CONCLUSION

ON THE HISTORICAL DIMENSION
OF "PROBABILITY"

It is quite generally believed that no meaningful or, better, significant rapprochement is possible between medieval thought and modern thought. For, the usual argument goes, the modern world is to a great extent a world made by man rather than by God—and made by him not only since the close of the Middle Ages but in large measure since the close of the nineteenth century. On this view, in other words, there is between us and the medieval an unbridgeable gap that has been forged among other things by the development of the mathematical and now more recently the logical sciences.

As a result of this historical transformation, it is contended, thought patterns have also been transformed, and to such an extent that they would no longer be understandable to the medieval man. For, in general, our thought is sophisticated whereas that of the medieval was naïve. The latter thought in terms of absolutes; we think in terms of approximations. He was fond of uniformity; we pride ourselves on being able to adapt ourselves to pluralism on all levels of life. He looked for simplicity in things; we remain ever conscious of complexity.

In short, we are thus presented with two radically different universes of thought. And, as Badi Kasm has observed, a particular universe of thought is systematically closed in upon itself and hence can only be judged on its own terms. But if this be the case, then it would seem to follow that any supposed rapprochement between medieval and modern thought is at best artificial and at worst misguided.

To put all this somewhat differently, the ghost of Jacob Burckhardt has not yet been laid to rest. Too willing to take some writers of the Renaissance at their word, this nineteenth century historian concluded that all that was good and noble about the "new birth" of intellect was due to a return to the Greeks. This view, to be sure, has been considerably modified by subsequent research. But to a great extent it remains the accepted conviction of most contemporary philosophers: the Muse of today's philosopher speaks not Latin but Greek—and perhaps even something more ancient than that.

It is interesting to note, therefore, that what some historians of philosophy have tried in vain to show the philosopher, historians of science are making ever more palatable to the interested scientist. The prodigious growth of the history of science in the past fifty years and, in particular, in the past ten years has clarified and qualified but never destroyed Pierre Duhem's thesis of continuity between medieval and Renaissance (or Newtonian) science. The results of research along these lines are well illustrated in the convincing work of John Henry Randall, Jr., entitled, curiously enough, The Career of Philosophy. Now whatever one may think of this gradually developing view of historical continuity between medieval and Renaissance science, he cannot fail to see that even if there be continuity during that period, it has only limited significance. For, it is anything but obvious that there is much important continuity between Newtonian science and the science of today. The revolutionary effect of Einstein's reformulation of celestial mechanics is a case in point. But no less important is the reformulation of terrestrial mechanics on the basis of the calculus of probability.

In briefest terms, it is generally felt that the introduction of "relativity" and "probability" into scientific thought has brought down the Newtonian absolutes and thus in effect cut the last tie between our world and the world of the medieval. In the place of absolutes, whether considered as conceptual or as propositional, man now deals with an "optique" or, if you will, a horizon of thought which is interpreted as a manifestation of his particular spatio-temporal condition. In the shadow of Einstein, all thought is described as being somehow or other "relative." And in the shadow of the quantum physicists, propositions are often viewed not as "true" but only as more or less effective approximations to truth. The absolute, however described, remains at best what Kant would call a transcendental ideal. In short, the new sophistication is upon us, and from it flow such bountiful blessings as freedom of conscience and a growing spirit of ecumenical rapproche-

1 Badi Kasm, L'Idée de Pouvoir en Métaphysique, Paris, 1939.
ment. But at the same time, it is felt, with the denouement of the absolute our last tie with medieval thought has been definitively cut.

The principal purpose of our study has been to question this supposed dichotomy between medieval and modern patterns of thought. This we have done by taking as our focal point the notion of probability, as expressed today and as expressed in the Middle Ages. To limit our task to the humanly possible, we have chosen to compare representative views of the twentieth century with the view of the best known of all medieval thinkers, Thomas Aquinas.

Our method has consisted primarily of studying what is said precisely in the hopes of describing the ideological universe which has made it acceptable to say such things. It is, if you will, two ideological universes which we have tried to describe, our own and that of the medieval. We recognize full well the differences between these two universes, and even more between the kinds of statements possible in each. But at the same time we claim to have found important similarities between these ideological universes which suggest, in turn, the possibility of an historical continuity with regard to the notion of probability.

To spell out in detail what has here been suggested, we propose to defend consecutively five major conclusions. Each of these conclusions, we think, can be drawn independently from the study which we have made; but some are more clear cut and obvious than others. Accordingly, we have staggered our conclusions from the most to the least obvious and thus from the most trivial and readily acceptable to the most important and controversial. In this way we hope to use the stronger in order to build support for the weaker. Our conclusions, then, are the following:

I. There is a similarity between the structure of Thomas's thought patterns and modern thought patterns.

II. There is a similarity between Thomas's notion of opinion, or probable knowledge, and modern notions of non-demonstrative knowledge.

III. There is a similarity between Thomas's disputation and the modern calculus of probability.

IV. There is a similarity (A) between Thomas's theory of probability and the contemporary logical theory of probability and (B) between Thomas's theory of contingency and the contemporary frequency theory of probability.

V. There is a relationship between (A) Thomas's distinction between scientia and opinion-probability and (B) the modern problem of probability in science.

ON THE HISTORICAL DIMENSION OF "PROBABILITY"

These, then, being our conclusions, we proceed at once to their elaboration.

1. There is a similarity between the structure of Thomas's thought patterns and contemporary thought patterns.

On the surface, at least, our approach to Thomas Aquinas has not differed remarkably from that of many other commentators on the thought of the medieval master. And, as for these commentaries, we quite readily admit that more often than not they will contain a far more thorough treatment of most of the topics which have entered into our discussion. From a logical point of view, at any rate, the presentation of these topics in scholastic manuals will manifest the results of centuries of reflection upon and development of principles and procedures set forth in the writings of Thomas himself. Precisely because of ideological trends since the time of Aquinas, his thought has undergone a great deal of refinement especially with regard to ontology and epistemology in general and the theory of science and of demonstration in particular. On the whole, no doubt, these developments of Thomas's thought were, at least for their time, all for the good; and, properly understood, they still have a contribution to make to contemporary thought.

