The EPR correlations and the chameleon effect

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Abstract

We describe an experiment in which two non communicating computers, starting from a common input in the form of sequences of pseudo–random numbers in the interval $[0, 2\pi]$, and computing deterministic $\{\pm 1\}$–valued functions, chosen at random and independently, produce sequences of numbers whose correlations coincide with the EPR correlations and therefore violate Bell's inequality.

The experiment is the practical implementation of a mathematical model of a classical, deterministic system whose initial state is chosen at random from its state space, with a known initial probability distribution, and whose dynamics exhibits the chameleon effect described below. Such a system satisfies the constraints of pre–determination, locality, causality, local independent choices, singlet law and reproduces the EPR correlations.

1 Introduction

Since the appearence of Bell's paper [10] in 1964 the following problem has played a central role in the debate on the foundations of quantum mechanics:

Do there exist classical systems which reproduce the EPR (or singlet) correlations under the constraints of pre-determination, locality principle, singlet condition and causality principle? (cf. Section (2) below for comments on the meaning of these terms).

The conceptual implications of this, apparently technical, question reside in the fact that the possibility (also called a hidden variable theory) of a classical interpretation of natural phenomena (e.g. the EPR type experiments), in which the properties of the physical objects are pre-determined, and not created at random by the act of measurement, is often identified to the possibility of having a realistic point of view on the natural phenomena. For this reason the above question is usually considered equivalent to the following:

Is it possible to simultaneously maintain: (i) a realistic point of view on the natural phenomena; (ii) the predictions of quantum mechanics (EPR correlations); (ii) the locality (or causality) principle?

It is a fact that the (practically) unanimous answer to this question, in the past 37 years, has been a flat: No!

The argument, supporting this unanimous answer, is that:

(i) the EPR correlations violate Bell's inequality

(ii) Bell's analysis proves that any system, satisfying pre-determination, locality and causality cannot violate Bell's inequality.

This argument is often condensed in the statement that there cannot exist local realistic hidden variable theories.

The quantum probabilistic point of view [9], challanged Bell's analysis on two different fronts namely:

1) by proving that it is theoretically inconclusive, i.e. that statement (ii) above is wrong (cf. [3] for a survey of these arguments)

2) by constructing a general mechanism (called "chameleon effect") [7] which shows how to construct classical systems violating the hidden mathematical assumption, implicit in the proof of Bell's inequality.

Yet, until recently, no attempt had been made to solve this controversy by means of an experiment. In other words, until recently, the general mechanism mentioned in item (2) above had not been substantiated into an example of a concrete classical physical system which satisfies pre-determination, locality and causality and yet violates Bell's inequality.

It is true that the list of experiments, some of which truly spectacular [12], confirming the emergence of the EPR correlations in several quantum mechanical phenomena is nowadays very long and continuously increasing.

However, being performed on systems with a strong quantum behavior, these experiments, while confirming the important statement that singlet correlations effectively appear in nature, cannot say anything on the main thesis of Bell, namely that no classical system can reproduce these correlations under the above constraints.

The first experiment in this direction was performed in april 1999 and reported in [5], [6]. Its results supported the point of view of quantum probability, in the sense that a local realistic violation of Bell's inequality was detected, however without reproduction of the EPR correlations.

The second experiment in this direction was performed in june 2001 and reported in [1], [2]. In it the EPR correlations were faithfully reproduced by local, independent and even deterministic, macroscopic, classical systems. As explained above, violation of Bell's inequality is a consequence of these correlations.

The third experiment will be performed publicly as a satellite activity of the "Japan-Italy Joint workshop on Quantum open systems and quantum measurement", (Waseda University, 27–29 September 2001) and consists of a sophisticated elaboration of the second one, based on three separated and non communicating computers. Thus, in this case, the classical macroscopic systems, violating by local independent choices the Bell's inequality, will be the computer themselves and the persons operating them. In this sense, although involving computers, the one described below is not a simulation of an experiment, but a real one.

In the following we describe the simple idea on which this experiment is based. To this goal, let us first of all state the problem more precisely.

