Peirce's "Most Lucid and Interesting Paper": An Introduction to Cenopythagoreanism[1]

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Mathematicians have always been the very best reasoners in the world; while metaphysicians have always been the very worst. Therein is reason enough why students of philosophy should not neglect mathematics. (MS 316)

INTRODUCTION

APPROXIMATELY SIX MONTHS before his death on April 14, 1914, in a long letter to his friend, F.A. Woods, Peirce reflected upon some of his previous work.

It was 17 years ago, lacking between 2 and 3 calendar months, that it first forcibly struck me. . .that my paper of Jan. 1897 ("The Logic of Relatives," *The Monist* 7:161–217)[2]. . .required considerable modification. Not that I had said anything *false* but that I had failed to state the matter in the simplest and best form. Thereupon, I wrote the most lucid and interesting paper I have ever written; but I had no way of publishing it. The editor to whom I sent it refused it on the ground that he was afraid I should soon make some further discovery; so he preferred the old mumpsimus to my sumpsimus.

Nine years later, [I] found that system of expressing assertions that I had given in my rejected article to have been a perfect Calumet mine in my own work (the letter was interrupted here, but continued on November 7). I undertook to write such an account of that system as no *stultus stultorum* could refuse to print; and as a natural consequence, it was far the worst I ever perpetrated (probably a reference to "Prologomena to an Apology for Pragmaticism," *The Monist*, 16:1903). (MS L 477:30–32)

Peirce is widely respected as America's most original and versatile intellect. It seems obvious that the best essay of a great scholar might be quite important for increasing our understanding of the true nature of that thinker's overall efforts. So when I first

[2]In this paper I will use '[]' to indicate editorial emendations in a text, while '()' will show my own inserted remarks. Peirce's Harvard MSS will be referenced according to a system presented in Richard R. Robin, *Annotated Catalogue of the Papers of Charles S. Peirce* (Amherst: Univ. of Massachusetts Press, 1967) as supplemented by Robin, "The Peirce Papers: A Supplementary Catalogue," *Transactions of the Charles S. Peirce Society*, 7 (1971), 35–37. Thus, MS 318:20 means number 318 in Robin's catalogue, sheet number 20. CP followed by numbers representing volume and paragraph refers to C. Hartshorne, P. Weiss, and A. Burke, eds., *Collected Papers of Charles Sanders Peirce* (Cambridge: Harvard Univ. Press, 1935, 1958).

read this passage, it immediately raised the prospect of locating a piece of writing that Peirce regarded very highly. Hoping for these results, I began searching for clues that might lead to identifying this work among the extant MSS. Because there are a number of places (in addition to the above letter) where Peirce mentioned and described that "rejected article," we know several of its properties. On the basis of an analysis of the contents[3] of the surviving MS 482 "On Logical Graphs," I have argued[4] that it fits Peirce's descriptions well, and that it is a nearly complete version of the essay Peirce regarded so highly.

It is clear from the beginning of MS 482 that topology is its context. Explanation of that background, and of a number of other important and relevant themes, must be reserved for later projects; the purpose of this one was first to undertake the collateral efforts of identifying the important MS and analyzing it. Then as a way of showing that it is indeed important for gaining a better understanding of Peirce, I have prepared an account of an important formal system implicit in it. On this basis we can vindicate Peirce's claim that the contents (this formal system, for one example) of his "most lucid and interesting paper," did indeed become a central part of the "perfect Calumet mine in [his own later] work."

The system presented here—which I name "Valency Analysis"—has been prepared by me using clues from MS 482. I have attempted to be consistent with what I take Peirce's insights to have been. He gave the system no special name. What I present is in the spirit of fleshing out and organizing (didactically) what he outlined.

One of the effects of this study of MS 482 will be an addition to our knowledge of the basis of Peirce's categoriology. Students of that topic have often lamented that there seems to be no place in the extant papers where he wrote out the proof he claimed to have that tetrads or higher order relations could be reduced to combinations of triads. Many persons have without success searched his writings for an account of this "reduction proof," presumably seeking something that is like a contemporary mathematical or logical proof, expressed algebraically. But perhaps they have been looking for the wrong thing. After having been one of the pioneer developers of logical algebras, Peirce came to favor a topological and diagrammatic approach in logic and logical analysis. Hence it would be quite natural for him to have expressed the reduction proof in some (nonalgebraic) diagrammatic form. If so, then students of the reduction proof ought to be familiar with Peirce's topological interests and with Existential Graphs, his system of logical diagrams. Indeed, if we were to use the contemporary name of the mathematical field that seems to lie under the reduction proof, it would

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[3]After a careful study of the folder for MS 482, I have concluded that internal evidence—such as Peirce's pagination, section numbering, and content—amply support the conclusion that the following represent a complete paper: 482:2–30 plus 482:38–45. This thesis is sustained by the continuity of content and of Peirce's section numbering. This was probably a hold-back rough copy. Persons who wish to review the argument in detail may write for a copy of my complete analysis.

[4]The arguments which indicate that MS 482 is indeed the "most lucid and interesting paper" begin with reviewing discussions about that paper in other Peirce writings, to wit: MS 500:02–03 (1911); MS 1589:02 (late); MS 479:02 (1903); MS 483:02 (1898–99); MS 485:02 (1898); MS 488:02 (1898). These passages together list several properties that the "most lucid paper" must possess. MS 482 indeed has all these properties. It is especially important that MS 482 includes the very development of Existential Graphs out of simplified entitative graphs. Again, the details of this argument are too lengthy to be presented here; I would be pleased to send a copy to any interested person.

be "Graph Theory," an offshoot of topology[5]. Peirce's awareness of, and participation in, the early development of graph theory is a story yet to be told.

