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The Concept of Energy and Its Early Historical Development

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The concept of energy, the premier concept of physics and indeed of all science, is here investigated from the standpoint of its early historical origin and the philosophical implications thereof. The fundamental assumption is made that the root of the concept is the notion of invariance or constancy in the midst of change. Salient points in the development of this idea are presented from ancient times up to the publication of Lagrange's Mécanique Analytique *(1788).*

1. INTRODUCTION

Of all the concepts or constructs of physics, energy, by its unifying capacity, has proved by all odds to be the most significant and successful. Its domain of application has indeed by now far transcended physics and covers all branches of science. Not only has it played a major role in the logical development of physics itself, but by common consent it is the physical construct which has proved to contain the greatest meaning for all aspects of human life. Under the misnomer "power," it is the stock in trade of the engineer and that which makes the wheels of the world go round. More and more, it is recognized by economists as the real wealth of nations. The interpretation of phenomena in terms of the transfer of energy from one place to another and the transformation of energy from one form to another is the most powerful single tool in human understanding of experience.

The impact of the concept of energy on society has been enormous in the past and will be even greater in the future. What is the nature of this impact? It has both ideological and technological aspects. The ideological influence consists largely in the fact that the concept serves as a unifying element in all scientific descriptions of experience, enabling all scientists to think more effectively about their various problems and thus promoting the fundamental unity of science. As knowledge of nature becomes more specialized, this role of energy becomes of increasing significance.

The technological aspect of the impact of the concept of energy on society scarcely needs emphasis. It is necessary only to remind ourselves of the stupendous increase in the average number of energy "slaves" per head of population on the earth in the last quarter century. This has correspondingly increased the well-being and comfort of many millions. At the same time, progress along this line has not been devoid of serious sociological problems. The energy supply available for transformation has not been well distributed, and many segments of the earth's population are going without their fair share. Moreover, even in those nations in which the energy supply available for human needs has vastly increased, this has been accompanied by unpleasant by-products like water and air pollution. To solve these problems will, of course, involve further skillful application of the energy idea, so that of its impact there appears to be veritably no end.

A concept like energy obviously has had a history. One cannot hope really to understand its present state or its future implications without some appreciation of this history. Closely associated with the historical development there is, moreover, the evaluation of what may be called the philosophical significance of the concept. The two aspects are strictly speaking inseparable.

It is the aim of the present essay to take a look at the origins and early development of the energy idea. This examination will be undertaken in the light of certain assumptions which are of essentially philosophical nature, namely, that the basis of the concept of energy as we use it today is the idea of *invariance*, which here means constancy in the midst of change. We think in this connection of what we now call the mechanical energy of a system of mass particles subject only to their mutual interactions: this quantity is a function of the velocities and positions of the particles (in some inertial reference frame) that stays *constant in time*, no matter what the motions of the particles may be.

Definitely implied in our procedure is the conviction that unless we can find in earlier notions a connection with the way we look upon the concept of energy today our search will be illusory. Of course, we must face the fact that not all scientists may agree that the notion of invariance in the midst of change is the key idea in energy. An example of an opponent of the idea is Ernst Mach,(1) who vigorously expressed the opinion that the actual root of the energy concept is to be found in the principle of the impossibility of perpetual motion. Mach was a searching critic of the philosophy and history of science and his views are entitled to great respect. Leaving aside the fact, however, that in any case the principle of the impossibility of perpetual motion is closely associated logically with the idea of invariance, we may note that Mach's extreme view, if followed, would have prevented the generalization of the idea of energy to all physical phenomena. For example, Mach would not accept the mechanical theory of heat. His polemic against it almost rivals in intensity his attack on the atomic theory. It seems clear that his positivistic leanings prevented him from seeing any advantage in imaginative scientific theorizing. He could hardly have become a successful theoretical physicist in the sense of Maxwell, Boltzmann, Gibbs, and their twentieth century successors.

2. ROOTS OF THE CONCEPT OF ENERGY IN ANTIQUITY. THE PHILOSOPHERS

Most scientific concepts are not easy to trace historically. Energy provides no exception. One plausible source of the idea is connected with the invention of machines, an important technological development in the life of early man. People early learned the social significance of the fact that human life is impossible without somebody's labor, but rather naturally sought to reduce the terrific burden of this labor. Eventually, some clever and imaginative folk discovered the possibility of taking the sting out of human labor by the use of such devices as the lever, the inclined plane, and various forms of pulley systems. These gadgets, which we now call simple machines, must have seemed to the ancients to be endowed with almost magical powers, they made it so much easier to raise heavy weights, for example, or to give an arrow greater speed, as by the use of the bow.

