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JOSEPHSON EFFECT AND CIRCLE MAP.

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ABSTRACT.

The Josephson effects associated with a thin insulating layer or a narrow constriction between two superconductors has important technological applications and also provides an instructive example of non-linear effects that may be studied with relatively simple methods. The system is dynamically analogous to a damped pendulum driven by a constant and an oscillating torque and may be simulated by solving a non-linear second order differential equation. The phenomena of phase-locking and transition to chaos are, however, much more conveniently studied by the discrete circle-map.

This paper discusses the connection between the continuous time Josephson equation and the discrete circle-map by the method of return-maps. It will be shown that the so called critical circle map in many cases (but not all) is associated with the transition to hysteretic behaviour of the Josephson junction. The critical circle map, exhibiting a fractal mode-locking structure, is an example of the relevance of the mathematical theories of rational numbers to physics and is recommended as a good example of the challenges of experimental mathematics.

1. The Josephson equation

Superconductivity is one striking example of a macroscopic quantum phenomenon. A crude explanation for metallic superconductivity is obtained by the observation by Cooper (1956) that the effective interaction between electrons through lattice vibrations (phonons) is attractive for states near the Fermi surface, so it becomes energetically favorable to form pair-states, the so called Cooper pairs. These pairs have spin zero and act therefore like particles obeying Bose-Einstein statistics, like e.g. the atoms of He^4 , and similarly to liquid Helium the electrons will condense in a superfluid state at sufficiently low temperatures. This is an example of a Bose-condensation due to the "social" nature of Bose-particles who, quite unlike the "exclusive" Fermi particles, prefer to occupy the same quantum state.

The common wave function for the condensed Cooper-pairs thus acquires the status of a macroscopic order-parameter for the superconducting state, and all of the most striking phenomena of superconductivity, like perfect diamagnetism (Meissner effect), flux quantization, and persistent currents, can be related to the quantum mechanical nature of this complex, scalar field that is coupled to the electromagnetic vector-field. The combined thermodynamic and quantum-mechanical theory leading to non-linear partial differential equations for these two fields was discovered already in 1950 by Landau and Ginzburg¹, and this phenomenological theory has later been able to account for the Josephson effects².

In 1962 it was suggested by Brian D. Josephson that a thin insulating layer between two bulk superconductors should be able to carry a tunnelling supercurrent in the absence of any voltage. The magnitude of this current is given by the dc-equation

$$I_s = I_c \cdot \sin\phi \quad (1)$$

where I_c is the critical current and ϕ is the phase difference between the superconducting wave functions on the two sides of the barrier. In the presence of a voltage difference, V , over the barrier, the phase difference will increase linearly with time in accordance with the ac-equation

$$d\phi/dt = 2eV/\hbar \quad (2)$$

where e is the numerical charge of the electron (thus, the charge of a Cooper-pair is $-2e$) and $\hbar = h/2\pi$, h being Planck's quantum of action. Eq.s (1) and (2) together show that a constant dc-voltage over the junction will produce an ac-supercurrent with vanishing mean value.

In practice, however, it is not easy to control the voltage across the junction. The experimental situation is one of current-control, because the resistance of the junction is much smaller than the resistance in the external wires. The junction-resistance is due to the normal (i.e. not condensed) electrons, and this normal fluid is thus providing an extra channel for the current, parallel to the super-channel described by eq. (1). The normal current is given by Ohm's law:

$$I_n = V/R_n \quad (3)$$

where R_n is the normal resistance. Finally, if the junction has a non-vanishing capacitance, C , the following displacement-current must be added to the super- and the normal current:

$$I_d = C \cdot dV/dt \quad (4)$$

The resulting model, called the resistively and capacitively shunted Josephson junction, is very successful in explaining the observed current-voltage characteristics, when a Josephson junction is exposed to a microwave field. The effect of the microwaves is then described by an ac-current source, $I_{AC} \cdot \sin \omega t$, added to the dc-current, I_{DC} . The model in fig. 1a corresponds to the equation:

$$I_d + I_n + I_s = I_{DC} + I_{AC} \cdot \sin \omega t \quad (5)$$

This equation can be reduced to a dimensionless form in the following way. First, we measure the currents in units of the critical current, I_c , putting

$$I = I_{DC}/I_c \quad ; \quad A = I_{AC}/I_c \quad (6)$$

Next, we note that the super-channel, described by eq.s (1) and (2), for small values of the phase ϕ acts like a self-inductance

$$L_s = \hbar/(2eI_c), \quad (7)$$

so the ratio between this quantity and the normal resistance, $\tau = L_s/R_n$, can be used as a unit of time. Thus,

we introduce the symbols

$$\dot{\phi} = \tau \cdot d\phi/dt \quad ; \quad \ddot{\phi} = \tau^2 \cdot d^2\phi/dt^2 \quad ; \quad f = \tau \cdot \omega/2\pi \quad (8)$$

and get the following second order differential equation:

$$Q^2 \cdot \ddot{\phi} + \dot{\phi} + \sin\phi = I + A \cdot \sin(2\pi fT) \quad (9)$$

where $T = t/\tau$, and Q is the dimensionless quantity

$$Q = R_n \cdot \sqrt{2eI_c C/\hbar} \quad (10)$$

As shown in fig. 1b the Josephson equation (9) may just as well describe a classical mechanical system: a damped and driven pendulum. The device may be constructed as metal disc with an excentric mass, M , and a friction unit, F , (e.g. a magnet). Through a torque-control unit, T , the experimenter should be able to apply a torque, consisting of a constant part superposed on a sinusoidally oscillating part, to the pendulum. The correspondence between the parameters and dynamic variables of the two analogous systems is shown in the table below.

<u>Josephson junction</u> .	<u>Pendulum</u> .
Phase ϕ	Angle from vertical
Supercurrent $I_s = I_c \sin\phi$	Torque from gravity
Normal current	Torque from friction
Capacitance C	Moment of inertia
Applied current $I_{DC} + I_{AC}$	Applied torque

In the following, however, we shall use "Josephson language", mainly because there is a wealth of experimental data to compare with for many different types of superconducting weak links. Before we go on to discuss such data we may remark the following on the different time scales in question. In addition to the inductive relaxation time τ , used as the time unit in eq. (9), and the period of oscillation τ/f of the ac-current (the microwave field), there are two other characteristic times. One is the capacitive relaxation time

$$R_n \cdot C = Q^2 \cdot \tau \quad (11)$$

and the other is the period of oscillation of the undamped "Josephson plasmon" or the undamped pendulum near the stable equilibrium in the absence of driving torques:

$$2\pi \cdot \sqrt{L_S \cdot C} = 2\pi Q \cdot \tau \quad (12)$$

The parameter Q is the quality factor for the plasmon-resonance; the motion will be oscillatory only for $Q > \frac{1}{2}$. A reason for using Q as the parameter instead of "the damping factor", $G = 1/Q$, is that it becomes easier to consider the case $Q = 0$, which is not a case of "infinite damping", but rather of vanishing capacitance. This case is particularly relevant for the case when the Josephson junction has the shape of a "Dayem-bridge", i.e. a narrow constriction in a thin superconducting film. The model in this case degenerates into "the resistively shunted junction", described by the first order differential equation (9) with $Q = 0$.

