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"Superconducting" Causal Nets

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The world is described as a relativistic quantum neural net with a quantum condensation akin to superconductivity. The sole dynamical variable is an operator representing immediate causal connection. The net enjoys a quantum principle of equivalence implying local Lorentz $SL(2, C)$ invariance and causality. The past-future asymmetry of its cell is similar to that of the neutrino. A net phase transition is expected at temperatures on the order of the W mass rather than the Planck mass, and near gravitational singularities.

1. INTRODUCTION

Physical time space is the space of events provided with temporal structure. This structure has been variously described by a chronometric tensor $g = (g_{mn}(x))$, or a spin form $\sigma = (\sigma_{\dot{A}Bm}(x))$, or a *causal relation* $x\mathbf{C}y$ ("x is causal to y") which tells when x is causally prior to y. Here, following up a suggestion of Sorkin 1987, it is described by a *causal net*, a set of events provided with a relation $x\mathbf{c}y$ ("x is connected to y") which means that x is an immediate causal antecedent to y.

The idea that local Lorentz invariance is a macroscopic quantum phenomenon has been aired, more or less seriously, at meetings on quantum time space structure for a decade and more. We can be more specific now: The world seems to be a quantum condensation of bosonlike event-pairs in a causal net of fermionic events. The spin form σ is a macroscopic ψ-vector of such pairs.

Net theory is a response to the singularities and infinities of field theories, including the Schwarzschild solution to Einstein's gravitational equations. That solution has two singularities, an outer one at $r = 2m$ and an inner one at $r = 0$. The outer singularity is merely the theory's way of telling us that we have laid on inappropriate coordinates, and in more appropriate coordinates it becomes a one-way membrane, permitting either

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inward or outward flow but not both. The inner singularity, however, is the theory's way of telling us that it is out of its depth. A deeper theory is needed to describe time space right at particles or singularities.

It was at first hoped that the one-way membrane, either of the point particle or of a gravitational topological charge, would be useful not only in astrophysics, but, because it breaks past-future symmetry T, also in particle physics, for the T-breaking weak interactions (Finkelstein, 1958). Astrophysical applications galore have materialized since the pioneering works of Penrose (1965) and Ruffini and Wheeler (1970), but not particle ones. In the first place, the one-way membrane is spherical, certainly an inappropriate starting point for a model of a spin-$\frac{1}{2}$ lepton. In the second place, Kruskal (1960) provides a T-symmetric universe of two sheets, combining an inward membrane with its primordial T-image, an outward one. Finally, Bekenstein (1973) and Hawking (1975) make it clear that small one-way membranes that are still large enough for semiclassical radiation theory are disastrously unstable. Only a quantum theory of gravity can tell whether there are smaller, yet more stable quantum versions of the classically unstable one-way membrane, as there are of classically unstable atoms.

This question, while still unanswered, is less compelling now, for according to the deeper net theory that is our response to the inner singularity, there is a more fundamental T asymmetry present at every event, and more relevant to particle physics; not the one-way *membrane*, which is contingent and global, but one-way *connections*, which are necessary and local, and constitute the world itself. Even in quantum nets whose cells have exact Lorentz SL_2 invariance, the elementary cell inevitably lacks T symmetry, being in fact described by a chiral spinor, and for the theory to survive, the T symmetry exhibited by the net at ordinary temperatures must arise from a coherent quantum condensation, as must the manifold topology, including dimensionality, and the unitary metric of the Hilbert space of the usual quantum theory.

The hypothecated "superconducting" phase of the net recalls most the transparent crystalline medium of enormous stiffness called *ether* by Isaac Newton; but Newton's ether, composed of tachyons, defines a rest frame, while this is a relativistic quantum ether and does not.

The primordial hot, disordered "normal" net phase that condenses into ether thus recalls both Chaos the father of Time and the *tohu v' bohu* (void and formless?) of the second verse of *Genesis*. Chaos being already a much used term, we may call the uncondensed phase *tohu*.

If we identify the T violation of the causal cell with that of the weak interactions, then the net would be expected to make a transition from "superconducting" to "normal," that is, from ether to tohu, at a temperature that may be as low as 100 GeV ($\sim 10^{15}$ K).

This makes ether a high-temperature "superconductor" by some standards, but not those of particle physics. Presumably, the continuum approximation seems to work down to the Planck length even though the causal cell is much larger, because the quantum condensation enormously suppresses incoherent net effects, as in superconductivity.

The dimensionality n of a cell in the quantum net, which is also the number of inputs to the cell, is a quantum number with nonnegative integer spectrum. We expect ether to have a definite dimensionality, restricted to some of the eigenvalues

$$n = N^2 = 0, 1, 4, 9, 16, \ldots$$

Tohu lacks such long-range order, but still has causal structure.

In order to maintain a correspondence of meanings between our various descriptions of the world, we move from manifolds to quantum time net in 6 steps, maintaining causality, locality, and local Lorentz invariance as we go. In succession we (1) causalize, (2) atomize, (3) algebrize, (4) quantize, (5) bosonize, and finally (6) condense, the structure of the world. The first four have been part of this program since 1965; the last two enter the program here for the first time.

1.1. Step 1: Causalize

That is, take the causal structure of the world as the unifying variable.

Einstein, following Riemann, does not causalize, but metrizes. The causal reformulation of Einstein's theory occurs in or is strongly suggested by the work of Robb (1936), Alexandroff (1956), Zeeman (1964), Kronheimer and Penrose (1965), Pimenov (1968), Finkelstein (1969), Latzer (1972), and Bombelli *et al.* (1987), among others. Bombelli *et al.* (1987) propose that time space is a *causal set*: "a locally finite set of points endowed with a partial order corresponding to the macroscopic relation that defines past and future." This partial order $x\mathbf{C}y$ is the *causal relation* $\mathbf{C}$.

Causal structure was first proposed to describe gravity alone. Kaluza, however, suggested that electromagnetism arises from a higher-dimensional Riemannian manifold or hyperspace enveloping our four-dimensional time space, a theory extended to quanta by Klein, topologized by Einstein and Bergmann, generalized to all gauge forces by deWitt (1964), and currently revived in string theory. Thus, it may be possible to express all forces, not only gravitational, in terms of causal structure.

My own previous attempts at a quantum theory of time start from the four-dimensional Minkowsian time space manifold M^4. But M^4 alone does not point clearly enough to the underlying quantum theory. Now, supported by the Kaluza hypothesis, we make a theory of variable dimensionality n, regarding n as a physical order parameter for time space, somewhat as it is for water, which may exist in volumes, bubbles, threads, or droplets. This offers hope of dealing with all forces at once, widens the question, and narrows the range of answers.

There are three conspicuous impediments to this first step.

First, the causal relation $x\mathbf{C}y$ is not local, but may hold for events as far apart as the birth and death of the universe. Since we have committed ourselves to local variables, we abandon **C** for a local causal relation **c** at the second step.

Second, the proper description of time space and gravity seems to be not a symmetric quadratic form $\|v\| = g_{mn}v^m v^n$ in a time-space tangent vector field v, but a linear form $[\![v]\!] = \sigma_m v^m$, the spin form; and the spin form of a manifold is not ordinarily expressed in terms of causal structure. Rather, the converse is standard since the work of Bergmann (1957). We first find causal roots for the spin form at the sixth step.

Third, the most successful theory of causal structure that we have, Einstein's, is globally acausal in that it admits chronometrics with timelike loops, such as Gödel's and Kerr's. Similar time loops occur in some net after the second step; we shed them after the third.

1.2. Step 2: Atomize

That is, construct the causal continuum from discrete physical finite elements or *cells.*

The project of a cellular time space has been undertaken in modern times by R. P. Feynman (see his Nobel address), R. Penrose,[2] Finkelstein (1969), Misner *et al.* (1973), t'Hooft (1979), Bombelli *et al.* (1987), Chew & Stapp (1987), and Żenczykowski (1987), among others; not to mention the well-known lattice theories that do not necessarily assume that the lattice physically exists, nor the fascinating medieval developments of discrete time, space, and matter in the *Kalam.* It occurs also in a report of the National Research Council (1985a,b):

> It may well be that the marriage of gravitation and quantum mechanics requires a few more drastic revisions of our ideas. For example, our description of space-time as a continuum may have to be replaced by a discrete granular structure at extremely short distance. (*Elementary Particle Physics*, page 97).
>
> It may be that local Lagrangian field theory is not the correct approach to quantum gravity. Perhaps, as some believe, the basic quantum quantities are not the variables describing a space-time continuum but a more discrete structure. (*Gravitation, Cosmology, and Cosmic Ray Physics*, p. 74).

[2]Private communication from R. Penrose on the theory of mops (1960). This is a precursor to Penrose's (1971) theory of spin networks, both of which influence the present work.

Such fundamental cellular theories are to be distinguished from those that assume a continuum of events within each cell, which may nevertheless be useful for some computations.

We localize the causal relation by taking as basic dynamical variable a relation $x\mathbf{c}y$ expressing *immediate* causal priority. We call $\mathbf{c}$ the (causal) *connection* relation, understanding that this connection is *one-way* ($x\mathbf{c}y$ does not necessarily imply $y\mathbf{c}x$) and *immediate* ($x\mathbf{c}y$ and $y\mathbf{c}z$ do not necessarily imply $x\mathbf{c}z$). Events in a continuum theory have no immediate causal relations, but only mediated ones.

A point set with a $\mathbf{c}$ relation, interpreted as events with a causal connection, is called a causal *net* $\mathbf{c}$. Every causal set $\mathbf{C}$ may also be regarded as a causal net $\mathbf{c}$, with the connection $x\mathbf{c}y$ defined to hold if and only if $x\mathbf{c}y$ and no event z exists with $x\mathbf{C}z\mathbf{C}y$ and $x \neq z \neq y$; but networks are more general than causal sets. (*Proof*: The causal network of two events x, y with

$$x\mathbf{c}y\mathbf{c}x$$

is not a causal set.)

An event α in a net, taken with all events $\{\delta\}$ such that $\delta\mathbf{c}\alpha$, defines a *cell* in the net $\mathbf{c}$, and is said to have *inputs* $\{\delta\}$ and *output* α (Figure 1). A *contra*cell, dually, is an event δ with all the events $\{\alpha\}$ such that $\delta\mathbf{c}\alpha$; its

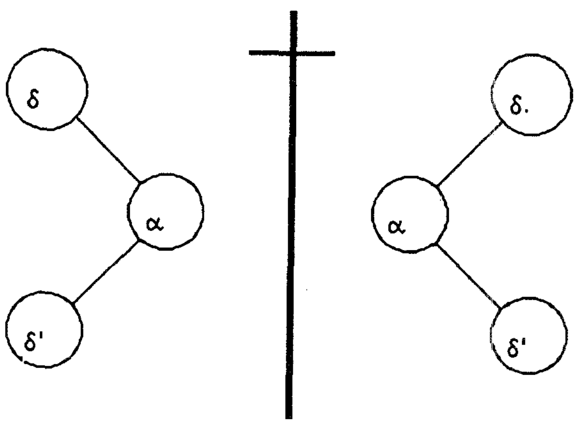

Fig. 1. Cells. Causality runs from left to right in all the figures. A dyadic cell 2 is shown on the left, its dual or †-image, a dyadic contracell $2^\dagger$, on the right. In the prequantum theory, the N inputs of a cell are permuted by a symmetric group S_N; in the quantum theory, by a linear group SL_N. The events δ and δ' thus correspond to ↑ (spin up) and ↓ (spin down). A second SL_2 acts upon the outputs of the contracell. Thus, SL_2 transformations are unicellular, while time reversal T, which maps $2 \to 2^\dagger$, is bicellular.

outputs are $\{\alpha\}$, its input δ. The cell is the net analog of the past light cone in the tangent space of the continuum theory; the contracell, the future light cone. An *n-adic cell* has n inputs; an n-adic contracell, n outputs. An *n-adic net* is one whose cells are n-adic. In the present theory the number of inputs to the cell, termed its *grade*, defines the dimensionality of the causal space.

Just as the classical principle of equivalence is expressed by Lorentz SL_2 invariance in the tangent space to the classical time space manifold, the quantum principle of equivalence asserts SL_N invariance of the quantum cell. We understand this group as mixing the inputs to the cell, its prequantum correspondent being S_N, the symmetric group on the N inputs to each N-adic cell. The quantum principle of equivalence asserts the equivalence of all the inputs to each cell. From the experimental spin-statistics connection we infer these equivalent entities obey odd (Fermi-Dirac) statistics.