The discoverers and users of such machines must have observed very early, however, that the mechanical advantage provided by them is always accompanied by a compensating disadvantage: nature is not inclined to give something for nothing. It was found, for example, that to raise a given weight by applying to a pulley system a force much less than the weight, the speed with which the pulley rope is pulled must be much greater than the speed with which the weight is raised. Alternatively, if one wishes to pull with low speed, the time needed for raising the weight is correspondingly increased. With the gain in ease of exertion in the performance of a given bit of labor provided by the machine there goes an inevitable loss of something represented in general by an increase in the time required to do the job. This fact was recognized explicitly in the writings on mechanics of Hero of Alexandria,[2] who flourished around 60 A.D. This peculiar principle of compensation, in which a certain gain in a vital effect is always balanced by a corresponding loss in an associated phenomenon, contained within itself the root of the concept of energy. The compensatory factor so evident in the behavior of machines implies that something stays constant in the midst of the obvious changes that take place in the operation of the machine. It is this constant "something" which later became quantified as energy.

At this point, we are tempted to look into Greek philosophy to see whether we can locate any reference to the general idea of constancy in the midst of change. As a matter of fact, it is there, though whether any Greek before Aristotle ever associated it with the behavior of machines is problematical. We can find what we are looking for in the alleged views of the two pre-Socratic philosophers Parmenides of Elea and Heraclitus of Ephesus (both of approximately the 6th century B.C.). Heraclitus is supposed to have taught that "all things flow" (panta rhei), or all is change. He was clearly impressed by the ever-changing flux of sensation characterizing our experience. Much of modern science is consistent with this point of view, as is shown in our concern for the changing behavior of physical systems with the passage of time. But acceptance of Heraclitus' idea in its extreme form would make all science a hopeless discipline, since we could never get a mental grip on anything before it became something else. As a matter of fact, some commentators on Heraclitus hold the view that in spite of his emphasis on the primacy of change, he also held that there is something invariant in the universe as a whole. This something he apparently took for *fire*, though he obviously did not mean fire in a modern sense, nor even in the ancient Greek practical sense. It was some ethereal essence which could be transformed into the common objects of our experience without net loss.

Parmenides comes definitely closer to the idea of constancy in the midst of change. Impressed (or possibly depressed) by the apparently chaotic sequence of events in human experience, he decided to treat change as merely an illusion. He felt that this is what men try to do when they invent names for things and so identify them continually throughout the flux of sensation. There is a strong human urge to extract from experience something that "stays put" long enough for effective observation and study; and this Parmenides emphasized. To be sure, his writings are fragmentary, and there is the obvious danger of reading too much into them. Nevertheless, the notion of invariance in the midst of change is there. If we seek an ancient patron saint of the concept of energy, it will surely be Parmenides.

Let us now return to machines and see what relevance to the concept of energy we can extract from the Greek attempts to explain their action.

It was Aristotle (384–322 B.C.) who wrote the first treatise on physics in the Western tradition. But this famous treatise *Physica*,[3] though it pays extensive attention to motion, says nothing about machines. However, there does exist a treatise attributed by some authorities to Aristotle, though others, including Marshall Clagett,[4] the well-known historian of mechanics, believe that the treatise was written by one of Aristotle's immediate successors. In the Latin version variously styled *Mechanica*, *Problemata Mechanica*, or *Quaestiones Mechanicae*, it may well be the first extant treatise on mechanics. At any rate, it contains probably the first attempt in Western science to explain how machines work. From the standpoint of the problem of the origin of the concept of energy, the importance of this treatise is that its treatment is based on a dynamical approach, in sharp contrast to the static method favored later by Euclid and Archimedes.

According to Pierre Duhem,[5] the author of *Mechanica* used the basic axiom taken from Aristotle's *Physica*: The "force" (*puissance* in French) exterted by the mover who moves a body is measured by the weight of the body and the velocity of the impressed motion. On this view, when the same "force" acts, the impressed velocity will be inversely proportional to the weight. If we represent velocity by V and weight by W, and "force" by F, Duhem expresses the content of the above axiom in the modern form:

$$F = kVW \tag{1}$$

where k is some constant. We may note in passing that the Greeks would not have used this form of expression, since they preferred always to use pure numbers in expressing mathematical relationships.

