

The Meaning of Bell's Theorem

Anthony Rizzi

Institute for Advanced Physics

Abstract

The import of Bell's Theorem is elucidated. The theorem's proof is illustrated both heuristically and in mathematical detail in a pedagogical fashion. In the same fashion, it is shown that the proof is correct mathematically, but it doesn't require, as is usually thought, one to abandon locality or realism.

PACS numbers: 03.65. Ud, 03.65.Ta, 01.70.+w

1 Introduction

Bell's 1964 paper[1] addressing the Einstein, Podolsky, Rosen's (EPR) paradox[2] peaked the debate between supporters of locality and/or realism on the one hand and the so-called orthodox, or Copenhagen,¹ interpretation of quantum mechanics on the other.² Here *locality* is the condition that one physical thing cannot influence another except by propagation of the effects in a finite time. *Realism* is the idea that the physical world, though we can affect it, exists fully independent of us. Both seem benign, indeed trivially true, to those not acquainted with quantum mechanics. However, for many familiar with quantum mechanics, the debate was capped in favor of

¹At its most generic, no one objects to the Copenhagen interpretation, for at that level it only expresses the complementarity of wave-particle aspects found in measurements. That is, at that level, it simply points to the fact that one type of measurement manifests wave aspects and another particle aspects. However, usually the Copenhagen interpretation carries, at least implicitly, much more philosophical baggage than this. It has, for example, been taken to include the negation of the principle of causality.

²In this paper, I do not discuss the Everett's so-called many worlds interpretation of Quantum mechanics.

Copenhagen when, in the early 80's, Alain Aspect did a series of experiments that confirmed that quantum mechanics accurately described the physical situation of the EPR paradox.[3] [4]

At the heart of the controversy stirred by Bell's theorem lies a continuing debate about the interpretation of the mathematical formalism of quantum mechanics. In the earliest days of the quantum theory, grappling with the interpretational problems plaguing the theory led to the traditional Copenhagen interpretation - a synthesis of Bohr's and Heisenberg's views (although neither of them used this term to refer to their joint views).[5][6][7][8] Even then, there were important differences between Bohr's view and Heisenberg's view that were never completely resolved. Heisenberg's view was more subjectivistic than Bohr's; he initially took the uncertainty relations that he derived to imply that certain objects do not exist in nature unless and until we observe them. This is not surprising considering the zeitgeist of the time in Germany. Contrary to Heisenberg, Bohr rejected the view that the experimental outcome is due to the observer. Instead, he chose to view the uncertainty principle in the larger framework of what he called complementarity. Bohr's complementarity was not particularly positivistic or subjectivistic; to a certain extent this is because he was quite vague about its descriptive content. Philosophically, he was also influenced by neo-Kantianism.[9] As evidenced by Bohr's conviction that the atom is real, he had an inkling that the natural world exists independently of experiments; however, precisely what sort of inkling he had is, at best, unclear. The subjectivist interpretation of the quantum theory set off one of Einstein's quotable remarks. He asked his friend A. Pais, if he "really believed that the moon exists only when [you] look at it?" [10]

Bell's theorem is controversial because it purports to show that physical reality must be non-local; furthermore some use it to bolster the subjectivist interpretation of quantum mechanics which claims a thing isn't there until you measure it. For Bell, his renowned theorem shows that even if one takes quantum mechanics to be incomplete and tries to expand it by use of "hidden variables" one must have, what Einstein called "spooky" action at a distance. We will see, by taking seriously the implications of the fact that all non-commuting observables are not simultaneously measurable, that one need not sacrifice locality.

2 Bell's Theorem and the EPR Paradox

For completeness and maximal usefulness for instruction, even at an introductory level, this section has three parts. First, to give the complete ground work for Bell's proof, the predictions of quantum mechanics are summarized for correlated spin 1/2 particles. The second part gives a concrete description of the EPR experiment and Bell's proof; it makes clear why the correlation predicted in the first part is inconsistent with Bell's inequality and hence apparently with locality as well. Again, such a non-locality means that a thing here instantaneously influences something on the other side of the universe. The final part gives the mathematical proof, relating it to the description and heuristic arguments of the second part.

2.1 The Quantum Mechanics of a pair of electrons in the Singlet State

In quantum mechanics, spin is a 3-vector quantity \mathbf{S} .³ We have $S^2 = S_x^2 + S_y^2 + S_z^2$ as well as the commutation relations $[S_x, S_y] \equiv S_x S_y - S_y S_x = iS_z$ (and other permutations of $\{x, y, z\}$) where we choose units so that $\hbar = 1$.

