CONCLUSION

ON THE HISTORICAL DIMENSION OF "PROBABILITY"

It is quite generally believed that no meaningful or, better, significant rapprochement is possible between medieval thought and modern thought. For, the usual argument goes, the modern world is to a great extent a world made by man rather than by God – and made by him not only since the close of the Middle Ages but in large measure since the close of the nineteenth century. On this view, in other words, there is between us and the medieval an unbridgeable gap that has been forged among other things by the development of the mathematical and now more recently the logical sciences.

As a result of this historical transformation, it is contended, thought patterns have also been transformed, and to such an extent that they would no longer be understandable to the medieval man. For, in general, our thought is sophisticated whereas that of the medieval was naive. The latter thought in terms of absolutes; we think in terms of approximations. He was fond of uniformity; we pride ourselves on being able to adapt ourselves to pluralism on all levels of life. He looked for simplicity in things; we remain ever conscious of complexity.

In short, we are thus presented with two radically different universes of thought. And, as Badi Kasm has observed, a particular universe of thought is systematically closed in upon itself and hence can only be judged on its own terms.[1] But if this be the case, then it would seem to follow that any supposed rapprochement between medieval and modern thought is at best artificial and at worst misguided.

To put all this somewhat differently, the ghost of Jacob Burckhardt has not yet been laid to rest. Too willing to take some writers of the Renaissance at their word, this nineteenth century historian concluded that all that was good and noble about the "new birth" of intellect was

[1] Badi Kasm, *L'Idée de Preuve en Métaphysique*, Paris, 1959.

due to a return to the Greeks.[1] This view, to be sure, has been considerably modified by subsequent research. But to a great extent it remains the accepted conviction of most contemporary philosophers: the Muse of today's philosopher speaks not Latin but Greek – and perhaps even something more ancient than that.

It is interesting to note, therefore, that what some historians of philosophy have tried in vain to show the philosopher, historians of science are making ever more palatable to the interested scientist. The prodigious growth of the history of science in the past fifty years and, in particular, in the past ten years has clarified and qualified but never destroyed Pierre Duhem's thesis of continuity between medieval and Renaissance (or Newtonian) science. The results of research along these lines are well illustrated in the convincing work of John Henry Randall, Jr., entitled, curiously enough, *The Career of Philosophy*.[2]

Now whatever one may think of this gradually developing view of historical continuity between medieval and Renaissance science, he cannot fail to see that even if there be continuity during that period, it has only limited significance. For, it is anything but obvious that there is much important continuity between Newtonian science and the science of today. The revolutionary effect of Einstein's reformulation of celestial mechanics is a case in point. But no less important is the reformulation of terrestrial mechanics on the basis of the calculus of probability.

In briefest terms, it is generally felt that the introduction of "relativity" and "probability" into scientific thought has brought down the Newtonian absolutes and thus in effect cut the last tie between our world and the world of the medieval. In the place of absolutes, whether considered as conceptual or as propositional, man now deals with an "optique" or, if you will, a horizon of thought which is interpreted as a manifestation of his particular spatio-temporal condition. In the shadow of Einstein, all thought is described as being somehow or other "relative." And in the shadow of the quantum physicists, propositions are often viewed not as "true" but only as more or less effective approximations to truth. The absolute, however described, remains at best what Kant would call a transcendental ideal. In short, the new sophistication is upon us, and from it flow such bountiful blessings as freedom of conscience and a growing spirit of ecumenical rapproche-

[1] Jacob Burckhardt, *The Civilization of the Renaissance in Italy*, New York, 1921.
[2] The full title of this work, already cited in the first chapter, is *The Career of Philosophy from the Middle Ages to the Enlightenment* (New York and London, 1962).

ment. But at the same time, it is felt, with the denouement of the absolute our last tie with medieval thought has been definitively cut.

The principal purpose of our study has been to question this supposed dichotomy between medieval and modern patterns of thought. This we have done by taking as our focal point the notion of probability, as expressed today and as expressed in the Middle Ages. To limit our task to the humanly possible, we have chosen to compare representative views of the twentieth century with the view of the best known of all medieval thinkers, Thomas Aquinas.

Our method has consisted primarily of studying what is said precisely in the hopes of describing the ideological universe which has made it acceptable to say such things. It is, if you will, two ideological universes which we have tried to describe, our own and that of the medieval. We recognize full well the differences between these two universes, and even more between the kinds of statements possible in each. But at the same time *we claim to have found important similarities between these ideological universes which suggest, in turn, the possibility of an historical continuity with regard to the notion of probability.*

To spell out in detail what has here been suggested, we propose to defend consecutively five major conclusions. Each of these conclusions, we think, can be drawn independently from the study which we have made; but some are more clear cut and obvious than others. Accordingly, we have staggered our conclusions from the most to the least obvious and thus from the most trivial and readily acceptable to the most important and controversial. In this way we hope to use the stronger in order to build support for the weaker. Our conclusions, then, are the following:

I. There is a similarity between the structure of Thomas's thought patterns and modern thought patterns.

II. There is a similarity between Thomas's notion of opinion, or probable knowledge, and modern notions of non-demonstrative knowledge.

III. There is a similarity between Thomas's disputation and the modern calculus of probability.

IV. There is a similarity (A) between Thomas's theory of probability and the contemporary logical theory of probability and (B) between Thomas's theory of contingency and the contemporary frequency theory of probability.

V. There is a relationship between (A) Thomas's distinction between *scientia* and opinion-probability and (B) the modern problem of probability in science.

These, then, being our conclusions, we proceed at once to their elaboration.

I. *There is a similarity between the structure of Thomas's thought patterns and contemporary thought patterns.*

On the surface, at least, our approach to Thomas Aquinas has not differed remarkably from that of many other commentaries on the thought of the medieval master. And, as for these commentaries, we quite readily admit that more often than not they will contain a far more thorough treatment of most of the topics which have entered into our discussion. From a logical point of view, at any rate, the presentation of these topics in scholastic manuals will manifest the results of centuries of reflection upon and development of principles and procedures set forth in the writings of Thomas himself. Precisely because of ideological trends since the time of Aquinas, his thought has undergone a great deal of refinement especially with regard to ontology and epistemology in general and the theory of science and of demonstration in particular. On the whole, no doubt, these developments of Thomas's thought were, at least for their time, all for the good; and, properly understood, they still have a contribution to make to contemporary thought.

It is our opinion, however, that studies of Thomas's thought have in general been overly absolutist in their interpretation of the Angelic Doctor. And, as a result, Thomas has perhaps been systematized far better than he has been understood. It has been our impression, at least, that the rationalistic formulations of many so-called Thomistic manuals make the thought of Thomas himself, when seen at first hand, seem by comparison the cautious estimates of a neophyte before the unknown.

In contrast to the view of Thomas which these manuals usually present, we maintain that (A) the basic distinction of Thomas's theory of knowledge is, broadly, that between creator and creature or, more narrowly, between God and man; and that (B) from this distinction flows the basic distinction of his theory of human knowledge; broadly, that between the certain and the probable or, more strictly, between the scientific and the opinionative, the demonstrated and the probable.