The experimental current-voltage characteristic with and without a microwave field present are sketched in fig. 2. (For a more detailed account, see ref. 3, and references therein). The figure applies to the cases $Q < \frac{1}{2}$; for greater values of Q the curves will exhibit hysteresis. The voltage measured is not the instantaneous value of V , but rather its mean value over a long period of time. A dimensionless measure of the voltage is the mean value of the phase derivative $\dot{\phi}$. This is related to the so called winding number, W , i.e. the average number of full turns of the phase per period of the ac-current (we always assume there is an ac-current with frequency f , even though its amplitude A may be zero):

$$W = \lim_{T \rightarrow \infty} \frac{\phi(T) - \phi(0)}{2\pi T \cdot f} = \frac{\langle \dot{\phi} \rangle}{2\pi f} \quad (13)$$

For $Q=0$, $A=0$ the IV-curve is a hyperbola approaching Ohm's law for the normal metal as I goes to infinity, but for $A>0$ there is a step in the current for every voltage that corresponds to an integral value $p/1$ of the winding number W . These are the so called harmonic steps, and their magnitude can be shown to go like the Bessel function J_p of the amplitude A . The supercurrent ($p=0$) should thus be reduced for $A>0$, but for real junctions there will often be an enhancement instead. This enhancement, the Dayem effect, is one of the few qualitative features that are not revealed by eq. (9) but must be explained by the microscopic theory⁴.

The subharmonic steps (not shown in fig. 2) for $W=p/q$, $q>1$, exist for all rational values of W and will be discussed in the following sections.

2. From continuous to discrete time .

The integration of the Josephson equation (9) is in principle straightforward, but very time consuming on a digital computer. A fast and reliable analog computer is the best tool, but this is very expensive, and the programming is rather intricate due to the presence of the $\sin\phi$. In this paper we discuss an alternative approach: the integration is done on an ordinary digital computer of a type that is common in high-schools and private homes (Amstrad 6128), but the results are presented graphically in a way that makes a transition to a discrete time model possible. Afterwards, the qualitative features of phase locking, bifurcations and chaos are investigated by the discrete model, the circle-map.

The method of integration is a 4th order Runge Kutta with variable step-length. When the four parameters Q , I , A and f have been chosen, the integration leads to a definite trajectory for every initial value of ϕ (in the interval from 0 to 2π) and $\dot{\phi}$. As the system is dissipative, the first part of the solution will be transient and has to be integrated out before the results start to have significance for the I-V characteristic. The determination of the observed voltage will then require a long period of integration before the mean value of $\dot{\phi}$ has attained a reasonably stable value. So a single I-V characteristic, where I is the only parameter to be varied is a tedious job. In the following we keep the ac-amplitude and the driving frequency fixed, $A = 1$, $f = 0.4$.

The period of the ac-current, $1/f$, is the only natural unit of discrete time. At all the times 0 , $1/f$, $2/f$, $3/f$, -

the situation with respect to the external world is the same. If we could find a simple mapping from the state $(\phi, \dot{\phi})$ at one of these times to the state at the next time in the discrete series, the whole investigation could be made much faster.

For the case $Q = 0$ the situation is even simpler, because ϕ is the only state variable. Instead of ϕ we use the reduced phase

$$y_n = \phi(n/f)/2\pi \quad (14)$$

so when ϕ is confined to the interval from 0 to 2π , y will lie between 0 and 1. The return map, F , defined by

$$y_{n+1} = F(y_n) \quad (15)$$

will then be well defined, and because there is a unique solution curve for every ϕ at a given time, we can be sure that F is a monotonous function. In some cases the discrete solutions y_n will repeat themselves after a small period and a graphical representation of y_{n+1} versus y_n will only show isolated dots, but in other cases a whole curve can be traced out. Fig. 3 shows such a case.

In general the return map for the Josephson equation, when it exists, can be shown to be a circle map⁵, i.e. a non-linear mapping of the unit circle on itself. The function F has two points of inflection and is reasonably well described as a case of Arnold's sine map⁶:

$$y_{n+1} = y_n + \Omega - (K/2\pi) \cdot \sin(2\pi \cdot y_n) \quad (16)$$

where the values of the dynamical variable y have to be reduced to the interval between 0 and 1. When comparing the return maps (15) to the sine map (16) one must allow for an arbitrariness in the choice of the phase of the ac-current at the discrete instants picked out. This means that there will be an arbitrary offset in the y -scale, so that the two points of inflection do not occur for $y=0$ and $y=1/2$, as they do for the sine map. This is, however, not essential.

The surprising thing is that the sine map fits the return maps so well, even when $Q > 0$ and the state space is two dimensional. The explanation is that the dissipative character of the system ensures that the discrete motion in the $(\phi, \dot{\phi})$ plane after the transient period will be restricted to an attractor of dimension lower than 2, and that in typical cases this attractor will define $\dot{\phi}$ as a single-valued function of ϕ . For $Q=0$ we have according to (9):

$$\dot{\phi}_n = I - \sin \phi_n \quad (17)$$

so for small values of Q we can expect something similar.

For $Q=0$ the return map is necessarily increasing, so the parameter K for the approximating sine map is less than 1. The circle map is then subcritical. The value of K is one minus the slope of the tangent at the point of inflection (comp. fig. 3). For $Q>0$ we may find a supercritical return map with $K>1$ and negative slope of the tangent, as shown in fig. 4. Correspondingly, in such cases the discrete attractor in the $(\phi, \dot{\phi})$ plane for some values of ϕ allows more than one value of $\dot{\phi}$, as can be seen in fig. 5. A closer inspection of the supercritical return

maps reveal small wiggles, so they are not strictly univalued⁵, but for a slightly supercritical case the wiggles are too small to be seen on a normal scale and are not important for the gross features of the I-V characteristics. We conclude that the return maps for the Josephson equation are well described by the sine map for the subcritical, critical and slightly supercritical cases.

