Crothers' Investigations into the Theory of Relativity

The General Theory of Relativity has now become a topic of household discussion, at least within the context of black holes, Big Bang cosmology and expansion of the Universe. These concepts have found their way into the curricula of high schools, deep into university physics courses, much research, and some pretty expensive experimental projects. Almost daily there are reports of discovery of another black hole and of physical evidence of the beginning of the Universe from the Big Bang of a cosmological singularity. So widespread now are these notions that they have taken on the mantle of verified scientific facts. Yet nothing can be further from the truth. Indeed, the evidence, both theoretical and physical, actually refutes black holes, big bangs and expansion of the Universe.

Has anyone ever found a black hole? The short answer to this is no, not a single one. According to the proponents of the black hole, the signatures of that bizarre object are:

(1) an infinitely dense singularity, a "point-mass";

(2) an event horizon.

Since nobody has ever identified an infinitely dense singularity anywhere, and since nobody has ever identified an event horizon anywhere, nobody has ever identified a black hole, anywhere. Furthermore, General Relativity is claimed to be a generalisation of Special Relativity, to non-uniform motion of material bodies. However, it is very easily proved that Special Relativity forbids the existence of infinite densities, and hence it forbids singularities, i.e. point-masses. So if General Relativity permits singularities (e.g. black holes), it does so in violation of Special Relativity. Indeed, according to Special Relativity, the dynamic mass m of a rest-mass m_0, moving with a speed $v < c$ along the x-axis, is

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}.$$

The dynamic volume of a cuboid rest-mass m_0 is $V = x_0^3\sqrt{1 - \frac{v^2}{c^2}}$, where x_0 is the length of the sides of m_0. Then the dynamic density D is

$$D = \frac{m}{V} = \frac{m_0}{x_0^3\left(1 - \frac{v^2}{c^2}\right)}.$$

This is infinite when $v = c$. But according to Special Relativity no material object can acquire the speed c, of light in vacuo (equivalently, this would require an infinite amount of energy, which is impossible). Therefore, point-masses are forbidden by Special Relativity, and hence also by General Relativity if the latter is to be consistent with the former. This is sufficient to invalidate the alleged black hole singularity and the alleged Big Bang cosmological singularity.

Another simple physical argument re-affirms this result; violation of Einstein's 'Principle of Equivalence' [1]. According to this Principle [2], in a freely falling inertial frame in a sufficiently small region of Einstein's gravitational field, Special Relativity must hold. Now Einstein's field equations are

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\kappa T_{\mu\nu}\,,$$

where $G_{\mu\nu}$ is Einstein's tensor, $R_{\mu\nu}$ the Ricci tensor, κ a constant, and $T_{\mu\nu}$ the energy-momentum tensor. Einstein claimed that for the static vacuum (i.e. empty) gravitational field, $T_{\mu\nu} = 0$, so that

$$R_{\mu\nu} = 0,$$

(since in this case the Ricci curvature invariant R is also zero). It is from a solution to $R_{\mu\nu} = 0$, the so-called "Schwarzschild solution", that the black hole is alleged. Now Special Relativity permits the presence of any number of arbitrarily large (but not infinitely large) masses, which can interact. Furthermore, the very definition of an inertial frame involves the presence of mass (and in the case of Special Relativity, two masses, viz., the mass of the observer and the mass of the observed, so that relative motion of material bodies is defined). But $R_{\mu\nu} = 0$ is a statement that there are no masses permitted, by definition, in the alleged gravitational field of $R_{\mu\nu} = 0$. Therefore, Special Relativity cannot be recovered in any "freely falling" inertial frame in the spacetime of $R_{\mu\nu} = 0$ and, indeed,

a "freely falling" inertial frame cannot even be present (since its very definition requires the presence of mass). Thus, Einstein's 'Principle of Equivalence' is violated by $R_{\mu\nu}=0$, and is therefore inconsistent with the General Theory of Relativity, which is based upon the validity of his 'Principle'. Therefore, the "Schwarzschild solution" violates the 'Principle' and is consequently invalid, thereby completely invalidating the black hole, even if the latter can be deduced from the "Schwarzschild solution" by some purely formal mathematical means. However, it has also been proved [3 – 21] that it is impossible to obtain the black hole from the "Schwarzschild solution" without violating the rules of differential geometry. This too is sufficient to invalidate the black hole.