A. *The basic distinction* of Thomas's theory of knowledge, in terms of which all else is to be judged, might *most properly* be described as *that between the absolutely necessary, the creator, and what is by comparison* *contingent, the creature*. But because of the specific bent of our investigations we prefer to speak of this distinction *somewhat more narrowly* as that *between the divine and the human*.

It is this distinction between the divine and the human which is at the heart of Thomas's division of all man's knowledge into natural (or, more loosely, reasoned) and revealed; and the latter, in turn, forms the basis for his evaluation of the two traditions, that of the saints and that of the philosophers. Still more broadly than this, we have seen that Thomas looks upon all human knowledge as imperfect by comparison to the divine, and for this reason paints a glowing picture of what is known by those who are closer than most to God: angels in general and such men as Adam, the prophets, and above all Christ. For Thomas, accordingly, the whole purpose of human specualtion is to approach as closely as possible in this life to that divine knowledge which is shared to perfection by the blessed in heaven. This orientation towards the divine (not surprising, of course, for a theologian) tends to distract Thomas from a closer investigation of the contingent in favor of a panoramic view of the way things must look to God. And thus Thomas's "frequency" approach to the contingent – not only in cosmology but also in his theory of disputation and of practical deliberation – is best seen within the context of God's providential knowledge of all particulars, past, present, and future. This is in no way intended to negate the value of what Thomas does say about these various human problems. Rather does it underline the element of relativity which permeates all that he says about such problems precisely because he is speaking as a mere man who does not have that clear vision which is the prerogative of God.

We realize full well that by introducing Thomas's theological views about God into a discussion of his theory of probability we satisfy neither the Thomist who likes his philosophy and theology neatly distinguished nor the probabilist who likes his science neatly isolated from "religious" considerations. But the place of probability in Thomas's thought is such that it cannot be adequately presented except within the full context of the divine and the human. For, in Thomas's view, the probable is proper to human, that is to say, to *merely* human knowledge; the range of the probable is reduced by scientific demonstration, and is ultimately transcended in the beatific vision.

At the risk of being criticized for hopelessly confusing areas of thought which are radically different one from another, we maintain that, *mutatis mutandis*, this theological vision of human knowledge is

not unlike the vision of many modern scientists who have expressed themselves on the subject. For, whether one talk about God or about a beatific vision or about an ultimate comprehension of the universe, the epistemic goal remains the same, and opinionative knowledge of the probable is man's most familiar means of approaching it. The scientist as such, of course, does not speak about God, nor does the theologian as such speak about degrees of confirmation or relative frequency. But each is in some way aware of a postulated culmination of human reasoning which, however he may care to describe it, gives finality to his intellectual endeavors. Indeed, it is only in the light of this postulated perfection of knowledge that he can speak at all meaningfully about the imperfections of what he already knows. In short, however others may choose to speak about cognitional limitations, Thomas does so within a theocentric context. Accordingly, if one wishes to grasp the full significance of what he is saying, one must be willing to accept him on his own terms (transposing, to be sure, if he is so inclined) – and these terms are theocentric.

B. In view, then, of the absolute superiority of divine knowledge over all merely human knowledge, Thomas maintains that whatever man knows, and in whatever way he knows it, his knowledge is but an imperfect approximation to God's comprehensive vision of all things.

However, within the horizon of the imperfect as such, some of man's knowledge is less imperfect than the rest. For, though man has only probable knowledge about many things, he does have certain knowledge about some things. Thus, without losing sight of God's epistemic superiority, Thomas still maintains a clear distinction *between that part of man's knowledge which is certain and that part of his knowledge which is only probable*.

To be sure, man's certainty may be unfounded, as in the case of heretics. But to the extent that man's certainty is founded in fact, it is due to his having to some extent approached the wisdom of God by determining the cause or causes of something through scientific demonstration. For, God's wisdom is, after all, a knowledge of the causes of things. Thus, again in view of the perfection of divine knowledge, the distinction in human knowledge between the certain and the probable *reduces to that between the scientific and the opinionative, the demonstrated and the probable*. This distinction, in turn, is hypothetically taken to be at least a rough approximation to that between the necessary and the contingent.

Applying metahistorical categories to history, unfortunately, Thomas uses these distinctions to sort out in the world those who have the truth and those who do not. The "extraneous" or "heretical" opinion is recognized from the fact that it is contrary to what is known to be true. Such over-zealous absolutism is, of course, easy to criticize; but the would-be critic could spend his time more profitably by trying to determine what are *his own* metahistorical absolutes. We are reminded, for example, of the case of the American who would dare to call himself a Communist or of the white South African who would dare to call himself an integrationist.

Be that as it may, as a corollary of this interpretation, we further maintain that for Thomas other distinctions between various branches of learning are of quite secondary importance. Even more, inasmuch as his notions of science and of opinion cut across the dividing lines of all human disciplines, he would find it difficult to understand a distinction between "philosophy" and "science" and impossible to understand a distinction between *philosophia* and *scientia*.

II. *There is a similarity between Thomas's notion of opinion, or probable knowledge, and modern notions of non-demonstrative knowledge.*

Having already noted a broad similarity between Thomist and modern thought patterns in general, we now wish to limit our attention to that part of these thought patterns which corresponds to Thomas's notion of opinion, or probable knowledge. This, in turn, restricts our attention to what might be called, in modern terms, the logic of science. Our purpose being once again to point out an important similarity, we take as our point of departure Thomas's notion of *probabilis*.

In Thomas's usage, *probabilis* applies in general to the class of all propositions which are (1) neither demonstratively false (2) nor demonstratively true. The adherence to such a proposition is an opinion, which accordingly is characterized precisely by the fact that it may be either true or false. Thus *the medieval notion of probability is essentially metascientific in that (1) it presupposes criteria of demonstration and (2) it implies with regard to a given proposition that these criteria are not fulfilled.*

In the second place, we find in modern thought, though not under the aegis of "probability," a recognition of the non-demonstrative which, *mutatis mutandis*, is not unlike that implied by Thomas's *probabilis*. To cite just a few examples of what we have in mind, we are reminded of Popper's characterization of science as *"doxa,"* Polanyi's search for "the personal" in science, and Perelman's analyses of argumentation in terms of "the preferable."

In the third place, we note Rudolf Carnap's insistence that one of what he considers the two basic meanings of *probability* which scientists have sought to explicate is that of "degree of confirmation." This sense of *probability*, he maintains, is the proper concern of what he calls "inductive logic." But inductive logic as understood by Carnap is precisely the logic of non-demonstrative reasoning. And thus the modern notion of probability is at least in part linked to the notion of the non-demonstrative.

From the foregoing, then, we see that it is historically unsatisfactory to consider "probability" simply and solely as an interpretation of one particular mathematical system. For, this would leave us with the conclusion that Thomas's view was much broader in that it took into account the whole range of the opinionative or non-demonstrative. And this, in turn, would make inexplicable the many and varied contemporary studies of the non-demonstrative which more often than not make no explicit reference to "probability."