It is our opinion, however, that studies of Thomas's thought have in general been overly absolutist in their interpretation of the Angelic Doctor. And, as a result, Thomas has perhaps been systematized far better than he has been understood. It has been our impression, at least, that the rationalistic formulations of many so-called Thomistic manuals make the thought of Thomas himself, when seen at first hand, seem by comparison the cautious estimates of a neophyte before the unknown.

In contrast to the view of Thomas which these manuals usually present, we maintain that (A) the basic distinction of Thomas's theory of knowledge is, broadly, that between creator and creature or, more narrowly, between God and man; and that (B) from this distinction flows the basic distinction of his theory of human knowledge, broadly, that between the certain and the probable or, more strictly, between the scientific and the opinionative, the demonstrated and the probable.

A. The basic distinction of Thomas's theory of knowledge, in terms of which all else is to be judged, might most properly be described as that between the absolutely necessary, the creator, and what is by comparison
CONCLUSION

ont the historical dimension of "probability" 283

not unlike the vision of many modern scientists who have expressed
themselves on the subject. For, whether one talk about God or about
a beatific vision or about an ultimate comprehension of the universe,
the epistemic goal remains the same, and opinionative knowledge of
the probable is man's most familiar means of approaching it. The
scientist as such, of course, does not speak about God, nor does the
theologian as such speak about degrees of confirmation or relative
frequency. But each is in some way aware of a postulated culmination
of human reasoning which, however, he may care to describe it, gives
finality to his intellectual endeavors. Indeed, it is only in the light of
this postulated perfection of knowledge that he can speak at all
meaningfully about the imperfections of what he already knows. In
short, however others may choose to speak about cognitional limi-
tations, Thomas does so within a theocentric context. Accordingly, if
one wishes to grasp the full significance of what he is saying, one must
be willing to accept him on his own terms (transposing, to be sure, if
he is so inclined) – and these terms are theocentric.

B. In view, then, of the absolute superiority of divine knowledge
over all merely human knowledge, Thomas maintains that whatever
man knows, and in whatever way he knows it, his knowledge is but
an imperfect approximation to God’s comprehensive vision of all things.
However, within the horizon of the imperfect as such, some of man’s
knowledge is less imperfect than the rest. For, though man has only
probable knowledge about many things, he does have certain knowledge
about some things. Thus, without losing sight of God’s epistemic
superiority, Thomas still maintains a clear distinction between that part
of man’s knowledge which is certain and that part of his knowledge which
is only probable.

To be sure, man’s certainty may be unfeigned, as in the case of
heretics. But to the extent that man’s certainty is founded in fact, it
is due to his having to some extent approached the wisdom of God by
determining the cause or causes of something through scientific
demonstration. For, God's wisdom is, after all, a knowledge of the
causes of things. Thus, again in view of the perfection of divine
knowledge, the distinction in human knowledge between the certain
and the probable reduces to that between the scientific and the opinionative,
de demonstrated and the probable. This distinction, in turn, is hy-
pothetically taken to be at least a rough approximation to that between
the necessary and the contingent.

Applying metahistorical categories to history, unfortunately, Thomas
uses these distinctions to sort out in the world those who have the truth and those who do not. The “extraneous” or “heretical” opinion is recognized from the fact that it is contrary to what is known to be true. Such over-zealous absolutism is, of course, easy to criticize; but the would-be critic could spend his time more profitably by trying to determine what are his own metahistorical absolutes. We are reminded, for example, of the case of the American who would dare to call himself a Communist or of the white South African who would dare to call himself an integrationist.

Be that as it may, as a corollary of this interpretation, we further maintain that for Thomas other distinctions between various branches of learning are of quite secondary importance. Even more, inasmuch as his notions of science and of opinion cut across the dividing lines of all human disciplines, he would find it difficult to understand a distinction between “philosophy” and “science” and impossible to understand a distinction between philosophia and scientia.

II. There is a similarity between Thomas’s notion of opinion, or probable knowledge, and modern notions of non-demonstrative knowledge.

Having already noted a broad similarity between Thomist and modern thought patterns in general, we now wish to limit our attention to that part of these thought patterns which corresponds to Thomas’s notion of opinion, or probable knowledge. This, in turn, restricts our attention to what might be called, in modern terms, the logic of science. Our purpose being once again to point out an important similarity, we take as our point of departure Thomas’s notion of probabilitas.

In Thomas’s usage, probabilitas applies in general to the class of all propositions which are (1) neither demonstratively false (2) nor demonstratively true. The adherence to such a proposition is an opinion, which accordingly is characterized precisely by the fact that it may be either true or false. Thus the medieval notion of probability is essentially metascientific in that (1) it presupposes criteria of demonstration and (2) it implies with regard to a given proposition that these criteria are not fulfilled.

In the second place, we find in modern thought, though not under the aegis of “probability,” a recognition of the non-demonstrative which, mutatis mutandis, is not unlike that implied by Thomas’s probabilitas. To cite just a few examples of what we have in mind, we are reminded of Popper’s characterization of science as “doxa,” Polanyi’s search for “the personal” in science, and Perelman’s analyses of argumentation in terms of “the preferable.”

ON THE HISTORICAL DIMENSION OF “PROBABILITY”

In the third place, we note Rudolf Carnap’s insistence that one of what he considers the two basic meanings of probability which scientists have sought to explicate is that of “degree of confirmation.” This sense of probability, he maintains, is the proper concern of what he calls “inductive logic.” But inductive logic as understood by Carnap is precisely the logic of non-demonstrative reasoning. And thus the modern notion of probability is at least in part linked to the notion of the non-demonstrative.

From the foregoing, then, we see that it is historically unsatisfactory to consider “probability” simply and solely as an interpretation of one particular mathematical system. For, this would leave us with the conclusion that Thomas’s view was much broader in that it took into account the whole range of the opinionative or non-demonstrative. And this, in turn, would make inexplicable the many and varied contemporary studies of the non-demonstrative which more often than not make no explicit reference to “probability.”

In the light of these considerations, then, we shall attempt to establish a similarity between Thomist and modern logic of science in terms of what we shall call opinion-probability. To do this, we shall proceed in three steps. First (A), we shall propose a general definition of the notion of opinion-probability which includes both Thomas’s probabilitas and the explicandum of Carnap’s probability. Secondly (B), we shall distinguish between the notion of probability and both explanations of it and instruments developed to deal with it. Thirdly (C), we shall use the first two steps as a basis for developing a criterion whereby the notion of opinion-probability can be recognized.