The cell suggests a fundamental atom of time or chronon, which we designate by ƞ. Here ƞ will not be interpreted formally, like a regularization procedure or like Planck's earliest conception of his quantum of action h, but physically. We expect ƞ to manifest itself in all sufficiently short-time physics. The limit ƞ $\to 0$ is no more physical than $\hbar \to 0$ or $c \to \infty$.

The cell and the net recall the outmoded Pitts–McCulloch theory of a neuron and a nervous system. The cell is a neuron; the output event, an axon; the input events, dendrons; the link $\delta c \alpha$ from an input δ to its output α, a fiber. Even the inhibitory dendron of the biological neuron has an analog, the input destruction operator or contra-input that arises as a necessary consequence of special relativity and quantum theory in Section 3.3.

The inputs of one cell may themselves be outputs of another; there seems to be no synaptic gap between event cells. Nor are there events between the output and the inputs in one event cell. While the quantum theory will admit quantum superpositions such as $3\alpha + 4\delta$, which could be said to be "between" α and δ, this betweenness holds not in the topological sense, but only in a quantum sense.

The neural analogy reminds us that there are connections from several inputs to one output within the cell, and that there are no connections between the inputs, but should not make us forget that the elements of the cell are events, momentary existences, not permanent bodies, which must be synthesized from events. We provide for SL_N invariance of the quantum theory by making the classical net theory invariant under S_N, the symmetric group on the N inputs to each N-adic cell. This leads to a local Lorentz invariance, part of what is meant by a relativistic net.

Actual computers and nervous systems also support nets, consisting of causally connected computational or neural events, and these nets abstract important features of the system. It is not inconceivable that the net algebra introduced below for the ultimately fine world nets will also have practical application to coarser nets.

Since we have taken a connection relation as fundamental dynamical variable, we may be said to be topologizing physics by these first two steps. The topology we use, however, is a novel kind of combinatory topology, a discrete version of the Alexandroff topology, based on asymmetric causal connections, not on the symmetric spatial connections of Euclidean topology (Section 5.3).

1.3. Step 3: Algebrize

Express the fundamental relations as algebraic relations.

This preliminary to algebraic quantization stems from Heisenberg. It is applied to causal structure in Finkelstein (1969). Here it calls for an algebraic theory of the causal connection, a *causal algebra* (Section 2.7). This seems nontrivial. At any rate, this step is still unfinished, in that here we still must decide among several plausible causal algebras.

Since the language of nets is set theory, we express the causal algebras in terms of a set algebra SET [Section 3.1 and D. Finkelstein (1987)], defined by the operator $\iota x = \{x\}$ (Peano's 1888 iota operator) and the disjoint union $x \vee y$ (C. S. Peirce's 1867 "arithmetic addition" $x + y$, Grassman's "Progressive Multiplication"). The fundamental variable describing the net is now an element of SET. The simplest of these nets, called dyadic, are made with only the one nonassociative dyadic operation $z = [x, y] = \iota x \vee \iota y$ giving the output z from two distinct inputs x and y in an elementary OR gate. The algebra of symmetric dyadic nets, where each event has two outputs as well as two inputs, stands out for initial study.

1.4. Step 4: Quantize

In the present formulation, to quantize is to adjoin quantum superposition + to the causal algebra (so that it becomes a ring). It is also convenient to adjoint the complex numbers $\mathbb{C}$ as well, so that the causal algebra becomes a linear algebra, interpreted as the Heisenberg algebra of the net, for the continuum of probability amplitudes never causes an infinity in the way that the continuum of a causal manifold does. In the quantum theory ι is a linear operator on ψ vectors and may be thought of as a quantum arrow of time, stripped of its associations with great statistical complexity and reduced to a microscopic elementary entity.

At the same time the spin form $[\![v]\!] = \sigma \cdot v$ is interpreted as a Hilbert space metric for the local quantum theory of an experimenter on spins with local time axis v. This quantum interpretation of the spin form and hence of the gravitational field, though crucial for the quantum theory of causal structure is not fully reconciled with the causal interpretation of the chronometric until the sixth stage.

This step makes all basic variables discrete, quantum, and topological in nature. In a quantum cosmogony it is unphysical to impose unitarity, which would express the eternal duration of the system under study.

We may then interpret spinors within the quantum net. In the affine (nonmetric) quantum theory (Section 3) of the N-adic net, there is a local linear group $SL_N := SL(N, \mathbb{C})$ interchanging the inputs to each cell. The inputs of each N-adic cell thus constitute a spinor whose components correspond to slightly different time space points, like the lattice fermion spinors of Susskin (1977). For $N=2$ this is the origin of local Lorentz invariance, but does not yet account for Poincaré invariance or T symmetry. In the classical net theory we have only a precursor of this SL_N invariance, the symmetric group S_N on the N inputs or outputs.

We thus rebuild the world out of 2-spinors (two-component spinors), as do von Weizsäcker (1955), Bergmann (1957), Penrose (see footnote 2), and Finkelstein (1969) in various senses; not classical spinors representing continuous classical observables such as Euler angles, but quantum ones representing probability amplitudes for two discrete alternatives, such as $\uparrow$ ("spin up") and $\downarrow$ ("spin down").

To parallel Bergmann's (1957) 2-spinor theory of gravity in a Kaluza hyperspace, we use a theory of higher dimensional gravity, or hypergravity, with structure group SL_N (the group that now arises as a symmetry of discrete quantum alternatives) instead of $GL(n, \mathbb{R})$ or $SO(n, \mathbb{R})$ (which do not). The N-component spinor on which SL_N acts is a ψ-vector for an N-fold alternative that we continue to call spin (or hyperspin) even when $N>2$. The generalization of the Bergmann (1957) 2-spinor time space to such N-spinors is called a Bergmann manifold B_N (Section 2.2).

Weinberg (1984) also asks whether hyperspace might have other than an orthogonal group. Since Weinberg postulates that all internal coordinates are Lorentz scalars, he answers no. This postulate may be unduly restrictive; for example, the Susskind (1977) lattice fermions even have a spinor part in their external coordinates, and spinorial internal coordinates are studied in Finkelstein (1955). More importantly, the distinction this postulate makes between scalar and spinor coordinates, while sound in the continuum of Weinberg's study, is too simple for nets. Each event in the world net has not one, but two mutually dual Lorentz groups, one on the inputs and one on the outputs, which commute with each other and which only merge in the continuum limit (Section 2.8.4). Much as with the two commuting space and body groups of Finkelstein (1955), what appears as spinor structure to one of these groups appears as scalar structure to the other. Accordingly, we proceed without Weinberg's postulate here, and with other than orthogonal structure groups.

1.5. Step 5: Bosonize

Choose the dynamics and the temperature to favor the dynamical formation of correlated bosonlike event pairs.

We do not work out an example of this step, but need it for the next.

1.6. Step 6: Condense

Specialize the dynamics and the temperature still further to permit quantum condensation of bosonlike event pairs.

We infer that ordinary Minkowski space is a macroscopic quantum condensation of the net from several features of experience that are otherwise incomprehensible within net theory:

1. Minkowski vectors. Macroscopic variables such as the components of tangent vectors v^m may then arise from microscopic ψ-vectors ψ_A consistent with the present symmetry groups and quantum superposition. We interpret a typical small tangent vector v^m of the classical differential manifold as a many-input ψ-vector $v_{\dot{A}B}$ describing a macroscopic number of coherent input pairs.

2. Reality of Minkowski space. The previously mysterious formal reduction of complex ambispinors to real sesquispinors (prefixes defined in Section 3.2) practiced since Cartan may now be understood as a spontaneous breaking of gauge invariance.

3. Field variables. This condensation relates anticommuting microscopic ψ-vectors to classical commuting macroscopic Minkowski vector components. The spin form σ, the only field variable in hypergravity, may be interpreted as the common wave function of an assembly of bosonlike input pairs. There is no σ in tohu.

4. Real gauge fields. Since the net theory has structure group SL_N, which would ordinarily lead to complex gauge vector fields, real gauge fields appear as a spontaneous breaking of gauge invariance.

5. Law of Inertia. We presumably will not see the net breaking translational invariance and absorbing momentum through Umklapp processes and scattering until we reach much higher total excitation energies than $1/\hbar$, for much the reason a superconducting lattice does not absorb momentum from its electron current: because of the quantum condensation of the elementary fermions. Propagation of excitations in tohu will be dissipative, through causal.

The idea that a vector is a great many coherent bosonlike fermion pairs hearkens back to de Broglie's idea that the photon is a pair of neutrinos and Feynman's that the graviton is a quartet of neutrinos, both of which have natural counterparts in this theory. Here, to be sure, the fermions and their bosonlike compounds are not particles, but events, microscopic quantum elements of the world net, and they undergo a bosonlike quantum condensation.

The proposed theory of classical n-dimensional causal space as a quantum condensation of a relativistic quantum net has an exact local SL_N invariance, reducing to Lorentz invariance for $N=2$ (Section 3.1). The fundamental spin form σ of Infeld and van der Waerden, Bergmann, and Penrose describing the gravitational field arises as the macroscopic quantum ψ-vector of a quantum condensation of bosonlike fermion pairs analogous to superconductivity (Section 4.3). We offer two relativistic topological candidates for the quantum action principle of the net, the causal Euler characteristic and the spin class (Section 5.1). There is likely a phase transition of the net from the ordinary vacuum to a chaotic or gaslike phase, at an energy or temperature closer to the W mass than the Planck mass (Section 4.3).

2. FROM CLASSICAL CAUSAL SPACE...

We begin the six-step journey outlined above with a brief critique of a preceding theory (Section 2.1) and a summary of our new classical starting point (Section 2.2).

2.1. Post Mortem of a Clifford-Algebraic Theory

This is to help a hypothetical reader of Finkelstein and Rodriguez (1986) with the transition to the present theory, and is not required for the present paper itself.

In the 1986 theory we still entertain a Riemannian theory of causal space and regard this as a smoothed aggregated description of a quantum net. To define the concept of a quantum net, we construct a quantum set theory that associates a quantum net with a real Clifford algebra generated by its vertices. Vertices of a quantum tetrahedron support the identity representation of its Clifford group, which is the Lorentz group $SO_+(1, 3, \mathbb{R})$. Simplices of arbitrarily high dimensions also occur, and also have orthogonal structural groups.

The 1986 theory is indeed quantum combinatoric and topological, and it indeed accounts for the Lorentz group; but it has no elementary spin-$\frac{1}{2}$ entities [supporting the $D(\frac{1}{2}, 0)$ representation of the Lorentz group]; spin-$\frac{1}{2}$ can enter only as a collective topological effect, like the spin-$\frac{1}{2}$ of skyrmions. In addition, its higher dimensional plexi have higher dimensional orthogonal groups $SO(n_+, n_-, \mathbb{R})$ with more than one timelike dimension, and causality fails beyond four dimensions. These aspects of the theory make it hard to relate to physics.

Now we replace the classical Riemannian causal manifold with its structure group $GL(n, \mathbb{R})$ by a Bergmann manifold with structure group SL_N, where $n = N^2$, sketched in Section 2.2. This results in the recent generalization of gravity called *hypergravity*, which is both starting point and guide for the present work. Then in Section 3.1, having discarded the Clifford-algebraic quantum set theory, we construct a Grassmann-algebraic one with SL_N symmetry. We are then in a better position to construct a Lorentz-invariant, spinor-based quantum net theory of the world.

2.2. The Chronometric Is the Determinant of the Metric

A B_N has first, like any manifold, an algebra of real coordinates defining its topology.

It does *not* have the usual quadratic chronometric form $g_{mn}(t)$ assigning a norm

$$\|v\| = g_{mn}v^m v^n = \tau^2$$

to each vector $v = (v^m)$ at each world point $t = (t^m)$.

Instead, it has a linear *spin form* $\sigma = (\sigma_{\dot{A}Bm}(t))$ assigning a Hermitian $N \times N$ spin form

$$[\![v]\!] = \sigma \cdot v$$

to each such v. The structure group for four-dimensional causal space is no longer $SO(1, 3, \mathbb{R})$, but its covering group SL_2, and for higher dimensions it is SL_N. It is possible to set $N = 2$ throughout this paper and thus specialize to the familiar four-dimensional world for a first reading; we then retrieve the theory of gravity of Bergmann (1957). The generalization of gravity for $N > 2$ is called hypergravity.