It should also be noted that the concept of the black hole did not come from any observations requiring a theoretical explanation. It was generated entirely from theory (and an erroneous theory at that). It is no wonder that nobody has ever found a black hole; and there is no theory which rightly predicts them. The black hole was stillborn, and has no place in science.

The Big Bang concept and its associated expansion of the Universe is in the same boat as its cousin, the black hole. First, as shown above, the alleged cosmological singularity, an infinitely dense point-mass containing all the matter and energy of the Universe, and spacetime itself besides, is inconsistent with Special Relativity and hence also with General Relativity. Once again, if General Relativity predicted a cosmological singularity it would do so in violation of Special Relativity. Furthermore, the Big Bang and expansion of the Universe are allegedly a consequence of the Friedmann-Lemaître-Robertson-Walker (FLRW) line-element. But it has been proved that the Big Bang and associated expansion of the Universe cannot be obtained from the FLRW line-element without a gross violation of differential geometry, and so they are invalid. In actual fact, the FLRW line-element predicts an infinite, unbounded Universe, independent of time [12] - no Big Bang and no expansion.

Another interesting fact is that "Schwarzschild's solution" is not Schwarzschild's solution [20 – 26]. It is also frequently claimed that Schwarzschild deduced the black hole from his solution, with an event horizon at the "Schwarzschild radius", R_s, given by

$$R_s = \frac{2Gm}{c^2}.$$

All these claims are patently false, because Schwarzschild did not breathe a single word about black holes, never "deduced" the alleged "Schwarzschild radius", of the so-called "event horizon", and in fact obtained a solution which precludes the black hole. Here is the "Schwarzschild solution", due to David Hilbert [20, 21, 25, 26] (using $c=G=1$),

$$ds^2 = \left(1-\frac{2m}{r}\right)dt^2 - \left(1-\frac{2m}{r}\right)^{-1}dr^2 - r^2(d\theta^2+\sin^2\theta d\varphi^2),$$

wherein r is alleged to go down to zero, one way or another. But here now is Schwarzschild's real solution [22],

$$ds^2 = \left(1-\frac{\alpha}{R}\right)dt^2 - \left(1-\frac{\alpha}{R}\right)^{-1}dR^2 - R^2(d\theta^2+\sin^2\theta d\varphi^2),$$

$$R = R(r) = \left(r^3+\alpha^3\right)^{\frac{1}{3}},$$

$$0 < r < \infty,$$

wherein α is an undetermined constant, supposed a function of the mass of the source of the alleged gravitational field associated therewith. Note that when $r=0$, Schwarzschild's line element is undefined, and there is no possibility of a black hole, which is alleged to occur in Hilbert's "Schwarzschild's solution" with infinitely dense singularity at $r=0$ and event horizon at $r=2m$ therein. Hilbert's "Schwarzschild's solution" violates the intrinsic geometry of the line-element, and is inconsistent with Schwarzschild's solution which does not violate the intrinsic geometry of the line-element. Also, one cannot assign a value to the constant α without introducing extraneous and *ad hoc* arguments, as Schwarzschild knew - and so he didn't. And even if Schwarzschild's solution or Hilbert's "Schwarzschild solution" were permissible, they conceive of the mass in terms of a centre of mass (i.e. a point-mass), and a centre of mass is not a physical object. There is no sense in asserting that an object and its centre of mass are identical, which is effectively what the proponents of the black hole do. In addition, the energy-momentum tensor contains all matter and energy that cause the gravitational field. Setting it to zero eliminates all causation of the gravitational field, and so causative mass cannot be introduced into the metric tensor *a posteriori* in the fashion of the proponents of black holes by their analogy with Newton's gravitational potential in the infinitely far field.

In the usual interpretation of Hilbert's "Schwarzschild's solution", the quantity r therein has never been properly identified. It has variously been called " the radius" [27, 28] of a sphere, the "coordinate radius"[29] or "radial coordinate" [30, 31] or "radial space coordinate" [32], the "areal radius" [29, 33], the "reduced circumference" [34], even "a gauge choice, which defines r" [35], but *never* for what it really is – the radius of Gaussian curvature. Being the radius of curvature it does not in fact determine the geodesic radial distance from the centre of spherical symmetry [1, 3 – 18, 33 – 38]. For a 2-D spherically symmetric geometric surface given by

$$ds^2 = R_c^2(d\theta^2 + \sin^2\theta d\varphi^2),$$

$$R_c = R_c(r),$$

the Riemannian curvature reduces to the Gaussian curvature K, given by [36, 42, 43, 44, 45],

