A Classical Derivation of Ritz's Electrodynamics

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Historical Introduction

Walter Ritz was born in Sion, Switzerland on February 22, 1878, the son of a landscape artist and grandson of an engineer. He began his studies at the Lycee communal of Sion and excelled academically. In 1897, he entered the Polytechnic school of Zurich where he began his studies in engineering. Disillusioned by the approximations and compromises of the field, he changed his concentration to physics, where he was a classmate of Albert Einstein. In 1901, he transferred to Gottingen Germany where he wrote his doctoral dissertation on a mathematical expression to predict frequencies in atomic spectral series. In 1903, he traveled to Leiden to attend a series of lectures on electrodynamics by H.A. Lorentz. After several moves and illnesses, he returned to Gottingen in 1908 where he wrote his magnum opus, *Recherches critiques sur l'Électrodynamique Générale*, in which he enunciated his view on the shortcomings of Maxwell-Lorentz theory and developed a new theory of electrodynamics based on the work of Gauss, Weber, Riemann and Clausis. In 1908-1909, he corresponded with Einstein in *Phsikalische Zeitschrift* over the proper way to represent blackbody radiation. He died several months later, on July 7, 1909, after a short but brilliant career.

Derivation

On the assumption of an emission theory, the force acting between two moving charges should depend on the density of the messenger particles emitted by the charges (D), the radial distance between the charges (ρ), the velocity of the emission relative to the receiver, (w_x), and the acceleration of the particles relative to each other (a_x). This gives us an equation of the form:

(1) $$F_x = eD\left[A_1 \cos(\rho x) + \frac{B_1 w_x}{c} + \frac{C_1 \rho a_x}{c^2}\right]$$

Let us consider the various terms in this equation. The number of the particles emitted in a time dt' is proportional to $e'dt'$ and the number of particles in an element of the surface of a sphere is $e'dS/\rho^2$. Hence

(2) $$D \propto \frac{dt'e'dS}{\rho^2}$$

To eliminate the time element, we consider the equation of the tangent plane to a sphere surrounding a stationary axis (X, Y, Z) relative to a particle moving in the frame (X', Y', Z') with $S(t')$. The radius in X is given by the equation:

$$\rho = c(t - t')$$

The position in the stationary coordinates is related as

$$X + x(t') = X' + x'(t') - (t - t')\, v_x'$$

The Jacobian of the transformation from X' to X gives the equation of the tangent plane of the emitted particles in the stationary frame of X as follows:

$$\frac{\partial\rho}{\partial n} = \frac{\partial(XYZ)}{\partial(X'Y'Z')} = \begin{vmatrix} \frac{\partial X}{\partial X'} & \frac{\partial X}{\partial Y'} & \frac{\partial X}{\partial Z'} \\ \frac{\partial Y}{\partial X'} & \frac{\partial Y}{\partial Y'} & \frac{\partial Y}{\partial Z'} \\ \frac{\partial Z}{\partial X'} & \frac{\partial Z}{\partial Y'} & \frac{\partial Z}{\partial Z'} \end{vmatrix}$$

(3)

$$= \begin{vmatrix} 1-(t-t')a_x'\frac{\partial t'}{\partial X'} & -(t-t')a_x'\frac{\partial t'}{\partial Y'} & -(t-t')a_x'\frac{\partial t'}{\partial Z'} \\ -(t-t')a_y'\frac{\partial t'}{\partial X'} & 1-(t-t')a_y'\frac{\partial t'}{\partial Y'} & -(t-t')a_y'\frac{\partial t'}{\partial Z'} \\ -(t-t')a_z'\frac{\partial t'}{\partial X'} & -(t-t')a_z'\frac{\partial t'}{\partial Y'} & 1-(t-t')a_z'\frac{\partial t'}{\partial Z'} \end{vmatrix}$$

$$= 1 - (t - t')\sum a_x' \frac{\partial t'}{\partial X'}$$

Now since

$$\rho = c(t - t')$$

$$t' = t - \frac{\rho}{c} = t - \frac{1}{c}\sqrt{X'^2 + Y'^2 + Z'^2}$$

$$\frac{\partial t'}{\partial X} = -\frac{1}{c}\cos(\rho X')$$

We find that

$$\frac{\partial\rho}{\partial n} = 1 + \frac{\rho}{c^2}a_\rho'$$

$$= -c\,\frac{\partial t'}{\partial n}$$

The density is then

$$D \propto \frac{dt'e'dS}{\rho^2} = -\frac{1}{c}\frac{e'}{\rho^2}\frac{\partial\rho}{\partial n}\,dS\,dn$$