In the light of these considerations, then, we shall attempt to establish a similarity between Thomist and modern logic of science in terms of what we shall call *opinion-probability*. To do this, we shall proceed in three steps. First (A), we shall propose a general definition of the notion of opinion-probability which includes both Thomas's *probabilis* and the *explicandum* of Carnap's *probability*$_1$. Secondly (B), we shall distinguish between the notion of probability and both explanations of it and instruments developed to deal with it. Thirdly (C), we shall use the first two steps as a basis for developing a criterion whereby the notion of opinion-probability can be recognized.

A. *The Notion of Opinion-Probability.* First of all, by "notion of opinion-probability" we shall mean *notion of the non-systematic. Notion* is here taken in a general sense broader than that of concept and is meant to imply, without further precision, awareness of or consciousness of. *Non-systematic* is also taken in a broad sense and is meant to imply non-necessary, or non-certain, or non-demonstrated, or even non-scientific in the Thomist sense which is not unrelated to the modern "indeterminate." Being negative, *non-systematic* is meant to imply also *"with respect to a given system."* In general, then, by "notion of opinion-probability" we mean *conscious or reflective awareness of the opinionative.*

B. *Explanations of and Instruments for Opinion-Probability.* Secondly, we wish to distinguish the notion of opinion-probability thus described both from explanations of the fact of opinion-probability and from instruments (conceptual or physical) developed to deal with it. For, it is one thing to recognize the non-systematic, it is another thing to attempt to explain or give the reason for the non-systematic thus recognized, and it is yet another thing to propose or develop an instrument to deal with the non-systematic.

To clarify what we mean here, we begin by recalling that we take "notion of the non-systematic" to imply *with respect to a given system*, S. In other words, the recognition of the non-systematic is essentially a recognition of the limits of S beyond which lies what is non-systematic, or non-demonstrated, with respect to S. And thus the recognition of the non-systematic suggests the need (1) to explain why there is a "non-systematic" with respect to S and (2) to develop some means – call it an instrument – of dealing with what is non-systematic with respect to S.

We deliberately avoid being too precise as to what constitutes a "system"; and, in particular, we avoid specifying whether "system" implies formalized or not, or whether it implies content or not. What is important, and all that is important in this context, is that *only what is "systematic" is considered demonstrated and that, accordingly, the "non-systematic" implies non-demonstrated.* Thus, what one will consider "non-systematic" is a function of what he considers "systematic." For example, if one takes Aristotelian physics as S, then any physical events not explained by that physics will be considered non-systematic *with respect to S.* Similarly, if one takes Newton's mechanics as S, then whatever relevant phenomena are not explained by Newton's system are non-systematic *with respect to S.* Recalling, finally, that systematic here implies demonstrated, we note that one might consider only formal theories in the strict logical sense to be "systematic" (in our sense) and hence anything extra-logical to be non-systematic in the sense of non-demonstrated.

Trusting, then, that we have sufficiently indicated the wide sense in which we take "systematic" and "non-systematic," we now wish to clarify somewhat what we mean by (1) an explanation of the non-systematic and by (2) an instrument for opinion-probability.

B. 1. *Explanation of Opinion-Probability.* An explanation of the non-systematic with respect to S is, in general, a meta-scientific reason for the fact of the non-systematic with respect to S. The reason given might refer to limits of S or to limits of its user or to planetary influences or to the divine will or whatever. What is important is that the reason is not itself a part of S but is a meta-judgment about S.

B. 2. *Instrument for Opinion-Probability.* Now, having recognized the non-systematic, non-S, with respect to a given system S, one might with or without explanation, propose or develop an instrument to deal with non-S. This instrument, physical or conceptual, might in principle be simply S itself but it is more likely to be some analogue or model of S, associated with S by more or less rigorous rules of correspondence, or even some modification of S. What is important here is that *since only S is considered demonstrative, the instrument non-S is not.* Thus, if we must refer to this instrument as being also a system, it is nonetheless, *qua* instrument for the non-systematic, *a non-demonstrative system* as opposed to the demonstrative S.

The distinctions thus made between the notion of, the explanation of, and the instrument for opinion-probability can be illustrated first from the example of Thomas Aquinas and then from the example of some modern writers.

Thomas Aquinas in recognizing the non-systematic sees it precisely as that about which one does not have demonstrative knowledge. That demonstration is not possible in all cases he explains physically in terms of contingency in terrestrial events and theologically in terms of man's lack of divine vision. Seeing that the contingent, unlike the necessary, is that which can be other than it is, he characterizes non-demonstrative knowledge as that which, unlike science, can be other than it is. Having thus pointed to the fact that the non-demonstrative is open to alternatives, he accepts as man's best instrument for dealing with the non-demonstrative a modification of demonstrative argumentation. This modified form of argumentation is dialectical disputation, in which, precisely, the two alternatives of any question are argumentatively opposed and evaluated. Since, finally, the practical order is concerned with the contingent as defined above, Thomas feels free to consider moral deliberation as a kind of disputation with regard to alternative courses of action. Aware, however, that both disputation and deliberation have to do with the non-demonstrative, Thomas notes that these methods arrive at the truth, somewhat like the occurrence of the physically necessary, only most of the time: *ut in pluribus.*

Among the moderns, Karl Popper's notion of *doxa* involves a recognition that the extra-logical is non-systematic; he explains this situation by appealing to the downfall of Newtonian absolutism; and, not unlike Thomas, he proposes the conjecture and refutation of logical theories as an instrument to deal with the non-systematic. Polya points to the non-systematic with respect to mathematics in terms of "plausibility" and, without explanation, elaborates a variety of logical techniques of "plausible reasoning." Perelman recognizes "the preferable," explains the need for recognizing it along the lines of Gonseth's "open philosophy," and proposes to deal with it by developing a theory of argumentation. Polanyi calls attention to the non-systematic with respect to physical science, explains it as being due to factors overlooked by those who exaggerate the ideal of "objectivity," and thus proposes the need to develop a social psychology of "the personal" in science. Others, more imbued with that very ideal of "objectivity," see the non-systematic simply as that which is still beyond the reach of logic and/or mathematics. Thus Borel, for example, urges prudent application of the calculus of probability to personal affairs and Carnap insists upon developing a logic of the non-demonstrative. Servien, finally, in recognizing the non-systematic as the extra-mathematical, proposes to deal with the latter by an elaboration of his distinction between the language of mathematics and the language of literature.

C. *How to Recognize the Notion of Opinion-Probability*. Turning now to our third step, we propose to elaborate a criterion on the basis of which the notion of opinion-probability can be recognized.

In preparation for this task, we note that though an instrument be addressed to "the non-systematic," it is nonetheless constructed according to the best available systematization of the non-systematic. The problem is simply that the non-systematic cannot in principle be *demonstratively* systematized. Whence it happens that an instrument addressed to the non-systematic will in principle encounter what are often referred to as non-systematic divergences. In view, then, of these non-systematic divergences, it is incumbent upon the constructor of the instrument to safeguard the efficacity of the instrument before the non-systematic by providing the instrument as much as possible with systematic means to adapt itself to non-systematic divergences. To do this, he adds to the instrument certain self-correcting devices by means of which non-systematic variations can be more or less effectively neutralized. These self-correcting devices amount to qualifications of the instrument and constitute the manifestation in that instrument of the notion of the non-systematic.

