# Balancing Necessity and Fallibilism: Charles Sanders Peirce on the Status of Mathematics and its Intersection with the Inquiry into Nature

**Ronald Anderson***

**Abstract** An interest in Charles Sanders Peirce and pragmatist thought in general emerged in the United States in the middle of last century to exert a powerful influence on a generation of American philosophers educated in the 1940s and 1950s, including Abner Shimony, whose thought is the occasion for this paper. Those threads in Peirce's work related to developing a scientifically informed worldview and metaphysics were the natural influences on Abner and this paper will begin by briefly reviewing a number of these threads and their influences in his writings. This sets the scene for the main project of the paper, an earlier historical project on a related aspect of Peirce's thought—his understanding of mathematics and its place in the description of nature. Mathematics was a foundational discipline for Peirce, one with qualities of necessity and certainty, features that stand in interesting contrast and tension to Peirce's view of an evolving nature which is governed by chance and our knowledge of which is always fallible and thus open to revision. Exploring these issues reveals deep background beliefs structuring Peirce's thought. The paper concludes in the contemporary realm with the speculation that due to the scientific developments of the 20th century, aspects of Peirce's work that formed a vision for

---

R. Anderson
Department of Philosophy, Boston College, Chestnut Hill, MA 02467, USA

*In Memory of Fr. Ronald Anderson, SJ.

It is with great sadness we note here that Fr. Ronald Anderson, SJ passed away soon after completing this essay in honour of his former mentor, Abner Shimony. Ron, who received a doctoral degree in particle physics in 1980 from the University of Melbourne, did his second doctorate in philosophy under Abner at Boston University; this was awarded in 1991.

It is likely that this essay will be the last work by Ron to appear in print. Readers should be aware that he did not have an opportunity to proof-read the final copy. The editors have had to edit a number of sentences in the text for grammar and punctuation, but the essay as printed here is essentially as he left it, and the voice in which it speaks is Ron's own. *The editors*.

W.C. Myrvold and J. Christian (eds.), *Quantum Reality, Relativistic Causality, and Closing the Epistemic Circle,* The Western Ontario Series in Philosophy of Science 73,

a scientific metaphysics for earlier generations may be less relevant now. Nevertheless, the naturalistic spirit and orientation of Peirce's work remains compelling and productive.

## 1 Peirce Scholarship and Peirce as a Resource for Forming a Scientifically Informed Metaphysics

An intricate and complex yet engaging and enticing task awaits the contemporary student seeking to enter the worlds and thought of Charles Sanders Peirce (1838–1914). One reason for this is simply: Peirce's vast output of texts. Estimates are given of published texts of around 10,000 printed pages with unpublished manuscripts of around 80,000 sheets.[1] Other reasons are the often technical nature of Peirce's writings, and their wide range. The topics addressed by Peirce include (in intersecting combinations) logic, semiotics, purely mathematical and scientific topics, the philosophical significance of these topics, and a sustained concern for building a comprehensive philosophical system informed by mathematics and science.[2] A further complexity arises from Peirce's thought evolving during his life, generating a subtle layered effect in his work that needs navigating carefully.

In addition, an equally intricate task awaits in tracing through scholarship on Peirce, a body of writings that matches the richness of Peirce's own writings. From the 1940s, when studies of his thought emerged (see [3] and [4] as early examples) to the present, the student is confronted with a range of academic styles and concerns reflecting the changing philosophical landscape of the 20th century.[3] During this period those reading Peirce as an historical project in classical American philosophy are together with those primarily concerned with locating resources within Peirce for informing present projects. Moreover, for a generation formed in and after the historiographic and sociological revolution in the study of science of the past few decades, it is instinctive to locate thinking deeply within its context, to attend to unique scientific practices of a time and culture, and to eschew seeking any abiding general essence to science. Thus for this generation factors such as the radical difference in present science (and mathematics) from that of Peirce's world lead to an unease with the project of earlier generations of Peirce scholars, of locating resources in Peirce for building a contemporary naturalized form of metaphysics or epistemology. Instead contextual projects suggest themselves, with force given the dense nature of Peirce's writings, with the attendant dangers of getting caught in projects that resist closure.

For exploring Peirce's life, Joseph Brent's biography [8] provides a perceptive and sympathetic reading, one that traces Peirce's upbringing in an academically and

---

[1] Ketner and Putnam, introduction to [1, p. 8].

[2] For a careful comment on the last mentioned of these projects in Peirce, see [2].

[3] For a general history in the 20th century of pragmatist thought relevant to the issues here, see [5–7].

socially privileged world in Cambridge, Massachusetts, through to his final years, isolated in Milford, Pennsylvania, at a property acquired in 1888.

Brent brings out vividly the complexity of Peirce's personal life and the degree it was marked with loss and tragedy through health problems, psychological struggles, financial difficulties, and the absence of a steady institutional context in which to work. For Brent:

> The beauty of the past arises from its permanence, from the impossibility of changing what was done. It is this forgiving permanence, suffusing even folly and tragedy with melancholy beauty, that transform the brilliant, bitter, humiliating, and above all tragic life of the American philosopher Charles Sanders Peirce into an odyssey of spirit which is at once fascinating, saddening, and compelling [8, p. 1].

In the spirit of Brent's assessment (and as Brent recognizes in this passage), the invariable experience of reading Peirce is to grow in admiration for his persistent creativity in generating ideas and texts in spite of the tragedy surrounding his life. Also on this point, Ketner and Putnam remark: "At times he must have written without stop: perhaps this explains at least in part his difficult nature" [1, p. 8]. The compelling quality is evident in the way Peirce is able to captivate and inspire others, reflected in the steady positive assessments by those who have encountered his thought. For example, his friend and supporter William James, writing to the President of Harvard in 1895 to recommend Peirce for teaching a course, remarked:

> He is the best man by far in America for such a course, and one of the best men living. The better graduates would flock to hear him—his name is one of mysterious greatness for them now—and he would leave a wave of influence, tradition, gossip, etc. that wouldn't die away for many years (quoted in [8, p. 243]).

And, as a long time scholar of Peirce's thought, Max Fisch, assessed the place of Peirce in glowing terms:

> Who is the most original and the most versatile intellect that the Americas have so far produced?" The answer "Charles S. Peirce" is uncontested, because any second would be so far behind as not to be worth nominating (quoted in [8, p. 2]).

The introductions to the two volume collection of Peirce's philosophical writings by Houser and Kloesel [9] and Houser [10] provide a concise entry to Peirce's thought, and the texts by Goudge [11], Murphey [12] and Hookway [13], and the more recent collection of studies edited by Houser et al. [14], together give a secure and comprehensive view of Peirce's thought and texts.

On venturing into Peirce's writings the now canonical multi-volume collection of his papers by Paul Weiss and Charles Hartshorne, joined later by Arthur W. Burks [15] is still the standard source and is now available on-line.[4] In addition, a chronological edition of his writings has been emerging [16]. Peirce has captivated many a student, not only by the power and breath of his ideas but also by his direct and immediate style as he develops ideas in an exploratory manner. His sustained use of vivid metaphors and tropes adds color, irreducible complexity, and subtlety to his

---

[4] References to the *Collected Papers* will be designated by the customary letters *CP x.y*, where *x* is the volume and *y* the paragraph number.

thinking. Indeed, for Peirce metaphor is intrinsic to philosophy, as we can see in a passage sounding distinctly contemporary, given recent studies of the intrinsic and important place of metaphor in writing and cognition:

> Metaphysics has been said contemptuously to be a fabric of metaphors. But not only metaphysics, but the logical and phaneroscopical [phenomenological] concepts need to be clothed in such garments. For a pure idea without metaphor or other significant clothing is like an onion without a peel [10, p. 392].

Given the power of Peirce's texts in these ways, the use of quotations from Peirce in the following is a perspicuous way to bring out the points I wish to make.

One of the centers for Peirce studies mid-century was Yale's philosophy department. Peirce scholars such as Paul Weiss and Rulon Wells had arrived in 1945 and John E. Smith in 1952. Abner's undergraduate years there in the 1940s, with teachers such as Weiss, and subsequent PhD studies in the early 1950s, were in the ambiance of these studies. Also, his Master's degree at Chicago in between undergraduate and PhD studies had another Peirce scholar, Charles Hartshorne, as director, who was familiar as well with the thought of Alfred North Whitehead. Later in the 1950s, when Richard Rorty and Richard Bernstein graduated from the department, it has been characterized as a "hotbed of pragmatist activity" [17, p. 97]. Rorty's PhD thesis on the history of concept of Potentiality for example, ended with an explicit pragmatist conclusion: "our descriptions of logical empiricism's difficulties ... suggest that we need to strive for the sort of rapprochement between formal logic, semiotics, and traditional epistemology which is found in the work of Peirce." (quoted in [17, p. 96]). When speaking of his earlier years, the captivating quality of Peirce's thought is evident in Abner's recollections:

> I read Peirce avidly and assented to almost everything that I understood of his semiotics, phenomenology, scientific methodology, pragmatism, critical common-sensism, and evolutionary metaphysics. Peirce's mixture of logical toughness, immersion in the history and practice of the natural sciences, and metaphysical speculation was inspiring to me then and continues to be so [18, vol. I, p. x].[5]
>
> I read lots of Peirce's papers, and I loved Peirce. I love Peirce to this day, and I think my point of view is closer to Peirce than to anyone else ([19, p. 15]).

As an illustration of the resources from Peirce's writings for forming a scientifically informed metaphysics, in the rest of this section I will outline a number of themes from Peirce's thought that figure in Abner's writings. This part of the paper will also serve to bring into relief aspects of the nature of Peirce's inquiry into the "facts of nature" (*CP* 2.750) as well as respect the occasion for the paper.