A. The Notion of Opinion-Probability. First of all, by “notion of opinion-probability” we shall mean notion of the non-systematic. Opinion is here taken in a general sense broader than that of concept and is meant to imply, without further precision, awareness of or consciousness of. Non-systematic is also taken in a broad sense and is meant to imply non-necessary, or non-certain, or non-demonstrated, or even non-scientific in the Thomist sense which is not unrelated to the modern “indeterminate.” Being negative, non-systematic is meant to imply also “with respect to a given system.” In general, then, by “notion of opinion-probability” we mean conscious or reflective awareness of the opinionative.

B. Explanations of and Instruments for Opinions-Probability. Secondly, we wish to distinguish the notion of opinion-probability thus described both from explanations of the fact of opinion-probability and
from instruments (conceptual or physical) developed to deal with it. For, it is one thing to recognize the non-systematic, it is another thing to attempt to explain or give the reason for the non-systematic thus recognized, and it is yet another thing to propose or develop an instrument to deal with the non-systematic.

To clarify what we mean here, we begin by recalling that we take "notion of the non-systematic" to imply with respect to a given system, S. In other words, the recognition of the non-systematic is essentially a recognition of the limits of S beyond which lies what is non-systematic, or non-demonstrated, with respect to S. And thus the recognition of the non-systematic suggests the need (1) to explain why there is a "non-systematic" with respect to S and (2) to develop some means -- call it an instrument -- of dealing with what is non-systematic with respect to S.

We deliberately avoid being too precise as to what constitutes a "system"; and, in particular, we avoid specifying whether "system" implies formalized or not, or whether it implies content or not. What is important, and all that is important in this context, is that only what is "systematic" is considered demonstrated and that, accordingly, the "non-systematic" implies non-demonstrated. Thus, what one will consider "non-systematic" is a function of what he considers "systematic." For example, if one takes Aristotelian physics as S, then any physical events not explained by that physics will be considered non-systematic with respect to S. Similarly, if one takes Newton's mechanics as S, then whatever relevant phenomena are not explained by Newton's system are non-systematic with respect to S. Recalling, finally, that systematic here implies demonstrated, we note that one might consider only formal theories in the strict logical sense to be "systematic" (in our sense) and hence anything extra-logical to be non-systematic in the sense of non-demonstrated.

Trusting, then, that we have sufficiently indicated the wide sense in which we take "systematic" and "non-systematic," we now wish to clarify somewhat what we mean by (1) an explanation of the non-systematic and by (2) an instrument for opinion-probability.

B. 1. Explanation of Opinion-Probability. An explanation of the non-systematic with respect to S is, in general, a meta-scientific reason for the fact of the non-systematic with respect to S. The reason given might refer to limits of S or to limits of its user or to planetary influences or to the divine will or whatever. What is important is that the reason is not itself a part of S but is a meta-judgment about S.

ON THE HISTORICAL DIMENSION OF "PROBABILITY" 387

B. 2. Instrument for Opinion-Probability. Now, having recognized the non-systematic, non-S, with respect to a given system S, one might with or without explanation, propose or develop an instrument to deal with non-S. This instrument, physical or conceptual, might in principle be simply S itself but it is more likely to be some analogue or model of S, associated with S by more or less rigorous rules of correspondence, or even some modification of S. What is important here is that since only S is considered demonstrative, the instrument non-S is not. Thus, if we must refer to this instrument as being also a system, it is nonetheless a new instrument for the non-systematic, a non-demonstrative system as opposed to the demonstrative S.

The distinctions thus made between the notion of, the explanation of, and the instrument for opinion-probability can be illustrated first from the example of Thomas Aquinas and then from the example of some modern writers.

Thomas Aquinas in recognizing the non-systematic sees it precisely as that about which one does not have demonstrative knowledge. That demonstration is not possible in all cases he explains physically in terms of contingency in terrestrial events and theologically in terms of man's lack of divine vision. Seeing that the contingent, unlike the necessary, is that which can be other than it is, he characterizes non-demonstrative knowledge as that which, unlike science, can be other than it is. Having thus pointed to the fact that the non-demonstrative is open to alternatives, he accepts as man's best instrument for dealing with the non-demonstrative a modification of demonstrative argumentation. This modified form of argumentation is dialectical disputation, in which, precisely, the two alternatives of any question are argumentatively opposed and evaluated. Since, finally, the practical order is concerned with the contingent as defined above, Thomas feels free to consider moral deliberation as a kind of disputation with regard to alternative courses of action. Aware, however, that both disputation and deliberation have to do with the non-demonstrative, Thomas notes that these methods arrive at the truth, somewhat like the occurrence of the physically necessary, only most of the time: ut in pluribus.

Among the moderns, Karl Popper's notion of duxa involves a recognition that the extra-logical is non-systematic; he explains this situation by appealing to the downfall of Newtonian absolutism; and, not unlike Thomas, he proposes the conjecture and refutation of logical theories as an instrument to deal with the non-systematic. Polya points to the non-systematic with respect to mathematics in terms of "plausi-
CONCLUSION

bility" and, without explanation, elaborates a variety of logical techniques of "plausible reasoning." Perelman recognizes "the preferable," explains the need for recognizing it along the lines of Gometh's "open philosophy," and proposes to deal with it by developing a theory of argumentation. Polanyi calls attention to the non-systematic with respect to physical science, explains it as being due to factors overlooked by those who exaggerate the ideal of "objectivity," and thus proposes the need to develop a social psychology of "the personal" in science. Others, more imbued with that very ideal of "objectivity," see the non-systematic simply as that which is still beyond the reach of logic and/or mathematics. Thus Borel, for example, urges prudent application of the calculus of probability to personal affairs and Carnap insists upon developing a logic of the non-demonstrative.Servien, finally, in recognizing the non-systematic as the extra-mathematical, proposes to deal with the latter by an elaboration of his distinction between the language of mathematics and the language of literature.

C. How to Recognize the Notion of Opinion-Probability. Turning now to our third step, we propose to elaborate a criterion on the basis of which the notion of opinion-probability can be recognized.

