From Time and Chance to Conciousness

Studies in the Metaphysics of Charles Peirce

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8

The First Flash of the Big Bang: The Evolution of Evolution

DAVID FINKELSTEIN

WHILE MANY HAVE PHILOSOPHIZED about the quantum revolution after the battle, a select prophetic few fought some of its campaigns in advance. The most famous examples are Francis Bacon, with his non-Aristotelian statistical logic of experiments, Newton, the non-Newtonian, with his interfering guide waves and his Old Unitarian deity of limited but evolving knowledge; and Charles Peirce, with his fundamental randomness, his evolutionism, and his real potentialities.

These three were well ahead of their century in this respect, but Peirce is possibly ahead of ours as well. Since physics does not yet have a unified cosmology, and Peirce attempted to make one, I was led to inquire whether we still have something to learn from him. In what follows I compare Peirce's philosophical cosmology with our present physical cosmology, to the extent that the differences in purpose, language, scope, and method permit. In several ways physics has become more Peircean than Peirce, and in one way physics still falls short of his expectations.

Classical versus Quantum Pragmatism

Since pragmatism is a theory of meaning, an epistemology, it must compete for survival with the quantum epistemology that Bohr proposed as the foundation of the quantum theory. Peirce's epistemology emphasizes the *pragma*, the deed, in the sense of a practice of the scientific community that gives a concept its meaning. He thus gives more weight to the deeds of scientists than to their words, as did Einstein later in his famous aphorism (roughly): Pay attention to what physicists do, not what they say. It was Einstein's emphasis on action that Heisenberg in turn consciously imitated when he invented quantum mechanics. Peirce's pragmatism is the forerunner of Einstein's and Heisenberg's operationalism, upon which Heisenberg and Bohr founded their quantum epistemology.

As a result, pragmatism and quantum epistemology share some family problems. Ordinarily there is a certain duality between actions and objects: we combine them in pairs, one action and one object, to make meaningful statements. Making *pragma* primary makes an object appear as a class of actions (those that produce it, say). This created the problem of objective beings for Peirce. How can one account for the remarkable suchness we find in nature, if things are merely classes of actions? Objective beings seemed indispensable to Peirce, but also implausible because of the acausal elements of his system (see tychism below). In one of his systems, an objective being arises when many infinite series of such actions converge to a consistent limit. He could not understand why such uniform limits should always exist. During his later years Peirce struggled mightily, but never satisfactorily, with the problem of objective beings.

This is not a predictive failure for him but a success. It makes him brother to all quantum physicists. Objective beings in the classical sense do not exist in quantum theory except as approximations. Quantum systems have a being that is relative, not objective. We face Peirce's problem today without the handicap of a dogmatic faith in the necessary existence of objective beings. Since there are no objective beings, we do not have to account for their existence but only for their simulation – an easier problem, to be sure, but one which is not yet fully solved.

Peirce's epistemology, however, is more deterministic and less stochastic ("tychastic," in Peirce's term) than Bohr's quantum epistemology in one critical respect. Peirce defines an individual as "determinate in respect to having or wanting each general character." Such determinate individuals are classical and do not exist in nature. The individuals that do exist, quanta such as photons or electrons, are not determinate in this ancient sense. At best they are determinate in respect to having or wanting a maximal family of general characters, a family which may vary from case to case, depending on what we actually determine about the photon. Such individuals are called sharp or pure cases in quantum theory.

It is well understood today how such pre-Peircean deterministic assumptions of determinate individuals can be removed from Peirce's system. Wherever pragmatism speaks in general terms of experimental operations or habits of action, one must refine it by taking into account the critical distinction between an input and an output operation, determinations before and after the fact, or productions and registrations. Transitions between these were assumed to be determinate by Peirce but are actually governed by the universal quantum statistical principle first noted as Malus's Law and later generalized as Born's Statistical Principle. For Peirce, as in classical thought, a variable quantity or coordinate is defined by a columnar table listing its values for each state of the object carrying the property. But in quantum theory, as one aspect of the input-output duality, a quantity is associated in general with transitions rather than states, and with a square table, as in the matrix mechanics of Heisenberg, rather than a columnar one.

