QUANTUM GRAVITY

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1 INTRODUCTION

Quantum gravity (QG) is the problem of finding a theory that describes the quantum effects on gravity. These effects escape the currently accepted physical theories. Our present knowledge of the basic dynamical laws is given by quantum mechanics (QM) and quantum field theory (QFT), general relativity (GR), and the standard model of particle physics. This set of theories has obtained an empirical success nearly unique in the history of science: so far there is no evidence of observed phenomena that clearly escape or contradict this set of theories — or a minor modification of the same. But these theories become meaningless in the regimes where relativistic quantum gravitational effects are expected to become relevant. These effects are not currently observed; they are negligible at currently accessible scales and are expected to become relevant only in extreme physical regimes. For instance, they should govern the end of the evaporation of black holes, the beginning of the life of the Universe near the Big Bang, and any measurement involving an extremely short length scale ($\sim 10^{-33}$ cm, the "Planck scale") or a very high energy. "Quantum gravity" is the name given to the theory-to-be-found that should describe these regimes.

The interest of the problem, however, goes far beyond the description of some so far inaccessible physics. The physics of the early twentieth century has modified the roots of our understanding of the physical world. It has changed the very meaning of the concepts we use to grasp it. GR, which is the field theory that describes gravity when we can disregard its quantum properties, has changed our understanding of space and time. QM, which has replaced classical mechanics as our general theory of motion, has modified the notions of matter, field, and causality. At present, we haven't yet found a consistent conceptual frame in which these modifications make sense together. Thus, our understanding of the physical world is currently badly fragmented. In spite of its empirical effectiveness, fundamental physics is in a phase of deep conceptual confusion. The problem of QG is to combine the insights of GR and QM into a conceptual scheme in which they can coexist. It is the problem of finding a novel picture of the world capable of bringing the twentieth century scientific revolution to an end. For this reason, many consider QG to be the most important open problem in fundamental physics.

In particular, QG is an investigation on the nature of space and time. The structure and the nature of physical space are expected to change radically at

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the Planck scale; and the conventional way of conceptualizing of time evolution is expected to cease to be viable at this scale. The theory is therefore likely to require us to revise the way we think of space and time.

Research in QG has developed slowly for several decades of the twentieth century, because general relativity had little impact on the rest of physics and the interest of many physicists was concentrated on the development of quantum theory and particle physics. In the last decade, the explosion of empirical confirmations and concrete astrophysical, cosmological and even technological applications of general relativity on the one hand, and the satisfactory solution of most of the particle physics puzzles in the context of the particle physics "standard model" on the other, have led to a strong concentration of interest in quantum gravity, and the progressed has become rapid. Research is currently very active.

A few tentative theories of QG have been proposed. The best developed of these are string theory [Green *et al.*, 1987] and loop quantum gravity [Rovelli, 2004]. Other very active directions of investigation include noncommutative geometry [Connes, 1994], dynamical triangulations [Ambjorn *et al.*, 1997], the spinfoam formalism [Perez, 2003] (strictly connected to the loop approach) and effective theories. Currently, none of these approaches has found any empirical corroboration, and none has won general theoretical consensus.

The problem of QG raises basic methodological issues and involves conceptual and foundational questions. Some of these are similar to the foundational questions that physics addressed at the time of other major conceptual shifts — the birth of classical mechanics, field theory, relativity, or quantum mechanics. Old problems demand new answers, in the light of the twentieth century's novel insights. A characteristic example is a revival of the cartesian-newtonian-leibnizian debate on the relational nature of space.

No exhaustive discussion on our current understanding of the physical world, and in particular on the current knowledge about space and time, can disregard the issues and questions raised by this search.

1.1 Quantum spacetime

GR and QM have widely extended our understanding of the physical world. They are solidly supported empirically, and have vast scientific and technological applications. But they have destroyed the coherent picture of the world provided by prerelativistic classical physics because each of the two is formulated under assumptions contradicted by the other theory. QM is formulated using an external time variable, the t of the Schrödinger equation — or, in the case of QFT, using a fixed, nondynamical background spacetime. Both an external time variable and a fixed background spacetime are incompatible with GR.

In turn, GR is formulated in terms of Riemannian geometry: the gravitational field is assumed to be a classical deterministic dynamical field, which can be identified with Riemann's metric field. But QM requires all dynamical fields to have quantum properties. At small scales a field appears as made up of discrete quanta and is governed by probabilistic laws.

Thus, GR and QM are formulated in terms of mutually contradictory assumptions. In spite of their empirical success, they offer a rather schizophrenic and confused understanding of the physical world.

Roughly speaking, we learn from GR that spacetime is a dynamical field and we learn from QM that a dynamical field is quantized. Therefore at small scales we might expect a "quantum spacetime" formed by "quanta of space", and allowing "quantum superposition of spaces". The problem of QG is to give a mathematical and conceptual meaning to such a notion of a quantum spacetime.

Some general indications about the nature of quantum spacetime, and on the problems this notion raises, can be obtained from elementary considerations based on GR and QM. The size of quantum mechanical effects is determined by Planck's constant $\hbar$. The strength of the gravitational force is determined by Newton's constant G, and the relativistic domain is determined by the speed of light c. By combining these three fundamental constants we obtain a length, called the Planck length $l_{\mathrm{P}} = \sqrt{\hbar G/c^3} \sim 10^{-33}$ cm. Quantum-gravitational effects are likely to be negligible at distances much larger than l_{P}, because at these scales we can neglect quantities of the order of G, $\hbar$ or $1/c$. Therefore we expect that the GR description of spacetime as a Riemannian space holds at scales larger than l_{P} and breaks down approaching this scale, where the full structure of quantum spacetime becomes relevant. QG is therefore the study of the structure of spacetime at the Planck scale.

Simple arguments indicate that l_{P} may play the role of a *minimal* length, in the same sense in which c is the maximal velocity and $\hbar$ the minimal exchanged action. For instance, the Heisenberg principle requires that the position of an object of mass m can only be determined with uncertainty x satisfying $mvx > \hbar$, where v is the uncertainty in the velocity; special relativity requires $v < c$; and according to GR there is a limit to the amount of mass we can concentrate in a region of size x, given by $x > Gm/c^2$, after which the region itself collapses into a black hole, removing itself from our observation. Combining these inequalities we obtain $x > l_P$; that is, gravity, relativity and quantum theory, taken together, appear to prevent position to be determined more precisely than the Planck scale. Various considerations of this kind have suggested that space might not be infinitely divisible. It may have a quantum granularity at the Planck scale, analogous to the granularity of the energy in a quantum oscillator. This granularity of space is fully realized in certain QG theories, such as loop quantum gravity, and there are hints of it also in string theory [Amati *et al.*, 1989]. Since this is a quantum granularity, it escapes the traditional objections to the atomic nature of space.

Time is affected even more radically by the quantization of gravity. In conventional QM time is treated as an external parameter and transition probabilities change in time. In GR there is no external time parameter. Coordinate time is a gauge variable which is not observable, and the physical variable measured by a clock is a complicated function of the gravitational field. Fundamental equations of QG might therefore not be written as evolution equations in an observable time variable. Strictly speaking, this is already true in classical GR: GR does not describe evolution of physical variables in time — it describes the relative evolution of physical variables with respect to one another. But a temporal interpretation is still available in classical GR, because spacetime appears as a solution of the dynamical equations of the gravitational field. However, a solution of the dynamical equation is like a "trajectory" of a particle and in quantum theory there are no physical trajectories: there are only transition probabilities between observable eigenvalues. Therefore in QG it may be impossible to describe the world in terms of a spacetime, in the same sense in which the motion of a quantum electron cannot be described in terms of a single trajectory. It is possible that to make sense of the world at the Planck scale, and to find a consistent conceptual framework for GR and QM, we may have to give up the notion of time altogether, and learn ways to describe the world in atemporal terms. Time might be a useful concept only within an approximate description of the physical reality.

The following section sketches the historical development of QG research and illustrates the main ideas, lines of research and current tentative theories. Various issues raised by this research are then illustrated in Section 3. Section 4 discusees in particular the changes in the notions of space and time forced by QG and Section 5 discusses the relation between the prolem of QG and other major open problems in fundamental phyiscs.

2 APPROACHES

A full account of the numerous ideas and approaches towards quantum gravity is outside the scope of this article. Only a few main research lines are illustrated here. For additional references, see the bibliographical note at the end of the article.

2.1 History and directions of research

Early ideas

The fact that the gravitational field should have quantum properties, and therefore we need a theory for describing these properties, was recognized very early. Already in 1916, one year after the birth of GR, Einstein pointed out that quantum effects must lead to modifications of GR [Einstein, 1916]. In 1927 Oskar Klein suggested that QG might ultimately modify the concepts of space and time [Klein, 1927].

In the early thirties Rosenfeld [Rosenfeld, 1930b; Rosenfeld, 1930a] wrote the first technical papers on QG, soon followed by Fierz and Pauli [Fierz, 1939; Pauli and Fierz, 1939] and later Gupta [Gupta, 1952]. The idea is to introduce a fictitious "flat space", to consider the small fluctuations of the metric around it — gravitational waves moving on flat space, described by the linearized Einstein equations — and to quantize these waves following the methods that had worked for the electromagnetic field. More precisely, the metric field $g_{\mu\nu}(x)$, which in Einstein's theory represents at the same time the spacetime metric and the gravitational field, is written as the sum of the two terms

$$(1) \quad g_{\mu\nu}(x) = \eta_{\mu\nu}(x) + h_{\mu\nu}(x).$$

$\eta_{\mu\nu}(x)$ is interpreted as the metric of a fixed background spacetime; $h_{\mu\nu}(x)$ is interpreted as the gravitational field, and quantized. A Hilbert space of states representing quantum states of gravitational waves is introduced, where $h_{\mu\nu}(x)$ is represented by a field operator $\hat{h}_{\mu\nu}(x)$. This is called the "covariant approach" to QG. The quantum of the field $h_{\mu\nu}(x)$, which is the gravitational analog of the photon, is called the "graviton", a name already in use in the early thirties.

In 1938, Heisenberg pointed out that the fact that the gravitational coupling constant is dimensional is likely to cause problems with the quantum theory of the gravitational field [Heisenberg, 1938]. In the mid thirties a young Russian physicist, Matvei Petrovich Bronstein, realized that the unique features of gravitation require a special treatment, when the full nonlinear theory is taken into account. He realized that field quantization techniques must be generalized in such a way as to be applicable in the absence of a background spacetime. Bronstein understandood that the limitation posed by GR on the mass density radically distinguishes the theory from quantum electrodynamics and would ultimate lead to the need to "reject Riemannian geometry" and perhaps also to "reject our ordinary concepts of space and time" [Bronstein, 1936].

A second line of investigation was opened in the forties by Peter Bergmann and his group [Bergmann, 1949a; Bergmann, 1949b; Bergmann, 1958; Bergmann, 1961; Bergmann and Komar, 1980]. The idea is to study and quantize the *hamiltonian* formulation of full GR, not just its linearization around flat space. This approach has the advantage that it does not assume a background spacetime on which to define the theory. The idea is that the states in the Hilbert space represent the quantum states of spacetime itself, and the full spacetime metric (maybe up to its nondynamical components) becomes a quantum operator

$$(2) \quad g_{\mu\nu}(x) \longrightarrow \hat{g}_{\mu\nu}(x).$$

This is called the "canonical approach" to QG. The same program was started independently by Dirac, who develops his constrained hamiltonian systems theory for this task [Dirac, 1950; Dirac, 1964; Dirac, 1958; Dirac, 1959].