One central characteristic of Peirce's thought is his "fallibilism." For Peirce, human inquiry is such that "... people cannot attain absolute certainty concerning questions of fact" (*CP* 1.149). Peirce's notion is woven into other themes such as

---

[5] Susan Haack [20] provides a similar list, attesting to the power of Peirce's thought to a generation: "Over time, it has been Peirce's work that has come to influence me the most: his formal fluency and logical innovations, of course, but also his distrust of easy dichotomies, his idea of the growth of meaning, his attractively naturalistic theory of inquiry, his constructive reconception of metaphysics and its role—not to mention his penchant for neologisms."

his critique of Descartes' foundational project of grounding knowledge, his position that chance is woven deeply and intrinsically into the universe, and his perspective on the evolutionary nature of the universe (e.g. *CP* 1.173 and 1.152). Peirce's fallibilism goes with a belief that although we have no assurance of the correctness about our knowledge of nature, over time, with inquiry, we converge closer to truth about reality.[6] For 20th century pragmatism, this quality continued as a spirit of anti-foundationalism and a rejection of secure absolutes that to Dewey figured in much of Western philosophy.[7] Moreover, as Bernstein observes: "It was Peirce who initially argued that fallibilism is essential for understanding the distinctive character of modern experimental science" [21].

For Abner this feature of Peirce's thought was associated with the use of probability theory and "... certainly prepared me for the point of view that probability is essential in our epistemology, and that judgments of very high probability in favor of one conjecture and against another are quite compatible with his overall fallibilism" [19, p. 18].

Peirce also has the elements of what is known as the propensity interpretation of probability in a notion of "would be," referred to by one commentator as a "watershed" separating the middle from the final years of his intellectual life [10, p. xx] and characterized by Abner succinctly:

> It is not surprising that two of the most eminent advocates of the frequency interpretation, Peirce (1932) [15, vol. II] and Karl Popper (1957) [73], abandoned the frequency interpretation in favor of a different ontic interpretation, or propensity. The propensity interpretation ascribes an ontological status to the tendencies of propensities of the various possible outcomes of a singular chance event, such as the toss of a coin or the decay of a nucleus [18, vol. II, p. 237 and associated discussion].

There is an interesting lineage of Peirce's idea to Abner's profound and striking development of the idea that quantum states prior to measurement can be characterized by a notion of "objective indeterminacy," a notion in continuity with Heisenberg's idea of potentiality. When asked on the origin of this idea in the AIP interview, he noted: "I was ripe for it. Because of my advocacy of Peirce's would-be analysis of probability I was ripe to accept Heisenberg's analysis of the wave function in terms of potentiality" [19, p. 7].

Related to his fallibilism is Peirce's abiding concern with the process of human knowing, particularly that associated with the sciences. He observed in 1897:

> From the moment when I could think at all, until now, about forty years, I have been diligently and incessantly occupied with the study of methods of inquiry. .... I have paid most attention to the methods of the most exact sciences, have intimately communed with some of the greatest minds of our time in physical science (*CP* 1.3).

---

6 See for example, Hookway [13, p. 73f] for a standard presentation of Peirce on these matters.

7 As representative of that perspective: "Pragmatism which arose in the first instance through Peirce's canonical critique of Descartes, has always been a very pluralist movement centered on a concern to continue the discussion of knowledge on a non-foundationalist basis..." [6, p. 467]. See also [22].

In an essay of Abner's in 1981, developing an naturalist epistemology where scientific investigations are drawn on, Peirce is nearby: "Among classical philosophers, Peirce seems to come closest to the integral epistemology which I envisage" [18, vol. I, p. 5] and in a later comment, that Peirce had "... the makings of a balanced epistemology. ... between dogmatism ... and excessive skepticism ... [and that] he also really anticipated so much of the epistemology of the latter half of the 20th century" [19, p. 18]. Further, on particularities of Peirce's characterization of methods of scientific inquiry:

> To summarize, I find at least four methodological ideas of great value in Peirce's paper on scientific inference: that the scientific method achieves its successes by submission to reality, that a hopeful attitude towards hypotheses proposed by human beings is indispensable to rational investigation of the unknown, that a usable criterion of fair sample involves subjective and ethical considerations, and that it is rational to make certain weak assumptions about the fairness of the data in order to permit inquiry to proceed [18, vol. I, pp. 234–235].

Two further aspects of Peirce's thought that one finds mention of in Abner's writings relate to Peirce's understanding of evolution and the notion that the laws of nature themselves are emerging and evolving features of the universe. Peirce expressed various doubts as to Darwin's account of the manner of evolution, although not doubting that evolution had taken place (see e.g., [11, p. 227f]), and Abner, on arguing for the non-existence of the principle of natural selection, sees an affinity with Peirce on this point:

> Peirce seems to subsume the theory of natural selection under the theory of probability. ... I believe that the my thesis of the non-existence of a principle of natural selection fits the main current of his thought. It is honorable to be an epigone of Peirce [18, vol. II, p. 245].

A more radical idea may be found in Peirce—that the fundamental laws themselves have an evolutionary explanation (*CP* 6.33), a speculation the nature of which to Abner reminds us of the "continuity of modern physics with metaphysics" [18, vol. I, p. 29]. In general Peirce posits the universe as evolutionary on its deepest level:

> The evolutionary process is, therefore, not a mere evolution of the existing universe, but rather a process by which the very Platonic forms themselves have become or are becoming developed (*CP* 6.194).

A sympathetic assessment of the idea in a range of thinkers (yet critical of Peirce) is given in [23], and Paul Davies has drawn on the idea in a number of general publications, e.g., most recently, [24].

The final idea of Peirce I wish to mention is that of proto-mentality or mentalism, referred to more generally as a position of panpsychism. The idea is found rather widely late 19th to the middle of the 20th century in the writings of figures such as James, Royce, Bergson, Teilhard de Chardin, Whitehead, and Hartshorne (for a impressive history of panpsychism, see [25]). For panpsychism, at a lower level of matter there is a dimension of mind or mentality throughout the universe, one that gets concentrated on higher levels such as in human consciousness. The idea is in conflict with that of contemporary notions of emergence, when consciousness can be seen as naturally emergent property, arising from the complexity of a pre-mental neurological matter. Debates on this topic continues, although as we increasingly understand how the brain generates the nature of consciousness, a "naturalist" perspective (and on that, in accord with the spirit of Peirce) would now seem to align with the notion of emergence. Peirce presented the idea in a series of papers (1891–1893) where he argued for a monism of mind and matter and a "dual aspect" theory of mind:

> The one intelligible theory of the universe is that of objective idealism, that matter is effete mind, inveterate habits becoming physical laws (*CP* 6.25) and ... what we call matter is not completely dead, but is merely mind hidebound with habits (*CP* 6.128).
>
> But all mind is directly or indirectly connected with all matter, and acts in a more or less regular way; so that all mind more or less partakes of the nature of matter. ... Viewing a thing from the outside, considering its relations of action and reaction with other things, it appears as matter. Viewing it from the inside, looking at its immediate character as feeling, it appears as consciousness (*CP* 6.268).

Abner mentions in his AIP interview how the idea was an attractive and important one for him, both religiously and intellectually, in his early encounter with Peirce as well as Whitehead, observing, in what constitutes the main reason for the notion, that if we evolved then our mental faculties are production of evolution, not just our bodies—and if they are, then "there must be something mental-like from which the faculties evolve" [19, p. 10]. More recently as well Abner has suggested that a naturalist "physicalism" can be a component in an epistemological naturalism when combined with a mentalism of a sort that "would have a fundamental status in nature, either coordinate with physical reality or yet more fundamental" [26, p. 306].

## 2 Peirce on Mathematics: Necessary and Hypothetical

While Peirce has been known for fields such as his studies on scientific as well as his work on logic and as one of the founders of American Pragmatism, the new wave of Peirce scholars from the 1960s onwards have drawn out and emphasized the central place of mathematics in his thought.[8] Benjamin Peirce, his father, was a well known mathematician in 19th century America and a powerful charismatic teacher at Harvard during Peirce's youth. He played an important role in Peirce's early education in a variety of ways. Peirce directly refers to the important influence of his father in his early education and in particular that "... without appearing to be so, he [Benjamin Peirce] was extremely attentive to my training when I was achild, and especially insisted upon my being taught mathematics according to his

[8] The work of Carolyn Eisele stands out here, both in numerous studies on Peirce's mathematics and scientific philosophy [28] as well as in editing the four volume *The New Elements of Mathematics* [27] containing Peirce's mathematical writings. Also, studies by Buchler [3], Goudge [11], Hookway [13], Joswick [29], Levy [30], Cooke [31] and Campos [32] have drawn out the importance and significance of aspects of Peirce's thought on mathematics.

directions …" (quoted in [27, vol. 4, p. v]). Brent's biography of Peirce emphasizes the weighty legacy of his father on his life, that his father had "draped on his shoulders the crushing mantle of genius" and engaged him in an exacting and intellectual training, the effects of which "were to aggravate his neurological pathologies, to nourish his arrogance, and to set his ambition afire" [8, p. 16].

Peirce first of all makes mathematics central in the priority he gives it in a classification of the disciplines. Moreover, it is a discipline in need of no other disciplines.[9] In one of his disciplinary mappings of the sciences, and the "architectonic character" of philosophy, Peirce observed:

> …. mathematics meddles with every other science without exception. There is no science whatever to which is not attached an application of mathematics. This is not true of any other science, since pure mathematics has not, as a part of it, any application of any other science, inasmuch as every other science is limited to finding out what is positively true, either as an individual fact, as a class, or as a law; while pure mathematics has no interest in whether a proposition is existentially true or not. In particular, mathematics has such a close intimacy with one of the classes of philosophy, that is, with logic, that no small acumen is required to find the joint between them (*CP* 1.245).
>
> …. It might, indeed, very easily be supposed that even pure mathematics itself would have need of one department of philosophy; that is to say, of logic. Yet a little reflection would show, what the history of science confirms, that that is not true. Logic will, indeed, like every other science, have its mathematical parts (*CP* 1.247).
>
> …. But mathematics is the only science which can be said to stand in no need of philosophy, excepting, of course, some branches of philosophy itself. It so happens that at this very moment the dependence of physics upon philosophy is illustrated by several questions now on the tapis (*CP* 1.249).