Also because of this duality, in quantum theory there is a conceptual unification at a basic level of time with energy, position with momentum, and in general of pairs of quantities which are complementary in the sense of Bohr. This unification halves the work of the speculative metaphysicist. For example, an understanding of position automatically leads to one of momentum in quantum theory, but not in classical. This unification has no counterpart in Peirce's system, an omission which is readily corrected.

The formulation we are most often given of pragmatism speaks of it as a theory of meaning based on practice. It also has an evolutionary aspect, which seems to me the blood that circulates through Peirce's entire system.

Meaning, Peirce insisted, is a triadic relation between (1) the entity *E* itself, (2) the symbol *S*, and (3) the user *U*. This leaves open how this relation is to be singled out physically from all other triadic relations among these things. What does it mean physically to say that S means E to U?Peirce solves this problem with the criterion of self-reproduction. When a new symbol is used successfully, it acquires a new user and is reused. Thus the *U-S-E* relation reproduces itself from one *U-S* pair to another. For Peirce, three elements become symbol, entity, and user when and only when their relation is self-perpetuating. His physicalistic theory of meaning thus fits equally well the communication of symbols and the transmission of genetic information. Peirce provides a theory not only of meaning but also of the evolution of meaning.

The unifying theme of Peirce's architecture is his evolutionism – his belief that everything evolves, including time itself.That he may never have Grecized the word "evolution" does not mean he did not consider the concept important. Peirce's first principle of scientific nomenclature is that the inventor of a concept has the exclusive right to name it. He declined to coin a word for evolutionism because he did not invent the concept.

In compensation, Peirce claimed three forms of evolution in his metaphysical system, corresponding to the biological evolutionary theories of Lamarck, Cuvier, and Darwin, respectively. He named these agapastic, anacastic, and tychastic, respectively, and defined them to be driven in three corresponding ways: teleologically from the future by agape or love, in the present by ordinary external forces, and in the past by original chance, respectively. His theory that we evolve to greater beauty through love seems a latter-day return of the Platonic Ideal and the Aristotelean Final Cause. Its teleology anticipates the anthropic principle that some propose today and is no more helpful in making predictions.

Tychism and Quantum Uncertainty

Given the famous successes of Newtonian mechanics, it was either notably perverse or remarkably prophetic for a practicing scientist of the nineteenth century to insist that all law is fundamentally statistical and that fundamental processes are random. To be sure, Boltzmann and Gibbs had shown how the law of large numbers could lead to apparently deterministic behavior of large random ensembles, and their work must have influenced Peirce. But they built on a foundation of deterministic mechanics and used statistics only to deal with complexity. Peirce was quite insistent on blasting out that foundation and replacing it by a fundamental tychism, his belief in the sovereignty of chance (Greek *tyche*). It is natural to ask what right Peirce had to be so right.

Peirce's principal argument for tychism was that we have feelings, and that deterministic systems, being machines, do not. Similar statements have appeared since the discovery of quantum theory, invoking quantum complementarity to explain free will and the like. The possibility of a literally friendly but still deterministic computer seemed absurd to Peirce. It does not to me.

His tychism was also a natural consequence of his evolutionism. The dynamical law was no more immune to evolution than anything else in his system. But in order for law to evolve, one might reason, there must be a prior lawless phase. Hence tychism. He also inferred his tychism from the existence of variety in nature, which he could explain only by random variation. These arguments seem to express prejudices rather than inductive reasoning. Under the heading of synechism below, I mention how one may also infer a species of tychism from Peircean continua, without insisting that Peirce used this argument.

Tychastic Logical Algebra

Perhaps because Peirce gave probability such a fundamental role in nature, early in his studies he founded an algebraic logic upon it which turned out to be a significant improvement upon Boole's, and which has a peculiar relation to the logic of Fermi-Dirac quanta.

There are three plausible algebraic operations expressing some kind of disjunction or union that are commonly used to combine predicates or classes, the OR (inclusive) of Aristotle, the POR (my abbreviation) of Peirce, and the XOR (exclusive OR) sometimes attributed to Boole.