In preparation for this task, we note that though an instrument be addressed to "the non-systematic," it is nonetheless constructed according to the best available systematization of the non-systematic. The problem is simply that the non-systematic cannot in principle be demonstratively systematized. Whence it happens that an instrument addressed to the non-systematic will in principle encounter what are often referred to as non-systematic divergences. In view, then, of these non-systematic divergences, it is incumbent upon the constructor of the instrument to safeguard the efficacy of the instrument before the non-systematic by adapting the instrument as much as possible with systematic means to adapt itself to non-systematic divergences. To do this, he adds to the instrument certain self-correcting devices by means of which non-systematic variations can be more or less effectively neutralized. These self-correcting devices amount to qualifications of the instrument and constitute the manifestation in that instrument of the notion of the non-systematic.

From these observations we now draw three conclusions which are subordinate one to the other. First of all, precisely insofar as the non-systematic is non-systematized, it will involve variables not systematically represented by the instrument addressed to it. Secondly, these unsystematized variables can and in many cases will diminish the effectiveness of the instrument as applied to the non-systematic. Thirdly, the effectiveness of the instrument before the non-systematic is therefore directly proportional to its ability to neutralize the effect of non-systematic variables.

In general, then, awareness of the non-systematic is manifested precisely by the fact of taking precautions against and thus attempting to neutralize the effect of non-systematic variables. This, in turn, reveals the non-demonstrative character of the system serving as an instrument and thus allows us to suggest the following as a criterion on the basis of which the notion of opinion-probability can be recognized:
The notion of opinion-probability is manifested whenever the results (or conclusions) obtained by utilization of an instrument are in some way qualified, thus qualifying indirectly the system on which the instrument is based.

That this criterion applies to Thomas's notion of opinion-probability has already been suggested, but it will be useful to spell out the suggestion in some detail. Thomas's basic presumption with regard to instruments addressed to the non-systematic is that the non-systematic can be represented disjunctively. Thus he divides contingent events into those which occur at in pluribus and those which occur at in paucioribus, he sets up a disputation according to opposite sides of a question, he portrays deliberation as a consideration of alternative choices. Yet in practice he often satisfies himself that the true opinion, theoretical or practical, is a golden mean between extremes. Because of the complexity of the problems involved, however, he is forced to admit (still, be it noted, within the confines of a dichotomous representation) that these instruments attain the truth only at in pluribus.

That this criterion applies to all modern notions of opinion-probability is, of course, more difficult to establish, since there are so many different formulations. Here, then, we presume no more than to point out that it applies both independently of the calculus of probability and in connection with the calculus of probability.

First of all, on the basis of Carnap's association of the non-demonstrative with "degree of confirmation," we identify as manifestations of opinion-probability Polya's reference to "plausibility" in connection with mathematics, Popper's reference to "doxa" and Polanyi's reference to "the personal" with regard to science, Perelman's reference to "the preferable" with regard to argumentative method, and so on. Secondly, we find manifestations of opinion-probability in dis-
cussions about the calculus of probability. We find it, for example, in
Gendre’s observations about the practical need to qualify Bernoulli’s
theorem with Stirling’s formula, in Russell’s breakdown of non-
mathematical meanings of probability and in particular in his reference
to probable probability with regard to applications of the calculus, in
Borel’s cautions about the applicability of the calculus to practical life,
in Polanyi’s insistence that as applied in these areas the calculus is a
maxim like other maxims, in Boll’s rather irresponsible statements
about probability as the law of the universe, in Reichenbach’s in-
sistence that all knowledge is probable, and, in general, in the innumer-
able discussions about the probability of induction.

III. There is a similarity between Thomas’s disputation and the modern
calculus of probability.

Having already proposed a similarity between Thomist and modern
tought-patterns in general and between Thomist and modern notions
of opinion-probability in particular, we now begin to specify similarities
involving directly the calculus of probability. And first of all we propose
that the calculus of probability, like medieval disputation, was originally
viewed as an instrument to deal with the non-demonstrative. The elabo-
ration of this proposal will amount to what we shall call the historical
meaning of “the calculus of probability.”

In brief, at first, we take “calculus” to refer to an instrument
and “probabilis” to refer explicitly to the notion of the non-systematic
and implicitly to a new way of expressing the non-systematic. To explain what
this involves, we shall: (A) extend the notion of the non-systematic
so as to make room not only for the qualification of an instrument but
also for the replacement of one instrument by another; (B) consider
abstractly the ideological universe in which the notion of a “calculus
of probability” originated; (C) consider concretely the evidence of this
ideological background in Laplace’s Philosophical Essay on Probabilities.

A. Replacement of one instrument by another. We have suggested in
the preceding discussion that the notion of the non-systematic tends
to generate an explanation as to why there is this non-systematic and
this in turn tends to generate an instrument to deal with the non-
systematic. We have further noted that the effectiveness of such an
instrument is directly proportional to its ability to neutralize the
effects of non-systematic divergences. Now we wish to add as a co-
rollary that if the neutralizing capacity of the instrument, however
qualified, is minimal with regard to a given problem, the need arises to
replace that instrument with another one.

ON THE HISTORICAL DIMENSION OF “PROBABILITY”

As examples of how this might apply to the contemporary history
of ideas, we refer to just three which are rather well known. First of all,
we call attention to the fact that repeated failures to establish the
Euclidean axioms led eventually to modifications of the axioms which
made possible non-Euclidean systems of geometry. Secondly, we note
that the inability of classical mechanics to deal effectively with certain
problems led to reformulations which we now know as quantum physics.
Thirdly, we recall that efforts to provide a perfect formalization of
arithmetic uncovered problems which eventually led to recognition
of both internal limitations of a formal system and of the need for richer
languages. Each of these examples in some way (more or less explicitly
according to the case) involves what might be called a recognition of
incompleteness. And thus on this level of replacement of one instrument
by another we are suggesting a connection between the notion of incompleteness and that of the non-demonstrated or non-systematic.

In what follows, then, we shall propose that the calculus of prob-
bility came to replace medieval dichotomous instruments as a more
effective means of dealing with the non-systematic. We shall also
observe, however, that this new-born instrument was in its childhood
considered precisely as an instrument of the non-systematic rather than
as a demonstrative system in its own right.