(4)

$$= \frac{ae'}{\rho^2}\left(1 + \frac{\rho}{c^2}a_\rho'\right)$$

Thus, the phenomenological equation of force is developed as:

$$F_x = eD\left[A_1\cos(\rho x) + \frac{B_1 w_x}{c} + \frac{C_1 \rho a_x}{c^2}\right]$$

(5)
$$= \frac{ee'\left(1+\frac{\rho}{c^2}a'_\rho\right)}{\rho^2}\left[A\cos(\rho x) - \frac{BU_\rho U_x}{c^2} - \frac{C\rho a'_x}{c^2}\right]$$

The coefficients of the terms in this expression, A, B and C, are independent of the coordinate system and functions of u^2/c^2 and U_ρ/c. Next, we expand the retarded positions and velocities of the moving charge in a Taylor series

$$x'\left(t-\frac{\rho}{c}\right) = x'(t) - \frac{\rho}{c}\, v'_x(t) + \frac{\rho^2}{2c^2}\, a'_x(t)$$

$$v'_x\left(t-\frac{\rho}{c}\right) = v'_x(t) - \frac{\rho}{c}\, a'_x(t)$$

If we consider the simultaneous distance $r_x = x - x'$ and omit the argument t, we obtain the following

$$\rho_x = x - x'\left(t-\frac{\rho}{c}\right) - \frac{\rho}{c}\, v'_x\left(t-\frac{\rho}{c}\right)$$

$$= x - x' + \frac{\rho^2}{2c^2}\, a'_x(t)$$

$$\rho^2 = \sum_{x,y,z} \rho_i^2 = r^2 + \frac{r\rho^2}{c^2}\, a'_r(t)$$

Solving for ρ we find

$$\rho^2 = \frac{r^2}{\left(1-\frac{ra'_r}{c^2}\right)} = r^2\left(1+\frac{ra'_r}{c^2}+\cdots\right)$$

$$\rho = r\left(1+\frac{ra'_r}{c^2}+\right)^{1/2} = r + \frac{r^2 a'_r}{2c^2}$$

$$\rho_x = r_x + \frac{r^2 a'_x}{2c^2}$$

And then we develop the first term of the force equation

(6)
$$\frac{\cos(\rho x)}{\rho^2} = \frac{\rho_x}{\rho^3} = \frac{r\cos(rx)+\frac{r^2 a'_x}{2c^2}}{r^3\left(1+\frac{ra'_r}{c^2}\right)^{3/2}} = \frac{\cos(rx)}{r^2}\left(1-\frac{3a'_r}{2c^2}\right) + \frac{a'_x}{2c^2 r}$$

We also develop the relative velocity

$$U_\rho = \sum_{x,y,z} \left[v_i - v_i'\left(t - \frac{\rho}{c}\right)\right] \frac{\rho_i}{\rho}$$

$$= \sum_{x,y,z} [\, v_i - v_i'] \cos(rx) + \frac{r a_r'}{c}$$

$$= v_r - v_r' + \frac{r a_r'}{c}$$

Since the velocity terms U^2 and U_ρ^2 occur with a factor of $1/c^2$, they do not need to be further developed and we may take

$$U^2 = u^2 = \sum_{x,y,z} (v_x - v_x')^2$$

$$U_\rho^2 = u_r^2 = \left(\frac{dr}{dt}\right)^2$$

Returning to our force equation, we obtain

$$F_x = \frac{ee'\left(1 + \frac{\rho}{c^2} a_\rho'\right)}{\rho^2} \left[A\cos(\rho x) - \frac{B U_\rho U_x}{c^2} - \frac{C \rho a_x'}{c^2} \right]$$

(7) $$= \frac{ee'}{r^2}\left(1 + \frac{r a_r'}{c^2}\right) \left[A\cos(rx)\left(1 - \frac{3 r a_r'}{2c^2}\right) + A\left(\frac{r a_x'}{2c^2}\right) - B\frac{u_x u_r}{c^2} - C\frac{r a_x'}{c^2} \right]$$

Next, we develop the coefficients as functions

$$A = \alpha_0 + \alpha_1 \frac{u^2}{c^2} + \alpha_2 \frac{u_r^2}{c^2} + \cdots$$

$$B = \beta_0 + \beta_1 \frac{u^2}{c^2} + \beta_2 \frac{u_r^2}{c^2} + \cdots$$

$$C = \gamma_0 + \gamma_1 \frac{u^2}{c^2} + \gamma_2 \frac{u_r^2}{c^2} + \cdots$$