$$K = \frac{R_{1212}}{g},$$

where R_{ijkm} is the Riemann tensor of the first kind and $g = g_{\theta\theta}g_{\varphi\varphi}$. Straightforward calculation gives

$$K = \frac{1}{R_c^2},$$

so that R_c is the inverse square root of the Gaussian curvature, i. e. the radius of curvature, and so r in Hilbert's "Schwarzschild's solution" is the radius of Gaussian curvature. The geodesic (or proper) radius, R_p, of Schwarzschild's solution is given by

$$R_p = \int \frac{dR}{\sqrt{1 - \frac{\alpha}{R}}},$$

and for Hilbert's black hole "Schwarzschild's solution", by

$$R_p = \int \frac{dr}{\sqrt{1 - \frac{2m}{r}}}.$$

Thus the proper radius and the radius of curvature *are not the same*; for the above, $R_p \neq R$ and $R_p \neq r$ respectively, in general [1, 3 – 18].

That Einstein's conception of the conservation and localisation of gravitational energy are erroneous easily follows from the fact that $R_{\mu\nu} = 0$ is inadmissible. Since the energy-momentum tensor can never be zero, Einstein's field equations can be written as

$$\frac{G_{\mu\nu}}{\kappa} + T_{\mu\nu} = 0,$$

where $G_{\mu\nu}/\kappa$ gives the components of a gravitational energy tensor. Thus, when $T_{\mu\nu} = 0$, $G_{\mu\nu} = 0$, i.e. $T_{\mu\nu}$ and $G_{\mu\nu}/\kappa$, *vanish identically*. Consequently, the total energy is always zero; there is no possibility of the localisation of gravitational energy; there are no Einstein gravitational waves. The LIGO project and its counterparts around the world, such as the AIGO, are destined to detect nothing.

Einstein's pseudo-tensor is alleged to describe the localisation of gravitational energy, gravitational waves, and the flow of energy and momentum. According to the foregoing this cannot be true. This is re-affirmed by the fact that Einstein's pseudo-tensor is a meaningless collection of mathematical symbols [46]. Einstein's pseudo-tensor, $\sqrt{-g}t_\nu^\mu$, is defined as [2, 42, 46, 47, 48],

$$\sqrt{-g}t_\nu^\mu = \frac{1}{2}\left(\delta_\nu^\mu L - \frac{\partial L}{\partial g_{,\mu}^{\sigma\rho}} g_{,\nu}^{\sigma\rho}\right)$$

wherein L is given by

$$L = -g^{\alpha\beta}\left(\Gamma_{\alpha\kappa}^{\gamma}\Gamma_{\beta\gamma}^{\kappa} - \Gamma_{\alpha\beta}^{\gamma}\Gamma_{\gamma\kappa}^{\kappa}\right).$$

Contracting the pseudo-tensor and applying Euler's theorem yields,

$$\sqrt{-g}t_\mu^\mu = L,$$

which is a 1st-order intrinsic differential invariant that depends only upon the components of the metric tensor and its 1st derivatives. However, the mathematicians Ricci and Levi-Civita proved in 1900 that such invariants *do not exist* [46, 49]. The invalidity of the pseudo-tensor is, of course, consistent with the invalidity of $R_{\mu\nu} = 0$. Consequently, everything built upon Einstein's pseudo-tensor is invalid. Connected with is the fact that Einstein's field equations cannot be linearised because linearisation implies the existence of a tensor that, except of the trivial case of being zero, *does not otherwise exist*, as proved by Hermann Weyl in 1944 [50].

The proponents of the Standard Model routinely ignore and attempt to suppress these facts [51, 52], because they completely invalidate their theories of black holes, big bangs and expansion of the Universe. Ironically, theoretically speaking, it is General Relativity itself which invalidates them. Observations also refute them.

References

[1] Crothers S. J. On Certain Conceptual Anomalies in Einstein's Theory of Relativity. *Progress in Physics*, v. 1, 52–57, 2008, (www.ptep-online.com/index_files/2008/PP-12-11.PDF).

[2] Einstein A. The Meaning of Relativity, Science Paperbacks and Methuen & Co. Ltd., 1967, pp.156-157.

[3] Crothers S. J. On the General Solution to Einstein's Vacuum Field and its Implications for Relativistic Degeneracy. *Progress in Physics*, v. 1, 68–73, 2005, (www.ptep-online.com/index_files/2005/PP-01-09.PDF).