A third approach to QG was introduced in the late fifties by Charles Misner [Misner, 1957], following a suggestion by John Wheeler. It is a quantization of general relativity à la Feynman, formally defined by the "path integral over geometries"

$$(3) \quad Z = \int Dg \; e^{-iS[g]}$$

where g is the metric field and $S[g]$ is the action of GR.

These three lines of research — covariant, canonical, and path integral — represented by equations (1), (2) and (3) respectively, still continue today. They have often influenced one another and have at times partially merged, but they have maintained a distinct flavor across more than half a century of research and are still clearly recognizable.

The basic program of the three approaches was already clearly established at the end of the fifties. The implementation of the initial programs has turned out to be a rather formidable task, but was accomplished during the sixties, in particular with the writing of the full set of Feynman rules in the covariant approach and the Wheeler-DeWitt equation in the canonical approach. Each approach, however, met serious stumbling blocks in the seventies: non-renormalizability in the covariant approach, ill-defined equations in the canonical and path integral approach. In the eighties, these stumbling blocks were overcome, in particular with the discovery of string theory in the covariant direction, and of loop quantum gravity and the (related) spinfoam formalism in the canonical and path integral directions.

The main lines of development are illustrated below.

Feynman rules and nonrenormalizability

The covariant formalism was developed during the sixties by Feynman [Feynman, 1963], DeWitt [DeWitt, 1964a; DeWitt, 1964b; DeWitt, 1965], Faddeev and Popov [Faddeev and Popov, 1967]. The technical difficulties derive from the gauge invariance of the Einstein equations, and is solved with the introduction of "ghost" particles, leading to the complete and consistent set of Feynman rules for perturbative quantum GR [DeWitt, 1967b; DeWitt, 1967c; Faddeev and Popov, 1967].

But in the early seventies the works of t'Hooft, Veltman, and then Deser and Van Nieuwenhuizen found indications that the theory does not work ['t Hooft, 1973; 't Hooft and Veltman, 1974; Deser and van Nieuwenhuizen, 1974a; Deser and van Nieuwenhuizen, 1974b], realizing Heisenberg's early fears. The reason is that the renormalization procedure, namely the technique used in QFT to remove the infinities that appear when considering the effects of arbitrarily small ("ultraviolet") field fluctuations, fails in the case of gravity. The definitive rigorous proof that the covariant quantization of general relativity fails because of nonrenormalizable ultraviolet divergences was obtained only later, in the late eighties, by Goroff and Sagnotti [Goroff and Sagnotti, 1985; Goroff and Sagnotti, 1986].

The interpretation of this failure is still controversial. There are two possibilities. One possibility is that the mistake was to assume, to start with, the existence of a background spacetime. The infinities come from short-distance fluctuations of the quantum field. These exist only if spacetime is continuous down to arbitrarily small scales. But the very fact that gravity is quantized questions the existence of such arbitrarily small scales. If, instead, spacetime has a quantized granular short-scale structure, the infinities might be just an artifact of the approximation taken in equation (1) considering $\eta_{\mu\nu}(x)$ (instead of the full $g_{\mu\nu}(x)$) as the spacetime metric. If so, the way out from the difficulty is to discard the background spacetime, and quantize the full gravitational field, as is done in the canonical or path integral approaches.

The alternative possibility is that it is GR which is not the correct theory. GR is strongly empirically supported, but only at large distances. At short distances, the world might be described by a modification of GR, with better ultraviolet behavior. There is a historical precedent: Fermi theory of weak interactions was an empirically successful but non-renormalizable theory. In the case of Fermi theory, the successful solution of the problem was to replace the theory with the Glashow-Weinberg-Salam electroweak theory, which is renormalizable and corrects Fermi theory at short scales.

Motivated by this analogy, the search for a short scale correction of GR, having better finiteness properties, has spanned several decades. After numerous attempts — some of which, like supergravity [Freedman *et al.*, 1976], and high derivative theories [Stelle, 1977] raised much hope, later disappointed — the search has lead to string theory.

Wheeler DeWitt theory

During the fifties and sixties, Bergmann's group and Dirac independently unraveled the hamiltonian structure of full GR, a rather complicated task. This structure was later clarified in the work of Arnowit Deser and Misner [Arnowitt *et al.*, 1962], using the metric $q_{ab}(x)$ of a constant-time spacelike surface, Ashtekar [Ashtekar, 1986; Ashtekar, 1987], using a connection field analogous to a Yang Mills field, and others.

In the early sixties, building on these results, Peres writes the Hamilton-Jacobi equations of GR [Peres, 1962]

$$(4) \qquad G^2(q_{ab}q_{cd} - \frac{1}{2}q_{ac}q_{bd})\ \frac{\delta S(q)}{\delta q_{ac}}\frac{\delta S(q)}{\delta q_{bd}} + \det q\, R = 0,$$

were R its Ricci scalar curvature of the metric q_{ab}, and $S(q)$ is the Hamilton-Jacobi functional. In 1967, Bryce DeWitt and John Wheeler wrote the "Einstein-Schrödinger equation" [DeWitt, 1967a] following the steps taken by Schrödinger in deriving the Schrödinger equation from the Hamilton-Jacobi equation, namely interpreting the Hamilton-Jacobi equation as the eikonal approximation of a wave equation obtained replacing derivatives with derivative operators:

$$(5) \qquad \left((\hbar G)^2(q_{ab}q_{cd} - \frac{1}{2}q_{ac}q_{bd})\frac{\delta}{\delta q_{ac}}\frac{\delta}{\delta q_{bd}} - \det q\, R\right)\ \Psi(q) = 0.$$

Today this is called the "Wheeler-DeWitt equation". In principle, this equation is expected to describe the full quantum dynamics of gravity. In practice, the equation remained very ill-defined for a long time: until the late eighties, when Ted Jacobson and Lee Smolin [Jacobson and Smolin, 1988] find some loop-like solutions to this equation, reformulated in Ashtekar's connection formalism, thus opening the way to loop quantum gravity.

Notice that the coordinate time variable t does not appear in the classical equation (4) nor in the quantum equation (5). This disappearance of the time variable has raised an intense debate, and will be discussed below.

Misner-Hawking sum-over-geometries

In the seventies, Steven Hawking and his group [Hawking, 1979] revived and developed the Wheeler-Misner path integral (3) in the form of a "Euclidean" integral over Riemannian (namely positive definite, as opposed to pseudo-Riemannian) metrics

$$(6) \qquad Z = \int Dg\; e^{-S[g]}.$$

The hope was that the Euclidean functional integral would prove to be a better calculational tool than the Wheeler-DeWitt equation. Hartle and Hawking [Hartle and Hawking, 1983] introduced the notion of the "wave function of the universe" and the "no-boundary" boundary condition for the Hawking integral, opening up a new intuition on QG and quantum cosmology. Jim Hartle [Hartle, 1995] developed the idea of a sum-over-histories formulation of GR into a fully fledged extension of quantum mechanics to the general covariant setting. The idea was later developed and formalized by Chris Isham [Isham, 1991].

But the Euclidean integral does not provide a way of computing genuine field theoretical quantities in QG any better than the Wheeler-DeWitt equation, and the atmosphere in QG in the middle of the eighties was rather gloomy.

The idea of a sum-over-histories definition of QG was revived again in the mid nineties by the spinfoam formalism, which offer a discretized definition of the integral (3) that appears to be better defined thanks to the same short-scale spacetime discreteness implemented in loop quantum gravity.

Black hole thermodynamics

In 1974 Hawking [Hawking, 1974; Hawking, 1975] announced a theoretical derivation of black hole radiation. A (macroscopic) Schwarzschild black hole of mass M emits thermal radiation at the temperature $T = \hbar/8\pi kGM$ (k is the Boltzmann constant). The result came as a surprise, anticipated only by the observation by Bekenstein, a year earlier, that entropy is naturally associated to a black hole [Bekenstein, 1972; Bekenstein, 1973; Bekenstein, 1974] and by the Bardeen-Carter-Hawking analysis of the analogy between laws of thermodynamics and dynamical behavior of black holes [Bardeen *et al.*, 1973]. In the light of Hawking's result, the Bekenstein entropy of a Schwarzschild black hole is

$$(7) \qquad S = \frac{kc^3 A}{4\hbar G}$$

where A is the area of the black-hole surface. Hawking's beautiful result is not directly connected to quantum gravity — it is a skillful application of QFT in curved spacetime, namely QFT interacting with a fixed, non quantized, gravitational field — but has a very strong impact on the field of QG. It opens a new field of research — "black hole thermodynamics" — and it opens the quantum-gravitational problems of understanding the statistical origin of the entropy (7). This is a challenge for any quantum theory of gravity.

Two years later, an influential paper by Bill Unruh [1976] convincingly argued that an observer that accelerates in the vacuum state of a conventional QFT interacts with the quantum field as if this was in a thermal bath. This shed light on black hole radiation, because an observer that remains at a fixed distance from a black hole is in constant acceleration (in order not to freely fall), and therefore black hole radiation can be interpreted simply as an Unruh effect. But at the same time this result appears to suggest that there is a deep general relation, which we do not yet understand, tying together gravity, thermodynamics and quantum theory.

In recent years, both the string and the loop approach to QG have been able to derive equation (7) from first principles [Strominger and Vafa, 1996; Rovelli, 1996a; Krasnov, 1997; Ashtekar *et al.*, 1998]. This has been considered a major success for both approaches. However, neither derivation is fully satisfactory. The string derivation does not work for conventional black holes such as a Schwarzschild black hole, but only for certain exotic black holes called extremal or nearly extremal; the loop derivation gives a finite result but the result depends on a free parameter of the theory (called the Immirzi parameter γ) that must be appropriately chosen in order to give the factor 1/4 in (7).

Noncommutative geometry

A geometrical space M admits two alternative descriptions. One is as a set of points x, the other is in terms of a commutative algebra A of functions on M. In particular, a celebrated result by Gelfand shows that a (compact Hausdorff) space M is determined by the abstract algebra A isomorphic to the algebra of the continuous functions on M. This algebraic point of view leads to a generalization of the notion of space, obtained by considering noncommutative algebras. In this sense, a noncommutative algebra defines a "noncommutative space".

Quantum theory is the discovery that the phase space of a dynamical system (the set formed by its classical states) must be replaced by a noncommutative space. In fact, the system's observables — that represent the ways we can interact with the system — form a commutative algebra of functions on the classical phase space, which becomes a noncommutative algebra in QM.

In the case of physical space, A can be identified with an algebra of coordinates, or with momentum space. If we interpret the elements of A as representing physical measurements, it is natural, in the light of quantum theory, to consider the possibility that the algebra be noncommutative. Accordingly, the hypothesis has been made that the short-scale structure of physical space might be described by a noncommutative geometry. This idea has been explored in a number of variants [Doplicher *et al.*, 1994; Doplicher *et al.*, 1995;

Doplicher, 1996].