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2 Predetermination, causality, locality and the chameleon effect

Consider a classical dynamical system composed of two particles $(1, 2)$. The term classical here means that

– each of the particles has a state space S

– observables are functions on S:

$$\tilde{S}_a^{(1)}, \tilde{S}_a^{(2)} : S \to \{\pm 1\}$$

– the measured values of the observables depend only on the initial state and on the dynamics (of each individual system).

The third requirement is the pre–determination requirement on which both EPR and Bell strongly insist as a crucial requirement of a realistic point of view. A crucial remark here is that the term "pre–determination" can be interpreted in the usual way of classical statistical mechanics: the value of the observable (e.g. the color of a ball) "is there" and we passively register it; or in an active way, like the color of a chameleon, which is "pre–determined" to become green on a leaf and brown on a piece of wood. This is why one should distinguish between "Einstein (or ballot–box) realism" and "chameleon realism".

The principle of causality asserts that:

– the state of any system at time 0 (present) is independent of the state of any other system at time $T > 0$ (future)

This means that:

– at time $t = 0$ the particles do not know which measurement will be done on them

and, mathematically it is translated in the fact that, at time 0, the global (i.e. system–apparatus) state ψ should be factorized as follows:

$$\psi = \psi_{system} \otimes \psi_{apparatus} \tag{1}$$

The locality principle, both in the EPR's and in Bell's words, means that:

"... the result B for particle 2 does not depend on the setting a, of the magnet for particle 1, nor A on b...." [10].

This means first of all that the two apparata make independent choices, i.e. that their preparations at time 0 are independent:

$$\psi_{apparatus} = \psi_{apparatus\ 1} \otimes \psi_{apparatus\ 2} \tag{2}$$

and, moreover, that two far away particles:

– don't feel their mutual interaction

– don't feel any interaction with a far away measurement apparatus

Obviously this does not exclude that:

"... the result B **for particle** 2 depends on the setting b, of the magnet **for particle** 2, and A on a...."

This statement expresses the essence of the **chameleon effect**. In fact, if we consider the two "chameleon observables": "color on the leaf" and "color on the wood", in analogy to "spin in direction a" and "spin in direction b", we see that the dynamical evolution of the chameleon, who becomes green when approaching the leaf and brown when approaching the wood, depends on the observable we measure.

More precisely (notice that the definition below is valid both for classical and quantum systems):

Definition. Let I be a set and, for each $x \in I$, let be given an observable $\tilde{S}_x$ of a system σ with state space S. The system σ is said to realize the chameleon effect with respect to the observables $\tilde{S}_x$ if: whenever the observable $\tilde{S}_x$ is measured, the dynamical evolution (discrete time) of the system

$$T_x : S \to S$$

depends on the measured observable $\tilde{S}_x$.

Remark. In our experiments the indices $x, y, \ldots$ are vectors in the unit circle in $\mathbf{R}^2$ (or angles in $[0, 2\pi]$).

The dynamics of the full system:

(particle 1, particle 2, apparatus 1, apparatus 2),

is unitary (an automorphism, at Heisenberg level). We will consider the reduced dynamics (cf. the Appendix), in Heisenberg representation, of the subsystem:

$(1,2) =$ (particle 1, particle 2)

which in general is not an automorphism, but a completely positive map (for classical systems this is equivalent to positivity).

The observables of the system $(1,2)$ are functions on $S \times S$ and we denote $f \otimes g$ those special observables of the form

$$f \otimes g(u,v) = f(u)g(v) \qquad ; \qquad u, v \in S$$

where f, g are functions on S. A local (reduced) dynamics for the system $(1, 2)$ is a positive map P, of the space of functions on $S \times S$ into itself, with the property that, for any two functions f, g on S,

$$P(f \otimes g) = P_1(f) \otimes P_2(g) \tag{3}$$

where P_1, P_2 are positive maps of the space of functions on S into itself. This means that particle 1 and particle 2 evolve with their own dynamics independently of each other. If condition (3) is satisfied, we will write

$$P = P_1 \otimes P_2 \tag{4}$$

For example, let $T_1, T_2 : S \to S$ be two reversible dynamics (invertible maps) and let $\tau_1, \tau_2 : S \to \mathbf{R}$ be two positive maps. Then the map

$$PF(u, v) := \tau_1(u) F(T_1 u, T_2 v) \tau_2(v) \quad ; \; u, v \in S \tag{5}$$

satisfies the locality condition (3) with

$$P_1(f)(u) := \tau_1(u) f(T_1 u) \quad , \quad P_2(g)(v) := g(T_2 v) \tau_2(v) \tag{6}$$

here F is a function on $S \times S$ and f, g functions on S. The reduced dynamics in our experiment will be of this form.