[4] Crothers S. J. On the Ramifications of the Schwarzschild Space-time Metric. *Progress in Physics*, v. 1, 74–80, 2005, (www.ptep-online.com/index_files/2005/PP-01-10.PDF).

[5] Crothers S. J. On the geometry of the general solution for the vacuum field of the point-mass. *Progress in Physics*, v. 2, 3–14, 2005, (www.ptep-online.com/index_files/2005/PP-02-01.PDF).

[6] Crothers S. J. On the Generalisation of Kepler's 3rd Law for the Vacuum Field of the Point-Mass. *Progress in Physics*, v. 2, 70–75, 2005, (www.ptep-online.com/index_files/2005/PP-02-05.PDF).

[7] Crothers S. J. On the vacuum field of a sphere of incompressible fluid. *Progress in Physics*, v. 2, 76–81, 2005, (www.ptep-online.com/index_files/2005/PP-02-06.PDF).

[8] Crothers S. J. On Isotropic Coordinates and Einstein's Gravitational Field.*Progress in Physics*, v. 3, 7–12, 2006, (www.ptep-online.com/index_files/2006/PP-06-02.PDF).

[9] Crothers S. J. On the Regge-Wheeler tortoise and the Kruskal-Szekeres coordinates. *Progress in Physics*, v. 3, 30–34, 2006, (www.ptep-online.com/index_files/2006/PP-06-06.PDF).

[10] Crothers S. J. Spherically Symmetric Metric Manifolds and the Black Hole Catastrophe, 2007, (www.aias.us).

[11] Evans M. W., Eckardt H., Crothers S. J. The Coulomb and Ampere Maxwell Laws in Generally Covariant Unified Field Theory, paper 93, (www.aias.us).

[12] Crothers S. J. On the 'Size' of Einstein's Spherically Symmetric Universe, *Progress in Physics*, v. 4, 69–73, 2007, (www.ptep-online.com/index_files/2007/PP-11-10.PDF).

[13] Crothers S. J. A Brief History of Black Holes. *Progress in Physics*, v. 2, 54–57, 2006, (www.ptep-online.com/index_files/2006/PP-05-10.PDF).

[14] Crothers S. J. Introducing Distance and Measurement in General Relativity: Changes for the Standard Tests and the Cosmological Large Scale. *Progress in Physics*, v. 3, 41–47, 2005, (www.ptep-online.com/index_files/2005/PP-03-07.PDF)

[15] Crothers S. J. A Short Discussion of Relativistic Geometry. *Bulletin of Pure and Applied Sciences*, v. 24E (No.2), 267-273, 2005, (www.geocities.com/theometria/shortgeo.pdf)

[16] Crothers S. J. On the general solution to Einstein's vacuum field for the point-mass when $\lambda \neq 0$ and its consequences for relativistic cosmology. *Progress in Physics*, v. 3, 7–18, 2005, (www.ptep-online.com/index_files/2005/PP-03-02.PDF).

[17] Crothers S. J. Relativistic cosmology revisited. *Progress in Physics*, v. 2, 27–30, 2007, (www.ptep-online.com/index_files/2007/PP-09-05.PDF).

[18] Crothers S. J. On line-elements and radii: a correction. *Progress in Physics*, v. 2, 25–26, 2007, (www.ptep-online.com/index_files/2007/PP-11-01.PDF).

[19] Crothers S. J. Gravitation On a Spherically Symmetric Metric Manifold. *Progress in Physics*, v. 2, 68–74, 2007, (www.ptep-online.com/index_files/2007/PP-09-04.PDF).

[20] Abrams L. S. Alternative space-time for the point mass. *Phys. Rev. D*, v. 20, 2474-2479, 1979, (arXiv:gr-qc/0201044).

[21] Abrams L. S. Black holes: the legacy of Hilbert's error. *Can. J. Phys.*, v. 67, 919, 1989, (arXiv:gr-qc/0102055).

[22] Schwarzschild K. On the gravitational field of a mass point according to Einstein's theory. *Sitzungsber. Preuss. Akad. Wiss., Phys. Math. Kl.*, 189, 1916, (arXiv: physics/9905030, www.geocities.com/theometria/schwarzschild.pdf)

[23] Schwarzschild K. *On the gravitational field of a sphere of incompressible fluid according to Einstein's theory.* Sitzungsber. Preuss. Akad. Wiss., Phys. Math. Kl., 424, 1916, (arXiv: physics/9912033).