The value of the spin form $[\![v]\!] = \sigma \cdot v$ is a metric (that is, is Hermitian positive-definite) exactly for future-timelike vectors. We do not give a fixed metric to the linear space of spinors associated with each point of a B_N, for that space is to be finite-dimensional and support a representation of SL_N, which has no finite-dimensional unitary representations. Each experimenter brings her own metric. We interpret $[\![v]\!]$ as the Hilbert space probability metric proper to any local quantum spin experimenter whose instantaneous time axis is v. Therefore $[\![v]\!]$ is called the (first quantized spin) *metric* of the future vector v. The spin metric for $N = 2$ is the sole dynamical variable of the Bergmann (1957) theory of gravity.

This reinterpretation of the gravitational field as quantum metric is not a total departure from the Riemannian theory, although it is not standard there. In a Riemannian theory of causal space, the first-quantization metric for scalar quanta moving in the space employs the chronometric volume element $\rho = \sqrt{(-\det g_{mn})}$, which is also a dynamical variable; and the first-quantization metric for vector fields employs the rest of the chronometric tensor, when we take all local experimenters into account. Thus, in the Riemannian theory, too, the first-quantization local Hilbert space metric (for integer spins at a point) may be considered to be the sole dynamical variable.

This quantum reinterpretation of the Riemannian variables g_{mn} and the spin form σ is crucial in our project of founding all time space concepts upon quantum ones.

Evidently, $[\![v]\!]$ and σ distinguish past v from future, while $\|v\|$ and g_{mn} do not, in the sense that time reflection changes the signs of $[\![v]\!]$ and σ, but not the signs of $\|v\|$ and g_{mn}. This is significant to the interpretation that follows (though in hypergravity $\|v\|$ and $g\ldots$ are odd in odd-dimensional spaces).

The usual chronometric norm $\|v\|$ of v is proportional to the determinant of $[\![v]\!]$. This is a polynomial of the Nth degree. Its importance is entirely due to the fact that it is the only SL_N invariant that can be made from v. The spin form σ of a Bergmann time space is thus a kind of Nth root of the chronometric tensor, not a square root as in Riemannian time space. Moreover, σ is the proper description of the hypergravitational field, not the chronometric tensor, which is subject to many algebraic constraints for $N > 2$.

Since the metric $[\![v]\!]$ is dimensionless (a probability) and the norm $\|v\|$ has dimensions of $(\text{time})^N$, the relation between metric and chronometric involves a fundamental time $\sqcap$:

$$\|v\| = \sqcap^N \det[\![v]\!] = \tau^N.$$

We tentatively suppose that $\sqcap$ is closer in order of magnitude to the W range than to the Planck length (Section 5.2), and use units with $\sqcap = 1$, called net units.

The space B_N has dimension N^2, causal structure (one timelike dimension), a chronometric tensor $g_{mn\ldots p}$ with N indices (not 2), an invariant Laplacian differential operator of differential order N (not 2), but geodesic equations still of differential order 2 (not N).

A B_N has a unique σ-preserving, torsion-free connection (Holm 1986, 1987), from which a curvature can be computed as usual. S. R. Finkelstein (1987) gives an action density R for a B_N, suitably reducing to the Einstein-Hilbert R for $N = 2$. In the case of higher dimensional causal spaces, following Kaluza, we expect four coordinates to serve as physical macroscopic coordinates, called *external*. The remaining coordinates, called *internal*, are available to describe "internal" structure, like electric charge in

Kaluza's theory and the most general gauge charges in the generalization of deWitt (1964).

2.3. Net

Now we atomize the Bergmann manifold. We first abstract from it the causal relation $x\mathbf{C}y$ ("x is causally prior to y" or "x before y"), which holds when there is a curve directed from event x to y whose tangent is everywhere in the closure of the future cone. This relation is usually assumed to be a partial order. (In a B_N there are N sheets to a light cone. The positive-definite spin forms constitute the future cone. The boundary of this cone is the "futuremost" of the N sheets.)

We next rework the concept of causal structure into a form more suitable to a quantum theory. We use the causal partial order to define a *connection relation* $x\mathbf{c}y$; and we use that to define a causal operator ι, which is the concept that we take with us into the quantum theory.

A partial order is a relation R that is transitive:

$$(xRy \cap yRz) \subset xRz$$

and asymmetric:

$$(xRy \cap yRx) \equiv (x = y)$$

hence reflexive:

$$xRx$$

There are serious problems with the relation of equality $x = y$ and hence with asymmetry, reflexivity, and partial order in quantum theory (Finkelstein, 1969); the lattice operations $\cup$ and $\cap$ are inappropriate for a coherent quantum theory (D. Finkelstein, 1987a); and the relation $\mathbf{C}$ is nonlocal. As a preliminary to quantization, we eliminate these concepts from the foundations of the theory.

A property of spaces is local if it may be verified for a space by verifying it for a neighborhood of each point in the space. A relation is local if it holds only for points that are near each other. We assume that the kinematics and the dynamics must be local; and also symmetric with respect to permutations of the N inputs to each cell, so that in the quantum theory we have SL_N invariance.

It is an involuted question whether the property of being a causal order itself. But certainly the causal order $x\mathbf{C}y$ is itself nonlocal, relating each event to all the past events in history back to cosmogenesis, and cannot be the basic variable of the world. However, each causal order on a finite space

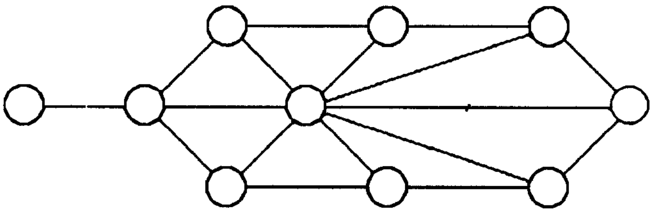

Fig. 2. A polyadic causal net. It has ten cells: one cenadic (no inputs), three monadic, four dyadic, and two triadic.

has within itself a germ that is local, a *connection relation* $x\mathbf{c}y$ defined next, the discrete analogue of the local light cone. The connection relation is what we describe by giving the cells of the network. Going from the global causal relation $x\mathbf{C}y$ to the local connection relation $x\mathbf{c}y$ is like going from integral to differential quantities.

If $x\mathbf{c}y$ we say that x (*causally*) *connects to* y, and also that x is an input to y, and that y is an output from x. Until further notice, **c** is our basic variable, not **C**. A set provided with a **c**, interpreted as the immediate causal connection, we call a (*causal*) *net* (Figures 1-3). To construct **C** given **c**, we use not the lattice operations $\cup$ and $\cap$, but the Grassmann ones $\vee$ (the disjoint union or *disjoin*) and $\wedge$ (the exhaustive conjunction or *conjoin*) obeying

$$x \wedge x = x \vee x = 0$$

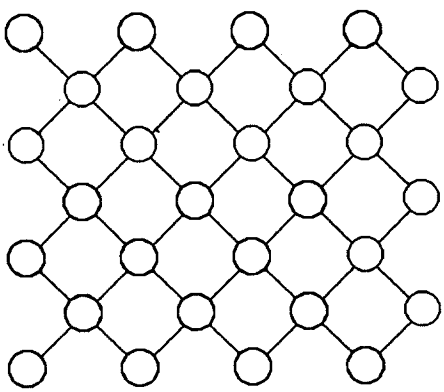

Fig. 3. Checkerboard. A segment of the infinite symmetric dyadic network $\mathbb{Z}^2$, giving the moves of a man in the game of checkers. Each pair of inputs to a cell supports a local SL_2 group identified with the Lorentz group. This is the network that appears in the Feynman and Hibbs (1965) model for the two-dimensional Dirac equation.

for all events x. The $\mathbf{C}$ induced by a given $\mathbf{c}$ is given by

$$\mathbf{C} = \mathbf{c}^* := \bigvee_{n=1}^{\infty} \mathbf{c}^n$$

$$x\mathbf{c}^n y := \bigvee_{\{z\}} (x\mathbf{c}z_1) \wedge \cdots \wedge (z_n \mathbf{c} y)$$

The $\mathbf{c}$ relation we assume to be *locally finite*: each input has but a finite number of outputs, and each output only a finite number of inputs.

2.4. Spin Net

We are interested in the net primarily as a conceptual precursor to the Bergmann manifold, which arises from the quantum net in a classical limit. But we may also form from a B_N an approximating n-adic causal net W^n whose events are closely and approximately uniformly spaced events of the manifold, and whose connection $x\mathbf{c}y$ is derived from the causal relation $\mathbf{C}$ of the manifold. The identity we impose here between the number of inputs to the cell and the dimensionality of the manifold first becomes clear in the quantum theory. For now it suffices to note that this cell is an abstract n-dimensional simplex.

The simplicial cells of an approximating causal net W^n generally do not form a manifold. About half is left out in the case of the checkerboard network $\mathbb{Z}^2$ of Figure 3, which is used as an approximation to the Minkowski plane M^2 by Feynman and Hibbs (1965).

To approximately represent the spin structure of a B_N in a causal net, we may then assign to each link $\delta\mathbf{c}\alpha$ of the net W^n the positive-definite Hermitian matrix $\sigma = (\sigma_{\dot{A}B})$ associated with the tangent vector at the mid-point of the geodesic from δ to α. We interpret this σ as the spinor Hilbert space metric to be used by a local macroscopic experimenter whose time axis is that tangent vector, doing spin experiments at that cell.

We do not take a spin net as the classical point of departure for quantum time space. A spin net appears only as a numerical approximation to a Bergmann manifold, which in turn is a large-quantum-number limit of a quantum net, which carries only the connection operator.

2.5. Relation to the Regge Cell

A *simplicial manifold* is a simplicial complex that results from a simplicial decomposition of a manifold. If each simplex in a simplicial manifold is assigned to a linear simplex in a Minkowski space M^n up to Poincaré transformations, and the assignments of any two simplices agree up to Poincaré transformations on their common faces, the simplicial manifold becomes a *Regge complex.* Then each edge has a chronometric norm consistent with a Minkowski geometry within each simplex, and these norms determine the Regge complex. This is the simplicial specialization of Regge's (1961) cellular theory.

It may be helpful to note a relation between the spin cell and the Regge cell. On its face, a spin cell differs from a Regge cell in carrying a spin metric on its timelike edges only, rather than chronometric norms on all its edges, and in not explicitly providing a Minkowskian geometry within its simplices. But the spin metric $[\![v]\!]$ may be extended from the timelike to the spacelike edges of the spin cell by linearity, every spacelike edge being the difference of two timelike ones:

$$[\![v]\!] = [\![v_1 - v_2]\!] = [\![v_1]\!] - [\![v_2]\!]$$

Then each edge v has a chronometric norm $\det[\![v]\!]$, and each spin cell defines a Regge cell. This indicates only one way in which Regge's theory may be extended from Riemannian to Bergmann manifolds.

2.6. Classical Net

To make a physical theory of nets we must:

1. give the domain of possible c relations and
2. define the action on that domain. These two steps give the network *kinematics* and *dynamics,* respectively.

We may describe them extrinsically, as by imbedding the net in some standard reference net, or intrinsically, without leaving the net itself.

The intrinsic and extrinsic approaches are perhaps coextensive in principle, but in practice they draw our attention to different theories and different problems. Since an extrinsic description dispenses with field variables, let us formulate an extrinsic net kinematics. We first construct a universal net.

Let us use Peano's operator ι for set formation:

$$\iota x = \{x\}$$

and Peirce's $\vee$ for the disjoint union, defined only for disjoint sets and given the symbolic value 0 when it is undefined:

$$x \vee x = 0$$

Classically we may represent an N-adic cell by the set of its N (distinct) inputs $\delta_1, \ldots, \delta_N$ alone, writing the output as

$$\alpha = \{\delta_1, \ldots, \delta_N\} = \iota\delta_1 \vee \cdots \vee \iota\delta_N$$

Here the same set α describes both the output and the inputs of the cell, and thus the entire cell. In this cell the output α contains its inputs within itself, and the connection relation $\delta c \alpha$ is expressed by $\delta \in \alpha$, "δ is an element of α." This α is invariant under the symmetric group S_N of the inputs. In the quantum theory it will be invariant under the group SL_N of the inputs.

2.7. The ∈ Connection

Any set represents a net in a similar way. The events of the net are the elements of the set, their elements, and so forth. The connection relation is again $\delta \in \alpha$. The net will be finite if all its events are finite; we call such a set *finitistic.* This means that the set, its elements, their elements, and so forth *ad finem,* are all finite. We designate the network W associated with a set S in this way by $W = \in^*(S)$.

The first natural choice for the space of possible nets W with a given set of inputs $\text{In} = \{\delta, \ldots, \delta'\}$ is then the infinite set of *all* finitistic sets constructed from In, which we designate by Set[In], taken with the $\in$ connection. A second kinematics that seems potentially useful would restrict W to the N-adic nets that can be imbedded in Set(In). One might finally specialize the kinematics to $N = 2$.

