THE RELATIVITY OF SIMULTANEITY: A CRITICAL ANALYSIS

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The purpose of this paper is to analyze the doctrine of the relativity of simultaneity, with respect both to its basic assumptions and to its observational foundations. To this end, it will be necessary to consider the logic of the argument, and to examine the relation of the doctrine to the principle of the constancy of the velocity of light, to the Lorentz transformation, and to the empirical evidence of the Michelson-Morley experiment.

1. Einstein's Technical Account of the Relativity of Simultaneity.

Einstein opens his paper, « Zur Elektrodynamik bewegter Körper », with a discussion of simultaneity. He points out that « all of our judgments, in which time plays a role, are always judgments about simultaneous events [1] ». Thus the statement that an event occurs at a certain time means that the event occurs simultaneously with the arrival of the hands of a clock at some particular position on the dial. If the event occurs in the neighborhood of the clock, no difficulty is experienced in thus establishing its time. But let two events occur at the points A and B, widely separated in space, and let each event be timed by a clock in its immediate vicinity. The times of the two events can then be compared only if some method is first established whereby to determine that the clocks are themselves synchronous. Einstein finds that such a method can be established if, by definition, the « time » which light signals take to travel from A to B is required to equal that which they take to travel from B to A. Let a light signal be sent from the clock at A at the time t_1, be reflected from the clock at B

[1] *Annalen der Physik* (1905), p. 893.

at the time t_2, and be received back at A at the time t_3. The two clocks are synchronous if $t_2 - t_1 = t_3 - t_2$; or, what amounts to the same thing, if

$$t_2 = \frac{t_1 + t_3}{2}.$$

It is, of course, assumed that the clocks here are not in motion. They are clocks placed in what Einstein calls the « stationary system ». Einstein notes that time must necessarily be defined by means of stationary clocks in the stationary system [1].

Now imagine the following situation. A rigid platform whose length, as measured in the stationary system, is L, moves relatively to the stationary system with a velocity v, in the direction AB. Clocks placed at the points A and B in the stationary system are synchronized by means of the above criterion, while clocks placed at the end-points of the moving platform, at A* and B*, are so regulated that when either of them coincides with one of the clocks fixed in the stationary system, the hands of the coincident clocks will have identical positions upon their respective dials. Observers located at A and B, who know the clocks in the stationary system to be synchronous, will therefore conclude that the clocks fixed at the end-points of the moving platform, at A* and B*, are likewise synchronous. However, observers on the moving platform who attempt to apply the criterion for synchronization directly to the clocks at A* and B*, will necessarily conclude otherwise. For let a light signal be sent from A* at the time t_1, be reflected from the clock at B* at the time t_2, and be received back at A* at the time t_3. Taking the length of the platform to be L, as measured in the stationary system, and remembering that light has a constant velocity c in the stationary system, it follows that

$$t_2 - t_1 = \frac{L}{c - v},$$

$$t_3 - t_2 = \frac{L}{c + v}.$$

Clearly, $t_2 - t_1 \neq t_3 - t_2$.

[1] *Annalen der Physik* (1905), p. 894.

Here the character of the argument must be made clear. It is assumed that there is but one time system, namely, that of the stationary system. These equations, therefore, represent time intervals between the indicated events as determined from the standpoint of observers in the stationary system. Einstein does not take these to be transformation equations. Hence he leaves the quantities unstarred, indicating thereby that they belong to the stationary system. Nevertheless, Einstein does hold that it is just these relations which underlie and determine the judgment made by observers on the moving platform, namely, the judgment that the clocks fixed at A* and B* are not synchronous.

Einstein himself carries the argument for the relativity of simultaneity only up to this point. However, the argument should be carried further. Solving the above equations simultaneously for t_2, and putting $\gamma = 1/(1 - v^2/c^2)^{1/2}$, it follows that

$$t_2 = \frac{t_1 + t_3}{2} + \frac{v}{c^2} L\gamma^2.$$

Thus the instant of reflection is shown to be subsequent to, rather than simultaneous with, the instant midway between the start and the return of the light signal. But observers on the moving platform must assume that the time required for a light signal to travel from A* to B* is the same as that required for it to travel

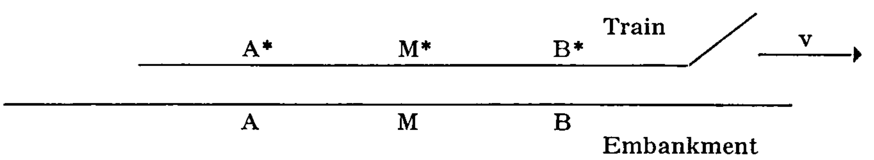

Figure 1

from B* back to A*, and that the clocks fixed at A* and B* are synchronized only if they satisfy the relation, $t_2 = (t_1 + t_3)/2$. Since the instant of reflection, as recorded by the clock at B*, is actually later than this by the amount given above, the observers on the moving platform must therefore conclude that the clock at B* is ahead of that at A*. That is to say, they must conclude that the two clocks do not strike a given hour simultaneously, but that the clock at B* strikes first, and then the clock at A*. [1]