B. Ideological Origins of “Calculus of Probability.” Having just
recognized the possibility of replacing one instrument by another, we
now prepare the way for a kind of meta-history of the calculus of prob-
ability by viewing it as a new instrument of the non-systematic parallel
with a new system gradually replacing the old on which had been based
medieval instruments of the non-systematic.

To begin with, we note that the notion of opinion-probability was
much more universally covered by probabilis than is the same notion
today by probable. Today, a variety of other terms (including “personal,”
“preferable,” etc.) substitute in one way or another for the medieval
probabilis. That this is largely due to expropriation of probability by
mathematicians is relevant but not directly to the point. The point is
rather that said expropriation had not yet taken place at the time when
the calculus of probability” took, as it were, its first baby steps. The
world of Cardano, even the world of Pascal and Fermat, and even the
world of the Bernoullis and of Laplace was still in some ways more
“medieval” than many of us would care to admit. For, Thomas’s picture
of man’s approximation to divine knowledge as well as his distinction
between the demonstrative and the probable were still at least implicitly
acknowledged. What gradually and sometimes dramatically changed was man’s view as to what was in fact “probable” and what was in fact “demonstrative.”

This, after all, was the very heart of the controversy over Copernican astronomy. Scholars like Bellarmine opposed Galileo not for favoring the Copernican system but for insisting that it was scientific (that is, demonstrative) rather than merely probable. Without approving of methods adopted to persuade Galileo, we nevertheless are today closer to Bellarmine’s view than to that of Galileo—and thus closer to Thomas’s evaluation of empirical science than to the post-Newtonian. But absolutism reigned in between. Galileo’s word in time became law with the triumph of Newton’s *Principia Mathematica Philosophiae Naturalis*. The general blueprint of natural motion had been definitively demonstrated not merely with regard to what happens *ut in pluribus* but with regard to what happens *semper*. The system, in short, was perfect: it was, as had been Aristotle’s cosmology before it, the new *scientia* of the macrocosm.

Though perfect, however, the system was not exhaustive. A realm of *ut in pluribus* and *ut in pascioribus* was still being subjected in the schools to the dichotomous instrument of disputation, which was becoming with each passing year more and more a stranger in a new world built by mathematics. Here, then, alongside of *scientia*, was the realm of the non-systematic, the non-demonstrative, the *probabilia*.

There was, then, a clear notion of *probabilis* in the schools. This notion, in turn, presupposed both a notion and a theory of demonstration. On the basis of the notion and theory of demonstration, the notion of the non-demonstrated was closely linked with that of the contingent, that is, that which can be other than it is. Operating on a principle of disjunction, the scholastic successors of Thomas Aquinas divided the contingent into what occurs *ut in pluribus* and what occurs *ut in pascioribus*, attacked the contingent with the dichotomous instrument of disputation, and proposed that one deliberate his practical decisions by consideration of alternative choices. Results obtained by these instruments, in contrast to those of the demonstrative syllogism, had to be qualified. And thus was kept alive the notion of *probabilis*, of the non-systematic.

In the course of time, Cardano and then Pascal and Fermat came to recognize that gambler’s rules already in existence might provide a more effective instrument with which to deal with the contingent. These gambler’s rules they and then others developed and systematized.

---

1. More specifically, we follow the translation into English of the sixth French edition by Frederick Wilson Truscott and Frederick Lincoln Emory entitled, *A Philosophical Essay on Probabilities* (New York, 1951). We have taken the liberty to correct their translation where we find it deficient. This work will be cited as *Philosophical Essay*. 

ON THE HISTORICAL DIMENSION OF "PROBABILITY" 293

That this more or less systematic instrument of the non-systematic came to be known as a *calculus* is due not only to its character as a mathematical instrument but to imitation and adulation of the great new instrument of the systematic, the calculus of Leibniz and Newton. (For Pascal, still under the influence of Descartes, it was rather a "geometry of chance.")

That this *calculus* of the non-systematic came to be called a calculus of *probability* is due to ingredients of the intellectual milieu which go back deep into the Middle Ages. To uncover in detail how these ingredients were kept before the minds of the first mathematical "probabilists," one might study in detail developments after Thomas with regard to (1) the Aristotelian theory of demonstration; (2) divine providence and foreknowledge in the face of man’s free will; and (3) moral systems of resolving practical doubt.

As for the calculus itself, the new instrument thus inaugurated was eventually systematized by Laplace according to standards of his day and by Kolmogorov and others according to standards of our day. But it is important to bear in mind that what is now a demonstrative system in its own right began as an instrument to deal with the non-systematic on the basis of a new theory about how to express the non-systematic: not disjunctively but in terms of a continuum of values between what happens always and what never happens.

**C. Historical Meaning of "Calculus of Probability."** We have just proposed that the notion of a "calculus of probability" is in part traceable to medieval ideology, and that the part which is medieval is precisely the "probability." It would require another book to prove that Thomas’s usage of *probabilis* remained current throughout the developmental period of the calculus of probability. In lieu of this, we shall here indicate only that the greatest nineteenth century "probabilist," Pierre Simon, Marquis de Laplace (1749–1827) not only addressed himself to the notion of opinion-probability but in effect saw his instrument as a replacement for the medieval method of disputation. Our remarks are based on his *Essai philosophique sur les probabilités* (1814), which served as an introduction to the third edition of his great *Théorie analytique des probabilités* (1820). Our purpose is to show that for Laplace (1) probability is a mark of imperfect knowledge; (2) proba-
bility is non-demonstrative knowledge; (3) the calculus of probability is an instrument of the non-systematic.