The main difference between the present kinematics and that of Finkelstein (1969) is that now we *connect* sets by $\in$ and there we *order* sets by $\subset$.

For example, the net $W = \text{Set}[\text{In}]$ with connection $\in$ lacks T symmetry. It has the inputs In, but no output, since every set is an element of some larger set. The kinematics also lacks T invariance at this stage, for we may imbed the net

$$\delta c \alpha, \qquad \delta' c \alpha$$

in Set[$\{\delta, \delta'\}$] as the set $\{\delta, \delta'\}$, while its T-image

$$\delta c \alpha, \qquad \delta c \alpha'$$

cannot be imbedded in Set[In] for any In, since a set is uniquely determined by its elements, implying $\alpha = \alpha'$. We may restore T symmetry to any of the kinematics mentioned so far by fiat if we wish; for example, by postulating that each event is uniquely determined by its outputs as well as by its inputs, or by describing W by an atlas rather than by one net.

Imbeddability in Set[In] implies the existence of a global causal order **C**, represented by the iterated relation $\in^*$.

If $x \in^* y$, we say that x is an *iterated element* of x.

The $\in^*$ *order* of a set S is the order $x \in^* y$ defined on the event set of $\in^*(S)$.

Then Rank(x), the set-theoretic rank of x, giving the number of nested ι's in x, appears as a timelike coordinate for the $\in^*$ order, in the sense that if $w \in^* x$, then $\text{Rank}(w) \leq \text{Rank}(x)$.

2.8. Classical Causal Algebra

Now we algebraize the connection relation, for the sake of subsequent quantization, as required in Section 1.3. The world is no longer to be expressed as a net of connection *relations*, but as a system of connection *equations*. In addition, these are to be local in their content rather than global. The resulting algebra we call the *connection algebra* of the world.

Let us follow the model of set theory. What is basic in quantum set theory is not the relation $\delta \in \alpha$, but the algebraic equation

$$\alpha = \iota\delta_1 \vee \cdots \vee \iota\delta_N$$

which implies $\delta_1 \in \alpha, \ldots, \delta_N \in \alpha$. We therefore take this as the basic equation of classical set theory as well. The algebraic environment for such equations is the algebra generated by

0 and 1	nonentity and nullset
the monadic operator ι	unit set formation
the dyadic operator $\vee$	disjoint union
a set of inputs $\text{IN} = \{\delta, \ldots, \delta'\}$	proper entities

subject to the equalities

$$\iota 0 = 0$$

$$\alpha \vee 0 = 0, \qquad \alpha \vee 1 = \alpha, \qquad \alpha \vee \alpha = 0$$

$$\alpha \vee (\beta \vee \gamma) = (\alpha \vee \beta) \vee \gamma$$

$$\alpha \vee \beta = \beta \vee \alpha$$

We now let $\text{SET}(\text{IN})$ designate this algebra of sets.

Similarly, instead of discussing the short-range net *relation* $\delta \mathrm{c} \alpha$, we take as basic the still more informative short-range *equation*

"α is the output from inputs $\delta_1, \ldots, \delta_N$"

We write this not in the relational language of classical logics, but in the equational language of set algebra:

$$\alpha = \iota\delta_1 \vee \cdots \vee \iota\delta_N$$

This is the algebraic expression for the cell of Figure 1, with output α and inputs $\delta_1, \ldots, \delta_N$. Here ι serves as the elementary *causal operator*; it represents the passage of one fundamental time transforming one input δ with one output $\alpha = \iota\delta$.

We single out a kinematic theory of nets for first study by the following process of successive specialization, leading to the concept of symmetric dyadic net (Section 2.8.4).

2.8.1. Polyadic Nets

In this kinematics each cell has any number of inputs, possibly varying from cell to cell. To simplify the notation, we may combine the two operations ι and $\vee$ into a bracket operator defined on any number of operands $\delta_1, \ldots, \delta_N$:

$$\alpha = [\delta_1, \ldots, \delta_N] := \iota\delta_1 \vee \cdots \vee \iota\delta_N$$

This bracket vanishes when an operand repeats:

$$[\delta, \delta] = 0$$

while Cantor's brace $\{\delta_1, \ldots, \delta_N\}$ ignores repeated operands, obeying the absorptive law

$$\{\delta, \delta\} = \{\delta\}$$

In the absence of such repetitions the two operations $[\cdots]$ and $\{\cdots\}$ agree.

The entire net is thus described by one expression giving the set of its final outputs in terms of its initial inputs. Such an expression, interpreted as a description of a net, is called a *product plexor*; more general plexors appear later. For example, the cell of Figure 1 and the net of Figure 2 are identified with the monadic and triadic plexors

$$\begin{gathered}[[\delta, \delta']] \\ [[[\delta, [\delta, \delta', \delta'']], [[\delta, \delta', \delta'']]], [[[\delta, \delta', \delta'']], \delta'']]\end{gathered} \tag{1}$$

respectively, while the contracell of Figure 1 is the plexor

$$[[\delta], [\delta]] = 0$$

the undefined.

2.8.2. N-adic Nets

This is the special class of the polyadic net in which each cell has N inputs, with N fixed once and for all. The bracket operation $[\cdots]$ of an N-adic net is N-adic; that is, operates on N operands exclusively.

2.8.3. Dyadic Nets

This is a still more special kinematic algebra, the N-adic net with $N = 2$. Its bracket operation $[\delta, \delta']$ is dyadic, and may be regarded as the simplest possible universal cell.

C. S. Pierce (ca. 1866) pointed out that nets of any dimensionality may be made from three-terminal cells, like those of Figure 1 for $N=2$. He associated such a cell with a general triadic *relation*; we specialize here to the triadic relation that is given by the dyadic *operation*

$$\alpha = [\delta, \delta']$$

Since polyadic nets can be simulated by dyadic ones and conversely, we cannot distinguish between these possibilities without more detailed calculation and further reference to experience. Simply to decide which branch to explore first, we appeal once more to the general philosophical principle that the operations that occur in nature are the most concrete and specific, the least general.

This decision conflicts slightly with our generalized Kaluza hypothesis, in giving a special role to $N=2$ and thus to four-dimensional causal space; but this can hardly be said to conflict with experience yet, and we have already made sure that higher dimensional networks may be synthesized out of dyadic ones.

2.8.4. Symmetric Dyadic Nets

Each of the above kinematics has a subkinematics with duality symmetry, where the number of outputs from each contracell is subject to the same restriction, if any, as the number of inputs to each cell. We call these *symmetric* kinematics. The symmetric dyadic kinematics, for brevity called the *X kinematics*, deals with nets made out of X-shaped pentads, as is the net $\mathbb{Z}^2$ of which Figure 3 is a segment. [In Finkelstein and Rodriguez (1985), where the causal structure is omitted, these pentads appear simply as 4-simplices, pentatopes, or "pentacles."] Since there is no evidence for the kind of microscopic irreversibility that would permit cells to have exactly two inputs but any number of outputs, we tentatively accept the X kinematics as the first to study. This also will simplify numerical computations.

It is rather scary that two events δ, δ' initially on opposite sides of the galactic network can be just two edges apart after the connection operation $[\delta, \delta']$ of the X kinematics. Will this not conflict with locality and causality? But the ι part of the operation $[\delta, \delta']$ guarantees that we never make time loops, and thus preserves causality; and $[\delta, \delta']$ is at least local after the fact: whatever events we multiply by that very act then belong to the same neighborhood in the net. The worst that this process can create is great curvature, pathological topology, and exponential computation time for computer experiments.

It is possible to avoid even these by some further kinematic postulate. For example, we might stipulate that only two events having a common input can have a common output. But this leads to a fixed checkerboard net with no significant dynamical behavior.

A softening of this postulate, permitting events with common inputs some fixed number of generations ago (say) to have common outputs, would permit dynamics and still speed up computer experiments enormously. But this seems artificial.

We therefore leave it to the action principle to suppress these long-range connections sufficiently for the world to exist, and retain the X kinematics. Two events may have a common output in the immediate future even though they are arbitrarily far apart in the immediate past, and dually.

We may think of the algebraic operations ι and $\vee$ as representing two morphogenetic processes, replication and union. Each event can replicate, its replica being one ℩ later; that process is represented by the operator ι. Then the replicas of N events can unite to form one offspring event; that process is represented by the product $\vee$. The process then iterates. Evidently N is something like the number of sexes at the microphysical level. This makes the dyadic net and binary Bergmann manifold an appealing hypothesis, but it does not prove that that is how the microphysical world actually works.

In the X kinematics there are two local symmetry groups at each event, one S_2 permuting the inputs to the event and a second the outputs from it. The two commute, since they act on different events. We call them the input and output groups. In the quantum theory they give rise to two commuting SL_2 groups.

2.9. Proper Time in Nets

The expression (1) of Section 2.8.1 for the net of Figure 2 is quartic in the inputs δ and δ'' and cubic in the input δ'. These degrees may be considered as measures of the future "cones" of these inputs in the net.

Theorem. The degree of an output α of a net in one of its inputs δ is the number of paths from δ to α in the net.

Proof. By induction. ∎

Let us designate the degree of α in δ by $\deg(\delta, \alpha)$. It is evident that $\deg(\delta, \alpha)$ is *supermultiplicative* in the sense that

$$\deg(\delta, \delta'') \geq \deg(\delta, \delta') \deg(\delta', \delta'')$$

Its logarithm is then superadditive in the obvious sense: It obeys the antitriangle inequality. This logarithm is thus a net correspondent to the manifold concept of proper time $\tau(\delta, \alpha)$ from δ to α, which is similarly superadditive:

$$\deg(\delta, \delta') \sim \exp[-\tau(\delta, \delta')/℩]$$

There are surely others, since the limiting transition $\eta \to 0$ blurs fine distinctions.

According to this measure, the proper time from an input to its output in a cell, or in any net where there are no other (causal) paths joining them, is exactly

$$\tau = \ln 1 = 0$$

In this sense, the causal connections is a null or lightlike one.

We have not yet defined time coordinates, as opposed to a proper time. One can go a certain way using transformation laws. We expect the four causal space coordinates to form a 4-vector near the origin. In the quantum cell of an X kinematics, the only SL_2 vector one can make is the usual Pauli spin form σ, whose time component is unity; to make a time, we multiply by η. This suggests that any experimenter will find the difference in time coordinate between input and output to be η. It would be doubly paradoxical if relativity and quantum theory thus conspire to provide an absolute time in the cell when there is none in the macrocosm.

3. ...TO QUANTUM CAUSAL SPACE...

We now proceed to the algebraic quantum theory of the net. The next section summarizes some essential algebra. We take for granted the parts of quantum theory that do not involve a metric and probablity; these constitute *affine quantum theory.* In affine quantum theory we still represent input channels by ψ-vector $\langle\psi] \sim (\psi^M)$ and output channels by contravectors (contragredient vectors) $[\varphi\langle = (\varphi_M)$, in such a way that the transition $\psi \to \varphi$ is forbidden just when the transition amplitude

$$A = [\varphi\langle\psi] = \varphi_M \psi^M$$

vanishes. Experimental variables are still represented by linear operators, and the eigenvalue principle still determines their possible values. We may still form tensor products and Fock spaces as in unitary quantum theory, and may form occupation number operators $N = \psi\partial_\psi$ relative to any basis. All these processes are invariant under the special linear group SL_N of the space of ψ-vectors.

3.1. Quantum Principle of Equivalence

Since a classical net is a classical set, let us assume that the quantum net underlying time space is a quantum set; that is, that the causal net is maximally described by a ψ-vector in the algebra SET of quantum sets constructed for that purpose (D. Finkelstein, 1987a) and summarized here. These ψ-vectors, interpreted as maximal descriptions of a quantum net, are called *plexors.*

The group SL_N, in contrast to the orthogonal groups, is a symmetry group of a complex Grassmann algebra; we suppose therefore that a quantum cell is the Grassmann product of its inputs. The commutativity $\alpha \vee \beta = \beta \vee \alpha$ postulated for the classical product $\vee$ is replaced by anticommutativity

$$\alpha \vee \beta = -\beta \vee \alpha$$

making redundant the nilpotence postulate $\alpha \vee \alpha = 0$. The inputs of a cell thus transform as spinors (the identity representation) of SL_N, leaving the output invariant. For $N = 2$ we require that this correspond to the usual double-valued action of the Lorentz group on Weyl spinors as SL_2, and for $N > 2$ it is required to include that physical SL_2 as a subgroup. This is the quantum root of the classical principle of equivalence. It constitutes the local Lorentz invariance of the theory.