They would have expressed the content of the axiom in the form

$$V_1/V_2 = W_2/W_1 \tag{2}$$

In any case, in modern physical terminology, if F is taken as the equivalent of what we now call force, Eq. (1) makes no sense. However, it could agree with modern physics if F is interpreted as *power* or the time rate of doing work, k being set equal to unity.

In the application of Eq. (1) to the behavior of a lever with weights W_1 and W_2 suspended from the ends of the weightless lever bar at distances l_1 and l_2 respectively from the fulcrum C, the further assumption is made in *Mechanica* that when the same "force" acts, the point of the lever *further* from the fulcrum C moves with *greater* velocity. The author convinced himself of this from the geometrical properties of the circle. But this is equivalent to the relations

$$V_1 = kl_1, \qquad V_2 = kl_2 \tag{3}$$

If these are combined with (1) or (2), the result is

$$l_1W_1 = l_2W_2 \tag{4}$$

which is the law of the lever. With VW treated as power rather than "force," the above "proof" is equivalent to that based on the modern principle of virtual velocities or virtual work. Of course, this amounts to reading into the Aristotelian treatment more than is actually there. This, however, is a fairly common procedure among historians of science. That the author of *Mechanica* preferred the dynamical method of establishing the law of the lever is significant. He evidently was impressed by the fact that *something* stays the *same* at both ends of the lever, in spite of the different weights.

These considerations gain in significance with respect to the origin of the concept of energy when we reflect that the explanation of the law of the lever by Archimedes, the greatest physicist in antiquity, proceeded on quite different lines. Archimedes shunned motion in his theoretical investigations and provided a "proof" based entirely on static equilibrium considerations. His method therefore sheds no light on the idea of energy.

3. THE MIDDLES AGES

Modern scholarship has shown that during the Middle Ages in Western Europe there was a great deal of interest in the attempt to explain the behavior of machines. Most of this was in the Aristotelian tradition. We shall not discuss it here, but merely call attention to the detailed studies by Hiebert,[6] Clagett,[4] and Moody and Clagett.[7]

3.1. Stevinus and Galileo

In looking for vestiges of the concept of constancy in the midst of change during the late 16th and early 17th centuries, we are confronted by two men, both of whom devoted much attention to the behavior of machines and endeavored to understand them from different points of view. The first was the famous Flemish engineer Simon Stevin (1548–1620), better known as Stevinus, and the second his contemporary, the even greater physicist, Galileo Galilei (1564–1642).

Stevinus was definitely a disciple of Archimedes rather than Aristotle. In his two great works, *De Beghinselen der Weeghconst* (Leiden, 1586) and *Hypomnemata Mathematica* (1608),[8] he showed complete disagreement with the Aristotelian method of understanding the behavior of a machine. He says, "The reason for the equilibrium of a lever does not reside at all in the arcs of the circle which its extremities describe." We have just seen that this motion was precisely the basis of the treatment in *Mechanica.* Disagreement could not have been more complete.

It is Stevinus' handling of the inclined plane that provides his chief claim to fame in the field of the operation of machines. His method here has a definite connection with the energy concept, since it makes use of the assumption of the impossibility of perpetual motion starting from rest. His famous scheme, of which he was so proud, imagines 14 equal balls fastened together in a single loop with inextensible strings of negligible mass and length and draped over two inclined planes of the same height placed back to back. One of the planes accommodates four of the balls on its surface and the other, of half the length, permits two balls to rest on it. The other eight balls hang symmetrically below the planes. Stevinus employs the logical principle of the excluded middle class to assume that the balls either start to move or do not move. But if they move at all, they must move indefinitely and this would be perpetual motion, which Stevinus discards as impossible. Hence, he concludes (after cutting off the eight balls hanging below the planes on the ground that they contribute nothing to the problem because of symmetry) that the balls on the plane must be in equilibrium. Therefore, the weight that can be supported on any plane is directly proportional to the length of the plane. This is essentially the law of the inclined plane as a machine. Stevinus was undoubtedly lucky in his specific set up. We are more concerned here, however, with his strong adherence to the idea of the impossibility of perpetual motion. He was probably familiar with the earlier views on this subject of Leonardo da Vinci[9] (1452–1519) and Girolamo Cardano[10] (1501–1576). There is no doubt these earlier scientists were convinced that it is not possible in terrestrial phenomena to get something for nothing, which would be what would happen if motion were to start by itself and persist indefinitely. This is indeed tied in with the modern energy concept and might well serve as an epigrammatic version of the general principle of conservation of energy or the first law of thermodynamics.