Given a local dynamics P and a state ψ, the Schrödinger evolution is defined by

$$\psi \mapsto \psi \circ P \tag{7}$$

and, since for any observable F one has

$$\psi \circ P(F) = \psi(P(F)) \tag{8}$$

it defines the Heisenberg evolution by

$$F \mapsto P(F) \tag{9}$$

The two representations are equivalent.

3 Physical idea of the experiment: general structure

We will choose the state space S (hidden variables) to be the interval $[0, 2\pi]$ (identified to the unit circle in the plane) and we will construct:

– for any pair of angles a, b, in $[0, 2\pi]$, two functions (observables)

$$\tilde{S}_a^{(1)}, \tilde{S}_b^{(2)} : S \to \{\pm 1\} \tag{10}$$

– for any pair of angles a, b, in $[0, 2\pi]$, two local (single particle) dynamics (local determinism) $P_{1,a}, P_{1,b}$

– a classical probability distribution ψ_o on the state space $S \times S$ (initial distribution of the hidden variables)

with the following properties: for any pair of angles a, b, in $[0, 2\pi]$, the correlations of the two random variables $\tilde{S}_a^{(1)}, \tilde{S}_b^{(2)}$, with respect to the measure ψ_o, evolved with the reduced dynamics $P := P_{1,a} \otimes P_{1,b}$ (cf. (3)), are exactly the singlet (EPR) correlations, i.e.

$$\int\int \tilde{S}_a^{(1)}(\mu_1)\tilde{S}_b^{(2)}(\mu_2)(\psi_o \circ P)(d\mu_1, d\mu_2) =: \langle S_a^{(1)} S_b^{(2)} \rangle = -cos(b-a) \tag{11}$$

Using (8) and (3) we can write the correlations (11) in Heisenberg representation:

$$\int\int P_{1,a}(\tilde{S}_a^{(1)})(\mu_1)P_{1,b}(\tilde{S}_b^{(2)})(\mu_2)\psi_o(d\mu_1, d\mu_2) \tag{12}$$

In order to complete our construction we want to introduce in our classical dynamical system the analogue of the singlet condition. In the standard (quantum) situation the singlet condition expresses the law of conservation of spin: the property of having spin zero is a constant of motion of the system.

Our classical analogue of the singlet condition will be the constraint that at time $t = 0$ (initial time) the state μ_1^0, of particle 1, and the state μ_2^0, of particle 2, coincide:

$$\mu^0 = \mu_1^0 = \mu_2^0$$

This means that the initial state space is not the whole phase space $S \times S$, but the surface (in $S \times S$) defined by the equation:

$$\mu_1 - \mu_2 = 0 \tag{13}$$

If we identify the unit circle in the plane to the interval $[0, 2\pi]$, this surface is simply the diagonal of the square $[0, 2\pi] \times [0, 2\pi]$.

We do not impose other constraints and we suppose that, on the initial surface (13), all the cells of the phase space with equal Lebesgue measure are equiprobable. Therefore our initial probability measure ψ_o will be:

$$\psi_o(d\mu_1, d\mu_2) = (2\pi)^{-1}\delta(\mu_1 - \mu_2)d\mu_1 d\mu_2 \tag{14}$$

where $d\mu$ is the Lebesgue measure.

