduction of essential properties, is as effective a solution now as it was when the *Metaphysics* was written.

Part 2: Essences and Space-time

Although the Scholastic metaphysics of substratum and substantial form, accident and essence, was resolutely rejected during the scientific revolution, fundamental questions of ontology, and hence of metaphysics, cannot but be revived by changes in the foundations of our theoretical understanding of the world. Newtonian dynamics challenged the mechanical world-view, special relativity rejected not only absolute space but the categorical distinction between spatial and temporal properties, and general relativity replaced the static space-time continuum with one dynamic and non-(pseudo)-Euclidean. But mathematical formulae do not immediately yield up ontologies, and the most sweeping changes in theory are also the most subject to varied interpretation. We are still trying to come to grips with a theory introduced over 70 years ago: Einstein's general theory of relativity. Despite the striking mathematical clarity and beauty of the theory, it can be rather sibylline in its pronouncements about the ultimate furniture of the world.

The special theory of relativity welded space and time into a unified space-time continuum which admits of no unique partition into spaces-at-a-time. Still, as a 4-dimensional object, Minkowski space-time has a fixed universal geometrical structure. The primary insight of the general theory was to allow the geometrical structure itself, the space-time metric, to become a dynamic entity influenced by the distribution of matter and energy in the universe. Einstein often motivated this change, in part, by an appeal to a sort of generalized version of Newton's third law: since the trajectories of massive particles are determined in part by the inertial structure of space-time, the inertial structure should itself be affected by the particles.[10] This seems to elevate space-time and the metric into *physical objects co-ordinate with material particles*. Having sacrificed the magisterial aloofness from mundane events which the space-time of special relativity enjoyed, the space-time of general relativity becomes a dynamic physical entity.

From this point of view, general relativity seems to establish space-time as a sort of physical substance, capable of entering into reciprocal causal interaction with the commoner class of objects such as tables and chairs. That Einstein so regarded it is suggested by his reference, in several articles and addresses, to the "aether" of general relativity.[11] One is tempted to regard the space-time of general relativity as an immaterial rubber block, contorted and twisted by the matter in it. Still, nothing amounting to an argument has been produced, and the mathematical entities introduced in a physical theory are always vulnerable to being analyzed away, shown to be fictions or abstractions which don't directly represent the physical reality. And recently John Norton and John Earman purport to have found an argument which demonstrates that if determinism is to have a chance, space-time *cannot* be a substance.[12] It is this argument which shall be our focus in this section. Before presenting it, we must briefly review the mathematical structure of the general therory.[13]

Our project is to construct a mathematical object which represents the universe. We shall do this by beginning with mathematical simples and adding on successive layers of structure. One must keep in mind that the (abstract) ontological structure of the mathematical representation may suggest, but does not entail, an analogous metaphysical analysis of the physical structure that it

[10]Cf., for example, A. Einstein, *The Meaning of Relativity*, 5th edn (Princeton: Princeton University Press, 1956), pp. 55–56. That Einstein took this generalized law of action and reaction seriously as a physical principle, and did not use it just as a rhetorical device, is evidenced in his research into unified field theories. In 1929 he rejected a set of field equations proposed by Elie Cartan because he believed that they allowed a causal influence of the metrical structure of space on its parallel structure without positing a causal connection in the other direction (E. Cartan and A. Einstein, *Letters on Absolute Parallelism 1929–1932* (Princeton: Princeton University Press, 1979) pp. 73–74, with responses on pp. 81 and 89).

[11]Three examples are cited by Norton in Norton, *op. cit.*, note 1, p. 183. I argue in the appendix that this impression is only created by taking certain passages out of context, where they refer not to the physical reality of space-time but to the physical significance of *co-ordinatizations* of space-time.

[12]The papers in which this line is developed are those cited in note 1. Norton, in his 1987 paper, claims to find this argument in Einstein's papers of 1913 and 1914. See the appendix of this paper for some observations on Norton's claims.

[13]Throughout this section I have striven to avoid technical detail, although some familiarity with the theory is presupposed in certain of the footnotes and in the appendix. I have no quarrel with the mathematical detail of Earman and Norton's argument, only with the ontological interpretation of it. I hope my somewhat intuitive sketch captures the fundamental structure of the argument simply and clearly.

represents. Indeed, one of my main contentions is that a failure to distinguish between the ontology of the mathematical representation and that of the thing represented has led to Norton and Earman's conclusion.

We begin with a simple collection of *points*. The points are pure abstract individuals, bare mathematical particulars which, initially, bear no properties and enter into no relations. Still, as mathematical objects they are distinct individuals. If you like, you can call these objects *space-time points*,[14] but one must bear in mind that they do not as yet have any spatio-temporal properties and that nothing yet suggests that the set is particularly suited to represent space-time. That is, as mathematical objects space-time points do not differ from any other kind of points—they only become connected with space-time because of further structure defined on them. Also, one might say that these space-time points are to represent *events*, minimal regions of (physical) space-time.[15] But this is a tendentious statement, since one of the questions raised by the argument we are considering is whether such points *do* or *can* represent events.[16]

The set of mathematical points can next be invested with successive levels of structure. A topology can be given by specifying which sets constitute open sets. Affine structure can be appended by providing an affine connection. And, at the highest level, a metric can be supplied compatible with the connection. The mathematical object so constructed exhibits the metaphysical structure of a collection of bare particulars with accidental properties. The various regions in the model are distinct because the mathematical points were already individuated *ab initio*. The very same set of points might have been given a different topological or affine or metrical structure, and the result would have been a distinct mathematical object. This fact is crucial for understanding Earman and Norton's argument.

Briefly, the argument turns on an observation about co-ordinate transformations. Classically, transformations such as rigid rotations can be interpreted in two ways. When taken *passively*, the transformation represents how the situation would have been described had we chosen a different co-ordinate system. If we had directed the x-axis out that way instead of this, then the co-

[14]There is an extremely unfortunate ambiguity in the term *space-time points:* sometimes it refers to elements of the mathematical object which represents space-time, and sometimes to physical events. This ambiguity should be kept in mind when reading this paper and when comparing the locutions here to other works. I hope that my intention will always be clear in context. This confusion of mathematical representation with thing represented infects much mathematical physics. For example, it is perfectly natural to say that temperature *is* a scalar field and that a scalar field *is* a mapping from space-time points (ambiguous!) into the reals, which seems to imply that temperature is a sort of mapping. *One* of the "is"s clearly must mean "is represented by", but I can't even say which because of the ambiguity of "field" as between physical entity and mathematical representation.

[15]Events are often likened to physical occurrences infinitesimally localized in space and time, like the explosion of an (infinitesimal) firecracker. However, this is slightly misleading since there need not be a physical happening in a region of space-time in order that there be an event there.

[16]Cf. Norton, *op. cit.*, note 1, p. 157.

ordinates assigned to the physical space-time points would have been different. But when taken *actively* the co-ordinate transformation generates a representation of a *physically distinct state of affairs*, a state described in terms of the same original co-ordinate system. The Earth receives a new set of co-ordinates under the transformation not because the co-ordinatization of space-time has changed but because we are considering the position of the Earth as having changed. Just as we might have directed the x-axis a different way, so God might have given the material universe a different orientation in space, or so Newton and Clarke maintained over Leibniz's objections.