The original disjunctive operation of Boole was written Á + ß and did not form an algebra. It was defined so that its laws paralleled those of ordinary arithmetic addition, with 2 + 2 = 4. Now the union of one dyad and another is a tetrad only if they are disjoint. So Boole never added two classes unless he first made sure they were disjoint. His + was not universally defined, therefore; he did not have a true algebra of logic, closed under its own basic operations. If I write HANDS for my two hands and LIMBS for my four limbs, Boole would not write HANDS + LIMBS = LIMBS, because that would break the law that 2 + 4 = 6 (stipulating for the sake of the example that I am a biped, not an insect). Instead Boole could write HANDS + (LIMBS - HANDS) = LIMBS. Today Boole's + is often called the disjoint union.

The modern operation XOR is defined as the union less the intersection of two given classes. I mention XOR only to make sure that it is not confused with Peirce's POR. Unlike Boole's Á + ß, XOR is universally defined, and even has the group property. Therefore some use it to set up a freer algebra of classes than Boole's. Returning to the above example, HANDS XOR LIMBS = FEET. By contrast, HANDS OR LIMBS = LIMBS.

Like Boole, Peirce took his disjunctive operation, which he too wrote as Á + ß and called arithmetic addition, to parallel the addition of the number of cases or possibilities, and so ultimately of probabilities. In the case of disjoint classes, Peirce's + coincides with Boole's partial +. The difference between Boole's + and Peirce's is that

Peirce insisted that Á + ß always have a symbolic value, and Boole did not. To accomplish this closure under +, Peirce introduced an ideal element š representing the undefined, and set Á + ß = š when a and ß are not disjoint. The invention of the logical š, Peirce recognized, corresponds for logic to the invention of the zero for arithmetic. He posited that the ideal element š obeys Á + š = š. The null class asserts too much to have elements, but š asserts nothing about anything. To avoid confusion among all the meanings of +, I write POR for Peirce's disjunction; for example, HANDS POR LIMBS = š.

In some measure this invention of Peirce is subsumed in an earlier double algebra of Grassmann. Peirce's logical addition $A + B$ is isomorphic to Grassmann's "progressive product" AB, better written $A \vee B$ by Peano some years later, taken over the binary numbers (0, 1) rather than Grassmann's real numbers. The Peirce law A + š = š (for all A) corresponds to Grassmann's $A \vee x = x$ and is satisfied by $x = 0$. Peirce's š is Grassmann's 0, the identically vanishing vector of quantum logics. (In the same way, XOR is subsumed in Clifford algebra, and OR in projective geometry.)

Grassmann, however, did not give a clear logical meaning to his progressive product, but only a geometric one, though he drew the analogy between the two, nor did he go on to found a general theory of sets on his algebra. Peirce did both, in his existential diagrams, for example.

In later work Peirce lapsed from his algebraic logic to a more relational one, founded on the inclusion relation Á ⊂ ß. He defined the OR and AND of Aristotle as the least upper bound and greatest lower bound in this partial order, as is done in the lattice theory of Dedekind. What are usually called von Neumann integers in set theory were earlier Peirce's.

There are important advantages in treating XOR as a product like Grassmann rather than a sum like Peirce. For Grassmann's double algebra has its own addition A + B, besides its two products ("progressive" and "regressive"), and while Grassmann never found a general logical interpretation for his sum, quantum theory does, in the theory of classes of fermions (quanta obeying the Pauli exclusion principle).

A quantum set theory combining Grassmann's double algebra with Peirce's logical interpretation is given in Finkelstein 1987. In this quantum alternative to Peirce's existential diagrams, the quantum OR unifying Peirce's POR with Grassmann's progressive product V is called QOR.

The First Flash and the Big Bang

Peirce's tychism led him also to a cosmogony which has little resemblance to our current cosmogonic theories, but may yet be vindicated. He describes "the first stages of development, before time existed," thus:

> Out of the womb of indeterminacy we must say that there would have come something, by the principle of Firstness, which we may call a flash. Then by the principle of habit there would have been a second flash. Though time would not yet have been, this second flash was in some sense after the first, because resulting from it. . . . We have no reason to think that even now time is quite perfectly continuous and uniform in its flow. (*CP* 1.354f.)
>
> We start, then, with nothing, pure zero. But this is not the nothing of negation. . . . The nothing of negation is the nothing of death, which comes second to, or after, everything. But this pure zero is the nothing of not having been born. There is no individual thing, no compulsion, outward or inward, no law. It is the germinal nothing, in which the whole universe is involved or foreshadowed. . . . It is boundless freedom. (*CP* 6.217f.)