It seems clear from an examination of the writings of Galileo that he fully grasped the significance of the compensatory factor in the operation of machines, which we now interpret in the light of the invariance involved in the concept of energy. By the time Galileo turned his attention to machines, the laws governing their behavior were rather well known. There is, curiously enough, no record that Galileo was familiar with the work of Stevinus, at any rate at the time when Galileo prepared his university lectures which led to the book *On Mechanics*[11] (first published in Italian in 1649, after the death of the author).

In the book just referred to, Galileo shows himself even more aware than his Aristotelian predecessor of the element of compensation involved in the action of a machine. In the very beginning, he comments on how so many mechanicians are deceived into thinking their machines can accomplish operations which are impossible. Quoting directly from the English translation of the book:

> "These deceptions appear to me to have their principal cause in the belief these craftsmen have and continue to hold of being able to raise very great weights with a small force, as if with their machines they could cheat nature whose instinct—nay, whose most firm constitution—is that no resistance may be overcome by a force that is not more powerful than it. How false such a belief is I hope to make most evident with true and rigorous demonstrations that we shall have as we go along."

This is not a completely clear and unequivocal statement, but taken in conjunction with what follows it seems to emphasize Galileo's grasp of the fundamental fact that in machines one cannot get something for nothing. A little later in the section from which the above quotation has been taken, he elucidates more extensively:

> "Now assigning any determined resistance [he means here the force to be exerted or the weight to be raised by the machine] and delimiting any force [he means here the *applied* force] there is no doubt that the given weight will be conducted by the given force to the given distance; for even though the [applied] force may be very small, by dividing the weight into many particles of which each shall not remain superior to the [applied] force and transferring them one at a time, the whole weight will finally be conducted to the appointed place; nor may it reasonably be said at the end of the operation that the great weight has been moved and translated by a force lesser than itself but rather by a force which has many times repeated that motion and space which will have been traversed only once by the whole weight. From which it appears that the speed of the force has been greater than the resistance of the weight [here the translator has followed Galileo in an illogical statement, for a speed cannot logically be compared with a resistance; what Galileo must have meant was the speed of the resistance of the weight] by as many times as that weight is greater than the force, since in the time in which the moving force has repeatedly traversed the interval between the endpoints of the motion, the thing moved has passed over this by a single time."

One is entitled to assume from this phraseology that Galileo grasped the essence of the principle of virtual velocities or virtual work. He felt so strongly the validity of this point of view that he repeated essentially the same statements on the next page of his treatise. He continually emphasized that though a machine does possess a decided mechanical advantage, it is only at the expense of the time required for it to carry out its function.

We pass over Galileo's attempts to explain the behavior of the lever and the inclined plane. In many ways he comes closest to an invariance concept in his famous pendulum experiment, devised in order to provide an experimental basis for his fundamental assumption that when a ball falls from rest at a given height from the ground, the velocity on arriving at the ground depends only on the height and is independent of the path of fall. It is not necessary to repeat the details here, as they are clearly set forth in Galileo's *Dialogues Concerning Two New Sciences.*[12] The important thing to note is Galileo's grasp of the fact that in spite of the different paths there is something which remains constant. It must have impressed the author of the ingenious experiment. Today we interpret it in terms of the invariant maximum potential energy associated with fall from a given height independently of path and time of descent.

Galileo's interest in pendulum experiments, as exemplified in the case just discussed, was undoubtedly stimulated by his very early discovery, as a young man, in the Cathedral of Pisa, of the isochronism of the small oscillations of a pendulum.