The force equation then becomes

$$F_x = \frac{ee'}{r^2} \left[\left(\alpha_0 + \alpha_1 \frac{u_x^2}{c^2} + \alpha_2 \frac{u_r^2}{c^2} \right) \cos(rx) + \alpha_0 \frac{r a_r'}{c^2} + \alpha_0 \left(\frac{r a_x'}{2c^2} \right) - \alpha_0 \frac{3 r a_r'}{2c^2} - \beta_0 \frac{u_x u_r}{c^2} - \gamma_0 \frac{r a_x'}{c^2} \right]$$

$$= \frac{ee'}{r^2} \left[\left(\alpha_0 + \alpha_1 \frac{u_x^2}{c^2} + \alpha_2 \frac{u_r^2}{c^2} \right) \cos(rx) - \beta_0 \frac{u_x u_r}{c^2} - \alpha_0 \frac{r a_r'}{2c^2} + \left(\frac{r a_x'}{2c^2} \right) (\alpha_0 - 2\gamma_0) \right]$$

This is Ritz's equation. Since this formula must reduce to Coulomb's law for stationary charges, we immediately obtain $\alpha_0 = 1$. We may obtain the last coefficient by considering the action of electrons on themselves in a charged body with density ρ at (xyz) and ρ' at (x'y'z').

$$mf_x = \int\int \rho\rho' R_x d^3x\, d^3x'$$

The electrostatic term gives zero because of equal action and reaction, and the velocity terms are all zero because all the elements have the same velocity. Hence

$$R_x = -\frac{1}{2c^2 r}[a'_r \cos(rx) + (2\gamma_0 - 1)a'_x]$$

$$= -\frac{1}{2c^2 r^3}\left[a'_x\{r_x^2 + (2\gamma_0 - 1)r^2\} + a'_y r_x r_y + a'_z r_x r_z\right]$$

Thus the body experiences a force which is a linear function of the accelerations

$$mf_x = -\frac{1}{2c^2}\left(A_x a'_x + B_x a'_y + C_x a'_z\right)$$

Where

$$A_x = \int\int \rho\rho' d^3x\, d^3x' \frac{(2\gamma_0 - 1)r^2 + (x - x')^2}{r^3}$$

$$B_x = \int\int \rho\rho' d^3x\, d^3x' \frac{(x - x')(y - y')}{r^3}$$

$$C_x = \int\int \rho\rho' d^3x\, d^3x' \frac{(x - x')(z - z')}{r^3}$$

These are identical with the results obtained by Lorentz formula if we take $2\gamma_0 - 1 = 1$, or $\gamma_0 = 1$. This results in an anisotropic inertial reaction for Ritz's formula, which leads to the electromagnetic mass added to the ordinary mass m

$$m' = \frac{1}{2c^2}\int\int \rho\rho' d^3x\, d^3x' \frac{r^2 + (x - x')^2}{r^3}$$

To determine the other coefficients, we may develop expressions for Ritz's equation and compare with Ampere's law. Consider the forces on a linear circuit as shown. At dl we have q moving with v and -1 moving with v-w. At dl' we have q' moving with v' and –q' moving with v'-w'.

To calculate the force from Ritz's equation we must calculate the sum:

$$F_x = \frac{\sum_{ij} e_i e_j{}'}{r^2}\left[\left(1+\alpha_1\frac{u_x^2}{c^2}+\alpha_2\frac{u_r^2}{c^2}\right)\cos(rx)-\beta_0\frac{u_x u_r}{c^2}-\alpha_0\frac{r a_r'}{2c^2}-\left(\frac{r a_x'}{2c^2}\right)\right]$$

Note that there are 4 forces from $\pm\, e'$ on $\pm\, e$. Consider the various terms

$$\sum_{i,j} e_i e_j = 0$$

$$\sum_{i,j} e_i e_j\, u_x^2 = qq'(v_x - v_x')^2 - qq'(v_x - v_x' + w_x')^2$$

$$-qq'(v_x - v_x' - w_x)^2 + qq'(v_x - v_x' - w_x + w_x')^2$$

$$= -2qq' w_x w_x'$$

$$\sum_{i,j} e_i e_j\, u_r^2 = -2qq' w_r w_r'$$

$$\sum_{i,j} e_i e_j\, u_x u_r = qq'(v_x - v_x')(v_r - v_r') - qq'(v_x' - v_x' + w_x)(v_r - v_r' + w_r')$$