2. Einstein's Popular Exposition of the Relativity of Simultaneity.

Any two events are simultaneous for a given observer if, occurring at equal distances from him, they are perceived by him at the same time. However, assume that lightning bolts have struck at two points, A and B, along a straight stretch of railway embankment, and at the corresponding points, A* and B*, on a train running at a constant velocity v along the embankment, in the direction AB. Let the lightning flashes be simultaneous for an observer located at M on the railway embankment, midway between A and B. Then these same flashes must appear to be successive, and not simultaneous, for an observer who is located at M* in the train, midway between A* and B*, but whose position at the instant—as judged from the embankment—that the lightning

[1] The essentials of this argument can, perhaps with advantage, be put in slightly different, and more concrete, form. Assume that clocks must be synchronized by means of light signals, and in accord with the criterion given above. Let a signal be sent from the clock at A at 1 o'clock, be reflected from B, and then be received back at A at 5 o'clock. Since $(1 + 5)/2 = 3$, it follows that observers must assume the clocks to be synchronized (granted sameness of rate) if, at the instant of reflection of the light signal at B, the clock located at B reads 3 o'clock. It is clear that this result does not depend in any sense upon the magnitude of the velocity of the signals, but only upon the fact that the signals have the same velocity from A to B as from B to A.

Now assume that the clocks have been found to be synchronous in the above test. But let the synchronization be checked by another set of observers. Let these observers send out their signal from A at 2 o'clock, and let them receive the signal back at 8 o'clock. However, assume that the signal used in this check has actually a variable velocity—a fact of which the observers are of course completely ignorant—and that the signal requires 4 hours elapsed time to travel from A to B, but only 2 hours to travel back from B to A. Applying the standard formula for the synchronization of clocks—and assuming, as they must, that the signal has a constant velocity—these observers will therefore conclude that the clocks at A and B are synchronized if, at the instant of reflection of the signal at B, the clock there reads $(2 + 8) / 2 = 5$ o'clock. Actually they will find, however, that at the instant of reflection of the signal, the clock at B reads 6 o'clock. Hence they will conclude that the clocks are not synchronous, but that the clock at B is ahead of that at A.

bolts struck, coincided with that of the observer at M on the embankment. As Einstein remarks with respect to the observer located in the train:

« Now in reality (considered with reference to the railway embankment) he is hastening towards the beam of light emitted from B, whilst he is riding on ahead of the beam of light coming from A. Hence the observer will see the beam of light emitted from B earlier than he will see that emitted from A. Observers who take the railway train as their reference-body must therefore come to the conclusion that the lightning flash B took place earlier than the lightning flash A [1]. »

This is Einstein's simplified argument for the relativity of simultaneity. The reference to the observer in the train as rushing toward the one beam of light while moving away from the other, is perhaps unfortunate [2]. However, a proper interpretation of the situation along the railway tracks has been given by Peter G. Bergmann [3]. Thus, Bergmann notes that the lightning bolts strike simultaneously—at least as determined from the standpoint of the embankment—at A, A* and B, B*. Light rays from these two bolts reach M simultaneously, but only after the lapse of a finite interval of time. Hence by the time the rays arrive at M, M* has moved on a certain distance. Hence too, the light rays which reach M* must travel over unequal distances, as measured along the embankment; and in so far as light has a constant velocity c relative to the embankment, these rays must therefore be seen successively at M*.

[1] Albert EINSTEIN, *Relativity: The Special and the General Theory* (New York, 1920), pp. 31-32.

[2] This point has been made by Andrew Paul Ushenko, « Einstein's Influence on Contemporary Philosophy », in *Albert Einstein: Philosopher-Scientist* (Evanston, III, 1949), p. 616. Ushenko notes that « the explanation is given in terms which would be entirely acceptable to a classical physicist, and therefore fail to make clear the new position ». In particular, Ushenko objects to Einstein's seeming implication « that the real reason why the passenger must see the two flashes in succession is his rushing towards one and away from the other ».

[3] *Introduction to the Theory of Relativity* (New York, 1947), p. 31. This book was written while Bergmann was in close association with Einstein, and presumably reflects Einstein's own considered judgment.