I believe that MS 482 represents at least one place where Peirce came very close to stating explicitly the reduction proof in terms of valency analysis. So what I present here (based on MS 482 and allied items) will include a reconstruction of what that proof might have been if it had been fully displayed.

Why did Peirce not present valency analysis or this proof in an explicit manner? The best speculation I can produce so far is that it was simple, as he often stated (for example, MS 292:80). Peirce was an accomplished mathematician, and it might have been a potential source of embarassment to him to have written out such a "simple" presentation—he may have thought other mathematicians might regard it, and him, as trivial. And perhaps it is in the context of research-level mathematicians, but the application of it in the theory of categories, and elsewhere in Peirce's system of science, is far from trivial. I should add that I have presented only one aspect of MS 482, and when the whole piece has been fully studied it may involve items that even a research mathematician would regard as important.

PART ONE: VALENCY ANALYSIS

Let

[Fig. 1]

represent an entity with one "loose end," and let it be named "Monad." Let an entity with two loose ends
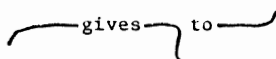
[Fig. 2]

be named "Dyad." Let

[Fig. 3]

an entity with three loose ends, be named "Triad." Let an entity with N loose ends be named a "spot." (Monads, dyads, and triads are also spots.) What I am here describing as an "entity" is some as yet unanalyzed relation name. For example, consider a triad: the entity could be specified as

[Fig. 4]

or as

[Fig. 5]

[5]N.L. Biggs, E.K. Lloyd, and R.J. Wilson, *Graph Theory 1736–1936* (Oxford: Oxford Univ. Press, 1976).

The point of concentrating upon Valency, and leaving the "content" of the relation un-specified is precisely to achieve results that will apply to any triad no matter what its content (or to any spot of valency N no matter what its content).[6]

Loose ends may connect with other loose ends to form a "bond" or bonds. Each bond always requires two and only two loose ends. Any series of spots with or without bonds is a "valental graph." "Valency" refers to the total number of loose ends in a valental graph. The valency of a monad is 1, of a dyad 2, of a triad 3, of a spot with N loose ends is N. (My terminology is sometimes not quite that used by Peirce, but I think the overall effect is the same.)

An "unbonded," or equivalently, a "simple" valental graph, is one that contains no bonds. This allows the statement of a rule.

alpha: V simple VG = Sum V spots

That is, the valency of a valental graph composed only of unbonded spots is equal to the sum of the valencies of all the spots in the graph. Here are three examples of simple valental graphs, I, II, III, together with their valencies:

I
Valency of I=3   **of II=5**   **of III=10**

[Fig. 6]
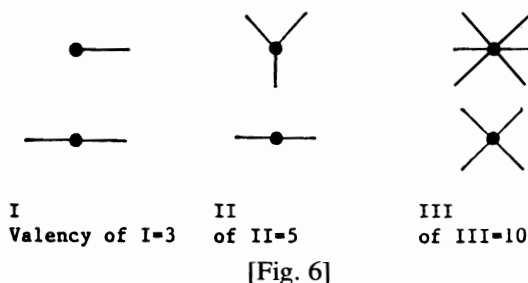
A valental graph that contains at least one bond we shall name a "complex valental graph." Here are eight examples of complex valental graphs:


[Fig. 7]
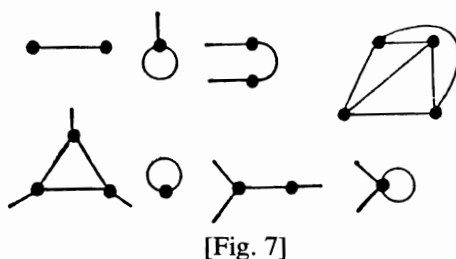
The valency of a complex graph is:

beta: V complex VG = sum V spots minus 2x sum Bonds

That is, the valency of a complex valental graph equals the sum of valency spots (im-agined as if they were simple and unbonded) minus 2x the sum of bonds. In MS 482, Peirce established this rule on the basis of general topological considerations. We can see, however, that one subtracts 2x sum bonds because each bond is seen as the result

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[6]Compare typical discussions by Peirce, such as that in J.M. Baldwin, *Dictionary of Philosophy and Psychology* (New York: Macmillan, 1902), vol. 2, 447–450.

of joining 2 loose ends, starting from an appropriate simple valental graph. That is, a complex valental graph can be understood as having been created from an appropriate collection of nonbonded spots plus the act of bonding (called "composition") 2 or more loose ends at a time. Because each bond consumes exactly 2 loose ends, each bonding act decreases overall valency (of the imagined simple spots) by 2. Here are some (eight) examples (letting "+" = compose).
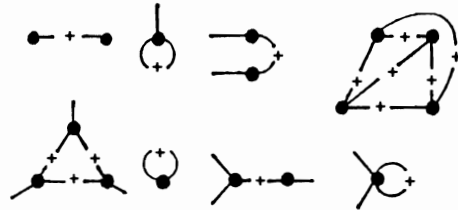
[Fig. 8]

A corollary of beta is:

gamma: It is not the case that any "perissid" (odd valency) spot can be composed exclusively from "artiads" (even valency) spots.

Gamma may be established from Beta by means of a *reductio ad absurdum*:

Assume as an hypothesis the contradiction of gamma, which would be: Some perissid of valency $2N+1$ = sum V spots minus 2x sum Bonds, and that "sum V spots" includes only artiads.