Rather strikingly Peirce gives mathematics a central role in developing a philosophy, as in a letter of 1894:

> My special business is to bring mathematical exactitude,—I mean *modern* mathematical exactitude, into philosophy,—and to apply the ideas of mathematics in philosophy (quoted in [27, vol. 4, p. x]).

Moreover in the development of thought itself, mathematics was the "earliest field of inquiry" as mathematics is the "most abstract of all the sciences" and the first questions asked are "naturally the most general and abstract ones" (*CP* 1.52–53).

The relationship between logic and mathematics forms an entangled thread in Peirce's thought. In various passages Peirce stressed the independence of mathematics from logic:

> I will not admit that the mathematician stands in any need of logic. The mathematician must reason, of course; but he needs no theory of reasoning, because no difficulties arise in mathematics which require a theory of reasoning for their resolution. The metaphysician *does* require a theory of reasoning; because in his science such difficulties *do* arise. All the special sciences (especially the nomological sciences) repose, more or less, on metaphysics, and therefore, at least indirectly, and some of them directly too, require a theory of logic. But pure mathematics can postpone such a theory [27, vol. 4, p. 98].

---

[9] For more on the manner in which mathematics is foundational in Peirce see [33] and [13, Chapter 6].

> It does not seem to me that mathematics depends in any way upon logic. It reasons, of course. But if the mathematician ever hesitates or errs in his reasoning, logic cannot come to his aid (*CP* 4. 228, 1902).

> Logic can be of no avail to mathematics; but mathematics lays the foundation on which logic builds … (*CP* 4.250).

He will also characterize mathematics as an activity of reasoning that is direct and intuitive. Logic, on the other hand, is a study of reasoning [34]. Referred to by Dipert [35, p. 46] as a "reverse-logicism," Peirce's priority of mathematics is a persistent strain in his writings. Yet, as commentators have noted, in other places Peirce comments on mathematics' dependency on logic.[10] And Peirce will note, when referring to Dedekind's work on numbers of 1888 that the "boundary between some parts of logic and pure mathematics … is almost evanescent" (*CP* 2.215).

One can see resonances in Peirce of an analogous distinction made in Whately's *Elements of Logic* [36]—a widely used logic book in the 19th century.[11] On several occasions Peirce noted that Whately's text, which he had first read as a youth, was of considerable influence on him, reflecting in a latter to Lady Welby in 1908 that "… from the day when at the age of 12 or 13 I took up, in my elder brother's room a copy of Whately's Logic … it has never been in my power to study anything—mathematics, ethics, metaphysics, gravitation, thermo-dynamics, optics, chemistry, comparative anatomy, astronomy, psychology, phonetics, economic, the history of science, whist, men and women, wine, metrology, except as a study of semeiotic" [37, p. 85]. Whately remarks that one can reason accurately prior to a study of logic, much as one can speak prior to the study of grammar he also likens logic to the "grammar of reasoning" [36, p. 11]. Analogously for Peirce, the ability to do mathematics is independent of a study of its methods of reasoning.

These foundational features of mathematics are woven into a number of other features Peirce ascribes to mathematics. First, stressing that he owes the idea to his father, Peirce often referred to mathematics as a science that draws "necessary conclusions":

> Of late decades, philosophical mathematicians have come to a pretty just understanding of the nature of their own pursuit. I do not know that anybody struck the true note before Benjamin Peirce, who, in 1870, declared mathematics to be "the science which draws necessary conclusions," adding that it must be defined "subjectively" and not "objectively" (*CP* 3.558).

> … It was Benjamin Peirce, whose son I boast myself, that in 1870 first defined mathematics as "the science which draws necessary conclusions." This was a hard saying at the time; but today, students of the philosophy of mathematics generally acknowledge its substantial correctness (*CP* 4.229).

The phrase that Peirce quotes is the opening sentence in Benjamin Peirce's well known study, "Linear Associative Algebra" [38]. Peirce argues in a number of places against a traditional definition of mathematics as the science of quantity (e.g. *CP* 3.554). Peirce also knew Boole's work well and there are echoes in Peirce

---

[10] Comprehensive discussions of this topic may be found in [30] and [35].

[11] For a study of the influence of Whately's text on Peirce see [39].

of Boole's same questioning of the significance of mathematics as the science of quantity as in his essay of 1847 [40, p. 4]. Similar notions of mathematics occur in the "Preface" of *Analytical Society Memories*, by Charles Babbage and John Herschel, where the power of a symbolic language for mathematical reasoning is celebrated and mathematics is characterize as examining "… the varied relations of necessary truth" [41, p. i]. Peirce also proposed a more general significance to mathematics in philosophy—all necessary *a priori* thinking is a form of mathematical thinking:

> Philosophy requires exact thought, and all exact thought is mathematical … I can only say that I have been bred in the lap of the exact sciences and I know what mathematical exactitude is, that is as far as I can see the character of my philosophical training (quoted in [27, vol. 4, p. x]).
>
> All necessary reasoning is strictly speaking mathematical reasoning [,] that is to say, it is performed by observing something equivalent to a mathematical Diagram …. [1, p. 116].

Perice refers positively to an analogous definition by George Chrystal in the ninth edition of the Encyclopedia Britannica (1883).[12] Hints of such a position may also be found in his father's writings [42]. Mathematics with a definition as the science that draws necessary conclusions is such that, to his father it "belongs to every inquiry, moral as well as physical." ([43, p. 97] and see also [44, p. 377]).

As one would suspect, to give such a foundational role for mathematics requires a rich conception of mathematics, which is indeed the case for Peirce. In particular for Peirce mathematical reasoning involves diagrams and a form of interior observation:

> … What then is the source of mathematical truth? For that has been one of the most vexed of questions. I intend to devote an early chapter of this book to it.1 I will merely state here that my conclusion agrees substantially with Lange's, that mathematical truth is derived from observation of creations of our own visual imagination, which we may set down on paper in form of diagrams (*CP* 2.77).
>
> … In mathematical reasoning there is a sort of observation. For a geometrical diagram or array of algebraical symbols is constructed according to an abstractly stated precept, and between the parts of such diagram or array certain relations are observed to obtain, other than those which were expressed in the precept. These being abstractly stated, and being generalized, so as to apply to every diagram constructed according to the same precept, give the conclusion. (*CP* 2.216).

Peirce's references to observation and mathematics occur shortly after a famous British Association for the Advancement of Science address by J.J. Sylvester in 1868 where a role is given for observation in the practice of the mathematics [45]. Perice quotes a phrase from Gauss that Sylvester had used in his address: "… for as the great mathematician Gauss has declared—algebra is a science of the eye—only it is observation of artificial objects and of a highly recondite character" (*CP* 1.34).

[12] Chrystal [42] characterized mathematics as: "any conception which is definitely and completely determined by means of a finite number of specifications, say by assigning a finite number of elements, is a mathematical conception. … As an example of a mathematical conception we may take "a triangle"; regarded without reference to its position in space, this is determined when three elements are specified, say its three sides …".

Peirce also associates perceptual judgments with mathematical proof noting the "... compulsiveness of the perceptual judgment is precisely what constitutes the cogency of mathematical demonstration" (*CP* 7.659, 1903). In this way the "compulsory" feature of mathematics is grounded.

For Peirce, this underlies a role mathematics can play in philosophy as errors will be reduced "to a minimum" in philosophy by:

> ... treating the problems as mathematically as possible, that is, by constructing some sort of a diagram representing that which is supposed to be open to observation by every scientific intelligence, and thereupon mathematically,—that is, intuitionally,—deducing the consequences of that hypothesis (quoted in [27, vol. 4, p. x]).

Other features of Peirce's notion of mathematics include taking mathematical reasoning as a form of experimenting with diagrams. A particularly bold statement of his position on this occurs in "Notes on Ampliative Reasoning" in 1902 that "Mathematical proof is probably accomplished by appeal to experiment upon images or other signs, just as inductive proof appeals to outward experiment" (*CP* 2.782). Such mathematical diagrams are "iconic" which leads to Peirce's rich and extensive work on semiotics that would take us to far a field to consider here (on this see [13, p. 189f]). That all thinking for Peirce involves signs is another way mathematics is linked deeply to general reasoning. Peirce in the following, on the practice of the reasoning, weaves all these threads together:

> ... he searches his heart, and in doing so makes what I term an abstractive observation. He makes in his imagination a sort of skeleton diagram, or outline sketch, of himself, considers what modifications the hypothetical state of things would require to be made in that picture, and then examines it, that is, observes what he has imagined, to see whether the same ardent desire is there to be discerned. By such a process, which is at bottom very much like mathematical reasoning, we can reach conclusions as to what would be true of signs in all cases, so long as the intelligence using them was scientific (*CP* 2.227).

In addition to characterizing mathematics as the discipline that draws necessary consequences, Peirce stressed (as in the last quotation), and increasingly as his thought developed, that mathematics is hypothetical. In particular, that "... all mathematicians now see clearly that mathematics is only busied about purely hypothetical questions" (*CP* 1.52). In this way mathematics is distinguished from an inquiry into nature:

> For all modern mathematicians agree with Plato and Aristotle that mathematics deals exclusively with hypothetical states of things, and asserts no matter of fact whatever; and further, that it is thus alone that the necessity of its conclusions is to be explained. This is the true essence of mathematics ... (*CP* 4.232, 1902).
>
> Mathematics is the study of what is true of hypothetical states of things. That is its essence and definition. (*CP* 4.333, 1902)

Peirce emphasizes in other places that hypotheses are creations of the mathematician and that this is the origin of the necessary nature of mathematics (*CP* 3.560, 8.110). This cluster of features then—mathematics as manipulating with and experimenting on diagrams, as observational, as working with hypothesis that are other than to do with facts about the world, as that which draws necessary consequences, as the discipline that is foundational and central in philosophy—together constitute Peirce's vision of mathematics. Ketner and Putnam go so far to remark that many of these features meant mathematics "was the inspirational source for the pragmatic maxim, the jewel in the methodological part of the semeiotic, and the distinctive feature of Peirce's thought" [1, p. 2].