The argument as thus set forth is qualitative. However, it can be formulated in quantitative terms. The lightning bolts strike at A, A* and B, B*, on embankment and train. These are single events; and from the standpoint of the embankment, they take place simultaneously, or at the time t_0. Now let the point M* in the train coincide with the point M on the embankment at the instant t_0, and let the flashes produced by the bolts as they strike at the coincident points A, A* and B, B*, be seen at M* in the train at the times, t_1 and t_2, respectively. Taking the distance between the points A* and B* to be L, as measured in the system of the embankment, it follows that

$$t_2 - t_0 = \frac{L/2}{c + v},$$
$$t_1 - t_0 = \frac{L/2}{c - v}.$$

Subtracting,

$$t_2 - t_1 = - \frac{v}{c^2} L \gamma^2.$$

The negative sign indicates that the lightning bolt which strikes at B, B* is seen by the observer at M* in the train prior to the bolt which strikes at A, A*.

The remainder of the argument may be omitted here, since it is similar to that in the technical account, already given. Besides, the character of the argument will be analyzed in detail in the next section. However, it should be noted that the result just gotten is the same as that derived from the technical account. Since the two results derive also from the same fundamental premise, namely, that the velocity of light combines with the velocity of a moving system in the manner, $c - v$ and $c + v$, it follows, therefore, that Einstein's two accounts of the relativity of simultaneity are identical in all essential respects.

3. The Character of the Argument.

As far back as 1922, the French philosopher, Henri Bergson, charged that Einstein, in his popular account of the relativity of simultaneity, had presupposed the existence of a privileged frame of reference, and that he had adopted, therefore, an anti-relativistic position.

In defense of Einstein, it must be pointed out that Bergson's charge is true only in a limited, and perhaps superficial, sense. Whereas classical physics had posited the ether as an absolute frame of reference, Einstein denies to any frame of reference a permanently privileged status [1]. In the given case, he argues the relativity of simultaneity from the standpoint of the railway embankment. However, this is a mere matter of choice. He could with equal right have argued the same conclusions from the standpoint of the train. Thus, taking the embankment as the stationary system, that is to say, as the system in which light has the constant velocity c, then from the standpoint of the embankment, light rays must move relatively to the train with the velocities $c - v$ and $c + v$; and conversely, taking the train as the stationary system, within which light has the constant velocity c, then from the standpoint of the train light rays must move relatively to the embankment with the velocities $c - v$ and $c + v$. In the first case, flashes seen simultaneously at M will be seen successively at M*, whereas in the second case, flashes simultaneous for M* will be successive for M. In short, the laws of physics are assumed to be the same, regardless of the frame of reference chosen. Certainly this represents a relativistic viewpoint.

Nevertheless, Einstein's mode of argument requires careful consideration. The argument, as already seen, is in two parts. To recapitulate: (1) The observer on the embankment sees the train move with a velocity v toward the one flash, and away from the other flash, the observer himself being so located along the embankment that he sees the two flashes simultaneously. It is therefore concluded that the observer in the train must actually see the flashes, not simultaneously, but successively. For the velocity of light being a constant c in the system of the embank-

[1] « The introduction of a 'luminiferous ether', » Einstein points out, « will prove to be superfluous, in so far as the view here developed will not require an [absolutely stationary space] endowed with special properties. » « Zur Elektrodynamik bewegter Körper », *op. cit.*, p. 892.

ment, it follows—from the standpoint of the embankment—that light rays from the flashes at A, A* and B, B* will overtake the observer at M* in the train only after the time lapses, $L / 2(c-v)$ and $L / 2(c+v)$, respectively; whence, as a consequence of this discrepancy, it must be inferred that the observer at M* in the train sees the flash from B, B* before he sees that from A, A*.

But (2) just as the observer at M takes the embankment to be his proper frame of reference, within which light has the constant velocity c, so too the observer at M* may—and indeed, must—take the train as his frame of reference. Hence for this observer, just as for the observer on the railway embankment, the velocity of light is a constant c; and in so far as he (the observer in the train) sees the flash which strikes at B, B* before he sees that which strikes at A, A*, yet finds these flashes to have occurred, in his own system, at points equidistant from him, he necessarily concludes that the one flash took place earlier than the other. What occurs simultaneously for the observer at M, occurs successively for the observer at M* in the moving train.

Now it is clear that Einstein has framed the first part of this argument from the standpoint of a single system—in this case, the system of the railway embankment. But on what grounds does Einstein thus assume that inferences, however legitimately drawn from the standpoint of one system, are necessarily binding upon the actual perceptions of observers located in the other system? Clearly, his assumption must be based upon some notion of causal dependence, or physical determinism, between events occurring in the two systems. Thus one may hold that it is a single light ray which issues from the flash at A, A*, and which moves along both the embankment and the train. In like manner, it is a single light ray which issues from the flash at B, B*, and which passes along both embankment and train. Now if these light rays from A, A* and B, B* arrive simultaneously at M, they are ascribed the constant velocity c in the system of the embankment; in which case, they must then be ascribed, from the standpoint of the embankment, the velocities $c - v$ and $c + v$ relative to the train. On the other hand, if the light rays from A, A* and

B, B* happen to meet simultaneously at M*, then they must be ascribed the velocity c in the system of the train, and, from the standpoint of the train, the velocities $c - v$ and $c + v$, relative to the embankment. Under no circumstances, however, will it be possible for light rays thus conceived to issue from A A*, and B, B*, and to converge simultaneously at both M and M*.