When investigating the dynamics of the sine map (16) one normally chooses a fixed value of K and determines the winding number

$$W = \lim_{n \rightarrow \infty} (y_n - y_0)/n \quad (18)$$

as a function of the parameter Ω . It then turns out that phase-locking occurs for every rational value $W = p/q$ in a finite Ω -interval. These phase locking intervals correspond to the steps on the Josephson I-V characteristics, both harmonic and subharmonic, but the correspondence is not straightforward, because when we vary I alone, both Ω and K change; $\Omega(I)$ is increasing, while $K(I)$ is decreasing. For some values of Q a transition takes place from supercritical to subcritical behaviour when I increases beyond a value $I_1(Q)$. The transition can only be seen on the I-V curves when $I_1(Q)$ is greater than the smallest current that allows a solution with nonzero voltage. For $A=1$, $f=0.4$ this will be the case in the interval $0.4 < Q < 1.1$; I_1 has a maximum ≈ 1.3 for $Q \approx 2$. No supercritical parts can be found on the I-V characteristics for these values of A and f with winding numbers above 0.44.

Fig. 6 shows three different return maps for the same

current $I = 1.05$ but three different Q -values, 0.3, 1, and 1.5. For $Q = 1$ we see only three isolated groups of points of three points each. This is a case of phase-locking and the winding number is $1/3$, but as there are nine points the mode structure is not $1/3$, but $3/9$. For a supercritical case there are several modes for some of the winding numbers that correspond to irreducible fractions p/q . This is because there is a local maximum on the circle map for $K > 1$, and therefore we can expect to find traces of the Feigenbaum route to chaos superposed on the mode locked steps.

For the subcritical circle map there is a finite measure of q -values giving an irrational winding number, but in the critical case, $K = 1$, this measure is zero and the values form a Cantor-set of fractal dimension 0.87, as will be discussed in the next section. For supercritical cases some of the phase-locked q -intervals overlap, meaning that we can find different winding numbers for the same q . This gives rise to hysteresis in the Josephson I - V curves, a feature that can be used for distinguishing supercritical regions from subcritical ones when looking at experimental data. However, there is another sort of hysteresis as regards the supercurrent for $Q > 1$. For $Q = 1.5$ we can lower the current from above 1 down to about 0.7 and find a subcritical circle map all the way, until it touches the line $y_{n+1} = y_n$ at which point the transition to zero voltage suddenly takes place. Raising the current again we stay on the supercurrent (the 0/1 step) until the current is almost 1, when the winding number changes discontinuously from 0 to 0.41. Apparently the return map between these two transitions includes an isolated point on the line.

3. The critical sine map .

The winding number W as a function of the driving parameter α for the sine map forms a "Devil's staircase". For the critical case $K = 1$ the staircase is complete, which means that the steps for rational W together have the same measure as the α -domain, although they do not cover it completely but leave a Cantor-set of zero measure. Considering α as a function of W we have the curious case of a function that is discontinuous for all rational values, but continuous for all irrational values of W . Fig. 7 shows a graph of the complete staircase for $0 < \alpha < \frac{1}{2}$. This is sufficient, because $W(1-\alpha) = 1 - W(\alpha)$.

The dynamical process for the critical sine map can be depicted as in fig. 8, where subsequent values of y (reduced to the interval between 0 and 1) have been plotted while α slowly increases from 0 to $\frac{1}{2}$. There is no real "chaos" in this picture, although it may look so. An irrational winding number corresponds to a quite regular motion on the circle, although it never repeats itself.

The critical sine map is particularly interesting because it shows a universal scaling structure common to all circle maps with a horizontal slope at the point of inflection, provided that the local behaviour is cubic. Shenker showed⁷ for the Fibonacci-fractions q_n/q_{n+1} , where the numerator and the denominator are consecutive members of the Fibonacci-sequence

$$1, 1, 2, 3, 5, 8, \dots, q_n = q_{n-1} + q_{n-2}, \dots \quad (19)$$

that the stability intervals asymptotically scale with the

power law

$$\Delta \Omega(q_{n-1}/q_n) \propto q_n^{-2.164} \quad (20)$$

If all stability intervals for $W = p/q$ scaled with the denominator in the same power, $-\alpha$, we could easily calculate the fractal dimension for the set of qs with irrational W . If $L(\Delta)$ denotes the measure seen when all stability intervals less than Δ are ignored, we have

$$dL(\Delta)/d\Delta \propto \tilde{q}(\Delta)^{1-\alpha} \cdot d\tilde{q}(\Delta)/d\Delta \quad (21)$$

where we have used that the number of irreducible fractions with denominator q is proportional to q , asymptotically. The quantity $\tilde{q}(\Delta)$ is the largest denominator whose stability interval can be seen with the Δ -scale, i.e. $\tilde{q} \propto \Delta^{-1/\alpha}$, and therefore

$$dL(\Delta)/d\Delta \propto \Delta^{-2/\alpha} \quad (22)$$

But by definition⁸ of D , the fractal dimension, $L \propto \Delta^{1-D}$, so we should have

$$D = 2/\alpha \quad (23)$$

Shenker's value, $\alpha = 2.164$ would thus give $D = 0.924$. A direct measurement using calculated stability intervals and the definition of D gives however⁹,

$$D = 0.870 \quad (24)$$

so, it seems that the Fibonacci fractions studied by Shenker are not very "typical". In fact they are extreme in the

sense that the Fibonacci intervals are always the largest for fractions with the same denominator. Another extreme are the fractions $1/q$ which always have the smallest intervals for the given q . These intervals are easily shown to go like q^{-3} , a consequence of the fact that N rises like a square root on the lowest part of the Devil's staircase.

The best way to define what is meant by "extreme" or "typical" fractions is based on the continued fraction expansion

$$p/q = 1/(a_1 + 1/(a_2 + 1/(a_3 + \dots + 1/a_N))) \dots \quad (25)$$

The number of terms, $N(p/q)$ can be shown to lie between the limits 1 and $N_{\max}(q)$, where $N_{\max}(q)$ is the number characterizing the Fibonacci-fractions for which all the terms are 1. Asymptotically for q large:

$$N_{\max}(q) = \frac{\ln[(3-\tau) \cdot q]}{\ln \tau} \quad (26)$$

where $\tau = (\sqrt{5}+1)/2$ is the golden mean ($1/\tau$ is the limiting value of the Fibonacci-fractions). An even better way of characterizing the fractions is to use the sum of terms

$$S = \sum a_n \quad (27)$$

For given q , S will lie between the maximum q (for $1/q$) and the minimum, attained by the Fibonacci-fractions

$$S_{\min}(q) = N_{\max}(q) \quad (28)$$

As a rule of thumb one can say that the length of the

stability intervals $\Delta\Omega(p/q)$ depends only on q and S , i.e. if two fractions have identical values of q and S we can be sure that the corresponding stability intervals have equal lengths within a few percent, even if their N values differ.