From these observations we now draw three conclusions which are subordinate one to the other. First of all, precisely insofar as the non-systematic is non-systematized, it will involve variables not systematically represented by the instrument addressed to it. Secondly, these

unsystematized variables can and in many cases will diminish the effectiveness of the instrument as applied to the non-systematic. Thirdly, the effectiveness of the instrument before the non-systematic is therefore directly proportional to its ability to neutralize the effect of non-systematic variables.

In general, then, awareness of the non-systematic is manifested precisely by the fact of taking precautions against and thus attempting to neutralize the effect of non-systematic variables. This, in turn, reveals the non-demonstrative character of the system serving as an instrument and thus allows us to suggest the following as *a criterion on the basis of which the notion of opinion-probability can be recognized: The notion of opinion-probability is manifested whenever the results (or conclusions) obtained by utilization of an instrument are in some way qualified, thus qualifying indirectly the system on which the instrument is based.*

That this criterion applies to Thomas's notion of opinion-probability has already been suggested, but it will be useful to spell out the suggestion in some detail. Thomas's basic presumption with regard to instruments addressed to the non-systematic is that the non-systematic can be represented disjunctively. Thus he divides contingent events into those which occur *ut in pluribus* and those which occur *ut in paucioribus*, he sets up a disputation according to opposite sides of a question, he portrays deliberation as a consideration of alternative choices. Yet in practice he often satisfies himself that the true opinion, theoretical or practical, is a golden mean between extremes. Because of the complexity of the problems involved, however, he is forced to admit (still, be it noted, within the confines of a dichotomous representation) that these instruments attain the truth only *ut in pluribus*.

That this criterion applies to all modern notions of opinion-probability is, of course, more difficult to establish, since there are so many different formulations. Here, then, we presume no more than to point out that it applies both independently of the calculus of probability and in connection with the calculus of probability.

First of all, on the basis of Carnap's association of the non-demonstrative with "degree of confirmation," we identify as manifestations of opinion-probability Polya's reference to "plausibility" in connection with mathematics, Popper's reference to "doxa" and Polanyi's reference to "the personal" with regard to science, Perelman's reference to "the preferable" with regard to argumentative method, and so on.

Secondly, we find manifestations of opinion-probability in discussions about the calculus of probability. We find it, for example, in Gendre's observations about the practical need to qualify Bernoulli's theorem with Stirling's formula, in Russell's breakdown of non-mathematical meanings of probability and in particular in his reference to *probable* probability with regard to applications of the calculus, in Borel's cautions about the applicability of the calculus to practical life, in Polanyi's insistence that as applied in these areas the calculus is a maxim like other maxims, in Boll's rather irresponsible statements about probability as the law of the universe, in Reichenbach's insistence that all knowledge is probable, and, in general, in the innumerable discussions about the probability of induction.

III. There is a similarity between Thomas's disputation and the modern calculus of probability.

Having already proposed a similarity between Thomist and modern *thought-patterns* in general and between Thomist and modern *notions of opinion-probability* in particular, we now begin to specify similarities involving directly the calculus of probability. And first of all we propose that *the calculus of probability, like medieval disputation, was originally viewed as an instrument to deal with the non-demonstrative.* The elaboration of this proposal will amount to what we shall call *the historical meaning* of "the calculus of probability."

In brief, at first, we take "calculus" to refer to an *instrument* and "probability" to refer explicitly to *the notion of the non-systematic* and implicitly to *a new way of expressing the non-systematic.* To explain what this involves, we shall: (A) extend the notion of the non-systematic so as to make room not only for the qualification of an instrument but also for the replacement of one instrument by another; (B) consider abstractly the ideological universe in which the notion of a "calculus of probability" originated: (C) consider concretely the evidence of this ideological background in Laplace's *Philosophical Essay on Probabilities.*

A. *Replacement of one instrument by another.* We have suggested in the preceding discussion that the notion of the non-systematic tends to generate an explanation as to why there is this non-systematic and this in turn tends to generate an instrument to deal with the non-systematic. We have further noted that the effectiveness of such an instrument is directly proportional to its ability to neutralize the effects of non-systematic divergences. Now we wish to add as a corollary that if the neutralizing capacity of the instrument, however qualified, is minimal with regard to a given problem, the need arises to replace that instrument with another one.

As examples of how this might apply to the contemporary history of ideas, we refer to just three which are rather well known. First of all, we call attention to the fact that repeated failures to establish the Euclidean axioms led eventually to modifications of the axioms which made possible non-Euclidean systems of geometry. Secondly, we note that the inability of classical mechanics to deal effectively with certain problems led to reformulations which we now know as quantum physics. Thirdly, we recall that efforts to provide a perfect formalization of arithmetic uncovered problems which eventually led to recognition both of internal limitations of a formal system and of the need for richer languages. Each of these examples in some way (more or less strictly according to the case) involves what might be called a recognition of incompleteness. And thus on this level of replacement of one instrument by another we are suggesting a connection between the notion of incompleteness and that of the non-demonstrated or non systematic.

In what follows, then, we shall propose that the calculus of probability came to replace medieval dichotomous instruments as a more effective means of dealing with the non-systematic. We shall also observe, however, that this new-born instrument was in its childhood considered precisely as an instrument of the non-systematic rather than as a demonstrative system in its own right.

B. *Ideological Origins of "Calculus of Probability."* Having just recognized the possibility of replacing one instrument by another, we now prepare the way for a kind of meta-history of the calculus of probability by viewing it as a new instrument of the non-systematic parallel with a new system gradually replacing the old on which had been based medieval instruments of the non-systematic.

To begin with, we note that the notion of opinion-probability was much more universally covered by *probabilis* than is the same notion today by *probable*. Today, a variety of other terms (including "personal," "preferable," etc.) substitute in one way or another for the medieval *probabilis*. That this is largely due to expropriation of *probability* by mathematicians is relevant but not directly to the point. The point is rather that said expropriation had not yet taken place at the time when "the calculus of probability" took, as it were, its first baby steps. The world of Cardano, even the world of Pascal and Fermat, and even the world of the Bernoullis and of Laplace was still in some ways more "medieval" than many of us would care to admit. For, Thomas's picture of man's approximation to divine knowledge as well as his distinction between the demonstrative and the probable were still at least implicitly acknowledged. What gradually and sometimes dramatically changed was man's view as to what was in fact "probable" and what was in fact "demonstrative."