Here a certain point must be made clear. Einstein has in no way committed himself, in this argument, either with regard to the manner in which the light rays are really propagated, or with regard to the order in which they actually issue from the points A, A* and B, B*. Let the light rays be propagated in any manner, and let them issue in any order, such that they arrive simultaneously at M. Then from the standpoint of the embankment, these light rays must be ascribed the constant velocity c, relative to the embankment, and consequently the velocities $c - v$ and $c + v$, relative to the train. So too, it must then be held, from the standpoint of the embankment, that the light rays issued from the point A, A* and B, B*, simultaneously with respect to the embankment, and successively with regard to the train. Similarly, if light rays are propagated in such manner, and issue from the points A, A* and B, B* in such order, that they converge simultaneously at M*, then from the standpoint of the train, they must be ascribed the velocity c relative to the train, and consequently the velocities $c - v$ and $c + v$, relative to the embankment. In which case it must be held, from the standpoint of the train, that the light rays issued from the points A, A* and B, B*, simultaneously in the system of the train, and successively in the system of the embankment.

Now this account obviously serves to render plausible the initial part of the above argument. Nevertheless, the resort to physical reasoning serves to raise certain rather serious difficulties. For the second part of the argument turns on the assertion that, from the standpoint of each observer, and relative to the observer's own system, the velocity of light is the constant c. Yet if, as was argued in the first part, light behaves consistently in such manner that the velocity c ascribed to a given light ray in one system entails the ascription of the velocity $c - v$, or $c + v$, to that same light ray relative to the other system, and if observations carried out in both systems are always consistent with these ascriptions, then it is difficult to understand how observers in both systems can be convinced, and on empirical grounds, that the velocity of light, in their respective systems, is always the constant c.

Actually, Einstein's mode of argument here is complex; and in the final analysis, it is probably illicit. Einstein first takes the embankment to be the stationary system, within which alone light is propagated with the constant velocity c, and in terms of which all physical relations are to be determined. But Einstein also accords to the train, at least for the observer at M*, the status of a stationary system. Hence for the observer in the train, just as for the observer on the railway embankment, the velocity of light is the constant c; and in so far as the observer in the train sees the flash which occurs at B, B* before he sees that which occurs at A, A*, yet finds the points A* and B* to be equal distant from him, this observer must necessarily conclude that the one flash took place earlier than the other.

However, in thus shifting from the hypothesis that the embankment alone is the stationary system, to the hypothesis that the train is also a stationary system, Einstein has retained a consequence dependent entirely upon his first hypothesis, to wit, he has retained the conclusion that the observer in the train does in fact perceive the flashes successively rather than simultaneously. For it was only from the standpoint of the embankment, taken as the stationary system, and with the train taken as the moving system, that the observer in the train was said to see the flashes successively. Nevertheless, it is just this conclusion, coupled with the further hypothesis that the train is itself a stationary system, which yields the result that simultaneity is relative.

But this mode of argument is illicit. Einstein can make any assumption that he wants, and he is at liberty to destroy any assumption by making a contrary assumption. But then he is not at liberty to retain in his final conclusion any part of the original assumption so destroyed. To do so is simply to shift hypotheses in the middle of the argument.

4. Simultaneity and the Principle of the Constancy of the Velocity of Light.

As seen in the last section, the argument that simultaneity is relative hinges almost entirely upon the principle that the velocity of light is a universal constant *c*. However, this principle is in no sense simple, and its full meaning and implications must now be analyzed.

In his paper of 1905, Einstein states the principle of the constancy of the velocity of light in these words: « Each light ray moves in the 'stationary system' of coordinates with the determinate velocity *c*, independent of whether the light ray has been emitted from a stationary or from a moving body [1]. » That is to say, the velocity of light does not compound in any way with the velocity of its source. However, this statement of the principle of the constancy of the velocity of light in no way satisfies the full meaning and scope of that principle as actually used in Special Relativity. On the contrary, the real meaning of the principle lies in the assertion that the velocity of light is always the same, whether measured in a given stationary system, or measured with respect to some frame or platform moving relatively to the stationary system. Thus Einstein speaks of « expressing in equations, that light (as required by the principle of the constancy of the velocity of light in conjunction with the principle of relativity) is also propagated with the velocity *c* when measured in the moving system [2] ».