The basic elements of structure of SET with their physical interpretation are

- 0 The scalar zero, representing nothing
- 1 The zero-grade unit, representing the null set
- $\vee$ Grassmann product, disjoint union
- ι Set formation; $\iota\alpha = \{\alpha\}$; interpretation varies with use
- $\mathbb{C}$ Complex numbers; quantum amplitudes
- $+$ Addition; quantum superposition

The symbols 1, $\vee$, and ι are common to the classical and quantum theory; the complex numbers and the symbol $+$ are adjoined in the quantum theory, and their adjunction is the quantization process. Thus, the most general plexor is a linear combination of plexi with complex probability amplitudes as coefficients. The usual symbol $\{\alpha, \beta, \ldots, \alpha\}$ for a finite set is replaced in this language by

$$[\alpha, \beta, \ldots, \alpha] := \iota\alpha \vee \iota\beta \vee \cdots \vee \iota\gamma$$

which vanishes if any of its operands coincide.

The *product* $\vee$, the quantum disjoin, is the product that Grassmann calls progressive and designates by $\vee$, recognizing it as a kind of union, and it is usually and inappropriately designated by $\vee$, as though it were a kind of intersection, following Cartan; see Barnabei *et al.* (1985) (and D. Finkelstein, 1987a) for this significant reinterpretation of the Grassmann product. In work that deals with the correspondence between classical and quantum concepts, the usual notation would be unacceptable. We reserve the sign $\wedge$ for Grassmann's regressive product, a kind of intersection, which we call the *coproduct* when it exists; this is the quantum conjoin.

We retain the continuum $\mathbb{C}$ of complex coefficients only to simplify the exposition. A plausible descent from $\mathbb{C}$ to $\mathbb{Z}$ (integer quantum theory) is sketched in D. Finkelstein (1987b). Here we work with complex coefficients, and adjoin to the operations $\vee$, $+$, ι of SET the operator $C: \psi \mapsto C\psi = \bar{\psi}$ of complex conjugation.

We do not give plexors an inner product, because the spinors of a Bergmann manifold have none. This is a fundamentally affine (metric-free, nonunitary, nonconservative) quantum theory. The conserved inner product arises from the condensation.

We allow for primitive input events $\delta, \ldots, \delta'$ by forming a linear algebra SET(IN) generated from inputs

$$\text{IN} := \{\delta, \ldots, \delta'\}$$

using the operations of SET. Evidently SET = SET({1}).

3.2. Quantum Causal Algebra

We use SET as universal quantum connection algebra. A maximal description of a quantum net is a plexor Ψ, an element of SET(IN). The most general such plexor is a linear combination of plexi. If Ψ is a plexus, its factors, stripped of their outermost braces or ι's, represent the outputs, the final events of the net. Their factors in turn, again stripped, represent cells one cell earlier; and so forth, *ad finem*. Ultimately, every event is expressed in terms of some input events IN as a plexor in the algebra SET(IN).

We may encapsulate the kinematics of the present theory of the world without being more specific than is presently justified:

P1. The world is a quantum causal net and ι is its connection operator.

Until now, ι served in quantum set theory only to generate a Grassmann algebra of high dimension from the number 1; it had no simple physical interpretation. One attraction of P1 is that it gives ι a physical interpretation as the connection operator.

As we have already seen (Section 2.8.4), it is plausible, *a priori*, that the world is a quantum net of X's. In ordinary dynamical problems, the quantum network has input and output surfaces with huge numbers of terminal events. For more speculative cosmogony, it is attractive to imagine that there are no primitive uncaused events $\alpha, \alpha', \ldots$ at all, and that the world is created from the null set 1 alone. This rather strictly limits the early stages of creation, however, and leaves no place for the experimenter.

One essential consequence of P1 is that an event with N inputs and M outputs supports the pinor representation of SL_N upon its inputs and the group SL_M on its outputs. In the binary continuum analog these correspond to two Lorentz SL_2 groups acting upon the past and future cones, and are identified by the postulate of differentiability at the origin. In the discrete theory we deal with finite event sequences, whose past and future directions vary independently, rather than smooth curves. We must therefore expect a doubling of quantum numbers associated with the Lorentz group.

3.3. Discrete Symmetries

We must also consider the discrete symmetries. A *theory* of space (that is, a class K of spaces) has some symmetry X if the X image Ψ^X of every theoretically possible space Ψ is also theoretically possible: $K^X \sim K$. A *space* Ψ has X symmetry if Ψ is isomorphic to its own X image: $\Psi \sim \Psi^X$.

In affine quantum theory, which lacks unitary structure, any physical entity ε that is maximally described by a complex ψ-vector with index structure ψ^α, where α is any index, composite or simple, is part of a quartet of entities with the following ψ-vector symbols, quantum amplitudes, and interpretations:

$\langle\alpha]$	ψ^α	the entity ε itself
$[\alpha\langle$	ψ_α	the contraentity, $\varepsilon^\dagger$
$\langle\bar{\alpha}]$	$\bar{\psi}^{\dot{\alpha}}$	the antientity or complex conjugate $C\varepsilon = \bar{\varepsilon}$
$[\bar{\alpha}\langle$	$\bar{\psi}_{\dot{\alpha}}$	the anticontraentity $\bar{\varepsilon}^\dagger$

The *contraentity* has the inputs and outputs of the entity interchanged, and is described by a contragredient or dual ψ-vector. We retain separate terms for the physical concept of contraentity and the mathematical one of the dual entity only because this ubiquitous duality also occurs in many mathematical contexts where no interchange of physical inputs and outputs is involved.

The *antientity* is a C image of the entity. While the "contra" process transforms inputs into *outputs*, the "anti" process maps inputs into C-imaged *inputs*.

The operations C and † generate the four-group

$$1, C, {}^\dagger, {}^\dagger C$$

In a unitary theory, with a conserved metric δ_{AB}, this becomes the two-group 1, †:

$$\varepsilon^\dagger = C\varepsilon, \qquad C(\varepsilon^\dagger) = \varepsilon$$

and then two symbols (like Dirac's bra and ket) suffice for the four concepts. In general the invariant *value* of contravector $[\beta\langle$ on vector $\langle\alpha]$ is written $[\beta\langle\alpha]$ and is interpreted as the complex transition amplitude, vanishing if and only if the quantum jump from initial channel α to final channel β is forbidden. A destructor of an entity with creator ψ is algebraically represented by a derivation ∂_ψ (defined only in a given basis).

When ψ^A is a spinor, supporting the identity representation of SL_N, the tensor $\psi^{\dot{A}B}$ is called an ambispinor, and the Hermitian ambispinors are called sesquispinors; these, too, are representations of SL_N, complex and real, respectively, of dimension N^2. If spinor ψ^A describes a quantum entity ε, then we say that the ambispinor describes an *ε pair*, and a sesquispinor a *real ε pair.*

P1 has C (set–antiset) symmetry, in that complex conjugation respects ι. Most networks in SET obviously lack C symmetry.

A problem connected with spin and statistics arises in Section 6.3.

P1 allows T symmetry of the theory, but not of any cell. To see this most simply, consider the linear net of two links

$$xcycz$$

whose plexor is

$$\iota 1 \vee \iota\iota 1 \vee \iota\iota\iota 1 = \iota 1 \vee \iota^2 1 \vee \iota^3 1$$

Its T image is presumably the isomorphic net

$$zcycx$$

The T image of the link xcy relative to the origin y must then be the link ycz on the other side of the origin, in order to go over well into the usual concept of T in the one-dimensional Minkowski space. (In an n-adic net that has a suitable order, T forms a new cell using n outputs from n neighboring cells as inputs, and taking as output one input common to these n cells.) This makes it clear that T is not defined on a cell xcy by itself, nor in most nets, and when it is defined, it maps the output of every cell to that of another. This difference between the unicellular SL_N transformations and the multicellular T is inherent in the net concept.

In general, therefore, even if the net is T-symmetric as in the above example, T and SL_N do not commute, and the noncommutativity is proportional to the step size ℏ. We cannot yet treat the question of T conservation, the noncommutativity of T, and time translation or energy, since we do not yet have the net correspondent of the continuum concept of energy, but it is plausible that it will fail.

In the quantum theory of the dyadic cell, the fact that T is not defined on the cell is expressed by the fact that T is not represented on 2-spinors. Since such spinors describe the neutrino, it is natural to consider whether the observed T asymmetry of the neutrino and the weak interactions is due to this T asymmetry in time space structure. On dimensional grounds, ℏ should then be about the characteristic time of the weak interactions, the reciprocal of the W mass M_W, the W time T_W. Yet it appears that the continuum is a good approximation down to the Planck time T_P. From the start, the Planck time and the W time contend over which shall be closer to the chronon ƞ. But the conclusion that ƞ $\approx T_P$ is drawn from the continuum theory itself. It is based on scattering cross sections, which involve expectation values, not eigenvalues, of time space parameters. To test this conclusion significantly, we must develop the quantum theory of nets—or any contending theory—to the point where it can do physics, and show that the net breaks T (and C and P) invariance at much lower energies than it does translational invariance. In general, the larger we set the value of ƞ, the more vulnerable the theory.

4. ... AND BACK

The problem now is to find our way home, from the algebra SET of quantum networks to ordinary classical causal space and quantum field theory. We must account for the emergence of continuous time space coordinates and the spin form σ from the algebra of plexors.

4.1. Four-Dimensional Checkers

We must not only cope with nets of variable dimensionality, but also reconcile two different dimensionalities at the same time in each, the complex and the real:

1. The structure group SL_2 is that of a quantum dyad; it must therefore act on the two inputs of a dyadic cell, which is a triangle. In the main, the cells of the world have the symmetry of dyadic cells from the macroscopic point of view, though there may be higher dimensional pyramids sitting on these triangular bases supporting the unitary particle groups. In the case of a B_N, the cell group is SL_N on the inputs of an N-adic cell. The outputs, inputs, cells, and net supporting the identity (complex, spinor) representation of SL_N are called the *complex* ones.

2. Yet the world has a four-dimensional topology, at least on a certain scale of sizes; on a smaller scale, its dimensionality may be even higher, as Kaluza pointed out. In general it should be made of n-adic cells with $n = N^2$, supporting the sesquispinor (real, "vector") representation of SL_N. I call these cells *real.*

We must reconcile these two contradictory-seeming requirements by forming the real n-adic cells as subnets composed of more fundamental complex N-adic ones. Since vectors of real dimension n are made from spinors of complex dimension N, it is formally simple to make a real cell of real dimension $n = N^2$ from a complex one of complex dimension N. Indeed, the n pairs formed from the inputs of a complex N-adic cell–anticell pair (Figure 4) support the ambispinor representation of SL_N, and the real pairs (those invariant under complex conjugation) support the sesquispinor or real vector representation. The output of the real cell is the "absolute

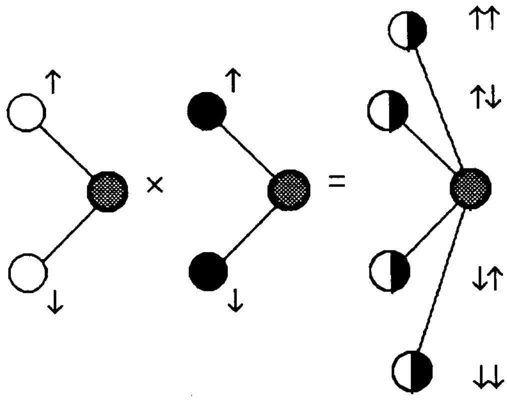

Fig. 4. $2\times\bar{2}=4$. Complex dyadic cell × dyadic anticell = real tetradic cell. Cell inputs (spinors) in white, anticell inputs (antispinors) in black, outputs (scalars) in gray, ambispinors in black and white. This product is not a dyadic net.

square" $\bar{\psi}\vee\psi$ of the output ψ of the complex one. The cell whose inputs are these n real pairs and whose output is the real output is a real cell.

But this kind of tensor product of dyadic cells is not a dyadic net. We can approach such a product within the X kinematics as a kind of product $V := 2[\bar{2}]$, where in general the product $W[V]$ of nets is defined by identifying each primitive input δ of the net V with the output of a replica of the net W. If W and V are dyadic nets, then so is $W[V]$.