Furthermore, we define the usual "ladder" operators S_+ and S_- such that

$$\begin{aligned} S_x &= 1/2(S_+ + S_-) \\ S_y &= -i/2(S_+ - S_-). \end{aligned} \tag{1}$$

Then, the following result holds: $S_{\pm} | S_z = m \rangle = (s(s+1) - m(m \pm 1)) | S_z = m \pm 1 \rangle$.

We now define a new operator $\sigma = 2\mathbf{S}$ and note that the eigenvalues of $\sigma_x, \sigma_y, \sigma_z$ are ± 1 . It is convenient to introduce more notation at this point:

$$\begin{aligned} | \alpha \rangle &= | \sigma_z = +1 \rangle \\ | \beta \rangle &= | \sigma_z = -1 \rangle \end{aligned} \tag{2}$$

Projecting the σ -components onto the $\{| \alpha \rangle, | \beta \rangle\}$ basis yields the representation

³In the following section we shall be using finite-dimensional Hilbert spaces to describe spin angular momentum of quantum mechanical systems.

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \quad (3)$$

These are the well-known Pauli spin matrices; note that the square of each is the unity matrix.

With this in mind, we proceed to remind ourselves of some facts about the angular momentum of a 2-particle composite system. \mathbf{S}_1 and \mathbf{S}_2 are the spin angular momenta of the two particles and $\mathbf{S} \equiv \mathbf{S}_1 + \mathbf{S}_2$. If s_1 and s_2 are the spin quantum numbers for the particles, the total spin quantum number s for the whole system ranges in integral steps from $|s_1 - s_2|$ to $s_1 + s_2$. For any one s , the eigenvalues of S_z (the z -component of total spin) range in integral steps from $-s$ to s . Finally note that $|S_z = m\rangle$ can be expressed in terms of $|S_{1z} = m'\rangle$ and $|S_{2z} = m''\rangle$ by referring to the Clebsch-Gordon/Wigner coefficients.

Now, for the singlet state of two spin- $\frac{1}{2}$ particles, $s = 0$ and $m = 0$; so the resulting state vector is:

$$|\Psi_{singlet}\rangle = |\alpha(1)\rangle|\beta(2)\rangle - |\beta(1)\rangle|\alpha(2)\rangle \quad (4)$$

We observe that it is rotationally invariant.

For later use, we will now calculate the correlation function which is defined as:⁴

$$C(\mathbf{a}, \mathbf{b}) = \langle \Psi_{singlet} | (\boldsymbol{\sigma}_1 \cdot \mathbf{a}) \otimes (\boldsymbol{\sigma}_2 \cdot \mathbf{b}) | \Psi_{singlet} \rangle \quad (5)$$

Each dot product of spin with a unit vector gives the component of the spin in that direction. The correlation function is just the average value of the product of the spin component for particle 1 with the spin of the component for particle 2; this can be easily checked by inserting completeness relations where appropriate.

Let the z -axis be along unit vector \mathbf{a} ; choose the x -axis so that \mathbf{b} lies in the $x - z$ plane, and let \mathbf{b} make an θ_{ab} with \mathbf{a} . We can then write

$$C(\mathbf{a}, \mathbf{b}) = \langle \Psi_{singlet} | (\sigma_{z1} \otimes (\sigma_{z2} \cos \theta_{ab} + \sigma_{x2} \sin \theta_{ab})) | \Psi_{singlet} \rangle \quad (6)$$

⁴Here \otimes denotes the tensor product of operators (over a tensor product of Hilbert spaces). Roughly speaking, if our composite system is made up of states of ψ (a system with Hilbert space \aleph) and states of ψ' (a system with Hilbert space \aleph'), then the Hilbert space of the composite system will be the tensor product $\aleph \otimes \aleph'$.

From equation 4 and 5, we can work out the action of the Pauli spin operators to give:

$$C(\mathbf{a}, \mathbf{b}) = -\cos \theta_{ab} \quad (7)$$

We will also need the probability that the spin component measured along the z-axis is opposite in sign to the spin component measured along the vector at angle θ_{ab} . To get this, we first solve the eigenvalue equation:

$$S_{\theta_{ab}} | vector \rangle = a | vector \rangle \quad (8)$$

where $S_{\theta_{ab}} = (\sigma_{z2} \cos \theta_{ab} + \sigma_{x2} \sin \theta_{ab})$, which gives eigenvalues of +1 and -1 with eigenvectors, respectively:

$$\begin{pmatrix} \cos[\theta_{ab}/2] \\ \sin[\theta_{ab}/2] \end{pmatrix} \text{ and } \begin{pmatrix} -\sin[\theta_{ab}/2] \\ \cos[\theta_{ab}/2] \end{pmatrix} \quad (9)$$