A connection with GR has appeared in the approach developed by Alain Connes [Connes, 1994]. Connes has noticed that in the algebraic framework the notion of distance is naturally encoded in the Dirac operator D. This is the derivative operator that appears in Dirac's spinor field equation for an electron. Let $\mathcal{H}$ be the Hilbert space formed by the spinor fields on a given Riemannian (spin) manifold M, D be the (curved) Dirac operator, and A an algebra of functions on M, seen as (multiplicative) operators on $\mathcal{H}$. From the triple $(\mathcal{H}, A, D)$, called a "spectral triple", we can reconstruct the Riemannian manifold. In particular, the distance between two points x and y can be obtained as

$$(8) \quad d(x,y) = sup_{\{f\in A, ||[D,f]||<1\}} \ |x(f) - y(f)|$$

a beautiful and surprising algebraic definition of distance. A non-commutative spacetime might be described by a spectral triple in which A is non-commutative. Connes suggests that this algebra may be chosen on the basis of the symmetries of the standard model, following the idea that the standard model might reveal the short-scale structure of spacetime in the same manner in which Maxwell theory revealed the structure of Minkowskian spacetime. The Connes-Chamseddine "spectral action", is simply the trace of the Dirac operator D, $S = Tr[f(D^2/(\hbar G))]$, where f is the characteristic function of the $[0, 1]$ interval. Remarkably, this action turns out to include the standard model action, including the poorly understood Higgs sector, as well as the action of GR [Chamseddine and Connes, 1996; Chamseddine and Connes, 1997]. The precise relation between the noncommutativity of noncommutative geometry and of QM has not yet been extensively investigated.

Other ideas and directions

A large number of other ideas and directions of investigation about QG have been proposed. Some of these research directions are still active. Only a few are mentioned below.

A project extensively explored is to define a quantum gravity in terms of the continuum limit of a discrete lattice theory, a technique that works in the case of quantum chromodynamics. Various attempts in this direction have failed in the past, because the lattice theory considered turned out not to have a continuum limit. One of the versions of this program, called *dynamical triangulation* is still very active, although no proof of the existence of a continuum limit exists yet.

Raphael Sorkin and his group have long explored a discrete model in which spacetime is replaced with a discrete set of points equipped with an ordering representing the causal relations [Sorkin, 1983]. Remarkably, the model has predicted a small but non-vanishing cosmological constant, of the correct order of magnitude, a prediction recently confirmed.

Roger Penrose and his group have developed twistor theory as a reformulation of metric geometry, with the hope of addressing the QG problem [Penrose, 1967]. So far, the results of twistor theory are more of mathematical than physical relevance.

Other research directions include Hartle's quantum mechanics of spacetime [Hartle, 1995], quantum Regge calculus [Williams and Tuckey, 1992; Williams, 1997], 't Hooft's deterministic approach ['t Hooft, 1996] and Finkelstein's theory [Finkelstein, 1997].

"Phenomenology" and Lorentz invariance

Until a few years ago, the research community was convinced that QG effects were certainly far outside our current observational reach. This conviction has been shaken by a number of recent suggestions that these effects might in fact be on the verge of being observable. The suggestion has even been made that certain data already observed, and which appear to be difficult to interpret with conventional physics, might be affected by QG effects. These suggestions concern for instance the cosmic propagation of high energy particles, fine details in the cosmological density spectrum, and others.

The issue appears to be related to the problem whether QG breaks Lorentz invariance. Small Lorentz noninvariant QG effects, if they exist, could be within or near observational reach. Lorentz invariant effects, on the other hand, are presumably far smaller, because Lorentz invariance forbids certain effects to happen. For instance, a small deviation from the Lorentz invariant dispersion relation governing light propagation could accumulate over cosmological travel times and yield observable frequency-dependent delays. In a Lorentz invariant context, light travel-time is a meaningless notion.

Naively one might expect that the existence of a minimal length in QG necessarily breaks Lorentz invariance. The argument is that the minimal length must be Lorentz contracted under a change of inertial frame, and therefore could not be minimal. But this argument is incorrect because it disregards quantum theory [Rovelli and Speziale, 2003]. In quantum theory a discrete quantity appears as the *eigenvalue* of an observable quantity, while a symmetry transformation transforms states, and therefore *means values*, not eigenvalues.

To illustrate this phenomenon, recall that in classical mechanics the z component L_z of the angular momentum transforms continuously under rotations. In the quantum theory, let a system be in the eigenstate $|\psi\rangle = |\hbar/2\rangle$ of L_z. Seen from a rotated reference frame, this system will appear to be in a superposition $|\psi\rangle = \alpha|\hbar/2\rangle + \beta| - \hbar/2\rangle$, where α and β vary continuously with the rotation angle. Therefore the expectation value $\overline{L_z} = |\alpha|^2\hbar/2 - |\beta|^2\hbar/2$ varies continuously in the rotation, but the eigenvalues remain the same. In physical terms: we always observe the discrete values $L_z = \pm\hbar/2$ in all reference systems — what changes continuously in a rotation is the probability of seeing one or the other. In the same fashon, in loop quantum gravity a (nonvanishing) minimal area is an eigenvalue. A surface which is in an eigenstate of the area will appear in a superposition of different area eigenstates if seeing from a boosted reference frame. The expectation value of the area of a surface can be smaller than the minimal area, but a (nonvanishing) measurement outcome cannot.

2.2 The main current tentative theories

The two currently most developed and most studied quantum theories of gravity are string theory and loop quantum gravity.

Strings

The major reason for the interest in string theory is that it is a fundamental theory of the world, including the gravitational field, which is likely to be free of ultraviolet divergences, and which encodes in a natural and strictly unified structure all the diverse ingredients we find in the world.

The starting point of the theory is the hypothesis that elementary objects are not point-like particles but rather strings, namely one-dimensional objects. The theory was initially studied as a tentative theory of the strong interactions, where it turned out to be incorrect. Quantum string theory is only consistent if spacetime has a certain dimension, called the critical dimension, which is 26 for the bosonic string and 10 for the supersymmetric string which includes fermions. The problem of reconciling the critical dimension with the fact that our world appears to be four dimensional is still open.

In 1984, Green and Schwarz introduced the idea that string theory might be a unified theory of all interactions, including gravity [Green and Schwarz, 1984]. In fact, one of the vibration modes of the proposed string has spin two, and can be identified with the graviton. Furthermore, a necessary (in general not sufficient) condition for string theory to be well-defined is that the background spacetime satisfies an equation that reduces to the Einstein equation in the large distance limit.

Consistency restricts the string models to a few alternatives. A supersymmetric model defined on a 10-dimensional flat spacetime using a large gauge group, appears to include all the ingredients of our world: the gauge group includes a subgroup which is the gauge group of the particle physics standard model, and the lowest energy vibration modes of the string include fermions, gauge bosons, and the graviton. Although no complete proof is available, the theory appears to have no ultraviolet divergences.

The idea is that six of the ten dimensions of spacetime may be invisible to us, because they are wrapped ("compactified") into a very small space (or because we are constrained to live on a four-dimensional surface). The effective physical theory in the four visible dimensions depends on the way the six extra dimensions are compactified. This can happen in a great number of different manners, giving rise to a huge number of effective four-dimensional theories. For the moment, no selection principle among this large number of possibilities has been found. Some of the resulting low energy models appear to have a strong resemblance to the standard model, but so far none seems to give precisely the physics we observe at low energy.

String theory is defined in terms of a perturbation expansion on a 10 dimensional fixed spacetime background. In the mid nineties, several nonperturbative aspects of string theory began to be investigated. Higher dimensional excitations, called "branes" (from "membrane") [Polchinski, 1995] appear to be needed in the theory for consistency, besides the strings themselves. (It has been suggested that the four-dimensional surface on which we live could be a four-dimensional brane.) The different string models appear to be related to one another (and to 11-dimensional supergravity) via simple transformations called "dualities", suggesting that all the different string models are actually different limits of a single unknown fundamental theory, tentatively-called "M-theory". The actual construction of this hypothetical fundamental theory — expected to be background independent — is still missing, and so far string theory exists only in the form of a number of (loosely) related models defined in terms of expansions over assigned background spacetimes.

In 1998, a certain conformal field theory was shown to include a sector that appears to be related to a supergravity theory on the product of Anti-deSitter spacetime and spheres. This led to the conjecture that the compactifications of string theory on an Anti-deSitter spacetimes is "dual" to a field theory on the spacetime boundary. In turn, this led to a new proposal for defining M-theory itself in term of a boundary theory: the idea is to reach background independence for M-theory using background dependent methods for the boundary theory.

The difficulties of the theory are many. No selection mechanism for the compactification is known — this is the problem of the selection of the "vacuum"; since each compactification gives different physical predictions, and there are hundreds of thousands of possible compactifications, string theory is effectively a collection of a huge number of different theories, each with different predictions and each with different physical parameters. As a result, the theory is incapable of computing the values of the standard model parameters and almost completely nonpredictive, in the sense that it can be compatible with almost any future experimental outcome. According to some critics, this lack of predictivity undermines the very nature of string theory as a scientific theory.

Even if we are willing to choose a compactification ad hoc, no compactification giving precisely the standard model in the low-energy limit is known. The theory requires supersymmetry, and the existence of observable supersymmetric particles has repeatedly been claimed as the distinctive prediction of the theory; but, in spite of several preliminary announcements, supersymmetric particles have not been found in experimental particle physics. Similarly, the possibility of detecting effects of the invisible dimensions has been considered, but experiments have given negative results. The theory requires a huge baggage of new physics (extra dimensions, an infinite number of fields with arbitrary masses and spins, supersymmetric particles ...) but so far none of this appears to be present, or have observable consequences, in the real world.

Loops

The main reasons for interest in loop quantum gravity are: that its physical assumptions are only QM and GR, namely well-tested theories; the fact that the theory is background independent; and that it is a well developed attempt to incorporate the general relativistic notions of space and time into QFT. The theory makes no claim of being a final "Theory Of Everything". It is ultraviolet finite, without requiring high-energy modifications of GR, supersymmetry, extra dimensions, or other unobserved physics.

Loop quantum gravity was introduced in 1988. The theory is the result of the merging of two lines of research, which turn out to solve each others difficulties [Rovelli and Smolin, 1988; Rovelli and Smolin, 1990].

The first of these was the Wheeler-deWitt theory. As in the Wheeler-deWitt approach, loop quantum gravity is a straightforward quantization of GR, with its conventional matter couplings, and is based on no specific physical assumption other than GR and QM. Following the basic rules of QM, the quantum states of loop quantum gravity are obtained from a representation of an algebra of field variables of GR; their physical interpretation is obtained by diagonalizing self-adjoint operators that represent physical quantities. The difference with respect to the old Wheeler-DeWitt theory is in the choice of an algebra of loop-variables as basic variables for the quantization. Thanks to this, the ill-defined Wheeler-DeWitt theory becomes a well-defined formalism where finite physical quantities can be computed.

The second input was the idea that gauge theories are naturally described in terms of loop-like excitations. This idea can be traced back to the very beginning of field theory, an intuition of Faraday's. Faraday understood electric and magnetic phenomena in terms of lines, the "Faraday lines", that fill up space. In the presence of charges, the Faraday lines can start and end on the charges; in the absence of charges, they close, forming "loops". Maxwell translated Faraday's intuition into mathematical physics, introducing the electric and magnetic field, which are vector fields everywhere tangent to the Faraday lines, thus opening the way to modern physics, which is entirely based on the notion of field. The idea that gauge field theories are better understood in terms of loops has been defended by many scientists, including Polyakov, Mandelstam, Wilson, and others. A quantum excitation of a single Faraday line is called a "loop state".

A formulation of a QFT in terms of loop states is viable and well understood in the context of the lattice approximation; but it faces difficulties when defined over a continuum spacetime background. However, these difficulties disappear in a background independent context. The reason is that in the presence of a background, the loop states are localized on the background spacetime: there is a distinct state for each position of the loop in space. In the case of gravity, instead, there is no background spacetime. The loop states themselves are the quantum excitations of space. Therefore loop states are not immersed in space: they "weave-up" physical space themselves, in the same manner in which an ensemble of threads can weave the fabric of a T-shirt.