A way of characterizing "typical" fraction is suggested by the result¹⁰ that the values of N for large q are normally distributed around the value

$$N_t(q) = \frac{12 \ln 2}{\pi^2} \cdot \ln q \quad (29)$$

with a standard deviation given by $\ln N_t$. The value N_t is about 40% of N_{\max} . If we look at the quantity

$$\sigma(q, S) = [1 + (S - S_{\min}) / (N_t \ln N_t)]^{-1} \quad (30)$$

we can see that it is confined to the interval between 0 and 1. Furthermore, it will have a rather stable distribution around the "typical" value $\frac{1}{2}$. An empirical formula for the phase-locking intervals is given by

$$\Delta\Omega(p/q) \approx b \cdot (q/r)^{-\xi(\sigma)} \quad (31)$$

where $b \approx 0.035$ and $r \approx 2.8$. The exponent ξ is given by the expression (30) using the interpolation formula

$$\xi(\sigma) \approx 2 + 1 / (5.1 \cdot \sigma + 1) \quad (32)$$

The expressions (30)-(32) summarize some of the main known features of the critical sine map and give within 10% uncertainties the correct magnitudes of the intervals, at least up to denominators about 100. However, it does not

pretend to give any exact insight in the self-similar structure. The self-similarity is not strict, but slightly modulated from place to place on the staircase, but still, the overall fractal dimension $D = 0.870$ seems very well defined. The universality of this result is confirmed by direct measurements of D on the critical steps of IV-characteristics for Josephson junctions simulated on an analog computer¹¹.

A proper theory for the scaling properties may lie hidden in the particular ordering of rational numbers known as the Farey-tree ¹². For a given value of the sum S ($S > 1$) there are exactly 2^{S-2} irreducible fractions between 0 and 1, which suggests that all these fractions may be ordered in a binary tree where each layer has a constant S . As shown in fig. 9 the construction is very simple: a fraction is formed by adding both the numerators and the denominators of the nearest adjacent fractions on the left and the right in the layers above. This will preserve the natural ordering of all the numbers from left to right. Fig. 9 shows also how to determine the number of terms in the continued fraction expansion by visual inspection: The branches of the tree are drawn alternating between solid and dashed lines. By starting in the point $1/1$ where $N=1$ and going to another point p/q , N is increased by one for every solid line and stays constant on the dashed lines.

The number theoretical mysteries hidden in such hard boiled physical problems as the Josephson junction and the pendulum have just begun to reveal themselves through the experimental mathematics of the circle maps.

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Figure captions .

Fig. 1 . Iconic models of physical analogies for the Josephson equation.

Fig. 2 . Current (I) - voltage (V) characteristics for Josephson junction. Solid curve: without ac-current. Dotted curve: with ac-current. Line: normal conductivity.

Fig. 3 . Return map for $Q=0$ showing how to determine K by the slope of the tangent at the point of inflection.

Fig. 4 . Supercritical return map.

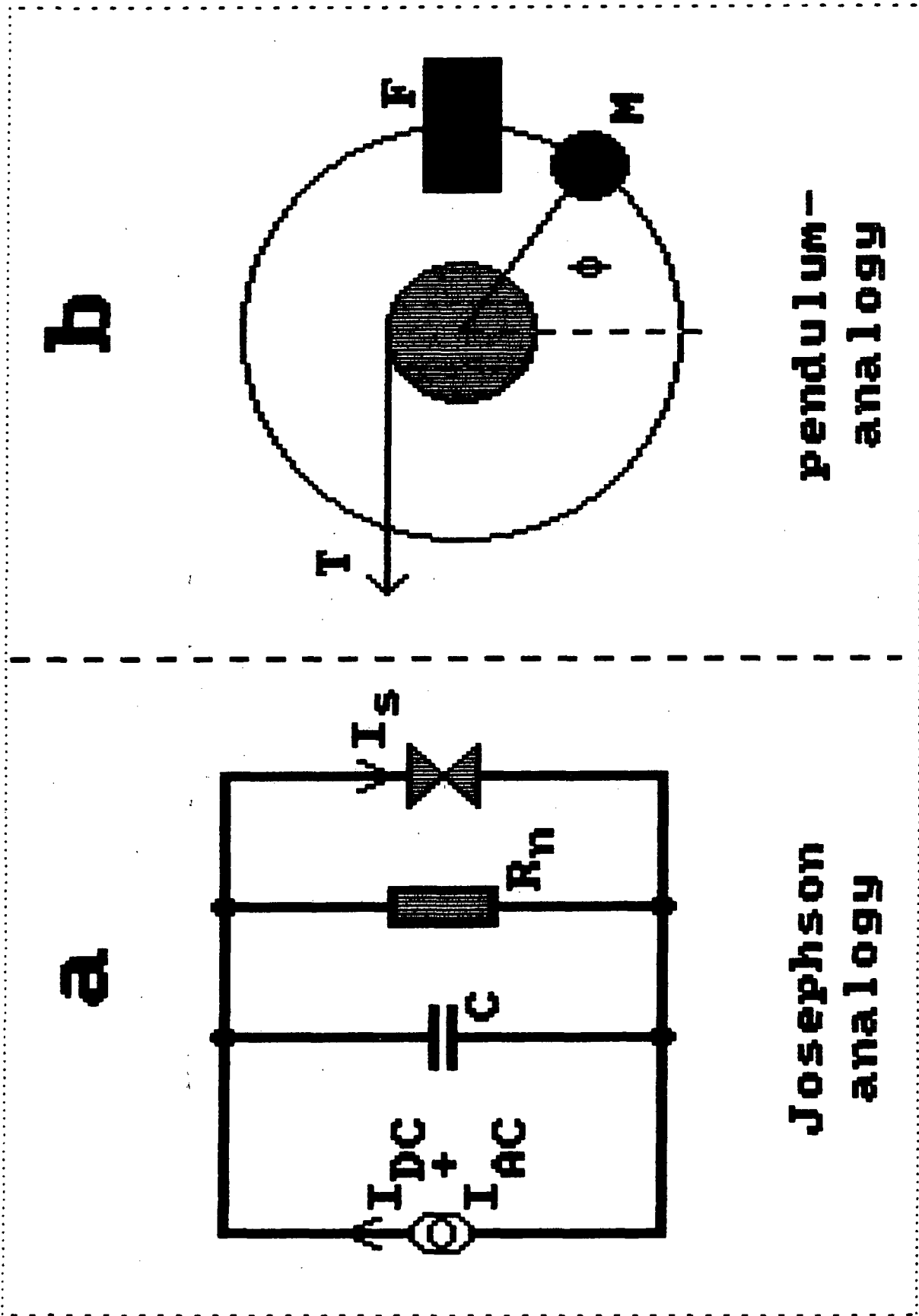
Fig. 5 . Discrete attractor corresponding to the return map of fig. 4. Horizontal: ϕ , vertical: $\dot{\phi}$. The arrow indicates a place where $\dot{\phi}$ is not uniquely determined from ϕ .

Fig. 6 . Three return maps for the same current.

Fig. 7 . Devil's staircase for the critical sine map.