Next, decompose the spin up eigenvector, $\begin{pmatrix} 1 \\ 0 \end{pmatrix}$ into the above eigenvectors of $S_{\theta_{ab}}$ and similarly for the spin-down one $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. This yields straightforwardly the probability that the particle in the pair encountering the z-axis detector will be measured with opposite eigenvalue to that encountering the detector set to measure the spin component at angle θ_{ab} ; that is, one is looking for the probability that the eigenvalues of the pair are (+1, -1) or (-1, +1). The probability is:

$$\cos^2 \frac{\theta_{ab}}{2} \quad (10)$$

2.2 Bell's Theorem: Heuristically

First, to make the proof accessible to those new to the subject and to render the problem sufficiently clear to others⁵, I'll lay out the experiment and then describe Bell's conclusions from it.⁶ I use the incarnation of the EPR paradox described by Bohm and Aharonov[12]. Let's consider a pair of electrons in the

⁵Those who have extensive background may want to skip to the mathematical proof in the next subsection.

⁶The main line of argument followed here is taken from my layman's level book "The Science Before Science." [11] The particulars have been modified to address physicists more directly.

singlet state. A set of these quantum mechanically entangled electrons are such that the mathematical relationships described above obtain between the measured values of the spin of each electron. The electron spins are precisely anti-correlated when measured at the same angle.

For concreteness, say the two correlated electrons are emitted from a nucleus of a particular atom, and they move in opposite directions. The experimental setup is shown schematically in figure 1. On the left and right, symmetrically placed around the atoms (nuclei) that emit the correlated electrons are two detectors. The left (right) detector can be set at some angle L (R) with respect to some vertical axis defined as “up,” i.e. the vertical will be considered zero degrees; this is a mere convention that does not effect the generality of the results. When set at a given angle, an electron striking a detector yields a binary value, which we designate as “+” for spin up and “-” for spin down, depending on whether the electron goes, respectively, along the direction in which the angle ray points or away from it (see figure 1).⁷ One thus can speak about a binary measured value for the spin component at any one angle. Quantum mechanics predicts that after a series of nuclear emission events, each of the resulting pairs of spin measurements has a probability of $\cos^2[(L - R)/2]$ of being anti-aligned (i.e. one electron + and its pair -), where, note, $L - R$ is the difference between the two angles (cf. equation 10). For example, if $L - R = 60$ degrees, then there is 75% probability of the two measurement results being anti-aligned. To see the problem take the following scenario:

1) With $L = 0$ degrees and $R = 60$ degrees, take 10 data runs, i.e., wait for 10 nuclear emission events. This experiment results in 10 electrons impinging on each detector and, hence, 10 outputs from each detector indicating whether an electron was measured to be up “+” or down “-”.

2) Now say that instead of the above, we did the experiment by taking 10 data runs $L = 0$ degrees and $R = 120$ degrees.

3) Fill the data into the corresponding parts of the table below. We, of course, pick data in such a way as to be in agreement with the predictions of quantum mechanics.

4) Knowing that the conservation of angular momentum will fix the remainder of $L = R = 0$ degrees, $L = R = 60$ degrees and $L = R = 120$ degrees portion of the table (that is when the angles, L and R , are the same). In

⁷Such an apparatus uses a magnetic field and is called a Stern-Gerlach experiment.

other words, “+” on one detector implies “-” on the other and vice-versa. Since these entries in the table were not obtained by experiment, we print them in a smaller font.

LEFT DETECTOR											RIGHT DETECTOR										
L	1	2	3	4	5	6	7	8	9	10	1	2	3	4	5	6	7	8	9	10	R
0	+	-	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-	+	0
60	-	+	-	-	+	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	60
120	-	+	-	-	+	-	-	+	-	+	+	-	+	+	-	+	+	-	+	-	120

Now, if we look at many of the various portions of the table, we see that they are correlated, as they should be, approximately according to $\cos^2[(L - R)/2]$. Specifically, for $L = 0$ degrees and $R = 60$ degrees the electrons in a pair are anti-aligned about 70% of the time, which is as close as one can get to the predicted 75% with just 10 runs. Similarly, for $L = 0$ degrees, $R = 120$ degrees, there is 30% anti-alignment, compared to the predicted 25%.