That is, by hypothesis there is at least one perissid that can be composed exclusively of artiads. We can see that "sum V spots" is even by hypothesis, and "2x sum Bonds" is necessarily even in any case. An even number subtracted from an even number always gives an even number. Therefore, the right side of the hypothesis equation would always be even. That creates a contradiction in the hypothesis, since by assumption, the left side is odd. This means the hypothesis is not the case. This completes a proof that establishes the truth of rule gamma.

Two valental graphs are "valency equivalent" if the two graphs have the same valency, no matter what other properties they might have. Thus, these four graphs are valency equivalent (the valency of each is 4):
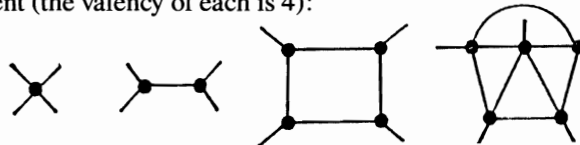

[Fig. 9]

While valental graph composition is accomplished by bonding loose ends, exactly two at a time, "decomposition" may be accomplished by breaking bonds, one bond at a time, each broken bond always creating exactly 2 loose ends.

The result of composing any collection of two or more monads, whether simple or complex, is that only "medads" (graphs of valency zero) are created. Simple monads cannot be decomposed, for there are no bonds to be broken. But complex monads

may be decomposed. For example (letting "—— com" = "may be composed from," and "—— decom" = "may be decomposed into"):
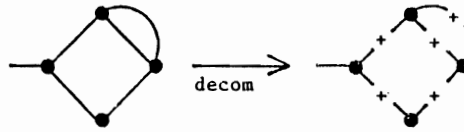
[Fig. 10]

If any simple or complex monad and any simple or complex dyad are composed, a monad results, thus:
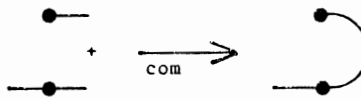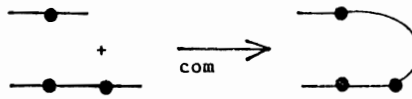

[Fig. 11]

Any simple dyad, like a simple monad, cannot be decomposed, because there are no bonds to break. But any complex dyad may be decomposed.

[Fig. 12]

If any dyad and any other dyad (either dyad being simple or complex) are composed, a dyad results, thus:


[Fig. 13]

From the definition of bonding by two loose ends at a time, it follows that no set of monads only can compose a triad (either simple or complex), and no set of dyads alone can compose a triad (either simple or complex). Hence it follows that:

delta: A triad (either simple or complex) cannot be composed of dyads exclusively, nor of dyads and monads exclusively, nor of just monads.

However,

epsilon: For any spot S which has a valency of four or more, a graph which will be the valency equivalent of S may be composed from triads exclusively.

Here are some examples (the sign '=V=' means "is valency equivalent to"):
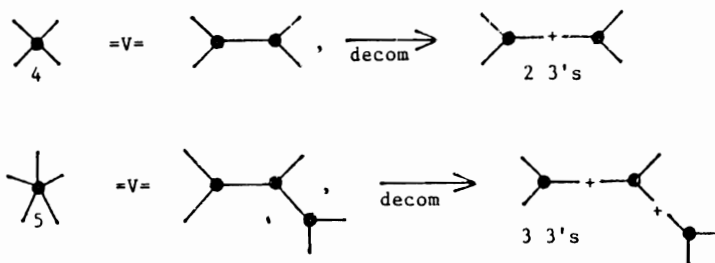

[Fig. 14]

We can see that rule epsilon is established by the "Fermatian Inference" (mathematical induction). From epsilon will follow:

zeta: Any spot of valency 4 or higher is decomposable into (may be seen as the valency equivalent composition from) N minus 2 triads, or: spot V (greater than or equal to 4) =V= comp (V minus 2) triads

That is, any spot of valency N, where N is greater than or equal to 4, is valency equivalent to the composition of some N minus 2 triads.

To summarize, WITHIN THIS SYSTEM OF VALENCY ANALYSIS: (1) triads may not be composed exclusively from only monads, or only monads and dyads, or only dyads; (2) but any tetrad or higher valency spot (of valency N) is valency equivalent to the composition of some series of N minus 2 triads; (3) thus tetrads and above are not indecomposable, but can be expressed as the valency equivalent composition of triads; (4) simple monads, dyads, and triads are, however, indecomposable.

Peirce's principal focus in MS 482 and elsewhere was on keeping track of bonding, or "putting things together." He was not especially interested in "medads," graphs with zero valency, except perhaps in how they came to be zero (how they came to be bonded as they are). This is the attitude of a chemist who sees a finished reaction (comparable to a medad) as the result of the activity of combining some more elemental compounds that have free ions (comparable to loose ends). We might object to this decision to bond two at a time. We might want to say, "let there be bondings greater than 2," bonding 5 at a time, for example:
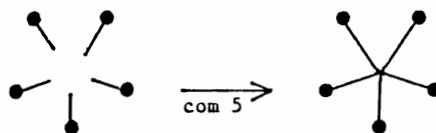
[Fig. 15]