Classical rigid co-ordinate transformations such as rotations and translations in space and time engendered the classical philosophical puzzles: could the universe have begun two years earlier or have been situated three meters North of its present location? But in the context of the General Theory, or, more exactly, when one demands general covariance of one's physical laws, the problem acquires a different cast. General covariance requires that the equations which describe the evolution of a physical system hold in *any* co-ordinate system. In particular, consider two co-ordinate systems that match outside of some region (the "hole") but differ inside. Then if a model expressed in terms of the first co-ordinate system constitutes a solution of the field equations, so must the model expressed in terms of the second. This is just the familiar passive co-ordinate transformation which yields no metaphysical problems.

But Earman and Norton have directed our attention to the fact that a second interpretation of this co-ordinate transformation is available, an active interpretation, the so-called *hole diffeomorphism*. As in the case of rigid rotation cited above, under the active interpretation the re-co-ordinatization is thought of as representing not the same physical situation described in a new frame, but a new physical situation described in terms of the same co-ordinate system. One defines a smooth map from the hole onto itself and uses the map to transport all of the mathematical objects to their image points under the diffeomorphism.[17] Since *all* of the mathematical machinery, including the metric itself, is moved, if the old model constituted a solution of the field equations, so will the new one.

The dilemma can now be posed. We have two distinct mathematical models of the field equations which agree everywhere outside the hole but give

[17]At this point I have radically oversimplified in order to avoid technical detail. The problem is that there is no way to compare tensors at different points of the manifold to see if they are "the same". You can't just pick up a tensor and drop it somewhere else. The trick is this: first express the tensors in co-ordinate form relative to the original co-ordinatization. Second, let the original co-ordinate curves generate a new set of co-ordinate curves which are the images of the old ones under the diffeomorphism. Of course, outside the hole the curves map onto themselves. Finally, define the *new* tensors as those which have the same functional form relative to the new co-ordinate curves as the old ones had in the old co-ordinates. These new tensors are called the *carry alongs* of the old tensors under the diffeomorphism. See Earman and Norton's papers for the mathematical details of this operation.

different solutions inside the hole. If one is obliged to regard each of these models as representing a distinct physical state of affairs, then the laws which govern the world would be radically indeterministic. For given any region, no matter how small, a complete specification of the physical state of the universe outside the region plus the field equations do not fix the physical state of the world within the region.

Of course, so far we only have a *conditional* dilemma. *If* one regards the diffeomorphically generated mathematical models as representations of distinct physical possibilities, then radical indeterminism follows. Not surprisingly, physicists do not so regard the models. They would consider that the two models represent the same physical state. But Earman and Norton argue that *if one regards space-time as a substance one must accept the models as representations of distinct physical states.* Here, according to them, the substantivalist faces a dilemma: either he must accept radical indeterminism or abandon his substantivalism.[18]

Now the mathematics which Earman and Norton present, and which I have crudely sketched, is quite unexceptionable. However, the lynchpin of the argument is not a piece of mathematics but a bit of metaphysics. It is here that we must focus our attention: What is it to regard space-time as a substance? How does such an ontology constrain one's interpretation of mathematical representations? For I do not believe that Earman and Norton establish that a substantivalist must be committed to regarding the models as representing distinct physical states. Indeed, I believe that they, lured on by the peculiar abstract ontology of mathematical representations, have fallen into the very problem that Aristotle uncovered in *Metaphysics* Z.3. But instead of attacking their arguments directly, I want to approach the problem a bit obliquely, by allowing the arch-substantivalist, Newton, to speak in his own defence.

We must first note that there is nothing about the hole argument which makes it particularly appropriate only to the general theory of relativity. Models of the special theory, and of Newtonian dynamics, and even of Newtonian dynamics with absolute space are also 4-dimensional manifolds with various geometrical objects defined on them.[19] All of these theories can be given covariant formulations. So if the indeterminism problem arises, *prima facie* it arises for everybody. This is somewhat surprising since Newton's theory, especially with absolute space, has been regarded as a paradigm of both substantivalism and of determinism.[20] Earman and Norton assert that a

[18]Cf. Earman and Norton, *op. cit.*, note 1, p. 524 and Norton, *op. cit.*, note 1, p. 180.

[19]A covariant presentation of the theories is given, for example, in M. Friedman, *Foundations of Space-Time Theories* (Princeton: Princeton University Press, 1983).

[20]John Earman has already revealed indeterminism of a rather different sort lurking around the edges of Newtonian theory, but even he regards the special theory of relativity as close to the ideal of deterministic theories. See J. Earman, *A Primer on Determinism*, Western Ontario Series in Philosophy of Science, vol. 32 (Dordrecht: D. Reidel, 1986), for details.

metaphysics which forces us to be indeterminists must be unacceptable: determinism must at least have a fighting chance of being true, contingent on physical facts.[21] Equally, it seems to me, an interpretation of the mathematical formalism which makes substantivalism *imply* indeterminism, irrespective of the details of the physical theory in question, is unacceptable. If not even Newtonian theory can be interpreted substantivally without radical indeterminism, substantivalism is not being given a fighting chance.[22] How, then, would Newton reply to the argument?

> As the order of the parts of time is immutable, so also is the order of the parts of space. Suppose those parts to move out of their places, and they will be moved (if the expression may be allowed) out of themselves. For times and spaces are, as it were, the places as well of themselves as of all other things. All things are placed in time as to order of succession; and in space as to order of situation. *It is from their essence or nature that they are places; and that the primary places of things should be movable, is absurd.*[23]

Newton insists that the parts of space and time have their spatio-temporal relations *essentially,* and any talk of such parts changing their spatio-temporal properties is evidently confused. For if they do bear these relations essentially, by what (metaphysical) means could one identify a displaced place (one which had changed its spatio-temporal position relative to other places) as the very same place? To hark back to Aristotle, one might say that places are themselves in places, or that places are subjects to which the predicate "occupying such-and-such a relative position with respect to these other places" truly applies. But that predicate applies to a place essentially. However, Earman and Norton's interpretation of the hole diffeomorphism requires the substantivalist to reject Newton's claims. Under their interpretation, space-time points (*physical,* not mathematical, space-time points) only possess their spatio-temporal relations *accidentally*. Points which were represented as being 2 metres apart in the original model, *those very points,* may be represented as being 8 metres apart in the diffeomorphically generated model. Points may be represented as changing their temporal sequence in different models: in one model p occurs in

[21] Earman and Norton, *op. cit.*, note 1, p. 524, Earman, *op. cit.*, note 1, p. 20.

[22] Earman does give Newton a proposed response, along the lines that since his space-time structure is fixed and unaffected by matter distributions, he can regard all models as employing one and the same space-time (Earman, *op. cit.*, note 1; also Earman and Norton *op cit.*, note 1, p. 519). As we will see below, the mutability of space-time, in Earman's sense, has no bearing on the question; and the immutability of places and times, in Newton's sense, is equally defensible in the context of the general theory of relativity.