So, what’s the dilemma? Note that the quantum mechanical prediction of the correlation (which, as defined in this section is $\cos^2[(L - R)/2]$) does not depend at all on the absolute angle of either detector; it just depends on the difference. On the other hand, the table shows that comparing $L = 60$ degrees and $R = 120$ degrees gives us only 60% anti-alignment where we needed 75% (i.e. should be 70 or 80%) by the prediction $\cos^2(60/2)$.

Now one may think the answer is simply that all statistics implies some variation, and with this small a sample, one can expect a lot of variation from the predicted value. However, try to make the situation better by manipulating the “+” and “-” signs in the table. You cannot; it is imposed on you by the fact that each angle must give on the average $\frac{1}{2}$ +’s and $\frac{1}{2}$ -’s and the correlation between measurements of all the various angle combinations must be given by the $\cos^2[(L - R)/2]$ rule. One cannot satisfy the correlation for all angles simultaneously. So, how can the electrons, which are in principle separated by great distance when they’re measured, “know” what angle to correlate to? For example, if the two detectors are set approximately equidistant from the source, then one measurement will be happening about when the other is; hence, one can set the time available for one measurement to affect the other arbitrarily small. Indeed, the experiments already actually done did not leave enough time for a signal to propagate at the speed of light from one electron to its pair. Hence, the electrons cannot be initially set to

do what they do, because we've seen that the correlation would not then be what it is observed to be. This is the idea of Bell's theorem.

2.3 Bell's Theorem: Mathematical Proof

One nice proof of Bell's theorem, which follows closely Bell's own proof[1], is given in the appendix (pg 233) of a text by Rae.[13] We will follow his proof in a notation slightly modified to be consonant with the heuristic treatment above. In his and Bell's proof, one assumes that the result of each measurement depends only on a mathematical description that includes only what's happening *near the electron being measured* not the far away electron. Assuming this, one then deduces that the resulting correlations will not agree with experiment.

Specifically, one starts by assuming that the outcome of each measurement at any angle is only a function of a random variable, " λ " associated with each pair. Bell points out that λ can actually be many variables or function of many variables. Since the measured behavior of any given electron in the experiment is quantified by λ which is "carried with" the given electron, assigning λ enforces the locality assumption. In other words, by assigning λ , we assume the measurement of one electron doesn't influence the state of its pair, only the locally determined λ does.

The proof proceeds as follows using the conventions defined above. λ is a hidden variable that determines S_{zleft} , the z -component of the spin of the particle moving to the left. More generally, the first subscript specifies the angle of the spin component measured and the second subscript specifies which detector the particle is approaching, the left or the right.

We define $p(\lambda)$ as the probability density so that $p(\lambda)d\lambda$ is the probability of a pair being produced with a value of λ between λ and $\lambda + d\lambda$. All the probabilities should add up to one so we lay down a normalization condition:

$$\int p(\lambda)d\lambda = 1 \tag{11}$$

We now consider an experiment in which the spin components corresponding to S_{zleft} and $S_{\phi right}$ are measured on a large number N of particle pairs.

Accordingly, the average of the values of the products $S_{zleft}(\lambda)S_{\phi right}(\lambda)$ will be given by $C(\phi)$ where

$$C(\phi) = \int S_{zleft}(\lambda)S_{\phi right}(\lambda)p(\lambda) d\lambda \quad (12)$$

This expression is clearly true since it is just the limit of the sum of the products multiplied by the probabilities.

If we now consider that *instead* of doing the previous experiment on the N electron pairs, we did a slightly different one. We keep the left apparatus as before, but the right apparatus is instead oriented to measure the component at an angle θ to the z -axis. We would thus obtain a similar expression for the quantity $C(\theta)$ and hence

$$C(\phi) - C(\theta) = \int [S_{zleft}(\lambda)S_{\phi right}(\lambda) - S_{zleft}(\lambda)S_{\theta right}(\lambda)] p(\lambda) d\lambda \quad (13)$$

Since the sum of the same (measured) component of spin of the left particle and the right particle is always zero, we get

$$S_{\theta right}(\lambda) = -S_{\theta left}(\lambda) \quad (14)$$

$$S_{\phi right}(\lambda) = -S_{\phi left}(\lambda) \quad (15)$$

Thus:

$$C(\phi) - C(\theta) = - \int S_{zleft}(\lambda) [S_{\phi left}(\lambda) - S_{\theta left}(\lambda)] p(\lambda) d\lambda \quad (16)$$

$$= - \int S_{zleft}(\lambda) S_{\phi left}(\lambda) [1 - S_{\phi left}(\lambda) S_{\theta left}(\lambda)] p(\lambda) d\lambda \quad (17)$$