This, after all, was the very heart of the controversy over Copernican astronomy. Scholars like Bellarmine opposed Galileo not for favoring the Copernican system but for insisting that it was scientific (that is, demonstrative) rather than merely probable. Without approving of methods adopted to persuade Galileo, we nevertheless are today closer to Bellarmine's view than to that of Galileo – and thus closer to Thomas's evaluation of empirical science than to the post-Newtonian. But absolutism reigned in between. Galileo's word in time became law with the triumph of Newton's *Principia Mathematica Philosophiae Naturalis*. The general blueprint of natural motion had been definitively demonstrated not merely with regard to what happens *ut in pluribus* but with regard to what happens *semper*. The system, in short, was perfect: it was, as had been Aristotle's cosmology before it, the new *scientia* of the macrocosm.

Though perfect, however, the system was not exhaustive. A realm of *ut in pluribus* and *ut in paucioribus* was still being subjected in the schools to the dichotomous instrument of disputation, which was becoming with each passing year more and more a stranger in a new world built by mathematics. Here, then, alongside of *scientia*, was the realm of the non-systematic, the non-demonstrative, the *probabilia*.

There was, then, a clear notion of *probabilis* in the schools. This notion, in turn, presupposed both a notion and a theory of demonstration. On the basis of the notion and theory of demonstration, the notion of the non-demonstrated was closely linked with that of the contingent, that is, that which can be other than it is. Operating on a principle of disjunction, the scholastic successors of Thomas Aquinas divided the contingent into what occurs *ut in pluribus* and what occurs *ut in paucioribus*, attacked the contingent with the dichotomous instrument of disputation, and proposed that one deliberate his practical decisions by consideration of alternative choices. Results obtained by these instruments, in contrast to those of the demonstrative syllogism, had to be qualified. And thus was kept alive the notion of *probabilis*, of the non-systematic.

In the course of time, Cardano and then Pascal and Fermat came to recognize that gambler's rules already in existence might provide a more effective instrument with which to deal with the contingent. These gambler's rules they and then others developed and systematized.

That this more or less systematic instrument of the non-systematic came to be known as a *calculus* is due not only to its character as a mathematical instrument but to imitation and adulation of the great new instrument of the systematic, the calculus of Leibniz and Newton. (For Pascal, still under the influence of Descartes, it was rather a "geometry of chance.")

That this *calculus* of the non-systematic came to be called a calculus of *probability* is due to ingredients of the intellectual milieu which go back deep into the Middle Ages. To uncover in detail how these ingredients were kept before the minds of the first mathematical "probabilists," one might study in detail developments after Thomas with regard to (1) the Aristotelian theory of demonstration; (2) divine providence and foreknowledge in the face of man's free will; and (3) moral systems of resolving practical doubt.

As for the calculus itself, the new instrument thus inaugurated was eventually systematized by Laplace according to standards of his day and by Kolmogorov and others according to standards of our day. But it is important to bear in mind that what is now a demonstrative system in its own right began as *an instrument to deal with the non-systematic on the basis of a new theory about how to express the non-systematic:* not disjunctively but in terms of a continuum of values between what happens always and what never happens.

C. *Historical Meaning of "Calculus of Probability."* We have just proposed that the notion of a "calculus of probability" is in part traceable to medieval ideology, and that the part which is medieval is precisely the "probability." It would require another book to prove that Thomas's usage of *probabilis* remained current throughout the developmental period of the calculus of probability. In lieu of this, we shall here indicate only that the greatest nineteenth century "probabilist," *Pierre Simon, Marquis de Laplace* (1749–1827) not only addressed himself to the notion of opinion-probability but in effect saw his instrument as a replacement for the medieval method of disputation. Our remarks are based on his *Essai philosophique sur les probabilités* (1819), which served as an introduction to the third edition of his great *Théorie analytique des probabilités* (1820).[1] Our purpose is to show that for Laplace (1) probability is a mark of imperfect knowledge; (2) proba-

[1] More specifically, we follow the translation into English of the sixth French edition by Frederick Wilson Truscott and Frederick Lincoln Emory entitled, *A Philosophical Essay on Probabilities* (New York, 1951). We have taken the liberty to correct their translation where we find it deficient. This work will be cited as *Philosophical Essay*.

bility is non-demonstrative knowledge; (3) the calculus of probability is an instrument of the non-systematic.

C. 1. *Probability as Mark of Imperfect Knowledge.* Laplace begins his *Philosophical Essay on Probabilities* by noting that "nearly all our knowledge is problematical" and that even "the small number of things which we are able to know with certainty... are based on probabilities."[1] After this humble beginning, which differs little from the (theocentric) attitude of a Thomas Aquinas, he goes on, in spite of his ignorance of medieval thought, to present a view of the cosmos not unlike that of Thomas. The old ideas of "final causes" or "chance," he says, have gradually been replaced by the idea of an orderly universe based upon Leibnitz's principle of sufficient reason.[2]

C. 2. *Probability as Non-Demonstrative Knowledge.* Of many examples in Laplace's work which compare favorably with Thomas's notion of probability, we cite just two.

First of all, speaking with regard to the tides, he notes that Kepler was aware of a tendency of waters towards the moon but "he was able to give on this subject only a probable idea. Newton," Laplace goes on, "converted into certainty the probability of this idea by attaching it to his great principle of universal gravity." [3] Laplace then goes on to say that his own calculations give

> a probability that the flow and the ebb of the sea is due to the attraction of the sun and moon, so approaching certainty that it ought to leave room for no reasonable doubt. It changes into certainty when we consider that this attraction is derived from the law of universal gravity manifested by all the celestial phenomena.[4]

Secondly, after observing that it is difficult to evaluate the probability of the results of induction, Laplace goes on to present a basically Thomist (Aristotelian) view of the preparatory character of induction. "Induction," he says,

> in leading to the discovery of the general principles of the sciences, does not suffice to establish them absolutely. It is always necessary to confirm them by demonstrations or by decisive experiments.[5]

[1] Laplace, *Philosophical Essay*, p. 1.
[2] Laplace, *Philosophical Essay*, pp. 3–4. This form of determinism, which for the objectivist Popper would amount to a "conspiracy theory" of ignorance, is briefly traced through history and defended by John Maynard Keynes in his *Treatise on Probability*, Part IV; chapters xxiv and xxv: pp. 281–323 (ed. New York, 1962).
[3] Laplace, *Philosophical Essay*, pp. 89–90.
[4] Laplace, *Philosophical Essay*, pp. 92–93.
[5] Laplace, *Philosophical Essay*, pp. 176–177.