Peirce places his First Flash, I infer from the unquoted context of this excerpt, in a preexisting space, though "time was not formed," leaving it to chance to determine where in space the First Flash occurs. But elsewhere Peirce makes it clear in general terms that all continua, including space, must have evolved and be evolving.

The idea that time had to be created at the beginning of the world is ancient but coming back into fashion. From the viewpoint of today, his theory is less radical than one that has the origin of the universe and of space-time in a single point event, Lemaître's primordial atom (or the first word of the space-time code of Finkelstein 1969). In these later theories space as well as time continua evolve. Nevertheless it is appropriate to call the original quantum event the First Flash in his honor. Peirce's First Flash is more radical than the vacuum fluctuation of Tryon 1973, which assumes a space-time continuum and preexistent Hamiltonian; and less radical than the pregeometry of Wheeler 1973 and Misner, Thorne, and Wheeler 1973, which lack the before-and-after assumed by Peirce.

The continuum theory of general relativity ignores the finite First Flash of Peirce. The Big Bang belongs to the later stage of cosmogony postulated by Peirce when continuum concepts begin to apply. I turn now to Peirce's concept of the continuum.

Synechism and Quantum Condensation

Peirce regarded his synechism (his theory that there are real physical continua) as the deepest of his principles. His continuum concept has little to do with that of Riemann. For example, he insists that a continuum is not to be exhausted by any collection of discrete points in it. It was recognized in classic antiquity that a line is more than its points because it has a principle of connection, which today we call its topology. Peirce's inexhaustibility of the continuum seems to be a stronger, more paradoxical, claim. It is false in ordinary set theory. It seems that Peirce's continuum consists of potentialities which (unlike his deterministic individuals) have no separate identity but merge continuously into each other and is nevertheless real. He presented his continuum as an instance of "real generality," a deliberate oxymoron in that a generality, ordinarily being an abstraction from reals, is not usually considered to be a candidate for reality.

There is a striking similarity between these real generalities with merging identities and the quantum *potentia* of Heisenberg with their continuous quantum superpositions. Indeed, Peirce's idea of a varying degree of identity among different entities resembles the concept that A. Landé later took as the foundation of quantum theory. Suppose (to speak in Peirce's terms) that Á and ß are two elements of a continuum so close that their identities have begun to merge and one first carries out an operation that produces Á and then an operation that tests for ß. On the one hand, this experiment fails to distinguish Á from ßunambiguously. On the other hand, it always gives either a positive or negative result. I would like to infer that it gives positive and negative results more or less at random, with the probability of a positive result increasing to unity as Á approaches ß. I suggest that this is a link between synechism and tychism implicit in Peirce's system, although Peirce does not seem to have worked it out in detail.

Quantum aggregates in general have an inexhaustibility that sounds like that of Peirce's continua; a property of a quantum aggregate is not a disjunction of conjunctions of properties of its elements in general, as properties of classical aggregates are. Laplace's reductionism breaks down both in quantum theory and in synechism.

The most paradoxical aspect of Peirce's continuum, its real potentialities, resembles an important phenomenon that occurs in quantum condensation, which we may illustrate with the example of liquid helium. The input of a single helium atom into a box may be described by a vector Ψ. This vector is a potentiality, not a reality, in the sense that

1. It describes how a helium atom may potentially be prepared.
2. Even if we are told that the atom has been injected in a way that is described by some vector, there is no experiment we can do on the atom which will tell us what that vector is. It is not meaningful to ask what is "the Ψ" of an atom encountered in nature.
3. Given Ψ, we can give probabilities for the outcome of every experiment we can do on the helium atom. The vector represents an indeterminate identity of the kind that Peirce denied to real individuals.