Although Einstein has nowhere deduced, nor even expressly formulated, this more general principle of the constancy of the velocity of light, its deduction is easily carried out. Consider, for instance, a source of light and a target, located some distance apart from one another in a stationary system. If a light ray is sent out from the source, it must arrive at the target with the velocity *c*. Indeed, this will be the case whether the source of light is at rest relative to the target, or is in motion either toward or away from

[1] « Zur Elektrodynamik bewegter Körper, » *op. cit.*, p. 895.

[2] *Ibid.*, p. 899. The principle of relativity is stated by Einstein in this manner: « The laws in accord with which the states of physical systems change, are independent of whether these changes of state be referred to one or to the other of two systems of coordinates moving, relative to each other, with uniform, translatory motion » (*Ibid.*, p. 895).

the target. But now let the target itself be given a velocity v, as measured in the stationary system, either toward or away from the source of light. From the standpoint of the stationary system, the light ray must now overtake the target with a relative velocity $c - v$, or $c + v$. However, from the standpoint of the target itself, it is the source of light, and not the target, which is in motion. For the target constitutes a frame of reference, or stationary system, in terms of which motion may be measured and determined. Since the velocity of light, relative to the target, is independent of any motion of its source, it follows, therefore, that the light ray must again arrive at the target with the velocity c.

Certainly it is evident that this represents a radical extension of the principle of the constancy of the velocity of light. By this new principle—which will be referred to as the general principle of the constancy of the velocity of light, in contrast to the original principle which was limited, or restricted, in scope—the velocity of light is in a sense absolute. Indeed, as Einstein himself remarks, in relativity theory, « the velocity of light... plays the role, physically, of an infinite velocity [1] ». The arguments for the relativity of simultaneity assume, of course, that the velocity of light combines vectorially with the velocity of any moving platform, or frame of reference. Nevertheless, if the velocity of light is in any sense infinite, as Einstein asserts to be the case, then certainly such a velocity cannot combine vectorially with the velocity of any material body.

Here it must be noted that the general principle of the constancy of the velocity of light can be illustrated, perhaps with advantage, from the relativistic law for the transformation of velocities. Let two material bodies move with velocities v_1 and v_2, and with the same sense, relative to a stationary system. By classical principles, the relative velocity between these two bodies should be $v^* = v_1 - v_2$. But in Special Relativity, the relative velocity between any two such bodies is somewhat greater than this simple difference, namely,

$$v^* = \frac{v_1 - v_2}{1 - \frac{v_1 v_2}{c^2}}.$$

[1] « Zur Elektrodynamik bewegter Körper, » *op. cit.*, p. 903.

Now this equation expresses in an interesting and instructive manner the full meaning of the constancy of the velocity of light. Let the velocity v_2 be that of the train, as measured relative to the embankment, and for v_1 substitute the velocity of light c, likewise as measured relative to the embankment. Then by this law, the light ray from the flash at A, A* will overtake the observer at M* in the train with a relative velocity $v^* = c$, whereas the light ray from the flash at B, B* will meet him with a relative velocity $v^* = -c$. Light rays which actually impinge upon the senses, or the instruments, of an observer can have, relative to that observer, only the constant velocity c.

The implication which the general principle of the constancy of the velocity of light has for simultaneity, is easily shown. In the railway train experiment, let the one lightning bolt strike at A, A* at the time t_{F_1}, and let the other bolt strike at B, B* at the time t_{F_2}. Now let the observer at M on the embankment see the flash which occurs at A, A* at the time t_1, and the flash at B, B* at the time t_2. Similarly, let the observer at M* in the train see these same flashes at the times t_1^* and t^*, respectively. Since light must have, in all systems of reference, the constant velocity c, the time intervals required for the light rays from A, A* and B, B* to reach the observers at M and M* will be,

$$t_1 - t_{F_1} = AM/c,$$
$$t_2 - t_{F_2} = BM/c;$$

and,

$$t_1^* - t_{F_1} = A^*M^*/c,$$
$$t_2^* - t_{F_2} = B^*M^*/c,$$

respectively. But $AM = BM$ and $A^*M^* = B^*M^*$. Therefore,

$$t_2 - t_1 = t_2^* - t_1^*.$$

Set $t_2 - t_1 = 0$. That is to say, let the flashes from the two bolts be perceived simultaneously at M. Then $t_2^* - t_1^* = 0$, and the bolts are likewise perceived simultaneously at M*.

Now in his contention that simultaneity is relative, Einstein intends, apparently, to make a statement of fact, namely, an assertion about how the observer at M* in the train will actually see the two lightning flashes. But if this is his intention, why then does he base his argument upon a provisional, restricted principle of the constancy of the velocity of light, rather than upon the final, general principle? Of course, had he based his argument upon this latter principle, it would have turned out that the two lightning bolts, if observed simultaneously at M, are also observed simultaneously at M* in the train.