Finally the local single particle dynamics $P_{1,a}, P_{1,b}$, which define the global dynamics through (3), will be (in Heisenberg representation) of the form

$$P_{1,a}(f)(u) := T'_{1,a}(u)f(T_{1,a}u) \quad , \quad P_2(g)(v) := g(T_{2,b}v)T'_{2,b}(v) \tag{15}$$

where $T_{1,a}, T_{2,b} : S \to S$ are differentiable functions with derivatives $T'_{1,a}, T'_{2,b}$ which are strictly positive almost everywhere and such that

$$\int_0^{2\pi}\int_0^{2\pi} \delta(\lambda_1 - \lambda_2)T'_{1,a}(\lambda_1)T'_{2,b}(\lambda_2)\frac{d\lambda_1 d\lambda_2}{2\pi} = 1$$

(cf. Section (4) below for an example of such functions). Notice that the local dynamics (15) have the form (6). In order to calculate the explicit form of the measure $\psi_o \circ P$, corresponding to the choices (14) of ψ_o and (15) of P, notice that, with these choices, the correlation $\langle S_a^{(1)} S_b^{(2)}\rangle$ (cf. (11)) becomes

$$\int\int \tilde{S}_a^{(1)}(T_{1,a}\lambda_1)\tilde{S}_b^{(2)}(T_{2,b}\lambda_2)\delta(\lambda_1 - \lambda_2)T'_{1,a}(\lambda_1)T'_{2,b}(\lambda_2)\frac{d\lambda_1 d\lambda_2}{2\pi} \tag{16}$$

and, with the notation

$$\tilde{S}_x^{(j)}(T_{j,x}\lambda_j) =: S_x^{(j)}(\lambda_j) \ ; \qquad j = 1, 2 \ ; \quad x = a, b$$

(notice that what we measure are the $\tilde{S}_x^{(j)}(T_{j,x}\lambda_j)$ and not the $\tilde{S}_x^{(j)}(\lambda_j)$) we finally obtain

$$\langle S_a^{(1)} S_b^{(2)}\rangle = (2\pi)^{-1}\int_0^{2\pi} S_a^{(1)}(\lambda)S_b^{(2)}(\lambda)T'_{1,a}(\lambda)T'_{2,b}(\lambda)d\lambda \tag{17}$$

Remark Notice that, if $p_o(\lambda_1, \lambda_2)$ is any probability density, then

$$\int\int \tilde{S}_a^{(1)}(T_{1,a}\lambda_1)\tilde{S}_b^{(2)}(T_{2,b}\lambda_2)p_o(\lambda_1, \lambda_2)T'_{1,a}(\lambda_1)T'_{2,b}(\lambda_2)d\lambda_1 d\lambda_2 =$$

$$= \int\int \tilde{S}_a^{(1)}(T_{1,a}\lambda_1)\tilde{S}_b^{(2)}(T_{2,b}\lambda_2)p_o(\lambda_1,\lambda_2)dT_{1,a}(\lambda_1)dT_{2,a}(\lambda_2)$$

Therefore, with the change of variables

$$\mu_1 =: T_{1,a}\lambda_1 \quad ; \quad \mu_2 =: T_{2,b}\lambda_2$$

the correlations become:

$$\langle S_a^{(1)} S_b^{(2)}\rangle = \int\int \tilde{S}_a^{(1)}(\mu_1)\tilde{S}_b^{(2)}(\mu_2)p_o(T_{1,a}^{-1}\mu_1, T_{2,b}^{-1}\mu_2)d\mu_1 d\mu_2$$

By taking limits the above identity continues to hold when p_o is a distribution. Therefore, if we choose $p_o(\mu_1,\mu_2) = (2\pi)^{-1}\delta(\mu_1-\mu_2)$, then an equivalent form for the correlations (16) is

$$\langle S_a^{(1)} S_b^{(2)}\rangle = \int\int \tilde{S}_a^{(1)}(\mu_1)\tilde{S}_b^{(2)}(\mu_2)(2\pi)^{-1}\delta(T_{1,a}^{-1}\mu_1 - T_{2,b}^{-1}\mu_2)\frac{d\mu_1 d\mu_2}{2\pi}$$

Which is the form originally used in [3]