[24] Brillouin M. The singular points of Einstein's Universe. *Journ. Phys. Radium*, v. 23, 43, 1923, (www.geocities.com/theometria/brillouin.pdf).

[25] Antoci S. David Hilbert and the origin of the "Schwarzschild" solution. 2001, (arXiv: physics/0310104).

[26] Loinger A. On black holes and gravitational waves. La Goliardica Paves, Pavia, 2002.

[27] Mould R. A. Basic Relativity, Springer–Verlag New York Inc., New York, 1994.

[28] Dodson, C. T. J., Poston, T. Tensor Geometry – The Geometric Viewpoint and its Uses, 2nd Ed., Springer–Verlag, 1991.

[29] Wald, R. M. General Relativity, The University of Chicago Press, Chicago, 1984.

[30] Carroll, B. W., Ostile, D. A. An Introduction to Modern Astrophysics, Addison–Wesley Publishing Company Inc., 1996.

[31] Misner, C. W., Thorne, K. S., Wheeler, J. A. Gravitation, W. H. Freeman and Company, New York, 1970.

[32] Zel'dovich, Ya. B., Novikov, I. D. Stars and Relativity, Dover Publications Inc., New York, 1996.

[33] Ludvigsen, M. General Relativity – A Geometric Approach, Cambridge University Press, Cambridge, UK, 1999.

[34] Taylor E. F. Wheeler J. A. Exploring Black Holes — Introduction to General Relativity, Addison Wesley Longman, 2000 (in draft).

[35] 't Hooft, G. (private communication).

[36] Levi-Civita T. The Absolute Differential Calculus, Dover Publications Inc., New York, 1977.

[37] Stavroulakis N. A statical smooth extension of Schwarzschild's metric. *Lettre al Nuovo Cimento*, Serie 2, v. 11, No. 8, 427–430, 26/10/1974, (www.geocities.com/theometria/Stavroulakis-3.pdf).

[38] Stavroulakis N. On the principles of General Relativity and the $\Theta(4)$-invariant metrics. *Proceedings of the* 3^{rd} *Panhellenic Congress of Geometry*, Athens, 169–182, 1997,(www.geocities.com/theometria/Stavroulakis-2.pdf).

[39] Stavroulakis N. On a paper by J. Smoller and B. Temple. *Annales Fond. Louis de Broglie*, v. 27, No. 3, 511–521, 2002, www.ptep-online.com/theometria/Stavroulakis-1.pdf

[40] Stavroulakis N. Non-Euclidean geometry and gravitation. *Progress in Physics*, v. 2, 68–75, 2006, (www.ptep-online.com/index_files/2006/PP-00-13.PDF).

[41] Stavroulakis N. On the gravitational field of a pulsating source. *Progress in Physics*, v. 4, 3–8, 2007, (www.ptep-online.com/index_files/2007/PP-11-01.PDF).

[42] Pauli W. The Theory of Relativity, Dover Publications, Inc., New York, 1981.

[43] Kay D. C. Theory and Problems of Tensor Calculus, Schaum's Outline Series, McGraw-Hill Book Company, 1988.

[44] McConnell A. J. Applications of Tensor Analysis, Dover Publications Inc., New York, 1957.

[45] Landau, L., Lifshitz, E. The Classical Theory of Fields, Addison-Wesley Publishing Company, Inc., Reading, Massachusettes, 1951.

[46] Levi-Civita T. Mechanics. - On the analytical expression that must be given to the gravitational tensor in Einstein's theory. *Rendiconti della Reale Accadmeia dei Lincei*, 26, 381, 1917. (English translation by S. Antoci and A. Loinger: www.geocities.com/theometria/Levi-Civita.pdf)

[47] Tolman R. C. Relativity Thermodynamics and Cosmology, Dover Publications Inc., New York, 1987.

[48] Dirac P. A. M. General Theory of Relativity. Princeton Landmarks in Physics Series, Princeton University Press, Princeton, New Jersey, 1996.

[49] Ricci, Levi-Civita T. Méthodes de calcul différential absolu et leurs applications, *Matematische Annalen*, B. 54, 1900, p.162.

[50] Weyl H. How far can one get with a linear field theory of gravitation in flat space-time?, *Amer. J. Math.*, 66, 591, 1944, (www.geocities.com/theometria/weyl-1.pdf)

[51] Crothers S. J. www.geocities.com/theometria/challenge.html

[52] Crothers S. J. www.geocities.com/theometria/Ricci.html