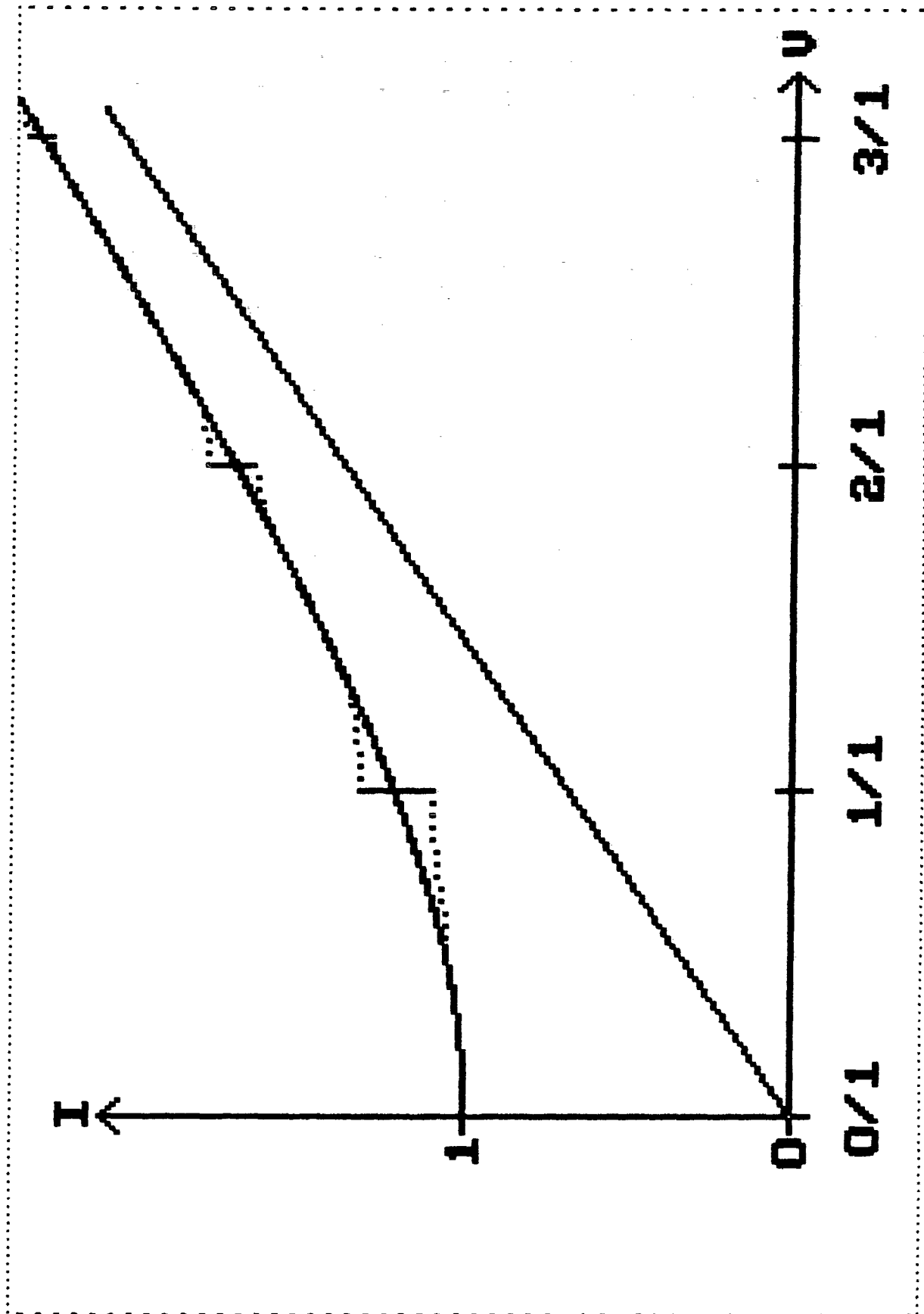
Fig. 8 . Dynamics of the critical sine map.

Fig. 9 . The Farey-tree.



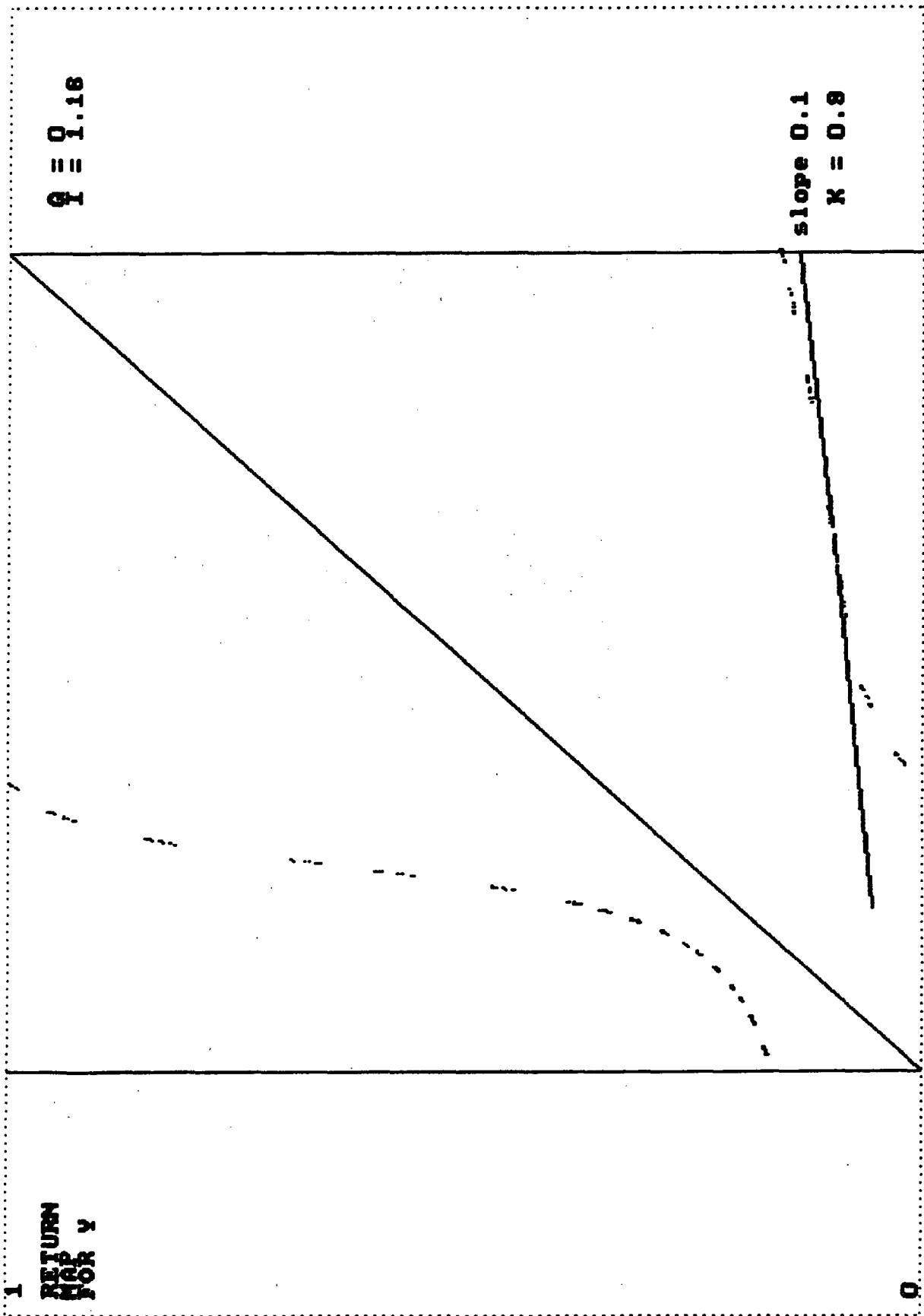
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Fig. 1