But Peirce could respond: "I can analyze that with my two at a time bonding and with my unique and elemental 1's, 2's, and 3's, especially my lovely 3's. Perhaps your bond 2N+1 at a time method will work, but my bond two at a time method works also, and mine is simpler."
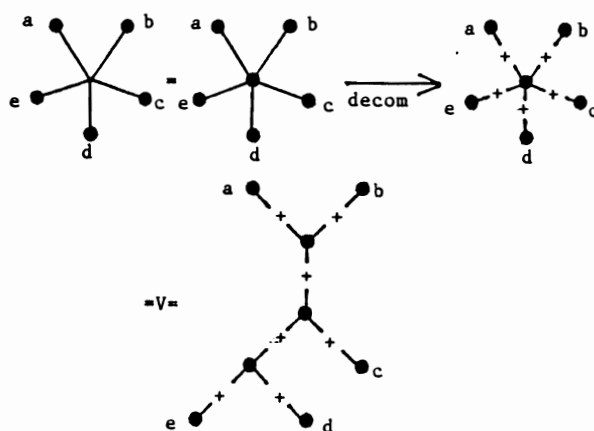
[Fig. 16]

I worked out the above before I first noticed Peirce's nearly identical solution.[7]

PART TWO: THE SIGNIFICANCE OF VALENCY ANALYSIS

One who has followed the discussion thus far might be inclined at this point to ask if Valency Analysis is of any importance, either in understanding Peirce's System of Science, or perhaps in a broader context. The germ of an answer to that very appropriate question lies within Peirce's 1913 letter to Woods, in his remark that "nine years later" (which would be 1897 + 9, or about 1906) he found the results of MS 482 to have been "a perfect Calumet mine in my own work." This suggests that the answer to part of this question might perhaps be found in works from 1906 or a few years earlier. A study of the whole period 1897–1906 is needed, especially the last two years. I have space only for briefly considering one of the more relevant groupings of essays. The principal published works of 1906 are: "Recent Developments of Existential Graphs and their Consequences for Logic," read before the Washington meeting of the National Academy of Sciences in April (MS 490 is probably a set of notes for this presentation); "Prologomena to an Apology for Pragmaticism," in the October *Monist* (MS 292 and 295 are earlier drafts of that article); and "Phaneroscopy, or Natural History of Signs, Relations, Categories, etc.: A Method of Investigating this Subject Expounded and Illustrated," a paper read before the Boston meeting of the National Academy of Sciences in November (MS 299 is probably a draft of this lecture). I think that we should take particularly careful notice of the title of this last mentioned presentation, and of the fact that in his description of it in *The Sun*, New York, Wednesday 28 November 1906, 6:5–7, Peirce revealed that it emphasized valency analysis and diagrammatic thought.[8]

In MS 292:33f, while applying valency analysis to a graphical analysis of logic, Peirce stated:

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[7]See Carolyn Eisele, *The New Elements of Mathematics by Charles S. Peirce* (The Hague: Mouton, 1976), Vol. 4, 307.

[8]See also a briefer account of it by Peirce in K.L. Ketner and J.E. Cook, eds., *Chares Sanders Peirce: Contributions to* The Nation *(Lubbock, Texas Tech Press, 1979), part 3, p. 269.*

> Every graph has a definite valency.... A number of dyads can only make a chain, and the compound will still be a dyad...unless the two ends are joined making it a medad.... But a number of triads can be joined so as to make a compound of any valency not exceeding the number of triads by more than two, and any odd number giving any odd valency under the same restriction. This shows that there are five natural classes of forms of graphs, namely, medads, monads, dyads, triads and higher perissids (odd valents), tetrads and higher artiads (even valents)....

It is noteworthy that here, nine years later, the discussion given by Peirce in outline form in part of MS 482 (which I have fleshed out in the first part of this essay) reappears virtually intact, and even with the same terminology. A new finding is added to Valency Analysis, namely that there are but five natural classes of forms of valental graphs.

Valency Analysis, according to Peirce, has an important application (continuing MS 292:34f):

> In classification generally, it may fairly be said to be established, if it ever was doubted (remember that Peirce was an expert in scientific classification), that Form, in the sense of structure, is of far higher significance than Material. Valency is the basis of all external structure; and where indecomposibility precludes internal structure—as in the classification of elementary concepts—valency ought to be made the first consideration. I term [this] the doctrine of *cenopythagoreanism* (compare 292:98).

That term was an appropriate choice, given Peirce's wish to honor his scientific ancestors, and given his own conclusions that mathematics is the most fundamental science, the ultimate basis for all intellectual activity. I do not recall seeing any essay in the literature on Peirce in which this doctrine of classification according to form, which he called Cenopythagoreanism, has been noticed accurately. Usually it is simply taken as an odd name for the categories. But it is clearly something much broader than that, for we find Peirce classifying according to external form in many areas of his thinking. In a number of places in his later work, Peirce gave a careful analysis of Form in terms of Icons, Diagrammatic Thought, and Mathematics. A late name for some of his more familiar pragmatic thought is indeed Cenopythagorean Pragmaticism.[9]

Now we are beginning to discover the significance of Peirce's "most lucid and interesting paper." For a further brief exploration of the classificatory doctrine of Cenopythagoreanism and how it can be further applied in cenoscopic philosophy, let us examine a parallel passage in MS 292 (sheets 56–81, beginning at 62).

> The chief purpose which governed the construction of [Peirce's logical] algebras and still more exclusively that of existential graphs has been the facilitation of logical analysis, and the resolution of problems in logic. . . . For example, one of the puzzling questions of logic is how concepts can be combined.

Peirce classified sciences of research according to the following hierarchy:[10]

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[9]CP 5.555.

[10]See K.L. Ketner, ed., "A Brief Intellectual Autobiography by Charles Sanders Peirce," *American Journal of Semiotics*, 2 (1983), 70.