In particular, $2[\bar{2}]$ is the four-input dyadic net of Figure 5. Its four inputs transform according to the "vector" (sesquispinor) representation of SL_2 provided we transform all three dyadic cells in V by the same SL_2,

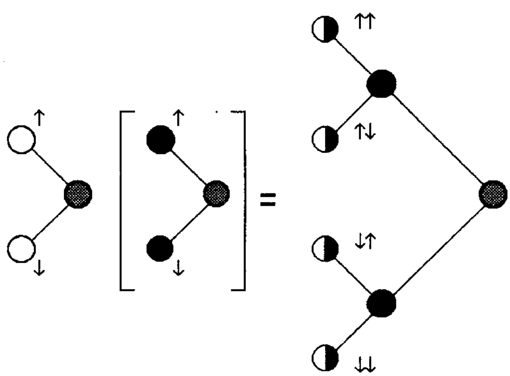

Fig. 5. $2[\bar{2}]=4$. A dyadic cell times a dyadic anticell equals a four-input dyadic net that transforms as a sesquispinor only if the groups of the two cells on the left are identified. Shading as in Figure 4. This is the unit cell of the net $\mathbb{Z}^4$.

suspending the gauge invariance that permits us to transform them separately. We say that these cells must have coupled spins.

The two-dimensional checkerboard $\mathbb{Z}^2$ of Feynman (Figure 3) is made up of pairs of integers $\{p, q\}$, with $\{p, q\}$ connected to $\{(p+1), q\}$ and $\{p, (q+1)\}$:

$$c\{p, q\} = \{\{(p+1, q\}\} \vee \{\{p, (q+1)\}\}$$

We make a four-dimensional causal net $\mathbb{Z}^4$ out of purely dyadic cells using the cell of Figure 5. Instead of $\mathbb{Z}^2$ we use $\mathbb{Z}^4$, the integer tetrads $\{p, q, r, s\}$. We suppose arbitrarily that the origin 0 is an event, not an antievent. We connect any event to two antievents obtained by stepping up either of the first two components p, q. We connect any antievent to an event obtained by stepping up either of the last two components r, s. That is, an event $\{p, q, r, s\}$ is connected to the two antievents $\{p+1, q, r, s\}$ and $\{p, (q+1)q, r, s\}$. An antievent $\{p, q, r, s\}$ is connected to the two events $\{p, q, (r+1), s\}$ and $\{p, q, r, (s+1)\}$.

It follows that the odd tetrads $\{p, q, r, s\}$ (those with $p+q+r+s$ odd) are antievents and the even are events proper. The cell is dyadic, and the net is clearly four-dimensional.

How do cells come to connect with anticells in this way? How do such pairs then connect to form a world like ours? These are dynamical questions, and we return to them in Section 4.3.

4.2. Breaking Superposition

We now know how to carry each of the following entities from the complex realm of N-spinors to the real one of n-vectors with $n = N^2$:

An individual N-spinor ψ becomes an n-vector $v = \bar{\psi} \vee \psi$.

The complex linear space of spinors becomes the real linear space of real vectors.

The N inputs of a complex cell become the n inputs of a real one.

The ray containing any product (or "simple") element of grade N in the Grassmann algebra of quantum sets becomes the ray containing a product element of grade n in the real Grassmann algebra over the real sets.

But there is no way to carry the general N-grade element, a sum or superposition of such products, or the general superposition of cells.

This seems to violate the principle of quantum superposition, an important clue to the further development.

4.3. The World As "Superconductor"

Let us now seek the quantum topological origin of the principle dynamical variable, the spin form σ: $v \mapsto [\![v]\!]$, giving the chronometric and gravitational structure of the world.

The transformation law of $[\![v]\!]$ is that of a ψ amplitude for a real input pair in the sense of Section 3.1. $[\![v]\!]$ itself is a dynamical variable—an observable, however, not a quantum ψ-vector; a crucial difference. The only way open to make such a variable from a ψ-vector is from a large set of quanta. There seem to be two routes, the incoherent and the coherent. We must choose whether the sesquispinor $v_{\dot{A}B}$ is:

1. An *incoherent* statistical description of a set of inputs, like a statistical operator $\rho_{\dot{A}B}$ or the Thomas–Fermi statistical density

$$\rho = \bar{\psi}\psi$$

constructed from the ψ functions of the electrons in a large atom. Then vector addition is incoherent superposition or mixing of ensembles. Or:

2. A *coherent* description of a set of real input pairs, like the macroscopially observable wave function ψ giving the ψ-vectors of the helium atoms in a superfluid, or the electrons in a superconductor. Then vector addition is coherent superposition of quantum ψ-vectors for an underlying quantum entity corresponding to a helium atom or electron pair.

A large set of inputs may "materialize" a ψ-vector for one of them into a macroscopic dynamical variable ψ. If the set is of input pairs, the emergent variable ψ may be coherent; if it is made up of single inputs, ψ must be incoherent.

The crudest way—and we are hardly equipped for anything refined, groping so far beyond our reach—to associate such a set of quanta with an edge is to let the edge *be* the set. Since a real edge transforms as a *pair* of complex inputs; this points to the coherent route, which likewise deals with pairs.

Moreover, there probably is no incoherent route at all. Even in affine quantum theories, variables X are still represented by linear operators X^A_B and a statistical operator must still have the index structure ψ^A_B, so that it may yield the expectation value $\langle X\rangle_\psi = \psi^A_B X^B_A$ for any variables X. In an affine theory an operator ψ^A_B is altogether different from a form $\varphi_{\dot{A}B}$. Nor is it permissible to use the experimenter's time axis $v_{\dot{A}B}$ to relate the two; such an explicit dependence on the experimenter is just what is meant by a violation of symmetry.

The coherent route offers much less resistance. It directly relates the Hermitian tensor $v_{\dot{A}B}$ to the set of pairs that makes up v.

In affine quantum theory there is no invariant way to go from the general plexor $\Psi_{\dot{A}B\dot{\Gamma}\Delta\cdots}$ for a quantum set of pairs, which has $2p$ lower indices, to such a one-pair tensor $v_{\dot{A}B}$, which has only 2 lower indices, just as the general many-pair electron ψ-vector does not determine a one-pair ψ-vector. So we do not have a system of time-space coordinates defined for all events in all nets.

But we should not expect such coordinates since only the quantum condensation of the network exhibits a manifold topology. It is sufficient to go the other way, from $v_{\dot{A}B}$ to a many-pair plexor; schematically

$$\Psi = \prod_n (v_{\dot{A}B}\varepsilon^{\dot{A}_n}\varepsilon^{B_n})$$

where $n \to \infty$ labels a sequence of nearly isomorphic replicas of the N^2 pairs $\varepsilon^{\dot{A}}\varepsilon^{B}$. Many-pair plexors $\Psi_{\dot{A}B\Gamma\Delta\cdots}$ that do not come from such a one-pair v simply do not represent vectors in a classical causal space, but some less ordered time space, though one that is still causal. The emergence of continuous coordinates on a net depends on the existence of a certain kind of long-range order in the net.

Again, the seeming violation of superposition is the counterpart of what happens in superconductivity when a microscopic ψ function becomes a "macroscopic ψ function." The emergence of the metric from the condensation is further discussed in Section 4.3.4.

We infer that the familiar causal space is a low-temperature (high-coherence) crystalline phenomenon akin to superconductivity; we call it ether. This is our second physical principle:

P2. Each world vector v statistically represents a large coherent ensemble of real pairs of complex events described by one sesquispinor $v_{\dot{A}B}$, which may be interpreted as the local spin metric.

Either the existence of such coherent structures will follow from dynamics, or it will be inconsistent with dynamics. In either case, P2 must disappear from the theory as a separate postulate, surviving only as a rule of interpretation, telling how this theory accounts for ordinary world vectors, their spin metrics, and the spin form σ that gives rise to gravity. In the meantime P2 has many favorable implications for further development:

4.3.1. Real Coordinates

We can at last hope to understand the mysterious reduction of complex ambispinors to real sesquispinors that one routinely performs at the very outset of spinor algebra ever since the spinorial studies of Cartan and of Infeld and van der Waerden. If Minkowski vectors are wave functions of a complex quantum theory, they should be complex. Now this break in gauge invariance signals spontaneous symmetry breaking and something like a superconducting transition.

4.3.2. Real Gauge Fields

The groups of these relativistic quantum models are all linear groups SL_N, while the particle groups are unitary SU_N. If one applies the usual

Lie-algebraic symmetry considerations, the gauge fields of the theory should all be complex bosons. We see the real parts daily in the laboratory; where are the imaginary parts?

This second break in gauge invariance also signals something like superconductivity at once. Like the reality of Minkowski space, the reality of the gauge fields is a spontaneously broken symmetry.

Nor is it any use looking for the imaginary parts of the gauge fields in tohu. That would be like looking for the Burgers vector of a crystal after the crystal has been gasified. The gauge fields describing transports of ψ-vectors from cell to cell are born of the long-range order of ether and disappear with it.

4.3.3. *The Law of Inertia*

The notable absence of any scattering by the net at available energies, leading us to believe that the cell size is much smaller than it actually is, is presumably also a result of this condensation.

This quantum condensation of bosonlike pairs somewhat resembles superconductivity, although an electron of a superconductor is paired with an electron with reversed momentum and spin some distance away, while an event input is paired with its C-transform (complex conjugate) in the same cell. Like a superconductor, ether defines no local rest frame for its excitations.

Presumably ether undergoes a phase transition at a temperature on the order of $1/\text{Ꞃ}$ to a differently ordered phase, and ultimately, if not at first, to the most disordered phase, tohu.

The analogy to superconductivity may be fertile. For example, it tells us to look for a Meissner effect. The Meissner effect gives a mass and range to the photon, and its analogue would give some quantum its mass, too, though which we cannot yet say.

Nevertheless, the phenomenon, of course, is not superconductivity, a persistent electrical flow in a material lattice, but a persistent material flow in the event net, or inertia. It is the law of inertia that fails in tohu; bodies do not follow geodesics there because bodies and geodesics cease to exist.

4.3.4. *The Quantum Metric*

We indicate qualitatively here how a quantum condensation of event-antievent pairs may give rise to an effective quantum metric and a unitary quantum theory. We consider a tangent vector $v=(v^m)$ as defining an excitation or defect described by a quantum condensation of pairs with a macroscopic ψ-vector $\mu=(\mu_{\dot{A}B})$. The spin form relates the ψ-vector μ to the tangent vector: $\mu = \sigma \cdot v = [\![v]\!]$. If a quantum pair (=quantum+antiquantum) creation $\mu^{\dot{A}B}$ is followed by a quantum destruction φ_B, the resultant is the creation of an antiquantum described by the ψ-vector

$$\psi^{\dot{A}} = \mu^{\dot{A}B}\varphi_B$$

The pair ψ-vector $\mu^{\dot{A}B}$ itself has the structure of a contrametric, mapping a contravector into an antivector. We suppose this mapping is a positive (definite) Hermitian tensor, so that it has a positive inverse $\mu_{B\dot{A}}$, which has the properties of a metric.

It remains to show that μ actually functions as a probability metric. By the quantum law of large numbers, we can express the general probability in terms of transitions of probability 1. Thus, it suffices to show that in a net of condensed pairs described by a macroscopic ψ-vector $\mu^{\dot{A}B}$, the transition $\psi \to \varphi$, representing an experiment with input ψ and output φ, is *compulsory* (has probability 1) when

$$\bar{\psi}^{\dot{A}} = \mu^{\dot{A}B}\varphi_B$$

or simply

$$\psi^A = \varphi^A$$

where we use μ to lower, μ^{-1} to raise, and C to conjugate indices as needed.

Now it takes more experimental repertory to verify that a transition has probability 1 than 0. It is necessary to have not only a quantum channel from input to output, but also a counting channel or its equivalent. The difference corresponds to that between an uncalibrated galvanometer, used for null detection, and a calibrated ammeter, used for intensity measurements. The concept of compulsory transition cannot be expressed in terms of affine (nonmetric) concepts without reference to the calibration process; an additional correspondence must be made between the mathematics and the physics to specialize from the affine to the unitary quantum theory.

We suppose here that the condensed medium plays the role of the extra channel. We stipulate that to verify that a transition has probability 1 means to perform the transition many times and verify that the network remains statistically unchanged. From this constancy we infer that all input quanta must be collected at the output, and that each transition therefore has probability 1.

This seems to mean that the input vector ψ and the output contravector φ in a compulsory transition must be a pair that is correlated by the condensation, so that the removal of any number of such pairs leaves the net μ unchanged. Further development of this proposal that the quantum metric is a macroscopic ψ-vector for pairs must wait upon a fuller description of the condensation.

5. TOPOLOGICAL DYNAMICS

The postulate P1 is kinematical, and P2 is statistical. We come now to dynamics.