The last step follows because $S_{\phi left}(\lambda) = \pm 1$

Taking the absolute value of both sides we get:

$$|C(\phi) - C(\theta)| = \left| \int S_{zleft}(\lambda) S_{\phi left}(\lambda) [1 - S_{\phi left}(\lambda) S_{\theta left}(\lambda)] p(\lambda) d\lambda \right| \quad (18)$$

$$\leq \int |S_{zleft}(\lambda) S_{\phi left}(\lambda) [1 - S_{\phi left}(\lambda) S_{\theta left}(\lambda)] p(\lambda)| d\lambda \quad (19)$$

Since $|S_{zleft}(\lambda)S_{\phi left}(\lambda)| = 1$, equation 19 can be written as

$$|C(\phi) - C(\theta)| \leq \int [1 - S_{\phi left}(\lambda)S_{\theta left}(\lambda)] p(\lambda) d\lambda \quad (20)$$

$$\leq 1 + \int S_{\phi left}(\lambda)S_{\theta right}(\lambda)p(\lambda) d\lambda \quad (21)$$

The integral in equation 21 is simply the correlation function between the measured value of two spin components that are at an angle of $(\theta - \phi)$ with each other and is therefore equal to $C(\theta - \phi)$. Thus we can write equation 21 as:

$$|C(\phi) - C(\theta)| - C(\theta - \phi) \leq 1 \quad (22)$$

This is one of the forms of the so-called Bell's inequalities. Substituting the quantum mechanically correct correlation function for $C(\alpha) = -\cos(\alpha)$ from equation 7, one finds a whole range of angles at which the inequality fails. For example, take the case we used in the section above where: $\theta = 120^\circ$, $\phi = 60^\circ$. In this case, Bell's inequality is written:

$$\frac{3}{2} \leq 1$$

This statement is clearly false. Hence, it is thus proved that the predictions of quantum mechanics, which are the ones found in experiment, cannot be true if the other assumptions we've made are true. By a reductio ad absurdum, we concluded that what we take to be our key assumption must be wrong, and each electron must be dependent on the state of the disconnected other.

Many in fact conclude that nature is non-local from this. They say there must be action at a distance, meaning instantaneous action of one body on another. Others, trying to avoid conceding action at a distance, conclude that spin properties and other properties that have non-commuting variables such as position and momentum, do not exist until they are observed; still many say there is no such escape.

3 The Meaning of Bell's Theorem

Now, locality is a cherished principle still even today in most of physics; most would rather not give it up. To certify that there is no escape route in such

a proof, one must make sure it contains no hidden assumptions. We will do this check, but first there are a couple signs, not proofs, that we might be on the wrong track in concluding that quantum mechanics implies non-locality. First, it is well known that quantum mechanics does not imply action at a distance in one very pragmatic sense. Philippe Eberhard showed that within quantum mechanics, there cannot be communication of information faster than the speed of light.[14] In other words quantum mechanics does not violate the letter of the law of special relativity. The second sign comes from the statistical nature of quantum mechanics.

3.1 Signs of Locality

Quantum mechanics is a statistical theory. The use of statistics is an admission of lack of knowledge. We only use statistics when we do not know (by choice or by reason of some impassable obstacle) the details of a particular phenomenon under consideration. In so doing, we leave out specifics and settle for averages. For example, in a coin toss, we say that there is a probability of one in two that it will be heads. In saying this we are relating very little about the individual coin toss. We are relating something about a large group of coin tosses. This allows us to not be concerned with the details of each toss, which are very complicated. Hence, it would be a great error to conclude that because we have an element of randomness in our description that the flipping of our coin is intrinsically random. This is to forget that we have deliberately left these details out.

Further, in taking averages as one does in statistics, it is easy to come to conclusions that take root in the failure to remember what you've included and what you've not included in your theory. Consider the case of a man who drowned in a lake with an average depth of two inches. Each member of a group of non-swimmers that walked across this lake could be told that his chance of drowning is very small. For anyone that actually does drown such a statement will have no importance. Specifics, not generalities, are in the end what reality is about. When we describe a group of things under consideration by statistics we are admitting our deficit of knowledge about those things, and yet making use of that knowledge to say what little we can about them.

Since quantum mechanics makes use of statistics, we need to be wary that the individuals may be left out of the account. Hence, concluding that there

is action at a distance, which means one electron of a given pair is acting instantaneously on the other of the same pair, from quantum mechanics seems problematic.

In particular, since the Eberhard proof says the statistics of measurements cannot be altered at a distance, i.e. they behave locally, and since it is measurement statistics that are directly described by quantum mechanics, it would be odd if the individual behavior were non-local. This oddity has not gone unnoticed, but it is nonetheless a key sign that something might be amiss with the non-locality proofs.