MATHEMATICS
PHILOSOPHY
  Phenomenology, or Ideoscopy (also Phaneroscopy)
  Normative Science
    Esthetics
    Ethics
    Logic (or Semiotic)
      Speculative Grammar
      Critic
      Methodeutic
  Metaphysics
IDIOSCOPY (Special Sciences, physics, psychics, etc.)

He also stated in 1904,[11] "In order to understand [my] doctrine, which has little in common with those of modern schools, it is necessary to know, first of all, how [I classify] the sciences." If we add to this Peirce's well-known identification of logic (the third of the normative sciences) with semiotic, we can see that Valency Analysis, which originated in Mathematics, has found its way down the scale of sciences, now appearing as the fuller Doctrine of Cenopythagoreanism, which is being used to analyze and explicate a puzzling problem in semiotic/logic. The proposed solution of the problem of concept combination, found at MS 292: 62–67, is too lengthy to consider now, but we note that Valency Analysis is a part of it. Indeed, the graphical representation of concept combination is "bonding of loose ends." The way this lies at the basis of Existential Graphs will be displayed shortly. This same manuscript sequence, by the way, contains evidence that the classification of signs is done according to the doctrine of Cenopythagoreanism.

A. VALENCY ANALYSIS IN THE EXISTENTIAL GRAPHS

The purpose now is to show how Valency Analysis (VA) and bonding is fundamental in Peirce's Existential Graphs (EG). In order to gain that understanding, we must quickly review the alpha part, roughly truth-functional logic. It contains no VA elements, but is essential for progress later. One should supplement this very condensed outline by studying some of the more complete accounts of EG.[12]

A single capital letter represents a simple proposition, and writing that letter on an appropriately designated surface called the sheet of assertion (SA) asserts the proposition. Writing two such letters side by side on SA conjoins them. Thus juxtaposition is

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[11]*Ibid.*, 69.

[12]See Don D. Roberts, *The Existential Graphs of Charles S. Peirce* (The Hague: Mouton, 1973); Kenneth L. Ketner, "The Best Example of Semiosis and its Use in Teaching Semiotics," *American Journal of Semiotics*, 1 (1981), 47–84; Ketner, "Peirce on Diagrammatic Thought," in *Zeichen und Realität*, ed. Klaus Oehler (Tübingen: Stauffenburg Verlag, 1984); Ketner, "Semiotic is an Observational Science," in *Iconicity: Essays on the Nature of Culture*, ed. P. Bouissac and R. Posner (Tübingen: Stauffenburg Verlag, 1986). See also J. Brunning, Peirce's Development of the Algebra of Relations, Diss. University of Toronto, 1981; Hans Herzberger, "Peirce's Remarkable Theorem," in *Pragmatism and Purpose: Essays Presented to Thomas A. Goudge*, ed. J. Slater, T. Wilson, and T. Sumner (Toronto: Univ. of Toronto Press, 1981). Also consider Alfred Tarski, "On the Calculus of Relations," *Journal of Symbolic Logic*, 6 (1941), 73–89; Tarski, *Introduction to Logic and to the Methodology of Deductive Sciences* (New York: Oxford Univ. Press, 1965), chapter 5.

the sign of conjunction. If a circle or other lightly drawn self-closing figure, called a cut, is drawn around a capital letter, then the proposition represented by the letter is understood to have been denied. With these two conventions, all the truth-functional connectives, and hence any truth-functional proposition, can be generated. Here are a few examples (where '&' is "and," and ÷ is "deny," 'im' is material implication, and 'eq' is material equivalence):

Algebra	Language	EG
A	atomic sentence	A
–A	deny sentence A	Ⓐ
––A	not not A	ⓐ
A & B	A and B	AB
A & –B	A and deny B	A Ⓑ
–(A & –B)	A materially implies B	(A Ⓑ)
A or B	A or (inclusive) B	(Ⓐ Ⓑ)
A eq B	A equivalent to B	(A ⒝(B Ⓐ)

[Fig. 17]

If one insists upon reading each EG graph by first translating back into the equivalent not-and algebraic form, the power of EG will be unnecessarily lost. To realize its considerable facility, one should learn to see the connectives directly as cut patterns. Here are several important ones (where $ and @ represent any truth functional sentence of any complexity).


Implies   Or   Equivalent
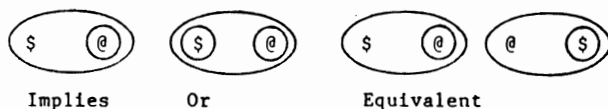
[Fig. 18]

Thus, one quickly learns to think of each "implication" connective as generating a kind of one-eyed binocular cut pattern, while an "or" connective generates a binocular pattern, and so on. To create the graph of a complex proposition, simply begin with the proper cut pattern of the principal connector in the proposition, then progress toward the least connectors, adding the appropriate graphs on top of cuts already drawn. Once one grasps this simple technique, the full computational power of the graphs is released.

Peirce developed five easily remembered and powerful transformation rules, but a discussion of them is not possible here.
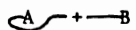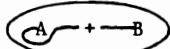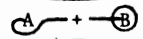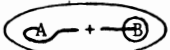
If we add a bit more, properties may be represented in the system. A required new symbol is a heavy line drawn from the left of a capital letter. The heavy line, called the line of identity, means "something exists." Moreover, we think of the line as composed of dots that touch. And we also understand it as asserting the identity of all dots within it. So, a simple unattached line states that something exists and that the dots on its extremities are identical to each other and to every other dot in the line. A capital letter having such a line attached to its left side represents a property. In VA terms, this

is a monad. By employing the "matrix" of truth-functional logic, we can generate the following parallels (where "SS" is the existential quantifier and "II" is the universal quantifier).