5.1. Action Principle

The problem that faces us now is a traditional inverse problem. The action density R for classical hypergravity is to be regarded as a phenomenological action for the ether, an infinitesimal and untypical sample of the entire phase space. We must infer a quantum action from this sample; we must, as it were, see through the transition to the microscopic action $S[\psi]$ valid for all nets. We have the form of R for all values of $N = 1, 2, \ldots$ to guide us; it is quasilinear of differential order N in the spin form σ. This suggests that the quantum action couples $N+1$ events, which presumably are those of an N-cell. Therefore, the quantum action may likely be expressed as a sum over cells. The volume element with which R is integrated is formed from the determinant of the spin form (S. R. Finkelstein, 1987).

There has been no progress with this inverse problem to report here. Let us turn to the direct problem, proposing a quantum action and deducing a phenomenological one from it.

What general form shall the dynamical law for a quantum net take? Since the fundamental variables, representing the outputs of the net are odd (Grassmann, Fermi), it is convenient to suppose that the creation and destruction operators for at least some odd quanta, whether quarks, leptons, or still more fundamental ones we cannot say, are linear combinations of output—and more generally, simplex—creators and destructors. That is, creating a certain quantum, say a neutrino, means adding an event to the world net.

The most powerful dynamical principle we have for a field ψ of odd quanta is the functional one: The vacuum-to-vacuum transition amplitude $[\text{vac}, +\infty\langle U[J]\langle \text{vac}, -\infty]$ in the presence of external odd sources $J = (J(x))$ is a Weyl–Fourier transform of the classical action $S = S[\psi, \psi^\dagger]$:

$$[\text{vac}, +\infty\langle U[J]\langle \text{vac}, -\infty] = \int\int [d\psi] \exp i\left[S + \int (dx)\psi J\right] \qquad (2)$$

with $J = -i\partial_\psi$. Here ∂_ψ, the Grassmann derivative with respect to ψ, is defined (like any partial derivative) not relative to the individual variable ψ, but only with respect to an entire first-grade basis. As ψ ranges over a first-grade basis for a Grassmann algebra, ∂_ψ ranges over the dual basis for the dual algebra. The dual variable $\psi^\dagger$ is cogredient with ∂_ψ but is a first-grade element of the same Grassmann algebra as ψ.

Let us adapt this dynamical principle to the present theory. We have already incorporated nonunitarity, replacing the usual Dirac bras and kets by a vacuum vector and covector.

What net corresponds to the vacuum in the action principle? The vacuum ψ-vector is Poincaré-invariant; this is appropriate to an infinite-time continuum, which we have already atomized, and which is bounded in cosmogony. There is a conspicuous net to take its place: the null set. In the Grassmann algebra SET the null set is represented by the zero-grade plexor 1 (the c-number unit). Let us replace both initial and final vacuum ψ-vectors by 1 in (1).

This means that at the beginning of an experiment a maximal experimenter creates the causal space in the experimental region event by event, and at the end of the experiment counts (or "destroys") these events one by one.

5.2. Action

Any candidate for the plexic action χ must account for the usual classical causal space. The action must account for bosonization and condensation.

To infer an action, let us consider the present theory as a "topological relativity" that relates to general relativity somewhat as general relativity relates to special, in the following sense.

The infinitesimal variations of general relativity vary the Minkowski chronometric of special relativity, and therefore demand chronometric variables, but respect the world topology. Similarly, those of a topological relativity vary the topology of the time space and therefore demand topological variables.

Now the action density R of gravity theory is sterile (leads to no equations of motion) in special relativity, where we fix the chronometry, but fertile in gravity theory, where we vary the chronometry. By analogy let us therefore seek an action $S[\psi]$ for quantum topology that is sterile in gravity theory, which fixes the topology during dynamical variations $\delta\sigma$ of the spin form, but fertile in quantum topology, which varies the topology. That is, just as the integral of R is a differential invariant in special relativity, $S[\psi]$ must be a topological invariant in general relativity. To make sense of this, we define this topological invariant first for the differential manifold, where it is required to be a functional of the curvature form, and then translate it into quantum net language.

Finally, in gravity theory we discover that what seems nongravitational, like the electromagnetic field, may simply be a gravitational variable restricted to a manifold of special form, Kaluza's. Analogously, we suppose that all the seemingly nontopological variables in the world are actually restrictions of topological ones. Connections are everything.

5.3. Causal Homology

It is therefore crucial to make use of the proper physical topology. The usual combinatory topology is founded on a symmetric concept of connection derived from experience with Riemannian and ultimately Euclidean geometry. It assumes the existence of spatial connections and puts them on the same footing as timelike ones, when (in the absence of any signs of tachyons) it is quite doubtful that either exist at all. A relativistic topology should deal with causal connections among events, not spatial connections among objects.

At the continuum level, the Alexandroff topology is already suitably relativistic. It is thus only necessary to construct a relativistic discrete topology and homology theory on the basis of the causal connection **c**.

The simplest way to make a relativistic homology is to restrict the concept of topological cell to what is supposed to exist: the causal cell. Henceforth, when we say edge, triangle, or simplex in general, we mean a *causal* edge, triangle, or simplex, with exactly one output vertex. The contracell is dually defined, with one input vertex. We call the resulting topology the *causal topology*. It is obvious now what definitions must be framed for the concepts of causal chain (linear combination of causal simplices), causal boundary (the part of the usual boundary composed of causal simplices), causal cycle (causal chain without causal boundary), and causal homology (causal cycles modulo causal boundaries) of a net.

Consider, for example, the simplest topological invariant that may be expressed in terms of the curvature form, the Euler (characteristic) class $\chi_E[\psi]$. It involves numbers of events, edges, triangles, and so forth. In the quantum theory these numbers are expressed as products of creators and destructors like $N = \psi\partial_\psi$. In the action principle, the destructor ∂_ψ is replaced by a surrogate variable $\psi^\dagger$ which anticommutes with ψ (unlike ∂_ψ, which obeys the canonical anticommutation relation with ψ). Similar surrogates are introduced for the higher rank elements of SET, representing the destruction of higher dimensional cells. The "classical" action χ_E that might appear in the quantum action principle is then the alternating sum

$$\chi_E = \sum (\psi\psi^\dagger) - \sum (\{\psi\varphi\}\{\psi\varphi\}^\dagger) + \sum (\{\psi\varphi\chi\}\{\psi\varphi\chi\}^\dagger - \cdots$$

over all faces $\{\psi\cdots\varphi\}$ of the net.

This is a promising start. In the quantum theory the Euler class appears as a free or bilinear action (the first sum), a direct Fermi interaction (the second sum), a direct sextic interaction (the third term), and so forth. From a distance the first two terms even resemble model actions for superconductivity.

But χ_E is too different in detail from the action principles for gravity and electrodynamics to survive. The electromagnetic action S_{EM}, for example, is the sum of a positive-definite term, the kinetic action $\sum E^2$ of the electric field E, and a negative-definite one, the potential action $-\sum B^2$ of the magnetic field B. In it timelike and spacelike simplices appear with opposite sign, the kinetic term arising from timelike triangles and the potential from spacelike. The same opposition of kinetic and potential actions appears in gravity theory between the electrogravitic field and the magnetogravitic field.

The Euler class ignores this basic difference between time and space. The first sum in χ_E is over *all* vertices; the second over *all* edges, although each N-simplex N has only N timelike ones and $N(N-1)/2$ spacelike; and so forth.

In many Riemannian theories this distinction between time and space is temporarily eliminated by analytic continuation $t \to it$, which turns a Minkowskian geometry into a Euclidean one. This device is limited to the Riemannian theory of causal space and fails in Bergmann manifolds B_N with $N > 2$, and so will not be used here. Our action principle must recognize how becoming differs from being, and time from space, and thus how the group SL_N acts upon net elements.

When the sums in the Euler characteristic are restricted to causal cells, we call the resulting class χ_C the *causal Euler characteristic.* The *causal* Euler characteristic is the relativistic concept; the usual Euler characteristic is the prerelativistic one.

This change in the concept of a cell has no effect on the first sum in χ_E; but greatly curtails the second, omitting all but N of the $N(N+1)/2$ classical edges; and still more the following sums.

Example. The *toroidal net* N_{pq} obtained by identifying the left-hand edge of the $p \times q$ checkerboard net with the right-hand one, and connecting the futuremost edge to the pastmost edge, has causal Euler characteristic $\chi_C = 0$ for all $p, q = 0, 1, 2, \ldots$.

It seems that the causal Euler characteristic of a net c is invariant under refinement in much the way that the nonrelativistic Euler characteristic is. Inserting an event in an edge increases the number of events and edges both by 1, leaving χ_C invariant. Connecting an input event to the output of a triangle to produce a tetrahedron increases both the number of edges and tetrahedra by one, but the number of triangles by two; and so forth. Evidently the Euler characteristic of a network may be decomposed into two terms, the causal Euler characteristic and what is left over, arising from spatial simplices, which we may call the spatial Euler characteristic.

A second possibility for the action is the *spin class* χ_S, the topological class that is associated with the present structure group SL_N and its curvature form, in much the way that the Chern and Pontrjagin classes χ_C and χ_P, for example, are associated with their respective structure groups $GL(n, \mathbb{C})$ and $GL(n, \mathbb{R})$ and their curvature forms. [For the latter classes see Eguchi *et al.* (1980).] One expects a relation of the form

$$\chi_S := \det\left[1 + \frac{1}{2\pi i} P\right]$$

to hold, where P is the spin curvature form, and the products implied by the symbol det are simultaneously Grassmann products $\vee$ for the time space indices and contractions for the spinor operator indices, as usual. (We provisionally copy the factor of $2\pi i$ from the known classes.) The relation of the spin class χ_S to the causal Euler characteristic χ_C or to the causal homology of the net is still unknown. It will ultimately be easier to express χ_S and χ_C in terms of yes-or-no net connections than, say, the Regge action.

More generally, any invariant polynomial in the curvature form gives rise to a topological invariant upon integration. A somewhat simpler action than χ_S is the determinant of the spin curvature Ρ (capital rho) itself,

$$S = \det \mathrm{P}$$

In any case, a proposed action principle for the world net has the form:

P3. $$[1\langle U[J]\langle 1] = \int\int [d\psi] \exp i\{\beta\chi[\psi] + \sum \psi J\}$$

where χ is a causal topological invariant of the Bergmann manifold expressed in terms of net variables, such as the causal Euler characteristic or spin class. Here β is a pure numerical constant.

We must return home to the dynamical principle [Section 5.1] of physics in Minkowski space when we replace the null set in P3 by the plexor representing the ether. It is clear how the interval $d\tau$ and the geodesic principle must arise from the net dynamics. In a net that approximates a B_N, the action for a net defect that is concentrated upon and defined by a world line, by SL_N invariance, will be proportional to some integral $\int S_{\text{eff}}\, d\tau$ of an invariant scalar effective action $S_{\text{eff}}(dx/d\tau)$. The matrix $v = (dx_{\dot{M}N}/d\tau)$ can be transformed by SL_N to the diagonal form $\text{diag}(+1, \ldots, +1, -1, \ldots, -1)$ and so no continuous invariant scalar can be made from $dx/d\tau$ except a trivial constant scalar $-M$, which appears as the effective negative mass of the defect; this is a generalization of Eyl's deduction of the geodesic principle for singularities in general relativity from the field equations for gravity.

Actually, Minkowski space requires an infinite of finite cells, which is somewhat against the spirit of this theory, and may even make some infrared properties blow up. Sometimes a compact universe is a safer place for particle experiments. The compact unitary group space U_N is an N-adic Bergmann manifold, an n-dimensional causal space, that is the most symmetric possible compact substitute for the noncompact Minkowskian space. It and its neutrino spectrum have been explored by Holm (1987) with interesting results.

5.4. Fundamental Time

Since σ_S is an integer, the constant β in P3 is defined only modulo 2π. It is tempting to set $\beta = \pi$ to eliminate the explicit appearances of i in P3, including the one in J. Then

$$\exp\{i\beta\chi_S[\psi]\} = (-1)^{\chi_S}$$

Let us leave β undetermined for now, noting, however, that β may be assumed to be of order unity.

The scale of the fundamental time ɲ then depends on which of the couplings or masses is also of order unity in net units, where $\hbar = c =$ ɲ $= 1$. Since the σ field leads so directly to the chronometric tensor field $g_{mn\ldots p}$, it is at first natural to expect that it is the gravitational coupling constant that is of order unity in natural units, of all the coupling constants we know. Then the fundamental time ɲ would be on the order of 1 planck. The other possibility suggested by the development so far is that the n is the scale on which T conservation breaks down, which is closer to 1 fermi than 1 planck. The absence of scattering from the net between 1 fermi and 1 planck is then to be explained as a result of the same quantum condensation that gives rise to the causal continuum. Presumably electrons in a superconductor hardly see their lattice, too. This decision rests ultimately on the action principle and its propagators, to which we now return.