That a significant feature of Peirce's characterization of mathematics is blended with actual practices of the mathematician provides further support for the place of mathematics in pragmatism. When commenting on the nature of mathematics Peirce often refers to the beliefs and practices of mathematicians, with attention frequently to historical contexts. The words "mathematician" and "mathematicians," for example, occur 202 times in the *Collected Papers*, and while less that "mathematics" (340) and "mathematical" (334) the number is significant. The usage accords with Peirce's pervading epistemological concern with the nature of human reasoning. Campos [32] has drawn attention to this dimension of mathematics for Peirce, noting Peirce's definitions of mathematics as necessary and hypothetical are "descriptions of mathematical activity" and observed in a comment that concisely sums up various points in this section:

> The practice of imagining hypothetical states of things and asking what would necessarily be true about them provides the context in which mathematical icons are conceived, created and recreated, so as to explore a myriad would-be worlds.

## 3 Balancing Mathematics and Inquiry into Nature

A long persistent thread in reflection on the empirical and natural sciences has been on the role of mathematics in such sciences.[13] As mathematics is a structured symbolic system with features of a natural language and long taken, as expressed by Galileo's famous trope, as the language of the book of nature, the issue in the broadest sense is one of the relationship of a language to reality, on the junction of "word" and "thing", an issue that has haunted modern philosophy. Locke's clear and direct separation of words, things and ideas in *An Essay Concerning Understanding* leaves the unsettling question of their relationship, and forms a textual monument to this question that has haunted modernity:

> We should have a great many fewer disputes in the world if only words were taken for what they are, the signs of our ideas only, and not for things themselves [46, vol. III, p. 10].

As applied to mathematics, the question appears as a semantic one of how the symbols and notation of mathematics embody mathematical concepts and refer either to mathematical objects or to features of empirical objects, such as properties and the laws of nature.[14]

---

[13] For the manner in which this topic can be addressed in tracing the history of physics, see [47,48].

[14] As an aside, Benjamin Peirce's study of Algebra of 1870 in various places uses the textual image for mathematics; mathematics as a language and with a grammar [43, p. 98, 105].

During the 19th century the question was sharpened as mathematics was increasingly seen as abstract and as a discipline separate from the sciences of nature. The development of abstract symbolic algebra (separate from arithmetic algebra) by Peacocke, Hamilton, and De Morgan in the early part of the century was part of this development in mathematics while the development of non-Euclidean geometry in the latter part was another.[15] Herschel's influential *Preliminary Discourse* [49] is representative of these moves and draws a sharp distinction between the abstract sciences of mathematics and the natural sciences concerned with causality and laws of nature. In Whately's logic, too, the text mentioned earlier, there is a persistent emphasis on how a proper understanding of Logic requires recognizing that logical matters to do with reasoning are distinct from "the observations and experiments essential to the study of nature" [36, p. 9; see also, p. 25, 338].

This stress on the unique features of mathematics brings into clear relief the question of relationship of mathematics and the natural sciences in a discipline such as mathematical physics. This multi-sided question can be posed generally as one about probing the nature of the meeting point of the abstract, necessary and symbolic with the concrete, contingent and empirical. This question will set the agenda for tracing Perice's texts on this topic.

By stressing in various places that mathematics has a distinct identity, different from the natural sciences, Peirce is part of these movements within 19th century mathematics. His emphasis on the hypothetical nature of mathematics is one such place where this occurs: "Mathematics is engaged solely in tracing out the consequences of hypotheses. As such, she never at all considers whether or not anything be existentially true, or not" (*CP* 1.247). And in some striking passages:

> The mathematician lives in another world from the rest of us, in a world of pure forms. Here he is domiciled and spends part of his time, but he is a mere sojourner; this is not the world that he knows or that he cares for. If you tell him that something in the world of mathematical forms corresponds to something in the real world, be cautious not to speak as if such a correspondence could impart any value to the mathematical object, or he may consider you impertinent. Of what consequence is that reality to him? [16, vol. 6, p. 258].
>
> There is no essential difference between pure and applied mathematics. The mathematician does not, as such, inquire into facts. He only develops ideal hypotheses. These hypotheses are all more or less suggested by observation and all depart from or transcend, more or less, what observation fully warrants. But if the hypotheses are developed with a view to ideal interests, it is pure mathematics. If they are made crabbed and one sided in the interest of truth it is applied mathematics. [27, vol. 2, p. vi].

Both of these passages, and the second one in particular, bear a resemblance to his father's almost Pythagorean vision of a fusion of mathematics and nature. For example, in a series of lectures published shortly after his death his father writes, with vivid metaphors:

> But in the frozen cave of geometry, the thoughts which may trickle in from the actual world are crystallized into glittering, passionless, and unsympathizing stalactites; and the

[15] For an exploration of this topic see [50].

> mathematical sage cares not whence they came,—whether they fell as dew from the quiet sky, or as rain from the clouds driven by the wind. Whatever their origin, they are ideal truth [43, p. 167].

And for his father, on the ready application of mathematics to the study of nature, the mathematics of quaternions to which the mathematician was led from imaginary numbers has become "the true algebra of space" that "clearly elucidates some of the darkest intricacies of mechanical and physical philosophy" (Ibid. p. 29).

In these passages and in his son's writings in particular there are hints of Cantor's view of "pure mathematics" as "free mathematics," presented in the *Grundlagen* of 1883. Such a mathematics is in opposition to that constrained by the empirical world, or "crabbed" in Peirce's phrase quoted above. As well Boole, in the text referred to above, remarks that mathematics considers operations in themselves, "independently of the diverse objects to which they can be applied" [40].

The spirit here is in accord with another characteristic of mathematics that Peirce stresses, viz., generalization, and this too is outlined in a context that places it in opposition to applied mathematics:

> Another characteristic of mathematical thought is that it can have no success where it cannot generalize. One cannot, for example, deny that chess is mathematics, after a fashion; but, owing to the exceptions which everywhere confront the mathematician in this field—such as the limits of the board; the single steps of king, knight, and pawn; the finite number of squares; the peculiar mode of capture by pawns; the queening of pawns; castling—there results a mathematics whose wings are effectually clipped, and which can only run along the ground (*CP* 4.236).

Interestingly Peirce then will often identify aspects of mathematics by placing them in contrast to science. The creative and free nature of forming hypotheses in mathematics, the necessary features of mathematics, and the pursuit of generalization in mathematics all stand in apparent contrast to the practices of the natural sciences.

The frequency with which Peirce places the intersection of mathematics and study of nature in this way is striking. It is a particular way of doing mathematics:

> The truths of mathematics are truths about ideas merely .... Thomson and Tait (Natural Philosophy §438) wisely remark that it is "utterly impossible to submit to mathematical reasoning the exact conditions of any physical question." A practical problem arises, and the physicist endeavors to find a soluble mathematical problem that resembles the practical one as closely as it may. ... The mathematics begins when the equations or other purely ideal conditions are given. "Applied Mathematics" is simply the study of an idea which has been constructed to look more or less like nature [27, vol. 4, p. xv].

Peirce continues this passage to mention that geometry is an example of "applied mathematics." The mathematician, will use a "space imagination" to form "icons of relations which have no particular connection with space." These are diagrams visually imagined of a space. But at the same time "space is a matter of real experience" (Ibid. p. xv). Elsewhere too, Peirce dwells on geometry's dual nature: non-Euclidean geometry is securely established in abstract mathematics, yet "geometry, while in its main outlines, it must ever remain within the borders of philosophy, since it depends and must depend upon the scrutinizing of everyday experience, yet at certain special points it stretches over into the domain of physics" (*CP* 1.249). Only measurements will tell the nature of the geometry of actual space. Peirce also intriguingly speculates on the existence of higher dimension, a topic of sustained interest in the latter part of the 19th century: "Thus, space, as far as we can see, has three dimensions; but are we quite sure that the corpuscles into which atoms are now minced have not room enough to wiggle a little in a fourth?" (*CP* 1.249). With practice a mathematician at home in universal geometry can adjust to a space of four dimensions: "Give a higher geometer sixty days to accustom himself to a four-dimensional space, and he would be ever so much more at home there than he ever can be in this perverse world" [51, vol. 3. p. 182].

The overall context for this seeming opposition between mathematics as the abstract hypothetical study and mathematics as practiced in the midst of the investigation into nature is one where Peirce is often addressing the practices of the mathematician and the practices of the scientist. It is here I propose we have a clue to a pervasive feature of Peirce's thought: that the apparently more systematic issue such as that posed above of the relationship of mathematics as a formal system to natural science and its objects, Western philosophy's old haunting issue of representation of thought to reality, appears invisible to Peirce. Instead it appears as steadily posed instead in terms of activities.[16] This is illustrated nicely in a passage where Peirce directly addresses the use of mathematics for physics:

> The complex plane is one of the meeting-grounds of mathematicians and physicists, and the latter are now quite at home in the presence of that coy handmaiden, the complex variable; indeed, the well-known transformation scene in which she and her image play such a prominent part, is now an important feature in the solution of some practical problems [27, vol. 3, p. 145].

Also mathematics is useful for the work of the physicist as, "First, it enables him to solve his own problems instead of employing a mathematician …. Secondly, it supplies him with fundamental conceptions and methods of thinking without which he never can rise from the ranks of the army of science" [27, vol. 3, p. 121].