[23] I. Newton, *Principia,* trans. by A. Motte, revised by F. Cajori (Berkeley: University of California Press, 1966), p. 8, my italics.

the absolute past of q while in another q occurs in the absolute past of p. Newton the substantivalist would clearly brand such assertions as absurd.[24]

Earman and Norton's difficulty arises from asserting that *the substantivalist must regard space-time as represented by the bare topological manifold.* Since the metric tensor, as a mathematical object, is an accidental property of the manifold, they can then infer that the spatio-temporal relations between physical regions must be accidental attributes of them. This results from *mistakenly reading the ontological structure of a mathematical representation into the object that it represents.* Since the mathematical structure is built up from bare particulars, *all* of whose properties and relations are accidental, one thereby arrives at a physical ontology of bare particulars all of whose physical features are accidental. That is, by denying that any physical attributes are essential to the entities which possess them one arrives at the prime matter of Z.3, with all of its attendant difficulties.

The solution to this dilemma is just that which, I have argued, Aristotle saw. Not all predicates of a subject are accidental features of the subject. The essential features cannot be removed from the subject while leaving it behind. Similarly, moving the metric tensor from one point to another is a mathematical operation with no correlate in the realm of physical possibility. Physical space-time regions cannot exist without, and maintain no identity apart from, the particular spatio-temporal relations which obtain between them.

Since the ontological structure of the physical universe does not mirror the ontological structure of the mathematical object representing it, the mathematics must be supplemented with a *metaphysical commentary*. Earman and Norton themselves have done this in identifying space-time with the bare topological manifold and in insisting upon using a diffeomorphism, which preserves topological structure. For if one were to just adopt the ontology of the mathematics whole-heartedly, one would be forced to regard physical space-time points as entirely like the bare particulars from which the mathematical structure is constructed. But if one were to adopt *this* interpretation, one would not need to go through all of the technicalities of the diffemorphism, for all one needs to do is *permute* the mathematical points to get a new mathematical object. That will generally alter the neighborhood structure, but that would just mean that the topological relations between physical space-time points are also accidental. Even Earman and Norton don't think the substantivalist must be committed to regarding permuted models as

[24]Howard Stein makes precisely this claim about Newton (in 'On Space-Time and Ontology: Extract from a Letter to Adolf Grunbaum', *Foundations of Space-Time Theories*, Minnesota Studies in the Philosophy of Science, vol. VIII, J. Earman, C. Glymour and J. Stachel (eds) (Minneapolis: University of Minnesota Press, 1977), p. 394). Stein, however, is more generous than am I in countenancing locutions in other contexts which would imply that space-time does not have its spatio-temporal metrical properties essentially.

representing physically different states: they accept that the topological relations are essential to physical points. Physical space-time, according to their version of substantivalism, is represented by a set of mathematical points plus a topology. The question is: Why stop there?

It is a most peculiar place to stop, for it commits the substantivalist to the belief that *space-time, and space-time points, have no spatio-temporal properties essentially*. If space-time is a substance in the universe, surely its appropriate mathematical correlate is an object which has enough mathematical structure to represent spatio-temporal properties! The substantivalist's natural response to the hole dilemma is to insist that space-time is represented not by the bare manifold but by the manifold plus metric, by the metric space.

Earman and Norton do not just presuppose that space-time is to be represented by the manifold; they provide arguments to that effect. Let us consider these arguments and some other conceivable objections to my substantivalist's position to see whether they establish their point.

Objection 1: The metric is mathematically just like the tensors representing other physical fields. The physical field it represents propagates, carries energy,[25] &c. Therefore the field must not be essential to space-time.

I answer that this is a *non sequitur*. It only shows that, *pace* Newton, space-time has physical features which make it quite akin to other physical objects. All the more reason to regard it as a substance on a par with tables and chairs. One might try to turn this objection into a dilemma as follows:

Objection 2: Since the metric is so similar to the other physical fields,[26] how can one assert that *it* is essential to space-time and the others not?

[25]Albeit in a somewhat peculiar way: the energy is represented by a pseudo-tensor rather than a tensor. See Earman & Norton, *op. cit.*, note 1, p. 519, for arguments that the energy associated with the metric is just like all other energy.

[26]There is a possible escape hatch even to the objection that the metric looks too similar to the matter fields, although it is not a position I would defend. A substantivalist could argue that space-time is represented by neither the metric space nor by the bare manifold, but by an intermediate object. The affine space, the manifold plus affine connection, is just such an object. The connection is not a tensor, and the supposed problem of the gravitational energy pseudo-tensor vanishes since no such tensor is associated with the connection. The weaker structure of the affine connection still overcomes the hole dilemma, since the only affine transformation which leaves the area outside the hole unchanged (if it includes a Cauchy surface) is the identity map. The simplest way to see this is as follows: suppose a region of some thickness surrounding the Cauchy surface is to remain unchanged by the diffeomorphism (as will be the case with a finite hole). Since the affine transformation (a transformation preserving the affine connection) maps geodesics onto geodesics, it will map any geodesic passing through this region onto itself. But any point within the hole will lie at the intersection of at least two geodesics which pass through the Cauchy surface. Since these geodesics are mapped onto themselves, each point in the hole is mapped onto itself, and the map is the identity map. This still allows the substantivalist to regard certain *global* transformations, such as moving everything 3 feet to the left (in a flat spacetime) or uniformly "stretching" space, as producing physically distinct situations, but these transformations do not give rise to any problems about indeterminism.

> But if *all* fields are essential, then space-time substantivalism becomes trivial: it only asserts that the whole universe is a substance.

I answer that despite the similarities, the metric has one distinguishing property: it represents spatio-temporal features. Since space-time has its spatio-temporal features essentially (cf. Newton above), the metric is essential to it and the matter fields not. Of course, if one could do physics without the metric, or show that the metric can be reduced to the matter fields, one could deny that space-time is a substance. As an aside, there are some peculiar *mathematical* features of the gravitational field equations which have been cited as evidence of a particularly intimate connection between the metric and space-time:[27]

> The Einstein equations are *non-linear*. Actually, in this respect they are not so different from other fields, for while the electromagnetic field, the scalar field, etc., *by themselves* obey linear equations in a given space-time, they form a non-linear system when their mutual interactions are taken into account. The distinctive feature of the gravitational field is that it is *self-interacting:* it is non-linear even in the absence of other fields. *This is because it defines the space-time over which it propagates.*[28]

Objection 3: If we do not classify such energy bearing structures as [a gravitational] wave as contained within spacetime, then we do not see how we can consistently divide between container and contained.[29]

I answer that the metaphor of container and contained is hopelessly obscure when applied to space-time. Space-time does not contain objects like a *box* contains objects (i.e. by surrounding them). It may contain objects like the space within the box contains objects, but that is not very enlightening. Two glosses of this objection are possible:

Objection 3a: Gravitational waves suggest that we cannot distinguish between the spatio-temporal and other aspects or properties of the universe.