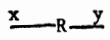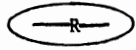| Algebra	Language	EG
SSxAx	Something is A	—A
IIx–Ax	There is no A	⬭ —A
SS–Ax	Something is not A	—Ⓐ
IIxAx	Everything is A	⬭ —Ⓐ

[Fig. 19]

If we introduce two monads involving two different properties, by using an act of assertion/bonding, we can produce the kind of medads traditionally known as the Aristotelian Categorical Propositions. Notice that the assertory act of bonding amounts to taking up two loose ends and asserting that the dots at their unattached extremities are identical, so that what before were two separate lines (two individuals) now through a new act of assertion (bonding) become one individual. In the examples to follow I will use "+" to show where bonding occurred. Try to imagine before and after bonding conditions by means of these + signs. Ordinarily one omits + in writing existential graphs.

Algebra	Language	EG
SSx(Ax&Bx)	Some A is B	A—+—B
IIx(Axim–Bx)	No A is B	⬭ A—+—B
SSx(Ax&–Bx)	Some A is not B	A—+—Ⓑ
IIx(AximBx)	All A is B	⬭ A—+—Ⓑ

[Fig. 20]

Notice that these are Boolean contradictories. A full Aristotelian square of opposition is obtained if one adds A as an extra premise to both the universal propositions. This of course is an advantage for a system designed to analyze as deeply as possible.
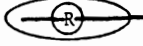
Dyadic relations may be introduced into EG in the following manner (where R is a dyadic relation).

Algebra	Language	EG
SSxyRxy	Some x is R to some y	x—R—y
IIxy–Rxy	No x is R to no y	⬭ —R—
SSxy–Rxy	Some x is not R to some y	Ⓡ
IIxyRxy	All x is R to all y	⬭ —Ⓡ—

[Fig. 21]

Notice that our convention will be that the left line of the R represents the x place-holder, and the right line represents that for y. Generally in EG in a given graph there

are as many individuals as there are separate lines of identity. Mixed quantification can be represented in ways illustrated by these examples.

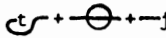| Algebra	Language	EG
SSxIIyRxy	Some x is R to all y	
IIxSSyRxy	All x is R to some y	

[Fig. 22]

It works out that a line of identity, the least enclosed part of which is on an even area, expresses existential quantification; while one, the least enclosed part of which is odd, expresses universal quantification. Odd and even enclosures can readily be ascertained by regarding the paper or other surface on which graphs are written (SA) as even. Then the area within one cut is odd, within two cuts is even, and so on.

The six examples just given are unbonded dyads. We may represent the bonding of dyads and monads in these ways (where a lower case letter with a line of identity at its left represents a definite individual).

Algebra	Language	EG
Rab	Alice is R to Bob	
SSy(Yy&-Rby)	Bob is not R to a Yankee	

[Fig. 23]

Two monads representing definite individuals may be bonded to represent their numerical identity. Nonidentity may be expressed by having a line of identity pass through a cut.

Algebra	Language	EG
t = c	Twain is Clemens	
-(t = j)	Twain is not Jones	

[Fig. 24]

Triadic relations may be represented in the following way, where, to provide a useful example, 'S' will mean the sign relation "_____ stands for _____ to _____."

Algebra	Language	EG
SSxyzSxyz	Some x stands for some y to some z	
IIxyz-Sxyz	No x stands for no y to no z	
SSxyz-Sxyz	Some x does not stand for some y to some z	
IIxyzSxyz	All x stands for all y to all z	

[Fig. 25]

We could bond three monads to the above to get a full sign relation (where r is a specific representamen, o is the object, and i the interpretant).

Algebra   Language   EG

Sroi   r stands for o to i
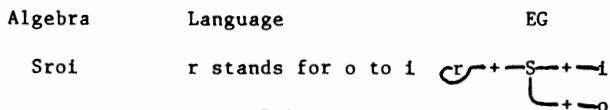
[Fig. 26]

Of course, given the results of VA, one does not need to introduce symbols for tetrads or relations of higher valency because according to VA they are in principle reducible to patterns of triads.

In terms of VA and EG, something in the spirit of the above is how one would represent sign relations. I can recall no place where Peirce represented a sign relation as a triangle such as this.

[Fig. 27]

This is often attributed to him, but falsely so, for this portrays a triadic relation as composed EXCLUSIVELY OF DYADS, something that was disproved in VA at rule gamma.
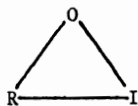
We can now use these ideas to experiment briefly with adapting the techniques of VA and EG to an important principle of Peirce's semiotic, that "Every $sign_1$ is interpretable in another $sign_2$." Consider the following graph (where S_1 is the first temporally occurring sign relation and S_2 is a later occurring sign relation).
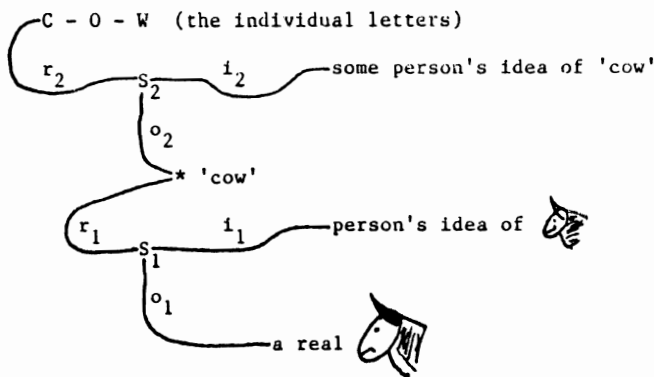
[Fig. 28]

At *, two elements that are the same entity in both of the two triadic relations are bonded, allowing lines to be joined. This would enable us to read Peirce's principle in the following way: "$sign_1$ means r_1 in the diagram, and "interpretable" means "in a second triadic relation like S_2," and "another $sign_2$" means the r_2 of the S_2 mentioned in the interpretable clause. There are a number of other permutations that one might try. Also, because each of the extremities is a sign, that means that there is potentially some additional triad bonded to each such sign, and triads on the signs of those ex-

tremities, and so on *ad infinitum*. So we see that we have made but a limited and tiny snapshot of the vast network of semiosis which is the intelligible world.