If many helium atoms are produced in the same box and cooled, however, then at very low temperature practically all the atoms are described by the same Ψ. The helium is then called a *quantum condensate*. We can then determine the Ψ by measurements on a small fraction of the population and use Ψ to make predictions about the rest. It becomes meaningful to ask what the Ψ of the aggregate is. The Ψ (up to an overall phase) has become a physical reality through the quantum condensation.

Peirce's synechism does not apply to any classical continuum but sounds rather like a quantum continuum. Although Peirce does not undertake to follow the evolution of the physical continua of his synechism, he clearly considers them as having evolved and even proposes a universal teleological principle, his agape (love), to win out over tyche (chance), and permit the emergence of cosmos from chaos. Thus Peirce seems to have included geometry in his evolutionism, as something that evolves, at least in principle. In a general way, physical cosmology today seems to support this evolutionism.

Peirce seems not to have resonated to the continuously evolving physical geometry of Riemann and Clifford, and to their space theory of matter, nor to Einstein's conceptual unification of space and time, which occurred in Peirce's later years.

It has long been accepted that quantum condensations occur in space-time, and they drive the inflationary phase of the Big Bang in some theories, but it is generally supposed that the condensation takes place in a preexistent space-time continuum. The first to suggest that such a quantum process is involved in the formation of the space-time continuum itself was Weizsacker (1951, 1955), who proposed that the isotropy of the space continuum is a macroscopic quantum effect. If his idea can be validated (as I am trying to do), this might be regarded as another posthumous victory for Peirce.

Peirce is a bit like Nostradamus: it is easier to interpret his predictions after the fact. But after correcting for this effect, it is clear that Peirce imaginatively anticipated some of the most surprising elements of modern physical cosmology. And in the problem of objective beings, and of objective continua in particular, he confronts the central problem of physics today, the synthesis of the theories of space-time and quanta. I turn now to one way in which his thought may still be ahead of ours.

Nomic Evolution

The belief in a fixed primordial dynamical law still dominates physics. This law is evidently a last relic of the Neoplatonic Logos. It puts the dynamical law outside the universe, beyond our influence. The alternative is autonomy, self-governance. Are we governed by a transcendent or an immanent dynamical law?

The dynamical law may have seemed more complicated in Peirce's time, before the simplifications of quantum theory and relativity, than it does now. It was therefore natural for Peirce to suppose that it must have evolved from something simpler.

Even today some speculate on an autonomous cosmology as Peirce did in the nineteenth century. For example, Wheeler (1973) proposes that "the only law is the Law of Large Numbers" (which is not a law). And Nielsen and Ninomiya (1984) propose that the quantum dynamic law is so complicated that it is effectively random.

While Peirce's synechism described an evolution of the laws of nature from a lawless state, no such Peircean autonomy has been attained in physical theory. Physics still runs on a law which is itself not part of the physical universe. In this section I would like to join those who speculate on a more autonomous and evolutionary and thus more Peircean physics.

To permit the evolution of physical law, Peirce relied on the universal habit-forming tendency of nature. First things happen by chance, then what has happened tends to recur, and so dynamical laws evolve instead of being imposed from outside, and autonomy is possible. Peirce's autonomy is thus rooted in an inheritance of acquired characteristics, a Lamarckian or agapastic evolution, not a Darwinian one.

In physics, however, we assume only one universal evolutionary principle, which is described by the equations of Heisenberg or Schrödinger and gives rise to a Darwinian or neo-Darwinian evolution that replaces all three of Peirce's evolutions under suitable conditions. This physical evolutionary principle is monistic and so resists Occam's Razor better than Peirce's trinity. It is interesting therefore to transplant Peirce's autonomy from Lamarckian to Darwinian soil and to seek an autonomous universe without habit-forming tendency.

At first, autonomy seems to imply an infinite regression of dynamical laws, in which the evolution of one dynamical law is governed by a higher-level law, whose evolution is governed by a still higher-level law, and so on ad infinitum, an infinite dynamical pluralism that is the antithesis of monism. This leads to the *self-governance problem*: to construct a finite self-governing cellular model in which the cellular structure encodes and coevolves with its own dynamical law.