I answer that in ordinary English, we have clear paradigms of spatio-temporal properties: distance, elapsed times, &c. We can easily determine which mathematical structures in a theory represent information about these

[27] I cite this passage for its suggestive character, although Hawking and Ellis do not flesh out the argument enough to establish its validity. As Robert Weingard has pointed out to me, non-Abelian gauge fields are also self-interacting, although they seem to have no intimate role in defining space-time.

[28] S. Hawking and G. F. R. Ellis, *The Large-Scale Structure of Space-Time* (Cambridge: Cambridge University Press, 1973), p. 227, my emphasis.

[29] Earman and Norton, *op. cit.*, note 1, p. 519.

properties, in this case the structure being the metric (as opposed to the electromagnetic field tensor, &c.). If this is an energy-bearing structure, all the better for substantivalism, as per the response to Objection 1.

If we interpret "container" and "contained" as respectively "subject" and "predicate", we obtain:

Objection 3b: If the metric is essential to space-time, we cannot distinguish between the predicate (metric) and that which *has* the predicate (space-time). But such a distinction is crucial to regarding space-time as a substance, and hence as a subject (bearer of fields).

I answer that regarding the metrical properties as essential to space-time no more obliterates the distinction between subject and predicate than does regarding my humanity as essential to me. I am not *identical with* my humanity, I am the subject of which my humanity is predicated, I am the bearer of that property. Still, that does not make my humanity an accidental attribute, nor does it make into a bare particular which might have been, say, a chair. Space-time may be a bearer of the metric field but still bears it essentially. (The relevance of *Metaphysics* Z.3 is, I hope, vindicated here.)

Objection 4: Classifying the metric as part of the container spacetime leads to trivialization of the substantivalist view in unified field theories of the type developed by Einstein, in which all matter is represented by a generalized metric tensor. For there would no longer be anything contained in spacetime, so that the substantivalist view would in essence just assert the independent existence of the entire universe.[30]

I answer that belief that space-time is a substance does not become trivialized if it could be argued that space-time is the only substance (Is Spinoza's substantivalism about God trivial?). Indeed, programmes such as Wheeler's "geometrodynamics" are correctly termed by Larry Sklar "supersubstantivalist".[31] The only sense in which unified field theories such as Einstein's could trivialize substantivalism is if the metric is *artificially* expanded and information about other structures just pasted on (in which case, the substantivalist reponse to Objection 2 would be undercut). Einstein's theory was arguably not of this sort: he attempted to represent electro-magnetism as an effect of the parallel structure of space-time, which is clearly a *geometrical* property. Distinctions can usually be drawn between theories which try to show that physical effects flow from the metrical structure and those which artificially inflate the metric or the dimensionality of space-time to code up more

[30]Earman and Norton, *op. cit.*, note 1, p. 519.
[31]L. Sklar, *Space, Time, and Spacetime* (Berkeley: University of California Press), p. 221 ff.

information.[32] If in certain cases such a judgement were disputable, it would still be fair to say that if the paradigm geometrical properties of space-time are ineliminably represented by the metric, then the substance which is the universe is as much correctly called "space-time" as it is "material body". Finally, the supersubstantivalist programme is certainly not trivial in the sense of being guaranteed to succeed: no one has yet made it work. Our best theories say that there is more to the universe than just space-time.

In arguing that substantivalism about the general theory is inescapably subject to the hole dilemma in a way that Newtonian and special relativistic theory may not be, Earman introduces the notion that in the general theory space-time is *mutable:*

> Finally, assume space-time *mutability* in the sense that the object fields which characterize the structure of space-time are not given *ab initio* but are regarded as dynamical objects on a par with the other fields. In any such theory substantivalism will be incompatible with Laplacian determinism. . .[33]

Earman acknowledges the charge that the notion of mutability is "so vague as to be useless", but feels that paradigm cases are clear enough to deploy the concept. The question we must address is twofold: In what sense is the space-time of the general theory mutable? and What bearing does mutability have on substantivalism?

It is unclear why Earman wants to designate the feature of being governed by local differential equations (rather than by global constraints) as "mutability". The only connection I can think of (which I would not attribute to Earman) is associated with a peculiar causal reading of the mathematics. The picture is that although the physical metric field is not given *ab initio,* the physical bare manifold *is*, and then the metric "propagates" along the manifold. This of course is nonsense: a model may *represent* objects propagating in space-time, and equations may state mathematical conditions relating the value of the metric tensor at some point to its value at points infinitesimally close, but the space-time metric does not propagate or "move". For the manifold and metric are not in time, time is in them.

The notion of *ab initio* which Earman invokes is obscure to me. A mathematician may, in explicating or constructing a mathematical object, begin by laying down some structure, but that does not mean that the object it represents is somehow metaphysically "laid down" first. "*Ab initio*" must here

[32]Cf. R. Weingard, "Early Unified Field Theories", typescript.
[33]Earman, *op. cit.*, note 1, p. 18, cf. also p. 19.

refer to the physical objects, not to the mathematical representation. But it cannot be given a temporal reading, and I don't know how to read it.[34]

It is true that according to the general theory of relativity the geometrical structure of space-time *nomically depends on* the matter fields in the universe. In this sense, as opposed to Newtonian theory and the special theory, one cannot specify what the correct mathematical representation of the space-time structure is before one knows the disposition of objects. But if *this* is the appropriate sense of "*ab initio*" then the manifold is also not given *ab initio:* matter distributions can also determine some of the topological structure of space-time, as when a black hole requires there to be a singularity.

The straightforward reading of "mutability" suggests that space-time as a whole, or regions of it, can alter its geometrical properties. Clearly, space-time cannot do this in the sense that a persisting 3-dimensional object can, i.e., by having *as a whole* different shapes at different times. Different *parts* of space-time may have different geometrical properties, but that no more implies mutability than does the fact that different parts of the Hope diamond have different shapes. Hence, the only viable reading of "mutable" here is counterfactual: space-time is mutable in the sense that parts of it, or it as a whole, *might have had* geometrical properties other than the ones they do have. But such a counterfactual is not implied by the mathematical form of the field equations! The substantivalist would evidently deny that, strictly speaking, this very space-time could have had different spatio-temporal properties. Of course, had the matter fields been different, a space-time very *similar* to this one might have existed. It is quite open to the substantivalist to be a counterpart theorist about the counterfactual locutions that physicists are wont to use. There might have been other space-times, as would have been described by the various non-isometric models of the theory. Some of these are sufficiently like our space-time to provide counterparts to regions of actual space-time and so to support counterfactuals about what would have been the case if... But those counterparts are not *these very* spatio-temporal regions.[35] In any case, counterfactuals

[34]We might note in association with this that it is rather difficult to clearly explicate the many causal locutions that are used about how the metric is *generated* or how it *interacts* with matter. Reconciling causal locutions with the God's eye view of the mathematician is deeply puzzling. This problem also arises in disputes about the coherency of talk about time travel into the past, and even raises its head in the passage from Hawking and Ellis cited above.