To resolve these issues, we now look more closely at the experiment and the proof.

3.2 A Second look at the Experiment

We will soon see that in our forming of the dilemma, we implicitly assumed that we could measure the spin state of one electron at two different angles without one measurement affecting the outcome of the other. The uncertainty principle testifies that we are allowed no such ability. Indeed, the whole standard interpretation of Bell's theorem hinges on the possibility of making measurements involving at least three different angles, which, in turn, means one must consider the measurements at two angles for one electron simultaneously. How so?

Recall that the measurements above were taken by changing the angle R of detection of the right detector from 60 degrees to 120 degrees, but the measurement at each angle was taken as if the experimenter had not done the other (see step one and two above). We did the first experiment with $R = 60$ degrees and then we said what happens if we did not do it, but did do $R = 120$ degrees. Indeed, we can say that one of these two experiments is really simply a hypothetical experiment, because we are in some sense refusing to allow *both* actually occurred. Yet, in our analysis we considered the data as if the experiments were both really done; thus, we implicitly assumed in our reasoning that it would yield the same result if both were really done on the same electron. This is a leap of logic; it leaps over the implicit assumption that one measurement would not affect the other.

In short, as long as we respect our *inability* to measure two *different* spin components of the *same* electron (not a statistical electron) *without* one measurement interfering with the result of the other, the conclusion will not

follow; that is, we will not be forced to assume action at a distance.

3.3 Reexamining the Mathematical Proof

In the mathematical proof given above, the case is clear. We twice assign *measured* values at two different angles to one electron. To manifest the hidden assumptions, take the λ as an index or label marking the pair under consideration.

Note that each electron measurement is associated with one value of the hidden variable. However, it may be that the association is not one-to-one, so that there is more than one electron with the same λ . Indeed, one could assume that the measured spin is a bounded smooth function of the hidden variable and thus will be repeated after enough runs, so that one can avoid having to measure the same one twice. However, such an assumption is by no means forced on you. Each pair can be completely unique; indeed each is already unique at least to the extent that each pair succeeds another in coming out of the source, and they come out of different nuclei in our scenario.

In any case, we are free to take the λ as an index marking which pair is under consideration. By assigning, in equation 14, $S_{\theta right}(\lambda) = -S_{\theta left}(\lambda)$, one is implicitly asking for the measured spin component for particle “*left*” at both θ and 0° (that is along the z – *axis*). This can be seen clearly in equation 16, where one sees that both $S_{z left}(\lambda)$ and $S_{\phi left}(\lambda)$ are used. The parallel equation 15, $S_{\phi right}(\lambda) = -S_{\phi left}(\lambda)$, similarly implies one is measuring the spin component for a single particle for both ϕ and z . Indeed, the problem already appears in equation 13 where both $S_{\phi right}(\lambda)$ and $S_{\theta right}(\lambda)$ are needed. Two hypothetical experiments are involved in getting these measured values. Either they are actually done or they are not actually done. If they are not done then we do not have the results. If they are done then first interferes with the result of the second; the second is no longer governed by $S(\lambda)$ in the same simple way.

Again, the only way we could know both values of the components of the spin is if the measured values were to be independent of each other, i.e. one not affected by the other. This is a fine starting assumption as long as one recognizes it as such. Then one is trying to prove that one cannot simultaneously know to arbitrary accuracy, by measurement, the value of two different spin components of one electron. If this is the case the proof

goes through flawlessly. That is, one can end up either concluding that such simultaneous exact knowledge by measurement cannot occur or that there is action at a distance; we choose the former, which squares with Heisenberg's uncertainty principle for non-commuting variables.

Of course, one's inability to simultaneously measure two aspects of a thing *exactly* does not mean that those aspects cannot exist simultaneously, so there is no real contradiction in this case. Indeed, if one chooses to make the possibility of exact measurement a necessary condition for the existence of a thing then *our mere* abilities determine nature. That is, we don't even have to exercise our abilities, *we somehow passively, by our mere existence*, cause things to be. Einstein insightfully remarked that all science depends on the negation of such a proposition, pointing to science's implicit reliance on the fact that the world exists independent of us.

So, how do we interpret the EPR experiment described above. First, we say the measurement of one of the electrons gives its "measure state" described by quantum mechanics. Once we know that state for a given angle, we can use our statistical knowledge of that "measure state" to predict the outcomes of further measurements on that electron. Further, by using the quantum predictions of the correlation of the pair, we also instantly know that the other electron will be in the same "measure state" with respect to the given angle; this fixes probabilities for measurement at other angles. The electron pairs carry their correlation from their emission from a common nucleus, but we are ignorant as to the specifics of that process and how it relates to the results of our measurements because of the limitations of our physical knowledge, at least at present.