A similar problem, the self-reproduction problem, was posed and solved earlier in the century by von Neumann. We may profit from von Neumann's example.

Von Neumann set out to model Darwinian biological evolution, which comprises self-reproduction, random variation, and natural selection. The critical process is self-reproduction. It has been suggested that each organic cell includes an internal symbolic representation of itself, which is used in self-reproduction. This implies a Peircean *U-S-E* triad in each cell.

At first. self-representation too seemed to imply an infinite regress. If the organism includes a representation of itself, must not this representation also be represented, and so on ad infinitum?

Von Neumann's strategy to break the infinite regression was to have the cell represent only as much of itself as was necessary to reproduce itself, avoiding redundancy. Evidently to reproduce a cell we do not need a representation of its internal representation, which would be redundant, but only of the rest of the cell. His model thus consists of

(I) An organism *E*, which includes:

- (II) a code symbol *S*,
- (III) a universal code-actuated assembler *U*, that produces not *E* but *E-S* when it is triggered by any valid code symbol *S*, and
- (IV) a code-transcriber *T*, which sometimes introduces random variations; and *E* is in turn included within

(V) a cellular universe *V* with a dynamical law and a nutritious environment supporting the operations of the previous elements (I-IV).

The structure (I)-(V) replaces Peirce's simple habit-forming tendency today. To reproduce itself, the organism E first inserts its own code *S* into *T* to transcribe *S*, then inserts the copy of *S* into *U* to produces a replica of *E-S*, and finally inserts the replica of *S* in the replica of *E-S*, producing a replica of *E*, which goes forth and does likewise. All life has this Von Neumann architecture.

The Peircean semantic *U-S-E* triad reappears as but the first three members of the above list. Here *S*, however, does not exactly mean *E* to *U*, but only *E-S*; from which *E* may be constructed, given *S*.

Self-reproduction resembles self-governance in more than their common element of self-reference. The dynamical law describes a replication in time, and reproduction describes a replication in space as well. Thus self-reproduction is a special case of self-governance.

Let us generalize the von Neumann strategy from self-reproduction to self-governance, and model self-governance without infinite regression much as von Neumann did self-reproduction, by eliminating redundancy. The first step in forming such a model is to bring the dynamical law from the supercelestial sphere into the universe itself, as genetics puts the source of our individual traits in our cells. This calls for a representation of the dynamical law, a dynamical code, distributed through space-time, determining dynamicals as the genome does genetics. This dynamical code then determines the dynamical law governing the ("anaclastic") evolution of everything but itself.

The dynamical law is then uniform across the universe only to the extent that all dynamical codes descend from a common ancestor formed shortly after the first flash. This theory was previously formulated and developed further by Lee Smolin (1991).

Such a cellular model automatically incorporates some of the insights of Einstein about space-time, such as locality and causality. The principles of quantum epistemology, however, have to be added to the previous discussion, and then they describe tychastic mutagenic processes. The quantum set theory already mentioned is a useful language for cellular structures incorporating the quantum epistemology.

Just as the von Neumann solution to the self-reproduction problem proved to be a reasonable prediction of processes that actually go on in the cell, we may hope that a solution to the self-governance problem will help us to discover new processes going on in nature. The solution proposed here assumes a small "seed" dynamical law (rather like the wired-in program of a computer that enables it to read in and execute more complicated programs) and allows for arbitrarily complex laws to evolve from that seed.

This general structure does not of itself guarantee significant autonomy. For example, inorganic reproduction such as crystal growth is a degenerate case in which *S* is the crystal unit cell and *E* is the crystal, and is too determinate to be called autonomous. The degree of autonomy depends on the information capacity of the code, and thus on the generality of the assembler *U* of (III). For autonomy it is important that *T* and *U* have relatively small complexity compared to the code symbol *S*; so we usually consider the limit as the complexity of *S* approaches infinity for fixed *T* and *U*. This construction will not be useful if the dynamical law of nature is actually simple, specifiable with just a few bits, but only if it is complex, requiring millions of bits for its specification. Deep down I have not given up the belief in a simple law or abandoned the search for a language that reveals this simplicity. And Peirce has left his footprints on this beach too.

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