Erratum: "Do we really understand quantum mechanics? Strange correlations, paradoxes, and theorems" [Am. J. Phys. 69 (6), 655–701 (2001)]

Franck Laloë
Laboratoire de Physique de L'Ecole Normale Superieure, Paris, France

[DOI: 10.1119/1.1466818]

Formulas (2) and (3) are incorrect, and should be:

$$\mathcal{P}_{+,+}=\mathcal{P}_{-,-}=\tfrac{1}{2}\sin^2\theta/2 \tag{2}$$

and

$$\mathcal{P}_{+-}=\mathcal{P}_{-,+}=\tfrac{1}{2}\cos^2\theta/2. \tag{3}$$

Also, the correct expression for (26) should be:

$$\alpha=1/\sqrt{2};\quad \beta=e^{-i\varphi}/\sqrt{2}. \tag{26}$$

This does not affect in any way the rest of the article. The author is grateful to Dr. Miguel Ferrero for pointing out these two errors.

Comment on "Role of the centrifugal force in vehicle roll," by Rod Cross [Am. J. Phys. 67 (5), 447–448 (1998)]

Antonio Alcaraz[a)]
Departamento de Ciencias Experimentales, Universitat Jaume I, E-12080 Castellón, Spain

Patricio Ramírez
Departamento de Física Aplicada, Universidad Politécnica de Valencia, E-46022 Valencia, Spain

(Received 20 December 2001; accepted 13 February 2002)

[DOI: 10.1119/1.1467911]

Cross has recently addressed an interesting problem about the physics of vehicle rollover including alcohol-intoxicated passengers.[1] The analysis is based on a previous paper by Swinson[2] that compares the toppling of a rectangular block subject to a horizontal force with vehicle rollover. Cross suggests that this analogy is incomplete unless one invokes centrifugal force.

The equilibrium conditions of a vehicle making a turn are considered in two cases: the torque acting about the center of mass, and the torque acting about the contact point of the outside wheel with the floor. Because different expressions are obtained, Cross proposes to add a horizontal centrifugal force acting through the center of mass to reconcile both results in a simple and intuitive way.

Although Cross' approach is certainly suggestive, the inclusion of inertial forces is frequently a source of trouble to elementary physics students (but apparently not to lawyers mentioned in the paper) when attempting to distinguish between real and fictitious forces.[3] Moreover, the problem can be easily solved without inertial forces, as we propose here. The difference between the two cases rests on taking moments about the point of contact P with the floor. This point is making a turn, so it is accelerated, and the familiar procedure of equating the net external torque to the rate of change of angular momentum is no longer valid.[4] Thus, taking an arbitrary inertial frame, the general relation of the time derivative of the total angular momentum ($\mathbf{L}_P=\Sigma_i m_i(\mathbf{r}_i-\mathbf{r}_P)\times(d/dt)(\mathbf{r}_i-\mathbf{r}_P)$) of a system of particles to the sum of torques ($\boldsymbol{\tau}_P=\Sigma_i(\mathbf{r}_i-\mathbf{r}_P)\times\mathbf{F}_i^{\text{ext}}$) with respect to the point P is[4,5]

$$\frac{d\mathbf{L}_P}{dt}+M(\mathbf{r}_{\text{c.m.}}-\mathbf{r}_P)\times\mathbf{a}_P=\boldsymbol{\tau}_P, \tag{1}$$

where $\boldsymbol{r}_{\text{c.m.}}$ and $\boldsymbol{r}_P$ denote, respectively, the position of the center of mass and the point P in this arbitrary inertial frame, and $\mathbf{a}_P$ is the acceleration of P in the same system. Note that Eq. (1) contains an additional term, that comes only from the systematic application of Newton's second law to each particle in the system. This additional term cancels if P coincides with the center of mass, if P is not accelerated, or if the vectors in the cross product are parallel. None of these conditions are observed at P.

Furthermore, the condition of absence of rolling is not that the net torque is zero, but that the time derivative of the total angular momentum is zero. When the additional term is calculated according to Fig. 1 in Ref. 1 (taking the figure in the plane yz and $\hat{i}$ being the normal axis), we obtain the "missing" factor without using centrifugal forces:

$$M(\mathbf{r}_{\mathrm{c.m.}}-\mathbf{r}_P)\times\mathbf{a}_P=M(x\hat{j}+H\hat{k})\times\frac{v^2}{R}\hat{j}=-\frac{Mv^2}{R}H\hat{\imath}. \tag{2}$$

Consequently, the result is the same as that obtained by taking a reference frame fixed to the center of mass. The approach we suggest here provides a more general view of rigid-body mechanics and avoids the use of inertial forces, a potential source of misunderstanding for undergraduate students.[3,6]

[a]Electronic mail: alcaraza@exp.uji.es

[1]R. Cross, ''Role of the centrifugal force in vehicle roll,'' Am. J. Phys. **67** (5), 447–448 (1998).

[2]D. B. Swinson, ''Vehicle rollover,'' Phys. Teach. **33**, 360–365 (1995).

[3]R. P. Baumann, ''What is centrifugal force?,'' Phys. Teach. **18**, 527–529 (1980).

[4]F. R. Zypman, ''Moments to remember,'' Am. J. Phys. **58** (1), 41–42 (1990).

[5]K. R. Symon, *Mechanics* (Addison–Wesley, Reading, MA, 1971).

[6]M. D. Savage and J. S. Williams, ''Centrifugal force: fact or fiction?,'' Phys. Educ. **24**, 133–138 (1989).