4 Conclusion

It's been said that EPR is the most cited paper in the literature.[15] This is an odd fact in one way, because it is often considered that such topics are on the boundary between physics and philosophy. In any case, EPR and Bell's theorem touch the heart of what physics is about. It reveals its strength and limitations. The key strength of modern physical theories, their ability to accurately predict experimental outcomes even when our understanding of the theory and intuition about it are underdeveloped and confused or even somewhat contradictory, is manifested by the quantum mechanics of Bell's

theorem discussed above. In this case, its strength also appears to be its weakness, because the theory's accuracy has induced serious and otherwise reasonable people to doubt the reality of the world independent of us by taking serious various interpretations of the theory.

The discussion above reveals somewhat the genesis of these kind of confusions. Confusions can arise when one does not give adequate attention to the step of advancing from the already existing mathematical structure of a theory and the accompanying physical situations that it describes to new theories and understandings. In particular, this step requires careful thinking about which direction one would like to go. There are two main paths to consider. If a physicist wants to delve more deeply into a theory's mathematical structure to find a more powerful theory that incorporates more experimental data and circumvents current weaknesses, he should continue in the generic line of thinking that brought the equations to the fore. This emphatically does *not* mean he should follow the exact same mathematical method and insights and avoid new ground; such is the definition of a sterile approach. I mean that philosophical sorts of considerations should not be allowed to control his thinking too much. By contrast, when the questions are about the ontological realities of the entities he is describing, he should step back a bit further and look carefully at the larger assumptions.

Not doing so can lead to unlikely or absurd conclusions. We saw this happen in our discussion of Bell's theorem; generically, we saw what looked fine mathematically was actually impossible physically.

It is instructive to recap the key point of our Bell's theorem discussion. To facilitate the discussion re-write $S_{\phi left}(\lambda)$, the measured value of the spin component in the ϕ direction of a particular particle as it moves toward the left detector, as: $S_{left}(\phi, \lambda)$. This way of writing it fully manifests that S is a function of two quantities. For a given electron of given λ , this function can be a smooth function of ϕ . That is, mathematically, one can think of all values of the function S existing at once, as does, for example, a plot of the function as a two dimensional surface in three dimensions. Despite this mathematical fact, however, physically, for fixed λ , one can only know the function's value for one value of ϕ at a time. Why? Because the act of measuring the electron in some way alters the other values, putting the particle in a different "measure state" therefore requiring a different functional description say $S_{left}(\phi_{1st}, \phi_{2nd}, \lambda)$, which now depends on what angle was measured first. Like a match, it can only be lit once, but can poten-

tially lit many different ways: e.g., striking on the side of the box, on a table surface, in the middle by a second match etc...

In the particular case considered above, we can determine two angles for one electron by measuring it and its pair each once. For example, I can measure the left particle at $\phi = 0^\circ$ and the right particle for $\phi = 60^\circ$ and will thus know the “measure state” of each. But now neither is in the same “measure state” as before the measurement, so the left will, in general, *not* measure at 60 degrees what the right measured and vice versa at 0 degrees.

I use the term “measure state” above to indicate that the reduction in the wave packet is simply an increase of our knowledge of the system. It is suggestive of the following interpretation of quantum measurement. Because the statistical nature of quantum mechanics, which is in turn due to our ignorance of the system, we must have one piece of information about the system before we can say anything further than just the correlation, i.e. the probability of the left detector measuring spin up at a given angle, given a measurement at a spin component at a given angle on the right detector. The jump is a jump in our knowledge, not in the reality of the thing measured. To manifest the absurdity of that position, the situation might be compared to the classical situation in which one needs the initial velocity and position of a projectile, say a ball, before one can use the laws of Newton to predict where it will land. One might say that until one observes the ball at a particular time, the ball is everywhere. After all, one needs measurements to say anything about a particular real ball. Equations giving statistical outcomes need initial data as well, just of a different type and interpretation. In either case, one can conclude measurement causes reality instead of revealing something of it. Of course, such a statement gets the facts exactly backwards; we know, for example, that the ball exists first and then experiment to obtain the laws of Newtonian physics that describe the dynamics of the ball.