5. Simultaneity and the Lorentz Transformation.

The principle of the constancy of the velocity of light, taken in its full sense, leads to the conclusion that two events, occurring at widely separated places in a given system, and simultaneously for observers in that system, are likewise simultaneous for observers moving with uniform velocity relative to the given system. However, it must now be shown that application of the Lorentz transformation leads to this same conclusion.

Consider again the situation involved in Einstein's technical argument for the relativity of simultaneity. Let the length of the platform, as measured in the stationary system, be L, and let the platform move with a velocity v, likewise as measured in the stationary system. If, therefore, a light signal leaves the point A* at the time t_1, is reflected from the point B* at the time t_2, and is received back at A* at the time t_3, the times required for the signal's journey, out and back, will be

$$t_2 - t_1 = \frac{L}{c - v},$$

and

$$t_3 - t_2 = \frac{L}{c + v},$$

respectively. This, of course, is exactly the time discrepancy used by Einstein to argue the relativity of simultaneity. However, this calculation is but the first step in a relativistic analysis of the given situation. Why, then, stop here?

Now assume that the light signal was sent out at the instant A* occupied, in the stationary system, the coordinate position x_1, and that the signal was reflected from B* at the instant B* occupied a coordinate position x_2, such that

$$x_2 - x_1 = v(t_2 - t_1) + L.$$

Furthermore, assume that the light signal returned to A* at the instant A* occupied the coordinate position x_3, such that

$$x_3 - x_2 = v(t_3 - t_2) - L.$$

From these data, taken entirely in terms of the stationary system, it is now possible to determine, in the system of the moving platform, the instant of reflection of the light signal at B*. Substitute the above values into the Lorentz transformation equations,

$$t_2^* - t_1^* = \gamma\left[(t_2 - t_1) - \frac{v}{c^2}(x_2 - x_1)\right],$$

$$t_3^* - t_2^* = \gamma\left[(t_3 - t_2) - \frac{v}{c^2}(x_3 - x_2)\right],$$

so as to eliminate the time intervals and coordinate differences on the right. Now put $L^* = L\gamma$, as required by the Fitzgerald-Lorentz contraction in the length of the moving platform, and the result is $t_2^* - t_1^* = t_3^* - t_2^* = L^*/c$; whence,

$$t_2^* = \frac{t_1^* + t_3^*}{2}.$$

The instant of reflection of the light signal at B*, as it occurs in the system of the platform, is therefore midway between the time that the signal was sent out from A*, and the time that it returned to A*.

Now consider the situation directly from the standpoint of the observers on the moving platform. For these observers, the length of the platform must be L*, and for them, as for observers in the stationary system, the velocity of light is the constant c. Hence for the observers on the moving platform, the times required for a light signal to travel out and back, between the points A* and B*, must be

$$t_2^* - t_1^* = L^*/c.,$$

and

$$t_3^* - t_2^* = L^*/c.$$

Therefore, the observers on the moving platform will assume that clocks located at A* and B* are synchronized if they satisfy the relation, $t_2^* = (t_1^* + t_3^*)/2$. Since the instant of reflection of the light signal at B*, as calculated from the Lorentz transformation, is identical with this relation, it follows that the criterion of synchronization necessarily assumed by observers on the moving platform, is in accord with the actual, physical situation as given by the Lorentz transformation, and is therefore justified.

The technical and the popular arguments for the relativity of simultaneity are essentially the same. However, it may be well here to carry through the analysis in terms also of the railway train experiment, and to do so in a somewhat concrete manner.

Let the flashes from A, A* and B, B* be perceived simultaneously at M, midway between A and B on the embankment. For convenience, take the velocity of the train, relative to the embankment, to be $v = (3/5)\ c$, with c taken to be unity, and let the distance between the points A* and B*, as measured in the system of the embankment, be $L = 16$. The times required for light rays from A, A* and B, B* to reach M*, will therefore be,

$$\Delta t_1 = \frac{L/2}{c - v} = \frac{8}{1 - 3/5} = 20,$$

and

$$\Delta t_2 = \frac{L/2}{c + v} = \frac{8}{1 + 3/5} = 5,$$

respectively.

Now at the instant ($t = 0$, in the system of the embankment) that the light rays leave A, A* and B, B*, let M* in the train be coincident with M, midway between the points A and B on the embankment. But M* is moving away from A and toward B. Hence by the time the light ray from A, A* reaches the observer at M* in the train, the point M* will have been displaced from A by the amount,

$$\Delta x_1 = v\Delta t_1 + L/2 = (3/5)\ 20 + 8 = 20.$$

Similarly, by the time the light ray from B, B* arrives at M*, M* will be displaced from B by the amount,

$$\Delta x_2 = v\Delta t_2 - L/2 = (3/5)\ 5 - 8 = -5,$$

the negative sign merely serving to indicate that M* is to the left of B, as viewed from the embankment.