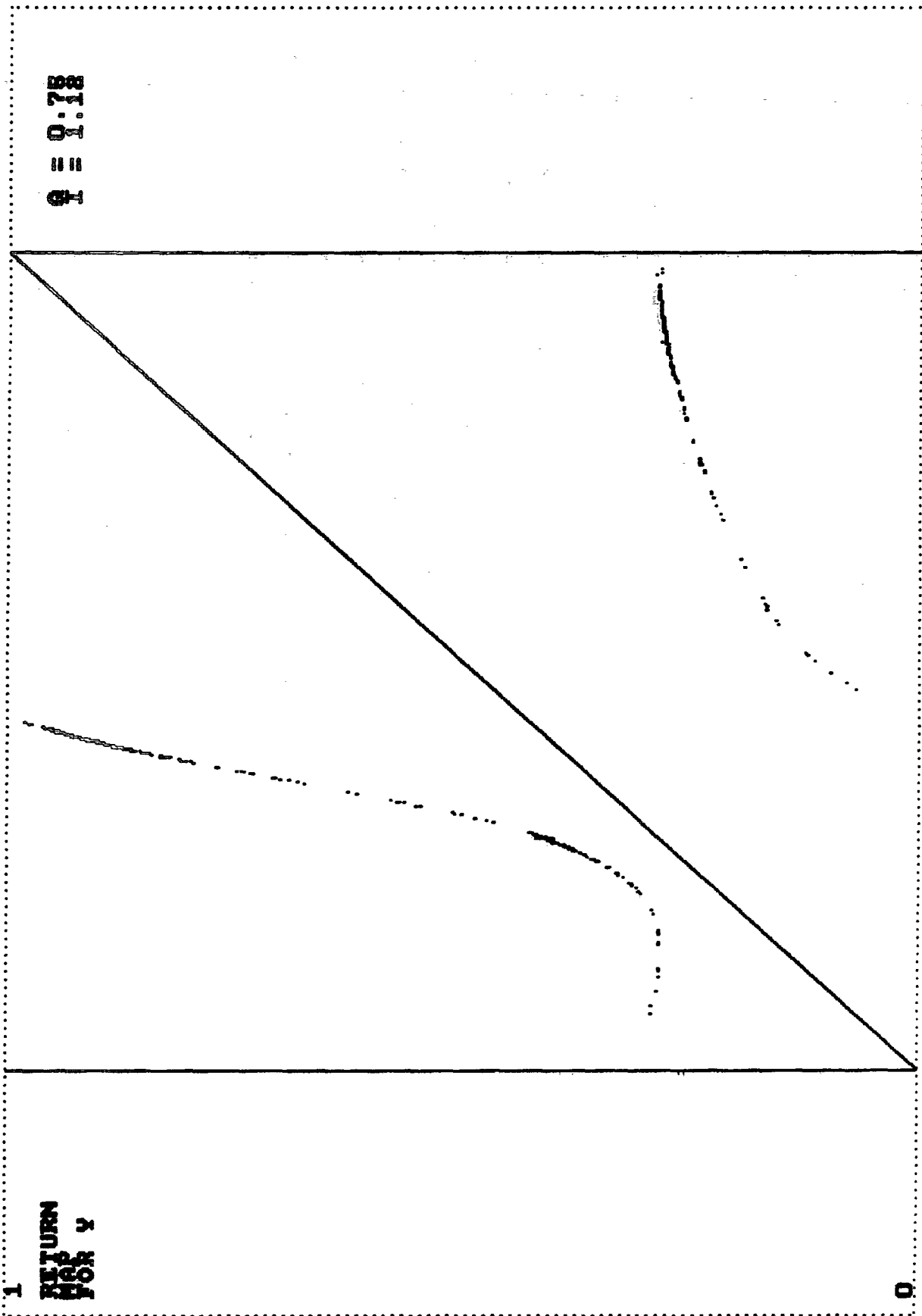
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Fig. 2



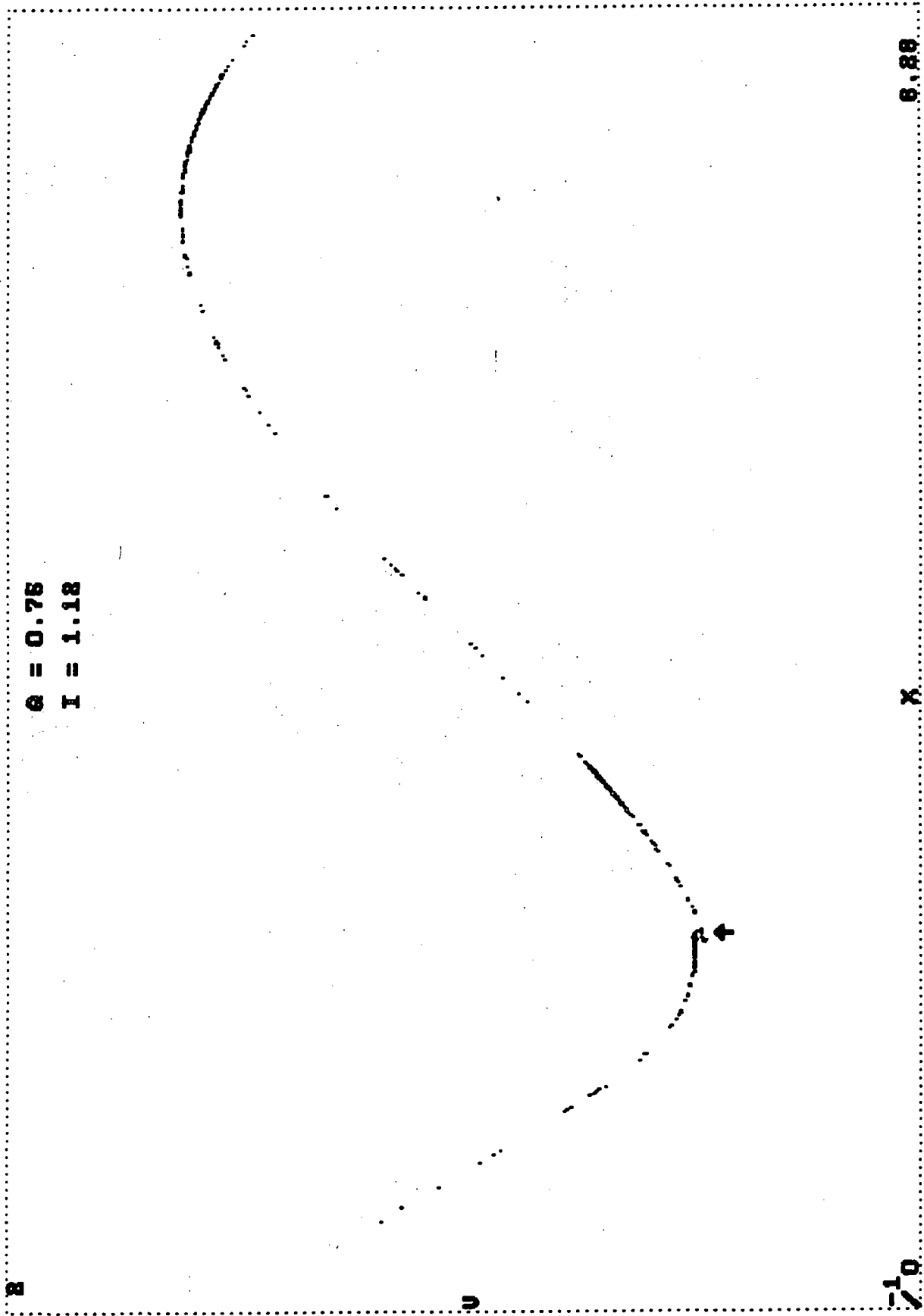
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Fig. 3