C. 3. *An Instrument of the Non-Systematic.* Given then, human fallibility and the resulting need for demonstration, Laplace tends to identify demonstration about the cosmos with Newton's mechanics. What the latter has not encompassed must be approached by instruments directed to what is non-systematic with respect to Newtonian mechanics. His own instrument, he finds, is particularly suited for this purpose. For, he points out,

> In the midst of numerous and incalculable modifications which the action of the causes receives... from strange circumstances these causes conserve always with the effects observed the proper ratios to make them recognizable and to verify their existence. Determining these ratios and comparing them with a great number of observations, if one finds that they constantly satisfy it, the probability of the causes may increase to the point of equalling that of facts in regard to which there is no doubt.[1]

Thus, says Laplace,

> The analytic formulae of probabilities... may be viewed as the necessary complement of the sciences... (and) ...are likewise indispensable in solving a great number of problems in the natural and moral sciences. The regular causes of phenomena are most frequently either unknown, or too complicated to be submitted to calculus; again, their action is often disturbed by accidental and irregular causes; but its impression always remains in the events produced by all these causes, and it leads to modifications which only a long series of observations can determine. The analysis of probabilities develops these modifications; it assigns the probability of their causes and it indicates the means of continually increasing this probability.[2]

In particular, Laplace notes that the analysis of probabilities has a very useful application in that it serves to determine "the mean values which must be chosen among the results of observations." [3] But, perhaps in keeping with the spirit of the French Revolution, he is most delighted with the possibilities of his instrument for the moral sciences. Thus, for example, not unlike Thomas Aquinas's moral statistics, Laplace rejoices in the utility of his instrument for determining "the probabilities of testimonies" [4] and "the probability of the judgments of tribunals." [5]

That Laplace is thereby putting in his own mouth the Thomist theory of contingency together with its corollary of a postulated necessity for what happens *ut in pluribus* is, we think, undeniable. Also

[1] Laplace, *Philosophical Essay*, p. 89.
[2] Laplace, *Philosophical Essay*, p. 195.
[3] Laplace, *Philosophical Essay*, p. 191.
[4] Laplace, *Philosophical Essay*, pp. 109–125.
[5] Laplace, *Philosophical Essay*, pp. 132–139.

undeniable is the fact that he wishes to apply his instrument to the same kinds of problems to which Thomas's theory of contingency was directed. That he places much more emphasis upon empirical observation than does Aquinas is also clear, and that the instrument which he addresses to these problems is superior to Thomas's bivalent system is not in question.

We need only add that there are clear indications in Laplace that he sees his mathematics as a replacement for medieval disputation. In one place, he cries forth an encomium of Francis Bacon for "insisting, with all the force of reason and eloquence, upon the necessity of abandoning the insignificant subtleties of the school, in order to apply oneself to observations and to experiments" and for "indicating the true method of ascending to the general causes of phenomena." [1] Yet at the same time Laplace admonishes:

> Let us enlighten those whom we judge insufficiently instructed; but first let us examine critically our own opinions and weigh with impartiality their respective probabilities.[2]

For Laplace, however, the best method of doing this is by use of "the theory of probabilities." For:

> It leaves no arbitrariness in the choice of opinions and sides to be taken; and by its use can always be determined the most advantageous choice. Thereby it supplements most happily the ignorance and the weakness of the human mind.[3]

To conclude this brief look at the ideology behind the "calculus of probability," we recommend most serious reflection upon the motives behind Laplace's name for his mathematics. In a chapter entitled "Concerning the Analytic Methods of the Calculus of Probability," he reviews the contributions of his predecessors, refers to all kinds of mathematical developments since Descartes, especially that of integral and differential calculus, and winds up with the most important historical observation of all:

> I have named the ensemble of the preceding methods the *Calculus of Discriminant Functions:* this calculus serves as a basis for the work which I have published under the title of the *Analytical Theory of Probabilities.*[4]

IV. *There is a similarity (A) between Thomas's theory of probability and the modern logical theory of probability and (B) between Thomas's theory of contingency and the modern frequency theory of probability.*

After having explained a similarity between Thomas's notion of

[1] Laplace, *Philosophical Essay*, pp. 179–180.
[2] Laplace, *Philosophical Essay*, p. 9.
[3] Laplace, *Philosophical Essay*, p. 196.
[4] Laplace, *Philosophical Essay*, p. 48.

probability and modern notions of non-demonstrative knowledge, we then showed that this notion of opinion-probability is very much in evidence in Laplace's views on probability. We also noted in Laplace, however, a similarity between his ideas about the cosmological basis of probability and Thomas's notion of contingency. For, in the view of Laplace as well as in Thomas's view a proposition about a contingent event is probable to the extent that it occurs with some determinable regularity. Thomas, of course, was content to say of such an event that it occurs (for example) *ut in pluribus*. But, with his mathematical sophistication as a guide, Laplace insisted on establishing with much more precision just what this *ut in pluribus* might be. Like his medieval predecessor, however, he was willing to grant that if an event occurs with sufficient regularity one might attribute exceptions to disturbing factors and thus postulate the existence of a necessary cause of such an event. But his conviction as to the absolutely demonstrative character of Newton's mechanics is such that he does not seem to admit what Thomas would call a demonstration *ut frequenter*.

Now it is of the utmost importance to note that in speaking about the frequencies with which more or less irregular events occur, Laplace refers quite often to their "probabilities." Though he does not seem to be consciously aware of what he is doing, he is in fact giving another sense to "probability" than the sense of opinion-probability which he explicitly discusses along lines not unlike that of Thomas Aquinas. This second sense of "probabilities" as relative frequencies gradually became, as Rudolf Carnap tells us in detail, a second *explicandum* for the interpretation of the calculus of probability.

Thus, while John Maynard Keynes and others continued to view "probability" as a characteristic of a proposition, as had Aquinas, others, including notably Richard von Mises, Hans Reichenbach, and the school of statisticians now represented by Ronald Fisher, have concentrated upon "probability" in the sense of relative frequency. Summing up the development, Carnap identifies "relative frequency in the long run" as *probability*$_2$ and identifies "degree of confirmation" as *probability*$_1$. The former constitutes the *explicandum* for the "mathematical" theory of probability, and the latter constitutes the *explicandum* for the "logical" theory of probability.

The differences between these two theories are not inconsiderable. To use the simple summary of Polanyi, the logical theory concentrates upon a *"probable" proposition* about events whereas the mathematical theory concentrates upon a proposition about *"probable" events*. The latter is a manifestation of the great modern ideal of "objectivity" in that it shuns any suggestion of "subjective" or "psychological" adherence. But the logical theory is no less "objective," since it is concerned with logical properties of a proposition and not with what a subject "thinks" or "feels" about that proposition.

In short, much has happened since the time of Laplace. And the most important thing that has happened is the formalization of the calculus of probability. For, as a result of this formalization it is now possible to introduce into a consideration of the calculus of probability several extremely important distinctions which were not clearly recognized at the time of Laplace. What these distinctions are can be summarized as follows. A careful analysis of formal systems has led to rather general agreement that (1) a formal system may be considered without regard to any interpretation, and that (2) a formal system of any importance is open to more than one interpretation. In more precise terms, these two points mean, respectively, that (1) there is an important distinction to make between a formal system and an interpretation of that system, and that (2) there is an equally important distinction to make between interpretation as such and a set of statements which interpret or are taken to interpret a given formal system.

We shall return to these points directly, but it will be useful beforehand to make three contrasting observations about Laplace. In the first place, Laplace seems to have viewed the calculus of probability somewhat naively (though not necessarily erroneously) as a direct representation of certain kinds of events now often referred to as aleatory. Secondly, he was aware of the fact but not of the significance of the fact that the mathematical instrument which he directed to such events was based upon the concepts and methods developed by Newton and others to represent the "systematic." Thirdly, he was implicitly involved in but not explicitly aware of two different interpretations of his instrument: the "logical" (probable propositions) and the "mathematical" (probable events).