6. PARTICLE SYMMETRIES

6.1. Inner versus Outer

It is natural in this scheme to ask: Do the internal unitary groups U_1, SU_2, and SU_3 of the standard model act on *inner* variables or on *deeper* variables?

The "inner" nonspatial symmetries are those inherent in the concept of the Bergmann manifold and in the Kaluza approach to unification (Section 2.2).

The "deeper" ones come from the decomposition of the inputs of the time-space net into their elements (Section 3.1). This possibility does not exist in the manifold theory, but arises in any combinatorial theory.

In the present theory, for example, SL_N groups indeed exist at levels below (in the sense of rank or elementhood) the causal space itself. If we expand an input event δ of a cell in the time-space net, we generally find that it is also an output of other cells:

$$\delta = \alpha' = \iota\delta'_1 \vee \cdots \vee \iota\delta'_{N'}$$

with some N' "deeper" inputs $\delta'_1, \ldots, \delta'_{N'}$. The deeper group $SL_{N'}$ acting on and mixing these deeper vertices leaves δ invariant.

Surely it is one charm of postulate P2 that it avoids this complication. For P2 tells us that deeper vertices are simply past events and deeper variables are past variables, and they split into internal and external ones just as the present ones do. The deeper variables that a nonspatial unitary group might act upon are merely the internal variables of the immediate past.

6.2. Unicellular versus Multicellular Transformations

Since rotations and boosts are represented within one cell while time translation and reflection are multicellular, it is natural to ask whether the unitary particle groups act within one cell or across many.

If the cell is binary, we have exhausted its resources with the Lorentz group. There is no room in the binary cell for another nontrivial unitary group commuting with SL_2. If the unitary group is monocellular, the cell is not binary.

Our first supposition, in accord with the idea of Kaluza, is that the cell has extra inputs which the unitary particle groups permute. This provides a monocellular representation.

To be sure, as the theory develops one becomes increasingly aware of multicellular entities such as defects and dislocations on which the unitary particle groups might act. The simplest such possibility is the contracell of Figure 1, composed of an event and its outputs, which generally belong to two other cells. Even in an X network, there is an entire SL_2 acting on these outputs, and an SL_3 permuting them with the event itself. This group tempts us to identify the two outputs with up and down flavors, and the event itself with a strange flavor. Unfortunately this hypothesis then requires us to explain the observed electromagnetic asymmetry between up and down flavors in the contracell, which, though slight at high energies, is much greater than the spatial asymmetry between up and down spins in the cell; and it leaves no equally natural place for the other flavors. This identification appears too simplistic.

6.3. Spectra

The propagators for the particles of physics must arise as matrix elements of the generating function of P3 in the Lorentz invariant ether ⟨vac] for external sources J of the particles. We shall take into account the inferred pairing only slightly in this first exploration, by confining ourselves to removing one, two, three, or four of the real inputs in a real cell, or equivalently, one or two each of the complex inputs and anti-inputs in the two complex cells that constitute the real cell. We shall grossly neglect the all-important pairing correlations within ether by casually adding and deleting events one at a time, leaving a more careful theory for the next time around.

To explore the need for inner or Kaluza inputs, let us first suppose all cells are dyadic.

The two complex inputs on each complex output we conventionally call *spin up* and *spin down.* The operation of destroying an input is, we recall, a contra-input. In a binary net, removing an input necessarily has the effect of removing an entire chain of events.

One complex contra-input is thus described relative to its cell by a chiral spinor or antispinor; the Lorentz-invariant propagators resemble those of a spin-$\frac{1}{2}$ chiral particle and its antiparticle.

Two complex contraevents in a single real cell may either be in the same complex cell, in which case the compound has spin zero, since one has input spin up and the other spin down, and the symmetric compound is excluded by the Fermi statistics; or they may be in separate cells, in which case they form an ambispinor, a complex vector, with both spin-0 and spin-1 parts.

The compound of three complex contraevents in a single real cell is defined by the fourth event and has the same transformation properties under the local Lorentz group. It is unlikely that single and triple excitations have the same mass spectrum.

A four-contraevent excitation in one real cell transforms with spin 0.

Four contraevent excitations in two neighboring real cells, however, create a spin-2 excitation if the four complex contra-inputs belong to four different complex cells.

It is natural to project that the single contra-input excitations are forerunners of a chiral lepton family, the double contrapinput compounds of a photon or vector boson family, the triple of a chiral quark family, with its three colors and flavors, and the quadruple of a graviton family. But clearly crucial degrees of freedom are missing, as anticipated.

We consider next the results of inserting rather than deleting inputs. This seems necessary here even without considering Kaluza dimensions, merely to support the symmetry between events and contraevents, and between spinors and contraspinors in spin-$\frac{1}{2}$ quanta like the electron.

Adding an input to a dyadic net creates an entire chain of events, beginning with a mating of the new input with one of the old ones. There is thus a pronounced asymmetry between input and contrainput within ether.

6.4. Spin–Statistics Problem

This asymmetry is aggravated in polyadic nets. If we add an input to (say) a dyadic cell, the original two inputs transform under SL_2 as a spinor, while the new one seems to be a scalar. The triad thus forms a *mixed multiplet*: one that combines representations of integer and fractional spin. (We do not speak of supermultiplets because all our events anticommute.) The $N+1$ events of an N-adic cell also form a mixed multiplet in this sense, since the output supports the scalar representation of SL_N and the N inputs the identity of hyperspinor representation. Moreover, the N inputs themselves are mixed from the point of view of SL_2, consisting of $N-2$ scalars and two spinor components. Several mixed multiplets appear in the standard model. Under weak SU_2, a lepton decomposes into a scalar dextrolepton (like the dextroelectron) and a spinor levolepton (like the levoneutrino and the levoelectron), all of Fermi statistics. Since that SU_2 has nothing to do with spin, the Fermi statistics raises no problems there as it does in the plexic model.

We must therefore doubt the Fermi statistics of all these components, which would violate the spin-statistics connection enough to kill the theory, now that these effects are associated with W masses rather than Planck masses.

One way to reestablish the spin-statistics connection is topological. For topological solitons in tensor (nonspinor) fields, the spin-statistics connection follows directly from topological arguments based only on continuity. For this fact to be directly applicable to our predicament, all laboratory particles would have to be topological solitons in the net.

This possibility is not yet readily excludable. All the spins $\frac{1}{2}$ in the world might in principle be global topological effects, like the spin-$\frac{1}{2}$ of some rigid point structures (Finkelstein, 1955), of skyrmions (Skyrme, 1961), or of topological charges [the "M-geons" of Finkelstein and Misner (1959)]; I had hoped so myself until 1965. But that would eliminate the deep origin provided by Bergmann (1957) for gravity and causal structure, and might mire us forever in the infinities of topological field theories. In any case, this possibility reminds us that the relation between the commutation relations of the variables and the statistics of the quantum is not always simple. It is further complicated here by the presence of two commuting Lorentz groups, on inputs and outputs, at each event. This makes it necessary to reexamine all the connections between spin, statistics, and algebraic commutation relations.

If we adhere to X networks, we avoid these vexed questions, but have to find a new substitute for Kaluza's internal coordinates. This is a critical point for further study.

7. DISCUSSION

We have set up an algebraic model of a relativistic quantum neural net and applied it to time space; not a solar plexus, but a cosmic one. Elsewhere (D. Finkelstein, 1987b) the net model of the world is compared to a relativistic quantum computer. If the Newtonian universe is likened to a cosmic clock, the net universe may be compared to a cosmic brain. Now we see that it must be a superconducting brain. For tangent vectors and spinors of classical differential geometry to arise from microscopic ψ-vectors that are cogredient but have a quantum interpretation, ordinary Minkowski space must be a quantum condensation, and one that is akin to superconductivity in pairing fermionic entities to form bosonlike ones, which then condense.

We propose a microscopic theory of the spin form and hence of gravity: the spin form σ gives the macroscopic pair ψ-vector associated with a tangent vector. The classical tensors derive their linearity, reality, and cogredience from being coherent macroscopic wave functions.

We have given a general form of action principle and some candidate actions for the net; one of these is a topological class associated with the structure group SL_N. When topology is a physical variable, it is important to choose the right one. We base the physical topology for a relativistic network on the directed causal connection, not on Euclidean symmetric or spacelike or timelike connections, which do not physically exist. This distinction is another reason to prefer the name *causal space* over the common term "space-time."

But the very form of this action principle is suspect in cosmogony, where different experimenters must inevitably work with different systems and yet must relate to each other. This seems to call for an extension of relativity (D. Finkelstein, 1987b), omitted from the theory presented here, for example the third relativity considered in Finkelstein 1987b.

Nevertheless it is possible that the present considerations, conservative as they may be, are a step towards such a further relativization. Extensions of relativity typically evoke new physical degrees of freedom—as a relativistic theory of the electric field calls in the magnetic field, and a relativistic theory of the electron brings out its spin. Since third relativity involves tearing systems apart, the new degrees of freedom it evokes may well be topological.

In any case, it seems *a priori* improbable that any of the action principles given here is right; they are simply the first we have been able to construct with plausible physical content and symmetry properties. They at least replace the daunting inverse problem by more amenable direct ones. If one of them turns out to work, this must be regarded as strong upport for the network theory; less so, if some linear combination with adjustable coefficients is required.

The quantum network is another quantization of gravity, an alternative to the continuum-based canonical quantization. Even if the net theory of gravity is right, it is still meaningful to quantize canonically the gravitational spin form σ, or even the chronometric tensor g_{mn}, if it can be done. It would be like quantizing hydrodynamics, which tells us about phonons, if not much about water molecules. When canonical quantizations of gravity yield finite results, they inform us about small gravitational disturbances of ether, if not of tohu, with periods much longer than п. But just as we need the quantum molecule for the physical constants and phase transitions of water, we need the quantum net for the physical constants and phase transitions of the time space net, including the particle masses and couplings.

Earlier discussions of vacuum phase transitions, as in inflationary cosmology, have generally been phenomenological, postulating an order-parameter field, but not the structure that is ordered. The present theory is fundamental rather than phenomenological, in that it gives the microscopic structure of the medium that is being ordered; the order parameters remain to be determined.

We have consolidated most of our theoretical debts with one promissory note, the quantum condensation. Now we must work to pay off the note. How do we return to the manifold description when it is valid? What are the possible phases of the net? What is the excitation spectrum of ether and to what particles do they correspond? Why is time space so stiff (or, as it is usually put, gravity so weak), with Planck time $T_P \ll п$?

We are somewhat in the position of electrons in a superconducting crystal trying to perceive the elementary cell. Above the transition temperature we would know our crystal first-hand because it would scatter us; below it we must proceed by rather indirect inference. It will be easier to find the excitation spectrum of tohu, where long-range correlations may at first be neglected.

The T asymmetry of the polyadic network may be merely a matter of mathematical inertia and lack of imagination; the asymmetric operator ι is simply the first one at hand. Bennet (1973) and Fredkin and Toffoli (1982) have defined T-symmetric computers, and we have here offered a relativistic T-symmetric dyadic ("X") quantum net kinematics.

The cell remains T-asymmetric. If the T violation of the weak interactions comes from the T asymmetry of the cell of this net, then the cell lifetime and size are the characteristic time and length for the weak interactions, not the Planck length and time, and the transition temperature is the characteristic temperature of the weak interactions, of order 100 GeV or 10^{15} K. A phase transition to ether of another dimensionality is conceivable. The tohu phase of the net is still a causal space, but might just as well have a continuous spectrum of fractal dimensions, depending on temperature and scale, as some definite manifold dimension. Phase transitions also prevent the singularities predicted by the Einstein equations from ever forming.

By ordinary standards ether has a reassuringly high transition temperature. Ordinary macroscopic mechanics is well tested in planetary mechanics at an ambient temperature of several degrees kelvin, which is about 10^{-14} on the weak scale. Even those ether properties that are not exactly zero but temperature dependent may be so small in interplanetary space that they could be present and yet be overlooked in experiments so far. It will be interesting to look for them, theoretically and experimentally.

By particle physics standards, however, 100 GeV is a chilly transition temperature indeed. It suggests that network phase transitions are going on in many experiments today, producing small, short-lived balls of tohu within the ambient ether.

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