[35]This is not to say that one must be a modal realist to be a substantivalist! Kripke might object that we can *stipulate* that we are counterfactually talking about *this very* region (cf. S. Kripke, *Naming and Necessity* (Cambridge, Mass.: Harvard University Press, 1980) pp. 43–47), but he runs afoul of the very problem of essential properties. For he admits that not every such stipulated possibility is metaphysically possible; there is, for example, no possibility that Nixon might have been a chair (cf. *ibid.*, pp. 110–116). Our substantivalist can adopt Kripke's view of counterfactuals, but must insist that the geometrical properties of an individual space-time be considered essential to it.

about space-time, like all counterfactuals, are puzzling beasts, and it is not obvious that the non-substantivalist will fare any better.[36]

The only non-question-begging sense in which the space-time of the general theory of relativity is *mutable*, then, is the sense in which its geometrical properties are posited to depend upon the matter fields via the Einstein field equations, so that different parts of space-time may have different metrics and models of the theory may be non-isometric. What bearing does this have on substantivalism? One possibility is:

Objection 5: Substances are supposed to be capable of independent existence. You say that space-time has its geometrical structure essentially. But then it should be capable of independent existence apart from the matter fields. But if we remove the matter fields, we change the metric (via the field equations) and so destroy the space-time.

I answer that this objection rests on a confusion of physical and metaphysical necessity. If all American families were increased by one member, then the average American family would as a *logical* consequence also be changed. The average family cannot exist independently of the flesh-and-blood ones, and its properties are entirely determined by theirs. No such *logical* or *necessary* connection is evident in the case of the metric.[37] The substantivalist can regard the field equations as *contingent* truths, so that it is *metaphysically* possible for a particularly curved space-time to exist even if all of the matter in it were annihilated. Similarly, I cannot physically exist without there being surrounding air, but this does not debar me from being a substance.

Objection 6: So far, you have only dwelt upon what the space-time substantivalist need not be committed to: he need not regard space-time as represented by the bare manifold, need not regard counterfactuals as indicating possible geometrical states of this very space-time, &c. Let us consider what the substantivalist must *accept*. Substantivalists, whatever their stripe, "must all agree concerning an acid test of substantivalism, drawn from Leibniz. If everything in the world were reflected East to West (or better, translated 3 feet East), retaining all the relations between bodies, would we

[36]Jeremy Butterfield has taken the question of evaluation of counterfactuals via counterpart relations to provide the key to solving the hole dilemma. In 'The Hole Truth', *British Journal for the Philosophy of Science* **40** (1989), 1–28, Butterfield adopts Lewis's account of modality to attack the problem, arguing that the most plausible counterpart relation between points is provided by the diffeomorphism itself. My own approach is more sympathetic to Kripke's account of counterfactuals than to Lewis's. Butterfield, Norton and I fight it out in our contributions in the forthcoming *PSA 1988* vol. 2, A. Fine and M. Forbes (eds) (East Lansing, Michigan: Philosophy of Science Association, 1989).

[37]There may be some such non-evident connection, as the geometrodynamicist would contend, but this must be argued for. Even for him, though, space-time is certainly capable of existing independently of anything else, it is just that there is nothing else for it to be independent from.

> have a different world? The substantivalist must answer yes since all the bodies of the world are now in different spatial locations, even though the relations between them are unchanged." But "the diffeomorphism is the counterpart of Leibniz's replacement of all bodies in space in such a way that their relative relations are preserved. ... In sum, substantivalists, whatever their precise flavor, will deny: *Leibniz equivalence*: Diffeomorphic models represent the same physical situation."[38]

I answer that although the substantivalist must acknowledge the movement of every *body* 3 feet East as resulting in a new physical situation, it in no way follows that the diffeomorphism is the appropriate generalization of such a change. Rather, it would be the movement of the *matter fields* which a contemporary Leibniz would have in mind, not moving the metric as well. The substantivalist *does* regard the mathematical object created by replacing all of the tensors *save the metric tensor* by their carry alongs under a diffeomorphism as representing a new physical situation. In general, of course, this new physical situation won't even satisfy the field equations. It is a special feature of Newtonian and special relativistic physics that moving all material bodies in a particular solution an equal amount will result in another solution. In general relativity there is no such automatic recipe for generating one solution on the metric space from another. Indeed, in general relativity the demand that one translate everything 3 feet East while retaining all the relations between bodies is not usually satisfiable. For example, if we move a physical triangle 3 feet East from a region of zero curvature to a region of positive curvature, the parts of the triangle *cannot* retain all of their geometrical relations to one another. Either the sides will no longer be straight or the interior angles will no longer sum to π.

There is no reason to suppose that the Leibniz experiment generalizes to moving the geometrical structure of space-time as well as the material bodies, and good reason not to suppose so. First of all, if it did include the geometrical structure, it would not be necessary to specify that one should move everything an *equal* distance. Indeed, only in the case in which the metric is left unchanged does it make sense to talk unequivocally about "the distance" that an object has been moved. For suppose that under a diffeomorphism such as Earman and Norton propose one moves all of the geometrical structure from point p to point q. It will not generally be the case that the spatio-temporal distance between p and q remains invariant under the transformation. Either there is no univocal distance between the two points at all or else, if one insists upon measuring distances according to the metric from which the move is made, it

[38]Earman and Norton, *op. cit.*, note 1, pp. 521–522; cf. also Earman, *op. cit.*, note 1, p. 17, and Norton, *op. cit.*, note 1, pp. 179–180.

will generally occur that a move 3 feet East from p to q followed by a move from q 3 feet West will not bring you back to p. It is doubtful that Leibniz could have had such a possibility in mind. But even if he did, the substantivalist can rightfully point out that it begs to the question to say that all spatio-temporal relations have been retained in such a move. For the spatio-temporal relations between p and q, or more generally between almost any pair of points in the hole, will have been changed. If, as Earman and Norton insist, the substantivalist must regard the bare manifold as a substance, then he can respond that the diffeomorphism does *not* leave all spatio-temporal relations unchanged. If, as I have maintained, he regards the metric space as the substance, then moving bodies around *in a given space-time* is a different mathematical operation than the one Earman and Norton present.

Thus our substantivalist does pass the acid test—appropriately construed. In Newtonian dynamics and special relativity he would accept the movement of all matter fields or particles 3 feet East as resulting in a new physical situation, one which still satisfies the field equations. In general relativity, he would accept the moving of matter fields globally also as creating a new physical situation. If the metric were sufficiently isotropic and homogeneous, a global transformation similar to moving everything 3 feet East might be definable and even lead to a new solution to the field equations. But accepting such *global* transformations as resulting in distinct physical states raises no spectre of indeterminism, for in such a case the Cauchy surface must be changed too. On the other hand, moving the matter fields around according to a *hole* diffeomorphism will not lead to a solution to the field equations unless it is the identity map. Accepting Leibniz's "acid test" does not need to imply indeterminism.

Objection 7: Your space-time substantivalist has escaped only temporarily by a semantical trick. Suppose we grant that the term "space-time" does refer to an entity best represented by the metric space rather than the topological manifold. Still, that entity does *have* topological features. If we consider it *qua* only its topological features, we are considering *something*, something which by definition does not have metrical features essentially. I will give you the term: space-time: let us call this new entity "topo-space". Now if space-time is a substance, so is topo-space, and we can just repeat the argument for indeterminism.