These data pertain entirely to measurements made in the system of the embankment. However, they may be transformed into the system of the train by means of the Lorentz transformation. Applying the transformation equation for time,

$$\Delta t_1^* = \gamma(\Delta t_1 - \frac{v}{c^2}\Delta x_1) = \frac{20 - (3/5)\ 20}{4/5} = 10,$$

and

$$\Delta t_2^* = \gamma(\Delta t_2 - \frac{v}{c^2}\Delta x_2) = \frac{5 - (3/5)\ (-5)}{4/5} = 10.$$

Thus $\Delta t_1^* = \Delta t_2^* = 10$. The times required for the light rays to reach M*, as transformed into the system of the train, are equal. If, therefore, the flashes from A, A* and B, B* are perceived simultaneously at M, then by the Lorentz transformation they must likewise be simultaneous for the observer at M* in the train.

As should be expected, this result is exactly the same as that got by applying directly to the system of the train the general principle of the constancy of the velocity of light. For the distance between the points A* and B*, as measured in the system of the embankment, is L = 16. But this is a measurement determined in accord with the Fitzgerald-Lorentz contraction in the length of the moving train, namely, $L = L^*/\gamma$. Hence the distance between the points A* and B*, as measured in the system of the train itself, must be $L^* = L\gamma = 16(5/4) = 20$. Now take the velocity of light to be the constant c, as measured in the system of the train. The times required for light rays to travel from A, A* and B, B* to M*, will therefore be,

$$\Delta t_1^* = \frac{L^*/2}{c} = 10,$$

and

$$\Delta t_2^* = \frac{L^*/2}{c} = 10,$$

respectively.

In short, if an observer at M on the embankment sees the flashes simultaneously, then on the basis of data taken in the system of the embankment, and by application of the Lorentz transformation, it must be concluded that the observer at M* in the train will likewise see the flashes simultaneously. This result, we now see, is in exact accord with what the observer at M* in the train should experience, assuming that the light rays travel over equal distances relative to the train, and at the constant velocity c.

6. Simultaneity and the Michelson-Morley Experiment.

The arguments for the relativity of simultaneity rest upon ideal experiments; nevertheless, when Einstein concludes, as in the railway train experiment, that it is impossible for both observers, at M and M*, to see the two lightning bolts simultaneously, he probably intends this to be a statement of physical fact. However, it is easily shown that Einstein's line of reasoning is closely analogous to that which led Michelson and Morley, in their celebrated experiment, to expect a positive result. Hence the question of physical fact can here be determined, at least with substantial probability, on the basis of analogy.

Consider the situation diagrammed in Fig. 2. It is assumed that the lengths, AB, A*B*, and A*C*, are all equal in the sense that a given measuring rod, if applied successively to each, will give the same result. Furthermore, it is assumed that light rays are propagated only in the system K, which is the stationary system, and that they have there the constant velocity c, in all directions.

Now let the system K* move with uniform velocity v, in the direction AB, and let the following events occur. At the instant A* coincides with A, let a flash of light be emitted from these coincident points, the light rays from this flash traveling out to the points B* and C*, and thence being reflected back to A*. Also, at the

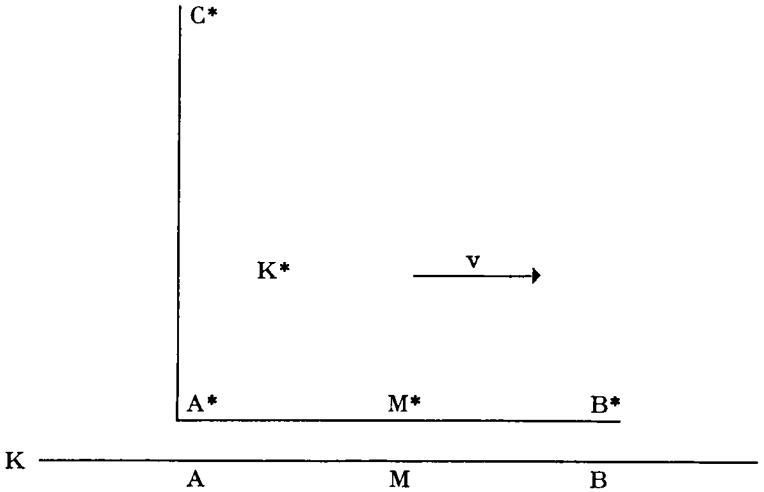

Figure 2

instant B* coincides with B, let a flash be emitted from these coincident points, and let this flash occur at a time such that it will be seen at M simultaneously with the flash from A, A*.