Now, then, to show that the contemporary view of formal systems brings considerable clarity into the muddled thinking of a Laplace, we shall use the distinctions made above in order to analyze a particularly relevant statement by Bertrand Russell. The latter, after noting general agreement about the calculus of probability as such and general disagreement about its interpretation, suggests the following as an escape from discord. "In such circumstances," he says,

> the simplest course is to enumerate the axioms from which the theory can be deduced, and to decide that *any concept which satisfies these axioms has an equal right, from the mathematician's point of view, to be called "probability."* If there are many such concepts, and if we are determined to choose between them, the motives of our choice must lie outside mathematics.[1]

What is to be noted in the first place about Russell's statement is a clear distinction between an uninterpreted (or, as it is sometimes called, abstract) formal system and an interpretation of that system. Considered precisely as uninterpreted, the formal system has no extralogical meaning. But, according to Russell, it can be given a meaning as it were indirectly by establishing a correspondence between statements in the formal system and statements which are "meaningful" or which have *content*. These latter, then, might be called *contensive* statements as opposed to the abstract statements of the formal system.

In the second place, we note that Russell allows for the possibility of more than one "interpretation" of the formal system. Thus, it is advisable to make a clear distinction between interpretation of a formal system and a particular set of contensive statements which "interpret" those in the formal system. For, a particular contensive statement which interprets a formal statement does not exhaust the possible interpretations that might be found for that same formal statement. For the sake of clarity, then, some logicians prefer to speak of an interpreting statement as an *interpretant*. Speaking somewhat loosely, we shall here refer to a set of interpreting statements as an interpretation.

In the third place, we note that Russell speaks about satisfying the axioms of the formal system, and thus in effect demands that the interpretation be *valid*. An interpretation is valid only if each contensive statement corresponding to a theorem of the formal system is true. And this, apparently, is what Russell demands when he says that a given "concept" must satisfy the axioms of the formal system.[2]

[1] Bertrand Russell, *Human Knowledge: Its Scope and Limits* (New York, 1962), p. 339 (italics added). The following analysis of Russell's statement is based upon the consistent position of Haskell B. Curry as stated in: *A Theory of Formal Deducibility* (Notre Dame, Ind., 1950), pp. 9–10; *Lecons de Logique Algébrique* (Louvain-Paris, 1952), pp. 26–27; "The Interpretation of Formalized Implication," *Theoria* 25 (1959): 13–16; *Foundations of Mathematical Logic* (New York-Toronto-London, 1963), pp. 48–49, 59–60. Similar though less developed views will be found in Alfred Tarski, *Introduction à la Logique* (Louvain-Paris, 1960), n. 37, pp. 106–115; Morris R. Cohen and Ernest Nagel, *An Introduction to Logic* (New York-Burlingame, 1962), pp. 137–142. The author is particularly indebted at this point to Jean Ladriere and to Madeleine Sergant for assisting him materially in the delicate task of expressing technical definitions with non-technical precision. He alone, however, assumes responsibility for the accuracy of his presentation.

[2] Earlier in the same work, Russell considers the notion of interpretation *ex professo*, and there makes it clear that what he demands of an interpretation is that it be *valid*. "Our formulas," he says, "are not regarded as 'true' or 'false,' but as hypotheses containing varia-

These demands, to be sure, are rigorous enough; but it is well to point out that they might be made even more rigorous. For one thing, the notion of a valid interpretation does not eliminate the possibility of having true contensive statements which, though relatable to a formal statement, do not correspond to a theorem of the formal system. In such a case, the interpretation would still be valid but it would not be *adequate*. This, in turn, suggests the possibility of a stronger formal system which could allow for an adequate interpretation. An interpretation is adequate, then, if each formal statement shown to correspond to a true contensive statement is a theorem of the formal system.

One might further inquire as to whether all possible valid interpretations of a given system are isomorphic. Stating the matter briefly, when a system is based on the first-order predicate calculus, it is possible to build a certain kind of interpretation which is called, in the strict sense of the word, a model. Between models it is possible to define a relation of isomorphism, that is to say, similarity of structure. A system admitting models is then said to be categorical if all its models are isomorphic or, in other words, if it determines its models up to an isomorphism.

That the calculus of probability is not a categorical system can be seen from the many and sometimes heated discussions between proponents of the "logical" and proponents of the "relative frequency" interpretation. Even more, relative frequency has been expressed in terms of both finite and infinite series. And thus is indicated in a general way that the reality in question is still too complex for the formal system (the calculus of probability) which is used in various ways to represent it.

We see, then, that the notion of interpretation is, among other things, a matter of degree. Interpretation as here used always involves a correspondence between statements. But the correspondence in question might be more or less exhaustive and thus, if you will, more or less perfect. To some extent, then, factors extrinsic to logic itself will determine how rigorous a correspondence shall be required. The degree of correspondence which one requires will then determine whether or not a given interpretation is *acceptable*.

bles. A set of values of the variables which makes the hypotheses true is an 'interpretation'" . . . The axioms consist partly of terms having a known definition, partly of terms which, in any interpretation, will remain variables, and partly of terms which, though as yet undefined, are intended to acquire definitions when the axioms are 'interpreted.' The process of interpretation consists in finding a constant signification for this class of terms." *Human Knowledge*, p. 343.

We may, however, leave the delicate problem of the standards for an acceptable interpretation of the calculus of probability to people such as the quantum physicists, for whom it is of more immediate importance. For, our concern for the moment is elsewhere.

What we want to draw out of the preceding considerations is the fact that an interpretation of a formal system does not of itself establish a "meaning" for the formal system. All it really establishes is a more or less perfect *logical correspondence* between contensive statements and the abstract statements of the formal system. And thus, we think, Russell is saying perhaps even more than he intends to say when he notes that "the motives of our choice must lie outside mathematics." The point at issue, then, is simply this: if a formal system has no extra-logical meaning, then an interpretant of an abstract statement in that formal system, *considered precisely as an interpretant*, has no meaning either. For, interpretation determines correspondence and not meaning. And thus, if a statement that is (or that is taken to be) an interpretant of an abstract statement has contensive meaning, this meaning is quite independent of the logical correspondence that is called interpretation.

Therefore, since the extra-logical meaning of the formal system entitled "the calculus of probability" comes neither from the formal system itself nor from its interpretation, the extra-logical meaning must come from some third source. What, then, is this source of the extra-logical meaning of the formal system entitled "the calculus of probability?"

The third source, we propose, is what we have called the historical meaning which is packed into the (extra-logical) name of the formal system called "the calculus of probability": namely, *the cultural tradition which has been associated with this instrument of the non-systematic from its origins*.