4 Complete specification of the experiment

Now let us make the following choices for $T_{1,a}$, $T_{2,b}$, $\tilde{S}_a^{(1)}$, $\tilde{S}_b^{(2)}$:

$$T'_{2,b}(\lambda) = \sqrt{2\pi} \qquad \text{(constant)} \tag{18}$$

$$T'_{1,a}(\lambda) = \frac{\sqrt{2\pi}}{4}|\cos(\lambda - a)| \tag{19}$$

$$S_x^{(1)}(\lambda) = \text{ sgn }(\cos(\lambda - x))$$

$$S_x^{(2)} = -S_x^{(1)} \qquad \text{the singlet condition}$$

With these choices, the correlations

$$\langle S_a^{(1)} S_b^{(2)}\rangle = (2\pi)^{-1}\int S_a^{(1)}(\lambda)S_b^{(2)}(\lambda)T'_{1,a}(\lambda)T'_{2,b}(\lambda)d\lambda$$

become

$$= -\int_0^{2\pi} \text{sgn}(\cos(\lambda - a)) \text{ sgn}(\cos(\lambda - b))\frac{1}{4}|\cos(\lambda - a)|d\lambda$$

$$= -\frac{1}{4}\int \text{sgn}(\cos(\lambda - b))\cos(\lambda - a)d\lambda = -\cos(b - a)$$

which coincide with the singlet (EPR) correlations

5 Conclusions

We have constructed:

(i) A classical state space for a system composed of two particles $(1, 2)$ (the product of two copies of the interval $[0, 2\pi]$)

(ii) a family of local, classical, reduced Heisenberg dynamics, for the system $(1, 2)$, parametrized by the angles in the interval $[0, 2\pi]$

(iii) for each of the two particles a family (also parametrized by the angles in the interval $[0, 2\pi]$) of classical binary observables (± 1 valued functions) denoted $\{\tilde{S}_a^{(1)}\}$ and $\{\tilde{S}_b^{(2)}\}$ respectively.

(iv) an initial distribution on the state space of the two particle system with the following property: for any pair of angles a, b, in $[0, 2\pi]$, the correlations of the two random variables $\tilde{S}_a^{(1)}, \tilde{S}_b^{(2)}$, with respect to the measure ψ_o, evolved with the local dynamics $P_{1,a} \otimes P_{2,b}$, are exactly the singlet (EPR) correlations, i.e. $-\cos(b - a)$.

This means that we have built a local, realistic, hidden variable model for the singlet (EPR) correlations.

This result reconciles the orthodox interpretation with physics, in the sense that it is true that "the act of measurement may determine the value of some observable", however this does not happen by virtue of weird collapses, mysterious objectifications and bizarre nonlocalities, but because of the quite natural and physically intuitive chameleon effect. On the other hand it also reevaluates the hidden variable theories by showing that they are not necessarily in contradiction with locality and confirming the point of view, expressed in [8], that the problem with hidden variable theories is not their existence but their wild non uniqueness.

Moreover, since, because of locality, the values of all the observables of particle 1 can be calculated withouth knowing anything about particle 2 and conversely, the above construction allows to realize the following experiments.

Two, separate and non communicating, experimenters agree that one of them will calculate only values of observables of particle 1 and the other only values of observables of particle 2.

For each point λ of the classical state space experimenter 1 chooses, arbitrarily and without communicating with the other experimenter, an angle a and computes the value of $\tilde{S}_a^{(1)}(\lambda)$. The other experimenter does the same and computes $\tilde{S}_b^{(2)}(\lambda)$.

After a long (30 or 40 thousands) series of choices, the two experimenters exchange their informations and compute the correlations for all possible choices (a, b): just as in the Eckert protocol for quantum cryptography [11].

Then they check a posteriori that, for appropriate choices of the angles (a, b), the Bell's inequalities are violated.

Another experiment, also realizable by the above construction, reproduces the scheme adopted in the experiments to measure the violation of the Bell inequality in quantum systems: two, separate and non communicating, experimenters make the same agreement as above with the following variants:

– they divide the input sequence of states (λ_n) into four subsequences (corresponding to four correlations), for example: the first 10.000 points, the second 10.000, and so on. This division is arbitrary, but the four subsequences must be the same for the two experimenters.

– each experimenter chooses, arbitrarily and without communicating with the other one, two angles, let us say that a, a' is chosen by 1 and b, b' by 2

– they agree a priori that 1 will choose one of the two angles, say a, in the 1–st and 2–d subsequence of points and the other angle, a', in the 3–d and 4–th; while 2 will choose one of the two angles, say b, in the 1–st and 3–d subsequence of points and the other angle, b', in the 2–d and 4–th.