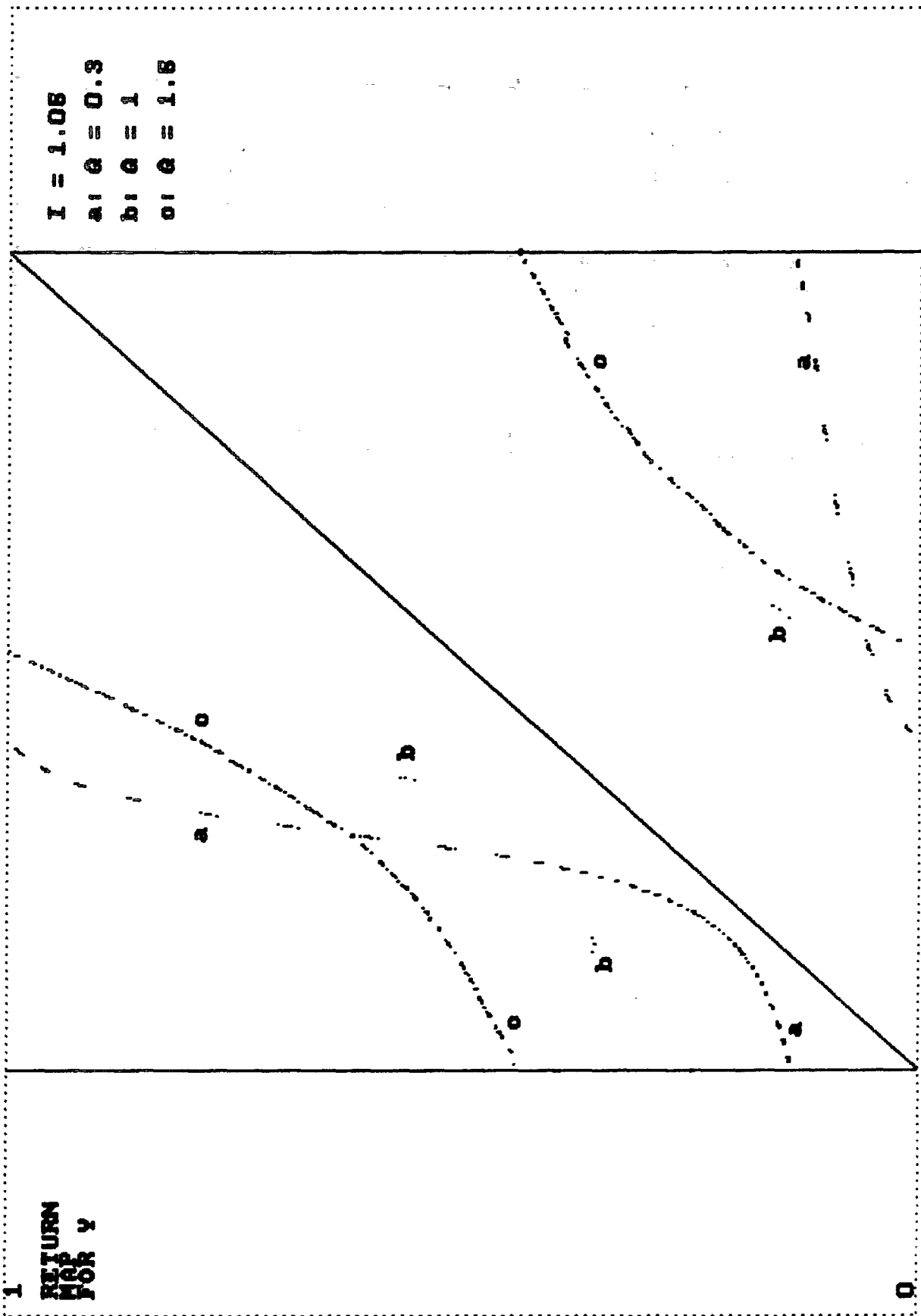
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Fig. 4