While posing the issue of the meeting places of mathematics and the natural sciences in terms of the practices of both disciplines is a persistent feature of Peirce's thought, there's a deeper more systematic question: how does mathematics' necessary and certain nature fits with Peirce's Fallibilism? The issue has been directly addressed by Haack [52] and Cooke [31] and to both there are unresolved tensions in Peirce's writings on this topic. For Haack the puzzle is that Peirce seems able to hold that our mathematical beliefs could be mistaken while still holding to a position that mathematical truths are necessary [52, p. 37]. Indeed Peirce in places stresses how mistakes can be made in doing mathematics and it is clear it is an uneasy problem for him (see, e.g., *CP* 1.149, *CP* 4.237). For Haack the tension resides in Peirce's failure to specify fully what is meant by fallibilism (a point other commentators have remarked on) and with a more elaborate specification, there are ways in which it could coexist for Peirce with mathematics necessary nature.

---

[16] There is an intriguing link between Peirce on this point and J.J. Sylvester and others in the British context such as James Clerk Maxwell that awaits further exploration. Maxwell, for example, in an British Association address in 1870, soon after Sylvester's address considers the relationship between mathematics and physics largely in terms of the activities of those in both disciplines.

Cooke argues that Peirce "can and should hold a position of fallibilism within mathematics, and that this position is more consistent with his overall pragmatic theory of inquiry and general commitment to the growth of knowledge" [31, p. 159]. In particular, for Peirce to hold for a type of theoretical infallibilism for mathematics would be deeply incompatible with his rejection of the separation of a science's intelligibility from its human knowers. Yet for Cooke Peirce could consistently allow error in the practice of the mathematician who for Peirce experiments with hypothetical truths via diagrams, and could be brought about by allowing a different form of fallibilism from that associated with investigating empirical features about the world. This would be a particular type of "internal fallibilism" such to allow for the obvious way mathematicians can make errors in doing mathematics, and further, recognizing such doubt in this realm for Cooke allows a general conclusion that it allows inquiring into new areas in mathematics—consequently discovering new relations and new systems [31, p. 174]. Such a position accords with Peirce remarks when commenting as indicated earlier on how deduction (or "analytical reasoning") involves perception and experimentation:

> Deduction is really a matter of perception and of experimentation, just as induction and hypothetic inference are; only, the perception and experimentation are concerned with imaginary objects instead of with real ones. The operations of perception and of experimentation are subject to error, and therefore it is only in a Pickwickian sense that mathematical reasoning can be said to be perfectly certain. It is so only under the condition that no error creeps into it; yet, after all, it is susceptible of attaining a practical certainty. (*CP* 6.595)

There is another deep issue here related to that to do with of foundations of knowledge. As those in the later pragmatist tradition of American thought have emphasized, Peirce's fallibilism can be seen as a form of anti-foundationalism, one that is not an either or sort where the opposite to foundationalism is a relativism (e.g. see [21, 53]). Moreover, we are now in the wake of a long sustained consideration in the 20th century of the pursuit of foundations in mathematics (see, e.g., [54, 55]). Peirce's famous critique of Descartes' grounding of the edifice of knowledge on an indubitable inner intuition is the basis of his anti-foundationalism (*CP* 5.264).

Also Peirce's metaphors have an anti-foundationalist flavor. Peirce will indeed use the metaphor of architecture, positively remarking when treating the classification of science and the "architectonic of philosophy" that the "... universally and justly lauded parallel which Kant draws between a philosophical doctrine and a piece of architecture has excellencies which the beginner in philosophy might easily overlook" (*CP* 1.176).[17] However, for Peirce the metaphor functions more as a way to comment on the texture and structure of a philosophical system: "that is why philosophy ought to be deliberate and planned out; and that is why, though pitchforking articles into a volume is a favorite and easy method of bookmaking it is not the one which Mr. Peirce has deemed to be the most appropriate to the exposition of the principles of philosophy ...." (*CP* 1.179). The architecture metaphor for

[17] Also, when characterizing philosophical systems Peirce will invoke the architectural metaphor: "There is a synchronism between the different periods of medieval architecture, and the different periods of logic. The great dispute between the Nominalists and Realists took place while men were building the round-arched churches ..." (*CP* 4.27).

knowledge therefore is not taken, as commonly taken in the philosophical tradition, to describe the building knowledge built on firm foundations.

Peirce also has various other powerful metaphors for knowledge which argue against knowledge being grounded on foundations, one being his famous metaphor of knowledge as on a bog and another, that of a bottomless lake.[18]

> The only end of science, as such, is to learn the lesson that the universe has to teach it. In Induction it simply surrenders itself to the force of facts. But it finds ... that this is not enough. It is driven in desperation to call upon its inward sympathy with nature, its instinct for aid, just as we find Galileo at the dawn of modern science making his appeal to il lume naturale. But in so far as it does this, the solid ground of fact fails it. It feels from that moment that its position is only provisional. It must then find confirmations or else shift its footing. Even if it does find confirmations, they are only partial. It still is not standing upon the bedrock of fact. It is walking upon a bog, and can only say, this ground seems to hold for the present. Here I will stay till it begins to give way. (*CP* 5.589)

> Consciousness is like a bottomless lake in which ideas are suspended at different depths. Indeed, these ideas themselves constitute the very medium of consciousness itself. Percepts alone are uncovered by the medium. We must imagine that there is a continual fall of rain upon the lake; which images the constant inflow of percepts in experience. All ideas other than percepts are more or less deep, and we may conceive that there is a force of gravitation, so that the deeper ideas are, the more work will be required to bring them to the surface. (*CP* 7:533)

Then there is Peirce's powerful and famous metaphor of knowledge as constituted by the fibers of a cable given when criticizing Descartes (adapted, as Haack [2] notes, from Thomas Reid):

> Philosophy ought to imitate the successful sciences in its methods, so far as to proceed only from tangible premisses which can be subjected to careful scrutiny, and to trust rather to the multitude and variety of its arguments than to the conclusiveness of any one. Its reasoning should not form a chain which is no stronger than its weakest link, but a cable whose fibers may be ever so slender, provided they are sufficiently numerous and intimately connected. (*CP* 5.265)

All these metaphors, which capture the spirit of Peirce's understanding of the inquiry into nature, are at odds with the spirit of mathematics. A chain metaphor in particular, one in opposition with that of a cable of fibers, has a long association with the deductive structure of mathematics in figures such as Descartes and Hume and in early 19th century writings on mathematics in the British context. Yet, as we have seen above, mathematics for Peirce has a foundational place in philosophy, it is acritical in that it stands in need of no other discipline to proceed and there is a necessary quality to its deductions. Moreover these qualities are often outlined

---

[18] Both Thagard [56] and Abrams [57] address Peirce's use of these metaphors. One may speculate too on the influence of Peirce's cultural context. As commentators on the anti-foundationalist dimension of pragmatist though have remarked, the world of the America following the civil war was one to encourage the development of "... a more flexible, open experimental way of thinking that would avoid all forms of absolutism and ideologies that result in intolerance" [21]. And in more general terms the expansionist spirit of a new country, with vast territory arguably lent itself to such thinking rather than the trend of European philosophy to search for secure foundations.

in contrast with the nature of the other sciences. There's a complex and apparent tension then, one that invites further consideration on the nature of mathematics.

A dimension of mathematics that mutes the foundationalist image is the role mathematicians play in the creation of hypotheses. Here, as Peirce stresses, they are not constrained by the nature of the world, and in this process lies a creative freedom for the mathematician. Thus a natural way to think of mathematics as foundational by virtue of its axioms and starting points is not immediately to the foreground in Peirce. However, and balancing this, Peirce is careful to note the process of hypothesis creation is not an arbitrary one. Peirce has hints in place of Platonist conception of mathematics, a potential foundation for mathematics. When addressing the issue that one would expect with arbitrary hypothesis creation, namely that "different mathematicians to shoot out in every direction into the boundless void of arbitrariness" Peirce remarks that this does not happen and this phenomena:

> ... is not an isolated one; it characterizes the mathematics of our times, as is, indeed, well known. All this crowd of creators of forms for which the real world affords no parallel, each man arbitrarily following his own sweet will, are, as we now begin to discern, gradually uncovering one great cosmos of forms, a world of potential being. The pure mathematician himself feels that this is so ... if you enjoy the good fortune of talking with a number of mathematicians of a high order, you will find that the typical pure mathematician is a sort of Platonist. Only, he is [a] Platonist who corrects the Heraclitan error that the eternal is not continuous. The eternal is for him a world, a cosmos, in which the universe of actual existence is nothing but an arbitrary locus. The end that pure mathematics is pursuing is to discover that real potential world. (*CP* 1.646)

Peirce here makes the commonplace observation that the practicing mathematician is a Platonist, and there's a hint of convergence of mathematics to a given form that parallels Peirce's notion of scientific investigators converging in time to truth about nature. Peirce's Platonist phrases can take lyrical form:

> That passage of the mathematician, Plato, strikes a sympathetic chord in every mathematicians' breast when he says that these heavens and earth we gaze upon are but the walls and floor of a dismal cavern which shut out from our direct view the glories of the world of forms beyond [16, vol. 6, p. 258].[19]

He leaves open however what this could mean for particular mathematical systems. It would take us to far a field to pursue the idea, but in giving a place to observation in mathematics, and experimentation on diagrams as part of mathematical reasoning, Peirce could be read as grounding a form of mathematical Platonism in a naturalist manner.[20] In general for Peirce the most likely source of inspiration for the mathematician's practices come for situations in the world, not a Platonic world of the beyond.

---

[19] As a further example of Peirce's balance of this mathematical world of the beyond with the study of nature, Peirce continues: "Yet, what would steam-engines, electric cables, turbine wheels, life-insurance and a thousand things be but for the hints which mathematicians have vouchsafed?" (*Ibid.* vol. 6, 258).

[20] Abner interestingly explores Gödel's Platonism in this manner, noting Gödel's fondness of "comparing intuition of mathematical objects with sensory perception of physical objects of ordinary experience" [26, p. 301].

The hypothetical nature of mathematics nevertheless dominates Peirce's account of mathematics, despite the hints of a grounding in a Platonic realm of potential form. Peirce also resists a Kantian move of grounding the axioms and starting points of mathematics in a metaphysical or otherwise foundation. In particular, Peirce denies any dependence of mathematics on space, time, or any form of "intuition" (*CP* 3.556).