I answer that it is true that one can inquire into the topology of a metric space, and determine what features it may have in virtue of that topology. Whatever conclusions one draws will equally apply to other metric spaces with the same topology even though their metrics may differ. The details of the metric may then *seem* to be irrelevant to the space-*qua*-topological. But this is not true if the entity only has its topology in virtue of its metric. For example, one may argue that the neighborhood structure is *derived* from the metrical features: sets of (physical) space-time points only form neighborhoods because of their spatio-temporal proximity relations. Thus the topology flows from the metric rather than the metric being imposed on the topological space. If one thinks of metric and manifold as form and matter, this is the ultimate case of the essential form making the matter what it is.

Nor is this response purely a philosopher's fantasy. Einstein expresses precisely such a view in a passage which Norton cites:

> If we imagine the gravitational field, *i.e.* the functions g_{ik}, to be removed, there does not remain a space of the type (I) [Minkowski space-time], but absolutely *nothing*, and also no "topological space". For the functions g_{ik} describe not only the field, but at the same time also the topological and metrical structural properties of the manifold.[39]

But whereas Norton, believing that the substantivalist must hold his view towards the topological space, takes this as evidence that Einstein rejected substantivalism, I contend that this is just the position the substantivalist must hold (see Appendix).

None of this constitutes a positive argument for regarding space-time as a substance. Like statements about the average American family, space-time locutions might yet turn out to be misleading ways of talking about material objects and their relational properties. But it does not follow from the fact that the space-time metric physically *depends on* material fields (via the field equations) that it can be *logically reduced to* relations between those objects. Nor, as we have seen, need the substantivalist accept radical indeterminacy; he need only insist that mathematically the metric space, not the topological manifold, represents space-time. One must resist the temptation to read the ontological structure of mathematical objects into the objects that they represent. And, as Aristotle saw, one must be extremely sensitive to the relationship between substances and their essential properties if one is to avoid the absurdity of bare physical particulars, or bare physical manifolds. With Aristotle's help, we have been able to turn back one set of objections to the view that space-time is capable of existence independent of material bodies. A close analysis of the positive arguments for that view must await another time and place.[40]

[39]A. Einstein, *Relativity: The Special and General Theory* (London: Methuen, 1977), p. 155.

[40]I address some of these issues in 'Buckets of Water and Waves of Space: Why Space-Time Probably is a Substance', typescript.

Appendix: Einstein and the Hole Argument

In *Einstein, the Hole Argument, and the Objectivity of Space,*[41] John Norton argues that Einstein had discovered the hole argument by 1913. Norton contends that the argument was presented in four papers by Einstein (two co-authored by Marcel Grossmann),[42] but that it has since been misunderstood. According to him, Einstein's argument has been misread as employing only a passive re-co-ordinatization of the manifold and as incorrectly trading on the fact that the same tensor is expressed as having different components in different co-ordinate systems.[43] One ought, he says, to read the transformation actively, as a diffeomorphism, rather than merely as a re-co-ordinatization. Thus, the hole argument was born over 70 years ago.

Although I agree with the general thrust of Norton's main conclusion, I do not believe that the texts will all bear the interpretation he gives them. First of all, I do not believe that all four passages are versions of the same argument. The first three, we are told, are essentially identical, and I shall so treat them.[44] But the fourth has a different, more complex structure. It is only the fourth which obviously employs an active redistribution of the metric in the manifold, although Norton is correct in pointing out that the first three (hereafter known as the "first argument") turn upon more than the possibility of different co-ordinate expressions for a tensor at a given point. Let us begin with the first argument, and come to the second argument in turn. I reproduce Norton's translation of the first argument:

> *If the reference system is chosen quite arbitrarily, then in general the g_{mn} cannot be completely determined by the T_{sn}* [i.e. the stress-energy tensor density]. For, think of the T_{sn} and g_{mn} as given everywhere, and let all T_{sn} vanish in a region Φ of four dimensional space. I can now introduce a new reference system, which coincides completely with the original outside Φ, but is different to it inside Φ (without violation of continuity). One now relates everything to this new reference system, in which matter is represented by T''_{sn} and the gravitational field by g'_{mn}. Then it is certainly true that
>
> $$T''_{sn} = T_{sn}$$
>
> everywhere, but against this the equations
>
> $$g'_{mn} = g_{mn}$$

[41] *Op. cit.*, note 1.

[42] The papers are: A. Einstein, 'Prinzipielles zur verallgemeinerten Relativitaetstheorie', *Physicalische Zeitschrift* **15** (1914), 176–180, and 'Die formale Grundlage der allgemeinen Relativitaetstheorie', *Sitzungsberichte der Preussichen Akademie der Wissenschaften* (1914), 1030–1085; A. Einstein and M. Grossmann, 'Entwurf einer verallgemeinerten Relativitaetstheorie und einer Theorie der Gravitation', *Zeitschrift fuer Mathematik und Physik* **62** (1913), 225–261, and 'Kovarianzeigenschaften der Feldgleichungen der auf die verallgemeinerte Relativitaetstheorie gegruendeten Gravitationstheorie', *Zeitschrift fuer Mathematik und Physik* **63** (1914), 215–225.

[43] Norton, *op. cit.*, note 1, p. 164.

[44] Cf. *ibid.*, p. 163.

will definitely not all be satisfied within Φ*. The assertion follows from this.
If one wants a complete determination of the g_{mn} (gravitational field) by the T_{sn} (matter) to be possible, then this can only be achieved by a limitation on the choice of reference systems.

*The equations are to be understood in such a way that each of the independent variables x'_n on the left hand side are to be given the same numerical values as the variables x_n on the right hand side.[45]

Norton goes on to claim that the only way to understand the footnote is to see Einstein as invoking a diffeomorphism rather than just a passive co-ordinate transformation.[46] But a much more straightforward reading is possible.

The argument that Einstein gives does employ a hole, but the first and last sentences make quite clear that the question at hand is not whether the entire state of the universe outside the hole determines the state inside. Rather, the question is whether the stress-energy tensor density T_{sn} (representing matter) defined *everywhere* determines the metric g_{mn} everywhere. Einstein thinks that it must, probably as a sort of application of Mach's principle that inertial structure is determined entirely by the disposition of masses. The proposed criterion for an acceptable theory is the following: if you lay down a co-ordinate system and specify T_{ns} everywhere as a function of those co-ordinates, then the g_{mn} expressed as a function of those co-ordinates should be fixed by the laws of the theory. Einstein shows by a *reductio ad absurdum* that this requirement cannot be fulfilled unless one restricts the reference systems which are allowed.