More precisely, the loop states of QG have self-intersection points called “nodes”. A node represents an elementary quantum excitation of space, or a single atom of space. Two nodes directly connected along a loop represent adjacent atoms of space. Nodes and links connecting nodes form a graph and carry quantum numbers. These quantum numbers determine the quantized volume of the atoms of space and the quantized area of the elementary surfaces separating adjacent nodes. A graph with these quantum numbers is called a “spin-network”, because the quantum numbers on the links turn out to be half-integers, or spins.

A spin network state does not have a position. Only combinatorial relations defining the graph are significant, not its shape or its position in space. In fact, a spin network state is not *in* space: it *is* space. Hence, in spite of its conservative basic assumptions (QM and GR), loop quantum gravity leads to a radically novel picture of space.

The possible values that the volume of a physical region or the area of a physical surface can take are determined by the spectra of the corresponding operators, following standard QM rules. These turn out to be discrete, giving the Planck-scale granular structure of space. These spectra have been computed and represent quantitative physical predictions of loop quantum gravity: a Planck-scale precision measurement of any area or volume is predicted to give as a result only the values in these spectra. For instance, the (main sequence of the) spectrum of the area is given by the expression [Rovelli and Smolin, 1995]

$$(9) \quad A = 8\pi\gamma\hbar G \sum_i \sqrt{j_i(j_i+1)},$$

where j_i is an n-tuplet of half-integers (corresponding to the quantum numbers of the links of the spin network state crossing the surface whose area is measured). γ is the Immirzi parameter, mentioned in Section 2.1.

The dynamics is determined by a Wheeler-deWitt equation on the space of spin network states. Its ultraviolet finiteness is a consequence of the granular structure of space. Different finite and well-defined versions of this equation have been constructed. At present it is not yet clear which of these, if any, is the physically correct one.

Applications of the theory include a derivation of the Bekenstein black hole entropy mentioned in Section 2.1, applications to the description of the classical singularities, such as the ones at the center of bloack holes, and applications to cosmology. The theory appears to be capable of controlling the black hole singularities and the initial Big Bang singularity. Indirect empirical evidence supporting predictions of the theory is actively searched in the astrophysical and cosmological domains.

The main difficulties of loop quantum gravity lie in recovering low energy phenomenology. Quantum states corresponding to the Minkowski vacuum and its excitation have not yet been constructed, and particle scattering amplitudes have not been computed. This deficiency weakens the strength of the finiteness claim, and bears on one of the key requirements on a quantum theory of gravity: full recovery of low energy physics. The dynamics is still poorly understood: the Wheeler-deWitt equation exists in more than one version. The lack of unitary evolution in time and the overall radical conceptual novelty of the results of the theory, where background spacetime is discarted altogether, are questioned by some.

The loop-string debate

A theory begins to be credible only when its original predictions are reasonably unique and are confirmed by new experiments. Neither loop quantum gravity nor string theory — nor any other current tentative theory of QG — are yet credible in this sense. Furthermore, in spite of much effort, both theories are still badly incomplete and far from being clearly understood. The problem of QG must therefore be considered still fully open.

Nevertheless, in both directions the research has progressed considerably in recent years: many problems that appeared too hard ten years ago have now been solved, and incomplete but *possible* solutions of the QG puzzle are now at hand.

However, the two theories differ profoundly in their hypotheses, achievements, specific results, and in the conceptual frame they propose. The issues they raise concern the foundations of the physical picture of the world, and the debate between the two approaches involves conceptual, methodological and philosophical issues.

The lesson of string theory appears to be that in order to remove the difficulties of the perturbative quantization of GR we have to couple the gravitational field to matter. Finiteness is achieved by replacing the pointlike Feynman vertices of conventional QFT with non-point-like interactions between strings, which are extended objects. The theory preserves the basic conceptual structure of QFT (background spacetime, unitarity, predictions in terms of an asymptotic S-matrix. . .) at the prices of renouncing a full implementation of the general covariance that characterizes GR, of huge extra baggage (extra dimensions, supersymmetry, infinite fields. . .) and of a dramatic decrease in predictiveness.

Loop quantum gravity, on the other hand, is rooted in the general covariance that characterizes GR. Ultraviolet finiteness is a consequence of the granular structure of space, which, in turn, is a standard quantum mechanical effect appearing when we regard GR as a theory of spacetime itself, and not as a theory of small perturbations around a background spacetime. The interest of the loop theory, therefore, is that it is a determined effort towards a genuine merger of QFT with the world view that we have discovered with GR. Furthermore, it leads to well-defined physical predictions which are in principle falsifiable. However, even disregarding the incompleteness of the theory, the conceptual price for this result is heavy: the theory gives up unitarity, time evolution, Poincaré invariance at the fundamental level, and the very notion that physical objects are localized in space and evolve in time.

Whether these radical conceptual steps are viable, and, if viable, whether they are justified, is a hotly debated issue.

3 METHODOLOGICAL ISSUES

3.1 Justification of the quantum gravity search

Absence of empirical data

The first obvious question about the search towards QG is whether the search is legitimate at all, given the total absence of empirical data directly about the regimes QG is concerned with. We have no direct empirical guidance in searching for QG — as, say, atomic spectra guided the discovery of quantum theory.

Some critics have argued that the QG search is futile, because anything might happen in QG regimes, at scales far removed from our experience. Maybe the search is impossible because the space of possible theories is too large.

At present, this worry is probably unjustified. If this were the problem, we would have plenty of complete, predictive and coherent theories of QG, and the problem would be the choice among them. Instead, the situation is the opposite: we haven't any. The fact is that we do have plenty of information about QG, because we have QM and we have GR. Consistency with QM and GR, plus internal consistency, form an extremely strict set of constraints. The problem currently debated is to find at least one complete and consistent theory of QG. If more will be found, we will have of course to resort to experiments to select the physically correct one.

Should gravity be quantized?

The possibility that quantum gravitational effects do not exist and gravity is intrinsically classical (non quantum) has been often suggested. The justification for this suggestion is that gravity can be seen as an interaction profoundly different from the others, since it admits a description in terms of spacetime geometry. This suggestion has also generated a research program, aiming at testing the consistency of a theory in which classical gravity interacts with QFT.

In its simpler form, this suggestion has today been largely abandoned. The reason is that, as was noticed in the early days of QM, an interaction between a classical and a quantum variable is always inconsistent. If Heisenberg uncertainty relations are violated for one dynamical variable, they are violated for all other variables as well. The idea of circumventing gravity quantization, however, has reappeared under various forms.

One suggestion is that the gravitational field may not represent true microscopic degrees of freedom, but only a collective, or "hydrodynamical", large scale description of these. This hypothesis is supported by phenomena such as the relations between gravity and thermodynamics revealed by the Unruh effect. Ted Jacobson [Jacobson, 1995] has even been able to derive the Einstein equations from (7) and standard thermodynamical relations, providing evidence that could be interpreted as supporting this idea. However, even if the gravitational field is just a collective variable, this does not mean that it will not display quantum effects. QM does not govern just elementary degrees of freedom; it governs all degrees of freedom, including collective ones. Thus, this possibility, even if realized, would not refute the need of a quantum theory of gravity.

Another suggestion is that gravity may be an emergent phenomenon induced by the other quantum fields. This idea is suggested by the fact that the renormalization process for a QFT on a curved spacetime generates terms in the action which are proportional to polynomials in the Riemann curvature, and the lowest order term is precisely the action of GR. The difficulty about this suggestion is that it is ambiguous as regards the dynamical status of the metric field. The variational principle states that dynamics is determined by the variation of the action with respect to the dynamical variables only, not with respect to anything appearing in it. If the metric field is assumed to be a dynamical variable, then it is a dynamical field like any other, and the fact that the dependence of the action with respect to it is modified by the renormalization of its interaction with other fields may change the details of its dynamics, but not the fact that it is a quantum field. If, on the other hand, the metric is not a dynamical field, then the action must not be varied with respect to it (as it is not varied with respect to it in the special relativistic context), and therefore the Einstein equations are not generated by the new terms in the action. In the first case the gravitational field needs to be quantized; while the second case is in contradiction with the empirical fact that the classical Einstein equations are satisfied

3.2 Research attitudes

Different attitudes can be distinguished in the physics community with respect to the methodology used for searching for a QG theory.

(a) The “pessimistic” attitude, already mentioned above, is that of those who worry that too many possibilities are open, anything might happen between here and the Planck scale, and the search for a quantum theory of gravity is therefore futile.

As mentioned, this worry is unfounded, because we do not have too many complete QG theories: we haven’t any.

(b) The view is often expressed that some totally new, radical and wild hypothesis is needed for QG. This “wild” attitude is based on the observation that great scientists had the courage to break with old and respected assumptions and to explore some novel “strange” hypotheses. From this observation, the “wild” scientist concludes that any strange hypothesis deserves to be investigated, even if it violates well established facts.

On historical grounds, this expectation is probably ill-founded. Wild ideas pulled out of the air have rarely made science advance. The radical hypotheses that physics has successfully adopted have always been reluctantly adopted because they were forced by new empirical data — Kepler’s ellipses, Bohr’s quantization, Planck's energy quanta — or by stringent theoretical deductions — Maxwell displacement current, Einstein's relativity. Generally, arbitrary novel hypotheses have led nowhere. This consideration leads to the next attitude in (c).

(c) Part of the research in QG is motivated by the hope that the knowledge of the world coded into GR and QM can be a good guide for finding a theory capable of describing physical regimes that we have not yet explored.

A motivation for this hope is that today we are precisely in one of the typical situations in which theoretical physics has worked at its best in the past. Many of the most striking advances in theoretical physics have derived from the effort to find a common theoretical framework for two basic and apparently conflicting discoveries. For instance, the aim of combining special relativity and non-relativistic quantum theory led to the theoretical discovery of antiparticles; combining special relativity with Newtonian gravity led to general relativity; combining the Keplerian orbits with Galilean physics led to Newton's mechanics; combining Maxwell theory with Galilean relativity led to special relativity, and so on. In all these cases, major advances have been obtained by "taking seriously" apparently conflicting theories, and exploring the implications of holding true the essential tenets of both theories. Today we are in one of these characteristic situations. We have learned two new very general "facts" about nature, expressed by QM and GR: we have "just" to figure out what they imply, taken together.

(d) A different point of view on the problem is held by those who accept that QM has been a conceptual revolution, but do not view GR in the same way. According to this point of view, the discovery of GR was "just" the writing of one more classical field theory. This field theory is likely to be only an approximation to a theory we do not yet know, and its teachings should not be overestimated. According to this opinion, GR should not be taken too seriously as a guidance for theoretical developments.

A possible objection to this point of view is that it derives from the confusion between (i) the specific form of the GR action and the GR field equations and (ii) the modification of the notions of space and time engendered by GR. The GR action could be a low energy approximation of something else. But the modification of the notions of space and time has to do with the diffeomorphism invariance and the background independence of the theory, not with its specific form. The challenge of QG is to incorporate this novelty into QFT, not the specific form of the GR action.

(e) A common attitude is the "pragmatic" attitude of the physicist who prefers to disregard or postpone these foundational issues and, instead, develop and adjust current theories. This style of research was effective during the sixties in the search for the particle physics standard model, where a long process of adjustment of existing QFT's led to a very effective theory.

It is questionable whether this attitude could be effective in a situation of foundational confusion like the present one. During the sixties empirical data were flowing in daily, to keep research on track. Today no new data are available. The "pragmatic" attitude may mislead the research: in the extreme case, the "pragmatic" physicist focuses only on the development of the theory at hand, without caring if the world predicted by the theory resembles less and less the world we see. Sometimes he is even excited that the theory looks so different from the world, thinking that this is evidence of how far ahead he has advanced in knowledge. But it is more likely that the difference between the theory and the world is only evidence of how much he is lost. Unfortunately similar excesses plague theoretical research today.