Here Peirce's view on mathematical truth and certainty has interesting resonances with the Scottish mathematician Stewart (1753–1828). Stewart stressed that the starting points of mathematics are assumed: "we have in view [...] not to ascertain truths with respect to the actual existences, but to trace the logical filiation of consequences which follow from an assumed hypothesis. If from this hypothesis we reason with correctness, nothing [...] can be wanting to complete the evidence of the result; as this result only asserts a necessary connexion between the supposition and the conclusion" [58, vol. II, p. 114]. Stewart's view was opposed by the Cambridge philosopher William Whewell, who sought to ground mathematical truths in broader metaphysical foundations of a Kantian nature.

Peirce's stress on the hypothetic nature of mathematics goes along as well with the spirit of characterizing logical inference in a hypothetical manner: "To say that an inference is correct is to say that if the premises are true the conclusion is also true; or that every possible state of things in which the premises should be true would be included among the possible state of things in which the conclusion would be true." (*CP* 2.710) It is also in accord with his support of a "Philonian" interpretation of conditional statements such "If A then B" as being true if A is either an empty class or A is untrue (for a discussion of this point see, Ketner and Putnam in [1]). What matters essentially is the structure of inference or mathematical or logical deduction, not its grounding in initial axioms or premises. Peirce's account has also later been associated with a position of "If-Thenism" or "deductivism" where truth as understood in this manner of connection and deducibility within a system (see [59], Chapter 10 for a modern discussion of this position).

In places when considering scientific investigations, Peirce sees that as hypothetical as well:

> Nothing is vital for science: nothing can be. ... The scientific man is not in the least wedded to his conclusions. He risks nothing upon them. He stands ready to abandon one or all as soon as experience opposes them. Some of them, I grant, he is in the habit of *calling established truths*; but that merely means propositions to which no competent man today demurs. It seems probable that any given proposition of that sort will remain for a long time upon the list of propositions to be admitted. Still, it may be refuted tomorrow; and if so, the scientific man will be glad to have got rid of an error. There is thus no proposition at all in science which answers to the conception of belief ([1], Lecture 1).

Here then an activity of science shares a feature of mathematics.

By dwelling on the hypothetical nature of mathematics (and science), and deductive relations Peirce on these issues appears as an early exemplification of the structuralism that was to flourish in the 20th century. Bourbaki's text, for example, *Elements of the History of Mathematics*, notes on that history that it would be

"... be tempting to say that the modern notion of "structure" is attained in substance around 1900; in fact it will need still another thirty years of apprenticeship before it appears in all its glory" [60, p. 21].

There remains the clear foundational nature of the "necessary" nature of inferences of the mathematician, and exploring this leads to a key distinction Peirce makes in mathematical reasoning. One type, "corollarial," involves immediate deductions in a straightforward way from axioms. They need not involve the iconic diagrams directly. The other type is "theorematic" reasoning, which involves a more active creation of strategies and experimentation with diagrams to achieve a result (*CP* 2.267 and *CP* 4.613 and for a discussion of this distinction see [29, 30]). And example of the latter would be a supplementary construction needed in a proof to bring about the conclusion. The significance of such reasoning to Peirce had been overlooked in the tradition, and Peirce's remarks here are part of his attention to activities of the mathematician.

As we have seen, Peirce will ground the necessary nature of mathematics in various ways. A further way for Peirce is in the intuition—to imply a type of mathematical intuitionism. Goudge perceptively remarks that while Peirce can be read this way it is "entirely out of harmony with his naturalism" [11, p. 259]. It is not, though, grounded in a psychological form of intuition, as, for Peirce, "the mathematician clothes his thought in mental diagrams, which exhibit regularities and analogies of abstract forms almost quite free from the feelings that would accompany real perceptions" [51, vol. 3, p. 258]. Among recent commentators on this point, Joswick [29] takes the semiotic dimension of Peirce's mathematics as providing of seeing how Peirce grounds mathematical necessity. Of the threefold types of signs for Peirce—symbols, icons and indexes—it is only an icon that can bring out the inferential nature of mathematics, as it exhibits the form of an object and thus presents the relationships in the object. For Joswick,

> The icon is the essential mathematical sign because by "direct observation of it other truths concerning its object can be discovered" (2.280). Through the direct examination of an icon necessary connections in the object can be seen and unexpected relations revealed. "The whole of inference," Peirce contends, "consists in *observation*, namely in the observation of icons" (7.557) [29, p. 111].

Such a position is in accord with Peirce's notion that all necessary reasoning involves the use of diagrams, stated strongly in manuscript notes of 1896: "All valid necessary reasoning is in fact thus diagrammatic" (*CP* 1.54). What appears as significant is that again for Peirce a foundational dimension is significantly grounded in the very activity of the mathematician, not in a formal independent feature of a mathematical knowledge or mathematical objects. In this way it parallels the quality of fallibilism that attends the inquiry into the facts of nature.

In tracing in Peirce's thought the qualities of fallibilism and necessity that attend the natural sciences and mathematics respectively, one can see a subtle overlap of both realms. Yet there is a persistent tension. In a recent essay, Cooke, on this very topic, remarks that on a "pragmatic level" as to "how it is practiced" as indicated here, mathematics is like the empirical sciences, even though Peirce "so frequently holds that mathematics and science must be conceived as separate" [61].

A further point where the apparent contradictory qualities appear in balance in Peirce is on the topic of abstraction in mathematics. For Peirce the abstract is an important feature of mathematics: "Another characteristic of mathematical thought is the extraordinary use it makes of abstractions" (*CP* 4.234) and "... it may be said that mathematical reasoning (which is the only deductive reasoning, if not absolutely, at least eminently) almost entirely turns on the consideration of abstractions as if they were objects" (*CP* 3.509). Yet for Peirce the use of abstractions are woven into everyday life as well as mathematics and science. In a rich play of metaphor Peirce weaves together these contexts:

> These examples exhibit the great rolling billows of abstraction in the ocean of mathematical thought; but when we come to a minute examination of it, we shall find, in every department, incessant ripples of the same form of thought, of which the examples I have mentioned give no hint (*CP* 4.235).

The point here is similar to an observation of Whitehead, that, as mathematics increasingly entered into ever greater extremes of abstract thought, it became at the same time increasingly relevant for the analysis of particular concrete facts [62, p. 47], and to Dewey's remark that the very power of mathematics in physics arises from its free and abstract nature [63, p. 412].

The final point I wish to address the question originally posed on how mathematics relates to nature: what is that meeting place of mathematics and nature? Here two commentators on Peirce can provide a way to focus two threads in Peirce's thought.

The first arises from Peirce discussion of how maps, as icons and diagrams represent (*CP* 5.329 and *CP* 8.122). To Hookway [13], this example, plus a consideration of how for Peirce a color sample may be taken to represent color schemes of a house, provide a way to understand what Peirce would take to be the applicability of mathematics to nature. Maps represent and require interpretation, and in a similar way mathematical systems represent when interpreted and applied to "state of affairs" of the same form as the relational structure of the mathematical system [13, p. 191]. Various phrases in Peirce support such a perspective, for example, as quoted above, to Peirce for a practical problem "... the physicist endeavors to find a soluble mathematical problem that resembles the practical one as closely as it may." In such a way then Peirce can be seen as using the old metaphor of representation theory: mathematics mirrors and maps a reality other than it. The perspective is surely one of the dominant ways mathematics is seen to function in a scientific theory.

Another thread though places the issue of the union between mathematics and the natural sciences in an activity associated with mathematics. For Peirce, mathematics with its observational nature and manner of experimenting on diagrams, as well as its hypothetical nature shares similar practices to those of the natural science. Plus as we have seen, any necessary type of thinking for Peirce is mathematical. Such a position has been argued recently by Daniel Campos:

> I would claim that for Peirce the most important application of mathematics does not consist in the deployment of this or that particular mathematicaltheory to solve this or that practical

> problem, but in the overall deployment of necessary reasoning to investigate problems in, say, phenomenology, aesthetics, ethics, logic, and the practical, physical and practical sciences [64, p. 73].

The abiding focus in Peirce is on the practices of the mathematician and scientist, plus the pervasive and central feature he gives to mathematics makes this a compelling perspective. It is one that sidesteps the long standing issue of how one realm of human endeavor, the mathematical and the resultant mathematical structures and theories, can represent a different realm that is implicated in notions of representation. It may be as well that in the background is Peirce's evocative expression that dealing with matters of representation entails further representations in an unending manner:

> The meaning of a representation can be nothing but a representation. In fact, it is nothing but the representation itself conceived as stripped of irrelevant clothing. But this clothing never can be completely stripped off; it is only changed for something more diaphanous. So there is an infinite regression here. Finally, the interpretant is nothing but another representation to which the torch of truth is handed along; and as representation, it has its interpretant again. Lo, another infinite series. (*CP* 1.399)

Moreover in a modern guise the position is similar to Hacking's proposal that traditional questions of realism when placed in the form of exploring classical issues to do with "representation" are intractable and a better perspective is obtained by exploring the instrumentality of our engagement with the world [65]. Hacking links his position directly to pragmatism: "The final arbitrator in philosophy is not what we think, but what we do" (Ibid. p. 31).

Further support from this position I'd suggest, although indirect, is related to Perice's panpsychism, his ascribing of a mental dimension to matter, and to a closely help belief that there is a natural mapping between mind and matter. That latter glides into a residue of idealism present in Peirce's writings, as, e.g., quoted above: "objective idealism, that matter is effete mind, inveterate habits becoming physical laws" (*CP* 6.25).

Here there is a blurring of traditional boundaries, not so much between the activity of the knower doing mathematics and the one involved in investigating nature, but between the knower doing mathematics and the realm which is the subject of that investigation, nature.