The *reductio* runs as follows. Consider a distribution of matter T such that $T=0$ within a region Φ. Consider two co-ordinatizations of the space-time x_n and x'^n which agree outside the hole but diverge within it. Then, since a zero tensor is expressed as having components equal to zero in all co-ordinatizations, the expression of T as a function of the x^n will be *functionally identical* to the expression of T in terms of the x'^n. That is, $T_{mn}=T'_{mn}$ *everywhere*, and "everywhere" can be taken in two senses. It is true that the co-ordinate components of T at any given point p in terms of the x co-ordinates are equal to the co-ordinate components of T in terms of x' co-ordinates *for that point p* $[(\forall p)(T_{mn}(p)=T''_{mn}(p)]$. But it is equally true that the co-ordinate components of T in terms of the x^n *at a point with co-ordinate values* $<N_1, N_2, N_3, N_4>$ *in the x co-ordinate system* is equal to the co-ordinate components of T in terms

[45]*Ibid.*, pp. 163–164, Einstein's italics and footnote.

[46]I do not want to reproduce the algorithm which Norton takes this footnote to indicate, because it is rather complex and, to me at least, not entirely clear. As far as I can tell, Norton is seeking an algorithm for comparing g_{mn} *(hp)* [the original metric at *hp* expressed in terms of the original co-ordinate system] with $h_* g_{mn}(hp)$ [the carried along metric at *hp* expressed in the original co-ordinate system]. If so, his algorithm should apply to $g_{mn}(p)$ the transformation for $x'^n \rightarrow x^n$ at *hp*, rather than the transformation $x^n \rightarrow x'^n$ at *p*. Norton's algorithm will probably compare $g_{m'n'}$ *(hp)* with $g_{m'n'}(p)$ (I say "probably" because he does not tell us in what co-ordinate system we are to express the original metric at *p*).

of the x'^n *at the point with the same numerical co-ordinate values in the* x' *co-ordinate system* $[(\forall N_1)(\forall N_2)(\forall\ N_3)(\forall N_4)(T_{mn}(<\mathrm{N}_1, \mathrm{N}_2, N_3, N_4>_x) = T'_{mn}(<N_1, N_2, N_3, N_4>_{x'}\ 0.$[47] The points referred to on either side of this equation are points which have a particular co-ordinate expression in each reference system, and since the systems diverge within the hole they will, in general, be different points of the manifold. It is this latter identity which, as his footnote illustrates, Einstein had in mind. It is this latter equation which correctly expresses the fact that the stress-energy tensor density has the very same functional form when expressed in terms of the x^n as it has when expressed in terms of the x'^n. But now an application of the Machian principle cited above yields that if the theory appropriately allows the T_{mn} to determine the g_{mn}, then the gravitational metric must *also* have the same functional form relative to each co-ordinatization, $g_{mn} = g'_{mn}$. This statement again is made for points with equal co-ordinate values, i.e. for different points in the manifold. But this clearly cannot be the case, since different points within the hole may have, e.g. metrics with different (scalar) curvatures: the gravitational field cannot generally take the same functional form when expressed in terms of the two co-ordinate systems.

Einstein's footnote makes clear that he is comparing the g_{mn} and the g'_{mn} at different points. Since the points are to have the same numerical co-ordinate values but are given in two different reference frames, they must in general be different points. In a scientific world in which changes of reference frame were common coin and diffeomorphisms not, the equations could not have been read otherwise. The argument is crisp and accurate, and it does indeed demonstrate that generally co-variant field equations cannot satisfy the Machian criterion stated above. This failure does not *simply* arise from gauge invariance,[48] nor is the point identical with the hole dilemma of Earman and Norton. It just is a fact that fixing T does not fix g in general relativity. There are, for example, solutions of the vacuum field equations with and without gravitational waves, and hence space-times with $T=0$ everywhere which are not isometries. The criterion that Einstein states at the beginning of the passage cannot be satisfied by the general theory.

The first argument, then, makes perfect sense in light of the Machian principle enunciated in the first sentence. No diffeomorphism need be invoked in understanding his footnote, only a re-co-ordinatization. Further, we later find Einstein holding to such a criterion and criticizing the equations of 1915 in its light. In 1917 Einstein finds fault with his own earlier analysis of planetary

[47]In these equations, I have used terminology consistent with Einstein's, as I interpret him. "T'_{mn}" refers to the co-ordinate expression of T as a function of the x'. A more perspicuous terminology would be $T_{m'n'}$, which would make evident that the prime indicates a change of co-ordinates, not a change in the tensor itself. In Einstein's second argument he had to introduce some such more perspicuous notation, indicative of a change in the mathematical operation being performed.

[48]*Pace* A. Pais, *Subtle is the Lord. . . :The Science and Life of Albert Einstein* (Oxford: Clarendon, 1982), pp. 221–222.

motion because it implies that "the inertia [of a body] is *influenced* by matter (at finite distances) but *not determined* by it. If only a single mass point existed it would have inertia... [but] in a consistent relativity theory there cannot be inertia relative to 'space' but only inertia of masses relative to each other".[49] And in 1918 he states as a fundamental principle that "The *G*-field is *without remainder* ("restlos") determined by the masses of bodies."[50] With the passage of time Einstein came to see that this Machian demand could not be met, and so he eventually abandoned it.[51]

Einstein's first argument does not employ an active transformation like Norton's diffeomorphism. Furthermore, it is not as general as Norton's since it requires that the stress-energy tensor density be identically zero within the hole, so that its co-ordinate representation under the re-co-ordinatization remains the same. His second argument, however, clearly does employ the notion of an active transformation. The extra complexity of this argument is reflected in the additional technical machinery he is required to use: whereas in the first argument he could unambiguously refer to g_{mn} and g'_{mn}, since one metric was at issue, albeit described in two reference frames, in the second argument he needs "$G(x)$" to refer to the original metric expressed in the original frame, "$G'(x')$" to refer to the original metric as expressed in the new co-ordinate system,[52] and $G'(x)$ to refer to the metric with the same functional form as $G'(x')$, but with the arguments now being the co-ordinates of the first system.[53] Thus although I don't think the texts support Norton's active reading of the first argument, there clearly is an active transformation in the second. It is not exactly the hole argument of Norton and Earman, but it is similar.

What did Einstein make of this argument? Originally, he inferred that general co-variance must be sacrificed. But he later came to accept, even embrace, that requirement. What had to be given up to remove the fangs from the hole argument?

Norton asserts that at first Einstein could only resolve the dilemma by adopting non-realism about space-time, by which he means the view that the English term "space-time" has "no referent in the physical world."[54] Norton thinks this inference of Einstein's to be unwarranted,[55] but accepts it as a temporary error on the way to a weakened "anti-substantivalist" view of

[49]Cited in *ibid.*, p. 286, Einstein's italics.
[50]Cited in *ibid.*, p. 287.
[51]Cf. *ibid.*, pp. 281–288.
[52]It is notable how Einstein uses his primes here, for it reflects the way he thought about the problem. To someone schooled in co-ordinate free presentations, $G(x')$ would be the natural way to denote the original metric as expressed in the new co-ordinate system, for the *metric* (G) is the same, only the reference frame has changed. But what was important to Einstein was the *functional form* of the metric expressed in terms of the co-ordinatization. Since that functional form changes under re-co-ordinatization, the same metric becomes $G'(x')$ rather than $G(x')$.
[53]Cf. Norton, *op. cit.*, note 1, p. 167.
[54]*Ibid.*, p. 177.
[55]*Ibid.*, pp. 176–183.

space-time (which we shall take up presently). Norton can cite some passages from 1915 to 1916 in which Einstein appears to take up this uncompromising position: "That this requirement of general co-variance, which takes away from space and time the last remnant of physical objectivity, is a natural one, will be seen from the following reflexion. . . ";[56] "The essential thing is: as long as the drawing paper, i.e. 'space', has no reality, then there is no difference at all between the two figures. . . ".[57] If we take these quotations in isolation, and ascribe to Einstein some understanding of referential semantics, Norton's claim appears to follow.