$$-qq'(v_x - v_x' - w_x)(v_r - v_r' - w_r)$$

$$+qq'(v_x - v_x' - w_x + w_x')(v_r - v_r' - w_r + w_r')$$

$$= -qq'(w_x w_r' + w_x' w_r)$$

$$\sum_{i,j} e_i e_j\, a_r' = \sum_{i,j} e_i e_j\, a_x' = 0$$

If we consider the differentials $dq_i v_i = I_i\, dl_i$, we may re-write these formula as follows:

$$\sum_{i,j} de_i de_j = 0$$

$$\sum_{i,j} de_i de_j u_x^2 = -2\, I\, I' ds ds' \cos\epsilon$$

$$\sum_{i,j} de_i de_j u_r^2 = -2\, II' ds ds' \cos(rds)\cos(rds')$$

$$\sum_{i,j} de_i de_j u_x u_r = -I\, I' ds\, ds'[\cos(xds)\cos(rds') + \cos(rds)\cos(xds')]$$

The second derivative of Ritz's equation is

$$d^2F_x = \frac{\sum_{ij} de_i de_j{}'}{r^2}\left[\left(1+\alpha_1\frac{u_x^2}{c^2}+\alpha_2\frac{u_r^2}{c^2}\right)\cos(rx)-\beta_0\frac{u_x u_r}{c^2}-\alpha_0\frac{r a_r'}{2c^2}-\left(\frac{r a_x'}{2c^2}\right)\right]$$

$$= \frac{I\, I' ds ds'}{r^2}\ [\{2\alpha_1\cos\epsilon + 2\alpha_2\cos(rds)\cos(rds')\}\cos(rx)$$

$$-\beta_0\cos(rds')\cos(xds) - \beta_0\cos(rds)\cos(xds')]$$

If we compare this with the original form of Ampere's law

$$d^2F = -\frac{I\, I' ds\, ds'}{2r^2}\cdot\{[(3-k)\cos\epsilon - 3(1-k)\cos(rds)\cos(rds')]$$

$$\cos(rx) - (1+k)\cos(rds')\cos(xds) - (1+k)\cos(rds)\cos(xds')\}$$

We obtain the coefficients in Ritz's equation

$$\alpha_1 = \frac{3-k}{4}$$

$$\alpha_2 = -\frac{3(1-k)}{4}$$

$$\beta_0 = \frac{(1+k)}{2}$$

Plugging this into Ritz's equation, we obtain an expression with one unknown.

$$F_x = \frac{ee'}{r^2}\left\{\left[1 + \frac{(3-k)}{4}\frac{u_x^2}{c^2} - \frac{3(1-k)}{4}\frac{u_r^2}{c^2} - \frac{ra_r'}{2c^2}\right]\cos(rx) - \frac{(1+k)}{2}\frac{u_x u_r}{c^2} - \frac{ra_x'}{2c^2}\right\}$$

Force of a Stationary Current on a Stationary Charge

To determine the constant k in Ritz's formula, we consider the force of a stationary current on a stationary charge. In this situation we have a charge e with no velocity, and in the wire, a charge q+ with no velocity and a charge q- with velocity w_x'. Evaluating the terms in Ritz's equation, we have

$$\sum_{i,j} e_i e_j = 0$$

$$\sum_{i,j} e_i e_j \, u_x^2 = -eqw_x'^2$$

$$\sum_{i,j} e_i e_j \, u_r^2 = -eqw'^2 \cos^2(rds')$$

$$\sum_{i,j} e_i e_j \, u_x u_r = -eqw'^2 \cos(rds')\cos(xds')$$

$$\sum_{i,j} e_i e_j \, a_r' = \sum_{i,j} e_i e_j \, a_x' = 0$$

Using the relation $dqw'/c = I'ds'$, we obtain the formula

$$F = -e\,I\frac{w'}{c}\int\frac{ds'}{r^2}\left[\cos(rx)\left\{\frac{3-k}{4} - \frac{3(1-k)}{4}\cos^2(rds') - \frac{1+k}{2}\cos(rds')\cos(xds')\right\}\right]$$

Since the angle between r and ds' is 90°, the integral is evaluated as

$$F = -\frac{eIw'}{c}\left(\frac{2\pi\cos(rx)}{a^2}\right)\left(\frac{3-k}{4}\right)$$

Which is zero if $k = 3$. Returning to the force formula, we find that

$$F_x = \frac{ee'}{r^2}\left\{\left[1 + \frac{3}{2}\frac{u_r^2}{c^2} - \frac{ra_r'}{2c^2}\right]\cos(rx) - 2\frac{u_x u_r}{c^2} - \frac{ra_x'}{2c^2}\right\}$$