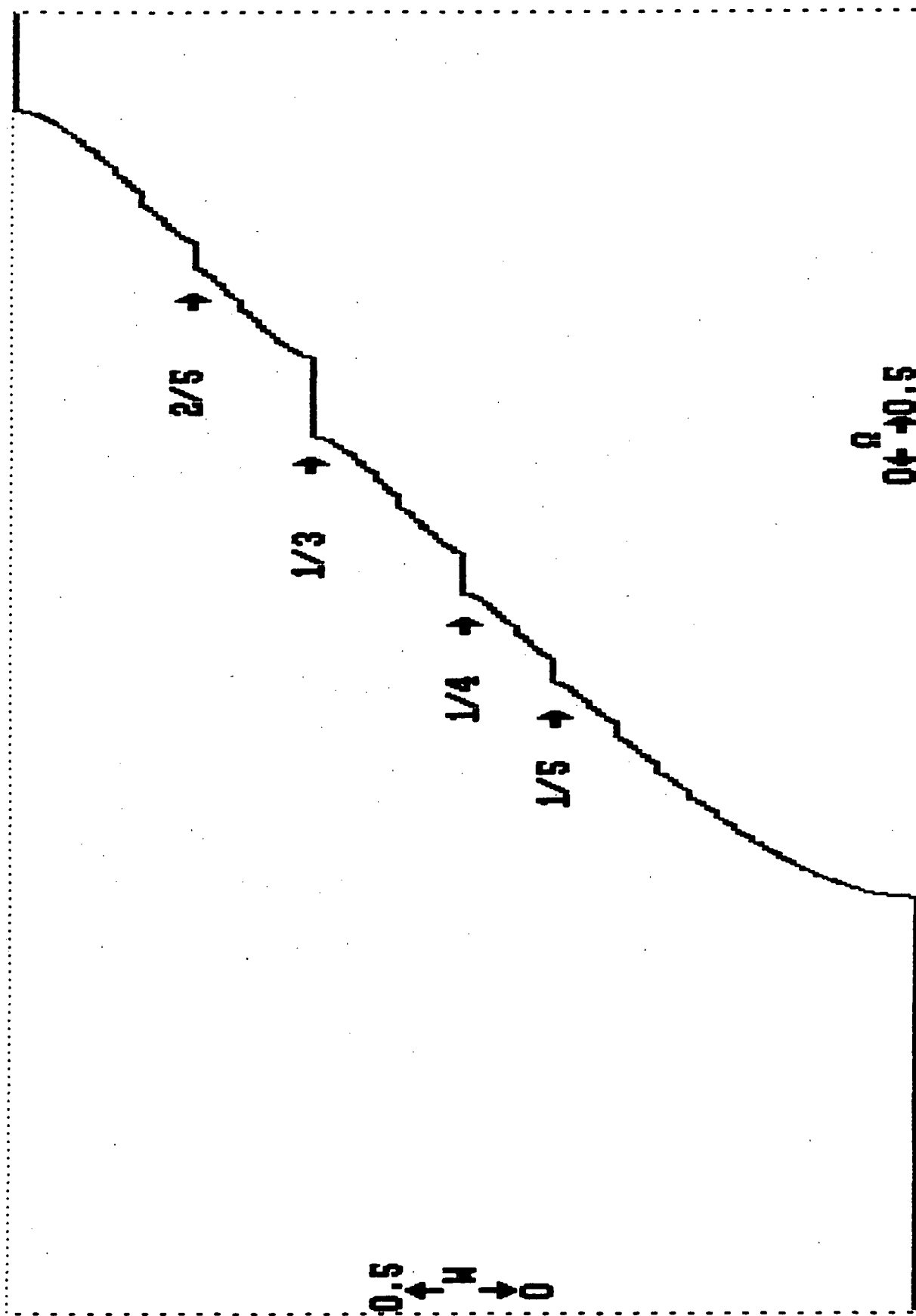
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Fig. 5



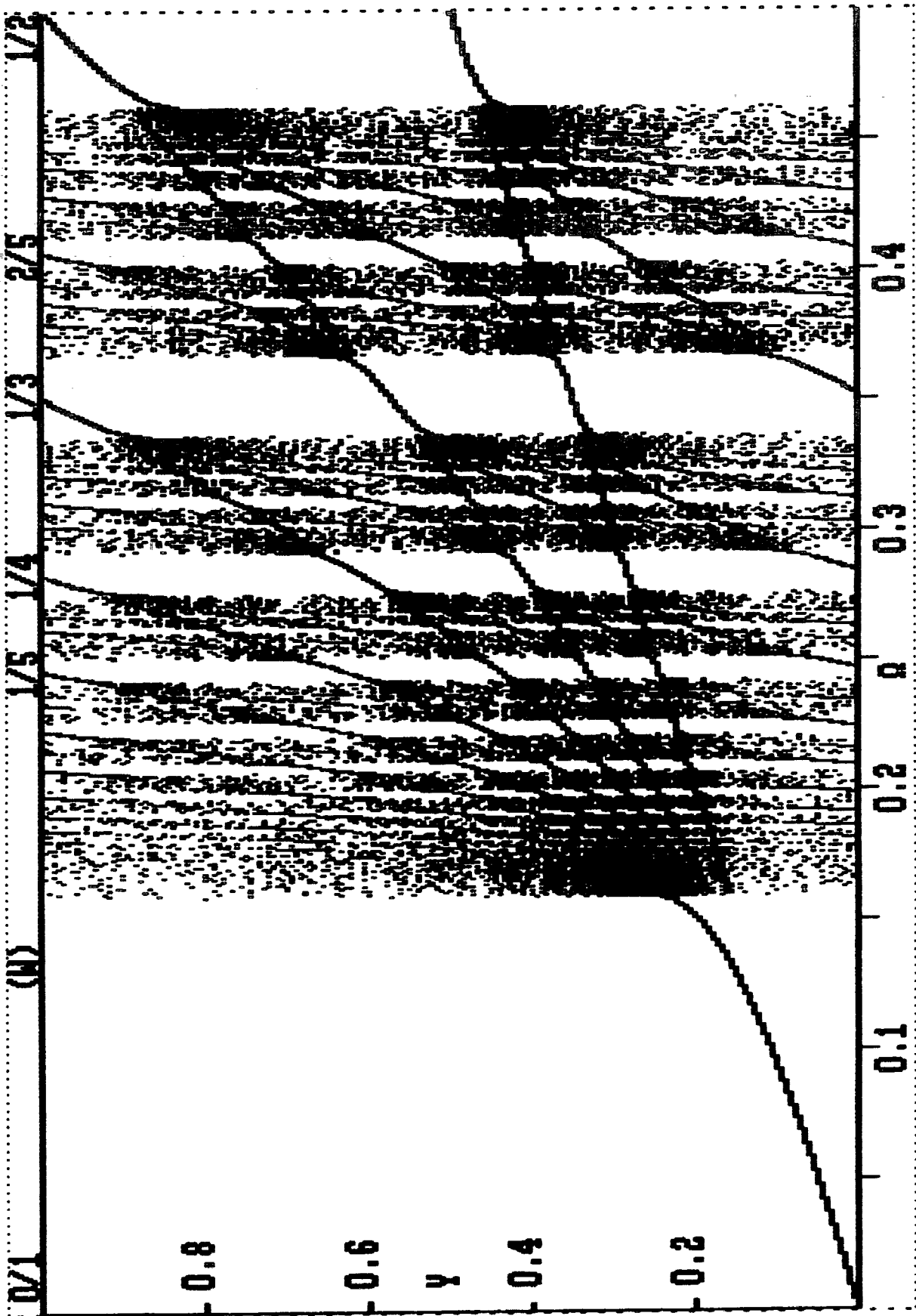
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Fig. 6



(up)

Fig. 7



(up)

Fig. 8

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