4. CONSERVATION IDEAS IN THE 17TH AND 18TH CENTURIES. DESCARTES, LEIBNIZ, AND LAGRANGE'S *MÉCANIQUE ANALYTIQUE*

After the death of Galileo and as the 17th century wore on, emphasis on the idea of conservation in physics became more marked. René Descartes (1596–1650) in France made much of it, particularly in connection with the laws of impact of bodies. His studies of these phenomena led him to what we now call the principle of the conservation of momentum or what he called conservation of quantity of motion. Descartes[13] was so impressed with this principle that he was led to the general assertion that the total momentum of the Universe is constant. He finally concluded that the proper measure of force as the entity responsible for the production of motion is the change in momentum per unit time. This view may well have had an influence on Newton when he came to systematize mechanics in his *Principia*.

Gottfried Wilhelm Leibniz (1646–1716) disagreed with the point of view of Descartes. In the year 1686, he published in the *Acta Eruditorum* (Leipzig) a brief paper[14] in which he termed the theory of Descartes a "perversion" of mechanics. He convinced himself that the "true" measure of the efficacy of a force is the product of the mass and the square of the velocity, which he termed the "*vis viva*" or "living" force, as contrasted to the "*vis mortua*" or "dead" force of statics. His argument, put in simple terms, is as follows. He imagines two masses m and $4m$. The first is assumed to be dropped from rest at the height $4h$ and the second a height h from the ground. Leibniz assumes that each mass in falling will acquire what he calls the "force" necessary to enable it to rise again to the same height. That is, the "force" involved in the fall of mass m through $4h$ will be sufficient to carry this mass up again to where it started and leave it there at rest, neglecting any friction or other resistance. But Leibniz also assumes that the same "force" is necessary to lift the mass m through the height $4h$ as to lift the mass $4m$ through the height h. We see that this is essentially treating the word "force" here as equivalent to "work" in the modern physical sense. Now, this clearly entails the result that the same "force" is involved in the fall of m through $4h$ as is involved in the fall of $4m$ through h. But the quantities of motion, in the Cartesian sense, acquired in these two falls are not the same; from the law of falling bodies, m in falling through $4h$ acquires a velocity twice as great as that which $4m$ acquires in falling through h. If we call the latter velocity v, the quantity of motion or momentum acquired by m in its fall is $m(2v)$, while that acquired by $4m$ is $4m(v)$, or twice as much. So, says Leibniz, there is no conservation of quantity of motion in this case and hence in general we should not speak of this kind of conservation. The problem remains, "What, if anything, *is* conserved here?" To Leibniz this is simple: It is the product of the mass times the square of the velocity acquired. For then, in the example under consideration $m(2v)^2 = 4mv^2$. This quantity Leibniz felt deserved a special name, and he called it the *vis viva*. It is, of course, related to what later became known in the 19th century as the *kinetic energy*, being twice the latter.

This difference in the views of Descartes and Leibniz gave rise to a celebrated controversy, which raged in scientific circles for some half a century. D'Alembert (1717–1783) felt he had finally solved it when he published his famous *Traité de Dynamique*[15] in 1743. Here, he emphasized that the apparently conflicting viewpoints are due essentially to a confusion in terminology, and that they can be readily reconciled by appropriate definitions. Descartes' concept of force involves assuming that the efficacy of a force is measured by its effect over time, or, as we should now express it, by the time integral of the force. But this is just the change in the momentum of the particle acted on by the force; to illustrate for a particle of mass m, from Newton's law of motion,

$$F = d(mv)/dt \tag{5}$$

and

$$\int_{t_0}^{t_1} F\,dt = (mv)_1 - (mv)_0 \tag{6}$$

where the right-hand side in (6) is the difference between the momentum values at the instants t_0 and t_1 between which the force is assumed to act.

On the other hand, as D'Alembert pointed out, it is perfectly possible to measure the efficacy of a force by its effect over space, and this is essentially what Leibniz had in mind. In modern notation (for the special case of the motion of a single particle along the x axis) we arrive at

$$\int_{x_1}^{x_2} F\,dx = (\tfrac{1}{2}mv^2)_1 - (\tfrac{1}{2}mv^2)_0 \tag{7}$$

or, in words, the cumulative effect of force over distance (the left side, which we now term the *work* done by the force), is equal to the change in the quantity $\frac{1}{2}mv^2$ between the two positions x_1 and x_0, brought about by the action of the force. We now call $\frac{1}{2}mv^2$ the *kinetic energy* of the particle and the equation (7) is known as the work-kinetic energy theorem.