As a sign of this cultural source of meaning, we point to an inconsistency in Russell's otherwise excellent analysis of "probability" as an interpretation of the formal system. According to Russell, we recall, whatever satisfies the axioms of the formal system can be called "probability." *But why, we should like to know, is it called "probability" if the system interpreted has in principle no meaning*? For all his logical clarity, Russell is caught in a vicious circle, from which, we think, the only escape is along the lines of Pius Servien's insistence that the formal system as such might just as well be called a Calculus of Sensations or, for that matter, "gindlegob." For, the name given to the formal system is *not* a part of the system but is rather a summary of the historical meaning given to that system in its developmental stages.

More specifically, while it is in principle true that any number of interpretations can be found for the calculus of probability, in actual fact only two important interpretations have been found:

(1) *the "logical" interpretation:* probability (degree of confirmation) of a proposition;

(2) *the "mathematical" interpretation:* probability (relative frequency) of a class of events.

It is possible to maintain, no doubt, that it is purely by chance that these two interpretations rather than others have been found for the calculus of probability. But the fact that the respective notions basic to these two interpretations are already present (one explicitly, the other implicitly) in Laplace's thoughts on the subject makes chance an unlikely explanation. Chance becomes even more unlikely when we realize that Laplace's two usages of *probability* correspond to Thomas Aquinas's usages of:

(1) *probabilis:* argumentatively supported (proposition);

(2) *contingens:* what happens (an event) either *ut in pluribus* or *ut in paucioribus.*

This being said, we consider our point as having been made. For, though it is perfectly obvious that neither of these notions served as the *explicandum* for an interpretation of a formal system during the Middle Ages, nevertheless the notions themselves, however refined they may have become, are essentially the *explicanda* of interpretations subsequently "found" for the calculus of probability.

V. *There is a relationship between Thomas's distinction between* scientia *and opinion-probability and the modern problem of probability in science.*

We have pointed out first in the abstract and then by a concrete consideration of Laplace that it was to a notion like Thomas's of opinion-probability that early probabilists directed their new instrument for the non-systematic. This new instrument, in turn, was felt to be concerned precisely with what was non-systematic with respect to the Newtonian system of celestial mechanics. The latter, in other words, was viewed as replacing medieval *scientia* and the former was viewed as replacing medieval disputation as a means of determining and increasing the probability of the opinionative. Moreover, since this new instrument was concerned primarily with physical events which fell short of the regularity requisite for *scientia*, its use in this regard gradually gave to "probability" a second meaning which embraced the *ut in pluribus* and *ut in paucioribus* whereby Aquinas characterized the *contingens.*

These points having been made, we are now in a position to bring out the full significance of our insistence that the calculus of probability was originated and developed *as an instrument to deal with the non-systematic.* In other words, we now want to make relevant to contemporary thought the fact that the calculus of probability was viewed during its formative years as a replacement for medieval disputation, that is to say, as the new preparation for or auxiliary to *scientia.*

In briefest terms, what is of the utmost importance about the present role of the calculus of probability is precisely the fact that it is no longer viewed as a preparation for or auxiliary to *scientia.* On the one hand, the *scientia* that was the Newtonian celestial mechanics has given way to Einstein's theory of relativity, and in the process man has lost his confidence in the absolutivity of *scientia.* On the other hand, and almost simultaneously, that which had been viewed as the propadeutic to *scientia* has suddenly found itself as the systematic representation of a large and important sector of *scientia* itself. And thus the new quantum physics has come to represent, from an historical point of view, a kind of wedding between *opinio* and *scientia.*

The resulting ideological crisis as to the meaning of this strangest of all weddings is still unresolved and will no doubt remain so for a long time to come. But it can already be observed that the crisis itself is due at least in part to an inadequate historical perspective and also in part to an exaggerated dichotomy between subject and object.

According to the traditional view – the view of Laplace as well as of Keynes – the imperfect, the merely "probable" was, qua imperfect, attributable to limitations on the part of the *subject.* The perfect, the "scientific," by contrast, achieved full *objectivity* or, so to speak, met the world on its own terms.

In the wake of quantum physics, this traditional view – essentially the same as that of Thomas Aquinas – was supposedly overturned. For, what had been for centuries two neatly distinct types of knowledge now seemed to be inextricably intermingled. The heretofore subjective "probable" was now projected upon the objective "scientific." Probability, science and objectivity were now thought to be all of one piece. And thus one could no longer say with Keynes and his forebears that the universe was determinate and that probability referred to gaps in our knowledge of that universe. One now had to say, rather, that the universe itself is indeterminate and that probability is simply an expression of that indeterminacy, without any reference to the (non-scientific) subject.

However, in spite of great dedication to the cause of objectivity, "the probable" has still not been successfully abstracted from "the subjective." Reichenbach, for example, still likes to insist that all knowledge is probable, and Popper tells us that it is *doxa* or verisimilitude. Thus, neither object nor subject qualifies any longer as the locus of certitude. This, for many, has withdrawn into a realm which presumably transcends the dichotomy of subject and object: the realm, namely, of logic as such. Certitude, if such there be, is to be found today only in the formal system.

This reaction to the calculus of probability, though unprecedented in its complexity, is nonetheless a familiar by-product of the introduction of a new mathematical instrument into man's efforts to harness the universe. The Pythagoreans and the Greeks in general, fascinated by the new geometry, saw geometrical design everywhere, and for this were eventually taken to task by Sextus Empiricus. The medieval followers of Ptolemy saw spheres and even epicycles in the heavens, and by way of reaction Kepler saw more heavenly harmony than was there. The founding fathers of the calculus tended to see "integrals" and "differentials" in the universe until Bishop Berkeley took time off from tar water to point out to them their inconsistencies.

Whatever the value of "the calculus of probability" as an instrument for nuclear research, this much at least seems clear. The universe is no more "determinate" or "indeterminate" today than it was a hundred years ago. Whether or not one considers the present formulation of quantum physics to favor one view over the other perhaps has something to do with the mathematics in question, but it has far more to do with one's views about the extent to which mathematics does more than merely measure. These views, in turn, are not derived from the formal system that persists in being called "the calculus of probability" but from a host of other factors which, *pace* positivists, can well be described as meta-physical.

In short, discussions about the calculus of probability and its applications have in one way or another been operating under the assumption that there has been in effect a wedding between *scientia* and *opinio.* And this assumption, which only now is beginning to be attacked at its roots, presupposes an ideology which goes back through Thomas Aquinas to the beginnings of Western thought.

This being said, we may consider as accomplished our task of pointing out the relevancy of medieval thought to post-medieval theories of probability. We have, to be sure, spoken as an interested layman about a subject that is not ours by profession. Since, therefore, we have surely failed in detail, we trust that we have not failed in perspective. For, we have been encouraged in our study by these century-old words of John Venn:

> No science can safely be abandoned entirely to its own devotees. Its details of course can only be studied by those who make it their special occupation, but its general principles are sure to be cramped if it is not exposed occasionally to the free criticism of those whose main culture has been of a more general character.[1]

[1] John Venn, *The Logic of Chance*, 1st ed. (London, 1866), Preface. Quoted by J. P. Day, *Inductive Probability* (New York, 1961), p. x.