The cumulative aspect of scientific knowledge and influence of the philosophy of science

The "pessimistic", "wild" and "pragmatic" attitudes illustrated above may have been influenced by a philosophy of science that under-emphasizes the cumulative aspect of scientific knowledge, and emphasizes, instead the "incommensurability" between an old theory and a new theory that historically supersedes it. More or less informed awarness of this long standing debate in philosophy of science has indeed affected the reserach attitude of many theoreticians.

On the other hand, attitude (c) described above is based on the expectation that the central physical tenets of QM and GR represent our best guide for accessing the unexplored territories of the quantum-gravitational regime. In a difficult research situation where cataclysmatic evolution is expected anyway in the consequences of the theory (for instance, the change of the nature of space), conservative assumptions based on the confidence on the cumulative aspect of knowledge can play an important role.

This faith in a cumulative aspect of scientific knowledge is based on the idea that there are discoveries that are "forever". For instance, the Earth is not the center of the universe, simultaneity is relative, absolute velocity is meaningless, and we do not get rain by dancing.

The fact that major aspects of a theory can have value outside the domain for which the theory was discovered may be at the root of much of the historical effectiveness of theoretical physics, and in particular of spectacular predictions such as Maxwell's radio waves, Dirac's antimatter or GR's black holes. This can perhaps be understood as just scientific induction: as a consequence of the fact that Nature has regularities. This is not the place to enter this discussion; but it is relevant to remark that the existence of these regularities is held by several researchers in QG as a source of confidence — althought, of course, not certainty — that the basic facts about the world found with QM and GR will be confirmed, not violated, in the quantum gravitational regimes that we have not yet empirically probed.

4 THE NATURE OF SPACE AND TIME

GR has modified the way we understand space and time. Combining GR with QM requires a further modification of these notions. It is important, however, to clearly distinguish the modifications of the notions of space and time required by QG from the ones already implied by GR alone. These are briefly summarized in Section 4.1 below. Section 4.2 and 4.3 then discuss the notions of space and time in QG.

4.1 The physical meaning of GR

GR is the discovery that spacetime and the gravitational field are the same entity. What we call "spacetime" is itself a physical object, in many respects similar to the electromagnetic field. We can say that GR is the discovery that there is no spacetime at all. What Newton called "space", and Minkowski called "spacetime", is nothing but a dynamical object — the gravitational field — in a regime in which we neglect its dynamics.

In newtonian and special relativistic physics, if we take away the dynamical entities — particles and fields — what remains is space and time. In general relativistic physics, if we take away the dynamical entities, nothing remains. The space and time of Newton and Minkowski are reinterpreted as a configuration of one of the fields, the gravitational field. This implies that physical entities — particles and fields — are not all immersed in space, and moving in time. They do not live on spacetime. They live, so to speak, on one another.

In classical GR it is customary to maintain the expressions "space" and "time" to indicate aspects of the gravitational field. But in the quantum theory, where the field can have quantized "granular" properties and its dynamics is quantized and therefore only probabilistic, most of the "spatial" and "temporal" features of the gravitational field are probably lost.

This absence of the familiar spacetime "stage" is called the *background independence* of the classical theory. Technically, background independence is realized by the gauge invariance of the GR action under (active) diffeomorphism. A diffeomorphism is a transformation that smoothly drags all dynamical fields and particles on the four-dimensional coordinate manifold. In turn, gauge invariance under diffeomorphism (or *diffeomorphism invariance*) is the consequence of the combination of two properties of the action: its invariance under arbitrary changes of coordinates (or *general covariance*) and the fact that there is no non-dynamical "background" field. Thus: *background independence* = *diffeomorphism invariance* = (*general covariance*+absence of non-dynamical background fields). These notions are illustrated in more detail below.

Diffeomorphism invariance

Pre-general-relativistic field theories are formulated in terms of a spacetime manifold M, and a set of fields $\varphi_1, \dots, \varphi_n$ on M. The manifold M is a (pseudo-)metric space whose points $P \in M$ represent the physical points of spacetime. Spacetime points are labelled by coordinates $x = (x^1, x^2, x^3, x^0)$ that represent the reading of measuring devices: clocks and distance-measuring devices ("rods"). More precisely, M is equipped with a (pseudo-)distance function $d(x, y)$ interpreted as the 4-interval between the two points x and y: a negative $d^2(x,y)$ gives the time measured by a clock in inertial motion between x and y; a positive $d^2(x,y)$ gives the proper length of a rod with the ends on x and y, in a state of inertial motion with respect to which x and y are simultaneous according to Einstein's definition of simultaneity; a null $d(x,y)$ indicates that light travels in vacuum from x to y. Notice that pre-general-relativistic physics deals with (relations between) two distinct types of measurements: (i) spacetime measurements measuring spacetime observables, performed by means of clocks and distance-measuring devices, and (ii) field measurements, measuring field observables, namely the values (or functions) of the fields $\varphi_1, \ldots, \varphi_n$.

This same interpretation framework is used in special-relativistic QFT. The only difference is that field observables can be quantized. The number of excited quanta has a particle interpretation. In a typical high energy scattering experiment, for instance, the field observable (ii) is the number of particles revealed by a particle detector (which is a field measuring device); while the spacetime observable (i) is the momentum of the particle, determined by measuring the spacetime position of the detector.

The theory does not predict the value of field observables φ alone, or spacetime observables x alone, but only combinations of the two, such as the value $\varphi(x)$ of a field φ at a certain spacetime location x. Spacetime and field observables are both quantities that have a direct operational interpretation; they can be called "partial observables". On the other hand, the quantities that can be predicted by the theory, such as $\varphi(x)$, for a given position x, can be called "complete observables".

The interpretation of a *general* relativistic field theory is different. In such a theory there is a field g representing the gravitational field and possibly other fields, representing other dynamical variables. These fields are defined on a differentiable manifold M, coordinatized by coordinates x. The formal structure of a general relativistic field theory is therefore similar to the structure of a pre-general-relativistic field theory. But two major differences force a different interpretation. First, the manifold M on which the fields are defined is not a metric manifold. The gravitational field g equips M with a metric structure $d_g(x,y)$.[1] Therefore clocks and distance-measuring-devices measure properties of the gravitational field g. It follows that the distinction between spacetime observables of the kind (i) and field observables of the kind (ii) is blurred. This blurring of the distinction between the two kind of partial observables is a crucial conceptual novelty of GR.

Second, the field equations are invariant under a transformation of the fields called active diffeomorphisms. An active diffeomorphism $g \to \tilde{g}$ is determined by (but should not be confused with) a smooth invertible function $f : M \to M$. Under a diffeomorphism transformation, the field g and all other fields are "dragged along" M by f. For instance, the transformed field $\tilde{g}$ defines a new distance

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[1] The length of a curve $\gamma^\mu(s)$ in M is $d_g(\gamma) = \int ds \sqrt{g_{\mu\nu}(\gamma(s)) \frac{d\gamma^\mu(s)}{ds} \frac{d\gamma^\nu(s)}{ds}}$ and $d_g(x,y)$ is a local extremum of $d_g(\gamma)$ over the curves that join x and y.

function $d_{\tilde{g}}(x,y)$ which is related to the one defined by g by

(10) $d_{\tilde{g}}(f(x), f(y)) = d_g(x,y)$.

In words: the distance between the two points $f(x)$ and $f(y)$, defined by the field $\tilde{g}$ is the same as the distance between the two points x and y defined by g.

The importance of this invariance of the field equations is due to the following. An active diffeomorphism may modify a solution of the GR equations in the future of a certain time surface t_0, without modifying it at all in the past of t_0. Therefore two distinct solutions of the equations of motion can be equal in the past and differ in the future. This fact gives us a choice: either (i) we interpret the theory as an indeterministic theory, where the future is not determined by the past. Or (ii) we interpret active diffeomorphisms as a "gauge invariance": that is, we postulate that the complete observables of the theory are only given by quantities that are invariant under this transformation. The alternative (i) is not viable, because experience shows that classical gravitational physics is completely deterministic. We are therefore forced to alternative (ii), which has heavy interpretative consequences.

To understand these consequences, let P be a point of M. Let $\varphi(P)$ be any property of the fields at P. For instance, $\varphi(P)$ may represent a value of the electromagnetic field at P or the spacetime scalar curvature at P, or something similar. None of these properties is invariant under active diffeomorphism. Therefore, it follows from the argument above that none of these properties can be predicted by the theory. Therefore the theory does not determine the physics at spacetime points $P \in M$.

At first this conclusion might sound bewildering: if physics does not predict what happens at spacetime points, what can it predict? In fact, historically, Einstein himself got at first confused and frustrated by this observation, to the point of stepping back from the diffeomorphism invariance he previously expected GR to have [Norton, 1984]. Einstein's version of the argument given above is called the "hole argument" (because Einstein considered a diffeomorphism affecting only a finite region of spacetime, empty of matter, or a "hole"), and was presented in [Einstein and Grossmann, 1914]. On this argument, and the discussion it has raised, see for instance [Earman, 1987; Earman, 1989; Belot, 1998; Earman, 2001; Pauri and Vallisneri, 2002] and references therein. Later however, Einstein changed his mind and accepted both diffeomorphism invariance and the conclusion (ii), realizing that this conclusion fully implemented his intuition on the very central physical meaning of the general relativistic conceptual revolution.

The way out of the puzzle is to understand that in the general-relativistic context the points of the manifold do not represent physical entities with an existence independent from the fields. Asking what are the properties of the fields at P is meaningless. Spacetime locations can only be determined by the fields themselves, or, by any other dynamical object we are considering. For instance, if the theory we consider includes two particles and the trajectories of the two particles happen to meet once, then the meeting of the particles determines a spacetime point. The theory is able to predict the value of the fields and any other physical properties at the spacetime point determined by the meeting of the particles. However, this point cannot be naively identified as a point of M, because the same physical situation can be represented by the set of dynamical variables obtained by an active diffeomorphism, where the particles meet now at a different point, say Q of M. The value of the fields at the point where the particle meet is invariant under such a transformation, because the particles' trajectories and the fields are dragged along M together. Einstein called this way of determining location in terms of the dynamical objects (fields and particles) of the theory itself, "spacetime coincidences".

Thus, a general relativistic theory does not deal with values of dynamical quantities at given spacetime points: it deals with values of dynamical quantities at "where"'s and "when"'s determined by other dynamical quantities.

Strictly speaking also in the pre-general-relativistic context physics deals with values of dynamical quantities at "where"'s and "when"'s determined by other physical quantities, because the times and distances used to determine location are physical quantities. But in the pre-general-relativistic context we can make a strict separation between: (i) "spacetime", viewed as a background entity, and measured by clocks and rods that one considered non-dynamical, and (ii) dynamical variables. In the general-relativistic context, on the other hand, this separation is lost, time and distance measurements are reinterpreted as measurements of the gravitational field, on the same footing as other field measurements, and there is no distinction between non-dynamical background and dynamical physical variables.

Physical meaning of the coordinates

A consequence of the above is that in the general-relativistic context the physical interpretation of the coordinates x is different from their interpretation in the non-general-relativistic context. In the general-relativistic context the coordinates x have no interpretation at all: observable quantities in GR correspond to quantities of the theory that are independent of the coordinates x. Recall that the non-general-relativistic coordinates represent the reading of clocks and rods: in the general-relativistic context, the reading of clocks and rods is represented by the non-local function $d_g(x, y)$ of the gravitational field. The fact that the non-general-relativistic coordinates x and the general-relativistic coordinates x are denoted in the same manner is only an unfortunate historical accident.