After this, the two experimenters exchange their informations and compute the correlations

$$\langle a, b\rangle \quad ; \quad \langle a, b'\rangle \quad ; \quad \langle a', b\rangle \quad ; \quad \langle a', b'\rangle$$

and they check that, for appropriate choices of the angles (a, a', b, b'), the Clauser–Horn–Shimony–Holt form of the Bell's inequalities are violated.

6 Appendix: A dynamical theory of measurement

In order to explain the emergence of local reduced dynamics, we summarize here the dynamical theory of measurement proposed in ([7]). This extends von Neumann's basic tenet that a good theory of measurement should keep into account the joint system–apparatus evolution, by introducing in it the notions of locality and causality.

All what said below, with appropriate interpretation of the symbols involved, is equally valid both for classical and quantum systems.

Let us introduce the following notations:
M measurement apparatus
S system
$T^t_{(S,M)}$ joint interacting evolution
T^t_S, T^t_M free evolutions
(S_o, M_o) initial state

The basic postulate of the classical theory of measurement is that the properties of the system are not modified by the measurement process (at least before the measurement itself). This means that the perturbations due to the interaction with the measurement apparatus are orders of magnitude less than the measured quantities. In formulae:

$$T^t_{S,M}(S_o, M_o) \text{ restricted to } S \ = \ T^t_S S_o \tag{20}$$

(for t before the measurement). On the contrary the properties of the measurement apparatus are modified due to the interaction with the system (otherwise there would be no measurement). In formulae:

$$T^t_{S,M}(S_o, M_o) \text{ restricted to } M \ \neq \ T^t_M M_o \tag{21}$$

(for t after the measurement).

We speak of *chameleon effect* when condition (20) is not justified. Thinking of the interaction system–apparatus, one could say that, in some sense, the surprising fact is that condition (20) is a very good approximation in many cases and not that it is violated in some other cases (e.g. quantum theory or, among classical systems, chameleons). Consider now a system

composed of two subsystems

$$S = (S_1, S_2) \ ; \quad M = (M_1, M_2) \tag{22}$$

Suppose that the two subsystems are spatially separated and subjected each to a different measurement. Therefore their evolution shall depend on several interactions. Let us write:

$$T^t_{(S,M)} = T^t_{(S_1,S_2,M_1,M_2)} \tag{23}$$

We want to introduce the locality condition in the dynamical evolution (23). It is reasonable to expect that the most general evolution (23) is nonlocal and that the class of local evolutions is very particular. The solution of this problem, which shall be given in several steps, confirms this expectation.

– i) It is convenient to use the Hamiltonian description of the dynamics:

$$T^t_{(S,M)} = e^{-itH} \tag{24}$$

because in it the various contributions to the interaction are better separated. Let us write therefore

$$H = H_S + H_M + H_I \tag{25}$$

separating the contribution of the system H_S, of the apparatus H_M, and of the interaction H_I.

– ii) Since $S = (S_1, S_2)$ we shall have

$$H_S = H_{S_1} + H_{S_2} + H_{S_1,S_2} \tag{26}$$

but the two systems are separated, therefore the **first locality assumption** is that their interaction is negligible, that is $H_{S_1,S_2} = 0$ or, equivalently:

$$H_S = H_{S_1} + H_{S_2} \tag{27}$$

– iii) Similarly $M = (M_1, M_2)$ and

$$H_M = H_{M_1} + H_{M_2} + H_{M_1,M_2} \tag{28}$$

If, as in the EPR experiment, we suppose that the only constraint between the two measurements is their simultaneity, then we arrive to the **second locality assumption** , that is $H_{M_1,M_2} = 0$ or, equivalently:

$$H_M = H_{M_1} + H_{M_2} \tag{29}$$

– iv) Eventually the interaction Hamiltonian shall have the form

$$H_I = H_{S,M} = H_{S_1,S_2,M_1,M_2} = H_{S_1,M_1} + H_{S_1,M_2} + H_{S_2,M_2} + H_{S_2,M_1} \tag{30}$$

and, again because of the spatial separation, it is natural to introduce the **third locality assumption** in the form

$$H_{S_1,M_2} = H_{S_2,M_1} = 0 \tag{31}$$

or, equivalently

$$H_I = H_{S_1,M_1} + H_{S_2,M_2} \tag{32}$$

Definition. A local dynamical law for the system (S_1, S_2, M_1, M_2), is given by an Hamiltonian of the form