I hope this outline will make plausible the claim that VA underlies EG, and that both are important tools which Peirce used in applying diagrammatic thought to study semiotic scientifically (using hypothesis, experiment, and observation). These considerations seem to vindicate remarks like those at CP 2.227.

B. VALENCY ANALYSIS AND PHANEROSCOPY

If we return to MS 292, we can find Peirce using Valency Analysis and Cenopythagoreanism to resolve one more issue within another of the sciences in his classification scheme: I mean the science of Phaneroscopy.

I invite you, Reader, to turn your attention to a subject which, at first sight, seems to have as little to do with signs as anything could. It is what I call the *Phaneron*, meaning the totality of all that is before or in your mind, or mine, or any man's, in any sense in which that expression is ever used. There can be no psychological difficulty in determining whether anything belongs to the Phaneron, or not; for whatever seems to be before the mind *ipso facto* is so, in my sense of the phrase. I invite you to consider, not everything in the Phaneron, but only its indecomposable elements, that is, those that are logically indecomposable, or indecomposable to direct inspection. I wish to make out a classification. or division. of these indecomposable elements; that is, I want to sort them into their different kinds according to their real characters. I have some acquaintance with two different such classifications, both quite true; and there may be others. Of these I know of, one is a division according to the Form or Structure of the elements, the other according to their Matter. The two most passionately laborious years of my life were exclusively devoted to trying to ascertain something for certain about the latter; but I abandoned the attempt as beyond my powers, or, at any rate, unsuited to my genius. I had not neglected to examine what others had done but could not persuade myself that they had been more successful than I. Fortunately, however, all taxonomists of every department have found classifications according to structure to be the most important.

A reader may very intelligently ask, How is it possible for an indecomposable element to have any differences of structure? Of internal logical structure it would be clearly impossible. But of external structure, that is to say, structure of its possible compounds (bondings!), limited differences of structure are possible; witness the chemical elements, of which the "groups," or vertical columns of Mendeleef's table, are universally and justly recognized as ever so much more important than the "series," or horizontal ranks in the same table. Those columns are characterized by their several valencies, thus. . . (here Peirce gave long lists of chemical elements that are medads, monads, dyads, triads, tetrads, pentads, hexads, heptads, and octads).

So, then, since elements may have structure through valency, I invite the reader to join me in a direct inspection of the valency of elements in the Phaneron. (MS 292:71–75)

If, then, there be any formal division of elements of the Phaneron, there must be a division according to valency; and we may expect medads, monads, dyads, triads, tetrads, etc. Some of these, however, can be antecedently excluded, as impossible. . . . In the present application, a medad must mean an indecomposdable idea, altogether severed logically from every other; a monad will mean an element which, except that it is thought as applying to some subject, has no other characters than these which are complete in it without any reference to anything else; a dyad will be an elementary idea of something that would possess such characters it does possess relatively to something else but regardless of any third object of any

category, a triad would be an elementary idea of something which should be such as it were relatively to two others in different ways, but regardless of any fourth, and so on. Some of these, I repeat, are plainly impossible. A medad would be a flash of mental "heat-lightning" absolutely instantaneous, thunderless, unremembered, and altogether without effect. It can further be said in advance, not, indeed, purely *a priori* but with the degree of apriority that is proper to logic, namely as a necessary deduction from the fact that there are signs, that there must be an elementary triad. For were every element of the Phaneron a monad or a dyad, without the relative of teridentity (which is, of course, a triad) it is evident that no triad could ever be built up. (That conclusion is a direct application of valency analysis.) Now the relation of every sign to its Object and Interpretant is plainly a triad. A triad might be built up of pentads or of any higher perissid elements in many ways. But it can be proved —and with extreme simplicity. . .that no element can have a higher valency than three (the reduction theorem from valency analysis). (MS 292:78)

Experienced students of Peirce will readily recognize that this long passage terminates in a short presentation of his famous categories, Firstness, Secondness, and Thirdness. It is quite enlightening to notice that he referred to them as his *Cenopythagorean Categories*.[13] Perhaps more importantly, what the foregoing shows can be summarized in the following way. It is well known that Peirce argued that mathematics was the most fundamental science, even more basic than philosophy. In MS 482, He developed in outline the mathematical system of Valency Analysis. In conjunction with his classification of the sciences, and with his own training as a scientist lurking in the background, he applied these fundamentals of Valency Analysis to sciences below mathematics, the first being philosophy. In other words, he interpreted this abstract mathematical system onto the structures of other subject matters, fundamentally a movement of diagrammatic reasoning (mathematical reasoning), but one that any expert mathematical physicist such as Peirce would regard as a mere routine, almost unconscious, aspect of scientific work.