It is of interest to note that there is now some doubt whether D'Alembert should be considered to have definitely settled the *momentum* vs. *vis viva* controversy. A recent historian of physics, Laudan,[16] has pointed out that historical evidence shows that arguments over the "true" measure of force continued long after 1743, and that many well-known writers on the subject made no mention of D'Alembert in their discussions. It seems that the 19th century writers who credited D'Alembert with the solution of the controversy did so because they were more familiar with his numerous accomplishments in mathematics and mechanics as well as his treatise on dynamics than with the works of his contemporaries and successors. The fact remains that D'Alembert did set forth the general argument that modern physics has found satisfactory.

A claim has been made in behalf of Christian Huygens (1629–1695) that he introduced the idea of *vis viva* before Leibniz. It is true that in his famous work

Horologium Oscillatorium[17] (1673), he discussed the compound pendulum and in his treatment he used effectively the product of mass times the square of the velocity for the various parts making up the pendulum. But nowhere did he single out this quantity for special attention or speak of it as a possible measure of the efficacy of a force, much less baptize it with a name to emphasize its significance in terms of invariance and conservation. It was later commentators who read the *vis viva* interpretation into Huygens' proof of the law of the compound pendulum.

D'Alembert was obviously impressed with the importance of the *vis viva* concept and devoted to it the final chapter of his *Traité de Dynamique*.[15] He entitled this chapter, "On the principle of the conservation of living force." He first states this for the perfectly elastic collisions of particles, in which it has the following form: When a number of particles collide elastically, the sum of the products of each mass times the square of its velocity remains constant. This is a true conservation law. D'Alembert does not deduce it. The modern deduction depends on the treatment of collisions by means of Newton's coefficient of restitution and the equating of this coefficient to unity to correspond to perfect elasticity. D'Alembert generalizes the principle to apply to a collection of particles held together by rigid connections, i.e., forming effectively a rigid body. If such a collection moves in such a way that no "accelerating force" (as he calls it) acts on any particle, the total *vis viva* remains constant, irrespective of the motions of the individual particles. Again, he does not demonstrate this, though he illustrates it with a number of special cases, based on the use of his well-known principle governing the motion of systems of particles subject to constraints. It corresponds of course to highly idealized and not very practical situations.

D'Alembert does indeed also discuss the case in which the masses of a system are acted on by accelerating forces and shows that then the total *vis viva* does *not* remain constant, but that the change in it is equal to the "effect" of the forces, which in modern terminology is the same as the work done by the forces. This result is equivalent to the work-kinetic energy theorem in modern mechanics. From the standpoint of our present concern, however, it is significant that nowhere deos D'Alembert interpret his result in terms of the conservation of a quantity made up of the sum of the *vis viva* and another quantity depending on the relative positions of the particles of the system. The value of introducing the notion of *potential energy* and hence the concept of the total mechanical energy was not at that time appreciated, though the germ of the idea was certainly there.

A closer approach to the energy construct as we employ it today is found in the famous treatise by Lagrange, *Mécanique Analytique*,[18] first published in 1788. One of the greatest landmarks in the history of physics, this constituted a systematic presentation of the science of mechanics from a mathematical point of view. In it, the author presented his celebrated method of generalized coordinates and derived the equations which still bear his name. In a chapter devoted to *vis viva* (or *force vive* in French), he finally showed in explicit fashion that in certain cases it is possible to set up a function of the coordinates of a system of particles which, when added to the *vis viva* of the system, yields a quantity constant in time. Of course, he does not call this the mechanical energy of the system, nor does he use the term energy anywhere in his treatise. Actually, he refers to the result as an example of the conservation of *vis* *viva*, for reasons which are not clear, since in the case he discusses the *vis viva* will certainly change in general as time passes. At any rate, the equation he writes corresponds to what we now call the energy equation for a dynamical system. In fact, Lagrange later recognizes this sort of equation as a first integral of the equations of motion. This certainly marks an epoch in the realization of the existence and availability of a unifying concept in the study of dynamical systems, though the time was not yet ripe for its complete exploitation. One reason for this may well have been the realization that the setting up of a first integral of the equations of motion was not always or even in general practical for terrestial dynamical systems.

From this point, the story of the evolution of the energy concept moves in the direction of other physical phenomena, notably heat.

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