$$H = (H_{S_1} + H_{M_1} + H_{S_1,M_1}) + (H_{S_2} + H_{M_2} + H_{S_2,M_2}) \tag{33}$$

From this it follows that the locality condition (33) is equivalent to the factorization condition

$$T^t_{(S_1,S_2,M_1,M_2)} = T^t_{(S_1,M_1)} \otimes T^t_{(S_2,M_2)} \tag{34}$$

which corresponds to the **factorization of the state space of the total system** given by

$$\mathcal{H}_{S_1} \otimes \mathcal{H}_{S_2} \otimes \mathcal{H}_{M_1} \otimes \mathcal{H}_{M_2} = \Big(\mathcal{H}_{S_1} \otimes \mathcal{H}_{M_1}\Big) \otimes \Big(\mathcal{H}_{S_2} \otimes \mathcal{H}_{M_2}\Big) \tag{35}$$

Let us now introduce the causality condition: at the initial instant, the system cannot know which measurement shall be done on it. This means that the initial preparation (state) of the system should be statistically independent on the state of the apparatus and mathematically it is expressed in the form:

$$\sigma_o = \text{ initial state of } (S_1, S_2, M_1, M_2) = \sigma_{1,2} \otimes \sigma_{M_1,M_2} \tag{36}$$

where σ_{S_1,S_2} is the initial state of (S_1, S_2) and σ_{M_1,M_2} the initial state of (M_1, M_2). Notice that **also causality is expressed by means of a factorization condition of the state space of the total system**. However, being given by:

$$\mathcal{H}_{S_1} \otimes \mathcal{H}_{S_2} \otimes \mathcal{H}_{M_1} \otimes \mathcal{H}_{M_2} = \Big(\mathcal{H}_{S_1} \otimes \mathcal{H}_{S_2}\Big) \otimes \Big(\mathcal{H}_{M_1} \otimes \mathcal{H}_{M_2}\Big) \tag{37}$$

this factorization is different from that of the dynamics. Summing up: both locality and causality are expressed by a factorization property, however they correspond to different factorizazions of the state space. The chameleon effect is a corollary of this difference: the state at the instant of measurement does not factorize, that is in general for $t > 0$, one cannot write it in the form

$$\sigma(t) = T^t \sigma_o = \sigma_{a,b}(t) \neq \sigma_{S_1,M_1}(t) \otimes \sigma_{S_2,M_2}(t) \tag{38}$$

In particular, if the apparatus $M_1 = M_a$ is predisposed for the measurement of S_a^1 and the apparatus $M_2 = M_b$ for the measurement of S_b^2, then one shall have

$$\sigma(t) = T^t_{(S,M)}\sigma_o = T^t\sigma_o = \sigma^t_{a,b} \tag{39}$$

Notice that **all what said up to now is equally valid both for classical and for quantum sistems** (to obtain the latter ones it is sufficient to substitute the Liouvillian for the Hamiltonian and the Poisson brackets for the commutator). This shows that the chameleon effect is a general characteristic both of classical and quantum physics.

Since the correlations between S_a^1 and S_b^2 are determined by the state $\sigma^t_{a,b}$ through the formula

$$E(S_a^1 S_b^2) = \int S_a^1(x) S_b^2(x) \sigma^t_{a,b}(dx) = \sigma^t_{a,b}(S_a^1 S_b^2) \tag{40}$$

in the classical theory, and through the formula

$$E(S_a^1 S_b^2) = \langle \sigma^t_{a,b}, S_a^1 \otimes S_b^2 \sigma^t_{a,b} \rangle = \sigma^t_{a,b}(S_a^1 S_b^2) \tag{41}$$

in quantum theory, it follows that in both cases the expectation value, which determines the correlations, also depends on the pair (a, b):

$$E(S_a^1 S_b^2) = E_{a,b}(S_a^1 S_b^2) \tag{42}$$

and it is well known that, **when the expectation value E depends on (a, b), it is impossibile to apply the Bell inequality**.