Now in so far as a light ray must travel, relative to the system K, a greater distance in going from A* to B* and back to A*, than in going from A* to C* and back to A*, the one trip should require a longer time than the other. This was of course the confident expectation of Michelson and Morley. In like manner, in so far as light rays, in order to reach M*, must travel, relative to the system K, a greater distance from the flash at A, A* than from that at B, B*, the one flash should be seen at M* after the other. This, of course, is the argument of Einstein. However, in the case of Michelson and Morley, the expectation was not fulfilled in experiment. Light rays were found to travel out and back along the rods A*B* and A*C* in the same time, exactly as though the rods were at rest. But this being the case, why then assume that the result anticipated in Einstein's railway train experiment would nevertheless be borne out in actual experience? Why assume that light rays which are indifferent to the motion of both rods, would nevertheless traverse the rod A*B* in a manner appropriate to its motion? Why assume that light rays which travel the courses A*B*A* and A*C*A* in the same time, would nevertheless require a greater time in order to travel from A* to M* than to pass from B* to M*?

The analogy between the reasoning involved in the Michelson-Morley experiment, and that involved in Einstein's ideal experiment along the railway tracks, is nearly perfect. If, therefore, it is the case that Einstein has actually demonstrated the relativity of simultaneity—if, that is to say, he has actually demonstrated that the observer at M* must necessarily *see* the flashes successively rather than simultaneously—then by a like necessity the Michelson-Morley result must have been positive rather than negative. But here it must be remembered that the Michelson-Morley result amounted to a single fact, namely, the fact that light rays require one and the same time in order to travel out and back over either arm of the interferometer.

7. Summary and Conclusion.

The results of this analysis of simultaneity can now be briefly summarized.

Einstein's two accounts of the relativity of simultaneity have been analyzed, and his popular exposition has been found to be consistent in every respect with his carefully reasoned technical account. The charge often made that Einstein misspoke himself in his popular work, is therefore without foundation.

However, it has been shown that the argument for the relativity of simultaneity is illicit, in that it depends upon a covert shift in hypotheses. Furthermore, it has been shown that the doctrine violates the principle of the constancy of the velocity of light, taken in its full, or general, sense; that it is contradicted by results obtained through application of the Lorentz transformation; and that it disregards the empirical evidence afforded by the Michelson-Morley experiment.

There is a final point which should be noted. The relativity of simultaneity is argued by Einstein at the very beginning of his paper of 1905. It is there argued in terms of the concepts available, such as the limited, provisional principle of the constancy of the velocity of light. One might say that it has somewhat the status of a premise in the deduction of the Lorentz transformation. But both the general principle of the constancy of the velocity of light and the Lorentz transformation itself, imply that simultaneity is absolute, and not relative. Certainly there is nothing illogical in this, since a false proposition can always entail a true one. Nevertheless, it is a curious fact that what is really a mere first step in the course of the argument has been consistently taken to be its conclusion, while the final conclusion to be drawn from the argument, namely, that simultaneity is absolute, has been overlooked, if not actually denied.

Résumé

Le raisonnement selon lequel la simultanéité est relative du point de vue de l'observateur est fondamental dans la Relativité restreinte. Pourtant, l'analyse de ce raisonnement mène aux critiques suivantes :

1. Le raisonnement appartient à la physique classique plutôt qu'à la physique relativiste, en ce qu'il adopte la conception classique de la propagation de la lumière et ignore le principe de la constance de la vitesse de la lumière.
2. Il entre en conflit avec l'évidence empirique, car son hypothèse de base et son mode de raisonnement sont presque exactement analogues à ceux qui amenèrent Michelson et Morlay à attendre un résultat positif lors de leur expérience d'interférence.
3. Il est logiquement inacceptable, car il repose à la fois sur l'affirmation et sur la négation d'un système de référence privilégié.

Bien plus, on montre que l'application correcte des transformations de Lorentz mène non pas à la conclusion que la simultanéité est relative, mais plutôt à celle qu'elle est absolue.

Abstract

The argument that simultaneity is relative to the viewpoint of an observer, is fundamental to Special Relativity. However, analysis of this argument leads to the following strictures: (1) the argument belongs to

classical physics rather than to relativity physics, in that it adheres to the classical view of the propagation of light, and ignores altogether the relativistic principle of the constancy of the velocity of light ; (2) the argument conflicts with empirical evidence, in that its basic assumption and mode of reasoning are almost exactly analogous to those which led Michelson and Morley to expect a positive result from their interferometer experiment ; (3) the argument is logically untenable, in that it rests on both the affirmation, and the denial, of a privileged frame of reference. Furthermore, it is shown that proper application of the Lorentz transformation, in actually transforming data from one system to another, must lead, not to the conclusion that simultaneity is relative, but rather to the conclusion that it is absolute.