Peirce's father is the likely influence here.[21] Peirce in 1889, in a dictionary entry on the topic of ideal-realism described his father's position as "the opinion that nature and the mind have such a community as to impart to our guesses a tendency toward the truth, while at the same time they require the confirmation of empirical evidence" (quoted in [2, p. xxv]). In various places in Benjamin Peirce's writings hints of such a fusion of mind and matter emerge such as from a textbook written when teaching at Harvard: "Every portion of the material universe is pervaded by the same laws of mechanical action, which are incorporated into the very constitutions of the human mind" [44, p. 30; 66, p. 495]. Then later, that the "identity between the laws of mind and matter" suggests their common origin, one that if it is "conceded

[21] For a discussion of the influences of Benjamin Peirce on Charles, see [67].

to reside in the decree of a Creator," ceases to be mystery (Ibid. p. 31). To suggest alternatively that consciousness was "evoked out of the unconscious" would fail to give an adequate cause for it. And in an address to the American Association for the Advancement of Science in 1853, noted that the sciences and geometry in particular show "the world to which we have been allotted is peculiarly adapted to our minds, and admirable fitted to promote our intellectual progress" [68, p. 12]. A striking poetic Pythagorean fusion of matter and mathematics occurs in the following:

> The highest researches undertaken by the mathematicians of each successive age have been especially transcendental ... but the time has ever arrived ... when the progress of observation has justified the prophetic inspiration of the geometers, and identified their curious speculations with the actual workings of Nature. [44, p. 29]
>
> Long before ... observation had begun to penetrate the veil under which nature has hidden her mysteries, the restless mind sought some principle of power strong enough and of sufficient variety to collect and bind together all parts of the found. This seems to be found, where one might least expect it, in abstract numbers. Everywhere the exactest numerical proportion was seen to constitute the spiritual element of the highest beauty. (Benjamin Peirce, quoted in [69, p. 101])

His father refers to his position as one of "ideality" and will write that "the whole domain of physical science is equally permeated with ideality" [43, p. 17].

Peirce was immersed in this world view from his earliest years and given the influence of his father overall in his life, this would account for beliefs that mathematics may be applied to nature and that the worlds of nature and mathematics cohere together. Moreover I would claim, they form background assumptions and beliefs in Peirce, a haunting presence from his father's world. They are invisible to him in the sense they are not to the foreground to be subject to philosophical investigation.

In addition, Peirce's ready and powerful use of metaphor is such to allow a background belief to persist, carried subtly in images beyond full explication. Against this backdrop, the tensions of the two fields of inquiry, mathematics and the natural sciences, as focused abstractly above, can remain invisible. This supplements the unity in the knower due to the overlapping similarities in the practices of the knower of both fields. The complexity here is in need of further elaboration and contextualization, but its presence is a pointer to the deep currents guiding Peirce's thought. If correct, there is exemplified what I would propose is a general lesson: that pressing the status of mathematics in a system of thought and its relationship to the study of nature is a sure path to the depth structures, often silent ones, that constitute that system of thought.

## 4 Concluding Reflections

Together, the topics of this paper leave us with the question of what resources from Peirce and his understanding of mathematics and its place in the natural science can we use to inspire and inform contemporary projects. In some ways Peirce sounds rather modern (and postmodern). The foundationalist projects of 20th century foundations of mathematics have receded. Long reflections on the implications of Gödel's incompleteness results have taught us that foundations in grounding deductive thought tend to recede and elude us. Also, naturalist movements in the philosophy of mathematics, which see similarities of mathematics with the empirical sciences have taken hold and have undertaken to explore the practices and activities of the mathematician (see, for example [70, 71]). On both of these points, Peirce appears as a fellow traveler who initiated new paths.

Other parts of Peirce's world now appear dated. The complexity of neurological structure as revealed by contemporary cognitive sciences have made projects of understanding consciousness possible in new ways, such as an emergent phenomena of (pre-mental) matter. John Searle, expresses this vision powerfully, if polemically:

> Some traditional philosophical problems, though unfortunately not very many, can eventually receive a scientific solution. This actually happened with the problem of what constitutes life. We cannot now today recover the passions with which mechanists and vitalists debated whether a "mechanical" account of life could be given. The point is not so much that the mechanists won and the vitalists lost, but that we got a much richer conception of the mechanisms. I think we are in a similar situation today with the problem of consciousness. It will, I predict, eventually receive a scientific solution. But like other scientific solutions in biology, it will have to give us a causal account. It will have to explain how brain processes cause conscious experiences, and this may well require a much richer conception of brain functioning than we now have [72].

In continuity with this perspective Peirce's (and Whitehead's) panpsychism, that placed a mental dimension on lower levels of matter, now, through the advances of science, appears superfluous. Also studies on the practices of mathematics with the resources of contemporary projects in the sociology of science and naturalist accounts of reasoning have surpassed what Peirce achieved. Both of these developments mean that Peirce's blend and balance of mathematics and the natural sciences that I've suggested are tied into deeply held beliefs on the unity of mind and matter inspired by his father, and grounded in commonality of practices, are similarly dated.

In addition, 20th century physics, with its new understandings of the nature of chance in nature arising from quantum theory have supplanted Peirce's worlds. Overall our emerging theories on the structure of matter and space and time from decades of particle physics and the more recent string theory and loop quantum loop gravity have revealed a complexity and richness of matter unknown in Peirce's time, and thus dated various of the themes mentioned in section I above. And again, Whitehead's elaborate metaphysics of the event appears as from an earlier time in physics, prior to our present micro-theory of fundamental reality (even if presently incomplete) that's of such a nature to supplant many features of Whitehead's metaphysics. As a lighthearted observation, the complexity and details of string theory then can be seen to rival and surpass the difficulties previous generations had in working though the elaborate structures of Whitehead's *Process and Reality*.

Still questions to do with the nature of mathematics tend to persist and the vigor and complexity of Peirce's thought on mathematics and the activity of the mathematician are such that the very exercise to enter into Peirce's texts and those of the

Peirce scholarship on this topic remains valuable. The exercise is valuable historically in order to understand a key part in American intellectual history and how that unfolded in 20th century thought, and its present configuration. Here though projects still await on contextualizing Peirce's thought in more complete ways than some of those touched on above. The exercise is also of value to develop a set of skills to explore analogous issues on the contemporary landscape. Moreover, as the work of both Peirce and Abner witness to: a naturalist vision of using the resources of the natural sciences to pursue the deep questions associated with our philosophical tradition remains productive. And something else, very rewarding, remains for all who encounter the writings of Peirce: the inspiring example of what it means to live the life of a scholar, on how, with persistence and single mindedness, to explore ideas in spite of personal struggles and setbacks and at the same time to write, steadily, persistently, and relentlessly.

**Acknowledgements** Various of the ideas in this paper where discussed with Abner Shimony for which I'm grateful and that is part of a larger gratitude to Abner for discussions, encouragement and inspiration over the years on all manner of issues to do with understanding the nature of science. The help of Alison Walsh, Elizabeth Cooke, David Skrbina and Daniel Campos for sources and ideas for the paper is gratefully acknowledge. I am also grateful to Joy Christian and Wayne Myrvold for the opportunity to be part of the conference honoring Abner's thought, from which this paper emerged, and to the American Institute of Physics for permission to quote from the recorded interviews with Abner made in 2002.

## References

1. Peirce, Charles S. (1992). *Reasoning and the logic of things: the Cambridge conferences lectures of 1898*. Cambridge, MA, Harvard University Press. Edited by Kenneth Laine Ketner; with an introduction by Kenneth Laine Ketner and Hilary Putnam.
2. Haack, Susan (1996). "Between Scientism and Conversationalism." *Philosophy and Literature* **20**(2): 455–474.
3. Buchler (1939), Justus (1939). *Charles Peirce's Empiricism*. New York: Harcourt Brace and Company.
4. Nagel, Ernest (1940). "Charles S. Peirce, Pioneer of Modern Empiricism." *Philosophy of Science* **7**: 69–80.
5. Kloppenberg, James T. (1996). "Pragmatism: An Old Name for Some New Ways of Thinking?" *Journal of American History* **83**: 100–138.
6. Rockmore, T. (2004). "On the Structure of Twentieth-Century Philosophy." *Metaphilosophy* **35**: 466–478.
7. Bernstein, Richard J. (1992). "The Resurgence of Pragmatism." *Social Research* **59**: 813–840.
8. Brent, Joseph (1993). *Charles Sanders Peirce: a life*. Bloomington, IN, Indiana University Press.
9. Peirce, Charles S. (1992). *The essential Peirce: selected philosophical writings*, Vol. 1. Bloomington, IN, Indiana University Press. Edited with an introduction by Nathan Houser and Christian J. W. Kloesel.
10. Peirce, Charles S. (1998). *The essential Peirce: selected philosophical writings*, Vol. 2. Bloomington, IN, Indiana University Press. Edited by the Peirce Edition Project; with an introduction by Nathan Houser.
11. Goudge, Thomas A. (1950). *The thought of C. S. Peirce*. Toronto, University of Toronto Press.