To illustrate in which sense observable quantities are independent of the coordinates x, consider a typical general-relativistic measurement. A standard application of GR is in precision measurements and precision modeling of solar system dynamics. In this context, partial observable quantities are the "instantaneous" distances d_p between the Earth and the different planets, defined as the proper time elapsed on Earth (measured by a clock at rest on Earth) while a radar signal goes from Earth to a planet p and back. Fixing an arbitrary initial event on Earth, one additional partial observable can be obtained as the proper time

τ from this event along the Earth trajectory. Complete observables are then the values $d_p(\tau)$ of the planet distances, at different local proper times τ. A general relativistic model of the solar system, with appropriately chosen initial data, can predict $d_p(\tau)$ for all p's and all τ's, and these predictions can be compared with experience. In building up this model, we choose an arbitrary coordinatization x of the solar system region, and express the gravitational field, the electromagnetic field, and the planets' positions in this coordinate system. The predicted quantities $d_p(\tau)$ are complicated non-local functions of the fields and planets' positions, which are independent of the coordinates x chosen. To be sure, the observable τ is introduced here only for convenience. We can equivalently express the predictions of the theory simply as a set of relations $f(d_p) = 0$ that must hold between the partial observables d_p.

General covariance and Kretschmann objection

The invariance of GR under active diffeomorphisms follows from two properties of the GR field equations. First, they are generally covariant. That is, they maintain the same form under any smooth change of coordinates $x \to x'(x)$ on M. This means that there is no coordinate systems on M which is preferred a priori. Second, there are no fixed non-dynamical fields in the field equations.

The first property, namely general covariance, is the property that Einstein most insisted upon, and that guided him in finding GR. The requirement of general covariance still plays a major role in selecting physical theories compatible with what we have understood about the world as a result of the general relativistic revolution.

However, general covariance *alone* is not excessively significative. Indeed, any field equation can be written in an arbitrary coordinate system. This fact was pointed out by Kretschmann shortly after Einstein wrote GR [Kretschmann, 1917], and has raised much discussion. As an example, consider the field equation

$$(11)\quad (\partial_T^2 - \partial_X^2 - \partial_Y^2 - \partial_Z^2)\ \varphi(X, Y, Z, T) = 0.$$

If we introduce arbitrary coordinates x (with components x^μ) as functions $x^\mu = x^\mu(X, Y, Z, T)$ (with inverse $X(x), Y(x), Z(x), T(x)$), the wave equation (11) becomes the generally covariant equation

$$(12)\quad \Box_g \varphi(x) \equiv \partial_\mu\ \sqrt{\det -g(x)}\ g^{\mu\nu}(x)\ \partial_\nu \varphi(x) = 0,$$

In this equation, the unknown is $\varphi(x)$, while $g^{\mu\nu}(x)$ and $\det g(x)$ are the inverse and the determinant of the *fixed* field

$$(13)\quad g_{\mu\nu}(x) = \frac{\partial X(x)}{\partial x^\mu}\frac{\partial X(x)}{\partial x^\nu} + \frac{\partial Y(x)}{\partial x^\mu}\frac{\partial Y(x)}{\partial x^\nu} + \frac{\partial Z(x)}{\partial x^\mu}\frac{\partial Z(x)}{\partial x^\nu} - \frac{\partial T(x)}{\partial x^\mu}\frac{\partial T(x)}{\partial x^\nu}.$$

The field theory for the scalar field φ defined by equation (12) is *not* diffeomorphism invariant, because in distinct coordinate systems the field equations for the unknown φ are different, in the sense that they are determined by different functions $g_{\mu\nu}(x)$.

Equation (12), on the other hand, can become one of the equations of a diffeomorphism invariant theory in which $g_{\mu\nu}(x)$ is also one of the unknown, namely a dynamical field. Therefore whether or not a theory is diffeomorphism invariant is not determined just by the aspect of an equation, but by the full specification of the dynamical quantities and their equations of motion.

Eistein's insistence on general covariance alone, however, should probably not be interpreted as a lack of clarity on his part, but only as his effort to emphasize the importance of a step that was *necessary*, not sufficient, to write a succeful relativistic field theory.

In addition, like all formal properties of physical theories, even full diffeomorphism invariance, should probably not be interpreted as a rigid selection principle, capable of selecting physical theories *just by itself.* With sufficient acrobatics, any theory can perhaps be re-expressed in a diffeomorphism invariant language. The same is true for any other formal invariance property. For instance, any theory can be rewritten as a rotational invariant theory, or with any other desired invariance property, by simply adding variables.

As an example, equation (11) can be viewed as physically equivalent to the diffeomorphism invariant system

$$(14) \quad \Box_g \varphi(x) = 0, \qquad\qquad Riem[g] = 0,$$

where $Riem[g]$ is Riemann's curvature and the unknowns are now the two fields φ and g. But there are prices to pay. First, this theory has a "fake" dynamical field, since g is constrained to a single solution up to gauges, by the second equation of the system. Having no physical degrees of freedom, g is physically a fixed background field, in spite of the trick of declaring it a variable and then constraining the variable to a single solution. Second, we can insist on a lagrangian formulation of the theory (14) [Sorkin, 2002], but to do this we must introduce an additional field, and it can then be argued that the resulting theory, having an additional field is different from (12) [Earman, 1989].

Diffeomorphism invariance is the key property of the mathematical language used to express the key conceptual shift introduced with GR: the world is not formed by a fixed non-dynamical spacetime structure, which defines localization and on which the dynamical fields live. Rather, it is formed solely by dynamical fields in interactions with one another. Localization is only defined, relationally, with respect to the fields themselves.

Relationalism and substantivalism

A non-dynamical background space was used by Newton. The first part of the *Principia*, Newton argues very explicitly that we must assume the existence of space as an entity. This part can be read as a polemic against the long dominant, and in particular Descartes's, relational understanding of space.

The two traditional views about space, absolute ("space is an entity") and relational ("space is a relation between entities"), suitably modified to take into account scientific progress, continue in contemporary philosophy of science under the names of *substantivalism* and *relationalism*.

We can say that Einstein has "unmasked" the entity introduced by Newton (which much disturbed Leibniz): Newton's space is nothing else than a field like the others, thought Newton considered it in a regime in which its dynamics could be neglected. Localization in space and in time, introduced by Newton against Descartes's relational localization, is revealed by Einstein to be, after all, still a relational location — in the sense of Descartes —, with respect to a specially chosen entity: the gravitational field. In a sense, we can therefore say that GR realizes a full return to a relational definition of space and time, after the Newtonian substantivalist parenthesis.

In other words, in prerelativistic physics, spacetime is a sort of structured container which is the home of the world. In general-relativistic physics, on the other hand, there is nothing of the sort. There are only interacting fields (including the gravitational field) and particles: the only notion of localization which is present in the theory is relative: dynamical objects (fields and particles) are localized only with respect to one another. This is the notion of relational space defended by Aristotle and Descartes, against which Newton wrote the initial part of the *Principia*. Newton had two points in his favor: the physical reality of inertial effects such as the concavity of the water in the bucket of his famous bucket experiment, and the immense empirical success of his theory based on absolute space. Einstein has provided an alternative interpretation for the cause of the concavity — the interaction with an entity: the local gravitational field — and a theory based on relational space that is empirically far more effective than Newton theory. Einstein has therefore reopened the possibility of a relational understanding of space and time, which was closed by Newton's bucket.

At the basis of Cartesian relationalism was the notion of "contiguity". Two objects are contiguous if they are adjacent to one another. Space is the order of things with respect to such contiguity relation. At the basis of the spacetime structure of GR there is a very similar notion: Einstein's "spacetime coincidence" is strictly analogous to Descartes' "contiguity".

The key to this novel relational understanding of space and time is Faraday's revolutionary idea that a field is a physical entity. Recall that Faraday visualized a field as a family of real lines filling up everything. Einstein's entire theoretical work fully implements this realistic interpretation of the fields. In Einstein popular-science writing, the gravitational field is a huge "jelly fish", a better metaphor than the lines of Faraday. Entities are not just particles, but also fields, the gravitational field is one field among the others. These entities are localized only with respect to one another.

A substantivalist position can nevertheless still be –and in fact is still– defended. Einstein's discovery that Newtonian spacetime and the gravitational field, are the same entity, can also be expressed by saying that there is no gravitational field: it is spacetime that has dynamical properties. This choice is not uncommon in the literature. The difference with the laguage used here is only a matter of choice of words. The substantivalist can therefore claim use the that, according to GR, "spacetime is an entity": indeed, it is the gravitational field, which is an entity. Since it is possible to define localization with respect to the gravitational field, the substantivalist can also say that "spacetime is an entity that defines localization".

However, this is an extremely weakened substantivalist position. To what extent is general-relativistic spacetime different from any other arbitrary entity with respect to which we can define a relational localization? We can call "spacetime" anything used to define localization. Newton's acute formulation of substantivalism contains a precise characterization of "space" [Newton, 1962]:

> "...So it is necessary that the definition of places, and hence of local motion, be referred to some motionless thing such as extension alone or "space", *in so far as space is seen to be truly distinct from moving bodies.*"

(My italic.) The characterizing feature of space, according to this substantivalist manifesto, is to be "truly distinct from moving" bodies. In modern terms and after the Faraday and Maxwell conceptual revolution, I believe this can only be translated as being "truly distinct from dynamical entities such as particles or fields". This is *not* the case for the spacetime of GR. The modern substantivalist can give up Newton's strong substantivalism ("spacetime is a non-dynamical entity") for the much weaker thesis "we call spacetime the gravitational field, which is a dynamical entity". But then what is the difference between this position and the relationalist one, if not just a choice of words?

To be sure, general relativistic relationalism doesn't fit comfortably with traditional relationalism either. E.g. observables of GR, conceived as coincidence quantities are non-substantival in that they don't require spacetime points to support them. But neither are they relational in the traditional sense of involving relations between "material" bodies or events in their histories.

The traditional substantivalist-relational alternative was formulated before the Faraday-Maxwell conceptual revolution, without taking the existence of the *fields* into account. After Farady and Maxwell, we understand the world also in terms of a new set or dynamical entities, the fields. Once we accept the existence of the fields, and Einstein's discovery that Newton's space is one of the fields, the distinction between substantivalism and relationalism is largely reduced to mere semantics.

When two opposite positions in a long-standing debate have come so close that their distinction is reduced to semantics, one can perhaps say that the issue is resolved. In this sense, it may be argued that GR has solved the long-standing issue of the relational versus substantivalist interpretation of space.

4.2 Background independence

Is QM compatible with the general relativistic notions of space and time sketched above? It is, but a sufficiently general formulation of QM must be used. For instance, the Schrödinger picture is only viable for theories where there is a global observable time variable t; this conflicts with GR, where no such variable exists. Therefore the Schrödinger picture makes little sense in a background independent context. But formulations of QM have been proposed that are more general than the Schrödinger picture. See for instance [Hartle, 1995] and [Rovelli, 2004]. Formulations of this kind are sometimes denoted “generalized quantum mechanics”, although they might be called “quantum mechanics” in the same sense in which “classical mechanics” is used to designate formalisms with different degrees of generality, such as Newton’s, Lagrange’s, Hamilton’s or symplectic mechanics.