C. 1. Probability as Mark of Imperfect Knowledge. Laplace begins his Philosophical Essay on Probabilities by noting that "nearly all our knowledge is problematical" and that even "the small number of things which we are able to know with certainty... are based on probabilities." 1 After this humble beginning, which differs little from the (theocentric) attitude of a Thomas Aquinas, he goes on, in spite of his ignorance of medieval thought, to present a view of the cosmos not unlike that of Thomas. The old ideas of "final causes" or "chance," he says, have gradually been replaced by the idea of an orderly universe based upon Leibniz’s principle of sufficient reason. 2

C. 2. Probability as Non-Demonstrative Knowledge. Of many examples in Laplace’s work which compare favorably with Thomas’s notion of probability, we cite just two. First of all, speaking with regard to the tides, he notes that Kepler was aware of a tendency of waters towards the moon but “he was able to give on this subject only a probable idea. Newton,” Laplace goes on, “converted into certainty the probability of this idea by attaching it to his great principle of universal gravity.” 3 Laplace then goes on to say that his own calculations give a probability that the flow and ebb of the sea is due to the attraction of the sun and moon, so approaching certainty that it ought to leave room for no reasonable doubt. It changes into certainty when we consider that this attraction is derived from the law of universal gravity manifested by all the celestial phenomena. 4

Secondly, after observing that it is difficult to evaluate the probability of the results of induction, Laplace goes on to present a basically Thomist (Aristotelian) view of the preparatory character of induction. “Induction,” he says, in leading to the discovery of the general principles of the sciences, does not suffice to establish them absolutely. It is always necessary to confirm them by demonstrations or by decisive experiments. 5

1 Laplace, Philosophical Essay, p. 1.
3 Laplace, Philosophical Essay, pp. 89-90.
4 Laplace, Philosophical Essay, pp. 92-93.
5 Laplace, Philosophical Essay, pp. 176-177.

ON THE HISTORICAL DIMENSION OF "PROBABILITY" 295

C. 3. An Instrument of the Non-Systematic. Given then, human fallibility and the resulting need for demonstration, Laplace tends to identify demonstration about the cosmos with Newton’s mechanics. What the latter has not encompassed must be approached by instruments directed to what is non-systematic with respect to Newtonian mechanics. His own instrument, he finds, is particularly suited for this purpose. For, he points out, in the midst of numerous and incalculable modifications which the action of the causes receives... from strange circumstances these causes conserve always with the effects observed the proper ratios to make them recognizable and to verify their existence. Determining these ratios and comparing them with a great number of observations, if one finds that they constantly satisfy it, the probability of the causes may increase to the point of equaling that of facts in regard to which there is no doubt. 6

Thus, says Laplace,
The analytic formulae of probabilities... may be viewed as the necessary complement of the sciences... and... are likewise indispensable in solving a great number of problems in the natural and moral sciences. The regular causes of phenomena are most frequently either unknown, or too complicated to be submitted to calculations; again, their action is often disturbed by accidental and irregular causes; but its impression always remains in the events produced by all these causes, and it leads to modifications which only a long series of observations can determine. The analysis of probabilities develops these modifications; it assigns the probability of their causes and it indicates the means of continually increasing this probability. 7

In particular, Laplace notes that the analysis of probabilities has a very useful application in that it serves to determine “the mean values which must be chosen among the results of observations.” 8 But, perhaps in keeping with the spirit of the French Revolution, he is most delighted with the possibilities of his instrument for the moral sciences. Thus, for example, not unlike Thomas Aquinas’s moral statistics, Laplace rejoices in the utility of his instrument for determining “the probabilities of testimonies” 9 and “the probability of the judgments of tribunals.” 10

That Laplace is thereby putting in his own mouth the Thomist theory of contingency together with its corollary of a postulated necessity for what happens ad in fasulas is, we think, undeniable. Also

1 Laplace, Philosophical Essay, p. 89.
2 Laplace, Philosophical Essay, p. 195.
3 Laplace, Philosophical Essay, p. 194.
4 Laplace, Philosophical Essay, pp. 199-225.
5 Laplace, Philosophical Essay, pp. 132-139.
undeniable is the fact that he wishes to apply his instrument to the same kinds of problems to which Thomas’s theory of contingency was directed. That he places much more emphasis upon empirical observation than does Aquinas is also clear, and that the instrument which he addresses to these problems is superior to Thomas’s bivalent system is not in question.

We need only add that there are clear indications in Laplace that he sees his mathematics as a replacement for medieval disputation. In one place, he cries forth an encomium of Francis Bacon for “insisting, with all the force of reason and eloquence, upon the necessity of abandoning the insignificant subtleties of the school, in order to apply oneself to observations and to experiments” and for “indicating the true method of ascending to the general causes of phenomena.” Yet at the same time Laplace admonishes:

Let us enlighten those whom we judge insufficiently instructed; but first let us examine critically our own opinions and weigh with impartiality their respective probabilities.¹

For Laplace, however, the best method of doing this is by use of “the theory of probabilities.” For:

It leaves no arbitrariness in the choice of opinions and sides to be taken; and by its use can always be determined the most advantageous choice. Thereby it supplements more largely the ignorance and the weakness of the human mind.²

To conclude this brief look at the ideology behind the “calculus of probability,” we recommend most serious reflection upon the motives behind Laplace’s name for his mathematics. In a chapter entitled “Concerning the Analytic Methods of the Calculus of Probability,” he reviews the contributions of his predecessors, refers to all kinds of mathematical developments since Descartes, especially that of integral and differential calculus, and winds up with the most important historical observation of all:

I have named the ensemble of the preceding methods the Calculus of Discriminant Functions: this calculus serves as a basis for the work which I have published under the title of the Analytical Theory of Probabilities.³

IV. There is a similarity (A) between Thomas’s theory of probability and the modern logical theory of probability and (B) between Thomas’s theory of contingency and the modern frequency theory of probability.

After having explained a similarity between Thomas’s notion of probability and modern notions of non-demonstrative knowledge, we then showed that this notion of opinion-probability is very much in evidence in Laplace’s views on probability. We also noted in Laplace, however, a similarity between his ideas about the cosmological basis of probability and Thomas’s notion of contingency. For, in the view of Laplace as well as in Thomas’s view a proposition about a contingent event is probable to the extent that it occurs with some determinable regularity. Thomas, of course, was content to say of such an event that it occurs (for example) ut in pluribus. But, with his mathematical sophistication as a guide, Laplace insisted on establishing with much more precision just what this ut in pluribus might be. Like his medieval predecessor, however, he was willing to grant that if an event occurs with sufficient regularity one might attribute exceptions to disturbing factors and thus postulate the existence of a necessary cause of such an event. But his conviction as to the absolutely demonstrative character of Newton’s mechanics is such that he does not seem to admit what Thomas would call a demonstration ut frequenter.