12. Murphey, Murray G. (1961). *The development of Peirce's philosophy*. Cambridge, MA, Harvard University Press.
13. Hookway, Christopher (1985). *Peirce*. London/Boston, Routledge & Kegan Paul.
14. Houser, Nathan, Don D. Roberts, and James Van Evra (1997) (eds.). *Studies in the logic of Charles Sanders Peirce*. Bloomington, IN, Indiana University Press.
15. Peirce, Charles S. (1931–1958). *Collected papers of Charles Sanders Peirce*. Vols. 1–8 Cambridge. Originally published: Cambridge, MA, Harvard University Press. Edited by Paul Weiss and Charles Hartshorne and A. W. Burk.
16. Peirce, Charles S. (1982). *Writings of Charles S. Peirce: a chronological edition*. Bloomington, IN, Indiana University Press. Edited by Max Harold Fisch and Christian J. W. Kloesel.
17. Gross, Neil (2003). "Richard Rorty's Pragmatism: A Case Study in the Sociology of Ideas." *Theory & Society* **32**: 93–148.
18. Shimony, Abner (1993). *Search for a naturalistic world view*. 2 volumes, Cambridge [England]/New York, NY, Cambridge University Press.
19. Shimony, Abner (2002). Transcript of oral history interview of Abner Shimony by Joan Bromberg, September 9 and 10, 2002. Niels Bohr Library, American Institute of Physics, 147 pp.
20. Haack, Susan (2005). "Formal Philosophy? A Plea for Pluralism" in Vincent F. Hendricks and John Symons, eds., *Formal Philosophy*, Automatic Press, pp. 77–98.
21. Bernstein, Richard J. (2006). "Pragmatic Reflections on Tolerance." Available online at http://www.pucp.edu.pe/eventos/congresos/filosofia/programa_general/lunes/plenaria/BernsteinRichard.pdf
22. Rockmore, T. (2005). "On Classical and Neo-Analytic Forms of Pragmatism." *Metaphilosophy* **36**: 259–271.
23. Shimony, Abner (1999). "Can the Fundamental Laws of Nature Be the Result of Evolution?" in C. Pagonis and J. Butterfield, eds., *From physics to philosophy*. Cambridge/New York, Cambridge University Press, pp. 208–223.
24. Davies, Paul (2007). "The Universe's Weird Bio-Friendliness." *The Chronicle of Higher Education* **53**(31): B14.
25. Skrbina, David (2005). *Panpsychism in the West*. Cambridge, MA, MIT Press.
26. Shimony, Abner (2002). "Some Intellectual Obligations of Epistemological Naturalism" in D. B. Malement, ed., *Reading natural philosophy: essays in the history and philosophy of science and mathematics*. Chicago, IL, Open Court, pp. 297–313.
27. Peirce, Charles S. (1979). *The new elements of mathematics*. 4 volumes. The Hague, Mouton/Humanities Press. Edited by Carol Eisele.
28. Eisele, Carolyn (1979). *Studies in the scientific and mathematical philosophy of Charles S. Peirce: essays by Carolyn Eisele*. The Hague/New York, Mouton. Edited by R. M. Martin.
29. Joswick, Hugh (1988). "Peirce's Mathematical Model of Interpretation." *Transactions of the Charles S. Peirce Society* **24**: 107–121.
30. Levy, Stephen H. (1997). "Peirce's Theoremic/Corollarial Distinction and the Interconnections between Mathematics and Logic" in N. Houser, D. D. Roberts, and J. Van Evra, eds., *Studies in the logic of Charles Sanders Peirce*. Bloomington, IN, Indiana University Press, pp. 85–110.
31. Cooke, Elizabeth F. (2003). "Peirce, Fallibilism, and the Science of Mathematics." *Philosophia Mathematica* **11**: 158–175.
32. Campos, Daniel (2007). "Peirce on the Role of Poetic Creation in Mathematical Reasoning." *Transactions of the Charles S. Peirce Society* **43**: 470–489.
33. Eisele, Carolyn (1971). "The mathematical foundations of Peirce's Philosophy" in *Studies in the scientific and mathematical philosophy of Charles S. Peirce: essays by Carolyn Eisele*. The Hague/New York, Mouton. Edited by R. M. Martin.
34. Hull, Kathleen (1994). "Why Hanker after Logic? Mathematical Imagination, Creativity, and Perception in Peirce's Systematic Philosophy." *Transactions of the Charles S. Peirce Society* **30**: 271–296.

35. Dipert, Randall R. (1994). "Peirce's Underestimated Place in the History of Logic: A Response to Quine," in Kenneth L. Ketner, ed., *Peirce and Contemporary Thought: Philosophical Inquiries*, New York: Fordham University Press, 32–58.
36. Whately, Richard (1826). *Elements of logic: comprising the substance of the article in the Encyclopedia Metropolitana; with additions*, &c. London, J. Mawman.
37. Peirce, Charles S. and Victoria Welby (2001). *Semiotic and significs: the correspondence between Charles S. Peirce and Victoria Lady Welby*. Elsah, IL, Arisbe Associates. Edited by Charles S. Harwick and James Cook.
38. Peirce (1870) Peirce, Benjamin (1881). "Linear Associative Algebra." *American Journal of Mathematics* **4**: 97–229. This paper is based on a memoir read before the National Academy of Sciences in Washington, 1870. It was published posthumously with notes by Charles S. Peirce.
39. Seibert, Charles H. (2005). "Charles Peirce's Reading of Richard Whately's 'Elements of Logic'." *History and Philosophy of Logic* **26**: 1–31.
40. Boole, George (1847). *The mathematical analysis of logic: being an essay towards a calculus of deductive reasoning*. Cambridge, England, Macmillan Barclay & Macmillan.
41. Babbage, Charles and John Herschel (1813). "Preface." *Memoirs of the Analytical Society*: i–xxii.
42. Chrystal, George (1883). "Mathematics". *The ninth edition, Encyclopaedia Britannica*. Vol. 15. Edinburgh, Adam & Charles Black.
43. Peirce, Benjamin (1881). *Ideality in the physical sciences*. Boston, FL, Little Brown & Co.
44. Peirce, Benjamin (1880). "The impossible in mathematics", in Mrs. John T. Sargent, ed., *Sketches and reminiscences of the Radical Club of Chestnut St. Boston*. Boston, FL, James R. Osgood, pp. 376–379.
45. Sylvester, James J. (1868). "Presidential Address to the British Association." reproduced in *Sylvester's mathematical papers*, vol. 2, p. 654.
46. Locke, John (1690). *An Essay Concerning Human Understanding*. London.
47. Anderson, Ronald and Girish Joshi (1993). "Quaternions and the Heuristic Role of Mathematical Structures in Physics." *Physics Essays* **6**: 308–319.
48. Anderson, Ronald and Girish Joshi (2008), "Interpreting Mathematics in Physics: Charting the Applications of SU(2) in 20th Century Physics." *Chaos, Solitons & Fractals*, **36**: 397–404.
49. Herschel, John F. W. 1831. *Preliminary Discourse on the Study of Natural Philosophy*. London: Longman, Rees, Orme, Brown, and Green.
50. Gray, Jeremy J. (2004). "Anxiety and Abstraction in Nineteenth-Century Mathematics." *Science in Context* **17**: 23–47.
51. Peirce, Charles S. (1975). *Charles Sanders Peirce: contributions to the nation*. Lubbock, TX, Texas Tech Press. Compiled and annotated by Kenneth Laine Ketner and James Edward Cook.
52. Haack, Susan (1979). "Fallibilism and Necessity." *Synthese* **41**: 37–63.
53. Bernstein, Richard J. (1989). "Pragmatism, Pluralism and the Healing of Wounds." *Proceedings and Addresses of the American Philosophical Association* **63**: 5–18.
54. Mancosu, Paolo (1997). *From Brouwer to Hilbert: the debate on the foundations of mathematics in the 1920s*. Oxford: Oxford University Press.
55. Giaquinto, M. (2002). *The search for certainty: a philosophical account of foundations of mathematics*. Oxford/New York, Clarendon Press/Oxford University Press.
56. Thagard, Paul and Beam, Craig (2004). "Epistemological Metaphors and the Nature of Philosophy." *Metaphilosophy* **35**: 504–516.
57. Abrams, Jerold J. (2002). "Philosophy After the Mirror of Nature: Rorty, Dewey, and Peirce on Pragmatism and Metaphor." *Metaphor and Symbol* **3**: 227–242.
58. Steward, Dugald (1854–1860). *The collected works of Dugald Stewart*. Edited by Sir W. Hamilton: 11 vols., Edinburgh, T. Constable: Vols. 2–4, Elements of the Philosophy of the Human Mind, vol. II first published 1814.
59. Chihara, Charles S. (2004). *A structural account of mathematics*. Oxford/New York, Clarendon/Oxford University Press.
60. Bourbaki, Nicolas (1994). *Elements of the history of mathematics*. Berlin/New York, Springer-Verlag.

61. Cooke, Elizabeth F. (2007). “Peirce’s General Theory of Inquiry and the Problem of Mathematics” (under review).
62. Whitehead, Alfred N. (1926). *Science and the modern world*. Cambridge: Cambridge University Press.
63. Dewey, John (1938), *The later works, 1925–1953*, Volume 12, Carbondale and Edwardsville, IL, Southern Illinois University Press, Edited by Jo Ann Boydston.
64. Campos, Daniel G. (2005). *The Discovery of Mathematical Probability Theory: A Case Study in the Logic of Mathematical Inquiry*. Ph.D. thesis, Pennsylvania State University, UMI AAT 3202479.
65. Hacking, Ian (1983). *Representing and intervening: introductory topics in the philosophy of natural science*. Cambridge/New York, Cambridge University Press.
66. Peirce, Benjamin (1855). *A system of analytic mechanics*. Boston, FL, Little Brown & Co.
67. Walsh, Alison (2000). *Relationships Between Logic and Mathematics in the Works of Benjamin and Charles S. Peirce*, Ph.D. thesis, Middlesex University.
68. Peirce, Benjamin (1853). Address of Professor Benjamin Peirce, president of the American Association for the year 1853, on retiring from the duties of president.
69. Emerson, Edward Waldo (1918). *The early years of the Saturday club, 1855–1870*. Boston, FL/New York, Houghton Mifflin Co.
70. Goodman, Nicolas (1991). “Modernizing the Philosophy of Mathematics.” *Synthese* **88**: 119–126.
71. van Kerkhove, Bart (2006). “Mathematical Naturalism: Origins, Guises and Prospects.” *Foundations of Science* **11**: 5–39.
72. Searle, John R. (2006). “Minding the Brain, Review of ‘Seeing Red: A Study in Consciousness’ by Nicholas Humphrey.” *The New York Review of Books* **53**.
73. Popper, Karl R. (1957). “The Propensity Interpretation of the Calculus of Probability, and the Quantum Theory,” in S. Körner, ed., *Observation and Interpretation*. London: Butterworths Scientific Publications, pp. 65–70.