But a close examination of the context reveals that the non-realist interpretation of these passages does not capture Einstein's meaning. For Einstein consistently interchanges assertions of the form "space and time are not physically real" with assertions of the form "spatial and temporal *co-ordinates* have no physical significance", and it is clearly the *latter* claim which concerns him. Consider the beginning of the section on general co-variance:

> In classical mechanics, as well as in the special theory of relativity, the co-ordinates of space and time have a direct physical meaning. To say that a point-event has the X_1 co-ordinate x_1 means that the projection of the point-event on the axis of X_1, determined by rigid rods and in accordance with the rules of Euclidean geometry, is obtained by measuring off a given rod (the unit of length) x_1 times from the origin of co-ordinates along the axis of X_1. To say that a point-event has the X_4 co-ordinate $x_4 = t$, means that a standard clock, made to measure time in a definite unit period, and which is stationary relative to the co-ordinates and practically coincident in space with the point-event, will have measured off $x_4 = t$ periods at the occurrence of the event.
>
> *This view of space and time* has always been in the minds of physicists, even if, as a rule, they have been unconscious of it. . .[58]

This "view of space and time" clearly has not to do with the *ontological status* of space-time but with the *operational significance* of spatio-temporal co-ordinates.[59] In Newtonian physics and in the special theory of relativity, once you associate an inertial frame with a physical body as its origin and fix some co-ordinate axes and a zero time, the assertion that an event will occur at, say $< 1, 3, 0, 7 >$ has *physical significance*. It tells you about the behavior of clocks and rigid rods connecting the body at the origin with the event. And co-ordinate differences between points have a similar meaning. Einstein's point in denying physical reality to space and time in a generally co-variant theory is that no such operations are associated with co-ordinate values in such a

[56] A. Einstein, 'The Foundation of the General Theory of Relativity', *The Principle of Relativity* (New York: Dover, 1952), p. 117. The essay was written in 1916.

[57] Cited in Norton, *op. cit.*, note 1, p. 174.

[58] Einstein, *op. cit.*, note 67, p. 115, my italics.

[59] Of course, given a sufficiently positivist outlook, questions of ontology or reality may simply reduce to questions of operational significance, and the question concerning us, whether space-time is a substance or not, cannot even be formulated.

theory. The most exhaustive information about the origin and axes of a reference frame in a generally co-variant theory will impart no information about how to physically locate particular co-ordinatized points. Co-ordinate differences also obviously will have no general physical significance.

Perhaps the physicist who Einstein speaks of as unconsciously holding this view of co-ordinatizations was himself several years earlier. That would in part explain why, in the first argument, he thought it significant that points with the *same co-ordinate value* would have different metrics. Since the axes of the reference frames could lie entirely outside of the hole, if some operational significance akin to that in the special theory could be ascribed to the co-ordinates here, general covariance would have to be curtailed. Once one shakes off the notion that co-ordinates have any operational meaning, a more complicated argument, such as the second one, is needed.

Einstein's concern with the operational significance of co-ordinates rather than the ontological status of space-time is also evident in the letter to Ehrenfest from which the second of the above "non-realist" quotations was taken. Earlier in the letter he writes "Your difficulty has its root in the fact that you instinctively treat the reference system as something 'real'."[60] But the "imaginary" character of the *reference system*, in this sense, need not impugn the reality of the object to which the system is attached. Differences in Dewey decimal numbers signify no real physical property, but that need not make us non-realists about the contents of the public library. If Einstein ever thought in terms of substances, non-substances, and non-existent entities, these passages would not commit him to non-realism about space-time itself.

Co-ordinate values in Newtonian theory and in the special theory can have a physical significance because what is co-ordinatized is the *metric space*, and because only a limited number of reference systems are allowed. These are two quite different points. Certain particular co-ordinatizations of these metric spaces yield especially simple co-ordinate expressions for the physical laws because these space-times exhibit a perfect isotropy and homogeneity. Curvilinear co-ordinate systems can be put down on these spaces, and the laws expressed in terms of them, but then the physical significance of the co-ordinate values becomes much more complicated, and no general significance can be ascribed to co-ordinate differences. Still, given any co-ordinate system laid down on a *metric space*, whether in special or general relativity, physical statements of the form "a clock moving from point p to point q along path l will register 5 ticks" can be made.

Once one moves to co-ordinates on the bare manifold, however, *no physical significance, no matter how complex, can be associated with the reference system.* Points in the bare manifold are no particular distance apart, stand in no causal relations, are not intrinsically connected by *any* physical process. Thus, if, in

[60]Norton, *op. cit.*, note 1, p. 173.

one way or another, physical processes are used to define reference systems, we find that *one can only meaningfully co-ordinatize space-time as a metric space.* This is one way of stating the hole dilemma.

Given Einstein's equation of the reality of space-time with the physical significance of frames of reference, it is not surprising that he should come to regard space-time as real only if it is invested with a metric. This is the position that Norton dubs "anti-realism", and is best captured by the following quotation:

> There can be no space [space-time] nor any part of space without gravitational potentials; for these confer upon space its metrical qualities, without which it cannot be imagined at all. The existence of the gravitational field is inseparably bound up with the existence of space.[61]

If this is taken to be the core of Einstein's position, then our metaphysical battles reduce to a mere squabble over terminology. Norton *calls* this position anti-substantival, but I hope to have shown that it is just what any red-blooded substantivalist, from Newton on, would insist upon! Space-time must be regarded as essentially a metric space, not a topological space which has its geometrical properties only accidentally. The real question is whether such a metrical space with intrinsic inertial structure must be considered to be capable of existing independently of material bodies (represented by the stress-energy tensor), or whether it can be analyzed away as a *façon de parler* about the relations that obtain between bodies. This is the issue which divided Newton on the one hand from Leibniz and Mach on the other, the issue which impelled Einstein to his original insistence that the stress-energy tensor *completely* determine the gravitational field. Einstein grew away from this demand, and in so doing abandoned the relationalist camp. I hope to have shown that Einstein was quite correct in his pronouncement, a year before his death, that "one should no longer speak of Mach's principle at all."[62] If one accepts the general theory of relativity as giving a literally correct picture of the physical world, one must stand with the Newtonian tradition and regard space-time as a substance.

[61]Cited in *ibid.*, p. 181.
[62]Cited in Paris, *op. cit.*, note 59, p. 288.