On the other hand, most of the conventional machinery of perturbative QFT is profoundly incompatible with the general relativistic framework. There are many reasons for this: (i) The conventional formalism of QFT relies on Poincaré invariance. In particular, it relies on the notion of energy and on the existence of the nonvanishing hamiltonian operator that generates unitary time evolution. The vacuum, for instance, is the state that minimizes the energy. But, in a general relativistic theory there is, in general, no global Poincaré invariance, no general notion of energy and no nonvanishing hamiltonian operator. (ii) At the roots of conventional QFT is the physical notion of particle. The theoretical experience with QFT on curved spacetime [Fulling, 1989] and on the relation between acceleration and temperature in QFT [Wald, 1994] indicates that in a generic gravitational situation the notion of particle can be quite delicate. (iii) Consider a conventional renormalized QFT. The physical content of the theory can be expressed in terms of its n-point functions $W(x_1, \ldots, x_n)$. We expect the n-point functions to be invariant under the invariances of the theory. In a general relativistic theory, invariance under an arbitrary coordinate transformation $x \to x' = x'(x)$ implies immediately that the n-point functions must satisfy

$$(15) \quad W(x_1, \ldots, x_n) = W(x'(x_1), \ldots, x'(x_n)).$$

Since any set of n (distict) points $(x_1, \ldots, x_n)$ can be transformed into any other set by a generic coordinate transformation, it follows that W is constant! It does depend on its arguments! Clearly we are in a very different framework from conventional QFT.

There is a possible escape strategy to circumvent these difficulties: write the gravitational field as the sum of two terms, as in equation (1), and assume that spacetime and causal relations are defined by the first term, rather than by the full gravitational field. This escape strategy brings back a background spacetime. A formulation of QG that does *not* take this escape strategy, and thus maintains the full symmetry of GR, is called *background independent.*

The divide

Different research directions are oriented by different evaluations given to the general relativistic spacetime conceptual revolution discussed above in Section 4.1. If this conceptual revolution is taken seriously, and understood as a feature of the world that we have learned, the problem of QG becomes the problem of understanding how to define and interpret a background independent QFT. This point of view orients a large part of the research in loop quantum gravity and similar approaches. Not surprisingly, this line of research is more strongly influenced by the GR research tradition.

On the other hand, if the GR conceptual shift is viewed as accidental, the motivation for developing QG comes more from other open problems such as the problem of the unification (see below). One argument often presented for this point of view is that since QG affects microphysics, we can always choose a scale which is sufficiently small to disregard macroscopic curvature effects and sufficiently large to disregard QG effects. At this scale the world is Lorentz invariant. Therefore in QG we can always assume the existence of an asymptotic Lorentz invariant region. This suggests that we can use techniques associated with asymptotic Lorentz invariance. This line of thinking, predominant in the string community, is more influenced by the particle-physics tradition, which is deeply wedded to Poincaré invariance and which has mostly neglected gravity throughout the twentieth century.

The cultural divide is sometimes very strong, in spite of repeated efforts to fill the gap. Both sides feel that the other side is incapable of appreciating something basic and essential: the structure of QFT as it has been understood in half a century of investigation, for the particle-physics side; the novel physical understanding of space and time that has appeared with GR, for the relativity side. Both sides expect that the other's point of view will turn out, at the end of the day, to be not very relevant. One side because GR is only a low energy limit of a much more complex theory, and thus cannot be taken too seriously as an indication about the deep structure of Nature. The other, because the experience with QFT is on a fixed metric spacetime, and thus is irrelevant in a genuinely background independent context.

4.3 The nature of time

Much has been written about the fact that the main equation of nonperturbative QG, namely the Wheeler-DeWitt equation (5) does not contain the time variable t. This presentation of the "problem of time in QG", however, is misleading, since it confuses the aspect of the problem that is specific to QG and the one which is already present in classical GR. Indeed, classical GR can be entirely formulated in the Hamilton-Jacobi formalism in terms of equation (4), where no time variable appears either.

In the classical general-relativistic context, the notion of time differs strongly from the one used in the special-relativistic context (and even more strongly from the one used in the pre-relativistic context). In the pre-relativistic context, following Newton, we assume that there is a universal physical variable t, measured by clocks, such that all physical phenomena can be described in terms of evolution equations in the independent variable t. In the special-relativistic concept, this notion of time is weakened. Clocks do not measure an universal time variable, but rather a proper time elapsed along inertial trajectories. If we fix a Lorentz frame, however, we can still describe all physical phenomena in terms of evolution equations in the independent variable x^0, even though this description hides the covariance of the system.

In the general relativistic context, we must distinguish two kinds of problems, that are often improperly confused. First, we can consider the problem of the dynamics of matter interacting with a given gravitational field, or, equivalently, on a given spacetime geometry. In this case, the fixed gravitational field still determines the value of the proper time τ elapsed along any (timelike) spacetime trajectory, measured by a clock moving along that trajectory. That is, a given gravitational field determines a local notion of time.

A distinct problem is given by the dynamics of the gravitational field itself, or by the interacting dynamics of gravity and matter. In this case, there is no external time variable that can play the role of observable independent evolution variable. The field equations are written in terms of an evolution parameter, which is the time coordinate x^0, but this coordinate, as explained above in section 4.1, does not correspond to anything observable. In general, the proper time τ along spacetime trajectories also cannot be used as an independent variable, as τ is a complicated non-local function of the gravitational field itself. Therefore, properly speaking, GR does not admit a description as a system evolving in terms of an observable time variable. This is particularly evident in the Hamilton-Jacobi formulation (4) of GR. This does not mean that GR lacks predictivity. Simply put, what GR predicts are relations between partial observables, which cannot in general be represented as dependence of dependent variables on a preferred independent time variable.

To be sure, the ontological status of the time variable t is far from being straightforward in Newtonian physics either. In Newtonian physics we describe the world in terms of physical variables $A(t), B(t), \ldots$ evolving in t. One may notice that in a sense we never directly access t, but only physical variables $A, B \ldots$, since the clock devices used to measure t are themselves physical systems with an observable time-dependent variable $C(t)$, such as the position of the clock's hand. Therefore, what we actually observe is always the relative evolution of observable variables $A(C), B(C), A(B) \ldots$ and never t itself. Newton makes this point clearly in the *Principia*, but also observes that the direct mathematicization of the apparent motions $A(C), B(C), A(B) \ldots$ becomes greatly simplified by hypostatizing the existence of t, and expressing all evolution in terms of t. This of course works excellently in the nonrelativistic and nongravitational context. But it is not illogical that Newton's strategy might fail in certain regimes. And in fact it fails in the relativistic gravitational regime, where no universal t can be introduced, and we can only describe the relative dependence of observable quantities. This is what happens in GR.

In a sense, any partial observable variable can be chosen as the independent one in GR. In general, none has the idealized properties assumed by the Newtonian time t, which grows monotonically irrespectively of the state of the system. For instance, in a closed cosmology the volume a of the universe and the proper time t_c since the Big Bang, along a galaxy worldline, are often used as independent variables. But a behaves badly if the Universe begins recontracting, and t_c is only defined in the approximation in which the Universe is assumed to be homogeneous (what is the value of t_c if two galaxies with different proper time from the big bang meet?)

Such a weakening of the notion of time in classical GR is rarely emphasized, because, after all, in classical physics we may disregard the full dynamical structure of the dynamical theory and consider only a single solution of its equations of motion. As mentioned, a single solution of the GR equations of motion determines a spacetime, where a notion of proper time is associated to each timelike worldline. In the quantum context, on the other hand, there is no single spacetime, as there is no trajectory for a quantum particle, and the very concept of time becomes fuzzy.

Attitudes towards the problem of time

Different attitudes can be found in the literature with regard to the problem of time. For technical overviews and references (not completely up to date), see for instance [Isham, 1992; Kuchar, 1992].

As already mentioned, a considerable part of QG research disregards the issue, and maintains that Minkowski space, Poincaré invariance, with its associated notion of time evolution (as a subgroup of the Poincaré group), should not be abandoned in building QG, notwithstanding the features of GR.

Other authors maintain that even if Poincaré invariance is lost in GR and the notion of time becomes more complex, still the idea that the world exists in time, and that its description is the description of systems evolving in time, is a primary notion that we cannot renounce.

Some of these authors have proposed minor modifications of GR, capable of reintroducing a fundamental notion of observable time evolution in the theory. One possibility is to choose a preferred gauge-fixing, in which diffeomorphism invariance is partially broken, and the time coordinate is gauge fixed to be equal to some function of the gravitational field. An example is York time, defined as the trace of the extrinsic curvature of a spacelike surface. Alternatively, the dynamics of GR can be modified, to get a theory with an independent time parameter.

Others accepts in full the challenge presented by GR of trying to conceptualize the world in the absence of a fundamental notion of time and time evolution, as illustrated in the following section.

Physics without space and time?

An illustrative example of how a formulation of mechanics might not use space and time as independent variables is provided by the following proposal (see [Rovelli, 2004], Chapters 4 and 6). Consider a finite spacetime region R bounded by a closed three-dimensional surface Σ. Let (φ, g) represent the value of all fields, including the gravitational field g, on Σ, and let (P_φ, P_g) represent the normal derivative of the fields out of Σ. In principle, all predictions of classical GR can be expressed as constraints on the possible values that the set $(\varphi, g), (P_\varphi, P_g)$ can take. Similarly, in principle all the predictions of QG can be expressed in terms of the probability amplitude $W(\varphi, g)$ of measuring the fields (φ, g). This is a generalization of Feynman's observation that the quantum dynamics of a particle is contained in the propagator $W(x', t'; x, t)$. Diffeomorphism invariance implies that $W(\varphi, g)$ does not depend on the way Σ is imbedded into M. In other words, the entire quantitative spatial and temporal dependence is encoded into the dependence of $W(\varphi, g)$ on the gravitational field g. If, for instance, we identify Σ with the surface of the initial, final and boundary values of a scattering experiment, then it is only the value of the gravitational field on the boundary that determines the time lapses between the initial and final surfaces. Recall indeed that in GR spatial distances and temporal intervals are functions of the gravitational field. $W(\varphi, g)$ can then be used in principle to determine all possible probabilistic predictions regarding the experiment, without using independent spatial or temporal variables.

In order to understand the quantum gravitational field, some of the emphasis on geometry should probably be abandoned. Geometry represents well the classical gravitational field, not quantum spacetime. This is not a betrayal of Einstein's legacy: on the contrary, it is a step in the direction of "relativity" in the precise sense meant by Einstein. The key conceptual difficulty of QG may therefore to find a way to understand the physical world in the absence of the familiar stage of space and time. What might be needed is to free ourselves from the prejudices associated with the habit of thinking of the world as "inhabiting space" and "evolving in time".[2]

Whether it is logically possible to understand the world in the absence of fundamental notions of time and time evolution, and whether this may be consistent with our experience of the world is an open question.

Unitarity

Absence of a fundamental notion of time evolution implies in particular that there is no unitary time evolution in the theory. Absence of unitarity is viewed with great suspicion by many physicists coming from the high energy tradition, where the requirement of unitarity has repeatedly played a major historical role. The argument is often put forward that a probabilistic theory without unitary time evolution is inconsistent. This is not correct, since inconsistency follows from lack of unitarity in the presence of a standard time evolution, and not in the absence of it. If, for instance, we describe the evolution of the universe using the volume of

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[2] If we take this extreme attitude, one problem is to recover the macroscopic notion of time evolution and the specific features of the macroscopic time observable, from an atemporal microscopic theory. It is well known that it is surprisingly hard to pin-point with precision what characterizes the time variable in a dynamical system; on the other hand, the thermodynamical and statistical behavior of physical systems is strongly temporally characterized. Accordingly, the hypothesis has been considered [Rovelli, 1993a; Rovelli, 1993b; Connes and Rovelli, 1994] that "temporal flow" is a feature of the world that appears only in the context of a statistical-thermodynamical description. In other words, "time" could be an artifact of our vaste ignorance of the microstate of the world.

the universe a as independent variable, there is no reason to require the probability for the universe to exist to be unit at all a. Indeed, there is a finite probability that the universe reach only a maximum value of a and then re contracts. On the other hand, the consistency of a probabilistic interpretation of QM in a context in which evolution is not expressed in terms of an external variable t is still unclear.