The insights and arguments laid down here I first produced and wrote independently in the previously cited manuscript “the Science Before Science,” and only later discovered that T. Brody had already said some of these things, though without all the detailed explications outlined above. Brody discusses the key issues[16][17] and even concludes with an example of an equation that reproduces the Bell’s inequality.⁸ He calls the inability of simultaneously knowing two variables that often attends violations of Bell’s

⁸He also references other such examples; confer [18][19] and [20].

inequality the joint measurability condition.

Some have either tried to avoid the simultaneous measurability condition or seem to have avoided it, but end up implicitly assuming it any way. Certain popular treatments sometimes seem to avoid the condition, but also implicitly assume it.

4.1 Acknowledgments

I'd like to thank Nicholas Teh of Princeton University for contributing most of section 2.1 (the qm of the electron pair singlet state), and assisting with exposition of the history in the introduction. I also thank him for his comments on the style and presentation of the arguments in the manuscript. Thanks to Stephen Barr and Juhan Frank for their comments on the paper and special thanks to Michael Romalis for helpful discussions on the manuscript.

References

- [1] J.S. Bell. On the einstein-podolsky-rosen paradox. *Physics*, 1:195–200, 1964.
- [2] B. Podolsky A. Einstein and N. Rosen. Can quantum-mechanical description of physical reality be considered complete? *Phys. Rev.*, 47:777–780, 1935.
- [3] A. Aspect. *Three Experimental Tests of Bell's Inequalities by the Measurement of Polarization Correlation Between Photons*. PhD thesis, Orsay (France), 1983.
- [4] J. Dalibard A. Aspect and G. Roger. Experimental tests of bell's inequalities using time varying analyzers. *Phys. Rev. Lett.*, 49:1804–1807, 1982.
- [5] N. Bohr. *Atomic Theory and the Description of Nature, Reprinted as The Philosophical Writings of Niels Bohr, Vol. I*. Woodbridge: Ox Bow Press, 1958/1987.

- [6] N. Bohr. *Essays 1932-1957 on Atomic Physics and Human Knowledge, Reprinted as The Philosophical Writings of Niels Bohr, Vol. II.* Woodbridge: Ox Bow Press, 1963/1987.
- [7] N. Bohr. *Essays 1958-1962 on Atomic Physics and Human Knowledge, Reprinted as The Philosophical Writings of Niels Bohr Vol III.* Woodbridge: Ox Bow Press, 1998.
- [8] N. Bohr. *Causality and Complementarity (Supplementary Papers) as The Philosophical Writings of Niels Bohr, Vol. IV.* Woodbridge: Ox Bow Press, 1999.
- [9] Jan Faye. *The Stanford Encyclopedia of Philosophy*, chapter Copenhagen Interpretation of Quantum Mechanics, pages URL = <http://plato.stanford.edu/archives/sum2002/entries/qm-copenhagen/>. Summer 2002 edition.
- [10] N. David Mermin. More experimental physics from EPR. In James T. Chushing and Ernan McMullin, editors, *Philosophical Consequences of Quantum Theory*, page 49, Notre Dame, IN, 1989. University of Notre Dame Press.
- [11] Anthony Rizzi. *The Science Before Science.*
- [12] D. Bohm and Y. Aharonov. *Phys. Rev.*, 108:1070, 1957.
- [13] Alastair M. Rae. *Quantum Mechanics.* Institute of Physics Publishing, Bristol and Philadelphia, 1992.
- [14] Philippe Eberhard. Bell's theorem and the different concepts of locality. *Nuovo Cimento*, 46:392-419, 1978.
- [15] F. Laloe. Do we really understand quantum mechanics? strange correlations, paradoxes and theorems. *Am J Phys.*, 69 No. 6:page 662, June 2001.
- [16] Thomas Brody. *The Philosophy Behind Physics.* Springer-Verlag, Berlin, Heidelberg, NY, 1994.
- [17] Pena L. Brody, T.A. Real and imagined nonlocalities in quantum mechanics. *Nuovo Cim.*, 58B:455, 1979.

- [18] G.C. Scalera. *Lett. Nuovo Cim.*, 38:16, 1983.
- [19] G.C. Scalera. *Lett. Nuovo Cim.*, 40:353, 1984.
- [20] S. Notarrigo. *Nuovo Cim.*, 83B:173, 1984.

Figure 1: The experimental setup as seen from an off-perpendicular angle. A particle decays into a pair of spin-entangled electrons, one moving to the right, the other to the left; each moves towards its own detector. One sets each detector to determine the spin component at a given an angle; spin along the angle vector is written as “+” and spin opposite as “-”

This figure "fig01_label.png" is available in "png" format from:

<http://lanl.arxiv.org/ps/quant-ph/0310098v1>