Finally let us remark that both (40) and (41) refer to the joint (system, apparatus) system. If we want that also the two apparata make local independent choices, then there should not be correlations between their states. This means that, in addition to the causality condition (36), we must also require the factorization

$$\sigma_{M_a,M_b} = \sigma_{M_a} \otimes \sigma_{M_b} \tag{43}$$

so that the initial state becomes:

$$\sigma_{a,b} = \sigma_{1,2} \otimes \sigma_{M_a} \otimes \sigma_{M_b} \tag{44}$$

Recalling now the factorization condition (34) and the fact that the symbol $S_a^1 S_b^2$ is a short–hand notation for

$$S_a^1 \otimes S_b^2 \otimes 1_{M_a} \otimes 1_{M_b} \tag{45}$$

and denoting $\overline{\sigma}_{M_a}$ (resp. $\overline{\sigma}_{M_b}$) the partial expectation over the M_a–algebra (resp. M_b–algebra), we see that both correlations (40) and (41) can be rewritten in the form

$$\sigma^t_{a,b}(S^1_a S^2_b) = (\sigma_{1,2} \otimes \sigma_{M_a} \otimes \sigma_{M_b})(T^t_{(1,2,M_a,M_b)}(S^1_a S^2_b)) = \tag{46}$$

$$= \sigma_{1,2}\left(\overline{\sigma}_{M_a}[T^t_{(1,M_a)}(S^1_a \otimes 1_{M_a})] \otimes \overline{\sigma}_{M_b}[T^t_{(2,M_b)}(S^2_b \otimes 1_{M_b})]\right) \tag{47}$$

Introducing the local reduced dynamics

$$P^t_{1,a}(S^1_a) := \overline{\sigma}_{M_a}[T^t_{(1,M_a)}(S^1_a \otimes 1_{M_a})] \quad ; \quad P^t_{2,b}(S^2_b) := \overline{\sigma}_{M_b}[T^t_{(2,M_b)}(S^2_b \otimes 1_{M_b})] \tag{48}$$

we finally obtain

$$\sigma^t_{a,b}(S^1_a S^2_b) = \sigma_{1,2}\left(P^t_{1,a}(S^1_a) \otimes P^t_{2,b}(S^2_b)\right) \tag{49}$$

Summing up we have proved that if: (i) the locality condition (34), (ii) the causality condition (36), (iii) the local independence condition (43), are satisfied, then also the reduced dynamics is local in the sense of (3).

Conversely, if we start from local reduced dynamics $P^t_{1,a}, P^t_{2,b}$ then, by taking unitary dilations, we can always construct larger (automorphic) dynamics $T^t_{(1,M_a)}$ and $T^t_{(2,M_b)}$ and therefore a global (automorphic) dynamics which satisfies the locality condition (34). Therefore to start from reduced dynamics is not a restriction.

Since, in the EPR type experiments, conditions (i), (ii), (iii) are always satisfied, the above arguments justify our use of local reduced dynamics.

In conclusion: we speak of "chameleon effect" when:

– i) the locality condition is satisfied

– ii) the causality condition is satisfied

– iii) the state of the total system can depend on the joint measurement (a, b) $(T^t_{a,b}\sigma_o = \sigma^t_{a,b})$

in these cases, since the state depends on the joint measurement, the Bell inequality is violated. Therefore the Bell inequality can be violated in full respect of locality and causality.

For those systems for which the application of the postulate (20) of classical measurement is justified one can apply the usual statistics.

Chameleons provide a simple example of classical system in which the above postulate is not justified. Therefore it is natural to expect that the statistics of the color of a set of pairs of chameleons will be different from that of a set of pairs of balls. The latter obey classical statistics (and the Bell inequality is satisfied), while our experiment proves that a set of pairs of "chameleon–like entities" can find an agreement on how to distribute their answers to binary questions (corresponding to $S^1_a, S^1_b, S^1_c, \ldots$) so to reproduce the EPR correlations hence violate Bell's inequality. It is conjectured that, by the above described chameleon effect, an arbitrary set of correlations for random variables with values in the interval $[-1, 1]$ can be obtained. However, at the moment, this is an open problem.

For these reasons, just as balls and dice are natural symbols for classical statistics, chameleons could be natural candidates to become the symbol of quantum statistics.

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