PART THREE: CONCLUSION

It is important to re-emphasize clearly that the doctrine of Cenopythogoreanism is not to be equated with Peirce's doctrine of the categories, as some scholars have done. It is a doctrine of classification based upon valency analysis of external form, developed within the science of mathematics, and deployed into the sciences that come later in Peirce's System of Science. Its appearance with the categories within phaneroscopy is but the first of many applications of it in a number of different sciences.

Indeed, application of the technique of classifying and analyzing in terms of the results of Valency Analysis, named the doctrine of Cenopythogoreanism, created a number of other results within Peirce's whole system, for example: the Existential Graph method of logical diagrammatization,[14] the doctrine of the categories which is a central part of Phaneroscopy; and the classification of signs. It has been possible to consider only some of these topics here briefly.

Given all these extensions and ramifications, it is clear that the doctrine of Cenopythagoreanism (classify in terms of external form by means of valency analysis)

[13]For example, see K.L. Ketner, ed. "A Brief Intellectual Autobiography by Charles Sanders Peirce," *American Journal of Semiotics*, 2 (1983), 77.

[14]For introductions to the Existential Graphs, see the works of Roberts and Ketner cited above.

occupied a central and fundamental position in Peirce's later works. That is a result which appears to vindicate our initial hypothesis that identifying and understanding the "most lucid and interesting article" would produce a more accurate understanding of Peirce.

My only goal has been to develop a truthful account of Peirce's thinking, or at least a part of it. I have not raised the other question, namely, "Are his hypotheses correct?" He thought they were on the basis of evidence that he sifted. Perhaps it seems odd to ask, within philosophy, "Is this hypothesis correct?" But Peirce argued that philosophy was a science, hence we should not simply say, about his hypotheses, that they are his views. They are that, of course, but if they are also correct, then they should become the view of every scientific intelligence.

And indeed, if Peirce's hypothesis in the present matter is correct, it would have far-reaching consequences. Among these would be that mathematics, the science of diagrammatic thought, provides a way to "see into" (comprehend) mind, which is a semiosis, a bundle of relations. As a telescope is used in astronomy, so mathematics is the "scope" of Phaneroscopy, and the means whereby mind can be scientifically observed and studied.[15] That was clearly expressed by Peirce more than once, the earliest I have found being from a review of Abbot's *Scientific Theism* in *The Nation* from 1886:[16]

> The knowledge of relations (according to Abbot) depends upon a special "perceptive use of the understanding." This view, although it is not adequately set forth, is the centre of all that is original in the book, and it is sure to excite a fruitful discussion of the question of the mode of discernment of relations. Of all the sciences—at least of those whose reality no one disputes—mathematics is the one which deals with relations in the abstractest form; and it never deals with them except as embodied in a diagram or construction, geometrical or algebraical. The mathematical study of a construction consists in experimenting with it; after a number of such experiments, their separate results suddenly become united in one rule, and our immediate consciousness of this rule is our discernment of the relation. It is a strong secondary sensation, like the sense of beauty. To call it a perception may perhaps be understood as implying that to understand each special relation requires a special faculty, or determination of our nature. But it should not be overlooked that we come to it by a process analogous to induction.

Moreover, this tool (diagrammatic thought, or general mathematical method) for objectively observing the highest kind of reality, is the technique for making scientific progress in semiotic, as Peirce clearly stated (for instance, at CP 2.227). This means that diagrammatic thought with its valency analysis background (especially as ultimately embodied in Existential Graphs) is a vital tool for Peirce's conduct of scientific inquiry in semiotic.

On a more general level, Peirce's conception of general mathematical method may be a means for our very survival as a civilization. Briefly, by this cryptic remark, I mean that because much of what now constitutes our civilization are systems of relations (food distribution systems, power grids, computer networks, and so on), and

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[15]See the works of Ketner cited above, plus Ketner, "How Hintikka Misunderstood Peirce's Account of Theorematic Reasoning," *Transactions of the Charles S. Peirce Society*, 21 (1985), 407–418.

[16]K.L. Ketner and J.E. Cook, eds., *Charles Sanders Peirce: Contributions to* The Nation (Lubbock: Texas Tech Press, 1975), part one, p. 73.

because we may be rapidly reaching the point where we may lose control of those systems (remember the shorter response times in missile defense computers, as one prominent example, not to mention other systems involving our environment), we may thus be blindly going about creating the kinds of systems which if they fail or get out of control in a certain kind of way, may bring our species to extinction. Indeed, we may be the first species on Earth to have created a kind of artificial environmental niche (our various systems) which has the potential to make extinct our whole species, either through being out of control or through collapsing. Systems of relations, so it is plausible to think, are more than the things and motions of things one associates with them—thus a functioning computer is more than a collection of parts, even electrified parts. Perhaps this "something more" could properly be called relations. It is clear that we know how to observe *things* and their motions. Peirce's Cenopythagoreanism and associated topics offer us, however, a way in which we might begin to develop a theory of how better to be able to observe and experiment upon relations. That, in turn, might yield a better understanding of the relational part of systems, a result which, if attained, might possibly allow us to control them, as opposed to the other unhappy possibility which now seems to be steadily drawing closer.

The foregoing point has been eloquently expressed by Walker Percy,[17] an independent discoverer of Cenopythagoreanism about one hundred years after Peirce found it, to whose essay I refer persons who would like to continue this line of thought, persons who, like me at this juncture, may be saying to themselves, "I think I may be a Cenopythagorean."

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[17]Walker Percy, *The Message in the Bottle: How Queer Man is, How Queer Language Is, and What One Has to Do With the Other* (New York: Farrar, Straus, and Giroux, 1975), pp. 3–45.