Now it is of the utmost importance to note that in speaking about the frequencies with which more or less irregular events occur, Laplace refers quite often to their “probabilities.” Though he does not seem to be consciously aware of what he is doing, he is in fact giving another sense to “probability” than the sense of opinion-probability which he explicitly discusses along lines not unlike that of Thomas Aquinas. This second sense of “probabilities” as relative frequencies gradually became, as Rudolf Carnap tells us in detail, a second explicandum for the interpretation of the calculus of probability.

Thus, while John Maynard Keynes and others continued to view “probability” as a characteristic of a proposition, as had Aquinas, others, including notably Richard von Mises, Hans Reichenbach, and the school of statisticians now represented by Ronald Fisher, have concentrated upon “probability” in the sense of relative frequency. Summing up the development, Carnap identifies “relative frequency in the long run” as probability and identifies “degree of confirmation” as probability. The former constitutes the explicandum for the “mathematical” theory of probability, and the latter constitutes the explicandum for the “logical” theory of probability.

The differences between these two theories are not inconsiderable. To use the simple summary of Polanyi, the logical theory concentrates upon a “probable” proposition about events whereas the mathematical theory concentrates upon a proposition about “probable” events. The

¹ Laplace, Philosophical Essay, pp. 179-180.
² Laplace, Philosophical Essay, p. 9.
³ Laplace, Philosophical Essay, p. 190.
⁴ Laplace, Philosophical Essay, p. 48.
latter is a manifestation of the great modern ideal of "objectivity" in that it shuns any suggestion of "subjective" or "psychological" adherence. But the logical theory is no less "objective," since it is concerned with logical properties of a proposition and not with what a subject "thinks" or "feels" about that proposition.

In short, much has happened since the time of Laplace. And the most important thing that has happened is the formalization of the calculus of probability. For, as a result of this formalization it is now possible to introduce into a consideration of the calculus of probability several extremely important distinctions which were not clearly recognized at the time of Laplace. What these distinctions can be summarized as follows. A careful analysis of formal systems has led to rather general agreement that (1) a formal system may be considered without regard to any interpretation, and that (2) a formal system of any importance is open to more than one interpretation. In more precise terms, these two points mean, respectively, that (1) there is an important distinction to make between a formal system and an interpretation of that system, and that (2) there is an equally important distinction to make between interpretation as such and a set of statements which interpret or are taken to interpret a given formal system.

We shall return to these points directly, but it will be useful beforehand to make three contrasting observations about Laplace. In the first place, Laplace seems to have viewed the calculus of probability somewhat naively (though not necessarily erroneously) as a direct representation of certain kinds of events now often referred to as aleatory. Secondly, he was aware of the fact but not of the significance of the fact that the mathematical instrument which he directed to such events was based upon the concepts and methods developed by Newton and others to represent the "systematic." Thirdly, he was implicitly involved in but not explicitly aware of two different interpretations of his instrument: the "logical" (probable propositions) and the "mathematical" (probable events).

Now, then, to show that the contemporary view of formal systems brings considerable clarity into the muddled thinking of a Laplace, we shall use the distinctions made above in order to analyze a particularly relevant statement by Bertrand Russell. The latter, after noting general agreement about the calculus of probability as such and general disagreement about its interpretation, suggests the following as an escape from discord. "In such circumstances," he says, "the simplest course is to enumerate the axioms from which the theory can be deduced, and to decide that any concept which satisfies these axioms has an equal right, from the mathematician's point of view, to be called "probability." If there are many such concepts, and if we are determined to choose between them, the motives of our choice must lie outside mathematics."

What is to be noted in the first place about Russell's statement is a clear distinction between an uninterpreted (or, as it is sometimes called, abstract) formal system and an interpretation of that system. Considered precisely as uninterpreted, the formal system has no extra-logical meaning. But, according to Russell, it can be given a meaning as it were indirectly by establishing a correspondence between statements in the formal system and statements which are "meaningful" or which have content. These latter, then, might be called contentive statements as opposed to the abstract statements of the formal system.

In the second place, we note that Russell allows for the possibility of more than one "interpretation" of the formal system. Thus, it is advisable to make a clear distinction between interpretation of a formal system and a particular set of contentive statements which "interpret" those in the formal system. For, a particular contentive statement which interprets a formal system does not exhaust the possible interpretations that might be found for that same formal system.

For the sake of clarity, then, some logicians prefer to speak of an interpreting statement as an interpretant. Speaking somewhat loosely, we shall here refer to a set of interpreting statements as an interpretation.

In the third place, we note that Russell speaks about satisfying the axioms of the formal system, and thus in effect demands that the interpretation be valid. An interpretation is valid only if each contentive statement corresponding to a theorem of the formal system is true. And this, apparently, is what Russell demands when he says that a given "concept" must satisfy the axioms of the formal system.

---


2 Earlier in the same work, Russell considers the notion of interpretation as *proposition*, and there makes it clear that what he demands of an interpretation is that it be valid. "Our formulas," he says, "are not regarded as 'true' or 'false,' but as hypotheses containing varia-
These demands, to be sure, are rigorous enough; but it is well to point out that they might be made even more rigorous. For one thing, the notion of a valid interpretation does not eliminate the possibility of having true contingent statements which, though relatable to a formal statement, do not correspond to a theorem of the formal system. In such a case, the interpretation would still be valid but it would not be adequate. This, in turn, suggests the possibility of a stronger formal system which could allow for an adequate interpretation. An interpretation is adequate, then, if each formal statement shown to correspond to a true contingent statement is a theorem of the formal system.

One might further inquire as to whether all possible valid interpretations of a given system are isomorphic. Stating the matter briefly, when a system is based on the first-order predicate calculus, it is possible to build a certain kind of interpretation which is called, in the strict sense of the word, a model. Between models it is possible to define a relation of isomorphism, that is to say, similarity of structure. A system admitting models is then said to be categorical if all its models are isomorphic or, in other words, if it determines its models up to an isomorphism.

That the calculus of probability is not a categorical system can be seen from the many and sometimes heated discussions between proponents of the "logical" and proponents of the "relative frequency" interpretation. Even more, relative frequency has been expressed in terms of both finite and infinite series. And thus it is indicated in a general way that the reality in question is still too complex for the formal system (the calculus of probability) which is used in various ways to represent it.

We see, then, that the notion of interpretation is, among other things, a matter of degree. Interpretation as here used always involves a correspondence between statements. But the correspondence in question might be more or less exhaustive and thus, if you will, more or less perfect. To some extent, then, factors extrinsic to logic itself will determine how rigorous a correspondence shall be required. The degree of correspondence which one requires will then determine whether or not a given interpretation is acceptable.