5 RELATION WITH OTHER OPEN PROBLEMS

In the history of physics, often two open problems have found a common solution. For instance, the problem of understanding the nature of light and the problem of unifying the electric and magnetic theory found a common solution in Maxwell theory. Often, however, the hope to solve two problems at once has been disappointed. For instance, in the sixties the hope was strong to find a theory for the strong interaction and at the same time get rid of renormalization theory; but QCD turned out to be a good solution of the first problem without addressing the second. The problem of QG has been suggested to be related to all sorts of open problems in theoretical physics.

5.1 Unification

The current description of the physical world is composed by a number of field theories: on the one hand GR, on the other hand the standard model which in turn is composed of the electroweak theory and quantum chromodynamics; in addition, fermions are present in several multiplets, and there are the Higgs scalars. The theory has more than a dozen elementary constants. In the wake of the successful unifications of electric and magnetic theory, and then of the electromagnetic and the weak interactions theories, research has long aimed to reduce the complexity of the standard model by providing a single coherent theory governed by a smaller number of elementary constants.

Opinions diverge on the relation between this “unification” problem and the problem of QG. *A priori*, there is no strict reason why the quantum properties of gravity should be understood only in conjunction with the other field theories; the quantum properties of electromagnetism, for instance, have been understood in the context of QED without reference to the other interactions, and so have the properties of the strong interactions.

Some arguments have been proposed to support the idea that the two problems must be solved together. I mention three: the first is speculative and I think weack. The second and third are technical and have some weight. First, there is a widespread expectation that a final “Theory Of Everything” should be at hand today. Historically, however, this expectation has been often present in theoretical physics, and so far always erroneously.

The second argument comes from the early history of the attempts at replacing GR with a renormalizable theory: supergravity has shown that the gravitational ultraviolet divergences are suppressed (although, at the end of the day, not cured) by an appropriate coupling between gravity and matter (a fermion field, in the case of supergravity).

The third argument supporting the relation between the two problems is the following. In the standard model, the coupling constants that determine the strength of the electromagnetic, weak and strong interactions depend on the scale of the phenomena considered. At normal scales, they are widely different in size, but they converge at a scale which is quite close to the Planck scale. This suggests that the scale at which unification might take place should be the same as the scale at which quantum gravitational effects become manifest, indicating that the two phenomena are likely to be related.

Concretely, QG is realized in string theory in the context of a tight unification, while loop quantum gravity proposes a solution of the QG problem unrelated to unification.

5.2 Interpretation of quantum mechanics

In spite of its enormous empirical success and its nowadays ubiquitous applications, QM is a theory which is viewed by many as not yet completely understood. The interpretation of the theory is relatively uncontroversial as long as we use it to describe physical systems interacting with an external system (the "observer") whose quantum properties can be disregarded. But a number of difficulties appear as soon as we take the quantum properties of the observer into account. In the physics community, the attitudes to this problem vary widely, ranging from a complete denial that a problem exists to various proposals for modifying QM in order to solve it. But the number of physicists who consider this a genuine open problem has been increasing in the last decade.

Various arguments have been proposed to tie the problem of the interpretation of QM to the QG problem. One is, once more, the expectation that a final theory might be at hand, and the final theory must be entirely self consistent.

Roger Penrose has proposed a specific mechanism via which quantum linearity might be broken by gravity: gravity might the a physical factor inducing a physically realized wave-function collapse [Penrose, 1986]. The proposal is in principle empirically testable.

In the context of Smolin's and Adler's attempts to derive quantum mechanics from the statistical behavior of a statistical dynamics of matrix models [Smolin, 2002; Adler, 2004], the suggestion has been made that one might seek a common origin for both gravitation and quantum field theory at a common deeper level of physical phenomena from which quantum field theory emerges.

Finally, the suggestion has been made that the relational aspect of spatiotemporal structure revealed by GR could be connected with the relational aspect of QM emphasized by the "relational" interpretations of QM [Rovelli, 1996b]. The first is determined by the relation of contiguity between systems; the second by the interaction between systems. But on the one hand locality implies that interaction happens only between contiguous systems, and on the other hand contiguity is only manifest via a physical interaction, suggesting a strict connection between the two relations. These ideas however, have not been developed beyond the stage of suggestions.

5.3 The cosmological constant

An elusive aspect of the current description of the universe is given by the cosmological constant, a constant introduced by Einstein, describing a long range gravitational coupling that can modify gravity at large distances. This constant plays a major role in cosmology, in QFT (where quantum field theoretical effects tend to make it unrealistically large), and recent cosmological observations seem to indicate that its value is very small but not, as previously expected, vanishing.

Although nothing clear has so far appeared in QG research concerning this constant, it must be noted that Raphael Sorkin's QG theory predicted a small value of the constant with the correct of order of magnitude, before its observation, as mentioned in Section 2.1.7.

5.4 Quantum cosmology

"Quantum cosmology" indicates the study of the Universe as a whole as a quantum system. There are two distinct problems that go under this name.

The first is the quantum version of the modelling of the dynamics of our Universe: in particular, the study of the quantum features of the dynamical systems obtained under the drastic simplification that the Universe is homogenous. These classical models, such as the Friedmann-Robertson-Walker model, play an important role in cosmology and are believed to give a good description of the large scale features of our Universe. Their quantization is of interest on several grounds. First, it provides a simplified framework in which many of the conceptual difficulties of QG can be examined and solutions can be tested. Second, they can be used to study what a quantum theory of gravity could us concerning the physics near and at the Big Bang itself, where quantum gravitational effects are expected to dominate.

The study of these models has been started in the sixties by Bryce DeWitt [DeWitt, 1967b; DeWitt, 1967c] and Charles Misner [Misner, 1969] and has seen a great development in the following decades. The limitation of these models, of course, is that they are based on the freezing of all the infinite numbers of degrees of freedom of GR, except for a finite number of them, and therefore they miss the entire field theoretical aspect of the QG problem.

A string cosmology has been developed by Gabriele Veneziano and collaborators, with the hope of finding observational consequences of string theory [Gasperini and Veneziano, 1993]. The application of loop quantum gravity to quantum cosmology ("loop cosmology") has recently led to a model which is finite and well-behaved at the initial singularity [Bojowald, 2001; Bojowald and Morales-Tecotl, 2006].

The second problem that goes under the name of “quantum cosmology” is the conceptual problem of describing a quantum system that forms the entire universe, and therefore for which there is no “external” observer: i.e., the study of quantum mechanics in the case in which the observer is inside the system.

This second problem is very loosely related to the problem of quantum gravity. It is true that it is impossible to be “external” with respect to the gravitational field, but one should not confuse “external” in the spatiotemporal sense with “external” in the dynamical sense. One cannot be “external” with respect to the electromagnetic field either, in the spatiotemporal sense; but we can nevertheless consider an electromagnetic system, viewed as a quantum system, interacting with an external system, viewed as a classical observer. The same can be done for a gravitational system. Therefore nothing *a priori* prevents us from using the standard Copenhagen interpretation of QM (whether or not this is satisfactory) in the context of QG. In other words, the problem of QG and this second problem of quantum cosmology are not necessarily related.

On the other hand, the difficulties raised by considering the observer as part of the system and the difficulties generated in QM by diffeomorphism invariance, in particular the absence of an external time, are of a similar nature, and both question the viability of the Copenhagen interpretation. A general scheme for addressing both kinds of difficulties, and defining a generalized formalism for QM, where there is no external time and no external observer, has been developed by Jim Hartle [Hartle, 1995].

6 CONCLUSION

After 70 years of research, there is no consensus, no established theory, and no QG theory has yet received any direct or indirect experimental support. In the course of 70 years, many ideas have been explored, fashions have come and gone, the discovery of the Holy Grail of QG has been several times announced, only to be later greeted by much scorn.

However, in spite of this, research in QG has not been meandering meaninglessly. On the contrary, a consistent logic has guided the development of the research, from the early formulation of the problem and the research directions in the fifties to nowadays. The implementation of the programs has been laborious, but has been achieved. Difficulties have appeared, and solutions have been proposed, which, after much difficulty, have lead to the realization, at least partial, of the initial hopes.

It was suggested in the early seventies that GR could perhaps be seen as the low energy limit of a theory without uncontrollable divergences; today, 30 years later, such a theory — string theory — is known. In 1957 Charles Misner indicated that in the canonical framework one should be able to compute eigenvalues; and in 1995, 37 years later, eigenvalues were computed — within loop quantum gravity. Much remains to be understood and some of the current developments might lead nowhere. We are not at the end of the road, we are only half-way through the woods. But looking at the entire development of the subject, it is difficult to deny that there has been substantial progress.

The progress cannot be just technical. The search for a quantum theory of gravity raises again old questions such as: What is space? What is time? What is the meaning of "being somewhere"? What is the meaning of "moving"? Is motion to be defined with respect to objects or with respect to space? Can we formulate physics without referring to time or to spacetime? And also: What is causality? What is the role of the observer in physics?

Questions of this kind have played a central role in periods of major advances in physics. For instance, they played a central role for Einstein, Heisenberg, Bohr and their colleagues. But also for Descartes, Galileo, Newton and their contemporaries, and for Faraday, Maxwell and their colleagues. Today some physicists view this manner of posing problems as "too philosophical". Most physicists of the second half of the twentieth century, indeed, have viewed questions of this nature as irrelevant. This view was appropriate for the problems they were facing. When the basics are clear and the issue is problem-solving within a given conceptual scheme, there is no reason to worry about foundations: a pragmatic approach is the most effective one. Today the kind of difficulties that fundamental physics faces has changed. To understand quantum spacetime, physics has to return, once more, to those foundational issues. We have to find new answers to the old foundational questions. The new answers have to take into account what we have learned with QM and GR. The problem of QG will probably not be solved unless these questions are carefully reconsidered.

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BIBLIOGRAPHICAL NOTE

For more details on the history of QG see the historical appendix in [Rovelli, 2004]; and, for early history see [Stachel, 1999b; Stachel, 1999a] and [Gorelik, 1992]. For orientation on current research on QG, see the review papers [Horowitz, 2000; Carlip, 2001; Isham, 1991; Rovelli, 1998a]. An interesting panorama of points of view on the problem and on philosophical issues it raises is in the various contributions to the book [Callender and Huggett, 2001]. See also the discussion in [Rovelli, 1997; Rovelli, 2000]. As a general introduction to QG ideas, see the old classic reviews, which are rich in ideas and present different points of view, such as John Wheeler 1967 [Wheeler, 1968], Steven Weinberg 1979 [Weinberg, 1979], Stephen Hawking 1979 and 1980 [Hawking, 1979; Hawking, 1984], Karel Kuchar 1980 [Kuchar, 1984], and Chris Isham's magisterial syntheses [Isham, 1984a; Isham, 1984b; Isham, 1997]. On string theory, classic textbooks are Green, Schwarz and Witten, and

Polchinksi [Green *et al.*, 1987; Polchinski, 1998]. On loop QG, see [Rovelli, 1998b; Rovelli, 2004]. For a discussion of the difficulties of string theory and a comparison of the results of strings and loops, see [Rovelli, 2003], written in the form of a dialogue, and [Smolin, 2003]. Smolin's popular book [Smolin, 2000] provides a readable introduction to QG.

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