... A set of values of the variables which makes the hypotheses true is an interpretation. ... The axioms consist partly of terms having a known definition, partly of terms which, in any interpretation, will remain variables, and partly of terms which, though as yet undefined, are intended to acquire definitions when the axioms are interpreted. "The process of interpretations consists in finding a consistent signification for this class of terms." *Human Knowledge*, p. 343.
CONCLUSION

More specifically, while it is in principle true that any number of interpretations can be found for the calculus of probability, in actual fact only two important interpretations have been found:

1. the "logical" interpretation: probability (degree of confirmation) of a proposition;
2. the "mathematical" interpretation: probability (relative frequency) of a class of events.

It is possible to maintain, no doubt, that it is purely by chance that these two interpretations rather than others have been found for the calculus of probability. But the fact that the respective notions basic to these two interpretations are already present (one explicitly, the other implicitly) in Laplace's thoughts on the subject makes chance an unlikely explanation. Chance becomes even more unlikely when we realize that Laplace's two usages of probability correspond to Thomas Aquinas's usages of:

1. probabili: argumentatively supported (proposition);
2. contingens: what happens (an event) either ut in pluribus or ut in paucioribus.

This being said, we consider our point as having been made. For, though it is perfectly obvious that neither of these notions served as the explicandum for an interpretation of a formal system during the Middle Ages, nevertheless the notions themselves, however refined they may have become, are essentially the explicanda of interpretations subsequently "found" for the calculus of probability.

V. There is a relationship between Thomas's distinction between scientia and opinio-probability and the modern problem of probability in science.

We have pointed out first in the abstract and then by a concrete consideration of Laplace that it was to a notion like Thomas's of opinion-probability that early probabilists directed their new instrument for the non-systematic. This new instrument, in turn, was felt to be concerned precisely with what was non-systematic with respect to the Newtonian system of celestial mechanics. The latter, in other words, was viewed as replacing medieval scientia and the former was viewed as replacing medieval disputations as a means of determining and increasing the probability of the opinonative. Moreover, since this new instrument was concerned primarily with physical events which fell short of the regularity requisite for scientia, its use in this regard gradually gave to "probability" a second meaning which embraced the ut in pluribus and ut in paucioribus whereby Aquinas characterized the contingens.

These points having been made, we are now in a position to bring out the full significance of our insistence that the calculus of probability was originated and developed as an instrument to deal with the non-systematic. In other words, we now want to make relevant to contemporary thought the fact that the calculus of probability was viewed during its formative years as a replacement for medieval disputatation, that is to say, as the new preparation for or auxiliary to scientia.

In briefest terms, what is of the utmost importance about the present role of the calculus of probability is precisely the fact that it is no longer viewed as a preparation for or auxiliary to scientia. On the one hand, the scientia that was the Newtonian celestial mechanics has given way to Einstein's theory of relativity, and in the process man has lost his confidence in the absolutivity of scientia. On the other hand, and almost simultaneously, that which had been viewed as the propaedeutic to scientia has suddenly found itself as the systematic representation of a large and important sector of scientia itself. And thus the new quantum physics has come to represent, from an historical point of view, a kind of wedging between opinio and scientia.

The resulting ideological crisis as to the meaning of this strangest of all weddings is still unresolved and will no doubt remain so for a long time to come. But it can already be observed that the crisis itself is due at least in part to an inadequate historical perspective and also in part to an exaggerated dichotomy between subject and object.

According to the traditional view – the view of Laplace as well as of Keynes – the imperfect, the merely "probable" was, qua imperfect, attributable to limitations on the part of the subject. The perfect, the "scientific," by contrast, achieved full objectivity or, so to speak, met the world on its own terms.

In the wake of quantum physics, this traditional view – essentially the same as that of Thomas Aquinas – was supposedly overturned. For, what had been for centuries two neatly distinct types of knowledge now seemed to be inextricably intermingled. The heretofore subjective "probable" was now projected upon the objective "scientific." Probability, science and objectivity were now thought to be all of one piece. And thus one could no longer say with Keynes and his forebears that the universe was determinate and that probability referred to gaps in our knowledge of that universe. One now had to say, rather, that the
universe itself is indeterminate and that probability is simply an expression of that indeterminacy, without any reference to the (non-scientific) subject.

However, in spite of great dedication to the cause of objectivity, “the probable” has still not been successfully abstracted from “the subjective.” Reichenbach, for example, still likes to insist that all knowledge is probable, and Popper tells us that it is doxa or verisimilitude. Thus, neither object nor subject qualifies any longer as the locus of certitude. This, for many, has withdrawn into a realm which presumably transcends the dichotomy of subject and object: the realm, namely, of logic as such. Certitude, if such there be, is to be found today only in the formal system.

This reaction to the calculus of probability, though unprecedented in its complexity, is nonetheless a familiar by-product of the introduction of a new mathematical instrument into man’s efforts to harness the universe. The Pythagoreans and the Greeks in general, fascinated by the new geometry, saw geometrical design everywhere, and for this were eventually taken to task by Sextus Empiricus. The medieval followers of Prolemy saw spheres and even epicycles in the heavens, and by way of reaction Kepler saw more heavenly harmony than was there. The founding fathers of the calculus tended to see “integrals” and “differentials” in the universe until Bishop Berkeley took time off from tar water to point out to them their inconsistencies.

Whatever the value of “the calculus of probability” as an instrument for nuclear research, this much at least seems clear. The universe is no more “determinate” or “indeterminate” today than it was a hundred years ago. Whether or not one considers the present formulation of quantum physics to favor one view over the other perhaps has something to do with the mathematics in question, but it has far more to do with one’s views about the extent to which mathematics does more than merely measure. These views, in turn, are not derived from the formal system that persists in being called “the calculus of probability” but from a host of other factors which, pace positivists, can well be described as meta-physical.

In short, discussions about the calculus of probability and its applications have in one way or another been operating under the assumption that there has been in effect a wedding between scientia and opinio. And this assumption, which only now is beginning to be attacked at its roots, presupposes an ideology which goes back through Thomas Aquinas to the beginnings of Western thought.

---